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"TACTICAL CHOICES WITH SMART BETA APPROACHES: AN ALTERNATIVE TO CAP-WEIGHTED ALLOCATION"

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Introduction

The investment management is divided in two different approach: the active management and the passive management. In the case of active management, the investment decision are based on analytical research, forecast and knowledge of the manager. The manager attempts to beat the market, that is, to achieve alpha. Alpha is a measure the performance of the active portfolio compared to a benchmark. In the case of passive management, the manager merely replicates the asset allocation of a benchmark in order to achieve the return of that index. Here the most important measure is the Beta, which reflects the sensitivity of the portfolio to the benchmark.

Traditionally, the active management was the widespread approach. In the publication “European Index Survey 2011” and “North America Index Survey 2011” Edhec Risk Institute proposed a survey to the institutional asset manager about the index quality, the key issues of the index and the future trends for the indexation. The most popular opinion was an increase attraction for passive management, probably due to high cost of the active management not justified with an adequate return.

Today, the most common benchmark are the Cap-Weighted indices. These indices allocate the securities according to their market capitalization. Many studies such as, Haugen and Baker (1991), Goltz and Le Sourd (2011), show that these indices are inefficient and with lack of diversification. As we will highlight in this work, the Cap-Weighted allocation have many issues that provoke an overall reduction of the performance of the index. In the context of the above-mentioned survey, the managers showed their interest in alternative indices, the so-called Smart Beta.

These indices, proposed by Edhec Risk Institute, aim to achieve better risk adjusted return by an alternative weighting scheme based on a well-diversified asset allocation.

The first generation of Smart attempted to achieve good performance by a single factor approach, tilting the index to a specific factor (e.g. value or small cap). This overexposure to a single factor leaded to lack of diversification, moreover, the investors weren’t aware of the risk they were bearing. On the contrary, the second generation of Smart Beta indices mix the application of a diversified weighting scheme with multi-factor selection stock stage, with the result to provide customizable benchmark based on the preference of the investors. In other words, this approach attempts to meet the requirement of the investors, offering a wide set of indices with different features.

In this thesis we will show how Smart Beta portfolios outperform the Cap-Weighted indices, providing an overall analysis of strength and shortcomings of the ten popular Smart Beta indices.

On one hand, in the Chapter 1 we introduce the Cap-Weighted approach with its theoretical background: the Markowitz's portfolio optimization and the CAPM model. On the other hand, in the Chapter 2 we analyze the Smart Beta approach. The first part highlight the risk exposure of these indices and describes the weighting scheme proposed in this work. In the third section of this chapter, we explain the main arguments proposed in the literature to solve the main shortcomings of this approach. The second chapter is completed with some brief considerations about the application of this approach in the passive management and in the active management.

The Chapter 3 concerns the empirical analysis. This part starts from a graphical analysis of the asset allocation. Then it follows a quantitative valuation of the performance of the portfolios. This valuation is performed with indicators, graphical support, turnover analysis and efficient frontier. In addition, in the last section of the Chapter 3 we break down the regression of the excess returns with two different model: CAPM model and Fama-French model.

The last Chapter propose two alternative benchmark that combines two different weighting scheme. The combination of Global Minimum Variance and Equally Weighted aim to confirm the findings of the Edhec Risk Institute proposed in the paper: “Smart Beta 2.0”, while the combination of Maximum Diversification Ratio and Semi-Diversified Minimum Variance has the purpose to verify if the combination of the winner portfolios dominates the single components.

1. The traditional approach: Cap-weighted index

1.1 Where is the origin and justification of Cap-Weighted Index? Modern Portfolio Theory, CAPM model.

Today, The Modern Portfolio Theory (MPT) is the basis of the majority of quantitative portfolio management. The theory is originally dated back to 1952, when Harry Markowitz has published his article: " *Portfolio selection*".

In this article he defines a framework, which describes portfolios of assets in terms of the means of their returns, the variance of their returns, and the correlation between the returns on the assets; his approach is also known as mean-variance optimization.

According to Markowitz, the problem of the investor is to maximize his wealth bearing the minimum risk. Each period the wealth of the investor changes, based on the yield of his wealth allocation as follows:

$$W_1 = W_O(1 + r)$$

Equation 1

Where W_1= wealth at time 1, W_O= wealth at time 0 and r = return.

The investor allocate his wealth based on his future expectation in order to maximize his utility with a mean-variance approach. His utility function is:

$$E[U(W_1)] = E(W_1) - \eta VAR(W_1)$$

Equation 2

Where W_1= wealth of the investor at time 1, η = risk aversion coefficient of the investor.

Therefore, the utility increase as increase the expected value of the wealth and decrease as increase the variance of the wealth. The risk aversion coefficient is increasing as increase the aversion of the investor to the risk, where the risk is the identified with the variance of the wealth.

In order to create a portfolio the investor allocate his wealth in different assets, where each asset contributes to the return of the portfolio. Therefore, the investor maximize his utility respect to the assets weights:

$$MAX_\omega\{E[U(W_1)]\} = MAX_\omega\{E(r_p) - \eta VAR(r_p)\}$$

$$under\ constraint \begin{cases} \sum \omega = 1 \\ \omega \geq 0 \end{cases}$$

Equation 3

where r_p= return of the portfolio.

Whit this in mind, let's have a better comprehension considering the main equations in Markowitz model. The portfolio expected return is as follow:

$$E(R_p) = \sum_{1}^{n} r_i\, w_i$$

Equation 4

,where r_i=asset return, w_i =asset weight, n = number of the assets.

The expected portfolio variance is:

$$E(V_p) = \sum_{i=1}^{n} w_i^2 \sigma_i^2 + 2 \sum_{i=1}^{n} \sum_{j=i+1}^{n} w_i w_j \sigma_i \sigma_j\, \rho_{ij}$$

Equation 5

Where σ_i=volatility asset, ρ_{ij}=correlation asset i and asset j.

According to MPT, the investor should decide the assets weights estimating the two moments: return and variance. It's important to focus on the 2nd equation, where ρ is the parameter for correlation across the assets and is positively correlated with the variance of the portfolio. In other words, when assets are positive correlated with each other, the variance of the portfolio increases, while when assets are negatively correlated the variance decreases. Therefore, diversification cannot be related just to the number of assets, since the portfolio variance and its risk can increase by adding high correlated assets. In conclusion, an investor could maximize his wealth by choosing the asset weights that maximize the trade-off between portfolio return and portfolio variance, taking care of the correlation across the assets or the so-called diversification.

Markowitz constructs the so-called efficient frontier, where efficient portfolio lie on.

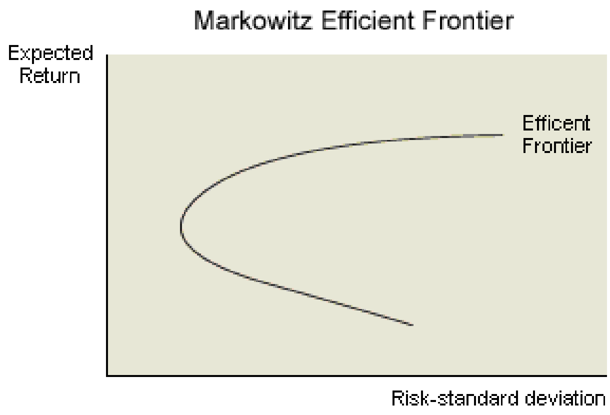


Exhibit 1 : Efficient Frontier. Source: Investopedia

An efficient portfolio is defined as the one, which maximizes the return for a given risk, and minimize the risk for a given return. Markowitz suggest a way to allocate the portfolio in efficient way that is to switch from a maximization of returns to a maximization of portfolio diversification. The diversification of a portfolio increase not just investing in many securities, but composing a strategic mix, paying attention on the level of correlation among his assets; a well-diversified portfolio is made up of low-correlated securities.

In conclusion, an investor maximize his wealth choosing the asset weights that maximize the trade-off between portfolio return and portfolio variance, taking care of correlation across the assets or better diversification.

CAPM theory (Sharpe,1964) introduces the distinction between specific (or idiosyncratic) risk and systematic risk. Specific risk, as its name suggests, relates to risks that are very specific to a security or a small group of security; the specific risk can be substantially reduced, or eliminated, by an adequate diversification. On the contrary, systematic risk is inherent to the entire market and it is affecting the whole financial investments. It is impossible to avoid this type of risk. The most efficient portfolio, the one that achieves the maximum trade-off between return and risk (Max Sharpe Ratio), is the market portfolio exposed just to the systemic risk and placed on the efficient frontier. A rational investor should invest in risk-free assets lending or borrowing and in the market portfolio. The market portfolio is a portfolio where the portion invested in each security is simply equal to the aggregate market value of the security divided by the sum of the aggregate market values of all securities.

Sharpe has also introduced a measure of the asset's sensitivity to market risk, the quantity often represented as Beta. Beta is calculated by regression analysis and can be identified as the tendency of the security's return to react to market fluctuation. The Equation 6 shows the CAPM model and the Equation 7 shows the equation of the Beta factor.

$$E[r_a] = r_f + \beta\left(E[r_m] - r_f\right)$$

Equation 6

$$\beta = Cov(r_a, r_m)/Var(r_a)$$

Equation 7

Where r_a =asset return, r_m =market return, r_f=risk free rate, $Cov(r_a, r_m)$= covariance between asset return and market return, $Var(r_a)$ = variance of asset return.

This equation claims that expected return assets are driven by a single-factor model, since the only macroeconomics factor influencing assets return is the correlation with expectation about market return.

Beta is the key point to better understand the CAPM. With a value of Beta bigger than 1, the asset will move (upside or downside) with a higher magnitude than the market, which can be translated in more standard deviation and, therefore, the asset is riskier compared to the market portfolio. This obviously is linked to a bigger risk premium. Moreover, if Beta is smaller than 1, the asset has less fluctuation and standard deviation relative to market portfolio. As a consequence, investors ask for a lower risk premium.

In other words, the beta of the portfolio is the coefficient providing the magnitude of the systematic exposure taken by an investor and the relative risk premium.

So his model predicts that the only risk rewarded by higher expected return is the systemic risk, since the expected (excess) return is a function of a measure of systemic exposure. Therefore an investor that invests in a portfolio different from the market portfolio would be exposed to unrewarded risk, since no one pay an excess return for idiosyncratic risk in CAPM world.

The proponents of Cap-weighted (CW) indices often propose as their main argument the CAPM theory. Mauldin (2006) notes that "*[the CAPM] is the basis for a number of index models, especially capitalisation weighted indexes like the S&P 500*".

1.2 Shortcomings and critic of Cap-weighted index

CAPM could work only in a theoretical world based on a range of heroic assumptions. The inability to hold those assumptions may lead to a biased prediction of an efficient market portfolio. According to the theory, the Cap-Weighted index is efficient when:

- Investors have identical preferences and investment horizons
- Investors can borrow without limits
- No taxes, no transaction costs

- All assets are tradable

Many researches reject those assumptions.

Boyer, Mitton and Vorkink (2009) say that the investors don't optimize their utility looking for mean-variance efficiency, instead they seek exposure to lottery-like payoffs; as a consequence homogeneous preferences are unfeasible. Moreover, empirically, it is straightforward the presence of noise traders in the market, which trade irrationally.

Transaction costs and taxes impact substantially the return of investors, although for most liquid and traded assets can be low, in particular when in an active strategy, where investor handles a portfolio with a high level of turnover.

Unlimited borrowing is not achievable for most of the traders. Although short selling could be a valid alternative, it's subject to restriction in many countries, especially for institutional investors.

Lilti et al.(2006), underlines that non-tradable assets, as human capital, represent a significant part of investor wealth.

Given the evidence, we can argue that CAPM hardly meets empirical validation.

Briefly, the theory predicting the efficiency of market portfolio is affected by many shortcomings from a theoretical and empirical point of view.

More doubts arise about Cap-weighted (CW) index as a good proxy for the market portfolio.

Roll (1977) argued that, although the true market portfolio could be mean-variance efficient, any constructed index is just a proxy for the market portfolio and so there is no reason to expect an efficient allocation, since these indices can be very different from each other. A true market portfolio should be able to include all risky assets, the non-tradable ones as well as the real estate, human capital and consumer durables.

Sinclair (1998) says that typical Cap-Weighted index, as S&P 500 cover 70% total market capitalisation without including bonds and small cap, providing a biased proxy of market portfolio. Goltz and Le Sourd (2010), following Roll's criticism to CAPM, gathered a list of indices composed by researchers who tried to find the "true" market portfolio, taking in consideration not just the classical stock index, but different types of investment or weighting scheme.

Table 1: Market portfolio proxies tested in empirical studies

Author(s)	Journal(s)	Description of the proxy	Assets included in the proxy			Conclusion
			Stocks	Bonds	Other assets	
Black, Jensen, and Scholes (1972); Fama and MacBeth (1973); Gibbons (1982)	*JPE; JFE*	Equal-weighted portfolio of all NYSE-listed stocks	*			Efficient
Frankfurter (1976)	*JF*	Dow Jones Ind. Average, S&P 500, Geometric mean market value index (522 stocks)	*			Not conclusive
Zhou (1981)	*JFE*	CRSP value-weighted index of NYSE stocks	*			Not efficient
Gibbons (1982)	*JFE*	Equally weighted index of NYSE stocks	*			Efficient
Stambaugh (1982)	*JFE*	NYSE common stocks, US corporate and government bonds and bills, residential real estate, house furnishings, and automobiles.	*	*	*	Efficient
Shanken (1985, 1987)	*JFE*	Equal-weighted CSRP index and/or long-term US government bond portfolio	*	*		Not efficient
Brown and Brown (1987)	*JPM*	Different market proxies including common stocks, fixed-income corporate issues, real estate, government bonds, municipal bonds	*	*	*	Not conclusive
Gibbons, Ross, and Shanken (1989)	*Emtrca*	Value-weighted CRSP index of all NYSE stocks	*			Not efficient
Harvey and Zhou (1990)	*JFE*	Value-weighted portfolio of all NYSE stocks	*			Not efficient
Harvey (1991)	*JF*	MSCI indices for a collection of countries.	*			Not efficient
Haugen and Baker (1991)	*JPM*	Wilshire 5000	*			Not efficient
Grinold (1992)	*JPM*	Commercial indices like S&P 500, FTA, All Ordinaries, TOPIX, DAX	*			Not efficient
Jagannathan and Wang (1993, 1996)	*FRBM, JF*	Market portfolio proxy including human capital (labour income)	*		*	Not conclusive
Fama and French (1998)	*JF*	MSCI global index	*			Not efficient
Dalang, Marty, and Osinski (2001/2002)	*JPM*	FT Global Equities Index	*			Not efficient
Kandel, McCulloch, and Stambaugh (1995)	*RFS*	Stocks listed on the NYSE and AMEX	*			Not efficient
Fama and French (2004)	*WP*	Value weighted portfolio of all NYSE, AMEX and NASDAQ stocks	*			Not conclusive

Journal abbreviations: JPE, Journal of Political Economy; JFE, Journal of Financial Economics; JF, Journal of Finance; JPM, Journal of Portfolio Management, Emtrca, Econometrica; RFS, Review of Financial Studies; FRBM, Federal Reserve Bank of Minneapolis; WP: working paper

Exhibit 2 : Market Portfolio proxies, Source: EDHEC-Risk Publication Capitalisation-Weighted Indexing

Analysing the Exhibit 2, we can derive the conclusion that most of the indices aren't efficient (or even conclusive). In spite of most common Cap-Weighted index, stock-only portfolio includes all the tradable stock in NYSE, or even NYSE, NASDAQ and AMEX. It's evident that researchers consider Cap-Weighted not sufficient to represent market portfolio. A further simple consideration comes up; adding assets to Cap-Weighted in order to have a better proxy of the market portfolio, is likely to change optimal weighting of the existing assets. Consequently, a subset of the true market portfolio, as Cap-Weighted, and the true market portfolio, can't be efficient at the same time.

It's true that three portfolios are efficient, but two of them use an equally-weighted allocation suggesting that for many researcher is a better proxy for market portfolio (Black, Jensen, and Scholes 1972; Fama and MacBeth 1973; Gibbons 1982; Shanken 1985, 1987); and Stambaugh (1982) propose a comprehensive market portfolio, taking into account stock, bonds and non-tradable assets.

The foregoing discussion has attempted to show that Cap-Weighted indices are considered as a component of market portfolio and, moreover, equally weighted indices are often used in the literature as a proxy for the market. To sum up, "commercial stock market indices are poor proxies for the market portfolio".

A persistent theme in Cap-Weighted index is concerning the unrewarded risk and lack of diversification.

With this in mind, let's illustrate what unrewarded risk is. Unrewarded risk means that the risk is not compensated by an adequate return. According to CAPM, the only rewarded risk is systematic risk, which is proportional to the sensitivity of the asset to the market fluctuation (β). In the modern age, the literature agreed that there are more factors influencing excess return. Some of these factors were illustrated first in the Fama-French three factors model and a forth factor were added later by Carhart.

$$r_i = \alpha + \beta_i RMK + \gamma_i SMB + h_{i_i} HML + m_i MO$$

Equation 8

Where r_i = excess asset return

This equation claims that excess asset return depends from 4 factors:

- RMK= Market risk exposure
- SMB= Small (Cap) Minus Big risk exposure
- HML= High Minus Low risk exposure
- MOM= Monthly Momentum risk exposure

The first risk was already introduced with CAPM, and it implies that excess return is correlated with sensitivity to market return.

SMB is identified as a size factor, because it depends from the asset capitalization. This factor was designed by Fama-French. SMB finds his own justification in the observation of persistence in outperformance of a firm with low capitalization on the market. In a quantitative manner is a zero-investment portfolio that is long on small capitalization stocks and short on big cap stocks.

HML is known as a value factor, since it's correlated with the ratio between the fundamental value and the market value. Also designed by Fama-French, HML came up from the examination of persistence outperformance of a firm with low book-to-market value on the market compared to the one with low book-to-market value. Again, HLM is a zero-investment portfolio that is long on high B\M firms and short on low B\M firms.

MOM is the so-called momentum factor, since it's relative to the inclination of a stock price that experienced increase to continue rise and a stock price that experienced decrease to continue decline. It was designed by Carhart (1997), who observed how stocks that had positive 12 month return continue to have positive return, while stocks that had 12 month negative return continue have negative return. MOM is the difference between: the equally weighted portfolio of the firm with the highest 30% of 11-month return lagged one month, and the equally weighted portfolio of the firm with the lowest 30% of 11-month return lagged one month.

Market Cap-Weighted indices are prone to risk of concentration, a shortcoming observed at a country level and at a sector level. In order to have a better idea about this, we can think about USA which has 19.3 trillion of market cap, a 52% of world capitalization, and its weight in MSCI world index is over 50%; about sector, concentration was more relevant in early 2000', when technology, telecommunications and media services composed one third of MSCI world. In other words, it is straightforward that a large concentration leads to lack of diversification, which is the main feature required for an efficient portfolio, according to all the portfolio allocations references on the relative literature (first of all the MPT of Markowitz).

Moreover, it is subject to a large cap bias and growth bias by construction, since asset weights are set based on market capitalization leading to concentration in large cap firm, which empirically shows often low B/M value (growth firm) compared to small cap. Furthermore, a larger capitalization leads to a higher exposure to momentum factor.

In addition, CW tends to be backward-looking while share prices are results of future expectations in terms of returns, therefore are forward-looking. By fact, a significant flaw is about mispricing, which in the extreme case could lead to capture the full effect of asset price bubble. Indeed, market capitalization is the product between price share and outstanding share, therefore if shares are mispriced, then CW is based on wrong data and is sub-optimal by construction.

2. The Smart Beta approach: alternative indexation

In the previous chapter, we introduced the MPT and the definition of diversification as the way to allocate efficient portfolio. Markowitz claimed that a well-diversified portfolio is not just plenty of securities, but an investor should take care of the correlation among the assets, since low correlation reduces variance of the portfolio and its Sharpe Ratio. Then, CAPM theory proposed the market portfolio as optimal diversified portfolio, which today is represented by the Cap-Weighted index.

Smart Beta attempts to enhance the quality of the portfolio diversification, creating a heuristic and scientific allocation scheme, which can be either complex or very simple (like Equally Weighted). Those allocations schemes, based on MPT mean-variance optimization, extend the traditional portfolio allocation by adding more flexibility, thanks not only to better diversified allocation but also to value-elements, such as specific factor exposure. Those weighting scheme relies on analytical tools and fundamental analysis, providing many different approaches that can meet different requirements of the investors. First generation of Smart Beta indices came out as a solution to the market-cap weighting methodology, and applied a single factor model in order to solve the main shortcomings of Cap-Weighted index. Amenc, Goltz and Martellini (2013) argue that Smart Beta 1.0 (referring to the first generation of smart beta) index was based on stocks' economic characteristics, such as value and growth, without distinguishing the stock methodology from the weighting methodology. Moreover, these indices forced the investors to be exposed to particular systematic risks that represented the source of their performance. In other words, investors weren't able to choose the kind of risk to be exposed, and that brings a lack of transparency of the index leading the Smart Beta approach to be commercially less attractive.

As a result many researchers have implemented a second generation of smart beta applying a multi-factor approach, following a more sophisticated weighting scheme, often called as Smart Beta 2.0. Likewise, as said by Amenc et al. (2013) in their work, the main shortcomings were surpassed, and now Smart Beta approach has two distinguished phases for stock selection and weighting, avoiding the undesired risk exposure.

In details, if an investor wants to benefit from a well-diversified portfolio replacing CW index, but he is not willing to face a huge liquidity risk (in the next paragraph we will analyse the most common risk in Smart Beta compared to CW), he can select the most liquid stock and then apply a smart beta optimization weighting methodology.

Briefly, Smart Beta 2.0 is an enhanced and more competitive alternative to Cap-Weighted indices that aims to substitute classical benchmark in the passive and active portfolio management.

2.1 Risks of Smart Beta Indices

Smart Beta suffers from both kind of risk: systematic risk and specific risk.

Systematic risk

The systematic risk can be measured in absolute terms or more often in comparison to Cap-Weighted indices. This risk concerns the exposure of the index on risk factors, depending on the methodology applied for the construction.

We have already seen for Cap-Weighted index that are tilted to large cap, momentum and growth factors. Furthermore, these indices are typically concentrated in highly liquid stocks, because firms with large capitalization are the most traded on the market. Instead, Smart Beta, as we explained before, are more often than not subject to small cap and value exposure, which are likely to have low level of liquidity compared to large cap.

Since the factor depends from how the index is constructed, different smart beta are tilted towards different factors. Given an index based on firm's economic size, it's likely to have an exposure on value or small cap factor which often has the best fundamentals because of the increased possibility of growth in spite of large cap or growth firms.

Furthermore, an index constructed according to a low-volatility factor, shows often a large concentration (particularly under no-short selling constraint) in some specific factors (as Utilities), with just few assets having a positive weight.

In order to have a better understanding of the factor exposure of Smart Beta indices, Amenc , Goltz and Martellini calculated the factor exposure of the major Smart Beta index, regressing the return of the stocks on the four factors model (Fama French 1993, Chachart 1997). The Exhibit 3 shows the results.

	FTSE RAFI U.S. 1000 Index	FTSE EDHEC Risk Efficient U.S. Index	MSCI USA Minimum Volatility Index	S&P 500 Equal Weight Index
Annualised Alpha	2.3%	3.3%	2.9%	2.7%
Market Beta	97.9%	93.1%	79.9%	102.4%
Size (SMB) Beta	14.8%	39.9%	-5.8%	40.5%
Value (HML) Beta	15.8%	0.4%	4.7%	1.5%
Momentum (MOM) Beta	-11.6%	-5.6%	-0.7%	-8.1%
Adjusted R-square	98.5%	98.7%	95.1%	98.9%

Exhibit 3 . Factor Exposures of Commercial Smart Beta Equity Strategies, Source: Edhec Risk Publication Smart Beta 2.0

As we can see, FTSE RAFI U.S 1000 shows exposure to the value factor, probably because is composed from huge number of equities of which a significant part has a high B/M value.

The common factor between the Smart Beta strategies is the negative correlation with momentum factor. This is highly unsurprising, since Cap-Weighted indices are typically exposed to momentum factor. Therefore any alternative strategies that deviate significantly from CW, should experience a negative exposure to momentum factor.

FTSE RAFI U.S. 1000, FTSE Edhec Risk Efficient and S&P EW show a significant exposure to SMB factor, confirming our previous explanation, that smart beta strategies tilt to rewarded risk as small cap.

On the contrary, MSCI USA minimum Volatility is the only index exposed to large cap, since its negative exposure to small cap. This is quite logical, given the fact that large cap firms are more stable and therefore experience less volatility compared to small cap, moreover, as we have already highlighted, low volatility index is subject to concentration as Clarke, de Silva and Thorley (2011) wrote about the long-only Global Minimum Variance (GMV): "*portfolio averages about 120 long securities, i.e., about 12% of the 1000-security investable set*".

Given this evidence for concentration in minimum volatility index, Amenc, Goltz and Stoyanov (2011) tried to quantify the magnitude of the lack of diversification in Minimum Variance portfolio through the dependency between weight of minimum volatility portfolio and correlation among assets. They took under consideration 100 stock, sorted by volatility into three groups of equal size. Let's consider the Exhibit 4.

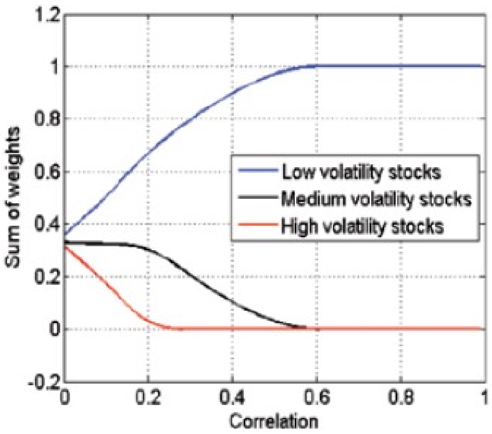


Exhibit 4 : Concentration of Minimum Volatility portfolios in low volatility stocks. Source: Edhec Risk Publication Smart Beta 2.0

Concentrations in low volatility index increase as correlation among assets increase. As we have previously stated if the correlations among assets increases, the benefit of diversification decrease. Consequently, the reduction of the portfolio's volatility is achieved just by concentrating the investment in low volatility index. Recently, the solution came up by bounding stocks weight (upper bound and lower bound) and imposing sector constraint, but more often than not investor experience a reduction in portfolio's performance.

Given this evidence for the importance of equity factor, Amenc, Goltz and Martellini provided in a recent research: "*Towards Smart Equity Factor Indices: Harvesting Risk Premia without Taking Unrewarded Risks*", a comprehensive theoretical and empirical explanation about equity factor and its fundamental role in Smart Beta. They deeply examine how the stock selection stage can strongly improve the performance and allows the investor to tilt the risk factor according to his preference. The results show that this smart beta approach leads not only to better performance, but also provides even more flexibility in active and passive management.

Specific Risk

The specific risk for the heuristic benchmark derives the assumptions of the model and on the parameter estimations, which can lead to lack of robustness and poor out-of-sample performance. As we explained earlier, Cap-Weighted index is constructed by weighting assets according to the market capitalization, justifying its optimality with the CAPM model.

The investor often relies on past performances in order to assess the quality of smart beta model, rather than verify by an accurate analysis, where its specific risk is bearing. Moreover, Smart Beta is a recent research and empirical tool derived from a relative recent period, so there is no possibility to verify a performance in the long term. In conclusion, a well-informed investor can't rely on short period past data performance if he wants to achieve an out-of-sample robustness of his Smart Beta Index.

To quantify the specific risk, we can sort it in two different dimensions:

- Parameter estimation risk
- Optimality risk

Every weighting scheme based on a parameter such as return, volatility and correlation, is subject to risk estimation. As a matter of fact, when a benchmark is constructed, the variables are taken as "expected" based on sample estimation, which obviously is a proxy for the true value. So risk estimation is due to the difference between the true value and the estimated value. This kind of risk increases according to the number of variables estimated. For instance, the Maximum Sharpe Ratio portfolio needs the estimation of return, volatility and correlation, therefore is exposed to a large estimation error. Furthermore, literature agrees that the parameter suffering the highest level of estimation error is the return, since past data hardly explain future return.

In the recent years, parameter estimation experienced a notable progress thanks to research in financial econometrics. The main methods are:

- Rolling methods, where parameters are estimated as an average of an interval of past data (normally 5 years)

- Exponential smoothing methods (moving averages with weights decreasing over time - most recent observations have a higher weight)
- Econometrics models as GARCH
- PCA analysis, which involve reducing the dimensionality of the set of parameter estimation
- Asset pricing model as CAPM, Fama French Model

A good estimation of a parameter requires a trade-off between the two components of parameter risk:

- Sample risk
- Model risk

Briefly, sample risk affects estimation based on sample-based information. For example when estimation is an average of past historical data, it can lead to a biased proxy out-of-sample. On the contrary, model risk is about the risk to use the wrong asset pricing model, for example when estimating a variable with a single-factor model while it depends from three factors.

The optimality risk consists of ignoring parameter estimates and replacing Max Sharpe ratio, the optimal portfolio by construction, with other optimal criteria. The new criteria are optimal under some conditions that can be more or less restrictive. Generally speaking, the investor is not looking for the best proxy for his parameter. On the contrary, he ignores parameter estimation and allocate his portfolio with an inefficiency cost related to his objective optimization, which is optimal (so coincides with Maximum Sharpe Ratio) just under heroic assumptions. In detail, low volatility strategies suffer from optimality risk since they rely on the assumption of negative correlation between risk and return, an assumption that recently found many criticisms amongst the financial literature.

A crucial case to point out is the case of fundamental strategies, which we have already spoked about. The supporter of this kind of strategy often claims that these strategies are not exposed to parameter risk and optimality risk since they don't rely on quantitative scheme, but a qualitative one. Instead, is showed empirically that fundamental indices have different performance according to the fundamental weighting variable or to the selection stock methodology, leading the investor to ignore the risks that is bearing and how manage them.

2.2 Strategy: Weighting Scheme and relative risk exposure

In this section, we are focusing on the most popular Smart Beta weighting scheme, taking a look to the trade-off between parameter risk and optimality risk. In addition, a pie chart show the difference between the average assets weight among the time interval considered for Cap-Weighted index and the specific Smart Beta considered. The assets weights considered are the FTSE 10 sectorial assets that will be employed for the empirical analysis of the next chapter.

Diversity Weights (DW)

The diversity weights index, proposed by Fernholz, Garvey and Hannon (1999), is composed on the basis of an alternative measure of the distribution of capital in the equity market: the stock market diversity.

$$D_p(w) = (\sum_{1}^{n} w_i^p)^{1/p}$$

Equation 9

This measure reaches his maximum value when $w_{i=}1/N$, where N = number of the assets and their minimum value with w_i=1, when there is a full investment in one single asset.

Fernholz applies his measure in comparison to Cap-Weighted index obtain as follow:

$$w_{DW} = \frac{w_{CW}^p}{1'w_{CW}^p}, 0 \leq p \leq 1$$

Equation 10

Where w_{cw}= weighted according to market capitalization. So for a value p=1 the results is a Cap-Weighted index, while for p=0 we obtain an Equally-Weighted Portfolio. In this research we consider p=1/2.It's quiet straightforward that this strategy isn't exposed to estimation error, since the weighting scheme is based on observable market capitalization. Instead, optimality risk is unclear, since the diversity measure is just a way to deviate from the market capitalization scheme in arbitrage of the investor.

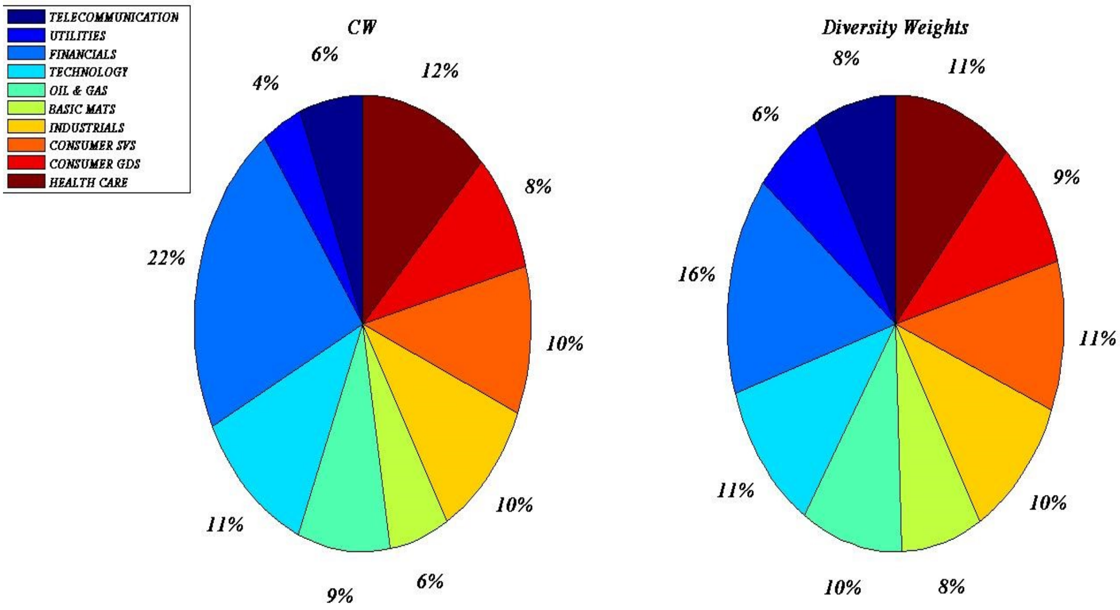


Exhibit 5 : Pie Charts of CW and DW average assets weights

Considering the Exhibit 5, the pie chart for Diversity Weight is similar to the one of Cap-Weighted, only financial sector has a significant lower weight. It's reasonable, since both weighting scheme are based on market capitalization.

Equally-weighted Portfolio or Max De-concentration (EW)

It's the most traditional "naïve" approach for Smart Beta Index. The diversification is simply based on an equal distribution among the assets, in order to avoid the main shortcomings of Cap-Weighted index as concentration and exposure to momentum factor, and of course is immune to estimation error. Although this may be true, an equally weighted approach has a highly cost of optimality since it reaches the Max Sharpe Ratio just if all the return, the volatility and the correlation of the assets are equal. In order to maintain the equal distribution of the portfolio the investor has to rebalance it often, having high cost of turnover. Moreover, EW experience tilting to small cap factor, resulting in a low level of liquidity. Imposing constraint on turnover and liquidity, EW can have a better performance compared to his Cap-Weighted counterpart. The pie chart is not relevant, since Equally Weighted scheme provides an equivalent weight to all the assets.

Equal Risk Contribution (also known as Risk Parity) (ERC)

Risk Parity is a particular case of risk budgeting optimization. Starting from a Markowitz mean-variance optimization, an investor can set some constraints about risk contribution of every assets. This is the so-called risk budget constraint. Mailard et al.(2008) proposed a portfolio based on risk budgeting method setting risk contribution equal across the assets. Recently, Clarke et al. (2013) provided an analytical solution, but in our empirical analysis we will use Mailard numerical solution as follows:

$$x^* = argmin\, f(x), \qquad f(x) = \sum_1^n \sum\nolimits_1^n (x_i(\Sigma x)_i - x_j(\Sigma x)_j)^2$$

$$u.c.\, 1^T x = 1 \; and \; 0 \leq x \leq 1$$

Equation 11

Where $RC = x_i(\Sigma x)_i$ is the risk contribution of one single asset to the overall risk of the portfolio, x_i = asset weight, Σ = covariance matrix. The Equally Risk Contribution is obtained by minimizing the difference between the risk contributions of the assets. This mathematical problem can be solved only numerically by a SQL (sequential quadratic programming) algorithm. This strategy is exposed to the parameter estimation risk, since volatility and correlation appear in the weighting scheme. Optimality risk is also consistent because this portfolio is optimal when all the Sharpe Ratio across the assets are equal and pairwise correlations too that is a very restrictive assumption.

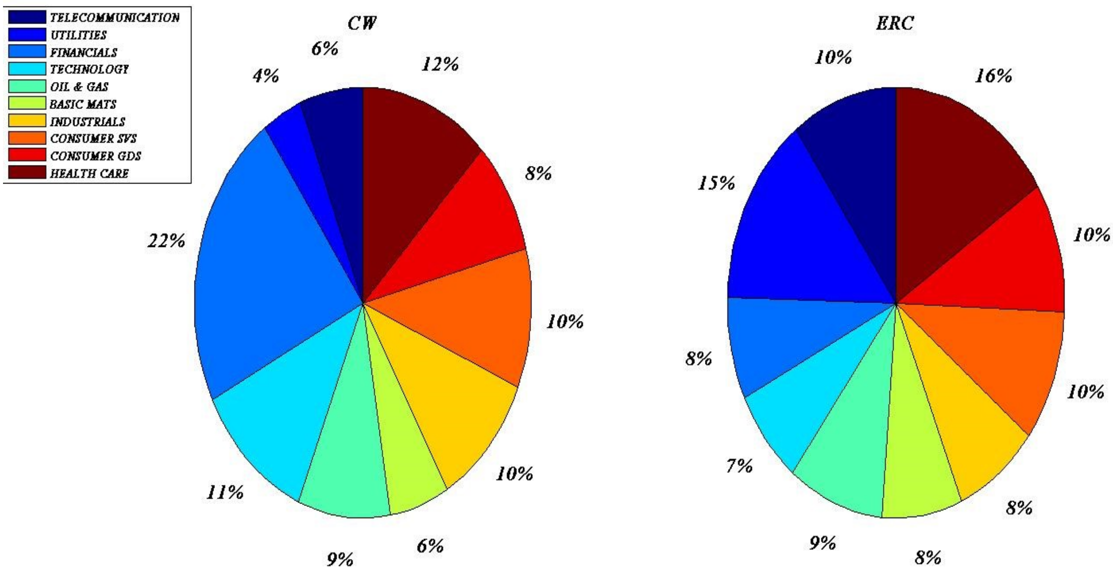


Exhibit 6 : Pie Charts of CW and ERC average assets weights

Exhibit 6 clearly shows that there is a significant change from Cap-Weighted allocation to Equally Risk Contribution allocation, consistently with the different approach in terms of assets distribution. Health Care sector and Utilities replace the leading position of Financial sector.

Diversified Risk Parity (DRP)

This strategy is an extension and simplification of ERC portfolio, which calculate uncorrelated risk contribution setting weights as the inverse of volatility.

$$w_{DRP} = \frac{\sigma^{-1}}{1'\sigma^{-1}}$$

Equation 12

The estimation risk is reduced compared to ERC portfolio, since correlation are assumed identical across the assets. The optimality risk is the same as ERC portfolio.

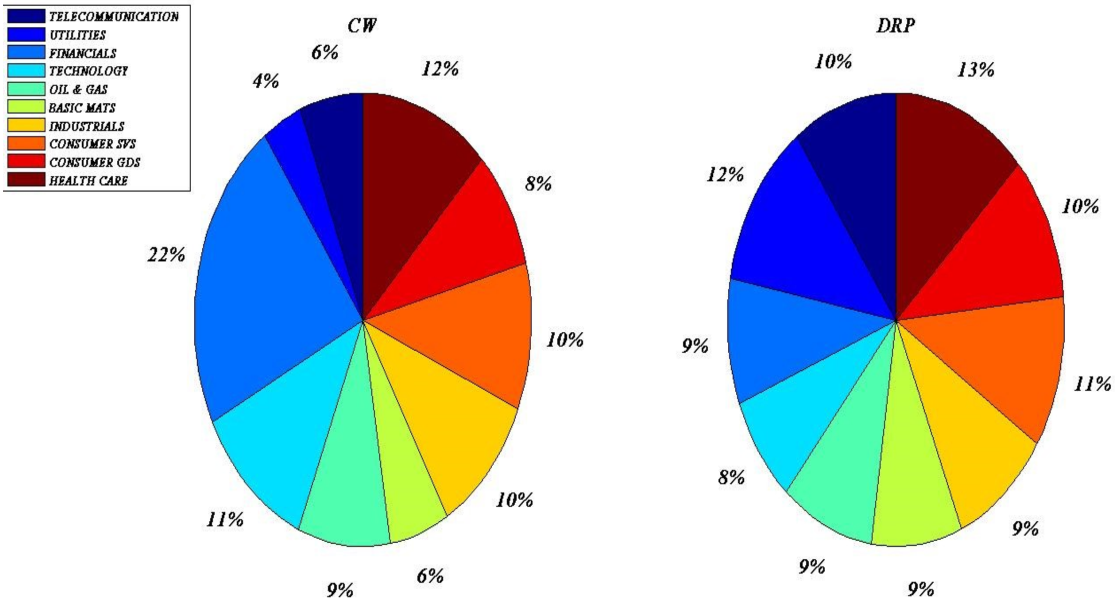


Exhibit 7 : Pie Charts of CW and DRP average assets weights

The pie chart of Diversified Risk Parity is similar to the one of Equally-Weighted contribution. Therefore, there is a lot of dispersion among the assets. In conclusion, it provides a better diversification in terms of number of assets compared to Cap-Weighted.

Maximum Diversification Ratio (MDR)

Choueifaty and Coignard (2008) propose a measure of portfolio diversification taking in account volatility and correlation of the assets. The result is the so-called Diversification Ratio:

$$DI = (\frac{\sum_i w_i \sigma_i}{\sqrt{\sum_{i,j} w_i w_j \sigma_{ij}}})$$

Equation 13

Where the numerator is the sum of the volatility of the single assets and the denominator is the volatility of the whole portfolio. In other words, this measure of diversification can be summarized as the distance between the individual volatility components and the volatility of the portfolio. The strategy maximizes this index as follow:

$$w^{MDR} = argMaxDR(w)$$

$$under\ contraint \begin{cases} \sum w = 1 \\ w > 0 \end{cases}$$

Equation 14

Choueifaty and Coignard underlined a particular feature of this strategy. MDR often lead to a concentration in few assets, creating doubts about the quality of the portfolio diversification. In order to clarify it, the researchers explain it as follow: "*For example, an MDR portfolio constructed using S&P500 stocks, may hold approximately 50 stocks. That does not mean however that this portfolio is not diversified, as the 450 stocks it does not hold are more correlated to the MDP compared the 50 stocks it actually holds*." Briefly, taking in account the correlation across the assets, MDP capture the assets which minimize the correlation reducing the volatility, leaving out the stock more correlated.

Estimation risk is considerable since volatility and correlation appear in the ratio. Optimality is under the condition that all Sharpe Ratio across the assets are equal.

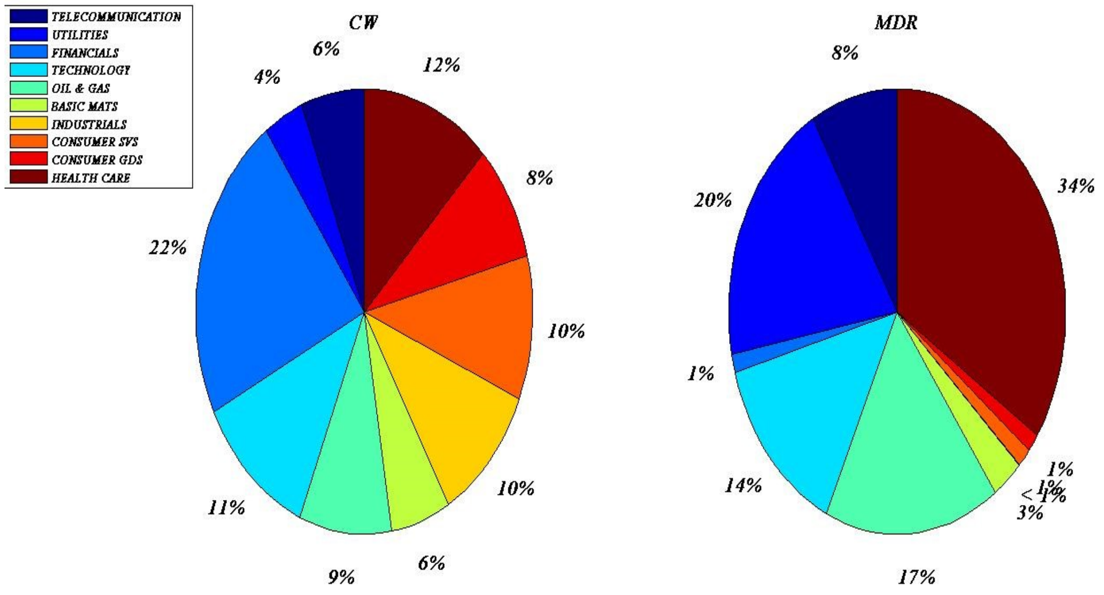


Exhibit 8 : Pie Charts of CW and MDR average assets weights

The pie chart of MDR shows that 5 assets have a relevant weight and the Health Care sector achieves the highest share. This strategy deviates substantially from the Cap-Weighted approach and the maximization of the diversification is achieved with a reduced number of assets, confirming the considerations of Choueifaty and Coignard.

Global Minimum Variance (GMV)

This portfolio minimizes the portfolio volatility.

$$MV = \begin{cases} \min w' \sum w \\ s.t\ w' 1_N = 1 \end{cases}$$

Equation 15

Σ= covariance matrix, w= assets weights. Normally constraints are set to no-short selling and the sum of the weights equal to 1. As we have already seen in the previous chapter, no-short selling leads to high level of concentration.

As MDR, the estimation risk is due to volatility and correlation estimation, while GMV is optimal when all the expected returns are identical across the assets.

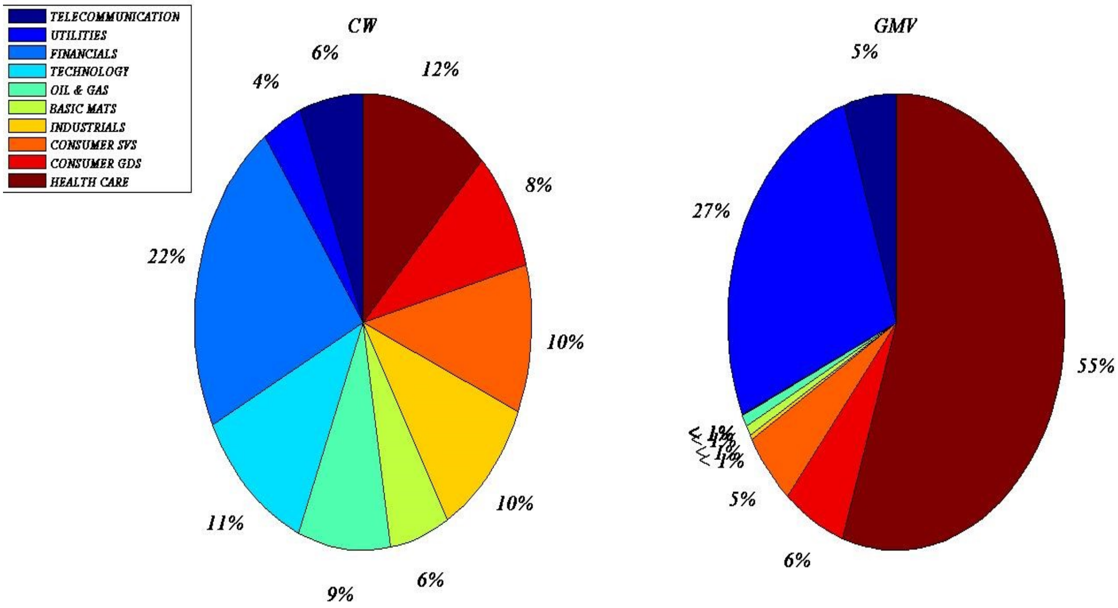


Exhibit 9 : Pie Charts of CW and GMV average assets weights

The Exhibit 9 confirms the main shortcoming of unbounded Global Minimum Variance: huge concentration in one or two assets. Global Minimum Variance is very far from the asset allocation of Cap-Weighted Index and shows a substantial lack of diversification.

Diversified Minimum Variance (DMV)

Coqueret (2014) proposed an alternative to GMV, minimizing portfolio volatility by setting a L^2 constraint on the portfolio weights.

The constraint is referred to a concentration index known as Herfindahl Index.

$$D(w) = w'w = \sum_{1}^{n} w_i^2 = \|w\|$$

Equation 16

Where w = asset weight.

This index decreasing more is the diversification level, reaching its minimum value for 1/N (Equally-Weighted Portfolio) and its maximum value 1 (single asset investment).

In this case the upper bound is the GMV asset allocation, since the portfolio is based on minimum variance optimization.

$$MV = \begin{cases} \min w' \sum w \\ s.t\ w' 1_N = 1 \\ w'w = \delta \\ \delta \leq 1/N \end{cases}$$

Equation 17

Where Σ=asset covariance matrix, w = asset weight. In the Diversified Minimum Variance the HHI index constraint is set at a proxy level of 1/N.

This constraint avoids concentration of GMV, with the same optimality and parameter risk.

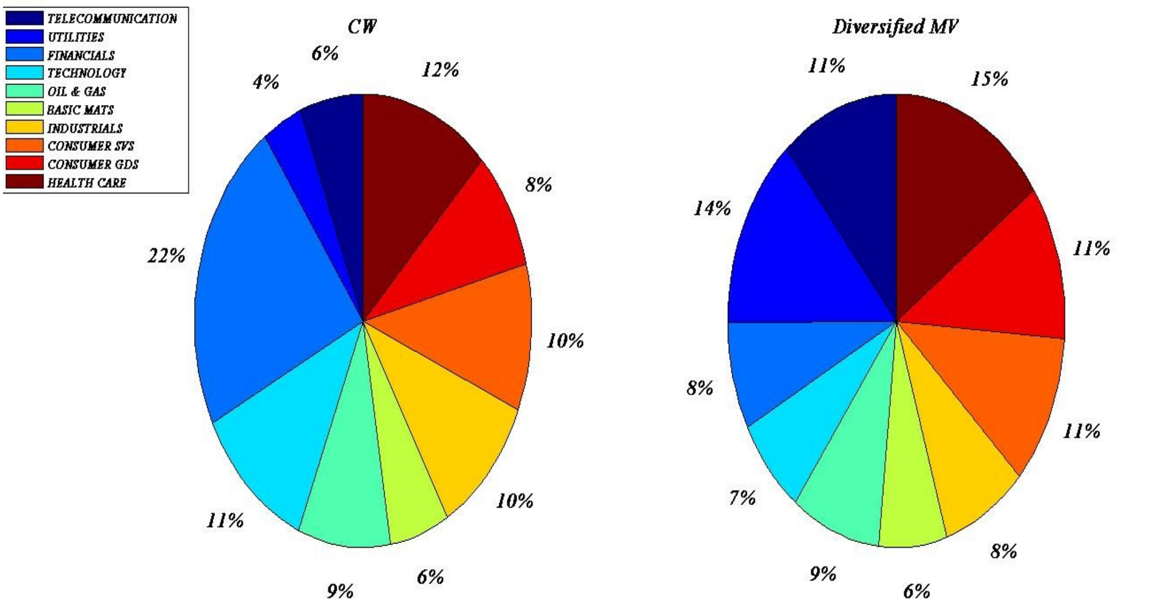


Exhibit 10 : Pie Charts of CW and DMV average assets weights

Considering the Exhibit 12, the Diversified Minimum Variance provides a well-diversified portfolio in terms of number of assets. In comparison to Cap-Weighted, Financial experience a significant reduction of share to the benefit of Telecommunication and Utilities.

Semi Diversified Minimum Variance (SD)

Coqueret proposed also an alternative to Diversified Minimum Variance, more aggressive.

In the Semi-Diversified portfolio the constrain is set at 2/N level of HHI index, because the author is looking for a median diversification pointing out that well-diversified portfolio have a level D $(w)^{-1}$ = 0,7 and low diversified have D $(w)^{-1}$= 0,3.

$$V = \begin{cases} \min w'\Sigma\, w \\ s.t\; w'1_N = 1 \\ w'w = \delta \\ \delta \leq \dfrac{2}{N} \end{cases}$$

Equation 18

where w =asset weight, Σ = asset covariance matrix.

Obviously, SD has the same optimality and parameter risk of GMV and DMV.

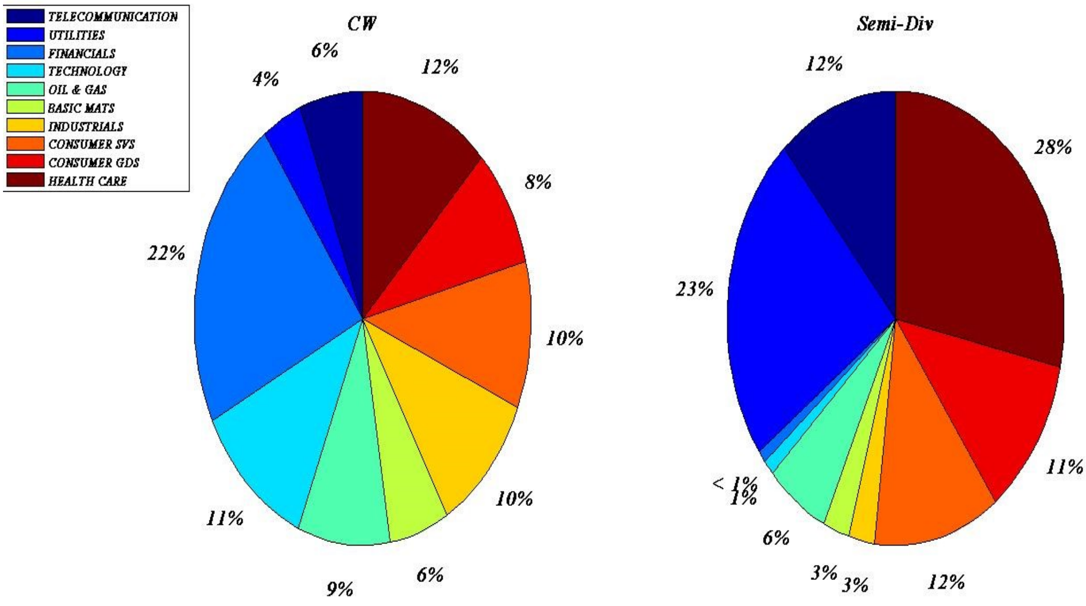


Exhibit 11 : Pie Charts of CW and SD average assets weights

The pie chart of the Semi-Diversified portfolio show that 6 assets have a relevant weight. Therefore, this approach is a "compromise" between the concentred Global Minimum Variance and the Diversified Minimum Variance. As the other strategy based on the minimization of the variance this portfolio deviates from the Cap-Weighted approach.

Max Decorrelation Portfolio (MDe)

This strategy is another alternative to minimum volatility optimization. Christoffersen et al. (2010) assumed constant volatility across the assets and minimized the portfolio correlation. He maximized a measure of diversification as follow:

$$\mathrm{w}^* = \mathrm{argmax}_{\mathrm{w}\epsilon\mathcal{R}}[1 - \sum_{\mathrm{i}}\sum_{\mathrm{j}} \mathrm{w_i w_j} \rho_{\mathrm{i,j}}]$$

Equation 19

Where w = asset weight, $\rho_{i,j}$= correlation parameter across the asset i and j.

It benefits from estimation risk since the only parameter required is correlation, bearing a higher optimality risk assuming constant volatility across the assets.

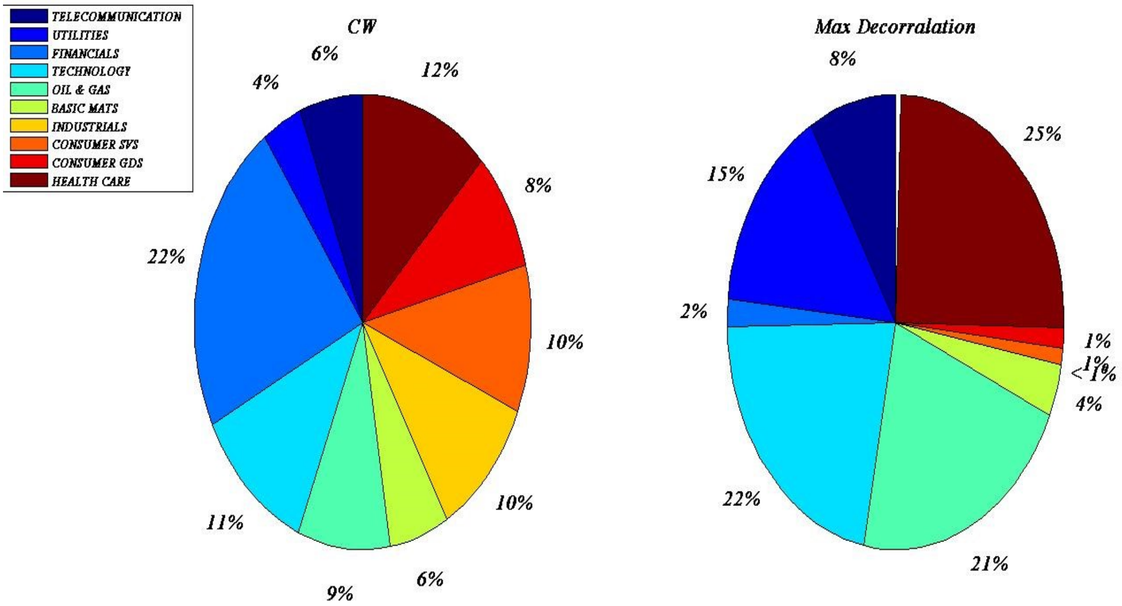


Exhibit 12 : Pie Charts of CW and MDe average assets weights

In the Exhibit 14, the pie chart of Max Decorralation strategy show that four assets covers more than 85% of the full investment. Therefore, this strategy maximize the diversification with a reduced number of assets.

Max Sharpe Ratio (MSR)

According with MPT, the tangency portfolio is the one maximizing the Sharpe Ratio. The Sharpe ratio is the average return earned in excess of the risk-free rate per unit of volatility or total risk. So the optimization is as follows:

$$w^* = argmax_{w \in \mathcal{R}} \frac{w'\mu}{\sqrt{w'\Sigma w}}$$

Equation 20

Where the numerator is the sum of the excess return and the denominator is the volatility of the portfolio. It’s straightforward to think that this strategy is highly exposed to the estimation risk parameter. Indeed, it requires the estimation of expected return, which suffers of a large estimation error. Amenc, Goltz, Martellini and Retkowsky (2011) tried to solve the problem by an indirect calculation of expected returns. They supposed that expected return is directly proportional to downside risk (the risk that the return goes below the expected value). Then, sorting the stock in group, based on this parameter, they distinguish stock in groups rather than stock by stock. On one hand, this method is appealing since it meets the risk-propensity of the investor, who can choose stock according to his risk aversion. On the other hand, low risk stock are disadvantaged since returns are proportional to the risk of the assets.

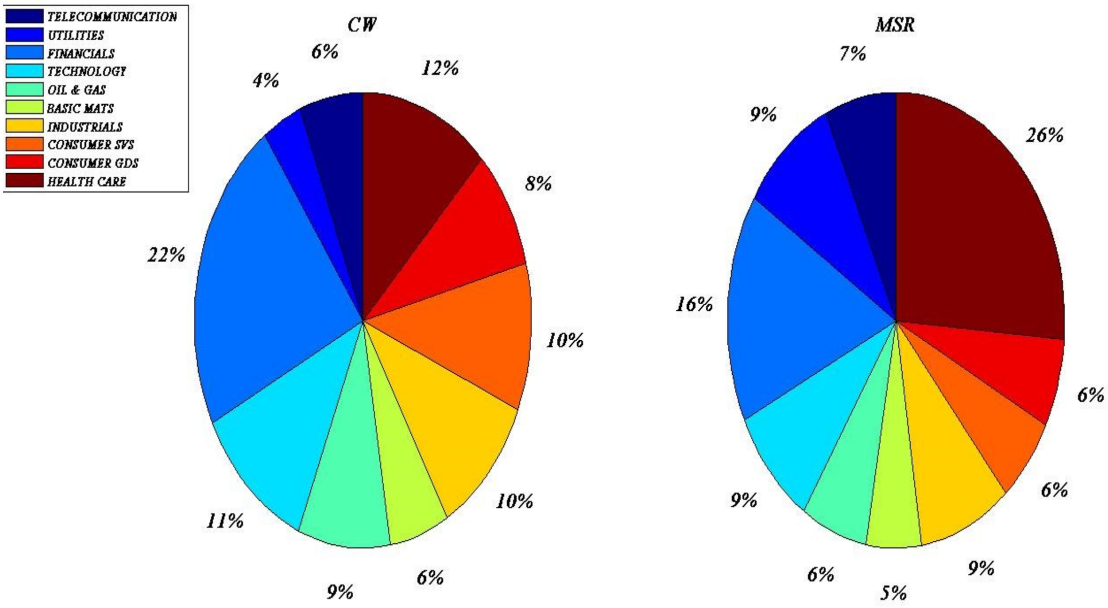


Exhibit 13 : Pie Charts of CW and MSR average assets weights

The pie chart of Max Sharpe Ratio show an interesting feature: it's the only Smart Beta approach where Financial sector achieves a share comparable to the one in the Cap-Weighted approach. Despite of this, this portfolio deviates significantly from Cap-Weighted scheme, particularly because of the substantial share of Health Care sector.

2.3 Controlling risk of Smart Beta Indices

In the section 2.1 we spoke about the risks of Smart Beta approach. The previously mentioned first generation of smart beta were affected by a lack of transparency and exposure to unrewarded risk leading to considerable drawdown for a long period. The Smart Beta 2.0 propose an innovative approach based on the control of both type of risk:

- the systematic risk by distinguish the selection stock stage from the application of the weighting scheme
- the specific risk by the combination of weighting scheme in order to reduce estimation

Controlling systematic risk

The features of Smart Beta strategy isn't particularly interesting for an institutional investor, unless investors focus on integration of the risks assets classes rather than qualitative approach based on economic fundamentals of the firms. We have briefly touched upon the solution to control the systematic risk, which imply a distinguished stock selection stage from the application of smart weighting scheme. Actually, an investor choose before his risk aversion, his expected return and the market. Thanks to this selection stage, it will be easier for him meet his requirement. For example if an investor wants to apply a Smart Beta scheme without incurring in low liquidity risk, he can apply the scheme just to the most liquidity assets. Moreover, if he wants a higher return with a well-diversified portfolio, he can tilt his Smart Beta index on value assets or small cap, which typical provide the higher returns. In other words, the second generation of Smart Beta allows both controlling the risk exposure and improving the performance.

First, we will show how the selection stock stage reduces (and even eliminates) the exposure to undesired factor.

	Global Minimum Volatility (GMV)				Maximum Sharpe Ratio (MSR)				Maximum Decorrelation (MDC)			
Universe	All stocks	Small size universe	Medium size universe	Large size universe	All stocks	Small size universe	Medium size universe	Large size universe	All stocks	Small size universe	Medium size universe	Large size universe
Market exposure of excess returns over CW	-26.20%	-25.41%	-28.03%	-23.92%	-21.92%	-23.69%	-23.94%	-20.09%	-8.60%	-7.93%	-10.57%	-6.59%
Size (Big – Small) exposure of excess returns over CW	-19.00%	-43.75%	-19.32%	1.83%	-21.13%	-46.28%	-21.40%	0.29%	-37.07%	-65.59%	-27.26%	-3.15%

Exhibit 14 : Size exposure of diversification strategies based on different size-based stock selection, Source: Edhec Risk Publication Smart Beta 2.0

The table shows the exposure to small cap factor of three Smart Beta indices, in comparison to S&P 500. The data are compared before and after stock selection stage. We can see easily that small cap exposure is mitigated by a large cap stock selection, which almost eliminate risk due to SML factor and reduces market exposure too. In addition, selection stage can be applied to avoid trading risk, as liquidity risk, selecting just the most liquid assets without any impact on the portfolio performance (Amenc et al. 2013).

Amenc et al. (2014) analyse in depth how selection stock impact on the portfolio performance. In the Exhibit 15 they compared the performance of CW index and Smart Beta (Diversified Multi-Strategy) with four different tilted factor: mid cap, high momentum, low volatility and value.

		Mid Cap		High Momentum		Low Volatility		Value	
	Broad CW	CW	Diversified Multi-Strategy	CW	Diversified Multi-Strategy	CW	Diversified Multi-Strategy	CW	Diversified Multi-Strategy
Ann Returns	9.74%	12.54%	14.19%	10.85%	13.30%	10.09%	12.64%	11.78%	14.44%
Ann Volatility	17.47%	17.83%	16.73%	17.60%	16.30%	15.89%	14.39%	18.02%	16.55%
Sharpe Ratio	0.24	0.39	0.52	0.30	0.48	0.29	0.50	0.35	0.54
Historical Daily 5% VaR	1.59%	1.60%	1.50%	1.64%	1.50%	1.42%	1.28%	1.59%	1.47%
Max Drawdown	54.53%	60.13%	58.11%	48.91%	49.00%	50.50%	50.13%	61.20%	58.41%

Exhibit 15 : Performance Comparison of U.S. Cap-Weighted Factor Indices and U.S. Multi-Strategy Factor Indices, Source: Edhec Risk Publication Toward Smart Equity Factor

It's straightforward that all the alternative indices, even CW with factor tilted, have higher Sharpe Ratio than Broad Cap-Weighted. In addition, the portfolios have similar volatility (less than Low Volatility, which obviously has lower standard deviation) but higher excess returns in comparison to market capitalization indices. Especially interesting are the data about downside risk, a well-known shortcoming of the first generation of Smart Beta. Two indicators are used to measure this risk: Var and Max Drawdown. Both indicators show similar value compared to CW index (even less), suggesting that, the selection stock stage solves the main problem of Smart Beta 1.0. Focusing on CW case, it's clear that the selection stage improved the performance *ceteris paribus*. Moreover, applying a Smart Beta approach to the stock selected, the risk/return properties improve again. In conclusion, selection stage helps the investor to extract value and reduce the risk, leading to a higher flexibility and better capacity to meet specific requirements.

Controlling specific risk

As we have already explained, specific risk is the sum of estimation risk and optimality risk. It's straightforward that in order to control the specific risk, an investor has to look for the best trade-off between optimality risk and estimation risk. Amenc et al. claims that "*It can perfectly happen that a "good" proxy (i.e., a proxy based on parameters with little estimation risk) for a "bad" target (i.e., a target a priori far from the true MSR based on true population values) eventually dominates a "bad" proxy (i.e., a proxy based on parameters plagued with substantial estimation risk) for a "good" target (i.e., a target a priori close to the true MSR based on true* *population values)”*. The following table can be a useful tool in order to obtain a comprehensive idea of this trade-off:

Stock Weighting Strategies	Optimality Risk	Estimation Risk
Max De-concentration	High	Low
Diversified Risk Parity		
Max De-correlation		
Minimum Volatility		
Max Sharpe Ratio	Low	High

Exhibit 16 : Smart Beta Scheme Comparison in terms of optimality risk and estimation risk Source: ETF Securities Publication Smart Beta 2.0 vs Smart Beta 1.0 vs Cap-Weighted

An EW portfolio (Max De-concentration) has the lowest estimation risk, since no parameter needs to be estimated. On the contrary, optimal risk is the highest because the EW is optimal under the condition of equality across variance, correlation and returns of all the assets. It is quite clear that a lower estimation risk has an opportunity cost in terms of optimality risk.

Amenc, Goltz and Martellini tried to quantify the specific risk. They split this risk in its two components for every weighting scheme, analysing every type of risk separately. Then they crossed the results and provided the following quantitative formula for the evaluation of specific risk:

Total distance (in terms of ex-ante Sharpe ratio based on true parameter values) of a given benchmark with respect to the true MSR portfolio

=

Distance of the given target benchmark with respect to the true MSR portfolio assuming away estimation risk (optimality risk in the absence of estimation risk)

+

Distance between the imperfectly estimated target and the true target (estimation risk)

Let’s focus on this equation.

The first addendum is the difference in terms of Sharpe Ratio between the heuristic benchmark and Max Sharpe Ratio portfolio using the true parameter. In other words, the authors apply the heuristic weighting scheme and the MRS weighting scheme to past data and calculate the difference in terms of Sharpe Ratio.

The second addendum is the difference in terms of Sharpe Ratio between the estimated benchmark and the true benchmark. To put it differently, the Sharpe Ratio of the benchmark is calculated with estimated parameter and true parameter, making a difference between those values for which we have a quantitative value of estimation risk. A useful example is provide by Amenc et al. (2013). In the Exhibit 17, five portfolios strategy are compared in terms of Sharpe Ratio. The first column shows the results considering the true parameter, while in the second column are taken in account the estimated parameter.

Portfolio strategy	Average Sharpe ratio with no estimation risk	Average Sharpe ratio with estimation risk	St. dev. of Sharpe ratio with estimation risk
MSR (Maximum Sharpe Ratio)	13.3377	0.5587	0.6114
GMV (Global Minimum Variance)	2.4904	0.8859	0.5743
EW (Equal-Weighted)	0.6048	0.6048	0.0000
CW (Cap-Weighted)	0.4972	0.4972	0.0000
50% GMV + 50% EW	1.0773	0.9443	0.3003

Exhibit 17 : Sharpe ratios for selected weighting schemes in the presence of estimation errors in expected excess returns and covariances Source: Edhec Risk Publication Smart Beta 2.0

It is a straightforward conclusion that MSR has the highest SR when there isn't estimation risk, since it is constructed to maximize it. It is significant that, when estimation risk is taken into account, the SR of the MSR portfolio plummet, suggesting that the estimation of expected returns contributes to offset the benefit coming from a optimality construction. EW and CW don't experience any changes from SR without estimation risk to SR with estimation risk, since no parameters are required. The most interesting case is the portfolio half weighted with GMV scheme and half Equally Weighted. This portfolio has the highest SR after taking into account estimation risk, suggesting that mixing smart beta strategy diversify away the specific risk. To put it in another way, a static strategy is hard to avoid (or mitigate) specific risk, instead, a dynamic diversification which mixes different strategies is the best way to reduce the optimality and parameter risks.

2.4 Passive Management

Before the implementation of Smart Beta, the passive investment was monopolised by Cap-Weighted Index, forcing the passive investors to have a very limited set of indices. Passive investors were able only to equal the market, without having the possibility to beat it. Moreover, passive investors and index providers were not taking any reputation risk, since the performance was the result of market condition. This situation caused an increasing popularity of this kind of passive investment because the investors didn't have any alternative.

Smart Beta represents the revolution of passive management (and not only), which aims to outperform the market. In other words, Smart Beta seeks a more efficient dimension, providing higher return, more controlled risk and well-diversified allocation compared to Cap-Weighted index. Smart Beta index mixes a stock picking strategy and well-diversified scheme in order to provide an alternative passive management, which implies features of active management. Basically, those heuristic indices have two main advantage compared to classical indices and active management.

First, as we have already explained in the previous chapter, Smart Beta creates value thanks both to the selection stock stage and the diversified multi-strategy. Many researchers confirmed how this index has high performance in long term. Second, those indices have an economic advantage since they provide the possibility for an investor to benefit from stock picking and tactical bets at a lower cost compared to active management. Furthermore, it is shown empirically that alpha of active manager are not persistent over the long period, since manager are incline to destroy value taking discretionary decision based on personal tactical bets or forecasts about stock price. On the contrary, Smart Beta leads to a persistent and cheap outperformance for passive management. In addition, the second generation approach guarantees transparency thanks to the specific and clear factor exposure, stimulated by esposure to risk reputation.

Amenc et al.(2013) addressed one important problem, which can lead many investors to get rid of using Smart Beta. They highlight how investors are very careful to the underperformance of the benchmark, and in other words, to downside risk. As an active manager, the performance of Smart Beta is compared to a benchmark: the Cap-Weighted index. In order to have a large consensus among the investors the heuristic index can't underperform the benchmark for a long time. The authors provide an easy and efficient solution for the control of tracking errors: the so-called relative risk control that apply a track record constraint to the weighing scheme.

Panel 1	Smart Beta *without* Relative Risk Control: Scientific Beta USA Indices			
	Efficient Max Sharpe	Efficient Min Volatility	Max Decorrelation	Max Deconcentration
Excess Returns over CW	1.72%	2.16%	1.53%	2.02%
Tracking Error	3.39%	4.60%	3.57%	3.62%
95% Tracking Error	5.28%	8.01%	5.58%	6.36%
Panel 2	**Smart Beta *with* Relative Risk Control: Scientific Beta USA Indices (3% Tracking Error)**			
	Efficient Max Sharpe	**Efficient Min Volatility**	**Max Decorrelation**	**Max Deconcentration**
Excess Returns over CW	0.68%	0.71%	0.99%	0.90%
Tracking Error	1.83%	2.10%	2.03%	1.86%
95% Tracking Error	3.01%	4.30%	3.55%	2.83%

Exhibit 18 : Relative risk of Scientific Beta USA strategy indices Source: Edhec Risk Publication Smart Beta 2.0

The Panel 1 shows four Smart beta Indices without any relative risk control, while in the Panel 2 is applied a 3% tracking error as constraint. We can see that the relative risk control erodes the outperformance of Smart Beta, although excess return over CW is still significant. In conclusion, the results show a trade-off between tracking error constraint and outperformance. Actually, another approach to mitigate tracking error is a core-satellite strategy, where an investor combines a heuristic portfolio with a traditional CW index, controlling the risk by assigning budget limits to risk contribution of the portfolio of the Smart Beta.

The authors analysed another relevant indicator for passive management, the turnover of heuristic benchmark. Since passive management just replicate the benchmark, many criticism come up for Smart Beta because of the high turnover required compared to Cap-Weighted index. The table below represents a comprehensive summary about how lower liquidity impacts the performance of the Smart Beta index compared to CW.

	USA Broad CW	USA Diversified Multi-Strategy			
		Mid Cap	High Momentum	Low Volatility	Value
Ann. 1-Way Turnover	2.65%	23.73%	63.46%	25.75%	23.83%
Relative Returns	-	4.45%	3.56%	2.90%	4.70%
Rel Returns net of 20 bps transaction costs	-	4.41%	3.43%	2.85%	4.65%
Rel Returns net of 100 bps transaction costs	-	4.22%	2.92%	2.65%	4.46%
Weighted Avg Mkt Cap ($m)	44,959	2,734	12,786	13,666	8,326
Days to Trade $1 bn Investment (95th percentile)	0.03	0.24	0.18	0.20	0.19

Exhibit 19 : Implementation Costs of U.S. Multi-Strategy Factor Indices Source: Edhec Risk Publication Toward Smart Equity Factor

The results are quiet clear, turnover is higher for Smart Beta but relative returns net of transaction cost keep a substantial level compared to CW. Even setting an unrealistic high transaction cost at 100 bps, the level of excess returns don't experience large decline. Moreover, the authors focused on an important indicator defined as "*Days to trade",* which is the average number of days required to trade the total stock position in a portfolio of $1 billion, assuming that 100% of average daily traded volume (ADTV) can be traded every day. All the heuristic strategies can be trade in less than a quarter of trading day. In conclusion, although Smart Beta may invest in less liquidity securities, it doesn't experience any significant reduction of performance or difficulty to be traded in the market.

2.5 Active Management

Smart Beta index can be also a useful tool for active management. For example, smart beta can be a substitute of a tilted factor strategy, as low-volatility, saving time and money for the active manager that normally should spent in order to have a balanced portfolio. Particularly interesting is the capacity of some factor to be "counter-cyclical" (minimum volatility) or "pro-cyclical" (momentum, size), providing a tool for hedging or gain profit from market timing in inexpensive way. Moreover, a Smart Beta index tilted to a specific factor can be used as complement of an actively management tilted to another specific factor. For instance a strategy oriented to value factor can be implemented with a momentum or low volatility Smart Beta index.

Furthermore, those heuristic benchmark can represent a more accurate and useful benchmark for active management. Again, a strategy oriented to a specific factor should be compared to an efficient strategy tilted to a specific factor too. Obviously, a fundamental strategy based on value factor should be evaluate in comparison to a benchmark tilted on value factor, which definitely will give a better proxy in order to evaluate the performance of the strategy.

In conclusion, active manager should think about smart beta index as a good opportunity to increase their alpha since they can benefit from benchmark, which reflect tactical bets, have more flexibility and so an high adaptability to desired risk exposure with transparency and efficiency.

3. Empirical Analysis

3.1. Dataset

In this empirical analysis are taken into account monthly time series of 10 sector indices, from 31/01/1994 to 30/10/2015. The sector indices are FTSE All World Cap-Weighted index, which represents large cap and mid cap of developed and emerging markets divided to sectors as follows:

TELECOMMUNICATION	Fixed Line Telecommunications Mobile Telecommunications
UTILITY	Electricity Gas Water & Multiutilities
FINANCIAL	Banks Nonlife Insurance Life Insurance Real Estate Investment & Services Real Estate Investment Trusts Financial Services
TECHNOLOGY	Software & Computer Services Technology Hardware & Equipment
OIL & GAS	Oil & Gas Producers Oil Equipment Services & Distribution Alternative Energy
BASIC MATERIALS	Chemicals Forestry & Paper Industrials Metals & Mining Mining
INDUSTRIALS	Construction & Materials Aerospace & Defence General Industrials Electronic & Electrical Equipment Industrial Engineering Industrial Transportation Support Services
CONSUMER STAPLES	Food & Drugs Retailers General Retailers

	Media Travel & Leisure
CONSUMER GOODS	Automobile & Parts Household Goods & Home Construction Leisure Goods Personal Goods Tobacco
HEALTH CARE	Health Care Equipment & Services Pharmaceuticals & Biotechnology

Table 1 : Description of the assets

Sector indices are used as assets for portfolio allocation with different weighting schemes in a monthly timing. The parameters are estimated based on real return using a rolling methods approach at monthly step with an interval of past data of 5 years, in other words, every data estimation is obtained by an historical average of the past 5 years data. The results are 201 vectors of weighting data for every weighting scheme. The main purpose of this analysis is to compare Cap-weighted scheme and Smart Beta scheme, focusing on the performance in the long term period. The weighting schemes are as follows:

WEIGHTING SCHEME
Global Minimum Variance
Max Sharpe Ratio
Equally Weighted
Max Decorrelation
Max Diversification Ratio
Equally Risk Contribution-Risk Parity
Diversified Risk Parity
Diversified Minimum Variance
Semi-Diversified Minimum Variance
Diversity Weight

Table 2 : List of the Smart Beta weighting schemes

3.2. Graphic results

The results take in account the evolution of a strategy allocation given 5 years of data information, from 31/01/1999 until 30/11/2015. It's important to underlying that for all the strategy is applied a positive weight constraint and leverage is not allowed (it's not possible to invest more than 100% of wealth). The following graphs show the weights of the 10 assets.

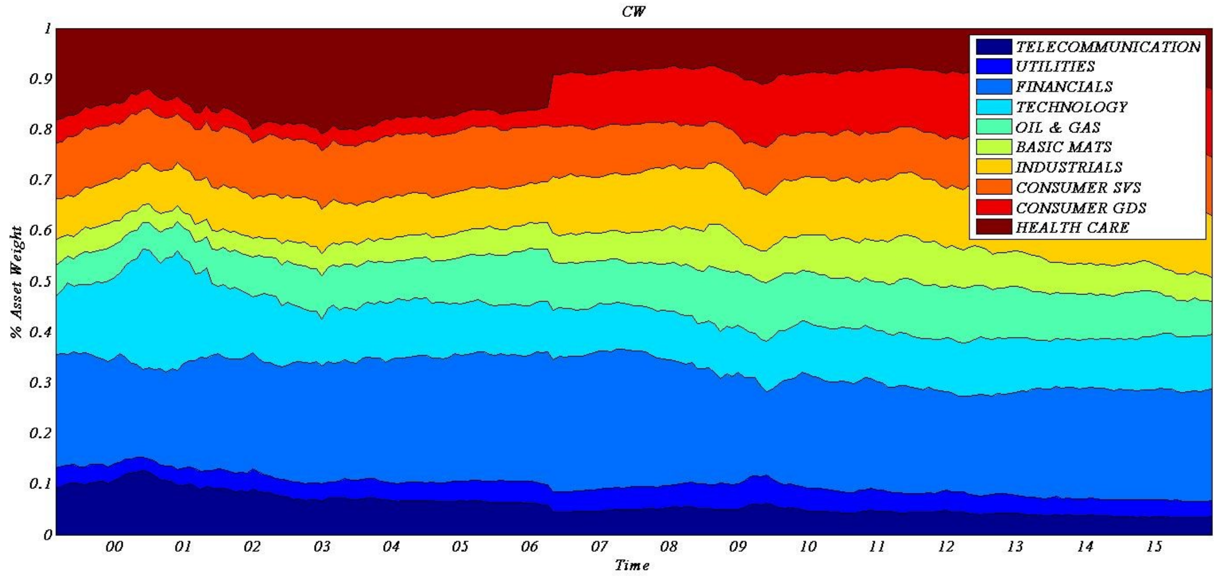


Exhibit 20 : Area Chart of the asset allocation for CW in the time interval 1999-2015

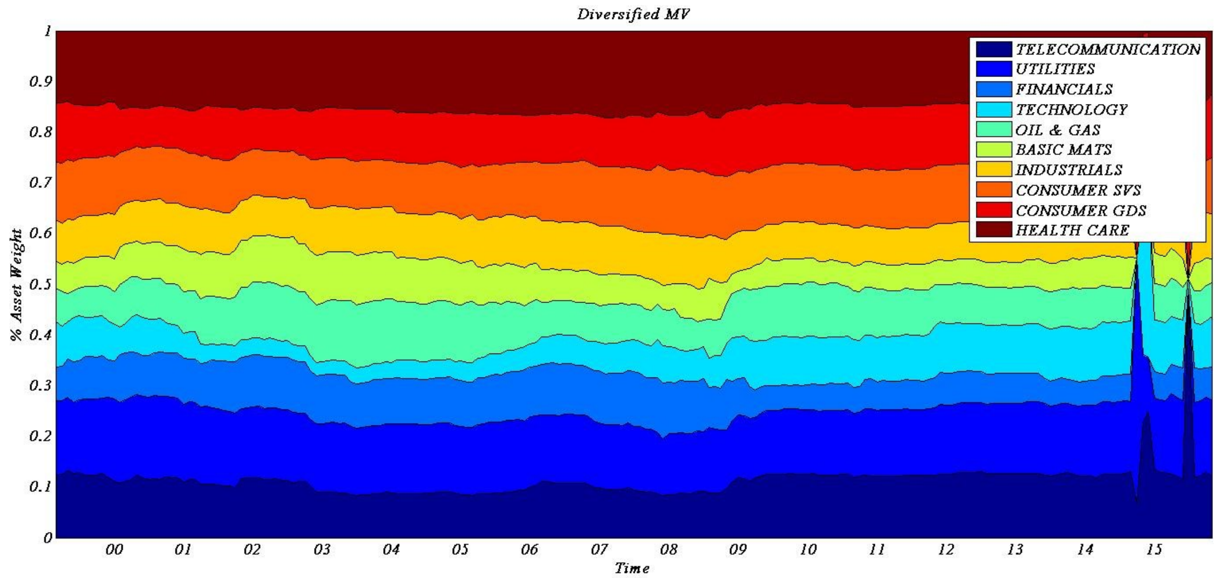


Exhibit 21 : Area Chart of the asset allocation for DMV in the time interval 1999-2015

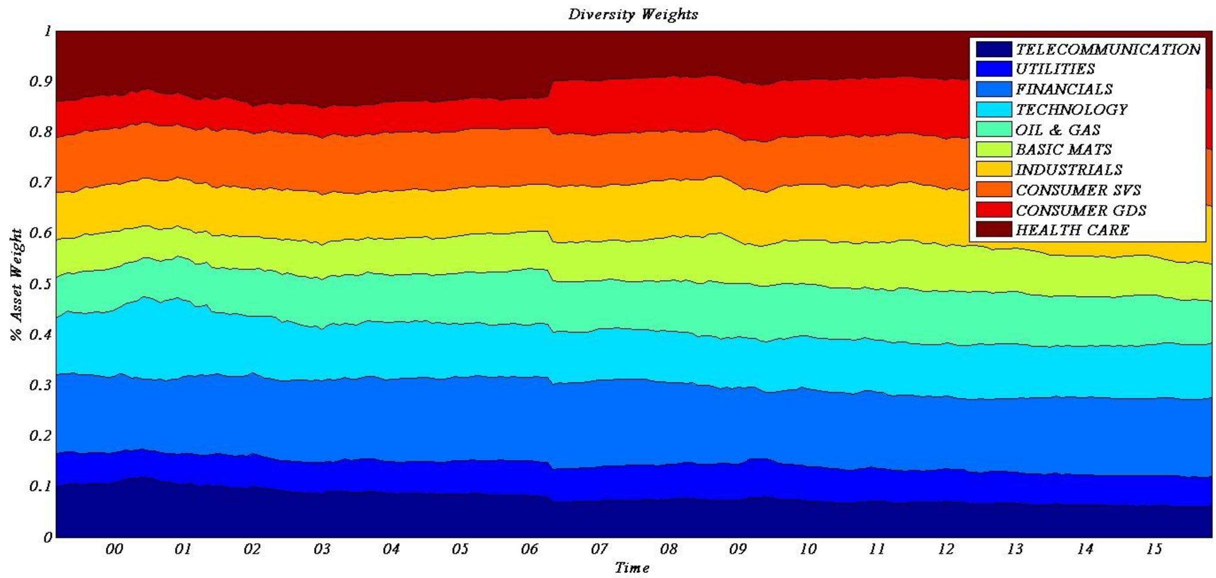


Exhibit 22 : Area Chart of the asset allocation for DW in the time interval 1999-2015

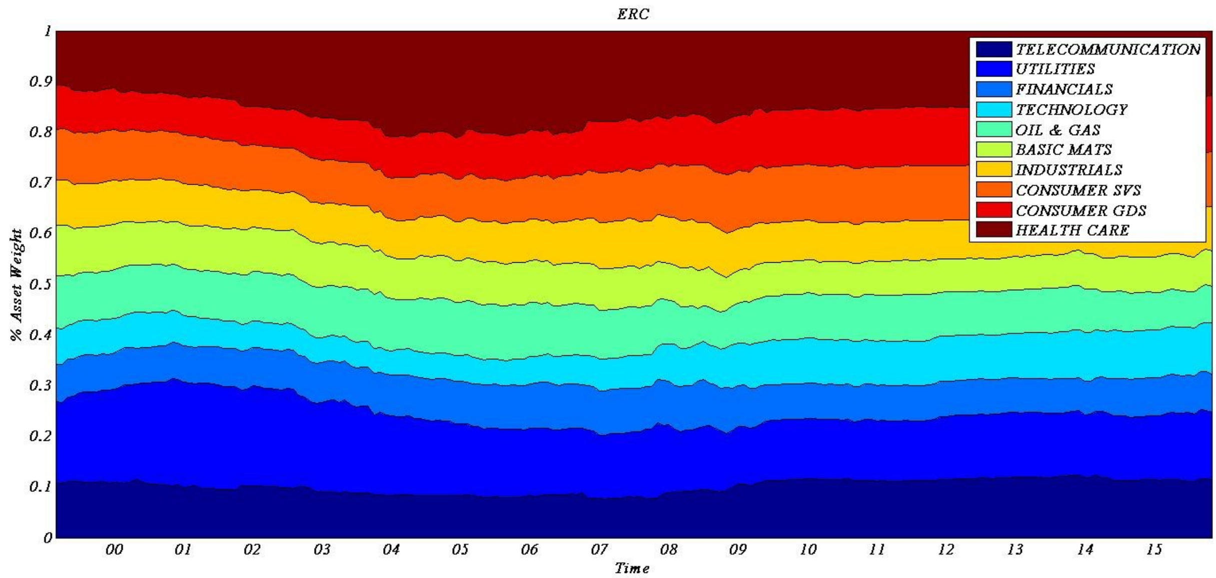


Exhibit 23 : Area Chart of the asset allocation for ERC in the time interval 1999-2015

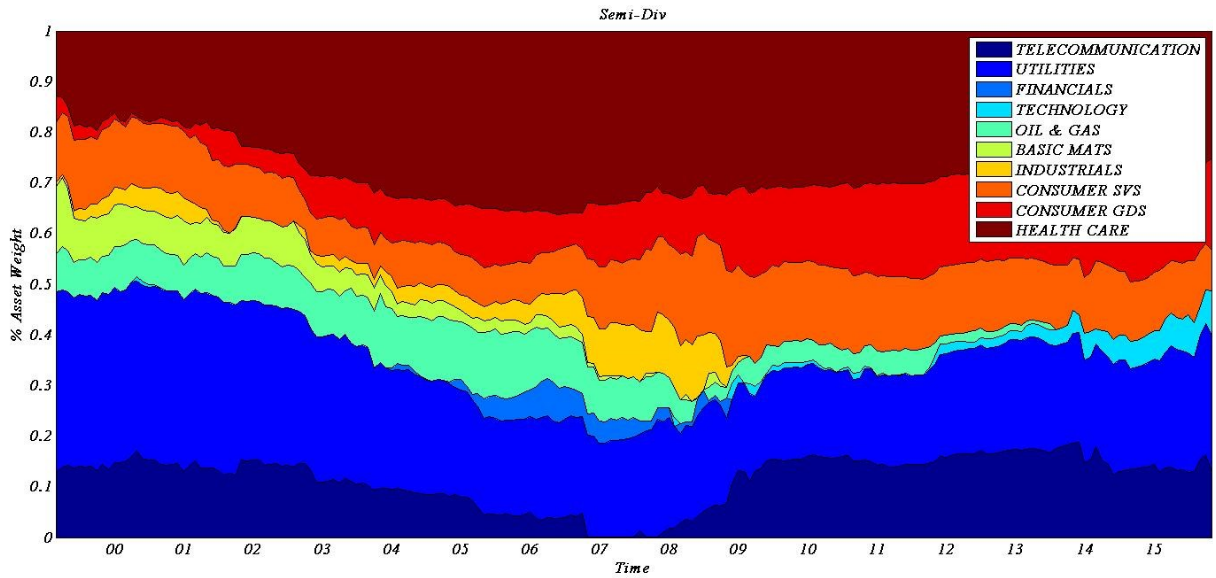


Exhibit 24: Area Chart of the asset allocation for SD in the time interval 1999-2015

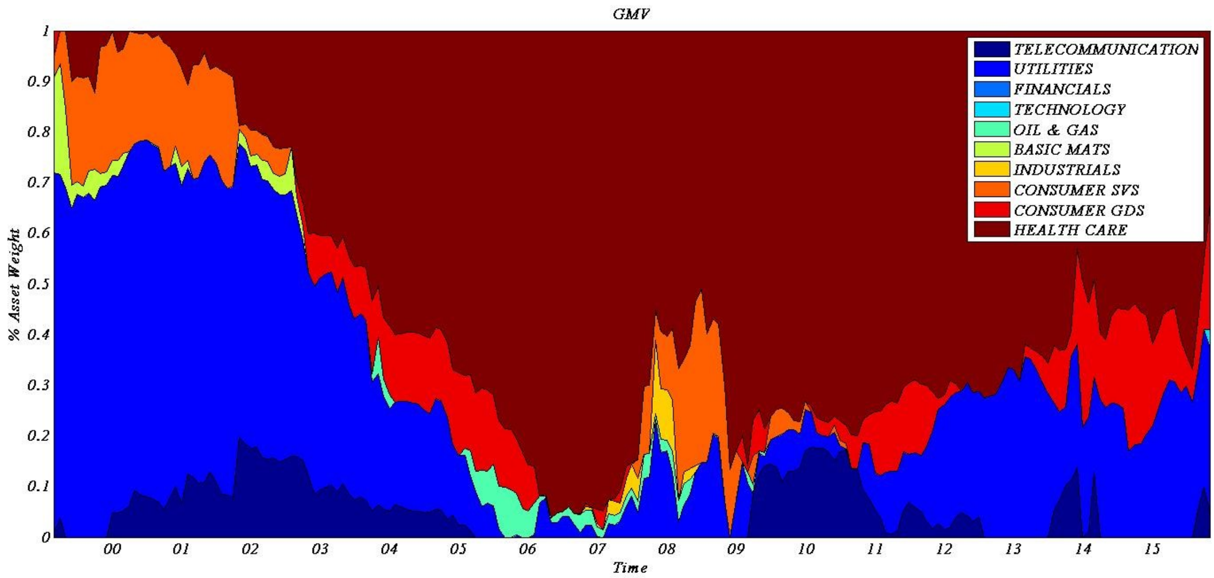


Exhibit 25 : Area Chart of the asset allocation for GMV in the time interval 1999-2015

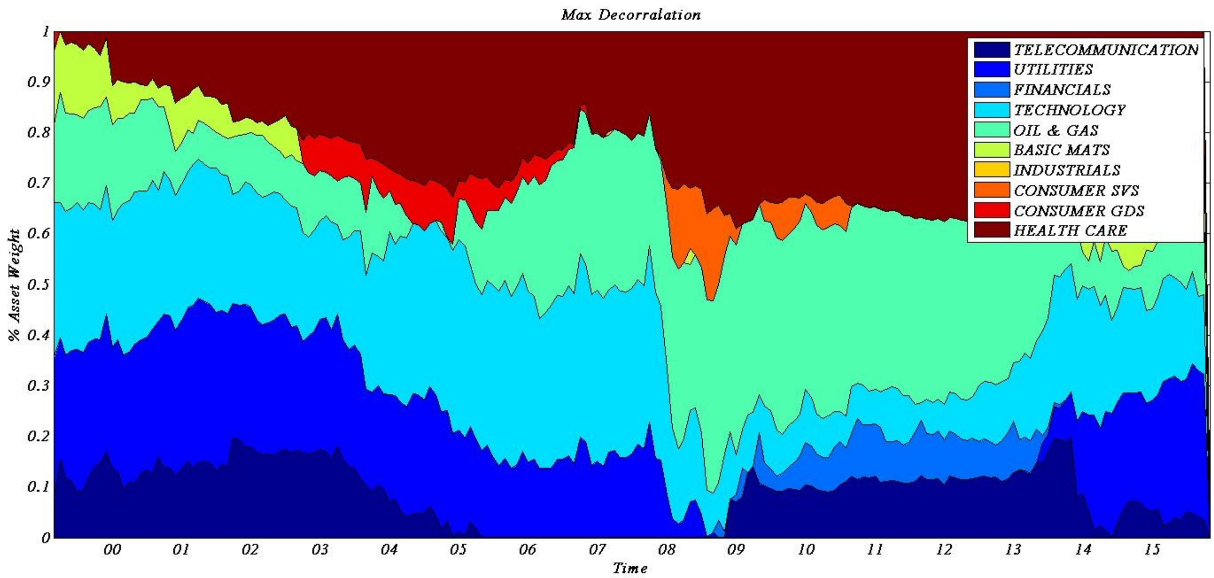


Exhibit 26 : Area Chart of the asset allocation for MDe in the time interval 1999-2015

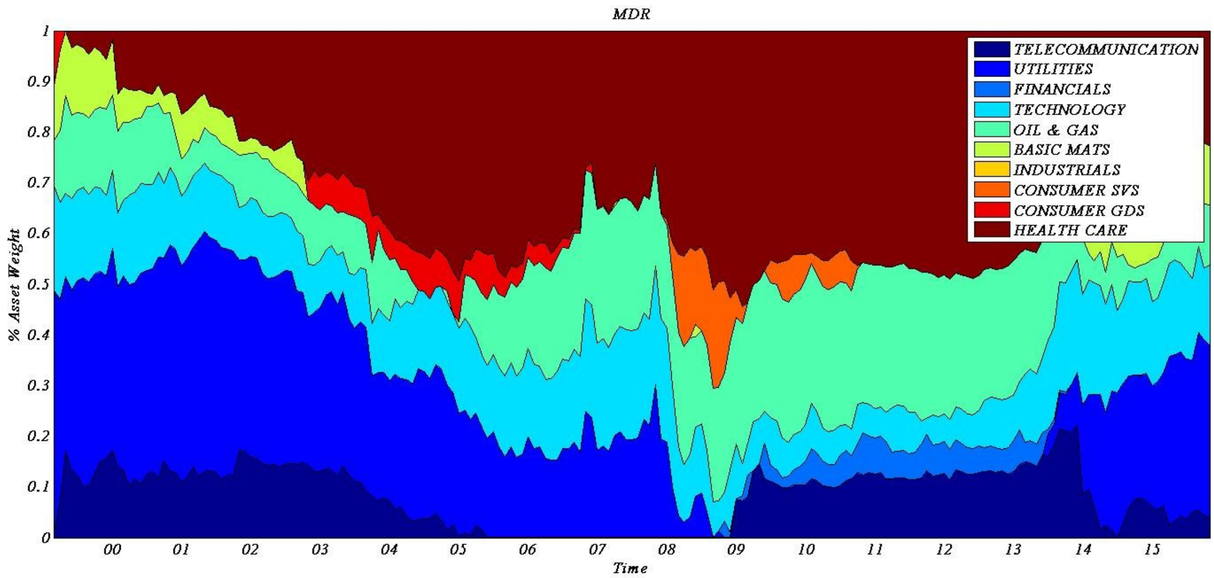


Exhibit 27 : Area Chart of the asset allocation for MDR in the time interval 1999-2015

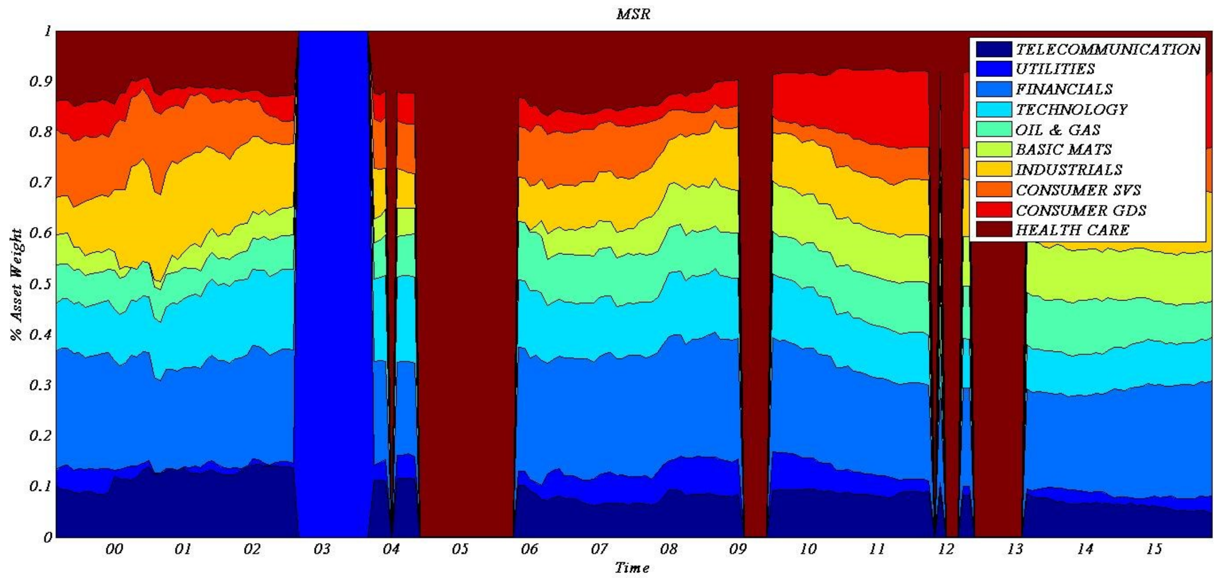


Exhibit 28: Area Chart of the asset allocation for MSR in the time interval 1999-2015

The reduced number of the assets allows us to break down clearly the graphs and derive some interesting remarks. By taking a comprehensive look on the different weighting schemes it's observed immediately a huge difference in terms of turnover and distribution. The weighting schemes Diversified Risk Parity, Diversified Minimum Variance, Equally Risk Contribution, Diversity Weights and Cap-Weighted show a more flat evolution of the weights amongst the assets, in my opinion for different reasons. Cap-Weighted and Diversity Weight are based on market capitalization, which obviously has less standard deviation compared to other parameters such as volatility or correlation, leading to a lower change in asset's weights. ERC is a particular case, since is demonstrated (Clarke et al.2013) that imposing positivity to the weights, the risk contribution is driven more by systematic risk compared to idiosyncratic risk, which has also a lower level of variation. DRP is based just on standard deviation and ignores correlation, giving a more uniform distribution of the weights compared to ERC. DMV has a low level of turnover by construction, since the limitative constraint, not influenced from the market condition.

On the contrary, Global Minimum Variance, Max Sharpe Ratio, Maximum Diversification Ratio, Semi Concentrated and Max Decorrelation show huge variance in the portfolio allocation. GMV, MSR, MDR and MD can have a common explication about their high level of turnover. All this portfolio maximizes or minimizes a specific quantity, without any constraints of diversification. Therefore, we can argue that the analytical construction of those portfolios leads to a high variance among assets weights. The motivation of the behavior of SD is not clear. Another interesting case is the GMV allocation. Empirically, this portfolio often ~~have~~ has a high level of sectorial concentration in Utilities, but this behavior is confirmed only until 2006. This sector is strongly influenced from market interest rate, since firms normally have high levels of debt. Probably, the financial collapse of 2007-2008, which brought a huge increase in the variance of market interest rate, leaded Utilities sector to be risky compared to the more stable Health Care sector. The main shortcoming of GMV, that is a very high level of concentration, is confirmed by this graph. During the previously cited financial crisis, Health Care reached almost the full concentration of the allocation, justifying the poor performance of GMV under no boundary constraints.

To give a data support to the previous comment, the Exhibit 29 and 30 represent the volatility of the assets during the period 2007-2009, when the financial crisis took place, and the correlation across the assets in the same period.

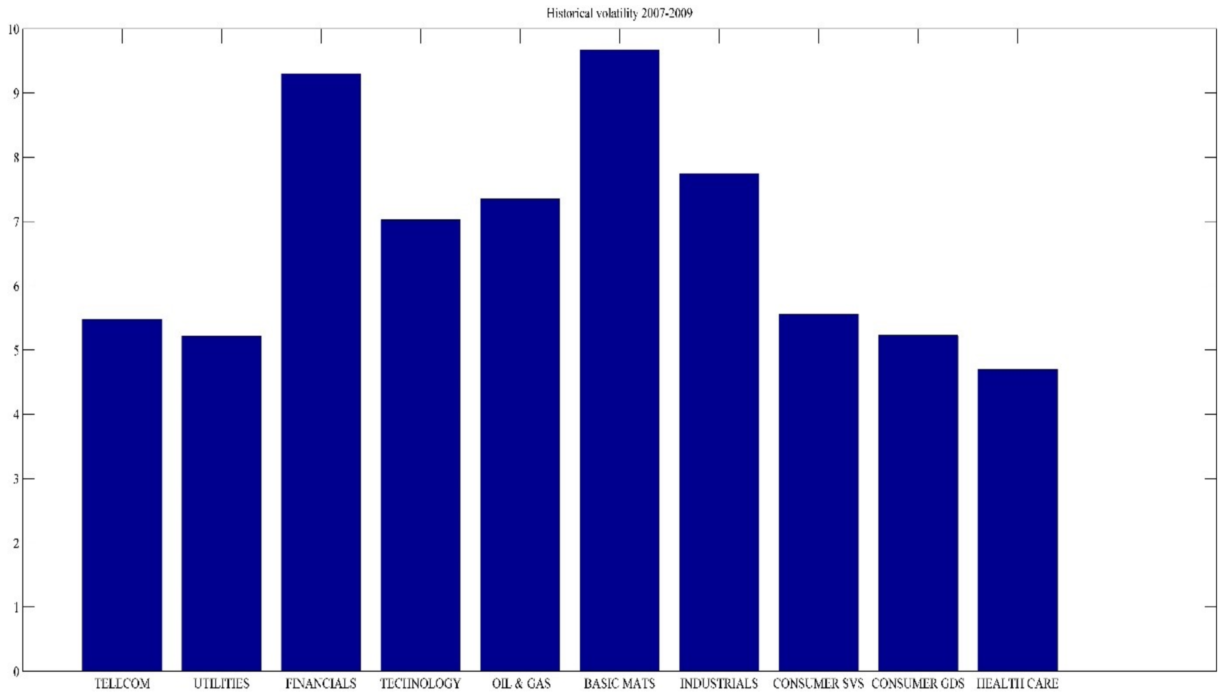
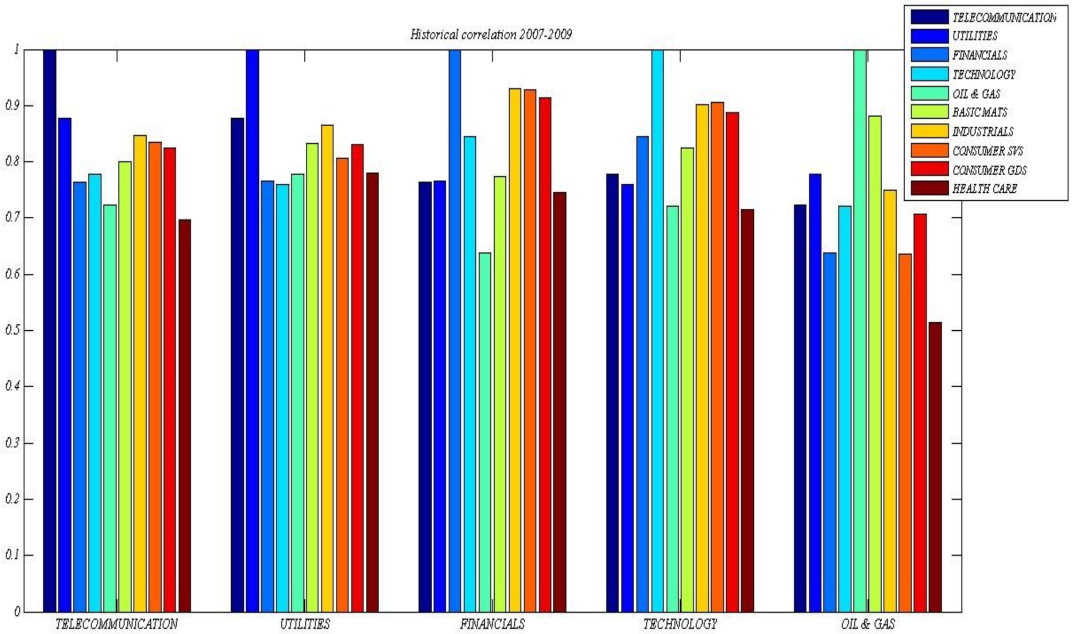


Exhibit 29 : Historical volatility of the assets in the period 2007-2009



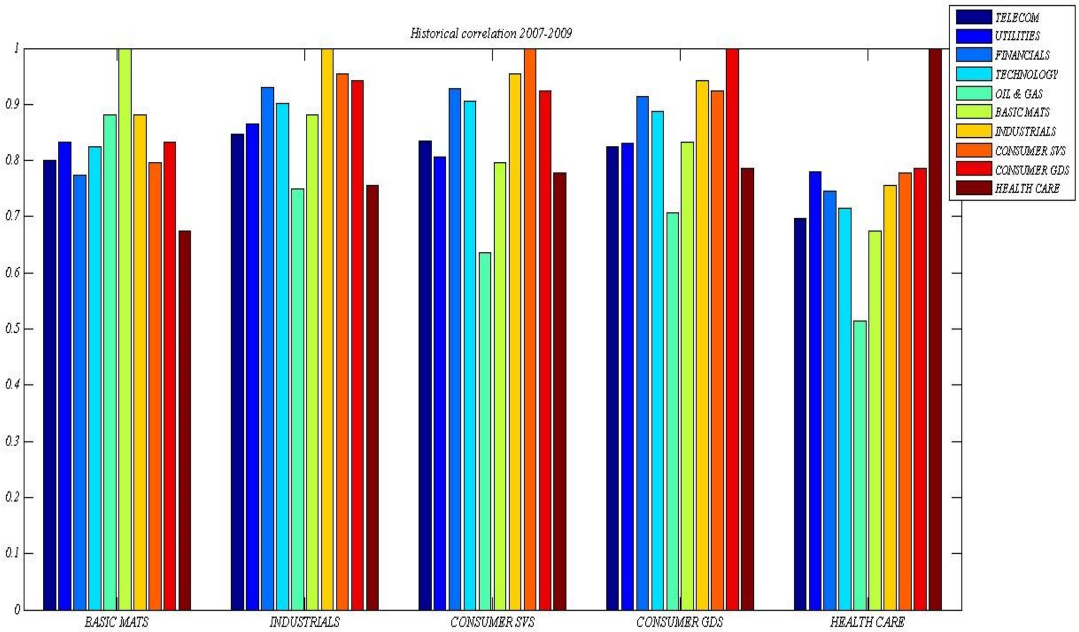


Exhibit 30 : Historical correlation across the assets in the period 2007-2009

As it was expected, the data shows that Health care has the lowest volatility as well as a relatively low correlation level. Moreover, it's interesting to take a look to Oil & Gas data. Although it has medium level of volatility, it benefits from a relative low correlation as Health Care, justifying his substantial presence in weighting schemes such as MDR and MDe in this time interval. As a matter of fact, both indices have a scheme which focus on diversification and where the correlation across the assets has an important role.

The persistence of Health Care asset in a leading position compared to the other assets, suggests that this asset provides a consistent contribution to diversification. Moreover, this sector probably is less exposed to systematic risk compared to the other ones, which represents a factor of good performance since the interval time considered includes three financial crisis (dot-com, Sub-prime, Sovereign Debt), increasing systematic risk.

This comment is justified from the following table, which shows the value of the beta of the 10 sectorial assets.

ASSET	***BETA***
TELECOMMUNICATION	0,871
UTILITY	0,571
FINANCIAL	1,179
TECHNOLOGY	1,316
OIL & GAS	0,932
BASIC MATERIALS	1,155
INDUSTRIALS	1,096
CONSUMER STAPLES	0,889
CONSUMER GOODS	0,893
HEALTH CARE	0,556

Table 3 : Systematic exposure of the assets (Beta) in the time interval 1999-2015

As a matter of fact, Health Care has less sensitivity to market fluctuations compared to the other assets, even less than Utilities.

It's interesting to take a look of MD and MDR, which have a similar allocation. Actually, they give a positive weight to the same assets in the same time interval, but with different proportion. Probably the absence of assets such as Industrials and Basic Materials is explained by the elasticity of the demand for these type of goods, which is high compared to the other sectors. A higher elasticity of the demand means a higher correlation to the market condition, since economic cycle is absorbed faster than other sectors. This is confirmed from the previous table where Industrials and Basic Materials are overexposed on systematic risk (Beta > 1). A particular case is the financial sector, which had an extreme extension in the last decade and, moreover, finance evolved as impetus of the modern economy. The huge market share of financial sector is definitely a factor of huge correlation with all the other economic sectors and with the market portfolio (Beta=1,17), explaining the motivation of absence in MDR and MDe. In addition, the comparison between MD, MDR and DMV and EQ confirms what was argued previously about the relation between diversification and the number of assets. Diversification can have different features, a naïve approach is based on the number of assets as DMV and EQ. A more scientific approach takes into account correlation and can show a concentration in relatively few assets, although it may be more diversified.

DW looks to solve the problem of concentration of CW, although sectorial concentration is not high as demonstrated also by the composition of FTSE ALL WORLD, which shows more concentration at country level than sector.

Semi-concentrated portfolio works similarly for MV, providing a more distributed allocation compared to MV, driven by the HHI index constraint.

3.3. Performance measures

After the graphical analysis of the asset allocations, it's important to break down quantitatively the weighting schemes, comparing these strategies with different indicators. These indicators are absolute values that provide a comprehensive valuation of the risk and return. In order to have a good

Sh= Sharpe Ratio: risk-adjusted return, calculated as the ratio between excess return (mean return minus risk-free return) and volatility.

$$\frac{\boldsymbol{E(r_k)}}{\boldsymbol{\sigma_k}}$$

Equation 21

Where $\boldsymbol{r_k}$ = return of the portfolio, $\boldsymbol{\sigma_k}$ = standard deviation of the portfolio.

So= Sortino Ratio: modification of Sharpe Ratio, calculated as the ratio between excess return and downside risk. Downside risk is the volatility of negative asset return.

$$\frac{\boldsymbol{R_p - R_f}}{\boldsymbol{\sigma_d}}$$

Equation 22

Where $\boldsymbol{R_p}$ = return of the portfolio, $\boldsymbol{R_f}$ = risk-free return, $\boldsymbol{\sigma_d}$ = negative standard deviation of asset return.

Tr= Treynor Ratio: calculated as the ratio between excess return and beta. Beta is the measure of systematic risk relative of the weighting scheme.

$$\frac{\boldsymbol{R_p - R_f}}{\boldsymbol{\beta}}$$

Equation 23

Where $\boldsymbol{R_p}$ = return of the portfolio, $\boldsymbol{R_f}$ = risk-free return, β = beta of the portfolio.

Cal= Calmar Ratio: risk-adjusted performance measure, calculated dividing rate of return and maximum drawdown normally in a given time interval.

$$\frac{\boldsymbol{R_p}}{\boldsymbol{Max\ DD_p}}$$

Equation 24

Where $\boldsymbol{R_p}$ = return portfolio, $\boldsymbol{Max\ DD_p}$ = maximum drawdown of the portfolio in the time interval considered.

Ste= Sterling Ratio: similar to Calmar Ratio, calculated as the ratio between rate of return and average maximum drawdown.

$$\frac{\boldsymbol{R_p}}{\boldsymbol{Av.Max\ DD_p}}$$

Equation 25

Where $\boldsymbol{R_p}$ =return portfolio, $\boldsymbol{Av.Max\ DD_p}$ = average among the maximum drawdown of the portfolio in the time interval considered.

Strategy	***Sh***	***So***	***Tr***	***Cal***	***Ste***
CW	**0,0679**	0,0908	**0,3149**	0,0056	0,0062
EW	0,0886	0,1205	0,4129	0,0075	0,0083

GMV	0,0849	0,1132	0,4745	0,0072	0,0075
MS	0,0319	0,0416	0,1595	0,0026	0,0027
ERC	0,0985	0,1318	0,4628	0,0081	0,0089
DRP	0,0971	0,1303	0,4556	0,0080	0,0088
MDe	0,0802	0,1129	**0,3878**	0,0068	0,0072
MDR	**0,1085**	**0,1487**	**0,5314**	**0,0097**	**0,0101**
SD	**0,1087**	**0,1426**	**0,5404**	0,0087	0,0095
DMV	0,1018	0,1360	0,4786	0,0082	0,0090
DW	0,0789	0,1065	0,3663	0,0066	0,0073
Main Index	0,0712	0,0951	0,3296	0,0059	0,0065

Table 4 : Performance of the portfolios in the time interval 1999-2015

First of all, Sharpe Ratios of the heuristic benchmark are all above the level of CW index, suggesting that just moving from the capitalization weighting scheme, a better return per unit of risk can be achieved easily. Even the DW scheme performances better than CW, although it uses capitalization as factor to allocate the assets. On the contrary, MSR portfolio has the worst the performance. This can be explained by the huge difficulty of the estimation of return assets, since they are calculated by ~~a~~ sample methods, which is backward looking, while the return comes from the expectations of the investor and so are forward looking.

Let’s consider the three downside risk indicators: Sortino Ratio, Sterling Ratio and Calmar Ratio. In the literature the main criticism to the Smart Beta were based on their risk of frequent negative return for long periods. Despite this, this data shows that Smart Beta indices achieve higher performance compared to Cap-Weighted when downside volatility or drawdown are taken in account. MDR achieves the best Sortino Ratio (0,1487), as the best Calmar Ratio (0,0097) as well as the best Sterling Ratio (0,0101).

The portfolios that achieve the best Sharpe Ratio are Maximum Diversification Ratio (0,1087), Diversified Minimum Variance (0,1018) and Semi-Concentrated (0,1085). MDR is the best one and has the 2nd higher Treynor Ratio (0,53) after the SD portfolio (0,54), suggesting that the good performance of this portfolio find its strength in the capacity to reward the systematic risk. It's interesting to notice how DMV and SD have a good performance compared to GMV, suggesting that HHI constraint, although is a very simple and intuitive constraint, improve the performance of portfolio based on the strategy of the minimization of the variance.

MDe has a poor performance compared to the other heuristic benchmarks, showing that in order to have a well-diversified portfolio is important to not split correlation and volatility and, moreover, the benefit to have less estimation risk is not rewarded with a higher performance. Looking at the performance of EW, we can argue that naïve approach works quiet good, and that in this case estimation risk is absent. In my opinion, given these evidence, estimation risk reduces considerable the performance only in the case of the estimation of the returns. In a nut shell, correlation and volatility have a low estimation risk compared to return, but a weighting scheme based only on one of this two parameter experience performance similar (e.g. DRP) or even lower (e.g. MDe) compared to weighting scheme based on both parameter.

About DRP and ERC is quiet straightforward that their performance are similar or, more precisely, ERC is slightly better than DRP. In light of this, ERC can be defined as a well-diversified portfolio, which doesn't need any enhancement about the level of diversification. Actually, looking at weighting graph in the previous paragraph, it can be noticed that these schemes are very similar and as a consequence have similar performance.

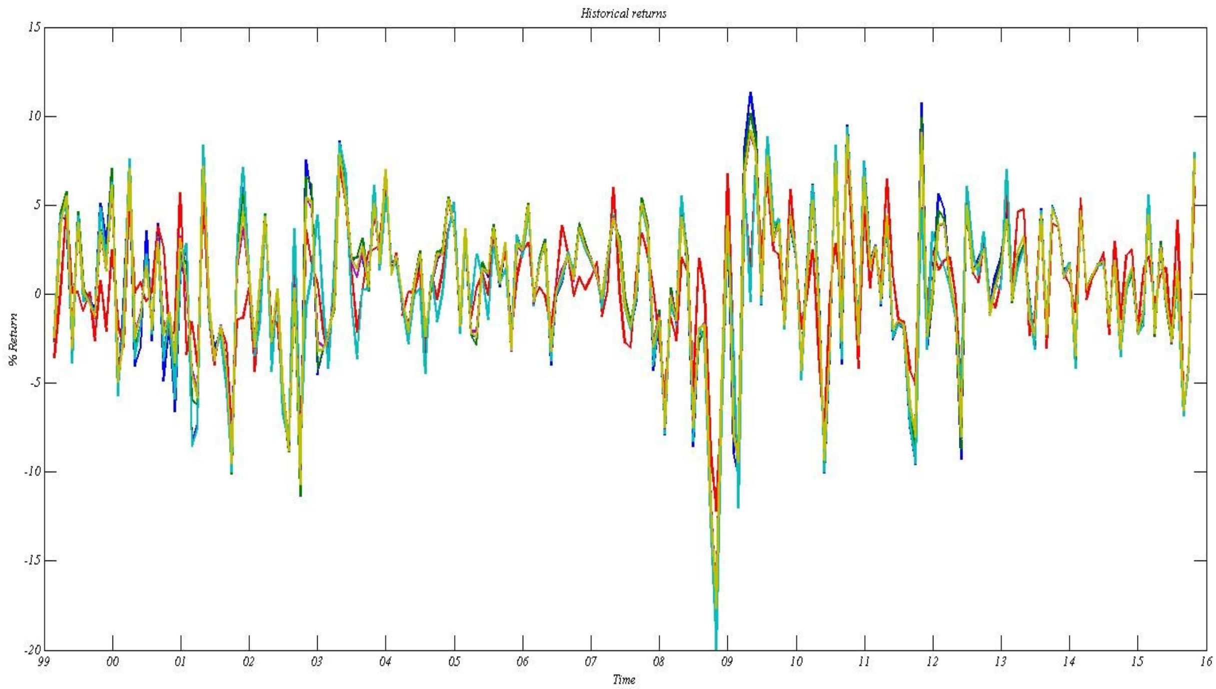
Returns

For an investor the first parameter that really matters is the return. In order to have a comprehensive analysis of returns strategies, it's important to mix a quantitative and qualitative approach. The Exhibit 31 shows the evolution of the return across the time, calculated as follows:

$$r_{tk} = w_{itk} * r_{itk}$$

Equation 26

Where w_{itk} = vector of assets weights at time t for every weighting scheme k, r_{itk} = vector of assets returns at time t for every weighting scheme k. The graph is divided in two parts. In the first part are represented the returns of: CW, EW, GMV, MS, ERC and DRP. In the second part: MDe, MDR, SD, DMV, DW and Main Index.



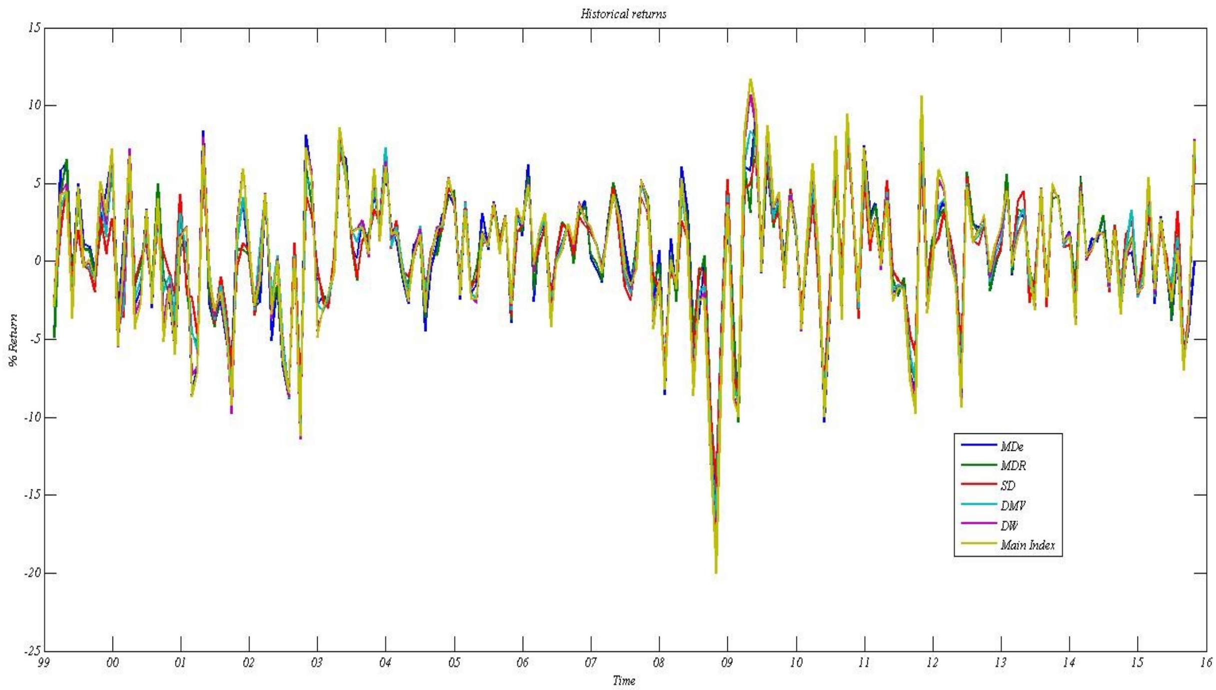


Exhibit 31: Historical returns of the portfolios in the time interval 1999-2015

From a first analysis of the graph it comes up that the variance of Cap-Weighted indices is higher than of the Smart Beta, since the relative line looks to be the "bound" of aggregate lines. It's interesting to notice how the returns moves frequently from positive to negative values in the time interval 1999-2003, 2007-2012, while in other periods are quiet stable across positive return. This is due to the increased volatility of these periods. Obviously, this effect has a higher magnitude in the period of the Sub-prime crisis of 2008. In order to have a clearer comparison, it's useful to use a quantitative approach. The table 5 provides four important data about the returns of the strategies:

- Mean of the returns
- Standard deviation of the return
- Max value of the returns
- Minimum value of the returns

Strategy	***Mean***	***StDev***	***Min***	***Max***
CW	0,3143	4,6271	**-19,8568**	**11,3519**
EW	0,3922	4,4289	-18,5431	10,1435
GMV	0,2955	**3,4822**	**-12,2041**	7,9238
MS	0,1398	4,3861	-19,988	9,4014
ERC	0,4044	4,1074	-17,4143	9,0533
DRP	0,4048	4,1680	-17,6374	9,1809
MDe	0,3394	4,2310	-15,5111	10,3429
MDR	**0,4221**	3,8927	-14,484	9,2819
SD	0,3852	3,5426	-14,6325	7,7434
MV	0,4145	4,0720	-17,2783	8,8072
DW	0,3572	4,526	-19,1609	10,7195
Main Index	0,3295	4,6315	**-20,0197**	**11,7265**

Table 5 : Mean return, standard deviation, minimum return and maximum return of the portfolios in the time interval 1999-2005

In light of the results, Maximum Diversification Ratio achieves the best average return, strengthening its leader position as the best performing portfolio. Also other portfolios, such as Diversified Minimum Variance, Equally Risk Contribution and Diversified Risk Parity realize a good performance in terms of average return, but these schemes experience a higher standard deviation, therefore, are riskier compared to MDR.

Being concerned about the risk, the best performing strategy is the Global Minimum Variance, which is able to reach its task to provide the less risky scheme. Also Semi-Diversified has a low standard deviation and it doesn't deny a considerable return compared to the other indices. On the contrary, Cap-Weighted Index achieves both low return and high standard deviation. We can explain this result by looking at the maximum and minimum return. The index based on market capitalization experiences the highest return in the time interval and, at the same time, the minimum return, which took place during the financial collapse of 2008. Given this evidence, we can argue that Cap-Weighted encounters huge loss when the systematic risk increases, while Smart Beta indices are capable of smoothing this loss, providing an overall higher return.

In light of the previous consideration, it's interesting to break down the data according to three time intervals, in order to take a look of the behavior in short term, medium-term and long term periods for every strategy. Given the 201 vector of assets weights, the periods are divided as follows:

- The short term is set from 26/02/1999 to 30/11/2004, $r_i \; with \; i = 1 \ldots .70$
- The medium term is set from 31/12/2004 to 30/09/2010, $r_i \; with \; i = 71 \ldots 140$
- The long term is set from 29/10/2010 to 30/10/2015, $r_i \; with \; i = 141 \ldots 201$

The table 4 shows the mean and standard deviation across these different times intervals.

Strategy	***MeanST***	***StDevST***	***MeanMT***	***StDevMT***	***MeanLT***	***StDevLT***
CW	0,1060	4,4641	0,1918	4,9076	**0,3143**	4,6157
EW	**0,2427**	4,3132	**0,3351**	4,7070	0,3922	4,4179
GMV	-0,1210	3,2683	0,0466	**3,6221**	0,2955	**3,4736**

MS	-0,1577	4,3167	-0,0187	4,6823	0,1398	4,3752
ERC	**0,2357**	3,8428	0,3260	4,3389	0,4044	4,0972
DRP	**0,2512**	3,9662	0,3277	4,4174	0,4049	4,1577
MDe	0,0782	4,3633	0,2560	4,4809	0,3395	4,2205
MDR	0,1056	3,7243	0,2727	4,0446	**0,4222**	3,8830
SD	0,1212	**3,2820**	0,2364	3,7126	0,3852	**3,5339**
DMV	**0,2409**	3,8616	0,3066	4,3118	0,4146	4,0619
DW	0,1732	4,3929	0,2654	4,8039	0,3573	4,5149
Main Index	0,1165	4,4219	0,2272	4,9120	**0,3296**	4,6200

Table 6 : Mean return and standard deviation of the portfolios in the short term (99-04), medium term (99-2010) and long term period (99-2015)

In the short term period, the best performing portfolio are Diversified Minimum Variance (0,24%), Equally Risk Contribution (0,23%) , Diversified Risk Parity (0,25%) and Equally-Weighted (0,42%). It's interesting to see how Maximum Diversification Ratio (0,10%) and Semi-Diversified (0,12%) portfolios experience a return similar to Cap-Weighted Index (0,10%) and Main Index (0,11%), even if with a lower standard deviation. Some portfolios such as MS (-0,15%) and GMV(-0,12%) have an average negative return, suggesting how these scheme aren't able to create value in the short term. Both GMV and SD achieve the lowest standard deviation, but it doesn't mean that GMV is not risky since it has negative average return.

In the medium term, which is the period with the highest volatility, Smart Beta shows generally less standard deviation compared to the Cap-Weighted Index.

Global Minimum Variance (3,6) and Semi-Diversified (3,7) succeed again in their task to provide the less risky allocation, while Diversified Minimum Variance (4,8) is riskier than another heuristic benchmark such as Maximum Diversification Ratio (4). Cap-Weighted indices are the riskier ones.

The best portfolio in terms of returns in this period are Equally Weighted (0,33%), Equally Risk Contribution (0,32%) and Diversified Risk Parity(0,32%). The Maximum Diversification Ratio achieves a good performance (0,27%), reducing the gap (differential in terms of return between two portfolios) with the above-mentioned portfolios:

- In the short term MDR returns 0,1 % of the investment, while EW, ERC and DRP 0,24%, therefore, a gap of 0,14%
- In the medium term MDR returns 0,27 of the investment, while EW,ERC and DRP around 0,33, therefore, a gap of 0,06%

Since this period include the financial collapse of the 2008, the performance of MDR in the medium term suggests that this index provide a good allocation when the market experience the most unstable time. Global Minimum Variance experience a huge increase in mean return (0,06%). Cap-Weighted indices are also the riskier ones.

In the long term, the difference between Cap-Weighted indices and the best portfolios, Maximum Diversification Ratio and Semi Diversified, reach his maximum value. In this time interval, MDR and SD achieve 0,42 % and 0,38% of mean return, while Cap-Weighted 0,32% and Main Index 0,31%. Furthermore, others Smart Beta portfolios are able to accomplish good performance: Diversified Minimum Variance (0,41%) Equally Risk Contribution (0,40%) ,Diversified Risk Parity (0,40%) and Equally Weighted (0,39%).

It’s very interesting to take a look to the Global Minimum Variance portfolio, which strongly recovers in terms of return since reach a 0,29%, achieving the highest positive differential with the medium term: +0,25%.

In conclusion, we can argue that the winner position of Maximum Diversification Ratio and Semi-Diversified portfolios is achieved in the long term period, while in the short period and medium period other portfolios reach the best performances, such as Equally Risk Contribution, Diversified Risk Parity, Diversified Minimum Variance. ERC and DRP have a more stable evolution across the time, while the above-mentioned MDR, SD and GMV achieve a substantial growth in the medium and long term.

Cumulated Returns

Furthermore, another way to measure performance of a portfolio is to break down the cumulated return across the time interval considered.

Cumulated return are compounded by a cumulative product of the return for every weighting scheme with initial value of 1, since it's suppose that the investment is 100% of investor's wealth (as a matter of fact, all the assets weights are normalized to 1).

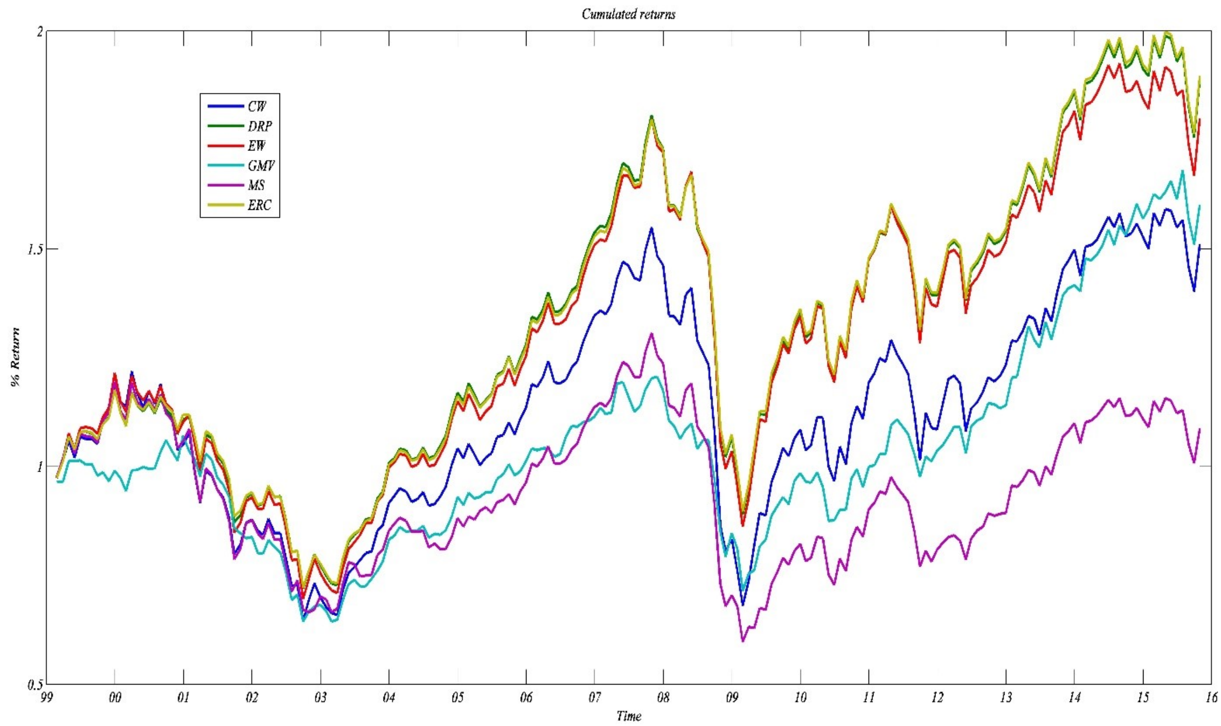
The cumulated returns are calculated at monthly basis as follow:

$$R_k = \prod_{j=1}^{201} r_j * w_{jk,}$$

Equation 27

Where r_j = vector asset return $w_{jk,}$= vector asset weight for every k weighting scheme.

Briefly, assets weights are calculated with rolling methods in a 5 years time interval and are applied to the observed return of the next month, supposing that an investor reallocate his portfolio every month on the basis of past 5 years data. The Exhibit 32 shows the cumulated returns for the portfolios considered. In order to realize a clearer analysis the graph is divided in 2 parts. In the first part the portfolio considered are: Cap-Weighted, DRP, EW, GMV, MS and ERC. In the second part are: Main Index, MDR, MDe, SD, DMV and DW.



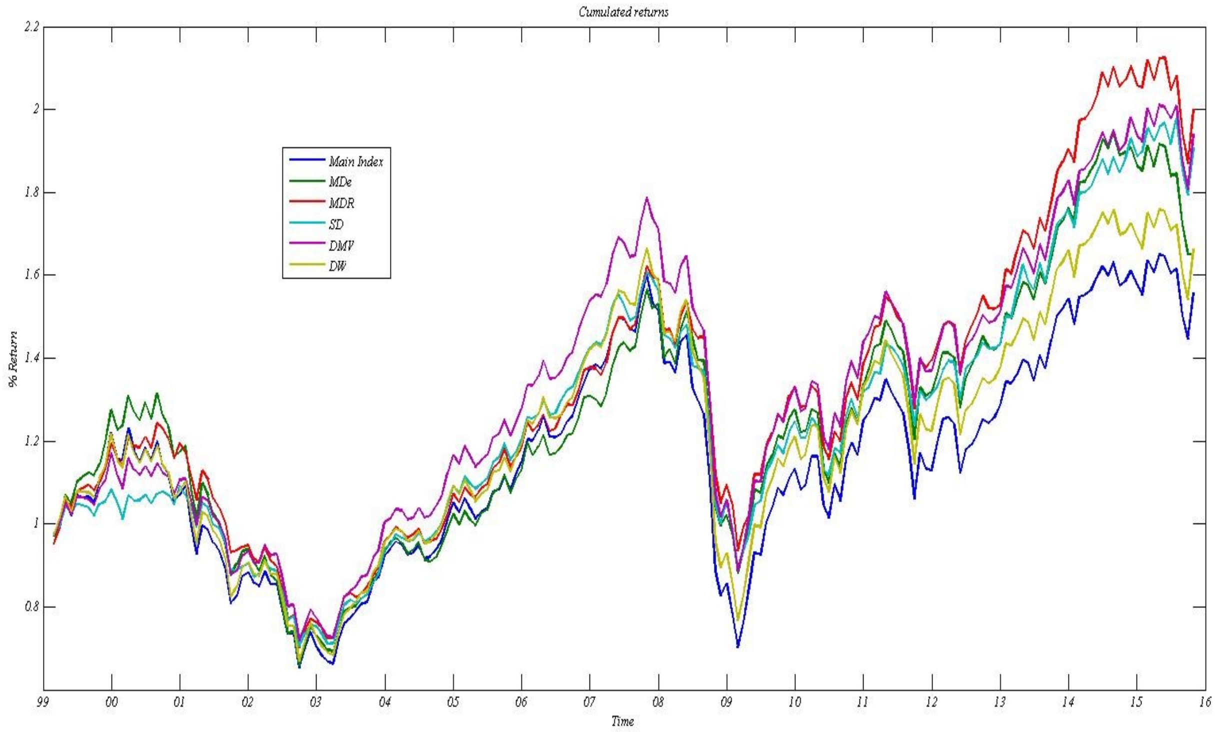


Exhibit 32 : Historical cumulated returns of the portfolios in the time interval 1999-2015

Firstly, the best performance is again achieved by MDR (2) meaning that in 15 years an investor will duplicate his wealth. The 2nd best is DMV (1.94) and 3rd SD (1.91). Cap-weighted index have a very poor performance: CW (1.52), DW (1.66) and Main Index (1,55), better only of MSR (1.08). The other heuristic benchmark have still good performance, ERC (1.89), DRP (1.88) , EW (1.79), MDe (1,64) and GMV (1,6).

As it could be expected, cumulated return are inclined to converge during the financial crisis, which is quiet reasonable since systematic risk increases so much that is quiet hard to avoid huge negative return.

Moreover, it's very important to highlight that in the first 5 years period the difference between Smart Beta and Cap-weighted is small, and even many heuristic benchmark have lower return than Cap-Weighted. Gradually, the performance of Smart Beta increases creating a significant gap from 2007 that Main Index, DW and CW hardly recover or even reduce after this date. May be that the low performance of CW index is due to the time interval taken in consideration, since CW are inclined to concentrate in the Financial Sector, which experienced huge negative returns during the financial collapse of 2008. Although this can be true, Smart Beta are tilted to sector less exposed to systematic risk, therefore, we can argue that these indices hedge against systematic risk. In conclusion, this confirms what was previously claimed: Smart Beta Index have better performance in the long term.

This plot also unmasks a wrong idea about Smart Beta. As it was already highlighted in the theoretical chapter, heuristic benchmarks are blamed to suffer of negative return for long period while Cap-Weighted have a faster recovery and therefore a more stable evolution across the time. On the contrary, looking at this graph we can see easily that there isn't significant difference between Smart Beta and Cap-Weighted about persistence of negative return and, actually, both indices experience similar negative return in terms of magnitude and according to the time frame. This suggests that Smart Beta can control the systematic risk, without being exposed to unrewarded risk.

Value-at-risk

The so-called VaR is a statistical measure commonly used in risk management to quantify the potential loss of an investment. In other words, it's a financial risk measure for the calculation of maximum loss of a specific portfolio over a certain time frame, given a confidence interval. This measure is based on loss distribution and so it's comparable across the different strategies.

There are many methods to estimate VaR. The most common is variance-covariance method, which suppose that losses (and profits) are distributed as a normal Gaussian distribution. This is the method implemented in this work. The equation for Var with 99% confidence is as follows:

$$Var(99\%) = NormInv(0{,}01) * \sigma_p + r_p$$

Equation 28

Where $NormInv(0{,}01)$ =normal inverse cumulative function with 99% of confidencen interval, σ_p = standard deviation portfolio, r_p = mean return portfolio.

Econometrics and statistical tools are employed to compound standard deviation of the portfolio. It's empirically proven that financial returns are affected by volatility clustering. Volatility clustering means that, large changes in price tend to cluster and follow-up change in price with the same magnitude. In order to capture volatility clustering, complex models such as GARCH, EGARCH and GJR are used to estimate standard deviation of financial returns. In this thesis, it's not necessary to use econometric tools. As a matter of fact, the dataset is breaking down at a monthly basis and it's empirically demonstrated that volatility clustering can be observed at weekly or daily basis, while there isn't heteroscedasticity for monthly return. Therefore, the variance of the portfolios is computed by the rolling method with a time interval of one year (12 observations).

To the end to have a clearer sketch, the VaR graph is divided in two parts. On one hand, the Exhibit 33 show the evolution of the VaR for the strategies: Cap-Weighted, Equally Weighted, Global Minimum Variance, Max Sharpe Ratio, Equally Risk Contribution and Diversified Risk Parity. On the other hand, the Exhibit 34 show: Max Decorralation, Maximum Diversification Ratio, Semi-Diversified, Diversified Minimum Variance, Diversity Weight and Main Index.

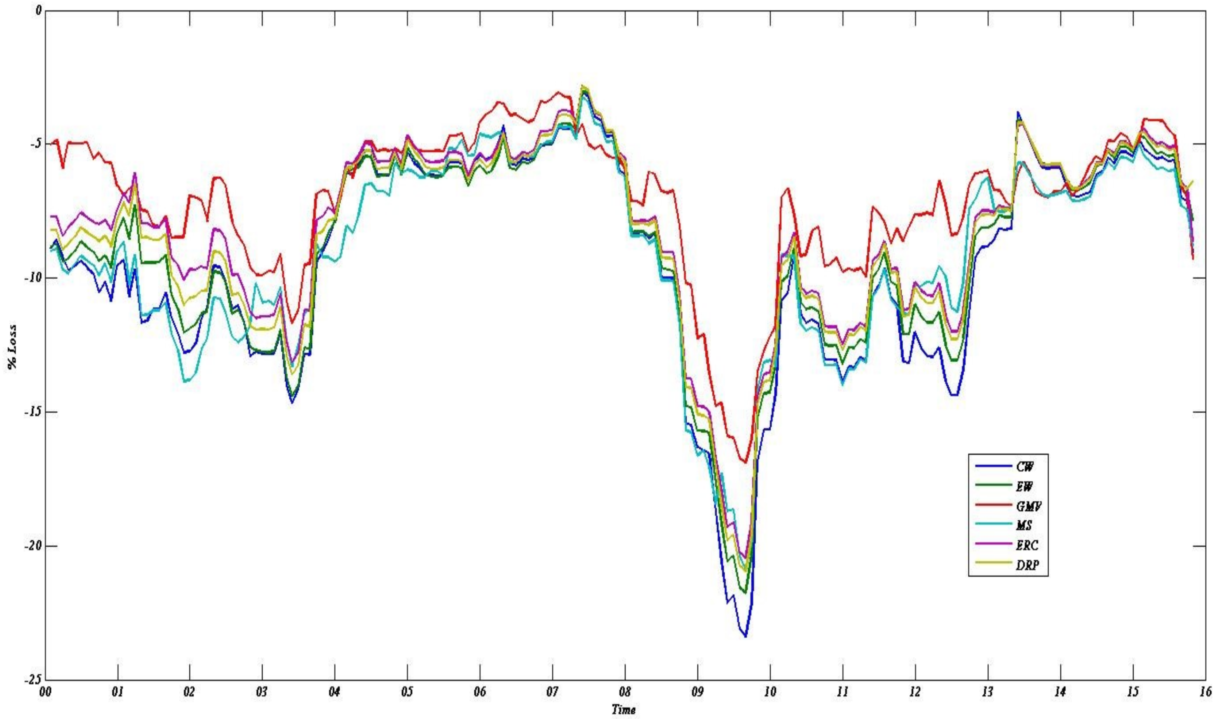


Exhibit 33: Historical VaR (99%) in the time interval 2000-2015

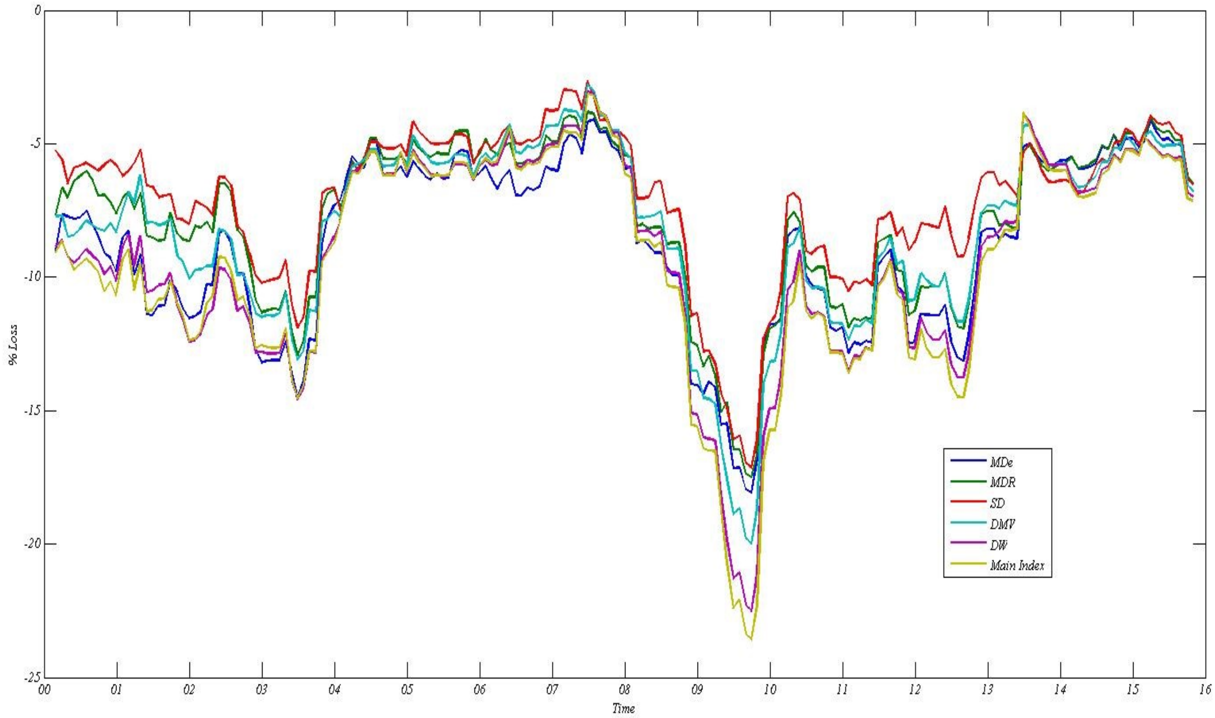


Exhibit 34: Historical VaR (99%) in the time interval 2000-2015

In order to have a better comprehension, it's useful to take a look of the average range of this graph, which is around 8%. In "financial" terms this means that the potential loss per month for these strategies is around 8% with 99% of the probability. Therefore, an investor quantifies his potential loss with almost certainty, since the probability to suffer higher loss is just 1%.

The difference of loss amongst the strategies are emphasized during the period of 2001-2003, 2007-2009 and 2012-2013, which are the period with the highest volatility. In addition, during these periods the variance of the losses is higher. Breaking down the graph during this period we can see that the line of losses for the Cap-Weighted index and Main Index is mostly below compared to the line of losses of the Smart Beta portfolios.

More interesting is the comparison in quantitative terms, comparing the average VaR across the time interval considered and the Maximum VaR. The Maximum VaR is experienced from all the strategies in the financial crisis of 2007-2009 how we can notice from the previous graph. Let's consider the Table 5.

Strategy	***AvNVar***	***MaxNVar***
CW	-9,43456	-23,3876
EW	-8,99254	-21,7674
GMV	**-7,11399**	**-16,8916**
MS	-9,1571	-20,8817
ERC	-8,28289	-20,4562
DRP	-8,54713	-20,95
MDe	-8,77026	-18,0839
MDR	-7,87241	-17,4966
SD	**-7,14771**	-17,1409
DMV	-8,22662	-19,9841

DW	-9,16198	-22,5194
Main Index	-9,37548	-23,5876

Table 7 : Mean VaR (99%) and Minimum VaR (99%) of the portfolios in the time interval 1999-2015

In the period of financial collapse of 2007-2009 the Cap-weighted index and the Main Index reach a maximum loss of more than 23%, while Maximum Diversification Ratio and Semi-Diversified produce a loss around 17%. In this period the best performing portfolio is Global Minimum Variance that reduce the maximum loss to 16,89%. Also Max Decorrelation portfolio has a good performance compared to Cap-weighted, since it reaches a maximum loss of 18% while in the other period it experiences a loss similar to Cap-Weighted. We can argue, that in the period of financial collapse the correlation parameter was very important for diversification. As a matter of fact, Equally Risk Contribution, Diversified Risk Parity and Diversified Minimum Variance suffer of a maximum loss around 20%, a negative result compared to the other Smart Beta. Looking at the weighting scheme of the portfolios, it can be supposed that ERC, DRP and DMV have less flexibility compared to the other ones that have a faster response to sudden change in market condition.

Equally Weighted and Diversity Weighted have performance similar to the Cap-Weighted index with a maximum loss of 21% and 22% during the period of financial collapse. It's important to underline that MDR, reaches an overall performance lower than SD and GMV, since its average maximum loss is around 7.8%, while SD and GMV have 7.1% Taking a look again on the graph, it comes up that MDR suffers of higher loss in the other period of high volatility, such as 2001-2003 and 2012-2013. Focusing on average performance, again Cap-Weighted indices experience the highest average VaR, while Smart Beta indices have an overall higher performance.

In conclusion, Global Minimum Variance can be considered a good portfolio for reduce loss as it could be expected, as well as Semi-Deversified Minimum Variance. MDR has also a good performance, even if it's lower than the GMV and SD. The Smart Beta has low performance, like Cap-Weighted indices.

3.4. Turnover

The promoters of Cap-Weighted index, often argue that Smart Beta needs a considerable amount of turnover which, taking into account commission cost, reduces the performance of the indices. In light of this, it's interesting to break down how turnover cost impacts the value of the Smart Beta portfolio.

Firstly, to that end is important to check the magnitude of turnover across time. In the Exhibit 35, it's shown the turnover of the 12 strategies calculated at monthly basis.

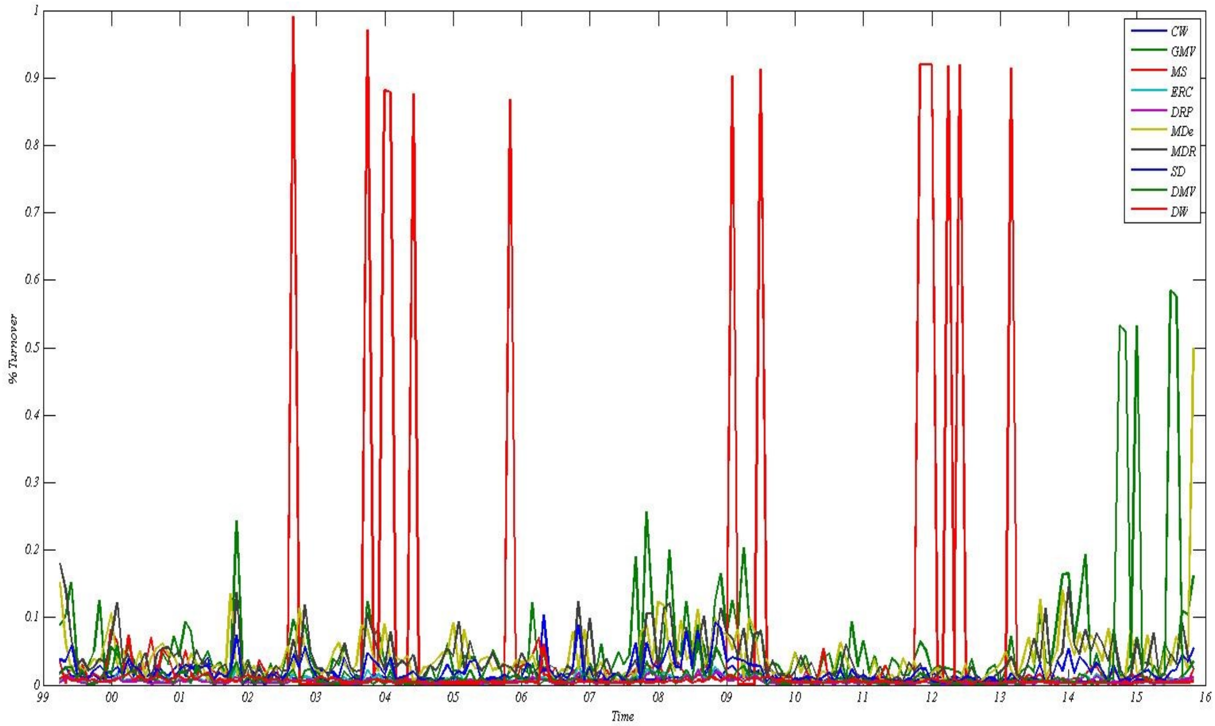


Exhibit 35: Turnover of the portfolios in the time interval 1999-2015

It's straightforward that the MS portfolio needs a high level of turnover, particularly during recession, confirming what it was seen in the weighting graph previously. The variance of the graph is substantial in the time interval 2007-2009 and 2013-2015. Excluding MSR, any strategy overtakes a level of 20% of turnover, less than DMV and GMV in the end of the time interval. It's true that CW has a lower and more stable turnover compared to heuristic benchmark, but it could be wrong think about it as significant just looking at the graph. With this in mind, an interesting point is to verify turnover in a quantitative view, looking for the cost of turnover in terms of performance and, especially, focusing on portfolio return.

Here turnover is compounded at monthly basis and cost commission are set at 20 base points. In the Table 8: Average Turnover is the average monthly turnover over time interval considered, Mean is the mean monthly return of the time interval considered and, then in the 3rd column there is the return net of transaction cost (av. Turnover * commission cost).

Strategy	*Average Turnover*	*Mean*	*RealMean (20 bp transaction cost)*
CW	0,011604	0,31432	0,314297
EQ	0	0,392233	0,392233
GMV	0,046239	0,295528	0,295435
MS	**0,076604**	0,139848	0,139695
ERC	0,006289	0,404412	0,4044
DRP	0,004824	0,339475	0,339465
MDe	0,040292	0,422176	0,422096
MDR	0,038618	0,385238	0,38516
SD	0,020053	0,414567	0,414526
DMV	0,023711	0,357278	0,357231
DW	0,005961	0,404877	0,404866

Table 8 : Turnover and Real Return of the portfolios in the time interval 1999-2015

It can be derived immediately how some of the portfolios don't have a significant turnover value. DRP, ERC and DW have a value lower than 1%, and this is reasonable since the weighting graph shows a stable evolution of the scheme across the time. Moreover, CW has a lower turnover level compared to the other Smart beta, with 1,1 % per month. The portfolio with the higher turnover level is MS with 7,6% turnover, which confirms the previous graph where MS had a far higher pick compared to the other portfolio. GMV (4,6%), MDe (4,%) and MDR 3,8%) have also a substantial turnover level. Interesting is the case of SC that, although is a winner portfolio, needs half of the other winner portfolio (MDR) with just 2% of turnover level per month.

When returns are netted of commission cost, it's straightforward that the turnover don't impact significantly the return. The reduction in portfolio return is very low, even for MS portfolio that experiences the highest turnover level.

3.5. Efficient Frontier

In order to have a graphical comparison between all the strategies, it was sketched the Constrained Efficient Frontier without risk-free. The Efficient Frontier is composed by the point of efficient portfolio in terms of trade-off between return and standard deviation, given the true moments of the 10 sector assets. In the Exhibit 36, the EF is represented together with the portfolio. The portfolios are set with mean and variance as average of the 15 years of data. The EF provide a useful tool for investor. As a matter of fact, a portfolio that have a lower performance indicator (Sharpe Ratio) compare to another one, maybe have lower standard deviation and so can be attractive for risk aversion investor. On the contrary, a portfolio have a high return and high standard deviation can be attractive for risk lover.

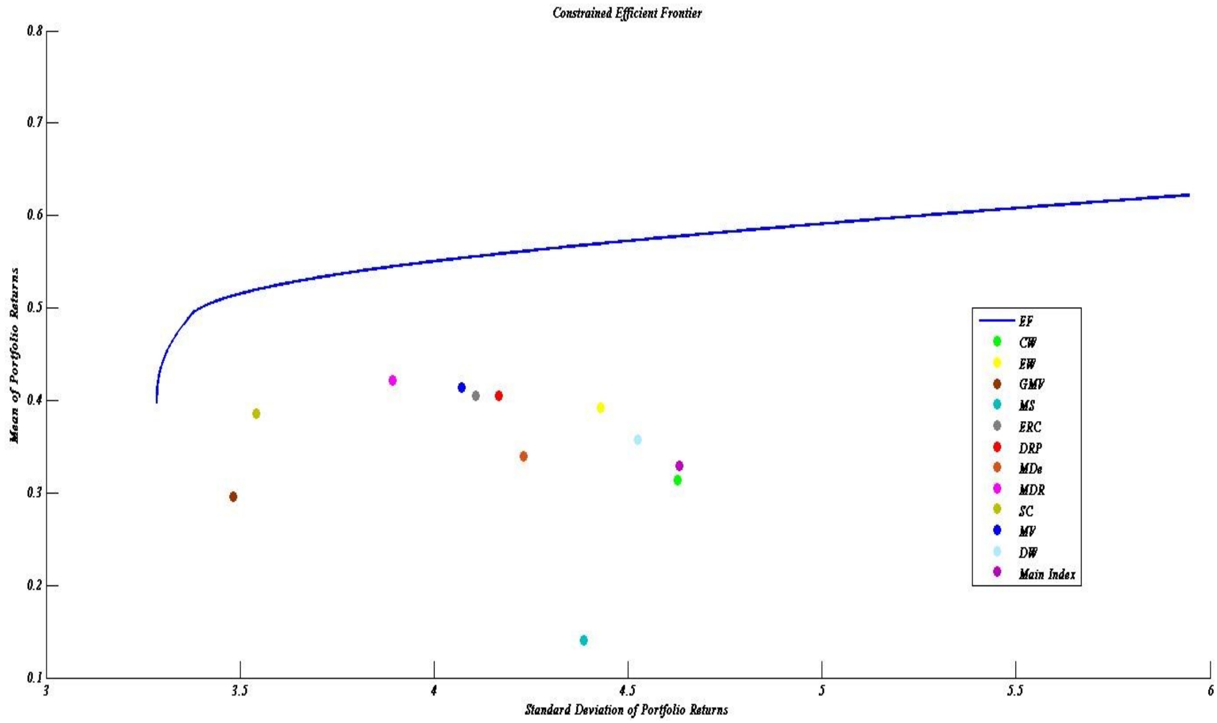


Exhibit 36: Constrained Efficient Frontier

The sketch is clear, the Cap-Weighted index have higher standard deviation compared to the Smart Beta and even lower return, therefore, aren't efficient portfolio relative to the other ones. Although Global Minimum Variance was noticed as a loser portfolio in the previous paragraphs, it produces the lowest standard deviation and so it's the portfolio less risky. This portfolio may be attractive for investor that aren't incline to risk.

Semi-Diversified has the 2nd lowest standard deviation, followed by Maximum Diversification Ratio, Diversified Minimum Variance, Equally Risk Contribution, Diversified Risk Parity, Max Decorrelation, Max Sharpe, Equally Weighted, Diversity Weight and Cap-Weighted.

The highest return is produced by Maximum Diversification Ratio portfolio, which is more efficient than many portfolios less than Semi-Diversified and Global Minimum Variance. The 2nd highest return is Diversified Minimum Variance, followed by ERC, DRP, EW, SD, DW, MDe, Cap-Weighted and MS.

Interesting is to break down ERC and DRP. The Equally Risk Contribution dominate the Diversified Risk Parity, confirming what it was claimed in the previous paragraph: the Risk Parity portfolio is already a well-diversified and performing portfolio and it's useless to improve the level of diversification looking for better performance.

3.6. Regressions

A further point is to break down the returns. Therefore, two different regression are applied in order to find a model that describe the data. Firstly, let's focus on single-factor model that it was already introduced: the CAPM model.

$$E[r_a] = \alpha + \beta\left(E[r_m] - r_f\right)$$

Equation 29

Where r_a = asset return, r_m = market return, r_f =risk free rate α = Jensen's Alpha, β = correlation parameter with the market return.

Before considering the findings, it's important to point out what is Jensen's Alpha. This is a measure derived from the CAPM model. It calculate the excess return of a portfolio above the return predicted from CAPM, given his systematic risk and the market portfolio return. This factor is used also for the evaluation of the performance of the active management. Together with Jensen's Alpha, it's computed Beta factor for all strategies. For every coefficient, it was verified his statistical significance trough his t statistic value. The t statistic value is the standardized value of the parameter, which is calculated as follows:

$$t_\beta = \frac{\hat{\beta} - \beta_o}{SE\left(\hat{\beta}\right)},$$

Equation 30

Where β_o=0, $\hat{\beta}$ = parameter estimation, $SE\left(\hat{\beta}\right)$ = parameter standard error.

The threshold is set at 95% confidence interval, therefore, the condition for parameter to be statistical significance is:

$$\|t_\beta\| > \|1{,}96\|$$

In addition, it's compounded the R^2 for all the strategies, which measures how much is able the model to explain the returns of the given strategy.

Strategy	***Alpha***	***t stat Alpha***	***Beta***	***t stat Alpha***	***R-Squared***
CW	-0,01469	-1,100	0,998223	346,352	0,998343
EW	0,079151	2,192	0,949895	121,834	0,986770
GMV	0,090265	0,653	0,622769	20,856	0,686104
MS	-0,14908	-1,267	0,876604	34,511	0,856835
ERC	0,116394	2,345	0,873852	81,535	0,97093
MDe	0,050943	2,413	0,875409	88,743	0,975353
MDR	0,16032	0,594	0,794477	47,299	0,918315
SD	0,150279	1,780	0,712869	40,867	0,893556
DMV	0,129085	1,651	0,866154	36,267	0,868584
DW	0,035817	2,606	0,975317	80,990	0,970554
DRP	0,111945	1,778	0,888762	224,279	0,996059

Table 9 : Regression of the excess returns of the portfolios, according to CAPM model

The value of Main Index are alpha=0 and Beta=1, since it's the benchmark.

Notably, there aren't any strategies that suffer from overexposure to systematic risk, since all the Beta value are less than 1. For all the strategies Beta coefficient is highly significant as we can see from the large value of t-statistic. On the contrary, for EW, ERC, MDe and DW the alpha value are statistical significance, while for the other strategy the t-statistic is too low.

All the value of alpha for the above-mentioned are significant bigger than 0.

As it could be expected, CW has a close value to 1. It's interesting to highlight how DW still has a high exposure to systematic risk as CW, although it aim to deviates from this weighting scheme.

Taking a comprehensive look of the table, it come up that a lower level of Beta allow to have higher level of alpha. But looking at MDe strategy, it is clear that a similar level of Beta with ERC lead to a low level of alpha compared to these strategy. This is a consequence of what we have already seen in the Table 4. Treynor Ratio for MDe is thoroughly lower than ERC. Therefore, if MDe rewards the systematic risk with lower return, this strategy will achieve lower alpha the beta being equals. This evidence is confirmed also looking at EW portfolio. Although has a high systematic exposure, similar to DW and CW, has a very much higher alpha, because this strategy achieves better Treynor Ratio.

Another key thing to point out is that GMV has a very low Beta level, compared to DMV and SD. This prove that minimize variance without any constraint allow to reduce the risk, since in this time interval systematic risk was very important. But SD has also a low level of Beta, while DMV has a level comparable to strategy that don't aim to minimize variance, such as ERC. Given this evidence, it can be claim that DMV is exposed to a too much restrictive constraint, which almost deny the possibility to minimize variance adequately. It's important to point out that the regression for alpha and beta, in the case of GMV, has the lowest value of R^2, therefore this model doesn't predict very well the excess return for GMV, suggesting a substantial idiosyncratic component affecting the return of GMV.

With the same presupposition, it was achieved another regression already introduce in the first chapter: the Fama-French Model (with Carhart add-on).

$$r_i = \alpha + \beta_i RMK + \gamma_i SMB + h_i HML + m_i MOM$$

Equation 31

Where r_i = excess asset return, β_i = correlation parameter to market return, γ_i = correlation parameter to small cap returns, h_i = correlation parameter to value firms returns, m_i = correlation parameter to momentum factor.

The intention is to provide a more accurate model.

Strategy	Alpha	MRKT	SML	HML	MOM	R-squared
CW	-0,006 -0,425	0,996 156,242	-0,018 -2,753	-0,005 -0,846	-0,005 -1,656	0,998
EW	0,057 1,629	0,952 91,355	-0,015 -0,900	0,067 4,835	-0,005 -0,556	0,989
GMV	0,004 0,028	0,646 21,66	-0,137 -2,061	0,188 3,436	0,051 1,530	0,713
MS	-0,218 -1,816	0,91 34,107	-0,082 -1,413	0,058 1,219	0,091 3,136	0,864
ERC	0,054 1,203	0,887 34,10	-0,043 -2,012	0,133 7,471	0,023 2,093	0,978
DRP	0,059 1,410	0,899 77,74	-0,037 -1,836	0,122 7,318	0,013 1,279	0,981
MDe	0,074 0,854	0,882 83,30	-0,032 -0,761	-0,101 -2,963	0,039 1,877	0,925
MDR	0,099 1,093	0,820 43,46	-0,094 -2,165	0,068 1,907	0,077 3,495	0,902
SD	0,063 0,723	0,737 39,98	-0,117 -2,798	0,175 5,080	0,055 2,612	0,890
DMV	0,076 1,642	0,880 77,33	-0,063 -2,854	0,112 6,172	0,028 2,505	0,977
DW	0,034 1,687	0,974 125,45	-0,018 -1,920	0,023 2,897	-0,006 -1,233	0,996

Table 10 : Regression of the excess returns of the portfolios, according to Fama-French model

Interesting is the alpha value. Alpha is not statistical significant for all the strategies, suggesting that multi-factor model succeed to explain return above the CAPM prediction for the strategies that it was statistical significant.

About SML, most of the strategies show a significant negative exposure to Small Cap factor. This is reasonable, since the asset proposed for this thesis are sectorial Cap-Weighted indices.

Therefore, it’s hard even for Smart Beta to capture Small Cap positive exposure, because the assets are already titled to a Large Cap exposure.

Some of the strategies like Equally-Weighted, Diversified Risk Parity, Max Sharpe Ratio and Max Decorralation are able to get rid of a negative exposure, having a no statistical significant exposure. Actually, these indices have low performance compared to other Smart Beta, moreover Max Diversification Ratio and Semi-Diversified have the most negative value. It can be argue that for this time interval and these asset, Small Cap factor wasn’t a source of higher performance according to the findings.

Looking at HML factor, it come up how heuristic benchmark tilted to this factor. All the Smart Beta strategies, less than Max Decorralation, show a positive exposure to this factor. Only MDR has a no significant exposure on this factor, but his t-statistic is border-line, since the value is 1.90.

In the case of MOM factor, the result are quiet reasonable. For the strategies Max Diversification Ratio, Max Sharpe Ratio, Equally Risk Contribution, Semi-Diversified and Diversified Minimum Variance the exposure to this factor is statistical significant and positive. The assets are Cap-Weighted leading to a positive momentum exposure. For Cap-Weighted there isn’t exposure, while normally these indices have a positive exposure. This is possible, since the market exposure is measured compared to FTSE All World, which is able to explain almost all the return of the Cap-Weighted index because CW have a very similar allocation.

In conclusion, the regression show that Smart Beta are able to benefit from exposure to HML factor compared to Cap-Weighted, but without selection stage the exposure to SML and MOM is similar to the Cap-Weighted indices.

4. Alternative Benchmark

In the 2nd chapter of this work, it was introduced the Smart Beta approach from a theoretical point of view. It was introduced the concept of specific risk for each weighting scheme, composed by estimation risk and optimality risk. In the section 2.7, an alternative combination between Global Minimum Variance and Equally-Weighted portfolio is proposed by Amenc et al. (2013) in order to reduce estimation risk and achieve better performance.

With this in mind, let's introduce the framework of the empirical analysis of this chapter. First, it's replicate the same combination proposed from Amenc (2013) with a portfolio 50% Global Minimum Variance and 50% Equally-Weighed (hereinafter GMV/EW).

In addition, this analysis aim also to verify if some benefit can be achieved combining winner portfolios. Given this, another alternative benchmark combine the two winners portfolios of the analysis performed in the previous chapter: 50% Maximum Diversification Ratio and 50% Semi-Diversified Minimum Variance (here in after MDR/SD).

In other words, the idea is to confirm the result of Goeltz (2013) and to verify if the combination effect works on the winner portfolio of this analysis.

4.1. Graphic results

First of all, it's interesting to break down the asset allocation graph. The Exhibit 37 show the evolution of the portfolio allocation across the time interval considered.

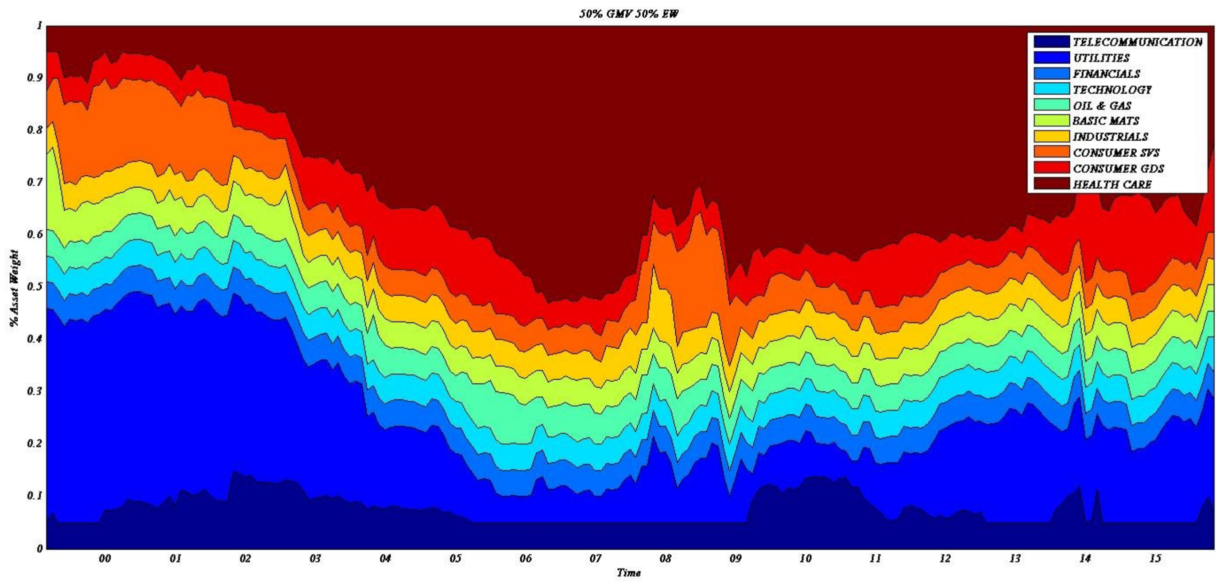


Exhibit 37: Area Chart for asset allocation of GMV/EW in the time interval 1999-2015

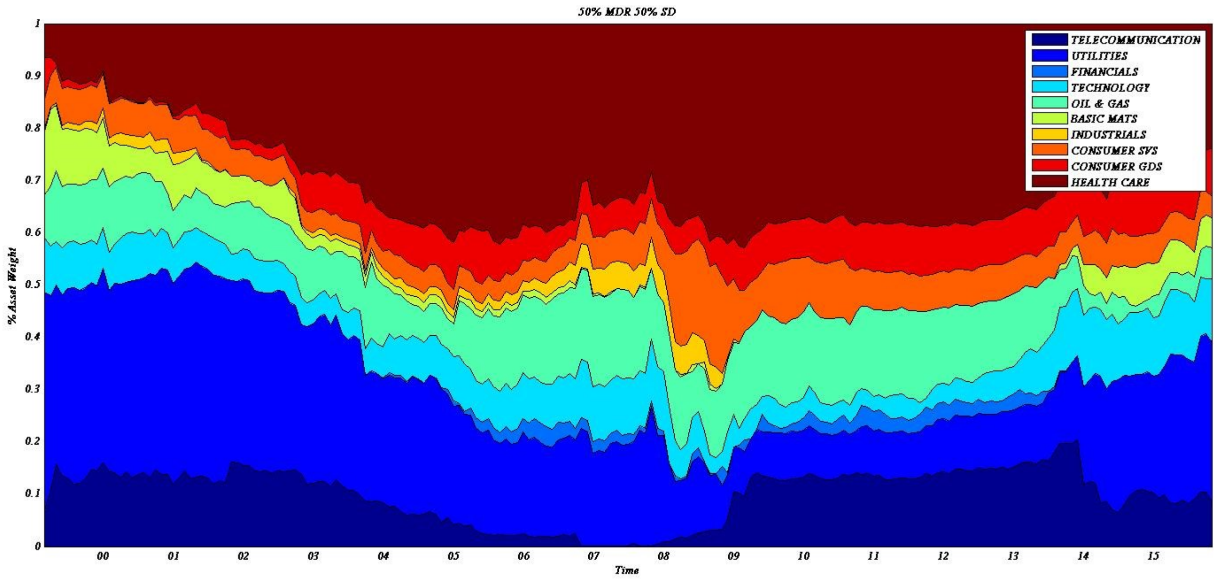


Exhibit 38: Area Chart for asset allocation of MDR/SD in the time interval 1999-2015

The GMV/EW allocation has two main feature:

- Health Care asset and Utilities asset are the only asset that experience a significant change in the portfolio weight; in the short term Utilities achieve a considerable share compared to Health Care, in the medium term Health Care benefits increases significantly its weight in the portfolio, and in the long term Utilities recovers a small amount of portfolio's share but still has a low weight compare to Health Care
- The weight evolution of the other assets is quiet stable

The MDR/SD allocation is tilted to Utilities and Health Care as GMV/EW.

In some time interval other asset, such as Oil&Gas during 09-12, have a substantial share of the portfolio.

More interesting is the comparison between the combined portfolio and their respective single components. Let's consider three graph:

- GMV/EW (above)
- Global Minimum Variance (below on the left)
- Equally-Weighted (below on the right)

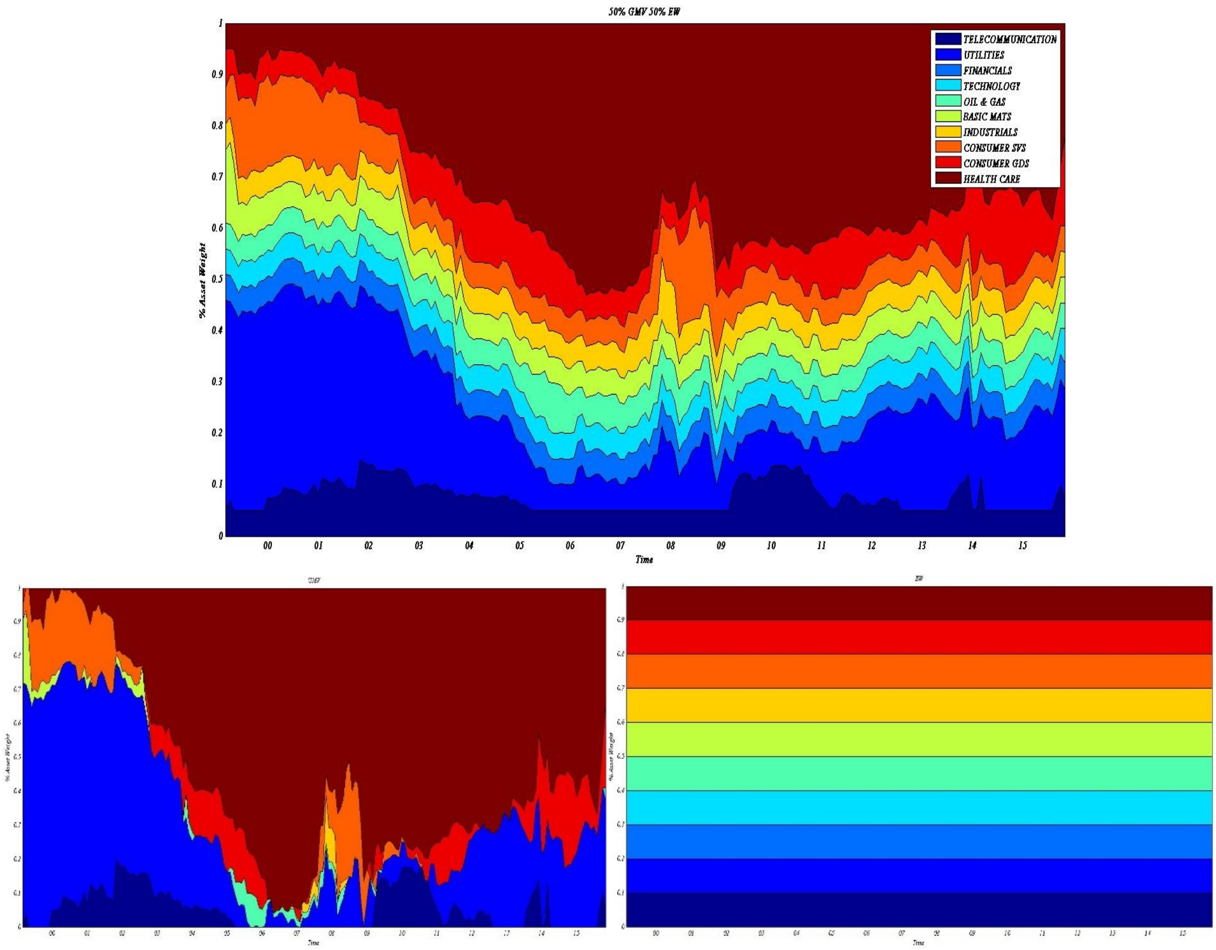


Exhibit 39: Area Chart for asset allocation of GMV/EW, GMV and EW in the time interval 1999-2015

The effect of the combination mitigate the huge concentration of the Global Minimum Variance Portfolio. As a matter of fact the combinated portfolio mark out the two optimization approach of the single component:

- Minimum variance optimization, since Utilities and Health Care have a considerable weight and have the lowest volatility as it was demonstrated in the section 3.2
- An naïve diversification approach where all the asset have the same weight, since excluding the above-mentioned assets, the other asset weight is very similar

The same analysis is done for the other alternative benchmark. The Exhibit 40 shows the three portfolios as follows:

- MDR/SD (above)
- SD (below on the left)
- MDR (below on the right)

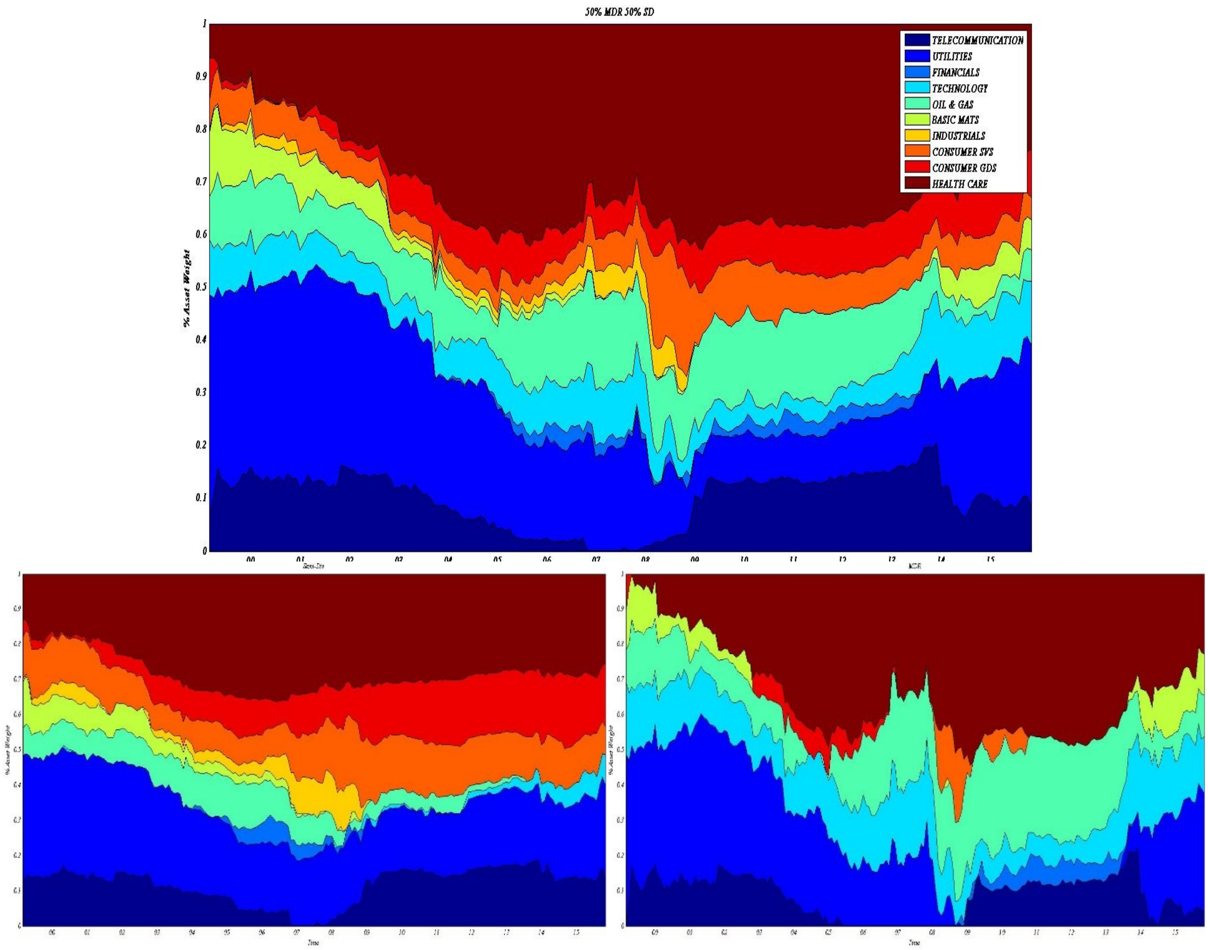


Exhibit 40: Area Chart for asset allocation of MDR/SD, MDR and SD in the time interval 1999-2015

The combined portfolio smooths the turnover level compared to Maximum Diversification Ratio, since it has a more stable evolution of the asset weight. MDR/SD still achieve a diversification without consider the number of assets, but the effect of Semi-Diversified mitigates the concentration of Maximum Diversification Ratio during the time interval 2007-2010 when only Health Care, Oil&Gas, Technology and Consumer Staples have a positive weight.

4.2. Performance measures

Given a naïve qualitative analysis, let's evaluate these portfolios in terms of performance.

The table 9 summarize the results of the same indicators as for the analysis of the previous chapter, the alternative benchmark are placed close their single component in order to have a more intuitive comparison.

Strategy	*Sh*	*So*	*Tr*	*Cal*	*Ste*
CW	0,0679	0,0908	0,3149	0,0056	0,0062
GMV/EW	**0,0904**	0,1188	0,4373	0,0074	0,0082
EW	**0,0886**	0,1205	0,4129	0,0093	0,0102
GMV	**0,0849**	0,1132	0,4745	0,0075	0,0083
MDR/SD	**0,1095**	0,1467	0,5357	0,0072	0,0075
MDR	**0,1085**	0,1487	0,5314	0,0026	0,0027
SD	**0,1087**	0,1426	0,5404	0,0081	0,0089
MS	0,0319	0,0416	0,1595	0,0080	0,0088
ERC	0,0985	0,1318	0,4628	0,0068	0,0072
DRP	0,0971	0,1303	0,4556	0,0097	0,0101
MDe	0,0802	0,1129	0,3878	0,0087	0,0095
DMV	0,1018	0,1360	0,4786	0,0082	0,0090
DW	0,0789	0,1065	0,3663	0,0066	0,0073
Main Index	0,0712	0,0951	0,3296	0,0059	0,0065

Table 11 : Performance of the portfolios and the alternative benchmark in the period 1999-2015

First of all, let’s consider the relative performance of alternative benchmark compared to their single component.

On one hand, GMV/EW achieve a higher Sharpe Ratio (0.0904%) compared to GMV alone (0.084%) and EW alone (0.088%). Given this evidence, it can be argue that:

- GMV/EW reach a better performance in terms of return per unit risk compared to the single components
- The difference between the performance of GMV/ EW and its single components is very small in terms of magnitude
- It's not clear the source of this small outperformance, since the other more specific indicators are just an average of the single components value.

On the other hand, MDR/SD achieve better Sharpe Ratio (0.1097%), but it's a very small amount compared to the magnitude of the single components SR, which for Maximum Diversification Ratio is 0.01085 and for Semi-Diversified is 0.01087. Again, the other indicators are an average between the single components value. In conclusion:

- MDR/SD reach a better Sharpe Ratio
- The difference between the performance of MDR/SD and its single components is very small, even smaller than the case of GMV/EW
- It's not clear the source of this small outperformance, since the other more specific indicators are just an average of the single components value.

As we could expect, comparing the portfolio to the overall set of weighting scheme, their performance are better than Cap-Weighted indices. But, GMV/EW has a lower performance compared to other Smart Beta, such as Equally-Risk Contribution, Diversified Risk Parity and Diversified Minimum Variance. Instead, MDR/SD is a winner portfolio as his single components.

In order to verify the effect of combination portfolio in terms of Sharpe Ratio over the time, the portfolios will be analyzed according to three time interval: short term period, medium term period and long term period. The time interval are the same as in the section 4.3 of this work.

Strategy	***SRshort***	***SRmedium***	***SRlong***
CW	0,0238	0,0391	0,0681
GMV/EW	**0,0401**	**0,0663**	**0,0904**

EW	**0,0563**	**0,0712**	**0,0888**
GMV	**-0,0370**	**0,0129**	**0,0851**
MDR/SD	**0,0327**	**0,0662**	**0,1097**
MDR	**0,0283**	**0,0674**	**0,1087**
SD	**0,0369**	**0,0637**	**0,1085**
MS	-0,0365	-0,0040	0,0320
ERC	0,0613	0,0751	0,0987
DRP	0,0633	0,0742	0,0974
MDe	0,0179	0,0571	0,0804
DMV	0,0624	0,0711	0,1021
DW	0,0394	0,0553	0,0791
Main Index	0,0263	0,0463	0,0713

Table 12 : Performance of the portfolios and the alternative benchmark in the short term period (99-04), medium term (05-10) and long term (11-15)

In the short term period, GMV/EW portfolio achieves a SR (0,04%) lower than EW (0,05%), but much higher than GMV (-0,03%). Proportionally, the difference negative difference with the single component Equally Weighted is small compared to the positive difference with the single component Global Minimum Variance. Instead, MDR/SD achieves an average SR (0,03%) between the two single components. In comparison with the other Smart Beta, the combined portfolios are dominated by ERC (0,06%), DRP (0,06%) and DMV (0,06%). On the contrary, Cap-Weighted portfolios achieve a lower performance (CW=0,02%, Main Index=0,02%) compared to combined portfolios.

In the medium term, the performance of GMV/EW (0,06%) converge to the one of EW (0,07%), while the distance with GMV (0,01%) is still substantial. MDR/SD achieves a Sharpe Ratio (0,66%) very close to the one of MDR (0,067%).

In the long term, GMV/EW portfolios achieves an higher SR (0,0904%) compared to Equally Weighted (0,088%) and to Global Minimum Variance (0,084%). Also MDR/SD achieves an higher SR (0,1097%) compared to Maximum Diversification Ratio (0,1087%) and Semi-Diversified (0,1085%).

This evidence suggest that combined portfolios achieve better Sharpe Ratio in the long term. Moreover, it can be considered another hypothesis. The performance of the single components converge only in the long term period, therefore, it may be probably that the combination effect works better when the performance of the single components are similar.

Conclusion

In the end, this work confirm many of the hypothesis expressed.

Firstly, a graphical support allowed to analyze the asset allocation of the different weighting schemes. The lack of diversification for the Cap-Weighted portfolio is not emphasized at sectorial level in the time interval considered, but still there is a concentration in the financial sector. The portfolios based on Smart Beta approach provide an asset allocation well-diversified. Some weighting scheme, such as ERC, DRP, DMV and EW, diversify assigning a significant positive weight to many assets, while other weighting scheme, such as MDR and MDe diversify taking in account the correlation across the assets and assigning positive weight to a reduced number of assets. Moreover, most of the heuristic benchmark assign the highest weighting share to Health Care, which experience the lowest volatility and the lowest correlation with the other assets considered.

Then, the portfolios were valuated with performance indicators, such as Sharpe Ratio, Sortino Ratio, Treynor Ratio. The Cap-Weighted index achieved the worst performance. Among the Smart Beta indices, the best portfolios were the Maximum Diversification Ratio and the Semi-Diversified.

In order to analyze the performance over the time, it was compared the mean and the variance of the portfolios in three different time intervals that mirror the short term period, the medium term period and the long term period. The results show that the traditional index outperforms some Smart Beta index, such as GMV, MSR and MDe, in the short term period. The other heuristic benchmark have a similar or above performance compared to the Cap-Weighted even in the short term. In the medium and long term period, the Smart Beta approach achieves higher mean and lower standard deviation, therefore, it's not only provide better return but also is less risky.

In addition, it was quantified the magnitude of the potential loss of the portfolios by using the Value-at-Risk measure. The results show that traditional index are exposed to higher potential losses, particularly, in the period of unstable market condition with high volatility. The weighting scheme that mainly reduce the potential losses is the Global Minimum Variance.

An interesting point was introduced in the theoretical chapter, about the dualism between CAPM model and Fama-French Model for the description of excess returns.

Firstly, the CAPM model was applied to the portfolios. The findings showed that this is mostly a good model for the description of the returns for almost all the portfolios, less than for Global Minimum Variance.

Then, all the excess returns of the portfolios considered were modelled with the Fama-French model. The most interesting finding is that, in general, the Smart Beta portfolios show a significant exposure on the Value factor. The negative exposure of the heuristic benchmark to the Small Cap factor is a consequence of the Cap-Weighted assets used for this analysis.

In the last chapter of this work were proposed two alternative benchmark. The first alternative benchmark combine a 50% of Global Minimum Variance and 50% of Equally Weighted. This benchmark aim to confirm the finding of Amenc et al.(2013), that the combination of one portfolios with estimation risk (GMV) and one portfolio without estimation risk (EW) achieves better Sharpe Ratio than the single components. The second alternative benchmark combines 50% Maximum Diversification Ratio and 50% Semi-Diversified with the purpose to verify if the combination of winner portfolios can outperform the single components. The results show that both portfolios achieve better Sharpe Ratio by a very small amount, the MDR/SD even lower than GMV/EW.

Ultimately, the traditional Cap-Weighted indexation is an old approach with a weak theoretical background, unable to meet the investment requirement and hard to adapt to the market condition.

On the contrary, the Smart Beta approach solve most of the problem of the traditional indices, providing a wide range of different solution for the investors with transparency and flexibility and reacting fast and adequately to the different market condition. Although, there are many studies on this topic (mainly by Edhec Risk Institute) this approach is still recent, suggesting that doing further researches may improve the quality of these indices.

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DATABASE

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