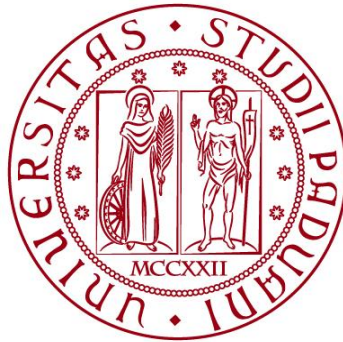


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TESI DI LAUREA

**Test and use of new Boussinesq wave model to analyse
wave run-up and overtopping**

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Preface

This thesis has been prepared within the Fluid Mechanics, Coastal, and Maritime Engineering Section at the Department of Civil and Mechanical Engineering, Technical University of Denmark (DTU), in partial fulfilment of the requirements for the degree of Master of Science in Engineering (MSc Eng). At the same time, it contributes to obtaining a Master's degree in Civil Engineering with a specialisation in Hydraulics from the University of Padua. The work was carried out as part of the double-degree programme T.I.M.E. (Top International Managers in Engineering).

The thesis corresponds to 30 ECTS at DTU and was developed in collaboration with fellow student Asger Harnung.

The internal supervisor at DTU was Professor Erik Damgaard Christensen, Head of Section, Fluid Mechanics, Coastal, and Maritime Engineering. From the University of Padua, the supervisor was Professor Paolo Salandin.

The research further benefited from the guidance and expertise of external supervisors from DHI A/S: Niels Gjør Jacobsen, Principal Engineer and Global Innovation Lead for Marine & Coastal, and Morten Eggert Nielsen, Scientific Software Specialist in Modelling Software & Engines.

It is assumed that the reader has a foundational knowledge of nearshore hydrodynamics and associated coastal engineering concepts.

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Abstract

This thesis investigates the capability of the MIKE21/3 Wave Model FM to predict wave run-up and overtopping. The model consists of a 2D version, MIKE21W, governed by Boussinesq-type equations, and a 3D version, MIKE3W, based on a Reynolds-averaged Navier–Stokes (RANS) solver. The study benchmarks both the newly implemented MIKE21W (with unstructured meshes and wetting–drying) and MIKE3W against established reference data. While a validation report for MIKE3W already exists, this work builds upon it by extending and further testing the models’ capabilities.

Validation is performed against laboratory and field experiments compiled in the CLASH database (De Rouck et al. (2004)), as well as the empirical design formulations of EurOtop (2018). A structured series of numerical experiments is conducted: solitary-wave and irregular-wave run-up cases evaluate the models’ ability to reproduce extreme uprush statistics (R_{max} and $R_{u2\%}$), while a broad selection of overtopping cases from CLASH provides mean overtopping discharges (q) across a wide range of freeboards, slopes, and wave climates, serving as a reference for overtopping accuracy.

Model performance is quantified using standard skill metrics (RMSD, scatter index, and relative bias), alongside sensitivity analyses of numerical parameters (mesh resolution, time step, and random-phase seed) and physical descriptors (wave steepness and breaker parameter $\xi_{m-1,0}$). Results show that MIKE21W accurately captures the general trends of run-up and overtopping, although with a slight tendency to underpredict in non-breaking wave conditions. MIKE3W generally yields improved predictions and performs very well overall, although with minor overpredictions in cases where breakers transition from plunging to spilling. This behaviour is linked to the absence of a turbulence closure model: while turbulence schemes such as $k-\epsilon$ were tested, they introduced excessive dissipation and were deemed unsuitable for this type of assessment. Both models demonstrate improved predictive skill at lower relative freeboards, whereas increasing freeboard is associated with greater scatter in the results. At larger freeboards, overtopping is increasingly governed by splash and spray processes, which are not explicitly represented in either model due to their underlying formulations.

The thesis concludes with practical guidance for coastal engineering design. MIKE21W is recommended for preliminary design and large-scale assessments, provided that mesh resolution and numerical settings are carefully controlled, as it offers substantial computational savings compared to MIKE3W. MIKE3W, in turn, is better suited for design-critical applications involving waves with higher nonlinearity or strong vertical bathymetric variations that violate the mild-slope assumptions of MIKE21W. Finally, recommendations are given for future model development and targeted experimental validation to reduce remaining uncertainties and strengthen the reliability of overtopping predictions in engineering practice.

Abstract (in italiano)

Questa tesi indaga la capacità del modello d'onda MIKE21/3 Wave Model FM di predire il run-up e la tracimazione (overtopping) delle onde. Il modello comprende una versione 2D, MIKE21W, governata da equazioni di tipo Boussinesq, e una versione 3D, MIKE3W, basata sulle equazioni note come Reynolds-averaged Navier–Stokes (RANS). Lo studio confronta sia la versione recentemente implementata MIKE21W (con mesh non strutturate e algoritmi di wetting–drying) sia MIKE3W, con dati di riferimento consolidati. Sebbene esista già un report di validazione per MIKE3W, questo lavoro ne estende l'analisi e verifica ulteriormente le capacità dei modelli.

La validazione viene effettuata confrontando i risultati con esperimenti di laboratorio e sul campo raccolti nel database CLASH (De Rouck et al. (2004)), oltre che con le formulazioni empiriche di EurOtop (2018). Viene condotta una serie strutturata di esperimenti numerici: casi di run-up con onde solitarie e irregolari valutano la capacità dei modelli di riprodurre statistiche di uprush estremo (R_{max} e $R_{u2\%}$), mentre una vasta selezione di casi di tracimazione dal database CLASH fornisce portate di tracimazione medie (q) su una gamma di altezze libere, pendenze e condizioni ondose.

La performance dei modelli è quantificata utilizzando metriche standard di accuratezza (RMSD, scatter index e bias relativo), insieme ad analisi di sensibilità dei parametri numerici (risoluzione della mesh, passo temporale e seme di fase casuale) e dei descrittori fisici (pendenza dell'onda e parametro di frangimento $\xi_{m-1,0}$). I risultati mostrano che MIKE21W cattura accuratamente le tendenze generali di run-up e tracimazione, sebbene tenda a sottostimare leggermente i casi di onde non frangenti. MIKE3W fornisce generalmente predizioni migliori e mostra ottime prestazioni complessive, sebbene in alcuni casi di transizione tra frangenti di tipo plunging e spilling si osservino leggere sovrastime. Questo comportamento è legato all'assenza di un modello di chiusura turbolenta. Schemi come $k-\epsilon$ sono stati testati, ma hanno introdotto dissipazione eccessiva e si sono rivelati inadatti a questo tipo di analisi. Entrambi i modelli mostrano maggiore accuratezza a bassi franchi relativi, mentre valori maggiori comportano una maggiore dispersione dei risultati. In queste condizioni, la tracimazione è sempre più influenzata da spruzzi e schiuma, fenomeni non rappresentati esplicitamente nei modelli a causa delle loro formulazioni di base.

La tesi si conclude con indicazioni pratiche per la progettazione in ingegneria costiera. MIKE21W è raccomandato per progettazioni preliminari e valutazioni su larga scala, a condizione che la risoluzione della mesh e i parametri numerici siano controllati. Questo modello offre inoltre notevoli risparmi computazionali rispetto a MIKE3W. MIKE3W, invece, è più indicato per applicazioni critiche in cui le onde presentano alta non linearità o variazioni batimetriche verticali significative che violano le ipotesi di mild-slope di MIKE21W. Infine, vengono fornite raccomandazioni per lo sviluppo futuro dei modelli e per la validazione sperimentale mirata, al fine di ridurre le incertezze residue e migliorare l'affidabilità delle predizioni di overflow in pratica ingegneristica.

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The authors of this work would like to express our sincere gratitude to their internal supervisor, Professor Erik Damgaard Christensen, for his invaluable guidance, support, and insightful feedback throughout this work. His expertise and encouragement were essential in shaping both the technical content and overall direction of this thesis.

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Acknowledgment is further given to DHI A/S for providing access to the MIKE 21/3 Wave Model FM modelling software license, which were indispensable for conducting the numerical simulations presented in this thesis.

Finally, sincere thanks are extended to all professors and staff who have contributed to the authors' education and supported their academic development during the MSc studies. Their dedication and expertise have provided a strong foundation for this research.

Table of Contents

Preface	ii
Abstract	iii
Abstract (in italiano)	iv
List of variables	viii
1 Introduction	1
2 Definitions and methodology	3
2.1 Run-up	3
2.1.1 Definition of run-up	5
2.1.2 Determination of $R_{u2\%}$	6
2.1.3 Statistical robustness of $R_{u2\%}$	7
2.2 Overtopping	8
2.2.1 Overtopping reduction measures	9
2.3 Wave theories	9
2.3.1 Regular wave	10
2.3.2 Irregular wave	13
2.4 Incident wave estimation	14
2.5 MIKE powered by DHI	14
2.5.1 Governing equations	15
2.5.2 Time integration	19
2.5.3 $k - \epsilon$ model	19
2.5.4 Flood and dry	19
2.5.5 Wave breaking	20
2.5.6 Type of outputs and their usage	21
2.6 Model performance evaluation	21
2.6.1 Root Mean Square Deviation (RMSD)	21
2.6.2 Scatter Index (SCI)	22
2.6.3 Relative Bias	22
3 Solitary wave run-up	23
3.1 Literature review and experimental comparisons	23
3.2 Model setup	24
3.2.1 Wave generation	25
3.2.2 Input convergence	26
3.3 Sensitivity analysis	27
3.4 Results	28
3.4.1 Results convergence	30
4 Irregular wave run-up	32
4.1 Literature review and experimental comparison	32

4.1.1	Current state of the art in wave run-up prediction	33
4.2	Validation of MIKE21W for run-up	34
4.2.1	Model setup	34
4.2.2	Wave decomposition	37
4.2.3	Results	39
4.3	Validation of MIKE3W for run-up	40
4.3.1	Model setup	40
4.3.2	Results	41
4.4	Discussion on the run-up results	43
4.4.1	Qualitative assessment of model performance	43
4.4.2	Surface rollers in MIKE21W	44
4.4.3	$k-\epsilon$ model in MIKE3W	44
4.4.4	Assessment of model applicability for engineering design	55
5	Irregular wave overtopping	56
5.1	Literature review and experimental comparison	56
5.1.1	CLASH database	58
5.1.2	Neural Network explanation and usage	59
5.2	Reproducing MIKE3W validation report with MIKE21W	61
5.2.1	Model setup	62
5.2.2	Parameters sensitivity	63
5.2.3	Results	64
5.3	Case selection from CLASH database	64
5.4	Validation of MIKE21W for overtopping	65
5.4.1	Model setup	65
5.4.2	Results	67
5.5	Validation of MIKE3W for overtopping	70
5.5.1	Model setup	70
5.5.2	Results	71
5.6	Discussion on the overtopping results	72
5.6.1	Qualitative assessment of model performance	72
5.6.2	Seed sensitivity in MIKE21W	73
5.6.3	Scatter index and relative bias	75
5.6.4	Influence of $\xi_{m-1,0}$	77
5.7	Assessment of model applicability for engineering design	79
6	Conclusion	81
6.1	Future work	83
A	Appendix	88
A.1	Convergence of solitary wave run-up	88
A.2	Convergence of MIKE21W to MIKE3W validation report	90
A.3	$k - \epsilon$ model: results of other test cases	90

List of variables

Symbol	Unit	Description
g	m/s^2	Gravitational acceleration
ρ	kg/m^3	Water density
ν	m^2/s	Kinematic viscosity
μ	$\text{Pa}\cdot\text{s}$	Dynamic viscosity
h	m	Still water depth
h_{deep}	m	Water depth at deep water
h_t	m	Toe depth
h_b	m	Water depth on the berm
η	m	Surface elevation
$\bar{\eta}$	m	Time-averaged free surface elevation
d	m	Total depth (local)
$\cot \alpha$ or $\tan \alpha$	[-]	Structure slope
B	m	Berm width
V	m^3	Water volume
R_c	m	Crest freeboard
B_t	m	Toe width
B_h	m	Width of schematised horizontal berm
G_c	m	Crest width of the structure
A_c	m	Armour crest freeboard
m	°	Foreshore slope
$\cot \alpha_d$	[-]	Slope angle below berm
$\cot \alpha_u$	[-]	Slope angle above berm
$\cot \alpha_{excl}$	[-]	Mean slope excluding berm
$\cot \alpha_{incl}$	[-]	Mean slope including berm
$\cot \alpha_B$	[-]	Berm slope
H_s or H_{m0}	m	Significant wave height
H_0	m	Deep-water wave height
$H_{m0,toe}$	m	Incident significant wave height at slope toe in CLASH database
$H_{m0,deep}$	m	Significant wave height in deep water
H_{max}	m	Maximum individual wave height

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List of variables - continued from previous page

Symbol	Unit	Description
H	m	Wave height (general)
H_1	m	Input wave height (general)
H_2	m	Effective wave height after propagation (general)
T	s	Wave period
T_p	s	Spectral peak period
$T_{m-1,0}$	s	Mean period based on m_{-1}/m_0
$T_{p,deep}$	s	Mean period in deep water
$T_{m,deep}$	s	Mean period in deep water
$T_{m-1,0,deep}$	s	Mean spectral period in deep water
$T_{p,toe}$	s	Mean period at toe
$T_{m,toe}$	s	Mean period at toe
$T_{m-1,0,toe}$	s	Mean spectral period at toe
β	°	Wave attack angle relative to structure normal
L	m	Wavelength (general)
$L_{m-1,0}$	m	Deep-water wavelength associated with $T_{m-1,0}$
L_s	m	Solitary wave wavelength
L_0	m	Deep-water wavelength
L_c	m	Cnoidal wave wavelength
λ	m	Wavelength (alternative notation)
k	m ⁻¹	Wavenumber
k_s	m ⁻¹	Solitary wave wavenumber
K_c	m ⁻¹	Cnoidal wave wavenumber
c_s	m/s	Solitary wave celerity
c_c	m/s	Cnoidal wave celerity
ω	rad/s	Angular frequency
f	Hz	Frequency
θ	deg / rad	Wave direction
C	m/s	Phase velocity
C_g	m/s	Group velocity
R_{max}	m	Maximum run-up elevation
R_u	m	General run-up level

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List of variables - continued from previous page

Symbol	Unit	Description
$R_{u1\%}$	m	Run-up exceeded by 1% of waves
$R_{u2\%}$	m	Run-up exceeded by 2% of waves
$R_{u10\%}$	m	Run-up exceeded by 10% of waves
$R_{u20\%}$	m	Run-up exceeded by 20% of waves
$R_{u50\%}$	m	Run-up exceeded by 50% of waves
R_u^*	m	Zero-padded list of run-up values
q	m ³ /s/m	Mean overtopping discharge per unit width
$\xi_{m-1,0}$	[-]	Surf similarity (Iribarren) parameter
p or P_i	[-]	Exceedance probability axis for run-up distribution
P_{ow}	[-]	Overtopping probability
γ_b	[-]	Berm influence factor
γ_f	[-]	Roughness influence factor
γ_β	[-]	Obliquity influence factor
γ_v	[-]	Influence factor for vertical wall at crest
γ^*	[-]	Composite influence factor for non-breaking waves
k	m ² /s ²	Turbulent kinetic energy (TKE)
$\bar{k}$	m ² /s ²	Time averaged turbulent kinetic energy (TKE)
ϵ	m ² /s ³	Dissipation rate of TKE
ν_t	m ² /s	Eddy viscosity
c_μ	[-]	Empirical constant in k - ϵ turbulence model
σ_c	[-]	Vertical distribution parameter controlling sigma-layer thickness
θ	[-]	Vertical stretching parameter for sigma-layers
b	[-]	Refinement parameter for vertical discretisation
Δt	s	Time step
$dt_{\max}$	s	Maximum timestep
$dt_{\min}$	s	Minimum timestep
Δx	m	Spatial grid/element size
CFL	[-]	Courant–Friedrichs–Lewy number
N_{waves}	[-]	Number of incident waves in a simulation
h_{wet}	m	Wetting depth threshold
h_{dry}	m	Drying depth threshold

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List of variables - continued from previous page

Symbol	Unit	Description
R	[-]	Riemann solver stabilisation factor
T_{sim}	[s]	Duration of the simulation
α	[-]	Spectrum scaling parameter
β	[-]	Spectral shape parameter
γ	[-]	Peak enhancement factor (JONSWAP)
σ	[-]	Spectral width parameter
$S_{\eta}(f)$	m ² /Hz	Wave energy spectrum
f	Hz	Wave frequency
f_p	Hz	Peak wave frequency
s	[-]	Spreading parameter in $\cos^{2s}$ directional distribution
η	m	Free surface elevation
u, v	m/s	Horizontal velocity components
U_r	[-]	Ursell number
m	[-]	Elliptic parameter in cnoidal wave theory
$K[m]$ and $E[m]$	[-]	Complete elliptic integral of first and second kind
$cn[z, m]$	[-]	Jacobi cosine elliptic function
$A[m]$ and $B[m]$	[-]	Elliptic coefficient used in cnoidal wave theory
m	[-]	Elliptic parameter in cnoidal wave theory
Q	m ³ /s	Discharge / volumetric flow rate
P	Pa	Pressure
F	N	Force
E	J	Energy
$RMSD$	[-]	Root mean squared deviation
SCI	[-]	Scatter index
RF	[-]	Reliability factor
CF	[-]	Consistency factor
$\bar{x}$	[...]	Mean of observed values or modeled values

1 Introduction

In recent decades, climate change and associated sea level rise have intensified the urgency of protecting low-lying coastal areas from flooding and storm impacts. Coastal structures such as dikes represent vital defences, but their performance depends critically on wave run-up and overtopping processes. Run-up determines whether waves exceed structural crests, while overtopping directly affects flooding hazards and structural safety. Accurate prediction of these processes is therefore essential for designing resilient coastal infrastructure and for managing flood risk in the face of increasingly extreme conditions.

A wide spectrum of modelling approaches exists for simulating wave–structure interactions. At one end, high-fidelity Computational Fluid Dynamics (CFD) models based on the Navier–Stokes equations can resolve detailed hydrodynamics but are computationally demanding and often impractical for large-scale or routine engineering applications. At the other end, depth-integrated models such as Boussinesq-type formulations provide a balanced alternative, capturing key nonlinear and dispersive effects with much greater efficiency. This trade-off between accuracy and computational feasibility makes them particularly attractive for engineering practice.

Within this framework, the MIKE 21/3 Wave Model FM, developed by DHI, has been enhanced with features such as unstructured mesh capability and wetting-and-drying functionality. These developments increase its potential for simulating wave run-up and overtopping over complex geometries and thus for application in design and assessment. While earlier validation efforts have examined the ability of MIKE3W to simulate overtopping for a limited set of cases, systematic benchmarking of the MIKE21W implementation is still lacking.

This thesis, therefore, undertakes a comprehensive validation of both MIKE21W and MIKE3W with respect to wave run-up and overtopping. The models are tested against the extensive experimental data available in the CLASH database (De Rouck et al. (2004)) and against the empirical formulations of EurOtop (2018), both of which are internationally recognised references in coastal engineering. The CLASH dataset provides a robust experimental foundation across a wide range of structures and conditions, while the EurOtop (2018) manual offers practical guidance derived from long-standing research and testing. Using both ensures a rigorous and multifaceted validation.

By combining validation against established benchmarks with a critical assessment of computational and practical aspects, the study aims to investigate the suitability and limitations of MIKE21W for engineering use. In doing so, it complements earlier MIKE3W validation work and contributes to improving the reliability of numerical overtopping assessments in design practice.

This thesis, therefore, aims to answer the following research questions:

- Under which conditions is MIKE21W suitable for accurately simulating wave run-up and overtopping, and when are more advanced models like MIKE3W required?
- How well do the MIKE21W and MIKE3W models reproduce benchmark data from the CLASH database and empirical formulations from EurOtop (2018), in terms of predicting wave run-up and overtopping?
- What are the computational and practical trade-offs between MIKE21W and MIKE3W in engineering applications, particularly with regard to setup complexity, simulation time, and accuracy?

These questions are explored through a structured program of numerical simulations, benchmark comparisons, and critical analyses of model applicability.

Chapter 2 establishes the theoretical foundation for the work. It introduces the governing principles of wave run-up and overtopping, defines the key physical and statistical parameters, and outlines the modelling framework used in the study. The chapter also provides detailed descriptions of the MIKE21W and MIKE3W models, highlighting their governing equations, numerical implementations, and turbulence closures. Furthermore, the performance evaluation metrics adopted in the thesis, such as RMSD, scatter index, and relative bias, are presented, together with a discussion of the reference datasets and empirical formulations used for benchmarking.

Chapter 3 focuses on the validation of run-up predictions under solitary wave conditions. This controlled setting allows for a systematic assessment of the models' ability to capture fundamental processes of wave uprush and rundown without the complexities of irregular wave fields. The simulations are compared against established experimental and numerical benchmarks, enabling a clear evaluation of how accurately MIKE21W and MIKE3W reproduce extreme run-up metrics, specifically R_{max} .

Chapter 4 extends the analysis to irregular wave conditions, which are more representative of real coastal environments. The chapter investigates the models' ability to reproduce statistical run-up measures under a wide range of wave spectra and structure slopes. Sensitivity tests are carried out to evaluate the influence of parameters such as random-phase seeding and mesh resolution. By confronting model predictions with empirical relationships and experimental data, this chapter provides insights into the robustness of MIKE21W and MIKE3W under more complex and stochastic forcing conditions.

Chapter 5 shifts the focus to overtopping, a process of direct relevance to coastal safety and design. Using the CLASH database and the EurOtop (2018) empirical formulations as benchmarks, the chapter evaluates the predictive skill of MIKE21W and MIKE3W in estimating mean overtopping discharge (q) for a variety of freeboard heights, structure slopes, and wave climates. The analysis also highlights model behaviour in cases dominated by a limited number of overtopping events and in conditions with high relative freeboard, where splash processes become dominant.

Chapter 6 synthesizes the findings of the thesis. It discusses the broader implications of the validation results for engineering practice, identifying where MIKE21W can be reliably applied and where more advanced solvers are required. The chapter also addresses the computational and practical trade-offs between depth-integrated and three-dimensional approaches, providing recommendations for model selection in design workflows.

2 Definitions and methodology

This chapter introduces the key quantities and concepts central to the thesis, including wave run-up and overtopping. The theoretical framework is outlined through regular and irregular wave theories, while the numerical tool "MIKE powered by DHI" is presented with its main features and output usage. Finally, the performance metrics applied to evaluate the model results are described.

2.1 Run-up

Wave run-up is generally defined as the maximum vertical extent of wave uprush above the still water level (SWL) in coastal areas. It consists of two components: swash oscillations and wave-induced setup (Holman (1986)). Accurate estimation of wave run-up is crucial in coastal engineering to ensure the safety, stability, and performance of both coastal and offshore structures.

Run-up plays a key role in flood protection design, as well as in the prevention of shoreline instabilities caused by scour and erosion, providing essential input for the planning, design, and management of waterfront infrastructure.

Run-up is generated when incident waves, whether wind-generated, infragravity, or long-period events, propagate into the nearshore zone and undergo transformation through shoaling, breaking, and bore formation in the surf zone. Upon reaching the beach face, the bore collapses and initiates an uprush of water, commonly called swash, that propagates shoreward as a thin, shallow supercritical sheet flow.

Following the definition provided by Christensen et al. (2002), the surf zone can be divided into four distinct horizontal regions, as illustrated in Figure 2.1. In the shoaling or deformation zone, waves increase in steepness and their profiles deform into an asymmetric shape, leaning forward. At the breaking point in the outer zone, waves begin to overturn, forming a jet whose strength depends on the breaker type, i.e., spilling or plunging. In the inner zone, the wave front can be described as a moving bore propagating toward the shoreline. Finally, the swash zone is defined as "the part of the beach that is alternately dry and wet due to the wave run-up" (Petti & Longo (2001)).

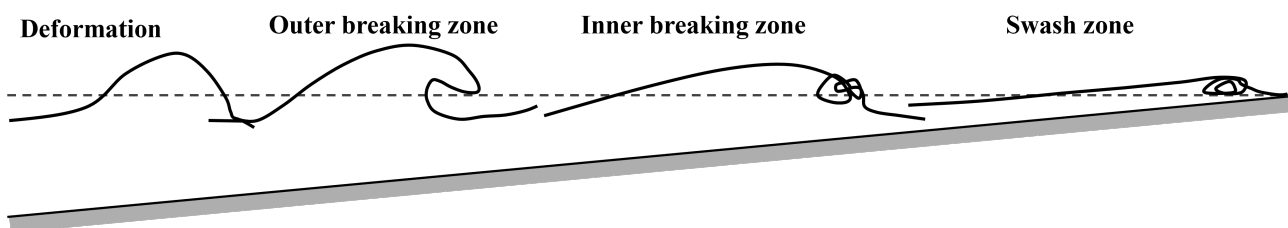


Figure 2.1: The surf zone divided into the shoaling/deformation zone, outer breaking zone, inner breaking zone, swash zone (from Christensen et al. (2002)).

The dynamics of the uprush are governed by a combination of factors, including the initial momentum of the bore, gravitational deceleration along the beach slope, bed friction, percolation losses in permeable substrates, and lateral divergence of the swash lens. Under these conditions, the run-up height can exceed the local water depth by several times, thereby violating the assumptions of linear wave theory and rendering depth-averaged

equilibrium concepts inapplicable (Svendsen (2006)). Moreover, the rapid variation of water depth over short horizontal scales prevents the use of locally constant depth approximations.

Hydrodynamically, the process is strongly dependent on the surf similarity parameter, ξ , which dictates whether swash motion is dominated by short-wave bores (reflective and intermediate beaches) or long-period infragravity waves (dissipative beaches).

From a hydrodynamic perspective, the characteristics of swash motion are strongly influenced by the surf similarity parameter, ξ . This parameter determines whether the uprush is dominated by short-wave bores, as typically observed on reflective and intermediate beaches, or by long-period infragravity waves, which are more relevant on dissipative beaches. Therefore, the surf similarity parameter provides a critical framework for predicting run-up behaviour across different coastal morphologies.

In this study, the surf similarity parameter, also referred to as the breaker parameter or Iribarren number, is defined following the convention used in EurOtop (2018). Specifically, it is expressed as:

$$\xi_{m-1,0} = \frac{\tan \alpha}{\sqrt{H_{m0}/L_{m-1,0}}} \quad (2.1)$$

where $\tan \alpha$ represents the slope of the structure's front face, H_{m0} is the significant wave height, and $L_{m-1,0} = gT_{m-1,0}^2/(2\pi)$ corresponds to the deep-water wavelength associated with the wave period at the toe of the structure, $T_{m-1,0}$.

It should be noted that the actual local wavelength near the structure's toe is not employed in this calculation; instead, the deep-water wavelength is used. As a result, the computed wave steepness represents a notional value rather than the true local steepness. This notional steepness serves primarily to calculate a “dimensionless wave period” and provides a convenient, standardised parameter for comparing different wave conditions and slope configurations, rather than representing the precise nearshore wave geometry.

The combination of structure slope and wave steepness determines the type of wave breaking, which can be classified according to EurOtop (2018), as illustrated in Figure 2.2. For values of the breaker parameter $\xi_{m-1,0} \gtrsim 2$, waves are generally considered non-breaking (surging type), although some degree of breaking may still occur. Conversely, for $\xi_{m-1,0} \lesssim 2$, waves are expected to break.

For wave run-up on slopes, EurOtop (2018) identifies the transition between plunging and surging conditions at $\xi_{m-1,0} = 1.8$, which is practically equivalent to the threshold value of 2. The mode of breaking also depends on the slope geometry: on gentle foreshores, waves predominantly break as spilling waves, often producing multiple breaker lines. Plunging waves occur for steeper slopes and are characterised by steep, overhanging fronts, with the wave tongue impacting either the structure face or the backwash from previous waves.

The intermediate regime between plunging and surging is referred to as collapsing waves. In this case, the wave front becomes nearly vertical, and the total water excursion on the slope (i.e. the combined run-up and run-down) is often larger than in purely plunging or surging conditions. It should be noted that this classification is not absolute: under generally plunging conditions, some individual waves may surge, and under generally surging conditions, some may still plunge.

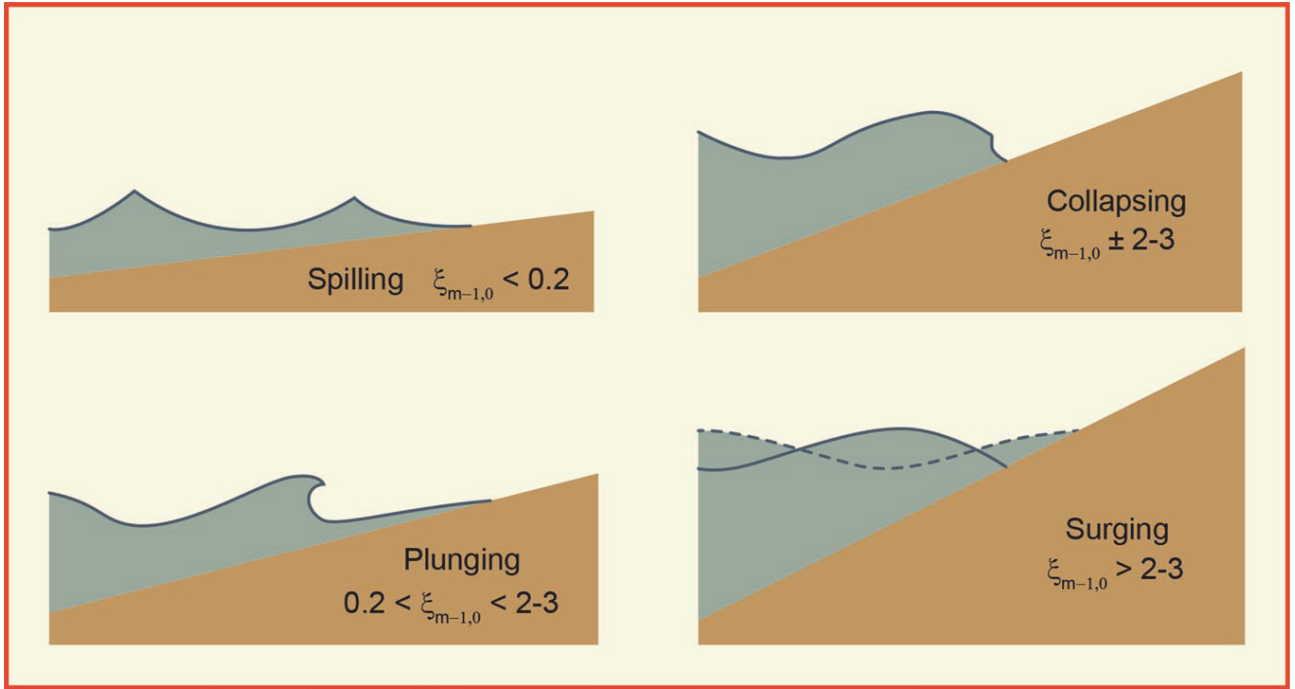


Figure 2.2: Type of breaking wave on a slope (from EurOtop (2007)).

In the uprush phase, the momentum of the incoming wave drives an upward-directed motion, which is gradually decelerated by gravity until the flow reverses and transitions into the backwash, a shallower, often longer-duration seaward return flow.

Swash–swash interactions, including collisions between incoming up-rushes and preceding backwashes, can limit run-up excursions, producing run-up saturation. This phenomenon, together with infragravity wave breaking, surf zone saturation, and bed infiltration, constrains the theoretical upper bound of run-up for a given wave climate. When the uprush exceeds the crest of coastal defences, overtopping and overwash occur.

2.1.1 Definition of run-up

Traditionally, empirical formulas are used to estimate run-up elevation as a function of wave characteristics (e.g., significant wave height and wavelength) and the foreshore slope. However, Atkinson et al. (2017) evaluated the performance of 11 run-up models against field observations and found significant variability among them. The models with the best accuracy had an RMSE of 25%. No single model consistently outperformed the others, highlighting the importance of site-specific calibration. In general, models derived from field data tended to yield more accurate estimates than those based on laboratory experiments, which often overpredict run-up heights.

Run-up can be quantified in different ways. The most direct method is through the maximum observed run-up elevation, R_{max} , which depends on the observation duration. Alternatively, a more statistically robust measure is the run-up level exceeded by 2% of the incident waves over a given period, denoted $R_{u2\%}$.

A definition sketch of the surface elevation during a run-up event can be seen in Figure 2.3.

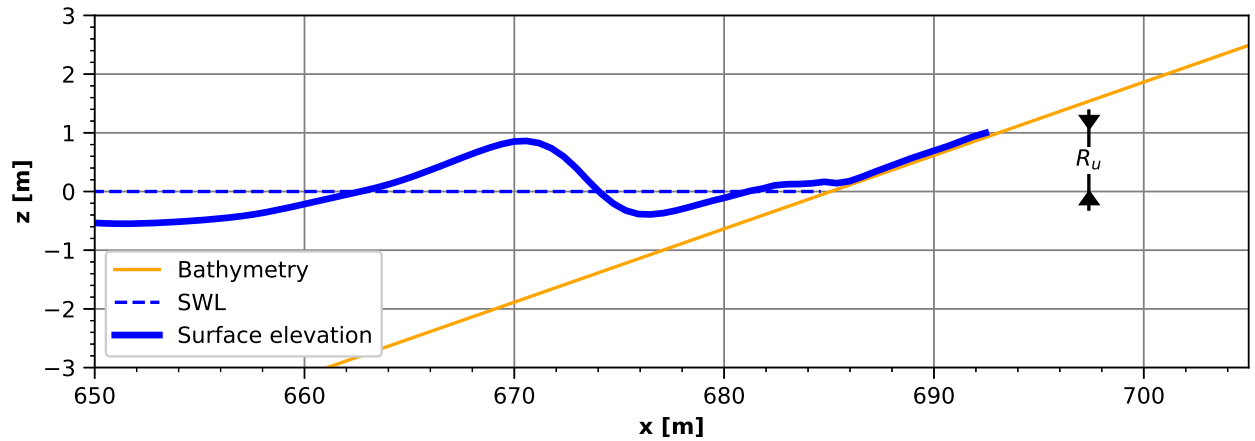
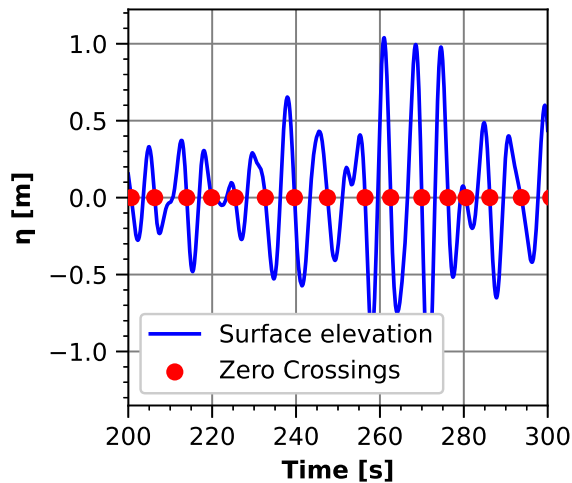


Figure 2.3: Schematic visualisation of the wave Run-up, R_u .

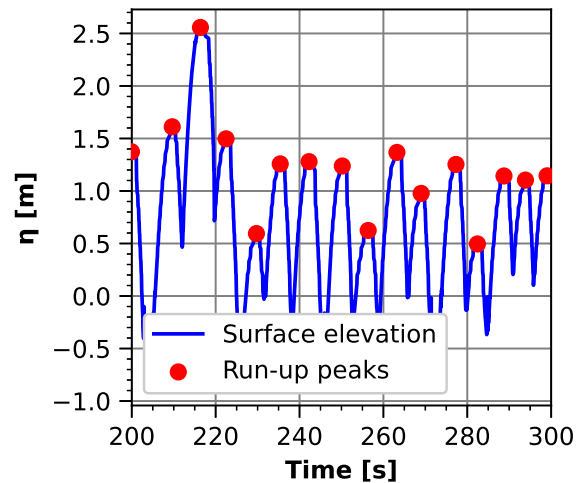
In practice, defining the run-up level also involves considerations regarding the minimum detectable water layer. Due to physical or numerical resolution limitations, very thin water tongues during run-up may not be measurable or simulated accurately. According to EurOtop (2018), the minimum detectable thickness typically ranges between 2 mm and 2 cm, depending on the scale of the experiment or model. However, these thin run-up tongues contain a very small amount of water.

2.1.2 Determination of $R_{u2\%}$

To estimate the run-up level exceeded by 2% of the waves, $R_{u2\%}$, the number of incident waves in the simulated time series was first determined. This was achieved by applying a zero-crossing analysis to the surface elevation signal, as described in Section 2.4. An illustrative example of the detected zero-crossings is provided in Figure 2.4a. Individual run-up peaks were then extracted from the run-up time series and filtered to remove minor oscillations, retaining only peaks larger than $0.3H_s$, where H_s is the significant wave height (Figure 2.4b).



(a) Example of the zero-downcrossing analysis.



(b) Example of individuation of run-up peaks.

Figure 2.4: Example of zero-crossing analysis and run-up peaks individuation used in the $R_{u2\%}$ estimation procedure.

The filtered peak values were arranged into a vector of length N , corresponding to the total number of incident waves, and sorted in descending order to form an empirical run-up exceedance distribution. Exceedance probabilities were assigned using $P_i = i/(N + 1)$, with $i = 1, \dots, N$. The vanishing probability at $P = 0$ was discarded to avoid an undefined run-up level associated with 0% exceedance. Finally, a linear interpolation was applied to the empirical distribution to estimate the run-up magnitude associated with a 2% exceedance probability, $R_{u2\%}$.

The explained procedure used to identify the $R_{u2\%}$ value is summarised in Algorithm 1.

Algorithm 1 $R_{u2\%}$ estimation procedure

- 1: Determine N_{waves} , using zero up-crossing analysis
 - 2: Let R_u be a list of run-up values, determined from run-up peak analysis
 - 3: Create R_u^* as a zero-padded list of length N_{waves}
 - 4: Fill the first entries of R_u^* with the values from R_u
 - 5: Sort R_u^* in descending order
 - 6: Generate probability axis p with $N_{\text{waves}} + 1$ equally spaced values from 0 to 1
 - 7: Remove the first probability value ($p[1] = 0$)
 - 8: Interpolate R_u^* at probability $p = 0.02$ to obtain $R_{u2\%}$
-

2.1.3 Statistical robustness of $R_{u2\%}$

The run-up height is commonly quantified using the parameter $R_{u2\%}$, as previously introduced. This definition, representing the run-up level exceeded by only 2% of the incoming waves, is widely adopted in coastal engineering practice. Nevertheless, its application in numerical modelling may be problematic when simulations cover relatively short durations. In such cases, the limited number of incident waves provides an insufficient statistical basis for the 2%-exceedance estimate. Moreover, the waves contributing to the 2% largest run-ups are typically the largest non-breaking waves, which are also the steepest and therefore among the most challenging for the model to resolve reliably.

As a more statistically robust alternative, the use of $R_{u10\%}$ can be considered. This parameter corresponds to the run-up level exceeded by 10% of the waves, thereby relying on a larger portion of the run-up distribution. The 10%-exceedance run-up thus provides a valuable complementary metric in situations where the simulation length does not permit a stable estimate of $R_{u2\%}$.

According to EurOtop (2018), several exceedance levels, including $R_{u1\%}$, $R_{u2\%}$, $R_{u20\%}$, and $R_{u50\%}$, have historically been reported for completeness in run-up experiments carried out at Delft Hydraulics (now Deltares), such as Report M422 (1953), Report M500 (1956), and Report M544 (1957). These values provide a broader statistical description of the run-up process, which is particularly valuable when comparing numerical model predictions with experimental or field data of different record lengths.

However, in the present study, these additional exceedance levels were not considered in the analysis. Despite efforts to retrieve the historical experimental data, the relevant values could not be accessed in a reliable and complete form through publicly available online sources or archives. Consequently, the comparison of numerical results with experimental reference data is limited to the 2% exceedance levels, reported in the EurOtop database.

2.2 Overtopping

Wave overtopping is defined as the average discharge of water passing over the crest of a coastal or hydraulic structure per unit width, denoted q , and typically expressed in either $\text{m}^3/\text{s per m}$ or in l/s per m (EurOtop (2018)).

The most common form, known as green water overtopping, occurs when the crest level of the structure is lower than the maximum wave run-up R_{max} , allowing a continuous sheet of water to pass over the crest. A second form arises when breaking waves on the seaward slope generate splashes, which may be carried over the crest either by their own momentum or by onshore winds. A third, less significant mechanism is spray, produced by wind acting on offshore wave crests; this contributes negligible volumes to the total discharge.

Empirical prediction formulae for overtopping generally account for both green water and splash, as these were measured during the model tests on which the methods are based. Wind effects are not explicitly included: while onshore winds have little influence on large green water events, they can increase discharges below 1 l/s per m by blowing spray droplets landward.

Understanding and accurately quantifying wave overtopping is of fundamental importance in coastal and hydraulic engineering, as it directly affects the design, safety, and functionality of coastal structures. Applications include:

- Designing and assessing the structural stability of dikes, seawalls, breakwaters, and revetments;
- Evaluating flood protection systems and risk to infrastructure located behind coastal defences;
- Ensuring the safe operation of coastal roads, railways, and promenades exposed to wave action;
- Managing erosion, scouring, and water intrusion that may affect materials or vegetation on the landward side of structures.

Wave overtopping is inherently stochastic in nature—varying randomly in time, space, and volume. Typically, only the largest waves contribute significantly to overtopping, carrying large volumes of water over the crest in short bursts, while smaller waves may not result in any overtopping. As a result, reliable overtopping estimates depend on adequate sampling durations and proper spatial averaging across the structure width.

EurOtop (2018) provides a widely adopted classification of the average overtopping discharge in terms of its potential consequences:

- $q < 0.1 \text{ l/s per m}$: Negligible effect on structural strength of the crest and rear slope;
- $q = 1 \text{ l/s per m}$: Poor-quality grass covers or clay may begin to erode; no damage expected for rubble mound structures;
- $q = 10 \text{ l/s per m}$: Significant overtopping for dikes and embankments; potential for severe erosion on the harbour side of rubble mound breakwaters under large wave conditions;
- $q = 100 \text{ l/s per m}$: Crest and landward slopes require protection (e.g., asphalt or concrete); rubble mound breakwaters may generate transmitted waves and require armour coverage on the landward side.

While the mean overtopping discharge provides a useful first estimate of overtopping severity, it is not sufficient on its own to fully characterise risk or design requirements. In particular, the wave height that causes

overtopping plays a critical role: for a given overtopping discharge, larger incident waves imply more energetic and potentially damaging overtopping events.

EurOtop (2018) therefore emphasises the need to consider not just the average discharge but also the incident wave conditions, the duration of overtopping, and the intended function of the structure. Acceptable overtopping limits vary significantly depending on the application—whether for structural integrity, asset protection, or the safety of pedestrians, vehicles, or operational facilities—and are discussed in detail in §3 of the EurOtop manual.

Zero overtopping

From a practical perspective, it may seem desirable to design a structure such that no water passes over the crest. However, in design practice, the concept of zero overtopping cannot be taken literally, as prediction methods never return a true value of zero discharge. Instead, very small mean discharges may be predicted that, in reality, correspond either to negligible amounts of water or to only one or two overtopping waves within an hour.

In physical model tests, limits on measurable discharges become important. Overtopping is commonly expressed in dimensionless form as:

$$\frac{q}{\sqrt{gH_{m0}^3}} \quad (2.2)$$

plotted against dimensionless freeboard. For typical datasets, the lower measurable limit is about 10^{-5} m³/s per m. This limit is often taken as a practical definition of “zero overtopping” in small-scale experiments. Translated to prototype conditions, it corresponds to mean discharges between 0.03 and 0.6 l/s per m for wave heights between 1 m and 7 m.

In reality, overtopping in this range may be entirely absent or up to an order of magnitude larger, depending on whether a few extreme waves overtop during the measurement period. While such small values may be detectable at full scale, they are generally not measurable in laboratory conditions.

2.2.1 Overtopping reduction measures

EurOtop (2018) emphasizes that increasing the crest freeboard is the most effective means of reducing overtopping, as discharges decrease exponentially with increasing freeboard. However, when raising the crest is not feasible, several supplementary measures can be considered.

One option is to increase permeability, for example by using porous armour layers or foreshores, which dissipate wave energy and thereby reduce run-up and overtopping. Another approach is the construction of a submerged breaker bar, which triggers wave breaking offshore and lowers wave heights at the structure toe. Finally, modifications to the crest geometry, such as the installation of a storm wall or a bullnose, can deflect or dissipate overtopping flows, particularly for steep or vertical structures.

Although these measures are important tools in practice, the design and optimisation of overtopping reduction strategies are beyond the scope of this thesis.

2.3 Wave theories

The representation of water waves is of primary importance in coastal and ocean engineering, as wave forcing governs run-up, overtopping, and the loading of coastal and offshore structures. Depending on the water depth, wave steepness, and the degree of nonlinearity, different theoretical descriptions have been developed to approximate wave motion. These theories can be broadly classified into regular wave theories, which describe deterministic, periodic or non-periodic (solitary) waveforms, and irregular wave theories, which represent the random nature of natural seas through spectral methods. In fact, real sea states are rarely regular. Instead,

they consist of a broad spectrum of wave components with varying frequencies and directions, driven by wind forcing and modified by wave–wave interactions. To account for this randomness, irregular wave theories are formulated in the spectral domain. One of the most widely used spectral models is the JONSWAP spectrum, which has become a standard representation of wind-sea conditions in engineering practice.

The following subsections present the main theoretical descriptions of regular and irregular waves employed in this study: solitary wave theory, cnoidal wave theory, and the JONSWAP spectrum for irregular waves.

2.3.1 Regular wave

Regular wave theories provide closed-form analytical solutions that capture essential nonlinear and dispersive effects in shallow water. Among these, solitary and cnoidal wave theories are particularly relevant. Solitary waves represent isolated, non-periodic disturbances that propagate without change of form, while cnoidal waves describe nonlinear but periodic long waves. Both theories are derived from the Korteweg–de Vries (KdV) equation and are applicable in shallow-water environments where dispersion is weak compared to nonlinearity.

Solitary wave

Solitary waves are non-periodic waveforms that maintain their shape while propagating over long distances. In shallow water, they are often modelled using analytical solutions derived from the Korteweg–de Vries (KdV) equation. In MIKE, the following analytical expressions are employed to describe the free surface elevation and associated horizontal fluxes of a solitary wave.

The surface elevation η of the solitary wave is given by:

$$\eta = H \operatorname{sech}^2(k_s((x - x_0)\cos(\theta) + (y - y_0)\sin(\theta))) \quad (2.3)$$

where H is the wave height, θ is the wave propagation direction, and (x_0, y_0) defines the initial crest location. The solitary wave moves in the direction defined by θ , with the wave amplitude concentrated around (x_0, y_0) . An example of the surface elevation computed with this formulation is illustrated in Figure 2.5, where the surface elevation is shown for different wave heights.

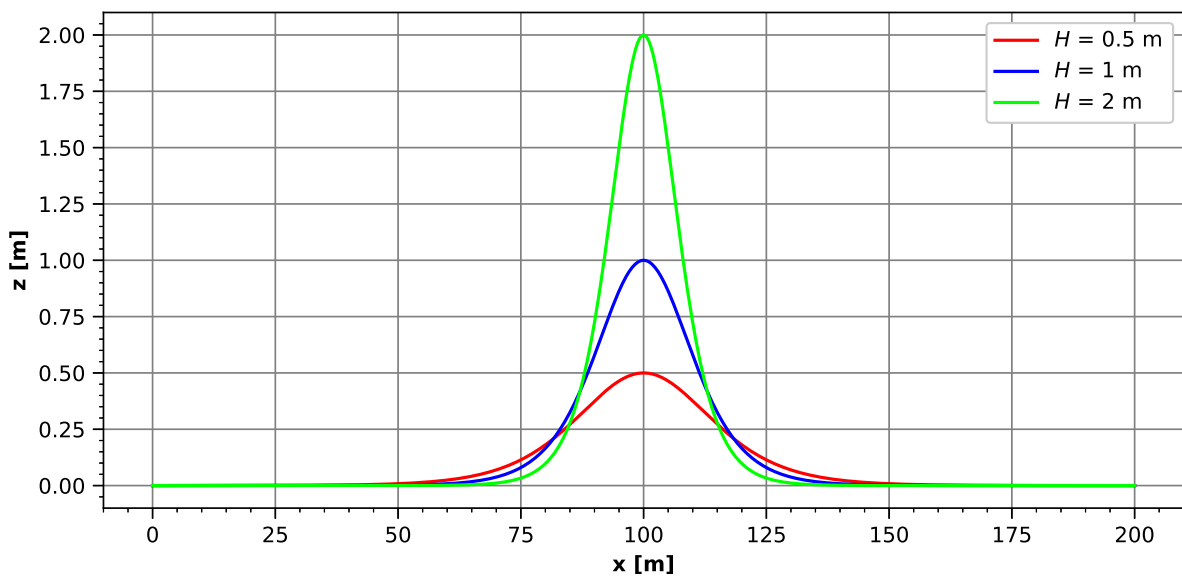


Figure 2.5: Surface elevation computed with the explained solitary wave formulation.

The corresponding depth-integrated horizontal fluxes in the x and y directions, denoted by P and Q , are computed as:

$$\begin{bmatrix} P \\ Q \end{bmatrix} = c_s \eta \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix} \quad (2.4)$$

Here, c_s is the celerity of the solitary wave, and k_s is the associated wavenumber. The wavenumber of the solitary wave depends on nonlinearity but not on frequency. These are given by:

$$k_s = \sqrt{\frac{3|H|}{4h^3}} \quad (2.5)$$

$$c_s = \sqrt{\frac{ghH^2(H + 3h)}{6h^2(H - h \ln(\frac{h+H}{h}))}} \quad (2.6)$$

where h is the still water depth and g is the gravitational acceleration.

Although a solitary wave is, in principle, infinitely long and associated with an infinite wave period, it is often useful to define an effective wavelength to characterise its spatial scale. Following common practice, the effective solitary wave length L_s can be expressed as:

$$L_s = \frac{2\pi}{k_s} = 2 \cdot \frac{\pi \cdot 1.165 \cdot h}{\sqrt{H/h}}, \quad (2.7)$$

This definition provides a convenient measure for comparing solitary waves with periodic waves, particularly when assessing scale effects or defining mesh resolution in numerical models.

Cnoidal wave

While solitary waves describe isolated disturbances, many practical applications require the modelling of periodic shallow-water waves. In this regime, the appropriate analytical framework is cnoidal wave theory, which generalises the KdV wave solution into a periodic constant form.

Cnoidal waves are expressed in terms of elliptic functions, specifically the complete elliptic integrals of the first, $K[m]$, and the second, $E[m]$, kind, as well as the Jacobi cosine-elliptic function, $cn[z, m]$, where m denotes the elliptic parameter.

The surface elevation η of the cnoidal wave is given by:

$$\eta[x, t] = H \left(B[m] + cn^2 [K_c(x - c_c t), m] \right) \quad (2.8)$$

where H is the wave height, K_c is the cnoidal wavenumber, and c_c is the cnoidal wave celerity. These parameters are defined as:

$$K_c = \sqrt{\frac{3}{4} \frac{H}{mh^3}} \quad (2.9)$$

$$c_c = \sqrt{g(h + A[m]H)} \quad (2.10)$$

with the corresponding wavelength given by $L_c = c_c T$, where T is the wave period.

The coefficients $A[m]$ and $B[m]$ are expressed in terms of the elliptic parameter as:

$$A[m] = \frac{1}{m} \left(2 - m - 3 \frac{E[m]}{K[m]} \right), \quad (2.11)$$

$$B[m] = \frac{1}{m} \left(1 - m - \frac{E[m]}{K[m]} \right), \quad (2.12)$$

A fundamental relation in cnoidal wave theory links the Ursell number, U_r , which measures the relative importance of nonlinearity to dispersion, to the elliptic parameter m :

$$U_r \equiv \frac{HL_c^2}{h^3} = \frac{16}{3} m K[m]^2. \quad (2.13)$$

This relation allows determination of the elliptic parameter m for a given depth h , wave height H , and wavelength L_c . Once m is known, the surface elevation η can be computed directly from eq. (2.8).

Figure 2.6 illustrates an application of the cnoidal wave formulation, where the surface elevation is computed for different wavelengths. It is evident that, as the wavelength increases, the cnoidal solution progressively approaches the shape of a solitary wave with the same height. This behaviour highlights the consistency between the two theories: solitary waves can be seen as the limiting case of cnoidal waves with infinite wavelength.

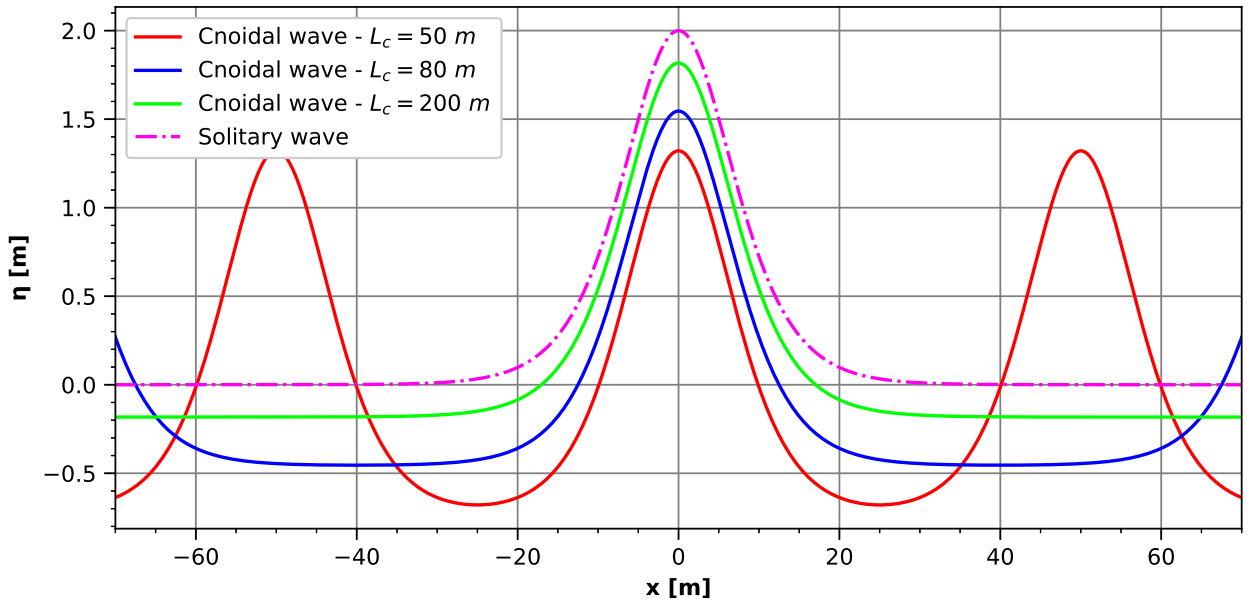


Figure 2.6: Comparison of surface elevation computed with cnoidal and solitary wave formulation.

It should be noted that the cnoidal wave formulation assumes a zero mean water level. In this thesis, its appli-

cation is restricted to the modelling of wave run-up, for the reasons outlined in Section 3.2.1.

2.3.2 Irregular wave

Natural sea states are inherently random, as they result from the superposition of a large number of wave components with different frequencies and directions. This stochastic nature of the ocean surface can be described completely and compactly using the wave energy density spectrum. The spectrum characterises the distribution of wave energy as a function of frequency, and, when combined with an appropriate directional spreading function, it fully defines the sea state.

Several spectral formulations have been developed over the years, with their parameters typically expressed in terms of the significant wave height, H_s , and the spectral peak period, T_p . Among the most widely used are the Pierson–Moskowitz spectrum Pierson Jr & Moskowitz (1964), which describes fully developed seas, and the JONSWAP spectrum Hasselmann et al. (1973), which accounts for the growth stage of wind-generated seas in fetch-limited conditions.

To extend the spectral description to directional seas, a directional spreading function is introduced. This function distributes the spectral energy across different propagation directions. Several forms exist, including the trivial case of a unidirectional distribution, the Gaussian distribution, and the commonly used $\cos^{2s}$ distribution Longuet-Higgins (1963).

In the present work, the JONSWAP spectrum is adopted in combination with a unidirectional distribution.

JONSWAP spectrum

The JONSWAP (Joint North Sea Wave Project) spectrum describes the spectral density of the sea surface elevation, $S_\eta(f)$, as a function of frequency f . The formulation is given by:

$$S_\eta(f) = \alpha H_s^2 f_p^4 f^{-5} \gamma^\beta \exp \left(-\frac{5}{4} \left(\frac{f_p}{f} \right)^4 \right) \quad (2.14)$$

where H_s is the significant wave height, f_p is the peak frequency, and γ is the peak enhancement factor, which accounts for the sharper peak observed in developing seas compared to fully developed ones. The parameters are further defined as:

$$\alpha \approx \frac{0.0624}{0.230 + 0.0336\gamma - 0.185(1.9 + \gamma)^{-1}} \quad (2.15)$$

$$\beta = \exp \left(\frac{-(f - f_p)^2}{2\sigma^2 f_p^2} \right) \quad (2.16)$$

$$\sigma = \begin{cases} 0.07, & f \leq f_p \\ 0.09, & f > f_p \end{cases} \quad (2.17)$$

Here, σ represents the spectral width parameter, which varies depending on whether the considered frequency lies below or above the peak frequency.

For numerical implementation, the frequency domain is truncated by specifying lower and upper cut-off frequencies, and the spectrum is discretised into a finite number of Fourier components. The choice of frequency discretisation directly determines the repetition period of the synthesised wave signal. Following recommendations from DHI, this repetition period should be of the order of the total simulation time minus the travel time of

waves from the boundary to the area of interest, in order to minimise artificial periodicity effects in the modelled sea state.

Finally, to ensure reproducibility of stochastic realisations, an initial random number (or seed) is specified. This seed governs the generation of random phases in the Fourier description. By fixing the seed, repeated simulations under identical input conditions yield consistent wave realisations, which is essential for controlled testing and comparison.

2.4 Incident wave estimation

The experimental databases used for the validation of the numerical models in this thesis report the incoming significant wave height at the slope toe, $H_{m0\ toe}$. To ensure direct comparability between the model results and the experimental data, a wave decomposition procedure based on linear wave theory was applied. This method, originally introduced by Zelt & Skjelbreia (1993), allows the separation of one-dimensional wave fields into incident and reflected components by analysing surface elevation time series recorded at multiple wave gauges.

The method relies on the known spatial distribution of wave gauges and uses the linear superposition of wave components to estimate incoming and reflected wave signals. For reliable decomposition, the spacing between the gauges must be selected to resolve the range of wavelengths containing most of the wave energy. In this study, gauge locations are chosen following the criteria outlined by Zelt & Skjelbreia (1993), ensuring that both the minimum and maximum spacing capture the shortest and longest wavelengths of interest, respectively. Intermediate gauges are placed using a geometric telescoping distribution to provide a smooth transition in spacing. An example of wave gauge spacing and placement is shown in Figure 5.5.

The decomposition procedure is implemented in Python (Jensen & Carstensen (2014) and Kramer et al. (2023)), where the surface elevation time series from the model output, combined with the known gauge positions, are used to calculate the incident and reflected wave signal. An example of decomposition of the wave time series is illustrated in Figure 4.4. From this, the incident significant wave height is derived and subsequently adopted as the representative significant wave height at the base of the slope.

Finally, to ensure consistency with the experimental database, an automated correction procedure was established. Cases in which the decomposed incident wave height deviated from the target database value were re-run iteratively with adjusted input wave conditions until convergence was achieved. The resulting matched wave heights were then used as the basis for all further analysis.

2.5 MIKE powered by DHI

In this thesis, the MIKE powered by DHI software suite is employed. The software is developed by the Danish Hydraulic Institute (DHI) and provides a versatile and robust framework for simulating water-related phenomena through physics-based numerical models.

Specifically, this work makes use of the MIKE 21/3 Wave Model FM system, a phase-resolving wave model formulated in the time domain, using a flexible mesh (FM) for detailed spatial resolution in complex geometries. This modelling system enables simulation of wave transformation from deep water to the nearshore, and is suitable for applications such as:

- Wave propagation and agitation in ports and coastal areas
- Surf and swash zone hydrodynamics

- Coastal flooding and wave run-up
- Nonlinear wave transformation and breaking
- Interaction with coastal structures, including reflection, transmission, and overtopping
- 3D wave kinematics for structural load calculations
- Input to dynamic mooring analyses (e.g., MIKE 21 MA)
- Tsunami and other transient wave event modelling

The two models utilised in this thesis are the MIKE 21 Wave Model FM, referred to as MIKE21W, and the MIKE 3 Wave Model FM, referred to as MIKE3W. Both models include a hydrodynamic module, which solves the governing flow equations and provides velocity and surface elevation fields. In MIKE3W, additional modules are available; namely, a turbulence module, which is required when using two-equation turbulence models for calculating vertical and horizontal eddy viscosity, and a sand transport module, which models the transport of non-cohesive sediments.

The MIKE 21/3 Wave Model FM system supports two alternative sets of governing equations:

- 2D Boussinesq-type equations as described by Madsen & Sørensen (1992). These equations account for nonlinear wave-wave interactions and provide accurate predictions of wave shoaling and dispersion for relative depth conditions up to $h/L_0 = 0.5$, where $L_0 = gT^2/(2\pi)$ is the deep water wave length;
- Fully nonlinear 3D Navier-Stokes equations, where the free surface is tracked using a height function. These equations allow for detailed simulation of three-dimensional wave kinematics and are particularly useful for highly nonlinear wave conditions and wave-structure interactions.

2.5.1 Governing equations

In the following, the governing equations for MIKE21W and MIKE3W are presented. Further details can be found in the scientific documentation available online (MIKE 21 Wave Model FM - Scientific Documentation (2025), MIKE 3 Wave Model FM - Scientific Documentation (2025))

MIKE21W

The two-dimensional enhanced Boussinesq equations are obtained by integrating the Navier-Stokes equations over the water depth and taking into account the effect that vertical accelerations have on the pressure distribution. For more details on the derivation of the two-dimensional enhanced Boussinesq equations, see Madsen et al. (1991) and Madsen & Sørensen (1992). The two-dimensional enhanced Boussinesq equations in conservative form can be expressed as:

$$\frac{\partial h}{\partial t} + \frac{\partial hu}{\partial x} + \frac{\partial hv}{\partial y} = 0 \quad (2.18)$$

$$\frac{\partial hu}{\partial t} + \frac{\partial hu^2}{\partial x} + \frac{\partial huv}{\partial y} = -gh \frac{\partial \eta}{\partial x} - \frac{\tau_{fx}}{\rho} - F_u - F_{vx} + \frac{\partial hT_{xx}}{\partial x} + \frac{\partial hT_{xy}}{\partial y} - \frac{\partial R_{xx}}{\partial x} - \frac{\partial R_{xy}}{\partial y} + \psi_1 \quad (2.19)$$

$$\frac{\partial hv}{\partial t} + \frac{\partial huv}{\partial x} + \frac{\partial hv^2}{\partial y} = -gh \frac{\partial \eta}{\partial y} - \frac{\tau_{fy}}{\rho} - F_v - F_{vy} + \frac{\partial hT_{xy}}{\partial x} + \frac{\partial hT_{yy}}{\partial y} - \frac{\partial R_{xy}}{\partial x} - \frac{\partial R_{yy}}{\partial y} + \psi_2 \quad (2.20)$$

Here t is the time; x and y are the Cartesian coordinates; $h = \eta + d$ is the total water depth, where η is the surface elevation and d is the still water depth; u and v are the depth-averaged velocity components in the x and y direction; g is the gravitational acceleration; ρ is the density of water. (τ_{fx}, τ_{fy}) are the x - and y -components of the stresses due to bottom friction; $F = (F_u, F_v)$ is the flow resistance due to porosity; $F_v = (F_{vx}, F_{vy})$ is the drag force due to vegetation; R_{xx}, R_{xy}, R_{yy} are additional terms due to the inclusion of wave breaking. The lateral stresses T_{xx}, T_{xy}, T_{yy} include viscous friction, turbulent friction and differential advection. They are estimated using an eddy viscosity formulation based on the depth-averaged velocities:

$$T_{xx} = 2\nu \frac{\partial u}{\partial x}, \quad T_{yy} = 2\nu \frac{\partial v}{\partial y}, \quad T_{xy} = \nu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \quad (2.21)$$

where ν is the eddy viscosity. The Boussinesq terms (ψ_1, ψ_2) are expressed as:

$$\begin{aligned} \psi_1 = & \left(B + \frac{1}{3} \right) d^2 \left(\frac{\partial^3 hu}{\partial x^2 t} + \frac{\partial^3 hv}{\partial x y t} \right) + d \frac{\partial d}{\partial x} \left(\frac{1}{3} \frac{\partial^2 hu}{\partial x t} + \frac{1}{6} \frac{\partial^2 hv}{\partial y t} \right) + \frac{1}{6} d \frac{\partial d}{\partial y} \frac{\partial^2 hv}{\partial x t} \\ & + B g d^3 \left(\frac{\partial^3 \eta}{\partial x^3} + \frac{\partial^3 \eta}{\partial y y x} \right) + B g d^2 \frac{\partial d}{\partial x} \left(2 \frac{\partial^2 \eta}{\partial x^2} + \frac{\partial^2 \eta}{\partial y^2} \right) + B g d^2 \frac{\partial d}{\partial y} \frac{\partial^2 \eta}{\partial y x} \end{aligned} \quad (2.22)$$

$$\begin{aligned} \psi_2 = & \left(B + \frac{1}{3} \right) d^2 \left(\frac{\partial^3 hv}{\partial y^2 t} + \frac{\partial^3 hu}{\partial y x t} \right) + d \frac{\partial d}{\partial y} \left(\frac{1}{3} \frac{\partial^2 hv}{\partial y t} + \frac{1}{6} \frac{\partial^2 hu}{\partial x t} \right) + \frac{1}{6} d \frac{\partial d}{\partial x} \frac{\partial^2 hu}{\partial y t} \\ & + B g d^3 \left(\frac{\partial^3 \eta}{\partial y^3} + \frac{\partial^3 \eta}{\partial x x y} \right) + B g d^2 \frac{\partial d}{\partial y} \left(2 \frac{\partial^2 \eta}{\partial y^2} + \frac{\partial^2 \eta}{\partial x^2} \right) + B g d^2 \frac{\partial d}{\partial x} \frac{\partial^2 \eta}{\partial x y} \end{aligned} \quad (2.23)$$

To give a conservative formulation, the gravity surface terms are split into two terms (see Chippada et al. (1998), Rogers et al. (2001), Quecedo & Pastor (2002), Liang & Borthwick (2009)):

$$g h \frac{\partial \eta}{\partial x} = \frac{1}{2} g \frac{\partial (h^2 - d^2)}{\partial x} - g d \frac{\partial d}{\partial x} = \frac{1}{2} g \frac{\partial (\eta^2 + 2\eta d)}{\partial x} - g d \frac{\partial d}{\partial x} \quad (2.24)$$

$$g h \frac{\partial \eta}{\partial y} = \frac{1}{2} g \frac{\partial (h^2 - d^2)}{\partial y} - g d \frac{\partial d}{\partial y} = \frac{1}{2} g \frac{\partial (\eta^2 + 2\eta d)}{\partial y} - g d \frac{\partial d}{\partial y} \quad (2.25)$$

It is easily seen that if η is constant, the two terms cancel exactly. In the discrete case, this is also true if the two derivatives are calculated using the same scheme.

In matrix form, the continuity equation and the momentum equations may be written:

$$\frac{\partial h}{\partial t} + \nabla \cdot \mathbf{F}^c = 0 \quad (2.26)$$

$$\frac{\partial \mathbf{V}}{\partial t} + \nabla \cdot \mathbf{F}^m = \mathbf{S}_0 + \mathbf{S}_f + \mathbf{S}_n + \frac{\partial \Psi}{\partial t} \quad (2.27)$$

where with $\mathbf{F}$ and $\mathbf{S}$ are indicated the flux components and the source terms.

Here $\mathbf{F}^c = (F_x^c, F_y^c)^T = (hu, hv)^T$, $\mathbf{V} = (hu, hv)^T$, and $\mathbf{F}^m = \mathbf{F}^{mc} - \mathbf{F}^{md} = (F_x^m, F_y^m)^T$.

MIKE3W

The model is based on the solution of the three-dimensional incompressible Reynolds-averaged Navier-Stokes equations. Thus, the model consists of continuity and momentum equations, and it is closed by a turbulent closure scheme.

The non-hydrostatic model is based on the incompressible Navier-Stokes equations, subjected to the assumptions of Boussinesq and with the free surface described by a height function. In a Cartesian coordinate system, the local continuity equation is written as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (2.28)$$

and the conservative form of the momentum equation can be written as:

$$\frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial x} + \frac{\partial vu}{\partial y} + \frac{\partial wu}{\partial z} = fv - \frac{1}{\rho_0} \frac{\partial q}{\partial x} - g \frac{\partial \eta}{\partial x} + F_u - F_{vx} + \frac{\partial}{\partial z} \left(\nu_t^v \frac{\partial u}{\partial z} \right) \quad (2.29)$$

$$\frac{\partial v}{\partial t} + \frac{\partial uv}{\partial x} + \frac{\partial v^2}{\partial y} + \frac{\partial wv}{\partial z} = -fu - \frac{1}{\rho_0} \frac{\partial q}{\partial y} - g \frac{\partial \eta}{\partial y} + F_v - F_{vy} + \frac{\partial}{\partial z} \left(\nu_t^v \frac{\partial v}{\partial z} \right) \quad (2.30)$$

$$\frac{\partial w}{\partial t} + \frac{\partial uw}{\partial x} + \frac{\partial vw}{\partial y} + \frac{\partial w^2}{\partial z} = -\frac{1}{\rho_0} \frac{\partial q}{\partial z} + F_w - F_{vz} + \frac{\partial}{\partial z} \left(\nu_t^v \frac{\partial w}{\partial z} \right) \quad (2.31)$$

Here t is the time; x and y are the Cartesian coordinates; η is the surface elevation; u , v , and w are the velocity components in the x , y , and z direction; q is the non-hydrostatic pressure; $f = 2\Omega \sin \Phi$ is the Coriolis parameter (Ω is the angular rate of revolution and Φ the geographic latitude); ν_t^v is the vertical eddy viscosity; g is the gravitational acceleration; ρ_0 is the density of water; $F_v = (F_{vx}, F_{vy}, F_{vz})$ is the drag force due to vegetation.

Equations (2.29) to (2.31) are obtained by splitting the total pressure, p , into a non-hydrostatic and a hydrostatic component, p_H , where:

$$p_H = p_A + \rho_0 g(\eta - z) + g \int_z^\eta (\rho - \rho_0) dz \quad (2.32)$$

The atmospheric pressure, p_A , at the free surface is assumed to be constant over the domain, and the density of water, ρ , is assumed to be constant.

The horizontal diffusion terms are described using a gradient-stress relation, which is simplified to:

$$F_u = \frac{\partial}{\partial x} \left(2\nu_t^h \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left(\nu_t^h \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right) \quad (2.33)$$

$$F_v = \frac{\partial}{\partial x} \left(\nu_t^h \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right) + \frac{\partial}{\partial y} \left(2\nu_t^h \frac{\partial v}{\partial y} \right) \quad (2.34)$$

$$F_w = \frac{\partial}{\partial x} \left(\nu_t^h \frac{\partial w}{\partial x} \right) + \frac{\partial}{\partial y} \left(\nu_t^h \frac{\partial w}{\partial y} \right) \quad (2.35)$$

where ν_t^h is the horizontal eddy viscosity.

The surface and bottom boundary conditions for u , v , and w are:

$$\begin{aligned} \text{at } z = \eta \\ \frac{\partial \eta}{\partial t} + u \frac{\partial \eta}{\partial x} + v \frac{\partial \eta}{\partial y} - w = 0, \quad \left(\frac{\partial u}{\partial z}, \frac{\partial v}{\partial z} \right) = (0, 0) \end{aligned} \quad (2.36)$$

$$\begin{aligned} \text{at } z = -d \\ u \frac{\partial d}{\partial x} + v \frac{\partial d}{\partial y} + w = 0, \quad \left(\frac{\partial u}{\partial z}, \frac{\partial v}{\partial z} \right) = \frac{1}{\rho_0 \nu_t^v} (\tau_{bx}, \tau_{by}) \end{aligned} \quad (2.37)$$

Here d is the still water depth, and (τ_{bx}, τ_{by}) are the x - and y -components of the bottom stresses.

The total depth, $h = \eta + d$, is obtained by vertical integration of the local continuity equation and taking into account the boundary conditions at the surface and the bottom:

$$\frac{\partial h}{\partial t} + \frac{\partial h \bar{u}}{\partial x} + \frac{\partial h \bar{v}}{\partial y} = 0 \quad (2.38)$$

where $\bar{u}$ and $\bar{v}$ are the depth-averaged velocities:

$$h \bar{u} = \int_{-d}^{\eta} u \, dz, \quad h \bar{v} = \int_{-d}^{\eta} v \, dz \quad (2.39)$$

In matrix form, the continuity equation and the momentum equations may be written

$$\frac{\partial \mathbf{h}}{\partial t} + \nabla \cdot \mathbf{F}^c = 0 \quad (2.40)$$

$$\frac{\partial \mathbf{U}}{\partial t} + \nabla \cdot \mathbf{F}^m = \mathbf{S}_h + \mathbf{S}_q \quad (2.41)$$

Here $\mathbf{F}^c = (F_x^c, F_y^c)^T = (h \bar{u}, h \bar{v})^T$, $\mathbf{U} = (u, v, w)^T$ and $\mathbf{F}^m = \mathbf{F}^{mc} - \mathbf{F}^{md} = (\mathbf{F}_x^m, \mathbf{F}_y^m, \mathbf{F}_z^m)^T$, where with $\mathbf{F}$ and $\mathbf{S}$ are indicated the flux components and the source terms.

If the hydrostatic pressure assumption is applied, the non-hydrostatic pressure will be zero. With this assumption, a three-dimensional, hydrodynamic model can be significantly simplified because the momentum equation in the vertical direction (eq. (2.31)) can be neglected.

Turbulence is represented through an eddy viscosity concept, where the effects of turbulence on the mean flow are parameterised by an effective viscosity. This eddy viscosity can either be prescribed using empirical formulations or obtained by solving semi-empirical turbulence closure models.

The software provides two alternative two-equation turbulence models:

- the k - ϵ model Larsen & Fuhrman (2018), Rodi (2017), which is widely used in engineering applications due to its robustness and computational efficiency;
- the k - ω model Larsen & Fuhrman (2018), Wilcox (2008), which offers improved near-wall treatment and is often preferred in cases with strong adverse pressure gradients or flow separation.

Both models compute turbulence based on transport equations for turbulent kinetic energy (k) and an additional variable (ϵ or ω), from which the eddy viscosity is derived.

2.5.2 Time integration

In explicit numerical schemes, the time step is constrained by the Courant–Friedrichs–Lewy (CFL) condition, which requires that the CFL number remain below unity to maintain stability. In this work, a variable time-stepping approach is applied to the solution of the Navier–Stokes equations, ensuring that the time step is automatically adjusted so that the CFL number does not exceed a prescribed critical value at any computational node.

For the Navier–Stokes equations in Cartesian coordinates, the CFL number is defined as:

$$CFL = \left(\sqrt{gh} + |u| \right) \frac{\Delta t}{\Delta x} \quad (2.42)$$

where h is the total water depth, u the velocity component in the x -direction, g the gravitational acceleration, Δx a characteristic element length in the x -direction, and Δt the time step. Here, Δx is approximated by the minimum edge length of each element, while h and u are evaluated at the element centre.

The software allows users to prescribe both a maximum and a minimum time step. The lower limit plays a key role, since very small values enable the model to dynamically adapt Δt to satisfy the CFL condition and thereby ensure numerical stability. In this thesis, a target CFL number of 0.5 was applied, with the minimum time step set to $dt = 10^{-10}$ s.

2.5.3 $k - \epsilon$ model

The eddy viscosities, ν_t and ν_t^h , appearing in the governing Reynolds-averaged Navier–Stokes (RANS) equations are, in this thesis, determined using the two-equation $k-\epsilon$ turbulence model.

In this model, the eddy viscosity ν_t is expressed as

$$\nu_t = c_\mu \frac{k^2}{\epsilon}, \quad (2.43)$$

where c_μ is an empirical constant, k denotes the turbulent kinetic energy, and ϵ is the dissipation rate of turbulent kinetic energy.

In the simulations carried out in this thesis, the turbulence model is applied using the standard parameter settings. A detailed description of the implementation and parameter values can be found in MIKE 3 Wave Model FM - Scientific Documentation (2025).

2.5.4 Flood and dry

To accurately simulate wave run-up and overtopping processes, the Flood and Dry option is enabled in the MIKE 21/3 Wave Model FM graphical user interface. This option manages the treatment of moving boundaries by reformulating the governing equations when the local water depth falls below the predefined wetting depth, h_{wet} . Specifically, when the water depth is less than h_{wet} , the momentum fluxes are set to zero, and only mass fluxes are considered in the computation. If the depth further decreases below the drying threshold, h_{dry} , the corresponding element or cell is excluded from the active simulation domain.

This numerical approach to handling wetting and drying fronts is based on the finite volume formulations presented by Zhao et al. (1994) and Sleight et al. (1998). Further technical details can be found in Section 6.3.1

of the MIKE 21/3 Wave Model FM Hydrodynamic Module and Sand Transport User Guide (2025). It is noted there that setting very small values for h_{wet} may lead to unphysically high flow velocities and, consequently, stability issues during the simulation.

Given these considerations, the influence of the wetting and drying parameters will be systematically investigated across different test cases. The aim is to evaluate their effect on the model's capability to reproduce run-up and overtopping accurately, and to identify an optimal range for h_{wet} that balances numerical stability with the need for physical precision.

2.5.5 Wave breaking

In MIKE21W, wave breaking is incorporated through the surface roller concept introduced by Svendsen (1984). Breaking is assumed to initiate when the local water surface slope exceeds a predefined critical angle. At this point, a surface roller is formed and modelled as a passive bulk of water that is detached from the main wave motion and advected with the wave celerity. The roller's presence introduces an additional convective momentum term, accounting for the vertical non-uniformity of the horizontal velocity distribution.

This approach is illustrated in Figure 2.7. A detailed description of the roller formulation and its implementation can be found in Madsen et al. (1997) and Sørensen et al. (2004).

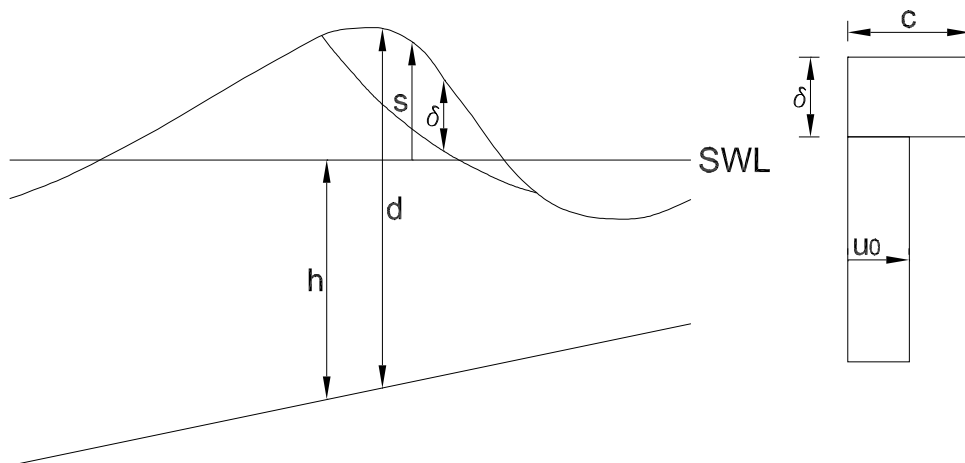


Figure 2.7: Cross-section of a breaking wave and the assumed vertical profile of the horizontal velocity components (from Schäffer et al. (1993)).

Although the model is primarily developed for spilling breakers, it has shown satisfactory performance in capturing wave height decay even in the presence of plunging breakers.

The main parameters governing the roller model and their default values are summarised below:

- Roller form factor: Defines the geometry of the roller. A default value of 1.5 is used for spilling breakers. For plunging breakers, a value of approximately 2.0 is recommended, as suggested by Ozanne et al. (2000).
- Roller celerity factor: The roller is assumed to travel at a celerity equal to the wave celerity multiplied by this factor. During the transition from initial breaking to the bore-like regime in the inner surf zone, the factor increases gradually from unity to the specified roller celerity factor. The default value is 1.3.

- **Initial breaking angle:** Defines the local surface slope threshold for breaking onset. A default value of 20° is applied for spilling breakers. For plunging breakers, Madsen et al. (1997) recommends increasing this value to 25° to improve accuracy.
- **Final breaking angle:** Represents the terminal critical slope during the transition to the bore regime. The default value is 10° .
- **Half-time for roller cut-off:** Governs the transition from the initial to the final breaking angle. It is typically set to $T_p/5$, where T_p is the peak wave period. For plunging breakers, a shorter time scale of $T_p/10$ has been found to yield better results (Madsen et al. (1997)).

This parametrisation enables a dynamic and physically informed representation of wave breaking behaviour, contributing to the accurate simulation of energy dissipation in the surf zone.

In MIKE3W, no explicit physical model for wave breaking is implemented. Instead, the dissipation of wave energy during breaking is implicitly handled by the numerical scheme. When the wave front steepens sufficiently, the higher-order harmonics are not adequately resolved on the computational mesh, leading to their numerical damping.

2.5.6 Type of outputs and their usage

To analyse the simulation results, several types of output files generated by the software are utilised. Each output format serves a specific purpose—whether for result verification, data extraction for further analysis, or purely for visualisation.

The following output types are employed:

- **Point output:** This format provides time series of selected variables at one or more specified locations throughout the simulation. It is particularly useful for extracting surface elevation data, which is used in the wave decomposition process described in section 2.4.
- **Line output:** This output type captures the spatial variation of variables along a defined transect within the domain. It is used to estimate the time series of run-up by identifying the furthest landward position reached by water during the simulation. Additionally, it can be used to visualise surface elevation profiles at specific time steps.
- **Discharge output:** This format is designed for calculating wave overtopping across a defined cross-section. It provides both the instantaneous discharge time series and the accumulated discharge volume. These outputs are essential for quantifying the total overtopping per simulation and for analysing the temporal evolution of overtopping. They also help identify potential overestimations due to numerical artefacts.

2.6 Model performance evaluation

The performance of the simulation model is assessed against benchmark datasets widely recognised in the literature. Evaluation is carried out through both visual inspection and the application of quantitative error metrics, which provide an objective basis for comparing model predictions with experimental observations.

2.6.1 Root Mean Square Deviation (RMSD)

The Root Mean Square Deviation (RMSD) is a widely used metric to quantify the average magnitude of the error between predicted and observed values. It is defined as the square root of the mean of the squared differences between predictions and observations:

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N (X_i - x_i)^2} \quad (2.44)$$

where X_i is the modelled value, x_i is the corresponding observed value, and N is the number of data points.

In this study, as all selected experimental cases to be modelled have Reliability Factor (RF) and Consistency Factor (CF) equal to 1 (see Section 5.3), it is reasonable to assign equal weights to all data points.

RMSE serves as a scale-dependent measure of accuracy and is useful for comparing the predictive performance of different models on the same dataset, though not across datasets of different magnitudes.

2.6.2 Scatter Index (SCI)

The Scatter Index (SCI) is a normalised version of the RMSE, obtained by dividing it by the mean of the observed values:

$$SCI = \frac{RMSE}{\bar{x}} \quad (2.45)$$

where $\bar{x}$ is the mean of the observed values x_i . SCI provides a dimensionless measure of the relative error and is especially useful for comparing performance across datasets of different scales.

A lower SCI indicates better agreement between predicted and observed values, whereas a higher SCI suggests greater deviation and model inconsistency.

2.6.3 Relative Bias

The relative bias quantifies the systematic error between predicted and observed values, expressed as a proportion of the observed mean:

$$\text{Relative Bias} = \frac{\bar{X} - \bar{x}}{\bar{x}} \quad (2.46)$$

where $\bar{X}$ and $\bar{x}$ are the mean modelled and observed values, respectively.

A positive bias indicates systematic overprediction by the model, while a negative bias denotes underprediction. The magnitude of the relative bias provides insight into the direction and severity of any systematic deviation between model and observations.

3 Solitary wave run-up

The solitary wave run-up scenario represents one of the most fundamental benchmark cases in coastal engineering. Its simplicity and well-defined physics make it an ideal starting point for understanding more complex interactions between waves and coastal structures.

This case is chosen specifically for its clarity in setup and interpretation. Compared to irregular or random wave scenarios, the solitary wave problem involves fewer variables and is well-supported by theoretical and empirical literature. Thus, it provides a solid foundation for validating numerical models and gaining initial insights into run-up behaviour, which will inform and contextualise more advanced cases later in this thesis.

In this chapter, the solitary wave run-up is modelled with MIKE21W and expressed using the maximum run-up R_{max} , since solitary waves represent single-wave events.

3.1 Literature review and experimental comparisons

Solitary wave interaction with coastal topographies and structures has been the focus of extensive research. Two primary configurations have garnered attention: vertical walls and sloped shores or structures. These serve as standard test cases for model calibration and validation due to their frequent use in analytical, numerical, and laboratory studies.

Vertical wall studies

One of the earliest comprehensive investigations into the reflection and run-up of solitary waves at vertical structures was carried out by Cooker et al. (1997), who studied waves incident perpendicularly on a fixed vertical boundary. Their work focused on the local pressure dynamics and overall reflection behaviour of high-amplitude solitary waves, from which they derived an analytical relationship between the incoming wave height and the maximum run-up:

$$\frac{R_{max}}{h} = 2\frac{H}{h} + \frac{1}{2}\left(\frac{H}{h}\right)^2 \quad (3.1)$$

where R_{max} denotes the maximum run-up, H is the incident wave height, and h is the still water depth. This formulation was later refined to include higher-order nonlinear effects, yielding:

$$\frac{R_{max}}{h} = 2\frac{H}{h} + \frac{1}{2}\left(\frac{H}{h}\right)^2 + \frac{3}{4}\left(\frac{H}{h}\right)^3 \quad (3.2)$$

which has been shown to improve accuracy for larger wave amplitudes.

More recently, Borsboom & Jacobsen (2021) validated this improved expression through high-fidelity CFD simulations performed with the waves2Foam module in OpenFOAM. Their numerical results exhibited strong agreement with the refined formulation in eq. (3.2).

Sloped beach studies

A substantial body of research has investigated the run-up behaviour of solitary waves on sloping beaches. One of the earliest systematic studies was conducted by Hall & Watts (1953), who performed pioneering laboratory experiments on concrete slopes of varying inclinations. Based on their observations, empirical formulas were

proposed to predict maximum run-up heights, which remain influential today and are often used as benchmarks for the validation of modern numerical models.

More recently, Liu & Cho (1994) applied a Boundary Integral Equation Method (BIEM) to investigate solitary wave run-up, incorporating bottom friction effects. Their results emphasised the importance of friction, especially on slopes less than 20°, and provided improved predictive accuracy for maximum run-up heights.

Further developments were presented by Han et al. (2015), who conducted comprehensive laboratory experiments on the interaction of solitary waves with both vertical structures and sloping beaches. For the run-up investigations, they examined slopes of 15°, 20°, and 45°, and systematically compared the measured run-up heights against existing empirical and numerical predictions.

From the cited literature, the maximum run-up height of a solitary wave on a plane slope can generally be expressed in the form:

$$\frac{R_{max}}{h} = a \left(\frac{H}{h} \right)^b \quad (3.3)$$

where a and b are empirical coefficients that depend on the beach slope. The corresponding values are listed in Table 3.1.

Table 3.1: Estimating coefficients of solitary wave run-up.

Deg [°]	Hall & Watts (1953)		Liu & Cho (1994)		Han et al. (2015)	
	a	b	a	b	a	b
15	3.75	1.120	3.561	1.109	3.38	1.066
20	—	—	3.530	1.133	3.74	1.154
45	3.10	1.150	2.906	1.129	3.75 ¹	1.128 ¹

3.2 Model setup

The model domain is constructed as a one-dimensional (1D) configuration by constraining the mesh to a single cell in the cross-shore direction. This effectively eliminates transverse variation and ensures that the problem remains strictly 1D. The domain length is set to 1000 m, with the bathymetry consisting of a 600 m flat region followed by either a vertical wall (in the flat-bed test case) or a sloped beach (in the inclined-bed test case).

The computational mesh is quadrangular, with varying spatial resolution to optimise simulation accuracy and computational efficiency. In the flat portion of the domain, the mesh cell size is uniform with an edge length determined by the number of elements per wavelength, n/L , the tested cases are shown in Table 3.2. To increase resolution in the region of interest—i.e., the sloped section where run-up occurs—the longitudinal cell size is refined to one-fifth the size of the cells of the flat zone. A transitional zone of 200 m is implemented at the interface between the flat and sloped regions to gradually reduce the cell length. This smooth refinement strategy avoids sudden changes in resolution, thereby preventing numerical instabilities while improving the accuracy of wave transformation and run-up prediction.

¹The line obtained by applying these coefficients in eq. (3.3) is not presented, as it does not align with the results reported by Han et al. (2015), indicating that a typo is likely present.

Table 3.2: Tested cases and relative parameters.

Bathymetry	Wave type	n/L	h [m]	h_{wet}	$H/h[-]$
Flat bed	Solitary	30	5 and 10	-	0.1 to 0.68
Slope	Solitary	30	5 and 10	0.05 and 0.1	0.12 to 0.32
Slope	Cnoidal	30	5 and 10	0.05	0.12 to 0.28

For the flat-bed scenario, the vertical wall is represented by a fully reflective land boundary. In the sloped-bed scenarios, varying beach inclinations are tested: 15°, 20°, and 45°, representing mild to steep beach profiles.

For the vertical wall case, the initial still water depth is set to $h = 5$ m and $h = 10$ m. For all cases involving a sloped beach, the initial still water depth is set to $h = 5$ m. This depth is applied to both solitary and cnoidal wave scenarios. In addition, solitary waves are also simulated with an increased water depth of $h = 10$ m.

For simulations involving sloped beaches, the drying depth is set to $h_{\text{dry}} = 0.005$ m, while two different values are tested for the wetting depth: $h_{\text{wet}} = 0.05$ m and $h_{\text{wet}} = 0.1$ m. This dual setting enables evaluation of how the wetting depth influences the model's ability to simulate run-up and rundown accurately.

A constant time step of $dt = 0.1$ s is used for all simulations. This value ensures sufficient temporal resolution to accurately resolve the wave propagation and interaction with the boundary.

To maintain stability, the Riemann solver stabilisation factor is set to $R = 0.3$. No additional physical processes are activated in the simulations—bed friction and viscosity are neglected, and wave breaking is not included. This allows for isolating the pure hydrodynamic behaviour of wave run-up without the influence of dissipative effects, ensuring the results are primarily driven by geometric and kinematic parameters.

3.2.1 Wave generation

At the offshore boundary, waves are generated using either solitary or cnoidal wave formulations, depending on the case. In both configurations, the wave direction is set perpendicular to the shoreline to maintain a purely 1D propagation.

Solitary wave

The solitary wave implementation in MIKE21W is a non-published feature (not directly accessible through the graphical user interface (GUI)), but embedded within the software's code. The formulation used follows the theoretical background presented in Section 2.3.1, which, given the water depth, defines the solitary wave solely by its wave height.

For simulations involving the vertical wall, the solitary wave height is specified in non-dimensional form as a ratio to the water depth, H/h , ranging from 0.10 to 0.68. For the sloped beach cases, the wave height is similarly normalised, with H/h ranging from 0.12 to 0.32. These ranges were chosen to ensure wave nonlinearity while avoiding wave breaking, which is not activated in the simulations.

Cnoidal wave

As said, the solitary wave formulation is not available directly in the GUI; therefore, as a tentative to model similar cases with one of the built-in formulations, the cnoidal wave formulation is tested. The cnoidal wave is generated by specifying both wave height and wave period. To simulate an effectively non-periodic, isolated wave event (similar to a solitary wave), the period was set to a large value, $T = 400$ s. This allows the wave to be treated as quasi-solitary in behaviour. The dimensionless wave height, H/h , for the cnoidal wave cases

ranges from 0.10 to 0.28. These values ensure that the wave remains nonlinear but within the validity range of the cnoidal theory.

3.2.2 Input convergence

A convergence study was carried out on the flat-bed configuration to investigate the sensitivity of the model to variations in input parameters. In particular, the number of elements per wavelength, denoted as n/L , was evaluated in relation to the Riemann solver factor. This analysis aimed to understand the extent of wave height damping caused by numerical discretisation and stabilisation.

Due to the computational demands of such a study, it was performed only for the solitary wave case. The matrix plot shown in Figure 3.1 compares the input wave height H_1 with the wave height that develops after approximately 500 meters of propagation, H_2 . The plot clearly shows that wave damping affects the entire tested parameter space. However, a trend of increased stability is observed for larger values of n/L , where the influence of the Riemann factor R becomes negligible.

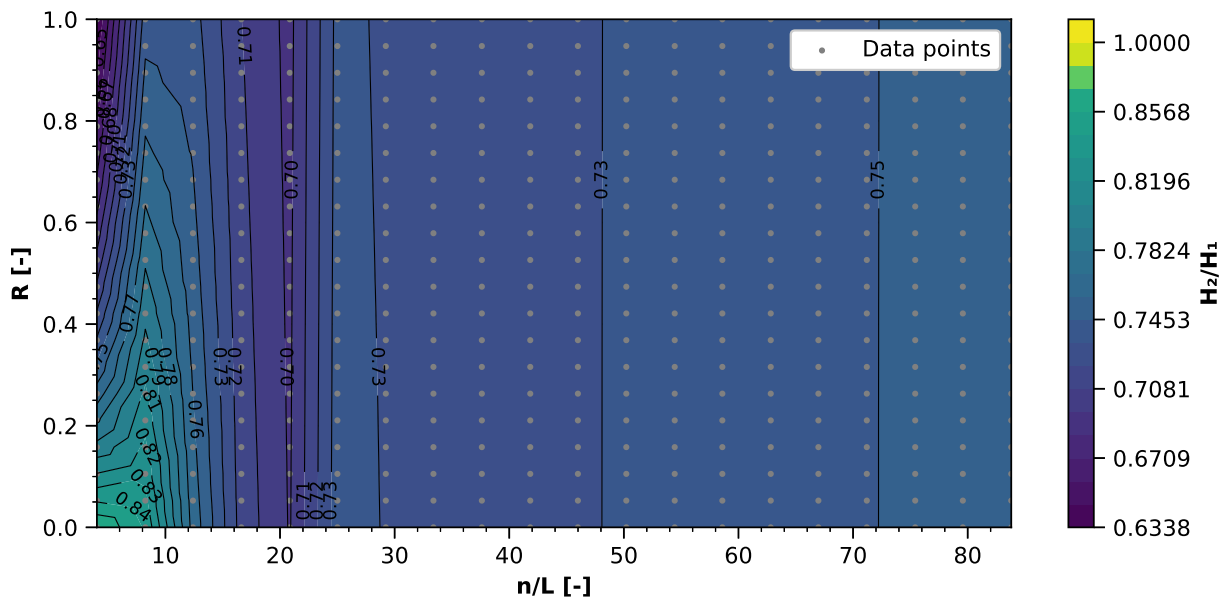


Figure 3.1: Convergence analysis of the wave decay H_2/H_1 for varying n/L and R .

To further investigate the behaviour of the wave along the flat portion of the domain, wave profiles at multiple time steps are compared in Figure 3.2. After an initial decay from the input value, the wave height stabilises. This suggests that a steady amplitude is reached before the wave encounters the slope or vertical wall.

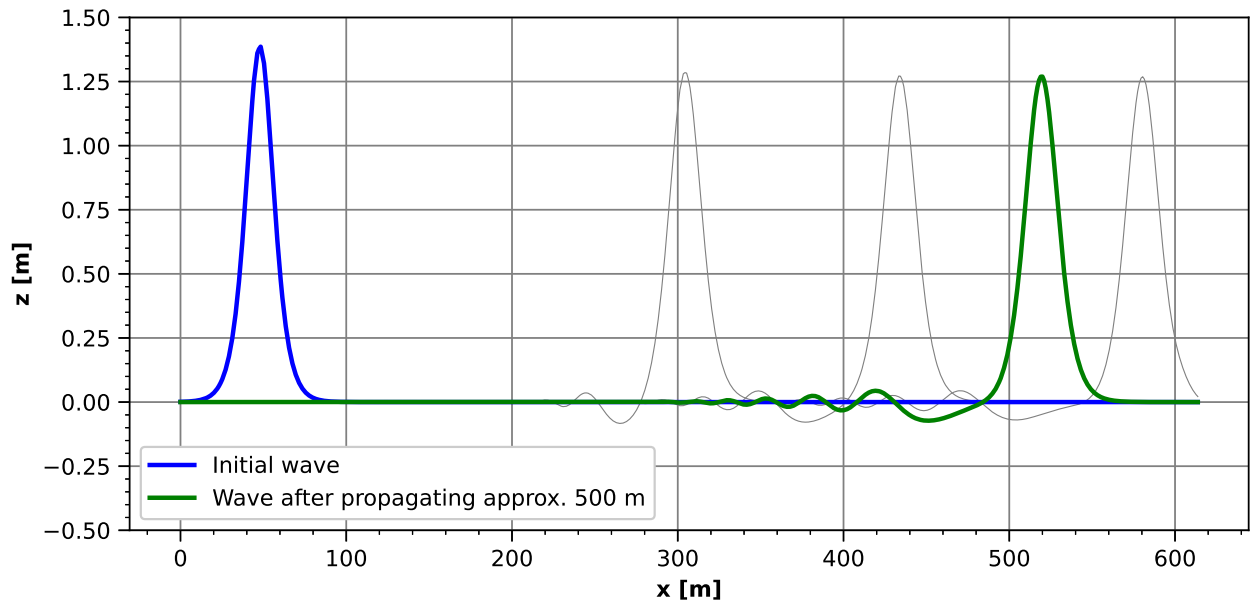


Figure 3.2: Comparison of the wave in the wave generation zone and after 500 meters of propagation. The grey lines represent different propagation steps.

Additionally, an unstable trailing oscillation—referred to here as a tail—is observed behind the solitary wave. This tail becomes more pronounced for larger wave heights and is reduced for smaller waves. It is likely a numerical artefact resulting from the limited-order approximation used in the solitary wave generation within MIKE21W.

A similar trend is noted in the cnoidal wave simulations, where the initially defined wave height also decreases during propagation. Eventually, the wave reaches a comparable amplitude to the solitary wave.

Given these observations, for all further analyses of wave run-up, the effective wave height is taken as the height measured at the toe of the slope rather than the input height, so the wave height corresponding to the green line in Figure 3.2. This approach ensures that the actual driving force behind the run-up is correctly represented.

3.3 Sensitivity analysis

A sensitivity analysis was performed on water depth (h), wetting depth (h_{wet}), and wave theory for slopes of 15° , 20° , and 45° . The results for the 15° slope are shown in Figure 3.3, while the analyses for the other slopes are provided in Appendix A.1.

The results were found to be particularly sensitive to the wetting depth, h_{wet} , highlighting the importance of this parameter in accurately capturing run-up dynamics. Based on the results in Figure 3.3, the parameter set adopted for the main analyses was $h = 10$ m, $h_{\text{wet}} = 0.05$ m, together with solitary wave theory. These choices provided the best overall agreement with experimental data.

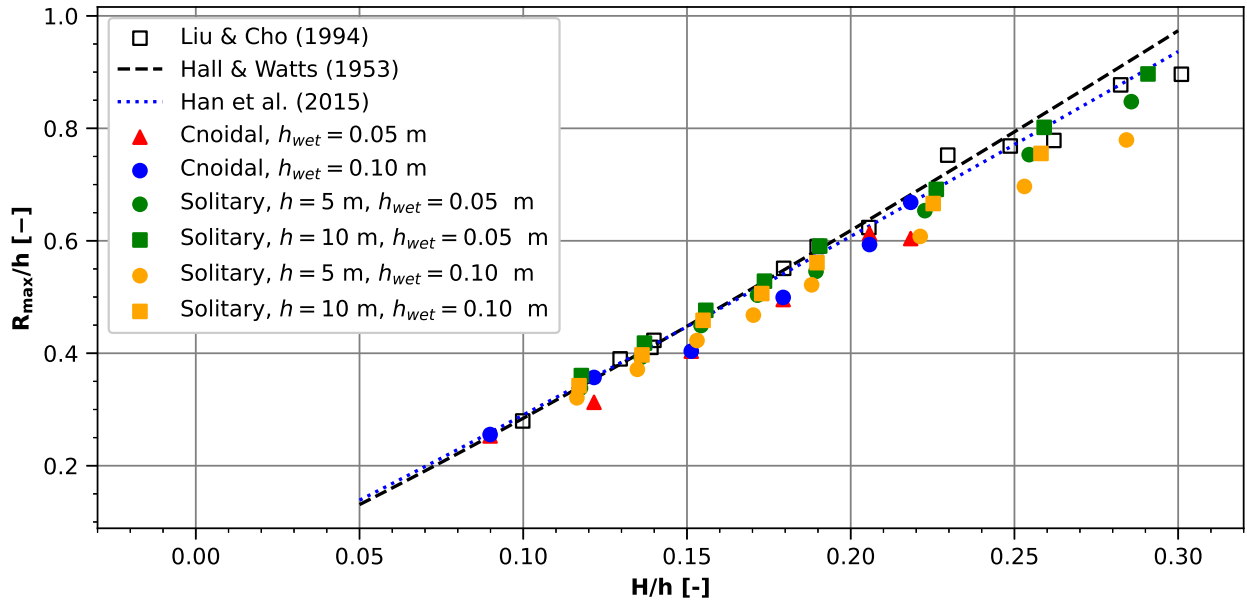


Figure 3.3: Sensitivity analysis for a 15° slope: influence of h , h_{wet} , and wave theory.

3.4 Results

The case of a solitary wave reflecting on a vertical wall is presented in Figure 3.4.

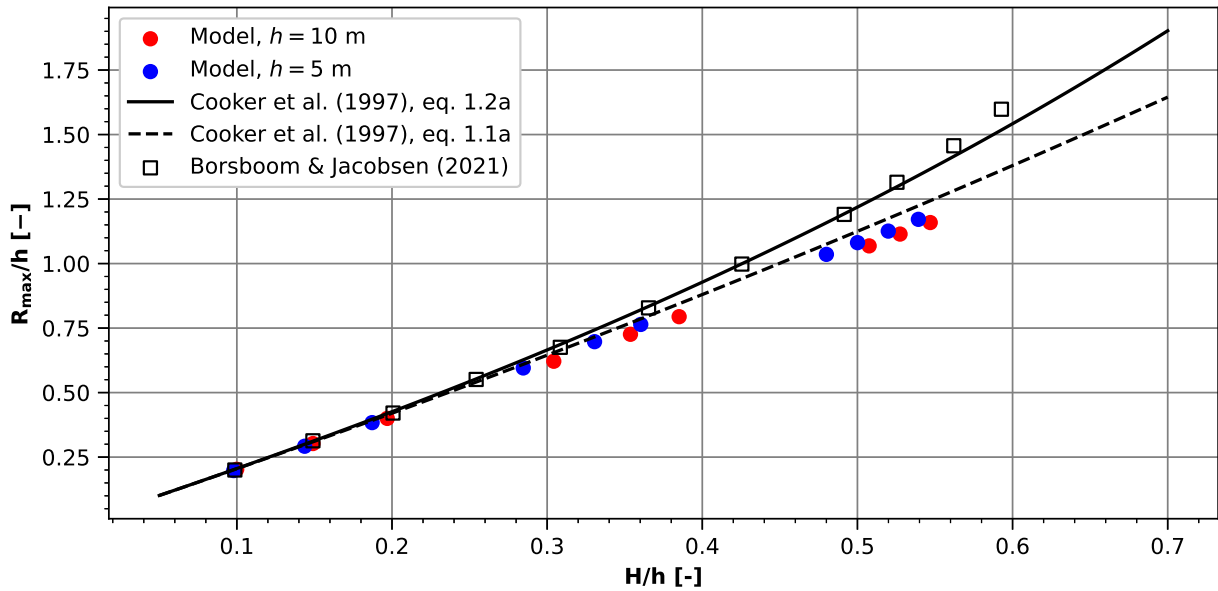


Figure 3.4: Predicted and modelled run-up for a solitary wave perpendicularly encountering a vertical wall.

The results in Figure 3.4 show good agreement with the analytical formulation given in eq. 1.1a (referred to in the text as eq. (3.1)), originally derived by Cooker et al. (1997). The refined expression in eq. 1.2a (referred to in the text as eq. (3.2)) extends eq. 1.1a by incorporating higher-order nonlinear effects, which explains the increasing deviation between the two equations at larger H/h values. The closer agreement with the lower-

order nonlinear expression is expected, since both the solitary wave formulation used here and the governing equations of the MIKE21W model are only weakly nonlinear.

The results concerning wave run-up on a sloped beach are shown in Figure 3.5, Figure 3.6, and Figure 3.7.

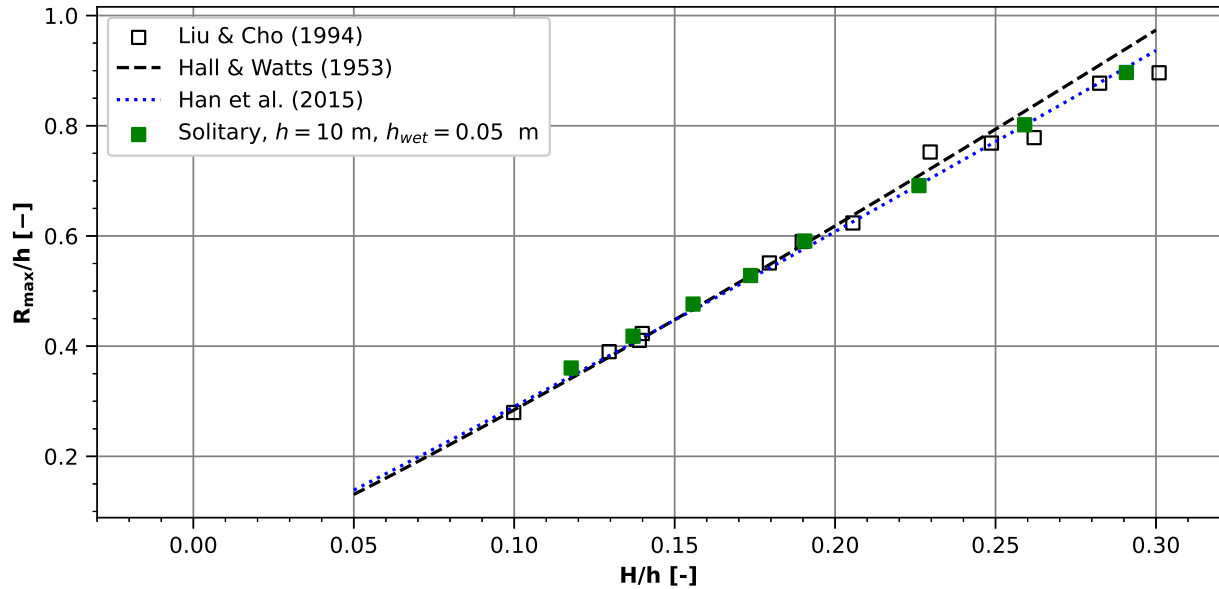


Figure 3.5: Predicted and simulated run-up of a solitary wave incident perpendicular to a 15° sloped beach.

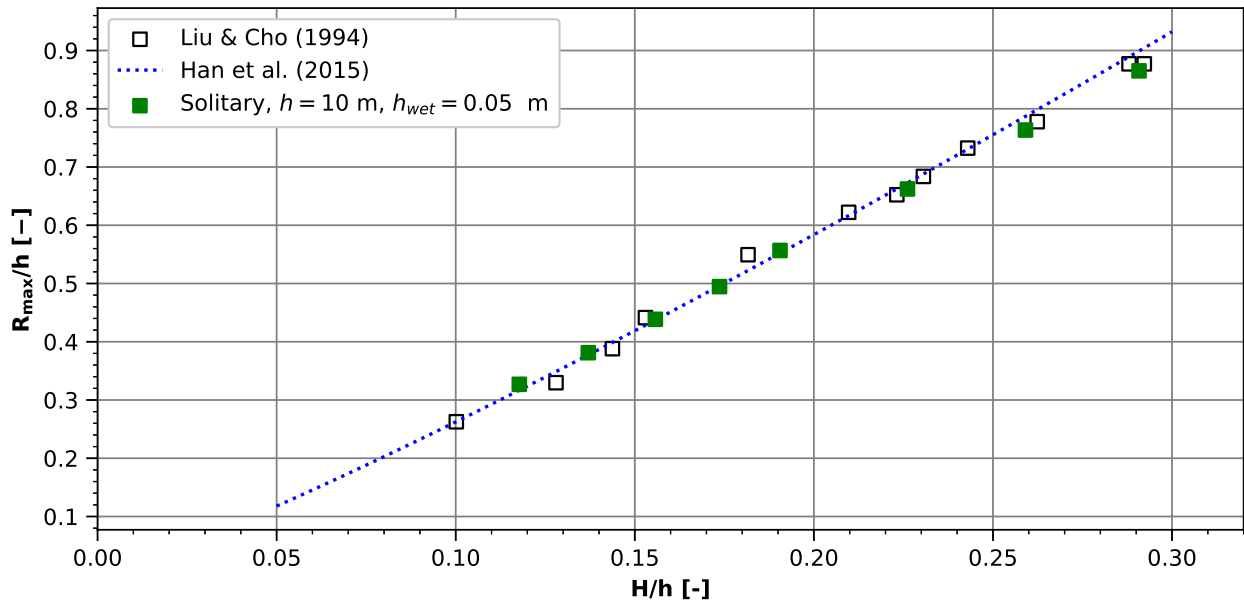


Figure 3.6: Predicted and simulated run-up of a solitary wave incident perpendicular to a 20° sloped beach.

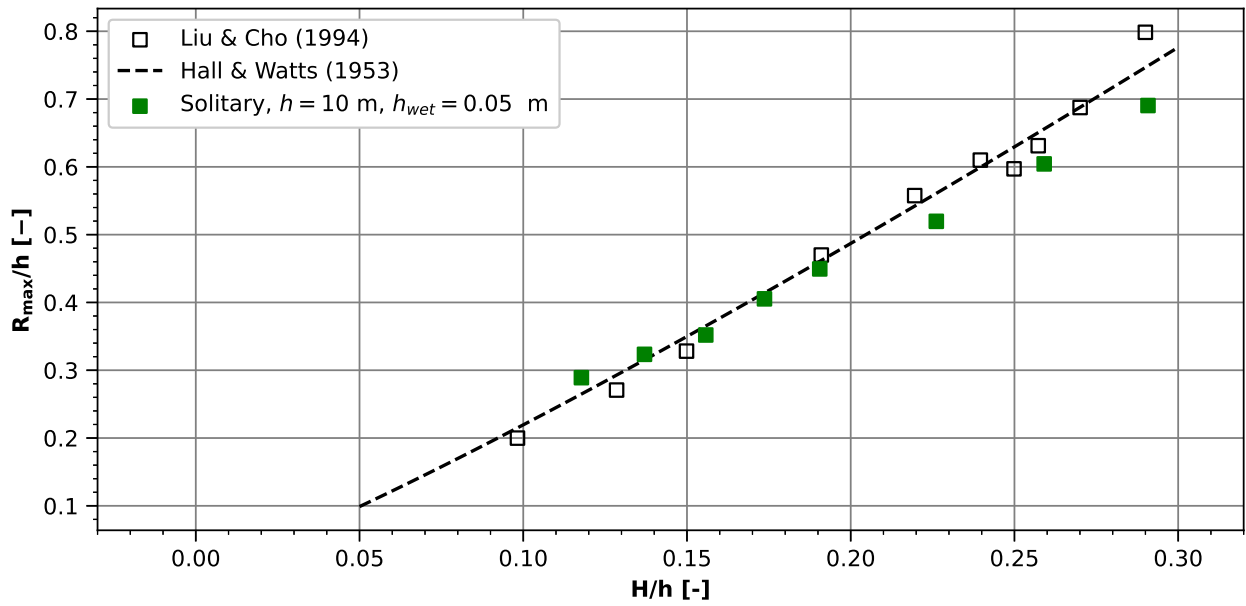


Figure 3.7: Predicted and simulated run-up of a solitary wave incident perpendicular to a 45° sloped beach.

Overall, the model produces very accurate predictions. The results across all cases show decreasing agreement with reference data (Liu & Cho (1994), Hall & Watts (1953), Han et al. (2015)) as the beach slope increases, with steeper slopes yielding less accurate predictions. This trend is consistent with the mild-slope assumption inherent in the governing equations.

3.4.1 Results convergence

A convergence analysis was made on the run-up results based on the discretisation of the mesh to ensure converged results. The discretisation of the mesh is stated in terms of elements per wavelength ($\frac{n}{L}$). Results of the analysis can be seen for the steepest slope of 45° in Figure 3.8.

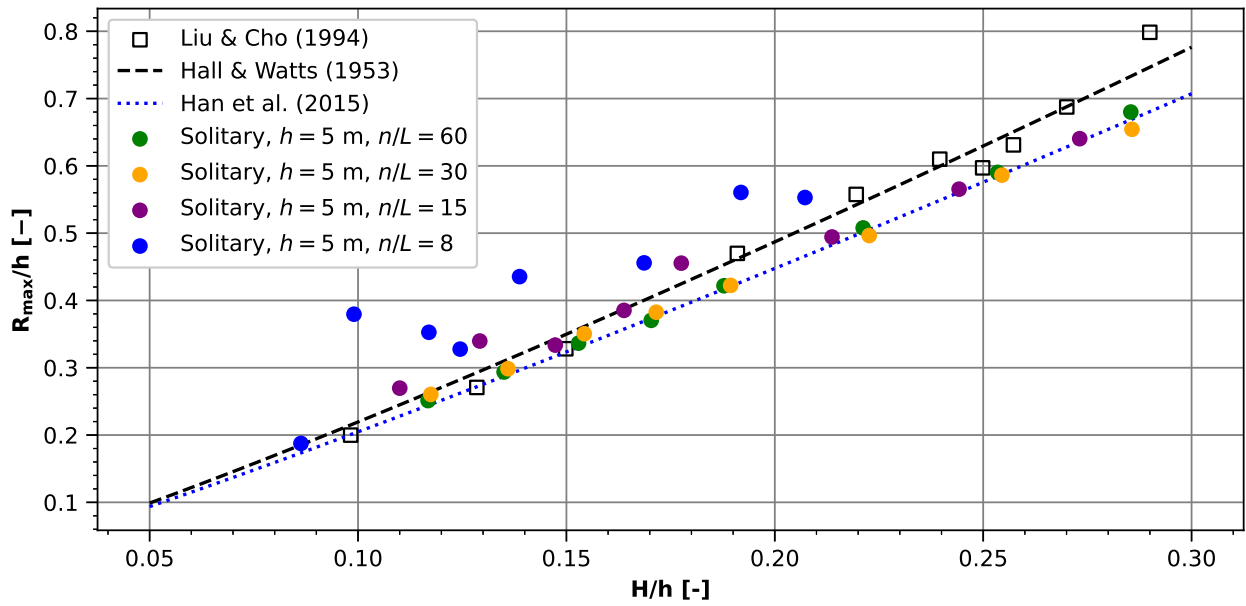


Figure 3.8: Convergence analysis. Solitary wave run-up, slope = 45°.

As illustrated in Figure 3.8, discretisations of $\frac{n}{L} = 8$ and $\frac{n}{L} = 15$ are too coarse to achieve a converged solution. However, a discretisation finer than $\frac{n}{L} = 30$ appears to result in converged outcomes. Similar convergence analyses were performed for slopes of 15° and 30°, which demonstrated the same trend. These additional results can be found in Appendix A.1.

4 Irregular wave run-up

In the previous chapter, the focus was on solitary waves, which represent an idealised and relatively simple case for investigating wave run-up processes. Solitary waves are useful for developing fundamental insight because they are characterised by a single crest, allowing for controlled studies of wave-structure interaction. However, solitary waves are rarely representative of natural sea states, which are inherently irregular and governed by a spectrum of wave components with varying frequencies, amplitudes, and phases.

The transition from solitary to irregular waves marks a significant increase in complexity. Irregular waves are composed of a superposition of individual wave components, typically described by statistical or spectral representations. This introduces phenomena such as wave-wave interaction, reflection from coastal structures, and constructive or destructive interference, all of which strongly influence the magnitude and variability of wave run-up. From an engineering perspective, it is precisely these irregular sea states that must be considered for reliable design and assessment of coastal and offshore structures, as they correspond to the most common and realistic loading scenarios encountered in practice.

The present chapter, therefore, extends the analysis to irregular waves, with the aim of bridging the gap between idealised wave conditions and realistic environmental forcing. Building on the convergence studies and sensitivity analyses performed for solitary waves, the parameter settings that provided the most accurate and stable results in the simpler case are adopted as a baseline for the irregular wave simulations. This ensures that the numerical framework is both robust and consistent, while also allowing the study to focus on the additional challenges introduced by the stochastic and spectral nature of irregular waves.

4.1 Literature review and experimental comparison

Historical development of wave run-up studies

The study of wave run-up has undergone a long evolution, moving from early qualitative observations to advanced numerical and probabilistic formulations. The development reflects broader progress in coastal engineering, experimental hydrodynamics, and computational modelling.

Early interest in wave run-up was motivated by practical concerns in harbour and coastal protection works. Initial descriptions were primarily observational, with measurements often taken in the field to guide design. A key breakthrough came with the work of Iribarren (1965), who introduced what later became widely known as the Iribarren number (also called the surf similarity parameter). This dimensionless quantity links wave steepness and slope angle, and remains a fundamental scaling parameter for run-up studies:

$$\xi = \frac{\tan \alpha}{\sqrt{H_0/L_0}} \quad (4.1)$$

where $\tan \alpha$ is the seaward slope of a beach or a structure, H is the wave height, and L_0 is the deep-water wavelength. Iribarren's formulation established that wave run-up depends strongly on both slope steepness and breaker wave type (spilling, plunging, or surging).

In the mid-20th century, systematic laboratory experiments provided the first empirical formulations of wave run-up. Saville Jr (1957) and Hunt Jr (1959) performed early flume studies, establishing relationships between

run-up and slope steepness. With Asbeck et al. (1953), for the first time in an international paper, a probabilistic description of run-up exceedance levels is introduced, such as $R_{u2\%}$, the run-up height exceeded by only 2% of incident waves. This probabilistic perspective was a turning point, recognising that design against maximum run-up requires statistical treatment of irregular wave trains.

The 1970s also marked the publication of the Shore Protection Manual (US), which consolidated empirical datasets into design charts and formulas. The SPM formulations expressed run-up as a function of the surf similarity parameter, explicitly accounting for breaker type. These semi-empirical relations were quickly adopted in engineering practice and became a standard reference for decades.

While laboratory studies dominated early research, the 1980s and 1990s saw a growing recognition that field conditions could differ substantially from laboratory flume results. Holman (1986), through video monitoring of natural beaches, provided important datasets on swash and run-up statistics. Similarly, Nielsen & Hanslow (1991) studied run-up variability on natural beaches, highlighting tidal influence and wave group effects. These works underlined the complexity of natural run-up processes, which are influenced not only by slope and wave steepness but also by beach morphology, infragravity waves, and wave groups.

During this period, probabilistic formulations became increasingly central. Design practice shifted towards quantiles of the run-up distribution, with $R_{u2\%}$ becoming widely adopted as a design parameter for coastal structures.

The need for broadly applicable predictive formulas for natural beaches led to the development of more refined empirical models. Among these, the widely cited work of Stockdon et al. (2006) proposed a predictive parameterisation of $R_{u2\%}$ based on a large dataset of field observations. This formula remains one of the most widely used in coastal hazard assessment, particularly in beach environments.

With increasing computational power, numerical models emerged as powerful tools for run-up prediction. Boussinesq-type models and nonlinear shallow water models allowed detailed simulation of wave shoaling, breaking, and swash dynamics. Later, fully three-dimensional Navier–Stokes solvers with Volume-of-Fluid (VOF) techniques enabled further study of turbulence and porous media interaction.

Modern studies increasingly combine empirical, numerical, and probabilistic approaches. EurOtop (2018), for instance, integrates laboratory data, field measurements, and numerical simulations to provide updated overtopping and run-up guidance for coastal engineers.

4.1.1 Current state of the art in wave run-up prediction

As mentioned, wave run-up is inherently a stochastic phenomenon, driven by the irregular and random nature of sea states. It is important to emphasise that the maximum run-up height is not necessarily associated with the absolute highest incoming wave, but rather with the highest non-breaking wave capable of propagating up the slope without significant energy dissipation. This implies that neglecting the probability of wave breaking can lead to systematic overprediction of run-up levels, particularly in sea states where breaking is expected to occur frequently.

In practice, the prediction of wave run-up on coastal structures is typically approached through empirical and semi-empirical relationships derived from physical model tests. The most widely applied formulation in Europe is presented in EurOtop (2018), where the run-up exceeded by 2% of the incoming waves, $R_{u2\%}$, is described by two main equations. The mean value approach is given in eq. (4.2), while a limiting maximum condition is presented in eq. (4.3).

$$\frac{R_{u2\%}}{H_{m0}} = 1.65 \cdot \gamma_b \cdot \gamma_f \cdot \gamma_\beta \cdot \xi_{m-1,0}, \quad (4.2)$$

with a maximum value of:

$$\frac{R_{u2\%}}{H_{m0}} = 1.0 \cdot \gamma_f \cdot \gamma_\beta \left(4 - \frac{1.5}{\sqrt{\gamma_b \cdot \xi_{m-1,0}}} \right), \quad (4.3)$$

where $R_{u2\%}$ is the wave run-up height exceeded by 2% of the incoming waves [m], γ_b is the influence factor for a berm [-], γ_f is the influence factor for roughness elements on the slope [-], γ_β is the influence factor for oblique wave attack [-], and $\xi_{m-1,0}$ is the breaker parameter [-].

Both eq. (4.2) and eq. (4.3) are based on an extensive database of both small- and large-scale physical model tests, and have been validated for a wide range of structural geometries and wave conditions. Nevertheless, the coefficients in these empirical formulations should not be interpreted as deterministic values. Instead, they should be considered stochastic variables that capture the inherent scatter in experimental data and the natural variability of wave run-up. Specifically, the coefficient 1.65 in eq. (4.2) has a mean value of 1.65 with a standard deviation of $\sigma = 0.10$, and for design purposes, a more conservative value of 1.75 is recommended. Similarly, the coefficient 1.0 in eq. (4.3) has a mean value of 1.00 and a standard deviation of $\sigma = 0.07$, with 1.07 recommended for design and assessment applications.

The stochastic interpretation is important, as it reflects that wave run-up does not follow a single deterministic path but rather exhibits a distribution of possible outcomes depending on sea state, structure geometry, and random phase interactions of the waves. In this context, design values represent a conservative shift within the statistical distribution, ensuring adequate safety margins for practical applications.

The findings of the European research programme OPTICREST (De Rouck et al. (2000)) align with the principles later adopted in the EurOtop methodology, showing that small-scale model tests yield reliable results for smooth slopes, with negligible scale or wind effects relative to prototype-scale observations. Consequently, in the present study, the focus will be on simple geometries with smooth and impermeable slopes. This approach reduces uncertainty by avoiding the additional complexities introduced by scale-dependent roughness and wind effects, thus isolating the core hydrodynamic processes governing the wave run-up.

4.2 Validation of MIKE21W for run-up

4.2.1 Model setup

As introduced in section 4.1, the computational domain was deliberately constrained to a simple and well-defined reference case, consisting of a uniform slope characterised by an impermeable and hydraulically smooth surface. This idealised configuration was selected to isolate the fundamental hydrodynamic mechanisms governing wave run-up, while at the same time minimising uncertainties that would otherwise arise from additional complexities such as surface roughness, material permeability, or irregular geometrical features.

The principal layout of the model geometry is illustrated in Figure 4.1, where the key parameters varied between simulations are highlighted. The figure also indicates the positions of the virtual wave gauges, which in MIKE 21 Wave FM are implemented as point output probes. The methodology for the spatial distribution and placement of these probes, which is essential for the subsequent decomposition of incident and reflected wave components, is described in detail in section 2.4.

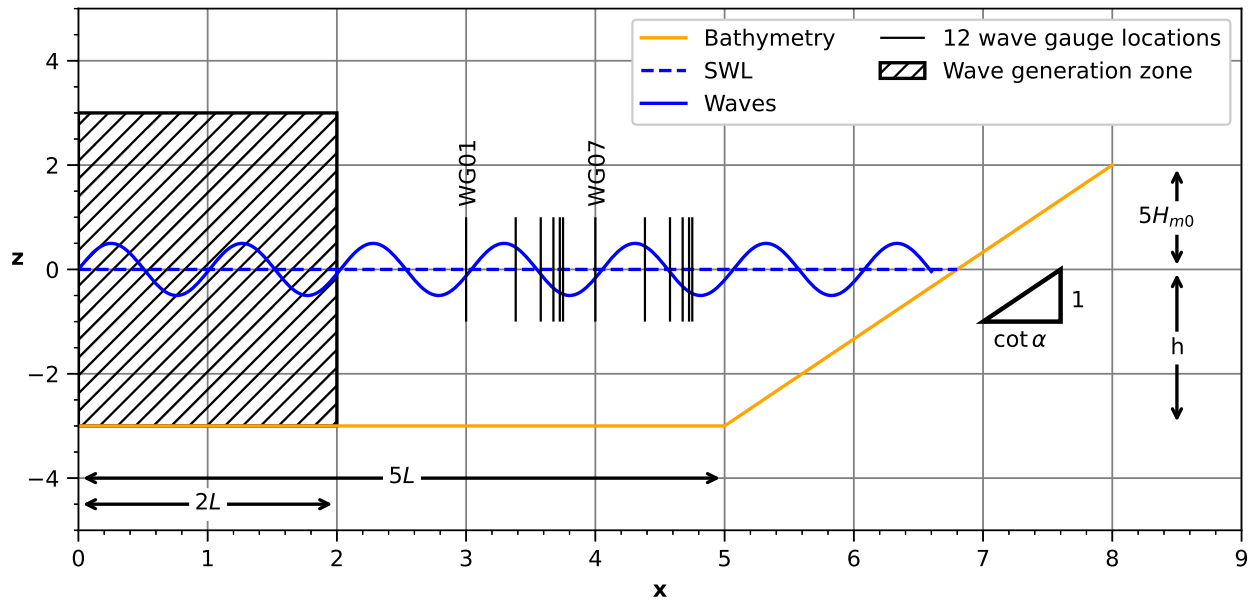


Figure 4.1: Principal sketch of run-up simulation geometry setup.

For all numerical experiments, the significant wave height was prescribed as $H_{m0} = 1.0$ m. To span a sufficiently broad parameter space and enable systematic comparisons, a total of 50 different cases were considered, distributed evenly across the selected range, as will later be illustrated in Figure 4.5. The Riemann solver factor was set to $R = 0.3$, following the convergence and sensitivity analyses carried out during the solitary wave simulations. These preliminary tests demonstrated that this choice provides an appropriate compromise between numerical stability and the accuracy of the computed free-surface evolution.

The temporal resolution of the simulations was controlled through a maximum time step of $dt = 0.025$ s. Each simulation was advanced for a total of 20000 time steps, corresponding to a simulated physical duration of 500 seconds. This simulation length was found to be sufficient to achieve statistical convergence of the run-up time series while avoiding unnecessary computational overhead.

Spatial discretisation was defined relative to the offshore wavelength, with the grid density at the offshore boundary varying between $\frac{n}{L} = 30$ and $\frac{n}{L} = 50$, depending on the slope of the structure. The grid was progressively refined in the shoreward direction, with the cell size near the slope toe reduced to approximately half of its initial offshore value. This adaptive refinement strategy was adopted based on observations from preliminary test simulations, which indicated that excessive nearshore resolution in combination with steep slopes could lead to numerical instabilities. The adopted discretisation scheme therefore represents a balance between computational efficiency, numerical stability, and the need to accurately resolve the nearshore hydrodynamics.

Flooding and drying processes were parametrically controlled by prescribing the wetting and drying thresholds as $h_{\text{wet}} = H_{m0}/18.5$ and $h_{\text{dry}} = h_{\text{wet}}/3$, respectively. These values are based on the sensitivity analysis performed in Section 5.2, and were selected to ensure a smooth and stable representation of the moving shore-line. Thereby, spurious oscillations were reduced in the free surface while maintaining an appropriate balance between numerical stability and physical accuracy.

To assess the influence of the wetting depth on the computed run-up, a simple convergence analysis was performed. The criterion was based on the cumulative maximum surface elevation attained at the most land-

ward wet point, which effectively represents the instantaneous maximum run-up on the slope. An example of the obtained results is shown in Figure 4.2, where $h_{\text{wet}} = 0.09$ m corresponds to the chosen wetting depth, $h_{\text{wet}} = H_{m0}/18.5$.

It can be observed that for the two lowest values of wetting depth, the evolution of the cumulative maximum run-up converges towards nearly identical results, indicating numerical robustness in this range. By contrast, larger wetting depths lead to a systematic underprediction of the maximum run-up. This discrepancy can be attributed to the delayed activation of successive shoreline-adjacent grid cells when larger values of h_{wet} are employed, effectively impeding the landward progression of the run-up. The influence of this effect is also evident at the initial stages of shoreline advance: as shown in Figure 4.2, higher wetting depths result in a delayed onset of positive run-up accumulation.

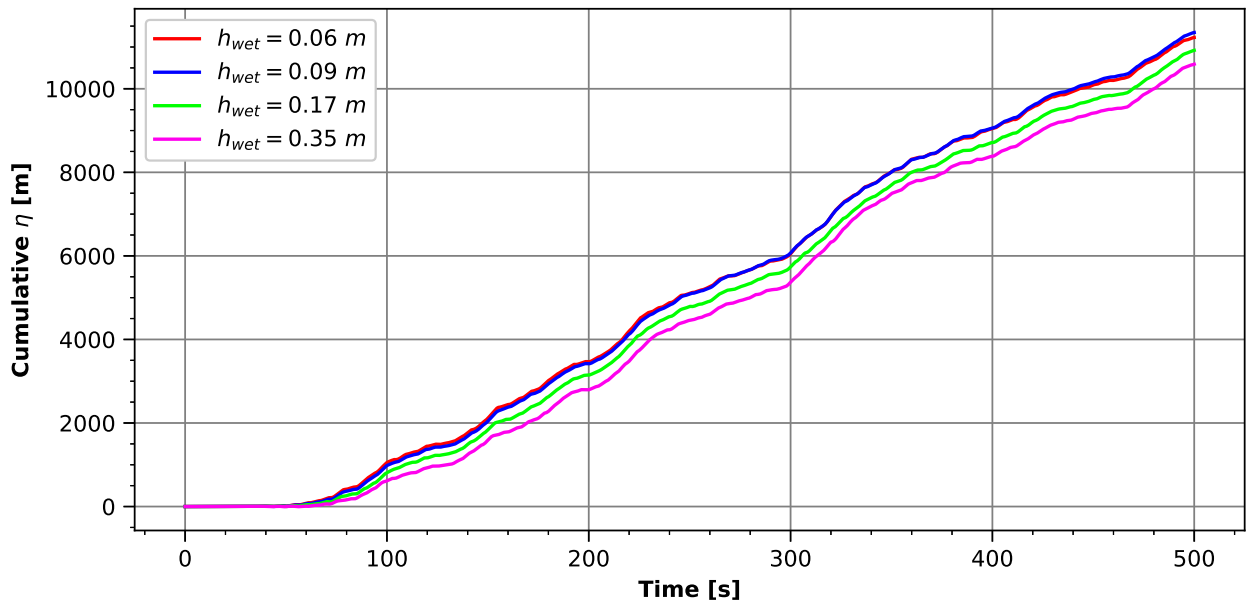


Figure 4.2: Evolution of the cumulative surface elevation for one example case, modelled with different values of wetting depth.

Further insight is provided by examining the position of the most landward activated cell, which corresponds to the highest surface elevation reached along the slope. This is presented in Figure 4.3 for a test case with structure slope $\cot \alpha = 8$. Over the limited time interval shown, the effect of the wetting depth is again evident: larger values restrict the landward progression of the water front, thus limiting the maximum run-up height that can be attained.

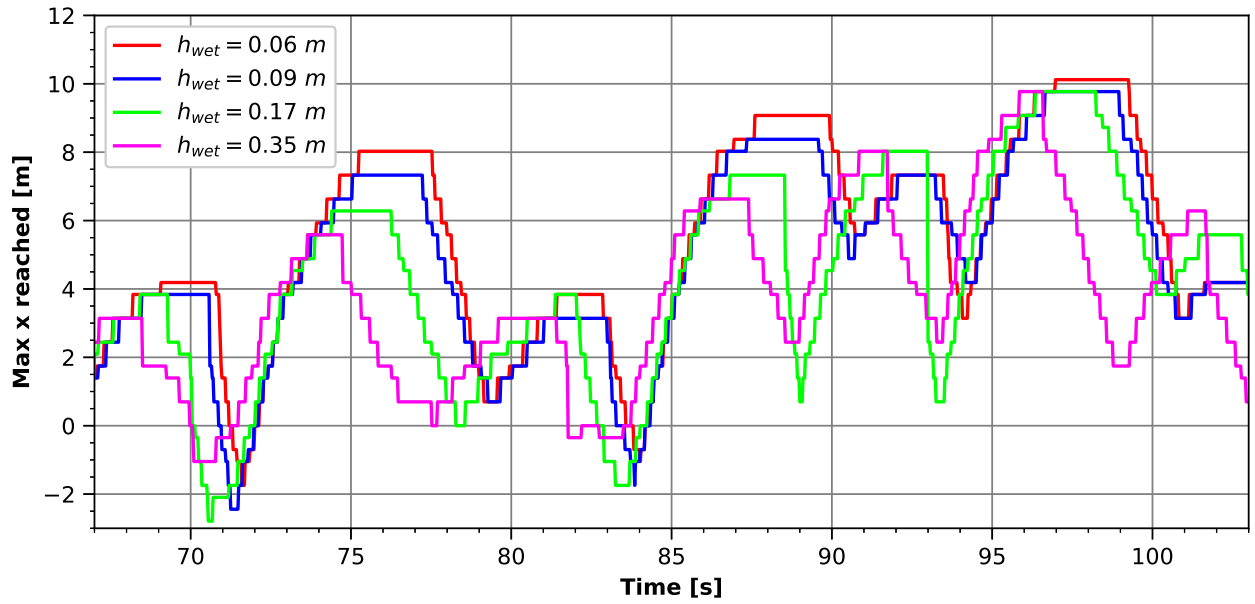


Figure 4.3: Close-up on the position of the most shoreward-activated cell.

When wave breaking was included in the simulations, the parameters used were based on the proposed values by Madsen et al. (1997) as presented in section 2.5.5.

The parameters used in the MIKE21W simulations are collected in table 4.1.

Table 4.1: Input parameters used for MIKE21W simulations.

Parameter	Values
Significant wave height, H_{m0} [m]	1.0
Number of simulation cases	50
Riemann solver factor, R [—]	0.3
Maximum time step, dt [s]	0.025
Number of time steps	20,000 (total duration 500 s)
Offshore grid density, n/L [—]	30 to 50
Nearshore grid refinement	Cell size ~ 0.5 of offshore value
Wetting depth, h_{wet} [m]	$H_{m0}/18.5$
Drying depth, h_{dry} [m]	$h_{wet}/3$
Wave breaking parameters	According to Madsen et al. (1997)

4.2.2 Wave decomposition

To achieve the prescribed target significant wave height of $H_{m0} = 1.0$ m within the computational domain, the input wave height was iteratively adjusted until the incident wave field converged to the desired level. This calibration procedure ensured that the simulated wave conditions were consistent with the intended design parameters.

The accuracy of the wave decomposition procedure described in section 2.4 was subsequently validated. For

this purpose, the time series of the incident and reflected wave components were extracted and analysed independently. The two signals were then recombined through linear superposition and compared against the directly measured time series at wave gauge WG01 (see Figure 4.1). The outcome of this comparison is presented in Figure 4.4.

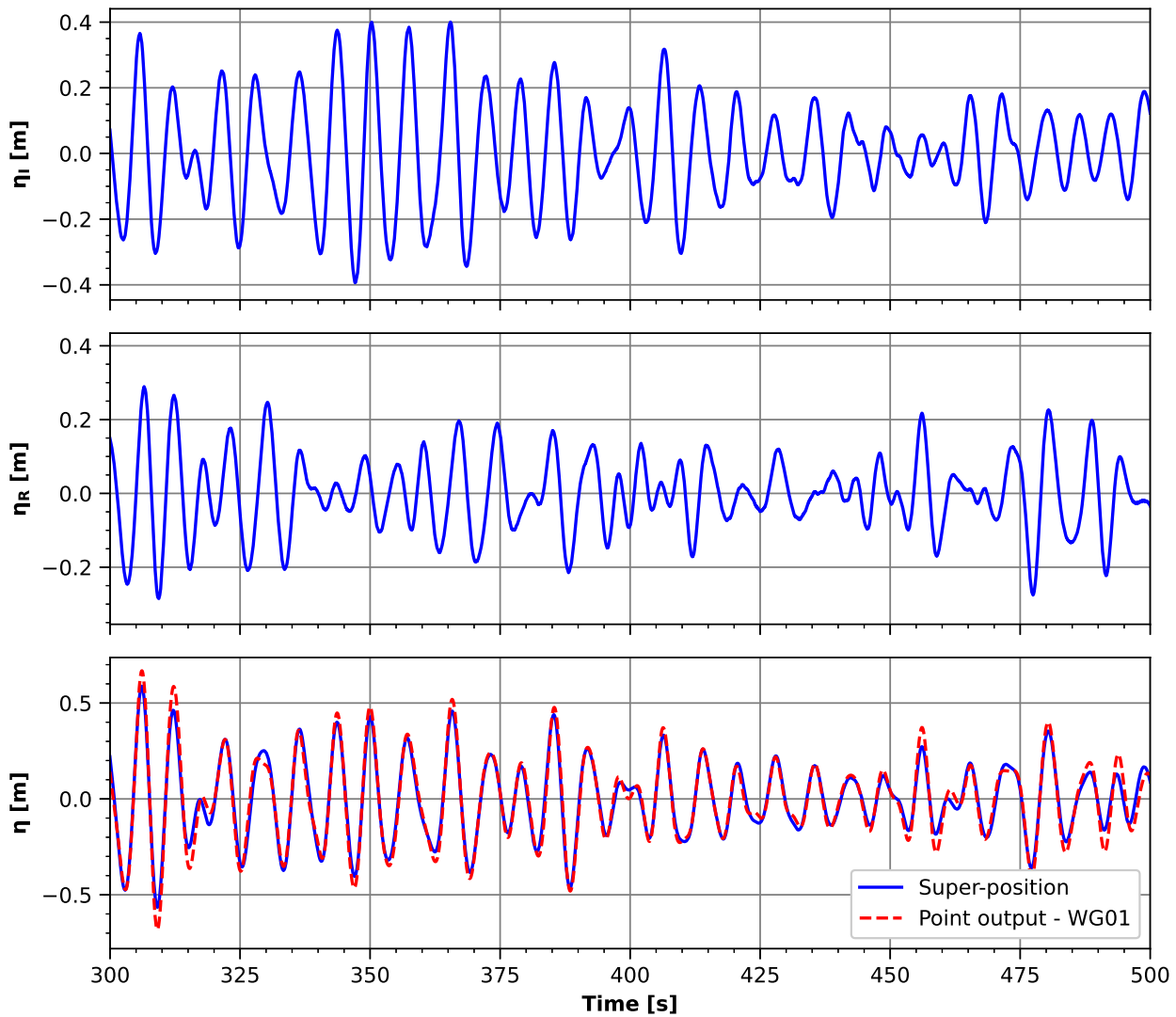


Figure 4.4: Decomposition of the wave time series in front of the structure into the incident (I) and reflected (R) wave fields. The lower panel illustrates the superposition of incident and reflected signals compared with the measured time series at WG01 (see Figure 4.1).

As illustrated in Figure 4.4, the reconstructed signal obtained from the superposition of the decomposed incident and reflected components reproduces the measured signal at WG01 with very good agreement, and only minor discrepancies were observed. This result confirms the accuracy and reliability of the applied decomposition methodology.

The ability to reliably separate and subsequently reconstruct the wave signal is particularly important in the context of run-up investigations on steep slopes, as it enables the incident wave to be clearly distinguished from the effects of wave reflection and structure–wave interactions.

4.2.3 Results

In Figure 4.5 and Figure 4.6, the numerical results obtained with and without the inclusion of wave breaking are compared against the EurOtop formulation. The 5% exceedance limits are also shown in both figures, and the region between the two curves represents the 90% confidence band of the EurOtop prediction.

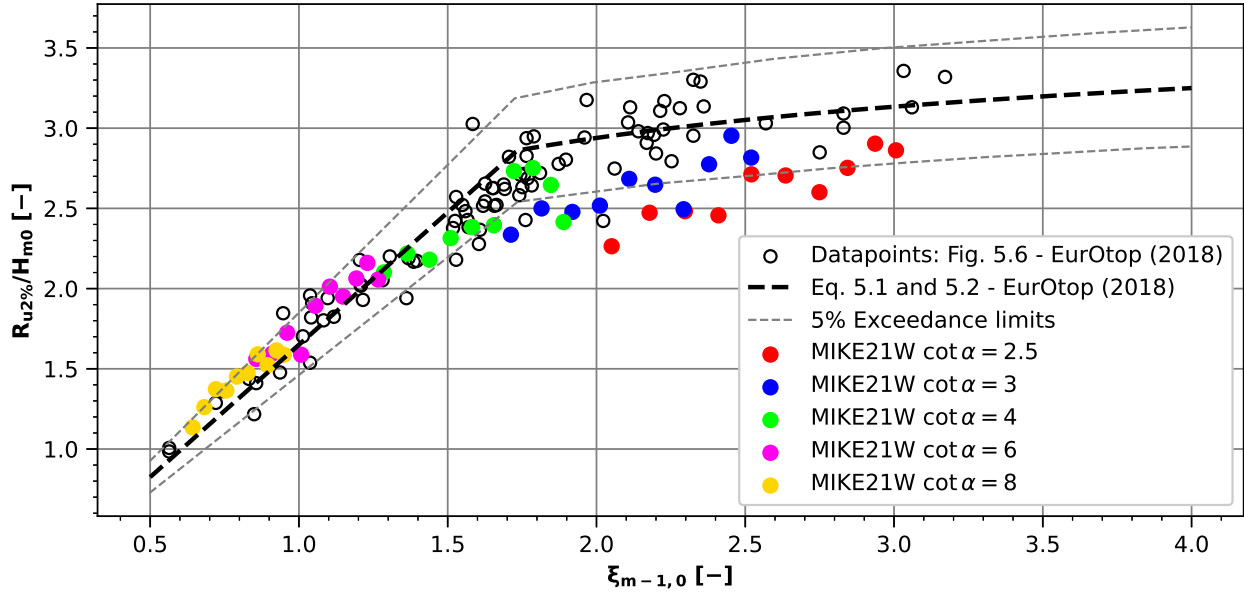


Figure 4.5: Run-up results (MIKE21W) without wave breaking included.

A comparison of the MIKE21W results (without the surface roller formulation) with the EurOtop prediction is shown in Figure 4.5. For $\xi_{m-1,0} \in [1.8, 3]$, where wave breaking is minimal, the model consistently produces slightly lower run-up values than predicted. In the lower range ($\xi_{m-1,0} \in [0.65, 1.8]$), which is dominated by wave breaking, the model results are closer to the EurOtop values. The discrepancy in the non-breaking regime, however, indicates that the model without wave breaking physics does not fully capture the run-up process, suggesting that the closer match in the breaking regime may not arise from the correct physical mechanisms.

In breaking-dominated cases, the absence of dissipative processes would be expected to lead to overprediction. The close agreement with EurOtop predictions for $\xi_{m-1,0} \in [0.65, 1.8]$ therefore suggests that this overprediction is counterbalanced by an inherent tendency of the model to underestimate.

This general underestimation may be linked to the model's theoretical foundation. The governing equations are based on a Boussinesq-type formulation that assumes a mild slope. The steep slopes used in this study may challenge this assumption, potentially leading to an underestimation of vertical accelerations and their contribution to run-up. This interpretation is supported by the comparison with MIKE3W results (Figure 4.8), which are not constrained by the mild-slope assumption and do not exhibit the same underestimation trend.

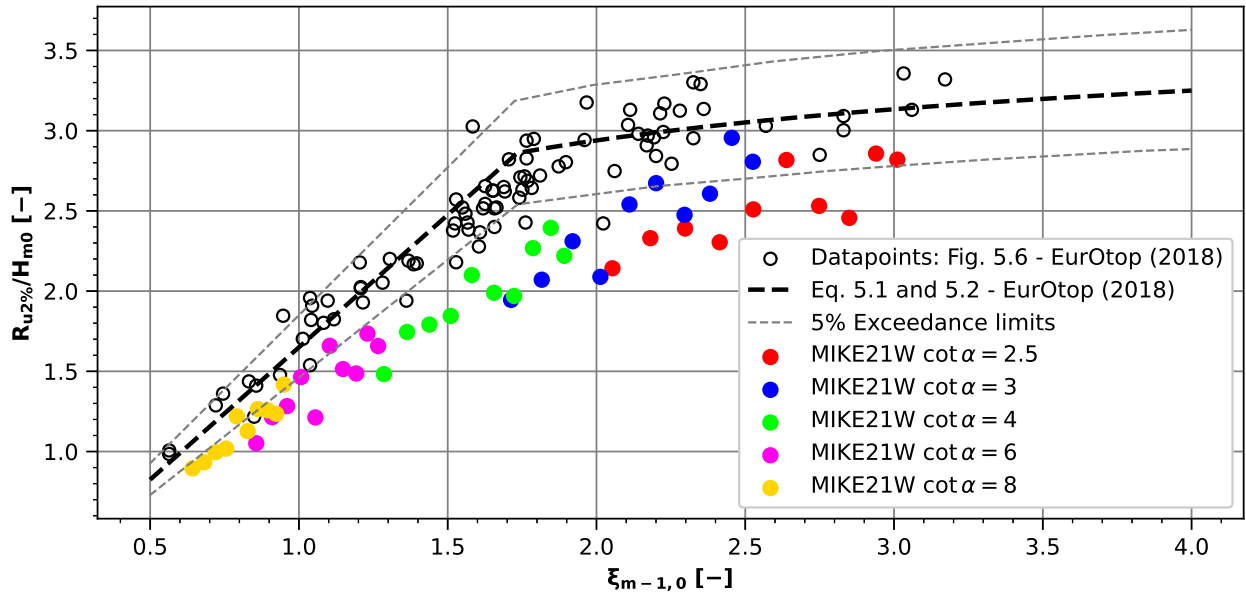


Figure 4.6: Run-up results (MIKE21W) with wave breaking included.

The results including wave breaking, obtained by activating the surface roller formulation in MIKE21W, are presented in Figure 4.6. The roller parameters were defined according to the recommendations of Madsen et al. (1997). As anticipated and expected from the findings in the non-breaking simulations, the inclusion of surface rollers did not improve the agreement with the reference formulation. Instead, it further amplified the underestimation of run-up across the parameter space, highlighting that the dissipative effects of the roller model exacerbate the inherent model bias rather than correcting it.

4.3 Validation of MIKE3W for run-up

4.3.1 Model setup

The numerical model setup is identical to the configuration applied in MIKE21W (see Table 4.1), except for the discretisation in the vertical (z -) direction and the turbulence model, namely the $k - \epsilon$ turbulence model as presented in Section 2.5.3. For the vertical discretisation, a total of ten sigma-layers were employed. The layer thickness varies according to the vertical distribution parameters $\sigma_c = 0.1$ and $\theta = 2$. The influence of the additional parameter b was examined by testing values of $b = 0$ and $b = 1$. When $b = 0$, the mesh is refined primarily near the free surface, whereas $b = 1$ yields a finer resolution both near the surface and the seabed. An illustration of the resulting meshes for the two cases is shown in Figure 4.7, with water depth $h = 18.5$ m and free-surface elevation $\eta = 1$ m.

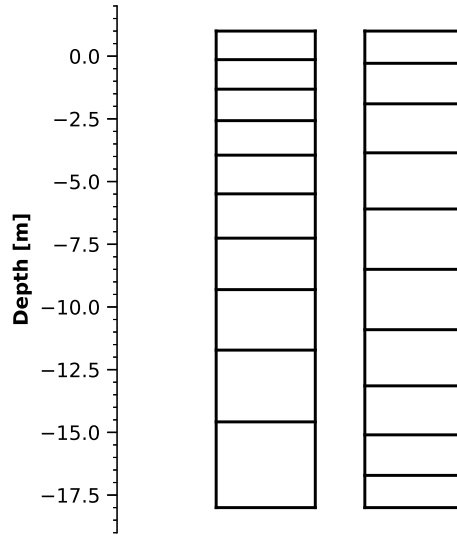


Figure 4.7: Example of vertical discretisation with $b = 0$ (left) and $b = 1$ (right).

The vertical discretisation with $b = 0$ primarily enhances the resolution near the free surface, which is important for accurately resolving wave kinematics that govern the run-up dynamics. In contrast, setting $b = 1$ provides increased refinement at both the surface and the seabed, though with slightly reduced resolution at the surface compared to the $b = 0$ configuration. The additional refinement near the seabed may be particularly relevant for capturing turbulence and bottom boundary layer processes in the swash zone. The implications of these discretisation settings will be further examined in Section 4.4.

4.3.2 Results

In Figure 4.8 and Figure 4.9, the numerical results obtained with and without the inclusion of the $k-\epsilon$ turbulence closure are compared against the EurOtop formulation. As in the previous analyses, the 5% exceedance limits are indicated in both figures, and the area enclosed between them represents the 90% confidence band of the EurOtop prediction.

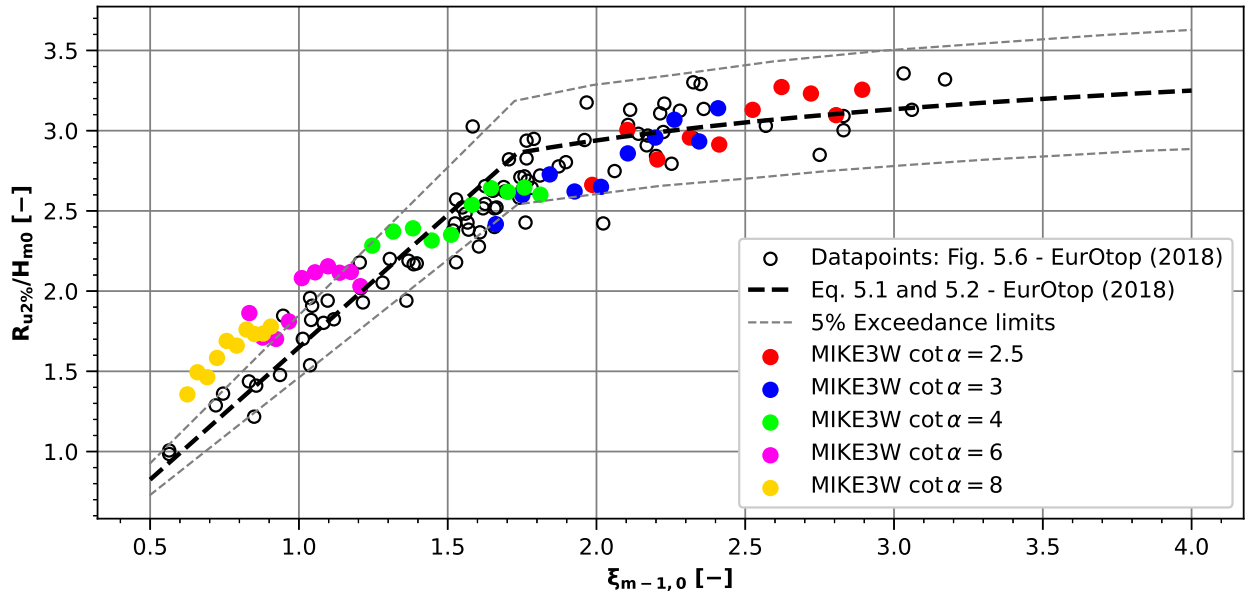


Figure 4.8: Run-up results (MIKE3W) without $k - \epsilon$ model included.

The results shown in Figure 4.8, obtained without turbulence modelling, display very good agreement with the EurOtop formulation for the range $\xi_{m-1,0} \in [1.4, 3]$. However, for breaker types corresponding to $\xi_{m-1,0} \lesssim 1.4$, the model begins to overpredict the run-up. This tendency is theoretically consistent, as the absence of turbulence production artificially inhibits energy dissipation, leading to higher shoreline excursion than expected under breaking-dominated conditions.

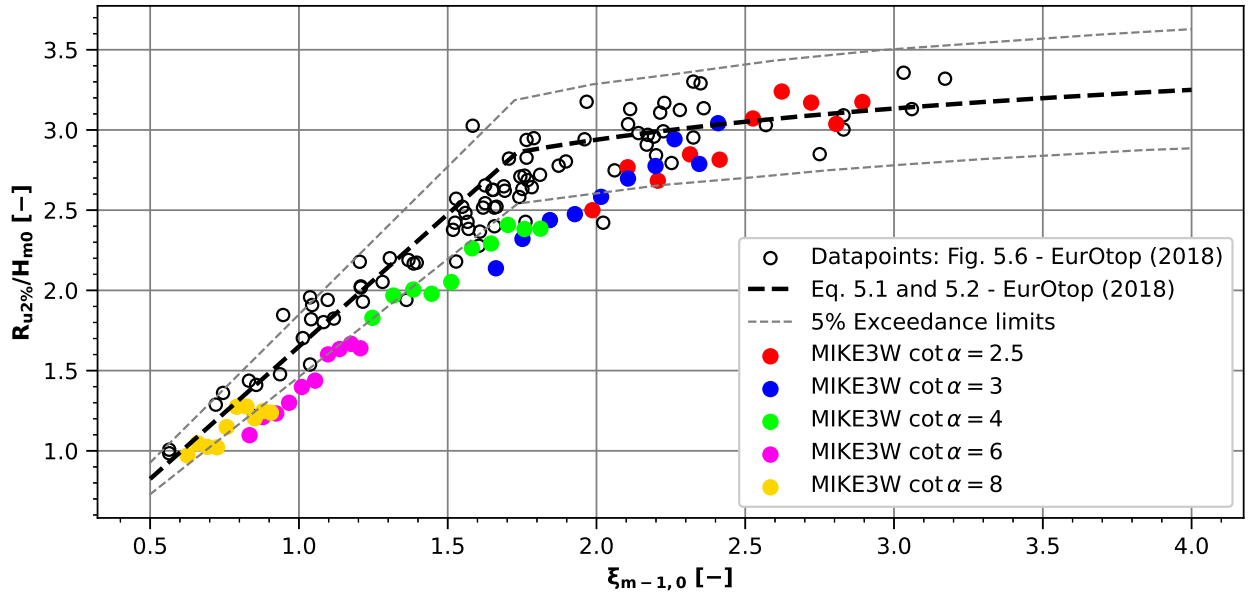


Figure 4.9: Run-up results (MIKE3W) with $k - \epsilon$ model included.

When the $k-\epsilon$ turbulence closure is activated, the results shown in Figure 4.9 are obtained. In this case, the

model exhibits a systematic underestimation of the run-up for $\xi_{m-1,0} \in [0.8, 2.3]$, which suggests that the turbulence scheme dissipates an excessive fraction of the incoming wave energy. By contrast, for $\xi_{m-1,0} \lesssim 0.8$, the predictions align well with the EurOtop formulation.

The improved agreement in the lower $\xi_{m-1,0}$ range, corresponding to the transition from plunging to spilling breaker types, indicates that the $k-\epsilon$ model may be more suitable for simulating conditions dominated by spilling breakers. Conversely, in regimes where plunging breakers are prevalent, the model appears overly dissipative, thereby reducing the predicted run-up below expected values.

The implications of these findings, as well as the broader performance and limitations of the turbulence closure in the context of run-up modelling, will be discussed in detail in Section 4.4.3.

4.4 Discussion on the run-up results

In this section, the results of each model are first discussed individually, followed by a direct comparison between the models. Finally, based on the insights gained from these analyses, an assessment of the applicability of the models for engineering design is presented.

4.4.1 Qualitative assessment of model performance

In Figure 4.10, a representative snapshot of the free surface during a maximum run-up event is presented.

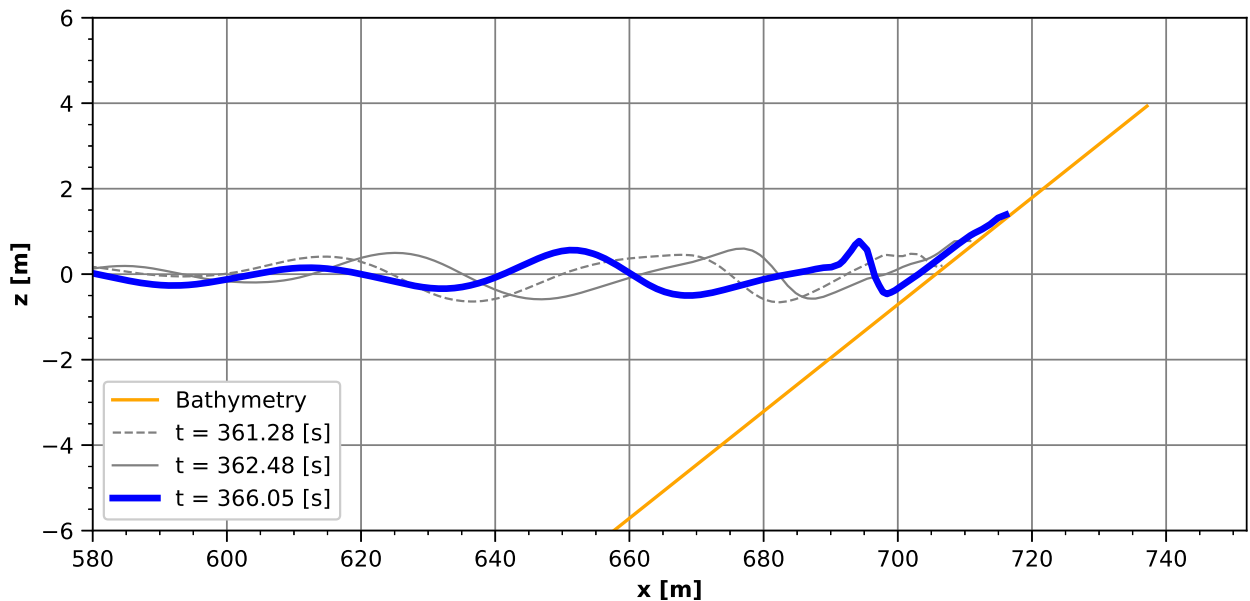


Figure 4.10: Surface elevation profiles during maximum runup in one representative case from MIKE21W without surface rollers included. Grey lines indicate snapshots taken prior to the event.

The surface elevation profiles shown in Figure 4.10 provide a qualitative assessment of the numerical model performance. The results demonstrate that the model is capable of reproducing physically realistic wave behaviour during run-up events. No indications of spurious oscillations or other numerical instabilities are observed, and the shoreline evolution appears dynamically consistent with expected swash-zone behaviour. A minor phase lag in the swash zone is noticeable, which is attributed to the wetting–drying algorithm employed in the model.

4.4.2 Surface rollers in MIKE21W

A qualitative assessment of the surface roller formulation in MIKE21W was conducted by comparing the free-surface elevation at a representative timestep with and without the inclusion of surface rollers, as shown in Figure 4.11.

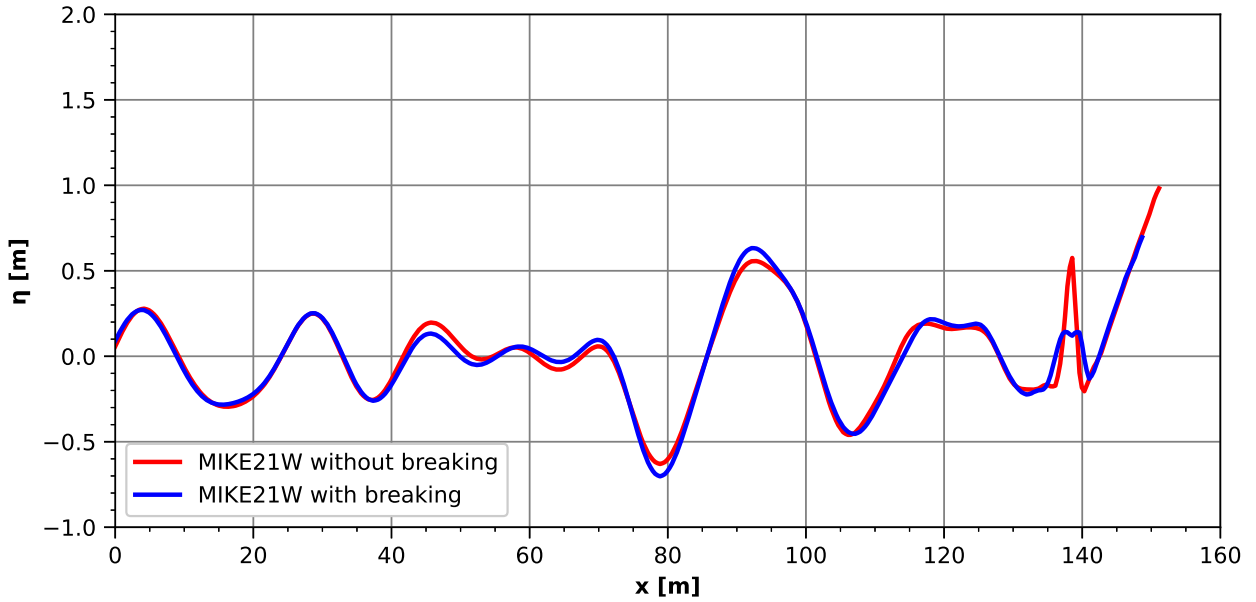


Figure 4.11: Comparison of surface elevation at a representative timestep with and without surface rollers included.

As illustrated in Figure 4.11, the surface elevation immediately upstream of the breaking point ($x \approx 138$ m) shows near-perfect alignment between the two simulations. This observation confirms that the surface roller formulation primarily acts within the breaking region to dissipate energy, as intended by the model design.

However, the evaluation of the overall performance of the surface roller model is complicated by the fact that the baseline simulation, without rollers, already exhibits a systematic underprediction of the run-up. Consequently, while the qualitative behaviour of the rollers is consistent with expectations, their effect cannot be fully assessed in terms of improving agreement with reference run-up data, as the underlying model bias dominates the observed discrepancies.

4.4.3 $k-\epsilon$ model in MIKE3W

The slight overprediction of run-up in MIKE3W without the $k-\epsilon$ model, for values of $\xi_{m-1,0} \lesssim 1.4$, compared with the underprediction obtained when the $k-\epsilon$ model is included, suggests that the turbulence model may be generating excessive dissipation.

The issue of turbulence overproduction in RANS models is well-documented in the literature (see Mayer & Madsen (2001)). This tendency is partly attributed to the conditional instability of turbulence models in regions that are close to potential flow. To address this, Larsen & Fuhrman (2018) proposed an extended formulation of the $k-\epsilon$ model. Kærgaard et al. (2019) investigated its application in MIKE3W and demonstrated that the modified formulation improves the representation of turbulence outside the surf zone and, as a result, yields a better reproduction of inner surf zone dynamics when compared with the classic observations by Ting & Kirby (1994). Nevertheless, the problem of excessive turbulent kinetic energy (TKE) production was still observed.

It should be noted that most of the above studies focused on undertow, turbulence, and surface elevation within the inner surf zone, rather than in the swash zone. In the latter, measurements of turbulence and free-surface motion have been provided, for example, by Petti & Longo (2001) and Sou et al. (2010).

Qualitative Assessment of the Spatial Distribution of TKE

To qualitatively assess the spatial distribution of turbulent kinetic energy (TKE) production, profiles of TKE are presented in Figure 4.12 for a representative maximum run-up event, and in Figure 4.13 as a cumulative distribution over the full simulation. The selected case corresponds to $\cot \alpha = 6$, $T_p = 7.09$ s, and $\xi_{m-1,0} = 1.21$, located in the middle of the expected breaking area of Figure 4.9. Four additional cases distributed across the breaking zone are reported in Appendix A.2.

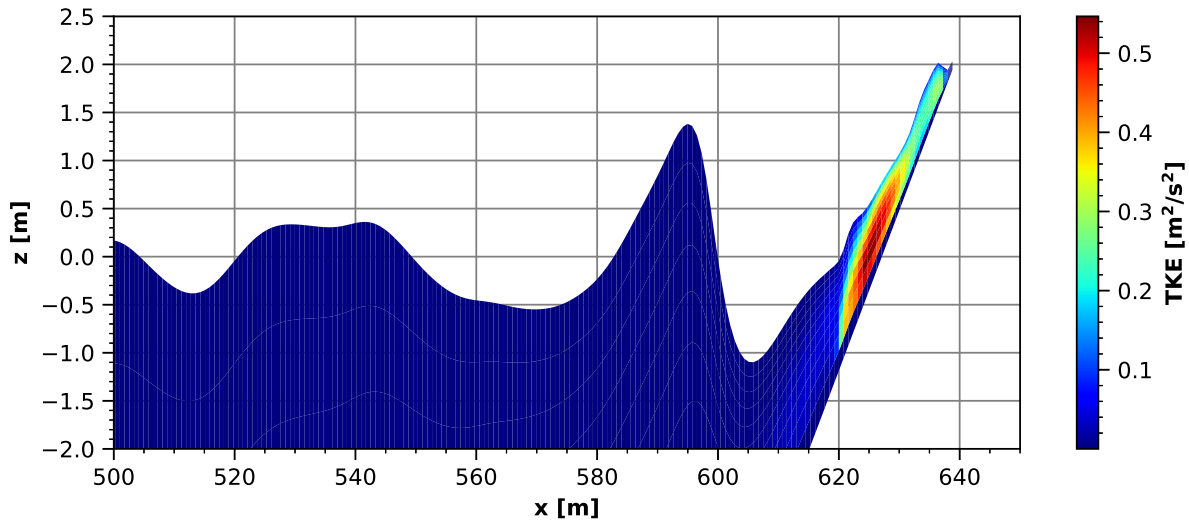


Figure 4.12: Instantaneous TKE during a maximum run-up event, $t = 214$ s, for a plunging wave having $T_p = 7.09$ s, and $\xi_{m-1,0} = 1.21$.

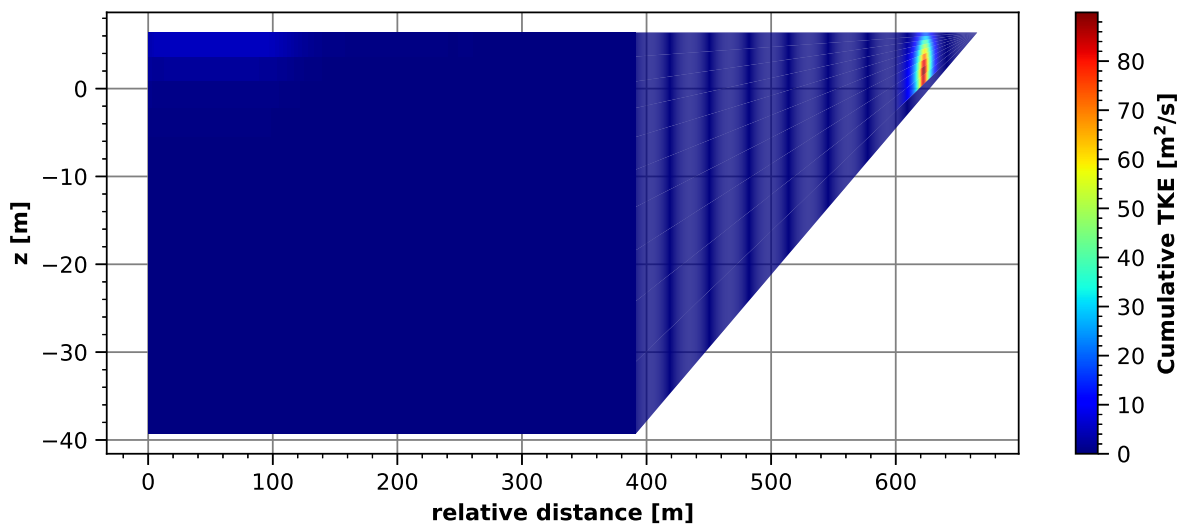


Figure 4.13: Cumulative TKE over the entire simulation duration, for a plunging wave having $T_p = 7.09$ s, and $\xi_{m-1,0} = 1.21$.

The TKE distribution is concentrated within a localised region around the inner surf and swash zones, consistent with physical expectations. In the cumulative plot (Figure 4.13), the accumulated TKE is obtained by summing, at each grid location, the values in corresponding vertical layers across successive timesteps. In other words, only like-with-like vertical contributions (e.g., surface to surface) are summed over time.

It should be noted that in the figure, the colour scale is stretched in the y -direction such that the darker blue shades appear to cover the entire domain, which may give a misleading impression of the spatial extent of the swash zone. Nevertheless, the x -coordinate of the TKE concentration coincides with the location of the swash zone.

This localised generation of TKE provides a positive indication of model performance. A comparison of surface elevations with and without the $k-\epsilon$ model (Figure 4.14) further supports this interpretation: good agreement is obtained up to the swash zone, after which the turbulence closure induces extra dissipation, leading to reduced run-up.

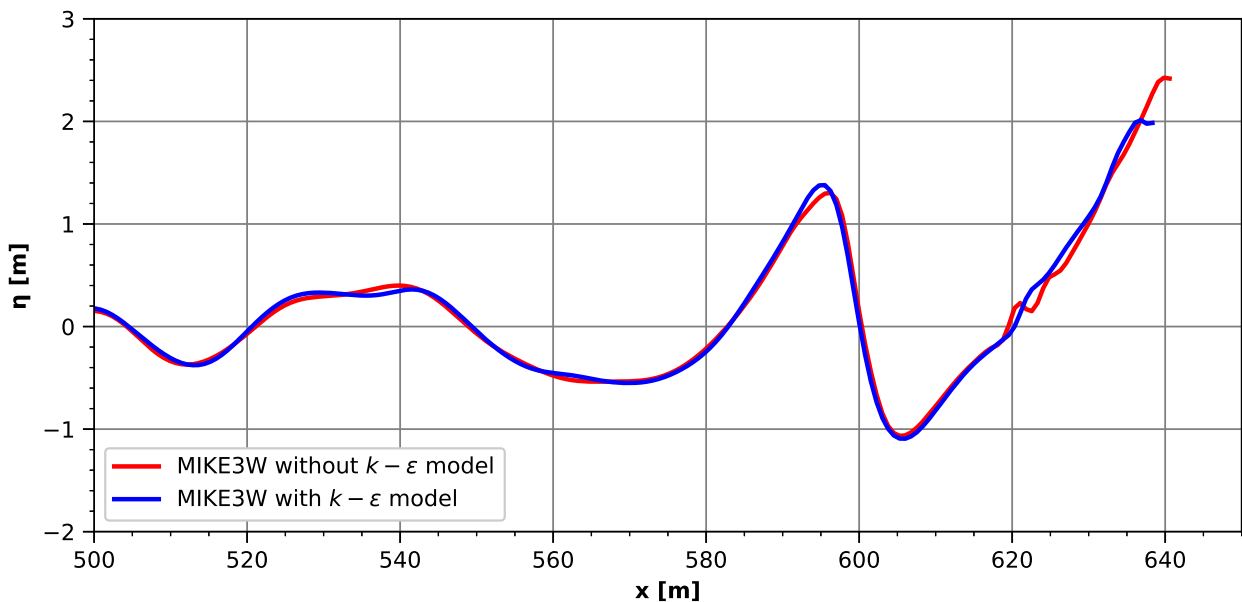


Figure 4.14: Time series comparison of MIKE3W with and without the $k-\epsilon$ model included for a representative run-up event, $t = 214$ s.

The temporal evolution of TKE for eight representative time steps (Figure 4.15) confirms that turbulence is advected shoreward atop the incoming wave crest, in agreement with Petti & Longo (2001) and Christensen et al. (2002), who noted that plunging breakers generate a large turbulence plume immediately after breaking, while turbulence levels remain relatively low outside this region.

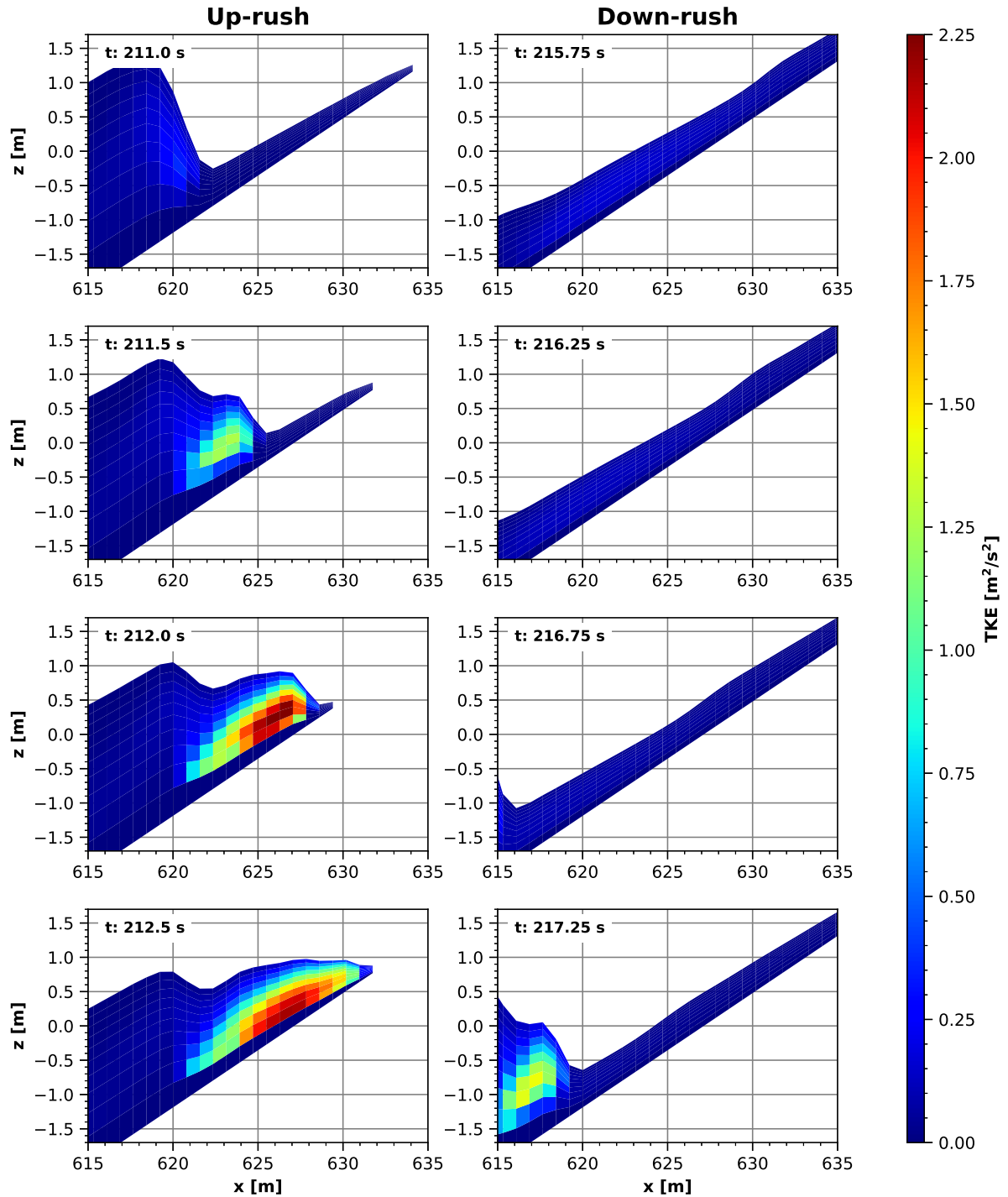


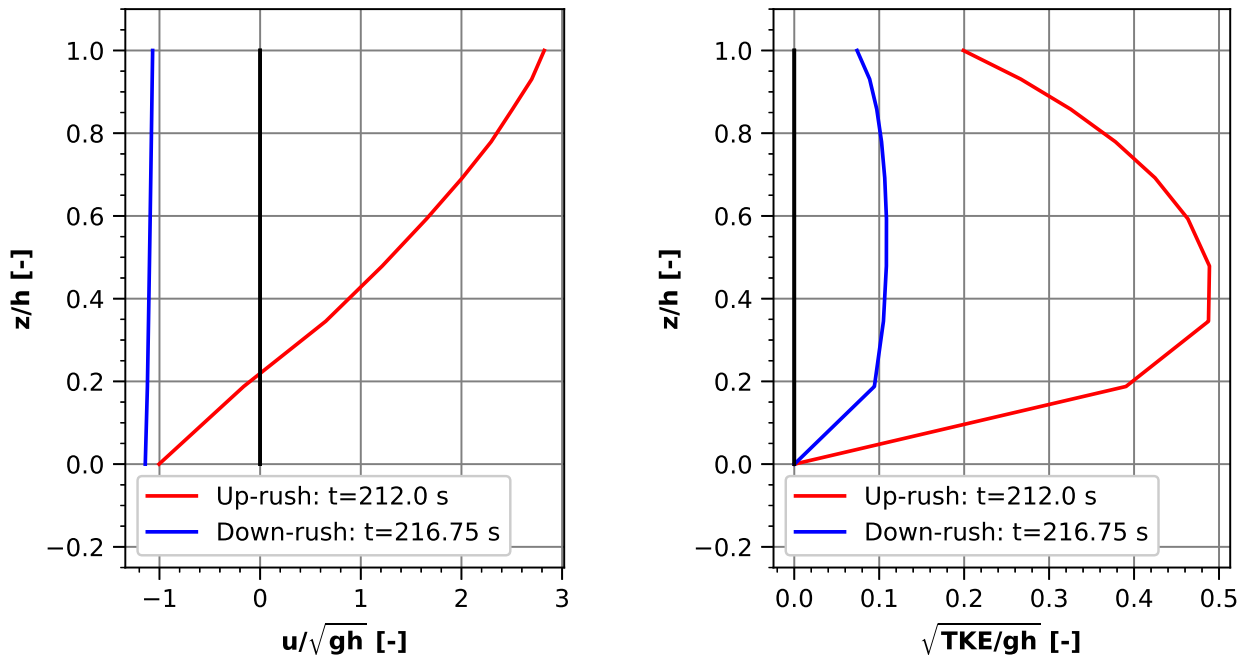
Figure 4.15: Snapshot of TKE at different time-steps of the up-rush (left) and down-rush (right) on the slope.

Quantitative assessment of TKE levels

The turbulence levels in the swash zone have been investigated for regular plunging waves by Petti & Longo (2001), who reported measurements at a cross-shore section located at the intersection between the bathymetry and the still water level (SWL). Their results, shown in Figures 12–14 of that work, indicate that the non-dimensionalised TKE reaches its maximum during the uprush at phase 60° , with values between 0.45 and 1, while during the downrush it decreases to 0.135–0.31.

These reference values are compared with one of the tested irregular-wave cases lying in the middle of the plunging-wave regime, namely the case with $\cot \alpha = 6$, $T_p = 7.09$ s, and $\xi_{m-1,0} = 1.21$, due to being in approximately the same range of $\xi_{m-1,0}$ as in their investigation. The intersection between the bathymetry and the SWL was identified in order to have the same location as presented in Petti & Longo (2001).

The corresponding profiles are shown in Figure 4.16, indicating that the model reproduces turbulence magnitudes consistent with laboratory observations.



(a) $\cot \alpha = 6$, $T_p = 7.09$ s, $\xi_{m-1,0} = 1.21$

(b) $\cot \alpha = 6$, $T_p = 7.09$ s, $\xi_{m-1,0} = 1.21$

Figure 4.16: Velocity and TKE profiles during the uprush and downrush at the SWL intersection.

It should be noted, however, that the results of Petti & Longo (2001) were obtained under regular wave conditions, where the distinction between uprush and downrush phases is sharper than in the present case with irregular waves.

When comparing the inner surf zone turbulence levels with those presented in Ting & Kirby (1994), the agreement is less satisfactory. The comparison was made by extracting the maximum values from the same cases shown in Figure 4.16 and evaluating them against the time-averaged non-dimensional maximum values reported by Ting & Kirby (1994) for plunging waves. In their study, the time-averaged non-dimensional maximum value is presented as 0.09.

In the present study, the time-averaged TKE was estimated by summing the values of the adimensionalised profiles and then dividing by the length of the simulation. An example of a time-averaged TKE profile for the reference case is depicted in Figure 4.17.

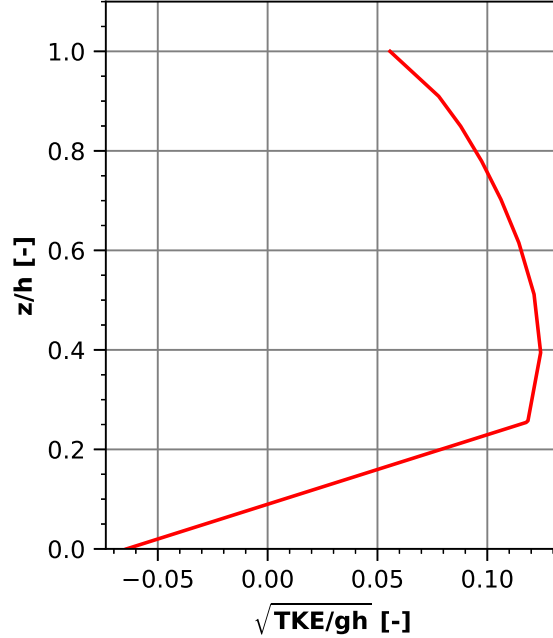


Figure 4.17: Time-averaged TKE profile for the case $\cot \alpha = 6$, $T_p = 7.09$ s and $\xi_{m-1,0} = 1.21$.

The computed maximum level of time-averaged TKE is checked manually by dividing the maximum value of the cumulative TKE from Figure 4.13 by the total simulation duration, i.e.

$$\bar{k} = \frac{\max\left(\int_0^t TKE(t) dt\right)}{T_{sim}} = \frac{90 \text{ m}^2/\text{s}}{500 \text{ s}} = 0.18 \text{ m}^2/\text{s}^2, \quad (4.4)$$

and subsequently non-dimensionalised with the mean depth as:

$$\sqrt{\frac{\bar{k}}{g(\bar{\eta} + h)}} = \sqrt{\frac{0.18 \text{ m}^2/\text{s}^2}{9.81 \text{ m}/\text{s}^2 \cdot 1.143 \text{ m}}} = 0.127. \quad (4.5)$$

Where $\bar{k}$ is the time-averaged TKE, T_{sim} is the duration of the simulation, g is the gravitational acceleration and $\bar{\eta} + h$ is the time-averaged water depth at the relevant location.

Given the maximum value of time-averaged TKE observed in Figure 4.17, an overproduction of TKE of around 40% is observed in comparison with the measurements of Ting & Kirby (1994). This level of overprediction is consistent with previous findings, where Lemos (1992) and Lin & Liu (1998) reported overestimations in the range of 25–50%.

Effect of Vertical Discretisation and Bed Resistance

The inclusion of bed resistance in the simulations has the potential to make turbulence production more realistic. However, it is also expected to contribute to the underestimation of run-up, since additional turbulence is

generated by bottom friction. As the influence of bed-induced turbulence is relatively limited in breaking wave conditions, only a minor effect on the overall results is anticipated. In this investigation, bed resistance was represented using a roughness height of $k_s = 0.004$ m, corresponding approximately to the hydraulic roughness of coarse sand or fine gravel.

The sensitivity to vertical discretisation was investigated in two ways. First, the choice of sigma parameters was varied by comparing $b = 0$ and $b = 1$, as illustrated in Figure 4.7. This adjustment was intended to enhance the resolution of turbulence processes near the boundary layer. Second, the number of vertical sigma layers was increased from 10 to 20 to verify numerical convergence of the model. The combined effects of these modifications are presented in Figure 4.18.

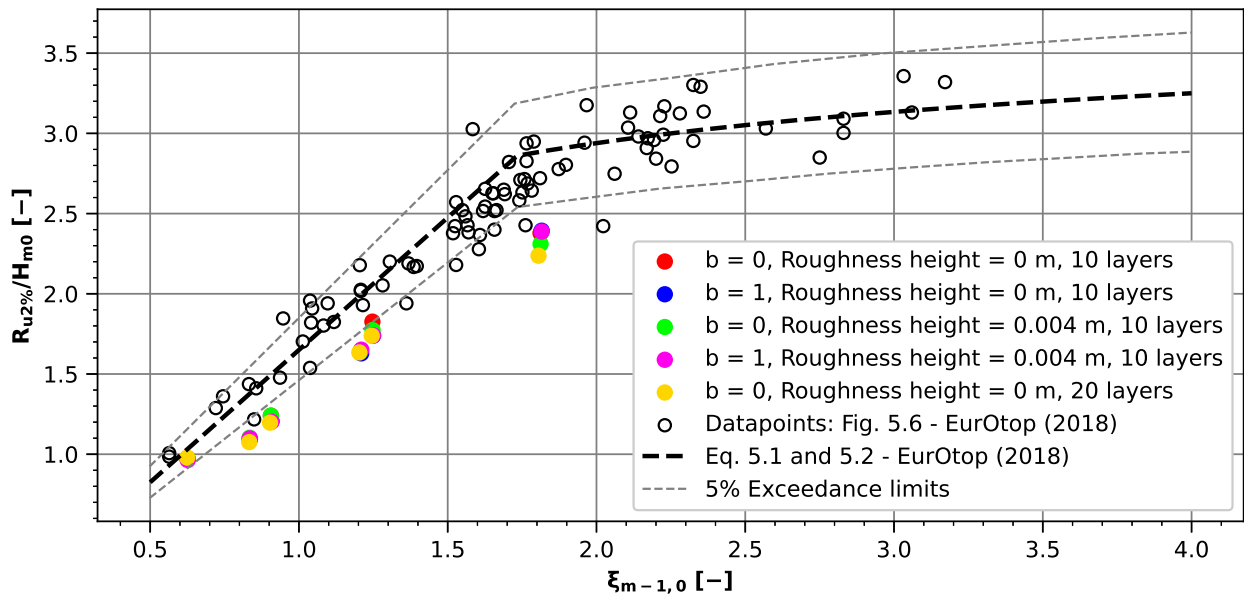


Figure 4.18: Effect of bed resistance and vertical discretisation on run-up results.

No significant differences are observed in the resulting run-up, as shown in Figure 4.18. The inclusion of bed resistance led, as expected, to increased energy dissipation and consequently a slightly reduced run-up height. Neither the application of $b = 1$ nor the doubling of the number of sigma layers produced a noticeable change, indicating that the model results are numerically converged with respect to vertical discretisation.

To further examine the combined effects of altered bed resistance and vertical discretisation, additional output quantities were analysed. Surface elevations across the full domain and with a zoom on the swash zone are shown in Figure 4.19 and Figure 4.20. These comparisons allow for identifying potential local differences.

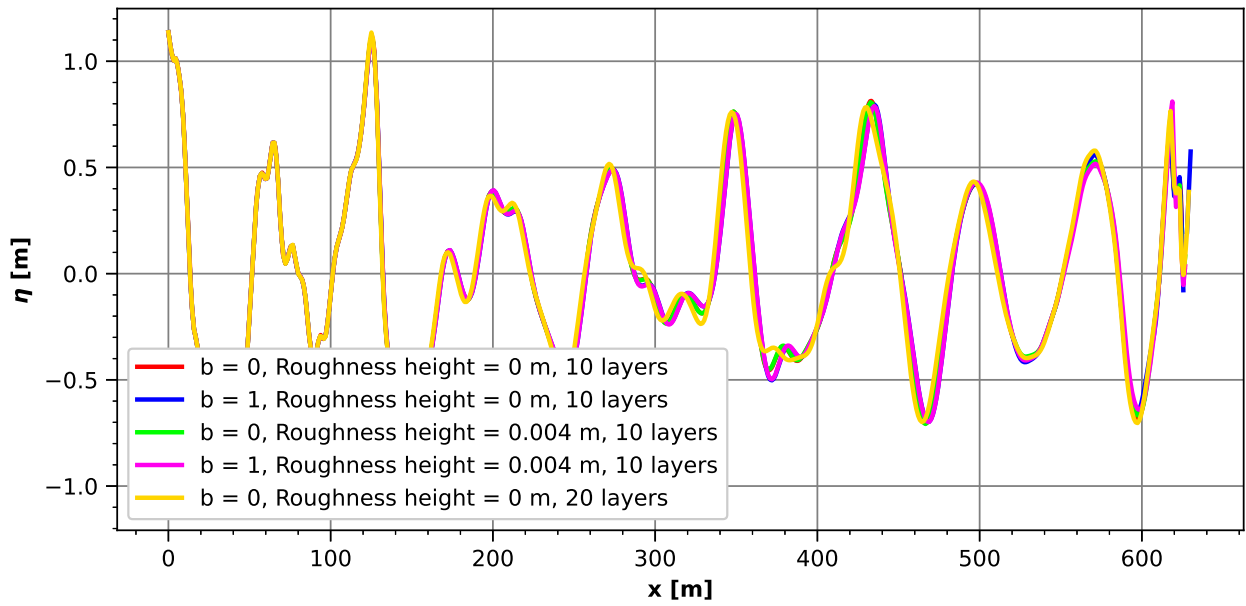


Figure 4.19: Surface elevation comparison for altered bed resistance and vertical discretisation.

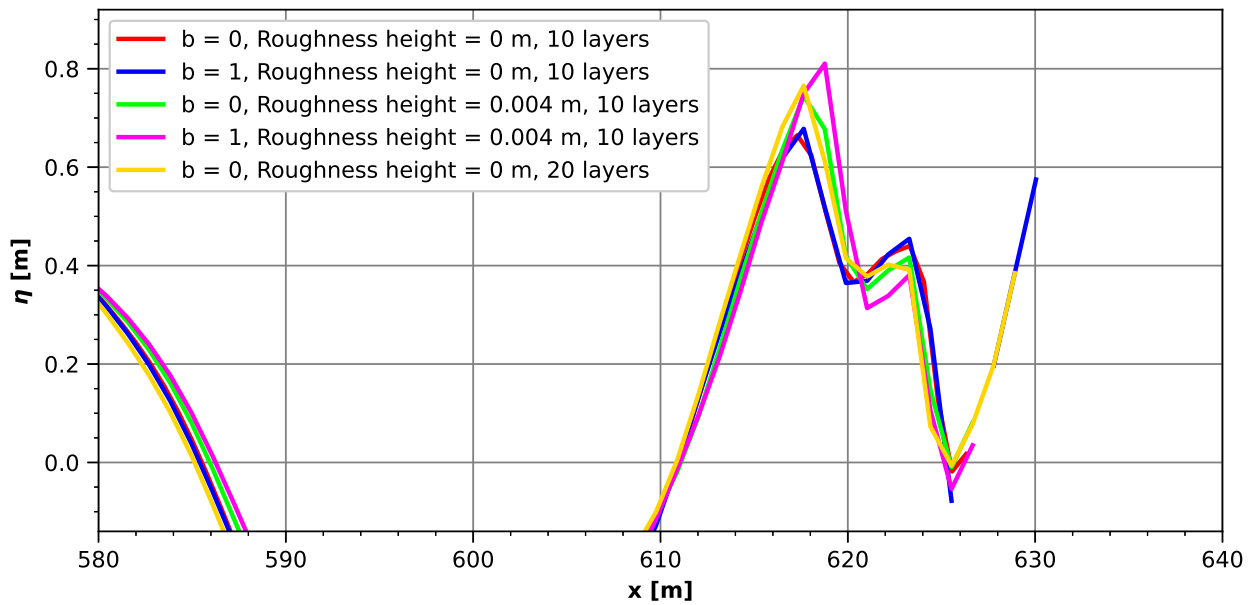


Figure 4.20: Surface elevation comparison for altered bed resistance and vertical discretisation, zoomed to the swash zone.

The comparison of surface elevations (Figures 4.19 and 4.20) confirms that no major differences arise from modifying either the vertical discretisation or the bed resistance. In Figure 4.20, minor discrepancies in run-up height appear at the specific timestep shown, but all simulations ultimately reached the same maximum run-up.

Velocity profiles were analysed to assess the influence of bottom friction and vertical resolution on the near-bed flow structure (Figure 4.21).

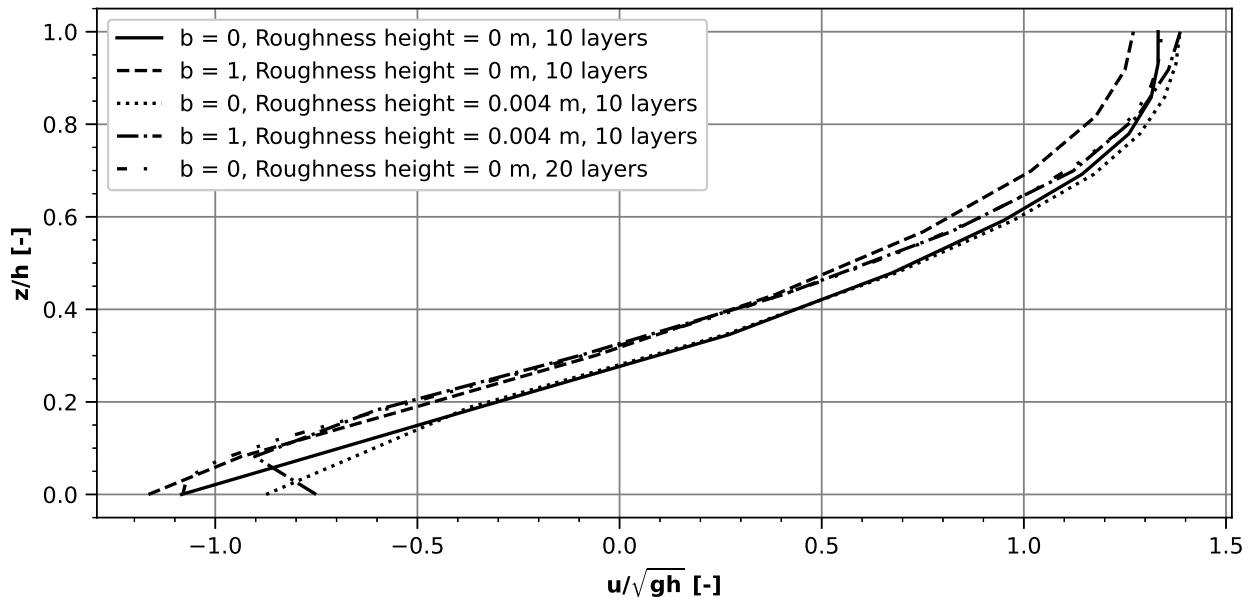


Figure 4.21: Velocity profile comparison for altered bed resistance and vertical discretisation.

The velocity profiles in Figure 4.21 are nearly identical across the different cases. As expected, including bed resistance leads to slightly reduced velocities near the bed due to frictional effects.

TKE profiles were also compared to investigate how the different settings influence the representation of turbulence, as shown in Figure 4.22.

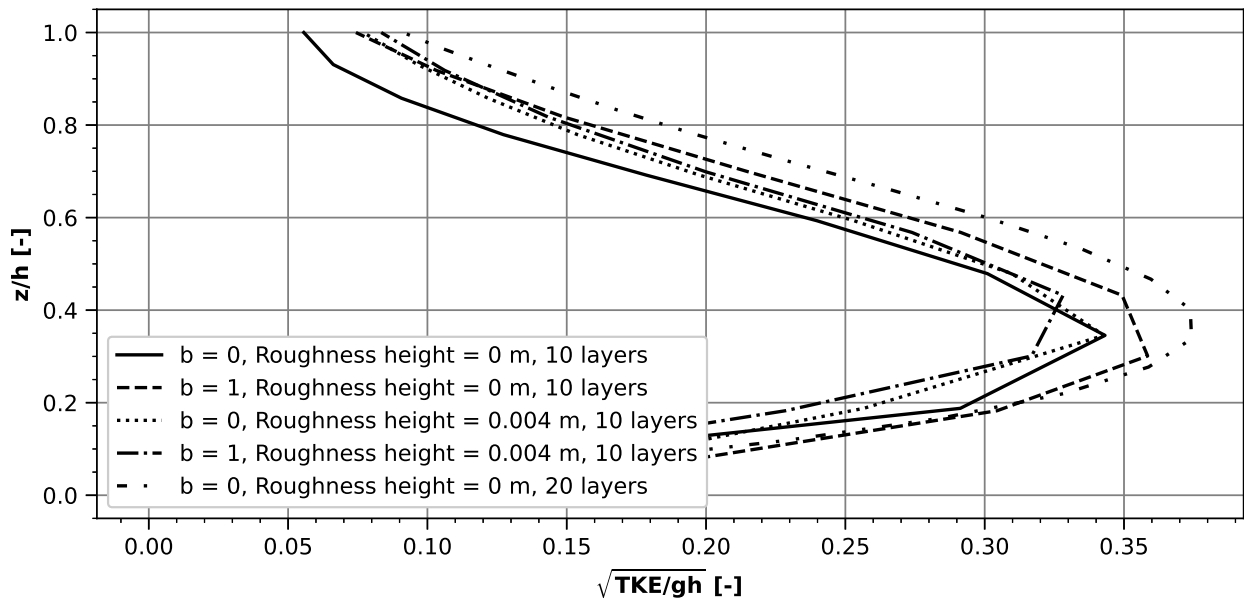


Figure 4.22: Turbulent kinetic energy (TKE) profile comparison for altered bed resistance and vertical discretisation.

The TKE profiles in Figure 4.22 are also very similar across the different cases. The highest TKE levels are found

in the 20-layer simulation, consistent with the slightly lower run-up observed in Figure 4.18. This suggests that increasing vertical resolution improves the representation of turbulence production, though the overall impact on run-up remains minor.

In summary, the analysis of run-up, surface elevations, velocity distributions, and TKE profiles shows that neither the choice of sigma parameter nor the doubling of vertical layers significantly alters the model outcomes. While including bed resistance introduces additional energy dissipation and slightly reduces run-up, the overall results indicate that the model is numerically converged with respect to vertical discretisation and relatively insensitive to the tested parameter choices.

Evaluation of scale effect on the TKE production

This section evaluates potential scale effects on the model results. To this end, five representative cases within the breaking zone ($\xi_{m-1,0} \lesssim 1.8$) are reproduced at a geometric scale of 1:20 and compared against the original model configuration described in Section 4.3.1.

The run-up results of this analysis are shown in Figure 4.23. The scaled model returns the same values of $R_{u2\%}$ as the original configuration, indicating no measurable impact of scale on run-up.

The run-up results of this analysis are presented in Figure 4.23.

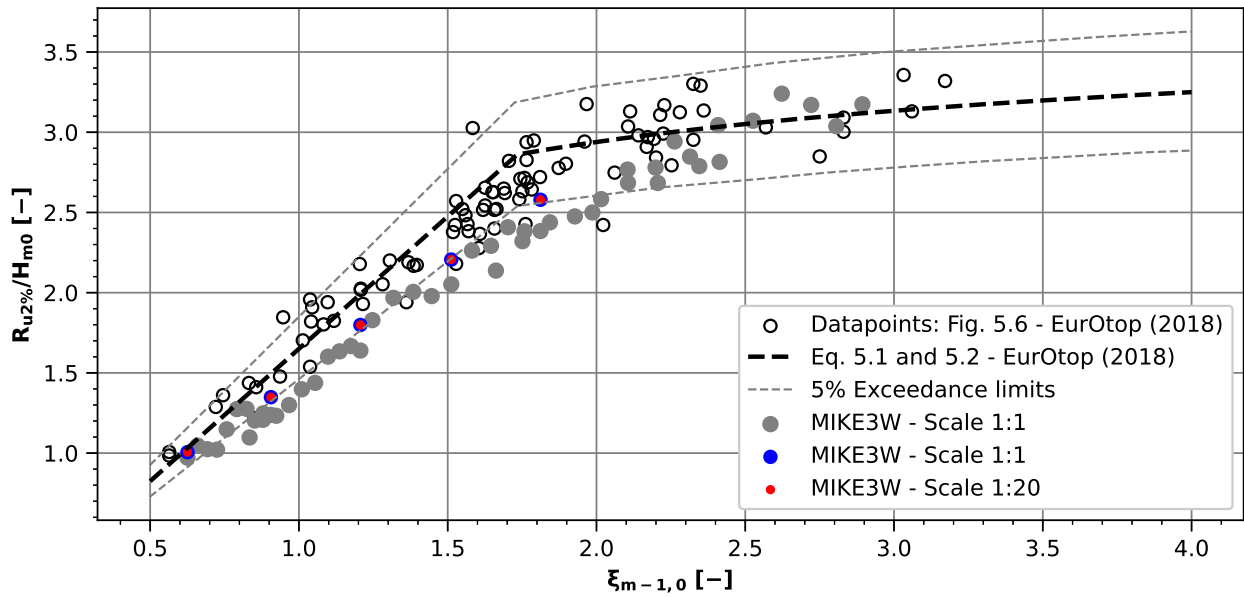


Figure 4.23: Run-up results evaluation of the scale effect on the production of TKE in MIKE3W.

To investigate this further, the period-averaged non-dimensional TKE was evaluated for a representative breaking wave event, selected from the time series to capture noticeable TKE production. The results are presented in Figure 4.24.

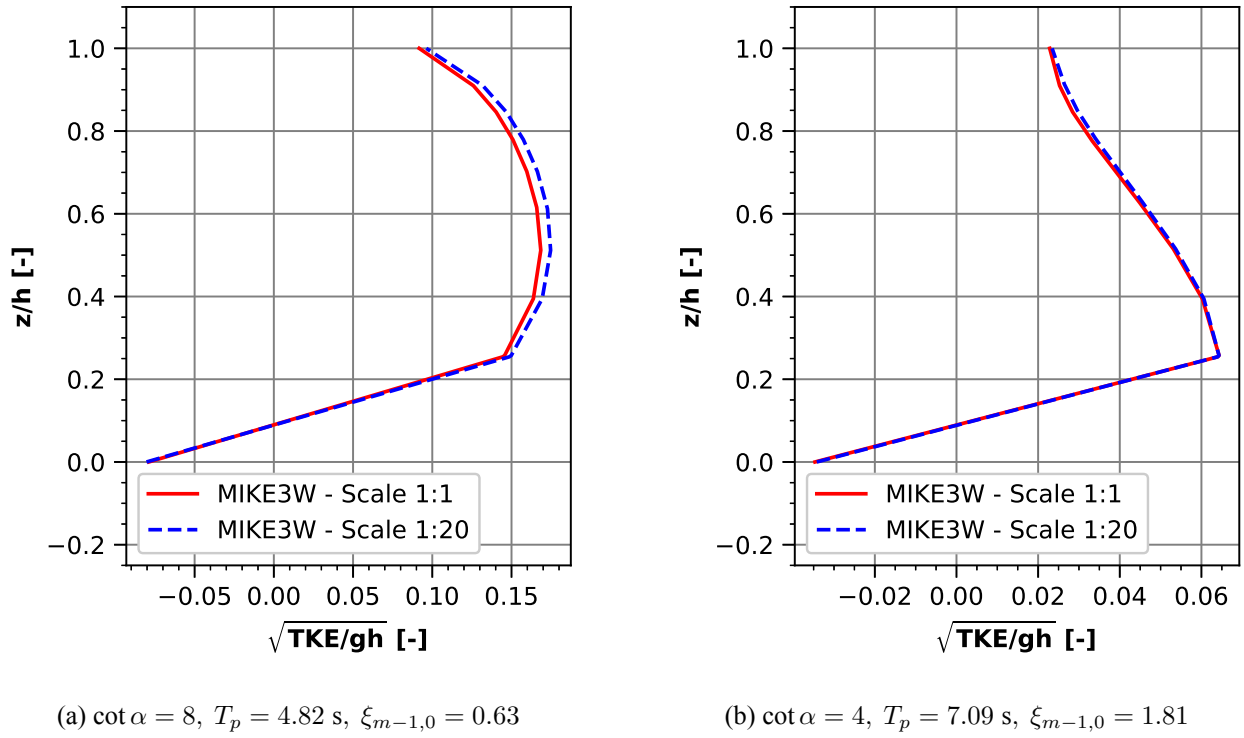


Figure 4.24: Time-averaged TKE profiles for different cases.

For both cases (left: $\xi_{m-1,0} = 0.63$, right: $\xi_{m-1,0} = 1.81$), the period-averaged non-dimensional TKE is consistent between the scaled and original models. This matches the identical run-up predictions obtained in Figure 4.23.

Overall, the results indicate that scale effects cannot explain the general overestimation of TKE observed in MIKE3W. This conclusion aligns with findings from the European research programme OPTICREST (De Rouck et al. (2000)), which reported little to no small-scale model effects for setups comparable to those considered in this thesis.

Concluding remarks of the $k - \epsilon$ model

The suspicion of turbulent kinetic energy (TKE) overprediction, inferred from the underestimation of wave run-up when applying the $k - \epsilon$ model, has been further investigated. A comparison of the time-averaged, non-dimensionalised turbulence levels in the inner surf zone with the experimental measurements of Ting & Kirby (1994) revealed an overprediction of approximately 40%. This magnitude of overprediction is consistent with previous findings, where Lemos (1992) and Lin & Liu (1998) reported values in the range of 25–50%. The turbulence level in the swash zone was seemingly comparable to the levels found by Petti & Longo (2001).

Several potential sources of the discrepancy were examined, including the number of sigma layers, sigma parameter settings, bed resistance, and scaling effects. However, none of these adjustments produced significant changes in the predicted TKE levels. It is therefore concluded that the overprediction of TKE cannot be attributed to inappropriate parameter choices, but rather originates from inherent limitations of the turbulence closure itself. This conclusion is in agreement with the broader literature, which highlights the challenges of accurately modelling turbulence in breaking waves and the general tendency of RANS models to overpredict TKE.

One contributing factor may be the neglect of air entrainment in the roller. The mixing of air and water within the roller reduces the effective density, which limits the downward penetration of turbulence. Consequently, a substantial portion of the turbulence is dissipated directly within the roller, rather than being transferred to the underlying layers. This mechanism is discussed in further detail by Christensen et al. (2002). Since the modification of turbulence models lies beyond the scope of this thesis, no further attempts to address this shortcoming are undertaken here.

4.4.4 Assessment of model applicability for engineering design

Both MIKE21W and MIKE3W demonstrate good capability in predicting wave run-up, with MIKE3W generally providing greater accuracy. MIKE21W exhibits a tendency to slightly underpredict run-up. However, in the plunging breaker regime, its results align closely with observations. As discussed in Section 4.2.3, this alignment is likely due to competing model behaviors: the artificial suppression of wave breaking, which increases run-up, offsets the model's general tendency to underpredict. Consequently, the good fit in this specific regime may not indicate a physically consistent representation, and its robustness under a wider range of conditions requires further investigation.

MIKE3W shows a slight tendency to overpredict run-up in the lower range of the plunging regime, approaching the spilling breaker regime. For both models, the inclusion of advanced physics, specifically the surface roller formulation in MIKE21W and the $k-\epsilon$ turbulence closure in MIKE3W, led to a decrease in agreement with the measurements. Therefore, the use of these specific options is not advised for run-up estimation in engineering design applications.

The strong performance of both models should be interpreted in the context of the relatively steep slopes used in this study. MIKE21W, which is based on Boussinesq-type equations optimized for milder slopes, demonstrated very good predictive capability. Its results suggest that the model's application can be extended to steeper slopes than those for which it was originally developed. Nonetheless, MIKE3W, with its ability to more fully resolve vertical processes, achieved a higher degree of accuracy under these challenging conditions.

This makes MIKE21W an excellent tool for preliminary studies and scenarios where computational efficiency is key, providing reliable estimates with low cost. MIKE3W, in turn, is the preferred choice for final, detailed designs where maximum accuracy is required. Its slight tendency to overpredict in some regimes is a beneficial trait for engineering, as it inherently falls on the side of safety.

5 Irregular wave overtopping

In this part of the study, the focus is placed on investigating overtopping caused by irregular waves using the MIKE21W and MIKE3W models, comparing their results with experimental data. Since DHI has already conducted a validation study for MIKE3W in similar conditions (MIKE 3 Wave Model FM - Validation report (2025)), this report will serve as a benchmark reference for evaluating the performance of MIKE21W. By comparing MIKE21W's overtopping predictions to those presented in the MIKE3W validation, the aim is to assess the model's reliability in simulating complex wave-structure interactions under irregular wave conditions.

The model setup applied in this chapter builds upon the findings of the preceding run-up analysis, ensuring consistency in wave generation, decomposition, and treatment of nearshore processes. Before proceeding to the broader comparison with experimental cases from the CLASH database, a preliminary phase is dedicated to tuning key modelling parameters within MIKE21W. This step is essential to ensure that the model is calibrated appropriately and that any observed discrepancies in performance are not simply due to suboptimal parameter settings. The outcome of this phase provides insight into MIKE21W's strengths and limitations in overtopping prediction and lays the groundwork for identifying the threshold at which a transition to more advanced modelling tools, such as MIKE3W, becomes necessary from an engineering perspective.

5.1 Literature review and experimental comparison

Early overtopping research, dating back to studies such as Tominaga et al. (1967), relied primarily on physical modelling and laboratory experiments on simple structures such as seawalls, breakwaters, and dikes. Foundational guidance documents emerged in the late 20th century, including the UK Environment Agency Manual on Overtopping Besley (1999), the Dutch TAW Technical Report on wave run-up and overtopping at dikes TAW (2002), and the German guidelines für Küstenschutzwerke (2002).

A major milestone was the publication of the first EurOtop manual in 2007 (EurOtop (2007)), which consolidated and expanded on earlier national guidelines. It incorporated significant new experimental data, notably from the CLASH project (De Rouck et al. (2004)), which compiled an extensive database of overtopping tests from different structure types and environmental conditions.

The second edition, EurOtop (2018), further refined prediction methods, introducing updated formulae that improved accuracy for very low and zero freeboard conditions, steep slopes, and vertical structures with complex geometries (e.g., foreshores, storm walls). It also provided enhanced treatment of individual overtopping wave volumes and tolerable overtopping limits, along with multimedia resources (e.g., videos) on its dedicated website. The continuous increase in overtopping-related publications reflects growing awareness of the problem, especially in relation to extreme storms and climate resilience (Romão et al. (2024)).

Several approaches are used to study and predict wave overtopping, each with advantages, limitations, and typical application contexts. The most widely applied in engineering practice are empirical models. They relate dimensionless mean overtopping discharge to relative crest freeboard using exponential-type relationships, originally proposed by Owen (1980) and later refined in EurOtop (2007, 2018).

EurOtop (2018) manual provides empirical formulations for the prediction of mean wave overtopping discharge at coastal structures. Two sets of equations are proposed: a mean value approach, representing a best fit to experimental data, and a design and assessment approach, which incorporates a degree of conservatism to account

for uncertainties. In this work, the mean value approach is adopted, as it offers a more accurate representation of observed data and is thus more suitable for validation purposes.

The wave overtopping discharge is described using two separate formulae: one applicable to breaking waves and one for non-breaking waves. These are presented in eq. (5.1) and eq. (5.2), respectively.

The general structure of the equations is derived from the original work by Owen (1980), in which overtopping discharge was shown to decrease exponentially with increasing freeboard R_c . This formulation was further developed in EurOtop (2007). However, subsequent studies, particularly van der Meer & Bruce (2014), demonstrated that the 2007 formulae tended to overpredict overtopping discharges in cases of low or zero freeboard, revealing the need for more flexible formulations. This led to the introduction of additional coefficients, ultimately resulting in the updated equations published in EurOtop (2018), which are now considered state-of-the-art.

The mean overtopping discharge for breaking wave conditions is given by:

$$\frac{q}{\sqrt{g \cdot H_{m0}^3}} = \frac{0.023}{\sqrt{\tan \alpha}} \cdot \gamma_b \cdot \xi_{m-1,0} \cdot \exp \left[- \left(\frac{2.7 \cdot R_c}{\xi_{m-1,0} \cdot H_{m0} \cdot \gamma_b \cdot \gamma_f \cdot \gamma_\beta \cdot \gamma_v} \right)^{1.3} \right] \quad (5.1)$$

with an upper bound defined by the non-breaking wave formula:

$$\frac{q}{\sqrt{g \cdot H_{m0}^3}} = 0.09 \cdot \exp \left[- \left(\frac{1.5 \cdot R_c}{H_{m0} \cdot \gamma_f \cdot \gamma_\beta \cdot \gamma^*} \right)^{1.3} \right] \quad (5.2)$$

where q [m³/m/s] is the mean overtopping discharge, g [m/s²] is the gravitational acceleration, H_{m0} [m] is the spectral significant wave height, $\tan \alpha$ [–] is the slope of the structure, $\xi_{m-1,0}$ [–] is the breaker parameter based on the spectral wave period $T_{m-1,0}$ [s], R_c [m] is the freeboard.

Furthermore, several influence factors are used. γ_b [–] is the influence factor for a berm, γ_f [–] is the influence factor for roughness elements on a slope, γ_β [–] is the influence factor for oblique wave attack and γ_v [–] is the influence factor for a wall at the end of a slope, γ^* [–] composite influence factor for non-breaking waves, accounting for complex geometrical features such as storm walls or promenades.

Full details of these coefficients and their recommended values are provided in §5.4 of EurOtop (2018). In particular, γ^* , introduced in the 2018 edition, addresses limitations of earlier formulations by incorporating the effects of multiple geometrical modifications into a single adjustment factor, as described in §5.4.7 of EurOtop (2018).

Recent developments have focused on using Machine Learning (ML) algorithms for overtopping prediction, such as Artificial Neural Network (ANNs), first developed during the CLASH project (De Rouck et al. (2004)) and trained on a large database of overtopping tests. The updated EurOtop ANN tool now draws from an extended database of over 17000 tests covering a wide range of structure types. Further details on ANN and their use are discussed in section 5.1.2.

Physical models remain the benchmark for overtopping research, especially for complex geometries or combined hydrodynamic and structural processes. Wave flume (2D) and basin (3D) experiments typically follow Froude scaling, with scales ranging from 1 : 20 to 1 : 60.

Advantages include direct measurement of mean discharge, wave-by-wave volumes, and flow velocities, while

accounting for real fluid–structure interactions. On the other hand, challenges include scale effects, such as those due to surface tension, viscosity, and Reynolds number that can distort results if not properly addressed, and model effects, such as limitations due to flume width, absence of wind, or boundary reflections. A deeper description of these phenomena is given in Frostick et al. (2011).

The usage of empirical models, usually referred to as overtopping prediction formulae, is constrained to the experimental range of their underlying datasets and simplified structure geometries. Extrapolation towards out-of-range parameter values or other structure types than those used for the fitting may result in inaccurate and even invalid overtopping predictions.

Numerical modelling constitutes a powerful approach for predicting wave overtopping, providing a flexible and detailed alternative to empirical formulations and physical model testing. By solving the governing fluid dynamic equations, most commonly Boussinesq-type or Navier–Stokes equations, through numerical discretisation techniques, these models enable the simulation of a wide range of hydrodynamic processes and structural configurations. Their capacity to resolve time-dependent flow fields, pressure distributions, and overtopping volumes makes them an indispensable tool for assessing overtopping under varying sea states and for structures of increasing geometric complexity.

A fundamental distinction in numerical modelling of hydrodynamics and coastal processes lies between depth-integrated and depth-resolving approaches. Depth-integrated models, such as the Nonlinear Shallow Water (NLSW) equations or Boussinesq-type solvers, simplify the vertical structure of the flow by integrating the governing equations over the water column. This reduces computational complexity and allows for efficient larger-scale simulations, at the cost of resolving detailed vertical dynamics. In contrast, depth-resolving models—typically based on the Reynolds-Averaged Navier–Stokes (RANS) equations and closed with turbulence models—resolve the full vertical structure of the flow. These are more computationally demanding but are essential for capturing vertical gradients in velocity, turbulence, and mixing, particularly in complex nearshore or structure-interaction environments. Both modelling types are employed in this thesis (see Section 2.5).

Other types of models not used in this thesis include extensions of the OpenFOAM CFD code, which are general-purpose, open-source computational fluid dynamics (CFD) tools capable of solving the full Navier–Stokes equations in 3D. These allow high-fidelity simulations of complex free-surface flows, turbulence, and wave-structure interaction, but typically require significant computational resources and user expertise.

5.1.1 CLASH database

As described in van der Meer et al. (2009), the CLASH (Crest Level Assessment of Coastal Structures by full-scale monitoring, neural network prediction and hazard analysis on permissible wave overtopping) project, funded by the European Union under the Fifth Framework Programme, developed one of the most extensive and homogeneous databases on wave overtopping ever assembled. The database was compiled between 2002 and 2004 and includes 10532 overtopping tests from 163 independent test series, sourced from more than 160 laboratory and field campaigns conducted worldwide.

The data incorporated into the CLASH database originates from a broad range of sources, such as:

- Historical overtopping tests from universities, research institutes, and engineering firms;
- Newly conducted tests under the CLASH project, specifically designed to fill “white spots” in the existing knowledge;
- Prototype measurements at full-scale coastal structures (e.g., at Ostia, Samphire Hoe, and Zeebrugge);

- Supplementary tests conducted to explore specific phenomena, such as the influence of oblique wave attack and the roughness of armour layers.

About 80% of the data comes from CLASH project partners, while the remaining data comes from other international contributors. Both public and confidential data are included, but no identifiable project-specific details are retained, ensuring general applicability.

Each test in the database is described using 31 parameters, which are grouped as follows:

- General parameters: Test ID, reliability factor (RF), complexity factor (CF);
- Hydraulic parameters (11 total)
- Structural parameters (17 total)

Reliability and Complexity Factors

To evaluate the quality of each test, two subjective but systematically assigned indices are included. The Reliability Factor (RF) ranges from 1 (very reliable) to 4 (unreliable), based on measurement quality and data completeness. The Complexity Factor (CF): Ranges from 1 (simple structure) to 4 (very complex), based on how well the structure can be represented with the selected parameters.

These factors help users assess which tests are most suitable for specific research or design purposes.

Hydraulic parameters

Among these parameters, some describes deep water conditions ($H_{m0,deep}$, $T_p\ deep$, $T_m\ deep$, $T_{m-1,0\ deep}$), some toe conditions ($H_{m0,toe}$, $T_p\ toe$, $T_m\ toe$, $T_{m-1,0\ toe}$). It is also specified the angle of wave attack relative to the normal on the structure (β), the mean overtopping discharge q and the overtopping probability P_{ow} .

Whenever some parameters were missing, because not measured or not noted during the tests, an acceptable value was searched for. This has been done by assumptions based on previous study or further calculations. To account to the introduced uncertainty of these values, a minimum RF of two was assigned.

Structural parameters

They include: the water depth at deep water h_{deep} , the measure of the foreshore slope m , the water depth in front of the toe of the structure h , the toe width B_t , the toe depth h_t and the roughness/permeability factor of the structure γ_f . The slope of the structure below and above the berm is indicated respectively by $\cot \alpha_d$ and $\cot \alpha_u$. The mean slope of the structure, excluding and including the berm, is indicated respectively by $\cot \alpha_{excl}$ and $\cot \alpha_{incl}$.

The berm is described by its slope $\tan \alpha_B$, the width B , the water depth on the berm h_b and the width of the horizontally schematised berm B_h .

Finally, the crest freeboard of the structure R_c , the armour crest freeboard A_c and the crest width of the structure G_c .

5.1.2 Neural Network explanation and usage

In addition to the available datasets such as the CLASH database, and the empirical formulations commonly found in the literature (e.g., EurOtop (2018)), predictive methods based on Artificial Neural Network (ANNs) modelling have been developed for wave overtopping estimation. Neural networks are data-driven models widely used as generalised regression tools for capturing complex cause-and-effect relationships. In practice, they have proven highly effective for solving difficult modelling problems across various scientific and

engineering fields—particularly when the interrelationships between input variables are unclear but sufficient experimental data is available to train the model.

Given the size and diversity of the CLASH database, applying neural networks to predict overtopping discharge has been a natural and successful approach. The development of a neural network model generally involves two main phases: a training (learning) phase, during which the model identifies relationships from known data, and a testing/validation phase, where the model's predictive performance is assessed using an independent subset of data.

The value of such tools becomes especially apparent when estimating overtopping in complex cases—for example, in structures with non-standard geometries, within the test cases used for the training of the model.

Based on the CLASH database, van Gent et al. (2007) developed a neural network model (hereafter referred to as NN), available at DELTARES (2007). This model reduces the number of input parameters compared to the full set previously described, using only those shown in Figure 5.1.

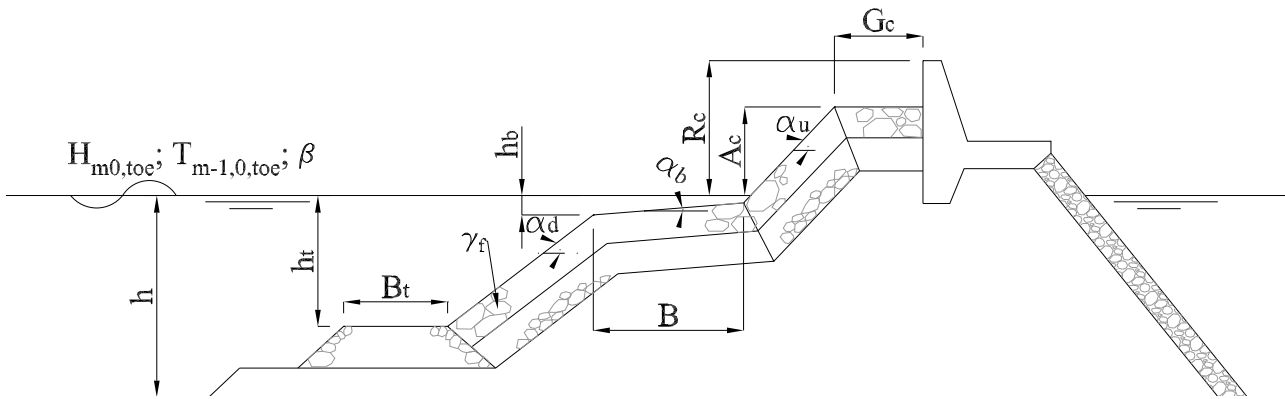
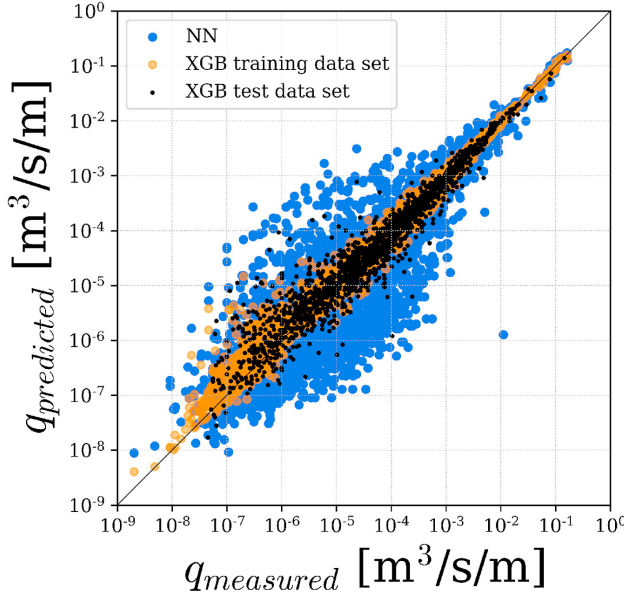


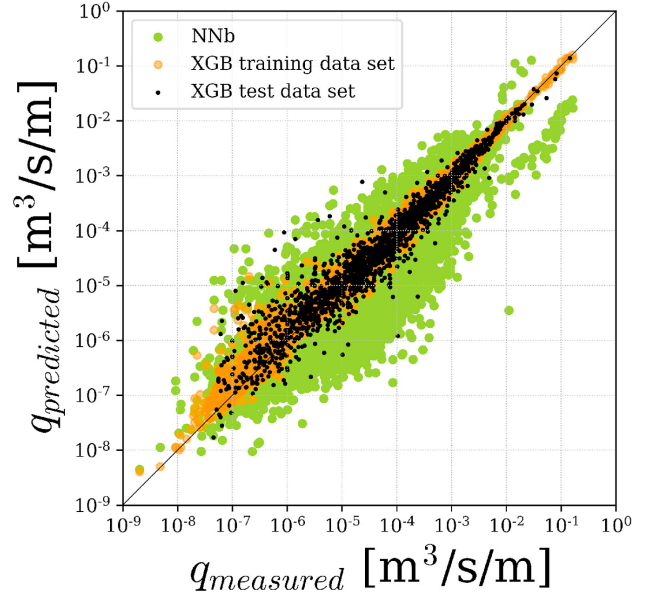
Figure 5.1: Input parameters used for NN-based prediction of wave overtopping discharges (from van Gent et al. (2007)).

A further development was presented by Zanuttigh et al. (2016), referred to here as NNb, which is based on a slightly different set of hydraulic parameters.

More recently, den Bieman et al. (2021) introduced a novel approach for predicting mean wave overtopping discharge using the advanced machine learning technique XGBoost (Chen & Guestrin (2016)). This model (referred to as XGB) has been shown to generally outperform other models when tested on datasets involving normally incident waves (see Figure 5.2).



(a) NN and XGB.



(b) NNb and XGB.

Figure 5.2: Predictions by the NN and NNb models for the CLASH database, compared with the XGB model for the training and test datasets (from den Bieman et al. (2021)).

Recent studies, such as Habib et al. (2023), suggest that other machine learning algorithms (e.g., Random Forests) can outperform ANNs in certain contexts, particularly for vertical seawalls, with crest freeboard and water depth at the toe emerging as dominant predictors.

All data-driven models show reduced accuracy under oblique wave conditions, likely due to the underrepresentation of such cases in the training datasets. Nonetheless, the performance of the XGBoost model improves significantly when a randomly selected subset of new oblique wave cases is added to the training data. Overall, the XGB model has been shown to reduce prediction errors by up to a factor of 5 compared to the NN and NNb models.

For the purposes of this thesis, the XGB model has been employed wherever Neural Network methods were required.

5.2 Reproducing MIKE3W validation report with MIKE21W

The validation study from MIKE 3 Wave Model FM - Validation report (2025) revolves around irregular waves propagating towards both an impermeable and permeable breakwater of varying geometry. In this study, only the impermeable breakwater is investigated. The geometrical setup of the breakwater can be seen in Figure 5.3.

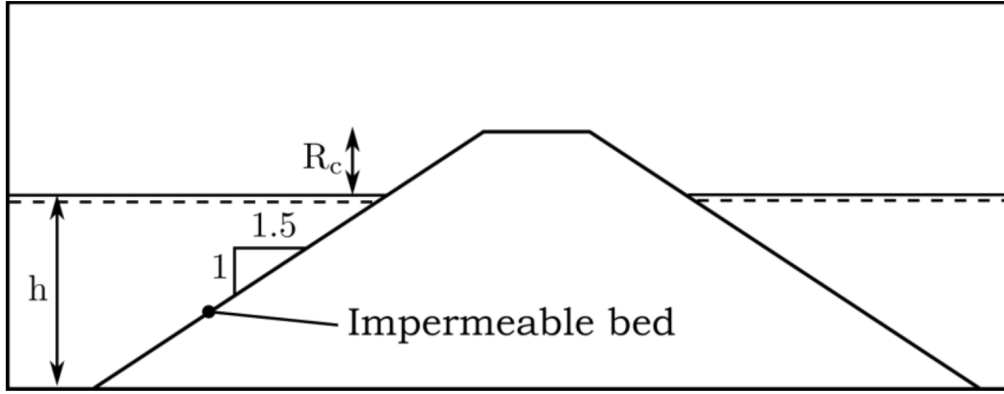


Figure 5.3: Sketch of the setup geometry from MIKE 3 Wave Model FM - Validation report (2025).

5.2.1 Model setup

The configuration of the MIKE21W model in this study is based on the input settings previously applied in MIKE3W simulations, as reported in the validation study by DHI. The MIKE3W simulations demonstrated promising skill in reproducing overtopping rates under irregular wave conditions, making them a natural reference point for establishing a comparable framework in MIKE21W.

Where feasible, boundary conditions, wave parameters, and bathymetric profiles have been directly adopted from the MIKE3W setup to ensure consistency between the two models. This includes the use of identical wave spectra, water levels, and structural geometries. While MIKE21W and MIKE3W differ in dimensionality and numerical formulation, aligning the input conditions allows for an evaluation of model performance that is less affected by variability in external inputs. The input parameters utilised in MIKE 3 Wave Model FM - Validation report (2025) are summarised in Table 5.1.

Table 5.1: Input parameters used in MIKE 3 Wave Model FM - Validation report (2025).

Parameter	Values
Water depth, h [m]	0.205, 0.237
Freeboard, R_c [m]	0.0762, 0.0442
Significant wave height, H_{m0} [m]	0.111, 0.074, 0.055
Peak period, T_p [s]	1.56, 1.43, 1.16
Simulation period [s]	1000
h_{dry} ; h_{wet} [m]	0.001; 0.003
Wave spectrum	JONSWAP, $\gamma = 3.3$
Turbulence model	$k-\epsilon$ formulation
Numerical solver	HLLC Riemann solver

Finally, the turbulence closure scheme, $k-\epsilon$ model, has not been applied in these simulations, since it is not part of the MIKE21W governing equations. Eddy viscosities have been set to zero, and bed resistance has likewise been neglected to maintain consistency with the simplified setup.

5.2.2 Parameters sensitivity

After configuring the model using parameters derived from the MIKE3W validation report, a sensitivity analysis was conducted to assess the influence of key parameters on simulation performance.

In Table 5.2, the sets of tested parameters are collected. The performance of the model with each of the parameters set was evaluated visually and by using the error metrics shown in the table.

Table 5.2: Model performance and configuration parameters for the tested cases.

Seed	$h_{dry}; h_{wet}$ [m]	Riemann	RMSD	SCI	Relative Bias
500	0.001; 0.003	0.5	0.00648	1.045	-0.864
500	0.001; 0.003	0.3	0.00655	0.993	-0.849
500	0.001; 0.003	0.1	0.00656	0.908	-0.752
500	0.0001; 0.002	0.5	0.00643	1.019	-0.848
500	0.0001; 0.001	0.5	0.00607	0.948	-0.826
600	0.001; 0.003	0.5	0.00625	1.031	-0.850
600	0.001; 0.003	0.1	0.00626	0.925	-0.788
600	0.001; 0.003	0.05	0.01024	2.232	3.225
600	0.0001; 0.002	0.1	0.00554	0.815	-0.755
600	0.0001; 0.001	0.5	0.00513	0.957	-0.808

The parameter configuration resulting in the best model performance is marked in bold in Table 5.2. This configuration demonstrates better performance relative to all other tested configurations (Appendix A.2). It achieves both a low relative bias and a small Root Mean Square Deviation (RMSD), indicating a high level of accuracy. In addition, the results exhibit minimal scatter around the reference values, confirming the robustness and consistency of the chosen parameter set. With a wave height of $H_{m0} = 0.037$ m, the final wetting depth chosen is $h_{wet} = 0.002$ m = $H_{m0}/18.5$.

Among the parameters tested, the Riemann number and flood/dry thresholds were found to significantly affect both model accuracy and stability. Reducing the Riemann number and lowering the flood and dry levels generally improved agreement with expected overtopping results. However, these adjustments also made the model more prone to numerical instability. In some cases, the instability was severe enough to cause the simulation to fail entirely. In other instances, the model remained technically operational but produced unphysically high overtopping rates due to isolated, unrealistically large discharge events triggered by singular extreme waves.

The minimum time step played a stabilising role and allowed for the use of lower Riemann numbers; however, this effect was limited, as the model still exhibited instability beyond a certain threshold despite the reduced time step.

Another important factor was the seed number used in the generation of the irregular wave spectrum. The seed number influences the random phase distribution of the wave components and, consequently, the timing and magnitude of individual waves. This was particularly impactful in low-discharge conditions, where overtopping could be dominated by a few large wave events that might occur in one realisation but not in another. To mitigate this variability, longer simulation durations could be employed to capture a more representative statistical distribution of overtopping events, but this is at the cost of increased computational demand. As such, the

sensitivity of overtopping predictions to the seed number will also be examined in more detail in the following analysis.

5.2.3 Results

To guarantee the correct analysis of the resulting overtopping discharge and comparison with the available experimental data and empirical formulation, the modelled wave field is decomposed into incident (right-going) and reflected (left-going) components, as described in Section 2.4, to calculate the incident wave height at the toe.

In Figure 5.4, the best-performing case simulated with MIKE21W is compared to the benchmark results presented in MIKE 3 Wave Model FM - Validation report (2025).

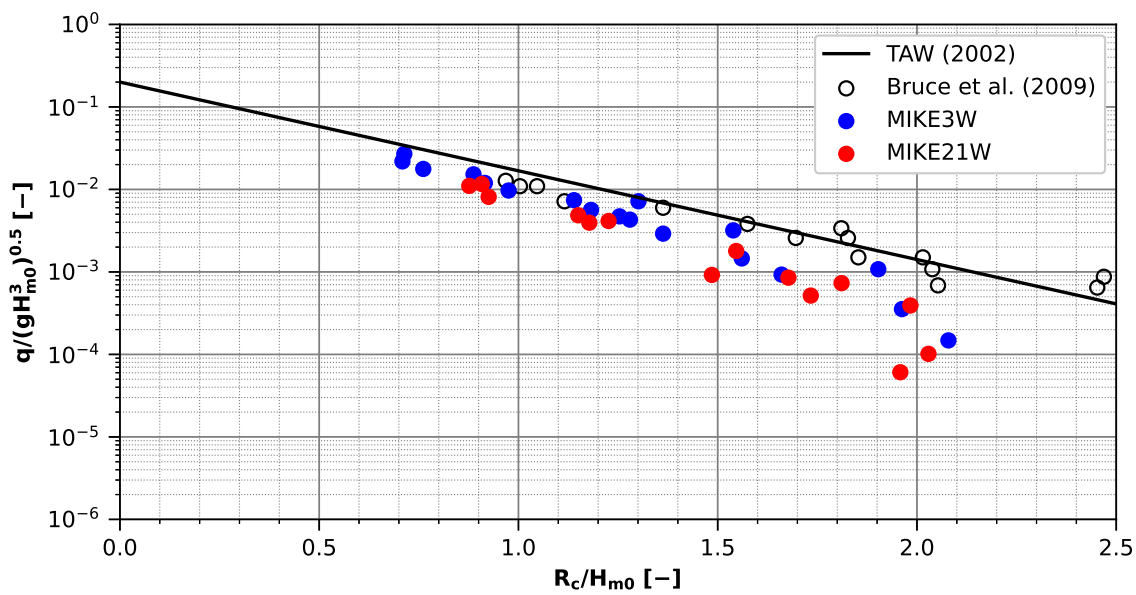


Figure 5.4: MIKE21W simulation results compared with results from MIKE 3 Wave Model FM - Validation report (2025), experimental data from Bruce et al. (2009) and the empirical formulation from TAW (2002).

The results in Figure 5.4 indicate that MIKE21W exhibits a slight underprediction relative to the MIKE3W simulations. This tendency is consistent with the observations made earlier when assessing the run-up performance of the two models. Moreover, both models show good agreement with the experimental data up to approximately $\frac{R_c}{H_{m0}} \approx 1.2$, beyond which they begin to increasingly underpredict the overtopping discharge.

5.3 Case selection from CLASH database

For validation purposes, two independent subsets of test cases were extracted from the CLASH database: one comprising simple geometries and another including more complex configurations with berms. This separation allows the analysis to first evaluate model performance under basic structural conditions before assessing the additional influence of berm geometry. The selection process involved a series of filtering steps, aimed not only at isolating reproducible and well-documented cases but also at reducing the dataset to a manageable size within the timeframe of this thesis.

The first filtering step targeted the reliability of the measured overtopping data. Only cases with both the Reliability Factor (RF) and the Consistency Factor (CF) equal to 1 were retained, ensuring a high level of confidence

in the underlying experiments.

Additional constraints were then applied to the structure geometry. In the simple subset, only cases with a uniform slope and a simple cross-sectional profile were considered. In the more complex subset, a berm width, B , was allowed. For both subsets, configurations featuring vertical walls on the crest were excluded. These geometrical properties can be seen illustrated in Section 5.4.1. To ensure hydraulically smooth conditions, only cases with a roughness reduction factor $\gamma_f = 1$ were included.

Finally, the analysis was limited to normal wave attack, with the angle of wave incidence fixed at 0° .

Applying these filtering criteria resulted in a final selection of 180 cases with simple geometry (hereafter referred to as "simple cases") and 95 cases with more complex geometry (hereafter referred to as "complex cases"). The corresponding ranges of key dimensionless parameters are summarised in Table 5.3.

Table 5.3: Range of main dimensionless parameters in the selected CLASH test cases.

Parameter	Range (simple cases)	Range (complex cases)
R_c/H_{m0} [–]	[0, 4.86]	[0.8, 2.72]
$q/(gH_{m0}^3)^{0.5}$ [–]	[0, $6.76 \cdot 10^{-2}$]	[0, $8.51 \cdot 10^{-3}$]
$\cot \alpha$ [–]	{1.19, 3, 4, 6}	{3, 4}
B/H_{m0} [–]	{0}	[2, 10]

Wave height matching procedure

Both in MIKE21W and MIKE3W, the model was calibrated to reproduce the target incoming wave height at the toe of the structure, $H_{m0, toe}$, as reported in the CLASH database. To achieve this, the modelled wave field was decomposed into incident (right-going) and reflected (left-going) components, as described in Section 2.4. The ratio between the desired and simulated incoming wave heights was computed and used to adjust the boundary wave height iteratively. The simulation was repeated with the corrected input until the modelled incoming wave height fell within $\pm 1\%$ of the target value.

This calibration procedure ensured that the numerical model accurately reproduced the experimental wave forcing conditions, thereby enabling meaningful comparison of the resulting cumulative overtopping discharges.

5.4 Validation of MIKE21W for overtopping

MIKE21W was employed to simulate the wave overtopping discharges for all the selected CLASH test cases, 180 simple and 95 complex. The objective was to reproduce the cumulative overtopping volume at the crest, enabling direct comparison between model results and measured data.

5.4.1 Model setup

The numerical domain was constructed as a one-dimensional (1D) stripe by limiting the mesh to a single cell in the cross-shore direction. The domain length was defined as a function of the wavelength L of the incoming wave conditions for each test case, with the offshore extent (from the boundary to the toe of the structure) set to $5L$, and the onshore extension beyond the crest was set to $3L$.

The geometry of the structure was reconstructed using the slope $\cot \alpha$, still water depth h , and crest freeboard R_c , as defined in the CLASH database. The crest width, being of secondary importance for overtopping processes in the simulated cases, was set to $(R_c + h)/10$.

A wave generation area having a width of $2L$ was utilised, and a sponge layer with a width equal to $1L$ was placed at the end of the domain.

A sketch of the bathymetry setup for the simple cases is depicted in Figure 5.5 and for the complex cases in Figure 5.6. In the same figure, an indication of the placement of the wave gauges used for the wave decomposition is also shown.

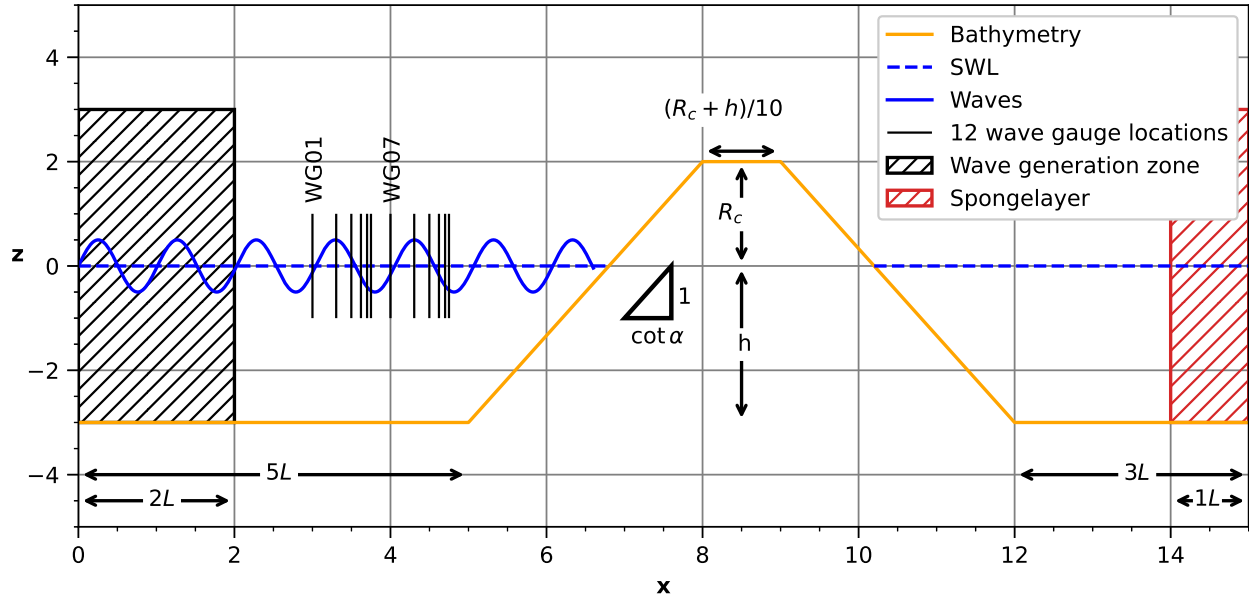


Figure 5.5: Simple cases. Sketch of the bathymetry input parameters used for defining the tested cases.

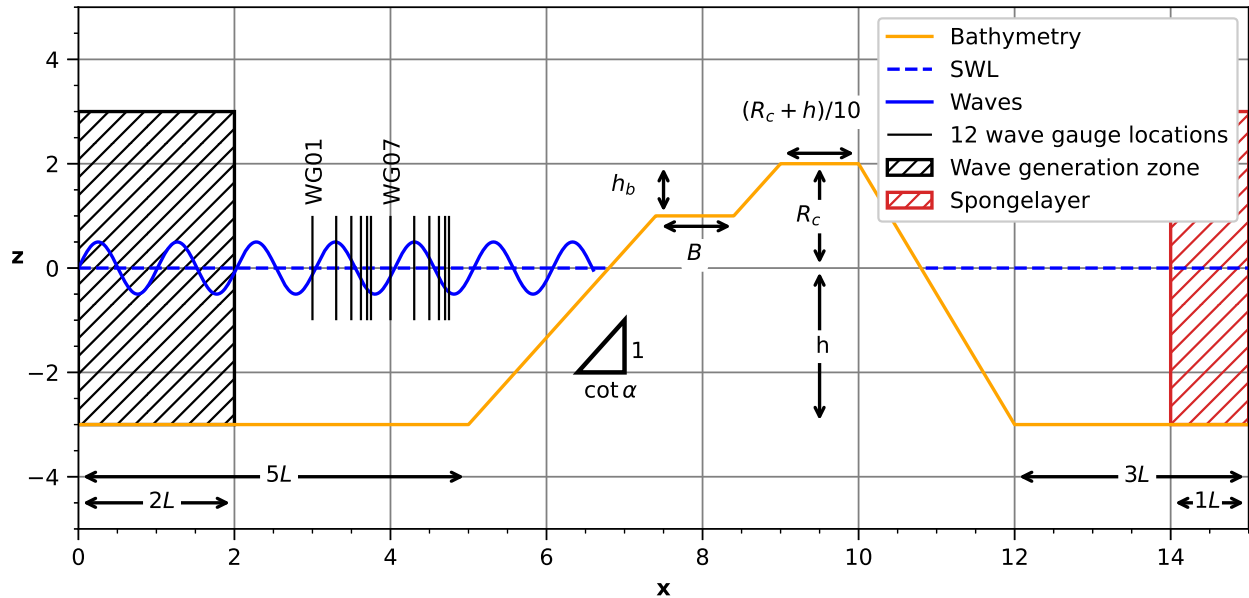


Figure 5.6: Complex cases. Sketch of the bathymetry input parameters used for defining the tested cases.

A uniform quadrilateral mesh was used, with a spatial resolution of $dx = L/40$.

Flooding and drying parameters were selected based on sensitivity analyses discussed in Section 5.2. Specifically, they were scaled with the significant wave height H_{m0} as $h_{\text{wet}} = H_{m0}/18.5$ and $h_{\text{dry}} = h_{\text{wet}}/3$.

A maximum timestep of $dt_{\text{max}} = 0.025$ s was adopted, while the minimum allowed timestep was set to $dt_{\text{min}} = 10^{-10}$ s. This very low minimum ensured that the model could adjust if necessary to maintain a satisfactory CFL number and achieve numerical stability (see Section 2.5.2). Each simulation was run for 20000 time steps, corresponding to a total simulation time of 500 s.

To balance numerical accuracy and stability, the Riemann factor was set to $R = 0.2$, which was found to offer a good compromise for overtopping discharge predictions.

Wave breaking was excluded from the simulations due to the results seen in the previous run-up analysis, see Section 4.2.3.

No additional physical processes, such as bed friction or eddy viscosity, were included in the simulations to isolate the effect of wave breaking and propagation dynamics on the overtopping process.

The parameters used in the MIKE21W simulations are collected in Table 5.4.

Table 5.4: Input parameters used in MIKE21W simulations.

Parameter	Values
Domain length	Offshore: $5L$; Onshore: $3L$
Wave generation zone width	$2L$
Sponge layer width	$1L$
Mesh type	Uniform quadrilateral
Spatial resolution, dx [m]	$L/40$
Structure geometry	$\cot \alpha$, h , R_c (from CLASH)
Crest width	$(R_c + h)/10$
Flooding/drying thresholds [m]	$h_{\text{wet}} = H_{m0}/18.5$; $h_{\text{dry}} = h_{\text{wet}}/3$
Maximum timestep, dt_{max} [s]	0.025
Minimum timestep, dt_{min} [s]	10^{-10}
Simulation period [s]	500
Riemann factor [–]	0.2
Wave breaking	Not included
Bed friction	Not included
Eddy viscosity	Not included

5.4.2 Results

Simple cases

An example of the model output is presented in Figure 5.7, which shows the time series of the instantaneous overtopping discharge as well as the corresponding accumulated discharge over time. In accordance with the model setup, where the structure has a width of 1 m, both quantities are expressed per unit width.

As expected for irregular wave conditions, the instantaneous overtopping discharge exhibits a highly fluctuating behaviour, with multiple peaks and no constant flow rate. This reflects the inherently random nature of the wave

overtopping process. The accumulated discharge curve shows a continuous increase over time, indicating that overtopped water is not returning seaward and all the flow is being retained.

To compute the mean overtopping discharge, the total accumulated volume is divided by the total simulation duration.

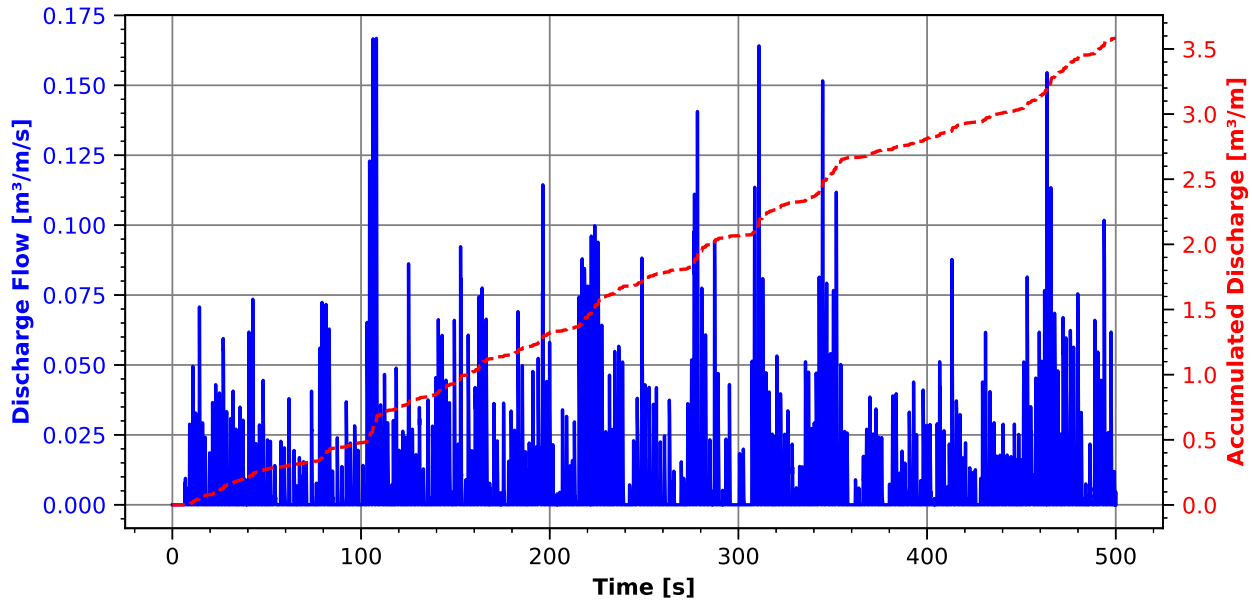


Figure 5.7: Example time series of overtopping discharge and accumulated discharge over time.

The comparison between simulated and measured mean overtopping discharges for the 180 selected simple CLASH test cases is presented in Figure 5.8.

To facilitate visualisation in logarithmic scale, observed or simulated values equal to zero have been replaced with a nominal value of 10^{-8} m³/s/m. The 1:1 agreement line (perfect fit) is shown in black, along with lines representing over- and under-prediction by a factor of 2 and 10, respectively. To highlight the influence of relative freeboard, data points are colour-coded according to the relative freeboard ratio, R_c/H_{m0} . Different markers are used to distinguish among the modelled structural slopes.

Overall, the model shows good agreement with experimental results: 54% of the cases fall within a factor of 2, and 72% within a factor of 10.

Several cases predict zero discharge, whereas the corresponding experiments report small but positive values, typically for $q_{\text{observed}} \lesssim 10^{-3}$ m³/m/s. A seed-sensitivity analysis (see Section 5.6.2) revealed that results in this range are controlled by just one or two overtopping events, causing significant scatter depending on the random phase realisation. Achieving statistical convergence for such small discharges would require much longer simulation durations and substantially higher computational cost, which was not pursued in this study. From an engineering design perspective, these very low overtopping rates are generally considered negligible.

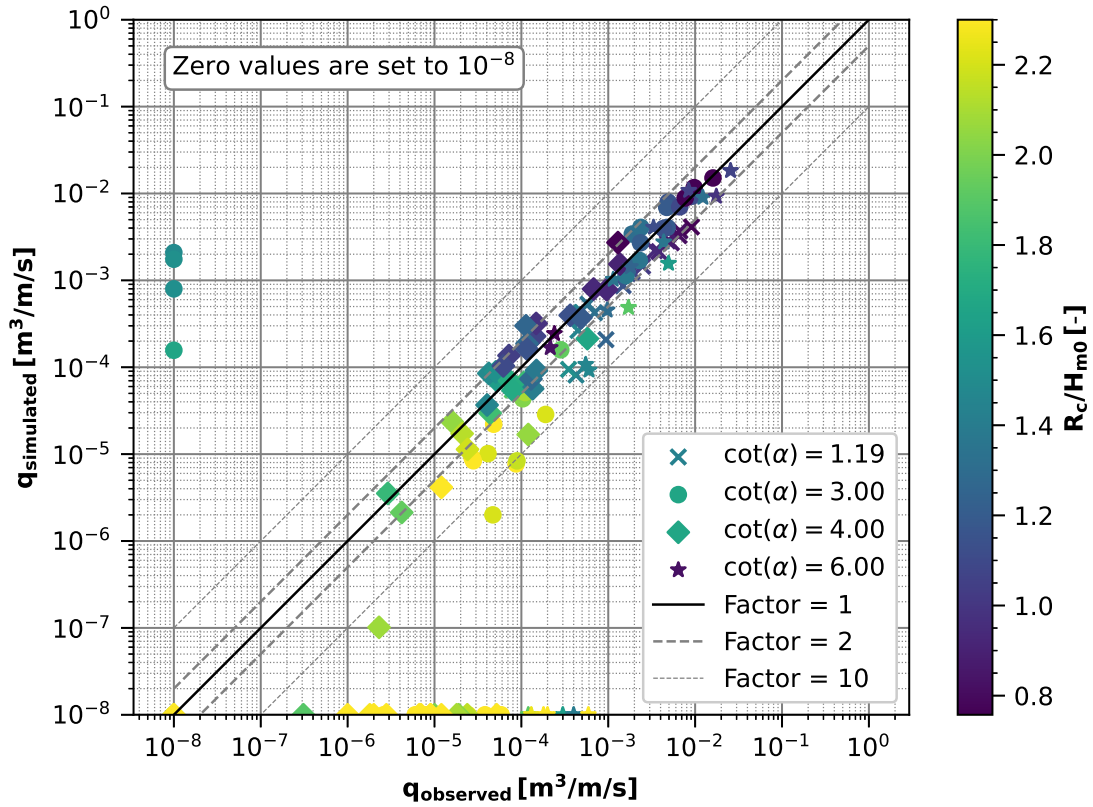


Figure 5.8: Comparison of measured and simulated with MIKE21W mean overtopping discharges for the selected simple cases.

In addition, at higher relative freeboards, the limited overtopping that does occur is increasingly dominated by splash and spray processes, where water crosses the crest as droplets rather than as coherent wave volumes. Such processes cannot be explicitly resolved by MIKE3W or MIKE21W due to their underlying formulations. Capturing them would require higher-fidelity CFD models.

Above this threshold ($q_{\text{observed}} \gtrsim 10^{-3} \text{ m}^3/\text{s}/\text{m}$), where overtopping is governed by multiple waves rather than isolated events, the model performance improves markedly. In this range, 83% of the predictions fall within a factor of 2 of the measurements, and 100% of the results lie within a factor of 10.

With respect to slope angle, the results appear evenly distributed around the 1:1 line across all marker types, suggesting no systematic slope dependency within the analysed dataset.

The few cases of pronounced overprediction in the upper left of Figure 5.8 are examined in greater detail in Section 5.5.2, where they are found to most likely result from uncertainties in the experimental dataset.

Complex cases

The results from the cases with inclusion of a berm are presented in Figure 5.9.

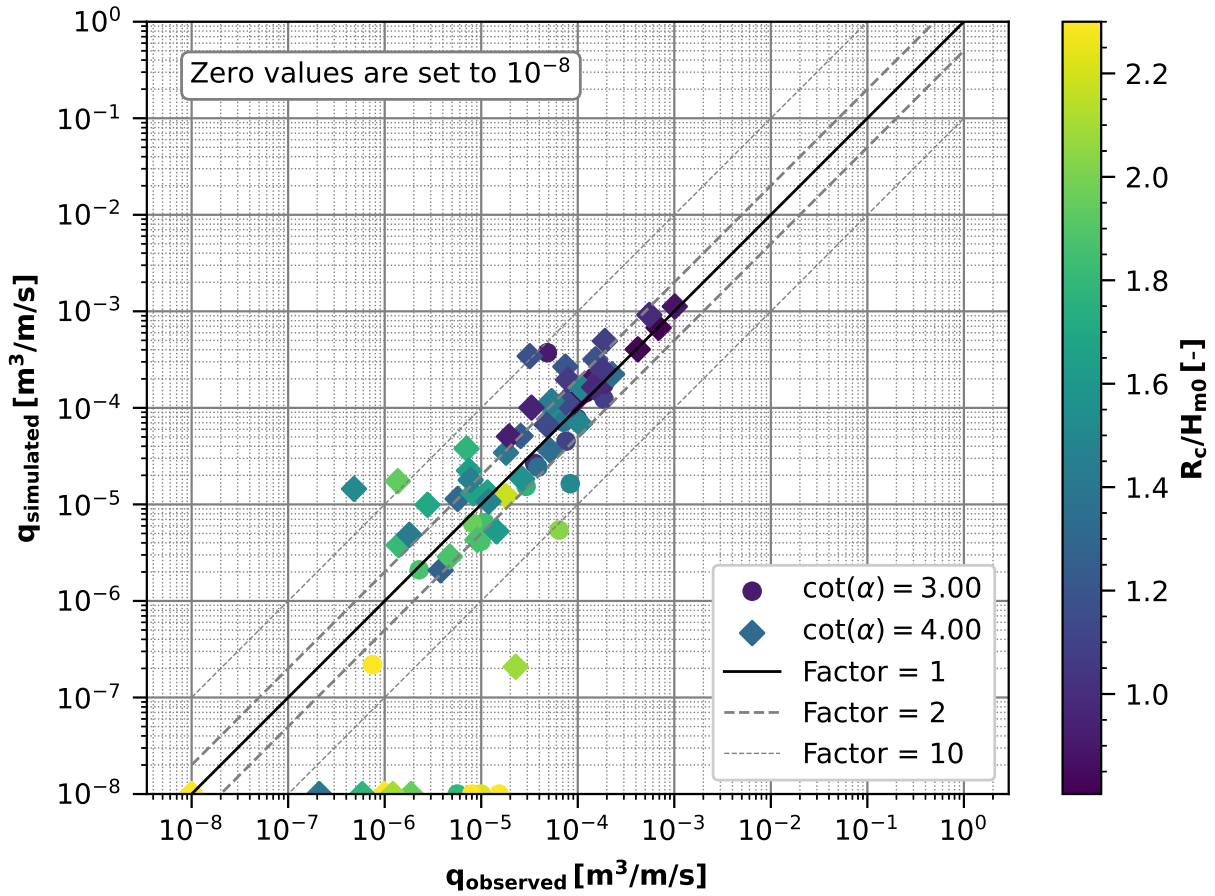


Figure 5.9: Comparison of measured and simulated with MIKE21W mean overtopping discharges for the selected complex cases.

The overtopping results for the complex cases show trends consistent with those observed for the simple cases. This suggests that the model is not only capable of reproducing overtopping behaviour for uniform slopes but also performs robustly for more complex geometries, such as the inclusion of a berm.

5.5 Validation of MIKE3W for overtopping

For validation of MIKE3W, only the simple cases from the CLASH database were tested; these cases are presented in Table 5.3.

5.5.1 Model setup

The numerical model setup is identical to the configuration applied in MIKE21W (see Section 5.4.1), except for the discretisation in the vertical (z -) direction. For the vertical discretisation, a total of ten sigma-layers were employed. The layer thickness varies according to the vertical distribution parameters $\sigma_c = 0.1$, $\theta = 2$ and $b = 0$.

Based on the findings in the run-up analyses (see Chapter 4), the $k - \epsilon$ model was not included.

5.5.2 Results

In Figure 5.10, the simulation results are compared with the corresponding cases from the CLASH database. A reference line is included in the plot, indicating deviations within a factor of 2 and 10.

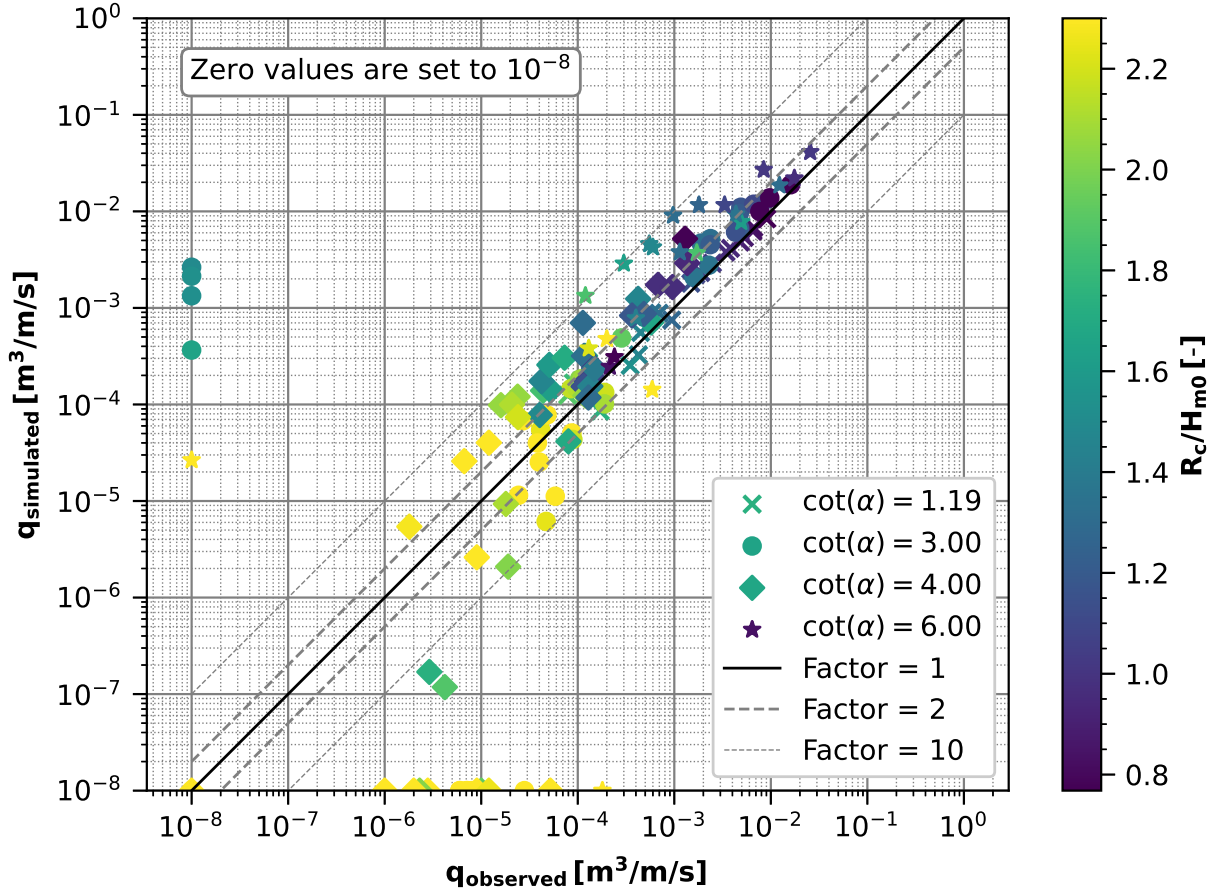


Figure 5.10: Comparison of measured and simulated with MIKE3W mean overtopping discharges for all selected cases.

Overall, a very good agreement is obtained for cases with lower relative freeboard, R_c/H_{m0} . For these cases, the predicted overtopping discharges closely match the CLASH measurements. At higher relative freeboard, where only very small overtopping volumes occur, a larger scatter is observed. This behaviour is expected, as the occurrence of overtopping becomes highly sensitive to whether one or two individual waves manage to pass the crest, as discussed in Section 5.6.2. These findings are consistent with the observations made for MIKE21W.

Across all cases, 52% of the simulations fall within a factor of 2 of the measurements, while 82% fall within a factor of 10. Compared to MIKE21W, MIKE3W produces fewer instances of null overtopping in situations where overtopping was observed in the CLASH experiments. Moreover, MIKE3W shows a slight tendency to overpredict discharges, whereas MIKE21W generally tends to underpredict them. The overprediction observed on the right side of the curve for the star-symbol cases ($\cot(\alpha) = 6$) is consistent with the overprediction observed in the run-up analysis (see Figure 4.8).

The five cases located on the left-hand side of Figure 5.10, where non-null overtopping was predicted despite null

overtopping being reported in the CLASH database, were investigated in more detail. A qualitative comparison of these cases was carried out against MIKE21W, MIKE3W, the XGB neural network model, and the empirical EurOtop formula. The results are summarised in Table 5.5. The case names correspond directly to those in the CLASH database.

Table 5.5: Comparison of discharges for selected cases.

Case		Discharge [$\text{m}^3/\text{m/s}$]				
Name	$\xi_{m-1,0}$	CLASH	MIKE21W	MIKE3W	XGB	EurOtop
103-001	2.157	0	0.0002	0.0004	0.0004	0.0005
103-002	2.585	0	0.0008	0.0013	0.0010	0.0010
103-003	3.042	0	0.0021	0.0026	0.0010	0.0011
103-004	3.725	0	0.0018	0.0021	0.0009	0.0009
110-018	1.571	0	0	0.00003	0.0005	0.00007

When comparing the four cases beginning with “103”, the MIKE21W and MIKE3W predictions are in the same range as those from XGB and EurOtop. This consistency suggests that the reported zero discharges in the CLASH database may reflect either experimental uncertainty or the high variability inherent to small overtopping events, rather than errors in the numerical models. In case 110-018, the predicted discharges differ more substantially across all approaches. However, the absolute discharge values are very small, and such discrepancies are not unexpected for low overtopping conditions.

5.6 Discussion on the overtopping results

In this section, the presented results of each model are discussed individually and compared directly. Based on the findings, an assessment of the applicability of the models for engineering design is presented.

5.6.1 Qualitative assessment of model performance

In Figure 5.11, a snapshot of the free-surface elevation is shown during one of the maximum overtopping events.

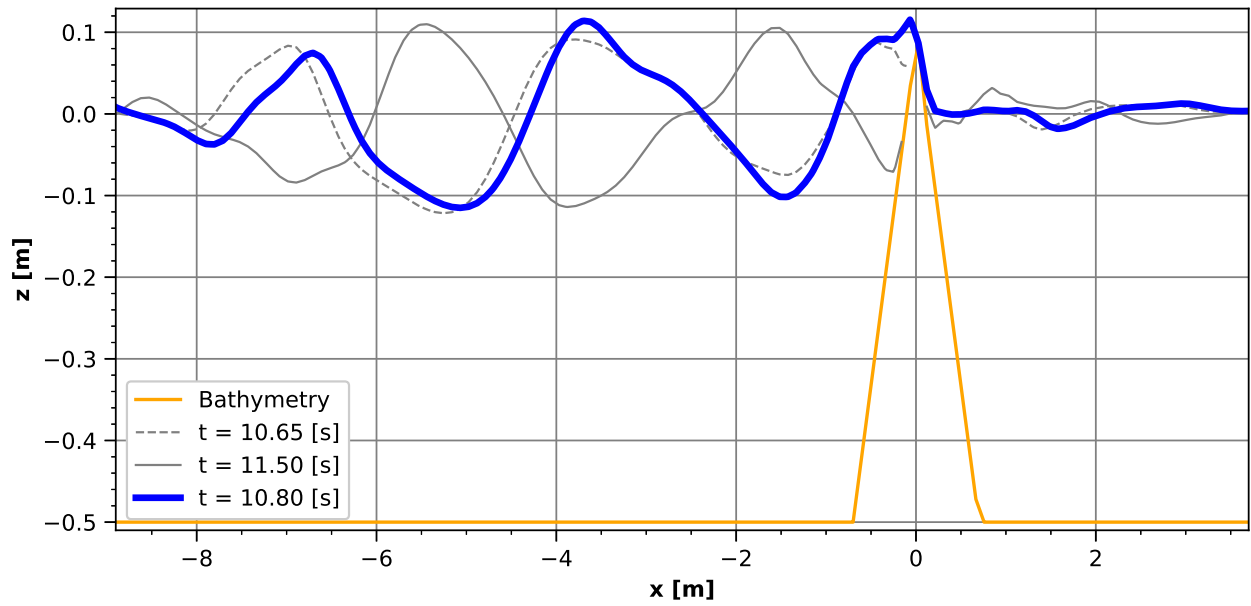


Figure 5.11: Water elevation profiles during an overtopping event in a representative case from MIKE3W. Grey lines indicate snapshots taken before and after the event.

In analogy with the assessment of the run-up time series (see Section 4.4.1), a qualitative evaluation is here made of the model performance under overtopping conditions. The surface elevation profiles in Figure 5.11, obtained with MIKE3W, demonstrate that the model is able to reproduce overtopping dynamics in a physically consistent manner, without indications of numerical instability. Similar to the run-up evolution, a slight phase lag is observed in the swash zone, which is attributable to the flood-and-dry formulation.

5.6.2 Seed sensitivity in MIKE21W

The simulation duration of 500 s was assumed sufficient to ensure statistical convergence. To verify this assumption, an analysis of seed sensitivity was carried out. The seed determines the random phase distribution used when generating the surface elevation from the wave spectrum. If significant scatter in the results appears when varying the seed, it indicates that the simulation length may be insufficient to achieve statistical convergence.

For the analysis, 30 cases were selected, evenly distributed across the observed discharge range (the x -axis in Figure 5.8). Each case was run with MIKE21W selecting 10 different seeds, and the results were compared. The outcome of this analysis is shown in Figure 5.12.

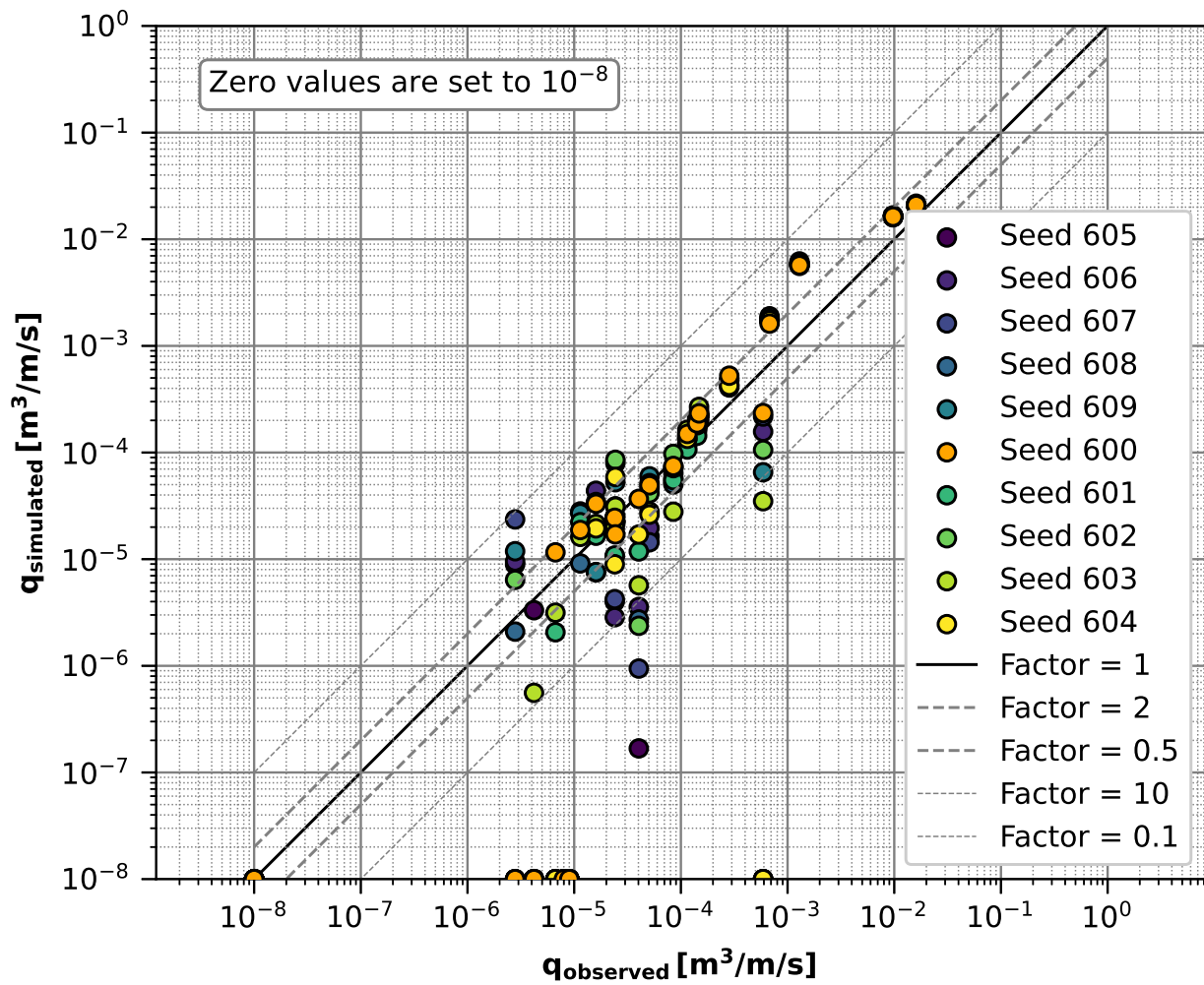


Figure 5.12: Seed sensitivity analysis for selected cases, showing the variability in predicted mean overtopping discharge across 10 different seeds.

As seen in Figure 5.12, the scatter introduced by varying the seed depends strongly on the magnitude of the expected discharge. For values of $q_{observed} \gtrsim 10^{-3} \text{ m}^3/\text{m/s}$, no clear dependence on the seed choice is observed, indicating statistical convergence. However, below this threshold, the scatter increases, with the largest deviation between seeds reaching approximately $10^{-4} \text{ m}^3/\text{m/s}$. This suggests that these low-discharge cases have not yet converged statistically and would require significantly longer simulation durations to do so.

Closer inspection of the instantaneous discharge time series for such cases reveals that overtopping becomes dominated by one or two individual waves rather than by contributions from multiple events, as it was for the case shown in Figure 5.7. An example is provided in Figure 5.13, where all of the discharge over 500 s originates from a single wave.

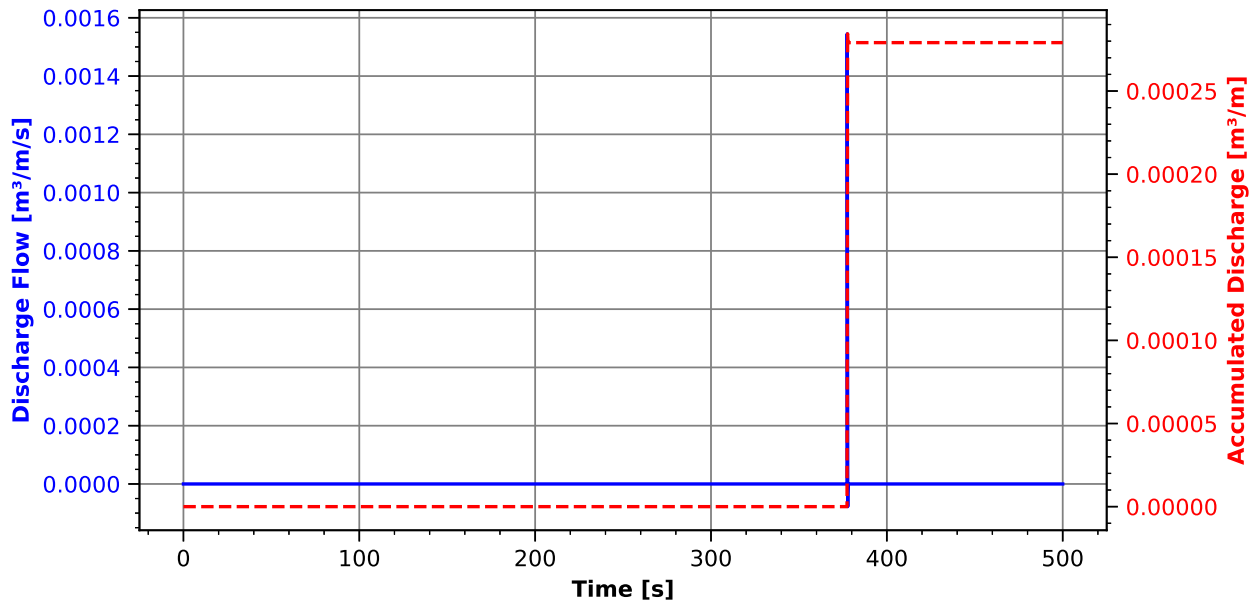


Figure 5.13: Example of seed sensitivity for Case 222-031 (seed 603), where overtopping is dominated by a single wave event over a duration of 500 s.

This behaviour makes it computationally impractical, within the timeframe of this study, to extend simulations of very low-discharge cases long enough to achieve full convergence. From an engineering design perspective, these cases are also of limited practical importance, as the resulting overtopping volumes are negligible compared to critical design thresholds. Nevertheless, designers need to be aware of this statistical sensitivity, particularly when working near the “zero overtopping” limit, as discussed in Section 2.2. It should be emphasised that this limitation is not specific to MIKE21W, but rather a general consequence of the stochastic nature of wave generation from spectra. Consequently, further pursuit of statistical convergence for these cases has not been undertaken.

5.6.3 Scatter index and relative bias

The results in Figures 5.8 and 5.10 indicate that the accuracy of the predictions varies with relative freeboard. To further investigate this influence, the Scatter Index (SCI) and relative bias, as defined in Section 2.6, were computed in bins of R_c/H_{m0} with a width of 0.5.

MIKE21W

The Scatter Index (SCI) and relative bias calculated using the results from MIKE21W simulations are shown in Figure 5.14.

A clear trend can be observed: with increasing R_c/H_{m0} , particularly above a threshold value of 1.7, the scatter of the results increases and the relative bias becomes more negative. This is partly related to the fact that higher relative freeboards are associated with lower overtopping discharges. In such situations, overtopping becomes controlled by a small number of individual waves, which makes the results more sensitive to statistical convergence rather than reflecting a systematic model limitation (see Section 5.6.2).

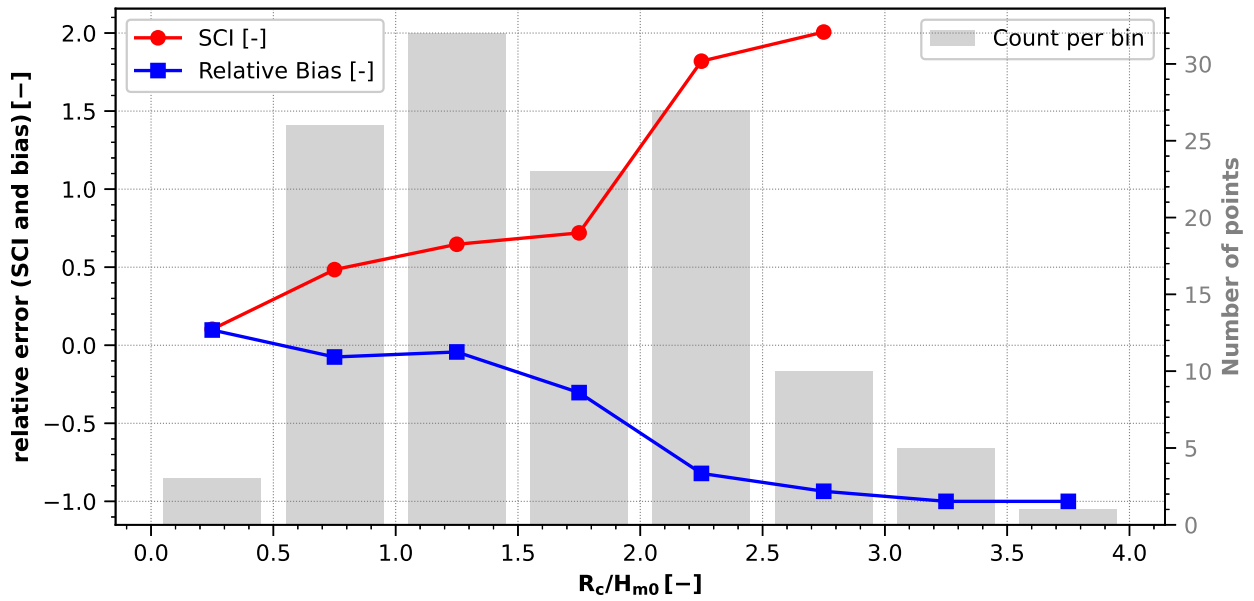


Figure 5.14: Scatter Index (SCI) and relative bias in overtopping prediction with MIKE21W as a function of R_c/H_{m0} .

Below this threshold, where overtopping discharges are less dependent on isolated events, the model shows good agreement with the experimental data, indicating that its performance is reliable once statistical effects are less dominant. Another contributing factor is that when the discharge depends on only one or two waves, these are typically the steepest and most non-linear events, which are inherently more difficult for the model to resolve.

MIKE3W

The scatter index and relative bias of the modelled overtopping discharge with MIKE3W are presented in Figure 5.15.

As for MIKE21W, the results show an increasing SCI with increasing relative freeboard. However, MIKE3W maintains a lower scatter over a wider range. For example, while MIKE21W exceeds an SCI level of 1 at approximately $R_c/H_{m0} \approx 1.7$, MIKE3W does not exceed this level until around $R_c/H_{m0} \approx 2.3$.

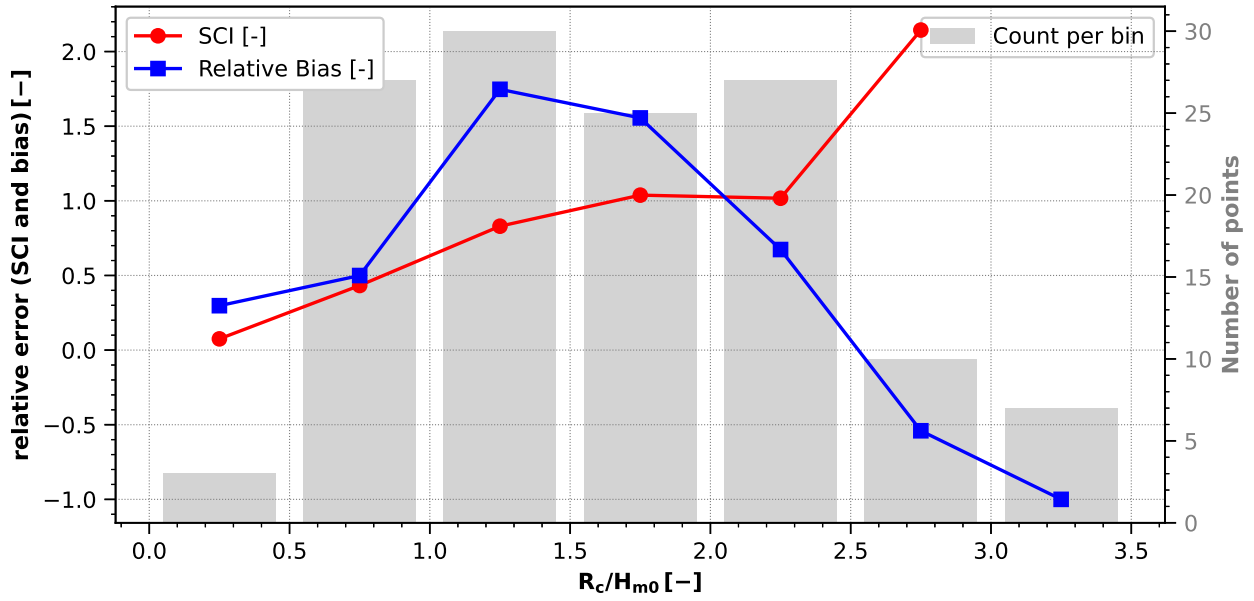


Figure 5.15: Scatter index (SCI) and relative bias in overtopping prediction with MIKE3W as a function of R_c/H_{m0} .

Another important distinction is found in the relative bias. Whereas MIKE21W shows a tendency toward negative bias (underprediction) in this range, MIKE3W exhibits a positive bias, meaning that overtopping discharges are slightly overpredicted. This behaviour results in a more conservative estimate, which is generally favourable for engineering design.

5.6.4 Influence of $\xi_{m-1,0}$

The present section investigates the sensitivity of the modelled overtopping predictions to the surf similarity parameter, $\xi_{m-1,0}$. The corresponding dependence is illustrated in Figure 5.16 and Figure 5.17, for MIKE21W and MIKE3W, respectively.

The few cases at the top of each figure correspond to the significant overpredictions of simulated discharge observed in the upper left of Figures 5.8 and 5.10. These cases are discussed in detail in Section 5.5.2, where they are attributed primarily to experimental uncertainties.

MIKE21W

Apart from the already mentioned outliers, Figure 5.16 shows very good agreement for $\xi_{m-1,0} \gtrsim 3$, while lower values show increasing underprediction and scatter. This range also contains the majority of cases, many of which still align well with the measurements. Notably, $\xi_{m-1,0} \approx 3$ marks the transition from surging to plunging breakers, as shown in Figure 2.2.

At first glance, this might suggest that the breaker parameter directly influences the underprediction of low-discharge cases. One hypothesis is that since the governing equations in MIKE21W, based on Madsen & Sørensen (1992), are only weakly nonlinear, underestimation of nonlinear effects might occur. Since wave nonlinearity, H/L , directly affects $\xi_{m-1,0}$ through $\xi_{m-1,0} = \beta/\sqrt{H_0/L_0}$, lower $\xi_{m-1,0}$ values (the range considered) correspond to higher nonlinearity. Nonlinearity affects the wave shape by crest steepening and trough flattening. As a result, the crest height, which governs overtopping discharges, could be underestimated when the nonlinearity is not fully captured.

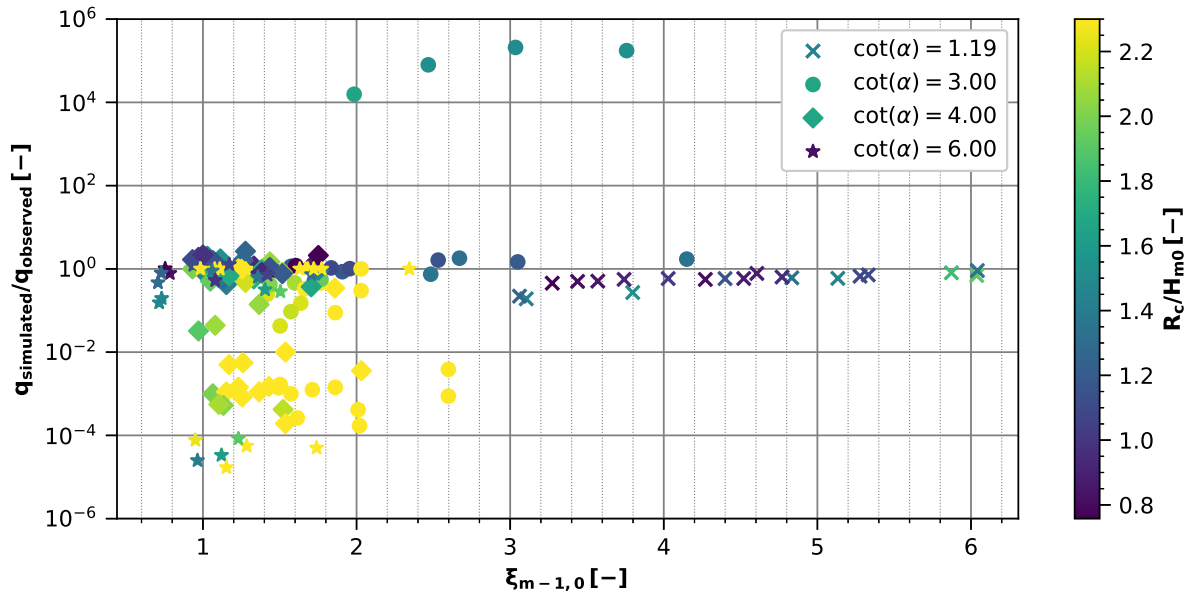


Figure 5.16: Sensitivity analysis of $\xi_{m-1,0}$ on the modelled overtopping discharge with MIKE21W.

However, comparison with MIKE3W results (Figure 5.17), which are not subject to this weakly nonlinear limitation in their governing equations, reveals a similar pattern. This indicates that the weakly non-linear formulation is not the primary cause of underprediction. Instead, the underpredicted cases with low $\xi_{m-1,0}$ typically have relatively high freeboards. Data points with similar $\xi_{m-1,0}$ but lower relative freeboards show very good agreement. As discussed previously, high relative freeboard is directly associated with low overtopping discharges. Therefore, the apparent dependence on $\xi_{m-1,0}$ largely reflects low discharge rates rather than an effect of the breaker parameter.

In summary, while low $\xi_{m-1,0}$ values appear correlated with underprediction, this is primarily due to the combination of high relative freeboard and associated low discharges. The model's performance remains robust as statistical effects become less dominant, and the breaker parameter itself appears to be a non-controlling factor in overtopping predictions.

MIKE3W

Apart from the previously noted outliers, Figure 5.17 shows that MIKE3W performs very well for $\xi_{m-1,0} \gtrsim 3$, whereas lower values are associated with increasing underprediction and scatter. This pattern mirrors the results from MIKE21W. Importantly, since MIKE3W does not share the same weakly non-linear limitations in its governing equations, the underprediction at low $\xi_{m-1,0}$ cannot be attributed to missing nonlinearity. Instead, the cases with low $\xi_{m-1,0}$ values generally coincide with relatively high freeboards, which, as previously discussed, are directly associated with low discharge rates and increased statistical variability.

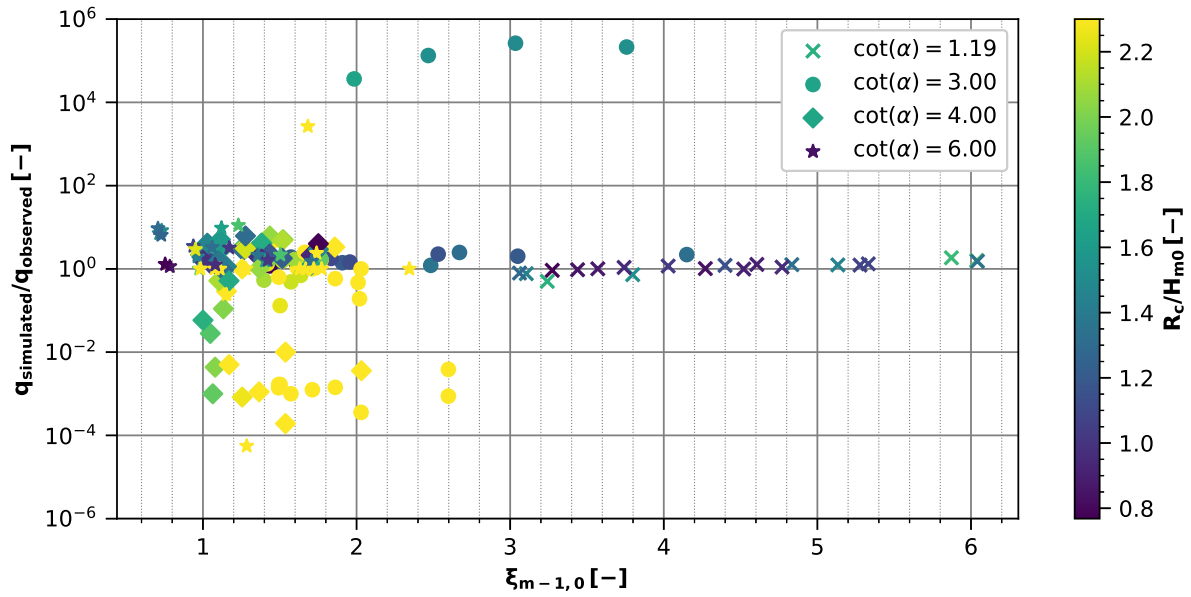


Figure 5.17: Sensitivity analysis of $\xi_{m-1,0}$ on the modelled overtopping discharge with MIKE3W.

In summary, the apparent influence of $\xi_{m-1,0}$ in MIKE3W is again interpreted as a secondary effect of low discharge cases rather than a direct dependence on the breaker parameter itself.

5.7 Assessment of model applicability for engineering design

Both MIKE3W and MIKE21W demonstrate strong capabilities in predicting overtopping, particularly in situations where the discharge is influenced by multiple overtopping waves and the relative freeboard is not excessively large. Under such conditions, the agreement with experimental results is excellent, underlining their applicability for most practical engineering cases.

At larger relative freeboards, overtopping increasingly occurs through splash and spray processes, where water passes the crest as droplets rather than as coherent wave volumes. Neither MIKE3W nor MIKE21W can explicitly resolve these processes due to their underlying formulations. Capturing such effects would likely require higher-fidelity CFD approaches specifically designed for free-surface fragmentation. This, however, is very computationally demanding, and the level of detail provided by such models is typically unnecessary for practical engineering design.

From a design perspective, it is important to note that MIKE3W generally exhibits a positive relative bias, leading to conservative predictions, while MIKE21W shows a slight negative relative bias, which may result in underprediction. This systematic difference should be taken into account when selecting the most appropriate tool for engineering assessments.

When the models are applied to zero-overtopping evaluations, as described in Section 2.2, longer simulation durations are required. This is not a shortcoming of the models themselves but rather a consequence of the stochastic nature of wave generation, where rare overtopping events demand sufficient simulation time for statistical convergence.

Another practical consideration is computational demand. For the relatively simple geometries used in this study, a typical MIKE21W simulation required about 3–4 minutes, whereas MIKE3W required approximately

15 minutes, i.e. four to five times longer. While this difference may appear modest for small domains, it becomes significant when modelling larger geometries, including effects such as wave spreading or more complex boundary conditions.

Ease of use is an important factor for practical applicability. The MIKE interface is structured such that transitioning from a two-dimensional to a three-dimensional configuration requires only minor modifications. This design facilitates the transfer of results from preliminary design phases, typically performed in MIKE21W, to more detailed analyses in MIKE3W with minimal additional effort.

Overall, both models perform very well in cases with significant overtopping discharges, making them suitable and reliable tools for engineering design. Based on SCI and relative bias, approximate applicability ranges are $R_c/H_{m0} \lesssim 1.7$ for MIKE21W and $R_c/H_{m0} \lesssim 2.3$ for MIKE3W. Beyond these thresholds, seed dependence becomes increasingly important and should be examined more carefully. It remains the user's responsibility to judge when the limitations of Boussinesq-type models are critical, such as under highly nonlinear wave conditions or steeply varying bathymetries. In many practical settings, the comparatively low computational cost of MIKE21W makes it an efficient and accurate option, especially in early design phases.

6 Conclusion

This thesis has evaluated the capability of the MIKE21W and MIKE3W numerical models in reproducing wave run-up and overtopping processes, with validation against experimental benchmark datasets. The results provide insight into the relative strengths, weaknesses, and applicability of the two modelling approaches in coastal engineering practice. The following addresses the research questions posed at the outset, followed by practical design considerations.

Under which conditions is MIKE21W suitable for accurately simulating wave run-up and overtopping, and when are more advanced models like MIKE3W required?

MIKE21W is appropriate when a computationally efficient, early-phase assessment is needed, particularly for moderate wave conditions and overtopping regimes where multiple waves contribute to the mean discharge. It captures the dominant physical mechanisms qualitatively and provides sufficiently accurate average overtopping estimates for moderate relative freeboards, making it useful for preliminary design and screening studies. MIKE21W tends to slightly underestimate run-up for non-breaking waves while showing close agreement in the plunging breaker regime. However, the model's alignment in the plunging breaker regime is likely due to suppressed breaking compensating for a general underprediction bias. Whether this agreement holds under different conditions is a key question for future work.

MIKE3W becomes necessary when higher fidelity is required across a broader range of conditions, such as in detailed final design, safety-critical assessments, or situations involving complex three-dimensional effects and rapidly varying bathymetries. The model produced robust run-up predictions for $\xi_{m-1,0} > 1.4$ and demonstrated more consistent reliability overall, making it the preferred choice when accuracy outweighs computational efficiency.

How well do the MIKE21W and MIKE3W models reproduce benchmark data from the CLASH database and empirical formulations from EurOtop (2018), in terms of predicting wave run-up and overtopping?

Both models successfully capture the main trends observed in the benchmark data, providing a solid basis for engineering use. However, systematic, model-dependent biases remain. For run-up, MIKE21W generally yielded slight underestimations. Its closer agreement in the plunging breaker regime should be interpreted cautiously, as it likely results from a compensating effect between two opposing tendencies, one leading to overprediction and the other to underprediction (as discussed previously). These tendencies may not consistently balance in other cases.

MIKE3W achieved good agreement for larger surf similarity parameters ($\xi_{m-1,0} > 1.4$) but tended to overpredict run-up at smaller $\xi_{m-1,0}$, owing to the absence of turbulence-induced dissipation in the tested configurations. Incorporation of the $k-\epsilon$ turbulence closure introduced excessive dissipation, thereby reducing accuracy. Analysis of the turbulent kinetic energy (TKE) further confirmed the well-documented RANS tendency to overproduce turbulence in breaking waves.

For overtopping, both models reproduced mean discharges well at moderate relative freeboards, showing excellent agreement with experimental data. Predictive skill declined at higher freeboards, where splash and spray processes, not explicitly resolved in both models, become increasingly important. A slight systematic bias was evident, with MIKE3W tending towards overprediction and MIKE21W towards underprediction. Differences

in predictive accuracy were also reflected in the scatter index (SCI), which exceeded unity at a relative freeboard, R_c/H_{m0} , of 1.7 for MIKE21W and 2.3 for MIKE3W.

Furthermore, cases with very low overtopping rates ($q \lesssim 10^{-3} \text{ m}^3/\text{s/m}$) proved sensitive to wave-phase randomness (random seed), requiring long-duration simulations to ensure statistical convergence. Overall, the results show good agreement with CLASH and EurOtop (2018) trends in regimes where the dominant physical processes are represented by the models, while discrepancies arise in regimes controlled by breaking turbulence, splash/spray processes, or stochastic variability.

What are the computational and practical trade-offs between MIKE21W and MIKE3W in engineering applications (setup complexity, simulation time, and accuracy)?

The trade-offs between the two models are clear and context-dependent. MIKE21W offers substantially faster runtimes: in this study, typical simulations required only $\approx 3\text{--}4$ minutes, compared with ≈ 15 minutes for MIKE3W with 10 sigma layers (a factor of 4–5). This computational efficiency makes MIKE21W attractive for parametric studies, early-phase design, or large-domain applications where many runs are needed.

By contrast, MIKE3W, although more computationally demanding, delivers higher fidelity and more reliable predictions across a broader set of conditions. The model's modest systematic overprediction can be conservative from a design safety perspective.

Practically, the GUI support for transitioning from 2D (MIKE21W) to 3D (MIKE3W) streamlines workflows; early MIKE21W results can be carried into MIKE3W for detailed follow-up with limited extra setup effort. However, the added setup complexity and run time of MIKE3W become important considerations for large and complex domains.

Practical recommendations for engineering design

- MIKE21W is recommended for preliminary design and situations where computational speed is important and the targeted wave/run-up regime is moderate.
- MIKE3W is recommended for final design, safety-critical assessments, or when three-dimensional effects, or higher-fidelity run-up/overtopping estimates are required.
- Avoid relying on the inclusion of MIKE21W's breaking treatment or on the $k\text{--}\epsilon$ closure model in MIKE3W for run-up and overtopping design without careful validation, since both can introduce excessive dissipation and reduce accuracy.
- For overtopping at very low discharges ($q \lesssim 10^{-3} \text{ m}^3/\text{s/m}$), plan longer simulations to overcome seed sensitivity and ensure statistical convergence. It should also be noted that splash and spray processes are not explicitly modelled.
- Approximate applicability thresholds are $R_c/H_{m0} \lesssim 1.7$ for MIKE21W and $R_c/H_{m0} \lesssim 2.3$ for MIKE3W, beyond which seed dependence becomes critical. Model selection, however, should not rely on a single freeboard or wave parameter. It should instead be guided by the design stage, problem complexity, available computational resources, and the required accuracy.

In summary, both MIKE21W and MIKE3W are valuable and defensible tools for coastal engineering when applied with an awareness of their respective limitations. MIKE21W offers an efficient and practical solution for early-stage assessments, while MIKE3W provides the fidelity required for detailed design and safety-critical

applications. The optimal workflow employs them complementarily, using fast two-dimensional exploration to guide targeted three-dimensional refinement where higher accuracy is needed.

6.1 Future work

The present study has established a foundation for the systematic validation of MIKE21W and MIKE3W in predicting wave run-up and overtopping, using both the CLASH database and the empirical formulations that underpin EurOtop (2018). Building on this work, several directions for future research can be identified.

A natural next step is to expand the validation effort by parametrically reproducing the full range of cases contained in the CLASH database. Automating the generation of computational meshes through a parametric mesh generator would make it possible to systematically test the models across all available structure geometries, slopes, and wave conditions. Such a framework would reduce human bias in case selection and provide a more comprehensive picture of model performance. Furthermore, coupling these systematic validation results with machine learning techniques could help to identify recurring patterns of model bias, quantify relative shortcomings, and propose correction factors or safety coefficients for engineering design.

Another promising line of investigation is the inclusion of structural porosity and permeability in the modelling framework. Many coastal defenses, such as rubble-mound breakwaters, dissipate wave energy through internal flow and turbulence within their porous cores. Accounting for this effect is essential for a more realistic representation of wave transformation and overtopping processes.

In addition, the present study has focused primarily on normally incident waves. In practice, wave obliquity and directional spreading play a significant role in governing both run-up and overtopping. Future model development and validation should therefore address multidirectional wave conditions, both to increase realism and to align more closely with design practice in coastal engineering.

Finally, as shown in Section 2.5.3, turbulence closure remains a key challenge. The standard $k-\epsilon$ model tends to overestimate turbulence in certain conditions, leading to discrepancies in overtopping predictions. Future work should explore improvement of the turbulence closure schemes. Although this extends beyond the specific scope of MIKE21W and MIKE3W, improvements in turbulence modelling would benefit a wide range of coastal and hydraulic applications where accurate overtopping prediction is critical.

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A Appendix

A.1 Convergence of solitary wave run-up

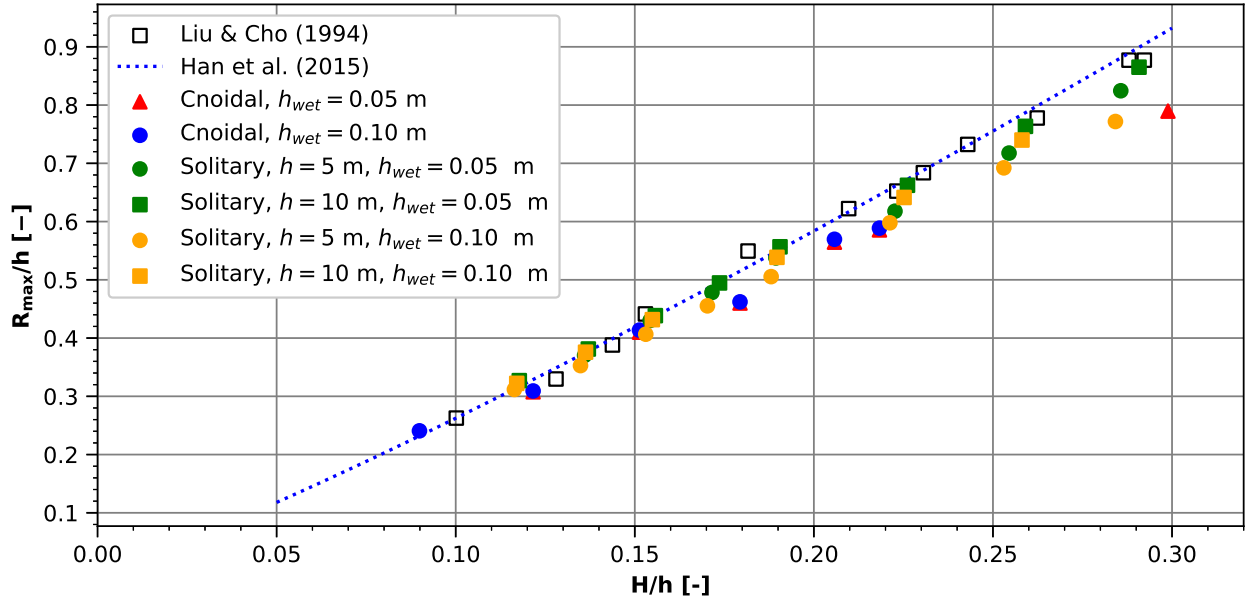


Figure A.1: 20° Slope. Sensitivity analysis to h , h_{wet} and theory applied.

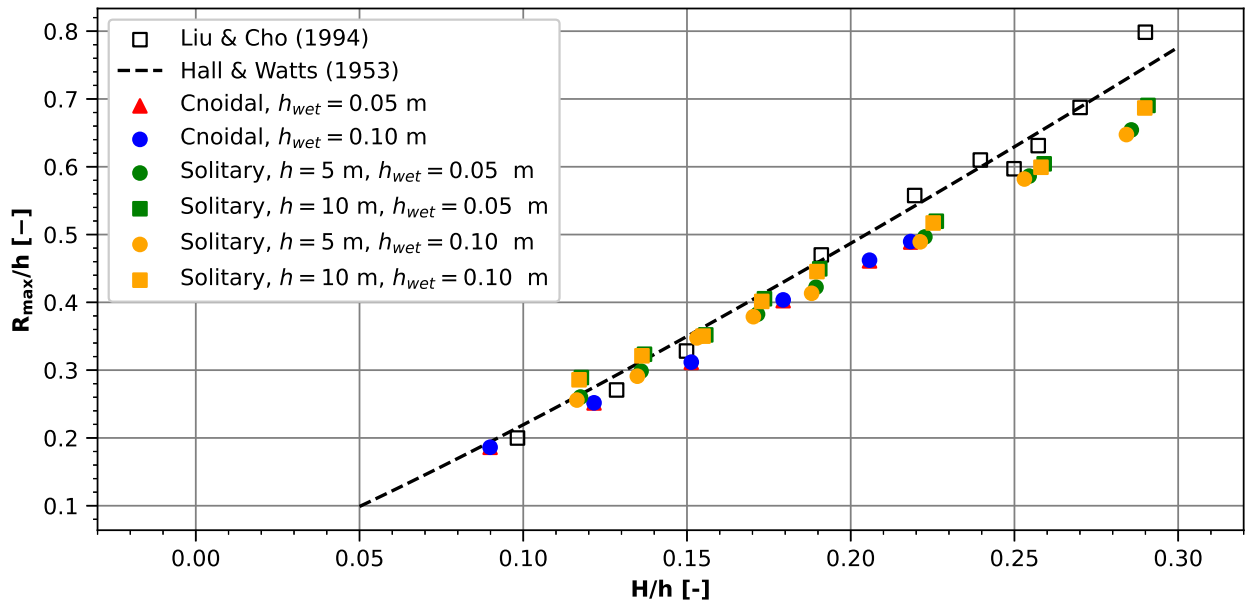


Figure A.2: 45° Slope. Sensitivity analysis to h , h_{wet} and theory applied.

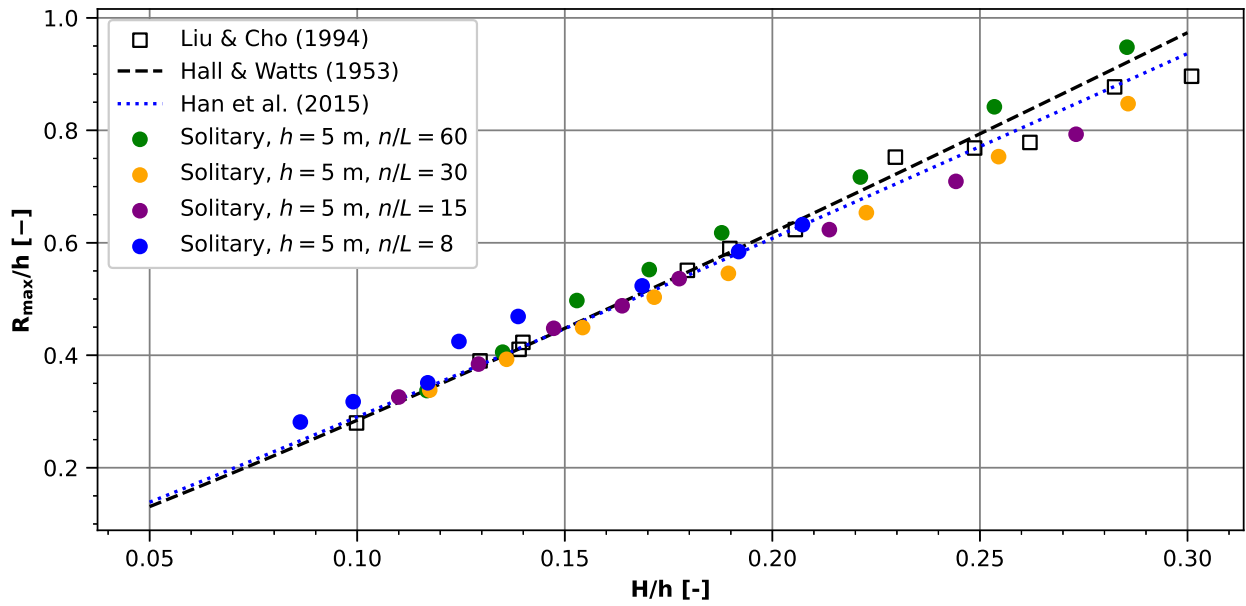


Figure A.3: Convergence of the cell size dx for the run-up of a solitary wave perpendicularly encountering a sloped beach of 15°

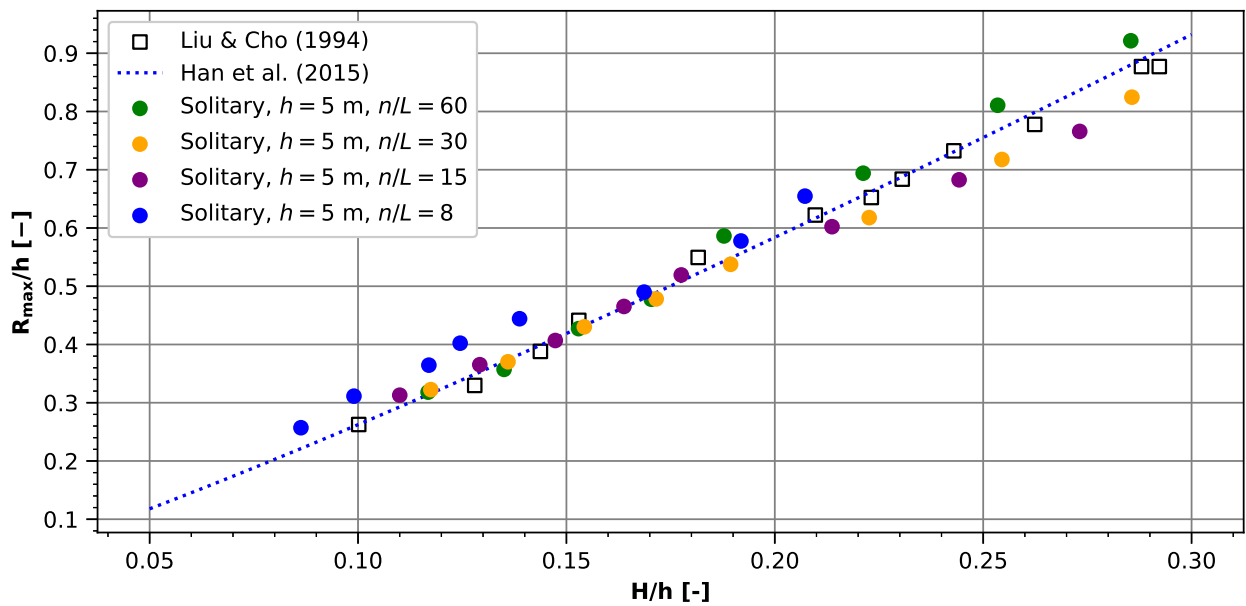


Figure A.4: Convergence of the cell size dx for the run-up of a solitary wave perpendicularly encountering a sloped beach of 20°

A.2 Convergence of MIKE21W to MIKE3W validation report

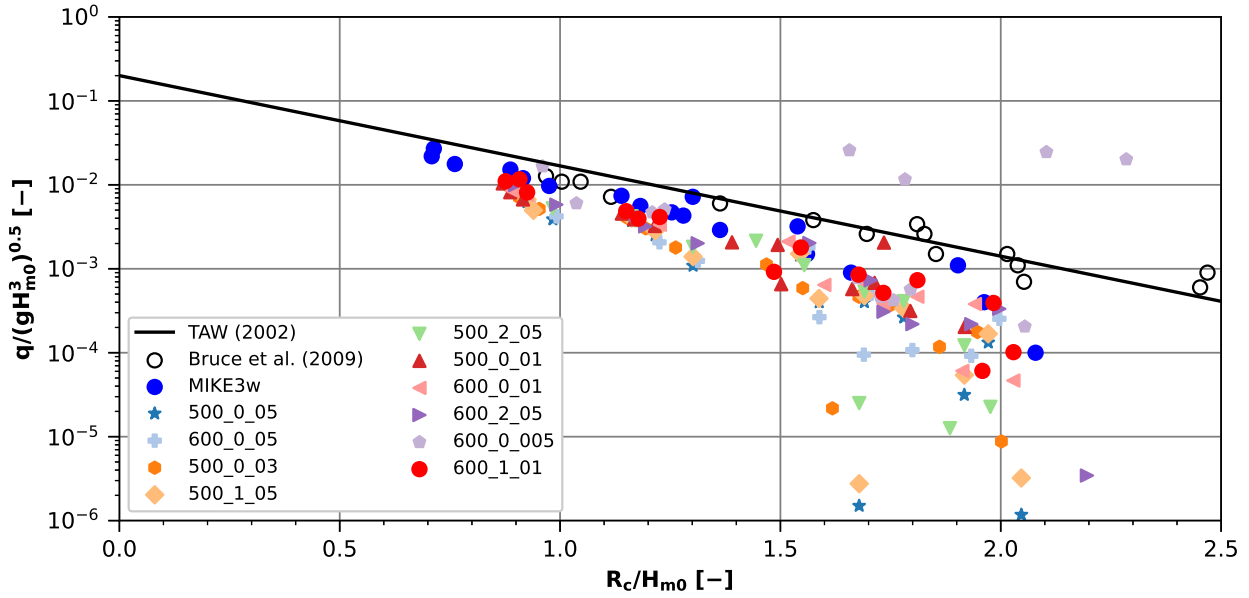


Figure A.5: Convergence of the mean overtopping discharge for all the tested cases of Table 5.2 to MIKE 3 Wave Model FM - Validation report (2025) validation report, with experimental data from Bruce et al. (2009) and the empirical formulation from TAW (2002).

A.3 $k - \epsilon$ model: results of other test cases

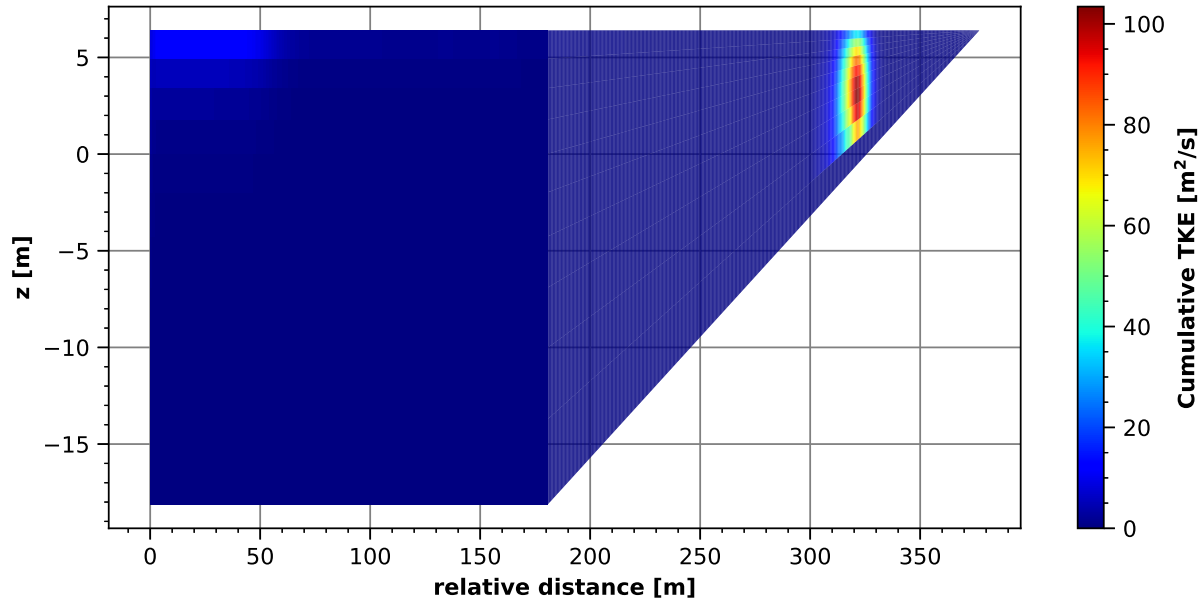
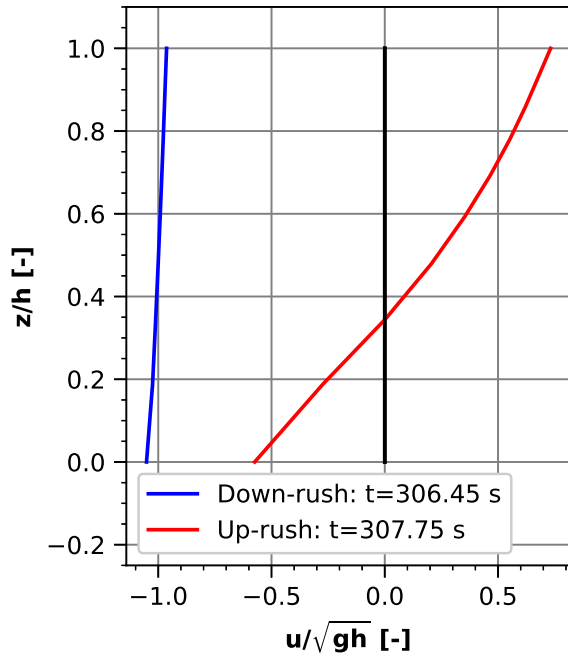
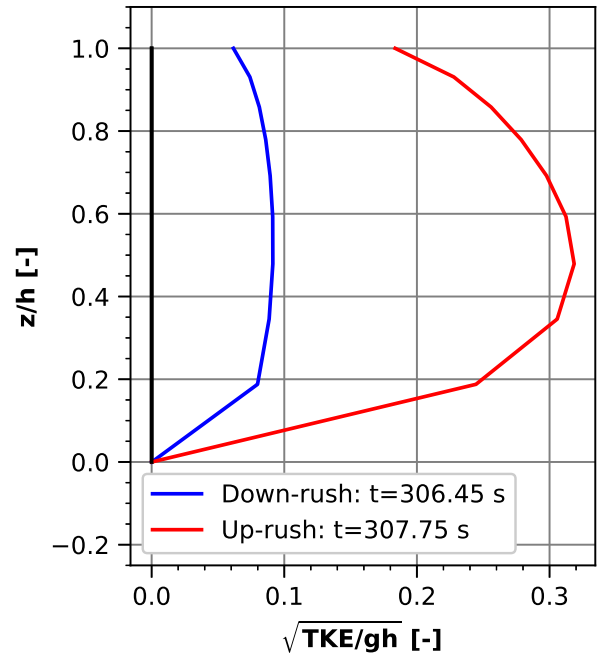


Figure A.6: Cumulative TKE over the entire simulation duration, for a plunging wave having $T_p = 4.82$ s, and $\xi_{m-1,0} = 0.63$.



(a) $\cot \alpha = 6$, $T_p = 4.82$ s, and $\xi_{m-1,0} = 0.63$.



(b) $\cot \alpha = 6$, $T_p = 4.82$ s, and $\xi_{m-1,0} = 0.63$.

Figure A.7: Velocity and TKE profiles during the uprush and downrush at the SWL intersection.

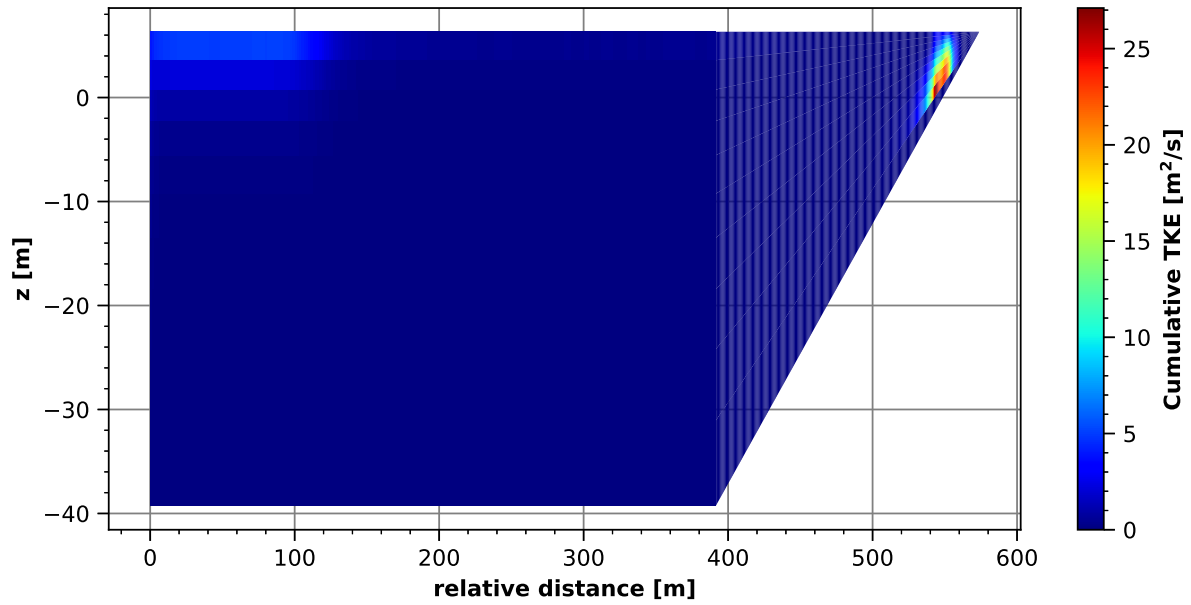
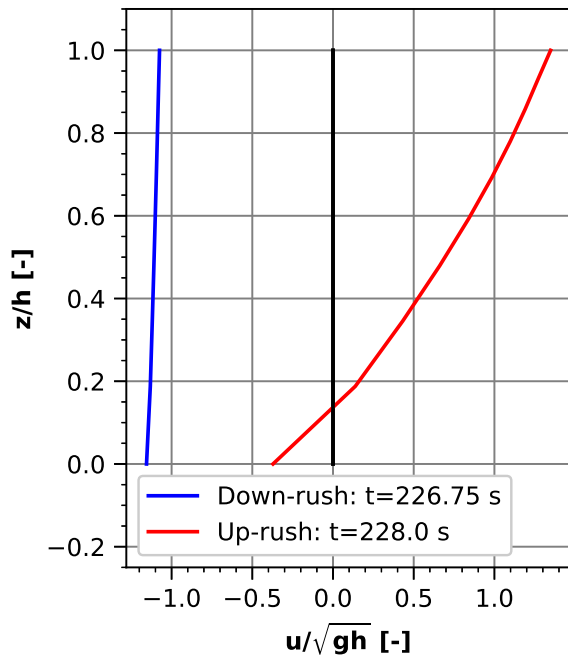
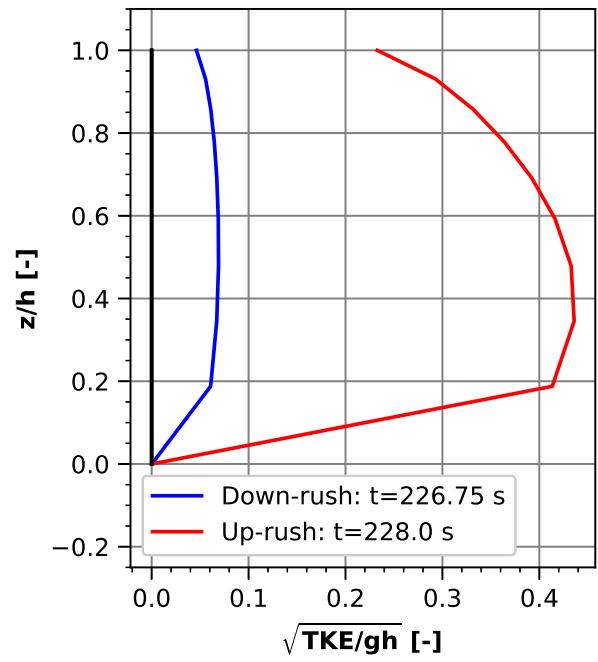


Figure A.8: Cumulative TKE over the entire simulation duration, for a plunging wave having $T_p = 7.09$ s, and $\xi_{m-1,0} = 0.91$.



(a) $\cot \alpha = 6$, $T_p = 7.09$ s, and $\xi_{m-1,0} = 0.91$.



(b) $\cot \alpha = 6$, $T_p = 7.09$ s, and $\xi_{m-1,0} = 0.91$.

Figure A.9: Velocity and TKE profiles during the uprush and downrush at the SWL intersection.

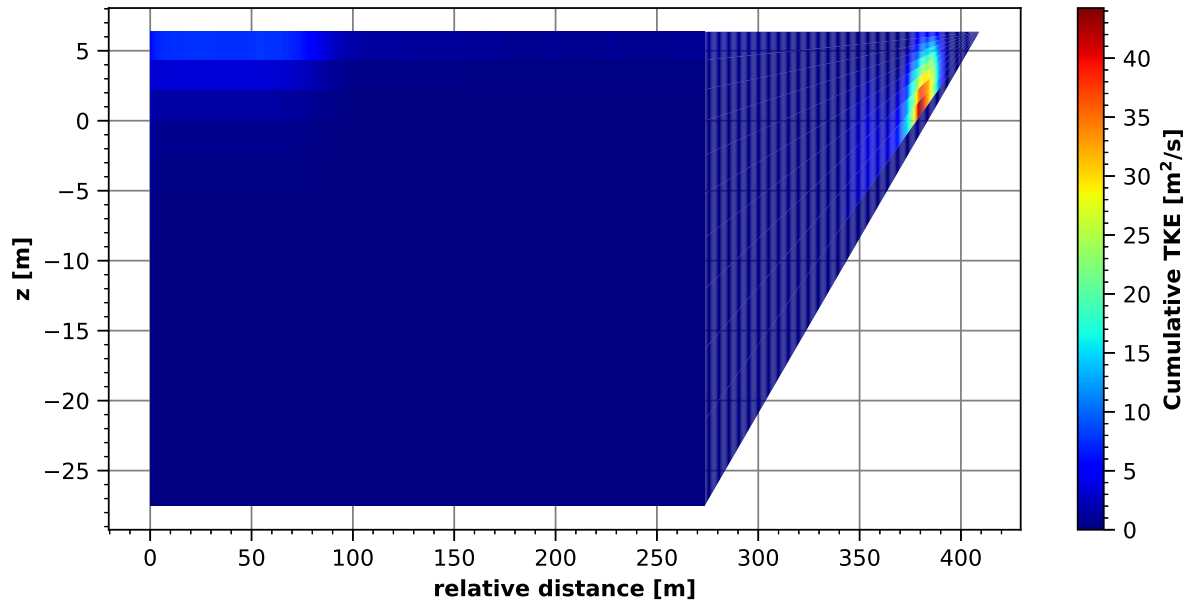
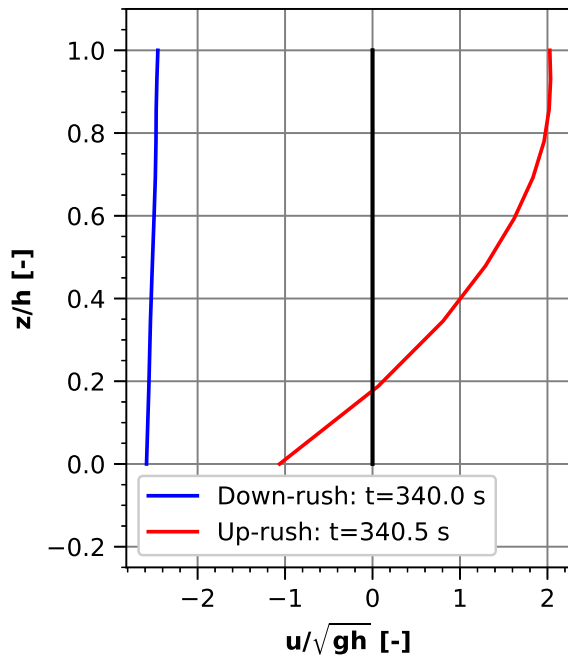
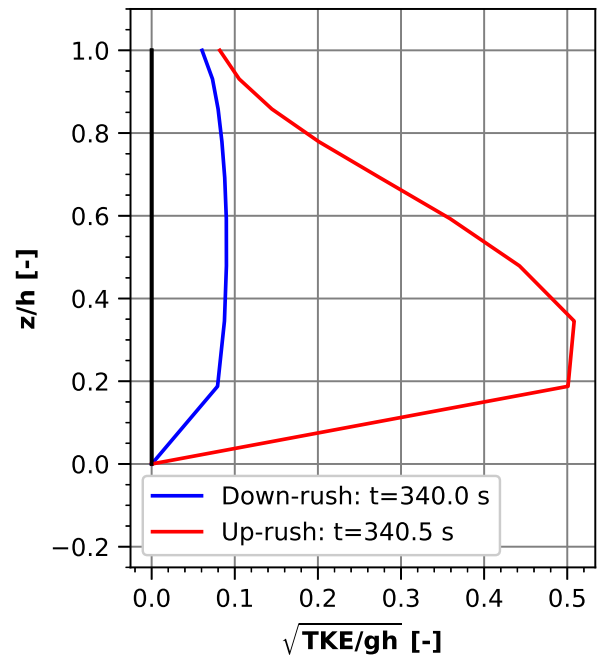


Figure A.10: Cumulative TKE over the entire simulation duration, for a plunging wave having $T_p = 5.94$ s, and $\xi_{m-1,0} = 1.51$.



(a) $\cot \alpha = 6$, $T_p = 5.94$ s, and $\xi_{m-1,0} = 1.51$.



(b) $\cot \alpha = 6$, $T_p = 5.94$ s, and $\xi_{m-1,0} = 1.51$.

Figure A.11: Velocity and TKE profiles during the uprush and downrush at the SWL intersection.

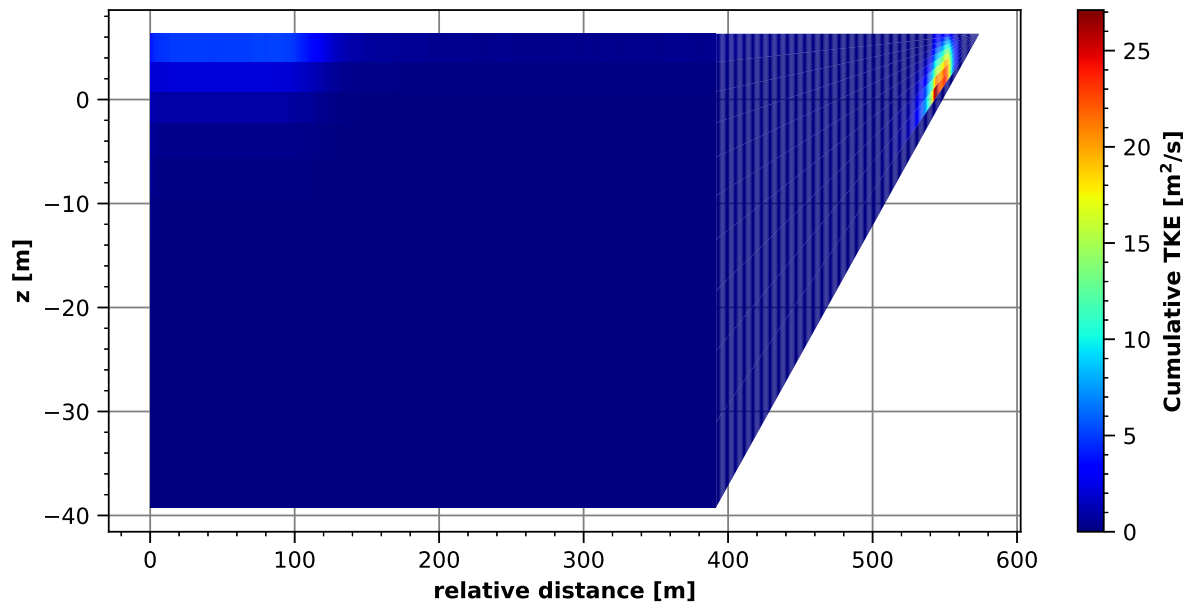
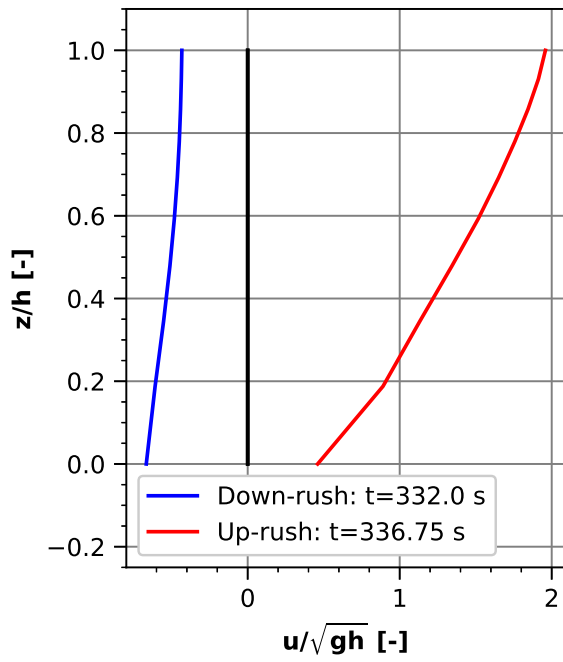
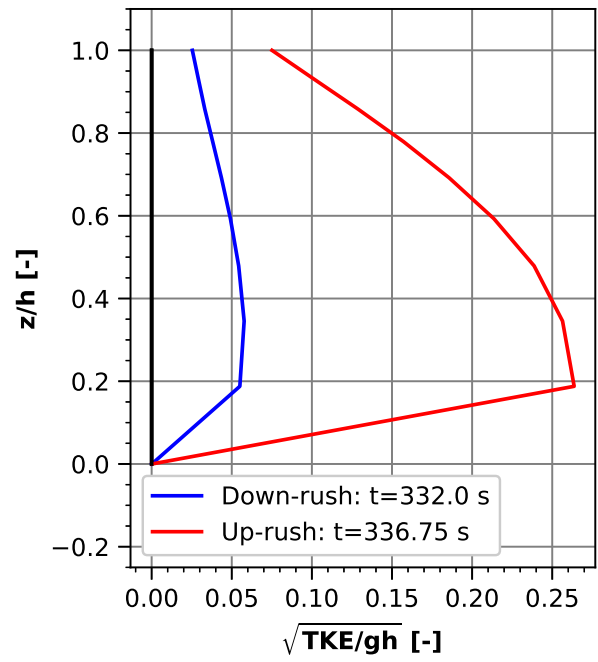


Figure A.12: Cumulative TKE over the entire simulation duration, for a plunging wave having $T_p = 7.09$ s, and $\xi_{m-1,0} = 1.81$.



(a) $\cot \alpha = 6$, $T_p = 7.09$ s, and $\xi_{m-1,0} = 1.81$.



(b) $\cot \alpha = 6$, $T_p = 7.09$ s, and $\xi_{m-1,0} = 1.81$.

Figure A.13: Velocity and TKE profiles during the uprush and downrush at the SWL intersection.

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