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Control algorithms for magnetically actuated microrobots

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A tutti coloro che mi hanno sostenuto in questo percorso

Sommario

In questa tesi si affronta il problema di levitazione di una piccola capsula magnetica tramite il campo magnetico generato da un secondo magnete permanente attuato da un manipolatore. Nel Capitolo 1 viene introdotto lo scenario del problema ed alcuni esempi di ambiti applicativi. Nel Capitolo 2 si analizza il sistema, ottenendo un modello in spazio di stato che lo descrive. Nel Capitolo 3 si presenta una strategia di controllo ed infine nel Capitolo 4 vengono riportati risultati e conclusioni.

Abstract

This thesis is focused on the problem of levitation of a small magnetic capsule using the magnetic field generated by a second permanent magnet actuated by a manipulator. In Chapter 1, the problem scenario and some application fields are introduced. In Chapter 2, the system is analysed, describing it through a state-space model. In Chapter 3, a control strategy is presented, and in Chapter 4, the results and conclusions are reported.

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Chapter 1

Introduction

Magnetic microrobots actuated by an external magnetic field are of particular interest in the medical field, since they potentially allow access to hard-to-reach areas of the human body, achieving diminished discomfort and faster recovery [1]; one example is colonscopy, where the camera can be carried by a small permanent magnet that moves under the influence of the field generated by a second permanent magnet placed at the end effector of a manipulator: this is the setup of the problem that will be studied.

Both the source magnet (SM) and the carrier magnet (CM) are modelled as small magnetic dipoles, and the CM has a cylindrical shape. The CM is immersed in intestinal mucus; therefore, in addition to the magnetic interaction and the gravitational force, it also experiences the viscous friction forces and the buoyancy force related to the fluid. The manipulator actuating the SM has 6 degrees of freedom.

Chapter 2

System analysis

2.1 Setup

2.1.1 Problem specification

A more detailed description of the system will now be provided. The carrier magnet (CM) is a small cylindrical permanent magnet that has a magnetic dipole moment $\boldsymbol{m_c}$; the source magnet (SM) is a permanent magnet placed at the end-effector of a 6-DoF manipulator, having a magnetic dipole moment $\boldsymbol{m_s}$. The CM is placed in a fluid (intestinal mucus) with no obstacles, underneath the SM. The CM is actuated by the magnetic field generated by the SM, while the SM is moved using the manipulator; the problem is to control the position in space of the CM by generating an appropriate trajectory for the SM.

2.1.2 Assumptions

Since the CM is immersed in a fluid (and assuming that it does not encounter obstacles), if it is actuated at small speeds and small accelerations, it tends to align itself with the field present in its position; therefore, it will be assumed (provided that the CM moves slowly) that the CM is always aligned with the field generated by the SM [2]. The actual control of the manipulator will not be studied; it will be approximated as a first-order system following the trajectory it receives as input.

As will be discussed in Chapter 3, to simplify the control and modeling, the SM dipole moment $\boldsymbol{m_s}$ will always be held horizontal (i.e., having null vertical component), remaining constant with respect to the world frame; this way, if the CM is kept aligned below the SM, then (since the CM is assumed to be always aligned with the field) the CM dipole moment $\boldsymbol{m_c}$ can also be assumed to always be horizontal and constant with respect to the world frame; in particular, it is assumed that $\hat{\boldsymbol{m}}_s = -\hat{\boldsymbol{m}}_c$ always. In fig. 2.1 the setup is shown schematically:

on top the SM; below it (vertically aligned) the CM whose dipole moment is aligned with the field line in its position.

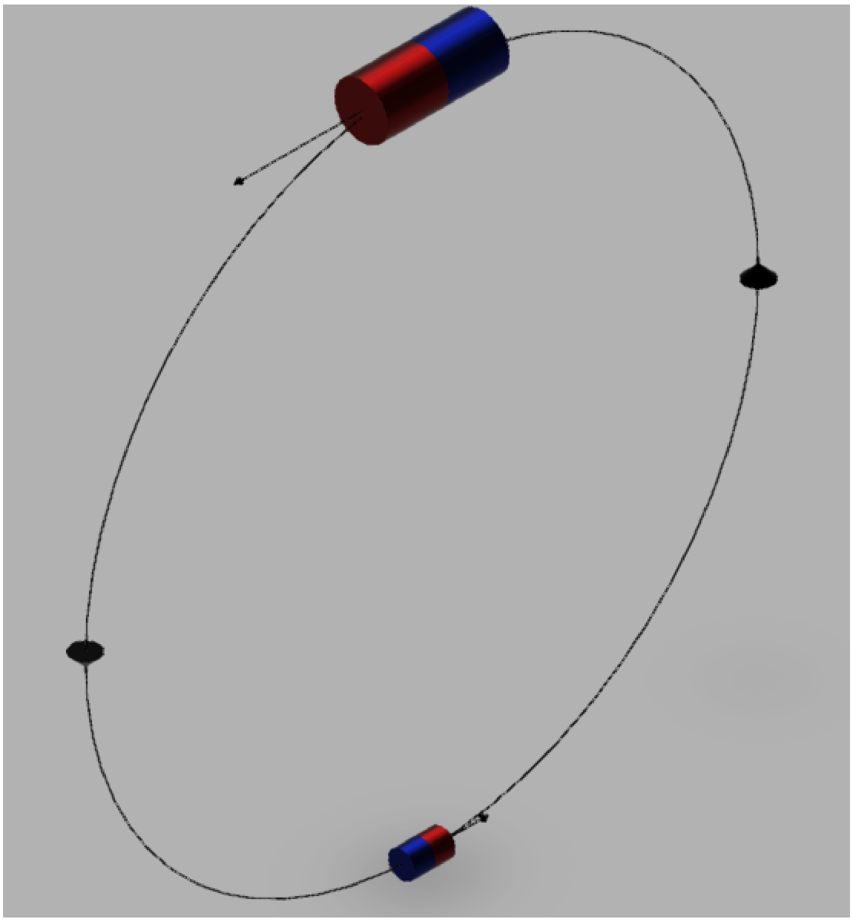

Figure 2.1: Schematic representation of the setup

2.2 State-space model

2.2.1 Generalized coordinates

The Lagrangian approach is used to obtain the equations governing the dynamics of the system. Since both the CM and the SM are constantly horizontal, it is sufficient to describe their position, and this can be done using the generalized coordinates

$$\boldsymbol{q} = (x,y,z)^T$$

$$\boldsymbol{u} = (u_x,u_y,u_z)^T$$

where $\boldsymbol{q}$ is used for the position of the CM, with z representing the vertical coordinate, while $\boldsymbol{u}$ is used for the position of the SM; $\boldsymbol{u}$ will also be the input (column) vector of the system.

2.2.2 Lagrangian

Since the CM does not rotate, the only kinetic energy associated with it is the translational kinetic energy; therefore,

$$K = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2)$$

where $\dot{x}^2 + \dot{y}^2 + \dot{z}^2 = ||\boldsymbol{v}||^2$ is the squared norm of the CM velocity.

The relative position between CM and SM is denoted by $\boldsymbol{r} = \boldsymbol{p_{CM}} - \boldsymbol{p_{SM}}$. The dipole moment of the SM will be controlled ensuring that $\boldsymbol{m}_s = (0,-M_s,0)$, where $M_s = ||\boldsymbol{m_s}||$; therefore, being $\hat{\boldsymbol{m}}_{\boldsymbol{s}} = -\hat{\boldsymbol{m}}_{\boldsymbol{c}}$, it holds $\boldsymbol{m}_c = (0,M_c,0)$, where $M_c = ||\boldsymbol{m_c}||$. The magnetic field generated by the SM in the position of the CM depends on their relative position $\boldsymbol{r}$ and on the dipole moment $\boldsymbol{m_s}$ of the SM, following the relation [2]

$$\boldsymbol{B}(\boldsymbol{r},\hat{\boldsymbol{m}}_{\boldsymbol{s}}) = \frac{M_s}{4\pi||\boldsymbol{r}||^3}D(\hat{\boldsymbol{r}})\hat{\boldsymbol{m}}_{\boldsymbol{s}}$$

where $D(\hat{\boldsymbol{r}}) = 3\hat{\boldsymbol{r}}\hat{\boldsymbol{r}}^T - I$, and $I \in \mathbb{R}^{3\times3}$ is the identity matrix.

The magnetic potential energy of the CM is given by $U_m = -\boldsymbol{m_c} \cdot \boldsymbol{B}(\boldsymbol{r},\hat{\boldsymbol{m}}_{\boldsymbol{s}})$, while the gravitational potential energy can be written (taking into account the buoyancy effect) as $U_g = (m - \rho V)gz$, where m is the mass of the CM, V is the volume of the CM and ρ is the density of the fluid in which the CM is immersed (we assume that $m > \rho V$). Therefore, the total potential energy of the CM is

$$U = U_m + U_g = -\boldsymbol{m_c} \cdot \boldsymbol{B}(\boldsymbol{r},\hat{\boldsymbol{m}}_{\boldsymbol{s}}) + (m - \rho V)gz.$$

The system also presents nonconservative forces given by the viscous friction of the intestinal mucus; since the CM does not rotate, the only friction of interest is the translational one, and for simplicity, it will be assumed that it is the same for all directions. Therefore, it can be expressed as $\boldsymbol{F}_\gamma = -\gamma\boldsymbol{v}$, thus providing the generalized forces

$$Q_x = -\gamma\dot{x},\ Q_y = -\gamma\dot{y},\ Q_z = -\gamma\dot{z}.$$

2.2.3 Dynamics equations

By writing the Lagrange equations associated with the Lagrangian $L = K - U$, i.e.

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_h} - \frac{\partial L}{\partial q_h} = Q_h$$

one can obtain the dynamics equations of the CM; denoting $k_m = \frac{\mu_0 M_s M_c}{4\pi}$, and $r = ||\boldsymbol{r}||$, where, as mentioned earlier, $\boldsymbol{r} = \boldsymbol{p_{CM}} - \boldsymbol{p_{SM}} = (x - u_x, y - u_y, z - u_z)^T$, the equations obtained are

$$\ddot{x} = \frac{k_m}{m}\left[\frac{15(x-u_x)(y-u_y)^2}{r^7} - \frac{3(x-u_x)}{r^5}\right] - \frac{\gamma}{m}\dot{x}, \tag{2.1}$$

$$\ddot{y} = \frac{k_m}{m}\left[\frac{15(y-u_y)^3}{r^7} - \frac{9(y-u_y)}{r^5}\right] - \frac{\gamma}{m}\dot{y}, \tag{2.2}$$

$$\ddot{z} = \frac{k_m}{m}\left[\frac{15(y-u_y)^2(z-u_z)}{r^7} - \frac{3(z-u_z)}{r^5}\right] - \frac{\gamma}{m}\dot{z} - \frac{m-\rho V}{m}g. \tag{2.3}$$

To allow for easy modification of the model and to ensure that the computations are correct, MATLAB code is also used to perform such symbolic calculations; the code is reported in Appendix A.

A state-space model can now be easily obtained, as is usually done with mechanical systems: 3 state variables are given by the coordinates x,y,z and 3 state variables are given by their time derivatives $v_x = \dot{x}$, $v_y = \dot{y}$, $v_z = \dot{z}$, providing the following 6 variables state-space model

$$\begin{aligned}
\dot{x} &= v_x \\
\dot{y} &= v_y \\
\dot{z} &= v_z \\
\dot{v}_x &= \frac{k_m}{m}\left[\frac{15(x-u_x)(y-u_y)^2}{r^7} - \frac{3(x-u_x)}{r^5}\right] - \frac{\gamma}{m}\dot{x} \\
\dot{v}_y &= \frac{k_m}{m}\left[\frac{15(y-u_y)^3}{r^7} - \frac{9(y-u_y)}{r^5}\right] - \frac{\gamma}{m}\dot{y} \\
\dot{v}_z &= \frac{k_m}{m}\left[\frac{15(y-u_y)^2(z-u_z)}{r^7} - \frac{3(z-u_z)}{r^5}\right] - \frac{\gamma}{m}\dot{z} - \frac{m-\rho V}{m}g,
\end{aligned}$$

however, the variables v_k will be referred to as $\dot{k}$.

Chapter 3

Control of the system

3.1 Control strategy

As anticipated, the control strategy of the CM revolves around keeping it vertically aligned with the SM, which not only is necessary for our assumptions (of the CM mantaining a constant horizontal dipole moment) to be valid, but is also key in the control approach, since it is an equilibrium configuration: therefore, a natural strategy is to linearize the system around said equilibrium and perform a linear control; in particular, pole placement will be employed. However, since the CM will be moving (as well as the SM), the equilibrium configuration will not be static but will move with the system; this must be taken into account while exploiting linearization. It is assumed that all states of the system are always observable.

3.2 Linearized state-space model

3.2.1 Equilibrium configuration

The dynamics equations derived in Chapter 2 are now used to show that the configuration described above is indeed one of equilibrium, but also to determine the exact vertical distance between CM and SM for which the net force on the CM is null. Recall that CM and SM are assumed to always be horizontal (i.e., with null vertical component), and having opposite versus ($\hat{\boldsymbol{m}}_s = -\hat{\boldsymbol{m}}_c$); for simplicity, it is assumed that among the possible configurations satisfying these assumptions, the one employed is $\boldsymbol{m}_s = (0,-M_s,0)$, $\boldsymbol{m}_c = (0,M_c,0)$.

From eq. (2.1) and eq. (2.2) it can be seen that when CM and SM are vertically aligned ($x = u_x$, $y = u_y$), then the horizontal forces are null, that is, $\ddot{x} = \ddot{y} = 0$. To determine the (vertical) distance $z_{eq} = (z - u_z)\big|_{equilibrium}$ (< 0) between CM and SM in the equilibrium configuration, one must solve eq. (2.3) imposing $\ddot{z} = 0$, $y - u_y = 0$, $r = |z - u_z| = |z_{eq}|$, and

$\dot{z} = 0$, that is

$$0 = \frac{k_m}{m}\left[-\frac{3z_{eq}}{|z_{eq}|^5}\right] - \frac{m-\rho V}{m}g;$$

taking into account that $z_{eq} < 0$, the equation to be solved is

$$0 = \frac{3k_m}{m{z_{eq}}^4} - \frac{m-\rho V}{m}g,$$

which provides

$$z_{eq} \overset{z_{eq}<0}{=} -\sqrt[4]{\frac{3k_m}{(m-\rho V)g}}.$$

For a given position $\boldsymbol{p_{SM}} = (u_x,u_y,u_z)^T$ of the SM, if $\boldsymbol{r_{eq}} = (0,0,z_{eq})^T$, then the equilibrium configuration of the CM is

$$\boldsymbol{p_{CM,eq}} = \boldsymbol{p_{SM}} + \boldsymbol{r_{eq}} = (u_x,u_y,u_z+z_{eq})^T.$$

3.2.2 Linearization around equilibrium

A linear state-space model is obtained next by linearizing the system derived in Chapter 2 around the equilibrium configuration found above; the velocities state-space equations are

$$\begin{aligned}
\ddot{x} &= f_1(x,y,z,\dot{x},\dot{y},\dot{z},u_x,u_y,u_z)\\
\ddot{y} &= f_2(x,y,z,\dot{x},\dot{y},\dot{z},u_x,u_y,u_z)\\
\ddot{z} &= f_3(x,y,z,\dot{x},\dot{y},\dot{z},u_x,u_y,u_z)
\end{aligned}$$

and (after some computations) their non-zero partial derivatives evaluated at the equilibrium are

$$\begin{aligned}
\partial_x f_1\big|_{eq} &= -\frac{1}{{z_{eq}}^5} = -\partial_{u_x} f_1\big|_{eq},\\
\partial_y f_2\big|_{eq} &= -\frac{3}{{z_{eq}}^5} = -\partial_{u_y} f_2\big|_{eq},\\
\partial_z f_3\big|_{eq} &= \frac{4}{{z_{eq}}^5} = -\partial_{u_z} f_3\big|_{eq},\\
\partial_{\dot{x}} f_1\big|_{eq} &= \partial_{\dot{y}} f_2\big|_{eq} = \partial_{\dot{z}} f_3\big|_{eq} = -\frac{\gamma}{m}\\
\partial_{\dot{x}}\dot{x} &= \partial_{\dot{y}}\dot{y} = \partial_{\dot{z}}\dot{z} = 1,
\end{aligned}$$

while the rest are null; by introducing the notation $a = \frac{1}{{z_{eq}}^5}$, these lead to the following linear state-space model

$$\boldsymbol{\delta}\dot{\boldsymbol{x}} = A(\boldsymbol{\delta x}) + B(\boldsymbol{\delta u})$$
$$\boldsymbol{\delta y} = C(\boldsymbol{\delta x})$$

$$A = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -a & 0 & 0 & -\frac{\gamma}{m} & 0 & 0 \\ 0 & -3a & 0 & 0 & -\frac{\gamma}{m} & 0 \\ 0 & 0 & 4a & 0 & 0 & -\frac{\gamma}{m} \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ a & 0 & 0 \\ 0 & 3a & 0 \\ 0 & 0 & -4a \end{pmatrix},$$

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix},$$

where the state vector $\boldsymbol{\delta x} = (\delta x, \delta y, \delta z, \dot{x}, \dot{y}, \dot{z})$ and the input vector $\boldsymbol{\delta u} = \boldsymbol{p_{SM}} - (\boldsymbol{p_{CM,ref}} - \boldsymbol{r_{eq}})$ represent, respectively, the deviation of the CM from the *current* equilibrium configuration and the deviation of the SM from the *current* equilibrium input; in particular $(\delta x, \delta y, \delta z) = \delta \boldsymbol{p_{CM}} = \boldsymbol{p_{CM}} - \boldsymbol{p_{CM,ref}}$. The meaning of *current* will be made more precise later and is related to the fact that the equilibrium keeps moving as the CM and the SM move; for now it suffices to say that the obtained linear state-space model describes, for a given equilibrium configuration, the behavior of the system close to said equilibrium. The linear state-space model is also obtained via a MATLAB code (for correctness and flexibility), which is reported in Appendix A.

3.2.3 Pole placement

To control the linearized state-space model, pole placement is employed; a MATLAB code is used first to verify controllability of the system and then to obtain the gains matrix and the reference gains matrix

```
disp(rank(ctrb(A,B)));  % verify that the system is controllable

P = [-50 -55 -60 -65 -70 -75];  % desired pole location
K = place(A,B,P);    % determine gains matrix to obtain
                     % the desired poles in closed loop
```

```
Acl = A - B*K;  % feedback matrix
syscl = ss(Acl,B,C,0);  % closed loop system

Kdc = dcgain(syscl);    % compute the static gains of the system
Kr = inv(Kdc);  % compute reference gains matrix
                % to obtain unit static gains in closed loop
```

Notice that the place command cannot work with symbolic matrices; therefore, this code can only be used once the system parameters are assigned; the poles are chosen to be stable and rapidly convergent.

3.3 Control implementation

3.3.1 System overview

The final system consists of

- reference generator: provides a desired trajectory for the CM;
- trajectory controller: given the CM states and the SM position, it generates a desired trajectory for the SM that allows for the tracking by the CM of the reference trajectory;
- manipulator control: approximates the manipulator behavior through a low-pass filter and converts an input desired trajectory into a time-dependent configuration vector (implemented through inverse kinematics algorithms);
- manipulator: given a joints configuration for the manipulator, it cinematically (i.e. instantaneously) converts it into a cartesian configuration of the SM;
- carrier magnet: simulates the dynamics of the CM; following our initial assumption, it constrains the CM to a fixed orientation, only allowing it to translate.

In fig. 3.1 the block diagram of the system is shown.

3.3.2 Linearization along a trajectory

The trajectory controller is responsible for the linearization of the system and the linearized control of the system close to the equilibrium. The equilibrium point around which the linearization is performed is moving as the CM and SM move; therefore, the linearized state-space model is valid close to the *current* equilibrium configuration, where *current* indicates the equilibrium

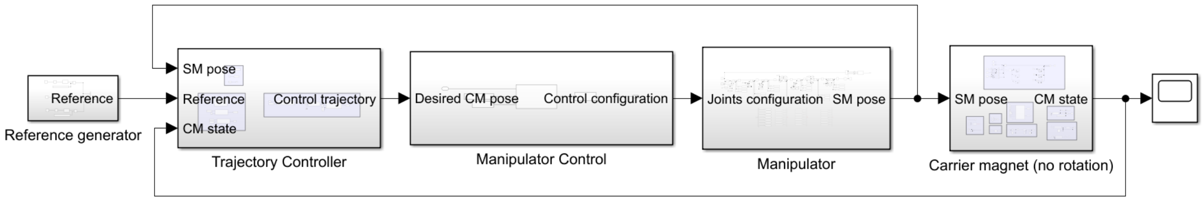

Figure 3.1: Simulink block diagram of the system

configuration determined by the SM position: at any time, given the SM position $\boldsymbol{p_{SM}}(t)$, the trajectory controller considers it as the input equilibrium configuration for the linearization, i.e.

$$\boldsymbol{p_{SM,eq}}(t) = \boldsymbol{p_{SM}}(t),\, \forall t \geqslant 0,$$

and therefore

$$\boldsymbol{p_{CM,eq}}(t) = \boldsymbol{p_{SM}}(t) + \boldsymbol{r_{eq}}(t),\, \forall t \geqslant 0.$$

From this it follows that the trajectory controller uses

$$\boldsymbol{\delta p_{CM}}(t) = \boldsymbol{p_{CM}}(t) - \boldsymbol{p_{CM,eq}}(t) = \boldsymbol{p_{CM}}(t) - (\boldsymbol{p_{SM}}(t) + \boldsymbol{r_{eq}}(t)),\, \forall t \geqslant 0$$

as positional deviation of the CM for the linearized system. If the trajectory is slow enough, the CM will stay very close to the current equilibrium position at all times, also allowing the linearization to remain valid.

The deviation reference used for the linearized system is given by the difference between the original reference and the current CM equilibrium configuration, since, as mentioned above, the actual CM position will be very close to the current equilibrium one;

$$\boldsymbol{\delta r}(t) = \boldsymbol{r}(t) - \boldsymbol{p_{CM,eq}}(t),\, \forall t \geqslant 0.$$

Finally, once the linear control produces a deviation control output $\boldsymbol{\delta u}(t)$, the trajectory controller provides to the manipulator control block the output

$$\boldsymbol{u}(t) = \boldsymbol{p_{SM}}(t) + \boldsymbol{\delta u}(t),\, \forall t \geqslant 0.$$

Chapter 4

Results and conclusions

4.1 Simulation setup

To test the effectiveness of the control, a Simulink model is used for simulation, in conjunction with Simscape to visualize the setup (see fig. 4.1); the Simulink block diagram can be seen in fig. 3.1, where each block is a subsystem as described in Chapter 3. The parameters used for the simulation are the following

- CM magnetic moment: $M_c = 0.064\,\mathrm{A\,m^2}$;
- SM magnetic moment: $M_s = 115.2\,\mathrm{A\,m^2}$;
- CM mass: $m = 5.2 \times 10^{-4}\,\mathrm{kg}$;
- CM height: $h = 15 \times 10^{-3}\,\mathrm{m}$;
- CM radius: $R = 3 \times 10^{-3}\,\mathrm{m}$;
- intestinal mucus viscous friction coefficient: $\gamma = 1 \times 10^{-3}\,\mathrm{kg\,s^{-1}}$;
- intestinal mucus density: $\rho = 1000\,\mathrm{kg\,m^{-3}}$.

With such parameters, the distance between the CM and the SM in equilibrium is $|z_{eq}| \simeq 0.220\,21\,\mathrm{m}$; the initial position of the SM is $\boldsymbol{p_{SM}}(0) = (0.35, -0.35, 0.24)^T\mathrm{m}$, and therefore the initial position of the CM is $\boldsymbol{p_{CM}}(0) = \boldsymbol{p_{SM}}(0) + \boldsymbol{r_{eq}} \simeq (0.35, -0.35, 0.01979)^T\mathrm{m}$

The manipulator used is a Kinova Gen3; for the choice of the manipulator time constant τ, several values are experimentally tested, since a too high value of τ leads to instability caused by the SM not being able to properly follow the trajectory needed to control the CM. Such tests consist of trying to maintain the CM still in the initial equilibrium position, with different values of τ, without the SM having to move unrealistically fast; after some iterations, the value $\tau = 0.04\,\mathrm{s}$ is chosen.

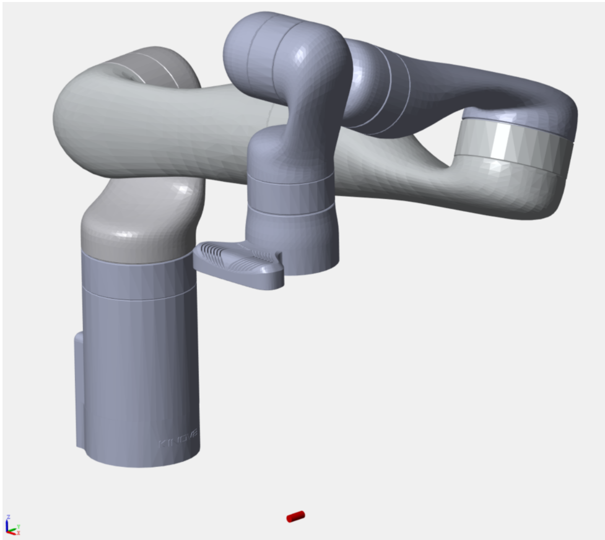

Figure 4.1: Simscape visualization of the setup

4.2 Translation test

Given the chosen time constant, it is interesting to test the speeds at which the CM can be moved without losing stability: after several tests, the system turns out to behave very well, managing also to move the CM fast; however, as the speed of the trajectory is increased, the motion of the manipulator becomes more unrealistic (since having too rapid oscillations for a real manipulator), highlighting the absence of a real dynamic control for the manipulator. To test translation, the reference motion is a linear displacement (low-pass filtered for smoothness) by $\boldsymbol{d} = (0.1{,}0.1{,}0.1)$m from the initial CM position; the motion is performed using different time intervals T to test different speeds. Two different tests are reported here for two different time intervals T, i.e., two different speeds v.

Test 1 $v = 0.1\,\mathrm{m\,s^{-1}}$ In this case the high speed requires the manipulator to perform very rapid small changes for a dynamically controlled manipulator, as can be seen from fig. 4.2

Test 2 $v = 0.01\,\mathrm{m\,s^{-1}}$ In this case, the motion of the manipulator is much smoother and more easily realizable for a dynamically controlled manipulator, as can be seen from fig. 4.4

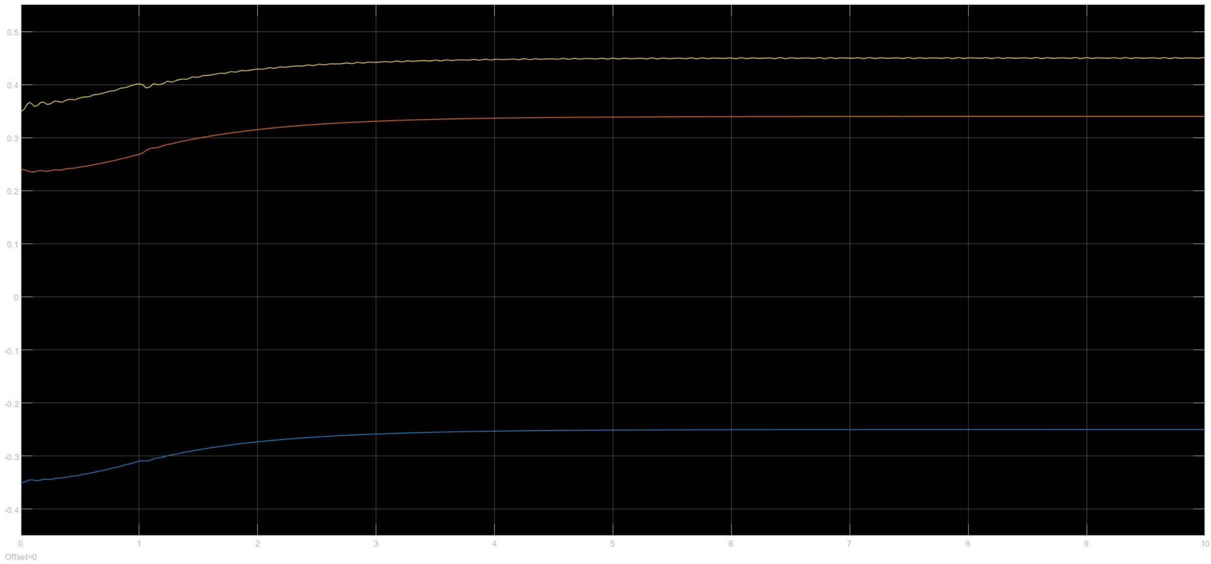

Figure 4.2: Motion of the SM for $v = 0.1\,\mathrm{m\,s^{-1}}$

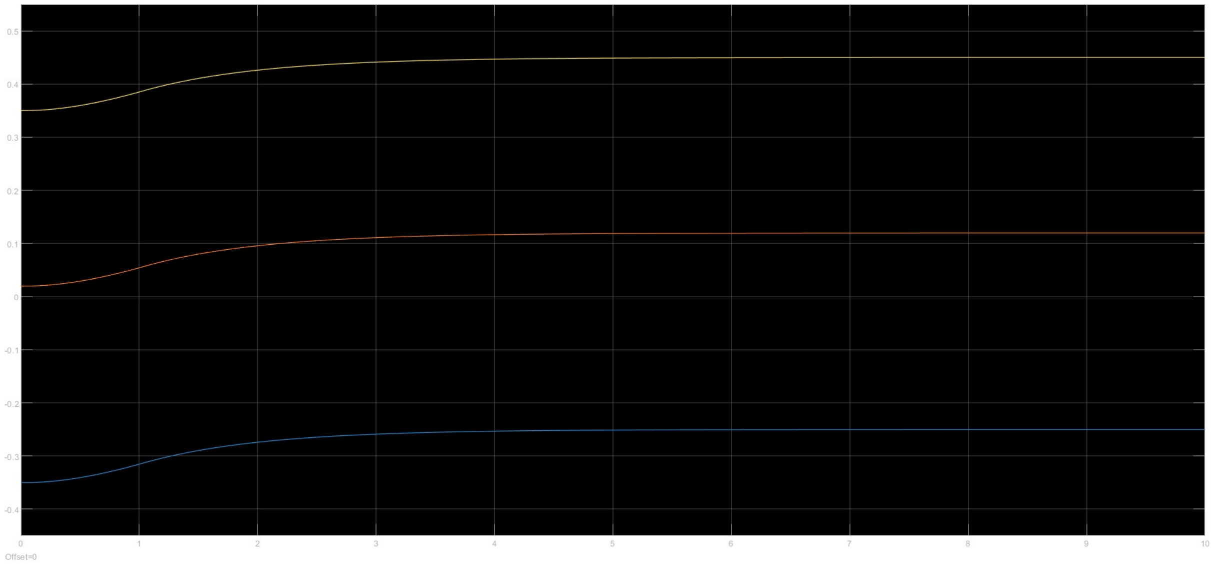

Figure 4.3: Motion of the CM for $v = 0.1\,\mathrm{m\,s^{-1}}$

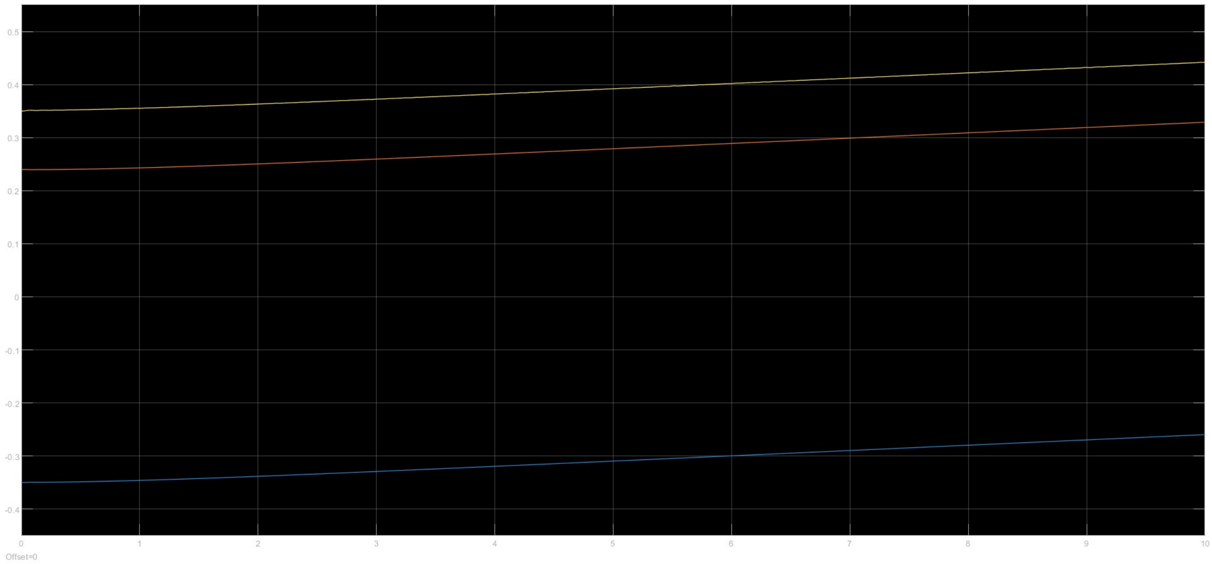

Figure 4.4: Motion of the SM for $v = 0.01\,\mathrm{m\,s^{-1}}$

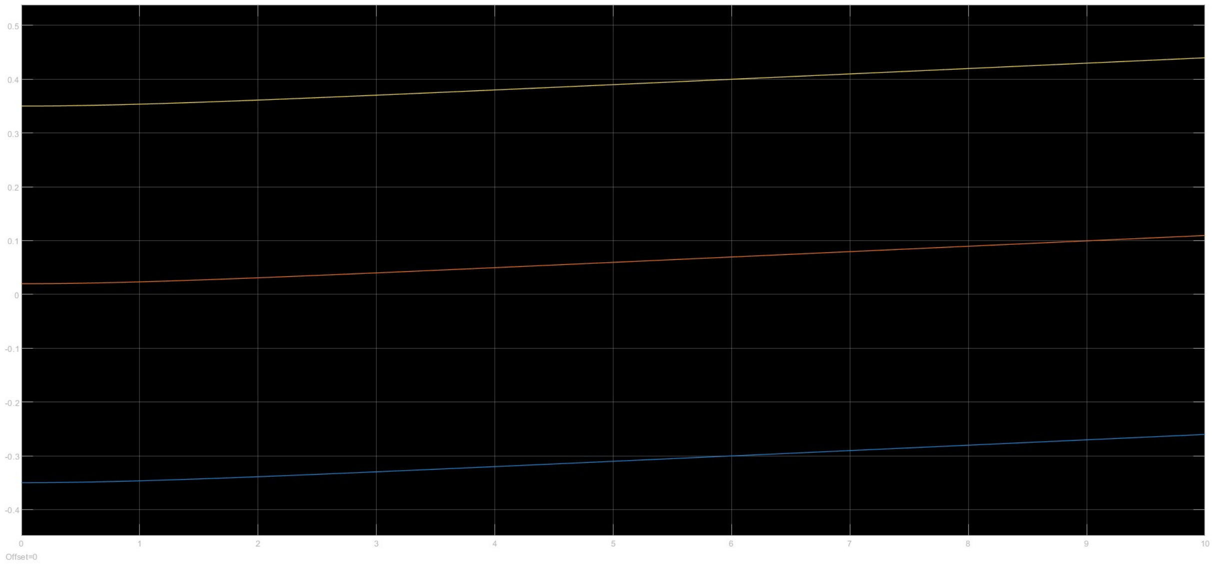

Figure 4.5: Motion of the CM for $v = 0.01\,\mathrm{m\,s^{-1}}$

4.3 Conclusions

The results show that the proposed approach is effective in controlling the system if actuated at low speeds; however, the presented analysis does not take into account the rotation of the CM, for which the controller may need an improvement, and it only approximates the actual control of the manipulator. A further study of the problem may lead to a deeper understanding of the limits of the proposed approach, and of the adjustments needed to this strategy in a scenario where the rotational dynamics and the manipulator control are included.

Appendix A

Codes

A.1 Symbolic dynamics equations code

```
%% system model (only translation)
syms Mc real     % magnitude of the CM dipole moment
syms Ms real     % magnitude of the SM dipole moment
syms m real      % mass of the CM
syms c_h real    % height of the CM
syms c_R real    % radius of the CM
syms c_V real    % volume of the CM
syms g real      % gravity acceleration
syms mu0 real    % vacuum permittivity
syms gamma real % viscous friction coefficient
syms rho real    % density of the intestinal mucus

%% capsule states
syms x y z dx dy dz ddx ddy ddz real    % position and derivatives

%% system inputs (SM position)
syms ux uy uz real           % position

%% relative pose between capsule and robot
r = [x-ux; y-uy; z-uz];      % relative position between CM and SM
dr = [dx; dy; dz];           % CM velocity
r_norm = norm(r);
r_hat = r / r_norm;
```

```
%% dynamic equations
m_c = Mc * [0; 1; 0];      % CM magnetic moment in the world frame

m_s = Ms * [0; -1; 0];     % SM magnetic moment in the world frame

% magnetic field generated by the SM in the CM position
B = (mu0 / (4*pi*r_norm^3)) * (3*dot(m_s, r_hat)*r_hat - m_s);

U = - dot(m_c, B) + (m - rho*c_V)*g*z;       % CM potential energy

K = 0.5 * m * sum(dr.^2); % CM translational kinetic energy

L = simplify(K - U);  % Lagrangian

% generalized coordinates
q = [x; y; z];
dq = [dx; dy; dz];
ddq = [ddx; ddy; ddz];
u = [ux; uy; uz];
Q = [-gamma * dq(1); -gamma * dq(2); -gamma * dq(3)];   % gen. forces

% derivation of Lagrange equations
for i = 1:length(q)
    dL_dq = jacobian(L,q(i));
    dL_ddq = jacobian(L, dq(i));
    ddL_dtddq = jacobian(dL_ddq, [q(i),dq(i)]) * [dq(i), ddq(i)]';
    EL_eqn(i) = simplify(ddL_dtddq - dL_dq - Q(i));
end

% equations of dynamics (isolation of x_ddot, y_ddot, z_ddot)
for i = 1:length(q)
    soln(i) = simplify(solve(EL_eqn(i), ddq(i)));
end

for i = 1:length(q)
```

```
    disp(soln(i));      % display equations
end
```

A.2 Symbolic linearization code

```
%% linearization
% compute equilibrium point
k_m = (mu0 * Ms * Mc) / (4 * pi);
z_eq = - ( 3*k_m / ((m-rho*V) * g) )^(1/4);

q_equilibrium = [0; 0; z_eq];
u_equilibrium = [0; 0; 0];

% jacobian of EL equations
Jq = jacobian(soln, [q; dq]);
Ju = jacobian(soln, u);

% evaluate at the equilibrium point
Jq_eq = simplify(subs(Jq, [q; dq; u], ...
                [q_equilibrium; zeros(3,1); u_equilibrium]));
Ju_eq = simplify(subs(Ju, [q; dq; u], ...
                [q_equilibrium; zeros(3,1); u_equilibrium]));

%% linearized state space model
A = [zeros(3), eye(3); Jq_eq];
B = [zeros(3); Ju_eq];
C = [eye(3), zeros(3)];
D = 0;
```

Bibliography

[1] L. Barducci, G. Pittiglio, J. C. Norton, K. L. Obstein, and P. Valdastri, “Adaptive dynamic control for magnetically actuated medical robots,” *IEEE Robotics and Automation Letters*, vol. 4, no. 4, pp. 3633–3640, 2019.

[2] A. W. Mahoney and J. J. Abbott, “Five-degree-of-freedom manipulation of an untethered magnetic device in fluid using a single permanent magnet with application in stomach capsule endoscopy,” *The International Journal of Robotics Research*, vol. 35, no. 1–3, pp. 129–147, 2016.