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Tesi di Laurea

Un rivelatore naturale per la luce strutturata: l’occhio umano

A natural detector for structured light: the human eye

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Abstract

The perception of polarization is widespread in the animal realm, enabling vital abilities in insects, arthropods, and cephalopods for orientation, biorecognition, or prey detection. Conversely, the visual system of mammals is essentially limited to the perception of light intensity (contrast) and wavelength (colours). Remarkably, however, the interaction of linearly polarized light with the central region of the macula can induce, even in human beings, the perception of a tiny bow-tie pattern oriented perpendicularly to the polarization plane.

This phenomenon, called Haidinger's brushes after the name of its discoverer in 1844, is maximally visible under blue light and originates from the spatial filtering of linear polarization by the protective layer of dichroic macular pigments, which are oriented in the fovea to form a sort of embedded radial polarizer for short wavelengths.

Illumination with structured beams of light, such as vector beams, exhibiting complex spatially-variant configurations of polarization, is expected to stimulate the perception of higher-order Haidinger's texture with a number of brushes related to the topological charge of the constituent orbital angular momentum beams. That extends further the detection capability of the visual system, including also the direct detection of phase with the naked eye, albeit under specific conditions.

In this thesis, we will consider the aforementioned entoptic phenomenon under the illumination with different vector beams generated via non-trivial combination of spin and orbital angular momentum states. An optical setup will be assembled and tested in the laboratory to prove the capability of assessing the orbital angular momentum content and vectorial state of a complex light beam with the naked eye.

Abstract

(italiano)

La percezione della polarizzazione è diffusa nel regno animale, consentendo a insetti, artropodi e cefalopodi capacità vitali per l'orientamento, il riconoscimento biologico o l'individuazione delle prede. Al contrario, il sistema visivo dei mammiferi è essenzialmente limitato alla percezione dell'intensità luminosa (contrasto) e della lunghezza d'onda (colori). Sorprendentemente, tuttavia, l'interazione della luce polarizzata linearmente con la regione centrale della macula può indurre, persino negli esseri umani, la percezione di un minuscolo motivo a farfalla orientato perpendicolarmente al piano di polarizzazione.

Questo fenomeno, chiamato "spazzole di Haidinger" dal nome del suo scopritore nel 1844, è visibile soprattutto sotto luce blu e ha origine dal filtraggio spaziale della polarizzazione lineare da parte dello strato protettivo di pigmenti maculari dicroici, che sono orientati nella fovea in modo da formare una sorta di polarizzatore radiale incorporato per le lunghezze d'onda corte.

L'illuminazione con fasci di luce strutturati, come i fasci vettoriali, che presentano complesse configurazioni di polarizzazione spazialmente variabili, dovrebbe stimolare la percezione di una struttura di Haidinger di ordine superiore con un certo numero di "spazzole" correlate alla carica topologica dei fasci di momento angolare orbitale costituenti. Ciò estende ulteriormente la capacità di rilevamento del sistema visivo, includendo anche il rilevamento diretto della fase a occhio nudo, seppur in condizioni specifiche.

In questa tesi, prenderemo in esame il suddetto fenomeno entottico mediante l'illuminazione con diversi fasci vettoriali, generati attraverso una combinazione non banale di stati di spin e momento angolare orbitale. Verrà assemblato e testato in laboratorio un apparato sperimentale per dimostrare la possibilità di caratterizzare il contenuto di momento angolare orbitale e lo stato vettoriale di un campo luminoso complesso mediante la sola osservazione a occhio nudo.

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Introduction

The purpose of this thesis is to investigate two topics belonging to different disciplines, structured light technology and human polarization sensitivity, and then bridge the gap between them, leveraging the properties of an optical structured beam to study this visual capability.

The starting point to describe this eye's property will be Haidinger's brushes, the first example discovered about the visual capability to discern polarized light. For humans this is an entoptic phenomenon (i.e., a visual effect generated by the interaction of light with internal structures of the eye), and currently there is no scientific evidence that it serves a functional purpose; indeed, due to neural adaptation, this perception fades after a few seconds, if it remains static. Instead, many species have evolved visual systems capable



Fig. 1: the mantis shrimp has one of the most complex visual systems. Each of the six rows in the midband of its ommatidium located along the equator of its visual organ is intended on specific properties or polarization direction of the light. From [3]

of exploiting polarized light. For example, the mantis shrimp, member of the Stomatopoda order, has a very complex visual system that enables communication between conspecifics and nearby species with polarized light. Honey bees use it as an orientation instrument. In Cephalopods such as squid, cuttlefish and octopuses, polarized light allows them to distinguish objects and signal aggressive or sexual behavior [1]. In the dense forests, where there are more degrees of polarized light, Panama butterflies belonging to the family of Heliconiidae also use it to communicate, reflecting

the iridescence of their wings. Furthermore, many species are under study, like some beetles and salmonids.

Consequently, the polarization vision is adapted by species that undergo a very different type of evolution [1]; in humans, it is generated by a combination of the natural dichroism of macular pigments in Henle's fibres, centered in the fovea (the most important region of the macula), while the corneal birefringence can modify the polarization state perceived [2].

The possibility of using optical polarized beam, and modulating its spatial complexity to adapt the beam to explore the configuration of the radial structure of the Henle's fibres, would be the perfect way to analyze accurately the characteristics of this human perception, like the limits on degrees of partial polarization, the spot size, the response as a function of type of wavelength, geometry and dynamics of

the beam. Structured light technology is designed to do exactly that: develop the method and instrument to spatially modulate an optical beam, in particular its properties: phase, polarization, amplitude, wavelength [4].

Structured light is used in a wide variety of fields: to transfer or decipher information respectively in optical fibres and quantum cryptography (included facial recognition); to concentrate and exploit with extreme precision the electromagnetic energy in optical manipulation and trapping techniques; to increase the resolution in optical microscopy, for material characterization and advanced laser applications in industry, and also in medical diagnostics.

A variety of optical elements have been developed to manipulate different properties of light. The polarization state of a beam can be controlled using linear polarizers, half-wave plates (HWPs), and quarter-wave plates (QWPs). These elements act uniformly across the beam profile and therefore cannot generate structured light on their own [5].

In contrast, the spatial structure of a beam can be engineered using refractive or diffractive optical elements, as well as more specialized devices such as spiral phase plates (SPPs), axicons, and spatial light modulators (SLMs), which enable precise control of the optical wavefront. More recently, metasurfaces and metalenses have emerged as compact platforms for wavefront engineering [6].

Some devices combine spatial shaping with polarization control. Among these, q-plates and liquid-crystal optical vortices (LCOVs) exploit spatially varying birefringence to generate structured beams, including vector beams, by coupling the spatial and polarization degrees of freedom [4].

The work is organized as it follows:

- Chapter 1 provides an overview of the phenomenology and etiology of human polarization perception.
- Chapter 2 introduces the theoretical framework for describing both the polarization of light and the spatial structure of optical beams, focusing in particular on beams carrying orbital angular momentum (OAM). The interplay between polarization and OAM is then explored, leading to the generation of the so-called vector beams, characterized by spatially varying polarization patterns.
- Chapter 3 presents the experimental work devoted to the design and testing of an optical setup for investigating the human perception of structured light beams with the naked eye. Finally, preliminary experimental results are discussed and compared with the predictions of the theoretical models.

Chapter 1: Human polarization sensitivity

1.1 Haidinger's brushes

In 1844 the Austrian physicist, geologist and mineralogist Wilhelm Karl von Haidinger (1795-1871) discovered that when observing the blue sky near the horizon, while the Sun is at its zenith, we perceive an entoptic image shaped like yellow brushes amounting approximately three degrees around the point of fixation [2]. This entoptic perception disappears quickly due to neural adaptation [7].

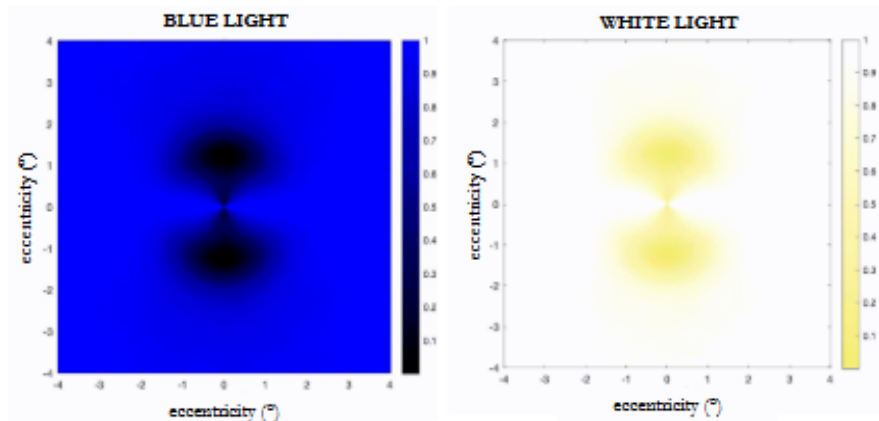


Fig 2: visualization of Haidinger's brushes on blue and white background. Adapted from [6]

This limitation can be overcome by moving the polarization pattern, for instance by rotating the head around the primary visual axis. Like all entoptic phenomena, it exhibits perceptible interocular and inter-subject variability, as will be discussed later. Haidinger suggested this phenomenon is caused by weak interaction between our eyes and light polarization induced by Rayleigh's scattering in the air: in particular, we obtain vertical polarization at the zenith of the sun.

We can obtain a polarized natural light also in other situations: through the reflection of light by the water, the refraction of light by glossy surface (a body of water, wet surface, some animal's skin and flora's protective layers) or in the dense woods [1].

Scientists advanced many hypotheses to describe Haidinger's brushes, however the correct approach for the description of Haidinger's brushes was advanced by James Clerk Maxwell in 1850, by the idea of a natural radial polarizer in the eye. The polarizer is precisely the Henle's Fibre Layer enabled by the presence of dichroic macular pigments [2]. Hochheimer (1978) removed a cornea and propagated a vertically polarized blue light beam perpendicularly to the retina, to study the transmitted and consequently reflected beam. He observed a Maltese cross shape, corresponding to the Henle Fibre layers, as expected. Moreover, Dodt and Kuba (1990) demonstrated this hypothesis electrophysiologically [2].

1.2 The etiology of the phenomenon

The fovea (1.5 mm diameter) is the central part of the macula lutea (5-5.5 mm diameter), the most specific and efficient part of the retina, responsible for phototransduction via cones and rods. In contrast to the direction of information propagation, the layers of neural cells stand in front of the photoreceptors. At an early age (see Fig. 3) there is a peripheral migration of neural cells away from the fovea's centre, resulting in a depression named the foveola (0.35 mm diameter). This region, devoid of neural cell obstruction, devascularized, and enhanced by a great density of cones, becomes the most precise part of the retina and responsible for the maximum visual acuity [8].

The foveola is almost completely devoid of S-cones (short-wavelengths sensitive cones), an arrangement believed to minimize the effects of chromatic aberration while maximizing spatial resolution, resulting in a natural chromatic blindness known as the S-cone scotoma, though neural interpretation compensates for this. Following their migration into the foveola, the cone axons are forced to stretch radially to reach the nearest lateral bipolar cells. This radial configuration, named the Henle's Fibre Layer, extends into the parafoveal region (2.5-3 mm in diameter) and is fundamental to the optical properties of the macula [8].

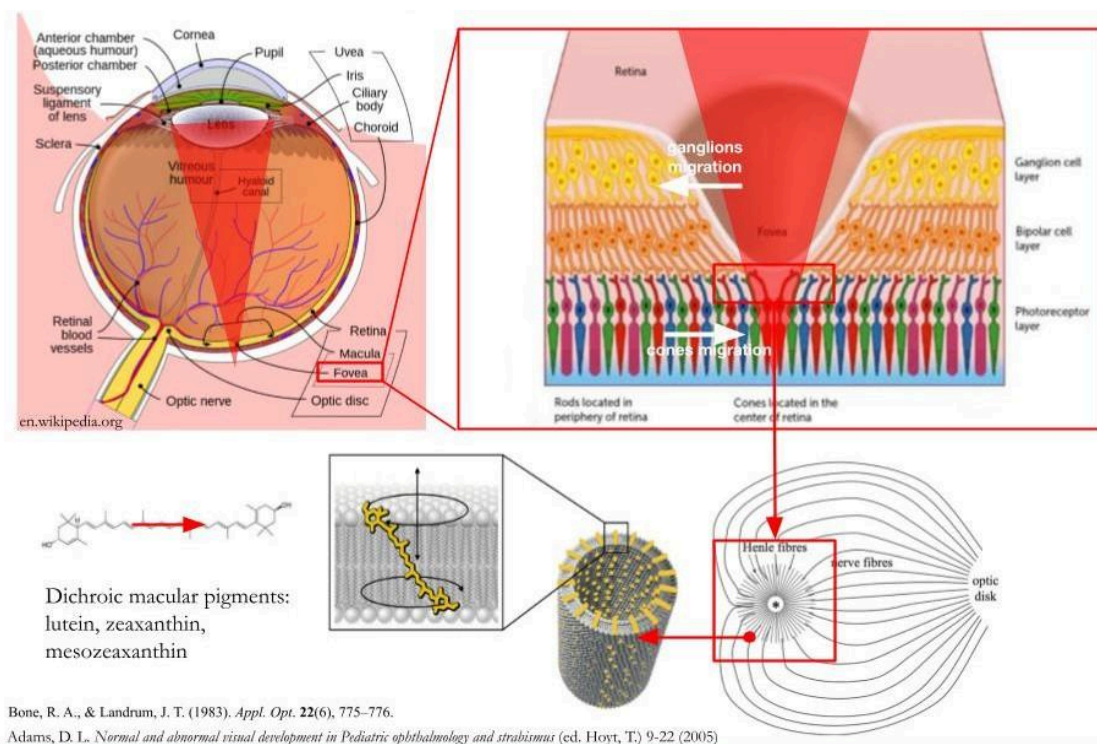


Fig 3: on the top left: a cross section of the human eye. On the top right: a cross section of the fovea and migration of ganglions, within the first 15 months of life, and cones, within the first .45 months of life. On the bottom, from right to left: representation of the Henle Fibre layers; macular pigments on a fibre; chemical structure of the dichroic macular pigments. Adapted from [9]

The macula is characterized by a high density of pigments. These elements are exogenous dietary carotenoids concentrated in the macula and they are fundamental for human polarized sensitivity due to their dichroic properties: they absorb more efficiently the electric field component parallel to their long axis, in particular in the wavelength range around $400 - 530nm$, corresponding to the blue spectrum [9]. As a consequence of higher energies of the shorter wavelengths of the visible spectrum, macular pigments, namely lutein and zeaxanthin, have a protective and preventive function for the human eye, since blue light is more prone to induce photochemical damage. These pigments are lipophilic and when present in the double membranes of the axons tend to align perpendicularly to them [9].

The dichroism of macular pigments derives in particular from the spatial configuration of their components: in fact, the long carbon chain produces an extended resonance system of electrons. Thus along this direction the delocalized electrons are highly sensitive to variations in the magnitude of the incident electric field, because the weaker binding force unconstrains them from any static configuration. In conjunction with the terminal OH- functional groups, lutein and zeaxanthin become reactive molecules in the interaction with free radicals or in general oxidizing agents, therefore they have antioxidant properties [9] (see Fig. 3).

Hence, the etiology of Haidinger's brushes, according to the Hypothesis of Maxwell, lies in the radial dichroism of the Henle's Fibre Layer. The vertically incident polarized blue light is absorbed by macular pigments when the polarization direction is parallel to their long axis (in the horizontal Henle's fibres in this case), while it is transmitted along the vertical fibres, which have macular pigments perpendicular to the oscillation of the electric field (see Fig. 4). In detail, for each pigment, only the vertical component of the polarized light is absorbed, so the absorption is a periodic function of the angle that a fibre forms with the direction of polarization, in accord with the law of Malus [9]:

$$I = I_0 \cos^2 \theta \quad (1.1).$$

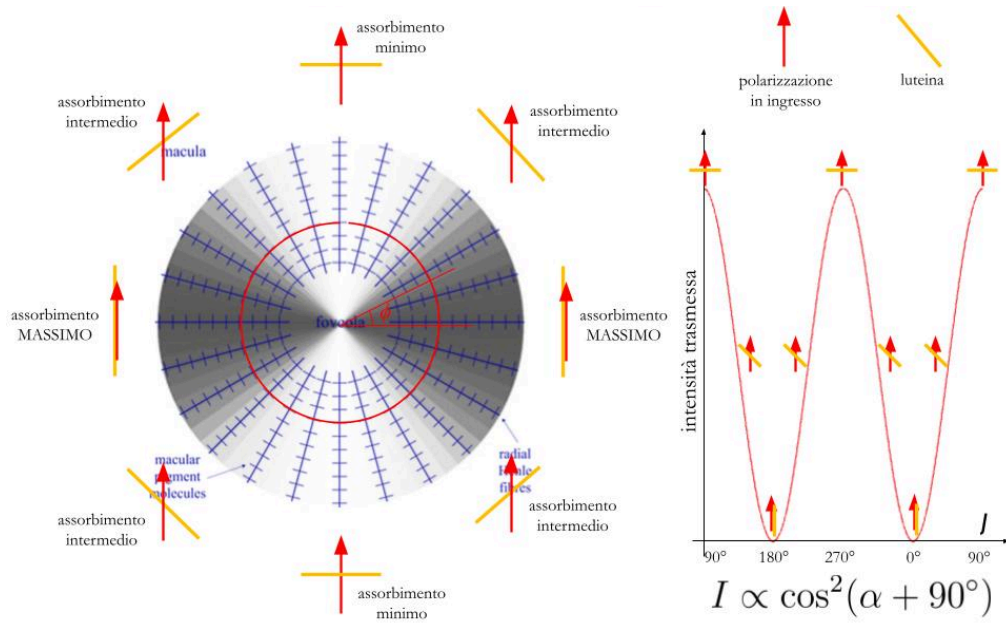


Fig 4: representation of the Henle's Fibre Layer, level of macular absorption in relation to the angle between long axis of macular pigment and direction of polarization of the incident light, in accord with Malus's law (1.1). Adapted from [9]

Thus, observing Haidinger's brushes in the blue sky, the color we perceive, in the site deprived of blue light via absorption by the pigments, is less intense and darker. Instead, if we have a white background, we perceive the complementary color of blue in the absorption region, the yellow, and a blue faded brush oriented perpendicularly. The presence of a yellow alone on a blue background and a blue alone on a white background is explained by the Theory of Opponent process, developed by Ewald Hering (1834-1918), the modern theory which describe the human perception of the colour in addition to trichromatic theory, through a complex mechanism of opposition between red and green, yellow and blue, white and black (for luminance) [10].

Another influencing factor for the characteristics of the polarized incident light in the human eye is the birefringence of the cornea, which can be modelled as an anisotropic transparent layer having different refractive indices n ($v = c/n$, with c speed of light in a vacuum) in two orthogonal directions. The cornea is the outermost layer of the eye, consequently it offers a first protection and it serves as the first lens to concentrate incident rays on the pupil. It has dimensions of 11.7 mm horizontally, 10.6 mm vertically, and a thickness of 0.52 – 0.67 mm; it is composed of 90% stroma, a structure of precise alignment of collagen fibres (which gives rise to intrinsic and form birefringence) and keratocytes [7].

The precise characteristics of corneal birefringence remain an active area of research. A study conducted by S. E. Temple et al. has exploited the formula derived by Rothmayer et al. to study the cornea birefringence, which is introduced here for the sake of completeness [7].

$$\theta_1 = \frac{1}{2} \arccos \left(\frac{\cos(2(\theta_0 + s))}{\sqrt{1 - \sin^2 \Delta \sin^2 2(\theta_0 + s)}} \right) - s \quad (1.2)$$

Where θ_1 and θ_0 are the perceived and incident angles of linear polarization, respectively; Δ is the retardance; s is the corneal azimuth (which the authors assume to be the slow axis, although they cannot demonstrate it). However, the correct and complete model of the cornea may follow a biaxial crystal model [7].

An incident non-linear polarized light can assume a linear polarization, if the term of corneal retardance changes the original phase shift into 0° or 180° . This phenomenon is named Schurcliff's brushes (discovered in 1955) and the resulting perception is a faded hourglass pattern similar to Haidinger's brushes, but here the incident light has circular or more common elliptic polarization (the electric field's vector rotate circularly or elliptically during the time).

However, the precise characteristics of corneal birefringence (as previously discussed) and the different spatial distribution of Haidinger's brushes between vertical and horizontal polarization, reported by Von Helmholtz (1924), are still not fully understood. In particular, he noted that the horizontal brushes are narrower than vertical ones [2].

Structured light contributed significantly to these studies. In the next chapter, the first part concerns a formal introduction and exposition of the fundamental physico-mathematical model used to describe this technology; starting from the polarization, and following the paraxial Helmholtz equation and illustrating its general solutions. It will conclude with the description of OAM (Orbital Angular Momentum) beams, vector beams and Jones's formalism.

Chapter 2 Structured light

2.1 Polarization and Jones formalism

The polarization of an electromagnetic wave is the direction of oscillation of its electric components, which are perpendicular to the axis of propagation of the wave due to Maxwell's law condition $\nabla \cdot \mathbf{E} = 0$. If the variation of the direction of oscillation can be expressed mathematically and deterministically in relation to the temporal and spatial coordinates, the wave is called polarized. In particular, polarization is a property arising from the phase and amplitude difference between the phase difference between the electric components, it is not solely time-dependent; there could be different polarization states at each point, such as vector beams, exactly the culmination of this theoretical analysis. Moreover, the light is called partially polarized if only a fraction of radiation is totally polarized (by definition, “totally polarized” and “polarized” are equivalent in this context). Thus, the degree of polarization and the direction of polarization can be defined as consequence. These definitions highlight the transition between the analytical formalism and the experimental world, because the phase relationship acquires physical significance only if the sampling time of the instrument which detects the electromagnetic wave is much shorter than the timescale of phase fluctuations [11].

Assuming z as the propagation axis, the electric field oscillates in the $x - y$ plane, therefore it can be expressed in terms of its two components, horizontal and vertical. Moreover assuming temporal and spatial coherence, then we can write:

$$E_x(\mathbf{r}, t) = E_{0,x}(\mathbf{r}, t) \cos(kz - \omega t + \phi_x(\mathbf{r}, t)) \quad (2.1a)$$

$$E_y(\mathbf{r}, t) = E_{0,y}(\mathbf{r}, t) \cos(kz - \omega t + \phi_y(\mathbf{r}, t)) \quad (2.1b)$$

Fixing a spatial coordinate and eliminating the temporal dependence from the spatial phase $\phi(\mathbf{r})$ (thus assuming a strictly monochromatic profile), the previous equations can be shown to be the parametric equations of an ellipse (obviously centered at the origin of $E_x - E_y$ axes). Precisely, by rewriting the total phase of E_y as $(kz - \omega t + \phi_x) + \Delta\phi$, with phase difference $\Delta\phi = \phi_y - \phi_x$, through the summation cosine formula, the fundamental trigonometric identity and appropriate substitutions, we can eliminate the propagation component $kz - \omega t$ and the individual phases ϕ_x and ϕ_y to obtain [12,13]:

$$\frac{E_x^2}{E_{0,x}^2} + \frac{E_y^2}{E_{0,y}^2} - 2 \frac{E_x E_y}{E_{0,x} E_{0,y}} \cos(\Delta\phi) = \sin^2(\Delta\phi) \quad (2.2)$$

All the information is contained in the phase-difference and the amplitudes of the two electric components. Usually the polarization ellipse is described by two angles: the orientation angle ψ that the major axis of ellipse defines with the E_x -axis; and the ellipticity angle χ defined in $[-\pi/4; \pi/4]$ between major and minor axis, giving the elliptical thickness:

$$\tan(2\psi) = \frac{2E_{0,x}E_{0,y}}{E_{0,x}^2 - E_{0,y}^2} \cos(\Delta\phi) \quad (2.3a)$$

$$\sin(2\chi) = \sin(2\alpha) \sin(\Delta\phi) \quad (2.3b)$$

By Equation (2.2), there are two important families of states of polarization:

$$\Delta\phi = 0, \pi \rightarrow \left(\frac{E_x}{E_{0,x}}\right)^2 + \left(\frac{E_y}{E_{0,y}}\right)^2 - 2\frac{E_x E_y}{E_{0,x} E_{0,y}} = 0 \rightarrow \left(\frac{E_x}{E_{0,x}} - \frac{E_y}{E_{0,y}}\right)^2 = 0 \rightarrow E_y = \pm \frac{E_y}{E_{0,x}} E_x \quad (2.4a)$$

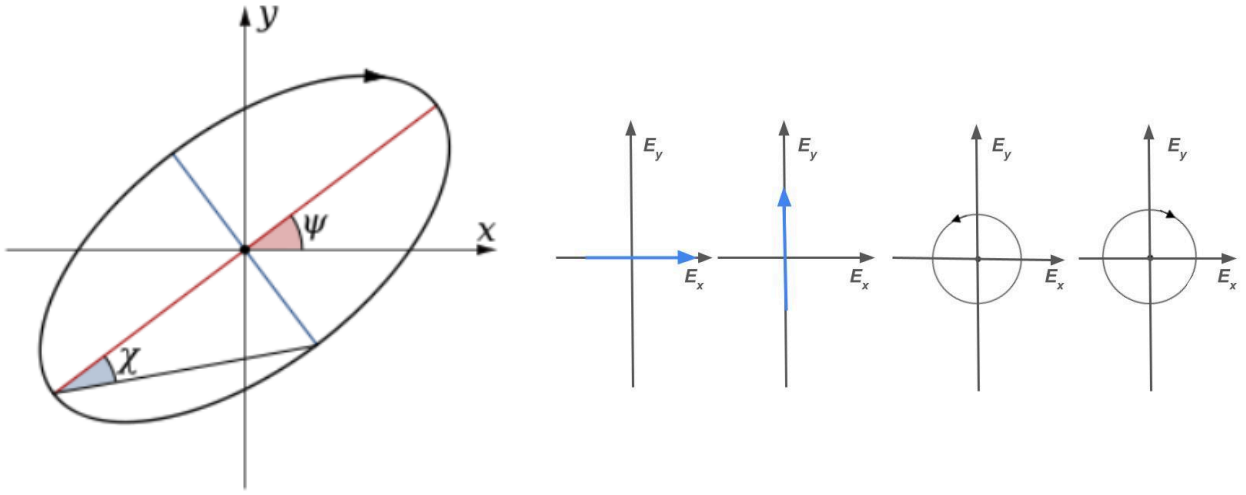


Fig 5: on the left, the ellipse of polarization, described via ψ and χ angles of orientation and ellipticity respectively (adapted from [13]). On the right, schematic representation of vertical and horizontal polarization (2.4a), right-handed and left-handed circular polarization (2.4c, 2.8b), respectively from left to right.

The ellipse collapses, yielding a linear polarization state along the angle defined by $\theta = \tan^{-1}\left(\frac{E_{0y}}{E_{0x}}\right)$.

$$\Delta\phi = \pm\pi/2 \rightarrow \frac{E_x^2}{E_{0,x}^2} + \frac{E_y^2}{E_{0,y}^2} = 1 \quad (2.4b)$$

results in the canonical ellipse. The additional condition $E_{0x} = E_{0y} = E_0$ gives the circular polarization state:

$$\frac{E_x^2 + E_y^2}{E_0^2} = 1 \rightarrow E_x^2 + E_y^2 = E_0^2 \quad (2.4c)$$

Without loss of generality, we briefly introduce the following formalism to support the subsequent discussion. George Gabriel Stokes in 1852 rewrote the fundamental Equation (2.2) in experimentally observable terms:

$$(E_{0x}^2 + E_{0y}^2)^2 = (E_{0x}^2 - E_{0y}^2)^2 + (2E_{0x}E_{0y} \cos \Delta\phi)^2 + (2E_{0x}E_{0y} \sin \Delta\phi)^2 \quad (2.5a)$$

And then in relation to the 4 Stokes parameters:

$$S_0^2 = S_1^2 + S_2^2 + S_3^2 \quad (2.5b)$$

The formalism of Stokes is useful for studying partially polarized light (evaluating the difference between left-hand side and right-hand side) and for the formalism of the quantum mechanics, where observables play a central role; in particular it is important to formally introduce the Poincaré Sphere, as we will be discussed [12].

To resolve the problems associated with the cumbersome trigonometric algebra, we prefer using complex phases through the fundamental Euler's identity $e^{i\phi} = \cos \phi + i \sin \phi$, in order to simplify calculations. This is the starting point to introduce Jones formalism: under the hypothesis underlying Equation (2.2), we can isolate the phase terms and amplitudes with a 2-dimensional complex vector, with the associated bra-ket notation, analogous to Dirac quantum notation:

$$|E(\mathbf{r})\rangle = \begin{pmatrix} E_{0x}(\mathbf{r})e^{i\phi_x} \\ E_{0y}(\mathbf{r})e^{i\phi_y} \end{pmatrix} = e^{i\phi_x} \begin{pmatrix} E_{0x}(\mathbf{r}) \\ E_{0y}(\mathbf{r})e^{i\Delta\phi} \end{pmatrix} \quad (2.6)$$

Here, spatial dependence has been explicitly reintroduced. Polarization states lie in the polarization plane, thus for each point there may be a different polarization state; this concept is fundamentally important for future discussions about vector beams. Jones formalism also necessitates having totally polarized light (in contrast to Stokes formalism). We recall that $|E(\mathbf{r})\rangle$ is not a true physical representation of an electric vector (technically, it is a phasor), because the real measurable electric field is given by $\text{Re}\{|E(r)\rangle\}$ (furthermore, it lacks dynamic terms $(k \cdot \mathbf{r} - \omega t)$, deliberately omitting these deterministic and common terms). The purpose of this notation is to study the relative phase shifts and the dynamic response to an optical element that modifies the electric field vector, consequently the global phase is not important. Thus it is possible to modify a Jones vector by a global phase and often normalise the vector to 1, in order to simplify the mathematical analysis.

The bra-ket notation is very useful to describe the intensity of the light beam, which is naturally defined by the inner product:

$$I \propto \langle E|E \rangle = (E_x^* \ E_y^*) \begin{pmatrix} E_x \\ E_y \end{pmatrix} = |E_x|^2 + |E_y|^2 = I_x + I_y \quad (2.7)$$

where the symbol $\propto$ refers to additional constants depending on the conventions adopted. The Cartesian basis becomes horizontal and vertical polarization respectively.

$$|H\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |V\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.8a)$$

Instead, the normalized circular basis can be obtained by imposing a phase shift of $\pm\pi/2$, for right-handed and left-handed circular polarization respectively, appropriately normalised to 1:

$$|R\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}, \quad |L\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix} \quad (2.8b)$$

Consequently each polarization state vector which satisfies Jones's hypothesis can be decomposed into a linear combination of the circular bases or the circular basis (or a generic complete vector basis). The following fundamental expression exhibits a generic linear combination on circular basis using the characterizing ψ and χ angles [12,13]:

$$|E(\psi, \chi)\rangle = \cos(\pi/4 - \chi)e^{-i\psi}|R\rangle + \sin(\pi/4 - \chi)e^{+i\psi}|L\rangle \quad (2.9)$$

Every optical element which transforms the polarization state of the Jones vector in accordance with the limits of Jones representation is mathematically a linear operator with domain and codomain elements belonging to $\mathbb{C}^2$, thus we can describe them with a 2×2 matrix. Now the optical elements that will be introduced work on the same Cartesian reference system of the electric field; we simply apply a rotation matrix $R(\theta)$ to generalize them for every a coordinate system rotated by an angle θ :

$$R(\theta) = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \quad (2.10)$$

The final matrix is calculated via $M_{new} = R(\theta)M_{old}R(-\theta)$. Coherently, $|E(\theta)\rangle = R(\theta)|E\rangle$.

A linear polarized filter is described by

$$P_x = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad (2.11)$$

Its function is to eliminate the vertical component, leaving the horizontal component invariant. Physically, the linear polarized filter is constituted by aligned dichroic polymer chains which absorb light oscillating parallel to their alignment axis, then the perpendicular component is transmitted [11]. By the application of a polarized generally rotated of an angle θ on a homogeneous electric field ($E_{0x} = E_{0y} = E_0$), with

appropriate trigonometric modulation is derived from the famous Malus law (1.1).

A linear retarder is represented by

$$W(\sigma) = \begin{pmatrix} 1 & 0 \\ 0 & e^{-i\sigma} \end{pmatrix} \quad (2.12)$$

namely a birefringent element. This object has two perpendicular axes named extraordinary and ordinary axes, with two different refractive indices, n_{st} and n_o respectively, with $n_{st} < n_o$. Consequently the speed of propagation of the electric wave varies between the two directions, introducing a phase shift between them. In particular, $v_{st} > v_o$, then extraordinary and ordinary axes are also called fast and slow axes respectively. By the invariance of global phase multiplication, it is possible to express the total phase shift in the vertical component, defining only one birefringence axis. The total phase shift σ is therefore derived as:

$$\sigma = \Delta k d = (k_{st} - k_o) d = (n_{st} - n_o) k_0 d$$

$$\text{from } k_{st,o} = \frac{2\pi}{\lambda_{st,o}} = \frac{2\pi\nu}{v_{st,o}} = n_{st,o} \frac{2\pi\nu}{c} = n_{st,o} \frac{2\pi}{\lambda_0} = n_{st,o} k_0 \quad (2.12a)$$

where d is the optical path, k_0 and λ_0 are wavenumber and wavelength in the vacuum respectively.

If $\Delta n d = \lambda_0/4 \rightarrow \sigma = \pi/2$, and the birefringent element is called quarter wave plate (QWP):

$$J_{QWP} = \begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix} \quad (2.12b)$$

which allows to convert a linearly polarized light at the angle $\theta = \pm\pi/4$ into a circularly polarized light, right-handed and left-handed respectively, and conversely.

If $\Delta n d = \lambda_0/2 \rightarrow \sigma = \pi$, and the birefringent element is called half wave plate (HWP):

$$J_{HWP} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (2.12c)$$

HWP converts the handedness of a circularly polarized light for any orientation of its axis. Generally, a linearly polarized light is transformed into a linearly polarized light with the direction of polarization which is symmetric to the birefringence axis [12].

It is important to emphasize the ideal nature of these matrices. In reality, there are always factors disturbing the correct and maximum efficiency of these optical components.

In conclusion, the Poincaré sphere is introduced as the most elegant representation of totally polarized

light states. Recalling equations (2.4) and (2.5), it is possible to describe S_1, S_2, S_3 in relation to S_0 and orientation and ellipticity angles:

$$\tan 2\psi = \frac{S_2}{S_1}; \quad \sin 2\chi = \frac{S_3}{S_0} \rightarrow S_1 = \cos(2\psi) \cos(2\chi), \quad S_2 = \sin(2\psi) \cos(2\chi), \quad S_3 = \sin(2\chi)$$

Introducing standard spherical coordinates $(r, \theta, \phi) = (1, \pi/2 - 2\chi, 2\psi)$ and normalizing the total intensity (S_0) to 1, we exactly obtain the parametric equation of a sphere:

$$(S_1, S_2, S_3) = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$$

Hence is established a bijective mapping between any polarized state (univocally defined by angles ψ and χ) and a point on the Poincaré sphere.

Right-handed and left-handed circularly polarized light result at north and south poles respectively, while linear polarization states are located on the equator, with horizontal and vertical polarization corresponding to x and $-x$ axes ($\chi = 0$ and $\psi = 0, \pi/2$ respectively) and in the other points the entirety of elliptical polarization states is present.

By substituting ψ and χ angles with θ and ϕ angles in Equation (12), we obtain:

$$|E\rangle = \cos(\theta/2)e^{-i\phi/2}|R\rangle + \sin(\theta/2)e^{i\phi/2}|L\rangle \quad (2.13)$$

This equation is valid for every generic state on the Poincaré sphere [12].

This expression describes a generic eigenstate spin-1/2 within the $SU(2)$ algebra in quantum mechanics. Hence there is a strict mathematical isomorphism between classical polarization state and quantum spin state [11].

We define the geometrical phase (or the Aharonov-Berry phase, or equivalently the Pancharatnam-Berry phase) has the total phase shift accumulated in a closed path on the Poincaré Sphere, independent from the time and the energy used, but in relation to the geometry of the path [12].

As will be seen later, the sphere is also useful for expressing Laguerre-Gauss spatial mode solutions with the Spatial Modes Poincaré Spheres (SMPS), then the vector beams through the Hybrid Poincaré Spheres (HPSs).

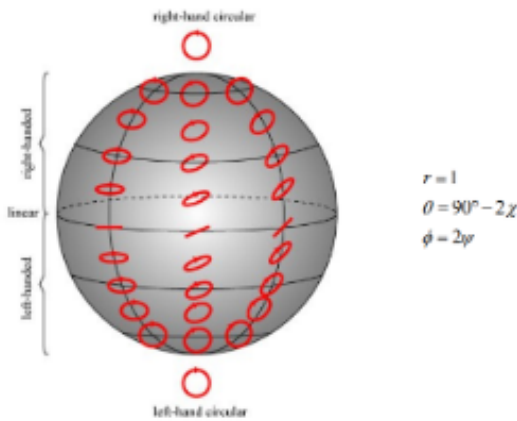


Fig 7: The Polarization Poincaré Sphere (PPS) and visual representation of the position of the polarization states on the PPS. Adapted from [14]

2.2 The Helmholtz Equation in the paraxial approximation and LG beams

$$U(\mathbf{r}, t) = A(\mathbf{r})e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t + \phi(\mathbf{r}))} \quad (2.14)$$

resolve d'Alembert equation (in the vacuum):

$$\left(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) U(\mathbf{r}, t) = 0 \quad (2.15a)$$

Using the relation $|k| = w/c$, the Helmholtz equation is obtained:

$$(\nabla^2 + k^2)U(\mathbf{r}) = 0 \quad (2.15b)$$

(where the new $U(\mathbf{r})$ is the spatial term of $U(\mathbf{r}, t)$). Under the paraxial approximation, the unit wave vector deviates negligibly from a fixed axis, conventionally z . Mathematically, we will use a wave of this form:

$$U(\mathbf{r}) = A(\mathbf{r})e^{ikz} \quad (2.16)$$

with conditions of the paraxial approximation which can be expressed as:

$$\left| \frac{\partial^2 A}{\partial z^2} \right| \ll |k^2 A|, \quad \left| \frac{\partial^2 A}{\partial z^2} \right| \ll |\nabla_{\perp}^2 A|, \quad \left| \frac{\partial^2 A}{\partial z^2} \right| \ll \left| k \frac{\partial A}{\partial z} \right| \quad (2.17)$$

By substituting the expression (2.16) in the Helmholtz Equation and applying paraxial conditions, the paraxial Helmholtz equation is derived:

$$\nabla_{\perp}^2 A + 2ik \frac{\partial A}{\partial z} = 0 \quad (2.18)$$

where a general family of solutions in cylindrical coordinates (ρ, ϕ, z) are the Laguerre-Gauss beams (namely LG beams), which can be written in full for sake of completeness:

$$A_{p,\ell}(\rho, \phi, z) = C_{p,\ell} \frac{w_0}{w(z)} \left(\frac{\sqrt{2}\rho}{w(z)} \right)^{|\ell|} L_p^{|\ell|} \left(\frac{2\rho^2}{w^2(z)} \right) \exp \left(-\frac{\rho^2}{w^2(z)} - i \frac{k\rho^2}{2R(z)} + i\ell\phi + i\Psi_{p,\ell}(z) \right) \quad (2.19)$$

where (ℓ, p) are the topological charge and the radial index respectively. The order of the solution is defined by $N = 2p + |\ell|$.

$L_p^{|\ell|}$ are generalized Laguerre polynomials:

$$L_p^{|\ell|}(x) = \sum_{m=0}^p (-1)^m \frac{(p + |\ell|)!}{(p - m)! (|\ell| + m)! m!} x^m \quad (2.19a)$$

$C_{p,\ell}$ is the normalization factor:

$$C_{p,\ell} = \sqrt{\frac{2p!}{\pi(p+|\ell|)!}}$$

And $w(z)$, w_0 , $R(z)$ and $\Psi_{p,\ell}(z)$ are the beam radius (or the spot size), the beam waist, the wavefronts curvature radius and the Gouy phase:

$$w(z) = w_0 \sqrt{1 + (z/z_R)^2} \quad (2.19b)$$

where z_R is the Rayleigh range:

$$z_R = \frac{\pi w_0^2}{\lambda} \quad (2.19c)$$

$$R(z) = z [1 + (z_R/z)^2] \quad (2.19d)$$

$$\Psi_{p,\ell}(z) = (|\ell| + 2p + 1) \arctan(z/z_R) \quad (2.19e)$$

Despite the complexity of the solution, the fundamental terms are redefined as:

$$U(\rho, \phi, z) = A_{\rho,\ell} \cdot e^{i\ell\phi} \cdot e^{ikz} \quad (2.19f)$$

where the envelope term, the azimuthal phase factor and the fundamental propagation direction have been respectively highlighted.

In $\rho = 0$ there is a mathematical singularity (because the azimuthal angle ϕ is undefined), resolvable only by imposing $U(0, \phi, z) = 0$. The other roots of LG solutions are determined by the p number and in particular to Laguerre polynomials, then they define radial profiles with zero intensity. Consequently, the radial index p corresponds to the number of nodal rings present in the intensity profile.

In quantum mechanics, these formulas are the general solutions of systems with rotation symmetry. Indeed the angular momentum operator along z -axis in cylindrical coordinates is given by:

$$L_z = -i\hbar \frac{\partial}{\partial \phi} \quad (2.20a)$$

its eigenvalues for the LG solutions can be found quite easily, due precisely to that phase term. Hence we obtain the orbital angular momentum quantization, characterized by the topological charge ℓ :

$$L_z|U\rangle = \ell\hbar|U\rangle \quad (2.20b)$$

Formally LG modes form an infinite-dimensional Hilbert space, representable with $L^2(\mathbb{R}^2, \mathbb{C})$. This formally bridges the gap between the beams following the characteristics of LG solutions, that we call OAM beams (Orbital Angular Momentum) and the physical quantity of angular momentum. Furthermore

the phase term gives to OAM beams a helicoidal wavefront, with a number of twists equal to the topological charge, right-handed and left-handed chirality for positive and negative values respectively. The intensity profile exhibits a radial dependence, resulting in an annular “doughnut-shaped” profile. Moreover, the topological charge influences Laguerre polynomials, spatially modifying the width of the concentric rings of intensity, in particular expanding the size of the central node [15].

Before proceeding with further formal derivations, a deeper physical and geometrical inspection of the helical wavefront is warranted.

A wavefront is formally defined as the geometric locus of points sharing the same instantaneous optical phase. For OAM beams, the non-planar structure is dictated by the coupling of the longitudinal propagation phase and the azimuthal phase factor, which forces the constant-phase points into a helical geometric locus. Consequently, in the xy -plane the electric field assumes different values in relation to the ϕ -coordinate. In structured optics, this topology is frequently visualized in the transverse plane by mapping the phase profile to a continuous color scale; this representation yields $|\ell|$ full color cycles that rotate in accordance with the sign of the topological charge ℓ . Furthermore, since the electric and magnetic fields must be strictly tangent to the wavefront, the helical tilt of this surface relative to the transverse xy -plane induces a proportional deviation of the local wave vector from the z -axis of propagation.

Consequently, the Poynting vector $\mathbf{S} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B}$ (in vacuum) acquires an azimuthal component. According to Poynting's theorem $\frac{dU}{dt} = - \oint_{\partial\Omega} \mathbf{S} \cdot \mathbf{n} d\Sigma$, this spiraling flow of electromagnetic energy is precisely the physical origin of the mechanical torque associated with OAM beams [11].

The physical significance of the OAM beam is hence exemplified: the deviation from the z -direction of the radiation pressure induced by the Poynting vector exerts a torque on an illuminated object, yielding an angular momentum.

Despite the particular geometry just described, it remains an azimuthally symmetric energetic contribution, because the intensity, proportional to the square of the electric field, yields $|e^{i\ell\phi}|^2 = 1$. We do not see any particular interference patterns unless we superpose more OAM beams, as will be shown later.

For $(\ell, p) = (0, 0)$ the fundamental mode is obtained, namely TEM₀₀. This is the Gaussian beam, which we will analyze:

$$U(\rho, z) = U_0 \frac{w_0}{w(z)} \exp\left(-\frac{\rho^2}{w^2(z)}\right) \exp\left(i\left(kz - \frac{k\rho^2}{2R(z)} + \Psi_{00}(z)\right)\right) \quad (2.21)$$

where U_0 is the amplitude of the beam. With appropriate algebraic substitutions, the expression can be written in a more compact form:

$$U(\rho, z) = \frac{U_1}{q(z)} \exp\left(-i\frac{k\rho^2}{2q(z)}\right) \exp(ikz) \quad (2.21a)$$

with $U_1 = iz_R U_0$ and the q -parameter is defined as:

$$q(z) = z + iz_R \quad (2.21b)$$

“Gaussian beam” is not an accidental name. In fact, the intensity of the wave assumes a Gaussian profile from the variation of ρ -coordinate, centered strictly on the z -axis.

The intensity of the Gaussian beam assumes the maximum value at $z = 0$ and the half-maximum at $z = z_R$, while ideal beams (such as Bessel solutions for Helmholtz Equation) have a constant profile in the direction perpendicular to the propagation axis, the intensity variation tends to spread over larger circumferences as z increases.

This phenomenon is modulated by the beam radius: at $\rho = w(z)$, the intensity reduces to $1/e^2$ of its peak value relative to z -coordinate ($w(0) = w_0$).

Moreover, the geometry of the wavefront can be derived by equating the sum of the phase components to $2\pi n$. The resulting equation is:

$$z - \frac{\rho^2}{2R(z)} \approx n\lambda - \frac{\lambda}{2\pi} \Psi_{p,\ell}(z) \quad (2.21c)$$

which is similar to a paraboloid equation with the radius of curvature given exactly by $R(z)$. It assumes a plane profile at $z = 0$ ($\lim_{z \rightarrow 0} R(z) = \infty$), its minimum at Rayleigh range ($R_{min} = 2z_R$) and it is asymptotic to z for long distances ($\lim_{z \rightarrow \infty} R(z) = z$), thus it presents a spherical wavefront [15].

The spatial modes of a generic solution described by the linear combination of LG solutions at the first order, $(\ell, p) = (\pm 1, 0)$, is mathematically a subspace of the infinite-dimensional space in two-dimensional complex space, can be represented through the Poincaré sphere, namely in this context Spatial Modes Poincaré Sphere (SMPS). We can construct it substituting $|R\rangle$ and $|L\rangle$ circular polarized

basis with

$$|LG_0^{+1}\rangle := | + 1 \rangle \propto e^{i\phi} \text{ and } |LG_0^{-1}\rangle := | - 1 \rangle \propto e^{-i\phi} \quad (2.22)$$

in accordance with respective handednesses and respecting the map given by Equation (2.8).

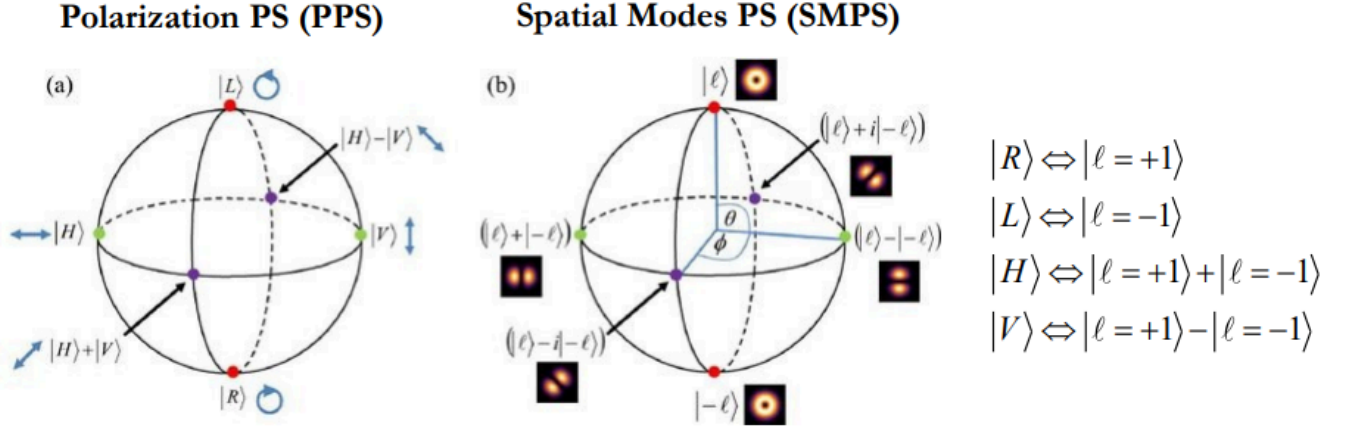


Fig 8: The polarization Poincaré Sphere (PPS) and Spatial Modes Poincaré Sphere (SMPS). The duality between them and visual representation of respective states is exemplified. Adapted from [14]

The result is a spatial intensity distribution of LG modes strictly analogous to the direction of polarization states. For instance, evaluating the azimuthal phase terms of the point given by $(r, \theta, \phi) = (1, \pi/2, 0)$, we find:

$$|H'\rangle = \frac{1}{\sqrt{2}}(| + 1 \rangle + | - 1 \rangle) \propto \cos(\phi) \quad (2.23)$$

Considering the intensity associated with $|H'\rangle$ state, we obtain:

$$I_{H'} \propto |H'|^2 \propto \cos^2(\phi) \quad (2.23a)$$

which corresponds to the intensity distribution of horizontally polarized light [13].

Furthermore, the polarization states description with Cartesian coordinates $|H\rangle$ and $|V\rangle$ maps directly onto the Hermite-Gauss functions (HG), another family of solutions of the Paraxial Helmholtz equation.

It is worth noting that a LG solution can represent only a component of the electric field. Consequently an electric field vector, using Jones formalism, can be expressed as a linear combination of polarization states and LG spatial modes. In particular, this description connects polarization and spatial structure in a non-separable way, as we will discuss in the following section [13].

2.3 Vector beams

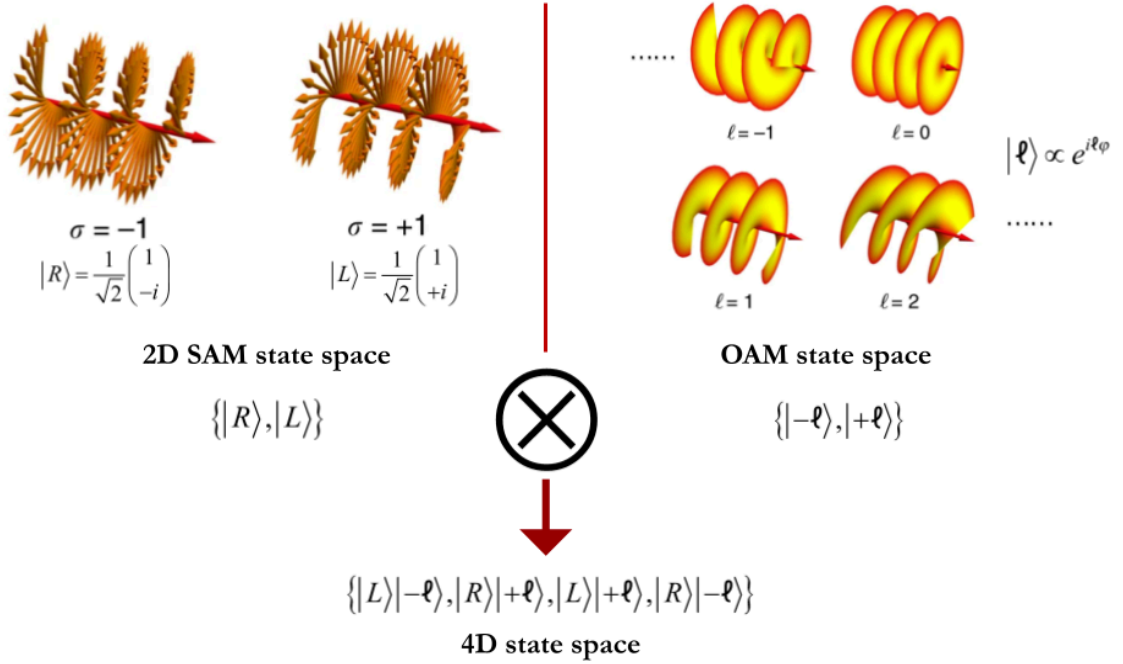
The Jones formalism can be generalized with tensor product between its spatial domain (2.22) and polarization space, then between the LG basis and circularly polarized basis (2.6) and the associated inner product (30c), following quantum mechanical formalism:

$$\mathcal{H} = \mathcal{H}_{\text{orb}} \otimes \mathcal{H}_{\text{spin}} \cong L^2(\mathbb{R}^2) \otimes \mathbb{C}^2 \quad (2.24a)$$

$$\{|+1\rangle, |-1\rangle\} \otimes \{|R\rangle, |L\rangle\} = \{|+1\rangle|R\rangle, |+1\rangle|L\rangle, |-1\rangle|R\rangle, |-1\rangle|L\rangle\} \quad (2.24b)$$

$$\langle \Phi | \Psi \rangle = \sum_{i=1}^2 \iint_{\mathbb{R}^2} \Phi_i^*(\rho, \phi) \Psi_i(\rho, \phi) \rho d\rho d\phi \quad (2.24c)$$

as the paraxial propagation is unitary, preserving the orthogonality of the modes. Therefore, it is sufficient to evaluate the scalar product on a fixed transverse plane, making it dependent solely on the radial and azimuthal coordinates.



Y. Shen, et al. Optical vortices 30 years on, *Light Sci. & Appl.*, **8**, 90, 1-29 (2019).

Fig 9: Visual representation of circular polarization states and helicoidal wavefront of OAM beams. The Cartesian product between their bases gives a base for vector beams. Adapted from [14]

The four bases define the aforementioned Hybrid Poincaré Spheres, namely HPS#1 and HPS#2, through $\{|+1, L\rangle, |-1, R\rangle\}$ and $\{|+1, R\rangle, |-1, L\rangle\}$ bases respectively. The elements of the HPS#1 yield

vortex vector beams, while the elements of the HPS#2 yield antivortex vector beams, due to the respective matching and mis-matching between the polarization handedness and the chirality of the spatial mode's helicoidal wavefront. These vectors are also named spin-orbit vectors.

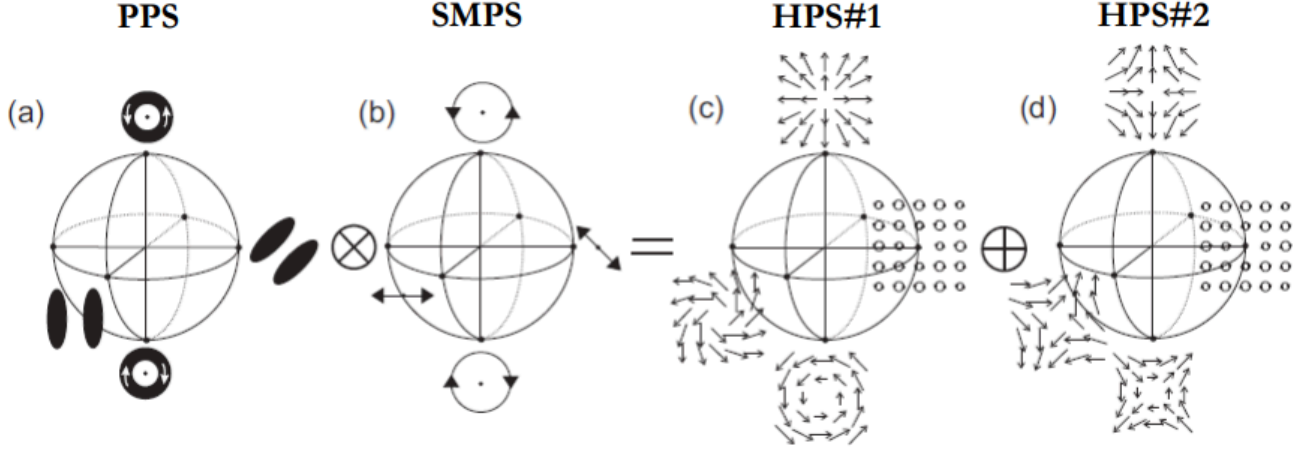


Fig 10: The Cartesian product between polarization Poincaré Sphere (PPS) and Spatial Modes Poincaré Sphere (SMPS) gives Hybrid Poincaré Spheres, the direct sum of HPS#1 and HPS#2, tractable via the $SU(2)$ algebra (formally isomorphous to the three-dimensional sphere S^3 algebra). In HPS lie non-separable combinations of polarization and spatial modes states. Adapted from [13]

A general vector in HPSs can be described recalling Equation (2.8):

$$|\Psi_{HPS1}(\psi, \chi)\rangle = \cos(\pi/4 - \chi)e^{-i\psi}|+\ell, L\rangle + \sin(\pi/4 - \chi)e^{+i\psi}|-\ell, R\rangle \quad (2.25a)$$

$$|\Psi_{HPS2}(\psi, \chi)\rangle = \cos(\pi/4 - \chi)e^{-i\psi}|+\ell, R\rangle + \sin(\pi/4 - \chi)e^{+i\psi}|-\ell, L\rangle \quad (2.25b)$$

For instance, if we work out the state $|R_v\rangle$ (radial vortex) in HPS#1 $(\psi, \chi) = (0, 0)$:

$$\begin{aligned} |R_v\rangle &= \frac{|+\ell, L\rangle + |-\ell, R\rangle}{\sqrt{2}} \propto \frac{e^{i\phi}(|H\rangle - i|V\rangle) + e^{-i\phi}(|H\rangle + i|V\rangle)}{2} = \\ &= \cos \phi |H\rangle + \sin \phi |V\rangle \equiv |\epsilon_\phi\rangle \end{aligned} \quad (2.26)$$

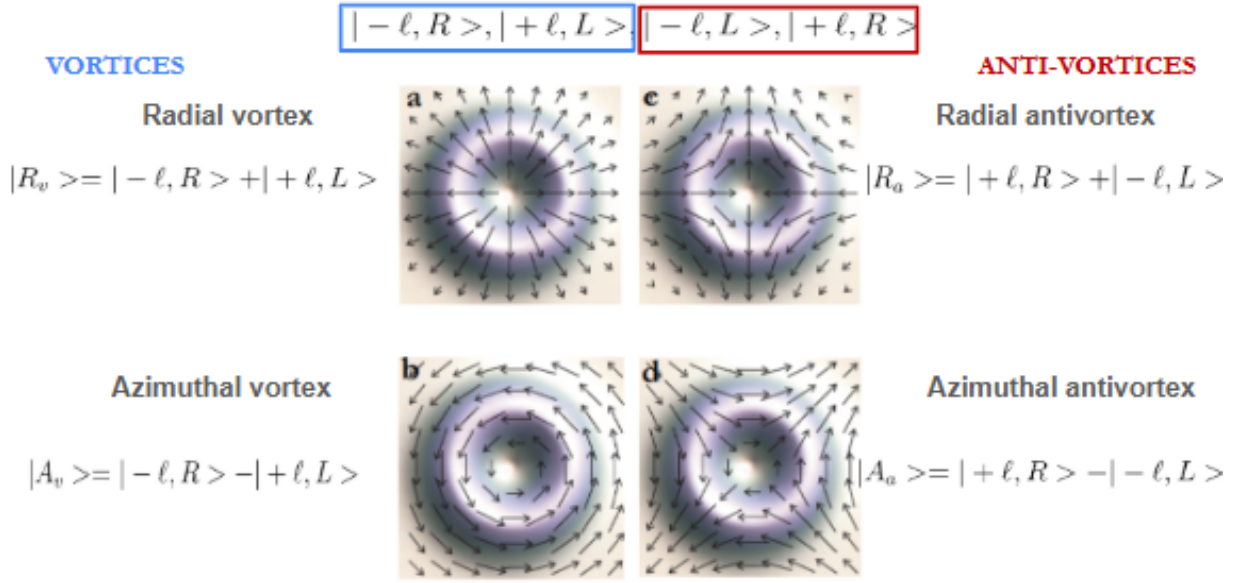


Fig 11: principal examples of the vectorial field generated by vortices and antivortices beams. Adapted from [14]

Knowing from the section 2.1 that a HWP inverts the handedness of polarization, this optical element becomes a simple physical link between HPS#1 and HPS#2.

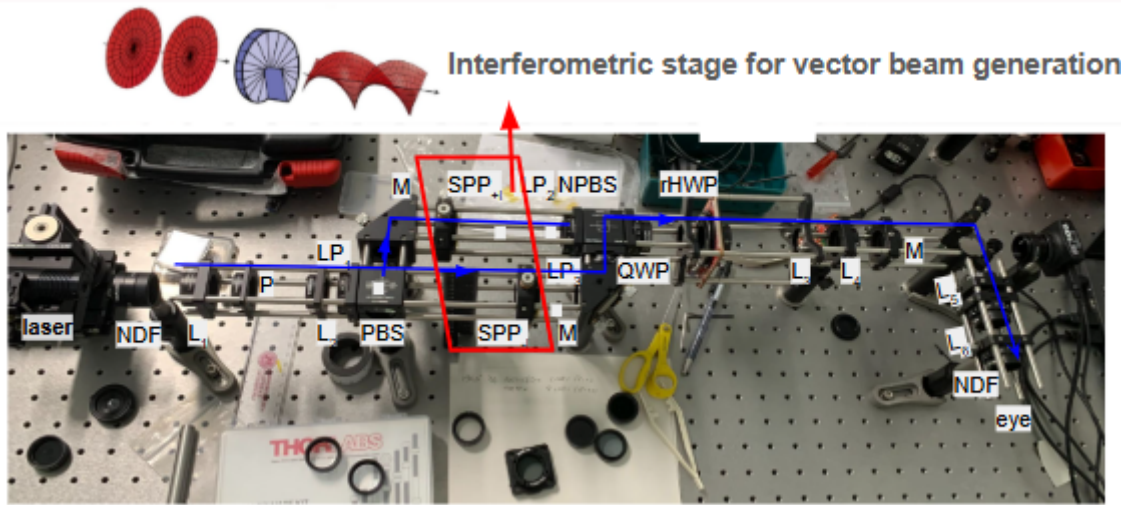
The following chapter describes the experimental setup arranged in the framework of this thesis, where there is the generation of vector beams produced through OAM beams with $|\ell| = 1, 2$ filtered using a linear polarizer, then to the human eye, to confirm human polarization sensitivity. This chapter formally describes a vortex beam produced through OAM beams at the first order. For $|\ell| = 2$ the mathematical description is analogous, then the superposition of azimuthal phase terms $e^{i2\phi}$ and $e^{-i2\phi}$ concerning two different orthogonal polarized states produces four constructive interference lobes after filtering. For superpositions of OAM polarized beams with $|\ell| > 1$ we talk of higher-order vector beams [13].

Hence the connection between classical electromagnetic properties of polarization and spatial configuration of OAM beams with semi-classical and quantum properties of intrinsic and extrinsic angular momentum, the spin and the orbital angular momentum respectively is totally defined.

Chapter 3: an experiment for human polarization sensitivity via structured light

3.1 Preliminary characterization of the research

In this experiment we have produced 4 vector beams through a Mach-Zehnder interferometer, by polarizing light vertically and horizontally along two different paths. Then we introduced a Spiral Phase Plate with $|\ell| = 1$ or $|\ell| = 2$ for each path, the essential element for the production of OAM beams (which will be explained shortly), always inserted with the same magnitude of topological charge but opposite sign on the two arms of the interferometric bench. The beams are reunited through a Beam Splitter and finally pass through a QWP oriented at 45° (defining horizontal polarization with 0°) which converts linear polarizations into circular polarizations. Then it is modified in a mechanically rotating HWP, whose role is to invert the polarization handednesses of the two orthogonal beams and impart a varying polarization between them, resulting into a rotating linear polarization at any point of the generated vector beams in order to bypass neural adaptation. The purpose of this work is to predict the interference profile intensity and confirm it with a polarizer and a camera first, then with the human eye, thanks to the macular dichroism of the Henle Fibre layers, as discussed before.



laser diode: 450nm, 10mW
P: pinhole

NDF: neutral density filter
 L_n : lens
M: mirror
LP: Linear Polarizer

(N)PBS: (Non-)Polarizing Beam Splitter
rHWP: rotating Half-Wave Plate
QWP: Quarter-Wave Plate
SPP ℓ : Spiral Phase Plate (ℓ topological charge)

Fig. 12: the optical table and experimental setup.

The experimental setup is powered by a 450 nm laser (TECBL-10GC-450-USB, World Star Tech) with a

controlled optical output power. A series of neutral density attenuators (NE03A, NE05A, NE10A, NE20A, NE30A, NE40A) are employed to reduce the intensity to values well below the exposure safety limit of the human eye. The beam conditioning is achieved through an optical system utilizing five plano-convex lenses (LA1951-A, LA1608-A, LA1509-A, LA1027-A) and one biconvex lens (LB1811-A-ML). Lenses 1 (LA1951-A) and 2 (LA1608-A) are configured as a 4f Fourier system, with a distance equal to the sum of their focal lengths, serving to expand and collimate the starting beam. A pinhole is positioned at the Fourier plane of this system to spatially filter the incident beam. Two additional pairs of lenses (lenses 3 and 4, lenses 5 and 6) are subsequently used to maintain the collimation of the Gaussian profile and adapt the beam width to the human pupil aperture. For polarization control and beam manipulation, the setup incorporates three linear polarizers on rotating supports (LPVISE100-A), a Polarizing Beam Splitter (CCM1-PBS251/M), and a Non-Polarizing Beam Splitter (CCM1-BS013/M). State control is facilitated by a motorized achromatic Half-Wave Plate (HWP), integrated with “Ello” software, and an achromatic Quarter-Wave Plate (QWP). The production of OAM beams is achieved using four Spiral Phase Plates (SPPs) with $\ell = 1, 2, 3, 4$, which constitute the essential elements of the setup and are the only components not provided by Thorlabs. The optical path is defined by a Mach-Zehnder interferometer configuration, employing three front-surface mirrors mounted on enclosed kinematic mirror mounts to deviate the final beam into a comfortable position for the human eye. Alignment is assisted by crosshair targets. Finally, the diagnostic stage consists of a power meter (PM16-120) for intensity measurements and a camera with a pixel size of $5.2 \mu m$ (DCC1545/M) to characterize the intensity profile.

The beam reached the human eye with a power of $P = (1.6 \pm 0.1) \cdot 10^{-3} \mu W$, a surface of $\Sigma = (0.28 \pm 0.02) mm^2$ and $\Sigma = (0.31 \pm 0.02) mm^2$ for the beams with $|\ell| = 1, 2$ respectively.

Consequently the irradiance are

$$\Phi(|\ell| = 1) = (5.7 \pm 0.4) \cdot 10^{-3} \mu W/mm^2, \Phi(|\ell| = 2) = (5.0 \pm 0.4) \cdot 10^{-3} \mu W/mm^2$$

in accordance with the ICNIRP Guidelines (International Commission on non-ionizing radiation protection) which allows for a maximum long-term exposure limit, in the blue frequency range, of $1 \mu W/mm^2$ [16].

3.2 Spiral Phase Plate (SPP) for the generation of OAM

The Spiral Phase Plate is an optical element with refractive index n_{SPP} and uniform spiral form, in particular exactly one turn of spiral, with a total longitudinal path accumulated of h_{tot} . The SPP imprints a helical phase term $e^{i\ell\phi}$ on the incident wave, producing the spatial structure of an OAM beam.

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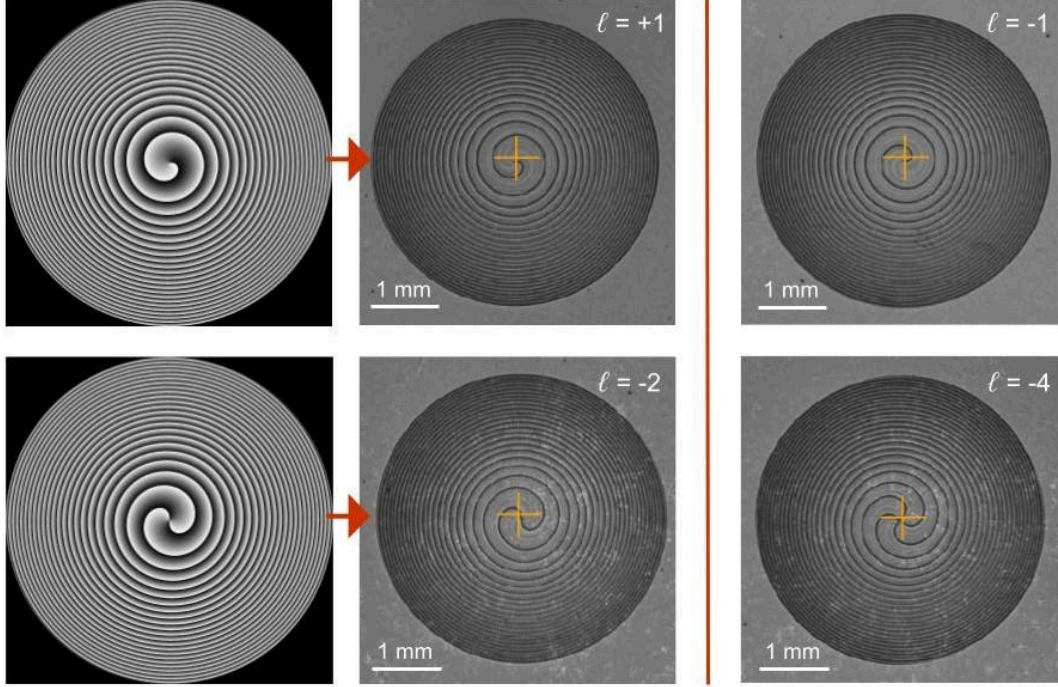


Fig 13: zoom-in of SPPs with $\ell = \pm 1, -2, -4$. Note the helicoidal structure (and its complexity increasing according to the topological charge value) and the different orientation in relation to $sgn(\ell)$. From [14]

The unique phase term influenced by the refractive index of the SPP is the spatial component of the phase. Then applying the Equation (2.12a) we obtain:

$$2\pi\ell = (n_{SPP} - 1)k_0 h_{tot} \rightarrow h_{tot} = \frac{\ell\lambda}{n_{SPP} - 1} \quad (3.1a)$$

And by the uniformity of the spiral:

$$h_{tot} : 2\pi = h(\phi) : \phi \rightarrow h(\phi) = \frac{\ell\lambda}{n_{SPP} - 1} \cdot \frac{\phi}{2\pi} \quad (3.1b)$$

The fabricated SPP are the diffractive version of a single-ramp spiral phase plate, since the phase is reported modulo 2π . Then, in the azimuthal direction the optical elements shows a number equal to ℓ of phase ramps from 0 to 2π , as shown in the figure. In addition, a focusing term is included.

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The SPPs employed in this work were fabricated by grayscale maskless optical lithography using a μ MLA laser writer (Heidelberg Instruments Mikrotechnik GmbH) present at the Department of Physics and Astronomy. In this technique, the desired surface relief is encoded as an 8-bit grayscale bitmap, in which each gray level corresponds to a specific UV exposure dose. The pattern is projected onto a photoresist layer - deposited over a transparent glass substrate - through a digital micromirror device (DMD), producing a spatially varying exposure, optimized for the working wavelength of 450 nm. After the development process, the exposed photoresist is converted into a continuous 3D surface relief, enabling the fabrication of high-quality diffractive SPP.

3.3 Mathematical representation of the construction of the beam

This central section describes step-by-step, using Jones formalism, the evolution of a laser beam incident on the optical path previously introduced. Before proceeding with the modeling, we recall the invariance of the polarization states by scaling the Jones vector by a global phase term. Moreover, we will not consider the absolute magnitude of the electric field, pointless for the purpose of this section, thus scaling generally global terms (as a consequence, there will be identifications between bases vectors and their scaling), to simplify the mathematical notation.

The beam, appropriately collimated and filtered by lenses 1 and 2 and the pinhole, should first have been split into two equal-intensity components orthogonally linearly polarized by the PBS, and subsequently acquiring the helical phase pattern through the SPP. The steps were inverted, due to inefficiency of the PBS. Consequently, the starting states are described by:

$$|\psi_+\rangle = |+\ell\rangle, |\psi_-\rangle = |-\ell\rangle \quad (3.2a), (3.2b)$$

where the spatial states $|\pm\ell\rangle$ are described by the imprinted helical phase factors $e^{\pm i\ell\phi}$ respectively, as previously discussed.

A polarizer before the defective PBS equally balances the two beams (the input laser is polarized); combining with two polarizers placed before the BS, they orthogonally polarize the beams, conventionally in x and y directions. We obtain:

$$|\Psi_+\rangle = |\psi_+, H\rangle = |+\ell, H\rangle, |\Psi_-\rangle = |\psi_-, V\rangle = |-\ell, V\rangle \quad (3.3a), (3.3b)$$

Then the beams are superposed in the BS. Introducing Jones formalism

$$|\Psi_0\rangle = |+\ell, H\rangle + |-\ell, V\rangle \propto e^{i\ell\phi}|H\rangle + e^{-i\ell\phi}|V\rangle \doteq \begin{pmatrix} e^{i\ell\phi} \\ e^{-i\ell\phi} \end{pmatrix} \quad (3.4)$$

Immediately after the BS, the QWP oriented at 45° relative to the polarized components imprints a phase shift of 90° between them. Using expression (2.10) and (2.12b) for Jones matrix, we exhibit principal steps of this derivation because it represents a comprehensive summary of the mathematics and the physics analyzed in this thesis:

$$\begin{aligned}
|\Psi_1\rangle &= J_{QWP}(\pi/4)|\Psi_0\rangle = R(\pi/4)J_{QWP}(0)R(-\pi/4)|\Psi_0\rangle \\
&\propto \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} e^{i\ell\phi} \\ e^{-i\ell\phi} \end{pmatrix} \propto \begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix} \begin{pmatrix} e^{i\ell\phi} \\ e^{-i\ell\phi} \end{pmatrix} \\
&= e^{i\ell\phi} \begin{pmatrix} 1 \\ -i \end{pmatrix} + e^{-i\ell\phi} \begin{pmatrix} -i \\ 1 \end{pmatrix} \doteq e^{i\ell\phi}|L\rangle - ie^{-i\ell\phi}|R\rangle
\end{aligned} \tag{3.5b}$$

Then formally:

$$|\Psi_1\rangle = |+\ell, L\rangle - i|-\ell, R\rangle \tag{3.5}$$

Expanding the results and using trigonometric identities we derive a measurable expression:

$$|\Psi_1\rangle \doteq \begin{pmatrix} e^{i\ell\phi} - ie^{-i\ell\phi} \\ -ie^{i\ell\phi} + e^{-i\ell\phi} \end{pmatrix} \propto \begin{pmatrix} \cos(\ell\phi) - \sin(\ell\phi) \\ \cos(\ell\phi) + \sin(\ell\phi) \end{pmatrix} = \begin{pmatrix} \cos(\ell\phi + \pi/4) \\ \cos(\ell\phi - \pi/4) \end{pmatrix} \tag{3.5a}$$

A $\pm 90^\circ$ rotation of the axis of the QWP converts a vortex into an antivortex and conversely. In particular for the rotation of $+90^\circ$, the resulting matrix is the reflection matrix of $J_{QWP}(\pi/4)$. Consequently, the vector beam thus produced is the spatial reflection of the first. Correspondingly, there is an inversion of the handedness of the starting vector. As a result, through calculations analogous to the previous ones, we obtain:

$$|\Psi_2\rangle = J_{QWP}(\frac{3\pi}{4})|\Psi_0\rangle \propto e^{-i\ell\phi}|L\rangle - ie^{i\ell\phi}|R\rangle = |-\ell, L\rangle - i|+\ell, R\rangle \tag{3.6}$$

And then:

$$|\Psi_2\rangle \propto \begin{pmatrix} \cos(\ell\phi - \pi/4) \\ \cos(\ell\phi + \pi/4) \end{pmatrix} \tag{3.6a}$$

The same symmetry can be analogously demonstrated between the antivortex induced by a QWP oriented at -45° and the vortex produced with a QWP at -125° , due to the π -axial symmetry of the QWP ($J_{QWP}(\pi/4) = J_{QWP}(-3\pi/4)$ and $J_{QWP}(-\pi/4) = J_{QWP}(3\pi/4)$). We note that the swap between SPPs exhibits the same result obtained through the inversion of handedness of the QWP previously expressed, because mathematically the swapping of the SPPs is exactly equal to the change of the helicoidal terms, the unique difference between the two spin-orbit states.

Now the beams pass through a HWP, which as explained in section 2.1 inverts the handedness of circular polarizations. However its useful function is to overcome neural adaptation through its rotation.

Mathematically an HWP generally rotated by an angle $\alpha(t)$ applied to a circular basis is represented by:

$$J_{HWP}(\alpha(t))|L\rangle = \begin{pmatrix} \cos 2\alpha(t) & -\sin 2\alpha(t) \\ -\sin 2\alpha(t) & -\cos 2\alpha(t) \end{pmatrix} \begin{pmatrix} 1 \\ -i \end{pmatrix} = \begin{pmatrix} e^{i2\alpha(t)} \\ ie^{i2\alpha(t)} \end{pmatrix} \propto |R\rangle \quad (3.7a)$$

$$J_{HWP}(\alpha(t))|R\rangle = \begin{pmatrix} \cos 2\alpha(t) & -\sin 2\alpha(t) \\ -\sin 2\alpha(t) & -\cos 2\alpha(t) \end{pmatrix} \begin{pmatrix} 1 \\ i \end{pmatrix} = \begin{pmatrix} e^{-i2\alpha(t)} \\ -ie^{-i2\alpha(t)} \end{pmatrix} \propto |L\rangle \quad (3.7b)$$

Thus the rotating HWP inverts the circular polarizations handedness and introduces a dynamic Pancharatnam-Berry relative phase shift of $2\alpha(t)$. For a vector beam this phase shift rigidly rotates the entire spatially-variant linear polarization pattern. When subsequently projected through a static polarizer, the transmitted interference maxima (lobes) rotate synchronously. This continuous dynamic sweeping of the visual stimulus is crucial to prevent the rapid onset of neural adaptation in human observers.

Finally, the following expressions show the four vector beams experimentally observed (with $|\ell| = 1, 2$):

$$|\Psi_{vortex,1}\rangle = | + 1, R\rangle - i | - 1, L\rangle \quad (3.8a)$$

$$|\Psi_{antivortex,1}\rangle = | - 1, R\rangle - i | + 1, L\rangle \quad (3.8b)$$

$$|\Psi_{vortex,2}\rangle = | + 2, R\rangle - i | - 2, L\rangle \quad (3.8c)$$

$$|\Psi_{antivortex,2}\rangle = | - 2, R\rangle - i | + 2, L\rangle \quad (3.8d)$$

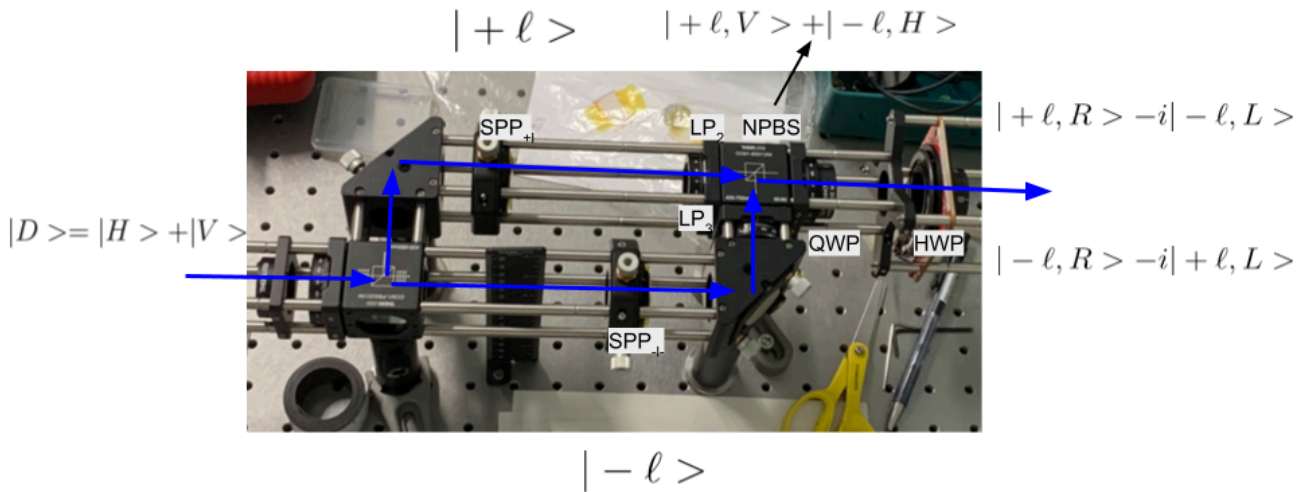


Fig. 14: visual description of vector beams generation through the optical table

Subsequently the camera has detected the effect of a linear polarizer on the beams, in order to confirm the correct construction of the vector beams. A polarizer rotated by a generic angle θ (by equations (2.10))

$$P(\theta) = \begin{pmatrix} \cos^2 \theta & \sin \theta \cos \theta \\ \sin \theta \cos \theta & \sin^2 \theta \end{pmatrix} \quad (3.9)$$

applied to general vortex and antivortex incident beams (following expressions (3.5) and (3.6) respectively)

$$P(\theta)|\Psi_1\rangle \propto \cos(\ell\phi - \theta + \pi/4) \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} \quad (3.10a)$$

$$P(\theta)|\Psi_2\rangle \propto \cos(\ell\phi + \theta - \pi/4) \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} \quad (3.10b)$$

Then the trigonometric elementary formulae for associated arcs and applying angle addition and subtraction formulas were exploited to achieve the result.

To calculate the number of intensity nodes, consequently the same number of intensity maxima (informally named “petals”), for instance in the case of a vortex, we must solve the following equation:

$$|\ell|\phi - \theta + \pi/4 = \pi/2 + n\pi \rightarrow \phi_n = \frac{\pi/4 + \theta + n\pi}{|\ell|} \quad (3.11a)$$

After solving the trivial equation with $n = 0$, by imposing the condition of periodicity, the previous expression is transformed into $\phi_N = \phi_0 + 2\pi$,

then substituting the expressions we obtain the result, equal for antivortex beams:

$$\frac{\pi/4 + \theta + N\pi}{|\ell|} = \frac{\pi/4 + \theta}{|\ell|} + 2\pi \rightarrow N = 2|\ell| \quad (3.11)$$

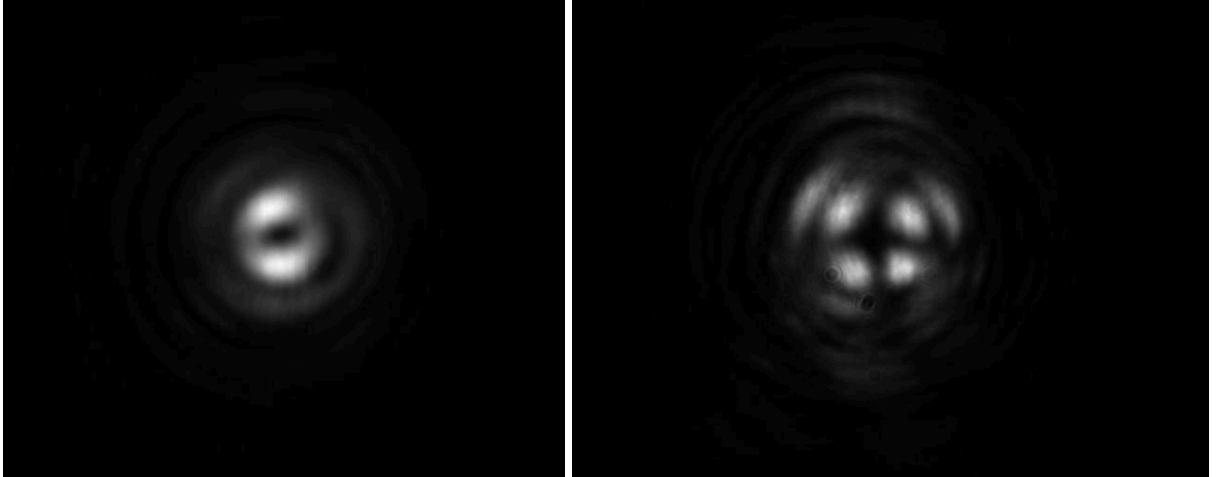


Fig. 15: Experimental acquisition of incident vector beams with $|\ell| = 1, 2$, respectively for two and four lobes, after filtering with a linear polarizer

Thus we should see two and four intensity maxima respectively for the states generated by SPPs with $|\ell| = 1$ and $|\ell| = 2$. We note that the sign of the angle θ in the expressions (3.11) highlights an interesting difference between the vortex and antivortex beams, which permits them to be easily distinguished experimentally: the rotation of the intensity extrema is in accordance with the rotation of the polarizer if the beam is a vortex, and conversely if it is an antivortex.

Finally, we show the expression of vector beams modified by a radial polarizer, which exactly physically describes the effect induced by macular pigments in the Henle's Fibre Layer. A radial polarizer can be represented by a linear polarizer with a global condition for the angle $\theta : \theta(\phi) = \phi$; locally, for each value of the helicoidal phase term, there is a Henle's Fibre parallel to it, which can be treated as a linear polarizer. Hence, by replacing the fixed angle θ with the dynamic angle ϕ in expressions (3.9) and then (3.11a), we conclude that the number of the intensity maxima that we should see is equal to

$$N_{vortex} = 2|\ell - 1| \text{ and } N_{antivortex} = 2|\ell + 1| \quad (3.12)$$

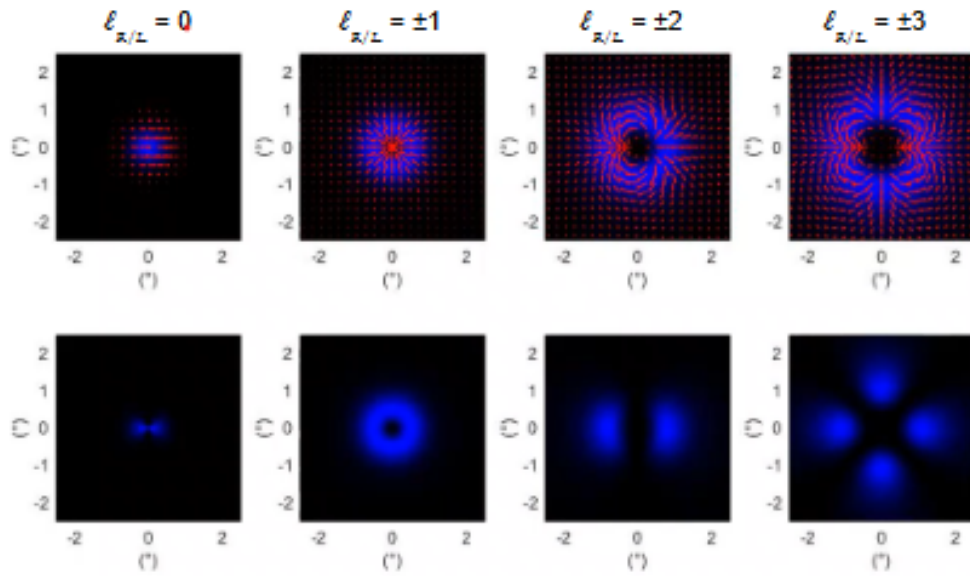


Fig. 16a: Visual representation of the human entoptic perception for **vortices**, in relation to the topological charge of the two superposed OAM beams. By counting the number of brushes, it is possible to infer the phase structure of the beam and detect orbital angular moment of light with the naked eye. Red arrows represent the vector field $E(\mathbf{r})$. Adapted from [14]

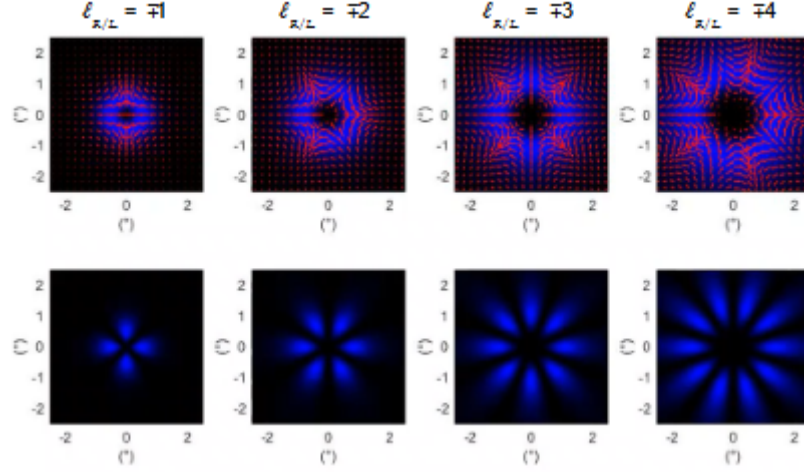


Fig. 16b: Visual representation of the human entoptic perception for **antivortices**, in relation to the topological charge of the two superposed OAM beams. By counting the number of brushes, it is possible to infer the phase structure of the beam and detect orbital angular momentum of light with the naked eye. Red arrows represent the vector field $E(\mathbf{r})$. Adapted from [14]

In particular for the vortex with $\ell = 1$, we have $N = 0$, physically a temporal pulsation of the intensity.

3.4 Results

Preliminary observations from a limited group of subjects confirmed the perception of the entoptic phenomenon. Hence, the experiment confirmed the predictions derived from the theories concerning human polarization sensitivity induced by the macular dichroism of the Henle's Fibre Layer, and the nature of OAM beams and vector beams, which have been analyzed in this dissertation. The major obstacles in the experiment, in addition to the inefficiency of the PBS previously described, were first achieving the equipartition of the intensity of the circularly polarized states via the setting of the QWP and the polarizers. However, the path with vertical polarization has maintained a range of three μW in the intensity of the linear polarization direction. The second principal difficulty was the centering of the SPP in the path and the elimination of the tilt between the two beams, due to the experimental stabilization limits of the SPPs in the lens holders and the optical table respectively. Regarding the observation of the pattern, the main problem was a granular haze that blurred the doughnut-shaped intensity profile, which was solved by resting the eye against the lens to prevent the focusing of the cornea and the crystalline lens. Finally, the great angular speed of the HWP made the phenomenon difficult to perceive. This last problem has been solved using constant rotation of the HWP controlled via a MATLAB script, rather than rotations at discrete angles, to create a clearly discernible visual pattern.

Conclusion

This study evidences the importance and the validity of structured light technology for investigating and analyzing Haidinger's brushes and, more generally, the human polarization sensitivity. Future work will aim to generalize our preliminary test into a group of subjects, in order to statistically verify the polarization perception through a psychophysical protocol.

Many scientists confirmed this potentiality of the structured light. For instance, D. Sarenac et al. (2020) have developed a similar experiment, where only one of the two generated beams passed on the SPP, while the other developed a relative phase shift: the test was successful, despite the higher value of the topological charge given by the SPP [17]. Moreover, D. Sarenac et al (2022) proposed a perception test with another instrument, the Liquid Optical Vortex (LOV) prism pair, incorporating a thin layer of liquid crystal inserts between the couple of prism, capable of modifying the geometrical phase (defined in section 2.1). The device acts as a spatially varying linear retarder, where the axis of birefringence is given by $\alpha(\phi) = q\phi + \alpha_0$, with $q = \pm 1/2$ named "charge" of the LOV prism. In particular this experiment managed to confirm the human sensitivity to static polarized stimuli (to overcome the neural adaptation, observers were instructed to continuously shift their focus between the beam and its optical centre) [18].

Equation (2.25) highlights the mathematical isomorphism between Jones vectors describing polarization states and the Hilbert space of spin-1/2 states. Under this correspondence, the Poincaré sphere is formally equivalent to the Bloch sphere, with right- and left-handed circular polarizations corresponding to the spin eigenstates along the quantization axis. This analogy extends naturally to vector beams, whose polarization and spatial degrees of freedom are classically non-separable. Such states provide a powerful platform for investigating spin-orbit coupling and classical analogues of quantum entanglement. In the quantum description of light, polarization and LG spatial modes correspond to the spin and orbital angular momentum degrees of freedom of single photons. In this context, the human visual system may represent an intriguing tool for probing classically non-separable spin-orbit states, while future psychophysical experiments could explore the perception of more complex structured states, including classical analogues of GHZ correlations [18].

In conclusion, the human eye can act as a detector for certain features of structured light, and the final scope is to use indirect information given by these type of tests to evaluate the distribution, density and correct functionality of macular pigments, the position of the fovea [17, 19], in order to aid in the early diagnosis linked to disease associated to these elements of the eye, such as macular degeneration (a condition very common among the elderly) central field visual dysfunction [17, 19, 20]. Moreover, structured light and its potentiality can be extended for studying and testing nearby visual human elements: the ocular media, all the transparent optical elements (consequently, for preventing the onset of cataracts), the ocular lag [21].

Perhaps the most remarkable result is that the human eye can act as a natural detector not only of polarization, but also of information encoded in the optical phase. Since orbital angular momentum is associated with the helical topology of the wavefront, the perception of structured vector beams demonstrates that, under appropriate conditions, the visual system can access phase-related information. This finding extends the conventional view of human vision, traditionally regarded as being limited to the perception of light intensity and wavelength (colours), revealing an additional sensitivity to the topology of structured optical fields.

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*“E se Dio esiste tu se nei un’evidenza,
i tuoi occhi non sono figli della scienza”*

