UNIVERSITA' DEGLI STUDI DI PADOVA

DIPARTIMENTO DI SCIENZE ECONOMICHE ED AZIENDALI "M.FANNO"

DIPARTIMENTO DI MATEMATICA

CORSO DI LAUREA MAGISTRALE IN
BUSINESS ADMINISTRATION

TESI DI LAUREA

Inventory Management with Raw Materials Costs Subject to Quotation:

The Analysis of the Jewellery Industry

RELATORE:

CH.MO PROF.SSA Alessandra Buratto

LAUREANDA: Denise Vicari

MATRICOLA N. 1081843

ANNO ACCADEMICO 2015 –2016

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Executive summary

This thesis has the objective to present the particular inventory management problem in case of procurement of raw materials subject to quotation, a subject that goes beyond traditional stock control policies proposed by literature, where purchase price is typically assumed as a constant and therefore not even considered in the decision of when and how much to order.

Inventory, in a more and more competitive environment, allows production and distribution processes to flow. Although stock inevitably ties up capital that could be profitably otherwise employed, if companies find a way to adequately manage it, they can move from considering it as a "necessary evil" to considering it as an assurance of a certain level of continuity and fluidity within business processes, ensuring at the same time the related control of the financial requirements. Inventory management is a complex subject, perceived to a more or less intense extent by all companies, and many operative researchers tried to deal with it, looking for the right balance between its benefits and disadvantages.

In the first chapter, a brief description of the functions of inventory is provided and the principal cost items that form the basis of every inventory management problem, highlighting the difficulties that can be encountered when appropriately measuring and quantifying them. Differently from traditional literature, greater attention is paid to the purchase cost, here considered as variable, which is a much more real situation for those companies that procure raw materials subject to quotation (i.e. commodities), whose price fluctuates over time.

In the second chapter, commodities are presented, together with the particular case of gold: its demand and supply are analyzed, as well as its determinants and uses. More than 50% of global gold demand is for goldsmith and jewellery industry and, because of that, deep analysis of this sector is conducted in the third chapter: its regulation, demand (of Italy and of the three major jewellery markets: India, China and U.S) and offer are studied, as well as the traditional procurement procedures and inventory management system applied.

In the fourth chapter, some of the models that deal with procurement of raw materials with fluctuating prices are presented. These can be divided into four groups based on what aspect of commodities procurement they focus on, their assumptions and conclusions; specifically the analyzed models can be ascribed into the following categories: order-up-to level models, minimum and maximum stock models, price thresholds models and E.O.Q.-based models.

Based on their characteristics' examination, a table is developed, aimed to benefit all those entities that operate with fluctuating price raw materials.

A possible modification of the model developed by Teng and Yang [2004] (Deterministic economic order quantity models with partial backlogging when demand and costs fluctuate over time) is also suggested in order to better fit the particular characteristics of the Italian jewellery industry and its typical procurement procedures, especially taking into account the "fashion" aspect which is inherent in the sector.

Chapter 1

Inventory:

Definition, Functions, Main Typologies and Problems

1.1 Definition of inventory and historical evolution of inventory systems

The majority of authors define *inventory* or *stock* as a reserve of resources waiting for a future use [Hillier, Lieberman 1995; Urgeletti Tinarelli 1992; Grando 1995], that can be both external (future sale) as well as internal (employment in subsequent productive stages) to the company.

Some other authors mean by stock what "exceed the normal requirement"; however, this definition seems far too vague to be effectively used, considering that it would be also necessary to be precise on what is meant by "normal requirement".

Ackoff, Sasieni [1968] moved from the traditional definition of resources as raw material, work-in-progress (WIP) and finished goods that, in a certain moment, wait to take part in a transformation or distributive process [Grando 1995] to embrace a wider one: usable and idling resources that can be of any type (e.g. men, materials, machines or money).

Similarly, Slack, Chambers, Johnston [2007] describe inventory as an accumulation of *transformed resources*, including not only materials, but also customers and information, as they flow through processes or networks; as a consequence, one will talk about physical inventory, queue and databases, respectively.

However, for the purpose of this analysis we will consider the inventory as all the physical entities that, in a certain moment in time, are under the control of who manages them [Urgeletti Tinarelli 1992].

Irrespective of the form of the resources taken into consideration, many authors identify stocks, the link between all productive and distributive activities, as the result of uneven flows: whenever there are differences between the timing or the rate of supply and demand, accumulation of resources occurs. A largely used analogy consists in comparing the inventory of a certain product to a tank, with a faucet and a hole for letting the water come in and out, respectively [Urgeletti Tinarelli 1992; Slack, Chambers, Johnston 2007].

Because of the uncertainty of the demand, a certain amount of product (water) will be constantly maintained as stock (tank) in order to ensure that customers' demand is satisfied when it occurs and avoid a *stock-out* situation. When rate of supply exceeds rate of demand, inventory increases; vice versa it decreases. The hole –i.e. the demand- is elastic, contracting and expanding according to unpredictable market forces and therefore it cannot be managed by the company. Then, the only choice left to the manager is to act on the faucet, controlling timing and volumes of entry; that is why inventory management decisions typically consist in deciding how much and when to order. Alternatively, only a *pull production* logic, a methodology requiring a component to be bought when it is needed and not before (*Just-in-time*) made famous by Toyota Motor Corporation, allows a substantial reduction of the stock, nearly to zero, without affecting the service offered.

The role played by inventory is central in all kind of companies, whether manufacturing, commercial or service companies, and they are often the key for their survival and success or failure, especially if we consider the uncertainty that affect firms, referring to both supply and demand as well as within the firm itself. Given the exogenous and endogenous variability that affects the different procurement/transformation/distribution stages, where exogenous is referred to the discontinuous development of procurement and final market and uniformity required by productive processes, and endogenous to the differences in timings and volumes between adjacent productive stages, Grando [1995] identifies the fundamental function of inventory as bringing back to unit these variabilities.

Because of this endogenous variability, it may not be possible to have all productive stages working in a perfectly synchronized manner; for example, this may be due to the existence of bottlenecks. This is the reason why the majority of authors consider the *decoupling* function as the main cause of inventory itself. Decoupling stocks, conveniently sized, in fact allows successive productive stages in the manufacturing and distribution process to operate independently from one another [Thierauf, Grosse 1970]. In this sense, inventory is often compared to a fly-wheel and a shock absorber, where former allows overcoming inactivity and dead times that can sometimes occur in the processes and the latter to soften the internal and external variability [Grando 1995, Urgeletti Tinarelli 1992].

On one hand, stocks are a necessary for the decoupling function they perform and because it would be physically impossible or even economically unsound to have goods arriving in a system in the exact moment when customers demand them, on the other hand, the problems they generate are far from being indifferent. Based on what said so far, it is clear that inventory management has to conciliate these contrasting requirements resulting in a typical trade-off situation between the relative advantages, among which the guarantee of a certain determined service level to customers, and disadvantages , that arise from keeping a certain amount of inventory (whether it is made of raw materials, WIP or finished goods).

Despite the competitive pressure posed to the offer of a company, e.g. quality, variety, short response time to both customers and market (sometimes referred to as *lead time* and *market time* respectively), that requires to keep stock, also in order to counteract a lack of flexibility of the productive process (later we will talk of *cycle inventory*), the minimization of costs still maintain its central relevance in all business decisions.

The correct definition of what and how much should be stored should therefore disregard subjective, psychological and emotional considerations [Urgeletti Tinarelli 1992], on the contrary, it should be based on costs and benefits relative to the existence or lack of inventory, although an equilibrium is not so easy to find, keeping in mind the objective that is pursued: the realization of the maximum profit, at least in the long run.

Historically, the attitude towards inventory and the methodologies for its efficient determination has been swinging between fatalism and pragmatic empiricism, leading to skepticism about the possibility of actually managing this phenomenon [de Witt 1978]. With the 20th century, however, attempts have been to apply analytical techniques to inventory management, maybe due to the simultaneous growth of manufacturing industries and industrial engineering. A first derivation is the well-known Wilson formula, which works mainly under deterministic conditions, but it is only until World War II that detailed attention was focused on the stochastic nature of inventory problems [Hadley, Whitin 1963]. While the scientific approach is what seems to be predominant nowadays in Western countries, the same cannot be said for Eastern ones; there the trend, brought into reality by Toyota, is to brings into question the necessity itself of keeping a stock of items, considered as a waste, by working on continuous improvement on being lean, with the ideal to meet demand instantaneously with perfect quality and no waste [Slack, Chambers, Johnston 2007].

1.2 Problems and objectives of inventory management

Based on what said so far, there is the need to keep only what is has to be sold or consumed, if goods are for internal use, as a stock and nothing more, if we do not want to incur in additional unnecessary costs and give up to other more profitable investments. Although different types of organizations (retail, wholesale, manufacturing) have different inventory requirements, the main questions to be answered are the same:

- how much do we order (or produce) of each product? – volume decision;
- when do we order (or start producing) each product and at which frequency – timing decision.

When dealing with these questions, we have to keep in mind that the level of stock is influenced by different productive processes, market variables and industry dynamics, as well as type of industry itself, and organizational structure. The firm's objectives have to be taken into consideration, that are customer service maximization, inventory investment minimization and efficiency (i.e. low-cost) operations.

The problem is that one objective is reached at the expenses of the others. Low-cost operations are maintained if production volumes are seldom changed, no overtime is incurred and set-up frequency for machines are kept at a minimum; a similar strategy results in large inventory and poor customer service. Inventory can be kept low if plant is flexible enough (where flexibility is considered both as mix flexibility, i.e. the ability to produce a wide range of products and services, as well as volume flexibility, i.e. the ability to change the level of output to produce different quantities of products and services over time [Slack, Chambers, Johnston 2007]) and customers are made to wait. If high levels of inventory are kept and plant is flexible in order to meet the demand whenever it occurs, maximum customer service is reached while both inventory and operations perspectives suffer.

Each of these objective is the goal of managers of different departments within the organization; since inventory control is at a crossroads in the activity of a company, and different mangers a certain role to play which involve the use of inventory, they all have different ideas about what the optimal stock level should be, giving rise to conflict within the company itself and sub optimization problems [Wild 1997, Tersine 1994].

Department	Requirement	Inventory goal	Inventory inclination
Marketing	Sell the product	Good customer service	High
Purchasing	Buy required material	Low cost per unit	High
Production	Make the product	Efficient lot size	High
Finance	Provide working capital	Efficient use of capital	Low
Engineering	Design the product	Avoiding obsolescence	Low

Table 1 Departmental orientation toward inventory [from Tersine 1994]

Each inventory goal cited in tab.1 has its associated cost. Fundamental element for stock control is the identification of these costs, so that everybody can realize the amounts involved and, most importantly, create and equilibrium among them in the light of broad purpose, profit maximization. As a consequence, we shall not think of inventory as separate from manufacturing, sales and other departments; on the contrary, its control should consider the entire organization [Magee, Boodman 1992]. In order to do so, inventory management will be much more simple, the more regular, and therefore predictable, the inflows and outflows are;

information are consequently needed about them, business and market constraints and costs and revenues associated with them.

1.3 Inventory typologies and functions

As stated in paragraph 1.1, the major function performed by stock is to decouple subsequent productive stages of the procurement/production/sale process that usually have different rhythms, allowing a more economical management for each of them. The reasons for these differences between supply and demand [Tersine 1994] or, broadly speaking, the motives [Graves 1993] that justify the existence of inventory within a company can be explained as follows:

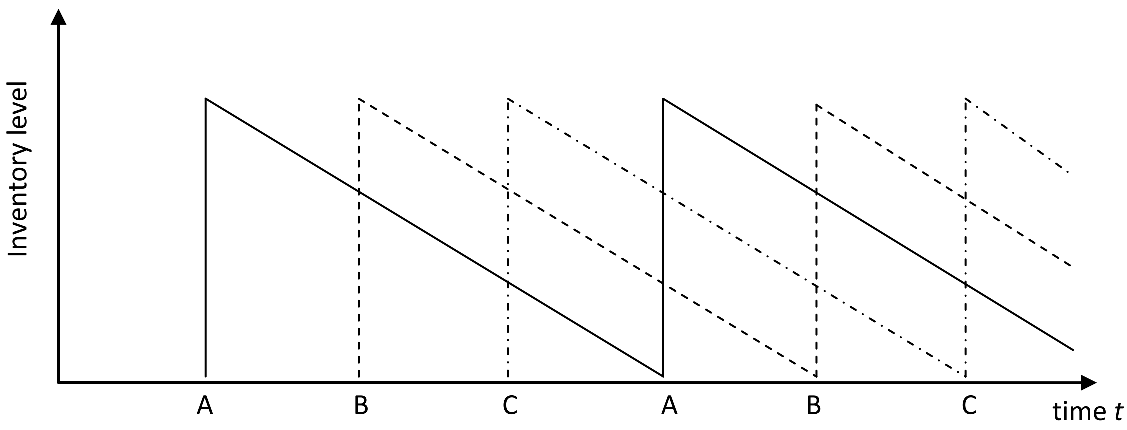
- *time factor* involves the production/distribution process that materials and components undertake, before the finished goods are made available. The length of this process can be considerable (time is needed to develop production schedule, ship raw material from suppliers first and to consumer or wholesaler then, produce the product) and not all costumers are willing to wait this long, they could actually move to competitors if they do not find what they look for. Inventory, by reducing shortage risk and consequently lost sales, enables the business to reduce the lead time in meeting demand immediately or within a reasonable time, fact that can even increase company's reputation and therefore profitability. They can be compared to the *transit inventories*, *according* to the classification proposed by Grando [1995] that accumulated to cover the demand during the production/transportation period; it optimizes productive process efficiency and it is proportional to the time taken to transfer a good from a transformation point to another;
- *discontinuity factor*, usually reconducted to the decoupling function of inventory, permits the treatment of the various dependent operations in an independent and economical manner and their scheduling at a more desirable performance level than if they were treated as a whole (*organizational inventory* according to Grando [1995]);
- *uncertainty factor* or *precautionary motive*: inventories are often retained as a hedge against uncertainty, relative to both the demand, most obviously, and supply. Problems that contribute to its determination are transportation anomalies and delays, failed, incomplete or delayed supply, unplanned machinery idle times and consequent product shortage, late, inaccurate, unreliable information further unexpected problems causing service level deterioration and suspension: they all may affect the ability of the company to meet the demand, as well as difficulties about the prediction of costs and the considerable price variation commodities, such as precious metals are subject to, make the presence of a *safety stock* [Grando 1995] necessary to protect the business firm

unanticipated and unplanned events, although leading to a higher overall inventory position, that ensures the balanced and uninterrupted execution of the operations. The amount of this kind of stock is determined by fixing a safety standard that takes into account the variability of the demand and the service level the company desires to offer to its customers, where the service level can be defined as percentage of the stock-out periods considered as acceptable compared to the totality of periods of the demand, or alternatively the amount of unsatisfied requirements in respect of the overall requirement;

- *economy factor* or *transaction motive* allows the organization to take advantage of cost reducing possibilities, by ordering or producing items in large lots (economic quantities). Due to purchases of quantities larger than what is actually needed and driven by price discount or transport optimization, these *lot size inventories* [Grando 1995] permit the creation of economies of scale that can reduce fixed costs (more notably, set-up and transportation costs) when the number of set-ups is reduced or the number of transactions is minimized, and can also smooth production and stabilize manpower levels in seasonal businesses;
- *speculative motive*, which can be divided into two sub motives. If the cost of purchasing or manufacturing an item is expected to increase in a determined period of time, it is advantageous to hold inventories in anticipation of such price increase. Companies that use commodities whose value may fluctuate would be better off stockpiling inventories. Held to take advantage of an expected price variation, in terms of both purchase and sale price, in a determined period of time, the mere *speculative inventory* finds the rationale for its existence in a comparison between risks and costs connected to and advantages and revenues following them. Stock may be also retained in advance of sales increases: if demand is expected to increase, it may be more economical to build up large inventories rather than increase production capacity at a future time [Graves 1993].

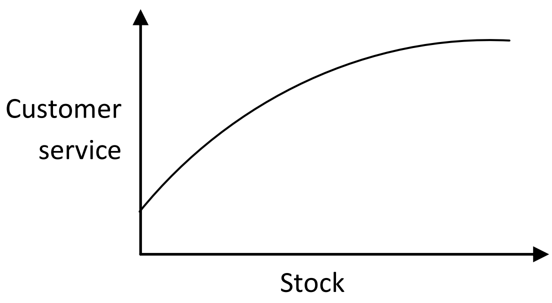
Based on the functions performed, inventory can be classified into the following typologies proposed by Tersine [1994]:

- *working stock* (*cycle* or *lot size stock*) is inventory acquired and held in advance of requirements so that ordering or production can be on a lot size rather than on an as needed basis. As already explained in the *economy factor/transaction motive*, lot sizing is done because minimization of ordering and holding costs and quantity discounts can be pursued. Sometimes it can be compared to *cycle inventory*, as defined by Slack, Chambers and Johnston [2007]: unless the operation is perfectly flexible, inventory is needed to ensure supply when it is engaged in other activities, given the impossibility of providing simultaneously a wide product range that forces to produce every item in distinct batches; cycle inventory compensates for intermittent supply of each type of product offered and should be sufficiently large to cover the demand of the period between two subsequent batches of the same product (pic.1).



Picture 1 Cycle inventory of products A, B, C [Slack, Chambers, Johnston 2007]

- *safety stock* (or *buffer* or *anticipation stock*), as illustrated by *uncertainty factor/precautionary motive,* is the inventory kept to protect against uncertainties of supply and demand. Unexpected occurrences like equipment breakdowns, shipping delays, unusual weather conditions, acts of God, errors in demand estimates and so on contribute to its formation, whose amount however depend on the combined valuation of the variability of the demand, reliability of supply and dependability of transport, keeping in mind that a trade-off between investment and availability of items exists [Wild 1997]. For each stocked item, the risk of stock-out is reduced by increasing the stockholding; the larger the investment the better the service. However, every additional stocked item has diminishing returns on the service level: as illustrated in pic.2, while it is relatively inexpensive to provide reasonable service levels, exceptionally good service levels are achieved by rapidly increasing inventories. The risk is that costs associated with such investment can greatly exceed the benefits perceived by consumers.



Picture 1 Trade off between stock and service level, expressed as stock availability [adjusted from Wild 1997]

Safety stock has two effects on company's costs: it decreases stock-out costs and increases holding costs. However, keeping a safety stock as assurance against the greatest possible demand and delay is not possible, since the costs associated with a policy of that kind would exceed the benefits that would be obtained with a complete elimination of the stock-out risk. The objective is therefore to find an equilibrium between the size of this extraordinary inventory (and the relative holding costs, capital costs and so on) and the assurance it provides: the higher the safety stock, the lower the risk of stock-out but the marginal assurance provided by each single unity of additional inventory decreases as safety stock itself increases. As a consequence, the problem is the identification of appropriate size of buffer inventory that could be considered economically justified [Magee, Boodman 1992; Tersine 1994]. Its level is therefore commensurate with demand's and lead time's stability and targeted service level and determined based on the comparison between the costs that the company would suffer because of additional stocked items for precautionary motives and the costs generated because of insufficient inventory [Grando 1995];

- *anticipation stock* is inventory kept to cope with large bur predictable fluctuation of the demand, as in the case of seasonality, erratic requirements (promotional campaigns, strikes, vacations shutdowns) and insufficient production capacity. Under these circumstances, it is convenient to order or produce in advance of demand requirement, leading to the formation of a stock, also known as *seasonal* or *stabilization stock*, that is depleted during peak demand periods, in this way it is in fact possible to level production rates and stabilize work force, therefore avoiding costs associated with changing production capacity (e.g. overtime);
- *pipeline stock* is inventory that is in transit, therefore not employed in any value-adding activity, to allow for the time materials take to be received, to flow through the production process where they are transformed into finished goods that are eventually delivered to consumers. It refers externally to inventory that is actually being shipped, internally to inventory that is being processed, waiting to be processed or being moved;
- *decoupling stock*, as already described, isolates one productive/distributive stage from the adjacent one to allow each one to operate independently;
- *psychic stock refers to amount of retail display inventory that stimulate demand and act* as salesperson, increasing the chances an item is seen and considered for the purchase. Its amount has a psychological effect on consumers' behavior: providing product visibility, full shelves increase sales exposing them as much product availability as possible, while lost sales and customers may be due to under-stocked shelves or stock-outs.

In addition to the previously stated reasons, current levels of inventory may be justified by exogenous phenomena and internal organization inefficiencies, these are indicated by Wild [1997] as follows:

- *market changes*: step or gradual changes (in the demand, entrance of new and/or substitute products, the loss of preexisting contractual agreements and so on) can contribute to both excesses and stock-outs, with their related obsolescence and shortage risks. Inputs from sales personnel are therefore critical;
- *insufficient communication between company's departments* can cause the accumulation of stocked items and their obsolescence, e.g. the marketing manager puts an item on promotion without informing the inventory controller; designers replace the old version of a product by an improved one, starting immediately selling it, with still large amounts of old product left unsold and so on;
- *poorly defined responsibilities* among departments, that may favor, in a myopic manner, the achievement of departmental objectives at the expenses of the overall company's ones.
- *layout and location of stores*: in companies where stores are inconveniently situated or far away from users, users themselves tend to keep their own stock, or if different departments order the same item independently, duplication of resources can occur;
- *low or inadequate systems and controls* can substantially affect the level of stockholding, if the time taken to process information is excessively long.

As emerged, there exist various types of inventories intended to serve a variety of purposes that, although being managed in different ways, should be consistent with the objectives established by the organization as a whole. Albeit functional reasons and classification previously explained justify the existence of inventory, this does not imply that companies should not attempt to reduce them, since they often tend to hide operational problems.

1.3.1 Benefits of inventory

From an accurate analysis of the function performed, the benefits of keeping stocked items within a company, whatever will be the classification considered, clearly emerge.

They counteract a lack of flexibility in production and purchase plan, avoiding production capacity modification from period to period, that would result in higher costs, and intermittent supply of finished goods, because of the constraints over cyclical production; allow operations to take advantage of short-term opportunities (for example, suppliers offering a particularly good deal or speculation on price/cost variations); can help in reducing overall costs (because of economic quantities and, when holding relatively large inventory, savings are greater than the cost of holding it, that happens for example when bulk-buying gets the lowest possible cost of inputs or when large order quantities reduce the number of order placed, the associated costs of administration and material handling [Slack, Chambers, Johnston 2007] and unitary transportation costs); can increase in value and become an investment (e.g. wine, gold); can help to face uncertainty, that affects the demand (by anticipating it, when its fluctuation are large but predictable, e.g. in seasonal businesses or when sales are expected to increase because of promotions; avoid stock-outs thanks to *buffer*/*safety stock*, thereby developing a reliable reputation) as well as the supply (e.g. unplanned production variation because of strikes, machine breakdowns, idle times, delays on procurements from suppliers, and so on).

Summing up, if conveniently employed, stock, like any other production mean, provides a return in terms of increased manpower productivity, by reducing labor and training costs as well the need for further training, machinery, plant and equipment investments; while providing requested products and services to customers. As a consequence, inventories are necessary to a production/distribution system as much as plants, machinery and transportation. [Magee, Boodman 1992].

1.3.2 Disadvantages of inventory

Although providers of indisputable advantages, these are counterbalanced by the drawbacks, already emerged in the dissertation, associated with keeping stocked items with a company. Even if appropriate actions can be taken to reduce them as much as possible, some of them will always affect the business entity since they are typically considered as "necessary evil".

When looking at the balance sheet of a company, inventory typically represents a major component of current assets. The greater the accounted amount, the greater the working capital. However, this has detrimental effects on the financials of a business.

It actually ties up large amount of capital, that could be otherwise more profitably invested (for profit making, debt servicing, dividend distribution), with the possibility of negative cash flow creation if it is not coupled with an accurate management of, among others, account payables and receivables. It limits the expansion of an organization through the lack of capital and reduces the return on investment by broadening the investment base. It also gives rise to actual costs, in the form of passive interests, if it is financed by third parties' capital. Large inventory can both limit the interest of investors, who look for attractive indexes and ratios (e.g.. low indebtedness and high invested capital rotation) that are not achievable with high levels of inventory, and consequently penalize the company in capital collection, as well as result into lower contracting power with creditors and debtors.

As for any other type of asset, there is an optimal level: having too much can impair income as much as having too little [Tersine, 1994]. Therefore, attention has to be paid in avoiding over-investments in inventory, which can become obsolete, deteriorate over time or damaged, especially in case of slow-moving items (i.e. with low turnover ratios) that can even increase in the event of entrance in the market of new substitute products, making the business sustain the related costs, in addition to the administrative, insurance and storage costs, far from being negligible. Moreover, with high capacity storehouse additional problems may arise because of inadequacy of the available space organization and management systems.

A not so evident disadvantage of stock is that it may hide problems within the organization: inventory can be seen as a "blanket of obscurity" that lies over production systems and prevents problems being noticed; by reducing stock accumulation between stages the operation can increase the chance of the intrinsic efficiency of the plant being improved [Slack, Chambers, Johnston].

1.4 Inventory control: definition, objectives and limitations

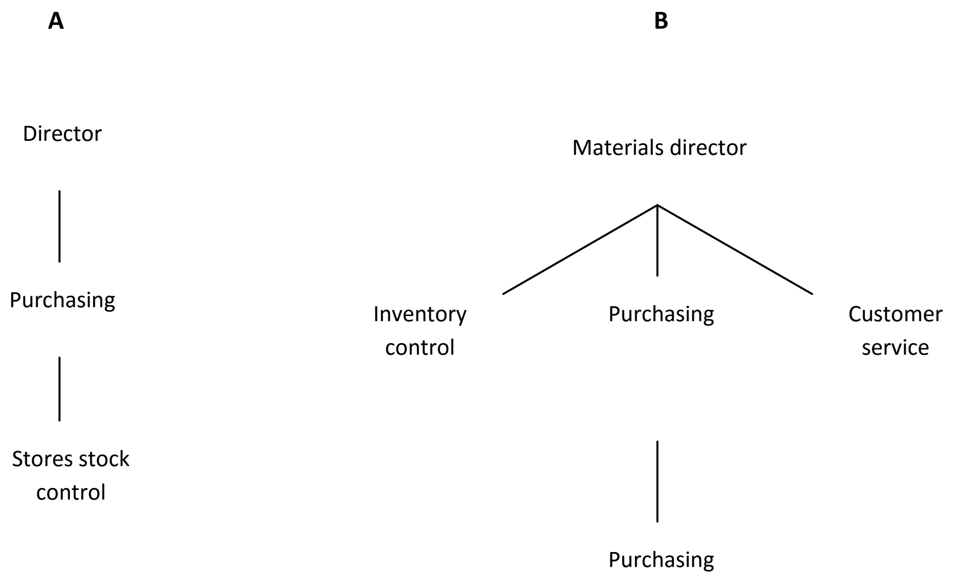
Given the actual competitive scenario, an appropriate inventory management is an essential element for business's survival, ensuring a correct balance between the conflicting requirements of an adequate production/distribution flow, achievable by means of considerable amount of stocked items, and a limited stock investment that deprives the company of the liquidity. Basically, the success of a venture depends on its ability to provide customers with the products or services they ask for, when they ask for, while being financially viable. In order to achieve this balance, *inventory control* is needed.

As sophisticated and efficient as inventory control systems can be, however, they cannot completely eliminate business risk, being a function of uncertainty; what they can actually do is to provide measures for such risk and develop a strategy that allows to face it in the most economical manner, consistent with company's objective.

Inventory control is defined as the activity that organizes the availability of items to consumers within a reasonable timescale, coordinating, overseeing and controlling the ordering, storage and use of components, consumables, spare parts, obsolescent items and all other supplies used in the production (each downstream department is seen as "internal client" of the upstream department), as well as of finished goods, whether they are current sale items or new products, intended for sale to final customer.

Stock control is therefore a key element in the achievement of company's aim, i.e. long-term profitability, although historically disregarded in the business practice, according to which, it had a secondary role, while greatest importance was attributed to production and marketing.

Gradually the inventory control has moved from store keeping to become a management discipline as its responsibilities have changed: if in the past the role of stock control was part of the physical store control and low-positioned in the organization, working for either purchasing, finance or manufacturing department (pic.3A), nowadays it is accepted as an operational activity working alongside purchasing (pic.3B). In case of separation of stock and stock control activities, the warehouse is responsible for managing the movement and holding of stock as well as for providing information to the rest of the company, where it is crucial for the information to be accurate.



Picture 3 Evolution of stock control in management structure [adjusted from Wild, 1997]

Inventory techniques (mainly demand forecasting, safety stock determination, supply pattern negotiation, slow-moving stock avoidance) are commonly applied in all kinds businesses; it is their appropriate mixing that allows different types of stocks to be managed.

Effective production control and marketing have to be strongly tied in with inventory control activities and all have to work together to satisfy the three company's objectives cited in paragraph 1.2, so interconnected that they cannot be isolated from one another without impairing the overall effectiveness of the business.

For example, if we consider a fluctuating demand, there are three main ways to react to aleatory customers' request. If production is very non-flexible (with reference to both volume and mix flexibility), all fluctuations will be absorbed by inventory, that will behave in the opposite way of the demand. Vice versa, whenever production is such flexible to absorb demand fluctuation, it will be synchronized with demand, while inventory will keep its level unchanged. These two situations represent the extremes within which a third case can be found: inventory and production share the costs related to the uncertainty of the demand.

As a consequence, depending on the rigidity of the processes and the costs associated to inventory, the choice between having production, inventory or both fluctuating will be made [de Witt 1978].

These objectives, maximum customer service, minimum inventory investment and efficient operations have to be translated into the following external, challenging but achievable, operational and financial targets to be applied into daily operations [Wild, 1997]:

- customer service, measured as:
  - stock availability or delivery on time
  - analysis of age distribution of lateness
  - assessment of customer satisfaction, as result of surveys
- inventory investment, measured as:
  - value
  - stock cover ratio
- operating costs, measured as:
  - warehousing and inventory operations costs
  - cost per transaction, movement and purchase.

More detailed, internal targets within inventory management department, also including personal objectives for individuals which all contribute to departmental targets, are needed to support the previously mentioned external targets.

However, if this is the ultimate ambition, effective inventory management is limited by the already mentioned contrasting, although legitimate, goals guiding the actions of the single functional area within the organization. It is therefore necessary to make explicit such inter-functional conflict in order to pursue the balance among them in business decisions and day-to-day operations [Magee, Boodman 1992].

The aim is not therefore to make all items available at all times, which would be detrimental to the finances of the company, but to meet the required demand at a minimum cost. Consequently, the most profitable strategy is not to optimize one of these functional objectives at the expenses of the others, but to make value judgments among them [Wild 1997, Plossl, Wight 1967].

1.5 The role of inventory in manufacturing and retail businesses

Although it is true that inventory control is a problem perceived by any type of company and control techniques are the same for all of them, it seems appropriate to draw a distinction between different issues and management models that characterize the role that stock plays in manufacturing and retail businesses. Actually, however, this distinction is not always as clear as it could seem at first sight, with the result that inventory control models, applied in retail companies, could be as effectively applied also in manufacturing companies, where materials management controls are typically employed for the management of stock of raw materials and components that feed productive processes.

For manufacturing companies, based on the different stages of the transformation process in which stock is and the functional purpose it has, the following categories can be identified:

- *raw materials*, purchased from suppliers, they are productive factors acting as inputs in the transformation process that feed the entire productive process. Also auxiliary materials fall within this category, with a mere subsidiary and accessory function, and can be further divided into consumables and spare parts;
- *work-in process (WIP)*, are partially completed final products that are still in the production process, representing both the accumulation of partially completed work and the queue of materials awaiting further processing. They link and balance the various productive stages, each with different productive capacity;
- *finished goods*, at the end of the productive process, are available for sale, distribution or storage.

This classification, useful for explanatory purposes, has to be qualified case by case, since the function that a certain stocked item has in the sequence of transformation/distribution process of company is not necessary the same it has for another one, for example up/downstream located.

Not all of the potentially useful items have to be kept, but only those that are needed for an actual and specific use and whose lack creates relevant damages to the company; as a consequence, if two substitute goods, i.e. destined to satisfy the same need, are stocked, it would be better to eliminate the duplicate, as well as all items obtainable with no additional costs.

However, the classification is useful to recognize the function played by inventory: the availability of raw materials and WIP ensure the regular the execution of production, finished goods try to make consumers' demand compatible with production, in an attempt to face its variability. Stock therefore represents a "lung" that absorbs disturbances generated by errors in demand forecasts and allows a more efficient plant and personnel utilization in case of demand fluctuations and an economic functioning of generally rigid production, in addition to decoupling function previously exposed[Magee, Boodman 1992]: that is why inventory management should not be treated as separate from production, on the contrary the existence of an inventory management policy strictly connected with the production programming, control and management is vital.

Differently from manufacturing companies, where stock problems can be solved together with production problems, in retail businesses they represent an autonomous problem. Moreover, the complexity of inventory management in retail businesses increases as the higher the sales volume is, the wider the range of products offered for sale, the more difficult their procurement and conservation is, the more variable the demand is [Urgeletti Tinarelli 1992]. Therefore, we will talk about merchandise management, defined as the process by which a retailer attempts to offer the right quantity of the right merchandise in the right place at the right time (i.e. carry out the intended task of inventory) while meeting company's financial goals [Levy, Weitz 2009]. When evaluating the performance of merchandise managers, the GMROI (i.e. gross margin return on inventory investment) is typically used, that measures how many gross margin dollars are earned on every dollar of inventory.

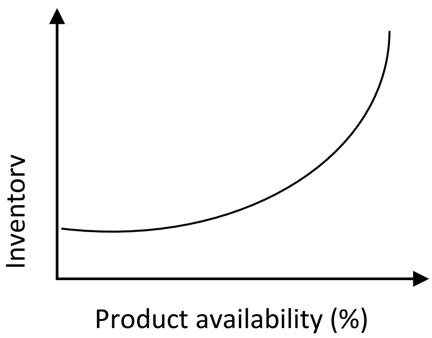
$$
\begin{aligned}
GMROI &= Gross\ Margin\ Percentage * Sales - to - stock\ ratio \\
&= \frac{Gross\ Margin}{Net\ Sales} * \frac{Net\ Sales}{Average\ Inventory\ at\ cost} \\
&= \frac{Gross\ Margin}{Average\ Inventory\ at\ cost}
\end{aligned}
$$

The unit of measure is the merchandise category, the assortment of items that customers perceives as substitute for one another; managing by category increase the probability of a store's assortment including the optimal combination (the one that gets more profit from the allocated space) of sizes and vendors.

Merchandise management process will be different depending on whether the merchandise category is staple (continuous demand over extended time period) or fashion-related (in demand only for relatively short period of time; within which also seasonal merchandise can be found, i.e. items whose sales fluctuate dramatically depending on the time of year) and, although in business practice the phases can be faced in different order, it can be articulated as follows:

- *forecast category sales*, based on historical sales and adjustments for controllable factors (e.g. openings and closings of stores, category's and complementary categories' prices and promotions, …) for staple merchandise, based on previous sales data, market research, fashion and trend services and vendors for fashion merchandise categories. The latter is typically more challenging for various reasons: buyers need to place orders and commit to buying specific quantities between 3/6 months before delivery and availability for sales, there is no opportunity to increase/decrease the quantity ordered before selling season has ended, some or all items are new and different from units offered in previous seasons or years;

- *develop assortment plan*, that is the set of SKUs (stock-keeping unit, the smallest unit available for inventory control) that a retailer offers in a merchandise category in each of its stores, reflecting his variety (or breadth; it is the number of different merchandise categories offered) and assortment (or depth, it is the number of SKUs within a category) decisions, considering the adopted retail strategy, assortments and GMROI, complementary merchandise, effects on buying behavior and stores' physical characteristics;
- *set inventory and product availability levels*; determined by the number of units of safety stock, product availability is the percentage of the demand of a particular SKU that is satisfied, also referred to as service level. The choice of the appropriate safety stock is crucial to successful assortment planning: if it is too high scarce financial resources, rather than being more profitably invested in increasing variety or assortment, are wasted on needless inventory; if it is too low, sales are lost as well as possible customers. Inventory investment and product availability are therefore characterized by a trade-off, as illustrated in pic.4.



Picture 4 Trade-off between inventory investment and product availability [adjusted from Levy, Weitz 2009]

The classification of merchandise categories or SKUs as A, B or C items (i.e. Pareto's or 80/20 law) may simplify the definition of safety stock. A-class items are best-sellers bought by many customers that provide the largest part of revenues, consequently retailers will perform greater effort to have those items on shelves, since their stock-outs would diminish retailer's image and customer loyalty, while for C-class items, purchased by small number of customers and contributing in lesser extent to company's revenue, a lower product availability is acceptable;

- *establish a control system for managing inventory*, with the objective of managing the flow of merchandise into the stores so that single merchandise category is minimized but still available when requested by customers. For staple category, an automated continuous replenishment control system is used, that monitors the level of each SKU and automatically triggers a reorder when the inventory falls below a predetermined level (that depends on several factors; among other we can mention the product availability the retailer wants to provide, the fluctuation of the demand, the lead time); for fashion category a merchandise budget plan is used, that specifies the planned inventory investment in dollars on the basis of how much merchandise will be ordered, delivered and sold each month during the selling season;

- *allocate merchandise to stores* divided into three sub-decisions: how much, what type and when to allocate merchandise to each to store; researches have demonstrated that this decision has more impact on profitability than does the one about the quantity to purchase;
- *analyze merchandise management performance* and take appropriate corrective actions, for example ordering a different quantity, lowering prices to increase sales, and allocating different assortments to specific stores;

1.6 Inventory management costs

Before the analysis of inventory models it is necessary to focus on inventory management costs, i.e. costs that are influenced by decisions made when dealing with inventory, for example those that increase/increase if high level of items are stocked or vice versa.

Because of scarcity of resources available to companies, appropriate and effective inventory policies should be developed and applied in each single particular case based on objective consideration, having a clear and deep understanding of costs revenues connected with different policies. The determination of the amount of inventory to keep should go beyond personal, affective and psychological motives coming from experience and past results as well as reasons why it should actually be kept, often conflicting against each other's since referring to different departments belonging to the same organization; on the contrary inventory control systems should pursue the long run profit maximization, with respect to the predetermined service level, at minimum costs while keeping an eye on technical, physical and financial constraints the business is subject to, considering that inventory management falls within the wider field of business management and control.

The identification of those costs that, among the various corporate costs, directly or indirectly influence stock control is therefore crucial. Unfortunately, this identification can be problematic and laborious since the costs items we are interested in are typically different than those reported in the accounting entries. Aim of accounting records is to provide a fair appreciation of assets and a representation of the flow of business resources, aim of operating costs is to provide a basis for managerial decisions. As a consequence, it is indispensable to operate a reclassification of the accounting records, sometimes applying also experimental or statistical methods. Two criteria should be met when performing such reclassification:

- costs have to reflect real expenses, in terms of amounts of money actually paid out or missed revenue opportunities;
- costs have to reflect only expenses or missed profits whose entity depends on the plan or program.

1.6.1 Purchase cost

It is the amount paid to the supplier: it is the unit purchase price if it is obtained from an external source or the unit production cost if it is produced internally. Manager should always consider the unit cost as the cost of an item as it is placed in inventory; therefore, for purchased items, it is the purchase price plus any freight costs; for manufactured items, it includes direct labor, direct material and factory overhead. Since this cost does not influence the decision of when and how much has to be ordered if it is constant (it does not vary because of the purchase quantity or timing), typically purchase cost is not considered in stock control models; this is because, in the long run, companies buy exactly what is needed and as a consequence the total purchase cost of a long period of time will be always the same, irrespectively of the policy applied.

On the contrary, we need to consider also the purchase cost when it varies because of quantitative/temporal variables.

For internally produced items, the purchase cost may be lower because of greater efficiency of men and machines in long continuous production runs.

For purchased items, it could not be constant (and therefore it should be modified for it) because of quantity discounts, price breaks, or changes over time due to currency devaluation or other causes. In such a case, bigger purchases are normally made in an increasing-price regime, and vice versa in the opposite case. Especially this latter case, is not usually dealt with in inventory management literature, maybe because it is believed that venture into speculative actions is not part of the job of an inventory manager.

Another different case is when items, typically raw materials, are purchased on fluctuating and unstable markets, such as commodity market. In such a situation, one would purchase and stock the commodity at times of low prices and use or sell the commodity when prices are high; "buy low and sell high" is an old business maxim. Rather than being known with certainty, as in the vast majority of models proposed by the literature, the price, quoted at time t is $C(t)=e^{c(t)}$, is assumed to be a random variable that evolves continuously, as a stochastic process, such as the geometric Brownian motion or the Ornstein-Uhlenbeck process. We denote μ(c) the drift of the process, which is a function of c, and $\sigma^2(c)$ its variance,

$$dc=\mu(c)dt+\sigma(c)W_t$$

where W_t is a Wiener process, i.e. a continuous time stochastic process with zero drift and unit variance per unit time (a Markov process).

If price evolves following a geometric Brownian motion, we will have $\mu(c)=\mu$ and $\sigma(c)=\sigma$, if it follows the Ornstein-Uhlenbeck process $\mu(c)= -\kappa(c-\bar{c})$and $\sigma(c)=\sigma$, where κ determines the speed at which prices revert to their long-run level, which is denoted $\bar{c}$ [Urgeletti Tinarelli 1992; Ackoff, Sasieni 1968; Berling, Martínez-de-Albéniz 2011].

1.6.2 Ordering cost

It originates from the expense of issuing a purchase order to an outside supplier or from an internal production setup costs. It includes the costs due the issuing of the order (mail, phone, invoicing expenses and so on), the receiving and control of the order (shipping, recording, quality control, etc) in case of purchases or, in case of internal production, due to planning, scheduling and launching activities.

It is made of two components: a constant one, independent on lot size and proportional to order frequency, and a variable one, dependent on lot size. In practice, the variable component is disregarded in favor of the constant one, because of its little incidence.

We define c_o the unit ordering cost including here also those costs that, although not being strictly ordering costs, depend on the action of issuing an order, e.g. transportation (in case of fees to be paid by the company and not by the supplier), processing and recording expenses, etc. [Grando 1995, Urgeletti Tinarelli 1992]

Ordering cost, expressed as Euro per order (€/order), include the following:

- *transportation costs,* if not included in purchase price and constant (independent on the load size);
- *order preparation and issuing costs*; these are an administrative type of expenses made mainly of clerical work connected with the choice of the supplier, planning and management, order issuing and its positioning in warehouses, possible solicitation of the order (even the case of internal orders), the cost of periodic review of the stock with the purpose of issuing new orders for replenishment. Clerical costs are particularly difficult to estimate and for this reason they are often not considered by companies. Following the order throughout all processing stages, noting the time that individuals intervene in each phase, the tools used, the encountered expenses (phone calls, mailing …), paying attention

on isolating the time that same individuals spend performing other tasks [Magee, Boodman 1992];

- *receiving costs*, relative to the control and positioning in warehouse control of the orders;
- *administrative costs*, recording the supply and prepare the payments.

Some considerations have to be made. Saying that ordering costs are constant for every order and Nc_o is the total ordering costs, being N the number of order made within an interval, is the same thing, thereby allowing for proportionality between total ordering costs and number of orders, which does not always hold. For example, labor costs are not proportional to N, since they stay constant within certain intervals of orders and do not decrease if N is reduced, since once hired, personnel cannot be fired. However, possibly personnel dimension is rationally made and proportional to the volume of work to be performed, while exceeding clerks can be devoted to other tasks, thereby justifying the assumption of proportionality between ordering costs and average number of orders issued in a certain interval.

We also need to add that, especially for items for ordinary use, it is not necessary to select the supplier, decide the quantity, etc whenever an order has to be issued thanks to advance technologies that make all the process more rapid and automatic.

Moreover, if we consider the possibilities of jointly ordering items from the same source and issuing big orders that can be spread out through time, the costs of incoming single orders reduce [Urgeletti Tinarelli, 1992].

1.6.3 Holding cost

It is the cost that is sustained to keep stoked items, arising from their need for physical space, maintenance, appropriate treatments for preservation of their characteristics and, first of all, they tie up capital. Carrying costs are mainly fixed, not dependent on inventory size, given that a certain space is dedicated to warehousing and need to be heated and efficiently kept. This assumption is verified as long as inventory does not exceed storage capacity; when this happens there extraordinary expenses (whose entity depends on the policy adopted) are incurred because of the rent of additional warehouses, that in turn need other operators, machinery, etc, or the use of some space that could be otherwise and more profitably used. There is also variable component of holding costs that is proportional to the value or volume of what is stocked (e.g. insurance, interests on invested capital, taxes, etc.).

However, with the same reasoning applied for ordering costs, it is assumed proportional to the quantity, to the volume or to the value of the inventory and to the holding time [Magee, Boodman 1992; Urgeletti Tinarelli 1992].

It can be expressed in two ways:

- as a percentage of the stocked quantity or volume within the considered time unit, expressed as Euro per quantity/volume and per time (we consider the year);
- as a percentage of the inventory value within the considered time unit, expressed as Euro per Euro and per time.

Holding costs is made of the following components:

- *storage costs*, real or figurative warehouse rent or its depreciation, if it is owned;
- *insurance expenses and taxes*, since stock, as other company's assets, needs to be protected against accidents such as fire, theft, special events, etc. Insurance coverage requirements are dependent on the amount to be replaced if property is destroyed, insurance premiums vary with the size of the inventory investments. Many states and municipalities have inventory taxes, some based on the inventory investment at the particular time of the year, while others are based on the average inventory investment for the entire year [Plossl, Wight 1967; Tersine 1994];
- *stationery, phones, general inventory record-keeping costs*;
- *handling costs*, i.e. cost of labor to move stock, or overhead cranes, gantries, forklift, trucks, and other equipment used for this purpose;
- *cost of capital*; reflecting lost earning power or opportunity cost, if the funds were invested elsewhere, a return on the investment would be expected; it is a charge that accounts for this unreceived return. Usually, it is one of the most important cost item and tends to proportionally increase with the interest rate, calculated or paid for every euro invested (interpreted as average cost of all financing means available to the company), the inventory value and the time stocked items wait in inventory. Although being a function of these three factors, inventory management decisions can be made making reference only to the first one, the interest rate, which in turn directly depends on the financial policy applied by the company [Magee, Boodman 1992].

  If capital employed is borrowed, cost of capital is equivalent to the passive interest rate due to the financing entity; while if we deal with own capital or granted by suppliers, it can be represented by the opportunity cost of investing the capital elsewhere, i.e. the return that could be obtained on its investments, at the best of its alternative use.

  Interest rate attributed to inventory investment can based on the interest rate a bank would ask for that amount or the rate that could be gained from a different use within the company; or alternatively on both of them, depending on the stock we consider and the company financial policy [Magee, Boodman 1992].

The usual simplifying assumption made in inventory management is that holding costs are proportional are proportional to the size of the inventory investments; ranging on an annual basis from 20% and 40% of the investment [Tersine, 1994].

Assuming that total holding cost is proportional to value of the stocked items in every time point in time of every interval is equivalent to saying that it is proportional to the value of the average stock in such interval.

Be:

- s_i stock level maintained in *i* days;
- T the number of considered days (such that $T = \sum n_i$);
- $\bar{s}$ the average stock in *T;*

The total holding cost in the time interval *T* will be

$$c_h = \frac{\sum s_i n_i}{T} = c_h \bar{s}$$

Moreover, conditions with purchase agreements with time deferred payments have to be carefully analyzed. One in fact could think that, being holding costs basically null throughout the considered period, carrying high inventory could be convenient because of the temporary reduction of holding costs in this period. This is clearly a wrong reasoning, for two reasons. First, deferred payments are granted in view of a higher purchase price, usually calculated based on current market rate; second, even in case of financing granted at zero rate from supplier, it is better to evaluate the opportunity cost of invested capital to verify the existence of alternative, more profitable, investments [Urgeletti Tinarelli 1992].

1.6.4 Depreciation, deterioration and obsolescence cost

Particularly relevant for fashion items or items that change chemically during storage, such as foods, photographic materials and pharmaceuticals, obsolescence and deterioration represent the risk that an item will lose value because of shifts in style or consumer preferences and change in properties due to age or environmental degradation, respectively. Stocked goods generally depreciate since it suffers a natural or artificial ageing that can be considered as a cost, allowing the expression this loss of value as a function of the time spent in inventory.

They assume different forms:

- total deterioration;
- the risk that a certain product, whether it is final product or a component, can become old-fashioned or technologically obsolete and therefore saleable only after price reduction or as spare parts;
- other reasons;

- because of thefts, damages, evaporation, etc.

Another hypothesis is that of inventory volume exceeding sales volume in the same period; in such a case all additional stock becomes unsalable. On the other hand, certain types of inventory, like those subject to fashion, preserve just a salvage value, as it happens frequently to seasonal markets goods.

Sometimes, it is thought as proportional to the inventory value and stock time, therefore it is often included within holding costs. Alternatively, it is considered as proportional to the value or quantity of inventory at the end of the period analyzed (think of fashion items). Assuming this second hypothesis, we will define c_p as the cost of every item at the end of the period. It can be calculated as difference between sale price and salvage value or as a difference between total cost (purchase cost plus percentages of overheads) and salvage value [Urgeletti Tinarelli, 1992].

1.6.5 Stock out cost (or shortage, penalty or depletion cost)

They are costs that arise when demand for a product exceeds its inventory availability in a certain moment so that the requested item is not available in stock, making impossible to quickly satisfy the demand because of stock outs, as the result of an external or internal shortage. An external shortage occurs when a customer's is not filled; an internal shortage occurs when an order of a group or of a department within the organization is not filled. External shortages can result into backorder costs, present profit loss, canceled orders and lost sales (potential sale) and future profit loss (goodwill erosion). Internal shortages can result in lost production (idle resources) and a delay in a completion date (disrupted schedule).

Assuming that emergency measures assuring that deliveries are made on or even before the customer required date, the extent of the cost depends on the reaction of the customer to the out-of-stock condition, depending on whether the shortage is satisfied by backordering, substitution with another product or canceled:

- customers wait: in monopoly condition this waiting could have no cost since customers are willing to wait this time because of absence of alternatives on the market. Similar circumstances are when the company has established such a strong brand loyalty that customer are extremely attached to it that or when products are of higher quality or with particular characteristics that cannot be found on the market, resulting in customers willing to wait (think of fashion goods). In more usual conditions the sale is only delayed a few days in shipments and extra costs are charged, like expediting costs, handling costs, frequently premium shipping and packaging costs. There are also increased transportation charges (airfreight instead of truck), increased setup or overtime costs, administrative costs or the cost of disrupting a planned production schedule. There may be also the cost of standby production facilities which are used only in an emergency, that could be obsolescent or worn out and have higher production costs than the regular equipment.

In this case we can express the shortage cost $\widehat{c_s}$ as the cost per unit of every missing unit and per unit of shortage time, proportional to both the quantity that is short and the duration of the shortage;

- customers do not wait: in such circumstances there is no backlogging and the company loses the sales and therefore the profit. The shortage cost for every missing unit, c_s, is equivalent to the lost profit for every unsatisfied order. The actual cost is less identifiable in this case but ranges from the apparent profit loss on the sale to loss of goodwill, which can be hard to specify.

  The quantification of this cost has long been a difficult and unsatisfactory resolved issue; for this reason, many organizations avoid the problem by specifying customer service levels, recognizing the difficulty of its determination. In case of actual lost sale, the base for its determination will be lost profit, i.e. contribution margin. Being a figurative cost whose only some of its components are recognized in the accounting records, there other factors that should be considered, like deterioration of company's image on the market, customers' lack of trust, etc that being characterized by subjectivity, justifies, to a certain extent, the opinion of who believes that this cost cannot be expressed monetary [Ackoff, Sasieni 1968; Tersine 1992; Urgeletti Tinarelli 1992].

Chapter 2

Raw materials subject to quotation:

commodities and the particular case of gold

When companies purchase from spot or foreign markets they are exposed to volatile prices and exchange rates fluctuations, which for some commodities and currencies can be very large, For this reason, they may resort to purchasing insurance against variations in prices to reduce their effect on profits (i.e. financial hedging) that, however, are not generally considered in the day-to-day procurement: on the contrary, procurement strategies should combine traditional inventory theory and financial price modeling [Berling, Martínez-de-Albéniz 2011]. As a consequence, there is the need for an operational hedging strategy in terms of purchase/inventory control decisions that minimizes the sum of purchase, holding and shortage costs that, differently from traditional models discussed earlier that ignore purchase price, assuming it as known or constant, also considers such price and its variability. Models of this kind will be dealt with in following chapters.

In this chapter, the characteristics of commodities are briefly presented, with deeper attention to gold, exploring its demand, its supply and uses and the determinants of its price.

2.1 Commodity

The term commodities indicates basic resources, like agricultural, energetic products or extracted through mining activities, often employed as inputs in the production of other goods and services used in commerce that are interchangeable with other commodities of the same type. This term actually denotes a good for which a demand exists but no qualitative differences are offered on the market, thus a fully or partially fungible good that has same intrinsic characteristics, irrespective of who produces it. Such standardized characteristics allow them to be traded on highly competitive international markets.

Soft and hard are the main categories commodities can be classified into. Soft commodities are grown, for example corn, wheat, sugar, cocoa beans. Hard commodities, that dominate the market place, are mined from the ground or taken from other natural resources, like gold, silver, oil, aluminum, and they are often refined in further commodities (e.g. oil refined into gasoline). Another category of commodities that of coal, iron, timber and so on that, however, do not have a liquid futures market. Furthermore, there are also “emerging commodities” that are expected to develop in the future and these are wind, solar, water, water and pollution

rights; for now however the only way for investors to access them is to buy stocks of companies that operate in these fields.
Commodity markets can therefore refer to a variety of different sectors, as specified in table 2.

Energy	Crude oil, natural gas, petroleum, coal, …
Precious metals	Gold, silver, platinum, palladium, iridium, …
Industrial metals	Aluminum, zinc, nickel, tin, …
Strategic metals	Mercury, selenium, titanium, chromium, vanadium, …
Plastics	Polypropylene, polyethylene, …
Cereals	Grains, corn, sorghum, peanuts, …
Tropical or colonial	Sugar, coffee, cocoa, pepper, tea, cotton, silk, …
Meat	Livestock, livestock feed, …

Table 2 Commodity typologies

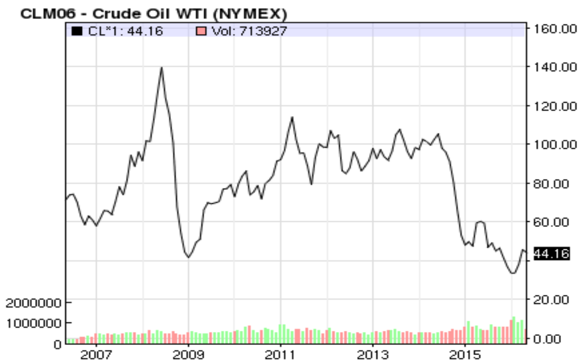
Within these sectors, gold and oil have their own peculiar market, because of their macroeconomic relevance: on one hand, gold is part of the monetary system, being the main reserve asset of leading nation's central bank, affected by the output of the mining industry and affecting also inflation levels; on the other hand oil market can be considered as oligopolistic (OPEC supply 77% of the total global oil) and its characteristics affecting income, climate change policies, etc. [Zolin 2016].
Commodity markets have a long history. Their origin can be traced down to the commerce of agricultural products in the 18th century, when buyers and sellers both wanted to limit the risks during crops' harvest and processing: buyers were interested in protection in case of scarce harvests and consequently high prices, sellers wanted guaranteed price for their products, thereby protecting themselves against high supply and relative low prices. However, quality and product delivery standardization levels were very low and a centralized stocking place did not exist. The Chicago Board of Trade (CBOT) is then founded, in 1848, acting as an intermediary between farmers and corn seed merchant; procedures for grain weighing and classification were established, as well as a centralized market where prices of future goods deliveries were fixed in advance, allowing all the interested parties to hedge against price variations. This lead to the modern futures market that rapidly expanded to other commodities, like metals, rubber, silk and hides.
However, as these markets developed, the hedging motive was increasingly followed by the speculative motive, often resulting into bubbles. The first one can be identified as the "tulip bubble": in Holland, by the end of the 16th century, a market dedicated to the trade of tulips

developed that attracted a lot of investors who however did not have a deep knowledge of the horticultural sector; prices then started to raise, beyond their reasonable physical value, until the bubble finally exploded, with notable negative consequences on the local economy.

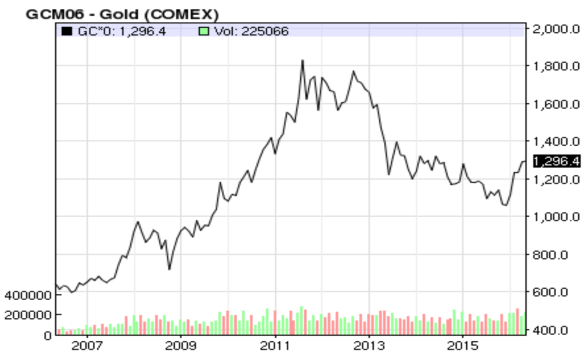
Nowadays, hedging and speculative motives often go together: markets are used both to reduce the effects related to the risk of price fluctuations and to take advantage of these movements. If at the beginning commodity markets represented a form of protection for raw materials suppliers and merchants, now a much wider public has access to them, including pension funds, hedge funds, investment banks, institutional investors and, with increasing frequency, also private individual investors. Currently, commodities cover an important role in lots of investment portfolios, and the increasing interest for them lead to the introduction of a wider range of negotiable commodities and investments methods; markets continues to give access to energetic resources, metals and agricultural products, but also to new categories of resources, such as carbon credits [Morgan Stanley 2007].

2.1.1 Commodity: characteristics, price volatility and types of contracts

Basic characteristics of commodity markets are not different from those of any other type of market, except for some aspects. The first one is seasonality that, even if it has greater effects for agricultural commodities, it also influence other commodities, like oil and natural gas; the second is the logistics: being basically goods with globalized demand and supply, the role of the transportation from production to consumption take quite always much more significance here than in other markets. However, the most interesting characteristics of commodity markets is price fluctuation.

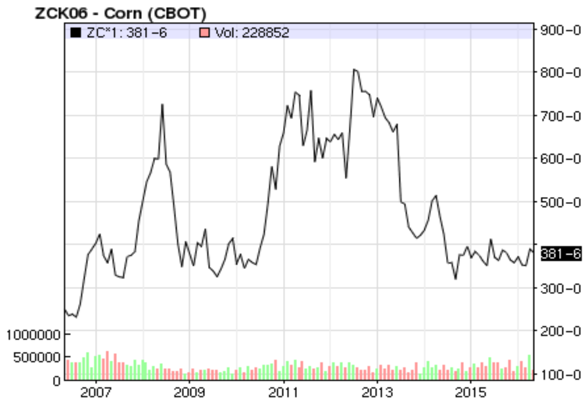


Picture 5 End of day commodity futures price quotes for crude oil WTI (NYMEX) from 2006 to April, 2016 [http://www.nasdaq.com/markets/crude-oil.aspx]



Picture 6 End of day commodity futures price quotes for CBOT Gold 100 oz (COMEX) from 2006 to April, 2016 [http://www.nasdaq.com/markets/gold.aspx?timeframe=10y]

Commodity market prices vary as a function of demand and supply, as it happens for any kind of good and service, but they also continuously change based on product availability, weather forecasts, actual weather conditions on production sites, reports and news on future production estimates, geo-political tensions, governmental choices such as subsidies, embargoes, duties and so on. Being so, their prices can be treated as an unknown variable or a highly volatile variable, characterized by, even daily, wide fluctuations, as it can be seen in pic.5, 6, 7.



Picture 7 End of day commodity futures price quotes for corn (CBOT) from 2006 to April, 2016 [http://www.nasdaq.com/markets/corn.aspx?timeframe=10y]

Commodity prices are also influenced by demand and supply dynamics such as restrictions of the supply (e.g. OPEC's policy of price targeting by engaging in supply management rather than letting markets determine the price) and adversary, alternative uses of the same commodities. From 21th century in fact, a large number of commodities is being used in ways different from the traditional ones; for example corn and other soft commodities, traditionally used for human food and animal feed production, are currently employed in the production of biofuels, because of the increasing awareness of negative effects of fossil fuels consumption on the environment. Therefore historically this kind of goods has proved to be among the most volatile assets. Companies replenishing from commodity markets and investors need to be aware that their recorded historical performance (typically lower than investments in stocks and positively related to inflation) and behavior are not necessary indicative of future performance and behavior [Morgan Stanley 2007].

Consequently, commodity producers suffers not only the market risk, that affects every economic operator, and specific risk (e.g. adverse weather conditions, labor cost, plants' epidemic, geopolitical risks, …), but he also has to face a high price volatility, finding himself in a situation where his commodity can be sold at a price particularly low or lower than the costs sustained to produce it. In order to reduce market risks, buyers could purchase commodities when the price is lower than it could be in high price hypothesis. In this case, however, opportunity costs have to be considered, arising from the stipulation of a contract for a product that will be only later delivered, adding the possibility of a further price reduction in the meanwhile, making the initial purchase inconvenient. On the contrary, he could cash purchase whenever the need for a particular quantity of commodity product is perceived that, if on one hand it could allow for the avoidance of inventory and capital tie up, on the other it could suffer a price higher than what could have been in other conditions.

In order to face this problem, starting from the 19th century physical trading (i.e. cash commodities or "actuals", referring to the physical goods, for example wheat, corn, soybeans, oil, gold, silver, etc, that someone is buying/selling/trading, where delivery and payment either takes place immediately or with a short lag, involving a visual inspection, carried out in physical markets such as farmers market) has been being accompanied by trading with derivatives, a peculiar type of financial instruments whose value is derived from a commodity termed an underling, that require the existence of agreed standards so that trades can be made without visual inspection. Currently, forward and futures contracts, together with spot prices and options on futures, represent the most common way to invest in commodities; rather than purchase and stock physical goods, for example oil or livestock, investors can use structured products to have financial access to them. These products can offer a return correlated to the underlying good, the so called spot price, or more commonly they are linked to commodity futures contracts or to a commodity index.

A forward contract is a private agreement between buyer and seller for the future delivery, at a fixed date, of a commodity at a price (forward price) defined when the contract is finalized. In this way, both parties know from the beginning how much has to be paid and how much has to be cashed in, respectively, when the goods will be successively delivered; activities can be planned with certainty, knowing in advance costs and income, without worrying about commodity's price variation; furthermore, all contract provisions (quantity, delivery time and methods) are freely agreed upon between the parties, through a private agreement. However, it is the private nature of forward contract itself that poses some limits. Especially in case of insolvency of one of the parties involved, there is no certainty about obligations' fulfillment, neither the solvent party is particularly protected. Although being a step forward in commodity trading, forward contracts are instruments that are not able to completely safeguard the interests of all the parties involved. This problem has been overcome with the introduction of futures contracts.

Future contracts are standard contracts, negotiated through commodity exchange, for the purchase or the sale of a commodity, with predetermined future delivery and price set in advance. Being so standardized, the commodity exchange where the futures itself is traded establishes the main provisions of the derivative contract; the only variable that is not set in advance is the price, that is determined by the law of demand and supply.

Voices	Meaning
Contract Symbol	C
Contract Size	5.000 bushel
Contract month	December, March, May, July, September
Trading hours	9.30-13.15
Price quotation	Cents$/bushel
Minimum price movement	¼ cent/bushel (12.50$/contract)
Daily price limit	0.20$ per bushel ($1.000/contract)
Last trading day	
Last delivery day	
Grade/standard/quality	
*Commodity exchange sets the amount of money that has to be deposited in advance for every contract (i.e. the margin)	

Table 3 Example of contractual voices of a futures contract (corn)

Future contracts are standard contracts, negotiated through commodity exchange, for the purchase or the sale of a commodity, with predetermined future delivery and price set in advance. Being so standardized, the commodity exchange where the futures itself is traded establishes the main provisions of the derivative contract; the only variable that is not set in advance is the price, that is determined by the law of demand and supply.

Basically, the price reflects the expectations on whether the price will increase or decrease in the future.

Swaps and exchange-traded commodities (ETCs) are other contracts used in commodity trading. Swaps are derivatives representing over-the-counter (OTC) agreements to exchange cash flows at regular intervals over an agreed period according to terms agreed on today, while ETCs (or commodity exchange-traded funds or commodity exchange-traded notes), similar to ETFs, traded and settled in the same way of stock funds, track the performance of an underlying commodity index including total return indices based on a single commodity [Heckinger 2013, Morgan Stanley 2007, Zolin 2016].

2.1.2 Commodity exchanges

Commodities and derivatives having a commodity as underlying asset are traded on commodity exchanges, open and organized market places where its members trade ownership titles of standardized quantities or volumes, at specified prices and delivery dates, of certain commodities. Commodity exchanges can be divided into three main categories: metals exchanges, fuels exchanges and soft (agricultural) commodity exchanges.

They play an important role in the commercial activities of manufacturers, farmers and entrepreneurs. Thanks to these institutions, trades are not anymore subject to the free will of the parties but regulated, meticulously defined and guaranteed by a publicly recognized structure that defines prices, quantities, quality, delivery time and methods of the product. Commodities exchanges in fact have been in existence as far back as history can tell, it is the typologies of contracts used that differentiate the modern exchange from the historical exchanges. Commodity exchanges' origin can be traced down to 500 B.C., in the roman Ceres's temple where mercantile assemblies occurred [Bon, 2007]. In the Middle Age purchases were performed by buyers before the delivery of the goods, ensuring a sale when traveling with large loads of commodities along dangerous routes, reducing the risk of losing the sale, thereby engaging into contracts similar to futures contracts nowadays available on the commodity exchanges.

On 1848 The Chicago Board of Trade has been established as a limited company with the purpose of gather together farmers and vendors; while the London Metal Exchange is the first forward and futures market for the negotiation of commodities.

Commodities exchanges have developed across the world, the main ones are in the United States (13) and Latin America (3), Asia (42), Europe (16) and Oceania (2).

In Italy, the *borse merci* are defined as meeting places intended for market negotiations, also through forward and futures (*contratti a termine*), of all goods and products that can be trade objects, except for those that are traded on the stock market. However, differently from the main foreign commodity exchanges, that deal mainly with futures, the *borse merci* focus on negotiations, both spot (*a pronti*) or delayed delivery, that usually take place out of the *borse merci* themselves. For this reason and since the deal is typically privately closed by the interested parties, Italian *borse merci* can be seen as structures for performing the exchanges rather than exchange systems, even if also here prices are transparent and information flow freely. Comparing them with major international exchanges, even if both come from the need of operators for a place (either physical and virtual) where trades can be performed, answers to this need came into different forms: Italian markets can be defined as man-dimension markets, where producers come into direct contact with their own buyers and entrust local quotations; while international exchanges represent real global markets that, through their quotations and volumes traded therein, are able to redistribute product quantities and ensure an equilibrium between supply and demand through fair and transparent prices [Bon 2007].

The highest performing and largest worldwide commodities figured in tables 4 and 5.

Exchange	Volumes traded on May, 3, 2016	Notes
New York Mercantile Exchange	2.182.306	The world's largest physical commodities exchange dealing in futures
London Metal Exchange	726.047	Offers an exchange platform for metals such as gold, silver, aluminum and copper
Chicago Mercantile Exchange	87.565	The first US commodities exchange market. The CME is still one of the world's largest commodities platforms
Chicago Board of Trade	5.132.319	Founded in 1973 as the first US options exchange, CBOT has a complex trading system and is still a world leader in options trading
Shanghai Metals Exchange	N/A	This is the largest metal exchange in China and one of the largest in the world

Table 4 The five highest performing and largest worldwide commodity exchanges [data adjusted from http://valuestockguide.com/guide-stock-commodities-exchanges/

http://www.lme.com/lme-clear/reports/daily-volumes/

http://www.cmegroup.com/market-data/volume-open-interest/exchange-volume.html

Table 5 Main International exchanges per commodity [adjusted from Morgan Stanley, 2007]

Commodity		Main exchanges	Most common futures contracts	Notes
Precious Metals	Silver	COMEX	Contract value: 5.000 troy ounces; quotation: dollars and cents/troy ounce	Although majority of paper contracts take place in the US, London remains the principal physical market; the *fixing* (spot price) takes place daily at 12.15. (1 troy ounce≈31.1 kg)
		CBOT	(Mini) Contract value: 1.000 troy ounces; quotation: dollars and cents/troy ounce;	
		TOCOM	Contract value: 60 kg; quotation: yen/troy ounce	
	Gold	COMEX	Contract value: 100 troy ounces; quotation: dollars and cents/troy ounce	*Fixing* is made twice a day, at 10.30 and 15.00 (time of London) by London Bullion Market Association; afternoon fixing is used as reference for gold price at worldwide level
		CBOT	Contract value: 33,2 troy ounces; quotation: dollars and cents/troy ounce;	
		TOCOM	Contract value: 1 kg; Quotation: yen/gram ounce	
	Platinum	NYMEX	Contract value: 50 troy ounces; quotation: dollars and cents/troy ounce	
		TOCOM	Contract value: 500 g; quotation: yen/gram ounce;	
	Aluminum	LME	Contract dimension: 25 tons; quotation: dollars per ton	Four daily outcry trading sessions, price is fixed during the second trading session (*ring*) of the trading day, between 12.55 and 13.00
	Nickel	LME	Contract dimension: 6 tons; quotation: dollars per ton	Four daily outcry trading sessions, price is fixed during the second trading session (*ring*) of the trading day,

Industrial Metals				between 13.00 and 13.05
	Lead	LME	Contract dimension: 25 tons; quotation: dollar per metric ton	
	Copper	LME	Contract dimension: 25 tons; quotation: dollars per metric ton	Four daily outcry trading sessions, price is fixed during the second trading session (*ring*) of the trading day, between12.30 and 12.35; outside this range copper contracts can be negotiated via LME Select (an electronic trading platform) or by phone
	Zink	LME	Contract dimension: 25 tons; quotation: dollars per ton	Four daily outcry trading sessions, price is fixed during the second trading session (*ring*) of the trading day, between 12.50 and 12.55
Energy	Natural gas	NYMEX	Contract dimension: 10.000 million BTU (British Thermal Units); quotation: dollars per million BTU	NYMEX price is the reference for global natural gas market
		ICE	Contract dimension: 1.000 million TU (Thermal Units); quotation: pounds per TU	
	Crude oil	WTI	Contract dimension: 1.000 barrels; quotation: dollars per barrel	Principal reference for American market
		ICE	Contract dimension: 1.000 barrels; quotation: dollars per barrel	Brent futures are the international reference for oil price (65% of oil is priced after Brent futures)
	Electricity		Although electricity contracts are treated similarly to those of other commodities, a most common futures contract does not exist since electricity cannot be stored; consequently its futures are often specific for a distribution area and consumption time	
			Contract dimension:	

	Carbon credits	CCX	equivalent to 100 metric tons of CO_2; quotation: dollars per ton
		ECX	Contract dimension: equivalent to 100 metric tons of CO_2; quotation: dollars per ton
Soft commodities	Cocoa	NYBOT	Contract dimension: 10 tons, quotation: dollar per pound
		Euronext LIFFE	Contract dimension: 10 tons, quotation: ₤ /pound
	Coffee	NYBOT	Contract dimension: 37.500 pounds, quotation: cents/pound
	Cotton	NYBOT	Contract dimension: 50.000 pounds, quotation: cents/pound
	Wheat	CBOT	Contract dimension: 5.000 pounds, quotation: cents/pound
	Corn	CBOT	Contract dimension: 5.000 pounds, quotation: cents/pound
	Soy	CBOT	Contract dimension: 5.000 pounds, quotation: cents/pound
		TGE	
	Sugar	NYBOT	Contract dimension: 112.000 pounds, quotation: cents/pound
		Euronext LIFFE	Contract dimension: 112.000 pounds, quotation: cents/pound
Dry freights	Dry freights	BIFFEX	

2.2 Gold as a commodity

Thanks to its beauty, its chemical properties and its use as commodity, gold has been considered for centuries as a precious good. For hundreds of years it was internationally used as currency. Between 1900-1933, period in which the gold standard was in place in the US and the cash money was convertible into gold at a fix price, all central banks had consistent gold reserves. By the end of the 20th century, with Bretton Woods, cash currencies were still anchored to the US dollar until Nixon suppressed dollar convertibility into gold, in 1971 [Morgan Stanley 2007].

Gold was among the first metals to be mined because it commonly occurs in its native form, because it is beautiful and imperishable and because exquisite objects can be made from it. The degree of purity of native gold, bullion (bars or ingots of unrefined gold), and refined gold is stated in terms of gold content. "Fineness" defines gold content in parts per thousand. For example, a gold nugget containing 885 parts of pure gold and 115 parts of other metals, such as silver and copper, would be considered 885-fine. "Karat" indicates the proportion of solid gold in an alloy based on a total of 24 parts. Thus, 14-karat (14K) gold indicates a composition of 14 parts of gold and 10 parts of other metals. Incidentally, 14K gold is commonly used in jewelry manufacture. The basic unit of weight used in dealing with gold is the troy ounce. One troy ounce is equivalent to 20 troy pennyweights. In the jewelry industry, the common unit of measure is the pennyweight (dwt.) which is equivalent to 1.555 grams [Kirkermo].

2.2.1 Supply

Gold has been mined starting from 2000 B.C., however highest gold fields' discoveries happened only on the 19th century, in California (where the Gold Rush began) and in South Africa. Especially in the African continent, gold mining is a high-capital intensive activity; as a consequence mining and searching activities demonstrated to increase when gold price is higher, and therefore profits are higher. In 2014 and 2015, major global gold producers were China, Australia and Russia, totaling 2.990 and 3000 metric tons globally produced. The top 6 producers between 2011 and 2015 were China, Australia, Russia, USA, Canada and South Africa, providing almost 50% of global mining production (elaboration of data by U.S. Geological Survey). With 490 tons, China was the largest producer in the world in 2015,

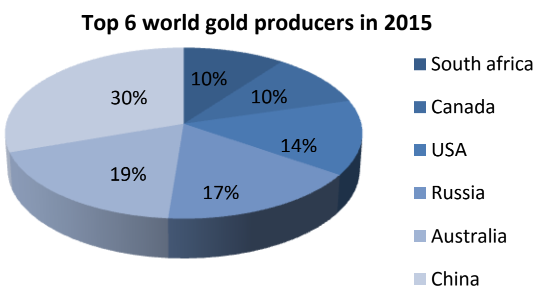


Chart 1 2015 World gold production [data adjusted from *Mineral commodity summaries 2016*]

accounting for around 16% of total production; Asia as a whole produces 23% of total newly-mined gold, Central and South America produces around 17% of the total, with North America supplying around 16% (data from WGC).

Gold market is supplied not only by mining activities but also from above-ground stocks, meaning jewelry, scraps, recycled gold and Central Banks' gold reserves (international agreements limited the sale of gold by Central Banks that were agreed on them. Among them, the Washington Agreements (1968), for example, established a double gold market: a free one , where gold price is subject to demand and offer, without intervention of Central Banks, and an official one, where gold price was fixed at 35 dollars/ounce; however, with gold and dollars not anymore anchored and fluctuating exchange rates, this double market does not exist anymore [Thomson Reuters GFMS 2014; *Mineral commodity summaries 2016* 2016; Morgan Stanley 2007]). As chart 3 shows, 71% of gold supply is mines, 27% is recycled, and 2% results from producer hedging. According to WGC (World Gold Council), recycled gold is gold sourced from fabricated products that have been sold or made ready for sale, which is refined back into bars; excluding gold traded-in for other gold products (e.g. by customers at jewellery stores) and process scrap. The majority (90%) of recycled gold is high-value gold, largely jewellery, while the 10% comes from industrial waste, including laptops, mobile phones, circuit boards, etc. Net producer hedging, on the other hand, measures the impact in the physical market of mining companies' gold forward sales, loans and options positions. In fact, hedging accelerates the sale of gold, releasing it to the market; over time, such hedging activity does not generate a net increase in the supply of gold because of the opposite impact given by de-hedging (process of closing out the hedged positions), which reduces the amount of available gold to the market [Thomson Reuters 2013, WGC].

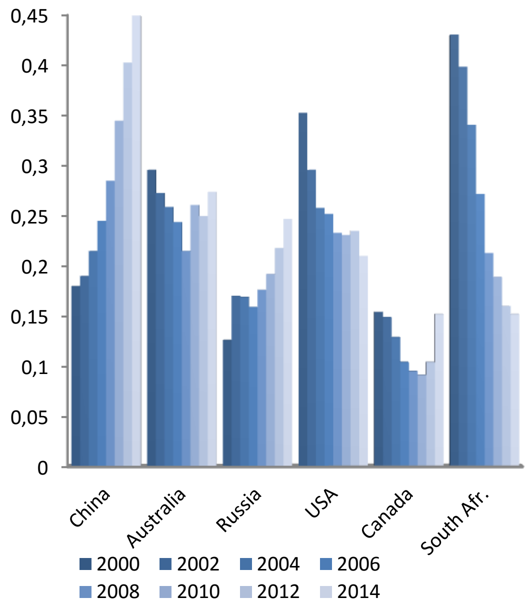


Chart 2 Thousands of metric tons of mined gold from 2000 to 2014 by major global producers [data elaborated from *Mineral commodity summaries 2002-2016*]

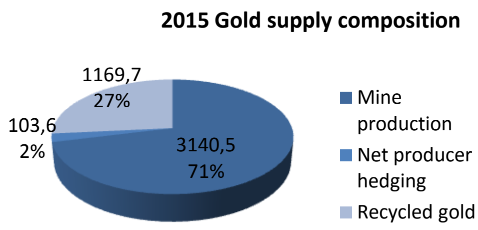


Chart 3 2015 Global supply composition [data adjusted from *Mineral commodity summaries 2016*]

2.2.2 Demand

With a 2015 demand of 4.212 tons, gold is used in many economic sectors, mainly jewellery, industrial fabrication and finance, because of its intrinsic characteristics. World Gold Council defines gold demand as the total of jewellery fabrication, technology, total bar and coin demand and demanded for ETFs and similar products.

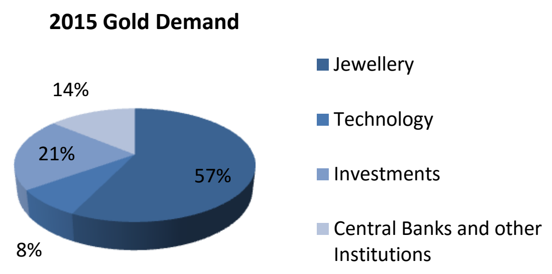


Chart 4 2015 Gold demand composition [data from WGC]

With 2.413,5 tons jewellery represents almost two thirds of 2015 world gold demand. Jewellery is described as the end-user demand for all newly-made carat jewellery and gold watches, whether plain gold or combined with other materials; second-hand jewellery, other metals plated with gold, coins and bars used as jewellery and purchases funded by the trading-in of existing carat gold jewellery are excluded. Jewellery fabrication (the first transformation of gold bar into semi-finished or finished jewellery) has to be distinguished from its consumption, as the former excludes the impact of imports/exports and stocking/destocking by manufacturers and distributors.

The world largest jewelry market is represented by India; consequently, demand for gold jewellery tends to reach its maximum levels during Indian holidays, such as "wedding season", Christmas and Hindu Diwali celebration [Morgan Stanley 2007, WGC]. Further details will be provided in the next chapter.

All uses of gold in the fabrication of electronics, dental, medical, decorative and other technological applications, including gold destined for plating jewellery, are captured by technology voice. Electronics, defined as the fabrication of gold into components used in the production of electronics, for example semiconductors and bonding wire, represent the largest component of this category. In fact, its conductivity and resistance to corrosion make gold the material of choice for manufacturers of high-specification components, especially in telecommunications and informatics sectors, and safety-critical applications. Thanks to its non-toxicity and biological biocompatibility, other uses can be found in dentistry, being a non-toxic and biologically inert, often as an alloy with platinum, palladium or copper. Within the "Other industrial" voice the following are included: high-technology (e.g. space industry and in fuel cells), automotive (as it proved to be a commercially viable alternative to other materials in catalytic converters), chemical and

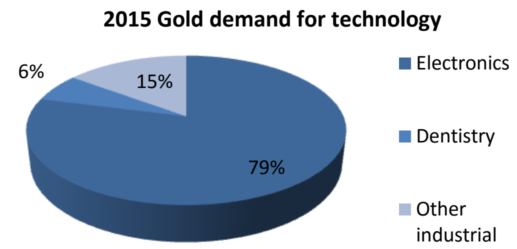


Chart 5 Gold demand for technology in 2015 [data from WGC]

nanotechnology (the demand for this latter category, although small, is expected to increase in the following years because of the growing numbers of patents being published) [Morgan Stanley 2007, WGC].

Gold has always attracted investors across the globe, whether institutional or private, because of its unique qualities that enhance risk management and capital preservation. Being the safe-haven asset par excellence, these qualities allow investors o hedge against inflation and dollar fluctuations, other than being particularly important during period of financial stress; researches in fact have demonstrated that a modest allocation of gold makes a vulnerable contribution to the performance of a portfolio by protecting against downside risk without reducing long-term returns. Recently it has become more accessible thanks to investment vehicles such as Exchange Traded Funds (ETFs). The demand of gold for investment is made of ownership of bars and coins (physical bar demand, official coins and medals imitation coins), or indirect via ETFs and similar products [Morgan Stanley 2007, WGC].

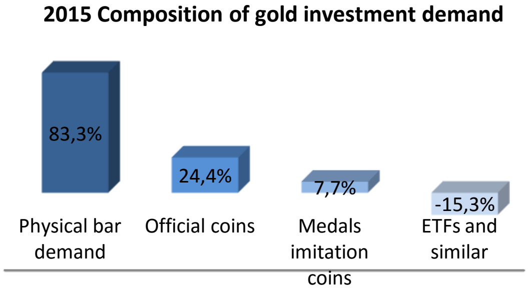


Chart 6 2015 Composition of gold investment demand [data from WGC]

Central Banks hold gold reserves as a store of value and as a guarantee to redeem promises to pay depositors, note holders, trading peers or to secure the currency. Their behavior with respect to gold has however changed over the past few years, reflecting a combination of slowing sales from European Central Banks and large purchases from emerging market countries in Latin America, the Middle East and Asia. Since 2010 in fact their demand has rapidly expanded, growing from less than 2% of total world demand in 2010 in 14% in 2014. This change can be due to the recognition of the reserve portfolio benefits that gold can provide; among others diversification of risk (especially from US dollar-denominated assets, with which gold has a strong negative correlation) and inflation hedging. Moreover, being one of the few assets being universally permitted by world’s Central Banks’ investment guidelines, it plays a prominent role in asset management, being its market deep and liquid [Morgan Stanley 2007, WGC].

2.2.3 Gold market

Gold market can be divided into two linked markets, the physical market and the market of gold claims (paper gold market that trades claims on physical gold rather than trading in physical gold itself), where the link is given by the underlying value of the claims that equals

the physical gold. Both are influenced by the same fundamentals but also by their participants; with physical market and gold claim market being mainly influenced by the demand of physical trade participants, mainly represented by gold producing firms, central banks and jewellery firms, and by the demand of investors using claims to get gold exposure, respectively [O'Gallaghan 1991].
Together with other precious metals like silver, platinum and palladium, price auctions for gold take place in London on a daily basis. The prices thereby obtained serve as a pricing mechanism for a variety of precious metal transactions and products. From November 2014, London Bullion Market Association (LBMA) gold price auction (*gold fixing*) takes place twice a day by ICE Benchmark Administration (IBA) at 10:30 and 15:00 with the price set in U.S. dollars per troy ounce (sterling and euro prices are also available but they are only indicative for settlement), before it was performed by the LBMA "Group of Five", made up by Bank of Nova Scotia Mocatta, Barclays Bank, Deutsche Bank, HSBC Bank USA and Société Générale.
Although physical market for the precious metal is all over the world, the majority of transactions occurs over-the-counter (OTC) at the LBM, with trading volumes, in millions of ounces, and traded values, in US dollar billions, equal to 219,6 and 277,8 in 2014 and 214,2 and 248 in 2015 [adjusted data from LBMA]. Other large gold OTC markets are New York Mercantile Exchange (NYMEX) and Tokyo Commodity Exchange (TOCOM). World most liquid gold futures contract is negotiated at NYMEX and it is used by big consumers, producers and financial operators. TOCOM and CBOT (Chicago Board of Trade) contracts are electronically traded [Morgan Stanley 2007, LBMA].

2.2.4 Gold price determinants

Gold prices are determined by several factors, either on the demand or on the supply side.
On the supply side, it can be found producers (either mining or recycling companies) and, since they started in the early 1980's to lease gold, also Central Banks.
Producers can provide their customers with gold by directly extracting from mines or by leasing it from Central Banks' reserves, through a bullion bank intermediary. The quantity supplied from extraction in any period is period is positively related to the gold price in prior periods since mines might take substantial time lag before reacting to a price change; and negatively related to the amount of extracted gold that is diverted to repay central banks for the gold leased in the previous period incremented by a physical interest rate in those cases where the Central Bank opts for interest to be repaid in gold [Levin, Wright 2006].

When Central Banks lease out gold, they give up a convenience yield (i.e. the benefit relative to physically holding gold for one period); in exchange of that they require the gold lease rate. By adjusting their gold reserves to the point where the physical rate of interest they receive is equal to the convenience yield forgone from holding gold, plus a default risk, Central Banks determine the quantity of gold supplied through leasing (in equilibrium the sum of convenience yield and default risk gives the lease rate required by banks). Consequently, the quantity that Central Banks lease to the industry is reduced in case of a fall in the physical interest rate or rise of convenience yield or default risk caused by political or financial distress. Current period supply from Central Banks depends also the previous period quantity of leased gold to be repaid at the previous physical interest rate, which in turn depends on previous period convenience yield and default risk [Levin, Wright 2006].

Total gold supply therefore depends positively depends on its current and lagged price, current gold lease rate, lagged default risk and lagged convenience yield, while it negatively depends on current default risk, current convenience yield and lagged lease rate.

On the other hand, demand for gold can be divided into two categories: "use" demand and "asset" demand.

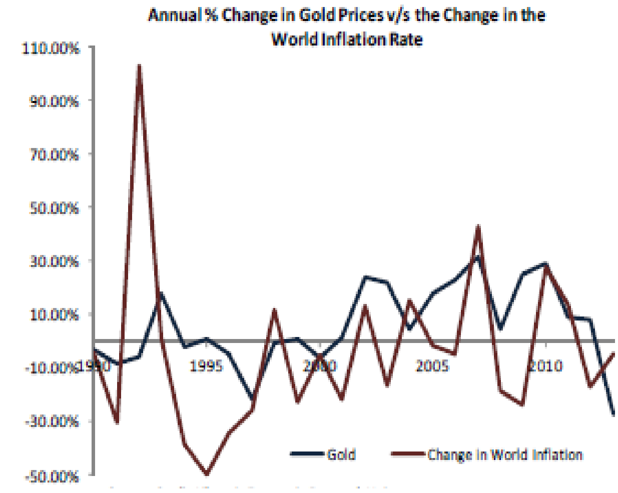
The use demand for gold (i.e. for jewellery, medals, electrical components, etc.) is affected by gold price and its volatility. In fact, if the price of gold increases in the current period, use demand for gold will decrease and if gold price is volatile, users delay their purchase of gold, resulting in a negative correlation between gold use demand and its price, as well as its volatility. However, the effects of gold price volatility on use demand are too short to be included in the studies carried out by WGC.

The second category treats gold as an investment, therefore affected by a wide range of factors including, among others, expected real interest rate, expectations for U.S. dollar exchange rate and inflation, "fear" (being gold considered a safe haven asset, it depends also on the perception of political and financial turmoil), and public announcements returns on other assets and correlation, or its lack, with other assets ("beta of gold").

Since gold prices are set in U.S. dollars per troy ounces, demand for gold is influenced by U.S. dollar exchange rate: if U.S. dollar becomes stronger non-U.S. denominated investors tend to be less favorable to gold purchases, since it becomes more expensive to buy gold which is quoted in dollars (and vice versa, if when U.S. dollar depreciates against other currencies, they find it cheaper to buy to gold).

Picture 8 Gold price and USD exchange rate in Euro from January, 2006 to May 2016 [http://www.infomine.com/ChartsAndData/ChartBuilder.aspx?z=f&gf=110575.USD.ozt&dr=5y&cd=1]

Moreover, investors and Central Banks view gold as a hedge against inflation, especially during period of monetary instability and in emerging countries whose economic situation can be highly uncertain. As the WGC explains, the long-run of gold is expected to rise in line with inflation essentially because it is related to the marginal cost of extraction and if, the



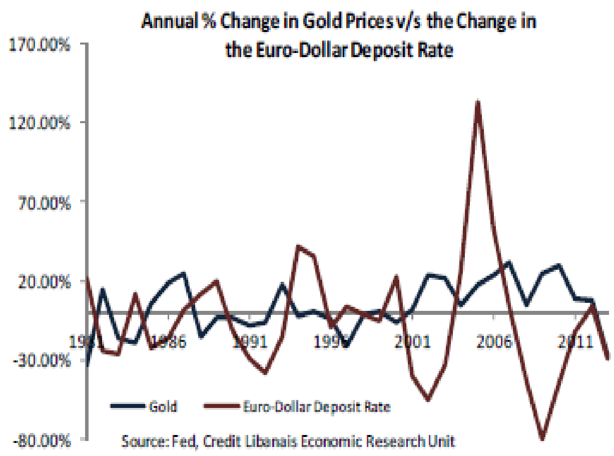
Picture 9 Gold as inflation hedge [Choueiri, Karawani 2014]

cost of production rises at the rate of inflation, the price of gold will rise at the same rate, irrespective of whether gold producers implicitly supply their customers by leasing from Central Banks as well as by extracting gold from mines, since in the latter case the gold has to be repaid it only affects supply in the short term. The long-run price of gold will be equal to the marginal cost of gold extraction if the market is competitive or proportional to the marginal cost of gold extraction if gold producers have market power. In either case the long-run price of gold will rise at the general rate of inflation. The theoretical basis that enables the gold lease rate to be used as a proxy for world real interest rates in the empirical analysis requires that the marginal cost of extraction rises at the general rate of inflation over the sample period [Levin, Wright 2006; Choueiri, Karawani 2014].

There has been a lot of debate about the assertion of gold reducing market and portfolio volatility, since the returns to holding gold have opposite sing to those of market portfolio, as the types of event that cause stock prices to collapse also tend to make gold price increase, therefore suggesting the existence of negative beta. A the more negative the beta is, the better diversification can be obtained and, since institutional investors have to diversify their portfolio to reduce risk, gold holdings are attractive, especially for short time periods when the stock market under-perform very badly, which raises demand for gold and thus its price; while if the beta of gold rises for a period of time, the portfolio demand for gold will fall during that period. Therefore, current beta negatively affects demand for gold as an investment, while it is positively related to lag values of beta [Levin, Wright 2006; Choueiri, Karawani 2014].

Picture 10 Gold price and S&P 500 from January, 2006 to May, 2016 [http://www.infomine.com/ChartsAndData/ChartBuilder.aspx?z=f&gf=110575.USD.ozt&dr=5y&cd=1]

Furthermore, precious metal's price is affected by interest rate, since asset demand for gold fluctuates in response to changes in the real interest rate as well as gold's beta; however, its impact on gold price is more complex to establish, depending on the nature of the modification. Price of gold should move inversely with interest rates (e.g. an expected increase of interest rates should cause negative adjustments of the price of gold, since it is relatively more convenient for investors to sell gold and obtain assets with higher interest rates); however there has been occasions in which they moved in line, for example if the rising interest rate reflect concerns over inflation or U.S. dollar. The relationship between interest rates and the gold price depends on a clear distinction between real and nominal interest rates and the precise cause of the rise in interest rates: if the rise in nominal interest rates aims to reduce inflation, it is highly likely



Picture 11 Gold and interest rate [Choueiri, Karawani 2014]

that gold prices will be negatively correlated with interest rates; while in the event the rise in interest rates aims to combat the formation of a financial bubble on stock markets, interest rates and gold prices could be positively correlated [Levin, Wright 2006; Choueiri, Karawani 2014].
Choueiri and Karawani (2014) computed the correlation coefficients between gold prices and all aforementioned variables, which are showed in table 6.

Correlation coefficients	Change in S&P 500 Stock Index	Change in World Inflation	Change in the USD Index	Change in 1 Month Euro-Dollar Deposit Rate
Change in Gold Prices	-0,30783141	0,22069402	-0,098801455	-0,4977280864

Table 6 Gold correlation coefficients [Choueiri, Karawani 2014]

Previously, it has been said that, since a significant part of available gold is constituted by above-ground stocks held by Central Banks and investors, a rising demand for gold can often be met without a mining activity intensification: this is one of the reasons why gold showed lower price volatility than many other commodities.

Picture 12 Gold and silver prices from January, 2006 to May 2016 [http://www.infomine.com/ChartsAndData/ChartBuilder.aspx?z=f&gf=110575.USD.ozt&dr=5y&cd=1]

Gold and silver have wide-ranging utility, from their use in art and jewelry to their use in the production of electronic equipment, but also as investment assets, their prices are therefore important in their allocation among competing uses.
Solt and Swanson [1981] and Reboredo and Ugolini [2015], based on gold and silver price data of 1971-1979 and 2001-2015 respectively, analyzed whether a correlation between the two precious metals' prices exist. Even if a positive relation among them exists, Solt and Swanson [1981] found that it was not stable. Reboredo and Ugolini [2015] found gold and silver highly dependent, with high time-varying averages, except at the upper tail and also documented silver's asymmetric downside and upside spillover effects on gold.

In the same way, also crude oil can be applied in a wide spectrum of activities. Crude oil is the world's most commonly traded commodity, of which the price is the most volatile and may lead the price procession in the commodity market. Investors in both advanced and emerging markets often switch between oil and gold or combine them to diversify their portfolios.

Picture 13 Gold and crude oil prices from January, 2006 to May, 2016 [http://www.infomine.com/ChartsAndData/ChartBuilder.aspx?z=f&gf=110575.USD.ozt&dr=5y&cd=1]

By studying the indirect impact of oil price on gold price through the inflation channel and studying their interactions with the US dollar index, Le and Chang [2011] found that they have long-run relationships, considered in pairs; in particular gold and oil market were found highly co integrated, implying that oil price could be use to predict gold price and vice versa. This could be explained by the role played by inflation: rising oil price generates higher inflation which strengthens the demand for gold and hence pushes up the gold price, thereby strengthening the role of gold as inflation hedge.

Chapter 3

The Italian goldsmith sector

3.1 Sector definition

Goldsmith sector is typically defined as the group of activities and companies that transform gold, silver and platinum, corals and precious stones for the production of goods, such as rings, earrings, trays, flatware and other decorative products; excluding activities that involve the use of gold and silver in industrial and medical sector, as well as for the production of coins[1]. The variety of products that this sector can create is very wide, also because of the various existing processing techniques that, coupled with the use of precious stones and gems that allow the creation of special and unique products. As a consequence, differentiation and flexibility are the keys for a successful strategy of companies operating in the gold sector.

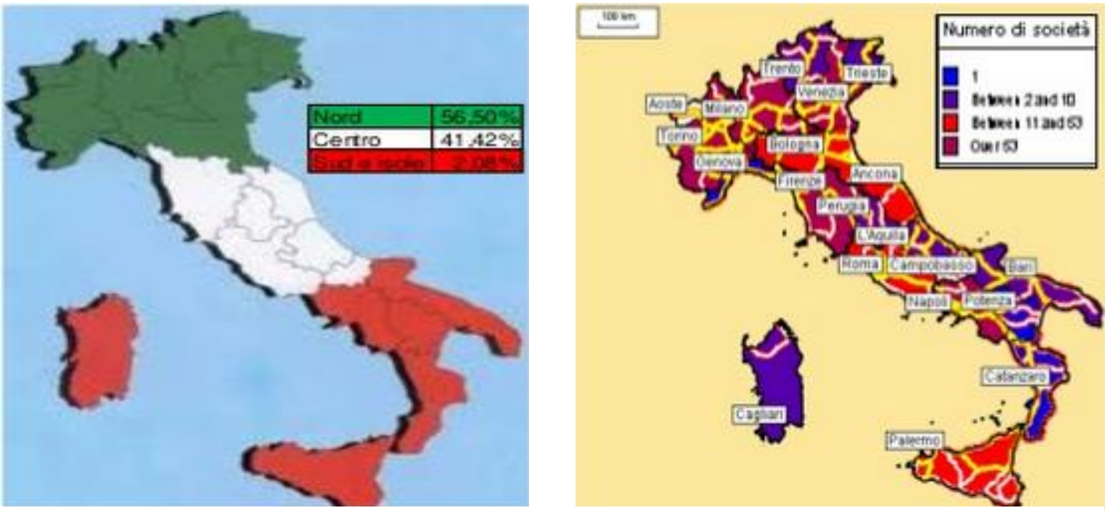
The quality of gold content, the exquisite design and the well-known craftsmanship make Italian gold and silver jewellery sector acknowledged as the manufacturing sector that most represent the quality of the so much praised Made in Italy. Italy has been famous throughout history as the manufacturer of finest gold jewellery, particularly in Europe; Italian jewellery production however, although still admired worldwide, has been slipping from its status as China, India and Turkey are demonstrating their roles in market.

Based on Federorafi's data[2], with 6.610 million Euro of sales (good figure, however 15/18% lower than pre-crisis levels), on 2013 the sector showed about 9.000 productive units in the country, with a significant reduction of 28% from 2001 because of closure of 3.400 companies, mainly due to the financial crisis. Providing jobs to 32.000 employees, the small-medium dimension of the average goldsmith enterprise can be appreciated, with 3,5 employees per productive unit, on average. About 10% of them represent the productive system of the sector, constituted by industrial companies, the remaining 90% is made of craftsmen. The structure of the sector is therefore made of a central unit made of bigger companies around which there are minor artisan enterprises that often work through outsourcing. Gold companies are characterized by an extremely high fragmentation, even more significant with respect to the Italian industry average, as a further proof of the artisanal trait of the production. Companies that distribute gold products (commercial units, considered

[1] www.vicenzaoro.com
[2] http://www.federorafi.it/allegati/scheda_sintetica_dati_settore_1463735505.pdf

as wholesalers, retailer and *compro oro*s) amount to about 23.000, with a reduction of 7% in respect to 2001, with 50.000 operators.

Artisan goldsmith is not equally distributed throughout the country, as illustrated in pic.14, and this is also due to three principal jewellery clusters. A cluster can be defined as a local productive system where the presence of small-medium enterprises can be observed, exchanging each other's semi-finished products and services, that belong to the same product group industry and share the same traditions, supported by economic associations and social organizations.



Picture 14 Distribution of revenues (on the left) and companies (on the right) relative to Italian jewellery sector on 2010 [IULM, 2011]

Historically, the three areas specialized in activities relative to the Italian jewellery sector are Arezzo, Vicenza and Valenza. This is also reflected in the revenues of 2010, the only ones to be higher than 500 thousands Euro.

Geographic Area	2010 Revenues, in thousands Euro
Arezzo	1.229.433,59
Vicenza	883.202,81
Valenza	630.880,51

Table 7 2010 Revenues of Italian jewellery clusters

3.2 Sector’s macroeconomic picture

3.2.1 GDP and gold price

Jewellery sector seems to be cyclical and highly correlated with GDP, but with more extreme fluctuations. In 2003 and 2004 there are particular cases of significant disequilibrium, a possible explanation can be drawn from the relation between the gold extraction and its price that reached its minimum in 2003 from 20 years and then started to vertiginously increase. The reduction of jewellery performance from 2007 to 2009 is not entirely attributable to national GDP reduction, but also to the entire world economic system and the deep crisis that determined a general slow-down, as confirmed by the contraction of export. The hike from 2011 to 2012 is surely attributable to an increase of gold price of at least 25%, not to an increment of the sold quantity, since if constant prices are considered, jewellery sales decreased by 4,3%.

The weight of the sector on the whole Italian company remains stable (≈0,16%), with a lightly positive trend.

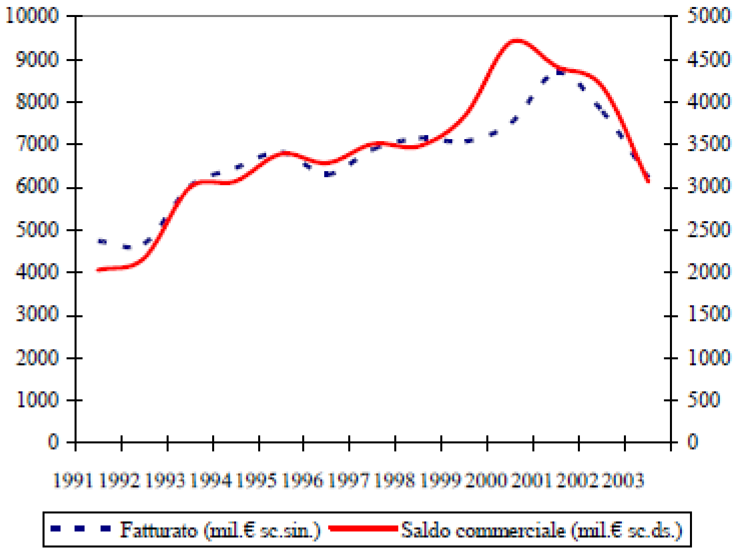
Export provide hope for recovery, being driving factor of the industry, with positive results from the various clusters, especially the one of Vicenza, signaling that Italian companies prefer growth and expansion opportunities offered by Eastern or other Mediterranean countries to unfavorable conditions and instability of the internal market.

While gold manufacturing gold companies are suffering, the peaks of gold price feed the boom of the *compro oro*s, specialized in the purchase of second-hand jewels, with an average annual turnover of 3 billion per year [IULM 2011].

Picture 15 Gold price from 2001 to 2016 [http://goldprice.org/gold-price-history.html]

3.2.2 Production

Within the industries typical of the Made in Italy, with 40% of European production and 35% of enterprises located in Italy, the goldsmith sector represented an important reality until 2000, recording a significant expansion during the 90's, with 1998 being the strongest year for Italian exports, and a sudden stop between 2002-2003 [Carcano, Catalani, Capello; 2005].



Picture 16 Long-run development of Italian goldsmith sector [Carcano, Catalani, Capello 2005]

In 1998, production reached 540 tons, of which 420 has been exported, mainly in Europe and North America, while national demand was 112,1 tons; while in 2012 only 62 tons were exported (about 15% of those of 1998) with national demand estimated to be one fourth of those the same year. This is an alarming signal, considering that exports have been the driving element of the sector throughout the 90's, however it is consistent with declines occurred in Europe and US.

This can be explained by market structural changes, among other the price of gold, the competition with new precious consumption goods represented by technology and the economic hardship [Metallis Consulting LTD, Metal Focus, 2013] that determined a two-tiered market: on one hand, we can find consumers that prefer entry-level, cheaper products, especially when they are innovative, with strong designs and combination with alternative materials such as leather, pottery and steel, on the other hand sales of luxury jewels, characterized by higher quality and price, still remain strong for those consumers that have incomes sufficient to allow their purchases [Wisniewski 2007], it is the mid-level market to suffer from this situation.

In 2010, in order to deal with the heavy financial crisis, a new commercial policy has been put in place, consisting in a general reduction of import/export duties to sustain the goldsmith sector in the international trade playground; this led to an increase in transparency and black market reduction; nevertheless Federorafi highlighted that duties imposed by certain countries still have an influence on export. In this sense, it started the Free Trade Agreement with South Korea and other countries (e.g. India, US, etc.), directing negotiation towards an agreement that allows its free trade throughout EC.

In the following year, negotiation between Confindustria and WTO (World Trade Organization) started, with the objective of liberalizing international trade.

Financial crisis had a deep impact on gold sector companies: 20% of them did not hire new personnel and 47% avoided productive investments, although already planned [Format Research 2015].

Since 2013, Italian gold sector revitalized, experiencing a production increase of 7,4%, mainly sustained by exports, that recorded an improvement of 7,2%, with United Arab Emirates, Switzerland, United States, France, Hong Kong and China as principal markets.

However, after a particularly brilliant biennium (2013-2014), with a +6,5% in first nine months of 2014), jewellery industry found itself in front of another slow-down, which physiological after the great growth of the previous year, and particularly evident for the United Arab Emirates market, also influenced by a negative price trend.

The situation of the sector remains extremely uncertain, suffering for a global demand slow-down, but it is expected to benefit from Euro devaluation. United States, once principal market for Made in Italy jewels, could be regain their part of driving market for luxury market segment, penalized in the latest years by competition with India and China [Club degli Orafi Italia & Intesa Sanpaolo, 2015].

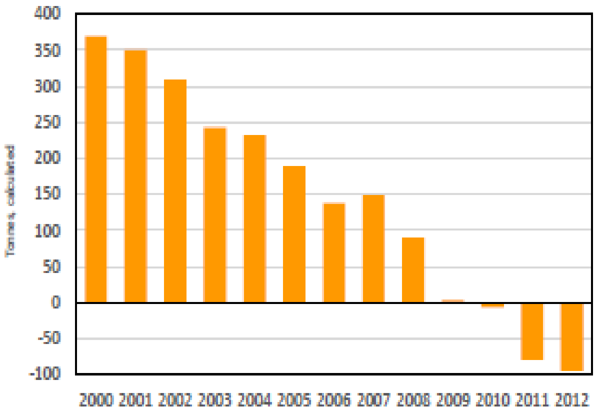
Opinions on whether the jewellery sector will finally see a recovery often depends on the type of company and geographic area considered. Small enterprises (<9 employees), more than medium-large companies (>9 employees) have suffered from crisis and turnover reduction, with less confidence about future positive prospects. As for the general economy, Northern companies have more positive expectation for future years, in respect of Southern ones. According to study performed by Federpreziosi Confcommercio, with Assocoral, Fiera di Vicenza and Assorologi, in the latest years jewelers found a reduction of both the “historical” customers (according to the 77% of companies interviewed) by 23% and of random customers (according to the 70% of companies interviewed) by 17%, as well as “Italian” customers (according to 72%)(which could be explained by: economic crisis, the maximum threshold of 1.000€ for the use of cash, the obligation to record and communicate purchasers’ data if they spent more than 3.600€) and “foreign” customers (according to 75%), particularly in Russia [Format Research 2015].

In the first six months of 2015, gold sector lived a sales increment (+5,7%), thanks to a positive push from foreign markets (Switzerland, Hong Kong, US and France, where the Valenza Jewels can be highlighted), although Chinese and Middle-Eastern demands were slowing down [Intesa San Paolo, 2015].

Augusto Ungarelli, President of *Club degli Orafi Italia*, stated that, despite the great volatility and international scenario uncertainty (war breeding grounds, fundamentalist movements, currencies volatility), jewel entrepreneurs keep on investing, confident in their product quality, handcraft, innovation, style and creativity [Club degli Orafi Italia & Intesa Sanpaolo, 2015].

3.2.3 Import and cost of labor

Italian gold sector exists thanks to imports of the commodity from other countries, since Italy is not provided with gold fields able to satisfy the entire demand; countries from which Italy imports more gold are Belgium and Switzerland. All around Europe, not more than 28 tons per year extracted, with only 5 kg from Italy.



Picture 17 Italian Net Gold Bullion imports [Metallis Consulting LTD, Metal Focus 2013]

In 2012 import has suffered a strong reduction of 7,8% clearly connected to the reduction of gold production in the country. Gold import can be, at least partially, explained by production off shoring conducted by Italian companies, especially in China, that allows the realization of jewels at costs greatly lower than Italian ones. Moreover, the possibility for consumers of comparison between the convenience among international products has facilitated their penetration in the national market.

Labor cost in Europe varies from a maximum of 51,2 €/h to a minimum of 4,1 €/h. Even if Italy, with 28,1 €/h, is more convenient than the average Eurozone, is more expensive than those countries that do not use Euro, and this could be move the Italian productive system in such countries. However, in this scenario, the fact that jewels are not anymore associated to the Made in Italy, the probable loss of value should be considered. Manpower still remains a barrier to entry for the gold sector. Nations like Turkey and India reproduce the same techniques, placing their products at global level, but at lower costs and this all works at Italy's disadvantage [IULM 2011].

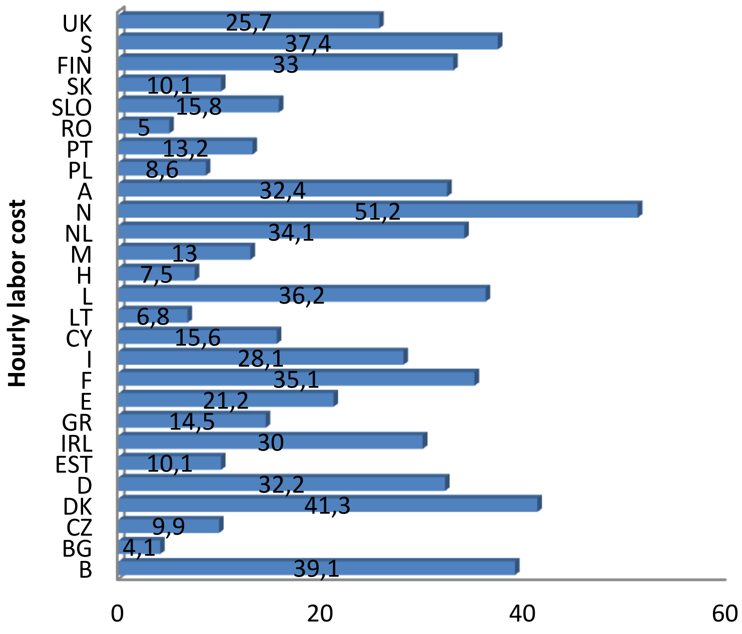


Chart 7 Hourly labor costs in 2015 [data adjusted from Eurostat]

Silver imports are increasing: 10% world silver jewellery market is Italian, with an increase of 14% of imports from Hong Kong and the decline of traditionally important markets such as US (-2%) and Germany (-19%) in 2013 [Metallis Consulting LTD, Metal Focus, 2013], which reflects a shift in silver jewellery consumption in each country.

Import	2013	2014
Value (million €)	5.019	4.690
Quantity (kg)	1.185.645	1.327.125

Table 8 Italian import of precious metals [Club degli Orafi Italia & Intesa Sanpaolo, 2015]

If destinations are considered, the areas that absorb more import are those that are most specialized in their manufacturing (Vicenza, Arezzo, Valenza), which is partially explained by the presence in these territories of producers but also of rich network of connected services, from specialized shipping agents to wholesalers and distributors [Carcano, Catalani, Capello; 2005] (see table 9).

Province	2013	2014
Alessandria	604,1	737,7
Milan	439,7	514,5
Rome	181,1	194,2
Vicenza	172,3	181,0
Florence	44,1	82,1
Arezzo	134,5	72,7
Turin	32,4	30,7
Padua	30,2	27,7
Varese	24,8	23,6
Naples	24,3	16,7
TOT.	**1912**	**2146,9**

Table 9 Italian import per province [Club degli Orafi Italia & Intesa Sanpaolo, 2015]

Country	2013	2014
Switzerland	405,9	460,7
Belgium	239,7	309,5
France	178,0	214,4
China	180,8	170,5
India	99,5	123,5
Germany	107,1	98,0
Thailand	66,2	95,1
UK	124,2	88,0
US	94,1	80,8
Romania	54,4	57,8
Austria	48,1	51,5
Israel	47,5	50,7
Hong Kong	38,4	40,5
Turkey	31,4	34,8
Other	19,5	59,8
TOT.	1.912,0	2.146,9

Table 10 Jewellery import (million Euros) by origin country [data from Club degli Orafi Italia & Intesa Sanpaolo, 2015]

3.2.4 Export

The significant role of export, which two thirds of production are destined to and 70% of it comes from Arezzo, Valenza and Vicenza districts, as driving element of Italian jewellery sector, especially during the 90's, has already been stated. In the first years of 21th century gold jewellery exports, predominant element in Italian export, showed a negative trend, while other products (e.g. platinum, precious stones, etc.) experienced a more vivid behavior. Analyzing exports per destination country, it emerges the role of US that represented for many years the principal international trade partner. Nevertheless, export to US dropped by 35% in 2003 after the downturn of 3% in 2002. In chart 8 the countries that received the 80% of export in 2001/2002/2003 are showed.

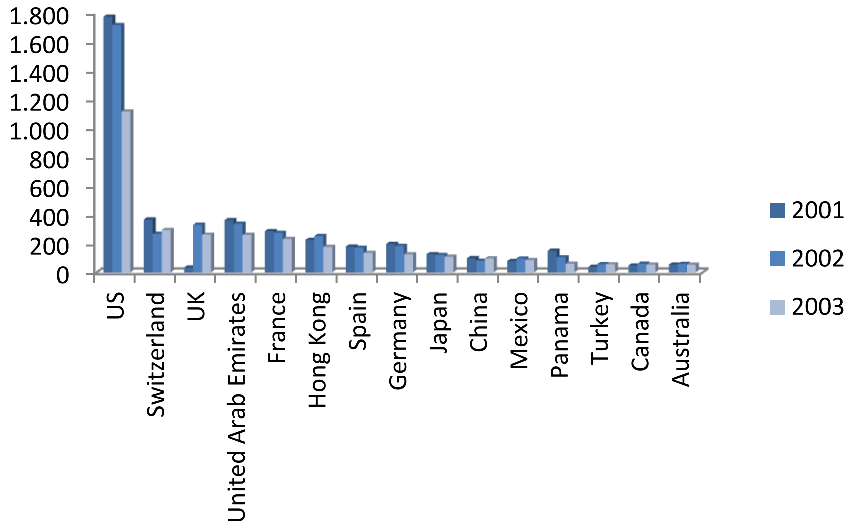


Chart 8 Export by principal destination (million Euro) [Carcano, Catalani, Capello; 2005]

It is likely that this negative development is linked to exchange effect; however the trend of Euro/USD exchange alone is not sufficient to explain export dynamics: also data about quantity exported signals a significant reduction (although less sharp, about 15%) [Carcano, Catalani, Capello; 2005].

The strong contraction of jewellery demand in the principal export countries (UAE -32,3% and US -42,8% in the two-year period 2007-2009) and the economic crisis of 2008, which caused strong fluctuations in commodity prices, can explain the negative trend of chart 9. Opposite trend was instead expected for 2010, thanks to significant increases of Swiss (+63,4%), Turkish (+49,8%), Chinese (+39,1%) and Hong Kong (+38,4%) demand.

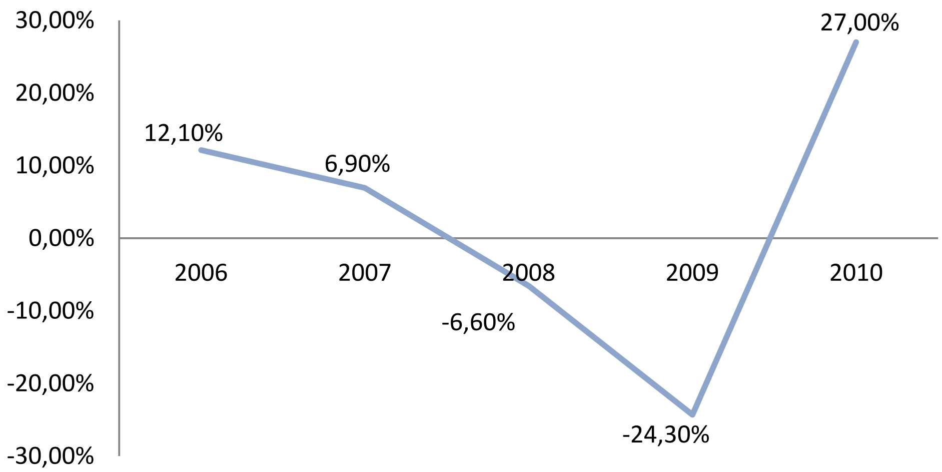
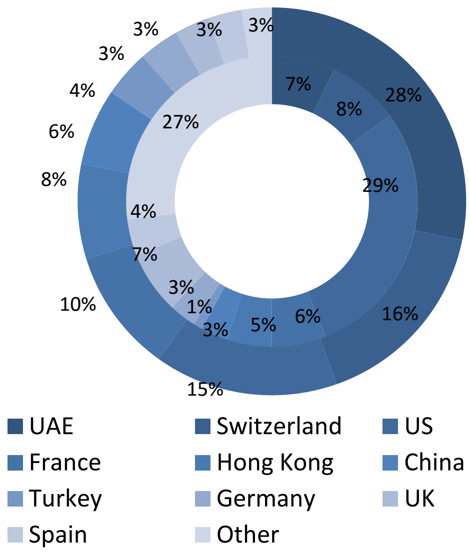


Chart 9 Italian export variation (value) 2006-2009 [adjusted from IULM 2011]

Italian export panorama is quite dynamic. If chart 10 is considered, the significant shift of the weights of 2003 (inner circle) and 2010 (outer circle) of the most important countries is interesting. In particular, UAE (United Arab Emirates) changed their position with the one of the US, and the same happened for France and Hong Kong; Switzerland quadrupled its own weight, China, turkey and Spain appeared. A general increase of the value of the single countries can be noticed, with consequent reduction of remaining percentage for the others. Contrasting data is the disappearance of Panama.

Italian exports are becoming more concentrated, with about 60% of them represented by the 4 major importers: UAE, Switzerland, Hong Kong and US. Observing the value of export, it emerges that they could in countertrend in respect of jewellery demand of the single States; this could be the result of Italian producers focusing on particularly interesting market niches that are not influenced by demand contraction.



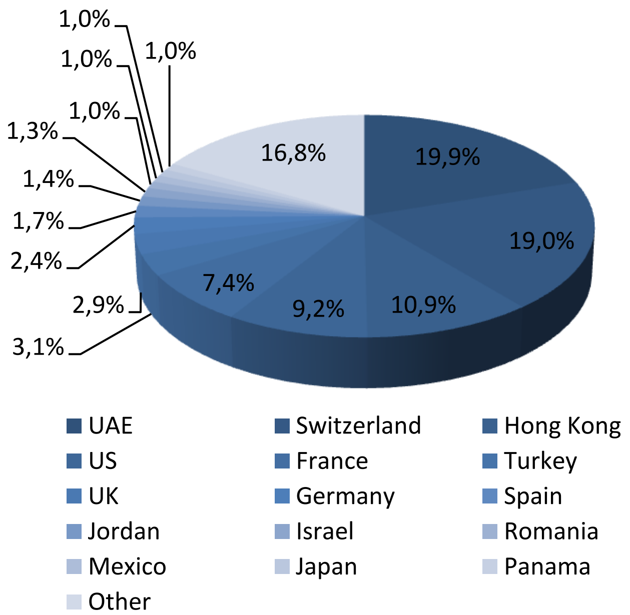


Chart 10 Italian export by country in 2003, in 2010 (on the left, inner and outer cirlce); and in 2014 (on the right) [data adjusted from IULM 2011, Club deglio Orafi Italia & Intesa Sanpaolo 2015]

3.2.5 Regulation

Gold trade and ownership is regulated by the law 17 January 2000, n.7 adjusting national and EU Directive (Directive 98/80/EC), which allows residents to buy and sell rough gold as investment and VAT-free. It also abolishes the 55-years long monopoly of Italian Exchange Office, which gold trade operations performed within the Italian territory for a value equal to or higher than 20 millions Lira have to be communicated to, irrespective of whether they are performed by professional operators acting on their own or in the name of and behalf of third parties. Bank of Italy's operations are excluded from this obligation.

Previously, Bank of Italy, through Exchange Office, was the only entity allowed to purchase and sell gold: this limitation not only determined the development of a black market for rough gold (proved by the discordance of between data relative to transformed quantity and officially purchased quantity; parallel imports have been estimated as 20% of transformed quantity [Gottardi 2001], but also made the acquisition of the precious commodity very expensive for manufacturing companies who were forced to suffer high intermediation costs.

Legislator also provided the definition of the term "gold" by distinguishing two typologies for the precious metal: gold as investment gold material, different from the previous one; and modified some regulation relative to VAT. Until 2000 in fact the purchase of a gold bar was permitted only with productive purposes; the taxation of capital gains for companies is 6%, while for private citizens is for 12,5%[3].

As implementation of EC Directive n.98/80, buying, selling and import of gold as investment is VAT-free and the mechanism of reverse change is applied in industrial trades, that basically postpones the payment of the value added tax to the moment in which the product itself is sold: this provided advantages to companies who were able to reduce their working capital requirement, relative to the stock of gold, by 20% [Gottardi 2001].

The fact that industrial use of gold is possible by means of alloys gives rise to a fraud risk. This is an old problem, in Italy there is the obligation on producers to place a mark on every goldsmith product that identifies the producer or importer, in order to easily trace the responsibility in case of disputes, as well as the title of fine gold contained therein. Legal titles for gold in Italy are 750 (18 kt), 585 (14 kt) and 375 (9 kt). A distinction between members and non-member of EU is made when dealing with the trade of foreign products within the national territory: if it is sufficient for the formers to have the indication of title and responsibility mark required by the origin country, the latter also need to exhibit importer's mark.

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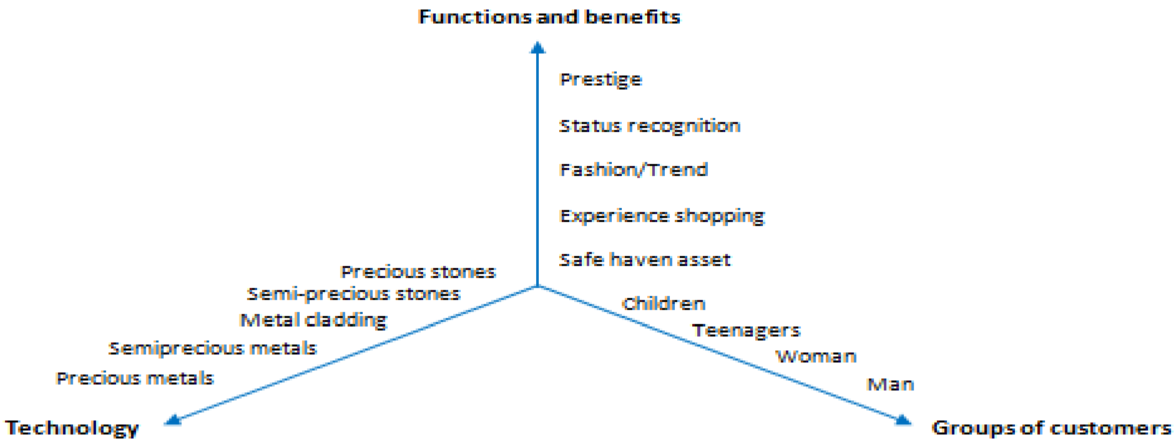
[3] See https://oro.bullionvault.it/Oro/Lingotti/Legge-Oro

Other than this regulation, the sector needs particular legal protection and guarantees. What is more and more required to professional operators is the product traceability throughout the entire supply chain, from the raw material to the finished product, in order to define origin, ethics and standard. Under this light, the voluntary certification system “TF-Traceability & Fashion” can be named, proposed by Unioncamere and recognized by EU, that allows to group all supply chains that relates with the Made in Italy that generate added value to apparel and fashion manufacturing. Once the standards are certified, the company is provided with a label and an identifying code that can be used to search for other information on the website www.tfashion.comcom.it.
There exist other initiatives, such as Federpreziosi Statute and “Patti di Sviluppo” of Vicenza jewellery cluster.

3.3 Demand

Italian jewellery is synonymous with great craftsmanship. Starting from the 80’s however a new market trend has been emerging, the so-called “fashion jewellery”. Precious and rare metals are substituted with common alloys and gems, products are characterized by standardized industrial manufacture products with intensive distribution, making the jewel losing its exclusive nature and becoming a real fashion accessory: these shifts are reflected on purchases and uses, gradually modifying also consumers’ demand [IULM 2011].
Two main segments can be consequently identified: on one hand the fashion jewellery that introduces less prestigious materials, accessible from everyone, thereby democratizing the jewel, on the other hand high quality products.
Another way to identify market segments is through homogeneous areas based on three dimensions strategic business units (SBU): technology (productive materials used), benefits (functions) and groups of customers, as showed in pic.20 [IULM 2011].
Depending on the gender of the acquirer, his age and the typology of the purchase (it is a gift or self-gift, a snap or pondered purchase, a bracelet or a pair of earrings), the main factors that influence the choice of a jewel are brand, style, price and point of sale. They way in which they interact and their weight in the final choice are not always the same.
The point of sale (POS) factor can be the most important if customers need suggestions about an economically demanding purchase or reassurance and guarantee, given that brands are not so diffused in jewellery. POS is also important for men, typically insecure when buying a gift,



Picture 18 The three dimensions of jewellery SBU [adjusted from IULM 2011]

providing reassurance, experience of consumers, etc.; for men great influence is played by the his relation with the person he is buying the gift for (wife, sister, mother,…) and the pressure the latter has posed on him; then other factors come, like jewel type, style, material, and price. For a woman, style and expert suggestions have lower importance, generally she has already obtained the decisional elements by window-shopping and advertising, she is mainly guided by product typology, price and brand, if she is weighing up a gift, or style, aesthetic, price and brand if she buys in the rush of the moment.

Based on age, these factors have different weight: if young people consider more the price factor, followed by material and brand, adult people instead value the most brand and style [Carcano, Catalani, Capello; 2005].

Factors that drive the buying process are described in the following paragraph.

3.3.1 Purchase driving factors

It has been said previously that the most important driver for a jewellery purchase are price, product typology, POS and brand.

Price is acquiring more and more importance in the choice of a jewel, typically consumers already have an idea of their budget even before entering the jewellery shop, those with a limited budget tend to focus on classic product that last for years or on accessory jewels; then it is up to seller's ability to move the choice to other factors. Buyers are often willing to revise their budget, often combining different purchase occasions. A highly-priced product however is often considered by distributors as a niche, being unable to reach significant volumes and role with respect to the whole market. Price poses limit also to the self-purchasing: the expensive is the piece of jewellery, the more the purchase is postponed and the fewer is the number of self-purchases, as people seem to expect to receive high-priced gifts rather than buy them themselves. This phenomenon is very common in the US, where the sum destined to it is around 2.500-3.000$ [Carcano, Catalani, Capello 2005; Sarett 1960].

Product typology (ring, necklace, bracelet, earring, etc.) guide the buying process in a relevant way. There exists a sort of unofficial code according such that a certain product type better conveys the message that a person desires to communicate when someone gives a present, for example a ring is a committing gift, almost a requirement for a proposal of marriage but much appropriate as a present between friends. The jeweler, once the typology id identified, can help the consumer by interpreting the style, which is much easier when the gift recipient or the buyer/recipient relation are known [Carcano, Catalani, Capello 2005].

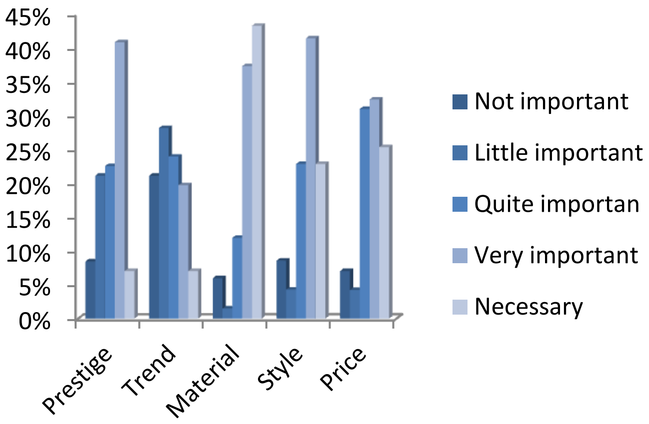


Chart 11 Results from a survey performed on a sample of 399 consumers by IULM. Respondents were asked to weigh the importance of prestige, trend, material, style and price when buying jewellery [IULM 2011]

Whether it is mono- or multi-brand, the point of sale (POS) provide the most important service to consumers through its assortment, whose creation is the result of an accurate and personal creation of brands system, each of them are appropriately represented in terms of products, in order to originate a virtuous relation between the chosen brands and POS sign. The jeweler, with his experience, reputation and tradition, reassures the buyer about the value and quality of the product, guiding him with information and advices [Carcano, Catalani, Capello 2005].

Based on Orafo Italiano's data, jewellery retail typologies can be of two types: typical small goldsmith artisan mono-brand shops or jewellery shop chains in shopping malls. The formers, located in town centers, recently experienced a significant sales decline, while the formers maintain a more frequent customer base, probably because the mall itself incentives spur-of-the-moment purchases of medium and medium-low range, by overcoming psychological barriers. Italian consumers seem to privilege buying branded products that can be easily found within a mall rather than typical workshop.

Only recently companies started giving attention to brand creation strategies, which in turn is a long, complex and expensive process. Brand is determinant in value creation, representing quality guarantee and social and ethical values. In order to have brand communicate them, it is necessary that they are confirmed by POS characteristics, so that the latter becomes the perfect place to come into contact with brand and its products. If brand can influence the choice of the POS, the final choice is guided by the jeweler who, suggesting an unbranded, moving and interestingly priced piece can make the buyer renounce to the branded one.

Differently, for those that perform a jewellery purchase for the first time or "committing" purchases (e.g. wedding band) brand becomes fundamental [Carcano, Catalani, Capello 2005].

3.3.1.1 The role of the "Made in Italy" in jewellery

Jewellery sector, notoriously artisan, has always been characterized by a demand for products with great craftsmanship, realized by well-known brands of the Made in Italy tradition, evoking intrinsic values, such as beauty, usefulness and well realized, as well as intangible values connected with life style: purchasing an Italian product means acquiring not just simple fashion good but also the access to a community that appreciates good tastes [Carcano, Catalani, Capello 2005], giving an aesthetic dimension to everyday life. A Made in Italy product is recognized around the world as guarantee, uniqueness and innovation (intended as creativity that leads to a durable improvement in functionality, usability, quality/price ratio).

In jewellery, the Made in Italy can be basically traced down to the artisan experience; it arises from the "*saper fare*", from the ability to innovate and the creativity of artisans, giving rise to equilibrium between the risks of artisan and industrial activities (i.e. inability of generating large volumes vs. standardization).

However, differently from the wider fashion system, in jewellery the incentive provided by stylists' image is missing, with the result that the name of origin is invested with scarce or null symbolic relevance for the end consumer, when compared with the previous cited sectors. As a result, Italian taste heritage and know-how risks to be not translated into a premium price. Not only the absence of established brands deprived Italian jewels of those symbolic and emotional values that determine the premium price, the Made in Italy was not even employed as quality certifier: if this value is perceived in the distribution (Made in Italy' excellence is indisputable in the supply chain), it scarcely felt by end consumers [Carcano, Catalani, Capello 2005].

United we stand, divided we fall: this is the key to revamp the value of Italian jewellery worldwide, considered of the fragmentation of the offer; Club degli Orafi operates in this sense.

Moreover, the Made in Italy is globally considered located in Milan, as creativity incubator and place where design becomes business; it may be time for goldsmith sector to abandon parochialism: it could be not anymore sufficient to bring the world to Vicenza, there is the need to bring Vicenza out [Carcano, Catalani, Capello 2005].

3.3.2 Buying occasions

Italian jewellery demand, as the function of jewel of fixing the events in the memories of those who buys or receives it, seems to be structured around precise occasions, such as birth, first communion, confirmation, graduation, engagement and wedding. These, in particular, allowed the development of specifically studied promotion policies, rather than being part of more generic collections.

Jewellery sales have a precise seasonality[4], with peaks at Christmas and on July. If the former is one of the occasions par excellence, July could be considered as the month of self-purchase, more connected to the happiness of summer.

Based occasions, three broad traditional markets can be identified:

- Christmas-gift market, accounting for nearly half of jewellery sales;
- bridal market, that does not include only engagement and wedding bands, but also the gifts to be exchange between bride and bridegroom, the "mementos" for ushers and bridesmaids, the home furnishings and many other gifts that family and friends give to the couple;
- gift market, generally speaking: birthdays, anniversaries, mother's and father's day, etc. Among festivities existing because of marketing reasons, S. Valentine's Day is the most important, particularly for young people [Carcano, Catalani, Capello 2005; Sarett 1960].

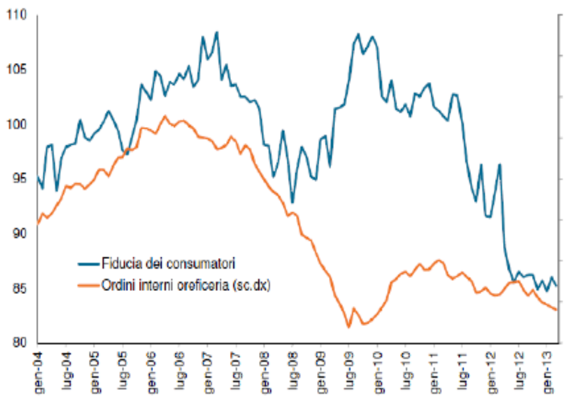
3.3.3 Italian demand

The negative behavior of exports of the last decade was not compensated by a satisfying performance of internal sales. Jewellery products' purchase has been negatively affected by the reduction of disposable income brought by financial crisis.

In recent years, the role of safe haven assets has been played by the house, as testified by the real estate boom, while goldsmith has been perceived as inessential and therefore penalized in favor of necessary or innovative substitute products, most of all technology and travels [C.R.E.I. 2007].

Moreover, goldsmith Italian market can be considered as mature, within which declining (e.g. pins and parures) and emerging segments (man, silver jewellery, or with other materials) co-exist [Carcano, Catalani, Capello; 2005].

[4] Retail sales are more concentrated in the last quarter of the year, with Christmas festivity, on February (S. Valentine's day) and on May (Mother's day). Signet and Finlay (two of the biggest goldsmith retailers worldwide) realize respectively 41% and 42% of their annual sales between September and December and, if profits are considered it is even more interesting how Signet gains 72% of its annual profit between November and January. For those companies that operate in the Chinese market, the most important seasonality is on the first quarter, during their New Year's Eve [Crestanello 2009].



Picture 19 Consumer confidence [Club degli Orafi & Intesa San Paolo, 2014]

The stationarity of the sector can be explained by both disaffection towards gold products and changed purchase habits, especially among young people and with respect to anniversaries and recurring events. The jewel is less and less considered as safe haven, but as snap purchase, with a consequently higher need to stimulate customers, with appropriate communication and image strategies and innovation policies; this phenomenon is shared by other countries, where a shift from food and apparel expenses to services and innovative products can be appreciated [Carcano, Catalani, Capello; 2005]. Moreover, if in the past the jewel represented a status, the achievement of a goal, nowadays the same role is played by travels and smart-phones and tablets which, together with increasing poverty, low consumer confidence (its level for the two-years period 2013-2014 has dropped in respect of the previous period, 2011-2012, resulting in a significant revenues reduction recorded by jewellery retailers) and unemployment, determined a general consumption decline (pic.25).

Reduced income and increasing inflation pose serious limits on the resources that customers devote to desirable products and a further shock is due the erosion of a part jewellery demand because of the completion of fashionable costume jewllery of low cost brand.

Goldsmith sector can be used as demonstration to illustrate how the crisis affected the country and the fashion industry, being the first to show purchase decline: as depicted in chart 12, since 2005 the Italian jewellery industry suffered a sharp decline, reaching its minimum in 2009 and losing 21% of internal demand from pre-crisis levels.

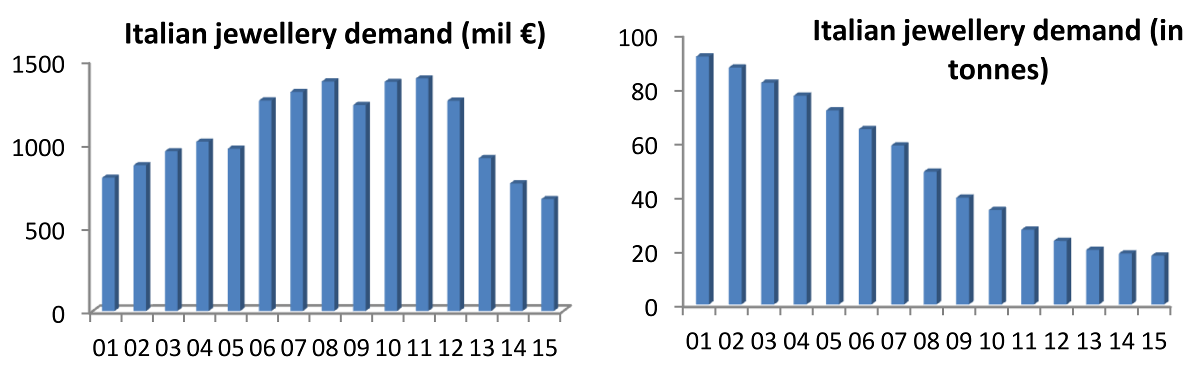


Chart 12 Italian jewellery demand in value and quantity [data adjusted from WGC]

3.3.4 Global demand

The aspiration to own and give gold jewellery, goes beyond national boundaries because of the belief of gold auspicious and enduring store of value, making it the ideal gold gift for festivals and weddings, celebrations, relationships, self-expressions and hopes for the future. The three largest markets for gold jewellery are China, India (60% of gold demand as at the end of 2015) and U.S [WGC].

Jewellery demand ended 2015 relatively firmly: fourth quarter little changed year-on-year at 674,1 tons (-6 tons), while third and fourth quarters produced together the strongest second-half year for gold jewellery since 2004. Considering full 2015 data, annual demand reduced by 3%: economic and socio-political factors caused significant declines across various markets, in particular Turkey, Middle East and Russia; nevertheless India was at the forefront of the most positive areas [WGC]

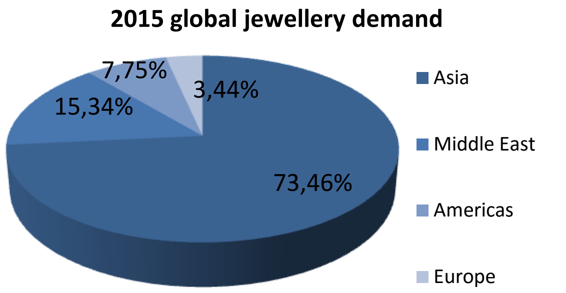


Chart 13 2015 global demand for jewellery by continent [data from WGC]

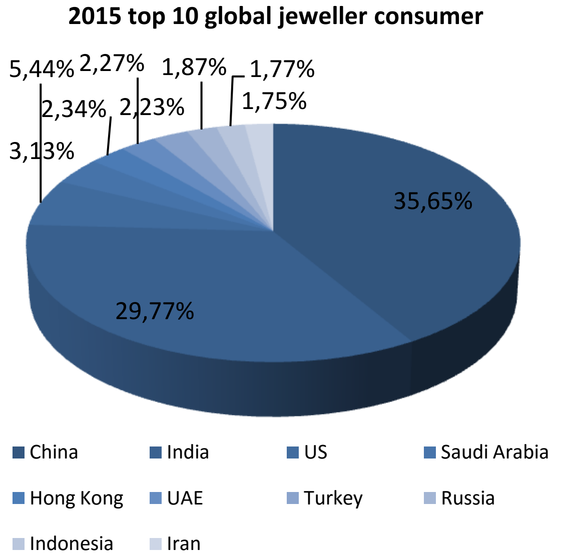
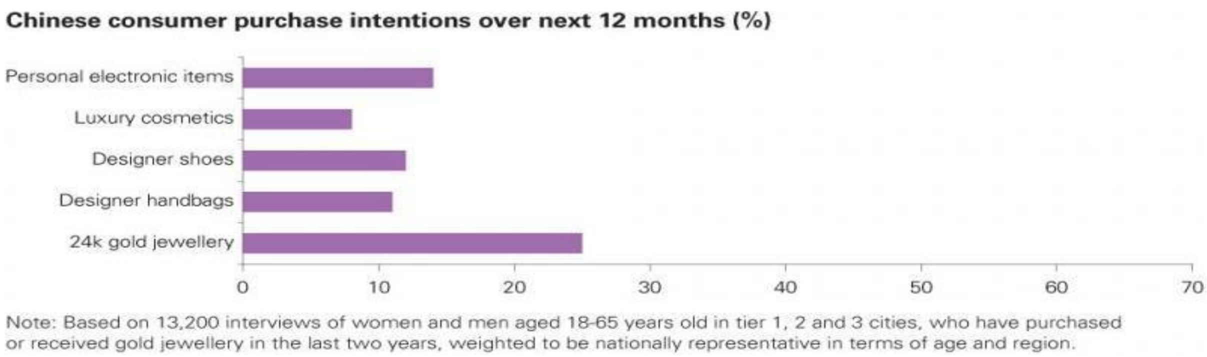


Chart 14 2015 top 10 global jewellery demand composition [data from WGC]

China

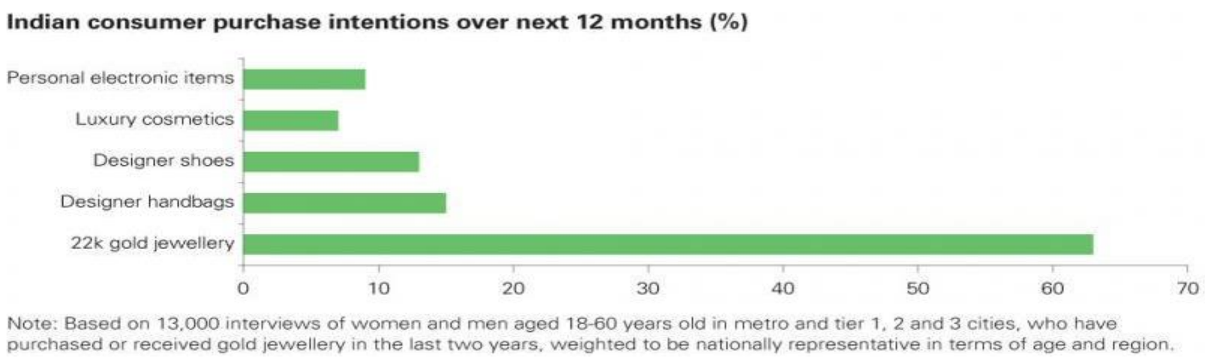
Despite global worries about China's economic wellbeing, gold jewellery demand was quite resilient in the face of slowing economic growth, with a decline demand for 2015 fourth quarter and entire year of just 1% and 3% respectively. This was due to economic downturn and stock market tumultuousness of the first half of the year through a damaging effect on consumers' sentiment. Competition is intensifying, putting pressure on margins and encouraging consolidation across the sector, at the expenses of small regional brands while larger retailers were better off, with better product range and deeper pockets. Inventories are however being managed very conservatively, given the lack of confidence that 2016 will provide significant demand growth. In China, gold has always been considered an aspiration gift and is believed to bring good fortune; it is often gifted to young family members for special occasions. The children of rich families are said to be born with "golden chopsticks" in their mouths rather silver spoons. For years China was the fastest-growing market for gold jewellery, with a steadily affluent society that holds that gold jewellery demonstrates sound financial foresight. The young Chinese have now become a dominant force in the gold and luxury goods market and look set to be the drivers of gold jewellery demand in the coming years [WGC, *Gold demand trends full year 2015*].



Picture 20 Data from WGC

India

Indian jewellery demand reached 654,3 tons (+5%), its highest value since 2010. November and December were particularly upbeat as Dhanteras was preceded by a drop in gold price, with price sensitive consumers taking the opportunity to make cheaper purchases. Dhanteras, the first day of Diwali, is a day for celebrating wealth. Devotees light the first lamps of the festival of lights to welcome Lakshmi, the goddess of prosperity, into their houses to bless them with fortune for the year ahead. In India, gold jewellery is considered as store of value, symbol of wealth and status and fundamental part of many rituals. In the country's rural population, a deep affinity for gold goes hand in hand with practical considerations of the portability and security of jewellery as an investment. This, in part, explains how India's appetite for gold defies market conditions: despite a 400 per cent rise in the rupee gold price over the last decade, gold demand from Indian consumers continues to grow. Being considered auspicious, gold ornaments should be worn for important ceremonies and occasions across the countries, as the ancient law decreed; gold is a fundamental part of marriage rituals in Indian society (weddings generate about 50% of annual gold demand) [WGC, *Gold demand trends full year 2015*].

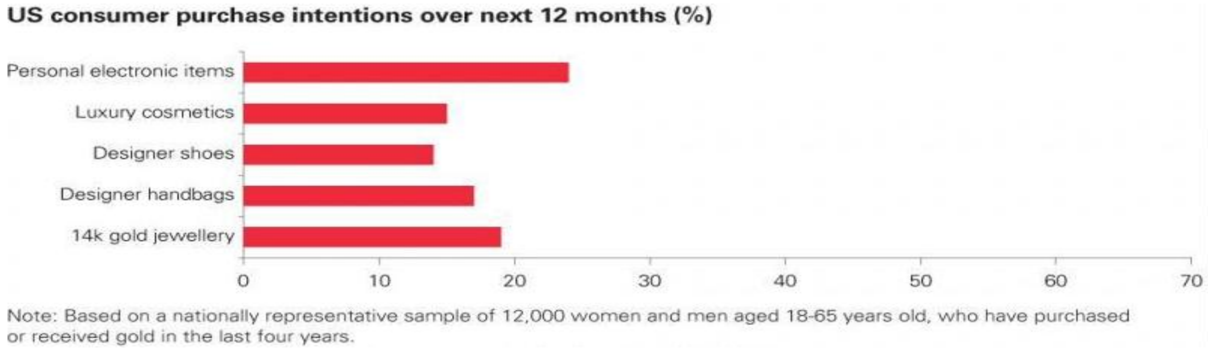


Picture 21 Data from WGC

U.S.

Demand for gold jewellery increased by 2.7% in the fourth quarter of 2015, from 44,4 tons increasing to 45,6 tons; matched by a cautious 3% growth in annual demand (from 116,6 to 119,6 tons) and benefitting from the drop in gold price during the third quarter, when retailers would be re-stocking in preparation for the seasonal surge in demand because of holidays season. The positive trend began in 2013 but now it seems more fragile: on one hand, consumers' disposable income has benefitted from lower oil and heating prices, on the other hand spending is shifting to travels and leisure rather than retail goods.

Drivers of U.S. demand for gold are majorly weddings: in 2011 U.S. market for engagement rings and wedding bands is more than 9,7 billion dollars. On average, couples spend between 1.500$ and 2.000$ on the bride's wedding band, while customized premium products are experiencing a growing trend [WGC, *Gold demand trends full year 2015*].



Picture 22 Data from WGC

Rest of the world

Globally, gold jewellery demand showed mixed results in 2015.

With demand falling by 26% (-5,2 tons), Turkey's fourth quarter results were in line with the weakness showed in the previous 9-months period. This can be explained by weak local currency, which kept local gold prices high, and domestic economic and political scenario (and its proximity to Middle Eastern countries' conflicts and terrorist incidents). Being so, consumer preferred to recycle their existing holdings of gold rather than purchasing more.

Middle Eastern markets behaved little better (-5%), in connection with declining tourist revenues in the UAE.

Asian markets, like Japan, Vietnam, Indonesia and South Korea grew in 2015 while Thailand, Malaysia, Taiwan and Hong Kong lose ground. In particular, Hong Kong showed the worst performance (-23% in the fourth quarter of 2015), suffering from its heavy dependence on Chinese tourists. On the other hand, demand in Vietnam increased by 31%, thanks to low inflation and strong economic growth that improved affordability among gold-consuming population.

Russian jewellery demand slumped to a 14-year low of 41,1 tons (-39%), collapsing since the middle of 2014, because of the effects of military intervention in Ukraine (e.g. freefalling currency and international sanctions) and plummeting oil revenues.

European regional demand contracted by 1% in both fourth quarter and full 2015; slight improvements are showed by UK (+1%) and Spain (+6%); France, Germany and Italy continued to shrink (-5%, -2% and -1%, respectively), but there are signs that at least Italy and Germany may be bottoming out, helped by slow-moving economic progress [WGC, *Gold demand trends full year 2015*].

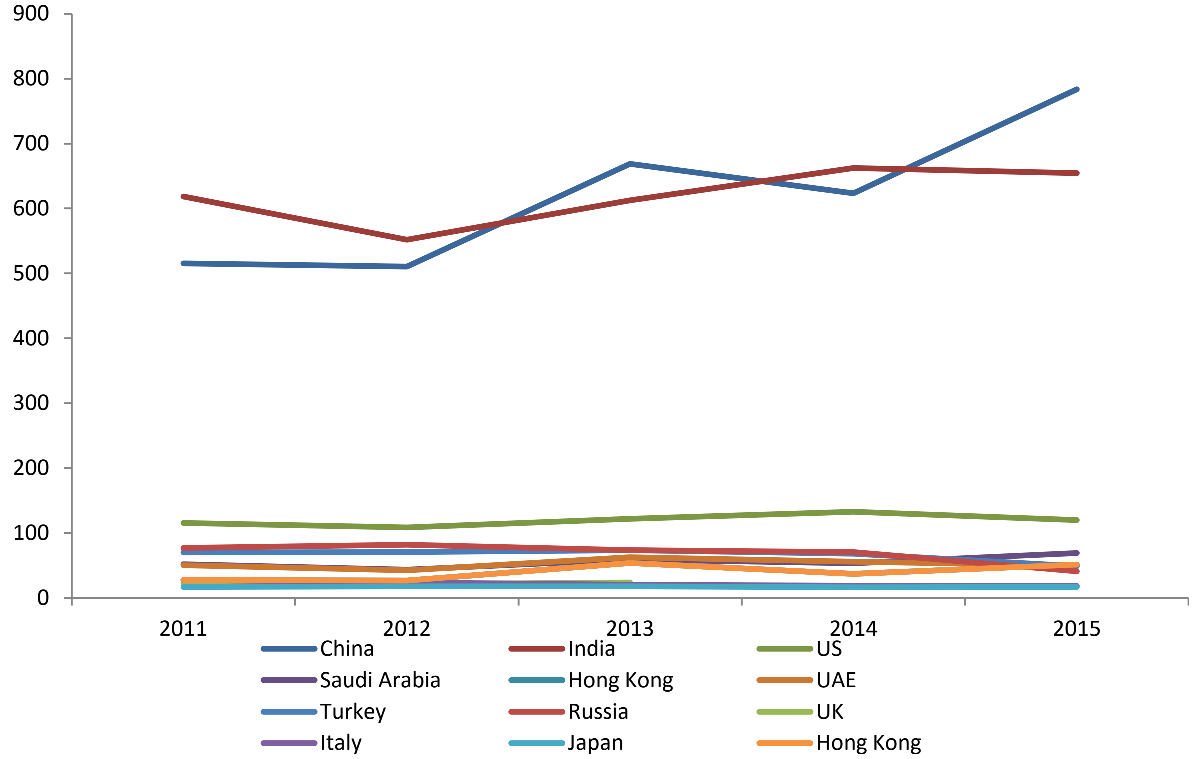


Chart 15 Gold jewellery demand (in tons) evolution from 2011 to 2015 [adjusted from WGC]

3.4 Offer

In Italy, goldsmith sector has always represented a linchpin of the Made in Italy, allowing its inclusion in the wider fashion system industry, given that jewels and other fashion items like apparel, glasses and so on all have in common a *plus* with respect to the value of a simple product. As the fashion system as whole, also jewellery industry has been characterized by certain stylized facts in the recent years: the artisan nature of the products, that brings together manual skills connected to the "*saper fare*" and the Italian taste; a highly fragmented entrepreneurial fabric, especially if compared with the average of national and local manufacturing industry; goldsmith districts, where external and agglomeration economies represents an important factor for cooperation and competition; high export orientation [SMILE TOSCANA].

Peculiarities of jewellery industry that distinguish it from other manufacturing activities are:

- high cost of raw materials and connected safety requirements;
- high intensity manpower, with extremely complex specific knowledge typical of artisan production;
- lower role played by economies of scale, especially in these situations where a certain "division" between the companies' relationships exist [SMILE TOSCANA].

Jewelley sector is in the maturity phase, characterized by market demand saturation. Competition is really strong, based on the different inputs employed in the manufacturing: the value perceived by the final consumer is actually the quality of the material, critical criterion per price definition. Product selection is seasonal, models change based on current collections while maintaining homogeneous some basic characteristics (e.g. the "remake" of successful pieces) [IULM 2011].

Carcani, Catalani and Capello [2005] identity common features shared by companies operating in the gold sectors:

- they operate in the medium-high level of goldsmith and jewellery market;
- mainly, they have small dimensions. Based on the definition provided by European Commission n.1442 of 6 May 2003[5], in 2010 companies were distributed as depicted in the following chart:

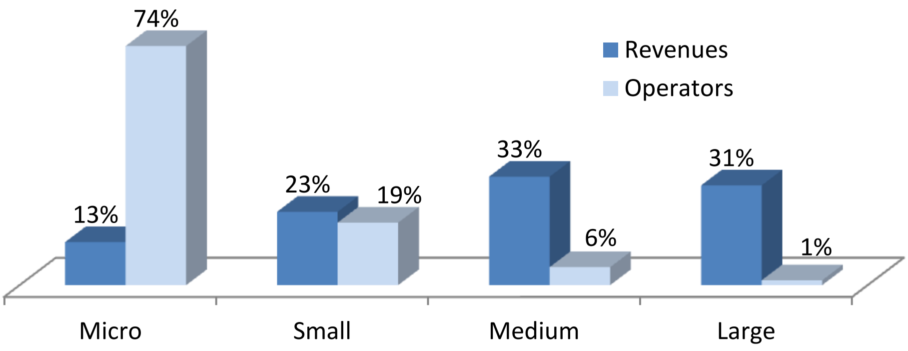


Chart 16 Companies classification based on dimension and number of employees in 2010 [adjusted from IULM 2011]

[5] Based on their dimensions, companies can be categorized as follows:

- micro: employees <10; annual turnover/balance sheet <2€ mil
- small: employees<50, annual turnover/balance sheet <10€ mil
- medium: employees <250, annual turnover <50€ mil or balance sheet <43€mil

Turnover seems to be inversely correlated to the number of operators. Historically, small businesses, whose number increased and experienced a positive trend until 2010, when mortality rate for the category jumps since they are not able to face the crisis; medium-sized companies see a contraction of sales in 2010, although showing more stale sales than small ones; big companies have largely fluctuating turnovers and are the first to perceive financial crisis's effects [IULM 2011];

- they are generally family-run businesses.

However, there are also some differences [Carcano, Catalani, Capello 2205]:

- integration degree: there are strongly vertically integrated companies that perform on their own all phases of the productive cycle as well as companies that instead are extremely specialized in one or more manufacturing stages. Companies directly relating with end market with their own braded point of sales tend to be limited;
- different technologies employed that result into different productive structures. Companies producing jewellery and gold products, where handmade still has a certain weight in the manufacturing, typically make use of outsourcing and internally realize the last stages of production, those most value-adding;
- ownership structure and managerial models; the majority of the entities is family-run while the presence of professionals not belonging to the controlling company is much less frequent;
- diversification degree: companies that operate in the sector can be specialized in the jewel industry as well as diversified in other businesses, such as watch making, leather and apparel industry.

3.4.1 Structural characteristics of goldsmith companies

Carcano, Catalani and Capello (2005) highlighted some structural characteristics of a sample of 429 goldsmith companies[6], irrespective of conjunctural elements. The cost of raw material, basically precious metals and stones, has a structural high incidence on revenues (about 60%), as well as subcontracting with third parties (about 19%); as a consequence, a certain flexibility of the cost structure can be denoted, with fixed costs (mainly personnel and depreciation) representing about 15% of sales, with depreciation lower than 2%..

Generally, they also have a relatively low level of long-term assets since the majority of assets are made by account receivables (about 42.3%) and inventory (28.7%) suggesting a behavior similar to the cycle of sales. Long-term liabilities have a small weight on the total financing

[6] Economic and financial performances from 2001 to 2003 have been analyzed.

resources, being total assets financed for 60% by current liabilities and for 20% by shareholders' equity, affirming in Italy the prevalence of highly capitalized companies, especially if they are family-run [Carcano, Catalani, Capello 2205].
Enterprises can adopt branded or unbranded strategies, where typically the distinction between them depends on brand awareness from end consumer. The study highlighted the importance of the brand as a driving factor of sales: a well defined and recognized brand is useful for increasing sales, even when the sector is suffering, while there exists a negative relation between branding strategies (effective, but expensive) and investments profitability, with a ROI of 1.8% against a ROI of 2.55% for unbranding strategies [Carcano, Catalani, Capello 2205]. This can be reasonable, since the formers have capital intensity (with more intangible assets) and labor costs higher than the latter, indicating their more structured reality. Branded companies tend to have bigger dimensions and significantly lower incidence of exports, given that they concentrate their investments on the Italian market. As a result of their strategies, they tend to have more liabilities and be more downstream vertically integrated, with one-brand retail shops [Carcano, Catalani, Capello 2005].

3.4.2 Labor

Traditionally, gold transformation is characterized by extremely high manual skills and appropriate manpower present only in areas with strong and long-established artisan traditions; this led to the development of a sector which is concentrated in particular contexts, i.e. the jewellery districts. Considering the principal profitability indicators, Italy find itself better positioned if compared with European competitors: sales per employee, production value per employee and purchase value per employee all show Italian figures above the European average and at the top of the classification, only after Belgium, France and Finland.
The situation is however evolving: some goldsmith products (e.g. chains) allow for a medium/high mechanization degree, with significant impacts in terms of capital intensity and labor productivity increase. Value added per employee is in fact lower than the European average: it demonstrates how in Italy value added, the most important result of productive activity, is obtained by sustaining a higher financial commitment in procurement than those observed in other European countries [Assicor 2008]. In other types of processes and products, a certain mechanization degree (however lower) and better technologies permitted labor rationalization, and a consequently better use of the productive factor, when comparing with typical artisan laboratories [Gottardi, Scarso 2001].
Labor cost is still very differentiated depending on the various countries of producers because of noticeable differences of salaries, as already seen in chart 7 (paragraph 3.2.3).

Competitiveness of some emerging countries, among others India, is becoming more and more worrying for Italy, because of these differences as well as the artisan traditions they have, that make them potentially intimidating competitors [Gottardi, Scarso 2001]. The manufacturing price is not anymore the basis for Italian competitiveness, once considered the gaps with labor costs in Turkey, India and China. Manufacturing price varies depending on product's complexity, manufacturing time and quality of the labor used: standardized products (e.g. chains) have low workmanship cost, while more sophisticated products that incorporate precious stones (fine jewellery) or that are sold through direct channel, bypassing the intermediary, the price is determined piece by piece. In a survey conducted by Crestanello [2009] it emerges that 35% of the interviewees declared a manufacturing price lower than 1€ while only for 16% of them it was higher than 5€ and/or was piece-determined and neither it was subject to particular changes throughout time: for 80% of the companies it remained unchanged between 2003-2005, for 14% it increased and for 6% it even reduced.

The objective of Italian jewellery industry has to be therefore the capability of proposing a product that is innovative, in terms of shapes and materials, and a better service, being the Italian creative capacity internationally acknowledged which leads foreign buyers to delegate to Italy the role of creative design for high quality products [C.R.E.I. 2006].

Production off-shoring processes with the objective of costs limitation in countries with lower labor costs do not represent a relevant phenomenon (although expected to increase) for goldsmith companies, differently for other industry belonging to the fashion system. Typical offshore activities (alternative still chosen only by some of the biggest and structured companies, especially towards Romania, Thailand, Jordan, South Africa, Turkey) are the most standardized, with lowest added value and requiring less technological competences. The reasons are not (only) explained by labor costs savings but by the existence of more favorable fees for certain consumption market, most of all U.S; sometimes this strategy is suggested by American importers themselves. Goldsmith off-shoring is also limited by the low number of companies with a brand: entrust foreign subcontractors with an unbranded production is risky since, once these businesses have acquired the competences necessary for the realization of a high quality products, they could more easily sell it to final customer and bypassing the intermediation of the Italian producer. Consequently, if on one hand production internationalization could increase companies' competitiveness, thanks to a cost reduction, on the other hand it transfers the skills and competences to other countries' producers that, someday, could become dangerous competitors [C.R.E.I. 2006, C.R.E.I. 2007, Crestanello 2009].

3.4.3 Technology and innovation

The industry is mainly still provided with conventional type of technology. Starting point for all processes is the realization of the alloy, made using gas or, more recently employed, induction furnaces, starting from the precious raw material presented in strand, as a sheet or it is obtained by molding under pressure. Although relatively recent, some processes are nowadays quite common, allowing for higher volumes but maintaining a low weight [Gottardi 2001].

According to Gottardi [2001], technological innovation has been mainly connected with chains production and the electrical molding. With the former, a substantial mechanization has been obtained, as well as highly specialized processes; while the latter is used for hollow jewels, allowing the direct realization of three-dimensional, even complex pieces, without too expensive machinery and combining them with other materials (other precious metals, but also textiles), giving birth to completely new articles.

Despite technology has never been so critical in goldsmith sector as in other manufacturing industries (greatest importance is given to the manual skills of the artisans) [Gottardi, 2001], this approach is changing, especially because Turkish and Chinese producers are becoming a serious threat: Crestanello [2009] states that Italian jewellery, especially the Vicenza district, has lost the opportunity to develop a highly capital intensive transformation process able to counteract the disadvantages of higher labor costs.

Further technological development could be represented by CAD/CAM applications and the use of rapid prototyping systems, also in consideration of the expensive nature of sample case creation and often subject to modifications suggested by the customer himself: these can provide important costs savings, postponing the expensive creation of the prototype at least to the moment in which some more precise indications on the type products demanded are obtained from customers. CAD/CAM and rapid prototyping offer also a reduction of time to market and an improvement of customer services [C.R.E.I. 2006, Crestanello 2009]. The phenomenon of 3D printing is becoming quite popular among goldsmith companies, since their production process requires the realization of prototypes, models and semi-finished products that can be advantageously created with this technology. Companies from Arezzo district were the first to implement it, making it fundamental for their competitiveness. This development, rather than providing benefits in the form of lower costs, they came as increased revenues because of the higher customers' willingness to pay [Boccardi et all, 2014].

Generally speaking, innovation can lead to two main results: cost reduction or product and service quality improvement that allows the repositioning in higher price ranges.

Innovation that provide cost savings are connected to the introduction of more modern machinery (process innovation) or productive re-organization policies (like waste reduction, better management control, set-up times reduction). Traditionally, innovation in the goldsmith sector occurs in a continuous and incremental way, by the implementation of more technologically advanced machinery as well by learning by using (i.e. when end users intervenes in the improvement of the productive process) [C.R.E.I. 2006, Crestanello 2009]. As already mentioned, among these innovation there is the electrical molding, a technique with high capital intensity (at least if only the goldsmith sector is considered) that permits the production of considerable piece of jewellery by using a limited amount of gold, which in times of precious metal's price huge fluctuation can be very advantageous [C.R.E.I. 2006, Crestanello 2009].

Innovation that enhances product and service quality and the consequent possibility of price increase are called up-grading; it can be due to new or better technical performances, the adoption of new and alternative materials, defectiveness reduction or better finish: these are all elements that are more and more important to the appearance of the final product through a more sophisticated design and better fitting market requirements, also considering that there is need of frequently renovate collections, given the shortened product life cycle [C.R.E.I. 2006, C.R.E.I. 2007, Crestanello 2009]. Other ways to pursue product up-grading are promotional and branding policies, product differentiation, exploitation of market niches (e.g. sophisticated consumers with high disposable income), flexibility increase and time to market reduction.

3.4.4 Procurement and inventory management

Most of the procedures for procurement and production of the goldsmith sector are different from the typical ones that are implemented in other manufacturing companies, this is mainly due to the volatile cost of the metal and normative restrictions. In this sector in fact, barriers to entry are not represented by long-term assets (except for highly automated processes) as well as economies of scale; on the contrary working capital has a great weight on the balance sheets of the companies belonging to this sector, due to deposits for the purchases of raw materials (gold and other precious metals and stones) and WIPs [Gottardi, Scarso 2001]. Gold price movements have contrasting effects: since jewel prices are determined based on the value of the raw material contained therein (i.e. the weight, the weighing in carats and the gold price), a reduction in gold price determines a incentives for both consumers, who can buy more, and businesses, in terms of working capital reduction. However, on the long-run, this drop of the gold price may have negative effects on gold jewellery demand (is able a gold piece of jewellery with a lower intrinsic value to generate interest among consumers?) [Gottardi, Scarso 2001].

Even if companies can go beyond these limits thanks to operators who offer specialized services able to diminish the negative impacts on capital of price variations, the management of procurement and working capital more often leads to a rationing of the precious metal because of the notable financial requirements. This subject is particularly important for those businesses that directly acquire the raw material who need a huge capital availability, which in turns determines a strong selection among producers, being very little the number of organizations able to operate in this way [Gottardi, Scarso 2001]. Capital is needed not only for procurement but also for gold being currently transformed, consequently the total financial requirement can suffer from severe fluctuations connected to the volumes of the orders and the length of the lead time: the longer the lead time, the longer financial resources are tied up and the higher the risks arising from metal's price movements.

These problems can be reduced using through the intermediation of metals banks or two legal institutions: contract work (*conto lavorazione*) and loan for use (*prestito d'uso*).

In the first case, metals banks, private companies with availability of consistent financial resources, are able to acquire significant quantities of gold bars[7] that are transformed into lighter sheets or strands and then sold. This permits medium-smaller producers to parcel out their procurement and reducing the stock, thereby limiting price risk. The price at which the precious metal is sold by metals banks will be however higher than the official price offered by the banking system [Cattaneo, Sabbadin, Virtuani 1993].

With contract work, big goldsmith companies (typically manufacturing or retail/distribution companies) provide in advance small producers with a gold quantity necessary for the realization of a predetermined quantity of products. The finished product is then returned to the contractor who pays only the costs of workmanship and not those related to the materials, since their ownership remains to the contractor. As indicated by Cattaneo, Sabbadin, Virtuani [1993], this solution gives the opportunity to small companies to operate even with scarce financial resources and, partially, to overcome the limitations connected to raw material procurement gold price fluctuations. On the other hand however it favors a strong dependence relation towards the contractor who, controlling the commercialization phase, can guide the activities of their subcontractors thereby hampering their dimensional development: if this legal institution is essential for a start-up goldsmith company, it turns into an obstacle for their renewal and qualification [Cattaneo, Sabbadin, Virtuani 1993].

[7] Standard gold bar: minimum fine gold content of 10,85 kg, maximum fine gold content 13,375 kg.

With loan for use, differently from the previous case, the metal is not provided ahead of time by an operator belonging to the field but by a banking company. With such a practice, quite common within the Arezzo district, gold is temporarily imported from a Swiss Bank (with their own gold reserves). In this way, small companies borrow a prearranged, small quantity of gold. At such contract's expiration date they pay the interests, typically favorable, on this quantity or the equivalent market value expressed in Euro (i.e. metal's quotation resulting from the London fixing at the moment of the invoicing). The interests paid on loan for use, generally intended for 30 or 90 days, are usually lower the official ones. Advantages and disadvantages of the loan for use are similar to those of the contract work: it allows small entrepreneurships to overcome the field's barriers to entry, by drastically reducing the necessary investment in working capital, while increasing their companies' dependence towards the intermediaries (whether distributors or bank) [Cattaneo, Sabbadin, Virtuani 1993; Gottardi, Scarso 2001].

What emerges from surveys conducted in Italy [Crestanello 2009] and U.S. [National Jeweler 2004, Shor, Huntington 2003] that, especially in smaller businesses where in the past scarce attention has been paid to inefficiencies, simple organizational modifications can provide significant cost savings. Management control is a problem that many artisan companies have in common and its introduction is made difficult because of a wide product range but also because of a lack of competences. In many enterprises reorganizational policies are focused mainly on personnel and cost reduction; however there is the risk that also expenses for investments, for example by postponing the acquisition of new machinery and not hiring qualified people able to provide new competences. In the U.S. , many jeweler do very little inventory planning, with a high percentage of the interviewees without a formal inventory management system in place and seem to basically rely upon guesses when it comes to calculating open-to-buy, also because they lack the fundamental skills.

It is also interesting to observe how procurement and inventory management vary throughout the supply chain. Typically, producers realize a single product line and sell big quantities while retailers sell piece by piece, with a much wider assortment. Given the intermediary role of wholesalers, they can reduce this discrepancy by purchasing big quantities of different models from lots of producers; their role becomes even more important when production and consumption markets are very far away from each others with a myriad of small enterprises, as in the case of the goldsmith sector [Crestanello 2009]. Wholesalers are also more specialized on providing services to the retailers, especially on foreign markets where a lower understanding of local consumption trends on the part of the small producer could turn into difficulties in the relationship with customers because of the language, the culture and so on.

As Crestanello [2009] observed, the higher competitiveness and gold price's increases lead many retailers to revise their strategies, especially their relationships with suppliers, and started to:

- reduce procurement costs by changing and reducing the numerousness of suppliers and, as well as initiating cooperative behaviors with them;
- implement product differentiation strategies;
- stimulate consumption through a higher productive variety and a more frequent renovation of the models;
- reduce inventory costs by implementing organizational policies such as just in time and quick response;
- invest into sales forecast planning and control system (ICT);
- take advantage of economies of scale through merge and acquisition.

As examined in depth before, transforming a raw material with a high specific value if compared to that of manufacturing, the goldsmith sector is prone to market risk, especially in situations of increasing variety of productive offer, length of product life cycle reduction, uncertainty in the demand and high seasonality [C.R.E.I. 2007, C.R.E.I. 2006, Crestanello 2009]. Traditionally, the reduction of the market risk is carried out by the retailer, who has the necessary information to identify consumers' trends. With the objective of having the right products at the right moment, retailers are starting to conduct quick reorder policies, asking for lead times more and more short and precautionary procurement strategies through orders fragmentation, and sharing market information (e.g. forecasts, sales trend, etc.) with their suppliers trying to guide product designing and planning, in order to better adapt them to market requirements. In a crisis situation, wholesalers and retailers know they are able to fetch a lower price especially on those products that are more repeatedly purchased throughout time. Some wholesaler have also started to extend payment dates and to introduce on consignment policies (*conto vendita*; which means that if retailers, once purchased the products are not able to sell them, can return them back to the wholesalers based on previous agreements) for procurement, which however induce an increase of risk and financial exposure. The resulting closer relationship between the actors in supply chain is finalized to a contraction of the time of value creation processes and consequently also the reduction of market risk [C.R.E.I. 2007, C.R.E.I. 2006, Crestanello 2009].

3.4.5 Distribution

As for other fashion system industries (e.g. apparel, footwear), the goldsmith sector's supply chain structure falls within those buyer-driven, where critical success factors are mainly design, brand and marketing activities, often under control of the large-scale retail trade, while productive functions are typically decentralized to subcontractors who perform merely executive tasks [Crestanello 2009]. In the goldsmith sector however, big branded goldsmith producers are still contending the coordination of the supply chain with big retail chains, e-commerce, telesales and companies belonging to the fashion industry that recently entered the jewellery sector. In fact design, being an activity with difficulty separable from production because of the strong connection between the ability of creating new products and the capacity of its industrialization, is still a prerogative of producers, even smaller ones. Currently, big retailers are assuming a more active role, bypassing wholesaler's intermediation and directly buying from the producer, especially because of the need of guiding design activity in order to meet specific market requirements and create new and exclusive products. Companies belonging to the entire goldsmith sector supply chain are therefore changing their strategies, leading to processes of vertical and horizontal integration because of the strong competitive pressure and market changes, among others:

- consumption reduction;
- competition from alternative luxury products (technology, travels, …);
- new entrants, in particular fashion brand companies, commercial television companies and ecommerce that are practicing downward pressure on prices, reducing market spaces for traditional retailers;
- possibility for consumers of comparing products are prices by visiting retailers' websites;
- wider offer;
- shortening product life cycle

[C.R.E.I. 2006, C.R.E.I. 2007, Crestanello 2009].

Retailers should report to producers customers' tastes and requirements, mainly quality, good value for money and delivery reliability; on their side, retailers also have their necessities. They actually ask for suppliers able to provide big orders and with big stock availability, this means that big retailers prefer to deal with big wholesalers, big importers or big producers, while smaller ones privilege procurement flexibility and the ability to quickly adapt to changes, often looking for assistance in procurement financing and keeping an eye on commercial reliability and product's quality. Speed and flexibility are becoming the keys in the supply chain, substituting price competition, margins reflecting added value and market power. Retailers currently ask for flexible and lean supply chains, which favors vertical integration processes [Crestanello 2009].

Boundaries between wholesalers and producers are not easy to identify, since many production companies also perform commercial activities, buying products from other companies in order to offer a wider assortment, and wholesalers are developing vertical integration strategies acquiring production companies in countries with low labor costs in order to safeguard flexibility and quality standards. In Italy, direct channels between goldsmith producers and consumers are quite rare; principal client of production companies are wholesalers or other producers. Traditionally, they provide to:

- reduce necessary contacts per transaction;
- offer wider variety to retailers;
- anticipate gold to producers by use of contract work;
- keep product stock;
- take back unsold items [Crestanello 2009].

The traditional wholesaler is however disappearing: quick response systems are becoming important for managing retailers' costs, minimizing their inventories and assuring flexibility for quickly adapt to changes of the demand; this requires high coordination between retailers and producers. In order to survive, wholesalers can drive their supply chain in two directions, downstream or upstream. At the moment bigger wholesalers have more power than smaller producers with respect to retailers, since they offer bigger markets and wider services, from counseling for design and collections' development to gold supply. They also have power against independent retailers, smaller specialized chains and sometimes big buyers (e.g. of department store) who did not develop a deep competence on goldsmith sector, offering them wide range of products, inventory management, promotions, merchandising services, information on consumers' preferences and agreement about returns of unsold items and credit extensions. They are proceeding to integration processes: upstream, to gain assured sources of procurement and negotiation power with independent producers; and downstream, to obtain coordination efficiency, price determination strategies and direct access to information about consumers' preferences [Crestanello 2009].

The market power in the supply chain gained by retailers and wholesalers came at the expenses of producers, in connection with the entrance in the field of low labor cost countries. When intermediaries are entrusted with product commercialization, for producers is more difficult to control the market and define price policies as well as promote autonomous promotional/advertising initiatives. Generally, producers reach final market under distributors brand, who autonomously define promotion and image, also penetration on actual and potential market is strongly influenced by their choices. Production capacity often remains unexploited, since the intermediaries do not guarantee adequate sales volumes. Goldsmith entrepreneurs find their decisional autonomy significantly limited, upstream by raw materials holders and downstream by commercial operators' control [Cattaneo, Sabbadin, Virtuani 1993]. Producers however could have opportunities to reinforce their role, by:

- creating barriers to entry, achievable by reinforcing the competitive advantage that new entrants do not have, such as economies of scale and scope, distinctive entrepreneurial competences, reserved information, monopoly conditions on products (design copyright, brand, exclusive sales relationships), product and technology innovation [Crestanello 2009];
- diversifying through vertical integration processes, that could lead to opportunities for better transactions efficiencies and planning, as well as strategic price policies for diminishing the surges of seasonal production and collection of information about purchase and consumption markets [Crestanello 2009];
- specializing on independent retailers, whole more suffer because of their small dimensions and market fragmentation;
- reorganizing productive activities.

3.5 Italian jewellery districts

As stated in paragraph 3.1 and 3.3, Italian goldsmith production is mainly constituted by medium-small enterprises, whose dimension and geographical position in quite limited territories allowed the development of industrial clusters or districts. As for furniture, leather and glasses, districts are characterized by a high level of know-how, design, creativity but typically not so technologically advanced. They are in industrial environments, where community and companies are indivisible elements, with the same values: this is what enables strong economies and competitiveness within the system, which is the result of a long evolution that permits values to be preserved and passed on.

As for gold jewellery production, Vicenza, Valenza and Arezzo are those with highest market shares; while for silver jewellery production Padua, Palermo, Florence and Marche Region can be named.

3.5.1 Arezzo

The district is exclusively placed in the province of Arezzo and its boundaries include the local economic systems of the area of Arezzo (Arezzo, Capolona, Castiglion Fibocchi, Civitella in Val di Chiana, Monte San Savino, Subbiano) and of Val di Chiana (Castiglion

Fiorentino, Cortona, Foiano della Chiana, Lucignano, Marciano della Chiana), which Laterina and Pergine Valdarno are added to.

Precious metals manufacturing is the specialization of Arezzo territory and developed, especially during the 70's and 80's (although its history can be traced down to the 14th century), thanks to crucial role its leader company, Uno A Erre, played for many years in activating gemmation processes through time within the area and innovation transfer. High concentration, long winning trend in terms of structural, occupational and market share expansion and its important incidence on national and global productive structure built the identity of the district. The number of actors involved continued to grow until the 90's, but with a slower pace than the previous decades.

Strengths of the districts are the tradition, the deep knowledge of the raw materials, a complete supply chain and the prestige of the systems.

This competitive environment, proved to be successful before, is now changing because of the difficult conjunctural situation: the acceleration of some processes (e.g. globalization, technological innovation, new consumption dynamics and distributive systems) influenced the district's structural character and business strategies. Being so the district, With about 1.300 businesses and 9.000 employees, finds challenges on costs, determining attention on production efficiency, as well as offer differentiation, while facing raw materials' price instability and a consequent accurate financial management.

There are three main business typologies: branded, subcontractors (*lavorazione c/terzi*) and mixed businesses. Branded companies have bigger dimensions and direct contact with end-consumers, selling jewels under their brand name. Subcontractors (*lavorazione c/terzi*) include companies that can be distinguished because of their different position along the supply chain, the number of processes, independence degree.

With the crisis, growth almost completely stopped,; therefore the only active markets are abroad, with connected risks: other than currency risk, there are also those connected to the financial crisis and geo-political factors and abroad competition.

District's vulnerability can be drawn from the predominance of small-medium enterprises, which does not allowed the presence of well-established brands: they are not always capable of employing instruments for winning on international markets, they give little attention to marketing activities and the difficulty they have on developing new distribution channels and offer differentiation [http://www.osservatoriodistretti.org, http://www.osservatoriodistretti.org/sites/default/files/rapporto-2014-distretto-orafo-di-arezzo.pdf].

Industrial Districts Institution of Arezzo (IDI)

Born in 2001, it aims to promote and guarantee district dynamics, develop plans for local development and represent the interests of the operators with region, with the State and with EU, with the objective of ensuring the planning and realization of productive policies functional to the development of the industrial districts of Arezzo: goldsmith, textile/apparel and leather/footwear. In order to do so, IDI tries to integrate local development plans with European policies devoted to employment, internationalization and innovation; it also officially acknowledges those businesses that most demonstrated innovation and ability to implement local infrastructures. Differently from other Italian goldsmith districts, Arezzo manages its own one in the wider regional environment, together with other fashion districts , in order to promote coordination, communication and collaboration for shared projects [http://www.osservatoriodistretti.org].

Arezzo Osservatorio oro-moda

Established by Arezzo Province, Arezzo CCIAA, University of Siena, Industrial Association, CNA and Confartigianto Vicenza, it permanently monitors the districts within the region, defines territorial priorities and assesses policies' effectiveness. It operates through two authorities: *Comitato d'indirizzo*, that defines objectives and guidelines for research and analysis of the districts and the laws to applied, and *comitato tecninco-operativo*, that defines operative methodologies, sampling guidelines, surveys and elaboration criteria, provides interpretation of results and handles external relationships.

It monitors the main economic variables at regional and single business level, providing precise pictures of districts' performance and performs a competitive benchmarking on principal national and international competitors [http://www.osservatoriodistretti.org].

Trade shows

Oroarezzo and Gold/Italy are among the most important Italian exhibitions and represents a fundamentally important support for Arezzo goldsmith district. Gold/Italy is dedicated to the Made in Italy goldsmith production, with the objective of enhancing Italian production characteristics (creativity, craftsmanship, innovation, design, flexibility) to international buyers. Oroarezzo is one of the most prestigious and established dates in the global gold environment. The annual fair occurring in spring dedicated to only sector operators, welcomes national and international wholesalers and jewels importers.

During these trade shows, two parallel events take place dedicated to particular themes: Bi-Jewel develops around of world of bijoux and fashion jewels, organized in collaboration with

Vogue Gioiello; Cash&Carry that allows Italian and foreign wholesalers and retailers that operate *sul pronto* (buying raw materials when orders are received) [http://www.osservatoriodistretti.org,].

3.5.2 Valenza

Goldsmith tradition of the area dates back to 1840 and before WWI, but the boom occurred in the postwar period. Companies have medium-large dimensions: the production district core is made by the 20% of big enterprises around which artisan businesses operates, often as a subcontractors. With an average of 5,6 employees per company and entrepreneurial turnover, there is an intensive social mobility. In 2007, the importance of Valenza district increased, with 13,8% of the total Italian jewellery exports, as a countertrend considering the general reduction of exports in the same year, able to be less subject to global crisis and win the competition with abroad fierce agents, such as India, China and Turkey.

Strengths of the district are the enormous technical and professional know-how, developed in 150 years of artisan goldsmith tradition (thanks to specialized operators passing on their experience by “training on the job”) that also resulted into brands that leader also abroad, connected with the presence of Politecnico and of the University that enforce the prestige of the territory; deep relationship among businesses and entrepreneurship and the ability to quickly adapt to market requirements.

Criticalities of the district, other than the lack of marketing investments (considerably low if compared with normal budgets of fashion companies), is its excessive dependence on big international jewellery brands. With modern organized distribution and brand reputation of international brands, there is less space for micro artisan gold businesses. The district has to strengthen itself in products marketing, a new challenge for those enterprises that face end-market on new basis: they have to change to priorities on market policies, favoring new competitive factors: quality, research, innovation, distributive channels and markets [http://www.osservatoriodistretti.org].

DIVALENZA

Many companies in territory adhere to a consortium, “Consorzio del marchio DIVALENZA”, in order to promote and market jewellery by establishing a company, DIVALENZA Srl (founded in 2012), own by the consortium, that manages the brand and uses it nationally and internationally. Only companies that design and produce jewels, having their main office and productive units within the Valenza district, can adhere to this consortium. It is a brand that links the tradition of artisan production and the territory and cultural elements of creative innovation of every piece of jewellery.

Consortium's mission can be summarized as follows:

- ensuring material and geographical origin quality of members' final product
- ensuring the compliance with environment protection and labor
- promoting members' production, also abroad, to sector's operators and end-consumers
- protect members' products against every abusive activity aimed to undue advantage from image and/or reputation of the brand
- develop, also qualitatively, members' production and its update to market tastes and requirements[8].

Trade shows

Taking place in October, Valenza Gioelli is the representative exhibition of goldsmith, precious stones and high jewellery. For years dedicated only to sector operators, the event had to be rethought in order to give an idea of revamp given the deep crisis: in order to make it more modern, innovative, up with times and provider of growth opportunities, the event is now open also to public. This will not damage professional operators, since the fair is basically divided in two: in Villa Scalcabarozzi, they have however moments specifically dedicated to them to create new collaboration, necessary for the business to develop. on the other hand, Valenza Gioielli Prestige welcomes goldsmith, luxury and quality companies and is dedicated also to public, which is useful for the companies that do not have a policy of openness to public to have a direct approach to end-consumers [http://www.oroportale.it/valenza-gioielli-98.php].

3.5.3 Vicenza

Precious metals manufacturing has ancient origins, with some found jewels dated back to 3th Century B.C but the evidence of the first goldsmith corporation is from 14th Century A.F. After first industrial revolution of the 19th Century A.F., the identity of the Palladian city experienced a deep transformation for the introduction of machinery in production processes.

It is in the half of 20th Century that the district starts to present characteristics showed also nowadays: a lot of small enterprises, with 1-5 employees, specialized in specific productive stages, working alongside largest companies of those times, such as Donnagemma, Biffi and Balestra. In the year of the economic boom, since production exceeded local and national

[8] http://www.divalenza.it/?page_id=2575

demand, producers looked for new potential markets, strengthening contacts with abroad, especially Germany, Switzerland and France: from this moment on, an attitude change among them occurred, who are more and more oriented to international markets. Peculiar role played in this internationalization process was that of Vicenzaoro exhibition that, born in 1954, became the most important Italian showcase for the sector and the second in Europe, after Basel; this allowed exports from 1966 to 1970 to double, reaching a volume of 33 million dollars. From the 80's and 90's this openness to international markets is a preponderant characteristic, as well as the small the dimensional pulverization of production (the 5 largest companies at the time employed less than 10% of operators). This is due to the scarce importance of economies of scale and the low barriers to entry that favored the proliferation of a variety of entrepreneurial realities [Unionacamere del Veneto 2012].

Center of district production is the *comune* of Vicenza, but goldsmith acidities expands also to Bassano del Grappa and Trissino, all with mechanized productive structures, a limited number of medium-size businesses and a lot of highly specialized small enterprises, often artisanal, with a strong link with the territory and the presence of tradition and culture integrating in the goldsmith product. Also satellite activities (technical and service activities) is very developed, mainly working as subcontractors, that play as support for marketing, by incentivizing diffusion on foreign markets (more than 50% of production is exported), especially in Middle East, U.S. and Japan. This connection is favored by trade fairs and events, such as Vicenzaoro and Orogemma, where producers and sector experts from more than 100 countries develop contacts; *Museo del gioiello*, the first permanent exposition in Italy and one the few ones around the world exclusively dedicated to jewels; education and research centers specialized for precious metals transformation and precious metals laboratories at Chamber of Commerce.

Most important commercial partners are US, UAE, Hong Kong and Jordan, with about 50% of all district exports, although their relative weight changed through time; if US and UK markets were the most important, now it is the one of the UAE (this is connected to the loss of competitiveness, in terms of both quality and price; in fact Italian products are threatened by Chinese, Indian, Thai and Turkish ones, where cost of labor is much lower); growing markets are also Turkey, China and Poland (exports to Poland increased by 6 times from 2006 to 2012): this strong orientation toward international markets is a strong point of the district. Together with Arezzo district, Vicenza has the leadership for Italian goldsmith production as in 2011 the 30% of imported gold is manufactured in and exported from the Palladian city.

During the last decade, by widening product range, the district is trying to adapt to market crisis and consumers' taste change (not anymore only high level jewellery but also medium-high).

Among district criticalities there is the low attention to customer and market requirements, due mainly to inefficiencies in terms of flexibility, delivery time and cooperation, and inadequacy of marketing and branding policies. The largest part of Vicenza producers in fact is unbranded and, without customer loyalty, distributors can easily change their suppliers' portfolio. Another problem can be identified in the shortened jewel life cycle, which makes it necessary to frequently renovate collections. A scarce capacity in strategic, research and product design planning has been noted in many small companies. The fact of being composed by a large number of small-sized companies and in particular, smaller than international competitors, mainly made by producers/wholesalers, is another difficulty for the district, which finds itself with lower contractual power. Chinese and Thai producers, not only offer more convenient prices, but also better services in terms of flexibility and quick response, not to mention the increasing importance of international jewellery trade shows in other countries (e.g. US, Far East). Moreover, the economic and financial situation, already worsened by appreciation of Euro/US dollar and continuous raw material price fluctuations, is creating a significant contraction in the credit to entrepreneurial system which, even if supported by a network of consortiums and guarantees, paralyzes a sector where the raw material has an important weight in relation to value added which could be not anymore sustainable for many businesses. Furthermore, figures depositary of distinctive competences, results of the long experience gained, are leaving the sector because of the difficulties posed by the recent crisis [Unioncamere del Veneto 2012; http://www.osservatoriodistretti.org].

Agreement for the development of Goldsmith sector of Vicenza

The Vicenza Goldsmith district is recognized by Veneto Region by regional law n.2502 of 08/08/2003 and the first Agreement is signed in the same year, then it has been re-proposed in 2006 and 2009-2012. Two committees are involved: the first one, mainly composed by trade association representatives and other members, defines district strategies and policies; the second one, composed by trade association delegates, implements and manages the initiatives. Main objective is helping the recovery from the severe crisis that hurt the sector, also in terms of employment. This is pursued with a wide range of initiatives: technological innovation, promotion, lobby creation and so on. Principal activities of the Agreement relates with sector knowledge research and transfer, through the analysis on raw materials to discover new productive models that shorten product life cycle and new prototyping process techniques.

The research relates also to logistics, looking for new methods for semi-finished and finished products inventories optimization, and marketing and communication, trying to give value to that *plus* at the origin of a Palladian city jewel and pushing for the promotion towards foreign countries, also through tourist promotion in Italy.

Companies have realized the problematics and grouped together to perform lobby activities by the Ministero per le Attività Produttive, in order to fight counterfeit. In this sense advantages in terms of lower taxes as incentives to businesses are being discussed, especially for those of small dimensions that more suffer for many difficulties, credit access among others. At national and EU level, there is the commitment to the elimination, or at least the reciprocity of duties with the US, through the different association representatives [http://www.osservatoriodistretti.org].

Trade shows

Reference point in the market worldwide for the goldsmith, Vicenza goldsmith exhibitions currently provide four annual appointments (VICENZAORO January and September, T-GOLD and VICENZAORO Dubai) for supporting gold sector operators, in order to make them grow and acknowledged globally. Vicenza represent the international heart for jewellery, with high technological and artistic level, as well as a goldsmith tradition characterized by creativity and constant innovation. The trade fair is an important show case for sector's producers, who can sign new contracts and develop important relations network. VICENZAORO January is typically the most rich and important event for businesses that presents their spring summer collections. Strong point is the organization of targeted workshops, convention and meetings with Italian and global sector representatives.

Simultaneously, T-GOLD, international exposition of machinery for goldsmith and jewels and gem instruments, presents global producers of machinery for gold processing and jewellery creation.

All events are supported by a significant communication activity, in particular during exposition periods, a TV channel dedicated to Vicenza trade show is active, accessible h24 throughout Europe and North Africa. Trade show institution also publishes a specialized magazine, distributed during exposition periods, with information about products, trends, emerging markets, interviews with important personalities and so on [http://www.osservatoriodistretti.org].

Chapter 4

Inventory models for

commodity markets and jewellery industry

Every business has its own peculiar set of characteristics, necessities and problems such that a particular decision-making system could be suggested with difficulty without a previous detailed analysis [Magee, Boodman 1992].

Operations research has been providing a vast number of models for inventory management, although they are ideal and simplified representations of the myriad of real situations. Because of their schematic and sometimes approximate nature, a consistency problem exists because every inventory has its own specific problem that cannot be easily framed and curbed into general rules due to a lot of reasons: the variety of stocked items, inventory management systems and organizational types, very close approximation of reality would be mathematically intractable, extremely accurate models can be so expensive that their final benefit does not justifies their cost. The relevance of a model to a given situation must be based on the reasonableness of its assumptions and limitations. When managers deal with inventory problems, they need to deeply analyze the situation and possibly create a model that better adapts to the real case; in doing so however the study of simple models is just a first step since, although approximate and schematic, they are valid and represent a starting point for the solution of more complicated problems.

All the developed models for inventory management can be described based on the following characteristics:

- repetitiveness of inventory decisions: there are repeat orders models, (repetitively consumed units are restocked on a recurring basis), and single order models, (orders are not repeated, at any rate regularly; e.g. seasonal, high fashion items, perishable food, etc.)
- parameters' regularity throughout time: there are static and dynamic models, where demand, prices, lead time are assumed constant or change over time, respectively;
- demand: with independence demand, there is no relation between the demand for an item and any other item; for example the demand for end items and final products. With dependent demand, the demand for an item is directly related to or the result of demand for a "higher level" item; for example raw materials, components, subassemblies.

- parameters' knowledge: models where parameters (demand, inventory costs and lead time) are assumed as known or at least easily computable are deterministic; if at least one parameter, instead of a unique value, can assume more alternative values, each of them associated to a certain probability, models are stochastic. Most of the literature in this case limited its interest to demand and lead time as random variables, with their own probability distribution functions: however, rather being considered as fixed, also purchase cost can be a random variable, as in the case of quoted commodities.

Companies purchasing from spot markets are exposed to volatile price fluctuations, which for some commodities, such as agricultural products, minerals and energy resources, are very large; fluctuations in exchange rates make companies that replenish from abroad suffer the same problem. The following models can be therefore applied to companies that face price uncertainty that affects commodity and foreign markets, as well as technology advance and changed market conditions.

As in more traditional models, the essential decisions in buying on a fluctuating market is the timing of the purchase and the decision of how much to purchase when the time arises. One would purchase and stock the commodity when its price is low and sell it when price is high, however the implementation of the old business maximum "buy low sell high" has been primarily qualitative, based on some combination of forecasting and guesses.

Given this high volatility, companies usually resort to financial hedging, purchasing insurance against variations in prices to reduce their effect on profits. However, these hedging strategies usually do not affect the day-to-day raw materials procurement that must take into account current price as well as its expected evolution: the procurement strategy must therefore combine traditional inventory theory and financial price modeling [Berling, Martínez-de-Albéniz 2011]. Whether customers are internal (e.g. uncertain production requirement in manufacturing) or external (unpredictable customers in retail), a safety stock, whose entity depends on demand forecasts, its variability and cost parameters (holding costs, backlogging penalties), must actually be kept so as to guarantee them a certain service level. However, also purchase prices have to be taken into consideration because of their volatility; this lead to modeling the price process and, based on whether price increases or decreases are expected, the safety stock will be higher or lower [Berling, Martínez-de-Albéniz 2011].

Other than hedging with derivatives, some companies also try to pass the fluctuation to their customers. However, these options are not free from problems: the stochastic demand implies that it is not known when future purchases shall be made and therefore they cannot match with the right derivatives; while passing the price change to customers entails a selling price based on what one paid for the goods rather than the current market price, whose acceptance from customers is hard to obtain [Berling, Xie 2014].

As a consequence, there is the need for an operational hedging strategy in terms of purchase/inventory control decisions that minimizes the sum of purchase, holding and shortage costs that, differently from traditional models discussed earlier that ignore purchase price, assuming it as known or constant, also considers such price and its variability. Models of this kind will be exposed in following paragraphs.

Despite the little interest towards the subject, some models have been developed throughout time and will be exposed in following paragraphs. Considering that the different assumptions, limitations and conclusions these models are build upon, they can be ascribed into the following groups:

- order-up-to level models;
- minimum and maximum policy (s,S) models;
- price thresholds models;
- E.O.Q.-based models.

The following table summarizes the characteristics of the analyzed models, as well as the communalities and the differences existing among them.

MODEL	Fabian, et al [1959]	Yang, Xia [2009]	Kalymon [1971]	Berling,Martínez-de-Albéniz[2011], Berling, Xie [2014]	Teng, Yang [2004]
AREA OF INTEREST	OPTMIZING TIMING/QUANTITY OF PROCUREMENT	OPTMIZING TIMING/QUANTITY OF PROCUREMENT	OPTIMALITY OF STATE-DEPENDENT POLICY (s, S)	HOW CURRENT PRICE, EXPECTED PRICE EVOLUTION AND PRICE THRESHOLDS AFFECT PROCUREMENT	OPTIMAL REPLENISHMENT SCHEDULE, CONVEX FUNCTION OF THE NUMBER OF REPLENISHMENT, MOVING FROM EOQ
OBJECTIVE	minimize the long-run average unit cost per period	maximize total discounted wealth (bank account + inventory)	minimize total discounted costs over an n-period horizon	minimize total discounted costs relative to order now or later	minimize total inventory costs
REVIEW	periodic	continuous	Periodic	continuous	periodic
DEMAND	considered in the form of usage rate r, random variable with normally distributed probability density function $\phi(r)$	Δ, compound Poisson process (λ demand arrival rate, vector $\boldsymbol{w}$ per-arrival demand distribution, δ average demand level per arrival	random variable D_i with probability distribution function $F(\cdot\|,p_i)$	D, Poisson arrival process with rate λ and arrival time T	$f(t)$, demand rate at time t, random variable
RAW MATERIALS PRICE	random variable P with normally distributed probability density function $\psi(p)$	π (inflation-adjusted) random variable evolving with a Markov process with mean-reverting and time continuity properties	random variable p_i evolving with a Markov process and probability distribution function $H(\cdot\|i\text{-}1, p_i)$	random variable $C(t)$ evolving with a Markov process, which can be a geometric Brownian motion ($\mu(c)=\mu$, $\sigma(c)=\sigma$) or Ornstein-Uhlenbeck process($\mu(c)=-\kappa(c-\bar{c})$, $\sigma(c)=\sigma$)	$c_v(t)$, random variable at time t

MODEL	Fabian, et al [1959]	Yang, Xia [2009]	Kalymon [1971]	Berling,Martínez-de-Albéniz[2011], Berling, Xie [2014]	Teng, Yang [2004]
DEMAND AND RAW MAT. PRICE RELATION	independent	independent	dependent: demand may depend on raw material price	independent	independent
DISCOUNT FACTOR	-	$\gamma=\alpha-\beta$, with α increasing rate of bank account, β inflation rate	α	r continuous and risk neutral	-
LEAD TIME	no: instantaneous replacement	no: instantaneous replacement	no: instantaneous replacement	lead time $L>0$	no: instantaneous replacement
ORDERING COSTS	-	-	$C(y_i-x_i)\|i, p_i$ (consider both ordering and purchase costs)	-	c_f
HOLDING COSTS	c_h unit holding cost	h unit holding cost	$L(y_i\|i, p_i)$, consider both holding and shortage cost	h unit holding cost per time unit	c_h unit holding cost per time unit
SHORTAGES & BACKLOGGING	shortages are allowed: c_s unit shortage cost, no backlogging (shortages are met immediately paying a premium)	shortages are allowed: b backlogging cost	shortages are allowed: $L(y_i\|i, p_i)$, consider both holding and shortage cost no backlogging	shortages are allowed: b backlogging cost per time unit	shortages are allowed: unsatisfied demand is backlogged (c_b unit cost per time unit) and decreases as time passes ($\beta(t)$), in case of lost sales, c_l is the opportunity cost of lost sales

MODEL	Fabian, et al [1959]	Yang, Xia [2009]	Kalymon [1971]	Berling,Martínez-de-Albéniz[2011], Berling, Xie [2014]	Teng, Yang [2004]
BANK ACCOUNT	-	$z(t)$ grows at rate α	-	-	-
DETERIORATION RATE	-	-	-	-	θ: a constant fraction of the on-hand inventory deteriorates per unit of time; no repair or replacement
RESULTS	the value of S (inventory at the beginning of the period), called S_n that minimizes $F_n(s, P, S)$ is s if $P \geq P_n(s)$, S'_n if $P_n \leq P_n(s_{max})$, s max if $P \leq Pn(s_{max})$; subject to $s_{max} \geq S \geq s$ where s is existing inventory at the end of the period and s_{max} is maximum storage capacity	optimal ordering policy, function of raw material price π_i and inventory position x, is of the form of π_i-dependent order-up-to type: for each π_i there is an inventory level $y^*(\pi_i)$ such that $y^*(\pi_i,x)=y^*$ if $x \leq y^*(\pi_i)$ or $=x$ if $x>y^*(\pi_i)$; $y^*(\pi_i)$ decreases in π_i; it studies the effect on the order-up-to levels of: the mean price level, the mean-reverting rate, the price standard deviation, per-arrival demand parameter	there exists an optimal policy Y^*_n, function of inventory before ordering x and market price p, such that $y^*_i(x,p)=S_i(p)$ if $x<s_i(p)$ or $y^*_i(x,p)=x_i$ if $x \geq s_i(p)$	the decision of when placing the order results from the comparison between the costs associated to order now or after of short period of time, such costs depend on whether, at the decision moment, there is already demand (k<0) or not (k>0), on current price and on price thresholds; depending on the price process considered (geometric Brownian motion or Ornstein-Uhlenbeck) upper and lower threshold can be determined	if $C'(t_i) \leq c'_v(t_i)$ for all ti the optimal solution is $n^*=1$ and $t^*_1=0$; if $c'_v(t_i) \leq S_{t_i}(t,t_i)$ for all t_i the optimal solution is $n^*=1$ and $t_1^*=H$; if $C'(t_i)>c'_v(t_i)>-S_{t_i}(t,t_i)$ then t^*_1 is unique as well as optimal value of s^*_i and t^*_i and be found by differentiating total costs, by s_i and t_i, and posing equal to zero, where $C'(t_i)$ is the marginal total inventory carrying cost (holding+deterioration), $c'_v(t_i)$ is the increasing rate of unit purchase cost and $S_{t_i}(t,t_i)$ is the expected shortage cost (baklogging+lost sales)

4.1 Order-up-to level models

Based on raw material price and inventory position at the decision time, inventory increases up to an optimal level or remains unchanged. The models considered are developed by Fabian, Fisher, Sasieni and Yardeni [1959] and Yang and Xia [2009].

4.1.1 Purchasing raw material on a fluctuating market [Fabian, et al 1959]

The model that Fabian, Fisher, Sasieni and Yardeni proposed in 1959 attempts to provide answer to the typical when/how much questions, based on a certain characteristics of the market, developing forecasts for the average price of the commodity one, two and three periods from the time of the decision making, where the period considered is the month.

The aim of the model is to determine the ordering rule (in terms of s and P), i.e. the amount to be ordered at any one decision-making date, given the state of the system (amount of stock on hand s, current market price of the commodity P, number of periods n for which a purchasing decision has to be made), so as to minimize the expected cost per period over a period of n months. The model considers a finite horizon problem with periodic review in which the demand and price in each period are independent and identically distributed.

The assumptions are the following:

- decisions on purchasing are made once a month;
- s is existing inventory, S is inventory at the beginning of the period, $\hat{s}$ is maximum storage capacity;
- price p and usage rate r (≈demand) are assumed random variables, independent of each other and of prices and usage rates of other months; p is subject to the density function $\psi(p)$, r to the density function $\phi(r)$;
- once purchased, the commodity cannot be resold ($S \geq s$) and shortages, are met by purchasing sufficient to meet demand at some fixed, high penalty cost per unit;
- delivery of the commodity is instantaneous (lead time $\tau=0$);
- holding and shortage costs $I(S)$ are known.

The expected cost of the first period of the n-period optimal policy, f_n, consists of:

- the immediate outlay (if any) on purchasing the optimal amount;
- the expected cost of holding and shortage costs over the first period;

while the expected cost of the remaining $n-1$ periods will depend on the new price and the new stock on hand at the beginning of the second period.

The stock on hand at the beginning of the second period will be the stock S less the actual usage r during the first period: if more items have been used than those available in stock

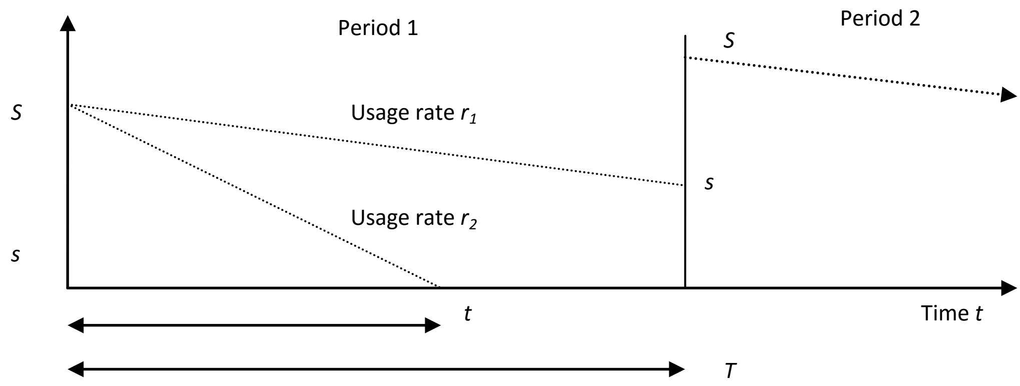
($S \leq r$) it will be zero (inventory does not assume negative values becauses shortages, although being admitted in the model, are met immediately by paying a premium), otherwise it will be $S-r$ ($S>r$). The minimum total expected cost when starting inventory is s and market price is p depends on two random variables, p and r, therefore their density functions have to be considered

$$f_n(s, P) = Min_{S \geq s}\{(S-s)P + I(S)$$

$$+\iint_{0,0}^{\infty,S} f_{n-1}(S-r,p)\phi(r)\psi(p)\,dr\,dp + \iint_{0,S}^{\infty,\infty} f_{n-1}(0,p)\phi(r)\psi(p)\,dr\,dp\} \quad (1.1)$$

Since the value of S is required monthly, an explicit function $S_n(P)$ is developed, relating S and the current price P of the commodity that minimizes the average cost per period, f_n/n ($S(P)$ is the form of $S_n(P)$ when the number of periods n tends to infinity).

Given that demand is a random variable, and therefore not known in advance, its possible developments have to be taken into consideration, by approximating it to a usage rate r. Pic.23 shows the stock on hand at the beginning of each of two subsequent periods considering two possible usage rates, r_1 and r_2: if r_1 is considered, S is not completely depleted by the end of the first period, differently from the situation with usage rate r_2, where the purchase of additional stock at time t is required.



Picture 23 Assumed stock behavior; S and s is stock on hand at the beginning and end of the period, respectively [Adjusted from Fabian, et al. 1959]

With c_h as unit holding cost and S the amount on hand at the beginning of the period, the holding cost of the average inventory over the entire period, with $r=r_1$ is

$$C_{h,r1} = c_h(S - 1/2\, r_1)$$

while, if $r \geq S$, holding costs are sustained only until the moment in which inventory level reaches zero (t) and not anymore, therefore they will be

$$C_{h,r2} = 1/2\, c_h S\,(t/T)$$

and, since $t/T = S/r_2$, it is

$$C_{h,r2} = 1/2\, c_h S^2/r_2 \,.$$

In the same way, shortage costs will be suffered only from moment *t* when $r \geq S$ and they are the made by unit shortage cost c_s times the number of units to be purchased for $r \geq S$,

$$C_s = c_s (r - S)$$

The expected inventory cost of the period is therefore the sum of the expected holding costs sustained over the entire period *T* (i.e. if $r \leq S$) and the expected holding costs and shortage costs suffered between *0-t* and *t-T* respectively (i.e. if $r \geq S$). Considering that the demand ($\approx r$) that will be experienced in the future is not known but only forecasted, and therefore subject to some uncertainty degree, the expected inventory costs are expressed as a function of its probability distribution:

$$I(S) = \int_0^S c_h \left(s - \frac{1}{2}r\right) \phi(r) + \int_S^\infty [\frac{c_h}{2}\frac{S^2}{r} + c_s (r - S)]\phi(r)dr \qquad (1.2)$$

Values of the decision function $S_n(P)$ are obtained iteratively and recuresively: first the optimal $S_1(P)$ is found, by minimizing $(S-s)P+I(S)$and equating to zero, then all other $S_2(P)$, $S_3(P)$. The function $f_n(s, P)$ reaches its minimum when market price of the commodity *P* equals the “marginal” inventory costs ($P=-dI(S)/dS$).

From Fabian, Fisher, Sasieni and Yardeni [1959] it follows that the value of *S* that minimizes average expected cost per period S_n, subject to $\hat{s} \geq S \geq s$, where $\hat{s}$ is the maximum storage capacity depends on *P* and is given by the following rule

$$S_n = \begin{cases} s & if\ P \geq P_n(s) \\ S'_n & if\ P_n(\hat{s}) \leq P \leq P_n(s) \\ \hat{s} & if\ P \leq P_n(\hat{s}) \end{cases} \text{[9]}. \qquad (1.3)$$

Differently from traditional inventory management models, the optimal amount to stock directly depends on the comparison commodity price at the decision date and the sum between that and the average expected inventory costs over the period, which in turn are a function of a random demand; which is why $S_n(P)$ will not be the same in all future periods but is the results of future prices.

This model is based on the assumption of demand and purchase price in the form of random variables, with a certain type of distribution (and parameters) for all the time periods 1, …, *n*. Using price forecasts for the first few periods ahead can decrease the expected cost of the optimal solution; however this possibility increases the computational difficulty since the solution involve *n+1* independent variables (the current market price and *n* forecasts) and not anymore only one single independent variable (the current market price). Things would be

[9] For mathematical proof, see Fabian, T.; Fisher, J. L.; Sasieni, M. W.; Yardeni, A. in *Operations Research*; Jan/Feb59, Vol. 7 Issue 1, pp. 117-122.

easier if forecasts are certain rather than being subject to error. In this case, whenever the forecast price in one future period is lower than the sum of current price and holding costs until that period, managers should stock enough to cover the demand expected up to next period with lower forecasted price, while if all forecast prices of the periods ahead are less than current price plus holding costs, an amount equal to the highest between the demand for these periods or *S(P)* should be stocked [Fabian, et al 1959].

4.2.2 Acquisition management under fluctuating raw material prices [Yang, Xia 2009]

As the previous model, the examination focuses on optimizing the timing/quantity of raw material acquisition and, although starting from different hypothesis (mainly relative to demand and price distributions), reaches a conclusion very similar to that of the previous model: the optimal ordering rule is function of raw material price and inventory level and has the form of an order-up-to-level policy. In particular, if current market price is P_0, the optimal amount to have in inventory should be $S(P_0)$: therefore, by comparing the level of the inventory at the decision date with $S(P_0)$ the decision about further procurement is made, in particular if it is lower than $S(P_0)$ additional purchases should be made and nothing otherwise. The most important assumptions for this model relate to the behavior of raw materials price: they are mean-reverting (i.e. prices are more likely to increase or decrease when the current price is low or high, respectively) and time continuous (i.e. price is not expected to change drastically in very short periods of time). These important assumptions, together with the Markovian behave of price, are realistic since commodity prices are influenced by worldwide supply/demand shifts, as well as geopolitical affairs that, although seemingly random, are characterized by long-term trends [Yang, Xia 2009].

Differently from Fabian et al however, it explicitly demonstrates that these order-up-to levels are a decreasing function of the current price and, being demand and market price random variables with their own specific parameters, analyzes them in correspondence of different characterizing parameters.

The following model considers a continuous-review acquisition problem, where raw material prices and demand are assumed to behave as Markov process and compound Poisson, respectively. The aim is to maximize the total wealth of the firm at some point in the distant future, which is equivalent to the firm's total discounted operating profit maximization, where the discount rate is the difference between interest rate and inflation rate. Since the total discounted revenue the firm can hope to generate is fixed, firm's total discounted operating costs have to be minimized.

Denoting time with t, the assumptions of the model are as follows:

- leaving out the inflation factor $e^{\bar{\beta}t}$ for some $\bar{\beta} \geq 0$, all prices and costs are stationary;
- the inflation-adjusted price $\Pi(t)$ of raw material follows a continuous-time, discrete-state Markov process;
- the inflation-adjusted unit profit $\bar{r}$ the firm can earn before accounting for the raw material purchasing cost is constant;
- the inflation-adjusted raw material holding cost rate $\bar{h}$ is constant and strictly positive;
- the inflation-adjusted backlogging cost rate $\bar{b}$ for unsatisfied demand is strictly positive;
- the firm has a limited warehouse space $\bar{x}_0$, and extra space can be obtained at a premium $\bar{h}'$;
- the cumulative demand $\Delta(t)$ up to time t follows a compound Poisson process, with the positive parameter $\bar{\lambda}$ and the positive vector $\bar{\boldsymbol{w}} = (\bar{w}(1), \bar{w}, \dots)$ describing respectively the demand arrival rate and the pre-arrival demand distribution, with $\sum_{d=1}^{+\infty} \bar{w}\,(d) = 1$. The average demand level per arrival is $\bar{\delta} = \sum_{d=1}^{+\infty} \bar{w}\,(d)d$;
- demand and raw material price processes are independent from each other;
- raw material inventory level at time t is $x(t)$ and it increases because of $q(t)$, the instantaneous acquisition quantity to be decided upon, and $\delta(t)$, the average demand quantity, as the following expression shows

$$dx(t) = dq(t) - d\delta(t) \;\; ; \tag{2.1}$$

- the bank account at time t is $z(t)$, with an initial deposit $z(0)=z_0$ for some z_0, and interests are earned upon it, with a rate equal to $\bar{\alpha}$, with $\bar{\alpha} > \bar{\beta}$. Since no fees are incurred for transferring money in and out of the bank account, it increases as a result of the interests and sales revenues earned, while it decreases because of procurement, holding and backlogging costs

$$dz(t) = \bar{\alpha}z(t)dt + e^{t}[\bar{r}d\delta(t) - \pi(t)dq(t) - \bar{h}\big(x(t)\big)^{+}dt - \bar{b}\big(x(t)\big)^{-}dt] \; ; \tag{2.2}$$

- with the realized inflation-adjusted raw material price at time t, $\pi(t)$, the total wealth $w(t)$ of the firm at time t is defined as

$$w(t) = e^{\bar{\beta}t}\pi(t)x(t) + z(t) \;\; . \tag{2.3}$$

The goal is to control the purchasing process in order to maximize the expected terminal wealth $E[w(T)]$ at some time $T \geq 0$, which is equivalent to maximizing the expected total discounted profit plus terminal inventory worth, where the discount rate is $\bar{\gamma} = \bar{\alpha} - \bar{\beta}$[10]. $(x + q')$

Let $f(i, x)$ be the total discounted expected profit that the firm can make when it starts with an inventory position *x* and a raw material price *i*,

$f(i,x) = [\bar{\lambda}\bar{\delta}\bar{r} + max_{q'=0,1\ldots}\{-\bar{\Lambda}\bar{\pi}(i)q' - \bar{h}(x+q') - \bar{b}(x+q') + \sum_j(i,j)f(j, x+q') +$
$\lambda d{=}1{+}\infty w d f(i{,}x{-}d{+}q')/\Lambda$ (2.4)

where $\bar{\Lambda} = \bar{\lambda} + \bar{\gamma}$ for convenience and $u^*(i,x) = \bar{\lambda}\bar{\delta}\bar{r}/\bar{\gamma} - f(i,x) + \bar{\pi}(i)x$ be the total cost plus the current value of inventory. From (2.4), the following recursive relation can be derived:

$$u^*(i,x) = min_{y=x,x+1,x+2,\ldots} v^*(i,y) \qquad (2.5)$$

where

$v^*(i,y) = [\bar{\lambda}\bar{\delta}\bar{\pi}(i) + (\bar{\gamma}\bar{\pi}(i) - \sum_j(i,j)(\bar{\pi}(j) - \bar{\pi}(i)))y + (\bar{h}y + \bar{b}y) + \sum_j(i,j)u^*(j,y) +$
$\lambda d{=}1{+}\infty w d u{*}(i{,}y{-}d)/\Lambda$. (2.6)

Assuming a current strictly positive *x* number of stocked items and *T(x)* the random amount of time it takes for all of them to be consumed by demand, if the company acquires one more item, the total holding cost it will pay for this item would be at least *h(x)*, with

$$h(x) = \bar{h}E\left[\int_0^{T(x)} e^{-\bar{\gamma}t}dt\right] = \bar{h}E\left[1 - e^{-\bar{\gamma}T(x)}\right]/\bar{\gamma} \qquad (2.7)$$

while the profit this purchase can generate would be at most *r(x)*, where

$$r(x) = \bar{r}E\left[e^{-\bar{\gamma}T(x)}\right] - \bar{\pi}^L \quad ; \qquad (2.8)$$

It can be demonstrated that there exists a maximum inventory level $\bar{x}^U$, with $x \geq \bar{x}^U$, such that $h(x) > r(x)$, thus the cost of ordering an additional unit would offset the benefits it offers. Similarly, there also exists a minimum inventory position $\bar{x}^L$. Assuming a current stock-out and that first-come-first-served is the approach used for dealing with backlog situations, if the latest backlogged unit disappears because of immediate unit acquisition, the overpayment *p* due to imprudent timing in acquisition will be at most

$$p = \left((\bar{\lambda} + \bar{\gamma})\bar{\pi}^U - \bar{\lambda}\bar{\pi}^L\right)/(\bar{\lambda} + \bar{\gamma}) \qquad (2.9)$$

On the other hand, if the unit is not purchased it causes at least a backlogging cost *b*, with

$$b = \bar{b}/(\bar{\lambda} + \bar{\gamma}) \quad . \qquad (2.10)$$

[10] For details, see Yang, Xia in *Production & Operations management*, Mar/Apr2009, Vol. 18 Issue 2, p212-225, Appendix A.

If we assume $\bar{b} > (\bar{\lambda} + \bar{\gamma})\bar{\pi}^U - \bar{\pi}^L$, we have that $b > p$: because the costs related to the acquisition of an additional unit are less than the shortage penalty costs, resolving any backlogging immediately when demand arrives is more convenient.

Therefore, a positive constant $\bar{x}^U$ and negative constant $\bar{x}^L$ exist, such that only inventory position $x \in \{-\bar{x}^L, \dots, -1, 0, +1, \dots \bar{x}^U\}$ can be considered [Yang, Xia 2009].

Yang and Xia [2009] demonstrated, iteratively applying (2.5) or (2.6), the existence of an optimal solution, $y^*(i,x)$: since, for *i*=0, ±1, ±2, …, $u^*(x)$ is a convex function of *x* and $v^*(i,y)$ is a convex function of *y*; the optimal ordering policy $y^*(i,x)$ is order-up-to type and depends on *i* and *x*. In particular, for each raw material price *i* and inventory position *x*, there exists a point $\bar{y}^*(i)$ such that $\bar{y}^*(i,x) = \bar{y}^*(i)$ or *x*:

$$\bar{y}^*(i,x) = \begin{cases} \bar{y}^*(i) & if\ x \le \bar{y}^* \\ x & if\ x > \bar{y}^* \end{cases} \quad . \tag{2.11}$$

This can be alternatively expressed as function of the instantaneous acquisition quantity/purchase rate *q*. If we consider q_t limited in the interval $[0, \bar{q}]$, a convex cost rate *H(.)*, the discount rate $\bar{\gamma}$, the constant demand arrival rate and level $\bar{\gamma}\bar{\delta}$ and the unit price per item $\bar{r}$, the function $f(\pi, x)$ can be the maximum total discounted expected profit the firm can make when it starts with raw material price and inventory position respectively equal to $\Pi_0 = \pi$ and $X_0 = x$, where

$$f(\pi,x) = \sup E_\pi\left[\int_0^{+\infty} e^{-\bar{\gamma}t}\left(\bar{\gamma}\bar{\delta}\bar{r} - \Pi_t Q_t - H(X_t)\right)dt\right]$$
$$\text{s.t. } Q_t \in [0, \bar{q}]$$
$$X_t = x + \int_0^t (Q_s - \bar{\lambda}\,\bar{\delta})ds \quad , \tag{2.12}$$

where E_π denotes that the probability space is set so that $\Pi_0 = \pi$; it can be demonstrated that total discounted expected profit function $f(\pi, x)$ is concave in the inventory position *x*, therefore there exists a certain $\bar{y}^*$ such that

$$\bar{y}^* = \sup\{x \in R \mid \partial_x f(\pi,x) \ge \pi\} \quad . \tag{2.13}$$

Provided that $f(\pi, x)$ has a continuous second-order derivative in π and a continuous first-order derivative in *x*, the optimal purchase policy $f(\pi, x)$ is

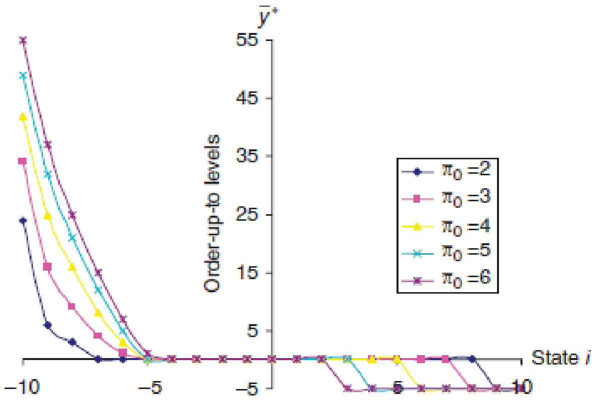
$$f(\pi,x) = \begin{cases} \bar{q} & when\ x \le \bar{y}^*(\pi) \\ 0 & when\ x > \bar{y}^*(\pi) \end{cases} \quad . \tag{2.14}$$

Equation (2.14) confirms that an order-up-to level $\bar{y}^*(\pi)$ exists so that purchase should be set at full capacity when the current inventory level *x* is below the order-up-to level, while purchase should not be made when *x* is above the order-up-to level [Yang, Xia 2009] [11].

[11] For detailed mathematical proof, see Yang, Xia in *Production & Operations management*, Mar/Apr2009, Vol. 18 Issue 2, p212-225.

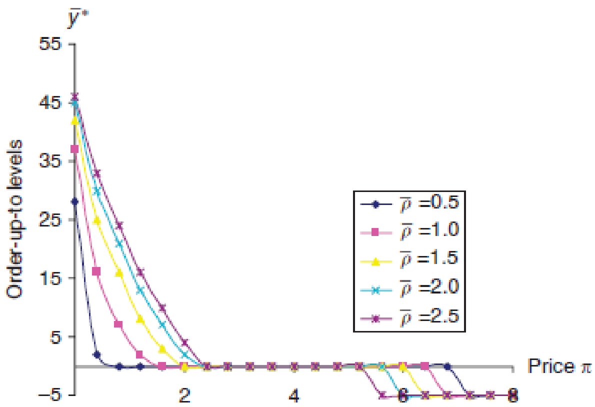
It can also be the demonstrated that, under the assumptions of mean-reverting prices and time continuity property, the order-up-to levels have monotone property: if u and v are convex and satisfy (2.6), the order-up-to levels $\bar{y}^*(i)$ are decreasing at the current price i, [Yang, Xia 2009].

Comparing results obtained from simulations, Yang and Xia [2009] analyzed how these order-up-to levels behave as parameters of both raw material price π and demand $\Delta(t)$ change, given that being random variable, they can assume different values, as well cost parameters. Order-up-to levels increase with the demand arrival rate $\bar{\lambda}$ and decrease as the discount rate $\bar{\gamma}$ climbs, which is expected since, as the discounting effect grows, the impact of the current commitment and investment for future use reduces. The levels decrease with unit holding cost rate $\bar{h}$ and increase with the unit backlogging cost rate $\bar{b}$. In the following graphs, the state i (which may be viewed also as the relative position of the current price with respect to the overall mean) is represented in the horizontal axis. Pic.24 depicts the behavior of order-up-to levels as the mean price level $\bar{\pi}_0$ varies. Looking at the graph, the company should react more dramatically to changes in the relative price position when the mean price $\bar{\pi}_0$ is larger. At the low end of the relative price position, the levels are higher for larger $\bar{\pi}_0$ values, while at the high end of the relative price, the levels are lower for larger $\bar{\pi}_0$ values. This is reasonable, since a larger $\bar{\pi}_0$ will lead to ordering costs being the major voice in the company's operating cost and the same relative price change involves a larger absolute price change. As a consequence, firm's decisions will be more sensitive to the relative price changes when $\bar{\pi}_0$ becomes larger [Yang, Xia 2009].



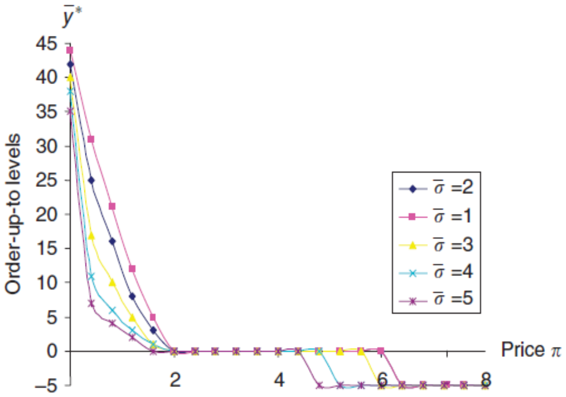
Picture 24 Acquisition policies at different $\bar{\pi}_0$ values [from Yang, Xia 2009]

Pic.25 represents the effects of changing mean-reverting rates $\bar{\rho}$ on the order-up-to levels.

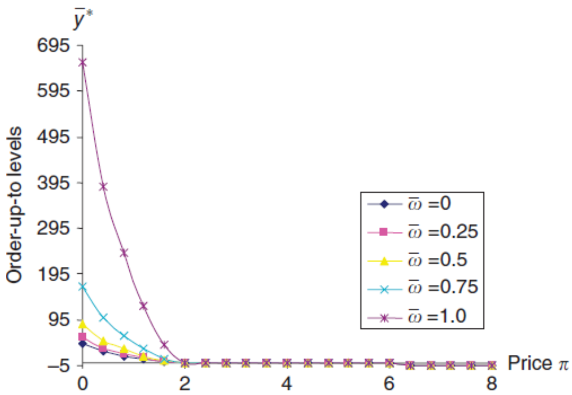


Picture 25 Acquisition policies at different $\bar{\rho}$ values [from Yang, Xia 2009]

It can be seen that as $\bar{\rho}$ increases, the level is decreasing at a faster rate in the current raw material price level. At the low end of the price, the levels are higher for larger $\bar{\rho}$ values, while at the high end of the price, the levels are lower for larger $\bar{\rho}$ values. It indicates, at higher $\bar{\rho}$ values, that the firm should much more appreciate a lower-than-usual price and less detest a higher-than-usual price. Consequently, the firm will even more when facing a lower price and order even less facing a higher price [Yang, Xia 2009].



Picture 26 Acquisition policies at different $\bar{\sigma}$ values [from Yang, Xia 2009]



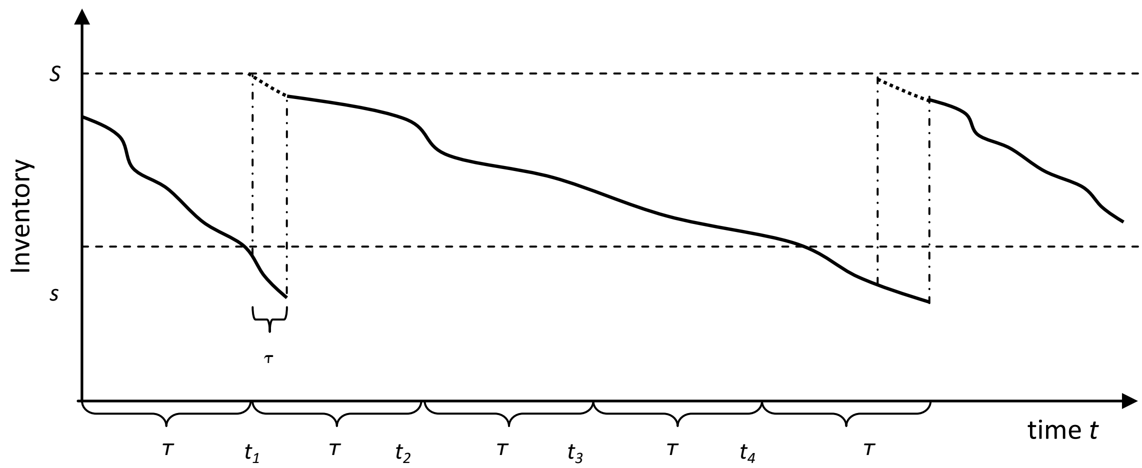
Picture 27 Acquisition policies at different $\bar{\varpi}$ values [from Yang, Xia 2009]

Pic.26 shows the effect of changing instantaneous standard deviation $\bar{\sigma}$ on the order-up-to levels. With an increasing $\bar{\sigma}$, the level at each price level diminishes: as reasonable, price uncertainty retains companies from ordering. Pic.27 expresses changes in the order-up-to levels with respect to changes in the per-arrival demand parameter $\bar{\varpi}$: as the latter increases, the levels increase as well since a higher $\bar{\varpi}$ implies larger, more variable arrival demand [Yang, Xia 2009].

4.2 Minimum and maximum stock model (*s*, *S*)

Based on raw material price, there exist a maximum (S or S_{max}) and a minimum (s or s_{min}) inventory level such that if stocked items are less than the minimum stock then an order up to the maximum stock should be placed; conversely, nothing is ordered. The model thereby analyzed is by Kalymon [1971] This methodology aims at maintaining the stock within certain threshold by issuing orders whose entity varies; the amount ordered is equal to the difference between the upper bound of the model, and the inventory availability at time t_i, whenever the stock available is less than the lower bound of the model (pic.28).

This method is particularly applied when inventory level review costs and those associated with issuing a new order are distinct and significant [Magee, Boodman 1992].With respect to s, it has to be sufficiently large to ensure that protection is provided, according to a certain service level, against the risk of stock-outs in a period of time equal to typical delay of the lead time and review period. Minimum stock s is given by two components: forecasted or average demand for the period previously said and a buffer that ensures against fluctuations within such period.



Picture 28 (s, S) policy inventory profile [adjusted from Grando 1995]

4.2.1 Stochastic prices in a single-item inventory purchasing model [Kalymon, 1971]

Kalymon studies a single-item, multi-period inventory model in which the distribution of purchase price is determined by a Markovian process ("memoryless" process: predictions for the future of the process can be made based solely on its present state just as well as one could knowing the process's full history. i.e., conditional on the present state of the system, its future and past are independent) and demonstrates the optimality of a state-dependent (*s*, *S*) policy: if the current price is *p*, then order up to *S(p)* if the inventory level is below *s(p)* and order nothing otherwise. Assumptions of the model are the following:

- inventory decisions are made at regular intervals;
- inventory level is known at the beginning of each period (x_i and y_i are respectively inventory before and after ordering in period *I*; since no disposals are permitted $y_i \geq x_i$) and current price is known before purchases are made;
- purchase price p_i evolves stochastically, according to a Markov process defined as $H(\cdot|i\text{-}1, p_i)$ (the probability distribution function of p_{i-1} given the value of p_i);
- demand D_i, for the period *i*, is a random variable whose probability distribution may depend on the current price (for all *i*, *D* is conditionally independent of D_{i+1}, given *p*); $F(\cdot|i, p_i)$ is the probability distribution function of D_i given the value of *p*; with *D* and p_{i-1} conditionally independent. Differently from the previous model where demand and purchase price were independent, this allows applications in which the increase/decrease

in price will be passed on to the next customer and may therefore decrease/increase demand;

- shortages and backlogging are allowed, therefore inventory levels can take on negative values;
- although instantaneous delivery is assumed, deterministic delivery lags are permitted ($\tau \neq 0$, differently from the other models considered in this thesis);
- ordering cost is $C(y_i\text{-}x_i|i,\ p_i) \equiv K_i\ \delta(y_i\text{-}x_i)+p_i(y_i\text{-}x_i)$ where K_i is a positive constant and $\delta(Z)\equiv 0$ for $Z=0$ and 1 for $Z>0$ ($\delta(Z)$ thus indicates whether or not a purchase order is made at time i);
- holding cost and shortage penalty cost are charged; their function, nonnegative convex in y_i, is $L(y_i|i,\ p_i)$;
- time value of money is recognized by means of a discount factor α per period, $0\leq\alpha\leq 1$.

The objective is to find a policy $Y_n=\{y_1,\ \ldots\ ,\ y_n\}$ function of existing inventory x_i and current price p_i that minimizes the total discounted cost over the n-period horizon.

The function $f_i(x,\ p|Y_i)$ represents the expected discounted costs for i remaining periods, given that $x_i=x$, $p_i=p$, $\beta_j=\alpha^{i-j}$ and policy Y_i will be followed; then

$$f_i(x,p|Y_i) \equiv E\sum_{i=1}^{j=i}\beta_j[C(y_i-x_i|j,p_i)+L(y_i|j,p_j)] \tag{3.1}$$

with $x_i=x$, $p_i=p$, $\beta_j=\alpha^{i-j}$ where α is the discount factor per period and $0\leq\alpha\leq 1$.

Let $f_i(x,p)$, the function representing the minimum of the previous equation, be defined by

$$f_i(x,p) = inf\ f_i(x,p|Y_i)\ , \tag{3.2}$$

Y_i^* is the optimal policy for the i-period horizon if (3.2) is achieved by Y_i^*.

The following is the mathematical proof that, supposing $K_{i+1}\geq\alpha K_i\geq 0$ for all $1\leq i\leq n$, there exists an optimal policy Y_n^* of the form $(s_i(p),S_i(p))$ in each period, where $S_i(p)>s_i(p)$ such that

$$y_i^* = \begin{cases} S_i(p) & if\ x<s_i(p) \\ x_i & if\ \ x\geq s_i(p) \end{cases}. \tag{3.3}$$

The function $f_i(x,p)$ defined in (3.2) satisfies the functional equation

$$f_i(x,p) = inf_{y\geq x}\left\{\begin{matrix} C(y-x|i,p)+L(y|i,p) \\ +\alpha\iint_{0,0}^{\infty,\infty} f_{i-1}(y-D,q)dH(q|i,p)dF(D|i,p)\end{matrix}\right\}. \tag{3.4}$$

Let be

$$G(y|i,p) = py+L(y|i,p)+\alpha\iint_{0,0}^{\infty,\infty} f_{i-1}(y-D,q)dH(q|i,p)dF(D|i,p). \tag{3.5}$$

Being $G(y|i,p)$ a K_i-convex function of y, the optimal policy for period i must be of the form (s_i,S_i) where S_i is the smallest number satisfying

$$G(S|i,p) = min_y G(y|i,p) \tag{3.6}$$

and s_i is the smallest number less than S_i for which

$$G(s_i|i,p) \leq G(S_i|i,p) + K_i, \tag{3.7}$$

with the s_i and S_i both varying with p, which is why this dependence is indicated with ($s_i(p)$, $S_i(p)$).

To develop the bounds on the optimal policy, we assume that prices are bounded by $p \leq \overline{p}$, and $L(y|i,\ p)=L(y|p)$, $K_i=K$, $H(q|i,\ p)= H(q|p)$, $F(D|i,\ p)=F(D|p)$ for all i, where $H(q|p)$ represents the probability distribution function of the next period price q, given that the current price is p.

We define $\underline{S}(p)$ as the smallest number such that

$$p\underline{S}(p) + L(\underline{S}(p)|p) = min_y\{py + L(y|p)\} \tag{3.8}$$

and let $\underline{S}(p) = inf_p\underline{S}(p)$. If we assume that $(py + L(y|p)) \to \infty$ as $|p| \to \infty$, being p restricted to the interval $[p, \bar{p}]$, $\underline{S}$ will be finite if $\underline{S}(p)$ is monotonic in p, continuous in p or if p takes only a finite number of discrete values, with $\underline{S} < S_n(p)$ for all n and p[12] [Kalymon, 1971].

We also define $\underline{s}(p)$ as the smallest number such that

$$p\underline{s}(p) + L(\underline{s}(p)|p) \leq p\underline{S} + L(\underline{S}|p) + K. \tag{3.9}$$

Since $(py + L(y|p)) \to \infty$ as $|y| \to \infty$ and $K \geq 0$, it can be seen that $\underline{s}(p) < \underline{S}$, with $\underline{s(p)} < s_n(p)$ for all n[13].

Moreover, defining $\underline{s}$ by

$$\underline{s} = \inf(\underline{s}(p)) \text{ with } p \leq p \leq \overline{p}\ , \tag{3.10}$$

we get

$$\underline{s} < s_n(p) \qquad \text{for all } n \text{ and } p. \tag{3.11}$$

As before, $\underline{s}$ is finite if $\underline{s}(p)$ is continuous or monotone in p, or if p takes on only a finite number of values.

These are the lower bounds of the *($s_i(p)$,$S_i(p)$)* policy, we now proceed with the definition of the upper bounds.

Be $S^*(p)$ the smallest number for which

$$(p - \alpha\mu_p)S^*(p) + L(S^*(p)|p) = min_y\{(p - \alpha\mu_p)p + L(y|p)\} = min_y\{Q(y,p)\} \tag{3.12}$$

where

$$\mu_p = \int_0^\infty q dH(q|p)\ . \tag{3.13}$$

[12] For detailed mathematical proof, see Kalymon, Basil A. in *Operations Research*, Oct71, Vol. 19 Issue 6, p1434-1458, Appendix A

[13] For detailed mathematical proof, see Kalymon, Basil A. in *Operations Research*, Oct71, Vol. 19 Issue 6, p1434-1458, Appendix A

Given that a linear function is both concave and convex , we have that $Q(y, p)$ is always convex, whether $(p - \alpha\mu_p)$ is positive or negative, and, since by assumption $Q(y, p) \to \infty$ as $|y| \to \infty$, $S^*(p)$ will be finite. From (3.8) and (3.10) we can prove that $\underline{S}(p) < S^*(p)$.

Define $\overline{s}(p)$ as the smallest number such that

$$Q(\overline{s}(p), p) \leq Q(S^*(p), p) + (1 - \alpha)K \quad . \tag{3.14}$$

An optimal $(s_n(p), S_n(p))$ policy exists for which $s_n(p) < \overline{s}(p)$ for all n.[14]

We also define $\overline{S}(p)$ as the smallest number greater than $S^*(p)$ for which

$$Q(\overline{S}(p), p) \geq Q(S^*(p), p) + \alpha K \, , \tag{3.15}$$

it follows that an optimal $(s_n(p), S_n(p))$ policy exists for which $S_n(p) < \overline{S}(p)$ for all n [15].

If we define $\bar{S}$ as

$$\bar{S} = sup\bar{S}(p), \quad p \leq p \leq \bar{p} \, , \tag{3.16}$$

then

$$S_n(p) \leq \bar{S} \text{ for all } n, p \, . \tag{3.17}$$

As before, being $\bar{S}(p)$ finite by the assumption $(p - \alpha\mu_p)y + L(y|p) \to \infty$ as $|y| \to \infty$, $\bar{S}$ is finite if $\bar{S}(p)$ in continuous or monotone in p or p is restricted to a finite set of values.

From all this, it follows that for all n and p, an optimal $(s_n(p),S_n(p))$ policy exists, such that

$$\underline{s} \leq s_n(p) \leq \overline{S} \, , \; \underline{s}(p) \leq s_n(p) \leq \overline{s}(p) \; , \; \underline{S}(p) \leq S_n(p) \leq \overline{S}(p) \; ;$$

[Kalymon 1971].

4.3 Price threshold models

Comparing current raw materials price with upper and lower price thresholds, the costs associated with procurement decisions (i.e. order now vs. wait and order later) are analyzed. Next the model of Berling, Martínez-de-Albéniz [2011] is presented, together with its development by Berling, Xie [2014].

4.3.1 Optimal inventory policy when demand and purchase cost are stochastic [Berling, Martínez-de-Albéniz 2011, Berling, Xie 2014]

First Berling and Martínez-de-Albéniz [2011], then Berling and Xie [2014] examine how price evolution determines optimal inventory policy; decoupling inventory problem into a set of sub-problems, each of which determines the optimal ordering time of the i-th unit, which are used to satisfy the i-th demand, where the study of 2014 is presented as an approximation

[14] For detailed mathematical proof, see Kalymon, Basil A. in *Operations Research*, Oct71, Vol. 19 Issue 6, p1434-1458, Appendix A

[15] For detailed mathematical proof, see Kalymon, Basil A. in *Operations Research*, Oct71, Vol. 19 Issue 6, p1434-1458, Appendix A

of the optimal purchasing/inventory policy, since procedures proposed in 2011 have a great computational complexity.

Optimal policy

The studies are conducted based on the assumptions of a continuous review setting with stochastic purchase price and demand. Moving from Kalymon's conclusions [1971], of state-dependent base-stock policy optimality, they characterize it as a series of threshold prices.

The assumptions are as the follows:

- commodity price quoted at time t is $C(t)=e^{c(t)}$, for simplicity the log-price c is used; and evolves stochastically as a Markov process of the form $dc=\mu(c)dt+\sigma(c)W_t$, where W_t is a continuous time stochastic process with zero drift and unit variance per unit of time, which can be either a geometric Brownian motion with $\mu(c)=\mu$ and $\sigma(c)=\sigma$ or a mean-reverting Ornestein-Uhlenbeck process with process $\mu(c)= -\kappa(c-\bar{c})$and $\sigma(c)=\sigma$, with κ being the speed at which price reverts to its mean $\bar{c}$;
- demand D follows a Poisson process with rate λ, with customers' arrival time T_i;
- since lead time L is constant, order i is always used to serve the i-th customers, minimizing holding and backordering costs
- there are no setup/ordering costs and once the order is placed, it cannot be cancelled;
- h is the out-of-pocket holding cost per time unit for units that arrive before they are demanded;
- shortages are permitted and are backlogged at a cost b per time unit. This is a reasonable assumption if the company quotes a fixed selling price (which is appropriate when the firm cannot pass cost increase to final customers because of the competitiveness of the downstream market and the existence of a substitute technology at a fixed price); it has however consequences on the service level offered: if prices are expected to increase, keeping the backlogging penalty b constant implies that eventually the firm, that strives to minimize costs, will find it convenient to provide very low service level and even stop selling the product; while, if prices, are expected to decrease, it will offer better service;
- expected costs are discounted using the continuous, risk-neutral, discount rate r.

The model considers the decision of when to place the order i. At each point in time t, inventory manager chooses, based on how many customer have arrived $(D_{[0,t]})$, i and the current log-price $c(t)$ whether or not to order. The optimal decision can be found by comparing the expected discounted costs associated with both options, and then choosing the one that minimizes them.

Berling and Xie use k = i – *number of demands that have already occurred* and update the notation each time a demand occurs, in this way the decision depends only on k and purchase cost $c(t)$.

With k future demand and $c(t)$, the expected discounted cost if the order is placed now are

$$J^O\big(k, c(t)\big) = \Pi(k) + C(t) \quad . \tag{4.1}$$

Depending on the value of k, the net present value $\Pi(k)$ can have two different forms (4.2).

If demand has already occurred ($k \leq 0$), $\Pi(k)$ is k-independent and equal to the discounted value of the backorder costs paid from now until the unit arrives after L time units. If $k>0$ (k has not been demanded) there are two elements of the net present value: holding costs, from the time the order arrives until demand occurs (if that is more than L time units from the decision date) and backorder costs, if L is longer than time between decision date and demand occurrence. Since demand is not known in advance but is a random variable, its distribution function has to be considered for expected discounted costs' calculation. The time until customer k arrives follows an Erlang distribution, with probability distribution function $g(u, k, \lambda) = \lambda^k u^{k-1} e^{-\lambda u} / \big((k-1)!\big)$ and a cumulative distribution function $G(u, k, \lambda) = \sum_{l=k}^{\infty} (\lambda u)^l e^{-\lambda u} / l!$. An Erlang distribution is a continuous probability distribution with support $x \in (0, \infty)$ and two parameters: a positive "shape" k and a real "rate" λ (sometimes its inverse, the "scale" μ, is used instead).

$$\Pi(k) = \begin{cases} \int_0^L b e^{-ru}\, du = \frac{b(1-e^{-rL})}{r} & if\ k \leq 0, \\ \int_{u=0}^{L} \left[\int_{v=u}^{L} b e^{-rv}\, dv\right] g(u,k,\lambda) du + \int_{u=L}^{\infty} \left[\int_{v=L}^{u} h e^{-rv} dv\right] g(u,k,\lambda) du = & \\ \quad \frac{b}{r}\left[\left(\frac{\lambda}{\lambda+r}\right)^k G(L,k,\lambda+r) - e^{-rL} G(L,k,\lambda)\right] + & if\ k > 0 \\ \quad \frac{h}{r}\left[e^{-rL}\overline{G}(L,k,\lambda) - \left(\frac{\lambda}{\lambda+r}\right)^k \overline{G}(l,k,\lambda+r)\right] & \end{cases}$$

(4.2).

If a short period of time Δ is waited before ordering, the expected discounted costs are

$$J^W\big(k, c(t)\big) = \begin{cases} b\Delta + e^{-r\Delta} E\big[j\big(0, c(t+\Delta)\big)\big] & if\ k \leq 0 \\ e^{-r\Delta}\big(\lambda\Delta E\big[J\big(k-1, c(t+\Delta)\big)\big] + (1-\lambda\Delta) E\big[J\big(k, c(t+\Delta)\big)\big]\big) & if\ k > 0 \end{cases}$$

(4.3).

For $k \leq 0$, the expected costs for already demanded units are equal to the expected discounted value of future price paid for such a unit and backorder costs sustained in time period equal to lead time plus the time until the unit is bought [Berling, Xie 2014].

Considering the relation between the drift of the price process μ and the risk-neutral interest rate r, it emerges that it is therefore optimal to order only if the price falls within an interval:

when price increases more slowly than interest rate ($\mu<r$) it is better to order a unit when the price is low enough. With $\mu>r$, when the price is sufficiently low, it is better not to order to satisfy future demand, since lower purchase price do not compensate the higher holding costs; while when the price is sufficiently high, it is better not to order and pay the backordering costs indefinitely [Berling, Martínez-de-Albéniz 2011].

Kalymon's state-dependent base stock policy can be combined with Berling's and Martínez-de-Albéniz's threshold-price policy, in particular there are two price thresholds, C_k^L and a C_k^H respectively, such that the units are purchased now ($J(k,c) = \Pi(k) + e^c = \Pi(k) + C$) if $C_k^L \leq C \leq C_k^H$, with C_k^H as non-increasing function of k and C_k^L as non-decreasing function of k, if price process has certain characteristics ($\frac{d\sigma}{dc} = 0$ and either $\frac{d^2\mu}{dc^2} + \frac{d\mu}{dc} < 0$ or $\frac{d\mu}{dc} = 0$) [Theorem 1 from Berling, Martínez-de-Albéniz 2011]. Moreover, the existence of the lower threshold is a more theoretical than real possibility: in fact, C_k^L exists ($\neq 0$) only when:

- expected holding and backorder costs are larger if the unit is ordered now than if additional demand is waited before the order being made (i.e. when $\Pi(k)>\lambda\Pi(k-1)/(\lambda+r)$);
- the expected increase in purchase price in absolute terms is not sufficient to offset the expected increase in holding and backorder costs when the price is low but will be so when the price is higher.

Based on theorem 1, equation (4.1) and (4.3), the expected discounted costs relative to the non-ordered unit that will be used to satisfy an already occurred demand ($k\leq 0$) are

$$J(0,c) = \begin{cases} \frac{b}{r} + \frac{1}{r}\left(\mu(c)\frac{\partial J(0,c)}{\partial c} + \frac{\sigma(c)^2}{2}\frac{\partial^2 J(0,c)}{\partial c^2}\right) & if\ c > c_0^H = \ln(C_0^H) \\ e^c + \Pi(0) & otherwise \end{cases} \tag{4.4}$$

while, for a non-ordered unit that will be used for the future demand k ($k>0$) they are

$$J(k,c) = \begin{cases} \frac{\lambda}{\lambda+r} J(k-1,c) & if\ c > c_k^H = \ln(C_k^H) \\ e^c + \Pi(0) & if\ c_k^L \leq c \leq c_k^H \\ \frac{\lambda}{\lambda+r} j(k-1,c) & \\ + \frac{1}{\lambda+r}\left(\mu(c)\frac{\partial J(k,c)}{\partial c} + \frac{\sigma(c)^2}{2}\frac{\partial^2 J(k,c)}{\partial c^2}\right) & if\ c < c_k^L = \ln(C_k^L) \end{cases} . \tag{4.5}$$

Given that *J(k,c)* continuously differentiable and that $\lim_{c\to\infty} J(k,c) = (b/r)(\lambda/(\lambda+r)^k)$ a sufficient number of border conditions can be derived to explicitly determine the thresholds C_k^L and C_k^H and therefore the optimal base-stock level to purchase up to at a given price C which is the largest k that fulfills $C_k^L \leq C \leq C_k^H$ [Berling, Xie 2014].

Approximate policy

Determine the thresholds following the optimal policy as proposed in the previous paragraph, even if possible, is often unfeasible because of computational complexity, that is why Berling and Xie [2014] developed heuristics by examining the relative increase in the expected discounted total cost of using the policy suggested the heuristics, near-optimal/approximate policy compared to that of the optimal policy.

By replacing *J(k-1, c)* with *Π(k-1)+e^c^* of eq. (4.5) and assuming that one replenishes unit *k*, if not now, as soon as a demand occurs, it becomes unit *k-1* and we obtain

$$\tilde{J}(k,c) = \begin{cases} \frac{\lambda}{\lambda+r}\left(\Pi(k-1)+e^c\right)+\frac{1}{\lambda+r}\left(\mu(c)\frac{\partial\tilde{J}(k,c)}{\partial c}+\frac{\sigma(c)^2}{2}\frac{\partial^2\tilde{J}(k,c)}{\partial c^2}\right) & if\ c>\widetilde{c_k^H} \\ \Pi(0)+e^c & if\ \widetilde{c_k^L}\le c\le\widetilde{c_k^H} \\ \frac{\lambda}{\lambda+r}\left(\Pi(k-1)+e^c\right)+\frac{1}{\lambda+r}\left(\mu(c)\frac{\partial\tilde{J}(k,c)}{\partial c}+\frac{\sigma(c)^2}{2}\frac{\partial^2\tilde{J}(k,c)}{\partial c^2}\right) & if\ c<\widetilde{c_k^L} \end{cases}$$

(4.6).

The differentiability of $\tilde{J}(k,c)$ and the following new border condition *k*>0 when *c*→∞

$$\lim_{c\to\pm\infty}\tilde{J}(k,c)=\frac{\lambda}{\lambda+r}\Pi(k-1)+\int_{t=0}^{\infty}\lambda E\left[e^{c(t)}|c(0)=c\right]e^{-(r+\lambda)tdt} \qquad (4.7)$$

give the conditions necessary to determine the thresholds $\widetilde{C_k^L}$ and $\widetilde{C_k^H}$, which is easier using (4.7) than (4.6); moreover all $\widetilde{C_k^L}$ and $\widetilde{C_k^H}$ do not have to be computed sequentially but only the threshold prices for *k*-values (base-stock levels) one is interested in. As said in paragraph 1.6.1, price process may evolve following a geometric Brownian motion or mean-reverting Ornstein-Uhlenbeck process: based on that, thresholds are determined in different ways.

Under geometric Brownian motion assumption, if *k*=0 or *k*>0, the upper threshold C_0^H or C_k^H in which one stops buying can be derived from (4.4) and the continuous differentiability of *J*(0, *c*) at $c_0^H=\ln(C_0^H)$, can be respectively approximated as

$$\widetilde{C_0^H}=C_0^H=e^{c_0^H}=\frac{\Pi(0)-D_0}{1+1/\alpha_0} \quad \text{and} \quad \widetilde{C_k^H}=\frac{\Pi(k)-D_k}{\left(1+\frac{1}{\alpha}\right)(B-1)} \qquad (4.8)$$

where $D_0=-b/r$; $\alpha_0=\left(\mu+\sqrt{\mu^2+2r\sigma^2}\right)/\sigma^2$, $B=\lambda/\left(r+\lambda-(\mu+\frac{\sigma^2}{2})\right)$ and $D_k=\lambda\Pi(k-1)/(\lambda+r)$.

This approximation for *k*>0 however may cause some problems because:

- it does not always provide a positive solution ($\widetilde{C_k^H}>0$);
- it does not guarantee the non-increasing property of the upper threshold.

Under Ornstein-Uhlenbeck process, the following lemma can be applied [Berling, Xie 2014]:

Lemma 1 *Regarding the existence and uniqueness of the approximate thresholds, the following properties hold:*

- *if* $\Pi(k)\le D_k$ *and one uses* μ_{OU} *and* σ_{OU}*, then the expression* $(\Pi(k)-D_k)/((1+\frac{1}{\alpha})(B-1)$ *will be negative for* $C<e^{\bar{c}-(r-\frac{\sigma^2}{2})\kappa}$ *and decreasing from infinity at* $C=e^{\bar{c}-(r-\frac{\sigma^2}{2})\kappa}$

towards 0 as C→∞. As a result, there will exist a unique solution to $\widetilde{C_k^H} = (\Pi(k) - D_k)/(1+\frac{1}{\alpha})(B-1)$*;*

- *if Π(k)>Dk ,then the right-hand side of (2.19) is strictly convex in C with a maximum at* $C = e^{\bar{c}-((r+\kappa)t-\frac{(1-e^{-2\kappa t})\sigma^2}{4\kappa}/(1-e^{-\kappa t})}$*. This implies that* $\widetilde{C_k^H}$ *is the minimum of* $e^{-rk/\lambda}\frac{b}{r} - \Pi(k)$ *and the largest C fulfills equality in (2.19), if such a C exists. If such a C does not exist, then* $\widetilde{C_k^H} = 0$.

4.4 E.O.Q-based models

As seen in paragraph 1.7.1, Wilson's model is one the most simple and long-standing methods. Even if its assumptions are very simple and far from reality, its first representation of the trade-offs of the decisions typical of stock control, constitutes a starting point for the development of more sophisticated variations. The classical inventory model assumes the idealized situation showed in pic.29, where *Q* is the order size. Upon order receipt, the inventory level is *Q*. At a constant demand rate, represented by the negative sloping lines, units are withdrawn from inventory. When inventory level is S_0, a new order is placed for *Q* units that is received, all at once, after a fixed time period, and then placed in inventory. For the definition of the cost function whose minimization allows the identification of the optimal value *Q*, it is necessary to present the costs to consider: based on the assumptions of the E.O.Q. model, stock out and obsolescence costs are not taken into consideration and the purchase cost is constant and therefore not included in the calculations; as a result total inventory costs will depend only on ordering and holding costs, in a way such that, as *Q* increases holding costs increase and ordering costs decrease, and vice versa. Taking the first derivative of the cost function ($y(Q) = c_o\frac{D}{Q} + c_h\frac{Q}{2}$) and setting it equal to zero we obtain $\frac{dy(Q)}{dQ} = \frac{c_h}{2} - \frac{c_o D}{Q^2} = 0$

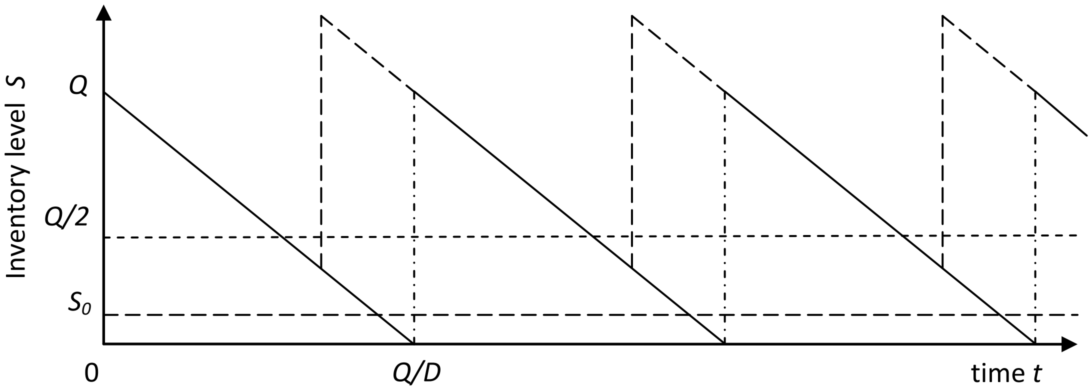
and solving for *Q* we get the E.O.Q. formula:

$$Q^* = \sqrt{\frac{2c_o D}{c_h}}\,.$$

We can therefore obtain T^* and n^* as

$$T^* = Q^*/\,D = \sqrt{\frac{2\,c_o}{c_h\,D}}$$

$$n^* = D/Q^* = \sqrt{\frac{c_h\, D}{2\, c_o}}\,.$$



Picture 29 E.O.Q. model inventory profile [adjusted from Ternsine 1994; Urgeletti Tinarelli 1992]

The E.O.Q. results in an item with a high unit cost being ordered frequently in small quantities (the saving in inventory investment pays for the extra orders); an item with a low unit cost is ordered in large quantities (the inventory investment is small and the repeated expense of orders can be avoided). If order cost c_o is zero, orders are placed to satisfy demand as it occurs, which results in no holding costs. If the holding cost c_h is zero, an order (only one) is placed for an amount that will satisfy the lifetime demand for the item [Tersine 1994]. Economic lot's size is increasing function of demand and unit order cost and decreasing function of carrying costs; this relation is clear: as order cost increases also lot size Q increases trying to minimize the number of orders to be issued within the same unit of time, vice versa lot size, and therefore the quantity stocked on average, decreases in the presence of high holding costs, in this case satisfying same demand with more frequent, smaller orders.

Many modifications have been made throughout time in order to better adapt this model to real cases (e.g. E.O.Q. with quantity discounts, with backlogging, with lost sales, so on).

Among these modification there is one proposed by Teng and Yang [2004] that considers both the variations of the purchase costs and the deterioration of the stocked items: an optimal replenishment schedule exists and total inventory costs can be represented by a convex function of the number of replenishments.

4.4.1 Deterministic E.O.Q. with partial backlogging when demand and cost fluctuate over time [Teng, Yang 2004]

As seen in chapter 2, in many circumstances demand rate and purchase cost (whether it is a production or purchase cost) are assumed to be constant. On one hand, this is a useful assumption since it allows for a simplification of the computations otherwise required; on the other hand however it does not consider the complexity of real life situations: as a far as demand is concerned, except for being known in advance, assumption often made under deterministic models set-ups, its rate stays stable only in the maturity phase of a product life cycle; while unit cost may change over time: the example brought by Teng and Yang [2004] involve the case of high-tech products, whose unit cost decrease significantly over its short product life cycle, while its demand increases. Moreover, the substantial incidence of purchase costs on sales (52% for all industry according to Teng and Yang [2004], citing Heizer and Render [2000]) makes it fundamental combining purchasing strategy into the E.O.Q model.

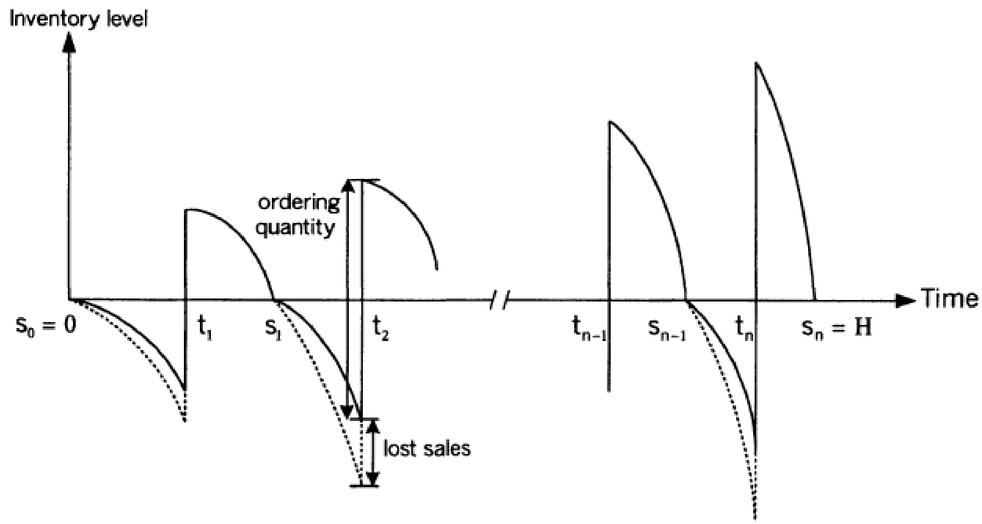
The following model proves that, under certain assumptions, the optimal replenishment schedule exists, it is unique and it is found by a one-dimensional iterative model. It is also demonstrated that the sum inventory costs (holding, backlogging, lost sales and purchase costs) is a convex function of the number of replenishments, which leads to the search of the optimal number of replenishments to find the local minimum.

The assumptions and the set-up of the model are as follows:

- planning horizon of the inventory problem is finite and it is H time units; initial and final inventory levels are both zero during H;
- instantaneous replenishment;
- lead time τ is zero;
- a constant fraction of the on-hand inventory deteriorates per unit of time (θ is the deteriorate rate) and there is no repair or replacement of the deteriorated items;
- shortages are allowed: unsatisfied demand is backlogged and the fraction of shortages backordered $\beta(t)$ is a decreasing function of time t, where t is the waiting time up to the next replenishment and $0 \leq \beta(t) \leq 1$ with $\beta(0)=1$; if $\beta(t)=1$ (or 0) for all t, then shortages are completely backlogged (or lost). In order to guarantee the existence of an optimal solution, it is assumed that $\beta(t)+t\ \beta'(t) \geq 0$, where $\beta'(t)$ is the first derivative of $\beta(t).c_b$ indicates the backlogging cost per unit per unit time, if shortages are backlogged; if they are not backlogged but lost, c_l is unit opportunity cost of lost sales, defined as sum of the revenue loss and the cost of lost goodwill; thus the opportunity cost is greater than the unit purchase cost, we assume therefore $c_v(t) < c_l$ for all t;
- c_f is the fixed ordering cost per order;
- $f(t)$ is the demand rate at time t, assuming that $f(t)$ is greater than zero in $(0, H)$ and continuous in the planning horizon $[0, H]$;
- $c_v(t)$ is the variable purchase cost per unit at time t; assuming it be greater than zero and differentiable in $(0, H)$;

- c_h is holding costs per unit per time unit;
- n is the number of replenishments over [0, H];
- t_i is i-th replenishment time, with i=1, 2, …, n;
- S_i is the time at which inventory level reaches zero in the i-th replenishment cycle, with i=1, 2, …, n;
- $C'(t)=c_h+\theta c_v(t)$ is the marginal total inventory carrying cost (i.e. the marginal sum of inventory and deterioration cost) per unit per unit time;
- $S(s, t)=c_b(t\text{-}s)\beta(t\text{-}s)+[c_l\text{-}c_v(t)][1\text{-}\beta(t\text{-}s)]$ is the expected shortage cost (the expected sum of backlogging and lost-sales cost) per unit at time t, where s is the beginning of the shortage and t is the replenishment time, with $s \leq t$. Since the longer the waiting time, the higher the expected shortage cost, it is also assumed that $S(s, t)$ is a non-decreasing function of t (the marginal expected shortage cost with respect to t is $S_t(s, t) \geq 0$.

A graphical representation of the model is provided in pic.30.



Picture 30 Graphical representation of inventory profile [from Teng, Yang 2004]

The i-th replenishment is made at time t_i; the quantity received at t_i is used partly to meet accumulated demand resulting in backorders in the previous cycle from time s_{i-1} to t_i (with $s_{i-1}<t_i$).The inventory at t_i gradually reduces to zero at s_i (with $s_i>t_i$).

The objective is to find the optimal number of replenishments n_i and the timing of the reorder points $\{t_i\}$ and the shortage points $\{s_i\}$ that minimizes total relevant costs.

Being $I(t)$, the level of inventory at time t, depleted by both demand and deterioration, the inventory level at time t during the replenishment cycle is governed by the following equation:

$$\frac{dI(t)}{dt} = -f(t) - \theta I(t)\ , \qquad \text{with } t_i \leq t \leq s_i \tag{5.1}$$

with the boundary conditions $I(s_i)=0$. Solving 5.1, considering that demand is a random variable, we have that

$$I(t) = e^{-\theta t}\int_t^{s_i} e^{\theta u} f(u)du = \int_t^{s_i} e^{\theta(u-t)} f(u)du \;\;, \text{ with } t_i \leq t \leq s_i. \qquad (5.2)$$

As a consequence the time-weighted inventory during the *i*-th cycle is

$$I_i = \int_{t_i}^{s_i} I(t)dt = \int_{t_i}^{s_i}\left[e^{-\theta t}\int_t^{s_i} e^{\theta u} f(u)du\right]dt = \frac{1}{\theta}\int_t^{s_i}\left[e^{\theta(t-t_i)} - 1\right]f(t)dt \;\;, \qquad (5.3)$$

with $i=1, 2, \ldots, n$.

Being demand a random variable, in the same way, the cumulative number of backorders $B(t)$ at time t during $[s_{i-1}, t_i)$ is

$$B(t) = \int_{s_{i-1}}^{t} \beta(t_i - u)f(u)du \;\;, \text{ with } s_{i-1} \leq t \leq t_i \qquad (5.4)$$

and the cumulative number of lost sales $L(t)$ at time t during $[s_{i-1}, t_i)$ is

$$L(t) = \int_{s_{i-1}}^{t}[1 - \beta(t_i - u)]f(u)du \;\;, \text{ with } s_{i-1} \leq t \leq t_i \qquad (5.5)$$

with $i=1, 2, \ldots, n$.

Thus, the time-weighted backorders due to shortages during the *i*-th cycle is B_i,

$$B_i = \int_{s_{i-1}}^{t_i} B(t)dt = \int_{s_{i-1}}^{t_i}(t_i - t)\beta(t_i - t)f(t)dt, \qquad (5.6)$$

and the total number of lost sales due to shortages during the *i*-th cycle is L_i

$$L_i = L(t_i) = \int_{s_{i-1}}^{t_i}[1 - \beta(t_i - t)]f(t)dt \;\; . \qquad (5.7)$$

From (5.2) and (5.4), the order quantity Q_i at t_i in the *i*-th replenishment cycle is

$$Q_i = B(t_i) + I(t_i) = \int_{s_{i-1}}^{t_i} \beta(t_i - t)f(t)dt + \int_{t_i}^{s_i} e^{\theta(t-t_i)}f(t)dt \;\; ; \qquad (5.8)$$

therefore, the purchase cost during the *i*-th replenishment cycle is P_i

$$P_i = c_f + c_v(t_i)Q_i = c_f + c_v(t_i)\left[\int_{s_{i-1}}^{t_i} \beta(t_i - t)f(t)dt + \int_{t_i}^{s_i} e^{\theta(t-t_i)}f(t)dt\right] \;. \qquad (5.9)$$

It follows that, if n replenishments orders are placed in $[0, H]$, the total relevant cost of this inventory system during the planning horizon H for the model is $TC(n,\{s_i\},\{t_i\})$, where

$$TC(n,\{s_i\},\{t_i\}) = \sum_{i=1}^{n}(P_i + c_h I_i + c_b B_i + c_l L_i) =$$

$$= nc_f + \sum_{i=1}^{n}\int_{s_{i-1}}^{t_i}\{[c_b(t_i - t) + c_v(t_i) - c_l]\beta(t_i - t) + c_l\}f(t)dt$$

$$+ \sum_{i=1}^{n}\int_{t_i}^{s_i}\left\{\left[\frac{c_h}{\theta} + c_v(t_i)\right]e^{\theta(t-t_i)} - 1) + c_v(t_i)\right\}f(t)dt \qquad (5.10)$$

with $0=s_0<t_i$ and $s_n=H$.

The objective is to identify $n,\{s_i\}$ and $\{t_i\}$, such that $TC(n,\{s_i\},\{t_i\})$ as expressed in (5.10) is minimized.

For a fixed value of n, the necessary conditions to do so are that $\frac{\partial TC(n,\{s_i\},\{t_i\})}{\partial s_i} = 0$ and $\frac{\partial TC(n,\{s_i\},\{t_i\})}{\partial t_i} = 0$ for $i=1, 2, \ldots, n$.

In this way we obtain, respectively,

$$[c_b(t_{i+1}-s_i)+c_v(t_{i+1})-c_l]\beta(t_{i+1}-s_i)+c_l-c_v(t_i)=$$
$$=\left[\frac{c_h}{\theta}+c_v(t_i)\right](e^{\theta(s_i-t_i)}-1) \quad (5.11)$$

and

$$[c_h+\theta c_v(t_i)-c'_v(t_i)]\int_{t_i}^{s_i} e^{\theta(t-t_i)}f(t)dt=$$
$$=\int_{s_{i-1}}^{t_i}\{[c_b+c'_v(t_i)]\beta(t_i-t)+[c_b(t_i-t)+c_v(t_i)-c_l]\beta'(t_i-t)\}f(t)dt\,. \quad (5.12)$$

From (5.11) and (5.12) we can deduct some important results:

- if $C'(t_i)\leq c'_v(t_i)$ for all t_i, then the optimal solution is $n^*=1$ and $t^*_1=0$; and if $c'_v(t_i)\leq S_{ti}(t,t_i)$ for all t_i, then the optimal solution is $n^*=1$ and $t^*_1=H$. It follows that, if the increasing rate of the unit purchase cost $c'_v(t)$ is higher than or equal to the marginal inventory carrying cost (intended as the sum of inventory and deterioration costs) per unit $C'(t)$ (i.e. when $0<C'(t)\leq c'_v(t)$), buying and storing a unit now is less expensive than buying it later. In the same way, when $c'_v(t)\leq -S_{ti}(t, t_i)<0$ (i.e. if the declining rate of unit cost $-c'_v(t)$ is larger than or equal to the marginal expected shortage cost per unit $S_{ti}(t, t_i)$) it is cheaper to delay the purchase until the end of the planning horizon H than buying it earlier [Teng, Yang 2004];
- if $C'(t_i)<c'_v(t_i)>-S_{ti}(t, t_i)$ for all t_i and t^*_1 is unique, then the solution to (5.11) and (5.12) is also unique (i.e. the optimal values of $\{s^*_i\}$ and $\{t^*_i\}$ can be uniquely determined); making equations (5.11) and (5.12) the necessary and sufficient conditions for finding the absolute minimum TC(n,{si},{ti});
- if $C'(t_i)<c'_v(t_i)>-S_{ti}(t, t_i)$, the total relevant cost $TC(n)$ is a convex function of the number of replenishments n, by Bellman's principle of optimality, where

$$TC(n)=TC(n,\{s_i^*\},\{t_i^*\})\,. \quad (5.13)$$

These results have the advantage to reduce the 2n-dimensional problem of finding $\{s^*_i\}$ and $\{t^*_i\}$ to a one-dimensional problem. Since $s_0=0$, it is only needed to find t^*_1 to generate s^*_1 by (5.12) and t^*_2 by (5.11) and then the rest of $\{s^*_i\}$ and $\{t^*_i\}$ uniquely by repeatedly using (5.11) and (5.12); therefore, for any chosen t^*_1 if $s^*_n=H$, then t^*_1 is chosen correctly; otherwise the optimal t^*_1 can be easily found by standard search techniques.

Moreover, being the total cost a convex function of the number of replenishments, the search for the optimal replenishment number n^* is reduced to find a local minimum. To do so, Teng and Yang [2004] assume that the length of $t_{i+1}-s_i$ is approximately 1. By using the average unit cost $\overline{c_v}$ to replace $c_v(t_i)$, they obtain the expected shortage cost as

$$S(s_i,t_{i+1})\approx c_b\beta(1)+(c_l-\overline{c_v})[1-\beta(1)] \quad (5.14)$$

and the marginal total inventory carrying cost as

$$C'(t) = c_h + \theta c_v(t) \approx c_h + \theta \overline{c_v}. \tag{5.15}$$

An estimate of *n* can be therefore expressed as

$$n_1 = rounded\ integer\ of$$

$$\left[\frac{(c_h+\theta\overline{c_v})\{c_b\beta(1)+(c_l-\overline{c_v})[1-\beta(1)]\}HQ(H)}{2c_f\{c_h+\theta\overline{c_v}+c_b\beta(1)+(c_l-\overline{c_v})[1-\beta(1)]\}}\right]^{1/2}, \tag{5.16}$$

where $Q(H) = \int_0^H f(t)dt$.

A proposal for the goldsmith sector

In the current competitive scenario characterized by the two contrasting needs of immediate demand satisfaction and cost reduction, inventory management has been acquiring more importance. From this work it emerges the central role played by stock in companies performances, which requires inventory control to be more accurate and precise as possible, being the result of modifications and adjustments to operations research models to each specific real case, in business, strategic and competitive terms, aiming to the simplification of procurement decisions coming from a full understanding of all the advantages and disadvantages related to the different alternatives available.

Although necessary in all businesses in order to survive, it is even more important in those circumstances characterized by a significant uncertainty degree, which can be relatively low if, as analyzed by most of the literature, relates to the unknown nature of demand, but also particularly consistent if uncertainty about raw materials purchase cost is added. This kind of uncertainty is typical for those companies that use in their operations materials subject to quotation, for example gold.

As seen in chapter 3, the jewellery industry, as well as the goldsmith sector in wider sense, has adopted various ways (procurement management and distributional organization) in order to reduce and even almost eliminate the risk associated to the price fluctuations, which sometimes can be significant, of raw materials employed in the production of magnificent piece of jewellery. As seen, the intermediation of metals banks, who offer specialized services and the rationing of the precious metal that allow to parcel out gold procurement and stock, permits the limitation of price risk; a similar function is carried out by the two legal institutions of contract work, i.e. advance provision of the necessary gold quantity for the realization of a predetermined quantity of products by an operator of the jewellery sector, and loan for use, i.e. the borrowing of the necessary gold quantity for a bank who asks for interest payment in return. This being considered, and the fact that in the sector only the cost of workmanship is paid while the cost of the precious metal is passed on, makes the model proposed by Teng and Yang [2004] more appropriate for application to the goldsmith sector than the other models presented in this work.

However, some modifications to this model have to be made to better fit the characteristics of the sector. In fact, as explained in paragraph 1.6, operations research inventory management models are ideal and simplified representation of the vast number of real situations that companies face and the relevance of a model to a certain situation is based on the reasonableness of its assumptions and limitations. In this case, the modifications mainly relate with the assumptions connected with the deterioration rate θ and the demand, while the overall structure of the model remains valid.

As seen in paragraph 1.8.4, obsolescence and deterioration represent the risk of an item losing value because of changes in consumers' styles or preferences or because of changes in properties due to age or environmental degradation, which is particularly relevant for those items that change chemically during storage, such as foods, photographic materials, medicine, volatile liquid and so on [Urgeletti Tinarelli 1992].

For deteriorating goods, product quantity available for sale decreases due to deterioration during their normal storage period [Tsao 2010]. Obsolescence can occur for many reasons: because once consumed, units are replaced by a substitute with similar or identical functionalities, because the function performed by that item is no longer asked for, or because of the existence of a program for its systematic replacement by a substitute product [Brown, Lu, Wolfson 1964]. This is not the case of gold and jewellery: while for example foodstuff like vegetables cannot be sold after a certain period of time because they have perished, stocked jewels do not deteriorate the more time they remain unsold, their physical characteristics are not affected by storage. As a consequence, the deterioration rate θ proposed in the model by Teng and Yang [2004] is not so appropriate. However, such deterioration rate θ cannot be placed equal to zero because equation (5.3), and all those resulting from the same, would otherwise lose significance. For these reasons, it could be appropriate to set θ very close to zero, but still not null; in this way the model would remain applicable, although with a certain degree of approximation.

The jewellery industry can be included in the wider fashion system: as different trends, styles etc affect the latter, they also affect the former, even if to a lesser extent. The fashion system is characterized by a highly volatile demand and continuously changing consumers' preferences. As Choi [2016] points out, inventory decisions are therefore subject to a high degree of risk. Companies of the sector found different ways to manage such risk. Choi brings the example of the postponement principle for the "dyeing garments" like sweaters and quick response policies adopted by Benetton and Zara, respectively. Instead of preparing inventory based on individual demand for each colored sweater for the upcoming season, Benetton plans the inventory by forecasting the aggregate demand and prepare the "un-dyed garments"; once consumers have committed their initial orders with specific the specific color, it quickly reacts by dyeing garments accordingly. Fast fashion companies like Zara adopt quick response policies to reduce lead time, through the adoption of a highly vertically integrated and in-housing production and distribution.

To accommodate obsolescence or other types of varying demand in the model thereby considered, it is possible to assume a demand distribution which vary over time, with a demand decreasing rate. The demand rate $f(t)$ of Teng and Yang [2004] can be substituted in two ways. Wang and Lin [2012] propose a demand $D_i(t)$ expressed as an exponential function of time t, where $D_1=Ae^{-\eta t}$, with $t\geq 0$, η is the demand decreasing rate and A is the initial market demand for a new product with a corresponding initial retailing price of p. Similarly, Tsao [2010] proposes a demand which is a function of price (the author distinguishes two retail prices depending on whether the product required is in stock, p_1, or out of stock, p_2) and decreases exponentially with time (the property of fashion goods) of the form $D(p_i,t)=(a_0-b_0p_i)e^{\lambda t}$, with $\alpha_0>0, b_0>0$ and i=1 or i=2, where λ is the market declining rate and a_0 and b_0 can be determined by regression analysis using historical transaction data for similar products.

This substitution seems reasonable given the "fashion" nature that jewellery has acquired, considering that companies are now required to frequently renovate collections, given the reduction of the product life cycle that jewels are experiencing, the increasing variety of the productive offer, the demand uncertainty and high seasonality that represent typical feature of the business, in connection with the quick response systems that retailers, in coordination with producers, have been implementing in order to ensure flexibility and speed when adapting to demand changes.

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Acknowledgements

20th June, 2016

I dedicate this accomplishment to my parents, for everything they have done for me and the support they gave me during all these years.

Heartfelt thanks goes to my sister, Chiara, who inevitably listened to all my complaints and nevertheless was able to make me smile to them. The interest and trust they provided me were important and pushed me to pursue this goal, trying to do all my best.

A special thank goes to Mauro, who supported me and inspired me from the very first moment we met, and to all my friends, new and old ones.

I thank my supervisor for her availability and politeness, and all teachers and professors who infused me the passion for my studies.