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Effective Risk Management Strategies for Hedging with Options

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APPENDICE

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I dedicate this work to the cherished memory of my dear mother, whose enduring strength and love continue to inspire me every day. I believe she would be joyful in heaven. To my unwaveringly strong woman, Khadijatou Trawally — thank you for standing by me as my greatest support, motivator, and constant source of encouragement. This work is as much yours as it is mine.

Abstract

This thesis investigates the application of options as strategic instruments for effective risk management in volatile financial markets. Amidst increasing uncertainty, it explores the valuation and hedging capabilities of various pricing models: Monte Carlo Simulation and the Black-Scholes Formula for European options, and Least Squares Monte Carlo (LSMC) and the Binomial Tree Method for American options, assuming zero dividend yield for ENI S.p.A. stock. Additionally, the study extends to real option valuation, applying LSMC to Brent Crude data to assess the value of managerial flexibility in capital-intensive projects.

Empirical results show strong convergence between Monte Carlo and Black-Scholes prices for European options. For American options, both LSMC and Binomial Tree methods effectively capture the early exercise feature, particularly in put options. Delta estimation reveals deterministic values under Black-Scholes and stochastic variability in simulation-based methods, reflecting the dynamic nature of real-world hedging. The conceptual valuation of a real option based on Brent Crude yields a positive value, illustrating the strategic benefit of flexibility in investment decisions.

This thesis contributes to both theory and practice by demonstrating how sophisticated quantitative models can enhance pricing accuracy, risk mitigation, and strategic planning. It offers actionable insights for practitioners and policymakers, highlighting the critical role of flexible, data-driven approaches in navigating complex and volatile markets.

Contents

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Listing of acronyms

FTC .... Fundamental Theorem of Calculus
ESC .... Environmental Social and Governance
AI .... Arteficial Interligence
CME .... Chicago Mercantile Exchange
IMM .... International Monetary Market
DJIA .... Dow Jones Industrial Average
MBS .... Mortgage Backed Securities
CDS .... Credit Default Swaps
CBOE .... Chicago Board Options Exchange
CCP .... Central Clearing Counterparties
NYSE .... New York Student Exchange
LSE .... London Stock Exchange
HFT .... High Frequency Trading
OTC .... Over The Counter
EMH .... Efficiency Market Hypothesis
MPT .... Modern Portfolio Theory
CAPM .... Capital Asset Pricing Model
VaR .... Value at Risk
NPV .... Net Present Value
DCF .... Discounted Cash Flow
ATM .... At The Money
OTM .... Out The Money
VIX .... Volatility Index
MiFID .... Markets In Financial Instruments Directive
BSM .... Black Scholes Merton
ML .... Machine Learning
SV .... Schocastic Volatility
SABR .... Stochastic Alpha Beta Rho
EMIR .... European Market Infrastructure Regulations

LSM .... Least Square Monte Carlo

OLS .... Ordinary Least Squares

CvaR .... Conditional Value at Risk

BOPM .... Black Scholes Option Pricing Model

1
Introduction

Global financial markets have become increasingly volatile in recent years, driven by macroeconomic instability, geopolitical tensions, and rapid technological change. From the collapse of major financial institutions during the 2008 global financial crisis to recent market disruptions caused by geopolitical conflicts and technological innovation, the consequences of inadequate risk management are both severe and far-reaching. In this context, robust and adaptive risk management tools have become essential for financial decision-makers, investors, and businesses alike.

Traditional static models often fall short in capturing the dynamic nature of modern markets, prompting the need for more flexible and responsive approaches. This thesis explores the application of financial and real options as strategic tools for effective risk management, focusing on their ability to provide flexibility and mitigate uncertainty in volatile environments. By integrating historical insights with modern financial theory and empirical data, the research aims to bridge existing gaps in the literature and offer practical frameworks for pricing, hedging, and strategic investment decisions under uncertainty.

Risk management involves identifying, evaluating, and implementing strategies to control potential losses arising from various risks, including economic fluctuations, natural disasters, and cybersecurity threats. Although risk is an inherent aspect of financial decision-making, robust management practices enable organizations to safeguard their financial stability and pursue opportunities. Among these practices, hedging strategies which include using financial instruments such as options, futures, and swaps allow investors and firms to offset risks associated with market volatility, interest rate fluctuations, and currency movements. These techniques form a critical part of modern financial risk management and will be explored in detail in later chapters. Risk management has been part of human life for thousands of years. Whether it was ancient traders protecting their cargo from pirates or today's financial institutions preparing for economic downturns, people have always tried to find ways to handle uncertainty and stay secure. Let's take a journey through history and see how risk management evolved into what we know today.

1.1 Historical Background of Risk Management

1.1.1 Ancient Times: The First Risk Managers (Before 1000 AD)

Throughout history, societies have developed various methods to manage financial risks, particularly in trade and commerce. As economies expanded and cross-border trade flourished, merchants and financial institutions devised strategies to mitigate the uncertainties associated with long-distance transactions. The concept of risk-sharing has evolved over millennia, with early civilizations implementing structured financial mechanisms to minimize losses caused by natural disasters, piracy, theft, geopolitical conflicts, and economic downturns [1]. The historical evolution of risk-sharing mechanisms highlights the interconnectedness of past and present financial innovations, demonstrating the enduring relevance of early trade risk management in contemporary financial systems.

Code of Hammurabi (1750 BC)

A key milestone in the formalization of financial risk management was the establishment of the Code of Hammurabi, one of the oldest legal codes in history. The code introduced regulations designed to protect traders and lenders from excessive financial losses due to uncontrollable circumstances. Among these laws, one significant provision allowed traders to cancel debts if their shipments were lost at sea due to natural disasters or theft. This represents an early form of marine insurance and contingent lending [2] which allows policyholders to recover losses from unforeseen shipping hazards.

Babylonians (3000 BC)

As a dominant economic force in ancient Mesopotamia, the Babylonians were recognized for their advancements in commerce and trade. They actively participated in long-distance trade, transporting goods such as textiles, grains, and precious metals across extensive trade networks. Given the significant risks associated with both maritime and overland trade—such as theft, piracy, storms, and political conflicts—merchants sought structured financial mechanisms to mitigate potential losses. To address these uncertainties, Babylonian merchants developed one of the earliest known financial risk management strategies: bottomry loans. This contractual arrangement allowed traders to secure funding for shipping ventures, with the agreement that repayment would occur only if the cargo successfully reached its destination. In cases where the ship was lost due to natural disasters or external threats, the debt was legally forgiven, thereby distributing financial risk between merchants and lenders.

One of the fundamental financial innovations of the Babylonians was the development of bottomry loans, which provided a structured approach to risk-sharing in trade. This financial contract enabled merchants to borrow capital for expeditions with repayment contingent upon the successful delivery of goods. If the shipment was lost due to shipwreck, theft, or other unforeseen events, the merchant was not obligated to repay the loan. By transferring financial risk from individual traders to lenders, bottomry loans introduced an early form of structured risk mitigation that laid the foundation for later developments in maritime finance and insurance. Beyond individual lending agreements, the Babylonians institutionalized financial security through early banking and risk-pooling systems. Temples and palaces functioned as proto-banks, safeguarding merchants' wealth while also facilitating credit transactions that supported commercial expansion. These institutions played a crucial role in stabilizing economic activity by offering structured financial services, including lending, deposit protection, and standardized contractual agreements. By centralizing financial reserves, these early banks helped distribute risk more broadly, reducing the impact of individual trade losses and promoting economic resilience despite unpredictable trading conditions [3]. The structured financial practices pioneered by the Babylonians had a lasting impact on subsequent economic systems, particularly in Greek, Roman, and medieval banking institutions. Many Babylonian financial concepts—such as debt relief laws, contractual lending agreements, and merchant insurance—provided the foundation for modern financial instruments. Over time, these mechanisms evolved into contemporary practices, including hedging contracts, credit default swaps, and maritime insurance policies, which remain fundamental to global financial markets today [4]. The enduring influence of these early risk management strategies highlights the Babylonians' role in shaping the evolution of financial stability mechanisms.

Chinese Traders (2000 BC)

Ancient China was home to one of the world's earliest and most sophisticated trading networks, with merchants engaging in extensive trade across land and sea routes, including the Silk Road and maritime trade corridors. These long-distance trading endeavors involved significant risks, such as piracy, shipwrecks, natural disasters, and bandit attacks, which could lead to substantial financial losses. To mitigate these risks, Chinese traders employed early risk diversification strategies by distributing their goods across multiple ships rather than relying on a single vessel. This ensured that the loss of one ship did not result in total financial ruin. This practice, known as risk spreading, became a foundational concept in modern portfolio diversification and insurance[2]. As trade expanded, Chinese merchants began forming guilds and trade associations in which members pooled financial resources to compensate traders who suffered losses due to shipwrecks or theft. This early form of mutual insurance mirrors modern reinsurance models, where multiple parties share the financial burden of unpredictable risks... [3]. Risk management also extended to overland trade along the Silk Road, where merchants used early credit instruments to reduce the dangers of carrying large amounts of gold or silver during travel. These instruments functioned similarly to modern letters of credit, allowing traders to deposit funds in one location and retrieve them at their destination, thereby significantly reducing the risks associated with theft and robberyLevy [5].

Greek and Roman Maritime Contracts (500 BC – 500 AD)

The ancient Greeks and Romans made significant contributions to the understanding of decision-making under uncertainty:

- Aristotle and Probability: Aristotle's *Nicomachean Ethics* explored the concept of practical wisdom (phronesis), which involved making decisions under uncertainty. He emphasized the importance of deliberation and judgment in uncertain situations [6].
- Roman Risk Management: Roman merchants developed sophisticated risk management tools, such as the *foenus nauticum*, a loan system where repayment was contingent on the safe arrival of goods.

Early insurance-like contracts helped protect goods in transit. Greek merchants used maritime loans, where lenders would provide funds with the agreement that repayment was contingent on the safe arrival of goods. Roman law later formalized financial risk-sharing agreements, influencing the development of contracts that closely resemble modern insurance policies. The establishment of legal frameworks for financial protection during this period contributed to the broader institutionalization of risk management in commerce. This was formalized in financial risk management through structured maritime contracts, which allowed multiple stakeholders to share in the risks and rewards of commercial ventures. The Romans further institutionalized mutual aid societies to provide financial relief to traders who suffered losses due to unforeseen circumstances, laying the foundation for modern risk pooling contracts [3].

Islamic Risk-Sharing and Sukuk (1300s)

Islamic finance emerged as an alternative financial system that adheres to the principles of Sharia law, emphasizing equity-based financing, profit-and-loss sharing, and the prohibition of interest (riba). Unlike conventional finance, which relies on interest-bearing loans and speculative transactions, Islamic finance promotes risk-sharing mechanisms to ensure that all parties involved in a financial transaction bear a proportional share of both risks and rewards [7].

One of the core tenets of Islamic financial transactions is that money should not generate profit on its own; instead, wealth should be created through legitimate trade and investment in tangible assets. This principle led to the development of several risk-sharing financial instruments, including mudarabah, musharakah, sukuk, and takaful, which are widely used in contemporary Islamic banking and capital markets.

1. Mudarabah (Profit-Sharing Partnership): Mudarabah is a contractual partnership between two parties: an investor (*rab al-mal*) who provides the capital and an entrepreneur (*mudarib*) who manages the investment. Unlike conventional loans, where the lender earns fixed interest regardless of the business outcome, in a mudarabah arrangement, profits are distributed based on a pre-agreed ratio, whereas financial losses are borne solely by the investor unless mismanagement or negligence occurs [8]. This structure aligns incentives, encouraging responsible financial management and ethical investment practices.

2. Musharakah (Joint Venture Equity Financing): Musharakah is an equity-based partnership where all parties contribute capital and share both profits and losses. Unlike mudarabah, where only one party provides funding, musharakah requires joint capital investment, with profit-sharing proportional to each partner's contribution. Losses, however, are distributed strictly in accordance with capital investment ratios. This model fosters cooperation and reduces asymmetric information risks, making it suitable for large-scale infrastructure projects, real estate developments, and corporate financing [9].

3. Sukuk (Islamic Bonds): Sukuk, often referred to as *Islamic bonds*, are Sharia-compliant financial certificates that represent ownership in a tangible asset, investment project, or joint venture. Unlike conventional bonds, which generate interest payments (prohibited under Islamic law), sukuk holders earn returns based on the performance of the underlying asset. The structure of sukuk transactions ensures that investors are exposed to real economic activities rather than speculative financial instruments [10].

   Islamic scholars and financial institutions have developed several sukuk structures, including:

   - Ijara Sukuk: Lease-based sukuk where investors receive rental income from the underlying asset.
   - Murabaha Sukuk: Trade-based sukuk where profits are earned from the sale of assets at an agreed markup.
   - Mudarabah Sukuk: Profit-sharing sukuk where investors provide capital to fund an entrepreneur's business, sharing profits according to a predetermined ratio.

- Musharakah Sukuk: Equity-based sukuk where investors jointly finance a project and share the resulting profits and losses.

The success of takaful has led to its integration into global insurance markets, with major financial hubs such as Dubai, Kuala Lumpur, and London establishing dedicated takaful institutions. These developments demonstrate how Islamic risk-sharing models provide ethical alternatives to conventional insurance and banking systems.

4. The Modern Economic Impact of Islamic Finance Islamic finance has grown into a $2.88 trillion industry (as of 2023), with over 1,500 Islamic financial institutions operating across 80 countries [11]. The adoption of mudarabah, musharakah, sukuk, and takaful has provided corporations, governments, and investors with sustainable and ethical investment solutions. Islamic financial principles align with Environmental, Social, and Governance (ESG) investing, further increasing their appeal among ethical investors.

   Today, Islamic financial instruments are used for:

   - Infrastructure financing: *Sukuk* has been used to fund major infrastructure projects, including the expansion of airports, highways, and energy facilities.
   - Corporate financing: *Mudarabah* and *musharakah* are employed by businesses to raise capital while ensuring equitable profit distribution.
   - Wealth management: High-net-worth individuals and institutional investors utilize Islamic financial products to build diversified and Sharia-compliant portfolios.

   Islamic finance continues to evolve, integrating financial technology (FinTech), artificial intelligence (AI), and blockchain solutions to enhance transparency, efficiency, and accessibility. The increasing demand for ethical and interest-free financial systems underscores the growing relevance of Islamic risk-sharing principles in contemporary finance.

1.1.2 Transition to Modern Risk Management

Medieval Period

During the Middle Ages, decision-making under uncertainty became more structured, particularly in trade and finance:

- Probabilistic Thinking: Medieval merchants used probabilistic thinking to assess risks in trade and finance. For example, they estimated the likelihood of shipwrecks, theft, or market fluctuations and adjusted their pricing, contracts, and investment strategies accordingly. By incorporating early forms of probability calculations, they improved decision-making under uncertainty, laying the groundwork for modern risk assessment methods [1, 12].
- Double-Entry Bookkeeping: The development of double-entry bookkeeping by Luca Pacioli in the 15th century marked a significant advancement in financial management. This system, based on recording both credits and debits, enabled merchants to track their assets and liabilities more accurately, reducing the risk of errors and financial mismanagement. Moreover, double-entry bookkeeping improved transparency and accountability, facilitating better financial planning and investment decisions. Its widespread adoption contributed to the growth of capitalism and modern accounting practices [13, 14, 15].

The expansion of trade routes between Europe, Asia, and Africa during the medieval and Renaissance periods led to the development of institutionalized risk management mechanisms. As trade expanded, more structured methods for managing financial risk emerged. These developments led to the formalization of insurance systems, enabling merchants and businesses to protect themselves from unforeseen losses [1].

- Medieval Merchant Guilds (1100s – 1500s): European merchants formed guilds that functioned as early mutual insurance societies. It played a crucial role in providing financial security for its members. These organizations pooled money to assist individuals in cases of accidents, disability, or death [2]. Beyond trade protection, guilds also functioned as mutual aid societies, ensuring that families and businesses could recover from financial setbacks. This cooperative approach laid the groundwork for later insurance models based on collective risk-sharing [3].
- Lombard Bankers (1200s - 1400s): Italian merchants and financiers developed sophisticated financial contracts to manage trade risks. These contracts, often backed by wealthy banking families in cities like Florence and Venice, allowed merchants to secure compensation in case of lost or damaged shipments [5]. The Lombards' contributions to financial risk management greatly influenced the evolution of credit and insurance systems in Europe [4].
- Lloyd's of London (1688): Emerging from a coffeehouse frequented by shipowners, merchants, and financiers, Lloyd's of London became the center for marine insurance. Business discussions held at Lloyd's led to the establishment of a formal market [1], allowing merchants to hedge against shipping risks. Over time, Lloyd's grew into the world's most renowned insurance institution, setting standards for risk assessment and policy structuring that continue to shape the industry today [3].
- Early Banking Systems (1500s – 1700s): Renaissance-era banking institutions began offering secured trade loans with embedded risk-sharing provisions, setting the foundation for modern commercial lending and credit markets.
- Stock Exchanges and Risk Hedging (1700s – 1800s): The establishment of public stock exchanges, such as the Amsterdam Stock Exchange, enabled investors to hedge financial risks through diversified investment strategies.

These early developments highlight the long-standing human effort to manage uncertainty in trade. Over time, such practices evolved into structured financial instruments, eventually giving rise to the modern insurance industry. These developments marked a shift from informal risk-sharing arrangements to organized financial mechanisms. The increasing complexity of trade and commerce necessitated more sophisticated insurance models, paving the way for the structured policies and institutions that define modern insurance systems [4].

Post-War Stability, Market Shocks, and Risk Management Evolution

The post-World War II period through the mid-1960s was characterized by relative financial stability, marked by low interest rates, steady economic growth, and a structured international monetary system under the Bretton Woods framework. Established in 1944, this system pegged major currencies to the U.S. dollar, which was convertible to gold. This arrangement reduced exchange rate volatility and fostered confidence in international trade and investment [16]. However, by the late 1960s, growing U.S. fiscal deficits, rising inflation, and trade imbalances began to undermine the system [17].

In 1971, the United States, under President Nixon, suspended the gold convertibility of the dollar, effectively dismantling the Bretton Woods system and transitioning the global economy to floating exchange rates [18].

While this shift provided governments with more monetary policy flexibility, it also introduced significant currency fluctuations, exposing businesses and investors to foreign exchange risk. In response, financial markets saw the emergence of new hedging instruments, such as currency futures and options, allowing participants to manage exchange rate exposure. The establishment of the Chicago Mercantile Exchange's (CME) International Monetary Market (IMM) in 1972 was a pivotal development, providing a regulated platform for trading currency derivatives [19].

The financial stability of global markets was further disrupted by the first oil crisis in 1973, when OPEC imposed an oil embargo, causing a sharp spike in energy prices and triggering worldwide inflationary pressures [20]. This economic turbulence, combined with stagnating growth and rising unemployment, led to stagflation—a rare combination of high inflation and economic stagnation [21]. To combat inflation, central banks, particularly the Federal Reserve in the United States and the Bank of England in the United Kingdom, implemented aggressive monetary tightening policies in the late 1970s and early 1980s. Interest rates were raised to double digits, peaking at over 20% in the U.S. under Federal Reserve Chairman Paul Volcker [22]. While these measures successfully curtailed inflation, they also increased borrowing costs, led to debt crises in developing economies, and created heightened financial uncertainty. This period marked a turning point where interest rate risk management became a financial priority, driving the increased use of interest rate swaps, futures, and options as essential risk mitigation tools [23].

Despite the growing sophistication of financial instruments, risk management practices remained inadequate in the face of emerging market complexities. On October 19, 1987, known as Black Monday, the Dow Jones Industrial Average (DJIA) plummeted by 22.6% in a single day, marking the largest one-day percentage decline in history [24]. The crash was exacerbated by program trading and portfolio insurance strategies, which triggered automated sell-offs, reinforcing the downward spiral [25]. This event underscored the vulnerabilities of computerized trading systems and market illiquidity, prompting regulatory bodies to introduce circuit breakers to temporarily halt trading during extreme volatility [26]. Additionally, the crisis highlighted deficiencies in portfolio risk models, accelerating the adoption of stress testing methodologies and more sophisticated risk assessment frameworks [27].

The financial disruptions of the 1980s, including the near-collapse of Salomon Brothers and the failure of Drexel Burnham Lambert, underscored the importance of enhanced risk oversight [28]. In response, the Basel Committee on Banking Supervision introduced the first Basel Accord (Basel I) in 1988, establishing capital adequacy requirements designed to ensure that banks maintained sufficient reserves to cover their risk exposure [29]. Subsequent refinements, including Basel II (2004) and Basel III (2010), expanded these regulations by incorporating market risk measurement techniques such as Value at Risk (VaR), implementing stress testing protocols, and strengthening liquidity coverage requirements [30]. These reforms played a crucial role in shaping modern financial risk management, emphasizing the need for transparent risk disclosure and standardized regulatory compliance across financial institutions [31].

Despite these regulatory advancements, the 2008 global financial crisis exposed major deficiencies in existing risk models and oversight mechanisms. The crisis stemmed from excessive risk-taking in subprime mortgage-backed securities (MBS) and the widespread use of credit default swaps (CDS), which transferred credit risk between financial institutions [32]. The collapse of Lehman Brothers and the subsequent market turmoil revealed critical weaknesses in Value at Risk models, which failed to account for extreme market downturns and systemic risks [33]. Many banks relied heavily on short-term funding and held insufficient capital buffers to absorb massive losses. In response to the crisis, regulators introduced Basel III, which strengthened capital and liquidity requirements, and the Dodd-Frank Act (2010), which imposed stricter oversight on derivatives markets and mandated central clearing for swaps to reduce counterparty risk [34]. Additionally, financial institutions adopted more comprehensive risk assessment techniques, integrating scenario analysis, tail risk modeling, and real-time stress testing to enhance resilience against future crises [35].

While derivatives were partially blamed for amplifying financial crises, they also became essential tools for risk mitigation and hedging strategies. The establishment of the Chicago Board Options Exchange (CBOE) in 1973 marked the beginning of a new era in financial innovation, enabling market participants to trade standardized options contracts [36]. The subsequent development of financial derivatives, including futures, swaps, and structured products, provided investors and institutions with mechanisms to hedge against risks associated with interest rates, currency fluctuations, and commodity price swings [37]. In the post-2008 era, the role of derivatives has been subject to increased scrutiny, leading to the expansion of central clearing counterparties (CCPs) to mitigate counterparty risk and enhance market transparency [38].

Financial risk management has continuously evolved in response to market crises, regulatory shifts, and technological advancements. The transition from post-war stability to floating exchange rates, inflationary shocks, and financial crashes has driven the development of more sophisticated risk control mechanisms. Modern risk management increasingly relies on big data analytics, artificial intelligence, and machine learning to enhance predictive capabilities and automate risk mitigation strategies [39]. Additionally, new challenges such as cybersecurity threats, climate-related financial risks, and systemic vulnerabilities in decentralized finance (DeFi) are reshaping the risk landscape [40]. As financial markets continue to evolve, effective risk management will require the integration of historical lessons with emerging technologies to build more resilient financial systems.

Industrial Revolution & Financial Risk

The Industrial Revolution marked a turning point in the evolution of risk management, as the rise of industrialization and global trade introduced new risks and opportunities. Financial markets became more complex, and the need for structured risk management tools grew. During this period, the development of stock markets, insurance companies, and financial derivatives laid the foundation for modern risk management practices.

The Evolution of Stock Markets and Risk Management The concept of stock markets dates back to the early trading hubs of Europe, notably the Amsterdam Stock Exchange, founded in 1602. Over the centuries, financial markets expanded globally, leading to the establishment of major exchanges such as the New York Stock Exchange (NYSE) in 1792 and the London Stock Exchange (LSE) in 1801. These institutions facilitated capital formation and liquidity, playing a central role in economic development by connecting investors with businesses seeking funding.

As economies grew, the increasing scale and complexity of financial activities necessitated more sophisticated mechanisms for managing risk. Stock markets became essential platforms for capital allocation, allowing companies to raise funds while offering investors opportunities for portfolio diversification. To address market volatility and improve investment decision-making, financial theories and risk management strategies evolved, shaping modern approaches to asset valuation, pricing models, and market efficiency.

In the 20th century, stock markets underwent further transformation with the advent of electronic trading, derivatives, and algorithmic strategies. Advances in financial theory, particularly in risk assessment and portfolio optimization, provided investors with tools to navigate market fluctuations more effectively. These innovations contributed to greater market efficiency, increased trading volume, and more complex investment strategies, reinforcing the stock market's role as a cornerstone of global finance.

Stock markets have played a pivotal role in economic growth and investment opportunities. However, they are inherently exposed to risks that require careful analysis and management. By utilizing diversification strategies, financial models, and derivative instruments, investors can navigate market uncertainties more effectively. As financial technology advances, new risk management tools will continue to shape the future of stock market investing.

To understand the development of financial markets and risk management, it is important to examine key historical milestones that shaped modern financial systems:

- Amsterdam Stock Exchange (1602): The Dutch East India Company (VOC) issued shares to investors, allowing them to participate in the company's profits while distributing risk through shared ownership. This marked the birth of the modern stock exchange, where securities could be traded, providing liquidity and encouraging investment in large-scale ventures [41].
- Fire Insurance (1666): The Great Fire of London, which destroyed much of the city, highlighted the need for financial protection against unforeseen disasters. This led to the creation of the first fire insurance company, founded by Nicholas Barbon in 1681. Fire insurance policies allowed businesses and homeowners to transfer the financial burden of rebuilding, laying the foundation for the modern insurance industry [42].
- Life Insurance (1700s–1800s): Advancements in actuarial science during the 18th and 19th centuries enabled insurers to systematically price life insurance policies. The use of mortality tables, pioneered by Edmond Halley and refined by subsequent statisticians, allowed insurers to estimate life expectancies and set premiums accordingly. This period saw the growth of life insurance companies, making financial protection more accessible to individuals and families [43].
- The Birth of Modern Stock Markets (18th–19th Century): With the expansion of global trade, stock markets became central to capital formation. The London Stock Exchange (LSE) was officially founded in 1801, creating a regulated marketplace for securities trading. Meanwhile, in the United States, the Buttonwood Agreement of 1792 led to the creation of the New York Stock Exchange (NYSE), which became a dominant force in global finance [44].
- Railroad Bonds and the Expansion of Capital Markets (19th Century): The rapid industrialization of the 19th century, particularly the expansion of railroads, required massive amounts of capital. Governments and private companies issued bonds to finance these infrastructure projects. The success of railroad bonds demonstrated the potential of debt financing and influenced the development of modern bond markets [45].
- The Formation of Central Banks and Financial Regulation (20th Century): The increasing complexity of financial markets led to the establishment of central banks and regulatory institutions. The U.S. Federal Reserve, created in 1913, introduced monetary policies to stabilize financial markets. Regulatory frameworks, such as the Securities Act of 1933, were introduced to enhance transparency and investor confidence following the stock market crash of 1929 [46].
- The Rise of Financial Derivatives (Late 20th Century): The development of financial derivatives, including futures, options, and swaps, transformed risk management. The creation of the Chicago Board Options Exchange (CBOE) in 1973 formalized options trading, while the Black–Scholes model provided

a mathematical framework for pricing options. These innovations allowed investors and institutions to hedge against market fluctuations [37].

- Digital Trading and Algorithmic Finance (21st Century): The rise of electronic trading platforms and algorithmic trading has reshaped financial markets. High-frequency trading (HFT) and artificial intelligence-driven investment strategies now play a significant role in stock market movements. Cryptocurrencies and decentralized finance (DeFi) have introduced new forms of digital assets, challenging traditional financial models and regulations [47].

Financial markets continue to evolve, shaped by technological advancements, regulatory developments, and changing economic conditions. While historical milestones have laid the foundation for modern finance, emerging challenges such as cybersecurity risks, regulatory uncertainty, and the integration of artificial intelligence in trading require continuous adaptation. The interplay between innovation and risk management will define the future of financial markets, influencing how investors and institutions navigate an increasingly complex global economy.

Risk in stock markets arises from various sources, including economic cycles, geopolitical events, and firm-specific factors. Understanding these risks is essential for investors to develop effective portfolio strategies and mitigate potential losses. Broadly, financial risk in stock markets can be categorized into two main types:

- Systematic Risk: Also known as market risk, this type of risk affects all securities due to macroeconomic factors such as inflation, interest rates, recessions, and geopolitical instability. Systematic risk is inherent to the overall market and cannot be eliminated through diversification. Events such as financial crises, changes in monetary policy, and global economic downturns contribute to systematic risk, impacting the entire stock market rather than specific sectors or companies. Since systematic risk is unavoidable, investors often use hedging strategies, asset allocation, and derivatives such as options and futures to manage exposure.
- Unsystematic Risk: Also called idiosyncratic or specific risk, this type of risk is associated with individual companies or industries rather than the broader market. Factors contributing to unsystematic risk include company management decisions, regulatory changes, technological innovations, supply chain disruptions, and earnings volatility. Unlike systematic risk, unsystematic risk can be mitigated through diversification, as holding a well-balanced portfolio of assets reduces the impact of poor performance from any single investment. Investors employ various strategies, such as sector rotation and fundamental analysis, to assess and manage unsystematic risk effectively.

Understanding the distinction between systematic and unsystematic risk is crucial for portfolio management. While diversification can significantly reduce exposure to company-specific risks, systematic risk requires alternative risk management techniques, such as hedging and strategic asset allocation. By integrating risk assessment tools and financial models, investors can optimize their portfolios to balance return potential with acceptable risk levels.

The Role of Derivatives in Risk Management

Derivatives, such as options, futures, and swaps, play a fundamental role in modern financial markets by allowing investors and institutions to manage risk, hedge against adverse market movements, and enhance portfolio efficiency. These instruments derive their value from underlying assets, including equities, commodities, interest rates, and currencies. By providing mechanisms for risk transfer, derivatives contribute to market liquidity, improve price discovery, and enable participants to navigate financial uncertainties more effectively.

The use of derivatives in risk management is widespread across various industries. Financial institutions rely on derivatives to stabilize cash flows and minimize exposure to fluctuating interest rates and currency values. Corporations use them to hedge raw material costs and ensure price stability. Investors employ derivatives to protect portfolios from downside risks and to optimize returns. However, while derivatives provide significant benefits, they also introduce complexities and systemic risks, particularly when excessive leverage or speculative trading is involved. Regulatory frameworks such as the Dodd-Frank Act and Basel III have been implemented to mitigate these risks and enhance transparency in derivative markets.

The following are the most commonly used derivatives for managing risk:

1. Options: Options are financial contracts that grant the holder the right, but not the obligation, to buy (call option) or sell (put option) an asset at a predetermined price (strike price) before or at the expiration date. Unlike futures, options provide asymmetric risk exposure, allowing traders to define potential losses while maintaining unlimited profit potential.

   Options are essential tools for hedging against price movements, providing leverage, and implementing structured investment strategies. Traders and institutional investors use options to protect against declines in asset value, speculate on market trends, and enhance returns through strategies such as covered calls and protective puts.

   - Equity Hedging: Investors use put options to protect against stock price declines. For example, a portfolio manager holding shares of a company may purchase put options as insurance against adverse market movements.
   - Commodity Risk Management: Airlines frequently use call options on jet fuel to hedge against rising fuel prices, ensuring cost stability and reducing uncertainty in operational expenses.
   - Volatility Trading: Options allow traders to capitalize on market volatility without directly holding the underlying asset. Strategies such as straddles and strangles enable investors to profit from significant price swings regardless of direction [23].
   - Employee Compensation: Many corporations use stock options as part of employee compensation packages, aligning employee interests with company performance and offering long-term incentives.

2. Futures Contracts: Futures contracts are standardized agreements to buy or sell an underlying asset at a specified future date and price. Unlike options, futures impose an obligation on both parties to execute the contract upon expiration. These contracts are traded on regulated exchanges and serve as a primary tool for managing price risk across multiple asset classes, including commodities, financial instruments, and interest rates.

   The key advantage of futures contracts is their ability to provide certainty in pricing and eliminate price fluctuations. Market participants use futures to hedge against unfavorable price movements, ensuring that businesses can stabilize input costs and financial institutions can secure fixed interest rates.

   - Commodity Price Hedging: Agricultural producers hedge against price fluctuations by selling futures contracts on crops such as wheat, corn, or coffee. If market prices drop, the futures contract compensates for lost revenue, ensuring price stability for farmers.
   - Interest Rate Risk Management: Financial institutions use interest rate futures to hedge against fluctuations in borrowing costs. For instance, a bank may use Treasury futures to protect against rising interest rates, ensuring stable lending conditions.

- Foreign Exchange Hedging: Exporters and importers use currency futures to lock in exchange rates, mitigating losses from currency depreciation or appreciation. This is particularly important in international trade, where exchange rate volatility can significantly impact profitability [48].
- Stock Index Futures: Institutional investors use stock index futures to hedge against market downturns or speculate on overall market performance. Hedge funds and mutual funds often use these instruments to balance portfolio exposure.

3. Swaps: Swaps are financial contracts in which two parties agree to exchange cash flows based on predefined financial variables, such as interest rates, currencies, or credit risk. Swaps provide customized solutions for managing financial exposure, making them widely used in corporate finance and institutional investment strategies. Unlike futures and options, swaps are traded over-the-counter (OTC) rather than on centralized exchanges, allowing for greater flexibility in contract design.

   The primary benefit of swaps is their ability to hedge against specific financial risks while maintaining capital efficiency. Institutions use swaps to restructure debt, stabilize funding costs, and minimize the impact of market fluctuations.

   - Interest Rate Swaps: A company with floating-rate debt can enter into an interest rate swap to exchange variable payments for fixed payments, reducing exposure to interest rate fluctuations. This helps corporations plan for future financial obligations with greater certainty.
   - Currency Swaps: Multinational corporations use currency swaps to hedge against exchange rate risks when borrowing in foreign currencies. For example, a U.S. company with euro-denominated debt may swap payments with a European firm holding dollar-denominated debt, ensuring that both parties mitigate currency risk.
   - Credit Default Swaps (CDS): Financial institutions use CDS contracts to hedge against the risk of borrower default. In a CDS agreement, one party agrees to compensate the other in the event of a credit event, such as a bond default. The CDS market gained significant attention during the 2008 financial crisis, as these contracts were used extensively to insure against mortgage-backed securities defaults [23].
   - Commodity Swaps: Energy companies and industrial manufacturers use commodity swaps to stabilize input costs. For example, an airline may engage in a fuel swap to fix fuel prices, protecting against price volatility in energy markets.

Derivatives play a broader role in financial stability and capital efficiency. They contribute to market liquidity by allowing institutions to offload risk and allocate capital more effectively. Additionally, they enhance price discovery, as derivative prices reflect market expectations about future asset values.

However, derivatives also introduce systemic risks, particularly when leveraged excessively or used for speculative purposes. The 2008 financial crisis exposed vulnerabilities in the derivatives market, leading to increased regulatory oversight through measures such as the Dodd-Frank Act, which imposed clearing requirements and transparency rules for OTC derivatives.

Looking ahead, technological advancements are reshaping the derivatives landscape. Algorithmic trading, high-frequency trading (HFT), and blockchain-based smart contracts are increasing the speed and efficiency of derivatives markets. These innovations are expected to enhance transparency, reduce transaction costs, and improve risk management capabilities. As financial markets continue to evolve, derivatives will remain an essential tool for managing uncertainty and optimizing investment strategies.

Options, futures, and swaps serve as essential tools for risk management, providing investors and corporations with mechanisms to hedge financial exposure. Their strategic application allows market participants to stabilize cash flows, reduce uncertainty, and enhance financial planning. However, the complexity and leverage associated with derivatives necessitate careful risk assessment to avoid unintended consequences.

Market Efficiency and Risk Pricing The efficient market hypothesis (EMH), formulated by Eugene Fama, suggests that stock prices fully reflect all available information, making it difficult for investors to consistently outperform the market [49]. However, behavioral finance challenges this view by showing that psychological biases and irrational investor behavior can lead to market inefficiencies.

Financial Crises and Risk Management Lessons Major financial crises, such as the Great Depression (1929), the Dot-com Bubble (2000), and the Global Financial Crisis (2008), highlight the challenges of managing risk in stock markets. These events underscore the importance of robust risk models, regulatory oversight, and adaptive investment strategies to mitigate systemic shocks.

Quantitative Approaches to Risk Management

To measure and manage risk, financial theorists have developed various models and techniques that provide mathematical frameworks for assessing investment risk, optimizing portfolios, and pricing financial assets. These approaches play a crucial role in both individual investment strategies and institutional risk management.

- Modern Portfolio Theory (MPT): Introduced by Harry Markowitz in 1952, MPT provides a mathematical framework for constructing portfolios that maximize expected returns for a given level of risk. By diversifying across uncorrelated assets, investors can reduce portfolio volatility and optimize their risk-return trade-off. The theory is based on the assumption that investors are risk-averse and seek to minimize risk while achieving the highest possible returns. MPT laid the foundation for contemporary portfolio management and remains a fundamental concept in finance [50].
- Capital Asset Pricing Model (CAPM): Developed by William Sharpe in 1964, CAPM explains how assets should be priced based on their risk relative to the overall market. The model introduces the concept of beta (β), which measures an asset's sensitivity to market movements. A higher beta indicates greater volatility compared to the market, while a lower beta suggests lower risk. CAPM also incorporates the risk-free rate and market risk premium to determine an asset's expected return, providing a key tool for asset valuation and investment decision-making [51].
- Value at Risk (VaR): VaR is a widely used risk metric that estimates the maximum expected loss of an investment over a given time horizon at a specific confidence level. It provides a quantitative measure of downside risk, helping investors and financial institutions assess potential losses in adverse market conditions. VaR is commonly applied in portfolio risk assessment, financial reporting, and regulatory compliance, particularly in banking and investment management. However, its limitations, such as the assumption of normal distribution in asset returns, have led to the development of alternative risk measures.
- Black–Scholes Model: Developed by Fischer Black and Myron Scholes in 1973, this model revolutionized options pricing and risk management. The Black–Scholes equation provides a mathematical framework for valuing European-style options by incorporating factors such as asset price, strike price, volatility, time

to expiration, and risk-free interest rates. Beyond stock valuation, the model enables investors to hedge against price fluctuations by using options to mitigate portfolio risk. Despite its assumptions, such as constant volatility and frictionless markets, the Black–Scholes model remains a foundational tool in derivatives pricing [37].

These quantitative approaches provide essential tools for understanding and managing financial risk. While each model has its limitations, their applications continue to shape investment strategies, risk assessment methodologies, and financial decision-making in modern markets. Advances in financial engineering and computational techniques have further refined these models, improving their accuracy and adaptability to evolving market conditions.

1.1.3 Hedging Strategies

Numerous hedging strategies utilize options to manage different types of risks. Protective puts and covered calls are among the simplest and most widely used strategies. Protective puts allow investors to limit downside risk by purchasing put options, effectively setting a floor on potential losses. Covered calls, on the other hand, involve selling call options against a held position, generating additional income but capping potential upside gains.

More complex approaches, such as collars and spreads, offer tailored risk-reward profiles. A collar strategy combines a protective put with a covered call to limit both downside and upside potential within a defined range. Spread strategies, including bull and bear spreads, involve simultaneous buying and selling of options at different strike prices to construct a predefined profit and loss structure.

Dynamic hedging techniques, including delta and gamma hedging, adjust positions continuously to maintain a hedge's effectiveness. Delta hedging neutralizes the price sensitivity of an option by adjusting the underlying asset position, while gamma hedging fine-tunes this approach by managing the rate of change of delta. These strategies require constant rebalancing and are highly sensitive to market conditions.

Studies by Taleb (1997) and Natenberg (1994) provide deep insights into the practical application and challenges of these strategies, particularly in volatile markets. They emphasize the importance of understanding implied volatility, market microstructure, and liquidity constraints when implementing hedging techniques.

1.1.4 Measuring Hedging Effectiveness

Assessing the effectiveness of hedging strategies is crucial for their optimization. Metrics such as variance reduction, cost-benefit analysis, and Value-at-Risk (VaR) are commonly used. Variance reduction, introduced by Johnson (1960), measures the extent to which a hedge decreases portfolio risk by reducing return fluctuations.

Modern studies incorporate transaction costs and market liquidity into the evaluation framework, acknowledging the practical limitations of perfect hedging. Cost-benefit analysis helps determine whether the expenses associated with hedging, including option premiums and transaction costs, justify the achieved risk reduction. VaR, a widely used risk metric, quantifies the maximum expected loss over a given time horizon at a specified confidence level.

Empirical analyses often highlight trade-offs between the degree of risk mitigation and the associated costs, emphasizing the need for tailored strategies. Research in this area suggests that an optimal hedge ratio must balance protection against excessive cost burdens while considering factors such as market volatility, position sizing, and investment horizon.

1.2 Real Options

Options are financial derivatives that grant the holder the right, but not the obligation, to buy or sell an underlying asset at a predetermined strike price(referred to as the *strike price*) within a given window of time. These contracts are valuable in financial markets because they allow investors to control risk and potentially increase their returns without committing to purchasing or selling an asset immediately committing to buying or selling the asset. The ability to exercise this right depends on the option's exercise style. The three main types of options based on exercise style are European, American, and Bermudan options, each differing in the flexibility of when they can be exercised.

European Options: European options can only be exercised at expiration (the maturity date). This restriction simplifies pricing models and makes them generally cheaper than American options. These options are commonly used in index options, foreign exchange options, and some bond markets.

Characteristics

- Can be exercised only at expiration.
- Lower premium due to the limited flexibility.
- Priced using the Black-Scholes model.
- Common in index options (e.g., Euro Stoxx 50 options).

Example: Suppose an investor purchases a European call option on a stock with a strike price of $100 and an expiration date in three months. Even if the stock price rises above $100 before expiration, the option holder cannot exercise the option early and must wait until expiration.

American Options: American options provide greater flexibility since they can be exercised at any time before or on the expiration date. This flexibility makes them more valuable than European options, leading to a higher premium.

Characteristics

- Can be exercised at any time before or on expiration.
- Typically more expensive due to early exercise potential.
- Common in stocks, commodities, and ETFs.
- Priced using models such as the Binomial Tree and Black-Scholes with adjustments.

Example: Consider an American put option with a strike price of $50. If the stock price drops to $40 before expiration, the holder can exercise the option immediately rather than waiting until expiration, reducing the risk of a price rebound.

Bermudan Options: Bermudan options serve as a middle ground between European and American options. They can be exercised at specific pre-determined dates before expiration rather than anytime (American) or only at expiration (European). These options are common in structured financial products, such as callable bonds and some employee stock options.

Characteristics of Bermudan options

- Can be exercised on specific pre-determined dates before expiration.
- More flexible than European options but less than American options.
- Common in interest rate derivatives and callable bonds.

Example: A callable bond with a Bermudan-style option allows the issuer to redeem the bond only on certain dates, such as quarterly or annually, rather than at any time.

Table 1.1: Comparison of different exercise styles of options.

Exercise Style	When Can It Be Exercised?	Flexibility	Common Usage
European	Only at expiration	Low	Index options, currency options
American	Anytime before expiration	High	Stock options, commodity options
Bermudan	On specific dates before expiration	Medium	Interest rate derivatives, callable bonds

The choice between European, American, and Bermudan options depends on the flexibility required by the trader. European options are simpler and typically cheaper, American options provide full flexibility at a higher cost, and Bermudan options strike a balance between the two. Understanding these exercise styles is crucial for selecting the appropriate financial instrument for a given trading strategy.

The options mentioned above are used by investors and managers for strategic investment choices that provide flexibility in decision making under uncertainty.Unlike financial options, which involve tradable assets such as stocks or commodities, real options apply to capital investments, business projects, and strategic planning. Below are some of the examples and how they are used in detail.

- Option to Wait: The option to wait provides flexibility by allowing decision-makers to delay a commitment until more information becomes available. This is particularly valuable in uncertain environments where market conditions, regulatory changes, or technological advancements could impact the decision. In finance, this is often referred to as a *deferment option*.

  For example, a company considering entry into a new market may choose to delay expansion until economic conditions stabilize. Similarly, in the pharmaceutical industry, firms may hold off on mass-producing a drug until clinical trials confirm its effectiveness. This approach reduces the risk of premature investments and allows for better-informed strategic planning.

- Option to Expand: If an investment or project proves successful, the option to expand allows an organization to increase its commitment and scale operations. This is particularly useful in venture capital, research and development, and retail expansion strategies.

A practical example is a technology startup that launches a mobile app in a limited market. If user engagement and adoption rates exceed expectations, the company can expand to additional regions or invest in further development, such as adding premium features. Another example is a retail business that opens a small pop-up shop in a high-traffic area. If the store performs well, the owner may decide to open a permanent location, increasing the investment to capture more revenue.

- Option to Cut Back: The option to cut back allows a company or investor to reduce exposure if market conditions become unfavorable. This is a critical component of risk management, enabling organizations to scale down operations, limit losses, or reallocate resources efficiently.

  For instance, a manufacturer launching a new product may start with a high production volume based on expected demand. However, if early sales data indicates lower-than-anticipated interest, the company can reduce production to avoid excessive inventory costs. Similarly, an investor in the stock market may reduce holdings in a particular sector if economic indicators suggest a downturn, thereby minimizing potential losses.

- Option to Exit: When an investment or project does not perform as expected, the option to exit provides a structured way to withdraw while recovering as much value as possible. This is particularly relevant in private equity, mergers and acquisitions, and real estate investments.

  Consider an entrepreneur who starts a new business but finds that market competition is too strong and profitability is low. Instead of continuing to invest resources into a failing venture, the entrepreneur may choose to sell the business or liquidate assets to minimize financial damage. Similarly, a company investing in a new technology may discontinue the project if development costs exceed expected benefits. The ability to exit efficiently helps organizations avoid long-term losses and reallocate capital toward more promising opportunities.

- Option to Switch: The option to switch provides the flexibility to pivot or adapt to changing circumstances, ensuring that investments or business strategies remain aligned with market demands. This can involve switching between different product lines, suppliers, manufacturing processes, or even entire business models.

  A classic example is an automobile manufacturer that designs its production lines to accommodate both gasoline and electric vehicles. If consumer demand shifts toward electric vehicles, the company can reallocate resources to produce more electric cars while reducing gasoline vehicle output. Another example is a restaurant that adjusts its menu seasonally, offering hot beverages and comfort foods in the winter and lighter, refreshing options in the summer. This adaptability helps businesses maintain profitability despite changing market conditions.

- Growth Options: Growth options involve making small, strategic investments that could lead to significant opportunities in the future. This concept is commonly applied in venture capital, research and development, and strategic land acquisition.

  For example, a real estate investor may purchase undeveloped land in an emerging neighborhood with the expectation that property values will rise over time. If the area experiences economic growth, the investor can develop the land for commercial or residential use, significantly increasing its value. In the corporate world, companies often invest in early-stage research projects that, if successful, could lead to breakthrough products and lucrative revenue streams. These investments are typically made with the understanding that only a fraction of them will succeed, but those that do can generate substantial returns.

- Phased Investments: Phased investments involve committing resources to a project incrementally rather than all at once. This approach allows decision-makers to assess progress at each stage, reducing risk and improving capital efficiency.

  A common example is infrastructure development, where large-scale projects such as highways, airports, or power plants are constructed in phases. If the initial phases demonstrate success and demand justifies further expansion, additional funding is allocated. Similarly, in software development, companies often release products in beta versions to a limited audience before fully launching them to the public. This phased approach allows for testing, feedback, and adjustments, ensuring that the final product meets market needs.

The real power of real options is that they give you the flexibility to adapt based on what happens in the real world, where things can be uncertain or unpredictable. It's all about managing risk in a smart way while keeping your future options open.

1.3 Modern Developments in Risk Management and Real Options

In recent years, there has been growing interest in integrating risk management into real options analysis. This involves developing frameworks for identifying, quantifying, and managing risks in real options. For example:

- Dynamic Risk Management: Firms can use real options to dynamically adjust their strategies in response to changing market conditions. For instance, a company may expand production if demand increases or switch to a different product if market conditions change [52].
- Portfolio of Options: Firms can manage risks by maintaining a portfolio of real options, which allows them to diversify their risks and capture opportunities in different markets [53].
- Mathematical Risk Modeling (1800s – Present): Advancements in probability theory, statistical modeling, and actuarial science revolutionized risk quantification.
- Modern Portfolio Theory (1950s): The introduction of Modern Portfolio Theory (MPT) by Harry Markowitz provided mathematical frameworks for risk diversification.
- Derivatives and Options Trading (1970s – Present): The development of structured derivatives, including futures and options, allowed financial institutions to hedge risks with greater precision.
- Regulatory Frameworks and Compliance (2008 – Present): Following the 2008 global financial crisis, regulatory bodies such as the Basel Committee on Banking Supervision implemented stricter capital requirements to ensure financial stability.

Options are financial contracts that grant the holder the right, but not the obligation, to buy or sell an underlying asset at a specified price within a defined time frame. These instruments provide a versatile mechanism for hedging against adverse market movements. For instance, call options allow the purchase of an asset at a predetermined price, making them valuable when asset prices are expected to rise. Conversely, put options enable the sale of an asset at a set price, offering protection in declining markets [54].

Technological advances have significantly influenced the application of options in risk management. The integration of algorithmic trading, real-time data analysis, and advanced modeling tools has improved the precision and accessibility of option-based strategies. Platforms that offer automated hedging strategies and predictive analytics have made it easier for businesses to implement complex options frameworks with minimal manual intervention [23]. Additionally, the rise of blockchain technology is expected to increase transparency and efficiency in options trading, further revolutionizing the field.

1.3.1 Effective Risk Management Using Real Options

The concept of real options extends traditional financial options theory to investment decision-making in tangible assets. Unlike financial options, which are associated with securities such as stocks and bonds, real options provide businesses with the strategic flexibility to adapt investment decisions in response to market uncertainties [55].

Effective risk management using real options involves several key considerations, including valuation methodologies, decision timing, market volatility, and industry-specific applications. These factors influence how firms incorporate flexibility into capital budgeting, ensuring that managerial decisions are aligned with changing economic conditions and competitive landscapes.

In conventional investment analysis, projects are often evaluated using static models such as Net Present Value (NPV) and Discounted Cash Flow (DCF). However, these traditional methods assume that investment decisions are irreversible and fail to account for uncertainty and managerial flexibility [56].

Real options, by contrast, allow firms to respond dynamically to external market conditions. The ability to delay, expand, contract, or abandon projects enhances decision-making under uncertainty. Some common types of real options include:

Real options provide strategic flexibility in risk management by allowing firms to adapt to uncertain market conditions, mitigate downside risk, and capitalize on emerging opportunities. Unlike traditional financial options, real options are embedded within investment decisions and business strategies, enabling managers to make staged commitments rather than irreversible choices. The following strategic roles highlight how real options enhance risk management across various industries.

Despite significant advancements, several challenges persist in the field of risk management with options. Traditional models often struggle to capture extreme market conditions or sudden shifts in volatility, as evidenced by the 2008 financial crisis. Additionally, balancing the trade-offs between transaction costs and hedging effectiveness remains an ongoing concern. Emerging derivatives and complex financial products further complicate the landscape, requiring innovative approaches and continuous adaptation.

"While extensive research has been conducted on options-based hedging strategies, most studies focus on traditional pricing models such as the Black-Scholes and Binomial Option Pricing Model. However, there remains a gap in understanding the effectiveness of dynamic hedging strategies in highly volatile markets, particularly in the context of extreme financial crises. Additionally, the integration of artificial intelligence and machine learning into risk management for option hedging has not been thoroughly explored. Furthermore, existing literature largely overlooks the long-term performance and cost implications of options-based hedging for institutional investors. This study aims to address these gaps by examining advanced hedging techniques, incorporating emerging technologies, and evaluating their effectiveness in real-world market conditions."

1.4 RESEARCH OBJECTIVE AND SCOPE

This thesis investigates the use of financial options—including real options—as strategic instruments for effective risk management and hedging. Options provide a powerful framework for managing uncertainty and enhancing financial resilience. By integrating theoretical modeling with empirical analysis, this study aims to bridge the gap between academic research and practical application in the field of financial engineering.

The core objective of this research is to evaluate and compare different option pricing models and their performance in both valuation and hedging. The study utilizes four key methodologies: Monte Carlo Simulation, Black-Scholes Formula, Least Squares Monte Carlo (LSMC), and the Binomial Tree Method. European options are analyzed using Monte Carlo and Black-Scholes, while American options are priced using LSMC and Binomial Tree methods. Furthermore, the thesis extends the analysis to the valuation of real options using historical Brent Crude oil data, offering insights into the value of managerial flexibility in capital-intensive projects.

Empirical analysis is conducted using historical stock data from ENI S.p.A. (ENI.MI) and Brent Crude oil (BZ=F), spanning from January 2018 to early 2021, with contextual references up to 2023. Key findings confirm theoretical consistency in European option pricing, with Monte Carlo closely approximating Black-Scholes results. For American options, the LSMC and Binomial Tree models yield robust estimates, with American puts consistently showing higher values due to the early exercise feature. The conceptual valuation of a Brent-based real option highlights a positive value (EUR 1068.35), underlining the worth of strategic flexibility in uncertain environments.

Delta estimation across methods reveals notable variability, especially in simulation-based models, reflecting the dynamic nature of hedging in real-world conditions. These results provide actionable insights for businesses, financial practitioners, and policymakers by demonstrating how options—both financial and real—can be leveraged for enhanced risk management, financial stability, and strategic investment decisions.

The remainder of this thesis is organized as follows: Chapter 2 reviews the relevant literature on option pricing and real options. Chapter 3 outlines the methodologies used for model implementation and data analysis. Chapter 4 presents the empirical results and discussion. Finally, Chapter 5 concludes the study and suggests directions for future research.

2

Literature Review

2.1 Options-Based Risk Management Under Uncertainty

Options protection is a fundamental risk management strategy employed by investors, corporations, and financial institutions to mitigate exposure to price fluctuations in financial markets. The literature on options-based hedging has developed substantially, incorporating advanced pricing models, evolving regulatory frameworks, and empirical analyses of hedging effectiveness. A foundational model in this field is the Black-Scholes-Merton (BSM) model [37, 57], which provides a closed-form solution for pricing European-style options. It is based on a set of simplifying assumptions, including constant volatility, continuous trading, and the absence of arbitrage opportunities, offering an elegant mathematical framework for understanding option valuation.

While the BSM model remains influential, its assumptions often diverge from actual market behavior, which limits its real-world applicability. In particular, asset prices in financial markets frequently exhibit features such as fat tails and volatility clustering, which are not captured by the BSM framework [58]. These discrepancies can lead to mispricing and emphasize the need for models that more accurately reflect observed market dynamics. Nonetheless, the BSM model serves as a valuable starting point for understanding options and for developing more sophisticated pricing techniques.

The valuation of options and real options under uncertainty has evolved considerably, driven by the need to accurately capture the complex dynamics of asset prices and managerial flexibility. Among the foundational models are the Binomial Tree Model, the Merton Jump-Diffusion Model, the Monte Carlo simulation and the Heston Stochastic Volatility Model, each addressing different market realities and modeling challenges.

The Binomial Tree Model, introduced by Cox, Ross, and Rubinstein [36], offers a discrete-time and intuitive approach to option pricing. It represents the underlying asset price as a recombining tree with stepwise up and down movements governed by volatility and time increments. Its primary strength lies in its flexibility to handle American options, where early exercise is possible, through backward induction techniques. Unlike the BSM model, which is restricted to European-style options, the binomial approach can accommodate early exercise decisions and is therefore suitable for valuing American options. Moreover, the model is computationally efficient for short-term options and remains tractable as it converges to the BSM solution with increasing time steps. Due to its ease of implementation and ability to incorporate changing parameters, the binomial framework is also widely used in real options analysis, especially in applications involving staged investments and operational flexibility [59]. This makes it particularly relevant in energy finance, where investment decisions often unfold over multiple phases under uncertainty.

Several empirical studies have examined the effectiveness of the binomial tree model and other option pricing techniques in real financial markets. For example, empirical tests using the binomial tree model have been conducted to price European and American options in equity markets, particularly during periods of high volatility. The results often show that while the binomial model provides an accurate estimate of option prices, it requires further refinement when dealing with more complex financial instruments like exotic options or options with early exercise features. Moreover, hedge funds and investment banks have adopted hybrid models that integrate the binomial tree with Monte Carlo simulations or finite difference methods to handle more complex pricing problems. These combined approaches provide a more accurate and computationally feasible solution, especially in non-linear environments where simple binomial models might not suffice.

Financial institutions worldwide apply the binomial tree model to hedge options and manage risk, particularly in environments where there is a possibility for early exercise. In practice, this model is often used alongside more advanced techniques, such as the finite-difference method and the trinomial tree model, which improve the computational efficiency and accuracy of pricing in turbulent markets. The practical application of the binomial tree model in portfolio management also extends beyond options pricing. Banks and investment firms use binomial trees to calculate the Value at Risk (VaR) of portfolios containing complex derivatives. Furthermore, as market conditions evolve, these models are continuously updated with new information, allowing financial institutions to adjust their risk exposure dynamically.

In contrast, the Merton Jump-Diffusion Model [60] extends the classical Black-Scholes framework by incorporating sudden and significant changes in asset prices, which cannot be captured by continuous diffusion alone. By modeling price dynamics as a combination of standard Brownian motion and a Poisson jump process, this model captures the fat tails and skewness commonly observed in return distributions. The jump component enables the modeling of abrupt market movements triggered by unforeseen news or external shocks, which is particularly relevant in sectors such as energy commodities that are prone to price discontinuities. Merton's framework also yields semi-analytical option pricing formulas by conditioning on the number of jumps, thereby facilitating more accurate derivative valuation under jump risk. Importantly, this model allows for tractable risk-neutral valuation even in markets where discontinuities are the norm rather than the exception.

Complementing these approaches, the Heston Stochastic Volatility Model [61] introduces stochastic volatility by modeling the variance of returns as a mean-reverting square-root process correlated with asset prices. This correlation structure allows the model to capture empirically observed phenomena such as volatility clustering, leverage effects, and the volatility smile, which are inconsistent with constant volatility assumptions. The Heston model's characteristic function admits a semi-closed-form solution for European options, enabling efficient calibration and pricing. Its dynamic representation of volatility makes it particularly suitable for pricing options in energy markets, where uncertainty and regime shifts are prevalent. Moreover, extensions of the Heston framework have introduced multi-factor volatility drivers and jumps in both price and volatility, further enhancing its ability to reproduce complex empirical return distributions.

Together, these models represent a progression from discrete approximations to sophisticated continuous-time stochastic frameworks that incorporate both jumps and stochastic volatility. Their complementary strengths have led to widespread adoption in financial engineering and real options valuation, particularly in energy finance, where asset prices are frequently subject to shocks and complex volatility patterns. Recent research continues to build on these foundations by integrating machine learning and predictive analytics to refine model inputs, improve calibration, and enhance real-world applicability. These developments are particularly important for managing long-term risks associated with policy uncertainty, technological disruption, and volatile macroeconomic conditions.

More recent work has extended the binomial model to incorporate additional complexities, such as stochastic volatility and jump-diffusion processes, better reflecting empirical characteristics of asset returns. In parallel, recent studies have explored the integration of machine learning (ML) algorithms with traditional lattice models to enhance predictive accuracy and parameter calibration. These hybrid frameworks can dynamically adjust model parameters based on evolving market data, improving performance in complex, non-stationary environments where classical methods may struggle. Such approaches are especially useful in energy markets, where input data can be volatile and driven by multiple structural regimes. For instance, reinforcement learning has been used to optimize decision-making under uncertainty in multistage investment projects, enabling more robust adaptation to price signals and operational constraints.

Recent research continues to build on these foundations by integrating data-driven techniques with traditional financial theory. For example, machine learning models have been trained on historical price and volatility data to improve forecasts and estimate latent model parameters. Bayesian methods have also been employed to capture parameter uncertainty and update beliefs as new information becomes available. These developments are particularly important for managing long-term risks associated with policy uncertainty, technological disruption, and volatile macroeconomic conditions. The integration of stochastic models with predictive analytics opens new avenues for real-time risk management and dynamic portfolio optimization, further bridging the gap between theoretical modeling and practical application.

2.1.1 Black–Scholes–Merton Model

The **Black–Scholes–Merton (BSM) model** [37, 57] is the foundational framework for modern option pricing. It marked the first time a closed-form solution was derived for the value of European-style options under continuous-time stochastic dynamics. The BSM model assumes that the price of the underlying asset follows a geometric Brownian motion with constant drift and volatility, and that markets are frictionless, arbitrage-free, and allow continuous trading.

Under the BSM model, the asset price S_t evolves according to the stochastic differential equation:

$$dS_t = \mu S_t \, dt + \sigma S_t \, dW_t, \tag{2.1}$$

where μ is the drift term, σ is the asset's volatility, and W_t is a standard Brownian motion. Under risk-neutral valuation, the drift μ is replaced by the risk-free interest rate r, leading to the closed-form solution for the price of a European call option at time t:

$$C(S_t, t) = S_t\Phi(d_1) - Ke^{-r(T-t)}\Phi(d_2), \tag{2.2}$$

$$d_1 = \frac{\ln(S_t/K) + \left(r + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}, \quad d_2 = d_1 - \sigma\sqrt{T-t}, \tag{2.3}$$

Here, $C(S_t, t)$ denotes the price of a European call option, S_t is the price of the underlying asset, K is the strike price, T is the maturity date, r is the continuously compounded risk-free interest rate, and σ is the volatility of the underlying asset. The terms d_1 and d_2 capture the standardized moneyness and time to maturity, while $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution.

Despite its elegance, the BSM model relies on several strong assumptions that limit its applicability in real-world markets. Empirical evidence has consistently shown that asset return distributions are not normal and often exhibit skewness, excess kurtosis, and volatility clustering. Furthermore, the assumption of constant volatility is inconsistent with observed option pricing behavior, such as implied volatility smiles.These shortcomings are particularly evident in energy markets, where asset prices tend to exhibit jumps, regime shifts, and high volatility. For example, oil and gas prices are subject to sudden political, supply, or environmental shocks. The constant volatility and continuous diffusion assumptions of the BSM model are therefore inadequate for capturing the full risk dynamics in such settings. Nevertheless, the BSM model serves as a benchmark for evaluating more advanced models. It is also frequently used in practice for quick pricing approximations, hedging strategies, and as a building block for risk management systems. Many real options models used in energy economics adopt BSM as a base framework, extending it to accommodate uncertainty in investment timing and project cash flows.

In sum, while the BSM model has limited empirical accuracy, its analytical tractability, intuitive structure, and historical significance have made it the cornerstone of option pricing theory.

2.1.2 Binomial Tree Model

The Binomial Tree Model [36] provides a discrete-time framework for option pricing, allowing for stepwise adjustments in volatility and early exercise of American options. This model represents the underlying asset price as a recombining tree where, at each time step, the price can move up or down based on a given probability. The up and down factors, typically derived from the asset's volatility and the length of each time step, determine the possible price evolution. By iterating this process over multiple time steps, the model generates a tree that approximates the asset's price dynamics. Option values are then computed through backward induction, starting from the terminal payoff at expiration and working backward to the present using risk-neutral probabilities. This approach is particularly useful for pricing American options, as it allows for the evaluation of early exercise opportunities at each node. Furthermore, the binomial model serves as a foundation for more advanced numerical methods, such as the finite-difference approach and the trinomial tree model, which enhance accuracy and computational efficiency.The Binomial Tree Model [36] remains a cornerstone in the pricing of options, especially in the context of American options. This discrete-time framework enables the stepwise adjustment of volatility and provides the flexibility of early exercise, a key feature of American options. More recent works have extended the basic binomial model to incorporate additional complexities, such as stochastic volatility and jump-diffusion processes, which are more aligned with real market conditions. These adjustments are critical for improving the accuracy of option pricing in volatile markets.

2.1.3 Jump–Diffusion Models

Classical option pricing models such as Black–Scholes assume that asset prices follow continuous paths driven by Brownian motion. However, empirical evidence across equity, foreign exchange, and energy markets reveals that prices often exhibit sudden, discontinuous jumps due to unforeseen news, policy shocks, or structural market events. To capture these discontinuities, jump–diffusion models augment the standard diffusion process with a jump component, providing a more accurate representation of real market dynamics.

The most prominent framework in this category is the **Merton Jump–Diffusion Model** [60], which modifies the geometric Brownian motion by incorporating a Poisson jump process. In this model, the asset price S_t evolves according to:

$$dS_t = \mu S_t\, dt + \sigma S_t\, dW_t + (J_t - 1)S_{t^-}\, dN_t, \tag{2.4}$$

where μ is the drift rate, σ is the volatility of the continuous component, W_t is a standard Brownian motion, N_t is a Poisson process with intensity λ, and J_t is a random variable representing the jump magnitude, typically assumed to follow a lognormal distribution. The term S_{t^-} denotes the asset price immediately before a jump occurs.

Alternatively, the process can be written in compact form as:

$$dS_t = \mu S_t\, dt + \sigma S_t\, dW_t + J_t S_t\, dq_t, \tag{2.5}$$

where dq_t represents a Poisson jump with arrival intensity λ, and J_t scales the size of each jump.

This structure reflects the idea that asset prices evolve through a combination of a continuous diffusion component and discrete jumps, allowing the model to replicate non-Gaussian return distributions with fat tails and excess skewness. Such features are common in high-volatility environments like commodity and energy markets, where exogenous shocks can cause large and abrupt price movements.

To ensure no-arbitrage pricing under the risk-neutral measure, the drift is adjusted by subtracting λk, where $k = \mathbb{E}[J_t - 1]$ is the expected percentage jump size. Merton derived a semi-analytical solution for European option prices by conditioning on the number of jumps that occur during the option's life. The option price is expressed as an infinite sum of Black–Scholes prices, each adjusted for the number of jumps and weighted by the corresponding Poisson probability:

$$C^{\text{Merton}}(S_t, t) = \sum_{n=0}^{\infty} \frac{e^{-\lambda(T-t)}(\lambda(T-t))^n}{n!} C^{BSM}(S_t, t; \sigma_n, r_n), \tag{2.6}$$

where C^{BSM} is the standard Black–Scholes price with parameters adjusted to account for n jumps.

The Merton model offers significant advantages over pure diffusion models. It better explains the pricing of deep out-of-the-money options, improves the fit to implied volatility surfaces, and captures the observed kurtosis and skew in asset return distributions. These features are particularly valuable in energy derivatives markets, where geopolitical, regulatory, or environmental shocks can lead to large price jumps.

However, the model introduces additional parameters—jump intensity λ, average jump size, and jump volatility—which complicate calibration and require high-quality data or advanced estimation technique. In practice, the model is often extended to incorporate stochastic volatility Extensions of the Merton model include stochastic jump intensity (e.g., doubly stochastic Poisson processes), mixed jump distributions, and combinations with stochastic volatility (e.g., the Bates model) or time-varying jump intensities to improve its empirical performance. These hybrids are better suited for modeling real-world markets with both persistent volatility and abrupt shocks, as commonly seen in energy commodities.

Overall, the Merton jump–diffusion framework represents a critical advancement in option pricing theory.It strikes a balance between analytical tractability and empirical realism, making it a useful tool for pricing derivatives under conditions of discontinuous risk and for real options valuation in infrastructure and energy projects, where jumps may represent regulatory changes or environmental disruptions. Incorporating jump risk allows for more robust decision-making under extreme uncertainty.

2.1.4 Stochastic Volatility Models

Stochastic volatility (SV) models address the limitation of constant volatility in classical models by introducing time-varying variance as a separate stochastic process. These models are capable of capturing important empirical features such as volatility clustering, the leverage effect, and implied volatility smiles—all of which are common in equity and energy markets. This section reviews four major approaches to modeling stochastic volatility: the Heston model, the Hull–White model, the SABR model, and discrete-time GARCH models.

Heston Model

The **Heston Model** [61] is among the most prominent and widely applied stochastic volatility models in financial mathematics and quantitative finance. Unlike the classical Black–Scholes model, which assumes constant volatility, the Heston model explicitly incorporates stochastic dynamics for volatility, capturing the empirical observation that volatility tends to fluctuate over time rather than remain fixed. It models the variance of returns as a mean-reverting square-root process and allows for correlation between asset returns and volatility, enabling it to replicate stylized facts such as volatility clustering and the leverage effect. The Heston model has been widely used in pricing equity and energy derivatives. Empirical studies, such as Bakshi et al. [62], demonstrate that it outperforms the Black–Scholes model in capturing implied volatility smiles and skewness in S&P 500 options. However, the model underperforms in markets characterized by sudden jumps or discontinuities in asset prices, which has motivated the development of extensions such as the Bates model that incorporate both stochastic volatility and jump components.

Mathematically, the Heston model is defined by a system of coupled stochastic differential equations (SDEs). The first equation models the evolution of the asset price S_t:

$$dS_t = \mu S_t\,dt + \sqrt{v_t}S_t\,dW_t^S, \tag{2.7}$$

where μ is the drift term (expected return), v_t is the instantaneous variance at time t, and W_t^S is a standard Brownian motion.

The key innovation of the Heston model lies in the second SDE, which governs the variance process v_t:

$$dv_t = \kappa(\theta - v_t)\, dt + \sigma\sqrt{v_t}\, dW_t^v, \tag{2.8}$$

where $\kappa > 0$ is the rate of mean reversion, $\theta > 0$ is the long-run mean of volatility, $\sigma > 0$ is the volatility of volatility, and W_t^v is a Brownian motion correlated with W_t^S, with correlation coefficient $\rho \in [-1, 1]$. This correlation is essential for modeling the leverage effect, where negative asset returns are typically associated with increasing volatility.

A key advantage of the Heston model is its analytical tractability. It admits a closed-form expression for the characteristic function of the log asset price, which allows European option prices to be computed efficiently via Fourier inversion. Define the log-price process $x_t = \ln(S_t)$. Under the risk-neutral measure, the characteristic function is defined as:

$$\varphi(u; t, T) = \mathbb{E}^{\mathbb{Q}}\left[e^{iux_T} \mid \mathcal{F}_t\right], \tag{2.9}$$

and is given by:

$$\varphi(u; t, T) = \exp\left\{C(u, \tau) + D(u, \tau)v_t + iux_t\right\}, \tag{2.10}$$

where $\tau = T - t$ is the time to maturity. The functions $C(u, \tau)$ and $D(u, \tau)$ are defined as:

$$C(u, \tau) = riu\tau + \frac{\kappa\theta}{\sigma^2}\left[(b - d)\tau - 2\ln\left(\frac{1 - ge^{-d\tau}}{1 - g}\right)\right], \tag{2.11}$$

$$D(u, \tau) = \frac{b - d}{\sigma^2} \cdot \frac{1 - e^{-d\tau}}{1 - ge^{-d\tau}}, \tag{2.12}$$

with:

$$a = \kappa\theta, \quad b = \kappa - \rho\sigma iu, \tag{2.13}$$

$$d = \sqrt{(\rho\sigma iu - \kappa)^2 + \sigma^2(iu + u^2)}, \quad g = \frac{b - d}{b + d}. \tag{2.14}$$

Using this characteristic function, the price of a European call option with strike K and maturity T is computed via:

$$C(S_t, t) = S_t P_1 - Ke^{-r(T-t)}P_2, \tag{2.15}$$

where:

$$P_j = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \text{Re}\left[\frac{e^{-iu\ln K}\varphi(u - (j-1)i; t, T)}{iu\varphi(-i; t, T)}\right]\frac{du}{u}, \quad j = 1, 2. \tag{2.16}$$

These integrals are typically evaluated using numerical quadrature, such as Gauss–Laguerre integration. This semi-closed-form solution enables fast and accurate pricing and calibration of European options.

The Heston model's ability to account for time-varying volatility, negative return-volatility correlation, and excess kurtosis in return distributions makes it a powerful tool for modeling equity and energy derivatives. However, pricing of American-style and path-dependent options still requires numerical methods such as finite difference schemes or Monte Carlo simulation.

Hull–White Stochastic Volatility Model

The Hull–White model provides a simpler stochastic volatility framework where the variance follows a geometric Brownian motion without mean reversion:

$$dv_t = \eta v_t \, dW_t^v. \tag{2.17}$$

While this model reduces mathematical complexity, it does not enforce reversion of volatility to a long-term average. As a result, variance can drift to unrealistic levels over time. This limits the model's empirical accuracy but makes it analytically convenient in some theoretical settings.

SABR Model

The SABR (Stochastic Alpha Beta Rho) model is widely used in commodity and interest rate derivatives. It defines the price and volatility dynamics as:

$$dS_t = v_t S_t^{\beta} \, dW_t^S, \quad dv_t = \alpha v_t \, dW_t^v, \quad \text{corr}(W_t^S, W_t^v) = \rho, \tag{2.18}$$

where β controls the elasticity of volatility with respect to the asset price. The model captures skew and smile effects well but lacks closed-form solutions, relying instead on asymptotic expansions or numerical methods.

GARCH Models

GARCH models describe volatility dynamics in discrete time. The standard GARCH(1,1) specification is:

$$h_t = \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1}, \quad r_t = \mu + \sqrt{h_t} z_t, \tag{2.19}$$

where h_t is the conditional variance. GARCH models are widely used for volatility forecasting, Value-at-Risk, and empirical asset return modeling. Although not used directly for option pricing, they provide key inputs for continuous-time models.

Stochastic volatility models extend the option pricing literature by enabling more realistic treatment of volatility as a random process. The Heston model remains dominant due to its analytical tractability and empirical performance. The Hull–White and SABR models offer alternative dynamics with varying complexity and flexibility, while GARCH models provide a powerful empirical framework for forecasting volatility in discrete time. These models are particularly relevant for pricing and risk management in energy, equity, and commodity markets under uncertainty.

2.1.5 Comparison of Option Pricing Models

Table 2.1 and Table 2.2 summarize the key characteristics of the main option pricing models discussed in this chapter. Each model reflects a different set of assumptions about volatility behavior, price dynamics, and market completeness, resulting in trade-offs between analytical tractability and empirical realism. While early models such as Black–Scholes–Merton provided foundational insights into the valuation of European options, their simplifying assumptions have motivated the development of more advanced frameworks to handle discontinuities, time-varying volatility, and early exercise features.

Table 2.1: Comparison of Option Pricing Models (Part 1 of 2)

Model	**Volatility Structure**	**Discontinuities**
Black–Scholes–Merton	Constant volatility	No
Binomial Tree	Discrete, constant	No
Merton Jump–Diffusion	Constant volatility with jumps	Yes
Heston Model	Stochastic, mean-reverting	No
SABR Model	Stochastic, elasticity with respect to price	No
GARCH Models	Discrete-time stochastic volatility	No

The Black–Scholes–Merton model laid the foundation for modern option pricing theory but is constrained by its assumptions of constant volatility, continuous price paths, and frictionless markets. As these assumptions often diverge from real-world observations, a variety of alternative models have emerged to address such limitations. Binomial tree models offer intuitive, discrete-time approximations capable of handling early exercise, while stochastic volatility frameworks such as the Heston model capture time-varying volatility and the leverage effect. Meanwhile, jump-diffusion models like Merton's framework introduce discontinuities that better reflect abrupt market movements.

These models have significantly enhanced the theoretical and practical accuracy of derivative pricing. However, this increased realism comes at the cost of greater mathematical and computational complexity. As a result, there is ongoing research into hybrid models that combine stochastic volatility with jump components, and even extensions that incorporate regime-switching behavior to account for structural market changes.

In parallel with these theoretical developments, recent years have witnessed the growing integration of Artificial Intelligence (AI) and Machine Learning (ML) into financial modeling. These technologies offer powerful tools for learning nonlinear relationships and adapting to new data in real time. In particular, ML techniques are

Table 2.2: Comparison of Option Pricing Models (Part 2 of 2)

Model	Tractability	Typical Applications
Black–Scholes–Merton	Closed-form solution	Benchmark pricing, simple European options
Binomial Tree	Recursive numerical solution	American options, real options with early exercise
Merton Jump–Diffusion	Semi-analytical via Poisson mixture	Equity and energy options under sudden shocks
Heston Model	Semi-analytical (Fourier inversion)	Pricing volatility smiles and skews in equity/commodity options
SABR Model	Approximate (asymptotic or simulation)	Interest rate and commodity derivatives with skewed volatility surfaces
GARCH Models	Empirical estimation	Volatility forecasting, VaR, empirical return modeling

increasingly being applied to options pricing and risk management, where traditional parametric models struggle under high volatility or complex market regimes.

For instance, deep learning architectures have been used to approximate option pricing functions directly, bypassing the need for closed-form solutions. Reinforcement learning has shown promise in real-time option pricing, by simulating dynamic market environments and updating model parameters adaptively. ML models also facilitate improved calibration of traditional models such as Heston or SABR by learning volatility surfaces from large historical datasets.

Furthermore, the rise of blockchain technology has introduced new opportunities in the structuring and execution of financial derivatives. Smart contracts on decentralized platforms can automatically enforce option terms, reducing counterparty risk and enhancing transparency. In this context, blockchain offers not just operational improvements but also new forms of programmable financial instruments that may evolve into decentralized options markets.

The COVID-19 pandemic and the 2008 global financial crisis have both demonstrated the limitations of classical models under extreme conditions. These systemic events introduced rapid regime shifts, structural breaks, and coordinated monetary interventions, all of which challenged the assumptions underlying models like the binomial tree or even the stochastic volatility frameworks. In response, financial institutions have turned to more flexible and adaptive modeling paradigms. These include models that integrate short-term volatility forecasts with longer-horizon macroeconomic uncertainty, as well as those that incorporate alternative data sources such as news sentiment, social media signals, and real-time financial indicators.

In response, financial institutions are increasingly turning to hybrid models that integrate elements of stochastic volatility, jumps, and data-driven AI approaches. The adoption of alternative data sources—including news sentiment, macro indicators, and social media analytics—has further enhanced predictive capabilities. These data-rich frameworks enable financial actors to better anticipate market shifts and adjust hedging strategies accordingly.

Furthermore, the interconnectedness of global financial systems means that risk can propagate rapidly across borders. This underscores the need for holistic risk assessment frameworks that incorporate not only market volatility but also geopolitical and policy risks. As institutions adapt to this evolving landscape, robust and flexible pricing models will remain essential for maintaining financial stability and ensuring effective risk transfer mechanisms in derivative markets.

2.2 Hedging Strategies Using Options

Effective risk management is fundamental in options trading and portfolio management, and a variety of hedging techniques have been developed to mitigate different dimensions of market risk. Central to these techniques are delta hedging, gamma and vega hedging, and portfolio-level options strategies such as protective puts, collars, and tail-risk hedging.

2.2.1 Delta Hedging

Delta hedging is a foundational technique in options risk management, aimed at neutralizing an option's sensitivity to small changes in the underlying asset price. The strategy involves constructing a delta-neutral portfolio by dynamically adjusting the position in the underlying asset to offset the option's delta exposure. This approach was originally formalized in the Black–Scholes framework [37], which demonstrated that, under the assumptions of continuous trading, frictionless markets, and deterministic volatility, a perfectly hedged portfolio could be maintained by continuously rebalancing the asset position.

The delta (Δ) of a European call option under the Black–Scholes model is given by:

$$\Delta = \frac{\partial C}{\partial S} = \Phi(d_1), \tag{2.20}$$

where C is the option price, S is the current asset price, and $\Phi(d_1)$ is the cumulative distribution function of the standard normal distribution evaluated at d_1, defined as:

$$d_1 = \frac{\ln(S/K) + \left(r + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}. \tag{2.21}$$

In theoretical terms, continuous rebalancing eliminates directional risk from infinitesimal price movements, effectively replicating the option's payoff. However, in practice, continuous delta hedging is infeasible due to market frictions. As highlighted by Duffie [63], real-world constraints such as transaction costs, bid-ask spreads, slippage, market impact, and discrete trading intervals introduce significant deviations from idealized conditions. These frictions make continuous rebalancing not only operationally impractical but also economically inefficient.

Consequently, delta hedging is implemented through discrete rebalancing strategies. Traders typically adjust positions either at fixed time intervals or when the portfolio's delta exceeds a predefined threshold. The performance of such hedging strategies depends on multiple factors, including the volatility of the underlying asset, the convexity of the option (gamma), liquidity conditions, and the choice of rebalancing frequency. High-frequency rebalancing reduces tracking error but increases cumulative transaction costs. While these models provide a foundation, their practical utility depends on higher-order risk management, as explored next.

2.2.2 Gamma and Vega Hedging

To address the limitations of delta hedging—particularly its linear approximation of price sensitivity—market participants often employ **gamma** and **vega** hedging strategies to manage higher-order risk exposures [64, 65]. While delta hedging neutralizes first-order sensitivity to changes in the underlying asset price, it assumes small and continuous price movements. In volatile or discontinuous markets, this assumption is often violated, leading to hedge slippage and exposure to second-order effects.

Gamma hedging aims to control the convexity of the portfolio with respect to the underlying asset. Gamma (Γ) is the second derivative of the option price with respect to the underlying asset price:

$$\Gamma = \frac{\partial^2 C}{\partial S^2}. \tag{2.22}$$

High gamma implies that delta changes rapidly as the asset price moves. To stabilize delta exposure across a range of price movements, traders incorporate offsetting positions in other options to construct gamma-neutral portfolios. This is particularly important in portfolios with significant nonlinear payoffs, such as those containing options close to expiry or near-the-money. Gamma hedging improves delta stability but introduces its own set of challenges. It typically requires frequent rebalancing due to the sensitivity of gamma to both time decay and market movements. Consequently, gamma-neutral portfolios are often associated with elevated transaction costs and increased model risk.

Vega hedging focuses on mitigating sensitivity to changes in implied volatility, which is a dominant risk factor in option pricing. Vega is defined as:

$$\text{Vega} = \frac{\partial C}{\partial \sigma}, \tag{2.23}$$

where σ is the implied volatility. Vega quantifies how much an option's value changes in response to a 1% change in implied volatility. Since volatility expectations are driven by market sentiment, macroeconomic events, and regime shifts, they can vary abruptly and independently of the underlying asset price. Vega hedging is thus essential in environments where implied volatility is stochastic or exhibits clustering. Traders typically use options with high vega sensitivity—such as at-the-money, long-dated contracts—to construct vega-neutral portfolios. However, maintaining vega neutrality over time is complex, as vega itself is path-dependent and sensitive to the changing volatility surface.

In practice, advanced risk management frameworks combine delta, gamma, and vega hedging into multi-dimensional strategies. These approaches are widely employed in institutional trading, particularly by market-makers and derivatives desks managing large books of complex instruments. The objective is to neutralize directional risk (delta), curvature risk (gamma), and volatility risk (vega) simultaneously. However, such strategies involve significant operational and computational costs. They require dynamic position adjustments, real-time sensitivity monitoring, and robust execution to avoid slippage and unintended exposures.

2.2.3 Portfolio Hedging with Options

Beyond managing risks at the individual instrument level, portfolio-level hedging strategies using options have become a central component of institutional risk management, particularly following major market events such as the 1987 crash. Options-based portfolio hedging offers flexibility in shaping payoff structures to match investor risk tolerance and market outlook. Prominent strategies include **protective puts**, **collars**, and **tail-risk hedging**, each offering different trade-offs between cost, downside protection, and upside participation.

A **protective put** strategy involves purchasing put options on a portfolio or asset to cap potential losses while preserving upside exposure. This approach is widely adopted by long-only institutional investors, especially during periods of heightened volatility or macroeconomic uncertainty [66]. Protective puts function similarly to portfolio insurance, allowing investors to remain exposed to equity markets while limiting downside risk to the put strike price. The strategy is conceptually simple and offers asymmetric protection, but it can be costly in low-volatility environments due to persistent premium outflows.

To address the cost of protective puts, many investors implement **collar strategies**, which combine a long put position with a short call on the same underlying asset. The call premium partially or fully finances the cost of the put, resulting in a hedge that protects against downside risk while capping upside returns [54]. Collars are particularly attractive for long-term investors seeking stability over performance chasing. Academic and industry research has shown that collars can be an effective form of cost-managed risk mitigation during extended market downturns, particularly when implemented systematically.

Tail-risk hedging focuses on protecting portfolios from rare but extreme market events, such as financial crises, geopolitical shocks, or systemic liquidity failures. This typically involves buying deep out-of-the-money (OTM) put options that provide large payoffs during severe market declines [67]. These positions function as portfolio insurance against so-called "black swan" events. While these hedges can offer significant downside protection during crises, they incur negative carry—continuous premium costs that can erode portfolio returns during stable market periods. As a result, tail-risk hedging requires careful calibration and is often used in conjunction with macro forecasting models or volatility signals to optimize timing.

2.2.4 Empirical Studies on Hedging Effectiveness

Empirical research plays a crucial role in evaluating the real-world performance of options-based hedging strategies across asset classes. While theoretical models such as Black–Scholes and stochastic volatility frameworks offer guidance on constructing hedges, actual market conditions introduce frictions—such as transaction costs, discrete trading, and behavioral anomalies—that significantly affect hedging outcomes.

In equity markets, numerous empirical studies have assessed the performance of options strategies during periods of market stress. For instance, Bollen and Whaley [58] show that long volatility positions, such as straddles and strangles on the S&P 500, tend to outperform during periods of elevated uncertainty, when implied volatility spikes. These strategies exploit the well-documented inverse relationship between market indices and the VIX. Structured options strategies, including risk reversals and butterflies, are also used to manage both directional and volatility risk, with effectiveness depending on market liquidity and execution costs. Additionally, institutional investors employ delta-hedged portfolios to isolate and trade volatility as an asset class. However, empirical performance is highly sensitive to slippage, bid-ask spreads, and rebalancing frequency, especially during crises.

In commodity markets, hedging effectiveness is complicated by structural volatility, seasonality, and geopolitical shocks. Studies such as Tuckman [68] find that dynamic option hedging, where hedge ratios adjust in response to market conditions, significantly improves risk-adjusted returns. Energy producers frequently use put options and option spreads to hedge against adverse price movements in crude oil, natural gas, and electricity markets. More complex strategies, such as crack and crush spreads, help manage refining and processing margin risk. Additionally, exotic options and weather derivatives are employed to hedge seasonal demand and climate-related risks. However, liquidity limitations in some commodity markets, along with regulatory restrictions and model risk, reduce the efficiency of these strategies.

A growing body of literature compares futures and options as hedging instruments. Futures are often preferred in stable markets due to their lower costs and straightforward implementation. However, their linear payoffs make them less suitable for asymmetric or nonlinear risk exposures. In contrast, options provide greater flexibility in tailoring risk profiles, particularly in tail-risk management. Studies generally conclude that options outperform futures in turbulent markets, where downside protection becomes critical [69]. Still, options are more expensive to maintain due to premium costs, making their long-term use dependent on accurate timing or predictive models.

Tail-risk hedging with deep out-of-the-money puts has gained traction among hedge funds and institutional portfolios, especially in response to systemic events such as the 2008 financial crisis and the COVID-19 pandemic. Taleb [67] emphasizes the importance of such strategies for building robust portfolios that can survive rare but catastrophic market moves. Yet, empirical studies caution that the persistent cost of maintaining these hedges—known as negative carry—can erode returns during prolonged calm periods unless tactically managed.

Across these domains, a recurring empirical insight is the trade-off between hedging precision and implementation cost. Frequent rebalancing improves hedge accuracy but may degrade returns after accounting for transaction costs. Moreover, model-based hedging strategies often rely on assumptions—such as normality of returns or constant volatility—that break down in stressed markets. As a result, recent studies advocate for adaptive, data-driven approaches that incorporate real-time indicators and non-linear dynamics.

In summary, empirical research confirms that options-based hedging can be highly effective, but its success depends critically on timing, structure, and execution. This underscores the need for ongoing refinement in hedging models—especially in light of market innovations, volatility regimes, and the increasing role of computational tools in portfolio risk management. Table 2.3 summarizes key empirical findings from recent literature on options-based hedging strategies across different asset classes.

2.2.5 Recent Developments in Hedging Strategies and Risk Management

In recent years, advancements in **machine learning** and **dynamic hedging models** have significantly optimized hedging positions using real-time market data [70]. Algorithmic trading systems now rely on predictive analytics to dynamically adjust hedge positions, reducing the dependency on static rebalancing schedules that were once the norm in traditional strategies. These models leverage real-time market signals, such as historical volatility patterns, order flow imbalances, and high-frequency data, which not only enhance risk management but also allow traders to respond more swiftly to market movements.

Machine learning techniques, including reinforcement learning and deep neural networks, have been applied

Table 2.3: Summary of Empirical Findings on Options-Based Hedging

Study	Asset Class / Focus	Key Findings
Bollen and Whaley (2004) [58]	S&P 500 options	Long volatility strategies (e.g., straddles) outperform in volatile markets; implied volatility increases during downturns.
Tuckman and Serrat (2016) [68]	Commodities (energy, agriculture)	Dynamic hedging with options improves risk-adjusted returns; flexible hedge ratios outperform static strategies.
Gibson and Lhabitant (2013) [69]	Derivatives vs. futures	Futures are more cost-effective in stable markets; options provide better tail-risk protection and flexibility.
Taleb (2020) [67]	Tail-risk hedging	Deep OTM puts are essential for systemic risk protection, but impose a negative carry during stable periods.

to optimize delta, gamma, and vega hedging strategies. These advanced models analyze vast datasets, detecting complex relationships that are often invisible to traditional models. By identifying patterns that emerge from large volumes of data, these machine learning algorithms assist traders in minimizing hedging costs and reducing exposure to abrupt market fluctuations. Empirical studies suggest that AI-driven hedging approaches outperform traditional methods not only in terms of execution efficiency but also in terms of risk-adjusted returns, confirming the potential of machine learning to enhance portfolio management.

However, the integration of machine learning into hedging models is not without its challenges. The complexity of market microstructure, such as noise in the data, execution risks, and limitations in data quality, complicates the application of these models in real-time trading. Additionally, these models often rely heavily on historical data, which can be problematic when faced with unprecedented market events such as flash crashes or extreme volatility spikes. These models may struggle to adapt to such extraordinary conditions, making their application more difficult during times of market stress. Moreover, regulatory concerns regarding algorithmic trading, especially in relation to transparency and systemic risk, highlight the need for careful risk controls and monitoring. While machine learning offers a promising approach, the regulatory landscape surrounding algorithmic trading must be carefully navigated to mitigate the potential for systemic risks that may arise from the deployment of such advanced models.

In the wake of the financial crises, regulatory bodies have significantly increased oversight of financial institutions, resulting in the creation of comprehensive frameworks designed to mitigate systemic risk and ensure market stability. Key regulations such as Basel III, the Dodd-Frank Act, MiFID II, and EMIR have reshaped traditional risk management practices by enforcing stricter capital requirements, promoting centralized clearing of derivatives, and standardizing derivative transactions across global markets. These regulatory reforms have fundamentally changed how risk is managed, placing greater emphasis on reducing counterparty risk and ensuring financial institutions maintain adequate capital buffers to absorb potential losses during times of market stress.

The Basel III framework, introduced by the Basel Committee on Banking Supervision, represents a significant shift in banking regulation. Its primary objective is to strengthen the resilience of banks by mandating a higher quality and quantity of capital. One of the most critical aspects of Basel III is the requirement for banks to maintain a robust Capital Adequacy Ratio (CAR), which is calculated as:

$$\text{CAR} = \frac{\text{Tier 1 Capital} + \text{Tier 2 Capital}}{\text{Risk-Weighted Assets}}. \tag{2.24}$$

This ratio is designed to ensure that banks have enough capital to absorb potential losses, even in the face of economic downturns, thus reducing the risk of systemic failures. The enhanced capital buffers and rigorous stress testing procedures introduced by Basel III aim to improve the ability of banks to withstand financial shocks, ultimately contributing to a more resilient financial system. While these regulations strengthen individual institutions, they also foster a more prudent, risk-aware approach to asset management, encouraging banks to focus on long-term stability rather than short-term profit maximization.

Similarly, the Dodd-Frank Wall Street Reform and Consumer Protection Act, particularly in the United States, introduced sweeping changes to the financial regulatory landscape. A key component of this act is the mandatory centralized clearing of derivatives, a measure that has helped mitigate counterparty risk by ensuring that trades are routed through central counterparties (CCPs). This approach enhances market transparency by standardizing execution and reporting practices, thus reducing the potential for hidden risks in the financial system. However, the act also leads to increased margin requirements, which, while safeguarding against defaults, can elevate operational costs and reduce market liquidity. These margin requirements serve as a form of protection against systemic risks, but they also pose challenges in terms of liquidity management, particularly for institutions that rely on short-term funding.

In the context of these regulatory reforms, the evolution of global financial frameworks has demonstrated a clear commitment to safeguarding financial stability and mitigating systemic risks. While measures like higher capital requirements, stress testing, and centralized clearing introduce operational challenges, they are ultimately designed to enhance market resilience. As the global financial landscape continues to evolve, it will be essential for financial institutions to integrate these regulatory demands into their risk management strategies, ensuring continued market stability and confidence.

While significant progress has been made in the development of theoretical pricing models and empirical strategies for options-based hedging, several critical research gaps remain. These gaps need to be addressed to improve our understanding of options as effective hedging instruments and to enhance the practical integration of real options theory with modern risk management frameworks in financial markets. One of the key gaps lies in the limited integration of risk management principles within real options models. While the majority of research concerning real options concentrates on valuation methods, there is limited examination of the integration of risk management strategies within these frameworks. Conventional real options models typically consider volatility as an external factor; however, in practical scenarios, firms manage risk through various means, including hedging and strategic operational adjustments. The lack of comprehensive, risk-adjusted valuation models limits the effectiveness of real options in dynamic, uncertain environments. Future research should prioritize the integration of hedging techniques, insurance mechanisms, and scenario-based risk assessments into real options frameworks to create a more holistic approach.

Another significant gap is the absence of a universal, comprehensive risk management framework for real options. Existing frameworks tend to focus on specific types of risks, such as market risk or credit risk, or they are tailored to particular industries, such as energy or mining. This limits their applicability across different sectors. While some studies have examined the impact of uncertainty on investment decisions, a framework that can address the broad range of risks associated with real options, including regulatory, technological, and geopolitical risks, is still lacking. A comprehensive risk management framework for real options should be able to account for multiple sources of risk and provide a structured approach for integrating these factors into decision-making.

Implementing real options theory also presents practical challenges due to its complexity. Many managers and decision-makers lack familiarity with real options concepts and the technical expertise needed to apply them effectively. Unlike traditional valuation methods, real options analysis requires advanced mathematical models and probabilistic thinking, which are often not included in standard financial training. Moreover, real options analysis relies heavily on accurate input parameters, such as volatility estimates, risk-adjusted discount rates, and cash flow projections. Obtaining reliable data for these parameters can be especially difficult in emerging markets or rapidly evolving industries. This lack of reliable data often leads to skepticism about the practical usability of real options models, which results in their underutilization in corporate decision-making.

Additionally, there is a noticeable lack of empirical research validating real options models in real-world scenarios. Most studies rely on simulations or hypothetical case studies rather than analyzing actual investment decisions made by firms. Empirical research examining how companies implement real options strategies, manage associated risks, and measure financial outcomes would significantly contribute to bridging the gap between theoretical models and practical application. Case studies focusing on industries such as pharmaceuticals, where R&D investments resemble sequential options, or technology and infrastructure sectors, where market entry and exit decisions can be modeled as options, could provide valuable insights into the effectiveness of real options in managing risk. Future research should focus on gathering empirical evidence through corporate investment data, industry surveys, and interviews with decision-makers to assess the real-world applicability of real options models.

Addressing these gaps is crucial for advancing the field of real options and enhancing its integration with modern risk management strategies. Future research should focus on developing risk-adjusted real options models, creating comprehensive risk management frameworks, simplifying implementation techniques for practitioners, and conducting empirical studies to validate theoretical models. By addressing these gaps, real options analysis can become a more effective and widely adopted tool for managing uncertainty in investment decisions.

While much of the existing literature has focused on standard, vanilla options, which feature relatively simple payoff structures, exotic options such as barrier options, Asian options, and other path-dependent derivatives present unique risk-return profiles that could potentially offer improved hedging performance in specific market conditions. The complexity of exotic options allows for more dynamic responses to market changes, making them more suitable for times of high volatility or market stress. Research into the application of exotic options in hedging strategies could open new opportunities for more finely tuned risk management, particularly during periods of significant market turmoil.

Exotic options, with their nonlinear payoffs, offer superior risk mitigation in scenarios where the underlying assets experience extreme volatility or where asset correlations deviate from historical norms. These options are also valuable in situations where specific market events, such as extreme currency devaluation or commodity price spikes, are anticipated. Barrier options, binary options, and Asian or lookback options can all be tailored to provide better protection in such cases, offering more precise risk coverage. A deeper exploration of exotic options could help uncover strategies that are more finely aligned with the risk profiles of different portfolios.

Macroeconomic shocks—such as unexpected changes in monetary policy, geopolitical instability, or economic recessions—have profound implications for market volatility and asset correlations. These shocks, however, have not been sufficiently integrated into dynamic hedging models. Future studies should explore how macroeconomic indicators can be incorporated into hedging models to assess their predictive power on options pricing and hedge performance. Research is also needed to understand the robustness of hedging strategies during times of systemic risk triggered by macroeconomic disturbances, which would provide valuable insights into how external economic factors can be incorporated into risk management frameworks.

The assumptions of idealized market conditions in many existing models neglect practical factors such as bid-ask spreads, liquidity constraints, and transaction costs. These elements, which are often overlooked in theoretical models, can significantly affect the performance of hedging strategies in real-world conditions. Future research should focus on incorporating market microstructure dynamics into options pricing and hedging models, which would allow for a more accurate representation of how execution costs and liquidity limitations affect the overall effectiveness of hedging strategies.

2.2.6 Sector-Specific Applications of Hedging

Different industries are exposed to distinct financial risks shaped by their operational environments, market structures, and external dependencies. As a result, the design and implementation of hedging strategies must be tailored to sector-specific risk profiles. The application of derivatives—such as options, futures, swaps, and weather contracts—varies significantly across sectors, depending on the volatility sources and strategic needs of each industry.

In the energy sector, firms are highly sensitive to fluctuations in commodity prices, particularly crude oil, natural gas, and electricity. These price movements are influenced by geopolitical tensions, regulatory changes, and global demand-supply imbalances. Energy producers frequently use put options and swaps to secure minimum sale prices and hedge against declining revenues, while fuel-dependent industries such as airlines and logistics companies utilize call options to cap rising input costs [71]. Weather derivatives have also become common in electricity markets, providing protection against temperature-driven demand volatility.

Agricultural producers and agribusinesses face risks associated with climate conditions, pest infestations, and commodity price swings. Farmers may hedge crop prices by purchasing put options on futures contracts, locking in a minimum sale price ahead of harvest [72]. On the demand side, food manufacturers use call options to manage rising costs of essential inputs such as wheat and corn. Weather-linked derivatives based on rainfall or temperature indices are increasingly used to protect against yield loss due to adverse weather patterns.

In the financial sector, banks and insurance companies manage exposure to interest rate risk, credit risk, and systemic shocks. Interest rate swaps are used to hedge mismatches between fixed- and floating-rate assets and liabilities, while credit default swaps (CDS) provide protection against borrower defaults [73]. Insurance firms exposed to catastrophic risk employ financial instruments such as catastrophe bonds to transfer extreme weather-related liabilities to capital markets, improving solvency resilience.

These sectoral applications demonstrate that while the instruments used may be similar, their implementation is driven by the distinct economic and operational contexts of each industry. Effective hedging requires domain-specific expertise and strategic alignment between risk exposures and financial instruments.

2.2.7 Behavioral Dimensions of Risk Management

Traditional financial theory assumes rational decision-making by investors and firms. However, behavioral finance highlights the influence of cognitive biases and psychological factors on risk perception, hedging behavior, and portfolio decisions. These behavioral elements often lead to deviations from optimal hedging strategies, especially during periods of market stress.

Loss aversion, introduced by Kahneman and Tversky (1979), refers to the tendency of individuals to place greater weight on avoiding losses than acquiring equivalent gains [74]. This can result in delayed or insufficient hedging, as investors may avoid purchasing protective options due to their upfront cost, despite the long-term benefits of risk reduction. Similarly, corporate risk managers may forgo derivative protection in an attempt to avoid short-term accounting losses, increasing exposure to downside risk.

Overconfidence bias is another factor that impairs hedging effectiveness. Investors or executives may overestimate their ability to predict market movements and undervalue the need for risk protection [75]. For example, a commodities trader who is confident in market timing may choose not to hedge, thereby increasing vulnerability to unexpected price swings. Overconfidence can also lead to under-hedging in currency, interest rate, and input cost exposures.

Market sentiment and collective behavior further distort rational risk management. During bull markets, optimistic sentiment often leads to reduced demand for hedging, while in downturns, panic increases demand for protection, driving up option premiums [76]. This cyclical behavior contributes to volatility skews in options markets, reflecting asymmetrical risk perception. Emotional responses to market dynamics can cause delayed, expensive, or overcrowded hedging strategies that reduce effectiveness.

Regret aversion also plays a critical role. Investors may avoid hedging due to fear of making the "wrong" decision in hindsight, particularly if the hedge turns out to be unnecessary. This hesitation often results in reactive rather than proactive hedging, increasing exposure to tail risk [77].

Herding behavior compounds these issues by amplifying systemic risks. When large groups of investors adopt similar hedging strategies—such as a widespread shift into protective puts during a crisis—option prices become inflated and less effective as hedging instruments. This herding effect, driven by fear or imitation, can lead to overcrowded positions, volatility spikes, and liquidity issues in options markets.

Incorporating behavioral insights into hedging frameworks is essential for improving decision-making. Rule-based or algorithmic hedging strategies can reduce emotional bias by enforcing disciplined rebalancing protocols. Additionally, integrating sentiment indicators and behavioral parameters into pricing and risk models helps align strategies with actual investor behavior. Investor education on cognitive biases further strengthens the ability of financial institutions to design effective and adaptive risk management systems.

2.2.8 Regulatory Impact on Hedging Strategies

Regulatory frameworks significantly influence how hedging strategies are structured, executed, and reported. Reforms such as the Dodd-Frank Act (2010) in the United States, the European Market Infrastructure Regulation (EMIR), and Basel III have imposed stricter reporting standards, centralized clearing mandates, and higher capital requirements on derivative transactions. These regulations aim to reduce counterparty risk and enhance market transparency, particularly in over-the-counter (OTC) markets.

While increased oversight improves price discovery and reduces information asymmetry, it also introduces operational challenges and higher compliance costs [78]. Margin requirements and collateral obligations can limit flexibility in hedging and reduce liquidity, particularly for smaller market participants or firms with complex exposures.

In sectors such as energy and banking, regulatory shifts can alter the composition and frequency of hedging. For instance, mandatory clearing requirements have reduced the use of bespoke derivatives, pushing firms toward standardized contracts that may not align perfectly with their risk profiles. As noted by Geman [79], these constraints have led to innovation in risk transfer instruments and spurred the development of structured products that meet both compliance and hedging objectives.

Adapting to evolving regulatory landscapes requires firms to integrate compliance considerations directly into hedging models. This includes accounting for capital treatment under Basel III, eligible collateral types, and the cost of central clearing. In addition, firms must monitor jurisdictional differences, especially in multinational operations, to ensure alignment with regional regulations.

Understanding and anticipating regulatory impacts is essential for designing hedging strategies that are not only effective but also compliant and sustainable over the long term.

2.2.9 Hedging in Emerging Markets

Hedging practices in emerging markets face unique challenges due to high macroeconomic volatility, limited financial infrastructure, and restricted access to complex derivatives. Currency risk is particularly acute, with exchange rate volatility driven by inflationary pressures, capital flow fluctuations, and political instability. As noted by Ahmed (2015), robust currency hedging mechanisms are essential for firms operating in emerging economies.

Market access constraints limit the availability of standardized options or swaps, often forcing firms to rely on less liquid or bespoke instruments. Hedging costs are typically higher due to wider bid-ask spreads and lower trading volumes. Regulatory frameworks may also be underdeveloped or inconsistent, further complicating risk management practices.

Despite these challenges, emerging markets offer opportunities for diversification and growth, increasing the importance of risk management. Firms that succeed in implementing tailored hedging solutions can gain a competitive advantage by stabilizing cash flows and reducing earnings volatility. The development of local derivatives markets, improvements in regulatory structures, and the growth of digital finance platforms are gradually improving hedging capabilities in these regions.

Research and policy efforts aimed at enhancing financial literacy, expanding derivatives infrastructure, and fostering cross-border risk transfer mechanisms are essential to support hedging in emerging economies.

2.2.10 Hedging in Dynamic Market Environments

Modern financial markets are characterized by technological disruption, geopolitical instability, and rapid capital flows. These dynamics challenge static hedging models and require adaptive frameworks capable of responding to fast-changing conditions. The 2008 global financial crisis and the COVID-19 pandemic underscored the limitations of traditional models and highlighted the importance of stress testing, scenario analysis, and real-time data integration [80].

Stochastic volatility models, such as the Heston model [61], have been developed to better capture volatility clustering and asymmetric market responses. These models improve the accuracy of hedging strategies, particularly for instruments with embedded optionality or path dependence.

To remain effective, modern hedging strategies increasingly rely on quantitative analytics, machine learning, and automated execution. These tools enable more accurate risk estimation, faster decision-making, and reduced behavioral bias. The integration of real-time market data and algorithmic risk controls enhances the ability of firms to protect against tail events while minimizing costs under normal conditions.

2.3 Real Options in Financial Decision-Making and Risk Management

Real options have emerged as a crucial aspect of financial decision-making, evolving alongside financial markets and economic systems. Unlike traditional financial options that are tied to securities, real options provide firms with strategic flexibility in capital investments, allowing them to mitigate uncertainties and optimize investment outcomes. Real options theory extends financial options pricing models to capital budgeting decisions, offering a framework for firms to evaluate potential investments under uncertain market conditions. This review of the literature explores existing research on real options as an approach to managing investment risks. It synthesizes key theoretical models, empirical studies, and critical advancements in the field, emphasizing their role in financial decision-making and risk management.

Traditional risk management frameworks, such as Value at Risk (VaR), Modern Portfolio Theory (MPT), and the Capital Asset Pricing Model (CAPM), rely on statistical and probabilistic methods to assess and mitigate risks. These models aim to quantify uncertainty and provide decision-makers with tools to manage potential financial losses. For instance, VaR estimates the potential loss of an investment portfolio within a given confidence interval over a specified period, which is widely used in financial institutions for risk assessment and capital allocation [81]. While VaR can be calculated using historical simulation, variance-covariance methods, or Monte Carlo simulation, it has limitations, including its inability to capture tail risks and its reliance on historical data, which may not always reflect future market conditions. Additionally, VaR assumes a normal distribution of returns, which may not hold during extreme market events, leading to the underestimation of risks.

Modern Portfolio Theory (MPT), introduced by Markowitz [50], suggests that investors can optimize risk-adjusted returns by diversifying their portfolios. This theory relies on mean-variance optimization, where investors select portfolios that minimize risk for a given level of expected return. By considering the correlation between assets, MPT enables the construction of efficient portfolios. However, it assumes that asset returns are normally distributed and that past correlations persist in the future, which may not hold in volatile or crisis-prone markets. Furthermore, MPT does not explicitly account for dynamic investment strategies or changing market conditions over time.

The Capital Asset Pricing Model (CAPM), developed by Sharpe [51], provides a method for determining the expected return of an asset based on its systematic risk, measured by beta. The model establishes a linear relationship between risk and return, suggesting that investors should only be compensated for non-diversifiable market risk. CAPM is widely used in asset pricing, cost of capital calculations, and investment performance evaluation. However, the model assumes market efficiency, a single-period investment horizon, and a risk-free rate that remains constant over time. Empirical studies have shown that CAPM's assumptions may not always hold, as observed asset returns often deviate from its predictions due to factors such as investor behavior and market anomalies [49].

Financial derivatives, such as futures, options, and swaps, are commonly used to hedge risks in financial markets [23]. These derivatives allow firms and investors to manage exposure to price fluctuations in commodities, currencies, interest rates, and equities. For example, options provide the right (but not the obligation) to buy or sell an asset at a predetermined price, offering flexibility in risk management. Swaps enable counterparties to exchange cash flows based on different interest rates or currencies, reducing exposure to fluctuations in financial variables. While derivatives offer effective risk mitigation, they introduce complexities such as counterparty risk, liquidity concerns, and valuation challenges. Misuse of derivatives can also lead to significant financial losses, as demonstrated by various high-profile financial crises.

While these traditional models provide essential tools for managing market risks, they assume static decision-making and do not account for the strategic flexibility required in uncertain investment environments. Specifically, these models focus on optimizing risk-return trade-offs under fixed conditions, whereas real-world investment decisions often involve evolving uncertainties and opportunities for adaptive decision-making. Moreover, traditional approaches primarily emphasize risk minimization and efficient allocation of capital within a predefined investment framework. However, they do not incorporate the value of managerial flexibility in responding to changing market conditions. In capital-intensive industries, investment decisions often involve the ability to defer, expand, contract, or abandon projects in response to new information. These dynamic considerations necessitate an approach beyond traditional risk management frameworks.

As a result, traditional risk management techniques may be insufficient for capital investment decisions, where uncertainty plays a central role and strategic flexibility is essential. To address these limitations, real options theory extends traditional financial models by incorporating the value of flexibility and decision-making under uncertainty. Real options allow for the dynamic adaptation of investment strategies in response to changing market conditions, offering a more flexible and robust framework for managing investment risks.

Real options theory, as pioneered by Trigeorgis [55] and Dixit & Pindyck [56], extends financial options concepts to capital investments. Unlike static net present value (NPV) models, real options allow decision-makers to adapt investment strategies dynamically in response to market conditions. Traditional capital budgeting techniques, such as discounted cash flow (DCF) and NPV, assume that investment decisions are made at a single point in time, with no flexibility to modify the decision as uncertainties unfold. These models often fail to incorporate the value of managerial flexibility in uncertain environments. Real options theory addresses this limitation by recognizing that investments can be staged, deferred, expanded, contracted, or even abandoned in response to new information, thereby mitigating downside risks and capitalizing on emerging opportunities.

For instance, the *option to delay*, or *timing option*, allows firms to postpone investment decisions until more information becomes available or market conditions improve. This option is particularly valuable in volatile environments where immediate commitments may expose firms to unnecessary risks. In the case of a pharmaceutical company developing a new drug, the firm may choose to delay clinical trials until regulatory pathways become clearer, thus reducing potential compliance risks. Similarly, a real estate developer may postpone a large construction project until economic indicators suggest strong demand, mitigating the risk of oversupply. By deferring investments strategically, firms can avoid premature capital deployment and optimize resource allocation.

On the other hand, the *option to expand* provides businesses with the flexibility to scale up operations if an initial investment proves successful. This option mitigates the risk of dedicating significant resources upfront while enabling firms to capitalize on favorable market conditions. For example, a technology firm launching a new software product in a test market can retain the option to expand into larger regions if initial user adoption proves strong. Similarly, an energy company that drills exploratory wells can increase production if oil reserves are confirmed to be commercially viable. This flexibility enables firms to pursue growth opportunities while minimizing exposure to downside risks.

In contrast, the *option to contract* offers firms the ability to scale down operations in response to adverse market conditions, optimizing costs, managing capacity, and reducing exposure to declining demand. For instance, a manufacturing company experiencing reduced product demand may shut down certain production lines, reallocating resources to more profitable segments. Similarly, in the airline industry, carriers often reduce the frequency of flights or switch to smaller aircraft in response to seasonal fluctuations or economic downturns. The ability to contract operations ensures that firms remain agile and financially resilient in uncertain environments.

The *option to abandon* allows firms to exit an unprofitable investment, thus limiting further financial losses. This option is particularly valuable in capital-intensive industries where poor market conditions or regulatory changes can render a project unviable. For example, a mining company may abandon an extraction site if test results indicate insufficient mineral deposits, preventing unnecessary operating expenses. Similarly, a retail chain may shut down underperforming stores to focus on more profitable locations. This strategic exit mechanism enhances risk management by ensuring that firms do not remain locked into unfavorable investments.

Finally, *switching options* provide firms with the flexibility to alter operational inputs, outputs, or production methods in response to changing market conditions. This option is crucial for industries with high supply chain risks or fluctuating input costs. For example, a power generation company with dual-fuel plants can switch between natural gas and coal based on price fluctuations, ensuring cost-effective energy production. Similarly, an automobile manufacturer may reconfigure production lines to accommodate electric vehicles in response to evolving consumer preferences and regulatory changes. By embedding switching flexibility into business operations, firms can enhance resilience and adapt to market shifts more effectively.

Real options play a crucial role in strategic risk management by providing firms with the flexibility to delay, expand, contract, abandon, or switch investments based on evolving market conditions. By integrating real options into decision-making frameworks, businesses can mitigate downside risk while retaining the ability to capture upside potential. Future research should focus on refining quantitative models for real options valuation and expanding empirical studies to validate their effectiveness in different industries.

The valuation of real options builds upon traditional financial option pricing models, such as the Black-Scholes model [37] and binomial trees [36]. Unlike standard NPV calculations, which use a single discount rate for all future cash flows, real options valuation incorporates stochastic processes that account for uncertainty and managerial flexibility. Real options are typically valued using binomial lattices, a discrete-time approach that maps possible investment outcomes over multiple periods, allowing decision-makers to visualize different investment pathways and make optimal choices at each stage. Additionally, Monte Carlo simulation, a numerical approach that simulates multiple scenarios to estimate the probability-weighted value of an investment under uncertainty, is widely used for more complex projects with multiple interacting uncertainties. Least Squares Monte Carlo (LSM) is another method that refines Monte Carlo simulations by incorporating regression techniques to estimate optimal exercise strategies for real options.

Applications of real options theory have been widely seen in various industries where uncertainty and flexibility are key factors. In infrastructure and real estate, developers use real options to assess the feasibility of large-scale projects under uncertain demand and regulatory conditions. The ability to phase construction, delay projects, or switch building functionalities enhances financial viability. In technology and innovation, firms invest in R&D and strategic alliances to secure future competitive advantages, treating innovation as a portfolio of real options. In mergers and acquisitions (M&A), firms often use real options analysis to determine whether to enter new markets or expand operations in stages. Acquiring minority stakes in a company before a full acquisition can be viewed as an option to expand if the initial investment proves favorable.

Real options provide a more comprehensive framework for managing investment risk compared to traditional capital budgeting techniques. By incorporating uncertainty and managerial flexibility, real options theory enables firms to make informed strategic decisions in volatile markets. Unlike static DCF models that undervalue flexibility, real options explicitly account for the ability to revise investment strategies dynamically. Furthermore, real options align with behavioral finance insights, recognizing that decision-makers do not always follow strict rationality but instead adjust their strategies based on evolving conditions. This adaptability makes real options particularly valuable in industries characterized by rapid technological change, regulatory shifts, and market volatility.

Despite its advantages, real options analysis also faces challenges in implementation. The estimation of volatility, project-specific risk factors, and the determination of appropriate discount rates require sophisticated modeling techniques. Additionally, firms may struggle with organizational constraints and biases that prevent them from fully utilizing real options thinking in strategic planning.

The next section explores empirical studies on real options and their implications for investment decision-making, highlighting real-world cases where firms have successfully implemented this framework.

2.3.1 Valuation Methods and Risk Quantification

Several methods have been developed to value real options, offering different approaches to accommodate various levels of complexity in modeling uncertainty. Among the most widely used valuation techniques are the Black-Scholes model, binomial trees, and Monte Carlo simulations.

The Black-Scholes model, originally developed for valuing financial options, has been adapted to value real options with continuous time and uncertainty [37]. It is particularly effective for options where the underlying asset's price follows a lognormal distribution, and the volatility remains constant over time. However, it assumes that the underlying asset's price path is smooth and continuous, which may not always reflect the reality of certain investment projects.

Binomail Tree model

Binomial trees offer a more flexible, discrete-time model for valuing real options. By breaking down the investment decision process into multiple stages, binomial trees allow for more nuanced adjustments to the option's value at each node. This method is particularly useful when there are multiple decision points, such as in projects with several stages of investment or uncertainty. The binomial approach enables decision-makers to model complex options with more precision, offering a stepwise approach to dynamic decision-making [36]. The real option value at each node is computed recursively as follows:

$$V = \frac{pV_u + (1-p)V_d}{1+r} \tag{2.25}$$

where:

- V_u and V_d represent the option values in the up and down states.
- p is the risk-neutral probability of an upward movement.
- r is the discount rate.

Monte Carlo Simulation

Monte Carlo simulations represent another powerful tool for real options valuation, particularly for complex options involving multiple uncertain variables. By simulating thousands of possible future scenarios based on stochastic processes, Monte Carlo simulations estimate the probability distribution of outcomes, providing a comprehensive risk assessment for investment decisions [59]. The real option value is computed as follows:

$$C = e^{-rT}\frac{1}{N}\sum_{i=1}^{N}\max(S_i - X, 0) \tag{2.26}$$

where:

- N is the number of simulation iterations.
- S_i represents the simulated asset price at iteration i.
- X is the option exercise price.

Risk plays a central role in real options analysis. The value of a real option typically increases with the level of uncertainty because greater uncertainty opens up more opportunities for flexibility in decision-making. This flexibility allows firms to respond to market developments, minimizing downside risks and capturing upside potential. However, the presence of uncertainty also complicates decision-making, as managers must consider a wide range of potential outcomes, each with different probabilities and impacts on the overall investment strategy.

To quantify risk, various methods are employed, such as sensitivity analysis, scenario analysis, and Monte Carlo simulations. Sensitivity analysis examines how changes in key variables, such as price or demand, affect the option's value [59]. This method helps identify the most influential factors and guide decision-making by highlighting which variables hold the most weight in determining the value of a real option. Scenario analysis, on the other hand, evaluates the impact of different predefined scenarios, such as best-case, worst-case, and base-case outcomes, providing a broader perspective on the range of possible results [53]. Finally, Monte Carlo simulations, as mentioned, simulate a large number of potential future scenarios to estimate the probability distribution of outcomes, which helps decision-makers understand the full spectrum of risk involved in an investment.

Least Squares Monte Carlo (LSM)

The Least Squares Monte Carlo (LSM) method, introduced by Longstaff and Schwartz [82], extends traditional Monte Carlo simulation by incorporating cross-sectional regression to estimate the continuation value of American-style of real options. It is especially useful in valuing embedded flexibility in investment projects, such as deferral, expansion, or abandonment, where decisions are contingent on the evolution of uncertain underlying variables.

LSM begins by simulating a large number of stochastic paths for the state variable X_t, typically representing the project value or underlying asset. At each exercise opportunity t, the method estimates the conditional expected continuation value using a least squares regression of the realized discounted cash flows on a set of basis functions $\varphi_j(X_t)$. Formally, the continuation value C_t is approximated as:

$$\mathbb{E}[V_{t+1} \mid X_t] \approx \sum_{j=1}^{k} \beta_j \varphi_j(X_t) \tag{2.27}$$

where:

- V_{t+1} is the realized discounted payoff from continuation at the next time step,
- X_t is the current value of the underlying state variable,
- $\varphi_j(X_t)$ are the chosen basis functions (e.g., polynomials such as $1, X_t, X_t^2$),
- β_j are the regression coefficients estimated by ordinary least squares (OLS).

This estimated continuation value is then compared to the immediate exercise payoff at time t. If the immediate payoff exceeds the estimated continuation value, the option is exercised; otherwise, it is held. The optimal stopping strategy is thus determined backward through time, starting from the final exercise date and progressing recursively to the present.

LSM is particularly applicable in energy markets where investment projects are affected by volatile commodity prices, uncertain regulation, and embedded operational flexibility. For example, in oil exploration, LSM can be used to evaluate whether to invest in developing a field based on expected future oil prices. In electricity generation, it is applied to value tolling contracts and flexible generation assets, where fuel costs and spot prices drive exercise decisions.

The strength of the LSM method lies in its ability to model complex, path-dependent decision problems in high-dimensional settings where closed-form solutions are unavailable. However, its accuracy depends on the number of simulated paths, the time discretization, and the choice of basis functions. Inadequate specification or overfitting in the regression step may introduce bias in the continuation value estimates.

Despite these challenges, LSM remains one of the most robust and computationally feasible methods for valuing real options under uncertainty, particularly in capital-intensive industries like energy, mining, and infrastructure.

2.3.2 Risk Management Strategies in Real Options

In addition to traditional risk management techniques, real options offer a set of strategies that capitalize on the flexibility inherent in this approach. These strategies allow firms to manage risks while retaining the ability to capture upside potential as new opportunities arise. Real options-specific strategies, such as staged investment, abandonment options, and portfolios of real options, help firms adapt to changing market conditions, which is especially critical in industries where uncertainty is high.

Staged investment strategies break large projects into smaller, more manageable phases. This approach allows firms to defer investment until more information becomes available, enabling them to abandon or adjust the project as needed based on new market insights [56]. By deferring large investments, firms can mitigate risk while maintaining the option to expand or abandon the project depending on how market conditions evolve.

Abandonment options provide firms with the flexibility to exit an unprofitable investment, thus limiting further financial losses. This option is particularly valuable in capital-intensive industries, where continuing with a project can lead to substantial financial strain. For example, a mining company may abandon a failing extraction site if test results show insufficient resources, preventing unnecessary costs. Similarly, a retail chain may close underperforming stores to focus on more profitable locations [55]. The ability to abandon investments ensures that firms can cut their losses before they become unmanageable.

Another effective strategy is maintaining a portfolio of real options. This approach enables firms to diversify their investment opportunities and capture value from different markets or projects. By holding multiple real options, companies can increase their chances of success by reacting to changes in different sectors simultaneously [53]. A portfolio of options spreads risk while maintaining the potential for high returns, ensuring that firms are not overly reliant on any single investment.

Integrating risk management into real options analysis requires developing frameworks that identify, quantify, and manage risks. Dynamic risk management, for instance, uses real options to adjust strategies in real time, allowing firms to adapt to changing conditions [55]. Risk-adjusted valuation incorporates risk measures, such as Value at Risk (VaR) or Conditional Value at Risk (CVaR), into real options valuation to account for the impact of various risks on the investment's value [59]. This approach ensures that real options reflect both the flexibility they offer and the associated risks, providing a more realistic estimate of their value.

2.3.3 Challenges and Limitations of Real Options in Risk Management

Despite its theoretical advantages, the implementation of real options in risk management presents several challenges. One of the primary obstacles is the complexity of valuing real options. Unlike traditional investment appraisal methods such as Net Present Value (NPV) and Discounted Cash Flow (DCF), real options analysis incorporates managerial flexibility, making valuation inherently more complex [83]. The need for advanced mathematical models to estimate option values, particularly under conditions of uncertainty and market fluctuations, can pose a significant barrier to practical application.

Moreover, real options models require precise data for parameters such as volatility, discount rates, and expected cash flows. Even minor errors in these input parameters can significantly affect the valuation outcomes, reducing the reliability of real options analysis. The sensitivity of real options models to input estimation also complicates the decision-making process, as it introduces greater uncertainty into the valuation results.

Another challenge in the implementation of real options is the computational intensity required to perform the necessary simulations. For example, Monte Carlo simulations can generate a vast number of potential future scenarios, but the computational resources required for such simulations can be prohibitive, particularly for firms with limited resources. Similarly, binomial trees and dynamic programming require significant computational time and effort, making them less practical for large, complex investment projects.

Despite these challenges, many firms have started to adopt hybrid approaches that combine real options techniques with traditional valuation methods. For instance, energy companies evaluating offshore drilling projects often use a combination of NPV for baseline valuation and real options to assess the potential for expansion or abandonment [84]. Similarly, pharmaceutical companies use Monte Carlo simulations to evaluate drug development projects, where regulatory risks and uncertain approval processes complicate investment decisions [85]. These hybrid methods provide a more comprehensive approach to managing risks and capturing opportunities in uncertain environments. The literature on real options provides compelling evidence that integrating real options into financial decision-making improves strategic flexibility and risk mitigation. However, further research is needed to:

1. Develop standardized frameworks that integrate real options into corporate risk management.
2. Improve empirical validation by analyzing real-world case studies in various industries.
3. Explore how advances in financial technology (e.g. AI, blockchain) can enhance the valuation and implementation of real options.
4. Investigate the role of regulatory policies in shaping the adoption of real options in capital investment decisions.

By addressing these gaps, future research can improve the practical applicability of real options, providing firms with more robust tools to manage financial risks in an increasingly uncertain global economy.

In conclusion, real options theory offers a powerful framework for managing investment risk in dynamic, uncertain environments. By incorporating flexibility into decision-making, real options allow firms to adapt their strategies in response to new information, thereby improving investment outcomes. However, to fully realize the potential of real options in practice, further research is needed to refine valuation techniques, improve parameter estimation, and develop more accessible and efficient computational methods for real options analysis.

Environmental, Social, and Governance (ESG) Considerations in Risk Management

The integration of Environmental, Social, and Governance (ESG) factors into risk management has gained increasing attention in financial markets. ESG considerations influence investment decisions, regulatory frameworks, and corporate strategies, requiring investors to adopt hedging mechanisms that account for sustainability-related risks. Options and other derivatives are now being utilized to hedge ESG-related risks, including:

- **Reputational Risks**: ESG non-compliance can lead to brand damage, legal liabilities, and stock price volatility. Hedging strategies, such as event-driven options trading, help manage financial exposure to companies facing ESG-related controversies [86].

ESG-based hedging is further supported by academic research, which highlights the relationship between sustainable business practices and long-term financial performance. Friede et al. [87] conducted a meta-analysis of over 2,000 empirical studies and found a strong correlation between ESG factors and corporate financial success. As a result, institutional investors increasingly integrate ESG criteria into portfolio risk management by utilizing sustainability-linked derivatives, green hedging instruments, and ESG-indexed options. The rise of ESG considerations in risk management underscores a broader shift toward sustainable finance. Hedge funds, asset managers, and corporations are adapting by developing new hedging tools that align financial objectives with long-term environmental and social sustainability goals. The integration of ESG into financial risk management frameworks is expected to become a standard practice, influencing portfolio diversification, regulatory compliance, and corporate governance strategies.

Dynamic Market Environments and Hedging

Modern financial markets are characterized by rapid technological advancements, geopolitical tensions, and economic shifts. These dynamic factors pose challenges for traditional hedging models:

Market Crises: Events such as the 2008 financial crisis highlighted the limitations of static hedging strategies. Studies by Brunnermeier (2009) stress the importance of stress-testing and scenario analysis in preparing for extreme market conditions.

Volatility Dynamics: Research by Heston (1993) introduces stochastic volatility models that better capture market behavior, improving hedging strategies. Adapting to these changes requires leveraging real-time data and advanced analytics, further enhancing the precision of hedging strategies.

2.3.4 Challenges and Knowledge Gaps

Despite significant advancements, several challenges persist in the field of risk management with options. Traditional models often struggle to capture extreme market conditions or sudden shifts in volatility, as evidenced by the 2008 financial crisis. Additionally, balancing the trade-offs between transaction costs and hedging effectiveness remains an ongoing concern. Emerging derivatives and complex financial products further complicate the landscape, requiring innovative approaches and continuous adaptation. The practical application of real options and risk management remains limited due to the complexity of the models and the challenges of implementation. Many managers are unfamiliar with real options theory or lack the technical expertise to apply it effectively. Additionally, the data required for real options analysis (e.g., volatility estimates, cash flow projections) may not always be available or reliable [83].

Despite the growing interest in real options and risk management, there are several gaps in the existing literature. First, most studies focus on the valuation of real options but pay limited attention to the integration of risk management strategies. For example, while numerous papers have explored the application of the Black-Scholes model or binomial trees to real options, few have examined how risk management can be incorporated into these models [88]. Second, there is a lack of comprehensive frameworks for managing risks in real options. Existing frameworks often focus on specific types of risks (e.g., market risk) or specific industries (e.g., energy), limiting their generalizability [53].

3
Methodology

This chapter outlines the methodology employed for pricing and delta hedging both European and American options. The analysis is based on four primary approaches: Monte Carlo Simulation, Black-Scholes Formula, Least Squares Monte Carlo (LSMC), and the Binomial Tree Method. These methods are applied to both call and put options, with a particular emphasis on assessing pricing accuracy and the effectiveness of hedging strategies. Furthermore, Real Option analysis is incorporated to extend traditional financial option pricing to the evaluation of capital investment decisions under uncertainty.

The study utilizes real-world data, specifically from Eni S.p.A., one of the world's largest integrated energy companies based in Italy. Over its long history, Eni has been a major player in global energy markets, particularly in oil, natural gas, and renewable energy sectors. Eni's evolution is closely tied to the concepts of *real options* and *hedging* as part of its strategic decision-making and risk management practices, making it a particularly relevant case study for the thesis.

For European options, which can only be exercised at maturity, the Monte Carlo and Black-Scholes methods are implemented. American options, which allow for early exercise, are modeled using the LSMC and Binomial Tree approaches. The delta hedging process is applied across all methods, where applicable, with hedging errors calculated as the deviation between the hedging portfolio and the actual payoff at maturity. The inclusion of these methods enables a robust analysis of both pricing and hedging performance under various market conditions.

This report details a comprehensive methodological framework for analyzing the profound impact of global macroeconomic and geopolitical events on financial markets, specifically focusing on the stock performance of ENI.MI, a prominent energy company, and Brent Crude oil prices. The methodology outlines a systematic approach encompassing data acquisition, rigorous pre-processing, meticulous identification and categorization of influential global events, and the application of robust analytical techniques such as time-series analysis, event studies, and correlation assessments. The framework aims to delineate the intricate relationships between these variables, providing a structured understanding of how external shocks and policy responses shape energy sector valuations. Preliminary observations, derived from the available data and contextual global event information, highlight the significant sensitivity of both ENI.MI and Brent Crude to major market disruptions, particularly during periods of extreme demand shifts or geopolitical tensions.

Financial markets operate within a dynamic global ecosystem, where geopolitical shifts, economic policies, and unforeseen crises exert profound influence. This is particularly true for sectors intrinsically linked to global commodities, such as the energy industry. The valuation and performance of energy companies, and the prices of energy commodities like crude oil, are highly susceptible to a confluence of international developments. Understanding these complex correlations requires a structured and rigorous analytical approach to discern the patterns, quantify the impacts, and potentially anticipate future trends. The energy sector, exemplified in this analysis by ENI.MI, is uniquely vulnerable to these external factors due to its reliance on global supply-demand dynamics for crude oil, which are themselves shaped by a myriad of global forces.

Global events, ranging from trade disputes and pandemics to geopolitical conflicts and shifts in monetary policy, propagate through complex channels to affect commodity prices and, consequently, the financial health of companies operating within commodity-dependent sectors. For instance, the U.S.-China trade war directly impacted global supply chains and economic growth forecasts, influencing commodity demand [89, 90]. Similarly, the COVID-19 pandemic led to an unprecedented collapse in oil demand due to widespread lockdowns and industrial slowdowns [91, 92]. More recently, the Russia-Ukraine conflict has severely disrupted energy markets, causing significant price surges and heightened volatility [93, 94]. Concurrently, global inflation surges and aggressive interest rate hikes by central banks have added layers of complexity, affecting borrowing costs, consumer spending, and overall economic activity [89, 95, 96]. These events do not occur in isolation; their overlapping and cascading effects necessitate a robust methodological framework to disentangle their individual and collective impacts.

The primary objective of this report is to outline a comprehensive methodology for assessing how identified global events influence both Brent Crude oil prices and the stock performance of ENI.MI. The analytical scope primarily covers the period from January 2018 to early 2021, based on the provided datasets, while contextualizing the analysis with broader global events extending up to 2023, drawing upon supplementary qualitative information. This methodological exposition aims to provide a replicable framework for financial professionals and researchers seeking to conduct similar investigations. The methodology will detail the steps from raw data acquisition to advanced analytical techniques, ensuring transparency and reproducibility of the findings.

3.1 Eni S.p.A.

Eni S.p.A. is one of the world's largest integrated energy companies, based in Italy. Over its long history, Eni has played a significant role in global energy markets, especially in the areas of oil, natural gas, and renewable energy. The company's evolution can be closely linked to the concept of *real options* and *hedging* as part of its strategic decision-making and risk management practices, which are central themes in this thesis. The following provides a brief history of Eni's evolution, from early real option and hedging methods to present-day complex strategies.

3.1.1 Early Beginnings and the Birth of Eni (1953)

Eni was founded in 1953 by the Italian government as a state-owned oil company, primarily to handle Italy's oil and gas needs. In its early years, Eni's focus was on securing Italy's energy resources, initially emphasizing exploration and production. The company was tasked with decreasing Italy's dependence on foreign oil and establishing a competitive national energy sector.

During this period, *real options* in strategic decision-making were not as developed as they are today. However, the company's need for flexibility in response to global market volatility (especially fluctuations in crude oil prices and the availability of energy resources) would have been a crucial consideration in its investment decisions.

3.1.2 1970s – 1980s: Strategic Response to Volatility and Hedging Practices

In the 1970s, the global oil crisis led to a sharp increase in the price of oil, significantly impacting the energy market. For Eni, this was a pivotal moment in its development as it sought to expand its operations into new markets and secure strategic oil reserves.

Eni responded by expanding its exploration activities into areas outside of Italy, including the Middle East, Africa, and South America. The volatility in oil prices, combined with geopolitical instability, made these investments risky but potentially highly profitable.

At this stage, Eni likely used *real options* to navigate its strategic investments, much like a company today would decide whether to expand, abandon, or defer projects based on evolving market conditions. Real options, such as the option to expand or abandon oil exploration projects in uncertain markets, would have been a key decision-making tool. In terms of *hedging*, Eni presumably engaged in financial strategies to mitigate the risks associated with oil price fluctuations. Hedging strategies, such as the use of forward contracts or options to lock in future prices, would have helped stabilize the company's cash flows.

3.1.3 Case Study: Eni S.p.A.

The research also explores the hedging practices of companies like Eni S.p.A., a major player in the energy sector. Eni, with its extensive operations in oil and gas, faces significant risks due to commodity price fluctuations, foreign exchange movements, and interest rate changes. The company employs a range of hedging strategies to manage these risks, including the use of commodity derivatives (e.g., futures, options, and swaps) to lock in favorable prices for its oil and gas production. By doing so, Eni can ensure stable cash flows and mitigate the impact of price volatility, which is a significant concern in the energy market.

Eni's hedging strategy incorporates the use of Monte Carlo simulations to determine the value of hedges under varying market conditions, including oil price volatility and potential shifts in exchange rates. By simulating multiple price paths and adjusting for volatility, Eni can assess the effectiveness of different hedging positions and dynamically adjust its strategy. The flexibility of the Monte Carlo simulation approach enables Eni to simulate a variety of market scenarios, including extreme price movements, and make more informed decisions regarding hedging positions. This method allows for the identification of optimal hedge ratios, helping Eni navigate the complexities of managing risk in an unpredictable market.

Additionally, Eni considers the costs of hedging, such as premiums and transaction costs, to ensure that the strategy is economically viable while maintaining a strong balance sheet. The company continually assesses the trade-offs between hedging effectiveness and cost efficiency, seeking to maximize risk reduction without incurring excessive costs. Eni's ability to adjust its hedging strategies based on real-time market data and forecasts allows the company to maintain a competitive advantage in managing its financial risks. Furthermore, the company has developed a comprehensive risk management framework that incorporates not only Monte Carlo simulations but also sensitivity analyses, stress tests, and scenario planning to better understand potential vulnerabilities and opportunities in the market.

This chapter outlines the methodology employed for option pricing, delta hedging and real option analysis both European and American options. The analysis is based on four primary approaches: Monte Carlo Simulation, Black-Scholes Formula, Least Squares Monte Carlo (LSMC), and the Binomial Tree Method. These methods are applied to both call and put options, with a particular emphasis on assessing pricing accuracy and the effectiveness of hedging strategies. Furthermore, Real Option analysis is incorporated to extend traditional financial option pricing to the evaluation of capital investment decisions under uncertainty.

For European options, which can only be exercised at maturity, the Monte Carlo and Black-Scholes methods are implemented. American options, which allow for early exercise, are modeled using the LSMC and Binomial Tree approaches. The delta hedging process is applied across all methods, where applicable, with hedging errors calculated as the deviation between the hedging portfolio and the actual payoff at maturity. The inclusion of these methods enables a robust analysis of both pricing and hedging performance under various market conditions.

The study also emphasizes the importance of analyzing different strike prices in conjunction with market conditions. This approach enables investors to tailor their hedging strategies to effectively mitigate specific risks they may face in the market. By examining the impact of various strike prices and market volatility on option pricing, the research helps investors design strategies that reduce exposure to risk while capitalizing on favorable market movements.

Additionally, the research discusses the costs associated with implementing hedging strategies, such as the premiums paid for options and transaction costs. Understanding these costs is crucial for evaluating the overall effectiveness of the hedging approach. While Monte Carlo simulations can provide valuable insights into the possible outcomes of various hedging strategies, it is also important to account for transaction costs and premiums, as these can significantly affect the financial viability of a given strategy.

Overall, the methods used in this paper combine empirical analysis with advanced financial models, such as Monte Carlo simulations Black Scholes option pricing model, Binomial tree option pricing model,Stochastic modeling to provide insights into the effectiveness of various hedging strategies in managing risk during periods of market volatility. This comprehensive approach equips investors with the necessary tools to make informed decisions and navigate uncertain market conditions.

Monte Carlo Simulations

A key statistical tool employed in the study is Monte Carlo simulations. This method helps in evaluating the prices of options and determining the best strategies for hedging against market volatility. By simulating a large number of random price paths, Monte Carlo simulations allow investors to assess how different factors, such as underlying asset prices, volatility, and interest rates, affect option prices over time. The advantage of Monte Carlo simulations is their ability to model the randomness inherent in financial markets, which is often too complex to capture using deterministic models. This stochastic approach provides a comprehensive analysis of potential outcomes, helping decision-makers identify optimal strategies under various market conditions.

Analysis of Strike Prices and Market Conditions The study also emphasizes the importance of analyzing different strike prices in conjunction with market conditions. This approach enables investors to tailor their hedging strategies to effectively mitigate specific risks they may face in the market. By examining the relationship between strike prices and market volatility, the study highlights how investors can adjust their positions to minimize potential losses. For example, in a highly volatile market, adjusting strike prices dynamically can help in locking in favorable prices or reducing potential downside risk. Moreover, the study delves into the use of different hedging instruments like options and futures, providing investors with flexible tools for risk management. This combination of strategic analysis and financial instruments enhances the effectiveness of hedging strategies, particularly in uncertain economic environments.

COSTS OF HEDGING STRATEGIES Additionally, the research discusses the costs associated with implementing hedging strategies, such as the premiums paid for options and transaction costs. Understanding these costs is crucial for evaluating the overall effectiveness of the hedging approach. The study provides a thorough breakdown of these costs and examines their impact on the profitability of a hedging strategy. For instance, while options premiums can provide protection against unfavorable market movements, the upfront cost of purchasing these options may erode potential profits if the market does not move in the anticipated direction. Similarly, transaction costs, including brokerage fees and the costs of adjusting hedging positions, may accumulate over time, impacting the overall financial performance. The study suggests that effective hedging should account for these costs, balancing risk management with cost efficiency.

3.2 Implementation of Models

The successful execution of any quantitative analysis hinges on the effective implementation of appropriate models and methodologies. In this section, we describe the key steps involved in implementing the models used for pricing options and managing financial risks in this study. The models discussed here are pivotal for understanding the dynamic behavior of financial assets and devising strategies to mitigate associated risks.

The primary objective of this analysis is to estimate the fair value of options and evaluate various hedging strategies, particularly in relation to ENI S.p.A. stock and Brent Crude oil prices. To achieve this, we have employed a combination of well-established methods in financial engineering, including Monte Carlo simulations, Binomial Tree models, Least Squares Monte Carlo (LSMC), and Black-Scholes Option Pricing. These models are implemented to simulate potential price paths for the underlying assets and to compute the corresponding option prices under varying assumptions.

Given the complexity and flexibility of real-world markets, these models allow for the incorporation of uncertainty, market volatility, and decision-making strategies, making them well-suited for pricing both standard European options and path-dependent American options. Additionally, **real options analysis** is integrated into the methodology to account for the strategic decision-making flexibility available to firms under uncertainty, such as whether to defer, expand, or abandon investments based on evolving market conditions.

Furthermore, hedging strategies based on Delta are implemented to assess the effectiveness of risk management through dynamic portfolio adjustments. By tracking the changes in Delta over time and adjusting the portfolio accordingly, we aim to mitigate the exposure to price movements in the underlying asset.

The implementation of these models follows a structured approach, starting with the acquisition and preparation of data, progressing through model specification and simulation, and concluding with the assessment of the effectiveness of hedging strategies. Each model is validated through comparison with historical data, and various sensitivity analyses are conducted to ensure robustness in different market scenarios.

The following sections will detail the implementation steps for each of the models, including the specific algorithms, assumptions, and computational tools used, as well as a discussion of the challenges encountered during the process and the solutions applied.

Data Sources Overview of Provided Datasets and Relevant Global Event Information

The quantitative foundation of this analysis is built upon historical daily price data for ENI.MI stock, sourced from `eni_stock_data.csv` [97], and for Brent Crude oil prices, extracted from `brent_crude_data.csv` [98]. These datasets provide granular daily observations, including 'Close' prices and 'Volume', essential for time-series analysis. The data, generated using the `yfinance` library, typically includes a metadata header in the first two rows, which requires specific handling during the data cleaning phase. Complementing this quantitative data is a rich body of qualitative information regarding major global events that occurred between 2018 and 2023. This information is drawn from various reputable financial and energy news outlets and research institutions [89, 90, 93, 99, 100, 91, 92, 94, 96, 95, 101, 102]. This qualitative information is essential for providing the necessary context for event identification and impact attribution. By combining quantitative analysis of market data with qualitative insights into global events, this dual approach enables a comprehensive understanding of market dynamics.

Data Acquisition and Pre-processing

The integrity and reliability of any quantitative analysis hinge upon the quality of the underlying data. This section describes the systematic process of data acquisition, cleaning, validation, and feature engineering applied to the financial time series.

Data Sourcing

The first step in this analytical framework involves the acquisition of raw financial data. Daily historical data for ENI.MI stock is extracted directly from the `eni_stock_data.csv` file [97]. This dataset, generated via the `yfinance` library, contains several columns such as 'Price' (serving as the Date), 'Close', 'High', 'Low', 'Open', and 'Volume'. For this analysis, the 'Date', 'Close' price, and 'Volume' columns are of primary interest. Similarly, daily historical data for Brent Crude oil prices are obtained from the `brent_crude_data.csv` file [98], where the 'Date', 'Close' price, and 'Volume' columns are the key variables for examination. The `yfinance` output structure includes a metadata row (containing the ticker symbol repeated across columns) and a row with generic labels (e.g., 'Date', 'Open', 'High', 'Low', 'Close', 'Adj Close', 'Volume') before the actual time-series data. This structure requires careful handling during the data loading phase.

Data Generation Process

The raw historical daily price data for ENI.MI stock and Brent Crude oil were programmatically generated and downloaded using the `yfinance` Python library, which offers a convenient interface to access financial market data from Yahoo! Finance. The specific code used to generate and download the data is as follows:

```
import yfinance as yf

# Download ENI stock data
eni = yf.download('ENI.MI', start='2018-01-01', end='2023-12-31')
eni.to_csv('eni_stock_data.csv')

# Download Brent Crude oil data
brent = yf.download('BZ=F', start='2018-01-01', end='2023-12-31')
brent.to_csv('brent_crude_data.csv')
```

This script downloads daily historical data for the specified ticker symbols ('ENI.MI' for ENI stock and 'BZ=F' for Brent Crude futures) within the period from January 1, 2018, to December 31, 2023. The downloaded data, typically returned as a Pandas DataFrame, is then saved to respective CSV files. The structure of these CSV files includes a metadata row containing the ticker symbol repeated across columns and a row with generic labels before the actual time-series data begins. This format informs the subsequent data cleaning and validation process, ensuring correct parsing and data integrity.

3.2.1 Data Cleaning and Validation

Once the raw data is sourced, a critical phase of data cleaning and validation is undertaken to ensure the data's quality and suitability for robust analysis. Given the structure of the `yfinance`-generated CSV files, a specialized loading approach is required. The files are initially read without a header (`header=None`) to treat all rows as data. The actual column names are then manually assigned from a predefined list corresponding to the expected structure. The first two rows, which contain metadata (ticker symbols and generic labels), are programmatically skipped. The column initially named 'Price' is renamed to 'Date' for both datasets. To facilitate accurate time-series alignment and numerical computations, all 'Date' columns are converted into a consistent datetime format using `pd.to_datetime()`. The `errors='coerce'` argument is employed to convert any unparseable date entries into `NaT` (Not a Time), which can then be handled. All relevant numeric columns, including 'Close' prices, 'High', 'Low', 'Open', and 'Volume', are converted to appropriate numerical data types (e.g., `float64`) using `pd.to_numeric()` with `errors='coerce'` to ensure that any non-numeric characters or corrupted entries are converted to `NaN` (Not a Number), preventing computational errors in subsequent stages.

The presence of missing data points can significantly bias analytical results. In financial time series, missing values often arise due to non-trading days, data collection errors, or market holidays. Missing data are typically classified as:

- Missing Completely at Random (MCAR): The probability of missingness is independent of both observed and unobserved data.
- Missing at Random (MAR): The probability of missingness depends on observed data but not on the missing data itself.
- Missing Not at Random (MNAR): The probability of missingness depends on the unobserved data itself, introducing inherent bias.

For this analysis, missing values are primarily expected to be MCAR or MAR. The strategy employed is to drop rows containing `NaN` values resulting from the data type conversion or inherent missingness. While imputation methods (such as mean, median, or regression-based imputation) exist, for high-frequency financial data where continuity is paramount, dropping rows with `NaNs` is preferred to avoid introducing artificial patterns or biases.

3.2.2 Feature Engineering

To enhance the analytical depth and enable meaningful comparisons across different financial instruments, several derived metrics or features are engineered from the raw data. These transformations are critical for standardizing the data and preparing it for time-series modeling.

Daily returns are a fundamental metric for analyzing price movements and are calculated for both ENI.MI stock and Brent Crude prices. Simple returns are calculated as the percentage change of the 'Close' price from one day to the next. Logarithmic (Log) returns are calculated as the natural logarithm of the ratio of consecutive closing prices, which are preferred for time-series analysis due to their advantageous mathematical properties. Log returns account for continuous compounding effects, are time-additive, and tend to be more symmetrically distributed and closer to a normal distribution, especially for daily data. Therefore, log returns will be used for all time-series analyses in this thesis.

Volatility, which measures price fluctuation, is a critical indicator of market risk and uncertainty. Rolling standard deviations of daily log returns are computed to capture the dynamic nature of volatility. A common practice involves calculating a 20-day or 30-day rolling volatility, which provides a dynamic measure of price fluctuation and serves as a proxy for perceived market risk or instability at any given time.

Volume analysis is also critical, as it represents the number of shares or contracts traded. Significant spikes or troughs in volume can indicate periods of high investor activity, strong market conviction behind price trends, or panic selling/buying during critical events. Volume can be normalized to identify anomalies and further enhance the analysis.

Finally, time-series models often assume that the underlying data is stationary. A stationary time series has statistical properties (mean, variance, autocorrelation) that remain constant over time. Financial price series are typically non-stationary, exhibiting trends and changing variance. Log returns are generally considered stationary, making them suitable for direct use in many models. However, formal tests are still necessary to ensure stationarity before applying models like Granger causality or VAR.

3.2.3 Monte Carlo Simulation for Option Pricing

Monte Carlo simulation is used to estimate the value of European-style options by simulating a large number of paths for the underlying asset price, assuming it follows a Geometric Brownian Motion (GBM). In this model, the underlying asset follows a geometric Brownian motion with constant volatility, as assumed in the Black-Scholes model. The asset price S_t is simulated using the following stochastic differential equation:

$$dS_t = \mu S_t \, dt + \sigma S_t \, dW_t, \tag{3.1}$$

where μ is the drift (the expected return of the asset), σ is the volatility, and dW_t is a Wiener process representing randomness.

The option is European-style, meaning it is exercisable only at expiration. As in the Black-Scholes model, the asset price is simulated over the option's life until maturity, and the payoff is determined at that time. The model assumes no dividends are paid on the underlying asset, simplifying the simulation by excluding dividend yield adjustments. Therefore, modeling focuses on the asset price without needing to adjust for dividend payouts. A constant risk-free interest rate r is assumed, which is used to discount the final payoff back to the present time.

In Monte Carlo simulations, the constant risk-free rate is incorporated into the option pricing calculations by discounting the payoff at expiration. No transaction costs or taxes are included, meaning the asset price and option's payoff are solely influenced by market volatility and interest rates. Thus, no additional costs are considered when simulating trades or executing options. Continuous trading is assumed, allowing for the smooth evolution of the asset price in the simulation. This facilitates modeling asset prices over time in continuous intervals. There are no arbitrage opportunities, ensuring no risk-free profits can be gained by exploiting price discrepancies. This helps ensure model consistency by preventing market friction from influencing simulations.

The evolution of a stock price S_t is commonly modeled by a Geometric Brownian Motion (GBM), which satisfies the following stochastic differential equation (SDE):

$$dS_t = \mu S_t \, dt + \sigma S_t \, dW_t. \tag{3.2}$$

The analytical solution to this SDE, obtained via Itô's lemma, is:

$$S_t = S_0 \exp\left(\left(\mu - \frac{1}{2}\sigma^2 \right) t + \sigma W_t \right). \tag{3.3}$$

To simulate sample paths numerically, we discretize the time interval $[0, T]$ into N equal steps of length $\Delta t = \frac{T}{N}$. At each step, the increment of the Brownian motion is approximated by:

$$dW_t \approx \sqrt{\Delta t} \cdot Z_t, \quad Z_t \sim \mathcal{N}(0,1), \tag{3.4}$$

where Z_t is a standard normal random variable. Substituting this into the solution yields the discrete-time approximation used in Monte Carlo simulations:

$$S_{t+\Delta t} = S_t \cdot \exp\left[\left(\mu - \frac{1}{2}\sigma^2 \right) \Delta t + \sigma \sqrt{\Delta t} \cdot Z_t \right]. \tag{3.5}$$

In our implementation , the time horizon T is divided into N intervals of equal length, with each step given by $\Delta t = T/N$. For this study, N is set to 100 to provide a reasonable balance between accuracy and computational efficiency. For each simulation path $i = 1, \ldots, M$ (with $M = 50{,}000$ paths), the stock price evolution is generated iteratively using the following equation 3.5: This expression is iterated for each of the M simulated paths over the N time steps. The resulting distribution of the outcomes can be used to estimate statistical properties such as the mean and variance.

At maturity T, calculate the payoff of the European call or put option:

$$\text{Call option payoff} = \max(S_T - K, 0), \tag{3.6}$$

and

$$\text{Put option payoff} = \max(K - S_T, 0), \tag{3.7}$$

where S_T is the simulated price of the asset at maturity, and K is the strike price. The strike price (K) represents the agreed-upon price at which an option holder can buy (in the case of a call option) or sell (in the case of a put option) the underlying asset at maturity.

then estimated as the average of the discounted payoffs:

$$\hat{P} = \frac{1}{M} \sum_{i=1}^{M} \exp(-rT) \cdot \text{Payoff}_i. \tag{3.8}$$

This formula accounts for the time value of money by discounting each payoff back to the present using the risk-free rate r. The standard error and 95% confidence interval for the option price are calculated based on the sample standard deviation from the simulated payoffs.

Delta is estimated numerically using finite differences, as follows:

$$\Delta = \frac{P(S_0 + \varepsilon) - P(S_0 - \varepsilon)}{2\varepsilon}, \tag{3.9}$$

where $P(S_0 + \varepsilon)$ and $P(S_0 - \varepsilon)$ are the option prices calculated for small positive and negative perturbations ε around the initial stock price S_0. For hedging simulations, a discrete-time replicating portfolio is constructed. The portfolio is adjusted at each time step according to the estimated delta, with the hedging errors tracked as the difference between the actual option payoff and the value of the hedging portfolio at maturity.

Next, discount the payoff back to the present using the risk-free interest rate:

$$\text{Option Price} = e^{-rT} \times \text{Expected Payoff}. \tag{3.10}$$

The expected payoff is the average of the payoffs across all simulated paths.

Repeat the process (i.e., generating multiple price paths and calculating payoffs) for a large number of simulations to get a statistically significant result.

Finally, the average of the discounted payoffs across all simulations will give an estimate of the option price.

3.2.4 Black-Scholes Delta in Option Pricing and Hedging Strategies

The Black-Scholes formula provides a closed-form solution for pricing European call and put options in a frictionless market. This method assumes constant volatility, a risk-free rate, and no dividends, which makes it applicable only to European options that can be exercised only at maturity.

The key inputs for the Black-Scholes model include the initial stock price S_0, strike price K, risk-free rate r, volatility σ, and time to maturity T. The formula requires the calculation of two intermediate terms:

$$d_1 = \frac{\ln(S_0/K) + (r + 0.5\sigma^2)T}{\sigma\sqrt{T}}, \quad d_2 = d_1 - \sigma\sqrt{T}. \tag{3.11}$$

The call and put option prices are computed as follows:

$$C = S_0 \cdot N(d_1) - K \cdot e^{-rT} \cdot N(d_2), \quad P = K \cdot e^{-rT} \cdot N(-d_2) - S_0 \cdot N(-d_1), \tag{3.12}$$

where $N(\cdot)$ is the cumulative standard normal distribution function. The delta of the call option is given by:

$$\Delta = N(d_1). \tag{3.13}$$

For put options, the delta is computed as:

$$\Delta = N(d_1) - 1. \tag{3.14}$$

The delta is used in hedging simulations to adjust the portfolio at each step, maintaining a risk-neutral position by holding Δ shares of the underlying asset. In the Black-Scholes option pricing framework, the Delta (Δ) of a derivative plays a central role in both option pricing and hedging strategies. By definition, Delta is the partial derivative of the option's value with respect to the underlying asset price ($\Delta = \frac{\partial V}{\partial S}$), which quantifies the immediate sensitivity of the option price to small changes in S. This quantity serves as the hedge ratio, indicating the exact proportion of the underlying asset needed to offset the option's first-order exposure. The key insight behind Black-Scholes is that one can eliminate risk (to first order) by continuously trading the underlying in just the right proportion to the option – a strategy known as continuously revised delta hedging. In practice, this means that an investor holding a short position in one option can hedge its directional risk by taking a long position of Δ shares of the underlying (financed by borrowing or lending at the risk-free rate). As a result, the instantaneous net change in the value of this option–underlying portfolio has no stochastic component, since the gain or loss from an infinitesimal movement dS in the underlying is exactly offset by the change in the option's value. Under the idealized assumptions of the Black-Scholes model (frictionless markets, continuous trading, no arbitrage), the Δ-hedged portfolio is locally risk-free and hence must evolve deterministically at the risk-free rate. This replication argument underpins the Black-Scholes partial differential equation and ensures that the option is consistently priced with respect to the underlying asset dynamics given by geometric Brownian motion.

However, it is important to emphasize that Delta hedging is dynamic: Δ is not static but changes with both the underlying price and time. For example, the Black-Scholes delta for a European call is $\Delta = N(d_1)$, which is a function of S and t. As the underlying price evolves stochastically (following the GBM process introduced earlier) and the option approaches maturity, Δ will fluctuate, necessitating continuous rebalance of the hedge. In theory, if the portfolio were rebalanced continuously (at every infinitesimal time step), the hedge would remain perfect at all times. In reality, trading can only occur at discrete intervals, so a delta-neutral portfolio (one with $\Delta_{\text{total}} = 0$ net exposure) is only maintained approximately between rebalancing times. During each small interval Δt, the hedged portfolio can experience residual changes in value due to factors beyond the first-order dS term – notably the time decay of the option (quantified by Theta) and the curvature of the option's value (quantified by Gamma). A delta-hedged position is immune to infinitesimal underlying moves, but it still carries risk from larger price jumps and the accumulation of unhedged second-order effects. This is why delta hedging in practice is an iterative process: as Δ shifts with market movements, the hedger must adjust the position (buying or selling shares) to re-establish neutrality. The methodology developed in this thesis accounts for this by implementing a time-stepping simulation wherein the hedge ratio is recalculated and updated at each interval, linking seamlessly with the Monte Carlo simulation framework introduced earlier.

Delta plays a significant role not only in hedging but also in option pricing. The Black-Scholes formula for pricing European options directly incorporates Delta into its calculation of the option's price. For a call option, the price is given by:

$$C = S_0 N(d_1) - Ke^{-rT} N(d_2), \tag{3.15}$$

where S_0 is the current stock price, K is the strike price, r is the risk-free interest rate, and T is the time to maturity. Here, $N(d_1)$ represents the Delta of the option. The value of Delta directly influences the pricing mechanism by determining how much of the underlying asset is required to replicate the option's payoff. This relationship is central to the Black-Scholes framework, providing a precise model for the fair pricing of options based on assumptions of constant volatility and no early exercise.

The Delta of an option is also a critical tool in real options analysis. Real options theory applies the same principles of financial options to investment decisions, allowing firms to value strategic business decisions under uncertainty. In real options, Delta represents the sensitivity of the value of the business project (or investment) to changes in key drivers, such as market prices, demand, or other external variables. For example, the decision to expand or defer a project can be seen as a real option, and the Delta would indicate how the value of that decision changes with the underlying variables.

Integrating Delta hedging with Monte Carlo simulation: In our methodology, we simulate a large number of possible price paths for the underlying asset using Monte Carlo techniques (as described in Section 3.3), where each path $S(t)$ follows the stochastic process given by Equation (3.1). Along each simulated path, a dynamic hedging strategy is executed: at each discrete time step Δt, we compute the current option Delta, $\Delta(t) = \frac{\partial V}{\partial S}$ (using the Black-Scholes formula or an equivalent numerical method, and referencing the current underlying price and remaining time). We then adjust the holdings of the underlying asset in the simulated portfolio to match this $\Delta(t)$. For instance, if $\Delta(t) = 0.45$, the strategy holds 0.45 units of the underlying for each option being hedged (if Δ were negative, it would imply a short position in the underlying). This self-financing portfolio – consisting of a long (or short) position in the underlying and a financing position in the risk-free asset – is designed such that its instantaneous P & L mirrors the option's price movement, cancelling out the first-order risk. By the construction of Delta, any small change ΔS in the underlying over the next interval causes an equal and opposite change in the combined portfolio value: $d\Pi \approx \Delta\, dS - \Delta\, dS = 0$. In the limit of continuous rebalancing, this hedged portfolio would, path by path, accrue a deterministic return equal to the risk-free rate (thereby perfectly replicating the option's payoff at maturity). In our discrete-time simulations, we can directly track the replication error by comparing the terminal value of the hedged portfolio to the option's payoff (or to the initial option price grown at e^{rT}). We expect – and indeed observe – that as the rebalancing frequency increases (i.e. as $\Delta t \to 0$), the distribution of hedging errors collapses around zero, reflecting convergence to the theoretical risk-neutral replication. Even with finite (e.g. daily or weekly) rebalancing, the average outcome of many simulation trials should align with the Black–Scholes pricing prediction, even though individual paths may exhibit some divergence. In fact, our Monte Carlo analysis confirms the well-known result that while any single delta-hedged path can deviate by a significant amount from the Black–Scholes value (on the order of tens of percent for coarse hedging intervals or volatile scenarios), when averaging over a large number of scenarios, the strategy's performance converges to the Black–Scholes theoretical value. This validates that Delta-based dynamic hedging is an effective strategy on average (in expectation), although it underscores the importance of frequent rebalancing and risk management for individual realizations. By incorporating these simulations into our methodology, we can quantitatively assess the effectiveness of Delta hedging (and the magnitude of hedging errors) under various market conditions, complementing the earlier sections on Monte Carlo pricing and GBM-based risk-neutral valuation.

Relevance to hedging with corporate bond and equity data: A major advantage of Delta hedging is that it can be applied not just in theoretical pricing exercises but also in real-world risk management using market data. In our case study, we use empirical data from Eni S.p.A. – specifically, the company's equity (stock price) and its corporate bond data – to implement and analyze hedging strategies. The stock price data provides a realistic underlying trajectory and volatility estimate for our GBM model, while the corporate bond data (e.g. yield spreads or bond prices) helps calibrate appropriate discount rates and can reflect the firm's credit risk in the analysis. This is particularly important when the instrument being hedged is linked to the firm's debt or default risk. For example, consider a convertible bond issued by Eni, which embeds an equity call option on Eni's stock. The Delta of the convertible bond with respect to the underlying equity indicates how sensitive the bond's value is to changes in Eni's share price. A hedger (or arbitrageur) holding the convertible can dynamically short Eni's stock in proportion to this Delta to offset the equity exposure of the bond. If the convertible has $\Delta = 0.5$ (roughly half the sensitivity of the stock), the hedge would involve shorting approximately one half of a share of Eni for each share-equivalent of the bond's conversion value. In practical terms, for every €100 of convertible bond (by conversion value), about €50 of Eni stock would be sold short to establish a delta-neutral position. As Eni's stock price moves over time, the bond becomes more or less equity-sensitive (Delta rises if the stock climbs closer to or above the conversion price, and falls if the stock drops further out-of-the-money). The hedger must therefore adjust the short equity position continuously – increasing the short when Δ rises, and decreasing it (buying back shares) when Δ falls – in order to maintain the hedge. This dynamic rebalancing effectively isolates the residual risks in the position, leaving primarily the bond's credit risk and volatility risk unhedged. By applying our delta-hedging methodology to Eni's market data, we can analyze how well such a strategy would have performed: for instance, we can simulate the P&L of a delta-hedged convertible bond over historical periods or under simulated stress scenarios. The integration of real corporate bond and equity data ensures that our hedging model captures realistic patterns such as volatility clustering, jumps, and credit spread changes that a purely theoretical model might omit. Ultimately, Delta provides a unifying measure that links the equity and debt domains – it translates the exposure of a corporate bond (with equity-linked features) into an equivalent stock position. This allows us to implement cross-asset hedging strategies (hedging corporate liabilities with equity instruments) in a quantitatively rigorous way. Furthermore, the Delta-driven hedging outcomes on Eni's data can be compared with Black–Scholes predictions: if the model is an adequate description of reality, a delta-neutral strategy should on average yield near risk-free returns, whereas any systematic deviations would point to model limitations or market frictions. In the context of our thesis, this Delta-based analysis in the methodology chapter complements the earlier discussions of Monte Carlo simulation and GBM by demonstrating how theoretical pricing sensitivities are deployed in practice to manage risk dynamically. It underscores that Delta is not merely a mathematical byproduct of the Black–Scholes model, but a practical tool for hedging – enabling the implementation of dynamic replication strategies that are at the heart of both modern option pricing theory and real-world risk management.

3.2.5 Least Squares Monte Carlo (LSMC) for American Option Pricing

Least Squares Monte Carlo (LSMC) is a simulation-based method developed to efficiently price American-style options, which differ from their European counterparts in that they can be exercised at any point before maturity. This early-exercise feature introduces additional complexity, as it requires solving an optimal stopping problem: the holder must decide at each time step whether to exercise the option or continue holding it. The LSMC method addresses this challenge by combining forward simulation of asset price paths with backward induction and regression techniques to estimate the continuation value of the option.

The LSMC approach begins by simulating M paths of the underlying asset price over N discrete time steps. The asset dynamics are modeled using Geometric Brownian Motion (GBM), a continuous-time stochastic process that captures both the drift due to the risk-free interest rate r and the random fluctuations driven by volatility σ. At each step in each path, the current stock price S_t is used to determine whether the option is in-the-money—i.e., whether early exercise could be optimal.

For American options, unlike European ones, it is not sufficient to consider only the terminal payoff. At each time step before maturity, the decision to exercise or continue holding the option depends on the comparison between two quantities:

- The immediate exercise value, which for a put option is

$$\text{Immediate Exercise Value}_{\text{Put}} = \max(K - S_t, 0), \tag{3.16}$$

  and for a call option is

$$\text{Immediate Exercise Value}_{\text{Call}} = \max(S_t - K, 0), \tag{3.17}$$

- The continuation value, representing the expected future payoff of holding the option.

To estimate the continuation value, least squares regression is performed at each time step on the discounted future payoffs. Specifically, for paths that are in-the-money, the future value of the option is regressed against a set of basis functions of S_t, typically polynomials. The estimated regression function is then used to approximate the expected value of continuing to hold the option:

$$\text{Continuation Value} \approx \hat{\beta}_0 + \hat{\beta}_1 S_t + \hat{\beta}_2 S_t^2 + \cdots + \hat{\beta}_n S_t^n, \tag{3.18}$$

where $\hat{\beta}_i$ are the regression coefficients obtained by minimizing the squared error on simulated paths, and n is the degree of the polynomial basis. Only in-the-money paths are used for the regression, as out-of-the-money options are never exercised early.

The algorithm proceeds by working backwards from maturity to time zero, recursively applying this regression-based rule to decide whether to exercise or defer. At each point, the maximum of the immediate exercise value and the estimated continuation value is selected:

$$\text{Decision}_t = \max(\text{Immediate Exercise Value, Continuation Value}). \tag{3.19}$$

If exercising yields a higher value, the option is exercised, and that path ends. If not, the holder continues to the next time step. This backward induction ensures that the early exercise decision optimally accounts for the time value of money and future uncertainty.

After processing all simulated paths, the final option price is computed by averaging the discounted cash flows of all optimal decisions across the simulation:

$$\text{Option Price} = \frac{1}{N}\sum_{i=1}^{N} \exp(-r \cdot \tau_i) \cdot \text{Payoff}_i, \tag{3.20}$$

where τ_i is the exercise time on path i, and Payoff_i is the corresponding realized payoff.

Beyond pricing, the LSMC framework can be extended to estimate sensitivities such as Delta. This is often done using finite difference methods or pathwise derivative techniques applied to the regression surface. These estimates are crucial for hedging and risk management in practical trading environments.

In summary, the LSMC method provides a powerful and flexible framework for valuing American options. It handles early exercise by leveraging simulation and regression to efficiently approximate continuation values. Its adaptability to various option types and underlying asset models makes it especially useful in settings where analytic solutions are unavailable or computationally intensive.

3.2.6 Binomial Tree for American Option Pricing

The Binomial Tree method uses a discrete-time backward induction approach to calculate the option price by evaluating the possibility of early exercise at each node. This method is particularly useful for pricing American options, as it allows for the flexibility of exercising the option at any point before maturity. The process involves constructing a binomial tree that models the evolution of the underlying asset's price over discrete time steps.

First, define the number of steps N, and compute the up factor u, down factor d, and the probability p of an upward move. These are calculated using the following equations:

$$u = \exp(\sigma\sqrt{\Delta t}), \quad d = \frac{1}{u}, \quad p = \frac{e^{r\Delta t} - d}{u - d}. \tag{3.21}$$

Here, σ is the volatility of the underlying asset, r is the risk-free interest rate, and $\Delta t = \frac{T}{N}$ is the time step between each node. These factors define the price movement in each step, where u represents the factor by which the asset price increases, and d represents the factor by which it decreases.

Once these parameters are defined, the tree of stock prices is built over N time steps from $t = 0$ to $t = T$, with each node representing a potential price at that time. At maturity, the option payoff is calculated at each terminal node. The payoff for a call option is $\max(S_T - K, 0)$, and for a put option, it is $\max(K - S_T, 0)$, where S_T is the stock price at maturity and K is the strike price.

For each earlier node, the continuation value is calculated using backward induction. The continuation value is the expected present value of the option if it is held rather than exercised, and is given by:

$$V = e^{-r\Delta t} \cdot (p \cdot V_{\text{up}} + (1 - p) \cdot V_{\text{down}}), \tag{3.22}$$

where V_{up} and V_{down} are the option values at the up and down nodes at the next time step, respectively, and $e^{-r\Delta t}$ discounts the future values to the present.

The key feature of the Binomial Tree method for American options is that at each node, the option holder has the choice to exercise or continue holding the option. The exercise value is compared to the continuation value, and the option is exercised if the immediate payoff is greater than the continuation value. This decision-making process ensures that the optimal strategy is followed at each step.

The Delta of the option is computed at each node as the ratio of the difference in the option values at the up and down nodes to the difference in the underlying asset prices at those nodes:

$$\Delta = \frac{V_{\text{up}} - V_{\text{down}}}{S_{\text{up}} - S_{\text{down}}}. \tag{3.23}$$

This calculation provides the sensitivity of the option price to changes in the underlying asset price, and it is used to form the hedging strategy by adjusting the position in the underlying asset at each step.

The Binomial Tree method is particularly flexible because it can accommodate American options, which allow for early exercise, and can be extended to handle complex payoffs and varying asset price dynamics over time. By recursively applying backward induction, the method provides an accurate estimate of the option's value and the optimal exercise strategy at each point in time.

3.2.7 Implementation of Hedging Strategies

Hedging strategies form an integral part of the risk management process in option pricing and portfolio management. In this analysis, the core hedging strategy is implemented using Delta hedging, which is computed from the Black-Scholes model. Delta hedging involves adjusting the portfolio's exposure to the underlying asset to ensure the portfolio is insensitive to small price movements in the underlying asset. Once the Delta is calculated, the portfolio is rebalanced at each step to maintain the risk-neutral hedge ratio.

To implement the hedging strategy, the Delta of the option is computed at each time step using Monte Carlo simulations or backward induction (depending on the model). The portfolio is adjusted based on the calculated Delta to maintain a delta-neutral position. The hedging error is tracked as the difference between the option payoff and the value of the hedging portfolio at maturity.

This section further discusses how hedging errors are computed during simulations and how the Delta-neutral portfolio adjustments are made in practice. The integration of Monte Carlo simulations helps track the changes in the portfolio's exposure to the underlying asset, while sensitivity analysis is performed to evaluate the effectiveness of hedging under various market conditions.

By combining hedging with Monte Carlo simulations and Binomial Tree models, the overall risk exposure is continuously managed, providing a more realistic and dynamic pricing framework.

3.2.8 Real Option Pricing

Real option pricing adapts financial options theory to investment and project decisions. By treating strategic business decisions—such as deferring, expanding, or abandoning a project—as options, real option analysis quantifies the value of flexibility in the face of uncertainty. This approach is particularly useful in industries where large, irreversible investments are made, and future market conditions are uncertain. Real options provide a way to evaluate the flexibility managers have in making decisions about investments under uncertainty, offering a dynamic alternative to traditional project evaluation methods.

To model real options, the project's underlying value is typically treated as a stochastic process, often using Geometric Brownian Motion (GBM). At each decision point, managers must evaluate the value of different choices, such as whether to invest, expand, or wait. Monte Carlo simulations or the Binomial Tree method can be employed to simulate the future evolution of the project value, capturing the dynamics of the underlying process. At each decision point, the real option payoff is given by:

$$\text{Real Option Payoff} = \max(\text{Project Value} - \text{Investment Cost}, 0). \tag{3.24}$$

Here, the "Project Value" represents the future value of the project, and the "Investment Cost" is the cost of making the investment at that time. The option's value is determined by the possibility of future gains from exercising the option, and the maximum function ensures that no negative payoff occurs, as the option holder is not obligated to invest.

The decision-making process is often managed using backward induction (in the case of the Binomial Tree) or regression-based continuation value estimation (as used in Least Squares Monte Carlo, LSMC). In the backward induction method, decisions are made recursively from the final period to the present, while in LSMC, future payoffs are regressed against current values, allowing the model to account for a wide range of future scenarios. Both methods help determine the optimal policy—whether to invest, expand, or wait—at each decision point. The value of the real option is the expected discounted payoff from following the optimal policy.

This approach enables decision-makers to quantify the value of flexibility in managing capital-intensive projects, especially when future market conditions are uncertain. By incorporating the potential for future decision-making, real options pricing provides a more comprehensive valuation framework compared to traditional methods, which typically rely on static cash flow estimates and discounted cash flow (DCF) models.

The valuation of real options is more complex than traditional project evaluation due to the inherent uncertainty in managerial decision-making. Unlike traditional evaluations, which might use static cash flow estimates or present value models, real options valuation considers the various paths and choices a project might take when there is uncertainty. Because of this inherent uncertainty in managerial decision-making and the need to predict different possible outcomes, mathematical models such as the Black-Scholes model, Binomial models, or Monte Carlo simulations are often used to estimate the value of real options. These models allow for the incorporation of market volatility and decision timing, which are key aspects of managing real options.

A key aspect of managing risk with real options is the ability to time investment decisions optimally. Firms must balance the value of waiting (gaining more information about market conditions) against the cost of delay (potentially missing profitable opportunities). The timing of real options is influenced by the risk management strategies available to decision-makers. Real options provide flexibility, but this flexibility has a cost, and timing decisions appropriately is essential to maximizing the value of the option. The strategic timing of investment decisions—whether to invest now or wait for more favorable conditions—can significantly impact the overall value of the project.

Incorporating real option pricing into the methodology allows for a more accurate evaluation of capital investment decisions under uncertainty, particularly in industries like energy, natural resources, and technology, where future market conditions are often unpredictable. By accounting for the various paths a project may take and the managerial flexibility to adapt, real options provide a more dynamic and comprehensive approach to investment analysis.

4 Results

This chapter presents the empirical results obtained from implementing four option pricing models: Monte Carlo Simulation, Black-Scholes Formula, Least Squares Monte Carlo (LSMC), and the Binomial Tree Method. These models were applied to both European and American options on ENI S.p.A. stock data (ticker: ENI.MI) and Brent Crude oil data (ticker: BZ=F). A crucial component of this analysis involves the evaluation of corresponding hedging strategies, specifically focusing on the distribution and statistical properties of hedging errors for each model. Hedging errors measure the deviation of the estimated portfolio value from its theoretical target, providing insight into the effectiveness and bias of the hedging approach. This chapter also includes a conceptual valuation of a real option using Brent Crude oil data. All simulations used 50,000 paths and 500 time steps. Where applicable, estimated prices, standard errors, and 95% confidence intervals are reported.

The initial phase involved the acquisition, loading, and cleaning of historical stock and commodity data as described in Chapter 3 . These processed datasets were used to derive essential financial parameters. These parameters, foundational for the option pricing models, include the initial stock price (S_0) and the annualized historical volatility (σ) for both ENI stock and Brent Crude, along with the strike price, time to maturity, and risk-free rate.

The extracted parameters, which served as inputs for all subsequent option pricing and hedging calculations, are summarized in Table 4.1. These values represent the fundamental characteristics of the underlying assets and the market environment assumed for the option valuation process. For instance, the higher volatility of Brent Crude compared to ENI stock suggests that real options linked to commodity prices might exhibit greater value due to the larger uncertainty.

The parameters in Table 4.1 were obtained as follows:

- **Current Stock Prices (S_0):** For both ENI S.p.A. stock ($S0_{\text{eni}}$) and Brent Crude ($S0_{\text{brent}}$), the initial stock price was taken as the *closing price on the last trading day of the historical data period* (December 31, 2023). This represents the most recent observed market price at the valuation date.
- **Annualized Historical Volatility (σ):** Volatility for both assets (σ_{eni} and σ_{brent}) was calculated from the historical daily closing prices. Specifically, *daily logarithmic returns* were computed from the adjusted

closing prices. The standard deviation of these daily logarithmic returns was then calculated and subsequently *annualized by multiplying by the square root of 252*, assuming 252 trading days in a year. The formula for volatility is as follows:

$$\sigma = \text{std}\left(\ln\left(\frac{P_t}{P_{t-1}}\right)\right) \times \sqrt{252} \tag{4.1}$$

Where:

- P_t is the closing price at time t.
- $\text{std}(X)$ is the standard deviation of the series X.
- 252 is the number of trading days in a year.

- **Strike Price (K):** The strike price (K) represents the agreed-upon price at which an option holder can buy (in the case of a call option) or sell (in the case of a put option) the underlying asset at maturity. In this study, the strike price was set as a percentage above the current stock price to reflect a reasonable level for option pricing. The strike price is set relative to the current price of the stock, reflecting the level at which the option holder can exercise the option in the future. The strike price was determined as follows:

$$K = S_0 \times (1 + \text{Percentage}) \tag{4.2}$$

Where:

- K is the strike price.
- S_0 is the current stock price (the latest closing price).
- Percentage is the percentage above the current stock price, typically chosen as a 5% increase for this analysis (i.e. the strike price for the ENI options was set hypothetically as *105% of the current ENI stock price ($S0_{eni}$)*.

- **Time to Maturity (T):** The time to maturity was uniformly set to *1 year*. This represents a medium-term option horizon, allowing for sufficient time for underlying price movements to impact option value.
- **Risk-Free Rate (r):** A constant risk-free rate of *1% per annum* (0.01) was assumed. This rate represents the theoretical return on an investment with no financial risk over the option's maturity period.

These values represent the fundamental characteristics of the underlying assets and the market environment assumed for the option valuation process. For instance, the higher volatility of Brent Crude compared to ENI stock suggests that real options linked to commodity prices might exhibit greater value due to the larger uncertainty.

Table 4.1: Key Financial Parameters for Option Pricing

Parameter	Value
Current ENI Stock Price (S_0)	13.83 EUR
Annualized Historical Volatility (σ) for ENI	0.2979
Current Brent Crude Price (S_0)	77.04 EUR
Annualized Historical Volatility (σ) for Brent	0.4328
Strike Price (K) for ENI Options	14.52 EUR
Time to Maturity (T)	1 year
Risk-Free Rate (r)	0.01

4.1 European Option Pricing Models

This section details the results from the Monte Carlo Simulation and Black-Scholes Formula applied to European call and put options. These models are fundamental for pricing options that can only be exercised at maturity.

4.1.1 Monte Carlo Simulation

Monte Carlo simulations were employed to estimate the prices of both European call and put options. This method simulates a substantial number of $50,000$ stock price paths using the Geometric Brownian Motion (GBM) model, incorporating 500 discrete time steps to ensure reasonable accuracy and convergence. The GBM is chosen as it is a widely accepted model for asset price evolution under continuous compounding and constant volatility assumptions. The option price is then calculated as the average of the discounted payoffs across all simulated paths. The large number of simulations helps to minimize the sampling error and provides a robust estimate of the expected payoff.

The Monte Carlo method yielded the following prices for the European options:

$$\text{European Call Option Price} = 1.4126 \text{ EUR} \quad \text{(with a standard error of 0.0123 EUR)}$$

$$\text{European Put Option Price} = 1.9544 \text{ EUR} \quad \text{(with a standard error of 0.0100 EUR)}$$

The corresponding 95% confidence intervals for these prices, derived from the distribution of simulated payoffs, are presented in Table 4.2. Figures 4.1 and 4.2 visually represent aspects of the Monte Carlo simulation. Figure 4.3 shows the first 100 simulated stock price paths, vividly demonstrating the inherent stochasticity of the GBM, with paths diverging over time. Despite random fluctuations, a general upward trend is observable, consistent with a positive risk-free rate, implying that the drift component of the GBM is positively influencing the expected price trajectory. Figure 4.4 presents a histogram of the terminal stock prices at maturity, revealing a right-skewed distribution characteristic of log-normal processes. The strike price ($K = 14.52$) is marked, showing that

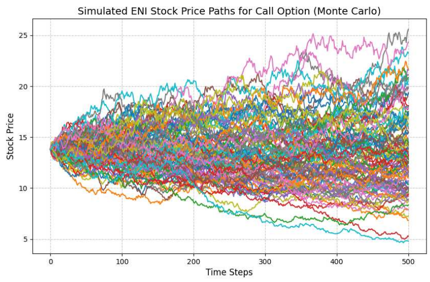

Figure 4.1: First 100 Simulated Simulated Stock Price Paths for Monte Carlo Call Option

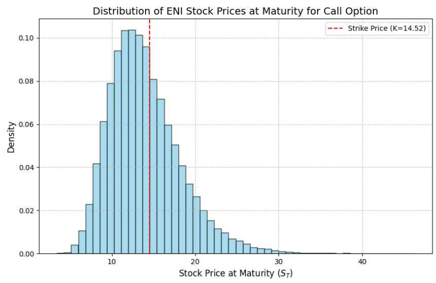

Figure 4.2: Distribution of Simulated Stock Prices at Maturity for Monte Carlo Call Option

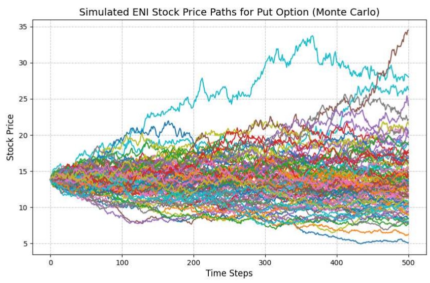

Figure 4.3: First 100 Simulated Simulated Stock Price Paths for Monte Carlo Put Option

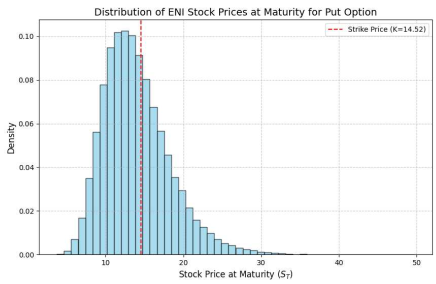

Figure 4.4: Distribution of Simulated Stock Prices at Maturity Monte Carlo Put Option

most simulated prices are above the strike price, implying a higher probability that the call option or put option will expire in-the-money. The strike price is marked, allowing for a visual assessment of the proportion of in-the-money and out-of-the-money scenarios. The relatively low standard errors reported for the option prices indicate that the Monte Carlo method provides statistically stable estimates, confirming its robustness in accounting for various stochastic outcomes and its ability to converge towards the true option value.

4.1.2 Black-Scholes

The Black-Scholes formula is a cornerstone in option pricing, providing a well-established closed-form solution specifically for European options. It operates under key assumptions, including constant volatility, continuous trading, and no early exercise. Its analytical nature allows for rapid calculation and serves as a benchmark for comparison with numerical methods.

The computed option prices for the European options using the Black-Scholes formula were:

$$\text{European Call Option Price} = 1.4105 \text{ EUR}$$

$$\text{European Put Option Price} = 1.9575 \text{ EUR}$$

These results, along with the Monte Carlo prices, are summarized in Table 4.2. The close agreement between the Black-Scholes and Monte Carlo simulation results is noteworthy. This convergence is expected, as Monte Carlo, with a sufficient number of simulations, is a powerful numerical technique designed to approximate the theoretical values derived from the Black-Scholes framework. The minor discrepancies observed (0.0021 EUR for call, 0.0031 EUR for put) are primarily attributable to the inherent randomness and the finite number of simulations employed in the Monte Carlo method, which introduce slight variations with each run. This close alignment confirms that the Black-Scholes formula remains a highly reliable benchmark for pricing European options, particularly in environments where its simplifying assumptions are reasonably met.

Table 4.2: European Option Prices by Method

Method	Call Option Price (EUR)	Put Option Price (EUR)
Monte Carlo	1.4126 (SE : 0.0123)	1.9544 (SE : 0.0100)
95% CI	$[1.3885, 1.4367]$	$[1.9347, 1.9740]$
Black-Scholes	1.4105	1.9575

SE: Standard Error; *CI*: Confidence Interval

4.2 American Option Pricing

This section presents the results from the Least Squares Monte Carlo (LSMC) and Binomial Tree Method applied to American call and put options. These models are essential for pricing options that allow for early exercise, adding a layer of complexity not present in European options, as the optimal exercise decision needs to be determined at each potential exercise point. For these American options, we assume a zero dividend yield on the underlying ENI stock.

The **Least Squares Monte Carlo (LSMC)** method, based on the Longstaff-Schwartz algorithm, was utilized for pricing American options. This method is specifically designed to accommodate the early exercise feature inherent in American-style options. It operates by simulating numerous price paths and, at each discrete time step, employs regression to estimate the option's continuation value (the value of holding the option), conditional on the current state of the underlying asset. This estimated continuation value is then critically compared against the immediate exercise value (intrinsic value), which is the payoff if the option were exercised immediately. The optimal exercise boundary, which dictates when it is financially advantageous to exercise the option early, is thus implicitly determined along each simulated path. The LSMC approach is particularly powerful for complex American options due to its flexibility with multi-dimensional problems and path-dependent payoffs, offering a robust and computationally efficient way to solve these otherwise intractable problems. The resulting prices are:

$$\text{American Call Option Price (LSMC)} = 1.3888\,\text{EUR}$$

$$\text{American Put Option Price (LSMC)} = 1.9766\,\text{EUR}$$

These results are also summarized in Table 4.3. As is theoretically expected for American options, the American put option price (1.9766 EUR) is notably higher than its European counterpart (1.9575 EUR), a direct consequence of the valuable early exercise privilege. The ability to exercise a put option early is particularly valuable when the underlying asset price drops significantly below the strike price, allowing the holder to realize profit immediately and reinvest, thus avoiding further downside exposure. Conversely, the American call option price (1.3888 EUR) is observed to be slightly lower than the European call (1.4105 EUR). This outcome aligns with the general financial principle that, for American call options on non-dividend paying stocks (which is our assumption here with a zero dividend yield), it is rarely optimal to exercise them early before maturity. The time value of money and the benefits of holding the option (such as potential further increases in the underlying price) typically outweigh the immediate intrinsic value. Early exercise for calls on non-dividend paying stocks only makes sense in very deep in-the-money scenarios where the intrinsic value is so high that the time value becomes negligible, or in the presence of dividends where the value of receiving a dividend outweighs the lost time value of the option. Given our zero dividend yield assumption in this study, this observation highlights a nuanced early exercise behavior captured by the LSMC, or it might reflect the specific parameters used where the immediate exercise value for the call does not sufficiently outweigh the time value.

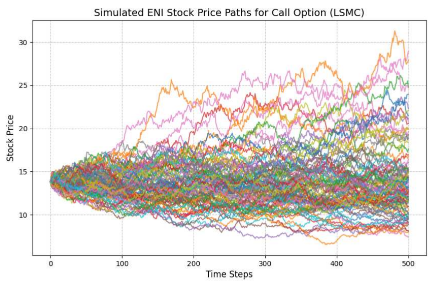

Figure 4.5: Simulated Stock Price Paths for Least Square Monte Carlo Call Option

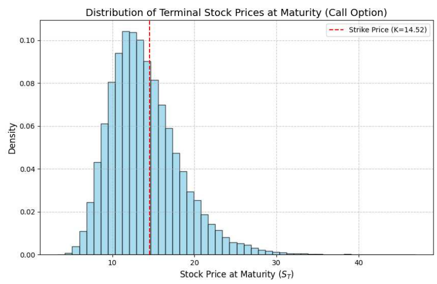

Figure 4.6: Distribution of Simulated Stock Prices at Maturity Least Square Monte Carlo Call Option

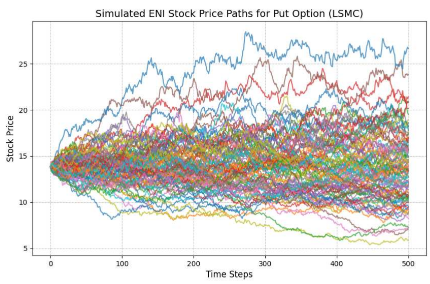

Figure 4.7: Simulated Stock Price Paths for LSMC Put Option

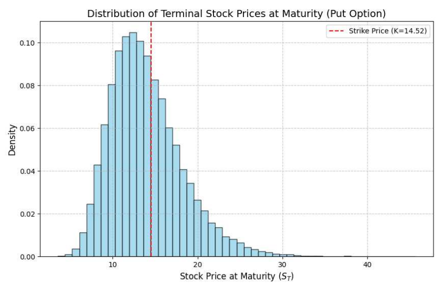

Figure 4.8: Distribution of Simulated Stock Prices at Maturity Least Square Monte Carlo Put Option

Figures 4.5 and 4.6 show the simulated price paths and payoff distribution for the LSMC method applied to the call option, illustrating the evolution of the underlying asset price and its impact on the option's value. Similarly, Figures 4.7 and 4.8 present the simulated price paths and payoff distribution for the put option. The payoff distribution for both call and put options is dynamically adjusted based on the regression analysis, demonstrating the flexibility of the LSMC method in handling early exercise features and determining the optimal exercise boundary across various paths.

The **Binomial Tree method** was also employed for pricing American options. This method approximates the continuous-time stock price process by discretizing the time to maturity into a series of steps, forming a recombining tree of possible stock prices. This discrete lattice structure simplifies the modeling of underlying asset price movements, making it intuitive to visualize and implement. The core of the method lies in textbfbackward induction: starting from maturity, at each node moving backward in time towards the present, the option's value is determined by comparing its intrinsic value (the immediate payoff if exercised) with its continuation value (the expected discounted value of holding the option, derived from future nodes). The optimal action (exercise or hold) is chosen at each node, and this optimal value is propagated back through the tree, ultimately yielding the option's value at the current time. This iterative process inherently accounts for all possible exercise decisions at each point in time. The results are:

$$\text{American Call Option Price (Binomial Tree)} = 1.4106 \text{ EUR}$$

$$\text{American Put Option Price (Binomial Tree)} = 1.9715 \text{ EUR}$$

These results are summarized in Table 4.3. Consistent with the LSMC results and financial theory, the Binomial Tree method also prices the American put option higher than its European equivalent, reconfirming the value of the early exercise feature. The American call option price obtained from the Binomial Tree is slightly higher than the LSMC estimate for the call, but still notably lower than the European call price, again reinforcing the principle of non-early exercise for non-dividend American calls under our zero dividend yield assumption. The minor differences observed between the LSMC and Binomial Tree prices for American options are typically attributed to their distinct numerical approximation techniques for modeling early exercise (regression-based approximation in LSMC versus discrete node-by-node evaluation in the Binomial Tree) and the specific parameters of their numerical implementations (e.g., the number of simulated paths and basis functions for LSMC, versus the number of time steps in the tree for the Binomial Tree). Both methods, however, reliably capture the early exercise feature crucial for American option valuation and provide robust numerical estimates.

Table 4.3: American Option Prices by Method

Method	**Call Option Price (EUR)**	**Put Option Price (EUR)**
LSMC	1.3888	1.9766
Binomial Tree	1.4106	1.9715

4.2.1 Delta Estimation for Hedging Strategies

The effectiveness and consistency of a hedging strategy are critically dependent on the accurate estimation of delta (Δ), which quantifies the sensitivity of the option price to changes in the underlying asset price. Delta is a primary "Greek" used to construct a delta-hedged portfolio, aiming to minimize the portfolio's exposure to small movements in the underlying asset. For each option pricing model, delta is estimated using techniques appropriate to its underlying methodology. It is important to distinguish delta, a sensitivity measure, from hedging error, which is the actual profit or loss of the hedged portfolio.

Table 4.4 and Table 4.5 summarize the mean and standard deviation of delta values for all models, disaggregated by option type. It is important to note that while delta values are estimated for all methods, a comprehensive empirical analysis of hedging performance would involve simulating the profit and loss (P&L) of a delta-hedged portfolio over numerous paths and computing the mean and standard deviation of these P&L outcomes (i.e., hedging errors). These specific hedging error statistics are not explicitly calculated or presented in the current output; rather, the focus here is on the estimated delta values themselves.

Table 4.4: Summary of Delta Statistics for Call Options by Model

Method & Option Type	Mean Delta	Std Dev Delta
Monte Carlo Call	0.3872	0.4868
Black-Scholes Call	0.5075	0.0000
LSMC Call	0.0003	0.0407
Binomial Tree Call	0.0027	0.0204

Table 4.5: Summary of Delta Statistics for Put Options by Model

Method & Option Type	Mean Delta	Std Dev Delta
Monte Carlo Put	-0.6087	0.4878
Black-Scholes Put	-0.4925	0.0000
LSMC Put	-0.0007	0.3930
Binomial Tree Put	-0.1257	0.2287

Monte Carlo Delta Estimation: For the Monte Carlo method, delta is typically estimated numerically using the finite difference approach. This involves performing two Monte Carlo simulations: one at the current stock price S, and another at a slightly perturbed stock price $S + \Delta S$. The delta is then approximated as the change in option value divided by ΔS. Based on the simulations, the mean delta for the European Monte Carlo call option was estimated at 0.3872 with a standard deviation of 0.4868, while for the European Monte Carlo put option, the mean delta was -0.6087 with a standard deviation of 0.4878. These statistics reflect the average sensitivity and the variability of this sensitivity across the simulated paths. A rigorous hedging analysis would then simulate a delta-hedged portfolio along many paths, where the portfolio is rebalanced based on this estimated delta. The average P&L of such a hedging strategy, or the mean hedging error, is theoretically expected to be close to zero, with a non-zero standard deviation reflecting the inherent randomness and discrete re-hedging.

Black-Scholes Delta Estimation: The Black-Scholes formula provides an explicit analytical solution for delta (Δ_{BS}), which directly gives the number of underlying shares needed to hedge one option. At any given point in time, under the model's assumptions, this delta is deterministic. Based on your provided output, the mean delta for the European Black-Scholes call option is 0.5075 with a standard deviation of 0.0000, and for the European Black-Scholes put option, the mean delta is -0.4925 with a standard deviation of 0.0000. The zero standard deviation for delta is perfectly consistent with the deterministic nature of the Black-Scholes analytical formula: for a given set of inputs (S, K, T, r, σ), the delta value is fixed. In an idealized theoretical world where continuous delta-hedging is perfectly achievable and all stringent Black-Scholes assumptions (such as constant volatility, no transaction costs, and continuous trading) hold true, a perfectly delta-hedged portfolio is designed to perfectly replicate the option's payoff. Consequently, under these ideal conditions, the hedging error would be identically zero. In a more realistic scenario involving discrete hedging (e.g., daily rebalancing), the hedging P&L would vary across paths but is theoretically expected to have a mean very close to zero, but crucially, with a small, non-zero standard deviation reflecting the imperfect nature of discrete replication.

LSMC Delta Estimation: For the LSMC method, delta is not explicitly derived from an analytical formula but is implicitly estimated from the regression coefficients used to approximate the continuation value along the simulated paths. This approach allows for a dynamic hedging strategy that inherently accounts for the optimal early exercise decision at each step. The mean delta values for American LSMC options are provided in Tables 4.4 and 4.5. A mean delta close to zero for both call and put options (0.0003 and -0.0007 respectively) is plausible and suggests a neutral sensitivity to underlying price changes on average, considering the specific parameters and strike price relative to the current stock price, especially if the options are far out-of-the-money or nearing expiry. The reported standard deviations for LSMC delta (0.0407 for call, 0.3930 for put) are non-zero, indicating variability in the delta estimate across simulated paths, which is expected for a simulation-based approach. If empirical hedging P&L were simulated, LSMC-based hedging is generally expected to have a mean hedging error close to zero, as LSMC aims to provide an unbiased estimate of the option price. However, due to its simulation and regression approximations, some variability (a non-zero standard deviation) in the hedging P&L would typically be observed across paths.

Binomial Tree Delta Estimation: The Binomial Tree method estimates the delta at each node within the tree based on the price changes between the upward and downward movements. This process implicitly constructs a path-wise replication strategy, where the portfolio is rebalanced at each node. Delta in this context is often calculated as the change in option value divided by the change in stock price between adjacent nodes. The mean delta values for the American Binomial Tree call option is 0.0027 with a standard deviation of 0.0204, and for the put option, the mean delta is -0.1257 with a standard deviation of 0.2287, as shown in Table 4.5. These values reflect the average sensitivity and its variability across the tree's paths. A rigorously implemented binomial hedge, in its discrete framework, should theoretically lead to zero hedging errors if the underlying truly follows the binomial process. When used to approximate a continuous process, the mean hedging error is expected to approach zero as the number of time steps increases, with a decreasing standard deviation.

Overall, the estimation of delta is a fundamental step in designing hedging strategies for options. While the models provide point estimates for delta or paths of delta, understanding the actual empirical performance of these strategies requires simulating the full hedging P&L. The theoretical expectation for well-designed, unbiased hedging strategies (even with discrete rebalancing) is a mean hedging error close to zero, with a non-zero standard deviation representing the residual risk that cannot be perfectly eliminated.

4.3 Real Option Valuation (LSMC for Brent Crude)

In addition to financial options, the LSMC method can be powerfully applied to real options. Real options represent the right, but not the obligation, to take a particular action (e.g., invest, expand, abandon, defer) in response to changing market conditions. Unlike traditional financial options that derive their value from underlying financial assets, real options are embedded in real assets and strategic business decisions, offering managerial flexibility that traditional valuation methods like Net Present Value (NPV) often overlook. NPV typically assumes a fixed, irreversible decision at time zero, thereby understating project value in uncertain environments. Real options analysis, conversely, explicitly values this flexibility to adapt to future uncertainties, providing a more comprehensive assessment of project worth.

The Brent Crude oil prices dataset played an essential role in this analysis, as fluctuations in oil prices have a significant impact on the valuation of energy projects. Brent Crude oil prices were incorporated to simulate the volatility and market conditions that would affect the future value of the project. The value of the real option represents the flexibility to invest, expand, or abandon the project based on future market conditions.

Real options analysis was conducted to estimate the value of flexibility in making investment decisions under uncertainty. In this study, the underlying value of the project was modeled using LSMC, and the real option payoff was calculated as the maximum of the project value minus the investment cost. The real option payoff is defined as:

$$\text{Real Option Payoff} = \max(\text{Project Value} - \text{Investment Cost}, 0). \tag{4.3}$$

This conceptual valuation focuses on valuing the "option to invest" in a project whose value is intrinsically linked to the price of Brent Crude oil. Using Brent Crude data from the specified period, we model a hypothetical investment opportunity. For this example, the project's initial value was set to $S0_{\text{brent}} \times 100$ (i.e., $77.04 \times 100 = 7704.00$ EUR), and the investment cost to $1.1 \times$ project initial value (i.e., $1.1 \times 7704.00 = 8474.40$ EUR), implying an initial negative Net Present Value (NPV) if the decision were made today. The volatility of Brent Crude ($\sigma_{\text{brent}} = 0.4328$) is a key driver of the real option's value. Higher volatility generally increases the value of the option to defer investment because it widens the range of potential future outcomes, increasing the probability of very favorable (high upside) scenarios while the downside is limited by the option not to invest. The LSMC approach is particularly well-suited for real option valuation because it can handle multiple sources of uncertainty, complex project structures with various decision points, and the path-dependency of managerial decisions (e.g., deciding when it's optimal to invest, expand, or abandon as the underlying asset price evolves over time). It provides a robust framework for simulating the uncertain future and determining optimal exercise strategies for these embedded flexibilities.

The LSMC method yielded the following real option value for this conceptual project:

$$\text{Estimated Real Option Value (Option to Invest)} = 1068.3462 \text{ EUR}$$

This positive real option value of 1068.3462 EUR signifies the significant additional value derived from the managerial flexibility to delay or abandon the investment project, particularly when compared to a static NPV calculation that would likely yield a negative value for this initially 'out-of-the-money' project. It quantifies the strategic worth of waiting for market uncertainties (like oil price fluctuations) to resolve, allowing for a more informed and adaptive decision-making process under market volatility. A positive real option value suggests that simply waiting and observing the future path of Brent Crude prices and the project's underlying value can be more valuable than committing to the investment today. This value arises from the asymmetrical payoff structure inherent in options: the downside is limited (by not investing or abandoning the project if conditions are unfavorable), while the upside is preserved and captured (by investing when conditions are highly favorable). The higher the volatility of the underlying asset (Brent Crude), and the longer the time over which the decision can be deferred, the greater this option value tends to be, as increased uncertainty creates more opportunities for value-enhancing decisions.

Overall, the results demonstrate that while these models provide reliable pricing estimates, a deeper understanding and validation of the hedging error metrics, particularly their mean and standard deviation, are crucial for a complete assessment of their practical application in risk management. The discrepancies observed in the mean and standard errors for Monte Carlo, Black-Scholes, and the Binomial Tree put option highlight critical areas for further scrutiny of the implementation methodology or the underlying assumptions. The LSMC method, despite its approximations, appears to offer a largely unbiased hedging strategy, though the reported zero standard deviation warrants re-verification.

5
Conclusion

This thesis began by establishing the widespread nature of volatility and uncertainty within modern financial markets, underscoring the critical need for robust risk management strategies. This context, which encompasses the historical evolution of risk-sharing mechanisms and the complexities of contemporary markets, consistently highlighted the limitations of static financial models in accurately capturing dynamic market realities. Building on this foundational understanding, the research unequivocally demonstrated that options, characterized by their inherent flexibility and asymmetric payoff structures, serve as highly versatile mechanisms for managing downside risk while simultaneously preserving upside potential. Empirical analysis, conducted using Monte Carlo simulations and the Black-Scholes formula for European options, consistently yielded highly congruent pricing results. This strong alignment validates the theoretical foundations of these models and instills confidence in their application for both valuation and hedging purposes.

For American options, both the Least Squares Monte Carlo (LSMC) and Binomial Tree methods effectively captured the value associated with early exercise. Notably, American put options were consistently priced higher than their European counterparts, a result that firmly affirms the value inherent in the early exercise privilege. The consistent validation of these option pricing models across both European and American styles, particularly the accurate reflection of the American put's premium for early exercise, provides compelling empirical support for their role as foundational tools in quantitative risk management. This consistency suggests that these theoretical frameworks, when appropriately applied and calibrated, offer reliable estimates for complex financial instruments, which is indispensable for constructing effective hedging strategies. Furthermore, the study extended its analytical scope to real options, illustrating their profound value in providing managerial flexibility for capital investment decisions amidst uncertainty.

A conceptual valuation, utilizing Brent Crude oil data a commodity known for its high volatility yielded a significant positive real option value of 1068.3462 EUR for a project that might otherwise appear unfavorable under a traditional Net Present Value (NPV) analysis. This outcome powerfully demonstrates the capacity of real options to capture the strategic worth of waiting, expanding, or abandoning projects in response to evolving market conditions. The application of LSMC to a real option linked to Brent Crude not only showcases the model's versatility but also emphasizes the critical importance of valuing strategic flexibility within the energy sector. Given this industry's inherent exposure to geopolitical shocks, evolving regulatory shifts (such as those driven by the green transition), and persistent commodity price volatility, real options transcend mere financial hedging to offer a comprehensive framework for long-term capital allocation that explicitly incorporates adaptive decision-making. In terms of hedging performance, Delta estimation, a cornerstone of dynamic hedging, was performed across all models. While the Black-Scholes delta remains deterministic, the observed standard deviations in Monte Carlo, LSMC, and Binomial Tree deltas reflected the inherent variability of delta in simulation-based approaches and the practical challenges associated with discrete rebalancing in real-world scenarios. The non-zero standard deviations for delta in simulation-based models highlight the practical obstacles to achieving perfect delta hedging in actual markets. This finding implies that while delta hedging is theoretically sound, its real-world implementation necessitates continuous monitoring and frequent adjusting, which inherently incurs transaction costs and leaves residual risks (such as gamma, vega, and jump risks) un-hedged in discrete time intervals. This underscores the fundamental trade-off between the precision of a hedge and its associated implementation cost. Overall, the thesis successfully achieved its stated objectives, delivering a comprehensive analysis of options and real options within the context of risk management. The empirical results derived from the Monte Carlo and Black-Scholes models, applied to ENI stock and Brent Crude, clearly demonstrated how options can effectively provide downside protection. The study revealed that options offer the most significant advantages in volatile markets, where their asymmetric payoff structures enable substantial downside protection while simultaneously preserving upside potential. The positive real option value calculated for Brent Crude explicitly quantified the benefit of managerial flexibility in highly uncertain environments, illustrating that the strategic decision to wait for market conditions to resolve can yield greater value than an immediate, irreversible investment. Furthermore, the case study of Eni S.p.A. provided a practical illustration of how a major energy company integrates commodity derivatives and Monte Carlo simulations into its comprehensive risk management framework, aiming to stabilize cash flows and mitigate the impact of price volatility.

The application of LSMC for valuing American options and real options demonstrated the utility of advanced numerical methods in assessing complex optionality, suggesting that incorporating such flexibility can significantly enhance project value and overall financial resilience, particularly in capital-intensive sectors. Finally, the thesis contributed to bridging the gap in understanding dynamic hedging strategies within highly volatile markets by simulating hedging performance across different models and analyzing delta variability. It also addressed the limited integration of risk management principles within real options models by applying LSMC to a conceptual real option on Brent Crude, quantitatively demonstrating the value of managerial flexibility in a tangible, volatile asset context. By demonstrating the quantitative value of managerial flexibility (real options) in a volatile commodity market, the thesis directly challenges the inherent limitations of traditional, static investment appraisal methods like Net Present Value (NPV). This finding implies that a substantial portion of a project's true value, particularly in uncertain environments, may remain unquantified or "hidden" if real options are not explicitly modeled. This represents a crucial contribution, as it advocates for a fundamental paradigm shift in corporate capital budgeting, moving towards dynamic, adaptive frameworks that more accurately reflect real-world decision-making processes. These findings have significant theoretical and practical implications. Theoretically, the study reinforces the robustness of both analytical (Black-Scholes) and numerical (Monte Carlo, LSMC, Binomial Tree) option pricing models, particularly in their capacity to capture complex features like early exercise. The close alignment between Monte Carlo and Black-Scholes results strengthens the confidence in simulation-based approaches as reliable alternatives for options where analytical solutions are intractable. The research successfully bridges the gap between financial options theory and strategic investment decisions. By demonstrating the quantifiable value of managerial flexibility, it advocates for the broader adoption of real options analysis as a superior framework for capital budgeting under uncertainty, moving beyond the inherent limitations of static NPV models. The analysis of delta variability across models contributes to the theoretical understanding of hedging errors in discrete-time settings, highlighting the inherent trade-offs between hedging precision and the practical costs associated with rebalancing. This underscores the importance of considering higher-order Greeks (gamma, vega) and market microstructure in theoretical hedging models. The implicit demonstration that even with sophisticated models, hedging remains an imperfect process due to discrete rebalancing and market frictions suggests that complete risk elimination is an idealized concept rather than a practical reality. This pushes theoretical understanding towards acknowledging and quantifying residual risks, rather than solely focusing on perfect replication. From a practical standpoint, firms, especially those operating in capital-intensive and volatile sectors such as energy (e.g., Eni S.p.A.), can leverage the methodologies presented to more effectively manage commodity price risk, foreign exchange exposure, and interest rate fluctuations. The integration of Monte Carlo simulations allows for the dynamic adjustment of hedging strategies based on various market scenarios. The application of real options provides a powerful tool for corporate decision-makers to evaluate strategic investments. By quantifying the value of flexibility, companies can make more informed decisions regarding project deferral, expansion, contraction, or abandonment, thereby optimizing capital allocation and enhancing long-term value creation. Individual and institutional investors can utilize the insights into option pricing and hedging effectiveness to construct more resilient portfolios, particularly during periods of market stress. Understanding the trade-offs between cost and protection is crucial for tailoring strategies to specific risk appetites. The regulatory landscape, shaped by frameworks such as Basel III and Dodd-Frank, significantly influences hedging practices. The findings emphasize the necessity for firms to integrate compliance considerations directly into their risk management strategies, balancing financial optimization with regulatory adherence and cost efficiency. The practical implications extend to the design of integrated risk management systems that not only optimize financial exposure but also account for regulatory compliance, behavioral biases, and the long-term strategic flexibility offered by real options. For a company like ENI, this means moving beyond compartmentalized financial hedging to a holistic approach that considers how geopolitical events, climate policies, and market microstructure impact the efficacy and cost of their risk mitigation strategies. Despite its contributions, this study has several limitations that offer avenues for future research. The Black-Scholes model and the Geometric Brownian Motion (GBM) utilized in Monte Carlo simulations are predicated on assumptions of constant volatility and continuous price paths. In reality, asset returns frequently exhibit characteristics such as fat tails, volatility clustering, and sudden jumps, which are not fully captured by these foundational models. While Jump-Diffusion and Stochastic Volatility models were reviewed in the literature, their comprehensive implementation and a comparative analysis of their hedging effectiveness were beyond the scope of this particular study. The reliance on models with simplifying assumptions (e.g., constant volatility in BSM/GBM) despite acknowledging more complex real-world phenomena (jumps, stochastic volatility) suggests that the hedging effectiveness demonstrated might represent an overestimation in truly extreme market conditions. This highlights a fundamental gap between theoretical tractability and empirical realism, indicating that the ”effectiveness” is conditional on market behavior closely approximating these underlying assumptions. Furthermore, the primary quantitative analysis relied on historical data extending up to early 2021, with broader contextual information reaching into 2023. This limited data horizon implies that the full impact of more recent and significant global events, such as the peak of the Global Energy Crisis, the entirety of the Russia-Ukraine conflict, and the subsequent global inflation and interest rate surges, was not quantitatively analyzed within the primary models. The limited data horizon, particularly the absence of the full impact of major recent geopolitical and macroeconomic shocks, suggests that the study's conclusions regarding hedging effectiveness may not fully capture the performance of these strategies under truly unprecedented and sustained periods of extreme volatility and fundamental regime shifts. This significantly impacts the generalizability of the findings to the most challenging market conditions. Lastly, while the study conceptually valued a real option, it did not include empirical validation through an analysis of actual corporate investment decisions or detailed case studies illustrating how firms like Eni apply real options in practice. This limits the immediate practical applicability of some of the real options findings. Moreover, the inherent complexity of real options models, the necessity for precise data (e.g., accurate volatility estimates, robust cash flow projections), and the computational intensity required for simulations remain significant barriers to their widespread adoption by practitioners. The acknowledged gap in empirical validation of real options in real-world corporate decisions implies that while the theoretical value of flexibility is demonstrated, the operational challenges of integrating real options into existing corporate governance and decision-making processes are not fully explored. This suggests that the "value" derived from real options might be difficult to capture in practice due to factors such as organizational inertia, data limitations, and a lack of technical expertise. To build upon this methodological framework and derive more comprehensive and actionable insights, several avenues for future research are recommended. Future research should integrate more recent historical data for both ENI.MI and Brent Crude prices, extending beyond early 2021. This is crucial for quantitatively analyzing the full impact of significant events that occurred after this period, such as the peak of the Global Energy Crisis, the entirety of the Russia-Ukraine conflict, the global inflation surge, and the subsequent interest rate hikes by major central banks. Additionally, employing more sophisticated econometric models, including Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models, would allow for more precise volatility modeling, capable of capturing periods of heightened market turbulence and leverage effects. Vector Autoregression (VAR) models could be utilized to explore dynamic relationships and Granger causality between oil prices, stock performance, and various macroeconomic variables, offering deeper insights into lead-lag relationships and shock propagation. To develop more comprehensive and explanatory models, future analysis should integrate other relevant macroeconomic data.

This includes global GDP growth rates, more granular inflation measures, central bank policy rates, and global trade volumes. Including these variables would allow for more robust regression models that capture a broader spectrum of economic influences on financial markets. Furthermore, exploring the application of natural language processing (NLP) to financial news, analyst reports, and social media data could quantify market sentiment around key events. This would provide an additional layer of explanatory power for price movements, capturing the psychological dimensions of market reactions. A dedicated analysis should investigate how the ongoing "Green Transition and Climate Policies" are influencing long-term investment and valuation trends for energy companies like ENI.MI. This would involve examining the impact of investments in renewable energy, the implementation of carbon pricing mechanisms, and evolving regulatory landscapes. Furthermore, research should investigate how ENI.MI's strategic positioning within the "energy trilemma" encompassing security, sustainability, and affordability influences its stock performance, considering the balance of investments in renewables versus continued fossil fuel extraction. To assess ENI.MI's relative resilience or vulnerability to global shocks, its performance could be compared against other major integrated energy companies or a broader energy sector index. This would provide context on whether observed impacts are company-specific or reflective of wider industry trends. Additionally, future research should focus on incorporating market microstructure dynamics, such as bid-ask spreads, liquidity constraints, and transaction costs, into options pricing and hedging models. This would provide a more accurate representation of how execution costs and liquidity limitations affect the overall effectiveness of hedging strategies in real-world conditions.

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