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Risk Budgeting and Frontier Markets: A Country Risk Indicator Approach For Risk Budgets Design

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Abstract

In questa tesi ho utilizzato un modello di allocazione innovativo, il Risk Budgeting, per costruire dei portafogli diversificati a livello internazionale, con un'esposizione variabile sui Mercati di Frontiera. Negli ultimi anni, i Mercati di Frontiera si sono imposti come asset class alternativa, grazie alla loro bassa correlazione ed integrazione con i mercati sviluppati ed emergenti. Tuttavia, sono mercati relativamente illiquidi e molto rischiosi. Il primo obiettivo di questa tesi è la costruzione dell'Indicatore di Rischio Paese, uno strumento quantitativo in grado di supportare il gestore nella scelta dei budget di rischio per i singoli Mercati di Frontiera. In secondo luogo, utilizzando 35 indici azionari forniti da Morgan Stanley Capital International, ho creato diversi portafogli utilizzando la metodologia del Risk Budgeting, con esposizione verso i mercati di frontiera al 5.00, 10.00, 20.00 e 40.00 per cento. Tali portafogli sono stati infine confrontati con dei portafogli generati implementando il modello di Markowitz e con un portafoglio di mercati sviluppati. La metodologia del Risk Budgeting, combinata all'Indicatore di Rischio Paese, permette la generazione di portafogli con un rischio sensibilmente inferiore rispetto ai portafogli generati attraverso il modello di Markowitz. In aggiunta, i Mercati di Frontiera si confermano un'ottima asset class alternativa, capace di apportare sensibili benefici attraverso la diversificazione del rischio.

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1 Introduction

During the last 18 months, stock markets reached their maximum historical quotations, closing a "bullish" period started with the recovery from the 2008 Financial Crisis. The European Sovereign Debt Crisis in 2011 dampened this positive trend for a few months, but it was not enough deep and widespread to revert it. Despite these last years of good positive returns and low volatility on stock markets, certainly eased by expansive monetary policy of the major Central Banks, the long-term effects on financial culture and financial industry of the subprime crisis are still remaining actual.

In their introductory considerations, Brurder and Roncalli (2012) highlighted the investors' lower risk tolerance. Investors turned to asset managers able to manage the crisis in a robust way, changing the core character of asset manager job. Nowadays, a good fund manager has to be able to manage risk. The central role of risk management in asset allocation, and more generally in financial and banking sector, is not only driven by financial culture considerations. After the financial crisis, legislators focused on risk control, introducing a framework that should limit the effect of future systemic financial crisis[1].

Another topic emerged between 2008 and 2009 is the renewed interest on integration and correlation between stock markets. Higher integration and positive correlation increase contagion risk, making a standalone country crisis a potential threat to the whole system. The Asian Crisis in 1997 clearly showed this aspect, highlighted again by the Subprime Crisis and, even more, by the European Sovereign Debt Crisis. There is huge literature on stock markets integration and the benefits of international diversification, starting from Levy and Sarnat (1970) and Solnik (1974). In particular, Solnik (1974) shows the beneficial effect of cross-market correlations achieved through international diversification (other important contributions are for example the ones of Jorion [1985] and Odier and Solnik [1993][2]).

In this thesis I want to combine these two considerations on risk management in asset allocation and the benefits from international diversification. In particular, I want to apply Risk Budgeting Approach to construct an internationally diversified portfolio including Developed, Emerging and Frontier Markets. As stated by Brurder and Roncalli (2012), Risk Budgeting is a widely used allocation method, with an increasing diffusion after the 2008 Financial Crisis.

[1] The most important example is the Basel framework, with Basel II (after the Subprime Crisis) and Basel III (after the European Sovereign Debt Crisis).

[2] Jorion (1985) shows that potential benefits of international diversification are more likely to accrue from a reduction in risk, rather than an improvement in returns. Finally, Odier and Solnik (1993) show that international diversification is still beneficial, even considering increasing informational integration and greater correlation during volatile periods, since overall international correlations remain low.

In the last years, Risk Budgeting properties were widely studied and there is a consistent literature on this topic (see for example the books of Rahl [2000], Meucci [2005], Scherer [2007] or Roncalli [2013] and the works of Sharpe [2002], Qian [2006], Maillard, Roncalli and Teiletche [2010], Asness, Frazzini and Pedersen [2012], Brurder and Roncalli [2012]). Even in the classic theory of portfolio allocation, risk considerations have their significance, generally simplified by the concept of volatility minimization given a certain return level. One of the main drawbacks of classic allocation theory is the construction of poorly diversified portfolio, especially in terms of risk diversification. Risk Budgeting Approach puts risk diversification as the core characteristic of the investment process, trying to solve the drawbacks of classic portfolio theory. About the second aspect, diversification benefits are widely admitted, especially in terms of risk reduction. In this work I want to focus on international diversification within the equity asset class. To achieve this goal, the investment universe includes the Frontier Markets, stock markets smaller, less accessible and less liquid than Developed and Emerging Markets. Recently, the interest over this markets increased, in order to exploit their growth opportunities and diversification benefits. In fact, forecasting the Emerging Markets of the future hides extremely high potential benefits and asset managers are seeking for non-conventional investment opportunities able to boost their performances. The literature on this topic is quite limited. Diversification benefits of Frontier Markets are studied in the work of Berger, Pukthuanthong and Yang (2011), where they found that these markets present low integration with the world market, increasing the potential benefits of international diversification (other works focused on a subset of Frontier Markets are the ones of Speidell and Krohne [2007], Jayasuriya and Shambora [2009], and Cheng, Jahan-Parvar, and Rothman [2010]).

The first aim of the thesis is to construct a tool, the Country Risk Indicator, useful to design the risk budgets on Frontier Markets. Secondly, I want to evaluate the performances of the portfolios obtained using Risk Budgeting against portfolios obtained using Markowitz model and a GDP-weighted portfolio of Developed Markets. I constructed the Country Risk Indicator using a simplified procedure of Country Risk Assessment. This is a necessary step when evaluating the opportunity to invest in a foreign market. I created four different portfolios on the basis of Risk Budgeting Approach, with an increasing risk contribution associated to the Frontier Market asset class (5.00, 10.00, 20.00 and 40.00 per cent). I analyzed these portfolios over a variable period between February 2005 and January 2015[3]. The investment universe varied across the investment period, since I added new Frontier Markets as soon as

[3] The four portfolios have different inception date, based on the number of Frontier Markets available. To ensure diversification, portfolios with 20.00% and 40.00% are launched when the number of Frontier Markets is acceptable. Given the relative illiquidity of this stock market asset class, I tried to avoid large bet on a single Frontier Market, reducing the potential liquidity risk.

they became available. In fact, I tried to simulate an asset manager activity, using only the information available at the time of the investment decision process. This simulation gives to my work an ex-ante perspective. I was not interested in testing ex-post, with the information available today, what an asset manager should have done ten years ago, since it was not possible for him to reason and to act as I could with a larger set of information.

The thesis is organized as follows. Section 2 discusses the classic model of asset allocation, the Markowitz Model, and its main drawbacks. Then, I present the Risk Budgeting Approach, showing its properties. In Section 3, I present the investment universe, focusing on the definition of Frontier Market provided by the different financial institutions. The second part includes some basic characteristics of the assets belonging to the investment universe. In Section 4, I illustrate the Country Risk Assessment procedure and the Country Risk Indicator. In particular, I define what country risk is and its main determinants. Then, I move to show the detailed computation of the Country Risk Indicator. Portfolio construction and evaluation are the heart of Section 5. I show how to shift from the Country Risk Indicator to risk budgets, adopting an automatic procedure. I present the different portfolios, highlighting basic characteristics, weights and risk contributions composition, turnovers, performance measures and tracking error analysis. Finally, I show the Value-at-Risk of my portfolios, comparing it with the Developed Market portfolio. Section 6 draws some conclusions.

2 Portfolio Theory

In the last decades, technological progress, financial innovation and globalization substantially increased the number of financial instruments and products and improved financial markets' diffusion and accessibility. The number of retail investors without or with a limited financial knowledge approaching financial markets increased, due to these new investment opportunities. However, we observed an increase in the complexity of investment choices, given to professional asset managers a central role in the new financial world.

Asset managers are responsible for asset allocation, which is the set of strategies and processes implemented to allocate investor's wealth over different markets or sectors (i.e., international or sectorial diversification) and asset classes (e.g., liquidity, fixed income, equity, commodities, real estate, private equity)[4]. Asset allocation includes the ex-ante analysis of investor's financial situation (current wealth and fiscal situation), risk aversion and investment objectives (e.g., time horizon, financial goals, etc.). The aim is to find the most suitable asset mix for the investor. It can be divided in three different steps. Given investor's risk aversion and investment objectives, the strategic asset allocation is the activity that determines the most suitable asset allocation over the medium-long term[5]. With the tactical asset allocation, the asset manager implements a periodical revision of the strategic portfolio composition in order to catch significant variations in risk-return trade-offs between and within the asset classes over the short and medium term (e.g., the adjustment of portfolio composition to the business cycle or to financial markets situation). Finally, with the operative asset allocation the asset manager moves from asset class to individual financial instruments, with the selection of the direct or indirect instruments (e.g., stocks, bonds, funds, ETFs, etc.) to build up the strategic and the tactical portfolio.

Asset allocation is a complicated process. It includes challenging aspects during the preliminary phase (definition of investor's risk aversion and objectives), the intermediary phase (returns forecast, risk assessment, strategic and tactical asset allocation) and the final phase (portfolio construction and evaluation). The problem of portfolio composition is complex and requires specific financial, mathematical and statistical skills, in order to ensure continuous long-term returns.

[4] An asset class is a macro-category including all the instruments with similar technical characteristics, returns and risks.

[5] An interesting paper on strategic asset allocation is the one of Eychenne, Martinetti and Roncalli (2011), in which he proposes a quantitative and systematical methodology to optimize portfolios using long-run fundamental pillars to forecast returns and assess risk for the different asset-classes.

The problem of portfolio selection is well developed in academic literature since the ‘50s. The first model was proposed in Markowitz (1952) and it is also known as the Mean-Variance model. Markowitz model played a central role in the future development of the Portfolio theory. Several works tried to improve or to test its findings, while other papers focused on the estimation of the inputs (expected returns and variance-covariance matrix) of the Mean-Variance optimizer. Moreover, the drawbacks of this model were the base for the development of alternative models for portfolio construction (see for example Michaud [1989], Black and Litterman [1991], Chow *et al*. [1999] and Tütüncü and Koenig [2004]) and for the Risk Budgeting Approach. Another important contribution to the Portfolio theory was the Capital Asset Pricing Model (CAPM), presented in Sharpe (1964). This model is more focused on the evaluation of the financial activities. It is a single-factor model on the relation between risk and return. There are several works that tried to test the validity of the CAPM and to improve the model (see for example Black, Jensen and Scholes [1972], Fama and MacBeth [1973], Roll [1977], Jagannathan and Wang [1996] and Lettau and Ludvigson [2001]). Different versions of CAPM were developed in the following years, as the ICAPM (see Merton [1973]) and the CCAPM. A criticism to the CAPM presented in Fama and French (1993) was the base for the development of the recent multiple-factor models, such as the Arbitrage Pricing Theory (APT).

CAPM and Markowitz model are still the most diffused models, especially in their corrected formulations. In the first part of this chapter, I present the Markowitz model and its main drawbacks. Except for its relevance in the Portfolio theory, it is the model against which I test the results of the Risk Budgeting Approach. In the second part, I present the Risk Budgeting Approach, which is the core methodology of portfolio construction adopted in this thesis.

2.1 Classic Portfolio Theory

2.1.1 The Markowitz Model

Markowitz (1952) introduced the idea that investment opportunities should be evaluated on the basis of their risk-return trade-offs and not only on the basis of their expected returns. Moreover, the model put the focus on diversification. Investing in multiple assets, investors can achieve diversification benefits through the reduction of risk given a certain level of return, exploiting assets that are negatively correlated.

The aim of Markowitz model is the selection of the efficient portfolios from a set of admissible portfolios (i.e., the portfolios that can be reached by an investor given its wealth and a particular set of assets). The efficient portfolios are those that maximize the return given a certain level of risk or, in other words, that minimize the risk given a certain level of return. This is known as the Mean-Variance Optimization Criterion. The set of efficient portfolios composed the efficient frontier. The Markowitz Model is based on strong assumptions[6]:

- Portfolio risk is characterized by the variability of the returns;
- Investors are rational and risk averse, with their utility function concave and increasing;
- Returns distributions are characterized by the first two moments[7].

The last assumption implies that the Mean-Variance Optimizer requires three different inputs in order to be implemented:

- The expected returns $E(R_i)$ of the different n assets ($i = 1,2,\dots,n$). This quantity is the mean of the distribution of the random variable R_i (asset returns);
- The variance σ_i^2 of the random variable R_i;
- The linear correlation coefficient ρ_{ik} for each couple of random variables (R_i,R_k), that is a standardized measure of the covariance between two different assets. This quantity is fundamental in diversification process, since diversification benefits strictly depends on the fact that assets are positive or negative correlated.

Given a vector of weights w it is possible to compute the portfolio expected return and portfolio variance, respectively in Eq.(2.1) and Eq.(2.2):

$$\mu_p = w'\boldsymbol{r} = \sum_{j=1}^{n} w_j \mu_j \tag{2.1}$$

$$\sigma_p^2 = w'\Sigma w = \sum_{j=1}^{n}\left(w_j^2\sigma_j^2\right) + 2\sum_{j=1}^{n}\sum_{k=1}^{n}\left(w_j w_k \sigma_{jk}\right) \qquad j \neq k \tag{2.2}$$

From Eq.(2.2) it can be seen the importance of the covariance between two assets included in the portfolio. If two assets are negatively correlated, diversification is beneficial through the reduction of portfolio variance. Markowitz (1952) was one of the first contribution that

[6] Several papers demonstrated that these assumption are not true. For example, on the investors rationality there is a growing literature on Behavioral finance, started from the criticism to Efficient Market Hypothesis made in Shiller (1981). On the side of returns distribution, there are several empirical studies that proved that these distribution are not normal or jointly elliptical (see for example Fama [1965] and Bouchaud and Chicheportiche [2012]).

[7] In the first formulation, the assumption was that asset returns are jointly normally distributed. However, Chamberlain (1983) and Owen and Rabinovitch (1983) release this assumption. They proved that it's sufficient that asset returns are jointly elliptically distributed. Thus, the distribution is still characterized by the first two moments, as for the normal distribution.

formalized the concept of diversification benefits of risk reduction through the investment in negatively correlated assets.

The set of efficient portfolios is determined by solving the following optimization problem:

$$min_w \;\; w'\Sigma w \tag{2.3}$$

$$s.to \;\; \mu_p = w'\boldsymbol{r}$$

$$w'\boldsymbol{1} = 1$$

- w is the $(1 \times n)$ vector of weights of the n assets;
- $\boldsymbol{r}$ is the $(1 \times n)$ vector of assets expected returns;
- Σ is the $(n \times n)$ variance-covariance matrix of the assets included in the portfolio;

In words, the solution of the model selects those portfolios that minimize the variance ($w'\Sigma w$ is portfolio return variance) for a certain level of portfolio expected return (first constraint), between the set of all admissible portfolios (second constraint). The analytical solutions of the model are presented in Eq.(2.5) and Eq.(2.6) and they come from the FOC on the minimization of the Lagrangian function in Eq.(2.4):

$$min_w \quad L(w) = \frac{1}{2} w'\Sigma w - \lambda_1(w'\boldsymbol{1} - 1) - \lambda_1\left(w'\boldsymbol{r} - \mu_p\right) \tag{2.4}$$

$$\sigma_p^2 = \frac{C}{\Delta}\mu_p^2 - \frac{2B}{\Delta}\mu_p + \frac{A}{\Delta} \tag{2.5}$$

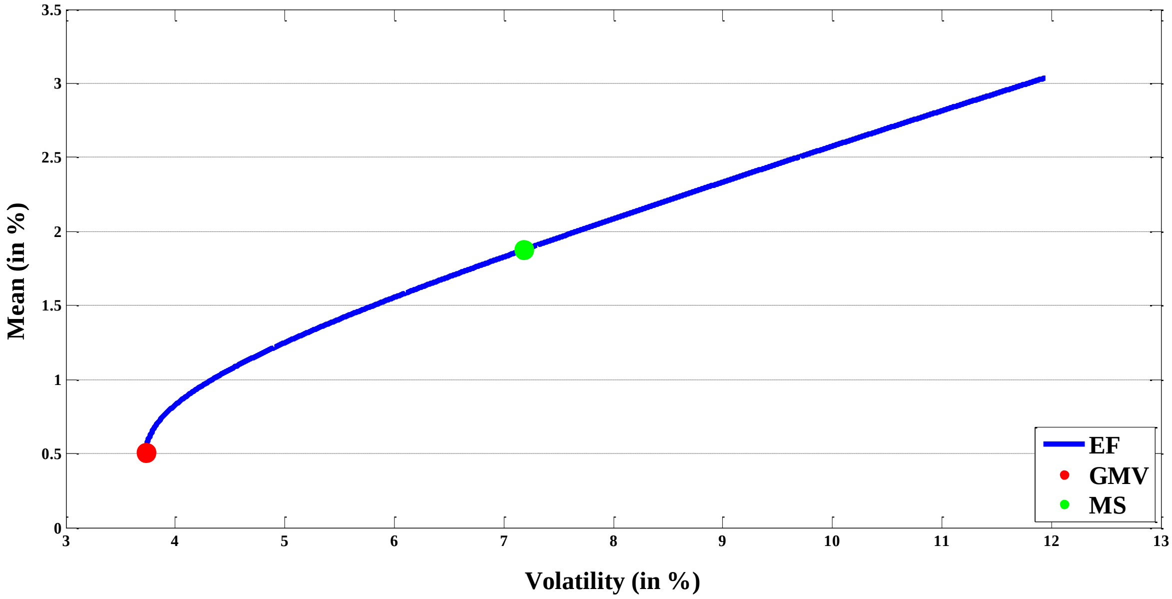
$$\widehat{w} = \boldsymbol{D} + \boldsymbol{E}\mu_p \tag{2.6}$$

$$A = \boldsymbol{r}'\Sigma^{-1}\boldsymbol{r} \qquad B = \boldsymbol{1}'\Sigma^{-1}\boldsymbol{r} \qquad C = \boldsymbol{1}'\Sigma^{-1}\boldsymbol{1} \qquad \Delta = AC - B^2$$

$$\boldsymbol{D} = \frac{A\Sigma^{-1}\boldsymbol{1} - B\Sigma^{-1}\boldsymbol{r}}{\Delta} \qquad \boldsymbol{E} = \frac{A\Sigma^{-1}\boldsymbol{r} - B\Sigma^{-1}\boldsymbol{1}}{\Delta}$$

As above-mentioned, the efficient frontier is the set of efficient portfolios obtained with the Mean-Variance optimization criterion. Graphically, it is represented by the concave function on the plane (σ,μ) that delimited the region of the admissible portfolios. In Figure 1, I provide a graphical example of the efficient frontier, using a set of fifteen MSCI European stock indices. Between the different efficient portfolios, the most relevant portfolios are the Global Minimum Variance (GMV) and the Maximum Sharpe (MS). The GMV portfolio is the portfolio with the lowest risk, and consequently the lowest return, between the efficient portfolios. It represents the lower bound of the efficient frontier. The MS portfolio is the portfolio with the highest trade-off between risk and return.

Figure 1 – Efficient Frontier Example



The last important thing about the efficient frontier is that amplifying the investment universe, it will shift always to the left. In fact, adding new assets to the initial set, we amplify the investment opportunities and we can reach higher returns given the same level of risk.

From the point of view of a rational investor, the optimal portfolio is the portfolio situated on the tangency point between his Mean-Variance utility function indifference curves and the efficient frontier. Risk averse investor utility function has the following form:

$$U = \mu_p - \frac{\gamma}{2}\sigma_p^2 \tag{2.7}$$

The indifference curves become flatter the lower the risk aversion coefficient. The optimal problem that the investor has to solve, given its utility function with a coefficient of risk aversion γ, is presented in Eq.(2.8) and the solution is presented in Eq.(2.9):

$$max_w \quad U = w'\boldsymbol{r} - \frac{\gamma}{2}w'\Sigma w \tag{2.8}$$

$$s.to \qquad w'\mathbf{1} = 1$$

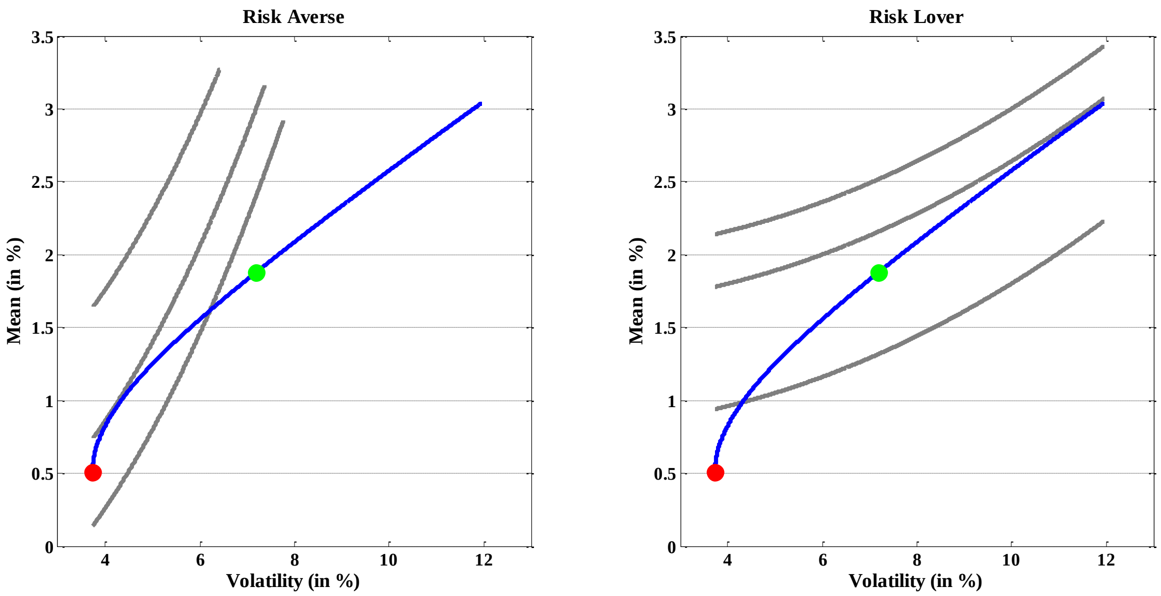
$$\widehat{w} = \frac{1}{\gamma}\Sigma^{-1}\boldsymbol{r} - \frac{B-\gamma}{C}\frac{1}{\gamma}\Sigma^{-1}\mathbf{1} = \frac{B}{\gamma}\widehat{w_{MS}} - \frac{B-\gamma}{\gamma}\widehat{w_{GMV}} \tag{2.9}$$

The optimal portfolio of the risk averse agent can be written as a combination of the weights of the MS and the GMV portfolios[8]. For extremely high risk aversion coefficient ($\gamma \to \infty$), the solution converges to the GMV portfolio. On the contrary, for extremely low

[8] See Appendix A for the demonstration.

risk aversion coefficient ($\gamma \to 0$), the solution is composed by extreme long position in MS portfolio and extreme short position on GMV portfolio. Figure 2 provides a graphical example of investor choice, with a comparison between two investors with a different risk aversion coefficient. As above-mentioned, the optimal choice will move to the right on the efficient frontier the lower the risk aversion.

Figure 2 – Optimal investor choice



An important development of the Markowitz model is the possibility to invest in a risk-free asset (i.e., the return is known) in addition to the *n* risky assets. Tobin (1958) added the notion of leverage incorporating this risk-free asset. Combining a risk-free asset with a portfolio on the efficient frontier, it is possible to construct portfolios with better risk–return trade-offs than portfolios on the efficient frontier. Given the return of the risk-free asset (r_f) the problem can be written as:

$$\begin{gathered} min_w \;\; w'\Sigma w \\ s.to \;\; \mu_p = w'\boldsymbol{r} + (1 - w'\mathbf{1})r_f \end{gathered} \tag{2.10}$$

The admissibility constraint is implicit in the target return constraint. One thing that is important to note is that the sum of the weights of the risky assets (w) is not equal to 1 as in the previous optimal problem. In fact, it's the sum between the weights of the risky assets and the weight of the risk-free asset that is equal to 1. The solution of the problem is presented in Eq.(2.11), while the variance of the efficient portfolios and the efficient frontier equation are presented in Eq.(2.12) and Eq.(2.13) respectively:

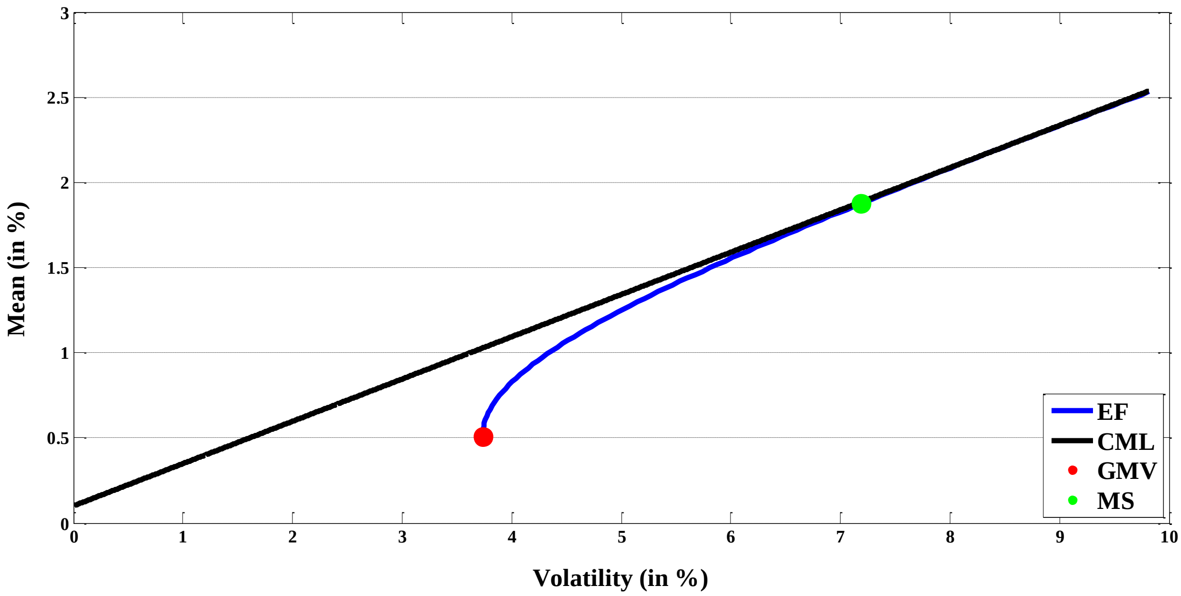
$$\widehat{w} = \frac{\Sigma^{-1}(\boldsymbol{r} - \mathbf{1}r_f)}{A - 2Br_f + Cr_f^2}(\mu_p - r_f) \tag{2.11}$$

$$\sigma_p^2 = \frac{(\mu_p - r_f)^2}{A - 2Br_f + Cr_f^2} \tag{2.12}$$

$$\mu_p = r_f + \sigma_p\sqrt{A - 2Br_f + Cr_f^2} \tag{2.13}$$

First of all, the solution of the problem is given by the weights of the risky assets, while the weight of the risk-free asset is given as a complement to 1. Secondly, as can be seen in Eq.(2.13), the efficient frontier is a straight line in the plane (σ,μ) representing the risk-return combinations obtained combining the risk-free asset and the risky assets. If the portfolio return is equal to the risk-free rate, the entire portfolio is invested in the risk-free asset. Moving to the right on the efficient frontier, we start to invest in the risky assets (i.e., $w'\mathbf{1} > 0$ and $1 - w'\mathbf{1} < 1$). Figure 3 provides an example of efficient frontier with risk-free asset.

Figure 3 – Efficient Frontier with Risk Free Asset Example



The line representing the efficient frontier with the risk-free investment is known as Capital Market Line (CML). By moving on the new efficient frontier, from the portfolio composed by the risk-free asset, we will move to a portfolio composed only by risky assets (i.e., $w'\mathbf{1} = 1$). This portfolio is located both on the efficient frontier and on the CML and its weights are:

$$\widehat{w} = \frac{\Sigma^{-1}(\boldsymbol{r} - \mathbf{1}r_f)}{\mathbf{1}'\Sigma^{-1}(\boldsymbol{r} - \mathbf{1}r_f)} = \widehat{w_{MS}} \tag{2.14}$$

This portfolio is known as Tangency portfolio or Maximum Sharpe portfolio, with return and risk presented in Eq.(2.15) and Eq.(2.16) respectively:

$$\mu_{MS} = \frac{A - Br_f}{B - Cr_f} \tag{2.15}$$

$$\sigma_{MS}^2 = \frac{(\boldsymbol{r} - \mathbf{1}r_f)\Sigma^{-1}(\boldsymbol{r} - \mathbf{1}r_f)}{\left(\mathbf{1}'\Sigma^{-1}(\boldsymbol{r} - \mathbf{1}r_f)\right)^2} = \frac{A - 2Br_f + Cr_f^2}{(B - Cr_f)^2} \tag{2.16}$$

The tangency portfolio between the efficient frontier and the CML is the portfolio that provides the highest Sharpe ratio. In particular, the intercept of the CML is equal to the risk-free rate and its slope is equal to the Sharpe ratio of the Tangency portfolio[9]:

$$\frac{\mu_{MS} - r_f}{\sigma_{MS}} = sgn(B - Cr_f)\sqrt{A - 2Br_f + Cr_f^2} \tag{2.17}$$

Focusing on investor choice, we start again from the mean-variance utility function in Eq.(2.7), with γ that represent the risk aversion coefficient. The optimal choice of the agent will be the portfolio located on the tangency between the indifference curves and the CML, which is the new efficient frontier. Eq.(2.18) formalizes the problem and Eq.(2.19) presents the solution:

$$max_w \quad U = (1 - w'\mathbf{1})r_f + w'\boldsymbol{r} - \frac{\gamma}{2}w'\Sigma w \tag{2.18}$$

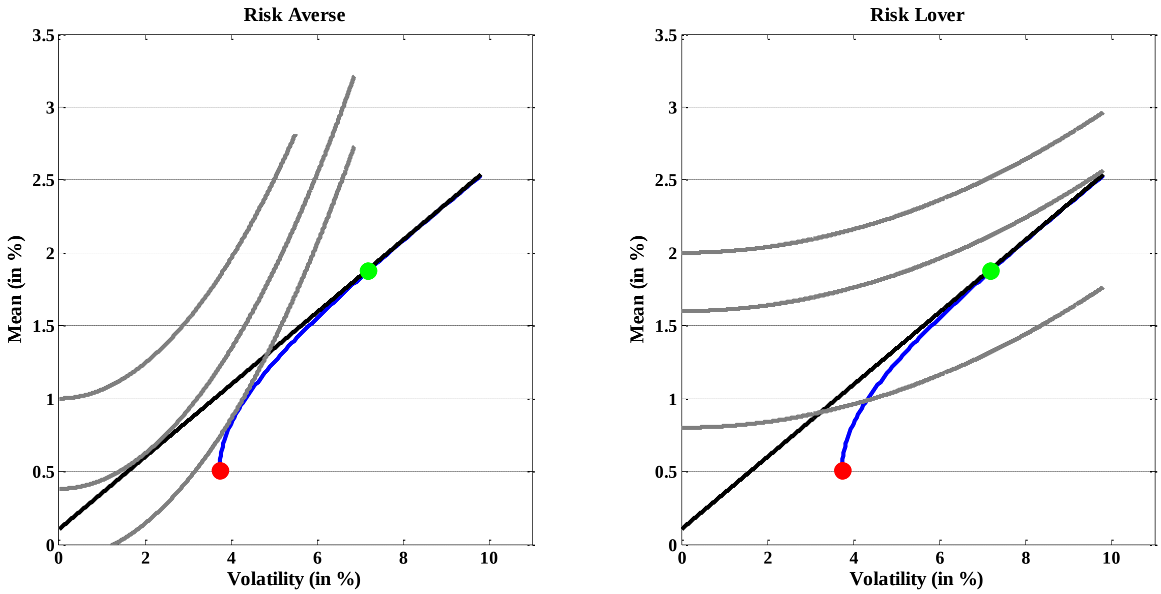
$$\hat{w} = \frac{1}{\gamma}\Sigma^{-1}(\boldsymbol{r} - \mathbf{1}r_f) = \frac{B - Cr_f}{\gamma}\frac{\Sigma^{-1}(\boldsymbol{r} - \mathbf{1}r_f)}{B - Cr_f} = \frac{B - Cr_f}{\gamma}\widehat{w_{MS}} \tag{2.19}$$

The solution is expressed in terms of the weights of the risky assets (i.e., $w'\mathbf{1} \neq 1$), while the weight on the risk-free asset is given as a complement to 1. An important thing to note is the fraction of the portfolio invested in the risky assets is proportional to the MS portfolio in this case[10]. The optimal portfolio is thus a combination of the risk-free asset and of the MS portfolio. For extremely high risk aversion coefficient ($\gamma \to \infty$), the solution converges to a portfolio composed only by the risk-free asset (i.e., $w'\mathbf{1} = 0$). On the contrary, for extremely low risk aversion coefficient ($\gamma \to 0$), the solution is composed by an infinitely long position in MS portfolio, financed by borrowing (i.e., shorting) at the risk-free rate. This is the concept of leverage introduced by Tobin. Using the risk-free asset, investors may leverage their posi-

[9] See Appendix A for the demonstration.
[10] Note that the Global Minimum Variance portfolio find in the case without risk-free asset does not belong to the CML. In fact, the new GMV portfolio is represented by the risk-free asset.

tion (shorting the risk-free asset) to increase the portion invested in the risky assets or de-leverage their position (selling part of the holdings in the risky assets) to invest the proceeds in the risk-free asset. Figure 4 shows an example of investor choice, comparing two different degrees of risk aversion. The tangency point between the CML and the indifference curve will move to the right, reducing the portion invested in the risk-free asset, the lower the risk aversion of the investor.

Figure 4: Optimal investor choice with risk-free asset



Tobin concluded that portfolio construction can be divided in two different steps. In the first step, investors should determine the risky portion of their portfolio. In the second step, they should leverage or de-leverage this portfolio to achieve their desired level of risk. The important thing is that the composition of the risky portfolio is independent of the investor's risk aversion and the two choices are independent of one another. This is known in literature as Tobin's separation theorem[11].

2.1.2 The Drawbacks of the Markowitz Model

As above-mentioned, the Markowitz model is one of the most studied and tested model in literature. Several papers found important drawbacks of this model and proposed important development. In this paragraph I present the main limitations of the Markowitz model, briefly discussing three important approaches proposed in Michaud (1989), Black and Litterman (1991) and Tütüncü and Koenig (2004).

[11] Sharpe (1964) with the CAPM showed that, given strong assumptions (e.g. Market Efficiency Hypothesis), the risky portfolio is the market portfolio. Thus, each investor should hold the market portfolio leveraged or de-leveraged in order to achieve their optimal level of risk.

The first important limit of the Markowitz model is the assumption that asset returns are jointly normally distributed. This assumption is crucial for the majority of the models of financial theory. However, asset returns distribution, especially for daily and weekly returns, are characterized by significant higher kurtosis (i.e., fat tails) and significant asymmetry with respect to the normal distribution. This characteristic of asset returns was widely investigated in empirical research (see for example Fama [1965], Kon [1984], Richardson and Smith [1993], Andersen *et al.* [2001] and Bouchaud and Chicheportiche [2012]). The result is coherent with empirical observations on the data that I used in this work. In fact, most of the indices failed the normality test even considering monthly returns[12]. However, this assumption was released by Chamberlain (1983) and Owen and Rabinovitch (1983). In particular, they proved that it's sufficient that asset returns are jointly elliptically distributed. In fact, joint elliptical distributions are still symmetrical and characterized by the first two moments, even if asset returns are not. This conclusion leaves the results of the Markowitz model unchanged. However, Bouchaud and Chicheportiche (2012) empirically reject this hypothesis.

The second concern is over the characteristics of the investor set behind the Markowitz model. In particular, the model assumes that investors are rational and risk averse, with a concave and increasing mean-variance utility function, initially proxied by a quadratic function. Investors' rationality is also the base of Market Efficient Hypothesis (MEH), but the growing literature in Behavioral economics and Behavioral finance argued that this assumption is too restrictive and doesn't hold in the real world. The first criticisms to investor rationality and MEH comes from Kahneman and Tversky (1979) and Shiller (1981). In particular, Kahneman and Tversky (1979) introduced the Prospect Theory, characterized by the asymmetry between gains and losses impact on investors' utility (i.e., loss aversion). This is one of the first criticism to rational behavior. Moreover, Shiller (1981) observed that stock market volatility is too high to be justified only by new information on dividends as predicted by the MEH. In the following years several studies proved the irrationality of investors, with the introduction of noise traders (see for example De Long *et al.* [1990]), investors' over- and under-reactions to news and the concept of overconfidence and biased self-attribution[13]. A lot of papers showed that investors' overconfidence is an important irrational component of investors' behavior (see for example Daniel, Hirshleifer and Subrahmanyam [1998]). Moreover, Daniel, Hirshleifer and Subrahmanyam (1998), quoting Campbell and Cochrane (1994), state that is neces-

[12] See Table 4 in Subparagraph 3.2.1.

[13] Investors' overconfidence is generally defined as the overestimation of the precision of a private information signal, that usually results in over-reaction. Biased self-attribution comes from attribution theory proposed by Bem (1965), for which individuals tend to attribute events that confirm the validity of their actions to high ability, while the events that disconfirm the action are attributed to external noise or sabotage (see Daniel, Hirshleifer and Subrahmanyam [1998]).

sary a utility function with extreme habit persistence in order to explain predictable variations in market returns. Quoting Griffin and Tversky (1992), they showed that experienced investors tend to be more overconfident than inexperienced investors. Odean (1998) shows that overconfidence about a private signal leads to investors' over-reaction, resulting in excess volatility. Finally, an interesting work is the one presented in Baker and Wurgler (2007), where they showed that stocks with low capitalization, low profitability, high volatility, low dividend payout, low growth or stocks of firms in financial distress, are more likely to be highly sensitive to waves generated by investors' sentiment. About the second investor's characteristic, several studies showed that quadratic preferences can generate implausible results, exhibiting increasing absolute risk aversion and displaying negative marginal utility after some finite wealth level (see for example the recent book of Levy [2011]). Thus, the assumptions behind the Markowitz model on investors' characteristics generally don't hold in real world.

Another important drawback of the Markowitz model is related to its outcomes, making its use in the pure form infeasible in practice. Markowitz' optimizers tend to create portfolios characterized by the presence of extreme positions, resulting in some cases in poorly diversified allocation. This is particularly evident in presence of short selling constraints. As mentioned in Section 1, this portfolios are poorly diversified especially in terms of risk. In reality, asset managers cannot take too large long and/or short positions, due to practical and legal limitations. In particular, institutional investors, with the exception of speculative portfolios (i.e., hedge funds), cannot create or they have strong limitations on short positions. This feature of the Markowitz model is well recognized in academic literature and some works tried to propose new models able to overcome this limit (see for example Michaud [1989], Black and Litterman [1991] and Tütüncü and Koenig [2004]). Michaud (1989) and Black and Litterman (1991) showed that Markowitz' optimizers tends to maximize errors. In particular, Michaud (1989) claimed that the habit of using historical data to produce a sample mean used as expected return contributes largely to the error maximization. In addition to that, according to Michaud (1989) and Scherer (2007), Markowitz model tends to select the securities that are the most prone to be subject to large estimation errors, maximizing the impact of estimation error on portfolio weights. These assets are those with the best (i.e. high return and low risk and/or correlation) and the worst features, resulting in too large long and short positions respectively. This is strictly related with another limit of the Markowitz model. This limit is the absence of assets' market capitalization weights considerations. Assets with a low level of capitalization usually present high expected returns and negative correlation with the other asset class, suggesting high portfolio weights. However, these assets are characterized by lower liquidity compared to mid- and large-cap securities. The overweight could lead to risk concentration on the small-cap asset class, consequently increasing dramatically the liquidity risk. Moreover, the quadratic programming optimization algorithm uses point estimates as inputs, treating them as if they were known with certainty, without differencing over different level of uncertainty. Finally, the mean-variance models are often unstable, in the sense that small changes in input might dramatically change the portfolio. The model is especially unstable in relation to the expected return input, since a small change in expected returns might generate a radically different allocation. This instability could dramatically increase transaction costs.

One of the first solution proposed to overcome the limit of the outcomes of the Markowitz model is to add constraints to the minimization problems presented in Eq.(2.3) and Eq.(2.10). These constraints can be imposed directly on the weights or indirectly through functions of the weights. The most simple and common constraint is the no short selling constraint. However, this constraint alone might lead to poorly diversified portfolios. To the pure model or to the no short selling constraint we can add lower and/or upper bound constraints to single asset to limit the minimum (including the maximum short position) and the maximum weight of this asset. Another common set of constraints are the group constraints, consisting in lower and/or upper bounds posed on a group of assets or an asset class (e.g., bonds, small-cap, emerging and frontier markets, etc.), instead of a single asset. Finally, an example of constraint not directly set on weights is the turnover constraint, that imposes a cap on the trading activity or on the portfolio rotation. This constraint helps to contain the impact of transaction costs on portfolio performance. In Appendix A, I provide some mathematical example of this solution.

Michaud (1989) proposed another solution, with the introduction of the concept of Resampled Efficiency. Portfolios on the resampled frontier are composed by assets weight vectors which are the average of the Mean-Variance efficient portfolios weight vectors given a certain level of portfolio return[14]. Unfortunately it has no economic justification and the resampled efficient portfolio is not Mean-Variance efficient. The process is divided in different steps. First of all, we start from a standard Mean-Variance optimization, dividing the portfolios of the efficient frontier into ranks. Secondly, we need to compute the resampled weights for a particular rank, starting from the estimate of the variance-covariance matrix and the mean vector of the historical returns.. We need to generate simulated returns, resampling the historical inputs taking a number of draws from the inputs distribution that should reflect the uncertainty in the inputs. The most common approach to generate simulated returns are

[14] This procedure ensures that after averaging the portfolio weights still sum up to 1.

Monte Carlo and Bootstrap simulation. Using the sample series we can compute a new variance-covariance matrix, used to produce a new efficient frontier that we have to divide in the same number of ranks used in the first step. We have to repeat the second step many times, in order to obtain many efficient frontiers with different weights for a certain rank of return. Finally, we have to compute the resampled frontier, that is composed by the portfolios obtained averaging the weights of the simulated efficient frontiers for a particular rank of return. The main advantage of resampled efficiency is the production of more intuitive portfolio allocations, less sensitive to input perturbations. This is because the resampled efficient portfolio is more diversified and less risky than simple Mean-Variance efficient portfolios. Moreover, as resampled efficiency is an averaging process, it is stable and small changes in the inputs are generally associated with small changes in the optimized portfolios. The biggest disadvantage of resampling comes from the fact that it does not have a sound theoretical foundation. First of all, the process creates statistically equivalent portfolios. Moreover, it cannot be argued theoretically that the resampled portfolio outperforms the mean-variance efficient portfolio. The problem is that a wide range of portfolios are statistically equivalent to the efficient frontier, and a straightforward revision of the managed portfolio could induce an exceptional increment in the transaction costs. Thus, we need a statistical inference procedure to transform the statistical equivalence region into a sample acceptance region to control for the type I error. By assumption, resampled frontier will represent the portfolio efficiency. In fact, a resampled efficient portfolio is a sample mean vector, with the useful statistical properties of the sample mean vector. Second, when compared to mean-variance efficiency, resampled efficiency has more practical investment value. The judgement over the efficiency of a portfolio is based on its proximity to the target resampled efficient portfolio.

Black and Litterman (1991) proposed another important solution. The Black-Litterman asset allocation model is a sophisticated method used to overcome the problem of unintuitive, highly-concentrated and input-sensitive portfolios. This model uses a Bayesian approach to combine the subjective views of the asset managers over the expected returns of one or more assets with the market equilibrium expected returns (i.e., the prior distribution). The resulting expected returns (i.e., the posterior distribution) is a complex weighted average of the investor's views and the market equilibrium. The basic and innovative idea is to combine the equilibrium with investor-specific views with different level of confidence set by the asset manager. In particular, the model allows both absolute and relative views (e.g., performance of an asset compared to another). In traditional Mean-Variance portfolio optimization, relative views cannot be expressed. The level of confidence on the view is expressed as the standard deviation around the expected return of the view, that is decreasing the higher the confidence of the manager. The confidence level affects the influence of a particular view in the portfolio construction. In fact, the lower the confidence on a view and the less this view affects the portfolio weights. This is a very an attractive feature, given the fact that the views are often incorrect. However, views indicate on which assets investors want to take a bet and in which direction this bet ought to be taken. As above-mentioned, the views are combined with the equilibrium returns, generally computed using the CAPM model. The positions are taken on the assets to which investors have expressed views in relation to the benchmark portfolio. The Black-Litterman model proposes a measure of neutral expected returns that is much less sensitive to extreme observations, thanks to the equilibrium return component.

The last important solution is the robust asset allocation proposed by Tütüncü and Koenig (2004). They start from a finite set of possible scenarios for expected returns and the variance-covariance matrix, using a box type uncertainty sets that may be obtained from confidence intervals on the elements of the expected return vector and variance-covariance matrix. The methods used to generate this set are different, like for example rolling method, EWMA and bootstrapping of historical data. Alternative method would use a statistical procedure (e.g. confidence intervals) built on top of the particular alpha and risk model. Given a choice for the decision variables, we are concerned about the worst-case realization of the data from the uncertainty set. The worst-case oriented robust optimization formulations seek to find the solution with the best worst-case guarantees (i.e., expected returns maximization between the worst framework). Robust efficient portfolios have significantly better worst-case behavior and are slightly inefficient with average inputs. Moreover, they remain relevant for long periods. Thus, they are suitable for buy-and-hold investors and present low turnover, and consequently lower transaction costs. However, the worst-case oriented model are conservative models. Thus they are not suitable for aggressive investors. Robust optimization models offer intuitive and useful approaches to interpret and manage uncertainty in parameters of optimization problems. Black-Litterman model and robust asset allocation help to introduce the uncertainty over the Mean-Variance optimization inputs (i.e., expected returns and variance-covariance matrix), solving a serious limit of the Markowitz model.

2.2 Risk Budgeting Approach

In this paragraph I present the risk budgeting approach. Risk budgeting is the core methodology adopted in this thesis to construct portfolios with Frontier Market asset class. It was designed to solve the drawbacks of the Markowitz model, in particular the tendency to produce poorly diversified portfolios, especially in terms of risk rather than in terms of weights. In the previous subparagraph I present some models that try to tackle the same problem. However, they continued to maintain the same framework of the Markowitz model, that is volatility minimization given a target expected return. Risk Budgeting Approach is completely different from those presented above, since it completely avoids any consideration on returns. In fact, the investors have to choose only the risk repartition between the assets included in the investment universe.

Risk Budgeting belongs to the family of the new risk-based investment style. These methodologies put risk diversification as their core feature and they don't use any performance forecast as input of the model. Thus, Risk Budgeting Approach helps to avoid the impact of return estimation error and return uncertainty on the construction of optimal portfolios. This aspect is not a secondary one. As above-mentioned, Michaud (1989) showed the importance of the estimation of the expected returns, given the high sensitivity of the outcomes of the optimization process (i.e., portfolio weights, asset allocation) to the changes in expected returns. Moreover, there is a broad consensus in literature that variance-covariance matrix is easier to estimate from historical data than expected returns, producing smaller estimation errors (see for example Merton [1980] and Chan, Karceski and Lakonishok [1999]). Thus, the implementation of Risk Budgeting methodology, with the consequent absence of expected return considerations, can help to solve serious limits of the Markowitz model, such as the lack of risk diversification, the tendency to maximize errors in expected return estimation and the production of unstable portfolios highly sensitive to changes in expected returns, potentially reducing transaction costs.

As mentioned in Section 1, despite its recent diffusion in asset managers' practice, Risk Budgeting Approach can count on a consistent literature. In this thesis, I use Bruder and Roncalli (2012) and Roncalli (2013) to develop the theoretical part on Risk Budgeting. In particular, I present a General Risk Budgeting Approach, where the risk budgets are set by the asset manager with a certain freedom. I'm not going to present the Equal Risk Contribution (ERC) portfolio, extensively studied by Maillard, Roncalli and Teiletche (2010)[15]. In this portfolio, the risk contribution for each asset are equal. However, the ERC portfolio is not suitable for investors that don't want to manage risk exposures in a uniform way[16]. This is precisely the situation in which I develop this thesis. Given their higher risk and relative illiquidity, Fron-

[15] In this paper they presented the main properties of the ERC portfolio. In particular, they showed that this portfolio is located between the Global Minimum Variance and the Equally Weighted portfolio.

[16] A classic example is the strategic asset allocation, where the asset manager has to allocate between different asset classes. In some cases these asset classes represent alternative or illiquid investment (e.g., small-cap in equity portfolios, high yield bonds in fixed income portfolios, commodities and real estate) where the asset manager needs to limit the risk exposure in a different way with respect to the other asset classes.

tier Markets cannot contribute to portfolio risk as Developed and Emerging Markets. Within the equity asset class, they represent an alternative investment and their weights should be constrained to a small portion of the portfolio. Given the illiquidity of the single Frontier Market, an ERC allocation will increase dramatically the liquidity risk.

2.2.1 The Theoretical Framework

As mentioned in Subparagraph 2.1.2, a way to solve the drawbacks of the Markowitz model is to add constraints to the pure optimization problem, ending up with the so called constrained efficient frontier. Within the various form of constraints, investors can impose a weight budgeting (e.g., upper and lower bounds), directly limiting the overall weight of asset classes or of a single asset. However, asset managers might constrain other aspects associated with asset classes, as for example their contribution to the portfolio return or to a performance measure, thus ending up with Performance Budgeting. With Risk Budgeting, investors impose constraints on the risk contribution of the asset classes or of a specific asset to the portfolio total risk.

I start from a portfolio of n assets, with w_i that define the exposure (i.e. the weight) of the i^{th} asset ($i = 1,2,\dots,n$). We denote $R(w_1,w_2,\dots,w_n)$ as the risk measure for the portfolio composed by the n assets with weights $(w_1,w_2,\dots,w_n)$. If the risk measure is coherent and convex we have the following Euler decomposition (see Artzner *et al.* [1999]):

$$R(w_1,\dots,w_n) = \sum_{i=1}^{n} w_i \cdot \frac{\partial\, R(w_1,\dots,w_n)}{\partial\, w_i} = \sum_{i=1}^{n} RC_i \qquad (2.20)$$

Following Eq.(2.20), the portfolio risk measure is the sum of the product between the exposure and its marginal risk. The risk contribution of the i^{th} asset is defined as:

$$RC_i(w_1,\dots,w_n) = w_i \cdot \frac{\partial\, R(w_1,\dots,w_n)}{\partial\, w_i} \qquad (2.21)$$

Risk contributions can be analyzed considering the marginal contributions to the overall portfolio risk. In particular, we observe that the marginal risk contribution of the i^{th} asset is defined as:

$$\frac{\partial\, R(w)}{\partial\, w_i} = \lim_{h\to\infty} \frac{R(w + h\boldsymbol{e_i}) - R(w)}{h}$$

The vector $\boldsymbol{e_i}$ is a vector of zeros apart for the element *i*. Then, assuming that *h* is small, we can obtain:

$$R(w + h\boldsymbol{e_i}) = R(w) + h\frac{\partial\, R(w)}{\partial\, w_i} \quad \rightarrow \quad MR_i = \frac{\partial\, R(w)}{\partial\, w_i}$$

The marginal risk contributions are part of the risk contributions. Risk Budgeting methods are based on the increase in the portfolio risk subsequent to a change in one of the assets risk.

Given a set of risk budgets b_i for the *n* assets, the Risk Budgeting portfolio is generally defined as the portfolio such that the risk contributions match their respective risk budgets:

$$\begin{cases} RC_1(w_1, \dots, w_n) = b_1 \\ \vdots \\ RC_i(w_1, \dots, w_n) = b_i \\ \vdots \\ RC_n(w_1, \dots, w_n) = b_n \end{cases} \tag{2.22}$$

The system presented in Eq.(2.22) is completely different from a Weight Budgeting portfolio (i.e., $w_i = b_i$), since it is composed by a set of nonlinear equations. The solution of this system represents the Risk Budgeting portfolio.

About the risk measures that can be used in Risk Budgeting Approach, we have several options. The most common risk measure is the volatility, used also in this thesis when I apply this approach to portfolio construction. Focusing on volatility risk measure, we obtain portfolio risk, marginal risk contribution and risk contribution in Eq.(2.23), in Eq.(2.24) and in Eq.(2.24) respectively[17]:

$$R(w) = \sigma(w) = \sqrt{w'\Sigma w} \tag{2.23}$$

$$\frac{\partial\, R(w)}{\partial\, w_i} = \frac{(\Sigma w)_i}{\sqrt{w'\Sigma w}} \tag{2.24}$$

$$RC_i(w) = w_i \frac{(\Sigma w)_i}{\sqrt{w'\Sigma w}} \tag{2.25}$$

A above-mentioned, volatility is not the only risk measure that can be used. Under the assumption of normally distributed returns, managing risk exposures with value at risk (VaR) and expected shortfall (ES) is equivalent to manage risk exposures with volatility (for VaR risk measure see for example Jorion [2007]) . In general, in a Gaussian world every risk

[17] In Appendix A, I provide the check using the Euler decomposition.

measure that doesn't depend on the mean of the distribution can be used. Of this class of risk measures, VaR and ES are the most common in financial markets practice.

Bruder and Roncalli (2012) proposed another formulation of the system in Eq.(2.22), where they define the portfolio in terms of weights, while the risk budgets are defined in relative terms. Moreover, they decided to obtain a long-only portfolio. However, risk budgets may be negative, with the concentration of risk on the other assets, depending on investors' objectives. Following Bruder and Roncalli (2012), the new system used to define the Risk Budget (RB) portfolio is presented in Eq.(2.26):

$$\begin{cases} w_i \cdot (\Sigma w)_i = b_i(w'\Sigma w) \\ b_i \geq 0 \\ w_i \geq 0 \\ \sum_{i=1}^{n} b_i = 1 \\ \sum_{i=1}^{n} w_i = 1 \end{cases} \tag{2.26}$$

Given the new formulation, we add two positivity constraints on risk budgets and weights and the usual constraints that risk budgets and weights have to sum up to 1. A problem connected to this specification of the problem may appear if the asset manager want to set some risk budgets equal to zero. Given σ_i the volatility of the asset *i* and ρ_{ij} the cross-correlation between asset *i* and *j*, we have that:

$$(\Sigma)_{ij} = \rho_{ij}\sigma_i\sigma_j \quad \Rightarrow \quad \frac{\partial\, \sigma(w)}{\partial\, w_i} = \frac{w_i\sigma_i^2 + \sigma_i \sum_{i\neq j} w_j\rho_{ij}\sigma_j}{\sigma(w)}$$

Eq.(2.27) represents the situation in which the risk budget for the asset *k* (i.e., b_k) is fixed equal to zero and Eq.(2.28) and Eq.(2.29) are the solutions of this constraint:

$$w_k\left(w_k\sigma_k^2 + \sigma_k \sum_{k\neq j} w_j\rho_{kj}\sigma_j\right) = 0 \tag{2.27}$$

$$w'_k = 0 \tag{2.28}$$

$$w''_k = -\frac{\sum_{k\neq j} w_j\rho_{kj}\sigma_j}{\sigma_k} \tag{2.29}$$

If the cross-correlation is positive, given the positivity constraint on w_k, the only possible solution is the one presented in Eq.(2.28), thus the weight has to be equal to zero. However, if the cross-correlation is negative, the weight on the asset *k* might be positive as presented in Eq.(2.29). The second solution may not satisfy the asset manager, in particular if he expects that the asset associated to a zero risk budget will not be included in the portfolio. If we want to set some risk budgets to zero with the objective to don't include some assets in the portfolio, it's better to impose strictly positive constraint on risk budgets (i.e., $b_i > 0$) and to eliminate ex-ante from the investment universe the assets associated to the zero risk contributions.

In the case with *n* assets it is not possible to find an explicit solution. It can be found for the case with two assets or, in the general case, introducing some strict restrictions, such as constant correlation[18]. Even from a computational point of view, the problem is not trivial and we can use Sequential Quadratic Programming algorithm to find the RB portfolio. There are two main formulations of the problem, presented respectively in Eq.(2.30) and Eq.(2.31):

$$w^* = \arg\min \sum_{i=1}^{n} \left(\frac{w_i(\Sigma w)_i}{\sum_{j=1}^{n} w_j (\Sigma w)_j} - b_i \right)^2 \tag{2.30}$$

$$s.to \quad \mathbf{1}'w = 1$$

$$\mathbf{0} \leq w = \leq \mathbf{1}$$

or

$$y^* = \arg\min \sqrt{y'\Sigma y} \tag{2.31}$$

$$s.to \quad \begin{cases} \sum_{\boldsymbol{i=1}}^{\boldsymbol{n}} b_i \ln y_i \geq c \\ y \geq \mathbf{0} \end{cases}$$

The formulation presented in Eq.(2.31) is similar to a constrained variance minimization problem. The optimal weights is not constrained to sum at 1, but we can satisfy the constraint by normalization of the optimal weights resulting from the solution of Eq.(2.31). Moreover, the constant *c* could be adjusted to obtain the same result and the weights will be expressed in relative terms:

$$\sum_{i=1}^{n} {y_i}^* = 1 \quad \rightarrow \quad w_i^* = \frac{{y_i}^*}{\sum_{i=1}^{n} {y_i}^*}$$

The formulation presented in Eq.(2.31) demonstrates that the RB portfolio, when the risk budgets are strictly positive, exists and is unique if the variance-covariance matrix is positive-definite. In fact, it corresponds to a minimization of a quadratic convex function with convex constraint. If there are some risk budgets that are null, there could be several solutions. More-

[18] In Appendix A, I provide the explicit solution for the case with two assets and constant correlation. However, even with constant correlation in some cases it is not possible to find an explicit solution.

over, it can be shown that previous results are valid for general convex risk measures R (e.g., VaR and ES), by replacing the marginal volatility by the marginal risk in the mathematical proofs. The optimization problem with a general risk measure can be written as:

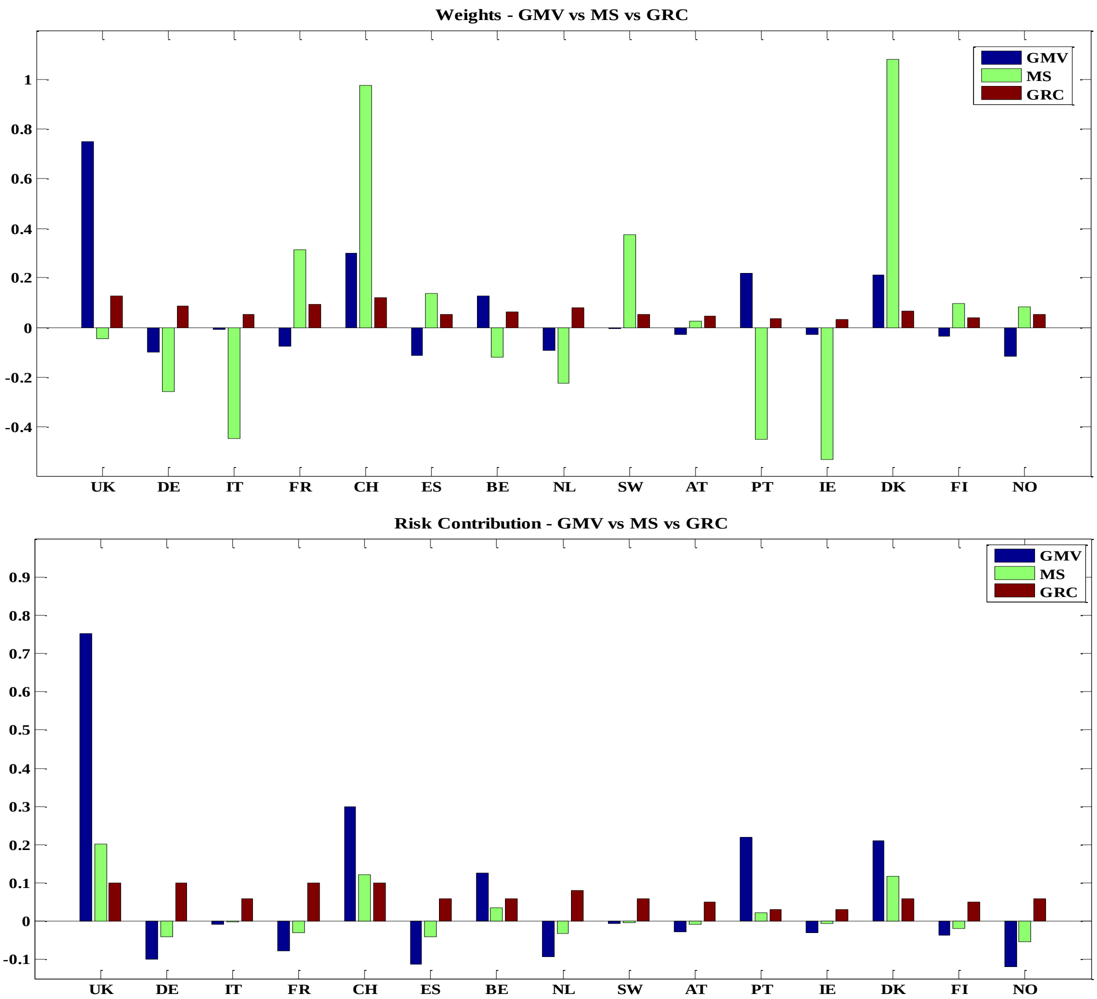
$$y^* = \arg\min\ R(y) \tag{2.31}$$

$$s.to \quad \begin{cases} \sum_{i=1}^{n} b_i \ln y_i \geq c \\ y \geq \mathbf{0} \end{cases}$$

Again, if the risk budgets b_i are strictly positive, the RB portfolio exists and is unique, while with some zero risk budgets there may be several solutions.

Figure 5 shows a comparison between two Mean-Variance Optimized portfolios (Global Minimum Variance and Maximum Sharpe) and a Risk Budgeting portfolio, in terms of weights and risk contributions. I constructed the portfolios with the same indices used to draw the examples of the efficient frontiers in Subparagraph 2.1.1.

Figure 5 – Weights and Risk Contributions Comparison (GMV vs MS vs RB)



The first panel of Figure 5 shows one of the limit of the Markowitz model, with the presence of extreme positions. The GMV and the MS portfolios are not realistic from a practical point of view. For the GMV, this feature is due to the low diversification in the long part and the aggregate size of the short part. For the MS, the problems arise especially from the short part, with very large positions and a consistent aggregate size. However, also the long part has some extreme positions[19]. From the second panel, we can notice that RB portfolio presents a higher degree of diversification, while the MVO portfolios, in particular the GMV, present a lower degree of diversification in terms of risk. In general, the figure shows clearly the properties of Risk Budgeting Approach, with the construction of realistic and well diversified portfolios in terms of weights and risk. Moreover, fixing the risk budgets, we can partially choose the degree of diversification of the RB portfolio. This choice is not possible with the MVO portfolios.

In conclusion, I want to present briefly other two important properties of Risk Budgeting showed by Bruder and Roncalli (2012). First of all, as the Equal Risk Contribution portfolio is located between the Global Minimum Variance and the Equally Weighted portfolios, the Risk Budgeting portfolio is located between the Global Minimum Variance portfolio and the Weight Budgeting portfolio (i.e., $\sigma_{GMV} < \sigma_{RB} < \sigma_{WB}$), where the Weight Budgeting is the portfolio such that the weights are set equal to the budgets (i.e., $x_i = b_i$). Secondly, as the Equal Risk Contribution portfolio, the Risk Budgeting Approach is an heuristic asset allocation method. Maillard, Roncalli and Teiletche (2010) showed that the ERC portfolio corresponds to the tangency portfolio when the correlation is the same and when the assets present the same Sharpe ratio. For the RB portfolio is more difficult to find such properties. However, considering a quadratic utility function with risk aversion in the form of the one presented in Eq.(2.7), the portfolio *w* is optimal if the expected returns satisfy:

$$\mu = \frac{2}{\gamma}\Sigma w \quad \Rightarrow \quad PC_i = w_i \mu_i = w_i \frac{2}{\gamma}(\Sigma w)_i \ \propto \ b_i \tag{2.32}$$

If the Risk Budgeting portfolio is optimal, the performance contribution of the i^th asset is proportional to its risk budget, as shown by Eq. (2.32). Thus, specifying the risk budgets, the investor can decide both the amount of risk and the amount of expected performance to assign to a particular asset.

[19] In the GMV portfolio almost 80.00 per cent is invested in MSCI UK, while the short part has a global weight in absolute value higher than 50.00 per cent. For the long part of the MS portfolio, MSCI CH and MSCI DK have weights close or above 100.00 per cent. The problem is particularly serious for MSCI DK, that is a relatively illiquid market. For the short part, we have weights in absolute value above 40.00 per cent and the size of the short part is above 200.00 per cent in absolute value.

3 Investment Universe

In this section, I present my investment universe, which included thirty-five stock market indices provided by Morgan Stanley Capital International (MSCI), covering Developed, Emerging and Frontier Markets. In the first part, I define what Frontier Markets are, presenting the main definitions given by different financial institutions, and stating the reason why I chose to use the MSCI indices. In the second part, I illustrate in detail my investment universe, including an analysis of its basic characteristics[20].

3.1 Frontier Markets

Frontier Market definition is not unique in financial world. Usually they represent small, relative illiquid, but investable, stock markets, mainly located in developing world[21]. When their market capitalization, accessibility condition and liquidity improve, Frontier Markets may be reclassified as Emerging Markets. The origin of this classification can be attributed to Standard and Poor's (S&P) in the 1990s. It gained importance when, in 2007, S&P launched the Select and Extended Frontier Indexes. In the same year, MSCI launched its Frontier Markets Index, followed in the next years by FTSE and Russell Investments. In the last years, mutual funds and ETFs on Frontier Markets were launched by the major investment companies and banks[22].

S&P Dow Jones provides a set of thirteen Frontier Markets indices[23]. Country composition varies according to the objective of the specific index and countries are chosen from a list of thirty-six Frontier Markets. The most representative index is the Frontier Markets Broad Market Index (BMI), which includes thirty-four countries. To be considered as Frontier Market, a stock market must meet at least two of the following requirements[24]:

- Full domestic market capitalization of over US$ 2.5 billion;
- Annual turnover value of over US$ 1 billion;
- A market development ratio of over 5 per cent[25].

[20] In particular I show basic characteristics of monthly returns (mean, volatility, skewness, kurtosis, etc.), cross-markets correlations (divided in sub-periods) and foreign currency movements.
[21] There are some exceptions, such as Estonia, Lithuania and some Arabic Gulf countries.
[22] One of the first fund was launched by Barclays Global Investors (BGI), with its BGI Frontier Fund in 2008. It was followed by Templeton, HSBC, Schroder International. ETFs were launched by Deutsche Bank, followed by Guggenheim Investments and RBS.
[23] For a complete list, see the S&P Dow Jones Frontier Indices Methodology note available at: us.spindices.com.
[24] See the S&P Dow Jones Indices' Country Classification Methodology available at: us.spindices.com.
[25] Market development ratio is defined as the ratio between the full domestic market capitalization of the exchange by the country's nominal GDP.

These constraints are the initial requirements for country inclusion in the S&P Dow Jones Indices' Global Equity Index (GBI). They can be considered as Frontier Markets constraints, since S&P Dow Jones requires more tight additional requirements for Emerging and Developed Markets. In addition to these quantitative constraints, the evaluation process includes other considerations about investor interest and market accessibility (e.g. turnover, number of listings, foreign investor interest) and market's development prospects about market breadth (i.e., listings), depth (i.e., market capitalization and turnover), and infrastructure (i.e., regulatory structure, custody, clearance and settlement). Finally, there are also some qualitative considerations about macroeconomic conditions, political stability, legal property rights, legal procedures, trading and settlement processes and conditions, opinions and experiences of institutional investors. In the construction of standalone country indices, S&P Dow Jones fixes some constraints on individual stock selection. The main variables observed are market capitalization, liquidity, days traded, public shares availability to foreign investors, investability (e.g. foreign investment restrictions), domicile, type of instrument. These requirements are variable, depending on the Frontier Market index considered[26].

FTSE provides two main indices, the FTSE Frontier Index Series and the FTSE Frontier 50, including twenty-six countries. The evaluation process followed to classify a country as Frontier, Emerging or Developed includes many steps and it's described in FTSE Country Classification Process paper[27]. It is based on the Quality of Markets Matrix, composed by different constraints divided in five groups:

- World Bank GNI Per Capita Rating;
- Market and Regulatory Environment (seven constraints);
- Custody and Settlement (six constraints);
- Dealing Landscape (seven constraints);
- Derivatives (one constraint).

Frontier Markets have to meet two constraints in Market and Regulatory Environment section ("Formal stock market regulatory authorities actively monitor market" and "No objection to or significant restrictions or penalties applied to the investment of capital or the repatriation of capital and income"), two constraints on Custody and Settlement section ("Settlement – Rare incidence of failed trades" and "Clearing and settlement – T + 3, T + 5 for Frontier") and one in Dealing Landscape section ("Transparency – market depth information / visibility and timely trade reporting process"). The process is mainly qualitative, with no specific quantita-

[26] For more details, see the S&P Dow Jones Frontier Indices Methodology note available at: us.spindices.com.
[27] All the documentation is available in FTSE website: www.ftse.com.

tive limits declared. Annually, FTSE revised country classification, publishing in March and in September a Watch List containing the countries monitored for promotion or demotion. In order to be included in the FTSE Frontier Markets Index Series, a country classified as Frontier must have at least three companies able to meet individual stock eligibility criteria[28].

Russell Investments provides a set of six Frontier Markets Indices, including thirty-six Frontier Markets. The most representative index is the Russell Frontier Index. In order to categorize countries between Frontier, Emerging and Developed, Russell Investments combines a set of macroeconomic and market-based criteria[29]. The first step is the definition of the Emerging/Frontier Market category, based on:

- Economic Criteria:
  - Relative Income, based on World Bank Income Category: "Less than high income";
  - Development Status, based on IMF classification: "Advancing";
  - Country Risk, based on Economist Intelligence Unit (EIU) Score: "Score greater than 40";
- Market (operational) criteria: Presence of FX Restrictions; Presence of Registration Restrictions; Stock transfer restrictions within fund complex not allowed; Relative Liquidity below the median; Foreign Ownership Limits moderate or restrictive; Segregate allowable accounts structure.

Then, Russell Investments defines Frontier Markets as "those that do not meet the established criteria for membership in Russell's Emerging Markets Indexes". Alongside the macroeconomic and country risk criteria, Russell Investment considers the following constraints on trading risks and challenges:

- Incomplete Regulatory Infrastructure;
- Absence of segregation in Trading and Custody accounts;
- Broader restrictions on Foreign Ownership Limits;
- Absence of Trade Confidentiality;
- Settlement Periods longer than T + 3;
- Market Liquidity lower than Emerging Markets;
- Presence of pre-deposit of shares requirements.

[28] For further information, see the Ground Rules for the FTSE Frontier Index Series note, available in FTSE website: www.ftse.com.
[29] For detailed information, see the Russell Global Indexes Construction and Methodology paper available in the Russell Investment website: www.russell.com. In particular, for Frontier Markets look at Section 9.

The country classification is revised annually, during the first quarter. As for S&P Dow Jones and FTSE, Russell Investments considers individual stocks eligibility criteria (e.g. individual market capitalization higher than US$ 1 million), in order to guarantee minimum liquidity standards.

The last provider of Frontier Markets indices is Morgan Stanley Capital International (MSCI), with a coverage of thirty-eight countries. Alongside the Frontier Markets Index, actually covering twenty-four countries, MSCI provides standalone country indices. To classify a country as Frontier Market, MSCI considers the equity markets not included in the MSCI Emerging Markets Index that respect the following principles[30]:

- Relative Openness to and Accessibility for foreign investors;
- Exclusion from Developed Market universe[31];
- Countries that are not experimenting a period of extreme economic instability (e.g. hyperinflation) or political instability (e.g. civil war);
- A minimum of two companies with securities eligible for the Standard Index.

In addition to these principles, MSCI considered the following qualitative and quantitative constraints:

- Economic Development:
    - Sustainability of economic development: no requirements[32];
- Size and Liquidity Requirements:
    - Number of the companies meeting this section requirements: 2;
    - Company size (full market capitalization): US$ 598 million;
    - Security size (float market capitalization): US$ 52 million;
    - Security liquidity: 2.5% Annual Traded Value Ratio (ATVR);
- Market Accessibility Criteria:
    - At least some Openness to foreign ownership;
    - At least partial ease of capital inflows/outflows;
    - Modest Efficiency of the operational framework;
    - Modest Stability of the institutional framework.

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[30] For detailed information, refer to MSCI Global Investable Market Indexes Methodology paper, available in MSCI website: www.msci.com.

[31] For example Luxembourg, Iceland or Cyprus, despite their small size, are included in the Developed Markets universe.

[32] For Frontier and Emerging Markets, MSCI doesn't require specific economic development level, given the wide heterogeneity between the countries belonging to these broad categories. For Development Markets the constraint is defined as "Country GNI per capita 25% above the World Bank high income threshold for 3 consecutive years".

The revision of country classification is semi-annual, in May and November. Frontier Markets are divided in sub-categories, based on more specific size and liquidity constraints. In addition to that, MSCI requires also individual stock eligibility criteria[33].

Table B.1 in Appendix B contains the whole list of Frontier Markets available, divided by the four providers (S&P Dow Jones, FTSE, Russell Investment and MSCI). As abovementioned, I decided to use MSCI Indices, because MSCI provides standalone country indices with daily quotations available in Datastream[34].

3.2 Investment Universe Basic Characteristics

The investment universe is composed by thirty-five stock market indices provided by Morgan Stanley Capital International (MSCI), covering Developed, Emerging and Frontier Markets:

- Developed Markets: three indices by geographical area (North America, Europe and Middle East, Pacific), covering twenty-three developed countries;
- Emerging Markets: single index, not divided by geographical area, covering twenty-three countries;
- Frontier Markets: thirty-one standalone country indices.

There are two main reasons that explain the choice to use geographical area or standalone country indices. First of all, they provide the representation of entire stock markets, covering different sectors (IT, Energy, Materials, Utilities, Consumer Staples and Discretionary, Industrials, Health Care, Communications), and they are characterized by a lower propensity to large movements typical of individual stocks. Thus, they represent real investment opportunities in a strategic and tactical asset allocation perspective. Secondly, given the illiquidity of Frontier Markets and the presence of capital controls, stock picking would amplify liquidity risk, constraining the possibility to close the position in the presence of serious downturn[35]. Asset managers practice is to enter in these markets through ETFs on stock market indices.

Another important feature of the investment universe is the composition of Frontier Markets asset class, based on standalone country indices instead of geographical area indices. As mentioned in Section 1, the aim of the thesis is to construct an individual Country Risk Indi-

[33] See note 30.
[34] Also S&P Dow Jones provides standalone country indices, but quotations on Datastream were not availbale for the major part of the countries.
[35] Frontier Markets are prone to large movements and are characterized by higher volatility compared to Developed and Emerging Markets. Moreover, their performances are strictly connected to international liquidity environment. The timing of investment is crucial, making the liquidity risk one of the most important threat to investment performance. Finally, stock picking in these Markets requires a daily or a weekly analysis, the implementation of technical analysis and operational allocation methodology, while in this thesis I used monthly data.

cator useful to compute the risk budgets, focusing on Frontier Markets and on their capacity to improve portfolio allocation through international diversification.

Table 1 – Investment Universe

MSCI INDICES	START	END	CURRENCY	ISO 4217
MSCI North America	31/12/1999	10/02/2015	US Dollar	USD
MSCI Europe & Middle East	31/12/1999	10/02/2015	US Dollar	USD
MSCI Pacific	31/12/1999	10/02/2015	US Dollar	USD
MSCI Emerging Markets	31/12/1999	10/02/2015	US Dollar	USD
MSCI Argentina	31/12/1999	10/02/2015	Argentina Peso	ARS
MSCI Jamaica	25/11/2008	10/02/2015	Jamaica Dollar	JMD
MSCI Trinidad & Tobago	25/11/2008	10/02/2015	Trinidad and Tobago Dollar	TTD
MSCI Bosnia Herzegovina	26/05/2010	10/02/2015	Bosnia and Herzegovina Marka	BAM
MSCI Bulgaria	31/05/2005	10/02/2015	Bulgaria Lev	BGN
MSCI Croatia	31/05/2002	10/02/2015	Croatia Kuna	HRK
MSCI Estonia	31/05/2002	10/02/2015	Euro	EUR
MSCI Kazakhstan	30/11/2005	10/02/2015	Kazakhstan Tenge	KZT
MSCI Lithuania	30/05/2008	10/02/2015	Euro	EUR
MSCI Romania	30/11/2005	10/02/2015	Romania New Leu	RON
MSCI Serbia	30/05/2008	10/02/2015	Serbia Dinar	RSD
MSCI Slovenia	31/05/2002	10/02/2015	Slovenian Tolar	SIT
MSCI Ukraine	31/05/2006	10/02/2015	Ukraine Hryvnia	UAH
MSCI Botswana	25/11/2008	10/02/2015	Botswana Pula	BWP
MSCI Ghana	25/11/2008	10/02/2015	Ghana Cedi	GHS
MSCI Kenya	31/05/2002	10/02/2015	Kenya Shilling	KES
MSCI Mauritius	31/05/2002	10/02/2015	Mauritius Rupee	MUR
MSCI Morocco	31/12/1999	10/02/2015	Morocco Dirham	MAD
MSCI Nigeria	31/05/2002	10/02/2015	Nigeria Naira	NGN
MSCI Tunisia	31/05/2004	10/02/2015	Tunisia Dinar	TND
MSCI Zimbabwe	30/11/2010	10/02/2015	Zimbabwe Dollar	ZWD
MSCI Bahrain	31/05/2005	10/02/2015	Bahrain Dinar	BHD
MSCI Jordan	31/12/1999	10/02/2015	Jordan Dinar	JOD
MSCI Kuwait	31/05/2005	10/02/2015	Kuwait Dinar	KWD
MSCI Lebanon	31/05/2002	10/02/2015	Lebanon Pound	LBP
MSCI Oman	31/05/2005	10/02/2015	Oman Rial	OMR
MSCI Saudi Arabia	31/05/2005	10/02/2015	Saudi Arabia Riyal	SAR
MSCI Bangladesh	30/11/2009	10/02/2015	Bangladesh Taka	BDT
MSCI Pakistan	31/12/1999	10/02/2015	Pakistan Rupee	PKR
MSCI Sri Lanka	31/12/1999	10/02/2015	Sri Lanka Rupee	LKR
MSCI Vietnam	30/11/2006	10/02/2015	Viet Nam Dong	VND

Table 1 presents the investment universe dataset, with the MSCI indices, starting and ending dates and currency denominations. Quotations were downloaded from Dastastream with daily frequency. Table B.2 in Appendix B presents the whole set of MSCI indices, with detailed country coverage for MSCI North America, MSCI Europe and Middle East, MSCI Pacific and MSCI Emerging Markets. Given the fact that the countries composing Developed and Emerging Markets indices have different legal tenders, the data are denominated in US Dollar. From the dataset, I excluded MSCI Western African Economic and Monetary Union (WAEMU) and MSCI Palestine indices. For MSCI WAEMU, I was not able to retrieve quotations from Datastream. In addition to that, it would be problematic to compute the Country Risk Indicator given the heterogeneity of countries belonging to WAEMU[36]. For MSCI Palestine, the inception date is 31/05/2013, too close to the ending date. Moreover, there are some concerns connected to the availability of macroeconomic data necessary to compute the Country Risk Indicator[37].

In portfolio construction process, I took a European investor perspective. Thus, I converted MSCI Price Series to monthly frequency and in Euro. The conversion in domestic currency was performed to replicate asset managers' practice on equity investment. Through the conversion in domestic currency, currency risk and performance are internalized by monthly returns. In this way, asset managers can profit from favorable exchange rate movements, taking specific bets on the currencies alongside the considerations on stock returns. For this reason, the Country Risk Indicator has a section dedicated to currency risk management. In the next subparagraphs, I'm going to present monthly return characteristics (in Local currency and in Euro), exchange rates and cross-country correlations. The graphs on MSCI indices monthly price and return time series, alongside exchange rate time series, are included in Appendix B.

3.2.1 Monthly Returns Descriptive Characteristics

For each MSCI index belonging to the investment universe, I computed some basics characteristics of monthly returns on the full sample period, both in Local currency and in Euro. Table 2 provides average, median, volatility, minimum and maximum, skewness and kurtosis of monthly returns in Local currency. Table 3 presents the same features of the distribution for monthly returns in Euro. As above-mentioned, I used monthly returns in Euro in the allocation process. Figure 6 and Figure 7 provide a scatter plot representation of the relation between average return and volatility for each index, divided by geographical area. In Table 4, I present the results of Jarque-Bera and Ljung-Box test on normality and autocorrelations of return distribution, for data in Local currency and in Euro.

[36] Standalone country indices for WAEMU were added during the May 2014 Semi-Annual Index Review and their quotations are not available.
[37] The major data-provider on Palestine is the World Bank. However, most of the time series start from 2013 only.

Table 2 – Monthly Returns Descriptive Characteristics (LOC)

MSCI INDEX	MEAN	MEDIAN	VOL	MIN	MAX	SKEW	KURT
North America	0.53	1.07	4.34	-18.07	10.78	-0.88	5.32
Europe And Middle East	0.26	0.49	5.83	-21.94	13.40	-0.68	4.45
Pacific	0.27	0.86	4.75	-17.92	11.03	-0.74	4.54
Emerging Markets	0.65	0.65	6.83	-27.50	16.66	-0.60	4.84
Argentina	1.23	2.11	11.09	-37.05	53.29	-0.53	3.75
Jamaica (6)	0.31	0.66	6.36	-16.96	19.57	0.24	4.21
Trinidad And Tobago (6)	0.45	0.17	2.96	-10.43	8.57	-0.23	5.87
Bosnia And Herzegovina (8)	-0.20	0.02	4.72	-9.40	13.00	0.40	3.67
Bulgaria (1)	-0.74	-0.76	9.14	-41.19	32.53	-0.52	8.07
Croatia	0.23	0.05	6.95	-23.90	27.82	0.24	7.23
Estonia	0.15	-0.14	9.55	-31.58	54.06	1.26	11.31
Kazakhstan (2)	0.42	0.22	11.73	-29.53	103.34	1.52	11.49
Lithuania (5)	0.47	0.14	7.93	-22.64	46.48	2.32	16.90
Romania (2)	0.55	1.46	9.76	-41.43	30.01	-0.79	6.82
Serbia (5)	-0.52	0.07	12.05	-42.27	42.65	-0.20	6.43
Slovenia	0.04	-0.12	6.03	-15.74	18.36	0.13	4.19
Ukraine (3)	-0.71	-1.11	12.19	-32.68	36.64	0.18	3.78
Botswana (6)	0.37	0.35	5.07	-18.55	15.86	0.06	4.38
Ghana (6)	1.55	0.96	7.20	-22.04	33.13	0.74	8.01
Kenya	1.34	1.91	6.65	-23.75	27.90	-0.62	5.71
Mauritius	1.24	1.09	5.95	-24.29	21.37	-0.27	7.47
Morocco	0.53	0.45	5.25	-16.84	20.66	0.52	4.53
Nigeria	0.90	0.70	9.37	-37.54	46.63	0.38	9.43
Tunisia	0.97	0.39	5.03	-12.01	23.85	0.58	6.31
Zimbabwe (9)	1.08	0.14	8.24	-27.01	25.61	-0.12	5.59
Bahrain (1)	-1.90	-1.46	6.88	-28.35	17.48	-0.64	5.89
Jordan	-0.57	-0.71	6.07	-23.19	19.89	-0.08	5.30
Kuwait (1)	-0.08	0.51	6.36	-17.84	21.06	-0.07	3.89
Lebanon	0.76	-0.39	9.23	-22.60	47.69	1.51	9.42
Oman (1)	-0.18	0.65	5.84	-29.86	13.44	-1.25	7.91
Saudi Arabia (1)	0.01	1.20	8.24	-25.64	20.50	-0.48	3.94
Bangladesh (7)	0.55	0.61	8.09	-28.96	16.75	-0.72	4.94
Pakistan	0.99	1.07	8.42	-50.35	31.72	-1.84	13.83
Sri Lanka	1.20	0.18	9.22	-23.44	53.12	1.71	11.20
Vietnam (4)	0.17	-0.48	11.08	-23.80	48.51	0.78	6.00

(1) From May 2005 ; (2) From November 2005 ; (3) From May 2006 ; (4) From November 2006 ; (5) From May 2008 ; (6) From November 2008 ; (7) From November 2009; (8) From May 2010; (9) From November 2010

Table 3 – Monthly Returns Descriptive Characteristics (EUR)

MSCI INDEX	MEAN	MEDIAN	VOL	MIN	MAX	SKEW	KURT
North America	0.64	0.80	3.77	-12.66	10.51	-0.49	3.41
Europe And Middle East	0.32	0.82	4.28	-13.76	13.62	-0.63	4.58
Pacific	0.37	0.06	3.98	-11.01	11.57	-0.21	3.79
Emerging Markets	0.71	0.80	5.65	-19.69	16.50	-0.51	4.28
Argentina	0.45	0.49	10.90	-36.77	53.77	-0.72	4.21
Jamaica (6)	0.11	-0.40	6.74	-20.28	21.72	0.11	4.33
Trinidad And Tobago (6)	0.78	0.74	4.60	-12.21	12.46	0.02	3.49
Bosnia And Herzegovina (8)	-0.20	0.02	4.72	-9.40	13.00	0.40	3.67
Bulgaria (1)	-0.74	-0.76	9.14	-41.37	32.53	-0.53	8.09
Croatia	0.21	-0.10	6.96	-23.34	27.29	0.34	7.11
Estonia	0.15	-0.14	9.55	-31.58	54.06	1.26	11.31
Kazakhstan (2)	0.10	-0.98	11.35	-26.25	103.87	2.24	16.33
Lithuania (5)	0.47	0.14	7.93	-22.64	46.48	2.32	16.90
Romania (2)	0.47	1.03	10.54	-45.00	31.98	-0.76	6.63
Serbia (5)	-0.99	-0.01	12.57	-47.58	42.45	-0.35	6.40
Slovenia	0.04	-0.12	6.03	-15.74	18.34	0.13	4.18
Ukraine (3)	-1.68	-3.68	12.35	-35.05	27.64	0.12	3.04
Botswana (6)	0.31	0.45	4.83	-21.37	16.05	-0.11	3.92
Ghana (6)	0.64	-0.27	9.06	-28.00	37.05	0.49	6.48
Kenya	1.37	1.79	7.39	-23.07	27.33	-0.43	5.03
Mauritius	1.28	1.77	6.13	-26.72	22.32	-0.85	7.58
Morocco	0.55	0.36	5.24	-17.21	20.38	0.51	4.47
Nigeria	0.76	0.57	9.55	-36.38	37.27	-0.01	6.04
Tunisia	0.73	0.29	5.37	-12.58	24.41	0.56	5.67
Zimbabwe (9)	1.41	1.91	7.88	-26.50	22.00	-0.42	5.36
Bahrain (1)	-1.85	-2.11	6.80	-24.55	17.68	-0.33	4.92
Jordan	-0.43	-0.59	6.11	-23.21	20.74	0.19	5.07
Kuwait (1)	-0.03	0.67	6.38	-25.83	23.26	-0.26	5.79
Lebanon	0.89	-0.24	9.14	-21.48	43.56	1.37	8.13
Oman (1)	-0.13	-0.26	5.76	-22.30	13.64	-0.56	5.09
Saudi Arabia (1)	0.07	0.97	8.30	-24.45	19.91	-0.42	3.64
Bangladesh (7)	0.84	2.12	9.11	-29.61	21.37	-0.53	4.43
Pakistan	0.75	1.65	8.96	-54.82	38.80	-2.03	14.42
Sri Lanka	1.12	0.37	9.46	-27.53	50.07	1.37	9.15
Vietnam (4)	0.07	0.10	11.37	-25.49	50.86	0.90	6.65

(1) From May 2005 ; (2) From November 2005 ; (3) From May 2006 ; (4) From November 2006 ; (5) From May 2008 ; (6) From November 2008 ; (7) From November 2009; (8) From May 2010; (9) From November 2010

Looking at the descriptive characteristics in Local currency, we can see that average return on Core indices (Developed and Emerging Markets) is 0.43 per cent, higher than Frontier indices' average at 0.34 per cent. In both subsets, there are important differences. MSCI North America and MSCI Emerging Markets exhibit average monthly returns two times larger than the other two. In Frontier subset, eight countries exhibit negative average monthly returns and three countries present average monthly returns below 0.20 per cent. With the exception of Vietnam, these countries are located in Europe and Middle East region. Alongside these poor performances, eight countries have average monthly returns close or above 1.00 per cent, between 25 and 90 bps above Emerging Markets. Six of these countries belong to Africa region. As expected, Frontier indices' volatility is higher than Core indices' volatility (7.83 vs 5.44 per cent). However, thirteen Frontier Markets present a volatility lower or very close to the one of Emerging Markets, while five of them exhibit very large volatility above 10.00 per cent[38]. Except for few indices, the skewness is sensibly different from 0, implying asimmetry in distribution. Core indices exhibit negative skewness, while sixteen Frontier indices present positive skewness. Finally, kurtosis is higher, on average, in Frontier indices (7.02 vs 4.79), suggesting heavier tails of the returns distribution compared to Core indices. Analyzing monthly returns in Euro, we can see that average return on Core indices increases to 0.51 per cent, suggesting a positive currency contribution. All of them are denominated in US Dollar, leaving the differences within the group unchanged. On the contrary, average return on Frontier indices declines to 0.24 per cent. In particular, fourteen countries present a negative currency contribution, with a difference close or above 50 bps for Argentina, Serbia, Ukraine and Ghana. For Middle East countries, the returns associated to exchange rate fluctuations help to reduce negative returns on stock indices. However, four of these countries and four European countries still exhibit negative average returns. For five countries, there is no variation in average return, due to the fact that they belong to the European Monetary Union or their currencies are pegged to Euro[39]. About volatility, we have a decrease in Core indices from 5.44 to 4.42 per cent and an increase in Frontier indices from 7.83 to 8.07 per cent. Skewness analysis didn't change between Local currency and Euro. Core indices still present negative skewness, while the Frontier indices that exhibit positive skewness increased from sixteen to seventeen. About kurtosis, it decreases for Core and Frontier indices, from 4.79 to 4.02 and from 7.02 to 6.69 respectively.

[38] Despite the Frontier Market status, Trinidad and Tobago presents the lowest volatility in the investment universe at 2.96, 138 bps below the North American one.

[39] Countries with indices denominated in Euro are Estonia and Lithuania. Slovenia belongs to the EMU, but the index is denominated in Slovenian Tolar. However, there is no currency contribution given the fact that, as country belonging to the EMU, the exchange rate between the old currency and the Euro is fixed. Finally, Bosnia and Herzegovina and Bulgaria have currencies pegged to Euro.

Figure 6 – Monthly Returns Mean and Volatility (LOC)

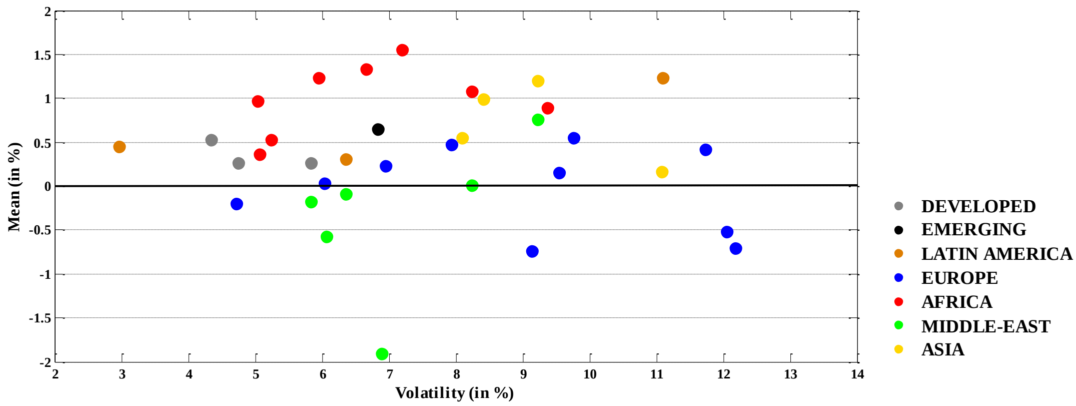
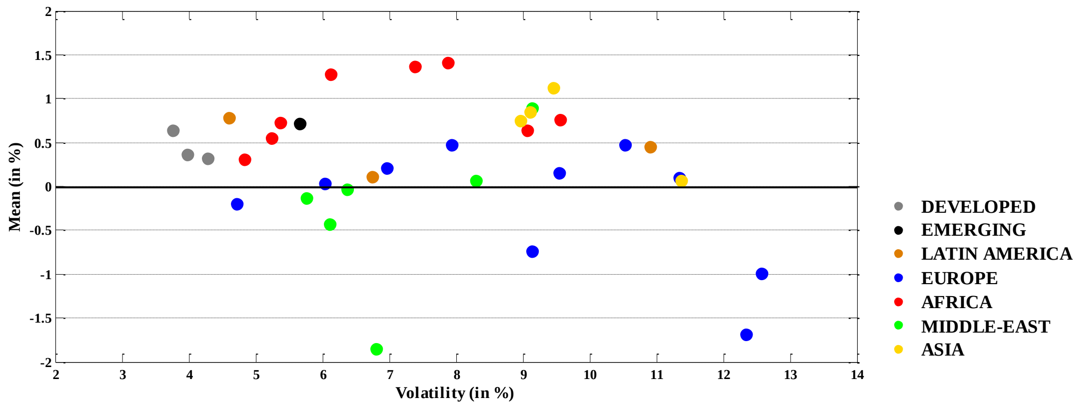


Figure 7 – Monthly Returns Mean and Volatility (EUR)



Reasoning by geographical area, the two figures show that Europe and Middle East generally provide the worst risk-return trade-off, while Africa seems to provide the best one. Given the same volatility, Asia and Latin America usually exhibit higher returns than European and Middle-East countries. As expected, Developed Markets present the lowest volatility, when considering monthly returns in Euro[40].

The following table presents the results of the Jarque-Bera test on normality of monthly return distribution and the Ljung-Box test on the autocorrelations. In this specific case, accepting the null hypothesis of the Jarque-Bera test means that monthly returns are normally distributed. For the Ljung-Box test, the acceptance of the null hypothesis implies to accept the evidence of no autocorrelation in the series of the residuals. The tests are conducted on each return series, both in Local currency and in Euro. The significance level is set at 5% for both. In Ljung-Box test, the number of lags considered is 20, which is the default value provided by Matlab function *lbqtest*.

[40] If we consider monthly returns in Local currency, there is the case of Trinidad and Tobago that exhibit the lowest volatility. Moreover, even considering only the countries with positive average returns, there are three African countries that present lower volatility and higher returns than MSCI Europe and Middle East.

Table 4 – Jarque-Bera and Ljung-Box Tests

MSCI INDICES	LOCAL CURRENCY		EURO	
	JB TEST	LB TEST	JB TEST	LB TEST
North America	X		X	X
Europe And Middle East	X		X	
Pacific	X		X	
Emerging Markets	X		X	
Argentina	X		X	
Jamaica	X			
Trinidad And Tobago	X			
Bosnia And Herzegovina	X			
Bulgaria	X		X	
Croatia	X		X	
Estonia	X		X	
Kazakhstan	X		X	
Lithuania	X		X	
Romania	X		X	
Serbia	X		X	
Slovenia	X		X	
Ukraine				
Botswana	X		X	
Ghana	X		X	
Kenya	X		X	
Mauritius	X		X	
Morocco	X		X	
Nigeria	X		X	
Tunisia	X		X	
Zimbabwe	X		X	
Bahrain	X		X	
Jordan	X	X	X	
Kuwait			X	
Lebanon	X		X	
Oman	X		X	
Saudi Arabia	X			
Bangladesh	X		X	
Pakistan	X	X	X	
Sri Lanka	X		X	
Vietnam	X		X	
X = Rejection of the null hypothesis				

As expected from the analysis of skewness and kurtosis, in most of the cases I reject the null hypothesis of normality. The exceptions are Bosnia and Herzegovina, Ukraine and Kuwait for data in Local currency and Pacific, Jamaica, Trinidad and Tobago, Bosnia and Herzegovina, Ukraine and Saudi Arabia for data in Euro. Evidences of autocorrelation are present for Jordan and Pakistan (Local currency) and in North America (Euro).

3.2.2 Currencies

In this subparagraph, I present in details the currency breakdown of the investment universe. Table 5 presents the different currencies, the exchange regime, their average monthly return and volatility over my investment period from February 2005 till January 2015. As above-mentioned, in Appendix B, I included the graphs on Spot Exchange Rate time series. The exchange rate regime is based on “De Facto Classification of Exchange Rate Regimes and Monetary Policy Frameworks” provided by the International Monetary Fund[41].

Table 5 – Currency Characteristics

ISO 4217	REGIME	MEAN	VOL	ISO 4217	REGIME	MEAN	VOL
USD	G - 1	0.07	3.10	MUR	A - 2	-0.02	2.34
ARS	A/E - 4	-0.85	3.70	MAD	C - 4	0.02	0.58
JMD	E - 2	-0.46	3.13	NGN	E - 2	-0.24	3.91
TTD	A - 4	0.06	3.18	TND	C - 4	-0.26	1.21
BAM	B - 5	0.00	0.00	ZWD	A - 4	0.25	2.70
BGN	B - 5	0.00	0.08	BHD	A - 4	0.07	3.10
HRK	B - 4	-0.02	0.71	JOD	A - 4	0.07	3.10
KZT	A - 4	-0.25	4.20	KWD	C - 4	0.08	2.65
RON	F - 2	-0.16	1.91	LBP	A - 4	0.07	3.10
RSD	F - 2	-0.38	2.14	OMR	A - 4	0.07	3.10
SIT (a)	B - 5	0.00	0.03	SAR	A - 4	0.07	3.10
UAH	A - 2	-0.92	4.76	BDT	A - 4	-0.11	3.26
BWP	C - 3	-0.57	2.93	PKR	G - 2	-0.37	3.07
GHS	F - 2	-1.37	4.80	LKR	A - 4	-0.17	3.23
KES	E - 2	-0.08	3.35	VND	A - 4	-0.19	3.35

Exchange Rate Anchor USD	A	Independently Floating	1
Exchange Rate Anchor EUR	B	Managed Floating with no pre-determined path for the exchange rate	2
Exchange Rate Anchor Composite	C		
Exchange Rate Anchor Other	D	Crawling Peg	3
Monetary Aggregate Target	E	Other conventional fixed peg arrangements	4
Inflation Targeting Framework	F		
Other	G	Currency Board Arrangements	5

(a) Slovenia adopted EUR on 1st January 2007. Until EUR adoption the currency was Slovenian Tolar (SIT).

[41] For further information, the “De Facto Classification of Exchange Rate Regimes and Monetary Policy Frameworks” is freely available in the International Monetary fund website www.imf.org in the section “Data and Statistics”.

As above-mentioned, MSCI North America, MSCI Europe and Middle East, MSCI Emerging Markets are denominated in USD, while MSCI Estonia and MSCI Lithuania are denominated in Euro[42]. On the thirty currencies, only US Dollar is independently floating. As expected, the majority of Frontier countries' currencies are pegged or in a Currency Board Arrangements (fourteen against USD, four against EUR and four against a composite basket). The other currencies are managed floating, following Monetary Aggregate or Inflation Targeting. The peg or the Currency Board Arrangements against the Euro explains why some currencies have return and volatility at or very close to 0.00 per cent. About currencies with positive contribution to returns, I highlighted in the previous paragraph the case of US Dollar and the currencies of the Middle East countries. In particular, Middle East countries are for the major part oil exporters and they benefit from their high current account surpluses.

An interesting aspect to notice is that some currencies exhibit large negative returns, in particular Argentina Peso, Jamaican Dollar, Ukraine Hryvnia, Botswana Pula, Ghana Cedi and Pakistan Rupee. Currency crises are not an extraordinary event in Developing countries and the topic is well developed in the literature (see for example Frankel and Rose [1996], Kaminsky, Lizondo and Reinhart [1998], Kumar, Moorthy and Perraudin [2003], and Laeven and Valencia [2008]). Argentina and Jamaica paid the consequences of currency crisis connected to Sovereign Debt Default[43]. Ukraine suffered the consequences of the civil war of 2014, with an exchange rate depreciation close to 65.00 per cent in the last year, while Pakistan Rupee mainly paid the economic and political instability of the last decades, alongside to civil conflicts that worsen the situation. Botswana experimented two main upward shifts of the EUR/BWP exchange rate. The first one occurred during the 2008 Financial Crisis and the second one occurred between 2013 and the first semester of 2014, where Botswana Pula depreciated against the Euro of about 14.00 per cent. However, during the last semester of 2014, the EUR/BWP exchange rate went back below 11.00, with an appreciation higher than 10.00 per cent. Finally, Ghana suffered the consequences of a deep currency crisis, due to wide current account and government balance deficits. Between 2013 and August 2014, Ghana Cedi moved from 2.62 GHS per EUR to 5.07 GHS per EUR, a depreciation of 93.00 per cent. In August 2014, Ghana government asked the intervention of the International Monetary Fund

[42] Estonia adopted EUR on 1st January 2011. Until EUR adoption the currency was Estonian Kroon (EEK), initially pegged to Deutsche Mark (DEM) (DEM/EEK = 8) and then to Euro (EUR/EEK = 15.6466. Lithuania adopted EUR on 1st January 2015. Until EUR adoption the currency was Lithuanian Litas (LTL) initially pegged to US Dollar (USD) (USD/LTL = 4) and from 2002 to Euro (EUR/LTL = 3.4528).

[43] For Argentina, the default occurred in 2001, but the effects were prolonged and leaded to Debt Restructuring in 2005. Moreover, in 2014 there was a second Selective Default. Jamaica default and Debt Restructuring happened in 2010.

and the exchange rate went back below 4.00 at 3.80 GHS per EUR, with an appreciation of 25.00 per cent.

3.2.3 Cross-country Correlations

As mentioned in Section 1, Berger, Pukthuanthong and Yang (2011) show that Frontier Markets present low integration with the world markets. In this subparagraph, I analyze the cross-country correlations between the MSCI indices included in the investment universe, in order to understand if international diversification can be beneficial. As presented in Table 1, Frontier Markets have different inception dates. Thus, the complete set of Frontier Markets were not available at the start of the investment period in February 2005. For this reason I computed the correlations over three non-overlapping subset, from May 2005 till October 2008, from November 2008 till October 2010 and from November 2010 till January 2015. In each period, I added the new Frontier Markets available. Figure 8, 9 and 10 present Correlation Heat-Maps for the three different subsets, based on Pearson's linear correlation coefficient.

Figure 8 – Cross-country Correlations Heat Map 05/2005-10/2008

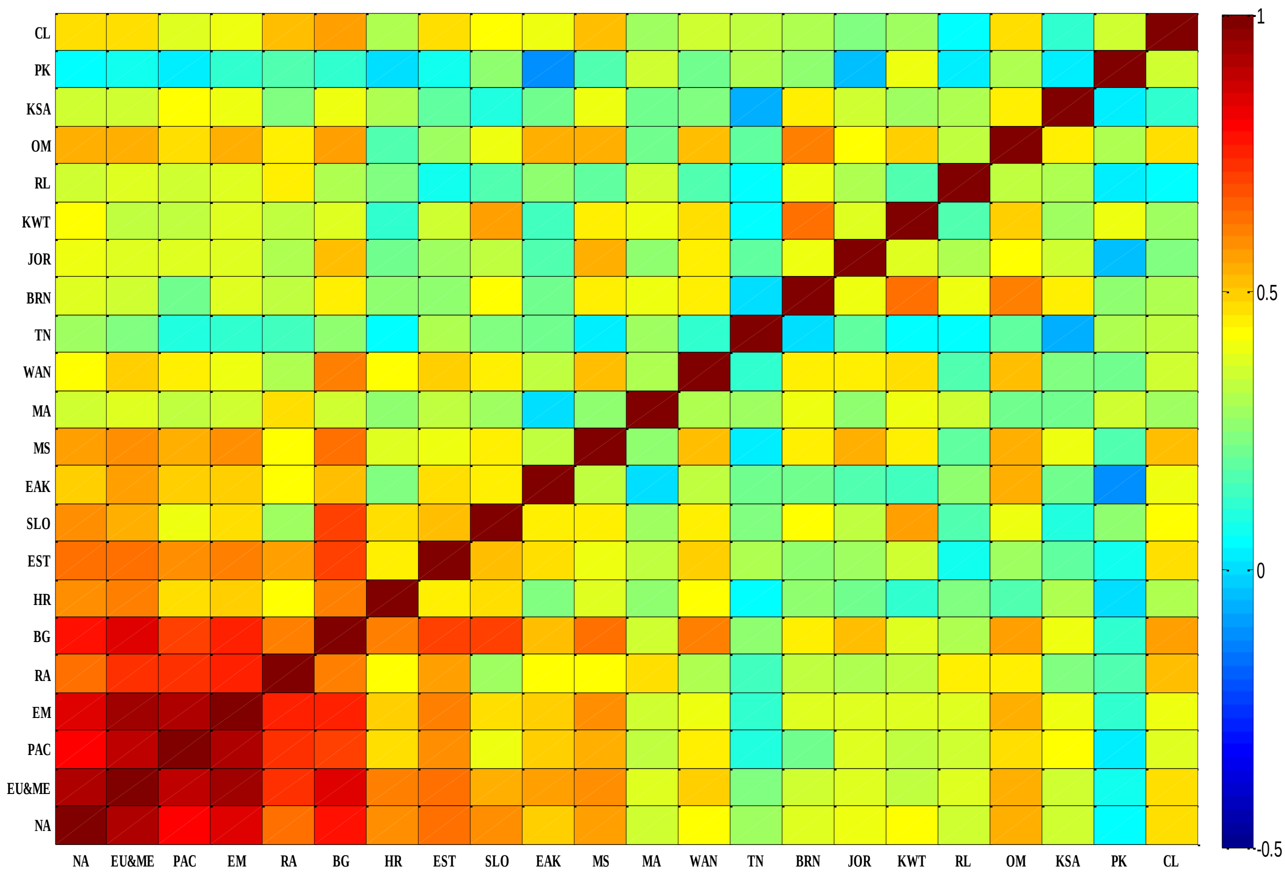


Figure 9 – Cross-country Correlations Heat Map 11/2008-10/2010

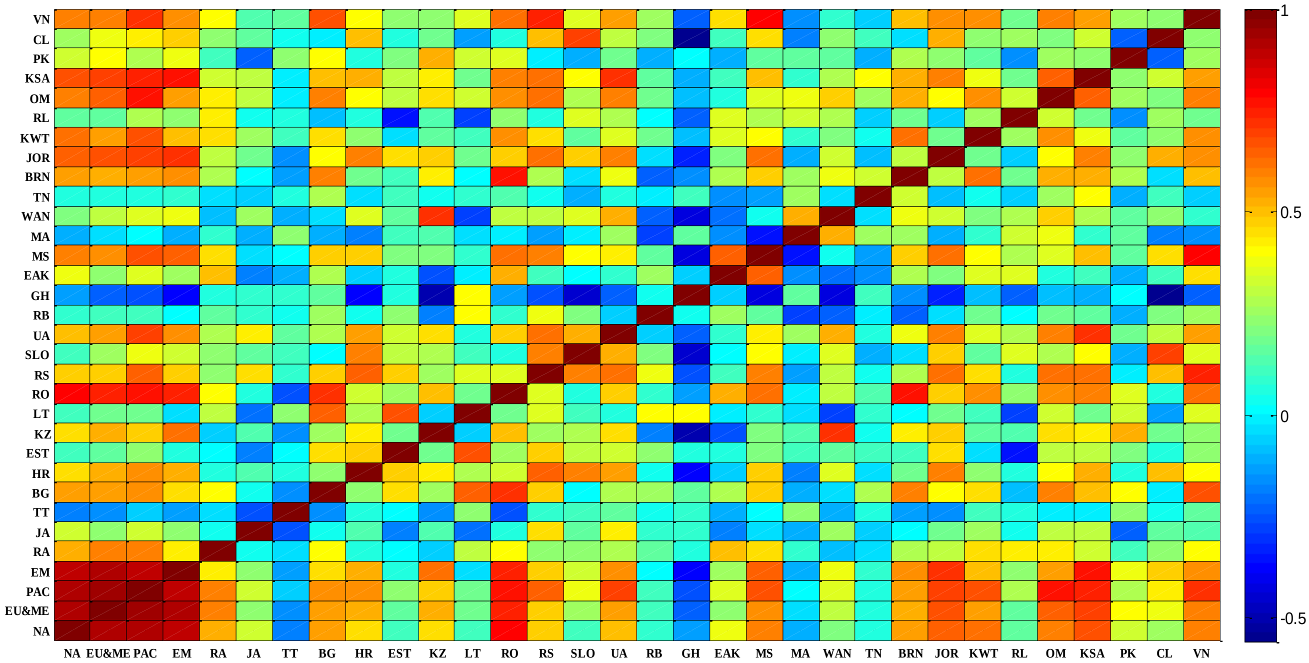
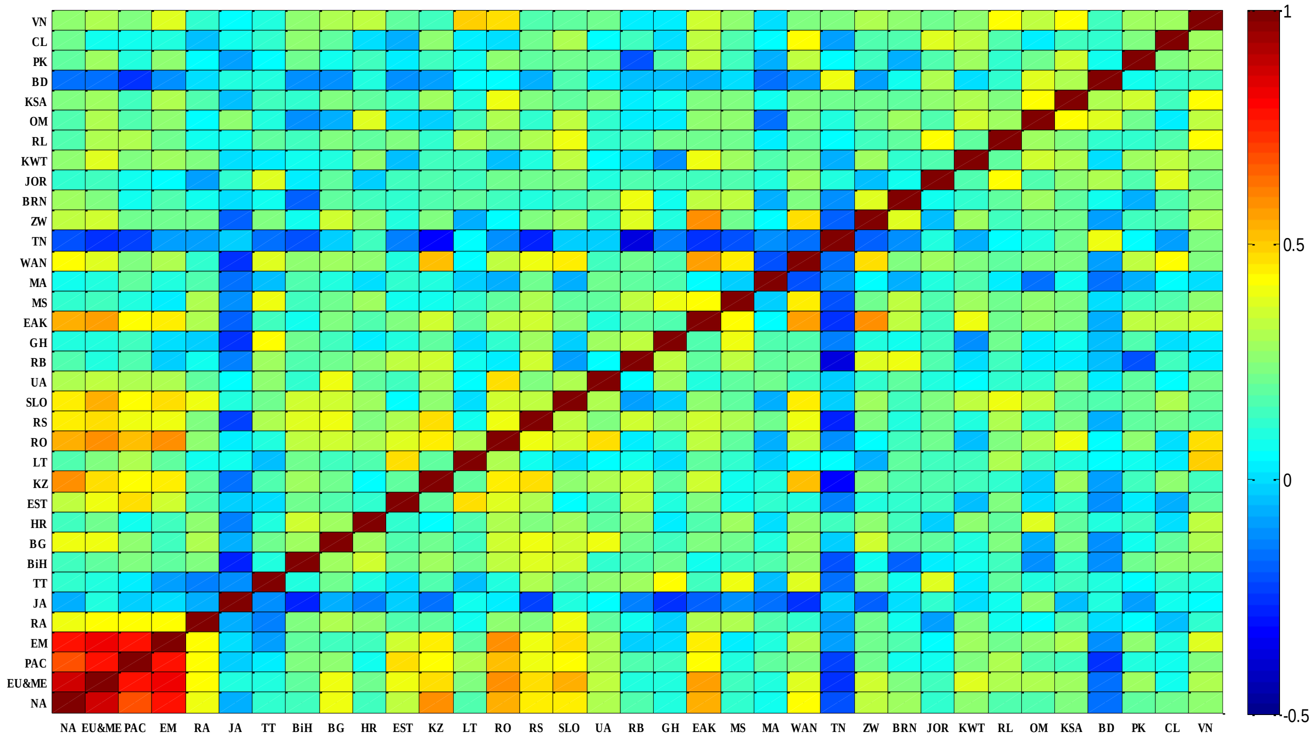


Figure 10 - Cross-country Correlations Heat Map 11/2010-01/2015



Correlation Heat-Maps are in line with expectations. The correlation between Frontier Markets and Core indices are generally low or negative in all the subsets. Within the group, Frontier Markets are less correlated, when they are not negatively correlated, than Developed and Emerging Markets. Analyzing the differences between the three periods, it can be noticed that correlations generally increased between November 2008 and October 2010. Frontier

Markets that showed very low or negative correlation during this period were not included in the previous subset, making impossible a comparison. This result is in line with literature findings of increasing correlation during stock markets turbulence (see for example Longin and Solnik [1995] and Boyer, Gibson and Loretan [1997]). Moreover, the 2008 Financial Crisis was particularly widespread and the contagion effect hit all equity markets around the world.

In the first period, the correlations between Developed and Emerging Markets were generally close or above 0.80, with the MSCI Pacific that appeared to be the less correlated within the group. Frontier Markets exhibited correlations generally around or below 0.50, with some special cases of negative correlation. Correlations above 0.50 are mainly exhibited by European countries, both with Core indices and within them. Despite their relative illiquidity and small size, this can be an evidence that these markets are more integrated or more accessible than other Frontier Markets. The countries with the lowest correlation were Tunisia and Pakistan. As above-mentioned, during the second period the correlations generally increased. In fact, the correlations between the Core indices were generally close or above 0.90. For Frontier Markets they generally remained between 0.00 and 0.50, but they increased especially for Middle East countries, where they reached value between 0.60 and 0.70, with the only exception of Lebanon[44]. African countries exhibited the lowest correlation, especially for Ghana (a new-added country), Morocco and Tunisia. Finally, in the last period correlations generally decreased. Between the Core indices they are close or above 0.70 and never above 0.90. For Frontier Markets, correlations are generally below 0.40, with a growth of negative correlation cases. Jamaica (a new-added country), Tunisia and Bangladesh exhibits the lowest correlations.

The analysis of cross-country correlations confirmed the initial hypothesis. Frontier Markets are generally less correlated within the group and with Developed and Emerging Markets. This is a fundamental aspect in order to benefit from international diversification.

[44] Lebanon is a particular case, since the country was recovering from the huge crisis generated by the war against Israel in 2006.

4 The Country Risk Indicator

Country Risk Assessment is the set of processes implemented to evaluate the risk associated to an investment (e.g., foreign direct investment, portfolio investment in equity, portfolio investment in the bond market, loans to a private or public counterparty) in a foreign country. Especially for Emerging and Frontier Markets, a serious procedure of Country Risk Assessment is necessary in order to evaluate the potential threats to investment profitability. For these countries the number of potential threats are generally higher compared to the ones present in Developed Markets. Moreover, these countries are more prone to serious downturn in presence of systemic crisis (e.g., currency crisis, banking crisis, debt restructuring, etc.)[45]. These characteristics, combined with their relative illiquidity and the problems behind the exit process (i.e., disinvestment) in presence of capital controls, are the economic and financial reasons behind the necessity of a correct country risk evaluation, in order to prevent the potential drop of the investment value, that can be severe in these markets[46]. Alongside the economic and financial reasons, there are also legal requirements connected to country risk evaluation, that became tighter after the application of Basel II and Basel III frameworks and after the European Sovereign Debt crisis.

One of the aim of this thesis is the construction of the Country Risk Indicator, a numerical rating to quantify qualitative information about country risk in investment. As mentioned in Section 1, the Country Risk Indicator is a tool useful to design the risk budgets on the Frontier Markets. With the Country Risk Indicator, I tried to move from the qualitative information coming from the Country Risk Assessment process to a quantitative tool able to quantify how much risk I can bear from investing in a particular Frontier Market. As I will explain in the dedicated paragraph, I tried to construct a rating based on free available information, without any access to professional not-free database. In this way, everybody can compute the Country Risk Indicator developed in this thesis. Moreover, I decided to use only the information available when the investment had to be done, in order to give to my work an ex-ante perspective.

In the first part of this Section, I present some definitions of country risk and its decomposition, alongside the basic process of Country Risk Assessment. In addition to that, I present the main literature on the evaluation of country risk and its impact on stock returns, that represents the theoretical base behind my Country Risk Indicator. In the second part, I present the

[45] With the exception of Greece, Ireland and Portugal, in the last fifteen years the systemic crisis hit only Emerging or Frontier Markets. Of course, we have to consider also the 2008 Financial Crisis, but it was particularly widespread and it cannot be considered as a standalone country crisis.

[46] Some of the Frontier Markets present minimum monthly returns between -35.00 and -50.00 per cent, compared to the average around -19.00 per cent for Developed Markets.

Country Risk Indicator, the choices behind the variable selection and the composition. Detailed tables on its composition are contained in Appendix C. Finally, I present a basic analysis of the Country Risk Indicator by geographical area. For the passage from the indicator to the risk budgets, I refer to the first part of Section 5 on portfolio construction and evaluation.

4.1 Country Risk and Country Risk Assessment

4.1.1 The Country Risk Definition

The definition of country risk is not unique, both in academic literature and in financial market practice. The notion is usually shaped according to operational needs, rather than a theoretical framework[47]. Moreover, it strictly depends on the knowledge of risks at a given point in time and on the type of the investment (e.g., FDI, portfolio investment, loans).

From an academic point of view, one of the first definition was provided by Haendel, Meadow and West (1975) and it was focused on political risk, defined as the “probability of occurrence of political events that will change the prospects for profitability of a given investment”. More recently, Meldrum (2000) defined country risk as the additional risks borne by cross-borders international transactions with respect to domestic transactions, able to decrease the expected profitability. The sources of these additional risks are varied and they include differences in economic structures, policies, socio-political institutions, geography and currencies. Oetzel, Bettis and Zenner (2001), defined the objective of country risk. In particular, country risk measures have to forecast political or economic events in a selected country that may affect the business climate, implying that investors will realize negative (or lower than expected) returns, when the investment was made. The definitions provided by Alexe *et al.* (2003) and Hoti and Mc Aleer (2004) are more focused on credit risk. In particular, in the first paper, country risk is the risk that a country default on its obligations. In the second one, country risk is referred to the “likelihood that a sovereign state or borrower from a particular country may be unable or unwilling to fulfill their obligations towards one or more foreign lenders and/or investors”. An example of definition provided by practitioners is the one developed by COFACE (Compagnie Française d'Assurance pour le Commerce Extérieur) in 2012, where the objective of the country risk is to measure the influence of a country’s macroeconomic and institutional evolution on company credit risk.

In order to arrive to a useful definition from an operational point view, it’s necessary to consider the multiple and systemic nature of the risks and the different types of transactions

[47] The definitions are extracted from the notes of Emerging Market and Country Risk Evaluation course, followed at IESEG School of Management (Université Catholique de Lille). Thus, the quotes are indirect.

threatened (e.g., FDI, portfolio investment, loans, etc.). Moreover, we need to take into account the global nature of country risk (i.e., contagion effect risk). In fact, even if country risk is mainly affected by country-specific factors, the impact of the international economic environment on these factors cannot be neglected[48]. One of the most wide definition, was recently provided by Bouchet, Clark and Groslambert (2003). They define country risk as "all the additional risks induced by doing business abroad, as opposed to domestic transaction". Another useful operational definition is the one of the Bank for International Settlement (BIS) in its "Core Principle for Effective Banking Supervision" of 2012. It defines country risk as the "risk of exposure to loss caused by events in a foreign country". This concept is broader than sovereign risk, as it involves all transactions (lending or investment activity) to or with individuals, corporates, banks or government. Thus, country risk could be defined as the whole set of factors related to the political, institutional or economic characteristics of a given country that could affect the profitability of an investment or the capacity of a private and/or public counterparty to meet its commitments in a way that could be measured ex-ante[49].

From the definition presented above, we deduce that country risk is a broad aggregate that includes different heterogeneous components, generated by different sources of risk. The main risks entering in the broad aggregate and their definitions are:

- political, legal and institutional risk is the risk that a change in the legal, political or institutional environment may affect, positively or negatively, the ability of a counterparty to meet its commitments or the profitability of an investment;
- macroeconomic risk is the risk that a brutal change in the macroeconomic environment may affect, positively or negatively, the ability of a counterparty to meet its commitments or the profitability of an investment;
- sovereign risk is the risk that a sovereign state may be unwilling or unable to meet its financial commitments (i.e., sovereign default);
- non transfer risk, or convertibility risk, is the risk that a large exchange rate depreciation, due to currency or balance of payment crisis, or a change in the legal environment (e.g., introduction of capital controls) may impact the capacity of a counterparty to meet its commitments in foreign currency.

[48] There are a lot of examples related to this issue. Think about the economic growth in developed countries, that strongly affects the economic performance of commodities exporters. Moreover, as shown by the recent drop in oil prices, international commodities prices are fundamental factors in the evaluation of commodities' exporters' economic stability. An example about financial markets is connected to international liquidity. A restrictive monetary policy, especially by the Federal Reserve (e.g., the recent Tapering), will lead to huge capital outflows from Emerging and Frontier Markets. This consistently increases the risk of severe downturn in the equity markets and a huge increase of interest rates in government bonds, threatening the government creditworthiness.

[49] The ex-ante perspective is fundamental in risk definition. In fact, risk measures are designed in order to capture the potential future threats to investment profitability or to creditworthiness.

Financial markets activity and the interaction between the main components of country risk generate three other important risks. Given the systemic nature of two of them, these risks hide serious potential threat for the investors:

- market risk is the risk that a change in market variables (e.g., interest rates, stock, bond and commodity prices, exchange rate) may affect the ability of a counterparty to meet its commitments or the profitability of an investment;
- systemic banking risk is the risk of a massive wave of defaults or liquidity crises in the banking system due to systemic reasons[50];
- systemic risk is the risk that a series of private-sector counterparties may default on their obligations due to systemic reasons[51].

In what follows, I want to present briefly political, legal and institutional risk, sovereign risk and the non-transfer risk. These risks are usually the base for the assessment of more complex type of risks. Moreover, they present peculiar features that are not present in macroeconomic and market risks. Finally, macroeconomic and market risk assessment is often hid in the assessment of the other risks.

As above-mentioned, political and institutional risks are the risks coming from heterogeneous institutional or political factors. Thus, they include:

- the risks surging from the institutional specificities of a given country, such as the institutional system (e.g., democracy, dictatorship) or the political system structure (e.g., multi-party system, political system stability);
- the risks resulting from internal social tensions and political struggles, such as revolutions, riots, civil war or inadequate policy reactions in face of economic shocks (government inefficiency);
- the risks surging from geopolitical and external tensions, such as war, diplomatic tensions and international sanctions or embargo[52];
- the risk coming from discretionary decisions of the government, such as nationalization, breach of contracts and discriminatory taxation.

[50] IMF provides a database of systemic banking crisis, based on the work of Laeven and Valencia (2008), updated by Laeven and Valencia (2012). They identify 147 episodes of banking crisis in the period 1970-2011. Moreover, the work is particularly interesting for the analysis of other important crisis that can be the base or the by-product of systemic banking crisis, such as currency crisis and sovereign default. I will develop this point in the literature part.

[51] Some examples of systemic reason are changes in the political (especially in the absence of democracy), legal (including the introduction of capital controls) or institutional (e.g., coupe d’etat) environment, deterioration of the macroeconomic environment, currency crisis, interbank liquidity crisis, sovereign default.

[52] An important recent example is the introduction of the sanctions against Russia following the Ukrainian war. Given Russian solidity, the sanctions took several months to be effective, but they are threatening now economic performances and the stability of the financial system.

Generally, they are not measured by specific ratings but they are an important component of sovereign ratings, convertibility ratings and systemic credit risk ratings[53]. These risks have direct impact on investment. First of all, we have to consider the loss risks. Losses directly related to political and institutional risks could be particularly high, especially in the case of disruptive events (e.g. wars, riots, etc.) and nationalizations or breach of contracts. Moreover, insurance against these risks is possible but very costly, given the difficult to forecast them and the potential amount to be covered. Secondly we have the reputational risks, generally related to international sanctions, institutional weaknesses (e.g., corruption) and the identity of the counterparties[54]. Finally, we have the operational risks, related to the security of the local staff, to the infrastructures and the possible disruption of the supply chain. Alongside the direct impact, we have also indirect impact of political and institutional risks. Political risks tend to lower growth and to exacerbate economic volatility, therefore increasing the risks sovereign default and currency crisis. The forecast of political risk is still challenging, given the absence of strong quantitative methodology and the difficulties to establish casual relationships in academic literature. Most of the times, it is based on subjective evaluation, considering a wide set of factors. The first one is history, observing the frequency of the troubles, riots and revolutions in the past, trying to identify crisis prone and politically instable countries[55]. Moreover, the sources of the past crises are useful to understand from which factors the risks might come from (e.g., geopolitical background, internal political situation, institutional features, social structure and tensions, economic situation). About geopolitical factors, we should evaluate the relationships with great and regional powers, unresolved frontier disputes, international sanctions and the degree of dependence toward foreign capital flows or foreign aid flows. The evaluation of the political system is complicated and even academic literature is not able to provide clear answer on which system should be preferred when considering portfolio investment[56]. For example, democracy is less prone to political frustration and presents better scoring for rule of law. On the contrary, it doesn't necessarily provide the right incentives for implementing policies to ensure stable long term growth (e.g., populism is a source of risk). The authoritarian regimes increase the risk of discretionary decisions, present high reputational risks and lower scoring for rule of law. Moreover, succession might be disrup-

[53] An important exception is the political risk service provided by the PRS Group.
[54] The identity of counterparties matters especially for companies controlled by the government, that usually represent the biggest firms in developing countries, controlling the strategic sector (e.g., commodities, banking system).
[55] The interpretation may be difficult and subjective opinion is fundamental. Past crisis can weaken the country institutions, increasing the instability and the risk of other turbulence in the future. On the other side, the troubled past may make population more tolerant and passive, increasing the stability of an authoritarian regime.
[56] An interesting work on the economic effect of institutions is the book of Acemoglu, Robinson and Woren (2012). However, it doesn't investigate the impact on investment performance.

tive. On the contrary, they present the positive feature that they could stabilize countries with weak institution. Another factor to monitor is the political structure, investigating the existence of appropriate check and balances and the structure of the political parties (e.g., political alternation, presence of religious party, etc.)[57]. In addition to that, investors must assess the stability of the legal environment, the level of protection of property and contractual rights, the guarantees over an equal treatment between national and foreigners operators. In particular, it's fundamental to investigate the presence of price controls, capital controls under an unstable exchange rate system and frequent nationalizations. Finally, corruption, governance, transparency and accountability are fundamental factors influencing the investors' propensity to invest. Some useful additional indicators to assess political risk are:

- the Corruption Perception Index computed by Transparency International[58];
- the Doing Business Indicators and the Governance Indicators provided by the World Bank[59].

A broad definition of sovereign risk is the risk that a sovereign state or one of its entity or agents fail to meet their financial commitments in a timely manner. When dealing with sovereign entities, we have to understand that sovereign are not ordinary counterparties. First of all, their big size and the powers and privileges of sovereignty increase the scope of risk assessment and complicate the analysis. Government have a huge impact on the national economy, through the legal framework and the policy actions. On the other side, the macroeconomic framework can impact the solvency of the sovereign[60]. Moreover, a state benefits from sovereignty on the domestic space and sovereign immunities, with consequent limited legal redress for creditors (for example see the legal suits following Argentinian default). The second important aspect is the importance of sovereign risk in Country Risk Assessment. Sovereign ratings usually serve as country ceiling, with only few private counterparties exhibiting better rating than their sovereign. This is reinforced by the strong correlation between yields on sovereign bonds and corporate bonds. In assessing sovereign risk, we have to distinguish between

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[57] High political noise can be translated in high market volatility. An important example is the one of Italy during the European Sovereign Debt Crisis, or, more recently, of Greece.

[58] To find more information about and the methodology behind the Corruption Perception Index, please visit the following website: www.transparency.org.

[59] Doing Business Indicators try to provide objective and comparable measures of business regulations across different countries, covering ten sectors. The Worldwide Governance Indicators are composed by six sectors, covering topics from accountability to government effectiveness to regulatory quality. For more information, visit the following websites: www.doingbusiness.org and www.govindicators.org.

[60] The most simple example is the reduction of revenues from direct and indirect taxation in presence of a recession. This could be problematic for a state with unstable financial situation. Another recent example is the one of banking bailouts, following the subprime crisis. Especially in Europe, some countries faced financial solidity problems generated by the costly bailouts of the banking system. Following Laeven and Valencia (2008), the median increase of public debt was equal to 23.90 percentage points, but for some countries (e.g. Ireland) was much more severe.

the willingness to pay and the ability to pay. Given the sovereignty privileges and the above-mentioned limited legal redress for creditors in the events of a default, sovereign can default even if they have enough resources to meet their financial commitments. The willingness to pay assessment is mainly based on qualitative and subjective evaluation. Some of the key variables to analyze is the frequency of past defaults (i.e., serial defaulter), the openness and the dependency towards external financing flows, institutions efficiency and the balance between costs and gains from default[61]. In addition to that, we have to conduct a qualitative analysis based on a deep knowledge of a country's institutions, political scene and economic situation. In the assessment of the ability to pay, we can introduce also quantitative analysis. First of all, it's important to analyze levels and trends of the key government finance ratios:

- fiscal ratios, such as primary balance, total fiscal balance and structural balance (in percent of GDP);
- debt ratios and the external debt ratios, such as total public debt stock (in percent of GDP or fiscal revenues) and total public and short term external debt (in percent of foreign reserves, exports or current account receipts);
- the debt servicing costs, i.e. the interests paid on public debt (in percent of GDP or fiscal receipts).

From this analysis, we can understand if the debt is on an unstable path and the relative solidity of government finance. Manasse, Roubini and Schimmelpfennig (2003) showed that the higher the public debt ratios and the higher is the probability of default. Moreover, the ratios related to external debt increase in the run-up to a crisis and are significantly higher than during non-crisis episodes. However, there is not a well-defined threshold indicating a risky area, given the challenges to find causal effect. Each countries have some specificities that make the interpretation challenging. It's more important to assess the stability of the debt path, rather than look at its level. Other important variables to observe in order to understand the source of potential shocks are the structure of fiscal receipts (e.g., tax rate, sensitivity of tax receipts to growth, tax basis), of fiscal expenditures, of public debt (e.g., maturity, indexation and sensitivity to exchange rate, interest rate growth or inflation shocks), external vulnerability and potential growth and short term growth prospects[62]. Finally, an important component

[61] The default will probably result in a suspension of external financing flows (i.e., attachment risks) or in a dramatically increase of the cost of external financing.

[62] About the structure of fiscal receipts, a recent example is the one of oil exporter countries. A sharp drop in oil revenues could undermine the stability of the government finance. On the structure of fiscal expenditure, it's important to analyze the aging-related expenditures (e.g., pensions, medical services, etc.). Given their size, especially in developed countries, the dynamic is fundamental to assess the future trajectory of the debt. Finally the structure of public debt is fundamental to assess the stability of government finance to shocks generated in the financial markets, such as a sharp increase in interest rates.

of sovereign risk is the liquidity risk. A government should continuously roll-over its maturing debt. The default occurs when the government has not enough resources to cover the gap between its financing needs and the financing resources it can get on the market[63]. Important variables to assess liquidity risk are the spreads on government bonds interest rates, that provide useful indications about the market willingness to roll-over the debt. Moreover, we should look at the size of financing needs, the assets that can be used in case of insufficient market financing, the public sector assets that can be quickly privatized in the case of a liquidity crisis, the public sector deposits and foreign currency reserves at the central bank and the aid flows from international financial institutions (e.g., IMF, World Bank).

The last risk I want to present is the non-transfer risk, that is the risk that local players and/or debtors cannot convert local currency into foreign currency and/or cannot transfer funds abroad to non-resident creditors. In practice, it translates into external debt default, arrears on import payments, impossibility to repatriate profits and large exchange rate depreciation[64]. When assessing non-transfer risk, we can decompose it in two risks. The first one is the so called risk de facto, that is the risk of a sharp drop in foreign exchange reserves and/or large exchange rate depreciation that makes the access to foreign currency liquidity too costly or impossible. This constitutes the base of balance of payment and currency crisis. The second one is the risk de jure, that arises with the introduction of capital controls. Sovereign risk and non-transfer risk have similar assessment procedure, with the evaluation of sustainability and liquidity risk of sovereign debt and external debt respectively. However, sovereign risk is strictly related to the public sector, while the non-transfer risk focuses on both public and private counterparties' financial commitments against foreign agents. Moreover, non-transfer risk can materialize even when the sovereign remains solvent on its debt[65]. On the contrary, a sovereign default usually occur simultaneously to the materialization of non-transfer risk. Forecast non-transfer risk is not simple. However, we can count on a consistent literature that tries to explain the insurgence of balance of payment and currency crises[66]. In particular, currency crises are usually preceded by a combination of numerous symptoms. Indicators that tend to perform well are the real exchange rate overvaluation, the adequacy of foreign exchange reserves coverage (relative to short term debt, imports or monetary aggregates), for-

[63] As shown by the European Sovereign Debt Crisis, liquidity crisis can occur even if public debt seems sustainable or the government appears solvent.
[64] The standard threshold for large exchange rate depreciation is set between 15.00 and 30.00 per cent.
[65] This can happen when sovereign debt is low or denominated mainly in local currency or when the government can count on the support of International Financial Institutions. Finally, this happens when exchange rate controls have been introduced to protect foreign exchange reserves.
[66] See the Subparagraph 4.1.3 on the relevant literature for the assessment of currency risk.

eign exchange reserves growth, current account deficit and domestic credit growth[67]. About the assessment of external liquidity risk, the key variables to analyze are again foreign exchange reserves in months of imports, external financing needs or short term debt or hot money relative to foreign exchange reserves, the Fitch's liquidity ratio and size, structure and liquidity of external assets[68].

4.1.2 The Country Risk Assessment

Country risk definition influences the processes underlying the Country Risk Assessment activity. As above-mentioned, this definition evolved during the last 50 years. It strictly depends on the particular historical period and on the crises that characterized that period. Thus, Country Risk Assessment is an activity continuously developed and past crises always left important lessons for the improvement of assessment procedures.

Rogoff and Reinhart (2008a) showed that sovereign default usually occurred in default waves typically spaced some decades apart. Laeven and Valencia (2008) highlighted the connection between different type of crises (currency crisis, systemic banking crisis, sovereign debt default, debt restructuring), the presence of twin or triple crises and the presence of these waves of crises. Political crises of the 60s and the 70s represent one of the first wave of crises. Decolonization and the emergence for the new sovereign states to renegotiate the business arrangement inherited from the past introduced severe political instability and institutional weaknesses. The crisis was characterized by the voluntary breach of contracts, with massive nationalizations, assets seizure, repudiation of sovereign debt and regulatory changes with damaging effects on foreign companies. According to Williams (1975), assets sized by emerging countries over the period 1956-1972 were worth the 25.00 per cent of 1972 FDI stock value. Jodice (1980) found 1535 expropriations in seventy-six countries from 1960 to 1976. These non-economic disruptive events had a huge negative impact on the business environment, influencing Country Risk Assessment procedures. First of all, practitioners learned that dealing with sovereign states entails specific risks that are not present with a private counterparty. Secondly, they understood that non-business factors may influence sharply business conditions, impacting on expected performance in a severe way. The second important series of crises is the wave of debt crises of the '80s. The imprudent macroeconomic management and borrowing by the debtor countries, the imprudent lending by the internation-

[67] A common threshold in financial markets practice for FX reserves relative to imports is 3 months of imports. For the current account deficit, the common threshold is 5.00 per cent of GDP.

[68] For short term debt and hot money ratio the common threshold is 100.00 per cent. Hot money is defined as the assets that can dry up or fly out of the country quickly in a stress framework (i.e., the most liquid assets). Fitch's liquidity ratio is the ratio between official FX reserves and foreign assets of commercial banks over external debt service and foreign liabilities of commercial banks. For more details, see Fitch's website: www.fitchratings.com.

al banks and adverse economic framework are the backdrop of these crises[69]. During this period, we observed a wave of sovereign defaults, with the consequent exclusion from financial markets access of the defaulting states and the fears of systemic banking crisis[70]. From this wave, practitioners learned the importance of the contagion effects. The recession moved from developed to emerging countries and the adverse financial conditions spread across developing countries[71]. Moreover, it showed the necessity of better risk management practices within the banking sector, more efficient supervision and higher capitalization. Another important wave of crises for Country Risk Assessment development was the multiple crises series of the late '90s. The boom in global capital flows, the combination of weaknesses in recipient countries, the unsustainable current account deficits, the balance sheet vulnerability and the weak banking systems constituted the backdrop of the crises[72]. Multiple or systemic crises characterized this wave, with a combination of a balance of payment crises, banking crises, a wave of corporate bankruptcies and, sometimes, sovereign defaults[73]. Practitioners learned that contagion effects go well beyond what is suggested by trade links, through herding behavior of foreign investors and exposure to common lenders. In addition to that, the series of crises highlighted the necessity of a better monitoring of the risks of the private counterparties and of a better monitoring of liquidity and currency risks. The 2008 Financial Crisis and the 2011 European Sovereign Debt Crisis gave the last important lessons for Country Risk Assessment. In a different way, the two crises showed the importance of contagion effect and the difficult to assess this type of risk, even if it was recognized by financial market practitioners since the Asian Crisis[74]. The crises highlighted again the importance of contagion effects and the global and multiple systemic nature of country risk. Moreover, we saw the relative inefficiency of macroeconomic analysis based on standard statistical indicators. This analysis, fundamental in assessing country risk, must be complemented by microeconomic evidences, balance sheet analysis, and information about interlinkage and off balance sheet

[69] Real interest rate increase and recession in industrialized countries due to counter-inflationary policies of the early '80s, alongside the decline in commodity prices, represented the adverse economic scenario.

[70] The majority of these repeated sovereign defaults involved Latin American countries: Peru (1978, 1980, 1984), Bolivia (1980, 1986, 1989), Honduras (1981), Costa Rica (1981, 1983, 1984), Argentina (1982, 1989), Mexico (1982), Dominican Republic (1982), Ecuador (1982), Brazil (1983), Uruguay (1983, 1987, 1990), Chile (1983) and Venezuela(1983, 1990).

[71] During the '80s, the tight monetary policy, implemented especially by the Federal Reserve, led to massive capital outflows from emerging economies. This situation was observed also during the 2008 Financial Crisis, with the flight to liquidity that hit also countries with good economic performance.

[72] The current account deficit led to real exchange rate overvaluation under pegged exchange rate system, undermining the competitiveness of these countries. Balance sheet vulnerability were mainly due to currency mismatches, maturity mismatches and high indebtedness.

[73] The most important crisis occurred in Mexico (1994-1995), East Asia (Thailand, Malaysia, Indonesia and Korea, 1997-1998), Russia (1998), Brazil (1998-1999) and Argentina (2001-2002).

[74] The difficult was related also to financial innovation, with the introduction of complex instruments. The subprime crisis moved from United States to all over the world, while the European Sovereign Debt Crisis spread across the weakest Eurozone countries (e.g., Piigs and some Eastern countries).

positions at firm-level. In addition to that, we learned the huge importance of subjective evaluation of the quantitative results obtained from the above-mentioned analysis.

The historical background is necessary to understand the modern structure of the assessment of country risk. In particular, Country Risk Assessment is the set of processes with the aim of assessing the different type of risks composing the broad aggregate of country risk, in order to:

- allow for an adequate remuneration of the risk (i.e., determination of the appropriate required rate of return);
- make easier global portfolio management and cross-border risk-mitigation;
- favor optimal asset allocation and investment choices, especially from a strategic asset allocation point of view;
- meet regulatory obligations[75].

It is a complex process, requiring a cross-disciplinary approach. Economics plays the major role in assessing country risk, especially with the implementation of quantitative tools. However, political sciences, history, sociology and geopolitics are fundamental for the interpretation of the results coming from quantitative analysis and for the assessment of political and institutional risk[76]. Given the absence of a consistent literature and the lack of strong model for assessing country risk, Country Risk Assessment is a combination of quantitative and qualitative tools. Qualitative judgment is not a subordinated aspect, especially when quantitative results suggest different interpretations. Even if country risk is often a subjective evaluation based on quantitative tools, agents should implement a clear and transparent methodology for comparability and counterfactual assessment.

There are multiple agents involved in Country Risk Assessment. The majority of banks, financial institutions and multinational firms have their country risk departments. In addition to that, there are several outsourced business services, such as rating agencies (e.g., Standard and Poor's, Moody's, Fitch), export credit agencies (e.g., COFACE, CESCE) and "thinks tanks" and consulting firms (e.g., Economist Intelligence Unit, Global Insight, PRS Group).

[75] Principe 21 of the Basel Committee "Core principle for effective banking supervision" states that "the supervisor determines that banks have adequate policies and processes to identify, measure, evaluate, monitor, report and control or mitigate country risk and transfer risk in their international lending and investment activities on a timely basis". Thus, the country risk is a component of counterparty credit risk assessment and, consequently, it influences the regulatory capital requirements.

[76] History is fundamental in country risk evaluation. For example, from Laeven and Valencia (2008), we know that crises tend to repeat (see for example the case of Argentina and, more recently, of Jamaica). Moreover, the knowledge of the history of social conflicts in a country helps to implement a more complete assessment of institutional risk.

These agents usually implement a country risk breakdown similar to the one presented above[77].

Between the above-mentioned agents, rating agencies present some peculiarities. First of all, they focus on borrowers' or specific financial instruments' creditworthiness, issuing individual ratings on these entities or instruments. In general, ratings are subjective opinion based on the analysis of publicly available information or provided by the issuer, with established criteria and methodology. They focus on issuers' creditworthiness and solvency, thus assessing credit risk. This assessment provides an ordinal measure of the likelihood of default, relative to the other issuers. In fact, a rating does not indicate a numerical probability of default, that can be retrieved only empirically. Their diffusion is also due to their standardization and their alpha-numerical symbol structure easy to understand. Even if they focus on credit risk, they consider also other sources of risk, as they can significantly impact the debtor creditworthiness. Rating agencies' reports provide a rationale for those ratings, including a complete assessment of country risk. The second fundamental characteristic is the oligopolistic feature of rating agency industry. This feature is mainly due to the legal framework. In particular, it comes from the role assigned to the rating agencies by regulatory institutions[78]. The third important characteristic is the revenue generation process. Except for sovereign entities, ratings are released at the request of the borrower, with the revenues generated mainly by the fees paid by the issuer. All these features are the foundation of the main criticisms to rating agencies work, especially after the 2008 Financial Crisis. The first concern is related to their market power and revenue generation process, with the existence of a conflict of interests that favors an inflation in ratings. The second concern is about the utility and accuracy of the rat-

[77] For example, Economist Intelligence Unit use the following breakdown of country risk in the construction of its country rating:

- sovereign risk is the risk of a build-up in arrears of principal and/or interest on foreign- and/or local-currency debt of a sovereign entity or guaranteed by the sovereign;
- currency risk is the risk of a devaluation against the reference currency of 25.00 per cent or more in nominal terms over the next twelve months;
- banking sector risk is the risk of a systemic crisis whereby banks holding 10.00 per cent or more of total assets become insolvent and unable to discharge their obligation to depositors and/or creditors.
- political risk is the risk connected to political stability and effectiveness that could affect a country's ability and or commitment to service its debt obligations and/or cause turbulence in the foreign exchange market;
- economic structure risk is the risk derived from a series of macroeconomic variables of a structural rather than a cyclical nature

[78] For example, there is the obligation for some regulated bodies (e.g., pension funds, insurance company) to invest in bonds with a certain minimum rating. Moreover, the eligibility of bonds as collateral in the operation with the central bank is strictly connected to the rating assigned. Finally, there is the possibility to use the ratings of recognized external credit assessment institutions to risk weight assets and compute capital requirements. Recognized external credit assessment institutions have to meet strict eligibility criteria. The necessity to respect these criteria poses huge entry barrier for potential incoming.

ings, especially given their pro-cyclicality and their role in the exacerbation of the crisis[79]. A forward-looking through the cycle risk assessment should be the base of the ratings, in order to maintain a certain degree of stability during the economic cycle. The final concern, and the most important for the objective of this thesis, is the country risk definition. Country risk is often proxied by sovereign risk (e.g. S&P, Fitch), with the sovereign rating serving as a ceiling for most of the ratings of the private counterparties domiciled in the same country. The reduction of country risk to sovereign risk is too limitative and this represents one of the reason behind the choice to don't use the ratings provided by the rating agencies.

4.1.3 Relevant Literature

There is not a significant literature over the effect of macroeconomic variables on stock market performances for Frontier Markets. Frontier Markets time series are limited and the majority of stand-alone country indices were launched after 2002. However, the aim of my Country Risk Indicator is to provide a quantitative tool to design the risk budgets on the Frontier Markets asset class. The relevant quantity behind the Risk Budgeting Approach is the risk and not the performance. On the risk side, the literature is more focused on the predictability of major crises (banking, financial or currency crises and sovereign default) rather than the relation between macroeconomic variables and stock market volatility. However, the Country Risk Indicator tries to assess country risk, with the primary objective of avoiding the insurgence of losses connected to these major crises. In fact, these crises represent disruptive events for investment. For this reason, I present some of the most relevant academic works over this topic. The majority of these works have their focus on Emerging Markets (EM), but some of their conclusions can be extended to Frontier Markets.

The first important work on the relation between macroeconomic variables and EM stock returns is the one of Bilson, Brailsford and Hopper (2001). It can be considered as an evolution of the work of Harvey (1995a), where he found that EM stock returns present little exposure to a set of global factors (world inflation, world GDP, world oil prices and a trade-weighted world exchange rate). They used twenty MSCI indices to proxy the EM stock markets. They implemented two different regressions. In the first one, they used as relevant macroeconomic factors the world market returns, the percentage change in one aggregate of money supply, the percentage change in a good price variable (e.g., inflation), the percentage

[79] Ratings tend to react slowly to the signals of a crisis. Moreover, a rating downgrade can worsen the debtor creditworthiness, exacerbating its crisis. Think for example to a sovereign entity that presents liquidity problems, with the interest rates on its bonds that increased. A downgrade that moves the rating from the investment grade class to the speculative class can exacerbate the crisis, since some institutional investors have to sell these bonds and they cannot be used as collateral in the operation with the central bank, threatening the stability of the country banking system. An example is the 2011 European Sovereign Debt Crisis.

change in a real activity variable (e.g., GDP or Industrial Production growth) and the percentage change in the exchange rate variable[80]. In the second regression, they implemented a Principal Component Analysis. Alongside the macroeconomic factors (money supply, good prices, real activity and exchange rates), they introduced a country political risk measure, the trade sector, interest rates and regional market returns[81]. Moreover, as suggested by Chen (1991), they introduced microeconomic variables, such as price-to-earning (PE) ratio and dividend yields. About the results, EM show little sensitivity to the return on the world market index, with only 10 markets that present significant coefficients (as expected from Harvey [1995a]). The sign is positive as expected. The exchange rate appears to be the most influential macroeconomic variable, with significance in twelve countries and a negative sign in most of the cases. The remaining macroeconomic variables perform relatively poorly, with only money supply that exhibit a little significance. The explanatory power is quite small, with low R-squared coefficients. In the second set of results, the global factor loses much of its significance, while the exchange rate still exhibits the highest significance. The conclusions over the other macroeconomic variables do not change. The results show the presence of a regional factor, with positive significant coefficients in some markets. The PE ratio is significant and positive in sixteen markets, as suggested by previous literature (see for example Bekaert and Harvey [2000]). Finally, the dividend yield is significant in ten markets, with negative coefficients. The explanatory power increases with an average adjusted R-squared of 60.00 per cent. However, the authors suggest to take carefully the results due to the large number of variables that can introduce multicollinearity concerns. Another important work is the one of Harvey (2004), in which he investigated the importance of political risk, financial risk and economic risk in portfolio and direct investment decisions. He used the measures provided by PRS International Country Risk Guide. The broad aggregate for country risk is composed for the 50.00 per cent by political risk measure and the remaining part is equally distributed between financial and economic risks. He found that trading strategies based on those ratings significantly improved portfolio returns. Moreover, the country risk measures are most useful for the analysis of Emerging rather than Developed markets. In fact, these markets face important non-diversifiable risk and the country risk is rewarded. Hooker (2004) used the Bayesian approach, developed by Cremers (2002), to investigate the relationships between the predictive power of some macroeconomic factors and EM equity returns. He considered six mac-

[80] As suggested by the IMF's 1996-1997 Annual Report, money supply and good prices are lagged by 1 month and real activity by 2 months.
[81] The politcal risk measure is the one provided by the PRS Group. The trade sector is proxied by the sum of imports and exports as percentage of GDP. The interest rate is proxied by the deposit rate. Finally, the regional market is proxied by an equally weighted index for a particular geographical area.

roeconomic factors and five financial factors[82]. The results provide strong evidence against the significance of most of the macroeconomic variables. The estimated posterior probabilities are well below the preset priors, with the only exception of the change in exchange rate. This is consistent with previous literature results, where the exchange rate appeared to be the only significant macroeconomic variable. Financial variables show higher significance. Among them, beta is a weak predictor, with posteriors always below the priors when all the financial variables are included. According to the previous literature findings, momentum, PE ratio and downside risk appear to be robust predictors of EM equity returns. Basher and Sadorsky (2006) used unconditional and conditional risk analysis to investigate the relationship between oil price movements and stock returns in twenty-one emerging stock markets. In both cases, oil price movements appear to have strong effects on EM returns. However, for conditional risk analysis this seems to depend on data frequency. Finally, Abugri (2008) used a six-variable vector autoregressive model to investigate whether dynamics in key macroeconomic indicators significantly explain market returns in four Latin American countries (Argentina, Brazil, Chile and Mexico). In particular, they focused more on macroeconomic shocks and volatility, rather than the magnitude of macroeconomic variables. The proxies for world macroeconomic factors are the US 3-month T-bill yield and the MSCI world index. The domestic macroeconomic variables are the nominal exchange rate, the money supply (M1 monetary aggregate), industrial productivity (industrial production index) and nominal interest rate (nominal lending or policy interest rate). The global variables appear to have the most consistent significant effects on all the four markets. However, it's difficult to extract a significant magnitude, since the shocks on macroeconomic variables tend to have different effects across the different markets.

The second topic includes the theoretical models behind the BoP and currency crises, that represent the core components of non-transfer risk. According to Krugman (1979), BoP crises are mainly based on structural weaknesses of the economy. Government bad economic policies, such as inconsistent fiscal, monetary and exchange rate policies, may lead to a deterioration of the fundamentals of the economy[83]. Usually, financial market practitioners' response to this inconsistency is the exit from the country, with the consequent capital outflows and speculative attacks. This will lead to the exhaustion of the central bank's foreign reserves. Thus, high fiscal deficit, decreasing foreign exchange reserves and rising external debt levels

[82] The six macroeconomic factors are: foreign currency exchange rate against USD, local interest rate, short term real interest rate (relative to the previous 36 months), change in expected GDP growth, inflation and a proxy of credit risk (JPM EMBI spreads). The five financial variables are: CAPM beta (60-months rolling), price momentum, price-to earnings ratio, price-to-book ratio, downside risk and market size.

[83] A common example between developing economies is the persistence of important fiscal deficits financed by indebtedness or printing money under fixed exchange rate regimes.

are warning signals of the insurgency of a crisis. In Obstfeld (1996), crises are the consequence of self-fulfilling expectations in theoretical settings with multiple equilibria. The government balances the costs and the gains from defending the exchange rate. If the speculators believe that the commitment of the government to defend the peg is not credible, they will ask for higher interest rates on domestic assets. The cost of defending the peg increases, as rising interest rates will translate into weaker growth and will threat a weak banking sector. This is the concept of self-fulfilling crises and they may be accelerate by the speculators' herding behavior. From the 90s, academic literature started to introduce the concept of multiple crises, especially focusing on the interaction between currency crises and systemic banking crises. According to McKinnon and Pill (1996), in an economy with insufficient banking sector regulation, deposit insurance and moral hazard problems, capital inflows result in over-lending cycles, consumption booms and current account deficits. Consequently, the real exchange rate tend to appreciate and growth slows. As the economy enters a recession, the excess lending during the boom makes banks more prone to a crisis. Meanwhile, the deterioration of the current account makes investors worried about the possibility of default on foreign loans. The systemic banking crisis, with the weaknesses of the banking sector, makes too costly the protection of the peg. Krugman (1999) and Aghion, Bacchetta and Banerjee (2001) stated that a currency depreciation weakens private sector balance-sheets, due to currency mismatches. This weakness threat the economic growth, with the consequent exchange rate depreciation, putting the economy in a vicious circle that could lead to a systemic banking crisis.

The last important topic I want to present is on the empirical literature over the major crises (i.e., financial, banking and currency crises and sovereign default) on developing economies. The scope for the analysis of these disruptive events is high. Laeven and Valencia (2008) identified 147 banking crises, 218 currency crises and 66 sovereign crises from 1970 to 2011. The first important contribution is the one of Frankel and Rose (1996). They tried to investigate the sources of currency crashes using a panel of annual data for 100 developing countries from 1971 to 1992. They defined currency crash as a large depreciation of the nominal exchange rate, translating in a substantial increase in the rate of change of nominal depreciation (i.e., 25.00 per cent, that is also at least a 10.00 percentage points increase in the rate of depreciation). They found that currency crashes tend to occur when the output growth is low, the growth of domestic credit is high and the level of foreign interest rates are high. Moreover, a low ratio of FDI to debt significantly increases the probability of a crash. The second pillar is the work of Kaminsky and Reinhart (1999), where they introduced the concept of twin crises[84]. They analyzed the links between banking and currency crises, finding that problems in the banking sector typically precede a currency crisis[85]. In fact, banking sector problems may threat the capacity of the central bank to defend the exchange rate. On the other side, the currency crisis worsens the banking crisis, activating a vicious spiral. In their work, they focused on a set of macroeconomic variables, in order to understand the sources of the crisis[86]. Usually, financial liberalization precedes banking crises. Moreover, a deterioration of the economic conditions after a prolonged boom in economic activity, financed with credit, can represent an early signal of the insurgence of a crisis. This situation is particularly important for currency crisis, since it leads to currency over-valuation. However, it is also the base of the banking system weaknesses, with an increase of deteriorated positions. Other two important works are Chang and Velasco (1998) and Kaminsky, Lizondo and Reinhart (1998), on financial and currency crisis respectively. These papers followed the first version of the above-mentioned Kaminsky and Reinhart (1999). In particular, Chang and Velasco (1998) used a simple small open economy version of the model of Diamond and Dybvig (1983). The illiquidity of the domestic financial system is the core problem, with banks' illiquidity representing the necessary condition for the insurgence of a crisis. They found that the short maturity of capital inflows can contribute to bank fragility more than their size. Moreover, domestic financial liberalization is positive, increasing banks' wealth, but it increases also the risks of the insurgency of a banking crisis. As a consequence of the crisis, the early liquidation of the assets causes a drop in asset prices, spreading the crisis to the whole economy and dramatically increasing its real costs. Distortive government policies, such as deposit guarantees and investment subsidies, may alter risk perception, causing over-investment. This will worsen the crisis when it will occur. Finally, the presence of a fixed exchange rate regime, alongside an illiquid banking system, can be disruptive. In fact, bank run represents a run on the currency if the central bank tries to act as a lender of last resort, causing a currency crises alongside the banking crises. Kaminsky, Lizondo and Reinhart (1998) investigated the empirical evidence on currency crises, trying to develop an early warning system able to predict the insurgence of these crises. Leading indicators are the base of this system. When an indicator exceeds a certain threshold, it is considered as a signal that a currency crisis may occur in the

[84] The oldest version of the work was published in 1996 and it was an important contribution to the literature.

[85] According to Laeven and Valencia (2008), this type of twin crisis occurred twenty-eight times in the period 1970-2011.

[86] Kaminsky and Reinhart (1999) used sixteen macroeconomic indicators: M2 multiplier (i.e. M2 over base money ratio), domestic credit over GDP ratio, real interest rate, lending-deposit rate ratio, excess M1 balances (real M1 less an estimated demand for money), M2 over reserves ratio, bank deposits, exports, imports, terms of trade, real exchange rate, reserves, real interest rate differential (country vs US or Germany, depending on whether the currency was pegged against USD or Deutsche Mark), output (usually the industrial production), stock returns (global indices for EM), consolidated public-sector deficit as a share of GDP.

following two years. They analyzed a wide set of variables, covering different areas, in order to select the indicators that proved to be the best in predicting currency crises. They selected the same sixteen indicators used in Kaminsky and Reinhart (1999)[87]. The indicators that proved to be successful in anticipating a currency crisis are the behavior of international reserves, the real exchange rate, the domestic credit, the public sector debt and the inflation. Other indicators that proved to have limited predictive power are trade balance, export, money growth, real GDP growth and fiscal balance[88]. Berg and Pattillo (1999) is an evolution of Kaminsky, Lizondo and Reinhart (1998). They compared the KLR leading indicators approach with a probit-based model of currency crisis prediction in order to understand which model would perform better in 1996 in the prediction of Asian crisis. They used the same variables of Kaminsky, Lizondo and Reinhart (1998). First of all, they implemented the KLR approach, computing the noise-to-signal ratios[89]. Secondly, they apply a probit regression technique to the same data and crisis. The probit model reproduces most of the KLR conclusions over the variables that are significant predictors of crisis. In particular, both the approaches show that the probability of a currency crisis increases when the real exchange rate is overvalued relative to the trend, reserve and export growth are low and the growth of money supply is high. Their analysis adds also large current account deficit and a high ratio of M2 to reserves. About the effectiveness in predicting the crisis, both models performed well. However, probit model provides slightly better forecast. Another important development on the prediction of currency crises is the work of Kumar, Moorthy and Perraudin (2003). They developed trading strategies in which an investor goes long or short in the currency depending on whether crash probabilities are low or high in order to test a currency crisis predictive logit model. They used data on thirty-two developing countries from January 1985 to October 1999, considering a wide set of global and country-specific macroeconomic and financial variables[90]. They found that logit forecasting models have significant explanatory power when

[87] The areas covered are: capital and current account, debt profile, international variables, financial variables, financial liberalization variables, real sector, fiscal variables, institutional and political factors. For the sixteen variables selected see Note 86.
[88] For example, negative factors are real exchange rate over-valuation, sharp domestic credit growth, large current account deficits, etc..
[89] Ratio of false signals, measured as a proportion of months in which false signals could have been issued (see Kaminsky, Lizondo and Reinhart [1998]).
[90] The variables considered are: foreign exchange reserves (12-month percentage change and as a ratio to imports), real GDP growth, real exchange rate, exports, budget balance over GDP ratio, dummy variable for high inflation regimes (unity if the percentage change in the level of the CPI over the last 2 months exceeds an annualized rate of 100%), FDI, portfolio investment , dummy for capital account liberalization (unity if liberalized), the ratio of official foreign debt to private foreign debt, dummies for currency crashes contagion effect (the first dummy takes unity if a country in the same region has experienced a currency crash in the last 3 months, the second one takes unity if a country export growth is closely correlated with the country experiencing a crisis experienced a crash in the last 3 months), a proxy for the external financial environment (global liquidity indicator), nonfuel commodity prices, a linear time trend and lagged monthly changes in the exchange rate (to catch momentum and overreaction to currency crisis).

estimated on two thirds of the sample and then used to predict crashes in the remaining third. The trading strategies proved to be successful. Consistently with the previous literature, the most important explanatory variables are the decrease in foreign exchange reserves, the current account balance worsening and weakening real activity. Moreover, contagion seems to play an important role in explaining the insurgence of a currency crash, both considering the regional factor and the export growth correlations. The last important contributions are the ones of Reinhart and Rogoff (2008b) and Laeven and Valencia (2008), updated by Laeven and Valencia (2012). In particular, Reinhart and Rogoff (2008b) showed that developing countries are more prone to default than developed countries. Moreover, sovereign defaults tend to occur in waves generally spaced some decades apart. Finally, I already quoted some of the conclusions of Laeven and Valencia (2008). This work is the base for the construction of the IMF Systemic Banking Crises Database. They considered banking, currency and sovereign crises. From 1970 to 2011, they found 147 banking crises, 218 currency crises, 66 sovereign crises, with 28 cases of currency-banking twin crises, 11 of banking-sovereign twin crises, 29 of currency-sovereign twin crises and 8 of triple crises. Alongside the definitions of the different type of crises and the analysis of initial conditions, they provide an estimation of fiscal costs and real effects of banking crisis. In particular, fiscal costs, net of recoveries, are quite high, with an average of 13.30 percent of GDP and a maximum 55.1 percent of GDP. Recoveries of fiscal aids vary widely across countries. The output losses, measured as deviations from trend GDP, due to systemic banking crises is about 20.00 percent of GDP on average during the first four years, with a maximum of 98.00 percent of GDP. This results show the level of danger of systemic banking crisis, that can represent disruptive events for investors. Their tendency to arise together with currency crises, increase the scope of non-transfer risk assessment.

4.2 The Country Risk Indicator Structure

The idea of using macroeconomic variables in allocation strategy is not a new concept. It constitutes the base of the strategic asset allocation. Following Eychenne, Martinetti and Roncalli (2011), strategic asset allocation requires long-term assumptions over asset risk and return characteristics as a key input, alongside macroeconomic models and forecasts of structural factors depending on the type of asset class[91]. This concept was extended to Risk Budgeting Approach, with the macroeconomic variables as the fundamental determinants of the risk budgets. For example, Bruder and Roncalli (2012), quoting Bruder, Hereil and Roncalli

[91] Some of the macroeconomic fundamentals observed are population growth, productivity, inflation, potential output growth, public debt path. Of course, these structural factors impact the different asset classes in a different way, depending on the structure of the instrument and of the returns.

(2011), present a methodology to manage sovereign bond portfolios where the risk budget for each country is proportional to its Debt or GDP. They compared four indexation methodology: debt weighting, fundamental indexation and risk-based indexation[92]. Bruder and Roncalli (2012) developed this issue, using the tools of the strategic asset allocation to design the risk budgets. The main limit of these procedures is that they focus only on a small set of macroeconomic variables. In the previous subparagraphs, I highlighted the importance of Country Risk Assessment procedure when dealing with developing countries. There is a great scope behind the evaluation of country risk. In particular, these countries are more prone to major crises than developed countries. These major crises can represent disruptive events for investors, carrying huge losses. Moreover, I showed that country risk is simply a broad aggregate, composed by different types of risk. Thus, it depends on a various set of factors and its evaluation cannot be reduced to the analysis of public debt and GDP only.

The aim of the Country Risk Indicator is to provide an homogeneous measure of individual country risk in order to compute the risk budgets on Frontier Market asset class. This tool tries to capture the different components of country risk and it is based on a set of various macroeconomic and financial variables. This indicator does not cover a complete Country Risk Assessment, but it maintains the same methodological procedure focusing on the main determinants of country risk. A complete Country Risk Assessment for the thirty-one Frontier Markets, over a period of ten years, will require a whole division of analysts and an incredible amount of resources. Moreover, the importance of subjective opinions in the evaluation of some aspects of country risk makes impossible the automatic computation of a quantitative measure useful to design the risk budgets. A deeper analysis of country risk should be conducted in the implementation of actively managed strategies, with the subsequent possibility to force the outcomes of the Country Risk Indicator algorithm. In fact, I want to highlight that the Risk Budgeting Approach is generally a base for the construction of alternative benchmarks or for the implementation of active management strategies. It does not ensure higher performance with respect to other methodologies in portfolio construction. For this reason, the introduction of subjective views should be performed month per month in the implementation of active strategies, improving the results of the automatic algorithm of the Country Risk Indicator.

As above-mentioned, the Country Risk Indicator is a measure of individual country risk. I computed the indicator for the thirty-one Frontier Markets present in my investment universe

[92] In debt weighting, the weights are proportional to the country debt portion of whole countries' debt. In fundamental indexation, the weights depend on the country GDP portion of the whole set of countries GDP. In risk-based indexation, the risk budgets, and not the weights, are proportional to debt or GDP portion.

from February 2005 to January 2015, on a monthly base. The indicator varies between 0 and 100 points (the higher the score the lower the risk) and it is given by the sum of three sub-indicators:

- the Economic Risk Indicator, with a maximum of 60 points;
- the Liquidity Risk Indicator, with a maximum of 25 points;
- the Political Risk Indicator, with a maximum of 15 points.

The Economic Risk indicator tries to assess the macroeconomic and the sovereign risk. It is important to note that some macroeconomic factors (e.g., current account deficits) and sovereign risk factors (e.g., government deficit) have an impact on non-transfer risk. The Liquidity Risk Indicator tries to capture the liquidity risk, especially related to the international liquidity, and the non-transfer risk, in particular from an exchange rate point of view. Note that the non-transfer risk assessment is present both in the Economic Risk and in the Liquidity Risk Indicator, with the evaluation of different aspects. The Political Risk Indicator tries to capture the political and institutional risk, with a specific section dedicated to serious turbulence (e.g., civil war). About the weights assigned to Economic, Liquidity and Political Risk Indicator (60.00, 25.00 and 15.00 per cent), I followed financial markets practice and I evaluated the reliability of the variables used to assess the different risks. In particular, the most important indicator is the Economic Risk Indicator. Academic literature demonstrated that macroeconomic variables tend to have no statistically significant effects on stock returns. However, the aim of Country Risk Indicator is not to forecast the markets that can potentially drive the highest returns. The indicator focuses on country risk and macroeconomic variables proved to be important in the prediction of major crises. In fact, country-specific economic framework is often a fundamental backdrop for the insurgence of these crises. Political and institutional risk is relatively important in Country Risk Assessment, due to the potential disruptive effects of negative shocks generated by this class of risk. However, I assigned a low weight to Political Risk Indicator because in the assessment of political and institutional risk, subjective evaluations and interpretations are fundamental. Subjective opinions cannot be taken into account in an automatic algorithm used for the computation of a quantitative measure of country risk. Thus, the reliability of the political risk measure significantly reduces when subjective interpretations are avoided.

The main limit of the Country Risk Indicator is the determination of the weights assigned to the single components. Unfortunately, they are mainly base on subjective evaluations, following, when it was possible, the theory and financial market practice. An econometric model able to produce more reliable weights is a necessary development for the future works. I cannot apply such a model, given the limited number of observations. As I will explain later, even an in-sample and out-of-sample analysis cannot be performed. Thus, if I want to test the significance of macroeconomic and financial variables preserving the investment period, I had to conduct a cross-country analysis from 2000 to 2004. At the time, I had only five countries and the cross-country model produced statistically significant coefficients only for the exchange rate and oil prices. This is consistent with the literature I presented above on the relation between stock returns and macroeconomic variables. However, in a Country Risk Assessment I had to consider other variables able to forecast major crises that are not captured by stock market returns in normal time. Conclusions are unchanged even introducing dummy variables for asset volatility and returns (e.g., increase in volatility, decrease in return) and for the various macroeconomic and financial variables (e.g., GDP growth above or below a specific threshold). In my opinion, future works could focus on the determination of Country Risk Indicator weights, thanks to longer time series and a greater availability of data.

One might ask why I decided to compute a new risk indicator to design the risk budgets when there are several measures proposed by various institutions. The main reason is that I want to provide an indicator of country risk using only freely available data. In this way, any investor can compute its own indicator without investing resources for the access to costly database and services. For this reason, I excluded the indicator proposed by the Economist Intelligence Unit and the PRS Group[93]. Between other diffused risk measures, the sovereign ratings issued by Credit Rating Agencies are freely available. However, as above-mentioned, this country risk measure focuses mainly on sovereign risk. I have already explained that country risk is composed by several components and sovereign risk is only one aspect. Even if behind sovereign ratings there is a complete Country Risk Assessment, the access to the papers is costly and they are not available for the whole investment period (i.e., 120 months). In addition to that, sovereign ratings issued by Credit Rating Agencies present several limits, that I have already discussed in the previous subparagraph.

Finally, I want to present the common principles behind the choice of the set of variables used for the computation of the Country Risk Indicator. I have already mentioned one of this principle: the macroeconomic and financial variables have to be freely available[94]. Secondly,

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[93] These services are too costly for non-professionals like me (in most of the cases more than € 3,000). EIU country risk indicator is available on Datastream only for a set of Frontier Markets. Moreover, it's present the broad aggregate, without the decomposition between the different type of risk. For these reasons, I decided to exclude it.

[94] For example, following this principle, I used the expectations on some macroeconomic variables provided by the IMF in its World Economic Outlook databases or in its International Financial Statistic database.

the variables have to be disposable for the whole set of Frontier Markets and for the whole investment period, excluding some exceptional cases[95]. The most important cases are:

- Foreign Exchange Reserves: in this case there are some missing during the investment period, but they never exceed the 10.00 per cent of the observations;
- Zimbabwe hyperinflation and currency crisis (2008-2009): it was not possible to retrieve estimations over some key variables, such as Real GDP Growth, Inflation, exchange rate;
- Ukraine (2014): Ukrainian civil war is the cause of the absence of some key variables;

The impact of these exceptional cases is limited. About Foreign Exchange Reserves, I will explain in Subparagraph 4.2.2 how I solved the problem of missing through their replacement with a coherent expectation. About Zimbabwe, MSCI Zimbabwe was launched later in 2010. Thus, the missing relative to Zimbabwe didn't influence the portfolio allocation. Finally, also in the Ukraine case, the missing didn't influence the portfolio allocation. In fact, if there is the presence of a serious conflict involving one country belonging to the investment universe, the risk budget is set automatically equal to 0.00 per cent. Finally, the most important principle: the variables had to be available when the investor had to make its investment choice. As mentioned in Section 1, I took the point of view of an investor that wanted to invest in Frontier Markets in February 2005. In the Country Risk Assessment, I cannot take an ex-post point of view, using the information available today. I had to conduct the country risk evaluation as an investor could do at the time in which the investment choice was made[96]. This strictly reduces the set of variables that can be used. For these countries, good sets of information are available only after 2013. Someone could argue that an in- and out-of-sample analysis would help to solve this point. However, given the limited number of observations for the return time series and the will to maintain a minimum diversification in subsequent portfolio allocation, makes impossible this methodology.

In the following subparagraphs, I present the structure of the Economic, the Liquidity and the Political Risk Indicator. For a detailed presentation of the points assigned to each variable, please refer to Appendix C, where I provide the tables with detailed specific case for each variable.

[95] An example is the choice to use government overall balance instead of primary balance in the assessment of sovereign risk.

[96] It is difficult to explain this point. An example is referred to Real GDP Growth. In 2005 I cannot use for the allocation process the Real GDP Growth of 2005. Now, in 2014, I know that data, but in February 2005 It was not available. Thus, I had to use the expectations on 2005 Real GDP Growth provided in the IMF WEO 2004 and 2005 databases.

4.2.1 The Economic Risk Indicator

The Economic Risk Indicator tries to assess the macroeconomic risk and the sovereign risk. It partially assesses the non-transfer risk, through the inclusion of current account balance and export structure evaluation. The indicator is given by the sum of:

- GDP Indicator (15 points);
- Inflation Indicator (5 points);
- Current Account Balance Indicator (5 points);
- Export Indicator (10 points);
- Government Finance Indicator (25 points).

The indicator presents a stable part and a sensitive part. The stable part is given by GDP Indicator, Inflation Indicator, Current Account Balance Indicator and Government Finance Indicator. The stability comes from the fact that the variables at the base of these indicators do not vary every month. The Export Indicator represents the sensitive part. It varies every month, since it is based on commodity prices. For the evaluation of the Economic Risk Indicator, I used, when it is possible, the data provided by the IMF World Economic Outlook (WEO)[97]. IMF provides expectations also in the International Financial Statistics (IFS) section. However, expectations are available only for some countries and only from 2015. IMF WEO section included also the older databases, up to 1999. In this way, I can retrieve the expectations that an investor could observe in the past years.

The reference variable for the Real GDP Indicator is the Real GDP Growth. This variable is fundamental in assessing macroeconomic risk, since it represents a signal of economic stability. Moreover, it is an important component of sovereign risk. In fact, government revenues and the sustainability of the debt strictly depends on economic growth. It is also connected to non-transfer risk, since economic recession could accelerate the insurgence of currency and banking crises. I used the expectations on Real GDP growth provided by the IMF in the WEO databases. World Economic Outlook papers are usually issued every April and October, thus their estimations are available for May and November. Using the most recent available expectations, for the first six months of the year I considered the expected Real GDP Growth for that year, while for the second half I considered an average between the Real GDP Growth for that year and the Real GDP Growth for the next year. In this way, I can take into account rel-

[97] World Economic Outlook databases are available in Data section of the IMF website: www.imf.org.

evant future expectations in asset allocation, as financial market practice suggests[98]. Real GDP Growth is examined under two aspects:

- the magnitude of the expected Real GDP Growth;
- the difference between the value of expected Real GDP Growth and the average of the Real GDP Growth of the previous two years.

The second component has the objective to detect signals of potentially upcoming downturn. However, it has also the objective to detect signals of economic recovery, with the consequent potential stock market upturn from the depressed quotations that usually characterized the recession phase. In this way, I can anticipate the choice to enter in a market that presents high growth opportunities, alongside a potential reduction in risk. If Real GDP Growth were not available (i.e., missing), the value assigned is 0, since it is a signal of high instability[99].

Annualized inflation in percentage term is the core variable of the Inflation Indicator. I took the expectations from the IMF WEO databases and I used the same procedure adopted for the Real GDP Growth in the computation of inflation expectations. Inflation is a base for the assessment of macroeconomic risk, being a relevant signal of economic stability. It is also relevant for non-transfer risk, since high inflation is a signal of currency depreciation. Finally, it is connected to sovereign risk, as it enters in the computation of the real interest rate. I analyzed the value of expected inflation and there are two relevant extreme cases. I assigned 0 value to the indicator in presence of extreme negative events, such as hyperinflation (more than 25.00 per cent) and deflation. The maximum was assigned in presence of stable prices. Price stability concept is broadened with respect to developed countries, with less strict thresholds. Again, to the missing I assigned a value of 0.

The reference variable for the Current Account Indicator is the current account balance as percentage of GDP. Current account balance is fundamental in the assessment of non-transfer risk. In fact, strong current account deficits are early signals of a currency crisis. It is also related to economic stability. I took the expectations from the IMF WEO databases and I used the same procedure adopted for the Real GDP Growth and inflation in the computation of my current account balance expectations. I examined current account balance under two aspects:

- the current value of the expected current account balance;
- the difference between the value of expected current account balance and the average of the current account balance of the previous two years.

[98] A simple example to explain this point is the one of a top-down investor that has to allocate its wealth in December 2005. This investor will not look only at the Real GDP Growth expected for 2005, but he will take into account especially the Real GDP Growth of 2006.

[99] I have already mentioned the currency crisis in Zimbabwe and the civil war in Ukraine.

The second aspect is much more important and takes into account the dynamic of the current account balance, in order to identify potential threats coming from a quick worsening of the economic framework. As in the previous cases, to the missing I assigned a value of 0.

The Export Indicator represents the sensitive portion of the Economic Risk Indicator, varying each month. The majority of the Frontier Markets are commodity exporters. Thus, their revenues strictly depends on international commodity prices. This indicator tries to capture the risk associated to the fluctuations of commodity prices. This risk threats economic growth, government revenues and current account balance position, representing a source of macroeconomic, sovereign and non-transfer risk. I used three commodity indices provided by the IMF with a monthly frequency: the Commodity Fuel (energy) Index, the Commodity Food and Beverage Price Index and the Commodity Metals Price Index[100]. I took annual data on exports from the World Trade Organization, computing the portion of agricultural and fuel and metal exports on the country's total export[101]. About fuel and metal exports, I divided the countries in three different class (fuel exporters, metal exporters and fuel and metal exporters) in order to select the right Commodity index to use[102]. The thresholds used in the evaluation of negative and positive monthly variations in commodity prices become tighter the higher the portion of commodities on total exports. In fact, if the incidence of commodities on country's revenues is high, even small changes in prices can affect significantly the revenues and, consequently, the economic framework[103].

The Government Finance Indicator tries to assess sovereign risk. Of course, the assessment refers to the ability to pay, since the assessment of the willingness to pay requires qualitative and subjective considerations. I used two variables for the assessment of this important risk:

- the general government overall balance as percentage of GDP;
- the general government gross debt as percentage of GDP.

In order to maintain the same source, I downloaded the data from the IMF WEO databases. However, government overall balance and gross debt data were completed in the April 2010 IMF WEO database. Thus, the expectations were not available till May 2010. In the assess-

[100] They are available in Research section of the IMF website: www.imf.org.
[101] The data are available in the World Trade Organization website: www.wto.org.
[102] For fuel and metal exporters, I used a combination between Commodity Fuel (energy) Index and Commodity Metals Price Index.
[103] For example if the percentage of agricultural export is between 10.00 and 25.00 per cent the minimum and maximum thresholds for commodity price variations are -10.00 and +10.00 per cent. If the percentage is above 50.00 per cent, the minimum and maximum thresholds for commodity price variations are -3.00 and +3.00 per cent.

ment of sovereign risk, I used the data of the previous year with a lag of six months[104]. In the selection of these variables, the above-mentioned principles played a central role. First of all, in assessing sovereign risk, primary balance is more efficient than overall government balance. However, for the majority of the countries included in the investment universe, primary balance data are not available. Even the more complete IMF IFS database does not cover all countries. Other important variables are the debt service and the interest rate spread on government bonds. In particular, the last variable could be useful in the assessment of the liquidity risk related to the roll-over of the debt. However, both the IMF IFS database and Datastream do not provide these data for a consistent number of countries. About public debt analysis, other useful variables are the net debt and the external debt. In particular, external debt is fundamental in the combined assessment of sovereign and non-transfer risk. There is a common opinion that external debt is much more important in the assessment of sovereign risk, since sovereign debt crisis are driven mainly by foreign investors' actions rather than domestic investors' ones. However, these variables were available only for a small number of countries in the IMF IFS database without significant missing. Alternatively, there are non-free databases that provide them. For these reasons I have to exclude them. The general government overall balance and gross debt are analyzed under two aspects:

- the value of the overall balance and the gross debt;
- the difference between the value of the overall balance and the gross debt and the average of the overall balance and the gross debt of the previous two years.

The first component tries to capture the risk associated to risky level of public deficit and debt. For general government overall balance the important threshold is -5.00 per cent of GDP. For the general government gross debt is 90.00 per cent of GDP. I didn't select this threshold following Reinhart and Rogoff (2010). I chose 90.00 per cent because it is close to the psychological barrier of 100.00 per cent and above this threshold the risk of capital flight from the country in adverse economic framework increases substantially. I gave more importance to the second component. In both cases, it tries to capture the dynamic of public balance and debt, in order to understand if the path is sustainable.

4.2.2 The Liquidity Risk Indicator

The Liquidity Risk Indicator tries to capture the liquidity risk and the non-transfer risk. I have already highlighted the importance of these two risks and the high scope for their assessment. The combination of illiquidity and currency crisis represents serious threats for the

[104] After six months (June), data on government balance and debt position of the previous year can be considered enough precise and they don't represent an estimate.

investment and they are the base of systemic banking crises. Currency crises and systemic banking crises, combined with the impossibility to close the position, can carry huge losses for the investors. The indicator is the sum of two sub-indicators:

- the International Liquidity Indicator, common for all Frontier Markets (15 points);
- the Foreign Exchange Reserve Indicator (10 pints).

I assigned an higher weight to the international liquidity indicator. Total reserves present some missing values and the confidence over the estimations is not complete. I conducted the assessment of the liquidity risk taking an international point of view. International liquidity conditions significantly affect the investment in the more risky assets. In particular, they affect the capital flows towards Emerging and Frontier Markets . Restrictive monetary policies in developed countries are usually associated to capital outflows from emerging economies[105]. Massive capital outflows usually generate currency and financial crisis, leading in some cases to a sovereign default too. I used Federal Reserve Policy Rate as a proxy for international liquidity. First of all, Federal Reserve monetary policy is a relevant variable in financial market agents' choices[106]. It affects liquidity conditions in the biggest financial market and it influences the performance of US Dollar, that still remains the most important currency in international transactions. Moreover, it affects massively the exchange rate of the currencies pegged to the USD. Between the Frontier Markets, fourteen countries on thirty-one present a currency pegged against USD. I decided to don't use a proxy of world money supply (given by the sum of Fed, ECB, BoE, BoJ and PBOC money supply), because in the last 15 years is always increasing. I downloaded Federal Reserve Policy Rate at monthly frequency[107]. I analyzed this proxy of the international liquidity under various aspects:

- the difference between the current policy rate and the 6-month moving average;
- the difference between the current policy rate and the previous month policy rate;
- the number of consecutive months with an increase in the policy rate.

I performed this evaluations considering also the level of the policy rate. In particular, when the policy rate exceeds the 2.00 per cent threshold, I assigned 0 value to the indicator. Especially during the dot-com bubble and the subprime crisis, this is a psychological barrier that signals the attempts to fight against a potential speculative bubble. In order to reduce the impact on the investment choices of the high variability of this indicator, the final value as-

[105] An example is the sovereign default wave that hit Latin America during the '80s. I briefly presented this crisis in the Subparagraph 4.1.2.
[106] Think for example to the capital outflows that hit Emerging and Frontier stock markets when the Federal Reserve announced the Tapering during the 2014.
[107] The time series of Federal Reserve Policy Rate can be found at the Board of Governors of the Federal Reserve System website: www.federalreserve.gov.

signed to the indicator is the 6-month moving average of the initial values computed. The first and the third indicator are particularly important in order to capture the objective of monetary policy. For example, two or three months of consecutive increases in policy rate can be viewed as important signals of a reinforcement of restrictive monetary policy.

The Foreign Exchange Reserve Indicator tries to assess the non-transfer risk. It can assess both the risk de facto and the risk de jure. First of all, with large capital outflows and the drop in foreign exchange reserves, the probability of the introduction of capital controls significantly increases. Secondly, large foreign exchange reserves decreases are usually associated to depreciation in the exchange rate and they represent early signals of a currency crisis. The variable used is the country-specific total reserves in US Dollar, given by the sum of FX reserves, gold reserves and the reserve position at the IMF[108]. Other fundamental variables are capital flows and the exchange rate. However, it is very difficult to find capital flow data for the countries included in my investment universe. About the exchange rate, the main concern arises for pegged exchange rates. For them, it's particularly difficult to forecast large depreciation looking at the value of the exchange rate, that remains stable until a currency crisis occurs. However, one of the objective of the Country Risk Indicator is to anticipate the insurgence of a currency crisis. For this reason, total reserves can capture both capital flows and exchange rate movements. I downloaded the data from the IMF IFS database. I replaced the missing values in two ways:

- if the data on total reserves were available in World Bank database, I replaced the missing with the value provided by the World Bank[109];
- if no data on total reserves were available in World Bank database, I replaced the missing with an estimated value[110].

I detected seasonality in total reserves time series for almost all countries. For this reason, I implemented the X-12-ARIMA seasonal adjustment program of the U.S. Census Bureau (see Findley *et al.* [1998]). Total reserves are analyzed under three points of view:

- the difference between the total reserves value and the 12-month moving average;
- the difference between the total reserves value and the previous month value;
- the number of consecutive months with a decrease in total reserves.

[108] The value of gold reserves is obtained multiplying the monthly quantity in ounce provided by the IMF IFS database with the monthly price of gold (3.00 PM, London Bullion Market).
[109] Total reserves data are available at the following website: data.worldbank.org .
[110] First of all, I estimate the FX reserve value using the sensitivity to exchange rate variation. Secondly, I maintain unchanged the quantity in ounce of gold reserves (gold reserves in quantity tend to be stable over time and changes were always reported). Finally, I assigned the same rate of variation in FX reserve to the IMF reserve position.

As in the previous case, the most important indicators are the first and the third. Total reserves significantly below the 12-month moving average are an early signal of currency crisis. Moreover, as suggested in the literature, it is one of the most powerful predictor of currency crisis. The number of consecutive months with a decrease in total reserves is a signal of massive capital flows and an early signal of a potential currency crisis. In general, total reserves below, even if not significantly, the 12-month moving average are a signal of potential currency depreciation. Finally, I assigned the maximum number of points to the countries belonging to the Euro Area, since they do not carry exchange rate risk.

4.2.3 The Political Risk Indicator

The Political Risk Indicator tries to capture the political and institutional risk. As abovementioned, the assessment of political and institutional risk requires qualitative interpretations of political and institutional factors. It's extremely difficult to quantify the risk associated to these factors. The automatic computation of the Country Risk Indicator does not allow qualitative interpretations. For this reason, I assigned a relative low weight to the Political Risk Indicator. Moreover, I decided to use an index computed by a third entity that captures relevant country's political and institutional characteristics. I used a personal scale only for the social conflicts and war component. The indicator is divided in two main components:

- the War Risk Indicator (5 points);
- the Political and Institutional Risk Indicator (10 points).

The War Risk Indicator tries to capture the risks coming from riots, social conflicts, civil wars and wars. The scope for the assessment of this risk is high, since these turbulences are disruptive events for foreign investors[111]. I created a scale to assign a score increasing with the importance of the conflict. The value of this score varies from 0 to 5[112]. On the basis of this scale, I designed the War Risk Indicator. If the conflict is particularly severe, I assigned a 0 value to the indicator.

In the computation of Political and Institutional Risk Indicator, I used the State Fragility Index provided by the Centre of Systemic Peace. The State Fragility Index is a quantitative indicator that tries to evaluate the stability of a country. It includes the evaluation of political,

[111] The most recent case is the Ukrainian civil war. From the beginning of the civil war to January 2015 (i.e., less than one year), MSCI Ukraine index in EUR lost the 43.49 per cent of its value.

[112] The score assigned are the following: 0 (no conflicts), 1 (riots), 2 (social conflicts), 3 (civil war), 4 (serious civil war or war against a foreign state outside country territory), 5 (civil war with the involvement of a foreign state or war against a foreign state on country territory).

social and institutional factors[113]. It is based on the effectiveness and the legitimacy of four performance dimensions (Security, Political, Economic and Social) and ranges from 0 (no fragility) to 25 (extreme fragility). The relevant thresholds used in the computation of Political and Institutional Risk Indicator are the same provided by the Centre of Systemic Peace[114].

4.3 The Country Risk Indicator Analysis

In this paragraph, I provide a quick analysis of the outcomes of Country Risk Indicator computation. Over the 120-month investment period, the geographical area that presents the highest average rating is the Middle-East, with a value close to 63.30. The good performance is driven especially by the Foreign Exchange Reserve Indicator. These countries are for the major part oil exporters and their total reserves increase significantly in the last years. This is consistent with the already discussed appreciation of the exchange rate[115]. In the middle part, they also benefit from a boost in oil prices during the 2008 Financial Crisis. On the other side, the average indicator decreased during the 2014, when the oil prices drop. Good performance of Foreign Exchange Reserve Indicator explains also the high average rating of European Frontier Markets, due to the presence of Eurozone countries. The value of 61.20 is close to the one of Latin America, at 61.00. Latin American value is mainly due to the good rating assigned to Trinidad and Tobago. Argentina and Jamaica present high variability, due to the sovereign defaults that characterized the performance of these countries in the last ten years. The lowest average values are the ones of Africa and Asia, with 59.30 and 57.50 respectively. However, African countries' ratings present high variability, with a big difference between the minimum and the maximum. For example, for more than three years, Zimbabwe had the lowest rating, due to the currency crisis that hit the country. The following figures present the time series of the average Country Risk Indicator by geographical area, with the dot-lines representing the minimum and the maximum Country Risk Indicator in that geographical area. I present both the overall indicator and the long-term indicator. The long-term indicator is the overall indicator minus the sensitive part, represented by the Export Indicator and the Liquidity Indicator.

[113] For an extensive presentation of the State Fragility Index, please visit the following website at the Analysis section: www.systemicpeace.org.
[114] The thresholds are: 0-3 (very low fragility), 4-7 (low fragility), 8-11 (medium fragility), 12-15 (medium high fragility), 16-19 (high fragility), 20-25 (very high fragility).
[115] See the Subparagraph 3.2.2.

Figure 11 – Latin America Average Country Risk Indicator

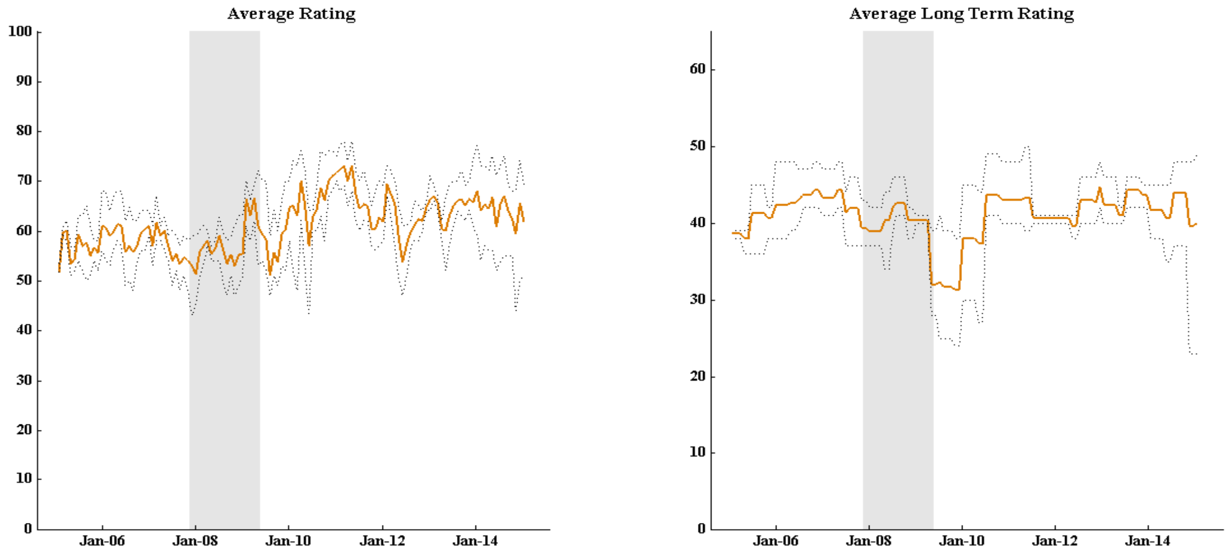


Figure 12 – Europe Average Country Risk Indicator

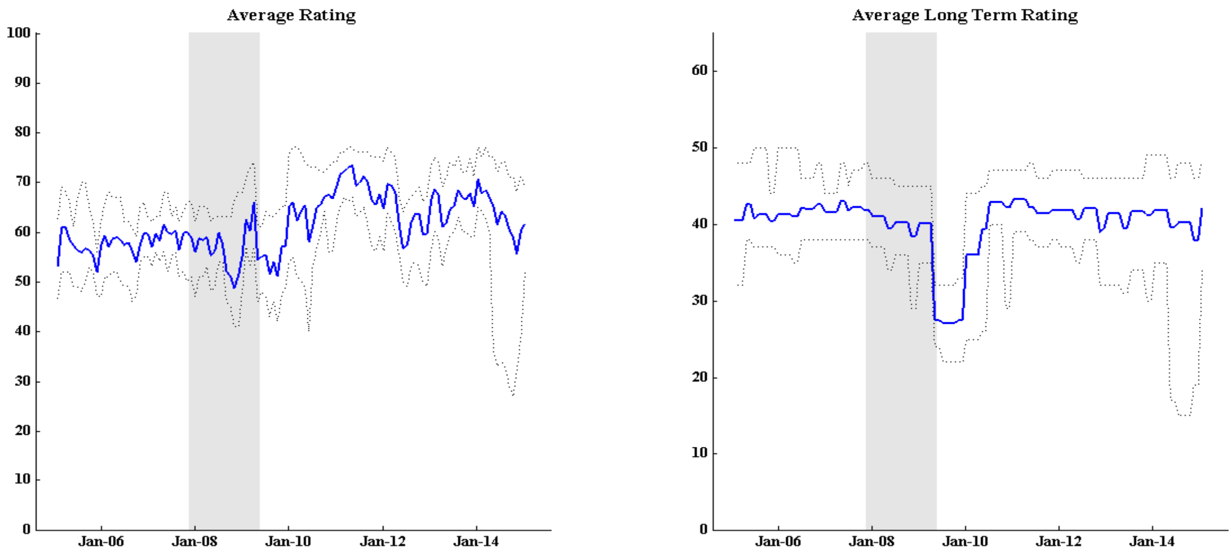


Figure 13 – Africa Average Country Risk Indicator

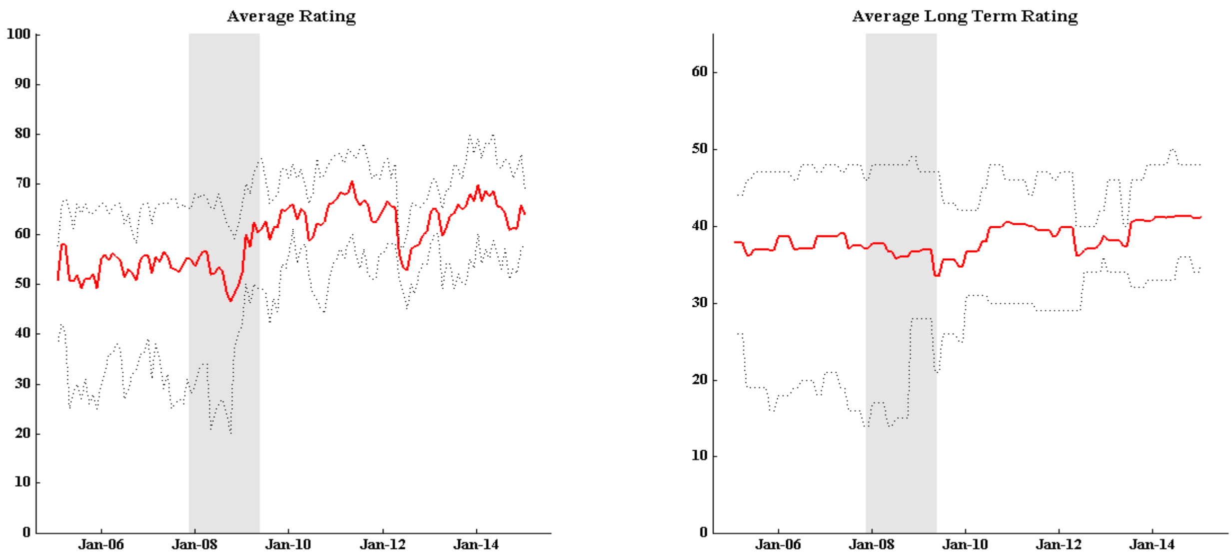


Figure 14 – Middle East Average Country Risk Indicator

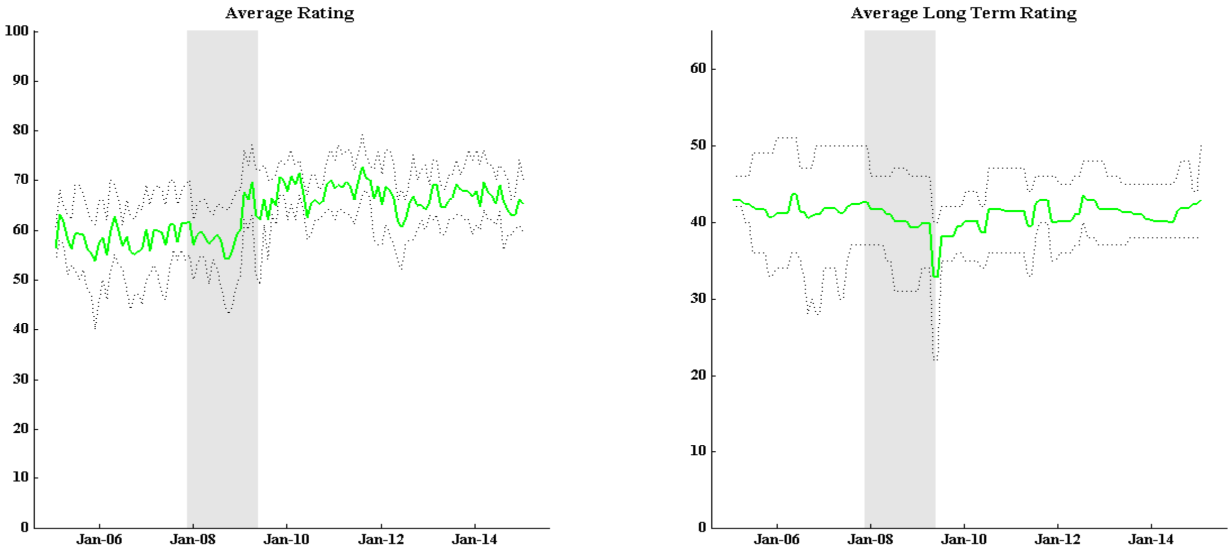
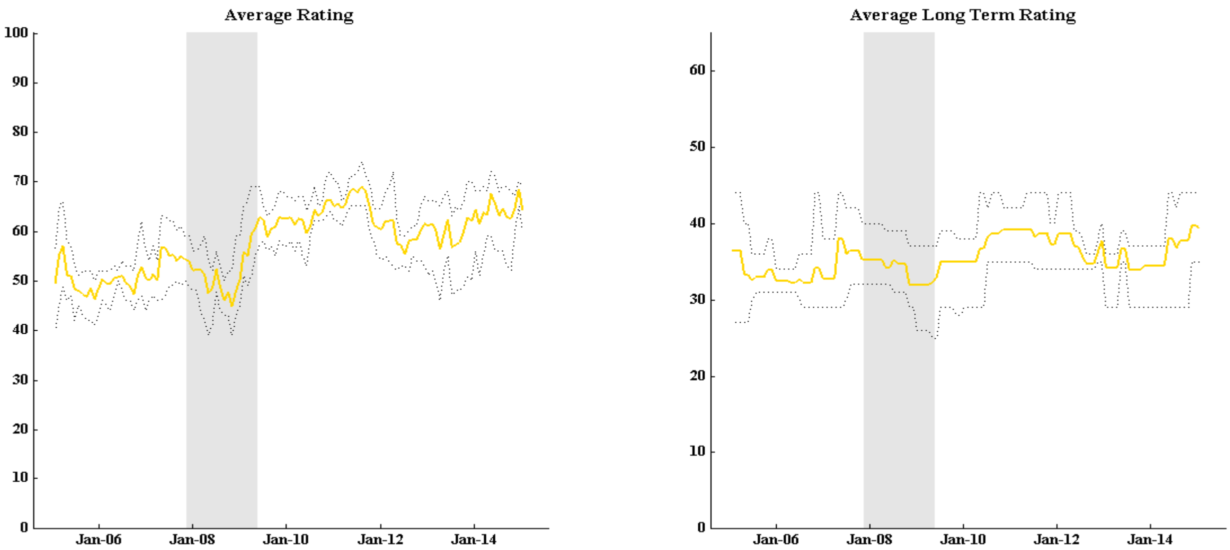


Figure 15 – Asia Average Country Risk Indicator



One important thing to note is that the long-term version of the Country Risk Indicator tend to react slowly to negative shocks, such as the 2008 Financial Crisis. This confirmed the positive feature of the inclusion of a sensitive portion. Moreover, without the Liquidity and the Export Indicator, Middle East countries present a rating very close to Europe and Latin America, with an average value of 41.00. Europe, with the exclusion of the Liquidity Indicator, presents an average rating lower than Latin America (40.00 and 41.00 respectively). This confirms the above conclusions that Europe and Middle East ratings are in some way inflated by the Liquidity Risk Indicator.

Europe, Africa and Asia Country Risk Indicators exhibit the highest heterogeneity, with important differences between the minimum and the maximum. For Africa, this variability slightly decrease after the 2008 Financial Crises. This is due to the jump of Zimbabwe, i.e. the country with the lowest Country Risk Indicator. This country was in a deep hyperinflation period, with a subsequent currency crises. This crisis was solved only in 2009. Looking at Europe, the high variability of the last months is given by the drop of the Ukraine Country Risk Indicator, following the start of the civil war.

Looking at the trend, in the last years African and Asian countries show the strongest positive trend. If we consider the last 12 months, African countries present an average rating close to Middle East countries, at 65.30 and 65.80 respectively. Asian countries reduced significantly the distance and overcome Latin American countries, with average ratings of 64.30 and 64.10 respectively. In the meanwhile, Europe lagged behind, with the lowest average rating at 63.40. The European Sovereign Debt Crises and the recent turbulence in Ukraine depressed significantly the average Country Risk Indicator for this geographical area.

5 Portfolio Analysis

In this Section, I apply the Risk Budgeting Approach to construct internationally diversified portfolios. A portion of these portfolios is invested in the Frontier Market asset class. As presented in Section 2, the inputs for the implementation of Risk Budgeting Approach are the risk measure and the risk budgets. In this thesis, I focus on volatility risk measure[116]. Thus, one of the input for the risk budgeting algorithm is the variance-covariance matrix. About the second input, I present how to use the Country Risk Indicator for the determination of the risk budgets. In general, the lower the country risk, and the higher the risk budget I assigned to a particular Frontier Market is. I applied this methodology only to Frontier Markets. In my investment universe, only Frontier Market asset class includes stand-alone country indices. Developed and Emerging Markets are broad aggregate of different and very heterogeneous countries, making impossible the computation of the Country Risk Indicator.

The investment period is 120 months, from February 2005 to January 2015. I included MSCI indices as soon as their variance-covariance matrix estimation became available. Thus, the investment universe varied during the ten years considered. I constructed four different portfolios, with a maximum risk exposure to Frontier Markets of 5.00, 10.00, 20.00 and 40.00 per cent[117]. However, the monthly risk budget assigned to the Frontier Market asset class varied during the investment period and it was not always equal to the maximum exposure. As I will show later, this risk exposure depends on international liquidity condition and the average risk perceived in Frontier Markets. Alongside risk budgeting portfolios, I used the Markowitz model to construct portfolios with a weight exposure of 5.00, 10.00, 20.00 and 40.00 per cent on Frontier Market asset class. Between the efficient allocations, I selected the most diffused ones, i.e. the Global Minimum Variance and the Maximum Sharpe portfolio. Then, I compared the risk budgeting portfolios with the portfolios obtained with the mean-variance optimization. For the comparison, I analyzed a wide set of basic characteristics and performance measures. I analyzed also portfolio composition, asset risk contribution, turnover, value at risk for the last year and tracking errors[118].

In the first paragraph, I present the computation of the inputs of the Risk Budgeting Approach (variance-covariance matrix and risk budgets) and Markowitz model (variance-

[116] In Section 2, I showed that volatility risk measure is not the only measure that we can use. In a Gaussian world, also Value-at-Risk and Expected Shortfall are coherent risk measure. Further development of this thesis can include different risk measures.

[117] As I will show later, the portfolios have different inception dates, based on the number of Frontier Markets available.

[118] I performed Tracking Error analysis on the basis of a GDP-weighted portfolio of Developed Markets.

covariance matrix and expected returns). I will briefly present also the constrained imposed to the mean-variance optimizer. In the second paragraph, I analyze and compare the portfolios obtained through the Risk Budgeting Approach and the Markowitz model. Appendix D presents the equations of the various performance measures used in this Section.

5.1 The Inputs of Risk Budgeting Approach and Markowitz Model

In order to provide the optimal allocation, the risk budgeting and the mean-variance optimization algorithms require two inputs. For the Risk Budgeting Approach these inputs are the risk measure and the risk budgets. For the Markowitz model, the inputs are the variance-covariance matrix and the expected returns. As above-mentioned, in this thesis I focused on volatility risk measure. Thus, the risk budgeting and the mean-variance optimization algorithm share one input (i.e., the variance-covariance matrix), while the second input is different on the basis of the approach implemented (i.e., the risk budgets and the expected returns respectively).

5.1.1 The Estimation of the Variance-Covariance Matrix and the Expected Returns

There are several ways to estimate the asset returns and the variance-covariance matrix. The most common approach in financial market practice are:

- sample estimators of mean and variance, including rolling method;
- exponential smoothing method;
- financial economic models, especially equilibrium models (e.g., CAPM, APT);
- econometric models (e.g., VARMA+GARCH);
- financial mathematic models (e.g., option pricing based models, stochastic volatility models).

Academic literature tried to answer to the question on which methodology provide the best and the most profitable estimation of returns and variance-covariance matrix. However, there is a broad consensus only on the inefficiency of sample estimators[119]. Most of the methodologies listed above were proposed to overcome the limit of the sample estimators. In particular, they tried to reduce the estimation error and its impact on the mean-variance optimization, that seriously affects the resulting allocation. However, the most diffused methodologies for the estimation of the inputs are the rolling method, the exponential weighted moving average (EWMA) and the equilibrium models. In some cases, we can have the combination of differ-

[119] I have already presented in Section 2 the criticism of Michaud (1989) and the solutions proposed by this work and Black and Litterman (1991).

ent approaches (e.g., equilibrium returns and EWMA variance-covariance matrix). Following Bruder and Roncalli (2012), other diffused approaches are:

- regularization of the objective function through resampling techniques (Tütüncü and Koenig [2004]);
- regularization of the covariance matrix, such as factor analysis (Hyvärinen and Oja [2000]), shrinkage methods (Ledoit and Wolf [2003]) and random matrix theory (Laloux *et al*. [1999]);
- regularization of the program specification by introducing some constraints on weights[120].

Only in the recent years, financial mathematic and econometric model increased there diffusion in the estimation of expected returns and variance-covariance matrix[121].

The implementation of the best and the most profitable way to estimate the risk measure and the inputs of the Markowitz model is outside of the objectives of this thesis. For this reason, I decided to estimate the expected returns and the variance-covariance matrix through the most diffused and classical methodologies. In particular, I used a 60-month rolling estimation and a 12-month exponential weighted moving average. About the rolling methodology, the relevant element to choose is the size of the evaluation window. Generally, it depends on the sample size and on the number of the assets for which I have to estimate the variance-covariance matrix. Sample estimator produces inconsistent result when the number of assets is greater than the elements of the return time series used to estimate the expected returns and variance-covariance matrix[122]. For this reason, the size of the rolling window has to be at least equal to the number of assets for which I want to estimate the variance-covariance matrix. As presented in Section 3, there are thirty-five assets in my investment universe. Thus, I chose a rolling window of sixty months, that is quite diffused in financial market practice. The drawback of this choice is that I require an initialization period of five years for each asset, in order to estimate the expected returns and the variance-covariance matrix. In this way, I cannot invest in the asset as soon as it is available, but I need to wait for five years. Given the short length of the time series, this procedure strictly reduces the investment opportunities in Fron-

[120] This is the most diffused procedure in financial market practice to obtain satisfactory solution. Moreover, Jagannathan and Ma (2003) show that this procedure is equivalent to shrink the covariance matrix.
[121] Before, these models were diffused in financial market practice in different areas. In particular, financial mathematic models were diffused mainly in derivative pricing, while econometric models were diffused in value-at-risk computation.
[122] Ledoit and Wolf (2003) declared that when the number of stocks is larger than the number of historical returns per stock, the sample covariance matrix is always singular, even if the true covariance matrix is known to be non-singular.

tier Markets[123]. For this reason, I used another methodology to estimate the inputs: the 12-month exponential weighted moving average. The EWMA gives more weight to recent observations and produces more variable estimations[124]. The EWMA is characterized by two elements: the size of the initialization window and the factor lambda (i.e., the smoothing parameter). In this case I lost only one year of observations. The aim of this procedure is to increase the investment opportunities reducing the number of observations lost for the initialization of the estimation methodology. Finally, I chose a lambda equal to 0.85. For monthly returns, JP Morgan suggests a lambda equal to 0.97 and this is quite common in financial market practice. However, the weight assigned to the last observation would be too small. For this reason, I reduced the value of lambda.

Especially the rolling estimations present some limits. Mean-variance optimizer is not robust to inputs computed using the sample moments. For risk budgeting portfolios, this is not a serious concern. In fact, Bruder and Roncalli (2012) show that the Risk Budgeting Approach produces satisfactory allocation even using the sample estimator for the variance-covariance matrix. About this concern, the Markowitz model implemented in this thesis presents some constraints on weights (i.e., constrained mean-variance model). This is one of the solution mentioned above and frequently used by financial practitioners to obtain satisfactory asset allocation. In this way, the concern about sample estimators is partially solved. In any case, further development of this work could focus on estimation procedures able to reduce the estimation errors. Even if the allocation obtained in this thesis will be satisfactory, the reduction of the estimation errors will improve the allocation process in any case.

5.1.2 The Risk Budgets Computation

The second input required by the Risk Budgeting Approach is the vector of risk budgets. In particular, I focus on the determination of the risk budgets assigned to Frontier Markets. As mentioned in Section 4, I used the Country Risk Indicator to design the risk budgets on Frontier Market asset class, while the risk budgets on Developed and Emerging Market asset class are designed in a different way. The process for the computation of the risk budgets on Frontier Markets is divided in two steps:

- the determination of the total risk budget for the Frontier Market asset class;
- the repartition of the total risk budget in individual risk budgets for the single Frontier Market.

[123] A consistent number of Frontier Market indices were launched after 2005. Thus, they became investable only after 2010, reducing the investment period on these assets to 4 years more or less.
[124] EWMA estimations adapt to the shocks quickly than rolling estimations. However, they can produce large estimation errors if this shock is isolated and it does not represent a change in the trend.

As above-mentioned, I created four different portfolios with a maximum risk budget for the Frontier Market asset class of 5.00, 10.00, 20.00 and 40.00 per cent. However, the risk budget is variable over the investment period. The risk budget assigned to this asset class in a particular month depends on the international liquidity condition and on the average risk framework in Frontier Markets. Excluding the choice to enter in one particular Frontier Market, that depends on country-specific condition, these are the main determinants behind investors' choice to invest in this alternative asset class. International liquidity condition is probably the most important one. When the international liquidity framework is adverse (e.g., restrictive monetary policies by the major central banks), investors tend to exit from the most illiquid markets, moving towards more secure markets[125]. On the contrary, when international liquidity framework is favorable, investors are more prone to bear risky activities as Frontier Market financial instruments. There are two main reasons behind this reasoning. First of all, when interest rates are low, the high differential between risky asset classes returns and the cost of raising funds generates good investment opportunity. Thus, investors are more willing to bear more risk in order to achieve these good return differentials. Secondly, an adverse liquidity framework can complicate the exit from illiquid assets. This happens especially when a speculative bubble bursts or a crisis arises, with massive capital outflows that can force the introduction of capital controls. The second determinant is the average risk framework. In particular, investors tend to exit from Frontier Market asset class when a series of common shocks increases the country risks of a large portion of these countries[126]. In this case, the adverse economic and financial framework forces the investors to exit before the insurgence of a serious crisis. The proxies for these two determinants are:

- the International Liquidity Risk Indicator for the international liquidity framework;
- the average Country Risk Indicator for the country economic and risk framework.

The value of the Liquidity Risk Indicator considered is the one of the month for which we want to compute the Frontier Market total risk budget. The average Country Risk Indicator considered is the average between all the Frontier Market Country Risk Indicators in the month for which we want to compute the Frontier Market total risk budget. The two determinants contribute to determinate the total risk budget in equal way (i.e., 5.00, 10.00 and 20.00 per cent for each determinant), with the exception of the 5.00 per cent risk budgeting portfolio (3.00 and 2.00 per cent for international liquidity and risk framework respectively). The relevant threshold for the Liquidity Risk Indicator is the half of its maximum value (i.e., 7 points).

[125] In Section 4, I made the example of Latin America crises during the '80s. Restrictive monetary policy of the Federal Reserve led to massive capital outflows, exacerbating a precarious economic framework.

[126] The most common examples are global recession, global financial crisis, default wave, etc..

The relevant thresholds for the Country Risk Indicator are 50 and 55 points. An average rating below 50 points represents very adverse condition, while an average rating above 55 points represents normal condition[127]. Another way to proxy economic and risk framework in Frontier Markets is to compute the number of countries behind a certain risky threshold. However, this procedure is already used in the determination of the individual risk budgets. Table 6 presents the constraints for the determination of the total risk budget. The total risk budget is given by the sum of the risk budget portion determined using the Liquidity Risk Indicator and the risk budget portion determined using the average Country Risk Indicator.

Table 6 – Total risk budget computation

TOTAL RISK BUDGET COMPUTATION				
DETERMINANTS	**PORTFOLIO**			
	5%	**10%**	**20%**	**40%**
LIQUIDITY RISK INDICATOR RISK BUDGET				
LRI lower than 7	0.00	0.00	0.00	0.00
LRI higher than 7	3.00	5.00	10.00	20.00
AVERAGE COUNTRY RISK INDICATOR RISK BUDGET				
Average CRI lower than 50	0.00	0.00	0.00	0.00
Average CRI between 50 and 55	1.00	2.50	5.00	10.00
Average CRI lower higher than 55	2.00	5.00	10.00	20.00
TOTAL RISK BUDGET = LRI RB + ACRI RB				

After the computation of the total risk budget for the Frontier Market asset class, I distributed it across the various Frontier Markets included in the investment universe in each month. I used the individual Country Risk Indicator to determine which is the country risk exposure that I'm willing to bear. The process has two steps:

- division of the countries in five categories on the basis of their investment quality;
- repartition of the risk budget assigned to a given class in equal parts between the countries belonging to that class.

In order to divide the various countries in the five different groups, I used their individual Country Risk Indicator. The higher the Country Risk Indicator, and the lower the country risk is. Thus, the country investment quality increases as the value of the Country Risk Indicator increases. The relevant thresholds for the classification change every month, on the basis of

[127] These thresholds are not relevant for an individual country risk measure. In fact, they are relevant only for the average between all Frontier Markets Risk Indicator.

the values assumed by the indicators of the thirty-one countries in that particular month. The groups are:

- low investment quality: bottom 10.00 per cent of the country indicator values;
- medium-low investment quality: between the 10th and the 40th percentile of the country indicator distribution;
- medium investment quality: between the 40th and the 60th percentile of the country indicator distribution;
- medium-high investment quality: between the 60th and the 90th percentile of the country indicator distribution;
- high investment quality: top 10.00 per cent of the country indicator values.

After the division of the countries in the five groups, I determined the individual risk budget dividing the portion of the total risk budget assigned to a particular group in equal parts across the countries belonging to that group:

$$b_i = \frac{x_j b_{FM}}{n_j} \tag{5.1}$$

- b_i = risk budget of country i
- x_j = portion of total risk budget assigned to the group quality j
- n_j = number of Frontier Markets belonging to the group quality j
- b_{FM} = total risk budget assigned to Frontier Market asset class

The portion of total risk budget assigned to low, medium-low, medium, medium-high and high quality groups are 0.00, 20.00, 25.00, 30.00 and 25.00 per cent respectively. The number of countries belonging to each group varies in each month. Of course the individual risk budgets for the low quality Frontier Markets are 0.00 per cent. For these countries, the country risk is too high compared to the others. I decided to assign a portion of total risk budget to medium-low quality Frontier Markets. Even if they present a consistent country risk, I want to preserve the diversification. Moreover, I take into account the possibility of estimation error in country risk. If the country risk is not particularly high, I can accept a small bet on these markets. In any case, the individual risk budgets assigned to these Frontier Markets is relatively low compared to the others. Generally this is the most numerous group, with a variable number between 8 and 11 countries. In this way, the individual risk budgets are contained and the impact on the portfolio of negative shocks in one of these countries will be sustainable. Finally, the portion assigned to the top quality group is 25.00 per cent, the same of the medium quality group. This could be strange, since someone can expect a more aggressive bet on these markets. However, I still have to take into account potential error in the assessment of the country risk. Moreover, the countries belonging to that group vary between 3 and 5. Thus, their individual risk budgets are relatively high compared to the others. In order to preserve the diversification and to control the liquidity risk, I cannot assign too large risk budgets on a single Frontier Market, even if I perceived that its risk is low. These markets still remain illiquid and the impact of negative shocks related to liquidity risk can be disruptive for the portfolio performance. Finally, I determined the risk budgets through this methodology for all the Frontier Markets. However, the investment universe varies over time and not all the Frontier Markets are present in some months. For this reason, I extracted the individual risk budgets of the countries included in the investment universe in each month and I normalized them in order that they sum to the total risk budget for the month considered.

I designed the individual risk budgets on the core part (i.e., Developed and Emerging Markets) in a different way. For these indices I don't have a Country Risk Indicator[128]. One of the aim of this thesis is to evaluate the impact of Frontier Market asset class on portfolio performance. For this reason, the only relevant condition on the core part is that the risk budgets are determined in the same way for all the portfolios, in order to ease the comparison between them. The risk budget assigned to Emerging Markets is equal to 10.00 per cent and it is fixed for all the portfolios. The individual risk budgets assigned to the three Developed Market indices are GDP-weighted risk budgets, computed on the residual risk budget available for a particular portfolio. In this way, the risk budgets are different in absolute term between the different portfolios, but they maintain the same relative structure. Eq.(5.2) presents the computation:

$$b_k = (1 - b_{EM} - b_{FM})wGDP_k \tag{5.2}$$

- b_k = risk budget for Developed Market index k (*North America, Europe and Middle East, Pacific*)
- b_{EM} = risk budget of Emerging Market index, equal to 10.00 per cent
- b_{FM} = total risk budget assigned to Frontier Market asset class
- $wGDP_k$ = GDP-weight of geographical area k on the total GDP of Developed Markets

I downloaded the data on GDP for the countries belonging to the Developed Market class from the IMF WEO databases.

[128] In Section 4 I explained the impossibility of computing the country risk for these indices, given the fact that they are an aggregate of very heterogeneous countries.

5.1.3 The Constraints on the Markowitz Model

The introduction of constraints on Markowitz model is one of the most diffused solution adopted by the asset managers in order to obtain satisfactory asset allocation[129]. In particular, asset managers usually set constraints on weights. These constraints can be individual or group constraints. In addition to that, I'm interested in posing constraints on mean-variance optimization in order to make comparable the portfolios generated with the Risk Budgeting Approach and the Markowitz model[130]. For this reason, I set weight constraints on all the asset classes presented in my investment universe.

For the Frontier Market asset class I introduced a group constraint, in order to limit the maximum exposure on the Frontier Markets. The constraints are equal to the maximum risk budgets, in order to ease the comparison between the different portfolio. Thus, I created four portfolios where the weights assigned to the Frontier Market group are 5.00, 10.00, 20.00 and 40.00 per cent. In this way, I can compare the two different approaches for asset allocation. In the risk budgeting portfolios, I set the constraint on the risk exposure, while in the Markowitz model, I set the constraint on the weight exposure. The above-mentioned constraints are fixed for all the investment period. In fact, for the Markowitz model I don't use the Country Risk Indicator. Thus, I cannot determine on the basis of this indicator a variable monthly exposure. The individual weights of the Frontier Markets are the result of the constrained model and I didn't impose individual constraints.

For the Emerging Markets index, I set a weight constraint equal to 10.00 per cent for all the investment period. Again, I used the same constraint adopted in the Risk Budgeting Approach. The difference is that in one case I constrained the risk exposure and in the other case I constrained the weight exposure. For the Developed Market asset class, I set individual constraints on each index. These constraints are lower and upper bounds that try to force the optimization algorithm to assign a weight close to the GDP-weight. In this way, I tried to use a methodology similar to the one behind the design of the risk budgets of the Developed markets. These constraints vary between the portfolios (the residual portions are different depending on Frontier Markets weights) and across the time, reflecting the variation in the GDP-weights.

[129] I have already discussed about it in Subparagraph 5.1.1. Moreover, in Section 2 and Appendix A, I presented the constrained Markowitz model from a theoretical point of view.

[130] For example, I cannot compare the 5.00 per cent risk budgeting portfolio with a general mean-variance optimized portfolio. The portion invested in Frontier Markets can be very different and the comparison will lose its informational power.

5.2 Portfolio Allocation and Analysis

In this paragraph, I present the allocations generated by the risk budgeting and the mean-variance optimization algorithms. I also analyze the different portfolios, with the comparison between risk budgeting and mean-variance optimized portfolios. However, I have to discuss an important topic before the presentation of the analysis.

As above-mentioned, the investment period is not the same for the various Frontier Markets. MSCI launched stand-alone country indices for Frontier Markets starting from the 80s, but a consistent number of these indices were introduced after 2005[131]. Moreover, I have to take into account that the rolling and the EWMA methodology require an initialization window. This causes a loss of observations equal to five years and to one year respectively. At the beginning of the investment period (i.e., February 2005), the size of the investment universe was limited (i.e., five countries for rolling methodology and twelve countries for EWMA methodology). For this reason, I could not launch the portfolios with an high total risk budget on Frontier Markets, such as the 20.00 and the 40.00 per cent portfolios. In fact, the resulting individual exposures would have been too large, increasing dramatically the liquidity risk[132]. The EWMA method was introduced exactly to increase the investment opportunities, easing the introduction of the portfolios with greater exposure on Frontier Market asset class. Table 7 presents the different investment period for each portfolio. Table 8 presents the number of countries included in each sub-periods (i.e., the evolution of the investment universe). About the inception date, there are no differences between risk budgeting and mean-variance optimized portfolios.

Table 7 – Portfolios investment periods

PERIOD	ROLLING			
	5%	10%	20%	40%
Starting date	01/02/2005	01/02/2005	01/06/2007	01/06/2010
Ending date	31/01/2015	31/01/2015	31/01/2015	31/01/2015
PERIOD	EWMA			
	5%	10%	20%	40%
Starting date	01/02/2005	01/02/2005	01/02/2005	01/06/2006
Ending date	31/01/2015	31/01/2015	31/01/2015	31/01/2015

[131] See Table 1 in Section 3 for more details.
[132] For example, the 40.00 per cent risk budgeting portfolio with rolling method could present individual risk budgets higher than 8.00 per cent. This is clearly an excessive exposure given the relative illiquidity of these stock markets.

Table 8 – Investment sub-periods

ROLLING SUB-PERIODS	NUM FM *	EWMA SUB-PERIODS	NUM FM *
02/2005 - 05/2007	5	02/2005 - 05/2006	12
06/2007 - 05/2010	12	06/2006 - 11/2009	18
06/2010 - 11/2011	18	12/2009 - 11/2011	28
12/2011 - 11/2013	22	12/2011 - 01/2015	31
12/2013 - 01/2015	28		
* NUM FM = Number of investable Frontier Markets per sub-period			

For the introduction of the 20.00 per cent and the 40.00 portfolios, I require the presence in the investment universe of a number of countries close to 10 and to 20 respectively. In this way, I can maintain a certain degree of diversification, avoiding too large individual exposures on these illiquid markets. Of course, the portfolios obtained using the EWMA inputs were launched before, given the smaller initialization window. Moreover, they are divided in less sub-periods, since the Frontier Markets became available for the investment more quickly. Finally, when I used the rolling method in the computation of the inputs, I was not able to invest in three Frontier Markets (i.e., Bosnia and Herzegovina, Zimbabwe and Bangladesh).

5.2.1 Portfolio Analysis and Comparison: RB and MVO - Rolling Method

In this paragraph, I present the portfolios obtained using the Risk Budgeting Approach and the Constrained Markowitz model described above, with the expected returns and the variance-covariance matrix estimated using a 60-month rolling methodology. As mentioned in Section 1, all the computations, the optimization algorithms and the analysis are implemented using the Matlab software. I analyzed twelve different allocations:

- four risk budgeting (RB) portfolios with maximum total risk exposure on Frontier Markets equal to 5.00, 10.00, 20.00 and 40.00 per cent;
- four mean-variance optimized portfolios with Global Minimum Variance (GMV) allocation and total weight exposure on Frontier Markets equal to 5.00, 10.00, 20.00 and 40.00 per cent;
- four mean-variance optimized portfolios with Maximum Sharpe (MS) allocation and total weight exposure on Frontier Markets equal to 5.00, 10.00, 20.00 and 40.00 per cent;

I performed the evaluation of the different portfolios over their entire investment periods. For the 5.00 and the 10.00 per cent portfolios, the investment period is 120 months, while for the 20.00 and the 40.00 per cent portfolios, it decreases to 92 and 56 months respectively. When I compare the portfolios with different risk or weight exposure on Frontier Market asset class, I perform the analysis over the same investment period[133]. Allocations' basic characteristics are the starting point of the analysis. In Table 9, I present the basic characteristics of the twelve portfolios over the different investment periods.

Table 9 – Portfolios basic characteristics (Rolling)

Investment Period 02/2005 - 01/2015							
Strategy	**Mean**	**Vol**	**Min**	**Max**	**Skew**	**Kur**	**IQR**
Risk Budget 5	0.51	3.64	-11.54	11.57	-0.70	4.62	3.72
Risk Budget 10	0.52	3.56	-11.54	11.22	-0.75	4.70	3.46
Markowitz 5 GMV	0.38	3.66	-11.74	11.73	-0.60	4.54	3.85
Markowitz 5 MS	0.47	3.75	-11.99	11.96	-0.69	4.59	3.70
Markowitz 10 GMV	0.39	3.60	-11.39	11.56	-0.61	4.49	3.79
Markowitz 10 MS	0.47	3.66	-12.45	11.66	-0.75	4.84	3.50
Investment Period 06/2007 - 01/2015							
Strategy	**Mean**	**Vol**	**Min**	**Max**	**Skew**	**Kur**	**IQR**
Risk Budget 5	0.21	3.85	-11.54	11.57	-0.57	4.33	3.75
Risk Budget 10	0.19	3.74	-11.54	11.22	-0.61	4.45	3.56
Risk Budget 20	0.16	3.58	-11.54	10.59	-0.68	4.61	3.21
Markowitz 5 GMV	0.10	3.89	-11.74	11.73	-0.49	4.29	3.93
Markowitz 5 MS	0.18	4.00	-11.99	11.96	-0.56	4.24	3.56
Markowitz 10 GMV	0.14	3.82	-11.39	11.56	-0.52	4.25	3.82
Markowitz 10 MS	0.18	3.90	-12.45	11.66	-0.64	4.50	3.53
Markowitz 20 GMV	0.05	3.65	-11.67	10.94	-0.60	4.50	3.54
Markowitz 20 MS	0.15	3.84	-13.36	10.97	-0.75	4.75	3.44
Investment Period 06/2010 - 01/2015							
Strategy	**Mean**	**Vol**	**Min**	**Max**	**Skew**	**Kur**	**IQR**
Risk Budget 5	0.75	2.46	-7.61	6.06	-0.59	4.22	2.96
Risk Budget 10	0.69	2.37	-7.26	5.97	-0.57	4.17	2.76
Risk Budget 20	0.60	2.23	-6.73	5.83	-0.53	4.05	2.49
Risk Budget 40	0.46	2.07	-5.90	5.63	-0.38	3.76	2.57
Markowitz 5 GMV	0.60	2.54	-7.51	6.24	-0.34	3.83	2.79
Markowitz 5 MS	0.75	2.68	-7.83	5.98	-0.72	3.90	2.74
Markowitz 10 GMV	0.68	2.51	-7.07	6.37	-0.36	3.58	3.07
Markowitz 10 MS	0.77	2.52	-7.71	5.76	-0.71	4.11	2.79
Markowitz 20 GMV	0.56	2.39	-5.86	6.32	-0.10	3.03	2.81
Markowitz 20 MS	0.72	2.49	-7.41	5.61	-0.75	4.05	2.51
Markowitz 40 GMV	0.30	2.17	-3.83	6.29	0.36	2.92	2.81
Markowitz 40 MS	0.52	2.56	-7.23	5.39	-0.70	4.42	2.16

[133] For example, I computed the characteristics of the 5.00 per cent portfolios, when I compared them with the 20.00 per cent portfolios, over the same investment period of 92 months.

First of all, I want to recall that Risk Budgeting Approach should ensure a reduction in risk, while it does not ensure higher returns. Bruder and Roncalli (2012) showed that, in expectations, the risk of the RB portfolio is between the GMV and weight budgeting (WB) portfolio risks. However, in my case, the volatilities of the RB portfolios are always lower than the volatility of the GMV portfolios. Of course, they are lower than the MS portfolios too. This condition is verified for all the portfolios, independently on their exposure on Frontier Markets. Moreover, this is verified in every investment period. The result is definitely better than what expected in theory. About average monthly returns, it is quite surprising that RB portfolios generally provide higher returns than MVO portfolios, in particular the MS portfolios. This situation normalized during the last investment period considered, when I introduced the 40.00 per cent portfolios. In this case the 10.00, 20.00 and 40.00 per cent MS portfolios present higher average returns than the 10.00, 20.00 and 40.00 per cent RB portfolios (0.69 vs 0.77, 0.60 vs 0.72 and 0.46 vs 0.52 per cent respectively). However, this is associated to an higher volatility (2.37 vs 2.52, 2.23 vs 2.49 and 2.07 vs 2.56 per cent respectively). Comparing the different investment periods, we can see that the last period is generally characterized by higher average monthly returns and lower volatility. In fact, this period started in June 2010, excluding the 2008 Financial Crisis. Moreover, it is a period of expansive monetary policies that led to massive capital flows towards the Frontier Markets. Generally, this boosted the performance of some of these markets and reduced the volatility. For the same reason, the period with the lowest return and the highest volatility is the second one. This period started at the beginning of the 2008 Financial Crisis. The crisis characterized almost one fourth of this period, amplifying its impact. About skweness, kurtosis and interquartile range (IQR), there are no clear conclusions, except for a reduction of kurtosis and IQR in the last investment period. This result is consistent with the above discussion on the financial framework that characterized the last period. About a comparison between the different portfolios, there is a surprising result. The volatility reduces as the total risk exposure on Frontier Markets increases. Considering the investment period from February 2005 to January 2015, the 10.00 per cent RB portfolio presents lower volatility and higher monthly average returns than the 5.00 per cent RB portfolio (3.56 vs 3.64 and 0.52 vs 0.51 per cent respectively). In the second investment period, the 20.00 per cent RB portfolio has a volatility lower than the 5.00 and 10.00 per cent RB portfolio (3.58 vs 3.85 and 3.74 per cent respectively). Finally, in the last investment period we pass from the 2.46, 2.37 and the 2.23 per cent of the 5.00, 10.00 and 20.00 per cent RB portfolio to the 2.07 per cent of the 40.00 per cent RB portfolio. Probably, this result is driven by the diversification. Increasing the exposure to Frontier Markets, we also increase the potential benefits in risk reduction coming from international diversification.

The second important analysis regards the weights and the risk contributions of the different asset classes. Figures from 15 to 19 show the allocation across the different asset classes, aggregating individual weights and risk contributions of the Frontier Markets. Figures from 20 to 23 focus on Frontier Markets only, presenting individual weights and risk contributions.

Figure 16 – Weights and Risk Contributions in 5% Portfolios (Rolling)

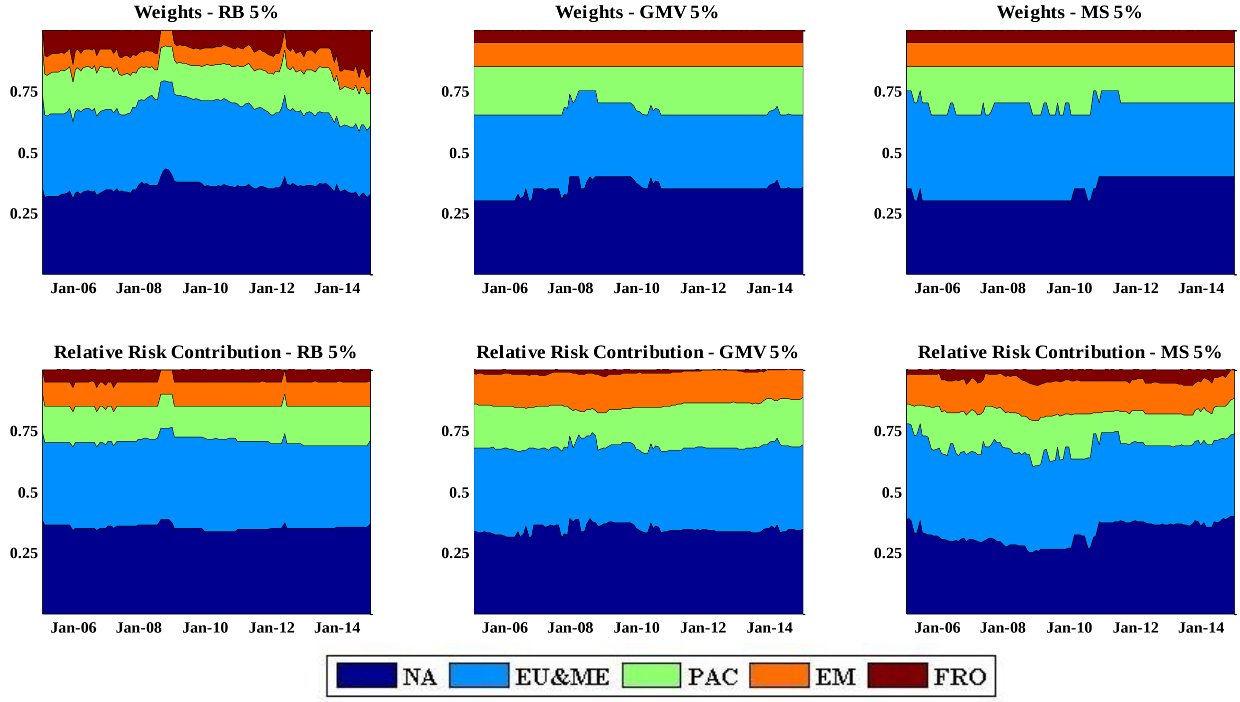


Figure 17 – Weights and Risk Contributions in 10% Portfolios (Rolling)

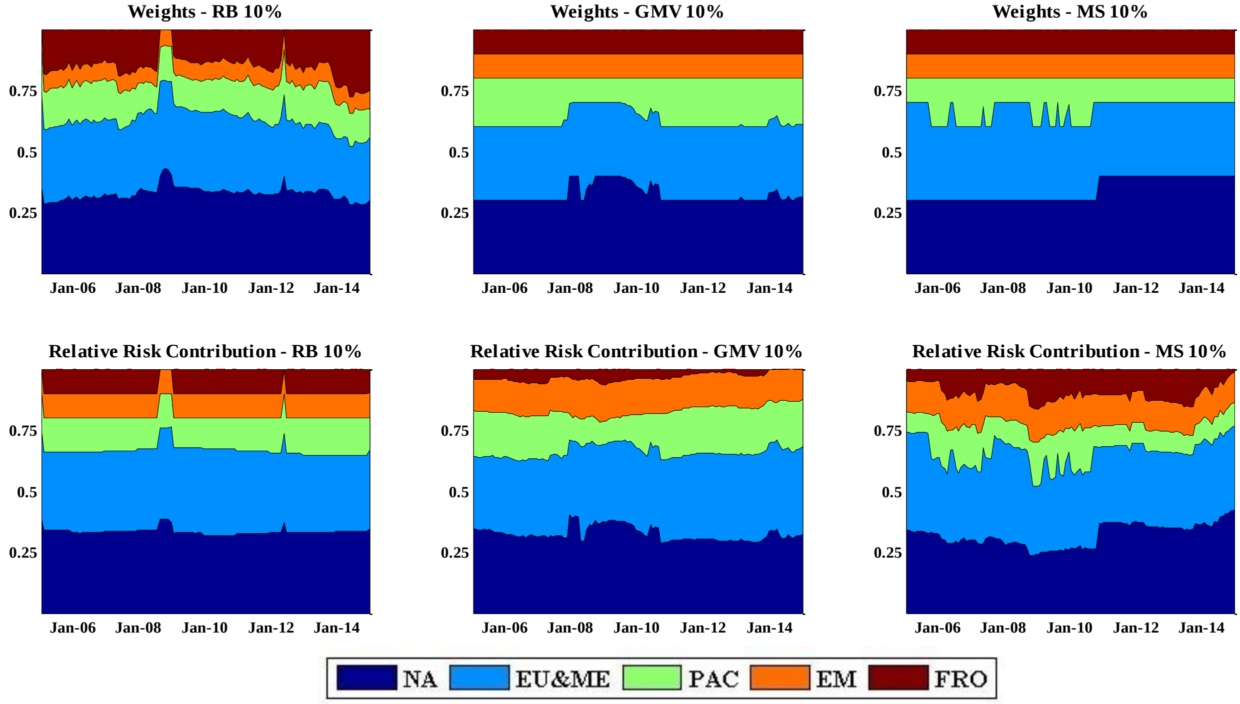


Figure 18 – Weights and Risk Contributions in 20% Portfolios (Rolling)

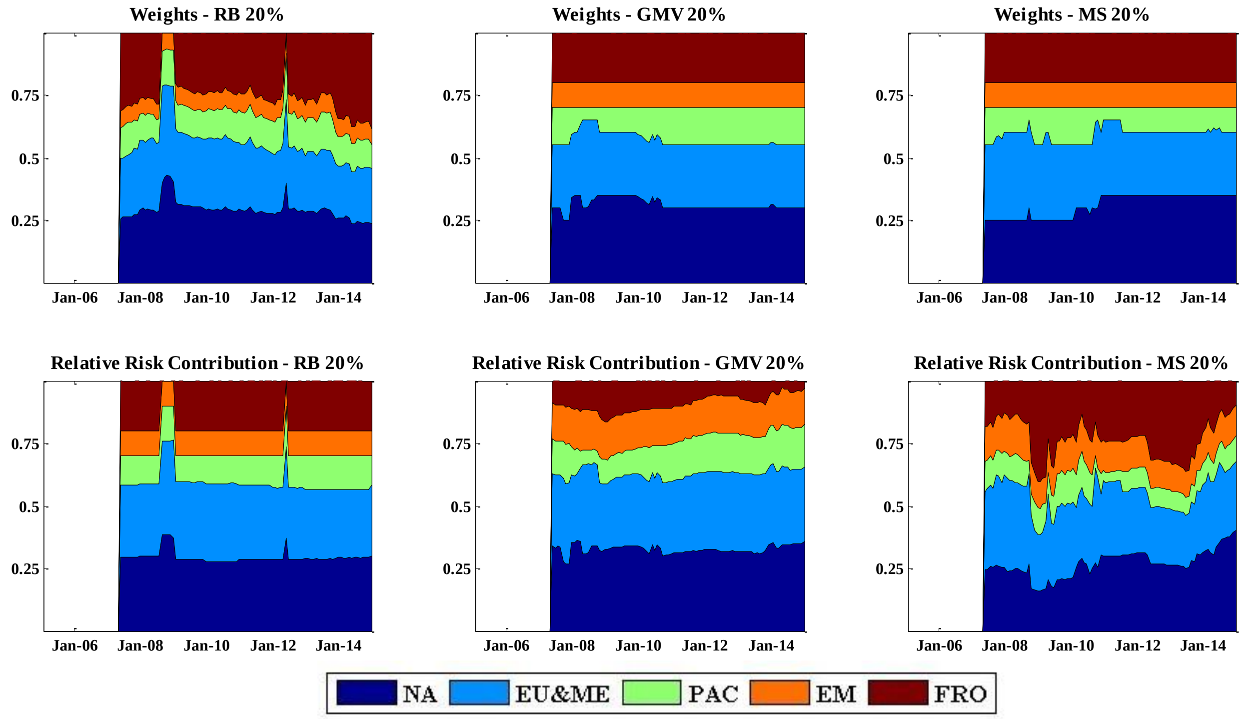
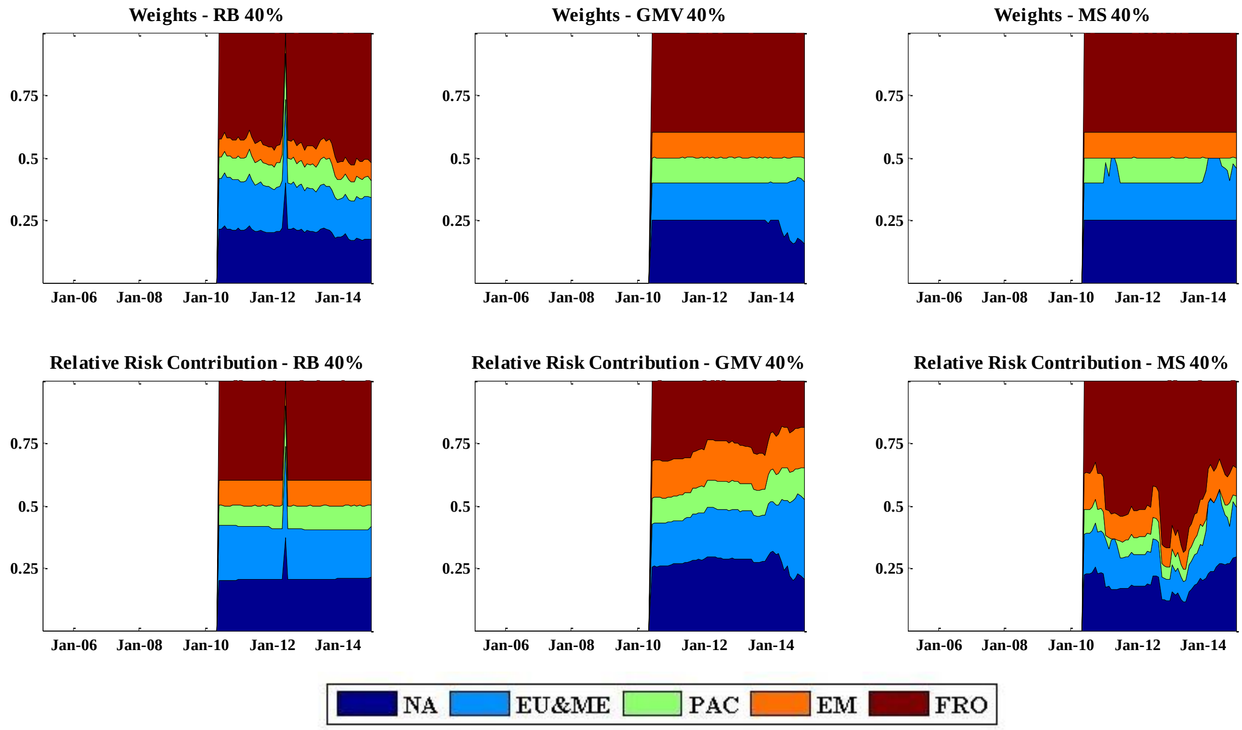
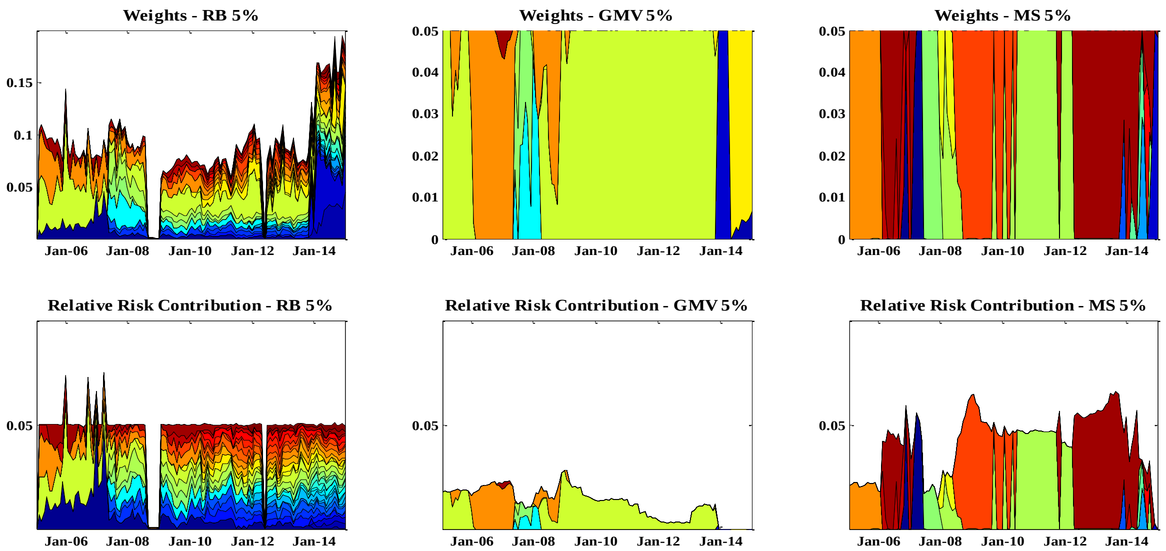


Figure 19 – Weights and Risk Contributions in 40% Portfolios (Rolling)



It is interesting the relation between the weights and the risk contributions for the Frontier Market asset class. The risk contributions for the GMV portfolios are always lower than the weights. On the contrary, for the MS portfolios the risk contributions are sensibly higher than the relative weights. RB portfolios have the ex-ante risk contributions fixed a priori. In this case, we can observe that to achieve the constraints on the risk budgets, the portfolio weight has to be larger than the risk exposure. Thus, except for the few cases in which the risk contribution on Frontier Market asset class is equal to 0.00 per cent, the weights of this asset class in RB portfolios are generally higher than the weights in GMV and MS portfolios. This is associated to higher risk contributions with respect to GMV portfolios, but lower risk contributions with respect to MS portfolios. MS portfolios present allocations that lead to higher contribution to the portfolio risk for the Frontier Market asset class, even if the weight exposures are lower than or equal to RB and GMV portfolios respectively. About the evolution over time, the risk contributions, given the weights, tend to increase during the crises and to decrease during booming periods. This is evident looking at 2008-2009 crisis period (i.e., subprime crisis) and at 2010-2014 booming period, with a break during 2012 in which risk contributions tended to increase again (i.e., European Sovereign Debt crisis). The result is largely expected, since volatility tend to increase and correlations tend to be stronger during the period of crisis[134]. For RB portfolios the point of view is different, but the conclusions are the same. In fact, in order to meet risk budget constraints, during booming period of low volatility and less strong correlations, the weights have to increase. This is clear observing the last four years, in which the positive international liquidity framework (e.g., expansive monetary policies) led to huge capital flows towards these markets, strongly reducing the volatility in Frontier Markets. Finally, it is interesting to note the effect of the determination of variable risk budgets. In particular, between 2008 and 2009 and in 2012, risk contributions and weights for RB portfolios are equal to 0.00 per cent. In this way, I avoided the most turbulent periods of the subprime and the sovereign debt crises.

Figure 20 – Frontier Markets weights and risk contributions in 5% Portfolios (Rolling)



[134] About the correlations, I discussed this aspect in Subparagraph 3.2.3.

Figure 21 – Frontier Markets weights and risk contributions in 10% Portfolios (Rolling)

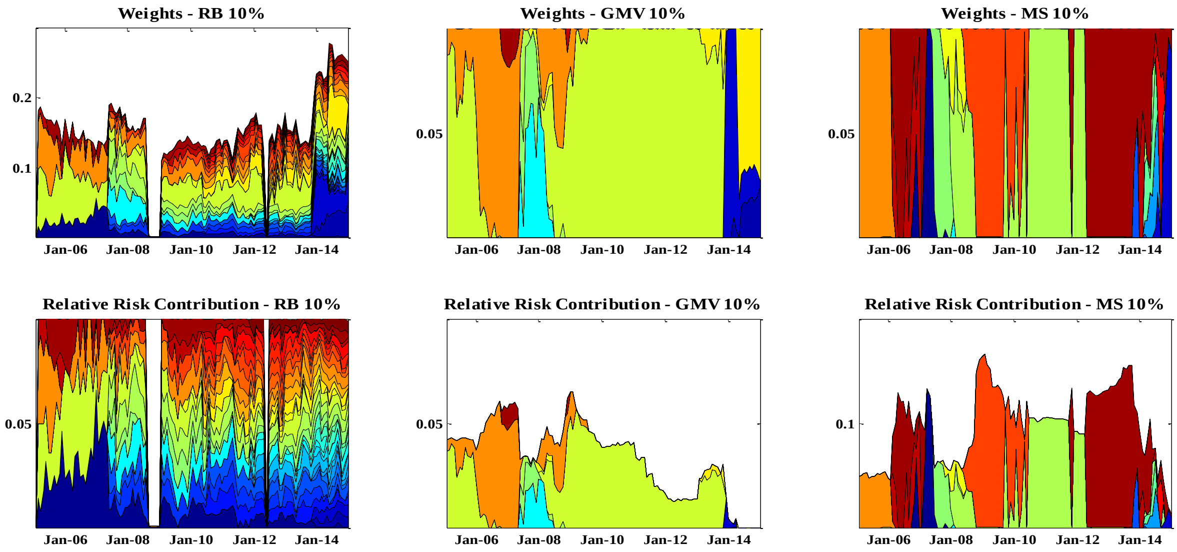


Figure 22 – Frontier Markets weights and risk contributions in 20% Portfolios (Rolling)

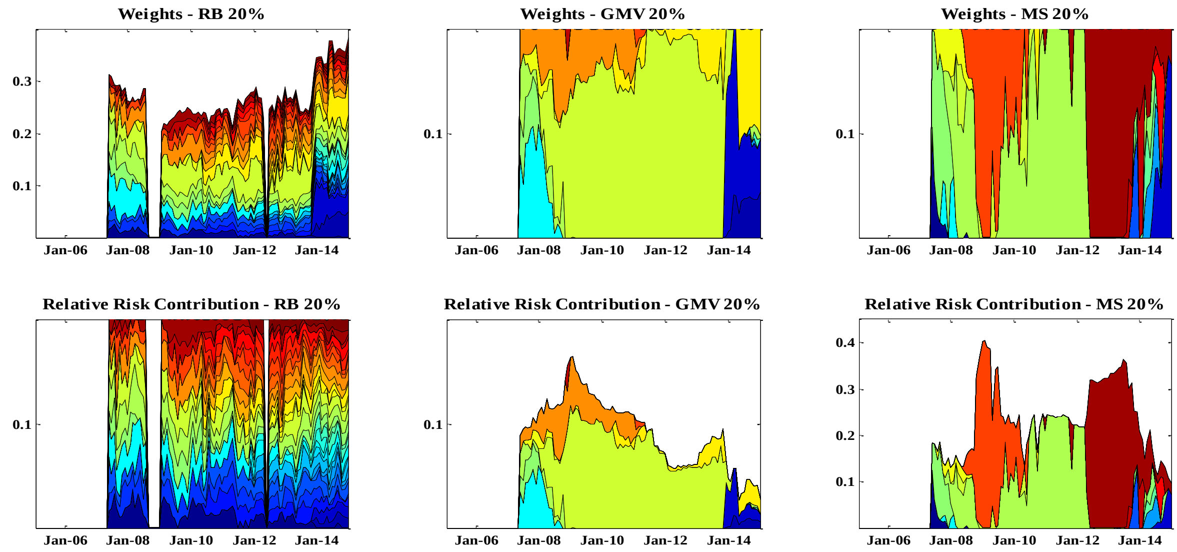
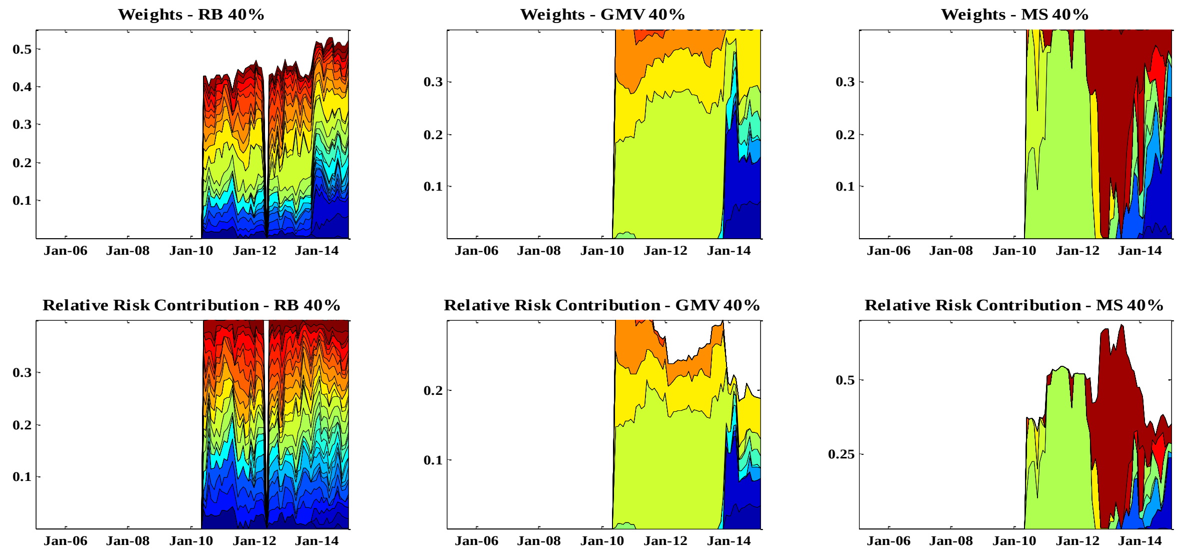


Figure 23 – Frontier Markets weights and risk contributions in 40% Portfolios (Rolling)



In Section 2, I have already mentioned that risk diversification is the core aspect of the Risk Budgeting Approach. In addition to that, I discussed about the poorly diversified outcomes produced by the mean-variance optimizer. The individual weights and risk contributions of the Frontier Market asset class show clearly these characteristics. As expected, RB portfolios always present an higher diversification, both in terms of risk and weight, than MVO portfolios. Of course, the number of Frontier Markets present in RB portfolios depends on the number of investable markets. In general, I have a coverage of the investable set between 75.00 and 90.00 per cent. On the contrary, MVO portfolios are poorly diversified, with a number of Frontier Markets included that are, usually, below five. Moreover, in a consistent number of months, I invested only in one asset. These conclusion are stable for every portfolio, regardless of the portfolio portion invested in Frontier Market asset class. Higher diversification is positive when dealing with illiquid and low correlated assets, as Frontier Markets. In fact, higher diversification increases the benefits from diversification in the reduction of portfolio risk. The low volatility presented by the RB portfolios can be explained by the high degree of diversification in these portfolios. Moreover, diversified exposures, especially in terms of risk, reduce the impact of country-specific negative shocks. Given the relative illiquidity of these markets, the impact of country-specific negative shocks are amplified and can lead to huge losses. A lower individual exposure helps to mitigate this important threat.

The third important portfolio characteristic that I want to analyze is portfolio turnover. Turnover represents the portion of the portfolio that changes (i.e., the securities bought and sold) over a certain period. It is expressed in percentage terms and it is a measure of portfolio rotation and activity. It is an important variable, since higher turnover could be associated to an higher impact of the transaction costs on portfolio performance. In this thesis I computed the monthly actual turnover. Actual turnover differs from the approximate one because it takes into account that portfolio weights change between the beginning and the end of the month, due to the returns realized on each asset within the month. Actual turnover is the turnover computed at the end of the month, and not at the beginning, considering the realized returns[135]. Table 10 presents the average monthly actual turnover for the twelve portfolios. Figures 24 and 25 present the evolution over time of the monthly actual turnover. The lighter lines represent the MVO portfolios, while the darker lines represent the RB portfolios[136].

[135] In Appendix D I present the relation between approximate turnover and actual turnover.

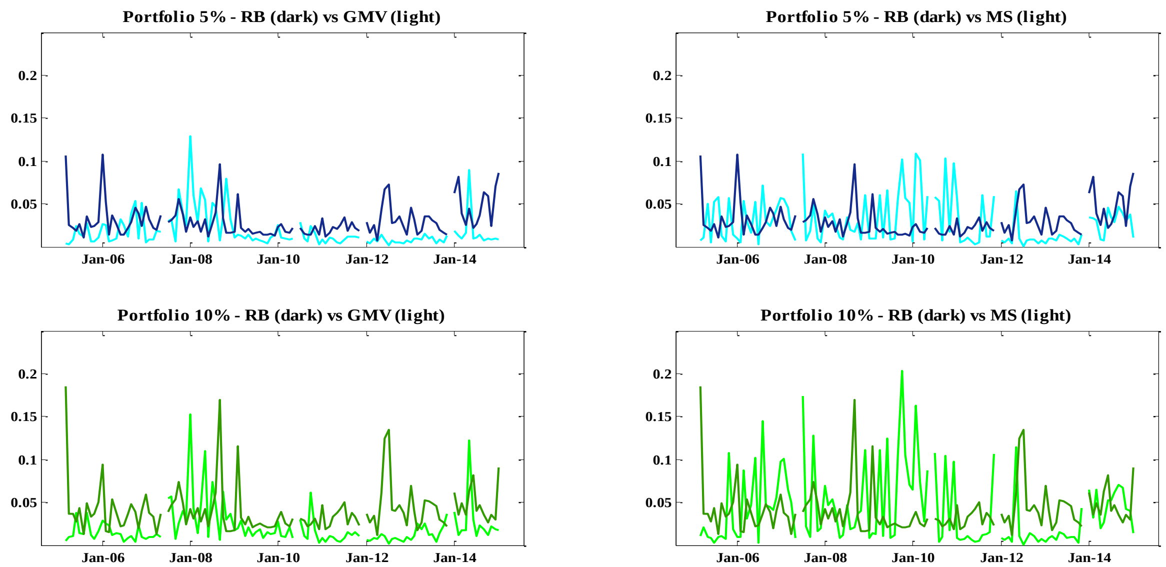
[136] Turnover is a discontinuous time-series. As already discussed, I divided the investment period in sub-periods, in order to add the new investable markets. This approach is necessary for the implementation of the algorithms in Matlab. When I introduced new assets I had to change the size of the matrices used as inputs. However, turnover is a measure of variation between two months. In this way I lost the first month and the change between the sub-periods, introducing discontinuity.

Table 10 – Average Monthly Actual Portfolio Turnover (Rolling)

ALLOCATIONS	PORTFOLIOS			
	5%	10%	20%	40%
Risk Budgeting	3.06	4.08	6.01	9.36
Global Minimum Variance	1.77	2.90	2.86	5.24
Maximum Sharpe	2.07	3.24	4.09	8.56

Average monthly actual turnover is quite limited in all the portfolios, with a minimum value of 1.77 per cent (5.00 per cent GMV) and a maximum value of 9.36 per cent (40.00 per cent RB). Portfolio turnover is mainly driven by the activity on the Frontier Market asset class. In fact, increasing the portion of portfolio assigned to this asset class, the turnover increases[137]. RB portfolios tend to present higher average actual turnover than MVO portfolios, with GMV portfolio exhibiting the lowest. A possible explanation is that RB portfolios present a contribution to turnover also from the Emerging Markets. In fact, in Risk Budgeting Approach I set a fixed constraint on the risk contribution of this asset class, but the weights are variable over time. Moreover, the monthly changes in Frontier Markets' risk budgets can contribute to slightly increases in the trading activity. However, we are observing relatively low turnovers and, consequently, small differences in absolute term. Thus the impact of transaction costs is very small[138].

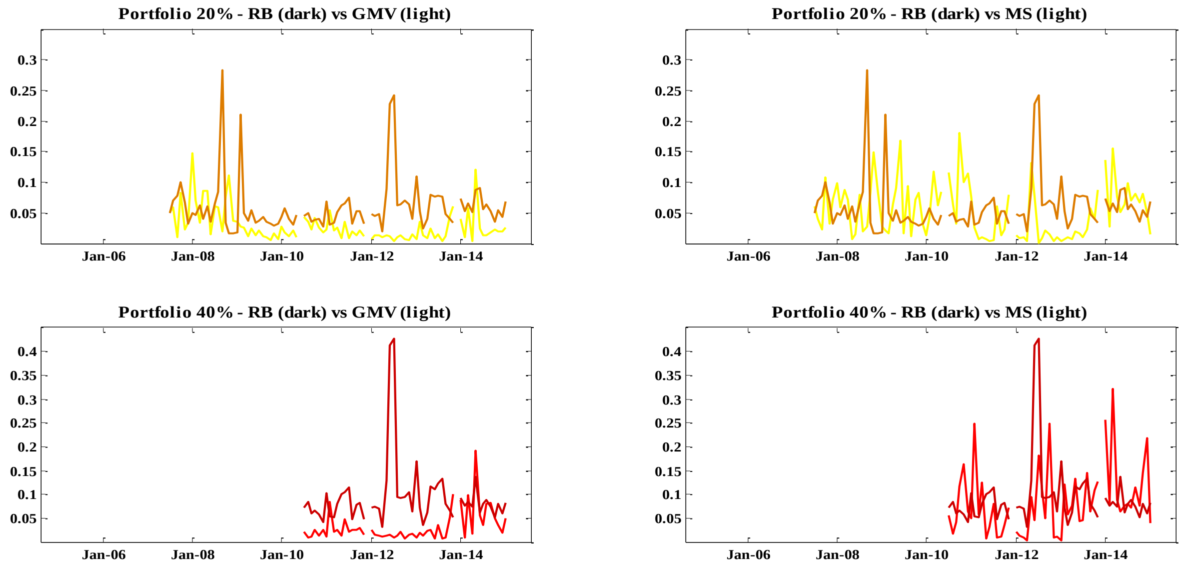
Figure 24 – Monthly Actual Turnover 5% and 10% Portfolios (Rolling)



[137] This is a byproduct of the determination of risk budget and weight constraints over the core portion. In Subparagraph 5.1.2 and Subparagraph 5.1.3, I presented a fix constraint on Emerging Markets and GDP-weighted constraints on Developed Markets. When imposed on the weights, a fix constraint determines a null contribution of Emerging Markets on portfolio turnover. The GDP-weighted constraints tend to produce stable weights at least within the year, with low contribution to turnover.

[138] Assuming a transaction cost between 0.10-0.15 per cent, the impact in most of the case will be close to 0.01 per cent.

Figure 25 – Monthly Actual Turnover 20% and 40% Portfolios (Rolling)



The spikes in the time series of monthly actual turnover represent changes in the inputs of risk budgeting and mean-variance optimization algorithms that justify a large change in the allocation. In normal conditions, RB portfolios tend to present a turnover higher than MVO portfolios. However, MVO portfolios present a consistently larger number of spikes than RB portfolios, with the exception of the comparison between 40.00 per cent RB and GMV portfolios. This is consistent with the theory presented in Section 2. Small changes in the inputs of the mean-variance optimizers (i.e., changes in risk-return trade-offs), especially changes in the expected returns, tend to produce large changes in the allocation. Thus, even in normal conditions, MVO portfolios present more spikes. On the contrary, RB portfolios tend to produce more stable portfolios. From the time-evolution of the monthly actual turnover for RB portfolios, we can see that the spikes are clustered around months of deep changes of the economic and financial conditions. This can be seen also from a preliminary analysis of the weights, where I showed that during the subprime and the sovereign debt crises, the RB portfolios had no exposure on the Frontier Market class. Thus the most important spikes for the RB portfolios are clustered across 2008-2009 (i.e., subprime crisis) and at the beginning of 2012 (i.e., sovereign debt crisis). These events dramatically changed the financial framework. The spikes represent the exit from these markets at the insurgence of the crises and the comeback when favorable financial conditions were restored.

The fourth step of portfolio analysis regards the analysis of the cumulated returns. Figures from 26 to 28 present the cumulated returns time series of RB and MVO portfolios. The first panel provides the comparison with the GMV portfolios, while the second one provides the comparison with the MS. Table 11 summarizes the results, with average annual returns between parenthesis.

Table 11 – Portfolios Cumulated Returns (Rolling)

Investment Period 02/2005 - 01/2015				
Strategy	**5%**	**10%**	**20%**	**40%**
Risk Budgeting	70.12 (5.46)	72.10 (5.58)	-	-
Global Minimum Variance	45.05 (3.79)	48.31 (4.02)	-	-
Maximum Sharpe	60.72 (4.86)	61.87 (4.93)	-	-
Investment Period 06/2007 - 01/2015				
Strategy	**5%**	**10%**	**20%**	**40%**
Risk Budgeting	13.10 (1.61)	11.70 (1.45)	9.45 (1.18)	-
Global Minimum Variance	2.60 (0.33)	6.31 (0.80)	-1.24 (-0.16)	-
Maximum Sharpe	9.85 (1.23)	10.16 (1.27)	7.63 (0.96)	-
Investment Period 06/2010 - 01/2015				
Strategy	**5%**	**10%**	**20%**	**40%**
Risk Budgeting	49.52 (8.81)	45.02 (8.12)	38.02 (7.00)	28.14 (5.34)
Global Minimum Variance	37.67 (6.94)	43.91 (7.94)	34.90 (6.49)	17.00 (3.35)
Maximum Sharpe	49.27 (8.78)	51.40 (9.10)	46.98 (8.42)	31.56 (5.93)

Considering the first and the second period, the RB portfolios provide the highest cumulated returns. The difference is important when we compare RB and GMV portfolios. In the last period, RB portfolios still exhibit cumulated returns higher than GMV portfolios, but they are lower than cumulated returns of their relative MS portfolios. Generally, for RB portfolios increasing the portion allocated to Frontier Markets, the cumulated returns decrease. The only exception is the first period, in which the 10.00 per cent RB portfolio presents a cumulated return over ten years of 72.10 per cent (5.58 per cent average annual return) compared to 70.12 per cent (5.46 per cent average annual return) of the 5.00 per cent RB portfolio. In order to compare the different periods, we have to look at the average annual returns. As expected, they are sensibly higher during the investment period started in June 2010. During this period, returns are above 6.00 per cent per year, with the only exception of the 40.00 per cent portfolios. In particular, for the 5.00 and 10.00 per cent RB and MS portfolios and for the 20.00 per cent MS portfolio, we have returns well above the 8.00 per cent per year. However, as above mentioned, we have to take into account that this period is characterized by the exit from the subprime crisis, with a very favorable monetary framework that boost the quotations. In any case, even considering the 10-year investment period, that includes both severe crises and booms, returns per annum are satisfactory, with the RB portfolios that present average annual returns well above 5.00 per cent (46 and 58 bps for the 5.00 and the 10.00 per cent RB portfolios respectively).

Figure 26 – Cumulated Returns from 02/2005 to 01/2015 (Rolling)

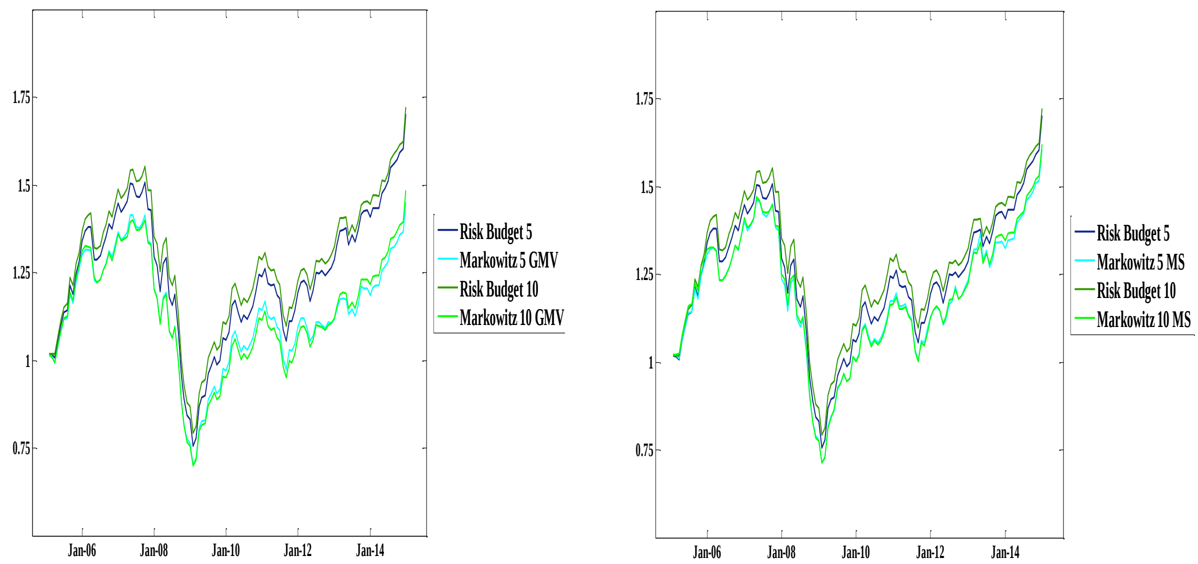


Figure 27 – Cumulated Returns from 06/2007 to 01/2015 (Rolling)

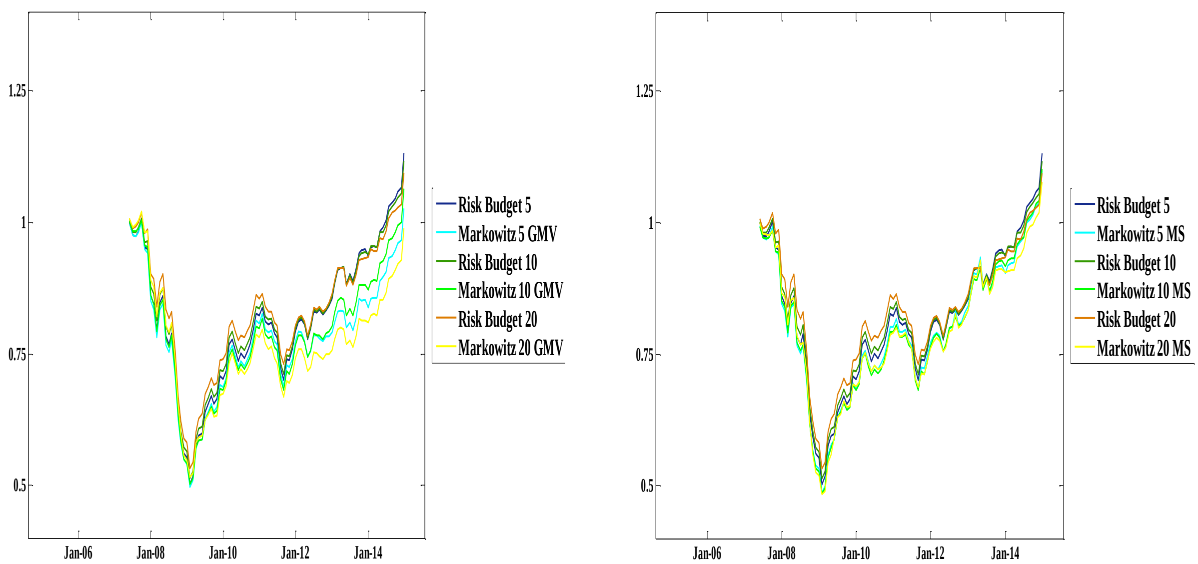
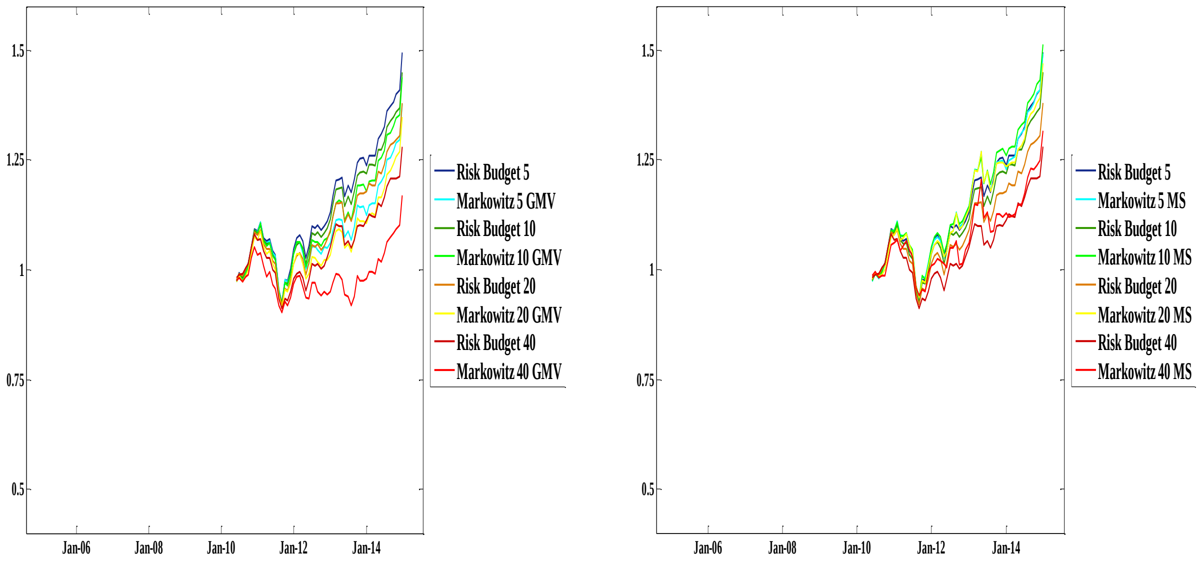


Figure 28 – Cumulated Returns from 06/2010 to 01/2015 (Rolling)



Another important step in portfolio analysis is the performance measure analysis. The rationale behind this type of analysis is the complexity of the selection of the best portfolios based on the basic characteristics. In general, the analysis of the basic characteristics can lead to different interpretations. Moreover, the analysis of average monthly returns do not provide a satisfactory answer, given the fact that we are not considering the risk borne for the achievement of these returns. It is better to analyze the portfolio performance relatively to the portfolio risk. In this way, we can draw some conclusions on which portfolio provides the best remuneration of the risk. In general, the best portfolio is the one that respects the risk constraint posed by the investor's risk aversion and that provides the highest reward of the risk borne. In this thesis, I used six performance measure. Two of them use a normal risk measure, while the other four use extreme risk measures. These measures are the Sharpe Ratio, the Sortino Ratio, the Return-VaR (Value at Risk) Ratio, the Return-ES (Expected Shortfall) Ratio, the Calmar Ratio and the Sterling Ratio[139]. These performance measures are very diffused in financial market practice. Table 12 presents the different performance measures for the twelve portfolios, with the same investment-period breakdown of Table 9.

[139] In Appendix D I provide the theoretical presentation of the risk measures used in this thesis.

Table 12 – Performance Measures (Rolling)

Investment Period 02/2005 - 01/2015						
Strategy	**Sh**	**So**	**Var**	**ES**	**Cal**	**St**
Risk Budget 5	0.14	0.17	0.07	0.06	0.04	0.05
Risk Budget 10	0.15	0.17	0.07	0.06	0.04	0.06
Markowitz 5 GMV	0.10	0.13	0.05	0.04	0.03	0.04
Markowitz 5 MS	0.12	0.14	0.06	0.05	0.04	0.05
Markowitz 10 GMV	0.11	0.13	0.05	0.04	0.03	0.04
Markowitz 10 MS	0.12	0.15	0.06	0.05	0.04	0.05
Investment Period 06/2007 - 01/2015						
Strategy	**Sh**	**So**	**Var**	**ES**	**Cal**	**St**
Risk Budget 5	0.05	0.07	0.02	0.02	0.02	0.02
Risk Budget 10	0.05	0.06	0.02	0.02	0.02	0.02
Risk Budget 20	0.05	0.05	0.02	0.02	0.01	0.02
Markowitz 5 GMV	0.03	0.03	0.01	0.01	0.01	0.01
Markowitz 5 MS	0.04	0.05	0.02	0.02	0.02	0.02
Markowitz 10 GMV	0.04	0.05	0.02	0.02	0.01	0.02
Markowitz 10 MS	0.04	0.05	0.02	0.02	0.01	0.02
Markowitz 20 GMV	0.01	0.02	0.01	0.01	0.00	0.01
Markowitz 20 MS	0.04	0.04	0.02	0.02	0.01	0.02
Investment Period 06/2010 - 01/2015						
Strategy	**Sh**	**So**	**Var**	**ES**	**Cal**	**St**
Risk Budget 5	0.30	0.41	0.22	0.15	0.10	0.18
Risk Budget 10	0.29	0.39	0.20	0.14	0.10	0.17
Risk Budget 20	0.27	0.36	0.19	0.13	0.09	0.15
Risk Budget 40	0.22	0.32	0.16	0.11	0.08	0.13
Markowitz 5 GMV	0.24	0.34	0.17	0.12	0.08	0.14
Markowitz 5 MS	0.28	0.36	0.21	0.13	0.10	0.16
Markowitz 10 GMV	0.27	0.39	0.19	0.14	0.10	0.16
Markowitz 10 MS	0.30	0.41	0.24	0.15	0.10	0.18
Markowitz 20 GMV	0.24	0.38	0.17	0.13	0.10	0.15
Markowitz 20 MS	0.28	0.36	0.21	0.13	0.10	0.16
Markowitz 40 GMV	0.14	0.28	0.11	0.09	0.08	0.10
Markowitz 40 MS	0.20	0.24	0.11	0.08	0.07	0.10

Sharpe and Sortino Ratio are performance measures that consider non-extreme risk measures (total risk and downside risk, both based on volatility). I prefer the Sortino Ratio, since it focuses on downside risk. In fact, investors are generally worried about an increase in volatility when the returns on their investment are negative. Looking at the evolution of these measures over the different investment period, we can draw the same conclusions derived from the analysis of the basic characteristics. In the last period, performance measures significantly increase for all the portfolios, while the second period is characterized by the worst level of the performance measures. Especially in the first two periods, RB portfolios present higher Sharpe and Sortino Ratio than their relative MVO portfolios. This result is very important, since the RB portfolios are able to achieve higher Sharpe Ratios than the MS portfolios, which is not expected in theory. Over the last investment period, the situation tends to normalize. Looking at the Sharpe Ratio, RB portfolios still present higher ratios than MVO portfolios. The only exceptions are 10.00 and 20.00 per cent RB portfolios when they are compared with their peer MS portfolios (0.29 and 0.27 vs 0.30 and 0.28 respectively). The analysis of Sortino Ratio produces the same conclusions. The RB portfolios that present the highest difference in performance measures with respect to their relative MVO peers are the 5.00 and the 40.00 per cent RB portfolios. In general, with the exception of the first period, increasing the portion allocated to Frontier Market asset class, we can notice a reduction in the performance measures. Especially in the last period, we can see that the highest performance relatively to the risk is the one of the 5.00 per cent RB portfolio, with a Sharpe and a Sortino Ratio equal to 0.30 and 0.41 respectively. The only portfolio able to share similar results is the 10.00 per cent MS portfolio. The other four performance measures (i.e., Return-VaR, Return-ES, Calmar and Sterling Ratio) focus on extreme-risk measures. The analysis of these measures leads to the same conclusions derived for the normal performance measure. The only interesting thing to note is that the distance between RB portfolios and MVO portfolios tend to be tighter when we consider performance measures based on extreme risk. Thus, RB and MVO portfolios tend to reward extreme risk equally, while RB portfolios tend to over-perform MVO portfolios when we consider non-extreme risk.

The last important step is the Tracking Error (TE) Analysis. The aim of this analysis is to compare a portfolio against its benchmark. TE are the deviations of portfolio returns from the benchmark returns. The other relevant quantity is the TE Volatility (TEV), that represents the volatility of the TE. If we compute the TEV only on downside deviations, we obtain the Semi-TEV. The ratio between TE and TEV (or Semi-TEV) gives an important index, the Information Ratio (or Semi-Information Ratio)[140]. This index is frequently used by fund managers in order to evaluate their ability in over-performing the benchmark. Sometimes is also used to design performance bonuses and management fees. Table 13 presents the TE analysis, with TE, TEV, Semi-TEV, IR and Semi-IR. The benchmark used is a GDP-weighted portfolio of Developed Markets only. This portfolio presents an average monthly return and volatility equal to 0.47 and 3.69 per cent respectively, over the 10- year investment period. Average monthly returns and volatilities move to 0.23 and 3.96 and to 0.81 and 2.63 per cent during the second and the last investment period respectively. Except for the first period, returns are

[140] See Appendix D for the mathematical formulation.

higher on average than the returns of the RB portfolios. About the volatility, the GDP-weighted Developed Market portfolio present always an higher volatility than RB portfolios.

Table 13 – Tracking Error Analysis (Rolling)

Investment Period 02/2005 - 01/2015					
Strategy	**TE**	**TEV**	**Semi-TEV**	**IR**	**Semi-IR**
Risk Budget 5	0.04	0.49	0.22	0.09	0.20
Risk Budget 10	0.05	0.68	0.33	0.07	0.15
Markowitz 5 GMV	-0.09	0.59	0.41	-0.15	-0.22
Markowitz 5 MS	0.00	0.76	0.54	0.00	0.00
Markowitz 10 GMV	-0.07	0.63	0.36	-0.11	-0.20
Markowitz 10 MS	0.00	0.77	0.46	0.00	0.01
Investment Period 06/2007 - 01/2015					
Strategy	**TE**	**TEV**	**Semi-TEV**	**IR**	**Semi-IR**
Risk Budget 5	-0.02	0.41	0.22	-0.06	-0.10
Risk Budget 10	-0.04	0.60	0.33	-0.07	-0.12
Risk Budget 20	-0.07	0.92	0.51	-0.07	-0.13
Markowitz 5 GMV	-0.13	0.61	0.43	-0.21	-0.30
Markowitz 5 MS	-0.05	0.83	0.57	-0.06	-0.08
Markowitz 10 GMV	-0.09	0.62	0.35	-0.15	-0.26
Markowitz 10 MS	-0.05	0.79	0.49	-0.06	-0.10
Markowitz 20 GMV	-0.18	1.02	0.56	-0.17	-0.32
Markowitz 20 MS	-0.08	1.37	0.75	-0.06	-0.10
Investment Period 06/2010 - 01/2015					
Strategy	**TE**	**TEV**	**Semi-TEV**	**IR**	**Semi-IR**
Risk Budget 5	-0.13	0.34	0.21	-0.38	-0.62
Risk Budget 10	-0.19	0.51	0.33	-0.36	-0.57
Risk Budget 20	-0.28	0.74	0.48	-0.37	-0.59
Risk Budget 40	-0.42	1.15	0.75	-0.36	-0.55
Markowitz 5 GMV	-0.28	0.63	0.46	-0.43	-0.60
Markowitz 5 MS	-0.13	0.92	0.63	-0.14	-0.20
Markowitz 10 GMV	-0.20	0.53	0.34	-0.37	-0.58
Markowitz 10 MS	-0.11	0.63	0.39	-0.17	-0.27
Markowitz 20 GMV	-0.32	0.83	0.52	-0.38	-0.61
Markowitz 20 MS	-0.16	1.12	0.67	-0.14	-0.24
Markowitz 40 GMV	-0.58	1.41	0.88	-0.41	-0.66
Markowitz 40 MS	-0.36	1.84	1.30	-0.19	-0.27

Except for the first period, TE are always negative, both for the RB and MVO portfolios. Consequently, also the Information and the Semi-Information Ratio are negative. Only the 5.00 and the 10.00 per cent RB portfolios evaluated over their 10-year investment period are able to produce acceptable positive Information and Semi-Information Ratio. Thus, the introduction of Frontier Markets in internationally diversified portfolios, with the methodologies proposed in this thesis, is not able to create portfolios that outperform a portfolio of Developed Markets only. It is important to notice that Tracking Error Analysis focuses on the excess return of a portfolio with respect to its benchmark. Thus, the outperformance considered is on the side of the returns. However, the aim of Risk Budgeting Approach is not to construct portfolios that guarantee an excess return with respect to other approaches. The aim is to diversify and reduce the risk. From this point of view, the RB portfolios created in this thesis are successful. I was not able to outperform a portfolio of Developed Markets in terms of return, that present stronger growth over the last four years, especially considering the US stock market, than Frontier Markets[141]. However, introducing Frontier Markets in internationally diversified portfolios, I was able through the Risk Budgeting Approach to construct portfolios with lower risks than a GDP-weighted Developed Market portfolio. Given the high risk and the relative illiquidity of these markets, the result can be considered satisfactory. With the RB portfolios I was able to amplify the diversification benefits coming from the investment in the Frontier Market asset class. Moreover, in two cases I was able to produce outperformance with respect to the benchmark, something that neither the MS portfolios were able to achieve.

5.2.2 Portfolio Analysis and Comparison: RB and MVO - EWMA Method

In this paragraph, I present the portfolios obtained using the Risk Budgeting Approach and the Constrained Markowitz model described in Paragraph 5.1, with the expected returns and the variance-covariance matrix estimated using a 12-month exponential weighted moving average. I analyzed the same twelve different allocations of the previous paragraph, i.e. four RB portfolios, four GMV portfolios and four MS portfolios with an increasing risk or weight exposure on Frontier Markets equal to 5.00, 10.00, 20.00 and 40.00 per cent. I performed the analysis of the different portfolios over their entire investment periods. As above-mentioned, with EWMA methodology, I increased the number of investable markets in each period, thanks to the shorter initialization window. In this way, I was able to launch the 20.00 per cent portfolios since the beginning and the 40.00 per cent portfolios after sixteen months. Thus, the investment period is equal to 120 months for the 5.00, the 10.00 and the 20.00 per cent portfolios. For the 40.00 per cent portfolios, the investment period is equal to 104 months. As I did in the analysis of the portfolios with the inputs estimated through the rolling method, when I compare the portfolios with different inception dates, I perform the analysis also over the same investment period . The analysis is structured in the same way as the analysis performed in the previous subparagraph. In particular, I used the same tools and steps. In the analysis, I will briefly compare the results of the portfolios generated with the two different methods for the computations of the inputs.

[141] This can be clearly seen in the price series graphs in Appendix B.

As in the previous subparagraph, allocations' basic characteristics are the starting point of the analysis. In Table 14, I present the basic characteristics of the twelve portfolios over the different investment periods.

Table 14 - Portfolios basic characteristics (EWMA)

Investment Period 02/2005 - 01/2015							
Strategy	**Mean**	**Vol**	**Min**	**Max**	**Skew**	**Kur**	**IQR**
Risk Budget 5	0.41	3.61	-11.61	12.16	-0.71	5.10	3.60
Risk Budget 10	0.45	3.56	-11.61	12.25	-0.77	5.36	3.57
Risk Budget 20	0.41	3.45	-11.61	11.61	-0.79	5.46	3.35
Markowitz 5 GMV	0.47	3.68	-11.53	11.77	-0.73	4.86	3.58
Markowitz 5 MS	0.57	3.68	-11.56	12.57	-0.58	4.66	3.72
Markowitz 10 GMV	0.44	3.62	-11.52	11.64	-0.83	5.28	3.32
Markowitz 10 MS	0.63	3.69	-11.57	13.25	-0.49	4.68	3.69
Markowitz 20 GMV	0.41	3.62	-14.92	11.17	-1.26	7.12	2.66
Markowitz 20 MS	0.74	3.73	-11.37	14.51	-0.34	4.86	3.82
Investment Period 06/2006 - 01/2015							
Strategy	**Mean**	**Vol**	**Min**	**Max**	**Skew**	**Kur**	**IQR**
Risk Budget 5	0.25	3.67	-11.61	12.16	-0.72	5.20	3.64
Risk Budget 10	0.30	3.62	-11.61	12.25	-0.78	5.46	3.34
Risk Budget 20	0.23	3.48	-11.61	11.61	-0.81	5.61	3.35
Risk Budget 40	0.22	3.36	-11.61	11.65	-0.77	5.86	3.45
Markowitz 5 GMV	0.34	3.73	-11.53	11.77	-0.74	4.95	3.54
Markowitz 5 MS	0.44	3.73	-11.56	12.57	-0.55	4.76	3.61
Markowitz 10 GMV	0.30	3.66	-11.52	11.64	-0.87	5.40	3.07
Markowitz 10 MS	0.51	3.75	-11.57	13.25	-0.45	4.77	3.55
Markowitz 20 GMV	0.24	3.68	-14.92	11.17	-1.33	7.24	2.55
Markowitz 20 MS	0.64	3.77	-11.37	14.51	-0.28	5.00	3.61
Markowitz 40 GMV	0.23	3.69	-15.61	12.96	-1.33	8.83	2.88
Markowitz 40 MS	0.76	3.96	-12.67	17.04	-0.02	6.10	3.74

As expected, the volatility of RB portfolios are lower than the volatility of MVO portfolios. Again, the surprising result is that RB portfolios present volatility lower than both the GMV and the MS portfolios. This condition is verified for all the different portfolios, independently on their exposure on Frontier Markets. Moreover it is verified for the whole set of investment periods. For RB portfolios, the volatility reduces as the total risk exposure on Frontier Markets increases. Considering the investment period from February 2005 to January 2015, the 20.00 per cent RB portfolio presents lower volatility than the 5.00 and 10.00 per cent RB portfolios (3.45 vs 3.61 and 3.56 per cent respectively). In the second investment period, the 40.00 per cent RB portfolio has a volatility lower than the other RB portfolios (3.36 vs 3.67, 3.62 and 3.48 per cent). Again, a possible interpretation is associated to the benefits in the reduction of risk coming from diversification. Increasing the exposure to Frontier Markets, we increase also the potential diversification benefits. About average monthly returns, the results are normal from a theoretical point of view and the opposite with respect to the one obtained with rolling mean method. RB portfolios always provide returns lower than or equal to MVO portfolios. The only exception is the 10.00 per RB portfolio compared to its peer GMV portfolio (0.45 vs 0.44 per cent respectively). This conclusion is valid for both the investment periods. Comparing the two periods, we can notice that the various portfolios tend to present a slightly increase in the volatilities (generally between 3 and 10 bps) during the second period. This is related to a consistent reduction in the average monthly returns, that in some cases are the half of the returns in the first period. About skweness, kurtosis and interquartile range (IQR), there are no clear conclusions. It is difficult to compare the portfolios obtained using the rolling method and these portfolios, since the investment periods are different. The portfolios that share the same investment period are the 5.00 and the 10.00 per cent portfolios. However, the portfolios obtained using the EWMA method for the computation of the inputs tend to present a similar risk profile than the portfolios analyzed in the previous paragraph. Moreover, they tend to have lower average returns and higher kurtosis. For the 5.00 and 10.00 per cent EWMA RB portfolios , the average monthly returns are 10 and 7 bps below the average monthly returns of their Rolling peers.

The second step is the analysis of the weights and the risk contributions. Figures from 29 to 32 show the allocation across the different asset classes, aggregating the individual weights and the individual risk contributions of the Frontier Markets. Figures from 33 to 36 focus on Frontier Markets only, presenting individual weights and risk contributions.

Figure 29 – Weights and Risk Contributions in 5% Portfolios (EWMA)

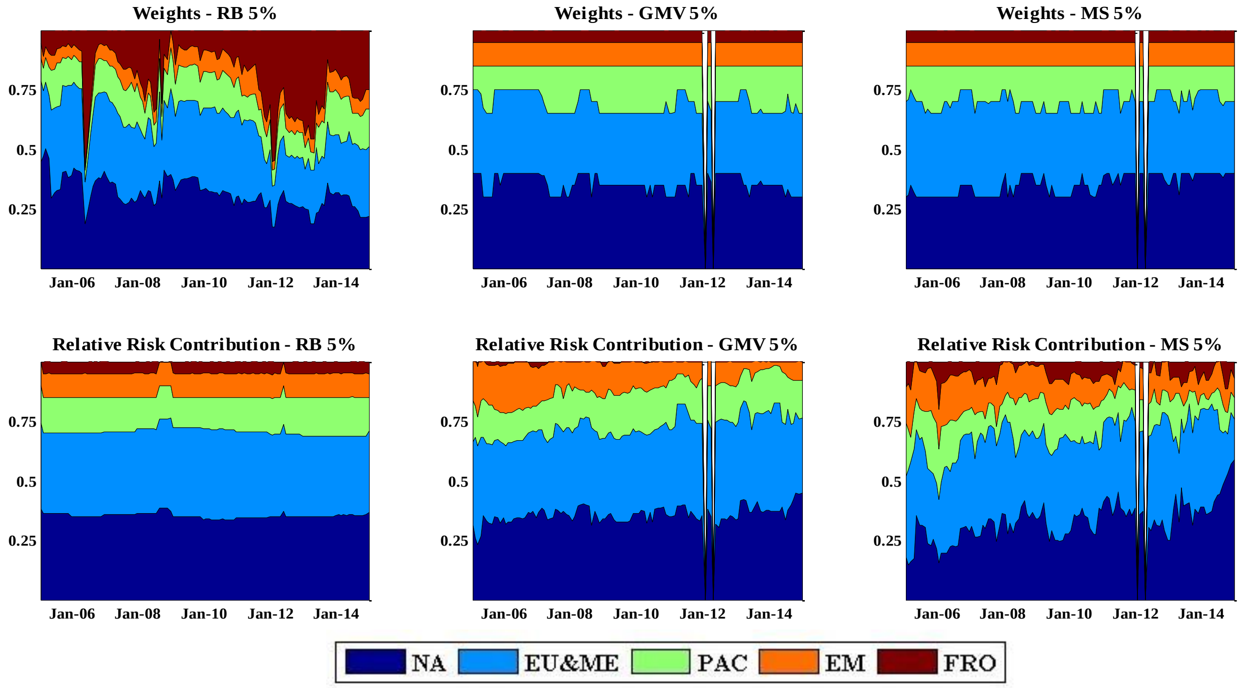


Figure 30 – Weights and Risk Contributions in 10% Portfolios (EWMA)

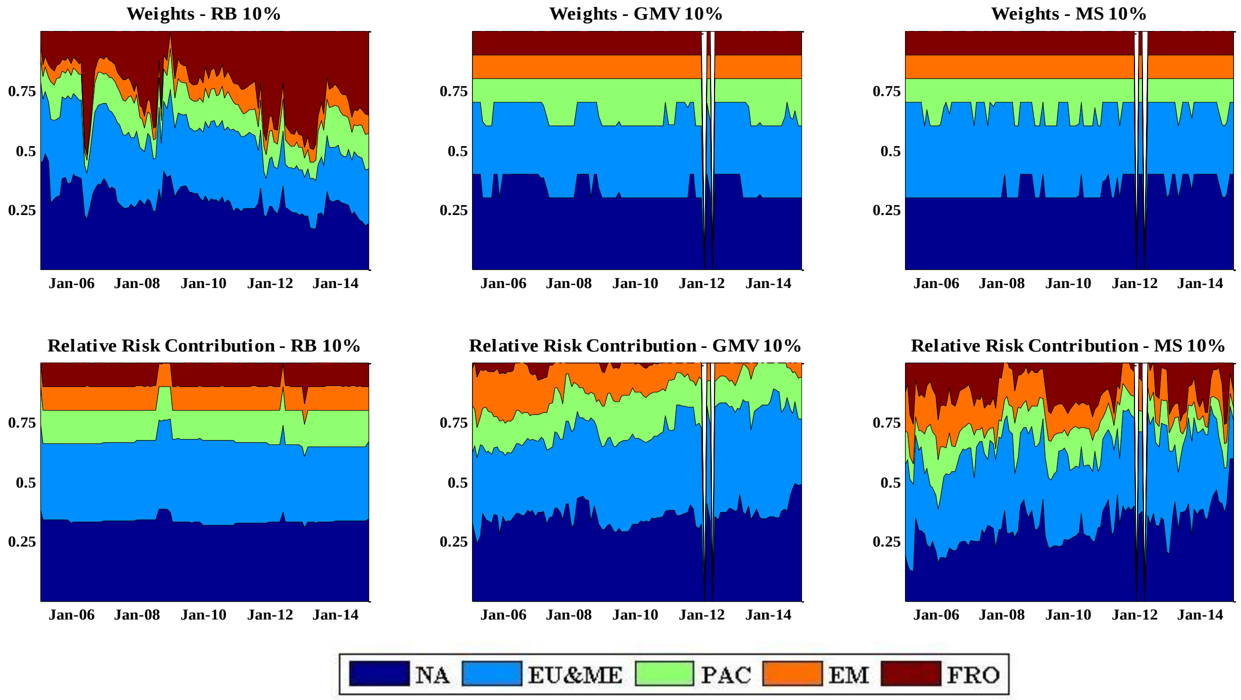


Figure 31 – Weights and Risk Contributions in 20% Portfolios (EWMA)

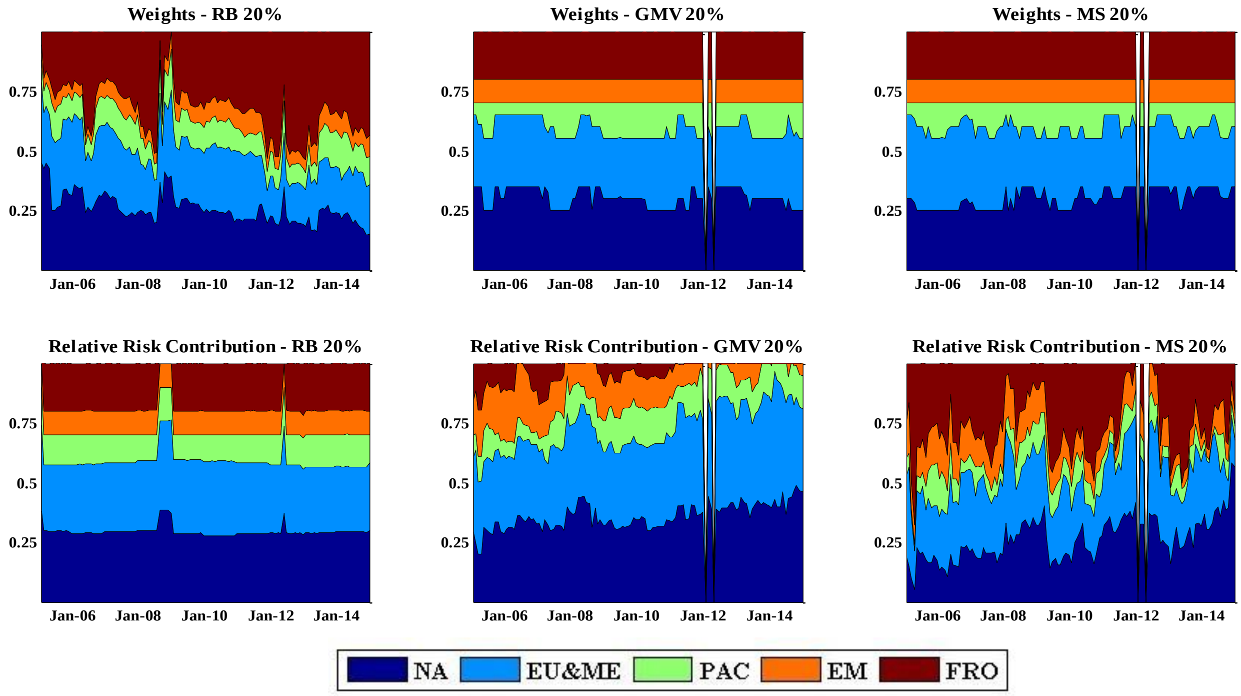
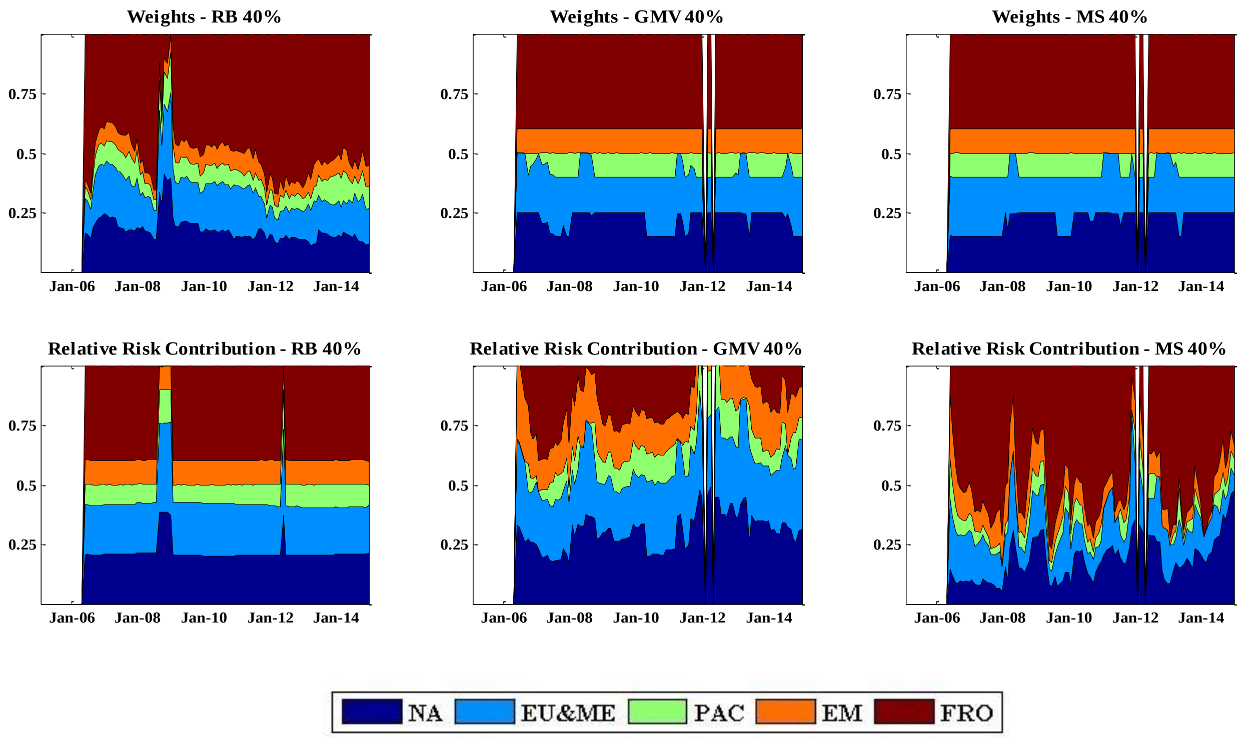


Figure 32 – Weights and Risk Contributions in 40% Portfolios (EWMA)



The first important thing to notice is the unusual behavior of the MVO portfolios during the 2012, when their weights went suddenly to 0.00. This problem comes from the optimization algorithm, due to an error in the estimation of the variance-covariance matrix. However, it characterized only two months, leaving the conclusions unchanged. As expected, the weights of the RB portfolios and the risk contributions of MVO portfolios are more unstable than their Rolling peers. The reason is that EWMA method, given the heavier weights assigned to the most recent observation, produces much more unstable (variable) estimations of the expected returns and of the variance-covariance matrix. This instability tend to produce unstable allocations and risk contributions. About the relation between weights and risk contributions for the Frontier Market asset class, the conclusions are similar to the ones presented in the previous subparagraph. The risk contributions for the GMV portfolios are always lower than the weights. In some special cases regarded the 5.00 and 10.00 per cent portfolios, the weights arrived to be 5 times larger than the risk contribution. On the contrary, for the MS portfolios the risk contributions are sensibly higher than the relative weights. When the Frontier Market risk contribution area disappeared, it doesn’t mean that the risk contribution is 0.00 per cent. For the MVO portfolios, this situation means that the risk contribution of this asset class to the portfolio risk is negative. This situation was present also in the previous paragraph, but in very limited and special cases. For this reason, I didn’t highlight the point. About the evolution over time, we can draw similar conclusions to the one of the previous subparagraph. In particular, for the MVO portfolios the risk contribution of the Frontier Market asset class given the weight, tends to increase during the crises and to decrease during booming period, especially in the last four years when it became negative. For RB portfolios the weight of this asset class, given the risk contribution, tends to increase during booming periods and to decrease during crises periods. Again, between 2008 and 2009 and in 2012, risk contributions and weights for RB portfolios are equal to 0.00 per cent. As mentioned in the previous subparagraph, in this way, I avoided the most turbulent periods of the subprime and sovereign debt crises.

Figure 33 – Frontier Markets weights and risk contributions in 5% Portfolios (EWMA)

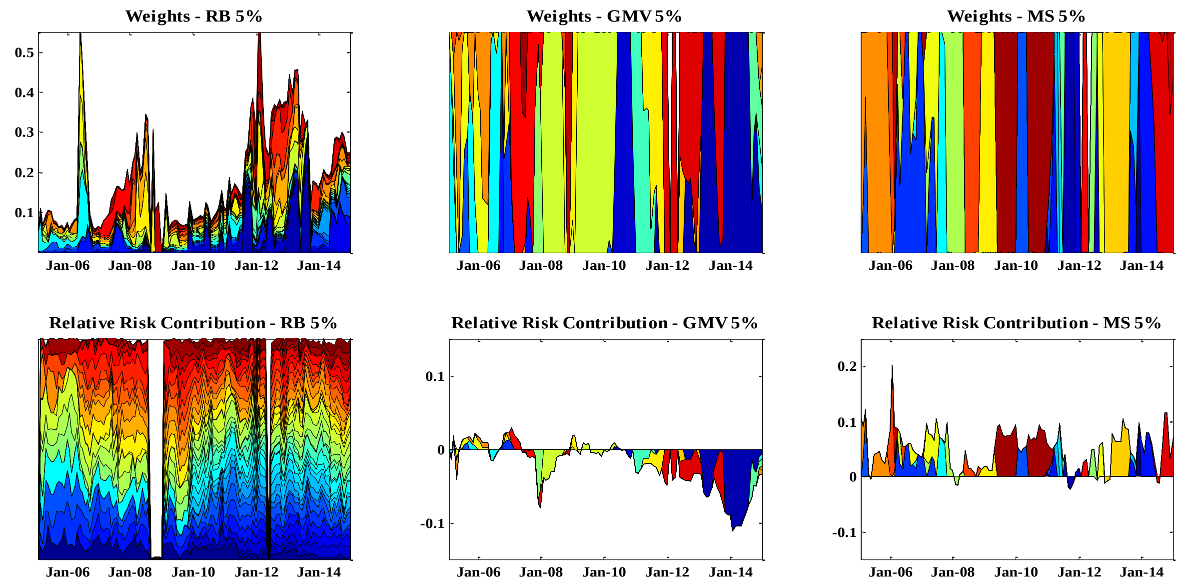


Figure 34 – Frontier Markets weights and risk contributions in 10% Portfolios (EWMA)

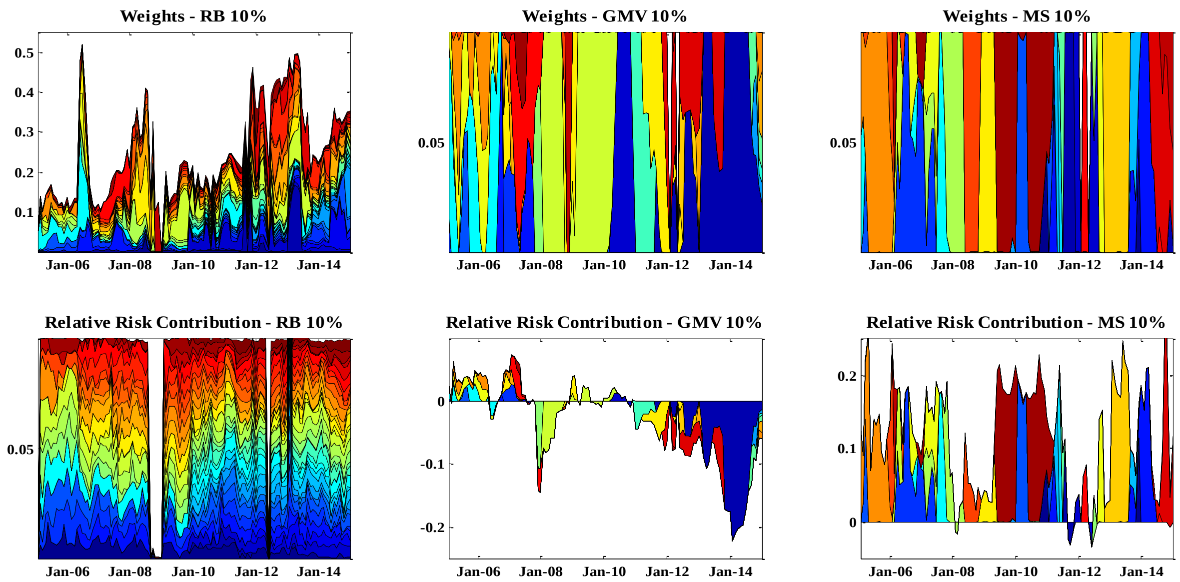


Figure 35 – Frontier Markets weights and risk contributions in 20% Portfolios (EWMA)

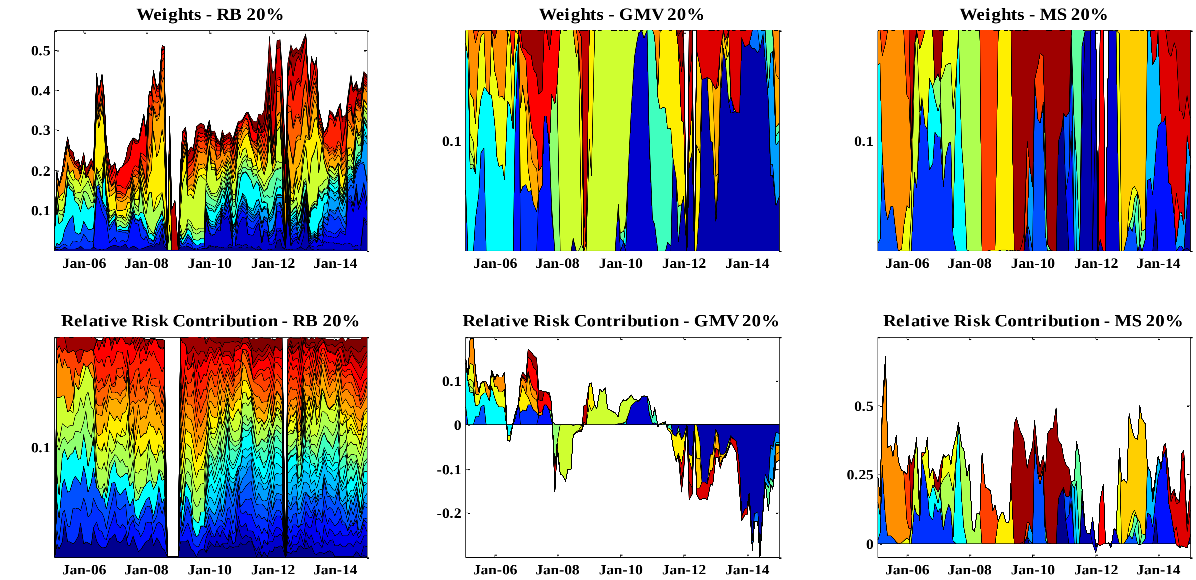
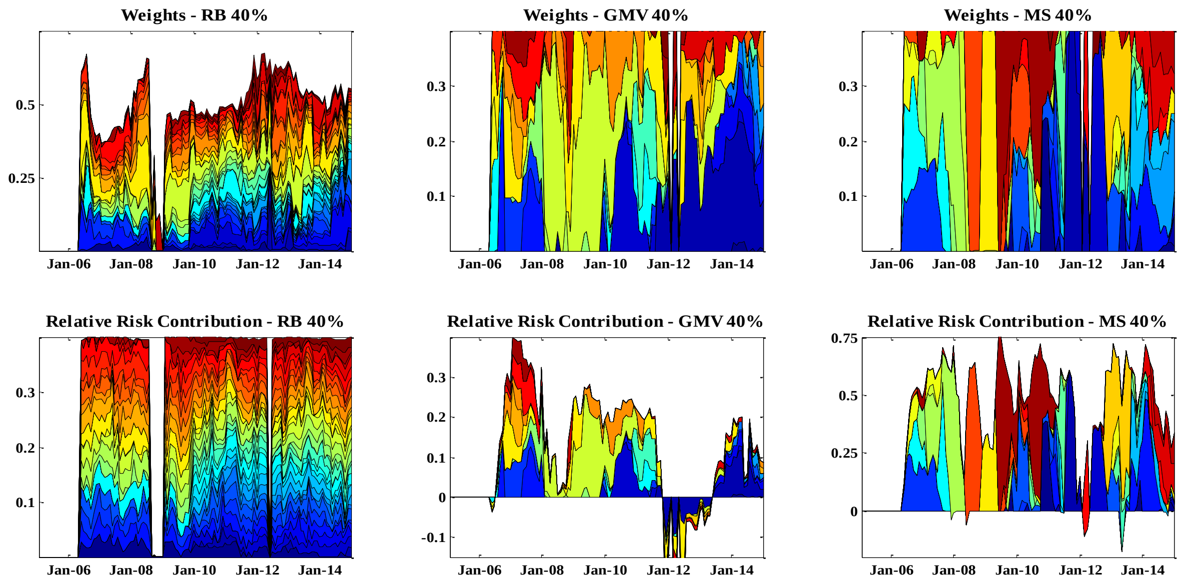


Figure 36 – Frontier Markets weights and risk contributions in 40% Portfolios (EWMA)



As in the previous subparagraph, RB portfolios always present higher diversification, both in terms of risk and weight, than MVO portfolios. Also in this case, I have a coverage of investable Frontier Markets between 75.00 and 90.00 per cent. On the contrary, MVO portfolios are poorly diversified, with a number of Frontier Markets included that usually are never above five. In a consistent number of months, MVO portfolios had only one Frontier Market. These conclusion are stable in every portfolio, regardless of the portfolio portion invested in Frontier Market asset class. However, there are two important differences. I have already mentioned the first, that is the higher variability in the weight and risk exposures. This is a byproduct of the methodology used to compute the inputs in this part (i.e., EWMA). The second is the risk contributions of the GMV portfolios. These portfolios exhibit negative risk contribution from the Frontier Market asset class, especially in the last months (the exception is the 40.00 per cent GMV portfolio that exhibits this feature in 2012). The negative risk contributions were higher than 10.00 per cent in some cases.

The third step is the analysis of the turnover. To perform the analysis, I used again the monthly actual turnover. Table 15 presents the average monthly actual turnover during the investment period for the twelve portfolios. Figures 37 and 38 present the evolution over time of the monthly actual turnover. Again, the lighter lines represent the MVO portfolios, while the darker lines represent the RB portfolios. Given the problem in the algorithm of the weights, I was not able to compute the turnover for the last 37 months for the MVO portfolios. In the computation of the average turnover, I used the first 83 months, assuming that the relation between the turnovers of the RB portfolios and MVO portfolios didn't change over the last period. This is consistent with the time evolution. In general, turnovers of the different approaches exhibit similar pattern during the whole investment period, maintaining the relative comparison stable.

Table 15 - Average Monthly Actual Portfolio Turnover (EWMA)

ALLOCATIONS	PORTFOLIOS			
	5%	10%	20%	40%
Risk Budgeting	9.74	10.37	11.11	14.14
Global Minimum Variance	5.73	10.38	6.55	12.35
Maximum Sharpe	7.29	16.85	9.40	18.39

Average monthly actual turnovers increased substantially with respect to the turnovers of the Rolling portfolios. The minimum value increases from 1.77 to 5.73 per cent (5.00 per cent GMV in both cases) and the maximum value increases from 9.36 to 18.39 per cent (40.00 per cent RB and MS respectively). In general, the turnovers for the portfolios generated using the EWMA method for the inputs computations are between two and six times larger than the turnovers of the portfolios presented in the previous subparagraph (i.e., rolling portfolios). The 5.00 and 10.00 per cent portfolios exhibit the highest increases. This result was largely expected. A preliminary analysis of the weights showed an higher variability in the portfolio allocations, driven by the higher variability of the inputs used in the algorithms. This is the byproduct of using EWMA estimates. RB portfolios present a turnover always higher than GMV portfolios. With respect to MS portfolios, RB portfolio turnovers are higher for the 5.00 and the 20.00 per cent portfolio. The explanation is still the activity on Emerging Market asset class, that is not present in MVO portfolios (the weight is fixed at 10.00 per cent). In any case, portfolio turnover is mainly driven by the activity on the Frontier Market asset class.

Figure 37 – Monthly Actual Turnover 5% and 10% Portfolios (EWMA)

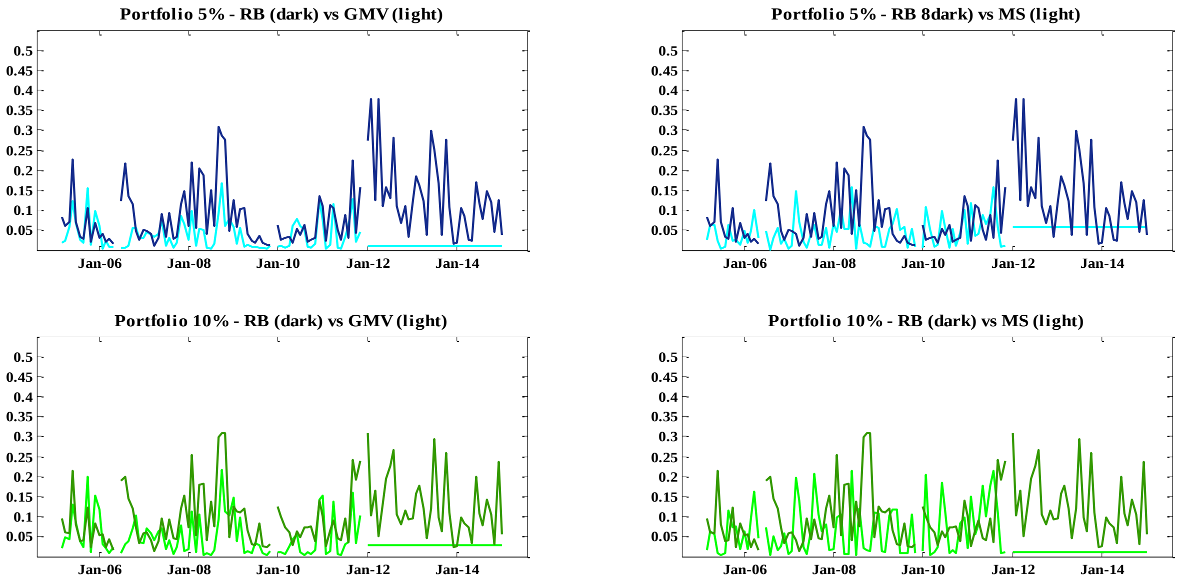
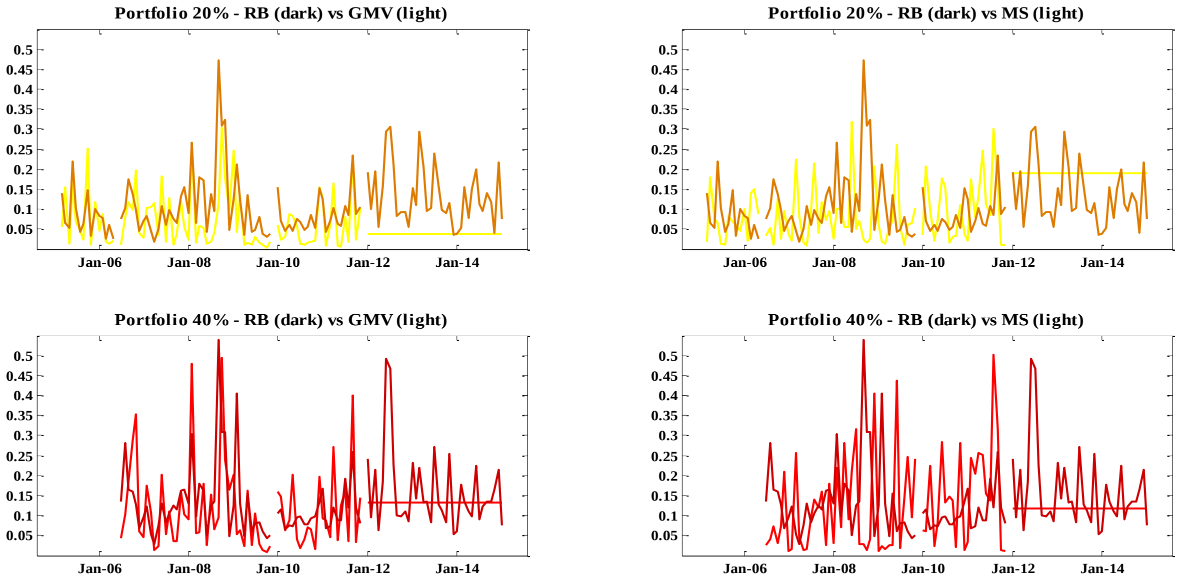


Figure 38 – Monthly Actual Turnover 20% and 40% Portfolios (EWMA)



The analysis of the time-evolution of the turnover cannot be done for the last 3 years. However, the conclusions would not change. It is interesting to compare Figures 37 and 38 with Figures 24 and 25. As expected, the use of EWMA estimates lead to a larger number of spikes with respect to the use of rolling estimates. EWMA estimates are much more variable than rolling estimates and the changes are generally larger. Consequently, the algorithms adjusted the allocations. However, this is done with a consistent change in portfolio weights, in order to reflect the new estimates. In this case, I think that EWMA method introduced too much variability in portfolio allocations, eliminating one of the most positive feature of Risk Budgeting Approach. This positive feature is the stability of the allocations.

The fourth step of portfolio analysis regards the analysis of the cumulated returns. Figures 39 and 40 present the cumulated returns time series of RB portfolios and MVO portfolios. Table 16 summarizes the results, with average annual returns between parenthesis.

Table 16 – Portfolios Cumulated Returns (EWMA)

Investment Period 02/2005 - 01/2015				
Strategy	**5%**	**10%**	**20%**	**40%**
Risk Budgeting	50.34 (4.16)	58.74 (4.73)	52.02 (4.28)	-
Global Minimum Variance	61.78 (4.93)	57.01 (4.61)	51.67 (4.25)	-
Maximum Sharpe	81.92 (6.17)	96.40 (6.98)	122.10 (8.31)	-
Investment Period 06/2006 - 01/2015				
Strategy	**5%**	**10%**	**20%**	**40%**
Risk Budgeting	21.45 (2.16)	27.12 (2.67)	19.27 (1.96)	18.97 (1.93)
Global Minimum Variance	31.83 (3.09)	26.98 (2.66)	19.95 (2.02)	18.00 (1.84)
Maximum Sharpe	46.60 (4.30)	58.16 (5.17)	81.32 (6.77)	101.91 (8.04)

RB portfolios tend to present cumulated returns, and relative average annual return, close to the GMV portfolios. MS portfolios provide significantly higher returns, that in some cases are more than two times the annual average return of RB portfolios. In particular, this is the case of the 20.00 and 40.00 per cent MS (1.96 vs 6.77 per cent and 1.93 vs. 8.04 per cent). It is difficult to compare EWMA and Rolling portfolios, given the fact that the investment periods are different, with the exception of 5.00 and 10.00 per cent portfolios over their entire investment period. In this case, Rolling RB portfolios present higher return than EWMA RB (5.46 and 5.58 vs 4.16 and 4.73 per cent, for 5.00 and 10.00 per cent RB portfolios respectively). We can derive the opposite conclusions when considering the MVO portfolios and, in particular, the MS portfolios. They present high annual average returns, even if the two periods include the subprime crises[142]. On the side of cumulated returns, it seems that the imple-

[142] With Rolling Portfolios I had to wait the last period in order to see returns above 8.00 per cent.

mentation of the EWMA method damaged the RB portfolios, while it had a positive effect on MS allocations. However, we will see later, with the analysis of the performance measures, if the risk is adequately remunerated. In fact, from the analysis of the basic characteristics, the higher returns of the MVO portfolios are associated to higher risk.

Figure 39 – Cumulated Returns from 02/2005 to 01/2015 (EWMA)

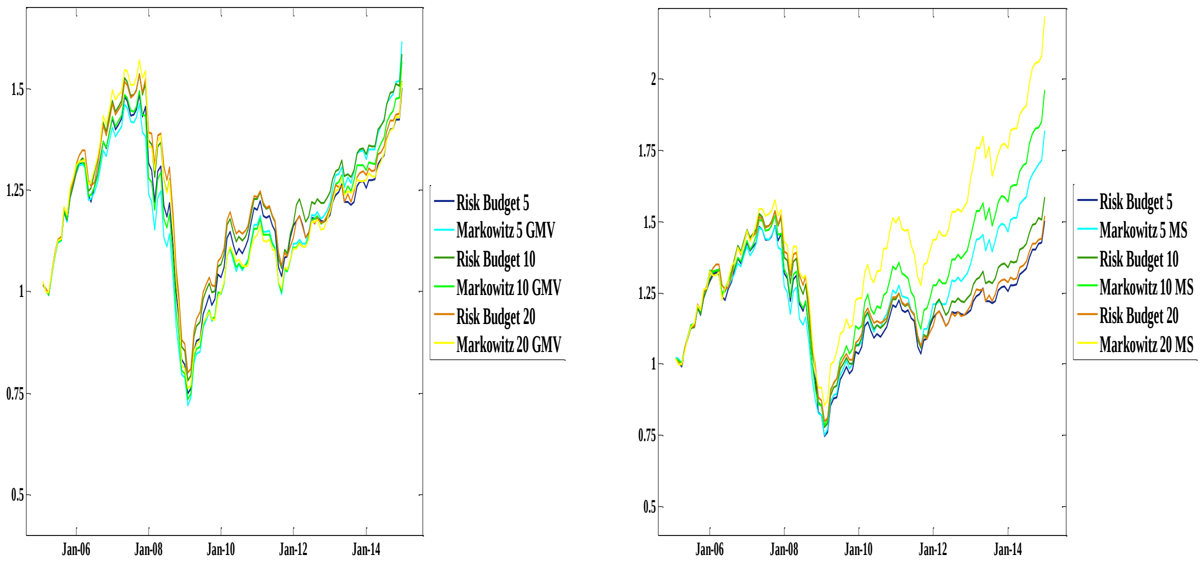
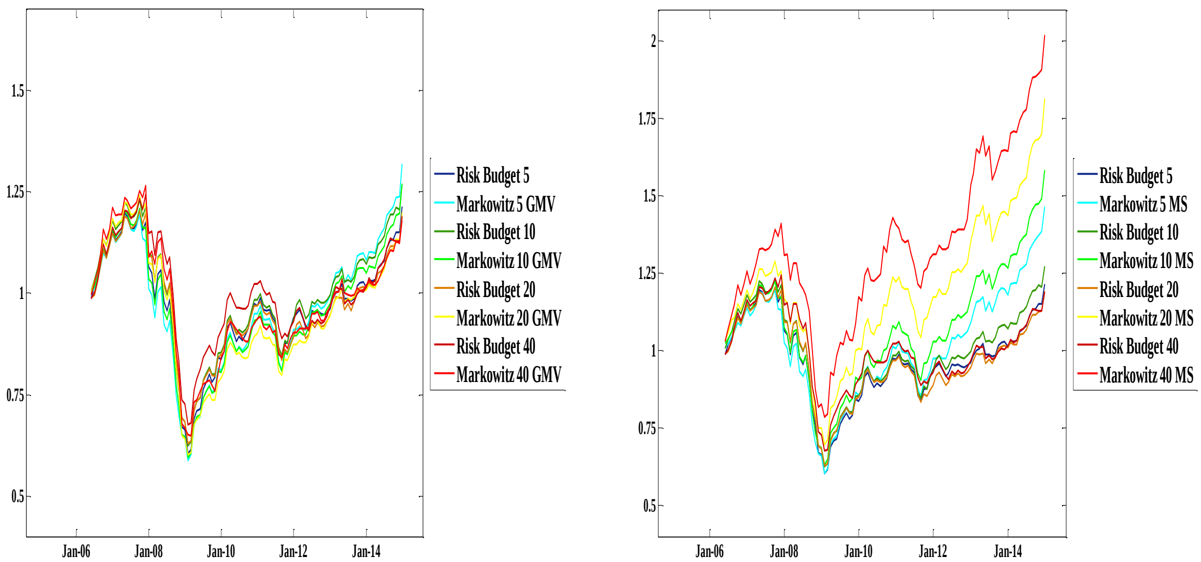


Figure 40 – Cumulated Returns from 06/2006 to 01/2015 (EWMA)



The fifth step is the performance measure analysis. I used the same performance measures analyzed in the previous paragraph and presented in Appendix D. To recap, these measures are the Sharpe Ratio, the Sortino Ratio, the Return-VaR (Value at Risk) Ratio, the Return-ES (Expected Shortfall) Ratio, the Calmar Ratio and the Sterling Ratio. Table 17 presents the different performance measures for the twelve portfolios, with the same investment-period breakdown of Table 14.

Table 17 – Performance Measures (EWMA)

Investment Period 02/2005 - 05/2006						
Strategy	**Sh**	**So**	**Var**	**ES**	**Cal**	**St**
Risk Budget 5	0.11	0.13	0.05	0.04	0.03	0.04
Risk Budget 10	0.13	0.14	0.06	0.05	0.04	0.05
Risk Budget 20	0.12	0.14	0.07	0.04	0.04	0.04
Markowitz 5 GMV	0.13	0.15	0.06	0.05	0.04	0.05
Markowitz 5 MS	0.15	0.19	0.08	0.06	0.05	0.06
Markowitz 10 GMV	0.12	0.14	0.06	0.05	0.04	0.04
Markowitz 10 MS	0.17	0.21	0.09	0.07	0.05	0.07
Markowitz 20 GMV	0.11	0.12	0.06	0.04	0.03	0.04
Markowitz 20 MS	0.20	0.25	0.11	0.09	0.06	0.09
Investment Period 06/2006 - 01/2015						
Strategy	**Sh**	**So**	**Var**	**ES**	**Cal**	**St**
Risk Budget 5	0.11	0.13	0.05	0.04	0.03	0.04
Risk Budget 10	0.10	0.12	0.05	0.04	0.03	0.04
Risk Budget 20	0.08	0.10	0.04	0.03	0.03	0.03
Risk Budget 40	0.10	0.11	0.05	0.04	0.03	0.04
Markowitz 5 GMV	0.11	0.13	0.05	0.04	0.04	0.04
Markowitz 5 MS	0.11	0.13	0.05	0.04	0.03	0.04
Markowitz 10 GMV	0.12	0.14	0.06	0.05	0.04	0.05
Markowitz 10 MS	0.11	0.13	0.05	0.05	0.04	0.05
Markowitz 20 GMV	0.12	0.14	0.06	0.05	0.04	0.05
Markowitz 20 MS	0.12	0.14	0.06	0.05	0.04	0.05
Markowitz 40 GMV	0.11	0.13	0.06	0.04	0.03	0.04
Markowitz 40 MS	0.11	0.13	0.06	0.04	0.03	0.04

First of all, I want to focus on Sharpe and Sortino Ratio. During the first investment period, MVO portfolios generally provide a significantly better reward of the non-extreme risk (i.e., better performance measures) with respect to their RB peers. The only exceptions are the 10.00 and 20.00 per cent GMV portfolios, with Sharpe and Sortino Ratio equal to or slightly below the ratios of their RB peers. The portfolio that provides the best performance measures is the 20.00 per cent MS portfolio, with a Sharpe and a Sortino Ratio equal to 0.20 and 0.25 respectively. During the second investment period, the difference between the two approaches tend to disappear and the portfolios present very similar performance measures. However, the MVO portfolios tend to exhibit slightly higher ratios than their RB portfolios (1 or 2 bps higher). This results are the opposite with respect to what I found in the previous subparagraph. This confirmed that MVO portfolios benefited from the implementation of the EWMA method in inputs estimation. The analysis of the measures focused on extreme risk leads to the same conclusions derived for the normal performance measure. In the first period, MVO portfolios tend to present better performance measures, while in the second period we have very similar rewards of the extreme risk. Looking at the time evolution, during the second period, GMV and RB portfolios present stable performance measures. A significant worsening is present only for the 20.00 per cent RB portfolio, with the Sharpe and the Sortino Ratio that moved from 0.12 and 0.14 to 0.08 and 0.10 respectively. On the contrary, MS portfolios present a significant drop in the level of the performance measures. The 10.00 per cent MS portfolio Sharpe and Sortino Ratios decrease from 0.17 and 0.21 to 0.11 and 0.13 respectively. Moreover, the 20.00 per cent MS portfolio exhibits performance measures that are close to the half of the level presented in the first period. Comparing rolling and EWMA portfolios, the performance measures of the rolling portfolios over the last investment period exhibit the highest level, as expected. If we compare the 5.00 and the 10.00 per cent RB portfolios over the same investment period, rolling portfolios exhibit better performance measures than EWMA portfolios. The Sharpe and the Sortino Ratio are 0.14 vs 0.11 and 0.15 vs 0.13, for the 5.00 per cent RB portfolios, and 0.17 vs 0.13 and 0.17 vs 0.14, for the 5.00 per cent RB portfolios. However, Rolling RB portfolios present worse performance measures than EWMA MS portfolios over the first investment period, as expected in theory. In fact, the situation presented in the previous subparagraph was anomalous from a theoretical point of view, since in expectations the Sharpe Ratio of a MS portfolio should be higher than the ratio of a RB portfolio. However, I want to clarify again that the aim of Risk Budgeting Approach is to reduce and diversify risk, and not to construct highly performing portfolios. Thus, the results still remain satisfactory.

Finally, the last step is the Tracking Error (TE) Analysis. I used the same tools presented in the previous paragraph (i.e., Tracking Error, Tracking Error Volatility, Semi-TEV, Information Ratio and Semi-IR). The benchmark considered is the GDP-weighted portfolio of Developed Markets. I recall that the portfolio presents an average monthly return and volatility equal to 0.47 and 3.69 per cent respectively, over the 10-year investment period. Average monthly return and volatility move to 0.36 and 3.78 during the second investment period. Average monthly return is higher than the average returns exhibited by RB portfolios. However, the risk is higher and especially in the second period this difference is important, usually higher than 10 bps. Table 18 summarizes the TE analysis.

Table 18 – Tracking Error Analysis (EWMA)

Investment Period 02/2005 - 01/2015					
Strategy	**TE**	**TEV**	**Semi-TEV**	**IR**	**Semi-IR**
Risk Budget 5	-0.06	1.01	0.96	-0.06	-0.06
Risk Budget 10	-0.02	1.08	0.92	-0.02	-0.02
Risk Budget 20	-0.06	1.23	0.92	-0.05	-0.06
Markowitz 5 GMV	0.00	0.67	0.45	0.00	0.00
Markowitz 5 MS	0.10	0.70	0.46	0.14	0.22
Markowitz 10 GMV	-0.02	0.92	0.72	-0.03	-0.03
Markowitz 10 MS	0.17	0.99	0.63	0.17	0.26
Markowitz 20 GMV	-0.05	1.54	1.32	-0.03	-0.04
Markowitz 20 MS	0.27	1.54	0.93	0.18	0.29
Investment Period 06/2006 - 01/2015					
Strategy	**TE**	**TEV**	**Semi-TEV**	**IR**	**Semi-IR**
Risk Budget 5	-0.10	1.07	0.98	-0.10	-0.11
Risk Budget 10	-0.06	1.14	0.94	-0.05	-0.07
Risk Budget 20	-0.13	1.29	0.93	-0.10	-0.14
Risk Budget 40	-0.13	1.76	1.23	-0.08	-0.11
Markowitz 5 GMV	-0.02	0.69	0.47	-0.03	-0.05
Markowitz 5 MS	0.08	0.69	0.47	0.12	0.17
Markowitz 10 GMV	-0.06	0.95	0.74	-0.06	-0.08
Markowitz 10 MS	0.15	0.98	0.64	0.16	0.24
Markowitz 20 GMV	-0.11	1.59	1.36	-0.07	-0.08
Markowitz 20 MS	0.29	1.53	0.95	0.19	0.30
Markowitz 40 GMV	-0.13	2.31	1.61	-0.06	-0.08
Markowitz 40 MS	0.40	2.54	1.41	0.16	0.28

The MS portfolios are the only portfolios able to outperform the benchmark, both in the first and in the second period. Looking at the Information Ratio, the 20.00 per cent MS portfolio present the highest value in both the periods, with 0.18 and 0.19. RB and GMV portfolios always exhibit a negative Information Ratio. 5.00 and 10.00 per cent EWMA MS portfolios exhibit higher Information Ratio than their Rolling RB peers (0.14 vs 0.09 and 0.17 vs 0.07 respectively). TE Analysis confirmed that EWMA inputs boost MS portfolios performance. In this way, the theoretical framework is partially restored. In any case, EWMA RB portfolios obtained the results expected in theory, and under some aspects better than the expectations. Moreover, these results are in line with the objectives of the Risk Budgeting Approach. In fact, EWMA RB portfolios are still the portfolios with the lowest risk between the EWMA portfolios. However, differently from the rolling RB portfolios, EWMA RB portfolios do not exhibit good performances when compared with EWMA MVO portfolios.

5.2.3 Daily Value at Risk: RB and MVO – Rolling and EWMA Method

The last topic I want to discuss is related to Risk Management procedures. In this paragraph, I present the computations of the daily Value at Risk (VaR). Given the number of portfolios, it was not possible to compute the daily VaR for all of them and for all the investment period. For this reason, I focused on the 5.00 per cent portfolios, considering both the RB and the MVO portfolios. The reason behind that choice is that they represent, with the 10.00 per cent portfolios, the most realistic and relevant portfolios in practice[143]. I computed the daily VaR over the last year and I compare the results between the different portfolios and the GDP-weighted Developed Markets portfolio. In this way, I can analyze if the introduction of Frontier Markets significantly impact, in a positive or in a negative way, the most diffused measure of extreme risk.

Financial risk management represents the set of concepts, tools, approaches and measures whose final purpose is to actively manage risks. Risk management in financial world is a fundamental activity, both for economic and regulatory reasons. First of all, the whole society has an interest in the stability of the financial system. Efficient and stable financial system is a fundamental condition for economic growth and for the wealth of the society. Proper risk management allows controlling the spill-over of financial risks that otherwise might undermine the stability and the functioning of the financial system. Secondly, the legal system requires the implementation of risk management procedures. The principles behind the intervention of the legislator are generally the preservation of the stability of the financial system and the protection of savings. In asset management activity, the most relevant risk is the market risk (i.e., the risk of a change in the value of a financial position due to changes in the value of the underlying components, such as stock and bond prices, exchange rates, commodity prices, etc.). The assessment of market risk can count on a regulated framework provided by the Basel Committee on Banking Supervision, with minimum requirements on the model choice, the adoption and the test of internal procedure[144]. The framework evolved during the years, but it maintained some core principles. The most important is the definition of the risk measure. This should represent the maximum loss a portfolio or an instrument can suffer at the 99.00 per cent probability in the holding period, which is the definition of the VaR. Thus,

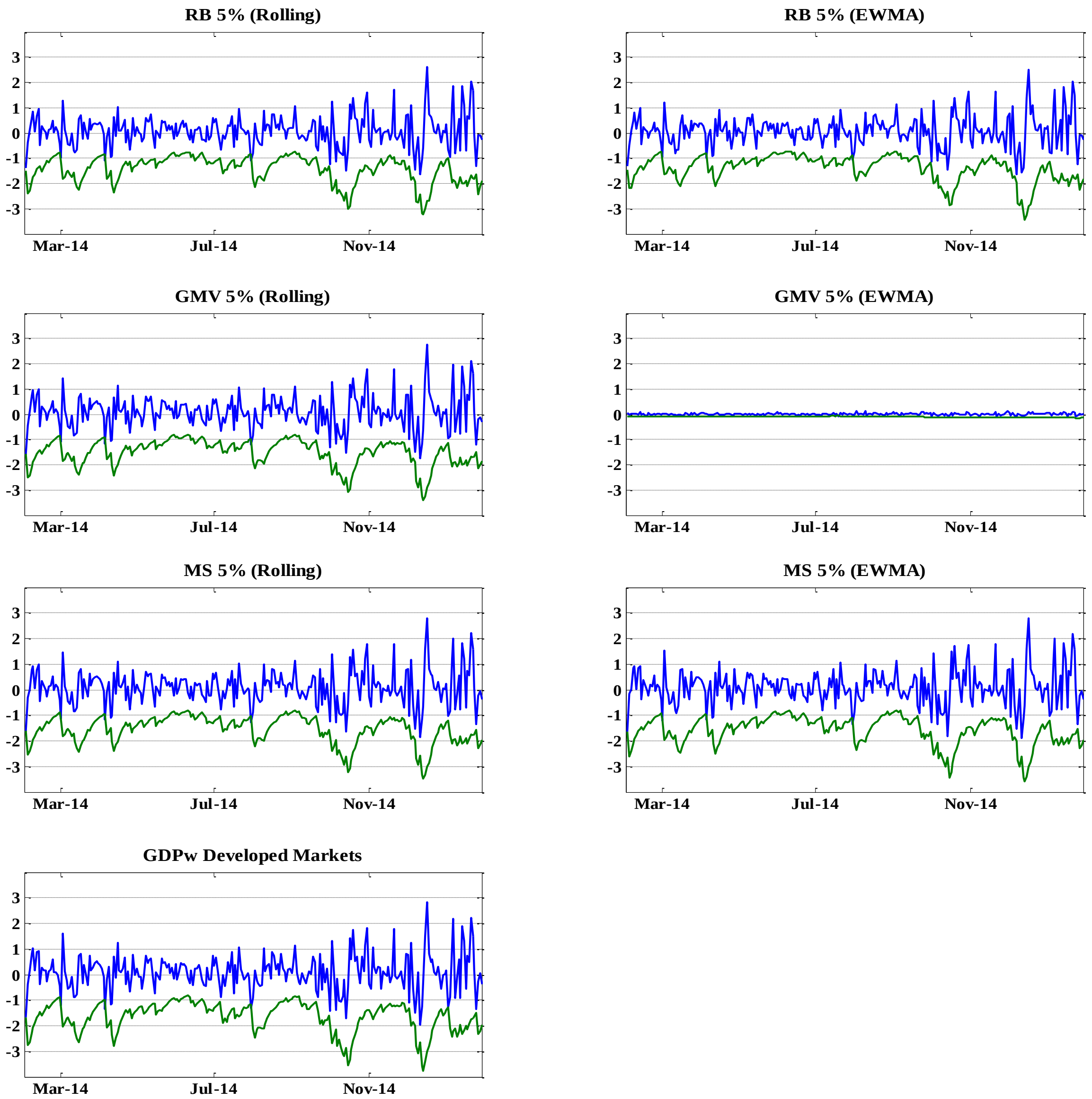
[143] In financial market practice it is difficult to find portfolios with a risk or a weight exposure on illiquid alternative asset class above 10.00 per cent. The exceptions are generally the thematic-portfolio or the hedge funds. However, in this thesis I wanted to provide a wider analysis, considering also larger exposures.

[144] For more details see Basel Committee on Banking Supervision (1996a,b) and the regulatory developments provided in the next Basel agreements (Basel II and Basel III) and available at the following website: www.bis.org/bcbs.

the VaR is fundamental in risk management procedures, since it is the most relevant risk measure for the assessment of market risk.

There is a consistent literature on the best approaches to compute the VaR[145]. In this thesis I used the EGARCH(1,1) model with Student Innovations. GARCH models have been introduced to capture the volatility clustering feature of financial returns time series (i.e., the persistence of volatility). In addition to that, the EGARCH specification allows for leverage effect (i.e., positive shocks decrease conditional variances). As above-mentioned, I computed the VaR for the 5.00 per cent RB and MVO portfolios and for the GDP-Weighted Developed Market portfolio over the last year, from February 2014 to January 2015. Figure 41 shows the time-evolution of the daily VaR for the seven different portfolios. The VaR time series is the green line. Table 19 presents the average daily VaR for the seven portfolios considered.

Figure 41 – Daily Value at Risk from 02/2013 to 01/2015



145 See for example the book of Jorion (2007).

Table 19 – Average Daily Value at Risk from 02/2013 to 01/2015

ALLOCATIONS	INPUT	
	Rolling	EWMA
5% Risk Budgeting	1.43	1.31
5% Global Minimum Variance	1.50	0.21
5% Maximum Sharpe	1.55	1.57
GDPw Developed Markets	1.64	

We have already seen that RB portfolios provide the lowest volatility (i.e., non-extreme risk measure) between the portfolios considered in this thesis. Looking at the average daily VaR, RB portfolios provide a lower VaR than the GDP-weighted Developed Market portfolio. Moreover, with the exception of the 5.00 per cent EWMA GMV, which is a peculiar case as can be seen in Figure 41, RB portfolios provide significantly lower average daily VaR than MVO portfolios. However, also the MVO portfolios present an average VaR lower than the GDP-weighted Developed Market portfolio. The introduction of Frontier Markets in internationally diversified portfolios, with a realistic exposure in terms of risk and weights, didn't increase the VaR, at least in the last year. Thus, Frontier Markets help to mitigate the portfolio risk, especially using the Risk Budgeting Approach in portfolio construction. This mitigation is present for both the non-extreme and the extreme risk. Moreover, a lower VaR implies a lower capital requirement associated to market risk for the bank. This represents costs reduction for the bank. In fact, minimal capital requirement are an immobilization of resources and liquidity, representing an opportunity cost of investing resources. Finally, the risk exposure measured by the VaR depends crucially on the underlying model used for the return series. One of the ways to test the accuracy of the model is the back-testing of the exceptions to the model, i.e. the number of times that the daily returns exceed the VaR. As can be seen in Figure 41, daily returns exceeded the VaR between three and four times over the last year (260 days) for all the portfolios. Thus, I can conclude that the model is quite accurate[146].

[146] The Market Risk Amendment of the Basel I agreement established a similar procedure for the testing the accuracy of internal procedures. The number of exceptions that I found in my portfolios are lower than the minimum threshold for the application of the sanctions, confirming the validity of the approach.

6 Conclusions

In this thesis, I implemented the Risk Budgeting Approach proposed by Bruder and Roncalli (2012) to construct internationally diversified portfolio, with a variable portion invested in the Frontier Market asset class. In particular, I constructed four risk budgeting portfolios, with a maximum risk exposure to Frontier Markets equal to 5.00, 10.00, 20.00 and 40.00 per cent. I simulated the activity of an asset manager that had to construct internationally diversified portfolios at the beginning of 2005. I used the same information available to him in each month, in order to give to my work an ex-ante perspective. The first aim of the thesis was to construct a tool, the Country Risk Indicator, useful to design the individual risk budgets on Frontier Markets. The second objective was to test the validity of the approach, comparing risk budgeting portfolios with their mean-variance optimized peers, with Global Minimum Variance and Maximum Sharpe allocation.

Risk budgeting portfolio is a generalization of the equally risk contribution portfolio (Maillard, Roncalli and Teiletche [2010]), with the risk budgets that are not necessarily the same. It is an heuristic method, since financial theory does not promote it as an optimal portfolio. However, risk budgeting portfolios are quite diffused in financial market practice. In particular they are used:

- as alternative benchmarks with respect to cap-weighted benchmarks;
- in risk management process;
- as a starting point for the implementation of active management strategy.

In this thesis, I focused on equity asset class. The investment universe is composed by three broad indices of Developed Markets, one broad index of Emerging Markets and thirty-one standalone-country indices of Frontier Markets, provided by Morgan Stanley Capital International. In the last years, asset managers are seeking for new investment opportunities able to boost portfolio performances, both in terms of returns and risk management. For this reason, I focused on Frontier Markets, since they represent an alternative asset class. These markets are smaller and less liquid than Developed and Emerging Markets. Section 3 discussed their basic characteristics. As expected, these markets generally exhibit higher risk than Core Markets (i.e., Developed and Emerging Markets). However, they present a very positive feature. In fact, they are less correlated within the group and with the other asset classes. Thus, they could be a source of potential diversification benefits, especially through the reduction of portfolio risk. The low correlation with traditional equity markets makes the Frontier Markets a perfect alternative asset class.

To design the risk budgets, I computed the Country Risk Indicator. This tool tried to use a simplified procedure of Country Risk Assessment to forecast individual country risk. As mentioned in the previous sections, the idea of using macroeconomic variable to manage the risk budgets is not new. However, previous works focused only on a small set of macroeconomic variable (i.e., GDP and public debt) able to partially assess only the sovereign risk. In Section 4, I highlighted that country risk is a broad aggregate of different types of risk and the sovereign risk is only one component. For this reason, my Country Risk Indicator is based on a larger set of variables. It is divided in sub-indicators, in order to assess the different types of risk (e.g., macroeconomic, sovereign, political-institutional and non-transfer risk). Through the assessment of country risk, I tried to avoid the major crises (i.e., financial, banking and currency crises) or other disruptive events (e.g., wars) that can lead to huge losses. These losses are amplified by the relative illiquidity of Frontier Markets, that complicates the exit process from these stock markets. Focusing on the geographical area, the Frontier Markets that exhibit the lowest country risk are the Middle-East countries. These countries benefit from positive Liquidity Risk Indicator, due to their consistent total reserves. However, during the last years, the distance between the different countries became tighter.

The total risk budget exposure on the Frontier Market asset class is variable over the investment period. It depends on the international liquidity framework and on the perception of the economic and financial risk over the Frontier Markets, proxied by the average Country Risk Indicator. After the construction of the risk budgets, I estimated the other inputs of the risk budgeting and the mean-variance optimization algorithms (i.e., expected returns and variance-covariance matrix). I implemented a 60-month rolling method and a 12-month exponential weighted moving average method. Using Matlab software, I constructed and compared the different allocations over a variable investment period (from 56 to 120 months), depending on the Frontier Market asset class exposure and the methodology adopted to estimate the inputs. Section 5 present the portfolio analysis, performed using a wide set of tools. Results are quite interesting. First of all, risk budgeting portfolio exhibit lower risk than mean-variance optimized portfolios (both GMV and MS portfolios). This result is valid independently from the investment period considered, from the risk/weight exposure on Frontier Markets and from the methodology used in the estimation of the inputs. The most surprising thing is that risk budgeting portfolios present a risk lower than global minimum variance peers. The result is better than what I expected from the theory, confirming the validity of the Country Risk Indicator Approach in the design of the risk budgets. About risk budgeting portfolios performances, I am not able to derive general conclusions. In fact, performance and tracking error analysis conclusions depend on the investment period considered, the input estimation method used, the risk/weight exposure on Frontier Market asset class and the mean-variance optimized allocation considered. In general, risk budgeting portfolios generated using rolling method for the estimation of the inputs, provide in some cases superior performances with respect to mean-variance optimized portfolios. In particular, they sometimes exhibit a Sharpe Ratio higher than the Maximum Sharpe portfolios. Again, this result is definitely better than what I expected. The introduction of the EWMA method penalized risk budgeting portfolios, with a general reduction of the performances and a slightly increase in risk. On the contrary, this method boost Maximum Sharpe portfolios performances. This methodology produces highly variable inputs, significantly reducing the stability of the allocation. Thus, it eliminates one of the most positive feature of the risk budgeting methodology, that is the stability of the asset allocation. About the comparison with a GDP-weighted portfolio of Developed Markets, the internationally diversified portfolios generated in this thesis are not able to provide higher performances. The only exceptions are the 5.00 and 10.00 per cent rolling risk budgeting portfolios and the EWMA Maximum Sharpe portfolios. The main constraints on internationally diversified portfolios performances are the modest returns of some Frontier Markets during the last five years. These years were characterized by the recovery from the 2008 Financial Crisis. However, only a small set of Frontier Markets were able to replicate the performance of the US stock market, that constitutes the major asset of the GDP-weighted Developed Market portfolio. On the risk side, the portfolios including Frontier Market asset class generally provide lower risk than the GDP-weighted portfolio of Developed Markets, regardless of the allocation considered. This result is consistent with the expectations. Frontier Markets are a source of diversification benefits, given their relatively low correlation. The reduction in risk is particularly relevant for the risk budgeting portfolios. As shown in Section 5, risk budgeting portfolios present an higher degree of diversification, both in terms of risk and weights, with respect to mean-variance optimized portfolios. This result was largely expected, since the core aspect of the Risk Budgeting Approach is the risk diversification. Moreover, the generation of poorly diversified portfolios is one of the main drawback of the Markowitz model. The higher degree of diversification of risk budgeting portfolios implied higher diversification benefits in risk reduction. In addition to that, it helps to reduce the negative impact of country-specific shocks, that can be particularly severe when dealing with illiquid asset. Finally, I completed the risk analysis with the computation of the daily VaR for the 5.00 per cent portfolios (both RB and MVO portfolios) over the last year. Except for one non-normal case, risk budgeting portfolios exhibit the lowest average daily VaR. This result is confirmed even considering the GDP-weighted portfolio of Developed Markets. Thus, the combination of risk budgeting techniques and Country Risk Assessment procedure, prove to be fundamental in risk management and led to satisfactory results, both in terms of non-extreme risk (e.g., volatility) and in terms of extreme risk (e.g., value at risk).

This thesis can represent a starting point for future works, given the fact that the literature on Frontier Markets is quite limited. An important further development regards the construction of the Country Risk Indicator. In particular, thanks to the increasing number of information available for developing countries, it will be possible to substitute some variables, or to integrate the set, with better determinants of the various type of risk assessed[147]. Moreover, it will be possible to determine the weights assigned to the various determinants of the risks on the basis of more consistent models, given the availability of longer returns time series. The second important development regards the use of Risk Budgeting Approach. As abovementioned, risk budgeting portfolios, constructed on the basis of the Country Risk Indicator, exhibit a consistent lower risk than mean-variance optimized portfolios. However, I was not able to derive any conclusion about the performance. I suggest to use the risk budgeting portfolios created in this work as a base for the implementation of an active management strategy. The aim will be to test whether the combination of risk budgeting, country risk evaluation and active management will be able to generate allocations able to outperform mean-variance optimized portfolios and Developed Market portfolios in a consistent way, maintaining the risk under control.

In conclusion, I showed that the introduction of Frontier Markets in an internationally diversified portfolio brings significant diversification benefits through risk reduction. These benefits are significantly higher when I created portfolios combining the Risk Budgeting Approach and the Country Risk Indicator (i.e., a Country Risk Assessment tool), confirming the validity of the approach used in this thesis. Unfortunately, I was not able to derive clear conclusions about the performance of these portfolios.

[147] Some examples are the substitution of the overall government balance with the primary balance and the integration of the set with debt service and external debt.

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Appendix A

A.1 Markowitz model optimal weights

As shown in Eq.(2.9) in paragraph 2.1.1, the optimal portfolio of the risk averse agent is a combination of the weights of the Maximum Sharpe and the global minimum variance portfolio. The solution of the simple Markowitz model can be written as:

$$\hat{w} = \boldsymbol{D} + \boldsymbol{E}\mu_p = \frac{A\Sigma^{-1}\mathbf{1} - B\Sigma^{-1}\boldsymbol{r}}{\Delta} + \frac{A\Sigma^{-1}\boldsymbol{r} - B\Sigma^{-1}\mathbf{1}}{\Delta}\mu_p$$

The expected return and the volatility for the MS and the GMV portfolio can be defined as:

$$r_{MS} = \frac{\boldsymbol{r}'\Sigma^{-1}\mathbf{1}}{\mathbf{1}'\Sigma^{-1}\boldsymbol{r}} = \frac{A}{B} \qquad \sigma_{MS} = \frac{\sqrt{A}}{|B|}$$

$$r_{GMV} = \frac{\boldsymbol{r}\Sigma^{-1}\mathbf{1}}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} = \frac{B}{C} \qquad \sigma_{GMV} = \frac{1}{\sqrt{C}}$$

Given the relation between weights and portfolio risk and return, the weights of the MS and the GMV portfolio are:

$$w_{MS} = \frac{\Sigma^{-1}\boldsymbol{r}}{\mathbf{1}'\Sigma^{-1}\boldsymbol{r}} \qquad w_{GMV} = \frac{\Sigma^{-1}\mathbf{1}}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}$$

From Eq.(2.9) we can show that starting from the weights of the optimal portfolio of the risk averse agent expressed in terms of the combination between the weights of the MS and the GMV portfolio, we end up with the general solution:

$$\hat{w} = \frac{B}{\gamma}\widehat{w_{MS}} - \frac{B-\gamma}{\gamma}\widehat{w_{GMV}} = \frac{B}{\gamma}\frac{\Sigma^{-1}\boldsymbol{r}}{\mathbf{1}'^{\Sigma^{-1}}\boldsymbol{r}} - \frac{B-\gamma}{\gamma}\frac{\Sigma^{-1}\mathbf{1}}{\mathbf{1}'^{\Sigma^{-1}}\mathbf{1}}$$

$$= \frac{\mathbf{1}'\Sigma^{-1}\boldsymbol{r}}{\gamma}\frac{\Sigma^{-1}\boldsymbol{r}}{\mathbf{1}'^{\Sigma^{-1}}\boldsymbol{r}} - \frac{\mathbf{1}'\Sigma^{-1}\boldsymbol{r} - \gamma}{\gamma}\frac{\Sigma^{-1}\mathbf{1}}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}$$

$$= \frac{1}{\gamma}\Sigma^{-1}\boldsymbol{r} - \frac{B-\gamma}{C}\frac{1}{\gamma}\Sigma^{-1}\mathbf{1}$$

A.2 The Capital Market Line

In paragraph 2.1.1, I presented the equations for the capital market line and the return and variance of the tangency (i.e., the Maximum Sharpe) portfolio. Recalling these equations, we can demonstrate that the slope of the CML is equal to the Sharpe ratio of the MS portfolio.

$$\mu_p = r_f + \sigma_p \sqrt{A - 2Br_f + Cr_f^2}$$

$$\mu_{MS} = \frac{A - Br_f}{B - Cr_f} \qquad \sigma_{MS}^2 = \frac{A - 2Br_f + Cr_f^2}{\left(B - Cr_f\right)^2}$$

From the CML we can see that the intercept is equal to the risk-free rate. The slope is exactly equal to the Sharpe ratio of the tangency portfolio:

$$\frac{\mu_{MS} - r_f}{\sigma_{MS}} = \left(\frac{A - Br_f}{B - Cr_f} - r_f\right) \frac{\left|B - Cr_f\right|}{\sqrt{A - 2Br_f + Cr_f^2}}$$

$$= \left(\frac{A - Br_f - \left(B - Cr_f\right)r_f}{B - Cr_f}\right) \frac{\left|B - Cr_f\right|}{\sqrt{A - 2Br_f + Cr_f^2}}$$

$$= sgn\left(B - Cr_f\right) \frac{A - 2Br_f + Cr_f^2}{\sqrt{A - 2Br_f + Cr_f^2}}$$

$$= sgn\left(B - Cr_f\right) \sqrt{A - 2Br_f + Cr_f^2}$$

A.3 Example of Constrained Markowitz model

As mentioned in Paragraph 2.1.2, a solution to the limits of the Markowitz model is to add constraints to the problem presented in Eq.(2.3), in order to obtain more realistic allocations. I'm going to present the formulations of some of these constrained Markowitz model.

A.3.1 No-short-selling constraint

$$min_w \;\; w'\Sigma w$$

$$s.to \;\; \mu_p = w'\boldsymbol{r}$$

$$w'\boldsymbol{1} = 1$$

$$w_i \geq 0$$

The no-short-selling constraint is one of the most common constraint, given the impossibility for some institutional investors to open short positions. However, in it's pure form it leads to poorly diversified portfolio. Thus, it has to be integrated with other constraints.

A.3.2 Linear Equalities or Inequalities

$$\min_w \; w'\Sigma w$$
$$s.to \;\; \mu_p = w'\boldsymbol{r}$$
$$w'\mathbf{1} = 1$$
$$Hw = h \quad or \quad Hw \le h$$

H is a matrix *qxn* and *h* is a vector *qx1* to set *q* linear constraints. It can be added to the no-short-selling constraint. It is mainly used to create group constraints, such as imposing upper and/or lower bound or a specific weight to a particular asset class or group of assets with homogenous characteristics (e.g., geographical area, sector, etc.).

A.3.3 Upper and Lower Bounds

$$\min_w \; w'\Sigma w$$
$$s.to \;\; \mu_p = w'\boldsymbol{r}$$
$$w'\mathbf{1} = 1$$
$$l \le w_i \le u$$

Lower and upper bound constraints are used to limit the minimum and the maximum exposure on a single asset. We can impose only lower bounds, only upper bounds or both lower and upper bounds (as in the example). They are useful in order to guarantee a minimum diversification. In fact the number of assets included in the portfolio will be equal to the reciprocal of the upper bound u. Moreover the lower bounds can be set in order to allow the short-selling, limiting the maximum exposure (in reality there are several limits for some institutional investors on the size of short selling positions).

A.3.4 Turnover constraint

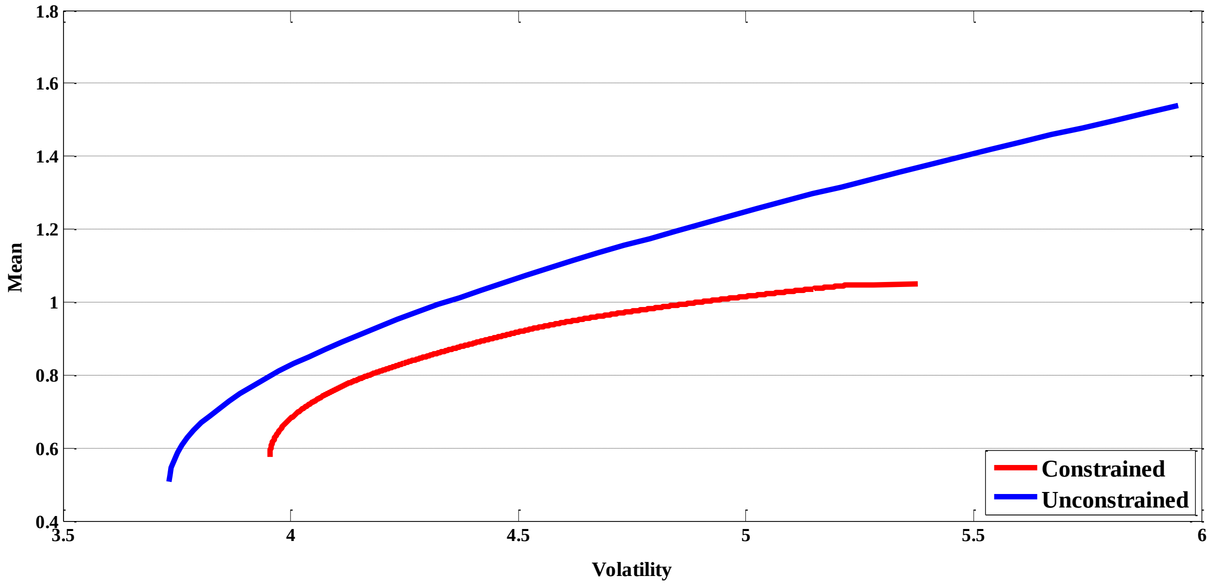
$$\min_w \; w'\Sigma w$$
$$s.to \;\; \mu_p = w'\boldsymbol{r}$$
$$w'\mathbf{1} = 1$$
$$\frac{1}{2}|w - \widetilde{w}|\mathbf{1} \le \tau$$

Turnover constraint is set to limit the trading activity or the portfolio rotation, consequently reducing the impact of transaction costs on portfolio performances. Given the initial allocation $\widetilde{w}$, τ is the maximum turnover allowed, representing the fraction of portfolio that can change in the re-allocation process. This constraint is very useful and common in real application.

A.3.5 The Efficient Frontier

For all of these optimal problems, the resulting efficient frontier will shift to the right with respect to the unconstrained efficient frontier, as shown in Figure A.1 for no short selling constraint:

Figure A.1: Unconstrained vs Constrained Efficient Frontier



A.4 Euler Decomposition of the Volatility Risk Measure

I want to show that the volatility risk measure satisfies the Euler decomposition of the portfolio risk presented in Eq.(2.20):

$$R(w_1, \dots, w_n) = \sum_{i=1}^{n} w_i \cdot \frac{\partial\, R(w_1, \dots, w_n)}{\partial\, w_i} = \sum_{i=1}^{n} RC_i$$

$$RC_i(w_1, \dots, w_n) = w_i \cdot \frac{\partial\, R(w_1, \dots, w_n)}{\partial\, w_i}$$

Using the volatility as the risk measure we obtain the Eq.(2.23), Eq.(2.24) and Eq.(2.25) and we can provide the check that volatility satisfies the Euler decomposition:

$$(w) = \sigma(w) = \sqrt{w'\Sigma w} \qquad \frac{\partial\, R(w)}{\partial\, w_i} = \frac{(\Sigma w)_i}{\sqrt{w'\Sigma w}} \qquad RC_i(w) = w_i \frac{(\Sigma w)_i}{\sqrt{w'\Sigma w}}$$

$$\sum_{i=1}^{n} RC_i = \sum_{i=1}^{n} w_i \frac{(\Sigma w)_i}{\sqrt{w'\Sigma w}} = w' \frac{\Sigma w}{\sqrt{w'\Sigma w}} = \sqrt{w'\Sigma w} = \sigma(w)$$

The same proof can be done for other risk measures. As mentioned in Paragraph 2.2.1, under the assumptions of normally distributed returns, value at risk and expected shortfall are admissible risk measures:

$$VaR(w;\alpha) = \Phi^{-1}(\alpha)\sqrt{w'\Sigma w}$$

$$ES(w;\alpha) = \frac{1}{1-\alpha}\sqrt{\frac{w'\Sigma w}{2\pi}}\, e^{-\frac{1}{2}\left(\Phi^{-1}(\alpha)\right)^2}$$

A.5 Risk Budgeting Approach explicit solutions

A.5.1 Explicit solution for the two-asset case (*n* = *2*)

We start from the case in which there are two assets and the weights are subjected to a positive constraint. Let *w* be the vector of weights (composed by *y* and *(1-y)*), (*b*,*1-b*) be the vector of budgets and ρ be the correlation.

$$\begin{pmatrix} RC_1 \\ RC_2 \end{pmatrix} = \frac{1}{\sigma(w)}\begin{pmatrix} y^2\sigma_1^2 + y(1-y)\rho\sigma_1\sigma_2 \\ (1-y)^2\sigma_2^2 + y(1-y)\rho\sigma_1\sigma_2 \end{pmatrix}$$

$$y^* = \frac{(b - {}^1\!/_2)\rho\sigma_1\sigma_2 - b\sigma_2^2 + \sigma_1\sigma_2\sqrt{(b - {}^1\!/_2)^2\rho^2 b(1-b)}}{(1-b)\sigma_1^2 - b\sigma_2^2 + 2\rho\sigma_1\sigma_2}$$

The solution is quite complex even with only two assets. To make easier the interpretation I show three extreme cases:

$$\rho = 0 \quad \Rightarrow \quad y^* = \frac{\sigma_2\sqrt{b}}{\sigma_1\sqrt{1-b} + \sigma_2\sqrt{b}}$$

$$\rho = 1 \quad \Rightarrow \quad y^* = \frac{\sigma_2 b}{\sigma_1(1-b) + \sigma_2 b}$$

$$\rho = -1 \quad \Rightarrow \quad y^* = \frac{\sigma_2}{\sigma_1 + \sigma_2}$$

When the assets are not correlated, the weight of the asset *i* is proportional to the square root of its risk budget and inversely proportional to its volatility. This conclusions are quite intuitive. In fact, the higher the budget assigned to a particular asset and the higher will be the weight, while the higher the volatility and the lower will be its weight. If the assets are perfectly correlated, the weight is proportional to the risk budget, instead of its square root. Finally if the assets are perfectly negative correlated the optimal weight doesn't depend on the risk budgets, but on the volatilities of the two assets.

A.5.2 General case (*n* > *2*) with constant correlation

To provide an explicit solution for the general case we have to introduce the assumption of constant correlation (i.e., $\rho_{ij} = \rho$). If the assets are not correlated (i.e., $\rho = 0$), we have the following equation for the risk contributions, and given the normalizing budget constraints, we can find the explicit solution:

$$RC_i = b_i \sigma(w) \quad \forall\, i \qquad \Rightarrow \qquad \sqrt{b_j} w_i \sigma_i = \sqrt{b_i} w_j \sigma_j$$

$$w^* = \frac{\sqrt{b_i}\sigma_i^{-1}}{\sum_{i=1}^{n} \sqrt{b_j}\sigma_j^{-1}}$$

The weight of asset *i* is proportional to the square root of its risk budget and inversely proportional to its volatility, as in the two-asset case. In the case of perfect correlation (i.e., $\rho = 1$), we have:

$$RC_i = \frac{w_i \sigma_i \left(\sum_{j=1}^{n} w_j \sigma_j\right)}{\sigma(w)} \quad \forall\, i \qquad \Rightarrow \qquad b_j w_i \sigma_i = b_i w_j \sigma_j$$

$$w^* = \frac{b_i \sigma_i^{-1}}{\sum_{i=1}^{n} b_j \sigma_j^{-1}}$$

Again, the solution is similar to the one provide in the two-asset case, with the weight of the asset proportional to its risk budget and inversely proportional to its volatility. The last explicit solution that can be found is the case in which the constant correlation is equal to the lower bound of the constant correlation matrix (i.e., $\rho = -1/(n-1)$). In this case the volatility of the portfolio is equal to zero and the solution is the equal risk contribution (ERC) portfolio:

$$w^* = \frac{\sigma_i^{-1}}{\sum_{i=1}^{n} \sigma_j^{-1}}$$

In this case the solution doesn't depend on the risk budgets, that are equal for all the assets. The proof of this last result is reported below, with the first equation defining the risk budgeting problem with constant correlation:

$$w_i \sigma_i \left((1-\rho) w_i \sigma_i + \rho \left(\sum_{j=1}^{n} w_j \sigma_j \right) \right) = b_i \sigma^2(w)$$

$$(1-\rho){w_i}^2{\sigma_i}^2+\frac{w_i\sigma_i}{w_j\sigma_j}\left(b_j\sigma^2(w)-(1-\rho){w_j}^2{\sigma_j}^2\right)=b_i\sigma^2(w)$$

$$if \quad \sigma^2(w)=0 \quad \Rightarrow \quad w_i\sigma_i=w_j\sigma_j=\bar{w}$$

$$\sigma^2(w)=\rho\sum_{i=1}^{n}w_i\sigma_i\left(\sum_{i=1}^{n}w_j\sigma_j\right)+(1-\rho)\sum_{i=1}^{n}{w_i}^2{\sigma_i}^2=\rho n^2\bar{w}^2+(1-\rho)n\bar{w}^2$$

If the correlation reaches the minimum (i.e., $\rho = -1/(n-1)$), it's possible to verify that the volatility of the portfolio is equal to zero. Thus, the ERC portfolio is the solution when the constant correlation reaches its lower bound. The weight is obviously inversely proportional to its volatility. Except for these three special and extreme cases, it is not possible to find an explicit solution for the general case. However, it is possible to find an implicit solution. Given the formulation of the risk budgeting problem with constant correlation matrix:

$$w_i\sigma_i\left((1-\rho)w_i\sigma_i+\rho\left(\sum_{j=1}^{n}w_j\sigma_j\right)\right)=b_i\sigma^2(w) \qquad if \quad W_i=w_i\sigma_i \quad B_i=b_i\sigma^2(w)$$

$$(1-\rho)W_i^2+\rho W_i\left(\sum_{j=1}^{n}W_j\right)=B_i \quad \Rightarrow \quad w_i=\frac{f_i(\rho,b)\sigma_i^{-1}}{\sum_{j=1}^{n}f_j(\rho,b)\sigma_j^{-1}}$$

The function f_i depends on the constant correlation and on the risk budgets and it generalizes the solution found in the three extreme cases. This function has to satisfy:

$$f_i(0,b)=\sqrt{b_i} \qquad f_i(1,b)=b_i \qquad f_i(-(n-1)^{-1},b)=1$$

In this way, from the implicit solution we can derive the explicit solution in the three special cases.

Appendix B

B.1 Investment Universe

Table B.1 – Frontier Markets Complete List

	MSCI		S&P		FTSE		RUSSELL	
LATIN AMERICA	Argentina Jamaica Trinidad & Tobago		Argentina Ecuador Jamaica Panama Trinidad & Tobago		Argentina		Argentina Trinidad & Tobago	
EUROPE	Bosnia Herzegovina Bulgaria Croatia Estonia Lithuania Kazakhstan Romania	Serbia Slovenia Ukraine	Bulgaria Croatia Cyprus Estonia Georgia Kazakhstan Latvia	Lithuania Romania Slovakia Slovenia Ukraine	Bulgaria Croatia Cyprus Estonia Lithuania Macedonia Malta	Romania Serbia Slovakia Slovenia	Bulgaria Croatia Cyprus Estonia Kazakhstan Kyrgyzstan Lithuania	Macedonia Malta Romania Serbia Slovakia Slovenia Ukraine
AFRICA	Botswana Benin Burkina Faso Côte d'Ivore Guinea Bissau Ghana Kenya	Mauritius Morocco Nigeria Senegal Togo Tunisia Zimbabwe	Botswana Côte d'Ivore Egypt Kenya Mauritius Namibia Nigeria	Tunisia Zambia	Botswana Côte d'Ivore Ghana Kenya Mauritius Nigeria Tunisia		Botswana Egypt Gabon Ghana Kenya Mauritius Namibia	Nigeria Tunisia Zambia
MIDDLE EAST	Bahrain Jordan Kuwait Lebanon Oman Palestine Saudi Arabia		Bahrain Jordan Kuwait Lebanon Oman		Bahrain Jordan Oman Qatar		Bahrain Jordan Kuwait Oman Qatar	
ASIA	Bangladesh Pakistan Sri Lanka Vietnam		Bangladesh Cambodia Pakistan Sri Lanka Vietnam		Bangladesh Sri Lanka Vietnam		Bangladesh Pakistan Papua New Guinea Sri Lanka Vietnam	
	38		36		26		36	

Table B.2 – Investment Universe Complete Set

DEVELOPED	EMERGING	FRONTIER
MSCI North America	**MSCI Emerging Markets**	**MSCI Argentina**
Canada	Brazil	**MSCI Jamaica (1)**
US	Chile	**MSCI Trinidad & Tobago (4)**
	Colombia	**MSCI Bosnia Herzegovina (2)**
MSCI Europe & Middle East	Mexico	**MSCI Bulgaria**
Austria	Peru	**MSCI Croatia**
Belgium	Czech Republic	**MSCI Estonia**
Denmark	Greece	**MSCI Lithuania**
Finland	Hungary	**MSCI Kazakhstan**
France	Poland	**MSCI Romania**
Germany	Russia	**MSCI Serbia**
Ireland	Egypt	**MSCI Slovenia**
Israel	Qatar	**MSCI Ukraine**
Italy	South Africa	**MSCI Botswana (1)**
Netherlands	Turkey	**MSCI Ghana (1)**
Norway	UAE	**MSCI Kenya**
Portugal	China	**MSCI Mauritius**
Spain	India	**MSCI Morocco**
Sweden	Indonesia	**MSCI Nigeria**
Switzerland	Korea	**MSCI Tunisia**
United Kingdom	Malaysia	**MSCI WAEMU (6)**
	Philippines	Benin
MSCI Pacific	Taiwan	Burkina Faso
Australia	Thailand	Côte d'Ivore
Hong Kong		Guinea Bissau
Japan		Senegal
New Zeland		Togo
Singapore		**MSCI Zimbabwe (3)**
		MSCI Bahrain
		MSCI Jordan
		MSCI Kuwait
		MSCI Lebanon
		MSCI Oman
		MSCI Palestine (5)
		MSCI Saudi Arabia
		MSCI Bangladesh
		MSCI Pakistan
		MSCI Sri Lanka
		MSCI Vietnam

(1) Added as stand-alone country at the November 2008 Semi-Annual Index Review

(2) Added as stand-alone country at the May 2010 Semi-Annual Index Review

(3) Added as stand-alone country at the November 2010 Semi-Annual Index Review

(4) Added as stand-alone country at the May 2011 Semi-Annual Index Review

(5) Added as stand-alone country at the May 2013 Semi-Annual Index Review

(6) Added as stand-alone country at the May 2014 Semi-Annual Index Review

B.2 MSCI Indices – Prices and Returns Time Series (EUR)

Figure B.1 – Core indices Price Series

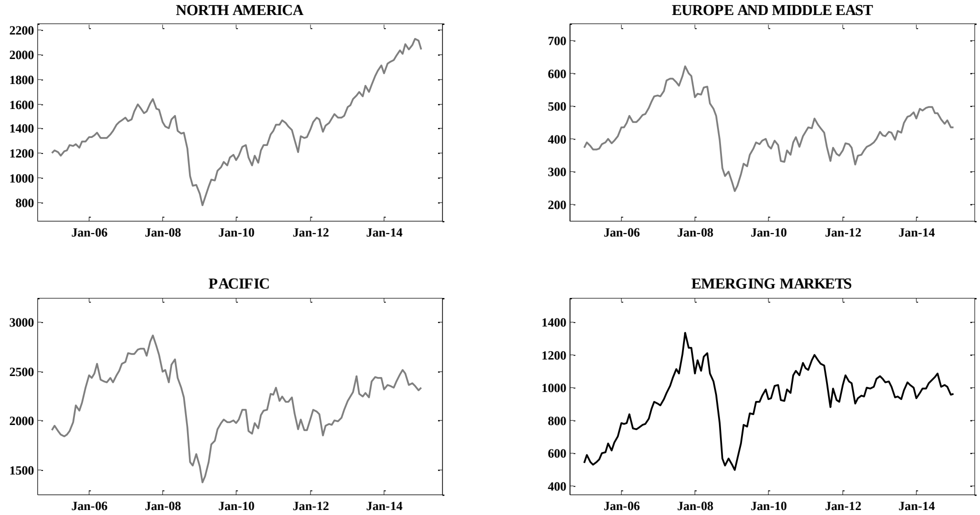
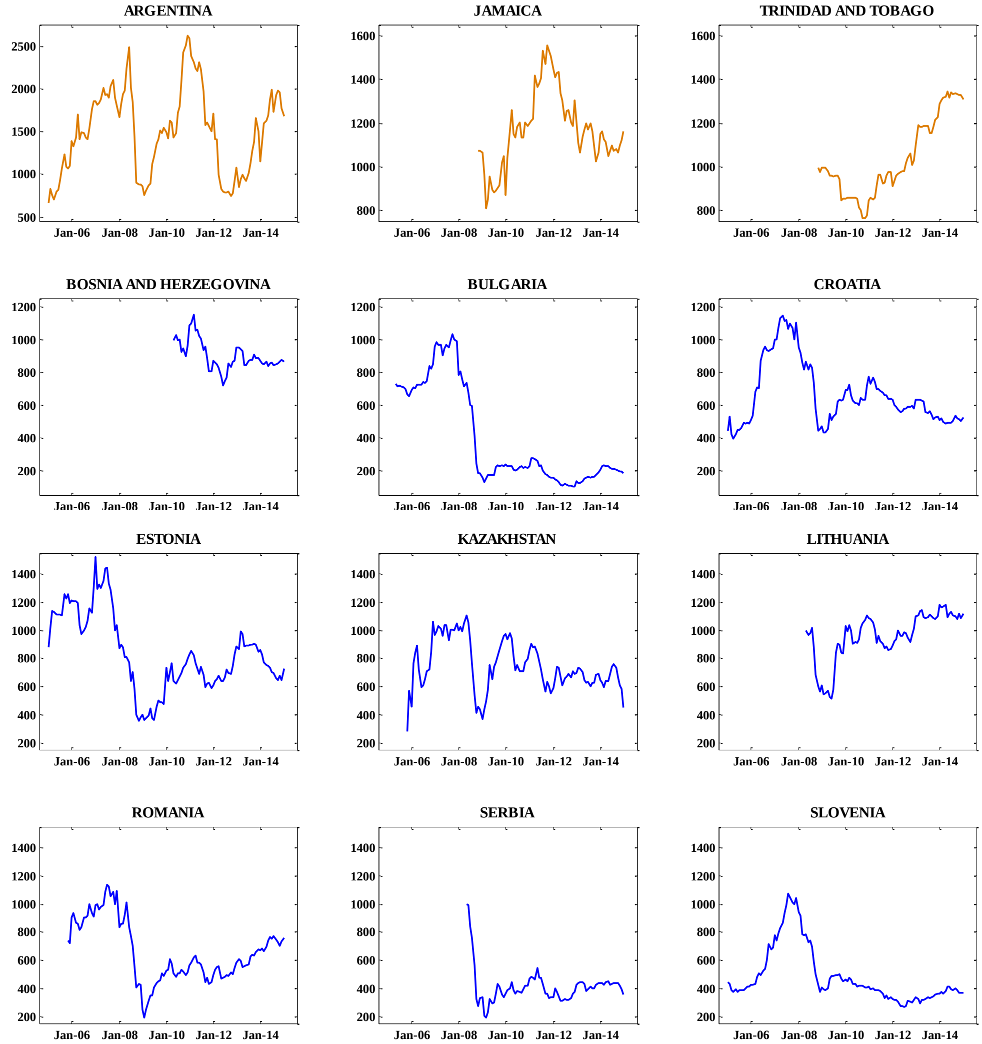


Figure B.2 – Frontier Markets indices Price Series



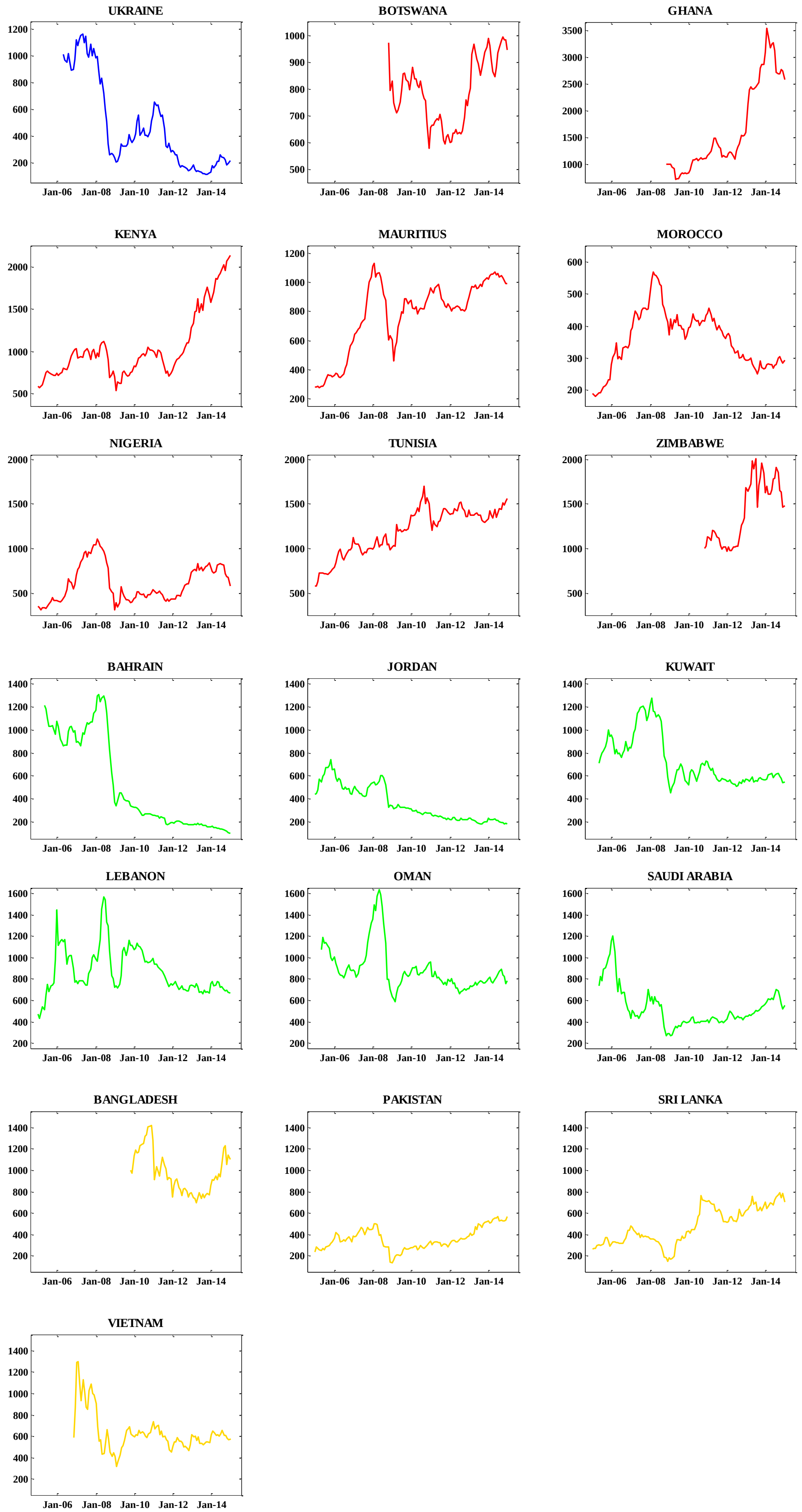

UKRAINE
BOTSWANA
GHANA
KENYA
MAURITIUS
MOROCCO
NIGERIA
TUNISIA
ZIMBABWE
BAHRAIN
JORDAN
KUWAIT
LEBANON
OMAN
SAUDI ARABIA
BANGLADESH
PAKISTAN
SRI LANKA
VIETNAM
Jan-06
Jan-08
Jan-10
Jan-12
Jan-14

Figure B.3 – Core indices Return Series and 12-Month Moving Average

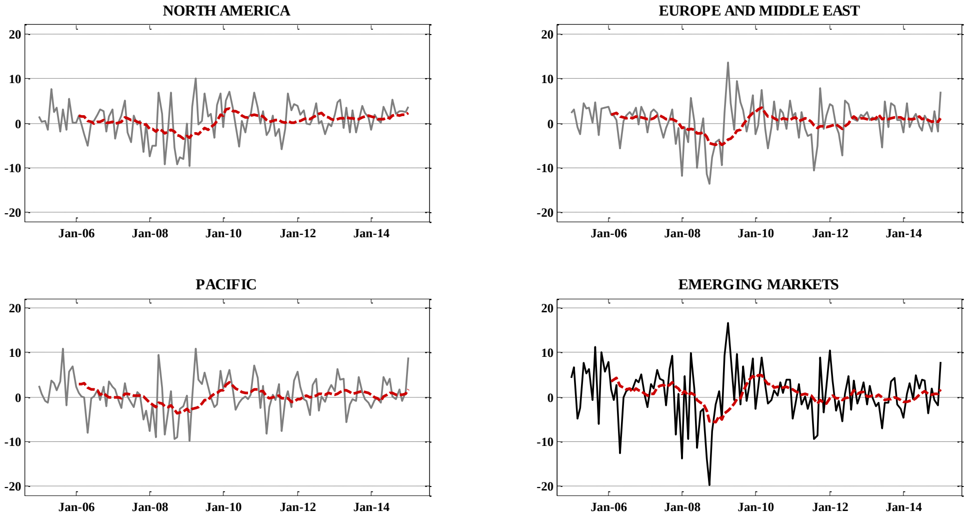
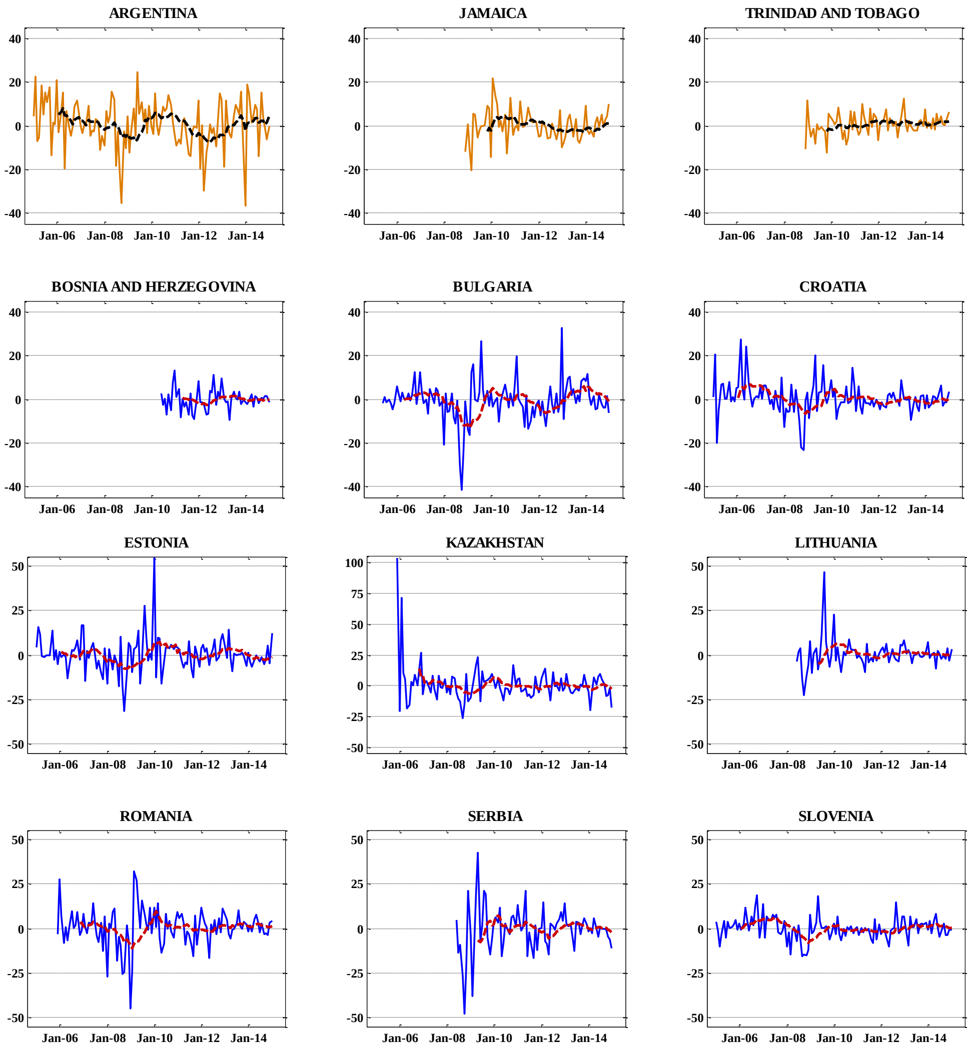
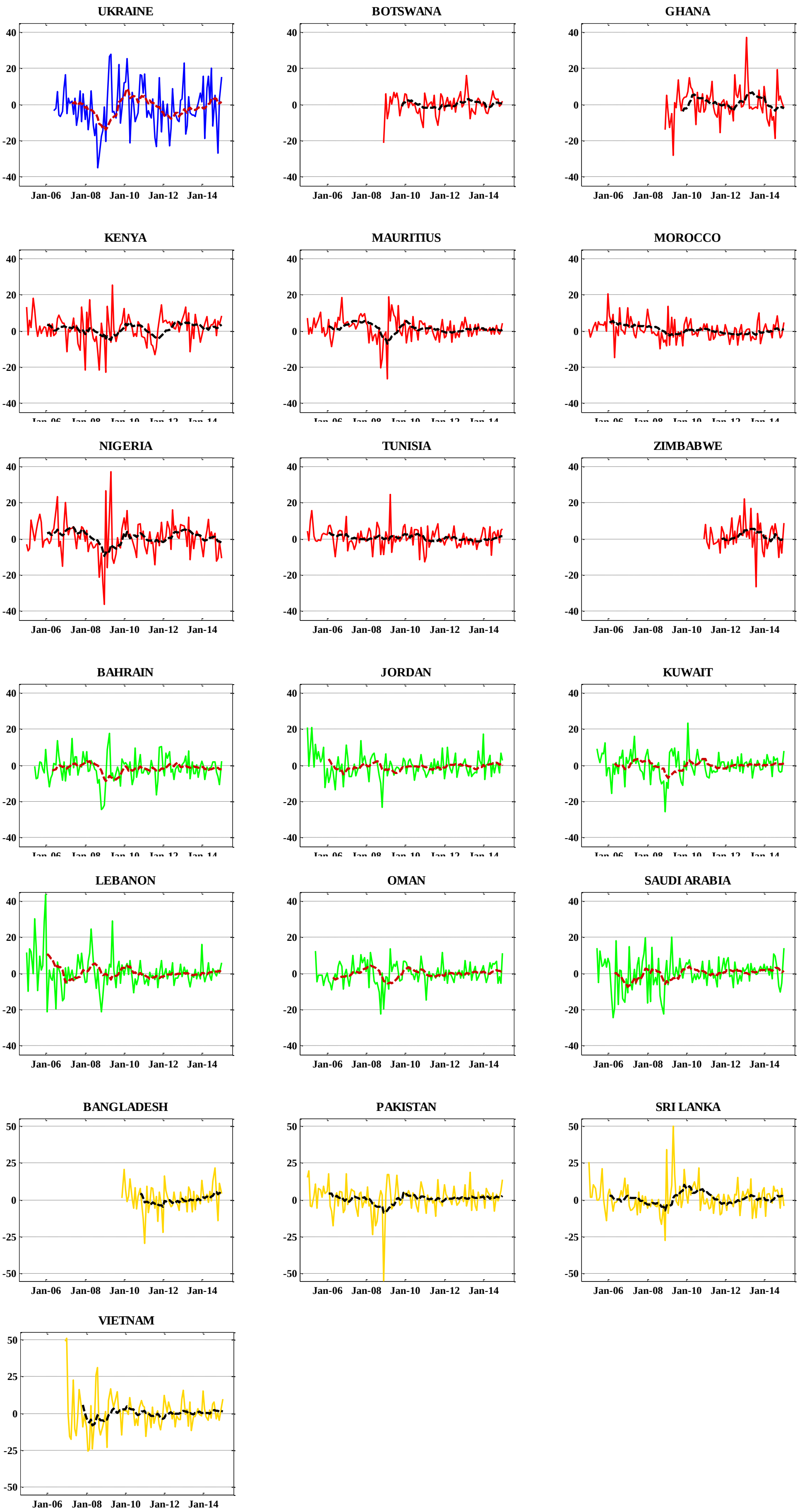


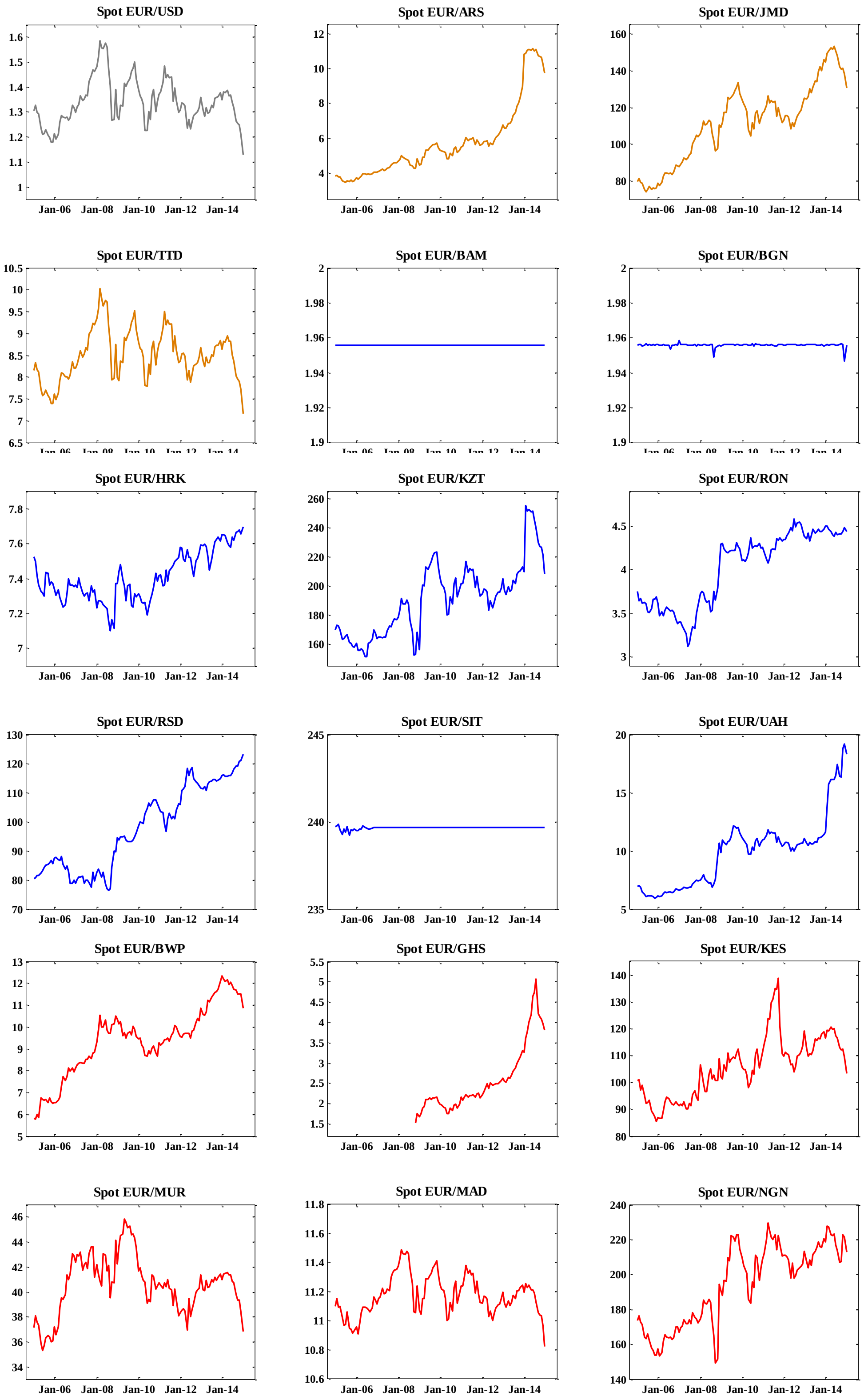
Figure B.4 – Core indices Return Series and 12-Month Moving Average

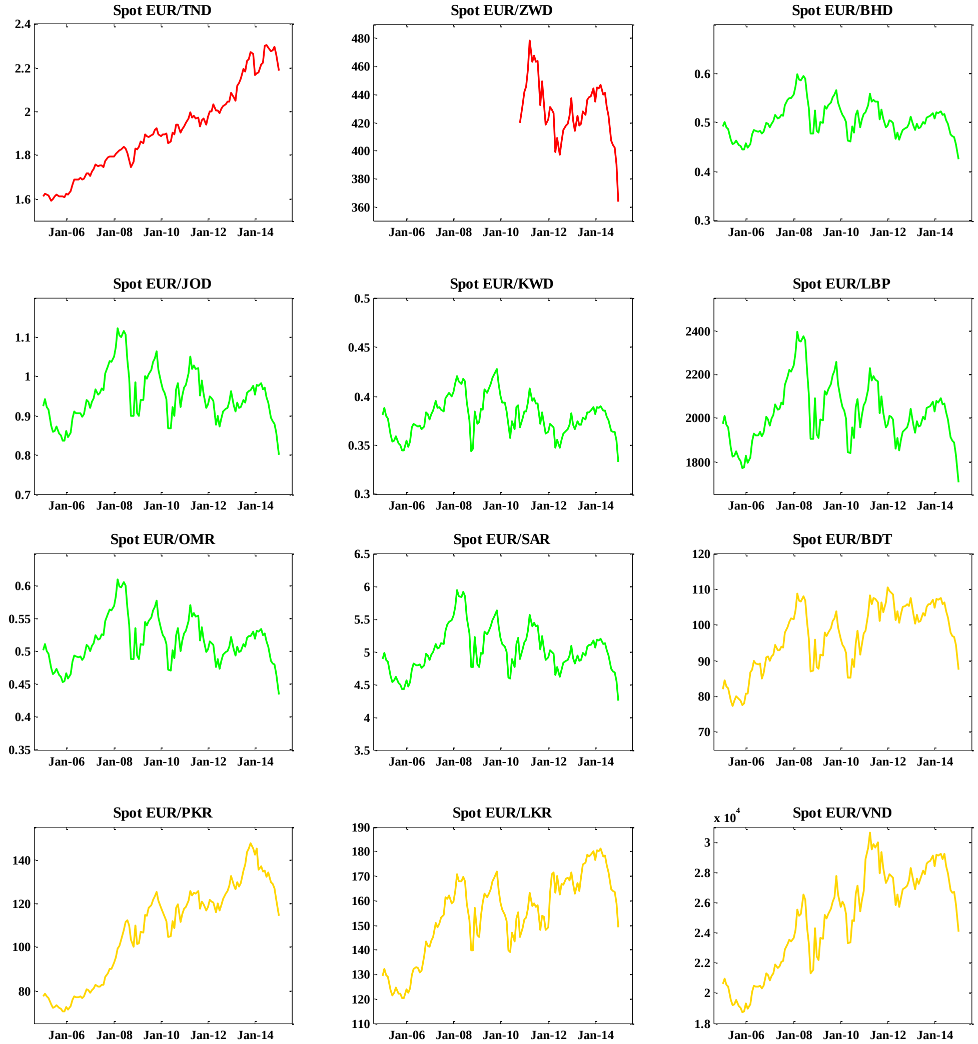



UKRAINE
BOTSWANA
GHANA
KENYA
MAURITIUS
MOROCCO
NIGERIA
TUNISIA
ZIMBABWE
BAHRAIN
JORDAN
KUWAIT
LEBANON
OMAN
SAUDI ARABIA
BANGLADESH
PAKISTAN
SRI LANKA
VIETNAM
Jan-06
Jan-08
Jan-10
Jan-12
Jan-14

B.3 Currency Composition

Figure B.5 – Spot Exchange Rate




Spot EUR/TND
2.4
2.2
2
1.8
1.6
Jan-06 Jan-08 Jan-10 Jan-12 Jan-14
Spot EUR/ZWD
480
460
440
420
400
380
360
Jan-06 Jan-08 Jan-10 Jan-12 Jan-14
Spot EUR/BHD
0.6
0.5
0.4
0.3
Jan-06 Jan-08 Jan-10 Jan-12 Jan-14
Spot EUR/JOD
1.1
1
0.9
0.8
0.7
Jan-06 Jan-08 Jan-10 Jan-12 Jan-14
Spot EUR/KWD
0.5
0.45
0.4
0.35
0.3
Jan-06 Jan-08 Jan-10 Jan-12 Jan-14
Spot EUR/LBP
2400
2200
2000
1800
Jan-06 Jan-08 Jan-10 Jan-12 Jan-14
Spot EUR/OMR
0.6
0.55
0.5
0.45
0.4
0.35
Jan-06 Jan-08 Jan-10 Jan-12 Jan-14
Spot EUR/SAR
6.5
6
5.5
5
4.5
4
3.5
Jan-06 Jan-08 Jan-10 Jan-12 Jan-14
Spot EUR/BDT
120
110
100
90
80
70
Jan-06 Jan-08 Jan-10 Jan-12 Jan-14
Spot EUR/PKR
140
120
100
80
Jan-06 Jan-08 Jan-10 Jan-12 Jan-14
Spot EUR/LKR
190
180
170
160
150
140
130
120
110
Jan-06 Jan-08 Jan-10 Jan-12 Jan-14
Spot EUR/VND
x 10^4
3
2.8
2.6
2.4
2.2
2
1.8
Jan-06 Jan-08 Jan-10 Jan-12 Jan-14

Appendix C

C.1 Economic Risk Indicator

REAL GDP GROWTH INDICATOR	POINTS
Current Value (A)	**8**
Missing Data	0
Negative Real GDP Growth	0
Positive Real GDP Growth	8
Comparison with previous years (B)	**7**
Missing Data	0
RGDPGrowth - AvRGDPGrowth (lag 1 and 2) ≤ -5.00%	0
RGDPGrowth - AvRGDPGrowth (lag 1 and 2) ≤ 0.00% & AvRGDPGrowth (lag 1 and 2) < 0.00%	0
-5.00% ≤ RGDPGrowth - AvRGDPGrowth (lag 1 and 2) ≤ -2.50% & AvRGDPGrowth (lag 1 and 2) > 0.00%	1
-2.50% ≤ RGDPGrowth - AvRGDPGrowth (lag 1 and 2) < 0.00% & AvRGDPGrowth (lag 1 and 2) > 0.00%	2
RGDPGrowth - AvRGDPGrowth (lag 1 and 2) ≥ 0.00% & AvRGDPGrowth (lag 1 and 2) < 0.00%	2
0.00% ≤ RGDPGrowth - AvRGDPGrowth (lag 1 and 2) < 5.00% & AvRGDPGrowth (lag 1 and 2) > 0.00%	6
RGDPGrowth - AvRGDPGrowth (lag 1 and 2) ≥ 5.00% & AvRGDPGrowth (lag 1 and 2) > 0.00%	7
Total (A + B)	**15**

INFLATION INDICATOR	POINTS
Missing Data	0
Deflation	0
Hyperinflation (Inflation ≥ 25.00%)	0
Inflation - AvInflation(lag1 and 2) ≥ 10.00%	0
Low Inflation (0.00% ≤ Inflation ≤ 0.50%)	3
High Inflation (5.00% ≤ Inflation ≤ 25.00%)	3
Normal Inflation (0.50% ≤ Inflation ≤ 5.00%)	5
Total	**5**

CURENT ACCOUNT BALANCE INDICATOR	POINTS
Current Value (A)	**1**
Missing Data	0
Negative Current Account Balance	0
Positive Current Account Balance	1
Comparison with previous years (B)	**4**
Missing Data	0
CA - AvCA (lag 1 and 2) ≤ -5.00% & AvCA(lag 1 and 2) < 0.00%	0
-5.00% ≤ CA - AvCA (lag 1 and 2) ≤ 0.00% & AvCA(lag 1 and 2) < 0.00%	1
CA - AvCA (lag 1 and 2) ≤ -5.00% & AvCA(lag 1 and 2) > 0.00%	1
0.00% ≤ CA - AvCA (lag 1 and 2) ≤ 5.00% & AvCA(lag 1 and 2) < 0.00%	3
-5.00% ≤ CA - AvCA (lag 1 and 2) ≤ 0.00% & AvCA(lag 1 and 2) > 0.00%	3
CA - AvCA (lag 1 and 2) ≥ 5.00% & AvCA(lag 1 and 2) < 0.00%	4
CA - AvCA (lag 1 and 2) ≥ 0.00% & AvCA(lag 1 and 2) > 0.00%	4
Total (A + B)	**5**

EXPORT INDICATOR		POINTS
Fuel and Metal Exports (A)	**Agricultural Exports (B)**	**5**
F&M Exp (% of Tot Exp) < 10.00%	***AG Exp (% of Tot Exp) < 10.00%***	**5**
Missing Data	*Missing Data*	0
No Missing Data	*No Missing Data*	5
10.00% ≤ F&M Exp (% of Tot Exp) < 25.00%	***10.00% ≤ AG Exp (% of Tot Exp) < 25.00%***	**5**
Return on IMF COM Index ≤ -10.00%	*Return on IMF COM Index ≤ -10.00%*	0
-10.00% < Return on IMF COM Index ≤ -5.00%	*-10.00% < Return on IMF COM Index ≤ -5.00%*	1
-5.00% < Return on IMF COM Index < 0.00%	*-5.00% < Return on IMF COM Index < 0.00%*	2
0.00% ≤ Return on IMF COM Index < 10.00%	*0.00% ≤ Return on IMF COM Index < 10.00%*	4
Return on IMF COM Index ≥ 10.00%	*Return on IMF COM Index ≥ 10.00%*	5
25.00% ≤ F&M Exp (% of Tot Exp) < 50.00%	***25.00% ≤ AG Exp (% of Tot Exp) < 50.00%***	**5**
Return on IMF COM Index ≤ -5.00%	*Return on IMF COM Index ≤ -5.00%*	0
-5.00% < Return on IMF COM Index ≤ -2.50%	*-5.00% < Return on IMF COM Index ≤ -2.50%*	1
-2.50% < Return on IMF COM Index < 0.00%	*-2.50% < Return on IMF COM Index < 0.00%*	2
0.00% ≤ Return on IMF COM Index < 5.00%	*0.00% ≤ Return on IMF COM Index < 5.00%*	4
Return on IMF COM Index ≥ 5.00%	*Return on IMF COM Index ≥ 5.00%*	5
F&M Exp (% of Tot Exp) ≥ 50.00%	***AG Exp (% of Tot Exp) ≥ 50.00%***	**5**
Return on IMF COM Index ≤ -3.00%	*Return on IMF COM Index ≤ -3.00%*	0
-3.00% < Return on IMF COM Index ≤ -1.50%	*-3.00% < Return on IMF COM Index ≤ -1.50%*	1
-1.50% < Return on IMF COM Index < 0.00%	*-1.50% < Return on IMF COM Index < 0.00%*	2
0.00% ≤ Return on IMF COM Index < 3.00%	*0.00% ≤ Return on IMF COM Index < 3.00%*	4
Return on IMF COM Index ≥ 3.00%	*Return on IMF COM Index ≥ 3.00%*	5
Total (A + B)		**10**

PUBLIC FINANCE INDICATOR	POINTS
General Government Gross Debt-to-GDP Ratio Current Value (A)	**2**
Missing Data	0
Debt-to-GDP Ratio ≥ 90.00%	0
Debt-to-GDP Ratio < 90.00%	2
General Government Gross Debt-to-GDP Ratio Comparison with previous years (B)	**4**
Missing Data	0
Debt-to-GDP Ratio - AvDebt-to-GDP Ratio (lag 1 and 2) ≥ 10.00%	0
0.00% < Debt-to-GDP Ratio - AvDebt-to-GDP Ratio (lag 1 and 2) < 10.00%	1
-5.00% < Debt-to-GDP Ratio - AvDebt-to-GDP Ratio (lag 1 and 2) ≤ 0.00%	3
Debt-to-GDP Ratio - AvDebt-to-GDP Ratio (lag 1 and 2) ≤ -5.00%	4
General Government Net Balance (% of GDP) Current Value (C)	**4**
Missing Data	0
Net Balance ≤ -5.00%	0
-5.00% < Net Balance < 0.00%	2
Net Balance ≥ 0.00%	4
General Government Net Balance (% of GDP) Comparison with previous years (D)	**5**
Missing Data	0
Net Balance - AvNetBalance (lag 1 and 2) ≤ -5.00% & AvNetBalance (lag 1 and 2) < 0.00%	0
-5.00% ≤ Net Balance - AvNetBalance (lag 1 and 2) ≤ 0.00% & AvNetBalance (lag 1 and 2) < 0.00%	1
Net Balance - AvNetBalance (lag 1 and 2) < 0.00% & AvNetBalance (lag 1 and 2) > 0.00%	2
0.00% ≤ Net Balance - AvNetBalance (lag 1 and 2) < 5.00%	4
Net Balance - AvNetBalance (lag 1 and 2) ≥ 5.00%	5
Total (A + B + C + D)	**15**

C.2 Liquidity Risk Indicator

INTERNATIONAL LIQUIDITY INDICATOR	POINTS
Comparison with 6-month Moving Average (A)	**5**
FedPR ≥ 2.00%	0
FedPR - 6-monthMA ≥ 10 bps & FedPR ≤ 2.00%	0
FedPR - 6-monthMA ≤ 10 bps & FedPR ≤ 2.00%	5
Comparison with previous month (B)	**5**
FedPR ≥ 2.00%	0
FedPR - FedPR (lag 1m) ≥ 25 bps & FedPR ≤ 2.00%	0
0 bps ≤ FedPR - FedPR (lag 1m) < 25 bps & FedPR ≤ 2.00%	2
-25 bps ≤ FedPR - FedPR (lag 1m) < 0 bps & FedPR ≤ 2.00%	4
FedPR - FedPR (lag 1m) < -25 bps & FedPR ≤ 2.00%	5
Number of Consecutive Months with an Increase in Fed Rate (C)	**5**
FedPR ≥ 2.00%	0
Number of Months = 4	0
Number of Months = 0 & C-Section Previous Month = 0	0
Number of Months = 3	1
Number of Months = 0 & C-Section Previous Month = 1	1
Number of Months = 2	2
Number of Months = 0 & C-Section Previous Month = 2	2
Number of Months = 1	3
Number of Months = 0 & C-Section Previous Month = 3	3
Number of Months = 0	5
Number of Months = 0 & C-Section Previous Month = 5	5
Total (A + B + C)	**15**

FedPR = Federal Reserve Policy Rate

6-monthMA = 6-month Moving Average Federale Reserve Policy Rate

C-Section = Number of Points for Section C

INTERNATIONAL LIQUIDITY INDICATOR	POINTS
Comparison with previous month (A)	**1**
TotRes - TotRes (lag 1) ≥ 0.00%	0
TotRes - TotRes (lag 1) ≥ 0.00%	1
Comparison with 12-month Moving Average (B)	**5**
TotRes - 12-monthMA ≤ -15.00%	0
-15.00% < TotRes - 12-monthMA ≤ -5.00%	1
-5.00% < TotRes - 12-monthMA < 0.00%	2
TotRes - 12-monthMA ≥ 0.00%	5
Number of Consecutive Months with a Decrease in Total Reserve (C)	**4**
FedPR ≥ 2.00%	0
Number of Months = 4	0
Number of Months = 0 & C-Section Previous Month = 0	0
Number of Months = 3	1
Number of Months = 0 & C-Section Previous Month = 1	1
Number of Months = 2	2
Number of Months = 0 & C-Section Previous Month = 2	2
Number of Months = 1	3
Number of Months = 0 & C-Section Previous Month = 3	3
Number of Months = 0	4
Number of Months = 0 & C-Section Previous Month = 4	4
Total (A + B + C)	**10**
TotRes= Deseasonalized Total Reserve (USD)	
12-monthMA = 12-month Moving Average Total Reserve	
C-Section = Number of Points for Section C	

C.3 Political Risk Indicator

WAR INDICATOR	POINTS
War Score = 5	0
War Score = 4	1
War Score = 3	2
War Score = 2	3
War Score = 1	4
War Score = 0	5
Total	**5**
0 = No Conflicts ; 1 = Riots ; 2 = Social Conflicts ; 3 = Civil War ; 4 = Serious Civil War or War against a Foreign State outside Country Territory ; 5 = Civil War with the involvement of a Foreign State or War with a Foreign State on a Country Territory	

POLITICAL RISK INDICATOR	POINTS
$20 \leq SFI \leq 25$	0
$16 \leq SFI \leq 19$	2
$12 \leq SFI \leq 15$	4
$8 \leq SFI \leq 11$	6
$4 \leq SFI \leq 7$	8
$SFI \leq 3$	10
Total (A + B)	**15**
SFI = State Fragility Index	

Appendix D

D.1 Approximate and Actual Turnover

The weights are the first relevant information for strategy comparison. Simply looking at their evolution over time, we can compute an approximate turnover. Portfolio turnover is the half of the sum of absolute weight deviations over two point in time. Eq.(D.1) presents the general formula, with *n* the number of assets included in the portfolio:

$$\tau = \frac{1}{2}|w_t - w_{t-1}|\mathbf{1} = \frac{1}{2}\sum_{\boldsymbol{i=1}}^{\boldsymbol{n}}|w_{i,t} - w_{i,t-1}| \qquad i = 1, \ldots, n \qquad (\mathrm{D.1})$$

However, this turnover is an approximation. In fact, the weights assigned to a specific asset class at the beginning of a period (e.g., month) t are not the same weights that we observe at the end of this period, just before implementing the optimal allocation for the following one (i.e., $t + 1$). This depends on the fact that the asset class have, in general, a non-null return. Given the vector of weights for period t (i.e., w_t) and the wealth at the beginning of period t (i.e., W_{t-1}, the wealth at the end of the previous period), Eq.(D.2) shows the monetary value of each of the portfolio components. Given the return of the asset during the period t (i.e., $R_{i,t}$), Eq.(D.3) presents the monetary value of each position at the end of the period.

$$v_{i,t}^{b} = W_{t-1} \times w_{i,t} \qquad (\mathrm{D.2})$$

$$v_{i,t}^{e} = v_{i,t}^{b}(1 + R_{i,t}) \qquad (\mathrm{D.3})$$

Given the Eq.(D.3), we can compute the portfolio overall value at the end of the period and the weights of the asset at the end of the period, presented in Eq.(D.4) and Eq.(D.5) respectively. The weights changed between the beginning and the end of the period because of the returns on each asset. From the weights at the end of the period, we can compute the real contribution of each asset to the portfolio turnover and, finally, the actual turnover. Eq.(D.6) contains the formula of the actual turnover (i.e., the turnover that takes in to account the changes in weights due to trading activity plus the contribution of asset returns).

$$W_t = \sum_{i=1}^{n} v_{i,t}^{e} = \sum_{i=1}^{n} v_{i,t}^{b}(1 + R_{i,t}) \qquad (\mathrm{D.4})$$

$$\overline{w}_{i,t} = \frac{v_{i,t}^{b}(1 + R_{i,t})}{W_t} \qquad (\mathrm{D.5})$$

$$\tau^{ac} = \frac{1}{2}\left|\overline{w}_{i,t} - w_{t-1}\right|\mathbf{1} = \frac{1}{2}\sum_{\boldsymbol{i=1}}^{\boldsymbol{n}}\left|\overline{w}_{i,t} - w_{i,t-1}\right| \qquad i = 1, \dots, n \qquad (\mathrm{D.6})$$

D.2 Performance Measures

There are several performance measures in order to compare different investment opportunities. It is not enough to focus only on the expected return, but we have to take into account also the risk associated to these opportunities. A correct evaluation process analyzes the returns, considering the underlying risk that the investor has to bear. In this way it is also possible to analyze different investment class. For this purpose, several risk-adjusted performance measures were developed in the past. The common characteristic of these measures is that they are a ratio between a return and a risk measure. In some cases, we consider non-extreme risk measure (e.g., volatility), in the others we consider extreme risk measure (e.g., value at risk). In this thesis I considered two performance measures with non-extreme risk and four performance measures with extreme risk. In general, higher ratios correspond to better rewards per unit of risk considered (non-extreme and extreme). Thus, the investor should select the asset or the portfolios that provide the highest values of performance measures.

D.2.1 Sharpe Ratio

The Sharpe Ratio is the most diffused performance measure in financial markets practice. Sharpe (1966) presented this ratio as the reward-to-variability ratio. The last version was formulated by Sharpe (1994). It is the ratio between the asset expected excess return with respect to the risk-free rate (i.e., return measure) and the volatility of the returns (i.e., risk measure). Eq.(D.7) shows the formula:

$$Sh = \frac{\mathrm{E}[r_i] - r_f}{\sigma_i} \qquad (\mathrm{D.7})$$

The base assumption, and the main limit, is the normality of the return distribution. The assumption of normality explained why the index focuses on the first two moments of the distribution. The main advantage is the easy computation and interpretation for each type of asset and return time series.

D.2.2 Sortino Ratio (or Index)

The Sortino Ratio, or Index, uses a non-extreme risk measure. It is similar to the Sharpe Ratio, but it substitutes the volatility with the downside risk, i.e. the volatility of the negative returns only. It is the ratio between the asset expected excess return with respect to the risk-free rate (i.e., return measure) and the downside risk (i.e., risk measure). Eq.(D.8) presents the formula of the Sortino Index:

$$So = \frac{\mathrm{E}[r_i] - r_f}{\sigma_i(D)} \tag{D.8}$$

$$\sigma_i(D) = \sqrt{\sigma^2(r_i \mid r_i < 0)} \tag{D.9}$$

Behind the formulation proposed by Sortino (2001), there is the idea that investors are not concerned about price movements in general. They cares but about the volatility (i.e., the risk) only when they are realizing returns on their investment below their minimum required return.

D.2.3 Return-VaR Ratio

The most diffused measure of extreme risk is the value at risk (VaR). Eq.(D.10) presents the ratio, while Eq.(D.11) presents the general formulation of the VaR, for a given confidence level α and time horizon:

$$VaRratio = \frac{\mathrm{E}[r_i]}{VaR(\alpha)} \tag{D.10}$$

$$VaR(\alpha) = inf\{l \in \mathbb{R} : P(L > l) \leq 1 - \alpha\} \tag{D.11}$$

Given a certain time horizon, the VaR of a portfolio at the confidence level α is given by the smallest number (threshold) l such that the probability that the loss L exceeds this threshold is at most (1- α). The main limit of the VaR is the lack of the sub-additivity property of a risk measure (i.e., the VaR of a portfolio should be smaller than the combination of the VaR of the underlying assets). This is also the main limit of using VaR as extreme risk measure.

D.2.4 Return-ES Ratio

The second extreme risk measure that I considered is the Expected Shortfall (ES). It was proposed to overcome the above-mentioned limit of the VaR. Simplifying, the ES is a conditional expectation that equals the mean of returns below the VaR. Eq.(D.12) presents the ratio, while Eq.(D.13) presents the general formulation of the ES, for a given confidence level α:

$$ESratio = \frac{\mathrm{E}[r_i]}{|ES(\alpha)|} \tag{D.10}$$

$$ES(R_t, \alpha) = E[R_t \mid R_t < VaR(\alpha)] \tag{D.11}$$

D.2.5 Drawdown-based Performance Measures: Calmar and Sterling Ratio

The Drawdown sequence monitors the losses. At a given starting point (t = 1), the Drawdown is set at zero (i.e., $D_1 = 0$). Then, the evaluation follows Eq.(D.12):

$$D_t = \min(0\,, (1 + D_{t-1})(1 + R_t) - 1) \tag{D.12}$$

The Drawdown sequence is graphically analyzed to identify the largest losses and the time to recover from losses. In general, a better strategy has smaller losses and a quick recovery from the minimums. The ratios based on the Drawdown sequence are two. Eq.(D.13) presents the Calmar Ratio and it uses as risk measure the absolute value of the maximum Drawdown. Eq.(D.14) shows the Sterling Ratio and it uses as risk measure the absolute value of the average of the Drawdowns. Given the Drawdown sequence (*DD*) for a given time period:

$$Cal = \frac{\mathrm{E}[r_i]}{|max(DD)|} \tag{D.13}$$

$$Str = \frac{\mathrm{E}[r_i]}{|mean(DD)|} \tag{D.14}$$

In financial markets practice, the Calmar Ratio is computed on a monthly basis (using a 36-month rolling window), while the Sterling Ratio is computed on an yearly basis.

D.3 Tracking Error Analysis

The previous measures are absolute measures, since they consider the returns of a given asset or strategy. In some cases, we need to perform a comparison between the performance of an asset or a strategy and a benchmark. To perform this comparison, we can implement the Tracking Error Analysis. In this analysis, the core aspects considered are the deviations of the asset or strategy returns from the ones of the benchmark. These deviations are called Tracking Errors. Eq. (D.15) and Eq.(D.16) present the computations of Average Tracking Error (TE) and Tracking Error Volatility (TEV), respectively:

$$TE = \mathrm{E}[R_t - R_t^B] \tag{D.15}$$

$$TEV = \sqrt{\sigma^2(R_t - R_t^B)} \tag{D.16}$$

The ratio between TE and TEV is also known as Information Ratio (IR). It is equivalent to a Sharpe Ratio computed on Tracking Errors, without the risk free. In financial markets practice is used to evaluate the performance of a portfolio with respect to the benchmark. It is also frequently used as the base for the computation of management fees or asset managers' bonuses, especially in hedge funds industry. As for the Sharpe Ratio, the TEV can be computed only on downside deviations. Using the Semi-TEV, we can compute the Semi-IR, that is equivalent to the Sortino Index. Eq.(D.17), Eq.(D.18) and Eq.(D.19) show the computation of the IR, the Semi-IR and the Semi-TEV, respectively.

$$IR = \frac{TE}{TEV} \tag{D.17}$$

$$IR = \frac{TE}{SemiTEV} \tag{D.18}$$

$$SemiTEV = \sqrt{\sigma^2(R_t - R_t^B \mid R_t - R_t^B < 0)} \tag{D.19}$$