General modelling and validation measurements for permantent magnet and reluctance sychronous machines

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Chapter 1

Introduction

In the last 20 years due to the more attention of the human kind about the ecological and carbon-free technologies and the rate of depletion of the conventional energy sources is increasing day by day, the study of the electrical machine is becoming even more important. For this reason, we need to better understand the machine with a complete model that can thoroughly describe this object. In this thesis we start from a general doubly-fed machine, we express all the important equation that describe it, and throughout the document we try to simplify this model adding in each step a new hypothesis. First of all we define the electric machine, that is a general term for electric motors and electric generators. They are electromechanical energy converters: an electric motor converts electricity to mechanical power while an electric generator converts mechanical power to electricity.

We decided to focus on the electric motor. Most electric motors operate through interacting magnetic fields and current-carrying conductors to generate rotational force. Motors and generators have many similarities and many types of electric motors can be run as generators, and vice versa.

Among the different types of electrical machines we focus on the synchronous machine, to be more precise about the IPM, SPM and reluctance machines.

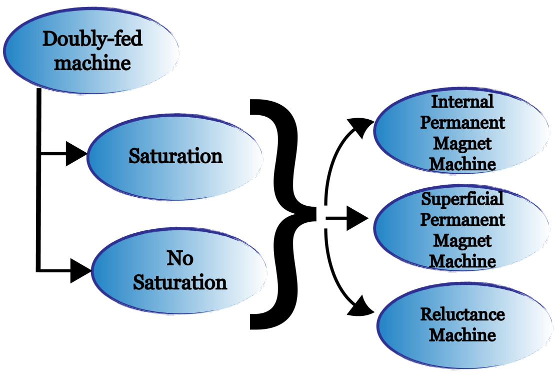

Figure 1.1: summary diagram of the steps performed in the thesis

Chapter 2

Basic Equations of an Electromechanical Energy Conversion System

In order to introduce the fundamentals of the Electromechanical Energy Conversion process, let us consider the simple variable reluctance system of fig:2.1. It has a single coil on the stationary element (stator) and an anisotropic rotating element (rotor). The position of the latter is given by the angle θ.
In this model it's assumed that hysteresis and eddy currents losses are null.

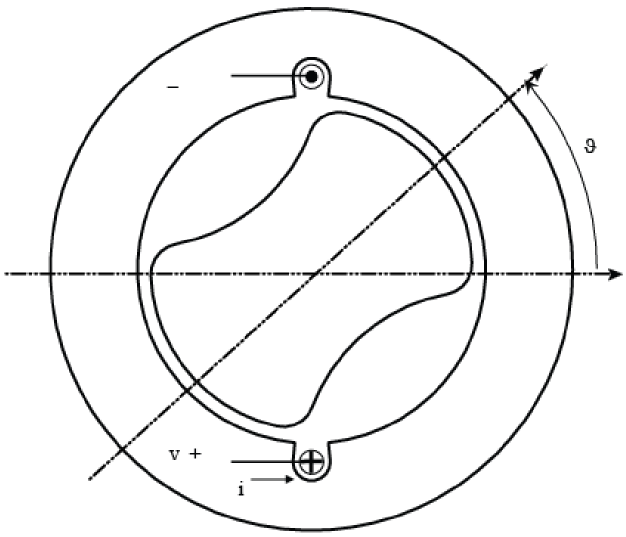

Figure 2.1: Graphical representation of the load sign convention

2.1 Voltage Equation

Assuming the passive terminal sign convention for voltage and current (load sign convention) the following expression can be written:

$$v = Ri + \frac{d\lambda}{dt} \tag{2.1.1}$$

Eq. 2.1.1 states that the applied voltage is balanced by the sum of the resistive voltage drop (Ri) and the back electromotive force due to the time variation of the flux linkage ($d\lambda/dt$).
In compliance with the physical assumptions above fixed, the flux linkage is a state function which depends on the current in the winding and the position of rotating element, according to a bi-univocal relationship between the current and the flux linkage for each rotor position given by the magnetization curve (shown in fig:2.2):

$$\lambda = \lambda(i, \theta) \tag{2.1.2}$$

the magnetization curve can be also reversed to obtain the current as a function of the flux linkage for each rotor position as

$$i = i(\lambda, \theta) \tag{2.1.3}$$

Voltage balance (2.1.1) can be then rewritten as:

$$\begin{aligned} v &= Ri + \frac{d\lambda}{dt} \\ &= Ri + \frac{\partial\lambda(i,\theta)}{\partial\theta}\frac{d\theta}{dt} + \frac{\partial\lambda(i,\theta)}{\partial i}\frac{di}{dt} \\ &= Ri + \underbrace{\frac{\partial\lambda(i,\theta)}{\partial\theta}\omega}_{\text{motional bemf}} + l(i,\theta)\frac{di}{dt} \end{aligned} \tag{2.1.4}$$

where $l(i, \theta) = \dfrac{\partial\lambda(i,\theta)}{\partial i}$ is the "differential inductance" which is the slope at the current level i of the magnetizing curve drawn for the position θ .
Flux linkage $\lambda(i, \theta)$ can be also expressed in the form:

$$\lambda(i, \theta) = L(i, \theta)i$$

where $L(i, \theta)$ is the "apparent inductance". Substituting in 2.1.1 we obtain:

$$\begin{aligned} v &= Ri + \frac{d}{dt}(L(i,\theta)i) \\ &= Ri + \frac{dL(i,\theta)}{dt}i + L(i,\theta)\frac{di}{dt} \\ &= Ri + \underbrace{\frac{\partial L(i,\theta)}{\partial\theta}\frac{d\theta}{dt}i}_{\text{bemf}} + \underbrace{\left[\frac{\partial L(i,\theta)}{\partial i}i + L(i,\theta)\right]}_{l(i,\theta)}\frac{di}{dt} \end{aligned} \tag{2.1.5}$$

By comparing 2.1.5 with 2.1.4 one can realizes that the last term gives the relation of the differental inductance vs. apparent inductance. The reverse equation can be derived observing that:

$$\lambda(i,\theta) = \int_0^i l(i,\theta)di = L(i,\theta)i \tag{2.1.6}$$

Then

$$L(i,\theta) = \frac{\int_0^i l(i,\theta)di}{i} \tag{2.1.7}$$

2.2 Energy and Coenergy Equations

For each angular position θ, magnetic energy is given by[1]:

$$w_m(\lambda,\theta) = \int_0^\lambda i(\lambda,\theta)d\lambda \tag{2.2.1}$$

Similarly coenergy is defined by[2]:

$$w'_m(i,\theta) = \int_0^i \lambda(i,\theta)di \tag{2.2.2}$$

Energy and coenergy can be represented by the areas of Fig: 2.2 from which it's easy to prove the relation[3]:

$$w_m + w'_m = \lambda i \tag{2.2.3}$$

2.3 Torque Equations

The energy balance can be obtain in the infinitesimal interval dt by multiplying both sides of 2.1.1 by idt:

$$\underbrace{iv\mathrm{d}t}_{\text{Electrical work absorbed}} = \underbrace{iR i\mathrm{d}t}_{\text{Joule losses}} + \underbrace{i\mathrm{d}\lambda}_{\text{Energy stored+Mechanical output}} \tag{2.3.1}$$

If dw_m is the infinitesimal variation of the energy stored in the system and $\tau d\theta$ the delivered mechanical work, the following balance equation can be written:

$$i\mathrm{d}\lambda = \mathrm{d}w_m + \tau d\theta \tag{2.3.2}$$

[1] The inverse of 2.2.1 is:

$$i(\lambda,\theta) = \frac{\partial w_m(\lambda,\theta)}{\partial\lambda}$$

[2] The inverse of 2.2.2 is:

$$\lambda(i,\theta) = \frac{\partial w'_m(i,\theta)}{\partial i}$$

[3] The relation can be proved mathematicaly by integrating both sides of:

$$\partial(\lambda,i) = \lambda\partial i + i\partial\lambda$$

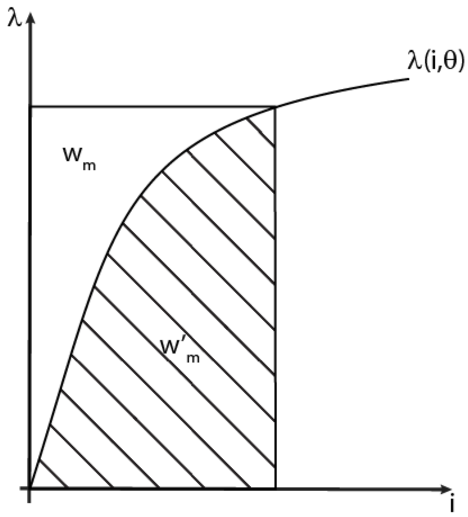

Figure 2.2: Graphical representation of energy and coenergy

The infinitesimal variation of the stored energy is given by the following differential form:

$$\mathrm{d}w_m = \frac{\partial w_m(\lambda,\theta)}{\partial \lambda}\mathrm{d}\lambda + \frac{\partial w_m(\lambda,\theta)}{\partial \theta}\mathrm{d}\theta \tag{2.3.3}$$

By substituting 2.3.3 in 2.3.2 it yields to:

$$i\mathrm{d}\lambda - \tau d\theta = \frac{\partial w_m(\lambda,\theta)}{\partial \lambda}\mathrm{d}\lambda + \frac{\partial w_m(\lambda,\theta)}{\partial \theta}\mathrm{d}\theta \tag{2.3.4}$$

Since the state variable are independent quantities it implies:

$$\underbrace{i(\lambda,\theta) = \frac{\partial w_m(\lambda,\theta)}{\partial \lambda}}_{\text{winding current}} \tag{2.3.5}$$

that confirms footnote[1] and

$$\underbrace{\tau(\lambda,\theta) = -\frac{\partial w_m(\lambda,\theta)}{\partial \theta}}_{\text{Torque expression}} \tag{2.3.6}$$

that gives the first form of the torque equation.
The torque can be also expressed as function of i and θ; to this purpose the coenergy $w'_m(i,\theta)$ is expressed by

$$w'_m(i,\theta) = i\lambda - w_m(\lambda,\theta) \tag{2.3.7}$$

By differentiating and substituting in 2.3.2:

$$dw'_m(i,\theta) = \lambda di + \tau d\theta \tag{2.3.8}$$

This equation can be used to get a new torque expression as a function of (i,θ):

$$\tau(i,\theta) = \frac{\partial w'_m(i,\theta)}{\partial \theta} \tag{2.3.9}$$

and also

$$\lambda(i,\theta) = \frac{\partial w'_m(i,\theta)}{\partial i} \tag{2.3.10}$$

that confirm footnote[2].

2.4 Linear(not saturated) System

In the case of not saturated system the relationship between flux linkage and current is linear for each position of θ

$$\lambda(i,\theta) = L(\theta)i$$

Therefore differential and apparent inductance coincide:

$$l(\theta) = L(\theta)$$

as well as energy and coenergy:

$$w_m(i,\theta) = w'_m(i,\theta)$$

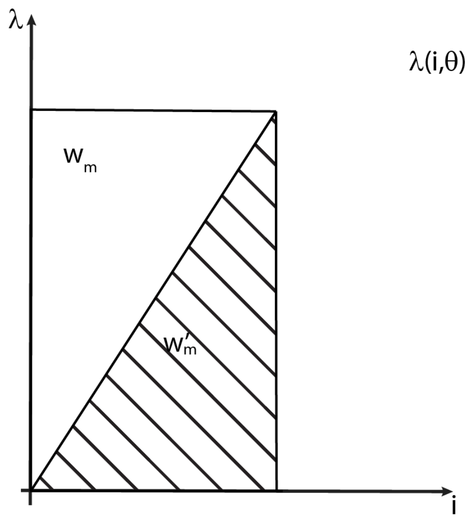

Figure 2.3: Graphical representation of energy and coenergy in linear machine

Consequently the voltage equation becomes:

$$\begin{aligned} v &= Ri + \frac{d}{dt}[L(\theta)i] \\ &= Ri + \frac{dL(\theta)}{dt}i + L(\theta)\frac{di}{dt} \\ &= Ri + \frac{\partial L(\theta)}{\partial \theta}\frac{d\theta}{dt}i + L(\theta)\frac{di}{dt} \end{aligned} \tag{2.4.1}$$

while energy and co-energy are now given by:

$$\begin{aligned} w_m &= \int_0^{\lambda} i(\lambda,\theta)d\lambda = \int_0^{\lambda} \frac{\lambda(i,\theta)}{L(\theta)}d\lambda \\ &= \frac{\lambda^2}{2L(\theta)} = \frac{L(\theta)i^2}{2} \\ &= \int_0^{i} [L(\theta)i]di = \int_0^{i} \lambda(i,\theta)di \\ &= w'_m \end{aligned} \tag{2.4.2}$$

and eventually the torque can be expressed by one of the following equations:

$$\begin{aligned} \tau(i,\theta) &= \frac{\partial w_m(i,\theta)}{\partial\theta} = \frac{\partial}{\partial\theta}\left[\frac{L(\theta)i^2}{2}\right] \\ &= \frac{1}{2}\frac{\partial L(\theta)}{\partial\theta}i^2 \\ &= \frac{1}{2}\frac{\partial\lambda(i,\theta)}{\partial\theta}i \end{aligned} \tag{2.4.3}$$

Chapter 3

General equation of the synchronous machine

The general equations of the electromechanical energy conversion introduced in Chapter 2 is herafter applied to a three-phase machine. The sole assumptions of no magnetic hysteresis and no eddy current are fixed in this Chapter. For the sake of generality a doubly-fed three-phase machine (a machine with three phase windings both on the stator and on the rotor) is at first consider. The machine will be described through actual stator and rotor voltages and currents as well as using quantities in different two axis reference frames. Then, by imposing appropriate conditions on rotor currents, the equations can be arranged for describing different types of synchronous machines, with Permanent Magnet (PM) excited rotor or with pure reluctance rotor. It is worth noticing that the description is an in-out modelling which uses only terminal and shaft quantities. This approach complies with the modern general theory of the electrical drives and prepare the basics for model identification by terminal measurements.

3.1 Equations of a doubly-fed three-phase machine

X	no hysteresis
X	no eddy currentes

We define a general machine called "doubly fed synchronous machine" which can be powered either through the stator or the rotor windings. This will be useful since it allows us to show all the steps from a general machine to a more specific machine like the reluctance one.
We'll use the subscript "sa,sb,sc" for all the quantities related to three stator phases and "ra,rb,rc" for those of the windings of the rotor.
The equations of a doubly-fed three-phase electrical machine can be expressed

for the three stator phasse as follows:

$$\begin{cases} v_{sa} = R_s i_{sa} + \dfrac{d\lambda_{sa}}{dt} \\ v_{sb} = R_s i_{sb} + \dfrac{d\lambda_{sb}}{dt} \\ v_{sc} = R_s i_{sc} + \dfrac{d\lambda_{sc}}{dt} \end{cases} \tag{3.1.1}$$

The same equations can be also expressed in a compact matrix form as:

$$\underline{v}_s = R\underline{i}_s + \frac{d\underline{\lambda}_s}{dt} \tag{3.1.2}$$

Similar equations can be written for the three-phase rotor windings simply replacing the index (subscript) s with r.
The two sets of equations can be merged to obtain a general form of a doubly-fed three-phase machine model. Assuming:

$$\underline{v} = \begin{bmatrix} \underline{v}_s \\ \underline{v}_r \end{bmatrix} \tag{3.1.3}$$

$$\underline{i} = \begin{bmatrix} \underline{i}_s \\ \underline{i}_r \end{bmatrix} \tag{3.1.4}$$

$$\underline{\lambda} = \begin{bmatrix} \underline{\lambda}_s \\ \underline{\lambda}_r \end{bmatrix} \tag{3.1.5}$$

with:

$$\underline{v}_s = \begin{bmatrix} v_{sa} \\ v_{sb} \\ v_{sc} \end{bmatrix}; \ \underline{v}_r = \begin{bmatrix} v_{ra} \\ v_{rb} \\ v_{rc} \end{bmatrix}$$

$$\underline{i}_s = \begin{bmatrix} i_{sa} \\ i_{sb} \\ i_{sc} \end{bmatrix}; \ \underline{i}_r = \begin{bmatrix} i_{ra} \\ i_{rb} \\ i_{rc} \end{bmatrix}$$

$$\lambda_s = \begin{bmatrix} \lambda_{sa} \\ \lambda_{sb} \\ \lambda_{sc} \end{bmatrix}; \ \lambda_r = \begin{bmatrix} \lambda_{ra} \\ \lambda_{rb} \\ \lambda_{rc} \end{bmatrix}$$

The general voltage equation can be then defined in matrix form as:

$$\underline{v} = \underline{R} \cdot \underline{i} + \frac{d\underline{\lambda}}{dt} \tag{3.1.6}$$

Because hysteresis and eddy current are supposed to be absent, there's a bi-univocal relationship between the current and flux linkage expressed by the relation $\underline{\lambda} = \underline{\lambda}(\underline{i}, \theta)$. According to the previous equation, each flux linkage depends on all the currents and on the position according to:

$$\underline{\lambda} = \underline{\lambda}(i_{sa}, ..., i_{ra}, ..., \theta) = \begin{bmatrix} \lambda_{sa}(i_{sa}, ..., i_{ra}, ..., \theta) \\ \vdots \\ \lambda_{ra}(i_{sa}, ..., i_{ra}, ..., \theta) \\ \vdots \end{bmatrix} \tag{3.1.7}$$

and also all the currents can be expressed as a function of the flux linkages and position as

$$\underline{i} = \underline{i}(\lambda_{sa}, ..., \lambda_{ra}, ..., \theta) = \begin{bmatrix} i_{sa}(\lambda_{sa}, ..., \lambda_{ra}, ..., \theta) \\ \vdots \\ i_{ra}(\lambda_{sa}, ..., \lambda_{ra}, ..., \theta) \\ \vdots \end{bmatrix} \tag{3.1.8}$$

Similarly to eq.(2.1.5) the eq.(3.1.6) can be transformed in:

$$\underline{v} = \underline{R} \cdot \underline{i} + \frac{\partial \underline{\lambda}}{\partial \theta}\frac{d\theta}{dt} + \underline{l}\frac{di}{dt} \tag{3.1.9}$$

where:

$$\begin{aligned} \underline{l} = \underline{l}(i_{sa}, ..., i_{ra}, ..., \theta) &= \begin{bmatrix} \frac{\partial \lambda_{sa}}{\partial i_{sa}} & \cdots & \frac{\partial \lambda_{sa}}{\partial i_{rc}} \\ \vdots & \ddots & \vdots \\ \frac{\partial \lambda_{rc}}{\partial i_{sa}} & \cdots & \frac{\partial \lambda_{rc}}{\partial i_{rc}} \end{bmatrix} \\ &= \begin{bmatrix} l_{sa,sa} & l_{sa,sb} & \cdots & l_{sa,rc} \\ \vdots & & & \vdots \end{bmatrix}_{6x6} \end{aligned} \tag{3.1.10}$$

is the differential inductance matrix.
Each of the differential inductance is a function of all the currents and of position(as an example:$l_{sa,sa} = l_{sa,sa}(i_{sa}, ..., i_{ra}, ..., \theta)$).

3.1.1 Energy Equations

The energy balance can be obtain by multiplying the 2.1.1 by $i\mathrm{d}t$:

$$\underbrace{\underline{i}^t \underline{v}\mathrm{d}t}_{\text{Electrical work absorbed}} = \underbrace{\underline{i}^t \underline{Ri}\mathrm{d}t}_{\text{Joule losses}} + \underbrace{\underline{i}^t \mathrm{d}\underline{\lambda}}_{\text{Energy stored+Mechanical output}} \tag{3.1.11}$$

If we call dw_m the variation of the energy stored in the system and $\tau d\theta$ the mechanical work delivered at the shaft, the equation can be written as follow:

$$\underline{i}^t \mathrm{d}\underline{\lambda} = \mathrm{d}w_m + \tau d\theta \tag{3.1.12}$$

The magnetic energy is a function of all the current and position, so that it can be written as $w_m = w_m(i_{sa}, ..., i_{ra}, ..., \theta)$.

The infinitesimal variation of stored energy can be then expressed by the following differential form:

$$\begin{aligned} \mathrm{d}w_m &= \frac{\partial w_m}{\partial \lambda_{sa}}\mathrm{d}\lambda_{sa} + \cdots + \frac{\partial w_m}{\partial \lambda_{ra}}\mathrm{d}\lambda_{ra} + \cdots + \frac{\partial w_m}{\partial \theta}\mathrm{d}\theta \\ &= \sum_{\forall \lambda} \frac{\partial w_m}{\partial \lambda}\mathrm{d}\lambda + \frac{\partial w_m}{\partial \theta}\mathrm{d}\theta \end{aligned} \tag{3.1.13}$$

By substituting 2.3.3 in 2.3.2:

$$\underline{i}\mathrm{d}\underline{\lambda} - \tau d\theta = \sum_{\forall \lambda} \frac{\partial w_m}{\partial \lambda}\mathrm{d}\lambda + \frac{\partial w_m(\underline{\lambda}, \theta)}{\partial \theta}\mathrm{d}\theta \tag{3.1.14}$$

Since the state variables are independent quantities:

$$\underbrace{\underline{i}^t(\underline{\lambda}, \theta) = \frac{\partial w_m}{\partial \lambda}}_{\text{winding current}} \tag{3.1.15}$$

that generalizes 2.3.5 for a multiphase machine and

$$\underbrace{\tau(\underline{\lambda}, \theta) = -\frac{\partial w_m(\underline{\lambda}, \theta)}{\partial \theta}}_{\text{Torque expression}} \tag{3.1.16}$$

which coincides to 2.3.6 that remain valid

The torque can be also expressed as function of i and θ; to this purpose we define again a state function called "coenergy" $w'_m(i, \theta)$:

$$w'_m(\underline{i}, \theta) = \underline{i\lambda} - w_m(\underline{\lambda}, \theta) \tag{3.1.17}$$

By differentiating and substituting in 2.3.2:

$$dw'_m(\underline{i}, \theta) = \underline{\lambda} d\underline{i} + \tau d\theta \tag{3.1.18}$$

This equation can be used to get a new torque expression in function of (i, θ):

$$\tau(\underline{i}, \theta) = \frac{\partial w'_m(\underline{i}, \theta)}{\partial \theta} \tag{3.1.19}$$

3.2 Equations of a doubly-fed two-phase machine

The equation of the doubly-fed machine can be expressed assuming equivalent two phase ststor and rotor windings that will be denoted by "$\alpha\beta$" hereafter. Of course, if the machine is really a two-phase machine this assertionis trivial. Otherwise the equivalent two-phase configuration can be obtained a coordinate transformation from "a,b,c" to "$\alpha\beta$" made by using the Clake matrix transformation that we'll call $\underline{C}$. The matrix can be simplified if we consider the homopolar component equal to zero, so $\underline{C}$ can be written as a 2x3 matrix as follows:

$$\begin{bmatrix} x_\alpha \\ x_\beta \end{bmatrix} = C \begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix} \tag{3.2.1}$$

where

$$C = \frac{2}{3} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \tag{3.2.2}$$

It can be also defined an "inverse Clark transformation" matrix $\underline{C}^{-1}$ that is a simplified inverse matrix of $\underline{C}$.

$$C^{-1} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \tag{3.2.3}$$

It appears that:

$$C^{-1}v_{s,\alpha\beta} = RC^{-1}i_{s,\alpha\beta} + \frac{d}{dt}(C^{-1}\lambda_{s,\alpha\beta}) \tag{3.2.4}$$

$$v_{s,\alpha\beta} = Ri_{s,\alpha\beta} + \frac{d\lambda_{s,\alpha\beta}}{dt} \tag{3.2.5}$$

Similar equations can be derived for the rotor quantities applying again the matrix $\underline{C}$.In other words stator and rotor are described in their actual two-phase variables.

3.2.1 Energy Equations

We'll use the symbol X_{xy} (with $x = s, r$ and $y = \alpha, \beta$) to indicate the vector which elements are:

$$\begin{bmatrix} X_{s\alpha} \\ X_{s\beta} \\ X_{r\alpha} \\ X_{r\beta} \end{bmatrix}$$

The general equation for an equivalent two-phase machine can be written as:

$$\underline{v}_{xy} = \underline{Ri}_{xy} + \frac{d\underline{\lambda}_{xy}}{dt} \tag{3.2.6}$$

The energy balance can be obtain by multiplying the 3.2.6 by $i^t_{xy}dt$:

$$\underbrace{\underline{i}^t_{xy}\underline{v}_{xy}\mathrm{d}t}_{\text{Electrical work absorbed}} = \underbrace{\underline{i}^t_{xy}\underline{Ri}_{xy}\mathrm{d}t}_{\text{Joule losses}} + \underbrace{\underline{i}^t_{xy}\mathrm{d}\underline{\lambda}_{xy}}_{\text{Energy stored+Mechanical output}} \tag{3.2.7}$$

If we call dw_m the infinitesimal variationof the energy stored in the system and $\tau d\theta$ the infinitesimal mechanical work delivered ate the shaft, the equation can be rewritten as follows:

$$\frac{3}{2}\underline{i}^t_{xy}\mathrm{d}\underline{\lambda}_{xy} = \mathrm{d}w_m + \tau d\theta \tag{3.2.8}$$

Let us remember that $\underline{\lambda}_{xy} = \underline{\lambda}_{xy}(i_{xy}, \theta)$ and $w_m = w_m(i_{xy}, \theta)$. The variation of the stored energy can be then expressed by the following differential form:

$$\begin{aligned} \mathrm{d}w_m &= \frac{\partial w_m}{\partial \lambda_{s\alpha}}\mathrm{d}\lambda_{s\alpha} + \cdots + \frac{\partial w_m}{\partial \lambda_{r\alpha}}\mathrm{d}\lambda_{r\alpha} + \cdots + \frac{\partial w_m}{\partial \theta}\mathrm{d}\theta \\ &= \sum_{\forall \lambda_{xy}} \frac{\partial w_m}{\partial \lambda}\mathrm{d}\lambda + \frac{\partial w_m}{\partial \theta}\mathrm{d}\theta \end{aligned} \tag{3.2.9}$$

The torque can be expressed as follow:

$$\tau(\underline{\lambda}_{xy}, \theta) = -\frac{\partial w_m(\underline{\lambda}_{xy}, \theta)}{\partial \theta} \tag{3.2.10}$$

or, through the co-energy:

$$\tau(\underline{i}_{xy}, \theta) = \frac{\partial w'_m(\underline{i}_{xy}, \theta)}{\partial \theta} \tag{3.2.11}$$

3.3 Permanent Magnet and Reluctance Machine

Equation derived in the previous two Sections can be particularized to the case of a PM (Permanent Magnet) excited machine.
To this purpose the PM in the rotor is modelled as a constant current excitation in the α-axis. Thus the currents of the rotor must be imposed as:

1. For a three-phase system (assuming homopolar current $i_{ro} = 0$)

$$\begin{cases} i_{ra} = I_{mg} = cost \\ i_{rb} = -\dfrac{I_{mg}}{2} \\ i_{rc} = -\dfrac{I_{mg}}{2} \end{cases} \tag{3.3.1}$$

2. For a two-phase system (homopolar current i_{ro} is zero for definition)

$$\begin{cases} i_{r\alpha} = I_{mg} = cost \\ i_{r\beta} = 0 \end{cases} \tag{3.3.2}$$

The model to describe a reluctance machine is obtained setting the excitation current of the rotor equal to zero. It follows that the currents are:

1. For a three-phase system

$$\begin{cases} i_{ra} = 0 \\ i_{rb} = 0 \\ i_{rc} = 0 \end{cases} \tag{3.3.3}$$

2. For a two-phase system

$$\begin{cases} i_{r\alpha} = 0 \\ i_{r\beta} = 0 \end{cases} \tag{3.3.4}$$

The torque equation shown in the previous paragraph can be also applied for PM and reluctance machine(eq.3.2.11 and eq.3.2.10).

3.4 Equation for synchronous machine in "d,q"

The equation for synchronous machine can be expressed according to different two-axis reference frames. It's defined a system called "dq" that is rotating with angular speed of ω_{dq}.
The coordinates transformation from "$\alpha\beta$" to "dq" can be made using the Park matrix denoted by $\underline{P}$.

$$\begin{bmatrix} x_d \\ x_q \end{bmatrix} = \underline{P} \begin{bmatrix} x_\alpha \\ x_\beta \end{bmatrix} \tag{3.4.1}$$

$$\underline{P} = \begin{bmatrix} \cos(\theta_{dq}) & \sin(\theta_{dq}) \\ -\sin(\theta_{dq}) & \cos(\theta_{dq}) \end{bmatrix} \tag{3.4.2}$$

while inverse Park transfomation from "dq" to "$\alpha\beta$" is

$$\underline{P}^{-1} = \begin{bmatrix} \cos(\theta_{dq}) & -\sin(\theta_{dq}) \\ \sin(\theta_{dq}) & \cos(\theta_{dq}) \end{bmatrix} \tag{3.4.3}$$

In order to distinguish the different two-axis reference frames, a superscript (apex) is used: "s" denotes stationary (fixed with the stator) reference frame for which $\omega_{dq} = 0$; "r" is for a reference frame fixed with the stator for which $\omega_{dq} = \omega_{me}$ and so on.

From eq.(3.2.4) using the transformation matrix P we obtain:

$$\begin{aligned} \underline{P}^{-1}\underline{v}_{s,dq} &= R\underline{P}^{-1}\underline{i}_{s,dq} + \frac{d}{dt}(\underline{P}^{-1}\underline{\lambda}_{s,dq}) \\ &= R\underline{P}^{-1}\underline{i}_{s,dq} + \underline{P}^{-1}\frac{d}{dt}(\underline{\lambda}_{s,dq}) + \underline{\lambda}_{s,dq}\frac{d}{dt}(\underline{P}^{-1}) \\ &= R\underline{P}^{-1}\underline{i}_{s,dq} + \underline{P}^{-1}\frac{d}{dt}(\underline{\lambda}_{s,dq}) + \underline{\lambda}_{s,dq}(\omega_{dq}J\underline{P}^{-1}) \end{aligned} \tag{3.4.4}$$

with:

$$J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \tag{3.4.5}$$

$$v_{s,dq} = Ri_{s,dq} + \frac{\mathrm{d}\lambda_{s,dq}}{\mathrm{d}t} + \omega_{dq}J\lambda_{s,dq} \tag{3.4.6}$$

It can be done the same procedures to obtain the voltage equation for the rotor.[4]
To derive a more general equation we'll define the transformation matrix $\underline{T}$ as:

$$\begin{aligned} \underline{T} &= \begin{bmatrix} \underline{P}(\theta_{dq}) & \underline{0} \\ \underline{0} & \underline{P}(\theta_{dq} - \theta_{me}) \end{bmatrix} \\ &= \begin{bmatrix} \cos(\theta_{dq}) & \sin(\theta_{dq}) & 0 & 0 \\ -\sin(\theta_{dq}) & \cos(\theta_{dq}) & 0 & 0 \\ 0 & 0 & \cos(\theta_{dq} - \theta_{me}) & \sin(\theta_{dq} - \theta_{me}) \\ 0 & 0 & -\sin(\theta_{dq} - \theta_{me}) & \cos(\theta_{dq} - \theta_{me}) \end{bmatrix} \end{aligned} \tag{3.4.7}$$

With subscript "xz"(x=s,r and z=d,q) we'll indicate the quantities in "dq" system.
Thus the general equation can be derived:

$$\begin{aligned} \underline{T}^{-1}\underline{v}_{xz} &= \underline{RT}^{-1}\underline{i}_{xz} + \frac{d}{dt}(\underline{T}^{-1}\underline{\lambda}_{xz}) \\ \underline{v}_{xz} &= \underline{TRT}^{-1}\underline{i}_{xz} + \underline{T}\frac{d}{dt}(\underline{T}^{-1}\underline{\lambda}_{xz}) \\ &= \underline{Ri}_{xz} + \underline{T}\frac{d\underline{T}^{-1}}{dt}\underline{\lambda}_{xz} + \underline{TT}^{-1}\frac{d\underline{\lambda}_{xz}}{dt} \\ &= \underline{Ri}_{xz} + \underline{G}_{xz}\underline{\lambda}_{xz} + \frac{d\underline{\lambda}_{xz}}{dt} \end{aligned} \tag{3.4.8}$$

[4]if the transformation rotates at the angular speed equal to ω_{me} then the system is fixed with the rotor

with:

$$\underline{G}_{xz} = \begin{bmatrix} 0 & -\omega_{dq} & 0 & 0 \\ \omega_{dq} & 0 & 0 & 0 \\ 0 & 0 & 0 & -(\omega_{dq}-\omega_{me}) \\ 0 & 0 & \omega_{dq}-\omega_{me} & 0 \end{bmatrix} \tag{3.4.9}$$

3.4.1 Energy Equations

The energy balance can be obtain by multiplying the 3.4.8 by $i^t_{xz}\mathrm{d}t$:

$$\underbrace{\underline{i}^t_{xz}\underline{v}_{xz}\mathrm{d}t}_{\text{Electrical work absorbed}} = \underbrace{\underline{i}^t_{xz}\underline{R}\underline{i}_{xz}\mathrm{d}t}_{\text{Joule losses}} + \underbrace{\underline{i}^t_{xz}\mathrm{d}\underline{\lambda}_{xz} + \underline{i}^t_{xz}G_{xz}\underline{\lambda}_{xz}}_{\text{Energy stored+Mechanical output}} \tag{3.4.10}$$

If we call dw_m the variation of the energy stored in the system and $\tau d\theta$ the mechanical work delivered at the shaft in the interval dt, the equation can be written as follow:

$$\underline{i}^t_{xz}\mathrm{d}\underline{\lambda}_{xz} + \underline{i}^t_{xz}\underline{G}_{xz}\underline{\lambda}_{xz}dt = \mathrm{d}w_m + \tau d\theta \tag{3.4.11}$$

The energy equation can be expressed as:

$$\tau = \frac{1}{\omega_m}[\omega_{dq}(\lambda_{sd}i_{sq} - \lambda_{sq}i_{sd}) + (\omega_{dq}-\omega_{me})(\lambda_{rd}i_{rq} - \lambda_{rq}i_{rd})] + \underline{i}^t_{xz}\frac{\partial\underline{\lambda}_{xz}}{\partial\theta} - \frac{dw_m}{d\theta} \tag{3.4.12}$$

Remembering that $w_m + w'_m = \lambda i$ then the equation became:[5]

$$\tau = \frac{1}{\omega_m}[\omega_{dq}(\lambda_{sd}i_{sq} - \lambda_{sq}i_{sd}) + (\omega_{dq}-\omega_{me})(\lambda_{rd}i_{rq} - \lambda_{rq}i_{rd})] + \frac{\partial w'_m}{\partial\theta} \tag{3.4.13}$$

We can define $(1 - \dfrac{\omega_{me}}{\omega_{dq}}) = k$ then the energy equation can be simplified:

$$\tau = \frac{p}{1-k}[\lambda_{sd}i_{sq} - \lambda_{sq}i_{sd} + k(\lambda_{rd}i_{rq} - \lambda_{rq}i_{rd})] + \frac{\partial w'_m}{\partial\theta} \tag{3.4.14}$$

If we suppose that $\theta_{dq} = \theta_{me}$ than the energy equation can be written as:

$$\begin{aligned} \tau d\theta &= p(\lambda_{sd}i_{sq} - \lambda_{sq}i_{sd})d\theta + i_{sd}d\lambda_{sd} + i_{sq}d\lambda_{sq} + i_{rd}d\lambda_{rd} + i_{rq}d\lambda_{rq} - \mathrm{d}w_m \\ \tau &= p(\lambda_{sd}i_{sq} - \lambda_{sq}i_{sd}) + i_{sd}\frac{\partial\lambda_{sd}}{\partial\theta} + i_{sq}\frac{\partial\lambda_{sq}}{\partial\theta} + i_{rd}\frac{\partial\lambda_{rd}}{\partial\theta} + i_{rq}\frac{\partial\lambda_{rq}}{\partial\theta} - \frac{\partial w_m}{\partial\theta} \end{aligned} \tag{3.4.15}$$

Remembering that $w_m + w'_m = \lambda i$ then the eq.(3.4.15) became:

$$\tau = p(\lambda_{sd}i_{sq} - \lambda_{sq}i_{sd}) + \frac{\partial w'_m}{\partial\theta} \tag{3.4.16}$$

[5]

$$\begin{aligned} w'_m &= \lambda i - w_m \\ \frac{\partial w'_m}{\partial\theta} &= i\frac{\partial\lambda}{\partial\theta} - \frac{\partial w_m}{\partial\theta} \end{aligned}$$

Chapter 4

Equation of the synchronous machine without saturation

4.1 Equations of a doubly fed three-phase machine

X	no hysteresis
X	no eddy currentes
X	no saturation

In the particular case that the system is linear, i.e. the reactance matrix $\underline{L}$ is function only of the position θ, the magnetization characteristic is written as follow:

$$\underline{\lambda}(i, \theta) = \underline{L}(\theta)\underline{i} \tag{4.1.1}$$

Where

$$\underline{L}(\theta) = \begin{bmatrix} \underline{L}_s(\theta) & \underline{M}_{sr}(\theta) \\ \underline{M}_{rs}(\theta) & \underline{L}_r(\theta) \end{bmatrix}_{6\text{x}6} \tag{4.1.2}$$

and

$$\underline{L}_r(\theta) = \begin{bmatrix} L_{r,a}(\theta) & M_{r,ab}(\theta) & M_{r,ac}(\theta) \\ M_{r,ba}(\theta) & L_{r,b}(\theta) & M_{r,bc}(\theta) \\ M_{r,ca}(\theta) & M_{r,cb}(\theta) & L_{r,c}(\theta) \end{bmatrix} \tag{4.1.3}$$

$$\underline{M}_{sr}(\theta) = \underline{M}^t_{rs}(\theta) = \begin{bmatrix} M_{sr,aa}(\theta) & M_{sr,ab}(\theta) & M_{sr,ac}(\theta) \\ M_{sr,ba}(\theta) & M_{sr,bb}(\theta) & M_{sr,bc}(\theta) \\ M_{sr,ca}(\theta) & M_{sr,cb}(\theta) & M_{sr,cc}(\theta) \end{bmatrix} \tag{4.1.4}$$

$$\underline{L}_s(\theta) = \begin{bmatrix} L_{s,a}(\theta) & M_{s,ab}(\theta) & M_{s,ac}(\theta) \\ M_{s,ba}(\theta) & L_{s,b}(\theta) & M_{s,bc}(\theta) \\ M_{s,ca}(\theta) & M_{s,cb}(\theta) & L_{s,c}(\theta) \end{bmatrix} \tag{4.1.5}$$

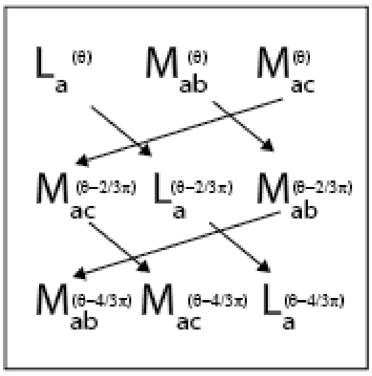

Figure 4.1: Graphical representation of the circulant matrix proprieties

The matrix $\underline{L}_s(\theta)$,$\underline{L}_r(\theta)$ and $\underline{M}_{sr}(\theta)$ are particular circulant matrix, where each row vector is rotated one element to the right relative to the preceding row vector and the argument is calculated in $-2/3\pi$ coordinates(Fig:4.1).

1. $M_{s,xy} = M_{s,yx}$ and $M_{r,xy} = M_{r,yx}$
2. $L_b(\theta) = L_a(\theta - \frac{2\pi}{3})$; $L_c(\theta) = L_a(\theta - \frac{4\pi}{3})$
3. $M_{bc} = M_{ac}(\theta - \frac{2\pi}{3}))$; $M_{ac} = M_{ac}(\theta - \frac{4\pi}{3}))$

According to the eq. 3.1.6 one can obtain:

$$\begin{cases} \underline{\lambda}_s = \underline{L}_s \underline{i}_s + \underline{M}_{sr} \underline{i}_r \\ \underline{\lambda}_r = \underline{M}_{rs} \underline{i}_s + \underline{L}_r \underline{i}_r \end{cases} \tag{4.1.6}$$

In a linear system can be demonstrated that energy and coenergy have the same value, so the following expression can be written:

$$\tau(i,\theta) = \frac{\partial w'_m(i,\theta)}{\partial\theta} = \frac{\partial w_m(\lambda,\theta)}{\partial\theta} \tag{4.1.7}$$

In addition we saw that the flux can be easily written as $\lambda = L(\theta)i$, as well the mechanical energy stored w_m is easily expressed as $w_m = \frac{1}{2} i^t \lambda$. Therefore the 4.1.7 become:

$$\tau = \frac{1}{2} i^t \frac{\partial\lambda}{\partial\theta} = \frac{1}{2} i^t \frac{dL}{d\theta} i \tag{4.1.8}$$

The torque can be splitted as:

$$\tau(i,\theta) = \underbrace{\frac{1}{2} \underline{i}_s^t \frac{d\underline{L}_s(\theta)}{d\theta} \underline{i}_s}_{\text{reluctance torque}} + \underbrace{\underline{i}_s^t \frac{d\underline{M}_{sr}(\theta)}{d\theta} \underline{i}_r}_{\text{electro-dynamic torque}} + \underbrace{\frac{1}{2} \underline{i}_r^t \frac{d\underline{L}_r(\theta)}{d\theta} \underline{i}_r}_{\text{cogging torque}} \tag{4.1.9}$$

The reluctance torque is related to the variations of the stator inductance matrix with the rotor position, that occurs every time the rotor has an anisotropic structure due to pole saliencies or flux barriers.

$$
\begin{aligned}
\tau_{rel}(\underline{i}_s, \theta) &= \frac{1}{2}\underline{i}_s^t \frac{d\underline{L}_s(\theta)}{d\theta}\underline{i}_s \\
&= \frac{1}{2}\frac{\partial \underline{\lambda}_s^t(\underline{i}_s, 0, \theta)}{\partial\theta}\underline{i}_s \\
&= \frac{1}{2}\underline{i}_s^t \frac{\partial \underline{\lambda}_s(\underline{i}_s, 0, \theta)}{\partial\theta} \\
&= \frac{\partial w_m(\underline{i}_s, 0, \theta)}{\partial\theta}
\end{aligned} \tag{4.1.10}
$$

where $\lambda_s(i_s, 0, \theta)$ is the stator flux linkage due to the stator current only, i.e. with $i_r \equiv 0$.

The electro-dynamic torque is related to the variation of the mutual inductances between the stator and rotor windings with the rotor position. The main reasons of such a variation is the movement of the rotor with respect to the stator.

$$
\begin{aligned}
\tau_{ed}(\underline{i}_s, \underline{i}_r, \theta) &= \underline{i}_s^t \frac{d\underline{M}_{sr}(\theta)}{d\theta}\underline{i}_r \\
&= \frac{\partial \underline{\lambda}_{sr}^t(0, \underline{i}_r, \theta)}{\partial\theta}\underline{i}_s = \underline{i}_s^t \frac{\partial \underline{\lambda}_{sr}(0, \underline{i}_r, \theta)}{\partial\theta} \\
&= \frac{\partial \underline{\lambda}_{rs}^t(\underline{i}_s, 0, \theta)}{\partial\theta}\underline{i}_r = \underline{i}_r^t \frac{\partial \underline{\lambda}_{rs}(\underline{i}_s, 0, \theta)}{\partial\theta}
\end{aligned} \tag{4.1.11}
$$

where $\lambda_{sr}(0, ir, \theta)$ is the stator flux linkages due to the rotor current only, i.e. with $i_s \equiv 0$, while $\lambda_{rs}(is, 0, \theta)$ is the rotor flux linkage due to the stator current only, i.e. with $i_r \equiv 0$.

The cogging torque is due to stator anisotropies mainly caused by stator slotting. It therefore does not occur in a slotless stator machine.

$$
\begin{aligned}
\tau_{cog}(\underline{i}_r, \theta) &= \frac{1}{2}\underline{i}_r^t \frac{d\underline{L}_r(\theta)}{d\theta}\underline{i}_r \\
&= \frac{1}{2}\frac{\partial \underline{\lambda}_r^t(0, \underline{i}_r, \theta)}{\partial\theta}\underline{i}_r \\
&= \frac{1}{2}\underline{i}_r^t \frac{\partial \underline{\lambda}_r(0, \underline{i}_r, \theta)}{\partial\theta} \\
&= \frac{\partial w_m(0, \underline{i}_r, \theta)}{\partial\theta}
\end{aligned} \tag{4.1.12}
$$

where $\lambda_r(0, ir, \theta)$ is the stator flux linkages due to the rotor current only, i.e. with $i_s \equiv 0$

4.2 Equations of a doubly-fed two-phase machine

As before, we'll use the symbol X_{xy} to indicate the vector which elements are:

$$
\begin{bmatrix} X_{s\alpha} \\ X_{s\beta} \\ X_{r\alpha} \\ X_{r\beta} \end{bmatrix} = \begin{bmatrix} X_{s,\alpha\beta} \\ X_{r,\alpha\beta} \end{bmatrix}
$$

The equation 4.1.9 can be rewritten for a bi-phase system as

$$\begin{aligned}\tau(i_{xy},\theta) =& \frac{1}{2}\underline{i}_{s,\alpha\beta}^{t}\frac{d\underline{L}_{s,\alpha\beta}(\theta)}{d\theta}\underline{i}_{s,\alpha\beta}+ \\ & \underline{i}_{s,\alpha\beta}^{t}\frac{d\underline{M}_{sr,\alpha\beta}(\theta)}{d\theta}\underline{i}_{r,\alpha\beta}+ \\ & \frac{1}{2}\underline{i}_{r,\alpha\beta}^{t}\frac{d\underline{L}_{r,\alpha\beta}(\theta)}{d\theta}\underline{i}_{r,\alpha\beta}\end{aligned} \tag{4.2.1}$$

We'll write the matrices in bi-phases system as:

$$\underline{L}_{s,\alpha\beta} = \begin{bmatrix} L_{s\alpha} & L_{s\alpha\beta} \\ L_{s\beta\alpha} & L_{s\beta} \end{bmatrix} \tag{4.2.2}$$

$$\underline{M}_{sr,\alpha\beta} = \begin{bmatrix} M_{sr\alpha} & M_{sr\alpha\beta} \\ M_{sr\beta\alpha} & M_{sr\beta} \end{bmatrix} = \underline{M}_{rs,\alpha\beta}^{t} \tag{4.2.3}$$

$$\underline{L}_{r,\alpha\beta} = \begin{bmatrix} L_{r\alpha} & L_{r\alpha\beta} \\ L_{r\beta\alpha} & L_{r\beta} \end{bmatrix} \tag{4.2.4}$$

The equation $\underline{\lambda}_s = \underline{L}_s\underline{i}_s + \underline{M}_{sr}\underline{i}_r$ can be transformed in the bi-phase system:

$$\begin{aligned} C^{-1}\underline{\lambda}_{s,\alpha\beta} &= \underline{L}_s C^{-1}\underline{i}_{s,\alpha\beta} + \underline{M}_{sr}C^{-1}\underline{i}_{r,\alpha\beta} \\ \underline{\lambda}_{s,\alpha\beta} &= C\underline{L}_s C^{-1}\underline{i}_{s,\alpha\beta} + C\underline{M}_{sr}C^{-1}\underline{i}_{r,\alpha\beta} \\ &= \underline{L}_{s,\alpha\beta}\underline{i}_{s,\alpha\beta} + \underline{M}_{sr,\alpha\beta}\underline{i}_{r,\alpha\beta} \end{aligned} \tag{4.2.5}$$

The same things can be done for $\lambda_{r,\alpha\beta}$ from which we obtain the expression of $M_{rs,\alpha\beta}$ and $Lr,\alpha\beta$.

If we analyse the matrix it appears that $L_{\alpha\beta} = L_{\beta\alpha}$

$$L_{s\alpha} = \frac{1}{6}(4L_a + L_b + L_c - 4M_{ab} - 4M_{ac} + 2M_{bc}) \tag{4.2.6}$$

$$L_{s\alpha\beta} = \frac{-L_b + L_c + 2M_{ab} - 2M_{ac}}{2\sqrt{3}} \tag{4.2.7}$$

$$L_{s\beta\alpha} = \frac{-L_b + L_c + 2M_{ab} - 2M_{ac}}{2\sqrt{3}} \tag{4.2.8}$$

$$L_{s\beta} = \frac{1}{2}(L_b + L_c - 2M_{bc}) \tag{4.2.9}$$

4.3 Equation for synchronous Machine in "dq"

In the section 3.4 we defeined the general equation for synchronous machine in "dq" system. With the hypothesis of no saturation the equation can be written as:

$$\begin{aligned} u_{xz} &= \underline{R}i_{xz} + \underline{J}_{xz}\underline{\lambda}_{xz} + \frac{d\underline{\lambda}_{xz}}{dt} \\ &= \underline{R}i_{xz} + \underline{J}_{xz}\underline{L}_{xz}\underline{i}_{xz} + \frac{d\underline{L}_{xz}}{dt}\underline{i}_{xz} + \underline{L}_{xz}\frac{d\underline{i}_{xz}}{dt} \end{aligned} \tag{4.3.1}$$

4.3.1 Energy Equations

Starting from: $\tau = p(\lambda_{sd} i_{sq} - \lambda_{sq} i_{sd}) + \dfrac{\partial w'_m}{\partial \theta}$ and considering the linearity of the system that we're studying, the co-energy can be evaluated as follow:

$$\frac{\partial w'_m}{\partial \theta} = \frac{\partial w_m}{\partial \theta} = \frac{1}{2} i^t \frac{\partial \lambda}{\partial \theta} = \frac{1}{2} i^t \frac{\partial L}{\partial \theta} i \tag{4.3.2}$$

That can be expressed as:

$$\begin{aligned} \frac{\partial w'_m}{\partial \theta} =& \frac{1}{2} \underline{i}^t_{s,dq} \frac{d\underline{L}_{s,dq}(\theta)}{d\theta} \underline{i}_{s,dq} + \\ & \underline{i}^t_{s,dq} \frac{d\underline{M}_{sr,dq}(\theta)}{d\theta} \underline{i}_{r,dq} + \\ & \frac{1}{2} \underline{i}^t_{r,dq} \frac{d\underline{L}_{r,dq}(\theta)}{d\theta} \underline{i}_{r,dq} \end{aligned} \tag{4.3.3}$$

Thus the torque equation became:

$$\begin{aligned} \tau =& p(\lambda_{sd} i_{sq} - \lambda_{sq} i_{sd}) + \\ & \frac{1}{2} \underline{i}^t_{s,dq} \frac{d\underline{L}_{s,dq}(\theta)}{d\theta} \underline{i}_{s,dq} + \\ & \underline{i}^t_{s,dq} \frac{d\underline{M}_{sr,dq}(\theta)}{d\theta} \underline{i}_{r,dq} + \\ & \frac{1}{2} \underline{i}^t_{r,dq} \frac{d\underline{L}_{r,dq}(\theta)}{d\theta} \underline{i}_{r,dq} \end{aligned} \tag{4.3.4}$$

The equation above display a particular case for which is supposed that $\theta_{dq} = \theta_{me}$. Therefore if we want a general equation it's easy enough to derive the following expression:

$$\begin{aligned} \tau =& \frac{p}{1-k} [\lambda_{sd} i_{sq} - \lambda_{sq} i_{sd} + k(\lambda_{rd} i_{rq} - \lambda_{rq} i_{rd})] + \\ & \frac{1}{2} \underline{i}^t_{s,dq} \frac{d\underline{L}_{s,dq}(\theta)}{d\theta} \underline{i}_{s,dq} + \underline{i}^t_{s,dq} \frac{d\underline{M}_{sr,dq}(\theta)}{d\theta} \underline{i}_{r,dq} + \frac{1}{2} \underline{i}^t_{r,dq} \frac{d\underline{L}_{r,dq}(\theta)}{d\theta} \underline{i}_{r,dq} \end{aligned} \tag{4.3.5}$$

4.4 IPM:Internal Permanent Magnet Machine

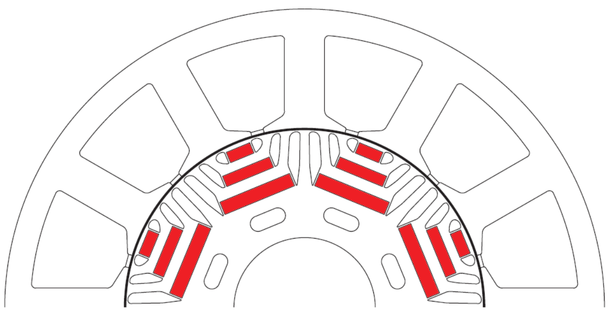

Figure 4.2: IPM example of lamination

The IPM machine is a particular electrical machine which has permanent magnet on the rotor in place of the winding.IPM can be analyzed as a double fed machine with his rotor current equal to $i_{r,\alpha} = I_{mg}$ and $i_{r,\beta} = 0$. Therefore the eq.4.2.1 can be written for an IPM machine as:

$$\begin{aligned}
\tau(i_{s,\alpha\beta},\theta) =& \frac{1}{2}\underline{i}^t_{s,\alpha\beta}\frac{d\underline{L}_{s,\alpha\beta}(\theta)}{d\theta}\underline{i}_{s,\alpha\beta} + && \text{reluctance torque}\\
& I_{mg}(i_{s\alpha}\frac{dM_{sr,\alpha}}{d\theta} + i_{s\beta}\frac{dM_{sr,\beta\alpha}}{d\theta}) + && \text{electro-dynamic torque}\\
& \frac{1}{2}I^2_{mg}\frac{dL_{r\alpha}}{d\theta} && \text{cogging torque}\\
=& \frac{1}{2}\underline{i}^t_{s,\alpha\beta}\frac{d\underline{L}_{s,\alpha\beta}(\theta)}{d\theta}\underline{i}_{s,\alpha\beta} + \\
& \frac{d\lambda_{\alpha,mg}}{d\theta}i_{s\alpha} + \frac{d\lambda_{\beta,mg}}{d\theta}i_{s\beta} + \\
& \frac{1}{2}I^2_{mg}\frac{dL_{r\alpha}}{d\theta}
\end{aligned} \tag{4.4.1}$$

with $M_{sr,\alpha}$, $M_{sr,\beta\alpha}$ elements of the matrix $\underline{M}_{sr}$ and $L_{r\alpha}$ of the matrix $\underline{L}_r$

The same equation can be evaluated in "a,b,c" system with $i_{ra} = I_{mg}$ and $i_{rb} = i_{rc} = -\frac{I_{mg}}{2}$:

$$\begin{aligned}
\tau(i_s,\theta) =& \frac{1}{2}\underline{i}^t_s\frac{d\underline{L}_s(\theta)}{d\theta}\underline{i}_s + && \text{reluctance torque}\\
& I_{mg}(\frac{dM_{sr,a}}{d\theta}i_{sa} + \frac{dM_{sr,b}}{d\theta}i_{sb} + \frac{dM_{sr,c}}{d\theta}i_{sc}) + && \text{electro-dynamic torque}\\
& \frac{1}{2}I^2_{mg}(\frac{dL_{ra}}{d\theta} + \frac{dL_{rb}}{d\theta} + \frac{dL_{rc}}{d\theta}) && \text{cogging torque}\\
=& \frac{1}{2}\underline{i}^t_s\frac{d\underline{L}_s(\theta)}{d\theta}\underline{i}_s + && \text{reluctance torque}\\
& \frac{d\lambda_{m,a}}{d\theta}i_{sa} + \frac{d\lambda_{m,b}}{d\theta}i_{sb} + \frac{d\lambda_{m,c}}{d\theta}i_{sc} + && \text{electro-dynamic torque}\\
& \frac{1}{2}I^2_{mg}(\frac{dL_{ra}}{d\theta} + \frac{dL_{rb}}{d\theta} + \frac{dL_{rc}}{d\theta}) && \text{cogging torque}
\end{aligned} \tag{4.4.2}$$

With:

$$\begin{aligned}
M_{sr,a}i_{sa} + M_{sr,b}i_{sb} + M_{sr,c}i_{sc} =& \\
& (M_{sr,aa} - \frac{1}{2}M_{sr,ab} - \frac{1}{2}M_{sr,ac})i_a + \\
& (M_{sr,ba} - \frac{1}{2}M_{sr,bb} - \frac{1}{2}M_{sr,bc})i_b + \\
& (M_{sr,ca} - \frac{1}{2}M_{sr,cb} - \frac{1}{2}M_{sr,cc})i_c
\end{aligned} \tag{4.4.3}$$

and

$$
\begin{aligned}
L_{ra} + L_{rb} + L_{rc} = & \\
& L_{r,a} - \frac{1}{2}M_{r,ab} - \frac{1}{2}M_{r,ac} + \\
& -\frac{1}{2}M_{r,ba} + \frac{1}{4}L_{r,b} + \frac{1}{4}M_{r,bc} + \\
& -\frac{1}{2}M_{r,ca} + \frac{1}{4}M_{r,cb} + \frac{1}{4}L_{r,c}
\end{aligned}
\tag{4.4.4}
$$

The equation of IPM can be also obtained in the "dq" system. We'll express the expression not considering the derivative quantities. Then the equation can be written as:

$$
\begin{aligned}
\tau &= p(\lambda_{sd} i_{sq} - \lambda_{sq} i_{sd}) \\
&= p[(L_{sd} i_{sd} + L_{sdq} i_{sq} + M_{sr,dd} I_{mg}) i_{sq} - (L_{sq} i_{sq} + L_{sqd} i_{sd}) i_{sd}]
\end{aligned}
\tag{4.4.5}
$$

if we call $M_{sr,dd} I_{mg} = \lambda_{mg}$ the flux linkage of the permanent magnet with the stator, we'll obtain the typical expression of the torque for IPM machines.

$$
\tau = p[\lambda_{mg} i_{sq} + (L_{sd} - L_{sq}) i_{sq} i_{sd} + L_{sdq}(i_{sq}^2 + i_{sd}^2)]
\tag{4.4.6}
$$

Remembering that $L_{sdq} = L_{sqd}$.

4.5 SyREL:Synchronous Reluctance Machine

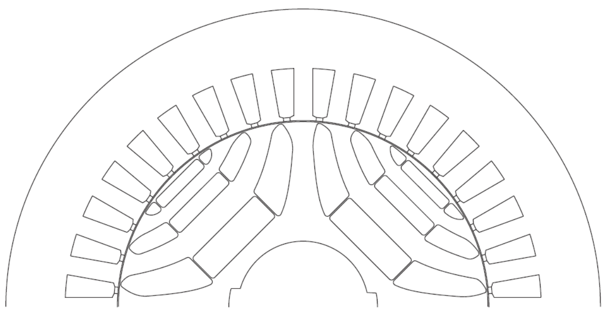

Figure 4.3: REL example of lamination

A reluctance machine is a type of electric machine that hasn't any windings or magnets on the rotor.This machine can described with the same equation of a bi-phase machine (eq.3.2.6) without rotor current $i_{r\alpha} = i_{r_\beta} = 0$

$$
\begin{aligned}
\tau(i_{s,\alpha\beta}, \theta) = & \frac{1}{2} \underline{i}^t_{s,\alpha\beta} \frac{d\underline{L}_{s,\alpha\beta}(\theta)}{d\theta} \underline{i}_{s,\alpha\beta} && \text{reluctance torque} \\
& + 0 && \text{electro-dynamic torque} \\
& + 0 && \text{cogging torque}
\end{aligned}
\tag{4.5.1}
$$

The same equation can be evaluated in "a,b,c" system with $i_{ra} = I_{mg}$ and $i_{rb} = i_{rc} = -\dfrac{I_{mg}}{2}$:

$$
\begin{aligned}
\tau(i_s,\theta) =& \frac{1}{2}\underline{i}_s^t\frac{d\underline{L}_s(\theta)}{d\theta}\underline{i}_s + && \text{reluctance torque} \\
& 0+ && \text{electro-dynamic torque} \\
& 0 && \text{cogging torque}
\end{aligned} \tag{4.5.2}
$$

The expression of the torque in "dq" for the reluctance machine can be obtained considering this machine an IPM without the magnetic flux form the permanent magnet so with $\lambda_{mg} = 0$. Thus from eq. 4.4.6 the torque can be written as:

$$
\tau = p[(L_{sd} - L_{sq})i_{sq}i_{sd} + L_{sdq}(i_{sq}^2 + i_{sd}^2)] \tag{4.5.3}
$$

4.6 SPM:Superficial Permanent Magnet Machine

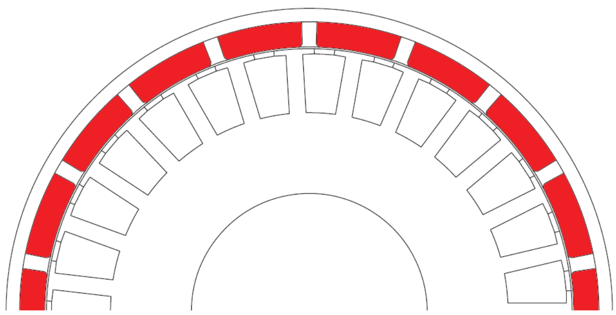

Figure 4.4: SPM example of lamination

The SPM machine can be studied as an IPM with an isotropic stator, thus $\dfrac{dL_{s,\alpha\beta}}{d\theta} \equiv 0$

$$
\begin{aligned}
\tau(i_{s,\alpha\beta},\theta) =& 0 && \text{reluctance torque} \\
& + \frac{d\lambda_{\alpha,mg}}{d\theta}i_{s\alpha} + \frac{d\lambda_{\beta,mg}}{d\theta}i_{s\beta} && \text{electro-dynamic torque} \\
& + \frac{1}{2}I_{mg}^2\frac{dL_{r\alpha}}{d\theta} && \text{cogging torque}
\end{aligned} \tag{4.6.1}
$$

In the "dq" system the isotropic stator proprieties are expressed by the relation $L_{sd} = L_{sq}$. Thus the torque equation can be obtained from the eq. 4.4.6.

$$
\tau = p\lambda_{mg}i_{sq} + L_{sdq}(i_{sq}^2 + i_{sd}^2) \tag{4.6.2}
$$

Chapter 5

Machine with Sinusoidally Distributed Winding

The winding distribution around the airgap of alternating current machines is normally designed to enhance the fundamental component of the airgap flux distribution, thus the flux linkage can be supposed to be almost sinusoidal with the rotor position.
Differently from other chapters in this case we'll focus on the PM and reluctance machine avoiding the general synchronous machine with rotor windings[6]. This choice was made because these type of machine are more interesting to analyze and furthermore are the most relevant to describe.
Therefore the inductances a three-phase machine can be defined as:

$$L_{a,b,c} = \frac{2}{3}\begin{bmatrix} L_\Sigma + L_\Delta \cos 2\theta & -\frac{L_\Sigma}{2} + L_\Delta \cos[2(\theta - 4/3)] & -\frac{L_\Sigma}{2} + L_\Delta \cos[2(\theta - 2/3)] \\ -\frac{L_\Sigma}{2} + L_\Delta \cos[2(\theta - 4/3)] & L_\Sigma + L_\Delta \cos[2(\theta - 2/3)] & -\frac{L_\Sigma}{2} + L_\Delta \cos(2\theta) \\ -\frac{L_\Sigma}{2} + L_\Delta \cos[2(\theta - 2/3)] & -\frac{L_\Sigma}{2} + L_\Delta \cos(2\theta) & L_\Sigma + L_\Delta \cos[2(\theta - 4/3)] \end{bmatrix} \tag{5.0.1}$$

With $L_\Sigma = \dfrac{L_d + L_q}{2}$ and $L_\Delta = \dfrac{L_d - L_q}{2}$.

The current can be assumed by the general expression:

$$\begin{cases} i_a = I_M \cos(\theta + \frac{\pi}{2} - \gamma) \\ i_b = I_M \cos(\theta + \frac{\pi}{2} - \gamma - \frac{2}{3}\pi) \\ i_c = I_M \cos(\theta + \frac{\pi}{2} - \gamma - \frac{4}{3}\pi) \end{cases} \tag{5.0.2}$$

and then the flux linkages can be derived from the expression $\underline{\lambda} = \underline{L}_{a,b,c}\underline{i}$, yielding to:

[6]In this chapter all the quantities will be reported to the stator therefore we'll avoid to use the subscript "s".

$$
\begin{cases}
\lambda_a = I_M(-L_\Sigma \sin(\theta-\gamma) + L_\Delta \sin(\theta+\gamma)) \\
\lambda_b = I_M(-L_\Sigma \sin(\theta-\gamma-\frac{2}{3}\pi) + L_\Delta \sin(\theta+\gamma-\frac{2}{3}\pi)) \\
\lambda_c = I_M(-L_\Sigma \sin(\theta-\gamma-\frac{4}{3}\pi) + L_\Delta \sin(\theta+\gamma-\frac{4}{3}\pi))
\end{cases}
\tag{5.0.3}
$$

The result is consistent with the assumptions that the flux linkages can be described as a sinusoidal function.

5.1 Equation of a two-phase machine

The same equation seen in the previous section can be derived for a two-phase machine using the $\underline{C}$ transformation (see section: 3.2).
For the inductances it results:

$$
L_{\alpha,\beta} = \begin{bmatrix} L_\Sigma + L_\Delta \cos 2\theta & L_\Delta \sin 2\theta \\ L_\Delta \sin 2\theta & L_\Sigma - L_\Delta \cos 2\theta \end{bmatrix} \tag{5.1.1}
$$

with $L_\Sigma = \dfrac{L_d + L_q}{2}$ and $L_\Delta = \dfrac{L_d - L_q}{2}$.
While transformation of eq.5.0.2 brings to:

$$
\begin{cases}
i_\alpha = -I_M \sin(\theta-\gamma) \\
i_\beta = I_M \cos(\theta-\gamma)
\end{cases}
\tag{5.1.2}
$$

and then the flux linkages become:

$$
\begin{cases}
\lambda_\alpha = -I_M L_q \cos(\gamma)\sin(\theta) + I_M L_d \sin(\gamma)\cos(\theta) \\
\lambda_\beta = I_M L_q \cos(\gamma)\cos(\theta) + I_M L_d \sin(\gamma)\sin(\theta)
\end{cases}
\tag{5.1.3}
$$

5.2 Equation for synchronous machine in "d,q"

In the "d,q" system the electric dynamics of the machine quantities can be expressed with easier equations thus it can be more useful to describe the machine in this particular reference frame. For instance the current, flux and inductance matrix will be expressed as follow:

$$
\begin{cases}
i_d = I_M \sin(\gamma) \\
i_q = I_M \cos(\gamma)
\end{cases}
\tag{5.2.1}
$$

$$
L_{d,q} = \begin{bmatrix} L_d & 0 \\ 0 & L_q \end{bmatrix} \tag{5.2.2}
$$

$$
\begin{cases}
\lambda_d = I_M L_d \sin(\gamma) \\
\lambda_q = I_M L_q \cos(\gamma)
\end{cases}
\tag{5.2.3}
$$

It's worth noticing that in this case the self inductances are constant and the mutual inductances $L_{dq} = L_{qd}$ disappear.

5.2.1 Torque Equation

The torque equation can be easily derived from 4.4.6 considering $L_{dq} = L_{qd} = 0$:

1. Internal Permanent Magnet machine[7]:

$$\tau = p[\lambda_{mg} i_q + (L_d - L_q) i_q i_d] \tag{5.2.4}$$

2. Superficial Permanent Magnet machine ($L_d = L_q$):

$$\tau = p\lambda_{mg} i_q \tag{5.2.5}$$

3. Reluctance machine ($\lambda_{mg} = 0$):

$$\tau = p(L_d - L_q) i_q i_d \tag{5.2.6}$$

[7]In the torque expression the quantities $\frac{1}{2} i^t \frac{dL}{dt} i$ are for the hypothesis null.

Chapter 6

Fourier Analysis of the Inductance Matrix

In this section we'll analyse a machine whose inductance matrix can be expressed as a Fourier series. Each inductances can be represented as:

$$L(\theta) = \sum_{n=-\infty}^{\infty} \dot{L}_n e^{\jmath n\theta} \tag{6.0.1}$$

with:

$$\dot{L}_n = L_n e^{\jmath\varphi_n} \tag{6.0.2}$$

Eq.6.0.1 can be applied to phase inductances yielding to:

$$\begin{aligned}
L_a(\theta) &= \sum_{n=-\infty}^{\infty} \dot{L}_{a,n} e^{\jmath n\theta} \\
L_b(\theta) &= \sum_{n=-\infty}^{\infty} \dot{L}_{b,n} e^{\jmath n\theta} = \sum_{n=-\infty}^{\infty} (\dot{L}_{a,n} e^{-\jmath\frac{2\pi}{3}}) e^{\jmath n\theta} \\
L_c(\theta) &= \sum_{n=-\infty}^{\infty} \dot{L}_{c,n} e^{\jmath n\theta} = \sum_{n=-\infty}^{\infty} (\dot{L}_{a,n} e^{-\jmath\frac{4\pi}{3}}) e^{\jmath n\theta}
\end{aligned} \tag{6.0.3}$$

Similarly it can be obtained for:

$$\begin{aligned}
L_{bc}(\theta) &= \sum_{n=-\infty}^{\infty} \dot{L}_{bc,n} e^{\jmath n\theta} \\
L_{ab}(\theta) &= \sum_{n=-\infty}^{\infty} \dot{L}_{ab,n} e^{\jmath n\theta} = \sum_{n=-\infty}^{\infty} (\dot{L}_{bc,n} e^{-\jmath\frac{4\pi}{3}}) e^{\jmath n\theta} \\
L_{ca}(\theta) &= \sum_{n=-\infty}^{\infty} \dot{L}_{ca,n} e^{\jmath n\theta} = \sum_{n=-\infty}^{\infty} (\dot{L}_{bc,n} e^{-\jmath\frac{2\pi}{3}}) e^{\jmath n\theta}
\end{aligned} \tag{6.0.4}$$

Then applying eq.4.2.6, 4.2.7 and 4.2.9 the harmonic contents of the inductance matrix in $\alpha\beta$ can be derived from the inductances in a, b, c as follow:

$$L_\alpha = \frac{1}{6}(4L_a + L_a e^{-jn\frac{2}{3}\pi} + L_a e^{-jn\frac{4}{3}\pi} - 4L_{bc}e^{-jn\frac{4}{3}\pi} - 4L_{bc}e^{-jn\frac{2}{3}\pi} + 2L_{bc}) \quad (6.0.5)$$

$$L_{\alpha\beta} = \frac{1}{2\sqrt{3}}(-L_a e^{-jn\frac{2}{3}\pi} + L_a e^{-jn\frac{4}{3}\pi} + 2L_{bc}e^{-jn\frac{4}{3}\pi} + 2L_{bc}e^{-jn\frac{2}{3}\pi}) \quad (6.0.6)$$

$$L_\beta = \frac{1}{2}(L_a e^{-jn\frac{2}{3}\pi} + L_a e^{-jn\frac{4}{3}\pi} - 2L_{bc}) \quad (6.0.7)$$

As we can see from the equations above, we can derive each element of the matrix from the value of L_a e L_{bc}. Since the transformation from the a, b, c to α, β system do not introduce any changes in the harmonic spectrum, we can say that:

$$\begin{aligned} L_{\alpha,n} &= f(L_{a,n}, L_{bc,n}) \\ L_{\alpha\beta,n} &= f(L_{a,n}, L_{bc,n}) \\ L_{\beta,n} &= f(L_{a,n}, L_{bc,n}) \end{aligned} \quad (6.0.8)$$

Using the transformation matrix $\underline{P}$ (see section 4.2) in the equation $\underline{PLP}^{-1}$, we'll obtain the matrix $\underline{L}_{dq}$ which elements are:

$$L_d = \frac{1}{2}[L_\alpha + L_\beta + \frac{1}{2}(L_\alpha - L_\beta)e^{2j\theta} + \frac{1}{2}(L_\alpha - L_\beta)e^{-2j\theta} - jL_{\alpha\beta}(e^{2j\theta} - e^{-2j\theta})] \quad (6.0.9)$$

$$L_{dq} = \frac{1}{4}[j(L_\alpha - L_\beta)(e^{2j\theta} - e^{-2j\theta}) + 2L_{\alpha\beta}(e^{2j\theta} + e^{-2j\theta})] \quad (6.0.10)$$

$$L_q = \frac{1}{2}[L_\alpha + L_\beta + \frac{1}{2}(-L_\alpha + L_\beta)e^{2j\theta} + \frac{1}{2}(-L_\alpha + L_\beta)e^{-2j\theta} + jL_{\alpha\beta}(e^{2j\theta} - e^{-2j\theta})] \quad (6.0.11)$$

We found that in this case the transformation from $\alpha\beta$ to dq changes how the n-harmonics is calculated in fact from the equation written above the n-harmonics in dq is function not only of the n-harmonics in $\alpha\beta$ but also of the n-2 and n+2 harmonics. This effect will be evident in the next section.

$$\begin{aligned} L_{d,n} &= f(L_{\alpha,n}, L_{\beta,n}, L_{\alpha,n-2}, L_{\beta,n-2}, L_{\alpha,n+2}, L_{\beta,n+2}, L_{\alpha\beta,n-2}, L_{\alpha\beta,n+2}) \\ L_{dq,n} &= f(L_{\alpha,n-2}, L_{\beta,n-2}, L_{\alpha,n+2}, L_{\beta,n+2}, L_{\alpha\beta,n-2}, L_{\alpha\beta,n+2}) \\ L_{q,n} &= f(L_{\alpha,n}, L_{\beta,n}, L_{\alpha,n-2}, L_{\beta,n-2}, L_{\alpha,n+2}, L_{\beta,n+2}, L_{\alpha\beta,n-2}, L_{\alpha\beta,n+2}) \end{aligned} \quad (6.0.12)$$

6.1 Machine with Distributed Winding

Figure 6.1: Example of machine with distributed winding

Using the equation above we can simulate the harmonic spectrum for each element of the matrix in each system starting from the harmonic spectrum of L_a and L_{bc}.
For the first analysis we decide to use the following description of L_a and L_{bc}, we supposed the presence of the zero and the second harmonic with the following coefficients:

$$\begin{aligned} L_a &= L_0 + L_2\cos(2\theta) \\ L_{bc} &= -\frac{L_0}{2} + L_2\cos(2\theta) \end{aligned} \tag{6.1.1}$$

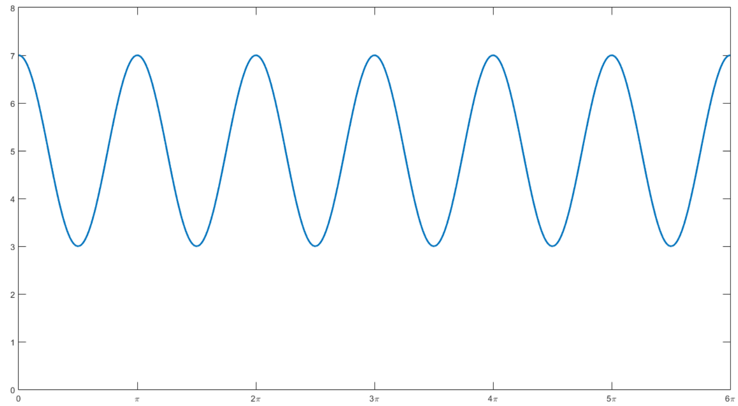

Figure 6.2: Phase Inductance L_a

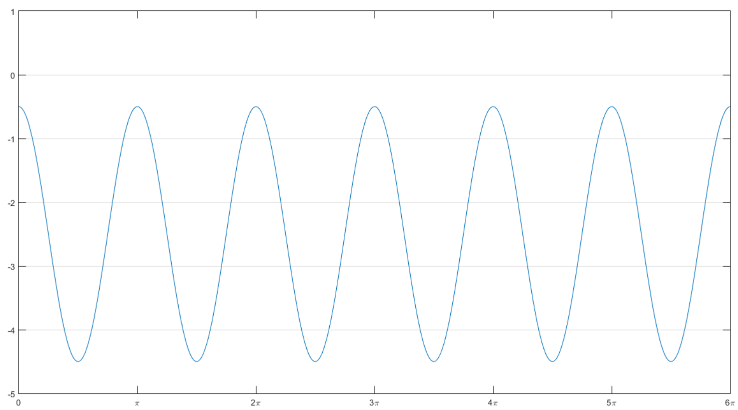

Figure 6.3: Phase Mutual inductance L_{bc}

This case represents a machine with sinusoidally distributed winding like in Chapter 5 thus we must obtain similar result.
Figs 6.4, 6.5 and 6.6 give the harmonic spectrum of L_α, L_β and $L_{\alpha\beta}$ derived form the equations described in the previous Section.

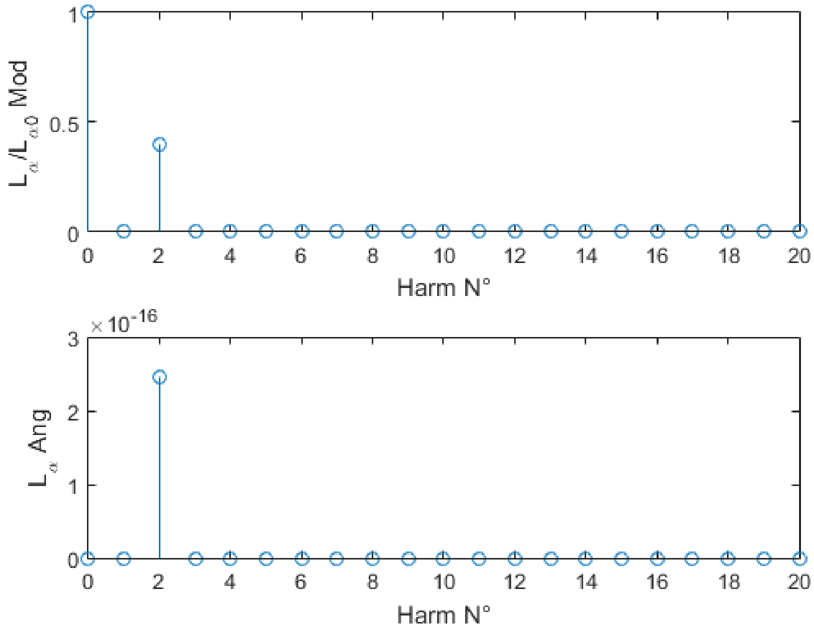

Figure 6.4: Harmonic Spectrum of L_α: amplitude and phase

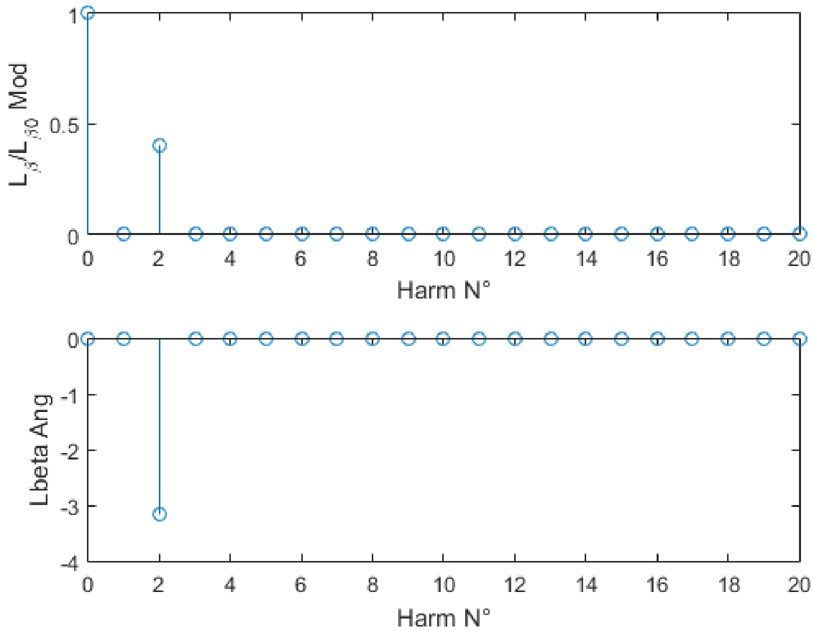

Figure 6.5: Harmonic Spectrum of L_β: amplitude and phase

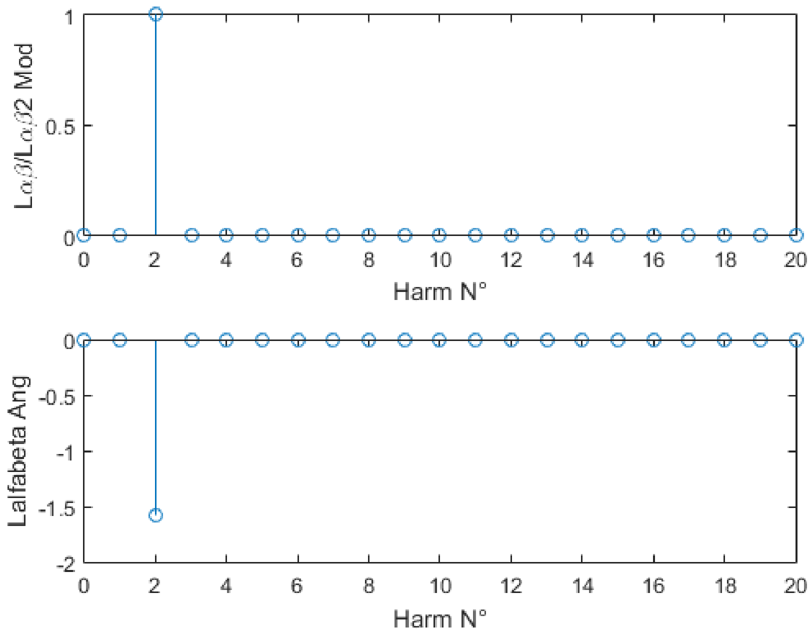

Figure 6.6: Harmonic Spectrum of $L_{\alpha\beta}$: amplitude and phase

As we can see we obtain that in L_α and L_β there are the zero and second harmonics but in $L_{\alpha\beta}$ only the second is present. If we compare the expression of the matrix $\underline{L}_{\alpha\beta}$ seen in the chapter 5 and the result obtained from this different method we find that they perfectly match; therefore the two method are equivalent for the analysis of the matrix.

In fact we have:

1. in L_α: $L_0 e^{j0}$ and $L_2 e^{j0}$
   $\Rightarrow L_0 + L_2 \cos(2\theta)$

2. in L_β: $L_0 e^{j0}$ and $L_2 e^{j\pi}$
   $\Rightarrow L_0 - L_2 \cos(2\theta)$

3. in $L_{\alpha\beta}$: $L_2 e^{j\dfrac{\pi}{2}}$
   $\Rightarrow L_2 \sin(2\theta)$

Next step is deriving the harmonic spectrum of the inductance matrix components in dq starting from the components in $\alpha\beta$.
Figs 6.7, 6.8 and 6.9 give the spectrum of L_d, L_q and L_{dq} respectively.

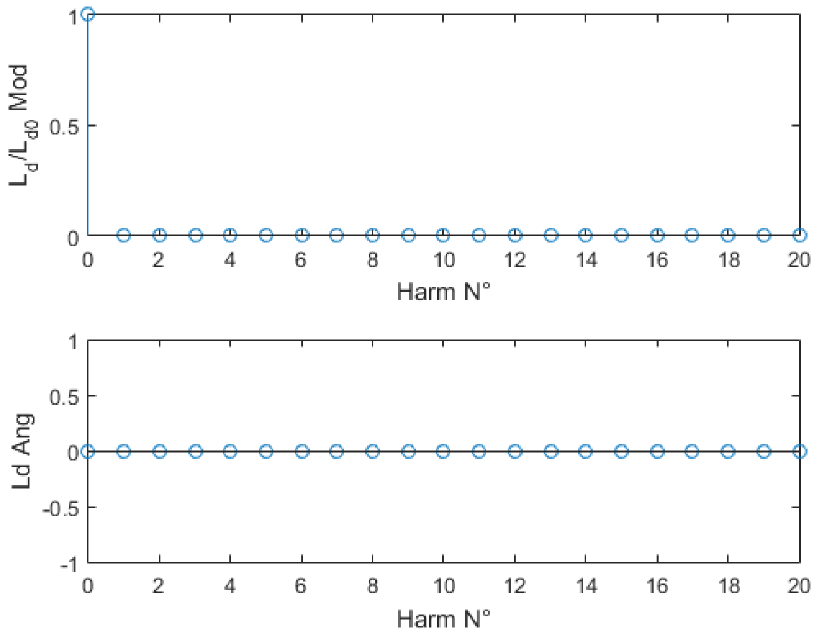

Figure 6.7: Harmonic Spectrum of L_d: amplitude and phase

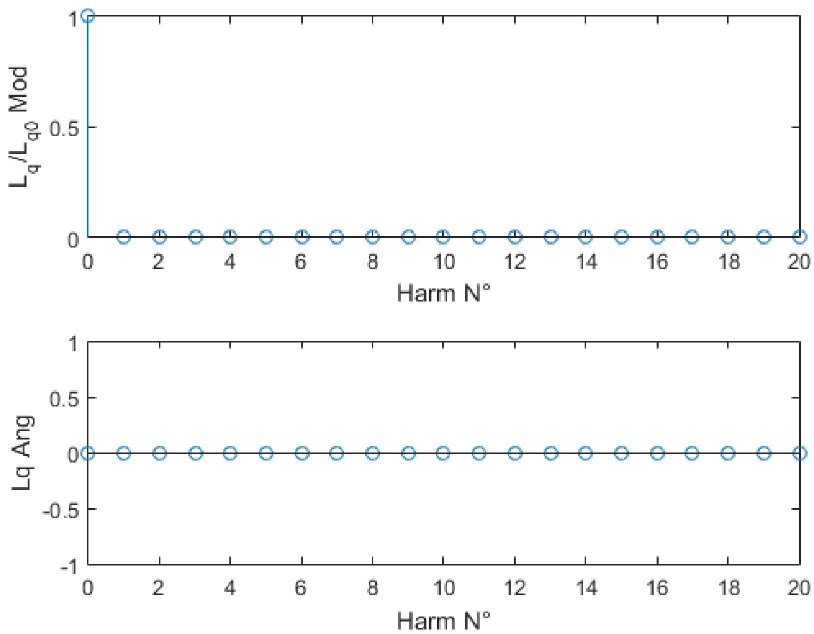

Figure 6.8: Harmonic Spectrum of L_q: amplitude and phase

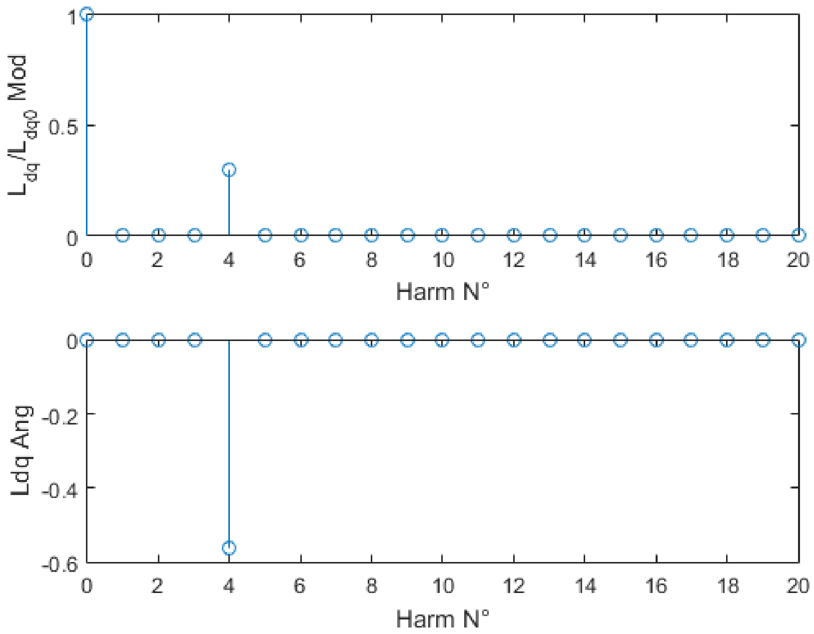

Figure 6.9: Harmonic Spectrum of L_{dq}: amplitude and phase

It appears that the L_d and L_q are constant value because the have only the zero harmonic; instead L_{dq} has all the harmonic equal to null. Therefore we can conclude that for a machine with sinusoidally distributed winding the current of d-axis does not influence the value of the flux on the q-axis and vice versa.

6.2 Machine with Concentrated Windings

As a second case we consider a machine with phase self and mutual inductances described in Fig 6.11 and 6.12.

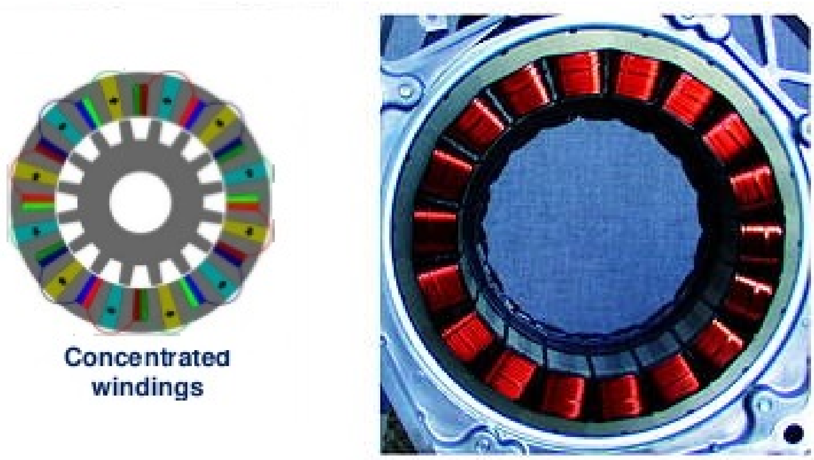

Figure 6.10: Example of machine with concentrated winding

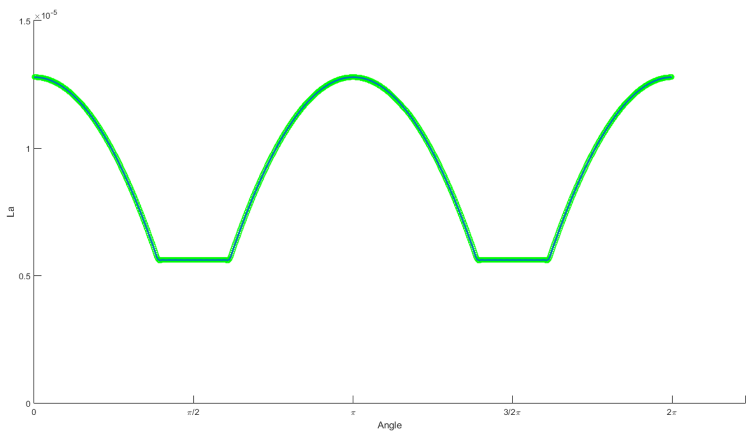

Figure 6.11: La

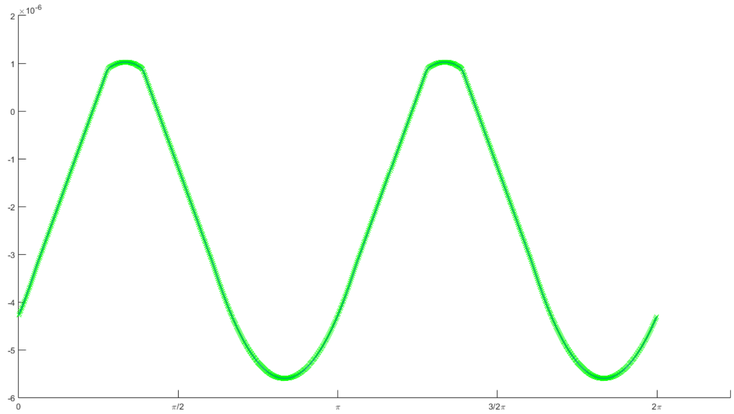

Figure 6.12: Lbc

In this case we cannot define a simple equation to describe the L_a and L_{bc} but it is easy enough to see that we have additional harmonics besides that of zero and the second order. These higher harmonics will change the spectrum of the matrix $\underline{L}_{\alpha\beta}$ and $\underline{L}_{dq}$ as shown in the next figures:

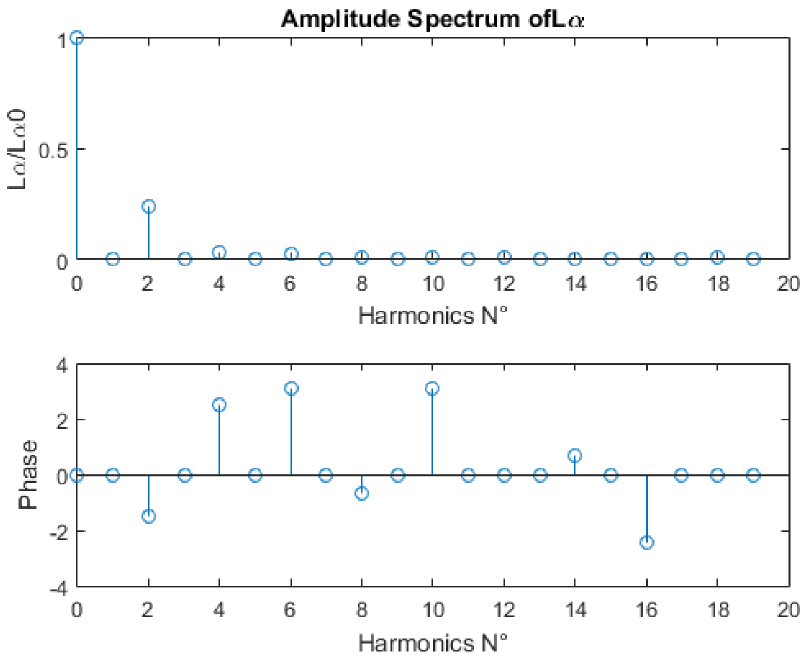

Figure 6.13: L_α Spectrum with $L_{\alpha 0} = 1.19 \times 10^{-2}$

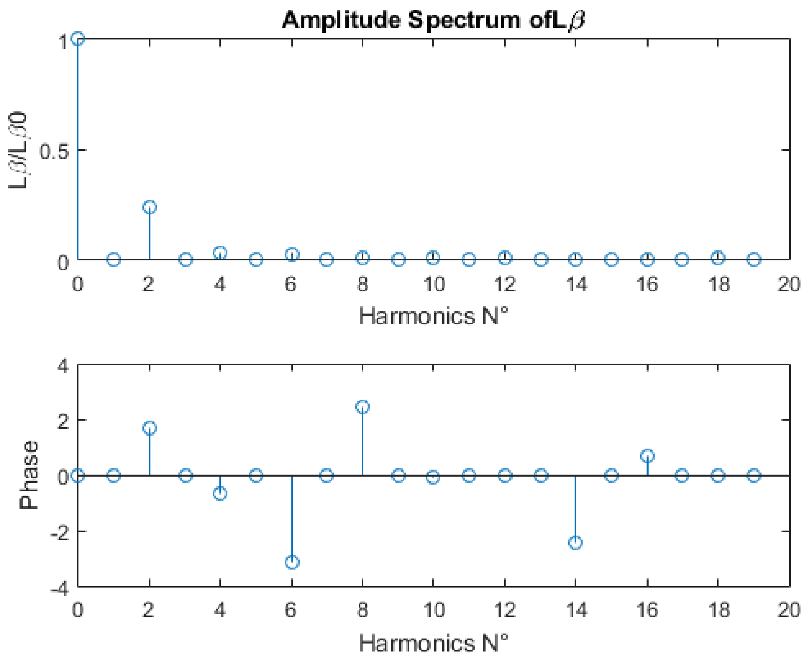

Figure 6.14: L_β Spectrum with $L_{\beta 0} = 1.19 \times 10^{-2}$

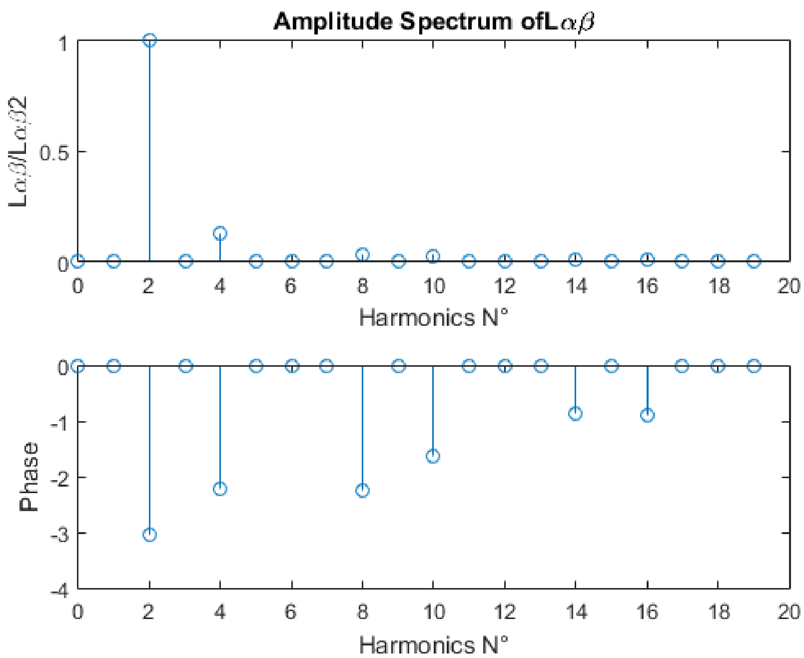

Figure 6.15: L_α Spectrum with $L_{\alpha\beta 2} = 2.83 \times 10^{-3}$

In $\alpha\beta$ there are the presence of $2k$-th harmonic and their amplitude decreases like $\frac{1}{k^2}$. Particular in $L_{\alpha\beta}$ there is not the zero harmonic but only the $2k$-th one.
If we compare the spectrum obtained in the first example (distributed windings) with this one (concentrated windings), we can deduce that in $L_{\alpha\beta}$ is never present the zero harmonic and those higher than the second order appear if the spectrum of L_a and L_bc also presents higher harmonics number.

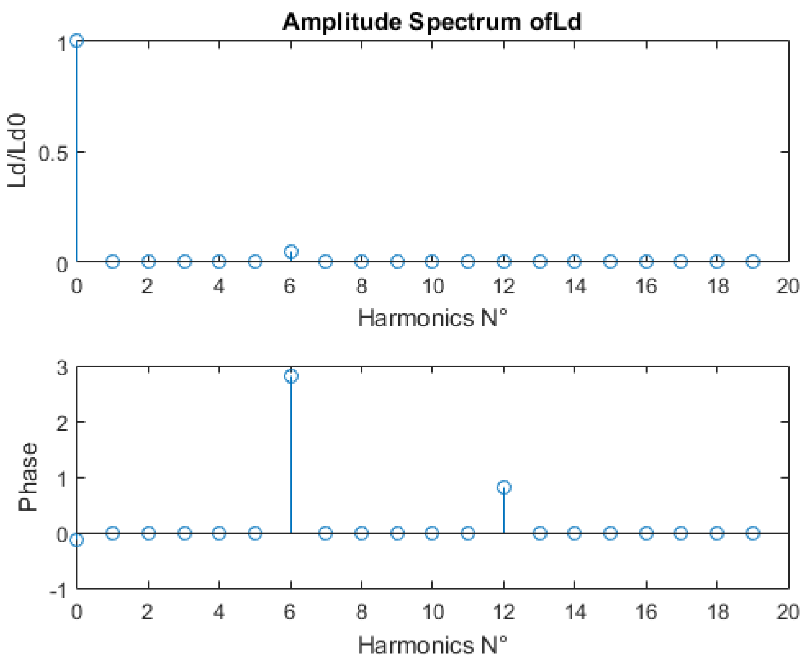

Figure 6.16: L_d Spectrum with $L_{d0} = 1.21 \times 10^{-2}$

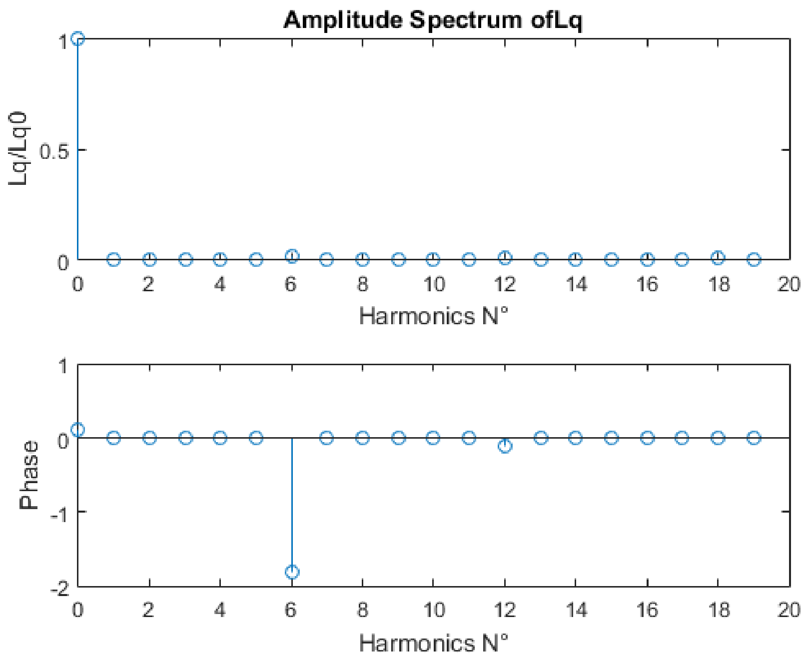

Figure 6.17: L_q Spectrum with $L_{q0} = 1.18 \times 10^{-2}$

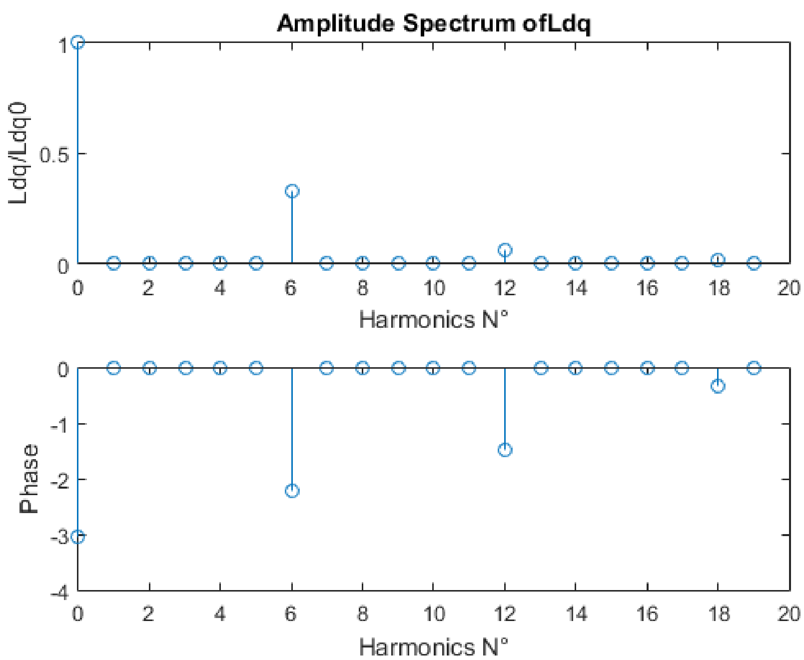

Figure 6.18: L_{dq} Spectrum with $L_{dq0} = 1.42 \times 10^{-3}$

In dq we find that the spectrum of the elements of inductance matrix is composed by the harmonics of $6k$ order. We examine the L_{dq} and find out that in this case a zero order harmonic is present although it's magnitude is 10 times smaller than the zero component in L_d or L_q.
Thus the L_{dq} is not null if in the "abc" system the spectrum of inductances presents harmonics higher than 2 and this happen when the windings are not perfectly distributed sinusoidally in the stator.
The presence of a not null L_{dq} entails that the fluxes are expressed in function of both the current $\lambda_d(i_d, i_q)$ and $\lambda_q(i_d, i_q)$

Chapter 7

Validation Measurement

In this chapter we'll analyse the measurement taken with different technique and we'll try to compare the result with the model described in the previous Chapters in order to see if there is correspondence between the real machine and its theoretical model.
The machine analysed is an IPM therefore we'll compare the results obtained from the measurements with the equation developed for this type of machine.

7.1 Test Benches

The measurements are obtained using two different test benches:

1. **Slow Speed Test Bench**:
this test bench is characterized by a master motor that through a mechanical gear can generate high torque at very low rotational speed. The master motor is a three-phase brushless with power equal to 1.24 [kW] and rotational speed of 3000[rpm]. Because of the mechanical gear the resulting speed is 50[rpm] and the torque can be almost 200[Nm]. On the motor a resolver and an encoder are installed.

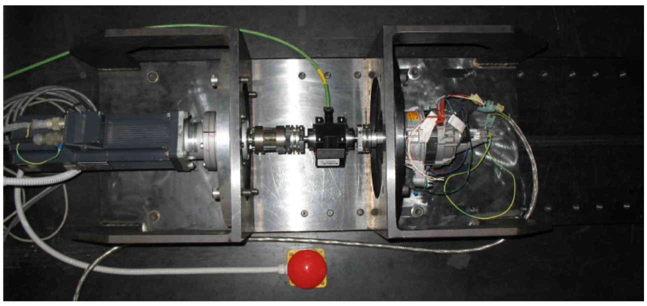

Figure 7.1: slow speed test bench structure: the master motor on the left side and the slave on the right. In this structure it's also installed a torquemeter to measure the torque.

2. **High Speed Test Bench**:
in this test bench the master motor is an asynchronous machine with

5.5[kW] rated power and a rotational speed of 2250[rpm]. On the motor is installed an encoder.

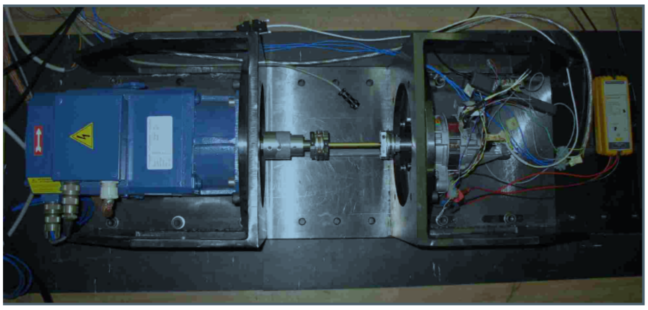

Figure 7.2: high speed test bench: the master motor on the left side and the slave on the right. In this configuration there is not the torque meter but it can be installed to measure the torque if necessary.

7.2 Measurement of Bemf

The measurements of the back electromotive forces are made using an oscilloscope and three differential voltage probes. The motor is rotated by an external machine that can be set to rotate at a specific speed.

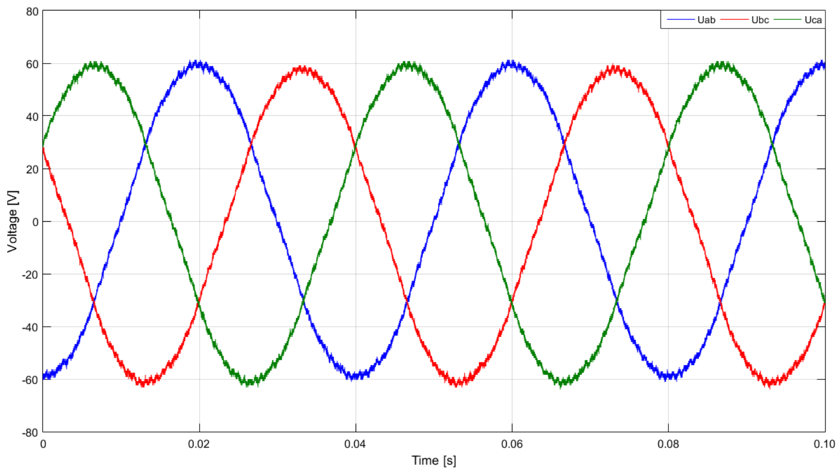

Figure 7.3: Bemf at 750 rpm with 1/20V probe

7.3 Measurement of the cogging torque

The cogging torque is the torque that is present even if the motor is not powered.

The test is performed in the slow speed test bench (described in the previous section) and a torque meter is installed to obtain the cogging torque measurement. In this test is important that the rotational speed is maintained constant throughout the entire duration of the test to avoid inertial torque that can invalidate the measurement.
The cogging depends on the configuration of the rotor and stator and the type of permanent magnet installed.

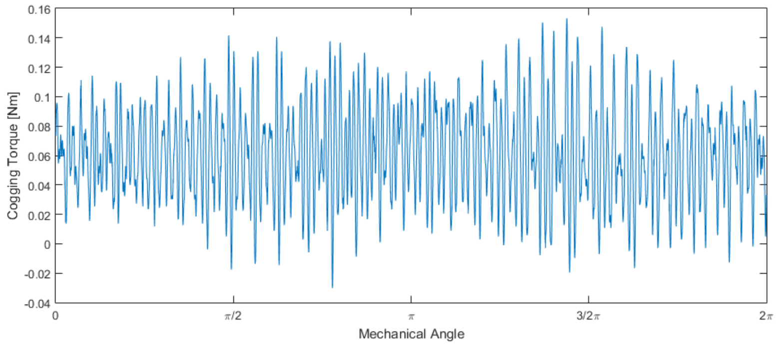

Figure 7.4: Cogging Torque Measured

Because the cogging is very small and there is a lot of disturbing signal in the measurement, we'll analyse with the FFT in MATLAB the output of torquemeter to find the different harmonics present in the data acquired.

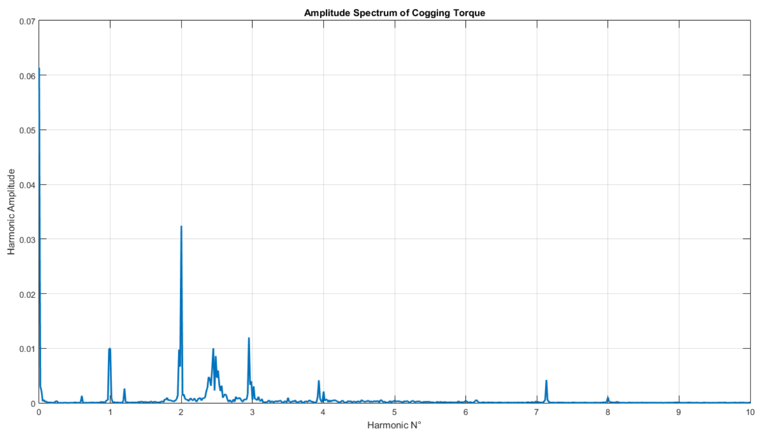

Figure 7.5: Harmonic Spectrum of the cogging torque the fundamental is considered at the frequency of 4.77[Hz] which complies with the rotational speed of 1 rad/s and the number of 30 slots.

The cogging torque is defined in the section 4.1 as:

$$\tau_{\text{cogg}} = \frac{1}{2}\underline{i}_r^t \frac{d\underline{L}_r(\theta)}{d\theta}\underline{i}_r \tag{7.3.1}$$

This formula is confirmed by the measurements; in fact the measurement is performed setting the stator current equal to zero and the equivalent current of the rotor due to the permanent magnet, because the machine is an IPM, is constant. So the cogging torque is caused by anisotropy of the rotor that make $\frac{d\underline{L}_r(\theta)}{d\theta} \neq 0$.

7.4 Measurement of the torque in MPTA condition

This type of measurements are made using the slow speed test bench, in this case we try to rotate the rotor with a constant speed of $1[\frac{\text{rad}}{\text{s}}]$. We repeat the measurement for 5 different current values: 1[A], 2[A], 3[A], 4[A] and 4.3[A]. All the measurement are taken using 20Nm torquemeter.
The outputs of the torquemeter are given in the following figures.

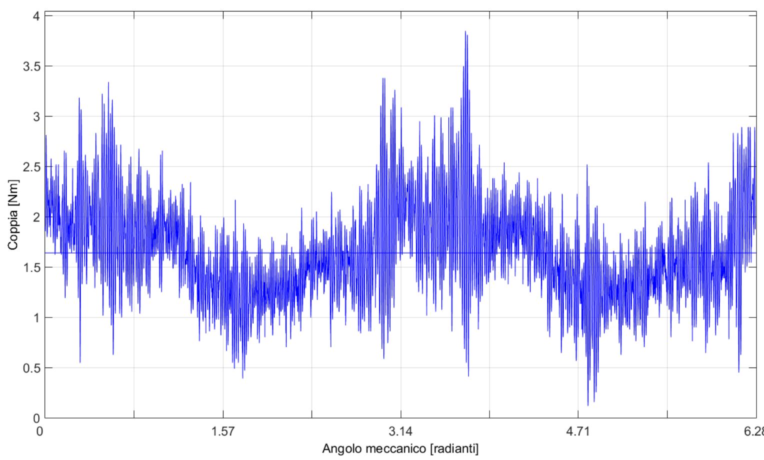

Figure 7.6: Torque I=1[A]

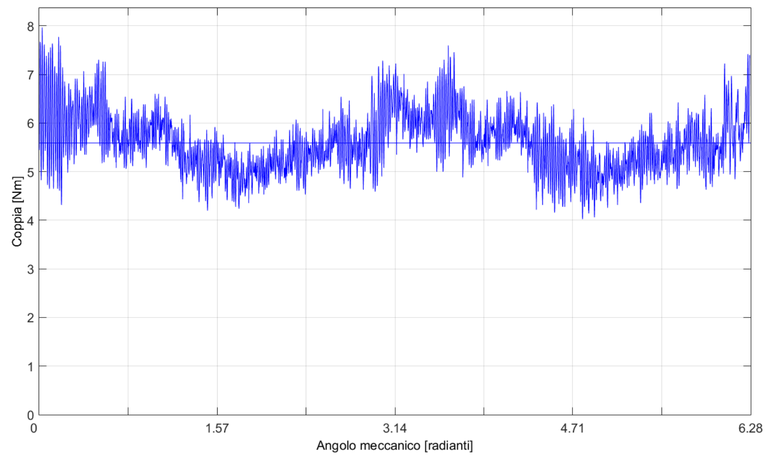

Figure 7.7: Torque I=2[A]

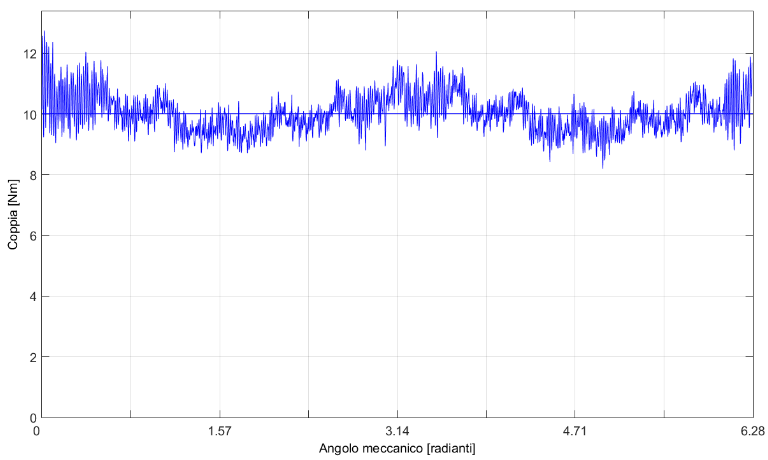

Figure 7.8: Torque I=3[A]

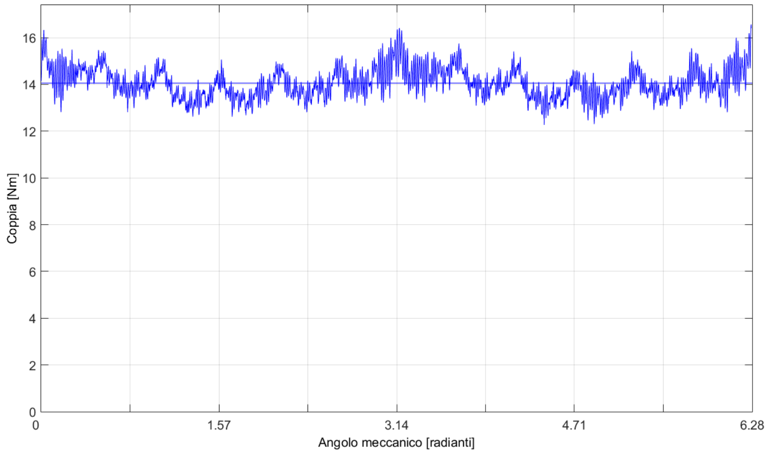

Figure 7.9: Torque I=4[A]

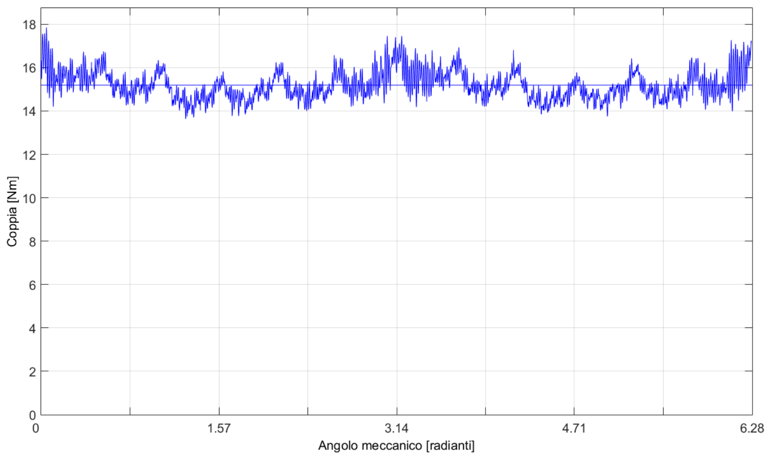

Figure 7.10: Torque I=4.3[A]

The first thing that is worth noticing is that the torque follow the equation seen in the previous chapters.

$$
\begin{aligned}
\tau(i_s,\theta) = & \frac{1}{2}\underline{i}_s^t\frac{d\underline{L}_s(\theta)}{d\theta}\underline{i}_s + && \text{reluctance torque} \\
& \frac{d\lambda_{m,a}}{d\theta}i_{sa} + \frac{d\lambda_{m,b}}{d\theta}i_{sb} + \frac{d\lambda_{m,c}}{d\theta}i_{sc} + && \text{electro-dynamic torque} \\
& \frac{1}{2}I_{mg}^2\left(\frac{dL_{ra}}{d\theta} + \frac{dL_{rb}}{d\theta} + \frac{dL_{rc}}{d\theta}\right) && \text{cogging torque}
\end{aligned}
\tag{7.4.1}
$$

In fact we see that at higher stator current we have higher torque. If we analyse the torque signal and distinguish each harmonic that it is composed of, we can isolate the cogging torque component and see that it's present as the theory predicts. In our case the cogging component is too small and it's hidden by the disturbs in the signal so we cannot isolate that frequency.

It's important to note that at the highest currents we are no longer in the linear state so the saturation begins to affect the average torque equation and the cogging torque, both becoming smaller then expected.

7.5 Torque Map

The torque map measurements are taken with the slow speed test bench, keeping a steady speed of $1[\frac{\text{rad}}{s}]$ and using a torquemeter of $20[Nm]$. The measurement are taken setting the maximum and minimum value of the i_d and i_q $[-4 \quad 4, -4 \quad 4]$; furthermore the current limit of $I = \sqrt{i_d^2 + i_q^2} < 4.3$ is respected.

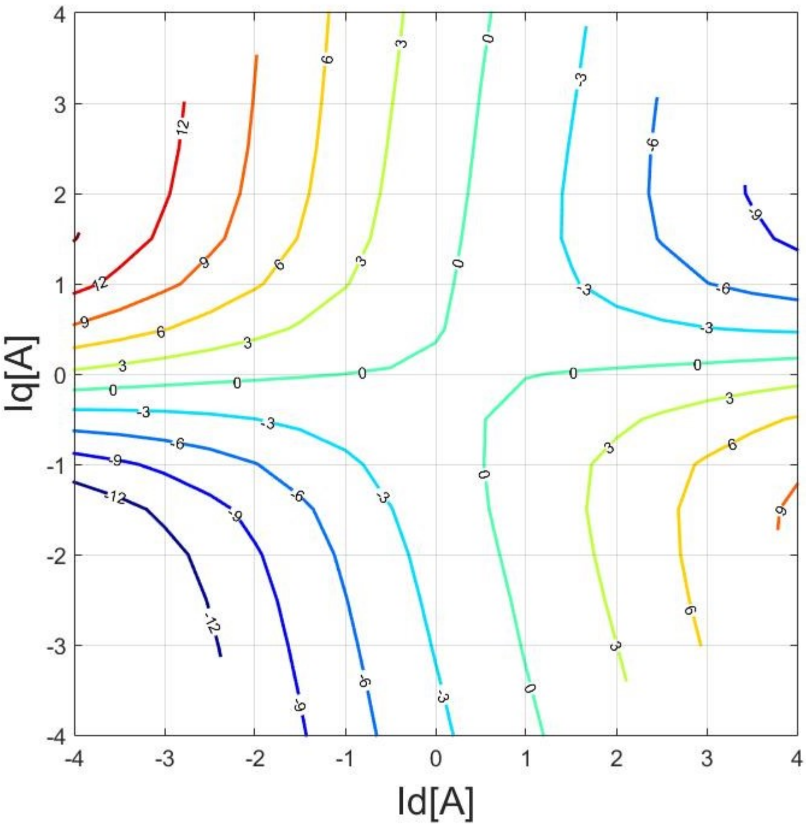

Figure 7.11: Torque Map

The current map looks like what we expected; the only minor difference

is that the map isn't perfectly symmetrical but results a little rotated this is caused by an error in the alignment during the measurements procedures.
A second aspect is the effect of the iron saturation on the vertical asymptote of the torque which is moved toward higher d-current when L_q decreases.

7.6 Magnetization Characteristic

The measurement to obtain the magnetization characteristic are carried out using the high speed test bench keeping a steady speed of $250[\frac{\text{rad}}{s}]$.
Fluxes in d and q-axis are obtained from d-q voltage and current measurements and applying steady state equation. In Figs 7.12 and 7.123 gives the results.

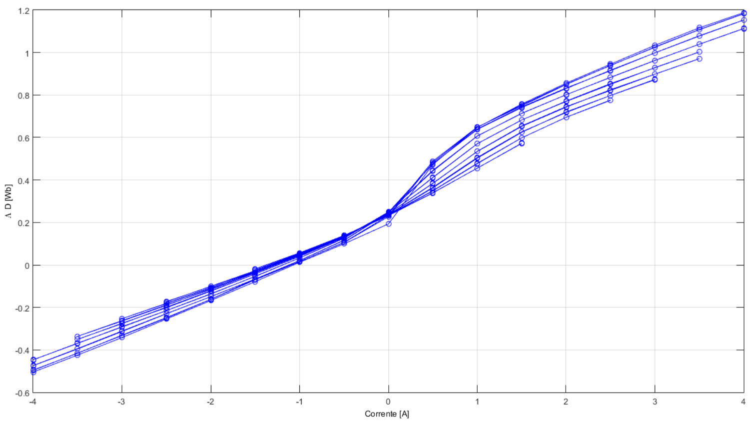

Figure 7.12: L_d

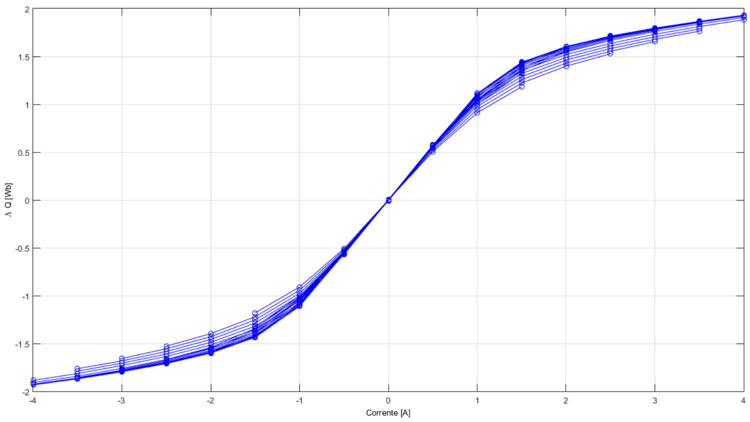

Figure 7.13: L_q

In these figures we represent the flux of each axis versus the current in d and q axis.
From the figures can be extract some propriety of the magnetization characteristic:

1. The $\lambda_d(0,0) \neq 0$ due to the permanent magnet in the machine.

$$\begin{aligned} \lambda_{is,d}(0,0) &= L_d i_d + L_{dq} i_q = 0 \\ \lambda_d(0,0) &= \lambda_{sd}(0,0) + \lambda_{mg} = \lambda_{mg} \end{aligned} \tag{7.6.1}$$

2. The λ_q is symmetrical and $\lambda_q(0,0) = 0$

$$\begin{aligned} \lambda_{is,q}(0,0) &= L_q i_q + L_{qd} i_d = 0 \\ \lambda_q(0,0) &= \lambda_{sq}(0,0) = 0 \end{aligned} \tag{7.6.2}$$

3. There is the presence of the inductance L_{dq} because we can see that λ_d also change with i_q and λ_q with i_d

These results confirm the model developed in the previous chapters.

Chapter 8

Conclusions

Our objective was to develop a general model that could describe thoroughly the electrical machine limiting assumptions. We started from a doubly fed machine and throughout the document at each step we added a new hypothesis to simplify the model created. After the model was developed, we took measurements on a machine to get information about it and compare the measure taken with the model created. We got that the real machine and the modeled are matched if the assumptions expressed are met during the measurement process. Therefore we can say that the model created is correct and can be used to describe the machine discussed in the document. Furthermore since the model is generic it can be developed for specific machines by adding appropriate hypothesis.
In conclusion the goal that we set has been reached and the model created thoroughly describe our machine.

Chapter 9

Riassunto In Italiano

[*] Lo scopo della tesi è quello di sviluppare un modello generale in grado di descivere le macchine elettriche che andremo a trattare. Siamo partiti dalla macchina alimentabile sia da rotore sia da statore, ne abbiamo descritto le equazioni di tensione, energia e coppia e da qui aggiungendo le ipotesi sulle correnti di rotore abbiamo ottenuto la descrizione anche per le macchine IPM, SPM e a riluttanza.
Tutte queste macchine vengono descritte in tre diversi sistemi di riferimento: "abc"," $\alpha\beta$ " e in "dq" e vengono ricavate le relative equazioni di tensione e di coppia.
Nel capitolo 4 viene descitta la macchina lineare ipotizzando che $\lambda = Li$. Questa ipotesi comporta che la descrizione della macchina può essere semplificata, e da qui otteniamo equazioni di coppia più utili rispetto al caso in cui vi era la saturazione del circuito magnetico.
Successivamente, è stata analizzata la macchina con l'ipotesi che i conduttori siano distribuiti sinusoidalmente alla'interno dello statore, ciò comporta che la macchina presenta una matrice delle induttanze i cui elementi L_{dq} risultano nulli. Questo risultato comporta che i flussi nel sistema di riferimento dq non sono funzione delle correnti dell'asse opposto, cioè λ_d è funzione solo di i_d e λ_q solo di i_q.
Abbiamo eseguito uno studio sulla componente armonica della matrice delle induttaze descritta nei capitoli precedenti, soprattutto il comportamento delle armoniche nei diversi sistemi di riferiento. Da qui, abbiamo ottenuto che, per avere equazioni i cui flussi in dq sono funzione della sola corrente del proprio asse (come nel capitolo precedente), è necessario che i conduttori siano sinusoidalmente distribuiti, altrimenti l'elemento L_{dq} risulta diverso da zero.
Per concludere, abbiamo eseguito misure su un motore IPM, e da qui abbiamo ottenuto che le componenti di coppia da noi ipotizzate nel modello trovano corrispondenza nella macchina reale. Perciò, il modello da noi creato trova riscontri con la macchina reale. Di conseguenza, è uno strumento valido e in grado di descivere le macchine elettriche.

Appendices

Appendix A

Nomenclature

Quantities	
i	Current
L	Apparent Inductance
l	Differential Inductance
R	Resistance
v	Voltage
w	Energy
w'	Co-energy
p	Pole Number
s	Slip
λ	Flux Linkage
τ	Torque
θ	Electric Angular Position
ω	Angular Speed

Index (subscripts)	
a, b, c	Quantities of a Three-phase windings
α, β	Two-phase Quantities of a Two-phase windings
d, q	Quantities of a two axis reference frame
s	Stator
r	Rotor
m	Mechanical
me	Electro-Mechanical
mg	Magnetic
x, y	Stator and Rotor Quantities in α, β (x=r,s; y=α, β)
x, z	Stator and Rotor Quantities in d, q (x=r,s; z=d, q)

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[2] Sebastian Ebersberger and Bernhard Piepenbreier. "identification of differential inductances of permanent magnet synchronous machines using test current signal injection". *Power Electronics, Electrical Drives, Automation and Motion (SPEEDAM) 2012 International Symposium on*, pages 1342 – 1347, June 2012.

[3] S.Bolognani. Profesor's note.

[4] Wikipedia.