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Efficient Street Sensor Selection For Traffic Flow Monitoring Leveraging Network Observability Metrics

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Abstract

For surveillance and safety purposes, the development of smart cities is currently focusing on traffic flow monitoring and congestion forecasting. These tasks frequently necessitate deploying a large network of sensors, but economic and environmental constraints may require starting with fewer devices. Network modeling in transport systems, which is crucial for selecting these sensors, often involves identifying supply, demand, and assignment models to capture network dynamics and parameters. Once a network model is available, sensor data can be gathered to refine traffic flow estimates. Additionally, this data allows for evaluating network conditions, such as vehicle travel times and traffic queue formations.

This work addresses efficient sensor selection for monitoring urban roads. To determine the most effective sensor placement, model-based techniques leveraging linear time-invariant systems and network observability are commonly used. Central to these strategies is the observability Gramian, which measures the energy induced by the initial state's free response. Higher energy levels signify greater observability and reduced sensing costs. Some metrics from the observability Gramian are here tested to evaluate the quality of state estimation, e.g. including dimensions of the observable subspace, energy distribution, mode balance, eigenvalue representation, and ellipsoid volume. These indicators are crucial for assessing system performance, and a comparison among them is provided in this thesis. Lastly, the theoretical findings are validated through real-world numerical simulations, contributing to enhance traffic flow monitoring in the city of Padua, Italy.

Sommario

Per scopi di sorveglianza e sicurezza, lo sviluppo delle città intelligenti si sta attualmente concentrando sul monitoraggio del flusso di traffico e sulla previsione della congestione. Questi compiti richiedono frequentemente il dispiegamento di una grande rete di sensori, ma vincoli economici e ambientali possono richiedere di iniziare con un numero ridotto di dispositivi. La modellazione della rete nei sistemi di trasporto, che è cruciale per la selezione di questi sensori, spesso comporta l'identificazione di modelli di offerta, domanda e assegnazione per catturare le dinamiche e i parametri della rete. Una volta disponibile un modello di rete, i dati dei sensori possono essere raccolti per affinare le stime del flusso di traffico. Inoltre, questi dati consentono di valutare le condizioni della rete, come i tempi di percorrenza dei veicoli e la formazione di code di traffico. Questo lavoro affronta la selezione efficiente dei sensori per il monitoraggio delle strade urbane. Per determinare il posizionamento dei sensori più efficace, vengono comunemente utilizzate tecniche basate su modelli che sfruttano sistemi lineari tempo-invarianti e l'osservabilità della rete. Centrale in queste strategie è il Gramiano di osservabilità, che misura l'energia indotta dalla risposta libera dello stato iniziale. Livelli di energia più alti indicano una maggiore osservabilità e costi di rilevamento ridotti. Alcune metriche del Gramiano di osservabilità vengono qui testate per valutare la qualità della stima dello stato, ad esempio includendo le dimensioni dello spazio sottospazio osservabile, la distribuzione dell'energia, l'equilibrio dei modi, la rappresentazione degli autovalori e il volume dell'ellissoide. Questi indicatori sono cruciali per valutare le prestazioni del sistema, e un confronto tra di essi è fornito in questa tesi. Infine, i risultati teorici vengono validati attraverso simulazioni numeriche reali, contribuendo a migliorare il monitoraggio del flusso di traffico nella città di Padova.

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IoT Internet of Things

IoV Internet of Vehicles

LTI Linear Time-Invariant

OSM OpenStreetMap

1 Introduction

Smart cities are an approach aimed at enhancing the sustainability and efficiency of cities by utilizing information and communication technologies. While the development of smart cities seeks to improve quality of life [8] by integrating technology into urban infrastructures, it has also brought about some significant challenges. One of the main issues is traffic congestion. The rapid increase in population and the number of vehicles in large cities has led to problems such as traffic congestion, energy consumption, and environmental pollution [16]. Addressing these issues has become critical to the success of smart cities. In this process, data management plays a vital role [1, 2]. Large amounts of data collected through sensors and IoT devices are analyzed and transformed into meaningful information. This data is used to monitor traffic flow, air quality, and energy consumption, enabling city administrations to make more informed, data-driven decisions. As a result, the goal of improving urban quality of life and utilizing resources more efficiently can be achieved.

Traffic data analysis is essential for understanding urban traffic dynamics and predicting future conditions. Realistic traffic models take into account key factors such as vehicle density, road usage, and travel times. Through these models, effective strategies to reduce traffic congestion can be developed [21]. Sensor networks play a fundamental role in the traffic management of cities [6]. Data collected through devices like cameras and traffic sensors provide city managers with real-time information on traffic flow and road conditions. This information supports decision-making processes for reducing traffic congestion, optimizing traffic lights, and improving emergency response times. The proper installation of sensor networks ensures that urban data is efficiently collected and made useful for city management. In this way, resources are managed more efficiently and environmental impacts are reduced.

In smart cities, sensor placement is a critical phase where limited resources must be utilized most efficiently [19]. During this process, it is crucial to position sensors at the most strategic points. A sensor placement plan must be developed based on factors such as traffic density and existing infrastructure, while also considering economic and environmental constraints.

In this study, further research is conducted on optimizing the placement of sensors. To achieve this, we focus on efficient sensor selection strategies for monitoring urban roads by examining how model-based techniques leverage linear time-varying systems and network observability [10]. One of these strategies, based on the observability Gramian, measures the energy resulting from the free response of the initial state and evaluates its effect on sensor placements. High energy levels increase observability while reducing detection costs, enabling more efficient system designs. Moreover, various metrics derived from the observability Gramian provide a critical foundation for evaluating system performance. We thus built on this knowledge to provide additional insights and validate such techniques on new real scenarios of interest.

1.1 Motivations and challenges

Optimal sensor placement is critical for enhancing the sustainability of smart cities and ensuring traffic safety. Placing sensors at strategic points is effective in collecting the data needed to *monitor and manage traffic flow* [6]. This provides the opportunity to track traffic density in real time and detect hazardous driving behaviors early. Especially during peak hours, thanks to sensors enable the prediction of traffic congestion, allowing for timely measures to alleviate traffic conditions. Additionally, it is important to manage increased vehicle density during big events. Sensors facilitate the control of traffic in such situations and ensure that emergency services are effectively directed. With optimal sensors placement can also reduce environmental impact; for example, regulating traffic flow can lower vehicle emissions, contributing to a cleaner urban environment. Therefore, the optimized placement of sensors is a significant motivation for enhancing both safety and the quality of urban city life.

The process of sensor placement faces various challenges. Firstly, *economic constraints* present a significant barrier. Sensor networks can be costly to install and maintain, which can strain cities' limited budgets. Therefore, urban planners must conduct careful analyses to identify the locations that will yield the highest efficiency. Additionally, *technological limitations* pose challenges. Sensors require adequate infrastructure to function properly. This increases the difficulty of integration with existing systems. Another challenge is the *accurate collection and management of data*. The data collected through sensor placement must be securely managed and analyzed. Furthermore, the *environmental conditions* where the sensors will be deployed can also affect performance; for instance, bad weather can hinder sensor functionality. Finally, the *longevity and durability* of the installed sensors is also an important consideration. This is necessary both to reduce costs and to ensure a continuous flow of data.

1.2 Thesis outline

The remainder of this thesis unfolds as follows.

Chapter 2 aims to analyze and synthesize the existing literature on the strategic placement and use of sensors in smart cities and urban management systems. It provides a comprehensive overview of the methodologies, technologies, and criteria related to sensor selection.

Chapter 3 presents the mathematical preliminaries for the models, estimators, and optimization methods used in sensor selection. The fundamental concepts such as discrete-time LTI systems, the observability Gramian, state estimator, luenberger observer gain, compartmental systems, set functions and submodularity are introduced.

Chaphter 4 addresses how sensors are to be placed on the road network, which data will be used, and how observability can be optimized. The fundamental steps to be taken within the proposed methodology for sensor selection are explained. Additionally, there is a discussion on which algorithms should be chosen for optimization.

Chapter 5 reports on how numerical simulations set up and collected data. First, it provides information about the model of the road network in Padua and the sensor placements. Next, it introduces a case study of the sensor placement strategies applied in the city of Padua, Italy. Simulation results are then presented, showing how these strategies help monitor traffic flow and enhance the system's observability. It is emphasized that proper sensor placement will make traffic monitoring and management more effective.

Chapter 6 summarizes the main findings of the research, highlighting the effectiveness of sensor placement strategies based on observability metrics. It points out the significant contributions of the work to enhancing traffic flow monitoring in smart cities. Additionally, future research directions are suggested, such as refining sensor selection techniques or applying these methods to different cities and various types of networks.

2

Literature review

The purpose of this section is to systematically analyze and synthesize the existing body of research related to the strategic placement and utilization of sensors in the context of smart cities and urban management systems. This literature review aims to provide a comprehensive overview of methodologies, technologies and criteria related to sensor selection, helping to understand their contribution to efficient urban management. Additionally, highlighting the importance of sensor selection in terms of data quality and accuracy is also one of the objectives. In this direction, the contributions of this thesis are elucidated.

2.1 State of the art

Various approaches have been developed to solve the sensor set selection problem. In [15], the optimal sensor placement problem in road transportation networks is addressed. Virtual variances are used for optimal sensor placement in road transport networks. This study aims to determine the most effective sensor placement to improve the efficiency of traffic monitoring systems. Virtual variances are a metric used for the estimation of traffic density at a given point. It develops a method to determine the best placement of sensors using these virtual variances. This method is expected to play an important role in improving the performance of traffic flow monitoring systems.

In [12], the issue of camera placement based on vehicular traffic for security surveillance of large cities is addressed. This work aims to develop effective camera placement strategies for city security. It presents a new approach to optimize the placement of cameras in areas with high vehicular traffic. This approach can help city safety authorities to optimize camera placements and enhance security surveillance in metropolitan areas. Camera placement is based on vehicle traffic data. This data is used to determine which areas are denser and where surveillance cameras can be placed most effectively. Optimization methods are used for camera placement. These methods use mathematical models and algorithms to determine the optimal camera locations based on available data. The priority needs of city security are assessed. Different factors are taken into account to determine which areas need more security surveillance. Simulations and analyses are performed to evaluate the effectiveness of the proposed camera placement strategies. These analyses are used to determine the most appropriate camera placement to increase the level of security in a given area.

In [17], an important topic in the field of control theory is addressed. By studying minimum controllability problems in a system, it is investigated whether it is possible to achieve a target state with minimum energy or minimum resource utilisation in the system. The mathematical definition of minimum controllability problems in a system and various techniques used to solve these problems are explained. It also discusses practical applications of minimum controllability problems and future directions of research in this area. Optimization techniques are used to minimize or maximize a given objective function. In minimum controllability problems, optimization techniques can be used to determine how to achieve a given target state with minimum energy or minimum resource use in the system. Linear algebra and matrix theory can be used to analyse and solve minimum controllability problems. Control theory is the study of how to control and manage systems. These approaches can be used to solve minimum controllability problems.

In [11], the use of observability and controllability Gramians or functions for determining optimal sensor and actuator locations in finite-dimensional systems is addressed. Observability and controllability matrices or functions are used to determine the most effective sensor and actuator locations for observing or controlling important variables in the system. Solution methods generally involve mathematical analysis and optimization techniques. Firstly, the system model and the targeted control or observation objectives are defined. Then, the focus is placed on the observability and controllability conditions in the system. Observability and controllability matrices or functions are computed, and the properties of these matrices or functions are examined. Determining optimal sensor and actuator locations often requires solving an optimization problem. These optimization problems can be solved using mathematical programming or numerical optimization techniques to optimize a specific objective function or find the best solution under certain constraints. The use of observability and controllability matrices or functions is emphasized, and the article explores how these matrices or functions can be used to determine optimal sensor and actuator locations. Solution methods typically involve mathematical analysis and optimization techniques.

The aim of [18] is to address the sensor placement problem to enhance security in networked control systems. It argues that the placement of sensors affects not only control performance but also the security of the system, suggesting that these two factors should be considered together. In particular, it focuses on the strategic placement of sensors to ensure system security while aiming to balance security and control. The study applies a game-theoretic framework to the sensor placement problem. This approach involves evaluating the placement of each sensor in relation to its interactions with other sensors and potential attackers. A game model has been developed to optimize sensor placement strategies, considering parameters such as sensor locations and numbers, as well as the strategies of potential attackers, to achieve security objectives. This enables a balance between control performance and security during sensor placement. The results demonstrate that the proposed game-theoretic framework effectively enhances the security of sensor placement strategies. The study is supported by various simulations and analyses, showing that optimal sensor placement can significantly improve the security of control systems. In this context, it concludes that while the sensor placement problem becomes more complex when security is taken into account, it also holds the potential for more effective solutions.

Real-time sensor selection processes in time-varying networks are discussed in [22]. The aim is to develop a framework to optimize the performance of sensors in dynamic systems. It seeks to enhance sensor efficiency and data quality under changing conditions, thereby improving the overall performance of the system. In this context, the sensor selection process is emphasized as having the potential to increase the reliability and efficiency of control systems. To address the real-time sensor selection problem, an algorithm is developed. This algorithm provides a selection process that guarantees system performance by taking into account the constantly changing states of the sensors and environmental conditions. While evaluating real-time data flow and system requirements, the algorithm uses mathematical models to predict the impact of the selections. This method provides the possibility to continuously select the most suitable sensors to manage uncertainties in time-varying networks and achieve performance targets. The results show that the proposed algorithm is effective in real-time sensor selection. Simulations and experiments demonstrate that the developed algorithm can significantly improve system performance under time-varying conditions.

The sensor selection problem are critically important for certain applications, especially when considering energy, cost and computational limitations. The issue addressed in [4] focuses on selecting a minimum number of sensors from a large sensor network to achieve the highest information gain or accuracy. Traditional methods can be computationally challenging and may become ineffective with large datasets. Therefore, according to this study, it is shown that probabilistic methods, such as randomized sampling, may be more suitable than deterministic methods for sensor selection. The complexity of sensor selection is emphasized and how the randomized sampling method can solve this problem is examined. Randomized sampling can work effectively with large datasets and simplify the sensor selection process. The proposed method develops randomized sampling strategies by considering the characteristics of the sensors and the network structure. These strategies ensure the efficient selection of sensors by taking into account the network's properties. This method can facilitate sensor selection in large-scale networks and can be considered an alternative to solve complex sensor selection problems.

One of the effective methods for solving the sensor selection problem is the convex optimization-based approach. In [14], it is mentioned that by simplifying the combinatorial structure, which is difficult to solve with traditional methods, sensor placement is optimized to best estimate system parameters. The authors combine convex relaxation and local optimization techniques, providing a sub-optimal sensor selection along with a bound that indicates how close this selection is to the global optimal solution. The obtained sub-optimal solutions are generally very close to the optimal one, and the method allows for fast and efficient selection among thousands of possible sensors. This study reduces computational costs by making sensor placement more efficient and can be applied to a wide range of applications.

In [3], it is presented a new approach to controlling complex networks based on energy awareness. It is indicated that traditional controllability concepts do not completely take into account the amount of energy required to manage the dynamic behavior of networks. Energy-oriented metrics have been developed to address this shortcoming. In this study, focuses on networks governed by linear dynamics, examining how the structure of the network impacts control energy. In particular, it highlights how the structural properties of a network can either increase or decrease the control effort and emphasizes the critical role of selecting the correct control nodes in this process. Additionally, the study analyzes various strategies for optimally determining control nodes and compares these strategies to minimize control energy. As a result, this work provides both theoretical and practical contributions to making complex networks more efficiently controllable and opens up new directions for future research.

Lastly in [5], the sensor selection problem with cost constraints in dynamic systems is addressed with the aim of enhancing system performance and optimizing costs. This study underscores the importance of strategically selecting sensors in dynamic systems. It focuses on the complexity of the sensor selection problem and the necessity of considering cost constraints. The study presents a framework for developing an effective sensor selection strategy for dynamically relevant bases. This framework aims to determine the optimal sensor configuration under a specific cost constraint. Both dynamic relevance and cost-effectiveness are taken into account in the sensor selection process.

2.2 Contributions

In previous studies concerning sensor selection, the techniques proposed in [9] and [23] were mainly utilized. According to these studies, model-based techniques used for sensor selection to monitor traffic flow are examined. The goal is to ensure the effective placement of sensors and to obtain maximum data with the a small number (possibly minimum amount) of sensors. In terms of traffic management, this is important to improve the efficiency of the system. Model-based techniques relying on observability were used for sensor selection. These techniques are based on methods such as graph theory, optimization, and observability theory to monitor traffic flow. According to the observability measures, the crucial roads and intersections where the sensors should be placed were identified. In this way, traffic flow can be monitored with maximum accuracy using the fewest sensors possible. The approach optimizes observability while minimizing the number of sensors. This method reduces costs and increases data accuracy in traffic monitoring systems. Research in [20] addresses sensor and actuator placement problems in complex dynamic networks, examining the structural properties for optimizing controllability and observability metrics. Specifically, it demonstrates that these optimization problems possess modularity and submodularity properties, allowing for globally optimal or approximately optimal solutions to be achieved using a simple greedy algorithm.

The aim for this research is to solve the sensor selection problem through the analysis of observability-based methods for control systems with limited sensing capabilities. Observability theory is indeed used as a well-known tool to better leverage the dynamic properties of the system and enhance sensor information quality. In [13], the authors discuss the observability-based guidance and sensor placement. This work explores how the concept of observability, used to predict the internal states of a system, can be applied in the process of sensor placement. The necessity of devising effective sensor placement strategies to accurately predict the internal states of a system is emphasized. The concept of observability plays a crucial role in the development of these strategies. In this thesis we thus examine how observability analyses can be utilized to determine which states of a particular system should be measured. We also suggest how to adopt an observability-based approach in sensor placement in order to get accurate estimates of system internal states, thereby enhancing system performance.

In this dissertation we exploit a cutting-edge method finalized at solving combinatorial optimization problems; particularly, the sensor selection problem. In this direction, observability-based metrics that are functions of the Gramian matrix associated to dynamic networks are optimized. Leveraging the fact that some of these metrics are submodular (e.g. rank and trace of the Gramian matrix), the sensor selection problem is tackled by adopting greedy algorithms. Such an approach allows to attain globally or approximately optimal solutions by incrementally find placements that maximize the considered observability metrics. On the other hand, when submodularity does not hold, other kinds of techniques are employed to reach the best configuration of sensors. For instance, in this thesis we show how to minimize the condition number of the Gramian matrix by means of a genetic algorithm.

Lastly, the findings of this study are validated through real-world simulations in Padua, Italy, offering practical recommendations for optimizing sensor selection to monitor urban traffic flow, thereby enhancing data-driven decision making processes for city management.

3 Mathematical background

This section provides mathematical preliminaries for models, estimators and optimization techniques used in sensor selection. The focus here will be on presenting the fundamental concepts related to discrete-time Linear Time-Invariant (LTI) systems, observability Gramian, set functions and submodularity.

3.1 LTI systems in the discrete-time domain

Discrete-time LTI systems are crucial for modeling and controlling dynamic systems. They have properties of linearity and time-invariance, making them predictable and easier to analyze. Represented by state-space equations or difference equations, these systems allow engineers to design, simulate, and control real-world systems effectively in the discrete-time domain. A state-space model that defines the dynamics of a discrete-time LTI system can be described by the following equations:

$$\Sigma : \begin{cases} x(t+1) = Ax(t) + Bu(t), \quad x(0) = x_0, \\ y(t) = Cx(t) + Du(t), \end{cases} \tag{3.1}$$

where the meaning of quantities in (3.1) is reported in the following list.

- $x(t)$: This is the state vector of the system and is of dimension n. It defines the state of the system at each time t.
- A : This is an $n \times n$ matrix that determines the transitions between the states of the system. The matrix A characterizes the internal dynamics of the system.

- B : This is an $n \times m$ matrix that defines how the inputs influence the state. It describes the effect of the input vector $u(t)$ on the state vector.

- $u(t)$: This is the input vector to the system and is of dimension m.

- $y(t)$: This is the output vector of the system and is of dimension p. It represents the results produced by the system externally.

- C : This is $p \times n$ matrix that determines how the states are transformed into outputs.it provides the measurement of the states.

- D : This is $p \times m$ matrix that represents the direct effect of the inputs on the outputs. It shows the impact of the input vector on the output.

- $x(0)$: This represents the initial condition of the system. That is, the state of the system at time zero.

Natural and forced responses can be derived by computing the general solution for system (3.1), which is given by

$$x(t) = \underbrace{A^t x(0)}_{x_l(t)} + \underbrace{\sum_{k=0}^{t-1} \left[A^{t-1-k} B u(k)\right]}_{x_f(t)}. \tag{3.2}$$

In particular, the *natural response* $y_l(t)$ of an LTI system is the behavior of the system when there are no external inputs acting on it. It is solely determined by the system's initial conditions and the internal dynamics defined by the system's state equations. The equation of natural response is indicated as

$$\begin{aligned} y_l(t) &= y(t) - y_f(t) \\ &= C x_l(t). \end{aligned} \tag{3.3}$$

While the *forced response* $y_f(t)$ of LTI system is the behavior of the system due to external inputs acting on it. This response reflects how the system reacts to these inputs and is independent of the initial conditions. The equation of forced response is indicated as

$$y_f(t) = C x_f(t) + D u(t). \tag{3.4}$$

3.2 The observability Gramian

Observability is a property that indicates whether the internal states of a dynamic system can be estimated from the external inputs and outputs. The more effectively the initial state $x(0)$ can be observed using inputs $u(t)$ and outputs $y(t)$, the more observable the system is considered to be. Observability is determined by the rank of the observability matrix. The definition of the observability matrix is given in equation (3.5):

$$O := \begin{pmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{pmatrix}. \tag{3.5}$$

In particular, the observability of a system depends on whether the rank of this matrix is $m := dim[x(t)]$. If $\text{rank}[O] = n$ the system Σ is observable, otherwise it is unobservable.

The *observability Gramian* is a matrix used to evaluate the observability of a system and reconstruct past state values. It measures the energy of the natural response induced by the initial state, representing how observable the dynamics of the system are. The observability Gramian is defined as

$$W := O^\top O = \sum_{k=0}^{n-1} (A^\top)^k C^\top C A^k. \tag{3.6}$$

This quantity measures the impact of the system's natural response: a non-zero Gramian matrix indicates that the system is observable. The inital state x_0 can be estimated as

$$\hat{x}(0) = W_t^{-1} O_t^\top \mathbf{y}_{l,t}, \tag{3.7}$$

where $O_t = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{t-1} \end{bmatrix}, \quad W_t := O_t^\top O_t, \quad \mathbf{y}_{l,t} := \begin{bmatrix} y_l(0)^\top & y_l(1)^\top & \cdots & y_l(t-1)^\top \end{bmatrix}^\top.$

The energy of the natural response measures how a system reacts based on its initial conditions, specifically the amount of power or energy it produces. This energy depends on the magnitude (amplitude) of the natural response. Systems can lose their initial energy levels over time, but in some cases, they can retain energy for a long time (as in an unstable system). The energy of the natural response is used to analyze the stability of an LTI system, to examine energy loss in mechanical, electrical, or thermal systems, and to assess system performance. Equation (3.8) is indeed used to define the energy in this sense:

$$E(x(0), t) = \|\mathbf{y}_{l,t}\|_2^2 = x(0)^\top W_t x(0). \tag{3.8}$$

The observability Gramian is thus used to assess the observability of a system using various metrics, such as

1. $rank[W]$: The rank of W is the dimension of the observable subspace. When applied to the observability Gramian matrix, the rank helps determine whether the system is observable.The observability Gramian provides more information about the observability of the system's states. If the rank of the Gramian is equal to n, this indicates that all states of the system are observable. If the rank of the Gramian is less than n, this means that only part of the system is observable, while some states cannot be observed.

2. $trace[W]/n$: The trace of W is directly related to the average energy. The trace of the Gramian provides an average measure of the observability of the system's states. If the trace value is high, all states of the system are highly observable, which occurs when the observability of the energy in different states is high. If the trace value is low, the observability of some states may be weak. In this case, only part of the system is observable, while other states may not be observable. If the trace is zero or close to zero, this indicates that the system is either completely unobservable or has very weak observability.

3. $\lambda_{\min}[W]$ and $\lambda_{\max}[W]$: The largest eigenvalue $\lambda_{\max}[W]$ of W corresponds to the most observable mode of the system. It is easier to gather information about this mode, and the energy required to estimate it is the highest. This indicates that the mode is strong in terms of observability. On the other hand, the smallest eigenvalue $\lambda_{\min}[W]$ of W corresponds to the least observable mode of the system. If this eigenvalue is very small, it becomes more difficult to gather information about that mode because it is weak in terms of observability and can be observed with less energy. $\lambda_{\min}[W]$ can be zero for unobservable systems.

4. $K[\mathrm{W}]$: The condition number ($K[\mathrm{W}] = \lambda_{\max}[W]/\lambda_{\min}[W]$) is an important metric that measures how balanced the observability of a system is. If all modes in the system are equally observable, $K[W]$ will be small, indicating

balanced observability. If some modes are weakly observable, then $K[W]$ becomes big. In this case, the observability is unbalanced. If there is an unobservable mode in the system, then $\lambda_{\min}[W]$ becomes zero, which means $K[W]$ grows infinitely. This indicates that the system is unobservable.

5. $\det[W]^{\frac{1}{n}}$: The determinant of W measures the volume of an ellipsoid in state space. This ellipsoid contains the initial states that can be observed with a specific prediction energy. The bigger the determinant, the more states can be observed with less energy. A small determinant indicates that it is more difficult to observe the system.

3.3 The Luenberger state estimator

A *state estimator* is a mathematical model used to predict and monitor the state of systems. These predictions are made to gain information about the system's state, to control it, or to monitor it. The main objective of state estimation is to estimate the internal state of the system from observable data. These estimates are important for controlling, managing and optimizing the system. In particular, given the system in (3.1), the luenberger state estimator takes the from

$$\hat{\Sigma} : \begin{cases} \hat{y}(t) = C\hat{x}(t), \quad \hat{x}(0) = \hat{x}_0, \\ \hat{x}(t+1) = A\hat{x}(t) + Bu(t) + L(\hat{y}(t) - y(t)). \end{cases} \tag{3.9}$$

The meanings of quantities in (3.9) are listed as follows.

- $\hat{y}(t)$: estimated output
- $\hat{x}(t)$: estimated system state
- L: *Luenberger observer gain*, a parameter used to correct the measurement error that comes into this estimator
- $\hat{y}(t) - y(t)$: measurement error. Also known as *innovation* (difference between measured output and estimated output)
- $\hat{x}_0$: estimate of the initial state of the system

The *Luenberger observer gain* is part of a control and observation method used for state estimation in dynamic systems. The Luenberger observer utilizes measurement data to make better predictions about the state of a system. The observer gain is a matrix used to adjust the output of the observer, combining the dynamics of the system with the measurement information. If the observer gain L is chosen correctly namely, so that the special radius[1] of $(A + LC)$ is strictly less than 1, then the *estimation error* $e(t) = \hat{x}(t) - x(t)$ decreases over time, allowing the observer to predict the system's state more accurately. Indeed the dynamics of the estimation error is formulated as in equation $e(t) = (A+LC)^t e(0)$ where $e(0)$ is initial estimation error: if the estimation error strictly decreases over time, $e(t)$ converges to zero, and the estimated state approaches the actual state.

3.4 Compartmental systems

A compartmental system is used to model the flows and movements within a small structure by dividing it into smaller compartments. These models are employed to describe and track the flow of matter, energy, or information between different compartments of a system. Each compartment represents a specific subdivision of the system, and flows occur between these compartments at certain rates. These flows symbolize the transfer of matter or information from one compartment to another. Compartment systems are an effective way to model and analyze traffic flow in road networks and traffic systems. This approach allows for the examination of vehicle movements between regions by dividing a road network into different compartments. One of the main points of this work is to model the road network as an LTI compartmental system [24]. In compartmental systems, the condition (3.10) is valid for each state variable:

$$x_i(t) \geq 0; \quad \forall t \geq 0. \tag{3.10}$$

This is justified by the fact that $x_i(t)$ usually represents a physical quantity such as amount or density in the i-th compartment. Discrete-time LTI compartmental models are used to describe the flow between two compartments. In these models, the flow from one compartment to another usually depends on the present

[1] I.e. the largest absolute value computed over all eigenvalues of a matrix

state in that compartment. The flow from compartment i to compartment j is expressed by $f_{ij}(t) = a_{ji}x_i(t); \quad j \neq i; \quad a_{ji} \in [-1, 1]; \quad 0 \leq \sum_{j=1}^{m} a_{ji} \leq 1$. The constraint on the coefficient a_{ji} is $\sum_{j=1}^{m} a_{ji} \in [0, 1]; \quad \forall i = \{1, \ldots, m\}$. The condition $\sum_{j=1}^{m} a_{ji} = 1$ correspond to most conservation. The a_{ji}'s are constants that determine how much matter is transferred from compartment i to compartment j. As $x_i(t)$ indicates that the amount of matter present in the i-th compartment at time t, $f_{ij}(t)$ denotes the flow between each pair of compartments is proportional to the current state of the i-th compartment.

3.5 Set functions and submodularity

In sensor placement problems, set functions are used to find the optimal sensor placement. Set functions measure the utility brought by placing sensors in different locations, allowing decisions to be made about where to place each sensor. A placement problem is approached as a problem where there are many potential locations (set S). The goal is to select a subset Q of these locations and ensure that this subset maximizes a specific objective function $f(Q)$. This objective function usually measures the observability and performance of the system. Set functions help solve an optimization problem to determine which sensor placement provides the best information gain. According to the set optimization problem

$$\mathcal{P}_0 : \; S^* = \underset{Q \subseteq S, |Q| = p}{\operatorname{argmax}} f(Q), \tag{3.11}$$

$f(Q)$ represents the amount of benefit obtained from the sensor placement, and $|Q| = p$ is the constraint on the number of sensor.

Submodularity refers to the fact that as the size of a set increases, the contribution of adding a new element decreases [20]. It is formally defined as follows: *Submodularity:* A set function $f : 2^S \rightarrow \mathbb{R}$ is called submodular if, for all subsets $\mathcal{A} \subseteq \mathcal{B} \subseteq \mathcal{S}$ and $s \notin \mathcal{B}$, it holds that

$$f(\mathcal{A} \cup \{s\}) - f(\mathcal{A}) \geq f(\mathcal{B} \cup \{s\}) - f(\mathcal{B}) \tag{3.12}$$

Adding a sensor to a small set has a large impact, but adding a sensor to a large set provides less contribution. Submodular functions are used to solve optimization problems more efficiently. If there is a submodular function, optimal or near-optimal results can be obtained using greedy algorithms. A greedy algorithm makes the best-looking choice at each step. That means, with each sensor addition, it selects the location that will provide the most contribution at that moment. For submodular functions, this method usually gives a result that is close to the best solution[2]. This enables fast and efficient placement of sensors, even in very large placement problems.

[2]The optimality gap is shown to be reduced to less than 36.8% if submodularity holds

4 Proposed methodology

This section addresses the problem of how sensors are to be placed on the road network, what data will be used, and how observability can be optimized. The placement of sensors is a critical step in effectively observing and managing the system's state. Therefore, proper modeling and optimization are essential for developing an effective sensor placement strategy.

4.1 Characterization of urban networks

The extraction of main network roads is a process for identifying the important features of roads in a specific region. This process is usually performed using information obtained from online vector map databases, particularly resources like OpenStreetMap [7]. The elements below explain the details of this process.

- **Urban Network Topology:**
  The topological structure of the urban network illustrates how roads connect with each other and how they form a network. This information is crucial for understanding the connections between roads and traffic flow.

- **Lengths and Widths of Roads:**
  The lengths of roads help analyze traffic flow in a specific area. Longer roads generally carry more traffic, while wider roads may have more vehicle capacity. This data is used to determine which roads are more congested and to understand what type of sensors are needed on these roads.

- **Presence of Intersections:**
  Intersections are points where traffic flow is most intense and where directional changes occur. These points are considered priority locations

for sensor placement. The characteristics of intersections are analyzed to understand how vehicles and pedestrians move.

- **Existence of Cul-de-sacs:**
  Cul-de-sacs are generally considered as less important roads in terms of traffic monitoring and management. Therefore, removing these types of roads from the map allows a focus on the analysis of significant roads.

- **Data Filtering:**
  All the information gathered is filtered to identify the main roads that constitute the road network. While important roads are identified, less significant roads, such as cul-de-sacs, are excluded. This process enables a focus on analyzing only the roads that are critical for traffic management and monitoring.

OpenStreetMap is a map database that allows us to obtain geographic information about all roads in a given area of interest $\mathcal{T}$. The roads obtained from this database can be generally defined as $\tilde{\mathcal{R}} = \{R_1, R_2, \ldots, R_M\}$. However, not all roads are equally important for specific applications, such as sensor placement. For example, cul-de-sacs and less-utilized roads may be considered insignificant for applications like traffic monitoring. Therefore, it is necessary to remove some roads from the initial road set $\tilde{\mathcal{R}}$. The street classification system of OSM is used for this extraction process. This system helps to identify different types of roads and to rank some roads in order of priority. Cul-de-sacs are typically deemed of secondary importance. These roads are removed from the analysis, allowing more important roads to be identified. As a result, the initial road set $\tilde{\mathcal{R}}$ is reduced to a set of principal roads, represented as $\mathcal{R} = \{R_1, R_2, \ldots, R_n\} \subseteq \tilde{\mathcal{R}}$, $n \leq M$. This process is crucial for analyzing the road network more efficiently and optimizing the sensor placement strategy.

4.2 Modeling of traffic flows

The underlying road network is represented as a compartmental model: this allows to investigate how traffic flows along the roads and how these roads function as compartments within the model. Thus, it becomes possible to examine the interactions between different roads and assess how traffic flows are distributed and the impact of the roads on this flow. Each path is denoted as $R_i \in \mathcal{R}$, and it is characterized by the following properties.

- Importance category of the path

  - Each road has an "importance score" that indicates how significant the road is.
  - According to OpenStreetMap data, roads are rated from 1 to 7:
    * 1: Least important road
    * 7: Most important road
  - These scores are used to understand the role of roads in traffic flow. A higher score means that the road is more busier or more critical.

- Roads connected to the external world
  - The set of roads connected to the external world is indicated by $\mathcal{P}$.

- Origin - destination matrix
  - The origin - destination matrix shows how the roads in the city are connected to each other. In the road network model, this matrix is used to understand how each road is connected to other roads.
  - This matrix indicates whether there is a transition between two roads.
    * $g_{ij} = 1$: if vehicles can pass from road R_i to road R_j, meaning there is a connection from R_i to R_j, then g_{ij} is equal to 1.
    * $g_{ij} = 0$: if vehicles cannot pass from road R_i to road R_j, then g_{ij} is 0.

    In this way, the matrix clearly shows which roads are connected and where transitions between roads are possible.
  - This matrix is crucial for modeling and managing traffic flow in the urban road network; indeed, from this matrix informations are obtained:
    * which roads are connected;
    * where one-way and two-way roads are located.
    * which roads are more critical for traffic.

    This information can be used to decide where sensors should be placed.

As mentioned above, the road network is modeled as a compartmental LTI system. This model represents the flow of traffic on roads at a given time and is expressed mathematically as in equation (3.1).
Within this context, the state vector $x(t) \in \mathbb{R}^n$ indicates that there are n roads in the network, while $x_i(t)$ indicates the number of vehicles passing through road R_i per unit time. Each road is defined as a state of the system, allowing for the analysis of traffic flow within the network.

Traffic Flow Splitting Ratios. The elements of matrix A determine the traffic flow between the roads in the network, calculated using the following formula:

$$a_{ji} = \begin{cases} \frac{\sum c_j}{\sum_{k:R_i R_k} c_k + c_i} & \text{if } R_i \to R_j \text{ and } R_i \notin \mathcal{P}; \\ \frac{\sum c_j}{\sum_{k:R_i R_k} c_k + 2c_i} & \text{if } R_i \to R_j \text{ and } R_i \in \mathcal{P}; \\ 0 & \text{otherwise;} \end{cases} \tag{4.1}$$

where

- a_{ji} represents the flow from road R_i to road R_j;
- $\mathcal{P}$ represents the set of roads connected to the external world;
- c_j represents the importance scores of the roads.

Principle of Mass Conservation. The diagonal elements a_{ii} are defined according to the principle of mass conservation. The following choice ensures that the number of vehicles on a road is equal to the sum of the number of vehicles entering and leaving that road.

$$a_{ii} = 1 - \left(a_{0i} + \sum_{j \neq i} a_{ji} \right), \tag{4.2}$$

where $a_{0i} \in [0, 1]$, $\quad \forall i = \{1, \ldots, n\}$.

In a traffic monitoring system, deciding where to place sensors on roads is crucial. These sensors measure the traffic flow and provide information about the system's state. However, placing sensors on every road is costly and impractical, so its important to strategically place sensors at key points. The selection of these points is based on the concept of observability. Suppose p sensors are deployed. The output equation is then defined as

$$y(t) = C(\mathcal{J})x(t), \tag{4.3}$$

where:

- $y(t)$ represents the measurement results from the sensors regarding traffic flow. This vector collects real-time system outputs from the p sensors.
- $C(\mathcal{J})$ is the binary diagonal matrix that determines on which roads the sensors are placed. This matrix is formed by choosing p rows from the $n \times n$ identity matrix, based on the index set $\mathcal{J} = \{s_1, \ldots, s_n\}$, with $s_i \in \{0, 1\}$. In other words, this matrix selects which roads the sensors are monitoring.

As clarified in (4.3), the matrix $C(\mathcal{J})$ determines which roads the sensors are placed on. The observability Gramian , defined from the A and $C(\mathcal{J})$ matrices is yelded by

$$W(\mathcal{J}) = \sum_{k=0}^{n-1} (A^\top)^k C(\mathcal{J})^\top C(\mathcal{J}) A^k. \tag{4.4}$$

The observability Gramian in (4.4) is thus used to reduce the number of deployed sensors, i.e. to best observe the traffic flow. Problem and approaches to solve it.

4.3 Sensor selection: formulation and solutions

This section details the formal definition for the sensor selection problem and some approaches to solve it.

Problem P 1 *Given the system matrices (A, B, D) in (3.1), find C by means of a polynomial-time algorithm such that any of the above metrics computed on the observability Gramian (3.5) is maximized (e.g., for rank, trace) or minimized (e.g., for condition number).*

In the context of selecting street sensors for traffic monitoring, the explanation provided in equation (3.1) is useful for describing the flow of vehicles in a urban area. Each component of the state $x(t)$ represents the number of vehicles passing through a specific road connection per second, while the components of the input $u(t)$ represent the number of incoming vehicles concerning the frontier nodes of the network. The state and input origin-destination matrices A and B perform state updates based on the previous values of $x(t)$ and the input $u(t)$. Additionally, the entries of the matrix C are generally binary and indicate the links where sensors are placed. Usually, the value of the matrix D is set to zero, and the output $y(t)$ observed through C is monitored. As stated in Problem P1, the matrix C needs to be found possibly by optimizing one of the observability-Gramian-based metrics.
Sensor selection involves trying many different combinations to optimize a specific objective. This often constitutes a combinatorial optimization problem. When sensors are placed on certain roads, the goal is to determine which sensor combinations can best observe those roads or provide the most information. The solution to this problem can be approached by using, e.g, the exhaustive search method. Exhaustive search means trying out all possible combinations of

available sensors. However, this method is impractical for large networks, as the number of combinations increases rapidly with the number of sensors. For this reason, two heuristic strategies are proposed to address problem P1. Heuristic methods aim to find good solutions more quickly than trying all possibilities. These approaches generally work as follows:

- **Local Search:** starting from a specific initial solution, it tries neighboring solutions to find a better one.
- **Complex Strategies:** more advanced algorithms, such as genetic algorithms, Greedy algorithm or swarm-based search methods, can attempt to optimize multiple solutions simultaneously.

The result obtained through heuristic methods is a suitable subset of sensors, denoted as $\mathcal{S}'$. The size of this subset is determined as $|\mathcal{S}'| = p$, meaning that the number of selected sensors must be p. However, it is important to note that the best solution obtained through exhaustive search, denoted as $\mathcal{S}^{\text{opt}}$, may not always be equal to the solution obtained through heuristics, $\mathcal{S}'$. Heuristic methods may get stuck in a local minimum in the case of a non-convex problem, making it sometimes difficult to find a better solution. The optimization performed through heuristic methods considers criteria related to observability. Here, the maximization of $rank[W]$, $tr[W]$ or the minimization of $K[W]$ are mainly pursued. These criteria help determine which sensors will provide better observation capabilities.

4.3.1 Greedy approach for the maximization $rank[W]$ and $tr[W]$

The greedy algorithm works by making sequential choices, adding sensors step by step to the road network. Each step selects the road that results in the greatest improvement in the observability measure, such as $rank[W]$ or $tr[W]$. Essentially, at each step, the algorithm places a sensor where it will provide the most significant increase in the objective function in Problem $\mathcal{P}_0$ (see 3.11). However, this greedy approach does not always guarantee finding the global optimal solution. Instead, it might find a local minimum due to the problems non-convex nature. Despite that metrics such as rank and trace obey submodularity; hence the greedy solution ensures convergence for the objective to a value that is off by at most 36.8% w.r.t. the global optimum. The pseudocode in Alg. 1 represents an implementation of this approach.

Algorithm 1 Greedy Approach

Require: Set of sensors S
Ensure: Set of selected sensors S^*
  $S_0 \leftarrow \emptyset$
  for $i = 0 : p$ **do**
    $\mathcal{S}_{i+1} \longleftarrow \mathcal{S}_i \cup \arg\max_{s \in \mathcal{S}} [f(\mathcal{S}_i \cup s) - f(\mathcal{S}_i) \mid s \in \mathcal{S} \setminus \mathcal{S}_i]$
  end for
  $S^* \leftarrow S_p$

4.3.2 Genetic algorithm for minimization of $K[\mathrm{W}]$

The condition number of the Gramian $K[\mathrm{W}]$ is generally not submodular, which means that using a greedy algorithm does not guarantee an optimal solution. Nonetheless, we can notice that

$$W(\beta) = \sum_{i=1}^{n} \beta_i \underbrace{\sum_{k=0}^{n-1} (A^\top)^k C_i^\top C_i A^k}_{W_i} = \sum_{i=1}^{n} \beta_i W_i, \tag{4.5}$$

where $\{\beta_i\}_{i=1}^{n}$ are activation variables, with $\beta_i = 1$ if sensor S_i is selected, $\beta_i = 0$ otherwise. Therefore, setting $\beta = \begin{bmatrix} \beta_1 & \beta_2 & \cdots & \beta_n \end{bmatrix}^\top$ the sensor selection problem can be expressed as a constrained optimization problem:

$$\begin{aligned} \mathcal{P}_1 : & \min_{\beta} K[W(\beta)] \\ & \text{s.t.} \ \sum_{i=1}^{n} \beta_i = p \\ & \beta_i \in \{0,1\}, \quad i = 1, \ldots, n \end{aligned} \tag{4.6}$$

The constraint in (4.6) requires the selection of exactly p sensors. Since the problem $\mathcal{P}_1$ is a nonlinear and nonconvex integer programming problem, it is not suited for standard optimization algorithms. Genetic Algorithm methods are utilized, where the fittest individuals from initial solutions are randomly selected and recombined to form a new generation of candidate solutions. The strategy given in Alg. 2 solves $\mathcal{P}_1$.

Algorithm 2 Genetic Algorithm

Require: Matrix A, Observation matrix C_matrix, Genetic algorithm weights β_bin_ga, Total number of individuals n

Ensure: Updated weight matrix W and selection C_i's

$W \leftarrow 0$

for each individual i from 1 to n **do**

Select the observation matrix for individual i: $C_i \leftarrow C\_matrix(i,:)$

Calculate: $W_i \leftarrow C_i' \cdot C_i$

for each k from 1 to $n-1$ **do**

Update W_i:

$W_i \leftarrow W_i + (A')^k \cdot (C_i') \cdot C_i \cdot A^k$

end for

Update W with the genetic algorithm weight:

$W \leftarrow W + \beta\_bin\_ga(i) \cdot W_i$

end for

return $[W, C_1, \ldots, C_m]$

5

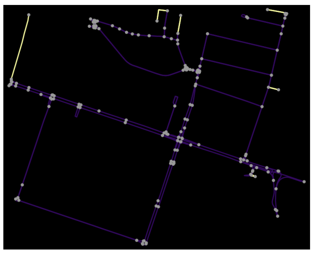
Numerical simulations

In this section, numerical simulations have been conducted to determine the most effective sensor placement by applying the techniques and models discussed in Chapters 3 and 4. The study area was chosen as "Corso Stati Uniti", located in the industrial area of Padua. A map network of this area was extracted and a comprehensive analysis of the optimum placement of the a small number of sensors was carried out. The sensor placement process is based on model-based techniques that utilize linear time-invariant systems and network observability. These techniques aim to enhance the system's performance and monitor traffic flow more effectively. In the analyses, various metrics obtained from the observability Gramian are tested. In particular, criteria such as dimensions of the observable subspace, energy distribution, mode balance provide significant indicators for evaluating the system's performance. The analyses performed demonstrate the effectiveness of the applied techniques and provide valuable data for achieving the optimal results in practice.

5.1 Setup description and data preprocessing

Padua industrial zone was chosen as the study area because of its high traffic density and the presence of different categories of roads. In this area, Corso Stati Uniti road, which is the main road, and the roads connected to it were selected to cover. In the selected area, there are 1,026 nodes and 1,406 edges connecting these nodes in the original road network. There are also duplicated and undefined roads in the original road network. To simplify the road network and make it suitable for analysis, the "simplify_graph" function from OSM was used. After this simplification, a road network model Figure 5.1a with 205 nodes and 348 edges was created. Simulation operations will be conducted on the remaining roads. Filtered from cul-de-sacs, only the boundaries of important roads are shown in Figure 5.1b. Here the edges are represented by colors and the graph is colored differently depending on whether the edges are on a border or not. Border roads are visualized in yellow and non-border roads in purple. After removing cul-de-sacs from the graph, 160 nodes and 269 edges remain. It can be observed that the nodes and edges are reduced compared to the previous graphs. In the following subsections, such a simplification is elucidated in detail. Moreover, the rationale behind network weighting is discussed.

(a) Network with cul-de-sacs

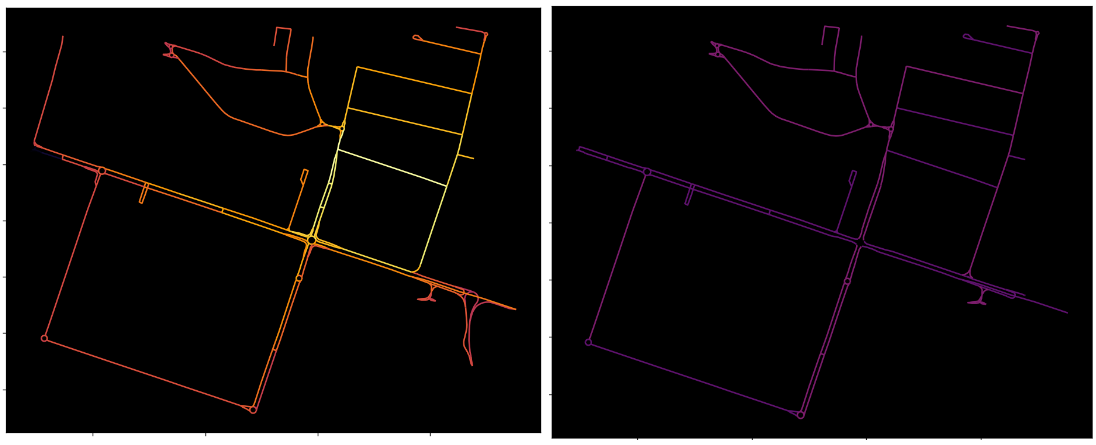


(b) Network without irrelevant cul-de-scas

Figure 5.1: Graph representation of the given road network (Corso Stati Uniti, Padua)

5.1.1 Edge centrality analysis

In order to remove cul-de-sac roads the following rationale based on edge centrality was adopted. Edge centrality refers to the average closeness (distance) value of each edge to other edges. Edge centrality is an important concept in network analysis, especially for assessing the significance of edges within the network structure. In Figure 5.2, the edge proximity centrality of a network is calculated, and a color distribution is applied accordingly. Low edge centrality values are represented by dark colors, while high edge centrality values are shown in bright colors. After removing cul-de-sacs, 219 edges remain in the road network. Before grouping, a map of the road network according to the edge centrality values of these remaining edges is shown in Figure 5.2a. The grouping process is carried out based on the names of the roads. After the grouping, 22 different road names are obtained. An important step in a road network is to analyze the roads that are divided into different segments. Edge centrality values and road segments are important in this analysis because more central roads usually contain more segments and this can help to understand the network topologically. Each road has a centrality value corresponding to the number of segments it contains, and these centrality values are converted into colors and represented on the map in Figure 5.2b.

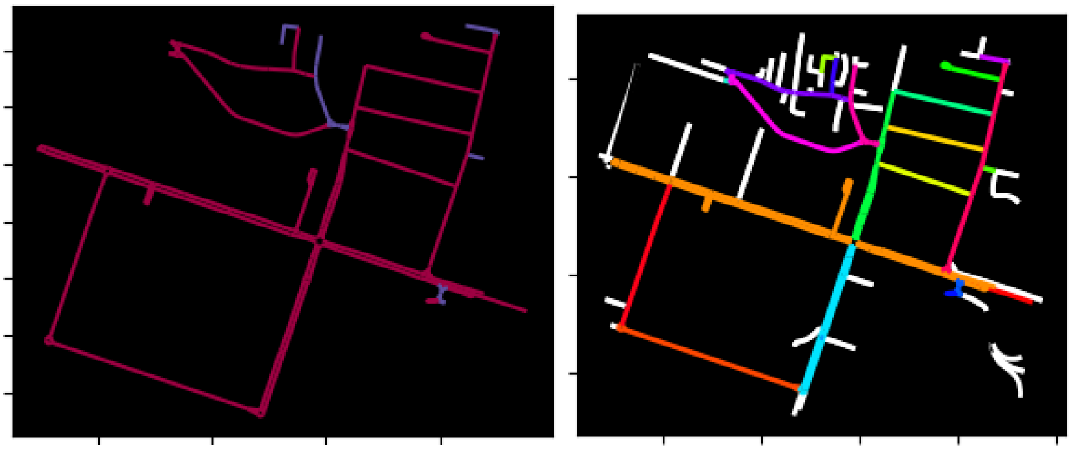


(a) Cul-de-sacs removal before grouping (b) Cul-de-sacs removal after grouping

Figure 5.2: Edge centrality analysis

5.1.2 Border detection

To compleate the cul-de-sacs removal, border detection has been performed. The graph in Figure 5.3a represents a data visualization that analyzes whether the important roads, filtered from cul-de-sacs, are borders or not. Different colors help identify whether a road is a border, while the black background enhances the visibility of the roads.The graph is used for determining borders and understanding which roads are borders. There are two prominent colors on the map: blue and purple. These colors are used to indicate whether the roads are borders or not based on the "isBorder" column. Blue represents the roads that are borders, while purple represents the roads that are not borders. Cul-de-sacs and the remaining road network are visualized in Figure 5.3b. The cul-de-sacs are depicted in shades of gray, while the remaining roads are highlighted with a more colorful palette. This allows for a clear visual understanding of which roads have been removed and which roads remain in the network.



(a) Border detection

(b) Cul-de-sacs removal

Figure 5.3: Road network visualization: cul-de-sac removal and border detection

5.1.3 Assignment of network weights

In order to emphasize the importance of highways and to make the road network structure more clear, the road network map is depicted in Figure 5.4. Highways are colored according to their type, with the highway value written in the middle of each road. Roads with high highway values are considered to be more critical in terms of traffic flow and connectivity. This visualization supports the analysis of the road network and makes it easier to understand which roads are more important.

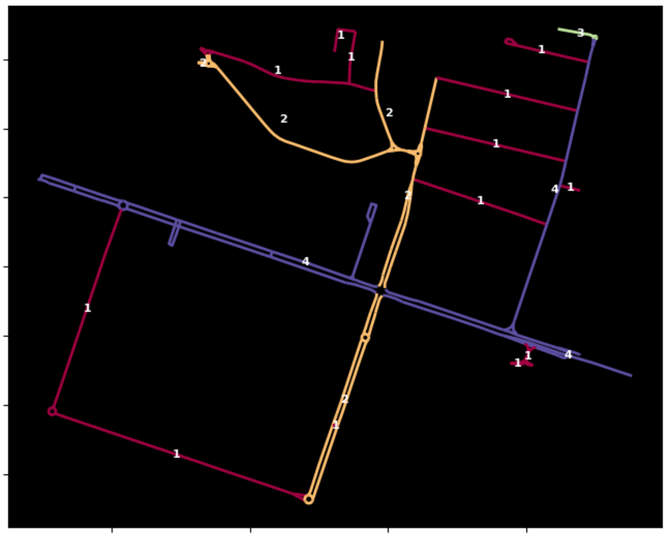


Figure 5.4: Highways importance

5.2 Discussion on the results obtained

This case study has been conducted to place $p \leq n = 22$ sensors on $\mathcal{R} = \{R_1, \ldots, R_n\}$ roads in the industrial zone of Padua. We then compare the solutions $\mathcal{S}^*_{rk}, \mathcal{S}^*_{tr}$ and $\mathcal{S}^*_{\mathcal{K}}$ of the three methods based respectively on rank, trace and condition number. This comparison is proposed to assess the effectiveness of sensor placement strategies and help identify the best sensor combinations aimed at enhancing the overall performance of the system. Define three state space models $\Sigma_{\text{rk}} = (A, B, C(\mathcal{S}^*_{\text{rk}}))$, $\Sigma_{\text{tr}} = (A, B, C(\mathcal{S}^*_{\text{tr}}))$, $\Sigma_{\mathcal{K}} = (A, B, C(\mathcal{S}^*_{\mathcal{K}}))$ and a state observer for each of them. The selection of sensors based on the metrics of the observability Gramian's dimensions of the observable subspace ($rank[W]$), energy distribution ($trace[W]/n$), mode balance ($K[W]$) is demonstrated in Figures 5.5, 5.6, and 5.7, respectively. The green lines represent the roads on which sensor are deployed (as determined by the solutions $\mathcal{S}^*_{rk}, \mathcal{S}^*_{tr}$ and $\mathcal{S}^*_{\mathcal{K}}$), while the blue lines represent the remaining roads of the underlying network.

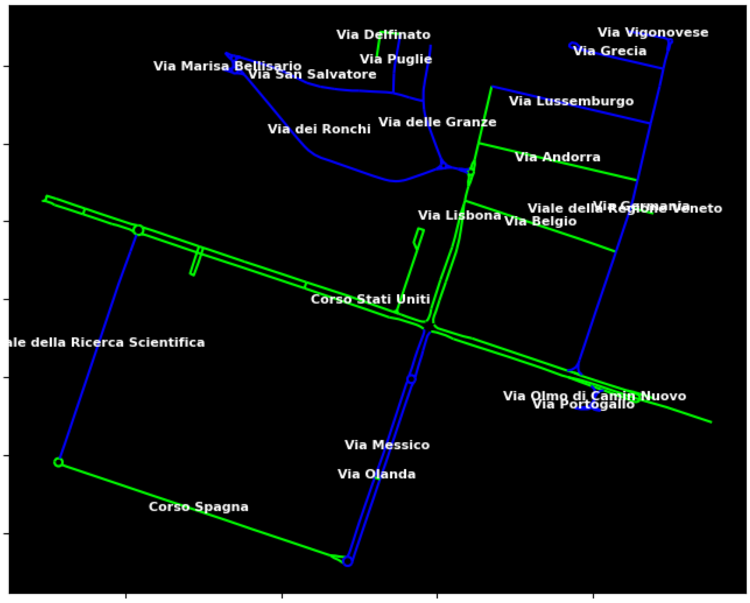


Figure 5.5: Selection based on rank maximization ($p = 8$)

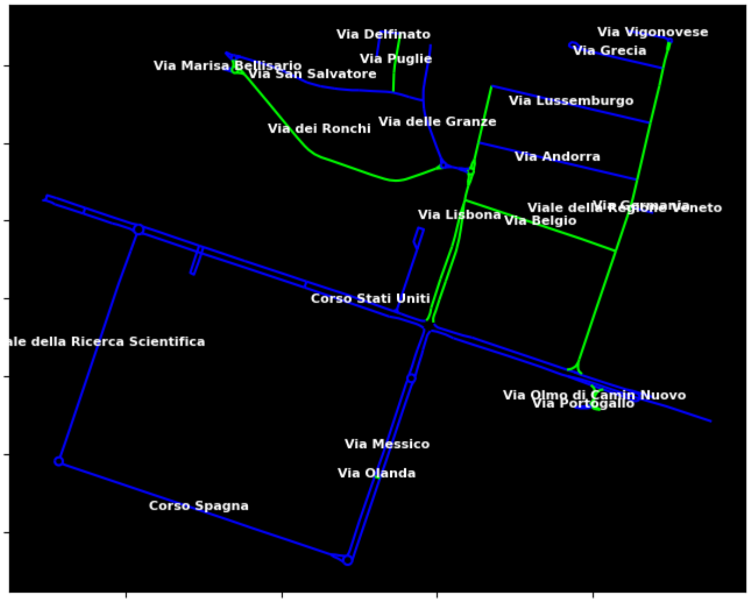


Figure 5.6: Selection based on trace maximization ($p = 8$)

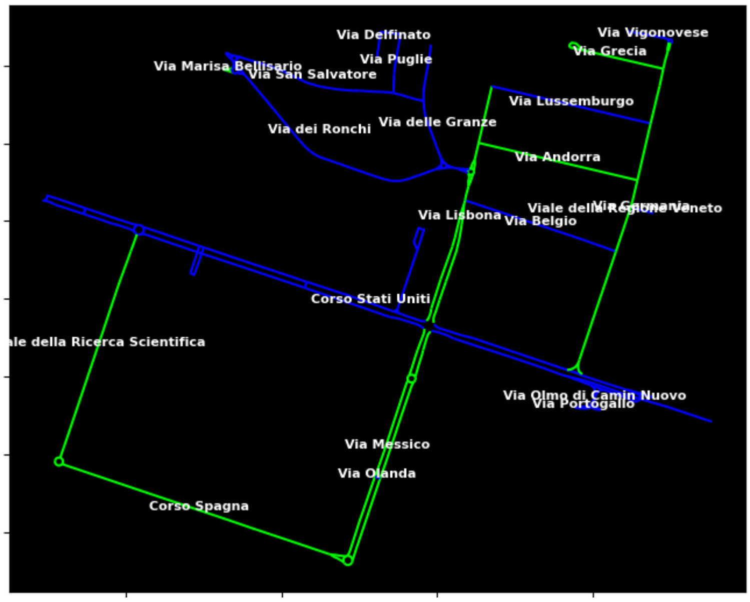


Figure 5.7: Selection based on condition number minimization ($p = 8$)

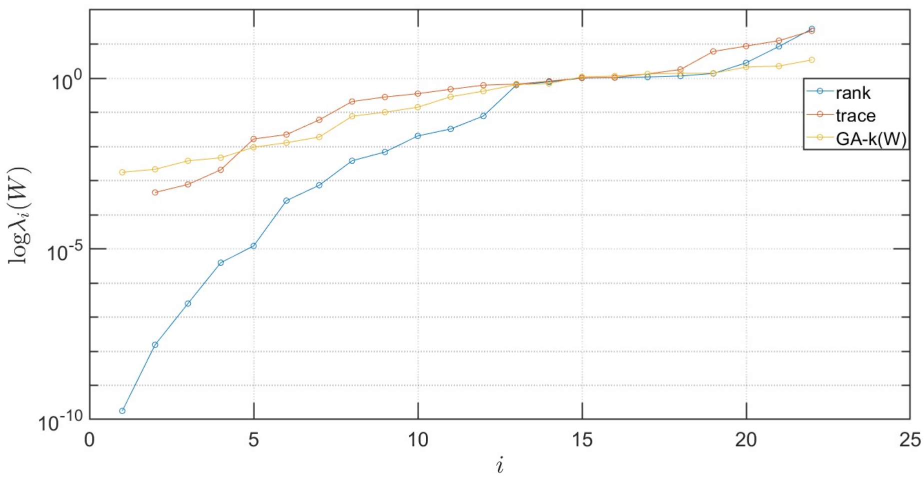


Figure 5.8: Eigenvalues distribution of the Gramian for the three selection cases ($p = 8$)

The eigenvalues of $W(\mathcal{S}^*_{rk})$, $W(\mathcal{S}^*_{tr})$ and $W(\mathcal{S}^*_{\mathcal{K}})$ matrices are shown in Figure 5.8. This diagram visually presents the eigenvalues of each Gramian matrix obtained, which reflect the properties of the corresponding objective function being optimized.

- The *rank* method (blue line) starts with a very low level of observability (approximately 10^{-10}) and slowly increases with respect to the other considered metrics. It guarantees higher observability with a low number of sensors.
- The *trace* (orange line) method starts with a higher initial level of observability, (approximately 10^{-4}), and increases more rapidly than the rank method. However, it does not guarantee the observability of all internal states, which can be a limitation in certain cases.
- The *condition number* method (yellow line) uses a genetic algorithm to balance observability and estimation performance, as we will see. It achieves results similar to the trace method but with a small number of sensors. Also, it allows to preserve a high rank condition, similarly to the rank method. However, the complexity of genetic algorithms and the fact that they sometimes require more computational effort can cause some challenges in practice.

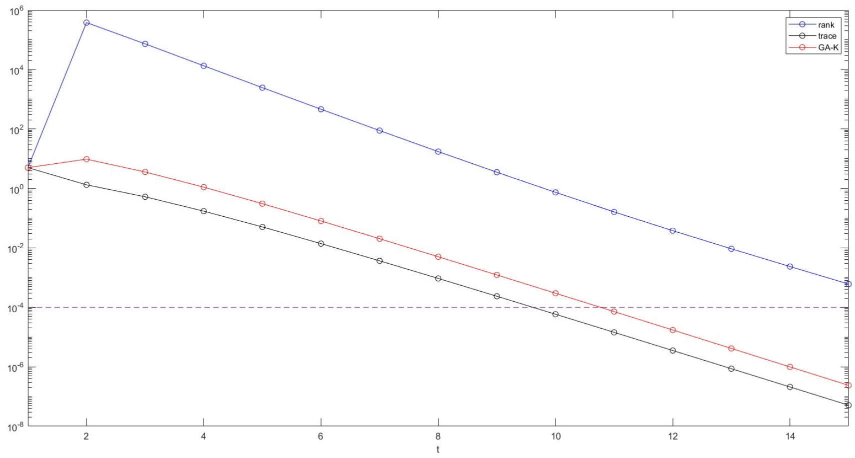


Figure 5.9: Estimation performance: average estimation error of the solution sets $\mathcal{S}^*_{rk}$, $\mathcal{S}^*_{tr}$ $\mathcal{S}^*_{\mathcal{K}}$ as the number of iterations grows ($p = 8$)

Figure 5.9 shows the change in estimation errors over time for the three different sensor selection methods. The following details can be observed.

- The rank method is characterized by the highest initial error compared to the other methods (starting at about 10^6). This indicates that the rank method has the worst performance at the beginning of the estimation procedure.
- The trace method initially starts with the lowest error compared to other methods (about 10^0). Over the time, the trace method steadily and consistently reduces the error more rapidly than other methods; thus, the trace method achieves better results earlier than rank and converges faster to a low error level.
- The condition number method (GA-K) demonstrates an error reduction which is slightly worse than the trace method but better than the rank method.

Even though the initial error of the three method is different, a similar rate of convergence characterizes them.

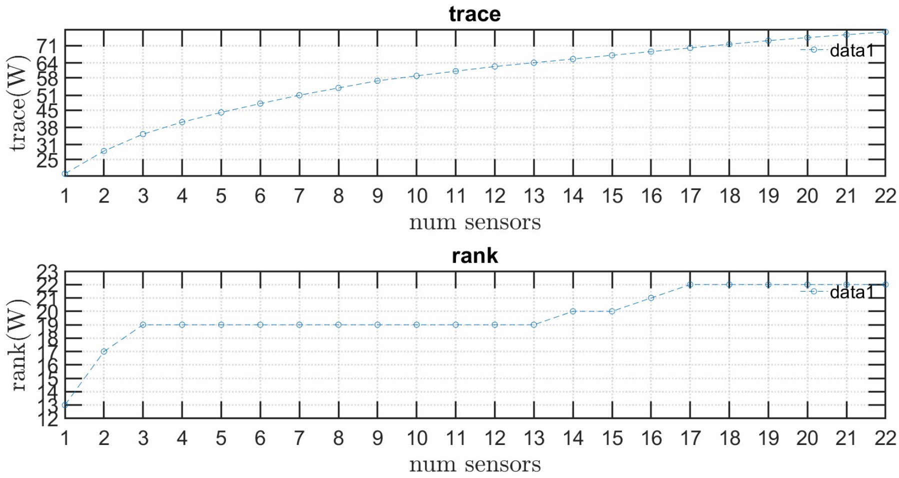


Figure 5.10: Performance of rank and trace methods as the number of sensors p increases and reaches the total number of roads n

Finally, Figure 5.10 illustrates how submodularity plays a role on the observability of a system as the number of sensors increases. Two main indicators are reported in these charts: the trace value of the Gramian and the rank of the Gramian. The following observations can be made.

- Top chart (trace vs. number of sensors): As the number of sensors increases, the observability of the system increases, but this increase is quite slow. This phenomenon occurs since maximizing the trace is equivalent to maximize the average of all the eigenvalues of W.
- Bottom chart (rank vs. number of sensors): As the number of sensors increases, the rank increases rapidly at first. After a certain number of sensors (3 sensors) the rank stabilizes. In addition, starting from 17 sensors, (number of sensors corresponding to the so-called *lag* of the network), it can be shown that the system becomes fully observable ($rank[W] = n$). This implies that adding more sensors no longer contributes to increase observability.

In conclusion, the rank method guarantees observability with a low number of sensors but it is characterized by a slower convergence for the estimation error; while the trace method provides fast and accurate estimates but does not guarantee observability, often requiring a higher number of sensors. The method based on condition number offers the best trade-off, optimizing both efficiency and accuracy, and providing high observability with a smaller number of sensors. On the other hand, the lack of submodularity for this metric requires using specialized algorithms to attain the desired minimum.

6

Conclusions and future work

In this study, the challenges encountered in sensor selection for urban traffic monitoring systems are addressed. It is assumed that at most one sensor can be installed on each street, and a subset of the total number of sensors is selected. In this context, geospatial data is obtained using the OpenStreetMap database, and this data is processed to model traffic routing dynamics. This study highlights the critical importance of proper sensor placement in enhancing the efficiency and accuracy of traffic flow monitoring. By utilizing mathematical foundations such as discrete-time LTI systems and the observability Gramian, effective optimization strategies have been devised, which enable informed decision-making in sensor deployment.

Numerical simulations have been conducted to determine the most effective sensor placement. The metrics obtained from the observability Gramian, such as rank, trace, and condition number are proven to be significant indicators of system performance. The findings indicate that the rank method ensures observability with a minimal number of sensors but tends to exhibit slower convergence in estimation error. In contrast, the trace method delivers fast and precise estimates but does not guarantee observability, possibly necessitating a larger number of sensors. The condition-number-based method strikes the best balance, optimizing both efficiency and accuracy while achieving high observability with fewer sensors. However, due to the non-submodular nature of this metric, specialized algorithms are required to achieve the desired minimum. Although the initial error varies across the three methods, they exhibit a similar rate of convergence.

In conclusion, this study aims to optimize traffic monitoring efficiency through strategic sensor placement, thereby improving traffic flow management. The analyses performed have validated the effectiveness of the proposed techniques and provided valuable data to achieve optimal sensor deployment real scenarios. Overall, this research contributes to the development of smart cities and reinforces the critical role of optimized sensor networks in urban traffic management.

Future work related to these topics may involve a more detailed review of these techniques and the investigation of new observability based metrics for time-varying or nonlinear systems. Additionally, adaptive sensor placement strategies that respond to real-time traffic conditions could be developed by integrating live data. The focus can be on energy efficiency and sensor placement optimization considering multiple objectives such as cost and communication delays. Improving algorithm efficiency will be necessary to scale these approaches for larger urban networks. Lastly, designing robust sensor networks capable of maintaining observability despite sensor failures is envisaged to strengthen the resilience of urban traffic management systems.

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References

[1] Andreas Allström et al. "Traffic management for smart cities". In: *Designing, developing, and facilitating smart cities: urban design to IoT solutions* (2017), pp. 211–240.

[2] Grigoris Antoniou et al. "Enabling the use of a planning agent for urban traffic management via enriched and integrated urban data". In: *Transportation Research Part C: Emerging Technologies* 98 (2019), pp. 284–297.

[3] Giacomo Baggio, Fabio Pasqualetti, and Sandro Zampieri. "Energy-aware controllability of complex networks". In: *Annual Review of Control, Robotics, and Autonomous Systems* 5.1 (2022), pp. 465–489.

[4] Shaunak D Bopardikar. "Sensor selection via randomized sampling". In: *arXiv preprint arXiv:1712.06511* (2017).

[5] Emily Clark, J Nathan Kutz, and Steven L Brunton. "Sensor selection with cost constraints for dynamically relevant bases". In: *IEEE Sensors Journal* 20.19 (2020), pp. 11674–11687.

[6] Sinem Coleri, Sing Yiu Cheung, and Pravin Varaiya. "Sensor networks for monitoring traffic". In: *Allerton conference on communication, control and computing*. Citeseer. 2004, pp. 32–40.

[7] OpenStreetMap Contributors. *OpenStreetMap*. `https://www.openstreetmap.org`.

[8] Julio Cesar Ferro De Guimarães et al. "Governance and quality of life in smart cities: Towards sustainable development goals". In: *Journal of Cleaner Production* 253 (2020), p. 119926.

[9] Marco Fabris, Riccardo Ceccato, and Andrea Zanella. "Efficient sensors selection for traffic flow monitoring: An overview of model-based techniques leveraging network observability". In: *arXiv preprint arXiv:2404.08588* (2024).

[10] Ettore Fornasini. *Appunti di Teoria dei Sistemi*. Editore Libreria Progetto, 2011.

[11] Didier Georges. “The use of observability and controllability gramians or functions for optimal sensor and actuator location in finite-dimensional systems”. In: *Proceedings of 1995 34th ieee conference on decision and control*. Vol. 4. IEEE. 1995, pp. 3319–3324.

[12] Yihui He et al. “Vehicle traffic driven camera placement for better metropolis security surveillance”. In: *arXiv preprint arXiv:1705.08508* (2017).

[13] Brian Thomas Hinson. “Observability-based guidance and sensor placement”. PhD thesis. 2014.

[14] Siddharth Joshi and Stephen Boyd. “Sensor selection via convex optimization”. In: *IEEE Transactions on Signal Processing* 57.2 (2008), pp. 451–462.

[15] Enrico Lovisari, Carlos Canudas de Wit, and Alain Y. Kibangou. “Optimal Sensor Placement in Road Transportation Networks using virtual variances”. In: *2015 54th IEEE Conference on Decision and Control (CDC)*. 2015, pp. 2786–2791.

[16] Juan Lu et al. “Expansion of city scale, traffic modes, traffic congestion, and air pollution”. In: *Cities* 108 (2021), p. 102974.

[17] Alex Olshevsky. “Minimal controllability problems”. In: *IEEE Transactions on Control of Network Systems* (2014), pp. 249–258.

[18] Mohammad Pirani et al. “A game-theoretic framework for security-aware sensor placement problem in networked control systems”. In: *IEEE Transactions on Automatic Control* 67.7 (2021), pp. 3699–3706.

[19] Martin Rodriguez-Vega. “Optimal sensor placement and density estimation in large-scale traffic networks”. PhD thesis. Université Grenoble Alpes [2020-....], 2021.

[20] Tyler H Summers, Fabrizio L Cortesi, and John Lygeros. “On submodularity and controllability in complex dynamical networks”. In: *IEEE Transactions on Control of Network Systems* 3.1 (2015), pp. 91–101.

[21] Erma Suryani et al. “Modelling and simulation of transportation system effectiveness to reduce traffic congestion: a system dynamics framework”. In: *Transportation Planning and Technology* 43.7 (2020), pp. 670–697.

[22] Reza Vafaee and Milad Siami. “Real-time sensor selection for time-varying networks with guaranteed performance”. In: *Automatica* 163 (2024), p. 111550.

[23] Luca Varotto, Alessandra Zampieri, and Angelo Cenedese. “Street sensors set selection through road network modeling and observability measures”. In: (2019), pp. 392–397.

[24] Gilbert G Walter and Martha Contreras. *Compartmental modeling with networks*. Springer Science & Business Media, 1999.