Università degli Studi di Padova – Dipartimento di Ingegneria Industriale

Corso di Laurea in Ingegneria meccanica

Relazione per la prova finale «Analisi e simulazione meccanismo di Theo Jansen»

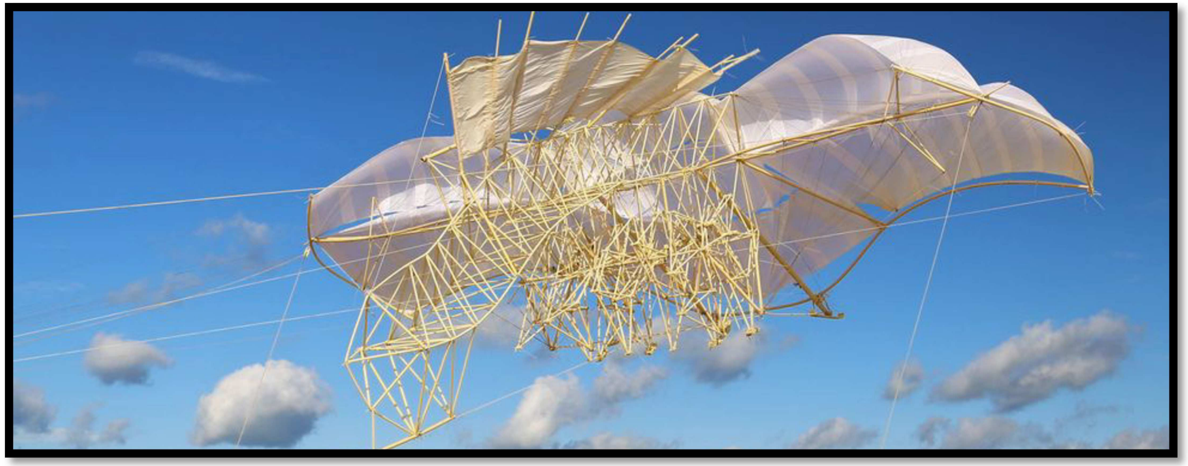
Tutor universitario: Ing. Bottin Matteo

Laureando: *Faccini Nicolò*

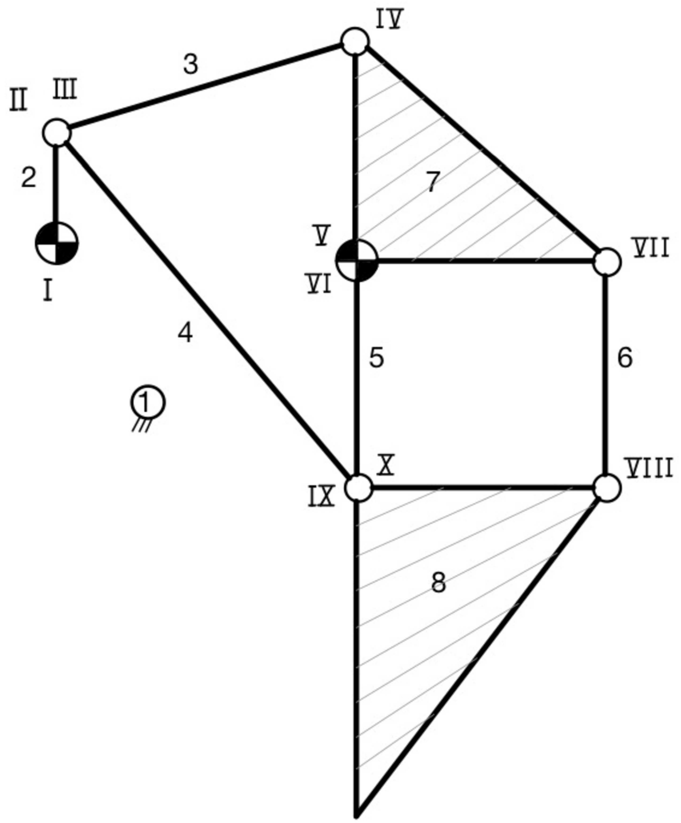
Padova, 06/07/2022

INTRODUZIONE

Il meccanismo in questione è utilizzato da Theo Jansen per il movimento delle sue sculture cinetiche.



OBIETTIVI DEL LAVORO



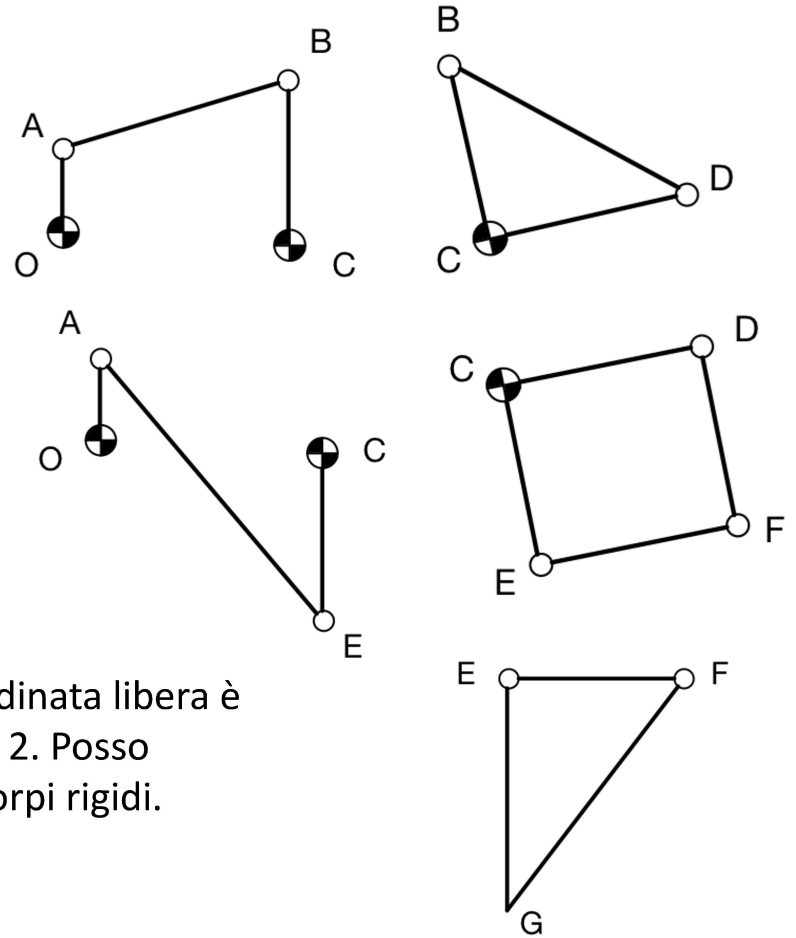
Calcolo dei gradi di libertà del meccanismo:

$$n = 3(m-1) - 2C_1 - C_2$$

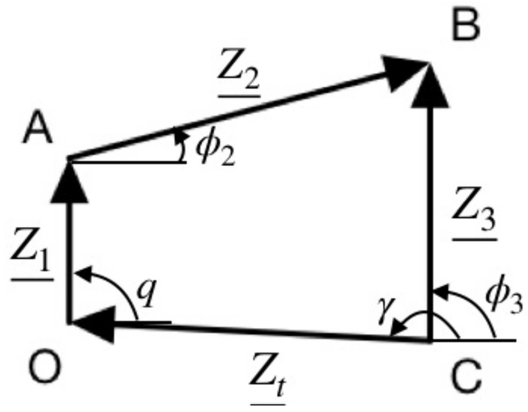
$$m = 8$$
$$C1 = 10$$
$$C2 = 0$$

$$n = 3(8-1) - 2 \cdot 10 = 1$$

Il meccanismo ha 1 grado di libertà. La coordinata libera è l’angolo della manovella, nonché il membro 2. Posso individuare 3 maglie, i membri 7 e 8 sono corpi rigidi.



ANALISI CINEMATICA DI POSIZIONE

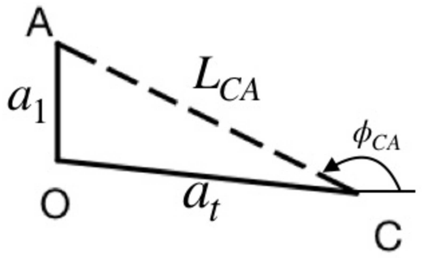


$$\underline{Z}_1 + \underline{Z}_2 - \underline{Z}_3 + \underline{Z}_t = \underline{0}$$

$$a_1 \begin{Bmatrix} cosq \\ senq \end{Bmatrix} + a_2 \begin{Bmatrix} cos\phi_2 \\ sen\phi_2 \end{Bmatrix} - a_3 \begin{Bmatrix} cos\phi_3 \\ sen\phi_3 \end{Bmatrix} + a_t \begin{Bmatrix} cos\gamma \\ sen\gamma \end{Bmatrix} = \underline{0}$$

$$\begin{Bmatrix} x_A \\ y_A \end{Bmatrix} = a_1 \begin{Bmatrix} cosq \\ senq \end{Bmatrix}$$

Grandezze ricavate:



$$L_{CA} = \sqrt{(x_C{}^2 - x_A{}^2) + (y_C{}^2 - y_A{}^2)}$$

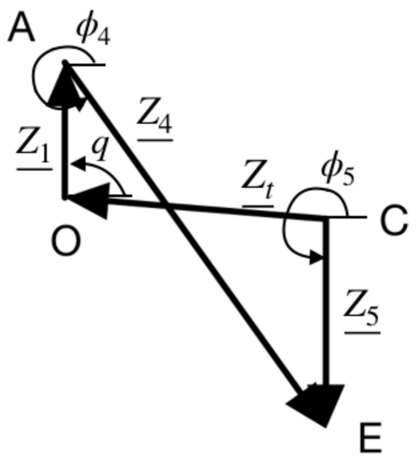
$$\phi_{CA} = tan^{-1}\left(\frac{y_A - y_C}{x_A - x_C}\right)$$

$$\phi_3 = \phi_{CA} - cos^{-1}\left(\frac{L_{CA}{}^2 + a_3{}^2 - a_2{}^2}{2 \cdot L_{CA} \cdot a_3}\right)$$

$$\begin{Bmatrix} x_B \\ y_B \end{Bmatrix} = \begin{Bmatrix} x_C \\ y_C \end{Bmatrix} + a_3 \begin{Bmatrix} cos\phi_3 \\ sen\phi_3 \end{Bmatrix}$$

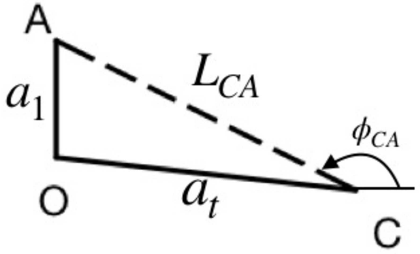
$$\phi_2 = tan^{-1}\left(\frac{y_B - y_A}{x_B - x_A}\right)$$

ANALISI CINEMATICA DI POSIZIONE



$$\underline{Z_1} + \underline{Z_4} - \underline{Z_5} + \underline{Z_t} = \underline{0}$$

$$a_1 \begin{Bmatrix} cosq \\ senq \end{Bmatrix} + a_4 \begin{Bmatrix} cos\phi_4 \\ sen\phi_4 \end{Bmatrix} - a_5 \begin{Bmatrix} cos\phi_5 \\ sen\phi_5 \end{Bmatrix} + a_t \begin{Bmatrix} cos\gamma \\ sen\gamma \end{Bmatrix} = \underline{0}$$



$$L_{CA} = \sqrt{({x_C}^2 - {x_A}^2) + ({y_C}^2 - {y_A}^2)}$$

$$\phi_{CA} = tan^{-1}\left(\frac{y_A - y_C}{x_A - x_C}\right)$$

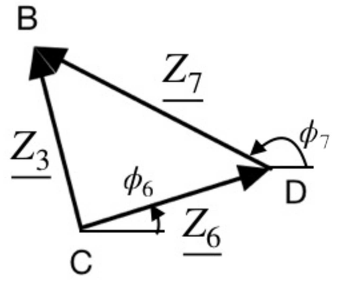
Grandezze ricavate:

$$\phi_5 = \phi_{CA} + cos^{-1}\left(\frac{{L_{CA}}^2 + {a_5}^2 - {a_4}^2}{2 \cdot L_{CA} \cdot a_5}\right)$$

$$\begin{Bmatrix} x_E \\ y_E \end{Bmatrix} = \begin{Bmatrix} x_C \\ y_C \end{Bmatrix} + a_5 \begin{Bmatrix} cos\phi_5 \\ sen\phi_5 \end{Bmatrix}$$

$$\phi_4 = tan^{-1}\left(\frac{y_E - y_A}{x_E - x_A}\right)$$

ANALISI CINEMATICA DI POSIZIONE



$$\phi_6 = \phi_3 - 90°$$

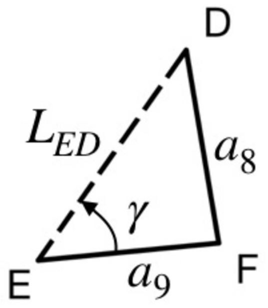
$$\begin{Bmatrix} x_D \\ y_D \end{Bmatrix} = \begin{Bmatrix} x_C \\ y_C \end{Bmatrix} + a_6 \begin{Bmatrix} cos\phi_6 \\ sen\phi_6 \end{Bmatrix}$$

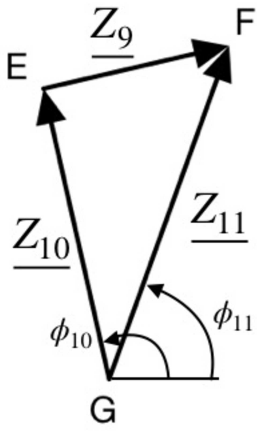
$$\phi_7 = tan^{-1}\left(\frac{y_B - y_D}{x_B - x_D}\right)$$

$$L_{ED} = \sqrt{({x_E}^2 - {x_D}^2) + ({y_E}^2 - {y_D}^2)}$$

$$\gamma = cos^{-1}\left(\frac{{L_{ED}}^2 + {a_9}^2 - {a_8}^2}{2 \cdot L_{ED} \cdot a_9}\right)$$

$$\phi_{ED} = tan^{-1}\left(\frac{y_D - y_E}{x_D - x_E}\right)$$





$$\phi_{10} = \phi_9 + 90°$$

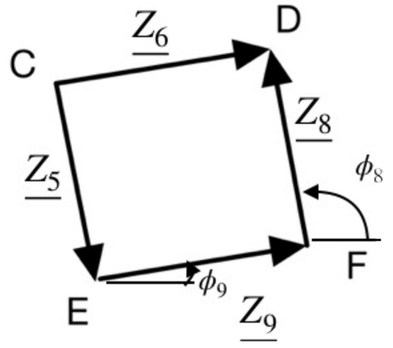
$$\begin{Bmatrix} x_G \\ y_G \end{Bmatrix} = \begin{Bmatrix} x_E \\ y_E \end{Bmatrix} - a_{10} \begin{Bmatrix} cos\phi_{10} \\ sen\phi_{10} \end{Bmatrix}$$

$$\phi_{11} = tan^{-1}\left(\frac{y_F - y_G}{x_F - x_G}\right)$$

$$\phi_9 = \phi_{ED} - \gamma$$

$$\begin{Bmatrix} x_F \\ y_F \end{Bmatrix} = \begin{Bmatrix} x_E \\ y_E \end{Bmatrix} + a_9 \begin{Bmatrix} cos\phi_9 \\ sen\phi_9 \end{Bmatrix}$$

$$\phi_8 = tan^{-1}\left(\frac{y_D - y_F}{x_D - x_F}\right)$$



ANALISI CINEMATICA DI VELOCITA'

Derivo l'equazione di maglia:

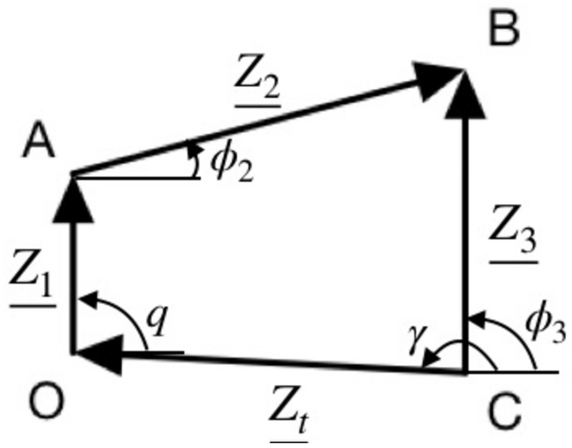
$$a_1 \begin{Bmatrix} -senq \\ cosq \end{Bmatrix} \dot{q} + a_2 \begin{Bmatrix} -sen\phi_2 \\ cos\phi_2 \end{Bmatrix} \dot{\phi}_2 - a_3 \begin{Bmatrix} -sen\phi_3 \\ cos\phi_3 \end{Bmatrix} \dot{\phi}_3 = \underline{0}$$

Definisco matrice Jacobiana e vettore dei termini noti:

$$J = \begin{bmatrix} -a_2 sen\phi_2 & a_3 sen\phi_3 \\ a_2 cos\phi_2 & -a_3 cos\phi_3 \end{bmatrix} \qquad A = \begin{Bmatrix} -a_1 senq \\ a_1 cosq \end{Bmatrix}$$

Calcolo rapporti di velocità:

$$\begin{Bmatrix} w_2 \\ w_3 \end{Bmatrix} = -J^{-1}A = \begin{Bmatrix} \dfrac{-a_1 sen\left(\phi_3 + q\right)}{a_2 sen\left(\phi_2 - \phi_3\right)} \\ \dfrac{a_1 sen\left(\phi_2 + q\right)}{a_3 sen\left(\phi_2 - \phi_3\right)} \end{Bmatrix}$$



Grandezze ottenute:

$$\begin{Bmatrix} \dot{\phi}_2 \\ \dot{\phi}_3 \end{Bmatrix} = \begin{Bmatrix} w_2 \\ w_3 \end{Bmatrix} \dot{q}$$

$$\begin{Bmatrix} \dot{x}_B \\ \dot{y}_B \end{Bmatrix} = a_3 \begin{Bmatrix} -sen\phi_3 \\ cos\phi_3 \end{Bmatrix} \dot{\phi}_3$$

ANALISI CINEMATICA DI VELOCITA’

Derivo l’equazione di maglia:

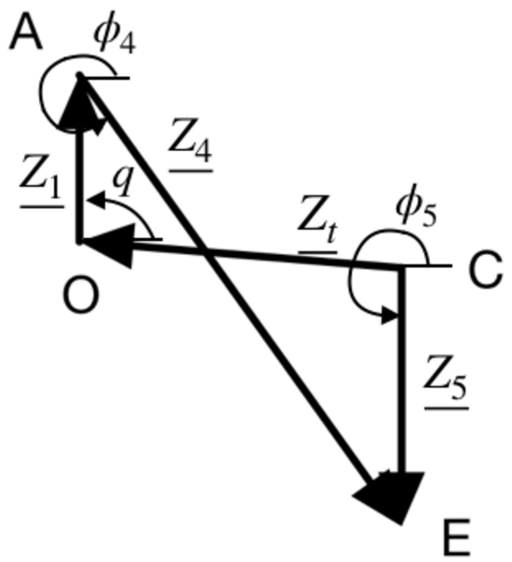
$$a_1 \begin{Bmatrix} -senq \\ cosq \end{Bmatrix} \dot{q} + a_4 \begin{Bmatrix} -sen\phi_4 \\ cos\phi_4 \end{Bmatrix} \dot{\phi}_4 - a_5 \begin{Bmatrix} -sen\phi_5 \\ cos\phi_5 \end{Bmatrix} \dot{\phi}_5 = \underline{0}$$

Definisco matrice Jacobiana e vettore dei termini noti:

$$J = \begin{bmatrix} -a_4 sen\phi_4 & a_5 sen\phi_5 \\ a_4 cos\phi_4 & -a_5 cos\phi_5 \end{bmatrix} \qquad A = \begin{Bmatrix} -a_1 senq \\ a_1 cosq \end{Bmatrix}$$

Calcolo rapporti di velocità:

$$\begin{Bmatrix} w_4 \\ w_5 \end{Bmatrix} = -J^{-1}A = \begin{Bmatrix} \dfrac{-a_1 sen\left(\phi_5 + q\right)}{a_4 sen\left(\phi_4 - \phi_5\right)} \\ \dfrac{a_1 sen\left(\phi_4 + q\right)}{a_5 sen\left(\phi_4 - \phi_5\right)} \end{Bmatrix}$$



Grandezze ottenute:

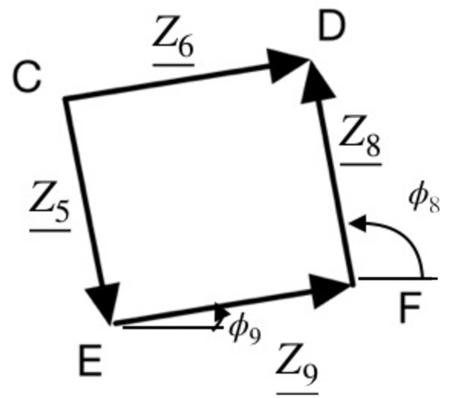
$$\begin{Bmatrix} \dot{\phi}_4 \\ \dot{\phi}_5 \end{Bmatrix} = \begin{Bmatrix} w_4 \\ w_5 \end{Bmatrix} \dot{q}$$

$$\begin{Bmatrix} \dot{x}_E \\ \dot{y}_E \end{Bmatrix} = a_5 \begin{Bmatrix} -sen\phi_5 \\ cos\phi_5 \end{Bmatrix} \dot{\phi}_5$$

ANALISI CINEMATICA DI VELOCITA'

Derivo l'equazione di maglia:

$$a_5 \begin{Bmatrix} -sen\phi_5 \\ cos\phi_5 \end{Bmatrix} w_5 \dot{q} + a_9 \begin{Bmatrix} -sen\phi_9 \\ cos\phi_9 \end{Bmatrix} \dot{\phi}_9 + a_8 \begin{Bmatrix} -sen\phi_8 \\ cos\phi_8 \end{Bmatrix} \dot{\phi}_8 - a_6 \begin{Bmatrix} -sen\phi_6 \\ cos\phi_6 \end{Bmatrix} w_6 \dot{q} = \underline{0}$$



Definisco matrice Jacobiana e vettore dei termini noti:

$$J = \begin{bmatrix} -a_8 sen\phi_8 & -a_9 sen\phi_9 \\ a_8 cos\phi_8 & a_9 cos\phi_9 \end{bmatrix} \qquad A = \begin{Bmatrix} -a_5 w_5 sen\phi_5 + a_6 w_6 sen\phi_6 \\ a_5 w_5 cos\phi_5 - a_6 w_6 cos\phi_6 \end{Bmatrix}$$

Calcolo rapporti di velocità:

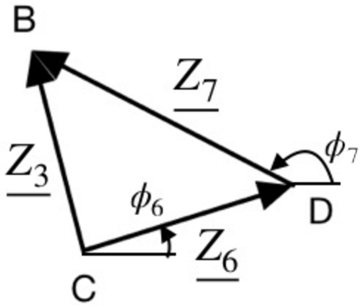
$$\begin{Bmatrix} w_8 \\ w_9 \end{Bmatrix} = \begin{Bmatrix} \dfrac{a_5 a_9 w_5 sen(\phi_5 + \phi_9) - a_6 a_9 w_6 sen(\phi_6 + \phi_9)}{a_8 a_9 sen(\phi_9 - \phi_8)} \\ \dfrac{a_5 a_8 w_5 sen(\phi_5 + \phi_8) - a_6 w_6 a_8 sen(\phi_6 + \phi_8)}{a_8 a_9 sen(\phi_9 - \phi_8)} \end{Bmatrix}$$

Grandezze ottenute:

$$\begin{Bmatrix} \dot{\phi}_8 \\ \dot{\phi}_9 \end{Bmatrix} = \begin{Bmatrix} w_8 \\ w_9 \end{Bmatrix} \dot{q}$$

$$\begin{Bmatrix} \dot{x}_F \\ \dot{y}_F \end{Bmatrix} = \begin{Bmatrix} \dot{x}_E \\ \dot{y}_E \end{Bmatrix} + a_9 \begin{Bmatrix} -sen\phi_9 \\ cos\phi_9 \end{Bmatrix} \dot{\phi}_9$$

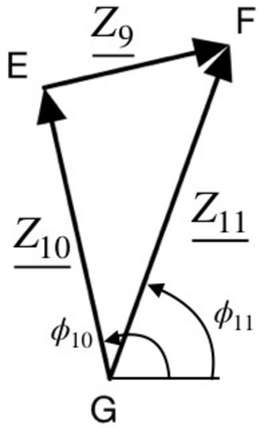
ANALISI CINEMATICA DI VELOCITA'



$$\dot{\phi}_6 = \dot{\phi}_3 \longrightarrow w_6 = w_3$$

$$\dot{\phi}_6 = w_6 \dot{q}$$

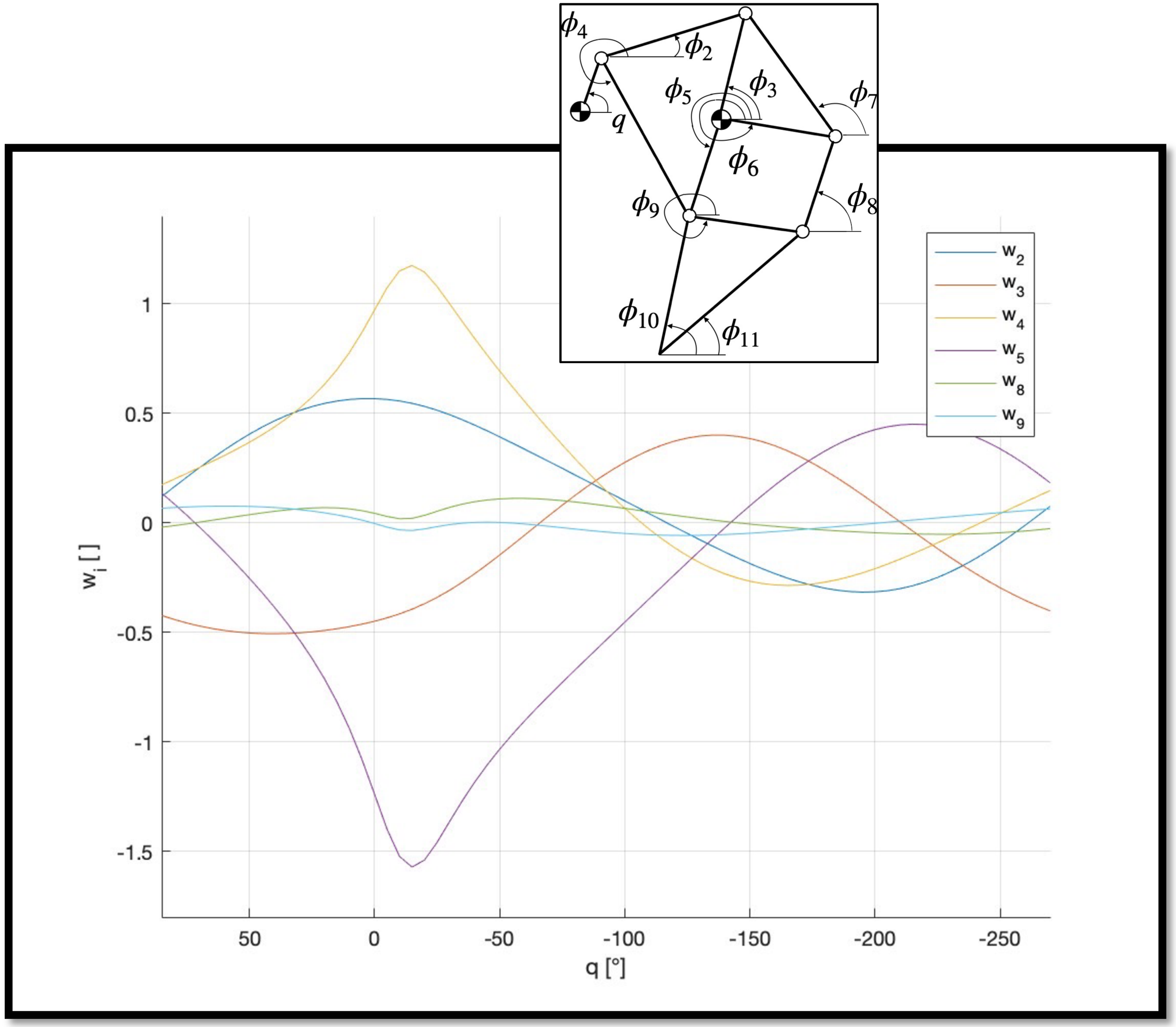
$$\begin{Bmatrix} \dot{x}_D \\ \dot{y}_D \end{Bmatrix} = a_6 \begin{Bmatrix} -sen\phi_6 \\ cos\phi_6 \end{Bmatrix} \dot{\phi}_6$$



$$\dot{\phi}_9 = \dot{\phi}_{10} \longrightarrow w_{10} = w_9$$

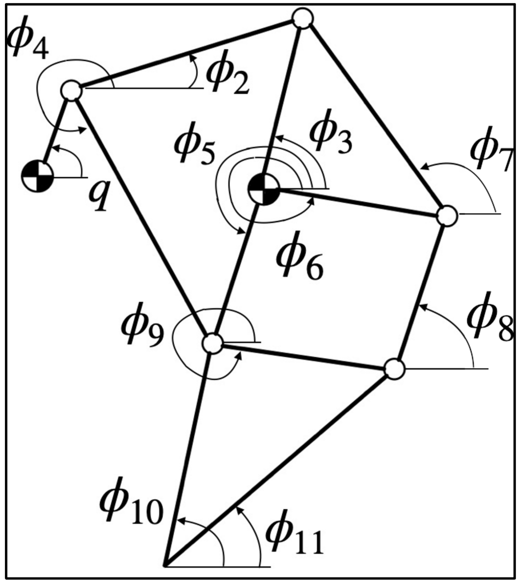
$$\dot{\phi}_{10} = w_{10} \dot{q}$$

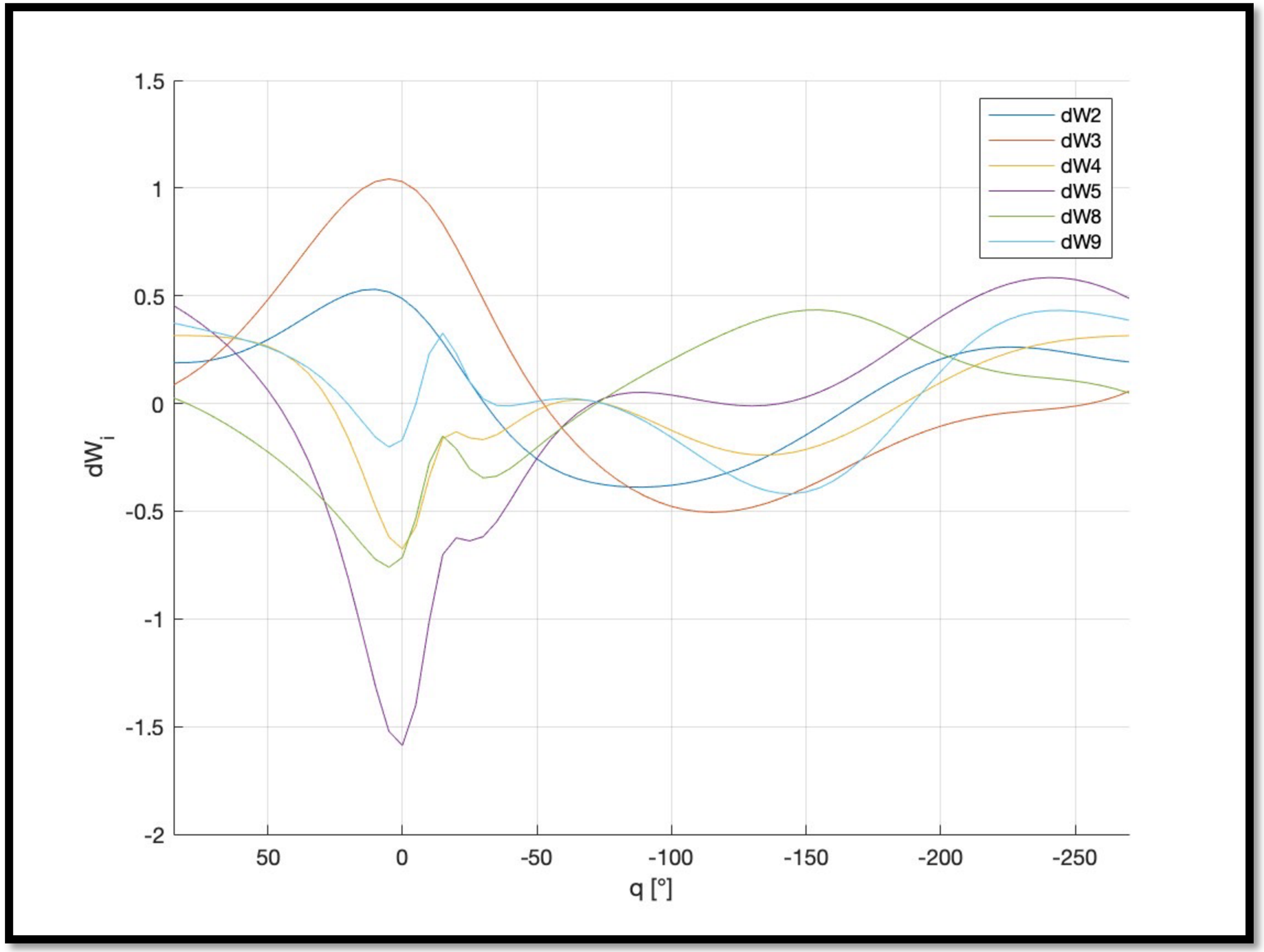
$$\begin{Bmatrix} \dot{x}_G \\ \dot{y}_G \end{Bmatrix} = \begin{Bmatrix} \dot{x}_E \\ \dot{y}_E \end{Bmatrix} - a_{10} \begin{Bmatrix} -sen\phi_{10} \\ cos\phi_{10} \end{Bmatrix} \dot{\phi}_{10}$$



Rapporti di velocità per una rotazione completa della manovella

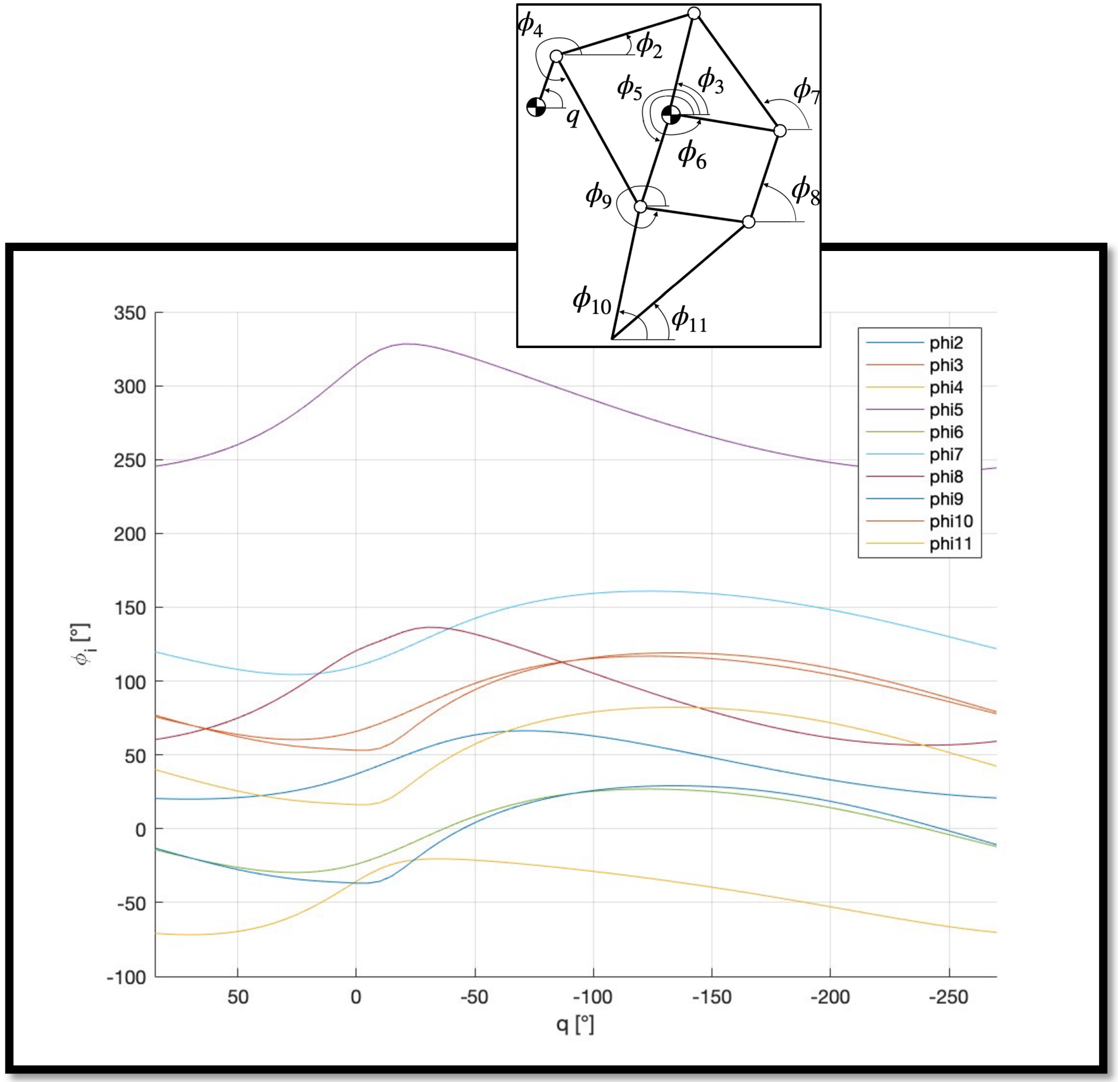
ANALISI DI ACCELERAZIONE



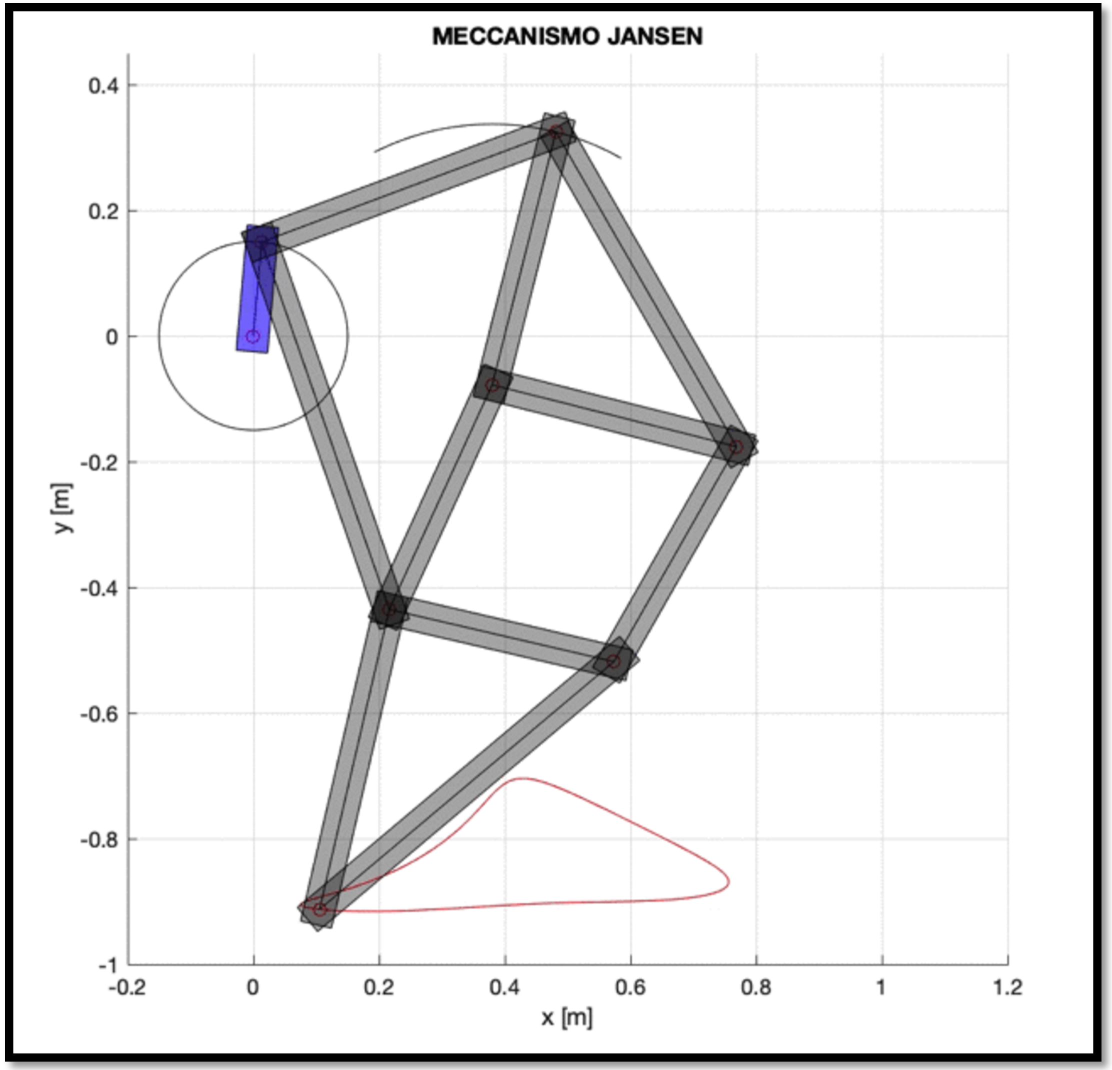


Rapporti di accelerazione per una rotazione completa della manovella

CONCLUSIONI

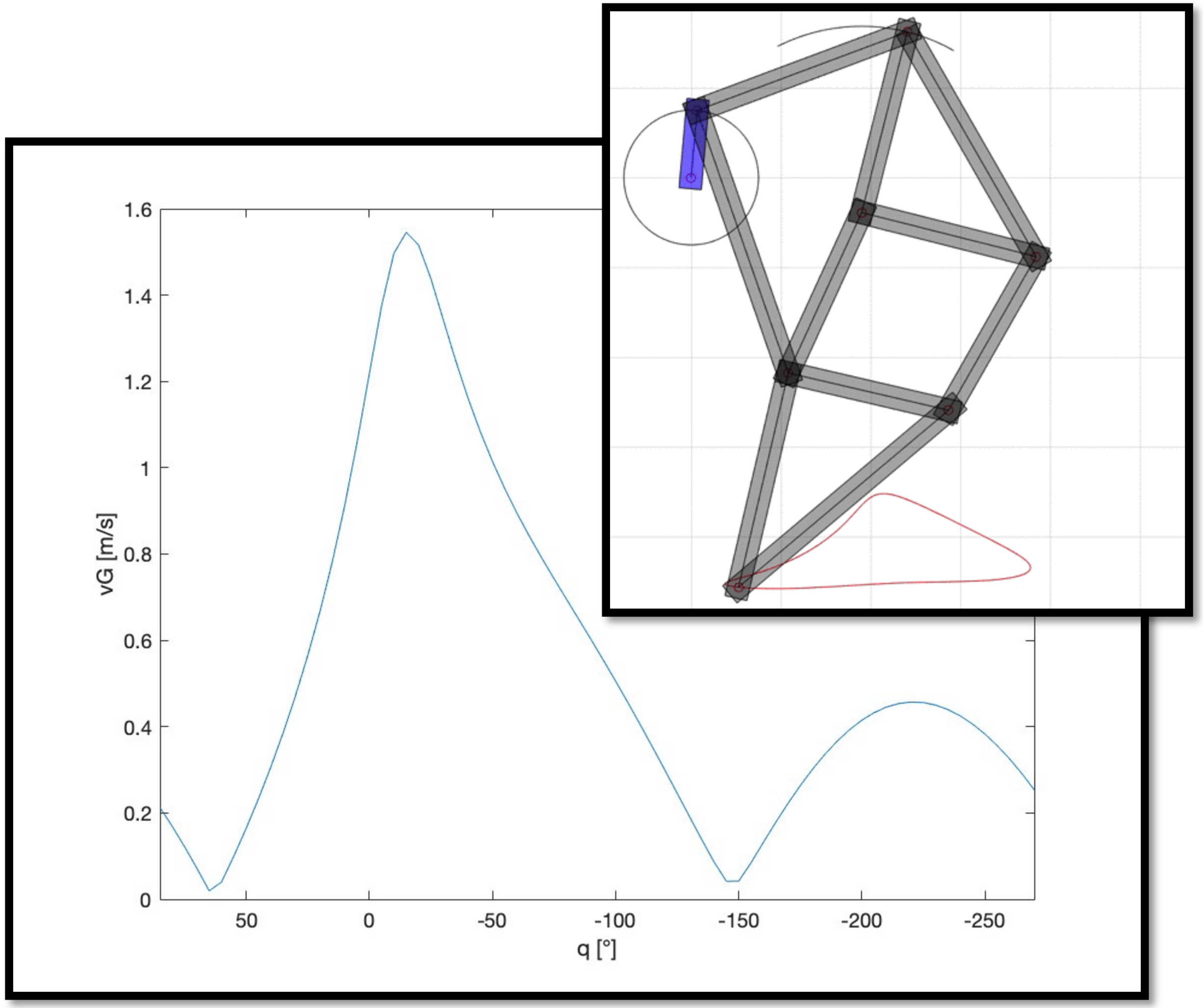


Angoli per una rotazione completa della manovella

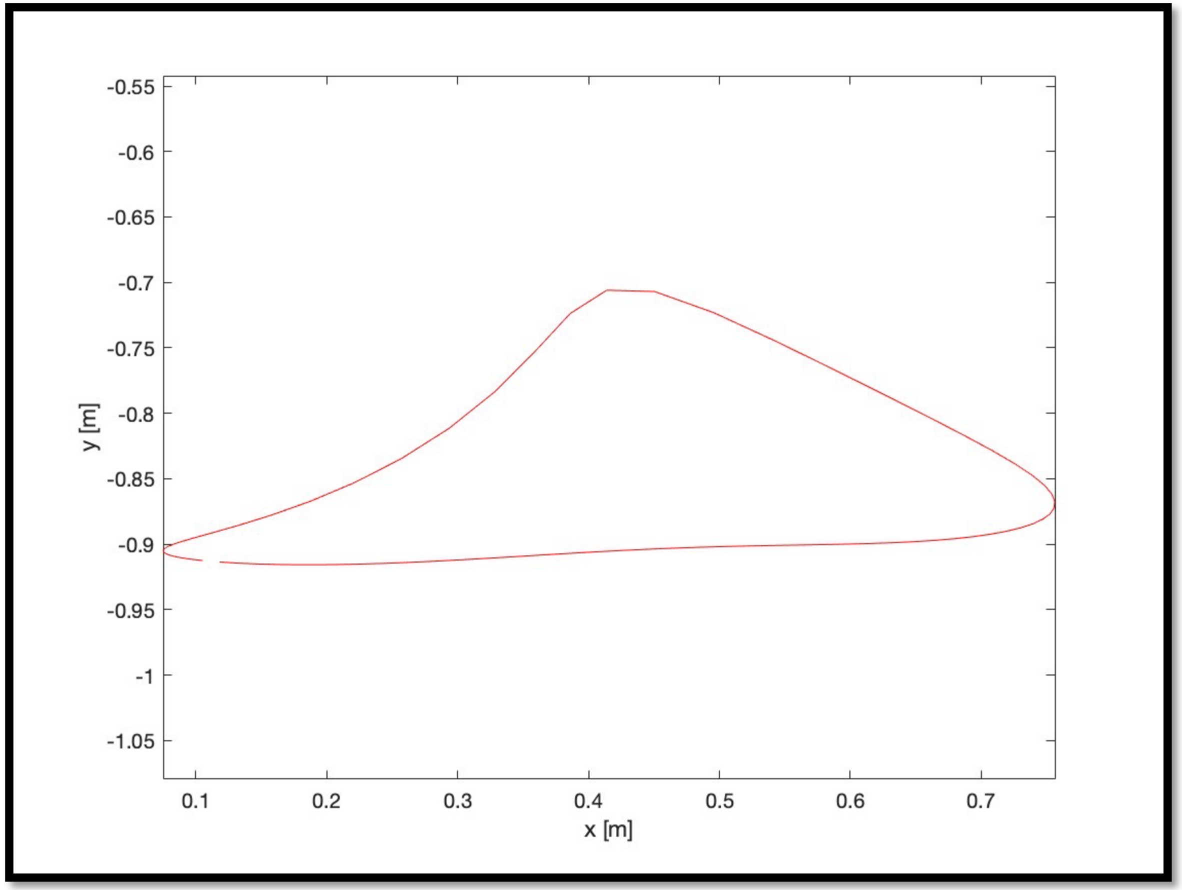


Simulazione del meccanismo

CONCLUSIONI



Velocità punto G per una rotazione completa della manovella



Traiettoria punto G