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New physics interpretations of the muon g-2 and W boson mass anomalies

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Abstract

The anomalous magnetic moment of the muon represents one of the most interesting and long-standing hint for new physics. Recently, the E989 experiment at Fermilab has confirmed previous results by the E821 experiment at BNL. Comparing the experimental average with the Standard Model prediction, leads to an interesting 4.2σ discrepancy. Moreover, the recent high-precision measurement of the W mass by the CDF collaboration is in sharp tension with the Standard Model prediction as obtained by the electroweak fit. If confirmed, this finding can only be explained in terms of new physics effects. The main goal of this thesis work is to develop compelling theoretical frameworks where both experimental results can be naturally explained, and derive predictions for other low- and high-energy observables, which will help to probe such models.

Contents

1 Introduction

The Standard Model theory (SM) is one of the greatest achievement obtained through half of the last century in particle physics. One Lagrangian describes all the dynamics, the properties and three of the four fundamental forces of nature (namely, the electromagnetism, the weak and strong interactions) applied to the *building blocks* of our universe: the elementary particles. Its success was also possible thanks to the experimental effort of many collaborations and teams around the globe that constantly put the SM up to the test: from the discovery of neutral current mediated processes to the massive carriers of the electro-weak force, from the experimental confirmation of the existence of color-charged states confined at low energies to the most recent discoveries of the top quark in 1995 at FermiLab [1,2], the tau neutrino in 2000 [3] and, finally, the responsible of the mass of all particles as well as the only scalar in the SM, namely the Higgs boson discovered for the first time in 2012 at LHC [4,5].
However, the fact that we discovered all the particles predicted by the SM sounds bittersweet: on one side we have a full self-consistent[1] quantum field theory that provides us all the computational tools to explain (almost) all elementary processes and theoretically evaluate them with great precision, on the other hand it is not a *complete* model, that is, it cannot explain everything we observe. For example, it does not include a quantum version of gravity, it does not explain why the electroweak scale $v \sim 10^2$GeV and the Plank scale $\Lambda_{\text{Pl}} \sim 10^{19}$GeV are so different, leading to the so-called *hierarchy problem*. It also lacks particle content to explain dark matter and dark energy in the ΛCDM model of cosmology.
As a consequence of increasingly precise experiments, the SM has difficulties to explain also some properties of its own particles. In this thesis we will focus our attention on two of them, namely the muon and the W-boson. This choice is dictated by the results obtained by two recent experiments: the M_W measurement by CDF-II collaboration in 2022 [31] and the $g-2$ muon magnetic moment at FNAL-E989 at FermiLab in 2021 [43]. In particular, they draw our attention because both experimental outcomes are significantly in tension with the SM prediction, confirming a long-standing discrepancy of about 4σ for the muon $g-2$, but also highlighting a surprising 8σ incompatibility for the M_W which results not only incompatible with the SM calculation, but also with the PDG world average [6].
The purpose of this thesis is to investigate these anomalies, understanding their origin and try to give them a reasonable explanation. To do so, we need to extend the SM in order to generate new possible corrections both at classical and quantum level, bearing in mind that whatever modification we generate must be punctually confronted with related experimental data, in order to discard any completion that would justify M_W and $g-2$ but would also generate new tension in other observables.
The present work is structured as follows: in Sec. 2 we introduce the M_W and $(g-2)_\mu$ anomalies from a historical viewpoint. We will briefly discuss the main results (both theoretical and experimental) obtained in the second half of the 20th century, eventually introducing the today state-of-the-art. At first, these anomalies will be studied individually in Sec. 3 and Sec. 4 respectively, within a model-independent theoretical framework, adopting effective field theory techniques such as SMEFT up to dimension-six operators. Possible UV completions of the SM that can account for both discrepancies will be outlined in Sec. 5. In particular, we will analyze three new physics scenarios: the first two assuming the existence of only one new (heavy) particle, the last one is the already known two-Higgs-doublet model, but implemented in a novel way to address both issues at the same time.
Finally, conclusions are drawn in Sec. 6.

[1] There are still long-standing mathematical issues at the foundations of the SM, e.g., its regularity once we remove regulators in correlation functions of fields or the legitimacy of using perturbation techniques in an interacting theory even if the interacting picture is not well defined. Despite this, the methods conventionally applied to extract observable estimates are self-consistent.

2 Precision test of the SM: state-of-the-art

Understanding the origin of an anomaly or, equivalently in our case, the deviation of an experimental quantity w.r.t. the theoretical expected one is as important as trying to solve the anomaly itself. Therefore, before explaining how to tackle M_W and $(g-2)_\mu$ theoretically (the main topics of our work), it is crucial to see how they arise from experimental evidences and which is the historical intercourse that brought us to discuss the results we find today in literature. This will be the very purpose of this section.

2.1 EWPO and the role of M_W

The birth of the Electro Weak (EW) sector of the Standard Model in the broken phase can be dated back to 1967 when Weinberg and Salam applied the Higgs mechanism to the unified $SU(2)_L \times U(1)_Y$ model made by Glashow [7] retrieving the physical carriers of the weak and electromagnetic interactions. The EW theory was further studied by t'Hooft and Veltman whose proved its renormalizability in 1972 [8]. In the same decade, experiments of electrons and neutrinos scattering were performed at Tevatron, Fermilab and CERN [9] aimed to investigate the neutral current sector and weak interactions mediated by massive vector bosons, but not yet able to produce them on-shell. Only in the 1980s, experiments as UA1 and UA2 [10, 11] performed using the CERN $Sp\bar{p}S$ (Super Proton Anti-Proton Synchrotron) with $\sqrt{s} \sim 540$GeV had enough energy in the centre of mass of the collision to produce W and Z boson on-shell, thus their discovery in 1983 and the experimental confirm of Glashow-Weinberg-Salam theory.
From this point onwards, several experiment were made to further invastigate the EW sector. Among them, we remember the *Large Electron-positron Collider* LEP, where in its first run from 1989 to 1995, it allowed the systematic study of Z-boson properties near its resonance thanks to the energy of the colliding beams $\sqrt{s} \sim 90$GeV; while its second run from 1995 to 2000 probed the non-abelianity of the EW gauge group by testing triple gauge couplings through on-shell production of W bosons pairs ($\sqrt{s} \in [161, 209]$GeV) [12]. We also want to cite the *Standford Linear Collider* (SLC) that started to operate in 1989 with the aim to measure properties of the EW sector depending on the polarization status of the initial states such as the left-right asymmetry $\mathcal{A}_{LR}$, by using polarized e^+e^- colliding beams. Finally, in the last decade, we remember the measurement of EW observables (among which, M_W) obtained in the *Large Hadron Collider* (LHC) using ATLAS detector in 2016 and further elaborated in 2017-2018 [16] with an energy in the proton center of mass of $\sqrt{s} \sim 7$TeV.
All the above mentioned experiments (and others) measured several quantities related to the EW sector of the SM. One may wonder if all those results: *(i)* were used to fix the SM free parameters (e.g., gauge couplings, masses of bosons) or, if not, *(ii)* to check SM predictions against experimental outcomes. To answer these questions, we must introduce the so-called electro-weak precision observables (EWPO) and the general electro-weak fit.
Let us return to the EW sector of the SM Lagrangian and its interactions with leptons. Neglecting vector boson self couplings, we know that it can be written in the form:

$$\mathcal{L}_{\text{SM}} \ni M_W^2 W_\mu^+ W_-^\mu + \frac{1}{2} M_Z^2 Z_\mu Z^\mu - \frac{g}{\sqrt{2}} \left(J_\mu^- W_+^\mu + J_\mu^+ W_-^\mu \right) - \frac{g}{c_W} J_\mu^{nc} Z^\mu \ , \tag{1}$$

with the notation used in Appendix A.2. M_W and M_Z are related by the Weinberg mixing angle θ_W which, in turn, can be expressed as a function of g and g', namely the $SU(2)$ and $U(1)$ gauge couplings. The only dimensional quantity in the SM is the quadratic coefficient μ of the scalar potential that can be expressed in terms of the Higgs vev $v = 246$GeV. Therefore, every coefficient in Eq. 1 is a function $-$ at tree level $-$ of three fundamental parameters: g, g' and v. As a consequence, every observable $\mathcal{O}$ depending purely on the EW sector can be written as a function $\mathcal{O}(g, g', v)$, thus we need to know only three observables to become predictive. Now, it is clear that in order to have precise theoretical estimates of the EW observable $\mathcal{O}$ we must be very precise in knowing the values of the parameters it depends by, thus the suggestion to trade the set (g, g', v) for another one $(\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3)$ made up by EWPO measured with very high precision. Those observables (called also *fiducial quantities* or *input*

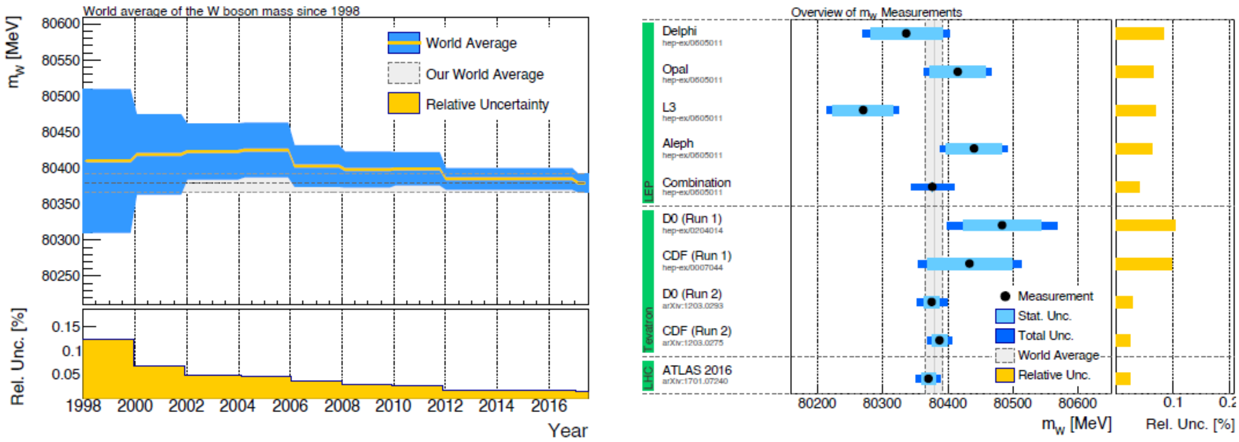


Figure 1: (Left): Evolution of the world average of M_W and its uncertainties vs. time. (Right): History of M_W measurements from different experiments. Both panels are taken from Ref. [14].

parameters) are the following:

$$\begin{aligned}
\mathcal{O}_1 = \alpha_{em}^{-1}(0) &= 137.035\ 999\ 180(10)\ \ [6]\ ; \\
\mathcal{O}_2 = G_F &= 1.1663788(6) \times 10^{-5} (\mathrm{GeV})^{-2}\ \ [6]\ ; \\
\mathcal{O}_3 = M_Z &= 91.1875(21)\mathrm{GeV}\ \ [35]\ ;
\end{aligned}$$

where α_{em} is measured through the giromagnetic factor of the electron, G_F is the Fermi constant measured by the decay of the muon and M_Z is the Z-pole mass measured in its resonance.
To see that we can indeed express other EWPOs in terms of $\mathcal{O}_{1\ldots3}$, let us take M_W as an example. From SM relation we know that $M_W = gv/2$. On the other hand, we know that at tree-level $\sqrt{2}G_F = v^{-2}$, $g = e/s_W$ and $M_Z = ev/(2s_W c_W)$ are valid relations. Thus, by rewriting g and v using the last identities, we obtain:

$$M_W(\alpha_{em}, G_F, M_Z) = \frac{M_Z}{\sqrt{2}} \sqrt{1 + \sqrt{1 - \frac{\sqrt{8}\pi\alpha_{em}}{G_F M_Z^2}}}\ . \tag{2}$$

If one replaces in Eq. 2 the values of α, G_F, M_Z given before, then it is found $M_W \approx 80.94$GeV, to be compared with the world average $M_W = (80.3545 \pm 0.0057)$GeV. This huge disagreement stands from the fact that tree-level relations are not enough to be compared with the extremely precise experimental data and therefore we need to improve the theoretical estimates by including loop effects. In particular, Eq. 2 can be corrected at loop order by introducing a parameter Δr [13] that takes into account the EW radiative corrections to the muon decay (that is, to G_F), leading to [14]:

$$M_W^2 = \frac{M_Z^2}{2} \left(1 + \sqrt{1 - \frac{\sqrt{8}\pi\alpha_{em}(1 + \Delta r)}{G_F M_Z^2}} \right)\ , \tag{3}$$

where Δr can be written as a function of SM parameters (among them, there is M_W so Eq. 3 can be solved iteratively [59]).
But if we need to introduce loop corrections in order to account for the experimental precision, then, even if we focus only on EWPO, we can no longer ignore the presence of other sectors in the SM other than the EW one, in particular the scalar and the strong ones since Higgs and quarks (especially the top) couple to the EW bosons, so they may modify significantly the theoretical estimates. As a result, other than $\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3$, we need to introduce other 4 observables measured at high precision [35]:

$$\begin{aligned}
\mathcal{O}_4 = \alpha_s(M_Z) &= 0.1177(10)\ ; \\
\mathcal{O}_5 = m_t &= 171.79(38)\mathrm{GeV}\ ; \\
\mathcal{O}_6 = M_H &= 125.21(12)\mathrm{GeV}\ ; \\
\mathcal{O}_7 = \Delta\alpha_{\mathrm{had}}^{(5)}(M_Z) &= 0.0276(10)\ ,
\end{aligned}$$

with $\alpha_s(M_Z)$ being the SU(3) strong coupling evaluated at the Z-pole, m_t and M_H the top and Higgs masses respectively and $\Delta\alpha^{(5)}_{\rm had}(M_Z)$ the five-flavour hadronic contribution (coming from non-perturbative QCD of u, d, s, c, b quarks in the photon propagator) to the running of the electromagnetic coupling α_{em}.
Given the base $\{\mathcal{O}_i\}_{i=1...7}$ of input parameters, then it is possible to perform a global EW fit, i.e., it is possible to make theoretical prediction within the SM about the values of all other EW observables at the desired loop-level precision (obviously if the task is computationally feasible) and finally compare the prediction with the experimental result [15]. As far as M_W is concerned, Fig. 1 reports a short history of the experimental outcomes obtained in the last decades. To this chronology, one should also add the recent CDF-II $M_W^{\rm CDF}$ measurement [31] which results incompatible both with the world average in Fig. 1 and with the SM prediction through EW global fit [27,32,35]. The only explanation to this discrepancy (assuming that $M_W^{\rm CDF}$ will be confirmed) is that the SM cannot account for all the corrections to the W propagator, thus the need for a new treatment to solve this issue as we are going to see in the following sections.

2.2 The muon $g-2$

In quantum mechanics (QM), the total angular momentum of a particle is given by $\vec{J} = \vec{L} + \vec{S}$ [2] with $\vec{L}$ the orbital angular momentum and $\vec{S} = (\hbar/2)\vec{\sigma}$ is the spin operator for spin 1/2 particles (with $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ where σ_i are the Pauli matrices). Analogously to the the magnetic dipole moment generated by a charged particle with orbital momentum $\vec{L}$, one can write the magnetic moment $\vec{\mu}_S$ associated to its *intrinsic* angular momentum, that is, the spin:

$$\vec{\mu}_S = g\frac{e\hbar}{2m}\vec{S} = g\mu_B\frac{\vec{\sigma}}{2} \ ,$$

where $\mu_B \approx 8.39 \times 10^{-8}\ ({\rm eV})^{-1}$ is known as the *Bohr magneton* and g is the gyromagnetic factor of the particle.
From a purely QM viewpoint, a charged lepton is described by a doublet $|\varphi\rangle$[3] and, once interacting with an external magnetic field $\vec{B}$, it obeys at the following Schrödinger-like equation:

$$i\dot{\varphi} = \left[\frac{\vec{p}}{2m} + V(x) + \frac{e}{2m}\vec{B}\cdot(\vec{L} + g\vec{S})\right]\varphi \ , \tag{4}$$

which, with minor modifications, is also known as *Pauli equation* written in 1927 [17]. The g-factor appearing in Eq. 4 was a free parameter until Dirac presented his relativistic equation for the electron in 1928 (and later in 1932) [18] that fixed $g = 2$. This conclusion was afterward confirmed by experiment aimed to analyze the fine structure of atomic emission spectra and the Zeeman effect.
The first hint for possible deviations from $g = 2$ dates back to 1948, when Kush and Foley [19] found the value of $g = 2.00238(10)$ for the electron while studying the hyperfine structure of atomic spectra. This tension with the Dirac prediction was solved in the same year thanks to the famous Schwinger, Feynman and Tomonaga 1-loop computation [20] in a quantum field theory (QFT) context (namely, the QED), leading to the result:

$$g_e = 2\left(1 + \frac{\alpha_{em}}{2\pi}\right) \simeq 2(1 + 0.00116) \ . \tag{5}$$

Other than explaining the anomaly, this calculation was fundamental for other two reasons: *(i)* it proved the success of QED as QFT in predicting observables and *(ii)* loop effects (i.e., pure quantum effects) have measurable physical consequences.

[2] We remember that in QM observables are operators acting on an Hilbert space $\mathcal{H}$. In particular, the angular momentum $\vec{L}$ acts on the space of equivalence classes of functions modulo square integrable $L_2(\mathbb{R}^3, d^3x)$ while $\vec{S}$ on a disjoint Hilbert space $\mathcal{H}_s \simeq \mathbb{C}^{2s+1}$, thus the proper writing of $\vec{J}$ acting on $\mathcal{H}_J \simeq L_2(\mathbb{R}^3, d^3x) \otimes \mathcal{H}_S$ should be $\vec{J} = \vec{L} \otimes \mathbb{I}_s + \mathbb{I}_L \otimes \vec{S}$. For simplicity, we will use the notation as in the main text.

[3] In QM there is no antimatter counterpart since we are working in the non-relativistic limit. Thus, from what will be the fermionic field $\psi(x) = (\varphi, \chi)^T(x)$ in quantum field theory, we must select only the matter part and write it as an element of a suitable Hilbert space $\mathcal{H}$.

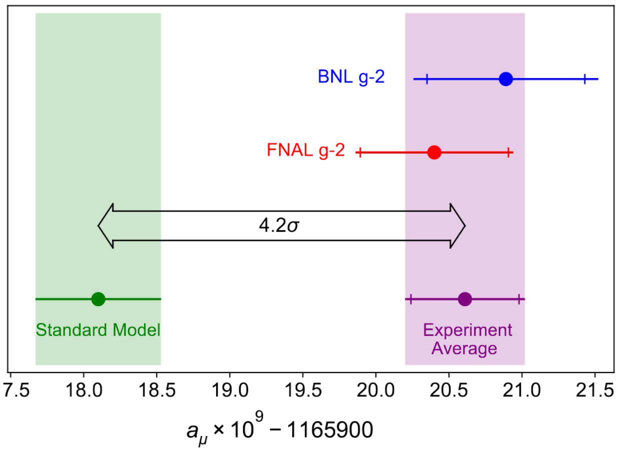


Figure 2: Experimental values of $a_\mu \equiv (g_\mu - 2)/2$ from BNL E821, FNAL-E989, and the combined average of the experiments. The inner tick marks indicate the statistical contribution to the total uncertainties. With a green band it is reported the SM prediction $\pm 1\sigma$ by Aoyama *et al.* [42]. Figure taken from Ref. [43].

For the muon g-factor, we need to wait until end 1950s – early 1960s, in particular after the discovery of parity violation in 1957, when it was understood how to use decaying pions at rest to generate polarized muons through the decay $\pi^\pm \to \mu^\pm + \nu_\mu(\bar{\nu}_\mu)$. In fact, the process conserves the total angular momentum and, since neutrinos (resp. antineutrinos) are always left (resp. right) handed, muons have only one way to arrange their helicity in order to have null total angular momentum in the pion rest frame. By this means, an experiment made by Garwin *et al.* permitted to measure the $g-2$ of the muon, finding $g_\mu = 2$ with a 10% precision [21]. This measurement was further improved in the same year by Cassels *et al.* finding $g_\mu = (2.004 \pm 0.014)$ [22] and again in 1960 by Garwin leading to [23]:

$$g_\mu = 2\left(1.00113^{+0.00016}_{-0.00012}\right) \ ,$$

that can be explained thanks to the Schwinger correction (notice that Eq. 5 does not depend on any masses or flavour-specific coefficients, hence it is valid for all leptons).
Several other experiments were performed since then (for an overview, see Ref. [25]) every time improving their resolution, hence the necessity to include in the $(g-2)_\mu$ theoretical prediction even more loop contributions from the EW, scalar, quark sectors of the SM (for a review, see Refs. [40, 42]; we will return on this in Sec.4.1). The most recent measurements on this topic have achieved a resolution of 0.54 part per million (ppm) as relative error in the BNL-E821 experiment at Brookhaven and 0.46 ppm at FermiLab-E989 [43] and it may be further reduced in new experiments as E34 at J-PARC [24]. This great precision calls for adequate theoretical estimates of the $(g-2)_\mu$: as it can be seen in Fig. 2 (and as we will see in this work), the SM cannot reproduce the E821 and E989 experimental results, showing a tension of 4.2σ, thus the need for SM extensions in which this discrepancy can be accounted for.

3 M_W within an EFT framework

First, we will focus on the M_W anomaly. Assuming a New Physics scale much above the electro-weak scale, we can employ the model-independent approach of Effective Field Therories (EFTs). In general, in order to define a EFT, one needs to:

(i) Define the dynamical fields at the given energy scale. They will be the particle content in our theory and the ones that will appear in all the effective operators.

(ii) Define the global and/or local symmetry group and how the fields transform under it. This step will influence the form taken by the operators (e.g., if we do not impose Lorentz invariance, we can have effective operators with not all Lorentz indexes contracted among themselves).

(iii) Define the possible symmetry breaking pattern of our EFT at the energy scale below Λ (or *cut-off* scale) at which we suppose some new physics may happen.

(iv) Define the expansion parameter related to the cut-off Λ. If we conduct experiments or make computations for energies $\mathcal{E} \ll \Lambda$, then we can re-express our results with a series expansion around $\varepsilon \equiv \mathcal{E}/\Lambda \to 0$.

To see point *(iv)* more concretely, one can use the path integral formalism. Let us suppose that an UV theory is described by a Lagrangian $\mathcal{L}(\phi, \Phi, g_{\rm NP})$, with ϕ being light fields ($M_\phi \lesssim \mathcal{E}$), Φ heavy fields ($M_\Phi \gg \mathcal{E}$) and $g_{\rm NP}$ some generic new physics coupling. Then the generating functional of such a theory would be:

$$Z_{\rm UV}[J,K] = \int_{\mathcal{E}=0}^{+\infty} D\phi D\Phi \exp\left(iS_{\rm UV}(\phi,\Phi,g_{\rm NP}) + i\left(\int d^4x(J\phi + K\Phi)\right)\right) \;, \tag{6}$$

where J, K generic external sources and $S_{\rm UV} = \int d^4x \mathcal{L}(\phi, \Phi, g_{\rm NP})$. Now, according to *(iv)* if we work at $\mathcal{E} \ll \Lambda \sim M_\Phi$ then we cannot generate Φ as final states, and they would appear only as internal propagators. However, no external legs for Φ implies no possible coupling with the external source K. Thus, in the infrared (IR) limit, one has:

$$\begin{aligned} Z_{\rm IR}[J] = Z_{\rm UV}[J,0] \;&=\; \int_{\mathcal{E}\lesssim\Lambda} D\phi\left(\int_{\mathcal{E}\sim\Lambda}^{+\infty} D\Phi \exp\left(iS_{\rm UV}(\phi,\Phi,g_{\rm NP})\right)\exp\left(i\int d^4xJ\phi\right)\right) \\ &\overset{!}{=}\; \int_{\mathcal{E}\lesssim\Lambda} D\phi \exp\left(iS_{\rm IR}(\phi, g_{\rm eff}) + i\left(\int d^4xJ\phi\right)\right) \;, \end{aligned} \tag{7}$$

where now $g_{\rm eff}$ are effective couplings that are related to $g_{\rm UV}$ through the integration of the heavy fields. Now, the problem is to find a $\mathcal{L}_{\rm IR}(\phi, g_{\rm eff})$ such that gives $S_{\rm IR}$ in Eq. 7. In general, this is a very hard task, but exploiting the fact that $\varepsilon \ll 1$ one can express $S_{\rm IR}$ trough a (potentially) infinite series of operators weighted by powers of $1/\Lambda$. Notice that the matching done in Eq. 7, assuming that $\mathcal{L}_{\rm IR}$ is perturbative, is consistent only if the UV theory holds perturbativity in the IR limit (e.g., if one wants an EFT for QCD for energies below the GeV, then a simple expansion around ε is not sufficient).
We will focus on a particular EFT: the so-called Standard Model Effective Field Theory (or SMEFT). Sec.3.1 will be dedicated to the introduction of the SMEFT as well as its impact on the definition of SM input parameters. Finally, in Sec.3.2 we provide the corrections to SM predictions of physical observables induced by the SMEFT (with special attention to M_W).

3.1 SMEFT general overview and results

The SMEFT is an effective field theory characterized by: *(i)* the same particle content of the SM; *(ii)* the same $SU(3)_C \times SU(2)_L \times U(1)_Y$ internal symmetry group and invariance under the Poincaré group. All the fields transforms as in the SM under those symmetries. Our expansion parameter will be always written in the form $\varepsilon = v^2/\Lambda^2$, where Λ is the cut-off scale, when our effective treatment would break down. Through our work we will assume $\Lambda \sim O(10\text{TeV})$ since no new particles have been

observed at LHC so far.
Following Ref. [26], let us write the effective SMEFT Lagrangian up to dimension 6 operators[4]:

$$\mathcal{L} = \mathcal{L}_{SM} + \sum_i \frac{\alpha_i}{\Lambda^2}\mathcal{O}_i \ , \tag{8}$$

where α_i (sometimes we will call them c_i) are dimensionless coefficients called *Wilson coefficients* related to the effective operator $\mathcal{O}_i$.

3.1.1 EW gauge sector

As a first example, let us consider the electro-weak (EW) gauge bosons that will be relevant in the following sections. In the SM, the kinetic terms and masses of gauge bosons arise, after spontaneous symmetry breaking (SSB), from the following Lagrangian:

$$\mathcal{L}_{SM} \ni -\frac{1}{4}W^I_{\mu\nu}W^{\mu\nu}_I - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + (D_\mu\phi)^\dagger(D^\mu\phi) \ , \tag{9}$$

where ϕ is the Higgs SU(2) doublet. To compare our findings with the ones reported in [26], in this subsection we adopt the same conventions, namely the Higgs vev $v \approx 174$GeV, the covariant derivative reads $D_\mu = \partial_\mu \mathbb{I} - ig_s\frac{\lambda_A}{2}G^A_\mu - ig\frac{\tau_I}{2}W^I_\mu - ig'YB_\mu$ (where λ_A and τ_I are the Gell-Mann and Pauli matrices respectively), the field strength tensor can be written as $X^a_{\mu\nu} = \partial_{[\mu}X^a_{\nu]} + g_\star f^{abc}X^b_\mu X^c_\nu$ where $X \in \{B, W, G\}$, a, b, c are suitable group indexes, f^{abc} and $g_\star$ the structure constants and the coupling respectively of the gauge symmetry group related to the specified gauge bosons.
Now, in the spirit of Eq. 8, we add dim-6 operators to Eq. 9. To do so, we first need to specify an operator base. In literature several bases have been proposed, adequately selected to simplify the study at hand (e.g., see the redundant Buchmüller and Wyler basis of Ref. [26], the Warsaw basis of Misiak *et al.* in Ref. [34] or the Strongly-Interacting Light Higgs basis of Ref. [47]): they are related by different ways to remove redundant operators by using the leading order Equations of Motions (EOMs) and integration by parts. For convenience, in this subsection the Buchmüller basis is adopted, while in the rest of the work the Warsaw one is preferred. Again, eventual changes in notation will be made explicit.
The dimension-six operators that modify Eq. 9 are the following:

$$\begin{array}{lll} \mathcal{O}_{\phi W} = \frac{1}{2}(\phi^\dagger\phi)W^I_{\mu\nu}W^{\mu\nu}_I & ; \quad \mathcal{O}_{\phi B} = \frac{1}{2}(\phi^\dagger\phi)B_{\mu\nu}B^{\mu\nu} & ; \quad \mathcal{O}_{WB} = (\phi^\dagger\tau^I\phi)W^I_{\mu\nu}B^{\mu\nu} \ ; \\ \mathcal{O}_{\phi,1} = (\phi^\dagger\phi)(D_\mu\phi^\dagger D^\mu\phi) & ; \quad \mathcal{O}_{\phi,3} = (\phi^\dagger D^\mu\phi)(D_\mu\phi^\dagger\phi) \ . & \end{array} \tag{10}$$

If we are interested in the mass modifications of the EW gauge bosons but not in the Higgs-Boson-Boson vertices, effectively the operators in Eq. 10 should be evaluated on the Higgs vev, i.e. $\phi \to \langle\phi\rangle = v(0,1)^T$. Remembering that:

$$D_\mu \langle\phi\rangle = \frac{-iv}{2}\begin{pmatrix} g\left(W^1_\mu - iW^2_\mu\right) \\ g'B_\mu - gW^3_\mu \end{pmatrix} \ ,$$

one finally obtains:

$$\begin{aligned} \mathcal{L}_{tot} \ = \ & \mathcal{L}_{SM} + \mathcal{L}_{eff} = -\frac{1}{4}W^I_{\mu\nu}W^{\mu\nu}_I - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \frac{v^2}{4}g^2W^I_\mu W^\mu_I + \frac{v^2g'^2}{4}B_\mu B^\mu - \frac{v^2gg'}{2}W^\mu_3 B_\mu \\ & + \frac{1}{\Lambda^2}\left[\frac{\alpha_{\phi W}v^2}{2}W^I_{\mu\nu}W^{\mu\nu}_I + \frac{\alpha_{\phi B}v^2}{2}B_{\mu\nu}B^{\mu\nu} - \alpha_{WB}v^2W^3_{\mu\nu}B^{\mu\nu} + \frac{\alpha_{\phi,1}v^4g^2}{4}W^I_\mu W^\mu_I + \frac{\alpha_{\phi,1}v^4g'^2}{4}B_\mu B^\mu \right. \\ & \left. - \frac{\alpha_{\phi,1}v^4gg'}{2}B_\mu W^\mu_3 + \frac{\alpha_{\phi,3}v^4}{4}\left(g'^2B_{mu}B^\mu + g^2W^3_\mu W^\mu_3 - 2gg'B_\mu W^\mu_3\right)\right] \ , \end{aligned}$$

[4]Notice that we did not include the dimension-five Weinberg operator since it will not be relevant for our purposes.

which can be rewritten as:

$$\begin{aligned}\mathcal{L}_{tot} &= -\frac{1}{4}\left(1-2\alpha_{\phi W}\frac{v^2}{\Lambda^2}\right)W^I_{\mu\nu}W^{\mu\nu}_I-\frac{1}{4}\left(1-2\alpha_{\phi B}\frac{v^2}{\Lambda^2}\right)B_{\mu\nu}B^{\mu\nu}-\alpha_{WB}\frac{v^2}{\Lambda^2}W^3_{\mu\nu}B^{\mu\nu}\\ &+\frac{1}{4}v^2\left(1+\alpha_{\phi,1}\frac{v^2}{\Lambda^2}\right)\left(g^2W^I_\mu W^\mu_I+g'^2B_\mu B^\mu-2gg'B_\mu W^\mu_3\right)+\frac{1}{4}\alpha_{\phi,3}\frac{v^4}{\Lambda^2}\left(g'B_\mu-gW^3_\mu\right)^2 \ .\end{aligned} \tag{11}$$

From Eq. 11 we notice the presence of *mass mixing* $W^3_\mu B^\mu$ and *current mixing* $W^3_{\mu\nu}B^{\mu\nu}$. As in the SM, B_μ and W^3_μ are not mass eigenstates, therefore we have to find the physical fields Z_μ and A_μ which cancel out the mixing terms. Before doing that, however, we also notice that the kinetic terms are not canonically normalized: we must perform a field redefinition in order to get back to the canonical form $-\frac{1}{4}X_{\mu\nu}X^{\mu\nu}$. The correct choice, up to $O(\Lambda^{-4})$ contributions is:

$$W^I_\mu\to\left(1-2\alpha_{\phi W}\frac{v^2}{\Lambda^2}\right)^{-1/2}W^I_\mu\ ;\quad g\to\left(1-2\alpha_{\phi W}\frac{v^2}{\Lambda^2}\right)^{1/2}g\ ;\ W^I_{\mu\nu}\to\left(1-2\alpha_{\phi W}\frac{v^2}{\Lambda^2}\right)^{-1/2}W^I_{\mu\nu} \tag{12}$$

$$B_\mu\to\left(1-2\alpha_{\phi B}\frac{v^2}{\Lambda^2}\right)^{-1/2}B_\mu\ ;\ g'\to\left(1-2\alpha_{\phi B}\frac{v^2}{\Lambda^2}\right)^{1/2}g'\ ;\ B_{\mu\nu}\to\left(1-2\alpha_{\phi B}\frac{v^2}{\Lambda^2}\right)^{-1/2}B_{\mu\nu}\ . \tag{13}$$

Notice that one needs to normalize also the coupling constants since we want the field strength tensors $X_{\mu\nu}$ to rescale as the fields X_μ. From now on, in order to distinguish renormalized fields and couplings from unrenormalized ones, we will denote the latter with the subscript *SM* [5].
Once one performs the shifts in Eqs. 12-13 and keeps terms linear in the Wilson coefficients, one gets:

$$\begin{aligned}\mathcal{L}_{tot} &= -\frac{1}{4}W^I_{\mu\nu}W^{\mu\nu}_I-\frac{1}{4}B_{\mu\nu}B^{\mu\nu}-\alpha_{WB}\frac{v^2}{\Lambda^2}W^3_{\mu\nu}B^{\mu\nu}+\frac{1}{2}v^2g^2_{SM}\left(1+(\alpha_{\phi,1}+2\alpha_{\phi W})\frac{v^2}{\Lambda^2}\right)W^+_\mu W^\mu_-\\ &+\frac{1}{4}v^2g^2_{SM}\left(1+(\alpha_{\phi,1}+\alpha_{\phi,3}+2\alpha_{\phi W})\frac{v^2}{\Lambda^2}\right)W^3_\mu W^\mu_3+\frac{1}{4}v^2g'^2_{SM}\left(1+(\alpha_{\phi,1}+\alpha_{\phi,3}+2\alpha_{\phi B})\frac{v^2}{\Lambda^2}\right)B_\mu B^\mu\\ &-\frac{1}{2}v^2g_{SM}g'_{SM}\left(1+(\alpha_{\phi,1}+\alpha_{\phi,3}+\alpha_{\phi W}+\alpha_{\phi B})\frac{v^2}{\Lambda^2}\right)B_\mu W^\mu_3\ ,\end{aligned} \tag{14}$$

where $W^\pm_\mu=\frac{1}{\sqrt{2}}\left(W^1_\mu\mp iW^2_\mu\right)$ and the field redefinition of $W^\pm_\mu$ is the same of W^I_μ because of linearity.
From Eq. 14 it is easy to read the new mass of the charged vector bosons:

$$\frac{1}{2}v^2g^2_{SM}\left(1+(\alpha_{\phi,1}+2\alpha_{\phi W})\frac{v^2}{\Lambda^2}\right)W^+_\mu W^\mu_-\stackrel{!}{=}M^2_WW^+_\mu W^\mu_-\to M^2_W=M^2_{W,SM}\left(1+(\alpha_{\phi,1}+2\alpha_{\phi W})\frac{v^2}{\Lambda^2}\right)\ , \tag{15}$$

where $M^2_{W,SM}=(g^2_{SM}v^2)/2$ is the usual SM mass of the charged vector bosons.
As anticipated, we cannot read so easily the masses of photon and Z because of the mixing terms. In order to pass to the physical base, one should perform a diagonalization procedure similar to the one explained in Ref. [28] (see also Appendix C.2 for our general approach). Here, we report only the final results, but the full diagonalization procedure is explained in some detail in Appendix A.1. Keeping only linear terms in the Wilson coefficients, the mass eigenstates can be written as:

$$\begin{aligned}M_\gamma &= 0\ ;\\ M_Z &= M_{Z,SM}\left(1+\left(\frac{1}{2}\alpha_{\phi,1}+\frac{1}{2}\alpha_{\phi,3}+c^2_w\alpha_{\phi W}+s^2_w\alpha_{\phi B}+2c_ws_w\alpha_{WB}\right)\frac{v^2}{\Lambda^2}\right)\ ,\end{aligned} \tag{16}$$

[5] In literature one can find different conventions, for example SMEFT fields and couplings are sometimes written with a bar whereas SM ones without it. See Ref. [30] as example.

where $M^2_{Z,SM} = \frac{v^2}{2}(g^2_{SM} + g'^2_{SM})$ is the pure SM Z-boson mass; $c_w(s_w)$ is the cosine (sine) of the SM Weinberg angle. Notice that the photon stay massless as it should be. Finally, B, W^3 and Z, A are related through the following relations[6]:

$$W^3_\mu = s_w\left(1+\alpha_{AA}\frac{v^2}{\Lambda^2}\right)A_\mu + \left(c_w\left(1+\alpha_{ZZ}\frac{v^2}{\Lambda^2}\right) - s_w\alpha_{AZ}\frac{v^2}{\Lambda^2}\right)Z_\mu \ ; \tag{17}$$

$$B_\mu = c_w\left(1+\alpha_{AA}\frac{v^2}{\Lambda^2}\right)A_\mu - \left(s_w\left(1+\alpha_{ZZ}\frac{v^2}{\Lambda^2}\right) + c_w\alpha_{AZ}\frac{v^2}{\Lambda^2}\right)Z_\mu \ ; \tag{18}$$

with:

$$\begin{aligned}
\alpha_{AA} &= s^2_w\alpha_{\phi W} + c^2_w\alpha_{\phi B} - 2s_w c_w\alpha_{WB} \ ; \\
\alpha_{ZZ} &= c^2_w\alpha_{\phi W} + s^2_w\alpha_{\phi B} + 2s_w c_w\alpha_{WB} \ ; \\
\alpha_{AZ} &= 2\left(s_w c_w(\alpha_{\phi B} - \alpha_{\phi W}) + (c^2_w - s^2_w)\alpha_{WB}\right) \ .
\end{aligned}$$

3.1.2 G_F constant

Another quantity which is worth analysing is the Fermi constant $G_F = \frac{1}{4\sqrt{2}}\frac{g^2_{SM}}{M^2_{W,SM}}$. In our effective treatment, G_F receives corrections in 3 different ways: *(i)* g_{SM} is modified due to the new coupling between the W boson and fermionic currents (the complete analysis is reported in Appendix A.2), *(ii)* M_W changes as described in Eq. 15 and *(iii)* there are new 4-fermion operators which directly contribute to processes were G_F is involved. In particular, the following four-fermion operator should be considered:

$$\mathcal{O}_{\ell\ell 3} = \frac{1}{2}\left(\bar{\ell_L}\gamma_\mu\tau^I\ell_L\right)\left(\bar{\ell_L}\gamma^\mu\tau_I\ell_L\right) = 2\left(\bar{\nu_L}\gamma_\mu e_L\right)\left(\bar{e_L}\gamma^\mu\nu_L\right) + \text{neutral current} \ , \tag{19}$$

where we omitted neutral currents of the kind $(\bar{\psi_L}\gamma_\mu\psi_L)(\bar{\chi_L}\gamma^\mu\chi_L)$ since they are not relevant for G_F. The SM tree-level Feynman amplitude $\mathcal{M}$ for the decay $\mu^- \to e^-\bar{\nu_e}\nu_\mu$ reads:

$$\mathcal{M}_{SM} = \frac{4G_{F,SM}}{\sqrt{2}}\left(\bar{\nu_{\mu,L}}\gamma_\mu\mu_L\right)\left(\bar{e_L}\gamma^\mu\nu_e\right) \ . \tag{20}$$

In order to obtain the SMEFT expression, the redefinitions that one should apply are:

$$M_{W,SM} \to M_{W,SM}\left(1+\left(\alpha_{\phi W} + \frac{1}{2}\alpha_{\phi 1}\right)\frac{v^2}{\Lambda^2}\right) \ ; \qquad (\bar{\nu_L}\gamma_\mu e_L) \to (\bar{\nu_L}\gamma_\mu e_L)\eta(\nu_L) \ ,$$

and add also the contribution from Eq. 19. Finally one gets:

$$\mathcal{M} = \frac{g^2_{SM}}{2M^2_{W,SM}}\left[\frac{\eta(\nu_L)^2}{1+(\alpha_{\phi 1}+2\alpha_{\phi W})\frac{v^2}{\Lambda^2}} + \frac{v^2}{\Lambda^2}2\alpha_{\ell\ell 3}\right]\left(\bar{\nu_{\mu,L}}\gamma_\mu\mu_L\right)\left(\bar{e_L}\gamma^\mu\nu_e\right) \stackrel{!}{=} \frac{4G_F}{\sqrt{2}}\left(\bar{\nu_{\mu,L}}\gamma_\mu\mu_L\right)\left(\bar{e_L}\gamma^\mu\nu_e\right) \ . \tag{21}$$

Notice that the contribution arising from $\mathcal{O}_{\ell\ell 3}$ is multiplied by $G_{F,SM}$ since this is already a $O(\Lambda^{-2})$ term. By linearizing in the Wilson coefficients α_i, the last equality of Eq. 21 leads us to the corrected definition of G_F:

$$G_F = \frac{g^2_{SM}}{4\sqrt{2}M^2_{W,SM}}\left(1+\frac{v^2}{\Lambda^2}(4\alpha_{\phi\ell 3} - \alpha_{\phi 1} + 2\alpha_{\ell\ell 3})\right) = G_{F,SM}\left(1+\frac{v^2}{\Lambda^2}(4\alpha_{\phi\ell 3} - \alpha_{\phi 1} + 2\alpha_{\ell\ell 3})\right) \ , \tag{22}$$

where we replaced $\eta(\nu_L) = \left(1+(\alpha_{\phi W}+2\alpha_{\phi\ell 3})\frac{v^2}{\Lambda^2}\right)$ as found at the end of Appendix A.2.

[6] To check that they hold, replace them in Eq. 14 and see that there are no mixed terms $A_\mu Z^\mu$ and no terms A^2_μ while the coefficient in front of Z^2_μ is $M^2_Z/2$ from Eq. 16.

3.1.3 The dipole Lagrangian $\mathcal{L}_M$

Finally, we would like to conclude with another important result that, although not directly related to M_W, it is connected to modifications to anomalous magnetic moments that will be our main topic for the whole Sec.4.
Anomalies in the magnetic couplings are possible within the SM theory thanks to loop contributions (see e.g. the $g-2$ factor of the electron which receives corrections in the QED 1-loop vertex). However, in the effective theory, new dimension-six operators (the so called *dipole operators*) may affect the magnetic couplings already at tree level. The operators we are interested in are the following (h.c. are understood):

$$\begin{array}{lll}
\mathcal{O}_{\ell W} = \left(\bar{\ell}_L \sigma^{\mu\nu}\tau_I e_R\right)\phi W^I_{\mu\nu}\ ; & \mathcal{O}_{\ell B} = \left(\bar{\ell}_L \sigma^{\mu\nu} e_R\right)\phi B_{\mu\nu}\ ; & \mathcal{O}_{uG} = \left(\bar{q_L}\sigma^{\mu\nu}\lambda_A u_R\right)\tilde{\phi} G^A_{\mu\nu} \\
\mathcal{O}_{uW} = \left(\bar{q_L}\sigma^{\mu\nu}\tau_I u_R\right)\tilde{\phi} W^I_{\mu\nu}\ ; & \mathcal{O}_{uB} = \left(\bar{q_L}\sigma^{\mu\nu} u_R\right)\tilde{\phi} B_{\mu\nu}\ ; & \mathcal{O}_{dG} = \left(\bar{q_L}\sigma^{\mu\nu}\lambda_A d_R\right)\phi G^A_{\mu\nu} \\
\mathcal{O}_{dW} = \left(\bar{q_L}\sigma^{\mu\nu}\tau_I d_R\right)\phi W^I_{\mu\nu}\ ; & \mathcal{O}_{dB} = \left(\bar{q_L}\sigma^{\mu\nu} d_R\right)\phi B_{\mu\nu}\ , &
\end{array}$$

where $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$ is completely antisymmetric in its indexes (one should remember this in order to simplify a lot the calculations).
In order to write the dipole Lagrangian, we need to expand the previous operators around $\langle\phi\rangle = v$. Notice that, since $\mathcal{O}_{ij}$ ($i \in \{\ell, u, d\}$ and $j \in \{W, B, G\}$) are already dim-6 operators, i.e., they are already weighted by Λ^{-2}, we can neglect differences between the new Higgs vev v and the SM one v_{SM} since they would contribute as another $O(\Lambda^{-2})$ factor. Similarly, one can use the SM definitions for B_μ and W^3_μ to relate with Z_μ and A_μ since differences are $O(\Lambda^{-2})$ as one can see in Eqs. 17-18. The same applies also for the gauge couplings. Hence, one gets:

$$\begin{aligned}
\mathcal{O}_{uG} &= \left(\left(\bar{u_L}, \bar{d_L}\right)\sigma^{\mu\nu}\lambda_A u_R\right)\begin{pmatrix}0\\1\end{pmatrix} G^A_{\mu\nu} = v(\bar{u_L}\sigma^{\mu\nu}\lambda_A u_R)G^A_{\mu\nu}\ ; \\
\mathcal{O}_{dG} &= v(\bar{d_L}\sigma^{\mu\nu}\lambda_A d_R)G^A_{\mu\nu}\ ; \\
\mathcal{O}_{uB} &= 2v(\bar{u_L}\sigma^{\mu\nu}u_R)\partial_\mu B_\nu = 2v(\bar{u_L}\sigma^{\mu\nu}u_R)(c_w\partial_\mu A_\nu - s_w\partial_\mu Z_\nu)\ ; \\
\mathcal{O}_{dB} &= 2v(\bar{d_L}\sigma^{\mu\nu}d_R)(c_w\partial_\mu A_\nu - s_w\partial_\mu Z_\nu)\ ; \\
\mathcal{O}_{\ell B} &= 2v(\bar{e_L}\sigma^{\mu\nu}e_R)(c_w\partial_\mu A_\nu - s_w\partial_\mu Z_\nu)\ ; \\
\mathcal{O}_{uW} &= 2v(\bar{u_L}\sigma^{\mu\nu}u_R)(s_w\partial_\mu A_\nu + c_w\partial_\mu Z_\nu + igW^-_\mu W^+_\nu) \\
&\quad +2\sqrt{2}v(\bar{d_L}\sigma^{\mu\nu}u_R)(\partial_\mu W^-_\nu - ig(s_w W^-_\mu A_\nu + c_w W^-_\mu Z_\nu))\ ; \\
\mathcal{O}_{dW} &= -2v(\bar{d_L}\sigma^{\mu\nu}d_R)(s_w\partial_\mu A_\nu + c_w\partial_\mu Z_\nu + igW^-_\mu W^+_\nu) \\
&\quad +2\sqrt{2}v(\bar{u_L}\sigma^{\mu\nu}d_R)(\partial_\mu W^+_\nu + ig(s_w W^+_\mu A_\nu + c_w W^+_\mu Z_\nu))\ ; \\
\mathcal{O}_{\ell W} &= -2v(\bar{e_L}\sigma^{\mu\nu}e_R)(s_w\partial_\mu A_\nu + c_w\partial_\mu Z_\nu + igW^-_\mu W^+_\nu) \\
&\quad +2\sqrt{2}v(\bar{\nu_L}\sigma^{\mu\nu}e_R)(\partial_\mu W^+_\nu + ig(s_w W^+_\mu A_\nu + c_w W^+_\mu Z_\nu))\ .
\end{aligned} \tag{23}$$

The effective dipole Lagrangian comes only from Eqs. 23 because the SM Lagrangian cannot have dipole operators because are not renormalizable. By multiplying each contribution by its Wilson coefficient α_{ij} and suitable gauge coupling (according to which gauge group we are considering) and by dividing each term by Λ^2 (for dimensional analysis), one finally obtains the following Lagrangian:

$$\begin{aligned}
\mathcal{L}_M &= g_s\frac{v}{\Lambda^2}\left(\alpha_{uG}\bar{u_L}\sigma^{\mu\nu}\lambda_A u_R + \alpha_{dG}\bar{d_L}\sigma^{\mu\nu}\lambda_A d_R\right)G^A_{\mu\nu} \\
&\quad +2e\frac{v}{\Lambda^2}\left[(\alpha_{uB}+\alpha_{uW})\bar{u_L}\sigma^{\mu\nu}u_R + (\alpha_{dB}-\alpha_{dW})\bar{d_L}\sigma^{\mu\nu}d_R + (\alpha_{\ell B}-\alpha_{\ell W})\bar{e_L}\sigma^{\mu\nu}e_R\right](\partial_\mu A_\nu - \tan\theta_w\partial_\mu Z_\nu) \\
&\quad +2g\frac{v}{\Lambda^2}\left[\alpha_{uW}\bar{u_L}\sigma^{\mu\nu}u_R - \alpha_{dW}\bar{d_L}\sigma^{\mu\nu}d_R - \alpha_{\ell W}\bar{e_L}\sigma^{\mu\nu}e_R\right](\partial_\mu Z_\nu + igc_w W^-_\mu W^+_\nu) \\
&\quad +2\sqrt{2}g\frac{v}{\Lambda^2}\left[\alpha_{dW}\bar{u_L}\sigma^{\mu\nu}d_R + \alpha^\star_{uW}\bar{u_R}\sigma^{\mu\nu}d_L + \alpha_{\ell W}\bar{\nu_L}\sigma^{\mu\nu}e_R\right](\partial_\mu W^+_\nu - ieA_\mu W^+_\nu - igc_w Z_\mu W^+_\nu) + \text{h.c.}\ .
\end{aligned} \tag{24}$$

The presentation we gave in this subsection is far from describing all the possible influences of dim-6 operators to the SM: we mainly focused on topics we will treat in some detail in this work. However, we made other computations on other sectors of the SM Lagrangian (especially concerning the Higgs boson) and all the results can be found in Appendix from A.2 to A.4 where we adopted the same notation as above.

3.2 M_W anomaly within SMEFT

We saw in general how to generate deviations from SM results by adding non-renormalizable effective operators. Now we want to exploit this methodology to bridge the gap between the SM prediction coming from the electroweak fit [14, 35]:

$$M_W^{\text{SM}} = (80.3545 \pm 0.0057)\text{GeV} ,$$

and the latest measurement made by CDF-II collaboration [31]:

$$M_W^{\text{CDF}} = (80.4435 \pm 0.0094)\text{GeV} ,$$

that leads to $\Delta M_W = M_W^{\text{CDF}} - M_W^{\text{SM}} = (89 \pm 11)\text{MeV}$ (errors are added in quadrature) which exhibit a discrepancy at $\sim 8\sigma$. To tackle this problem, we choose Ref. [27] as a guide, therefore we will use its notation in this context, namely the Higgs vev $v \approx 246\text{GeV}$, the covariant derivative reads $D_\mu = \partial_\mu \mathbb{I} + ig_s \frac{\lambda_A}{2} G_\mu^A + ig_2 \frac{\tau_I}{2} W_\mu^I + ig_1 Y B_\mu$, the field strength tensor is $X_{\mu\nu}^a = \partial_{[\mu} X_{\nu]}^a - g_\star f^{abc} X_\mu^b X_\nu^c$ with $X \in \{B, W, G\}$. These modifications do not alter the results found in the previous subsection, up to factors $1/\sqrt{2}$ due to the different Higgs vev. Furthermore, although physical results are basis independent, we will use the Warsaw basis that is still related to the one used in the previous subsection as highlighted in Sec.4 of [34].
In this section we write the SMEFT Lagrangian as:

$$\mathcal{L} = \mathcal{L}_{SM} + \sum_i c_i \mathcal{O}_i,$$

where c_i are dimensionful Wilson coefficients, as one can see by dimensional analysis $[c_i] = -2$ and not considering the dim-5 Weinberg operator as previously mentioned.

3.2.1 $\Delta M_W(S, T, U)$

First, we would like to express ΔM_W in terms of universal quantities in order not to rely to any explicit UV completion of the SM. In particular, it turns out that a convenient way to take into account new physics deviation from SM predictions in the global electro-weak fit is to consider the so-called Peskin-Takeuchi (or oblique) parameters S, T, U, W, Y, X which are vanishing in the SM, therefore accounting for pure new physics effects. As far as ΔM_W is concerned, the following relation holds [60]:

$$M_{W,\text{CDF}}^2 - M_{W,\text{SM}}^2 \equiv \Delta M_W^2 = \frac{\alpha_{em} c_W^2}{c_W^2 - s_W^2} M_Z^2 \left(-\frac{1}{2} S + c_W^2 T + \frac{c_W^2 - s_W^2}{4 s_W^2} U \right) . \tag{25}$$

There is also another notation in the literature (e.g., see Refs. [27, 32, 33]) for the oblique parameters, where they are denoted by a *hat* and related to the ones in Eq. 25 by a normalization factor $S = 4 s_W^2 \hat{S}/\alpha_{em}$, $T = \hat{T}/\alpha_{em}$ and $U = -4 s_W^2 \hat{U}/\alpha_{em}$.
Now, our first step is to explicitly evaluate $\hat{S}, \hat{T}, \hat{U}$ and write them as linear combination of SMEFT Wilson coefficients c_i. They are defined as [32, 33]:

$$\hat{S} \equiv \frac{c_w}{s_w} \Pi'_{W_3 B}(0) \; ; \quad \hat{T} \equiv \frac{1}{M_w^2} \left(\Pi_{W_3 W_3}(0) - \Pi_{W_+ W_-}(0) \right) \; ; \quad \hat{U} \equiv -g_2^2 \left(\Pi'_{W_3 W_3}(0) - \Pi'_{W_+ W_-}(0) \right) . \tag{26}$$

To understand the meaning of the terms $\Pi_{ij}(0)$ with $i, j \in \{W_3, B, W_+, W_-\}$, one should recall the general expression for a vector boson propagator:

$$\Pi_{ij}^{\mu\nu}(q^2) = \Pi_{ij}(q^2) \eta^{\mu\nu} + \Delta_{ij}(q^2) q^\mu q^\nu , \tag{27}$$

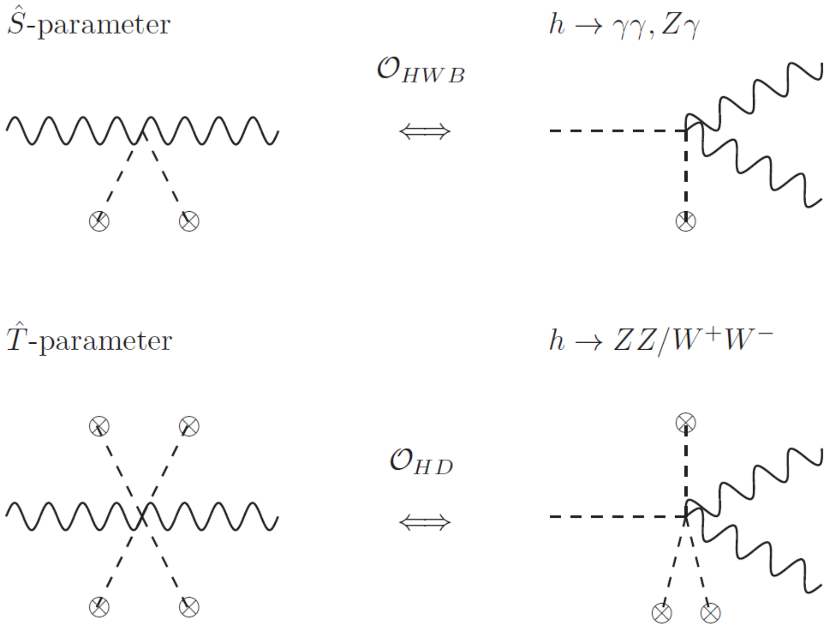


Figure 3: Tree-level Feynman diagrams of processes mediated by the effective operators $\mathcal{O}_{HWB}$ and $\mathcal{O}_{HD}$, in particular, how they can generate corrections to vector boson propagators (left) and genuine new interactions that modify observables as the Higgs decay rate in different vector bosons (right). Picture taken from Ref. [27].

where q^μ is the 4-momentum of the (virtual) propagating boson. Since $\hat{S}$, $\hat{T}$ and $\hat{U}$ are generated by oblique corrections, they do not depend on the particular choice of initial and final states of the process that gives rise to their contribution, but only on kinematic parameters such as $q^2 \sim m^2$ the masses of the initial state particles. Usually, light fermion scattering processes are studied (e.g., e^+e^- scattering at LEP), therefore one can set $q^2 \to 0$ and Taylor expand[7] the (Lorentz) scalar term $\Pi_{ij}(q^2)$ in Eq. 27 while neglecting the $q^\mu q^\nu$ contribution. The final structure of the propagator is:

$$\Pi^{\mu\nu}_{ij}(q^2 \to 0) \approx \left(\Pi_{ij}(0) + \Pi'_{ij}(0)q^2 + \frac{1}{2}\Pi''_{ij}(0)q^4 + O(q^6)\right)\eta^{\mu\nu} , \tag{28}$$

and the coefficients of the Taylor expansion in the r.h.s. are just the ones used in Eq. 26.

Now, we should compute the SMEFT corrections to the W_3B and W_+W_- propagators. The former is influenced by $\mathcal{O}_{HWB} = \left(H^\dagger \tau_I H\right) W^I_{\mu\nu}B^{\mu\nu}$ and the latter by $\mathcal{O}_{HD} = \left(H^\dagger D_\mu H\right)\left((D^\mu H)^\dagger H\right)$. Since now we are not interested in the Higgs-gauge boson coupling, we evaluate these operators in the Higgs vev $H \to \langle H \rangle = \frac{v}{\sqrt{2}}(0,1)^T$. With a little bit of algebra, one finds:

$$\left(H^\dagger \tau_I H\right) W^I_{\mu\nu}B^{\mu\nu} \longrightarrow \frac{v^2}{2}\left((0,1)\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}\right) W^3_{\mu\nu}B^{\mu\nu} = -\frac{v^2}{2}W^3_{\mu\nu}B^{\mu\nu} ; \tag{29}$$

$$\left(H^\dagger D_\mu H\right)\left((D^\mu H)^\dagger H\right) \longrightarrow \frac{v^4}{16}\left(g_1 B_\mu - g_2 W^3_\mu\right)^2 . \tag{30}$$

As one can see from Fig. 3, SMEFT contributions arise at tree-level, so the leading order correction to the amplitude is simply given by the Feynman rules of the operators $\mathcal{O}_{HWB}$ and $\mathcal{O}_{HD}$.

- **Computing $\hat{S}$:** the amplitude related to Eq. 29 is:

$$\Pi^{\mu\nu}_{W_3B}(q^2) = v^2 c_{HWB}(q^2\eta^{\mu\nu} - q^\mu q^\nu) . \tag{31}$$

[7] $\Pi_{ij}(q^2)$ is analytical in $q^2 \to 0$ because we are considering only tree level contribution in SMEFT theory, while potential loop contributions may spoil the low-momentum regularity since poles coming from light particles that contribute to the loop must be considered. See also [33].

Notice that $\mathcal{O}_{HWB} \ni W^3_{\mu\nu}$ contains a self-coupling between W_+ and W_- because SU(2) is a non-abelian group. However, we did not consider this term since it would give rise to a BW_+W^- vertex which do not contribute to Π_{W_3B}. Comparing Eq. 31 with Eq. 28, it is easy to see that $\Pi'_{W_3B}(0) = v^2 c_{HWB}$, thus:

$$\hat{S} = \frac{c_w}{s_w} v^2 c_{HWB} \ . \tag{32}$$

• **Computing $\hat{T}$:** the procedure is similar to $\hat{S}$. From Eq. 30 one retrieves the following intermediate expression:

$$-ic_{HD}\frac{v^4}{16}\left(g_1^2\eta^{\mu\nu}B_\mu B_\nu - 2g_1g_2\eta^{\mu\nu}B_\mu W^3_\nu + g_2^2\eta^{\mu\nu}W^3_\mu W^3_\nu\right) \ ,$$

where we intentionally left explicit the fields in order to better visualize how the first term would contribute to $\Pi_{BB}(0)$; the second term to $\Pi_{W_3B}(0)$ (and *not* to $\Pi'_{W_3B}(0)$ so it does not affect $\hat{S}$); the third term to $\Pi_{W_3W_3}(0)$ which is of our interest since it appears in the definition of $\hat{T}$. Therefore, focusing only on the third term, one realizes that:

$$\Pi_{W_3W_3}(0) = -c_{HD}\frac{v^4}{8}g_2^2 \ ,$$

where we multiplied the above expression with a symmetry factor of 2. Hence:

$$\hat{T} = -\frac{1}{M_w^2}c_{HD}\frac{v^4}{8}g_2^2 \ .$$

Notice that $\hat{T}$ depends only by a dimension-six operator, so we can use as values of v, M_W and g_2 the SM ones as done many times in Sec.3.1. We can rewrite the previous result as:

$$\hat{T} = -\frac{v^2}{2}c_{HD} \ . \tag{33}$$

Finally, we want to explicitly show that $\hat{S}$ and $\hat{T}$ are vanishing in the SM, as previously stated. As a fact, only $(D_\mu H)^\dagger(D^\mu H)$ (evaluated at the Higgs vev) may contribute to the gauge boson propagators, but its contribution is vanishing for $\hat{T}$ and it does not influence $\hat{S}$. To see this, we can compute:

$$(D_\mu H)^\dagger(D^\mu H)\Big|_{H=\frac{v}{\sqrt{2}}} = \frac{v^2}{8}(2g_2^2W^+_\mu W^\mu_- + (g_1B_\mu - g_2W^\mu_3)^2) \ .$$

By selecting only the terms which may contribute to the propagators in Eq. 26, one finds the following amplitudes:

$$\Pi^{\mu\nu}_{W_+W_-}(q^2) = -\frac{v^2}{4}g_2^2\eta^{\mu\nu} = \Pi^{\mu\nu}_{W_3W_3}(q^2) \ ,$$

hence $\hat{S}$ does not receive contributions since there is no dependence in q^2 while $\hat{T}$ goes to zero because $\Pi_{W_+W_-}(0) = \Pi_{W_3W_3}(0)$.

• **Computing $\hat{U}$:** concerning the case of $\hat{U}$, we remark that $\hat{U}$ depends on the same propagator $\Pi_{VV'}(0)$ as $\hat{T}$ but with an additional q^2. Such an extra q^2 can be obtained by the same operators generating $\hat{T}$ by adding two additional derivatives. Therefore, effectively, adding two derivatives to $\mathcal{O}_{HD}$ would generate a dim-8 operator which we systematically neglect. Thus, hereafter, we can safely neglect $\hat{U}$ in Eq. 25. One may question if this approach, that is theoretically consistent, is also experimentally justified. It turns out that this is indeed the case, as one can see in Ref. [32]: the global EW fit after the CDF-II measurement imposing $U = 0$ or $U \neq 0$ does not significantly alter the χ^2 of the global fit.

Finally, replacing Eqs. 32-33 in Eq. 25, one has the ΔM_W dependence written in terms of SMEFT Wilson coefficients. For completeness, we also computed W and Y oblique parameters at tree-level using the same strategy as here. Because they will not be useful in the following discussion, we leave the results in Appendix A.5.

3.2.2 $\hat{S}$ and $\hat{T}$ dependent observables

In the previous subsection we outlined the definitions of the $\hat{S}$ and $\hat{T}$ parameters as well as their dependence on the SMEFT Wilson coefficients. However, we would like to find a correlation between those parameters and experimental quantities which may give bounds on their values. First, let us write the effective interaction Lagrangian in the EW broken phase [27]:

$$\begin{aligned}\mathcal{L}^{\rm int}_{\rm SMEFT} \ni\ & g^{(1)}_{hWW} h W^+_\mu W^\mu_- + g^{(2)}_{hWW} h W^+_{\mu\nu} W^{\mu\nu}_- + g^{(1)}_{hZZ} h Z_\mu Z^\mu + g^{(2)}_{hZZ} h Z_{\mu\nu} Z^{\mu\nu} + g_{h\gamma\gamma} h F_{\mu\nu} F^{\mu\nu} \\ & + g_{h\gamma Z} h F_{\mu\nu} Z^{\mu\nu} + g_{hhh} h^3 + \left(g_{he} h \bar{e}_L e_R + g_{hu} h \bar{u}_L u_R + g_{hd} h \bar{d}_L d_R + \text{h.c.}\right) + \dots\ ,\end{aligned} \tag{34}$$

where the dots denote the presence of other interactions that we will not use. Using $Z_h^{1/2} = 1 + \left(c_{H\Box} - \frac{1}{4} c_{HD}\right) v^2$ as Higgs field renormalization, we get the following coefficients:

$$\begin{aligned}
g^{(1)}_{hWW} &= \frac{2M_W^2}{v}\left(1 - \frac{v^2}{4}(c_{HD} - 4c_{H\Box})\right)\ ; \\
g^{(2)}_{hWW} &= 2c_{HW} v\ ; \\
g^{(1)}_{hZZ} &= \frac{M_Z^2}{v}\left(1 + \frac{v^2}{4}(c_{HD} + 4c_{H\Box})\right)\ ; \\
g^{(2)}_{hZZ} &= v\left[c_{HW}\left(\frac{M_W^2}{M_Z^2}\right) + c_{HB}\left(\frac{M_Z^2 - M_W^2}{M_Z^2}\right) + c_{HWB}\left(\frac{g_1 g_2}{g_1^2 + g_2^2}\right)\right]\ ; \\
g_{h\gamma\gamma} &= v\left[c_{HW}\left(\frac{M_Z^2 - M_W^2}{M_Z^2}\right) + c_{HB}\left(\frac{M_W^2}{M_Z^2}\right) - c_{HWB}\left(\frac{g_1 g_2}{g_1^2 + g_2^2}\right)\right]\ ; \\
g_{h\gamma Z} &= 2v\left[c_{HW}\left(\frac{g_1 g_2}{g_1^2 + g_2^2}\right) - c_{HB}\left(\frac{g_1 g_2}{g_1^2 + g_2^2}\right) + \frac{c_{HWB}}{2}\left(\frac{g_1^2 - g_2^2}{g_1^2 + g_2^2}\right)\right]\ ; \\
g_{hhh} &= -\frac{M_H^2}{2v}\left(1 - \frac{v^2}{4}(c_{HD} - 4c_{H\Box}) - \frac{2v^4}{M_H^2} c_H\right)\ ; \\
g_{h\psi} &= -\frac{m_\psi}{v}\left(1 - \frac{v^2}{4}(c_{HD} - 4c_{H\Box})\right) + \frac{v^2}{\sqrt{2}} c_{H\psi}\ ,
\end{aligned} \tag{35}$$

with $\psi \in \{\ell, u, d\}$. A comprehensive study of all the coefficients appearing in Eq. 34 with all the calculations needed to arrive at Eqs. 35 is reported in Appendix B and also in Appendix C with a more general approach and a new Higgs field renormalization.

The most relevant processes to be considered are $h \to \gamma\gamma$, $h \to \gamma Z$, $h \to ZZ$ and $h \to W_+ W_-$ as shown in the r.h.s. of Fig. 3. Concerning $h \to ZZ$, $h \to \gamma\gamma$ and $h \to \gamma Z$, we note that they are all proportional to $\hat{S}$ which depends on the $W_3 B$ propagator by definition. Indeed, expanding around the Higgs vev, one obtains the coupling $h W_3 B$ which gives rise, in the physical basis, to the hZZ, $h\gamma\gamma$ and $h\gamma Z$ vertices. Now, we should remember that $\hat{S}$ is written in terms of $\Pi'_{W_3 B}$ so we need two derivative couplings in order to get a term of order q^2, hence the Higgs should couple not to the gauge bosons but to their field strengths. In particular:

- $h\gamma\gamma$ stems from $\mathcal{L}^{\rm int}_{\rm SMEFT}$ as $h F_{\mu\nu} F^{\mu\nu}$ which leads the decay $h \to \gamma\gamma$,
- $h\gamma Z$ stems from $\mathcal{L}^{\rm int}_{\rm SMEFT}$ as $h Z_{\mu\nu} F^{\mu\nu}$ which leads the decay $h \to \gamma Z$,
- hZZ stems from $\mathcal{L}^{\rm int}_{\rm SMEFT}$ as $h Z_{\mu\nu} Z^{\mu\nu}$ which leads the decay $h \to ZZ$.

$\hat{T}$ is defined through the $W_3 B$ and $W_+ W_-$ propagators and expanding as done for $\hat{S}$ around the Higgs vev, one finds the vertices $h\gamma\gamma$, $h\gamma Z$, hZZ, hWW. Now we recall that $\hat{T}$ depends on the leading term of $\Pi_{W_3 B}$ and $\Pi_{W_+ W_-}$ so we must not consider derivative couplings which would otherwise give rise to contributions in q^2. In particular:

- $h\gamma\gamma$ cannot contribute to $\hat{T}$ since in $\mathcal{L}^{\rm int}_{\rm SMEFT}$ there is not the $h A_\mu A^\mu$ term because the photon is massless. The $h \to \gamma\gamma$ decay depends on $\hat{S}$ only,
- $h\gamma Z$ cannot contribute to $\hat{T}$ as there is no $h A_\mu Z^\mu$ term. The $h \to \gamma Z$ decay depends on $\hat{S}$ only,

- hW_+W_- stems from the Lagrangian as $hW^+_\mu W^\mu_-$ which contributes to the decay $h \to WW$. Since there is no $\hat{S}$ contribution, this decay is mainly related to $\hat{T}$,
- hZZ stems from the Lagrangian as $hZ_\mu Z^\mu$ which contributes to the decay $h \to ZZ$. We saw that also $\hat{S}$ contributes to this process but only at the next-to-leading order in the q^2 expansion of $\Pi_{W_3B}(q^2)$, so the leading contribution is given by $\hat{T}$,

and this justifies our initial claim.

To study deviations from the pure SM prediction, we define the Higgs signal strength as:

$$\mu_{VV'} \equiv \frac{\Gamma(h \to VV')}{\Gamma_{SM}(h \to VV')} \ , \tag{36}$$

where $V, V' \in \{\gamma, Z, W_+, W_-\}$, $\Gamma(\dots)$ is the decay rate predicted by the SMEFT Lagrangian while $\Gamma_{SM}(\dots)$ is the one predicted by the Standard Model only.
Before proceeding with the calculations, we notice that Eq. 36 can be simplified: in fact, we can write in full generality the decay rate as:

$$\Gamma(h \to VV') = \Phi\, |\mathcal{M}|^2 \ ,$$

with Φ containing all the phase-space and kinematic contributions and $\mathcal{M}$ is the (unpolarized) Feynman amplitude of the process. Since $\mu_{VV'}$ is a ratio between processes with identical initial and final states, the Φ factor is the same and simplifies, leading to:

$$\mu_{VV'} = \left| \frac{\mathcal{M}}{\mathcal{M}_{SM}} \right|^2 \ . \tag{37}$$

The SMEFT amplitude is $\mathcal{M} = \mathcal{M}_{SM} + \mathcal{M}_{NP}$, where $\mathcal{M}_{NP}$ denotes the contribution of tree level dimension-six operators. Performing a linear expansion on the Wilson coefficients, one gets:

$$\mu_{VV'} = \left| 1 + \frac{\mathcal{M}_{NP}}{\mathcal{M}_{SM}} \right|^2 \approx 1 + 2\mathrm{Re}\left[\frac{\mathcal{M}_{NP}}{\mathcal{M}_{SM}} \right] \ , \tag{38}$$

so we do not need to compute all the decay rates, but only the Feynman amplitudes of the process.

- $\boldsymbol{\mu_{\gamma\gamma}}$**:** to compute $\mathcal{M}(h \to \gamma\gamma)_{NP}$, we write the Feynman rule associated with the term $g_{h\gamma\gamma} h F^2_{\mu\nu}$, which is the only dim-6 operator that contributes to the process:

$$g_{h\gamma\gamma} h F^2_{\mu\nu} \longrightarrow 4 i g_{h\gamma\gamma} (p_1 p_2 \eta_{\mu\nu} - p^2_\mu p^1_\nu) \ ,$$

where p^1_μ, p^2_μ are the 4-momenta of the two outgoing photons in the final state. Then, the complete Feynman amplitude reads:

$$\mathcal{M}(h \to \gamma\gamma)_{NP} = 4 i g_{h\gamma\gamma} (p_1 p_2 \eta_{\mu\nu} - p^2_\mu p^1_\nu) \epsilon^\mu(p_1, \lambda_1) \epsilon^\nu(p_2, \lambda_2) \ , \tag{39}$$

with $\epsilon^\mu(p_i, \lambda_i)$ being the polarization vector of the i-th photon and λ_i its helicity. Now, we must compute $\mathcal{M}(h \to \gamma\gamma)_{SM}$ which is loop-induced. In particular, the result of the loop integral is given by:

$$\mathcal{M}(h \to \gamma\gamma)_{SM} = A^\gamma_{SM} \epsilon_\mu(p_1, \lambda_1) \epsilon_\nu(p_2, \lambda_2) (p_1 p_2 \eta^{\mu\nu} - p^\mu_2 p^\nu_1) \ , \tag{40}$$

with:

$$\begin{aligned}
A^\gamma_{SM} &= A^\gamma_{W,SM} + A^\gamma_{f,SM} = \frac{i g_{2,SM} \alpha}{\pi M_{W,SM}} I_\gamma \ ; \\
A^\gamma_{W,SM} &= \frac{i g_{2,SM} \alpha}{\pi M_{W,SM}} I^\gamma_W \left(\frac{M^2_{H,SM}}{4 M^2_{W,SM}}, 0 \right) \ ; \\
A^\gamma_{f,SM} &= \frac{i g_{2,SM} \alpha}{\pi M_{W,SM}} N_c \sum_f Q^2_f I_f \left(\frac{M^2_{H,SM}}{4 m^2_{f,SM}}, 0 \right) \ ,
\end{aligned}$$

in agreement with Ref. [29]. Replacing everything in Eq. 40 and computing the ratio as in Eq. 38 one gets:

$$\mu_{\gamma\gamma} = 1 + 2\left(\frac{4\pi M_{W,SM} g_{h\gamma\gamma}}{g_{2,SM}\alpha I_\gamma}\right) . \tag{41}$$

Now we recall $g_{h\gamma\gamma}$ from Eq. 35: being already a linear combination of c_i, we can use as gauge boson mass values and as Higgs vev the SM ones, so we can write:

$$g_{h\gamma\gamma} = v_{SM}\left(c_{HW}s_w^2 + c_{HB}c_w^2 - c_{HWB}s_w c_w\right) ,$$

and remembering that $M_{W,SM} = \frac{v_{SM}g_{2,SM}}{2}$, one gets from Eq. 41:

$$\mu_{\gamma\gamma} = 1 + \frac{4\pi v^2}{\alpha I_\gamma}\left(c_{HW}s_w^2 + c_{HB}c_w^2 - c_{HWB}s_w c_w\right) .$$

Since, as a first estimate, we want to highlight the dependence on $\hat{S}$, we set all the Wilson coefficients to zero, but c_{HWB}. Thus:

$$\mu_{\gamma\gamma}|_{c_{HWB}} \approx 1 + \frac{4\pi v^2}{\alpha I_\gamma}(-c_{HWB}s_w c_w) = 1 + \frac{4\pi}{\alpha I_\gamma}\hat{S}s_w^2 . \tag{42}$$

- $\boldsymbol{\mu_{\gamma Z}}$**:** the procedure is analogous the the $\mu_{\gamma\gamma}$ case, so we will omit some steps. The Feynman rule for $g_{h\gamma Z}hZ_{\mu\nu}F^{\mu\nu}$ reads:

$$g_{h\gamma Z}hZ_{\mu\nu}F^{\mu\nu} \longrightarrow 2ig_{h\gamma Z}(p_1p_2\eta_{\mu\nu} - p_\mu^2 p_\nu^1) ,$$

thus the Feynman amplitude reads:

$$\mathcal{M}(h\to\gamma Z)_{NP} = 2ig_{h\gamma Z}(p_1p_2\eta_{\mu\nu} - p_\mu^2 p_\nu^1)\epsilon^\mu(p_1,\lambda_1)\epsilon^\nu(p_2,\lambda_2) , \tag{43}$$

where $\epsilon^\mu(p_1,\lambda_1)$ is the polarization vector associated with the photon, while $\epsilon^\nu(p_2,\lambda_2)$ is the one associated with the Z-boson. The SM process is loop induced, and the loop integral is:

$$\mathcal{M}(h\to\gamma Z)_{SM} = A_Z\epsilon_\mu(p_1,\lambda_1)\epsilon_\nu(p_2,\lambda_2)(p_1p_2\eta^{\mu\nu} - p_2^\mu p_1^\nu) , \tag{44}$$

with:

$$\begin{aligned}
A_Z &= A_f^Z + A_W^Z = \frac{ig_{2,SM}\alpha}{\pi M_{W,SM}} I_Z ; \\
A_W^Z &= \frac{ig_{2,SM}\alpha}{\pi M_{W,SM}} I_W^Z\left(\frac{M_{H,SM}^2}{4M_{W,SM}^2}, \frac{M_{Z,SM}^2}{4M_{W,SM}^2}\right) ; \\
A_f^Z &= \frac{ig_{2,SM}\alpha}{\pi M_{W,SM}} \sum_f Q_f g_f(\theta) I_f\left(\frac{M_{H,SM}^2}{4m_{f,SM}^2}, \frac{M_{Z,SM}^2}{4m_{f,SM}^2}\right) ,
\end{aligned}$$

in agreement with Ref. [29]. Replacing the above expressions into Eq. 44 and computing Eq. 38, one gets:

$$\mu_{\gamma Z} = 1 + \frac{4\pi M_{W,SM}g_{h\gamma Z}}{g_{2,SM}\alpha I_Z} \overset{\text{Eq. 145}}{=} 1 + \frac{4\pi v_{SM}^2}{\alpha I_Z}\left[c_{HW}s_w c_w - c_{HB}s_w c_w - \frac{1}{2}c_{HWB}(c_w^2 - s_w^2)\right] .$$

As we did for $\mu_{\gamma\gamma}$, we keep only linear terms in c_{HWB}, obtaining:

$$\mu_{\gamma Z}|_{c_{HWB}} \approx 1 - \frac{2\pi}{\alpha I_Z}\hat{S}(c_w^2 - s_w^2)\tan\theta_w . \tag{45}$$

- $\boldsymbol{\frac{\mu_{ZZ}}{\mu_{WW}}}$**:** by linearizing in the Wilson coefficients, we can rewrite the ratio as:

$$\frac{\mu_{ZZ}}{\mu_{WW}} \sim 1 + 2\mathrm{Re}\left[\frac{\mathcal{M}(h\to ZZ)_{NP}}{\mathcal{M}(h\to ZZ)_{SM}} - \frac{\mathcal{M}(h\to WW)_{NP}}{\mathcal{M}(h\to WW)_{SM}}\right] . \tag{46}$$

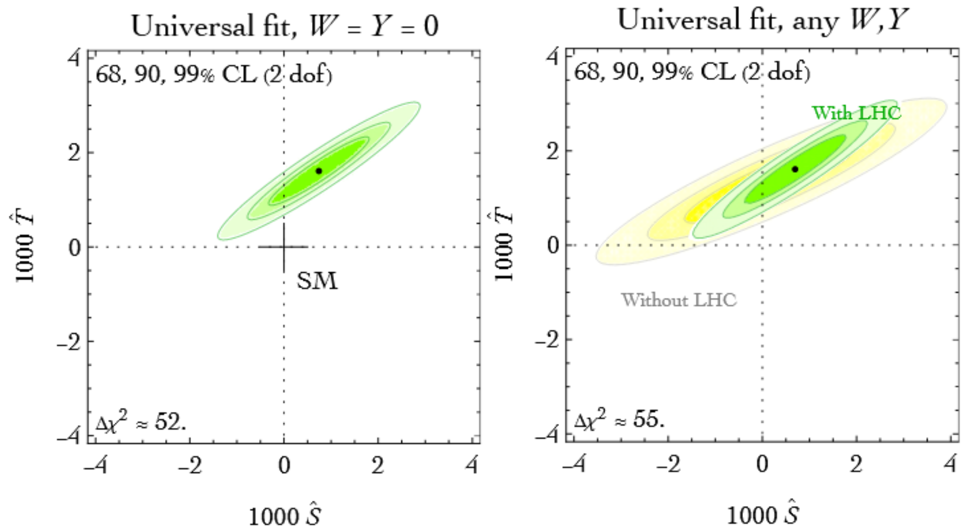


Figure 4: Global fit of the universal $\hat{S}, \hat{T}, W, Y$ electroweak parameters. The yellow regions show the analogous results computed without including the new LHC bounds on W, Y. Picture taken from [32].

This time, the leading order contributions in the SM amplitudes are already given at tree level since the Z and W bosons are massive, so they can couple directly with the Higgs. Therefore, for our estimate we will consider only tree level contributions. The ratio of the Feynman amplitudes is given by the ratio of the effective couplings $g^{(1)}_{hWW}$, $g^{(2)}_{hWW}$ (resp. $g^{(1)}_{hZZ}$, $g^{(2)}_{hZZ}$) with the tree level SM coupling between Higgs and massive vector bosons. In particular, we remember that:

$$g_{hW,SM} = \frac{2M^2_{W,SM}}{v_{SM}} \quad ; \quad g_{hZ,SM} = \frac{M^2_{Z,SM}}{v_{SM}} \ ,$$

while $g^{(1)}_{hWW}$, $g^{(2)}_{hWW}$ and $g^{(1)}_{hZZ}$, $g^{(2)}_{hZZ}$ were given in Eq. 35. Notice that we explicitly wrote the "SM" subscript whenever the Standard Model relations hold: we remember that in general, the effective couplings do not contain the SM masses and couplings due to their normalization. However, in order to highlight the $\hat{T}$ dependence of $\frac{\mu_{ZZ}}{\mu_{WW}}$, we keep only linear terms in c_{HD} neglecting all the others. Doing so, one arrives at:

$$g_1|_{c_{HD}} = g_{1,SM} \ ; \qquad g_2|_{c_{HD}} = g_{2,SM} \ ; \qquad v|_{c_{HD}} = v_{SM} \ ;$$

$$M_W|_{c_{HD}} = M_{W,SM} \ ; \qquad M_Z|_{c_{HD}} = M_{Z,SM}\left(1 + \frac{v^2}{4}c_{HD}\right) \ ,$$

so we need to pay attention only to M_Z. Notice also that, keeping only c_{HD}, the contributions arising from $g^{(2)}_{hWW}$ and $g^{(2)}_{hZZ}$ vanish. We can now compute all the needed Feynman amplitudes:

$$\begin{aligned}
\mathcal{M}(h \to ZZ)_{NP}|_{c_{HD}} &\sim -2i\frac{M^2_{Z,SM}}{v_{SM}}\frac{3}{4}c_{HD}v^2_{SM}\eta_{\mu\nu}\epsilon^\mu(p_1,\lambda_1)\epsilon^\nu(p_2,\lambda_2) \ ; \\
\mathcal{M}(h \to ZZ)_{SM} &= -2i\frac{M^2_{Z,SM}}{v_{SM}}\eta_{\mu\nu}\epsilon^\mu(p_1,\lambda_1)\epsilon^\nu(p_2,\lambda_2) \ ; \\
\mathcal{M}(h \to WW)_{NP}|_{c_{HD}} &\sim -i\frac{2M^2_{W,SM}}{v_{SM}}\left(-\frac{v^2_{SM}}{4}c_{HD}\right)\eta_{\mu\nu}\epsilon^\mu(p_1,\lambda_1)\epsilon^\nu(p_2,\lambda_2) \ ; \\
\mathcal{M}(h \to WW)_{SM} &= -2i\frac{M^2_{W,SM}}{v_{SM}}\eta_{\mu\nu}\epsilon^\mu(p_1,\lambda_1)\epsilon^\nu(p_2,\lambda_2) \ .
\end{aligned} \tag{47}$$

Then, by replacing Eq. 47 into Eq. 46 one gets:

$$\left.\frac{\mu_{ZZ}}{\mu_{WW}}\right|_{c_{HD}} \sim 1 + 2\left(\frac{3}{4}v^2c_{HD} - \left(-\frac{v^2}{4}c_{HD}\right)\right) = 1 + 2v^2c_{HD} \overset{\text{Eq. 26}}{=} 1 - 4\hat{T} \ . \tag{48}$$

In order to justify the M_W value as measured at CDF, we need $\hat{T} = (0.84 \pm 0.14) \times 10^{-3}$ and a less constrained $\hat{S} \sim 10^{-3}$ [35] (see Fig.4). From Eqs. 42, 45 and 48 this means that:

(i) $\mu_{\gamma\gamma} \approx 1.2$ for $\hat{S} \sim 10^{-3}$, which is a remarkable modification and eventually measurable at LHC thanks to its present 10% experimental resolution.

(ii) $\mu_{\gamma Z} \approx 1.08$ for $\hat{S} \sim 10^{-3}$, which is similar to the experimental resolution so, at present, no conclusive results may be drawn by the $h \to \gamma Z$ decay.

(iii) $\mu_{ZZ}/\mu_{WW} \approx 0.997$ for $\hat{T} = 0.84 \times 10^{-3}$ that is far below the experimental resolution, thus this ratio cannot provide useful constraints at present.

4 Muon g-2 anomaly

In this section, we will address the muon $g-2$ anomaly by means of new physics contributions within an EFT approach. First, let us briefly review the SM prediction for the muon $g-2$.

4.1 The SM prediction for the muon g-2

In general, given a lepton ℓ interacting with a photon, one can write the vertex amplitude as:

$$\mathcal{M} = -ieQ_\ell \bar{u}(p')\Gamma^\mu(p,p')u(p)\epsilon_\mu(q) \ , \tag{49}$$

where Q_ℓ is the electric charge of the lepton and the 4-momenta are assigned as in Fig. 5. All the information concerning the vertex structure as well as the magnetic dipole moment contributions are inside the $\Gamma^\mu(p,p')$ expression which, in full generality, can be written as (we use the notation of Ref. [39]):

$$\Gamma^\mu(p,p') = \underbrace{F_E(q^2)\gamma^\mu}_{\text{standard vertex term}} + \underbrace{i\frac{\sigma^{\mu\nu}}{2m_\ell}q_\nu F_M(q^2)}_{\text{magnetic dipole term}} + \underbrace{\frac{\sigma^{\mu\nu}q_\nu}{2m_\ell}\gamma_5 F_D(q^2)}_{\text{electric dipole term}} + \underbrace{\frac{q^2\gamma^\mu - q^\mu \not{q}}{m_\ell^2}\gamma_5 F_A(q^2)}_{\text{removed by EOMs}} \ , \tag{50}$$

with m_ℓ being the mass of the considered lepton. For the purposes of our work, we will consider only the form factor $F_M(q^2 \to 0)$ since the anomalous magnetic moment a_ℓ is defined in the limit of no momentum transfer as:

$$a_\ell = \frac{1}{2}(g_\ell - 2) = F_M(0) \ ,$$

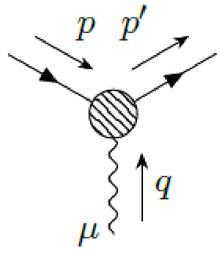
where g_ℓ is the spin g-factor of the considered lepton. From now on, we will focus only on the muon case ($\ell = \mu$).

Within the SM, it is possible to compute perturbatively the EW contributions to a_μ with a high resolution. On the other hand, the hadronic contributions to a_μ can be evaluated either through first principle based techniques, such as lattice QCD, or by means of a dispersive approach which exploits the $e^+e^- \to$ hadrons experimental data [40].
The QED contribution arise from diagrams involving the three charged leptons (e, μ, τ) interacting with the photon. Since the a_μ is dimensionless, the lepton-mass dependence appears in the form of the ratio between lepton masses. Thus, the QED contribution can be written as [42]:

$$a_\mu^{\rm QED} = A_1 + A_2\left(\frac{m_\mu}{m_e}\right) + A_2\left(\frac{m_\mu}{m_\tau}\right) + A_3\left(\frac{m_\mu}{m_e}, \frac{m_\mu}{m_\tau}\right) \ , \tag{51}$$

where the A_i $(i = 1,2,3)$ can be written as a perturbative expansion in powers of the fine-structure constant $\alpha \approx 1/137.035$ as:

$$A_i = \left(\frac{\alpha}{\pi}\right)A_i^{(2)} + \left(\frac{\alpha}{\pi}\right)^2 A_i^{(4)} + \left(\frac{\alpha}{\pi}\right)^3 A_i^{(6)} + \cdots + \left(\frac{\alpha}{\pi}\right)^n A_i^{(2n)} \ , \tag{52}$$



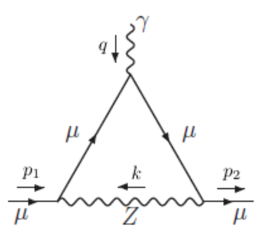


Figure 5: (left): Generic Feynman diagram for a lepton-lepton-photon vertex. (right): Z-boson contribution to a_μ^{EW}.

where each $A_i^{(n)}$ is computed in the renormalized perturbation theory. All the contributions related to the L loop-order are proportional to $(\alpha/\pi)^L$. Therefore, using Eqs. 51-52, we can rewrite:

$$a_\mu^{\rm QED} = \sum_{L=1}^{5} \xi_L \left(\frac{\alpha}{\pi}\right)^L \ , \tag{53}$$

where

$$\xi_L = A_1^{(2L)} + A_2^{(2L)}\left(\frac{m_\mu}{m_e}\right) + A_2^{(2L)}\left(\frac{m_\mu}{m_\tau}\right) + A_3^{(2L)}\left(\frac{m_\mu}{m_e}, \frac{m_\mu}{m_\tau}\right) \ .$$

Notice that in Eq. 53 we considered corrections up to five loops since this is the highest fully-evaluated order. The coefficients ξ_L $(L = 1\ldots5)$ are known and read:

$$\begin{aligned}
\xi_1 &= \frac{1}{2} \qquad \text{(Schwinger computation)} \ , \\
\xi_2 &= 0.765857423(16) \ , \\
\xi_3 &= 24.05050982(28) \ , \\
\xi_4 &= 130.8734(60) \ , \\
\xi_5 &= 751.917(932) \ .
\end{aligned}$$

Replacing in Eq. 53 one gets the final result:

$$a_\mu^{\rm QED} = 116\ 584\ 718.842(34) \times 10^{-11} \ . \tag{54}$$

The SM weak contribution to a_μ is known up to two-loop precision and it can be written as:

$$a_\mu^{\rm EW} = a_\mu^{(2)\ \rm EW} + a_\mu^{(4)\ \rm EW} \ . \tag{55}$$

The one-loop contribution $a_\mu^{(2)\ \rm EW}$ is given by triangular-shaped Feynman diagrams where one of the EW bosons or the Higgs runs in the loop. As an example, we illustrate how to compute the Z-boson correction to the $(g-2)_\mu$ since this result will be helpful in the following Sec.5.1.1. The Feynman diagram illustrating the process is shown in Fig. 5. In the dimensional regularization approach, we can write the vertex function as:

$$\begin{aligned}
\Gamma_\mu \ = \ & \int \frac{d^dk}{(2\pi)^d}\frac{ig\mu^\epsilon}{c_w}\gamma_{a2}(g_LP_L + g_RP_R)\frac{i}{\not{p}_2 + \not{k} - m_\mu}(e\mu^\epsilon)\gamma_\mu\frac{i}{\not{k} + \not{p}_1 - m_\mu}\frac{g\mu^\epsilon}{c_w}\gamma_{a1}(g_LP_L + g_RP_R) \times \\
& \times\frac{i}{k^2 - M_Z^2}\left(\eta^{a1,a2} - \frac{k^{a1}k^{a2}}{M_Z^2}\right) \ ,
\end{aligned} \tag{56}$$

where $P_{L,R}$ are the chirality projectors, g is the $SU(2)_L$ gauge coupling, c_w is the cosine of the Weinberg angle, $\epsilon = \frac{4-d}{2}$, $g_L = -1/2 + s_w^2$ and $g_R = s_w^2$. First, we observe that we are just interested in the terms of Γ_μ of the form $\sigma^{\mu\nu}q_\nu$ since only the latter contribute to F_M. To solve Eq. 56, we use the public Mathematica package Package-X [41] and, by expanding for $m_\mu/M_Z \ll 1$, we get the leading term:

$$-\frac{g^2e}{c_w^2}\frac{m_\mu}{96\pi^2M_Z^2}(\cos(2\theta_W) + (\sin(2\theta_W))^2) \stackrel{!}{=} \frac{e}{2m_\mu}F_M(q^2 \to 0) \ .$$

Using $G_F/\sqrt{2} = \frac{g^2}{8M_W^2}$ and $M_Zc_w = M_W$ we finally find:

$$F_M(0) = a_\mu^{(2)\ \rm EW}(Z) = \frac{G_F\sqrt{2}}{16\pi^2}m_\mu^2\frac{(1-4s_w^2)^2 - 5}{3} \approx -193.89(2) \times 10^{-11} \ . \tag{57}$$

Similarly, it can be computed the one-loop W-boson contribution $a_\mu^{(2)\ \rm EW}(W) = 388.70(0) \times 10^{-11}$ and the Higgs contribution $a_\mu^{(2)\ \rm EW}(H) \sim 10^{-14}$ (negligible, because suppressed by the Yukawa coupling $y_\mu \sim m_\mu/v$). Thus, the one loop EW correction is [40]:

$$a_\mu^{(2)\ \rm EW} = a_\mu^{(2)\ \rm EW}(W) + a_\mu^{(2)\ \rm EW}(Z) + a_\mu^{(2)\ \rm EW}(H) = (194.82 \pm 0.02) \times 10^{-11} \ . \tag{58}$$

The two-loop EW contribution $a_\mu^{(4)\ \mathrm{EW}}$ arises from QED corrections or fermionic loop insertions in the one-loop diagrams. The latter contains also hadronic contributions from charged pions and kaons that are treated non perturbatively. The final two-loop contribution is known [42]:

$$a_\mu^{(4)\ \mathrm{EW}} = (-42.08 \pm 1.5[m_H, m_t] \pm 1.0[\mathrm{had.}]) \times 10^{-11} \ , \tag{59}$$

where the first error is due to uncertainties on Higgs and top masses, while the second to hadronic uncertainties. Therefore, the total EW contribution to a_μ reads:

$$a_\mu^{\mathrm{EW}} = (150.7 \pm 1.5[m_H, m_t] \pm 1.0[\mathrm{had.}]) \times 10^{-11} \ . \tag{60}$$

Finally, the hadronic contribution a_μ^{had} stems from QED diagrams with the insertion of light quark loops. From data-driven evaluations [40, 42] at the next-to leading order, it is found:

$$a_\mu^{\mathrm{had}} = 6845(40) \times 10^{-11} \ . \tag{61}$$

Finally, we need also to include an additional hadronic light-by-light contribution which reads:

$$a_\mu^{\mathrm{LbL}} = 78.7(30.6)_{\mathrm{stat}}(17.7)_{\mathrm{syst}} \times 10^{-11} \ . \tag{62}$$

Now, we can compute the full SM contribution to the muon anomalous magnetic moment:

$$a_\mu^{\mathrm{SM}} = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{EW}} + a_\mu^{\mathrm{had}} + a_\mu^{\mathrm{LbL}} = 116\ 591\ 810(43) \times 10^{-11} \ . \tag{63}$$

Notice that the a_μ^{SM} error comes mostly from the hadronic contribution. On the other hand, the recent E989 experiment at Fermilab confirmed the previous E821 experiment at BNL, with a combined final result of [43]:

$$a_\mu^{\mathrm{EXP}} = 116\ 592\ 061(41) \times 10^{-11} \ . \tag{64}$$

Hence, defining $\Delta a_\mu = a_\mu^{\mathrm{EXP}} - a_\mu^{\mathrm{SM}} = 251(59) \times 10^{-11}$ (the errors are summed in quadrature), it is found a discrepancy at 4.2σ level. In order to account for this tension, one can invoke New Physics (NP) contributions, provided they do not spoil the agreement between several experimental data with SM predictions. For example, one can notice that $\Delta a_\mu \sim 3 \times 10^{-9}$ is of the same order as $a_\mu^{\mathrm{EW}} \sim 2 \times 10^{-9}$. If the NP sector is weakly coupled and $\Delta a_\mu^{\mathrm{NP}}$ scales with lepton masses as the SM weak contribution, then by Naive Dimensional Analysis (NDA):

$$a_\mu^{\mathrm{NP}} \sim \frac{g_{\mathrm{NP}}^2}{16\pi^2} \frac{m_\mu^2}{\Lambda_{\mathrm{NP}}^2} \ .$$

Here, the experimental value of Δa_μ can be reproduced, being $g_{\mathrm{NP}} \sim 1$, only if $\Lambda_{\mathrm{NP}} \sim v$. For such low NP energy scale the SMEFT approach cannot be used since $v/\Lambda_{NP} \ll 1$ does not hold. Moreover, the lack for new particles at LHC further disfavours this scenario. As a result, there are only two possibilities to solve the $g-2$: *(i)* $\Lambda_{\mathrm{NP}} \ll v$ and NP particles are very weakly coupled to the SM ($g_{\mathrm{NP}} \ll 1$), or *(ii)* $\Lambda_{\mathrm{NP}} \gg v$ and strongly coupled to the SM ($g_{\mathrm{NP}} \sim 4\pi$). In the following, we choose the latter possibility.

4.2 SMEFT approach to the muon g-2

Our ansatz is that Λ_{NP} is in the multi-TeV range. At this scale, only few SMEFT operators contribute significantly to the $g-2$: the dipole operators $\mathcal{O}_{eB}^\mu = (\bar{\ell}_L \sigma_{\mu\nu} \mu_R) H B^{\mu\nu}$, $\mathcal{O}_{eW}^\mu = (\bar{\ell}_L \sigma_{\mu\nu} \mu_R) \tau^I H W_I^{\mu\nu}$ and the four fermion operator $\mathcal{O}_T^{\mu q} = (\bar{\ell}_L^j \sigma_{\mu\nu} \mu_R) \epsilon_{jk} (\bar{q}_L^k \sigma^{\mu\nu} u_R)$ (with $j, k \in \{1, 2\}$ being the $SU(2)_L$ group indexes). Thus, we need to account for only three out of 59 Warsaw basis operators, and their leading contributions to Δa_μ are graphically given in Fig. 6. From the Buchmüller and Wyler magnetic Lagrangian given in Eq. 24 we need to select only the following terms (we switch to the Warsaw basis and add the $\mathcal{O}_T^{\mu q}$):

$$\mathcal{L}_M \ni \frac{c_{eB}^\mu}{\Lambda^2} \mathcal{O}_{eB}^\mu + \frac{c_{eW}^\mu}{\Lambda^2} \mathcal{O}_{eW}^\mu + \frac{c_T^{\mu q}}{\Lambda^2} \mathcal{O}_T^{\mu q} + \mathrm{h.c.} \ , \tag{65}$$

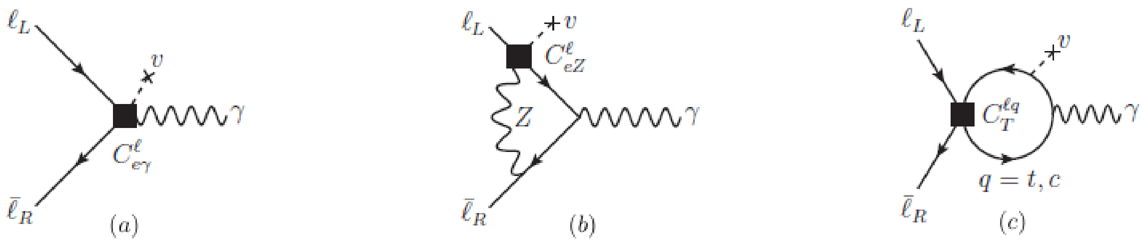


Figure 6: Feynman diagrams contributing to the leptonic g-2 up to one-loop order in the SMEFT. Effective interactions are depicted as a solid black square and to get correction to the muon just replace $\ell = \mu$. Picture taken from Ref. [44].

which is the same Lagrangian considered in [44] but a minor difference: in there, results are expressed entirely on the physical gauge bosons A_μ and Z_μ. To solve the issue, we can expand the operators around the Higgs vev and rewrite the gauge states in the physical basis using the SM relations, since Eq. 65 is already at $O(\Lambda^{-2})$ order. Thus:

$$\begin{aligned} c^{\mu}_{eB}(\bar{\ell}_L\sigma^{\mu\nu}\mu_R)HB_{\mu\nu} &= \frac{2(v+h)}{\sqrt{2}}c^{\mu}_{eB}(\bar{\ell}_L\sigma^{\mu\nu}\mu_R)(c_w\partial_\mu A_\nu - s_w\partial_\mu Z_\nu)\ ; \\ c^{\mu}_{eW}(\bar{\ell}_L\sigma^{\mu\nu}\mu_R)\tau_I HW^I_{\mu\nu} &= -\frac{2(v+h)}{\sqrt{2}}c^{\mu}_{eW}(\bar{\ell}_L\sigma^{\mu\nu}\mu_R)(s_w\partial_\mu A_\nu + c_w\partial_\mu Z_\nu + igW^-_\mu W^+_\nu)\ ; \end{aligned}$$

and collecting terms in the same gauge field and where the Higgs takes the vev $v = 246$GeV, we get:

$$\begin{aligned} \mathcal{L}_M \ni\ & -\frac{\sqrt{2}v}{\Lambda^2}(\bar{\ell}_L\sigma^{\mu\nu}\mu_R)\left[\left(c^{\mu}_{eB}c_w - c^{\mu}_{eW}s_w\right)\partial_\mu A_\nu - \left(c^{\mu}_{eW}c_w + c^{\mu}_{eB}s_w\right)\partial_\mu Z_\nu\right] \\ & -ig\frac{\sqrt{2}v}{\Lambda^2}c^{\mu}_{eW}(\bar{\ell}_L\sigma^{\mu\nu}\mu_R)W^-_\mu W^+_\nu + \text{h.c.}\ . \end{aligned} \tag{66}$$

By imposing $c^{\mu}_{eB}c_w - c^{\mu}_{eW}s_w = c^{\mu}_{e\gamma}$ and $c^{\mu}_{eW}c_w + c^{\mu}_{eB}s_w = -c^{\mu}_{eZ}$ one obtains the notation used in [44]. Actually, in the second line of Eq. 66 it appears a $\mu\mu W^-W^+$ vertex which is not considered in Fig. 6. The reason is that it would not be chirally enhanced: as we will see computing explicitly the a_μ contributions, they scale as $\frac{m_\mu v}{\Lambda^2}$ whereas the $\mu\mu WW$ vertex would contribute as $\frac{m^2_\mu}{\Lambda^2}$ (it would be a correction to the EW Standard Model contribution to the $g-2$ where charged bosons run in the loop, thus the final mass dependence should be the same, but the replacement $M^2_W \to \Lambda^2$) with a suppression factor of $\frac{m_\mu}{v} \sim 10^{-3}$.

Before computing the diagrams in Fig. 6, we would like to better address the Δa_μ dependence. The fact that Δa_μ depends mainly on $\mathcal{O}^{\mu}_{e\gamma}$, $\mathcal{O}^{\mu}_{eZ}$ and $\mathcal{O}^{\mu q}_T$ in the multi-TeV scale is also due to loop induced contributions. In fact:

(i) They create new (w.r.t. the SM) loop Feynman diagrams contributing to Δa_μ as in Fig.6 diagrams (b) and (c),

(ii) They modify couplings at tree-level because of new processes induced by dim-6 operators (e.g., see Eq. 116 for the electric charge or Eq. 22 for G_F), but also at loop-level because of the Renormalization Group Equations (RGEs). In fact, a generic SM coupling or Wilson coefficient c_i obeys at the following equation:

$$16\pi^2\mu\frac{dc_i(\mu)}{d\mu} \equiv \dot{c}_i = \gamma_{ij}c_j(\mu)\ , \tag{67}$$

where γ_{ij} is the anomalous matrix element related to the coupling c_i, which is usually computed in perturbation theory at the desired order (in this section we consider always 1-loop corrections if not stated differently). It is straightforward to see that the μ-running of the generic c_i coupling is correlated to (potentially all) other couplings c_j, and to solve Eq. 67 one must compute the full anomalous dimension matrix.

The most complex part of the work is keeping track of *(ii)*. To do so, we use some known results in the literature concerning the analytic form of the RGEs (as in Refs. [36,45,46]), the anomalous dimension matrix elements linearized in the Wilson coefficients as in Ref. [47] and Mathematica computational packages as DSixTools [48] to solve numerically the RGEs up to 1-loop order. In our multi-TeV scenario, we must evolve the couplings and Wilson coefficients up to $\Lambda_{\text{NP}} = 10\text{TeV}$[8]. In order to solve the RGEs we must give a starting value to all SMEFT Wilson coefficients, and to do so we must define the matching procedure, i.e., an energy scale μ_{match} at which both SM and SMEFT are valid theories and fix the unknown Wilson coefficients in such a way that they yield the same low-energy Feynman amplitudes: $\mathcal{M}^{\text{low}}_{\text{SM}} \overset{!}{=} \mathcal{M}^{\text{low}}_{\text{SMEFT}}$. Doing so, we obtain the Wilson coefficients $c_i(\mu = \mu_{\text{match}})$ evaluated at the matching scale, which we can finally evolve at the desired $\Lambda = 10\text{TeV}$. Fortunately, DSixTools automatically perform the matching procedure (we choose the default $\mu_{\text{match}} = M_Z \approx 91.18\text{GeV}$) given the SM input parameters evolved at the Z-pole scale. Since trying to solve even numerically the RGEs for all the Wilson coefficients is a very long computational task (for the non-redundant Warsaw basis it requires to solve 2499×2499 coupled differential equations, considering the multiplicity of flavour-index two- and four-fermion operators), we refer to Ref. [39] and take the result:

$$\Delta a_\mu^{10\text{TeV}} \approx \frac{(10\text{TeV})^2}{\Lambda^2}\left(1.7 \times 10^{-6} c^{\mu}_{eB} - 9.2 \times 10^{-7} c^{\mu}_{eW} - 2.2 \times 10^{-7} c^{\mu q}_{T} + O(10^{-9})\right) , \tag{68}$$

where $c_i = c_i(\mu = 10\text{TeV})$ are the real (dimensionless) Wilson coefficients of the respective dim-6 operator $\mathcal{O}_i$. Eq. 68 justifies also from a RG viewpoint the operator choice made in Eq. 65.
Now, we perform the computation of the Feynman amplitudes of the diagrams in Fig. 6 and take only the part concerning the a_μ correction. We report briefly some results, where we adopted the same strategy as the one used to get to Eq. 57 and the same software Package-X.

• $\boldsymbol{\mathcal{O}^{\mu}_{e\gamma}}$: its contribution is shown in Fig. 6(a). Let be p_1 the 4-momentum of the incoming muon, p_2 the one of the outgoing muon and q the one of the ingoing photon. The Feynman rule related to the SMEFT $\mu\mu H\gamma$ interaction (with H evaluated at its vev $v = 174\text{GeV}$) reads:

$$\frac{2v}{\Lambda^2} c^{\mu}_{e\gamma} \sigma^{\mu\nu} P_R q_\nu \; . \tag{69}$$

Other than the diagram in Fig.6(a) one should also consider its hermitian conjugate. The tree-level amplitudes are:

$$\begin{aligned} \mathcal{M}_A &= \frac{2v}{\Lambda^2} c^{\mu}_{e\gamma} \bar{u}(p_2)\sigma^{\mu\nu} P_R u(p_1) q_\nu \; ; \\ \mathcal{M}_A^{\text{h.c.}} &= \frac{2v}{\Lambda^2} c_{e\gamma} \bar{u}(p_2)\sigma^{\mu\nu} P_L u(p_1) q_\nu \; . \end{aligned}$$

Therefore:

$$\mathcal{M}_A^{\text{tot}} = \mathcal{M}_A + \mathcal{M}_A^{\text{h.c.}} = \frac{2v}{\Lambda^2} c^{\mu}_{e\gamma} \bar{u}(p_2)\sigma^{\mu\nu}(P_L + P_R) u(p_1) q_\nu \overset{!}{=} \frac{e}{2m_\mu} F^A_M(0) \bar{u}(p_2)\sigma^{\mu\nu} q_\nu u(p_1) \; ,$$

from which it can be easily read:

$$F^A_M(0) = \frac{4vm_\mu}{e}\frac{c^{\mu}_{e\gamma}}{\Lambda^2} \; . \tag{70}$$

• $\boldsymbol{\mathcal{O}^{\mu}_{eZ}}$: its contribution is shown in Fig.6(b). The choice of the external momenta is the same as in the previous case. Let be k the virtual 4-momentum of the Z boson in the loop. The Feynman rule related to the SMEFT $\mu\mu HZ$ vertex (again with H evaluated at $v = 174\text{GeV}$) is:

$$\frac{2v}{\Lambda^2} c_{eZ} \sigma^{\mu\nu} P_R q_\nu \; . \tag{71}$$

[8]Notice that the NP energy scale, in general, is not equivalent to the mass of the heavy NP particle M_{NP}. In fact M_{NP} would be a dimensionful parameter of a UV completion of the SM and it would be radiatively corrected by RGEs as the one in Eq. 67 after being matched at low-energy with the SM.

In this case there are 4 topologically equivalent Feynman diagrams which contribute: the one in Fig.6(b) and its h.c., the one where the effective interaction is in the lower external fermion line and its h.c. We omit the explicit form of h.c. diagrams since it is the similar as done before. Using the dimensional regularization, the loop integral takes the form:

$$\begin{aligned}
\mathcal{M}_B^{\rm up} &= \frac{ige}{c_W}\frac{2v}{\Lambda^2}c^{\mu}_{eZ}\mu^{4\epsilon}\int\frac{d^dk}{(2\pi)^d}\bar{u}(p_2)\frac{\gamma_{a2}(g_RP_R+g_LP_L)[\not{p}_2-\not{k}+m_\mu]\gamma_\mu[\not{p}_1-\not{k}+m_\mu]\sigma_{a1,\nu}k^\nu}{[(p_2-k)^2-m_\mu^2][(p_1-k)^2-m_\mu^2][k^2-M_Z^2]}u(p_1)\times \\
&\times\left(\eta^{a1,a2}-\frac{k^{a1}k^{a2}}{M_Z^2}\right)\epsilon^\mu(q)\ ; \\
\mathcal{M}_B^{\rm down} &= -\frac{ige}{c_W}\frac{2v}{\Lambda^2}c^{\mu}_{eZ}\mu^{4\epsilon}\int\frac{d^dk}{(2\pi)^d}\bar{u}(p_2)\frac{\sigma_{a2,\nu}k^\nu[\not{p}_2-\not{k}+m_\mu]\gamma_\mu[\not{p}_1-\not{k}+m_\mu]\gamma_{a1}(g_LP_L+g_RP_R)}{[(p_2-k)^2-m_\mu^2][(p_1-k)^2-m_\mu^2][k^2-M_Z^2]}u(p_1)\times \\
&\times\left(\eta^{a1,a2}-\frac{k^{a1}k^{a2}}{M_Z^2}\right)\epsilon^\mu(q)\ ,
\end{aligned}$$

with the label *up* or *down* indicating the position of the $\mu\mu HZ$ vertex. Summing up the two contribution and selecting $\sigma^{\mu\nu}q_\nu$ type terms, we get:

$$-\frac{e^2}{s_Wc_W}\frac{2v}{\Lambda^2}\frac{c^\mu_{eZ}}{16\pi^2}\log\left(\frac{\Lambda^2}{M_Z^2}\right)(-1+4s_W^2)\stackrel{!}{=}\frac{e}{2m_\mu}F_M^B(0)\ ,$$

where we evolve the contribution at the energy scale $\Lambda\sim 10$TeV. By making explicit the magnetic form factor, we finally arrive at:

$$F_M^B(0)=\frac{4vm_\mu}{e}\frac{c^\mu_{eZ}}{\Lambda^2}\frac{\alpha}{2\pi}\frac{3s_W^2-c_W^2}{s_Wc_W}\log\left(\frac{\Lambda}{M_Z}\right)\ , \tag{72}$$

with $\alpha=e^2/(4\pi)$.

• $\mathcal{O}_T^{\mu q}$: its contribution is shown in Fig.6(c). External momenta are taken as the previous ones. Let be k the virtual 4-momentum of the up-type quark in the loop. The Feynman rule related to the SMEFT $\mathcal{O}_T^{\mu q}$ operator is:

$$\frac{ic_T^{\mu q}}{\Lambda^2}\sigma^{\mu\nu}\sigma_{\mu\nu}P_R\ , \tag{73}$$

and its h.c. is obtained by replacing $P_R\to P_L$ (remember that we assumed real Wilson coefficients, so there is no need to take its complex conjugate). Also in this case there are 4 Feynman diagrams which contributes to $F_M(0)$: one where the Higgs vev v is taken before the interaction with the photon (following the fermion line in Fig.6) and its h.c, one where v is taken after the photon interaction (and its h.c.). Notice that, from a computational point of view, the Higgs vev insertion is equivalent to a mass insertion of the quark in the loop (modulo the $\sqrt{2}$ normalization factor). Using dimensional regularization, we find the following amplitude:

$$\begin{aligned}
\mathcal{M}_C=&-N_c\frac{2}{3}e\frac{c_T^{\mu q}}{\Lambda^2}m_q\mu^{3\epsilon}\int\frac{d^dk}{(2\pi)^d}\left[\frac{\bar{u}(p_2)\sigma_{a1,a2}\sigma^{a1,a2}u(p_1){\rm Tr}[(\not{k}+\not{q}+m_q)(\not{k}+m_q)\gamma_\mu(\not{k}+m_q)]}{[(k+q)^2-m_q^2][k^2-m_q^2][k^2-m_q^2]}\right. \\
&\left.+\frac{\bar{u}(p_2)\sigma_{a1,a2}\sigma^{a1,a2}(P_L+P_R){\rm Tr}[(\not{k}+\not{q}+m_q)\gamma_\mu(\not{k}+m_q)(\not{k}+m_q)]u(p_1)}{[(k+q)^2-m_q^2][k^2-m_q^2][k^2-m_q^2]}\right]\epsilon^\mu(q)\ ,
\end{aligned}$$

with $N_c=3$ being the quarks color number. Now, we sum up all these contributions (for $q=t,c$ even if we know that the dominant contribution is typically from the top quark) and take the magnetic moment related terms, obtaining:

$$\sum_{q=c,t}\frac{c_T^{\mu q}}{\Lambda^2}\frac{2}{\pi^2}\log\left(\frac{\Lambda}{m_q}\right)\stackrel{!}{=}\frac{F_M^C(0)}{2m_\mu}\ ,$$

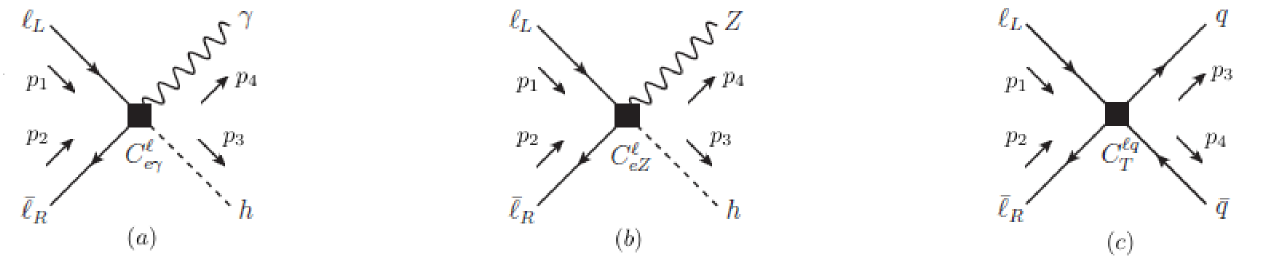


Figure 7: Feynman diagrams describing SMEFT interactions coming from operators used to correct the $(g-2)_\mu$ anomaly (see text for details). To get correction to the muon just replace $\ell = \mu$. Picture edited from Ref. [44].

from which we get:

$$F_M^C(0) = \sum_{q=c,t} \frac{c_T^{\mu q}}{\Lambda^2} \frac{4 m_\mu m_q}{\pi^2} \log\left(\frac{\Lambda}{m_q}\right) . \tag{74}$$

Before proceeding, a few comments are in order. Notice that the 1-loop contributions to F_M are log-dependent on the cut-off energy scale Λ of our theory. This should not be surprising, since the scale dependence of the Wilson coefficients is captured by the RGEs which, at the leading order, are governed by the anomalous dimensions γ_{ij} as exposed in Eq. 67. A leading order solution to these RGEs (which is enough for our purposes) leads to the following result [49]:

$$c_i(\mu) \approx \underbrace{c_i(\Lambda)}_{\text{tree-level}} \underbrace{- \frac{1}{16\pi^2} \log\left(\frac{\Lambda}{\mu}\right) \gamma_{ij} c_j(\Lambda)}_{\text{1-loop}} , \tag{75}$$

which is exactly the kind of dependence we found in our calculations. Finally, the final result is obtained by the sum $\Delta a_\mu = F_M^A(0) + F_M^B(0) + F_M^C(0)$ which explicitly reads:

$$\Delta a_\mu = \frac{4 m_\mu v}{e \Lambda^2} \left(c^\mu_{e\gamma} + \frac{\alpha}{2\pi} \frac{3 s_W^2 - c_W^2}{s_W c_W} c^\mu_{eZ} \log\left(\frac{\Lambda}{M_Z}\right)\right) - \sum_{q=c,t} \frac{c_T^{\mu q}}{\Lambda^2} \frac{4 m_\mu m_q}{\pi^2} \log\left(\frac{\Lambda}{m_q}\right) . \tag{76}$$

4.3 Linking the muon g-2 to high-energy observables

As stated in Sec.4.1, introducing new interactions to solve Δa_μ may modify also other observables. In this case we require that such a modification is not in tension with experimental results. In particular, from the interactions used in Fig.6 we may expect to have also effective Higgs couplings with fermions and vector bosons, as depicted in Fig.7 which may modify scattering cross-sections.
The aim of this subsection is to analytically evaluate the leading order contributions of the diagrams in Fig.7 to cross-sections of the kind $\mu^+\mu^- \to X$ in a hypothetical muon collider with $\sqrt{s} \gg$TeV and then compare our findings with the ones in Ref. [44].

• $\boldsymbol{\mu\bar{\mu} \to h\gamma}$**:** the process $\mu\bar{\mu} \to h\gamma$ arise from the diagram in Fig. 7(a) and its differential cross section can be written in the CM (center-of-mass) reference frame as:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{CM}} = \frac{1}{64\pi^2} \frac{|p_1'|}{|p_1|} \frac{|\mathcal{M}|^2_{\text{CM}}}{s} .$$

Since we are working in the ultra-relativistic (UR) limit, then we can take $m_\mu \sim 0 \sim m_H \ll \sqrt{s}$. Then the previous formula is further simplified as:

$$\left(\frac{d\sigma}{d\cos(\theta) d\phi}\right)_{\text{CM}} = \frac{1}{64\pi^2} \frac{|\mathcal{M}|^2_{\text{CM}}}{s} . \tag{77}$$

The 4-momenta choice is exposed in Fig.7 and the Feynman rule related to the $\mathcal{O}^\mu_{e\gamma}$ operator is the one in Eq. 69 without v (since here the Higgs field is an on-shell particle) and with a factor $1/\sqrt{2}$

because we used the Higgs field expansion: $H = v + \frac{h}{\sqrt{2}}$ where $v = 174$GeV. The Feynman amplitude reads:

$$\mathcal{M} = \frac{\sqrt{2}c^{\mu}_{e\gamma}}{\Lambda^2}\bar{v}(p_2)\sigma_{\mu\alpha}(P_R + P_L)u(p_1)p_4^{\alpha}\epsilon^{\mu}(p_4,\lambda) \ ,$$

where λ is the photon polarization versor. Thus, the unpolarized modulus square is:

$$|\mathcal{M}|^2 = \frac{2\,|c^{\mu}_{e\gamma}|^2}{\Lambda^4} \underbrace{\frac{1}{4}}_{\mu \text{ polariz.}} \mathrm{Tr}\left[\not{p}_1\sigma_{\beta\nu}\not{p}_2\sigma_{\alpha\mu}\right] p_4^{\alpha}p_4^{\beta} \underbrace{\sum \epsilon^{\mu}(p_4,\lambda)\epsilon^{\nu\star}(p_4,\lambda)}_{=-\eta^{\mu\nu}} \ . \tag{78}$$

Using the following CM 4-momenta:

$$p_1 = (\mathcal{E},\mathcal{E},0,0) \ ; \qquad p_2 = (\mathcal{E},-\mathcal{E},0,0) \ ;$$
$$p_3 = (\mathcal{E},-\mathcal{E}\cos\theta,-\mathcal{E}\sin\theta,0) \ ; \qquad p_4 = (\mathcal{E},\mathcal{E}\cos\theta,\mathcal{E}\sin\theta,0) \ ,$$

and the usual Mandelstam invariants s, t, u, we get:

$$|\mathcal{M}|^2_{\mathrm{CM}} = \frac{1}{2}\frac{|c^{\mu}_{e\gamma}|^2}{\Lambda^4}s^2\left(1-\cos^2\theta\right) \ .$$

Replacing everything in Eq. 77, we obtain:

$$\frac{d\sigma}{d\cos\theta} = \frac{1}{64\pi^2}\frac{|c^{\mu}_{e\gamma}|^2}{\Lambda^4}s\left(1-\cos^2\theta\right) \longrightarrow \sigma(\mu\mu\to h\gamma) = \frac{|c^{\mu}_{e\gamma}|^2}{48\pi}\frac{s}{\Lambda^4} \ . \tag{79}$$

Notice that $\mu\bar{\mu}\to h\gamma$ is already generated at tree level within the SM, however, the Higgs coupling to the muon scales as the Yukawa $y_\mu \sim \frac{m_\mu}{v}$, thus is very suppressed. As a fact, the SM leading contribution is due to top-quark loop where the loop suppression factor $1/(16\pi^2)\sim 10^{-2}$ is subdominant w.r.t. the top to muon mass ratio $m_t/m_\mu \sim y_t/y_\mu \sim 10^3$. We remark also that the result in Eq. 79 would be the same if we computed $\sigma(\mu\bar{\mu}\to\gamma Z)$. In fact, since we are working in the UR limit where all the SM masses could be set to zero, we can use the *Goldostone Boson Equivalence Theorem* (GBET) [50] which states that we can replace the massive vector bosons with their longitudinal degree of freedom, that is, the (massless) boson eaten from the Higgs doublet after the EWSB. But before the symmetry breaking, the Higgs SU(2) doublet is $H = (\phi_+, v + \frac{h+i\phi_0}{\sqrt{2}})^T$ and ϕ_0 (the Z-boson longitudinal mode) couples exactly with the same coefficient of h.

• $\boldsymbol{\mu\bar{\mu}\to hZ}$: the process $\mu\bar{\mu}\to hZ$ is shown in the Feynman diagram Fig.7(b). The computational procedure is similar to the previous case, so we will skip some intermediate steps. The relevant Feynman rule is related to Eq. 71 with the caveats mentioned above. Remembering that the sum over all the Z-boson polarizations is:

$$\sum \epsilon_\mu(p_4,\lambda)\epsilon^{\star}_{\nu}(p_4,\lambda) = -\left(\eta_{\mu\nu} - \frac{p_{4,\mu}p_{4,\nu}}{M_Z^2}\right) \ ,$$

the unpolarized amplitude square reads:

$$|\mathcal{M}|^2_{\mathrm{CM}} = -\frac{|c^{\mu}_{eZ}|^2}{\Lambda^4}\mathrm{Tr}\left[\not{p}_1\sigma_{\beta\nu}\not{p}_2\sigma_{\alpha\mu}\right]p_4^{\beta}p_4^{\alpha}\left(\eta_{\mu\nu}-\frac{p_{4,\mu}p_{4,\nu}}{M_Z^2}\right) = \frac{1}{2}\frac{|c^{\mu}_{eZ}|^2}{\Lambda^4}s^2(1-\cos^2\theta) \ .$$

By replacing this result in Eq. 77 we get:

$$\frac{d\sigma}{d\cos\theta} = \frac{1}{64\pi^2}\frac{|c^{\mu}_{eZ}|^2}{\Lambda^4}s\left(1-\cos^2\theta\right) \longrightarrow \sigma(\mu\bar{\mu}\to hZ) = \frac{|c^{\mu}_{eZ}|^2}{48\pi}\frac{s}{\Lambda^4} \ . \tag{80}$$

Notice that this is exactly the same result of Eq. 79 but the replacement $c^{\mu}_{e\gamma}\to c^{\mu}_{eZ}$: this is also a consequence of GBET at high energies.

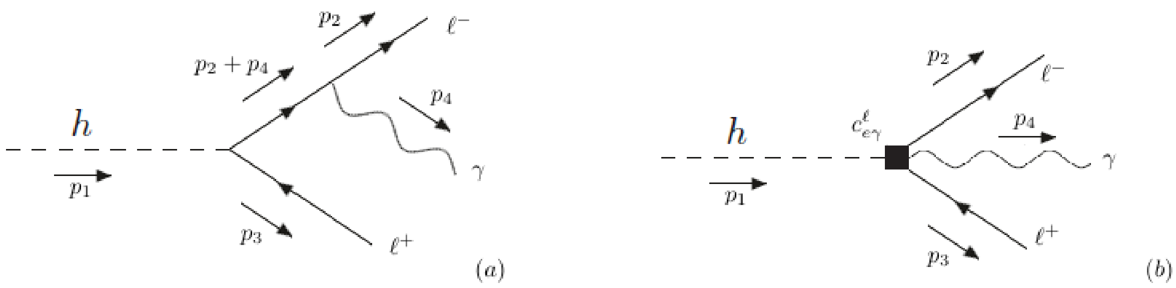


Figure 8: (a): Tree-level Feynman diagram of the decay process $h \to \ell^+\ell^-\gamma$ using pure SM interactions. Another possibility for the photon is to couple with the other fermion (see text). (b): Tree-level contribution of the SMEFT operator $\mathcal{O}^\ell_{e\gamma}$ to $h \to \ell^+\ell^-\gamma$ decay (its h.c. is understood). Effective interactions are denoted by a solid square.

• $\boldsymbol{\mu\bar{\mu} \to q\bar{q}}$: the process $\mu\bar{\mu} \to q\bar{q}$ is shown in the Feynman diagram of Fig. 7(c). We report the result for the generic up-type quark $q = u, c, t$: it is clear that for our purposes, the top quark would be the leading contribution followed by the charm. Starting from the Feynman rule in Eq. 73, we compute the unpolarized amplitude square in the CM reference frame:

$$|\mathcal{M}|^2_{\text{CM}} = \frac{\left|c_T^{\mu q}\right|^2}{\Lambda^4}\frac{1}{4}\text{Tr}\left[\not{p}_1\sigma_{\alpha\beta}\not{p}_2\sigma_{\mu\nu}\right]\text{Tr}\left[\not{p}_4\sigma^{\alpha\beta}\not{p}_3\sigma^{\mu\nu}\right] = 8\frac{\left|c_T^{\mu q}\right|^2}{\Lambda^4}s^2\cos^2\theta \ .$$

Thus:

$$\frac{d\sigma}{d\cos\theta} = \frac{N_c}{4\pi}\frac{\left|c_T^{\mu q}\right|^2}{\Lambda^4}s\cos^2\theta \longrightarrow \sigma(\mu\bar{\mu} \to q\bar{q}) = \frac{N_c}{6\pi}\left|c_T^{\mu q}\right|^2\frac{s}{\Lambda^4} \ . \tag{81}$$

Notice that Eqs. 79-81 lead to cross sections linearly growing with the center of mass energy squared s, therefore spoiling the unitarity bounds at high energies. Actually, this is exactly what happens in EFT, as one can easily see from cross-sections computed in the Fermi effective theory of weak interactions, where $\sigma \propto G_F^2 s$: in our case the Fermi constant is replaced by the Wilson coefficient c_i of interest and the information of heavy new particles is hidden in the NP scale Λ. At tree-level, when no RGE applies, we can safely assume $\Lambda \sim M_{\text{NP}}$ and thus $\sigma \propto M_{\text{NP}}^{-4}$ which is the contribution coming from the (NP) virtual heavy particle propagating between initial and final states. At leading order the interaction is punctual[9] (a Dirac delta in the 4-coordinates), so the effective representations in Fig.7 are justified; at next-to-leading order the heavy particle propagator can be expanded around $s/M^2_{\text{NP}} \ll 1$ generating a series of derivative operators whose mass-dimension is greater than 6 (and so we neglect them, since we are considering dim-6 SMEFT theory). Therefore the unitarity is not broken since $\sigma \propto s/M^4_{\text{NP}}$ only for $s \ll M^2_{\text{NP}}$ otherwise the effective approach is not applicable.

• $\boldsymbol{h \to \ell^+\ell^-\gamma}$: we want to highlight how the operator $\mathcal{O}^\ell_{e\gamma} = (\bar{\ell}_L\sigma^{\mu\nu}e_R)HF_{\mu\nu}$ not only contributes to cross-sections as in Fig.7 but also to decay width as in Fig.8. To do so, we analytically compute the new physics contribution to the partial decay width $\Gamma(h \to \ell^+\ell^-\gamma)$ coming from Fig.8(b) and the interference between Fig.8(a) and Fig.8(b). However, remind that the SM one-loop top contribution is not subleading thanks to the enhancement factor of 10^3 given by the top-to-muon mass ratio.

The Feynman amplitudes read:

$$\begin{aligned}
\mathcal{M}^{\text{SM}}_A &= -ie\frac{m_\ell}{v}\bar{u}(p_2)\gamma_\mu\frac{\not{p}_2 + \not{p}_4 + m_\ell}{(p_2+p_4)^2 - m_\ell^2}v(p_3)\epsilon^\mu(p_4,\lambda) \ ; \\
\mathcal{M}^{\text{NP}}_B &= -\frac{\sqrt{2}c^\ell_{e\gamma}}{\Lambda^2}\bar{u}(p_2)\sigma_{\mu\alpha}v(p_3)p_4^\alpha\epsilon^\mu(p_4,\lambda) \ .
\end{aligned}$$

[9] To see this, given a UV theory, one can integrate out the heavy fields through the EOM method and then write the Wilsonian effective action: the heavy propagator will naturally emerge since, by definition, is the Green function of the free-theory operator which we use to get rid of the field. Alternatively, one can consider the indetermination principle $\Delta\mathcal{E}\Delta t \lesssim \hbar$: for a process with $\mathcal{E} \ll M_{\text{NP}}$ the allowed excitation of the heavy field must last $t \to 0$. Therefore it cannot propagate since the maximum speed is limited by c, thus the punctual interaction in the Minkowski spacetime.

In order to get the decay width, we must compute the unpolarized amplitude square $|\mathcal{M}|^2_{\text{tot}} = |\mathcal{M}_A + \mathcal{M}_B|^2$, which can be expressed as:

$$|\mathcal{M}|^2_{\text{tot}} = \left|\mathcal{M}_A^{\text{SM}}\right|^2 + \left|\mathcal{M}_B^{\text{NP}}\right|^2 + 2\text{Re}\left(M_B^{\text{NP}} M_A^{\text{SM}\star}\right) \ ,$$

hence, to get the NP contributions, we need to compute the second and third terms of equation which give rise to the following differential decay rate:

$$d\Gamma = \frac{|\mathcal{M}_B^{\text{NP}}|^2 + 2\text{Re}\left(M_B^{\text{NP}} M_A^{\text{SM}\star}\right)}{2M_H} d\phi^{(3)} \equiv d\Gamma_1 + d\Gamma_2 \ , \tag{82}$$

where $d\phi^{(3)}$ is the 3-body phase space. In the UR limit for the final states (which is a good assumption since $M_H \gg m_\ell$) it can be written as:

$$d\phi^{(3)} = (2\pi)^4 \delta^{(4)}\left(p_1 - \sum_{i=2}^{4} p_i\right) \prod_{i=2}^{4} \frac{d^3 p_i}{(2\pi)^3 2\mathcal{E}_i} \ . \tag{83}$$

Focusing on $d\Gamma_1$, the NP unpolarized amplitude square evaluated in the Higgs rest reference frame reads:

$$\left|\mathcal{M}_B^{\text{NP}}\right|^2 = -\frac{2|c_{e\gamma}^\ell|^2}{\Lambda^4} \text{Tr}\left[\not{p}_2 \sigma_{\beta\mu} \not{p}_3 \sigma_{\alpha\mu}\right] p_4^\beta p_4^\alpha = 32 \frac{|c_{e\gamma}^\ell|^2}{\Lambda^4} (p_2 p_4)(p_3 p_4) \ ,$$

and

$$d\Gamma_1 = \frac{|c_{e\gamma}^\ell|^2}{16\pi^5 \Lambda^4 M_H} (p_2 p_4)(p_3 p_4) \delta^{(4)}\left(p_1 - \sum_{i=2}^{4} p_i\right) \prod_{i=2}^{4} \frac{d^3 p_i}{\mathcal{E}_i} \ .$$

To integrate the 3-body phase space, we use the *integral scalar decomposition method.* First, we consider the CM reference frame of the leptons in the final state. Let use define:

$$I^{\mu\nu}(q) = \int d^3 p_2 d^3 p_3 \frac{p_2^\mu p_3^\nu}{\mathcal{E}_3 \mathcal{E}_4} \delta^{(4)}(q - p_2 - p_3) \ , \tag{84}$$

with $q = p_1 - p_2$. We can decompose the previous integral, in full generality, as:

$$I^{\mu\nu}(q) = A(q)\eta^{\mu\nu} + B(q) \frac{q^\mu q^\nu}{q^2} \ ,$$

so that

$$\begin{aligned} I_1(q) &\equiv \eta_{\mu\nu} I^{\mu\nu}(q) = 4A(q) + B(q) \ ; \\ I_2(q) &\equiv q_\mu q_\nu I^{\mu\nu}(q) = q^2[A(q) + B(q)] \end{aligned}$$

are Lorentz scalar integrals, so they does not depend on the chosen reference frame. Applying those definitions in Eq. 84 led us to:

$$I_1(q) = \pi q^2 \ ; \qquad I_2(q) = \frac{\pi}{2} q^4 \ ,$$

thus:

$$I^{\mu\nu}(q^2) = \frac{\pi}{6} \eta^{\mu\nu} q^2 + \frac{\pi}{3} q^\mu q^\nu \ .$$

Finally we can compute $d\Gamma_1$:

$$\begin{aligned} d\Gamma_1 &= (\text{integrate out } p_2, p_3) = \frac{|c_{e\gamma}^\ell|^2}{16\pi^5 \Lambda^4 M_H} p_4^\alpha p_4^\beta \frac{d^3 p_4}{\mathcal{E}_4} I_{\alpha\beta}(q^2) \\ &= \frac{|c_{e\gamma}^\ell|^2}{16\pi^5 \Lambda^4 M_H} \mathcal{E}_4 d\mathcal{E}_4 d\Omega_\gamma \frac{\pi}{6} [q^2 \underbrace{p_4^2}_{=0} + 2(p_4 q)(p_4 q)] \overset{(\star)}{=} \frac{|c_{e\gamma}^\ell|^2}{12\pi^3 \Lambda^4} M_H \mathcal{E}^3 d\mathcal{E}_4 \ , \end{aligned}$$

Process	NP contribution [ab]	pure SM contribution [ab]
$\sigma(\mu\bar{\mu}\to h\gamma)$	$0.07\left(\frac{\sqrt{s}}{10\text{TeV}}\right)^2\left(\frac{\Delta a_\mu}{3\times10^{-9}}\right)^2$	$2\times10^{-3}\left(\frac{10\text{TeV}}{\sqrt{s}}\right)^2$
$\sigma(\mu\bar{\mu}\to hZ)$	$38\left(\frac{\sqrt{s}}{10\text{TeV}}\right)^2\left(\frac{\Delta a_\mu}{3\times10^{-9}}\right)^2$	$122\left(\frac{10\text{TeV}}{\sqrt{s}}\right)^2$
$\sigma(\mu\bar{\mu}\to t\bar{t})$	$58\left(\frac{\sqrt{s}}{10\text{TeV}}\right)^2\left(\frac{\Delta a_\mu}{3\times10^{-9}}\right)^2$	$1700\left(\frac{10\text{TeV}}{\sqrt{s}}\right)^2$

Table 1: Comparison of the contribution between SMEFT operators and pure SM prediction to different muon scattering processes. The NP contributions are extrapolated from the results found (see text), setting $\Lambda \approx$ 10TeV; the SM ones are given in [44].

where in the step $(\star)$ we used the kinematics in the Higgs rest frame $(p_4 q) \sim M_H \mathcal{E}_\gamma = M_H \mathcal{E}_4$ and we integrate out the photon angular distribution $\int d\Omega_\gamma = 4\pi$. Now we need to integrate over all the possible photon energies in the final state. It is straightforward to see that the minimum energy is $\mathcal{E}_{4,\min} = 0$ and the maximum $\mathcal{E}_{4,\max} = M_H/2$ for 4-momentum conservation. Thus:

$$\Gamma_1 = \frac{|c^\ell_{e\gamma}|^2}{12\pi^3\Lambda^4} M_H \int_0^{M_H/2} \mathcal{E}_4^3 d\mathcal{E}_4 = \frac{|c^\ell_{e\gamma}|^2 M_H^5}{768\pi^3\Lambda^4} \ . \tag{85}$$

To compute $d\Gamma_2$ we adopt the same strategy and notation as for $d\Gamma_1$ so we will skip some intermediate steps. First, notice that Fig.8(a) depicts only one SM contribution: the other one is when the photon propagator is attached to the other external fermion line. The sum of these two contributions is (we also take the complex conjugate since it appears in Eq. 82):

$$\mathcal{M}_A^{\text{SM}\star} = ie\frac{m_\ell}{v}\bar{v}(p_3)\left[\gamma_\mu \frac{\not{p}_3 + \not{p}_4}{(p_3+p_4)^2} - \frac{\not{p}_2 + \not{p}_4}{(p_2+p_4)^2}\gamma_\mu\right] u(p_2)\epsilon^{\mu\star}(p_4,\lambda) \ ;$$

hence the interference between SM and SMEFT amplitudes:

$$\begin{aligned}
\mathcal{M}_A^{\text{SM}\star}\mathcal{M}_B^{\text{NP}} &= -\frac{i\sqrt{2}em_\ell}{v}\frac{c^\ell_{e\gamma}}{\Lambda^2}\left[\frac{\text{Tr}[\not{p}_3(\not{p}_2+\not{p}_4)\gamma_\mu \not{p}_2\sigma_{\nu\alpha}]}{(p_2+p_4)^2} - \frac{\text{Tr}[\not{p}_3\gamma_\mu(\not{p}_3+\not{p}_4)\not{p}_2\sigma_{\nu\alpha}]}{(p_3+p_4)^2}\right](-\eta^{\mu\nu}) \\
&= 8\sqrt{2}\frac{em_\ell}{v}\frac{c^\ell_{e\gamma}}{\Lambda^2}[(p_2p_3)+(p_3p_4)+(p_2p_4)] \ .
\end{aligned}$$

The differential decay rate reads:

$$\begin{aligned}
d\Gamma_2 &= \frac{\sqrt{2}}{32\pi^5}\frac{em_\ell}{vM_H}\frac{\text{Re}[c^\ell_{e\gamma}]}{\Lambda^2}[(p_2p_3)+(p_3p_4)+(p_2p_4)]\delta^{(4)}\left(p_1 - \sum_{i=2}^4 p_i\right)\prod_{i=2}^4 \frac{d^3p_i}{\mathcal{E}_i} = \text{scalar integrals} \\
&= \frac{\sqrt{2}}{32\pi^5}\frac{em_\ell}{vM_H}\frac{\text{Re}[c^\ell_{e\gamma}]}{\Lambda^2}\left[I_1(q^2) + I_1(q^2)\frac{p_4 q}{q^2/2}\right]\frac{d^3p_4}{\mathcal{E}_4} = \frac{\sqrt{2}}{8\pi^3}\frac{em_\ell}{v}\frac{\text{Re}[c^\ell_{e\gamma}]}{\Lambda^2}M_H\mathcal{E}_4 d\mathcal{E}_4 \ ,
\end{aligned}$$

thus:

$$\Gamma_2 = \frac{\sqrt{2}}{8\pi^3}\frac{em_\ell}{v}\frac{\text{Re}[c^\ell_{e\gamma}]}{\Lambda^2}M_H\int_0^{M_H/2}\mathcal{E}_4 d\mathcal{E}_4 = \frac{\sqrt{2}em_\ell M_H^3}{64\pi^3 v}\frac{\text{Re}[c^\ell_{e\gamma}]}{\Lambda^2} \ , \tag{86}$$

where $v = 246$GeV has been used (just replace $v \to \sqrt{2}v$ in the final equation to get the result with $v = 176$GeV). Finally $\Gamma(h \to \ell^+\ell^-\gamma)$ is given by adding together Eqs. 85-86. Notice that the final expression depends only on the $c^\ell_{e\gamma}$ coefficient. Therefore, we can relate $c^\mu_{e\gamma}$ with Δa_μ by using Eq. 70. Let us now suppose that the anomalous magnetic moment of the muon is entirely corrected by $\mathcal{O}^\mu_{e\gamma}$. Then we can re-express the new-physics contribution to the decay width as:

$$\Gamma(h \to \ell^+\ell^-\gamma)_{\text{NP}} = \frac{\alpha M_H^3 \Delta a_\mu}{64\pi^2 v^2}\left(\frac{M_H^2}{48m_\mu^2} + 1\right) \ .$$

Now, setting $\alpha = 1/137$, $v = 174$GeV, $M_H = 125$GeV, $\Delta a_\mu = 250 \times 10^{-11}$ and $m_\mu = 0.105$GeV, we get: $\Gamma_{\text{NP}} \approx 2 \times 10^{-9}$MeV. Remembering the total Higgs decay width experimentally measured

$\Gamma(h \to X) \approx 3$MeV we get a branching ratio of the NP contribution of $O(10^{-9})$ which is at least 4 order of magnitude less than the today experimental resolution [6]: as for now, this rare decay mode cannot be used to significantly constrain the SMEFT parameter space. Following the same logic, we can also re-write the cross-sections found in Eqs. 79-81 as a function of Δa_μ to see if NP effects are nowadays detectable. The main results are reported in Table 1. Also in this case we notice that all the NP contributions are too small to be detected: they are of order $O(10)$ab while the integrated luminosity of LHC runs (aimed to discover new heavy particles and rare events) is only $O(10)\text{fb}^{-1}$. New multi-TeV muon colliders seem to be the most promising way in the foreseeable future to increase the luminosity and the available energy in the CM of collisions [51].

5 New Physics scenarios

Until now, we worked only in a model-independent way through the systematic use of SMEFT to see which effective operators may explain M_W anomaly and Δa_μ individually. Now, we want to identify a common scenario which potentially can explain both the discrepancies (this will be the heart of our work) starting from the results found in previous sections, and then implementing them in a concrete UV completion of the SM: the two-Higgs doublet model (2HDM).

5.1 BSM heavy particles

UV completions of a model are mainly characterized by two factors: *(i)* the (eventual) extension of the symmetry group of the theory and *(ii)* the number and properties of the new field content, especially how they transform under the symmetries identified in *(i)*. For our purposes, we will not extend the SM symmetry group nor the Lorentz one, so our UV completion will be invariant under $(SU(3)_c \times SU(2)_L \times U(1)_Y) \oplus \mathcal{P}_+^\uparrow$[10]. There are several reviews which classify the new particle content according to their transformation properties (e.g., see Ref. [52]): in Table 2 we report only few cases which will be useful for our discussion.

Particle	$\mathcal{P}_+^\uparrow$	$SU(3)_c \times SU(2)_L \times U(1)_Y$	SMEFT ops.
ω_1	scalar	(3,1,-1/3)	$\mathcal{O}_T^{\mu q}$
Π_7	scalar	(3,2,7/6)	$\mathcal{O}_T^{\mu q}$
Δ_1	Dirac fermion	(1,2,-1/2)	$\mathcal{O}_{eB}^{\mu}$, $\mathcal{O}_{eW}^{\mu}$
E	Dirac fermion	(1,1,-1)	$\mathcal{O}_{eB}^{\mu}$
Σ_1	Dirac fermion	(1,3,-1)	$\mathcal{O}_{eW}^{\mu}$
$\mathcal{B}$	vector	(1,1,0)	-
$\mathcal{L}_1$	vector	(1,2,1/2)	$\mathcal{O}_{eB}^{\mu}$, $\mathcal{O}_{eW}^{\mu}$

Table 2: List of some possible new particles in a extension of the SM. The first column uses the same notation of [52], the second indicates the transformation property under the proper orthocronous Poincaré group, the third, using the notation (A, B, C), describes the transformation under SU(3), SU(2), U(1) respectively (in particular, given a SU(N) group, transforming as N [resp. N^2-1] is equivalent to the fundamental [resp. adjoint] representation) and the fourth lists the SMEFT operators generated at tree-level and significant at high-energies for Δa_μ.

Let us start by considering $(g-2)_\mu$. From Eq. 68 and Eq. 76 we saw that only 3 operators significantly contribute to the muon anomaly. To get a first idea of viable NP scenarios, we look for heavy particles which may generate already at tree-level the above mentioned operators[11]. Those particles are listed in Table 2. By looking at how they transform under the gauge group, we expect that the operator $\mathcal{O}_T^{\mu q}$ is generated by ω_1 or Π_7 since they non-trivially transform under $\mathrm{SU}(3)_c$. The other four particles ($\mathcal{B}$ is excluded) contribute both to $\mathcal{O}_{eB}^{\mu}$ and $\mathcal{O}_{eW}^{\mu}$, fact that does not emerge immediately but only integrating out these fields using leading order EOMs (see Ref. [52], Appendix D). Now, we wonder how many of the particles listed in Table 2 generate *also* the $\hat{S}$, $\hat{T}$ oblique parameters introduced in Sec.3.2.1 in order to justify the CDF-II measurement too. To answer this question, we need to find those particle that at tree-level can generate the Wilson coefficients c_{HWB} and/or c_{HD} related to $\mathcal{O}_{HWB} = \left(H^\dagger \tau_I H\right) W^I_{\mu\nu} B^{\mu\nu}$ and $\mathcal{O}_{HD} = \left(H^\dagger D_\mu H\right)\left((D^\mu H)^\dagger H\right)$ respectively, since those are the

[10] The direct sum is a consequence of the *Coleman-Mandula* theorem which states that the generators of the internal symmetry group and the Poincaré group must commute. Only supersymmetric scenarios, with the introduction of Grassman-odd generators Q_α - namely supercharges - provide a non-trivial enlargement of the symmetry group, but this is beyond the scope of the present work.

[11] Notice that this approach is valid only for rough estimates but not for precise results since $(g-2)_\mu$ was computed at loop-level precision and inferences using only this tree-level matching would be inconsistent in our perturbative treatment.

leading contributions to $\hat{S}$, $\hat{T}$ as saw in Eq. 32 and Eq. 33. From the tree-level dictionary, we read:

$$\begin{aligned} Z_\phi C_{\phi WB} \equiv Z_H c_{HWB} \;&=\; -\frac{gg'(\gamma_{\mathcal{L}_1})^\star_r(\gamma_{\mathcal{L}_1})_r}{4M^4_{\mathcal{L}_{1,r}}} - \frac{g(g^B_{\mathcal{L}_1})_{rs}(\gamma_{\mathcal{L}_1})^\star_r(\gamma_{\mathcal{L}_1})_s}{4M^2_{\mathcal{L}_{1,r}}M^2_{\mathcal{L}_{1,s}}} - \frac{g'(g^W_{\mathcal{L}_1})_{rs}(\gamma_{\mathcal{L}_1})^\star_r(\gamma_{\mathcal{L}_1})_s}{4M^2_{\mathcal{L}_{1,r}}M^2_{\mathcal{L}_{1,s}}} \\ &+\frac{1}{f}\left[-\frac{\mathrm{Im}\left((\tilde{\gamma}^B_{\mathcal{L}_1})_r(\gamma_{\mathcal{L}_1})^\star_r\right)g}{2M^2_{\mathcal{L}_{1,r}}} - \frac{\mathrm{Im}\left((\tilde{\gamma}^W_{\mathcal{L}_1})_r(\gamma_{\mathcal{L}_1})^\star_r\right)g'}{2M^2_{\mathcal{L}_{1,r}}}\right] + \dots \,, \end{aligned} \tag{87}$$

where g, g' refer to the SM gauge couplings for SU(2) and U(1) respectively, f is an effective energy scale for dimension-five operator contributions in the UV theory and all the other couplings of the kind g^j_i or γ_i refers to NP couplings of new particles between themselves or the SM ones (we will explicit later only the couplings we need). The dots indicate the presence of other terms generated by particles that are not listed in Table 2. Notice that, together with the Wilson coefficient we also considered the renormalization of the Higgs field Z_H. This is similar to what done in Appendix B.1 and C.1 for the Higgs sector: this time, its kinetic term is not canonically normalized because of interactions with NP particles, therefore a field shift $H \to Z_H^{-1/2}H$ is in order. Similarly, we read the c_{HD} coefficient:

$$\begin{aligned} Z^2_\phi C_{\phi D} \equiv Z^2_H c_{HD} \;&=\; \frac{g'(g^B_{\mathcal{L}_1})_{rs}(\gamma_{\mathcal{L}_1})^\star_r(\gamma_{\mathcal{L}_1})_s}{M^2_{\mathcal{L}_{1,r}}M^2_{\mathcal{L}_{1,s}}} - \frac{\left(h^{(2)}_{\mathcal{L}_1}\right)_{rs}(\gamma_{\mathcal{L}_1})^\star_r(\gamma_{\mathcal{L}_1})_s}{M^2_{\mathcal{L}_{1,r}}M^2_{\mathcal{L}_{1,s}}} + \frac{2\mathrm{Re}\left(\left(h^{(3)}_{\mathcal{L}_1}\right)_{rs}(\gamma_{\mathcal{L}_1})^\star_r(\gamma_{\mathcal{L}_1})^\star_s\right)}{M^2_{\mathcal{L}_{1,r}}M^2_{\mathcal{L}_{1,s}}} \\ &-\frac{\mathrm{Re}\left((\hat{g}^H_{\mathcal{B}})_r(\hat{g}^H_{\mathcal{B}})_r\right)}{M^2_{\mathcal{B}_r}} - \frac{(\hat{g}^H_{\mathcal{B}})^\star_r(\hat{g}^H_{\mathcal{B}})_r}{M^2_{\mathcal{B}_r}} + \dots \,, \end{aligned} \tag{88}$$

with the same notation as before. From the equations above, we expect that $\mathcal{L}_1$ contributes both to $\hat{S}$ and $\hat{T}$ while $\mathcal{B}$ contributes only to $\hat{T}$. At this point, we still cannot conclude how they contribute to the M_W anomaly, so further considerations are carried out in the next two subsections.

5.1.1 $\mathcal{B} \sim (1,1,0)$ vector-like particle

Let us assume in our scenario that $\mathcal{B}$ is the only NP particle. We add to the SM Lagrangian also the following terms:

$$\begin{aligned} \mathcal{L}_{\mathrm{NP}} = \mathcal{L}^{(2)}_{\mathcal{B}} + \mathcal{L}^{\mathrm{int}}_{\mathcal{B}} \;&\ni\; \frac{1}{2}\left[(D_\mu\mathcal{B}_\nu)^\dagger D^\nu\mathcal{B}^\mu - (D_\mu\mathcal{B}_\nu)^\dagger D^\mu\mathcal{B}^\nu + M^2_{\mathcal{B}}\mathcal{B}^\dagger_\mu\mathcal{B}^\mu\right] \\ &-g^H_{\mathcal{B}}\mathcal{B}^\mu H^\dagger iD_\mu H - \mathcal{B}^\mu\left[(g^L_{\mathcal{B}})_{ij}\bar{\ell}_{L,i}\gamma_\mu\ell_{L,j} + (g^e_{\mathcal{B}})_{ij}\bar{e}_{R,i}\gamma_\mu e_{R,j}\right] + \text{h.c.} \,, \end{aligned} \tag{89}$$

where in the fist line we wrote the quadratic terms, in the second line the interactions of our interest (i.e., the ones that appear in Eq. 88 and the one relevant for the $(g-2)$ as we will see in a moment). If $\mathcal{B}$ is the only new particle, then c_{HD} can be written as:

$$c_{HD} = -2\frac{\mathrm{Re}\left(g^H_{\mathcal{B}}\right)^2}{M^2_{\mathcal{B}}} \,,$$

hence

$$\hat{T} \overset{\text{Eq. 33}}{=} -\frac{v^2}{2}c_{HD} = \frac{\mathrm{Re}\left(g^H_{\mathcal{B}}\right)^2 v^2}{M^2_{\mathcal{B}}} > 0 \,. \tag{90}$$

Therefore, $\mathcal{B}$ provides a positive deformation of the $\hat{T}$ parameter which is exactly what we need to justify the CDF-II measurement (remember that $\hat{S}$ is preferred positive, but also $\hat{T} > 0$ and $\hat{S} = 0$ can justify the M_W anomaly). What about Δa_μ? $\mathcal{B}$ do not generate at tree-level operators in Eq. 68, however we notice that its transformation properties are very similar to the Z-boson ones. In particular $\mathcal{B}$ is a color and $SU(2)_L$ singlet and its hypercharge is zero, meaning that it is also electrically neutral. Moreover, from the second line of Eq. 89 we notice that its couplings with SM fermions are really similar to the Z-boson ones (just replace $gg^f_L/c_W \to g^L_{\mathcal{B}}$ and $gg^f_R/c_W \to g^e_{\mathcal{B}}$). Therefore we can expect that $\mathcal{B}$ gives a 1-loop level contribution to Δa_μ with a diagram analogous to Fig.5 where instead of

Z there is a $\mathcal{B}$ propagating in the loop. Hence, the result previously found in Eq. 56 turns out to be applicable also in this context with few changes. The loop Amplitude reads:

$$\begin{aligned}\mathcal{M} &= -\sum_{i=e,\mu,\tau} e\mu^{3\epsilon}\int\frac{d^dk}{(2\pi)^d}\bar{u}(p_2)\gamma_{a2}\left((g^L_{\mathcal{B}})_{\mu i}P_L+(g^e_{\mathcal{B}})_{\mu i}P_R\right)\frac{(\not{p}_2+\not{k})+m_i}{(p_2+k)^2-m_i^2}\gamma_\mu\frac{(\not{p}_1+\not{k})+m_i}{(p_1+k)^2-m_i^2}\gamma_{a1}\times\\ &\quad\times\left((g^L_{\mathcal{B}})^\star_{\mu i}P_L+(g^e_{\mathcal{B}})^\star_{\mu i}P_R\right)u(p_1)\frac{1}{k^2-M^2_{\mathcal{B}}}\left(\eta^{a1,a2}-\frac{k^{a1}k^{a2}}{M^2_{\mathcal{B}}}\right)\epsilon^\mu(q,\lambda)\ . \end{aligned}\tag{91}$$

Let us assume for simplicity that $(g^L_{\mathcal{B}})_{ij}$, $(g^e_{\mathcal{B}})_{ij}\in\mathbb{R}$. By expanding the previous integral around $m_i/M_{\mathcal{B}}\ll 1$ and selecting only terms contributing to $(g-2)_\mu$ (we use again Package-X) we find:

$$\Delta a^{\mathcal{B}}_\mu=\sum_{i=e,\mu,\tau}\frac{m_\mu}{12\pi^2M^2_{\mathcal{B}}}\left[3(g^e_{\mathcal{B}})_{\mu i}(g^L_{\mathcal{B}})_{\mu i}m_i-\left((g^e_{\mathcal{B}})^2_{\mu i}+(g^L_{\mathcal{B}})^2_{\mu i}\right)m_\mu\right]\ .\tag{92}$$

Notice that, in general, $g_{\mathcal{B}}$ may not be diagonal in the lepton flavour space, hence in Fig.5 it is possible for electrons and taus to run in the virtual fermion lines inside the loop. This has consequences in flavour physics since already at tree-level the coupling $\mathcal{B}\bar{\ell}_1\ell_2$ can lead to lepton number violation. To solve the issue, one may set $g_{\mathcal{B}}$ to be diagonal so that only the muon can run in the loop, simplifying the previous result in:

$$\Delta a^{\mathcal{B}}_\mu=-\frac{m^2_\mu}{12\pi^2M^2_{\mathcal{B}}}\left((g^e_{\mathcal{B}})^2-3g^e_{\mathcal{B}}g^L_{\mathcal{B}}+(g^L_{\mathcal{B}})^2\right)\ ,$$

where the subscript $\mu\mu$ in $g_{\mathcal{B}}$ is understood. Assuming all the couplings of $O(1)$, we have that to get $\Delta a_\mu\approx 250\times10^{-11}$, we need $M_{\mathcal{B}}\sim 200$GeV. Replacing this estimate in Eq. 90, we obtain $\hat{T}\sim 1$ which is too large: we would require an higher mass for $\mathcal{B}$ in the multi-TeV scale. Hence, using this simplified model, is hard to accommodate both $(g-2)_\mu$ and ΔM_W.

5.1.2 $\mathcal{L}_1\sim(1,2,1/2)$ vector-like particle

Now, let us assume that $\mathcal{L}_1$ is the only NP particle (in this subsection we will call it $\mathcal{L}$ for simplicity). We add to the SM Lagrangian the following terms:

$$\begin{aligned}\mathcal{L}_{\rm NP} &= \mathcal{L}^{(2)}_{\mathcal{L}}+\mathcal{L}_{\mathcal{L}}\ni k\left[(D_\mu\mathcal{L}_\nu)^\dagger D^\nu\mathcal{L}^\mu-(D_\mu\mathcal{L}_\nu)^\dagger D^\mu\mathcal{L}^\nu+M^2_{\mathcal{L}}\mathcal{L}^\dagger_\mu\mathcal{L}^\mu\right]+\\ &\quad-(\gamma_{\mathcal{L}}\mathcal{L}^\dagger_\mu D^\mu H+\text{h.c.})-ig^B_{\mathcal{L}}\mathcal{L}^\dagger_\mu\mathcal{L}_\nu B^{\mu\nu}-ig^W_{\mathcal{L}}\mathcal{L}^\dagger_\mu\tau^I\mathcal{L}_\nu W^{\mu\nu}_I-h^{(2)}_{\mathcal{L}}\mathcal{L}^\dagger_\mu HH^\dagger\mathcal{L}^\mu\\ &\quad-(h^{(3)}_{\mathcal{L}}(\mathcal{L}^\dagger_\mu H)^2+\text{h.c.})-\frac{1}{f}\left(\mathcal{L}^{\mu\dagger}\left[(\tilde{g}^{eD\ell}_{\mathcal{L}})_{ij}\bar{e}_{R,i}D_\mu\ell_{L,j}+(\tilde{g}^{De\ell}_{\mathcal{L}})_{ij}D_\mu\bar{e}_{R,i}\ell_{L.j}\right]+\text{h.c.}\right)\ ,\end{aligned}\tag{93}$$

with $k=1/2$ if $\mathcal{L}$ is real, $k=1$ if complex. Notice that $\mathcal{L}$ couples to the Higgs through derivative couplings, so the renormalization Z_H must depend also on $\gamma_{\mathcal{L}}$. This is possible because $\mathcal{L}$ transforms under the SM gauge group as the Higgs doublet, so we can replace $H\to\mathcal{L}$ in some SM interactions without breaking gauge invariance (obviously H is a scalar while $\mathcal{L}$ is a vector, so one needs to contract properly all the Lorentz indexes). From Eqs. 87-88, we read the following corrections:

$$c_{HD}=\frac{1}{M^4_{\mathcal{L}}}\left(g'g^B_{\mathcal{L}}|\gamma_{\mathcal{L}}|^2-(h^{(2)}_{\mathcal{L}})|\gamma_{\mathcal{L}}|^2+2\text{Re}\left((h^{(3)}_{\mathcal{L}})\gamma^\star_{\mathcal{L}}\gamma^\star_{\mathcal{L}}\right)\right)\ ,$$

hence

$$\hat{T}\overset{\text{Eq. 33}}{=}-\frac{v^2}{2}c_{HD}=-\frac{v^2}{2M^4_{\mathcal{L}}}\left(g'g^B_{\mathcal{L}}|\gamma_{\mathcal{L}}|^2-(h^{(2)}_{\mathcal{L}})|\gamma_{\mathcal{L}}|^2+2\text{Re}\left((h^{(3)}_{\mathcal{L}})\gamma^\star_{\mathcal{L}}\gamma^\star_{\mathcal{L}}\right)\right)\ .\tag{94}$$

Analogously

$$c_{HWB}=-\frac{|\gamma_{\mathcal{L}}|^2}{4M^4_{\mathcal{L}}}\left(gg'+gg^B_{\mathcal{L}}+g'g^W_{\mathcal{L}}\right)\ ,$$

therefore

$$\hat{S}\overset{\text{Eq. 32}}{=}\frac{c_W}{s_W}v^2c_{HWB}=-\frac{c_W}{s_W}\frac{|\gamma_{\mathcal{L}}|^2v^2}{4M^4_{\mathcal{L}}}\left(gg'+gg^B_{\mathcal{L}}+g'g^W_{\mathcal{L}}\right)\ .\tag{95}$$

We notice that, differently from Sec.5.1.1, in this case we generate both positive or negative $\hat{T}$ (however, to explain ΔM_W, we must select the positive case) and also we get non-vanishing $\hat{S}$ contribution. As far as $(g-2)_\mu$ is concerned, $\mathcal{L}$ generates $\mathcal{O}_{eB}$ and $\mathcal{O}_{eW}$. From the tree-level matching, we read that:

$$\begin{aligned} Z_H^{1/2} c_{eB}^{\mu} &= \frac{g'}{8fM_{\mathcal{L}}^2}\left((\tilde{g}_{\mathcal{L}}^{De\ell})^\star - (\tilde{g}_{\mathcal{L}}^{eD\ell})^\star\right)\gamma_{\mathcal{L}} \ ; \\ Z_H^{1/2} c_{eW}^{\mu} &= \frac{g}{8fM_{\mathcal{L}}^2}\left((\tilde{g}_{\mathcal{L}}^{De\ell})^\star - (\tilde{g}_{\mathcal{L}}^{eD\ell})^\star\right)\gamma_{\mathcal{L}} \ . \end{aligned} \tag{96}$$

To see where we stand, let us do a quick estimate. We notice that all Eqs. 94-96 depend by $\gamma_{\mathcal{L}}$, so through this parameter we can correlate directly ΔM_W and Δa_μ. Let us start from Eq. 68 and consider only c_{eB}^{μ} and c_{eW}^{μ}: we can replace them using the tree-level dictionary (again, this is only for rough estimates, remember that Δa_μ was computed using 1-loop RGEs). For simplicity, let us assume the coefficients to be real. Then:

$$\Delta a_\mu^{10\text{TeV}} \sim \frac{(10\text{TeV})^2}{8fM_{\mathcal{L}}^2}\left((\tilde{g}_{\mathcal{L}}^{D\mu\mu}) - (\tilde{g}_{\mathcal{L}}^{\mu D\mu})\right)\gamma_{\mathcal{L}}(1.7\times 10^{-6}g' - 9\times 10^{-7}g) \ ,$$

and assuming a $\mathcal{L}$ weakly interacting (all the coefficients of $O(1)$), we arrive at:

$$\Delta a_\mu \sim 10^{-9}\left(\frac{10\text{TeV}}{M_{\mathcal{L}}}\right)\frac{\gamma_{\mathcal{L}}}{f} \xrightarrow{M_{\mathcal{L}}\sim 10\text{TeV}} \Delta a_\mu \sim 10^{-9}\frac{\gamma_{\mathcal{L}}}{f} \xrightarrow{\Delta a_\mu \sim 3\times 10^{-9}} \frac{\gamma_{\mathcal{L}}}{f} \sim 3 \ .$$

Analogously, we can estimate c_{HD} as:

$$c_{HD} \sim \left(\frac{\gamma_{\mathcal{L}}}{f}\right)^2 \frac{f^2}{M_{\mathcal{L}}^4} \xrightarrow{\gamma_{\mathcal{L}}/f\sim 3} c_{HD} \sim \left(\frac{f}{10\text{TeV}}\right)^2 \frac{1}{10\text{TeV}^2} \ .$$

To explain ΔM_W we require $c_{HD} \sim (0.2/\text{TeV})^2$, hence $f \sim 6$TeV, which is reasonable since f is an energy scale which is related to the NP scale Λ and we found them to be of the same order. In our case, the contribution of $\mathcal{L}$ to the $(g-2)_\mu$ is depicted in Fig. 9. Notice that, from the second line of Eq. 93 one may expect that couplings as $\mathcal{L}_\mu \partial^\mu H$ are possible, hence eventual mixing between $\mathcal{L}$ and H. However, this is not the case since we applied the canonical normalization Z_H, thus derivative couplings of the (normalized) Higgs would not emerge.

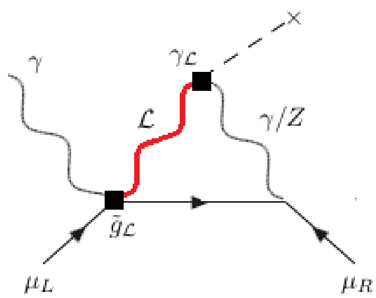


Figure 9: Feynman diagram of the $(g-2)_\mu$ correction coming from the new particle $\mathcal{L}$ at 1-loop level (its h.c. is understood). The $\mathcal{L}$ propagator is shown in red, while NP interactions with a solid black square.

To conclude, we recognise that both $\mathcal{B}$ and $\mathcal{L}$ have the potential to explain ΔM_W and Δa_μ. However, the former gives rise to a too small correction to Δa_μ while the latter, even if do account for both anomalies, it requires a non-renormalizable UV completion which is still not satisfactory, since it would only be a different instance of EFT of a more complex UV theory, conceptually not too far from the SMEFT treatment done in previous sections. We conclude also that the tree-level matching used so far may be too rough to explain anomalies that can receive sizable loop-level corrections. Therefore, our study will be focused on a precise, renormalizable extension of the SM, whose contribution to M_W and $(g-2)_\mu$ will be considered systematically at loop level precision: the 2HDM.

5.2 Two-Higgs Doublet Model (2HDM)

Before exposing the computations we have done, we briefly summarize some features of the 2HDM that we will use in our analysis and also some results that can be found in literature.

5.2.1 General overview

The 2HDM is an extension of the SM: while the latter has only one Higgs SU(2) doublet, in the former we add another $SU(2)_L$ doublet that transforms as (1,2,1/2) under the SM gauge group. For now, let us call Φ_1 and Φ_2 those doublets keeping in mind that, in general, neither Φ_1 nor Φ_2 are identifiable as the SM SU(2) one (we will shortly see this). The general form of the scalar potential in the unbroken phase, is given by the following expression [53]:

$$\begin{aligned}\mathcal{V} \;=\;& m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - [m_{12}^2\Phi_1^\dagger\Phi_2 + \text{h.c.}] \\ &+\frac{\lambda_1}{2}\left(\Phi_1^\dagger\Phi_1\right)^2 + \frac{\lambda_2}{2}\left(\Phi_2^\dagger\Phi\right)^2 + \lambda_3\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right) + \lambda_4\left(\Phi_1^\dagger\Phi_2\right)\left(\Phi_2^\dagger\Phi_1\right) \\ &+\left[\frac{\lambda_5}{2}\left(\Phi_1^\dagger\Phi_2\right)^2 + \left[\lambda_6\left(\Phi_1^\dagger\Phi_1\right) + \lambda_7\left(\Phi_2^\dagger\Phi_2\right)\right]\left(\Phi_1^\dagger\Phi_2\right) + \text{h.c.}\right] ,\end{aligned} \tag{97}$$

where $\{m_{11},\, m_{22},\, \lambda_{1\ldots4}\} \in \mathbb{R}$ and $\{m_{12},\, \lambda_{5\ldots7}\} \in \mathbb{C}$. However, since we will focus on CP-conserving models, we can take $\{m_{12}, \lambda_{1\ldots7}\} \in \mathbb{R}$. The doublets can be written in the unbroken phase similarly to the SM one:

$$\Phi_{j=1,2} = \begin{pmatrix} \phi_j^+ \\ \frac{1}{\sqrt{2}}(v_j + \phi_j + ia_j) \end{pmatrix} , \tag{98}$$

and similarly the vev of the doublets can be written as:

$$\langle\Phi_1\rangle = \frac{v}{\sqrt{2}}\begin{pmatrix} 0 \\ \cos\beta \end{pmatrix} , \langle\Phi_2\rangle = \frac{v}{\sqrt{2}}\begin{pmatrix} 0 \\ e^{i\xi}\sin\beta \end{pmatrix} ,$$

where ξ is a complex phase that we can set to zero if there are no complex parameter in Eq. 97 as we assumed. We define $v_1 = v\cos\beta$, $v_2 = v\sin\beta$ so that $(246\text{GeV})^2 = v^2 = v_1^2 + v_2^2$ and $\tan\beta \equiv v_2/v_1 = \langle\Phi_2\rangle / \langle\Phi_1\rangle$, with β conventionally chosen to be in the interval $[0, \pi/2]$.
In general, a relevant problem in 2HDMs is the possibility of tree-level flavour-changing neutral current processes (FCNC). In fact, SM fermions can interact with two scalar doublet, thus a generic Yukawa interaction can be written as:

$$\mathcal{L}_{\text{yuk}} \ni y_{ij}^1\bar{\psi}_i\psi_j\Phi_1 + y_{ij}^2\bar{\psi}_i\psi_j\Phi_2 , \tag{99}$$

with $i, j \in \{1, 2, 3\}$ flavour indexes. Given the previous form of the doublet vevs, interactions in Eq. 99 generates fermion mass terms of the kind:

$$m_{ij} = y_{ij}^1\frac{v_1}{\sqrt{2}} + y_{ij}^2\frac{v_2}{\sqrt{2}} .$$

Now, if we use the same strategy as in the SM, namely rotate the fields in such a way that m_{ij} is diagonal, we immediately notice that in 2HDMs with at least two Higgs doublets (as in our case) not all the Yukawa matrices may be diagonalized simultaneously thus, the possibility for flavour violation once we use the mass basis. Since FCNC are phenomenologically strictly constrained, Yukawa matrices as in Eq. 99 would require some sort of fine tuning in their elements which may be deemed to be a non-natural solution. To solve this issue, one can relate to the Glashow-Weinberg theorem [56] which states that tree-level FCNC are absent in any theory where a given fermion does not couple to more than one Higgs doublet. To do so, it is usually introduced a Z_2 symmetry where to each right-handed fermion is associated a Z_2 charge (+ or −). Depending on the combination with which these charges are assigned, several scenarios take form: they are reported in Table 3. It is clear that with this (conventional) choice of Z_2 charges, right-handed singlets can only couple with one Higgs doublet, thus each fermion mass matrix will depend only by one Yukawa matrix and their diagonalization is possible and analogous to the SM one. Actually, from the potential in Eq. 97, we notice that

Model	Φ_1	Φ_2	u_R	d_R	e_R
Type I	$-$	$+$	$+$	$+$	$+$
Type II	$-$	$+$	$+$	$-$	$-$
Type-X	$-$	$+$	$+$	$+$	$-$
Type-Y	$-$	$+$	$+$	$-$	$+$

Table 3: Z_2 charge assignment for right-handed SM fermions and definition of four 2HDM models. All other fields are assumed to be even under Z_2 symmetry. Type-X is also known as *lepton specific* model and Type-Y as *flipped* model.

m_{12}^2, λ_6 and λ_7 terms violate this discrete symmetry. However, while λ_6, λ_7 hardly violate Z_2 (the symmetry cannot be restored), it is only softly broken by m_{12}^2 meaning that it can be restored in the UV limit [57]. Therefore, we impose $\lambda_6 = \lambda_7 = 0$ while keeping $m_{12}^2 \neq 0$ in general.

So far, we introduced two complex doublet with a total of 8 real degrees of freedom (dof). After the ElectroWeak symmetry breaking, 3 of them will be eaten by the Z and W bosons through the Higgs mechanism, one dof will represent the Higgs h with $m_h = 125$GeV discovered at LHC, one will represent another CP-even scalar H while another one a CP-odd scalar A. The remaining two dof represent two charged Higgs that we will denote as $H^\pm$. To pass from the gauge base to the physical one, it is needed to rewrite the potential in Eq. 97 (with $\lambda_6 = \lambda_7 = 0$) using the notation in Eq. 98 and then diagonalize the 2×2 matrices of the terms quadratic in the fields. The procedure is fully reported in Sec.2.1 of Ref. [53]: we only highlight the introduction of the mixing angle α to disentangle the two CP-even mass eigenstates h and H such that:

$$\begin{aligned} H &= \cos(\alpha)\phi_1 + \sin(\alpha)\phi_2 \ , \\ h &= -\sin(\alpha)\phi_1 + \cos(\alpha)\phi_2 \ . \end{aligned}$$

Now, we can finally rewrite the generic Yukawa Lagrangian of Eq. 99 in a more complete form, in the mass basis and applying the Z_2 discrete symmetry [55]:

$$\begin{aligned} \mathcal{L}_{\text{yuk}}^{\text{2HDM}} &= -\sum_{\psi=u,d,\ell} \frac{m_\psi}{v}\left(\xi_h^\psi \bar{\psi}\psi h + \xi_H^\psi \bar{\psi}\psi H - i\xi_A^\psi \bar{\psi}\gamma_5\psi A\right) \\ &\quad - \left[\frac{\sqrt{2}V_{\text{CKM}}^{ij}}{v}\bar{u}_i\left(m_u\xi_A^u P_L + m_d\xi_A^d P_R\right)d_j H^+ + \frac{\sqrt{2}m_\ell\xi_A^\ell}{v}\bar{\nu}_L e_R H^+ + \text{h.c.}\right] \ , \end{aligned} \quad (100)$$

where V_{CKM} is the Cabibbo-Kobayashi-Maskawa matrix and $P_{L,R}$ the chiral projectors. All the couplings modifier ξ_h^ψ, ξ_H^ψ, ξ_A^ψ with $\psi \in \{u, d, \ell\}$ are related to the type of 2HDM since they depend on the Z_2 charge assignment (i.e., which lepton couples to which doublet): for completeness, they are reported in Table 4, using the same α and β angles previously defined. As far as doublets-gauge fields couplings are concerned, they would come from the following gauge-kinetic Lagrangian:

$$\mathcal{L}_{g-k} = (D^\mu\Phi_1)^\dagger(D_\mu\Phi_1) + (D^\mu\Phi_2)^\dagger(D_\mu\Phi_2) \ ,$$

with D_μ the SM covariant derivative. By writing $\Phi_{1,2}$ in terms of their physical states, one arrives at:

$$\begin{aligned} \mathcal{L}_{g-k} \ni\ & \frac{g^2+g'^2}{8}v^2 Z_\mu Z^\mu\left(1 + \frac{2h}{v}\sin(\beta-\alpha) + \frac{2H}{v}\cos(\beta-\alpha)\right) \\ & + \frac{g^2v^2}{4}W_\mu^+ W_-^\mu\left(1 + \frac{2h}{v}\sin(\beta-\alpha) + \frac{2H}{v}\cos(\beta-\alpha)\right) \ . \end{aligned} \quad (101)$$

Notice that they are nothing but the SM couplings modified by a factor $\sin(\beta-\alpha)$ or $\cos(\beta-\alpha)$. Since gauge fields trivially transform under Z_2, these couplings are the same in all 2HDM scenarios. In particular, one realizes that in the limit $\sin(\beta-\alpha) \to 1$ [resp. $\cos(\beta-\alpha) \to 1$] vector bosons only couple with h [resp. H] through the same coupling of the SM Higgs. This is not a case, because those CP-even scalars are related to the SM Higgs field by:

$$h_{\text{SM}} = \sin(\beta-\alpha)h + \cos(\beta-\alpha)H \ . \quad (102)$$

Modifier	Type-I	Type-II	Type-X	Type-Y
ξ_h^u	$\cos\alpha/\sin\beta$	$\cos\alpha/\sin\beta$	$\cos\alpha/\sin\beta$	$\cos\alpha/\sin\beta$
ξ_h^d	$\cos\alpha/\sin\beta$	$-\sin\alpha/\cos\beta$	$\cos\alpha/\sin\beta$	$-\sin\alpha/\cos\beta$
ξ_h^ℓ	$\cos\alpha/\sin\beta$	$-\sin\alpha/\cos\beta$	$-\sin\alpha/\cos\beta$	$\cos\alpha/\sin\beta$
ξ_H^u	$\sin\alpha/\sin\beta$	$\sin\alpha/\sin\beta$	$\sin\alpha/\sin\beta$	$\sin\alpha/\sin\beta$
ξ_H^d	$\sin\alpha/\sin\beta$	$\cos\alpha/\cos\beta$	$\sin\alpha/\sin\beta$	$\cos\alpha/\cos\beta$
ξ_H^ℓ	$\sin\alpha/\sin\beta$	$\cos\alpha/\cos\beta$	$\cos\alpha/\cos\beta$	$\sin\alpha/\sin\beta$
ξ_A^u	$\cot\beta$	$\cot\beta$	$\cot\beta$	$\cot\beta$
ξ_A^d	$-\cot\beta$	$\tan\beta$	$-\cot\beta$	$\tan\beta$
ξ_A^ℓ	$-\cot\beta$	$\tan\beta$	$\tan\beta$	$-\cot\beta$

Table 4: Yukawa modifiers of SM fermions to the 2HDM neutral fields h, H, A in the four scenarios of Table 3. The couplings to $H^\pm$ are not reported since they follow the relation in Eq. 100.

Only in the *alignment condition* $h_{\rm SM}$ corresponds to one of the CP-even scalars, making $\beta-\alpha$ an interesting parameter for our model[12]. From Eq. 101, notice also that AW^+W^- and AZZ vertexes are absent (and obviously h, H, A being neutral, cannot couple to the photon).

5.2.2 Constraints and results in the literature

Let us start again from Eq. 97 where $\lambda_6 = \lambda_7 = 0$ and $m_{12}^2 \in \mathbb{R}$ is understood. The model presents 8 real parameter, hence identifying $\mathbb{R}^8$ as its natural parameter space. However, not all of those parameters are independent: in particular we can relate m_{11} and m_{22} to the other couplings through the following relations [53]:

$$\begin{aligned} m_{11}^2 &= m_{12}^2\tan\beta - \frac{v^2}{2}(\lambda_1\cos^2\beta + (\lambda_3+\lambda_4+\lambda_5)\sin^2\beta)\ ; \\ m_{22}^2 &= m_{12}^2\cot\beta - \frac{v^2}{2}(\lambda_2\sin^2\beta + (\lambda_3+\lambda_4+\lambda_5)\cos^2\beta)\ , \end{aligned}$$

thus reducing the number of independent dof to 7: $\{m_{12}^2, \lambda_{1\ldots5}, \tan\beta\}$. One can further manipulate this base by rewriting the quartic couplings in terms of $h, H, A, H^\pm$ masses, getting [58]:

$$\begin{aligned} \lambda_1 &= \frac{1}{v^2\cos^2\beta}\left(\cos^2\alpha M_H^2 + \sin^2\alpha M_h^2 - \tan(\beta) m_{12}^2\right)\ ; \\ \lambda_2 &= \frac{1}{v^2\sin^2\beta}\left(\sin^2\alpha M_H^2 + \cos^2\alpha M_h^2 - \cot(\beta) m_{12}^2\right)\ ; \\ \lambda_3 &= \frac{1}{v^2}\left(2M_{H^\pm}^2 + \frac{\sin(2\alpha)}{\sin(2\beta)}(M_H^2 - M_h^2) - \frac{m_{12}^2}{\sin\beta\cos\beta}\right)\ ; \\ \lambda_4 &= \frac{1}{v^2}\left(M_A^2 - 2M_{H^\pm}^2 + \frac{m_{12}^2}{\sin\beta\cos\beta}\right)\ ; \\ \lambda_5 &= \frac{1}{v^2}\left(\frac{m_{12}^2}{\sin\beta\cos\beta} - M_A^2\right)\ . \end{aligned} \tag{103}$$

In our analysis, we will use the following non-redundant base: $B = \{M_h, M_H, M_A, M_{H^\pm}, \lambda_1, \sin(\beta-\alpha), \tan\beta\}$. However, we must identify h (or H) as the Higgs found at LHC, thus setting M_h (or M_H) to 125GeV: this is defined as *normal* (or *inverted*) scenario. Eventually, our parameter space will be a subset of $\mathbb{R}^6$, but we can still do some work to further reduce it. In fact, before comparing our model with experimental results, it must be theoretically consistent, meaning that it should satisfy three important constraints:

(a) *Perturbativity*: couplings in Eq. 97 must be always $\lesssim 4\pi$ both at tree and loop-level when running by RGEs (the cut-off scale Λ can be set in the multi-TeV scale for our purposes), otherwise the perturbative treatment does not apply,

[12] All the couplings in Table 4 can be rewritten in terms of $\sin(\beta-\alpha)$, $\cos(\beta-\alpha)$ and $\tan\beta$ using goniometric relations (e.g., see Ref. [53]).

(b) *Vacuum stability*: the scalar potential must be bounded from below in all field space directions and also for asymptotically large values of the fields. For $|\Phi_{j=1,2}| \to +\infty$ the potential is lead by the quartic couplings, meaning that the stability is asymptotically given by [59]:

$$\lambda_1 > 0 \qquad \lambda_2 > 0\ ; \qquad \lambda_3 + \lambda_4 - |\lambda_5| > -\sqrt{\lambda_1 \lambda_2}\ ;$$
$$m_{12}^2(m_{11}^2 - m_{22}^2\sqrt{\lambda_1/\lambda_2})(\tan\beta - (\lambda_1/\lambda_2)^{1/4}) > 0\ . \tag{104}$$

(c) *Unitarity*: at high energies, the probability to have a two-body scattering process $1+2 \to 1'+2'$ must be normalizable to 1. In perturbation theory, unitarity is usually imposed order by order: for our purposes we implement tree-level unitarity, which is given by the following relations [53]:

$$\begin{aligned}
8\pi &\geq \left|\frac{3}{2}(\lambda_1+\lambda_2) \pm \left(\frac{9}{4}(\lambda_1-\lambda_2)^2 + (2\lambda_3+\lambda_4)^2\right)^{1/2}\right|\ ; \\
8\pi &\geq \left|\frac{1}{2}(\lambda_1+\lambda_2) \pm \left(\frac{1}{4}(\lambda_1-\lambda_2)^2 + \lambda_4^2\right)^{1/2}\right|\ ; \\
8\pi &\geq \left|\frac{1}{2}(\lambda_1+\lambda_2) \pm \left(\frac{1}{4}(\lambda_1-\lambda_2)^2 + \lambda_5^2\right)^{1/2}\right|\ ; \\
8\pi &\geq |\lambda_3 + 2\lambda_4 \pm 3\lambda_5|\ ; \\
8\pi &\geq |\lambda_3 \pm \lambda_4|\ ; \\
8\pi &\geq |\lambda_3 \pm \lambda_5|\ .
\end{aligned} \tag{105}$$

Next step is to implement the constraints coming from the M_W measured by CDF collaboration and the Δa_μ from FermiLab:

(d) (S,T) *check*. In Sec.3.2 we always related the M_W anomaly to $\hat{S}$ and $\hat{T}$ because are connected through the relation in Eq. 25, where we already explained that the U dependence is negligible. Hence, fixing $U = 0$, S and T (notice, without *hat*) are experimentally constrained as follows [32, 61]:

$$\frac{(S-S_0)^2}{\sigma_S^2} + \frac{(T-T_0)^2}{\sigma_T^2} - 2\rho_{ST}\frac{(S-S_0)(T-T_0)}{\sigma_S\sigma_T} \leq R^2(1-\rho_{ST}^2)\ , \tag{106}$$

with $R^2 \approx 2.3,\ 4.61,\ 5.99,\ 9.21,\ 11.83$ at 68.3%, 90%, 95%, 99% and 99.7% confidence level respectively. We remember also that the central values measured are: $(S_0 \pm \sigma_S) = (0.15 \pm 0.08)$ and $(T_0 \pm \sigma_T) = (0.27 \pm 0.06)$ with a correlation of $\rho_{ST} = 0.93$. To implement the bound in Eq. 106 we need to see how S and T are corrected by 2HDMs. Their full analytic expressions are quite long, but they are known (e.g., see Eq.38 and Eq.45 of Ref. [54] or Eqs.3.27-3.28 of Ref. [53]) and they carry an important information: in order to generate a non-vanishing T, the CP-even scalars and the CP-odd one *must not* be degenerate in mass with the charged Higgs (but h, H and A may be degenerate in mass between themselves). This result is independent by the type of scenario considered, since T is an oblique correction, then flavour-blind by definition, therefore it does not depend by the explicit choice of Yukawa modifiers specified in Table 4.

(e) Δa_μ *check*. 2HDMs provide at loop level corrections to the muon anomaly through the new fields $H, A, H^\pm$. The kind of diagrams contributing to the process at 1-loop are shown in Fig.10 and its correction is given by [59]:

$$\Delta a_\mu^{\text{2HDM}}(1\text{ loop}) = \frac{G_F m_\mu^2}{4\pi^2\sqrt{2}} \sum_{j=h,H,A,H^\pm} (\xi_j^\mu)^2 x_j^\mu F_j(xj^\mu)\ , \tag{107}$$

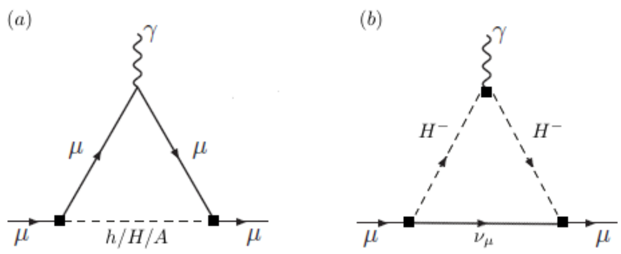


Figure 10: Feynman diagrams of contributing to $\Delta a_\mu^{\rm 2HDM}(1\text{ loop})$. (a): To get the result in Eq. 107, 4 of this kind must be considered: two where a muon mass insertion is in one of the external μ propagators; two with a muon mass insertion in one of the internal propagators. (b): Analogously, 2 diagrams of this kind must be considered, each one with a muon mass insertion in the external propagators. 2HDM new interactions are denoted by a solid square.

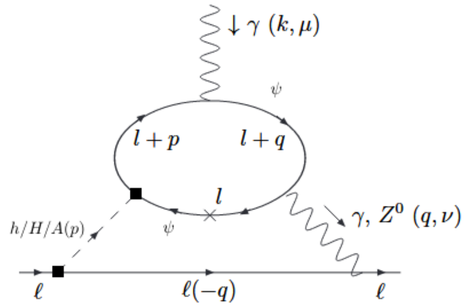


Figure 11: Barr-Zee diagram contributing to $\Delta a_\mu^{\rm 2HDM}(2\text{ loop})$. 2HDM new interactions are denoted by a solid square. Picture taken and re-edited from [62].

where ξ_j^μ are the couplings listed in Table 4; $x_j^\mu \equiv m_\mu^2/M_j^2$; and the functions $F_j(x_j^\mu)$ are:

$$\begin{aligned} F_{h,H}(x) &= \int_0^1 \frac{t^2(2-t)}{1-t-xt^2}dt \ , \\ F_A(x) &= \int_0^1 \frac{-t^3}{1-t+xt^2} \ , \\ F_{H^\pm}(x) &= \int_0^1 \frac{-t(1-t)}{1-(1-t)x} \ . \end{aligned}$$

However, because of the suppression in $(m_\mu/v)^4$ for $x \ll 1$ (to see this, just compute the integrals for $x \to 0$ and then take the limit outside the sum in Eq. 107[13]), heavy fermion loops may become no more sub-dominant contribution at 2-loop level, describing Barr-Zee diagrams of the kind Fig.11. Their contribution is also analytically known and reads [59]:

$$\Delta a_\mu^{\rm 2HDM}(2\text{ loops}) = \frac{G_F m_\mu^2}{4\pi^2\sqrt{2}}\frac{\alpha_{em}}{\pi}\sum_{j,\psi} N_\psi^C Q_\psi^2 \xi_j^\psi \xi_j^\mu x_j^\psi G_j(x_j^\psi) \ , \tag{108}$$

with $j \in \{h, H, A\}$ and Q_ψ, N_ψ^C the electric charge and the color number of the fermion ψ in the loop (typically $\psi \in \{t, b, \tau\}$ is enough for our purposes). The function $G_j(x_j^\psi)$ is defined as:

$$G_j(x) = \int_0^1 \frac{\mathcal{N}_i(t)}{t(1-t)-x}\log\left(\frac{t(1-t)}{x}\right)dt \ ,$$

where $\mathcal{N}_{h,H}(t) = 2t(1-t)-1$ and $\mathcal{N}_A(t) = 1$. By adding together the contributions in Eqs. 107-108 one gets the desired $\Delta a_\mu^{\rm 2HDM}$ to confront with the data.

[13] No mathematical problems arise here because of the sum convergence.

Reference	Issue addressed	Scenario-Type	$M_{H^\pm}$ [GeV]	M_H [GeV]	M_A [GeV]
[58]	M_W	NS-I	[87, 1091]	[130, 1092]	[22, 1098]
[58]	M_W	NS-II	[598, 1138]	[419, 1128]	[459, 1125]
[58]	M_W	NS-X	[99, 1091]	[132, 1092]	[30, 1098]
[58]	M_W	NS-Y	[595, 1139]	[419, 1128]	[453, 1125]
[58]	M_W	IS-I	[144, 455]	[16, 120]	[38, 429]
[58]	M_W	IS-II	impossible	impossible	impossible
[58]	M_W	IS-X	[166, 446]	[62.5, 120]	[16, 420]
[58]	M_W	IS-Y	impossible	impossible	impossible
[61]	M_W	NS-I	< 1000	< 1000	< 1000
[63]	M_W	NS-I	[100, 1000]	[150, 1000]	[200, 1000]
[63]	M_W	NS-II	[600, 1500]	[500, 1500]	[600, 1500]
[64]	M_W	IS-I	[160, 600]	[20, 120]	< 600
[64]	M_W	IS-X	[160, 600]	[60, 120]	< 600
[65][(†)]	M_W	NS-I	[320, 440]	[200, 500]	[200, 500]
[65][(†)]	M_W	NS-II	impossible	impossible	impossible
[66]	$g-2$ and M_W	NS-X	M_H+ [50, 80]	≥ 300	[10, 60]
[67]	$g-2$ and M_W	NS-X	[280, 600]	[250, 600]	[10, 40]

Table 5: Mass range estimates found in literature for the new scalars in 2HDM in different scenarios-type combinations. In normal scenario (NS) H is the heavy CP-even neutral scalar so $M_H > 125$GeV; in the inverted scenario (IS) H is the lighter CP-even scalar so $M_H < 125$GeV. Except for Ref. [58], other mass ranges estimates were extrapolated by the graphs present in their respective papers. Since masses are correlated between themselves, please refer to the paper of interest for further information.
[(†)] Here a scale-invariant 2HDM is considered, where the mass of the scalars are only radiatively generated through a Coleman-Weinberg like potential: at tree-level all the scalars are massless.

Because of the recent M_W result from CDF collaboration and the confirmed BNL measurement of Δa_μ, several recent papers can be found in the literature discussing how different models (among which the 2HDM) can account for the observed discrepancies. In particular, we found different papers analysing thoroughly only one of those anomalies at once, but very few addressing both the issues. All of them applied the theoretical constraints described at points (a), (b), (c) of this subsection and, under different assumptions and scenarios, tried to find reasonable mass ranges for the new scalars, taking into account other experimental data as *(i)* Higgs precision data implemented in codes as HiggsSignals [69], *(ii)* Higgs direct search bounds coming from Tevatron, LHC, LEP experiments implemented in HiggsBounds [70] and *(iii)* constraints from flavour physics, in particular flavour changing processes as the decay of the meson $B \to X_s \gamma$. We summed up the main results of the references we took into consideration in Table 5 (they are mainly related to M_W since this is the latest discovery). We did not consider Ref. [60] for the final results since it assumed lepton flavour violating processes at tree-level that we would like to avoid. On the other hand, we considered Ref. [59] addressing the $(g-2)_\mu$ anomaly even if it is not listed in Table 5.

5.2.3 Our computation

Our way to proceed is inspired by Ref. [58]: we would like to systematically analyze each of the 8 possible 2HDM scenarios considering all the constraints from (a) to (e) of Sec.5.2.2; then we confront our findings with references at our disposal and finally guess possible regions of the parameter space that satisfy all the given bounds. To perform the calculations, we use the doublet-Higgs-model calculator 2HDMC v.1.8.0 [54] (last compiled 5 Aug 2022) and interface it with a C++ program that realise graphics through ROOT v.6.22/02 [68] (last compiled 12 Nov 2020).
In Sec.5.2.2 we identified a base B of 6 independent coefficients, however scanning over all the parameter space would take a very long computational time. To overcome the problem, we decided to fix *a priori* three of the six parameters (namely $\lambda_1, \tan\beta, \sin(\beta-\alpha)$) and let only the physical masses

ID	$\tan\beta$	$\sin(\beta-\alpha)$	λ_1
1	50	1	1
2	50	0.999	1
3	30	1	1
4	30	0.001	1

Table 6: Different set of input parameters used in our computations. The sequential number associated to each set is only for clarity.

RUN	Scenario-Type	Input ID	M_A [GeV]	incr.	M_H [GeV]	incr.	$M_{H^\pm}$ [GeV]	incr.
1	NS-II	1	[10, 100]	2GeV	[130, 160]	1GeV	[80, 300]	5GeV
2	NS-X	1	[10, 50]	1GeV	[130, 160]	1GeV	[80, 300]	5GeV
3	NS-X	2	[10, 50]	1GeV	[130, 160]	1GeV	[80, 300]	5GeV
4	IS-X	3	[5, 20]	0.5GeV	[10, 120]	2GeV	[80, 350]	5GeV
5	IS-X	4	[5, 20]	0.5GeV	[10, 120]	2GeV	[80, 300]	5GeV

Table 7: Different set of mass intervals used in our computations. The sequential number associated to each set in the first column is only for clarity; the ID in the second column refers to the set in Table 6; incr. is what we called incremental step in point *(ii)* (see text).

$M_H, M_A, M_{H^\pm}$ to vary. The choice of our input parameters is not random, but an educated guess coming from the results of Refs. [58, 66, 67] and they are reported in Table 6. Also the mass intervals we used in our program take into account the findings in all references in Table 5. For each scenario (normal (NS) or inverted (IS)) and type of Yukawa couplings, our flow chart can be summarized as:

(i) Initialize the kind of 2HDM to study: choose a set of input parameters from Table 6; set the Yukawa type; set the NS ($M_h < M_H$) or IS ($M_H < M_h$) condition.

(ii) First scanning strategy: in order to have a general overview of the behaviour of our model, we scan through a large region of the mass space but with a lower resolution to save computational time. In particular, for NS we used:

$$\begin{aligned} M_A &\in [10, 800]\text{GeV} && \text{incremental step: } 10\text{GeV}\ ; \\ M_H &\in [130, 800]\text{GeV} && \text{incremental step: } 10\text{GeV}\ ; \\ M_{H^\pm} &\in [80, 800]\text{GeV} && \text{incremental step: } 10\text{GeV}\ , \end{aligned}$$

and for IS:

$$\begin{aligned} M_A &\in [10, 800]\text{GeV} && \text{incremental step: } 10\text{GeV}\ ; \\ M_H &\in [10, 120]\text{GeV} && \text{incremental step: } 2\text{GeV}\ ; \\ M_{H^\pm} &\in [80, 800]\text{GeV} && \text{incremental step: } 10\text{GeV}\ . \end{aligned}$$

(iii) Implementation of all the constraints from (a) to (e) described in Sec.5.2.2. We accepted points in the parameter space that generate the experimental ΔM_W and Δa_μ at 99.7% C.L.

(iv) Second scanning strategy: for those scenario-type combinations that have points in the mass space that pass all the constraints, we re-launched the program, setting mass intervals near the allowed region and with an higher resolution than the first scan (see Table 7 for more details). Step *(iii)* is applied again.

From Table 7 we see that, besides RUN-1, every other scenario that passes all the tests has the pseudoscalar mass $M_A < M_h/2 \approx 63$GeV. According to Ref. [66] this is troublesome, since the decay $h \to AA$ (with h here we denote the Higgs discovered at LHC) is kinematically possible, but there is an experimental upper bound for its branching ratio $\text{Br}(h \to AA) < 5\%$ valid almost in general for light pseudoscalars with $M_A \geq 10$GeV. A possible solution is to set to zero the hAA coupling as done in [67], further correlating the six free parameters of our model, but this is not the way we like to take, since $h \to AA$ may receive important contributions at loop level by fermion loops (notice from Table 4 that for Type-X Yukawas, the coupling $A\bar{\ell}\ell$ scales as $\tan\beta$ which is relevant considered our input parameters) and also because the fine tuning of free coefficients such that hAA is vanishing at tree level could be deemed as *ad hoc* solution. We prefer to compute the branching ratio. Fortunately, 2HDMC provides the theoretical estimates of all the main decay modes for the new scalars and the full decay width of the particles (see Sec.2.4 of Ref. [54] for further details about its implementation). Thus, the final step of our program is:

(v) For each point in the mass space that passes step *(iv)*, check if $\mathrm{Br}(h \to AA) < 5\%$: if yes, accept the point otherwise reject it.

Finally, we confront our final output list of points in the parameter space and confront it with results known in literature. As far as our choice of initial coefficients[14] is concerned, we conclude the following:

NS-I : Do not provide any valid point because of Δa_μ constraint. In particular, the contributions are either too low or with the wrong sign (we require $\Delta a_\mu > 0$).

NS-II : There are points that pass all the steps of our algorithm and are restricted in the region:

$$M_H \in [220, 280]\text{GeV} \;, \qquad M_A \in [20, 70]\text{GeV} \;, \qquad M_{H^\pm} \in [270, 320]\text{GeV} \;,$$

with input parameter ID = 2. However, from Ref. [59], it is clear that $M_{H^\pm} < 380\text{GeV}$ is forbidden because of FCNC bounds, in particular from $\bar{B} \to X_s\gamma$, and that this constraint holds $\forall\beta$. This is because quarks couple to the charged Higgs according to the Yukawa interactions in Eq. 100 and they depends on ξ_A^u and ξ_A^d. However, from Table 4 one can easily read that for Type-II scenarios $\xi_A^u = (\xi_A^d)^{-1} = \cot\beta$, hence they simplify once they are inserted in the Feynman amplitude of the FCNC process. Hence, the final decay rate $\Gamma(\bar{B} \to X_s\gamma)$ would surely depend on $M_{H^\pm}$ propagating in the loop, but not on β, therefore the constraint coming from this decay is valid $\forall\beta$ as previously claimed. Everything considered, we can conclude that this scenario is not viable.

NS-X : If we consider the perfect alignment case with input parameter ID = 1, then there is a region of the mass space that pass the point *(iv)* of our algorithm, but none of them pass the branching ratio check (this result is confirmed by Ref. [67]). On the other hand, using the input ID = 2 we found some good intervals that pass all the tests. In particular:

$$M_H \in [270, 280]\text{GeV} \;, \qquad M_A \in [20, 30]\text{GeV} \;, \qquad M_{H^\pm} \in [300, 320]\text{GeV} \;,$$

that seems reasonable considering NS-X results from Table 5.

NS-Y : There are no valid points for reasons similar to NS-I.

IS-I : There are no valid points for reasons similar to NS-I, NS-Y.

IS-II : It is impossible to justify M_W in this case, because of theoretical constraints. In particular, because of the heavy charged Higgs masses required, the perturbativity of quartic couplings is spoiled once one consider the RGEs [58]. Hence, IS-II has not been considered in our work in the first place.

IS-X : Both RUN = 4, 5 gave similar results. In particular there is a very narrow region in the mass space that satisfies all the constraints, namely for:

$$M_H \sim 90\text{GeV} \;, \qquad M_A \in [7, 15]\text{GeV} \;, \qquad M_\pm \in [160, 165]\text{GeV} \;.$$

Comparing with Ref. [58] and Table 5, we conclude that IS-X is possible, but less likely than NS-X because of the (too) light pseudoscalar.

IS-Y : Impossible for the same reasons of IS-II.

[14] We are aware that we analyzed only a subspace of null measure of the initial 6-dim parameter space, nonetheless, we are also convinced that our analysis helped at least to identify more viable scenarios than others.

We conclude that **NS-X** in the non-alignment condition and with a light pseudoscalar is the most viable scenario-type combination to explain M_W, Δa_μ given both theoretical and experimental bounds and we would like to further investigate it.

First, we report in Fig. 12 how the mass space restricts after different bounds are subsequently implemented. In particular, the up-right panel shows the possibility for the charged Higgs to be the lightest particle of the model and yet satisfying the ΔM_W bounds. This is possible because the necessary S, T parameters depend significantly on the difference of new scalar masses, i.e., $M_H - M_H^\pm$ and $M_A - M_H^\pm$ rather than their absolute values (e.g., see Fig.1 of Ref. [59]). Thus, light neutral states and heavy charged ones are as acceptable as heavy neutrals with light $H^\pm$, as the region with $M_H > 550$GeV highlights. Notice also that, at this stage, H and A can still be degenerate in mass, as stated in point (d) of Sec.5.2.2. By adding the $(g-2)_\mu$ constraint, we obtain the upper-right panel in Fig.12: there, most of the previous accepted points are no longer present (notice the different scale in the y-axis) and only light pseudoscalars are selected. The fact that lepton-specific models can generate such a correction is evident from Eqs. 107-108: loop corrections to Δa_μ depend on the modifiers ξ_j^ψ and ξ_j^μ both at 1- and 2-loops. Reading Table 4, one can see that type-II and type-X have ξ_A^ℓ boosted by $\tan\beta \gg 1$ and, since type-II has been rejected, only type-X can give sizeable contributions. But the most drastic cut of the parameter space is in the bottom panel of Fig.12 because of the $\mathrm{Br}(h \to AA)$ test. In fact, as stated in Ref. [66] the hAA vertex must be suppressed in order to avoid a great branching fraction. Thus, only a narrow region of the parameter space has the right combination of coefficients to satisfy this requirement.

To better visualize it, we decided to plot $\mathrm{Br}(h \to AA)$ by varying $\tan\beta$ and one mass or $\sin(\beta-\alpha)$ at a time. The plots are given in Fig. 13. From the up-left and bottom-left panels, we realize that there is little dependence on M_A (as far as it is sufficiently distant from $M_h/2$) and on $M_{H^\pm}$. On the other hand, M_H and $\sin(\beta-\alpha)$ influence significantly the branching ratio. In particular, considering Fig. 3 of [66], we notice that for fixed values of M_H and M_A, to have $\mathrm{Br}(h \to AA) < 5\%$ we need the coupling modifier ξ_h^ℓ to be almost constant. For example, given the input parameters of bottom-right panel of Fig.13, we need $\xi_h^\ell \sim 1.2$. But this is all what we need in order to theoretically predict the *locus* of low branching ratio in the $(\sin(\beta-\alpha), \tan\beta)$ plane. In fact, with a few goniometric relations, one gets:

$$\sin\alpha = \frac{1}{1+\tan^2\beta}\left(-\frac{\sin(\beta-\alpha)}{\cos\beta} + \tan\beta\sqrt{1-\frac{\sin^2(\beta-\alpha)}{\cos^2\beta}+\tan^2\beta}\right) ,$$

where we express all the dependences in β and $\sin(\beta-\alpha)$. If $\xi_h^\ell \approx \text{const.} \equiv k$, then it translates in the $(\sin(\beta-\alpha), \tan\beta) \equiv (x,y)$ plane as the implicit curve:

$$(1+y^2)\cos(\arctan(y))\cdot k \approx -\frac{x}{\cos(\arctan(y))} + y\sqrt{1-\frac{x^2}{\cos^2(\arctan(y))}+y^2} . \tag{109}$$

Just setting $k \approx 1.22$ adequately interpolate the narrow blue region in Fig.13 bottom-right panel.

Finally, for completeness, in Fig.14 it is reported the plot in the $(\Delta a_\mu, \Delta M_W)$ of the points in mass space that satisfied all the constraints imposed by our algorithm (namely, the one in bottom panel of Fig. 12). Please, be aware that in the figure $\Delta M_W \neq M_W^{\mathrm{CDF}} - M_W^{\mathrm{SM}}$ *but* $\Delta M_W \equiv \sqrt{\Delta M_W^2} = \sqrt{M_{W,\mathrm{CDF}}^2 - M_{W,\mathrm{SM}}^2} = (3.78 \pm 0.23)$GeV. It seems there is no particular correlation left between those two quantities once all the constraints have been applied.

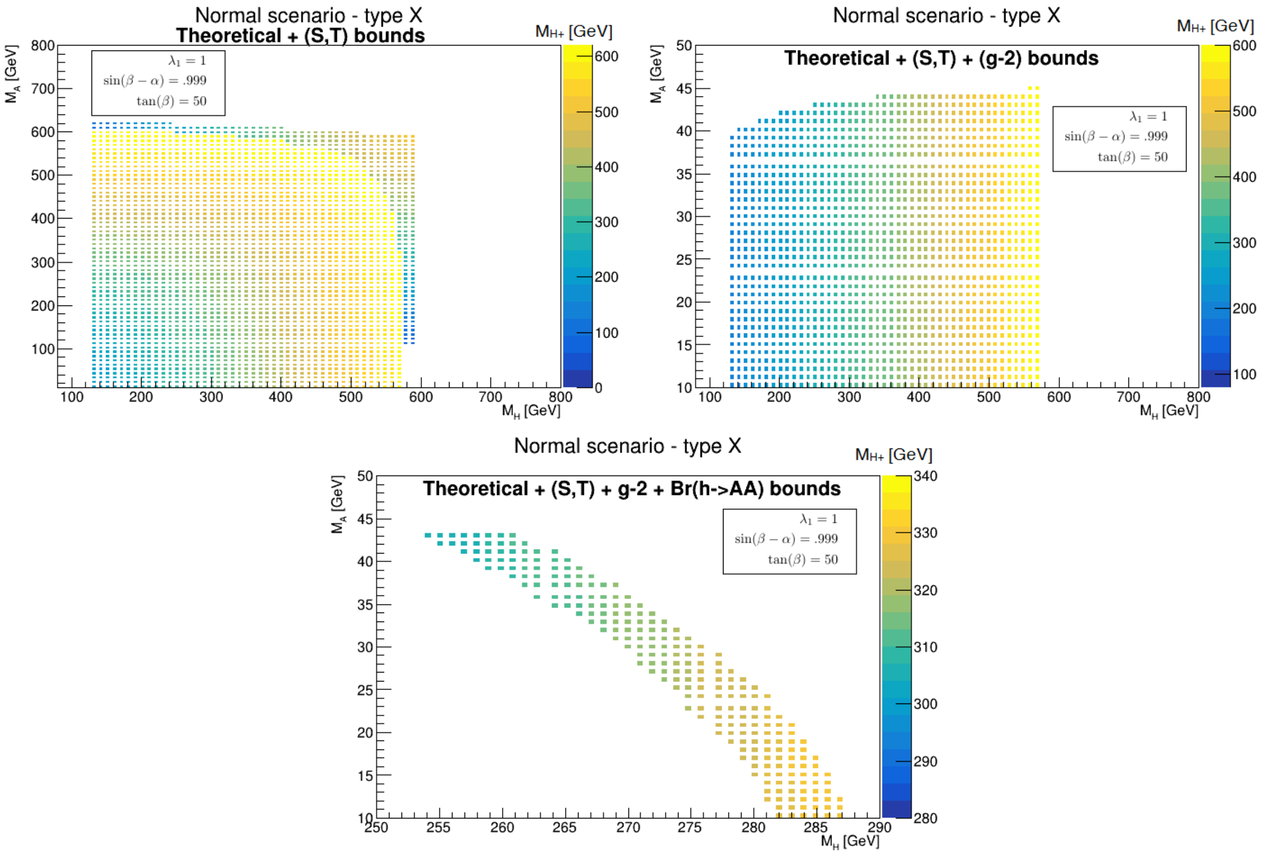


Figure 12: Plots of accepted points in the mass space after that different constraints are imposed in the NS-X scenario. The input parameters are shown in each graph and $M_{H^\pm}$ is shown in color code. (up-left): Parameter space after points (a) to (d) of Sec.5.2.2 are implemented. (up-right): Parameter space after points (a) to (e) of Sec.5.2.2 are implemented. (down): Parameter space after point *(v)* of our algorithm.

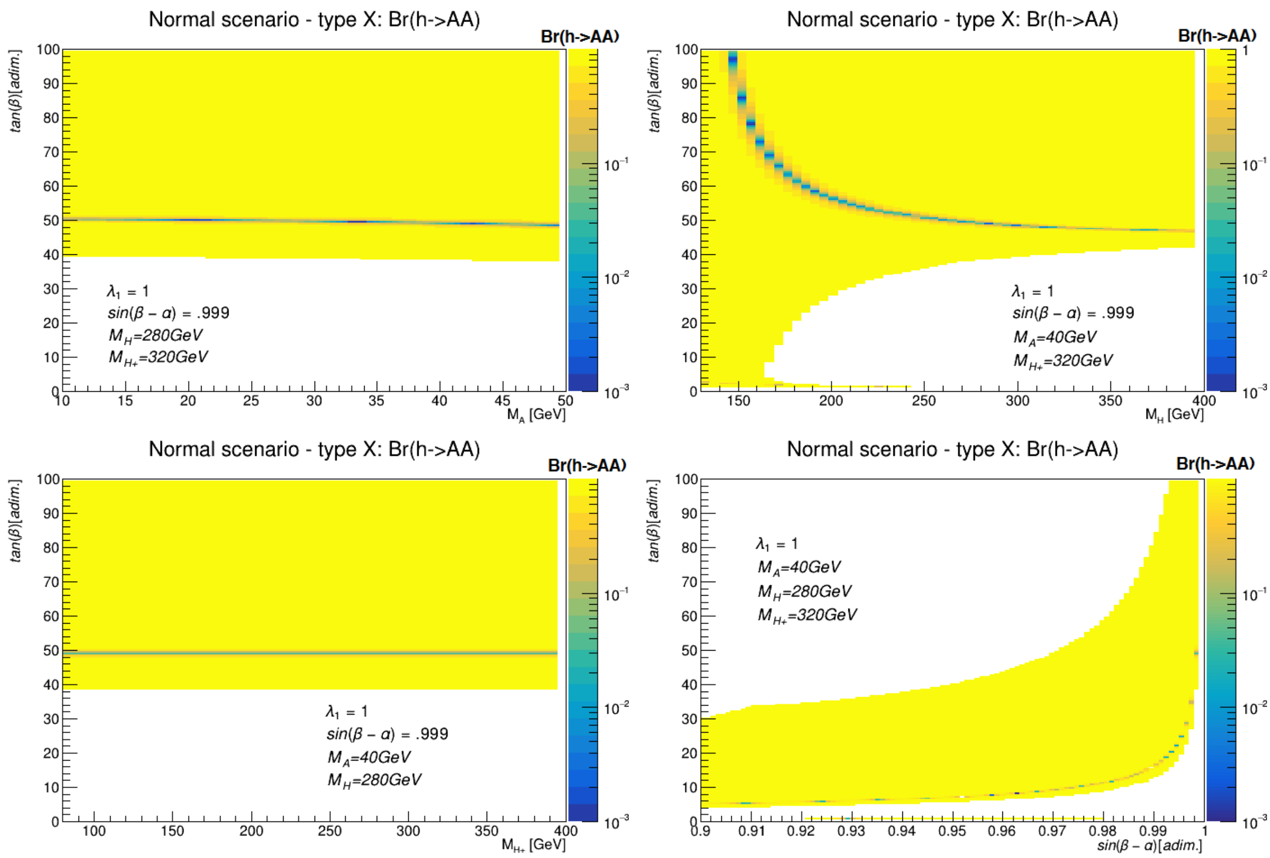


Figure 13: Plots of Br($h \to AA$) in (M_j, $\tan\beta$) or ($\sin(\beta-\alpha)$, $\tan\beta$) planes with $j \in \{H, A, H^\pm\}$ in the NS-X scenario. The input parameters are shown in each graph and the branching fraction is shown in log-scale with color code. Only theoretical constraints have been applied and white regions do not satisfy them.

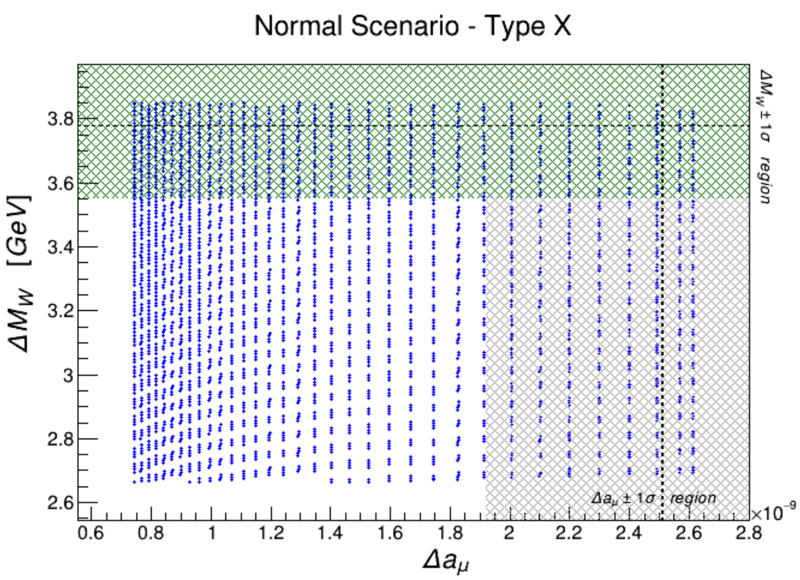


Figure 14: Plot in the (Δa_μ, ΔM_W) of the points in mass space that satisfied all the constraints imposed by our algorithm (namely, the one in bottom panel of Fig. 12) in NS-X scenario. Gray and green dashed bands illustrate the 1σ regions around the central value of Δa_μ and $\Delta M_W \equiv \sqrt{\Delta M_W^2} = \sqrt{M_{W,\mathrm{CDF}}^2 - M_{W,\mathrm{SM}}^2}$ respectively.

6 Conclusions

The recent M_W value measured by CDF-II as well as the muon anomalous magnetic moment $(g-2)_\mu$ measured at BNL and FNAL-E989 show remarkable discrepancies with their experimental predictions, thus they may call for BSM physics. Following a *bottom-up* approach, we considered Effective Field Theory extensions of the Standard Model (SMEFT) which allowed us to provide model-independent predictions without relying on any specific UV complete model. We considered all the SMEFT dimension-six operators that modify the theoretical predictions of $(g-2)_\mu$ and M_W up to next-to leading order corrections.
New physics contributions to M_W can be accounted for by the so-called oblique corrections that are well described by the $\hat{S}$, $\hat{T}$, $\hat{U}$ Peskin-Takeuchi parameters. Within SMEFT, $\hat{S}$ and $\hat{T}$ can be generated at tree level by the $\mathcal{O}_{HWB}$ and $\mathcal{O}_{HD}$ operators, respectively, while $\hat{U}$ can be neglected as it arises from dim-8 operators. Interestingly enough, new contributions to M_W unavoidably affect the Higgs boson properties. In particular, we proved that $\hat{S}$ modifies the decay rate of $h \to \gamma\gamma$ and $h \to \gamma Z$ while $\hat{T}$ contributes to $h \to ZZ$ and $h \to W^+W^-$. We found that an explanation of the M_W anomaly through $\hat{S}$ is associated with large effects in $h \to \gamma\gamma$ that are already challenged by the current experimental resolution at LHC. On the other hand, if the M_W anomaly is dominantly accounted for by $\hat{T}$, the modifications to the $h \to ZZ$ and $h \to W^+W^-$ decay rates are well below the current experimental bounds.
Then, we proceeded to analyze the $(g-2)_\mu$ in a similar fashion. First, we identified those SMEFT operators which may contribute sizeably to the $(g-2)_\mu$ even for a new physics scale in the multi-TeV, that are $\mathcal{O}^\mu_{eB}, \mathcal{O}^\mu_{eW}$ and $\mathcal{O}^{\mu q}_T$. An explanation of the muon $g-2$ anomaly in terms of the above operators, would unavoidably induce non standard effects for $\mu\bar{\mu} \to h\gamma/hZ/q\bar{q}$ and $h \to \ell^+\ell^-\gamma$. We computed the NP contributions to all these processes, leading us to the conclusion that visible effects in $\mu\bar{\mu} \to h\gamma/hZ/q\bar{q}$ would require a muon collider running at energies $\sqrt{s} \sim 10$ TeV with an integrated luminosity of order $L \sim 10\text{ab}^{-1}$.
Finally, we analyzed specific extensions of the SM that may account for both ΔM_W and Δa_μ. As a first approach, we considered heavy NP particles generating the operators $\mathcal{O}^\mu_{eB}, \mathcal{O}^\mu_{eW}$ and $\mathcal{O}^{\mu q}_T$ at tree level. We found that only two vector-like particles, namely $\mathcal{B} \sim (1,1,0)$ and $\mathcal{L} \sim (1,2,1/2)$, may contribute simultaneously to M_W and $g-2$. However, the corrections to Δa_μ are always too small even in the regions of the parameter space where the M_W anomaly is accomodated. Hence, we moved to consider a fully renormalizable UV model, the 2HDM. By imposing a softly broken Z_2 discrete symmetry to avoid FCNC at tree-level, we found eight viable models containing six parameters each. We systematically analyzed the allowed parameter space by imposing all the theoretical and experimental bounds stemming from low- and high-energy data. Eventually, we found that only the lepton specific scenario can satisfy all these constraints while simultaneously explaining the ΔM_W and Δa_μ anomalies. In particular, we found that the relevant parameter space is such that the heavy Higgs sector of our 2HDM includes a scalar boson with mass $M_H \in [270, 280]$GeV, a light pseudoscalar with $M_A \in [20, 30]$GeV and charged Higgses with $M_{H^\pm} \in [300, 320]$GeV. We found that one of the most stringent bound on our 2HDM parameter space is given by the searches $h \to AA \to 4\tau$ and $h \to AA \to 2\mu 2\tau$. An improvement of the current resolution on the branching fractions of $h \to 4\tau/2\mu 2\tau$ could probe or falsify the analyzed scenario.

A SMEFT useful results

A.1 EW gauge and mass states: the diagonalization procedure

Here, we explicit some steps similar to what done in Ref. [28] in order to pass from the gauge states W^3_μ, B_μ to the Z and γ mass eigenstates, and to retrieve their masses.
The main idea is to rewrite the terms in Eq. 14 containing B and W_3 fields in the following form:

$$\mathcal{L}_{tot} \ni -\frac{1}{4}\Phi^T_{\mu\nu} K \Phi^{\mu\nu} + \frac{1}{2}\Phi^T_\mu M^2 \Phi^\mu \ , \tag{110}$$

where K, M^2 are 2×2 matrices, $\Phi_{\mu\nu} = \left(W^3_{\mu\nu}, B_{\mu\nu}\right)^T$ and $\Phi_\mu = \left(W^3_\mu, B_\mu\right)^T$. In our case, K and M^2 read:

$$K = \begin{pmatrix} 1 & 2\alpha_{WB}\frac{v^2}{\Lambda^2} \\ 2\alpha_{WB}\frac{v^2}{\Lambda^2} & 1 \end{pmatrix} \ ;$$

$$M^2 = \frac{1}{2}v^2 \begin{pmatrix} g^2_{SM}\left(1 + (\alpha_{\phi,1} + \alpha_{\phi,3} + 2\alpha_{\phi W})\frac{v^2}{\Lambda^2}\right) & -g_{SM}g'_{SM}\left(1 + (\alpha_{\phi,1} + \alpha_{\phi,3} + \alpha_{\phi W} + \alpha_{\phi B})\frac{v^2}{\Lambda^2}\right) \\ -g_{SM}g'_{SM}\left(1 + (\alpha_{\phi,1} + \alpha_{\phi,3} + \alpha_{\phi W} + \alpha_{\phi B})\frac{v^2}{\Lambda^2}\right) & g'^2_{SM}\left(1 + (\alpha_{\phi,1} + \alpha_{\phi,3} + 2\alpha_{\phi B})\frac{v^2}{\Lambda^2}\right) \end{pmatrix} .$$

Now, following Ref. [28], we need to find a 2×2 matrix D such that $DKD^T = \mathbb{I}_2$. One can show that, according to the structure of our K, the following matrix do the job:

$$D = \begin{pmatrix} \frac{1}{\sqrt{1-4\alpha^2_{WB}\frac{v^4}{\Lambda^4}}} & -\frac{2\alpha_{WB}\frac{v^2}{\Lambda^2}}{\sqrt{1-4\alpha^2_{WB}\frac{v^4}{\Lambda^4}}} \\ 0 & 1 \end{pmatrix} \overset{\text{Linearize}}{=} \begin{pmatrix} 1 & -2\alpha_{WB}\frac{v^2}{\Lambda^2} \\ 0 & 1 \end{pmatrix} \ , \tag{111}$$

and the eigenvalues of DM^2D^T are the masses of the physical particles while the respective normalized eigenvectors written as row-vectors give the orthogonal matrix R which diagonalize DM^2D^T, hence the normalized physical gauge bosons A_μ and Z_μ written in terms of B_μ and W^3_μ. First, we compute DM^2D^T (notice that it is symmetric) and keep only linear terms in the α's:

$$DM^2D^T = \frac{1}{2}v^2 \begin{pmatrix} g^2_{SM}(1 + A_{13W}) + 2g_{SM}g'_{SM}A_{WB} & -g_{SM}g'_{SM}(1 + A_{13WB} + g'^2_{SM}2\alpha_{WB}\frac{v^2}{\Lambda^2}) \\ \dots & g'^2_{SM}(1 + A_{13B}) \end{pmatrix} \ ,$$

where:

$$A_{13W} = 1 + (\alpha_{\phi,1} + \alpha_{\phi,3} + 2\alpha_{\phi W})\frac{v^2}{\Lambda^2} \ ; \qquad A_{13WB} = 1 + (\alpha_{\phi,1} + \alpha_{\phi,3} + \alpha_{\phi W} + \alpha_{\phi B})\frac{v^2}{\Lambda^2} \ ;$$
$$A_{13B} = 1 + (\alpha_{\phi,1} + \alpha_{\phi,3} + 2\alpha_{\phi B})\frac{v^2}{\Lambda^2} \ .$$

Then the eigenvalues are given by the characteristic polynomial $\det\left(DM^2D^T - \lambda\mathbb{I}\right) = 0$:

$$\lambda\left(\lambda - \frac{v^2}{2}\left(g^2_{SM}(1 + A_{13W}) + g'^2_{SM}(1 + A_{13B}) + 4g_{SM}g'_{SM}\alpha_{WB}\frac{v^2}{\Lambda^2}\right)\right) \overset{!}{=} 0 \ . \tag{112}$$

Keeping only terms linear in the Wilson coefficients, Eq. 112 provides the masses of the physical particles written in Eq. 16.

A.2 Coupling of W and Z to fermions

We start by considering the SM Lagrangian concerning the coupling of vector bosons to fermionic currents:

$$\mathcal{L}_{J,SM} = \frac{g_{SM}}{\sqrt{2}}\left(J^-_\mu W^\mu_+ + J^+_\mu W^\mu_-\right) + \frac{g_{SM}}{c_w}J^{nc}_\mu Z^\mu \ , \tag{113}$$

where we remember that, for the SM:

$$J^+_\mu = \bar{d_L}\gamma_\mu u_L + \bar{e_L}\gamma_\mu \nu_L \ ; \qquad J^-_\mu = \bar{u_L}\gamma_\mu d_L + \bar{\nu_L}\gamma_\mu e_L \ ; \qquad J^{nc}_\mu = g_L\bar{\psi}\gamma_{\mu,L}\psi + g_R\bar{\psi}\gamma_{\mu,R}\psi$$
$$g_L = T_{3L} - s^2_w Q \ ; \qquad g_R = -s^2_w Q,$$

where T_{3L} is weak isospin quantum number, Q the EM charge of the particle.
Similarly to what done for M_W, M_Z in Sec.3.1, we must consider all the dimension-six operators that may modify the coupling between the weak gauge bosons and a fermionic current. Hence, the following operators should be included:

$$\mathcal{O}_{\phi\ell 1} = i\left(\phi^\dagger D_\mu \phi\right)\left(\bar{\ell_L}\gamma^\mu \ell_L\right)\ ;\quad \mathcal{O}_{\phi\ell 3} = i\left(\phi^\dagger D_\mu \tau^I \phi\right)\left(\bar{\ell_L}\gamma^\mu \tau_I \ell_L\right)\ ;\quad \mathcal{O}_{\phi e} = i\left(\phi^\dagger D_\mu \phi\right)\left(\bar{e_R}\gamma^\mu e_R\right)\ ;$$
$$\mathcal{O}_{\phi q 1} = i\left(\phi^\dagger D_\mu \phi\right)\left(\bar{q_L}\gamma^\mu q_L\right)\ ;\quad \mathcal{O}_{\phi q 3} = i\left(\phi^\dagger D_\mu \tau^I \phi\right)\left(\bar{q_L}\gamma^\mu \tau_I q_L\right)\ ;\quad \mathcal{O}_{\phi u} = i\left(\phi^\dagger D_\mu \phi\right)\left(\bar{u_R}\gamma^\mu u_R\right)\ ;$$
$$\mathcal{O}_{\phi d} = i\left(\phi^\dagger D_\mu \phi\right)\left(\bar{d_R}\gamma^\mu d_R\right)\ ;\qquad \mathcal{O}_{\phi\phi} = i\left(\phi^\dagger \epsilon D_\mu \phi\right)\left(\bar{u_R}\gamma^\mu d_R\right)\ ,$$

where $\epsilon = \epsilon_{ij}$ with $i, j \in \{1, 2\}$ is the Levi-Civita tensor, $\ell_L = (\nu_L, e_L)^T$ and $q_L = (u_L, d_L)^T$. Since Eq. 113 is written in terms of physical fields, we should rewrite the previous operators accordingly. In particular, since we are interested in the boson-current coupling and not in boson-Higgs or Higgs-current coupling, we can evaluate the previous operators in the Higgs vev $\langle\phi\rangle = (0, v)^T$. Then, using the results from Eqs. 17-18 and remembering the normalizations from Eqs. 12-13 we get:

$$\begin{aligned}
i\left(\phi^\dagger D_\mu \phi\right) &= -\frac{v^2 g_{SM}}{2c_w}\left(1+\alpha_{ZZ}\frac{v^2}{\Lambda^2}\right) Z_\mu\ ;\\
\mathcal{O}_{\phi\ell 3} &= \frac{2v^2 g_{SM}}{\sqrt{2}}\left(1+\alpha_{\phi W}\frac{v^2}{\Lambda^2}\right) W^-_\mu\left(\bar{e_L}\gamma^\mu \nu_L\right) + \frac{v^2 g_{SM}}{2c_w}\left(1+\alpha_{ZZ}\frac{v^2}{\Lambda^2}\right) Z_\mu\left(\bar{\nu_L}\gamma^\mu\nu_L - \bar{e_L}\gamma^\mu e_L\right)\ ;\\
\mathcal{O}_{\phi\phi} &= -\frac{v^2 g_{SM}}{\sqrt{2}} W^+_\mu\left(1+\alpha_{\phi W}\frac{v^2}{\Lambda^2}\right)\left(\bar{u_R}\gamma^\mu d_R\right)\ .
\end{aligned}\tag{114}$$

Operators containing quarks instead of leptons can be obtained by replacing $\ell_L \to q_L$, $e_R \to d_R$, $\nu_R \to u_R$ in the expressions in Eq. 114.
Notice that we canonically normalized the vector bosons but not the fermions. This is because the covariant derivative is invariant under the rescaling in Eqs. 12-13, so the quadratic part in the fermions is not changed and their kinetic term is still canonically normalized.
Before summing the dimension-six contributions, we should rewrite the SM fields in Eq. 113 using the new normalizations, finding:

$$\mathcal{L}_J = \frac{g_{SM}}{\sqrt{2}}\left(1+\alpha_{\phi W}\frac{v^2}{\Lambda^2}\right)\left(J^+_\mu W^\mu_+ + J^+_\mu W^\mu_-\right) + g_{SM} J^{nc}_\mu\left(W^\mu_3 - \tan\theta_w B^\mu\right)\ . \tag{115}$$

The neutral current sector can be expanded as:

$$\begin{aligned}
\mathcal{L}_J \ni\ & g_{SM} T_{3L}(\bar{\psi_L}\gamma^\mu\psi_L) W^3_\mu + g'_{SM}\left(Q\bar{\psi}\gamma_\mu\psi - T_{3L}\bar{\psi_L}\gamma_\mu\psi_L\right) B^\mu\\
\stackrel{(\star)}{=}\ & g_{SM} A_\mu s_w\left(1+\alpha_{AA}\frac{v^2}{\Lambda^2}\right) Q\left(\bar{\psi}\gamma^\mu\psi\right) + g_{SM} Z_\mu\left[\left(c_w\left(1+\alpha_{ZZ}\frac{v^2}{\Lambda^2}\right) - s_w\alpha_{AZ}\frac{v^2}{\Lambda^2}\right) T_{3L}(\bar{\psi_L}\gamma^\mu\psi_L)\right.\\
&\left. - \left(\frac{s_w^2}{c_w}\left(1+\alpha_{ZZ}\frac{v^2}{\Lambda^2}\right) + s_w\alpha_{AZ}\frac{v^2}{\Lambda^2}\right)\left(Q\bar{\psi}\gamma_\mu\psi - T_{3L}\bar{\psi_L}\gamma^\mu\psi_L\right)\right]\\
=\ & eQA_\mu(\bar{\psi}\gamma^\mu\psi) - \frac{g_{SM}}{c_w} Q Z_\mu\left(s_w^2\left(1+\alpha_{ZZ}\frac{v^2}{\Lambda^2}\right) + s_w c_w\alpha_{AZ}\frac{v^2}{\Lambda^2}\right)(\bar{\psi}\gamma^\mu\psi)\\
& + \frac{g_{SM}}{c_w} Z_\mu T_{3L}(\bar{\psi_L}\gamma^\mu\psi_L)\left(1+\alpha_{ZZ}\frac{v^2}{\Lambda^2}\right)\ ,
\end{aligned}\tag{116}$$

where in the step $(\star)$ we substitute B_μ and W_μ by the expressions given in Eqs. 17-18. Notice that in the final expression the EM coupling is not denoted with the subscript SM: this is because $e = g_{SM}s_w\left(1+\alpha_{AA}\frac{v^2}{\Lambda^2}\right)$ is the new EM coupling corrected with the dimension-six contributions (if one neglects corrections of $O(\Lambda^{-2})$ then he gets the usual $e_{SM} = g_{SM}s_w$).
According to Ref. [26], the final Lagrangian can be written as:

$$\mathcal{L}_{tot,J} = \mathcal{L}_J + \mathcal{L}_{(6)} = \frac{g_{SM}}{\sqrt{2}}\left(J^-_\mu W^\mu_+ + h.c.\right) + \frac{g_{SM}}{c_w} J^{nc}_\mu Z^\mu\ , \tag{117}$$

where now the charged currents receive corrections from Eq. 115, neutral currents from Eq. 116 and both from dimension-six operators. Exploiting the results from Eq. 114 and summing up, we get the following results (over the arrow we denote which dim-6 operator(s) should be considered):

$$\begin{aligned}
(\bar{\nu_L}\gamma_\mu e_L) &\xrightarrow{\mathcal{O}_{\phi\ell 3}} && (\bar{\nu_L}\gamma_\mu e_L)\left(1+(\alpha_{\phi W}+2\alpha_{\phi\ell 3})\frac{v^2}{\Lambda^2}\right) \equiv (\bar{\nu_L}\gamma_\mu e_L)\eta(\nu_L)\ ;\\
(\bar{u_L}\gamma_\mu d_L) &\xrightarrow{\mathcal{O}_{\phi q 3}} && (\bar{u_L}\gamma_\mu d_L)\left(1+(\alpha_{\phi W}+2\alpha_{\phi q 3})\frac{v^2}{\Lambda^2}\right) \equiv (\bar{u_L}\gamma_\mu d_L)\eta(u_L)\ ;\\
(\bar{u_R}\gamma_\mu d_R) &\xrightarrow{\mathcal{O}_{\phi\phi}} && -(\bar{u_R}\gamma_\mu d_R)\frac{v^2}{\Lambda^2}\alpha_{\phi\phi} \equiv (\bar{u_R}\gamma_\mu d_R)\eta(u_R)\ ;\\
(\bar{\nu_L}\gamma_\mu \nu_L) &\xrightarrow{\mathcal{O}_{\phi\ell 1},\mathcal{O}_{\phi\ell 3}} && (\bar{\nu_L}\gamma_\mu \nu_L)\frac{1}{2}\left(1+(\alpha_{ZZ}-\alpha_{\phi\ell 1}+\alpha_{\phi\ell 3})\frac{v^2}{\Lambda^2}\right) \equiv (\bar{\nu_L}\gamma_\mu \nu_L)\epsilon(\nu_L)\ ;\\
(\bar{e_R}\gamma_\mu e_R) &\xrightarrow{\mathcal{O}_{\phi e}} && (\bar{e_R}\gamma_\mu e_R)\left(s_w^2+\frac{v^2}{\Lambda^2}\left(s_w c_w\alpha_{AZ}+s_w^2\alpha_{ZZ}-\frac{1}{2}\alpha_{\phi e}\right)\right) \equiv (\bar{e_R}\gamma_\mu e_R)\epsilon(e_R)\ ;\\
(\bar{u_R}\gamma_\mu u_R) &\xrightarrow{\mathcal{O}_{\phi u}} && (\bar{u_R}\gamma_\mu u_R)\left(-\frac{2s_w^2}{3}+\frac{v^2}{\Lambda^2}\left(-\frac{2s_w^2}{3}\alpha_{ZZ}-\frac{2s_w c_w}{3}\alpha_{AZ}-\frac{\alpha_{\phi u}}{2}\right)\right) \equiv (\bar{u_R}\gamma_\mu u_R)\epsilon(u_R)\ ;\\
(\bar{d_R}\gamma_\mu d_R) &\xrightarrow{\mathcal{O}_{\phi d}} && (\bar{d_R}\gamma_\mu d_R)\left(\frac{s_w^2}{3}+\frac{v^2}{\Lambda^2}\left(\frac{s_w^2}{3}\alpha_{ZZ}+\frac{s_w c_w}{3}\alpha_{AZ}-\frac{\alpha_{\phi d}}{2}\right)\right) \equiv (\bar{d_R}\gamma_\mu d_R)\epsilon(d_R)\ ;\\
(\bar{e_L}\gamma_\mu e_L) &\xrightarrow{\mathcal{O}_{\phi\ell 1},\mathcal{O}_{\phi\ell 3}} && (\bar{e_L}\gamma_\mu e_L)\frac{1}{2}\left(-1+2s_w^2+\frac{v^2}{\Lambda^2}(2s_w c_w\alpha_{AZ}-\alpha_{ZZ}+2s_w^2\alpha_{ZZ}-\alpha_{\phi\ell 1}-\alpha_{\phi\ell 3})\right) \equiv\\
& && \equiv (\bar{e_L}\gamma_\mu e_L)\epsilon(e_L)\ ;\\
(\bar{u_L}\gamma_\mu u_L) &\xrightarrow{\mathcal{O}_{\phi q 1},\mathcal{O}_{\phi q 3}} && (\bar{u_L}\gamma_\mu u_L)\left(\frac{1}{2}-\frac{2}{3}s_w^2+\frac{v^2}{\Lambda^2}\left(-\frac{2}{3}s_w^2\alpha_{ZZ}-\frac{2}{3}s_w c_w\alpha_{AZ}+\frac{1}{2}\alpha_{ZZ}-\frac{1}{2}\alpha_{\phi q 1}+\frac{1}{2}\alpha_{\phi q 3}\right)\right) \equiv\\
& && \equiv (\bar{u_L}\gamma_\mu u_L)\epsilon(u_L)\ ;\\
(\bar{d_L}\gamma_\mu d_L) &\xrightarrow{\mathcal{O}_{\phi q 1},\mathcal{O}_{\phi q 3}} && (\bar{d_L}\gamma_\mu d_L)\left(-\frac{1}{2}+\frac{1}{3}s_w^2+\frac{v^2}{\Lambda^2}\left(\frac{1}{3}s_w^2\alpha_{ZZ}+\frac{1}{3}s_w c_w\alpha_{AZ}-\frac{1}{2}\alpha_{ZZ}-\frac{1}{2}\alpha_{\phi q 1}-\frac{1}{2}\alpha_{\phi q 3}\right)\right) \equiv\\
& && \equiv (\bar{d_L}\gamma_\mu d_L)\epsilon(d_L)\ .
\end{aligned}$$

By replacing these expressions inside J^-_μ, J^+_μ and J^{nc}_μ of Eq. 117 one finally gets the correct W-current, Z-current couplings. Notice that, thanks to $\mathcal{O}_{\phi\phi}$ there is a genuine new interaction between the W boson and a right-handed quark current (there is no lepton counterpart because right handed neutrinos are needed and in our approach we use only SM fields).

A.3 The fermion-Higgs coupling

Dimension-six operators that contribute to Yukawa-type interactions are (h.c. operators are understood):

$$\mathcal{O}_{\ell\phi} = \left(\phi^\dagger\phi\right)\left(\bar{\ell}_L e_R \phi\right)\ , \qquad \mathcal{O}_{\phi\phi} = \left(\phi^\dagger\phi\right)\left(\bar{q_L} u_R \tilde{\phi}\right)\ , \qquad \mathcal{O}_{d\phi} = \left(\phi^\dagger\phi\right)\left(\bar{q_L} d_R \phi\right)\ , \tag{118}$$

where $\tilde{\phi} = i\tau_2\phi^\star$. The SM Yukawa interactions read:

$$\mathcal{L}_{SM} \ni Y_e \bar{\ell_L} e_R \phi + Y_u \bar{q_L} u_R \tilde{\phi} + Y_d \bar{q_L} d_R \phi + \text{h.c.}\ , \tag{119}$$

where Y_i are 3×3 Yukawa matrices. If we want to write Higgs-fermion interactions considering also dim-6 operators, then we simply add to Eq. 119 the terms in Eq. 118, and then we expand around the Higgs vev $\langle\phi\rangle = (v+h)(0,1)^T$. Notice that in this case we consider also the fluctuations around the minimum since we are interested in terms like $h\bar{\psi}\psi$. Hence:

$$\begin{aligned}
\mathcal{L}_y &= v\left(Y_e+\frac{v^2}{\Lambda^2}\alpha_{e\phi}\right)\bar{e_L}e_R + \left(Y_e+3\frac{v^2}{\Lambda^2}\alpha_{e\phi}\right)h\bar{e_L}e_R + v\left(Y_u+\frac{v^2}{\Lambda^2}\alpha_{u\phi}\right)\bar{q_L}u_R + \left(Y_u+3\frac{v^2}{\Lambda^2}\alpha_{u\phi}\right)h\bar{q_L}u_R\\
&\quad +v\left(Y_d+\frac{v^2}{\Lambda^2}\alpha_{d\phi}\right)\bar{q_L}d_R + \left(Y_d+3\frac{v^2}{\Lambda^2}\alpha_{d\phi}\right)h\bar{q_L}d_R + \text{h.c.} + O(h^2\bar{\psi}\psi)\ ,
\end{aligned} \tag{120}$$

where we omitted interactions between 2 fermions and 2 or 3 Higgs (they come entirely from dimension-six operators). From Eq. 120 it is easy to read the corrected mass m_i of fermions (assuming mass basis, i.e. the Yukawa matrices are diagonal) and the new couplings g_{iH} between Higgs and 2 fermions[15]:

$$m_i = v\left(Y_i + \frac{v^2}{\Lambda^2}\alpha_{i\phi}\right) ; \qquad g_{iH} = Y_i + 3\frac{v^2}{\Lambda^2}\alpha_{i\phi} ,$$

where $i \in \{e, u, d\}$. Notice that now the fermion masses are no longer directly proportional to the Higgs vev and that the Higgs-fermion-fermion interactions may be no longer diagonal in the mass basis since, if it is true that Y_i matrices are diagonal, the 3×3 matrices in flavour space $\alpha_{i\phi}$ may be not, leading to the possibility to couple the Higgs boson to different generations of fermions (unless Y_i and $\alpha_{i\phi}$ are simultaneously diagonalized, but *a priori* this would be a particular case).

A.4 The Higgs mass and self-interactions

According to the SM Lagrangian, the scalar potential reads:

$$V(\phi^\dagger\phi) = m^2\phi^\dagger\phi + \frac{\lambda}{2}(\phi^\dagger\phi)^2 .$$

The only dimension-six operator containing only Higgs doublets and without derivatives contributing to the previous potential is $\mathcal{O}_\phi = \frac{1}{3}(\phi^\dagger\phi)^3$. Then the SMEFT scalar potential is:

$$V(\phi^\dagger\phi)_{tot} = m^2\phi^\dagger\phi + \frac{\lambda}{2}(\phi^\dagger\phi)^2 + \frac{\alpha_\phi}{3\Lambda^2}(\phi^\dagger\phi)^3 . \tag{121}$$

The modified Higgs vev is given by minimization of Eq. 121. Once we set $|\phi|^2 = v^2 \equiv x$, the solution is:

$$x = \frac{\Lambda^2}{2\alpha_\phi}\left(-\lambda + \sqrt{\lambda^2 - \frac{4m^2\alpha_\phi}{\Lambda^2}}\right) .$$

As already done several times in this part of the Appendix, we consider only linear terms in the Wilson coefficients, obtaining the modified Higgs vev:

$$v^2 = -\frac{m^2}{\lambda}\left(1 + \alpha_\phi\frac{m^2}{\lambda^2\Lambda^2}\right) \longleftrightarrow m^2 = -\lambda v^2\left(1 + \frac{\alpha_\phi}{\lambda}\frac{v^2}{\Lambda^2},\right) , \tag{122}$$

where the two relations can be obtained one from the other by inversion.
We focus now on the Higgs quartic coupling. The SMEFT terms we are interested in are:

$$\mathcal{L}_H \ni \frac{\lambda}{2}(\phi^\dagger\phi)^2 + \frac{\alpha_\phi}{3\Lambda^2}(\phi^\dagger\phi)^3 \xrightarrow{\phi^\dagger\phi=(v+h)^2} \frac{\lambda}{2}(v+h)^4 + \frac{\alpha_\phi}{3\Lambda^2}(v+h)^6 \xrightarrow{(\star)} \frac{1}{2}h^4\left(\lambda + 10\frac{v^2}{\Lambda^2}\alpha_\phi\right) \stackrel{!}{=} \frac{\lambda_H}{2}h^4 , \tag{123}$$

where in the $(\star)$ step we kept only h^4 terms. From the last equality it is clear that the effective quartic coupling reads $\lambda_H = \lambda + 10\alpha_\phi\frac{v^2}{\Lambda^2}$.
We proceed in a similar way also for the Higgs mass. Starting from Eq. 121, expanding around the Higgs vev and keeping quadratic terms in the Higgs field, we obtain:

$$h^2\left(m^2 + 3\lambda v^2 + 5\alpha_\phi\frac{v^4}{\Lambda^2}\right) . \tag{124}$$

Now we should remember that v (equivalently m) is modified as described in Eq. 122, thus by inserting in Eq. 124 and keeping only linear terms in α_i, one obtains:

$$h^2\left(2\lambda v^2 + 4\alpha_\phi\frac{v^4}{\Lambda^2}\right) \stackrel{!}{=} \frac{1}{2}M_H^2h^2 \longrightarrow M_H^2 = 4\lambda v^2 + 8\alpha_\phi\frac{v^4}{\Lambda^2} . \tag{125}$$

[15] Here flavour indexes are understood.

A.5 W and Y parameters

The approach is analogous to Sec.3.2 so we will skip some steps.
The W and Y parameters are defined as [32]:

$$W \equiv \frac{1}{2} M_W^2 \Pi''_{W_3 W_3}(0) \ ; \qquad\qquad Y \equiv \frac{1}{2} M_W^2 \Pi''_{BB}(0) \ . \tag{126}$$

W receives SMEFT corrections by $\mathcal{O}_{WW} = \frac{1}{2}(D_\rho W^I_{\mu\nu})^2$. By expanding it, one finds:

$$\mathcal{O}_{WW} \ni (\partial_\rho \partial_\mu W^3_\nu - \partial_\rho \partial_\nu W^3_\mu)(\partial^\rho \partial^\mu W^\nu_3 - \partial^\rho \partial^\nu W^\mu_3) \ ,$$

where we selected only the terms which contributes to $\Pi^{\mu\nu}_{W_3 W_3}(q^2)$ and ignored all the vertices of the kind WWW or higher order. The propagator corrected by $\mathcal{O}_{WW}$ reads:

$$\Pi^{\mu\nu}_{W_3 W_3}(q^2) = -2 c_{WW} q^4 \eta^{\mu\nu} \longrightarrow \Pi''_{W_3 W_3}(0) = -4 c_{WW} \ . \tag{127}$$

By replacing Eq. 127 in the definition of W, one gets $W = -\frac{1}{2} g_2^2 v^2 c_{WW}$.
The procedure for Y is the same: one should replace $W_3 \to B$ and $c_{WW} \to c_{BB}$ in Eq. 127, obtaining $Y = -\frac{1}{2} g^2 v^2 c_{BB}$.

B The effective $\mathcal{L}^{\rm int}_{\rm SMEFT}$

In this part of the Appendix we want to explicit some calculations made in order to arrive to the coefficients in Eq. 35 and make some discussion about them. The way to proceed is similar to what explained in Sec.3.1 and explicit in Appendix A. All the notation used in this section is related to the Lagrangian written in Eq. 34 and Sec. 3.2 in the main body of this work.

B.1 The effective Higgs sector

Other than the SM terms, in our effective approach we must consider also these dimension-six operators:

$$\mathcal{O}_H = (H^\dagger H)^3 \; ; \qquad \mathcal{O}_{H\Box} = (H^\dagger H)\Box(H^\dagger H) \; ; \qquad \mathcal{O}_{HD} = \left|H^\dagger D_\mu H\right|^2 \; .$$

Summing up SM and dimension-six operators, the effective Lagrangian of the Higgs sector is:

$$\mathcal{L}_H = (D_\mu H)^\dagger (D^\mu H) + m^2 (H^\dagger H) - \frac{\lambda}{2}(H^\dagger H)^2 + c_H (H^\dagger H)^3 + c_{H\Box}(H^\dagger H)\Box(H^\dagger H) + c_{HD}\left|H^\dagger D_\mu H\right|^2 \; . \tag{128}$$

First, we need to find the new Higgs vev. Similarly to what done in Appendix A.4, we rewrite the scalar potential:

$$V_H(H^\dagger H) = m^2(H^\dagger H) - \frac{\lambda}{2}(H^\dagger H)^2 + c_H (H^\dagger H)^3 \; .$$

Next we find its minimum through a minimization procedure. The condition to have absolute minimum is (set $\langle H^\dagger H \rangle = \frac{v^2}{2} \equiv x$):

$$\frac{\partial V_H}{\partial x} \stackrel{!}{=} 0 \longrightarrow m^2 = \lambda x - 3 c_H x^2 \; .$$

Solving the second degree equation in x and keeping only linear terms in c_H one obtains[16]:

$$v^2 = \frac{2m^2}{\lambda} + 6\frac{c_H m^4}{\lambda^3} \longrightarrow v = \sqrt{\frac{2m^2}{\lambda}} + \frac{3}{\sqrt{2}}\frac{m^3}{\lambda^{5/2}} c_H \; . \tag{129}$$

On the other hand, one remembers $v^2_{SM} = \frac{2m^2}{\lambda}$, hence one can rewrite Eq. 129 as:

$$v^2 = v^2_{SM}\left(1 + \frac{3}{2}\frac{c_H}{\lambda}v^2\right) \; . \tag{130}$$

Differently from what we did from Appendix A.1 to A.4, we would like to compute not only new Higgs vev, the normalized couplings and fields etc. but also the effective couplings between Higgs and other particles and Higgs self-interactions. To do so, we can no longer evaluate the operators in $\langle H \rangle$ but we need to explicit the physical field too, i.e. $H \to \frac{v+h}{\sqrt{2}}(0,1)^T$ where h is the fluctuation around the vev (we are using the unitary gauge to simplify our calculations). Thus, expanding the operators in Eq. 128, one has:

$$\begin{aligned}
(D_\mu H)^\dagger (D^\mu H) &= \frac{1}{2}(\partial_\mu h)^2 + \frac{v^2 g_2^2}{4} W^+_\mu W^\mu_- + \frac{v g_2^2}{2} h W^+_\mu W^\mu_- + \frac{g_2^2}{4} h^2 W^+_\mu W^\mu_- + \frac{(v+h)^2}{8}(g_1 B_\mu - g_2 W^3_\mu)^2 \; ; \\
m^2 (H^\dagger H) &= \frac{\lambda v^2}{4}\left(1 - \frac{3}{2}\frac{c_H}{\lambda}v^2\right)(v+h)^2 \; ; \\
\frac{\lambda}{2}(H^\dagger H)^2 &= -\frac{\lambda}{2}\frac{(v+h)^4}{4} \; ; \\
c_H (H^\dagger H)^3 &= \frac{c_H}{8}(v+h)^6 \; ; \\
c_{H\Box}(H^\dagger H)\Box(H^\dagger H) &= -c_{H\Box}(v+h)^2(\partial_\mu h)^2 \; ; \\
c_{HD}\left|H^\dagger D_\mu H\right|^2 &= \frac{c_{HD}}{4}(v+h)^2(\partial_\mu h)^2 + c_{HD}\frac{(v+h)^4}{16}(g_1 B_\mu - g_2 W^3_\mu)^2 \; .
\end{aligned} \tag{131}$$

[16]Cfr. Sec.3 of Ref. [30].

Notice that, because of the covariant derivatives, there already appear the EW gauge bosons and their couplings with the Higgs. We will not use them now, since they will be treated separately in the next subsection. Focusing only on the Higgs terms, we notice that $\mathcal{O}_{H\Box}$ and $\mathcal{O}_{HD}$ contribute to the Higgs kinetic term $\frac{1}{2}(\partial_\mu h^2)$ making it not canonical. To make it canonical, we should redefine $h \to Z_h^{1/2} h$ with Z_h such that:

$$\frac{1}{2}(\partial_\mu h)^2 \left[1 + \left(\frac{1}{2}c_{HD} - 2c_{H\Box}\right) v^2\right] Z_h \stackrel{!}{=} \frac{1}{2}(\partial_\mu h)^2 \longrightarrow Z_h^{1/2} = 1 + \left(c_{H\Box} - \frac{1}{4}c_{HD}\right) v^2 \; . \tag{132}$$

By applying this shift in all the operators in Eq. 131 and linearizing in the Wilson coefficients, we obtain:

$$\mathcal{L}_H \ni \frac{1}{2}(\partial_\mu h)^2 + \frac{1}{2}(\partial_\mu h)^2 \left(\frac{1}{2}c_{HD} - 2c_{H\Box}\right)\left(\frac{2h}{v} + \frac{h^2}{v^2}\right) v^2 + g_h h + g_{hh} h^2 + g_{hhh} h^3 + g_{4h} h^4 + g_{5h} h^5 + g_{6h} h^6 \; , \tag{133}$$

where g_i are given by (cfr. Sec.5 of Ref. [36]):

$$\begin{aligned}
g_h &= 0 \; ; \\
g_{hh} &\equiv -\frac{1}{2}M_H^2 = -\frac{\lambda v^2}{2}\left(1 + v^2\left(2c_{H\Box} - \frac{1}{2}c_{HD} - 3\frac{c_H}{\lambda}\right)\right) \; ; \\
g_{hhh} &= -\frac{M_H^2}{2v}\left(1 - \frac{v^2}{4}(c_{HD} - 4c_{H\Box}) - \frac{2v^4}{M_H^2}c_H\right) \; ; \\
g_{4h} &= -\frac{\lambda}{8}\left(1 + (4c_{H\Box} - c_{HD}) - 15\frac{c_H}{\lambda}v^2\right) \; ; \\
g_{5h} &= \frac{3}{4}c_H v \; ; \\
g_{6h} &= \frac{c_H}{8} \; .
\end{aligned} \tag{134}$$

• **About g_{hhh}**
We briefly focus on how we computed g_{hhh}. If one groups all the terms in h^3, he should find:

$$g_{hhh} = -\frac{\lambda v}{2}\left(1 - \frac{3}{4}v^2(c_{HD} - 4c_{H\Box}) - \frac{5v^2}{\lambda}c_H\right) \; .$$

Then, we exploited the g_{hh} coefficient, i.e., the new Higgs mass, in order to rewrite λv^2 as:

$$\lambda v^2 = M_H^2\left(1 - v^2\left(2c_{H\Box} - \frac{1}{2}c_{HD} - 3\frac{c_H}{\lambda}\right)\right) \; ,$$

and finally get g_{hhh} in Eq. 134 (the same that appears in Eq. 35). Notice that M_H^2 at denominator of c_H can be replaced by $M_{H,SM}^2 = \lambda v_{SM}^2$ since distinguishing the new Higgs mass with the SM one (equivalently v^2 from v_{SM}^2) would be a higher order correction $O(\Lambda^{-4})$.

B.2 The effective EW gauge boson sector

This subsection is similar to Sec.3.1 and computations done in Appendix A.1 so we will skip some explanations. The dimension-six operators involved are:

$$\mathcal{O}_{HW} = (H^\dagger H)W^I_{\mu\nu}W_I^{\mu\nu} \; ; \qquad \mathcal{O}_{HB} = (H^\dagger H)B_{\mu\nu}B^{\mu\nu} \; ; \qquad \mathcal{O}_{HWB} = (H^\dagger \tau_I H)W^I_{\mu\nu}B^{\mu\nu} \; ,$$

and $\mathcal{O}_{HD}$ defined in the previous subsection. The effective EW gauge boson Lagrangian reads:

$$\mathcal{L}_{EW} = -\frac{1}{4}W^I_{\mu\nu}W_I^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + (D_\mu H)^\dagger(D^\mu H) + c_{HW}\mathcal{O}_{HW} + c_{HB}\mathcal{O}_{HB} + c_{HWB}\mathcal{O}_{HWB} + c_{HD}\mathcal{O}_{HD} \; . \tag{135}$$

First, we expand every operator around $H = \frac{v+h}{\sqrt{2}}(0,1)^T$ obtaining:

$$\begin{aligned}
(D_\mu H)^\dagger (D^\mu H) &= \text{see Eq. 131} ; \\
c_{HW}(H^\dagger H) W^I_{\mu\nu} W^{\mu\nu}_I &= \frac{1}{2} v^2 \left(1 + \frac{2h}{v} + \frac{h^2}{v^2}\right) W^I_{\mu\nu} W^{\mu\nu}_I ; \\
c_{HB}(H^\dagger H) B_{\mu\nu} B^{\mu\nu} &= \frac{1}{2} c_{HB} v^2 \left(1 + \frac{2h}{v} + \frac{h^2}{v^2}\right) B_{\mu\nu} B^{\mu\nu} ; \\
c_{HWB}(H^\dagger \tau_I H) W^I_{\mu\nu} B^{\mu\nu} &= -\frac{1}{2} c_{HWB} v^2 \left(1 + \frac{2h}{v} + \frac{h^2}{v^2}\right) W^3_{\mu\nu} B^{\mu\nu} ; \\
c_{HD} \left| H^\dagger D_\mu H \right|^2 &= \text{see Eq. 131} .
\end{aligned} \tag{136}$$

Now we notice that due to $\mathcal{O}_{HW}$ (resp. $\mathcal{O}_{HB}$) the kinetic term for W^I_μ (resp. B_μ) is no longer canonically normalized. Analogously to what done for the Higgs, we would like to redefine $W^I_{\mu\nu} \to W^I_{\mu\nu} Z_W^{1/2}$ and $B_{\mu\nu} \to B_{\mu\nu} Z_B^{1/2}$ so that:

$$-\frac{1}{4} W^I_{\mu\nu} W^{\mu\nu}_I (1 - 2c_{HW} v^2) Z_W \stackrel{!}{=} -\frac{1}{4} W^I_{\mu\nu} W^{\mu\nu}_I \longrightarrow Z_W^{1/2} = 1 + c_{HW} v^2 ; \tag{137}$$

$$-\frac{1}{4} B_{\mu\nu} B^{\mu\nu} (1 - 2c_{HB} v^2) Z_B \stackrel{!}{=} -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} \longrightarrow Z_B^{1/2} = 1 + c_{HB} v^2 . \tag{138}$$

Now we remember that only fields can be rescaled, not the field strengths, thus, if we want Eqs. 137-138 to hold, then we must redefine also the couplings (see Eq. 12 and Eq. 13) as:

$$g_2 \to Z_W^{-1/2} g_2 ; \qquad g_1 \to Z_B^{-1/2} g_1 .$$

By replacing Eq. 136 into Eq. 135 and rescaling the fields (also the Higgs), one obtains the full Lagrangian:

$$\begin{aligned}
\mathcal{L}_{EW} = {} & \frac{v^2 g_2^2}{4} W^+_\mu W^\mu_- + \frac{v g_2^2}{2} Z_H^{1/2} h W^+_\mu W^\mu_- + c_{HD} \frac{(v+h)^4}{16} (g_1 B_\mu - g_2 W^3_\mu)^2 \\
& - \frac{1}{4} W^I_{\mu\nu} W^{\mu\nu}_I - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{2} c_{HWB} v^2 W^3_{\mu\nu} B^{\mu\nu} + \frac{1}{2} c_{HW} v^2 \left(\frac{2h}{v} + \frac{h^2}{v^2}\right) W^I_{\mu\nu} W^{\mu\nu}_I \\
& + \frac{1}{2} c_{HWB} \left(\frac{2h}{v} + \frac{h^2}{v^2}\right) B_{\mu\nu} B^{\mu\nu} - \frac{1}{2} c_{HWB} \left(\frac{2h}{v} + \frac{h^2}{v^2}\right) W^3_{\mu\nu} B^{\mu\nu} .
\end{aligned} \tag{139}$$

As expected, we see mixing terms $W^3_{\mu\nu} B^{\mu\nu}$ and $W^3_\mu B^\mu$ as in Eq. 14. To get rid of them, we should apply the same procedure as did in Appendix A.1, namely the diagonalization procedure. Adopting the same notation of Eq. 110, we have:

$$K = \begin{pmatrix} 1 & c_{HWB} v^2 \\ c_{HWB} v^2 & 1 \end{pmatrix} ; \qquad M^2 = \begin{pmatrix} \frac{v^2 g_2^2}{4} \left(1 + \frac{v^2}{2} c_{HD}\right) & -\frac{v^2 g_1 g_2}{4} \left(1 + \frac{v^2}{2} c_{HD}\right) \\ \cdots & \frac{v^2 g_1^2}{4} \left(1 + \frac{v^2}{2} c_{HD}\right) \end{pmatrix} .$$

The matrix D such that $D M^2 D^T = \mathbb{I}_2$ is:

$$D = \begin{pmatrix} \frac{1}{\sqrt{1-a^2}} & -\frac{a}{\sqrt{1-a^2}} \\ 0 & 1 \end{pmatrix} \stackrel{\text{Linearize}}{=} \begin{pmatrix} 1 & -a \\ 0 & 1 \end{pmatrix} \qquad \text{where } a = c_{HWB} v^2 .$$

Next we compute $D M^2 D^T$ whose eigenvalues are the physical masses of Z-boson and photon:

$$D M^2 D^T = \begin{pmatrix} \frac{v^2}{4} \left(g_2^2 + v^2 \left(\frac{1}{2} c_{HD} g_2^2 + 2 c_{HWB} g_1 g_2\right)\right) & -\frac{v^2}{4} \left(g_1 g_2 + v^2 \left(\frac{1}{2} c_{HD} g_1 g_2 + c_{HWB} g_1^2\right)\right) \\ \cdots & \frac{v^2}{4} \left(g_1^2 + \frac{v^2}{2} c_{HD} g_1^2\right) \end{pmatrix} .$$

The characteristic polynomial:

$$\lambda\left(\lambda-\frac{v^2}{4}(g_1^2+g_2^2)\left(1+v^2\left(\frac{1}{2}c_{HD}+2c_{HWB}\frac{g_1g_2}{g_1^2+g_2^2}\right)\right)\right)\stackrel{!}{=}0\ ,$$

hence the masses:

$$M_\gamma^2=0\ ;\qquad M_Z^2=\frac{v^2}{4}(g_1^2+g_2^2)\left(1+\frac{v^2}{2}\left(c_{HD}+4c_{HWB}\frac{g_1g_2}{g_1^2+g_2^2}\right)\right)\ . \tag{140}$$

From Ref. [30] one can rewrite W^3_μ and B_μ in terms of A_μ, Z_μ and the corrected Weinberg angle $\bar\theta$ (let be $\bar c\equiv\cos\bar\theta$ and $\bar s\equiv\sin\bar\theta$) as:

$$\begin{aligned}W^3_\mu &= Z_\mu\left(\bar c+\frac{1}{2}\bar s c_{HWB}v^2\right)+A_\mu\left(\bar s-\frac{1}{2}\bar c c_{HWB}v^2\right)\\ &= Z_\mu c_w\left(1+s_w^2v^2\xi_W\right)+A_\mu s_w\left(1-c_w^2v^2\xi_W\right)\quad\text{with } \xi_W=c_{HW}-c_{HB}+\tan\theta_W c_{HWB}\ ;\end{aligned}\tag{141}$$

$$\begin{aligned}B_\mu &= -Z_\mu\left(\bar s+\frac{1}{2}c_wc_{HWB}v^2\right)+A_\mu\left(\bar c-\frac{1}{2}s_wc_{HWB}v^2\right)\\ &= -Z_\mu s_w\left(1+c_w^2v^2\xi_A\right)+A_\mu c_w\left(1-s_w^2v^2\xi_A\right)\quad\text{with } \xi_A=c_{HB}-c_{HW}+\cot\theta_W c_{HWB}\ ,\end{aligned}\tag{142}$$

where we can relate goniometric functions of $\bar\theta$ to the SM ones c_w and s_w through the following relations (they are retrieved by using trigonometric identities and linear expansion in the Wilson coefficients):

$$\begin{aligned}\bar s &= s_w\left(1+c_w^2v^2\left(c_{HB}-c_{HW}+\frac{1}{2}c_{HWB}\frac{g_{2,SM}^2-g_{1,SM}^2}{g_{1,SM}g_{2,SM}}\right)\right)\ ;\\ \bar c &= c_w\left(1-s_w^2v^2\left(c_{HB}-c_{HW}+\frac{1}{2}c_{HWB}\frac{g_{2,SM}^2-g_{1,SM}^2}{g_{1,SM}g_{2,SM}}\right)\right)\ ;\\ \tan\bar\theta &= \tan\theta_W\left(1+\left(c_{HB}-c_{HW}+\frac{1}{2}c_{HWB}\frac{g_{2,SM}^2-g_{1,SM}^2}{g_{1,SM}g_{2,SM}}\right)\right)\ .\end{aligned}$$

Before proceeding any further, we can do a consistency check of our calculations. If they are right, then we should expect that by expanding the terms $(g_1B_\mu-g_2W^3_\mu)^2$ in Eq. 139 linearly in the Wilson coefficients and by applying the redefinitions in Eqs. 137-138, we should retrieve the masses of the photon the Z-boson and, hopefully, there should be no mixed term $Z_\mu A^\mu$. In particular, the general form is:

$$\mathcal{L}_{EW}\ni\frac{v^2}{8}\left(1+\frac{c_{HD}}{2}v^2\right)(g_1B_\mu-g_2W^3_\mu)^2=\alpha_1A_\mu^2+\alpha_2A_\mu Z^\mu+\alpha_3Z_\mu^2\ .$$

By computing the coefficients α_i, one gets:

$$\begin{aligned}\alpha_1 &= 0\ \text{(correct, photons are massless)}\ ;\\ \alpha_2 &= 0\ \text{(correct, we are in the mass basis)}\ ;\\ \alpha_3 &= \frac{v^2}{4}\left(1+2c_{HWB}v^2\frac{g_1g_2}{g_1^2+g_2^2}\right)(g_1^2+g_2^2)\left(1+\frac{c_{HD}}{2}v^2\right)=\text{(linearize)}\\ &= \frac{v^2}{8}(g_1^2+g_2^2)\left(1+\frac{v^2}{2}\left(c_{HD}+4c_{HWB}\frac{g_1g_2}{g_1^2+g_2^2}\right)\right)\stackrel{!}{=}\frac{1}{2}M_Z^2\ \text{(correct, as in Eq. 140)}\ .\end{aligned}$$

Now we express the field strength tensors $W^I_{\mu\nu}$ and $B_{\mu\nu}$ in terms of the physical fields strength tensors $F_{\mu\nu}=\partial_{[\mu}A_{\nu]}$, $Z_{\mu\nu}=\partial_{[\mu}Z_{\nu]}$ and $W^\pm_{\mu\nu}=\partial_{[\mu}W^\pm_{\nu]}$. With some algebra, one should obtain the following results:

$$
\begin{aligned}
W^1_{\mu\nu}W_1^{\mu\nu} + W^2_{\mu\nu}W_2^{\mu\nu} &= 2W^+_{\mu\nu}W_-^{\mu\nu} + 4g_2^2\left(W^+_\mu W^\mu_-(W_3)^2 - W^3_\mu W^\mu_+ W^3_\nu W^\nu_-\right) \\
&\quad +4ig_2\left(\partial_\mu W^+_\nu(W^\mu_- W^\nu_3 - W^\nu_- W^\mu_3) + \partial_\mu W^-_\nu(W^\nu_+ W^\mu_3 - W^\mu_+ W^\nu_3)\right) \ ; \\
W^3_{\mu\nu}W_3^{\mu\nu} &= \left(\left(\bar{c} + \frac{1}{2}\bar{s}c_{HWB}v^2\right)Z_{\mu\nu} + \left(\bar{s} - \frac{1}{2}\bar{c}c_{HWB}v^2\right)F_{\mu\nu}\right)^2 - g_2^2\left(W^+_\mu W^-_\nu - W^-_\mu W^+_\nu\right)^2 \\
&\quad +2ig_2^2\left(\left(\bar{c} + \frac{\bar{s}}{2}c_{HWB}v^2\right)Z_{\mu\nu} + \left(\bar{s} - \frac{\bar{c}}{2}c_{HWB}v^2\right)F_{\mu\nu}\right)\left(W^\mu_+ W^\nu_- - W^\mu_- W^\nu_+\right) \ ; \\
B_{\mu\nu}B^{\mu\nu} &= \left(\left(-\bar{s} - \frac{\bar{c}}{2}c_{HWB}v^2\right)Z_{\mu\nu} + \left(\bar{c} - \frac{\bar{s}}{2}c_{HWB}v^2\right)F_{\mu\nu}\right)^2 \ ; \\
W^3_{\mu\nu}B^{\mu\nu} &= \left(-\bar{c}\bar{s} - \frac{\bar{c}^2 + \bar{s}^2}{2}c_{HWB}v^2\right)Z^2_{\mu\nu} + (\bar{c}^2 - \bar{s}^2)Z_{\mu\nu}F^{\mu\nu} + \left(\bar{s}\bar{c} - \frac{\bar{s}^2 + \bar{c}^2}{2}c_{HWB}v^2\right)F^2_{\mu\nu} \\
&\quad +ig_2(W^+_\mu W^-_\nu - W^-_\mu W^+_\nu)\left(\left(-\bar{s} - \frac{\bar{c}}{2}c_{HWB}v^2\right)Z^{\mu\nu} + \left(\bar{c} - \frac{\bar{s}}{2}c_{HWB}v^2\right)F^{\mu\nu}\right) \ .
\end{aligned}
\tag{143}
$$

Finally, replacing everything into Eq. 139 and keeping only kinetic terms and Higgs-Boson-Boson interactions, one has:

$$
\begin{aligned}
\mathcal{L}_{EW} = \ & \frac{v^2g_2^2}{4}W^+_\mu W^\mu_- + \frac{vg_2^2}{2}Z_h^{1/2}hW^+_\mu W^\mu_- + \alpha_3 Z_\mu Z^\mu + \frac{v}{4}(Z_h^{1/2} + c_{HD}v^2)h(g_1B_\mu - g_2W^3_\mu)^2 \\
& -\frac{1}{2}W^+_{\mu\nu}W_-^{\mu\nu} - \frac{1}{4}Z_{\mu\nu}Z^{\mu\nu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + 2c_{HW}vhW^+_{\mu\nu}W_-^{\mu\nu} \\
& +c_{HW}vh\left(\left(\bar{c} + \frac{1}{2}\bar{s}c_{HWB}v^2\right)Z_{\mu\nu} + \left(\bar{s} - \frac{1}{2}\bar{c}c_{HWB}v^2\right)F_{\mu\nu}\right)^2 \\
& +c_{HB}vh\left(\left(-\bar{s} - \frac{\bar{c}}{2}c_{HWB}v^2\right)Z_{\mu\nu} + \left(\bar{c} - \frac{\bar{s}}{2}c_{HWB}v^2\right)F_{\mu\nu}\right)^2 \\
& -c_{HWB}vh\left(-\bar{c}\bar{s}Z^2_{\mu\nu} + \bar{c}\bar{s}F^2_{\mu\nu} + (\bar{c}^2 - \bar{s}^2)Z_{\mu\nu}F^{\mu\nu}\right) \ .
\end{aligned}
\tag{144}
$$

Notice that the new mass of the W-boson is $M_W^2 = \frac{v^2g_2^2}{4}$ with v and g_2 the new vev and normalized coupling respectively (they do not coincide with the SM ones).
From Eq. 144 one can read all the Higgs-boson-boson interaction coefficients. In particular, if we write (using the same notation of Eq. 34):

$$
\mathcal{L}_{EW} \ni g^{(1)}_{hWW}hW^+_\mu W^\mu_- + g^{(2)}_{hWW}hW^+_{\mu\nu}W_-^{\mu\nu} + g^{(1)}_{hZZ}hZ_\mu Z^\mu + g^{(2)}_{hZZ}hZ_{\mu\nu}Z^{\mu\nu} + g_{h\gamma\gamma}hF_{\mu\nu}F^{\mu\nu} + g_{h\gamma Z}hF_{\mu\nu}Z^{\mu\nu} \ ,
$$

then by matching (and using Eq. 140) we get:

$$
\begin{aligned}
g^{(1)}_{hWW} &= \frac{2M_W^2}{v}\left(1 - \frac{v^2}{4}(c_{HD} - 4c_{H\Box})\right) \ ; \\
g^{(2)}_{hWW} &= 2c_{HW}v \ ; \\
g^{(1)}_{hZZ} &= \frac{M_Z^2}{v}\left(1 + \frac{v^2}{4}(c_{HD} + 4c_{H\Box})\right) \ ; \\
g^{(2)}_{hZZ} &= v\left[c_{HW}\left(\frac{M_W^2}{M_Z^2}\right) + c_{HB}\left(\frac{M_Z^2 - M_W^2}{M_Z^2}\right) + c_{HWB}\left(\frac{g_1g_2}{g_1^2 + g_2^2}\right)\right] \ ; \\
g_{h\gamma\gamma} &= v\left[c_{HW}\left(\frac{M_Z^2 - M_W^2}{M_Z^2}\right) + c_{HB}\left(\frac{M_W^2}{M_Z^2}\right) - c_{HWB}\left(\frac{g_1g_2}{g_1^2 + g_2^2}\right)\right] \ ; \\
g_{h\gamma Z} &= 2v\left[c_{HW}\left(\frac{g_1g_2}{g_1^2 + g_2^2}\right) - c_{HB}\left(\frac{g_1g_2}{g_1^2 + g_2^2}\right) + \frac{c_{HWB}}{2}\left(\frac{g_1^2 - g_2^2}{g_1^2 + g_2^2}\right)\right] \ ,
\end{aligned}
\tag{145}
$$

that is, coefficients in Eq. 35. Notice that all the masses and couplings are not written with the *SM* subscript, hence they assume values corrected with dimension-six operators. In $g^{(2)}_{hWW}$, $g^{(2)}_{hZZ}$, $g_{h\gamma\gamma}$ and $g_{h\gamma Z}$ such values can be replaced with the SM ones since their difference would be a higher order correction.

B.3 The effective fermion-Higgs coupling

This subsection is identical to Appendix A.3, but the coefficients name. The effective dimension-six operators which influence the Yukawa couplings are (hermitian conjugated are understood):

$$\mathcal{O}_{He} = (H^\dagger H)\bar{\ell}_L e_R H \; ; \qquad \mathcal{O}_{Hu} = (H^\dagger H)\bar{q}_L u_R \tilde{H} \; ; \qquad \mathcal{O}_{Hd} = (H^\dagger H)\bar{q}_L d_R H \; .$$

By expanding these operators around the Higgs vev (keeping also the physical scalar h) and summing to the SM Higgs-fermions interactions, the effective Yukawa Lagrangian reads:

$$\begin{aligned} \mathcal{L}_Y \;\; = \;\; & -\frac{v + Z_h^{1/2} h}{\sqrt{2}}(\bar{e_L} Y_e e_R + \bar{u_L} Y_u u_R + \bar{d_L} Y_d d_R + \text{h.c.}) \\ & + \frac{(v + Z_h^{1/2} h)^3}{2\sqrt{2}}(c_{He}\bar{e_L} e_R + c_{Hu}\bar{u_L} u_R + c_{Hd}\bar{d_L} d_R + \text{h.c.}) \; , \end{aligned} \tag{146}$$

where we remembered to add the Higgs field normalization $Z_h^{1/2}$ while the fermions do not need it because they remain canonically normalized.
The following calculations are identical for the quarks and the charged leptons, so let be $\psi \in \{e, u, d\}$ and work in generality with ψ.
The fermion mass receives correction from $\mathcal{O}_{H\psi}$, so, considering only quadratic terms of the kind $\bar{\psi}\psi$ in Eq. 146, one gets:

$$\left(-\frac{vY_\psi}{\sqrt{2}} + \frac{v^3}{2\sqrt{2}} c_{H\psi}\right) \bar{\psi_L}\psi_R \stackrel{!}{=} -m_\psi \bar{\psi_L}\psi_R \longrightarrow \frac{vY_\psi}{\sqrt{2}} = m_\psi + \frac{v^3}{2\sqrt{2}} c_{H\psi} \; , \tag{147}$$

hence Eq. 146 can be rewritten as:

$$\begin{aligned} \mathcal{L}_Y \quad & = \quad -m_\psi \bar{\psi_L}\psi_R + \left(-\frac{Y_\psi}{\sqrt{2}} Z_h^{1/2} + \frac{3}{2\sqrt{2}} v^2 c_{H\psi}\right) h\bar{\psi_L}\psi_R + h.c. + O(h^2\bar{\psi}\psi) \\ & \stackrel{\text{Eq. 147}}{=} \quad -m_\psi \bar{\psi_L}\psi_R + \left[-\frac{m_\psi}{v}\left(1 - \frac{v^2}{4}(c_{HD} - 4c_{H\Box})\right) + \frac{v^2}{\sqrt{2}} c_{H\psi}\right] h\bar{\psi_L}\psi_R + h.c. + O(h^2\bar{\psi}\psi) \; , \end{aligned} \tag{148}$$

from which one can easily read the Higgs-fermion coupling $g_{H\psi}$ (appearing in Eq. 35):

$$g_{H\psi} = -\frac{m_\psi}{v}\left(1 - \frac{v^2}{4}(c_{HD} - 4c_{H\Box})\right) + \frac{v^2}{\sqrt{2}} c_{H\psi} \; . \tag{149}$$

C Generalization of the EFT treatment

In Appendix A, B but also in all Sec.3, we made a specific choice of dim-6 operator bases and wrote our results (couplings, masses etc.) accordingly. Now we want to generalize the procedure and perform the calculations without choosing a specific base of operators: deviations from the SM will be denoted as a dimensionless quantity δ_i where the subscript i will indicate the coupling we are referring to. In general, the structure of δ_i is a linear combination of Wilson coefficients c_j as:

$$\delta_i = \sum_j c_{i,j} \frac{v^2}{\Lambda^2} \ ,$$

and its explicit form will depend on the explicit 6-dim operator basis choice that one makes.

C.1 Higgs sector

In general, we can write the Higgs sector of the SMEFT Lagrangian in the EW broken phase, using the unitary gauge, as:

$$\begin{aligned} \mathcal{L}_H \ &= \ \frac{1}{2}(\partial_\mu h)^2(1+\delta_h+\delta_{HD}) + \delta_{\partial h3}\frac{1}{2}(\partial_\mu h)^2\frac{h}{v} + \delta_{\partial h4}\frac{1}{2}(\partial_\mu h)^2\frac{h^2}{v^2} \\ &\quad -\frac{\lambda v^2}{2}(1+\delta_{mh})h^2 - \frac{\lambda v}{2}(1+\delta_{h3})h^3 - \frac{\lambda}{8}(1+\delta_{h4})h^4 + O(h^5) \ . \end{aligned} \tag{150}$$

Usually, one normalises the Higgs field in order to have a canonical kinetic term. By redefining $h \to Z_h^{1/2} h$, one should impose:

$$\frac{1}{2}(\partial_\mu h)^2(1+\delta_h+\delta_{HD})Z_h \stackrel{!}{=} \frac{1}{2}(\partial_\mu h)^2 \longrightarrow Z_h = (1-\delta_h-\delta_{HD}) \ . \tag{151}$$

By using this normalisation and keeping only linear terms in the Wilson coefficients (equivalently, linear terms in δ_i), we get:

$$\begin{aligned} \mathcal{L}_H \ &= \ \frac{1}{2}(\partial_\mu h)^2\left(1+\delta_{\partial h3}\frac{h}{v}+\delta_{\partial h4}\frac{h^2}{v^2}\right) - \frac{\lambda v^2}{2}(1+\delta_{mh}-\delta_h-\delta_{HD})h^2 \\ &\quad -\frac{\lambda v}{2}\left(1+\delta_{h3}-\frac{3}{2}(\delta_h+\delta_{HD})\right)h^3 - \frac{\lambda}{8}(1+\delta_{h4}-2(\delta_h+\delta_{HD}))h^4 + O(h^5) \ . \end{aligned} \tag{152}$$

Notice that the δ_i are not necessarily independent, i.e., chosen a base of non-redundant dim-6 operators, it is possible to rewrite some deltas in function of other deltas. For example, if δ_{HD} comes from the modification induced by the operator $\mathcal{O}_r = (H^\dagger H)\,|D_\mu H|^2$, then after expanding it around $H = \frac{v+h}{\sqrt{2}}(0,1)^T$, it becomes clear that it will contribute to $\delta_{\partial h3}$ and $\delta_{\partial h4}$. On the other hand, derivative couplings also arise from the operator which contributes to δ_h, e.g., $\mathcal{O}_{H\Box} = (H^\dagger H)\Box(H^\dagger H)$. So $\delta_h, \delta_{HD}, \delta_{\partial h3}$ and $\delta_{\partial h4}$ can be expressed as linear combination of two Wilson coefficients $c_{H\Box}$ and c_r, therefore these deltas are not independent.

C.2 Gauge sector

As done for the Higgs sector, we start by looking at the general form of the EW gauge Lagrangian in the SMEFT framework:

$$\begin{aligned}
\mathcal{L}_{EW} &= -\frac{1}{4}W^I_{\mu\nu}W^{\mu\nu}_I(1+\delta_W) + \delta_{Wh}W^I_{\mu\nu}W^{\mu\nu}_I\frac{h}{v} + \delta_{Wh2}W^I_{\mu\nu}W^{\mu\nu}_I\frac{h^2}{v^2} \\
&\quad -\frac{1}{4}B_{\mu\nu}B^{\mu\nu}(1+\delta_B) + \delta_{Bh}B_{\mu\nu}B^{\mu\nu}\frac{h}{v} + \delta_{Bh2}B_{\mu\nu}B^{\mu\nu}\frac{h^2}{v^2} \\
&\quad -\frac{1}{2}W^3_{\mu\nu}B^{\mu\nu}\delta_{WB} + \delta_{WBh}W^3_{\mu\nu}B^{\mu\nu}\frac{h}{v} + \delta_{WBh2}W^3_{\mu\nu}B^{\mu\nu}\frac{h^2}{v^2} + (D_\mu H)^\dagger(D^\mu H)\left(1+\delta_{HD}\left(1+\frac{h}{v}\right)^2\right) \\
&= -\frac{1}{4}W^I_{\mu\nu}W^{\mu\nu}_I(1+\delta_W) + \delta_{Wh}W^I_{\mu\nu}W^{\mu\nu}_I\frac{h}{v} + \delta_{Wh2}W^I_{\mu\nu}W^{\mu\nu}_I\frac{h^2}{v^2} \\
&\quad -\frac{1}{4}B_{\mu\nu}B^{\mu\nu}(1+\delta_B) + \delta_{Bh}B_{\mu\nu}B^{\mu\nu}\frac{h}{v} + \delta_{Bh2}B_{\mu\nu}B^{\mu\nu}\frac{h^2}{v^2} \\
&\quad -\frac{1}{2}W^3_{\mu\nu}B^{\mu\nu}\delta_{WB} + \delta_{WBh}W^3_{\mu\nu}B^{\mu\nu}\frac{h}{v} + \delta_{WBh2}W^3_{\mu\nu}B^{\mu\nu}\frac{h^2}{v^2} \\
&\quad +\left(1+\delta_{HD}\left(1+\frac{h}{v}\right)^2\right)\frac{v^2g_2^2}{4}W^+_\mu W^\mu_-\left(1+\frac{h}{v}Z_h^{1/2}\right)^2 \\
&\quad +\frac{v^2}{8}\left(1+\delta_{HD}\left(1+\frac{h}{v}\right)^2\right)(g_1B_\mu - g_2W^3_\mu)^2\left(1+\frac{h}{v}Z_h^{1/2}\right)^2 + (\mathcal{L}_H \text{ terms})\,.
\end{aligned} \tag{153}$$

Notice that the gauge bosons kinetic terms are not canonical and we will not renormalize them in order to keep track how δ_W and δ_B affect the explicit form of the physical fields and their masses. Note also the last two lines of Eq. 153: modifications to the gauge bosons masses and couplings with the Higgs are both dependent by δ_{HD} and $Z_h^{1/2}$. This is not the most general form we could write this terms: for example, we could write $\frac{v^2g_2^2}{4}W^+_\mu W^\mu_-\left(1+\delta_{mw}+(1+\delta_{mWh})\frac{h}{v}+(1+\delta_{mWh2})\frac{h^2}{v^2}+O(h^3)\right)$. However, doing so, we cannot see explicitly how the Higgs field renormalization and the SMEFT contributions to $|D_\mu H|^2$ modify the gauge bosons masses and couplings because they would be hidden in the definition of δ_{mw}, δ_{mWh}, δ_{mWh2}. Therefore, we prefer the notation as in Eq. 153.
In order to rewrite the fields W^3_μ and B_μ in terms of Z_μ and A_μ, we must perform the diagonalization procedure of the kinetic and mass terms as described in [28]. Using the same notation of Eq. 110, we have:

$$K = \begin{pmatrix} 1+\delta_W & \delta_{WB} \\ \delta_{WB} & 1+\delta_B \end{pmatrix}; \qquad M^2 = \frac{v^2}{4}(1+\delta_{HD})\begin{pmatrix} g_2^2 & -g_1g_2 \\ -g_1g_2 & g_1^2 \end{pmatrix}.$$

We now have to find a matrix D such that $DM^2D^T = \mathbb{I}_2$. A convenient choice is:

$$D = \begin{pmatrix} \frac{(1+\delta_B)^{1/2}}{(1+\delta_B+\delta_W)^{1/2}} & -\frac{\delta_{WB}}{(1+\delta_B)^{1/2}(1+\delta_B+\delta_W)^{1/2}} \\ 0 & \frac{1}{(1+\delta_B)^{1/2}} \end{pmatrix} \overset{\text{Linearize}}{=} \begin{pmatrix} 1-\frac{1}{2}\delta_W & -\delta_{WB} \\ 0 & 1-\frac{1}{2}\delta_B \end{pmatrix}.$$

Next we compute DM^2D^T and retrieve its eingenvalues which are the physical masses of Z-boson and photon. We get:

$$M_\gamma^2 = 0\,; \qquad M_Z^2 = \frac{v^2}{4}(g_1^2+g_2^2)\left(1+(\delta_{HD}-\delta_B s_w^2-\delta_W c_w^2+2s_wc_w\delta_{WB})\right), \tag{154}$$

with $s_w = \sin\theta_W$ and $c_w = \cos\theta_W$ where θ_W is the SM Weinberg angle. The eigenvectors of DM^2D^T can be written as:

$$\begin{aligned}
v_\gamma &= \left(g_1\left(1+\tfrac{1}{2}(\delta_W-\delta_B)\right),\ g_2\left(1+\tan_w\delta_{WB}\right)\right); \\
v_Z &= \left(g_2(1+\tan_w\delta_{WB}),\ -g_1\left(1+\tfrac{1}{2}(\delta_W-\delta_B)\right)\right);
\end{aligned}$$

where we did not name them as A or Z since they still need to be properly normalized. To do so, we construct the matrix R defined by v_γ and v_Z used as row vectors and then we impose that R is a orthogonal matrix. If we want $R^T R = \mathbb{I}_2$ to hold, then we have to choose:

$$R = \begin{pmatrix} \frac{g_1}{\sqrt{g_1^2+g_2^2}}\left(1+\frac{1}{2}c_w^2(\delta_W-\delta_B)-s_w c_w \delta_{WB}\right) & \frac{g_2}{\sqrt{g_1^2+g_2^2}}\left(1-\frac{1}{2}s_w^2(\delta_W-\delta_B)+\delta_{WB}(\tan_w - s_w c_w)\right) \\ \frac{g_2}{\sqrt{g_1^2+g_2^2}}\left(1+\delta_{WB}(\tan_w - s_w c_w)-\frac{1}{2}s_w^2(\delta_W-\delta_B)\right) & -\frac{g_1}{\sqrt{g_1^2+g_2^2}}\left(1+\frac{1}{2}(\delta_W-\delta_B)c_w^2-s_w c_w \delta_{WB}\right) \end{pmatrix} .$$

At this stage, one may compute $RDM^2D^TR^T$ as consistency check. If the calculations are correct, then one has to find a diagonal matrix with the eigenvalues of Eq. 154 on the diagonal (this is our case). Finally, it is possible to write the physical states as:

$$\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} = (RD)^T \begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} ,$$

obtaining:

$$\begin{aligned} W_\mu^3 &= \frac{g_2}{\sqrt{g_1^2+g_2^2}} Z_\mu \left(1+\frac{1}{2}s_w^2\delta_B - \left(s_w^2+\frac{1}{2}c_w^2\right)\delta_W + \tan_w s_w^2 \delta_{WB}\right) \\ &\quad + \frac{g_1}{g_1^2+g_2^2} A_\mu \left(1-\frac{1}{2}c_w^2\delta_B - \frac{1}{2}s_w^2\delta_W - s_w c_w \delta_{WB}\right) ; \qquad (155) \\ B_\mu &= -\frac{g_1}{\sqrt{g_1^2+g_2^2}} Z_\mu \left(1-\delta_B\left(c_w^2+\frac{1}{2}s_w^2\right)+\frac{1}{2}c_w^2\delta_W + \cot_w c_w^2 \delta_{WB}\right) \\ &\quad + \frac{g_2}{\sqrt{g_1^2+g_2^2}} A_\mu \left(1-\frac{1}{2}c_w^2\delta_B - \frac{1}{2}s_w^2\delta_W - s_w c_w \delta_{WB}\right) . \qquad (156) \end{aligned}$$

We can perform the same consistency check done in Appendix B.2 to see if Eqs. 155-156 are correct. We know that in the mass basis there are no terms of the kind A_μ^2 and no mixing $A_\mu Z^\mu$. From the Lagrangian in Eq. 153 those terms may arise from $(g_1 B_\mu - g_2 W_\mu^3)^2$, but if we compute it using the previous decomposition in physical fields, we find:

$$\begin{aligned} &(g_1 B_\mu - g_2 W_\mu^3)^2 = X_1 A_\mu^2 + X_2 A_\mu Z^\mu + X_3 Z_\mu^2 \quad \text{with:} \\ &X_1 = 0 \qquad \text{correct ;} \\ &X_2 = 0 \qquad \text{correct ;} \\ &X_3 = g_1^2(1-\delta_B) + g_2^2(1-\delta_W) + 2g_1 g_2 \delta_{WB} \quad \text{correct, it leads to } M_Z^2 . \end{aligned}$$

Now, using the following algebraic results:

$$\begin{aligned} W_{\mu\nu}^1 W_1^{\mu\nu} + W_{\mu\nu}^2 W_2^{\mu\nu} &= 2W_{\mu\nu}^+ W_-^{\mu\nu} + 4ig_2(W_{\mu\nu}^+ W_-^\mu W_3^\nu - W_{\mu\nu}^- W_+^\mu W_3^\nu) \\ &\quad + g_2^2[(W_\mu^2 W_\nu^3 - W_\mu^3 W_\nu^2)^2 + (W_\mu^3 W_\nu^1 - W_\mu^1 W_\nu^3)^2] ; \\ W_{\mu\nu}^3 W_3^{\mu\nu} &= (\xi_{w3z} Z_{\mu\nu} + \xi_{w3a} F_{\mu\nu})^2 + 4ig_2(\xi_{w3z} Z_{\mu\nu} + \xi_{w3a} F_{\mu\nu}) W_+^\mu W_-^\nu \\ &\quad + g_2^2 (W_\mu^1 W_\nu^2 - W_\mu^2 W_\nu^1)^2 ; \\ B_{\mu\nu} B^{\mu\nu} &= (\xi_{bz} Z_{\mu\nu} + \xi_{ba} F_{\mu\nu})^2 ; \\ W_{\mu\nu}^3 B^{\mu\nu} &= (\xi_{w3z} Z_{\mu\nu} + \xi_{w3a} F_{\mu\nu})(\xi_{bz} Z^{\mu\nu} + \xi_{ba} F^{\mu\nu}) + 2ig_2(\xi_{bz} Z_{\mu\nu} + \xi_{ba} F_{\mu\nu}) W_+^\mu W_-^\nu ; \end{aligned}$$

where:

$$W_\mu^1 = \frac{W_\mu^+ + W_\mu^-}{\sqrt{2}} ; \qquad W_\mu^2 = \frac{i}{\sqrt{2}}(W_\mu^+ - W_\mu^-) ;$$
$$W_\mu^3 = \xi_{w3z} Z_\mu + \xi_{w3a} A_\mu ; \qquad B_\mu = \xi_{bz} Z_\mu + \xi_{ba} A_\mu ;$$

we can finally rewrite the Lagrangian in Eq. 153 as (we omit 4-vertex gauge coupling $O(W^4)$ and higher order couplings between Higgs and gauge fields, but they can be retrieved from the original

Lagrangian by applying the decomposition in Eqs. 155-156):

$$\begin{aligned}
\mathcal{L}_{EW} \;=\; & -\frac{1}{2}(1+\delta_W)W^+_{\mu\nu}W^{\mu\nu}_- - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}Z_{\mu\nu}Z^{\mu\nu} + 2\delta_{Wh}W^+_{\mu\nu}W^{\mu\nu}_-\frac{h}{v} + 2\delta_{Wh2}W^+_{\mu\nu}W^{\mu\nu}_-\frac{h^2}{v^2} \\
& + F^2_{\mu\nu}\frac{h}{v}\left(c_w^2\delta_{Bh} + s_wc_w\delta_{WBh} + s_w^2\delta_{Wh}\right) + F^2_{\mu\nu}\frac{h^2}{v^2}\left(c_w^2\delta_{Bh2} + s_wc_w\delta_{WBh2} + s_w^2\delta_{Wh2}\right) \\
& + Z^2_{\mu\nu}\frac{h}{v}\left(s_w^2\delta_{Bh} - s_wc_w\delta_{WBh} + c_w^2\delta_{Wh}\right) + Z^2_{\mu\nu}\frac{h^2}{v^2}\left(s_w^2\delta_{Bh2} - s_wc_w\delta_{WBh2} + c_w^2\delta_{Wh2}\right) \\
& + F_{\mu\nu}Z^{\mu\nu}\left[\frac{h}{v}\left((c_w^2 - s_w^2)\delta_{WBh} - 2s_wc_w(\delta_{Bh} - \delta_{Wh})\right) + \frac{h^2}{v^2}\left((c_w^2 - s_w^2)\delta_{WBh2} - 2s_wc_w(\delta_{Bh} - \delta_{Wh})\right)\right] \\
& + \left(1 + \delta_{HD}\left(1 + \frac{h}{v}\right)^2\right)\frac{v^2g_2^2}{4}W^+_\mu W^\mu_-\left(1 + \frac{h}{v}\left(1 - \frac{1}{2}(\delta_h + \delta_{HD})\right)\right)^2 \\
& + \frac{v^2(g_1^2+g_2^2)}{8}\left(1 + \delta_{HD}\left(1 + \frac{h}{v}\right)^2\right)Z^2_\mu(1 - s_w^2\delta_B - c_w^2\delta_W + 2s_wc_w\delta_{WB}) \\
& \times\left(1 + \frac{h}{v}\left(1 - \frac{1}{2}(\delta_h + \delta_{HD})\right)\right)^2 + \mathrm{TGC} + O(W^4)\;;
\end{aligned} \tag{157}$$

with TGC the following triple gauge couplings:

$$\begin{aligned}
\mathrm{TGC} \;=\; & \frac{-ig_1g_2}{\sqrt{g_1^2+g_2^2}}\left[F_{\mu\nu}W^\mu_+W^\nu_-\left(1 - \frac{1}{2}c_w^2\delta_B + \frac{1}{2}(1+c_w^2)\delta_W + \cot_w c_w^2\delta_{WB}\right)\right. \\
& \left. + A_\mu\left(W^+_\nu W^{\mu\nu}_- - W^-_\nu W^{\mu\nu}_+\right)\left(1 - \frac{1}{2}c_w^2\delta_B + \frac{1}{2}(1+c_w^2)\delta_W - s_wc_w\delta_{WB}\right)\right] \\
& \frac{-ig_2^2}{\sqrt{g_1^2+g_2^2}}\left[Z_{\mu\nu}W^\mu_+W^\nu_-\left(1 + \frac{1}{2}s_w^2\delta_B + \frac{1}{2}c_w^2\delta_W - s_wc_w\delta_{WB}\right)\right. \\
& \left. + Z_\mu\left(W^+_\nu W^{\mu\nu}_- - W^-_\nu W^{\mu\nu}_+\right)\left(1 + \frac{1}{2}s_w^2\delta_B + \frac{1}{2}c_w^2\delta_W + \tan_w s_w^2\delta_{WB}\right)\right]\;;
\end{aligned} \tag{158}$$

where $X_{\mu\nu} = \partial_{[\mu}X_{\nu]}$. Notice that $F^2_{\mu\nu}$ and $Z^2_{\mu\nu}$ are canonically normalized as a consequence of the diagonalization procedure, but the fact that both W^I_μ and B_μ were not normalized is still reminiscent in the Z^2_μ terms and in Eqs. 155-156. Notice also that we can normalize the charged boson kinetic term by applying $W^\pm_\mu \to Z_W^{1/2}W^\pm_\mu$ with $Z_W = (1-\delta_W)$ and, if we want the field strength tensor to transform as $W^\pm_{\mu\nu} \to Z_W^{1/2}W^\pm_{\mu\nu}$ then also the coupling g_2 must be redefined as $g_2 \to Z_W^{-1/2}g_2$ (hence the combination $g_2W^\pm_\mu$ is unchanged). Obviously, other than the TGC terms in Eq. 158, one can also add new genuine triple gauge bosons vertices, coming from $SU(2)_L\times U(1)_Y$ invariant dim-6 operators written in the EW broken phase.

As final check, let us match the result from Eq. 157 with a coefficient from Eqs. 35 (namely one of those used in the main body of this work, where we made an explicit base operator choice), for example the $hZ_\mu Z^\mu$ coupling. For this exercise, we can take $\delta_{HD} = \frac{c_{HD}v^2}{2}$ and $\delta_h = -2v^2c_{H\Box}$[17]. We are interested in the terms of the last two lines of Eq. 157 and, by keeping only terms proportional to hZ^2_μ we get:

$$\begin{aligned}
& \frac{v^2(g_1^2+g_2^2)}{8}\frac{2h}{v}\left(1 - \frac{1}{2}\delta_h + \delta_{HD} + \frac{1}{2}\delta_{HD}\right)(1 - s_w^2\delta_B - c_w^2\delta_W + 2s_wc_w\delta_{WB})Z^2_\mu \\
= \; & \underbrace{\frac{v^2(g_1^2+g_2^2)}{4}(1 + \delta_{HD} - s_w^2\delta_B - c_w^2\delta_W + 2s_wc_w\delta_{WB})}_{=M_Z^2 \text{ (see Eq. 154)}}\left(1 - \frac{1}{2}(\delta_h - \delta_{HD})\right)\frac{h}{v}Z^2_\mu \\
= \; & \frac{M_Z^2}{v}\left(1 - \frac{1}{2}(\delta_h - \delta_{HD})\right)\frac{h}{v}Z^2_\mu\,.
\end{aligned}$$

[17]This result comes from the mathcing between Eq. 151 of this subsection with Eq. 132 from Appendix B.1.

Now, by replacing δ_{HD} and δ_h as suggested before, we get:

$$\frac{M_Z^2}{v}\left(1+\frac{v^2}{4}(4c_{H\Box}+c_{HD})\right)hZ_\mu^2 \stackrel{!}{=} g_{hZZ}^{(1)}hZ_\mu^2 \, ,$$

as in Eq. 35.

C.3 Another Z_h for the Higgs field

In the previous subsections we used as Higgs field renormalizarion $Z_h^{1/2}=1+\left(c_{H\Box}-\frac{1}{4}c_{HD}\right)v^2$, but this is only a possible choice. In particular, thanks to the S-matrix equivalence theorem, the physical results (e.g., cross sections, decay rates) that one can compute from a Lagrangian are independent from the particular choice of field renormalization $\phi\to F(\phi)$, provided that the function $F(\phi)$ is local. This does not mean that also the couplings and the Feynman rules are written in the same way as we are going to see.

The purpose of this subsection is to review briefly the results from Appendix B.1 to Appendix B.3 using the same notation, but a different normalization for the Higgs field, namely the one proposed in Ref. [38] where not only the Higgs kinetic term results canonically normalized, but also the derivative Higgs couplings are vanishing. The Higgs redefinition is:

$$h \longrightarrow h\left(1+\left(c_{H\Box}-\frac{1}{4}c_{HD}\right)v^2\left(1+\frac{h}{v}+\frac{h^2}{3v^2}\right)\right) \, . \tag{159}$$

By using this shift, the terms proportional to $(\partial_\mu h)^2$ in Eq. 133 become:

$$\frac{1}{2}(\partial_\mu h)^2\left[1+2\left(c_{H\Box}-\frac{1}{4}c_{HD}\right)v^2\left(1+\frac{h}{v}\right)^2\right]\left[1+v^2\left(\frac{1}{2}c_{HD}-2c_{H\Box}\right)\left(1+\frac{h}{v}\right)^2\right]=\frac{1}{2}(\partial_\mu h)^2 \, ,$$

where there are no derivative couplings as claimed before. Notice that the redefinition in Eq. 159 is identical to our redefinition in Eq. 132 if we keep only linear terms in the Higgs field. As a consequence, we expect that the new renormalization will *not* modify the couplings involving the lowest power of the Higgs fields[18], namely h^2 terms for the scalar potential; $hX_\mu X^\mu$ and $hX_{\mu\nu}Y^{\mu\nu}$ terms for the gauge sector, where we named as X_μ ($X_{\mu\nu}$) a generic EW boson (field-strength).

Now, we can re-compute the coefficients of Eq. 134 by using the new redefinition and we get:

$$\begin{aligned}
g_h &= 0\,;\\
g_{hh} &\equiv -\frac{1}{2}M_h^2=-\frac{\lambda v^2}{2}\left(1+v^2\left(2c_{H\Box}-\frac{1}{2}c_{HD}-3\frac{c_H}{\lambda}\right)\right)\,;\\
g_{hhh} &= -\frac{M_h^2}{2v}\left(1-\frac{3v^2}{4}\left(c_{HD}-4c_{H\Box}\right)-\frac{2v^4}{M_h^2}c_H\right)\,;\\
g_{4h} &= -\frac{\lambda}{8}\left(1+\frac{14}{3}v^2\left(4c_{H\Box}-c_{HD}\right)-\frac{15v^4}{M_h^2}c_H\right)\,;\\
g_{5h} &= \frac{M_h^2}{4v}\left(c_{HD}-4c_{H\Box}+\frac{3v^2}{M_h^2}c_H\right)\,;\\
g_{6h} &= \frac{M_h^2}{24v^2}\left(c_{HD}-4c_{H\Box}+\frac{3v^2}{M_h^2}c_H\right)\,.
\end{aligned} \tag{160}$$

In a similar way, we start from the first line of Eq. 144 and use the new Higgs redefinition. We are not interested in the other terms because Z_h does not appear since there are already first order corrections in the Wilson coefficients and we want to keep only linear terms in c_i according to our perturbative expansion. This statement implies, together with the fact that couplings with at least two Higgs fields in the gauge sector (analogously the Higgs-fermion sector) are modified, that none of the coefficients

[18]We exclude in this discussion terms without Higgs fields because they are trivially left invariant since there are no Higgs to redefine.

in Eq. 145 and Eq. 149 change.
For completeness, we report some higher order Higgs-gauge couplings with the new redefinition:

$$\begin{aligned}
O(h^2): \quad & \Bigg[\frac{1}{8}\left((g_1 B_\mu - g_2 W^3_\mu)^2 + 2g_2^2 W^+_\mu W^\mu_-\right) \\
& + \frac{v^2}{4}\left((2c_{H\Box} + c_{HD})(g_1 B_\mu - g_2 W^3_\mu)^2 + (4c_{H\Box} - c_{HD})g_2^2 W^+_\mu W^\mu_-\right)\Bigg] h^2 \ ; \\
O(h^3): \quad & \frac{v}{6}\left[\left((2c_{H\Box} + c_{HD})(g_1 B_\mu - g_2 W^3_\mu)^2 + (4c_{H\Box} - c_{HD})g_2^2 W^+_\mu W^\mu_-\right)\right] h^3 \ ; \\
O(h^4): \quad & \frac{1}{24}\left[\left((2c_{H\Box} + c_{HD})(g_1 B_\mu - g_2 W^3_\mu)^2 + (4c_{H\Box} - c_{HD})g_2^2 W^+_\mu W^\mu_-\right)\right] h^4 \ .
\end{aligned} \tag{161}$$

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