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Experimental study on the identification of coherent structures in turbulent free surface flows

Studio sperimentale sull'identificazione delle strutture coerenti in moti turbolenti a superficie libera

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Abstract

The coherent structures within open-channel flow over either gravel or spheres bed was investigated in a tilting laboratory flume, starting from Particle Image Velocimetry (PIV) measurements of the instantaneous velocity fields. A comparison between different hydraulic conditions was carried out in order to study how different flow depths and bed materials could influence the turbulent properties of the structures identified by applying the U-level and modified Phase-Space techniques. The experiments reveal that there are no appreciable differences by using gravel or spheres bed, for a similar Reynolds numbers. By increasing the flow depth, the turbulent structures increase in length and frequency, but the results are affected on the number of components considered to reconstruct the signal. The U-level technique seems to be quite appropriate to the experimental data set used in the present thesis, because the results approximately agree with earlier works. On the contrary, the modified Phase Space technique, besides it would be a more rigorous and more sensitive method to real flows than U-level, does not seem to work. This fact is probably due to a too high spatial resolution of the data set, and/or to the innovation consisting on adding a second (internal) ellipsoid to detect turbulent events from the velocity fields.

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Nomenclature

С	Correlation value	[-]	
ds	Gravel uniform thickness		
dt	Time step	[s]	
dx	PIV streamwise spatial resolution	[mm]	
D	Uniform water depth	[mm]	
D ₅₀	Gravel mean grain size	[mm]	
f	Sampling frequency	[Hz]	
$\mathbf{f}_{\mathbf{S}}$	Spatial frequency of the turbulent structures	[Hz]	
\mathbf{f}_{t}	Temporal frequency of the turbulent structures	[Hz]	
g	Acceleration due to gravity	[m/s ²]	
h	Mean flow depth	[m]	
k	Universal Von Kàrmàn constant	[-]	
K_{U}	U-level conditional threshold	[-]	
Ks	Strickler roughness coefficient	$[m^{1/3}s^{-1}]$	
L _E	Length of individual eddies	[m]	
L	Spatial length of each time interval	[m]	
Ls	Length of turbulent structures	[m]	
$n_{\rm E}$	Number of turbulent events detected by for each row	[-]	
O _{A,B}	Overlapping area of the image A,B	[m/s]	
$p_{\rm U}$	U-level end event probability threshold	[-]	
Re	Reynolds number	[-]	
R_h	Hydraulic radius	[m]	
\mathbf{S}_0	Bed slope	[-]	
$\mathbf{S}_{\mathbf{u}}$	Standard deviation of streamwise velocity series	$[m \ s]$	
S_u	Standard deviation of vertical velocity series	$[m \ s]$	
u	Instantaneous local streamwise velocity	$[m \setminus s]$	
u'	Instantaneous local streamwise velocity fluctuations	$[m \setminus s]$	
Ut	Depth-local mean streamwise velocity	$[m \ s]$	
U*	Shear velocity	$[m \ s]$	
U^{+}	Normalised depth-local mean streamwise velocity	[-]	
V	Vertical velocity	$[m \setminus s]$	
v'	Instantaneous local vertical velocity fluctuations	$[m \ s]$	

V	Vertical mean velocity	$[m \ s]$
W	Lateral velocity	$[m \ s]$
W _{A,B}	Weighting used in the image A,B	[-]
W	Width of the frame	[mm]
х	Streamwise axis	[m]
У	Vertical axis	[m]
ϕ	Diameter spheres	[mm]
ΔC	Difference in correlation values	[-]
$ ho_g$	Gravel particle density	[kg/m ³]
$ ho_s$	Spheres density	[kg/m ³]

INTRODUZIONE IN ITALIANO DEL LAVORO DI TESI

L'obbiettivo di questo lavoro di tesi è l'identificazione e lo studio delle strutture di flusso coerenti all'interno di moti turbolenti a superficie libera. Ciò che si intende realizzare è un confronto dei risultati tra diverse condizioni idrauliche sperimentate in laboratorio.

I dati sperimentali in esame erano già stati ottenuti da attività di ricerca condotte negli ultimi anni, che avevano fatto uso della tecnologia PIV (Particle Image Velocimetry) per misurare il campo di velocità della corrente all'interno di una canaletta. Da questa serie di precedenti misurazioni, sono state scelte tre condizioni idrauliche caratterizzate dall'avere il fondo in ghiaia e altrettante con il fondo rivestito in sfere, aventi rispettivamente lo stesso tirante idraulico e all'incirca lo stesso Numero di Reynolds.

Il primo punto è consistito nella rimozione da ogni istantanea di velocità (longitudinale) PIV delle parti del campo di moto giacenti al di sotto del livello medio del fondo e al di sopra della superficie libera, poiché si era interessati a studiare il solo flusso della corrente. Alcune di queste immagini PIV sono state in seguito unite assieme, tramite una speciale procedura che verrà descritta in seguito, allo scopo di ottenere, per ciascuna condizione di flusso scelta, sei serie temporali abbastanza lunghe da poter identificare almeno tre o quattro macro-strutture turbolente, che in base alla letteratura dovrebbero avere una lunghezza caratteristica nella direzione principale della corrente di circa 3 - 5 volte la profondità della corrente.

Il secondo punto ha previsto la rimozione della velocità media da ogni intervallo temporale, ottenendo misure delle fluttuazioni di velocità. Ciascun campo di moto così ottenuto è stato quindi decomposto in 500 componenti energetiche applicando la procedura SSA (Singular Spectrum Analysis). Questi sono stati poi ricostruiti utilizzando differenti combinazioni di modi (Roussinova et. al.; 2010): i primi modi, presi in numero tale affinché racchiudano circa il primo 50 % dell'energia cinetica totale del segnale, sono stati utilizzati al fine di esporre le strutture turbolenti più grandi (strutture coerenti); le componenti seguenti, in numero tale da ricoprire un altro 33 % dell'energia, sono stati classificati come un rumore del segnale e quindi scartati da ulteriori analisi.

Il terzo punto della presente tesi è stata l'identificazione delle grandi e piccole strutture di flusso turbolenti, dalle fluttuazioni di velocità e dai segnali ricostruiti post decomposizione, applicando due differenti procedure: il metodo U-level ed il metodo Phase-Space.

La tecnica U-level sembra fornire risultati abbastanza soddisfacenti. Applicato alle fluttuazioni di velocità originali, mostra tante strutture di flusso di dimensioni estremamente variabili, da piccolissime a molto grandi. La decomposizione SSA aiuta a separare queste strutture, anche se non considerando implicitamente la loro dimensione come elemento discriminante ma piuttosto il loro contenuto energetico. Le strutture dal più alto contenuto energetico tendono ad essere più grandi, ma assieme a loro appaiono anche diverse strutture di dimensioni nettamente più contenute ma anch'esse fortemente energetiche. Quando viene utilizzato un numero sufficientemente basso di modi SSA per esporre le strutture coerenti, queste appaiono visivamente più nitide, senza molte di quelle strutture di piccole dimensioni menzionate pocanzi.

La frequenza spaziale delle strutture turbolenti, identificate tramite il metodo U-level, diminuisce all'aumentare della profondità della corrente, e lo stesso trend si verifica anche per quanto riguarda la frequenza temporale delle strutture più grandi. La situazione opposta la si riscontra invece sulla

loro lunghezza caratteristica, che aumenta col crescere del tirante idraulico. Le strutture turbolenti risultano mediamente di dimensioni 10 - 20 % maggiori per le condizioni idrauliche aventi il letto in ghiaia; di conseguenza, la frequenza spaziale risulta leggermente più alta per i casi di letto in sfere (all'incirca della stessa percentuale).

Il metodo Phase-Space è stato applicato con l'aggiunta di una modifica, rispetto a quanto illustrato da Nikora (2005), che è consistita nell'aggiunta di un ulteriore ellissoide nella rappresentazione dei dati nel piano di fase: sono stati dichiarati come eventi turbolenti di interesse quei punti compresi tra l'ellissoide più esterno, che elimina i dati più estremi e presumibilmente errati, e l'ellissoide più interno, il quale invece è stato ipotizzato individui al suo interno le zone di flusso non caratterizzate da eventi turbolenti rilevanti. Malgrado questo metodo debba teoricamente fornire un'identificazione più precisa degli eventi turbolenti, in quanto, contrariamente alla tecnica U-level, è in grado di riconoscere le accelerazioni estreme del fluido, non ha fornito risultati ritenuti accettabili.

Si è convenuto che questo metodo Phase-Space, modificato nel suddetto modo perché possa identificare eventi turbolenti nel campo di moto, in realtà non sia in grado di funzionare. Un'ulteriore causa di questo insuccesso, altre alla particolare modifica apportatasi, potrebbe risiedere nell'eccessiva risoluzione spaziale utilizzata dalle istantanee di velocità PIV, o anche agli errori commessi nel misurare la velocità delle particelle di tracciante. Le caratteristiche delle strutture turbolenti rilevate mediante tale metodo, non sembra possano fornire informazioni attendibili.

1. Introduction

The aim of this thesis is the identification and study of coherent flow structures in turbulent free surface flows, in order to make a comparison of their properties between different hydraulic conditions.

The velocity data set had already been obtained from previous experiments that had used the Particle Image Velocimetry (PIV) technique during laboratory flume experiments. From these measurements, it was decided to choose three hydraulic conditions for both gravel and sphere beds, having respectively the same flow depths and approximately the same Reynolds Number.

The first step has been to crop the streamwise velocity snapshots (single instantaneous velocity fields) under the bed surface elevation and above the water free surface, in order to examine just what happen within the flow. Afterwards, a few of these instantaneous snapshots have been stitched together, obtaining (for each flow condition) six velocity time series long enough to identify at least three or four macro-turbulent structures, that according to the literature should have a mean length between 3 to 5 times the flow depth.

The second step has consisted in subtracting the mean velocity from each row of the time intervals, so obtaining the velocity fluctuations. They have been decomposed into 500 energetic components by applying the Singular Spectrum Analysis (SSA) technique. The velocity fields have been reconstructed by using different combinations of modes (Roussinova et al.; 2010): the first modes corresponding to about 50% of the turbulent kinetic energy to expose the large (energetic) structures; a second group of modes, recovering about 33% of the energy, to expose the small (less energetic) structures. The remaining modes have been dealt as a noise, and then were not considered.

The third step has been to identify the large and the small structures from both the original and the reconstructed signals, by applying two different techniques: U-level and Phase-Space.

The U-level technique seems to provide quite good results. The raw fluctuations show lots of structures ranging from small to large, and the SSA decomposition helps to separate them, even if not just by size but by energy content. The most energetic structures tend to be bigger, but also several "small" structures are detected because strongly energetic. When a low number of SSA modes is used to expose the large structures, they appear more clearly visible.

The spatial frequency of the turbulent structures, and the temporal frequency of the large structures, decreases as the flow depth increases. The opposite situation occur for their characteristic mean length. The turbulent structures result 10 - 20 % larger for the cases of gravel bed. As a consequence, the spatial frequency results slightly higher for the cases of spheres bed (approximately by the same quantity).

The Phase-Space technique was applied by introducing an innovation, compared to what reported by Nikora (2005), consisting on a second (internal) ellipsoid: turbulent events are detected as the events contained between the internal ellipsoid and external one. Besides it would be a more rigorous and more sensitive method to real flow than U-level, usually shows confused clusters of points instead of quite clear structures. As a conclusion, this Phase-Space method modified to detect turbulent events, does not seem to work. Another reason might also be the too high spatial resolution adopted to collect PIV velocity instantaneous, and/or the errors committed to measure the velocity of the tracer particles. The detected turbulent properties seem do not give very reliable information.



The following flow chart shows the scheme of the main points of this thesis.

Figure 1-1: Scheme of the present thesis

2. Large scale turbulent flow structures

Turbulence is one of the main factors determining the character and intensity of many river processes. These include erosion and sediment transport, resistance to flow, diffusion of matter, heat transfer, and the genesis of bed and channel forms. In the field of turbulence, there is a great deal of research activity in open channel flows, particularly in the area of *coherent flow structures*. These are generally considered to be repetitive quasi-cyclic large-scale (of the order of the flow depth) turbulent motions.

Until the 1950s, turbulent fluctuations were considered random and chaotic, but after then it was inferred (Velikanov; 1949) that flow turbulence might be a "structural" and "quasiperiodic" phenomenon. This structural concept of flow turbulence has been later experimentally confirmed by several researchers, who studied the kinematic structure of the flow in flumes with smooth and rough beds (Fidman; 1953). By using a camera moving with the flow to record images of fluid motion within visualized turbulent flow, Fidman concluded that the maximum energy of turbulence is contained in the low-frequency turbulent fluctuations caused by the largest turbulent disturbances, the vertical and longitudinal dimensions of which scale with the flow depth.

Experiments leaded in the 1960s using similar technique revealed the existence in the turbulent flow of *large-scale eddies* with the vertical size close to the flow depth h and the length varying from about 7h for smooth beds to 4h for rough beds (Klaven; 1966, 1968). These eddies are the most stable structural features and can be considered as quasiperiodic elements of the open-channel flow turbulence.

Thanks to more recent studies (Shvidchenko et al., 2001), that produced detailed measurements of flow velocity vectors fluctuations in flumes with smooth and gravel beds, it has been found that the turbulent flow appears to consist of ordered sequence of long-living three-dimensional large-scale turbulent eddies. These eddies move downstream at the bulk flow velocity. This causes quasiperiodic high forward speed downwelling ("*sweeps*") and burst-like upwelling ("*ejections*") fluid motions throughout the entire flow depth, resulting in quasiperiodic fluctuations of the flow velocity vector. The movement of the eddies takes place along relatively stable paths ("*macrojets*"), with a width close to 2h, and it is responsible for the existence of alternating high speed and low-speed regions in both the streamwise and spanwise directions.

It has been hypothesized that the depth-scale (or "macroturbulent") eddies are closely linked to what is called the "*bursting*" *phenomenon in boundary layers*. Intensive experimental research on bursting processes in open-channel flows near both smooth and rough beds was performed by many investigators (Grass et al., 1991; Nezu and Nakagawa, 1993; Best, 1993). It has been found that the ejected low-momentum fluid travels across the entire flow depth up to the water surface while high-momentum fluid moves from the water surface toward the bed giving rise to *rolling structures*, which are similar to the large-scale eddies mentioned before. In addition, the streamwise spacing of the bursting events has been found to be between 2h and 7h, which is close to the observed length of large-scale eddies and thus partly supports the hypothesis about close linkage between these two phenomena.

Figure 2-1 report a picture of the macroturbulent flow structures detected by Shvidchenko et al. (2001), by using a digital camera with an exposure of 4 Hz. Figure 2-2 show a 3D schematic representation of these large-scale turbulent structures.



Figure 2-1: Picture and two dimensional scheme of large-scale turbulent structures of open-channel flow over gravel bed. Camera is moving with mean flow velocity. The length of individual eddies Le results approximately 4.5 times the flow depth. (Shvidchenko, 2001).



Figure 2-2: Model of three dimensional large-scale turbulent flow structure of open-channel flow over a mobile bed. (Shvidchenko, 2001).

3. Experimental facilities

3.1. Flume set-up

The experiments were carried out in a 12.6 m long, sloping rectangular flume which is 459 mm wide (Figure 3-1).



Figure 3-1: Photograph of the flume (Nichols, 2013)

The flume was simply supported on a pivot joint at the inlet end, and on a pivot joint attached to a screw thread jack at the outlet end. Adjustment of the screw thread allowed the gradient of the flume to be varied. In these tests, the flume was tilted to a slope which varied from $S_0 = 0.001$ to 0.004 in 0.001 increments.

A constant head pump was used to recirculate water in the flume. Control of the discharge from the pump was achieved with an adjustable valve in the flume inlet pipe. The magnitude of the discharge was determined using a u-tube manometer connected to a standard orifice plate assembly (BS5167-1, 1997). The manometer could be read to the nearest mm, and this means that the flow rate was measured to an accuracy of 0.5 l/s.

The depth of the flow was controlled with an adjustable gate at the downstream end of the flume to ensure uniform flow conditions throughout a section as long as possible, in particular in the measurement section of the flume.

The uniform flow depth was measured with point gauges which were accurate to the nearest 0.5 mm. This measurement was conducted at 4 positions, situated 4.4 m to 10.4 m from the upstream

flume end in 2 m increments. At the start of each of these measurements the gauge was reset to zero datum which corresponded to the mean bed level and then it was raised until its tip was just in contact with the water surface.

Temperature was measured before and after each test using a digital thermometer accurate to ± 0.5 ^oC, placed 0.5 m downstream of the measurement sections. In this way, it was able to record a representative temperature for the water in the flume, and without influencing the flow or free surface structure in the measurement section. For each flow condition, the temperature changed by less than one degree centigrade over the course of the measurement. This fact ensured stable thermal conditions and therefore constant viscosity and surface tension.

3.2. Bed types

Two bed conditions had been examined during this study, see Nichols (2013). During the first phase of testing, the flume had a bed of well-mixed washed river gravel. The gravel particles, scraped to a uniform thickness of $d_s = 50$ mm (nominal), had a density of $\rho_g = 2600$ kg/m³ and mean grain size of $D_{50} = 4.4$ mm.

During the second phase of testing, the bed was composed of a hexagonally packed arrangement of $\phi = 25$ mm diameter spheres. This bed type was selected to give a similar flow resistance to the gravel bed, but with significantly different physical bed shape. Two layers of spheres were used in order to give a bed thickness similar to that of the gravel bed, and to allow realistic interfacial flows into and out the porous bed. These spheres were manufactured by plastic injection moulding, and had a density of $\rho_s = 1400 \text{ kg/m}^3$.

Figure 3-2 show these two bed materials.



Figure 3-2: The two bed substrates used: washed river gravel with mean grain size of 4.4 mm, and 25 mm diameter polymer spheres (Nichols, 2013)

3.3. Particle Image Velocimetry (PIV)

3.3.1. Introduction

In order to have measurements of the velocity flow field, two-dimensional Particle Image Velocimetry (PIV) had been used to collect such data.

Particle Image Velocimetry (PIV) is a non-intrusive technique for fluid flow measurement and provides instantaneous velocity fields over global domains. It records the position over time of small tracer particles introduced into the flow to extract the local fluid velocity.

PIV was developed in the 1970's after continuous efforts to optimize the Laser Doppler Velocimetry technique (LDV), and in the end of the 1980's started to be very used. In the last years, PIV has become very popular, mainly because it is not particularly difficult to use and because of the huge quantity of information that it possible to obtain from a single measurement.

The basic requirements for a PIV system are the following:

- a proper flow velocity tracer, that would have the same density of the water
- an optically transparent test-section
- an illuminating light source (laser): tracer particles are lighted in the transparent test-section thanks to an illuminating light source generated by a laser
- a recording medium: tracer particles light reflection is recorded in one or a series of images by using one or more cameras with CCD sensors
- a computer for image processing: the acquired images are processed by using specific software, that allow to know information concerning the velocity field from the tracer particles displacements.



3.3.2. System set-up and Calibration

Figure 3-3: Overview photograph of PIV system set-up (Nichols, 2013)



Figure 3-4: Diagram of camera arrangements for flow visualisation

Figures 3-3 and 3-4 show respectively a photograph of the PIV system and a diagram of the camera arrangement for the flow measurements used in these tests.

The laser emitter unit projects a beam of two concentric lasers over the top of the flume where it contacts a 45° mirror, sending the beam vertically downwards at the centre of the flume. The beam then passes through optics designed to form and focus a light sheet. The laser sheet illuminated a volume approximately 220 mm long in the streamwise direction and approximately 3 mm thick in the lateral direction.

Two CCD, each with an image area of 1600 x 600 pixels, were focused on the laser sheet, and were synchronized with the laser pulses. The cameras were situated a distance of 1.25 m from the light sheet, with an angle of 30° between them. The use of two cameras improves the accuracy of the vertical and streamwise velocity measurements, and also allows for transverse velocities to be calculated.

The total field of view of the cameras allowed to capture data from an area between 2.5 and 6 water depths long in the streamwise direction. This length was expected to be suitable for capturing at least one large scale turbulent event.

The resolution of the images was approximately 6.5 pixels per mm in either direction, equivalent to $42 \text{ pixels per mm}^2$, allowing a strong particle definition and a high spatial resolution of the resulting velocity vector.

For the PIV measurements, the neutral tracer introduced to the flow to act as seeding particles was Plascoat Talisman 30. This polymer, used with a diameter of around 150μ m, is able to follow the flow path representatively during each measurement.

In order to obtain the velocity field, a pair of particle images separated by a time delay of 10^{-3} seconds was captured on each camera and this was repeated at a fixed frequency of 26.9 Hz. Since the measurement duration was 300 seconds, 8070 vector maps were constructed per flow regime.

The next figure show one of the PIV cameras with its view of the measurement plane, along with an example image of the seeding particles in a flow captured by this camera.



Figure 3-5: One PIV camera and its view of the seeding particles in the flow field (Nichols, 2013)

An important step, necessary when using the PIV technique, is the *calibration* of the images.

This procedure allows the output of the PIV analysis to be represented in terms of m/s instead of pixels/s concerning the velocities, and in mm rather than pixels concerning the spatial locations.

In order to do this, velocity images were captured of a calibration plate consisting in an orthogonal grid of circular markers at know spatial position. This calibration plate had dimensions of 200 mm by 200 mm, and its horizontal axis was always parallel with the flume bed. The datum position was defined by a unique large marker located at the centre of the array.

Figure 3-6 shows the calibration plate used, where it is visible the marker that defines the datum position.



Figure 3-6: Calibration target plate used for PIV calibration

3.4. Data pre-processing and validation of PIV data

Each image from the two PIV cameras was divided into interrogation areas of 16×16 pixels. This interrogation area size corresponds to a physical area of around 2.5 x 2.5 mm. This interrogation area is considered small enough to assume uniform flow inside it.

Because of the overlapping area between the cameras equal to 50 %, the spatial resolution of the measurements results around 1.36 mm in the streamwise direction and 1.27 mm in the vertical direction. These settings would ensure that there was sufficient particle density within each interrogation area to allow accurate estimation of local velocity.

A two dimensional *cross-correlation technique* determined the velocity vector for each interrogation area by comparing the image captured in two frames separated by 10^{-3} seconds, to determine the most likely average motion of the particles (Bastiaans, 2000), as illustrated in Figure 3-7.



Figure 3-7: Cross-correlation to particle images to obtain velocity vector

More specifically, the cross-correlation function is a statistical tool that gives information about the dependence of the value of a random unknown in one point, compared to the value assumed to another unknown in another point. Each interrogation area of the first frame is switched to the second one, along x and y directions, and for each position the value of the function is calculated.

It will result in a distribution where the maximum peak corresponds to the statistically maximum overlapping between the position of the particles within one interrogation area, in the first and in the second frame. This value represents the mean displacement of the particles within an interrogation area. This cross-correlation procedure is repeated for each interrogation area of the two frames captured by the cameras.

As a result, a map with displacement vectors is obtained, and by using the time interval between the two captured images, this will give the entire flow field.

The next figure show an example of the correlation plane for one interrogation area. It is evident a strong dominant peak along the streamwise axis. This was typical of most interrogation areas for all flow conditions, since the flow is predominantly in the streamwise direction, with some vertical variation. The correlation peak was determined to be valid if it was more than 20% higher than the next most significant peak (Siegel, et al., 2001; Dantec Dynamics, 2002). Interrogation areas with an invalid peak (typically < 2 % of vectors) were identified for reconstruction during the *moving average validation*.



Figure 3-8: Cross-correlation plane for one interrogation area (Nichols, 2013)

At this point, the vector maps have underwent *range validation* and *moving average validation* in order to correct any spurious data points. Less than 5 % of vectors have been replaced.

The *range validation* was configured on an individual basis for each flow condition to remove only the large, obviously incorrect vectors.

The *moving average validation* comes from the idea that the measured velocity field would be slowly changing inside the interrogation area and then the adjacent vectors would not be very different from each other.

A 3 x 3 interrogation area window over the vector field was clean to identify vectors which were significantly different from adjacent vectors. Three times this operation was performed. Each time, any vectors which differed by more than 10 % from the mean value of the vectors surrounding it was replaced by the mean value. This also provided a vector for any interrogation areas whose vectors were removed by the previous validation stages.

Finally, the vector maps from the two PIV cameras were combined to form the final vector field. Figure 3-8 report an example of a PIV vector field, before and after correcting any spurious or 'bad' data point.



Figure 3-9: Example of PIV vector field, before and after applying Data Validation.

By following this procedure, a time series of vector maps was constructed for each of the flow conditions described in the next paragraph. This data was then exported in a numerical format to allow detailed analysis using Matlab.

4. Choice of the hydraulic conditions

The hydraulic conditions analysed in this study came from PIV data set previously collected and used for other experiments (Nichols, 2013). From these measurements, it was decided to choose three hydraulic conditions for both gravel and sphere beds, having respectively the same flow depths and approximately the same Reynolds number.

The following two tables report all the conditions previously collected and then the ones used in this study have been pointed out.

Condition	Bed slope	Depth	Depth Mean Velocity	Reynolds Number
	So	D [mm]	V [m/s]	Re
1	0.004	40	0.41	17300
2	0.004	50	0.50	26500
3	0.004	60	0.55	35200
4	0.004	70	0.60	42600
5	0.004	80	0.64	54800
6	0.004	90	0.69	67200
7	0.004	100	0.74	86000
8	0.003	50	0.36	19700
9	0.003	60	0.41	25800
10	0.003	70	0.47	35500
11	0.003	80	0.52	45800
12	0.003	90	0.57	55200
13	0.002	60	0.32	19500
14	0.002	70	0.35	25900
15	0.002	80	0.40	31800
16	0.001	70	0.26	18500

 Table 4-1: Measured hydraulic conditions for gravel bed flows

Table 4-2: Measured hydraulic conditions for spheres bed flows

Condition	Bed slope	Depth	Depth Mean	Reynolds Number
	C	D [mm]	V clocity	Do
	S 0	D, [iiiii]	<i>V</i> ,[III/S]	Re
17	0.004	40	0.28	10800
18	0.004	50	0.36	15100
19	0.004	60	0.43	24500
20	0.004	70	0.50	32700
21	0.004	80	0.57	38800
22	0.004	90	0.65	47300
23	0.004	100	0.71	59700
24	0.003	50	0.26	11000
25	0.003	60	0.35	19500
26	0.003	70	0.44	27900
27	0.003	80	0.49	32100
28	0.003	90	0.57	42400
29	0.002	60	0.22	12000
30	0.002	70	0.32	19700
31	0.002	80	0.41	30800
32	0.001	70	0.21	14300

V is the depth average mean velocity, and Re is the Reynolds number.

As it is evident, the bed slopes are not the same for gravel and sphere beds flow, because otherwise the Reynolds Numbers will have been too different if tests with the same slope had been used. In this study, only the streamwise velocity component has been considered.

4.1. Chop off the air and bed from the velocity snapshots

The first step of dealing with the data set was to chop off the unwanted portions of the PIV data snapshots. These portions are the areas under the bed surface and above the water free surface.

This operation is really necessary, because for this study it is important to examine just what happen within the flow depth.

The next figures show this concept with an instantaneous image for each flow condition. On the left are reported the full velocity snapshot, on the right the versions that are chopped off to erase the bed and air. The zero datum of the bed surface is located at the average elevation of the bed surface, the zero datum of the streamwise position is located at the most upstream position of the flume measurement area.



Figure 4-1 Gravel bed, uniform flow depth = 60 mm, slope 0.002



Figure 4-2: Gravel bed, uniform flow depth = 70 mm, slope 0.002



Figure 4-3: Gravel bed, uniform flow depth = 80 mm, slope 0.002



Figure 4-4: Spheres bed, uniform flow depth = 60 mm, slope 0.003



Figure 4-5: Spheres bed, uniform flow depth = 70 mm, slope 0.003



Figure 4-6: Spheres bed, uniform flow depth = 80 mm, slope = 0.003

It is evident, thanks to the colour maps, that the flow velocity progressively increases from the bottom to the free surface. This is in accordance with the theory of hydraulic boundary layers.

Concerning the case of bed in gravel and flow depth equal to 80 mm, the PIV snapshots did not investigate the highest approximately 9 mm of the flow close to the free surface. This means that it was possible to study just 71 mm of the total depth of 80 mm.

This did not happen for the other 5 flow conditions, where the velocity snapshots cover the entire area of interest.

4.2. Data quality

Before using the data set in order to study the required turbulent characteristics, it was necessary to prove qualitatively that these data are valid. This means that they are supposed to be without any systematic error, due for example to the application of the PIV technique.

In order to do this, a few singular columns of random snapshots have been plotted. The next figures show 5 streamwise velocity profiles, represented by 5 different colours, for each chosen hydraulic conditions.



Figure 4-7: Streamwise velocity profiles over the depth, for all the chosen flow conditions

From these images, it is possible to observe the typical velocity profile of two-dimensional uniform flows. These results are actually equivalent to the ones observed thanks to the colourmap in the previous figure.

As an assessment of accuracy, all the instantaneous PIV velocity fields, for each flow condition, were used to calculate the time averaged profiles of velocity (U) and turbulence fluctuations (u'). In particular, U was calculated by first averaging all the velocity snapshots (8070 in total) and then the values along the streamwise direction, for each flow condition. By using the standard deviation instead of the streamwise velocity, the same calculation was performed to obtain the turbulence

fluctuations.

In accordance with Nezu and Nakagawa (1993), the time average velocity with depth, for 2D turbulent boundary layer, should follow the next non-dimensional relationship:

$$U^{+} = \frac{1}{\kappa} \ln\left(\frac{y}{k_s}\right) + A_r \qquad \text{for} \quad \frac{y}{h} < 0.2 \qquad (4.1)$$

where: $U^+ = U/U_*$

$$U_* = \sqrt{g R_h S_0} =$$
Shear velocity

K = 0.41 is the universal Von Karman constant, k_s is the Strickler roughness coefficient, A_r is a constant of integration, g is the acceleration due to gravity, R_h is the hydraulic radius, S_0 is the bed slope.

 k_s is here taken equal to 4.4 mm, because this value correspond to the gravel mean grain size and to the maximum height above the spheres bed mean elevation ; A_r is adjusted to provide a good adaptation between the measured and the expected profile.

Figure 4-8 show the normalised depthwise location y/k_s plotted against the normalised mean velocity U^+ , and also the expected profiles estimated from equation (4.1).

The following values of A_r have been calculated: 7.5, 8.0 and 8.5 for the cases of gravel bed and flow depth equal respectively to 60, 70 and 80 mm; 6.0, 5.5 and 5.5 for the cases of spheres bed and flow depth equal respectively to 60, 70 and 80 mm. This means that, according to van Driest (1956), only the gravel bed can be considered as completely rough.



Figure 4-8: Normalised streamwise velocity profiles, calculated from PIV measurements (blue colour) and expected from Nezu and Nakagawa formula (red colour), for all the flow conditions.

The velocity profiles take an expected form for all the analysed flow conditions, in particular for the highest depths and for the cases of spheres bed.

Nezu and Nakagawa (1993) proposed also an important formula concerning the turbulence fluctuations. They should follow the next non-dimensional relationship:

$$u_{rms+} = u'/U_* = 2.30e^{(-y/D)}$$
(4.2)

where D is the uniform flow depth, u_{rms+} is the normalised streamwise turbulence intensity, u' is the instantaneous local streamwise velocity fluctuations.

Figure 4-9 reports, as with figure 4-8, the comparison between the measured and expected results. The quantity y/D is plotted against u_{rms+} .



Figure 4-9: Normalised streamwise velocity fluctuations profiles, calculated from PIV measurements (blue) and expected from Nezu and Nakagawa formula (red), for all the flow conditions.

The profiles seem to do not take an expected form, for all the analysed flow conditions. This could be due to the analysis for high spatial resolution, causing more error and "noise" in the velocity vectors. In order to overcome this inconvenient, it would be useful to adjust the moving average step of the analysis, or to apply a median filter to remove erroneous data.

5. Method of stitching vector fields

5.1. Introduction

The idea of stitching instantaneous velocity snapshots together is to produce in the streamwise direction series of data with a spatial length long enough to identify at least three or four macro-turbulent structures, in order to study afterwards their turbulent properties. According to the literature, each coherent structure should be 3 to 5 times the flow depth. This indication will be used later in order to choose a proper number of these snapshots to stitch together.

For each flow condition analysed, the relative data set (from PIV technique) contains measurements of:

- streamwise (u),vertical (v) and lateral (w) velocity;
- horizontal (x) and vertical (y) position of the grid of points where the velocity components are measured.

The width (W) of the frame, along the streamwise direction, has been calculated as the number of measured points times the distance between each point. It has resulted in a streamwise length of 244 mm.

The sampling frequency (f) of the snapshots, used in the PIV technique, was 26.9 Hz. The total number of collected images, afterwards added in 3D matrices, was 8061, as the duration of the data set was 300 seconds. The spatial length of each image resulted equal to 242.7 mm.

The next figures show a series of these velocity snapshots. Each image follows the previous one by dt = 1/f, and all of them have been added to form a 3D matrix.



Figure 5-1: PIV velocity snapshots, added to form a 3D matrix

Starting from the first image of each time series, the mean flow velocities and the time intervals between this snapshot and the following (20) ones have been calculated. By multiplying these mean velocities by the time intervals dt, 2*dt, 3*dt, ..., 20*dt, respectively, the distances travelled between the first instantaneous and the following snapshots have been found.

By using equations notation:

- U(i) = mean velocity between image 1 and i-th
- $t(i) = time interval between image 1 and i-th = i \cdot dt$
- $ds(i) = distance travelled = U(i) \cdot T(i)$

This procedure is obviously not rigorous, because it uses the total mean velocity of the snapshots without considering its real distribution. But, because the sampling frequency is quite high, the errors will be small enough to make it sensible.

The *first method* that has been used, in order to stitch together the snapshots, considers as the next image to stitch the one closest to the position W/2 or W/4. As a result, the overlapping area between the images results equal to half or one quarter of the frame width.

The *second method* instead carries out a linear interpolation between the images that lie immediately before and after the position corresponding to half or one quarter of the frame width. Therefore, it is supposed to be more accurate than the previous one.

The following formula has been considered to find out the correlation value (C) in the overlapping area between initial and following instantaneous velocity:

$$C = \frac{Cov(A,B)}{std(A)*std(B)}$$
(5.2)

where: A = velocities in the overlapping area of the first snapshot

B = velocities in the overlapping area of the second snapshot

Cov = covariance

std = standard deviation of the overlapping area in snapshot i and i+1

The covariance between two jointly distributed real-valued random variables x and y with finite second moments is defined as the expected value of the product of their distances from the mean:

$$Cov(X,Y) = E[(X - E[X])(Y - E[Y])]$$
(5.3)

The standard deviation here used consist actually in calculating the standard deviation for each column of the matrix corresponding to the overlapping area, and then the standard deviation of the resulting row. In this way, a single number is obtained each time.

This correlation value (C) represents a measure of how much the velocity changes between the overlapping area of the snapshots. If it is high (0.7 - 0.9), the mean velocity at locations in the overlapping area is supposed to well represent the local velocities, because it means that the two images are quite similar in the overlapping area. On the contrary, whether the correlation value is low therefore the resulting stitched image might not be very rigorous in the area of overlapping.

In order to stitch the images together and create a continuous visualization over time of the streamwise velocity, a weighting of the overlapping areas has been used. In this way, the transition from one to other image has been made smoother. The weightings used are sinusoidal functions, whose equations are:

$$w_A = \frac{1}{2} a \sin(2\pi x) + 1$$

 $w_B = 1 - w_A$ (5.4)

where x = f(y) = (1/overlapping area), a = f(y), w_A and w_B are the weightings used respectively in the first and in the second snapshots.



Figure 5-2: Weightings used in the overlapping areas between following images

The weighting 1 is referred to the n-th image (on the right), the weighting 2 to the (n+1)-th one (on the left).

By using, for instance, an overlapping area equal to 50% of the snapshots, the resulting new image will be composed by three parts:

- the rightmost portion of the first image on the right
- the leftmost portion of the second image on the left
- the weighting part from both images in the middle, consisting in the following sum:

$$\mathbf{m} = (\mathbf{w}_{\mathrm{A}} \cdot \mathbf{o}_{\mathrm{A}}) + (\mathbf{w}_{\mathrm{B}} \cdot \mathbf{o}_{\mathrm{B}}) \tag{5.5}$$

where m = middle part; o_A , $o_B = overlapping areas of the first and second image, respectively.$

Figure 5-3 show this mechanism for two instantaneous, where the image to stitch to the first one results, in this case, the number 6 of this time series. The resulting vector field will be composed by images 1 and 6.



Figure 5-3: Method of stitching vector fields applied to two snapshots; 50% of overlapping area

This procedure has been repeated several times, within the same data set, obtaining a twodimensional image that contains the streamwise flow velocity of a long time series.

The Figure 5-4 is an extension of the previous one: another instantaneous, the number 11 of the time series, is added to the number 1 and 6.



Figure 5-4: Method of stitching vector fields applied to more snapshots

These particular sinusoidal functions (equations 5.4) has been chosen because it allows a smooth stitching between snapshots, where for the previous one has more weight the portion close to the bed, for the next one the portion close to the free surface. This idea come from the streamwise velocity profile, where definitely it increases from the bottom (very low velocities) to the free surface (almost the maximum velocities).

Figure 5-5 show this scheme of stitching snapshots applied to a real data set, composed by 3 images.



Figure 5-5: Three snapshots stitched together by using the method previously described
5.2. Choice of the overlapping area between snapshots

In order to find out an overlapping area between the instantaneous that might lead to good correlation values, it was decided to prove the technique with 50% overlap at first and afterwards to use 25% of the frame width. This comparison has been done by considering the stitching of just two instantaneous images, where the first one vary along all the time series. This means that more than 8000 correlation values has been calculated, for both methods, concerning couples of stitched images.

The next image report for simplicity just the difference, ΔC , of the values obtained by using these two different overlapping areas between the instantaneous images. More specifically:

$$\Delta C = C_{25\%} - C_{50\%} \tag{5.6}$$

All the flow conditions have been considered.





Figure 5-6: Difference in correlation value (ΔC) related to the overlapping area between couples of stitched snapshots; these overlapping area are 50% and 25% of the frame width. All the hydraulic conditions are here reported.

The correlation value between following snapshots does not appear significantly higher when an overlapping area of 25% is used instead of 50%, because the differences in the correlation value are on average very close to zero. As a consequence, in order to reduce the number of necessary snapshots, this last value (50%) will have used for all the following calculations.

5.3. Stitching of following snapshots with and without interpolation

In order to have a model that could better represent, if possible, what really happened in the flows here analysed, it was proposed to operate a linear interpolation of the snapshots close to the position corresponded to 50% of the frame width.

The previous method, instead, used as the image to stitch the one closest to 50% of the frame width, thus it was a simpler procedure.

The next figure show this concepts with an example, in which the results obtained by both methods are pointed out. For this example, the instantaneous images to stitch with the first one results the number 10 whether no interpolation is applied. Otherwise, with the second procedure, a linear interpolation between the velocity snapshots 10 and 11 is required.



Figure 5-7: Distance of following snapshots to the 50% of the frame width against the number of these snapshots; both the results obtained, with and without interpolation of the instantaneous, are pointed out.

Again, the comparison between these two methods has been done by using the correlation value in the overlapping area between the velocity snapshots. In particular, this time:

$$\Delta C' = C_{interp} - C_{no \ interp} \tag{5.7}$$

Instead of the entire data sets, for brevity only 100 couples of stitched images are reported for every flow condition.





Figure 5-5-8: Difference in correlation value ($\Delta C'$) related to the overlapping area between couples of stitched snapshots, with and without interpolation of the images.

These difference of correlation value in the overlapping area between couples of snapshots does not appear significant when a liner interpolation between them is applied. Actually, it is sometimes negative. As a consequence, it was decided to use, as the next image to stitch, just the nearest one to a distance of 50% of the frame width.

5.4. Choice of the time intervals

As a brief summary, the final decision concerning the method of stitching instantaneous velocity snapshots consisted of:

- Using an overlapping area between the snapshots as equal as possible to 50 % of the frame width, by choosing as the follow image to stitch the one closest to this particular position (half of the frame width), without any interpolation of the images;
- Using the weightings functions described by equations (5.4) and (5.5) in the overlapping area between the velocity instantaneous, in order to stitch them together.

Six time intervals have been chosen from the data set. They were selected looking at the flow condition having on average the lowest values of correlation in the overlapping area between

couples of following images (8070 - 1 couples, in total). This case correspond to the first flow condition: gravel bed, uniform depth of 60 mm, slope 0.002.

At this point, the six time intervals have been identified looking at the times having on average the highest correlation values for ranges of at least 100 consecutive velocity snapshots.

The next figure shows, by using dashed lines, the location of these intervals along the entire time series. They have been found in the beginning and in the end of this time series.



Figure 5-5-9: Correlation value related to the overlapping area between couples of stitched snapshots; case of gravel bed, uniform flow depth = 60 mm and slope = 0.002

As it is possible to observe, these correlation values are approximately 0.68, actually not particularly high.

The next three figures represent the velocity field of these chosen time intervals, six for each flow condition, for the cases of gravel bed. The scales of representation are distorted: along the horizontal axis, the distances are approximately 10% than along the vertical axis.



Figure 5-10: Gravel bed, uniform flow depth = 60 mm, slope = 0.002; time intervals



Figure 5-11: Gravel bed, uniform flow depth = 70 mm, slope = 0.002; time intervals



Figure 5-12: Gravel bed, uniform flow depth = 80 mm, slope = 0.002; time intervals

The next table describe more in details the six detected time intervals for the first analysed flow condition (see Table 4-1).

Interval N.	First velocity snapshot N.	Last velocity snapshot N.	Duration (s)
1	320	425	3.90
2	550	660	4.09
3	940	1050	4.09
4	6940	7042	3.79
5	7730	7838	4.01
6	7900	8006	3.94

Concerning the remaining flow conditions (Table 4-1 and 4-2), it was decided to keep the start of each time interval and the same total spatial length (approximately 1585 mm) as the ones reported in the Table 5-1. This means that the end of the time intervals will be different from each other, according to the mean flow velocity.





Figure 5-13: Method of stitching velocity instantaneous; scheme

5.4.1. Comparisons between the chosen time intervals and flow conditions

In order to check whether all these time intervals are representative of their own flow condition, it was decided to plot their mean velocity over depth. The next figure shows this kind of comparison, where each graph consists in one particular hydraulic condition.



Figure 5-14: Mean velocities over depth of the chosen time intervals, for each flow condition; comparisons

For each flow condition, the mean velocity of the six time intervals follow a similar pattern. As a consequence, they can be taken to be representative for the entire time series.

It is evident, in particular for the lower flow depths, that the maximum of the velocity lie a few millimetres under the free surface. This result is in accordance with the theory of turbulent free surface flows.

By considering the velocity mean profile over the depth of all the time intervals, for each flow condition, the Reynolds number have been calculated. The figure below shows the Reynolds Number profiles over the depth, by considering separately the cases of gravel and spheres bed.



Figure 5-15: Reynolds number mean profiles over depth; the cases of gravel bed are reported on the left, the cases of spheres bed on the right.

According to Figure 5-15, by varying the flow depth there is almost the same relationship for the gravel and spheres beds. The Reynolds Number appear slightly higher for the cases of gravel bed, thanks to a stronger increase within the first millimetres near the bottom.

5.5. Correlation value over depth

In order to prove that the method of stitching vector field previously explained would be sufficiently accurate, it was decided to report how the correlation value (concerning the overlapping area between the snapshots) over depth changes according to the number of rows considered in the calculation of the covariance.

The equation (5.2) provided a single number for the entire section, by considering all the rows together to calculate the covariance and the standard deviation. Now, starting by calculating these quantities in every single row, more and more rows will be considered together, in order to identify how many of them would be necessary to have good correlations values over the depth. As a consequence, these values will be obtained for shorter depth as a higher number of rows will be considered.

The next equation explains this mechanism:

$$C(n) = \frac{Cov(A,n;B,n)}{std(A,n)*std(B,n)}$$
(5.8)

A,B = velocities in the overlapping area respectively of the first and second snapshot, by considering *n* rows over the total depth;

Cov = mean value of covariance of the overlapping area in snapshot i and i+1, by considering *n* rows over the total depth;

std = mean value of standard deviation of the overlapping area in snapshot i and i+1, by considering *n* rows over the total depth;

n = number of rows considered, that vary from one to the total number of rows in the velocity fields.

The next figure show the results concerning a few stitched snapshots. It is representative for all the hydraulic conditions here studied.



Figure 5-16: Correlation values along the depth in the overlapping area between velocity snapshots : starting by calculating the correlation value for each row, more and more rows are progressively considered, up to the entire flow depth

It is clearly visible that the correlation value increases as a higher number of rows in the covariance calculation is considered. The general pattern reveals a shape with the highest values in the middle, and by considering 15 rows these values results very close to the one concerns the entire flow depth and obtained by applying equation (5.2).

This means that the mean flow velocity between following instantaneous is able to represent the local velocities in a quite efficient way.

6. A few decomposition methods for use on the data set

6.1. Proper Orthogonal Decomposition (POD)

6.1.1. Introduction

The proper orthogonal decomposition (POD) is a method for data analysis aimed at obtaining lowdimensional approximate descriptions of high-dimensional processes. POD provides a basis for the modal decomposition of an ensemble of functions, such as data obtained in the course of experiments or numerical simulations. The basic functions it yields are commonly called empirical eigenfunctions, empirical basis functions, empirical orthogonal functions, proper orthogonal modes, or basis vectors. The most striking feature of the POD is its *optimality*: it provides the most efficient way of capturing the dominant components of an infinite-dimensional process with only a finite number of "modes", and often surprisingly few "modes".

There are three POD methods: *Karhunen-Loève decomposition* (KLD), *principal component analysis* (PCA), and *singular value decomposition* (SVD).

Regarding turbulent flow structures, POD is also relevant for studying inhomogeneous turbulent flow fields in which the dominant modes are believed to represent *coherent structures*.

6.1.2. Methodology

The main idea of POD is to find a set of ordered orthonormal basis vectors in a subspace (\mathbb{R}^m) where a random vector takes its values, such that the samples in the sample space can be expressed optimally using the selected first *l* basis vectors. The mean square error can be used as a measure for the optimal problem:

$$\mathsf{E}\{\|\mathbf{x} - \mathbf{x}(l)\|^2\} \le \mathsf{E}\{\|\mathbf{x} - \hat{\mathbf{x}}(l)\|^2\}$$
(6.1)

where x(l) is the approximate expression of a random vector x using the first l basis vectors of the undetermined set of orthonormal basis vectors, and $\hat{x}(l)$ is the approximate expression of x using arbitrary *l* basis vectors in \mathbb{R}^m .

By assuming $x \square \mathbb{R}^m$ as a random vector and $\{\phi_i\}_{i=1}^m$ as a set of arbitrary orthonormal basis vectors, than *x* can be expressed as:

$$x = \sum_{i=1}^{m} y_i \phi_i = \phi_y \tag{6.2}$$

where:

$$y_i = \phi_i^T x \ (I = 1, 2, ..., m),$$

 $y = (y_1, y_2, ..., y_m)^T,$
 $\Phi = [\phi_1, \phi_2, ..., \phi_m)]$

The objective of the POD is to find a set of basis vectors that satisfied the following extreme value problem:

$$min_{\phi_i}\varepsilon^2(l) = \mathsf{E}\{\|\mathbf{x} - \mathbf{x}(l)\|^2\}$$

such that $\phi_i^T \phi_j = \delta_{ij}$ $l, j = 1, 2, ..., m,$ (6.3)

where $\varepsilon^2(l)$ is the mean square error, $\mathbf{x}(l) = \sum_{i=1}^l y_i \phi_i (l \le m)$.

6.1.3. The use of POD in turbulent flows

POD provides a mathematical definition of energy-relevant structures and a method for their extraction from stochastic statistically steady turbulent velocity fields. The structures do not need to correspond to coherent structures, but to events that contribute most, in a statistical sense, to the energy of the turbulent. POD attempts to minimize the square of the quantity being analysed. Therefore, in the particular case of the velocity field, the quantity to be minimized is the *kinetic energy*. Following Sirovich (1987), the POD procedure within the context of experimental fluid mechanics can be summarized as follows. Assume that an ensemble of N random, non-homogeneous velocity or vorticity fields is represented by

$$V = V(x, t_z), \qquad z = 1, 2, ..., N,$$
 (6.4)

where t_z is the time index and x is the vector position, herein for a 2D field. Also, assume that

$$\{\varphi_n(\mathbf{x})\}, \qquad n = 1, 2, \dots, N,$$
 (6.5)

is a collection of N vector functions used to describe the random field. These functions are taken to be orthonormal:

$$(\varphi_n, \varphi_m)_{\mathbf{x}} = \int_{\Omega} \varphi_n(\mathbf{x}) \varphi_m(\mathbf{x}) \, d\mathbf{x} = \delta_{nm} \tag{6.6}$$

where δ_{nm} is the Kronecker delta and Ω is the spatial domain. The POD technique finds the collection of eigenfunctions $\{\varphi_n(x)\}$ combined with time functions $\alpha_n(t)$ that best fits the ensemble V(x, t) in a least square sense. Thus, the problem is to find, for any fixed integer N, the minimum of

$$\langle \| V(\mathbf{x},t) - \sum_{n=1}^{N} \alpha_n(t) \varphi_n(\mathbf{x}) \|_{\mathbf{x}}^2 \rangle_t$$
(6.7)

where the summation term is the POD of V and $\langle . \rangle_t$ represents time-averaging. To solve the problem, therefore to determine the best collection of $\{\varphi_n(\mathbf{x})\}$ and $\alpha_n(t)$, the POD method solves the *Eigenvalue problem*

$$\int_{\Omega} R\varphi_n \, dx = \lambda_n \varphi_n \tag{6.8}$$

where λ_n are the associated Eigenvalues and R is the two-point spatial correlation tensor, defined as

$$\mathbf{R}(\boldsymbol{x}, \boldsymbol{x}') = \langle \boldsymbol{V}(\boldsymbol{x}, t) \boldsymbol{V}(\boldsymbol{x}', t) \rangle$$
(6.9)

It can be shown that the best collection $\{\varphi_n(\mathbf{x})\}\$, than the POD modes, can be determined from the Eigenfunctions of $\mathbf{R}(\mathbf{x}, \mathbf{x}')$ after ordering the Eigenvalues as

$$\lambda_n \ge \lambda_n \ge \lambda_n \ge \dots \tag{6.10}$$

The *POD coefficients* $\alpha_n(t)$ are determined by projecting the velocity ensemble onto the calculated *POD modes* $\varphi_n(\mathbf{x})$,

$$\alpha_n(t_z) = \int_{\Omega} \varphi_n(x) V(x, t_z) \, dx \,, \tag{6.11}$$

and are proportional to a set of orthonormal functions in time and space a_n ,

$$\alpha_n = \lambda_n a_n \tag{6.12}$$

It can be further shown that the energy contribution of the *n*th POD-mode is provided by λ_n , and that the relative contribution E_n of each mode follows from

$$E_n = \frac{\lambda_n}{\sum_{n=1}^N \lambda_n}$$
(6.13)

If a low Reynolds number flow is analysed or if the flow is highly coherent, it is possible to represent it using only a few modes. For large Reynolds numbers, however, the energy can be distributed into a large number of modes. As discussed by Kostas et al. (2005), POD has gained large use for constructing low-dimensional models that describe the main flow characteristics using, if possible, a low number of modes.

The interpretation of the POD results is a combination of the information obtained by the POD modes and that provided by the POD coefficients. A common situation arises if a strong connection between two or more POD modes exists. A usual situation occurs if a coherent structure is convected at roughly at a constant speed. In that case, the POD modes corresponding to the structure are paired with roughly equal Eigenvalue. It was recently observed that such a connection between POD modes can indicate that part of the structure described by one mode contains part of the structure described by the other mode, which had decayed in energy and scale.

In addition to the POD of the velocity field, it was also observed that an unambiguous description of coherent structures using the POD technique is obtained when the vorticity field is analysed. Similar to the velocity field analysis, the POD modes of vorticity correspond to vortical structures that contribute most to the flow entropy.

The snapshot method of Sirovich (1987) is used to reduce the number of calculations required during the POD procedure. This is useful if the amount of spatial information is much higher than the available number of snapshots. In that case, the Eigenvalue problem is written in terms of the temporal correlation tensor $C = \langle \mathbf{V}(\mathbf{x},t) \mathbf{V}(\mathbf{x},t') \rangle$ as

$$C \, \Phi_n = \lambda_n \Phi_n \tag{6.14}$$

where Φ_n is the *n*th Eigenvector corresponding to the *n*th Eigenvalue λ_n . The Eigenfunctions φ_n are determined by

$$\varphi_n(\mathbf{x}) = \sum_{z=1}^N \Phi_n(t_z) V(\mathbf{x}, t_z)$$
(6.15)

Once the Eigenfunctions are obtained, the POD coefficients are determined using Eq. (6.11).

6.2. Dynamic Mode Decomposition (DMD)

6.2.1. Introduction

DMD is a decomposition method based on snapshots of the flow only but that yields fluid structures that accurately describe the motion of the flow. It is equally applicable to experimental and numerical flow field data. It is also able to deal with sample flow visualization and with time-resolved PIV measurements.

It is important to realize that Dynamic Mode Decomposition does not contain any averaging process, neither in time nor in space. As a consequence, the temporal and spatial information is fully preserved.

DMD algorithm relies on the reconstruction of a low-dimensional inter-snapshot map from the available flow field data. The spectral decomposition of this map results in a eigenvalue and eigenvector representation of the underlying fluid behaviour contained in the processed flow fields. This dynamic mode decomposition allows the breakdown of a fluid process into dynamically relevant and coherent structures and thus aids in the characterization and quantification of physical mechanisms in fluid flow.

6.2.2. Methodology

Preprocessing of the experimental data may be necessary in order to eliminate inherent measurement noise. The data shall be represented in the form of a snapshot sequence, given by a matrix V_1^N ,

$$\boldsymbol{V}_{1}^{N} = \{ \boldsymbol{v}_{1}, \, \boldsymbol{v}_{2}, \, \boldsymbol{v}_{3}, \dots, \, \boldsymbol{v}_{N} \}$$
(6.16)

where v_i stands for the *i*th flow field. In the above definition, the subscript 1 denotes the first member of the sequence, while the superscript N denotes the last entry in the sequence, i.e. the first and last columns of the matrix V_1^N , respectively. It is further assumed an ordered sequence of data separated by a constant sampling time Δt .

In the first step, it is assumed that a linear mapping *A* connects the flow field v_i to the subsequent flow field v_{i+1} , that is,

$$v_{i+1} = A v_i,$$
 (6.17)

and that this mapping is approximately the same over the full sampling interval $[0,(N-1) \Delta t]$. If the flow fields stem from a nonlinear process, this assumption amounts to a linear tangent approximation. For slowly varying systems, a multiple- scale argument can provide a foundation for the above assumption. In the special case of a purely linear process, no approximation is invoked by assuming a constant mapping. In any case, the assumption of a constant mapping between the snapshots v_i will allow to formulate the sequence of flow fields as a *Krylov sequence*:

$$\boldsymbol{V}_{1}^{N} = \{ \boldsymbol{v}_{1}, \boldsymbol{A}\boldsymbol{v}_{1}, \boldsymbol{A}^{2}\boldsymbol{v}_{1}, \dots, \boldsymbol{A}^{N-1}\boldsymbol{v}_{1} \}$$
(6.18)

The purpose then is the extraction of the dynamic characteristics (eigenvalues, eigenvectors, pseudoeigenvalues, energy amplification, resonance behaviour, etc.) of the dynamical process described by A based on the sequence V_1^N .

As the number of snapshots increases and the data sequence given by V_1^N captures the dominant features of the underlying physical process, it is reasonable to assume that, beyond a critical number of snapshots, the vectors given by (6.17) become linearly dependent. In other words, adding further flow fields v_1 to the data sequence will not improve the vector space spanned by V_1^N . When this limit is reached, it is possible to express the vector v_N as a linear combination of the previous, and linearly independent, vectors v_1 , i=1,...,N-1 according to

$$\mathbf{v}_{\rm N} = \mathbf{a}_1 \mathbf{v}_1 + \mathbf{a}_2 \mathbf{v}_2 + \ldots + \mathbf{a}_{{\rm N}-1} \mathbf{v}_{{\rm N}-1} + \mathbf{r} \tag{6.19}$$

or in matrix form:

$$v_{\rm N} = V_1^{N-1} a + r \tag{6.20}$$

with $\boldsymbol{a}^{\mathrm{T}} = \{a_1, a_2, \dots, a_{\mathrm{N-1}}\}$ and \boldsymbol{r} as a residual vector. According to Ruhe (1984):

$$A\{v_1, v_2, v_2, \dots, v_{N-1}\} = \{v_2, v_3, v_4, \dots, v_N\} = \{v_2, v_3, v_4, \dots, V_1^{N-1}a\} + r e_{N-1}^{T}$$
(6.21)

or in matrix form:

$$AV_1^{N-1} = V_2^N = V_1^{N-1}S + re_{N-1}^{\rm T}$$
(6.22)

with $\boldsymbol{e}_{N-1} \square \mathbb{R}^{N-1}$ as the (N - 1)th unit vector.

A simple calculation shows that the matrix S is of companion type with:

$$\mathbf{S} = \begin{pmatrix} 0 & & & a_1 \\ 1 & 0 & & a_2 \\ & \ddots & \ddots & & \vdots \\ & & 1 & 0 & a_{N-2} \\ & & & & 1 & a_{N-1} \end{pmatrix}$$
(6.23)

whose subdiagonal entries reflect the fact that, by design, the vector in the *i*th column of V_2^N is identical to the vector in the (i + 1)th column of V_1^{N-1} for i =1,...,N-2. The only unknowns in *S* are the coefficients { $a_1, a_2, ..., a_{N-1}$ } which constitute the above-mentioned (N -1)-component linear representation of the last sample v_N in terms of the previous samples { $v_2, v_3, v_4, ..., v_N$ }.

The eigenvalues of *S* then approximate some of the eigenvalues of A. The well-known *Arnoldi method* is closely related to the decomposition above but successively orthogonalizes the vectors of V_1^N resulting in a decomposition of the form $AQ \approx QH$ with $V_1^{N-1} = QR$ and $H = RSR^{-1}$ as a *Hessenberg matrix*. Again, the eigenvalues of H approximate some of the eigenvalues of A. In practice, the reduction of A to *Hessenberg* form by the *Arnoldi method* is not accomplished by a simple QR-decomposition of V_1^{N-1} , but rather by a sequence of projections onto successive Krylov subspaces. This yields a more stable algorithm, but for these projections the matrix A has to be available which makes the classical Arnoldi method unattractive. Rather, the idea is to have less favourable stability (and convergence) properties of the algorithm in order to gain a numerical technique that is exclusively based on flow fields and is thus equally applicable to experimental data and large-scale numerical simulations.

The computation of *S* then proceeds as follows: the last element of a given data sequence v_N is expressed as a linear combination of the previous elements of the sequence as stated in (6.20) whose least-squares solution, for a full-rank matrix V_1^{N-1} , is given by

$$\boldsymbol{a} = \boldsymbol{R}^{-1} \boldsymbol{Q}^H \boldsymbol{v}_N \tag{6.24}$$

with $QR = V_1^{N-1}$ as the economy-size QR-decomposition of the data sequence V_1^{N-1} . The (N – 1)component vector *a* then forms the last column of the companion matrix *S*.

Even though the above decomposition based on a companion matrix S is mathematically correct and is often used to prove convergence properties of the *full Arnoldi method*, a practical implementation yields an ill-conditioned algorithm that is often not capable of extracting more than the first or first two dominant dynamic modes. This is particularly true when the data stem from an experiment and are contaminated with noise and other uncertainties. For this reason, it is a good idea to choose a more robust implementation that results in a 'full' matrix \tilde{S} – related to S via a similarity transformation. Robustness is achieved by a preprocessing step using a singular value decomposition of the data sequence $V_1^{N-1} = U\Sigma W^H$. Substituting the singular value decomposition $U\Sigma W^H$ into (6.22) and rearranging the resulting expression, the result is $U^H A U = U^H V_2^N W \Sigma^{-1} \equiv \tilde{S}$. By recognizing that the matrix U contains the *proper orthogonal* modes of the data sequence V_1^{N-1} , the above operation amounts to a projection of the linear operator A onto a *POD* basis. A further advantage of this operation, besides a more robust calculation of the low-dimensional representation to a limited projection basis U given by the non-zero singular values of Σ (or by singular values above a prescribed threshold).

The *modal structures* are extracted from the matrix \tilde{S} in a manner analogous to recovering the global modes from the eigenvectors of the *Hessenberg matrix* H of the standard *Arnoldi method*. The dynamic modes Φ_i present the following expression:

$$\boldsymbol{\Phi}_i = \boldsymbol{U} \boldsymbol{y}_i \tag{6.25}$$

with yi as the ith eigenvector of \tilde{S} , i.e. $\tilde{S}y_i = \mu_i y_i$, and U as the right singular vectors of the snapshot sequence V_1^{N-1} .

The above decomposition method, whether in its mathematical form based on a companion matrix or in its implementation based on a full matrix, is able to extract coherent structure from a sequence of data fields only.

6.3. Singular Value Decomposition (SVD)

6.3.1. Introduction

Singular value decomposition (SVD) is a method for:

- transforming correlated variables into a set of uncorrelated ones that better expose the various relationships among the original data items;
- data reduction, because identifying and ordering the dimensions along which data points exhibit the most variation;

• data reduction, because once it has identified where the most variation is, it is possible to find the best approximation of the original data points using fewer dimensions.

The basic ideas behind SVD is to reduce a high dimensional and variable set of data points to a lower dimensional space that exposes the substructure of the original data more clearly and orders it from most variation to the least. It is possible simply to ignore variation below a particular threshold to massively reduce the data and be assured that the main relationships of interest have been preserved.

6.3.2. Methodology

SVD is based on a theorem from linear algebra which says that a rectangular matrix **A** can be broken down into the product of three matrices:

- an orthogonal matrix \mathbf{U} ($\mathbf{U}^{\mathrm{T}}\mathbf{U} = \mathbf{I}$)
- a diagonal matrix **S**
- the transpose of an orthogonal matrix $\mathbf{V} (\mathbf{V}^{\mathrm{T}} \mathbf{V} = \mathbf{I})$,

obtaining the following expression:

$$\mathbf{A}_{\mathrm{mn}} = \mathbf{U}_{\mathrm{mm}} \mathbf{S}_{\mathrm{mn}} \boldsymbol{V}_{nn}^{T} \tag{6.26}$$

The columns of U are orthonormal eigenvectors of AA^{T} , the columns of V are orthonormal eigenvectors of $A^{T}A$, and S is a diagonal matrix containing the square roots of eigenvalues from U or V in descending order (they are called the *singular values* of A).

6.4. Singular Spectrum Analysis (SSA)

6.4.1. Introduction

Singular spectrum analysis (SSA) is a technique of time series analysis and forecasting. It combines elements of classical time series analysis, multivariate statistics, multivariate geometry, dynamical systems and signal processing.

The aim of SSA is to make a decomposition of the original series into the sum of a small number of independent and interpretable components such as a slowly varying trend, oscillatory components and a structureless noise.

It is based on the singular-value decomposition of a specific matrix constructed upon time series.

The birth of SSA is usually associated in 1986 with the publication of papers by Broomhead and King, while the ideas of SSA were independently developed in Russia (St. Petersburg, Moscow) and in several groups in the UK and USA.

SSA has proved to be very successful, and has already become a standard tool in the analysis of climatic, meteorological and geophysical time series. More recent areas of application of SSA include engineering, medicine, econometrics and many other fields.

The most important aspect of SSA are that it is a *nonparametric technique* that works with arbitrary statistical processes, whether linear or nonlinear, stationary or non-stationary, Gaussian or non-Gaussian. Moreover, contrary to the traditional methods of time series forecasting (both

autoregressive or structural models that assume normality and stationarity of the series), SSA method is non-parametric and makes no prior assumptions about the data.

6.4.2. Methodology

Consider the real-valued nonzero time series $Y_T = (y_1,...,y_T)$ of length T. The main purpose of SSA is to decompose the original series into a sum of series, so that each component in this sum can be identified as either a trend, periodic or quasi-periodic component (perhaps, amplitude-modulated), or noise. This is followed by a reconstruction the original series. The SSA technique consist of two complementary stages: *decomposition* and *reconstruction*, both of which include two separate steps. At the first stage the series is decomposed and at the second stage the original series is reconstructed and used (without noise) for forecasting new data points.

6.4.3. Stage 1: Decomposition

First step: Embeddind

Embedding can be regarded as a mapping that transfers a one-dimensional time series $Y = (y_1,...,y_T)$ into the multi-dimensional series $X_1,...,X_K$ with vectors $X_i = (x_i,...,x_{i+L-1})^T \in \mathbb{R}^L$, where K = T - L + 1. Vectors X_i are called *L-lagged vectors* (or, simply, *lagged vectors*). The single parameter of the embedding is the *window length L*, an integer such that $2 \le L \le T$. The result of this step is the *trajectory matrix* $\mathbf{X} = [X_1,...,X_K] = (x_{ij})_{i,j=1}^{L,K}$.

By using the extended form:
$$\mathbf{X} = \begin{pmatrix} x_1 & x_2 & x_3 & \dots & x_K \\ x_2 & x_3 & x_4 & \dots & x_{K+1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_L & x_{L+1} & x_{L+2} & \dots & x_T \end{pmatrix}$$
.

The trajectory matrix **X** is a *Hankel matrix*, which means that all the elements along the diagonal i+j = const are equal. Embedding is a standard procedure in time series analysis. With the embedding performed, future analysis depends on the aim of the investigation.

Second step: Singular Value Decomposition

The second step makes the singular value decomposition of the trajectory matrix **X** and represents it as a sum of rank-one bi-orthogonal elementary matrices. Denote by $\lambda_1, ..., \lambda_L$ the eigenvalues of $\mathbf{X}\mathbf{X}^T$ in decreasing order of magnitude ($\lambda_1 \ge ... \lambda_L \ge 0$) and by $U_1, ..., U_L$ the orthonormal system (that is, $(U_i, U_j) = 0$ for $i \ne j$ (the orthogonality property) and $||U_i|| = 1$ (the unit norm property)) of the eigenvectors of the matrix $\mathbf{X}\mathbf{X}^T$ corresponding to these eigenvalues. (U_i, U_j) is the inner product of the vectors U_i and U_j and $||U_i||$ is the norm of the vector U_i . Set:

$$d = \max(i, \text{ such that } \lambda_i > 0) = \operatorname{rank} \mathbf{X}.$$

If it is denoted Vi = \mathbf{X}^{T} Ui/ $\sqrt{\lambda_{i}}$, then the SVD of the trajectory matrix can be written as:

$$\mathbf{X} = \mathbf{X}_1 + \dots + \mathbf{X}_d, \tag{6.27}$$

where $\mathbf{X}_i = \sqrt{\lambda_i} U_i V_i$ (i =1,...,d). The matrices \mathbf{X}_i have rank 1; therefore they are elementary matrices, U_i (in SSA literature they are called *'factor empirical orthogonal functions'* or simply EOFs) and V_i (often called *'principal components'*) stand for the left and right eigenvectors of the trajectory matrix. The collection ($\sqrt{\lambda_i}$, U_i , V_i) is called the *i-th eigentriple* of the matrix \mathbf{X} , $\sqrt{\lambda_i}$ (i = 1,...,d) are the *singular values* of the matrix \mathbf{X} and the set { $\sqrt{\lambda_i}$ } is called the *spectrum* of the matrix \mathbf{X} . If all the eigenvalues have multiplicity one, then the expansion (6.27) is uniquely defined.

SVD (6.27) is *optimal* in the sense that among all the matrices $\mathbf{X}^{(r)}$ of rank r < d, the matrix $\sum_{i=1}^{r} \mathbf{X}_i$ provides the best approximation to the trajectory matrix \mathbf{X} , so that $\|\mathbf{X} - \mathbf{X}(r)\|$ is minimum. Note that $\|\mathbf{X}\|^2 = \sum_{i=1}^{d} \lambda_i$ and $\|\mathbf{X}_i\|^2 = \lambda_i$ for i = 1,...,d. Thus, it is possible to consider the ratio $\lambda_i / \sum_{i=1}^{d} \lambda_i$ as the characteristic of the contribution of the matrix \mathbf{X}_i to expansion (6.27).

Consequently, $\sum_{i=1}^{r} \lambda_i / \sum_{i=1}^{d} \lambda_i$, the sum of the first r ratios, is the characteristic of the optimal approximation of the trajectory matrix by the matrices of rank r.

6.4.4. Stage 2: Reconstruction

First step: Grouping

The grouping step corresponds to splitting the elementary matrices \mathbf{X}_i into several groups and summing the matrices within each group. Let $I = \{i_1, ..., i_p\}$ be a group of indices $i_1, ..., i_p$. Then the matrix \mathbf{X}_I corresponding to the group I is defined as $\mathbf{X}_I = \mathbf{X}_{I_1} + \cdots + \mathbf{X}_{I_m}$. The split of the set of indices J = 1, ..., d into the disjoint subsets $I_1, ..., I_m$ corresponds to the representation

$$\mathbf{X} = \mathbf{X}_{\mathrm{I}_{1}} + \dots + \mathbf{X}_{\mathrm{I}_{\mathrm{m}}} \tag{6.28}$$

The procedure of choosing the sets $I_1,...,I_m$ is called the *eigentriple grouping*.

For given group I the contribution of the component X_i into the expansion (6.28) is measured by the share of the corresponding eigenvalues: $\sum_{i \in I} \lambda_i / \sum_{i=1}^d \lambda_i$.

Second step: Diagonal Averaging

Diagonal averaging transfers each matrix I into a time series, which is an additive component of the initial series Y_T . If z_{ij} stands for an element of a matrix **Z**, then the k-th term of the resulting series is obtained by averaging z_{ij} over all i,j such that i + j = k + 2. This procedure is called *diagonal averaging*, or *Hankelization* of the matrix **Z**. The result of the Hankelization of a matrix **Z** is the Hankel matrix *H***Z**, which is the trajectory matrix corresponding to the series obtained as a result of the diagonal averaging. The Hankelization is an optimal procedure, in the sense that the matrix *H***Z** is the nearest to **Z** (with respect to the matrix norm) among all Hankel matrices of the corresponding size. In its turn, the Hankel matrix *H***Z** uniquely defines the series by relating the value in the diagonals to the values in the series. By applying the Hankelization procedure to all matrix components of (6.28), another expansion is obtained:

$$\mathbf{X} = \widetilde{\mathbf{X}}_{\mathbf{I}_1} + \dots + \ \widetilde{\mathbf{X}}_{\mathbf{I}_m} \tag{6.29}$$

where $\widetilde{\mathbf{X}}_{I_1} = H\mathbf{Z}$. This is equivalent to the decomposition of the initial series $Y_T = (y_1, ..., y_T)$ into a sum of *m* series:

$$y_t = \sum_{\substack{k=1\\55}}^{m} \tilde{y}_t^{(k)}$$
(6.30)

where $\widetilde{Y}_{T}^{(k)} = (\widetilde{y}_{1}^{(k)}, ..., \widetilde{y}_{T}^{(k)})$ corresponds to the matrix $\mathbf{X}_{I_{k}}$.

6.5. Comparisons and final decision

It was eventually decided to apply the Singular Spectrum Analysis decomposition, that makes use of the Singular Value Decomposition as well, instead of the Proper Orthogonal Decomposition and the Dynamic Mode Decomposition techniques.

The main reason is that SSA method is *modal free* and makes no prior assumptions about the data. This does not happen, for instance, for the POD method. It requires a data pre-processing in order to "prepare" the data set to be applied to the real decomposition.

Moreover, SSA procedure appears a bit less complicated, even if it uses several linear algebra tools.

7. Application of SSA method to laboratory data set

In order to identify the more and the less energetic turbulent structures, it was eventually decided to apply the Singular Spectrum Analysis (SSA) technique to the laboratory data.

For each flow condition previously described, the mean flow velocity for each row was subtracted, in order to study just the fluctuations of the velocity data field. The signal was than decomposed into 500 SSA modes (the eigenvectors of the original matrix), and three groups of modes ware chosen in order to identify, respectively, the main coherent structures, the small structures and noise. The 500 resulting eigenvalues represent the kinetic energy of the corresponding eigenvectors. The velocity fields ware reconstructed by using different combinations of modes (Roussinova et al. 2010): the first modes corresponding to about 50% of the turbulent kinetic energy to expose the large (energetic) structures; a second group of modes, recovering about 33% of the energy, to expose the small (less energetic) structures.

The following images show a few results, obtained by applying the SSA decomposition to the time intervals for the chosen flow condition (Tables 4-1 and 4-2).

7.1. Gravel Bed Surface, Uniform Water Depth 60 millimetres

7.1.1. SSA modes (eigenvalues) over the depth

In order to visualize the distribution of the kinetic energy over the depth, the figure below has been reported. It shows, for each time interval, the mean of the 500 SSA eigenvalues over the depth.



Figure 7-1: Energy distribution with depth, for each analysed time interval; bed gravel, uniform flow depth = 60 mm, slope = 0.002; the values on the horizontal axis are not representative, they just compare the time intervals

The kinetic energy of fluctuation data is almost constant in the central part of the flow depth, and increases towards the bed and the surface in the last millimetres. The pattern of the energy is similar

for all the six time intervals in this experiment, but within the first 10 - 15 mm near the bottom, corresponding to the maximum of the energy.

7.1.2. Spectrum of the SSA modes

The next logarithmic graphs reports the depth-mean SSA eigenvalue of the velocity fluctuations matrix, for each analysed time interval.



Figure 7-2: fractional kinetic energy; gravel bed, flow depth = 60 mm, slope = 0.002



Figure 7-3: cumulative kinetic energy; gravel bed, flow depth = 60 mm, slope = 0.002

It is evident that the contribution of each mode is getting smaller going from the first modes to the last ones. All the intervals seem to have approximately the same behaviour, both concerning fractional and cumulative kinetic energy. However, the energy contained in the very first component vary from 5 to 8 %, and this is quite a big difference.

As it is clear from the Table 7-1, approximately the first 50 % of the energy is contained within the first 37 - 47 modes, for this flow condition. The reconstructed signal based on these modes represent the *coherent* (large) *structures*.

The following ~140 modes contain 33 % of the energy and expose the *turbulent small structures*. The remaining modes are identified as a noise. Their contribution is almost irrelevant and as a consequence they are not considered in the reconstructed signal.

	Coherent (large) structures		Small structures		Noise	
intervals	modes	energy[%]	modes	energy[%]	modes	energy[%]
1	1_41	49,96	42_176	33,09	177_500	16,95
2	1_47	50,08	48_185	32,93	186_500	16,99
3	1_46	50,10	47_180	32,87	181_500	17,03
4	1_42	49,95	43_175	33,09	176_500	16,96
5	1_45	49,95	46_183	32,99	184_500	17,06
6	1_37	49,87	38_170	33,19	171_500	16,94

Table 7-1: SSA groupings,; gravel bed, uniform flow depth = 60 mm, slope = 0.002

7.1.3. Reconstructed signals by summing the SSA eigenvectors

For each of the six chosen time intervals (paragraph 5.4), the next images show: velocity fluctuations, the reconstructed signal using the first group of modes (large structures) and then using the second group of modes (small structures).

Even if the second group of modes represent about 33 % of the total kinetic energy, the velocities appear very lower than for the first group of modes, which contains the most significant part of the original signal.

• First time interval



Figure 7-4: Velocity fluctuations, and reconstructed signals using the main energetic modes and the less energetic modes after SSA decomposition; gravel bed, uniform flow depth = 60 mm, first time interval: frame n° 320 to 425



• Second time interval

Figure 7-5: Velocity fluctuations, reconstructed signal using the main energetic modes and the less energetic modes after SSA decomposition; gravel bed, uniform flow depth = 60 mm, second time interval: : frame n° 550 to 660

• Third time interval



Figure 7-6: Velocity fluctuations, reconstructed signal using the main energetic modes and the less energetic modes after SSA decomposition; gravel bed, uniform flow depth = 60 mm, third time interval: : frame n° 940 to 1050



• Fourth time interval

Figure 7-7: Velocity fluctuations, reconstructed signal using the main energetic modes and the less energetic modes after SSA decomposition; gravel bed, uniform flow depth = 60 mm, fourth time interval: : frame n° 6940 to 7042

• Fifth time interval



Figure 7-8: Velocity fluctuations, reconstructed signal using the main energetic modes and the less energetic modes after SSA decomposition; gravel bed, uniform flow depth = 60 mm, fifth time interval: : frame n^o 7730 to 7838



• Sixth time interval

Figure 7-9: Velocity fluctuations, reconstructed signal using the main energetic modes and the less energetic modes after SSA decomposition; gravel bed, uniform flow depth = 60 mm, sixth time interval: : frame n^o 7900 to 8006

For each time interval, it is possible to identify the existence of alternating high speed and lowspeed regions, even if they are not always very clear. The velocity fluctuations exposed by the second group of energetic modes appear generally close to zero, but in very small regions along the entire flow field.

7.2. Comparison between all the different flow conditions

The purpose of this chapter is to find out how the flow kinetic energy could changes by having a different type of bed (gravel and spheres) but the same Reynolds number and flow depth, and a different flow depth but the same material for the bed and same Reynolds number.

The next images show a comparison between the different flow conditions, instead of between the time intervals as previously proposed.

The eigenvectors are used to reconstruct the signal. The eigenvalues represent, at each spatial location over the depth, the energetic contribution of the corresponding eigenvector. The sum of all the energetic contributions correspond to the unity.

7.2.1. SSA modes (eigenvalues) over the depth

The next figure refers (as Figure 7-1) to the mean value of all the eigenvalues (modes) at each depth, considering again the velocity fluctuations of the vector fields. The horizontal axis represent the kinetic energy, but the values are not representative: they just compare the patterns.

Only the first three time intervals are reported, because they are representative also for the other ones.



Figure 7-10: Energy distribution with depth, for all the flow conditions and for the first three time intervals; the values on the horizontal axis (energy) are not representative, they just compare the different flow conditions.

The interpretation of these results is not very clear, because they seem to quite depend on the specific time interval. However, the energy slightly increases toward the depth, and the maximum values lie near the free surface and especially near the bottom. For the cases of gravel bed, the peach of energy close to the bottom appear more important than for the cases of spheres bed. On average, the energy appear to increases as the flow depth increases.

The Figure 8-11 shows the depth-mean values of these patterns, in order to have a clear comparison between the different flow conditions.



Figure 7-11:Depth-mean values of the kinetic energy, compared to the highest K.E. detected; along the vertical axis are reported the different flow conditions, along the horizontal axes the K.E. mean values over the depth for each time interval

The kinetic energy increases as the flow depth increases, for both the cases of gravel and spheres bed, according to the classic hydraulic theory (Bernoulli). The energy results slightly higher (approximately 10 %) when gravel is used as the bed material (by considering that, as previously reported, the flow in the third flow condition is only 71 mm depth, instead of 80 mm), compared to the cases of spheres bed.

7.2.2. Spectrum of the eigenvalues



Figure 7-12: Spectrum of the SSA components, for all the flow conditions and for the first three time intervals.

The kinetic energy contained in the first component can varies from 4 to more than 20 %, according to the specific time interval and flow condition. It progressively decreases in the following modes, with a similar gradient. Between the modes 20 and 100, the energy follows a power law having the same exponent of -0.8 ± 0.03 , for all the flow conditions.

By considering now the cumulative kinetic energy, the figure below report an average between the time intervals for each flow condition.



Figure 7-13: Cumulative kinetic energy of the SSA components, average value for each flow condition

It is quite clear that the percentage of energy contained in the first few modes is getting higher as the flow depth increases, and it appears slightly higher for the cases of spheres bed than for the cases of gravel bed.

This fact affected the number of SSA components used to reconstruct the signals. As reported in the Table 7-2: the mean number of modes used to expose the large structures is higher for the lower depths, and slightly higher for the cases of spheres bed.

In general, the depth seems to effect the results much greater than the type of bed.

Table 7-2: Mean value, for each flow condition, of the number of SSA modes used to expose the large and the small structures

Flow condition	Coherent structures		Small structures		Noise	
	modes	energy[%]	modes	energy[%]	modes	energy[%]
Gravel bed, D = 60 mm	43	49.99	178	33.03	279	16.99
Gravel bed, D = 70 mm	27	50.15	155	32.87	318	16.99
Gravel bed, D = 80 mm	22	50.03	148	32.99	330	16.98
Spheres bed, D = 60 mm	46	49.98	191	33.05	263	16.97
Spheres bed, D = 70 mm	35	49.91	174	33.12	291	16.97
Spheres bed, D = 80 mm	17	49.92	143	33.08	340	17.00

8. Identification of the turbulent flow structures via U-level

8.1. Introduction

The goal of this chapter is to examine the properties of the turbulent structures such as spatial frequency, temporal frequency and mean characteristic length. For this reason, it was decided to apply the *U-level* technique for objectively identifying these turbulent structures within the time series at each spatial location. This means that this method is objective in terms of defining what constitutes the beginning and end of a turbulent event.

The U-level technique identifies turbulent events as having velocity temporal fluctuation components greater in magnitude than a given threshold, which is usually some fraction of the standard deviation of the time series. Luchik et al. (1987) presented a modification whereby the end of an event was defined by a different threshold to the start of an event. In the modified U-level scheme, the *start of an event* is detected from the streamwise velocity fluctuations, u', or vertical velocity fluctuations, v', when:

$$|u'| > k_U S_u$$
, or $|v'| > k_U S_v$ (8-1)

and the *end of an event* is defined by:

$$|u'| > p_U k_U S_u$$
, or $|v'| > p_U k_U S_v$, (8-2)

 k_U is a threshold value, S_u and S_v are the standard deviations of the streamwise and vertical velocity time series respectively, and p_U is a probability between 0 and 1. A probability of $p_U = 0.25$ is the most commonly used (Luchik & Tiederman, 1987; Shah & Antonia, 1989; Krogstad, et al., 1992), and $k_u = 1.3$ has been shown to give sensible results for data collected in laboratory flumes (Bogard & Tiederman, 1986). It was decided to use these values proposed by the literature. A U-level algorithm was written in MatLab to transform any continuous time series vector into a discrete binary form, whereby a value of unity indicates the presence of a turbulent event and a

value of zero indicates no event. This was applied to the velocity fluctuations, at each spatial location over the full time series, before and after applying the SSA decomposition (Chapter 7).

An example is given in Figure 8.1, where the absolute values in a time series are shown. The red and green lines show the thresholds for detecting respectively the start and end of a turbulent event, and the black line shows the resulting binary series. The binary series of the fluctuating terms u' and v' are defined as:

$$u'_U = U(u')$$
, and $v'_U = U(v')$, (8-3)

respectively, where U represents the U-level analysis function. The number of turbulent events (n_E) detected in the Figure 8-1 results equal to 10.



Figure 8-1: U-level turbulent event detection. Event start (green line) is detected when the time series exceeds 1.3 standard deviations, and event end (red line) is when the value drops below 25% of the start threshold. Binary detected event data (black line) have been scaled down to improve clarity

In the next section, the results obtained from the application of the U-level technique to the matrices of velocity fluctuations, before and after applying the SSA decomposition, are shown. From the binary matrices that identify the coherent structures, the following properties have been calculated:

- spatial frequency [1/m]
- temporal frequency [1/s]
- mean length [mm]

Afterwards, a comparison between different time intervals concerning the same flow condition, and between different flow conditions for the same time interval has been conducted, concerning these characteristics of the turbulent structures.

8.2. Gravel Bed Surface, Uniform Water Depth 60 millimetres

8.2.1. Visualization of the turbulent structures

By using a threshold value of 1.3, the U-level technique was applied to all the time intervals, before and after applying SSA decomposition as reported in Chapter 7.

The next figures show the velocity fields and the turbulent structures detected for each time interval (in black colour). The first two images of each figure are referred to the velocity fluctuations; the second two to the first group of SSA modes (Chapter 7) that is supposed to represent the large turbulent structures; finally, the last two images are referred to the second group of SSA components that represent the small turbulent structures.

• First time interval



Figure 8-2: Identification of the flow structures by using U-level technique; gravel bed, uniform flow depth = 60 mm, slope 0.002, first time interval: frame n° 320 to 425.
• Second time interval



Figure 8-3: Identification of the flow structures by using U-level technique; gravel bed, uniform flow depth = 60 mm, slope 0.002, second time interval: frame n° 550 to 660.

• Third time interval



Figure 8-4: Identification of the flow structures by using U-level technique; gravel bed, uniform flow depth = 60 mm, slope 0.002, third time interval: frame n° 940 to 1050.

• Fourth time interval



Figure 8-5: Identification of the flow structures by using U-level technique; gravel bed, uniform flow depth = 60 mm, slope 0.002, fourth time interval: frame n^o 6940 to 7042.

• Fifth time interval



Figure 8-6: Identification of the flow structures by using U-level technique; gravel bed, uniform flow depth = 60 mm, slope 0.002, fifth time interval: frame n^o 7730 to 7838.

• Sixth time interval



Figure 8-7: Identification of the flow structures by using U-level technique; gravel bed, uniform flow depth = 60 mm, slope 0.002, sixth time interval: frame n^o 7900 to 8006

The turbulent events identified by U-level technique usually appear to be small and scattered. This happens in particular for the velocity fluctuations and the turbulent small structures, where a lot of very small events appear on the flow visualization.

However, this technique seems to provide quite good results. The raw fluctuations show lots of structures ranging from small to large, and the SSA decomposition helps to separate them, even if not just by size but by energy content.

In fact, the most energetic structures tend to be bigger, but also several "small" structures are detected because strongly energetic.

The second group of modes (that is supposed to expose the small structures), on the contrary, shows smaller and more frequent structures than the first group of modes.

Concerning the large structures, they are more clearly visible as a smaller number of SSA modes is used to reconstruct the signal (for instance, 37 instead of 47, as reported in Table 7-1). For the flow condition of gravel bed and uniform flow depth equal to 60 mm, the number of the modes used is on average quite high (43, from Table 7-2). As a consequence, a lot of small turbulent event are detected in the identification of the coherent structures. This fact might also be caused by the stitching procedure (Chapter 5).

Looking at the Table 7-2, this situation might appear better for the other flow conditions, that will be reported later in this chapter.

8.2.2. Structures spatial frequency

From the binary matrices obtained by applying the U-level technique, a count of the turbulent events (n_E), has been done for each row. The spatial frequency (fs) was then obtained by dividing the number of events (n_E) by the length (L_t) of the reconstructed field of view (approximately 1.58 metres, as reported in paragraph 4.3):

fs (row) =
$$n_E$$
 (row) / L_t [1/m] (8.4)

The analysed time intervals seem to have a similar behaviour compared to each other, for the fluctuations, large structures and small structures.



Figure 8-8: Structures spatial frequency, for fluctuations, large sructures and small structures; gravel bed, uniform flow depth = 60 mm, slope 0.002; the depth-mean values of these quantities are reported in the lower right of the figure

The spatial frequency increases from the bottom to the free surface, concerning the velocity fluctuations. Starting from values of 30 - 40 structures per metres, it arrives up to 55 - 60 structures per metre. The increase seems to occur in the bottom 10 - 20 mm of the flow, and there appears to be a reduction close to the water surface. The mean value over the depth is about 53 structures per metre for the raw fluctuations.

It is possible to see 25 - 25 big structures per metre, but without a specific pattern: the values appear reasonably constant with depth.

Concerning the small structures, these values are approximately of 60 - 70 per metre in the main part of the flow and near the free surface, while they are about 50 per metre near the bottom.

8.2.3. Structures temporal frequency

The temporal frequency (f_t) was obtained by multiplying the spatial frequency (f_s) by the mean flow velocity for each row of the relative time interval (U_t) :

$$f_t (row) = f_s (row) \cdot U_t (row) \qquad [1/s] \qquad (8.5)$$



Figure 8-9: Structures temporal frequency, for fluctuations, large sructures and small structures; gravel bed, uniform flow depth = 60 mm, slope 0.002; the depth-mean values of these quantities are reported in the lower right of the figure

The temporal frequency increases slightly toward the free surface, for velocity fluctuations, large and small structures, especially in the first 10 mm of the flow. The mean values over the depth are respectively 18, 8 and 23 structures per second, approximately. The difference between the time intervals do not seem very important.

8.2.4. Structures mean length

Finally, the third detected property was the mean length of the turbulent structures (L_s), calculated by dividing the sum of the length of all identified structures by the number of events (n_E), for each row. The spatial resolution (dx) of the measurements resulted about 1.356 mm, along the streamwise direction. The symbol $\Sigma(1)$ refers to the sum of the binary 'ones'.

$$L_{s} (row) = \sum(1) \cdot dx / n_{E} (row) \qquad [mm] \qquad (8.6)$$

The mean length of the turbulent events generally decreases away from the bed. The mean values are approximately 5 millimetres for small structures and fluctuations.



Figure 8-10: Structures mean length, for fluctuations, large sructures and small structures; gravel bed, uniform flow depth = 60 mm, slope 0.002; the depth-mean values of these quantities are reported in the lower right of the figure

Concerning the large structures, these results of 13 - 15 mm as their mean length are not in accordance with the literature (between 2 to 12 depths, 4-5 depth on average, according to Shvidchenko et al., 2001; between 3 to 5 depth, according to Roy et al., 2004), even if appear some spikes of 25 - 28 mm. This happen, as previously introduced, because a large number of very small structures are identified, as shown in the figures 8-2/8-7.

8.3. Some other results

In this section a few results about the remaining flow conditions are reported. The purpose is to show potential difference concerning the visual identification of the turbulent structures when different numbers of SSA modes (see Table 8-1) are used to reconstruct the signal, for large and small structures.

The next figures show the turbulent structures detected in the first time interval for the remaining five analysed flow conditions (see Tables 4-1 and 4-2).



Figure 8-11: Identification of the flow structures by using U-level technique; gravel bed, uniform flow depth = 70 mm, slope 0.002, first time interval, frame no. 320 to 409;



Figure 8-12: Identification of the flow structures by using U-level technique; gravel bed, uniform flow depth = 80 mm, slope 0.002, first time interval, frame no. 320 to 414



Figure 8-13: Identification of the flow structures by using U-level technique; spheres bed, uniform flow depth = 60 mm, slope 0.003, first time interval, frame no. 320 to 419



Figure 8-14: Identification of the flow structures by using U-level technique; spheres bed, uniform flow depth = 70 mm, slope 0.003, first time interval, frame no. 320 to 409



Figure 8-15: Identification of the flow structures by using U-level technique; spheres bed, uniform flow depth = 80 mm, slope 0.003, first time interval, frame no. 320 to 403

		Coherent structures		Small structures		Noise	
N.	Flow condition	modes	energy[%]	modes	energy[%]	modes	energy[%]
1	Gravel bed, D = 60 mm	1_41	49,96	42_176	33,09	177_500	16,95
2	Gravel bed, D = 70 mm	1_19	50,20	20_146	32,82	147_500	16,98
3	Gravel bed, D = 80 mm	1_16	50,29	17_134	32,69	135_500	17,02
4	Spheres bed, D = 60 mm	1_57	49,83	58_202	33,18	203_500	16,99
5	Spheres bed, D = 70 mm	1_41	50,10	42_184	32,94	185_500	16,96
6	Spheres bed, D = 80 mm	1_9	49,94	10_125	33,02	126_500	17,04

Table 8-1: SSA groupings; first time interval of all the chosen hydraulic conditions

From these last five figures (8-11 / 8-15) and according to the Table 8-1, it appears evident that, as hypothesised in the paragraph 8.2.1, looking at a smaller portion of the dominant energy gives a clearer picture of the bigger structures.

These large structures appear quite vary in size and position.

The small structures, on the contrary, seem to have almost the same size and distribution along the streamwise and vertical directions, for all the cases here analysed.

8.4. Comparison between the different flow conditions

The next figures show depth-mean values of the turbulent structures properties previously showed (spatial frequency, temporal frequency and mean length), by applying again the U-level technique.

Raw fluctuations, the main energetic modes (large structures) and the less energetic modes (small structures) are considered separately. In this way, a comparison between the different flow conditions has been possible, in order to identify potentially different behaviours of these coherent structures.

8.4.1. Spatial frequency of the turbulent structures

According to the Figure 8-16, the spatial frequency of the turbulent events detected from velocity fluctuations increases as the flow depth decreases, and this happen for the large and small structures as well, looking at the Figures 8-17 and 8-18. The spheres bed generally has a slightly higher frequency than the gravel bed.

These considerations referred to depth-mean values, looking at all the six time intervals.



Figure 8-16: Spatial frequency of the turbulent structures detected from velocity fluctuations, by applying U-level technique; depth-mean value for all the six flow conditions and time intervals



Figure 8-17: Spatial frequency of the large turbulent structures, detected by applying U-level technique; depth-mean value for all the six flow conditions and time intervals



Figure 8-18: Spatial frequency of the turbulent small structures, detected by applying U-level technique; depth-mean value for all the six flow conditions and time intervals

The number of large structures can vary from 5 - 10 to 25 - 30 per metre, from 50 - 60 to about 70 per metre concerning the small structures and with a bit lower values looking at the turbulent events detected from the fluctuations, according to the particular flow condition.

The variability between time intervals seems to increase as the flow depth is increased.

8.4.2. Temporal frequency of the turbulent structures



Figure 8-19: Temporal frequency of the turbulent structures detected from velocity fluctuations, by applying U-level technique; depth-mean value for all the six flow conditions and time intervals



Figure 8-20: Temporal frequency of the turbulent large structures, detected by applying U-level technique; depth-mean value for all the six flow conditions and time intervals



Figure 8-21: Temporal frequency of the turbulent small structures, detected by applying U-level technique; depth-mean value for all the six flow conditions and time intervals

According to the Figure 8-20, the temporal frequency of the large structures decreases as the flow depth increases, and the values are slightly higher for the cases of spheres bed. These results are in accordance with the results achieved by Nichols (2013).

The number of large structures can vary from 1 - 5 to 8 - 12 per second.

No pattern trends are evident looking at the figures 8-19 and 8-21, concerning fluctuations and small turbulent structures, whereas their spatial frequency decreases as the flow depth increases, as previously reported. This fact is a consequence that, as reported in the Figure 5-13, the mean velocity increases as the flow depth increases.

8.4.3. Mean length of the turbulent structures

According to the next three figures, the mean length of the raw fluctuations, large and small structures increases as the flow depth increases. This result is in accordance with the results reported by Nichols (2013).

These values are slightly higher for the cases of gravel bed.



Figure 8-22: Mean length of the turbulent structures detected from velocity fluctuations, by applying U-level technique; depth-mean value for all the six flow conditions and time intervals



Figure 8-23: Mean length of the turbulent large structures, detected by applying U-level technique; depth-mean value for all the six flow conditions and time intervals

The more interesting results concern the flow structures mean length regards the coherent (large) structures.

According to the literature, this quantity should be between 2 to 12 depths, 4-5 depth on average, according to Shvidchenko et al. (2001); between 3 to 5 depth, according to Roy et al. (2004).

From the Figure 8-23, it appears evident that the results are quite different compared to the specific time interval, especially for the higher flow depths.

The mean length of the coherent structures, looking at the Figure 8-23, can vary from 10 - 15 to 30

- -100 millimetres, according to the particular flow condition. This means that they are around 0.50
- -1.5 flow depths, because the U-level method detect several small energetic structures.



Figure 8-24: Mean length of the turbulent small structures, detected by applying U-level technique; depth-mean value for all the six flow conditions and time intervals

The figure 8-25 show the depth-mean length of the large structures dimensionless with the flow depths, for every considered hydraulic condition.



Figure 8-25: Non-dimensional depth-mean length of the large structures, detected by applying U-level technique

The non-dimensional mean length of the coherent structures, according to the results given by the U-level method, increases as the flow depth increases. Also the variability of the results seems to follow this same trend, suggesting that deeper, faster flows may require longer time series (or wider reconstructed field of view) in order to obtain a good representative average.

8.5. Considerations

If a threshold higher than 1.3 had been used, a smaller number of turbulent events would have been detected. This would mean to have a lower number of "small" structures, but at the same time a reduced size for the "large" structures. The same effect had been produced by using less SSA modes to describe the large structures.

The opposite situation would happen by using a threshold lower than 1.3., and more SSA modes to expose the large structures than the ones that was used in this chapter.

As reported in the beginning of the present and of the previous chapters, all the decisions concerning the SSA groupings and the U-level threshold values came from previous study and experiments. For this reason, it was decided to do not vary these parameters, but just to check their validity on the data set used in the present thesis.

According to the findings reported in this chapter, U-level technique seems to show quite reliable results, even if it is not as rigorous a method.

9. Identification of the turbulent flow structures via Phase-Space

9.1. Introduction

The goal of this chapter is again to examine the properties of the turbulent structures such as spatial frequency, temporal frequency and mean characteristic length, by applying a different technique than the one reported in the previous chapter.

Phase-Space Thresholding method, as with U-level, objectively identified these turbulent structures within the time series at each spatial location.

The concept springs up from the problem to detect erroneous spikes from Acoustic Doppler Velocimetry (ADV) data sets, caused for instance by the noise floor and aliasing of the Doppler signal. Hence the idea of seeing how velocity and its derivatives appear in phase space had started to emerge.

This method uses the concept of a phase space plot in which the variable and its first and second derivatives are plotted against each other.



Figure 9-1: Phase Space showing cloud of data from PIV measurements.

In this work the derivative is taken with respect to the streamwise distance rather than over time, since each vector field represents a wide snapshot at one instant in time.

U represents the instantaneous velocity at point, $\frac{dU}{dx}$ is the first order derivative and $\frac{d^2U}{dx^2}$ the second order derivative. In particular, U corresponds to the streamwise velocity.

It is apparent, from Figure 9-1, that most of the data cluster in an ellipsoid cloud. The size of this ellipsoid is defined by a certain threshold, and the points outside it are designated as spikes.

9.1.1. Procedure

For the present thesis, it was decided to add a modification compared to what reported was by Nikora (2005): instead of considering a single ellipsoid, two ellipsoids will be used in order to identify turbulent events from PIV instantaneous:

- the points outside the external one will be considered as spikes and removed (as Nikora suggest)
- the points within the internal ellipsoid will be considered as "not extreme events" and removed;
- finally, the points contained between the two ellipsoids will be marked as "extreme events", and as a consequence they will represent the turbulent events.

Theorically this method should provide a more sensitive detection of turbulent events, as it can recognise extreme accelerations, jerks and shears, where U-level only considers velocity.

The first step consist of reducing the values on the z axis, $\frac{dU^2}{dx^2}$, by the best fit of a plane through it, such that the mean plane lies on the plane on the x-y axes.



Figure 9-2: Phase Space, plane fit removed from the data points. For this particular example, there are no evident difference in the cloud shape compared to the previous step (see Figure 9-1).

As the next step, the values of acceleration, $\frac{dU}{dx}$, are reduced by the best fit of a line through the data, such that the line of best fit now lies on the x axis. This allows the two threshold ellipsoids to be constructed with the same mean position.



Figure 9-3: Phase Space, semi-ellipsoid; plane fit and line fit removed

The length, width, and height of the ellipsoids are based respectively on the standard deviation of U, $\frac{dU}{dx}$ and $\frac{dU^2}{dx^2}$ achieved in this last step. In particular, these quantities are multiplied by certain values of threshold:

$$a = \text{length} = \text{threshold} \cdot \text{std}(U)$$

$$b = \text{width} = \text{threshold} \cdot \text{std}\left(\frac{dU}{dx}\right)$$

$$c = \text{height} = \text{threshold} \cdot \text{std}\left(\frac{d^2U}{dx^2}\right)$$
(9-1)

The threshold that has been used to define the external ellipsoid was suggested by Nikora (2005), with a value equal to 1,5.

Concerning the internal ellipsoid, the value of threshold was adjusted in order to obtain approximately the same results of temporal frequency of the turbulent fluctuations achieved by applying the U-level technique. This value has been obtained to be 0.2.

9.2. Gravel Bed Surface, Uniform Water Depth 60 millimetres

9.2.1. Visualization of the turbulent structures

By using a threshold value for the internal ellipsoid of 0.2 and for the external one equal to 1.5, the modified Phase-Space scheme was applied to all the time intervals, before and after applying the SSA decomposition as reported in the Chapter 7.

The next figures show the velocity fields and the turbulent structures detected from the chosen time intervals (Chapter 5). The first two pictures of each figure are referred to the raw fluctuations; the

second two to the first group of SSA modes (Chapter 7) that is supposed to represent the large turbulent structures; finally, the last two pictures are referred to the second group of SSA components that represent the small turbulent structures.



• First time interval

Figure 9-4: Identification of the flow structures by using Phase Space technique; gravel bed, uniform flow depth = 60 mm, slope 0.002, first time interval: frame no. 320 to 425.

• Second time interval



Figure 9-5: Identification of the flow structures by using Phase Space technique; gravel bed, uniform flow depth = 60 mm, slope 0.002, second time interval: frame no. 550 to 660.

• Third time interval



Figure 9-6: Identification of the flow structures by using Phase Space technique; gravel bed, uniform flow depth = 60 mm, slope 0.002, third time interval: frame no. 940 to 1050.

• Fourth time interval



Figure 9-7: Identification of the flow structures by using Phase Space technique; gravel bed, uniform flow depth = 60 mm, slope 0.002, fourth time interval: frame no. 6940 to 7042.

• Fifth time interval



Figure 9-8: Identification of the flow structures by using Phase Space technique; gravel bed, uniform flow depth = 60 mm, slope 0.002, fifth time interval: frame no. 7730 to 7838.

• Sixth time interval



Figure 9-9: Identification of the flow structures by using Phase Space technique; gravel bed, uniform flow depth = 60 mm, slope 0.002, sixth time interval: frame no. 7900 to 8006.

The turbulent events identified by Phase-Space technique usually appear very confused, concerning velocity fluctuations, large and small structures. Actually, just looking at the figures 9-4 / 9-9, this method does not seem to provide good results.

The turbulent events appear very small even after the SSA decomposition. Moreover, a lot of them are removed from the positions corresponding to the higher and lower streamwise velocities regions, where the coherent structures are more probably supposed to be located.

The number of SSA modes used to reconstruct the large structures for the case reported in this paragraph (gravel bed, uniform flow depth equal to 60 mm) is on average quite high (43, from the table 8-2). It will be interesting to examine some other time intervals, concerning different flow conditions, where this number would be appreciably lower, in order to observe if some difference might occur. This work will be done in the paragraph (9.3.).

9.2.2. Structures spatial frequency

By applying equation number (8.4) to the binary matrices obtained from Phase Space technique, the following results have been achieved.



Figure 9-10: Structures spatial frequency, for fluctuations, large sructures and small structures; gravel bed, uniform flow depth = 60 mm, slope 0.002; the depth-mean values of these quantities are reported in the lower right of the figure

All the six time intervals seem to have a similar behaviour compared to each other, for the fluctuations, large structures and small structures.

The spatial frequency for fluctuations and small structures is almost constant for mostly of the depth. It present a local minimum point corresponding to about 10 % of the depth, and important reductions occur close both to the bed and the water surface.

The pattern concerning the large structures is roughly constant, with mean values between 34 - 38 structures per metre.

According to the Figure 9-10, there are approximately 44 - 47 turbulent fluctuations and 31 - 35 small structures per metre.

Unfortunately, these results do not appear to have any obvious sense, and looking again at the Figures 9-4 / 9-9, the modified Phase-Space technique does not seem to work (or at least with the present data set).

However, the following paragraph show other achieved results, and the final decision concerning the efficiency of this method is postponed to the end of this chapter.

9.2.3. Structures temporal frequency

The temporal frequency was obtained multiplying the spatial frequency (f_s) by the mean flow velocity for each row of the relative time interval (U_t) , as reported in the equation (8.2).



Figure 9-11: Structures temporal frequency, for fluctuations, large sructures and small structures; gravel bed, uniform flow depth = 60 mm, slope 0.002; the depth-mean values of these quantities are reported in the lower right of the figure

The temporal frequency increases toward the free surface, especially within the first 10 millimetres, and quickly decreases in the last few millimetres.

From this last figure, there are per metre about 15 - 18 turbulent fluctuations, 12 - 14 large structures and 12 - 13 small structures. The higher variability between the time intervals seems to concern the raw fluctuations.

9.2.4. Structures mean length

The mean length of the turbulent structures was calculated by using equation (8.3).



Figure 9-12: Structures mean length, for fluctuations, large sructures and small structures; gravel bed, uniform flow depth = 60 mm, slope 0.002; the depth-mean values of these quantities are reported in the lower right of the figure

The mean length of the turbulent events appears reasonably constant with depth, but for few isolated spikes as the one concerning the large structures detected 10 mm under the free surface in the first time interval.

The mean values are approximately 4 - 4.5 millimetres for large and small structures, 3 millimetres concerning the velocity fluctuations.

As previously reported, these results do not appear very reliable, and neither in accordance with the literature (Shvidchenko et al., 2001; Roy et al., 2004).

9.3. Some other results

In the present paragraph are reported a few results about the remaining flow conditions. The purpose is to show potential difference concerning the visual identification of the turbulent structures when different numbers of SSA modes (see Table are used to reconstruct the signal, for large and big structures.

The next figures show the turbulent structures detected in the first time interval for the remaining five analysed flow conditions (see Tables 4-1 and 4-2).



Figure 9-13: Identification of the flow structures by using Phase Space technique; gravel bed, uniform flow depth = 70 mm, slope 0.002, first time interval, frame n° 320 to 409



Figure 9-14: Identification of the flow structures by using Phase Space technique; gravel bed, uniform flow depth = 80 mm, slope 0.002, first time interval, frame n^o 320 to 414



Figure 9-15: Identification of the flow structures by using Phase Space technique; spheres bed, uniform flow depth = 60 mm, slope 0.003, first time interval, frame n^o 320 to 419



Figure 9-16: Identification of the flow structures by using Phase Space technique; spheres bed, uniform flow depth = 70 mm, slope 0.003, first time interval, frame n° 320 to 409



Figure 9-17: Identification of the flow structures by using Phase Space technique; spheres bed, uniform flow depth = 80 mm, slope 0.003, first time interval, frame n^o 320 to 403

From these last five figures (9-13 / 9-17) and according to the Table 8-1, it is evident that, in spite of different numbers of SSA modes are used to expose the turbulent structures in the several time intervals, the results still appear do not make any sense.

9.4. Comparison between the different flow conditions

The next figures show depth-mean values of the turbulent structures properties previously showed (spatial frequency, temporal frequency and mean length), by applying the modified Phase-Space technique. The raw fluctuations, the main energetic modes (large structures) and the less energetic modes (small structures) are considered separately. In this way, a comparison between the different flow condition has been possible, in order to identify potentially different behaviours of these coherent structures.

9.4.1. Spatial frequency of the turbulent structures



Figure 9-18: Spatial frequency of the turbulent structures detected from velocity fluctuations, by applying Phase Space technique; depth-mean value for all the six flow conditions and time intervals

The spatial frequency concerning the velocity fluctuations results between 45 and 48 structures per metre, for all the six chosen flow conditions.



Figure 9-19: Spatial frequency of the large turbulent structures, detected by applying Phase Space technique; depth-mean value for all the six flow conditions and time intervals

According to the Figure 9-19, the spatial frequency of the large structures decreases as the flow depth increases, in a similar way for both gravel and spheres bed. The mean values are about 35 structures per metre for the lower depths, up to 25 for the higher ones, with a higher variability of the results.



Figure 9-20: Spatial frequency of the turbulent small structures, detected by applying Phase Space technique; depth-mean value for all the six flow conditions and time intervals

Finally, the spatial frequency for the small structures seems to slightly increase as the flow depth increases. The values are a bit higher for the cases of gravel bed.

According to the Figure 9-20, the number of small structures can vary from 33 - 35 per metre up to 37 - 40 per metre.
9.4.2. Temporal frequency of the turbulent structures



Figure 9-21: Temporal frequency of the turbulent structures detected from velocity fluctuations, by applying Phase Space technique; depth-mean value for all the six flow conditions and time intervals



Figure 9-22: Temporal frequency of the turbulent large structures, detected by applying Phase Space technique; depth-mean value for all the six flow conditions and time intervals



Figure 9-23: Temporal frequency of the turbulent small structures, detected by applying Phase Space technique; depth-mean value for all the six flow conditions and time intervals

The temporal frequency concerning both velocity fluctuations and small structures increases as the flow depth increases. These mean values vary from 15 to 22 turbulent fluctuations per seconds, and from 12 to 18 small structures per second.

Instead, there are on average 12 large structures per second for all the hydraulic conditions, but the results present more variability for the highest flow depth.

9.4.3. Mean length of the turbulent structures



Figure 9-24: Mean length of the turbulent structures detected from velocity fluctuations, by applying Phase Space technique; depth-mean value for all the six flow conditions and time intervals

The mean length of the raw fluctuations increases as the flow depth increases, with higher values for the cases of gravel bed. However, from the Figure 9-24, these lengths are approximately 3 mm for every flow condition.



Figure 9-25: Mean length of the turbulent large structures, detected by applying Phase Space technique; mean values over the depth of all the six time intervals

As already introduced, according to the literature (Shvidchenko et al., 2001; Roy et al.; 2004), the mean length of the coherent structures should be approximately 3 to 5 times the flow depth.

Looking at the Figure 9-25, it appears evident that these calculated values are completely wrong, because it results a mean length of about 10% of the flow depth, for each case. The only positive thing, is that the mean length increase as the flow depth increase, but all the other information are probably unreliable.



Figure 9-26: Mean length of the turbulent small structures, detected by applying Phase Space technique; mean values over the depth of all the six time intervals

According to the Figure 9-26, the mean length of the small structures results about 4 mm for all the hydraulic conditions.

10. Comparison of the results achieved from U-level and Phase – Space techniques

A comparison between the results obtained by applying U-level and Phase Space techniques to the same data set is now presented.

The aim of this chapter is to show how the identification and the properties of the turbulent structures might depend on the specific method applied, or if some similarities might be observed. This comparison will be done by matching vector fields relative to the same time interval and flow condition, that obviously represent the same physical quantity (velocity fluctuations, large structures or small structures). Moreover, in a similar way will be compared the turbulent properties (characteristic mean length, spatial and temporal frequency) of these structures, calculated by using both the methods.

10.1. Visualization of the turbulent structures

Some of the vector fields (binary matrices) previously used in the chapters 8 and 9 to show a comparison between different flow conditions and time intervals, obtained respectively by applying U-level and Phase Space techniques, are now used to compare these two methods.

In particular, the binary matrices corresponding to the detected turbulent events, concerning the first time interval are now reported for the following hydraulic conditions:

- gravel bed, uniform flow depth equal to 60 mm
- gravel bed, uniform flow depth equal to 80 mm
- spheres bed, uniform flow depth equal to 80 mm

The reason is that respectively a high (41), a medium (16) and a low (9) number of SSA components were used to reconstruct the signal concerning the large structures. In this way, only these three examples should be representative for a valid visual comparison.



Figure 10-1: First time interval: instantaneous no. 320 to 425, gravel bed, uniform flow depth = 60 mm, slope = 0.002; turbulent fluctuations detected by applying U-level and Phase-Space techniques



Figure 10-2: First time interval: instantaneous no. 320 to 425, gravel bed, uniform flow depth = 60 mm, slope = 0.002; turbulent large structures detected by applying U-level and Phase-Space techniques



Figure 10-3: First time interval: instantaneous no. 320 to 425, gravel bed, uniform flow depth = 60 mm, slope = 0.002; turbulent small structures detected by applying U-level and Phase-Space techniques



Figure 10-4: First time interval: instantaneous no. 320 to 414, gravel bed, uniform flow depth = 80 mm, slope = 0.002; turbulent fluctuations detected by applying U-level and Phase-Space techniques



Figure 10-5: First time interval: instantaneous no. 320 to 414, gravel bed, uniform flow depth = 80 mm, slope = 0.002; turbulent large structures detected by applying U-level and Phase-Space techniques



Figure 10-6: First time interval: instantaneous no. 320 to 414, gravel bed, uniform flow depth = 80 mm, slope = 0.002; turbulent small structures detected by applying U-level and Phase-Space techniques



Figure 10-7: First time interval: instantaneous no. 320 to 403, spheres bed, uniform flow depth = 80 mm, slope = 0.003; turbulent fluctuations detected by applying U-level and Phase-Space techniques



Figure 10-8: First time interval: instantaneous no. 320 to 403, spheres bed, uniform flow depth = 80 mm, slope = 0.003; turbulent large structures detected by applying U-level and Phase-Space techniques



Figure 10-9: First time interval: instantaneous no. 320 to 403, spheres bed, uniform flow depth = 80 mm, slope = 0.003; turbulent small structures detected by applying U-level and Phase-Space techniques

From these last figures, it is quite evident that the two methods do not lead to similar results, concerning all the three types of detected turbulent events. This fact seems to happen for every flow condition.

Looking in particular at the figures 10-7 and 10-8, the modified Phase-Space scheme seems to provide opposite results than the U-level technique. This might be a further prove than the Phase-Space method here used is not actually able to detect turbulent events.

10.2. Properties of the turbulent events

Looking at the Figures 8-8, 8-9 and 8-10 for U-level, 9-10, 9-11 and 9-12 for Phase-Space, concerning the turbulent properties for fluctuations, large and small structures, the following considerations can be proposed about their patterns along the flow depth:

- The pattern of the spatial frequency, concerning fluctuations and small structures, is similar except within the first few millimetres near the bottom, because a local minimum point can be observed by using the second method;
- A maximum point concerning the spatial frequency of the large structures seems to lie near the middle of the depth by applying the Phase-Space, a bit nearer the free surface by applying the U-level;
- A similar situation can be observed for the temporal frequency of the large structures;
- The temporal frequency for fluctuations and small structures increases within the first few millimetres quicker by applying U-level than Phase-Space method. The remaining part looks like similar for both of them.

From the Figures 8.10 and 9.12, the characteristic mean length of the turbulent events appears quite constant along the depth. It slightly increases toward the bed when the first method is used and present a spike close to the bed according to the second method.

Now are reported a few images, already used in the past two chapters. The aim is to give an objective comparison, again between the U-level and the Phase-Space techniques, concerning the depth-mean values of these turbulent properties.



Figure 10-10: Spatial frequency of the turbulent events detected by applying U-level (on the left) and Phase Space technique (on the right); mean values over the depth of all the six time intervals

The Phase-Space technique tends to identify more large structures and less small structures than the U-level method. Instead, the number of the raw fluctuations is approximately the same, even if there are some difference between the several flow conditions.



Figure 10-11: Temporal frequency of the turbulent events detected by applying U-level (on the left) and Phase Space technique (on the right); mean values over the depth of all the six time intervals

As explained in the previous chapter, the threshold of the internal ellipsoid adopted by Phase-Space technique was adjusted in order to obtain, compared to U-level, approximately the same values of temporal frequency concerning the turbulent fluctuations. Nevertheless, by using the Phase-Space method, these values would be still on average slightly lower even if a smaller threshold would be adopted.

The Phase-Space technique detect per metre more large structures and less small structures. Moreover, the trends concerning the different flow conditions appear different compared to the specific method adopted.



Figure 10-12: Characteristic mean length of the turbulent events detected by applying U-level (on the left) and Phase Space technique (on the right); depth-mean values of all the six time intervals

The most relevant difference between the two adopted methods lie on the characteristic mean length of the large structures. Actually, the Phase-Space technique provides results much lower than the ones given by the literature, as reported in the previous paragraph. Instead, the U-level seems to give quite good results, as noted in the Chapter 8.

By using Phase-Space, even the mean length of fluctuations and small structures is lower than the one found by applying U-level technique.

10.3. Considerations

Looking at the trends concerning the mean-depth values of the turbulent structures properties, the U-level technique seems to apply quite reliable results, although several small turbulent events are detected in the identification of the coherent structures.

On the contrary, the Phase-Space technique, besides it would be more rigorous and more sensitive to real flow than U-level, always shows confused clusters of points.

The reasons might be four:

- The introduction of the internal ellipsoid cannot guarantee a valid detection of the turbulent events from velocity fields;
- Instead of discarding data outside the outer ellipsoid as erroneous, these points might be replaced with an average of the points around them, by applying a moving average validation procedure;
- The sides of the interrogation area of the two PIV cameras used to collect velocity instantaneous was to 16 pixels instead of 32, as generally adopted. This major resolution might have led to a loss of accuracy of the vector fields, because of a too low number of tracer particles detected within each pixel.
- The inevitable errors committed to measure the velocity fields, also because of what reported in the previous point, might have been led to sensible most important errors in the calculation of the first and especially second order derivative, used in the algorithm.

More specifically, compared to the velocity measurements, the errors can be up to double for the first order derivatives, up to four times for the second order derivatives.

11. Conclusions

PIV measurements have been conducted in a sloping rectangular flume, having different flow conditions. Some of these velocity instantaneous snapshots have been stitched together by using particular weightings functions, with an overlapping area between the images equal to 50% of the frame width, obtaining six time intervals long enough to contain at least 3 – 4 large scale turbulent flow structures. The Singular Spectrum Analysis technique has been applied to these time intervals, and the signal was reconstructed to expose the large structures and then the small structures, by using respectively 50 % and 33 % of the total kinetic energy. Finally, the U-level and the Phase Space techniques have been applied to identify the large and the small structures from the reconstructed signals.

The aim of this work was the identification and the study of the flow structures for turbulent free surface flows, in order to compare their properties between different hydraulic conditions. Moreover, a purpose was to establish which of the two methods better follows the results from the literature, mainly concerning the large-scale turbulent structures.

The conclusions may be summarised as follow:

- The method of stitching velocity instantaneous seems to be quite rigorous for its purpose to obtain time series longer than just a single snapshot, because the correlation values in the overlapping area between the snapshots are usually high (more than 0.7).
- The Singular Spectrum Analysis decomposition technique is able to decompose velocity fields into energetic components that are used to reconstruct the signal. It helps to separate the raw fluctuations by energy content. The size of these structures, as a consequence, quite depend on the specific time interval that is being studied, and in particular to the number of components used to reconstruct the signal.
- The kinetic energy increases as the flow depth increases, for both the cases of gravel and spheres bed. The energy results are slightly higher when gravel is used as the bed material.
- The percentage of kinetic energy contained in the first modes is getting higher as the flow depth increases. Moreover, this quantity appear more important for the cases of spheres bed than for the cases of gravel bed.
- The U-level technique seems to provide quite good results. The raw fluctuations show lots of structures ranging from small to large. But looking at the binary matrices, concerning the signal reconstructed to expose the coherent structures, the most energetic structures tend to be bigger, although several "small" structures are detected because strongly energetic. When a low number of SSA modes is used to expose the large structures, the large structures appear more clearly visible.

The spatial frequency of the turbulent structures decreases as the flow depth increases, the opposite situation occur for the characteristic mean length. Concerning the temporal frequency, it is not evident a particular pattern.

The turbulent structures result 10 - 20 % larger for the cases of gravel bed. As a consequence, the spatial frequency results a bit higher for the cases of spheres bed (approximately by the same quantity).

• The Phase-Space technique, besides it would be a more rigorous and more sensitive method to real flow than U-level, usually shows confused clusters of points instead of quite clear structures. This fact might be due to the innovation consisting on adding a second (internal)

ellipsoid to detect turbulent events from the velocity fields. Another reasons might also be the too high spatial resolution adopted to collect PIV velocity instantaneous, and/or the errors committed to measure the velocity of the tracer particles.

Finally, instead of discard data outside the outer ellipsoid as erroneous, it would be a good idea to replace these points with an average of the points around them, by applying a moving average validation procedure.

It would be interesting, for a further work, to use both the streamwise and depthwise velocity data, in order to prove if the techniques here used might lead to similar results by using both the velocity components. Moreover, it would be better to use more water depths and also more velocity instantaneous to stitch together to get clearer patterns of the properties of the turbulent structures. Instead of the SSA groupings proposed by Roussinova et al. (2010), it would be useful to select different ratios to distinguish the difference between the large and the small turbulent structures. Finally, in a further work a lot of the noise detected from the velocity data set could be removed by using a median filter.

References

- Baltzer, J. R., Adrian, R. J., & Wu, X. (2010). Turbulent boundary layer structure identification via POD. *Center for turbulent research*, 55-64.
- Baker, K. (2005). Singular Value Decomposition Tutorial.
- Brevis, W., & Garcia-Villalba, M. (2011). Shallow-flow vusialization snalysis by proper orthogonal decomposition. *J. Hydraulic Res.*, 49, 586-59.
- Broomhead, D.S., & King, G. (1986). Extracting qualitative dynamics from esperimental data. *Physica D*, 20, 217-236
- Goring, D. G., & Nikora, V. J. (2002). Despiking Acoustic Doppler Velocimeter Data. J. Hydraulic. Eng., 128, 117-126.
- Hassani, H. (2007). Singular Spectrum Analysis: Methodology and Comparison. J. Data Science, 5, 239-257.
- Hassani, H. (2010). A Brief Introduction to Singular Spectrum Analysis.
- Jia, Y. B. (2013). Singular Value Decomposition.
- Kostas, J., Foucaut J.M., Stanislas, M. (2005). Application of double spiv on the near wall turbulence structure of an adverse pressure gradient turbulent boundary layer. In: 6th International Symposium on Particle Image Velocimetry.
- Liang, Y. C., Lim, S. P., Lee, H. P., Lin, W. Z., Lee, K. H., & Wu, C. G. (2002). Proper Ortoghonal Decomposition and it's applicationd–Part 1: Theory. *J. Sound and Vibr.*, *252(3)*, 527-544.
- Luchik, T. S., & Tiederman, W. G. (1987). Timescale and structure of ejections and bursts in turbulent channel flows. *J. Fluid Mech.*, *174*, 529-552.
- Nezu, I. & Nakagawa, H. (1993). Turbulence in Open-Channel Flows. Rotterdam, Netherlands: Balkema.
- Nichols (2013). *The interaction mechanism of airborne acoustic fields with turbulent shallow flows*. Ph.D. thesis, Univ. of Bradford, Bradford, UK.
- Nikora, V. (2005). Flow turbulence over mobile gravel bed: spectral scaling and coherent structures. *Acta Geophisica*, *53*(4), 539-552.
- Roussinova, V., Shinneeb, A. M., & Balachandar, R. (2010). Investigation of Fluid Structures in a Smooth Open-Channel Flow Using Proper Orthogonal Decomposition. J. Hydraulic. Eng., 136, 143-154.

- Roy, A. G., Belanger, T. B., Lamarre, H., & Kirkbride, A. D. (2004). Size, shape and dynamics of large-scale turbulent flow structures in a gravel-bed river. *J. Fluid Mech.*, *500*, 1-27.
- Ruhe, R. V. (1984). Loess-derived soils, Mississii Valley Region: 1. Soil-sedimentation system. *Soil Sci. Soc. Am. J.*, 48, 859-863.
- Schmid, P. J. (2010). Dynamic mode decomposition of numerical and experimental data. J. Fluid Mech., 656, 5-28.
- Schmid, P. J., Li, L., Juniper, M. P., & Pust, O. (2010). Applications of the dynamic mode decomposition. *Theor. Comput. Fluid Dyn.*
- Schmid, P. J., Meyer, K. E., & Pust, O. (2009). Dynamic Mode Decomposition and Proper Orthogonal Decomposition of flow in a lid-driven cylindrical cavity. δth International Symposium on Particle Image Velocimetry.
- Shvidchenko, A. B., & Pender, G. (2001). Macroturbulent structure of open-channel flow over gravel beds. Water Resources Research, 37, 709-719; Dep. of Civil Eng., Univ. of Glasgow, Glargow, UK.
- Sirovich, L. (1987). Turbulence and the dynamics of the coherent structures, Parts 1, 2 and 3. *Quarterly of Applied Mathematics, 45*, 561-590

Van Driest, E. R. (1956). On turbulent flow near a wall. J. Aeronaut. Sci., 23, 1007-1011

Zhiglijavsky, A. (2010). Singular spectrum analysis for time series.