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"EVALUATING MARKET TIMING OCCURRENCE WITH QUANTILE REGRESSION. MODEL DESIGN AND APPLICATION TO U.S. MUTUAL FUNDS"

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Firma dello studente

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SUMMARY

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INTRODUCTION

In finance, the performance of the investment manager is a really discussed topic, indeed for many years this argument has been the content of several studies and discussions.

The main goal of the portfolio manager is to obtain higher returns in relation to the stock market, but how can the manager outperform it? Can he predict the market changes and gain higher returns? In this thesis we try to answer to these questions studying a particular phenomenon called market timing.

Market timing is a superior skill that the managers might have and it consists in forecasting the market changes, and, use these forecasts to gain an excess return modifying the portfolio composition accordingly to those predictions. In particular in our empirical analysis we have analyzed 23 mutual funds trying to verify whether the managers of these funds have market timing skills or not, that is if they are able to predict exactly the market changes and consequently exploiting those forecasts to obtain higher returns.

In order to spot the actual presence of market timing we have applied for each fund two of the most used models which study this phenomenon: the Henriksson-Merton (1981) model and the Treynor-Mazuy (1996) procedure. Applying these models, two regressions have been built: a simple one referred to the Henriksson and Merton procedure and a quadratic one referred to the Traynor-Mazuy model.

For each procedure we have estimated the regressions in the first place with the Ordinary Least Squares estimator and in a second moment the regression results have been calculated with the usage of quantile regression. We have decided to implement the QR too because this regression is able to explore the relationship between the independent variables and the conditional quantiles of the dependent one (not only the conditional mean as in the OLS) allowing to gain further insight and to obtain a much more complete statistical picture than the Ordinary Least Squares. Therefore in our study we are interested in analyzing the market timing phenomenon much more in detail exploring how it is changing throughout the quantiles (from the lower to the upper ones) and not just on the conditional mean. The goal of our investigation is to determine whether or not, with both OLS and QR applied to the H-M and T-M models, the managers of the fund have the ability to predict the market changes and consequently modify the portfolio composition accordingly to those forecasting in order to outperform the market gaining a higher return.

In the first chapter we introduce quantile regression explaining its features and properties. In particular focusing our attention on the differences between the QR and the Ordinary Least Squares estimator, showing how the usage of quantile regression could be really useful to analyze the market timing phenomenon and how it is possible to have a much more complete statistical picture of the phenomenon through QR than OLS.

Furthermore it is shown how quantile regression can overcome some problems that might occur with the Ordinary Least Squares.

In the second chapter we introduce the market timing and style analysis concepts reviewing the literature pertaining to our study. Furthermore we describe in details the Henriksson-Merton model showing how this procedure is built and highlighting how this model can test whether or not the managers have market timing abilities.

In the last part of the chapter we describe the Treynor-Mazuy models showing how the market timing is tested in this procedure. The authors, in their paper, did not present a specific regression equation (which, anyway, can be derived from their explanations), indeed they decided to explain the market timing phenomenon and their test through some graphical representations. Anyway in the chapter both regression equation and graphical representations will be shown.

In the third chapter we introduce the empirical data: twenty-three U.S. mutual fund and the Standard & Poor's 500 which is our benchmark in the empirical analysis. The sample period goes from 6 January 2006 to 27 December 0f 2019.

In the first part of the chapter we describe the U.S. mutual funds (which all belong to the "Large Blend Funds" category of Morningstar) highlighting their investment policy and the sectors in which they invest the most. While in the second part we calculate the descriptive statistics of our data.

In the fourth chapter we explain the empirical analysis that we have done. In particular the chapter can be dived in four sections. In the first one we study the market timing phenomenon by estimating the Henriksson and Merton model through the usage of Ordinary Least Squares estimator, trying to spot whether or not the managers of the fund were able to predict the market and therefore to increase the fund's return.

The second section is characterized by the usage of the Treynor and Mazuy model, in which the quadratic regression is estimated through the OLS. Of course also in this case our goal is to find some evidence of the market timing phenomenon, analyzing the managers skills.

In the third section we apply to the Henriksson and Merton model the quantile regression in order to study the manager's prediction skills not only on the conditional mean but throughout all the quantiles considered (in our case 19 quantiles).
In the fourth part of the chapter we have applied also to the Treynor and Mazuy model the quantile regression. As in the previous paragraph we tried to study the market timing phenomenon in all the quantiles considered.
In the last paragraph we spot a problem in our sample: the data are influenced by heteroskedasticity and their volatility changes over time, furthermore the variance of the funds is really related to the market one, and all of these conditions can have a negative impact on our results (they could be not reliable). In order to overcome this problem we have applied a GARCH model calculating the "new" returns and then applying the quantile regression on them in order to search for market timing skills in the "purified" data.

In the fifth chapter we summarized the empirical results of the thesis trying to give a final answer to the questions "do the managers have market timing abilities ? Can they actually predict the market changes and gain higher returns ?.

1 CHAPTER - QUANTILE REGRESSION

1.1 INTRODUCTION TO QUANTILE REGRESSION

Quantile regression was introduced in order to offer “the opportunity for a more complete view of the statistical landscape and the relationships among stochastic variables”. (Koenker (2005)).

Indeed, the standard linear regression has a limit: it just analyzes the relationship between a set of independent variables and the conditional mean of a dependent variable Y.
Since the linear regression curve gives just “a grand summary for the averages of the distributions corresponding to the set of xs” (Mosteller and Tukey (1997)), other tools as histograms, kurtosis, boxplots, etc. are usually applied to statistical analysis in order to gain further insight.
It’s quite clear that the linear regression is able to offer just an incomplete picture for statistical analysis, exploring, only the mean of the distributions.

A new statistic technique was proposed by Koenker in order to overcome this problem: the Quantile Regression (QR). This tool manages to model the relationship between the independent variables and the conditional quantiles of the dependent one, indeed it allows to estimate the entire conditional distribution of a outcome variable.
In other words quantile regression is a statistic tool able to complete the linear regression picture gaining further insight.

1.2 ORDINARY LEAST SQUARES VS QUANTILE REGRESSION

Linear regression is one of the most used models in applied statistic, its purpose, as already pointed, is to explore the relationship between a response variable Y and one or more explanatory variables X_i. In particular it studies the conditional mean function: the function that analyzes how the mean of the dependent variable changes with the covariates.
The relationship between the variables can be studied applying the so called estimating method: Ordinary Least Squares (OLS).
The OLS estimates the unknown parameters in a linear regression by minimizing the sum of the square of the differences between the observed values Y_i and the predicted ones.

This estimating method is considered the most suitable one to explore linear models because as long as the Gauss-Markov theorem's assumptions are satisfied, the Ordinary Least Squares is the Best Linear Unbiased Estimator (BLUE).
The OLS assumption are specified below. Notice that ε is the error term. In particularly:

- The explanatory variable X_i is non-stochastic
- The expectations of the error term have to be zero in order to get an unbiased estimator: $E[\varepsilon_i] = 0$
- Homoscedasticity condition. The variance of the error terms is constant: $Var[\varepsilon_i] = \sigma^2$
- No autocorrelation, the error terms are independent and identically distributed: $Cov[\varepsilon_i, \varepsilon_j] = 0$ with $i \neq j$

Whenever one of these assumptions are violated the Ordinary Least Squares estimates can be misleading and the OLS cannot be considered anymore the best, linear, unbiased estimator. It should be noted that the Gauss-Markov's assumptions are quite strict, therefore this makes the OLS not a very flexible estimating method.
Quantile regression on the contrary is significantly more flexible and offers a clearer picture of the relationship between the variables than the Ordinary Least Squares.
It is straightforward that QR and OLS have different features, in the section below this differences are briefly analyzed.

For instance, one of the problems encountered with the usage of OLS is related to the homoscedasticity assumption. Indeed, it is possible that in a data set the variance of the error terms is not constant in all the distribution, therefore the homoscedasticity assumptions is violated. In this case the Ordinary Least Squares' results are not reliable and appropriate anymore. In order to overcome the problem it is possible to apply the quantile regression method in that the QR is able to provide reliable estimates also in presence of heteroscedasticity.

The sensitivity of the OLS to extreme outliers is another issue that can be solved by applying the quantile regression.
The outliers can distort significantly the Ordinary Least Squares' results, misleading which is the real relationship between the covariates and the dependent variable. Quantile regression, on the other hand, is more robust to extreme outliers and can offer good estimates.

Another issue that arises with the OLS is that it explores just the impact of a covariate on the conditional mean, providing a partial view of the data.
The QR explores the relationship between the dependent variable and the covariates on the entire distribution, at any quantile of the conditional distribution, giving a more complete picture of the data set.

The differences which have been mentioned above, between the Ordinary Least Squares and the QR, will be analyzed more thoroughly at the end of this chapter.

1.3 ESTIMATION OF QUANTILE REGRESSION

In this paragraph it will be explained from a statistical and mathematical point of view the estimating methods of conditional quantile functions.
Consider X as any real-valued random variable, characterized by its distribution function:

$$F(x) = P(X \leq x)$$

The τth quantile of X is defined as: $F^{-1}(\tau) = inf\ \{x : F(x) \geq \tau\}$ for any $0 < \tau < 1$
Note that the median, $F^{-1}(1/2)$ plays an important role.

Koenker and Basset (1978) have the credit of creating an innovative method for the calculation of quantiles. Their intuition was to consider the Quantile regression as an extension of the Ordinary Least Squares, using it as a template for estimating the quantiles. In particular they extended the Ordinary Least Squares estimation of the conditional mean to the estimation of the conditional quantile functions.
With their intuition Koenker and Basset were able to calculate quantiles using a simple optimization problem and the least squares becomes a guide for this development.
As Koenker and Basset (1978) stated, what makes this intuition really important "is the fact that we have expressed the problem of finding the τth sample quantile, a problem that might seem inherently tied to notion of an ordering of the sample observations, as the solution to a simple optimization problem" . In other words they were able to find a simple way to estimate the quantiles, replacing sorting by optimizing.

Moreover, in support to the above theory there are the statements expressed by Hallock and Koenker (2001): "just as we can define the sample mean as the solution to the problem of minimizing a sum of squared residuals, we can define the median as the solution to the problem of minimizing a sum of absolute residuals". Thus, it is more than clear that the estimation of the quantiles derives from a manageable optimization problem.

As we have already asserted, the symmetry of the absolute value yields the median, therefore similarly, minimizing the sum of asymmetrically weighted absolute residual it is possible to get the quantiles.

Thus given a random sample $\{y_1, y_2 \dots . y_n\}$ we can write a minimization problem.

$$min_{\xi \epsilon \mathbb{R}} \sum_{i=1}^{n} \rho_\tau (y_i - \xi)$$

Where the function $\rho_\tau(\cdot)$ is illustrated in the figure 1.

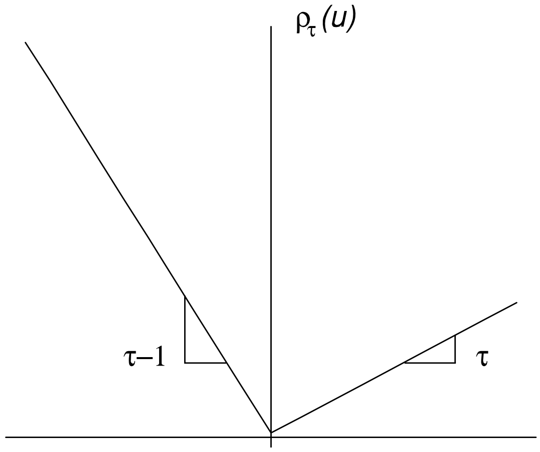


Figure 1: Quantile regression p function

Solving the minimization problem above we can get the τth sample quantile.

After defining quantiles as a minimization problem the question that needs to be answered is: how can the conditional quantile function be estimated? Ordinary Least Squares offers a template for this development.

As we know by solving $min_{\mu \epsilon \mathbb{R}} \sum_{i=1}^{n} (y_i - \mu)^2$ we get the sample mean. Similarly, replacing μ by $\mu(x) = x'\beta$ we obtain an estimate of the conditional expectation function $E(Y| X = x) = x'\beta$ solving:

$$min_{\beta \epsilon \mathbb{R}} \sum_{i=1}^{n} (y_i - x'\beta)^2$$

In quantile regression we can follow the same process. Since the τth quantile $\hat{\alpha}(\tau)$ solves

$$min_{\alpha \epsilon \mathbb{R}} \sum_{i=1}^{n} \rho_\tau (y_i - \alpha)$$

The τth conditional function $Q_y(\tau|X = x) = x'\beta(\tau)$ can be estimate through the following minimization problem:

$$min_{\beta \epsilon \mathbb{R}} \sum_{i=1}^{n} \rho_\tau(y_i - x'\beta)$$

The above expression (linear function of parameters) can be easily solved by linear programming methods.

1.4 QUANTILE REGRESSION PROPERTIES

Quantile regression has some crucial properties (called equivariance properties) which makes it a more useful method than the Ordinary Least Square.
In order to understand the idea behind these properties it could be useful to present a brief example. Suppose that, using a statistical model, we are analyzing a particular liquid's temperature and that we decide to switch from Centigrade to Fahrenheit that is changing the scale of the measurement. How is going to change the interpretation of the results? They will be invariant. Indeed, whenever the data are changed in a completely predictable way the interpretation of the estimates does not change. This type of property can be grouped together whit others, under the name of "equivariance properties".

Let's know analyze the equivariance properties of quantile regression.
Defining a τth regression quantile as $\hat{\beta}(\tau; y, X)$ based on observations (y, X). It is possible to detect four equivariance properties.
Let A be any $p \times p$ nonsingular matrix, $\gamma \epsilon \mathbb{R}^p$, and $\alpha > 0$. For any $\tau \epsilon [0, 1]$

- Scale equivariance:
  $\hat{\beta}(\tau; \alpha y, X) = \alpha\hat{\beta}(\tau; y, X)$
  $\hat{\beta}(\tau; -\alpha y, X) = -\alpha\hat{\beta}(1 - \tau; y, X)$

- Shift equivariance or regression equivariance:
  $\hat{\beta}(\tau; y + X\gamma, X) = \hat{\beta}(\tau; y, X) + \gamma$

- Equivariance to reparameterization of design:
  $\hat{\beta}(\tau; y, XA) = A^{-1}\hat{\beta}(\tau; y, X)$

Moreover, quantile regression owns another important equivariance property: the equivariance to monotone transformations. This property is much more powerful than those presented above. Let Y be any random variable and $h(\cdot)$ a non decreasing function on $\mathbb{R}$. Then we have

$$Q_{h(Y)}(\tau) = h\big(Q_Y(\tau)\big)$$

The above formula suggests that the quantiles of the transformed random variable *h(Y)* are the transformed quantiles of the original Y.

It is important to highlight that the mean does not own this property, indeed: $Eh(Y) \neq h\big(E(Y)\big)$

1.5 DIAGNOSTIC TESTS FOR QUANTILE REGRESSION

1.5.1 The Wald Test

The traditional methodology of linear regression supposes that the coefficients of distinct quantiles have the same slope throughout the entire distribution: geometrically it means that the conditional quantile functions are parallel to each other.
On the other hand, in Quantile regression this do not happen, indeed usually the slopes' parameters differ across the quantiles. The main difference between the linear and the quantile regression is that in the first the slopes coefficient are the same across the quantile and in the latter they differ.

In order to assert whether in a model it is necessary to apply quantile regression or not, it has to be tested the equality of slopes across the quantiles. The Wald test is suitable for doing this. In order to get the main idea behind this test it is useful to present a simple example.

Let $Y_i = \alpha_1 + \alpha_2 x_i + u_i$ be the two-sample model

$x_i = 0$ for n_1 observations in the first sample.
$x_i = 1$ for n_2 observations in the second sample.

The Wald test is going to test the equality between the interquantile ranges of the two samples. The null hypothesis is:

$$\begin{aligned}\alpha_2(\tau_2) - \alpha_2(\tau_1) &= \big(Q_2(\tau_2) - Q_1(\tau_2)\big) - \big(Q_2(\tau_1) - Q_1(\tau_1)\big) \\ &= \big(Q_2(\tau_2) - Q_2(\tau_1)\big) - \big(Q_1(\tau_2) - Q_1(\tau_1)\big) \\ &= 0\end{aligned}$$

Here it is tested if the parameter α_2 across quantiles τ_1 e τ_2 is significantly different.
The test can be written as:

$$T_n = \big(\widehat{\alpha_2}(\tau_2) - \widehat{\alpha_2}(\tau_1)\big)/\sigma^2(\tau_1, \tau_2)$$

The above example express quite good the main idea behind the Wald test, let's now express it with more general hypothesis.

Let $\zeta = (\beta(\tau_1)', \ldots., \beta(\tau_m)')'$ be a vector and the null hypothesis equal to: H_0: $R\zeta = r$ where q is the rank of R.

And the statistic test : $T_n = n\left(R\hat{\zeta} - r\right)'[RV^{-1}R']^{-1}\left(R\hat{\zeta} - r\right)$
Where V_n is a $mp \times mp$ matrix.
Notice that T_n is asymptotically χ^2_qunder the null hypothesis.

The Wald test is really powerful because, unlike the OLS' heteroscedasticity test , it is robust to outlying observation and moreover it is able to test various coefficients across several quantiles.
Furthermore this test is considered such a useful tool because is able to assert if it is necessary to implement quantile regression or not. If the coefficients' slope are equal across all the quantiles is not make sense to apply QR, indeed Ordinary Least Squares will be used.

1.5.2 The symmetry test

Another important diagnostic test was introduced by Newey and Powell (1987): the symmetry test.
Newey and Powell suggested an estimator called "Asymmetric Least Squares" analogue of regression quantile estimation, that is able to give information about symmetry of the conditional distribution of y_i given x_i. (Newey and Powell (1987)).

In the symmetry test the null hypothesis checks if the distribution is symmetric, in particular if the sets of coefficients for symmetric quantiles around the median will equal the value of the coefficients at the median. Thus the Newey and Powell test can be formulated as:

$$H_0: [\beta(\tau) + \beta(1 - \tau)/2] = \beta(1/2)$$

Considering more general hypothesis the null hypothesis can be written as:

$$H_0 = H\xi = h$$

The statistic test (for general hypothesis) can be written as:

$$T = n\left(H\hat{\xi} - h\right)'\left[H\widehat{W^{-1}}\hat{V}\widehat{W^{-1}}H'\right]^{-1}\left(H\hat{\xi} - h\right)$$

1.5.3 The PseudoR^2

Quantile regression, similarly to the Ordinary Least Squares, can be tested for the goodness of fit through the so called pseudoR^2 (or half R^2). This index follows the same general idea that leads to the typical R^2 in the classical least squares regression, indeed the latter could be used as a template for the calculation of the former.
In order to formulate the psuedoR^2 index it is useful to recall how the typical R^2 is expressed:

$$R^2 = 1 - \frac{RSS}{TSS}$$

Where RSS is the residual sum of squares and TSS corresponds to the total sum of squares.

For the half R^2 must be taken into account that the quantile regression is different from the OLS, indeed the latter is based on unweighted sum of squares, on the contrary the first one is built on the absolute weighted sum minimization.
Therefore, in the QR the RSS corresponds to RASW$_\theta$, that is the "residual absolute sum of weighted differences between the observed dependent variable and the estimated quantile conditional distribution" (Koenker (2005)), for each quantile θ. On the other hand, TSS in the quantile regression is expressed as TASW$_\theta$: " the total absolute sum of weighted differences between the observed dependent variable and the estimated quantile" (Koenker(2005)), for any quantile θ.
Let consider the following regression model:

$$Q_\theta(\hat{y}|x) = \widehat{\beta_0}(\theta) + \widehat{\beta_1}(\theta)x$$

Therefore RASW$_\theta$ and TASW$_\theta$ can be formulated as follows:

$$RASW_\theta = \sum_{y_i \geq \widehat{\beta_0}(\theta) + \widehat{\beta_1}(\theta)x_i} \theta\left|y_i - \widehat{\beta_0}(\theta) - \widehat{\beta_1}(\theta)x_i\right| + \sum_{y_i < \widehat{\beta_0}(\theta) + \widehat{\beta_1}(\theta)x_i} (1-\theta)\left|y_i - \widehat{\beta_0}(\theta) - \widehat{\beta_1}(\theta)x_i\right|$$

$$TASW_\theta = \sum_{y_i \geq \theta} \theta |y_i - \hat{\theta}| + \sum_{y_i < \theta} (1-\theta)|y_i - \hat{\theta}|$$

Accordingly with the above formulas the pseudoR^2 can be written as:

$$Pseudo\ R^2_\theta = 1 - \frac{RASW_\theta}{TASW_\theta}$$

The half R^2 is an index that measures how good the regression fits with the data, in particular it is an indicator of goodness of fit of the considered model. It is important to highlight that the pseudoR^2 does not provide information for the whole regression model. Indeed the index is associated to a specific quantile and for each given quantile it will be calculated at local level, illustrating if the considered quantile is affected by the covariates.

1.6 THE QUANTILE CROSSING

Through the Quantile regression it is possible to estimate, independently, multiple conditional quantile functions.

Hypothetically the estimated quantile curves should not cross each other but when this occurs the rule which asserts that the distribution function and its related inverse function has to be monotone increasing will be violated, causing the so called quantile crossing.

In other words having a quantile crossing problem it means that, for example, a given certain point $(x_0; y_0)$ might be located above the 40th and low the 30^{th} percentile, which is clearly impossible.

Should be noted that if there are several observation points which violate the rule mentioned above, the covariates effects can be considered misleading and erroneous.

A possible solution to the quantile crossing is to enforce monotonicity in a stronger way across the quantile functions. Indeed, He (1997) suggested a model based on this idea. In particular he considered the location-scale shift model:

$Y_i = x_i'\beta + (\gamma x_i')u_i$ where u_i is iid.

He proposed to estimate the model in three different steps:

1. a median regression of y_i on $x_i$to obtain $\hat{\beta}$ and associated residuals $\hat{u_i} = y_i - x_i'\hat{\beta}$
2. a median regression of $|\hat{u}_i|$ on $x_i$to obtain $\hat{\gamma}$ and associated fitted values, $s_i = x_i'\hat{\gamma}$
3. a bivariate quantile regression of $\hat{u}_i$ on s_i constrained through the origin to determine coefficients $\hat{\alpha}(\tau)$

From the previous steps we get an estimation of the conditional quantile functions that is for sure monotone in τ at all x since $\hat{\alpha}$ is monotone. We can formulate the conditional quantile function as:

$$\widehat{Q_Y}(\tau|x) = x'\left(\hat{\beta} + \hat{\alpha}(\tau)\hat{\gamma}\right)$$

considering that s_i are nonnegative.

It is important to highlight that the s_is might be negative, therefore imposing a constraint to step 2. to obtain nonnegative estimations could be a possible solution to this problem. However, He suggests on the contrary to use the unconstrained approach because it might be used as a diagnostic test for the misspecification of the model.

1.7 DIFFERENCES BETWEEN OLS AND QUANTILE REGRESSION

At the beginning of the chapter the main differences between OLS and Quantile regression have been briefly mentioned, let's analyze them now more in detail.

The Ordinary Least Squares, as has been widely stated, explores just the impact of a set of covariates on the conditional mean, therefore it cannot provide information regarding non-central location.
The main problem of the conditional-mean model is that the information about the tails are lost. For example, let's consider a study for economic inequity: of course the attention is paid to the lower (the poor) and the upper (the rich) tails, but the Ordinary Least Squares cannot give proper information about them and so it may be useless and inefficient in analyzing this kind of studies. On the other hand Quantile regression analyzes the relationship between the dependent variable and the covariates on the entire distribution and not just on the conditional mean, giving a more complete picture than the OLS. Indeed, exploring all the distribution allow the Quantile regression to provide information about the tails as well, gaining further insight.

Another advantage deriving from the usage of Quantile regression is that, unlike the OLS, it provides reliable estimates even in presence of heteroscedasticity. Indeed, QR is much more flexible and robust than the Ordinary Least Squares. In fact, if the homoscedasticity assumption is violated (the variance of the error terms is not constant) consequently the OLS cannot be considered the BLUE estimator anymore: its results are no longer reliable and appropriate.
It should be noted that the Gauss-Markov's assumptions (i.e. the homoscedasticity condition) on one side makes the OLS the Best Linear Unbiased Estimator (BLUE), but on the other, makes it a really inflexible one.

Applying the Ordinary Least Squares method, an addition problem may occur: the estimates could be misleading because the high sensitivity of the OLS to extreme outliers.
Barnett and Lewis (1994) defined the outliers as " an observations in a data set which appears to be inconsistent with the remainder of that set of data". This outlines' definition makes pretty clear that if an observation is rather far from the others, the sample mean may be widely effected and therefore the estimates will be not reliable. This problem is overcome thanks the usage of the Quantile regression. Indeed the QR estimates are not influenced (as much as in the OLS) by outlying observations because quantile regression is more robust to outliers and thus its results are considered acceptable.

The differences spotted above, show how much useful can be the Quantile regression. The main difference between the OLS and the QR is that the first one in order to provide good estimates has to be subject to very strict assumptions and this makes it not flexible and not a suitable estimator in many cases. On the other hand, the latter, thanks to its features is considered a much more flexible and robust estimator which could overcome the problems due to the usage of the Ordinary Least Squares.

2 CHAPTER - MARKET TIMING

2.1 MARKET TIMING DEFINITIONS AND OVERVIEW

In finance, the performance of the investment manager is a really discussed topic, indeed for many years this argument has been the content of several studies and discussions.
The main goal of the portfolio manager is to obtain higher returns in relation to the stock market, using his superior abilities.
This superior skills can be dived into two categories: the microforecasting, also known by the name of “security analysis”, and the macroforecasting also known as “market timing”.

The security analysis predicts the price changes of determine individual stocks. In particular this process consists in identifying which stocks are over- or under-valued and using them to obtain excess returns. In other words, the manger is trying to spot which individual stocks have the expected returns that lie above or below the SML (Security Market Line). Concretely, the manager will include in the portfolio the under-valued stocks, waiting for the market to value them in the proper way, and on the contrary he will exclude from the portfolio those stocks which are over-valued.

The market timing consists in forecasting the market changes, and, accordingly the forecasts modify the composition of the portfolio. Thus, when the manager predicts an up-market he will decrease the level of riskless assets (which generate lower returns) in the portfolio and consequently he will increase the quantity of risky assets which have higher returns. On the contrary if the manager forecasts that the market is going to fall his goal will be to decrease the volatility of the portfolio, increasing the level of bonds and therefore decreasing the risky assets in the portfolio.
The market timing, thus, is the ability to predict market changes and therefore modify the portfolio accordingly to these predictions.

As we already pointed, several academics studied these two phenomena. Jensen (1972b) and Fama (1972) proposed several models in which they tried to evaluate the macro and the micro forecasting abilities of investment managers. In this models the prediction’s skills can be tested by comparing the ex post portfolio returns with the market one.

The Jensen model is based on the idea that the manger has to predict the market return and it is assumed that both predicted return and real market return have a normal distribution. Therefore, under these conditions it is possible to measure the market timing abilities by analyzing the correlation which exists between manager's predictions and the realized market return.

Another really important model which studies the market timing phenomenon is the Treynor and Mazuy model (1966). The authors studied 57 mutual funds and tried to define if the managers of these funds had market timing skills. In particular they proposed an adjusted version of the CAPM in which they added a quadratic term to test for forecasting skills. The idea on which is based this model is that if the manager is able to predict the market changes, he will increase the volatility of the portfolio whether he forecasts an up-market and, on the contrary, he will decrease the volatility of the portfolio when the market fall. The model results showed that there was no evidence that, the managers through their abilities, could beat the market.

Henriksson and Merton (1981) studied the forecasting abilities too. The authors presented two different statistical procedures (a non-parametric and a parametric one) able to define whether or not there is market timing skill.
In the non-parametric procedures the market timer's predictions are observable and the test can be used without taking into account any assumptions about the market's returns distribution.
On the other hand the parametric procedure (which is an adjusted version of the CAPM) can test the market timing skills even without being able to observe them and this is possible thanks to the usage of the return data alone.
Henriksson-Merton and Treynor- Mazuy models are analyzed more in detail in this chapter.

Because both H-M and T-M models use an adjusted version of the CAPM to test for market timing skills could be useful to present a briefly overview about it.

The Capital Asset Pricing Model is a mathematical model which has the purpose to determine if there is a correlation between the portfolio return (or stock return) and its level of risk β. The CAPM is formulated in the following way:

$$R_p - R_f = \alpha + \beta\left(R_M - R_f\right) + \epsilon_p$$

Where:

R_p= return of the considered portfolio

R_f= return of the risk-free assets

R_M= return of the market

It is possible to determine two type of risks: the unsystematic and the systematic risk. The former could be reduced through portfolio diversification, while the latter (represented by β) cannot, thus investors are not able to decrease this type of risk, hence they can just bear it. The parameter β is defined as the ratio of the covariance between the portfolio returns and the market returns, divided by the variance of the market return, therefore $\beta = \frac{Cov(R_p, R_M)}{Var(R_M)}$.

Higher is β higher is the risk of the portfolio, but at the same time higher are the achievable level of returns, of course the potential losses too. We will see in the next sections that the parameter β is fundamental in both H-M and T-M.

The parameter alpha (α) (proposed by Jensen) corresponds to the excess return obtained by security analysis. Indeed, α reflects the possibility that the manager has superior predictions skills. These skills will be used by the manager to select stocks which can earn more than (R_M – R_f), that is the risk premium, for their level of risk in CAPM.

As we will see in the next paragraphs, the Capital Asset Pricing Model will be used as a framework for building tests about security analysis and market timing and both the parameters which have been mentioned above will be useful to explain the models.

2.2 HENRIKSSON AND MERTON MODEL

The Henriksson and Merton's procedures are based on the previous market timing model proposed by Merton (1981). In this model the author states that an investor can divide his portfolio between stocks (risky assets) and bonds (risk-free assets), and, he can change over time the split according to his predictions on the two assets' return. Indeed, in the model Merton studied market timing assuming just two possible scenarios: the market timer predicts either if stocks outperform bonds, or vice-versa, if bonds outperform stocks. The main result of this analysis is summarized by Merton as follows: "the pattern of returns from successful market timing will be shown to have an isomorphic correspondence to the pattern of returns from following certain option investment strategies". In other words Merton shows that, it is impossible to distinguish the return patterns achieved by option strategies (i.e. protective put) from the one obtained by a successful market timing process. This isomorphic correspondence it is fundamental for Merton's model since he used this relation to derive a theory which is able to determine and assess the value of market timing predictions skills. The main concept behind this model is that an investor can divide his portfolio between stocks and bonds and he can change over time the split according to his predictions on the two assets' return. Furthermore in the model Merton has proved, exploring how investors would modified their beliefs because of the market timer's predictions, that for forecasts to have a positive value, the probability of a correct prediction conditional to the market's return, is a necessary and sufficient condition. Although this model offers the possibility to analyze the value of market timing skills, it is not able to detect the magnitude of these superior forecasting capabilities.

Based on the Merton model Henriksson and Merton (1981), therefore, proposed two statistical procedures capable of testing the market timing skills: a non-parametric test used when the manager's predictions are observable and a parametric one in which the manager's forecasts are not observable.

First of all, let R_M(t) be the return of the market and R_f(t) the return per dollar of the risk free assets. The manager can forecast if $R_M(t) > R_f(t)$ or $R_M(t) \leq R_f(t)$

$$\gamma(t) = \begin{cases} 1 & if \quad R_M(t) > R_f(t) \\ 0 & if \quad R_M(t) \leq R_f(t) \end{cases}$$

Where $\gamma(t)$ is the forecast variable of the market timer. The probabilities conditional to the market's return for $\gamma(t)$ are defined as follows:

$$p_1(t) \equiv prob\big(\gamma(t) = 0 \big| R_M(t) \leq R_f(\text{t})\big)$$
$$1 - p_1(t) \equiv prob\big(\gamma(t) = 1 \big| R_M(t) \leq R_f(\text{t})\big)$$

and

$$p_2(t) \equiv prob\big(\gamma(t) = 1 \big| Z_M(t) > R_f(\text{t})\big)$$
$$1 - p_2(t) \equiv prob\big(\gamma(t) = 0 \big| Z_M(t) > R_f(\text{t})\big)$$

Notice that $p_1(t)$ and $p_2(t)$ represent the conditional probabilities of an exact prediction, respectively given $R_M(t) \leq R_f(\text{t})$ and $R_M(t) > R_f(\text{t})$.

Furthermore it should be noted that $p_1(t)$ and $p_2(t)$depend solely whether or not $R_M(t) > R_f(\text{t})$, indeed the conditional probabilities are not affected by the magnitude of $\big|R_M(t) - R_f(\text{t})\big|$.

Considering this innovative formulation of the marketing timing problem, as we already said, Henriksson and Merton were able to develop two tests. Here below the two statistical procedures will be presented.

2.2.1 Non-parametric test

The non-parametric procedure evaluates the forecasting skills of managers. The most notable features of this test are that the market timer's predictions are observable and that the test can be used without any assumptions about the market's returns distribution.

Merton (1981) proved that when $p_1(t) + p_2(t) = 1$ the manager's forecasts have no value (it is a sufficient and necessary condition). In particular the investors, in this case, would not change their previous estimates about the market's returns relying on the manager's predictions since they are worthless and without any value. On the other hand, to have a positive value for the market timing forecasts is necessary to meet the following condition: $p_1(t) + p_2(t) \neq 1$.

It is quite clear now that the idea behind the non-parametric test is to assert whether $p_1(t) + p_2(t) = 1$ or not. In particular if the previous formula it is equal to one then there are not forecasting skills, on the other way around, if $p_1(t) + p_2(t) \neq 1$ the manager has market timing abilities.

The hypothesis used in this test are defined as follows:

$H_0 : p_1(t) + p_2(t) = 1$
$H_1 : p_1(t) + p_2(t) \neq 1$

Where, as we already said, H_0 defines the case in which there are no forecasting skills and, on the other hand, H_1 determines the opposite case (market timing ability). The advantage of building the test around this null hypothesis is that $p_1(t)$ and $p_2(t)$ are sufficient statistics to estimate prediction skill and that they do not depend on the market's returns distribution. Henriksson and Merton stated that the focus of this test is to " determine the probability that a given outcome from our sample came from a population that satisfies the null hypothesis". This probability is determined through the following procedure.

1. The variables are defined as :

- $N_1 \equiv$ number of observations where $R_M \leq R_f$
- $N_2 \equiv$ number of observations where $R_M > R_f$
- $N \equiv N_1 + N_2 =$ total number of observations
- $n_1 \equiv$ number of exact forecasts when $R_M \leq R_f$
- $n_2 \equiv$ number of wrong forecasts when $R_M > R_f$

2. $n \equiv n_1 + n_2 =$ number of times forecast that $R_M \leq R_f$
3. Considering E as the expected value, we have that: $E\left(\frac{n_1}{N_1}\right) = p_1\,;\;\; E\left(\frac{n_2}{N_2}\right) = 1 - p_2;$
4. From the way H_0 is constructed, it follows that:

$$E\left(\frac{n_1}{N_1}\right) = p_1 = 1 - p_2 = E\left(\frac{n_2}{N_2}\right) \Rightarrow E\left(\frac{n_1+n_2}{N_1+N_2}\right) = E\left(\frac{n}{N}\right) = p_1 \equiv p\,;$$

As we can see, under the null hypothesis, p is the expected value for both $\frac{n_1}{N_1}$ and $\frac{n_2}{N_2}$ therefore will be sufficient to estimate just one of them, since they own the same expected value.
Note that n_1 and n_2 are sums of IID random variables with binomial distributions and consequently the probability that $n_i = x$ from a sample of N_i is:

$$p(n_i = x|\ N_i, p) = \binom{N_i}{x} p^x (1-p)^{N_j - x} \qquad i = 1{,}2$$

Using the Bayes' s theorem we can calculate the probability $p(n_i = x|\ N_1, N_2, n)$ given H_0. Let now denote as A the case in which the market timer predicts m times that $R_M \leq R_f$ (i.e. $n = m$), and as B the event where x are the times in which he predicts correctly that $R_M \leq R_f$ (i.e $n_1 = x$) and $m - x$ the times in which his predictions are wrong (i.e. $n_2 = m - x$). Therefore $p(n_i = x|\ N_1, N_2, m) = P(B|A)$ and solving through the Bayes's theorem it is possible to write:

$$P(B|A) = \frac{P(B\ +\ A)}{P(A)} = \frac{P(B)}{P(A)}$$

$$= \frac{\binom{N_1}{x}\binom{N_2}{m-x} p^x (1-p)^{N_1 - x}\, p^{m-x} (1-p)^{N_2 - m + x}}{\binom{N}{m} p^m (1-p)^{N-m}}$$

$$= \frac{\binom{N_1}{x}\binom{N_2}{m-x}}{\binom{N}{m}}$$

The above formula defines the distribution of n_1 (under H_0) given N_1, N_2 and n. It should be noted that n_1 is a hypergeometric distribution.

The range of the admissible values for n_1 is given by:

$$\underline{n_1} \equiv\ max(0, n - N_2) \leq n_1 \leq min(N_1, n) \equiv \overline{n_1}$$

We can use the previous formulas to derive the confidence intervals in order to test H_0 that is the absence of prediction skills.

Let's denote with c the probability confidence level and consider a two-tail test, in this case the null hypothesis will be rejected if $n_1 \geq \overline{x}(c)$ or if $n_1 \leq \underline{x}(c)$

$\overline{x}$ and $\underline{x}$ are respectively the results of the following equations:

$$\sum_{x=\bar{x}}^{\overline{n_1}} \binom{N_1}{x}\binom{N_2}{n\ - x} / \binom{N}{n}\ =\ (1 -\ c)/2$$

And

$$\sum_{x=\underline{x_1}}^{x} \binom{N_1}{x}\binom{N_2}{n-x} / \binom{N}{n} = (1-c)/2$$

On the other hand with a one-tail test, where c is the confidence level, the null hypothesis will be rejected whether $n_1 \geq x^*(c)$

$x^*(c)$ is expressed as follows:

$$\sum_{x=x^*}^{\overline{n_1}} \binom{N_1}{x}\binom{N_2}{n-x} / \binom{N}{n} = 1-c$$

The confidence intervals are pretty easy to calculate when the sample available is small, on the contrary when the size of the sample is large the calculation becomes a problem. However for massive samples the normal distribution can be considered a good approximation for the hypergeometric distribution.

In summary, this non-parametric test proposed by Henriksson and Merton is able to test if the manager has prediction abilities or not. Moreover this procedure shows that there is the possibility that the market timer do not have the same forecasting skills in predicting up and down markets ($p_1(t) \neq p_2(t)$). Indeed, for how the test is constructed, the conditional probabilities do not have to be equal to each other, what really matters in this model is that the sum of $p_1(t) + p_2(t)$ has to be stationary and equal to one under null hypothesis, which, if it is true proves that there are no market timing skills.

The market timer, thus, could have better forecasting skills in predicting an up of the market $R_M(t) > R_f(t)$ and therefore have a “higher” $p_2(t)$, or vice-versa he can make better predictions for down markets $R_M(t) \leq R_f(t)$).

However one really important requirement of the non-parametric test is that the manager’s predictions have to be observable, when this it is not possible another procedure could be used to test the investment performance: the parametric test of market timing.

2.2.2 Parametric Test

As we already pointed the non-parametric test is based on the idea that the forecasts are observable. However it is rare for an investor, who attempts to assess the performance of a portfolio, to get to know the manager's predictions and to observe them. There is the possibility, though, under specific conditions, to figure out, analyzing just the portfolio return series, which were the market timer's predictions. Unfortunately, these kind of deductions usually provide biased and misleading estimates, in particular they are extremely noisy if the manager's predictions are affected by micro-forecasts about certain individual stocks. In order to overcome this problem Henriksson and Merton proposed a new procedure called parametric test. This model can test the market timing skills even without being able to observe them and this is possible thanks to the usage of the return data alone. It is straightforward to understand that not being able to utilize the time series of the predictions has a cost, which is that, particular assumptions about how the returns on securities are generated, has to be met.

On previous studies about the market timing abilities it was always assumed that the trend of stocks returns was compatible with the Security Market Line, that is:

$$R_p(t) - R_f(t) = \alpha + \beta x(t) + \epsilon(t)$$

Where $x(t) \equiv R_M(t) - R_f(t)$ which is the market excess returns, $R_p(t)$ represents the portfolio return, and $\epsilon(t)$ it is residual term which satisfy the following conditions:

$$E[\epsilon(t)] = 0$$
$$E[\epsilon(t)|x(t)] = 0$$
$$E[\epsilon(t)|\epsilon(t-i)] = 0, \qquad i = 1,2,3$$

As Jensen (1972b) proved when are only utilized the return data, this regression does not allow to distinguish if the incremental performance is due to the market timing skills of the manager or to the micro-forecasts about individual stocks, thus, it is not possible to separate market timing from security analysis and vice-versa.

Henriksson and Merton's parametric test overcome this problem, indeed it is able, not only to determine if there are forecast abilities without observing them, but also to distinguish the micro and macro forecasting.

Before studying the procedure in details let's consider which are the assumptions on which the test is based: the securities are valued following the CAMP and the manager has the power to choose, for the portfolio, distinct systematic levels of risk.
The main idea behind this last assumption is that there are two different systematic levels of risk, and each of them depend on the market's return predictions that is if it is expected that the returns on market outperform the risk-free assets or not. In other words the market timer will have one beta when he forecasts that $R_M(t) \leq R_f(t)$ and another value of beta when he expects that $R_M(t) > R_f(t)$.
Let's now define the two possible values of β:

$$\beta(t) = \begin{cases} \eta_1 & if\ the\ manager\ predicts\ that \quad R_M\ (t) \leq R_f\ (t) \\ \eta_2 & if\ the\ manager\ predicts\ that \quad R_M\ (t) > R_f\ (t) \end{cases}$$

Therefore, considering $\beta(t)$ as the beta of the portfolio, if we have $\beta(t) = \eta_1$ it means that the market timer predicts a down market, on the contrary, if $\beta(t) = \eta_2$ he forecast an up market.
It should be noted that if it is possible to observe beta over time to find out if there are predictions skills or not, we can simply use the non-parametric test mentioned before, on the contrary if β is a random variable (it is not observable) then the parametric test has to be applied.
Let's define b as the unconditional expected value of $\beta(t)$ which is not observable. Then considering that q is the unconditional probability that $R_M(t) \leq R_f(t)$, we have that:

$$b = q[p_1\,\eta_1 + (1-p_1)\,\eta_2] + (1-q)[p_2\,\eta_2 + (1-p_2)\,\eta_1]$$

Let $\theta(t)$ be a random variable, determined as $\theta(t) = [\beta(t) - b]$
$\theta(t)$ can be considered as the unanticipated part of β, and its distribution conditional on $x(t)$ is described in the following formulas:

- When $x(t) \leq 0$

$\underline{\theta} = \underline{\theta}_1$

$\underline{\theta}_1 = (\eta_1 - \eta_2)[1 - qp_1 - (1-q)(1-p_2)]$ with $prob = p_1$

$= (\eta_2 - \eta_1)[qp_1 + (1-q)(1-p_2)]$ with $prob = 1 - p_1$

- When $x(t) > 0$

$$\underline{\theta} = \underline{\theta}_2$$

$$\underline{\theta}_2 = (\eta_2 - \eta_1)[qp_1 + (1-q)(1-p_2)] \quad \text{with } prob = p_2$$

$$= (\eta_1 - \eta_2)[1 - qp_1 + (1-q)(1-p_2)] \quad \text{with } prob = 1 - p_2$$

Therefore we can calculate the expect value of θ conditional to $x(t)$ as:

$$\mathrm{E}(\Theta|\mathrm{x}) = \begin{cases} \overline{\theta_1} = (1-q)(p_1 + p_2 - 1)(\eta_1 - \eta_2) & when\ x(t) \leq 0 \\ \overline{\theta_2} = q(p_1 + p_2 - 1)(\eta_2 - \eta_1); & when\ x(t) > 0 \end{cases}$$

The market timer portfolio's returns it is expressed in the following formula, in which λ it is the expected increment of the return due to the usage of the security analysis (manager's selection abilities).

$$R_p(t) = R_f(t) + [b + \theta(t)]x(t) + \lambda + \epsilon(t)$$

Given the above formula it is possible to derive an adjusted version of the CAPM which could be used not only for testing the presence of forecasting skills but also to detect when the incremental performance is due to market timing or to the microforcasting. The regression can be define as:

$$R_p(t) - R_f(t) = \alpha + \beta_1 x(t) + \beta_2 y(t) + \epsilon(t)$$

and $y(t) \equiv max\left[0, R_f(t) - R_M(t)\right] = max[0, -x(t)]$

It should be noted that the same level of returns achieved by the market timing strategy defined above, could be obtained applying a partial protective put strategy, in which the put options can be acquired with a strike price of $R_f(t)$ on the market portfolio.
Furthermore in this regression we can see that there is a separation between the manager's selection abilities which are represented by α, and the market timing skills defined by the coefficient β_2 , therefore, as already mentioned, this parametric procedure is able to measure and distinguish when the performance increases thanks to the market timing skills or thank to the stock selections abilities.

Let's now derive the OLS estimates of β_1 and β_2 , where β_1 represents the systematic risk :

$$plim\hat{\beta}_1 = \frac{\sigma_{px}\sigma_y^2 - \sigma_{py}\sigma_{xy}}{\sigma_x^2\sigma_y^2 - \sigma_{xy}^2}$$
$$= b + \overline{\theta_2}$$
$$= p_2\eta_2 + (1 - p_2)\eta_1$$

$$plim\hat{\beta}_2 = \frac{\sigma_{py}\sigma_x^2 - \sigma_{px}\sigma_{xy}}{\sigma_x^2\sigma_y^2 - \sigma_{xy}^2}$$
$$= \overline{\theta_2} - \overline{\theta_1}$$
$$= (p_1 + p_2 - 1)(\eta_2 - \eta_1)$$

If the value of β_2 is statistically equal to zero it means that the manager is not able to predict the market evolutions, in particular that $p_1(t) + p_2(t) = 1$, therefore we can conclude that the manager does not have forecast skills.

On the other hand if the value of β_2 is statistically less or greater than 0 it means that the manager has positive (or negative) market timing abilities. It should be noted, though, that a negative value of β_2 would make any sense, therefore it would mean that the manager has market timing ability, but he is using it in an irrational way (i.e. raise the market risk when he predicts a down market). Furthermore a negative value of β_2 would violate the condition of $p_1(t) + p_2(t) \geq 1$.

Henriksson and Merton developed another regression equation used to test for market timing. In other words this new regression it is equivalent to the one proposed before, but it differs in the way how the market timing problem is expressed. The advantage of this new specification is that it is easier to understand and is more intuitive than the previous one.

In this new regression we have that $x_1(t) \equiv \min[0, x(t)]$, $x_2(t) \equiv \max[0, x(t)]$:

$$Z_p(t) - R(t) = \alpha' + \beta_1' x_1(t) + \beta_2' x_2(t) + \epsilon$$

If $x(t) > 0$ we will have $x_1(t) = 0$ and $x_2(t) = x(t)$, therefore, intuitively β_2' will represent the "up- market" beta of the portfolio. In the same way, if $x(t) \leq 0$ then $x_1(t) = x(t)$ and $x_2(t) = 0$ the interpretation of β_1' is the "down-market" beta.

In this new specification to verify if there are market timing abilities, $\widehat{\beta_2'}$ must be significantly greater than $\widehat{\beta_1'}$. In other words the expected "down-market" beta has to be smaller than the up-market one. It is important to highlight that testing $\widehat{\beta_2'}$ greater that $\widehat{\beta_1'}$ or to test $\beta_2 > 0$ (as in the first regression proposed) is equivalent, both tests verify if there are predictions skills, the difference is just about how the tests are build, not about what they are testing.

The estimates of $\widehat{\beta_1'}$ and $\widehat{\beta_2'}$ can be written as follows:

$$
\begin{aligned}
plim\widehat{\beta_1'} &= E[\beta(t)|x(t) \leq 0] \\
&= p_1\eta_1 + (1 - p_2)\,\eta_2
\end{aligned}
$$

$$
\begin{aligned}
plim\widehat{\beta_2'} &= E[\beta(t)|x(t) > 0] \\
&= p_2\eta_2 + (1 - p_2)\,\eta_1
\end{aligned}
$$

2.2.3 Example of H-M model

Could be useful to present a brief example to explain more in details how the market and the market timing affect the funds' returns. In particular let's consider the Henriksson and Merton regression in which for convenience we do not take into account the value of $R_f(t)$.

$$R_p(t) = \alpha + \beta_1 R_M(t) + \beta_2[max(0, -R_M(t))] + \varepsilon_t$$

Furthermore let be $\alpha = 0$ and $\varepsilon_t = 0$

For $R_M(t) > 0$ we have that $R_p(t) = \beta_1 R_M(t)$, therefore the fund's returns depends positively from the market. If the market is increasing then we expect that the fund's returns increase too. There is a strong positive correlation between the two variables: if the market is going good the fun's returns are positive.

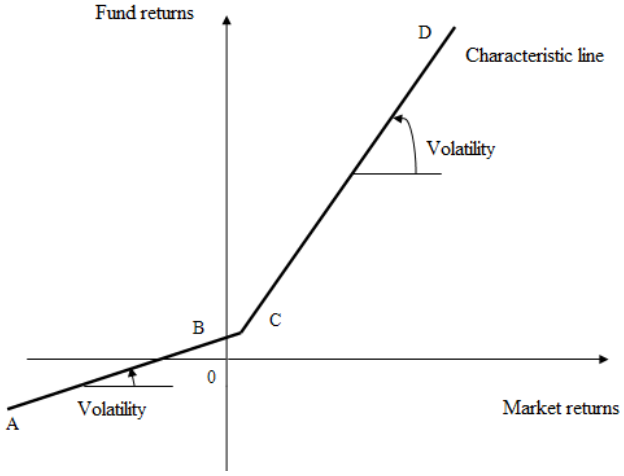
On the other hand for $R_M(t) < 0$ we have that, the fund's returns depends on both β_1 and β_2 in particular: $R_p(t) = (\beta_1 - \beta_2)R_M(t)$.

Therefore from the previous formula we can see that when $\beta_2 > 0$ the market timing coefficient is decreasing the value of β_1, that is to decrease the level of risk exposure of the portfolio in order to "protect" the fund trying to reduce the losses.

On the other hand when the $\beta_2 < 0$ the market timing coefficient is influencing the value of β_1 increasing it, and therefore increasing the level of risk exposure of the fund: the managers are trying to get higher fund's returns.

2.3 THE TREYNOR AND MAZUY MODEL

Treynor and Mazuy in their model proposed a statistical test to verify if the managers are able to predict the market changes and therefore if they have market timing abilities.
In particular they applied this test to the performance of 57 funds, asking themselves "is there any evidence that the volatility of the fund was higher in years when the market did well than in years when the market did badly?"
For the authors, having market timing skills means that the manager is able to forecast if the market is going to fall or to increase and then, accordingly to his predictions he will change the portfolio's structure. In particular, if the manager predicts that the market is going to go up he will increase the volatility of the portfolio, going from less to more volatile stocks. On the other hand he would decrease the volatility of the portfolio if a market fall is expected.
Treynor and Mazuy, in their paper, did not present a specifically regression equation (which, anyway, can be derived from their explanations), indeed they decided to explain the market timing phenomenon and their test through some graphical representations. In this paragraph both regression equation and graphical representations will be shown.
As we mentioned above, Treynor and Mazuy analyze through several graphics the market timing phenomenon and how the predictions of the manager about the market changes affect the composition and therefore the volatility of the portfolio. Let's analyze one of the most meaningful graphics proposed by the authors.



Source: Treynor and Mazuy (1966)

Figure 2: Fund that has consistently outguessed the market

The figure above shows the situation in which the manager predicts correctly the market changes at every period. Therefore, it is possible to see that the forecaster has chosen to increase

the volatility of the portfolio when the market is increasing (see the segment CD of the characteristic line). On the contrary, when the manager predicts that the market is going to fall, his decision is to decrease the volatility of the portfolio to reduce the risk exposure from the market (segment AB). As we can see from the graphic the slope of the characteristic line represents the volatility of the portfolio. Of course the slope will be steep (higher volatility of the portfolio) when the market is expected to go up and vice-versa the slope will be flat (lower volatility of the portfolio) when the market is expected to fall.
It should be noted that the figure 2 is actually the graphical representation of the Henriksson and Merton model. Indeed Treynor and Mazuy started their own interpretation of the market timing phenomenon from a reasoning which was similar and comparable to the H-M one. From that reasoning they had developed their own methodology, obtaining a characteristic line which is a curve (as we will see) and not a half-line anymore as in the H-M model.

In a "real world" would be impossible for a forecaster to predict the market changes perfectly (as in the case above), but still, he can have some forecasting skills. Indeed, if the market return increases over time it is likely that the manager through his forecasting skills has foreseen good performance and consequently has raised the volatility of the portfolio by choosing appropriately different assets. This process will gradually change the characteristic line which will become a concave upward line. Indeed, there will be a transition from the left part of the graphic (low volatility and flat slope) to the right part in which the slope is steep and the volatility is higher, furthermore the slope between these two extremes will vary in a more or less continuous way. Therefore if the manager has good forecasting skills and the number of the predictions which are right are more than the once which are wrong, than, the characteristic line will be curve.
In order to use the predicting skills in a useful way it is necessary that the mangers change the portfolio volatility systematically to create a curve characteristic line.

The picture below shows the transition process described before, in which the characteristic line from straight becomes curve. This happened as we said, when the predictions of the manager are more often right than wrong. Furthermore the degree of curvature of the characteristic line depends on the manager's decisions about the volatility. Indeed if he bets heavily on his predictions (i.e. he expects an up-market) the manager will change drastically the fund volatility (i.e. increasing it) according to his forecasting and therefore the characteristic line will be much more steeper (the degree of curvature has changed).

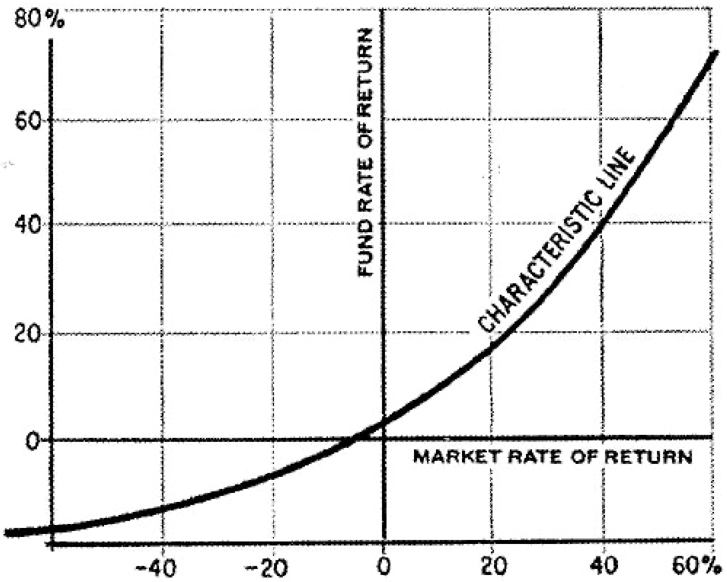


Figure 3: Fund that has outguessed the market with better-than-average success

As already mentioned above there is also a more mathematically approach to the Treynor and Mazuy model which involves the usage of a regression equation and the introduction of a quadratic term in the classical CAPM version.

Let's now present the regression which describe the model:

$$R_P - R_f = \alpha_P + \beta(R_M - R_f) + \gamma(R_M - R_f)^2 + \epsilon$$

Where:

R_P =defines the considered portfolio return

R_f =defines the return on the risk-free securities

α_P =defines a selectivity skill

R_M =defines the return market

As we can see the above specification is an adjusted version of the CAPM, in which it is introduced a quadratic term of the excess market return that allows to test for market timing abilities.

The parameter γ reflects the ability of an investment manager to anticipate the market trend and accordingly to his prediction to adjust the portfolio composition. If γ is significantly positive we can assert that the manager has forecasting abilities, on the contrary if the parameter is significantly negative the manager does not have market timing skills and he is not able to predict the changes of the market and therefore modify the structure of the portfolio.

The parameter α, on the other hand, as in the Henriksson and Merton model, denotes the ability of the manager to select the right stocks for the portfolio.
If α is significantly positive then the manger has selection abilities, on the contrary if the parameter is negative the manager is not able to build the optimal portfolio: he does not have selection skills.

In both cases, the mathematical and the graphical one, the main idea behind this model is that the manager is able to affect the portfolio volatility changing its composition in relation to his forecasting regarding the trend of the market (up or down) . The goal of an investment manager is to try to anticipate the market trend and consequently to modify the portfolio in order to reduce the risk exposure and therefore to realized less losses in comparison to the market (when the market fall), or, when the market is rising, to increase the volatility to obtain greater portfolio returns in relation to the market.

2.3.1 Example of Treynor and Mazuy model

Could be useful to present a brief example to explain more in details how the benchmark and the market timing affect the funds' returns. In particular let's consider the Treynor and Mazuy regression in which for convenience we do not take into account the value of $R_f(t)$.

$$R_P = \alpha_P + \beta(R_M) + \gamma(R_M)^2 + \epsilon$$

Let α and ε_t be both equal to zero.

If $R_M > 0$ the market returns R_P depends on the two component $\beta(R_M)$ and $\gamma(R_M)^2$. Furthermore each component depends on their coefficient (β, γ), in particular if both market and market timing coefficients are greater than zero consequently their components are positive and they have a positive impact on the fund's returns. Noticed that whether the market is increasing $(R_M > 0)$ is reasonable to think that the coefficient related to the market timing will be greater than zero because if not the market timing component decreases the fund's returns when the market is bullish, not allowing the fund to gain higher returns compared to the market. Thus, a positive γ makes the fund's returns higher than the one obtained just following the benchmark.

On the contrary if $R_M < 0$ and both the coefficients β and γ are positive the quadratic component $\gamma(R_M)^2$ reduces the losses due to the negative value of $\beta(R_M)$: the market timing coefficient is reducing (in somehow compensating) the negative impact of the market on the fund's returns.

On the other hand if γ is negative too the market timing component is amplifying the losses which occurs when the market is bearish, indeed the fund's returns are decreasing even more because of the negative impact of the market timing coefficient.

3 CHAPTER- EMPIRICAL DATA

In this section applying the methodologies introduced in the previous chapters, we will build and explore an empirical analysis with the aim to verify whether or not there is the presence of market timing phenomenon in the samples considered. The main goal is to investigate if the managers who run the funds taken into account for the study are able to forecast the market changes and then accordingly to the predictions modify the composition of the portfolio to get higher returns or to avoid possible losses. In order to do this we will use the H-M and T-M methods introduced in the second chapter. Applying these procedures to our empirical data and studying the coefficients estimates we will be able to assert if there is market timing or not.

The analysis described above will be developed in two different phases, in the first one the estimation method used will be the Ordinary Least Squares regression, while, the second phase will be characterized by the usage of the Quantile regression.
In particular we will apply the H-M and the T-M models to the returns of several U.S mutual funds with respect to a chosen benchmark and through the usage of the OLS we will estimate the coefficients of the regression, studying whether or not the managers had market timing skills. In a second moment we will estimate again the models mentioned previously (H-M and T-M) but this time through Quantile Regression in order to study the presence of market timing in different quantiles and to verify if the timing skills are changing through them.

Noticed that the usage of the QR it is useful to gain further insight with respect to the Ordinary Least Squares, studying the coefficients estimates in different quantiles allows to have a much more complete picture than the OLS and to explore for each quantile the market timing phenomenon and studying how it changes through all the distribution. Furthermore it could be interesting to compare the results obtained by the Ordinary Least Squares and the Quantile Regression and to verify how the outcomes change because of the different estimating method applied.
In the next paragraph the data used for the empirical analysis will be introduced.

3.1 EMPIRICAL DATA

In the empirical analysis we have considered 23 U.S. mutual funds which all belong to the category "Large Blend Funds" of Morningstar. The Large Blend funds is a category of funds which are quite representative of the whole U.S. stock market in growth rates, price and size. This type of funds are considered "blend" when there is an equilibrium between growth and value characteristics, in other words none of these features predominate the other.

The index "Standard and Poor's 500" (S&P 500) has been used as the benchmark and the period taken into consideration for the analysis goes from 6 January 2006 to 27 December of 2019.

Furthermore it should be noted that for the analysis the funds' prices and returns have been considered on a weekly basis, as well as the benchmark's data.
The data referred to the prices have been obtained by the usage of the data-stream "Refinitiv Eikon" and consequently the returns have been calculated through the following formula:

$$r = \frac{P_t - P_{t-1}}{P_{t-1}}$$

Where r represents the return and P the price.
The funds chosen for the study are described below.

- **VFTNX**: the Vanguard FTSE Social Index is a well-diversified fund and it is considered one of the most socially conscious ones. Therefore it does not include stocks which are implicate in arguable business (i.e. nuclear power, fossil fuels, gambling etc.) and furthermore it excludes companies which do not promote the diversity too. The managers fund have been able to find an equilibrium between the diversification strategy and the aim of avoiding controversial business.
  The holdings of the VFTNX fund between different sectors are defined as follows (just the most important ones are shown): Technology 28.20%, Healthcare 14.49%, Consumer Cyclical 13.42%, Financial Services 13.23%, Communication Services 13.08%, Consumer Defensive 6.36% , Industrials 5.33% etc.
  The first 10 stocks which have the highest weight in the fund are: Apple Inc, Microsoft Corp, Amazon.com Inc, Facebook Inc A, Alphabet Inc A, Alphabet Inc Class C, Tesla Inc, JPMorgan Chase & Co, Visa Inc Class A, Procter & Gamble Co.

- **HAIAX**: the Hartford Core Equity Fund aim is to seek a growth of the capital and to pursue a widely diversification strategy not just about the industries but about the companies too. The managers fund invest most of the assets (minimum 80%) in common stocks and in particular they focus on large capitalization companies.

  The holdings of the HAIAX fund between different sectors are defined as follows: Technology 24.90%, Healthcare 14.39%, Financial Services 12.70%, Consumer Cyclical 11.53%, Communication Services 11.00%. etc.

  The first 10 stocks which have the highest weight in the fund are: Apple Inc, Amazon.com Inc, Alphabet Inc A, Microsoft Corp, Facebook Inc A, Procter & Gamble Co, The Walt Disney Co, JPMorgan Chase & Co, UnitedHealth Group Inc, Merck & Co Inc.

- **CSXAX**: the Calver US Large-Cap Core Responsible Index Fund seeks to follow the Calvert Principles for Responsible Investment, widely investing in stocks which meet these principles. The strategy that managers pursue might introduce some risk in the fund, but unfortunately this risk is not always compensate.

  The holdings of the CSXAX fund between different sectors are defined as follows: Technology 26.92%, Healthcare 15.09%, Financial Services 13.23%, Consumer Cyclical 12.49%, Communication Services 9.82% etc.

  The first 10 stocks which have the highest weight in the fund are: Apple Inc, Microsoft Corp, Amazon.com Inc, Alphabet Inc A, Procter & Gamble Co, Visa Inc Class A, JPMorgan Chase & Co, NVIDIA Corp, Tesla Inc, The Home Depot Inc.

- **PRDGX**: the T Rowe Price Dividend Growth Fund is managed with the aim to increase in the long-term the capital growth and to increase the current and dividend income. Therefore to reach this goals the managers are investing most of the assets (at least 65%) in stocks of dividend-paying firms.

  The holdings of the PRDGX fund between different sectors are defined as follows: Technology 19.66%, Healthcare 18.22%, Financial Services 15.12%, Industrials 12.89%, Consumer Defensive 9.54% etc.

  The first 10 stocks which have the highest weight in the fund are: Microsoft Corp, Apple Inc, Visa Inc Class A, Danaher Corp, JPMorgan Chase & Co, Thermo Fisher Scientific Inc, UnitedHealth Group Inc, Accenture OLC Class A, Dollar General Corp, Becton, Dickinson and Co.

- **GESSX**: the GE RSP US Equity Fund aim is to increase the long term capital and income growth. In order to do that the managers invest most of the fund's net assets (minimum 80%) in common and preferred stocks of U.S companies.
  The holdings of the GESSX fund between different sectors are defined as follows: Technology 24.94%, Financial Services 15.63%, Healthcare 13.77%, Communication Services 11.97%, Consumer Cyclical 10.80% etc.
  The first 10 stocks which have the highest weight in the fund are: Apple Inc, Microsoft Corp, Amazon.com Inc, JPMorgan Chase & Co, Alphabet Inc A, Facebook Inc, Visa Inc Class A, The Walt Disney Co, Qualcomm Inc, Merck & Co Inc.

- **TISCX**: the TIAA-CREF Social Choice Eq Fund seeks to follow the performance of the whole U.S stock market trying to obtain a long-term favorable total return, investing more than 80% of its assets in equity securities. Furthermore the managers invest just in those companies whose business is consistent with the criteria of the fund (fund's ESG criteria).
  The holdings of the TISCX fund between different sectors are defined as follows: Technology 25.77%, Healthcare 15.39%, Consumer Cyclical 11.32%, Financial Services 10.91%, Industrials 9.12% etc.
  The first 10 stocks which have the highest weight in the fund are: Apple Inc, Microsoft Corp, Alphabet Inc A, Alphabet Inc Class C, Procter & Gamble Co, NVIDIA Corp, Tesla Inc, The Home Depot Inc, Verizon communications Inc, Adobe Inc.

- **PRBLX**: the Parnassus Core Equity Fund is considered as a large-capital growth fund in which the managers invest minimum 75% of the assets in dividend-paying stocks. Furthermore, it is considered one of the most socially conscious funds. Therefore it does not include stocks of companies which gain significant revenues from arguable business (i.e. weapons, tobacco etc.)
  The holdings of the PRBLX fund between different sectors are defined as follows: Technology 27.38%, Industrials 18.23%, Communication Services 11.35%, Healthcare 10,39%, Consumer Cyclical 10.22% etc.
  The first 10 stocks which have the highest weight in the fund are: Microsoft Corp, Amazon .com Inc, Comcast Corp Class A, Danaher Corp, Deere & CO, Verizon Communications Inc, Applied Materials Inc, FedEx Corp, CME Group Inc Class A, Linde OLC.

- **GQEFX**: the managers of GMO Quality Fund Class IV invest the large part of the fund's assets in companies tied economically with the U.S seeking total returns.

The holdings of the GQEFX fund between different sectors are defined as follows: Technology 35.21%, Healthcare 24.27%, Financial Services 10.10%, Consumer Cyclical 9.02%, Communication Services 8.01% etc.

The first 10 stocks which have the highest weight in the fund are: Apple Inc, Microsoft Corp, UnitedHealth Group Inc, Accenture OLC Class A, Coca-Cola Co, Oracle Corp, Johnson & Johnson, Facebook Inc A, Medtronic OLC, U.S. Bancorp.

- **CMNWX**: the strategies which characterize the Principal Capital Appreciation Fund are the diversification and the low-turnover strategies which allowed the fund to obtain a satisfying level of long-term results. The managers invest the large part of the fund's net assets (minimum the 80%) in small, medium and large capitalization firms.

  The holdings of the CMNWX fund between different sectors are defined as follows: Technology 23.94%, Financial Services 13.59%, Healthcare 12.71%, Consumer Cyclical 11.74%, Communication Services 11.43% etc.

  The first 10 stocks which have the highest weight in the fund are: Microsoft Corp, Apple Inc, Amazon. Com Inc, Alphabet Inc A, Jpmorgan Chase & Co, Visa Inc Class A, Adobe Inc, Pepsico Inc, T-Mobile Us Inc, Facebook Inc A

- **AWEIX**: the managers of CIBC Atlas Disciplined Equity Fund seek to get long-term capital appreciation and current income. Minimum the 80% of the net assets are invested in equity stocks of U.S companies, furthermore the managers might invest a part in debt securities too.

  The holdings of the AWEIX fund between different sectors are defined as follows: Technology 24.63%, Healthcare 17.65%, Financial Services 12.97%, Consumer Cyclical 11.50%, Industrials 10.57% etc.

  The first 10 stocks which have the highest weight in the fund are: Amazon.com Inc Microsoft Corp, Apple Inc, Alphabet Inc Class C, Visa Inc Class A, UnitedHealth Group Inc, Danaher Corp, Qualcomm Inc, Thermo Fisher Scientific Inc, Johnson & Johnson.

- **WMLIX**: the Wilmington Large-Cap Strategy Fund invests a considerable part of the fund's net assets (a least 80%) in large capitalization companies with the aim to achieve a long-term capital appreciation.

  The holdings of the WMLIX fund between different sectors are defined as follows: Technology 24.43%, Healthcare 13.55%, Financial Services 12.89%, Consumer Cyclical 12.19%, Communication Services 10.76% etc.

The first 10 stocks which have the highest weight in the fund are: Apple Inc, Microsoft Corp, Amazon.com Inc, Facebook Inc A, Alphabet Inc A, Alphabet Inc Class C, Berkshire Hathaway Inc Class B, Tesla Inc, Johnson & Johnson, JPMorgan Chase & Co.

- **DFEOX**: the DFA US Core Equity 1 Portfolio widely invests in common stocks of small capitalization companies seeking to obtain a long-term capital appreciation. However this strategy might increase the fund's risk.
  The holdings of the DFEOX fund between different sectors are defined as follows: Technology 22.50%, Financial Services 14.59%, Consumer Cyclical 13.12%, Industrials 12.89%, Healthcare 12.42% etc.
  The first 10 stocks which have the highest weight in the fund are: Apple Inc, Microsoft Corp, Amazon .com Inc, Facebook Inc A, Verizon Communications Inc, Johnson & Johnson, JPMorgan Chase & Co, Berkshire Hathaway Inc Class B, Visa Inc Class A, Alphabet Inc Class C.

- **BRLIX**: the Bridgeway Blue Chip Fund aim is to achieve a long-term total return on capital and to reach this goal the managers' fund invest minimum 80% of its net assets in the blue-chip category minimizing the costs and the distributions of capital gains. The managers choose in which blue-chip stocks to invest through a statistical technique based on the market capitalization.
  The holdings of the BRLIX fund between different sectors are defined as follows: Technology 21.19%, Communication Services 17.24%, Financial Services 16.48%, Industrials 11.50%, Consumer Defensive 11.01%, Healthcare 9.29% etc.
  The first 10 stocks which have the highest weight in the fund are: Microsoft Corp, Visa Inc Class A, Facebook Inc, Amazon.com Inc, Apple Inc, Qualcomm Inc, Procter & Gamble Co, United Parcel Service Inc Class B, JPMorgan Chase & Co, PepsiCo.

- **TIGRX**: the TIAA-CREF Growth & Income Fund invest the large part of its net assets in income-producing equity seeking to obtain a long-term capital appreciation and income.
  The holdings of the TIGRX fund between different sectors are defined as follows: Technology 25.75%, Healthcare 14.72%, Consumer Cyclical 12.61%, Communication Service 11.38%, Financial Services 11.38% etc.
  The first 10 stocks which have the highest weight in the fund are: Apple Inc, Microsoft Corp, Amazon.com Inc, Alphabet Inc Class C, Facebook Inc A, Procter & Gamble Co, JPMorgan Chase & Co, NVIDIA Corp, The Home Depot Inc, Bank of America Corp.

- **AFDAX**: the American Century Sustainable Equity Fund usually invest a considerable part of its net assets (at least 80%) in equity securities and in particular in large-size companies. The fund seeks to achieve long-term capital growth.
  The holdings of the AFDAX fund between different sectors are defined as follows: Technology 24.88%, Financial Services 13.93%, Healthcare 13.92%, Consumer Cyclical 12.42%, Communication Services 9.32% etc.
  The first 10 stocks which have the highest weight in the fund are: Microsoft Corp, Apple Inc, Amazon.com Inc, Alphabet Inc A, Procter & Gamble Co, Prologis Inc The Home Depot Inc, Facebook Inc A, NextEra Energy Inc, NVIDIA Corp.

- **BTEFX**: the aim of the Boston Trust Equity Fund is to obtain a long-term capital growth investing in firms of any size but having an investing preference for large capitalization companies.
  The holdings of the BTEFX fund between different sectors are defined as follows: Technology 21.69%, Industrials 16.18%, Financial Services 15.56%, Healthcare 12.94%, Consumer Defensive 9.37% etc.
  The first 10 stocks which have the highest weight in the fund are: Apple Inc, Microsoft Corp, Alphabet Inc Class C, Accenture PLC Class A, Visa Inc Class A, Costco Wholesale Corp, UnitedHealth Group Inc, Union Pacific Corp, Nike Inc B, Starbucks Corp.

- **JDEAX**: the JP Morgan US Research Enhanced Equity Fund invests mostly in securities of U.S companies which are include in the Standard & Poor's 500 index seeking to obtain high level of total return. Noticed that the fund might also invest in stocks which are not in the S&P 500 index.
  The holdings of the JDEAX fund between different sectors are defined as follows: Technology 24.30%, Financial Services 14.59%, Healthcare 13.67%, Consumer Cyclical 11.83%, Communication Services 11.42% etc.
  The first 10 stocks which have the highest weight in the fund are: Apple Inc, Microsoft Corp, Amazon.com Inc, Alphabet Inc A, Facebook Inc A, Alphabet Inc Class C, Mastercard Inc A, Berkshire Hathaway Inc Class B, PayPal Holdings Inc, UnitedHealth Group Inc.

- **VTSMX**: the Vanguard Total Stock Market Index Fund seeks to replicate the performance of the CRSP U.S Total Market Index. In other words the fund aim is to track the performance of the whole U.S. stock market which are almost all include in the index mentioned above.

The holdings of the VTSMX fund between different sectors are defined as follows: Technology 23.51%, Healthcare 14.10%, Financial Services 13.01%, Consumer Cyclical 12.10%, Communication Services 10.36% etc.

The first 10 stocks which have the highest weight in the fund are: Apple Inc, Microsoft Corp, Amazon.com Inc, Facebook Inca A, Alphabet Inc A, Alphabet Inc Class C, Berkshire Hathaway Inc Class B, Tesla Inc, Johnson & Johnson, JPMorgan Chase & Co.

- **IGIAX**: income investment and blended growth strategy are used to manage the Integrity ESG Growth & Income Fund. In particular using these strategies the managers seek to get first of all a long-term capital growth and secondarily a dividend income. The fund invests widely in domestic common stocks trying to find an equilibrium between growth and dividend-paying stocks. It should be noted that the managers might invest also in non-paying dividend stocks and in companies of any size.

  The holdings of the IGIAX fund between different sectors are defined as follows: Technology 25.51%, Financial Services 17.44%, Healthcare 14.29%, Industrials 11.26%, Consumer Cyclical 11.02% etc.

  The first 10 stocks which have the highest weight in the fund are: Apple Inc, Qualcomm Inc, Thermo Fisher Scientific Inc, NVIDIA Copr, Starbucks Corp, Visa Inc Class A, S&P Global Inc, PepsiCo Inc

- **SNAEX**: the Schroder North American Equity Fund tries to provide a long-term growth of capital. The fund includes investments in stocks of large and small companies as well.

  The holdings of the SNAEX fund between different sectors are defined as follows: Technology 24.36%, Healthcare 16.99%, Financial Services 12.67%, Consumer Cyclical 11.50%, Communication Services 11.37% etc.

  The first 10 stocks which have the highest weight in the fund are: Apple Inc, Microsoft Corp, Amazon.com Inc, Facebook Inc A, Alphabet Inc A, Alphabet Inc Class C, Johnson & Johnson, Procter & Gamble Co, Visa Inc Class A, UnitedHealth Group Inc.

- **QAACX**: the managers of the Federated Hermes MDT All Cap Core Fund invest primarily in U.S. companies' common stocks, choosing in which stocks to invest from companies which are included in the Russell 3000 Index. This investment policy seeks to get long-term capital appreciation.

The holdings of the QAACX fund between different sectors are defined as follows: Technology 23.37%, Healthcare 15.60%, Consumer Cyclical 14.79%, Financial Services 13.32%, Industrials 9.56% etc.

The first 10 stocks which have the highest weight in the fund are: Alphabet Inc A, Apple Inc, Amazon.com Inc, Microsoft Corp, Domino's Pizza Inc, Colgate-Palmolive Co, Cadence Design System Inc, The Travelers Companies Inc, Kimberly-Clark Corp, Otis Worldwide Corp Ordinary Shares.

- **MEFOX**: the Meehan Focus Fund seeks to achieve long-term capital growth. The fund specially invests in common stocks of companies which have a great growth potential over a period of at least three years.

  The holdings of the MEFOX fund between different sectors are defined as follows: Technology 29.80%, Consumer Cyclical 21.06%, Financial Services 16.60%, Communication Services 12.41%, Healthcare 10.65& etc.

  The first 10 stocks which have the highest weight in the fund are: Microsoft Corp, Apple Inc, Lowe's Companies Inc, Berkshire Hathaway Inc Class B, United Rentals Inc, Alphabet Inc Class C, Amazon.com Inc, ONC Financial Services Group Inc, CVS Health Corp, Alphabet Inc A.

- **SUWAX**: the DWS Core Equity Fund invests a large part of its net assets in equity (mainly common stocks) attempting to get long-term capital growth. It should be noted that the fund invests in companies of any size.

  The holdings of the SUWAX fund between different sectors are defined as follows: Technology 25.37%, Healthcare 14.84%, Consumer Cyclical 12.35%, Financial Services 12.05%, Communication Services 11.21% etc.

  The first 10 stocks which have the highest weight in the fund are: Apple Inc, Microsoft Corp, Amazon.com Inc, Visa Inc Class A, Alphabet Inc Class C, Roku Inc Class A, Amgen Inc, T-Mobile Us Inc, Oracle Corp, Qualcomm Inc.

- **Standard & Poor's 500**: the S&P 500 is one of the most important North American stock index, therefore it is the most used stock benchmark for Wall Street listed securities and it is the underlying benchmark for an incredibly wide range of derivative products such as option and futures.

  The S&P 500 was created in the 1957 from Standard & Poor's and it seeks to follow the trend of an equity basket made up of 500 U.S. large capitalization companies.

Furthermore it should be noticed that the funds VTFNX,CSXAX and VTSMX are characterized by passive management, on the contrary all the other funds have an active approach. This information will be really useful for our analysis because will help us to interpret some of the empirical results.

When we talk about active funds we refer to those funds in which the managers try to outperform a reference benchmark (in our case the S&P500) predicting the market and consequently changing the funds composition (selling and buying assets) in order to gain excess returns. On the contrary in the passive approach the managers' aim is to replicates as closely as possible the performance of the benchmark chosen as reference. In this type of management the managers do not undertake active positions and therefore the portfolio composition tries to mimic the weights assigned to the stocks of the benchmark .

3.2 Descriptive statistics

Before proceeding with the estimation of the H-M and T-M regression models, it could be useful to analyze from a statistical point of view the 23 mutual funds and the benchmark taken into consideration for this study during the sample period.

In particular, in this section we will explore the funds' price changes through a graphical representation (figure 4) and furthermore descriptive statistics (i.e. mean, median, standard deviation, kurtosis etc.) of the benchmark and the funds will be calculated to have a first statistical analysis about the empirical data.

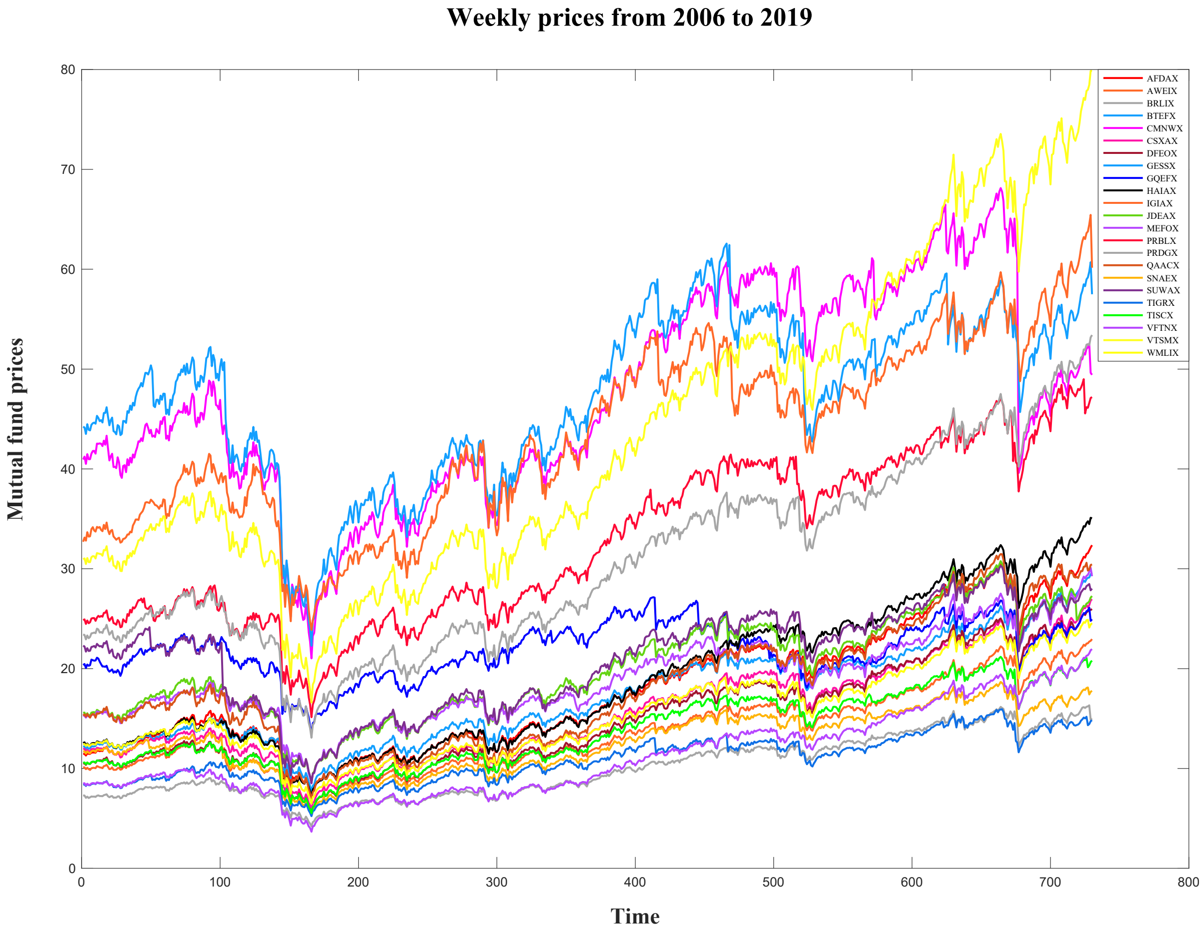


Figure 4: Weekly prices of the funds from 2006 to 2019

As we can see from the graphical representation in the 2009 all the funds' prices had suffered a large fall. This huge decrease was caused by the "U.S. subprime mortgage crisis" which in turn resulted in the 2008 financial crisis: one of the worst crisis which the world had to face in the last decades. Of course the U.S. economy was widely affected by all these events to such

and extend that a great recession in all the economy sectors occurred, decreasing therefore the funds' prices in those years (in particular in the 2009).
After the 2009 we can see from the graphic that the prices were gradually increasing, showing an upward trend with some downward peaks until the end of 2018 in which another large fall in prices occurred. In the 2018 there were several reasons that caused the decrease of stock prices, but one of the most important one could be traced in the commercial war undertaken by Trump against China. This commercial war affected negatively not just the Asian country but the United States too, causing therefore the falling of stock prices.

Let now focus our attention on the descriptive statistics: the table 1 shows the statistical calculation.

First of all, in order to have an overview of the whole empirical data we can focus our attention on the range of returns during the sample period. This range goes from a minimum value of - 0.352 (CMNWX fund) to a maximum one of 0,159 (VFTNX fund).
However, to gain further insight it is useful to analyze and explore the mean's returns of the funds and the benchmark.
As we can see from the table below, just five of the twenty-three funds taken into account (VFTNX, HAIAX, DFEOX, AFDAX,VTSMX) have a higher mean return than the benchmark (S&P 500).
This means that the advisors' funds through their strategies were able to get higher return than the benchmark that is to perform better than the market.
On the contrary, we can see that most of the funds have a lower mean return than the index which means that the managers were not able to outperform the market (i.e. the benchmark) and therefore the returns of the funds are lower.

Another feature in which could be interesting to focus our attention is the standard deviation (SD). From the table's results we can assert that eight funds have a lower standard deviation than the benchmark. This means that these funds are less risky than the considered benchmark. On the contrary of course, those funds which have a higher standard deviation can be considered more risky than the Standard and Poor's 500.

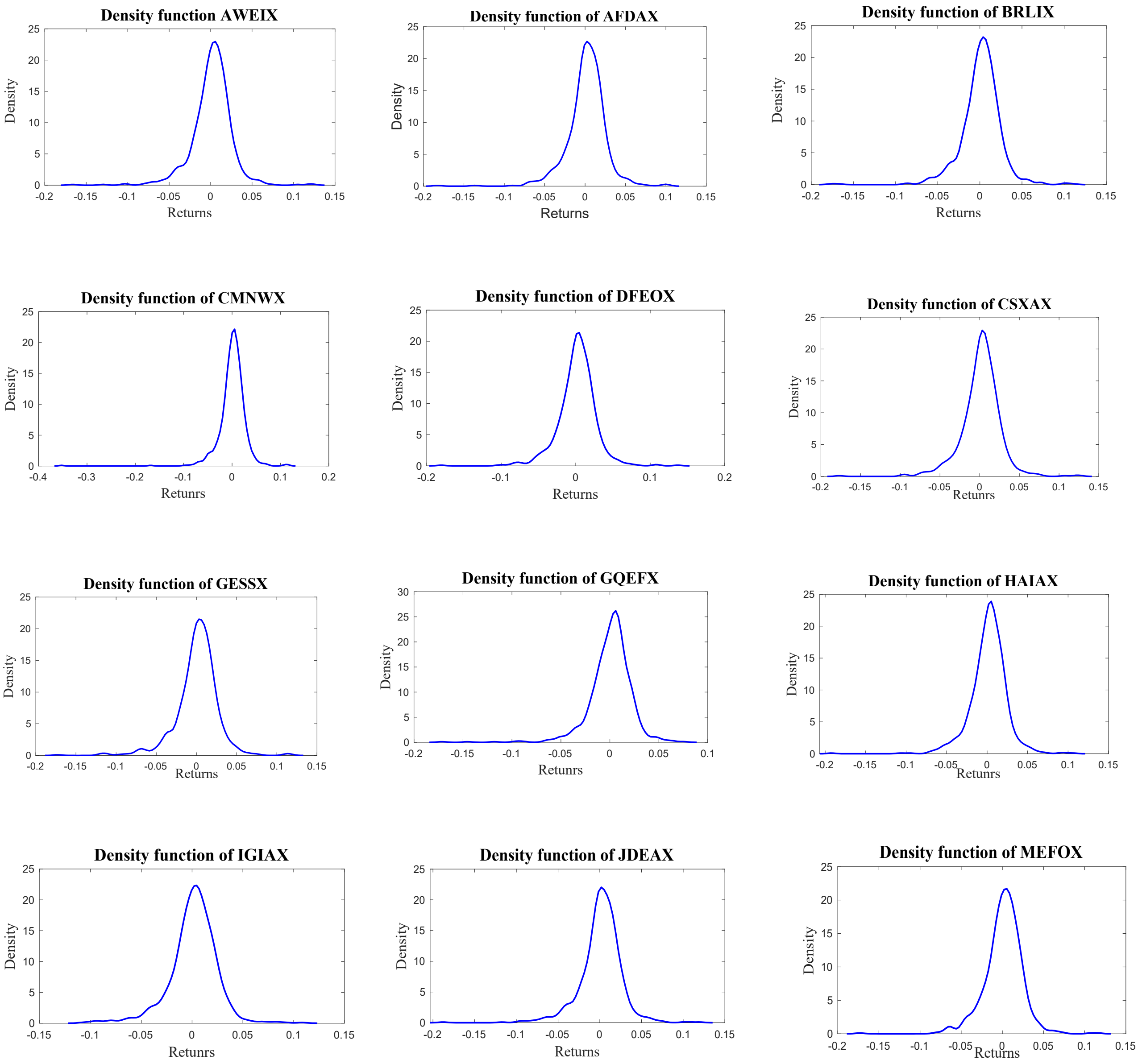
In relation to our analysis could be useful to explore and analyze the skewness too.
As we can see from the table all the funds (including the benchmark) have negative skewness.
The negative skewness is particularly noticeable in the CMNWX fund with a value of

-3, 270.

Furthermore from the column which reports the kurtosis it is possible to assert that all the funds have leptokurtic distributions (all the kurtosis statistics are positive).

The below graphics (figure 5) support the results obtained by the descriptive statistics.

Figure 5: Density function of the funds’ returns



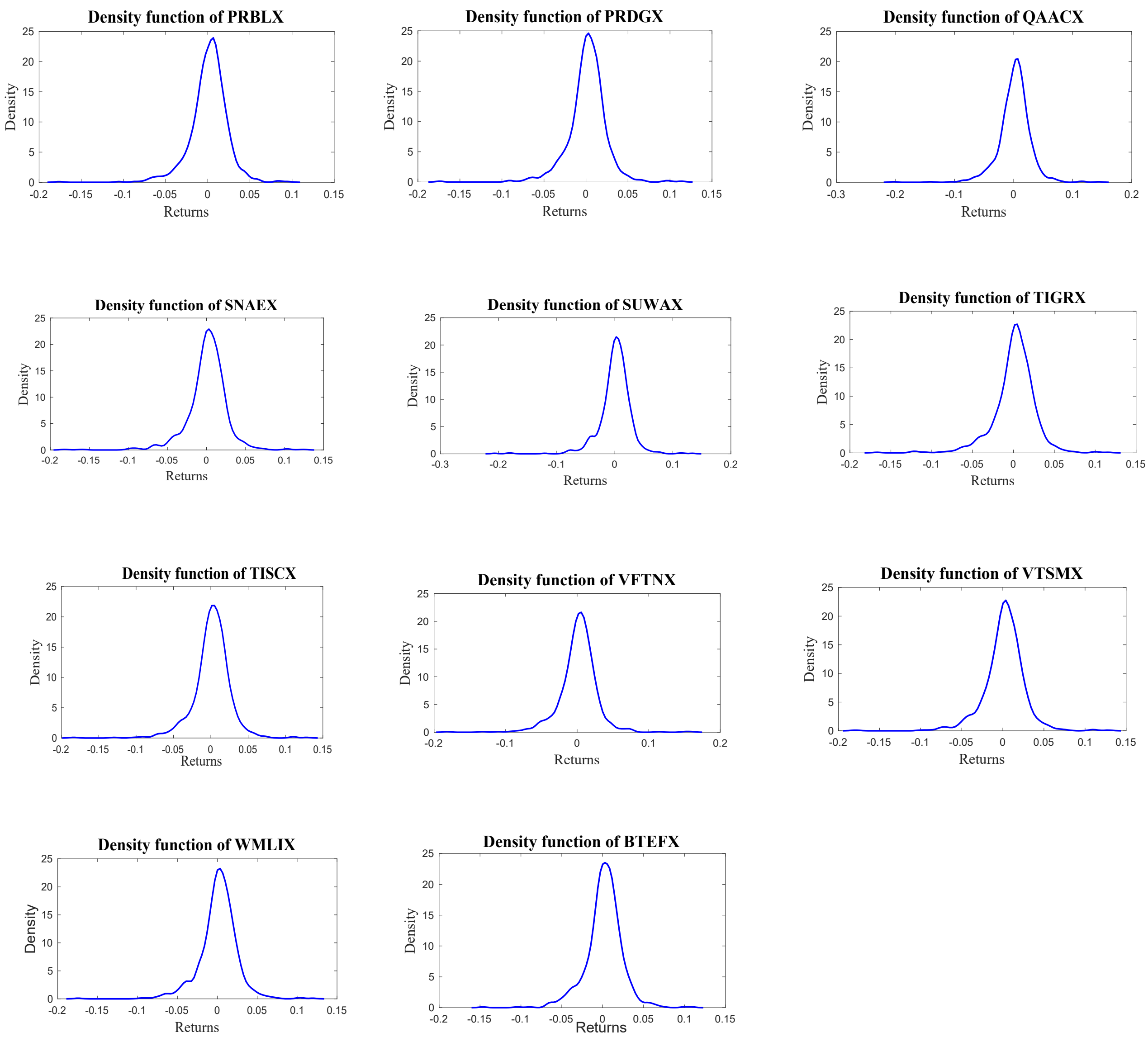

Density function of PRBLX
Density function of PRDGX
Density function of QAACX
Density function of SNAEX
Density function of SUWAX
Density function of TIGRX
Density function of TISCX
Density function of VFTNX
Density function of VTSMX
Density function of WMLIX
Density function of BTEFX
Density
Returns

Table 1: Descriptive statistics of the funds (1/2)

Mutual Fund	Min	Max	Mean	Median
VFTNX	-0,18168	0,15925	0,00169	0,00349
HAIAX	-0,19202	0,10633	0,00172	0,00365
CSXAX	-0,17665	0,12592	0,00147	0,00297
PRDGX	-0,17409	0,11347	0,00142	0,00248
GESSX	-0,17298	0,11746	0,00075	0,00322
TISCX	-0,18392	0,12809	0,00129	0,00294
PRBLX	-0,17634	0,09573	0,00118	0,00311
GQEFX	-0,17149	0,07584	0,00055	0,00273
CMNWX	-0,35236	0,11626	0,00070	0,00265
AWEIX	-0,16667	0,12335	0,00144	0,00307
WMLIX	-0,17446	0,11930	0,00125	0,00237
DFEOX	-0,18040	0,13672	0,00161	0,00329
BRLIX	-0,17667	0,11111	0,00131	0,00270
TIGRX	-0,16688	0,11592	0,00117	0,00319
AFDAX	-0,18271	0,10180	0,00174	0,00328
BTEFX	-0,14657	0,10923	0,00150	0,00276
JDEAX	-0,18820	0,12012	0,00113	0,00245
VTSMX	-0,17919	0,12821	0,00164	0,02426
IGIAX	-0,10697	0,10833	0,00114	0,00268
SNAEX	-0,18140	0,12340	0,00093	0,00234
QAACX	-0,20352	0,14433	0,00135	0,00326
MEFOX	-0,17355	0,11647	0,00119	0,00282
SUWAX	-0,20708	0,13333	0,00074	0,00261
S&P 500	-0,18195	0,12026	0,00159	0,00258

Table 2: Descriptive statistics of the funds (2/2)

Mutual Fund	S.D	Kurtosis	Skeweness
VFTNX	0,026	10,764	-0,386
HAIAX	0,023	12,073	-1,022
CSXAX	0,025	9,543	-0,589
PRDGX	0,022	10,643	-0,809
GESSX	0,026	9,064	-0,957
TISCX	0,025	10,255	-0,819
PRBLX	0,022	10,431	-1,099
GQEFX	0,022	13,502	-1,742
CMNWX	0,027	42,566	-3,270
AWEIX	0,024	10,220	-0,776
WMLIX	0,024	9,421	-0,573
DFEOX	0,025	9,212	-0,779
BRLIX	0,024	12,262	-1,039
TIGRX	0,026	8,966	-0,996
AFDAX	0,024	10,941	-0,975
BTEFX	0,023	7,741	-0,603
JDEAX	0,025	9,918	-0,903
VTSMX	0,003	9,870	-0,611
IGIAX	0,023	6,475	-0,630
SNAEX	0,025	11,221	-1,086
QAACX	0,027	10,716	-0,765
MEFOX	0,023	9,218	-0,701
SUWAX	0,027	14,109	-1,425
S&P 500	0,024	10,378	-0,670

4 CHAPTER – EMPIRICAL ANALYSIS

As we already mentioned before our empirical analysis will be developed in two phases characterized by the usage of the OLS and the Quantile Regression . In particular in this section we will focus our attention on the Ordinary Least Squares estimating method.
The market timing phenomenon will be studied creating two regression models: a simple one referred to the Henriksson and Merton procedure and a quadratic one referred to the Traynor and Mazuy model. These two regressions will be estimated and analyzed in order to define whether or not the managers of the funds had market timing skills.

4.1 HENRIKSSON AND MERTON MODEL: OLS ANALYSIS

The linear regression which has been estimated for the Henriksson and Merton model it was build using the following formula:

$$R_p(t) - R_f(t) = \alpha + \beta_1 x(t) + \beta_2 y(t) + \varepsilon_t \qquad p = 1, \dots, 23$$

Where:
$R_p(t)$ = returns of each fund considered
$R_f(t)$ = returns of risk-free securities
$x(t) = R_M(t) - R_f(t)$ = excess returns realized on the market
$y(t) = max[0, -x(t)]$
$R_M(t)$ = returns of the benchmark considered (S&P500)

It should be noted that in this model the return of risk-free security that is $R_f(t)$ is not taken into consideration.

The regression proposed above has been estimated through the usage of MATLAB software, by which we calculate the coefficients of the regression applying an OLS estimator.
What we are really interested in, is to verify whether or not the coefficient β_2 (for each fund) is significantly different from zero, in this case we can assert that there is market timing and therefore the managers have forecasting skills. In the opposite situation, in which the β_2 is not significantly different from zero the advisories do not have market timing abilities.

What do we expect from the OLS results is to find some evidence of market timing in the funds that we analyzed, but at the same time it is important to highlight that the OLS estimator is not the best one for this type of empirical analysis. Therefore it gives a partial statistical pictures of the phenomenon focusing just on the conditional mean.

Furthermore as we have already explained in the chapter 3, in our sample there are three funds (VFTNX, CSXAX, VTSMX) which are characterized by passive management. The managers of these funds do not undertake active positions for managing the fund: they do not attempt to outperform the market, on the contrary, they have to replicate the performance of a chosen benchmark. Because of this passive approach the managers do not try to implement market timing and therefore, what do we expect from our results is that in the three passive funds will be no evidence of market timing exactly because the managers are not even trying to predict the market and obtain higher returns. From a statistic point of view this means that in our results the coefficients related to the market timing shouldn't be statistically significant for the three passive funds.

4.1.1 Analysis of OLS results

Analyzing the OLS estimates (table 4) we can assert that the coefficients related to the intercept (α) are not always statistically significant. Indeed just in four mutual funds out of twenty-three the coefficient are statistically different from zero. As we already explained the coefficient α in the H-M model refers to the abilities of the managers to build an optimal portfolio, selecting those stocks which can increase the fund's return (security selection).
In these empirical analysis, from the four funds which have a significant intercept just two of them has a positive value of α. This means that the managers of these funds were able through an accurate selection of the stocks to gain a higher return compared to the one obtained just by following the market. On the contrary in the fund in which the coefficient related to the security analysis is negative the manager does not have selection abilities indeed he could not select properly the stocks of the funds: with his wrong selection the manager gained a lower fund's return compare to the one obtained by following the benchmark's trend.

If we explore the coefficients associated to the market's return (S&P500) we can see that β_1 is highly significant for each mutual fund taken into consideration in our analysis. In particular the values related to the benchmark goes from 0,7289 of the GQEFX fund to the maximum value 1,1275 of VFTNX.

Last but not least, we focus our attention on the H-M coefficients (see table 3).
Exploring β_2 that is the coefficients associated to the market timing phenomenon we can see that just in three funds (HAIAX, AFDAX, GESSX) the coefficient is statistically significant.
In particular we have the following coefficients' values: -0,0667 (HAIAX), -0,0665 (AFDAX) and 0,0593 (GESSX.). It should be noted that just one of the coefficients has a positive value while the other ones are negative. The negative values of the market timing have of course a negative impact on the fund's return, indeed it means that the managers will get a lower return compared to the one obtained by simply following the market (benchmark). Furthermore it should be noted that by assumption the Henriksson and Merton model (chapter 2) does not consider negative β_2. Therefore in their reasoning the two authors assert that having negative forecasting skills would mean that the managers have market timing abilities but they are using those skills in an irrational way that is reducing the fund's return which of course would have make any sense.
On the other hand, considering the positive value of GESSX (figure 6) we can assert that the advisor has market timing skills and that through his abilities he was able to get a fund's return which was greater than the possible return obtained just by following the trend of the market's index (S&P500). In this last case we found evidence of market timing phenomenon: the manager was able to forecast exactly the market changes and therefore accordingly to his prediction he changed the composition of the portfolio gaining a higher excess return.
Taking now into consideration the three passive funds we got the expected results: none of the funds has a significant coefficient associated to the market timing. The managers are not trying to outperform the index predicting it and consequently changing the portfolio composition but they are just attempting to replicate its performance. We can conclude that as we expected there is no evidence of market timing for the passive funds.

Below are shown in table 3 the summary of market timing coefficients estimates.
The all OLS results in table 4, and the graph of GESSX fund in figure 5.

Figure 6: OLS regression on H-M model of GESSX fund

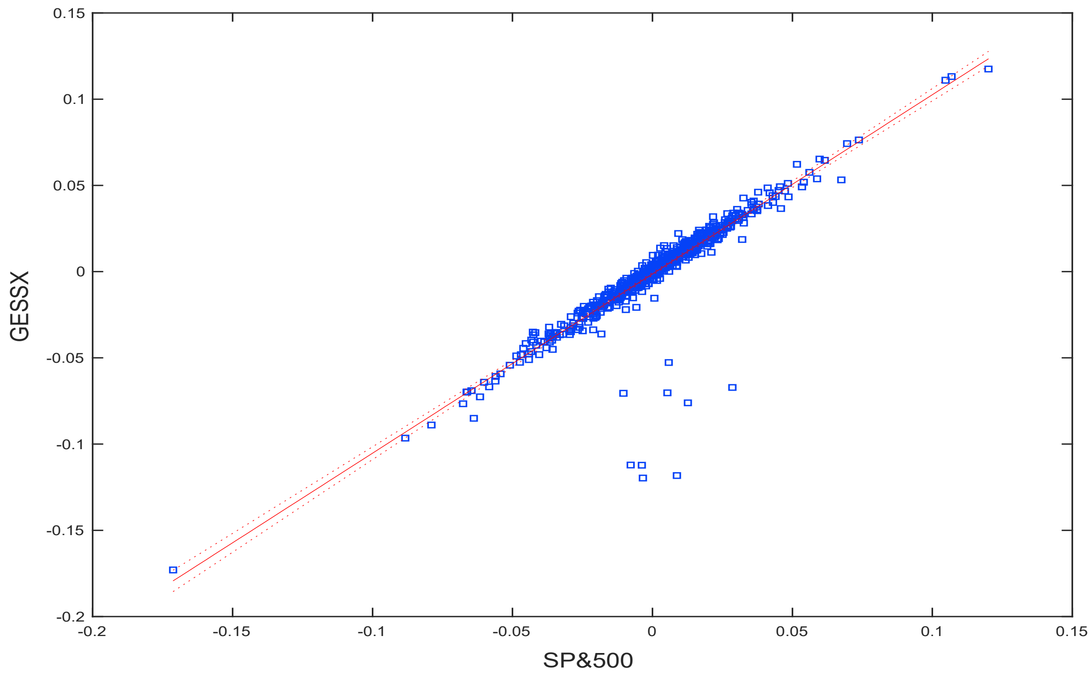


Table 3: Summary of market timing coefficients estimates for H-M.

Mutual Fund	H.M coeff.
DFEOX	0,0069
BRLIX	-0,0091
TIGRX	-0,0245
AFDAX	-0,0665*
BTEFX	-0,0105
JDEAX	0,0046
VTSMX	0,0034
IGIAX	0,0073
SNAEX	-0,0201
QAACX	-0,0043
MEFOX	-0,0409
SUWAX	-0,0587

Mutual Fund	H.M coeff.
VFTNX	0,0476
HAIAX	-0,0667*
CSXAX	-0,0078
PRDGX	-0,0192
GESSX	0,0593*
TISCX	-0,0254
PRBLX	-0,0418
GQEFX	-0,0387
CMNWX	-0,1093
AWEIX	-0,0113
WMLIX	0,0051

Table 4: H-M estimates whit OLS. *p<0,1; **p<0,05; ***p<0,001

Coefficients	OLS	SE	Tstat	pValue
		VFTNX		
Intercept	-0,0004	0,0003	-1,5921	0,1118
SP& 500	1,1275***	0,0320	35,2648	0,0000
H-M coeff.	0,0476	0,0361	1,3179	0,1879
		HAIAX		
Intercept	0,0007**	0,0003	2,9177	0,0036
SP& 500	0,9189***	0,0143	64,0739	0,0000
H-M coeff.	-0,0667*	0,0344	-1,9407	0,0527
		TISCX		
Intercept	-0,0001	0,0002	-0,6117	0,5409
SP& 500	1,0217***	0,0140	73,0552	0,0000
H-M coeff.	-0,0254	0,0305	-0,8330	0,4051
		BRLIX		
Intercept	-0,0001	0,0004	-0,3183	0,7503
SP& 500	0,9477***	0,0170	55,9004	0,0000
H-M coeff.	-0,0091	0,0595	-0,1535	0,8781
		MEFOX		
Intercept	4,26E-05	0,0004	0,1180	0,9061
SP& 500	0,9126***	0,0324	28,1308	0,0000
H-M coeff.	-0,0409	0,0376	-1,0903	0,276
		CSXAX		
Intercept	-0,0001	0,0002	-0,4621	0,6442
SP& 500	1,0225***	0,0192	53,1342	0,0000
H-M coeff.	-0,0078	0,0291	-0,2676	0,7891
		QAAXC		
Intercept	-0,0004	0,0004	-0,8798	0,3792
SP& 500	1,0977***	0,0222	49,4411	0,0000
H-M coeff.	-0,0043	0,0453	-0,0953	0,9241
		GQEFX		
Intercept	-0,0003	0,0006	-0,5177	0,6048
SP& 500	0,7289***	0,0408	17,8666	0,0000
H-M coeff.	-0,0387	0,0534	-0,7254	0,4684
		AFDAX		
Intercept	0,0007**	0,0002	3,0541	0,0023
SP& 500	0,951***	0,0163	58,2761	0,0000
H-M coeff.	-0,0665*	0,0362	-1,8362	0,0667

Coefficients	OLS	SE	Tstat	pValue
		IGIAX		
Intercept	-0,0003	0,0008	-0,3607	0,7184
SP& 500	0,8734***	0,0587	14,8691	0,0000
H-M coeff.	0,0073	0,1015	0,0717	0,9428
		VTSMX		
Intercept	-1,69E-05	0,0001	-0,1411	0,8878
SP& 500	1,0268***	0,0072	142,2161	0,0000
H-M coeff.	0,0034	0,0123	0,2757	0,7829
		BTEFX		
Intercept	0,0001	0,0004	0,3406	0,7335
SP& 500	0,9163***	0,0309	29,6394	0,0000
H-M coeff.	-0,0105	0,0491	-0,2143	0,8304
		PRBLX		
Intercept	0,0001	0,0005	0,2715	0,7861
SP& 500	0,852***	0,0272	31,3623	0,0000
H-M coeff.	-0,0418	0,0438	-0,9531	0,3409
		DFEOX		
Intercept	-0,0001	0,0002	-0,6032	0,5465
SP& 500	1,069***	0,0137	78,1528	0,0000
H-M coeff.	0,0069	0,026	0,2655	0,7907
		SNAEX		
Intercept	-0,0005	0,0003	-1,5946	0,1112
SP& 500	0,9931***	0,0136	72,8451	0,0000
H-M coeff.	-0,0201	0,0232	-0,8664	0,3866
		TIGRX		
Intercept	-0,0003	0,0004	-0,6602	0,5094
SP& 500	1,0129***	0,0183	55,3226	0,0000
H-M coeff.	-0,0245	0,0490	-0,5001	0,6171
		AWEIX		
Intercept	0,0000	0,0004	-0,0432	0,9655
SP& 500	0,9725***	0,0229	42,5565	0,0000
H-M coeff.	-0,0113	0,0422	-0,2671	0,7895
		JDEAX		
Intercept	-0,0005*	0,0003	-1,6680	0,0957
SP& 500	1,0363***	0,0126	82,2967	0,0000
H-M coeff.	0,0046	0,0218	0,2104	0,8334

Coefficients	OLS	SE	Tstat	pValue
		WMLIX		
Intercept	-0,0004	0,0003	-1,4012	0,1616
SP& 500	0,9917***	0,0094	105,2221	0,0000
H-M coeff.	0,0051	0,0154	0,3343	0,7382
		CMNWX		
Intercept	-2,94E-05	0,0008	-0,0376	0,9700
SP& 500	0,9702***	0,0305	31,7590	0,0000
H-M coeff.	-0,1093	0,1392	-0,7849	0,4328
		PRDGX		
Intercept	0,0001	0,0002	0,5337	0,5937
SP& 500	0,9176***	0,0091	100,6737	0,0000
H-M coeff.	-0,0192	0,0187	-1,0260	0,3052
		GESSX		
Intercept	-0,0013**	0,0006	-2,2145	0,0271
SP& 500	1,0376***	0,0247	41,9519	0,0000
H-M coeff.	0,0593*	0,0360	1,6486	0,0997
		SUWAX		
Intercept	-0,0005	0,0005	-0,9932	0,3209
SP& 500	1,0412***	0,0240	43,4563	0,0000
H-M coeff.	-0,0587	0,0599	-0,9810	0,3269

4.2 TREYNOR AND MAZUY MODEL: OLS ANALYSIS

The linear regression which has been estimated for the Treynor and Mazuy model it was build using the following formula:

$$R_p(t) - R_f(t) = \alpha + \beta_1\big[R_M(t) - R_f(t)\big] + \gamma\big[R_M(t) - R_f(t)\big]^2 + \varepsilon_t \qquad p = 1, \dots, 23$$

Where:
$R_p(t)$ = returns of each fund considered
$R_f(t)$ = returns of risk-free securities
$R_M(t)$ = returns of the benchmark considered (S&P500)

As in the H-M procedure, it should be noted that also in the T-M model the return of risk-free securities ($R_f(t)$) is not taken into consideration.
Exactly as in the previous section we have calculated the regression proposed above through MATLAB software applying an OLS estimator. The main goal of this calculation is to determine the coefficients of the regression for each fund and in particular to establish whether or not there is market timing. In the Treynor and Mazuy model the coefficient related to the market timing phenomenon is γ, indeed we are interested in verifying if γ is significantly different from zero or not. In the case in which the market timing coefficient is not significant the managers clearly do not have prediction skills, on the contrary if it is significant they have forecasting abilities.
As in the H-M model, also in this procedure (T-M) we will estimate the passive funds too. What do we expect is to not find evidence of market timing for those funds.

4.2.1 Analysis of OLS results

Analyzing the table above (table 6) in which are summarized the T-M model's estimates we can see that the coefficient associated to the intercept (α) is significantly different from zero in nine funds. As in the H-M model, α represents the selection abilities of the managers. In this case from the nine funds just in two of them the coefficient has a positive value (AFDAX, HAIAX): the advisors of these funds were able, by choosing wisely the stocks to gain a higher return through their abilities.

On the contrary we have that in 7 funds the coefficient associated to the selection skills is negative. In this case the managers clearly did not have the ability to select properly the fund's stocks, furthermore because of their wrong selection the returns of the fund is lower compared to the one that would have been obtained just by following the market's trend.

Exploring the market's return coefficient we can see, exactly how happened for the H-M model, that all the funds have really high significant β_1. The values related to the S&P500 goes from a minimum value 0,868 of PRBLX fund to a maximum value 1,108 of VFTNX fund.

Focusing now our attention on the coefficients associated to the market timing (γ) we can assert that γ is statistically significant in seven funds. In particular just in two of them GESSX (figure 7), VFTNX the market timing coefficient has a positive value.

Having a positive value of γ means that the managers have market timing skills and that they forecasted correctly the market changes and accordingly to their prediction they have changed properly the composition of the portfolio in order to increase the fund's return in respect with the one obtain through the market.

On the other hand in the other five funds the market timing coefficients are all negative. This means clearly that the managers do not have prediction skills. In particular, thinking that they were able to predict exactly the market changes, the managers decided to modify the portfolio composition, but this alteration of the portfolio has led to gain lower returns instead of increasing them. The managers clearly failed in predicting the market's changes.

Taking now in consideration the three passive funds we didn't get exactly the expected results. Indeed for VFTNX we have found that the coefficient associated to the market timing phenomenon is statistically significant. How is this possible? Because of the management fees. In particular for the VFTNX fund there is a cost of 0.11% per year for the management expenditures, this means that in order to pay this fee the returns of the fund has to exceed the market about that 0.11% : the funds is not just following the market but is outperforming it. For this reason it is plausible that VFTNX has a significant coefficient of market timing even if is a passive funds.

Considering the other two funds, as we expected there is no evidence of market timing in both of them.

Below are shown in table 5 the summary of market timing coefficients estimates.

The all OLS results in table 6, and the graphics of GESSX and GQEFX fund in figure 7.

Figure 7: OLS regression on T-M model of GESSX and GQEFX fund

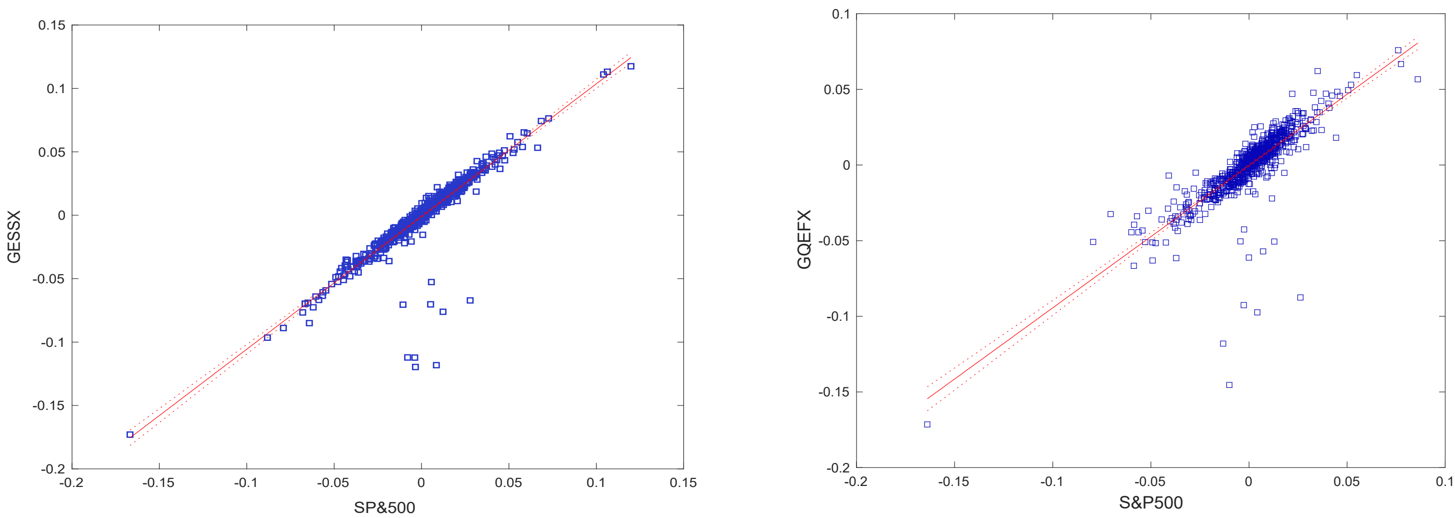


Table 5: Summary of market timing coefficients estimates for T-M.

Mutual Fund	T-M coeff.
DFEOX	0,1481
BRLIX	0,0378
TIGRX	0,1386
AFDAX	-0,3544*
BTEFX	0,1739
JDEAX	0,0282
VTSMX	0,0944
IGIAX	0,5182
SNAEX	0,0091
QAACX	-0,1042
MEFOX	-0,2098
SUWAX	-0,0601

Mutual Fund	T-M coeff.
VFTNX	0,5119**
HAIAX	-0,4936***
CSXAX	0,152
PRDGX	-0,1211**
GESSX	0,2757**
TISCX	-0,0091
PRBLX	-0,5103***
GQEFX	-0,5779**
CMNWX	-0,3035
AWEIX	0,0927
WMLIX	0,0448

Table 6: T-M estimates whit OLS. *p<0,1; **p<0,05; ***p<0,001

Coefficients	**OLS**	**SE**	**Tstat**	**pValue**
		AFDAX		
Intercept	0,0004**	0,0001	2,7503	0,0061
SP& 500	0,9823***	0,0138	70,9524	0,0000
SP& 500^2	-0,3544*	0,1976	-1,7938	0,0733
		BTEFX		
Intercept	-0,0001	0,0003	-0,2426	0,8084
SP& 500	0,9242***	0,0193	47,9374	0,0000
SP& 500^2	0,1739	0,3251	0,5351	0,5928
		DFEOX		
Intercept	-0,0002	0,0002	-1,0347	0,3012
SP& 500	1,0672***	0,0081	131,9158	0,0000
SP& 500^2	0,1481	0,1643	0,9016	0,3676
		HAIAX		
Intercept	0,0005**	0,0002	3,1301	0,0018
SP& 500	0,9485***	0,0099	96,1149	0,0000
SP& 500^2	-0,4936***	0,1127	-4,3811	1,35E-05

Coefficients	OLS	SE	Tstat	pValue
		PRBLX		
Intercept	0,0001	0,0003	0,2610	0,7941
SP& 500	0,868***	0,0146	59,3384	0,0000
SP& 500^2	-0,5103***	0,1481	-3,4463	0,0006
		QAACX		
Intercept	-0,0003	0,0003	-1,2860	0,1988
SP& 500	1,0987***	0,0153	71,6839	0,0000
SP& 500^2	-0,1042	0,1788	-0,5826	0,5603
		TIGRX		
Intercept	-0,0005	0,0003	-1,5936	0,1115
SP& 500	1,0278***	0,0217	47,3561	0,0000
SP& 500^2	0,1386	0,2642	0,5245	0,6001
		VTSMX		
Intercept	-4,35E-05	0,0001	-0,5011	0,6165
SP& 500	1,0262***	0,0046	223,7593	0,0000
SP& 500^2	0,0944	0,0667	1,4168	0,1570
		AWEIX		
Intercept	-0,0002	0,0003	-0,6180	0,5367
SP& 500	0,9797***	0,0138	70,9995	0,0000
SP& 500^2	0,0927	0,1939	0,4784	0,6325
		CMNWX		
Intercept	-0,0008*	0,0004	-1,9116	0,0563
SP& 500	1,0251***	0,0454	22,6011	0,0000
SP& 500^2	-0,3035	0,6649	-0,4565	0,6482
		GESSX		
Intercept	-0,001**	0,0004	-2,27140	0,0234
SP& 500	1,0092***	0,0115	87,48590	0,0000
SP& 500^2	0,2757**	0,1208	2,28130	0,0228
		IGIAX		
Intercept	-0,0005	0,0005	-1,0155	0,3102
SP& 500	0,876***	0,0365	23,9795	0,0000
SP& 500^2	0,5182	0,7133	0,7265	0,4678
		MEFOX		
Intercept	-0,0002	0,0003	-0,67850	0,4977
SP& 500	0,9319***	0,0182	51,26740	0,0000
SP& 500^2	-0,2098	0,1965	-1,06790	0,2859

Coefficients	OLS	SE	Tstat	pValue
		SNAEX		
Intercept	-0,0007**	0,0003	-2,0218	0,0436
SP& 500	1,0041***	0,0135	74,3728	0,0000
SP& 500^2	0,0091	0,0668	0,1357	0,8921
		TISCX		
Intercept	-0,0003	0,0002	-1,5658	0,1178
SP& 500	1,0352***	0,0127	81,6251	0,0000
SP& 500^2	-0,0091	0,0984	-0,0925	0,9263
		WMLIX		
Intercept	-0,0003*	0,0002	-1,8083	0,071
SP& 500	0,9895***	0,0059	166,3201	0,0000
SP& 500^2	0,0448	0,0685	0,6543	0,5131
		BRLIX		
Intercept	-0,0002	0,0002	-0,8889	0,3744
SP& 500	0,953***	0,0200	47,6890	0,0000
SP& 500^2	0,0378	0,2376	0,1593	0,8735
		CSXAX		
Intercept	-0,0003	0,0002	-1,4815	0,1389
SP& 500	1,0286***	0,0109	94,3041	0,0000
SP& 500^2	0,152	0,1239	1,2268	0,2203
		GQEFX		
Intercept	-0,0003	0,0005	-0,6315	0,5279
SP& 500	0,7425***	0,0260	28,532	0,0000
SP& 500^2	-0,5779**	0,2366	-2,443	0,0148
		JDEAX		
Intercept	-0,0005*	0,0003	-1,8618	0,063
SP& 500	1,0342***	0,0105	98,7109	0,0000
SP& 500^2	0,0282	0,0739	0,3820	0,7025
		PRDGX		
Intercept	1,68E-05	0,0002	0,0943	0,9249
SP& 500	0,9264***	0,0082	113,2612	0,0000
SP& 500^2	-0,1211**	0,0587	-2,0641	0,0394

Coefficients	OLS	SE	Tstat	pValue
SUWAX				
Intercept	-0,0009**	0,0004	-2,2508	0,0247
SP& 500	1,072***	0,0229	46,8203	0,0000
SP& 500^2	-0,0601	0,2834	-0,2121	0,8321
VFTNX				
Intercept	-0,0004*	0,0002	-1,9356	0,0533
SP& 500	1,1083***	0,0212	52,2311	0,0000
SP& 500^2	0,5119**	0,1892	2,7062	0,0070

4.3 CONCLUSION

After exploring in both H-M and T-M models the managers forecasting skills we can conclude that there is no particular evidence of market timing phenomenon.
Therefore in all our analysis we were able to spot just three funds in which the market timing coefficient was positive and significant, this led us to think that the managers able to predict exactly the market are just a few and that much more often the managers make wrong predictions about the market's changes, decreasing the fund's return. Because all of these proof we can conclude that, at least in our empirical analysis, the presence of market timing phenomenon is it really low.

However it is important to highlight that the proposed studies about market timing are influenced by the type of estimator that we have chosen, therefore the Ordinary Least Squares estimator just gives us an overview about the phenomenon. As we have widely discuss in the previous chapters the OLS explores the relationship between a set of independent variables and the conditional mean of a dependent variable Y not giving therefore a complete and detailed picture of the phenomenon. Consequently it stands to reason that with the OLS estimates we did not find an important evidence of market timing. Furthermore in support of this analysis it is important to say that the same empirical studies have been done with the monthly fund's returns but the result weren't significant at all so it was necessary to take into consideration a larger sample (weekly returns of the funds).
What do we expect from the usage of Quantile Regression is to spot much more significant coefficients in particular those referred to the market timing phenomenon as the QR explores the relationship between the independent variables and the conditional quantiles of the dependent one, allowing to gain further insight and to obtain a much more complete statistical picture than the OLS. Because all of these reasons we believe that the results which will be obtained through the usage of the QR will show much more evidence of the market timing phenomenon.
Furthermore considering the three passive funds we expect that the quantile regression calculation will show some evidence of market timing even in these funds, in particular for two reason. The first one because we have shown that even if the funds are run with a passive approach it is possible that it exceeds the market, while, the second one is because QR analyzes the phenomenon at each quantile that we consider, therefore its analysis is much more detailed than the OLS and consequently could be possible to finds some evidence of market timing even in the passive funds.

4.4 HENRIKSSON AND MERTON MODEL: QUANTILE REGRESSION ANALYSIS

As we have already mentioned, in this section our goal is to estimates the H-M model through the usage of quantile regression hoping to analyze the model more in detail focusing our attention on the market timing phenomenon. In order to do this we have applied the QR regression to the H-M model estimating for each fund, nineteen equidistant quantiles (0.05,0.1,0.15…0.90,0.95). Noticed that, as in the OLS case, α , β_1, β_2 represent respectively the intercept (security analysis coefficient), the coefficient associated to the market's returns (S&P500) and the market timing coefficient. The results that we have found are presented below in the table 7 and in the figure 11-13.

4.4.1 The Wald Test

Before proceeding with the explanation of our results it is fundamental to introduce the Wald test.
As we have already mentioned in the previous chapters the traditional methodology of linear regression (the OLS regression) supposes that the coefficients of different quantiles have the same slope throughout the entire distribution but this does not happen for Quantile Regression. Indeed one of the main features of the QR is that the slopes' parameters differ from each other across the quantiles: each quantile "contains" different information about the distribution which allows to gain more information from the quantile regression distribution than the OLS one.
The Wald test verifies if the null hypothesis (same slope for the coefficients through the quantiles) is rejected or not: in the case H_0 is accepted the quantile regression shouldn't be applied (the OLS gives all the necessary information), on the contrary if the null hypothesis is rejected than QR has to be used.

Applying the Wald test to our empirical data we have tried to verify if the coefficient's slopes for each quantile considered (in our study 19) are the same or not. In particular we found that in the following funds is not necessary to implement quantile regression (H_0 is accepted): SNAEX, PRDGX, AWEIX, BRLIX and AFDAX.
The Wald test statistics for each funds are respectively: 1.1972, 1.1964, 1.1887, 1.0219, 1.2535. With these statistics we have to accept the null hypothesis of equality slopes through the quantiles and therefore for these elements the quantile regression is not necessary to be applied: the OLS can well explain the regression.

4.4.2 The intercept (security analysis coefficient)

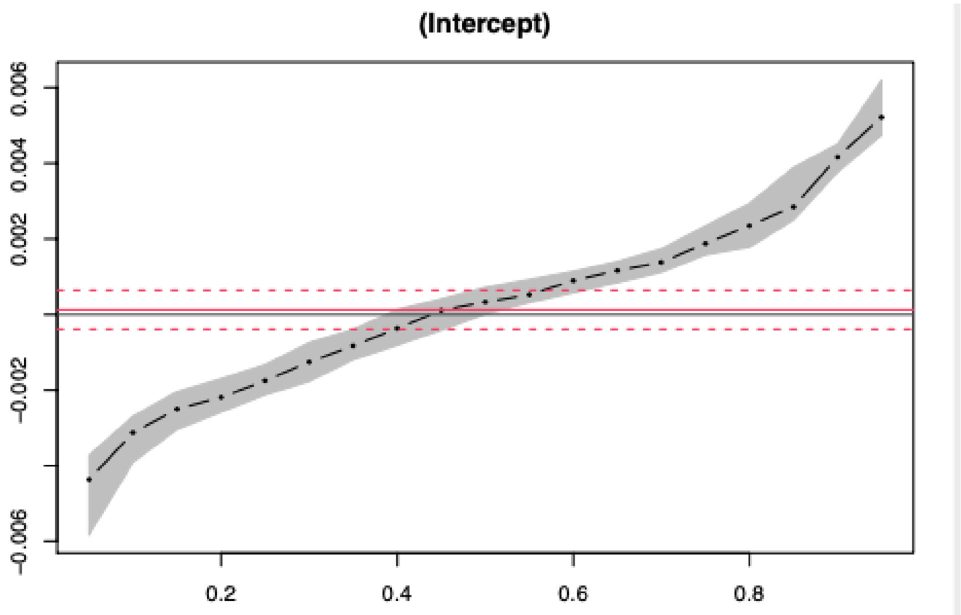
Analyzing the QR estimates it is straightforward to verify that almost always the coefficients associated to the intercept (α) are significantly different from zero, but this does not happen around the central quantiles (from 0,40 to 0.55 quantile), in which the coefficients are not significant.

If we focus our attention on the trend of the intercept we can notice that it is increasing as the order of the quantiles is increasing. In particular this upwards trend could be really interesting to be analyzed from an economic point of view because studying the coefficients' trend allows us to explore how the selection abilities of the managers are changing through the quantiles of all the distribution. In particular for each fund, we can see that in the first quantile (τ = 0,05), the intercept's coefficient is always negative: the managers do not have selection skills, on the contrary they are decreasing the fund's returns with their wrong choices. Going through the quantiles in the distribution we can see that for each fund there is a point in which α becomes positive (i.e. for BTEFX α is positive in τ = 0,45). This means that the managers are choosing properly the stocks of the funds and that through their selection abilities they are increasing the fund's returns. It should be noticed that form the τ = 0,5 quantile all the funds have a positive value of the intercept. In conclusion we can assert that the coefficients related to the selection skills have an increasing trend (positive value from τ = 0,5) and therefore the managers choices have a good impact on the fund and its returns are increasing. Furthermore talking about the trend it should be noted that in the central quantiles, as we expected, the intercept value is nearly zero: this happens because the returns on average are really close to zero.

If we make a brief comparison with the results got from the OLS and the ones obtained with the QR is clear that the latter can give much more information that the first. Indeed, in the OLS case we have just four funds in which the intercept is statistically significant and consequently the studying which we can do on those coefficients is limited. On the other hand in the QR case most of the coefficients are significant through all the quantiles and because of this we have the possibility to study more and get much more information about the security analysis phenomenon.

In the figure 8 is shown the intercept of the BTEFX fund: it should be noted the upwards trend of the coefficient through the quantiles and that from the 0.5 quantile the coefficient becomes positive.

Figure 8. BTEFX intercept.



4.4.3 The benchmark's coefficient (S&P500).

Exploring the market's return coefficients we can see, exactly as happened with the OLS estimates, that the coefficients are always high significant for each fund at each quantiles.
What is interesting to explore about the market's return coefficient it is its development through all the distribution. In particular we can see that the coefficients' trend is not well defined: it is increasing as the order of quantiles is increasing but it has some downward peaks all over the distribution. So, in contrast to α we are not able to give a precise description of the trend but could be useful to show some examples about it in the above figures.

BTEFX: S&P500 coeff. HAIAX: S&P500 coeff.

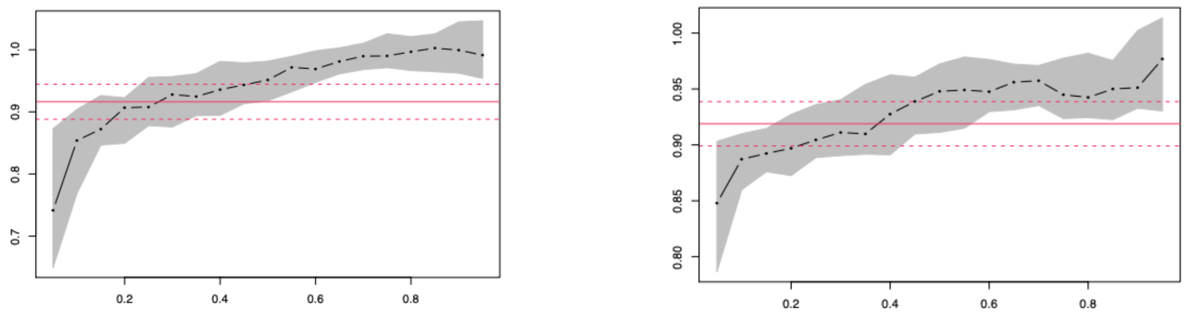


Figure 9. S&P500 coefficient of BTEFX,HAIAX

As we have seen, β_1 is increasing as the order of quantiles is increasing that is, higher the quantiles considered higher is its value. But what does it mean from an economic point of view? When beta has a high and positive value the coefficient can be interpreted as a first sign of managers' market timing abilities. Therefore as we have already pointed, the manager who are able to predict the market trend, changes the value of the benchmark's coefficient accordingly to his prediction of an up or a down-market. Consequently a high and positive level of β_1 is an indicator of manager's abilities in predicting an up-market, and thus increasing the risk exposure of the fund to gain higher returns. Furthermore, of course a positive and high value of the index's coefficient have consequently a positive impact on the fund's returns.

On the contrary when there is a down-market the managers try to reduce the risk exposure of the fund, decreasing the value of β_1, but the low value of the benchmark's coefficient has of course a negative impact on the fund's returns which becomes negative (usually in the lower quantiles). In this case, in which the market is bearish and the fund's returns are negative the impact of the market timing coefficient is very relevant. But as we have said in a down-market the managers try to reduce the risk for the fund and therefore they are less active and risk less, consequently the fund's returns will decrease even more.

4.4.4 The H-M coefficient

Last but not least, we focus our attention on the H-M coefficients.
Exploring β_2 it is possible to notice that the coefficient, especially in the central quantiles is not always statistically significant. Therefore exploring carefully our empirical results we can see that generally the market timing coefficients are significantly different form zero especially in the lower (from τ = 0,05 to τ = 0,3) and the upper quantiles (from τ = 0,75 to τ = 0,95), while in the central ones there is no significance (in particular from τ = 0,4 to τ = 0,65) . Of course this is just a general analysis, indeed each fund has its own distribution and significance level of the coefficients, for example the fund BTEFX has a significant H-M coefficient even in the central quantile τ = 0,5.

What we are really interested in interested in exploring the development of the market timing phenomenon through the quantiles, in particular, exactly as the S&P500 coefficients, we cannot define precisely the trend of the market timing phenomenon. Therefore it has not developed in a regular way: it is increasing as the order of quantiles is increasing but it has many downward peaks all over the distribution. In particular exploring the results we can see that generally (of course with some exceptions) in the low quantiles market timing has a negative impact on the fund's returns: the managers did not predict exactly the market changes (clearly they do not have market timing skills) and consequently they affected negatively the fund. On the other hand going through the distribution, increasing the order of quantiles it is possible to notice how the market timing coefficients increase. In particular what can we spot is that from τ = 0,55 until the last quantile considered (0.95) in all the funds in which the H-M coefficient is significant we can see that the market timing coefficients have a positive value too and consequently a positive impact in the fund's return. This means that going through the quantiles the market timing abilities of the managers are changing: they are able to predict the market properly and consequently to change the portfolio composition in order to gain higher returns. Although it should be noted that we can find a strong evidence of market timing phenomenon especially from τ = 0,75 to τ = 0,95 because in this quantiles a large part of the funds have significant H-M coefficients and has we already said positive values too, while in the other quantiles the level of significance is limited just to a small number of funds.

Could be interesting to make a brief comparison between the OLS results and the QR ones. In particular what is really important to highlight is that with the Ordinary Least Squares regression we were able to spot just two funds in which the managers had market timing skills.

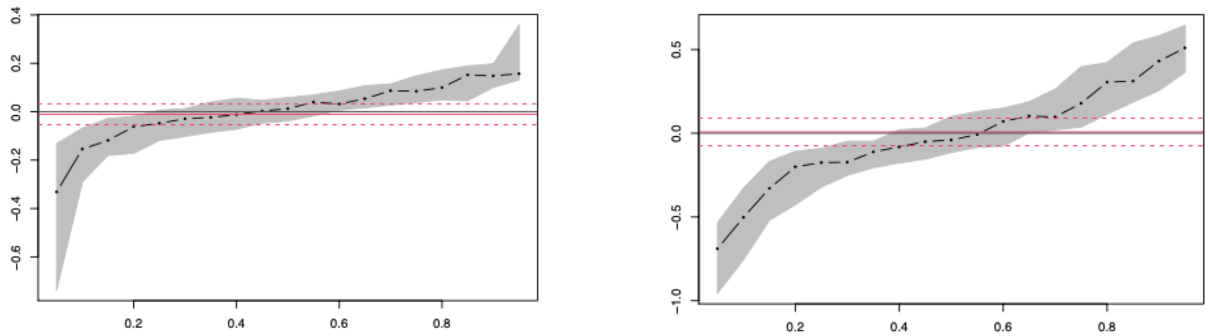
In the other estimated funds either the coefficient was not significant or it was negative and this was the proof that the managers did not have prediction abilities. Furthermore having so few significant and positive coefficients on which build our studies has made a bit complicated to explore the phenomenon as there wasn’t a great evidence of market timing. On the other hand with the quantile regression estimates we found anyway non significant or negative β_2 but at the same time we found many positive and significant coefficients which allow us to study more in detail the market timing: we were able to explore the phenomenon all over the distribution and verify the changes over the quantiles.

In the figure below (10) we can see the market timing coefficient development for BTEFX and IGIAX funds: notice that they have an upward trend with many downward peaks all over the distribution.

Figure 10. H-M coefficient of IGIAX, BTEFX

BTEFX: H-M coeff

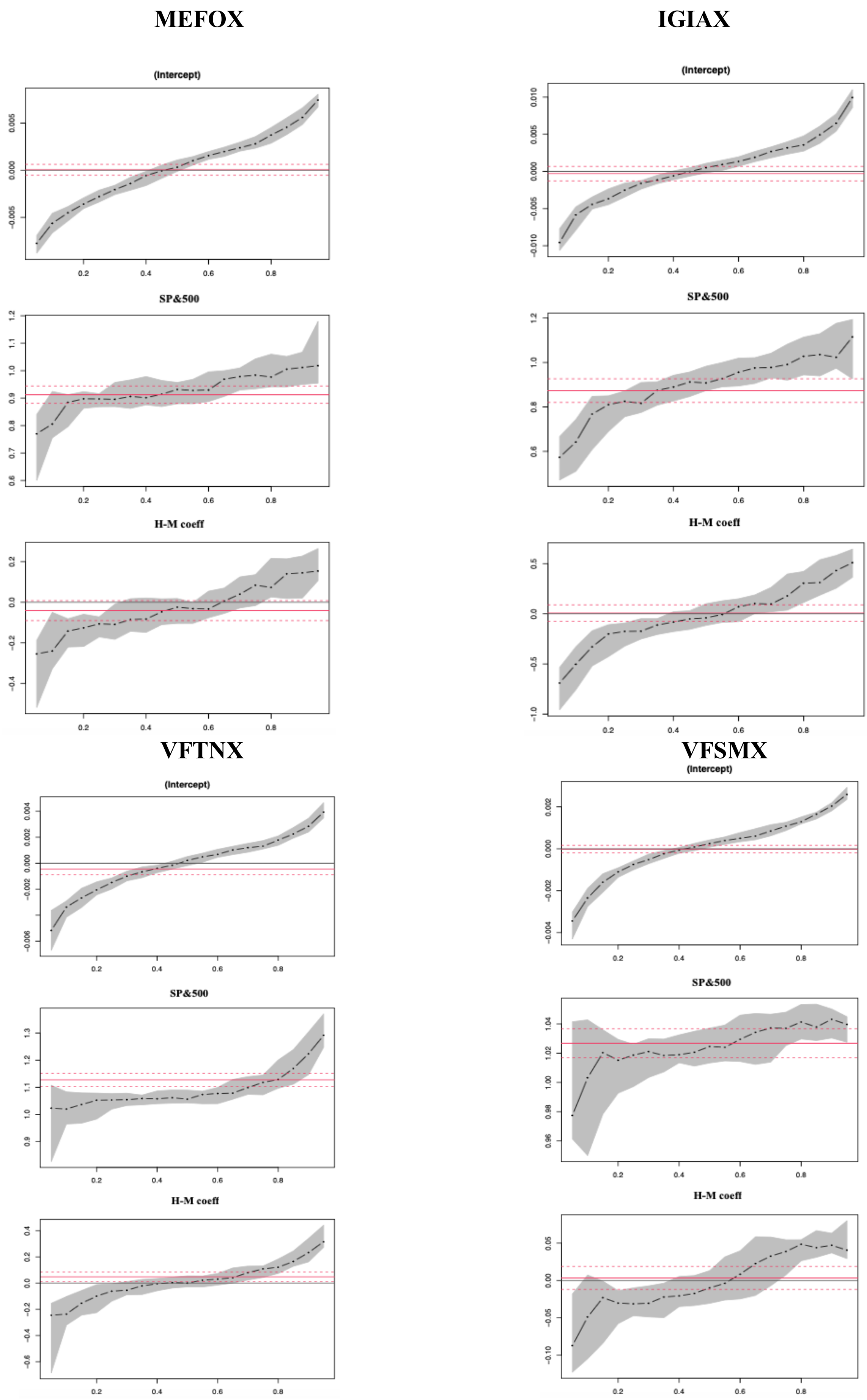
IGIAX: H-M coeff.



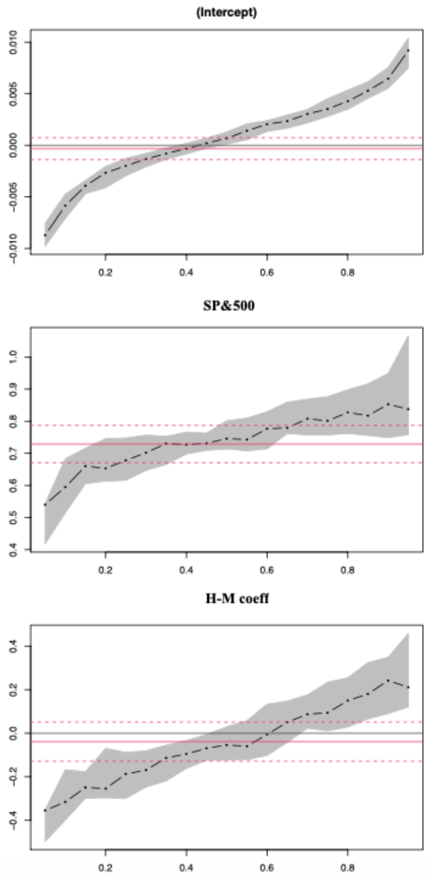
The figure 11 below shows the graphics for quantile regression: each fund distribution is represented by three different pictures in which are represented the intercept, the benchmark and the H-M coefficients. In the figure are shown the graphics of those funds in which is possible to appreciate more the quantile regression distribution and the coefficient’s trend that we have analyzed before.

Notice that the table 7 represents the QR estimates but not in all the distribution just in some quantiles ($\tau = 0.05, 0.3, 0.5, 0.8, 0.95$) in order to give a general idea about the empirical data. The figure 12 is a scatterplot of QR regression of IGIAX fund in which each line represents a quantile.

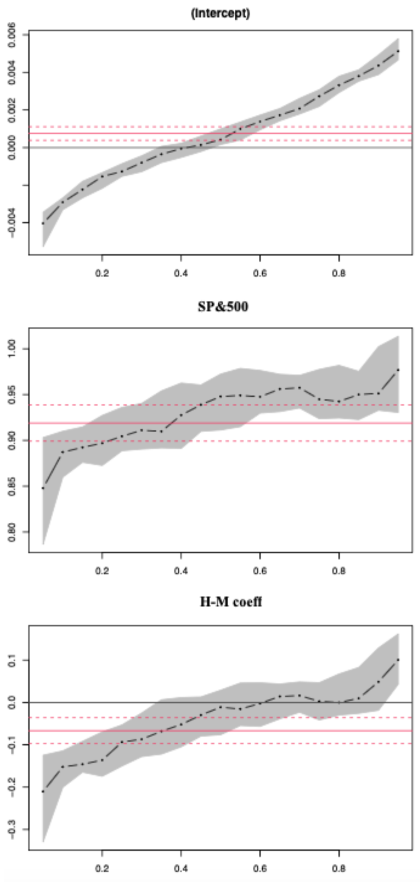
Figure 11. Coefficient’s estimates with QR (H-M model).



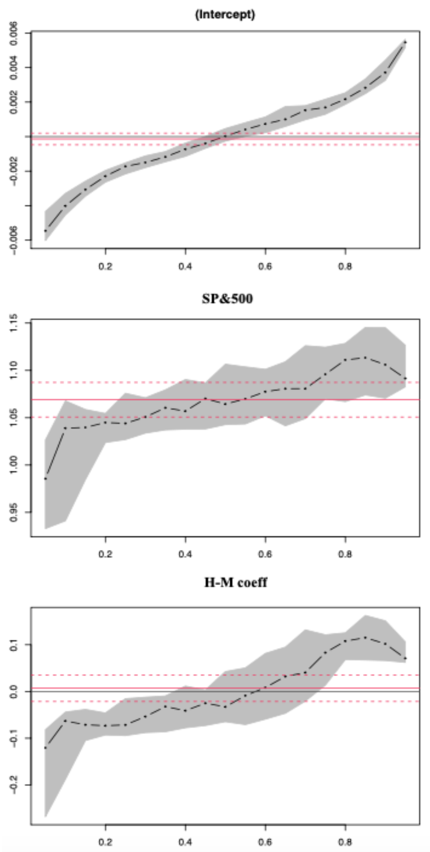
GQEFX



HAIAX



DFEOX



CSXAX

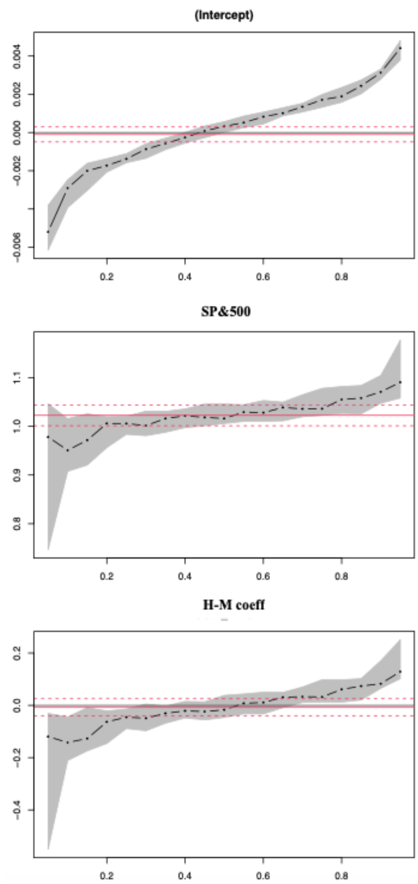
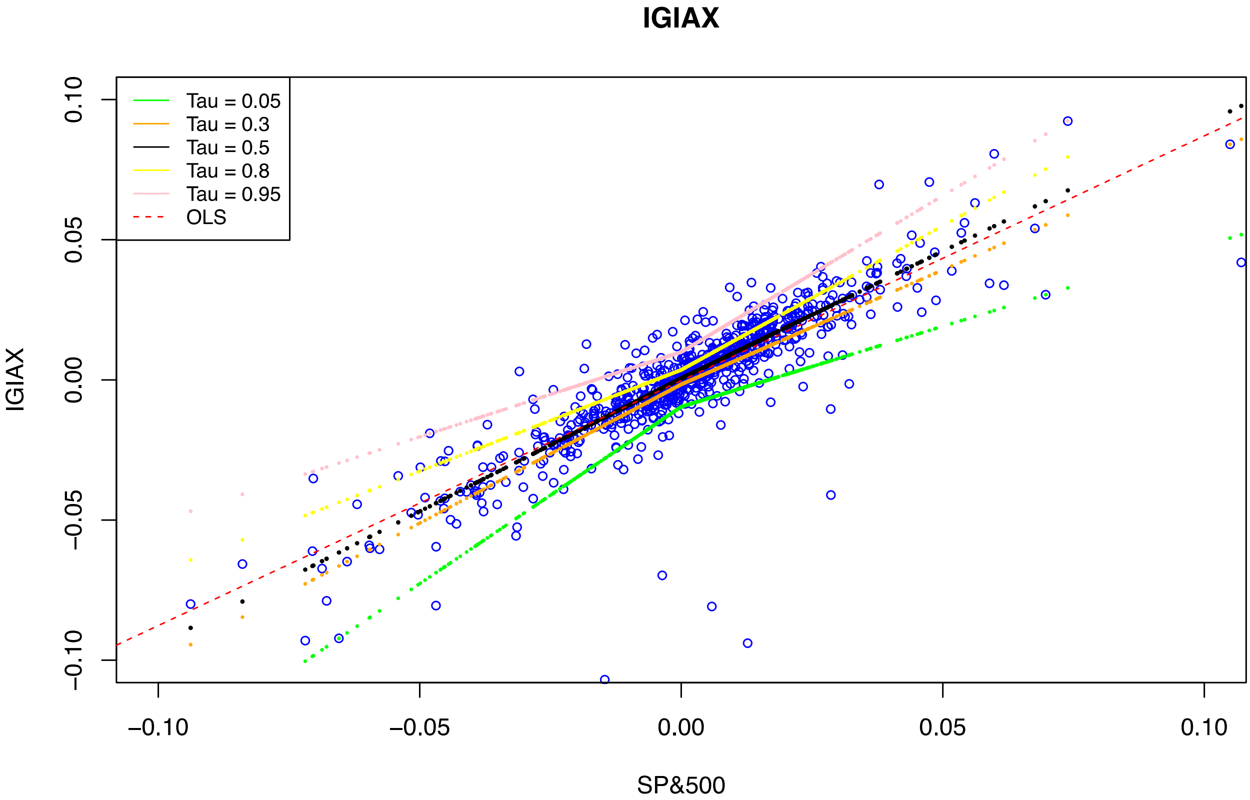


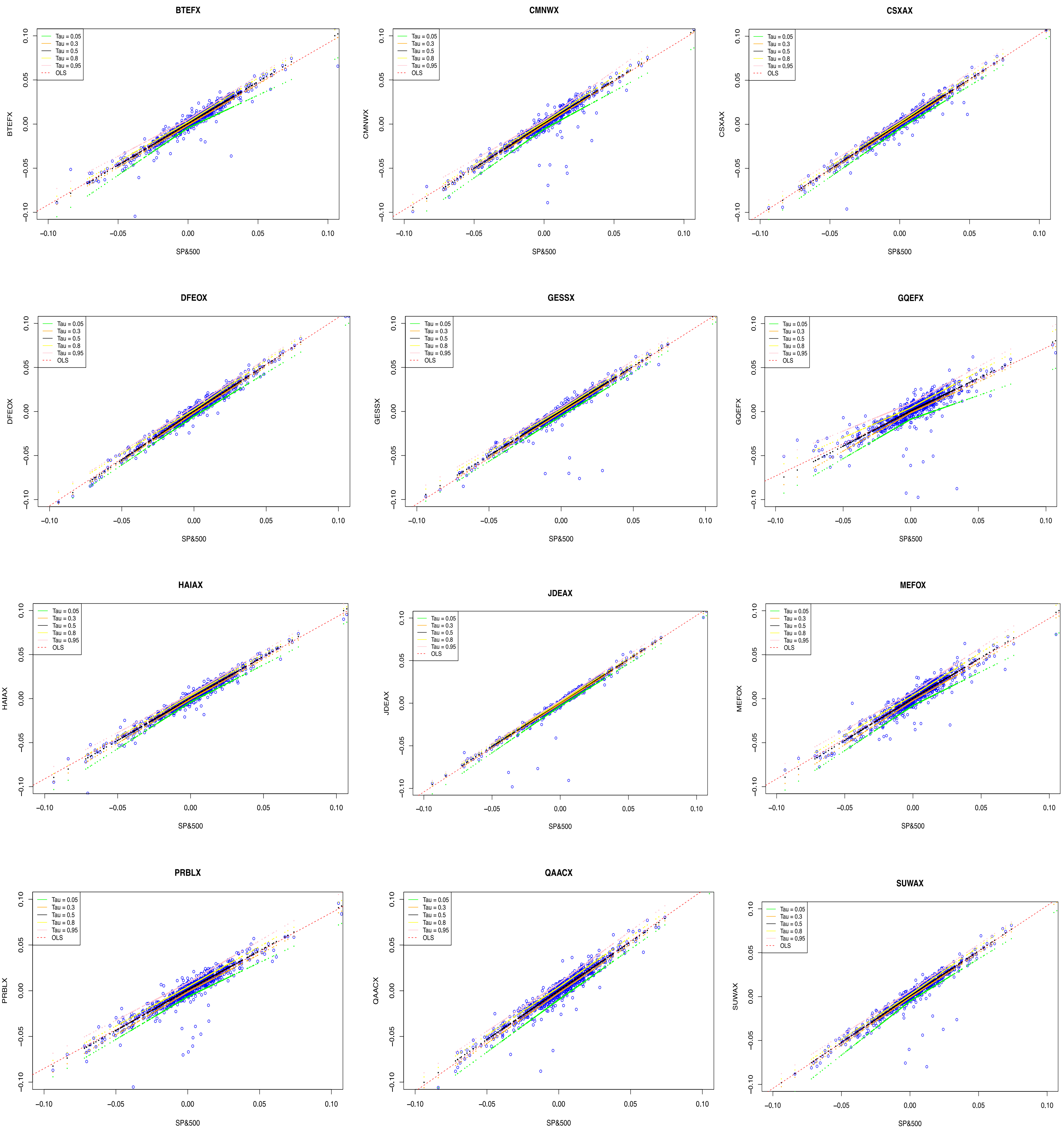
Figure 12. Scatterplots of the fund IGIAX: quantile regression estimates (H-M model)

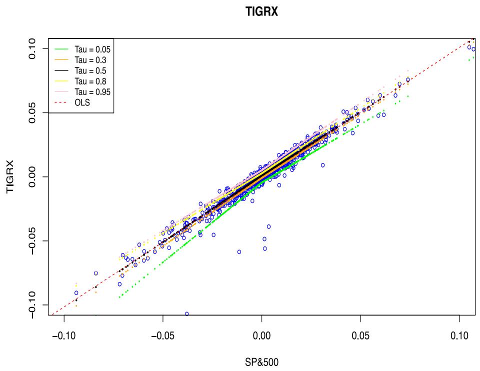


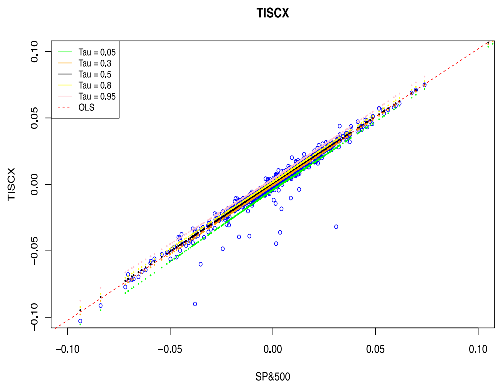
This figure shows the quantile regression estimates where each line correspond to a defined quantile. In this case we have considered τ =0.05,0.3,0.8,0.95.

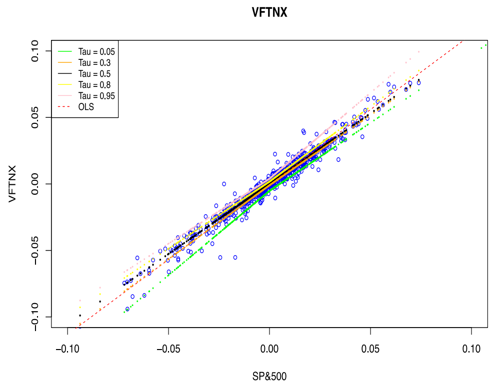
Could be interesting to notice that for high or lower level of market's returns (in upper and lower quantiles) the dispersion of the funds' return increases. This could depend on an increase in volatility when the returns of the market are located in extreme values of the distribution. Therefore the extreme quantiles depends on the variance of the funds' returns which depends on the variance of the market which changes over time. This phenomenon in which there is a problematic volatility could lead to bias estimates. Because the data of our sample are affected by heteroskedasticity and their volatility changes over time and the variance of the funds is really related to the market one, we could have bias results (they could be not reliable). In order to overcome this problem, that is to be sure that the returns are not affect by problematic volatility, it is possible to use a GARCH model to calculate the "new" returns (of the funds and the benchmark) and to estimates once more the quantile regression in order to have more reliable data.

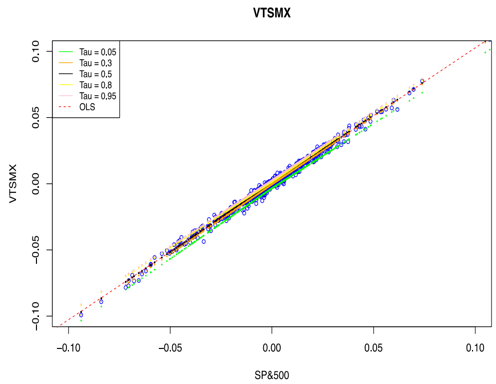
Figure 13. Scatterplots of the funds: quantile regression estimates (H-M model)




TIGRX
Tau = 0.05
Tau = 0.3
Tau = 0.5
Tau = 0.8
Tau = 0.95
OLS
TIGRX
SP&500


TISCX
Tau = 0.05
Tau = 0.3
Tau = 0.5
Tau = 0.8
Tau = 0.95
OLS
TISCX
SP&500


VFTNX
Tau = 0.05
Tau = 0.3
Tau = 0.5
Tau = 0.8
Tau = 0.95
OLS
VFTNX
SP&500


VTSMX
Tau = 0.05
Tau = 0.3
Tau = 0.5
Tau = 0.8
Tau = 0.95
OLS
VTSMX
SP&500

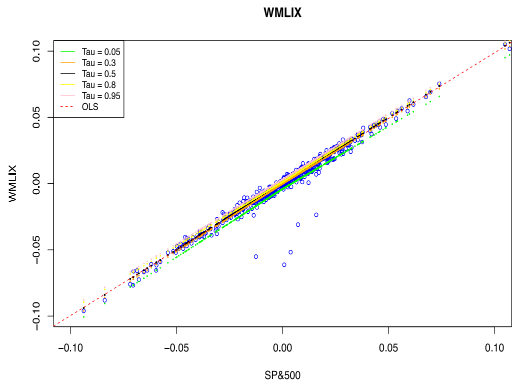

WMLIX
Tau = 0.05
Tau = 0.3
Tau = 0.5
Tau = 0.8
Tau = 0.95
OLS
WMLIX
SP&500

Table 7: H-M estimates whit QR. *p<0,1; **p<0,05; ***p<0,001

Coefficients	τ = 0.05	τ = 0.3	τ = 0.5	τ = 0.8	τ = 0.95
BTEFX					
Intercept	-0,0043***	-0,0012***	0,0003	0,0023***	0,0051***
SP& 500	0,7417***	0,9278***	0,9513***	0,9952***	0,9919***
H-M coeff.	-0,3313***	-0,0292	0,0126	0,0983***	0,1580**
CMNWX					
Intercept	-0,0057***	-0,0009***	0,0003**	0,0022***	0,0043***
SP& 500	0,8600***	0,9870***	0,9844***	1,0256***	1,1164***
H-M coeff.	-0,2452**	-0,0426**	-0,0255	0,055**	0,2133***
CSXAX					
Intercept	-0,0051***	-0,0008***	0,0003*	0,0018***	0,0044***
SP& 500	0,977***	1,0013***	1,016***	1,0553***	1,090***
H-M coeff.	-0,1194**	-0,0491**	-0,0169	0,0618**	0,1290**
DFEOX					
Intercept	-0,0054***	-0,0014***	0,0000	0,0021***	0,0054***
SP& 500	0,9855***	1,0507***	1,064***	1,1015***	1,0916**
H-M coeff.	-0,1205***	-0,0538**	-0,0329*	0,0980***	0,0703**
GESSX					
Intercept	-0,0051***	-0,0011***	0,0001	0,0022***	0,0045***
SP& 500	0,9947***	1,0056***	1,0245***	1,0362***	1,0661***
H-M coeff.	-0,0365	-0,0128	0,0195	0,0733**	0,1273***
GQEFX					
Intercept	-0,0087***	-0,0013**	0,0006*	0,0042***	0,0090***
SP& 500	0,5403***	0,7020***	0,7458***	0,8257***	0,8486***
H-M coeff.	-0,3542**	-0,1687***	-0,0525	0,1478***	0,2241***
HAIAX					
Intercept	-0,0040***	-0,0008**	0,0004*	0,0033***	0,0051***
SP& 500	0,8479***	0,911***	0,9479***	0,9418***	0,9768***
H-M coeff.	-0,2102***	-0,0869***	-0,0112	-0,0008	0,1003***
IGIAX					
Intercept	-0,0095***	-0,0016***	0,0005	0,0035***	0,009***
SP& 500	0,5733***	0,8163***	0,908***	1,027***	1,1149***
H-M coeff.	-0,6894***	-0,1733***	-0,0404	0,3051***	0,5096**
JDEAX					
Intercept	-0,0033***	-0,0006***	0,0001	0,0014***	0,0026***
SP& 500	0,9904***	1,0099***	1,0269***	1,028***	1,076***
H-M coeff.	-0,1080**	-0,0205	0,0095	0,0211**	0,081***

Coefficients	τ = 0.05	τ = 0.3	τ = 0.5	τ = 0.8	τ = 0.95
MEFOX					
Intercept	-0,0077***	-0,002***	0,0003	0,0037***	0,0074***
SP& 500	0,7701***	0,8956***	0,9316***	0,9767***	1,0183***
H-M coeff.	-0,2542***	-0,1089**	-0,0239	0,0731**	0,152**
PRBLX					
Intercept	-0,0066***	-0,0013***	0,0008**	0,0042***	0,0085***
SP& 500	0,7492***	0,825***	0,857***	0,881***	0,917***
H-M coeff.	-0,1832**	-0,109**	-0,0299	0,0244	0,0973**
QAACX					
Intercept	-0,0097***	-0,002***	0,0000	0,0038***	0,007***
SP& 500	1,1059***	1,067***	1,0701***	1,105***	1,154***
H-M coeff.	-0,0411	-0,0382	0,0018	0,057*	0,114**
SUWAX					
Intercept	-0,005***	-0,001***	0,0001	0,002***	0,006***
SP& 500	0,961***	1,011***	1,037***	1,087***	1,0661***
H-M coeff.	-0,271***	-0,071***	-0,0098	0,0633**	0,0550
TIGRX					
Intercept	-0,004***	-0,0008***	0,0007***	0,0027***	0,005***
SP& 500	0,909***	0,989***	0,996***	1,015***	1,025***
H-M coeff.	-0,334***	-0,075***	-0,038**	0,079***	0,0826***
TISCX					
Intercept	-0,004***	-0,0007***	0,0003**	0,0019***	0,0038***
SP& 500	1,026***	1,011***	1,014***	1,025***	1,046***
H-M coeff.	-0,053**	-0,024*	0,0016	0,0224	0,0728
VFTNX					
Intercept	-0,0051***	-0,0009***	0,0002	0,0017***	0,0038***
SP& 500	1,0227**	1,054***	1,056***	1,129***	1,292***
H-M coeff.	-0,246***	-0,054**	-0,0003	0,121***	0,317***
VTSMX					
Intercept	-0,0034***	-0,0005***	0,0002**	0,001***	0,0025***
SP& 500	0,977***	1,021***	1,024***	1,041***	1,040***
H-M coeff.	-0,086***	-0,030**	-0,0100	0,0485***	0,041*
WMLIX					
Intercept	-0,0039***	-0,0001	0,0004***	0,0013***	0,003***
SP& 500	0,944***	0,970***	0,9890***	0,997***	0,994**
H-M coeff.	-0,085***	-0,052***	-0,016*	0,029**	0,025**

4.5 TREYNOR AND MAZUY MODEL: QUANTILE REGRESSION ANALYSIS

In this section our goal is to use quantile regression to estimates the T-M model, calculating for each fund the QR in nineteen equidistant quantiles (0.05,0.1,0.15…0.90,0.95), hoping to get from the estimates more information about the market timing phenomenon than the OLS.
It should be noticed that exactly as in the OLS case, α, β_1, γ, represent respectively the intercept (security analysis), the coefficient associated to the market's returns (S&P500) and the market timing coefficient.

4.5.1 The Wald test

We have applied the Wald test to our empirical data in order to determine whether or not is necessary to implement the Quantile regression. In particular we have tried to verify if the coefficient's slopes for each quantile considered (in our study 19) are the same (that is H_0).
If the null hypothesis is rejected it means that the coefficient's slope are not the same and that quantile regression can be applied, on the contrary if H_0 is accepted there is no need to use QR because the OLS gives all the necessary information.

Calculating the Wald test we have accepted the null hypothesis for the following funds: TISCX, AWEIX, BRLIX, AFDAX, JDEAX, SUWAX, CSXAX.
The Wald test statistics for each fund are respectively: 1.0562, 1.16, 0.9711, 1,2953, 0.9578, 0.9953, 0.7508, these thresholds makes us to accepts H_0.
Accepting the null hypothesis we determine that there is equality coefficient's slopes through the quantiles and therefore quantile regression is not necessary to be applied for the mentioned funds.

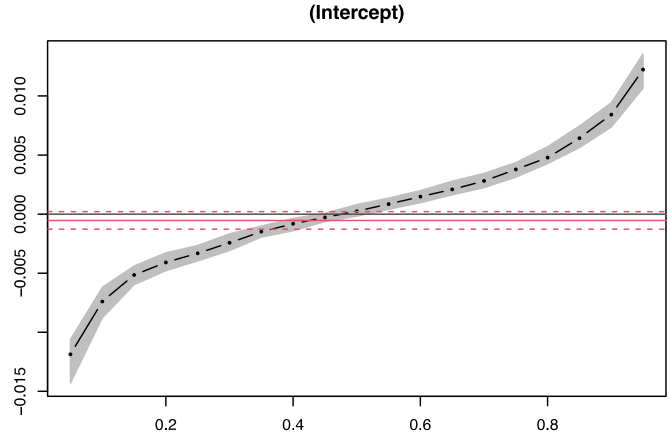
4.5.2 The intercept (security analysis coefficient)

Analyzing the QR estimates it is straightforward to verify that almost always the coefficients associated to the intercept (α) are significantly different from zero, but this does not happen in the central quantiles in particular in τ = 0.45 in which the coefficients associated to the intercept are not statistically significant for eleven funds. However in all the other quantiles α riches a very good level of significance.

If now we focus our attention on the values and on the trend of the intercept we can notice that it is increasing as the order of the quantiles is increasing. In particular we can see that starting from τ = 0.05 all the funds have negative α, it means that the managers do not have selection skills, but going through the quantiles it is possible to see that the coefficients values are slowly increasing and becoming positive. In particular in the lower quantiles α has still negative values for some funds (not all of them), while in the upper quantiles (from τ = 0.55 to τ = 0.95) all the coefficients are increasing and positive. When the coefficient value becomes positive means that the managers are able to select properly the stocks and thanks to this selection ability they are increasing the fund's return compared to the one which would have been obtained just following the benchmark. Therefore we can conclude that generally, the security analysis coefficient are statistically significant throughout all the distribution for each fund except for τ = 0.45, and, furthermore it has an upward trend in which α is increasing as the order of the quantiles is increasing. In the lower quantiles there is no evidence of selection abilities (coefficients are negative): the managers are decreasing the fund's returns with their wrong choices. While in the upper ones we can find evidence of security analysis (positive coefficients) because the managers have selection abilities and they are using them to gain an excess return in respect to S&P500.

If we briefly compare this results with the OLS ones it is clear that quantile regression give much more information and allows to study the security analysis phenomenon. Indeed in the OLS estimates just in two funds were possible to have evidence of the managers' selection skills. Ones again QR has proven to be more useful than OLS for this type of studies.
In the figure 14 is shown the intercept of the IGIAX fund: it should be noted the upwards trend of the coefficient described above.

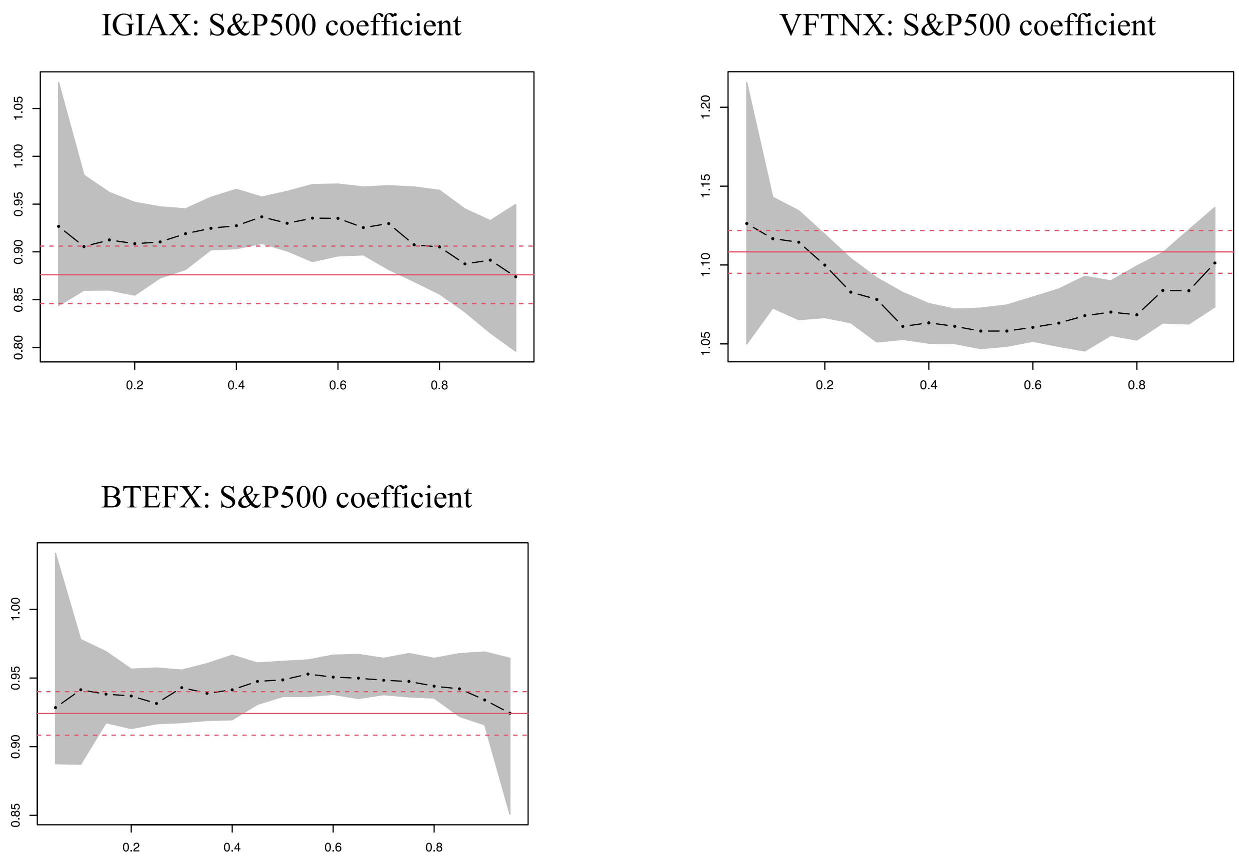
Figure 14. IGIAX intercept (T-M model)



4.5.3 The benchmark's coefficient (S&P500)

Analyzing the QR estimates for β_1 we can see that the coefficient is always high significant for each fund at each quantile. In this case the trend of the coefficients associated to the market is not well defined, indeed it seems to have for some of the funds an upward trend with several downward peaks all over the distribution, while for others it has a downward trend with up peaks. It is impossible to defined and describe in a general way the S&P500 coefficient development because it is not the same for all the funds. To understand better what we are talking about could be useful to show the below graphics (figure 15)

Figure 15. S&P500 coefficient of IGIAX, VTFNX, BTEFX



How is possible to see from the graphics the coefficient does not have the same trend for all the funds: for IGIAX and BTEFX it has a concave development, on the contrary in VTSMX it has a convex trend. Therefore what we can conclude is that β_1 is always statistically significant for each fund and quantile, but its development is irregular and each funds have its own trend of the coefficient.

4.5.4 The T-M coefficient (S&P500^2)

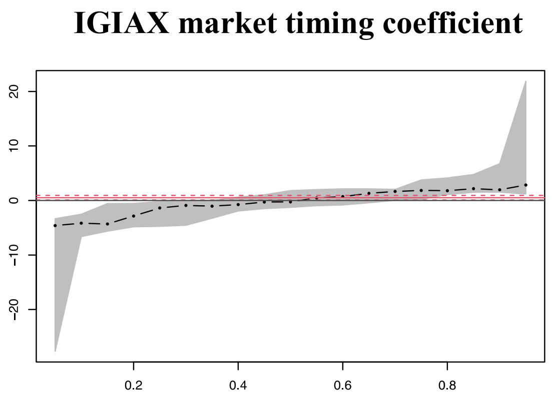
Exploring the coefficient associated to the market timing it is possible to notice that the coefficients are almost always significantly different from zero in both lower and upper quantiles (with just few exceptions) while especially in the in the central (τ = 0.5- 0.55) and the middle-upper ones (τ = 0.75-0.8) there are many funds in which the T-M coefficient is not significant.

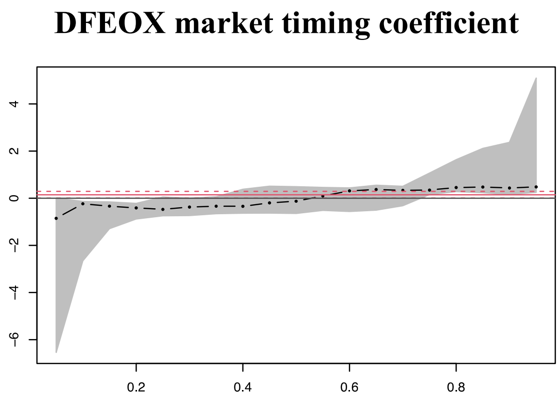
Focusing now our attention on the development of the market timing coefficients it is difficult to define precisely its trend indeed it is not regular . As for the S&P500 coefficient, also here the developments of the coefficient is different for each fund even if the general tendency is to have an upwards trend with several peaks all over the distribution. In particular exploring the results we can see that in the lower and central quantiles (from τ = 0.05 to τ = 0.55), of course with some exceptions, in the large part of the funds the coefficients have a negative value: the managers do not have market timing skills indeed they have done wrong prediction which decreased the fund's returns. But if we increase the order of the quantile it is possible to noticed that the coefficients are increasing too and that γ at one point becomes positive for each fund (in different quantiles). When the coefficient associated to the market timing is positive we can assert that the managers has market timing skills and that they are gaining excess return in respect to the market through their abilities. Generally we can conclude that the market timing coefficient is statistically significant approximately in all the quantiles except for τ = 0.5-0.55 and τ = 0.75-0.8 in which respectively seven and five funds do not have a significant coefficient. The trend of the coefficient is generally an upward trend in which the coefficients has negative values (no presence of market timing) in the lower and central quantiles and positive values (evidence of market timing) on the upper ones (generally from and τ = 0.6) . In the following figure we can see an example of upwards trend about the market timing coefficient in the BTEFX fund.

If we make a comparison of these results with the one obtained through the usage of the OLS we can conclude that with quantile regression it was much easier to find evidence of market timing, indeed in the T-M model estimates with the OLS just two funds out of twenty-three had significant and positive coefficients. Of course with quantile regression we found anyway non significant or negative γ but at the same time we found many positive and significant coefficients which allow us to the market timing phenomenon more in detail.

In the figure 16 below it is show the market timing coefficient trend of the IGIAX and DFEOX fund.

Figure 16. Market timing trend of IGIAX, DFEOX



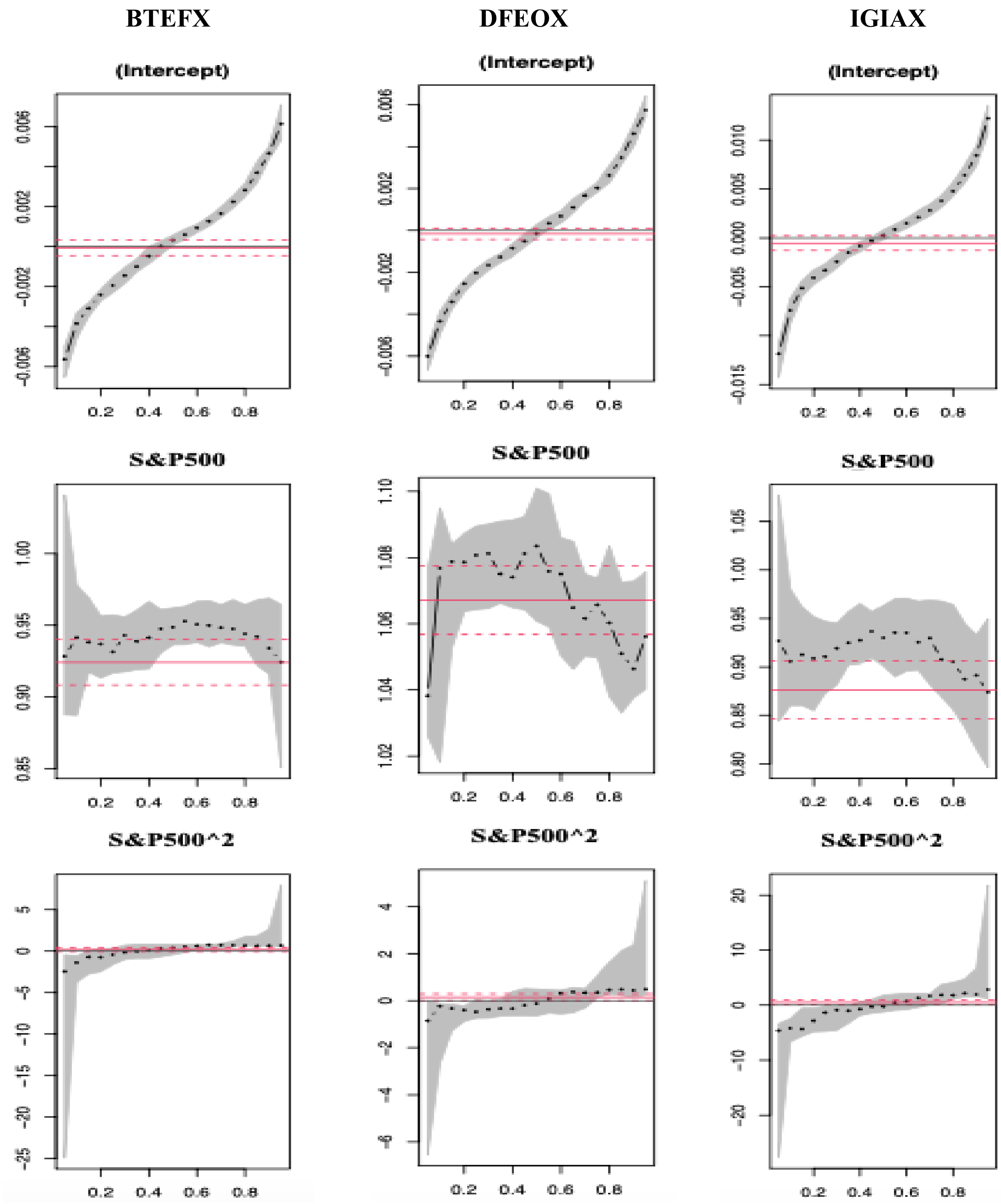


The figure17 shows the graphics for quantile regression. In the figure are shown the graphics of those funds in which is possible to appreciate more the quantile regression distribution and the coefficient's trend that we have analyzed before.

Notice that the table 8 represents the QR estimates but not in all the distribution just in some quantiles ($\tau = 0.1, 0.3, 0.5, 0.7, 0.9$) in order to give a general idea about the empirical data.

The figure 18 is a scatterplot of QR regression of BTEFX fund in which each line represents a quantile.

Figure 17. Coefficient’s estimates with QR (T-M model).



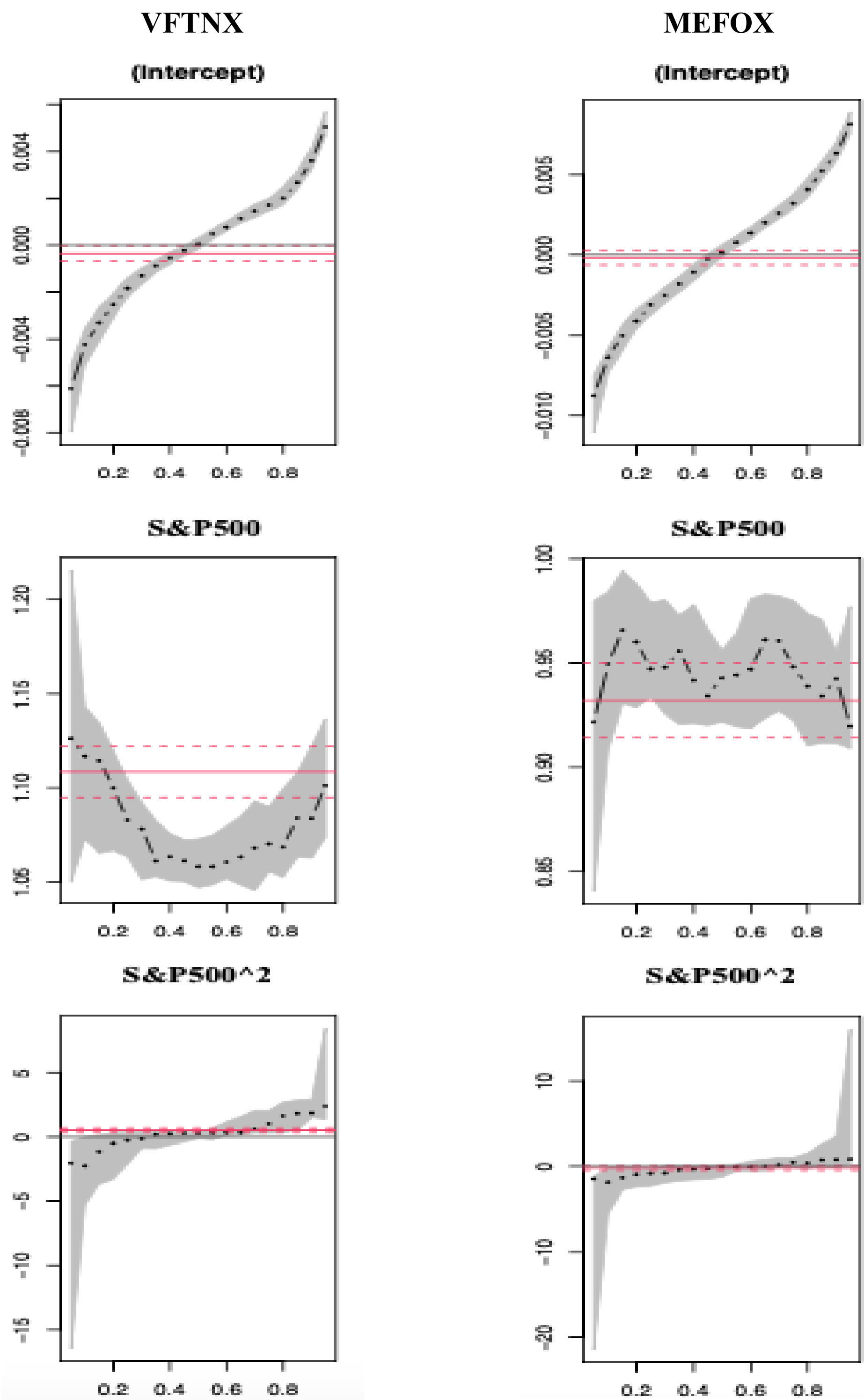
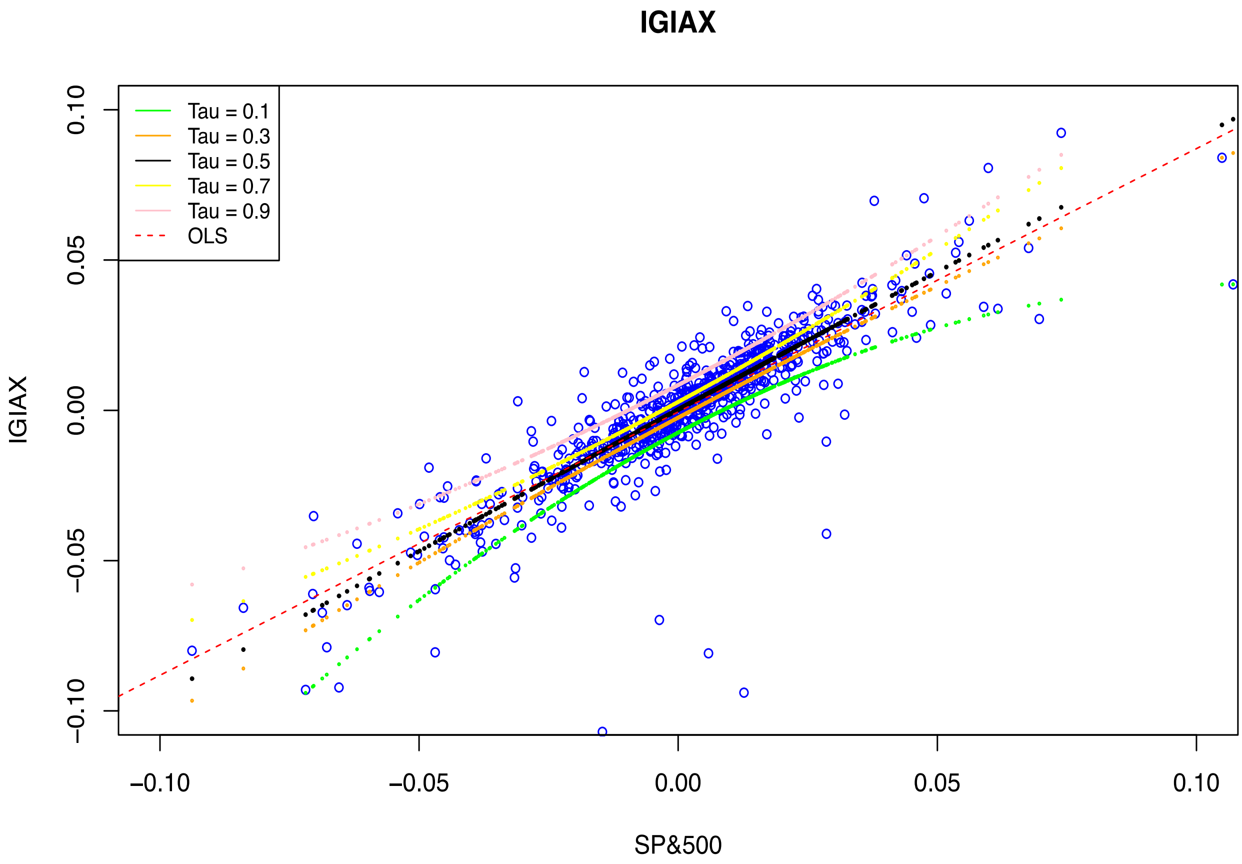

VFTNX
MEFOX
(Intercept)
(Intercept)
S&P500
S&P500
S&P500^2
S&P500^2
0.2
0.4
0.6
0.8
-0.008
-0.004
0.000
0.004
-0.010
-0.005
0.000
0.005
1.05
1.10
1.15
1.20
0.85
0.90
0.95
1.00
-15
-10
-5
0
5
-20
-10
0
10

Table 8: T-M estimates whit QR. *p<0,1; **p<0,05; ***p<0,001

Coefficients	τ = 0.1	τ = 0.3	τ = 0.5	τ = 0.7	τ = 0.9
BTEFX					
Intercept	-0,003***	-0,001***	0,0003*	0,001***	0,004***
SP& 500	0,941***	0,942***	0,948***	0,948***	0,933***
SP& 500^2	-1,388***	-0,1437	0,408***	0,736***	0,657***
CMNWX					
Intercept	-0,003***	-0,001***	0,0002*	0,001***	0,003***
SP& 500	1,002***	1,005***	0,993***	0,984***	1,001***
SP& 500^2	-0,12694041	-0,240**	0,0403	0,286**	0,366***
DFEOX					
Intercept	-0,004***	-0,001***	-0,00014952	0,001***	0,004***
SP& 500	1,076***	1,080***	1,083***	1,061***	1,046***
SP& 500^2	-0,235*	-0,357***	-0,127	0,332**	0,436***
GESSX					
Intercept	-0,003***	-0,001***	0,00009	0,001***	0,004***
SP& 500	1,023***	1,011***	1,017***	1,012***	1,005***
SP& 500^2	-0,474***	-0,146*	0,269**	0,291**	0,214**
GQEFX					
Intercept	-0,006***	-0,001***	0,0006**	0,003***	0,006***
SP& 500	0,743***	0,775***	0,776***	0,766***	0,770***
SP& 500^2	-1,784***	-0,858***	-0,585***	0,670***	2,336***
HAIAX					
Intercept	-0,003***	-0,001***	0,0005**	0,002***	0,004***
SP& 500	0,964***	0,953***	0,953***	0,947***	0,931***
SP& 500^2	-0,798***	-0,494***	-0,575***	-0,095	0,467***
IGIAX					
Intercept	-0,007***	-0,002***	0,00024954	0,002***	0,008***
SP& 500	0,905***	0,919***	0,929***	0,927***	0,889***
SP& 500^2	-4,155***	-0,904***	-0,260	1,582***	1,945***
MEFOX					
Intercept	-0,006***	-0,002***	0,00013277	0,002***	0,006***
SP& 500	0,949***	0,947***	0,942***	0,961***	0,943***
SP& 500^2	-1,835***	-0,782***	-0,064	0,124	0,807***
PRBLX					
Intercept	-0,005***	-0,001***	0,0008***	0,003***	0,006***
SP& 500	0,837***	0,876***	0,866***	0,870***	0,884***
SP& 500^2	-0,812***	-0,693***	-0,589***	-0,200	0,561***

Coefficients	τ = 0.1	τ = 0.3	τ = 0.5	τ = 0.7	τ = 0.9
PRDGX					
Intercept	-0,003***	-0,001***	0,0004***	0,001***	0,003***
SP& 500	0,922***	0,917***	0,920***	0,910***	0,903***
SP& 500^2	-0,082	-0,183**	-0,209**	-0,056	0,061
QAACX					
Intercept	-0,006***	-0,002***	7,6466E-05	0,002***	0,006***
SP& 500	1,111***	1,088***	1,072***	1,073***	1,086***
SP& 500^2	-0,354**	-0,177	-0,257*	0,299*	0,520**
SNAEX					
Intercept	-0,001***	-0,0005***	0,0003***	0,001***	0,002***
SP& 500	1,002***	0,989***	0,985***	0,987***	0,990***
SP& 500^2	-0,193***	-0,076	-0,071	-0,082*	0,139*
TIGRX					
Intercept	-0,004***	-0,001***	0,0005***	0,001***	0,004***
SP& 500	1,050***	1,029***	1,010***	1,004***	0,991***
SP& 500^2	-0,734***	-0,518***	0,101	0,419***	0,483***
VFTNX					
Intercept	-0,004***	-0,001***	5,356E-05	0,001***	0,003***
SP& 500	1,116***	1,078***	1,058***	1,067***	1,083***
SP& 500^2	-2,276***	-0,133	0,283***	0,599***	1,867***
VTSMX					
Intercept	-0,002***	-0,0006***	0,0002**	0,0009***	0,002***
SP& 500	1,029***	1,031***	1,029***	1,024***	1,016***
SP& 500^2	-0,263***	-0,141***	-0,072	0,188**	0,260***
WMLIX					
Intercept	-0,003***	-0,0003***	0,0003***	0,0009***	0,002***
SP& 500	0,986***	0,997***	0,998***	0,994***	0,982***
SP& 500^2	-0,309***	-0,362***	-0,079***	0,166***	0,061

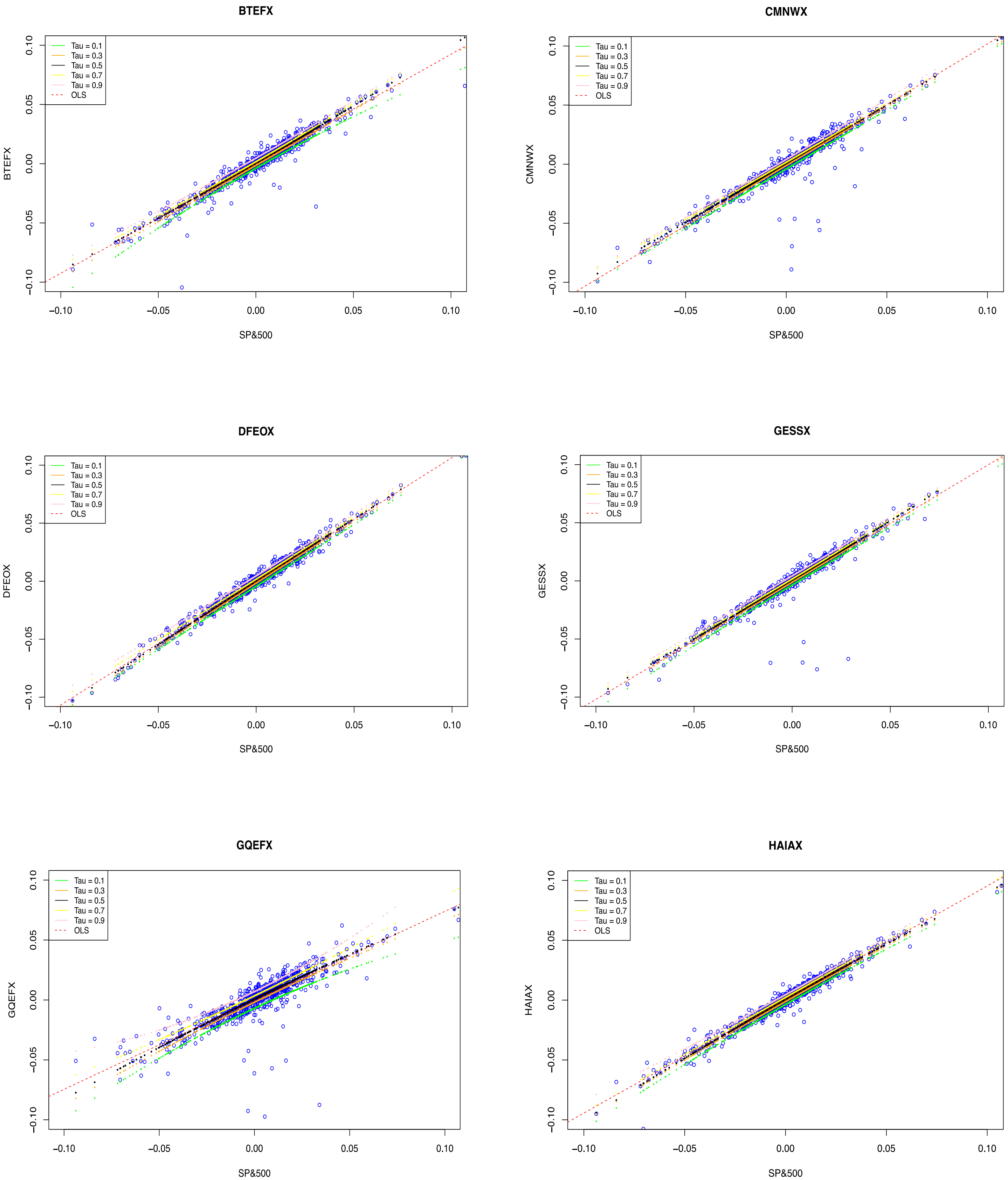
Figure 18. Scatterplots of the fund IGIAX: quantile regression estimates (T-M model)

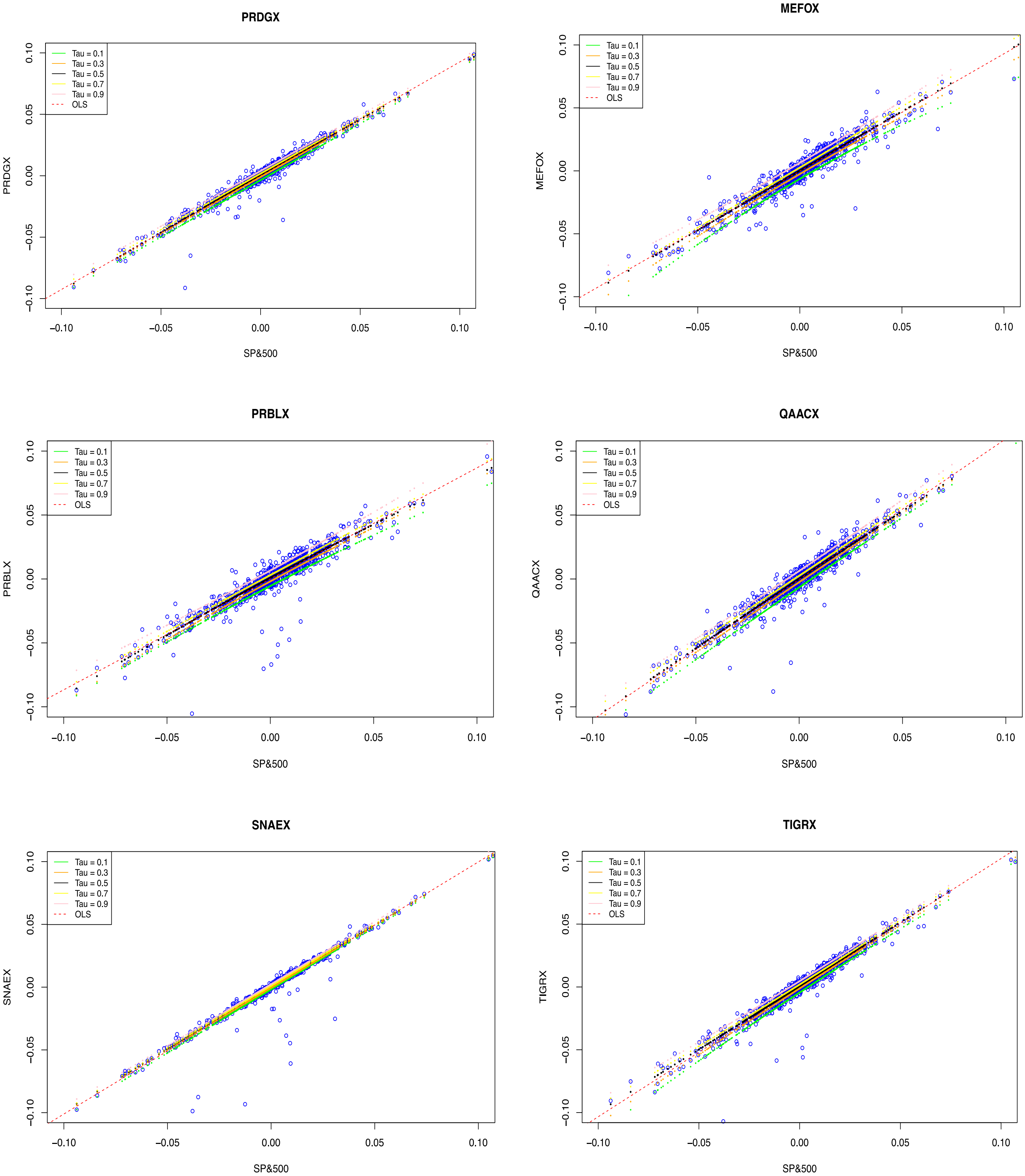


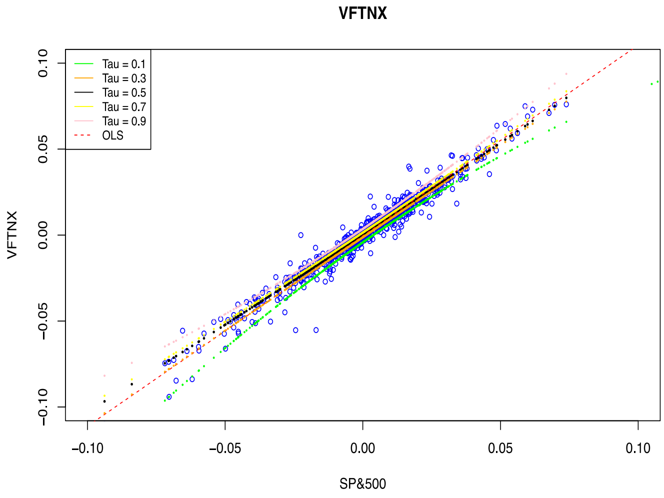
This figure shows the quantile regression estimates where each line correspond to a defined quantile. In this case we have considered τ =0.1,0.2,0.7,0.9.

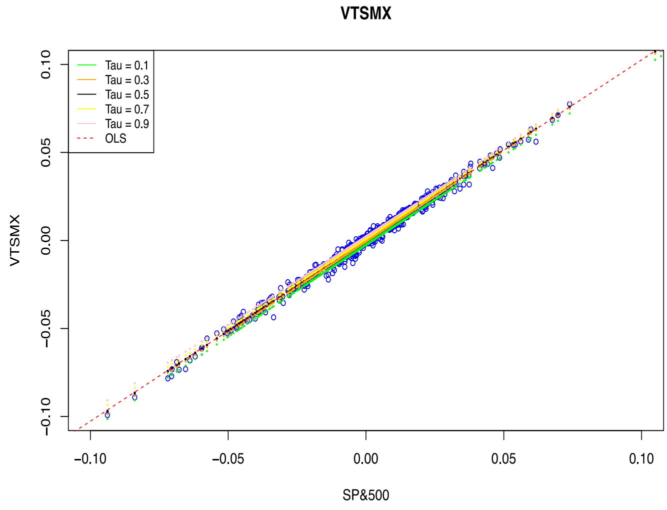
Furthermore for convenience the scatterplots of the other funds are shown in the figure 19 with reduced measures.

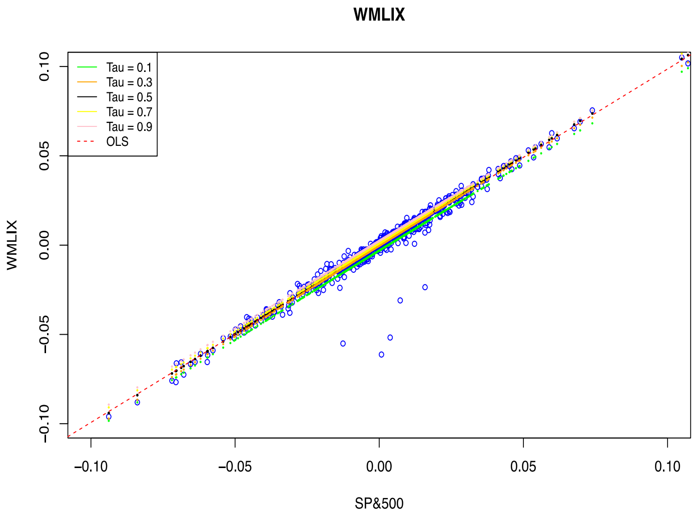
Figure 19. Scatterplots of the funds: quantile regression estimates (T-M model)




PRDGX
MEFOX
PRBLX
QAACX
SNAEX
TIGRX
Tau = 0.1
Tau = 0.3
Tau = 0.5
Tau = 0.7
Tau = 0.9
OLS
SP&500
−0.10
−0.05
0.00
0.05
0.10


VFTNX
Tau = 0.1
Tau = 0.3
Tau = 0.5
Tau = 0.7
Tau = 0.9
OLS
VFTNX
SP&500
−0.10
−0.05
0.00
0.05
0.10


VTSMX
Tau = 0.1
Tau = 0.3
Tau = 0.5
Tau = 0.7
Tau = 0.9
OLS
VTSMX
SP&500
−0.10
−0.05
0.00
0.05
0.10


WMLIX
Tau = 0.1
Tau = 0.3
Tau = 0.5
Tau = 0.7
Tau = 0.9
OLS
WMLIX
SP&500
−0.10
−0.05
0.00
0.05
0.10

4.6 QUANTILE REGRESSION ANALYSIS WITH GARCH PRE-FILTERING

As we have already pointed out in our results we can see that for high (or lower) level of market's returns (in upper and lower quantiles) the dispersion of the funds' return rises, this increment in dispersion depends on an increase in volatility. The rise of volatility usually occurs when the returns of the market are located in extreme values of the distribution (in the extreme quantiles). This component of volatility which characterized our empirical data cannot be overlooked. Therefore as we have anticipated in the previous section, in order to remove the volatility component from our sample it is possible to apply the generalized autoregressive conditional heteroscedasticity model (GARCH).

In this paragraph we have applied the GARCH model to both funds and benchmark returns, removing from them the volatility component. Consequently we have estimated the quantile regression for each fund in order to spot, once more, the presence of market timing abilities and to see if there are consistent differences between the results obtained with GARCH model and the one obtained just applying the QR.
Furthermore in this case, we decided to not estimate the OLS regression for two reason: first of all is not so interesting from an economic point of view to analyze it because as we have widely explained the OLS studies just the conditional mean while we are interested in all the distribution. Secondly we decided to not implement the OLS also because we do not expect to find particular evidence of market timing in the Ordinary Least Squares estimates as happened in the previous analysis.

4.6.1 Henriksson and Merton model: QR analysis with GARCH pre-filtering.

As we have done in the above cases we have estimated the quantile regression in nineteen quantiles (τ = 0.05, 0.1, 0.15 . . . . . 0.95) of course this time using the data "purified" from the volatility. The results that we have got are shown and explained above (table 9 and figure 20). Noticed that, as in the past cases, α , β_1, β_2 represent respectively the intercept (security analysis coefficient), the coefficient associated to the market's returns (S&P500) and the market timing coefficient.

Before proceeding with the explanation of the results it should be highlighted that we have applied the Wald test to the new data in order to verify in which funds the quantile regression should be applied.

Calculating the Wald test we have accepted the null hypothesis (H_0: same slope's coefficient throughout all the distribution) for the following funds: AFDAX,PRBLX,QAACX and VFTNX.
The Wald test statistics for each fund are respectively: 1.2446, 1.2733, 0.8952,1.0535.
Consequently, accepting the null hypothesis we determine that quantile regression is not necessary to be applied for the mentioned funds.

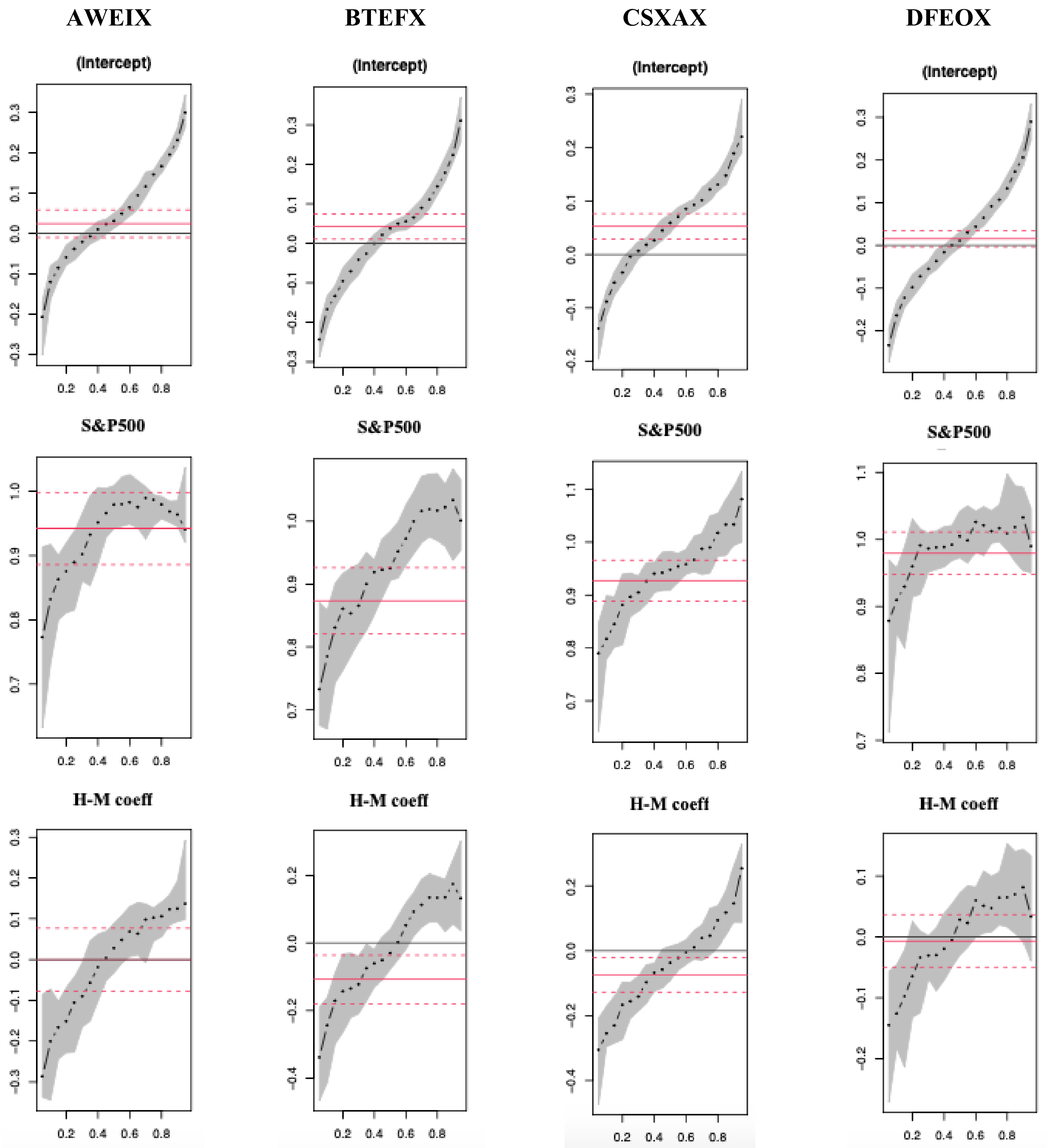
Focusing now our attention on the coefficient which is associated to the intercept (security analysis), we can verify that it is significant in most of quantiles for each fund, with the exception of the low-central quantiles (from $\tau = 0.35$ to $\tau = 0.45$). In these quantiles a notable numbers of funds do not have the intercept's coefficient significant. Of course, as always it should be taken into account that each fund has its own distribution and therefore its own quantiles in which the coefficients are significant or not.
Considering the trend of α we can notice that it is increasing as the order of the quantiles is increasing for each fund. This means that the managers in the lower quantiles do not have selection abilities indeed the fund's coefficients are negative: the advisors are decreasing the fund's returns with their wrong choices.
Going through the quantiles in the distribution we can see that all the funds from $\tau = 0.55$ till the last quantile considered (0.95) have a positive value of the intercept: the managers are choosing properly the stocks of the funds and through their selection abilities they are increasing the fund's returns. We can conclude that in the lower quantiles (from $\tau = 0.05$ to $\tau = 0.3$) the managers do not have any selection ability while from $\tau = 0.55$ to $\tau = 0.95$ the intercept becomes positive for all the funds which means that the managers are able to obtain excess returns through their skills.
If we make a brief comparison with the results obtained in the previous section (H-M model estimated with QR), we can see that the they are not that different. Indeed, in both the models we have an upward trend for the coefficient associated to the intercept and in particular in both cases we can see that in the upper quantiles the managers reach to have selection skills. What differs from the simple QR model and the one calculated with the GARCH pre-filtering is that in the former there were more significant coefficients related to the intercept, while in the latter the significance of the coefficients is slightly less.
At the end we can conclude that there are no particular differences between the two models.

Exploring now the β_1 we can see that it is always high significant for each fund at each quantiles. Considering the trend of the benchmark coefficient it has an upward trend but with few downward peaks all over the distribution.
If we compare the results about S&P500 with the one obtained in the previous section we can see that the results are pretty similar: in both model the coefficients are always significant and they are increasing as the order of quantiles is increasing.

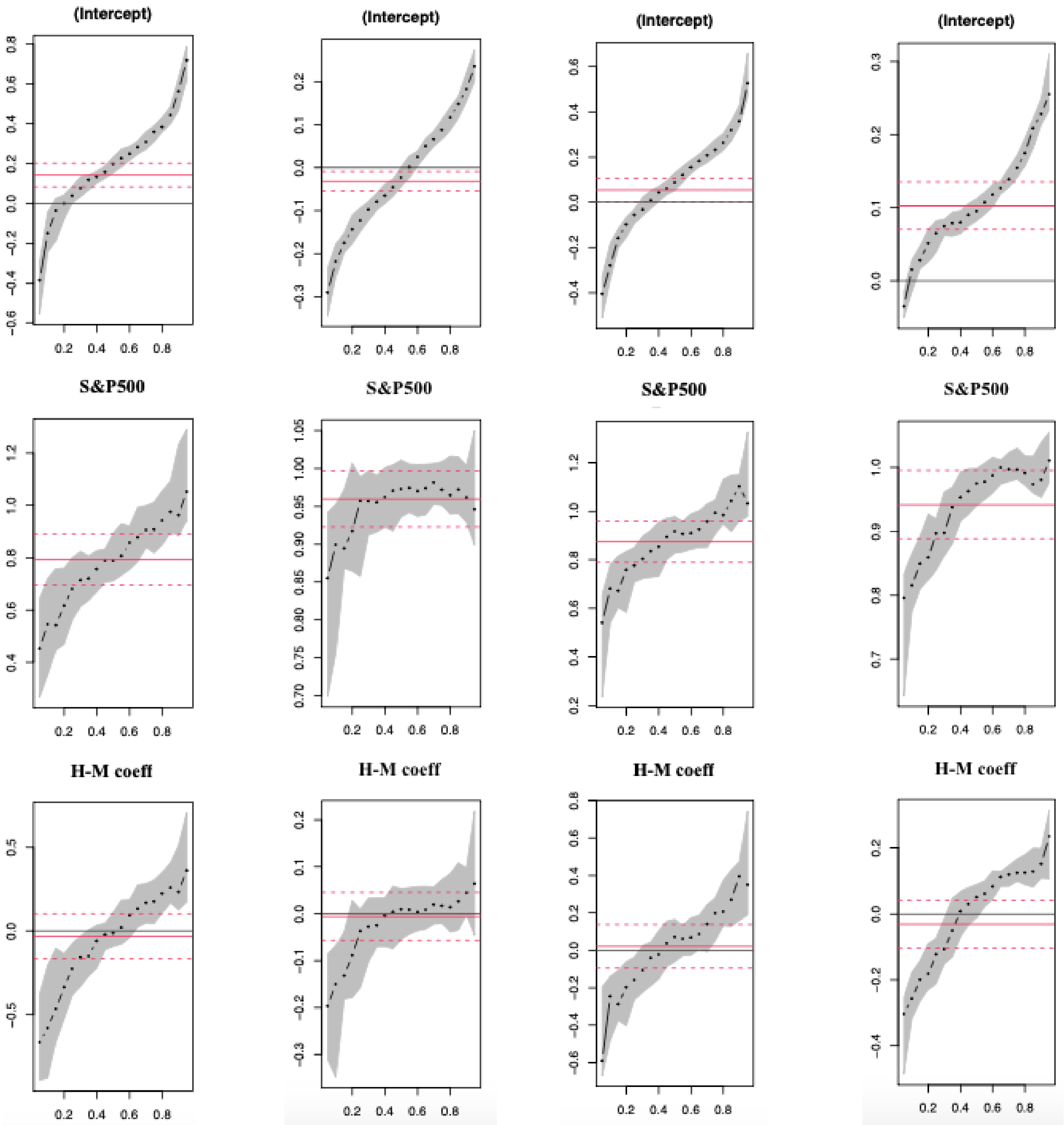
Last but not least we focus our attention in the H-M coefficient.
Exploring β_2 it is possible to notice that the coefficient generally is not significant from $\tau = 0.35$ to $\tau = 0.75$ quantiles, while is significant in particular in the upper (from $\tau = 0.8$ to $\tau = 0.95$) and lower quantiles (from $\tau = 0.05$ to $\tau = 0.3$).
Analyzing the trend of market timing coefficients we can notice that it is increasing as the order of quantiles is increasing but it has many downward peaks all over the distribution. In particular in the lower quantiles the coefficient is negative, which means that the managers do not have market timing abilities, on the contrary they are reducing the fund's returns even more with their wrong forecast. On the other hand in the upper quantiles the value of β_2 is not just significant but it is also positive (for each fund the coefficient becomes positive in different quantiles), which means that the managers are predicting correctly the market and that through their abilities they are increasing the funds' returns. In particular we can spot strong evidence of market timing phenomenon especially from τ = 0,7 to τ = 0,95 because in this quantiles the majority of the funds have significant and positive H-M coefficients while in the other quantiles the level of significance is limited just to a small number of funds.
Comparing this results with the previous model we can assert that the results are quite similar, it changes just the range in which the funds are more significant, but the quantiles in which we could spot presence of market timing are almost the same.

The figure 20 below shows the graphics for quantile regression. In the figure are shown the graphics of those funds in which is possible to appreciate more the quantile regression distribution and the coefficient's trend that we have analyzed before.
Notice that the table 9 represents the QR estimates but not in all the distribution just in some quantiles ($\tau = 0.05, 0.3, 0.5, 0.8, 0.95$) in order to give a general idea about the empirical data.

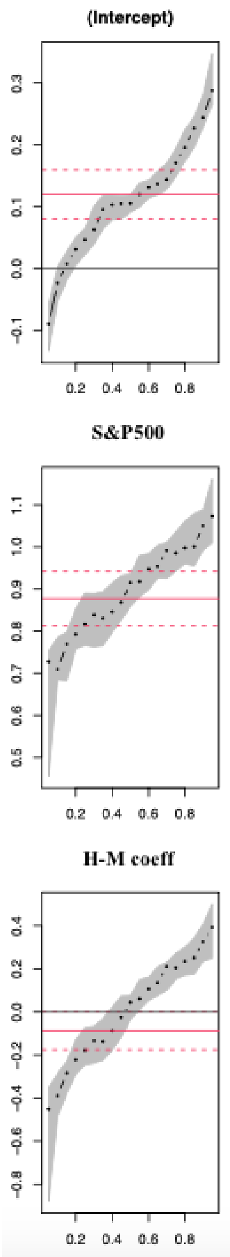
Figure 20. Coefficient's estimates with GARCH pre-filtering (H-M model).



GQEFX **HAIAX** **IGIAX** **JDEAX**



TIGRX **CMNWX**


(Intercept)
-0.1
0.0
0.1
0.2
0.3
0.2 0.4 0.6 0.8
S&P500
0.5
0.6
0.7
0.8
0.9
1.0
1.1
0.2 0.4 0.6 0.8
H-M coeff
-0.8
-0.6
-0.4
-0.2
0.0
0.2
0.4
0.2 0.4 0.6 0.8

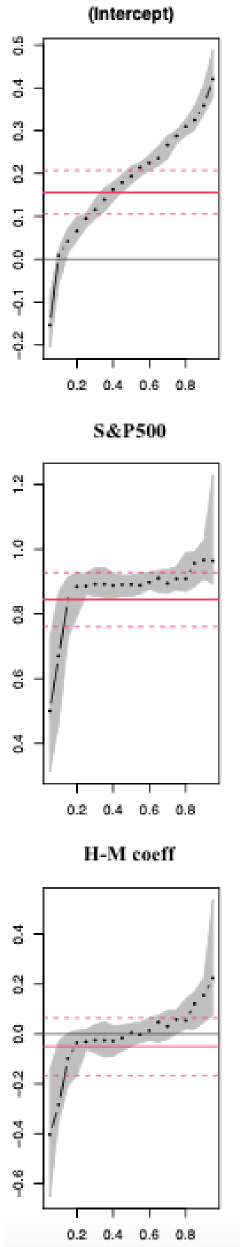

(Intercept)
-0.2
-0.1
0.0
0.1
0.2
0.3
0.4
0.5
0.2 0.4 0.6 0.8
S&P500
0.4
0.6
0.8
1.0
1.2
0.2 0.4 0.6 0.8
H-M coeff
-0.6
-0.4
-0.2
0.0
0.2
0.4
0.2 0.4 0.6 0.8

Table 9: H-M estimates whit HR. *p<0,1; **p<0,05; ***p<0,001

Coefficients	τ = 0.05	τ = 0.3	τ = 0.5	τ = 0.8	τ = 0.95
AWEIX					
Intercept	-0,206***	-0,0202	0,031**	0,166***	0,299***
SP& 500	0,773***	0,902***	0,979***	0,979***	0,940***
H-M coeff.	-0,285***	-0,089**	0,0277	0,105**	0,137**
BRLIX					
Intercept	-0,265***	-0,042**	0,054**	0,182***	0,350***
SP& 500	0,852***	0,891***	0,924***	0,986***	0,972***
H-M coeff.	-0,190**	-0,0332	0,014	0,128**	0,141**
BTEFX					
Intercept	-0,243***	-0,041**	0,038**	0,145***	0,311***
SP& 500	0,733***	0,865***	0,925***	1,017***	1,001***
H-M coeff.	-0,338***	-0,122**	-0,029	0,134***	0,132**
CMNWX					
Intercept	-0,154***	0,115***	0,194***	0,310***	0,420***
SP& 500	0,500***	0,893***	0,892***	0,909***	0,965***
H-M coeff.	-0,405***	-0,026	0,004	0,054*	0,223***
CSXAX					
Intercept	-0,138***	0,007	0,059***	0,131***	0,221***
SP& 500	0,790***	0,905***	0,948***	1,015***	1,081***
H-M coeff.	-0,305***	-0,142***	-0,037	0,091**	0,254***
DFEOX					
Intercept	-0,234***	-0,055***	0,011	0,133***	0,287***
SP& 500	0,878***	0,987***	1,004***	1,009***	0,993***
H-M coeff.	-0,145**	-0,031	0,028	0,065**	0,037
GESSX					
Intercept	-0,081**	0,103***	0,170***	0,283***	0,331***
SP& 500	0,807***	0,900***	0,913***	0,919***	1,112***
H-M coeff.	-0,289***	-0,020	0,062**	0,142***	0,485***
GQEFX					
Intercept	-0,386***	0,076**	0,198***	0,384***	0,719***
SP& 500	0,454**	0,715***	0,790***	0,943***	1,053***
H-M coeff.	-0,665**	-0,158**	-0,012	0,224***	0,362**
HAIAX					
Intercept	-0,290***	-0,097***	-0,023	0,116***	0,237***
SP& 500	0,855***	0,957***	0,973***	0,965***	0,944***
H-M coeff.	-0,197**	-0,025	0,009	0,014	0,053

Coefficients	τ = 0.05	τ = 0.3	τ = 0.5	τ = 0.8	τ = 0.95
IGIAX					
Intercept	-0,405***	-0,032	0,088***	0,263***	0,526***
SP& 500	0,541***	0,804***	0,917***	0,985***	1,032***
H-M coeff.	-0,590***	-0,098	0,072	0,206***	0,351**
JDEAX					
Intercept	-0,035*	0,075***	0,095***	0,175***	0,256***
SP& 500	0,796***	0,897***	0,975***	0,991***	1,010***
H-M coeff.	-0,303***	-0,107***	0,051**	0,125***	0,233***
MEFOX					
Intercept	-0,293***	-0,024	0,099***	0,297***	0,556***
SP& 500	0,782***	0,894***	0,901***	0,963***	0,915***
H-M coeff.	-0,240**	-0,111**	-0,092**	-0,001	-0,052
PRDGX					
Intercept	-0,222***	-0,039**	0,026**	0,133***	0,274***
SP& 500	0,894***	0,937***	0,960***	0,963***	0,992***
H-M coeff.	-0,240**	-0,030	0,023	0,058*	0,169**
SNAEX					
Intercept	0,032	0,124***	0,154***	0,210***	0,300***
SP& 500	0,692***	0,876***	0,910***	0,962***	0,963***
H-M coeff.	-0,435***	-0,088***	0,016	0,150***	0,205***
SUWAX					
Intercept	-0,069**	0,135***	0,189***	0,310***	0,431***
SP& 500	0,714***	0,820***	0,885***	0,936***	1,060***
H-M coeff.	-0,300***	-0,140***	-0,028	0,099**	0,286***
TIGRX					
Intercept	-0,090**	0,063***	0,105***	0,196***	0,288***
SP& 500	0,728***	0,839***	0,915***	0,998***	1,073***
H-M coeff.	-0,451***	-0,133***	0,044	0,235***	0,392***
TISCX					
Intercept	-0,060**	0,042***	0,073***	0,152***	0,258***
SP& 500	0,750***	0,913***	0,982***	0,999***	0,973***
H-M coeff.	-0,359***	-0,080***	0,052**	0,124***	0,156***
VTSMX					
Intercept	-0,203***	-0,024***	0,007	0,068***	0,141***
SP& 500	0,990***	0,984***	1,003***	1,009***	0,983***
H-M coeff.	-0,054	-0,031*	0,014	0,044**	0,031

Coefficients	τ = 0.05	τ = 0.3	τ = 0.5	τ = 0.8	τ = 0.95
			WMLIX		
Intercept	-0,134***	0,049***	0,096***	0,158***	0,226***
SP& 500	0,891***	0,982***	0,983***	0,988***	1,015***
H-M coeff.	-0,122**	-0,037*	-0,021	0,014	0,110***

4.6.2 Treynor and Mazuy model: QR analysis with GARCH pre-filtering.

Exactly as we have done for the Henriksson-Merton model we have estimated the quantile regression (using the GARCH pre-filtering) in nineteen quantiles (τ = 0.05, 0.1, 0.15 . . . . . 0.95). The results obtained are shown and explained above (figure 21 and table 19). Noticed that, as in the past cases, α, β_1, γ, represent respectively the intercept (security analysis), the coefficient associated to the market's returns (S&P500) and the market timing coefficient.

As we have done for the H-M model, we have calculated the Wald test, and the funds for which the null hypothesis is accepted are: MEFOX, QAACX, VFTNX and PRBLX.
The Wald test statistics for each fund are respectively: 0.8314, 0.9102, 1.081,1.689.
Consequently, accepting the null hypothesis we determine that quantile regression is not necessary to be applied for the mentioned funds.

Considering the coefficient associated to the intercept we can assert that it is almost always significant in all the distribution except in τ = 0.45 in which there are several funds which do not have a significant α. As in the T-M model (without GARCH pre-filtering) the trend of the coefficient is increasing as the order of quantiles is increasing, indeed, in the lower quantiles the coefficients are negative, while, going through the distribution they become positive. This means that in the lower part of the distribution the managers do not have selection abilities, while in the central and upper quantiles (in particular from τ = 0.5 to τ = 0.95) all the coefficients are increasing and positive: the managers have selection abilities and they are using it to gain higher return in respect to the one obtained just following the benchmark. Comparing these results with the one obtained applying just the quantile regression we can assert that they are similar to each other: the manager do not have ability in the lower quantiles while in the central and upper he has it and both trends are upwards.

Exploring now the benchmark coefficient it is straightforward to verify that it is high significant for each fund at each quantiles. Regarding the development of the coefficient through the quantiles, it is not well defined: each fund has its own trend which differs from the others. But what can we see from the graphics is that a significant amount of funds have a concave trend indeed the benchmark coefficient is increasing through the quantiles and then from a certain point (different for each funds) the values of the coefficients is decreasing, however never becoming negative.

Comparing this results with the one obtained just applying QR we can see that they are quite similar but the difference that we can spot is that the coefficient trend is more pronounced in this model than in the previous one. Indeed in this case it is possible to see much more the convex trend of the benchmark, while in the other study the trend was less evident.

Taking into consideration the T-M coefficient we can notice that the coefficient is significant especially in the lower (from τ = 0.05 to τ = 0.25) and upper quantiles (from τ = 0.75 to τ = 0.95), while from τ = 0.3 till τ = 0.7 there are a relevant number of funds in which the coefficient is not significant.

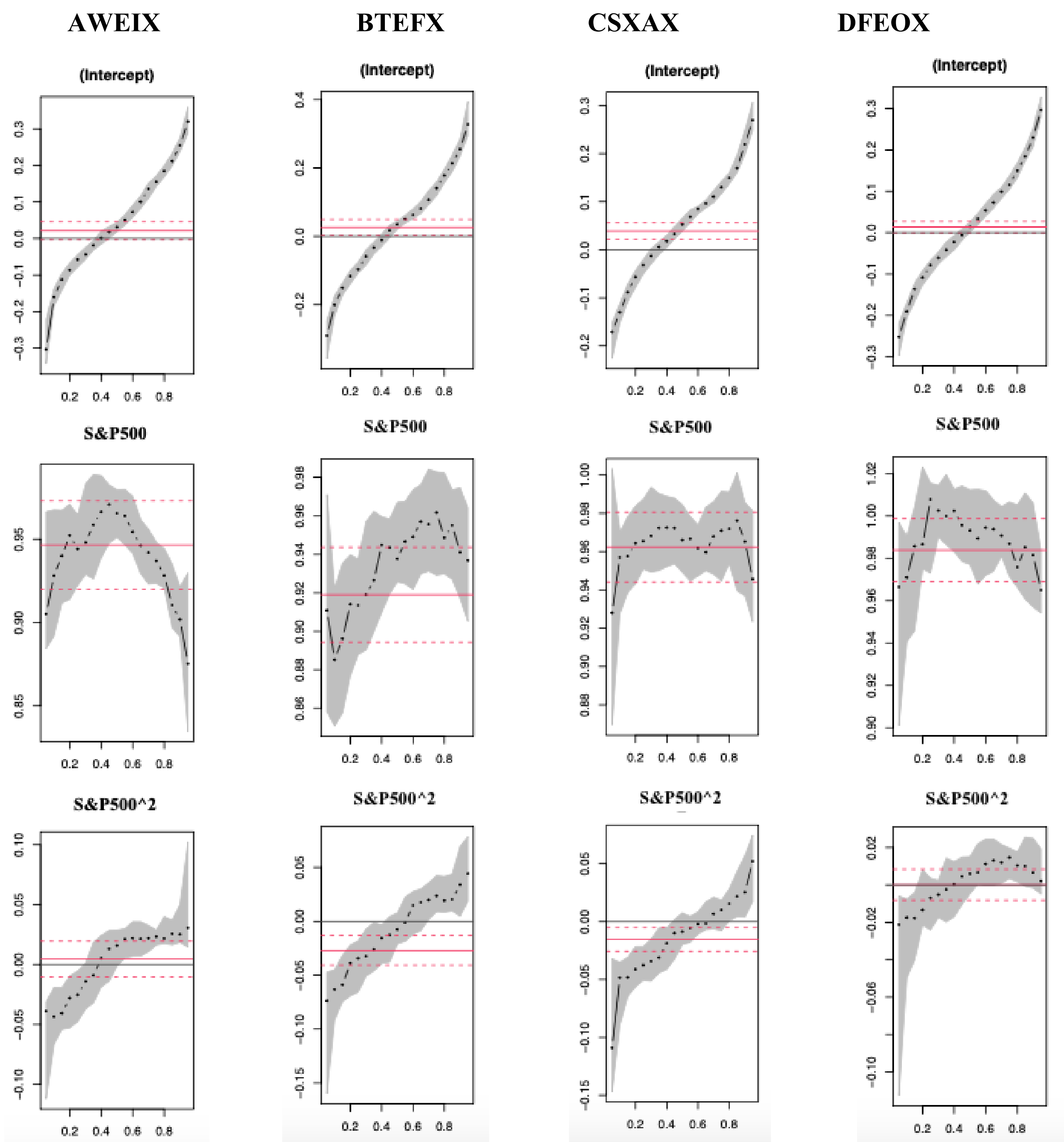
Focusing now our attention on the development of the market timing coefficients we can assert that generally the trend is upward (with some peaks): the coefficient tends to increase as the order of quantiles is increasing. In particular in the lower quantiles the market timing coefficient is negative for all the funds, which means that the managers do not have market timing skills. On the other hand going through the distribution, as we said, the coefficient value is increasing. In particular from τ = 0.5 until the last quantile considered (0.95) in all the funds in which the H-M coefficient is significant we can see that the market timing coefficients have a positive value too and consequently a positive impact in the fund's return. Thus we can conclude that in the lower quantiles there is no evidence of market timing skills, while especially in the upper ones (from τ = 0.75 to τ = 0.95), in which the majority of the funds have significant and positive value of T-M coefficient, we can spot the presence of market timing skills. The managers are predicting correctly the market and are gaining higher returns.

If we analyze together the benchmark coefficient and the market timing coefficient we can see that in almost all the funds (see figure 21) there is a point in which the index coefficient is decreasing and on the contrary the market timing coefficient is increasing. This means that as the quantiles are increasing the impact of the benchmark on the fund's returns decreases, on the contrary, as the quantiles are increasing the impact of the market timing on the fund's returns increases allowing the funds to get higher excess returns.

Comparing the results of this model and the one estimated just with the QR we can see that in the former there are much more non significant T-M coefficients than the latter. As a matter of fact we have to highlight that the final conclusion in both the models are that in the upper quantiles there is presence of market timing skills, while in the lower there is not. Furthermore another difference that we can spot is that in this case the upward trend of the H-M coefficient is much more pronounced that the one in the other model, therefore there is a greater increase in the market timing coefficient value through the distribution.

The figure 21 below shows the graphics for quantile regression. In the figure are shown the graphics of those funds in which is possible to appreciate more the quantile regression distribution and the coefficient's trend that we have analyzed before. Furthermore from this figures is it possible appreciate both the benchmark and the market timing trend and to see how they are developing also in relation to each other.

Figure 21 Coefficient's estimates with GARCH pre-filtering (T-M model).



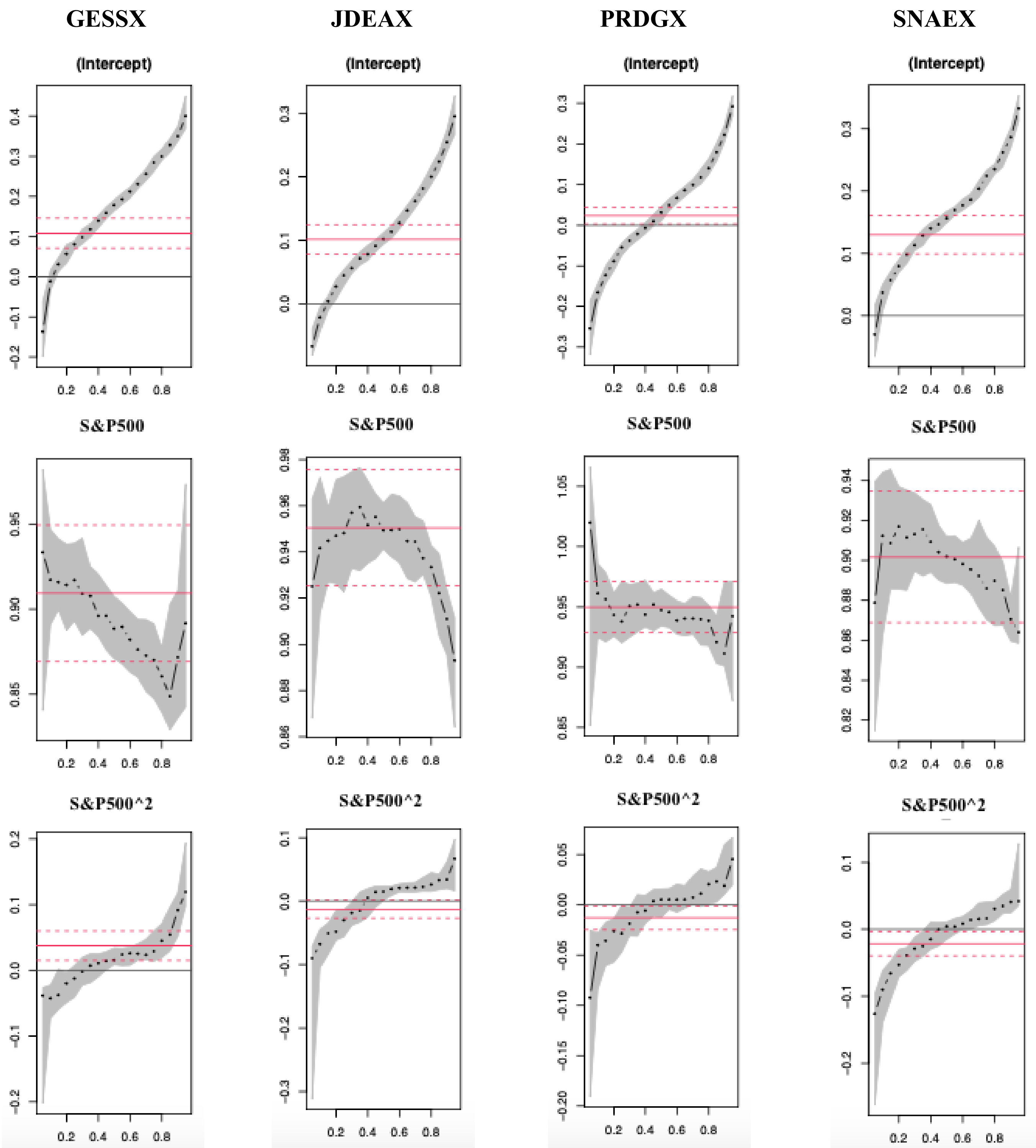

GESSX
JDEAX
PRDGX
SNAEX
(Intercept)
(Intercept)
(Intercept)
(Intercept)
S&P500
S&P500
S&P500
S&P500
S&P500^2
S&P500^2
S&P500^2
S&P500^2
0.2 0.4 0.6 0.8

Table 10: T-M estimates whit QR. *p<0,1; **p<0,05; ***p<0,001

Coefficients	τ = 0.05	τ = 0.25	τ = 0.5	τ = 0.75	τ = 0.95
AFDAX					
Intercept	-0,214	-0,078	-0,016	0,061	0,228
SP& 500	0,967	0,960	0,963	0,972	0,937
SP& 500^2	-0,045	-0,018	-0,010	0,002	0,002
AWEIX					
Intercept	-0,303	-0,058	0,031	0,156	0,318
SP& 500	0,905	0,944	0,966	0,937	0,888
SP& 500^2	-0,039	-0,025	0,016	0,023	0,023
BRLIX					
Intercept	-0,285	-0,089	0,050	0,176	0,365
SP& 500	0,932	0,914	0,924	0,933	0,903
SP& 500^2	-0,058	0,002	0,008	0,026	0,045
BTEFX					
Intercept	-0,291	-0,097	0,035	0,140	0,327
SP& 500	0,911	0,914	0,938	0,960	0,937
SP& 500^2	-0,074	-0,034	-0,008	0,023	0,044
CMNWX					
Intercept	-0,248	0,089	0,194	0,296	0,453
SP& 500	0,719	0,902	0,890	0,883	0,873
SP& 500^2	-0,074	-0,006	0,002	0,010	0,057
CSXAX					
Intercept	-0,171	-0,032	0,053	0,130	0,269
SP& 500	0,928	0,965	0,966	0,971	0,946
SP& 500^2	-0,109	-0,038	-0,009	0,010	0,052
DFEOX					
Intercept	-0,252	-0,079	0,014	0,116	0,296
SP& 500	0,966	1,008	0,993	0,987	0,965
SP& 500^2	-0,021	-0,007	0,006	0,015	0,002
GESSX					
Intercept	-0,136	0,081	0,179	0,285	0,400
SP& 500	0,933	0,917	0,889	0,871	0,892
SP& 500^2	-0,039	-0,012	0,016	0,029	0,119
GQEFX					
Intercept	-0,543	-0,002	0,197	0,389	0,750
SP& 500	0,795	0,814	0,795	0,828	0,925
SP& 500^2	-0,099	-0,033	-0,002	0,040	0,110

Coefficients	τ = 0.05	τ = 0.25	τ = 0.5	τ = 0.75	τ = 0.95
HAIAX					
Intercept	-0,321	-0,125	-0,023	0,087	0,238
SP& 500	0,936	0,973	0,972	0,962	0,920
SP& 500^2	-0,053	-0,014	0,003	0,010	0,041
IGIAX					
Intercept	-0,515	-0,083	0,101	0,268	0,641
SP& 500	0,826	0,844	0,883	0,901	0,868
SP& 500^2	-0,115	-0,031	0,013	0,042	0,043
JDEAX					
Intercept	-0,067	0,045	0,102	0,182	0,296
SP& 500	0,925	0,948	0,949	0,937	0,893
SP& 500^2	-0,090	-0,030	0,015	0,023	0,066
PRDGX					
Intercept	-0,254	-0,055	0,031	0,118	0,290
SP& 500	1,020	0,938	0,947	0,937	0,943
SP& 500^2	-0,092	-0,028	0,005	0,010	0,046
SNAEX					
Intercept	-0,030	0,098	0,157	0,224	0,332
SP& 500	0,879	0,911	0,902	0,886	0,864
SP& 500^2	-0,126	-0,039	0,004	0,016	0,042
SUWAX					
Intercept	-0,097	0,088	0,185	0,290	0,499
SP& 500	0,833	0,882	0,898	0,886	0,913
SP& 500^2	-0,088	-0,039	-0,008	0,017	0,036
TIGRX					
Intercept	-0,117	0,025	0,114	0,212	0,373
SP& 500	0,934	0,906	0,894	0,885	0,881
SP& 500^2	-0,236	-0,049	0,007	0,036	0,072
TISCX					
Intercept	-0,105	0,020	0,082	0,158	0,285
SP& 500	0,931	0,951	0,957	0,939	0,911
SP& 500^2	-0,108	-0,034	0,012	0,022	0,027
VTSMX					
Intercept	-0,220	-0,045	0,008	0,059	0,149
SP& 500	1,008	0,996	0,999	0,992	0,970
SP& 500^2	-0,003	-0,007	0,005	0,010	0,002

WMLIX

Intercept	-0,160	0,022	0,091	0,146	0,240
SP& 500	0,975	0,998	0,994	0,982	0,965
SP& 500^2	-0,010	-0,012	-0,004	0,001	0,035

5 CHAPTER – CONCLUSION

As we have widely explained the goal of this thesis is to verify if the managers of the funds have superior skills which allow them to gain higher return with respect to the stock market. There are two different type of superior abilities: the market timing and the security analysis, in this thesis we have dealt with both of them but in particular we focused our attention on the market timing phenomenon, therefore the analyzed models have been proposed in the first place to explore the market timing skills. In order to summarized the results of this dissertation it could be useful first of all to recap the results obtained by each section of the chapter four.

In the first section we have applied the Ordinary Least Squares regression to the Henriksson-Merton model. The empirical results that we have found did not show particular evidence of market timing skills. Therefore from the twenty-three fund just in three of them the coefficient associated to the market timing were significant, and from them, just one was significant and positive. The manager of that fund has prediction skills and he is able to forecast correctly the market exploiting those predictions to gain an excess return.
We can conclude that analyzing the H-M model through the usage of OLS estimator, the managers do not have market timing abilities (except for one): there is no much evidence of the phenomenon.

In the second section the Treynor-Mazuy model has been estimated with the OLS ,and, also here, exactly as happened for the H-M model the evidence of market timing is very limited. Therefore in this case it has been found seven funds in which the market timing coefficient is significantly different from zero, but just in two of them the coefficients are significant and positive. This means that from the twenty-three funds only two managers are able to predict the market and gain higher returns: it is clear that considering the all sample the presence of market timing abilities is very limited.

In the third section we have calculated the Henriksson-Merton model through the usage of quantile regression estimating the regression in 19 different quantiles. As we expected in the QR we found much more significant coefficients than in the OLS regression. In particular from the results we can see that generally the market timing coefficients are significant and have positive values in the upper quantiles

In particular from the results we could find evidence of market timing phenomenon especially from τ = 0,75 to τ = 0,95 because in this quantiles a large part of the funds have significant and positive H-M coefficients, while in the other the level of significance is limited just to a small number of funds. The funds which have positive and significant market timing coefficients are those in which the managers have prediction skills, in particular the advisors are forecasting exactly the market and consequently changing the portfolio composition to outperform the market and gain higher returns. We conclude that in the H-M model estimated with quantile regression there is evidence of market timing skills especially in the upper quantiles. It should be noted how even if we are studying the same model (H-M) the results change whether we are using the OLS or the QR. If we use the former there is no particular evidence of market timing phenomenon, on the contrary if we use the second estimating method we find several proof that the managers has market timing skills.

In the fourth section of the chapter we have estimated the Treynor-Mazuy model with quantile regression in 19 different quantiles (as in the H-M model). From the results that we have obtained we can see that the coefficient associated to the market timing are almost always significant in both lower and upper quantiles (with just few exceptions), while especially in the in the central (τ = 0.5 - 0.55) and the middle-upper ones (τ = 0.75 - 0.8) there are many funds in which the market timing coefficient is not significant. The fact that in the lower and upper quantiles the T-M coefficients are significant does not necessary mean that the managers of the funds have market timing abilities, therefore we can confirm the presence of market timing just when the T-M coefficients are both significant and positive. This happens in our study, generally from the quantile τ = 0.6, of course with some exception because each funds have its own distribution). Consequently we can conclude that in the T-M model estimated through the usage of the QR there is evidence of market timing, in particular the phenomenon can be spotted in the upper quantiles of the distribution.
If we compare the estimates with the Ordinary Least Squares ones we can see how much the results differ from each other: with the OLS estimator we almost did not find any presence of market timing abilities indeed we spotted prediction skills just in two funds out of the twenty-three analyzed, while with the quantile regression we found some proof of market timing presence through the distribution.

After our empirical analysis we can conclude that the proposed studies about market timing are influenced by the type of estimator that we have chosen. Indeed as we have widely discuss the

OLS explores just the relationship between a set of independent variables and the conditional mean of a dependent variable Y not giving therefore a complete and detailed picture of the phenomenon. Consequently it stands to reason that with the OLS estimates we did not find any important evidence of market timing. On the other hand the QR is able to study the conditional quantiles in all the distributions and therefore is obvious that using this estimator we were able to analyze much more in detail the prediction skills of the managers.

We can conclude that the managers could have market timing skills but as we have seen from our study is not that easy to find this particular abilities in the advisors. Indeed even in the QR, in all the lower quantiles there were never evidence of market timing: the phenomenon has been spotted just in the upper ones. This means that the managers might have this particular ability but unfortunately is not that common.

Furthermore as we have mentioned in the previous section our data are influenced by heteroskedasticity and their volatility changes over time, furthermore the variance of the funds is really related to the market one, and all of these conditions can have a negative impact on our results (they could be not reliable). In order to overcome this problem we have calculated a GARCH model estimating the "new" returns and then we have applied quantile regression on them searching, once more, for market timing skills.

The results that we have obtained are not that different from the previous calculation. In particular the main differences that we have spot are: in the GARCH model there are more "non-significant" coefficients than in the previous T-M and H-M models.

Furthermore in the GARCH model the coefficients' trend (especially the benchmark and market timing one) were more pronounced than in the previous models and therefore was more intuitive to understand how the two coefficients developed through the quantiles.

But as a matter of fact we have to highlight that the final conclusion for both the models (T-M and H-M) calculated just with QR are the same which we obtained from the GARCH one: that in the upper quantiles there is presence of market timing skills, while in the lower and central ones there is not, and therefore that the presence of market timing is not that strong, exactly as happened in the past estimations.

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