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Early Warning Indicators for Systemic Risk: A MIDAS Quantile Regression Approach

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Firma del candidato

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Contents

Introduction

In finance, the systemic risk can be defined as the probability that a given set of events and circumstances undermines the stability of an interconnected financial system. In this regard, three basic elements that characterize the systemic risk can be identified:

- Probability: it is a number between 0 and 1 associated to a given event, trying to describe its likelihood of occurrence.
- Triggering event: it is the unexpected unfavorable event that hits a single component of the financial chain and then stretches throughout the entire process. It can be derived from the public system (e.g. a liquidity constraint imposed by the central bank), from an external shock (e.g. a natural disaster) or from an internal endogenous shock (e.g. the failure of a bank).
- Propagation dynamic: it is a phenomenon that depends on the structure of the financial system and it is strictly related to the level of interconnection among institutions; hence, to the intensity of contagion and to the depth of spillover effects, through a sequence of consequent events (e.g. a cascading failure).

The importance of the systemic risk is high and ever increasing for at least three categories of subjects.

- Policy makers and financial regulators deal with this topic in order to implement restrictive measures and provide instructions to the agents, pursuing the minimization of the probability of unexpected systemic occurrence, and therefore the impact of adverse events on the system.
- Academic researchers have recently channeled their attention to the systemic risk as an object of study, in order to identify its deep determinants and to elaborate increasingly complicated models for the phenomenon description and for the forecasting.
- Business practitioners, i.e. risk managers and institutional investors, need to measure, monitor and manage the exposures to systemic risk, and all that it is related to, given that this kind of risk tends to reduce the benefits of diversification, and may bring painful financial experiences.

It is important for all these subjects to measure, control and manage systemic risk, and further understand the dynamic of occurrence of risk spillovers across institutions and markets. For this purpose, it is pertinent to make a distinction about the notion of systemic risk:

- first of all, the systemic risk contribution, i.e. the negative externality that a large and strongly interconnected financial institution may exert on other institutions by just undertaking additional risk, and then by undermining the financial stability;
- second of all, the financial system risk, which refers to the overall dimension of risk, i.e. the probability to experience a systemic event.

It is also useful to frame the systemic risk in accordance with its specific source.

- Firstly, an internal idiosyncratic problem of a big single institution may easily spread to other institutions which keep many financial connections to it involving the entire system.

- Secondly, we could observe a group of institutions that share a common risky exposure toward a product, a market or a country, and that may materialize itself in an adverse event, striking the whole system.
- Lastly, systemic risk can arise endogenously inside the financial market because of financial imbalances, and unfold itself in a sudden and harmful way for the system (i.e. the case of a speculative market bubble, or the same circumstance which led to the 2008 financial crisis).

Naturally, a source of systemic riskiness is not independent from other sources and a systemic financial disruption can have more sources or can be the ultimate point of a sequence of more scenarios.

Taking into account all these features, the main involved institutions have recently concentrated their efforts in the deepening of this topic. In particular, since the 2008 financial crisis, new definitions and new measures of systemic risk have been implemented, as well as new tools in risk management and risk modeling have been created. We have witnessed significant developments about the ways in which systemic risk is being measured and assessed by public and private institutions.

The need and the usefulness to employ accurate and complete tests and reliable early warning indicators to assess the presence and the intensity of systemic risk is out of the question. Using reliable indicators is very helpful to gauge the early warning signals. In this case, the test reliability can be defined as the ability of a given model to issue signals with relatively limited out-of-sample forecast errors. In other words, the reliability of a model can be tested by comparing its forecasting performance against other models through an out-of-sample validation, which means using the known sample data to get the model parameters, and then proceeding to using the model to make predictions about unknown data, independent from the sample. These procedures are essential for assessing systemic risk, and then to implement the right corrections in the portfolios of customers, or to timely implement macro-prudential policies.

The aim of the latest research has been to create and to identify those indicators, risk measures and risk modeling that provide the best early warning signals about systemic financial risk.

The impossible challenge is to implement a comprehensive model that manages to capture and to disclose any detail and any signal of possible increasing systemic risks inside a system, like the economic and financial one, which is highly interconnected with a very complex structure that is rapidly and constantly evolving.

Considering these points, the purpose of this thesis is to try to develop and to apply a test procedure to evaluate the presence of systemic risk inside the financial markets at a given time. Especially, for the identification of signals related to systemic risk, an early warning model will be built by combining two different statistical tools: the quantile regression analysis (whose reference contribution comes from the American econometrician Roger William Koenker, which published in 1978, together with Gilbert Bassett, the book “Regression Quantiles”, a work in turn based on other past ideas and approaches not yet been explored in depth until then) and the mixed frequency data sampling regression model (MIDAS model was firstly introduced by Eric Ghysels *et al.* in 2002, in its “The MIDAS touch: Mixed data sampling regression models”, and then developed in depth by Elena Andreou *et al.* in 2010, in its “Regression Models With Mixed Sampling Frequencies”).

Each one of these approaches produces some advantages and brings specific characteristics which allow a unique and differentiated analysis of systemic risk.
Quantile regression analysis is performed for the estimation of the conditional quantiles (i.e. the median and any other percentile of the population distribution) of the variable of interest through a set of predictor variables, differently from the famous Ordinary Least Square method used to estimate simple linear regression models, and as a result, to get the conditional mean of dependent variable. Hence, quantile regression models the conditional quantile of a dependent variable, such as the first decile (q=0,10) or the ninety-ninth percentile (q=0,99), describing the impact of a set of explicative variables on different points of the conditional distribution of the same dependent variable. This analysis provides many advantages.

- First of all, quantile regression allows the evaluation of the impact of a shock on each specific part of the dependent variable distribution, while the OLS estimates give us information linked to the change of the conditional mean of the distribution. Moreover, the quantile approach is more appropriate to estimate the asymmetric impact of a shock on a distribution: it is used to investigate the relation between systemic risk and macroeconomic framework.
- Secondly, the quantile estimates are known to be more robust to extreme values with respect to OLS estimates, for the same reason the median is not sensitive to extreme values of the sample distribution, as opposed to the mean. This is very useful in the financial analysis: since heavy financial stresses are rare events, we can model linkages, interdependences and any other financial pattern in a more reliable and stable way.

Rather than running a regression among data sampled at the same frequency, MIDAS allows to combine data with different sampling frequencies, especially for those cases in which the variable of interest is sampled at a lower frequency and the relevant explicative data is sampled at high frequency. An example can be seen in some macroeconomic models, based on variable of interests sampled quarterly or annually, like the GDP growth, and explicative variables sampled monthly, such as inflation. Other examples can be found in financial economics, as we deal with abundant data at intra-daily sampling frequencies, like stock returns or daily volatilities, used to explain data with lower frequencies. The most significant advantage of MIDAS approach is to allow to efficiently exploit all available information extractable from sampling data, whatever the sampling frequency, and then avoiding the series aggregation.
By combining these two approaches, the target is to implement and to fine-tune a procedure to test the presence of systemic risk in a reliable way.
The thesis is structured as follows. Chapter 1 provides an overview of the systemic risk in every basic aspect: its definition, its origin and the relevant literature. Chapter 2 presents a description of the most important tools to measure systemic risk, by providing a categorization of them. Chapter 3 will describe the theoretical framework of the main tools needed for the empirical analysis: all the conceptual and mathematical characteristics of the two basic approaches used to implement the test, that is the MIDAS regression model and the quantile regression. The detailed description of the methodology for the empirical analysis will be described as well. Chapter 4 will show the application of the theoretical knowledge with an empirical analysis, that is the presentation of the main results. The conclusion is a brief summary of the work, with other possible extensions. The detailed description of dataset and all MATLAB codes will be reported in the Appendix A and Appendix B, respectively. The very last section will be dedicated to the list of figures and tables and to the references.

Chapter 1
Systemic Risk: An Overview

This chapter provides an overview about the nature of the systemic risk, in each main aspect: its definition, its origin and the relevant literature.

1.1 Definition of Systemic Risk

Defining systemic risk might be difficult, as the circumstances in which it materializes itself vary so much that identifying common patterns and common characteristics among situations is quite difficult. For this reason, the systemic risk lacks a univocal interpretation, and its underlying concept is wide and not clearly definable.

At the same time, the importance to find a useful and comprehensive definition is out of question. A unique and overall operating definition of systemic risk would allow the implementation of coherent models, requirements, measures and monitoring systems from regulators and financial managers, and then providing a common standard for all agents. More homogenous and more precise measures of risk in turn allow a more optimal portfolio management and the implementation of less ambiguous public policies. It would be the best target for public authorities, who, over the past two decades, introduced the financial stability of the whole financial system among the principal macroeconomic objectives.

A first definition comes from the *Bank of International Settlement*, which in 1994 defined the systemic risk as “the risk that failure of a participant to meet its contractual obligations may in turn cause other participants to default with a chain reaction leading to broader financial difficulties”. Kaufman in 1995 defined systemic risk as the "probability that cumulative losses will accrue from an event that sets in motion a series of successive losses along a chain of institutions or markets comprising a system […], that is, systemic risk is the risk of a chain reaction of falling interconnected dominos". In both these definitions, there are at least three common elements which should characterize any other definition of risk: some financial difficulties which hit an entity, the failure of the same entity and the spreading of all negative financial consequences as a chain reaction to other interconnected entities.

In 2000, Freixas, Parigi and Rochet related the concept of systemic risk to that of spreading of financial crisis from one country to another. They analyzed the role of the payment systems and the response of the central banks in dealing with financial distress due to lack of liquidity.

Furthermore, in 2000, Allen and Gale highlighted the key role of contagion in the spreading of adverse events, stating that a shock “may cause a bank to go bankrupt and liquidate its assets. This causes other banks which have deposits in it to also go bankrupt and so on. Eventually all banks are forced to liquidate their assets at a considerable loss”.

The first authors to ever provide a structured schematization of all possible scenarios of systemic crisis were De Bandt and Hartmann. They defined systemic risk as “the risk of experiencing systemic events in strong sense”, i.e. those events which cause the failure of some institution inside the market. In particular, they pointed out the two main components of the

Systemic events in the financial system

Type of initial shock	Single systemic events (affect only one institution or one market in the second round effect)		Wide systemic events (affect many institutions or markets in the second round effect)	
	Weak (no failure or crash)	Strong (failure of one institution or crash of one market)	Weak (no failure or crash)	Strong (failures of many institutions or crashes of many markets)
Narrow shock that propagates				
– Idiosyncratic shock	✓	✓contagion	✓	✓contagion leading to a systemic crisis
– Limited systematic shock	✓	✓contagion	✓	✓contagion leading to a systemic crisis
Wide systematic shock			✓	✓systemic crisis

Note: ✓ means that the combination of events defined by the cell is a systemic event. The shaded area describes cases of systemic events in the narrow sense. Systemic events in the broad sense also include the cells with ✓ in the last row.

Figure 1.1: Systemic events in the financial system

Source: ECB Working Paper no. 35 by De Bandt and Hartmann, "Systemic Risk: A Survey", 2000

systemic risk: the initial unexpected shock and the propagation mechanisms. They very clearly explained the difference between the idiosyncratic shock (which affects only a single clearly a single institution) and the systematic shock (which affects the whole market, e.g. a sudden increase of inflation rate, a stock market crash or a sudden liquidity shortage in a given financial market), and provided a clear description of the propagation mechanism, that is the mechanism through which shocks propagate through the entire system. They stated that "the transmission of shocks is a natural part of the self-stabilizing adjustments of the market system to a new equilibrium".

In 2004, Kupiec and Nickerson defined the systemic risk in relation to the negative financial consequences of its materialization, that is, the possibility that an economic shock could bring a substantial increase in the volatility of stock prices, significant reduction in corporate liquidity, potential failures and loss of efficiency.

In 2005, Chan *et al.* described systemic risk as "the possibility of a series of correlated defaults among financial institutions (typically banks) that occurs over a short period of time, often caused by a single major event". The correlated defaults are the demonstration of the contagion, a direct consequence of the level of interdependence and of the intensity of network among financial institutions. For that, authors report the typical example of the bank run, i.e. the banking panic for which most depositors decide to simultaneously withdraw their funds from the bank, creating a run that can ultimately cause bank failures.

Similarly, Billio *et al.* in 2010 argued that "systemic risk can be realized as a series of correlated defaults among financial institutions, occurring over a short time span and triggering a withdrawal of liquidity and widespread loss of confidence in the financial system as a whole". In this definition, the new element is the loss of confidence, which in the financial system might cause a violent break of the whole financial market (as it has happened in the interbank market during the recent financial crisis), with consequent reset of exchanged volumes and consistent losses for actors.

As we can see, each definition of systemic risk adds some details and some additional characteristics that mark the specific concept. In all cases, the common points are two: an unfavorable event, that is the shock, and a set of consequent negative impacts, that is the propagation dynamic, which in turn puts in trouble the whole financial system.

The systemic risk is the probability that all the above-mentioned situations will take place, or more in general, the probability that an entire market or financial system collapses, as a consequence of the materialization of some specific risk combined with a strongly interconnected system, with many interdependencies and interlinkages.
At this point, it is useful to make some distinctions across different facets of the argument.
The first distinction to keep in mind is between uncertainty and risk. The uncertainty is a situation in which one does not have a background information of the event. Hence, the distribution of the phenomenon is unknown. The risk, on the contrary, is the known probability of a phenomenon, derivable from a known probability distribution. A Knight's publication of 1921 stated that "risk is present when future events occur with measurable probability", while "uncertainty is present when the likelihood of future events is indefinite or incalculable". This distinction is relevant because the research of Epstein and Wang in 1994 proved that uncertainty, unlike risk, may lead to indeterminate equilibria, that is a set of continuum equilibria for some given fundamentals. In this context, actually not so rare, the final equilibrium price is determined by "animal spirits", and this might explain the wide volatility of stock prices.
The second basic distinction is between systemic risk and specific (or idiosyncratic) risk. As described earlier, systemic risk is the overall, not diversifiable, risk which characterizes the entire financial system, that can be summarized with the beta measure of the activity. Specific risk is the risk inherent to a specific individual asset (a particular company's stock or other security), group of securities or at most to a given asset class. This last component of risk can always be eliminated by portfolios diversification, since investors do not need to be rewarded to bear that. Therefore, this type of risk is not a crucial determinant of the expected portfolio return. It depends on the specific characteristics of the issuer company, and it is essential to appreciate its capital solidity and its economic prospect, as well as taking into account the characteristics of the sector. Therefore, specific risk should be based on the fundamental analysis of the issuer company, and it is relevant in equity pricing and in the interest rate determination of corporate bonds.
The last distinction is between systemic risk and systematic risk. Duan and Zhang, in 2013, made the following distinction: "systematic risk arises from exposures to common risk factors, and systemic risk is purely due to interconnections" and "although large systematic risk may lead to systemic risk, they are not synonymous". Consequently, the systemic risk is the probability to have a shock at company level with relative contagion due to connection and consequent break of the system, while the systematic risk can be thought as the market risk, that is the probability of experiencing losses due to common factors, including the economy (recession and expansion), interest rates change, natural disasters, geopolitical issues. For instance, in 2008, the failure of Lehman Brothers (the shock) caused a wave of financial distress throughout the whole financial system (the propagation), because of its high integration and connection. This was the realization of a systemic risk. As a consequence of the financial markets' break, a huge recession involved the world, so that whoever invested in risky assets during the previous years underwent a heavy reduction of investments value, whatever the portfolio combination. This was the realization of a systematic risk. It is evident that the realization of a systematic risk could determine an increase of systemic risk, because of the lower cash flows for firms and the lower possibilities to get funds in the financial markets. Nevertheless, as it happened in 2008, depending on the intensity of the financial break, systemic risk might lead to an increase of systematic risk as well.

1.2 Origins of Systemic Risk

The understanding of the fundamentals of systemic risk is crucial for any setting of risk management and test procedure. In this section, we will try to identify those basic factors from which systemic risk originates, involving all other institutions.

As seen, systemic risk involves two joint components: the shock and the transmission mechanism. In 2012, Billio, Getmansky, Lo and Pelizzon identified four sources of systemic risk, known as "*the 4 Ls*": losses, leverage, liquidity and linkages. While from an alteration of the first three *Ls* could stem a shock, the last *L* expresses the potential of a systemic involvement through a transmission mechanism.

Losses of a company are an important vehicle of idiosyncratic risk, which can easily trigger a self-fulfilling mechanism of instability spreading. For instance, a bank that takes a lot of downside risk through investments in risky assets, with highly volatile returns and unstable price dynamics, may face a consistent increase in the probability of losses. Losses must be cushioned within the bank: a reduction in the credit supply, an increase of the debt issue or the reduction of the bank capital value. When these types of imbalances come up, the probability of materialization of a systemic risk considerably increases (Lang and Forletta, 2019).

Leverage is a central component of systemic risk, and it must be continuously monitored, as the financial risk it entails is huge. Financial history shows that most financial systemic crisis erupted in a context of highly leveraged institutions and excessive optimism. The systemic crisis caused by an excessive systemic leveraging could lead to a consistent and persistent deterioration of the systemic health and of living standard of agents. This is due to the very strong pro-cyclicality of the leverage. When everything goes well for companies and households, the healthy balance sheet incentives the landing expansion for asset purchases, the increased asset demand push up prices and, as a result, the assets value in the balance sheet. When asset prices are widely above the intrinsic value of the asset, i.e. that explained by its fundamentals, the release of some news about the presence of the bubble or about the real solidity of a company is sufficient to trigger harmful deleveraging processes, in which massive assets sales and assets prices drop and impairments of balance sheets cause the stoppage of financial markets. This is the materialization of a systemic risk, in accordance with the basic scheme, shock and propagation. However, the financial leverage of a firm roughly measurable as the ratio between its total assets and its equity remains an easy way to increase the business profitability and the investment opportunities.

Lastly, liquidity plays a central role in the explanation of systemic risk. Generally speaking, liquidity can be defined as the possibility of an economic agent to exchange part of their existing wealth with cash, financial assets or other types of goods. In particular, there exists a standard classification of the concept of liquidity.

- Monetary liquidity is the liquidity delivered by the central bank to the financial system, in accordance with its need. In other words, it is the monetary base of the economy, given by the sum of cash and bank reserves. Through the central bank operations, monetary liquidity is provided in order to balance the eventual lacks of liquidity of the system.
- Funding liquidity is the ability to settle obligations with immediacy when due. The International Monetary Fund defines it as "the ability of a solvent institution to make agreed-upon payments in a timely fashion". Generally speaking, funding liquidity may also refer to the capacity of financial operators to get financing in a timely fashion in

terms of cash and capital. The two concepts are related. In general, as long as a company manages to maintain cash inflow higher than cash outflow at any instant of time, this company will be considered a liquid company. For a bank, cash inflow sources are new deposits, new assets sales, securitization, new debt issue or equity issue, new lending from interbank markets or directly from the central bank.

- Market liquidity is a measure of the liquidity of an entire market. A market is liquid whenever trades can be executed at very low costs, in a timely manner and with limited impacts on prices. So, time needed for the conclusion of the operation, burden of transaction costs and exchangeable amount at a given moments are the three basic components to define the market liquidity. An asset is liquid when it can be quickly sold in the market without high costs, and then "liquidated" in cash. As far as market liquidity, there is a set of stylized facts: market liquidity can suddenly dry up; market liquidity has some commonality across securities, and it is strictly related to the volatility; market liquidity is subject to 'flight to quality', that is the movement of capitals from risky assets issued by distressed companies (less liquid) towards safer assets issued by solid companies; finally, market liquidity co-moves with the market, and in particular, a negative relationship between the market liquidity and the assets returns exists (Acharya and Pedersen, 2005).

Therefore, low liquidity is a source of price volatility and market failure, and an early indicator for market stress. From that, liquidity risk is defined as the probability to become illiquid, and in particular it is a function of the availability of financing sources and of immediacy in the assets exchange. As a result, the liquidity risk is included in the asset pricing as a premium, called liquidity premium, and in the overall final yield of a security as a spread, called liquidity spread. When an asset is expected to be highly liquid, the liquidity spread will be very low, and this reduces the overall return rate and increases the price of the security.

Actually, there is evidence that the liquidity risk is stably present but has a limited impact on the return rate, and relatively much less relevant than the credit risk. Anyway, situations in which this risk is high are as rare as potentially disruptive for the whole system: when it materializes itself, the resources allocation undergoes many inefficiencies, the lack of liquidity increases the systemic risk, and the consequent financial instability could turn into a systemic financial crisis. Therefore, illiquidity and systemic risk are strictly linked: holding an illiquid portfolio entails a large price impact of a potential forced liquidation, and this affects the company's own capital. If the phenomenon involves many parties, all of them will suffer the consequences of a high correlation and of a potential global financial crisis. This is the essence of the systemic risk: the involvement of more institutions in the bearing of a shock, which aligns the correlations among assets and among equities. That is the reason why, when a systemic risk materializes itself, the "flight to quality" occurs, i.e. portfolios and wealth adjustments consisting of massive sales of more risky assets and purchase of safer assets like valuables, unrelated to the financial markets (just think of the strong increase in the gold price during a systemic distress). For that reason, market liquidity is the first alarm that reports to authority some shortcomings inside the market, and that could degenerate in a systemic distress.

With that being said, the real danger behind the liquidity risk is the default risk. A financial institution is insolvent when it can no longer entirely meet its financial obligations when due, and in particular unable to pay its medium and long-term bonds. It happens when the value of total assets becomes irredeemably lower than the value of total claims, and the cash generated

by activities is insufficient to repay debtors. The endpoint is the closing of the involved company. The issue consists in distinguishing institutions that are just illiquid from those that are insolvent. During the period of economic expansion and financial stability of the market, it is quite easy to identify insolvent institutions and to understand when a company is facing just a temporary problem of liquidity. But when the recession comes and the financial market crashes, the widespread liquidity problems that many companies must face make it very difficult to identify the real insolvent firms from those which have no problem with long-term cash flows and repayments. There is no clear separation between liquidity problems and insolvency cases, and the systemic risk becomes more intense and heavier.
At the basis of liquidity shortages for a firm there is a mixture between market illiquidity and the so called “maturity mismatching”: it is the situation in which a firm holds more short-term liabilities than short-term assets, and equivalently more long-term liabilities than long-term assets. A related concept is the “liquidity mismatching”, when at any time cash flows generated by financial assets does not match with cash flows needed to repay financial liabilities when due. As long as a company manages to repay any debt whenever it is due, it means that there is no internal liquidity problem, and no financing problem, even though the leverage is high. The real problem comes up when financing becomes too costly and the market liquidity quickly dries up. An interesting case is the liquidity management of a bank. A bank borrows funds from deposits, that are short-term liabilities, and lends funds in the long term, asking a higher interest rate which allows an interest spread: this is the net interest margin, and it is an important source of profit for a bank. Hence, the only maturity mismatch is not a danger for the system, but problems arise when market liquidity dries up and depositors demand all their funds back. This is the case of a bank run, that can heavily undermine the solidity of a bank and triggers a systemic crisis. Diamond and Dybvig explained in a famous model how liquidity mismatch may cause a self-fulfilling panic.
Taking into consideration the above-mentioned points, the capacity to get financing is a crucial aspect for an institution, and it is a decisive factor to monitor in order to maintain financial stability.
The last fundamental element, in order to have a complete general understanding about the origins of systemic risk, is the linkage. There would not be any systemic risk in a context without linkages among institutions, because the propagation mechanism would be missing. About this last component, the starting point of the analysis is the financial contagion theory.
The financial system can be imagined as a very interwoven chain, where assets of an institution towards others correspond to liabilities from the same institutions, issued to finance other activities. If a bank undergoes a reduction in the value of its asset, it will probably have a more prudent behavior, by reducing further exposures and risks, in order to maintain itself financially solvent. This means to reduce loans granted to other institutions, to sell or securitize some assets, to convert deposits and other liquid assets in cash, and finally to cushion a loss on equity. Each one of these operations produces a simultaneous effect on the balance sheets of institutions which are in some way linked to the bank. For example, if a bank declares default on its obligations, then all claim-holders will face a loss equal to the total amount lent to the bank; if one of them undergoes such a high loss that its own capital is insufficient to absorb it, the solution is in turn to declare default. The transmission mechanism could easily involve the whole system, as a consequence of a so-called “domino effect”.
The concept of contagion has a basic role in this dynamic. The interdependence among institutions of the financial chain is related to both payments system and direct loans. Studies

have been conducted on the mechanism of contagion and what emerges is that its impact is not necessarily strong, but it is mitigated by the fact that financial institutions do not sit and watch, rather, they obviously counteract to the incoming distress. Methods to predict and respond to potentially systemic shocks are constantly being developed and updated. Therefore, although they cannot entirely eliminate the risk of contagion, they still provide tools for risk mitigation.
A contagion can stem from a default, but also from the variability of asset prices. Fluctuations of asset prices can entail losses big enough to create a shock to some institutions, and trigger a propagation, even without direct reciprocal loans among them. The consequent losses may make it more difficult to get financing, forcing further constriction in the activities and in the assets value. Reductions in the equity value are equivalent to reductions in the assets value of other financial intermediaries, and asset sales may bring to lower prices, which will cause further balance sheet effects, and then higher losses for an increasing number of institutions (Brunnermeier and Pedersen, 2008). In general, a leveraged institution faces a stronger reduction in its value: when its assets value decreases by a given percentage, the net value of the institution might decrease by an even higher percentage, because of a greater reduction for the equity, that is for that part of balance sheet available to absorb losses.
There are three main ways to recapitalize the bank and to restore a sustainable leveraging: it is possible to ask for a capital infusion by stockholders, therefore increasing the capital; it is possible to ask for a debt restructuring, which means reducing the liabilities; it is possible to sell part of own assets, by selling them and getting liquidity to repay liabilities. This last action can trigger the so-called asset price effects and the above-mentioned balance sheet effects, and if these negative impacts are discounted in the market valuation by economic agents, the chances for a materialization of a systemic risk increase considerably.
As earlier mentioned, the initial condition of leveraged institution may amplify the asset price effects on the balance sheets. The two basic financial mechanisms which describe these systemic distressed situations are named “loss spiral” and “margin spiral”, and both identify the more general “liquidity spiral”.
In a loss spiral, an initial external shock, like a loss, a default or an unexpected cost increase, could cause important liquidity problems inside the internal liquidity circuit of a company. As said, liquidity problems may force the asset liquidation, which may trigger some downward price fluctuations and further losses on existing positions, then generating a systemic involvement.
The margin spiral strengthens the loss spiral. When an investor buys an asset, they can use this asset as collateral and borrow money against it. The difference between the security’s price and its value taken as collateral (because a loan is never granted for the total price) is called margin, and must be covered by the equity. Equivalently, the total asset value of a firm is financed by a given amount of debt, but not completely, because a part must be covered by its own capital. Hence, the total margin can never exceed the total own capital at any time, and it determines the maximum leverage. When a loss, as a default, overwhelms a company, along with the loss spiral, the margin spiral triggers as well: the assets reposition got through sales causes an overall price reduction, making prices move away from their fundamentals; this process, other than to make losses on existing assets increase, (causing further funding problems) causes an increase of the margin, which forces a leverage reduction and further asset sales. Moreover, when funding liquidity becomes stricter, institutions becomes more adverse to taking long positions in high-margin securities, because of the high capital required. But lower market liquidity leads

to more price fluctuations, higher volatility, higher financing costs and consequently higher financing risk.

In this context, risk managers are more likely to wish a dejection of their leveraging, but when many institutions de-leverage their positions, the market liquidity suddenly dries up. Losses spiral and margin spiral are self-perpetuating pro-cyclical processes that heavily amplify an initial shock like a default, as well as the burden of the systemic risk. In particular, Adrian and Shin show that leverage is pro-cyclical in a financial system in which balance sheets are continuously marked to market, because assets' prices fluctuations stimulate continuous adjustments, corrections and reactions by financial institutions. In this framework, aggregate liquidity can be seen as the growth rate of the aggregate financial sector balance sheet: when asset prices increase, balance sheets quickly become stronger and the leverage degree declines; the accumulated surplus capital is progressively employed in expanding balance sheets, through more asset purchases and loans and more short-term debt. When the borrowers research is very intense, the risk to deal with sub-prime borrowers considerably increases, up to the downturn of the credit cycle and the materialization of the systemic risk.

Finally, there is one last more theoretical issue that is strictly related to the propagation of a systemic risk. It concerns the externality, which plays a key role in the risk spreading. In economics, an externality is a negative or positive impact on the wealth of someone of an action performed by a third party. In finance, negative externalities due to the behavior of an institution can more easily affect the wealth of other institutions, their actions and the overall financial stability.

There are many examples about financial externalities and their dynamics.

One of them is the rapid and generalized spreading of negative information about the solidity of an institution, in a context of strong information asymmetry. The only failure of a bank could trigger a bank run towards other banks, even if performing and totally solid, for the sole reason that depositors fear the real solidity of their own bank, perceived as a failing bank as well. A correct, transparent and continuous communication between the financial intermediaries and their customers could reduce the fear because it would reduce the asymmetric information.

The same goes for those clients who were debtors of the failing bank: it may become more difficult for them to get financing by other banks, because a bank failure causes more burdensome restrictions on credit lines, and less trust towards former failed bank's clients. The failure of a bank leads to a loss of information on relationships and creditworthiness, and alert the entire network that the failed bank was not doing a correct screening and monitoring of its own clients. The consequence is a negative externality on former clients and stakeholders.

Another example is the individual decision of a financial intermediary to become universal and to sell any kind of financial product and financial service for any type of clientele. This decision implies the necessity to enter the financial chain in such a way that it can forge the highest number of relationships and contacts. The expansion, the internationalization and the universalization of a company and of its activities may create a focal point inside the financial system, so that the well-being of such institution determines the stability of the entire system. It is the so-called "too big to fail" and "too interconnected to fail" situation, and the complexity of these linkages considerably increases the systemic risk. On this specific matter, there are important monitoring policies and moral hazard issues for public authorities, whose main target is financial stability.

A last example is provided by the strict relation between the financial system and the real productive economy. When a bank faces a shock, rather than selling assets, it can decide to perform a credit rationing and increase interest rates on new debts; this decision unavoidably falls back on consumption, saving and investment possibilities and desires of firms and households. The result is a probable reduction of transactions, output and prices. The consequent recession involves the entire system, and this is a very big negative externality which in turn hits the financial markets. It is a task of public authorities to mitigate both the intrinsic systemic risk of the system (through a set of provisions and regulations and through a correct setting of monetary policies by the central bank), and the possible vicious cycle that a recession may cause on financial safety of economic agents, through a right implementation of economic policy and public intervention.

1.3 Brief Literature Review on Systemic Risk

This section presents a review of the main contributions and the main conclusions concerning systemic risk in the literature of the past decades. It is important to note that the more consistent progress about this topic have come true over the last two decades, and this is probably due to a deeper understanding of financial dynamics, to the introduction of more complicated and complete econometric tools and to the necessity to provide explanations on relatively recent global phenomena and events, such as the global financial crisis.

The introduction of systemic issues dates back to the Great Depression, when the classical economic theory faced some difficulties in explaining the persistence and the harmfulness of the crisis. Keynes first, in his *General Theory*, found a possible interpretation of the crisis, and first dealt with the topic of basic uncertainty and its role in the disequilibrium of the capitalistic systems. Later, other authors, such as Kindleberger and Minsky, argued that the financial structure endogenously leads to the increase of the fragility of the system, with an increase in the probability of crisis; these authors implicitly pointed out the endogeneity of systemic crises and, therefore, the presence of a systemic risk component. During the '70s and '80s, due the occurrence of a sequence of international economic instabilities and further financial crisis over the world, more contributions to the theory on systemic risk were provided by economists. In particular, adjustments mechanisms of financial systems to external shocks was analyzed, in order to formalize an overall view about crisis, which can be generated either endogenously and exogenously. The investigation about behavioral aspects and about their role in the materialization of a systemic crisis began and made progress. Nonetheless, up to 1980s, the research about systemic issue remained limited to a few pieces of work, and mostly focused on descriptive economic and historical analysis of systemic events, such as crisis.

From the '80s onwards, contributions increased in number and improved in quality. The systemic phenomenon was studied by economists from different perspectives, without necessarily a common direction of interest, and through the usage of different backgrounds and different methodologies. The massive application of new mathematical and econometric models and the analysis performed by means of financial data and empirical evidence gave a strong boost in this field. The consequent proliferation of working papers about systemic risk over the last twenty years of the XX century was very heterogenous.

It is possible to follow three directions of investigation in the literature about systemic risk (as suggested by Bazzana and Debortoli, 2002), identifiable in accordance with the origin place of the systemic event: the payment system, the financial system and the banking system.

The decision to use a given payment system and settlement procedure for the financial transactions has been seen as an important characteristic for systemic events. Authors have focused on the features of settlement structures and of payment systems as determinants of systemic crisis. The study of these systems implies a deep analysis of the network of relationships among financial institutions, and the investigation of basic concepts as the shock propagation and contagion. In particular, the payment system can be a source of systemic risk because of a direct and fast transmission of shocks to others or because of an unexpected malfunction of the same payment system. Therefore, researchers focused on the definition and identification of some types of risk, whose monitoring and measuring nowadays are essential for risk managers: the credit risk, the liquidity risk, the operational risk and the legal risk.

Simultaneously, the financial system began to be analyzed as a vehicle of systemic shocks that lead to systemic crisis. Contagion models derived starting from investors' portfolio decisions and asset price dynamic models have been implemented.

In general, the discussion developed on the fact that the financial market constitutes a mean of propagation, but also a source of systemic events. For example, as it has been mentioned before, large price fluctuations and market liquidity crisis can affect single financial institutions, and from that spread through the whole market, or can directly affect large part of the market, as a large number of operators are involved.

Researches of Calvo and Mendoza showed that the globalization of risky assets markets may reduce incentives to collect costly country-specific information and increase incentives to hold an arbitrary market portfolio. All of this strengthens the contagion among investors, and therefore the systemic risk; with short-selling constraints, the gain to get information at a fixed cost could reduce as markets grow.

Schinasi and Smith found that the role of leverage and of diversification is essential and sufficient to explain the contagion: for investors it is optimal to sell risky assets when a shock comes up, and the portfolio rebalancing leads to the spreading along the financial chain. Therefore, contagion can be explained without assumptions of market imperfections, but just through the standard portfolio theory.

Kodres and Pritsker argued that the contagion is due to the portfolio rebalancing on international markets after a macroeconomic shock, and the information asymmetries amplify this impact. Moreover, they provide explanations and evidence about the particular suffering state of emerging countries (at that time, many crises were involving emerging countries, above all Russia, Mexico and Argentina).

Allen and Jagtiani studied the effects of bank portfolio diversification. When an institution includes non-banking activities in its asset portfolio, it reduces its exposure to sectorial idiosyncratic risk but increases exposure to systemic risk. This reduces the same potential of diversification, and since this risk does not appear in the risk premium, the systemic risk exposures are enhanced.

Das and Uppal found that the effect of systemic risk on the composition of the portfolio is limited, and that systemic risk slightly reduces the benefits of diversification suggested by standard theory. They started from two stylized facts: returns on international equities are characterized by jumps; the jumps simultaneously occur across countries, creating the systemic risk. They implemented a model of equity returns through a multivariate system of jump-diffusion processes, where the arrival is simultaneous across assets. They concluded that systemic risk does affect the allocation between the riskless and risky assets, but there is a small impact on the composition of the risky portfolio, and this reduces the benefits from international diversification.

A fundamental contribution to the economic and financial theory has been given by that line of research that tried to overcome all those unrealistic assumptions of "perfection" and homogeneity of the world in the standard theory. During '80s, many authors discovered important features used to elaborate more realistic assumptions and models, in order to catch the imperfections of the world. In particular, the causes of the assets price dynamics were investigated, as a sudden fall or unjustified growth, with models based on the hypothesis of heterogeneous agents instead of representative agent, and the pieces of work that emphasized psychosocial and cognitive aspects.

Grossman and Stiglitz proved that prices just partially reflect all the information available, and for that reason markets are not always efficient. Haltiwanger and Waldman pointed out that agents are heterogenous in preferences and in ability to analyze information, and formulate their own expectations with limited rationality for the most. Hart and Kreps demonstrated that the idea that rational speculative activity should bring to more stable price is not always correct.

As a result of these new important economic principles, following researches in finance focused on the real and evident discrepancies between theory and evidence: many investors don't follow the advice of financial theory, don't hold the market portfolio, don't diversify correctly and buy a limited number of stocks, selected after a personal deepening, a given public announcement, or after seeing a mass phenomenon, then combining the rational approach with the emotionality. Authors defined these investors as noise traders: they act on the basis of partial information and in general of what they consider useful to give some individual advantages, when in reality it is imprecise and irrelevant information.

De Long *et al.* highlighted the importance of these agents because they are the majority, and their actions heavily affect the price formation process. Based on this hypothesis of "noise", many models on prices dynamics have been implemented.

An important conclusion has been provided by Calvo and Mendoza in 1999: within a financial market, investors can be divided between informed and uninformed, where uninformed ones tend to imitate the informed ones; therefore, causing a problem of signal extraction when informed investors act in accordance with information not related to fundamentals, and this favors the propagation of eventual shocks through the financial markets.

Brock and Hommes implemented a model based on the assumptions of heterogenous agents and with limited rationality, and they demonstrate how their choices affect the price fluctuations and the market trends.

Camerer provided important contributions on the distancing of assets prices from their fundamental value. In particular, he defined three phenomena: growing bubbles, information bubbles and fads. Growing bubbles are explosive trends of assets prices in a context of rational expectations, where investors, in a social mechanism of coordinated opinions, expect further increases. Information bubbles are price deviations from fundamentals due to some market failures which impede the price to correctly embed all available information. Fads are price deviations from fundamentals due to social and psychological forces following a change in risk perceptions or in the perceived utility. In general, a financial bubble is a powerful and dangerous vehicle of instability and systemic risk. Though there is a wide literature about causes and consequences of bubbles, and the mechanisms that lead to their formation and explosion, in this context it sufficient to say that possible causes of bubbles are: excessive monetary liquidity in the financial system, which incentives inconsiderate leveraging and excessive credit supply by the bank, causing more asset price volatility; social and psychological factors as investors' herd behavior, investors' irrationality and incapacity to derive the exact assets' fundamental value, moral hazard (which induces agents to undertake an excessive level of risk).

The last line of investigation on systemic risk is linked to the banking system, and in particular to the transmission mechanisms of a shock from a single bank to the whole system.

Generally speaking, systemic risk is an unavoidable and intrinsic part of the financial sector, which is more susceptible to shocks and contagions than other sectors of an economy, mainly at banking level. The vulnerability of the banking system unfolds in those cases of run, and it is due to the intrinsic nature of the banking activity: savings collection through deposits and funds lending, which consists mainly of illiquid and long-term operations.

With this structure, a sudden and unexpected increase in the withdrawals would force the bank to liquidate its assets, incurring in significant losses. As said, the difficulty of a single bank may involve the entire banking system by transmitting it through some channels, and from that it may reach the entire economy. These propagation channels, which have been widely studied throughout years, are the same ones as the priorly mentioned: the direct exposure channel, from which the domino effect occurs, and the information channel, from which the bank run.

In this framework, the reference point is the work by Diamond and Dybvig, who, in a famous model, tried to explain how the banks' holding of illiquid assets like loans, and liquid liabilities like deposits, may cause a self-fulfilling panic among depositors. In fact, banks hold only a fraction of the deposits and lend the remaining part, while a sudden increase in withdrawal requests, met on the basis of the first-come-first-served rule, may prompt all bank's depositors to withdrawal as well, because of the increased fear about bank's insolvency. Potentially, the liquidity shortage could drag the bank into a deep crisis.

Gorton confirmed the same intuitions: a banking crisis occurs when, due to informative problems, depositors decide to withdraw their funds and use them in alternative and more profitable way, fearing a poor performance of the bank's assets.

Chen made remarks saying that banking panic and contagion occurs when, in a situation of first-come-first-served rule, and under information lack and imprecisions about the bank's patrimonial health, depositors get scared and decide to withdrawal.

Aghion, Bacchetta and Banerjee went even further and discovered that, when a bank is unable to find funds necessary to satisfy the higher withdrawal requests, the probability of bankruptcy raises; this triggers panic because of the spreading of beliefs of lack of liquidity, and this boosts contagion.

Allen and Gale deepened the liquidity shock transmission among banks. In particular, they explained how the strength of a contagion depends on the structure of intermediaries' relationships in the various regions. Since liquidity shocks are not perfectly correlated across regions, an optimal practice for a bank is to hold assets from institutions located in other regions. This would provide an insurance against liquidity preference shocks. It is proved that the more interwoven the network of connections, and the more complete the structure of interregional claims, the more stable and robust the system. Theoretically, this important conclusion, later confirmed by the financial network theory, is a way out from a systemic point of view, with the purpose of reducing the systemic risk.

Freixais, Paris and Rochet argued that banks face liquidity needs as depositors do not know the place where they will need to consume. This encourages the creation of interbank credit lines in order to better cope with liquidity shocks. Unfortunately, this interbank exposition increases systemic exposition due to coordination failure, even if all banks are solvent. A bankruptcy due to some kind of shock will affect the entire system in accordance with the pattern of payments across areas, and this can also affect healthy banks.

Peek and Rosengren demonstrated that, while in the past local shocks were more contained within the country of origin, today, due to globalization, advanced technology and new structures, they quickly spread at an international level.

Over the last twenty years, the literature about every faceting of systemic risk has grown rapidly, involving many different aspects, methodologies and interpretations.

A line of investigation tried to develop new mathematical and statistical measures of the intensity of systemic risk, by inferring it directly from assets prices and their correlations.

A great contribution in this regard has been given by Adrian and Brunnermeier. At first, they elaborated the concept of Conditional Value at Risk (CoVaR), that is the value at risk of financial institutions conditional on an entire distressed financial system. Then, they measured the institution's marginal contribution to systemic risk as the difference between CoVaR and the financial system's VaR. Finally, they proposed a measure of the overall systemic risk by taking the difference in the VaR of the financial system conditional on a distressed institution with respect to median state: the ΔCoVaR. They showed that all the main determinants of systemic risk, as foreseen by the theory, (leverage, liquidity, maturity mismatching, dimension and price bubbles) are significant in the explanation of ΔCoVaR; this also implied a strong predictive power.

Billio, Lo, Pelizzon and Getmansky edeveloped several econometric measures of connectedness, and then useful proxies for systemic risk, based on Granger-causality networks theory and principal-components analysis applied to financial institutions' monthly returns. They discovered that all financial sectors have become increasingly connected over the last decades, therefore increasing the systemic risk with complicated varying network of relationships; moreover, there is an evident asymmetry toward the banking sector, which turns out to be a core element in the transmission of shocks.

Acharya *et al*. defined useful measures regarding each financial institution's contribution to systemic risk: the systemic expected shortfall (SES) is the tendency of an institution to be undercapitalized when the whole system is undercapitalized; SES increases with its Marginal Expected Shortfall (MES), that is the expected loss in the tail of systemic loss distribution.

Recently, Brownlees and Engle presented a measure of systemic risk contribution for a company, called SRISK. It measures the firm's capital shortfall when a heavy market prices decline occurs, as a function of company's size, leverage and long run marginal expected shortfall. Moreover, they proved that it is able to capture early warning sign of a crisis.

Furthermore, many other researchers developed other important systemic risk, by using market and balance sheet information (e.g. by Chan Lau and Gravelle, Avesani, Duan and Wei, Huang).

Meanwhile, the importance of the deepening and modeling at best the role of systemic correlations among institutions, prices and assets classes arose.

Lee, Lin and Yang found that the assets correlations increase with company size but reduce with its default probability. Additionally, they proved that assets correlations are industry specific, asymmetric and with a pro-cyclical impact on real economy, rising during economic downturns and declining during economic upturns. In general, many studies reported that financial crisis are associated with increase of both cross-correlations among stocks and the level of systemic risk.

Allen *et al*. created a model to show how asset commonality and short-term debt of banks interact to generate an excessive systemic risk. In fact, when banks exchange assets to reduce individual risks, two asset structures may emerge: a clustered asset structure, where groups of banks hold common asset portfolios and default together, or an uncluttered asset structure, where defaults are more dispersed. In this framework, information contagion can more likely be found in the clustered structure, unless the bank debts are more long-term.

Das and Uppal created a model to determine the investor's optimal portfolio through a multivariate system of jump diffusion processes. They based this model on a set of confirmed stylized facts: returns of international equities are characterized by jumps; jumps tend to occur simultaneously, and this generates the systemic risk; systemic risk reduces the benefits of diversification and hits more leveraged positions. They found that, while losses from reduced diversification may be smaller, the loss from highly leveraged positions may be larger.

Busse *et al*. once again confirmed, through a probabilistic approach, that the systemic risk does reduce the gain from diversification. In particular, they tried to compute the risk loading on the portfolio premium due to the presence of systemic risk, by using measures as VaR and Tail VaR.

A very important historical event, which marked the western economies forever, the political institutions, hence the academic research on systemic risk as well, was the 2008 financial crisis. This event was taken as an object of study by many authors, as a case either for deepening new branches and analysis methodologies, and for further applications to investigate in order to fulfill shortcomings of the theory. In particular, indirect spillover effects, common exposures and informational contagion played an important role in the crisis, because they triggered liquidity spirals and the blackout of the financial system. The failure of such a big, relevant, interconnected and central investment bank, as Lehman Brothers Holdings Inc., is considered one of the strongest shocks ever hitting advanced economies in recent times, and that caused the materialization of a huge systemic risk. In general, the 2008 financial crisis demonstrated that factors for financial distress of large parts of the economy strongly depend on the interconnections among financial institutions. Moreover, an increasingly set of new financial instruments emerged in order to maximize returns with minimal specific risk for financial institutions. Unfortunately, by acting optimally at an individual level, nobody gave attention to the possible effects on the stability of the entire system, and a huge systemic risk grew.

Diamond and Rajan showed how a bank failure becomes contagious, not through the typical channel of bank run, but rather through the consequent liquidity shortage. They proposed a set of possible government interventions which take into account the fact that liquidity and solvency problems are endogenous and not perfectly identifiable.

As early as 2009, during the Great Recession, academic research focused all its efforts on the study and on a better understanding of the ongoing events.

Hellwig first tried to analyze the intrinsic causes of the global financial system crisis, and how the subprime securities crisis in US turned into a worldwide crisis. The securitization, that is the procedure with which a bank pools a set of contractual debts (such as mortgages or other assets) into one new security (MBSs, or ABSs, of which CDOs) whose cash flows are linked to that of underlying debts, and sells it to other investors, played a basic role in the explanation of the huge propagation of the shock in the financial system. The author argued that the incidence of systemic risk in the system was huge because of an excessive maturity and liquidity transformation operated by financial institutions through the shadow banking system (where Structured Investment Vehicles, Special Purpose Entities, Hedge Funds and others operated):

when the system broke down in 2007, the overhang of ABS caused additional downward pressure on securities prices. When operators began to recognize the defaults in US mortgages, a mix of interaction between market malfunctioning and insufficiency of equity capital in financial institutions caused a detrimental downward spiral in the global financial system.
Brunnermeier *et al.* highlighted the importance of bank's capitalization and the level of liquidity in the detrimental spirals triggered by a crisis. Before the crisis, an asset price boom of housing occurred; after the burst, banks faced many difficulties to raise funds: the excessive lack of confidence of banks in the downward phase is an explanation of pro-cyclical balance sheet movements. Authors tried to investigate the reaction of the banking sector to monetary impulses, controlling for level of liquidity and capitalization. The main result is that the less capitalized and liquid banks face more pro-cyclical effects.
Acharya and Merrouche investigated in depth the liquidity issue, both before and during the crisis. They found that, just after August 2007, when money markets froze, the liquidity demand on the interbank markets faced an increase of 30% for precautionary motives. In particular, this increase involved banks with higher credit risk in a period of high payment activity, driving up interbank rates.
More recently, Acharya and Thakor analyzed the "dark side of liquidity creation". The linkage between leverage, liquidity creation and systemic risk gives rise to some questions that their study tries to address. They consider a model in which both debt financing and equity financing discipline the bank managers in order to create an ex-ante liquidity: debts does it by the credible threat that, if the made investments will not earn enough return to cover the expense for interests, the company might be forced to inefficiently liquidate its assets, and *in extremis* it fails; instead, equity financing disciplines bank managers by providing compensation-based incentives to select the most efficient projects. However, since these incentives involve payments from ex-post cash flows, equity financing may reduce the ex-ante bank liquidity. Consequently, the optimal capital structure of the bank is affected by the trade-off between the ex-ante efficiency of leverage relative to incentives for managers given by equity and the ex-post cost of inefficient liquidations due to high leverage. With uncertainty about aggregate risks, bank creditors take their cue from liquidation decisions of other banks, but this behavior may lead to contagious liquidations, such as bank runs. Authors proved that, under given conditions, banks choose excessive leverage relative to the socially optimal level, and this justifies the public intervention through capital requirements.
Gai *et al.*, by using a network approach, found that systemic breakdowns of the interbank market can be explained with a precautionary provision by banks because of more concerns about future liquidity needs, and more fears to undergo a liquidity drying. In fact, during the crisis, the interbank markets froze up because banks stopped lending at all, and this led to devastating effects on the whole financial system and on the real economy. Authors highlighted the contribution that stricter liquidity requirements for SIFI (Systemically Important Financial Institutions) can reduce contagion through financial markets.
Kritzman *et al.* identified the main drivers of the financial system breakdown in the failure of prudent regulation of financial markets and in the excessive risk taking by institutions. Moreover, securitization, shadow activities and a flexible accounting prevent researchers from directly observing the deep interdependencies of financial institutions, and this made it difficult to correctly monitor the systemic risk. For that reason, authors introduced the absorption ratio, a measure of implied systemic risk which captures the extent to which markets are unified.

When markets are strictly linked, they become more fragile, and negative shocks propagate more quickly and broadly.
Another important case of study was the European debt crisis, in which different patterns and different evaluations of the role of systemic risk (in this framework linked to the concept of sovereign relevance) were implemented.
A basic contribution was given by Pagano and Sedunov. They showed the existence of a positive correlation between the aggregate systemic risk taken by financial institutions and sovereign debt yields of particular European countries. For that, they suggested that the systemic risk of a country's financial system (especially in Europe where there are such heterogenous systems) should be included in the sovereign debt return. Moreover, they also discovered a flight-to-quality effect towards stronger and safer countries, such as Germany, and at the same time a spillover effect across weaker financial systems: this instability inside a strong and large commercial and monetary integrated area like the EU led to a heavy and dangerous systemic risk.
A more recent and very successful line of investigation, that can be seen as the natural evolution of a branch that was taking into consideration matters such as contagion and interconnectedness, is represented by the network theory applied in finance. The network theory is a mathematical approach which studies the graphs, namely representations of relations among points (individuals), and that introduced new analytical elements, such as social structure, edge, vertex, links. In this case, this approach allowed the analysis of the structure of the connections among financial institutions, providing new models and new methods for the identification and the quantification of the systemic risk.
Nevertheless, early models of financial risk already put emphasis on networks, describing direct domino effects caused by defaults on claims between financial institutions: Furfine (2003) studied the network structure of the financial system in the US, Upper and Worms (2004) in Germany, Agnes Lubl´oy (2006) in Hungary, van Lelyveld and Liedorp (2006) in the Netherlands, Elsinger *et al*. (2006a) in Austria, Wells (2004) in the UK, Mistrulli (2007) in Italy.
For instance, Elsinger *et al*. tested a new approach to assess systemic risk by using a network model of interbank loans, and found that the correlation among banks' asset portfolios is the main channel of contagion. Even if the systemic contagion is an outstanding event, it can heavily strike the financial system, and authors proved that, to prevent contagion, it suffices a small amount of fund by a lender of last resort.
Alentor *et al.* performed a complete analysis with a network model purpose-built to evaluate the financial stability and to understand how the structure of financial system affect the systemic risk. They modeled a banking system, where a set of banks (the nodes) are connected by interbank linkages (the edges) and then proceeded to evaluate the likelihood of contagious defaults by making some key parameters defining the structure vary (such as capitalization level, interconnectedness degree, exposures size, systemic concentration). They came to some important conclusions: better capitalized banks are more able to face contagious defaults, in a non-linear fashion; connectivity degree is non-monotonic, that is at lower levels of connectivity, an increase in it makes the contagion effect stronger, but at higher levels of connectivity, a further increase in connectivity makes bank more capable to face shocks; the size of the interbank liabilities raises the probability of contagious defaults; the higher the systemic concentration of the banking systems the larger the systemic risk.

Cont and Moussa highlighted the need to use the network structures theory in order to reduce the bias of estimates on contagion and systemic risk. Moreover, they proved further basic results: contagion is sensitive to changes in the network structure and to the level of connectivity; more heterogeneous networks are more resilient to contagion; a double conflicting effect when increasing the connectivity of a network exists: on the one hand there is a reinforcement of potential channels for the propagation a financial distress, on the other hand there is a stabilizing element due to higher risk sharing; the prevailing effect depends on the level of capitalization of the whole network: in undercapitalized networks, a higher connectivity makes the network more sensible to instable contagions.
Battiston *et al.* modeled a network of credit relations among financial agents through a system of stochastic processes describing the dynamics of individual financial robustness (that is the ability to cope with changes due to some shocks). The density of a network is a proxy of the level of diversification within the system, and it is used as an explicative variable for the probability of individual default and for the probability of systemic default. Authors proved that the risk diversification may create some instability as the number of subjects in the network arises. This is due to the fact that more financially fragile actors inside the network do amplify an initial shock and this worsens the intensity of a systemic crisis.
Amini *et al.* proposed a new framework to test the resilience of a financial network to shocks. They used an analytical criterion for resilience to contagion based on the analysis of default cascades in heterogeneous networks. It is observed that the size of a default cascade generated by a shock may be wide when the depth of the shock achieves a given threshold.
Battiston *et al.* defined the systemic risk as the probability of default of a large portion of the financial system, and it is a function of the network structure. Under this perspective, they identified financial institutions as nodes and edges as lending relations among them, weighted by outstanding debt; then created a new measure, the Debt Rank, which is the fraction of the total economic value potentially affected by the distress of each single node. This method allows to identify the systemically important nodes inside the network, and can be used to categorize the so-called SIFIs from not SIFI. Authors were so able to identify the key American financial institutions during the 2008 financial crisis. Their result remarked the importance to integrate the issue of "too-connected-to-fail" and "too-central-too-fail" to the classical "too-big-to-fail" by policy makers and academics.
Poledna and Thurner re-used the Debt Rank measure to assess the systemic risks in the financial networks by each bank. They argued that the systemic risk in financial networks may be heavily reduced by increasing transparency, i.e. by making public the estimates of Debt Rank of each individual bank to all other banks, and then by forcing the reduction of the interbank borrowing from risky SIFIs. This ideal framework would favor a more homogeneous risk sharing within the system, because of a massive reduction of cascading failures.
Cellai *et al.* constructed a financial network model that combines the default-related and the liquidity-related contagion mechanisms, such that it was possible to quantify the impact of the illiquidity and the default of an institution on the overall systemic level of liquidity and others' defaults. The basic element of this model is the concept of "cascade": when an institution becomes insolvent, this shock on the asset side of creditors may propagate and cause further insolvencies to others, generating a "cascade", i.e. an "accumulation" of defaults of banks in the system. The same goes for the liability side: an eventual illiquidity and difficulty to get funding for an institution may turn out as a shock for its debtors, and accumulate itself in an

"illiquidity cascade". The conclusion is that, without fire sales, the mean level of defaults in the financial network is negatively linked to the capacity of the bank to be liquid.

Hautsch *et al.* created the realized systemic risk beta to measure the financial companies systemic risk contribution, conditional on network spillover effects and market information. This beta is calculated as the total time-varying marginal effect of a firm's VaR on the total systemic VaR.

Acemoglu *et al.* investigated the resilience of financial networks to financial contagion under different conditions. They found that a more densely connected financial network, which undergoes a negative shock, seems to be more financially stable, but only up to a certain point, beyond which density may make the propagation more fluid. Then, they pointed out that the same determinants that contribute to resilience under certain assumptions, do contribute as well to higher systemic risk under other assumptions.

An advanced approach has been provided by Billio, Caporin, Panzica and Pelizzon in their 2015 work "Network connectivity and systematic risk". They defined the "systematic risk" as the risk that an investor's well diversified portfolio is exposed to, due to the dependence of returns to common variables. At the same time, the systemic risk is strictly linked to the concept of contagion risk and spillover effect, and to the linkages between institutions. They highlighted the need to separate channels through which risk can propagate: exposures to common factors in case of systematic risk, contagion and spillover in case of systemic risk. For that purpose, the most feasible model to capture systemic risk exposures and to describe features of a network of connections is just a network model. Their goal was to analyze the strict relation between systematic risk and systemic risk, and in particular to estimate the feedbacks among network exposures and common factors, by using network-based methods to get information on linkages among institutions. Their model is a variant of the CAPM/APT model in which networks are used to infer exogenous links among assets. With this framework, authors were able to identify four components of the asset risk: the structural idiosyncratic risk, the structural systematic risk, the endogenous risk strictly derived from asset interconnections and network exposures which is reflected in the systematic risk, and the endogenous risk derived from effects of interconnections on the idiosyncratic risk. By using this risk structure, it is possible to identify three sources of the risk premium: the common factors exposure, the impact of asset interconnections to common factors and the amplification effect of idiosyncratic risk. Authors tried to estimate the impact of the network exposures and common factors on risk exposures and risk premia of stock. The main results are basically four: the systematic component is the prevailing driver of the total risk of a diversified portfolio; the idiosyncratic risk has a low impact on the total portfolio risk, while the impact of network exposures on the idiosyncratic risk is irrelevant; the risk absorption due to negative correlations among assets has a relevant role; a systematic risk component due to network exposures is present but varying over time.

Roukny, Battiston and Stiglitz showed how the networks structure could make it more difficult to assess the real level of systemic risk in the credit markets. They introduced a model to compute specific and systemic probability of default in a banking network based on credit relations and affected by external shocks. After identifying network conditions that lead to multiple equilibria, it is proved how these equilibria increase uncertainty in the estimation of the default probability and of the expected losses.

One final note for the role of systemic risk in policy making. The issue of the financial systemic risk has been crucial for policy makers since the 1970s, when the Basel Committee on Banking Supervision was established by advanced economies. Since then, many international institutions, common regulations and policies have been studied and adopted in order to try to govern and address the systemic risk and to maintain the financial stability, at least in the first world. In general, it is possible to classify preventive policies, that try *ex ante* to minimize the systemic risk, and resolution policies, that try *ex post* to minimize the negative impacts following the materialization of a systemic risk. We can identify macro-prudential policies, monetary policies and network infrastructure policies.

Macro-prudential policies consist of *ex-ante* targeting measures on the banks' balance sheets, and aim to enhance the resilience of financial institutions to external shocks. They provide a set of anti-cyclical capital requirements, leveraging requirements, margin requirements and liquidity requirements that have the objective to disincentive excessive risks taking, whatever the types of assets classes involved. In particular, the countercyclical capital buffer policy plays a crucial role: it requires banks to hold more capital when credit is increasing quickly, and it allows them to use it when losses arise in times of recession and credit crunch. These policies are very important in order to continue supplying credit to the real economy. In Europe, the ultimate responsible for macro-prudential policies monitoring is the European Systemic Risk Board, as a part of the European System of Financial Supervision, but there are many other institutions, such as the Basel Committee, the Financial Stability Board and national authorities.

Monetary policies have the basic task to maintain the price stability, but can contribute for financial stability through a powerful set of tools, which heavily affect the decisions of economic agents and investors. In particular, the definition of reference interest rates, of liquidity, of mandatory reserve requirements and the implementation of given purchase programs have a strong impact on the systemic risk, both perceived one and actual one within the system. In the Eurozone, the European Central Bank, along with the governors of central bank of member states, are the ultimate responsible of the monetary policy.

Network infrastructure policies are a set of rules that all operators within the financial market must comply with in order to ensure the correct functioning of markets and infrastructures. They provide a wide set of regulations of safe conduct of business and competition policy, such as transparency and information requirements for financial intermediaries towards authorities and consumers, product quality requirements, reporting and disclosure, client protection, bankruptcy procedures, other procedures on clearing, settlement and recording of payments and financial transactions. The purpose of these policies is to pursue the systemic stability and the soundness of all operators in the financial markets, so that all transaction-related risks can be mitigated. At an international level, the general regulatory principles have been published by the Committee on Payment and Settlement Systems in the "IOSCO Principles for Financial Market Infrastructures". In Europe, there are many regulations and directives on markets infrastructures, such as EMIR, MIFIR, MIFID, MAR and MAD, SFD and so on.

Chapter 2
Measurement of Systemic Risk

The complexity of the financial system consists of deep interconnections among a huge number of economic agents with different characteristics and interests, legal contracts with many types of provisions and enforcement, economic practices, financial operations and decisions, and in general of the market environment built up by authorities and management companies: the combination of all these inputs and their simultaneous interactions determine the outputs and the outcomes of the financial markets. The necessity to monitor and to evaluate the progress by all financial actors, from public authorities, fund's managers and intermediaries to individual investors or households, requires the creation of a very large variety of models and measures, in order to make informed and optimal decisions about investments and activities. In particular, the monitoring and the correct measurement of systemic risk have always played a central role for the institutions, and its modeling, as seen, was affected by the discoveries in the literature, based on different evolving assumptions.

As some studies confirm (Danielsson, Shin and Zigrand, 2010), market participants' actions mostly depend on perceived risk: when they believe that financial trouble is coming, they react by taking hedging actions that are reflected by an effective increase of realized volatility. Hence, the conclusion is that investors' preferences and expectations are not independent but they affect each other, making a systemic distress come true in a self-fulfilling way. This generates an endogenous systemic risk because it arises within the market, and it is opposed to exogenous risk, which takes into account a shock that is external to the financial system, and prompts investors to react in order to protect themselves. For instance, we can consider the case of an endogenous increase of the systemic risk within the financial system, such as a speculative bubble. In this case, we can presume the existence of schematic dynamics which periodically occur in the system, and that can be modeled and measured, allowing the construction of predictive models for systemic risk. It is shown that, as a price bubble builds up, the investors' perceived risk declines and the actual intrinsic risk accordingly increases, while, after the burst, the contrary occurs: a quick drop in the intrinsic risk and an unjustified huge rise in the perceived risk, which amplifies the disruptive force of the shock. Therefore, there is a double result: perceived risk and actual risk have a negative correlation, and the irrational behavior is a characteristic of the financial market.

On the contrary, if we observe an unpredictable exogenous shock hitting the financial market, then the previous systemic risk response model would be inadequate, and different settings and measures should be considered. In fact, given that these shocks are infrequent, unpredictable and unknown, and given that there is no common pattern to be studied and analyzed, it is much more difficult to create an empirical and statistical basis on which to build a model for measurement of financial risks and for forecasting probable financial crisis.

Moreover, further elements complicate the evaluation: when a financial crisis occurs, it hits economies in different ways, with different triggering factors, different channels of propagation and toward different parts of the economy. The huge heterogeneity adds degrees of complexity in the implementation of good predictive and systemic risk control system. According to some more pessimist authors, there is not even a possibility to reliably and consistently identify actions, dynamics within the financial systems and common schemes that are always valid, because of the heterogeneity across systems and over time, and the uniqueness of such events. In any case, it is vital for institutions and authorities to perform unbiased evaluations through the use of trustworthy tools and measures: the former in order to make optimal decisions for own business and own clients, the latter in order to pursue public mandates, such as financial stability and consumer protection, but also to limit government bailouts and to implement financial reforms. For this reason, the pursuit of new quantitative tools has never stopped over time.

2.1 Classifications of Systemic Risk

In the literature, the efforts focused on systemic risk assessment developed along two dimensions: the time dimension of the systemic risk, that is the pro-cyclicality strictly linked to the "aggregation" of risk over time due to systemic factors, and the cross-sectional dimension, which analyses how the systemic risk is deployed within the financial system at a given moment. This last dimension is strictly related to the network and linkages across institutions, and an unbiased evaluation of the systemic risk must include both dimensions. The final purpose is to create assessment instruments which are consistently able to capture signals about systemic trends that could make financial markets vulnerable to unpredictable shocks.

About the systemic risk assessment, there are mainly three aspects that an analyst should take into account:

- the first one is based on the study of the risk arising from the asset side of institutions' balance sheets, such as the default risk, country risk and market risk;
- the second one is based on the study of the risk arising from the liability side, such as business risk and funding risk;
- the last one is based on those risks deriving from interactions between the two sides, such as liquidity risk, maturity or currency mismatch.

For each aspect there are three possible approaches to consider:

- the first approach focuses on balance sheets' linkages, trying to discover and to measure the size of shocks, the intensity and the direction of propagation;
- the second approach makes use of market data and tries to exploit the information conveyed by returns and assets prices, such as volatility, correlations, credit spreads, liquidity spreads, risk premia and so on, to estimate systemic risk and shocks correlations;
- the last approach is based on the analysis of a set of indicators that allows simulations in order to evaluate the probabilities that an initial unstable situation combined with possible incoming shocks, may result in a heavy systemic crisis.

Moreover, systemic risk may emerge in the cyclical dimension or in the structural dimensions:

- the cyclical dimension of the systemic risk is strictly related to the temporary risk perceived by institutions at any given moment of the economic cycle, and in particular emerges with too much risk appetite during economic growth periods, and too much risk aversion during recessions; when this risk materializes, the system will suffer periods of low liquidity, fire sales, pronounced price reduction, weak balance sheets, credit crunch;

- the structural dimension of the systemic risk emerges in case of structural problems in the financial infrastructures, in the relationships among institutions, due to lack of appropriate regulations, excessive public monetary interventions, financial innovations or other country issues; it occurs with more intensity in presence of too-big-to-fail and too-interconnected-to-fail companies, and may heavily undermine the financial stability.

It can be useful to keep in mind some distinctions about the relevant economic risks. In general, the risk strictly linked to the systemic conditions of the markets, that is to market prices of traded securities, is the so-called "market risk", and it is function of systemic factors. It is the risk that investors bear because of volatility in the market value of financial assets, and then in held portfolios value, and it is due to factors affecting the entire market. Sometimes, market risk is referred to the systematic risk, because it cannot be eliminated by diversification, but only partially hedged. In accordance with the type of price, there are four different categories of this risk:

- equity price risk: it refers to all those positions affected by changes in the stock prices; as equity, it is quite most to any change in the economy, and it is one of the relevant parts of the market risk;
- interest rate risk: it refers to all those positions whose market price is affected by the evolution of the long-term interest rates prevailing in the market; it involves assets as bonds, forward, futures and swaps, and comprises sub-categories of risk, as yield curve risk, basis risk and repricing risk;
- exchange rate risk: it refers to all those positions whose price is affected by fluctuations in the exchange rates between the domestic currency and the foreign currency; it hits mainly all those institutions which operate in the international markets;
- commodity price risk: it hits all those assets whose price is affected by the fluctuations in the prices of commodities traded in the markets, like oil, gold, silver, and it involves assets as derivatives and repurchase agreements.

A last relevant component of market risk is the volatility risk: it is the risk of variations in the prices of assets as a consequence of changes in the volatility of other risk components. For example, the equity risk is related to the change in the stock price, but these changes do not follow a constant standard deviation, since it is possible to face periods with higher market volatility of the same stocks. The volatility risk is particularly relevant in portfolios of derivatives, where the volatility of the underlying price is a relevant determinant of the derivative price. The measure of the sensitivity of asset prices to changes in the volatility of underlying asset price is called *Vega*.

In the risk assessment and management for the derivatives, there are many other types of useful risk measures: *Delta* measures the sensitivity of the market portfolio value to the change in the underlying asset price; *Gamma* measures the non-linearity between the market derivative price and the corresponding underlying asset price; *Rho* (or discount rate) measures the sensitivity of the market portfolio value to the change in the discount rate used to discount the cash flows; *Theta* is the sensitivity of the market portfolio value to the incoming of maturity, therefore to the passage of time.

For a general classification of all measures of systemic risk, the most recent complete work is by Bisias, Flood, Lo, Valavanis, who, in 2012, published “A Survey of Systemic Risk Analytics”. They provided a review of thirty-one quantitative measures of systemic risk from

the literature, focusing on those issues particularly relevant for risk measurement and management. It is quite interesting to see the performed classification proposed by them and the relative groupings of all measures, that allows to rapidly and efficiently identify measures that optimally satisfy the finality and the scope that the reader pursues.
They performed a classification of the main systemic risk measures by basing it on four criteria that reflect four perspectives in the usage of indexes: the supervisory perspective, the research perspective, the required datatypes and the reference time horizon.
We are going to describe briefly each perspective, focusing mainly on those issues of particular interest for this thesis.

2.1.1 Systemic Risk Measures by Supervisory Scope

The classification on the supervisory perspective has been thought for supervision and monitoring of systemic risk from government authorities in dealing with public issues. In particular, the taxonomy proposed by authors is useful for each public authority that operates in a given field or given financial sector, and wants to identify those measures that best suit their scope. A given systemic risk measurement system may be more or less appropriate for each public supervisor depending on its mandate; and since a financial crisis is always characterized by shocks and triggering events affecting specific institutions and specific sectors, the existence of customized measurement schemes can help identify the weakness and intervene accordingly.
In this classification, the main distinction is between micro-prudential measures and macro-prudential measures: this distinction recalls an important international regulatory standard, which provides for the partition of levels of supervision. On the one hand, the macro-prudential regulation and supervision occurs at a system-level, and aims to mitigate risk for the financial system as a whole. Macro-prudential regulators and supervisors need reliable indicators of systemic risk to pursue their mandate, and intervene with appropriate macro-prudential tools, such as capital requirements, necessary to prevent financial pro-cyclicality. On the other hand, the micro-prudential regulation and supervision focuses on specific companies' activities, and operates at a firm-level. The aim is to ensure the robustness of institutions' balance sheets to shocks, and in particular to ensure their solvency, the correct conduit of business and consumer protection. While doing that, the micro-prudential authority significantly contributes in keeping the systemic risk under control. For that reason, this perspective provides the two mentioned categories of measures, macro-prudential and micro-prudential ones. Within this latter, there are further sub-categories, pertaining to the reference financial sector to which measures relate: securities and commodities, banking and housing, insurance and pensions, in addition to "general application" measures.
About this perspective, Bisias *et al.* analyses an important aspect relative to the reaction of human behavior to economic policies, recalling the famous Lucas critique, according to which econometric models' predictions lose their effectiveness when a new policy is implemented, due to the reactions and the self-fulfilling expectations of economic agents. Anyway, the monitoring of the level and of the dynamics of the systemic risk is a necessary operation for the objectives of authorities.

The following figure shows in detail all sub-categories with the list of measures. Further explanation about each measure will be provided in the following sections.

Systemic Risk Measure

Microprudential Measures—Securities and Commodities:

Crowded Trades in Currency Funds
Equity Market Illiquidity
Serial Correlation and Illiquidity in Hedge Fund Returns
Broader Hedge-Fund-Based Systemic Risk Measures

Microprudential Measures—Banking and Housing:

Network Analysis and Systemic Financial Linkages
Simulating a Credit Scenario
Simulating a Credit-and-Funding-Shock Scenario
Bank Funding Risk and Shock Transmission
The Option iPoD
Multivariate Density Estimators
Simulating the Housing Sector
Consumer Credit
Lessons from the SCAP
A 10-by-10-by-10 Approach
Distressed Insurance Premium

Microprudential Measures—Insurance and Pensions:

Granger-Causality Networks
Mark-to-Market Accounting and Liquidity Pricing

Microprudential Measures—General Applications:

The Default Intensity Model
Contingent Claims Analysis
Mahalanobis Distance
CoVaR
Co-Risk
Marginal and Systemic Expected Shortfall
Risk Topography
The Leverage Cycle

Macroprudential Measures:

Costly Asset-Price Boom/Bust Cycles
Property-Price, Equity-Price, and Credit-Gap Indicators
Macroprudential Regulation
Principal Components Analysis
GDP Stress Tests
Noise as Information for Illiquidity

Figure 2.1: Classification of systemic risk measures based on the supervisory perspective
Source: "A Survey of Systemic Risk Analytics" by Bisias, Flood, Lo and Valavanis, 2012

2.1.2 Systemic Risk Measures by Research Method

The classification based on research perspective is focused on theoretical models and econometric methods, from which systemic risk measures are developed. The research taxonomy as proposed by Bisias *et al.* has been thought to be user-friendly for researchers, allowing them to quickly point out common algorithms and data structures within each category. Authors identified in a synthetic and very useful way the origin of systemic risk in the already mentioned four "L": when economic agents overuse leverage to increase returns, the volatility of outcome is amplified, because a small loss may easily turn into a heavy liquidity shortage, due to a negative loop of fire sale of illiquid positions throughout the linkages network. From this scheme of the financial crisis, they classified systemic risk measures into

five groups: loss probabilities distribution measures, default likelihood measures, illiquidity measures, network effects measures, and macroeconomic conditions measures.
The following figure shows the complete list of measures in accordance to this criterion. Each sub-category and the most relevant measures will be described in the next sections.

Systemic Risk Measure

Probability Distribution Measures:
- Mahalanobis Distance
- Multivariate Density Estimators
- CoVaR
- Co-Risk
- Marginal and Systemic Expected Shortfall

Contingent-Claims and Default Measures:
- The Default Intensity Model
- Contingent Claims Analysis
- The Option iPoD
- Simulating the Housing Sector
- Consumer Credit
- Distressed Insurance Premium

Illiquidity Measures:
- Mark-to-Market Accounting and Liquidity Pricing
- Noise as Information for Illiquidity
- Crowded Trades in Currency Funds
- Equity Market Illiquidity
- Serial Correlation and Illiquidity in Hedge Fund Returns
- Broader Hedge-Fund-Based Systemic Risk Measures

Network Analysis Measures:
- Network Analysis and Systemic Financial Linkages
- Granger-Causality Networks
- Bank Funding Risk and Shock Transmission
- Principal Components Analysis

Macroeconomic Measures:
- Costly Asset-Price Boom/Bust Cycles
- Property-Price, Equity-Price, and Credit-Gap Indicators
- Macroprudential Regulation
- Simulating a Credit Scenario
- Simulating a Credit-and-Funding-Shock Scenario
- GDP Stress Tests
- Lessons from the SCAP
- A 10-by-10-by-10 Approach
- Risk Topography
- The Leverage Cycle

Figure 2.2: Classification of systemic risk measures based on the research perspective
Source: "A Survey of Systemic Risk Analytics" by Bisias, Flood, Lo and Valavanis, 2012

Probability distribution measures

The loss probability distribution measures, also called "tail measures", calculate the systemic risk by analyzing the co-dependence among distributions of appropriate variables of interest: they are based on the joint distribution of outcomes of a set of financial institutions, and are able to provide information from estimates of correlated losses. In particular, these measures are cross sectional, since they examine the dependence of a group of financial institutions at a given moment of time in a transversal way, and make typically use of equity returns. Anyway, measuring the dependency between two distributions requires the overcoming of some hurdles,

such as the inaccuracy of assumptions on the distribution of returns, the lack of a sufficiently large set of historical data, the necessity to deal with extreme values.
A brief description of some measures, that is CoVaR, Co-Risk, MSE, SES and the Mahalanobis distance will follow.
As we know, the Value at Risk (VaR) is a risk measure of loss for investments, trying to estimate with a given probability how much a portfolio may lose over a certain time period. It is a measure that quantifies the risk of a portfolio, useful both for regulatory purposes and for internal management, in order to limit excessive exposures to losses. Since VaR enables comparisons across portfolios and assets classes, it has become a reference measure, that is a benchmark for risk managers in the asset allocation processes and for researchers in the creation of more structured and complicated risk models. In terms of the measured loss, reference is made either to the total value of a position or to the risk per euro invested (return), and the basic question VaR answers to is: what is the highest hypothetical loss such that there is a low probability (5%) that the effective loss is greater than this amount over a given time horizon? Statistically, VaR measures how much a financial institution can lose on a financial asset in terms of market value or return rate, with a given probability and over a given time horizon. It is therefore defined as the q-quantile of the asset return distribution which satisfies:

$$Pr\left(X^i \leq VaR_\alpha^i\right) = \alpha$$

where X is the return of the institution i, and α is the significance level, the prefixed probability which usually takes value 0,05. In other terms, the VaR is the quantile of the returns density which satisfies:

$$\int_{-\infty}^{VaR(\alpha)} R_t f(R_t) dR_t = \alpha$$

where α is the probability that losses will be larger that VaR.
So, at this point, the first listed measure, that is the Conditional Value at Risk (CoVaR), will be more intelligible. While the VaR is referred to the risk of a single institution, the CoVaR measures the systemic risk as the VaR applied to the whole financial system conditional on the situation of strong common distress of all other institutions. In fact, the risk associated to one bank does not necessarily reflect the systemic risk, that is the risk of financial instability in the entire system. At the same time, a systemic risk measure should identify the risk brought by each institution to the whole system, because of its deep interconnection and consequent externalities. Moreover, risk measures are effective if they focus on forms of imbalances, bubbles or liquidity constraints, that is the real drivers of systemic risk.
Given these conditions, $CoVaR_\alpha^{j|i}$ is the VaR of the institution j (or of the whole financial system), conditional on an event G (X^i), function of the return X of the institution i. If we assume that the event is that the return of institution i achieves its VaR, then
$G\left(X^i\right) = \{X^i = VaR_\alpha^i\}$, and CoVaR is explicitly defined as the q-quantile of the joint probability distribution such that:

$$Pr\left(X^j \leq CoVaR_\alpha^{j|i} | X^i = VaR_\alpha^i\right) = \alpha$$

Therefore, the CoVaR corresponds to the VaR of the institution j when the return of the institution i achieves its VaR, and it allows to study the consequences within a network of financial institutions. In fact, when we consider the whole system, the $CoVaR_\alpha^{system|i}$ points out which institutions contribute more to the systemic risk.

From that, it is possible to get the difference between the VaR of the institution j when the return of the institution i is at its VaR (then, conditional on the distress of i) and the VaR of the same institution j when the return of the institution i is at its median state.

Hence, $\Delta CoVaR_{\alpha}^{j|i}$ quantifies how much a bank i contributes to increase the risk for the institution j:

$$\Delta CoVaR_{\alpha}^{j|i} = CoVaR_{\alpha}^{j|X^{i}=VaR_{\alpha}^{i}} - CoVaR_{\alpha}^{j|X^{i}=Median^{i}}$$

Focusing on the systemic dimension, $\Delta CoVaR_{\alpha}^{system|i}$ is the difference between the VaR of the financial system conditional on the situation of distress of institution i, and the VaR of the financial system when the performance of the institution i is at its median value of the distribution. This measure quantifies spillover effects by measuring how much an institution adds to the overall risk of the financial system.

It can be equally useful to derive $CoVaR_{\alpha}^{j|system}$: this is the VaR of the institution j when the whole system is at its VaR, that is when the return of the market portfolio of all institutions' assets is at its VaR, and the entire financial system is suffering. It helps to identify those institutions that are most at risk in case of a financial crisis. Hence, $\Delta CoVaR_{\alpha}^{j|system}$reflects the increase in the VaR of the institution j given a financial crisis, and measures the extent to which a single bank is affected by a systemic risk.

These measures based on the VaR have some shortcomings:

- they do not provide any information about the size of losses;
- they are not coherent measures, since they do not fulfill the sub-additivity principle.

Without going into the mathematical details, Artzner published in 1999 a famous paper in which a set of four desirable properties for measures of risk was defined, calling "coherent" all measures which comply with them. These properties are:

- the positive homogeneity: if a position is multiplied by a scalar, the risk measure will also be a multiple of that scalar, and it implies that the risk of a position is proportional to its size;
- the monotonicity: if a portfolio has a better value than another portfolio under any possible scenario, then the risk of first one must be smaller than the risk of the second one; it implies that, if the losses of a portfolio are smaller than the portfolio Y, the risk measure of X should be smaller than that of Y;
- the translational invariance: it implies that addition of a sure amount of capital in a portfolio should reduce the risk by the same amount;
- the sub-additivity: the risk measure of a portfolio should never be higher than the sum of the risk measure values of each single position of the portfolio; this means that the risk of a portfolio can be lower or at most equal to the sum of the risks of the individual positions, and it ensures portfolio diversification principle.

It has been shown that VaR is not a coherent risk measure as it does not respect the sub-additivity property: at sufficiently low probability levels, VaR of a portfolio may be lower than the sum of the VaR of the single positions, and some works highlighted that this happens because the return distribution exhibits fat tails.

Problems described above are overcome by implementing a risk measure called Expected Shortfall (ES), which fulfill all the properties. This is the expected value of losses undergone by an institution given the situation of distress (losses beyond the VaR, fixed a level of confidence). Therefore, the Expected Shortfall at α-level is the expected portfolio return in the

worst α% of cases, and it is an excellent alternative to VaR, because more sensitive to the shape of the tail of the return distribution.
Then, ES is the average of returns R of a firm or of a portfolio, when its loss exceeds its VaR:

$$ES_\alpha = -\mathbb{E}[R|R \leq VaR_\alpha]$$

ES is either sub-additive and homogeneous measure, and then complies with all properties. This makes this risk measure appropriate and very useful, to be used as a criterion for the composition of optimal portfolios. It is possible to note that, while the VaR provides the expected capital needed for a financial institution to limit the probability of failure, the difference between ES and VaR represents the expected value of the cost to face to save the bank from bankruptcy, in the case in which its capital is not enough. ES represents also the premium that an insurer would ask an institution if he wants to insure against the risk of losses higher than the VaR.
The two tail measures strictly linked to ES are the Marginal Expected Shortfall (MES) and the Systemic Expected Shortfall (SES).
For the derivation of the MES, there is a useful theoretical exemplification. A big financial company with a large organizational structure, composed by many territorial units and many target markets, could need to know the contribution of each individual operating unit to the total firm's profitability. The similarity is with a financial portfolio: the institution is a big portfolio, with assets and liabilities, that is with many positions on different financial products traded by units. It is possible to get the total return of the institution, R, as the sum of the returns, *r*, of each individual unit *i*, weighted by the relative contribution to the total return, *y*:

$$R = \sum_i y_i \, r_i$$

In this case, the ES of a financial portfolio, with significance level α will be:

$$ES_\alpha = -\sum_i y_i \, \mathbb{E}[r_i|R \leq VaR_\alpha]$$

Finally, we can take the first derivative of ES with respect the weight y of each institution, and this allows us to compute the impact of an increase in the weight of a given asset return on the expected total return in case of distress:

$$\frac{\delta ES_\alpha}{\delta y_i} = -\mathbb{E}[r_i|R \leq VaR_\alpha] \equiv MES_\alpha^i$$

This is the generic definition of the Marginal Expected Shortfall for the unit I, given α.
So, MES measures the assets' expected loss when the portfolio returns fall below a certain threshold, the VaR, over a given time horizon; likewise, MES measures the expected return of a firm's unit when the whole company has a return below the VaR. Finally, MES can be adopted to a systemic context: in this case, it measures a firm's expected equity loss when market return falls below its VaR over a given time horizon. It can be defined as the average return of a firm during the worst days for the market, and represents the company's contribution to a systemic crisis. Then, the expected marginal loss conditional on a distress case is a proxy of the contribution of a bank to the overall systemic risk. Bisias *et al.* calculates MES as the mean of the returns of a firm's equity during the 5% worst cases for the overall market return, proxied by the CRSP Value Weighted Index.

Marginal risk measure	**Conditional on institution**	**Conditional on system**
VaR	MVaR	CoVaR
ES	MES	CoES

Table 2.1: Classification of marginal systemic risk measures conditional on a given event

A little more complicated fashion is required to implement the Systemic Expected Shortfall (SE) measure. SES provides another way to measure the contribution of single institution to the systemic risk, by calculating the propension of a firm to be undercapitalized when the whole system is undercapitalized. Let e_0^i be the equity of the firm i at time zero, e_1^i the equity at time one; k^i a fraction of the total asset a^i; $E_1 = \sum_i^N e_1^i$ the total market capitalization at time zero, in a market with N institutions; $A = \sum_i^N a^i$ the value of the total assets in the market. Hence, it is possible to define the SES of firm i as the expected amount of equity capital falling below a given target threshold, that is the fraction of total assets of institution i, all conditional on the systemic crisis, that is the situation of systemic undercapitalization, when the total market capitalization is less than a fraction of total market assets:

$$SES^i = \mathbb{E}[e_1^i - k^i a^i | E_i \leq kA]$$

At this point, we let $r^i = \frac{e_1^i}{e_0^i}$ be the stock return of firm *i*; $l^i = \frac{a^i}{e_0^i}$ be the leverage of firm *i*; $R = \frac{E_1}{E_0}$ be the total return of the market; $L = \frac{A}{E_0}$ be the aggregate leverage of the whole system. It is possible to calculate the percentage return measure of SES:

$$SES^i(\%) = \mathbb{E}[r^i - k^i l^i | R \leq kl]$$

It has been shown that SES increases with the degree of leverage *l* of a financial institution and with its expected loss, calculated in the tail of the losses distribution of the system.

Since SES is a theoretical construct, it requires some proxies (on equity, expected falls for a firm and for the system and so on) and leading indicators (as MES, leverage) in order to be implemented and estimated.

An interesting systemic risk measurement, based on stochastic processes, networks and conditional probability of default, is the CoRisk (Giudici and Parisi), which measures the change in the institutions' probability of default due to spillovers and contagion effect. To derive this measure, three steps have been implemented:

- first of all, the countries' economy is divided in three macro-economic sectors, sovereign sector, productive sector and financial sector;
- secondly, for each sector, a spread measure is linearly modelled as function of an country-related idiosyncratic component and a common systematic component, in order to control for different sources of risk;
- thirdly, these spreads are used to derive correlation networks, identifying the most relevant contagion channels, and to calculate the probabilities of default for each economic sector in each country;
- finally, the default probabilities are combined with the correlation network in order to get the CoRisk.

In particular, the CoRisk-in measures the change in the default probability of an economic agent due to the contagion from an external shock, while the CoRisk-out measures the impact of this change towards other external firms and sectors.

Without going into the analytical details in depth, here the two main formulas for this measure will be provided.
We define PD_t^m as the probability of default for the institution *m* at time *t*, and TPD_{t+1}^m the total probability of default at time t+1 for the same institution, which embeds both sector-specific and contagion risk components. The $CoRisk_{in}$ can be considered as the percentage variation of the survival probability (1-PD_t^m) of an institution *m,* when a potential contagion from external shock occurs:

$$CoRisk_{in,t}^m = \frac{(1 - PD_t^m) - (1 - TPD_{t+1}^m)}{(1 - PD_t^m)}$$

It is then shown that if $CoRisk_{in,t}^m$>0 (or <0), the total default probability of institution m increases (declines) after a contagion effect. In those cases in which the institution is damaged in terms of default probability by positive linkages within network, then the contagion is said to be negative (since TPD>PD, then CoRisk-in>0); on the other hand, there is a positive contagion when an institution obtains advantages from negative linkages with other neighbors (since TPD<PD, then CoRisk-in<0).
By deriving the partial correlation coefficients between interest spreads of two institutions, *m* and *n*, and by controlling for the systematic component S, we get $\rho_{mn|S}$, and with that we can calculate $CoRisk_{out}$:

$$CoRisk_{out,t}^m = 1 - (1 - PD_t^m)^{\sum_{n \neq m} \rho_{mn|S}}$$

As said, $CoRisk_{out}$ measures the impact exerted by the institution m on its near partners it is linked to. It is interesting to note that the incoming contagion is different from the outcoming contagion, and it is due to the different impacts of the shocks on the single institutions' default probabilities. Under this perspective, CoRisk-in is a good proxy of the vulnerability of a firm, while the CoRisk-out is a proxy of its systematic financial importance.
By applying this model to the Eurozone countries during last decade, it was discovered that the sovereign sector distress increased the systemic component more than the financial sector, and the propagation didn't favor the risk sharing, but made weaker the weakest institutions and stronger the strongest institutions. This synthetic risk measure appeared to be quite flexible, because it allows to evaluate the relevance on the systemic risk of each country, of each main economic sector, for type of risk, both in the cross-sectional and temporal dimension.
Ultimately, the Mahalanobis distance is deepened. Mathematically, it is the measure of the distance between a point A and a distribution D, in terms of standard deviations away A from the mean of D: if this distance is zero, then point A is at the mean of D, otherwise it is moved away from the mean. Kritzmann and Li re-used this mathematical measure in finance, in order to calculate the financial turbulence.
A financial turbulence indicator measures the intensity of violent or unsteady movements in the global financial market across time, and provides some important implications for the financial asset allocation. Not to be confused with the concept of systemic risk measure: this is linked to the fragility or robustness of the financial system, and measures its susceptibility to shocks, before the materialization of a turbulence. The turbulence occurs when asset returns and prices behave in an uncharacteristic way given their historical pattern, showing extreme movements, decoupling of correlations and increased volatility.
In particular, Chow *et al.* showed in their important works how to use the squared Mahalanobis distance to compute the financial turbulence, and created a model to base on it the construction of appropriate portfolios. The market turbulence index proposed by them is the following:

$$d_t = (y_t - \boldsymbol{\mu})\Sigma^{-1}(y_t - \boldsymbol{\mu})'$$

where d_t is a scalar indicating the market turbulence for a particular time period t;y_t is a vector of observed asset returns of n assets during t; $\boldsymbol{\mu}$ is the vector of historical mean returns; Σ^{-1} is the inverse of the sample variance-covariance matrix of historical returns.
The observation of the outcomes of this measure highlighted some basic features: the structure variance-covariance of asset returns is not constant over time; the economy oscillates between a steady low-volatility expansive state and a panic-driven high-volatility recessive state; during the turbulent periods, the return-to-risk ratio significantly reduces, since the increased volatility; the turbulence is always unexpected and persistent across time; there exists a mean reverting behavior of the variable of interest after the turbulence. Moreover, the turbulence index is much widespread in the financial industry, and has many usages. Portfolio managers use it to stress-test portfolios, or to construct regime-dependent investment strategies and portfolios in such a way as to be unsusceptible to turbulence, and then more resilient to shocks. Finally, they can use it to improve some risky strategies, in order to reduce the risk exposure.

Contingent claims and default measures

On the list, the next sub-section comprises contingent claims and default measures.
A contingent claim is a contract whose future payoff depends on the value of an underlying asset, "contingent" on the realization of some given uncertain event. This is also the general definition of derivative, like the option, an instrument that gives the right to buy or sell the underlying asset at a specified exercise price by a given expiration date. In finance, the contingent claims have been widely used to address some financial economics issues, by developing models and mathematical constructs, and in corporate finance as an innovative valuation method. In fact, it has been noted that the firm's equity can be associated to a call option on firm's assets as underlying: the contingent claims analysis (CCA), which uses the same derivatives pricing models, allows the evaluation of relevant items in the firm's balance sheet, like equity and debt, by exploiting information from the balance sheet and the market. In particular, assuming a stochastic process for the market value of the firm's total, equity is represented by a call option, which gives shareholders the right to acquire bank business, while liabilities can be symmetrically represented by a put option, which allows a bank's creditors to sell the assets of the institution in case of failure.
This approach is a flexible framework applicable to many types of analyses: it allows to estimate sovereign risk and to analyze the impact of banking system risk on the sovereign risk; to estimate the relationship between macroeconomic factors and the time pattern of implied bank assets, distance to distress, default probability or expected losses; to project banking risks under stress scenarios; and to analyze the impact of the government guarantees on bank funding costs.
Another application of contingent claim analysis is to measure and analyze liquidity risk and systemic risk by considering banks' short-term assets and liabilities. With this information, it is possible to construct measures of default likelihood for each institution and then link them either directly or indirectly through their joint distribution. In particular, literature focused on the application of CCA to evaluate the systemic risk of the financial sector. By combining bank's balance sheet information and forward-looking market data, new systemic risk measures, based on the impact of eventual government guarantee against losses related to banks' debts, have been introduced. Systemic risk is modeled by evaluating the expected losses

of a set of too-big-to-fail institution in financial destress: variations in market prices, and consequent changes in firms' perceived risk derived from its equity volatility, affect the individual sensitivity to common risk factors, and then the dependence structure of expected losses among institutions; analyzing this dependence and its effect on the joint expected losses helps to identify the joint tail risk of multiple entities. The 'tail dependence' measuring is fundamental to study systems with dense linkages among many institutions. These methods allow to identify the marginal contribution of each financial institution to the dynamics of the overall systemic risk and to quantify the risk transferred from banks to the government.
It follows a brief mathematical description of the basic model.
The CCA aims to adjust the balance sheet for risk, by assuming that at any moment t, the market value of a bank's assets A, is equal to the sum of the market value of the equity E, and market value of total debt D (which differs from the nominal value F to be paid at maturity T):

$$A_t = E_t + D_t$$

To assess the option incorporated in the values of debt and equity, it is necessary to estimate the dynamic of the market asset value. Typically, it is assumed that the asset value follows a Geometric Brownian Motion process, with risk-neutral dynamics given by the stochastic differential equation:

$$\frac{dA_t}{A_t} = rdt + \sigma_A dW$$

As said, the equity value can be associated to the price of a call option: if, at maturity of debt, the asset value allows to repay the face value to debtholders, then $A_T > F$, and the shareholders' equity will get $A_T - F$; if at T the firm defaults on its debt, due to $A_T < F$, then debtholders have the first claim on residual asset A_T and shareholders get nothing. The payoff of the equity for shareholders will be:

$$E_t = max(A_T - F; 0)$$

At this point, it has been shown that, by applying the Black-Scholes formula for European call option, equity value can be calculated as:

$$E_t = A_t N(d_1) - Fe^{-r(T-t)} N(d_2)$$

where N represents the cumulative distribution function of a normal standard, and then:

$$d_1 = \frac{ln\left(\frac{A_t}{F}\right) + (r + \frac{\sigma_A}{2})(T-t)}{\sigma_A \sqrt{(T-t)}}$$

$$d_2 = d_1 - \sigma_A \sqrt{(T-t)}$$

On the other hand, the actual value of the debt is equal to the difference between assets value and equity value, and re-formulating the previous formulas, the final result will be:

$$D_t = A_t N(-d_1) + Fe^{-r(T-t)} N(d_2)$$

Anyway, the value of the liabilities can be measured as the difference between the value of the debt in a risk-free world and the expected value of the losses due to the bank's default, which is equal to the price of a put option on the institution's assets, P_t:

$$D_t = Fe^{-r(T-t)} - P_t$$

from which:

$$P_t = Fe^{-r(T-t)} N(-d_2) - A_t N(-d_1)$$

In other words, the value of the risky debt is equal to the value of the riskless bond and the value of a put option on the firm's assets, with strike price equal to the face amount. Note that $N(-d_2)$is the risk-neutral probability that the company will default on the debt (i.e. the likelihood that the value of the company will be less than the face value of debt at its maturity), while d_2 is known as distance to default, since represents the difference between the expected value of the firm's assets and the firm's default point. The idea that corporate debt and equity can be viewed as derivatives written on the firm's assets is the basis of the structural approach, used to analyze credit risk.

The contribution for the calculation of the systemic risk consists of the combination of this put option with other data and procedures. In particular, under the hypothesis that guarantees against the failure provided by the government do not affect the equity value, the spreads observable in the Credit Default Swap market should capture the potential expected losses faced by the institutions. From this, the price of the put option written on the CDS, $P_{CDS,t}$, is calculated in order to derive the fraction of the total loss due to the firm's default coverable by the government's guarantees: $\alpha_t = 1 - \frac{P_{CDS,t}}{P_t}$

Of course, the public guarantee reduces the CDS spreads, which reflects the probability to default, already implicit in the put option. Then, $\alpha_t P_t$ is the fraction of the put option price which reflect the default risk covered by the public guarantee, while the complementary fraction $(1 - \alpha_t)P_t$ represents that part of default risk not covered by the public system, and then fully borne by the institution. The systemic risk measure, ϱ_t, can be calculated as the total losses incurred by the government during a systemic crisis, equal to the sum of the amount given to the whole system as a guarantee, composed by N companies:

$$\varrho_t = \sum_{i=1}^{N} \alpha_t^i P_t^i$$

By adopting a similar framework, many other measures have been implemented, such as the Distress Insurance Premium (DIP). Simply, DIP is the hypothetical insurance premium needed to cover the risk of losses in a distressed banking system, where the distress occurs when at least 15% or more of total liabilities of the financial system defaulted. In particular, the systemic risk measure is the premium of that insurance policy which protects against losses of a hypothetical portfolio composed by the total liabilities in all the banking system. It is calculated as the risk-neutral expectation of portfolio credit losses, L, conditional on total losses equal or higher than its minimum value (that is a minimum share of the sector's total liabilities):

$$DIP = \mathbb{E}^Q[L | L \geq L_{min}]$$

This indicator can be obtained through Monte Carlo simulations, given key variables such as banks' liability value, probability of default, loss given default and correlations.

One can notice that the definition of DIP is quite similar to that of ES (expected shortfall), since both of them indicate the expected loss conditional on the overcoming of a given threshold. The main difference is that this threshold is a percentile distribution in the ES, while it is the not normalized minimum value of the underlying portfolio in the DIP.

As done for other measures, it can be useful to identify the SIFIs, by decomposing DIP into a sum of marginal risk contributions to the overall systemic risk (in this case the hypothetical insurance premium) from each institution of the banking system. The marginal risk contribution is the expected loss of the bank i, conditional on a large loss of the full portfolio:

$$\frac{\partial DIP}{\partial L_i} = \mathbb{E}^Q[L_i | L \geq L_{min}]$$

It has been shown that bank's contribution to the systemic risk is roughly linear in its default probability, but non-linear with respect to the institution size and the assets correlation.

Illiquidity measures

Illiquidity and insolvency measures tries to capture the mechanisms and the probability that liquidity risk arises following negative shocks on the structure of assets and liabilities of financial institutions, bringing to negative responses by the companies and the financial system as a whole. Measuring systemic risk for illiquidity and insolvency requires the study of the exposures of the companies and of the susceptibility to systemic propagation.

Contrary to what happens during stable periods on the markets, during a financial crisis an institution selling many quantities of assets may negatively affect market conditions, generating the so-called market liquidity risk. The materialization of the market liquidity risk becomes evident by looking at five important indicators:

- the bid-ask spread, that is the difference between the highest price that a buyer is willing to pay (bid price) for an asset and the lowest price (ask price) that a seller is willing to ask, reflects the transaction costs, as well as the tightness between demand and supply of a given asset; when this indicator increases, it means that the quantities the counterparties are willing to trade at that prices decline, and then the asset becomes illiquid, difficult to exchange to a fair price;
- the trading volumes and the frequency of trading orders for each asset and for each price on both buy/sell side, are an indicator of the degree of market depth; they highlight the distance to possible scenarios of asset illiquidity;
- the market resilience measures the capacity of a given asset market to face and to withstand to external shocks, and then reflects the sensitivity of assets prices to systemic changes, as well as the speed with which these prices revert to its equilibrium fundamental value after a shock; a low market resilience indicates an intrinsically riskier market, with more probability to face liquidity distress;
- the market breadth is the fraction of overall market participating in up or down price movements, and gives an idea of the consistency of liquidity within asset classes;
- the immediacy indicates the time needed to execute a transaction in the market, and it is function of the number of market makers and participants, as well as the technology available for the trading.

There are also other important issues to be covered when assessing illiquidity.

The high asymmetric information among operators along with the widespread fear to face losses due to market movements may bring to imitative and herding behaviors, for which smaller institutions imitate the larger ones, causing self-fulfilling and self-sustaining market instability. Moreover, the already discussed liquidity spiral might amplify this isolated liquidity risk to the whole system, increasing the global systemic risk: a bank facing distress will be forced to liquidate some assets and to accept fire sales on the market, and if the recovered funds are not enough, the probability to default, and then to become insolvent, becomes dramatically high. Important policy implication can be derived, such as the limitation of individual liquidity risk to prevent systemic distress.

As said, banks are particularly exposed to funding illiquidity, given their activity of maturity transformation. For a systemic perspective, the interaction between the funding illiquidity and the asset illiquidity is quite relevant, since funding shortages feed asset fire sales which cause further funding shortages to other institutions, materializing the systemic propagation.

A brief description of two important systemic risk measures follows: the “Noise as Information of Illiquidity” and the “Equity Market Illiquidity”.

The Noise as Information of Illiquidity is a deviation measure for the systemic liquidity risk which is based on a set of assumptions and stylized facts. First of all, the point of view from the treasury bonds market is taken into account, because of its importance and high liquidity, in order to evaluate the liquidity condition of the overall market. It is assumed that the aggregate liquidity is strictly connected to the amount of available arbitrage capital, that is the amount of extra capital that institutions accumulate to be used in case of distress in order to provide liquidity: during stable and calm scenarios, banks accumulate abundant capital in the form of liquid assets such as treasury bonds, and for this reason, arbitrage forces entirely eliminate large price deviations from their fundamental values. During market crises, the accumulated capital declines and institutions will wish liquidate positions: the aggregate liquidity quickly dries up, the arbitrage forces in the market becomes weaker, and the prices move far away from their fundamentals. In this case, temporary price deviations highlight the arbitrage capital shortage and the lack of systemic liquidity, while the dynamics of the arbitrage capital is reflected in the position and shape of the Treasury yield curve. The survey of this "noise" in the price of treasury bonds is the basis for a new measure of market-wide liquidity risk, and then an indicator for an important component of the systemic risk. In fact, given its systematic nature, this measure should be informative on asset returns in those markets particularly sensitive to global liquidity conditions. Then, noise in the treasuries market is informative about global market liquidity, because of its central role and its high liquidity and low credit risk, which makes the noise intrinsically low.
The daily liquidity noise measure at time *t*, given n_t the number of treasury bonds available on moment *t*, is calculated as:

$$Noise_t = \sqrt{\frac{1}{n_t}\sum_{i=1}^{n_t}[y_t^i - y^i(b_t)]^2}$$

where y_t^i are the observed market yields of the bond *i* on day *t*, and $y^i(b_t)$ are the fitted yields implied by a bond price model, whose b_t is the vector of parameters of a parameterized forward curve, backed out from the data. In particular, b_t are derived by minimizing the weighted (for duration D_i) sum of the squared deviations between the observed bonds prices P_t^i and the fitted model-implied prices $P^i(b)$:

$$b_t = argmin_b \sum_{i=1}^{n_t} [\left(P^i(b) - P_t^i\right) \times \frac{1}{D_i}]^2$$

Measures about Equity Market Illiquidity have been proposed by Khandani and Lo (2011), which analyzed a trading strategy consisting of buying loser stocks and selling winner stocks, and then providing liquidity by correcting temporary imbalances between demand and supply. In particular, the performance in terms of profitability of this strategy has been observed over time: whenever the profitability was higher, then there was less liquidity in the market, and the liquidity premium of trade increased. Authors established a relationship between the illiquidity and the positive autocorrelation in asset returns among equity portfolios. Moreover, they found evidence of a significant positive autocorrelation among returns in less liquid securities portfolios (such as small-cap stocks, mortgage-backed securities, and so on) emerging-market investments. Within this framework, authors presented two types of liquidity measure.
The first measure is the "Contrarian Strategy Liquidity Measure", based on the simple high-frequency mean-reversion strategy described above, where buying and selling occur over

lagged m-minute returns, that is portfolios are composed at time t by considering the stocks returns over the previous 5 (up to 60) minutes. Long positions, on those stock with the lowest return, and short positions, on those stock with the highest returns, are equally weighted,
and then the overall portfolio is a neutral strategy, and rebalanced each minute. As said, cases with higher profitability are associable with less market liquidity, and this is reflected in the higher liquidity premium. Authors noted that its profitability has been decreasing for last decades due to the increasing number of market actors providing even more liquidity, thus reducing the liquidity premium.
A second measure of market liquidity is the "Price Impact Liquidity Measure", and is related to the Kyle's "lambda", an inverse proxy of liquidity: the higher the value of lambda the lower the liquidity and the market depth, and vice-versa. In particular, it measures the liquidity through a linear regression of the trading volume required to move the security price by one unit. Authors estimate this measure by exploiting information from the transactions during trading hours on each day, such as the sequence of intra-day returns R, prices p and quantities v for each security i in each moment t. From this set of variables, they estimate the equation:

$$R_{it} = \hat{a}_i + \hat{\lambda}_i \cdot Sign\, log(v_{it} p_{it}) + \varepsilon_{it}$$

where the sign of the logarithm indicates the sign of the position, buying (+) or selling (-). In particular, if R is positive, the transaction will have the sign + (net buying), while if R is negative, the transaction will have the sign − (net selling). The average of the estimated price impact coefficients λ_i for each one of n assets provides the "Price Impact Liquidity Measure":

$$PILM = \frac{\sum_{i=1}^{n} \lambda_i}{n}$$

Network analysis measures

So far, only aggregate systemic risk measures have been discussed. Measures of an aggregate nature typically tend to provide the average risk or the dispersion of a dataset, and this is a shortcoming. Nowadays, the global economy and the financial systems are increasingly complex and evolving, and a single index based on the mean is not enough informative of the real state of the system. Unlike the macroeconomic measures, risk measures in the "Network Measures" group exploit tools and knowledges specifically that seems to be more appropriate to detect the systemic risk. In fact, they are able to explain how systemic events unfold over time, and to track them evolution until the realization of the systemic crisis. Then, in this framework, relationships and connections among institutions acquire the most importance.
Networks models and networks measures have proved to be valid and useful in explaining the increased impact of shocks on the systemic stability due to the increased complexity and weaving of the financial system. Two fundamental elements significantly contributed to make relevant the role of the networks: the innovations over the last decades and the intense financial globalization. The innovations increased the involvement of an always higher number of new institutions, instruments, contracts, practices, new funds, new sectors, and above all new technologies, significant stuff for the determination of the systemic risk. On the other side, the globalization has strengthened and stepped up the network of economic relationships among financial institutions over sectors and over countries, as never before. As said, the bankruptcy of a single too-central-to-fail institutions may trigger a system crisis, due to its interconnectedness with a high number of operators.

Billio, Getmansky, Lo and Pelizzon firstly formalized the relation between the degree of correlation among institutions in the market and the capacity of a financial crisis to have a systemic scope, and that depends on the concentration of the risk, the intensity of the connections and the sensitivity of relevant variables to strong changes. Authors proposed many econometric measures of connectedness based on the Principal Components Analysis (PCA), which allows the detection of the commonalities among asset returns of institutions, by decomposing the variance-covariance matrix of returns for each financial sector. PCA is used as an exploratory tool of the data, since it allows to identify the intrinsic distance and links among units and sectors. PCA can be executed through eigenvalue decomposition (i.e. the factorization of a matrix into a standard form, whereby the matrix is represented in terms of its eigenvalues and eigenvectors; λ is an eigenvalue of (n×n) matrix A if there exists a non-zero vector v such that Av=λv, where v is the eigenvector of A corresponding to λ) of a covariance matrix.

A known related measure that deserves to be mentioned is the Absorption Ratio, described by Kritzmann *et al.* in 2010. This measure is the fraction of the total variance of a set of N asset returns explained (or absorbed) by a fixed number of eigenvectors, that are the first n < N principal components. A high AR indicates that the market is weak and vulnerable to negative shocks, and then the systemic risk is high. Letting N be the number of assets (or financial institutions), n be the number of eigenvectors used, $\sigma_{e\,i}^2$ be variance of the *i*-th eigenvectors, $\sigma_{A\,i}^2$ be variance of the *j*-th asset, it is possible to obtain:

$$AR = \frac{\sum_{i=1}^{n} \sigma_{e\,i}^2}{\sum_{j=1}^{N} \sigma_{A\,i}^2}$$

A leading distress indicator is the difference between AR calculated for long and short estimation windows:

$$\Delta AR(n) = AR(n)_{short} - AR(n)_{long}$$

Another important tool for the systemic risk measurement consists in the application of Granger-Causality on Networks. This measure includes tests which provide information about the relevance of correlation degree among institutions, not conditionally on shock occurrence. Moreover, this approach allows as well to define the direction of the link among units, highlighting the source of propagation and the dynamics. The Granger causality is a statistical notion of causality that determines whether one time series is useful in forecasting another: series X is said to "Granger-causes" series Y if values of X are informative on the evolution of Y, and then X is useful in forecasting Y. The mathematical formulation of the test is based on the linear regressions of X on Y and of Y on X.

In particular, it is possible to model the framework needed to develop this kind of tests.

Let R_t^A and R_t^B be the stock returns of institutions A and B, two stationary time series with zero mean and a linear inter-relationship described by the following autoregressive model:

$$R_t^A = \sum_{j=1}^{m} \alpha_j R_{t-j}^A + \sum_{j=1}^{m} \beta_j R_{t-j}^B + \varepsilon_t^A$$

$$R_t^B = \sum_{j=1}^{m} \theta_j R_{t-j}^A + \sum_{j=1}^{m} \lambda_j R_{t-j}^B + \varepsilon_t^B$$

where m is the maximum lag j chosen, $\alpha_j, \beta_j, \theta_j, \lambda_j$ are the coefficients, and ε_t^A and ε_t^B are uncorrelated error terms, specifically two white noise with zero mean and unit variance. R_t^B is said to Granger-causes R_t^A when β_j is statistically different from zero, while R_t^A Granger-causes

R_t^B when θ_j is statistically different from zero. When both variables reciprocally Granger-cause each other, then there is simultaneity. The test for the Granger causality is an F-test with following null hypothesis:

$$H_0: \beta_1 = \beta_2 = \cdots = \beta_m = 0 \; that \; is \; R_t^B \nRightarrow R_t^A$$
$$H_0: \theta_1 = \theta_2 = \cdots = \theta_m = 0 \; that \; is \; R_t^A \nRightarrow R_t^B$$

For instance, there should not be any Granger causality between actual asset price changes and lagged prices under the assumption of informationally efficient market, while a positive Granger causality should exist in not efficient markets, because of the presence of frictions, transaction costs and other constraints. Moreover, the width of the Granger causality in evaluating the correlation among asset returns can be considered a proxy of spillover effects and interconnectedness among market participants: the stronger the Granger causality, the stronger the level of interconnections and integration among financial institutions, and then the relevance of an eventual systemic distress. Thus, generalizing, the identification of causal relationships in the sense of Granger among institutions of the financial network is a very useful procedure to study the propagation of the excesses of returns variability in the financial system. In this framework, a causality indicator can be defined:

$$(A \Rightarrow B) = \begin{cases} 1 & if \; A \; Granger\_causes \; B \\ 0 & otherwise \end{cases}$$

This indicator variable is the basis for other five important interconnectedness measures, which are defined below. The generalized case will be considered: a system with N institutions that interact each other, generically named *j* and *i*, instead of A and B.

1) Degree of Granger Causality (DGC): it is the percentage of statistically significant Granger-causality relationships within a system of N institutions, for a total of N(N−1) possible relationships (pairs):

$$DCG = \frac{\sum_{i=1}^{N} \sum_{j \neq i} (j \Rightarrow i)}{N(N-1)}$$

When DCG is higher than a given threshold K, than there is a strong interconnectedness and interdependence among institutions' returns, and a systemic event is more likely to occur. This risk can be calculated through Monte-Carlo simulations. In particular, Monte-Carlo simulation is useful to understand whether Granger causal relationships among institutions are due to randomness. Specifically, assuming independence among financial institutions, a certain number of time series representing each financial institutions' returns are simulated, and on each simulated relationship the test for Granger causality is performed to identify significant connections. By repeating this procedure many, many times, it will be possible to represent the resulting distribution, whose center will be the fraction of significant connections under the null hypothesis of no statistical relation among institutions.

Another similar related measure is the Dynamic Causality Index (DCI), which tries to capture the level of interconnection among financial institutions by computing the fraction of relevant Granger causality relations (that is significant at p-value < 0.05) among their returns over the total number of relations.

2) Number of connections: it is useful to assess the presence of Systemically Important Financial Institutions, because allows to survey the relevance degree of each institution by simply counting the number of its connections with others. Letting S be a variable representing the whole system, #In be the number of institutions in the system that Granger causes the institution i, #Out be the number of financial institutions in the system Granger caused by a

given institution i, #In+Out is the sum of the two last measures. In particular, the Granger causality is considered to be significant if the connectivity measure exceeds a given threshold K, and only those cases are useful to identify risks of systemic crisis starting from a shock. Moreover, #In+Out gives an idea about the centrality of the institutions.

$$\#In: \quad (S \Rightarrow i)|_{DGC \geq K} = \frac{\sum_{j \neq i}(j \Rightarrow i)|_{DGC \geq K}}{N-1}$$

$$\#Out: \quad (i \Rightarrow S)|_{DGC \geq K} = \frac{\sum_{j \neq i}(i \Rightarrow j)|_{DGC \geq K}}{N-1}$$

$$\#In + Out: \quad (i \Leftrightarrow S)|_{DGC \geq K} = \frac{\sum_{j \neq i}(j \Rightarrow i) + (i \Rightarrow j)|_{DGC \geq K}}{2(N-1)}$$

3) Sector-Conditional Connections: these measures are similar to those of point 2), with the only difference that the significant causality is obtained among institutions belonging to different sectors. Hence, the counting of the number of significant connections is conditional on the type of sector. Letting M be the type of sector (banks, insurers, funds, brokers, and so on) indexed by α,β=1,…M, it is possible to get three measures: #In-from-Other is the number of other types of financial institutions in the financial system that significantly Granger-cause institution i, #Out-to-Other is the number of other types of financial institutions in the system that is significantly Granger-caused by institution i, while #In+Out-Other is the sum of the last two.

$$\#In.from.Other: \quad [\sum\nolimits_{\beta \neq \alpha}(S|\beta) \Rightarrow (i|\alpha)]|_{DGC \geq K} = \frac{\sum_{\beta \neq \alpha}\sum_{j \neq i}[(j|\beta) \Rightarrow (i|\alpha)]|_{DGC \geq K}}{(M-1)N/M}$$

$$\#Out.to.Other: \quad [(i|\alpha) \Rightarrow \sum\nolimits_{\beta \neq \alpha}(S|\beta)]|_{DGC \geq K} = \frac{\sum_{\beta \neq \alpha}\sum_{j \neq i}[(i|\alpha) \Rightarrow (j|\beta)]|_{DGC \geq K}}{(M-1)N/M}$$

$$\#In + Out: \quad [(i|\alpha) \Leftrightarrow \sum\nolimits_{\beta \neq \alpha}(S|\beta)]|_{DGC \geq K} = \frac{\sum_{\beta \neq \alpha}\sum_{j \neq i}[((j|\beta) \Rightarrow (i|\alpha)) + ((i|\alpha) \Rightarrow (j|\beta))]|_{DGC \geq K}}{2(M-1)N/M}$$

4) Closeness: it measures the shortest distance between a financial institution and all other institutions directly or indirectly reachable from it. In particular, an institution j is weakly causally C-connected to i if there exists a causality path of length C between i and j. Thus, there should exist a sequence of nodes $k_1, \dots, k_{c-1}$, where each node represents an institution, such that the impact from j to i, through the C nodes, is unitary:

$$(j \Rightarrow k_1) \cdot (k_1 \Rightarrow k_2) \cdot (k_{c-1} \Rightarrow i) \equiv \left(j \overset{C}{\Rightarrow} i\right) = 1$$

As said, C is the shortest distance from j to i, for which if $j \overset{C}{\Rightarrow} i = 0$ then $C_{ji} = N - 1$ for all $C \in [1, N-1]$:

$$C_{ji} = \min_{C}\left\{C \in [1, N-1] : (j \overset{C}{\Rightarrow} i) = 1\right\}$$

Given a percentage of statistically significant Granger-causality relationships within a system of N institutions higher than a threshold K, the closeness measure for institution j is defined as an average of the number of possible shortest distances with the rest of the system S, and represents just the "proximity" in terms of connection among institutions:

$$C_{jS}|_{DGC \geq K} = \frac{\sum_{i \neq j} C_{ji}(j \overset{C}{\Rightarrow} i)}{N-1}|_{DGC \geq K}$$

5) Eigenvector Centrality: it measures the relevance of a financial institution in a network in accordance with its level of connection. It assigns a scores to each financial institution based its centrality and importance inside the network. The measure is the eigenvector v of the

adjacency matrix $[A]_{ij} = (j \Rightarrow i)$ associated with eigenvalue 1 (that is $Av = v$), and can be written as the sum of the eigenvector centralities of institutions caused by j, conditional on a network with a significant number of Granger-causality relationships:

$$v_j|_{DGC \geq K} = \sum_{i=1}^{N} [A]_{ji}\, v_i|_{DGC \geq K}$$

Macroeconomic measures

Macroeconomic measures of systemic risk try to put in relation the probability of systemic distress in the financial system and the dynamics of macroeconomic aggregates, as presented by the main economic and monetary policy models. In fact, the strict relation between the financial system and the so-called "real economy" is known from a theoretical perspective. A break in the productive economy, where firms and households operate, is reflected in the financial system as a shock due to the stoppage of liquidity providing from operators, bank runs or defaults cascade in the debt chain. On the other side, a financial crisis due to bubble bursting, irrational behavior or debt overhangs can easily involve other productive field of the economy, causing a recession. Therefore, it is possible to link financial variables, based on information on financial institutions, to other "real" macroeconomic variables, such as real GDP, inflation and public debt, in order to extrapolate common patterns useful to explain the systemic risk.

Considering all the above-mentioned criteria, systemic risk measures should be associated with real macroeconomic outcomes, especially in issues of policy decisions: in order to evaluate variations in the distributions of crucial economic variables to change in systemic risk, it is important to test the ability of a given risk measure to predict shifts in the expected quantiles following macroeconomic shocks. By performing a similar analysis, Giglio, Kelly and Pruit (2015) demonstrated three important stylized facts: the systemic risk measures show a particularly strong association with the downside risk (the risk that actual return falls below the expected return, in a context of uncertainty about the size of this fall) of macroeconomic shocks; financial sector equity volatility is quite informative about the future real activity, much more than the volatility of non-financial sectors; financial market distresses precede a monetary policy responses, even if this response could be insufficient to slow down an increased downside risk.

Any macroeconomic measure of systemic risk tries to take into account the fact that fragility within the financial system tends to be exacerbate during a crisis, and the instability tends to dramatically increase its macroeconomic impact. Therefore, macroeconomic effects of so-called "financial frictions" take place in the system, that Brunnermeier, Eisenbach and Sannikov (2012) tried to analyze in their "Macroeconomics with Financial Frictions: A Survey". The literature on the frictions is wide, and can be subdivided into four sections: the works about the role of Persistence (Carlstrom and Fuerst), Amplification (Bernanke, Gertler and Gilchrist) and Instability (Brunnermeier and Sannikov); the credit quantity constraints, like credit rationing (Stiglitz and Weiss), endogenous constraints (Geanakoplos and Fostel) and margin spirals (Brunnermeier and Pederson); the demand for liquid assets and bubbles; the financial intermediaries' theory and the money theory.

In general, the financial frictions are the set of difficulties and "stickiness" elements involved in conducting a transaction, and include both monetary and non-monetary costs. In fact, the overall process of making transactions includes time, effort, money, and tax for gathering information and performing all the operations required. For instance, buying a stock or borrowing money requires a set of delicate operations such as conducting research to get information, determining the price, complying with all regulations and bureaucratic procedures, spending time to materially execute the transaction. All these financial frictions, stemming from the financial sector, have been identified as a key element affecting fluctuations of relevant economic variables (like GDP growth), that is exerting an impact on the real economy.

Implications of financial frictions have been analyzed. A temporary adverse shock is said to be "persistent" if its effects last for a long time, and a long time is required for a complete rebuilding of the previous capital from productive agents. The persistence property is a function of the feedback effects of the frictions in the financial system: a negative shock affecting the value of a business network intensifies the present financial frictions and forces entrepreneurs to invest less. In particular, the persistence of a shock will be as more relevant as deeper and more serious is the situation of illiquidity and the necessity to fire-sell, namely the channels by which an initial shock is amplified. As known, fire sales depress the capital price, causing loss spirals (the net worth of agents further reduces), margin spirals (productive agents must reduce their leverage ratio) and other cuts. This is the amplification effect: the reduction in capital caused by negative shocks to the network reduces the cost of capital, and may bring to illiquidity and fire sales which further reduce the price of capital, amplifying the effect of the initial shock. Finally, a time dimension of the amplification effect makes it dynamic: the unexpected persistence of a temporary shock reduces expected future asset prices, and in turn this is reflected into lower actual asset prices. As a consequence, the capital of productive agents even further is eroded, and more fire sales are required. The presence of dynamic non-linear amplification effects is the main reason of the existence of wide volatility dynamics, and is the basis of the intrinsic unavoidable instability of the financial system. Therefore, the intrinsic financial instability is the consequence of endogenous risks resulting from interactions in the system, such as the liquidity spirals, which may cause large discontinuous drop in the prices and funding drying up. It is at such times that a demand for liquid assets strongly emerges.

From the perspective of the policy maker, the macro-economic aspect is fundamental, in order not to implement distorted policies and regulations, and to perform a correct monitoring for the prevention of the system. On this topic, Borio (2010) developed a complete macro-prudential framework, based on four well-defined dimensions: the criterion of success of given policies in limiting the risk of systemic financial distress; the degree to which systemic risk should be tracked; the right balance between an aggregate approach and a cross-sectional sectoral view; the right balance between rules and discretion.

As said, the macro-prudential perspective requires a setting of regulatory and supervisory arrangements suited to the system as a whole, rather than to single institutions. The essence of the macroeconomic view is the top-down approach, consisting of the definition of general standard for the entire system and, from there, the derivation of standards for the individual institutions. The assumption is that risk drivers depend on the distorted collective behavior of financial institutions, which in turn depend on risk perceptions of responses to it. Therefore, macroeconomic perspective investigates endogenous risks that emerges at system level. The objective of this approach is to limit the risks of systemic financial distress, and then to contain the possible risks and the costs for the real economy. In general, the final purpose of macro-prudential policy is to promote financial stability and limits systemic risk.

In this framework, two dimensions are taken into account: the time dimension and the cross-sectional dimension.

The time dimension analyzes the evolution of aggregate systemic financial risk over time, focusing on the central role of the pro-cyclicality of the financial system, that is the set of those elements and mechanisms within the economy that help to oversize the natural output cyclicality and fluctuations. From a policy perspective, pro-cyclicality is contrasted through countercyclical buffers in order to stabilize the system.

The cross-sectional dimension analyzes the risk generation and allocation within the financial system at a given point in time, and in particular it studies the risks and vulnerabilities stemming from common exposures, interlinkages and failures. To mitigate the risks stemming from common exposures and interlinkages, prudential tools and requirements regarding the contribution of each institution to systemic risk are implemented, such that each institution pays for the externality its activities exert on the system.

Finally, Borio points out the necessity to link each systemic risk measure with the purpose and the dimension analyzed. It is unrealistic and dangerous to base a macroeconomic policy or a theoretical model entirely on a single systemic risk measure, which can never capture all relevant aspects: the minimum distinction suggested by Borio should be between the time dimension and the cross-sectional dimension. In particular, according to Borio, in the time dimension, the ideal measure would be a robust leading indicator of financial distress, which allows to take remedial actions well in advance (at least over one year): they are the so-called early warning indicators. In the cross-sectional dimension, the ideal risk measure should be able to consistently quantify the individual marginal contribution of each institution to systemic risk. Of course, policy makers inevitably must face the trade-off between the accuracy of measures and models and their precision, and then between generality of information required and specificity of adopted measures. In general, it is believed by analysts that it is better to be approximately right than precisely wrong.

	Macroeconomic perspective	**Microeconomic perspective**
Operational objective	To limit systemic financial distress	To limit financial distress of single institutions
Ultimate objective	To stabilize output	To protect savers and investors
Risk nature	Endogenous risk (dependent on collective behavior)	Exogenous risk (external to each single institution)
Central risk source	Correlations and common exposures across institutions; procyclicality	Solvency, liquidity, leverage, confidence
Prudential control	Top-down	Bottom-up

Table 2.2: Comparison between macroeconomic and microeconomic perspectives

2.1.3 Systemic Risk Measures by Data Requirements

The complexity of the financial system consists as well of a huge set of different participants, market practices, tools, relationships characteristics and many other factors. At the same time, many important differences intervene among measures and models that try to capture different facets of systemic risk. In general, a correct framework for the detection and the management of the financial risk, both for policy makers and institutions, should consider different perspectives and different tools in order to be able to capture the continuous evolving structure of the financial system and to adequately use the most appropriate measures of systemic risk. Hence, it is important to consider how the changeability of financial systems should be included in the used approaches: a given approach suitable today might not be in the future. Furthermore, the practical implementation of certain systemic risk measures requires a set of precise and

Systemic Risk Measure
Macroeconomic Measures:
Costly Asset-Price Boom/Bust Cycles
Property-Price, Equity-Price, and Credit-Gap Indicators
Macroprudential Regulation
Granular Foundations and Network Measures:
The Default Intensity Model
Network Analysis and Systemic Financial Linkages
Simulating a Credit Scenario
Simulating a Credit-and-Funding-Shock Scenario
Granger-Causality Networks
Bank Funding Risk and Shock Transmission
Mark-to-Market Accounting and Liquidity Pricing
Forward-Looking Risk Measures:
Contingent Claims Analysis
Mahalanobis Distance
The Option iPoD
Multivariate Density Estimators
Simulating the Housing Sector
Consumer Credit
Principal Components Analysis
Stress-Test Measures:
GDP Stress Tests
Lessons from the SCAP
A 10-by-10-by-10 Approach
Cross-Sectional Measures:
CoVaR
Distressed Insurance Premium
Co-Risk
Marginal and Systemic Expected Shortfall
Measures of Illiquidity and Insolvency:
Risk Topography
The Leverage Cycle
Noise as Information for Illiquidity
Crowded Trades in Currency Funds
Equity Market Illiquidity
Serial Correlation and Illiquidity in Hedge Fund Returns
Broader Hedge-Fund-Based Systemic Risk Measures

Figure 2.3: Classification of systemic risk measures based on the data requirements
Source: “A Survey of Systemic Risk Analytics” by Bisias, Flood, Lo and Valavanis, 2012

crucial decisions by the analysts, such as the institutions’ characteristics, the frequency and the timespan of the adopted measure, the levels of accuracy and granularity of the used dataset, the eventual necessity to transform the raw inputs of the measure. From this set of choices, it is possible to derive another classification of the risk measures, approximatively based on the type and quantity of required data. The figure shown below lists the measures and the approaches in an increasing order of the level of detail and information of the data required to implement them. It can be useful both for regulators and financial institutions.

Anyway, it is important to take into account the approximation of this classification when one is dealing with it: each single measure must be evaluated individually and adapted to the context it is applied, on the basis of its advantages and weaknesses, of the data available with respect to those required for the computation, of the foresight and the sensitivity of the adopted model to given aspect of the systemic risk. Therefore, the measure must be adequately chosen and adapted to the specific situation (e.g. the precise characteristics of the analyzed financial crisis). Under this perspective, using more approaches at the same time may be only partially a good choice, because it adds further difficulties and may not be fit to correctly evaluate the systemic risk. Below, a brief description of those sub-sections of measures not previously discussed.

Forward-looking risk measures

Backward-looking measures rely on historical data, used to estimate probabilities and size of future tail events, but presents some shortcomings. In particular, they seem to be inadequate in preparing authorities and institutions for eventual shocks that lie ahead. Furthermore, making portfolio decisions and policy making on the basis of past returns may contribute to the pro-cyclicality of risk management, and then to consequent negative impacts.

As an alternative solution, that tries to overcome these problems, it is useful to introduce the forward-looking measures, which extract information from current data about the possible evolutions of the variables of interest, and then of the systemic risk. Anyway, these measures should not be considered as forecasting tools, but as alternative tools for the evaluation of portfolio value and other patterns within the financial system, based on a forward-looking method. Moreover, even for these measures the non-linearity of relationships increases the degree of complication for their correct implementation. In order to better survey the risk dimension, forward-looking risk measures are oriented to future cash flows of portfolio positions under different scenarios, helping to focus the attention of the operators on the potential risk factors in the system.

A useful example is given by the measure "option i-PoD" (Christian Capuano, 2008). It is the probability of default implied by option prices, that is inferred from equity options by applying the principle of minimum cross-entropy (Cover and Thomas, 2006). In particular, the probability of default is defined as the probability that the underlying asset value will fall below a given threshold, which is not prefixed but it is endogenously determined. It is shown how this framework provides robustly informative results on the implied expected evolution of balance sheet items, such as assets, equity and leverage. Moreover, this measure allows to determine also the implied asset volatility and the Greek letters, useful elements in the risk management.

Stress-test measures

The stress test is the assessment of the capacity of institutions to face economic shocks and to prove to robustly withstand under adverse scenarios. They are performed by public authorities mainly towards on SIFIs within the banking system, and try to plot 'what if?' scenarios, playing a crucial role in the systemic risk monitoring process.

Accordingly, the stress test tries to answer two basic questions, by taking the perspective of a banking portfolio:

1) which economic and financial scenarios would lead to significant losses?
2) how big could these losses be?

The answer to the first question requires the identification of those possible future extreme circumstances that may negatively damage the value of the institutions, while the estimate of the expected loss at the worst-case scenario, through the usage of a portfolio loss function, is the answer to the second question.

Recently, stress tests gradually became a central element of risk management, mainly for supervisory purposes and financial stability analysis by policy makers. For this reason, the development of the most important stress testing frameworks has been implemented by central banks and other financial supervision authorities. In the U.S. the Comprehensive Capital Analysis and Review Process and the Dodd-Frank-Act-Stress Tests are performed; in Europe, the Stress tests of the European Banking Authority is performed.

Within this framework, the stress test measure provides a tool to evaluate the probability of big losses of a portfolio, of an institution or of the whole system. This risk measure is usually based on the assessment of that amount of capital needed to make the probability of big losses sufficiently small (e.g. VaR, ES).

The ultimate role of these tests is to allow risk reducing actions, since they bring to a better understanding of risk factors of each bank, and it provides indications for portfolio adjustments in order to avoid detrimental effects in case of bad scenarios.

As example, the 10-by-10-by-10 network-based approach is shortly described. This approach is based on the analysis of the risk exposures of a selected group of SIFIs under a set of distress scenarios: in particular, 10 designated institutions are required to report their gains or losses, in terms of market value and cash flows, for each one of the 10 described stressful scenarios (for example, the default of a counter-party, a shift in the yields curve, an increase in credit spreads, a variation in housing prices), and also to provide the identities of those 10 counter-parties whom gains and losses are the biggest in magnitude for each scenario. The institutions at the top are those whose failure would produce the heaviest loss for the reporting bank. This process will allow policy makers to assess those nodes with higher tension in terms of liquidity and value, and to give a shape to the systemic risk. In fact, 10-by-10-by-10 is a scenario measure that may help to map joint exposures of the system and to identify further SIFIs.

Cross-sectional measures

As said, two dimensions of systemic risk can be identified: the time dimension, inherent to the pro-cyclicality of the financial system, and the cross-sectional dimension, inherent to the linkages among financial institutions, which affect the sensitivity to risk spreading over the whole system. These measures are useful to monitor the degree of fragility of the system and its resilience to shocks at each point of time, trying to examine the co-dependence among institutions on the basis of their financial health conditional on particular circumstance. The already discussed CoVaR, SES and the Co-Risk measures belong to this group.

2.1.4 Systemic Risk Measures by Event or Decision Time Horizon

This last classification takes into account the time horizon with respect to given events or decisions related to a systemic financial distress. In particular, it is possible to identify different systemic risk measurement tools with respect to three temporal moments.

Systemic Risk Measure
Ex Ante Measures—Early Warning:
Costly Asset-Price Boom/Bust Cycles
Property-Price, Equity-Price, and Credit-Gap Indicators
The Default Intensity Model
Network Analysis and Systemic Financial Linkages
Simulating the Housing Sector
Consumer Credit
GDP Stress Tests
Distressed Insurance Premium
The Leverage Cycle
Serial Correlation and Illiquidity in Hedge Fund Returns
Broader Hedge-Fund-Based Systemic Risk Measures
Ex Ante Measures—Counterfactual Simulation and Stress Tests:
Simulating a Credit Scenario
Simulating a Credit-and-Funding-Shock Scenario
Lessons from the SCAP
A 10-by-10-by-10 Approach
Marginal and Systemic Expected Shortfall
Contemporaneous Measures—Fragility:
Granger-Causality Networks
Contingent Claims Analysis
The Option iPoD
Multivariate Density Estimators
CoVaR
Co-Risk
Contemporaneous Measures—Crisis Monitoring:
Bank Funding Risk and Shock Transmission
Mahalanobis Distance
Principal Components Analysis
Noise as Information for Illiquidity
Crowded Trades in Currency Funds
Equity Market Illiquidity
Ex Post Measures—Forensic Analysis:
Macroprudential Regulation
Mark-to-Market Accounting and Liquidity Pricing
Ex Post Measures—Orderly Resolution:
Risk Topography

Figure 2.4: Classification of systemic risk measures based on the time horizon

Source: "A Survey of Systemic Risk Analytics" by Bisias, Flood, Lo and Valavanis, 2012

1) *Ex-ante* measures try to capture financial signals about a potential systemic distress over short/medium-term period. Their objective is to provide early warning of

increasing imbalances and risks for the whole system, and a good measure of their performance and reliability is the ratio between the correctly predicted historical episodes (the signal from the systemic risk measure) and false alarms (its noise). This measure of signal strength relative to background noise is the signal-to-noise ratio. Since the core of this thesis is the implementation of an experimental early warning indicator for the systemic risk, the following subsection of this chapter will be dedicated to a deeper description of this category.

2) Contemporaneous measures try to quantify the level of outstanding disturbance, confusion and uncertainty within the actual system, and the big advantage of using them is to allow a better and adequate asset re-allocation and a prompt view for policy makers during emergency situations. The reaction time is very important, since it can determine the fate of a firm or of the system as a whole, especially when a crisis occurs.
3) *Ex-post* measures refer to the "after-the-event" situation, where probability and projections no longer matter, and the final outcome can be directly observed. This kind of measures, which the ex-post analysis is based on, is useful for many aspects: it allows to clearly report the events and to maintain accountability for policy makers, and in particular, since regulation is a repeated game, the monitoring incentives diligent behavior; it allows to evaluate the performance of other measures compared to what the regulator or institution initially projected, and to determine the accuracy of the other risk assessment methods; it can be used by scholars and researches to clarify certain events and their causes, and to identify new theoretical patterns.

As seen, the main critical usefulness of a classification of systemic risk measurement related to events is the timeliness provided to the decision-making process of institutions and policy makers, which must decide whether, when and how to carry out a given operation on the financial markets. The main characteristics of the three categories are described below.

<u>*Ex-ante* measures of systemic risk</u>

Within the financial system there are many threats to financial stability, because of the existence of a set of unpredictable possible shocks, herding behaviors, expectations and many other dynamics that make the market instable and continuously changing. In this context, the detection and the monitoring of so many possible threats are possible just through a wide set of monitoring methods and *ex-ante* measures. In fact, these measures allow the detection of the systemic risk in a probabilistic sense, in order to provide an overview about increasing imbalances or imminent dangers inside the financial network.

This classification identifies two further sub-groups of *ex-ante* measures: those consisting of early warnings and those based on counterfactual simulations and stress tests.

Early warning measures are endowed with a reliable forecasting power about possible evolutions of the systemic risk by a given not short time horizon, in order to allow the identification of eventual imbalances. Therefore, these techniques attach a given probability to future triggering events, that in turn are based on a dataset of observations of specific systemic features. The next sub-section will discuss more in detail this category.

The second subset of models, suited to analyze the systemic risk before a given adverse event, comprises all those measures that focus on the behavior of institutions and networks in hypothetical distressing conditions, trying to estimate the systemic vulnerability, intended as the sensibility in the institutions' performance to an adverse combination of external factors. In particular, counterfactual simulations and stress tests are analysis based on a set of specific scenarios, outlined through past occurred episodes, suitably built circumstances or extreme hypothetical situations, by considering some extreme values that given economic quantities may assume. At the same time, the reverse stress test can be performed in order to identify those circumstances and scenarios that cause certain pre-specified outcomes and level of distress for institutions.

These kinds of measures are very useful also to categorize vulnerable institutions and networks. For example, the Supervisory Capital Assessment Program (SCAP) is a stress testing conducted by policy makers in the U.S. to determine if the largest U.S. financial institutions have sufficient capital buffers to face a financial market turbulence. This stress test applies two macroeconomic scenarios, described in terms of GDP growth, unemployment and housing prices, one based on standard assumption and the other based on a negative expected circumstance. The consequent analysis of each institution's risk profile, by using models, internal dataset, and various estimation methods, brought to the mapping of the systemic risk in different types of macroeconomic scenarios.

At the same time, policy makers have recently intensified the usage of counterfactual simulations in order to evaluate the systemic risk due to contagion and common exposures in the financial market. In fact, the adverse contagion is a rare event, that occur in few extreme scenarios, but that can produce a very heavy negative impact on the soundness of each involved institution. Therefore, counterfactual simulations provide information about relevant institutions, whose financial strength affects the systemic financial stability, and about the nature of the connections, whose structure impact on the extent and the intensity of a contagion. Therefore, they are able to give an indication about whether or not, and how, a contagion may be a problem of systemic scope. Nevertheless, there are two main theoretical shortcomings of counterfactual simulations as systemic risk measure: they are based on too strong underlying assumptions, and their models present a lack of behavioral foundations.

<u>Contemporaneous measures of systemic risk</u>

Contemporaneous measures of systemic risk can be categorized by their declared task, in accordance with the necessity of the interested institution: the quantification of the fragility of the system, or the monitoring of the actual evolution of a financial crisis.

Measuring financial fragility has multiple purposes: signaling the intensity of an ongoing distress, and in general informing about the present state of the system; identifying the financially weak or failing institutions, sectors and markets; allowing the implementation of appropriate public interventions from those authorities which have a public mandate and must comply with some duties of communication with the media. The focus, given the nature of the systemic risk, and the way it unfolds, is always to operate within compressed time frames, in order to properly face and work out any systemic imbalance. For that reason, these contemporaneous measures allow to evaluate signals about the fragility of the system on a daily or even intra-daily basis. Among these frequently updated measures, there are SES measure, CoVaR measure, Co-Risk and the contingent claims analysis.

The second need satisfied by contemporaneous measures is the monitoring of the evolution of a financial crisis. In particular, they allow to track the evolution of a distress when it unfolds, and they are a powerful tool for policy makers dealing with a crisis, and with the necessity to work out appropriate policies in rapid times. For instance, they include the Mahalanobis distance, the Noise as Information for Illiquidity and other tools from the Principal Component Analysis.

Ex-post measures of systemic risk

Ex-post measures of systemic risk are a fundamental component of a more general *ex-post* analysis, very useful for the identification of some weaknesses in the financial sectors and some defects in the regulatory and supervisory system, allowing for any important reform. Moreover, by deepening and quantifying the most relevant structural vulnerabilities of financial markets it is possible to better understand the gaps in the regulation and in the control system, and to remedy accordingly. For these purposes, the *ex-post* measurement is the starting point, and this is continuously evaluated, even after a systemic event: its role for a coordinated transparent response by policy makers and for addressing the choices of market participants, especially in case of panic, is fundamental.

These measures have two further sub-categories: measures for forensic analysis and measure for orderly resolution.

Forensic analysis is based on a method of investigation whose objective is to detect and record causes, consequences and sequence of events in cases of adversity in the financial system, in order to identify institutions' liabilities (in general, it is related to courts of law for criminal matters). This analysis includes the usage of a set of tools and technical expertise, including the measurement, for the collection of information about the system working, the participants' integrity and the completeness of regulation. It is a rational procedure to deal with adverse events, and to proceed with resulting recovery of damages, as the redefinition of the regulatory framework.

The second subgroup refers to those *ex-post* systemic risk measures which play an important role for an orderly resolution of failing institutions. In particular, Brunnermeier, Gorton, and Krishnamurthy proposed a real "risk topography", that is the deep and complete study of the shape and features of the whole financial system, seen as a network of contractual connections. In particular, risk topography is a network-based model, based on the inference from each financial institution of the "sensitivity" of their capital account and liquidity to a set of pre-specified possible scenarios and factors (for instance, they should report the amount of profit and loss, and the variation of their liquidity position, stemming from a unitary change in the Liquidity Mismatch Index, or in the house price, or in the interest rate, and so on). The topography is centered on these two dimensions, capital gain and liquidity change, because they are considered the most significant determinants of the behavior of financial institutions during financial crisis. Then, a panel dataset is created by pooling all the responses from each institution to the same scenarios on each reporting date (quarterly or monthly). Authors highlighted two main advantages of this approach:

1) it allows to enhance the regulatory risk assessment and the macro-prudential supervision performed both by public authorities and the same institutions, if the dataset is made publicly available; in particular, the analysis of the dataset may uncover the presence of risk and liquidity "pockets" within the financial system, that is, of excessive imbalances and risk exposures, unawares undertaken by each firm, that may result in a large systemic concern;
2) it would provide to the researchers the complete essential data needed for current macroeconomic models that still do not embed components of financial sector.

2.2 Early Warning Indicators for Systemic Risk

Financial risk analysis requires a complete collection of data and the knowledge of a set of tools and techniques for the assessment of vulnerabilities and risks; at the same time, the monitoring and warning system require the detection and the analysis of causal links among those relevant variables determining the systemic risk. In general, the ultimate objective of an Early Warning System (EWS) is not to predict the exact timing of a crisis, but to estimate at each moment the probability of given adverse events to occur within a specific time horizon, and then to quickly and clearly communicate information and warnings about probable incoming dangers for the institutions, which can consequently respond and prudently act to protect itself with right policies.

EWIs is able to anticipate extreme movements in the financial cycle, that is, as defined by Borio, the "self-reinforcing interactions between perceptions of value and risk, risk-taking, and financing constraints". These interactions between investors' perceptions and risk-taking give life to a sequence of financial expansions and contractions, which can be reflected in the economic cycle as well. The theoretical focus underlying this type of tools applied on the financial market is the endogenously determined systemic risk. In particular, the outsize financial booms are the right background to create the conditions of a financial crisis, with high risk propensity from economic agents, dangerously increasing credit supply and asset prices well away from assets fundamentals. Precisely, a standard EWS can be based on the setting of specific thresholds for the evolution of the private debt or asset prices: when the actual price significantly deviates from its long-run trends, overcoming prefixed threshold, it is reasonable to assume an ongoing increase in systemic risk, that is in probability of a financial crisis due to a financial bubble. Among variables that can be used for the detection of countries' financial systemic risk, there are:

- the credit-to-GDP ratio gap is the difference between the credit-to-GDP ratio and its long-term path; if this gap widens, then a financial imbalance is emerging, with the consequent increase in the systemic risk of disruption;
- the house price gap is the deviation of inflation-adjusted property prices from their long-term path; its working is similar to the previous one;
- the Debt Service Ratio (DSR) is the ratio between the debt service (interest payments and amortizations) and the income from the international trade; a DSRs increase signals a credit expansion, as credit growth is reflected into a higher debt service;
- other ratios of the type of DSR, but based on sub-groups of debt, such as foreign currency debt or household debt, can be considered.

For instance, the household debt issue has been studied in depth in recent literature. Countries where household debt is particularly high and increasing, such as in Netherlands, face periods with high consumption and a continuing GDP growth: this economic overheating can be detrimental for the long-run structural economic growth and for the banking system health, that chances a crisis. Another type of DSR considers the foreign currency debt or the cross-border debt, in order to evaluate the role of current account deficits or exchange rate evolution in the boost of systemic risk. Many studies have proven that levels of household debt and foreign debt are effective in surveying presence of increasing vulnerability in the system, since the respective indicators tend to show abnormal values over their trend during those pre-crisis phases in which systemic risk is increasing. These variables, even more if combined together, may be rather useful to extract information about probable future cumulative distress in the banking system. Moreover, combining information from different variables remarkably improves the precision of these indicators. In fact, they showed some peaks before recent crisis, and for that they can be considered as good predictors of systemic risk. For all these reasons, policy makers have focused on studying and monitoring the household debt and the foreign debt as crucial variables to look at for financial stability purposes.

In particular, policy makers should integrate EWIs within a wider and more complete analysis, and must be certain of the utility and robustness of the indicators. There are some specific properties and characteristics that EWIs should comply with to be useful in detecting the presence of systemic risk (Drehmann and Juselius, 2014).

1) A statistical forecasting power of EWIs based on real time evolution of the variable of interest is needed for a correct implementation of time-varying macro-prudential policies.
2) The timing of the signal issue is fundamental: it should emerge before a crisis, early enough to be effective and useful for a correct implementation of remedial policies, but not too early in order to avoid some detrimental and unnecessary restrictions.
3) The stability of the signal is required to be sure of the feasibility of a new policy. In fact, policy makers should implement a new policy on the basis of clear long-term evolutions and trends of crucial variables, and not on the basis of outliers and unstable indicators. Therefore, EWIs should issue stable and persistent signals, reducing uncertainty on trends and not reducing the forecasting power when a crisis is approaching.
4) EWI signals should be transparent, easy to interpret and linked to the financial cycle theory. In fact, there is evidence (Lawrence *et al.*, 2006) that counterintuitive and not easily interpretable forecasts and signals are ignored by policy makers, and this makes EWIs less useful. Moreover, the low complexity of EWIs contributes to reducing the risk of overfitting. An overfitted model is a very complex statistical model which is adapted to observed data just because it has an excessive number of parameters with respect to the number of observations. In fact, even a completely wrong model can perfectly fit and explain dataset, but it produces unreliable predictions.

As said, a basic element of these indicators is the critical threshold, which can be derived through different methodologies, and that allow to identify warning signal when the variable crosses it. In general, it needs to find, within a range of potential thresholds applied on a large panel dataset on more countries over a long time, the one which allows to significantly signal a warning when it is crossed. A crisis is correctly predicted if warning signals emerge whenever the threshold is crossed in the periods immediately preceding the crisis. It is possible to identify the right threshold by minimizing the noise-to-signal ratio, that is the ratio between the number of false alarms (warnings appearing even when no crisis occurs) and the number of correct warning signals (warnings appearing when the crisis occurs). In fact, being a minimization problem, there is an error term whatever the chosen threshold: it should be set in order to be able to correctly predict the highest number of crisis. The higher the number of predicted crisis, the higher the reliability of EWI, given the lowest number of false alarms.

There are some caveats to consider in this framework.

As said, the false alarms occur whenever warning signals are issued but no crisis followed. This does not necessarily indicate a malfunction of the indicator, since there two possibilities: either there are specific factors that affect the variable of interest without being linked to the systemic risk (and in this case the signal launched by the indicator is not significant), or the threshold is crossed because of a real increase of systemic risk, but the crisis does not occur since the

imbalance automatically disappears before the possible materialization of the crisis. Then, some warning signals are not necessarily succeeded by a crisis, and for that, in order to evaluate the performance of the EWI, it is useful to look at the fraction of crisis effectively occurred within a certain period of time, conditional on the breach of the threshold.
In any case, even if the systemic risk is correctly identified by EWI, the conveyed information does not refer to the precise timing of the future crisis, but just to the increased probability of systemic distress. More precisely, EWIs are not able to signal an intensification of financial imbalances, but only the presence of dangerous imbalances, through a dichotomous outcome.
Moreover, these indicators need to be interpreted with caution: their calibration is based on the historical evolution of the variable of interest, and for that they do not take into account more recent innovations in institutions and economic structure. In fact, the overall resilience to shocks of the system is a function of the regulatory environment (microeconomic and macroeconomic policies) and of the same technological and financial innovations, and this would require continuous re-calibrations of the indicators. Therefore, it would be better if EWIs were considered just a part of a more complete set of tools used to detect systemic risk and other vulnerabilities under different facets, and then just the first step of a broader analysis in the assessment of financial risk.
Finally, there are four important shortcomings related to EWIs:

- these indicators are constructed on historical dataset and specific relations among variables; the consequence is that, if a random innovation or a structural break occurs, a new specification of the underlying EWIs model is requires, in order to not lose the initial predictive power and to maintain the consistency of the indicator;
- the heterogeneity of the financial systems across countries makes the EWIs' intrinsic scheme of "one-size-fits-all" not valid; then, the thresholds set for an economy may not be the best one for other economies, because of countries' specific characteristics; in general, it seems not possible to construct a standard indicator to be applied to countries based on the patterns provided by a single country, given the asymmetries of the systems and the impacts of adverse events;
- EWIs are constructed to test the presence of systemic risk and vulnerabilities strictly referred to the economic and financial cycle, that is the endogenous alternation of phases of expansion and contraction within the system, and not to structural breaks and other random factors that may trigger a crisis without being linked to the natural cycle (such as those crises due to unsustainable sovereign debts, bad management of the monetary policy, fraudulent behaviors of important institutions);
- since crises are rare events, there may be problems of data limitation and significance, even in presence of a wide coverage; the consequence is the impossibility to create a robust and complete framework for the application of EWIs, and hence a particular focus is required in the selection of dataset.

Another fundamental point is the evaluation of the performance of the EWI, that is its robustness and capacity to issue right early warnings and to detect an increase of systemic risk. As usual in the statistical testing, two different possible kinds of errors should be analyzed:

- type I error is the rejection of the null hypothesis when it is true; it is also known as "false positive", and emerges when the systemic risk has really increased, and a crisis is effectively oncoming, but the warning indicator fails to signal it;

- type II error is the acceptation of the null hypothesis when it is false; it is also known as "false negative" or “false alarms”, and emerges when the warning indicator signals an increase in systemic risk and in probability of an incoming crisis even though this risk is not real.

Since the complete elimination of both kinds of errors is not possible, and since the reduction of one of them implies the increase of the other one, the evaluation of EWIs’ performance consists of the assessment of the trade-off between the error types, looking at their joint minimization. In particular, the procedure for the construction of a robust EWI requires the achievement of an optimal trade-off between the correct predictions and the false alarms: a perfect and totally informative EWI does not exhibit any error, and then provides the right positive signals whenever a crisis is really incoming, and negative signals the rest of the time. At the other extreme, a totally uninformative EWI provides signals without any reliability, not very different from the outcomes of a coin toss. In reality, plausible EWIs are between two extremes, and their position can be assessed through appropriate tools. For example, a common and useful instrument for the evaluation of EWIs’ performance is the Receiver Operating Characteristic (ROC) curve, that is a graphical plot representing the true positive rate (i.e. the probability of correct predictions) against the false positive rate (i.e. the probability of false alarms) for a set of thresholds. ROC curve provides a map for type I and type II errors, which gives indications on the informativeness of a test: by plotting the cumulative distribution function of probability of correct predictions in the y-axis and the cumulative distribution function of the probability of false alarms on the x-axis, the ROC curve is represented. The area under the ROC curve is a proxy of the quality of the signal issued by the indicator: for a given implied threshold, if this area is equal to 1, the indicator is totally informative (since its probability of correct predictions is 1, and its probability of false alarms is 0), while if it is equal to 0.50, then the indicator is totally uninformative (with a true positive rate of 50% and a false positive rate of 50%).

There is a large literature on EWIs for financial system that analyzes their crucial theoretical aspects and tries to elaborate new frameworks and new methodologies of construction and calculation. Kaminsky, Lizondo, and Reinhart (1998) and Kaminsky and Reinhart (1998) developed operational Early Warning System for the survey of systemic risk through the "signal" approach. This approach is just that one described above: the evolution of a set of indicators is monitored in order to identify unusual paths, signaling a probable incoming crisis when one of them overcomes some prefixed thresholds. This approach, widely used by policy makers, showed to be useful in anticipating some crises (such as currency crises in 1997).

Demirgüç-Kunt and Detragiache (1999) developed a multivariate empirical probability model for banking crises in order to monitor banking sector fragility, and created an EWS that issues a signal whenever a probability of a crisis crosses a specific threshold. Therefore, there are two basic methodologies to assess the risk of incoming crisis: the signal approach and the probability model.

Davis and Karim (2008) analyzed the usefulness and the robustness of some approaches applied to a comprehensive dataset, and found that the most appropriate model to assess EWI signal for a crisis is the logit model, both at global level and a country level. Moreover, they highlighted the importance to take into account policy makers preferences, through a loss function, during the construction of an indicator and the setting of thresholds.

Bussiere and Fratzscher (2006) created an EWI based on a multinomial logit model, since it is shown that the binomial dependent variable models are not able to distinguish among more states of the world, like pre-crisis periods and the crisis/post-crisis periods, when the variable of interest undergoes an adjustment process towards a new sustainable and stable trend. By solving this bias, the multinomial logit model allows to improve the forecasting ability.
As variables of interest, it is very common to consider both macroeconomic variables and financial variables, like it has been done by Alessi and Detken (2011), who investigated the relationship between short-term interest rates and bank risk, finding that unusually low interest rates over a long period of time make bank risk increase. They used a global measure of liquidity, that is the global private credit gap, for the construction of a real time signaling approach for asset price cycle.
Lo Duca and Peltonen (2013) developed a new framework to assess the probability of financial distress, providing several important contributions to the literature. Firstly, they constructed a new Financial Stress Index (FSI), a country-specific composite index which tries to capture the beginning and the evolution of a crisis by grouping more stress measures referred to different segments of the financial system. In fact, when a negative shock occurs (such as a bubble burst, a currency crisis or a default), each segment of the financial system faces a distress, and this amplifies the shock all over the economy: the broader the distress, the stronger the co-movements of financial variables and the more systemic the crisis. The considered segments of financial system are four: the equity market, the banking sector, the foreign exchange market, and the bond market. The components j of FSI, for each country i at each quarter t, are basically of two types, risk premia and implied volatilities: they are transformed into integers that range from 0 to 3, such that, when a given component value drops to the fourth quartile $q_{j,i,t}$ of the distribution, then it takes value 3. Therefore, the higher the value the higher the stress level, and the FSI is the simple average of the transformed variables:

$$FSI_{i,t} = \frac{\sum_{j=1}^{n} q_{j,i,t}(Ind_{j,i,t})}{n}$$

FSI is useful to identify the beginning of a financial crisis. In particular, to identify systemic events, authors studied the relationship between the FSI and other measures of real economic activity. Afterwards, they created a model for the prediction of out-of-sample systemic financial crises by considering jointly both national and global indicators of macro-financial vulnerabilities in a multivariate framework. Finally, both macro-prudential indicators of vulnerabilities and multivariate indicators are evaluated through the usage of discrete choice models, and by taking into account policy makers' preferences. In fact, many macro-prudential measures aim to identify and to prevent accumulating imbalances, being endowed with some forecasting power for systemic events. In particular, many authors (Borio and Drehmann in 2009, or Alessi and Detken in 2009) found that global measures of liquidity are the best performing indicators for this purpose, since they allow policy makers to get useful information and to timely react to growing imbalances.

Chapter 3
Theoretical Framework for New EWIs

After having accurately presented and described the argument in the first two chapters, and provided a critical review of the relevant literature about the systemic risk, this third chapter will be dedicated to the theoretical description of framework and tools needed for the empirical analysis. In fact, the core of this thesis is the implementation of early warning indicators for systemic risk, and in particular the design of a test procedure for the survey of signals related to systemic risk in financial markets. This is done by performing a quantile regression on a Mixed-Data Sampling (MIDAS) model: these two approaches are combined to allow the building of this test for the identification of systemic risk, and it will consist of an application of the nonparametric test of Granger causality in quantile, proposed by Jeong, Hardle and Song (Econometric Theory, 2012).

The purpose is to create an analytical tool that monitors the evolution of those macroeconomic situations where risks to financial stability become relevant. As said, the ultimate objective is multiple, and depends on the user's aims:

- to quantify the contagion risk within the financial market in order to survey SIFIs and their potentially dangerous interlinkages;
- to provide signals of systemic distress from each segment of interest within the financial system;
- to measure the resilience and the health of the financial system as a whole and of its key sectors;
- to forecast and to anticipate crises, so that mangers can rebalance and adjust positions in their financial portfolios.

However, the real advantage of an EWS is to strengthen the ability to identify incoming troubles within the financial system with timeliness. In general, the creation of an EWI requires the definition and the extent of the framework (definition of variables of interest and explanatory variables, data coverage across countries and markets, covered timespan), the definition of systemic distress and the typology of mathematical and statistical tools used in the analysis for the signal survey. The following sections of this chapter will describe and define the statistical approaches used for the design of the EWI: the quantile regression model, the MIDAS model and the nonparametric test of Granger causality-in-quantiles. The final part will describe in detail the methodology for the empirical analysis.

3.1 Quantile Regression Models

The quantile of order τ (or τ-quantile) is that value q_τ of the distribution of a random variable x that divides it into two parts, and that includes first τ*N observations in ascending order, with τ a real number in the range [0,1] and N the number of observations. For instance, with N=50, the 0.1-quantile of the distribution is that value of x that includes first 5 observations starting from the last one. Also, quartiles refer to those three cut quarters of the distribution, which divide a dataset into four equal-sized groups; the decile is the tenth part, the median is the 0.5-quantile and the τ'th quantile is the 100*τ'th percentile.

When the cumulative distribution function of the random variable x is known, the τ-quantiles are outputs of the quantile function: it returns the value q_τ of the random variable x such that the probability of the variable being strictly less than q_τ equals the given probability, which corresponds to τ:

$$\tau = Pr(x \leq q_\tau) \equiv F_x(q_\tau)$$

Therefore, quantile function is the inverse of the cumulative distribution function, given τ:

$$q_\tau = F_x^{-1}(\tau)$$

Quantile regression is an approach to modeling the relationship between a dependent variable and one or more explanatory independent variables, within a framework based on the concept of quantile and alternative to the more common linear regression estimated through OLS: whereas the method of least squares estimates the mean of the dependent variable conditional on a set of regressors, quantile regression estimates its conditional quantiles. In other words, this type of analysis allows to estimate the quantile of the distribution of y given x, and then how the τth'quantile (e.g. the median) of the distribution of y given x changes with x.

This type of analysis was established by Koenker and Bassett (1978), who first introduced the linear quantile regression and worked out a first algorithm for the computation of the proper coefficients. Their fundamental contribution to quantile regression was based on important past works in the field, mainly by Bošković, Laplace, and Edgeworth. In 1760, the mathematician Roger Bošković, in an attempt to confirm a suggestion by Isaac Newton, tried to estimate the Earth's ellipticity by combining different measures and information on locations and latitudes through an alternative mathematical framework. His intuition consisted of the minimization of the sum of absolute deviances from respective medians, in order to obtain median regression slopes, and all this happened half century earlier the first formulation of the least square method by Legendre. Afterward, Laplace studied in deep and formalized the Bošković's ideas, and elaborated the so-called "method of situation". In particular, Laplace remarked that, algebraically, the solution to the ellipticity problem is nothing but the computation of a weighted median: by minimizing the sum of absolute deviances subject to the constraint that the errors sum to zero, and then by imposing that the fitted line goes through the center of observations (average of variables), the problem becomes a regression analysis through the origin; by rotating the line through this artificial origin, it is possible to find that slope which minimizes the sum of absolute errors. In that way, Laplace suggests to estimate the intercept as a mean and the slope coefficient as a median. Finally, Francis Edgeworth, resuming the same ideas almost a century later (in 1888), gave some crucial contributions to this theoretical framework. He highlighted the problems related to the sample mean, whose adequacy as estimator heavily depends on the implausible normality assumption of data: about that, he was able to prove that median and mean may be very divergent under different characteristics of data distribution, and showed in which cases median may have smaller asymptotic variance than the mean. Anyway, his central work consisted of the adoption of a new geometric approach to median regression: he minimized the sum of absolute errors for both intercept and slope coefficients, discarding the Bošković -Laplace constraint of zero errors sum. Furthermore, from this 'double median' approach, he even proposed an extension for a 'plural median' approach in the multivariate case, but those few and not very powerful technological tools he had at his time didn't allow him to overcome many tedious computational problems intrinsic to this

setting. With the widespread adoption of computers and linear programming over the last century, more technologically advanced and more computationally powerful tools enabled the completion of this approach.
Koenker and Bassett, by resuming those previous works, started from the Edgeworth's median regression estimator and 'generalized' it, obtaining the quantile regression estimator.
Next subsection will present more in deep the analytical aspects of this analysis.

3.1.1 Fundamentals of Quantile Regression

Quantile regression analysis provides a very powerful explanatory and predictive statistical model, useful in dealing with the increasing complexity of data in a robust and flexible way. There are several advantages provided by the application of this type of models: on the one hand, they are presented as more flexible and they weaken assumptions (for instance, they fit conditional quantiles of the dependent variable with a general model which does not assume any particular parametric form for its distribution); on the other hand, they provide conclusions and information that no other framework is able to give (for instance, they are very useful in risk management since they allow to deal with problems linked to tails of conditional distributions or to modeling a whole conditional distribution).
The first step is the description of the quantile regression model under an analytical point of view, starting from the list of required assumptions. There are three basic assumptions to be met for the usage of quintile regression.

- The zero conditional quantile assumption. Similarly to the Ordinary Least Square estimator of the Linear Regression Model, even in the quantile regression the error term must not be correlated with the set of explanatory variables. In particular, as OLS estimator is unbiased and consistent if the model $\mathbb{E}(y|\boldsymbol{x}) = \boldsymbol{x\beta}$ is well-specified, that is if $\mathbb{E}(\varepsilon|\boldsymbol{x}) = 0$, similarly the quantile estimator is consistent if the τ-quantile of ε_τ is zero at any point of the $\boldsymbol{x}$ distribution:
$$\mathbb{Q}(\varepsilon_\tau|\boldsymbol{x}) = 0$$
such that $\mathbb{Q}(y_\tau|\boldsymbol{x}) = \boldsymbol{x\beta}$. Therefore, the unbiasedness of estimated quantiles of order τ of the dependent variable, conditional on the vector of explanatory variables, is satisfied when the zero conditional quantile condition of the error term is met.
- Linearity of the model. Even though non-linear quantile regression analyses are available, in this framework, for simplicity, it is assumed a linear dependence between the response variable and other explanatory variables. In fact, the linear relationship may be an accurate specification of the actual relationship among variables, especially if no specific form is surveyed.
- For problems related to efficiency of the estimator, it needs to take into account very large samples and a very continuous response variable, as well as independence of observations.

It is worth noting that these assumptions are fewer and much less stringent than those required by OLS estimator: the distribution of y_τ conditional on $\boldsymbol{x}$ is not assumed to have a normal distribution; sample observations of dependent and independent variables are not required to be independent and identically distributed (I.I.D.) draws from a joint distribution, and other specific assumptions, like homoskedasticity, do not apply; finally, the nonzero finite fourth moments of observations are not required, because quantile regression is robust to outliers of

dependent variable (but it is less robust to outliers and extremes of the explanatory variables, and for these cases the weighted quantile regression becomes more appropriate).
A further issue refers to the assumptions on the error term. As known, if the least squares assumptions hold and if the errors are homoscedastic, uncorrelated and have expected value of zero (whatever the type of distribution), then OLS is BLUE (Best Linear Unbiased Estimator), since it is the estimator with the lowest sampling variance within the class of linear unbiased estimators. In the quantile regression model, the assumption of I.I.D. errors can be useful to simplify the underlying mathematics, allowing quickly derivation of some asymptotic properties. Unfortunately, the hypothesis of I.I.D. errors is actually likely to be violated, and for this reason, other tools can be implemented for both the OLS estimates and quantile estimates: the 'robust' option (that is the regression performed with modified standard errors, robust to heteroskedasticity and clustering), or resampling techniques (such as the bootstrap).
The basic model of ordinary quantile regression is now described. Consider a target random variable Y as a scalar real macroeconomic factor characterized by the following distribution function:

$$F(y) = Pr\ (Y \leq y)$$

The τ-th quantile of Y, for any $\tau \in (0, 1)$, is its inverse probability distribution function, defined as:

$$\mathbb{Q}_\tau(y) = inf\ \{y: Pr\ (Y \leq y) \geq \tau\}$$

that is the infimum of y such that the distribution function of y takes a value that is strictly greater than the prefixed order τ of quantile. The quantile function $\mathbb{Q}_\tau(y)$, such as the distribution function $F(y)$, completely characterizes the random variable Y.
The τ-th sample quantiles can be seen as solutions to an optimization problem. As shown by Koenker, the quantile can be interpreted as the minimizer of some "check function", or "loss function", denoted as $\rho_\tau(\cdot)$. This different point of view based on an optimization problem is analogous for other statistical measures, as the mean. Indeed, given a sample $\{y_1, \dots, y_n\}$ from a single distribution $F(y)$, it can be shown that the sample mean is the solution to the problem:

$$\overline{y} = arg\ min_\xi \sum_i (y_i - \xi)^2$$

the sample median is the solution to:

$$\hat{\mathcal{Q}}_{0.5}(y) = arg\ min_\xi \sum_i |y_i - \xi|$$

and the sample τ-th quantile is the solution to:

$$\hat{\mathcal{Q}}_\tau(y) = arg\ min_\xi \sum_i \rho_\tau(y_i - \xi)$$

with quantile loss function:

$$\rho_\tau(u) = u(\tau - I_{u<0})$$

In other words, quantiles are identified as those points ξ of the domain of the function $\sum_i \rho_\tau(y_i - \xi)$ at which its values are minimized. The extension from the sample problem to the regression framework consists of replacing the ξ by the regression function $x_i'\beta_\tau$, where x_i is a $K \times 1$ vector of regressors, in order to minimize the total "loss" of residuals defined by $\rho(\cdot)$. Hence, the check function is an asymmetric absolute loss function that retrieves the τ-th sample quantiles, whose u, that is the argument of the function ρ, is just the model residuals, namely the difference between the observations y_i and the fitted values $x_i'\beta_\tau$. In particular, $\rho(\cdot)$ assigns weights τ if the error is positive and $(\tau - 1)$ if the error is negative, being I(.) an indicator function equal to 1 whenever u<0, and zero otherwise. Therefore, check function gives asymmetric weights to the error, depending on the quantile and the sign of the error.

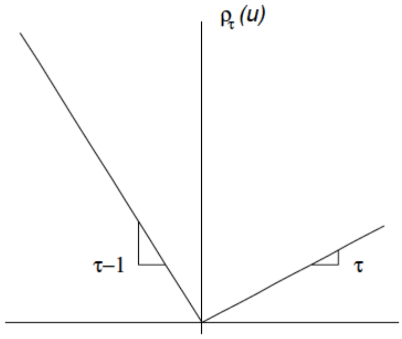


Figure 3.1: Quantile Regression loss function ρ

The quantiles can be written as solutions to the optimization problem:

$$\hat{Q}_\tau(y) = \arg\min_{\xi\in\mathbb{R}} \mathbb{E}[\rho_\tau(Y-\xi)]$$

which, by applying the analogy principle, becomes:

$$\hat{Q}_\tau(y) = \arg\min_{\xi\in\mathbb{R}} \sum_{i=1}^{n}\rho_\tau(y_i-\xi)$$

whereby, for some $\tau \in (0, 1)$, we have to find $\hat{y} = x\hat{\beta}$ to minimize the expected loss:

$$\mathbb{E}[\rho_\tau(Y-\hat{y})] = (\tau-1)\int_{-\infty}^{\hat{y}}(y-\hat{y})dF(y) + \tau\int_{\hat{y}}^{\infty}(y-\hat{y})dF(y)$$

By differentiating with respect to $\hat{y}$, the optimality condition is:

$$(\tau-1)\int_{-\infty}^{\hat{y}}dF(y) - \tau\int_{\hat{y}}^{\infty}dF(y) = F(\hat{y}) - \tau = 0$$

Any element of y such that $F(y) = \tau$ minimizes the expected loss, given the monotonicity of F. If the solution is unique, then $\hat{y} = F^{-1}(y)$, otherwise there exists a set of τ-th quantiles from which the smallest one will be chosen.

Given the usual linear regression model $y_i = x_i'\beta_\tau + u_i$, and the quantile restriction condition $Q_\tau(u_\tau|\boldsymbol{x}) = 0$ described above, it is possible to re-write the distribution function of y as:

$$F(\tau - x_i'\beta_\tau|x_i) = \Pr\,(y_i \le \tau|x_i)$$

and we get the linear conditional quantile function:

$$Q_{\tau|X=x}(y_i) = x_i'\beta_\tau$$

In the Koenker and Bassett's specification, expectation-based quantile representation is used for handling conditioning information sets, and the future quantiles of y_{t+1}, conditional on information set I_t, are functions of observables $\boldsymbol{x_t}$:

$$Q_\tau(y_{t+1}|\mathrm{I}_t) = \beta_{\tau,0} + \boldsymbol{\beta'_\tau x_t}$$

Solving the optimization problem implemented on this conditional quantile function allows us to estimate the τ-th regression quantile coefficient:

$$\widehat{\beta_\tau} = \arg\min_{\beta\in\mathbb{R}^K}\sum_{i=1}^{n}\rho_\tau(y_i - x_i'\beta_\tau)$$

It is interesting to compare the quantile regression model with the standard linear regression model. As known, the standard regression model allows to estimate the conditional expectation of the dependent variable, that is its average response given a set of covariates:

$$\mathbb{E}(y_i|x_i) = \beta_0 + \beta_1 x_{i1} + \cdots + \beta_p x_{ip}$$

and the β_j are estimated by solving the least squares minimization problem:

$$\min_{\beta_0,\ldots,\beta_p}\sum_{i=1}^{n}[y_i - (\beta_0 + \sum_{j=1}^{p}x_{ij}\beta_j)]^2$$

In contrast, the conditional quantile regression model for the τ-th quantile of the response variable, given a set of explanatory variables and the zero conditional quantile assumption, is:

$$Q_\tau(y_i|x_i) = \beta_{\tau,0} + \beta_{\tau,1}x_{i,1} + \cdots + \beta_{\tau,p}x_{i,p}$$

and the β_j are estimated through the least absolute deviations method, which minimizes the sum of absolute errors:

$$min_{\beta_{\tau,0},\dots,\beta_{\tau,p}} \sum_{i=1}^{n} \rho_\tau(y_i - (\beta_{\tau,0} + \sum_{j=1}^{p} x_{ij}\beta_{\tau,j}))$$

with:

$$\rho_\tau(e) = \tau max(e; 0) + (1-\tau)max\,(-e; 0)$$

For each quantile τ, the solution to this minimization problem produces a set of quantile regression coefficients. Moreover, they are no more constant, like in the linear regression model, but are now functions of the respective quantiles.
As showed, the estimations are the result of specific optimization problems. In particular, the mean of a distribution is represented as that point that minimizes the average squared distance over population, while the quantile is that point that minimizes the same average distance, not squared but with weights equal to τ for points above the fitted line and (1-τ) for points below the line. To each value of τ, interpretable as the proportion of the sample having values below the respective quantile, can be associated a specific fitted conditional quantile function, unlike the linear regression model. In fact, if the quantile regression model is built for 10 quantiles, the 10 resulting equations bring to 10 different coefficients of the explanatory variables, one for each conditional quantile.
It can be noted that for different values of the τ-th quantile of the response variable, the error terms of individual I are related, such that the distribution of ε_τ and of $\varepsilon_{\iota\neq\tau}$ are shifts of one another. In case of $\varepsilon_\tau \sim i.i.d.$, the τ-th quantile of the error term is not zero, but it is a constant c which depends on the different order considered τ and ι, such that:

$$Q_\tau(y_i|x_i) = Q_\iota(y_i|x_i) + c_{\tau,\iota}$$

Then, under I.I.D. assumptions for the error term, the conditional quantile functions are just some shifts of one another, and the response variable distribution is not affected by any shape change.
If we want to sum up the main difference between the linear regression model and the quantile regression model, it can be said that the former one focuses on the conditional mean of a response variable without taking into account its conditional distributional properties, that are "standardized" away with a set of strict assumptions, while the latter allows the complete analysis of the conditional distributional properties of the variable of interest. In particular, there are at least three features to be mentioned.

- The inadequacy of the conditional mean from a distributional point of view. In the linear regression model, the mean of the distribution is used to represent its central tendency and describe the relative impact of some covariates, but, in cases of asymmetric distributions, it may be inadequate to identify both shape shifts and the right impact on variable of interest. Indeed, in case of symmetric distributions, mean and median coincide, but with skewed distribution the median becomes more appropriate to represent the central tendency. For these cases, the conditional median regression model, and, more in general, the conditional quantile regression model, provide the best analysis for modeling location changes.

- The violation of homoskedasticity assumption. In the linear regression model, the homoskedasticity assumption requires the variance of the response variable y_i, conditional on the explanatory variable x_i, to be constant for any observation i. Anyway, if the homoskedasticity does not hold, and there are different probability density functions for different values of explanatory variable, the conditional mean may become not useful to understand how x_i affects y_i. The quantile regression model overcomes this shortcoming.
- The violation of one-model assumption. One of the conditions of OLS estimator is that the data have nonzero finite fourth moments, that is large outliers are unlikely. This assumption is needed in order to avoid undue influence on the fitted regression line, but it has a cost: the necessity to eliminate eventual outliers reduces the reliability and the precision of the analysis, since information is lacking (this aspect becomes relevant in those study about social stratifications or distributions, like income distribution). The quantile regression model allows to evaluate any aspect of the distribution, without renouncing any information.

It can be useful to summarize the main differences between linear regression framework and quantile regression framework, as done in the following table.

Linear Regression	**Quantile Regression**
It models the conditional mean $\mathbb{E}(Y\|X)$	It models the conditional quantiles $Q_\tau(Y\|X)$
Sensitive to outliers	Robust to outliers
Assumption on distribution is desirable	Unknown distribution
Computationally inexpensive	Computationally intensive
No large dataset required	Large dataset required

Table 3.1: Comparison between Linear Regression and Quantile Regression

The coefficients estimation resulting from the quantile optimization problem benefits from many important properties, as shown by Koenker and Bassett. Several such properties of quantile regression estimators are defined by the concept of "equivariance": when the data is altered in some predictable way, no fundamental effect on quantile estimation is expected to occur, that is the regression estimates should change such that the interpretation of the results is invariant. The classical example is the rescaling in a model for a temperature of a liquid: if the scientists decide to change the scale of the temperature from Fahrenheit to Centigrade, the model should not provide different substantial results. Authors managed to prove this. Four basic equivariance properties are presented below.
Let $\hat{\beta}(\tau; y; X)$ be the τ-th regression quantile coefficient obtained from y and X; let $a > 0$ be a real number and $\gamma \in R^p$ be a unit basis vector; let A be a $p \times p$ nonsingular matrix; finally, the order $\tau \in [0,1]$. Then, three specific properties are showed.

1) Scale equivariance:
   $\hat{\beta}(\tau; ay; X) = a\hat{\beta}(\tau; y; X)$
   $\hat{\beta}(\tau; -ay; X) = a\hat{\beta}(1-\tau; y; X)$
2) Shift equivariance:
   $\hat{\beta}(\tau; y + X\gamma; X) = \hat{\beta}(\tau; y; X) + \gamma$
3) Equivariance to reparameterization of design:
   $\hat{\beta}(\tau; y; XA) = A^{-1}\hat{\beta}(\tau; y; X)$

Another equivariance property is described by authors as very important to show the great potential of quantile regression. Let $h(.)$ be a nondecreasing monotonic function on R, and Y a random variable, then the quantiles of the transformed random variable $h(Y)$ are simply the transformed quantiles of the original Y (property not always shared with the mean). This property is formalized in the following point.

4) Equivariance to monotone transformations:

$$Q_{\tau|x}(h(Y)) = h(Q_{\tau|x}(Y))$$

This property follows from observing that:

$$\Pr(Y < y|x) = \Pr\,(h(Y) < h(y)|x)$$

and has many important implications. When considering a transformation of the response variable y, such as $h(y)$ (e.g. a logarithmic transformation), some important assumptions in linear regression model (e.g. linearity in model specification, homoskedasticity of the new dependent variable, or the normality assumption for residuals) may be violated, because the OLS estimator does not enjoy the property 4), while quantile regression estimator does. For instance, if a transformation of y is performed, such as $F = e^{y}$, and if $Q_\tau(y|X) = X\beta_\tau$, then: $Q_\tau(F|X) = F(Q_\tau(y|X)) = e^{X\beta_\tau}$.

In the multiple regression, another basic condition, called "subgradient optimality condition" was analytically described by Koenker and Bassett. The details will not be covered in this context, but the general conclusions, since this condition crucially characterizes the quantile regression. As shown by authors, the observations (y, X) are said to be 'in general position' if, for any $h \in \mathcal{H}$:

$$y_i - x_i b(h) \neq 0 \qquad \forall i \notin h$$

where h is an index p-element subsets of the first n integers, $X(h)$ is a submatrix of the nonsingular matrix X with rows $\{x_i : i \in h\}$, and $y(h)$ is a p-vector with coordinates $\{y_i : i \in h\}$. Then, coefficients vector in its basic form is $b(h) = X(h)^{-1}y(h)$. If (y, X) are in general position, there exists a solution to quantile regression optimization problem of the form $b(h) = X(h)^{-1}y(h)$, if and only if:

$$(\tau - 1)\mathbf{1}_p \leq \xi_h \leq \tau\mathbf{1}_p$$

where $\xi_h = \sum_{i\in\bar{h}} \psi_\tau(y_i - x_i' b(h))x_i' X(h)^{-1}$ and $\psi_\tau = \tau - I_{u<0}$, with $\bar{h}$ the complement of h. Moreover, if the inequalities are strict, $b(h)$ is a unique solution.

Finally, there is another important characteristic of sample quantile regression, that is its robustness: an alteration in the order statistics above the median in such way that they remain above the median, does not change the position of the same median. Robustness properties are very important for quantile estimation and inference, whose outcomes and distributions are not influenced by the order statistics or specific observations, but rather by the local behavior of the conditional distribution of the response near the specified quantile. Therefore, any of the y observations may be arbitrary altered without causing changes in the initial solution $\widehat{\beta_\tau}$. At the same time, there is a higher sensitivity of quantile regression estimates to the sign of the residuals, which matter in affecting it, and to the observations $\{x_i\}$. Authors explained that y can be freely moved up or down provided that the fitted τ-th quantile regression plane is not crossed without altering the fit, and this highlights that observations are never neglect, but they equally contribute in the estimation process. This property has been formalized in such a way:

$$\hat{\beta}(\tau; y; X) = \hat{\beta}(\tau; X\hat{\beta}(\tau; y; X) + D(y - X\hat{\beta}(\tau; y; X))y; X)$$

where D is a diagonal matrix with nonnegative elements d_i. This is an important feature, even for the interpretation of the quantile regression.
Moreover, in those cases in which conditional densities of the response variable are heterogenous and change with observations, there might be loss of robustness and efficiency of the estimator. The introduction of a suitably weighted quantile regression (WQR) model, which combines strengths across multiple quantile regressions by using data-dependent weights at different quantiles obtained from a sparsity function, may lead to an efficiency improvement. WQR estimates are more robust and efficient, but computationally more costly. In particular, the weighted estimator is the solution to the optimization problem:

$$\beta_\tau^{WQR} = \arg\min_{\beta\in\mathbb{R}^K} \sum_{i=1}^{n} f_i(\xi_i)\rho_\tau(y_i - x_i{}'\beta_\tau)$$

where $f_i(\xi_i)$ represents the weights associated to ξ_i for each observation $i = 1, \dots, n$.
Let $\varepsilon = (\varepsilon_1, \dots, \varepsilon_n)^T$ be a vector of random errors, $F(\varepsilon)$ its distribution function, $f(\varepsilon)$ its density function, and $\{\tau_k,\ k = 1, \dots, K\}$ a set of K quantiles. Then, WQR weights w_k can be defined as:

$$w_k = \frac{f(F(\tau_k)^{-1})}{\sqrt{\tau_k(1-\tau_k)}}$$

where $1/f(F(\tau_k)^{-1})$ is called "sparsity function", or "quantile-density function, and it is the density of each observation i at the quantile of interest k. Hence, the WQR estimator is more efficient and reliable than the simple QR estimator since the sample information is processed in a more effective way through the inclusion of normalized weights w_k^N into estimation:

$$w_k^N = \frac{w_k}{\sum_k w_k}$$

Given $\tau = \tau_k$, the solution to the previous optimization problem is the WQR estimate, which can be computed as:

$$\hat{\beta}_\tau^{WQR} = \sum_k w_k^N \beta_{\tau k}$$

3.1.2 Interpretation and Inference in the Quantile Regression

As said, while OLS regression is the most widely used parametric model, relying on assumptions not often met, quantile regression requires no particular assumption on the distribution of the residuals, and allow to survey different aspects of the relationship between the response variable and the explanatory variables. Therefore, quantile regression model and coefficients require a different interpretation with respect to linear regression model.
Indeed, in a standard linear regression model, such as:

$$\mathbb{E}(Y|X = x) = x'\beta$$

β is the expected change in y due to a unit change in x, when all the other covariates are held constant. It is the expected value of the partial derivative of y with respect to x_j:

$$\frac{\partial\mathbb{E}(Y|X = x)}{\partial x_j} = \beta_j$$

On the other hand, a quantile regression model with a monotone transformation of the response variable (remind the property of equivariance to monotone transformations: $Q_{\tau|X=x}(h(Y)) = h(Q_{\tau|X=x}(Y))$), such as:

$$Q_{\tau|X=x}(h(Y)) = x'\beta_\tau$$

will have the following partial derivative:

$$\frac{\partial Q_{\tau|X=x}(h(Y))}{\partial x_j} = \frac{\partial h^{-1}(x'\beta)}{\partial x_j}$$

For example, with a logarithmic transformation:

$$Q_{\tau|X=x}(log(Y)) = x'\beta_\tau$$

there will be:

$$\frac{\partial Q_{\tau|X=x}(h(Y))}{\partial x_j} = \beta_j e^{x'\beta_\tau}$$

The interpretation of the coefficients in the quantile regression is analogous to that in the linear standard regression: while in the latter, coefficients express the impact of a unit change in the explanatory variable on the conditional mean of response variable, in the former one, they express the impact on the conditional τth quantile. Typically, predictive models of socio-economic phenomena tend to focus on the central tendency through the conditional mean of the variable of interest, often by assuming a steady distribution over time and over observations. Actually, the relationship at different points in the conditional distribution of the variable of interest does change within many frameworks, and quantile regression model fits very well with the description of it. Therefore, while the mean regression helps to understand how the conditional mean of y is affected by covariates X, quantile regression helps to identify an impact of covariates on y at each quantile of its conditional distribution: this allows to get a complete description and view about the distribution of y conditional on each value of X, and then to understand the deep relationship among variables. This makes the quantile estimation method a very effective tool for exploration of the potential effects of a set of covariates X on the whole distribution of y, not only on its mean. For this reason, unlike standard regression, the quantile regression allows to model a family of curves, that need to be interpreted, and to analyze particular segments of the conditional distributions. Moreover, it allows to test the I.I.D. error assumption of OLS estimator: if the slope quantile regression coefficients randomly fluctuate around a constant level, and at the same time the intercept systematically increases with x, then there is evidence of homoskedasticity (otherwise, there is heteroskedasticity).
Quantile regression methods have been widely used in economics and finance (e.g. to study the impact of some specific determinants and policies on wages, on students' performance or on income distribution) as a flexible statistical tool. The point is that explanatory variables are unlikely to affect the response variable so as to shift the entire distribution equally by a fixed quantity, but rather so as to hit in different ways and with different intensities each quantile class of the dependent variable. Such pattern can be surveyed within the quantile regression framework.
It may be useful to understand the utility of the quantile regression, and the best circumstances to adopt it. In general, the quantile regression is a valid option whenever the conditional mean fails to fully and reliably capture the data pattern:

- in case of skewed data and asymmetric distribution, it allows to study the true distributional relationship of variables;
- in case of multimodal data and data with outliers, it maintains its robustness;
- in case of heteroskedasticity, it fits very well in dealing with it.

Therefore, quantile regression allows to get a more comprehensive analysis of the relationship among variables, and its biggest advantage is the robustness to outliers in the response variable. Moreover, it may be quite useful in those cases with high complexity of interactions but a weak

relationship between the means of variables (e.g. in ecology), as well as when percentile curves and impacts on extreme values requires a particular focus (e.g. in finance).
In economics, the predictive quantile regression model allows to investigate potentially nonlinear dynamics among macroeconomic variables, without assuming a parametric distribution, but letting the shape of the distribution fully depend on the predictors. Indeed, the quantile model detects the effects of predictors on different parts of the response distribution, and this allows to predict the quantiles. In this regard, this model seems to be particularly suitable for investigating the impact of systemic risk on macroeconomic shocks' tail, and useful for the issue of reliable early warning signals for systemic risks at appropriate forecasting horizons.
After showing the utility of quantile regression and the characteristics in the interpretation of coefficients, it is necessary to understand how to elicit robust and reliable statistical conclusions from data analysis. In this regard, conditional quantile regression seems to offer an easier interpretable objective for statistical analysis. Koenker and Bassett described the relevant literature on the asymptotic theory of quantile regression, and used it to present reliable tools for inference.
The basic asymptotic property is the consistency. Given the parametric form of the conditional quantile function of Y:

$$Q_{\tau|X=x}(Y) = g(x, \beta_\tau)$$

then, its estimator:

$$\widehat{\beta_\tau} = \arg\min_{\beta \in \mathbb{R}^K} \sum_{i=1}^{n} \rho_\tau(y_i - g(x, \beta_\tau))$$

converges in probability to β_τ:

$$\| \widehat{\beta_\tau} - \beta_\tau \| \to 0 \text{ as } n \to \infty$$

In an ordinary univariate sample quantile framework, with:

$$\widehat{\xi_\tau} = \arg\min_{\xi \in \mathbb{R}^K} \sum_{i=1}^{n} \rho_\tau(y_i - \xi_\tau)$$

it can be proved from the monotonicity of the sub-gradient condition that:

$$\widehat{\xi_\tau} \to \xi_\tau \text{ as } n \to \infty$$

given the random sample $\{y_i, \dots, y_n\}$ got from the distribution F with a unique τ-th quantile, namely $\xi_\tau = F_\tau^{-1}$. The form of the distribution of $\widehat{\xi_\tau}$ depends on the behavior of ξ_τ near τ, and it is shown that, if density of F, $f(F_\tau^{-1})$, is bounded away from 0 and ∞ near τ, then:

$$\sqrt{n}\left(\widehat{\xi_\tau} - \xi_\tau\right) \sim N(0, \omega^2)$$

with $\omega^2 = \tau(1-\tau)/f(F_\tau^{-1})^2$.
Under I.I.D. sampling assumption, in order to have convergence, $\widehat{\xi_\tau} \to \xi_\tau$, a sufficient condition is:

$$F(\xi_\tau - \varepsilon) < \tau < F(\xi_\tau + \varepsilon) \quad \text{for all } \varepsilon > 0$$

Under independent but not identical distributed sampling, with the sequence of random variables $\{X_i\}_{i=1}^{n}$ distributed as $\{F_{ni}\}_{i=1}^{n}$, the consistency of the τ-th sample quantile holds if:

$$\sqrt{n}(a_n(\varepsilon) - \tau) \to \infty$$

$$\sqrt{n}(\tau - b_n(\varepsilon)) \to \infty$$

with:

$$a_n(\varepsilon) = \overline{F}_n(\xi_\tau - \varepsilon)$$

$$b_n(\varepsilon) = \overline{F}_n(\xi_\tau + \varepsilon)$$

where $\overline{F}_n = \sum_{i=1}^{n} F_{ni}/n$.

In a linear quantile regression model, $Q_{\tau|x}(Y) = x'\beta_\tau$, with the τ-th conditional quantile function $Y|x$, necessary and sufficient conditions to have $\widehat{\beta_\tau} \to \beta_\tau$ are $\sqrt{n}(a_n(\varepsilon) - \tau) \to \infty$ and $\sqrt{n}(\tau - b_n(\varepsilon)) \to \infty$, where:

$$a_n(\varepsilon) = \sum F_{ni}(x_i\beta_\tau - \varepsilon)/n$$
$$b_n(\varepsilon) = \sum F_{ni}(x_i\beta_\tau + \varepsilon)/n$$

The description of the finite sample theory of the quantile regression estimator and the asymptotic theory is usually preparatory to the theory of inference for the univariate quantiles. Koenker and Bassett described main inference tools for quantile regression: Wald tests, rank tests, likelihood ratio type tests and other resampling methods.

The Wald test is designed to test for the equality of slope coefficients across quantiles, through the survey of inter-quantile ranges of two samples. Indeed, while linear regression model assumes identical conditional distributions of the dependent variable, implying no variation in the slope coefficients of different quantiles, the quantile regression allows them to vary across quantiles. In a two-sample model, with n_1 observations in the first sample and n_2 observations in the second sample, and a binary variable $x_i = 1$ for the second sample, such that:

$$Y_i = \beta_1 + \beta_2 x_i + u_i$$

the τ-th quantile regression coefficient, β_2, is the difference between the τ-th sample quantiles of the two samples. Then, the null hypothesis requires the equality of the slope coefficients across τ_1 and τ_2, namely the $(\tau_2 - \tau_1)$-interquantile ranges to be equal for each sample:

$$\begin{aligned} \mathrm{H}_0\text{:}\ \ \beta_2(\tau_2) - \beta_1(\tau_1) &= \big(Q_2(\tau_2) - Q_1(\tau_2)\big) - \big(Q_2(\tau_1) - Q_1(\tau_1)\big) \\ &= \big(Q_2(\tau_2) - Q_2(\tau_1)\big) - \big(Q_1(\tau_2) - Q_1(\tau_1)\big) = 0 \end{aligned}$$

The Wald test, based on the asymptotic normality, is:

$$T_n = \frac{\hat{\beta}_2(\tau_2) - \hat{\beta}_1(\tau_1)}{\hat{\sigma}(\tau_1, \tau_2)}$$

where the asymptotic variance of $\hat{\beta}_2(\tau_2) - \hat{\beta}_1(\tau_1)$ is:

$$\sigma^2(\tau_1, \tau_2) = \left[\frac{\tau_1(1-\tau_1)}{f(F^{-1}(\tau_1))^2} - 2\frac{\tau_1(1-\tau_2)}{f\big(F^{-1}(\tau_1)\big)f\big(F^{-1}(\tau_2)\big)} + \frac{\tau_2(1-\tau_2)}{f(F^{-1}(\tau_2))^2}\right]\left[\frac{n}{nn_1 - n_1^2}\right]$$

In general, there are several tests that try to capture the significance of the treatment effect in these two-sample models. For instance, some tests, like the Mann-Whitney-Wilcoxon tests, survey the location shift alternatives, others survey the scale shift alternatives, and still others evaluate non-parametric alternatives.

Rank-based inference, that is rank tests based on the dual quantile regression process, seems to be particularly useful for a wide variety of quantile regression inference problems, including the creation of confidence intervals for specific quantile regression coefficient estimates. Koenker and Bassett studied in deep the details and properties of this kind of tools for inference.

An alternative to the Wald test and rank test for quantile regression is the likelihood ratio test, based on the value of the objective function under null and alternative models. Given the linear model $y_i = x_i'\beta + u_i$ with $u_i \sim i.i.d.$, the median regression coefficient can be tested such that:

$$\mathrm{H}_0\text{:}\ R\beta = r$$

Letting $\hat{V}_\tau$ be the value of the objective function under the unrestricted minimizing estimator $\hat{\beta}_{0.5}$:

$$\hat{V}_\tau = \min_{\beta\in\mathbb{R}^p} \sum \rho_\tau(y_i - x_i'\beta_\tau)$$

and $\tilde{V}_\tau$ the function under the restricted estimator $\tilde{\beta}_{0.5}$:

$$\tilde{V}_\tau = \min_{\beta\in\mathbb{R}^p | R\beta = r} \sum \rho_\tau(y_i - x_i'\beta_\tau)$$

then, it is applied the statistic:

$$T_n = \frac{8(\tilde{V}_{0.5} - \hat{V}_{0.5})}{s_{0.5}} \sim \chi_q^2 \text{ (under H}_0\text{)}$$

where $s_{0.5}$ is the value obtained by the sparsity function when τ =0.5, and q is the rank of R. Finally, many resampling methods, like the bootstrap method, have been described in literature for the construction of confidence intervals for quantile estimators. These methods are very powerful because they allow the estimation of confidence intervals and other elements without special assumptions, and this is particularly useful in case of small sample sizes or when the asymptotic approximation on the variable of interest is difficult to elicit.

A last point to be described concerns the goodness of fit and the model performance in the quantile regression framework. There are at least three important measures that are worth to be mentioned.

The first measure is the coefficient of determination R^2, estimated as 1 minus the ratio between the sum of absolute deviations in the fully parameterized models and the sum of absolute deviations in the unconditional quantile model (unlike the standard linear regression, where variances of squared deviations are taken into account):

$$R^2 = 1 - \frac{\sum_{i=1}^{n} \rho_\tau (y_i - \hat{\alpha} - x_i{}' \hat{\beta}_\tau)}{\sum_{i=1}^{n} \rho_\tau (y_i - \hat{q}_\tau)}$$

This formula measures the typical "quantile loss" in terms of absolute deviations based on conditioning information relative to the "losses" based on the historical unconditional quantile estimate. It should be noted that for the in-sample fit, R^2 lies between 0 and 1, while for the out-of-sample fit, R^2 may be negative, if an unsuitable model is worse than a constant unconditional quantile fit.

A second measure is the average absolute error of prediction ($ATAE$), which can be interpreted as the mean squared error (MES) of the standard linear model:

$$ATAE = \sum_{i=1}^{n} \rho_\tau (y_i - \hat{\alpha} - x_i{}' \hat{\beta}_\tau)/N$$

Therefore, the smaller the $ATAE$ the better the performance.

The last measure is the ratio of quantile exceedance ($RQEX$):

$$RQEX = \sum_{i=1}^{n} \mathbf{1}_{y_i < \hat{\alpha} + x_i{}' \hat{\beta}_\tau}/N$$

This is a measure of calibration, and a well calibrated model has a $RQEX$ close to τ.

3.2 Mixed-Data Sampling Regression Models

The second basic class of tools needed to implement the early warning indicator is represented by the Mixed Data Sampling regression models, or MIDAS models, introduced by Ghysels *et al.* ("The MIDAS Touch: Mixed Data Sampling Regression Models", 2002). MIDAS models, used both in macroeconomic analysis and in financial applications, are derived by combining elements from temporal aggregation literature and distributed lag models, with a weight function which tracks the high frequency lags of covariates.

The distributed lag model, which is used to estimate current values of a response variable based on both the current values and the lagged values of covariates, after the aggregation of higher frequency values, usually takes the form:

$$y_{t_q} = a + B(L) x_{t_q} + \varepsilon_{t_q}$$

where $B(L)$ is the lag polynomial operator, and $t_q = 1, \dots T_q$. The response to high frequency explanatory variable is modeled using distributed lag polynomials in order to avoid the problem of parameters proliferation. MIDAS models are quite similar to distributed lag models, but there is a crucial difference: the dependent variable, sampled at lower frequencies, is regressed on distributed lags of the covariates, sampled at higher frequencies, and no aggregation is performed. In fact, variables sampled at higher frequencies contain valuable information with higher predictive power than that got from aggregated models. Therefore, MIDAS models are tightly parameterized, reduced form regression models for time series that involve processes and data sampled at different frequencies, where explanatory variables have higher frequency. This type of regression allows to address those common situations where data are not sampled at the same frequency, and in particular when the variable of interest is sampled at lower frequency. Usually, in these cases, an aggregation of higher frequency data is done in order to handle data at the same frequency, but this causes an information loss that makes the estimation less efficient. For instance, some macroeconomic data is sampled monthly (e.g. many monetary variables and price) while other data is sampled annually or quarterly (e.g. real GDP). If the analyst wants to study the relationship between inflation and economic growth, they can either aggregate inflation data to a quarterly sampling frequency or apply a MIDAS regression which combines monthly and quarterly data.
Here, a brief description of the MIDAS model under an analytical point of view. Let Y_t be the dependent variable sampled at some interval of reference, that is a fixed sampling frequency (annually, monthly, etc.), and let $X^{(m)}$ be the explanatory variable sampled at a higher frequency, namely m times faster than the frequency of Y_t (e.g., if Y_t are annual data, and $X_t^{(m)}$ are quarterly data, then it is sampled m=4 times faster than Y_t). Given the lag operator $L^{1/m}$ such that $L^{j/m}X_t^{(m)}$=$X_{t-j/m}^{(m)}$, and given a polynomial of length j^{max} as a weighting function, $B(L^{1/m};\theta) = \sum_{j=1}^{j^{max}} B_j(\theta)L^{j/m}$, where $L^{j/m}$ makes values of $X_t^{(m)}$ lag by j/m periods, then the simplest linear MIDAS regression can take the form:

$$Y_t = \beta_0 + \beta_1 B(L^{1/m};\theta)X_t^{(m)} + \varepsilon_t^{(m)}$$

For example, this MIDAS model allows to regress yearly Y_t on quarterly or monthly $X_t^{(m)}$, up to j^{max} lags. The parametrization of lag coefficients in $B(L^{1/m};\theta)$ requires the usage of a parameter vector function θ (Almon lag function, Almon exponential lag function, Beta polynomial function) and a suitable information criterion (Akaike, Schwarz or Hannan-Quinn). Usually, the number of lags of $X_t^{(m)}$ may be really large, and one might be forced to deal with a huge dataset and many frequencies. Within this framework, the parameter proliferation problem arises: if the coefficients of the lagged polynomial were not restricted, such that B were not function of θ, then the number of parameters to estimate would be very high, and their interpretation may be unfeasible. For example, to capture the impact of daily data on a dependent variable sampled annually, then 365 coefficients would be required. It is clear that some sort of restriction upon the structure of the coefficients must be applied. As said, a first solution is to aggregate the highest frequency data in order to have the whole dataset sampled with the same (lowest) frequency, at a cost of not fully exploiting all available information. For this reason, the coefficients of the polynomial in $L^{1/m}$ are introduced by a known function $B(L^{1/m};\theta)$ of a few parameters summarized in a vector θ, and this function requires a

specification. Finally, β_1 expresses the overall impact of lagged $X_t^{(m)}$ on Y_t. In general, the MIDAS regression requires a nonlinear least squares estimation (NLS).
A more explicit specification for the weighting function $B(L^{1/m};\theta)$ is:

$$B(L^{1/m};\theta) = \sum_{j=0}^{J} c(j;\theta) L_m^j = \sum_{j=0}^{J} \frac{\varphi(j;\theta)}{\sum_{j=1}^{J}\varphi(j;\theta)} L_m^j$$

with $B(1;\theta) = \sum_{j=0}^{J} c(j;\theta) = 1$, where the lagged coefficients θ of c need a parametrization. The smart and optimal parameterization of the lagged coefficients is one of the main key MIDAS features. Many types of parametrizations have been proposed in accordance with the number of coefficients and the function shaping:

- the linear scheme:

$$c(j;\theta) = 1/J;$$

- the hyperbolic scheme, like the normalized exponential Almon lag polynomial, expressed with an exponential function:

$$c(j;\theta) = \frac{e^{\theta_1 j + \cdots + \theta_Q j^Q}}{\sum_{j=1}^{J} e^{\theta_1 j + \cdots + \theta_Q j^Q}}$$

- the not normalized Almon lag polynomial specification of order P, where the sum of individual weights are not equal to 1:

$$\beta_1 c(j;\theta = [\theta_0, \dots, \theta_P]) = \sum_{p=1}^{P} \theta_p j^p$$

  or in matrix form:

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ \vdots \\ c_J \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & & 1 \\ 1 & 2 & 2^2 & \cdots & 2^P \\ 1 & 3 & 3^2 & & 3^P \\ & \vdots & & \ddots & \vdots \\ 1 & J & J^2 & \cdots & J^P \end{bmatrix} \begin{bmatrix} \theta_0 \\ \theta_1 \\ \vdots \\ \theta_p \end{bmatrix}$$

  where Almon lags can be computed via OLS estimation after having properly transformed high frequency data regressors (slope coefficients can then be computed by rescaling weights);

- the normalized beta probability density function, unrestricted (Un) and restricted (R) cases with non-zero (nz) and zero (z) last lag (the best fit with a high number of MIDAS lags):

$$c_j^{Un,nz} = c(j;\theta = [\theta_1, \theta_2, \theta_3]) = \frac{d_j^{\theta_1 - 1}(1-d_j)^{\theta_2 - 1}}{\sum_{j=1}^{J} d_j^{\theta_1 - 1}(1-d_j)^{\theta_2 - 1}} + \theta_3$$

$$c_j^{Un,z} = c(j;\theta = [\theta_1, \theta_2, 0])$$

$$c_j^{R,nz} = c(j;\theta = [1, \theta_2, \theta_3])$$

$$c_j^{R,z} = c(j;\theta = [1, \theta_2, 0])$$

  with $d_j = j/(J+1)$;

- the not normalized polynomial specification with step functions:

$$\beta_1 c(j;\theta = [\theta_1, \dots, \theta_P]) = \theta_1 I_{j\in[\alpha_0,\alpha_1]} + \sum_{p=2}^{P} \theta_p I_{j\in(\alpha_{p-1},\alpha_p]}$$

  with $\alpha_0 = 1 < \alpha_1 < \cdots < \alpha_P = J$ and $I_{j\in[\alpha_{p-1},\alpha_p]} = \begin{cases} 1 & as\ \alpha_{p-1} \le j \le \alpha_p \\ 0 & otherwise \end{cases}$

In any case, the shape of the weighting function can be derived by data: weights attached to different lags can be hump-shaped, strongly declining or moderately increasing for different values of parameters, and this flexibility is a strength point of the MIDAS framework. If the parametrization is based on a simple linear distributed lag function, there are no restrictions on the MIDAS model, which becomes U-MIDAS model (Unrestricted MIDAS model): one estimates the individual unconstrained coefficients by using a simple regression program. It has been shown that U-MIDAS model works better for small values of m (e.g. quarterly/monthly data). The linear lag polynomial is:

$$c(L)\mathscr{y}_\tau = \delta(L)x_{\tau-1}^m + \epsilon_\tau$$

where $\{\mathscr{y}_\tau\}_\tau$ is the disaggregated process $\{y_t\}_t$, while $c(L) = 1 - c_1L^1 - \cdots - c_pL^p$ and $\delta(L) = (\delta_0 + \delta_1L^1 + \cdots + \delta_dL^d)$ are the polynomials of order, respectively, p and d. There is evidence that the U-MIDAS model fits quite well with data when the gap between lower frequency and higher frequency is not large (e.g. month to quarter). In fact, if frequency mismatch is small (e.g. quarterly to monthly), distributed lag function $B(L^{1/m};\theta)$ can be removed, since the introduction of higher frequency variables does not increase massively the parameter space (as it does with daily variables). A general linear U-MIDAS model to implement in this case is:

$$y_{t,\tau} = \beta_{0,\tau} + \beta_{1,\tau}x_{t,1} + \beta_{2,\tau}x_{t,2} + \beta_{3,\tau}x_{t,3} + \varepsilon_t$$

where the regressand is regressed at the quarterly frequency on the set of low-frequency regressors up to three months earlier. As seen, the big advantage of MIDAS model is the possibility to exploit the within-period information and, in the unrestricted framework, without dealing with the proliferation problem.

The MIDAS model can be extended to the multivariate framework, in which n explanatory variables are sampled regardless of frequencies:

$$y_t = \beta_0 + \sum_{i=0}^{n}\beta_iB(L;\theta_i)X_{t,i}^{(m_i)} + \varepsilon_t^{(m)}$$

or in matrix form:

$$Y = X(\theta)\beta + \varepsilon$$

with parameters vectors $\beta = (\beta_0, \beta_1, \dots, \beta_n)'$ and $\theta = (\theta_1, \dots, \theta_n)$, the residual vector $\varepsilon = (\varepsilon_1, \dots, \varepsilon_T)'$, and the $T \times (n+1)$ matrix of explanatory variable:

$$X(\theta) = \begin{pmatrix} 1 & B^{(m_1)}(L;\theta_1)x_{1,1}^{(m_1)} & \cdots & B^{(m_n)}(L;\theta_n)x_{1,n}^{(m_n)} \\ & \vdots & \ddots & \vdots \\ 1 & B^{(m_1)}(L;\theta_1)x_{T,1}^{(m_1)} & \cdots & B^{(m_n)}(L;\theta_n)x_{T,n}^{(m_n)} \end{pmatrix}$$

The forecasting model, which depends on forecasting horizon h, can be expressed as:

$$\hat{y}_{T+h|T} = \hat{\beta}_0 + \hat{\beta}_1B(L^{1/m};\hat{\theta})x_T^{(m)}$$

As expected, MIDAS regression is a powerful tool in the hands of analysts, because it enables the interdependence analysis in case of variables with different frequencies, such as the high-frequency returns with other macro-financial data observed at lower frequencies. As it has been proven, the regression resulting from the aggregation of all available data to common least frequencies will always be less efficient than a MIDAS regression that fully exploits the information from $X_t^{(m)}$, and this is the big advantage of MIDAS framework. Of course, this framework is perfectible, and many researchers improved it and adapted it according different needs and circumstances. For example, Engle and Rangel elaborated in 2008 the GARCH-MIDAS models, that allowed to incorporate information on the macroeconomic environment into the long-run component. In 2012, Conrad and Loch used GARCH-MIDAS model to investigate the relationship between the long-term market risk and the macroeconomic environment, showing how macroeconomic variables reflect information on market risks.

There are many empirical studies that link mixed data frequency variables, especially in a predictive regression framework. For instance, in the macroeconomics, low frequency macro variables, such as GDP growth, observed annually or quarterly, may be regressed on high frequency financial variables, such as the volatility of stock returns, observed daily. These high frequency variables become leading indicators of economic cycle, and MIDAS models seem to be quite adequate in dealing with those. In financial economics, MIDAS models have been implemented to link high frequency risk measures with low frequency returns. A wide empirical literature on predictive regression found links between low frequency excess stock returns and high frequency volatility predictors.
The real novelty item proposed by this thesis is the application of the quantile regression to a set of variables with different sampling frequencies. Therefore, this approach should combine the two techniques described above, giving rise to the MIDAS quantile regression. As said, quantile regression models the conditional quantile of the dependent variable, describing the relationship at different points in the conditional distribution of the variable of interest, and moving the focus from the conditional mean to the full distribution. In particular, previous within-period information, got by a set of low-frequency explanatory variables, is used to estimate current values of the high-frequency variable of interest, usually published by statistical office with a given lag. The MIDAS quantile regression model used in these cases has been described by Ghysels:

$$\mathcal{Q}_\tau(y_t|\Omega_t) = \beta_{\tau,0} + \beta_{\tau,1}[B(L^{1/m};\theta)X_t^{(m)}]$$

with quantile order $\tau \in (0,1)$, sampling period of the low-frequency variable $t = 1, \dots, T$, and weighting function $B(L^{1/m};\theta) = \sum_{k=1}^{K} b(k;\theta)L^{(k-1)/m}$, where K is the order of the lag polynomial and $L^{(k-1)/m}x_t^{(m)} = x_{t-(k-1)-m}^{(m)}$.
MIDAS quantile regression exhibits the parameters proliferation problem as well, since many high-frequency regressors produce different impacts on different quantiles over time, and this makes data transformations unfeasible for quantile forecasting. Some authors (Lima and Menf, 2018, or Mazzi and Mitchell, 2019) proposed alternative models to overcome the problem (e.g. the penalized quantile regression or the Bayesian approach).
In the next chapter, a MIDAS quantile regression model will be treated in order to build some early warning indicators. In particular, high frequency information, provided by daily stock returns, will be employed to predict conditional quantiles of low frequency information, like some macroeconomic indicators, following a MIDAS quantile model:

$$\mathcal{Q}_\tau(y_t|\Omega_t) = \beta_{\tau,0} + \beta_{\tau,1}\sum_{d=0}^{D} w_d r_{t-1-d}^{(m)}$$

3.3 A Nonparametric Test of Granger-Causality in Quantiles

The third pillar for the building of the EWI is the nonparametric test of Granger causality in quantile, and in particular the test presented by Jeong, Härdle e Song (Econometric Theory, 2012).
While a parametric test assumes that data comes from a given parametric family of probability distributions (e.g. a normal distribution), parameterized by mean and standard deviation, the nonparametric test does not assume anything about the underlying distribution. Therefore, the parametric test is more accurate and has greater statistical power when the assumption of

normally distributed data is not violated, and in general when the underlying distribution is known. When the data are not normal, nonparametric tests can perform better.

The Granger causality is a statistical concept useful to understand not whether the time series X causes the time series Y, but whether X is able to forecast Y, that is if past values of X contain information useful to predict current values of Y. There are many econometric tools for investigating co-movements and causality between changes in two or more time series, and this type of analysis can provide important information about risk spillovers among financial markets. For instance, the Granger causality in risk allows to survey whether past history of highly risky events on a market is able to predict other risky events occurring on other markets. To test this prediction ability of a time series X_t on the time series Y_t, the Granger causality test in the distribution tail can be performed:

$$\mathrm{H}_0 : \Pr\left(Y_t < -VaR_t \middle| I_{y;\,t-1}\right) = \Pr\,(Y_t < -VaR_t | I_{y;\,t-1}, I_{x;\,t-1})$$

where $I_{y;\,t-1}$ is the information set available at time $t-1$ for Y_t. Rejecting null hypothesis means that time series X_t does Granger-cause the time series Y_t in risk at level α with respect to I_{t-1}, and then information from past events in X_t can be used to predict occurrence of risky events in Y_t. Granger causality can be investigated in mean, in variance (for the detection of volatility spillovers among financial markets) or in terms of the entire conditional probability distribution. X_t is said to Granger-causes Y_t in mean if:

$$\mathbb{E}(Y_t | Y_{t-1}, \dots, Y_{t-p}, X_{t-1}, \dots, X_{t-q}) \neq \mathbb{E}(Y_t | Y_{t-1}, \dots, Y_{t-p})$$

In particular, the Granger causality in conditional mean is mostly used in research, even though it has a relevant shortcoming: conditional mean is just one element used for an overall summary about the conditional distribution, while the causal relationship in tail area may be very different from that in the center of the distribution. For this reason, a Granger causality detected in tail quantiles may produce results different from a Granger causality detected in the center of the distribution. Recent literature has focused on the concept of Granger causality in quantiles, which allows to address flaws due to non-Gaussian distributions with asymmetry, non-linearity and fat tails. In these cases, information content provided by the quantiles about distributions is wider and more precise than the information provided by the mean. X_t is said to Granger-causes Y_t in quantile if:

$$Q_\tau(Y_t | Y_{t-1}, \dots, Y_{t-p}, X_{t-1}, \dots, X_{t-q}) \neq Q_\tau(Y_t | Y_{t-1}, \dots, Y_{t-p})$$

For example, a general important result of economic research, confirmed by strong empirical evidence from the point of view of the mean regression, is that, on average, people with more education are more likely to get higher earnings over their lifecycle; however, the impact of higher education is not constant over conditional income distribution since, from quantile regression, there is evidence that high education is significantly associated with higher earnings mainly for the upper tails of income. The Granger causality test in quantiles tries to understand whether education significantly Granger-causes income over different conditional quantiles. The big advantage of this test is that, since the conditional quantile is insensitive to outliers, a set of conditional quantiles can define more in detail and more precisely the whole distribution. The nonparametric test for Granger causality in quantile presented by Jeong, Härdle e Song aims to test conditional quantile restrictions through nonparametric estimation methods in dependent data situations. This procedure, described below, should not be confused with that presented by Hong, Liu, and Wang (Journal of Econometrics, 2007): their test determines if an extreme downside movement of a given time series, i.e. a tail event, has predictive content for

(or equivalently, can be considered a lagged indicator for) an extreme downside movement of another time series.
Here a formalization of the nonparametric test for Granger-causality in quantile.
Denote $w_t \equiv (y_{t-1}, \dots, y_{t-p})$, $z_t \equiv (y_{t-1}, \dots, y_{t-p}, x_{t-1}, \dots, x_{t-q})$ and $v = (w_t, z_t)$; the conditional distribution function y_t given $z_t(w_t)$ as $F_{y|z}(y_t|z_t) * F_{y|w}(y_t|w_t)$, with $F(y_t|z_t)$ absolutely continuous in y for all v; the conditional quantile functions $Q_\tau(z_t) \equiv Q_\tau(y_t|z_t)$ and $Q_\tau(w_t) \equiv Q_\tau(y_t|w_t)$, whereby $\Pr\big(F_{y|z}(Q_\tau(z_t)|z_t) = \tau\big) = 1$. Then, the hypotheses to be tested consist of quintile restrictions:

$$\mathrm{H}_0\text{: } \Pr\big(F_{y|z}(Q_\tau(w_t)|z_t) = \tau\big) = 1$$

$$\mathrm{H}_1\text{: } \Pr\big(F_{y|z}(Q_\tau(w_t)|z_t) = \tau\big) < 1$$

Accepting the null hypotheses means that each variable x does not Granger-cause y in the specific τ-th quantile at each specific moment. It is true iff $\mathbb{E}\big(\mathbf{1}_{y_t \le Q_\tau(w_t)|z_t}\big) = \tau$, given $\mathbf{1}_{y_t \le Q_\tau(w_t)} = \tau + \varepsilon_t$ with $\mathbb{E}(\varepsilon_t|z_t) = 0$. To consistently test H_0, given $f_z(z_t)$ the marginal density function of z_t, a proposed nonparametric test is the distance measure J:

$$J = \mathbb{E}([F_{y|z}(Q_\tau(w_t)|z_t) - \tau]^2 * f_z(z_t))$$

whereby $J = 0$ under H_0, and $J > 0$ under H_1. Given $\mathbb{E}(\varepsilon_t|z_t) = F_{y|z}(Q_\tau(w_t)|z_t) - \tau$, the distance measure can be expressed as:

$$J = \mathbb{E}(\varepsilon_t * \mathbb{E}(\varepsilon_t|z_t) * f_z(z_t))$$

Estimation of each components of the test requires specific formulations based on the so-called kernel methods and kernel functions. Denoting the dimension of z as $m = p + q$, the kernel function as $K_{ts} = K(\frac{z_t - z_s}{h})$ with bandwidth h and time dimension T, the estimated weighted conditional expectation $\mathbb{E}(\varepsilon_t|z_t) * f_z(z_t)$ is:

$$\widehat{\mathbb{E}(\varepsilon_t|z_t) * f_z}(z_t) = \frac{\sum_{s \neq t}^{T} K_{ts}\varepsilon_s}{(T-1)h^m}$$

And then, the test statistic is:

$$\hat{J} = \frac{\sum_{t=1}^{T}\sum_{s \neq t}^{T} K_{ts}\varepsilon_t\varepsilon_s}{T(T-1)h^m}$$

with $\varepsilon_t\varepsilon_s = \big[\mathbf{1}_{y_t \le Q_\tau(w_t)} - \tau\big]\big[\mathbf{1}_{y_s \le Q_\tau(w_s)} - \tau\big]$. Furthermore, given the kernel function $L_{ts} = L(\frac{w_t - w_s}{a})$, it is possible to estimate all other components of the test:

$$\hat{\varepsilon}_t = \mathbf{1}_{y_t \le \hat{Q}_\tau(w_t)} - \tau$$

with:

$$\hat{Q}_\tau(w_t) = \hat{F}_{y|w}(\tau|w_t)^{-1}$$

where:

$$\hat{F}_{y|w}(y_t|w_t) = \frac{\sum_{s \neq t} L_{ts} * \mathbf{1}_{y_s \le y_t}}{\sum_{s \neq t} L_{ts}}$$

The asymptotic properties of the test statistic are based on two basic assumptions:

- process $\{y_t, x_t\}_{t=1}^{T}$ is strictly stationary and absolutely regular with a geometric decay rate;
- functions f_y, f_z and f_w are all bounded and belong to the class of functions $\mathfrak{U}_\mu^\alpha$, $\alpha > 0$, $\mu > 0$, that are $(d-1)$-times partially differentiable for $d - 1 \le \mu \le d$;
- error term ε_t is a martingale difference process with finite fourth moments;

- a set of technical conditions are required in order to have a uniform convergence rate of the nonparametric kernel estimator of conditional cumulative density function and conditional quantile with mixing data (for any detail, look at the paper by Jeong, Härdle e Song);
- a set of technical conditions (e.g. the order k and l of kernel functions $K(\cdot)$ and $L(\cdot)$ must be nonnegative) are required in order to bound estimation bias.

Assuming the compliance of all assumptions, and defining the stochastic process $L_t = (\varepsilon_t, z_t)^T$, the conditional variance of the error term $\sigma_\varepsilon^2(z) = \mathbb{E}[\varepsilon_t^2|z_t = z]$ and the test $J = \frac{\sum_{t=1}^{T}\sum_{s\neq t}^{T} K_{ts}\varepsilon_t\varepsilon_s}{T(T-1)h^m}$, it is possible to get the following result:

$$Th^{m/2}J \longrightarrow N(0, \sigma_0^2) \text{ in distribution}$$

with $\sigma_0^2 = 2 * \int K^2(u)du * \mathbb{E}(\sigma_\varepsilon^4(z) * f_z(z_t))$ and its consistent estimator $\hat{\sigma}_0^2 = 2\tau^2(1-\tau)^2 \frac{1}{T(T-1)h^m}\sum_{s\neq t} K_{ts}^2$. Under the null hypothesis $Th^{m/2}\hat{J} \longrightarrow N(0, \sigma_0^2)$ in distribution, while under the alternative hypothesis $Th^{m/2}\hat{J} \longrightarrow N(\mu, \sigma_1^2)$ in distribution, with $\mu = \mathbb{E}(f_{y|z}^2(Q_\tau(z_t)|z_t) * l^2(z) * f_z(z))$, $\sigma_1^2 = 2 * \int K^2(u)du * \mathbb{E}(\sigma_v^4(z) * f_z(z_t))$, $\sigma_v^2(z) = \mathbb{E}[v_t^2|z_t]$ and $v_t = \mathbf{1}_{y_t \leq Q_\tau(w_t)} - F(Q_\tau(w_t)|z_t)$.

The power performance of the hypothesis test for different combinations of time series length T, quantile order τ and quantile coefficient β_τ, such that the higher $\beta_\tau \in [0,1]$ the stronger the causality in quantile of x_t on y_t, have been analyzed by authors. The power of a test expresses the probability to correctly reject the null hypothesis when the alternative hypothesis is true: the higher the power, the lower the probability of making a type II error (wrongly accepting the null hypothesis when it is false). There are three main results:

- the larger T given τ and β_τ, the larger the power, since more data allow to get a more consistent evidence about causality;
- the higher β_τ given τ and T, the larger the power, and then the stronger the causality in quantile of x_t on y_t;
- given β_τ and T, power of test is usually higher for quantile orders closer to the median, and lower for quantile orders closer to the extremes.

3.4 Methodology for the Empirical Analysis

The description of the methodology is a crucial step for the construction and the development of EWIs. In particular, by applying more methods and models with different characteristics it is possible to discover the best one, that one which fits better with the ultimate purpose of an EWI.

As shown, the test is an extension of the Jeong's nonparametric test for the Granger causality in quantiles, in which information from time series data sampled at different frequencies is fully exploited. This procedure will test the following hypotheses:

$$\mathrm{H}_0: \Pr\big(F_{y|z}(Q_\tau(w_t)|z_t) = \tau\big) = 1$$

$$\mathrm{H}_1: \Pr\big(F_{y|z}(Q_\tau(w_t)|z_t) = \tau\big) < 1$$

It has been proved that, under the null hypothesis

$$Th^{m/2}\hat{J}_T \longrightarrow N(0, \sigma_0^2) \text{ in distribution}$$

where σ_0^2 can be estimated as $\hat{\sigma}_0^2 = 2\tau^2(1-\tau)^2 \frac{1}{T(T-1)h^m} \sum_{s \neq t} K_{ts}^2$. Hence, the test implemented for the empirical analysis has the following extended form:

$$Th^{m/2}\hat{J}_T / \hat{\sigma}_0 = \sqrt{\frac{T}{(T-1)}} \frac{\sum_{t=1}^{T}\sum_{s \neq t}^{T} K(\frac{z_t - z_s}{h})[\mathbf{1}_{y_t \leq \hat{Q}_\tau(w_t)} - \tau][\mathbf{1}_{y_s \leq \hat{Q}_\tau(w_s)} - \tau]}{\tau(1-\tau)\sqrt{2}\sqrt{\sum_{s \neq t} K_{ts}^2}}$$

Since $Th^{m/2}\hat{J}$ tends in distribution to $N(\mu, \sigma_1^2)$ under the alternative hypothesis, then J will be zero if and only if H_0 is true: in this case, given the quantile order τ, there is no Granger causality in quantiles between the two variables.

The test creation in MATLAB, as shown in the Appendix B, requires the implementation of six steps.

1. The first step is the simple calculation of the first factor of the test, $w = \sqrt{T/(T-1)}$.
2. The second step is the creation of the error matrix $\varepsilon_t\varepsilon_s$, that is the product $[\mathbf{1}_{y_t \leq \hat{Q}_\tau(w_t)} - \tau][\mathbf{1}_{y_s \leq \hat{Q}_\tau(w_s)} - \tau]$ for each quantile order τ and each moment t, needed to identify those observed values of y exceeding their estimated conditional quantiles. $\mathbf{1}$ is an indicator function, equal to 1 when the underlying condition is true, and zero otherwise. In this case, it produces a vector of T elements (zeros and ones) for each quantile order. The error matrix is a $(T \times T \times q)$ matrix.
3. The third and the most complicated step is the construction of the kernel function $K(.)$. A kernel distribution is a nonparametric representation of the probability density function of a random variable, that is estimated as a "generalization" of the histogram density. But unlike histograms, that figure discrete objects, a kernel distribution groups each single distribution applied to each observed value in order to construct a unique smoothed continuous probability curve, and it is very useful when there is no parametric distribution that fits dataset and no specific distribution assumption can be done. While the kernel density estimator K is the estimated pdf of the random variable, the kernel k is a smoothing weighting non-negative function which defines the shape of the curve used to generate the pdf. The most common used kernel functions are the uniform function, the Epanechnikov function, the Gaussian function, the triangle function and the quartic function. The smoothness of the resulting density distribution is controlled by a smoothing parameter, called bandwidth h, whose value strongly affects the final result. Since the resulting density is strongly sensitive to the value of h, this should be chosen so as to optimize the trade-off between estimator bias and estimator efficiency. There are many optimality criteria to compute h, but the most used one is the minimization of the mean integrated squared error (or simply some rules-of-thumb accordingly derived, such as Silverman's rule, suitable with a Gaussian function).
   In the test application, the kernel function is a weighting probability function based on the additional information that could be embedded by the high frequency observations (not used for the construction of the error matrix). In particular, a multivariate kernel distribution function, that is the estimated pdf of a vector of random variables, is implemented. Since multivariate framework is applied to a random vector, it requires the definition of a square diagonal bandwidth matrix, with main diagonal elements (h_1, $h_2, \dots, h_T$), and of a product kernel $K(.) = k(z_1)k(z_2) \cdots k(z_T)$, where $k(.)$ is a one-dimensional kernel smoothing function. The kernel smoothing functions used for the test are the uniform function:

$$k(z) = \frac{1}{2}\boldsymbol{I}(-1 \leq z \leq 1)$$

and the Gaussian function:

$$k(z) = \frac{1}{\sqrt{2\pi}} e^{\frac{-z^2}{2}}$$

The bandwidths h applied to inputs are computed from the Silverman's rule of thumb for multivariate data, described in MATLAB documentation:

$$h_t = \sigma_t \left(\frac{4}{(T+2)m}\right)^{\frac{1}{T+4}}$$

for $t = 1,2,\dots,T$, where m is the number of lags of explanatory variable with respect to the dependent variable, and σ_i is the standard deviation of the explanatory variable in each period t. Given $K(\frac{z_t - z_s}{h})$, the last component to be defined is the input $z_t - z_s$ on which the product kernel is applied. Four different computations have been implemented: in the first one, $z_t - z_s$ is the difference among high frequency explanatory data collected over any two time periods; in the second one, the simple moving average of high frequency data has been implemented, instead of the pure observations; the third one is the exponentially weighted moving average with a window size of m and decreasing coefficient $l = 0{,}95$; the last one is the exponentially weighted moving average of variance, used in order to take into account the information provided by the variance of high frequency variables (i.e. the market risk). The formula for the computation of the exponentially weighted moving average $EWMA$ is:

$$EWMA_t = l * EWMA_{t-1} + (1-l) * x_t$$

At the end, the output required for the test is available: $z_t - z_s$is divided by h_t, then the Gaussian function $k(z_t)$ is applied to each value $\frac{z_t - z_s}{h_t}$, and the product along the time dimension among all $k(z)$ will give as output the kernel value K needed for the test.

4. The fourth step consists of the computation of the test numerator: it is the sum of all values given by the products between each kernel function value K and the error matrix $\varepsilon_t \varepsilon_s$, for each $t \neq s$.
5. The fifth step consists of the computation of the test denominator, which requires just the quantile order, its complement and the squared values of K: $\tau(1-\tau)\sqrt{2}\sqrt{\sum_{s \neq t} K_{ts}^2}$.
6. The ultimate step is the computation of the test statistic $Th^{m/2}\hat{J}_T / \hat{\sigma}_0$ by assembling all the pieces: $test\ statistic = w.* (test\ num./test\ den.)$. The resulting value is compared with the critical value of 1,96: as said, the test statistic tends to a Normal distribution as number of observations goes to infinity, and given a significance level α=0,05, the critical value is 1,96. If the test statistic is lower than this critical value, then the null hypothesis of lack of Granger causality for that specific quantile is accepted. Finally, this procedure is repeated for each quantile order, and all values are represented in appropriate figures.

The innovative point in the application of this test comes from the estimation of $\hat{Q}_\tau(w_t)$, that is the set of estimated quantiles of the dependent variables conditional on the lagged high frequency explanatory variables at any moment. $\hat{Q}_\tau(w_t)$ is estimated through the MIDAS quantile regression, applied to the following model:

$$Q_\tau(y_t) = \beta_{\tau,0} + \beta_{\tau,1}[B(L^{1/m};\theta)X_t^{(m)}] + e_t$$

The implementation of an optimization problem based on an explanatory variable weighted in accordance with the MIDAS approach, and in particular the search of a minimizer x of a nonlinear constrained multivariate function, allows the estimation of each parameter (intercepts and slopes), and then the estimation of the conditional quantiles of y_t, used for the test. In the MIDAS quantile regression, there are basically two relevant points to be clarified under a computational point of view: the definition of the optimization problem for the quantile regression, and the definition of the MIDAS weights to be applied to X.

As said, the sample τ-th quantiles are identified as those points ξ of the domain of the function $\sum_i \rho_\tau(y_i - \xi)$ at which its values are minimized:

$$\hat{Q}_\tau(y) = \arg\min_\xi \sum_i \rho_\tau(y_i - \xi)$$

in which the quantile loss function is defined as:

$$\rho_\tau(u) = u(\tau - I_{u<0})$$

By replacing ξ with the regression function $x_i'\beta_\tau$, where x_i is a vector of K regressors, it is possible to minimize the total "loss" of residuals defined by $\rho(\cdot)$. u is just the model residuals, namely the difference between the observations y_i and the fitted values $x_i'\beta_\tau$. Therefore, quantiles can be written as solutions to the optimization problem:

$$\hat{Q}_\tau(y) = \arg\min_{\xi\in\mathbb{R}} \sum_{i=1}^{n} \rho_\tau(y_i - \xi)$$

whereby, for some $\tau \in (0, 1)$, we have to find $\hat{y} = x\hat{\beta}$ to minimize the expected loss. Computationally, this procedure can be carried out on MATLAB with the functionalities provided by "*fmincon*", that is a nonlinear programming solver which allows to find the minimum of a nonlinear multivariable function under a set of constraints, such as bounds, linear equality or non-linear inequality. It is very useful for the optimization of ratios and trade-offs through the setting of a criterion function, or objective function. To do that, an anonymous function has been used: this is a function not stored in a program file but associated to variables whose data type is *function_handle*. Therefore, the first step is to define the objective function in a specific M-file:

```
function [fval,condQuantile] = objFun(params,y,X,q,smoother)
```

in which input '`params`' contains relevant elements, such as '`intercept`' and '`slope`', which allow to compute the conditional quantiles:

```
condQuantile = intercept + slope .* (X * weights).
```

On the basis of the estimated conditional quantiles, the loss is defined as:

```
loss = y - condQuantile
```

and so even the asymmetric loss function, needed for the implementation of regression:

```
fval = loss' * (q - (loss<0)).
```

Of course, there are a set of bounds, optimization options and smoothers to take into account, and that can be found in the Appendix B. The core part of the code is given by the numeric minimization:

```
estParams = fmincon(@(params) objFun(params, EstY, EstX, q, smoother),...
```

This method allows to get the required parameters that satisfies the optimization problem: estimated slopes and intercepts, in '`estParams`', are those '`params`' which minimize the constrained objective function '`objFun`' through the usage of '`fmincon`'. Upper bounds and lower bounds are previously set, and the very initial parameters are estimated by OLS.

The second problem to deal is about the definition of the MIDAS weights to be applied to X:

```
condQuantile = intercept + slope .* (X * weights)
```

where `weights` are obtained through a function specifically created, '`midasBetaWeights`':

```
function weights = midasBetaWeights(nlag,param1,param2).
```

Inputs of this function are quite simple to describe: '`nlag`' is the number of lags of the explanatory variable used to explain the low frequency dependent variable detected for the subsequent period; '`param1`' (or '`k1`') and '`param2`' (or '`k2`') are the first and the second parameter used in the one-parameter Beta polynomial, respectively. As a consequence, there is a specific weight attached to each lag of the independent variable. After having subtracted from an array of ones a linearly spaced vector ψ between 0 and 1 (excluding these extremes), with `nlag` evenly spaced points, obtained values are raised to the parameter (`k2-1`), if `k1=1`, and then multiplied by the same spaced vector to the power of (`k1-1`), whenever `k1≠1`. Weights are finally divided by the sum of all weights:

```
weights = weights./nansum(weights).
```

Analytically, MIDAS weights w_k show the following direct proportionalities:

$$w_k \propto (1-\psi)^{(\mathrm{k2}-1)} \;\; if \;\; \mathrm{k1}=1$$

$$w_k \propto (1-\psi)^{(\mathrm{k2}-1)} * (\psi)^{(\mathrm{k1}-1)} \;\; if \;\; \mathrm{k1} \neq 1$$

Appendix B contains the details of this function. The following figure shows the output of function '`midasBetaWeights`' applied in the quantile regression between the industrial production index and daily stock returns of the main countries. Almost anywhere, weights attached to independent variables are strongly decreasing as lag increases.

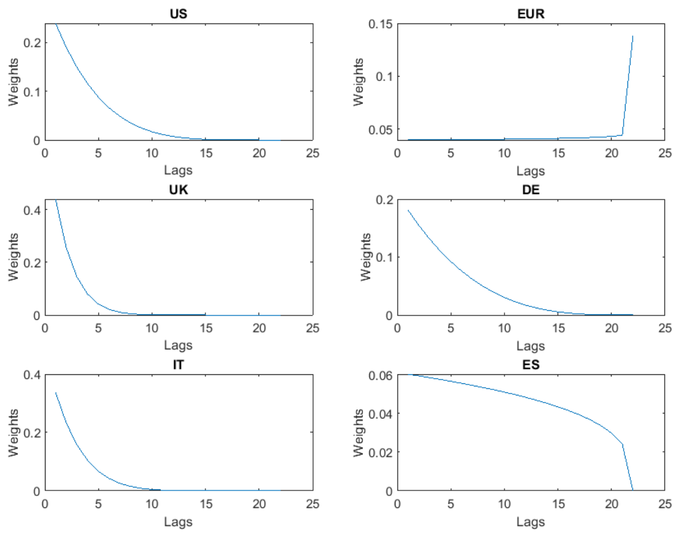


Figure 3.2: MIDAS Beta weights attached to each lag

As widely discussed, final purpose of this thesis is to create an EWI, that is a test for the detection of systemic risk in an economy. Other than the description of models and tools used in the analysis, the selection and the specification of suitable variables and dataset are a basic part of the methodology. Hence, it is crucial to identify that set of variables that can be useful to anticipate and predict to some extent incoming risks in the economic system. In order to achieve this objective, there exists a stylized fact that comes to our help: in the financial markets, the most operators carry out transactions trying to anticipate outcomes from real economy and to raise more or less rational expectations about changes in financial variables. A financial crisis caused by endogenous systemic mechanisms will always produce some consequences on the real economy, i.e. on the credit supply of banks, the saving and investment decisions of firms and households, the buying decisions of consumers, the prices fixed by sellers, the quantities of goods produced by producers, and the levels of income, wealth and wellbeing perceived by the society. It goes without saying that an increase of systemic risk in the financial system, that can be promptly detected by early warning indicators based on the evolution of specific financial variables, can be a caveat of an incoming heavier crisis, in which real variables are going to be strongly affected. The analysis will be performed evaluating a set of relationships between a high frequency variable, reflecting the market sentiment, the expectations, the financial stability and the possible increases of some relevant risks, and a low frequency variable, that reflects the macroeconomic state, in order to capture the materialization of the systemic risks. In particular, there are four relevant sources of systemic risk that will be examined.

1. The stock market risk: the systemic impact stemming from stock price variations, stock financial risk, changes in the Value-at-Risk and sectorial risk (financial sector and industrial sector) will be firstly evaluated.
2. The geopolitical risk, of which a big part can be approximated by changes in the crude oil prices, is another important source of systemic risk.
3. The currency risk, represented by fluctuations in the exchange rates, allows to get a wide view about the systemic risk generated at international level.
4. The sovereign risk, which shows a more national relevance, is the last source of systemic risk to be analyzed, looking at changes in the Credit Default Swap premia.

The first relation refers to the stock market risk. Indeed, since it is considered forward looking, the stock market provides the most common leading indicators on future economic activity, widely used in the literature. While the increase of systemic risk can be analyzed starting from the evolution of stock prices and returns, the materialization of the same systemic risk into an adverse event can be measured by a macroeconomic indicator linked to the productive activity, such as an industrial production index. The first application of the test will be based on the relationship between daily stock returns r as predictive variable, and the changes in the monthly Industrial Production Index IPI, as the affected variable:

$$Q_\tau(IPI_t) = \beta_{\tau,0} + \beta_{\tau,1} Z_{t-1}(\mathrm{k})$$

$$Z_{t-1}(\mathrm{k}) = \sum_{j=0}^{J} w_{k,j} r_{t-1-j}$$

In the model above, the conditional quantiles of IPI, $Q_\tau(IPI_t)$, for each order τ and for each month t, are obtained through a linear quantile regression on the set Z_{t-1} of lagged variables, that are function of the Beta polynomial parameters k; Z_{t-1} is the set of lagged daily stock returns r_{t-1-j}, with lag j up to $J = 22$, weighted by MIDAS weights $w_{k,j}$. In other words, each country's IPI, published by national statistical institutes on the 15^{th} of each month, is regressed-in-quantile on the 22 previous daily stock returns, occurred on the stock market of the same country (they are 22 because each month has on average 22 observations, since financial markets are closed on weekends and on holidays). Furthermore, each IPI change is regressed on the daily returns of the previous month, and not on the 22 daily returns immediately preceding the date of publication (the 15^{th}): this time horizon of 15 days (about 10 daily observations of r) is needed in order to take into account the delays in the publication of statistical bulletin, and make the estimates less biased (in other words, when a statistical bulletin, published on the 15^{th} of current t^{th} month, refers to an output of the previous $(t\text{-}1)^{\text{th}}$ month, then lagged high frequency regressor should start 15 days before the publication date). Stock returns (and returns of all other variables used in this analysis) are computed as simple percentage changes, excluding other financial sources of income from the underlying assets:

$$r_t = \left(\frac{P_t}{P_{t-1}} - 1\right) * 100$$

There are many studies that try to examine the relationship between current stock returns and future production, based on the assumption of strong predictive ability of stock markets. Indeed, whereas variations in IPI are a good proxy of incoming changes in the overall economic activity, variations in the stock returns often anticipate the same variations in IPI. The underlying economic logic is easy: an increase in the industrial production of an economy is a symptom of an ongoing expansive phase of the economic cycle, in which most firms invest and sell much more, accounting revenues that are continuously increasing. Under these circumstances, generated cash flows increase and are expected to increase, and so also the profitability. Since market operators discount all the relevant information about productive sectors, they will anticipate higher outcome of future IPI in their stock prices' evaluations, and this is automatically and instantly reflected in current stock prices and returns, which go up. This process is probably more evident before an incoming crisis, that is a recessive phase: if, as a result of some negative events, market operators expect a reduction of future cash flows to the industrial sectors, and accordingly a reduction in the expected profitability, then they will adjust downwards their evaluation on stock value, causing an immediate lowering of stock returns. Therefore, the correlation between future industrial production and current stock returns is expected to be positive.

Obviously, some flaws about the usage of this correlation for the construction of an EWI should be remarked: first of all, it is a correlation, not a univocal causal relationship, and variables may determine each other; moreover, if the services and the primary activities compose the largest part of GDP, then IPI may not be a good proxy of the overall macroeconomic state, and may not be useful to totally capture the presence of systemic risk, which can be generated by other sectors; of course, this procedure cannot take into account increase of systemic risk caused by completely unpredictable exogenous source (e.g. a sudden pandemic of coronavirus disease), and by those factors other than expectations on macroeconomic state affecting stock market movements; finally, one must be careful to different measurement criteria of variables, like IPI, since it is not standardized across countries. In general, advanced economies best suits for the application of this test, since they are highly monetarized and have a strong industrial sector and many highly capitalized companies listed on the stock markets: in this case, the correlation between industrial production and stock market is more likely to be stronger. Anyway, it should be reminded that the regression is a quantile regression, and the test does note detect the causality in mean, but the causality in quantile: therefore, evidence from correlations in mean are not the object of this study.

By the same logic as above, it is interesting to evaluate the relationship of a risk measure on the quantiles of the production index. For this purpose, the daily Value at Risk of stock returns used as regressor can be quite explicative:

$$Z_{t-1}(\mathrm{k}) = \sum_{j=0}^{J} w_{k,j} VaR_{t-1-j}$$

In the Appendix A, the daily Value-at-Risk of stock returns is represented for each country. Because of lack of intra-daily information about stock returns, daily VaR has been estimated through the variance-covariance method applied with conditional volatilities inferred from a GARCH (1,1) model with Student-t innovations. Four basic steps have been taken.

1. First of all, each time series of daily stock returns were demeaned, in order to have a set of zero mean data.

2. Secondly, the parameters of a regression model with GARCH time series errors were estimated by using the maximum likelihood criterion, so as to fit the model to the response data (i.e. the return time series). In MATLAB:

```
Model = garch('GARCHLags',1,'ARCHLags',1,'Distribution','t');
[EstMdl, estParamCov, logL] = estimate(Model, Returns).
```

3. Given the fully specified conditional variance model, each daily conditional variance is inferred from the response data. Daily conditional volatilities are then computed.

```
CondVar = infer(EstMdl, Returns);
CondVol = sqrt(CondVar).
```

4. Finally, daily Value-at-Risk with a 95% confidence level is computed through the parametric variance-covariance method, which is based on the assumption of normally distributed stock returns with zero mean:

$$VaR_{t,0.05} = -1{,}645 * \sigma_t$$

The Generalized Autoregressive Conditional Heteroskedasticity process is a common approach used to estimate returns' volatility in financial markets, that are characterized by heteroskedasticity and volatility clustering, precisely. GARCH process allows to model the change in variance over time as a function of the lagged residual errors from a mean process (or innovations, that is the stochastic part of the process) which is supposed to have a Student's t distribution, and the autoregressive component given by the lagged error variance terms. Therefore, a GARCH model has two key components: a GARCH polynomial with degree p, composed of p lagged conditional variances, and an ARCH polynomial with degree q, composed of q lagged squared innovations. p and q are the maximum nonzero lags of each respective polynomial. In this case, the model is a GARCH (1,1):

$$\varepsilon_t|\psi_{t-1} \sim N(0, \sigma_t^2)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

A differentiation among sectors of the stock market may be useful to identify the true origin of the systemic risk, and to obtain an EWI based on a stronger and more specific relation. Indeed, the common stock market index measures the overall market performance, which is calculated on the set of many sectors and industries. Albeit there exists a correlation among stocks and sectors of a stock market, the market sentiment originated in a specific sector could become crucial for the anticipation of the systemic risk. The test will be also applied in order to consider the relevance of two leading sectors within a stock market: the financial sector and the industrial sector. Hence, the quantile models will consider the industrial stock returns ri and the financial stock returns rf, as follows:

$$Z_{t-1}(\mathrm{k}) = \sum_{j=0}^{J} w_{k,j} ri_{t-1-j}$$

$$Z_{t-1}(\mathrm{k}) = \sum_{j=0}^{J} w_{k,j} rf_{t-1-j}$$

A stronger correlation is expected to occur between industrial stock returns and Industrial Production Index, since value investors do take into account the evolution of industrial output as a relevant factor of profitability for their sectorial investments.

Another fundamental relationship to which to apply the test of Granger causality in quantile, and which may allow to promptly identify an increasing systemic risk, refers to the impact of financial market risk on the macroeconomic stability. Indeed, financial market risk is a source of systemic risk, and may be able to anticipate it. The common measure for financial market risk, used within this framework, is the variance of demeaned stock returns, which convey very useful information about the current financial stability, the current expectations on future macroeconomic state and the discounting of an increasing systemic risk as a factor related to the business cycle. Using the same notation as before, the MIDAS quantile model becomes:

$$Q_\tau(IPI_t) = \beta_{\tau,0} + \beta_{\tau,1} Z_{t-1}(\mathrm{k})$$

$$Z_{t-1}(\mathrm{k}) = \sum_{j=0}^{J} w_{k,j} r_{t-1-j}^2$$

Variance of returns measures the variability of demeaned stock returns, and helps to evaluate the financial risk in asset allocation processes. In the Appendix A, the exponentially weighted moving variances of daily returns are showed for each country. These charts are useful to identify the most turbulent periods on each stock markets, and the most volatile markets.

The high variability in stock returns seems to show a countercyclical behavior: during recessions, volatility increases much above its overall mean, while during expansions it is slightly lower. What really matters is to assess whether variance of stock returns may be useful to anticipate the business cycle: in fact, the literature confirms the existence of a correlation between current financial volatility and future real macroeconomic variables, such as the same Industrial Production Index, even if this predicting power is not likely to be stable and equally strong over time. Anyway, there is no doubt that stock market variability is able to convey information about the overall macroeconomic environment: in accordance with the efficient market hypothesis, any relevant information, included that about future trends of real economy (e.g. the production), is discounted by rational market operators during the stock evaluation phase, and its uncertainty is necessarily reflected in the variance of returns. There are four reasons why the stock market volatility might anticipate the economic activity.

1. There exists a component of procyclicality, since the stock price volatility may directly affect the macroeconomic environment: an increase in stock volatility makes riskier the value of collateral provided by firms; this widening of market frictions reduces the operators' availability to provide funds and to lend, causing a stoppage of financial markets, and therefore a slowdown in the economic activity.
2. The increase of the perceived risk, due to negative events or incoming crisis, makes market operators more risk averse: it is reflected in the risk premia, and then in the asset prices, which fluctuate more.
3. The increase of perceived risk provides a disincentive to invest and to commit money to longer term projects, and this may cause a slowdown of economic activities.
4. Finally, there are some behavioral explanations, like representativeness and anchoring, whereby stock market movements create some beliefs from firms and households about the general state of an economy, and this strengthens the mutual interaction between financial markets and real economy. Furthermore, the participants' preference for speculative trading or for long-term investments strongly depends on the type of stock market and its riskiness: for instance, swing trading and short-term positions are relatively prevailing in the riskier markets, such as emergent markets and derivatives markets. If this is the case, the correlation between stock market prices and macroeconomic variables will be much more significant in advanced economies.

A useful variable that embeds relevant information exploitable to detect systemic risk is the price of crude oil, which has been considered in the literature as a leading indicator of changes in economic phases. The quantile model will include IPI changes and changes in the crude oil price (WTI and Brent) ro:

$$Z_{t-1}(\mathrm{k}) = \sum_{j=0}^{J} w_{k,j} ro_{t-1-j}$$

The market sentiment and the relative balance between global demand and global supply of oil are the main determinants of oil price movements, and these movements cause a set of consequences and different impacts on the national economies. In general, demand and supply of oil are strongly affected by geopolitical issues and events of global significance: that's why oil price changes are useful to capture increases of geopolitical risk, and then of systemic risk. There is not a unique direction: past research shows that oil price changes produce both negative and positive effects on macroeconomic variables depending on the type of economy. In particular, in oil importing advanced countries, a price increase tends to cause a slowdown of economic growth, an increase in inflation and unemployment and an overall reduction in the value of the financial assets; in oil importing emerging countries, in addition to above mentioned negative effects, a severe overall impoverishment of households and small producers may cause a heavy reduction in the levels of investments, consumption and well-being, other than an increase in the probability of capital flight, massive debt defaults and speculative attacks; oil exporting countries are the main beneficiary of an oil price increase, because of higher revenues got by oil industries, which may result in higher national income if the global demand is inelastic. Therefore, oil price increase implies a wealth transfer from importing to exporting countries, and always causes some costly resource reallocation. Historically, increases of oil prices has been followed by recessions and debt defaults, because of higher costs for households and firms, which must face declining cash flows and lower income. As early as in 1983, the economist J. D. Hamilton showed that, since WWII, all recessions (but one) occurred in U.S. has been preceded by an increase in oil price, and this confirmed the existence of a Granger-causality between oil prices and output. Same evidence was found for the 2008 financial crisis: the acceleration in oil prices during the years priors to 2008 was a signal of increased systemic risk, which materialized itself as the house bubble burst and the global economy fell into a heavy recession. Hence, the oil price is a good detector of systemic risk at global level, since it embeds market sentiments on global economic stability.
Of course, there are some flaws: as said, oil price changes produce different impact on each country, and this makes the test results neither universal nor unambiguous. Moreover, the reliability of the test is function of the strength of the correlation between oil price and industrial production. Since oil volumes is going to decline over next decades, it is likely that oil price will become less relevant for global economy and global financial patterns.
Another source of systemic risk is represented by the foreign exchange market. The exchange rate variations generate the so-called currency risk, which may produce heavy negative consequences in many economies, despite the attempts to implement and preserve a fixed exchange rate regime: unavoidable global shocks or nefarious public policies may be the cause of detrimental capital outflows, speculative attacks and currency crisis. In particular, there exists a non-linear significant relationship between exchange rates and industrial production of a country: this impact, become stronger with the globalization process, occurs mainly through the trade channel, and subsequent variations of prices. It is widely acknowledged that the depreciation of currency tends to be expansionary, while its appreciation tends to be contractionary. Anyway, the evidence is not unique and unambiguous, rather it is as complicated as the reality: short-run and long-run effects of positive and negative changes of exchange rate on industrial production can be distinguished, with different impact on different industrial sectors, and different elasticities of import and export, depending on the type of examined economy, its degree of openness, the management of the exchange rate regime and the extent of exchange rate variations. All of this without taking into account the role of

financial flows, and the fact that there exist some endogeneity problems, such as simultaneity bias. However, the existence of non-linear effects of exchange rates on the industrial production, because of impacts on imported and exported quantities, can be tested with our nonparametric test, and used as EWI. The model to be implemented will be:

$$Z_{t-1}(\mathrm{k}) = \sum_{j=0}^{J} w_{k,j} re_{t-1-j}$$

where re are the lagged changes of the exchange rate for each country, expressed in terms of dollars.

A last important source of systemic risk is the country risk, that is the uncertainty faced by investors when dealing with a specific country, and in particular the sovereign risk, that is the risk to lose money because of the default on sovereign debt. The institutional investors, which buy Treasury bonds issued by the government, tend to ask higher sovereign risk premia, reflected in the interest rate, whenever the fiscal sustainability seems to be compromised. Changes in the sovereign risk premia could be caused by dangerous fiscal policies, and may trigger adverse loops and heavy macroeconomic implications, through the banking channel and the public channel: the reduction in the value of Treasury bonds held by banks as safe assets and the increase in the interests paid by taxpayers are a signal of an increase in the systemic risk due to public system, that can finally result in an economic downturn. It is evident that changes in sovereign risk premia are somehow reflected in the whole financial system, and then in the domestic private economic system. The tricky challenge is given by the existence of simultaneity: on the one hand, pre-existing weaknesses of economic fundamentals gradually exacerbate the fiscal health, and this causes increase in the risk premium; on the other hand, higher risk premia asked on public bonds in turn worsen credit conditions offered by financial operators, and this brings to a reduction in the firms' investments and a slowdown in the economic activity. There is much empirical evidence on the negative effects produced by higher sovereign risk: higher borrowing costs, higher risk of private capital outflows, reduction in private investments, lower equity prices, and finally lower industrial production. Of course, it is interesting to evaluate the impact of a change in the sovereign risk on different quantiles of the industrial production, and to detect signals of incoming distress. In order to do that, changes in the Credit Default Swap premium issued on government's debts, rp, are considered the regressors of the model:

$$Z_{t-1}(\mathrm{k}) = \sum_{j=0}^{J} w_{k,j} rp_{t-1-j}$$

Indeed, the CDS spread is a good measure of the sovereign risk perceived by operators, and may anticipate the incoming of an economic crisis, promptly signaling a higher systemic risk.

Chapter 4
Test Application: An Empirical Analysis

After having described the theoretical framework and the main tools, and therefore the nonparametric test to use as Early Warning Indicator, the core part of this thesis is going to be presented: the practical implementation and application of the test under an empirical point of view. The nonparametric test for causality in quantiles needed for signals detection related to systemic risk is applied by means of the programming platform MATLAB, and the accurately selected dataset provided by the financial data platform Thomson Reuters Eikon. As already specified, the test is an extension of the nonparametric Granger causality-in-quantile test, proposed by Jeong, Hardle and Song (Econometric Theory, 2012), based on conditional quantile estimates of the dependent variable obtained by a MIDAS (Mixed Data Sampling) quantile regression.

This analytical test could be useful to monitor the evolution of systemic risks and to identify incoming troubles with timeliness. Of course, information provided by an EWI should always be considered within a probabilistic framework, with all its own limitations and flaws, beginning with the fact that the future can never be foreseen with total precision. This is just a starting point, to be improved and to be integrated with other tools, and then to take with a grain of salt.

This chapter and following Appendices will define all the elements required for a valid and useful empirical analysis: the dataset about countries and markets, the definition of the variables, the covered timespan, the definition of systemic distress under an analytical point of view and the main empirical findings. In the last part, conclusions will be drawn, about the presented work and about future possible extensions.

4.1 Empirical Analysis: Results

The execution of the methodology and its application with real-world data are the crucial step to get empirical evidence and reach a conclusion. The results of the empirical analysis are described and graphically presented in this paragraph, while the dataset description is reported in the Appendix A.

As widely explained, the common linear framework, which models the conditional mean of a normally distributed response variable as a linear combination of predictor variables, is not very useful when there exists the possibility of nonlinear dependence among variables. Indeed, as showed in the following figure, linear regression between the mean monthly stock returns and the Industrial Production Index changes leads to unreliable results, because of a null goodness of fit, which proves that the regression fails to accurately model the data.

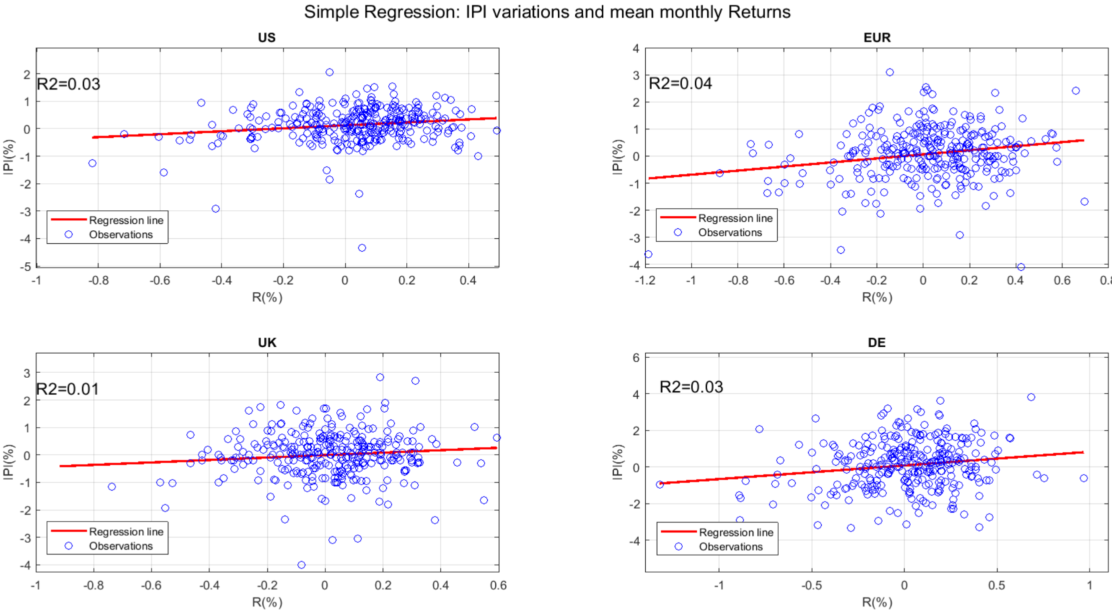


Figure 4.1: Simple linear regression lines

Misspecification problem makes any indicator and model built on those data not valid for the causality detection. Given an evidence of nonlinearity, the nonparametric causality-in-quantile test gives the most robust and reliable results, especially against outliers, breaks and jumps, because it evaluates each quantile of the distribution, and not only its center. As reported in the descriptive tables of Appendix A, both response and independent variables exhibit non-normal characteristics: negative values of the skewness point to higher probability of large decrease, while kurtosis much higher than three point to higher peaks and fatter tails. Therefore, IPI returns have a fat-tailed left-skewed distribution, that is a non-normal distribution which justifies the use of nonparametric models. The quantile regression model can be used when some conditions of linear regression, such as normality and linearity, are not met, and when the analyst is interested not in what affects the expected value of response variable, but in what affects its whole quantile distribution. MIDAS quantile regression allows to estimate conditional quantiles of industrial production in the presence of high frequency (daily) explanatory variables, such as stock returns. The output argument reports the results of a specific approach: the smoothed asymmetric loss function minimization. It is characterized by:

- no use of specific analytic gradient in MLE;
- no use of Global Optimization Toolbox;
- the use of a starting smoother, that is the average absolute residuals, that allows to smooth the non-differentiable objective function;
- the use of FMINCON options for numerical optimization.

Following tables show the results of a first application of MIDAS quantile regression on the dataset about Eurozone and UK, at the 5th quantile order.

EUR

```
Method: Smoothed Asymmetric loss function minimization
Minimized function value:    34.8658
Quantile order:         0.05
```

	Coeff	StdErr	tStat	Prob
Intercept	-1.2055	2.0085e-15	-6.0021e+14	0
Slope	1.3527	2.0085e-15	6.7349e+14	0
k2	0.96547	1.339e-15	7.2105e+14	0

UK

Method: Smoothed Asymmetric loss function minimization
Minimized function value: 32.9829
Quantile order: 0.05

	Coeff	StdErr	tStat	Prob
Intercept	-1.0961	1.1158e-15	-9.8237e+14	0
Slope	-0.1629	3.1243e-15	-5.2139e+13	0
k2	12.166	2.2316e-15	5.4516e+15	0

Table 4.1: MIDAS quantile regression coefficients for Eurozone and UK, q=0,05

Since the quantile regression is based on the minimization of an asymmetric loss function, each minimized function value is reported. For each coefficient (intercept, slope and the parameter used in the one-parameter Beta polynomial, `k2`), the standard error and the statistical significance are listed. Therefore, for Eurozone at 5th quantile order, the estimated quantile model is:

$$Q_{0.05}(IPI_t) = -1{,}205 + 1{,}353 * Z_{t-1}(0{,}96)$$

with $\beta_{0;0.05} = -1{,}205$ and $\beta_{1;0.05} = 1{,}353$.

The interpretation of coefficient is quite simple, analogous to that of OLS coefficients: the estimated 5th percentile of European IPI change is equal to -1,205% whenever stock return rates were null (it is the unconditional quantile), and increases by 1,353 basis percentage points whenever stock return rates increased by 1% during the previous month. These relationships seem to be strong because the standard deviations of each parameter are very low, and the p-values are very close to zero: coefficients at quantile order q=0,05 are significantly different from zero, and the relations are stable, for both Eurozone and United Kingdom. The standard errors of coefficients are the standard deviation of simulated parameters, obtained through a bootstrapping with the estimated residuals.

In order to implement the nonparametric test, the function created on MATLAB to perform the quantile regression automatically computes the conditional quantiles for each quantile order, from 0,01 up to 0,99. An illustration is showed below: the quantile coefficients are estimated from the quantile model applied to IPI changes of Unites States, for the 5th percentile, the 95th percentile and the median (q=0,5).

US

Method: Smoothed Asymmetric loss function minimization
Minimized function value: 24.9142
Quantile order: 0.05

	Coeff	StdErr	tStat	Prob
Intercept	-0.54656	5.5791e-16	-9.7967e+14	0
Slope	-0.29955	6.137e-16	-4.8811e+14	0
k2	5.7388	8.9265e-16	6.4289e+15	0

Method: Smoothed Asymmetric loss function minimization
Minimized function value: 69.1767
Quantile order: 0.5

	Coeff	StdErr	tStat	Prob
Intercept	0.17007	1.9527e-16	8.7095e+14	0
Slope	-0.012618	9.7634e-17	-1.2924e+14	0
k2	6.6937	1.1158e-15	5.9989e+15	0

Method: Smoothed Asymmetric loss function minimization
Minimized function value: 18.0639
Quantile order: 0.95

	Coeff	StdErr	tStat	Prob
Intercept	0.83305	1.5621e-15	5.3327e+14	0
Slope	0.14964	1.3948e-17	1.0728e+16	0
k2	13.472	6.4271e-14	2.0962e+14	0

Table 4.2: MIDAS quantile regression coefficients for US, q=0,05; q=0,50; q=0,95

The unconditional th-quantile, that is the intercept, increases with the order quantile, by definition; the slope is more impacting for the extreme estimated quantiles, and closer to zero for the median. The direction is not unique: in US data, estimated quantiles of IPI changes increase with stock return rates for higher quantile orders, and decreases for lower quantile orders. It means that, when there is a heavy fall in stock returns during a given month, the estimated quantile distribution of IPI change for the following month is expected to be more asymmetric, defined over a wider range of values and with a longer left tail. All the coefficients are statistically significant. Hereinafter, results from regression will be figured in terms of estimated quantiles of IPI changes, which is the basic variable needed for the nonparametric test. The following figures plot the monthly observations of IPI change, together with the 5th percentile for both US and Eurozone.

US ***EUR***

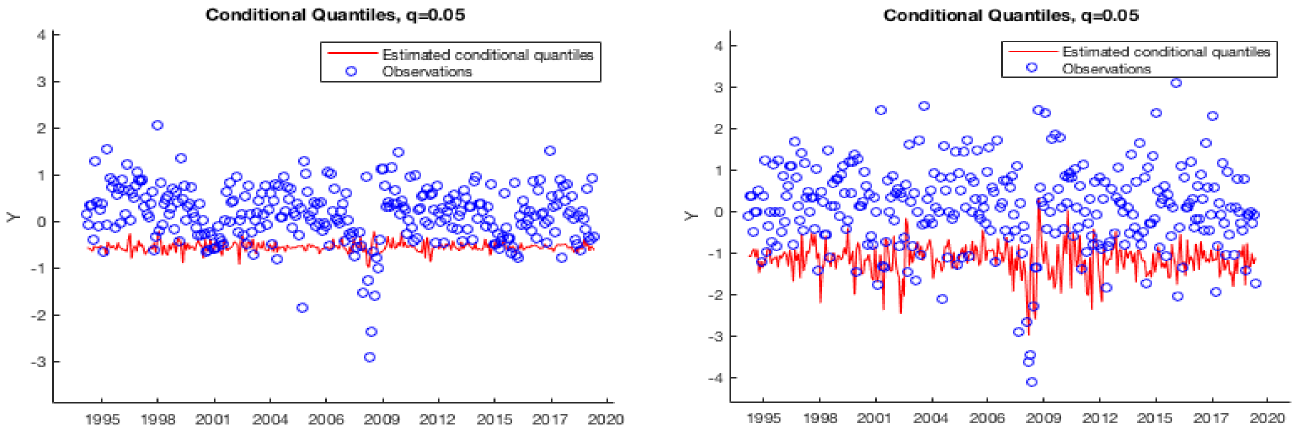


Figure 4.2: IPI changes (%) and conditional quantiles (q=0,05) of US and Eurozone, 1995-2019

It is evident the different degree of statistical dispersion of observations between the two graphs: in US, monthly variations in IPI are more concentrated on a range of values between -1% and +1%, with very few outliers. This is reflected in a flatter red line representing the estimated conditional quantiles at q=0,05. Contrarily, a wider and more volatile picture characterizes the Eurozone: conditional quantiles are more sensible to the market performance, and IPI changes cover a wider range of values.

EUR

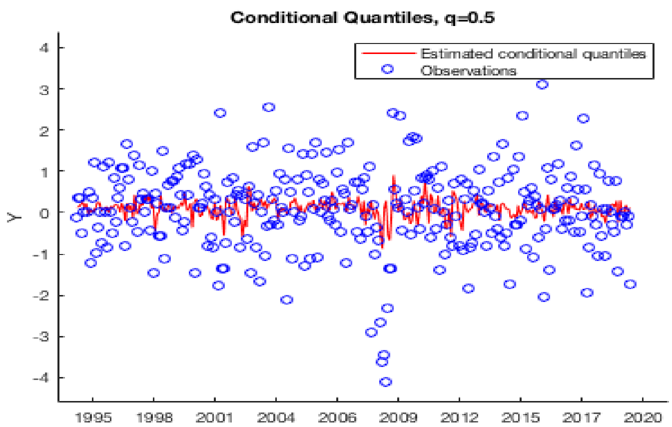


Figure 4.3: IPI changes (%) and conditional median of Eurozone, 1995-2019

The median of distribution of European IPI returns, conditional on daily stock returns of MSCI EMU Index, is plotted in the figure above. Of course, the evolution of estimated median is different from the evolution of estimated mean, but it fluctuates just above zero, similarly to the expected value of returns. In any case, the strongest oscillation occurs in correspondence of the 2008 financial crisis, whatever the quantile order.

The following charts plot observed monthly IPI changes and estimated conditional quantiles at q=0,05 for each country.

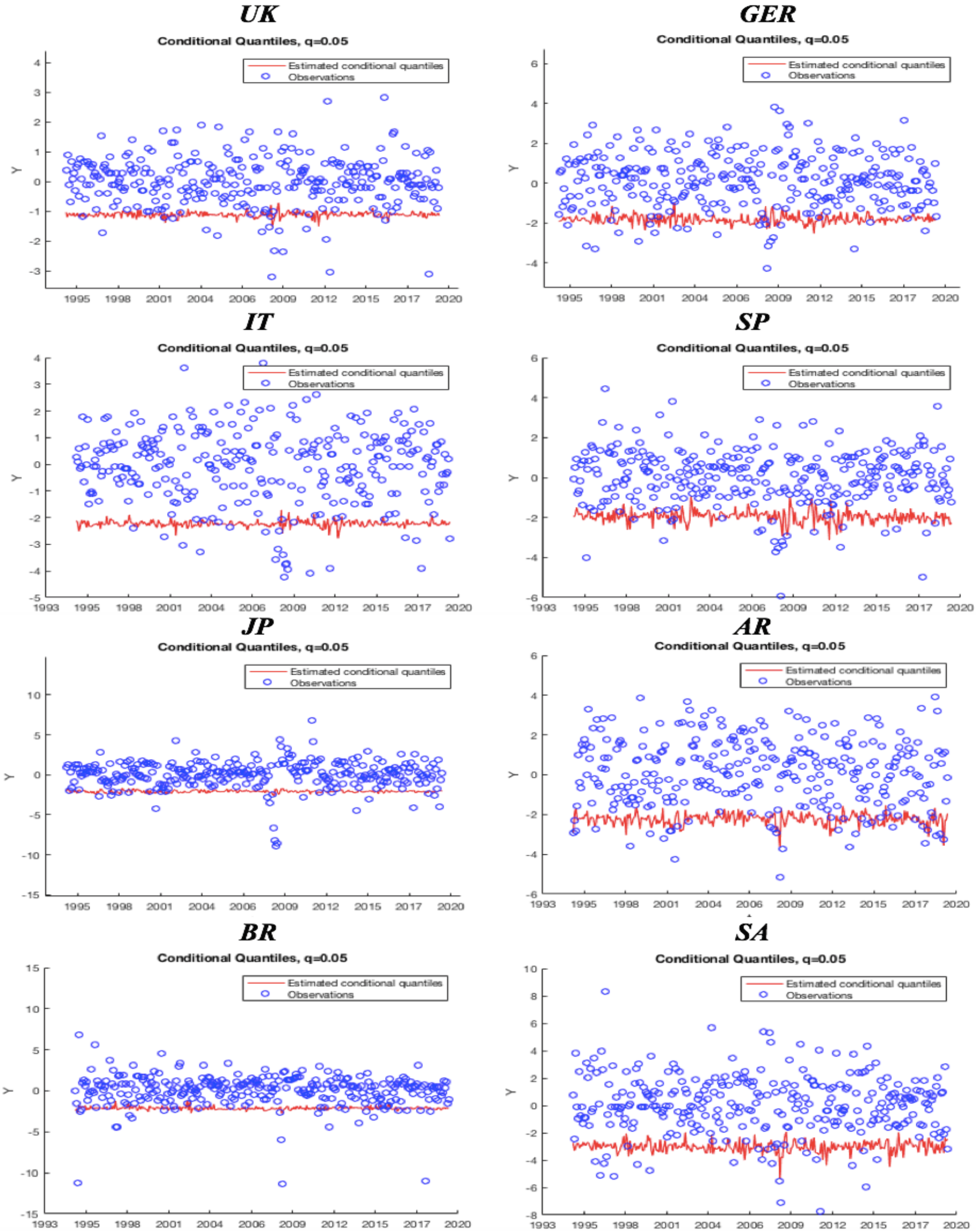
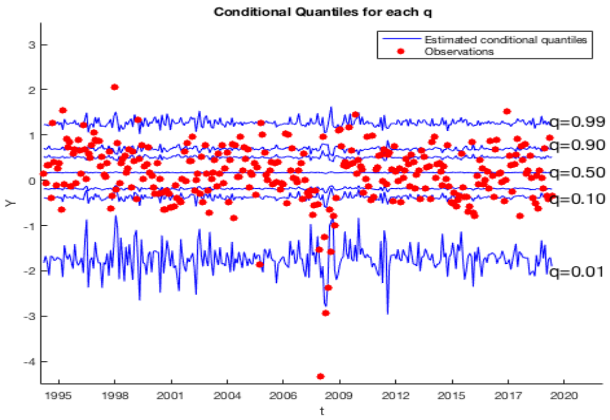


Figure 4.4: IPI changes (%) and conditional quantiles (q=0,05) for all countries, 1995-2019

It is interesting to take a look at every relevant conditional quantile for US and Eurozone, in order to understand the estimated evolution of the whole conditional distribution: the 1th percentile, the 10th percentile, the median, the 90th percentile and the 99th percentile.

US



EUR

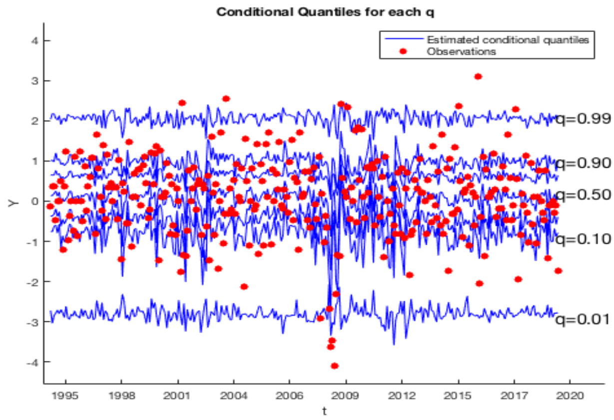
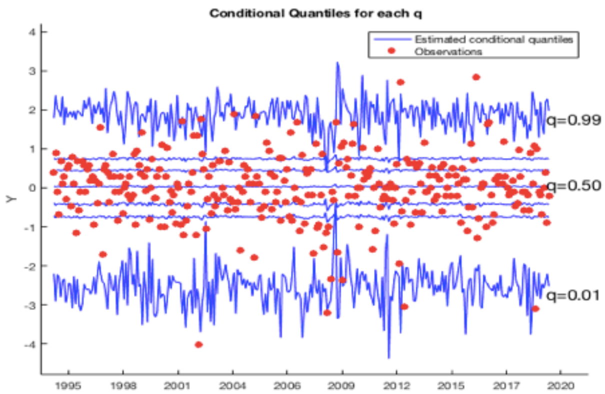


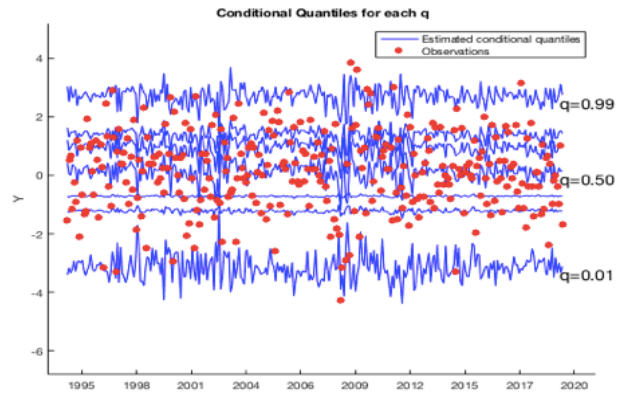
Figure 4.5: IPI changes (%) and conditional quantiles (relevant q) for US and Eurozone

It can be noted that the evolution of the first estimated quantile of IPI change, at q=0,01, is much more volatile in US than in the Eurozone, while all other conditional quantiles are more volatile in Eurozone. This reflects the higher dispersion in the European data about IPI change, and the different impacts of stock return rates on the industrial production. Moreover, it is evident the presence of a longer left tail in the IPI change distribution: the longer distance between the first estimated percentile and all other relevant estimated percentiles is a common characteristic of each country, as shown in the following charts.

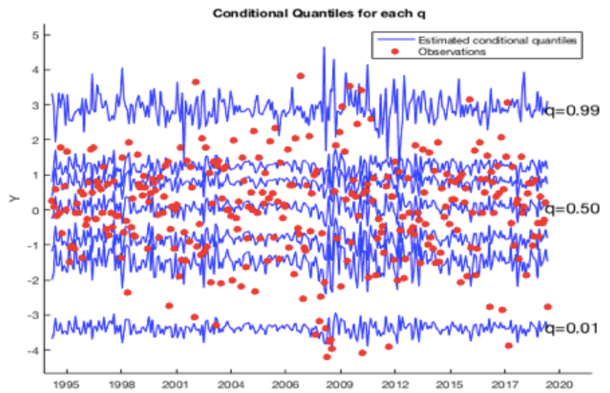
UK



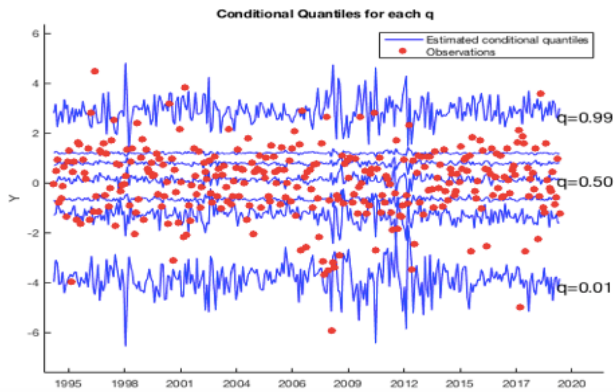
GER



ITA



SP



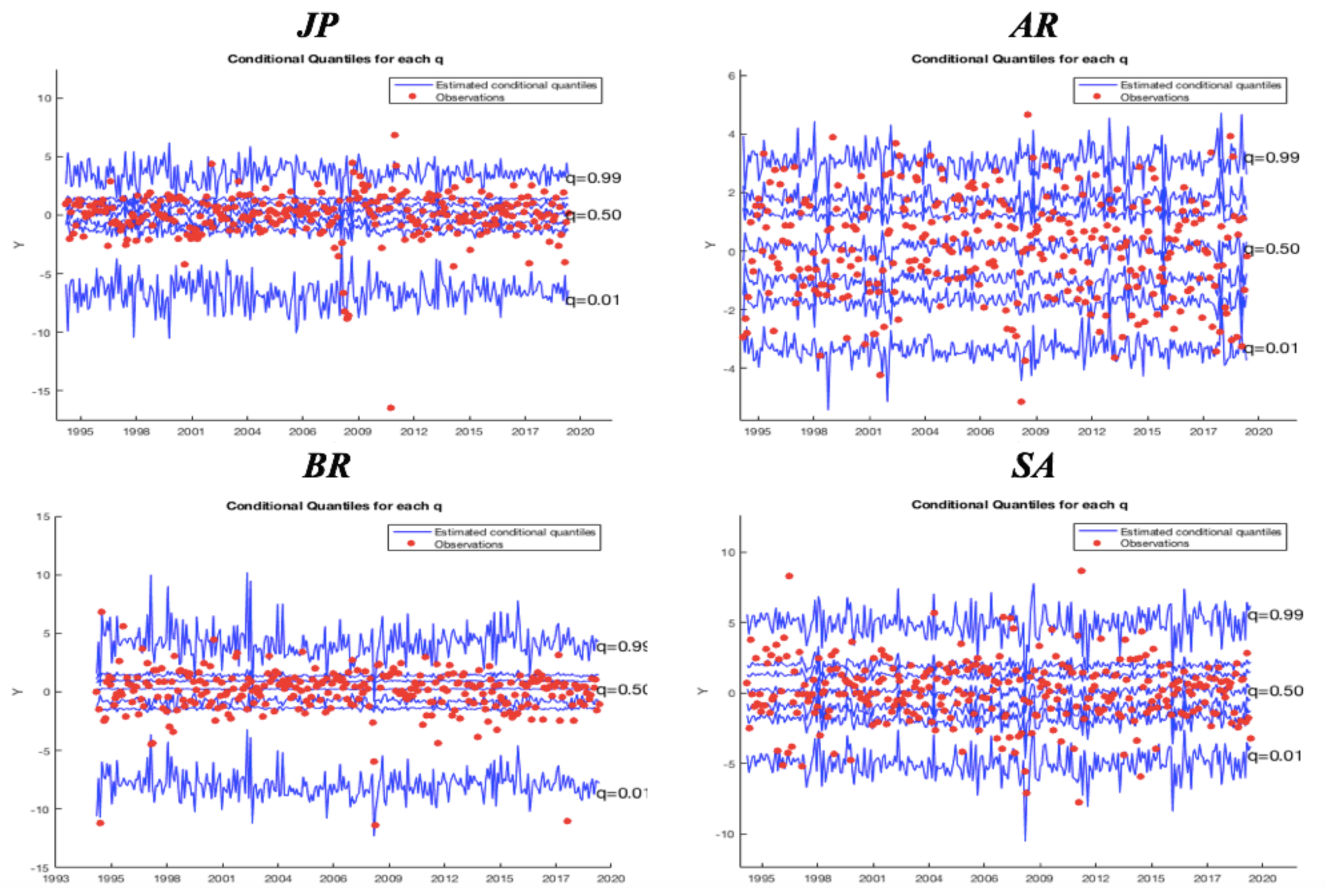


Figure 4.6: IPI changes (%) and conditional quantiles (relevant q) for all countries

Final step consists of the application of the causality-in-quantiles test, which is justified by the fat-tailed non-normal distributions of variables of interest. Indeed, nonparametric causality-in-quantiles test takes into account all quantiles of the distribution, not only the center of the distribution as the most common tests do: it is evident that, in many phenomena, the behavior in the tails is different from that of the rest of the distribution.
Here we are going to test the Granger causality from stock returns to IPI returns quantiles of each country. The following tables show the test statistics and the p-values of relevant quantile orders for US (left side) and the results about the 5th percentile of each country (right side).

Nonlinear Granger causality test, US

	Test_stat	**p_value**
Q1	0	0.5
Q10	5.1	3.0208e-07
Q20	4.21	1.691e-05
Q30	2.88	0.002132
Q40	1.54	0.062309
Q50	1.72	0.043234
Q60	1.2	0.11554
Q70	0.58	0.28118
Q80	1.47	0.071307
Q90	2.38	0.0089697
Q99	0	0.5

Nonlinear Granger causality test, q=0.05

	Test_stat	**p_value**
US	3.3	0.00054192
EUR	0.31	0.37839
UK	2.67	0.004
DE	1.05	0.14728
IT	3.56	0.00021562
ES	2.82	0.0025616
JP	4	3.995e-05
AR	3.97	4.504e-05
BR	0.01	0.49601
ZA	0	0.5

Table 4.3: Results of causality test, for US and other countries (at q=0,05)

The reading of these tables is easy: stock returns significantly affect (or rather significantly correlate with) specific quantiles of the monthly industrial production whenever the test statistic is higher than 1,96 (or equivalently, the p-value is less than 0,05). Therefore, it is clear that US stock returns Granger cause IPI change at the 10th, the 20th, the 30th, and the 90th percentile, while they remain not useful to predict the 40th, the 50th, the 60th, the 70th and the 80th percentile. Furthermore, stock returns of domestic stock markets seem to be relevant to predict the 5th percentile in UK, Italy, Spain, Japan and Argentina.
For an immediate understanding and visualization of Granger causality, all the results of the test will be graphically presented in the following figures. The vertical axis shows the test statistic J, the horizontal axis shows the quantile orders from 0,01 up to 0,99, and the flat red line represents the 5% critical value, that is 1,96. Therefore, the null hypothesis that stock return does not Granger cause IPI changes for a given quantile order is rejected whenever the value J is higher than 1,96, that is whenever the p-value is lower than 0,05.

US ***EUR***

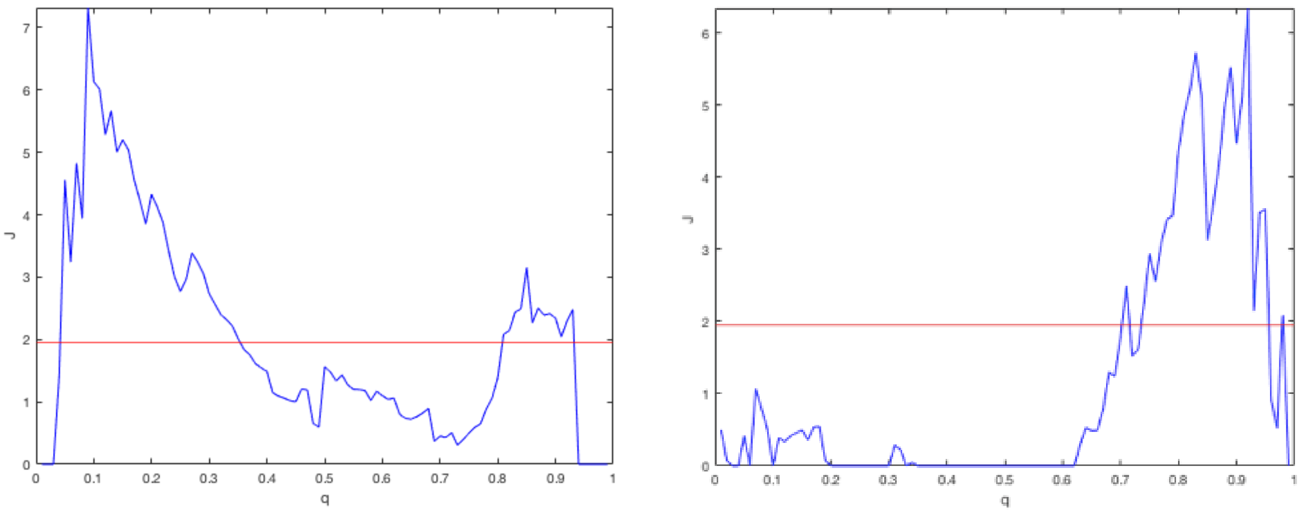


Figure 4.7: Nonparametric Granger-causality test for all quantiles, stock returns on IPI

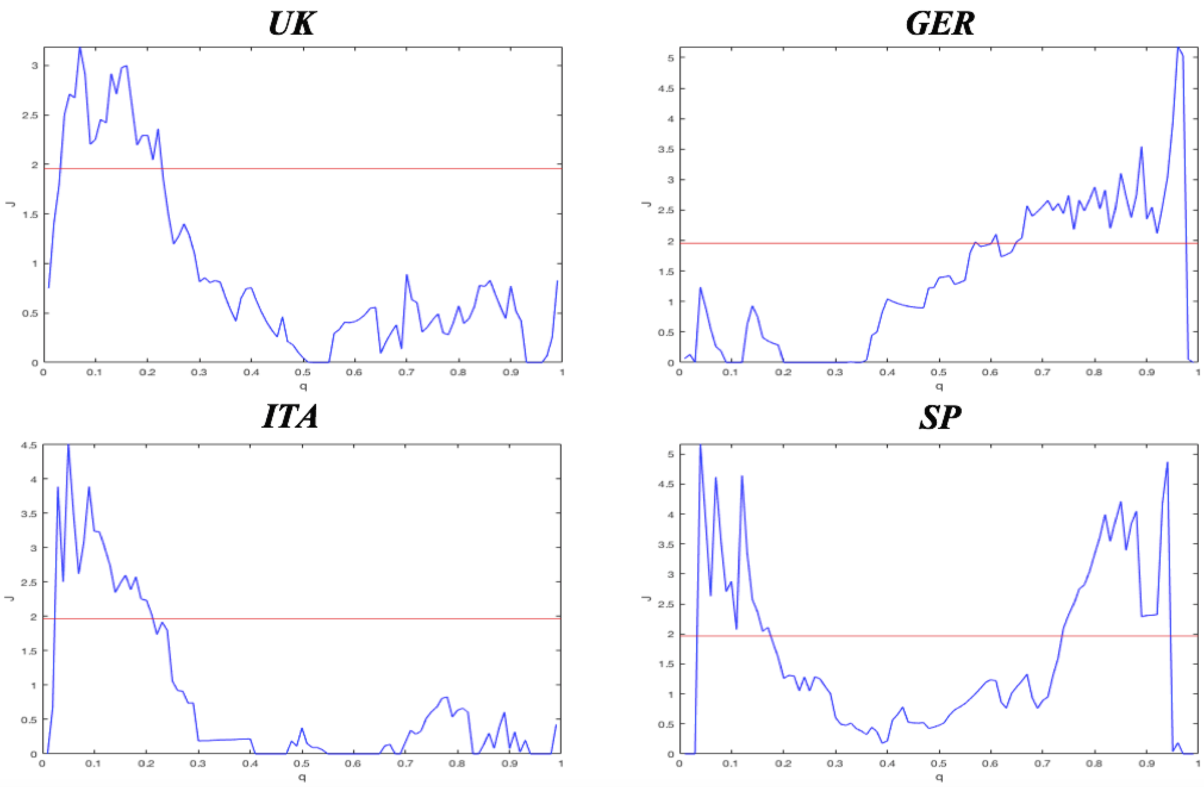
Graphically, it is possible to effectively identify and immediately evaluate relevant quantiles of the IPI distribution Granger-caused by stock returns movements.
Given a critical value of 1,96 the causality-in-quantile test applied to US data does reject the null hypothesis roughly within the quantile range of 0,05 to 0,35 and the range of 0,80 to 0,95, while there is no Granger causality within the range of 0,35 to 0,80. In the Eurozone, the null hypothesis is rejected roughly within the quantile range [0,70; 0,95]: this means that a fall in the stock market returns Granger causes a significant movement in the upper quantiles of the IPI returns distribution, and therefore stock returns have a strong predictive power for those quantiles only. In any case, there is no predictive power for central quantiles, like the median: a confirmation of this lack of correlation is given by the null goodness of fit in the linear regression models, which try to estimate the expected value (that is a central measure just like the median) of the variable of interest (i.e. IPI returns). This means that a stock market crush does not necessarily imply a proportionate fall in the production, even though its probability distribution undergoes relevant changes, maintaining the same expected mean as before. This lack in the detection of an impact in the central tendency can be due to:

- errors in the evaluation of the overall economic performance of stocks (behavioral explanations, panics, bubbles, wrong evaluations of relevant information like a shock or new public policies);
- recovery time of the crash in the stock market (it often happens that a sudden crash is followed by a fast recovery, even within the same month, and hence no change occurs in the industrial production);
- other factors, such as reverse causality, expectations, time framing.

An important conclusion for US data is that extreme reductions in industrial production are quite significantly predicted by changes in stock returns, and in general the noncausality part is longer on the central positions of the distribution.

The next figure shows the results for all other countries. Shapes of the blue lines are very uneven, asymmetrical and different from country to country, but there is a clear characteristic common to each chart: causality is not significant, or it is at least much weaker, on the central quantiles with respect to external quantiles, where the relationship with stock returns tends to be stronger for each country. Hence, given many dissimilarities, another important conclusion is that the relationship on the tails, both the left and the right one, is relevant: extreme changes in industrial production are significantly anticipated by stock returns, while the relation with central tendencies is unclear and more ambiguous, and it requires further deepening.

The left tail of the distribution of IPI returns, which corresponds to the probability of significant IPI decreases, is represented by lower quantiles, and, as showed by graphs on test statistics, it is the most relevant part of the distribution in almost every country: there exists a significant Granger causality in IPI return distribution of UK, Italy, Spain, Japan, Argentina, Brazil and South Africa roughly within the quantile range [0,05; 0,30]. At the same time, upper quantiles of IPI returns are significantly Granger-caused by stock returns in Germany, Spain and Argentina.



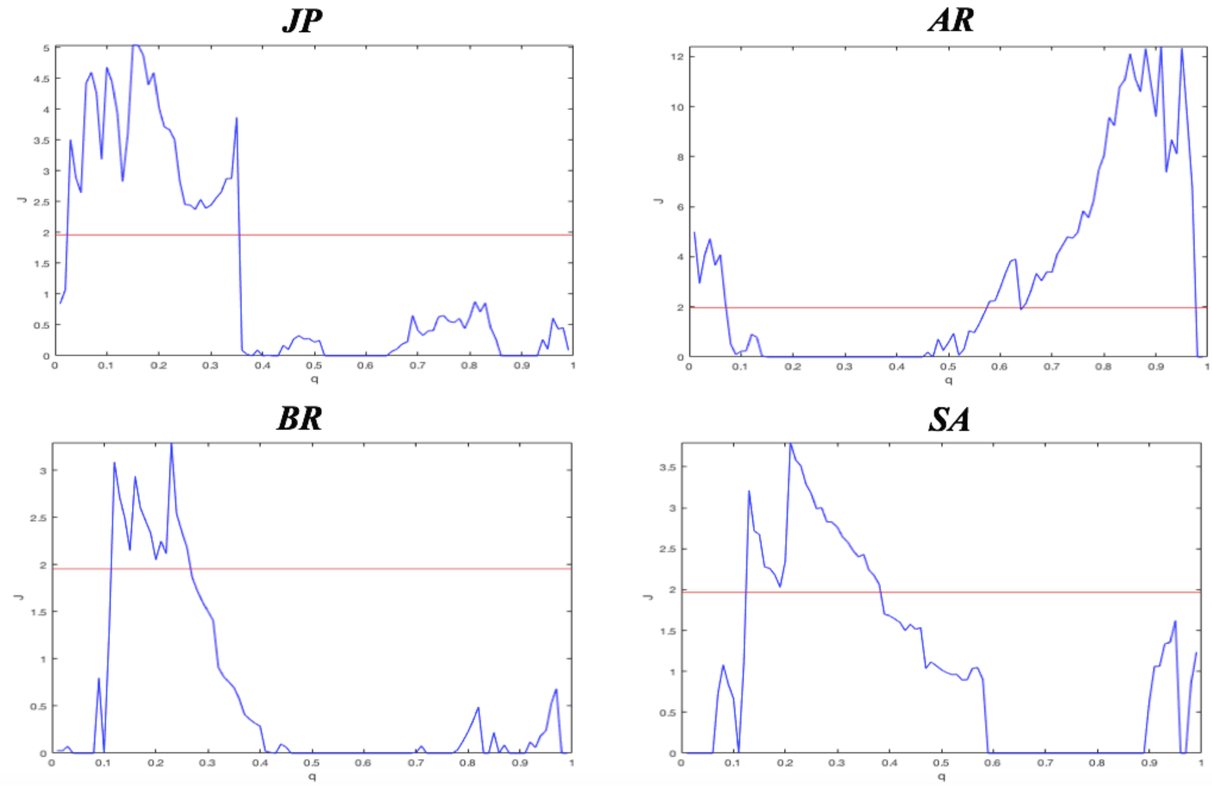


Figure 4.8: Nonparametric Granger-causality test, stock returns on IPI, all countries

Looking at chart of Italy and UK, since the test statistic exceeds the critical value (red line) roughly when 0,05 ≤ q ≤ 0,22, then it stands to reason that a relevant change in total stock returns may reflect a change in the perception of systemic risk, and then an increase in the probability to deal with lower negative IPI returns (which corresponds to lower quantiles).
The evaluation of a risk measure, like Value at Risk, on the quantiles of the production index provides even more interesting results. The daily Value at Risk of stock returns, estimated through the variance-covariance method and a GARCH volatility model, is used as the explicative variable instead of the same stock returns. Charts representing VaR of daily returns, reported in Appendix A, highlight the perfect correspondence of VaR with an extreme lower quantile of the distribution (as explained, the VaR is a conditional quantile).
The next figures present results of testing whether daily Value at Risk of stock returns may predict IPI returns at the various quantiles. Unlike the previous figures, which showed very dissimilar graphs, in the case of VaR it seems that a common pattern among countries could be found. Indeed, the evidence of causality seems to exhibit a reverse hump-shaped pattern across quantiles in almost every country: the conclusion of a higher impact on extreme quantiles of IPI distribution, and a null impact on central positions, is confirmed again.
In US, the test statistic exceeds the critical value roughly when q < 0,35 and 0,85 ≤ q ≤0,95 respectively: there is no Granger causality for 0,35 ≤ q ≤0,85 and for very extreme q, even if the test statistic is very close to the threshold for almost the entire distribution. A very similar pattern is showed by UK, Germany, Spain, Japan, Argentina and South Africa. The Eurozone IPI is mostly affected on the left tail, while Italy IPI is mostly affected on the right tail of the distribution.

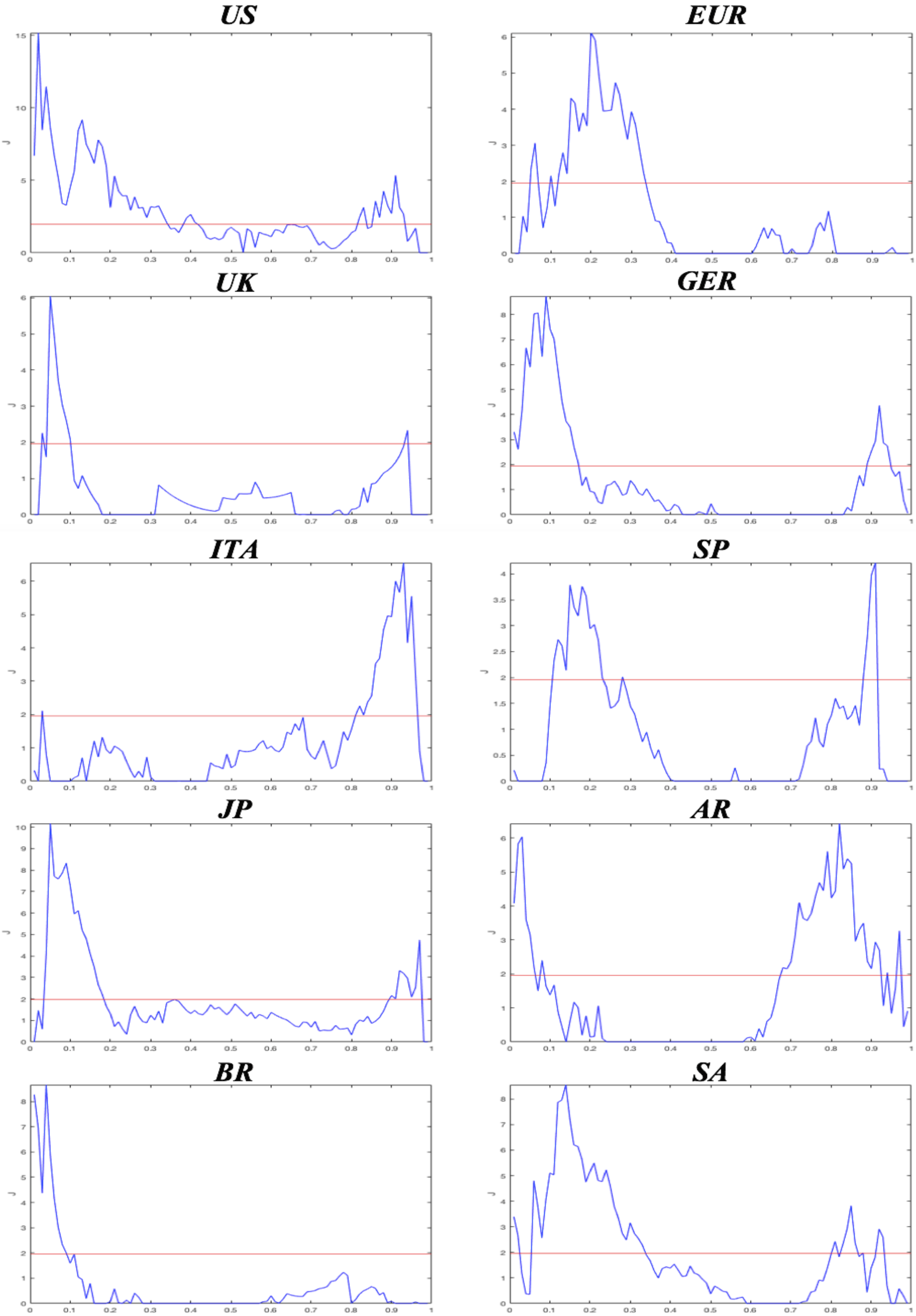


Figure 4.9: Nonparametric Granger-causality test, VaR on IPI, all countries

The next figures present results of testing whether a Granger causality can be detected in different sectors of a stock market: industrial sector and financial sector.

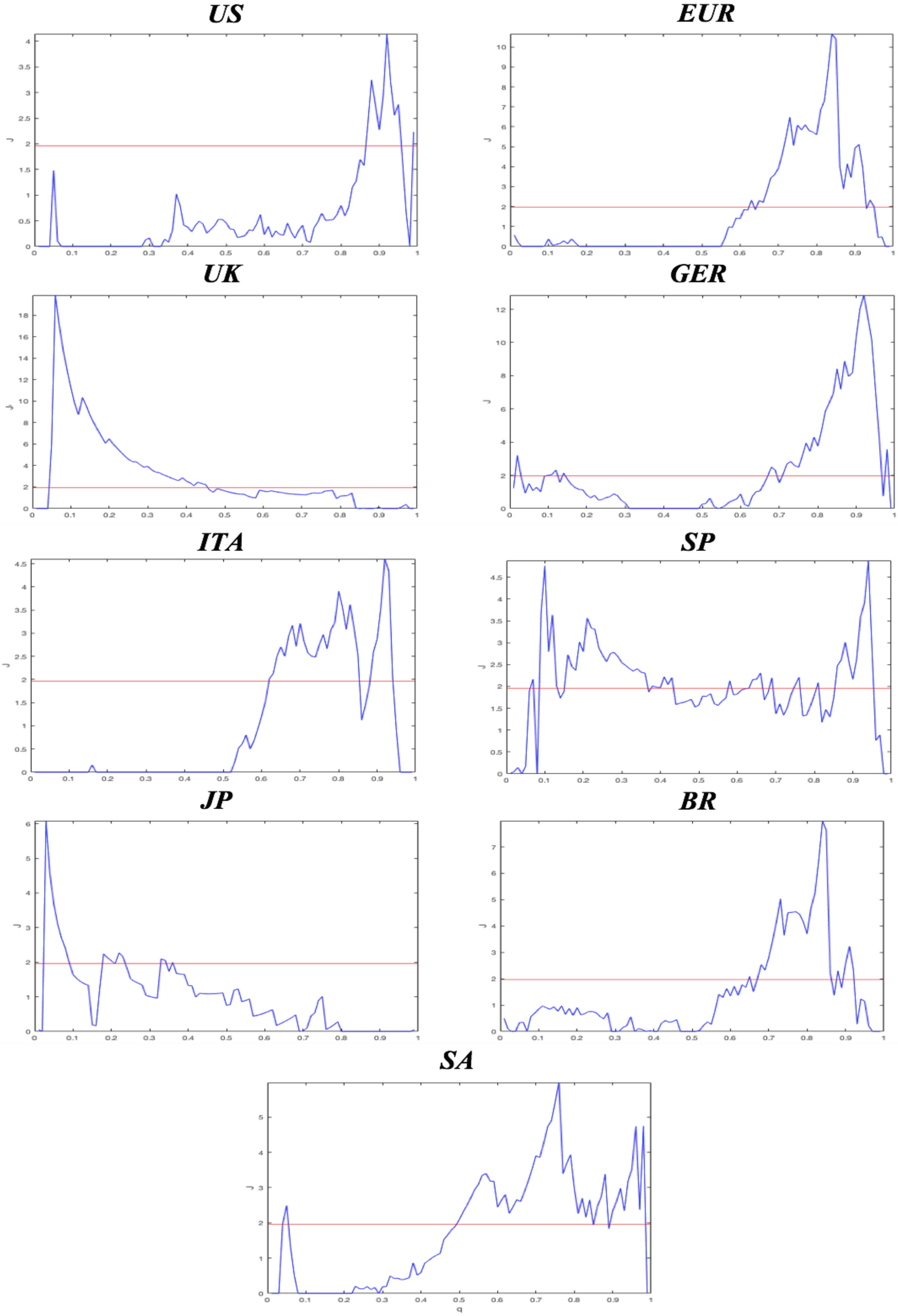


Figure 4.10: Nonparametric Granger-causality test, stock returns of industrial sector

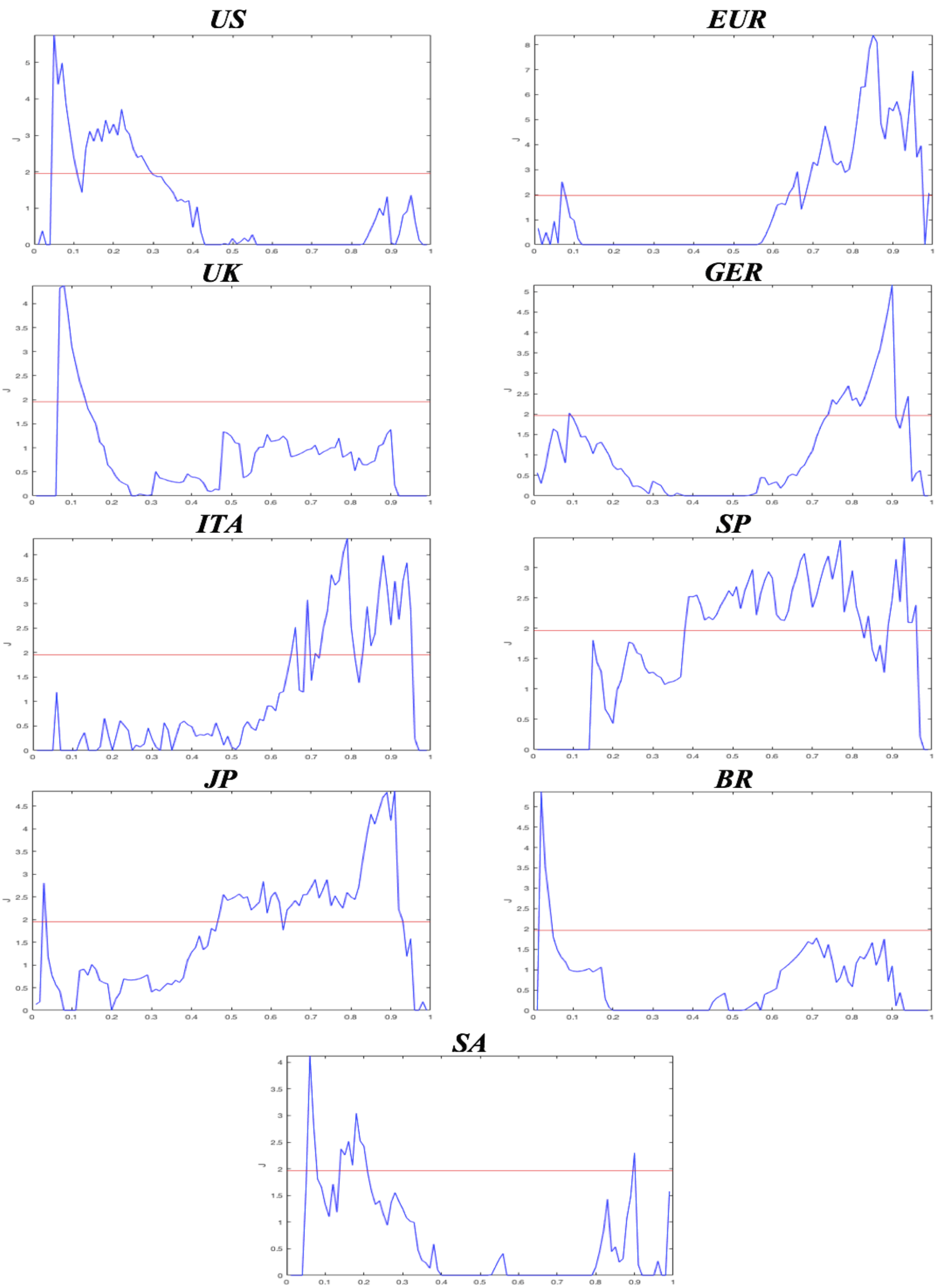


Figure 4.11: Nonparametric Granger-causality test, stock returns of financial sector

Charts for each country are various and different from each other, and it is not easy to identify any common patterns and to draw any conclusions. In general, there are not remarkable differences in Granger-causality detection between the two sectors in the Eurozone, UK, Germany, Italy and Spain. About other countries, the tail of IPI returns impacted by stock

returns changes in accordance with the sector: for instance, industrial stock returns in US Granger cause IPI change in the upper quantiles of the distribution, while the financial stock returns Granger cause the distribution in the lower quantiles.
Results of the next relationship refer to the impact of market risk on the industrial production. Moving variance of daily returns, that allows to identify the most turbulent periods on stock markets, is showed for each country in the Appendix A. As said, volatility increases much above its overall mean during recession phases, and it is slightly lower during expansive phases. A correlation between current stock return volatility and real macroeconomic variables, such as IPI, is not likely to be stable and equally strong over time, but it may still have some predicting power. The results from MIDAS quantile regression, showed below for US and Eurozone, clearly suggest that significant impacts on the IPI returns might occur at the very least quantiles of the distribution.

US ***EUR***

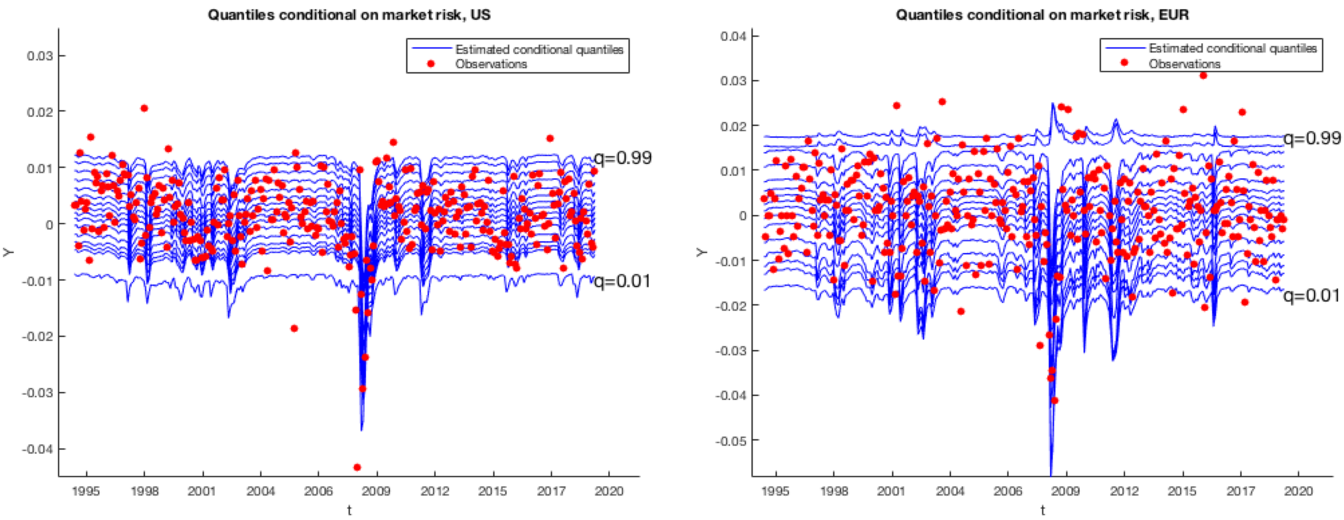


Figure 4.12: IPI changes (%) and quantiles conditional on market risk, US and Eurozone

Every conditional quantile over time seems to be equally spaced from each other, but the first one, that is the conditional quantile at quantile order q=0,01. Moreover, the most curves in correspondence of each quantile order show very similar fluctuations and equal trends: this means that returns variance does not correlate with some particular quantile of the IPI change distribution, and it is not useful to make predictions. Therefore, as we can see from illustrations above, daily volatility in stock market is not relevant to explain changes in the distribution of the IPI returns, except for its very extreme positions only. This is true for US and Eurozone at least: only deep changes in IPI returns could be partially predicted by stock market volatility, perhaps during turbulent periods, financial crisis and under persistent uncertainty.
Anyway, as showed in the next figure, results from nonparametric Granger-causality test are not consistent with the theory, and are very different than as expected. This may be due to some lack or error in the construction of the test for this type of variable, or maybe to a different interpretation.

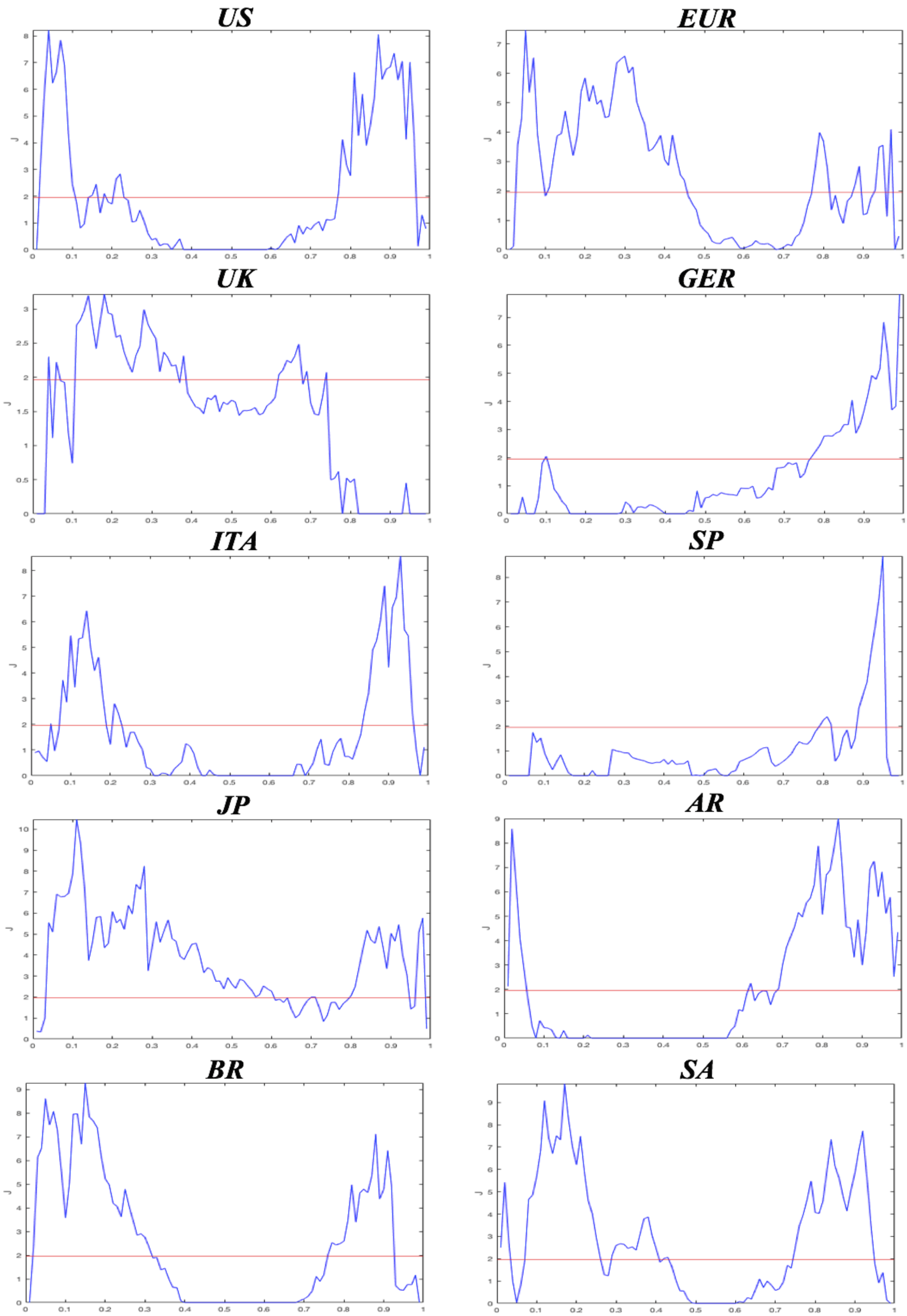


Figure 4.13: Nonparametric Granger-causality test, market risk on IPI, all countries

The next figure present results of the Granger causality-in-quantile test between the IPI and the oil price. For US, IPI change is regressed on returns of WTI price, while for all other countries it is regressed on returns of Brent price. Further explanations about data are given in the Appendix A.

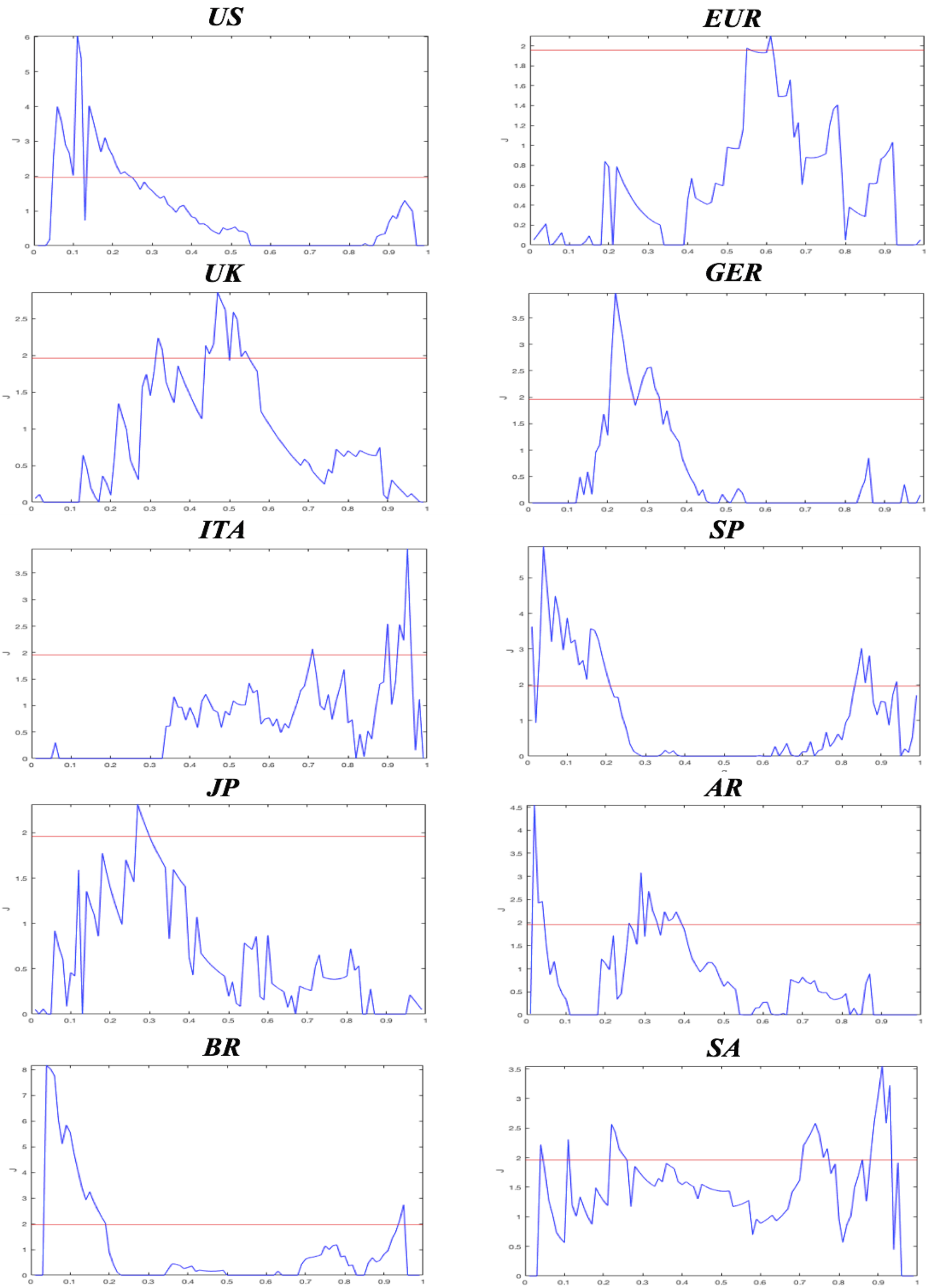


Figure 4.14: Nonparametric Granger-causality test, oil price on IPI, all countries

In this case, interpretation of charts is as interesting as puzzling. In US, a Granger causality is detected around lower quantiles, roughly within the quantile range [0,05; 0,25], while there is no Granger causality for $q < 0,05$ and for $q > 0,25$. Moreover, there is an inexplicable low peak within the significant quantile range, roughly on $q=0,13$, for which no Granger causality exists. On the other side, at least over the last 25 years (the timespan of the dataset) in Europe, variations in the oil price did not anticipate at all changes in IPI distribution, whatever the quantile order: probably, the US economy tends to be more sensible to the performance of the petroleum industry than European economy as a whole. In any case, even in all other countries, the quantile range in which a Granger causality is detected tends to be quite narrow or too small to be somehow significant: in UK, in Germany, in Italy, in Japan, in Argentina, in South Africa. Spain and Brazil show the most relevant significance in extreme quintiles, while in UK oil price tends to affect changes in the central positions of IPI distribution.
Results of the next relationship refer to the impact of currency risk, proxied by variations in exchange rates, on the industrial production. Exchange rate and exchange rate variations are graphically reported in the Appendix A. Even though big exchange rate variations may be cause and consequence of capital outflows and currency crisis, there is no linear and stable relationship between exchange rates and industrial production, that is affected mainly through the trade channel. Results are very different, depending on the country and the currency involved, the type of underlying economy and the quantiles of the distribution.

EUR

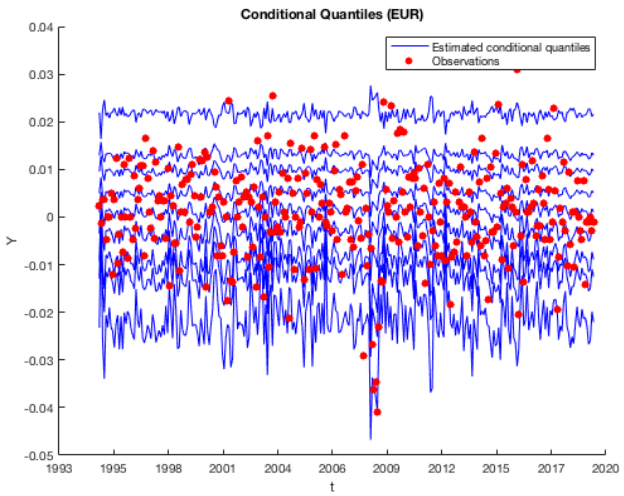


Figure 4.15: IPI changes (%) and quantiles conditional on exchange rate, Eurozone

The results from MIDAS quantile regression on the Eurozone are showed in the graph above. The estimated conditional quantiles of IPI distribution seem to be more sensible to variations of the exchange rate USD/EUR at its lowest quantile order than at higher quantile orders. A fall in the European industrial production may occur after an appreciation of euro, but it is necessary to understand how significant this relationship is.
The next figure present results of the Granger causality-in-quantile test between the IPI returns and the respective exchange rates, USD to national currency. It is evident that in the Eurozone there is no Granger causality-in-quantile, whatever the quantile order. Essentially, currency risk, given by the fluctuations in the value of the euro, is not a source of systemic risk, and this is probably due to the strength of European economy and the credibility of the ECB. In all other analyzed countries, a Granger causality is detected mainly for upper quantile orders: exchange

rate variations Granger cause IPI change roughly for 0,70 ≤ q ≤ 0,90 in UK, for 0,70 ≤ q ≤ 0,99 in Argentina, for 0,60 ≤ q ≤ 0,85 in Brazil, for 0,80 ≤ q ≤ 0,95 in South Africa. In Japan alone, the Granger causality is detected for lower quantile orders. It is interesting the case of Argentina, that is a country known to be financially and monetary instable and periodically hit by some currency crisis: the test rejects the null hypothesis almost for every quantile order, with the exception of very central quantiles of IPI distribution.

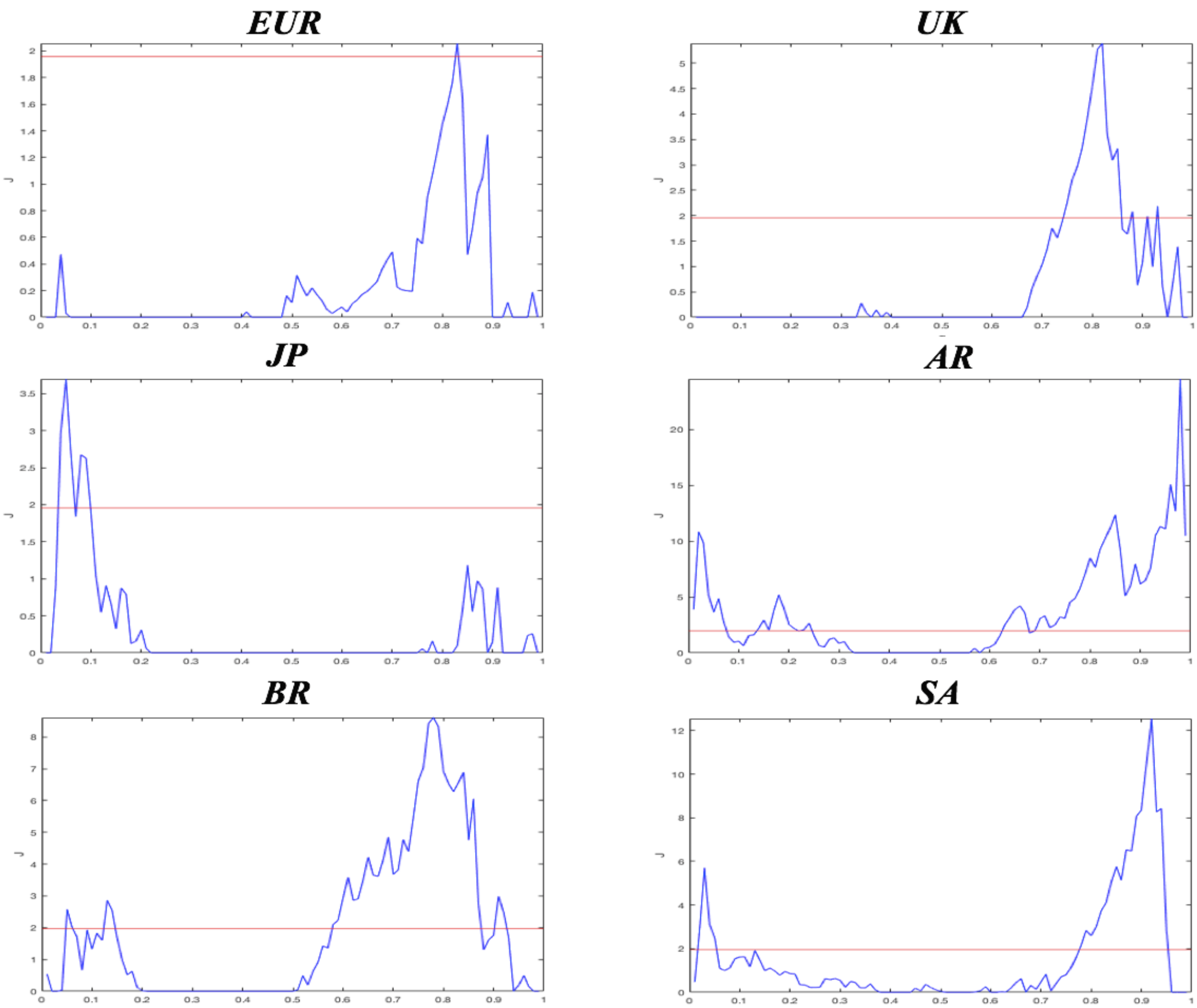


Figure 4.16: Nonparametric Granger-causality test, exchange rate on IPI, all countries

The next figure present results of the Granger causality-in-quantile test between the IPI change and the Credit Default Swap spread change, obtained from CDS contracts on public bonds with maturity of 5 years. The detailed description of this variable is given in the Appendix A. In this case, the variations in CDS premium is the variable of interest since it should reflect variations in the sovereign risk. In fact, changes in sovereign risk premia are somehow reflected in the financial system, and then in the domestic private economic system, and for this reason the CDS spread may signal a higher systemic risk.
Actual test results are as expected, in one way or another. No Granger-causality is detected in US, in UK, in Germany, in Brazil and in South Africa, and this is an evidence that, at least in those countries, the sovereign risk has not contributed in increasing the systemic risk over the last 25 years. On the other side, CDS spread change seems to Granger-cause some quantiles of IPI return distribution in Italy, Spain and Argentina: null hypothesis is rejected for higher quantile orders in Italy and Argentina, and for more central quantile orders in Spain.

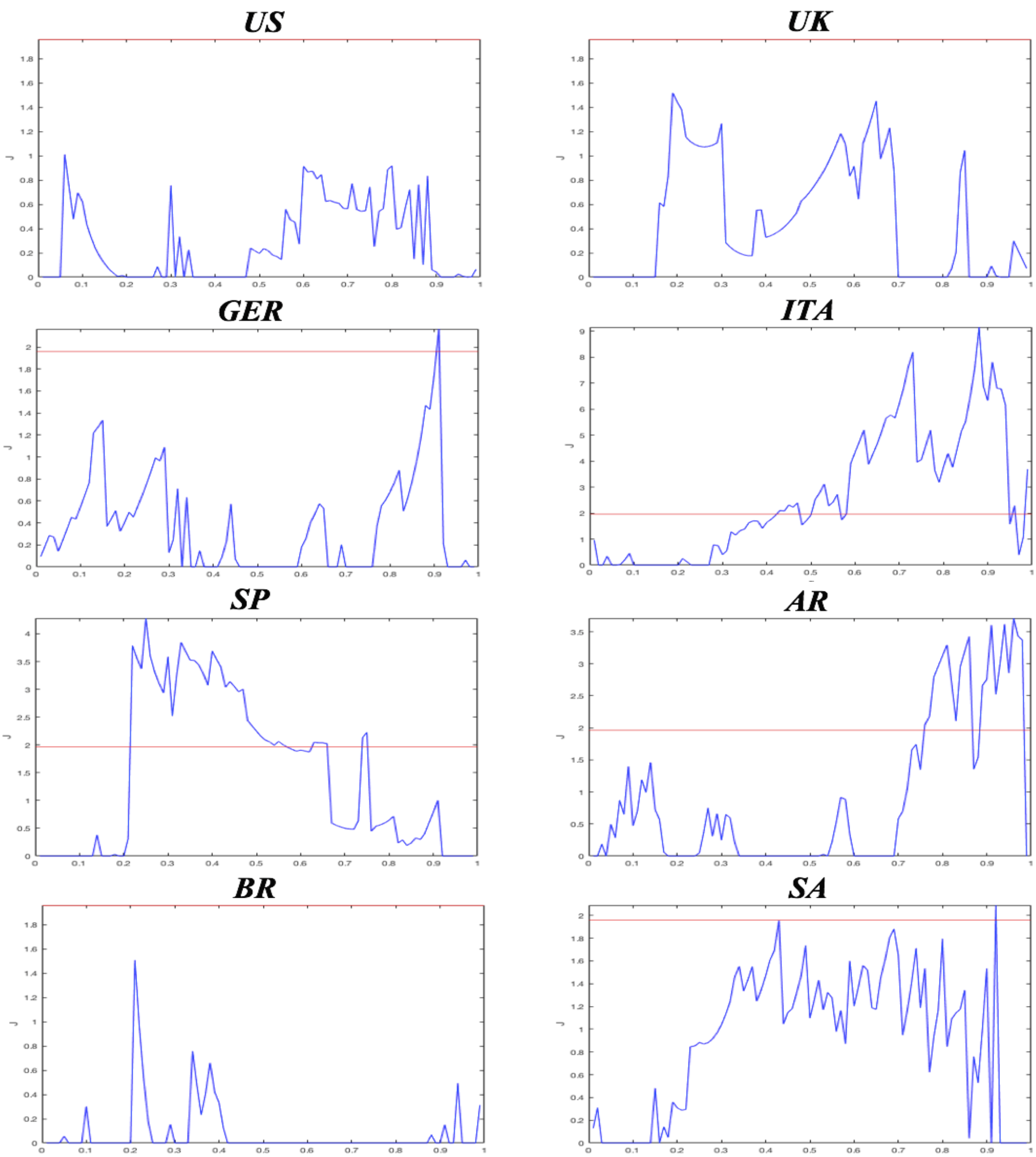


Figure 4.17: Nonparametric Granger-causality test, CDS premium on IPI, all countries

4.2 Conclusion

The ultimate purpose of this thesis was to implement a new test procedure to evaluate the presence of systemic risk within the financial markets. In order to pursue this goal, the essay has been divided into two main macrostructures, as the same thesis title suggests: the first one includes Chapter 1 and Chapter 2, and deals with systemic risk, measurement and early warning indicators in general; the second one includes Chapter 3 and Chapter 4, and deals with the MIDAS quantile regression approach used to implement the nonparametric test.
Chapter 1 provided an overview of the systemic risk in every basic aspect: its definition, its origin and the relevant literature. The leading concepts to keep in mind for the definition of

systemic risk are essentially two: the probability of an unfavorable event, that is the economic shock, and the possible set of consequent negative impacts, that is the propagation dynamic, which in turn puts in trouble the whole financial system. Therefore, the systemic risk is defined as the probability that an entire market collapses as a consequence of the materialization of some specific risk combined with a strongly interconnected system. As widely explained, there are many possible sources of systemic risk, but all of them can be traced back to the 4 Ls: from alterations of losses, leverage or liquidity may stem a shock, while the degree of linkage among institutions expresses the potential of systemic involvement through a propagation dynamic. Moreover, the concepts of externality and contagion, which can derive, for instance, from a default or from the asset price variability, are strictly linked to the concept of propagation dynamic. The interdependence among institutions in the financial chain is evident if we look at the payments system and at the direct loans. The magnitude of a shock in the financial system strongly depends on the reactions of financial institutions, that counteract to the expectations of incoming distress, and with their choices may mitigate or amplify the contagion. The last paragraph of the first Chapter described the main contributions and the main conclusions concerning systemic risk in the literature of the past decades. In fact, a very consistent progress in the literature took place over the last decades because of a deeper understanding of financial dynamics, a wider use of more complicated and complete econometric tools and the necessity to provide explanations on relatively recent global phenomena and events.

Chapter 2 presented a description of the most important tools to measure systemic risk, by providing a categorization of them. Indeed, the monitoring and the correct measurement of systemic risk has always played a central role for the institutions and all financial actors: it is crucial for institutions and authorities to perform unbiased evaluations through the use of trustworthy measures, in order to make optimal decisions for own business and to pursue public mandates, such as financial stability and consumer protection. The systemic risk assessment requires the analysis of three basic aspects: risks arising from the asset side of balance sheets, risks arising from the liability side and risks deriving from interactions between the two sides. Financial risk management aims to handle exposures to a set of risks linked to financial and business operations: default risk, funding risk, liquidity risk, business risk, market risk (that includes the equity price risk, the interest rate risk, the exchange rate risk, the commodity price risk). After having generally described the main characteristics of each risk, a general classification of all measures of systemic risk, as defined by Bisias, Flood, Lo, Valavanis (2012), has been reported. The proposed classification of systemic risk measures is based on four criteria, that reflect four different perspectives in the usage of them: the supervisory perspective, the research perspective, the required datatypes and the reference time horizon. Each category contains sub-sections of measures. For instance, among probability distribution measures, Value-at-Risk, Expected Shortfall, Conditional VaR, Co-Risk and Mahalanobis distance have been presented. Other subcategories are also described, such as illiquidity measures, network measures, default measures, contingent claim-based measures, macroeconomic measures, forward-looking measures and stress tests. All these measures allow the implementation of Early Warning System, whose purpose is not to predict the exact timing of a crisis, but to estimate the actual probability of adverse events to occur within a specific time horizon, and then to quickly signal warnings about probable incoming distress for the institutions. An efficient signal emerges whenever a threshold is crossed in the periods immediately preceding the crisis. Among variables used for the detection of systemic risk, there are the credit-to-GDP ratio gap, the house price gap, the Debt Service Ratio (DSR).

Chapter 3 described the theoretical framework of the main tools needed for the empirical analysis: Mixed Frequency Data Sampling regression model (introduced by Eric Ghysels *et al.* in "The MIDAS Touch: Mixed Data Sampling Regression Models) and quantile regression model (described in "Regression Quantiles" by Koenker and Bassett). The implementation of Early Warning Indicators for systemic risk was done by performing a quantile regression on a MIDAS model, and then by applying the nonparametric test of Granger causality-in-quantiles proposed by Jeong, Hardle and Song (Econometric Theory, 2012).
Quintile regression requires three basic assumptions: the zero conditional quantile assumption, the linearity of the model, and large samples with independent observations. There is no assumption on the distribution since quantile regression models the conditional quantiles of the response variable, and therefore it is robust to outliers, differently from linear regression. The τ-th regression quantile coefficients β are estimated by solving an optimization problem on the conditional quantile function through the least absolute deviations method (which minimizes the sum of absolute errors). Regression quantile coefficients own the following properties: scale equivariance, shift equivariance, equivariance to reparameterization of design, equivariance to monotone transformations and the subgradient optimality condition. The interpretation of coefficients is quite simple: while the mean regression helps to understand how the conditional mean of y is affected by covariates X, quantile regression helps to identify an impact of covariates on y at each quantile of its conditional distribution. In general, the quantile regression is a valid option whenever the conditional mean fails to fully and reliably capture the data pattern: in case of skewed data and asymmetric distribution, in case of multimodal data and data with outliers, in case of heteroskedasticity.
In the MIDAS models, the dependent variable, sampled at lower frequencies, is regressed on distributed lags of the covariates, sampled at higher frequencies. Hence, MIDAS models are parameterized reduced form regression models for time series that involve data sampled at different frequencies, where explanatory variables have higher frequency. The parametrization of lag coefficients requires the usage of a parameter vector function (Almon lag function, Almon exponential lag function, Beta polynomial function) and a suitable information criterion (Akaike, Schwarz or Hannan-Quinn). The smart and optimal parameterization of the lagged coefficients is one of the main key MIDAS features. Many types of parametrizations have been proposed in accordance with the number of coefficients and the function shaping. In general, the MIDAS regression requires a nonlinear least squares estimation. The real novelty item proposed by this thesis is the application of the quantile regression to a set of variables with different sampling frequencies: therefore, this approach should combine the two models described above, giving rise to the MIDAS quantile regression.
After having estimated the conditional quantiles of the response variable, the nonparametric test of Granger causality in quantile was applied in order to identify Granger causality in various conditional quantiles of the dependent variable distribution. The tested hypotheses consisted of a quintile restriction: accepting the null hypotheses means that the set of lagged independent variables does not Granger-cause the dependent variable in the specific τ-th quantile at each given moment. Granger causality in quantiles is designed to deal with non-Gaussian distributions, with asymmetry, non-linearity and fat tails: in these cases, information content provided by the quantiles about distributions is more precise than the information provided only by the mean.
Chapter 4 showed the empirical analysis, with a detailed presentation of the main results. First of all, the nonparametric test was created in MATLAB on the basis of the test procedure

introduced by Jeong, Hardle and Song (Econometric Theory, 2012): under the null hypothesis, the test statistic tends in distribution to $N(0, \sigma_0^2)$, and then it is zero if and only if the null hypothesis is true, that is if there is no Granger causality in quantiles between the two variables given a quantile order τ. In this kernel-based test, the causality detection required the estimation of $\hat{Q}_\tau(w_t)$, that is the set of quantiles of the dependent variables, estimated through the MIDAS quantile regression, conditional on the high frequency lagged explanatory variables. Parameters are estimated as solutions to an optimization problem, for which the sample τ-th quantiles are identified as those points ξ of the domain of the function $\sum_i \rho_\tau(y_i - \xi)$ at which its values are minimized. In MATLAB, the functionalities provided by "*fmincon*" together with a *function_handle* was used. Explanatory variables were weighted according to the MIDAS approach: the parametrization of lag coefficients made use of a Beta polynomial function, with parameters k1 and k2 and a specific weight attached to each lag.

In the empirical analysis, the selection of suitable variables useful to anticipate increasing risks was based on the assumption that the most financial operators carry out transactions trying to anticipate outcomes from real economy and to raise more or less rational expectations about changes in financial variables. The analysis will be performed evaluating a set of relationships for more countries between a low frequency variable, reflecting the market expectations, and a high frequency variable, that reflects the macroeconomic state such as the industrial production. Examined sources of systemic risk are stock market risk (stock price variations, stock return volatility, daily Value-at-Risk, sectorial risk), geopolitical risk (oil price variations), currency risk (exchange rate variations) and sovereign risk (CDS premium variations). Main conclusions of the empirical analysis are listed below.

Extreme reductions in industrial production are quite significantly Granger caused by changes in stock returns, while the noncausality part is longer on the central positions of the distribution. The relationship on the tails, both the left and the right one, is relevant: extreme changes in industrial production are significantly anticipated by stock returns, while the relation with central tendencies is unclear and more ambiguous, and it requires further deepening.

Granger causality between VaR and industrial production seems to exhibit a reverse hump-shaped pattern across quantiles, with a higher impact on extreme quantiles of IPI distribution, and a null impact on central positions.

Granger causality detection within both financial sector and industrial sector of the stock market is difficult to analyze, given the heterogeneity of results.

Results from Granger causality test on the impact of the stock return variance are very different than as expected, and no conclusion can be drawn.

US economy tends to be more sensible to the performance of the petroleum industry than European economy as a whole. In any case, even in all other countries, the quantile range in which a Granger causality is detected tends to be quite narrow or too small to be somehow relevant.

No Granger causality due to exchange rate is detected in the Eurozone, whatever the quantile order. In all other analyzed countries, a Granger causality is detected mainly for upper quantile orders, especially for UK data, for Argentina data and for Brazil data.

Sovereign risk has not been a source of systemic risk over the last 25 years at least in US, in UK, in Germany, in Brazil and in South Africa. On the other side, CDS spread change seems to Granger cause some external quantiles of IPI return distribution for Italy, Spain and Argentina.

The main finding is that some relationships present nonlinear characteristics which invalidate any linear specification or any test based on linearity assumption. The nonparametric causality-in-quantiles test allows to overcome this problem by highlighting the causal effects on specific parts of the conditional distribution of the variable of interest. The strength of Granger causality may differ across the upper and lower quantiles of the conditional distribution, and for this reason, it should be considered the entire conditional distribution of the variations of a response variable, not only the center of the distribution. The same systemic risk should be detected by testing the significance on more external quantiles, instead of analyzing the mean of the conditional distribution.
However, information provided by the EWI should always be considered within a probabilistic framework, since the future can never be foreseen with full precision. The proposed test is just a starting point, to be improved and to be integrated. In particular, further studies may focus on two relevant aspects of the EWI, remained unsolved.
The first one refers to its reliability. The reliability of a test is its ability to issue signals with timeliness and relatively limited out-of-sample forecast errors. It can be tested by comparing its forecasting performance against other models through an out-of-sample validation, that is by using the known sample data to get the model parameters, and then by using the same model to make predictions about unknown data, independently from the sample. Besides, in the implementation of the test, other more reliable kernel functions, such as the Epanechnikov function, and other bandwidths, like that computed with the leave-one-out least squares cross validation (proposed by Racine and Li, 2004), may improve the results.
The second aspect to be deepened refers to the strength and reliability of the MIDAS quantile regression analysis. In particular, econometric tools should be employed in order to investigate a regularity and a robustness of the relationships used in the empirical analysis. The direction of beta coefficients should be stable and robust, even across quantiles. Furthermore, the results should be more accurate by including the lagged dependent variables among the covariates in the MIDAS quantile model specification, instead of considering the equity-related part only.
The causality test with conditional quantiles based on the MIDAS quantile model without the lagged dependent variables provides results strictly linked to the information obtained by the higher frequency data only. In this case, the rejection of the null hypothesis may be due to the lack of the lagged dependent variable to estimate the conditional quantile. The solution would be to estimate the conditional quantiles applying the MIDAS quantile regression on a sort of autoregressive distributed lag model with mixed frequency data (e.g. an ADL (1, 22), with one-period-lagged value for IPI and the usual 22 lagged daily stock returns) in this case, the regression equation used to predict current values of the dependent variable is based on both the past values of low frequency explanatory variables and the lagged value of the response variable. Moreover, a further test can be applied in order to detect the relevance of each component of the ADL model on the final results: by comparing the results of the nonparametric test obtained from conditional quantiles estimated through the lagged dependent variable only with the results obtained from the complete ADL model, one can assess whether and how much both information sources, lagged dependent variable and independent variables, are relevant to predict the response variable.

Appendix A

Data Selection and Descriptive Analysis

In this appendix, figures and tables for the descriptive analysis of the dataset will be showed. All data have been taken on the platform Eikon Thomson Reuters, before being elaborated with MATLAB. For the most variables, return rates (percentage changes), cumulative returns and related descriptive statistics are showed. Return rates of each variable are computed as simple percentage changes, excluding other financial sources of income from the underlying assets:

$$r_t = \left(\frac{P_t}{P_{t-1}} - 1\right) * 100$$

Here the list and the description of variables used to build the Early Warning Indicators. Every time series has a time span of about 25 years, from 1995 until the end of 2019, except for the CDS premia, which start in 2008.

- The Industrial Production Index (IPI) is a monthly economic indicator that measures the real output of secondary sectors of a country, relative to a base year. These indices are computed by national statistical offices, and for this reason they are not homogeneous and standardized measures, but vary across countries. Typically, they are Fisher indices with the weights based on periodic estimates of value added, and the value in the base year (e.g. 2012) arbitrarily set at 100. Their periodic variations express changes in the production volumes. Chosen IPIs are referred to the same countries of each stock market index but, as showed by figures, just ten of them are "comparable" (i.e. covering the whole time-span, purified by seasonality, with a base value of 100 and with similar composition of the examined industrial output), and then exploitable for the analysis: IPI USA, IPI EMU (European Monetary Union), IPI UK, IPI Germany, IPI Italy, IPI Spain, IPI Japan, IPI Argentine, IPI Brazil, IPI South Africa. For the analysis, the growth of industrial production is considered. The broadest index type is used, including manufacturing, energy and construction sectors.
- The real Gross Domestic Product at constant market prices, expressed in $, may be useful to make up for the lack of IPI, and especially to fill the countries' missing data. Anyway, it is a good alternative to IPI, since it is the most wide and important macroeconomic variable used to measure the market value of all the ultimate goods and services produced in a country over a given period. Easy to understand the reason why GDP and IPI do show a strong correlation, especially in industrialized and widely monetarized countries. Moreover, GDP is published quarterly by national statistical institutes, and therefore operators in the financial markets, like the stock market, have a larger timespan to discount all relevant information about the trends and the macroeconomic state. Missing countries in the dataset are Australia, Argentina, Brazil, Russia, India and China. The most valuable information for the analysis is given by the countries' economic growth, computed as the percentage change of real GDP in each quarter.
- The stock market indices, and in particular the time series of stock prices and stock returns, have been selected from the same provider, MSCI. The selected stock market

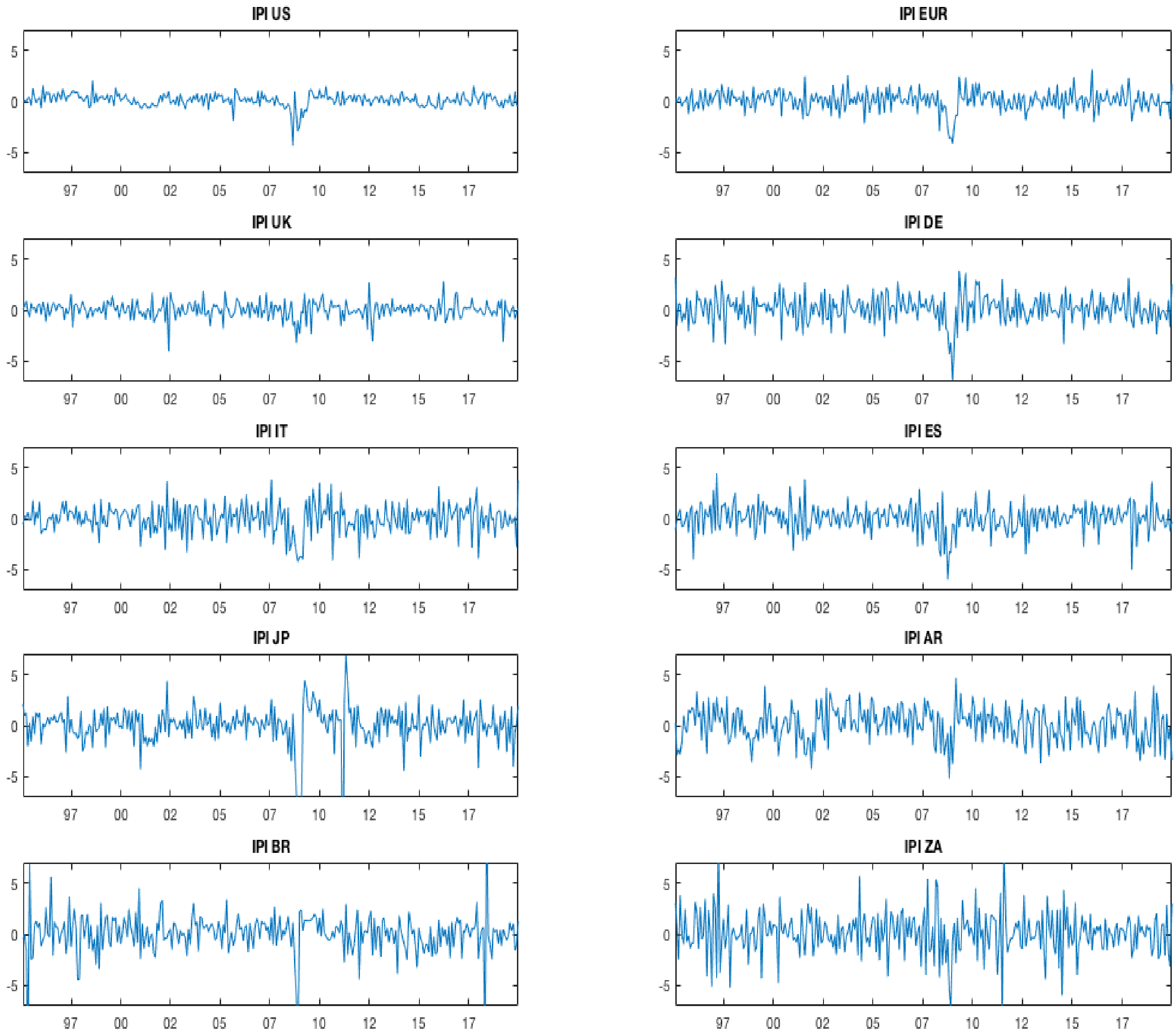
indices are: MSCI USA for United States, MSCI EMU for European Economic and Monetary Union, MSCI UK, MSCI Germany, MSCI Italy, MSCI Spain, MSCI Greece, MSCI Japan, MSCI Australia, MSCI Argentina, MSCI Brazil, MSCI Russia, MSCI India, MSCI China, MSCI South Africa (the first indices cover advanced economies, while the last 5 indices cover BRICS countries). Each MSCI Index measures the performance of the large and mid cap segments of the respective stock market, covering approximately 85% of the respective free float-adjusted market capitalization. In the implementation of the test, all indices have been used but those from countries without an acceptable production index.

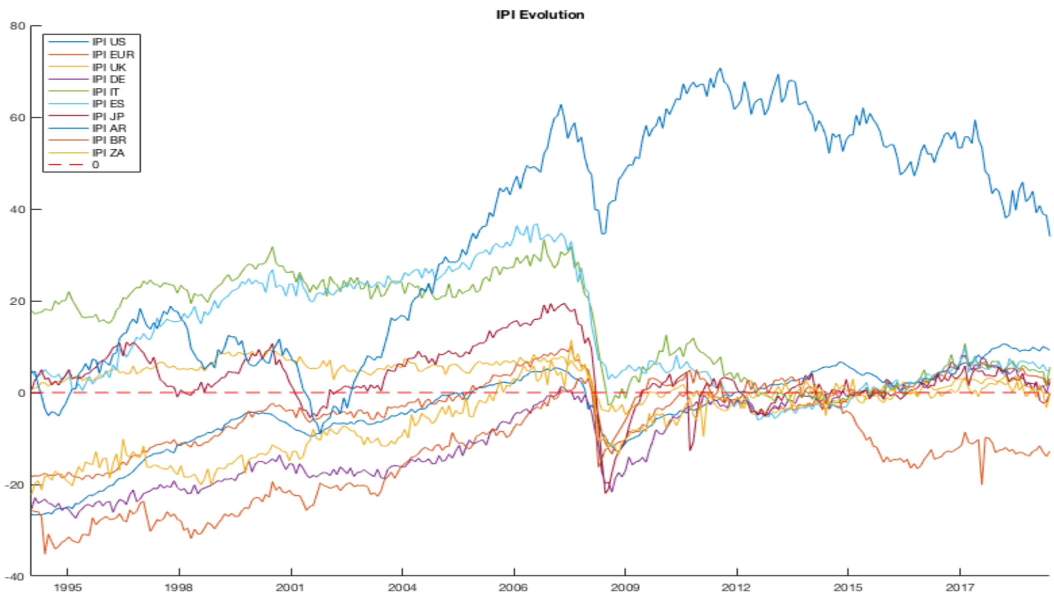
- The sectorial analysis is performed through the decomposed parts of the FTSE price indices of each country: financial sector and industrial sector. They are market-capitalization weighted index representing the performance of large and mid cap stocks in financial sector and industrial sector, respectively. For example, industrials and financials have a relevant position within the FTSE USA Index: 98 and 130 constituencies each, out of a total of 609, and 10% and 14% respectively as weights on the total index. Dataset is not complete for all countries: Greece, Argentina and Russia have missing observations, and for this reason they have been excluded.
- The oil prices used in the test are the West Texas Intermediate spot crude oil price (or WTI) and the Brent crude oil price. These prices are global benchmark for oil pricing, and mainly differ in place of extraction and production (Texas and other U.S. countries for the first one, the North Sea of Northwest Europe for the second one), lightness and sweetness (WTI is sweeter and lighter), main trading market (the New York Mercantile Exchange, or NYMEX, for the first one, and the Intercontinental Exchange, in London, for the second one) and global relevance (WTI price is the benchmark for US markets, while Brent price is the international benchmark used by OPEC, and for this reason it seems to be more sensible to geopolitical issues). Although the transport costs of Brent oil are typically lower, its price is typically higher than the WTI price, but the difference is very small.
- The exchange rates convey very useful information about currency risk, and then about a component of systemic risk which has proven in the past to be very relevant and disruptive for many economies. Indeed, even if many countries try to keep their currency value pegged to another stronger currency, it is not uncommon that some global shocks or some nefarious public policies may cause detrimental capital outflows, speculative attacks and currency crisis, which bring to an economic downturn. These shocks can be clearly seen on the figures representing variations of exchange rates, where outliers and strong changes highlight some currency devaluation or public intervention. Dataset on exchange rates provides the time series for national currency prices expressed in terms of dollars. Being the dollar the reference currency, data on U.S. currency value have been excluded. Moreover, since Italy, Germany, Greece and Spain share the same currency, they are not listed, and just the conversion euros-to-dollar is figured. Australia is the last missing country.
- The Credit Default Swap premium (or spread) is the annual payment made by the CDS buyer to the CDS seller in order to have protection against the default risk of a reference entity over the length of the contract. The premium is expressed in basis points (that is the percentage of the debt’s face value), and it is linked to the default risk on the public

debt issued by each national government. The spread, based on contracts issued in $, is a good measure of the sovereign risk perceived by operators. For example, a CDS spread of 80 basis points (0,8%) on the Italian Treasury bonds means that, in order to get protection against default risk of a public bond with $10 million par value, the CDS buyer must pay $80.000 a year. The CDS maturity refers to the initial length of a contract upon its beginning, and in general the higher the maturity, the higher the riskiness undertaken by the CDS seller, and the higher the premium paid by the CDS buyer. The CDS premium used for the analysis refers to contracts with a maturity of 5 years. Missing countries are Greece, Japan, Australia and India. Variations in CDS premium are the variable of interest, since they should reflect variations in the sovereign risk.

A.1: Industrial Production Indices

A.2: Industrial Production Index - Change





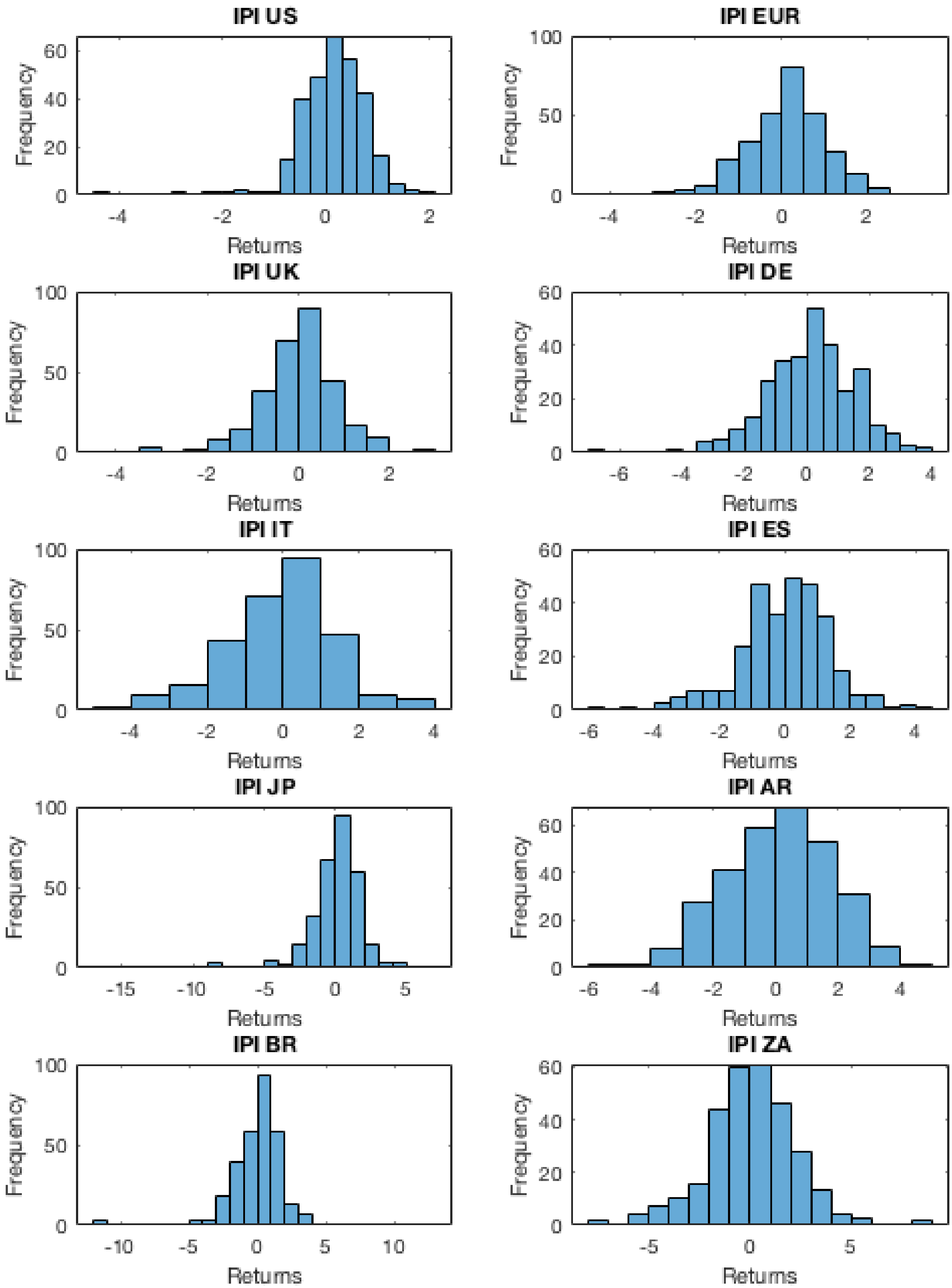
A.3: Descriptive Statistic of IPI Change

	Mean	Median	IQ	StDev	Min	Max	Skew	Kurt
IPI US	0.13446	0.16168	0.77813	0.6478	-4.3368	2.0525	-1.6107	11.913
IPI EUR	0.082602	0.10395	1.2038	1.0026	-4.1053	3.1031	-0.51233	4.7616
IPI UK	0.00070181	0	0.87237	0.85867	-4.0149	2.8254	-0.63543	5.9046
IPI DE	0.11523	0.20608	1.8626	1.4171	-6.8771	3.8314	-0.47245	4.6035
IPI IT	-0.026192	0.084152	1.6743	1.453	-4.2105	3.8132	-0.32381	3.506
IPI ES	0.0089871	0.14929	1.6856	1.3997	-5.9258	4.454	-0.53523	4.6369
IPI JP	0.017676	0.19831	1.8698	1.9808	-16.459	6.8386	-2.7058	21.486
IPI AR	0.098447	0.14505	2.5129	1.731	-5.1587	4.6684	-0.15655	2.6581
IPI BR	0.074662	0.30918	1.9588	2.0656	-11.335	12.891	-0.80732	15.498
IPI ZA	0.10652	0.099068	2.6639	2.2004	-7.7551	8.6283	-0.056071	4.6392

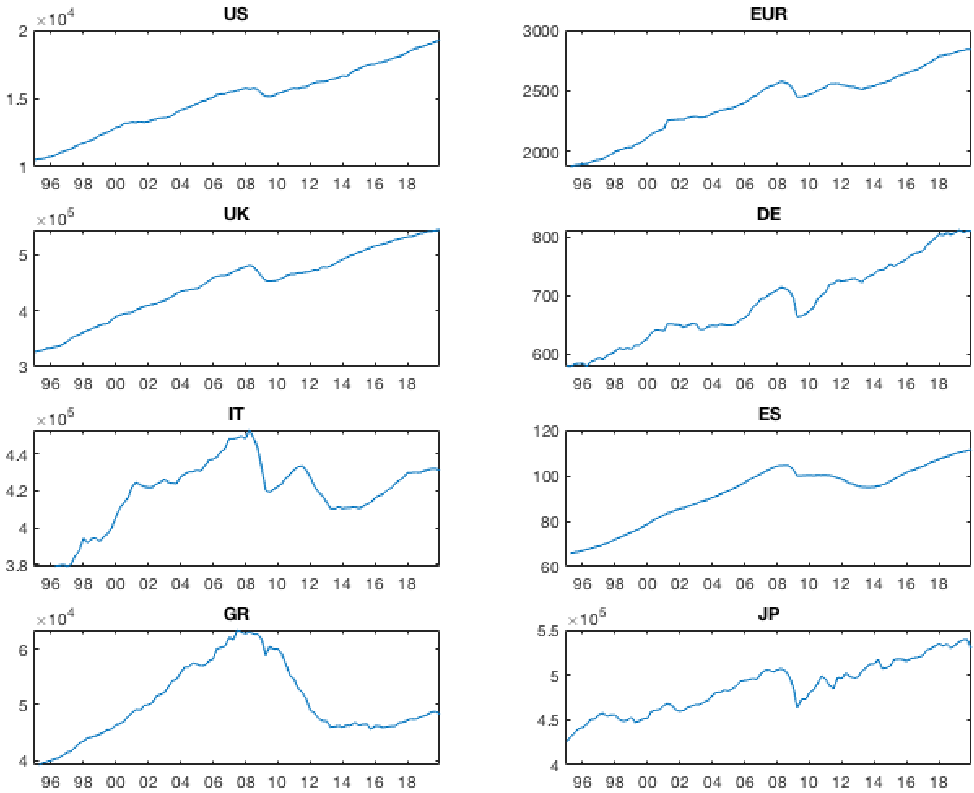
A.4: Relevant Quantiles of IPI Change

	Q1	Q5	Q50	Q95	Q99
IPI US	-2.1209	-0.69835	0.16168	1.0007	1.4881
IPI EUR	-3.1809	-1.4591	0.10395	1.6561	2.4294
IPI UK	-3.0658	-1.2454	0	1.3251	1.8631
IPI DE	-3.3031	-2.1904	0.20608	2.3551	3.1794
IPI IT	-3.9243	-2.7467	0.084152	2.0622	3.5827
IPI ES	-3.8476	-2.6621	0.14929	2.0755	3.3608
IPI JP	-8.3967	-2.5491	0.19831	2.2867	4.2748
IPI AR	-3.6901	-2.874	0.14505	2.8086	3.7804
IPI BR	-8.5003	-2.4773	0.30918	2.4717	5.0423
IPI ZA	-5.7483	-3.9546	0.099068	3.446	5.5369

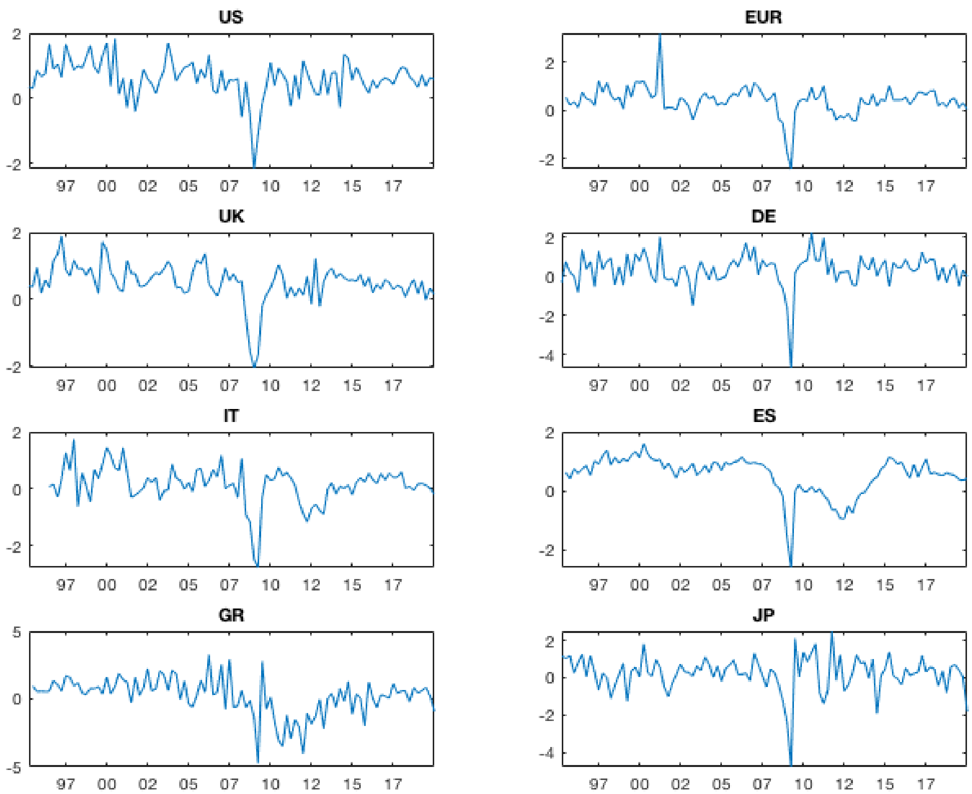
A.5: Frequencies of IPI Returns for each Country



A.6: Real GDP at Constant Market Price ($, Quarterly Data)



A.7: GDP Growth Rates (%)

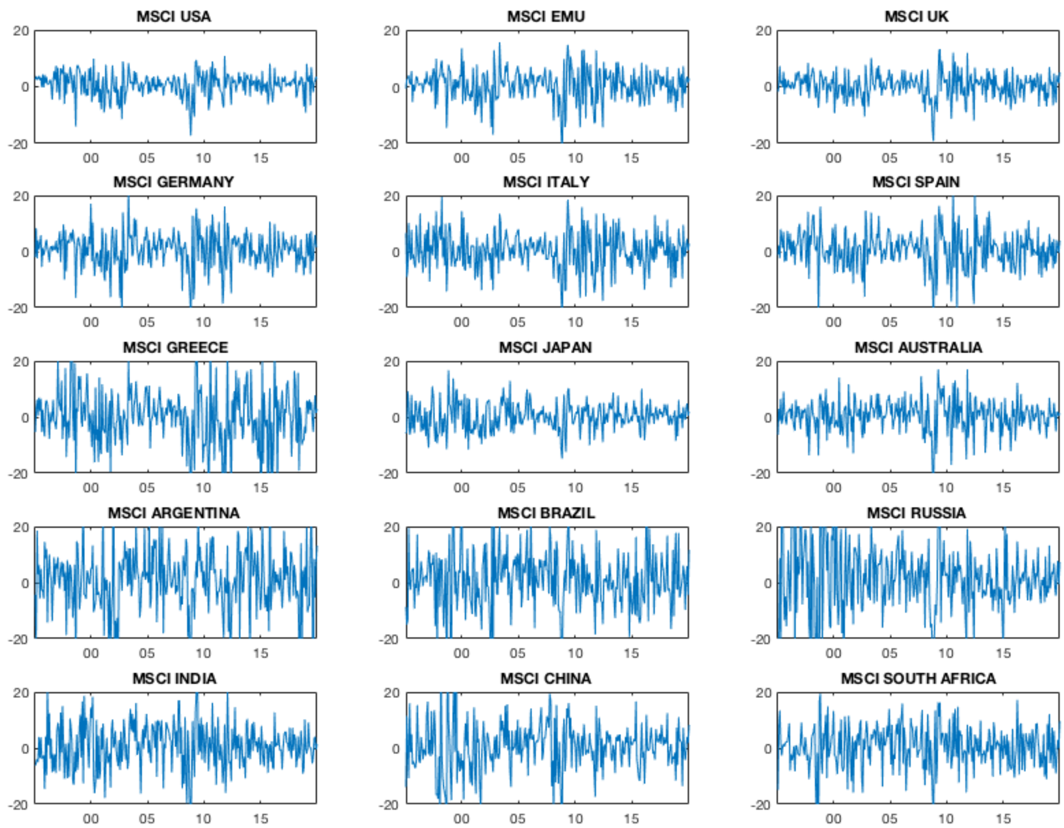


A.8: Descriptive Statistic of GDP Growth Rates

	Mean	Median	IQ	StDev	Min	Max
US	0.60924	0.63038	0.56174	0.58152	−2.1638	1.8312
EUR	0.42139	0.453	0.52211	0.60334	−2.4116	3.211
UK	0.51212	0.54609	0.49389	0.57375	−2.0605	1.9029
DE	0.33698	0.42637	0.88098	0.83433	−4.6745	2.2297
IT	0.13715	0.18536	0.61297	0.67863	−2.7756	1.7412
ES	0.53428	0.68438	0.6002	0.6498	−2.6056	1.5997
GR	0.22164	0.44226	1.4192	1.4103	−4.7477	3.2583
JP	0.22243	0.27148	0.8632	0.96726	−4.7865	2.4925
AU	0.76554	0.75195	0.57331	0.53691	−0.48451	2.9551
AR	0.94082	−1.1761	10.046	8.4742	−11.513	19.968
BR	2.5359	3.3537	5.9724	4.0975	−8.4404	8.3212
RU	0.69265	0.62271	1.6764	1.2627	−3.5273	3.1942
IN	1.7012	2.0753	7.1995	3.8698	−6.7741	8.922
CN	3.7152	8.7753	14.983	11.959	−22.782	20.651
ZA	0.62628	0.66029	0.81566	0.62402	−1.5555	1.873

A.9: Stock Market Indices

A.10: Monthly Returns (%) of each Index



A.11: Descriptive Statistic of Daily Returns (%)

	Mean	Median	IQ	StDev	Min	Max	Skew	Kurt
MSCI USA	0.036559	0.035811	0.97937	1.1337	-9.0752	11.675	-0.10776	11.936
MSCI EMU	0.025154	0.045672	1.3162	1.3673	-9.916	11.481	-0.0031432	9.5942
MSCI UK	0.017988	0.037144	1.2006	1.2455	-10.834	12.931	-0.024485	13.113
MSCI GERMANY	0.02779	0.046435	1.4784	1.4853	-9.1875	12.287	0.029348	8.3879
MSCI ITALY	0.017108	0.024084	1.6001	1.5798	-14.524	13.281	-0.0098787	9.3112
MSCI SPAIN	0.030841	0.030976	1.5445	1.5656	-14.823	17.357	0.11784	11.337
MSCI GREECE	-0.007129	0.0086933	2.0595	2.2547	-22.167	18.736	-0.087855	10.921
MSCI JAPAN	0.0098047	0	1.4438	1.3717	-9.0747	13.057	0.15495	7.9807
MSCI AUSTRALIA	0.026154	0.039884	1.4091	1.3647	-14.764	9.2076	-0.47835	11.537
MSCI ARGENTINA	0.033756	0	2.1447	2.3653	-40.029	17.752	-1.0383	24.877
MSCI BRAZIL	0.043171	0.055209	2.2341	2.2458	-16.742	18.928	0.20207	10.753
MSCI RUSSIA	0.069935	0.042237	2.259	2.7513	-26.665	27.405	0.16074	15.503
MSCI INDIA	0.034389	0.022396	1.5726	1.6024	-11.345	21.515	0.16508	11.887
MSCI CHINA	0.019555	0.0031072	1.6436	1.8281	-13.448	15.078	0.24143	9.6594
MSCI SOUTH AFRICA	0.027032	0.058444	1.8187	1.7271	-12.686	13.148	-0.19666	7.4625

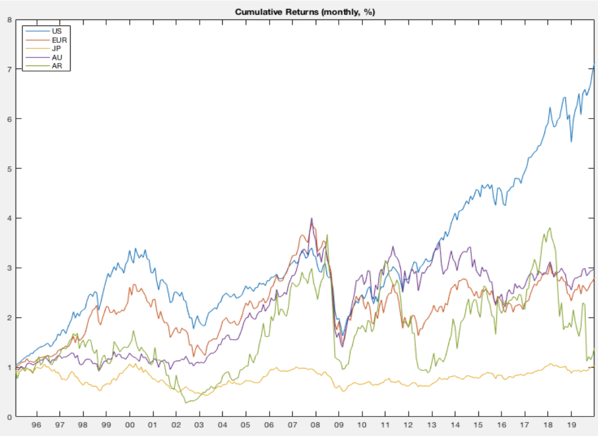
A.12: Descriptive Statistic of Monthly Returns (%)

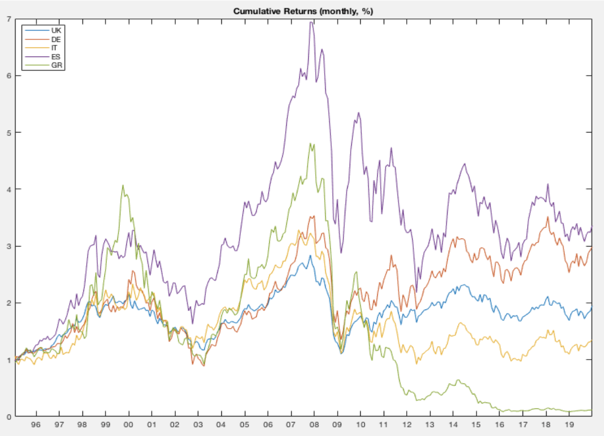
	Mean	Median	IQ	StDev	Min	Max	Skew	Kurt
MSCI USA	0.74741	1.1617	5.2106	4.225	-17.247	10.833	-0.69432	4.1899
MSCI EMU	0.51142	0.83734	6.72	5.7271	-23.984	15.66	-0.5724	4.3011
MSCI UK	0.32244	0.51521	5.302	4.4493	-19.127	13.241	-0.39926	4.3541
MSCI GERMANY	0.57256	0.88968	7.3911	6.3767	-24.351	22.389	-0.5009	4.6011
MSCI ITALY	0.33247	0.29122	8.018	6.7847	-23.624	19.556	-0.1787	3.321
MSCI SPAIN	0.63802	1.0446	7.5517	6.7815	-25.517	21.421	-0.31483	4.2331
MSCI GREECE	-0.14677	0.18437	11.168	10.374	-36.732	29.686	-0.3123	4.1408
MSCI JAPAN	0.13181	0.35332	6.4793	4.9651	-14.788	16.688	0.026272	3.2012
MSCI AUSTRALIA	0.53606	0.72396	6.2203	5.7826	-25.563	16.999	-0.50599	4.7333
MSCI ARGENTINA	0.84749	1.0426	12.109	11.898	-50.532	52.917	-0.14609	5.7449
MSCI BRAZIL	0.95613	1.012	12.187	10.504	-39.003	36.504	-0.20137	4.1755
MSCI RUSSIA	1.6878	1.656	13.787	13.96	-60.572	61.133	0.28787	6.2661
MSCI INDIA	0.80004	0.65503	10.283	8.1569	-28.556	36.633	0.12105	4.1017
MSCI CHINA	0.48515	0.79782	9.0196	9.2852	-27.672	46.497	0.55296	6.5636
MSCI SOUTH AFRICA	0.55361	0.78778	9.3574	7.5297	-30.844	19.396	-0.46957	4.0148

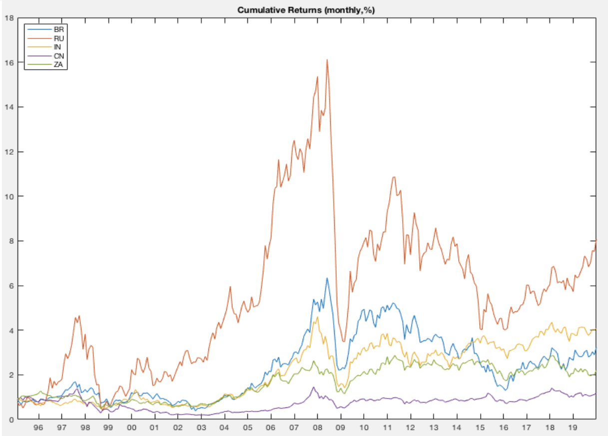
A.13: Relevant Quantiles of Daily Returns (%)

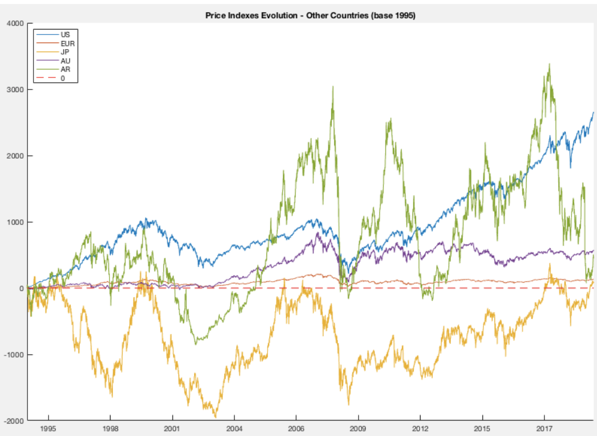
	Q1	Q5	Q50	Q95	Q99
MSCI USA	-3.1389	-1.771	0.035811	1.6503	3.3089
MSCI EMU	-3.8986	-2.1273	0.045672	2.0186	3.6012
MSCI UK	-3.3982	-1.9048	0.037144	1.7907	3.2956
MSCI GERMANY	-4.3392	-2.3334	0.046435	2.2677	4.049
MSCI ITALY	-4.5152	-2.4372	0.024084	2.3929	4.1371
MSCI SPAIN	-4.4122	-2.3688	0.030976	2.3009	4.2821
MSCI GREECE	-6.2878	-3.5408	0.0086933	3.3525	6.3459
MSCI JAPAN	-3.6166	-2.1613	0	2.1466	3.6395
MSCI AUSTRALIA	-3.672	-2.0314	0.039884	2.0065	3.7306
MSCI ARGENTINA	-6.1189	-3.4547	0	3.3926	6.2592
MSCI BRAZIL	-6.0737	-3.4294	0.055209	3.2195	6.1326
MSCI RUSSIA	-8.095	-3.8827	0.042237	3.9611	7.9731
MSCI INDIA	-4.5369	-2.4368	0.022396	2.408	4.4483
MSCI CHINA	-5.0029	-2.8241	0.0031072	2.7758	5.4364
MSCI SOUTH AFRICA	-4.5968	-2.7888	0.058444	2.657	4.4251

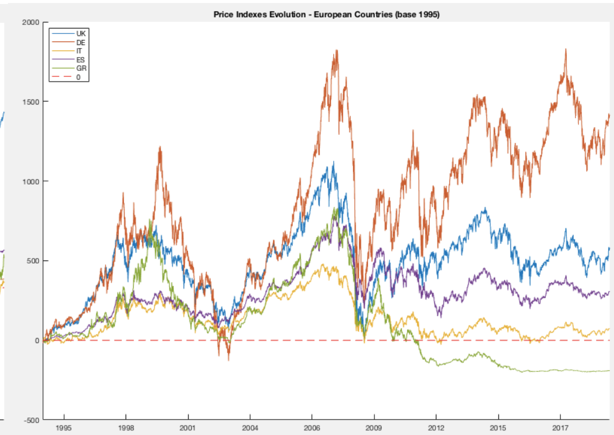
A.14: Cumulative Stock Returns (%) and Price Index Evolution

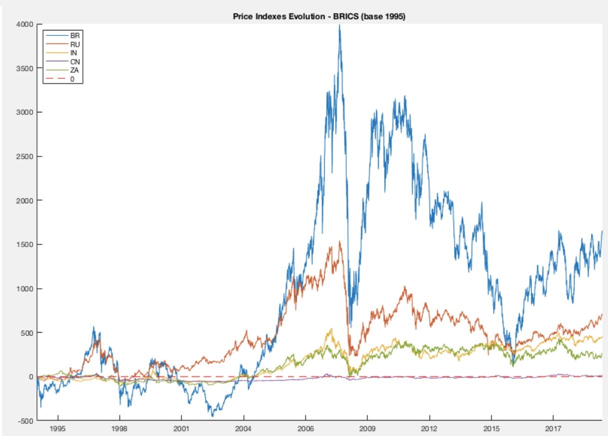




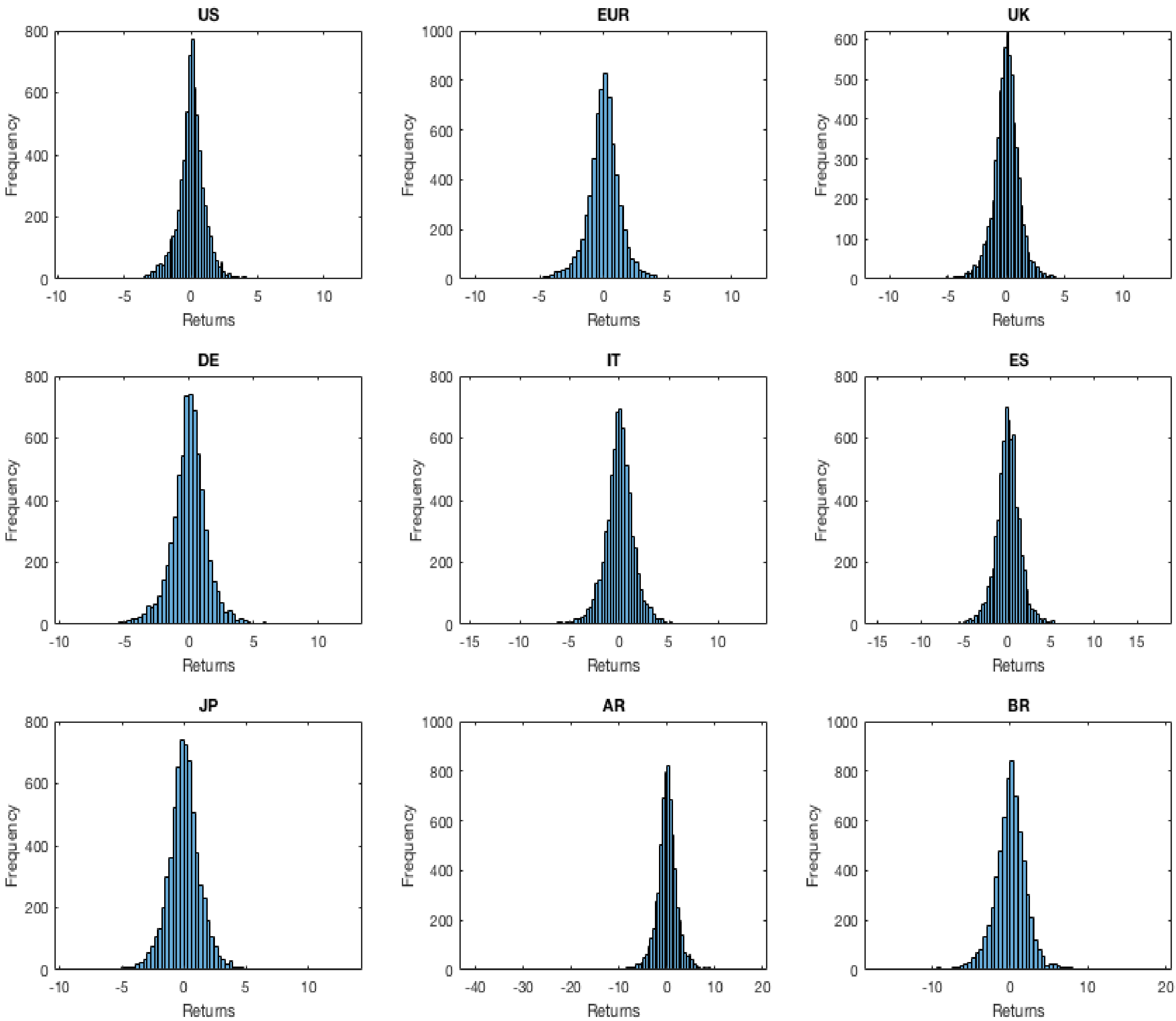








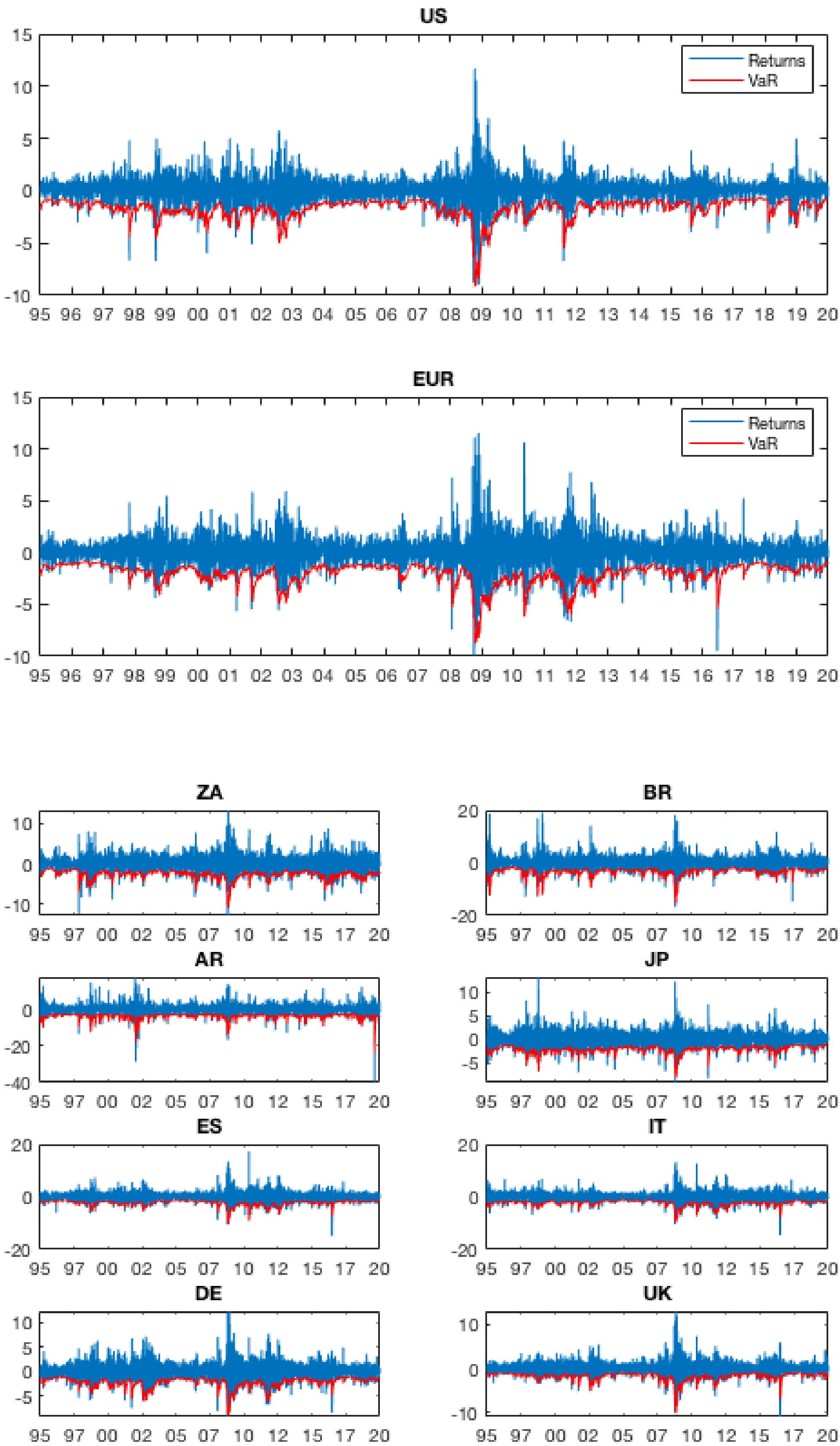
A.15: Frequencies of Daily Returns for each Country



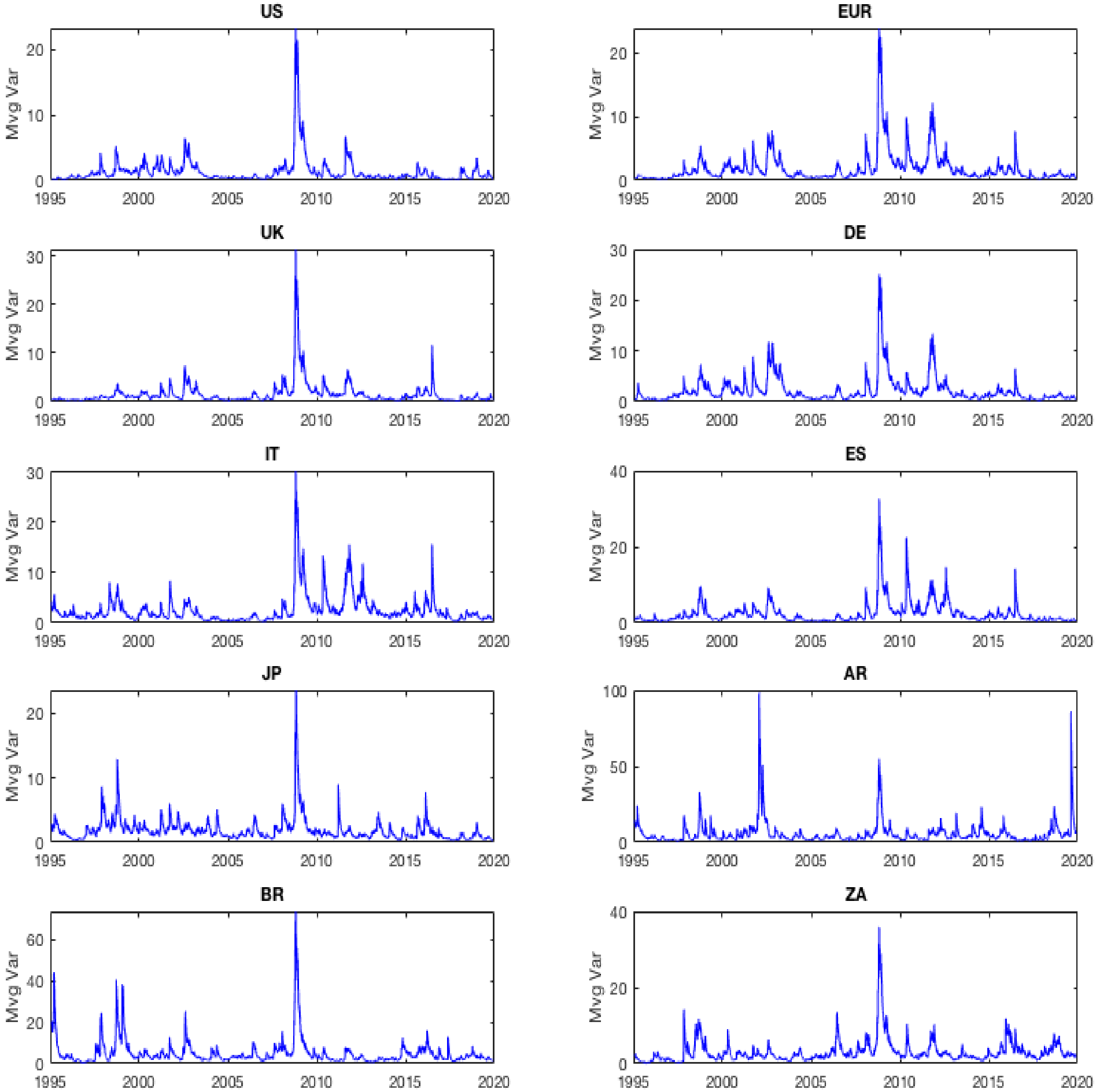
A.16: Value at Risk and Expected Shortfall of Daily Returns

	VaR	ES
MSCI USA	0.020644	0.013435
MSCI EMU	0.011825	0.0076652
MSCI UK	0.0094436	0.0061119
MSCI GERMANY	0.011909	0.0078368
MSCI ITALY	0.0070193	0.0046062
MSCI SPAIN	0.013019	0.0084791
MSCI GREECE	−0.0020134	−0.0013444
MSCI JAPAN	0.0045366	0.0031615
MSCI AUSTRALIA	0.012875	0.0081425
MSCI ARGENTINA	0.0097712	0.0061662
MSCI BRAZIL	0.012589	0.0083266
MSCI RUSSIA	0.018012	0.010815
MSCI INDIA	0.014113	0.0090885
MSCI CHINA	0.0069243	0.0046161
MSCI SOUTH AFRICA	0.0096932	0.0066518

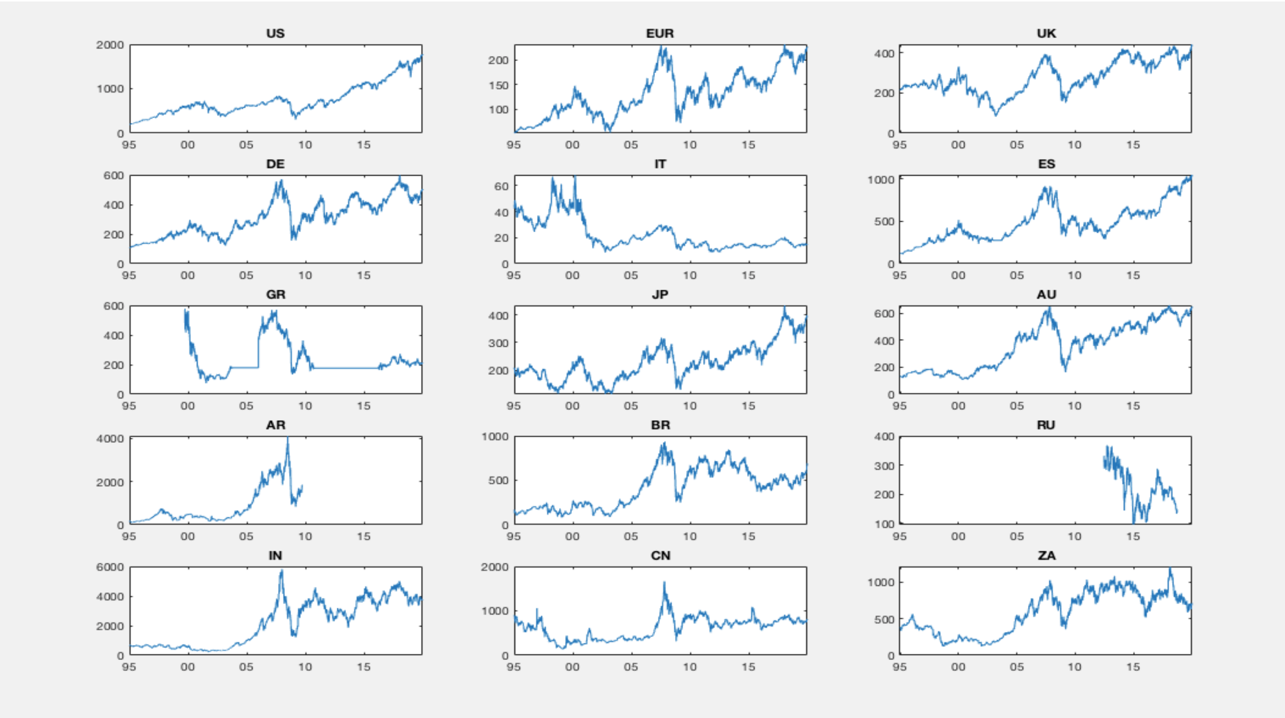
A.17: Value-at-Risk of Daily Returns (%, alpha=0.05)



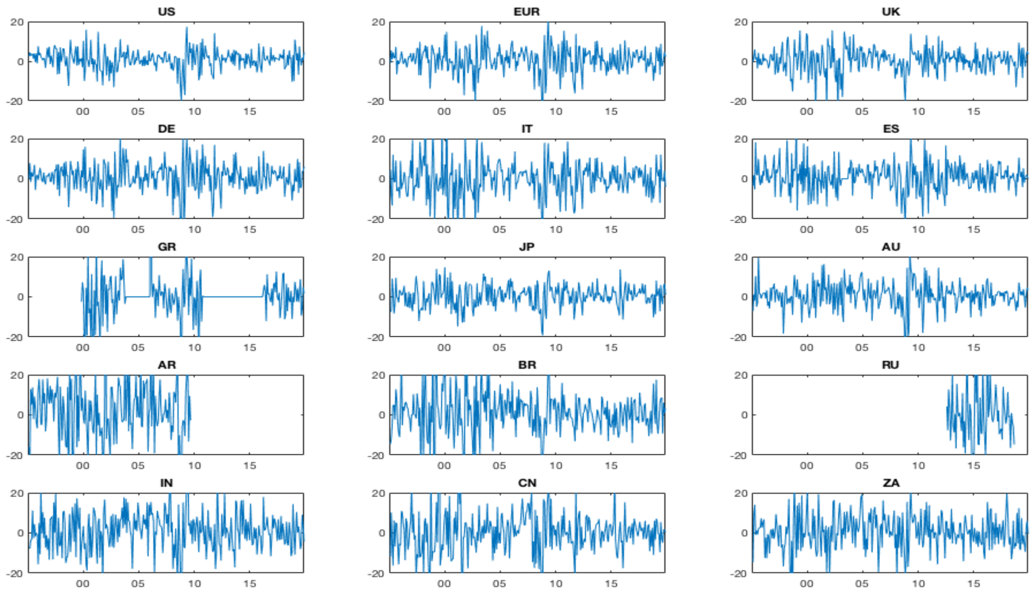
A.18: Moving Variance of Daily Returns (%)



A.19: Price Indices - Industrial Sector



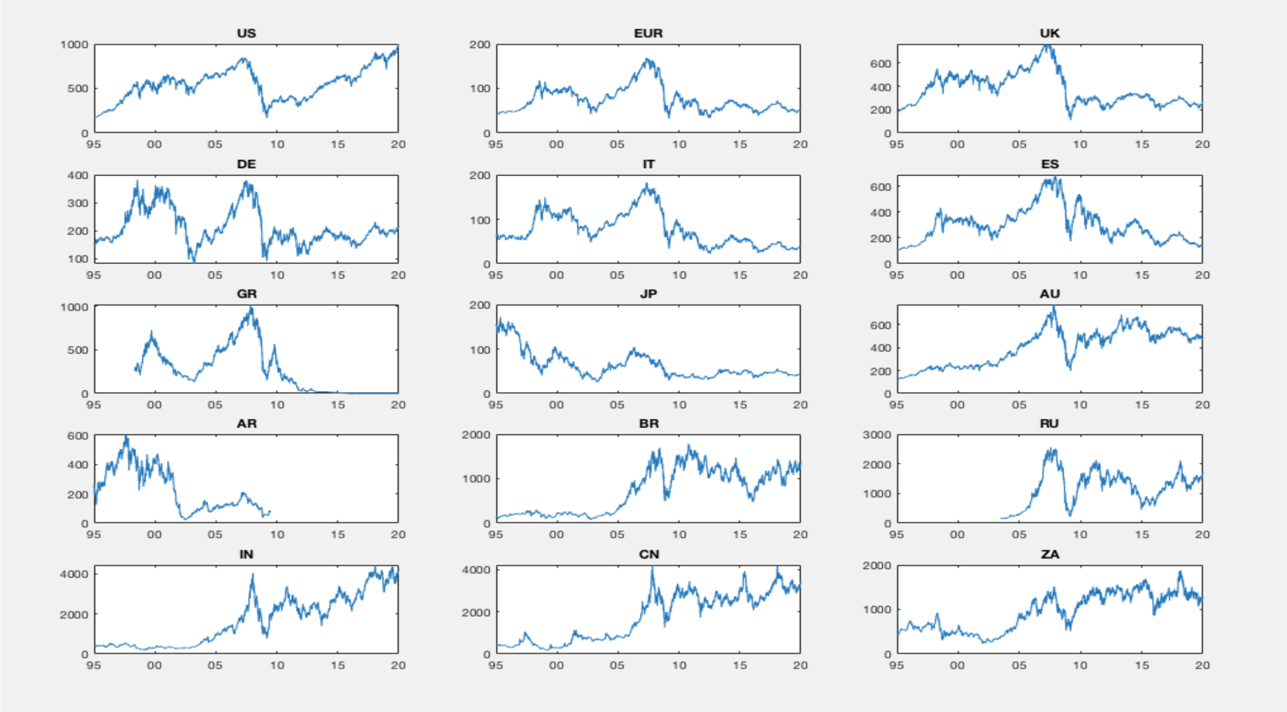
A.20: Monthly Returns (%) – Industrial Sector



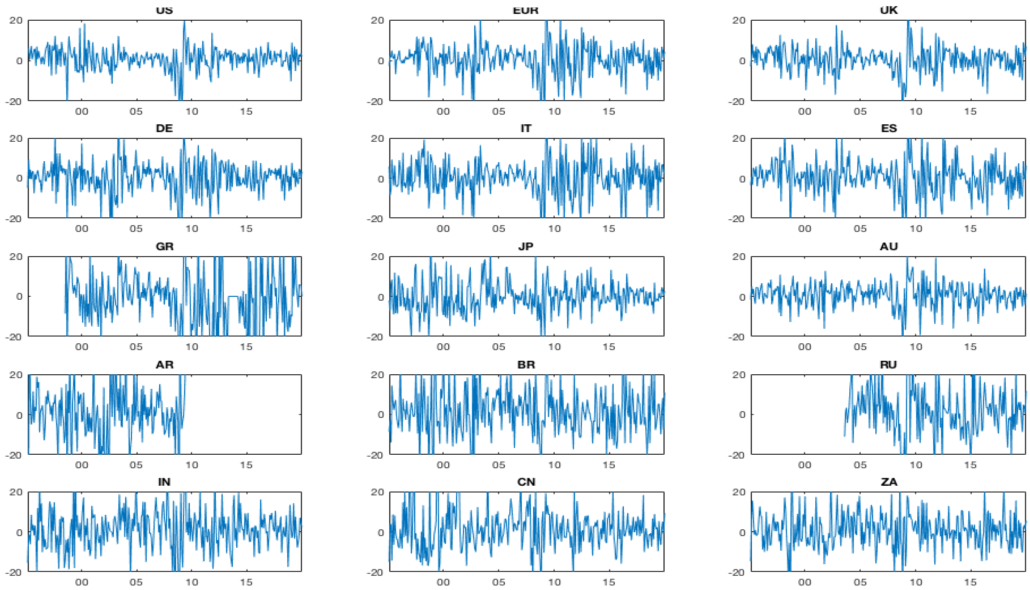
A.21: Descriptive Statistic of Daily Returns (%) – Industrial Sector

	Mean	Median	IQ	StDev	Min	Max	Skew	Kurt
US	0.041749	0.035616	1.1709	1.267	−9.1679	10.074	−0.17197	8.8605
EUR	0.034416	0.055874	1.5184	1.568	−10.91	15.478	0.089334	10.069
UK	0.021336	0.035075	1.4407	1.4349	−15.418	10.572	−0.37794	10.654
DE	0.038457	0.036278	1.6967	1.725	−13.254	21.687	0.23903	13.339
IT	0.0011327	0	1.928	1.8794	−11.471	10.923	0.0424	6.2469
ES	0.046005	0.02037	1.5646	1.6405	−10.612	12.717	0.083704	7.7699
GR	0.0066757	0	0.72961	2.4691	−14.694	111.68	NaN	NaN
JP	0.021503	0.020105	1.6467	1.535	−11.94	13.58	0.10784	7.5423
AU	0.034635	0.038763	1.529	1.4927	−14.635	12.089	−0.23148	9.9255
AR	0.10527	0.027678	2.7962	2.9186	−28.572	17.374	NaN	NaN
BR	0.047269	0	2.1844	2.2715	−19.225	19.033	−0.046223	9.65
RU	−0.0050696	0	2.7144	2.97	−23.587	13.996	NaN	NaN
IN	0.042761	0.0053945	1.8369	1.8864	−10.729	23.079	0.24016	9.3917
CN	0.021914	0	1.7838	2.1676	−14.918	19.733	0.54138	10.902
ZA	0.025819	0.039846	1.9523	1.8272	−14.994	12.478	−0.16218	7.5987

A.22: Price Indices - Financial Sector



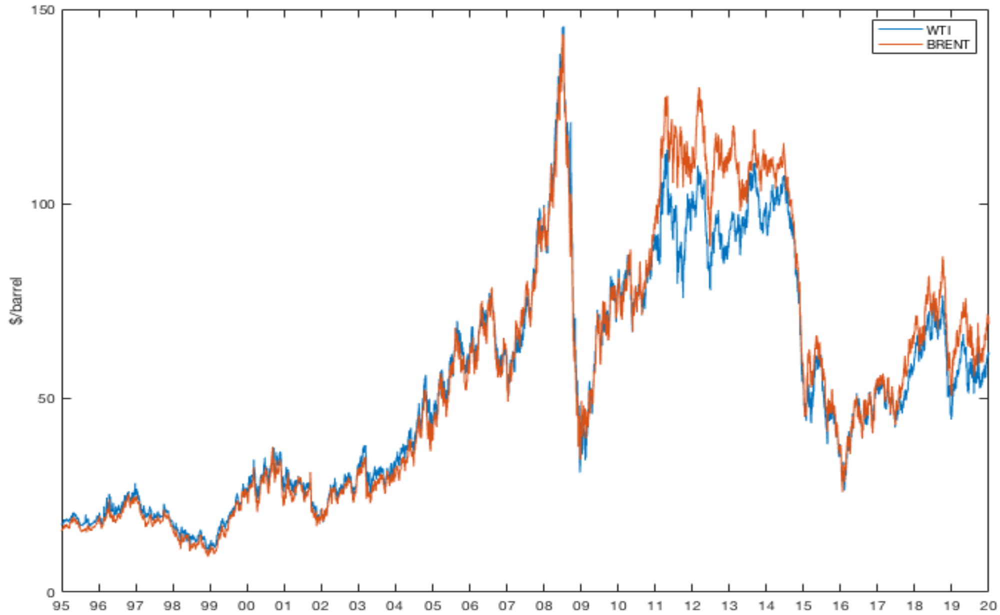
A.23: Monthly Returns (%) – Financial Sector



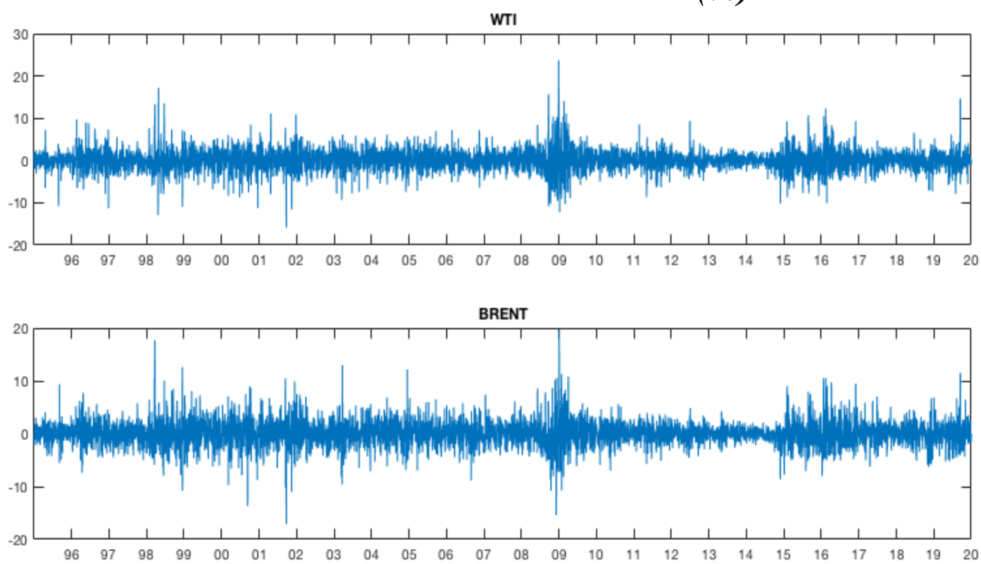
A.24: Descriptive Statistic of Daily Returns (%) – Financial Sector

	Mean	Median	IQ	StDev	Min	Max	Skew	Kurt
US	0.040917	0.016352	1.2751	1.6216	−15.622	15.521	0.23442	17.517
EUR	0.01869	0.030788	1.5738	1.7523	−15.389	18.371	0.26021	12.813
UK	0.020346	0.01827	1.5245	1.7031	−18.1	19.75	0.14518	16.894
DE	0.022578	0.010029	1.6687	1.879	−13.742	20.792	0.3752	13.263
IT	0.012422	0	1.856	1.9683	−21.389	18.439	−0.043839	10.215
ES	0.022828	0.014482	1.8775	1.9685	−19.602	22.936	0.27515	11.84
GR	−0.057915	0	2.2698	3.7496	−33.333	30.97	NaN	NaN
JP	−0.0024774	−0.037074	1.8677	1.8374	−11.999	17.236	0.47779	8.3012
AU	0.03086	0.041154	1.4327	1.4302	−14.91	12.88	−0.2722	11.697
AR	0.012963	0	2.6593	2.907	−28.566	30.057	NaN	NaN
BR	0.063322	0.0095388	2.4651	2.4878	−17.343	24.772	0.30706	10.085
RU	0.092725	0	2.4384	2.8094	−20.377	46.496	NaN	NaN
IN	0.053125	0	1.9213	2.0086	−15.19	23.516	0.31465	10.368
CN	0.052386	0	1.7767	2.1338	−12.725	14.906	0.42047	8.9817
ZA	0.033731	0.022009	2.0359	1.9431	−14.674	13.556	−0.11907	7.7739

A.25: Oil Prices



A.26: Returns on Oil Price (%)



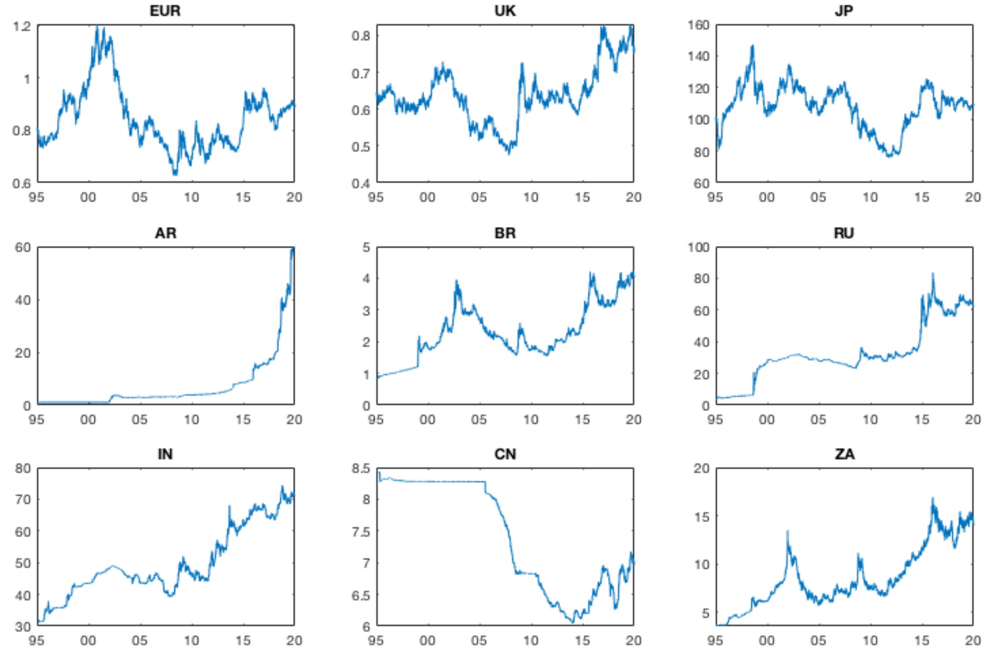
A.27: Descriptive Statistic of Oil Price

	Mean	Median	IQ	StDev	Min	Max	Skew	Kurt
WTI	53.225	50.105	46.98	29.094	10.73	145.66	0.48627	2.2411
BRENT	55.301	51.325	50.295	33.098	9.14	143.6	0.53378	2.1511

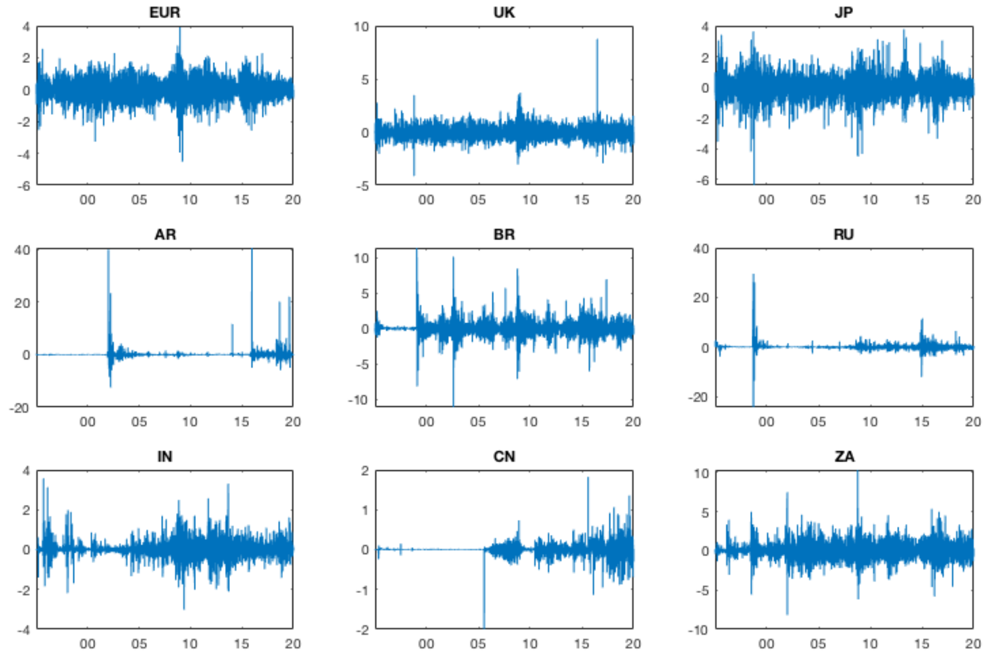
A.28: Descriptive Statistic of Returns on Oil Price

	Mean	Median	IQ	StDev	Min	Max	Skew	Kurt	TotRet
WTI	0.047249	0	2.3954	2.3709	−15.816	23.709	0.24975	8.6619	3.5003
BRENT	0.048071	0	2.3642	2.2779	−17.076	19.685	0.22285	7.6367	4.2495

A.29: Exchange Rates for each Country, in terms of $



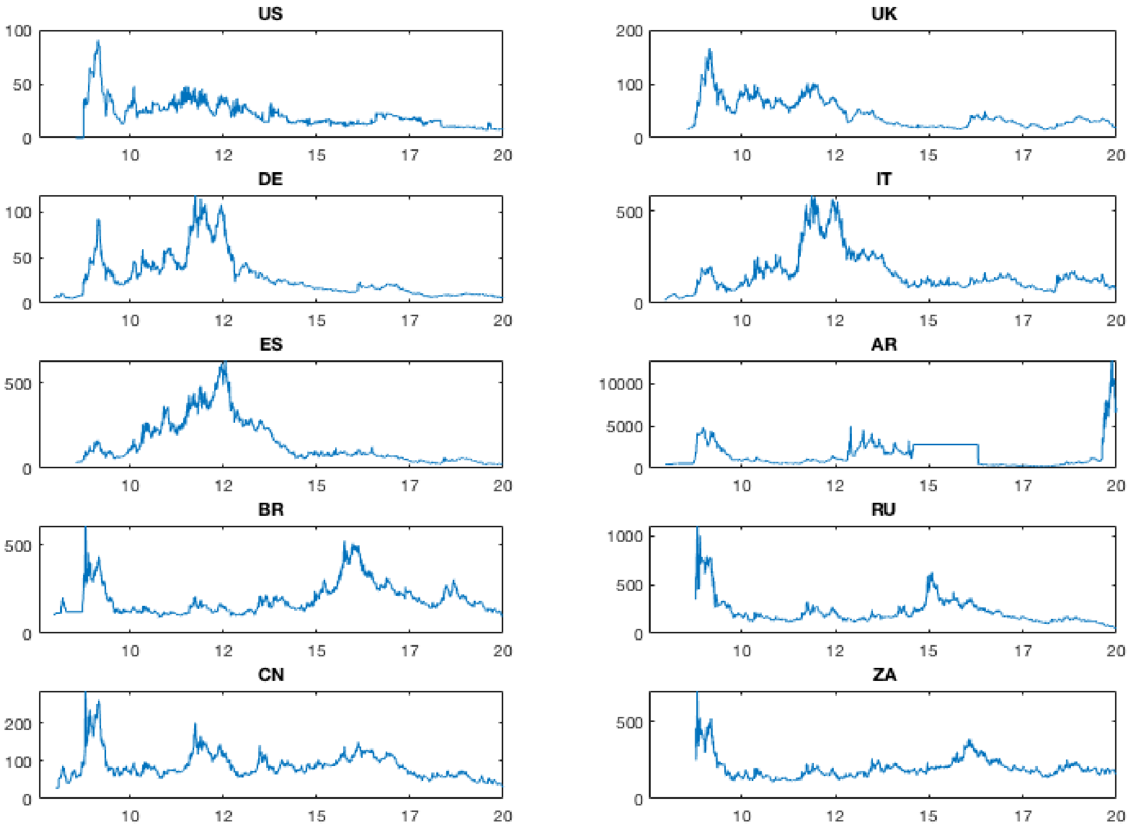
A.30: Variations of Exchange Rates (%)



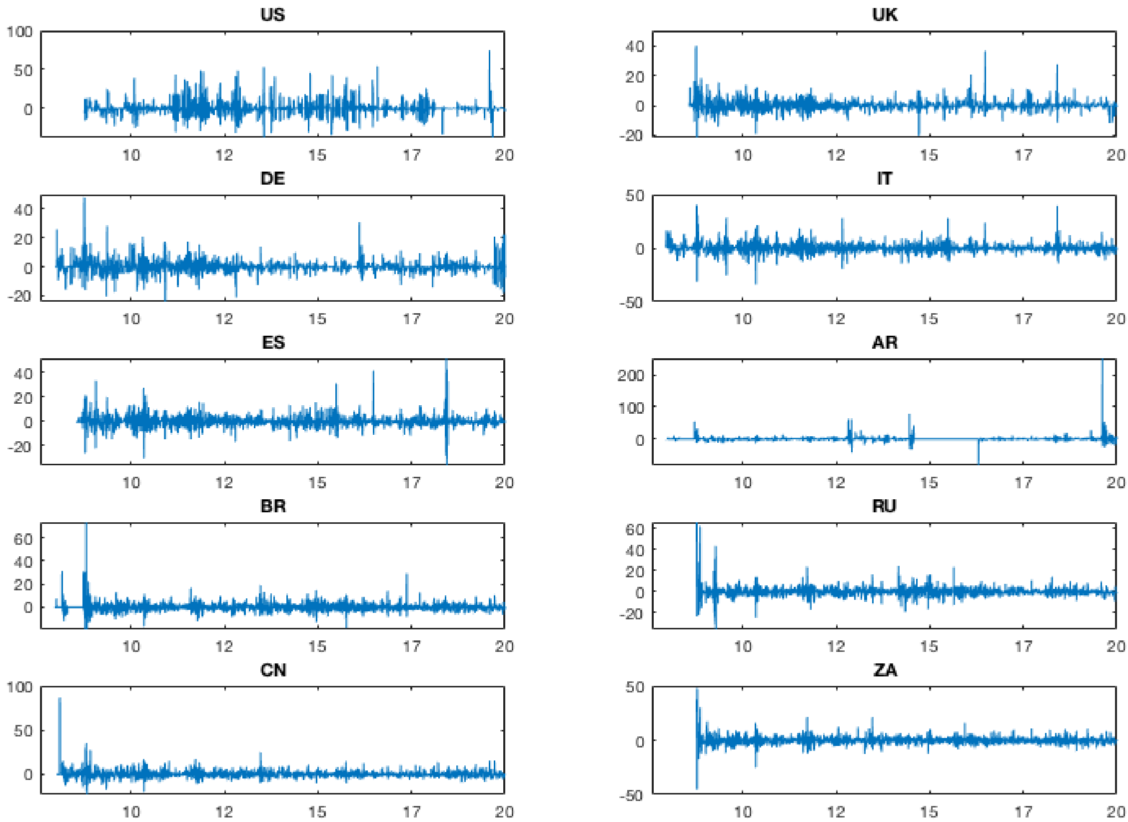
A.31: Descriptive Statistic of Exchange Rates

	Mean	Median	IQ	StDev	Min	Max	Skew	Kurt
EUR	0.84559	0.82422	0.14842	0.11823	0.62584	1.2067	0.86193	3.375
UK	0.63676	0.6294	0.0754	0.07477	0.4747	0.8308	0.34153	2.9515
JP	108.01	109.51	16.29	13.65	75.76	147.14	-0.46681	2.9723
AR	7.0141	3.1538	4.7983	10.665	0.987	59.986	3.0323	12.568
BR	2.2799	2.1477	1.1862	0.85885	0.833	4.2667	0.34892	2.3118
RU	32.42	29.346	7.3476	17.75	3.55	83.591	0.5507	2.837
IN	49.774	46.465	16.665	10.724	31.355	74.364	0.55102	2.2713
CN	7.4206	7.6044	1.648	0.8554	6.0412	8.4462	-0.18451	1.3017
ZA	8.5146	7.6154	4.2806	3.1621	3.5258	16.935	0.61319	2.4229

A.32: CDS Spread of each Country (b.p., in $)



A.33: CDS Premium Change (%)



A.34: Descriptive Statistic of CDS Premium

	Mean	Median	IQ	StDev	Min	Max
US	21.521	18.17	14.14	12.619	0	90
UK	44.598	35.415	37.851	26.545	15.89	165
DE	28.144	20.06	27.263	23.487	5.2	118.38
IT	161.93	126.05	87.147	111.79	16.5	586.7
ES	147.42	87.665	172.09	129.31	21.91	634.35
AR	1627.3	972.83	2203.2	1603.9	229.62	12750
BR	191.35	162.07	104.48	87.737	91.16	606.31
RU	221.91	175.33	109.86	137.2	54.64	1106
CN	86.956	79.5	39.67	35.176	29.236	284
ZA	199.37	184.99	58.06	68.287	106	700.9

A.35: Descriptive Statistic of CDS Premium Change

	Mean	Median	IQ	StDev	Min	Max
US	0.26078	0.005223	2.7071	8.2901	-37.736	74.704
UK	0.055201	0.013478	1.6222	3.2156	-21.739	40
DE	0.09414	-0.019368	3.113	4.3783	-24.044	47.887
IT	0.14531	-0.0047336	3.4331	4.2737	-34.042	40.171
ES	0.092977	-0.01217	3.2586	4.5827	-36.302	51.915
AR	0.33199	-0.003383	2.9833	7.5661	-82.093	250.7
BR	0.06048	-0.11203	3.0671	3.6722	-19.168	73.439
RU	0.023959	-0.13921	3.1669	4.342	-35.573	65.75
CN	0.083496	-0.18012	3.3684	4.1543	-23.256	86.876
ZA	0.035306	-0.017654	2.9599	3.4911	-45.004	47.694

Appendix B

MATLAB Codes

In this appendix, the implemented codes are reported. In particular, there are two main blocks of codes: the first one describes the function used to perform the MIDAS quantile regression on the dataset; the second one describes the function used to implement the nonparametric test. Function for the MIDAS quantile regression has been created starting from an analogous function written by Eric Ghysels *et al.*, and specifically adjusted in accordance with the purposes of this thesis. Function for the test is the precise implementation of the mathematical formula described in the previous sections.

```
% MIDAS QUANTILE REGRESSION
function [estParams, EstY, EstYdate, EstX, EstXdate, CondQuantile, fval,
resid, yLowFreqSim, se] = MidasQuantsimple (Y, Ydates, X, Xdates, q, Xlag,
horizon, estStart, estEnd)

% Description:
% MIDAS quantile regression estimates the conditional quantile of n-period
% Y, given a conditioning variable (predictor) X sampled at higher
% frequency, with MIDAS weights; basic model is:
%               Yt = b0 + b1*[B(L(1/m))*Xt,m] + et
%
% Input Arguments:
%   Y           T-by-1 observation data for the low frequency variable,
%               inscluding NaN, from a full time series (double)
%
% Ydates        T-by-1 dates (including Saturdays, Sundays and holidays)
%               (cell, format ['01/15/1995'], mm/dd/yyyy)
%
%   X           T2-by-1 time series with T2>T; it is the high frequency
%               conditioning variable (predictor), which have the same
%               timespan of Y but much more high frequency observations
%               (double)
%
% Xdates        T2-by-1 dates of high frequency variable
%               (including Saturdays, Sundays and holidays)
%               (cell,mm/dd/yyyy)
%
%   q           A scalar between zero and one that specifies the orders
%               (i.e., alpha or tau) of the quantile
%
%  Xlag         A scalar integer that specifies the number of lags for the
%               high frequency predictor, to which MIDAS weights are
%               assigned (e.g., for regression of monthly data (y) on daily
%               data (x), the number of lags may be 30, or 22 trading days)
%
% horizon       Number of (high frequency) lags from which lagged high
%               frequency regressor starts (e.g., when a statistical
%               bulletin of i-th month is published every 15th of following
%               (i+1)-th month, then horizon should be 15)
%
%  estStart     Start date for parameter estimation
%               (char, format yyyy-mm-dd,'1995-01-15')
%
```

```
%  estEnd       Terminal date for parameter estimation
%               (char, format '1995-01-15')
%
%
% Output Arguments:
% Note: each output variable reports the results of a specific approach: no
% gradient, no search, yes smoother (a non-negative scalar that specifies
% how to smooth the non-differentiable objective function; default is
% average absolute residuals), yes options.
%
%  EstParams    Estimated parameters for [intercept; slope; k],
%               where intercept and slope are the coefficients of the
%               quantile regression, and k is the parameter in the
%               MIDAS Beta polynomial
%
%   EstY        Modified output variable employed for the regression
%
%  EstYdate     Serial dates for the output variables
%
%   EstX        Modified explanatory variable employed for the regression
%
%  EstXdate     Serial dates for the explanatory variables
%
% CondQuantile  Estimated conditional quantiles. This is the fitted value
%               of Y, given by the right-hand-side of the quantile
%               regression model
%
%   fVal        Value of the non-differentiable loss function:
%               fval = loss'*(q-(loss<0)), where loss = Y-CondQuantile
%
%   Resid       Residuals: residual = EstY-CondQuantile
%
%   YSim        Simulations of low frequency variable:
%               ySim = condQuantile+resid
%
%  StandError   Standard error of each parameter [intercept; slope; k]
%
%  Figures representing data and estimated quantiles;
%  Tables showing main regression outputs: coefficients, standard errors
%  and p values of slope, intercept and parameter k.

% CODES FOR FUNCTION
% Set Data
mask = ~isnan(Y);
DataY = Y(mask);
DataYdate = Ydates(mask);
DataY = DataY(:);
DataYdate = DataYdate(:);

maskx = ~isnan(X);
DataX = X(maskx);
DataXdate = Xdates(maskx);
DataX = DataX(:);
DataXdate = DataXdate(:);

DataYdateVec = datevec(DataYdate);
DataYdateNum = datenum(DataYdateVec);
DataXdateVec = datevec(DataXdate);
DataXdateNum = datenum(DataXdateVec);
estStart = datenum(estStart);
estEnd = datenum(estEnd);

% Minimum and maximum dates that data support
minDateY = DataYdateNum(1,:);
```

```
minDateX = DataXdateNum(max(1,Xlag+horizon),:);
if minDateY > minDateX
    minDate = minDateY;
else
    minDate = minDateX;
end

maxDateY = DataYdateNum(end,:);
maxDateX = DataXdateNum(end,:);
if maxDateY > maxDateX
    maxDate = maxDateX;
else
    maxDate = maxDateY;
end

% Check and set default sample period
if estStart < minDate
    estStart = minDate;
end
if estEnd > maxDate
   estEnd = maxDate;
end

% Construct Y data
tol = 1e-10;
locStart = find(DataYdateNum >= estStart-tol, 1);
locEnd = find(DataYdateNum >= estEnd-tol, 1);
EstY = DataY(locStart:locEnd);
EstYdate = DataYdateNum(locStart:locEnd);
nobs = locEnd - locStart + 1;

% Construct lagged X data
EstX = zeros(nobs,Xlag);
EstXdate = zeros(nobs,Xlag);
for t = 1:nobs
    loc = find(DataXdateNum >= EstYdate(t)-tol, 1);
    if isempty(loc)
        loc = length(DataXdateNum);
    end
    if loc-horizon > size(DataX,1)
        nobs = t - 1;
        EstY = EstY(1:nobs,:);
        EstYdate = EstYdate(1:nobs,:);
        EstLagY = EstLagY(1:nobs,:);
        EstLagYdate = EstLagYdate(1:nobs,:);
        EstX = EstX(1:nobs,:);
        EstXdate = EstXdate(1:nobs,:);
        maxDate = EstYdate(end);
        break
    else
        EstX(t,:) = DataX(loc-horizon:-1:loc-horizon-Xlag+1);
        EstXdate(t,:) = DataXdateNum(loc-horizon:-1:loc-horizon-Xlag+1);
    end
end

% ESTIMATION
% Initial parameters estimated by OLS
k0 = 5;
X0 = [ones(nobs,1), EstX * midasBetaWeights(Xlag,1,k0)'];
OLS = X0 \ EstY;
params0 = [OLS;k0];
resid0 = EstY - X0 * OLS;

% Setting for numerical optimization
```

```
smoother = mean(abs(resid0)); % Smoother function
lb = [-Inf;-Inf;0];          % Low Bounds
ub = [Inf;Inf;100];          % Upper Bounds

% Optimization options
     options = optimoptions('fmincon','Algorithm','interior-
point','Display','off');

% Numeric minimization
% minimize non-differentiable function by Newton method, finite-difference,
no gradient, no search, yes smoother, yes options
estParams = fmincon(@(params) objFun(params, EstY, EstX, q, smoother),
params0, [], [], [], [], lb, ub, [], options);

% Conditional quantiles
% no gradient, no search, yes smoother, yes options
[fval,CondQuantile] = objFun(estParams,EstY,EstX,q,0);

% Bootstrap standard errors
nsim = 100;
ind = randi(nobs,[nobs,1]);
resid = EstY - CondQuantile;
for i = 1:nsim        %  with bootstrap standard error method: 'Residual'
    yLowFreqSim = CondQuantile + resid(ind);
    paramSim(:,i) = fminsearch(@(params)objFun(params, yLowFreqSim, EstX,
q, smoother), estParams);
end
se = std(paramSim,0,2);
zstat = estParams./se;
pval = 0.5 * erfc(0.7071 * abs(zstat)) * 2;
pval(pval<1e-6) = 0;

% PLOTTING
% Display the estimation results
columnNames = {'Coeff','StdErr','tStat','Prob'};
rowNames = {'Intercept';'Slope';'k2'};
fprintf('Method: Smoothed Asymmetric loss function minimization\n');
fprintf('Minimized function value: %10.6g\n',fval);
fprintf('Quantile order: %10.6g\n',q);
Table2 = table(estParams, se, zstat, pval, 'RowNames',
rowNames,'VariableNames', columnNames);
disp(Table2);

% plot quantiles
figure
hold on
plot(EstYdate, CondQuantile, 'LineWidth',0.7);
dateaxis;
scatter(EstYdate,EstY,'MarkerEdgeColor',[0 0 1]);
legend('Estimated conditional quantiles','Observations');
xlabel('t');
ylabel('Y');
title(sprintf('Conditional Quantiles, q=%2.6g', q));
hold off

end

% LOCAL FUNCTIONS
%----------------------------------------------------------------------------
% 1) Objective function
function [fval,condQuantile] = objFun(params,y,X,q,smoother)

% Allocate parameters
intercept = params(1);
```

```
slope = params(2);
k2 = params(3);

% Compute MIDAS weights
nlag = size(X,2);
k1 = 1;
weights = midasBetaWeights(nlag,k1,k2)';

% Conditional quantile
condQuantile = intercept + slope .* (X * weights);

% Asymmetric loss function
loss = y - condQuantile;
if smoother == 0
    % Non-differentiable loss function
    fval = loss' * (q - (loss<0));
else
    % Piecewise linear-quadratic smoothing loss function
    maskSmall = loss < (q-1)*smoother;
    maskBig = loss > q*smoother;
    maskMedian = ~maskSmall & ~maskBig;
    fvalSmall = -0.5*(q-1)^2*smoother + (q-1)*loss(maskSmall);
    fvalBig = -0.5*q^2*smoother + q*loss(maskBig);
    fvalMedian = 0.5/smoother .* loss(maskMedian).^2;
    fval = sum(fvalSmall) + sum(fvalBig) + sum(fvalMedian);
end

end

%-------------------------------------------------------------------------
% 2) MIDAS beta polynomial weights
function weights = midasBetaWeights(nlag,param1,param2)

seq = linspace(eps,1-eps,nlag);
if param1 == 1
    weights = (1-seq).^(param2-1);
else
    weights = (1-seq).^(param2-1) .* seq.^(param1-1);
end
weights = weights ./ nansum(weights);

end
```

```
% NONPARAMETRIC GRANGER-CAUSALITY IN QUANTILES TEST
function [JJ] = Jtest(Y,X,Qy,qq)

% Input Arguments:
%    Y      Tx1 vector of dependent variables (e.g. monthly observations)
%
%    X      mxT matrix, with m lags of explanatory variable (e.g. absolute
%           returns) and T periodic samples (e.g. 22 daily obs. for 12
%           months); T must be equal to the number of observations of Y
%           (sampled at lower frequency); time sequence goes from
%           (1,1)->(22,1)->(1,2)->(22,2)-> and so on.
%
%    Qy     conditional quantiles of Y given X, obtained after running a
%           MIDAS quantile regression between X and Y; it is a Txq matrix
%           (usually q=99, that is 99 quantile orders, from 0.01 to 0.99 )
%
%    qq     1xq vector which specifies each quantile order (it should
%           be equally spaced)
%
%
% Output Arguments:
%    JJ     8xq matrix representing Jeong test values for each quantile
%           order of qq;
%           there are 8 rows, i.e. 8 different combinations of methods:
%                     2 Kernel functions used (Gaussian and Uniform) x
%                     4 types of inputs for the Kernel function (absolute
%                     data; simple moving average; exponential moving
%                     average; exponential moving variance) x
%                     1 bandwidth (source: MATLAB documentation)
%
% Figures representing the combinations; they allow the immediate
% visualization of the results, by identifying those quantiles orders in
% which x is statistically significant, at significance level of 0.05
% (t>1.96), to Granger-cause Y;
% Tables showing the test statistic and the p value for the following
% quantiles orders: q=0.01, q=0.10, q=0.20, q=0.30, q=0.40, q=0.50, q=0.60,
% q=0.70. q=0.80. q=0.99. q=0.99.


% CODES FOR FUNCTION
% 0.1) Quantile orders
q = size(qq,2);

% 0.2) Time horizon
T = size(Y,1);

% 0.3) Lags
m = size(X,1);

% 1) First factor of the test
W = sqrt(T/(T-1));

% 2) Product of epsilons (errormat) for each quantile order
onet = [];
for i = 1:q
    for t = 1:T
        if Y(t,1)-Qy(t,i)<0
            onet(t,i) = 1;
        elseif Y(t,1)-Qy(t,i)==0
            onet(t,i) = 1;
        else
            onet(t,i) = 0;
        end
    end
```

```
end

theta = ones(size(onet,1),size(onet,2));
for i = 1:q
    for t = 1:T
        theta(t,i) = theta(t,i)*qq(1,i);  % vector of quantile order
    end
end

epsilont = onet-theta;                    % espilon at time t
errormat = ones(size(epsilont,1),size(epsilont,1),size(qq,2));
for i = 1:q
    errormat(:,:,i) = epsilont(:,i)*epsilont(:,i)'; % errors matrix
end

% 3.1) Kernel functions: Zt-Zs (4 methods)
% First method: X as Z
X11 = X(:);
Xmean = mean(X11);

In1 = [];
ZtZs1 = [];
for i = 1:T
    In1 = X(:,i);
    for j = 1:T
        ZtZs1(:,j,i) = X(:,j)-In1;
    end
end

% Second method: Simple Moving Average of X
MAS = movavg(X11,'simple',m);
MAS = reshape(MAS,m,T);
MAS1 = MAS(:);
MAS11 = MAS(m,:);

In2 = [];
ZtZs2 = [];
for i = 1:T
    In2 = MAS(:,i);
    for j = 1:T
        ZtZs2(:,j,i) = MAS(:,j)-In2;
    end
end

% Third method: Exponential Moving Average of X, 'manual'
% (fix weighting decreasing coefficient lambda = 0.95)
MAEm0 = Xmean;
MAEm1 = zeros(size(X11,1),1);
lambda = 0.95;
for j = 1:size(X11,1)
    MAEm1(j,:) = lambda*MAEm0+(1-lambda)*X11(j,:);
    MAEm0 = MAEm1(j,:);
end
MAEm = reshape(MAEm1,m,T);
MAEm11 = MAEm(m,:);

In3 = [];
ZtZs3 = [];
for i = 1:T
    In3 = MAEm(:,i);
    for j = 1:T
        ZtZs3(:,j,i) = MAEm(:,j)-In3;
    end
end
```

```
% Fourth method: Exponential Moving Average of Variances, 'manual'
MAEV0 = var(X11);
MAEV1 = zeros(size(X11,1),1);
for j = 1:size(X11,1)
    MAEV1(j,1) = lambda*MAEV0+(1-lambda)*(X11(j,:).^2);
    MAEV0 = MAEV1(j,1);
end
MAEV = reshape(MAEV1,m,T);
MAEV11 = MAEV(m,:);

In4 = [];
ZtZs4 = [];
for i = 1:T
    In4 = MAEV(:,i);
    for j = 1:T
        ZtZs4(:,j,i) = MAEV(:,j)-In4;
    end
end

% 3.2) Kernel function: Bandwidth h (from MATLAB doc, 'mvksdensity')
w = (4/((T+2)*m))^(1/(T+4)); % h for method 1
for i = 1:size(X,2)
    se1(:,i) = std(X(:,i));
end
h1 = (se1.*w);

for i = 1:size(X,2)          % h for method 2
    se2(:,i) = std(MAS(:,i));
end
h2 = (se2.*w);

for i = 1:size(X,2)          % h for method 3
    se3(:,i) = std(MAEm(:,i));
end
h3 = (se3.*w);

for i = 1:size(X,2)          % h for method 4
    se4(:,i) = std(MAEV(:,i));
end
h4 = (se4.*w);

% 3.3) Kernel function: ZtZs/h
for s = 1:size(ZtZs1,3)     %inputs of method 1
    for i = 1:size(ZtZs1,2)
        for j = 1:size(ZtZs1,1)
            inp11(j,i,s) = ZtZs1(j,i,s)./h1(1,i);
        end
    end
end

for s = 1:size(ZtZs1,3)     %inputs of method 2
    for i = 1:size(ZtZs1,2)
        for j = 1:size(ZtZs1,1)
            inp21(j,i,s) = ZtZs2(j,i,s)./h2(1,i);
        end
    end
end

for s = 1:size(ZtZs1,3)     %inputs of method 3
    for i = 1:size(ZtZs1,2)
        for j = 1:size(ZtZs1,1)
            inp31(j,i,s) = ZtZs3(j,i,s)./h3(1,i);
        end
```

```
    end
end

for s = 1:size(ZtZs1,3)      %inputs of method 4
    for i = 1:size(ZtZs1,2)
        for j = 1:size(ZtZs1,1)
            inp41(j,i,s)=  ZtZs4(j,i,s)./h4(1,i);
        end
    end
end

% 3.4) Kernel function: k(ZtZs/h)  (2 functions)
for j = 1:size(inp11,3)        % gaussian (=1)
    [k111(:,:,j)] = gaussian(inp11(:,:,j));
end

for j = 1:size(inp21,3)
    [k211(:,:,j)] = gaussian(inp21(:,:,j));
end

for j = 1:size(inp31,3)
    [k311(:,:,j)] = gaussian(inp31(:,:,j));
end

for j = 1:size(inp41,3)
    [k411(:,:,j)] = gaussian(inp41(:,:,j));
end

for j = 1:size(inp11,3)        % uniform (=2)
    [k112(:,:,j)] = uniform(inp11(:,:,j));
end

for j = 1:size(inp21,3)
    [k212(:,:,j)] = uniform(inp21(:,:,j));
end

for j = 1:size(inp31,3)
    [k312(:,:,j)] = uniform(inp31(:,:,j));
end

for j = 1:size(inp41,3)
    [k412(:,:,j)] = uniform(inp41(:,:,j));
end

% 3.5) Kernel function: K(ZtZs/h)  (product kernel)
for i=1:size(k111,3)                     %ZtZs1
    pK111(:,:,i)=prod(k111(:,:,i),1);    %function1
end
pK111=squeeze(pK111);

for i=1:size(k112,3)                     %ZtZs1
    pK112(:,:,i)=prod(k112(:,:,i),1);    %function2
end
pK112=squeeze(pK112);

for i=1:size(k211,3)                     %ZtZs2
    pK211(:,:,i)=prod(k211(:,:,i),1);    %function1
end
pK211=squeeze(pK211);

for i=1:size(k212,3)
    pK212(:,:,i)=prod(k212(:,:,i),1);    %function2
end
pK212=squeeze(pK212);
```

```
for i=1:size(k311,3)                        %ZtZs3
    pK311(:,:,i)=prod(k311(:,:,i),1);      %function1
end
pK311=squeeze(pK311);

for i=1:size(k312,3)
    pK312(:,:,i)=prod(k312(:,:,i),1);      %function2
end
pK312=squeeze(pK312);

for i=1:size(k411,3)                        %ZtZs4
    pK411(:,:,i)=prod(k411(:,:,i),1);      %function1
end
pK411=squeeze(pK411);

for i=1:size(k412,3)
    pK412(:,:,i)=prod(k412(:,:,i),1);      %function2
end
pK412=squeeze(pK412);

% 4.1) Numerator of the test: Product
for i=1:size(errormat,3)              %ZtZs1
    kee111(:,:,i)=pK111.*errormat(:,:,i);
end

for i=1:size(errormat,3)
    kee112(:,:,i)=pK112.*errormat(:,:,i);
end

for i=1:size(errormat,3)              %ZtZs2
    kee211(:,:,i)=pK211.*errormat(:,:,i);
end

for i=1:size(errormat,3)
    kee212(:,:,i)=pK212.*errormat(:,:,i);
end

for i=1:size(errormat,3)              %ZtZs3
    kee311(:,:,i)=pK311.*errormat(:,:,i);
end

for i=1:size(errormat,3)
    kee312(:,:,i)=pK312.*errormat(:,:,i);
end

for i=1:size(errormat,3)              %ZtZs4
    kee411(:,:,i)=pK411.*errormat(:,:,i);
end

for i=1:size(errormat,3)
    kee412(:,:,i)=pK412.*errormat(:,:,i);
end

% 4.2) Numerator of the test: Sum of products
for i=1:size(kee111,3)                   %ZtZs1
    kee111(:,:,i)=kee111(:,:,i)-diag(diag(kee111(:,:,i)));
    skee111(:,i)=sum(kee111(:,:,i),'all');
end

for i=1:size(kee112,3)
    kee112(:,:,i)=kee112(:,:,i)-diag(diag(kee112(:,:,i)));
    skee112(:,i)=sum(kee112(:,:,i),'all');
end
```

```
for i=1:size(kee211,3)                %ZtZs2
    kee211(:,:,i)=kee211(:,:,i)-diag(diag(kee211(:,:,i)));
    skee211(:,i)=sum(kee211(:,:,i),'all');
end

for i=1:size(kee212,3)
    kee212(:,:,i)=kee212(:,:,i)-diag(diag(kee212(:,:,i)));
    skee212(:,i)=sum(kee212(:,:,i),'all');
end

for i=1:size(kee311,3)                %ZtZs3
    kee311(:,:,i)=kee311(:,:,i)-diag(diag(kee311(:,:,i)));
    skee311(:,i)=sum(kee311(:,:,i),'all');
end

for i=1:size(kee312,3)
    kee312(:,:,i)=kee312(:,:,i)-diag(diag(kee312(:,:,i)));
    skee312(:,i)=sum(kee312(:,:,i),'all');
end

for i=1:size(kee411,3)                %ZtZs4
    kee411(:,:,i)=kee411(:,:,i)-diag(diag(kee411(:,:,i)));
    skee411(:,i)=sum(kee411(:,:,i),'all');
end

for i=1:size(kee412,3)
    kee412(:,:,i)=kee412(:,:,i)-diag(diag(kee412(:,:,i)));
    skee412(:,i)=sum(kee412(:,:,i),'all');
end

% 5) Denominator of the test
a=ones(1,size(qq,2))-qq;
b=a.*qq;
c=sqrt(2);
d=c.*b;           % first factor, common to all

dpK111=sqrt(sum(pK111.^2-diag(diag(pK111.^2)),'all')).*d;%ZtZs 1
dpK112=sqrt(sum(pK112.^2-diag(diag(pK112.^2)),'all')).*d;

dpK211=sqrt(sum(pK211.^2-diag(diag(pK211.^2)),'all')).*d;%ZtZs 2
dpK212=sqrt(sum(pK212.^2-diag(diag(pK212.^2)),'all')).*d;

dpK311=sqrt(sum(pK311.^2-diag(diag(pK311.^2)),'all')).*d;%ZtZs 3
dpK312=sqrt(sum(pK312.^2-diag(diag(pK312.^2)),'all')).*d;

dpK411=sqrt(sum(pK411.^2-diag(diag(pK411.^2)),'all')).*d;%ZtZs 4
dpK412=sqrt(sum(pK412.^2-diag(diag(pK412.^2)),'all')).*d;

% 6) J TEST RESULTS
J111=W.*(skee111./dpK111); %ZtZs1
J112=W.*(skee112./dpK112);

J211=W.*(skee211./dpK211); %ZtZs2
J212=W.*(skee212./dpK212);

J311=W.*(skee311./dpK311); %ZtZs3
J312=W.*(skee312./dpK312);

J411=W.*(skee411./dpK411); %ZtZs4
J412=W.*(skee412./dpK412);

% Correction of approximated J
for i=1:size(qq,2)       %ZtZs1
```

```
    if J111(1,i)<=0
       J111(1,i)=0;
    else
       J111(1,i)=J111(1,i);
    end
end

for i=1:size(qq,2)
    if J112(1,i)<=0
       J112(1,i)=0;
    else
       J112(1,i)=J112(1,i);
    end
end

for i=1:size(qq,2)      %ZtZs2
    if J211(1,i)<=0
       J211(1,i)=0;
    else
       J211(1,i)=J211(1,i);
    end
end

for i=1:size(qq,2)
    if J212(1,i)<=0
       J212(1,i)=0;
    else
       J212(1,i)=J212(1,i);
    end
end

for i=1:size(qq,2)      %ZtZs3
    if J311(1,i)<=0
       J311(1,i)=0;
    else
       J311(1,i)=J311(1,i);
    end
end

for i=1:size(qq,2)
    if J312(1,i)<=0
       J312(1,i)=0;
    else
       J312(1,i)=J312(1,i);
    end
end

for i=1:size(qq,2)      %ZtZs4
    if J411(1,i)<=0
       J411(1,i)=0;
    else
       J411(1,i)=J411(1,i);
    end
end

for i=1:size(qq,2)
    if J412(1,i)<=0
       J412(1,i)=0;
    else
       J412(1,i)=J412(1,i);
    end
end

% Figures
```

```
figure   %ZtZs1
plot(qq,J111,'b');
hold on
ylim([0 inf]);
yline(1.96,'r');
ylabel('J111');
xlabel('q');
hold off

figure
plot(qq,J112,'b');
hold on
ylim([0 inf]);
yline(1.96,'r');
ylabel('J112');
xlabel('q');
hold off

figure   %ZtZs2
plot(qq,J211,'b');
hold on
ylim([0 inf]);
yline(1.96,'r');
ylabel('J211');
xlabel('q');
hold off

figure
plot(qq,J212,'b');
hold on
ylim([0 inf]);
yline(1.96,'r');
ylabel('J212');
xlabel('q');
hold off

figure   %ZtZs3
plot(qq,J311,'b');
hold on
ylim([0 inf]);
yline(1.96,'r');
ylabel('J311');
xlabel('q');
hold off

figure
plot(qq,J312,'b');
hold on
ylim([0 inf]);
yline(1.96,'r');
ylabel('J312');
xlabel('q');
hold off

figure   %ZtZs4
plot(qq,J411,'b');
hold on
ylim([0 inf]);
yline(1.96,'r');
ylabel('J411');
xlabel('q');
hold off

figure
```

```
plot(qq,J412,'b');
hold on
ylim([0 inf]);
yline(1.96,'r');
ylabel('J412');
xlabel('q');
hold off

% Ultimate function Output
JJ = cat(1,J111,J112,J211,J212,J311,J312,J411,J412);

% Display the final results: test statistic and p value
Test = [J211(1,1); J211(1,10); J211(1,20); J211(1,30); J211(1,40);
J211(1,50); J211(1,60); J211(1,70); J211(1,80); J211(1,90); J211(1,99)];
pval = (1-tcdf(Test, T-1));

columnNames = {'Test_stat','p_value'};
rowNames =
{'Q1';'Q10';'Q20';'Q30';'Q40';'Q50';'Q60';'Q70';'Q80';'Q90';'Q99'};
fprintf('Nonlinear Granger causality test\n');
Table = table(Test, pval,'RowNames',rowNames,'VariableNames',columnNames);
disp(Table);

end

%-----------------------------------------------
%% Kernel functions
% Gaussian function
function [k] = gaussian(X)
a = X.^2;
b = (-1/2).*a;
c = exp(b);
k = c./(sqrt(2*pi));
end

% Uniform function
function [k] = uniform(X)
c = zeros(size(X,1),size(X,2));
for i = 1:size(X,2)
    for j = 1:size(X,1)
        if X(j,i)>=-1 && X(j,i)<=1
            c(j,i) = 1;
        else
            c(j,i) = 0;
        end
    end
end
k = (1/2).*c;
end
```

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References

Bibliography

Adrian T. and Brunnermeier M. K. (2011), *CoVaR* New York, pag. 1-44.

Aghion P.R., Bacchetta P., Banerjee A. (1999), *Capital Markets and the Instability of Open Economies*, in Agenor P.R., Miller M., Vines D., Weber A.A. edited by *The Asia Financial Crisis: Causes, Contagion and Consequences*, Cambridge University Press, Cambridge.

Acemoglu D., Ozdaglar A. and Tahbaz-Salehi A. (2015), *Systemic Risk and Stability in Financial Networks,* American Economic Review*,* Vol. 105, No. 2, pag. 564-608.

Acharya V. and Merrouche O. (2013), *Precautionary Hoarding of Liquidity and Interbank Markets: Evidence from the Subprime Crisis,* Review of Finance, vol. 17, issue 1, pag. 107-160.

Acharya V. and Pedersen L.H. (2005), *Asset pricing with liquidity risk*, Journal of Financial Economics, Volume 77, Issue 2, pag. 375-410.

Acharya V., Pedersen L.H., Philippon T. and Richardson M. (2016) *Measuring Systemic Risk*, The Review of Financial Studies, Vol. 30, Issue 1, pag. 2–47.

Acharya V. and Thakor A.V. (2016), *The dark side of liquidity creation: Leverage and systemic risk,* Journal of Financial Intermediation, vol. 28, issue C, pag. 4-21.

Alentor A., Nier E., Yang J. and Yorulmazer T. (2008), *Network Models and Financial Stability*, Bank of England Working Paper No. 346, 29 pages.

Alessi L. and Detken C. (2011), *Quasi real time early warning indicators for costly asset price boom/bust cycles: A role for global liquidity*, European Journal of Political Economy, vol. 27, issue 3, pag. 520-533.

Allen F. and Gale D. (2000), *Financial Contagion*, Journal of Political Economy, pag. 1-33.

Allen L. and Jagtiani J. (2000), *The Risk Effects of Combining Banking, Securities and Insurance Activities*, Journal of Economic and Business, 52, pag. 485-497.

Amini H. and Cont R. (2010), *Stress Testing the Resilience of Financial Networks*, SSRN Electronic Journal 15(01):1250006-1.

Artzner P, Delbaen F., Eber J.M. and Heath D. (1999), *Coherent Measures of Risk*, Mathematical Finance 9(3), pag. 203 – 228.

Avesani R.G. (2005), *A Market-Based Approach to Evaluate Financial System Risk and Stability*, IMF Working Papers (05/232).

Bank for International Settlement (1992), *Recent Developments in International Interbank Relations*, Report prepared by a working group established by the Central Banks of the Group of Ten countries.

Battiston S., Delli Gatti D., Gallegati M., Greenwald B. and Stiglitz J.E. (2011), *Default Cascades: When Does Risk Diversification Increase Stability?*, ETH Risk Center Working Paper No. 11-006, available at SSRN.

Battiston S., Tasca P. and Deghi A. (2017), *Portfolio diversification and systemic risk in interbank networks*, Journal of Economic Dynamics and Control, Elsevier, vol. 82(C), pag. 96-124.

Bazzana F. and Debortoli F. (2002), *Il rischio sistemico in finanza: una rassegna dei recenti contributi in letteratura*, Alea Tech Reports 017, Department of Computer and Management Sciences, University of Trento, Italy.

Bernanke B., Gertler M. and Gilchrist S. (1996), *The Financial Accelerator and the Flight to Quality,* The Review of Economics and Statistics, The MIT Press*,* Vol. 78, No. 1, pag. 1-15.

Billio M., Caporin M., Panzica R. and Pelizzon L. (2015), *Network connectivity and systematic risk*, Technical Report, University Ca' Foscari.

Billio M., Getmansky M.S., Lo A. and Pelizzon L. (2010) *Measuring Systemic Risk in the Finance and Insurance Sectors*, MIT Sloan School Working Paper.

Billio M., Getmansky M., Lo A. and Pelizzon L. (2012), *Econometric Measures of Connectedness and Systemic Risk in the Finance and Insurance Sectors*, Journal of Financial Economics, 2012, vol. 104, issue 3, pag. 535-559.

Bisias D., Flood M.D., Lo A.W. and Valavanis S. (2012), *A Survey of Systemic Risk Analytics*. U.S. Department of Treasury, Office of Financial Research No. 0001, available at SSRN.

Borio C. (2010), *Implementing a macroprudential framework: Blending boldness and realism,* Bank for International Settlements.

Borio C. and Drehmann M. (2009), *Assessing the risk of banking crises – revisited,* Bank for International Settlements Quarterly Review.

Brock W.A. and Hommes C.H. (1998), *Heterogeneous beliefs and routes to chaos in a simple asset pricing model*, Journal of Economic Dynamics and Control, 22, pag. 1235-1274.

Brownlees C. and Engle R.F. (2017) *SRISK: A Conditional Capital Shortfall Measure of Systemic Risk,* The Review of Financial Studies, Vol. 30, Issue 1, pag. 48–79.

Brunnermeier M., Eisenbach T. and Sannikov Y. (2012), *Macroeconomics with Financial Frictions: A Survey*, Advances in Economics and Econometrics, Tenth World Congress of the Econometric Society, New York Cambridge University Press.

Brunnermeier M., Gorton G. and Krishnamurthy A (2012), *Risk Topography*, NBER Macroeconomics Annual 26, 26, pag.149-176.

Brunnermeier M.K., and Sannikov Y (2014), *Monetary Analysis: Price and Financial Stability*, ECB Forum on Central Banking, Sintra, Portugal.

Brunnermeier M. and Pedersen L.H. (2008), *Market Liquidity and Funding Liquidity*, Published by Oxford University Press on behalf of The Society for Financial Studies.

Busse M., Dacorogna M. and Kratz M. (2013), *The Impact of Systemic Risk on the Diversification Benefits of a Risk Portfolio,* Article for Risks 2(3).

Bussiere M. and Fratzscher M. (2006) *Towards a new early warning system of financial crises,* Journal of International Money and Finance, vol. 25, issue 6, pag. 953-973.

Calvo G. and Mendoza E. (1999), *Rational Contagion and the Globalization of Securities Markets*, Journal of International Economics.

Camerer C. (1989), *Bubbles and fads in asset prices*, Journal of Economic Surveys, 3, pag. 3-41.

Capuano C. (2008), *The Option-iPoD: The Probability of Default Implied by Option Prices Based on Entropy*, IMF Working Paper No. 08/194, available at SSRN.

Carlstrom C.T. and Fuerst T.S. (1998), *Agency costs and business cycles,* Economic Theory volume 12, pag. 583–597.

Cellai D., Hurd T.R., Melnik S. and Shao Q. (2014), *Illiquidity and Insolvency: A Double Cascade Model of Financial Crises*, available at SSRN.

Chan N., Getmansky M., Haas S., and Lo A. (2004), *Systemic Risk and Hedge Funds*, Working Paper by National Bureau of Economic Research.

Chan-Lau J. and Gravelle T. (2005), *The End: A New Indicator of Financial and Nonfinancial Corporate Sector Vulnerability*, IMF Working Paper No. 05/231, available at SSRN.

Chen Y. (1999), *Banking Panics: The Role of the First-come, First-served Rule and Information Extenalities*, Journal of Political Economy, pag. 946-968.

Chow G., Jacquier E., Lowrey K. and Kritzman M. (1999), *Optimal Portfolios in Good Times and Bad." Financial Analysts*, Journal, vol. 55, no. 3, pag. 65–73.

Conrad C. and Loch K. (2012), *Anticipating Long-Term Stock Market Volatility,* Journal of Applied Econometrics 30(7).

Cont R., Moussa A. and Santos E. B. (2010), *Network Structure and Systemic Risk in Banking Systems*, available at SSRN.

Cover T.M. and Thomas J.A. (2006), *Elements of Information Theory (Wiley Series in Telecommunications and Signal Processing),* Wiley-Interscience, 605 Third Avenue New York, NY United States.

Danielsson J., Shin H.S. and Zigrand J.P. (2010), *Risk Appetite and Endogenous Risk,* FMG Discussion Papers from Financial Markets Group.

Das D., Uppal R. (2001), *Systemic Risk and International Portfolio Choice*, Working Paper.

Davis E. and Karim D. (2008), *Comparing early warning systems for banking crises,* Journal of Financial Stability, vol. 4, issue 2, pag. 89-120.

De Bandt O. and Hartmann P. (2000), *Sistemic Risk: A Survey*, ECB working papers series.

De Long J.B., Shleifer A., Summers L.H., Waldmann R. J. (1990), *Noise Trader Risk in Financial Markets*, Journal of Political Economy, 98, pag. 703-738.

Demirgüç-Kunt A. and Detragiache E. (1999), *Monitoring Banking Sector Fragility A Multivariate Logit Approach*, IMF Working Paper No. 99/147, available at SSRN.

Diamond D.V. and Dybvig P. (1983), *Bank Runs, Deposit Insurance, and Liquidity*, Journal of Political Economy, pag. 401-419.

Diamond D.W. and Rajan R.G (2001), *Liquidity Risk, Liquidity Creation, and Financial Fragility: A Theory of Banking,* Journal of Political Economy*,* Vol. 109, No. 2 pag. 287-327.

Drehmann M. and Juselius J. (2014), *Evaluating early warning indicators of banking crises: Satisfying policy requirements,* International Journal of Forecasting, 2014, vol. 30, issue 3, pag. 759-780.

Duan J.C. and Zhang C. (2013), *Cascading Defaults and Systemic Risk of a Banking Network*, Paper available at SSRN.

Duan J.C. and Wei J.Z. (2007), *Systematic Risk and the Price Structure of Individual Equity Options*, Review of Financial Studies, available at SSRN.

Engle R. and Rangel J. (2008), *The Spline-GARCH Model for Low-Frequency Volatility and Its Global Macroeconomic Causes,* Review of Financial Studies, vol. 21, issue 3, pag. 1187-1222.

Elsinger H., Lehar A. and Summer M. (2006), *Risk Assessment for Banking Systems,* Management Science, vol. 52, issue 9, pag. 1301-1314.

Epstein L. and Wang T. (1994), *Intertemporal Asset Pricing Under Knightian Uncertainty*, Econometrica, vol. 62, issue 2, pag. 283-322.

Freixais X., Parigi B.M. and Rochet J. (2000), *Systemic Risk, Interbank Relations and Liquidity Provision by the Central Bank,* Journal of Money Credit and Banking, pag. 611-640.

Furfine C.H. (2003), *Interbank Exposures: Quantifying the Risk of Contagion,* Journal of Money, Credit and Banking, 2003, vol. 35, issue 1, pag. 111-28.

Gai P. and Kapadia S. (2019), *Networks and systemic risk in the financial system*, Oxford Review of Economic Policy, Oxford University Press, vol. 35(4), pag. 586-613.

Geanakoplos J. and Fostel A. (2014), *Endogenous Collateral Constraints and the Leverage Cycle,* Annual Review of Economics, Vol. 6, pag. 771-799.

Ghysels E., Andreou E. and Kourtellos A. (2010), *Regression Models with Mixed Sampling Frequencies*, Journal of Econometrics, 158, pag. 246-261.

Ghysels E., Santa Clara P. and Valkanov R. (2004), *The MIDAS Touch: Mixed Data Sampling Regression Models*, CIRANO Working Papers.

Giglio S., Kelly B.T. and Pruitt S. (2015), *Systemic Risk and the Macroeconomy: An Empirical Evaluation,* Fama-Miller Working Paper, Chicago Booth Research Paper No. 12-49, Available at SSRN.

Giudici P. and Parisi L. (2016), *CoRisk: Measuring Systemic Risk Through Default Probability Contagion*, Paris December 2016 Finance Meeting EUROFIDAI - AFFI, available at SSRN.

Gorton G. (1985), *Bank Suspension of Convertibility*, Journal of Monetary Economics, 15, pag.177-193.

Grossman S.J. and Stiglitz J.E. (1980), *On the Impossibility of Informationally Efficient Markets*, American Economic Review, 70, pag. 393-408.

Haltiwanger J., Waldman M. (1985), *Rational Expectations and the Limits of Rationality: An Analysis of Heterogeneity*, American Economic Review, 75, pag. 326-340.

Hart O.D., Kreps D. M. (1986), *Price Destabilizing Speculation*, Journal of Political Economy, 94, pag. 927-952.

Hautsch N., Schaumburg J. and Schienle M. (2013), *Financial Network Systemic Risk Contributions*, available at SSRN.

Hellwig M.F. (2009), *Systemic Risk in the Financial Sector: An Analysis of the Subprime-Mortgage Financial Crisis*, De Economist 157, pag. 129–207.

Hong Y., Liu Y. and Wang S. (2009), *Granger causality in risk and detection of extreme risk spillover between financial markets*, Journal of Econometrics.

Huang Q. (2019), *Systemic risk and financial regulation*, University of Groningen, SOM research school.

Jeong K., Hardle W.K. and Song S. (2012), *A Consistent Nonparametric Test for Causality in Quantile,* Article in Econometric Theory.

Kaminsky G., Lizondo S. and Reinhart C. (1998), *Leading Indicators of Currency Crisis,* IMF Staff Papers, Vol. 45, Number 1.

Kaminsky G. and Reinhart C. (1998), *On Crisis, Contagion and Confusion,* mimeo.

Kaufman G. G. (1994), *Bank Contagion: A Review of the Theory and Evidence*, Journal of Financial Services Research.

Keynes J. M. (1936), *The General Theory of Employment, Interest and Money*, McMillan, London.

Khandani A.E. and Lo A.W. (2011), *Illiquidity Premia in Asset Returns: An Empirical Analysis of Hedge Funds, Mutual Funds, and US Equity Portfolios,* Quarterly Journal of Finance (QJF), vol. 01, issue 02, pag. 205-264.

Kindleberger C.P. (1978), *Manias, Panics and Crashes: A History of Financial Crisis*, MacMillan, London.

Knight F.H. (1921), *Risk, Uncertainty and Profit*, Houghton Mifflin Company.

Kodres L. and Pritsker M. (2001), *A Rational Expectation Model of Financial Contagion*, Working Paper.

Koenker R. and Bassett G. (1978), *Regression Quantiles*, Econometrica, Vol. 46, No.1, pag. 33-50.

Koenker R. (2005), *Quantile Regression*, Cambridge University Press.

Kritzman M., Li Y., Page S. and Rigobon R. (2010), *Principal Components as a Measure of Systemic Risk*, available at SSRN.

Kupiec P. and Nickerson D. (2004), *Assessing Systemic Risk Exposure from Banks and GSEs Under Alternative Approaches to Capital Regulation*, The Journal of Real Estate Finance and Economics, Springer, vol. 28(2_3), pag. 123-145.

Lang J.H. and Forletta M. (2019), *Bank capital-at-risk: measuring the impact of cyclical systemic risk on future bank losses*, ECB Macroprudential Bulletin, vol. 9.

Lawrence M., Goodwin P., O'Connor M. and Onkal D. (2006), *Judgmental forecasting: A review of progress over the last 25years*, International Journal of Forecasting, 22, pag. 493-518.

Lee S.C., Lin C.T. and Yang C.K. (2011), *The Asymmetric Behavior and Procyclical Impact of Asset Correlations,* Journal of Banking & Finance 35(10):2559-2568.

van Lelyveld I. and Liedorp F. (2006), *Interbank Contagion in the Dutch Banking Sector: A Sensitivity Analysis*, International Journal of Central Banking, vol. 2(2).

Lima L. R. and Menf F. (2018), *Quantile forecasting with mixed-frequency data*, International Journal of Forecasting. Forthcoming.

Lo Duca M. and Peltonen T. (2013), *Assessing systemic risks and predicting systemic events,* Journal of Banking & Finance 37(7): pag. 2183–2195.

Lublóy Ágnes (2004), *Systemic Risk Implications of the Hungarian Interbank Market,* Working Papers WP 2004/10.

Mazzi G.L. and Mitchell J. (2019), *Nowcasting Euro Area GDP Growth Using Quantile Regression,* EU Working paper.

Minsky H.P. (1977), *A Theory of Systemic Fragility*, in Altman E. I. and Sametz A. W., *Financial Crises*, Sharp, New York.

Minsky H.P. (1982), *Can It Happen Again? Essay on Instability and Finance*, Wiley, New York.

Mistrulli P.E. (2007), *Assessing financial contagion in the interbank market: Maximum entropy versus observed interbank lending patterns*, Temi di discussione (Economic working papers) 641, Bank of Italy, Economic Research and International Relations Area.

Pagano M.S. and Sedunov J. (2016), *A Comprehensive Approach to Measuring the Relation Between Systemic Risk Exposure and Sovereign Debt*, Journal of Financial Stability, Vol. 23, pag. 62-78, available at SSRN.

Peek J. and Rosengren E.S. (1997), *The International Transmission of Financial Shocks: The Case of Japan*, American Economic Review, 87, pag. 495-505.

Poledna S. and Thurner S. (2013), *DebtRank-transparency: Controlling systemic risk in financial networks,* Scientific Reports vol. 3, Article number: 1888.

Roukny T., Battiston S. and Stiglitz J.E. (2016), *Interconnectedness as a Source of Uncertainty in Systemic Risk*, Columbia Business School, Research Paper No. 16-14, available at SSRN.

Schinasi G. and Smith R. T. (2000), *Portfolio Diversification, Leverage, and Financial Contagion*, IMF Staff Papers, 47, pag. 159-176.

Stiglitz J.E. and Weiss A. (1981), *Credit Rationing in Markets with Imperfect Information,* American Economic Association, The American Economic Review, Vol. 71, No. 3, pag. 393-410.

Upper C. and Worms A. (2004), *Estimating bilateral exposures in the German interbank market: Is there a danger of contagion?,* European Economic Review, vol. 48, issue 4, pag. 827-849.

Wells S. (2004), *UK Interbank Exposures: Systemic Risk Implications*, Journal of Monetary Economics II(1).

Wooldridge J.M. (2010), *Econometric Analysis of Cross Section and Panel Data,* MIT Press Ltd*,* The MIT Press*.*

Sitography

MIDAS MATLAB Toolbox written by Eric Ghysels and collaborators, and repacked by Hang Qian: https://it.mathworks.com/matlabcentral/fileexchange/45150-midas-matlab-toolbox.

Principles for Financial Market Infrastructures (PFMI): https://www.bis.org/cpmi/info_pfmi.htm.

MATLAB Documentation to implement each part of the test: https://it.mathworks.com/help/matlab/.