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Stability and uplift of supergravity vacua

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1 Introduction

As we all know, consistency between modern cosmological observations and the Einstein field equations of General Relativity necessarily requires the existence of a positive cosmological constant Λ . Its introduction, on the one hand, is fundamental for explaining the dominance of the dark energy, which is the actual source of the current accelerated expansion of the Universe. On the other hand, even more importantly, on a positive cosmological constant relies also the *inflation* phenomenon, a phase of accelerated expansion of the Universe occurred 10^{-36} s after the Big Bang and widely accepted as the solution of some flaws of the hot Big Bang models, such as the flatness and horizon problems.

Recent cosmological observations on type Ia supernovae [1, 2] have however shown that the rate of expansion during the inflationary epoch was much more consistent than nowadays. This incongruity, together with other reasons related to the inflation dynamics, suggested that the constant quantity put by hand into the Einstein-Hilbert action cannot be the only contribution to Λ and that some other mechanism that generates an effective cosmological constant must exist. Among the many feasible candidates for this mechanism, the most intriguing one is based on the introduction of one or more scalar fields with a nonvanishing scalar potential $V(\phi)$, whose Lagrangian, $-\frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi)$, to be added to the Einstein-Hilbert Lagrangian $\frac{1}{16\pi G}(R - 2\Lambda)$, clearly provides the net cosmological constant $\Lambda + 8\pi GV(\phi_0)$ when the scalar fields sit in the extremum ϕ_0 of the scalar potential. Since in the Standard Model of Fundamental Interactions no fundamental scalar particle is present apart from the Higgs (whose vacuum energy $V(\phi_0)$ however induces a cosmological constant 120 orders of magnitude larger than the observed one), such a scenario inevitably involves some new physics beyond the Standard Model.

As a consequence, any theory that aspires to consistently extend the Standard Model by means of new scalar fields *must* allow for the possibility of a positive cosmological constant, dictated by a positive vacuum energy. Moreover, its value should not be random, and should be possibly adjusted (also at the quantum level) to the very small cosmological constant currently observed, without enforcing unreasonable fine tunings.

At the present time, the most promising candidate for such an extended theory is certainly String Theory [3, 4, 5]. It was born as a theoretical framework in which the point-like particles of particle physics are replaced by one-dimensional vibrant objects called *strings*, and has proven to be a very robust model for describing the unification of all four interactions in nature in a consistent way with Quantum Mechanics and General Relativity. Due to the strong internal coherence constraints, nowadays are known just five 10-dimensional supersymmetric string theories, which are actually deeply tied together by many duality relations and are even connected to a different model in eleven dimensions, the so-called M-theory, which does not involve strings at all.

Although the particle spectrum of string theories and M-theory is made of a finite number of light states and an infinite tower of very massive excitations,

fortunately both theories have a low-energy limit (describing just the lightest modes) given by supergravity theories living in ten and eleven dimensions respectively. Since the space-time on which we usually base our experience is 4-dimensional, however, it is necessary to assume that the six or seven additional dimensions are small and “compactified” in a finite volume, which is the reason why they have not been detected yet. In particular, depending on how compactification is performed, we obtain several 4-dimensional effective supergravity theories, which differ by both matter content and amount of supersymmetry. This abundance of lower-dimensional models underlies the rich content of String Theory and, most importantly, allows to arrange many pleasant phenomenological features.

For example, the most trivial compactification of 11-dimensional supergravity, in which the internal manifold is assumed to be the product of seven circles, leads to a 4-dimensional supergravity theory with the maximum ($N = 8$) amount of supersymmetry whose matter content and Lagrangian are completely fixed. However, due to the quite unrealistic spectrum and the absence of a scalar potential, this compactification is sufficient neither to provide phenomenologically viable models nor to possibly support a positive effective cosmological constant. On the other hand, one may consider more complicated compactifications, in which for instance the product of seven circles is replaced by manifolds with more structure (such as the seven-sphere). In this case higher-dimensional forms may acquire nontrivial background fluxes, which lead to more complicated effective theories in four dimensions, typically coming with non-abelian gauge symmetries and with a scalar potential as a result of the more complicated geometry. The 4-dimensional supergravities arising from such compactifications are indeed more suitable for providing interesting phenomenology and their study is essential to understand what specific compactification has been chosen by nature.

Unfortunately, however, not all 4-dimensional supergravities derive from the 11-dimensional landscape and, thanks to the many restrictions imposed by supersymmetry invariance, the theories with a big number of supersymmetries turn out to be the ones with a more traceable stringy origin. On the contrary, supergravity models with just one supersymmetry are rather flexible and can accommodate many interesting phenomenological features (also because are the only that include chiral fermions), but have a mostly unclear connection to String Theory. As a consequence, despite their unrealistic phenomenology, theories with a large amount of supersymmetry play an important role in the context of flux compactifications and are still at the center of current investigations, especially to obtain new information about the issues String Theory is currently suffering from.

Among the problems that afflict String Theory, one of the most prominent concerns the selection of a vacuum state compatible with a positive cosmological constant, as required by the initial cosmological considerations. In fact, not only we do not know the full vacuum structure of String Theory, but the simple task of constructing a stable de Sitter vacuum in a supergravity theory with a traceable stringy origin has proved to be extremely challenging. An intuitive reason for the abundance of Minkowski and Anti de Sitter vacua and this apparent lack of de Sitter ones is that supersymmetry can be possibly preserved only in presence of negative cosmological constants, and thus a positive cosmological constant is always a signal

of supersymmetry breaking. Throughout the years several attempts were made to find stable de Sitter vacua in 4-dimensional compactified theories, and many of the most studied models in the literature include contributions to the scalar potential motivated by nonperturbative effects. For example, in the compactification of type IIB supergravity, an explicit $N = 1$ framework [7], also known as KKLT scenario, have been assembled and has shown to easily provide de Sitter solutions: however, their stability is ensured only as long as both nonperturbative effects and D-branes are introduced. Conversely, all known examples of de Sitter solutions in extended supergravity models with a clear higher-dimensional origin (maximal supergravity included) display tachyons, which, besides, are too large to guarantee a sufficiently long inflationary period.

Therefore, analyzing the existence of stable de Sitter vacua in supergravity theories with a traceable stringy origin, whether phenomenologically viable or not, may give new information about the difficulty we are currently facing to generate them in low energy string models. In particular, a possible discovery of stable de Sitter solutions can eventually reveal which elements are actually really necessary for guaranteeing positive cosmological constants and stability at the same time. The scope of this thesis is thus to study a particular supergravity model to understand whether stable de Sitter vacua may arise or not.

As a first step, we will focus on a very special model, namely the maximal $SO(8)$ gauged supergravities arising from compactifications of M-theory on the seven-sphere and a consistent $N = 1$ truncation of them. Despite the unrealistic spectrum, these theories are an efficient playground for understanding the occurrence of de Sitter vacua, because they are really constrained and their matter content is completely determined by supersymmetry, and thus many calculations can be done analytically until the end. We will then extend the analysis to the so-called STU-models, which are generalizations of the $N = 1$ models previously studied and which mostly derive from M-theory compactifications as well. In particular, we will first proceed with a general analysis, searching for stable dS vacua without caring about the possible uplift to eleven dimensions. Secondly, we will restrict to a subclass of these models for which the correspondence with maximal supergravity as well as M-theory is nowadays clear. Relevantly, in contrast with the recent literature [38, 42], our search will be based on scalar potentials containing only on perturbative contributions, and nonperturbative effects will be totally excluded.

However, let us stress that the present work completely neglects many phenomenological features that should be certainly present in a theory that aims to fully reproduce all experimental observations. Apart from the unrealistic spectrum and the lack of chiral fermions, we do not control whether the size of the cosmological constant is compatible with the desired hierarchy $\Lambda \ll m_{\text{gravitino}}^2 \ll m_{\text{scalars}}^2$ required by current cosmological experiments. Moreover, the actual value of the found cosmological constants is only classical and may acquire nontrivial quantum corrections, which might actually produce radical changings. Although these and many other issues are ignored, we will see firsthand that reproducing stable dS vacua with a traceable uplift will be anyway extremely difficult. The thesis will be structured as follows.

- In the second chapter we provide a very basic introduction to supergravity theories in four and eleven dimensions, paying particular attention to the supersymmetry breaking mechanism and the production of effective cosmological constants by matter couplings.
- In the third chapter we give a brief overview of the general method that allows to reduce a higher-dimensional theory to four dimensions, specifying then the discussion to the compactification leading to our STU-model.
- The fourth chapter contains a preliminary analysis of the STU-models, in which the results for the truncations of the $SO(8)$ supergravities are exposed and matched with the recent literature.
- Finally, in the fifth chapter, which is the core of the thesis, we present the original material on the search of stable dS vacua in STU-models, both in the general and in the upliftable case.

2 Supergravities in four and eleven dimensions

In this chapter we provide a very brief introduction to supergravity theories in four and eleven dimensions. In summary, we first introduce minimal $D = 4$ supergravity and discuss how a cosmological constant naturally arises in this context from the coupling to matter fields. We then describe pure and gauged maximal supergravities in four dimensions, ending with an overview of the low-energy limit of M-theory, 11-dimensional supergravity.

The scope of these sections is, on the one hand, to exhibit the conditions under which a generic $N = 1$ supergravity theory coupled to chiral multiplets admits stable (Anti) de Sitter vacua, either preserving or breaking supersymmetry. On the other hand, the discussion aims to prepare a theoretical framework for the description of how our 4-dimensional supergravity model derives from the 11-dimensional theory upon dimensional compactifications, which will be the main content of the next chapter.

2.1 Supersymmetry and supergravity

One of the most prosperous ideas for extending the Standard Model of Fundamental Interactions relies on the concept of *supersymmetry*, i.e. a symmetry relating the bosonic and fermionic degrees of freedom of a theory [8, 9, 10]. In fact, although there is actually no direct evidence that supersymmetry is an exact symmetry of nature, many Standard Model's supersymmetric extensions are interesting candidates for solving the so-called hierarchy problem, for guaranteeing the unification of the Standard Model's couplings at the GUT scale and for providing dark matter candidates.

We remind that supersymmetry is defined in terms of N generators, acting on the Hilbert space of a certain field theory and called *supercharges* Q^I , $I = 1, \dots, N$, which exchange bosonic and fermionic single-particle states as

$$Q^I |\text{boson}\rangle = |\text{fermion}\rangle, \quad Q^I |\text{fermion}\rangle = |\text{boson}\rangle. \quad (2.1)$$

These operators, obeying for consistency anti-commutation relations among themselves, transform as spin-1/2 objects, and, even though they satisfy a trivial commutation relation with Poincaré space-time translations, they do *not* commute with Lorentz generators. Therefore, they produce a space-time symmetry and cannot be treated independently of the Poincaré generators, differently, for example, from internal symmetries.

The anti-commuting algebra the supercharges enjoy admits irreducible representations via fermionic operators, the *supermultiplets*, whose dimension depends on N . By (2.1), it is straightforward that each supermultiplet contains both bosons and fermions, which have the same number of degrees of freedom and, if supersymmetry is not spontaneously broken, even the same mass. The analysis in terms of supermultiplets is particularly useful because, if a theory is invariant under supersymmetry, its field content can be subdivided into supermultiplets and fully classified according to them.

Examples. By exploiting the explicit form of the supersymmetry algebra, one can show that a generic *massless* supermultiplet of a theory invariant under N supersymmetry transformations contains 2^N on-shell complex degrees of freedom, out of which 2^{N-1} are bosonic and 2^{N-1} are fermionic. In particular, the more the number of supercharges increases, the longer the supermultiplets become.

- For instance, $N = 1$ supersymmetry admits chiral multiplets, whose (on-shell) degrees of freedom are those of one Weyl fermion and one complex scalar. Apart from multiplets with spin higher-than-2 particles, then, $N = 1$ supersymmetry presents vector multiplets, containing one vector and one Weyl fermion, and graviton multiplets, consisting of one graviton and one spin-3/2 particle, the gravitino. Even gravitino multiplets, made of one gravitino and one vector, are included. One can easily check that each of the mentioned multiplets is characterized by 2 real bosonic and 2 real fermionic degrees of freedom.
- In contrast, an $N = 8$ supersymmetric theory presents a more constrained matter content and contains just one supermultiplet with at most spin-2 particles, whose degrees of freedom include 70 real scalars, 56 spinors, 28 vectors, 8 gravitinos and one graviton. Likewise, this multiplet has 2^8 real bosonic and 2^8 real fermionic degrees of freedom.

Even though in principle the number of supersymmetry generators can assume any integer value, by just taking into account the restrictions on the particles' spin, N cannot be arbitrarily large. In fact, in four dimensions any supermultiplet contains particles with spin at least as large as $\frac{1}{4}N$ and thus, to describe local and interacting theories, N can be at most as large as 4 for theories with maximal spin 1 (like gauge theories) and as large as 8 for theories with maximal spin 2 (like gravity). For this reason, $N = 4$ and $N = 8$ are, respectively, upper bounds for non-gravitational and gravitational supersymmetric theories and represent the maximum amount of supersymmetry those theories can enjoy, whence the term *maximal* supersymmetric theories. Instead, supersymmetric theories (either including gravitons or not) with just one supersymmetry are said to be *minimal*. In this thesis we will deal with just minimal and maximal supersymmetry.

By definition, then, a supersymmetric model which is also invariant under general coordinate transformations is called *supergravity* model [11, 12]. Even if it is not trivial, it turns out, thanks to the supersymmetry algebra, that a theory invariant under both supersymmetry and general coordinate transformations is equivalent to a theory having *local* supersymmetry. In fact, the anti-commutator between two supersymmetry transformations, which reads

$$\{Q_{\alpha}^I, \bar{Q}_{\dot{\beta}}^J\} = 2\delta^{IJ}\sigma_{\alpha\dot{\beta}}^{\mu}P_{\mu}, \quad (2.2)$$

is proportional to a space-time translation. Hence, in theories with local supersymmetry (where the spinorial infinitesimal supersymmetry parameters ε^{α} depend on x^{μ}) this anti-commutator is an infinitesimal translation whose parameters depend on the space-time point. This means that locally supersymmetric models are automatically invariant under local Lorentz transformations, and are nothing but

theories of gravity. Thus, theories with local supersymmetry and General Relativity are intimately tied together.

The spread interest for local supersymmetric theories is therefore justified by the fact that, if supersymmetry is actually realized in nature, the theory that correctly describes *all* interactions among elementary particles must be necessarily a supergravity theory. Moreover, supergravity theories have a better ultraviolet behavior than General Relativity, especially in theories with a large number of supersymmetries.

Analogously to Yang-Mills theories where the gauging of a bosonic internal symmetry requires the introduction of a gauge (vector) field, gauging supersymmetry demands the introduction of a new suitable gauge connection. This field, on the one hand, has to appear in proper covariant derivatives which eventually restore the invariance of the action under local transformations. On the other hand, in analogy with the gauge vector fields, it should transform into the derivative of the infinitesimal supersymmetry parameter, which is actually a spinorial quantity. Thus, the gauge field must be a vector-spinor ψ_μ^α , which (on-shell) propagates just two helicity 3/2 states. The field ψ_μ^α is therefore an on-shell gravitino and the supersymmetry of the theory further requires that a graviton sits in the same supersymmetry multiplet. For this reason, local supersymmetry necessarily implies the presence of at least a graviton multiplet, which is another way to understand the already mentioned intimate connection between local supersymmetry and gravity.

We now expose the main features of the simplest supergravity model, $N = 1$ supergravity, which is the theory with the lowest possible number of local supersymmetries.

2.2 $N = 1$ supergravity

Let us first analyze the most general field content an $N = 1$ supergravity model can accommodate. Since we cannot build any locally supersymmetric Lagrangian without including the gravitino, $N = 1$ supergravity must contain at least¹ a **graviton multiplet**, which we denote by $(g_{\mu\nu}, \psi_\mu)$. If the graviton multiplet is the sole content, the theory is also known as *pure* minimal supergravity.

In addition to the graviton multiplet, we can also insert other fields which will be naturally coupled to gravity. As we will see, however, local supersymmetry imposes many restrictions to the form of the total Lagrangian and gives rise to particular interaction terms, in such a way that the whole theory is eventually specified just by a little number of data. Apart from the graviton one, the multiplets which enter $N = 1$ supergravity are the following.

- There can be n_c **chiral multiplets**, each one made of one complex scalar and one Majorana spinor, denoted by (ϕ^n, χ^n) , $n = 1, \dots, n_c$. The fermionic degrees of freedom should play the role of ordinary matter fields in this theory, while their superpartners, i.e. the corresponding scalars, are additional

¹As we will explain soon, when supersymmetry is local, the gravitino cannot be contained in a gravitino multiplet.

particles not yet observed. Besides, chiral multiplets should also provide the Higgs, which is the only scalar particle detected.

- There can be n_v **vector multiplets**, made of one gauge boson (which should reproduce ordinary Yang-Mills interactions) and one Majorana fermion, also known as gaugino. However, as we will discuss in the next chapter, our $N = 1$ truncated model does not include any vector multiplet, and therefore vector multiplets will no longer be considered in this section.
- Although in principle gravitino multiplets should be present in minimal supergravity thanks to form of the supersymmetry algebra, it has been shown that adding local supersymmetric interactions to the free gravitino multiplet leads to undesired pathologies, like the existence of interactions propagating with speed bigger than the speed of light, the improper set to zero of some degrees of freedom or the need for the inclusion of higher-spin particles. Hence, no gravitino multiplet can be inserted.

Once established which fields actually play a role in minimal supergravity, we illustrate the most general locally supersymmetric Lagrangian (with at most two space-time derivatives) regulating their dynamics. The total Lagrangian has the form

$$\mathcal{L}_{\text{tot}}(g_{\mu\nu}, \psi_\mu, \phi^n, \chi^n) = \mathcal{L}_{\text{pure}}(g_{\mu\nu}, \psi_\mu) + \mathcal{L}_{\text{matter}}(g_{\mu\nu}, \psi_\mu, \phi^n, \chi^n), \quad (2.3)$$

where $\mathcal{L}_{\text{pure}}$ is a function of the sole graviton multiplet and is the only contribution to \mathcal{L}_{tot} in pure supergravity (and therefore it is independently invariant under local supersymmetry transformations). In particular, this part includes the graviton and the gravitino kinetic terms and appropriate interactions between them, mainly deriving from the space-time covariantization of the kinetic part. It should be stressed that, in four dimensions, the requirements of local supersymmetry invariance and of at most two space-time derivatives completely fix $\mathcal{L}_{\text{pure}}$, which does not depend on any modifiable parameter.

On the contrary, $\mathcal{L}_{\text{matter}}$ arises in presence of chiral supermultiplets and contains the scalars and the spinors kinetic and interaction terms. In contrast to $\mathcal{L}_{\text{pure}}$, fortunately, these couplings are *not* completely determined by supersymmetry and, as we will see, this relative freedom allows the existence of a quite arbitrary cosmological constant.

We now examine independently the two pieces contributing to \mathcal{L}_{tot} .

2.2.1 Pure supergravity action

In order to write a Lagrangian displaying also spinor derivatives in an invariant form under general coordinate transformations, we should be able to covariantize the spinor derivatives in analogy with the usual space-time covariantizations of General Relativity. This procedure is necessary for the pure minimal supergravity Lagrangian $\mathcal{L}_{\text{pure}}$, due to the presence of a gravitino derivative.

The covariantization can be achieved by utilizing the so-called Cartan formalism [13], in which the metric $g_{\mu\nu}$ is expressed in terms of the *vielbein* $e_\mu^a(x)$, defined

by $g_{\mu\nu}(x) = e_\mu^a(x)e_\nu^b(x)\eta_{ab}$. For each point x^μ , these matrices encode the (always existing) coordinate transformations mapping to the local inertial frame at that point. Moreover, this formalism allows the definition of the *spin connection* $\omega_\mu^a{}_b(x)$, which can be expressed as a function of the vielbein and satisfies $\omega^{(ab)} = 0$ for consistency.

Out of the spin connection we can construct the spinor covariant derivative

$$D_\mu\chi \equiv \partial_\mu\chi + \frac{1}{4}\omega_\mu^{ab}\gamma_{ab}\chi, \quad (2.4)$$

which, thanks to the spin connection's properties, transforms covariantly under both local Lorentz and general coordinate transformations. Since the gravitino is a vector-spinor field and supergravity is invariant under general coordinate transformations, the supergravity Lagrangian exhibits a gravitino covariant derivative, $D_\mu\psi_\nu$, which contains, in addition to the usual Levi-Civita connection term linked to the vector index ν , also the right hand side of (2.4).

The pure supergravity Lagrangian can be written in a form manifestly invariant under local space-time transformation as (up to 4-Fermi terms)

$$e^{-1}\mathcal{L}_{\text{pure}} = \frac{M_P^2}{2}R - \frac{1}{2}\bar{\psi}_\mu\gamma^{\mu\nu\rho}D_\nu\psi_\rho, \quad (2.5)$$

where $e = \sqrt{-\det[g_{\mu\nu}]}$, R is the Ricci scalar and $M_P = (8\pi G_N)^{-1}$ is the (reduced) Planck Mass. The former term is the usual Einstein-Hilbert Lagrangian, while the derivative contribution within the latter reproduces the free gravitino action. Because of the symmetry of $\Gamma_{\mu\nu}^\rho$, just the spin connection further contributes to the covariant derivative and induces a graviton-gravitino coupling, which is the only present interaction apart from the 4-Fermi terms we neglected. Besides, one can prove that this Lagrangian is invariant (up to total derivatives) under the infinitesimal local supersymmetry transformations

$$\delta_\varepsilon\psi_\mu = M_P D_\mu\varepsilon, \quad \delta_\varepsilon e_\mu^a = \frac{1}{2M_P}\bar{\varepsilon}\gamma^a\psi_\mu. \quad (2.6)$$

We should note that no cosmological constant appears into $\mathcal{L}_{\text{pure}}$ unless we try to modify the supersymmetry algebra into the (A)dS superalgebra. This modification (which is actually feasible) leads to a new contribution proportional to ε into the supersymmetry transformations (2.6) and gives rise, in addition to a cosmological constant, also to a gravitino mass. In particular, it can be proved that *only* for AdS we can write a consistent supersymmetric completion with a single supercharge. However, in our $N = 1$ model a cosmological constant (and suitable modifications of (2.6) giving rise to an (A)dS algebra) are dynamically generated by the coupling to matter fields.

2.2.2 Matter couplings

The contribution $\mathcal{L}_{\text{matter}}$ in (2.3) is completely specified by a few data which, interestingly, are the same ones that fully determine a theory with the same matter

content (except the graviton multiplet) in global supersymmetry. Therefore, the imposition of local supersymmetry does *not* introduce neither other freedom nor other restrictions on the choice of the theory parameters and just yields additional couplings. Before explaining which kind of data is necessary to state, let us give a few information about the scalar sector of $\mathcal{L}_{\text{matter}}$.

First, since we regard our supersymmetric theories as low-energy effective models arising from a more fundamental theory, we do not have to worry about the restrictions imposed by renormalizability. If renormalizability is not an issue, it turns out that in a generic supersymmetric theory the scalar fields describe a nonlinear σ -model based on a (Riemannian) target manifold \mathcal{M}_{sc} . Their Lagrangian is therefore characterized by non-minimal kinetic terms with the structure $-g_{m\bar{n}}(\phi)\partial^\mu\phi^m\partial_\mu\phi^{\bar{n}}$. Besides, the scalar fields must be interpreted as coordinates on \mathcal{M}_{sc} with metric $g_{m\bar{n}}$ and the total Lagrangian must be eventually invariant under general scalar coordinates transformations $\phi^n \rightarrow \tilde{\phi}^n(\phi^m)$. However, \mathcal{M}_{sc} is not a generic complex manifold of dimension n_c and its structure is highly restricted by supersymmetry: for $N = 1$ supersymmetry, for instance, \mathcal{M}_{sc} must be a Kähler manifold (and a Kähler-Hodge manifold in supergravity). This means that its metric g_{mn} must satisfy

$$g_{mn} = g_{\bar{m}\bar{n}} = 0 , \quad (2.7)$$

$$g_{m\bar{n}} = g_{\bar{n}m} = \partial_m\partial_{\bar{n}}K \equiv K_{m\bar{n}} , \quad (2.8)$$

where $K(\phi^m, \phi^{\bar{n}})$ is a (locally existing) *real* function called **Kähler potential** with dimension two in mass. Hence in minimal supersymmetry (and supergravity) the structure of the scalar manifold \mathcal{M}_{sc} is entirely expressed by the function $K(\phi^m, \phi^{\bar{n}})$, which is one of the information to specify in order to construct the matter Lagrangian. If renormalizability of the (global) theory is requested, however, the kinetic term must be minimal, i.e. $g_{m\bar{n}} = \delta_{m\bar{n}}$ and \mathcal{M}_{sc} must be flat, and this implies that the Kähler potential assumes the simple form $K(\phi^m, \phi^{\bar{n}}) = \sum_{i=1}^{n_c} \phi^i\phi^{\bar{i}}$.

In both local and global supersymmetry, the masses of the chiral fields and all their self-interaction terms (Yukawa as well as cubic and quartic scalar potentials) are fully encoded in a single holomorphic function $W(\phi^n)$, the **superpotential**, whose mass dimension is three. Renormalizability restricts $W(\phi^n)$ to be a polynomial of at most cubic order. It is relevant that

- the Kähler potential $K(\phi^m, \phi^{\bar{n}})$ and
- the superpotential $W(\phi^m)$

are actually the *only* information that completely determines $\mathcal{L}_{\text{matter}}$.

The simplest way to obtain the matter Lagrangian in supergravity is to start from the corresponding globally supersymmetric Lagrangian and to perform certain modifications on its terms. In fact, it can be shown that the total supergravity Lagrangian (2.3) in presence of matter fields can be written as the sum between the pure supergravity Lagrangian (2.5) and the global matter Lagrangian, after having space-time covariantized the derivatives in the latter, “Kähler covariantized” all derivatives, replaced the superpotential with the combination $e^{K/2M_P^2}W$ and added new terms (out of which even a gravitino mass and new interactions) which

eventually ensure the invariance of the total action under local supersymmetry transformations. The end result of these modifications is schematically

$$e^{-1}\mathcal{L}_{\text{tot}} = \frac{M_P^2}{2}R - \frac{1}{2}\bar{\psi}_\mu\gamma^{\mu\nu\rho}D_\nu\psi_\rho + \dots - V(\phi), \quad (2.9)$$

where the dots stand for the kinetic terms of the chiral multiplet, a gravitino mass and interaction among all fields, except for the non-derivative scalar self-interactions which are encoded in the scalar potential

$$V \equiv V_F - V_G = e^{K/M_P^2} \left[g^{m\bar{n}} (D_m W) (D_{\bar{n}} W^*) - \frac{3|W|^2}{M_P^2} \right]. \quad (2.10)$$

In the last formula, the derivative $D_m = \partial_m - \partial_m K/M_P^2$ is covariant under Kähler transformations

$$K(\phi^m, \phi^{\bar{n}}) \rightarrow K(\phi^m, \phi^{\bar{n}}) + h(\phi^m) + h^*(\phi^{\bar{n}}), \quad (2.11)$$

and enters just in the first contribution to the potential, V_F , often called F-term, because $F_m \equiv D_m W$ is exactly the auxiliary field (conventionally called F) that is needed to have the on-shell closure of the supersymmetry algebra. Both the contributions V_F and V_G to the scalar potential are semi-positive definite (the former thanks to the positive definiteness of the Kähler metric), and thus, in general, V is neither positive nor negative definite.

We should note that the supergravity scalar potential (2.10), and even the total Lagrangian (2.9), reduce to the corresponding global quantities in the global limit $M_P \rightarrow \infty$. Hence, the exponential and the term V_G proportional to $|W|^2$ in (2.10) are supergravity corrections that vanish in global supersymmetry. However, as we will discuss later, they crucially modify the vacuum selection and the cosmological constant in local supersymmetry. Of these two correction, phenomenologically speaking, the most relevant is the (semi-negative definite) term $\sim |W|^2$ which makes the scalar potential no longer semi-positive definite (as it is in global supersymmetry, since just V_F contributes).

For the sake of completeness, let us mention that the introduction of vector multiplets just adds a new contribution V_D , called D-term, into the right hand side of (2.10). This term, which is semi-positive definite as V_F , depends on how the vectors are coupled to the chiral multiplets by gaugings and its explicit structure will be discussed in the example of section 2.4.1, once we will have introduced gauged supergravity.

We have now all the elements to discuss supersymmetry breaking and to explain how a cosmological constant is effectively produced in $N = 1$ supergravity by matter couplings.

2.2.3 Supersymmetry breaking and cosmological constant

As we noticed, in a supersymmetric theory (based on a Minkowski vacuum state) all the particles belonging to the same supermultiplet must exhibit the same mass. Therefore, since we do not see any mass degeneracy in the elementary particle

spectrum at least at energies of order 10^2 GeV, if supersymmetry is at all realized in nature, it must be *broken* at low enough energy. There are in general two ways supersymmetry can be broken: either spontaneously or explicitly.

- Supersymmetry is *spontaneously* broken [10, 11] if the theory is supersymmetric but admits a (stable or metastable) supersymmetry breaking vacuum state, i.e. a vacuum state $|\Omega\rangle$ that is not left invariant by all the supersymmetry transformation: $Q^I |\Omega\rangle \neq 0$ for some I . In such a vacuum, one or more scalars acquire a vacuum expectation value of the order of the supersymmetry breaking scale.
- Supersymmetry is *explicitly* broken if the Lagrangian contains terms which do not preserve supersymmetry by themselves. These terms should have positive mass dimension, in order to be irrelevant in the far UV and so to not break the nice UV properties of supersymmetric theories. In such a scenario, called *soft* supersymmetry breaking, the supersymmetry breaking scale enters explicitly in the Lagrangian.

Here we focus on the spontaneous way to break supersymmetry in the minimal theory, also because soft supersymmetry breaking models can actually arise as low energy descriptions of models where supersymmetry is broken spontaneously.

We first recall that a (Minkowski) vacuum is a Lorentz invariant state configuration. This means that all field derivatives and all fields but scalar ones should vanish in a vacuum state. For this reason, the only non trivial part of the Hamiltonian which can be different from zero in a vacuum is the non-derivative scalar part, which, by definition, is the scalar potential V . Therefore, the vacua of a theory, which are the states where the energy is a critical point, are in one-to-one correspondence with the critical points of V . In particular, the stable (or metastable) vacua are in correspondence with the global (or local) minima of V .

Let us now study the condition a vacuum configuration should satisfy to break the supersymmetry, both in the global and local case. We remind that the supersymmetry transformation of a field operator $\Phi(x)$ can be written as $[\bar{\epsilon}Q, \Phi(x)] = \delta_{\text{susy}}\Phi(x)$. By applying the above definition, thus, a vacuum field configuration $\Omega = \{\Phi_i\}$ (in which at most the scalars are nonvanishing) breaks supersymmetry if and only if it contains at least one field $\tilde{\Phi}$ whose supersymmetry variation $\langle \Omega | \delta_{\text{susy}} \tilde{\Phi} | \Omega \rangle = \langle \Omega | [\bar{\epsilon}Q, \tilde{\Phi}] | \Omega \rangle \equiv \langle \delta_{\text{susy}} \tilde{\Phi} \rangle$ is not zero. Since bosonic fields transform in fermionic ones, for a bosonic field $\tilde{\Phi} = \Phi_{\text{bos}}$ we automatically have $\langle \delta_{\text{susy}} \Phi_{\text{bos}} \rangle = \langle \text{fermions} \rangle = 0$, and so $\tilde{\Phi}$ cannot be bosonic. This implies that all supersymmetry breaking can only come from the vacuum configurations that produce a nonvanishing $\langle \delta_{\text{susy}} \Phi_{\text{fer}} \rangle$.

By simply looking at the supersymmetry transformations of the chiral fermions (proportional to F-terms) and the gauginos (proportional to D-terms), one deduces that spontaneous supersymmetry breaking can only occur via a nonvanishing F-term and/or a nonvanishing D-term, i.e. for a vacuum field configuration satisfying $V_F \neq 0$ and/or $V_D \neq 0$ (see (2.10)). If, as in our model, we restrict to a theory displaying just chiral multiplets, supersymmetry is spontaneously broken if and only if $\langle D_m W \rangle \neq 0$ for some m .

Many comments are in order at this point. First of all, since in global supersymmetry the scalar potential is $V = V_F + V_D$, and since V_F and V_D are both semi-definite positive, a vacuum state preserves the supersymmetry if and only if V evaluated at the vacuum, $\langle V \rangle$, is zero. Conversely, supersymmetry is broken if and only if its vacuum energy $\langle V \rangle$ is bigger than zero. Hence, supersymmetric vacua are indeed described by the zero's of the scalar potential and the potential itself is an useful order parameter of supersymmetry breaking.

As we anticipated, the situation in supergravity is extremely different due to the scalar potential correction V_G proportional to $|W|^2$ in (2.10). In fact, in supergravity, supersymmetry is preserved if both V_F and V_D are vanishing, but, due to the presence of V_G , this does not imply that V is zero. Thus, the only useful information for establishing the spontaneous breaking is the F-term and D-term values at the vacuum, while the whole scalar potential no longer fully determines the breaking. However, if V is positive, we certainly can conclude that supersymmetry is broken, because, due to the semi-negative definiteness of $-V_G$, in this case necessarily at least one between V_F and V_G must be different (and bigger) than zero. Instead, if V is zero or negative no useful information can be extracted.

Let us now turn to the discussion about the cosmological constant, whose introduction is actually indispensable for phenomenology. We recall that, in a gravity theory described by a Lagrangian \mathcal{L} , the cosmological constant Λ is defined as the *constant* term that contributes to $e^{-1}\mathcal{L}$:

$$e^{-1}\mathcal{L} = \frac{M_P^2}{2}R + \dots - \Lambda . \quad (2.12)$$

The appearance of a cosmological constant in a theory admitting a certain vacuum state $|\Omega\rangle$ is intimately tied to the presence of a non-trivial scalar potential. In fact, if we denote by ϕ_0^m the vacuum expectation value (VEV) of the scalar fields ϕ^m in $|\Omega\rangle$, the scalar potential expressed in terms of their vacuum fluctuation $\tilde{\phi}^m = \phi^m - \phi_0^m$ is

$$V(\phi_0 + \tilde{\phi}) = V(\phi_0) + \left[\partial_m V(\phi_0) \tilde{\phi}^m + \frac{1}{2} \partial_m \partial_n V(\phi_0) \tilde{\phi}^m \tilde{\phi}^n + \dots + \text{h.c.} \right] . \quad (2.13)$$

Hence, by comparing (2.9) and (2.12), it is straightforward that a cosmological constant is effectively generated around the vacuum $|\Omega\rangle$ if $V(\phi_0)$ does not vanish, and its value is exactly the scalar potential evaluated at the vacuum: $\Lambda = V(\phi_0)$.

By the previous discussion on supersymmetry breaking, we conclude that a globally supersymmetric theory can just admit either Minkowski stable vacua ($\Lambda = 0$) preserving the supersymmetry or dS metastable vacua ($\Lambda > 0$) breaking the supersymmetry.

As we mentioned, however, the coupling to gravity drastically modifies also this scenario. Thanks to the contribution V_G , supergravity can indeed exhibit even AdS vacua ($\Lambda < 0$), either preserving or breaking supersymmetry. On the contrary, since $V > 0$ implies supersymmetry breaking, a dS vacuum can just break supersymmetry, which is a further confirmation of what we said at the end of section 2.2.1.

We should notice that, at least in the $N = 1$ theory, there is a strict connection between the cosmological constant and the scale of supersymmetry breaking, which is

actually a great problem in global supersymmetry. In fact, since the supersymmetric partners of the particles in the Standard Model have not been observed yet, we have to take a large scale of supersymmetry breaking, for example 1 TeV. This gives a cosmological constant whose value is many orders of magnitude larger than the observed one. This problem may be solved in supergravity theories, because, thanks to V_G , with an appropriate choice of the superpotential the resulting cosmological constant may be small at will (although quantum corrections may radically change its value).

Examples. Let us now provide two simple examples of Minkowski vacua, which, as we will see in the last chapter, will be particularly relevant for the search of stable dS vacua in our model. We focus in particular on a theory without vector multiplets, in such a way that supersymmetry breaking can only occur when $D_n W \neq 0$ for some n .

- (i) The simplest Minkowski vacuum is indeed a *Minkowski supersymmetric* vacuum, ϕ_s , which satisfies

$$\begin{cases} V(\phi_s) = 0 \\ D_n W(\phi_s) = \partial_n W(\phi_s) + \frac{\partial_n K(\phi_s)}{M_P^2} W(\phi_s) = 0 \quad \text{for any } n . \end{cases} \quad (2.14)$$

A quick inspection of (2.10) tells us that also $V_G(\phi_s)$ must vanish, and so it must be $W(\phi_s) = 0$. Therefore ϕ_s enjoys

$$\begin{cases} W(\phi_s) = 0 \\ \partial_n W(\phi_s) = 0 \quad \text{for any } n . \end{cases} \quad (2.15)$$

Interestingly, Minkowski supersymmetric vacua are always minima of the scalar potential. To prove this fact, we notice that the Hessian matrix

$$V'' = \begin{pmatrix} \partial_i \partial_{\bar{j}} V & \partial_i \partial_{\bar{j}} V \\ \partial_i \partial_{\bar{j}} V & \partial_i \partial_{\bar{j}} V \end{pmatrix} \quad (2.16)$$

evaluated at $\phi = \phi_s$ is block diagonal, given that each term in $\partial_i \partial_{\bar{j}} V(\phi_s)$ presents a vanishing factor due to (2.15). For the upper-left diagonal block we obtain, again taking account of (2.15) and that $g^{m\bar{n}} = K^{m\bar{n}}$, $\partial_i \partial_{\bar{j}} V(\phi_s) = e^{K(\phi_s)} K^{m\bar{n}}(\phi_s) W_{im} W_{\bar{j}\bar{n}}^*$, where for compactness we wrote $W_{mn} = \partial_m \partial_n W(\phi_s)$.

Therefore, using a matricial notation, $\partial_i \partial_{\bar{j}} V(\phi_s) = e^{K(\phi_s)} \left(W K(\phi_s) W^\dagger \right)_{i\bar{j}}$, and so $\partial_i \partial_{\bar{j}} V(\phi_s)$ is semi-positive definite because $e^{K(\phi_s)} > 0$ and, for any $v \in \mathbb{C}^3$, $v \neq 0$, we have $v^\dagger W K(\phi_s) W^\dagger v = (W^\dagger v)^\dagger K(\phi_s) (W^\dagger v) \geq 0$ due to the positiveness of $K(\phi_s)$ (remember that \mathcal{M}_{sc} is Riemannian). Obviously also the lower-right block $\partial_{\bar{i}} \partial_j V$ is semi-positive definite and hence the Hessian matrix is semi-positive definite. Let us notice that, if $\det [W_{ij}] \neq 0$, $(W^\dagger v)^\dagger K(\phi_s) (W^\dagger v) > 0$ and the diagonal blocks are positive definite: in this case the Minkowski vacuum is even a strict minimum of the potential.

This result was however expected from the stability of generic supersymmetry-preserving vacua.

- (ii) Looking at (2.10), it is rather clear that the only other possible way to obtain a vanishing $V(\phi_0)$, in this case breaking supersymmetry, is to impose that at the vacuum V_F and V_G are both different from zero and that they compensate each other: $V_F = V_G \neq 0$. This means that it must be

$$K^{m\bar{n}}(\phi_0)D_m W(\phi_0)D_{\bar{n}}W^*(\phi_0) = \frac{3|W(\phi_0)|^2}{M_P^2} . \quad (2.17)$$

The simplest method to solve this equation is to suppose that that supersymmetry is broken just along one fixed direction, for example ϕ^1 , i.e. $D_1 W(\phi_0) \neq 0$ and $D_m W(\phi_0) = 0$ for $m \neq 1$, in such a way that only one term survives in the sum in the left hand side. With this assumption, (2.17) becomes indeed

$$K^{1\bar{1}}(\phi_0) \left| \partial_1 W(\phi_0) + \frac{\partial_1 K(\phi_0)}{M_P^2} W(\phi_0) \right|^2 = \frac{3|W(\phi_0)|^2}{M_P^2} . \quad (2.18)$$

This equation is automatically satisfied for any value of $W(\phi_0)$ if $\partial_1 W(\phi_0) = 0$ and $K^{1\bar{1}}(\phi_0) |\partial_1 K(\phi_0)|^2 = 3M_P^2$. Therefore, in an $N = 1$ model where there is at least one scalar field ϕ^T for which the Kähler potential fulfills the requirement

$$K^{T\bar{T}} |\partial_T K|^2 = 3M_P^2 , \quad (2.19)$$

a vacuum state ϕ_n that enjoys the three conditions

$$\begin{cases} D_T W(\phi_n) \neq 0 \\ D_m W(\phi_n) = 0 \quad \text{for } m \neq T \\ \partial_T W(\phi_n) = 0 , \end{cases} \quad (2.20)$$

is always a Minkowski vacuum which breaks supersymmetry just along the T direction. Let us note that, by exploiting the explicit form of the Kähler covariant derivative, (2.20) can be easily rewritten as

$$\begin{cases} W(\phi_n) \neq 0 \\ D_m W(\phi_n) = 0 \quad \text{for } m \neq T \\ \partial_T W(\phi_n) = 0 . \end{cases} \quad (2.21)$$

Besides, if the Kähler potential has the form

$$K(\phi) = \tilde{K}(\phi^T) + \hat{K}(\phi^{n \neq T}) \quad (2.22)$$

where \tilde{K} depends only on ϕ^T and \hat{K} depends only on the other fields, the condition (2.19) on the Kähler potential must be checked just for \tilde{K} .

Such vacua, called *no-scale* vacua [11, 12, 14], will frequently appear in our model because, as we will prove, our Kähler potential will satisfy the conditions (2.19) and (2.22). In particular, their simple characterization in terms of the theory parameters will be our starting point for finding stable dS vacua. Let us note that the F-term evaluated at a no-scale vacuum is a function of ϕ^T , and the

gravitino mass (which determines the supersymmetry breaking scale) is an arbitrary parameter as long as the vacuum expectation value of ϕ^T is not fixed, whence, indeed, the name “no-scale”.

Even more interestingly, also no scale vacua are always minima of the scalar potential, fact that can be shown with a demonstration similar to the previous one. Differently from the supersymmetric case, however, the stability of no-scale vacua is a nontrivial fact, given that they are not supersymmetry-preserving.

Although by now we mentioned just Minkowski vacua, in $N = 1$ supergravity obtaining even stable dS vacua is relatively easy, due to the huge freedom on the definitions of the Kähler potential and the superpotential. However, it is difficult to identify the origin of generic $N = 1$ theories in terms of an high energy string model. Since our eventual aim is to find the uplift of supergravity vacua, we will focus on the maximal theories, whose origin is well understood and whose structure is very restricted by the requirement of supersymmetry invariance. Anyway, the above analysis about minimal supergravity was not useless at all, because a consistent truncation of our maximal theory will lead to an $N = 1$ model, which is the one we will eventually examine.

2.3 $N = 8$ supergravity

In this section we provide a very short overview of ungauged maximal supergravity in four dimensions [16, 19]. When no gauge interaction is present, the field content and even the action of maximal supergravity are uniquely determined by the (strong) requirement of local supersymmetry invariance, and, in contrast with minimal supergravity, no scalar potential appears in the Lagrangian.

Let us first analyze which particles play a role in this theory. As we mentioned in the example of section 2.1, the only possible (massless) $N = 8$ supermultiplet with at most spin-2 particles is made of 70 real scalars, 56 spinors, 28 vectors, 8 gravitinos and one graviton, which therefore constitute the (completely fixed) field content of maximal supergravity. We denote these fields by

$$\left(\phi^{ijkl}, \chi^{ijk}, A_\mu^\Lambda, \psi_\mu^i, e_\mu^a \right), \quad (2.23)$$

where we left as understood the anti-symmetrization with respect to the indices i, j, \dots , which assume 8 values and identify the transformation properties of ϕ^{ijkl} , χ^{ijk} and ψ_μ^i under the R -symmetry group (SU(8) for the maximal theory [10]). On the contrary, both the 28 vectors, labeled by the index $\Lambda = 1, \dots, 28$, and the graviton are real fields² and therefore singlets of R -symmetry.

For being able to write down explicitly the maximal supergravity Lagrangian, we have to provide a few indispensable information about the scalar and the vector sectors of the theory.

The scalar sector of maximal supergravity (as of all supergravity theories with $N \geq 3$) is described by a G/H coset space σ -model, where $G = E_{7(7)}$ is the global

²Actually, the vectors can be written also as A^{ij} and complex conjugates, with the self-duality condition.

isometry group of the theory and $H = \text{SU}(8)$ ($\text{SU}(8)/\mathbb{Z}_2$, strictly speaking) is its maximal compact subgroup [19]. In a convenient formulation of this σ -model (particularly useful for applying the gauging procedure), the 70 scalar fields ϕ^{ijkl} parametrize an $E_{7(7)}$ -valued matrix $\mathcal{V} = \mathcal{V}(x)$ taken in the fundamental **56** representation of the group. However, due to the redundancy in parameterizing the coset manifold G/H by an $E_{7(7)}$ -valued matrix, we have to require that the Lagrangian of the theory is eventually invariant not only under global $E_{7(7)}$ transformations of \mathcal{V} (as in any σ -model), but also under local $\text{SU}(8)$ transformations. Thanks to this advantageous formulation, both of them act linearly on \mathcal{V} and, respectively, from the left and from the right as

$$\delta \mathcal{V}_M^{\underline{N}} = \Lambda^\alpha (t_\alpha)_M^{\underline{K}} \mathcal{V}_K^{\underline{N}} - \mathcal{V}_M^{\underline{K}} k_{\underline{K}}^{\underline{N}}, \quad (2.24)$$

where Λ^α are constant parameters, $(t_\alpha)_M^{\underline{N}}$ are the 133 generators of $E_{7(7)}$ in the **56** representation, $k_{\underline{M}}^{\underline{N}}$ is a generic (x -valued) vector belonging to the $\text{SU}(8)$ algebra and the underlined indices refer to the behaviour under the subgroup $\text{SU}(8)$. An useful (although somewhat unconventional) parametrization of \mathcal{V} , which facilitates the coupling to fermions [16], makes use of the decomposition of the fundamental **56** representation as $\mathbf{56} \rightarrow \mathbf{28} + \mathbf{28}'$ and defines 56-dimensional complex vectors $\mathcal{V}_M^{ij} = (\mathcal{V}_\Lambda^{ij}, \mathcal{V}^{\Sigma ij})$, $M = 1, \dots, 56$, $\Lambda, \Sigma = 1, \dots, 28$, labeled by the (28 independent) anti-symmetrized indices $i, j = 1, \dots, 8$. Together with their complex conjugates denoted by $\mathcal{V}_{Mij} = (\mathcal{V}_{\Lambda ij}, \mathcal{V}^{\Sigma}_{ij})$, these $28 + 28$ vectors constitute the 56×56 matrix

$$\mathcal{V}_M^{\underline{N}} = (\mathcal{V}_M^{ij}, \mathcal{V}_{Mkl}) = \begin{pmatrix} \mathcal{V}_\Lambda^{ij} & \mathcal{V}_{\Lambda kl} \\ \mathcal{V}^{\Sigma ij} & \mathcal{V}^{\Sigma}_{kl} \end{pmatrix}, \quad (2.25)$$

which transforms under global $E_{7(7)}$ from the left (with the index M) and under local $\text{SU}(8)$ from the right (with the indices ij in the **28** representation).

Obviously, since the $\text{SU}(8)$ transformation is local, the derivative $\partial_\mu \mathcal{V}_M^{ij}$ (which must enter somehow in the Lagrangian, for instance in the scalar kinetic term) does not transform covariantly under $\text{SU}(8)$. However, its covariantization is required to eventually write the Lagrangian in a manifestly invariant form under local $\text{SU}(8)$. To render it covariant, we should define the composite $\text{SU}(8)$ scalar connection

$$\mathcal{Q}_{\mu ij}{}^{kl} = \delta_{[i}^{[k} \mathcal{Q}_{\mu j]}{}^{l]}, \quad \text{with} \quad \mathcal{Q}_{\mu i}{}^j = \frac{2}{3} i \left(\mathcal{V}_{\Lambda ik} \partial_\mu \mathcal{V}^{\Lambda jk} - \mathcal{V}^{\Lambda}_{ik} \partial_\mu \mathcal{V}_\Lambda{}^{jk} \right), \quad (2.26)$$

which is indeed such that the $\text{SU}(8)$ covariant derivative

$$\mathcal{D}_\mu \mathcal{V}_M^{ij} = \partial_\mu \mathcal{V}_M^{ij} - \mathcal{Q}_{\mu kl}{}^{ij} \mathcal{V}_M{}^{kl} \quad (2.27)$$

transforms under local $\text{SU}(8)$ (and global $E_{7(7)}$) as the matrix \mathcal{V}_M^{ij} . Out of this derivative, we can construct an $\text{SU}(8)$ (self-dual) tensor

$$\mathcal{P}_{\mu ijkl} = i \left(\mathcal{V}_{\Lambda ij} \partial_\mu \mathcal{V}^{\Lambda}_{kl} - \mathcal{V}^{\Lambda}_{ij} \partial_\mu \mathcal{V}_{\Lambda kl} \right), \quad (2.28)$$

which will enter in the kinetic term of the scalars and can be shown to be invariant under global $E_{7(7)}$.

In a supergravity theory with $N \geq 3$, the R -symmetry group is contained in H , i.e. it must be $H = H' \times R$ where H' is some complement. In particular, in maximal supergravity the two groups even coincide up to the discrete \mathbb{Z}_2 factor which will play no role in the following. Therefore, the fermionic fields ψ_μ^i and χ^{ijk} sit in local representations of $H = \text{SU}(8)$ of dimensions 8 and 56 respectively, which must be taken into account for writing a Lagrangian invariant local $\text{SU}(8)$.

The discussion about the vector sector is somewhat complicated due to the so-called electromagnetic duality in four dimensions [15, 18, 16], which we now briefly summarize. Let us mention that a fundamental ingredient in the construction of supergravity theories in a generic space-dimension D is the on-shell duality between massless p -forms and $(D - p - 2)$ -forms. In particular, in 4-dimensional maximal supergravity, this fact translates in an on-shell duality between the 28 vector fields A_μ^Λ previously introduced, conventionally called *electric*, and their 28 on-shell duals, called instead *magnetic*. To discuss this duality, we denote by $F_{\mu\nu}^\Lambda$ the 28 field strengths of the vectors A_μ^Λ , and we define the corresponding 28 dual field strengths by

$$G_{\mu\nu\Lambda} = i\varepsilon_{\mu\nu\rho\sigma} \frac{\partial \mathcal{L}}{\partial F_{\rho\sigma}^\Lambda} . \quad (2.29)$$

For instance, in the free case of with canonical kinetic terms, these field strengths are proportional to the Hodge duals of $F_{\mu\nu}^\Lambda$, defined by $\tilde{F}_{\mu\nu\Lambda} = \frac{1}{4!} \varepsilon_{\mu\nu\rho\sigma} F^{\rho\sigma\Lambda}$. The definition of $G_{\mu\nu\Lambda}$ is particularly convenient because, in general, by making use of both $F_{\mu\nu}^\Lambda$ and $G_{\mu\nu\Lambda}$, the Bianchi identities and the equations of motion of \mathcal{L} can be cast in the same form and written respectively as

$$\partial_{[\mu} F_{\nu\rho]}^\Lambda = 0 , \quad (2.30)$$

$$\partial_{[\mu} G_{\nu\rho]\Lambda} = 0 . \quad (2.31)$$

The last equation moreover implies that $G_{\mu\nu\Lambda}$ is a closed two-form (obvious in the free case), which allows the introduction of the above-mentioned dual magnetic fields $A_{\mu\Lambda}$, defined at least locally as $G_{\mu\nu\Lambda} = 2\partial_{[\mu} A_{\nu]\Lambda}$.

It can be interesting to analyze which transformations (called duality transformations) on the field strengths leave invariant the preceding construction. Since the two equations (2.30) and (2.31) are also homogeneous, they are invariant under the following infinitesimal linear combination of the field strengths and their duals

$$\begin{pmatrix} \delta F^\Lambda \\ \delta G_\Lambda \end{pmatrix} = \begin{pmatrix} A^\Lambda_\Sigma & B^{\Lambda\Sigma} \\ C_{\Lambda\Sigma} & D_{\Lambda}^\Sigma \end{pmatrix} \begin{pmatrix} F^\Sigma \\ G_\Sigma \end{pmatrix} , \quad (2.32)$$

where A, B, C, D are constant 28×28 matrices which give rise to a (nonsingular) 56×56 matrix belonging to $\text{GL}(56, \mathbb{R})$. However, imposing that the sole equations (2.30) and (2.31) are invariant is not sufficient (and actually meaningless), since G_Λ is defined in terms of F^Λ (and of the Lagrangian) by equation (2.29), whose invariance under duality transformations (2.32) must be required for consistency. Besides, in general, we should allow also the other fields within the Lagrangian to transform under duality transformations and impose that their field equations are invariant as well. It can be shown that, if we require the invariance under

duality transformations (2.32) of (2.29), (2.30), (2.31) and of all possible equations of the other fields, the 56×56 duality matrix must belong to a subgroup G_{dual} of $\text{Sp}(56, \mathbb{R})$, which is the group of matrices preserving the skew-symmetric matrix

$$\Omega_{MN} = \begin{pmatrix} 0 & \mathbf{1}_{28} \\ -\mathbf{1}_{28} & 0 \end{pmatrix}. \quad (2.33)$$

In maximal supergravity, in particular, the duality group G_{dual} is exactly the isometry group of the scalar manifold, $G = E_{7(7)}$, properly embedded into $\text{Sp}(56, \mathbb{R})$. Moreover, it turns out that the vector fields and the (already discussed) scalar fields are the only to be subject to the $E_{7(7)}$ duality transformations: the vectors $A_\mu^M \equiv (A_\mu^\Lambda, A_{\mu\Sigma})$ collectively transform in the **56** representation of $E_{7(7)}$, and the scalars as already specified in (2.24).

It should be noted that, although a duality transformation preserves by definition all equations of motion, in general it does not preserve the Lagrangian: for this reason, these dualities are said to be on-shell. More precisely, starting from a certain Lagrangian, duality transformations define *equivalence classes* of Lagrangians that lead to the same field equations and Bianchi identities.

We have now all ingredients to write the ungauged Lagrangian of $N = 8$ supergravity. As we just said, the Lagrangian is not invariant under the full electromagnetic duality group $E_{7(7)}$, but at least its invariance should persist up to terms proportional to the field equations. Actually, the Lagrangian invariance turns out to be eventually respected only by a subgroup of $E_{7(7)}$ [18]. Moreover, the Lagrangian will only contain the electric potentials A_μ^Λ and not their magnetic duals $A_{\mu\Lambda}$ (fact that manifestly breaks the full $E_{7(7)}$ duality group, but not necessarily the invariance of the equations of motion).

A particularly simple formulation of the theory makes use of the complex (anti-)self dual combinations of the field strength $F_{\mu\nu}^\Lambda$, denoted by $F_{\mu\nu}^\mp{}^\Lambda$ and defined by

$$F_{\mu\nu}^\Lambda = F_{\mu\nu}^{+\Lambda} + F_{\mu\nu}^{-\Lambda}, \quad \tilde{F}_{\mu\nu}^\pm{}^\Lambda = \pm F_{\mu\nu}^\pm{}^\Lambda. \quad (2.34)$$

In fact, in terms of $F_{\mu\nu}^\pm{}^\Lambda$, the ungauged Lagrangian [18] can be written as (using the mostly minus metric signature, setting $M_P = 1$ for compactness and neglecting 4-Fermi terms)

$$\begin{aligned} e^{-1}\mathcal{L} = & -\frac{1}{2}R - \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma} \left(\bar{\psi}_\mu{}^i \gamma_\nu D_\rho \psi_{\sigma i} - \bar{\psi}_\mu{}^i \overleftarrow{D}_\rho \gamma_\nu \psi_{\sigma i} \right) \\ & - \frac{i}{4} \{ \mathcal{N}_{\Lambda\Sigma} F_{\mu\nu}^{+\Lambda} F^{+\mu\nu\Lambda} - \bar{\mathcal{N}}_{\Lambda\Sigma} F_{\mu\nu}^{-\Lambda} F^{-\mu\nu\Lambda} \} \\ & - \frac{1}{12} |\mathcal{P}_\mu^{ijkl}|^2 - \frac{1}{12} \left(\bar{\chi}^{ijk} \gamma^\mu D_\mu \chi_{ijk} - \bar{\chi}^{ijk} \overleftarrow{D}_\mu \gamma^\mu \chi_{ijk} \right) \\ & - \frac{\sqrt{2}}{12} \left\{ \bar{\chi}_{ijk} \gamma^\nu \gamma^\mu \psi_{\nu l} \left(\mathcal{P}_\mu^{ijkl} + \hat{\mathcal{P}}_\mu^{ijkl} \right) + \text{h.c.} \right\} \\ & + F_{\mu\nu}^{+\Lambda} \mathcal{O}_\Lambda^{+\mu\nu} + F_{\mu\nu}^{-\Lambda} \mathcal{O}_\Lambda^{-\mu\nu} \\ & - i \left[(\mathcal{N} - \bar{\mathcal{N}})^{-1} \right]^{\Lambda\Sigma} \left[\mathcal{O}_{\mu\nu\Lambda}^+ \mathcal{O}_\Sigma^{+\mu\nu} + \mathcal{O}_{\mu\nu\Lambda}^- \mathcal{O}_\Sigma^{-\mu\nu} \right], \end{aligned} \quad (2.35)$$

where:

- The matrices $\mathcal{N}_{\Lambda\Sigma}$ and $\mathcal{O}_{\Lambda}{}^{\mu\nu}$ depend in general on the fields (in particular, $\mathcal{O}_{\Lambda}{}^{\mu\nu}$ contains just fermion bilinears and the possible superscripts “+” and “-” mean the (anti-)self dual decomposition (2.34)). For a generic choice of $\mathcal{N}_{\Lambda\Sigma}$ and $\mathcal{O}_{\Lambda}{}^{\mu\nu}$, the sum between the second, the fifth and the sixth lines represents the most general gauge field Lagrangian which is at most quadratic in the field strengths. For the maximal theory, however, $E_{7(7)}$ and $SU(8)$ covariance imposes many restrictions on $\mathcal{N}_{\Lambda\Sigma}$ and $\mathcal{O}_{\Lambda}{}^{\mu\nu}$, which turn out to be completely determined in terms of $\psi_{\mu}{}^i$, χ^{ijk} and $\mathcal{V}_{\Lambda}{}^{ij}$ (see [18] for their explicit form).
- Since fermions belong to a nontrivial representation of $SU(8)$, the derivatives $D_{\mu}\psi_{\mu}{}^i$ and $D_{\mu}\chi^{ijk}$ are both space-time and $SU(8)$ covariantized.
- The caret on $\hat{\mathcal{P}}_{\mu}^{ijkl}$ indicates the addition of other terms (that we do not report here) which are necessary to eventually ensure local supersymmetry invariance.

This Lagrangian is manifestly invariant under local $SU(8)$ transformations and the interaction terms that appear (in addition to the kinetic ones) are uniquely fixed by local supersymmetry. Interestingly, the strong restriction of supersymmetry invariance forbids the introduction of a scalar potential, which could have been present if we had imposed just the other invariance constraints. Indeed, it can be shown that the ungauged action is invariant under the following local supersymmetry variations, parametrized by 8 infinitesimal spinorial parameters ε^i :

$$\begin{aligned}
\delta\psi_{\mu}{}^i &= \mathcal{D}_{\mu}\varepsilon^i + \frac{1}{4}\sqrt{2}\hat{F}_{\mu\nu}{}^{-ij}\gamma^{\rho\sigma}\gamma_{\mu}\varepsilon_j + \frac{1}{4}\bar{\chi}^{ikl}\gamma^a\chi_{jkl}\gamma_a\gamma_{\mu}\varepsilon^j \\
&\quad + \frac{1}{2}\sqrt{2}\bar{\psi}_{\mu k}\gamma^a\chi^{ijk}\gamma_a\varepsilon - \frac{1}{576}\varepsilon^{ijklmnpq}\bar{\chi}_{klm}\gamma^{ab}\chi_{npq}\gamma_{\mu}\gamma_{ab}\varepsilon_j , \\
\delta\chi^{ijk} &= -2\sqrt{2}\hat{\mathcal{P}}_{\mu}^{ijkl}\gamma^{\mu}\varepsilon_l + \frac{3}{2}\hat{F}_{\mu\nu}{}^{-[ij}\gamma^{\mu\nu}\varepsilon^{k]} - \frac{1}{24}\sqrt{2}\varepsilon^{ijklmnpq}\bar{\chi}_{lmn}\chi_{pqr}\varepsilon^r , \\
\delta e_{\mu}{}^a &= \bar{\varepsilon}^i\gamma^a\psi_{\mu i} + \bar{\varepsilon}_i\gamma^a\psi_{\mu}{}^i , \\
\delta\mathcal{V}_M{}^{ij} &= 2\sqrt{2}\mathcal{V}_{Mkl}\left(\bar{\varepsilon}^{[i}\chi^{jkl]} + \frac{1}{24}\varepsilon^{ijklmnpq}\bar{\varepsilon}_m\chi_{npq}\right) , \\
\delta A_{\mu}{}^M &= \Omega^{MN}\mathcal{V}_N{}^{ij}\left(\bar{\varepsilon}^k\gamma_{\mu}\chi_{ijkl} + 2\sqrt{2}\bar{\varepsilon}_i\psi_{\mu j}\right) + \text{h.c.} .
\end{aligned} \tag{2.36}$$

In these formulas, again, the caret on the field strengths stands for the addition of suitable terms needed to guarantee local supersymmetry.

2.4 Gauged supergravity

As we noticed in the previous section, not only maximal supergravity has a field content completely fixed by supersymmetry, but it does not admit even a scalar potential. For this reason, this theory has a Minkowski (supersymmetric) ground state and (A)dS vacua are totally absent.

However, it is always possible to deform a supergravity theory (maximal supergravity included) by coupling the already present (non-)abelian vector fields to

charges assigned to the elementary fields, maintaining at the same time supersymmetry invariance. This procedure, also known as *gauging*, gives rise to the so-called *gauged supergravities* [15, 16], which are still supersymmetric theories, but much more flexible than the initial ones thanks to the huge freedom on how to perform the gauge couplings. For instance, gauged maximal supergravity admits a scalar potential, which may eventually support an effective cosmological constant, provide mass terms for the fields and describe possible spontaneous supersymmetry breaking scenarios. Moreover, as we will discuss in section 3.2, gauged maximal supergravity still has a clear 11-dimensional origin, which is essential to establish the connection between its possible vacua and M-theory.

The nice mentioned properties triggered our interest for these theories. Therefore, in this section we briefly review the gauging procedure in supergravity and introduce the embedding tensor formalism [15], which is completely general and applies almost in the same way to all supergravity theories, independently of the space-time dimension or the number of supercharges. In the last subsection we finally specify the discussion to maximal supergravity, showing in particular that a scalar potential and mass terms are generated by gaugings.

2.4.1 The gauging procedure

In this subsection we summarize the main features of the gauging procedure, without entering in details about the construction of the diverse parts of the Lagrangian or about the possible issues that arise from the modifications we introduce. In the next subsection, instead, we will address the explicit constructions by exploiting the embedding tensor formalism.

Let us start from a generic ungauged supergravity theory, i.e a supergravity theory invariant under a generic number N of supersymmetry transformations in which, however, the possible vectors are not coupled to the scalar fields by gaugings. We should keep in mind mainly the reference example of maximal supergravity, or the $N = 1$ supergravity model discussed in section 2.2 with the proper inclusion of n_v free vector multiplets (A_μ^I, λ^I) , $I = 1, \dots, n_v$, described by the minimal kinetic term

$$\mathcal{L}_{\text{vec}} = \delta_{IJ} \left[-\frac{1}{4} F_{\mu\nu}^I F^{\mu\nu J} - \frac{1}{2} \bar{\lambda}^I \not{D} \lambda^J \right], \quad (2.37)$$

to be added to (2.3) together with some interaction terms with the graviton multiplet in order to eventually guarantee the local supersymmetry invariance (see for instance [11, 12] for more details).

As we have seen, the scalar fields of a supergravity theory parametrize a nonlinear σ -model based on the target manifold \mathcal{M}_{sc} . Besides, it is quite common that \mathcal{M}_{sc} exhibits the structure of a coset space G/H (obligatory for $N \geq 3$): in this particular case the isometry group of the scalar manifold is simply given by G . Maximal supergravity is indeed included in this context, given that its scalar manifold is $\mathcal{M}_{\text{sc}} = \text{E}_{7(7)}/\text{SU}(8)$, with isometry group $G = \text{E}_{7(7)}$. We recall that, thanks to the σ -model properties, if we denote by $\hat{\xi}_\alpha^i(\phi)$, $i = 1, \dots, n_c$, $\alpha = 1, \dots, \dim G$ the $\dim G$ killing vectors of G (which generate the infinitesimal isometries), the

supergravity Lagrangian is invariant under the scalar isometry transformations

$$\phi^n \rightarrow \phi^n + \delta\phi^n \equiv \phi^n + \Lambda^\alpha \hat{\xi}_\alpha^n(\phi) , \quad (2.38)$$

where Λ^α is a constant parameter. In analogy with Yang-Mills theories, the gauging procedure simply consists in choosing a subgroup G_0 of the isometry group G and in promoting the invariance of the Lagrangian under the corresponding isometry transformations (2.38) from global to local. In other terms, we have to select the subset of $\dim G_0$ ($\leq \dim G$) Killing vectors ξ_I^i generating the subgroup G_0 (which, in general, are linear combinations of $\hat{\xi}_\alpha^i$) and to impose the invariance of the Lagrangian under the (more general) scalar isometry transformations

$$\phi^n(x) \rightarrow \phi^n(x) + \Lambda^I(x) \xi_I^n(\phi(x)) , \quad (2.39)$$

where now the parameters $\Lambda^I(x)$ depend on the space-time point.

Obviously, the new transformations (2.39) are not symmetries of the initial theory, due to the presence of derivatives in the original Lagrangian. However, it is sufficient to perform certain modifications on its terms to actually ensure the theory invariance under both local isometries (2.39) and local supersymmetry.

- (i) First, to guarantee the invariance under local isometries, we need to substitute ordinary derivatives with gauge covariant derivatives. For instance the scalar ones $\partial_\mu \phi^n$ become

$$D_\mu \phi^n \equiv \partial_\mu \phi^n + g A_\mu^I \xi_I^n(\phi) , \quad (2.40)$$

where g is a dimensionless coupling constant and A_μ^I are $\dim G_0$ vector fields already present in the theory. The covariant derivative (2.40) transforms under local isometries exactly as the initial derivatives if and only if we require that under local isometries the vectors transform as well, according to

$$\delta A_\mu^I = \partial_\mu \Lambda^I + g A_\mu^M X_{MN}^I \Lambda^N , \quad (2.41)$$

where X_{MN}^I are constants depending on how G_0 is embedded into the global group G and on G itself. However, the fact that the transformation law (2.41) is different from the usual Yang-Mills one (namely, X_{MN}^I is no longer anti-symmetric in $[MN]$), ruins the invariance of the vector kinetic terms, which, as we will see, can be recovered by performing appropriate modifications on the field strengths. Apart from this distinction, (2.41) shows that the index I should be treated as an index in the adjoint representation of G_0 .

- (ii) The application of (i) alone ensures the theory invariance under local isometries, but produces a deformed Lagrangian no longer invariant under the original local supersymmetry. In fact, for instance, under a supersymmetry transformation extra contributions arise from the variations of the vector fields in the covariant derivatives. In order to restore this invariance, we have to introduce into the Lagrangian new fermionic mass terms (proportional to g) and a scalar potential (proportional to g^2) which, however, do not have to break the local gauge symmetry (2.39) and (2.41). Besides, the supersymmetry variations of the fermionic fields have to be modified as well, by inserting

terms proportional to the gauge coupling g , whose structure must be strictly connected to the inserted fermionic terms. Interestingly, these modifications are sufficient and we do not need to introduce any other term proportional higher powers of g for guaranteeing supersymmetry invariance.

Example. As an example, let us describe the result of the previous procedure applied to minimal supergravity and write the gauge contribution V_D to the scalar potential, which was already quoted in section 2.2.2. For the sake of simplicity, we suppose that $G_0 = G$ in such a way that all Killing vectors are gauged and that the embedding of G_0 into G is trivial.

First we should notice that, in presence of couplings between chiral and vector multiplets, the most general vector kinetic term, in place of (2.37), assumes the non-minimal form

$$\mathcal{L}_{\text{vec}} = \text{Re } f_{IJ}(\phi^m) \left[-\frac{1}{4} F_{\mu\nu}^I F^{\mu\nu J} - \frac{1}{2} \bar{\lambda}^I \not{D} \lambda^J \right], \quad (2.42)$$

where $\text{Re } f_{IJ}(\phi^n)$ is the real part of a holomorphic function $f_{IJ}(\phi^n)$, called *gauge kinetic function*. Moreover, supersymmetry invariance requires also a (topological) term proportional to the imaginary part of $f_{IJ}(\phi^n)$ (see [10, 11] for details on it), which is obviously absent in the minimal case (2.37) in which $f_{IJ}(\phi^n) = \delta_{IJ}$. For $G_0 = G$, the index I in (2.39-2.41) assumes $\dim G = \dim G_0$ values and should be regarded in the adjoint representation on G . Besides, in this simple case the constants X_{MN}^I turn out to be the structure constants of G , as will be clear by the embedding tensor formalism.

In order to write the explicit form of V_D , let us define the *Killing prepotentials*

$$P_I = M_P^2 i \xi_I^n \frac{D_n W}{W}, \quad (2.43)$$

which are $\dim G$ real functions of the scalar fields (one for each Killing vector). With this premise, it can be shown that the D-term contribution to the scalar potential can be expressed as

$$V_D \equiv \frac{g^2}{2} \text{Re } f_{IJ} D^I D^J, \quad (2.44)$$

where the quantities $D^I = [\text{Re } f]^{-1 \ IJ} P_J$ assume the name D^I because they are the on-shell auxiliary fields (conventionally called D^I) needed for the off-shell closure of the supersymmetry algebra. We notice that V_D is proportional to g^2 (as it should be) and that, as anticipated in the previous section, it is semi-positive definite.

Despite this clear construction, the gauging procedure presents some nontrivial issues we have neglected so far. In the first place, many modifications have to be made on the vector field strengths: not only because of the (mentioned) unusual transformation law (2.41) of the vectors appearing in the covariant derivatives, but also due to the fact that the other vectors entering in the Lagrangian may belong to nontrivial representations of the gauge group G_0 . Besides, not all the combinations of Killing vectors can be consistently gauged preserving the supersymmetry, i.e.

just particular embeddings of G_0 into G are feasible for obtaining a supersymmetric theory. Fortunately, these complications can be effectively faced (and solved) by utilizing the embedding tensor, which fully describes the embedding of the gauge group into the global symmetry group and entirely parametrizes the action of gauged supergravity.

2.4.2 The embedding tensor formalism

The embedding tensor formalism [15] allows to formulate the gauging of a supergravity theory, i.e. the above steps (i) and (ii), in a way which is manifestly covariant under the global isometry group G . This formalism is particularly efficient in maximal and half-maximal supergravities, thanks to the many restrictions that supersymmetry imposes in these cases.

Let us firstly summarize the general transformations of the bosons under the global isometry group G in a generic *ungauged* theory. As we have seen, the scalars ϕ^n transform under G as $\delta\phi^n = \Lambda^\alpha \hat{\xi}_\alpha^n(\phi)$, which is a nonlinear representation of G as long as the Killing vectors are not proportional to ϕ^n . On the contrary, the transformation of the vector fields A_μ^I is linear and is given by $\delta A_\mu^I = -\Lambda^\alpha (t_\alpha)_J^I A_\mu^J$, where $(t_\alpha)_I^J$ denote the generators of $\mathfrak{g} = \text{Lie } G$ in a fundamental representation \mathcal{R}_v of dimension n_v . In general, the bosonic sector can even contain higher-rank p -forms (whose rank is however restricted by supersymmetry), which, analogously, transform in particular linear representations of G .

As we said, gauging corresponds to promoting a subgroup $G_0 \subset G$ to a local symmetry. In particular, this subgroup G_0 can be defined by selecting within the global symmetry algebra \mathfrak{g} a subset of $\dim G_0 \leq n_v$ generators which give rise to a subalgebra of \mathfrak{g} . Since we want to keep the procedure as general as possible, we denote the general linear combination of generators of \mathfrak{g} spanning this subalgebra by

$$X_I = \Theta_I^\alpha t_\alpha, \quad I = 1, \dots, n_v, \quad (2.45)$$

by means of a constant tensor Θ_I^α , the *embedding tensor*, which describes the explicit embedding of the gauge group G_0 into the global symmetry group G . We can simply imagine this object as a constant $(n_v \times \dim G)$ matrix whose indices α and I transform respectively in the adjoint representation and in the fundamental representation \mathcal{R}_v of G . Since in general $\dim G_0 \leq n_v$, just $\dim G_0$ out of the n_v generators X_I are linearly independent, and therefore the rank of the matrix Θ_I^α , which is $\dim G_0$, does not always assume its maximum value n_v . Let us notice that, using the notation of the previous subsection, Θ_I^α are equivalently the coefficients that determine the gauged Killing vectors ξ_I^i in terms of the original ones $\hat{\xi}_\alpha^i$, i.e. $\xi_I^i = \Theta_I^\alpha \hat{\xi}_\alpha^i$.

There are actually many advantages in utilizing the embedding tensor in order to describe the gauging of a supergravity theory.

- First, this formalism allows to keep the entire construction formally G -covariant. In fact, the deformed equations of motion remain manifestly G -covariant if the embedding tensor is treated as a spurionic object that simultaneously transforms under G according to the structure of its indices.

Just upon specifying a particular gauge group of G_0 we select a particular set of Θ_I^α , and the global symmetry group is broken.

- Moreover, the gaugings are *entirely* parametrized in terms of the constants Θ_I^α , which completely determine the deformed Lagrangian.

Before turning to the modifications to be done to the Lagrangian, we should notice that the gauging procedure does not work for an arbitrary choice of the embedding tensor. In fact, for instance, consistency with G_0 being a group requires that the generators (2.45) close into a subalgebra of \mathfrak{g} and this leads to nontrivial (quadratic) constraints on the constants Θ_I^α . It turns out that there are in general two sets of constraints the embedding tensor should satisfy for ensuring both the gauge and the supersymmetry invariance of the theory. Besides, these constraints can be formulated as G -covariant homogeneous equations in Θ , which allows to construct their solutions by purely group-theoretical methods. The first set of constraints, linked to the closure of the gauge algebra, is quadratic in Θ , while the second one, required by supersymmetry invariance, is just linear. Let us analyze in detail each of them.

1. The first requirement (related to the consistency of the gauge theory) is the invariance of Θ under the action of the local gauge symmetry group G_0 . This request is highly nontrivial, because in general the indices I and α of Θ_I^α transform in two different representations of the gauge group (except in $D = 3$ where the vector fields, and thus the index I , transform in the adjoint representation). Since G_0 is precisely defined as the projection with Θ , the invariance of Θ under G_0 can be expressed as

$$0 = \delta_J \Theta_I^\alpha \equiv \Theta_J^\beta \delta_\beta \Theta_I^\alpha = \Theta_J^\beta (t_\beta)_I^K \Theta_K^\alpha + \Theta_J^\beta f_{\beta\gamma}^\alpha \Theta_I^\gamma, \quad (2.46)$$

where we used the fact that the generators in the adjoint representations are given by the structure constants, i.e. $(t_\alpha)_\beta^\gamma = -f_{\alpha\beta}^\gamma$. The meaning of this constraint becomes obvious if we contract (2.46) with a generator t_α , obtaining the equivalent relation

$$[X_I, X_J] = -X_{IJ}^K X_K. \quad (2.47)$$

By considering the anti-symmetric part in $[IJ]$ on both side, it is straightforward that the commutator between the generators X_I and X_J must be still a generator belonging to the set $\{X_I\}$, which therefore forms an algebra with structure constants $-X_{[IJ]}^K$. On the other hand, upon symmetrization in (MN) the left hand side trivially vanishes, while the right hand side does not, and therefore we get the non-trivial relation

$$X_{(IJ)}^K X_K = 0. \quad (2.48)$$

Thus, the constraint (2.46) also implies that the symmetrized constants $X_{(IJ)}^K$ vanish upon contraction with another generator, which means that (2.46) is in general stronger than the simple closure.

2. In addition to the quadratic constraint (2.46), Θ must satisfy another linear constraint required by the compatibility with supersymmetry. Its specific form however depends on the number of space-time dimensions and supercharges considered. In order to see how it may act, we notice that in general the embedding tensor Θ_I^α lives in the tensor product

$$\mathcal{R}_{v^*} \otimes \mathcal{R}_{\text{adj}} = \mathcal{R}_{v^*} \oplus \dots , \quad (2.49)$$

where \mathcal{R}_{v^*} is the representation conjugate to \mathcal{R}_v and the dots symbolize irreducible representations of G whose precise form depends on the particular group and representations considered. Therefore, the linear constraint restricts Θ to some of the representations appearing in the right hand side of (2.49), as we will see explicitly in the case of maximal supergravity in the following subsection.

However, interestingly, in many cases the linear constraint can be derived by purely bosonic considerations related to the consistency of the deformed tensor gauge algebra (2.47). For example, in $D = 4$ dimensions, it can be proved [18] that the embedding tensor must enjoy

$$X_{(MN}{}^P \Omega_{K)P} = 0 , \quad \Omega_{MN} = \begin{pmatrix} 0 & \mathbf{1}_{n_v} \\ -\mathbf{1}_{n_v} & 0 \end{pmatrix} . \quad (2.50)$$

While it is rather straightforward to verify that the constraints are necessary, it has to be checked case by case (i.e. for each dimension and number of supercharges) that indeed they are sufficient to define a consistent gauging.

Let us now explain how the steps (i) and (ii) of the previous subsection can be implemented to build the gauged Lagrangian according to the embedding tensor formalism. As we anticipated in (i), the global symmetries associated to the generators X_I can be made local by substituting the derivatives with the standard covariant derivatives. More precisely, for the scalars (which transform in nonlinear representations of G), these derivatives are defined by (2.40), while for the fields transforming in a linear representation \mathcal{R} of G they can be constructed according to

$$D_\mu \equiv \partial_\mu - g A_\mu^I X_I , \quad (2.51)$$

where X_I are the generators of the gauge group in the linear representation \mathcal{R} . Let us notice that (2.40) reduces to (2.51) if also the scalars belong to a linear representation of G . Having introduced the covariant derivatives, the theory should be automatically invariant under the standard combined gauge transformations

$$\begin{aligned} \delta \phi^n(x) &= \Lambda^I(x) \xi_I^n(\phi(x)) , \\ \delta A_\mu^I(x) &\equiv (D_\mu \Lambda(x))^I = \partial_\mu \Lambda^I(x) + g A_\mu^M(x) X_{MN}{}^I \Lambda^N(x) , \end{aligned} \quad (2.52)$$

where $\Lambda^I(x)$ is a local parameter and, by definition, $X_{MN}{}^I \equiv (X_M)_N{}^I = \Theta_M^\alpha (t_\alpha)_N{}^I$.

Apart from the minimal couplings introduced by (2.51), as we mentioned in (i), we need to modify the original vector kinetic terms, which are no longer invariant

under (2.52) since they were invariant under global transformations. Thanks to the fact that the new vector transformation laws are similar to the ones of a Yang-Mills theory with structure constants $-X_{[MN]}^I$, the natural expectation for the new field strength is

$$\mathcal{F}_{\mu\nu}^I = 2\partial_{[\mu}A_{\nu]}^I + gX_{[JK]}^I A_{\mu}^J A_{\nu}^K . \quad (2.53)$$

However, due to the presence of a symmetric part in the commutator algebra (2.47), it can be shown that the structure constants $X_{[MN]}^I$ fail to satisfy the Jacobi identities, which are indeed verified if and only if $X_{(MN)}^I = 0$. As a consequence, the standard non-abelian field strength (2.53) turns out to be not fully covariant (the violation being proportional to the tensor $X_{(MN)}^I$) and the standard kinetic term is not invariant. This issue has been solved (even in a covariant way, see [15]) by introducing a set of two-form vector fields of the type $B_{\mu\nu}^{IJ} = B_{[\mu\nu]}^{(IJ)}$ and by defining new field strengths $\mathcal{H}_{\mu\nu}^I = \mathcal{F}_{\mu\nu}^I + gX_{(MN)}^I B_{\mu\nu}^{MN}$, which transform covariantly under gauge transformations if we impose a proper transformation law for the two-form $B_{\mu\nu}^{MN}$. Obviously, these two-forms cannot simply be added to the field content of the theory, since the number of degrees of freedom is carefully balanced by supersymmetry. Thus, a priori, the presence of extra fields in the gauged theory is an obstacle for the supersymmetric construction. Instead, miraculously, the various contributions to the two-forms (coming from the field strengths and possible additional topological terms) precisely combine into the Lagrangian to give rise to a first-order field equation, which corresponds to the fact that the two-forms are auxiliary fields that do *not* provide additional on-shell degrees of freedom (but are just the on-shell duals of the (scalar) fields already present in the ungauged theory). This construction holds even in presence of higher-rank p -forms in the ungauged theory: the deformation leads to an entanglement of the p -forms and the $(p+1)$ -forms via the corresponding field strengths, and it turns out that the additional fields are the on-shell duals of the fields already present in the ungauged theory.

The discussed modifications produce a deformation of the original Lagrangian which is compatible with the new local gauge group. As we said, the next step (ii) is introducing further modifications that eventually render the Lagrangian invariant also under (a possible deformation of) the original supersymmetry, which is obviously broken due to extra contributions coming, for example, from the vector supersymmetry variations in the covariant derivatives.

By applying the Noether procedure, the extra (unwanted) contributions can be canceled by introducing the bilinear fermionic terms [15]

$$\mathcal{L}_{\text{ferm-mass}} = g \left(\bar{\psi}^i A_{ij} \psi^j + \bar{\chi}^A B_{Ai} \psi^i + \bar{\chi}^A C_{AB} \chi^B \right) , \quad (2.54)$$

where ϕ^i and χ^A stand, respectively, for the gravitinos and the spin 1/2 fermions and we have suppressed the Lorentz indices and the γ -matrices. The matrices A_{ij} , B_{Ai} and C_{AB} are functions of the scalar fields (thus, they can contain fermionic mass terms when the scalars assume a VEV) and should transform under proper representations for ensuring the supersymmetry invariance of the theory (as we will

see in the explicit case of maximal supergravity). Besides, it can be shown that the additional terms (2.54) are exactly sufficient to delete all supersymmetry violating contributions in linear order of g if the supersymmetry transformations are modified as (schematically)

$$\delta\psi^i = \delta_0\psi^i - gA^{ij}\varepsilon_j, \quad \delta\chi^A = \delta_0\chi^A - gB^{Ai}\varepsilon_i, \quad (2.55)$$

where δ_0 are the covariantized supersymmetry variations of the ungauged theory. In fact, the fermionic shifts now present in the supersymmetry variations (2.55) are needed to eliminate the $D_\mu\varepsilon$ contributions arising from the variations δ_0 on (2.54).

Eventually, supersymmetry invariance at the order g^2 implies the addition of a scalar potential of the form

$$\mathcal{L}_{g^2} \propto -V = -g^2 \left(B^{Ai}B_{Ai} - A^{ij}A_{ij} \right), \quad (2.56)$$

needed to cancel the g^2 contributions descending from the action of the fermionic shifts in (2.55) on the bilinear fermionic terms (2.54) previously added into the Lagrangian. As can be easily noticed, in general the scalar potential V is not positive definite, and may in particular support dS and AdS vacua. However, the consistent cancellation of all supersymmetry variations at the order g^2 requests that the fermionic matrices A_{ij} , B_{Ai} and C_{AB} satisfy some nontrivial algebraic identities, and in particular the traceless condition

$$g^2 \left(B^{Ai}B_{Aj} - A^{ik}A_{jk} \right) = \frac{1}{N}\delta_j^i V. \quad (2.57)$$

Relevantly, the introduction of the terms (2.54) and (2.56) not only is necessary for ensuring local supersymmetry invariance, but even sufficient, since it turns out that the stated transformations close without producing any term of order g^3 , which, otherwise, should have been canceled by inserting into the Lagrangian some contribution proportional to higher powers of g .

2.4.3 Gauged maximal supergravity

We now specify the preceding formalism to the case of maximal supergravity [16, 15], explicitly identifying the constraints on the embedding tensor and the extra terms to be added to the Lagrangian (2.35) in order to consistently perform the gauging and maintain at the same time the supersymmetry invariance.

We start from the ungauged theory described in section 2.3, where the electric and magnetic vector fields $A_\mu^M = (A_\mu^\Lambda, A_{\mu\Sigma})$ transform in the fundamental **56** representation of the $E_{7(7)}$ duality group with generators $(t_\alpha)_M^N$, $\alpha = 1, \dots, 133$. As we have already noted, in the conventional supergravity Lagrangian only the 28 electric potentials A_μ^Λ appear, but for the gauging procedure we base ourselves on all 56 gauge fields (this is furthermore indispensable in order to achieve a duality covariant description of flux compactifications). Nevertheless, the correct balance of degrees of on-shell physical freedom will be indeed realized in the end result.

Following what we said in the previous subsection, the gauge group G_0 must be a subgroup of the global isometry group $E_{7(7)}$, and its generators X_M , $M = 1, \dots, 56$, which couple to the gauge fields A_μ^M , are decomposed in terms of the 133 independent $E_{7(7)}$ generators t_α as $X_M = \Theta_M^\alpha t_\alpha$, by means of the embedding tensor Θ_M^α belonging to the $\mathbf{56} \times \mathbf{133}$ representation of $E_{7(7)}$. The rank of Θ_M^α , which equals $\dim G_0$, is expected to be smaller than 28, because the ungauged Lagrangian should be based just on 28 vector fields to describe the physical degrees of freedom (and this bound will turn out to be indeed satisfied).

An admissible embedding tensor is subject to the quadratic and linear constraints, which ensure respectively that a proper subgroup of $E_{7(7)}$ is selected and that the corresponding supergravity action remains supersymmetric. As we mentioned, these constraints can be even characterized group theoretically. In fact, in maximal supergravity, the embedding tensor transforms in the representation

$$\mathbf{56} \times \mathbf{133} = \mathbf{56} + \mathbf{912} + \mathbf{6480} \quad (2.58)$$

of $E_{7(7)}$, and it can be shown [17] that the linear constraint required by supersymmetry restricts Θ_M^α to the $\mathbf{912}$ part in this decomposition. This means that, as a matrix, Θ_M^α has 912 independent components. Besides, this condition on the representation is equivalent to the linear (and $E_{7(7)}$ covariant) equations

$$(t_\alpha)_M{}^N \Theta_N^\alpha = 0, \quad (t_\beta t^\alpha)_M{}^N \Theta_N^\beta + \frac{1}{2} \Theta_M^\alpha = 0, \quad (2.59)$$

where the index α is raised by the inverse of the $E_{7(7)}$ invariant metric $\eta_{\alpha\beta} = \text{tr}[t_\alpha t_\beta]$. On the other hand, by the general discussion of the previous section, the quadratic constraint (2.46) takes the $E_{7(7)}$ covariant form

$$C_{MN}{}^\alpha \equiv f_{\beta\gamma}{}^\alpha \Theta_M^\beta \Theta_N^\gamma + (t_\beta)_N{}^P \Theta_M^\beta \Theta_P^\alpha = 0, \quad (2.60)$$

where $f_{\alpha\beta}{}^\gamma$ are the structure constants of $E_{7(7)}$. Obviously, $C_{MN}{}^\alpha$ can be assigned to irreducible representations contained in the $\mathbf{56} \times \mathbf{56} \times \mathbf{133}$ representation and, by making use of the conditions given in (2.59), one can prove [16] that $C_{MN}{}^\alpha$ must belong to the $\mathbf{133} + \mathbf{8645}$ part in that product. Therefore, the equations (2.59) and (2.60), or, equivalently, the mentioned conditions in terms of group representations, explicitly determine the values that the constants Θ_M^α can assume to ensure a feasible gauging for maximal supergravity.

Let us now briefly discuss the modifications to be made on the ungauged Lagrangian (2.35) and on the supersymmetry transformations (2.36) to guarantee gauge and supersymmetry invariance. First of all, all field derivatives should be substituted with the gauge covariant derivatives

$$D_\mu = \partial_\mu - g A_\mu^M X_M = \partial_\mu - g \left(A_\mu^\Lambda \Theta_\Lambda^\alpha + A_{\mu\Sigma} \Theta^{\Sigma\alpha} \right) \tau_\alpha, \quad (2.61)$$

where τ_α are the generators of $E_{(7)7}$ in the representation in which the fields transform and we explicitly distinguished between electric and magnetic charges,

denoted respectively by Θ_Λ^α and $\Theta^{\Sigma\alpha}$. In this way, the theory is automatically invariant (except for the terms containing the field strengths $\mathcal{F}_{\mu\nu}^M$) under the standard combined gauge transformations

$$\begin{aligned}\delta\mathcal{V}_M^{\bar{N}}(x) &= \Lambda^K(x)X_{MK}{}^P\mathcal{V}_P^{\bar{N}}(x), \\ \delta A_\mu^M(x) &\equiv (D_\mu\Lambda(x))^M = \partial_\mu\Lambda^M(x) + gA_\mu^N(x)X_{NP}{}^M\Lambda^P(x).\end{aligned}\tag{2.62}$$

In order to render invariant also the terms containing $\mathcal{F}_{\mu\nu}^M$, by following the outlined general strategy, we need to define modified field strengths $\mathcal{H}_{\mu\nu}^M$, functions of $\mathcal{F}_{\mu\nu}^M$ and of a set of two-forms $B_{\mu\nu}^\alpha$. These two-forms $B_{\mu\nu}^\alpha$ must be subject to suitably chosen gauge transformation rules to ensure the covariance of $\mathcal{H}_{\mu\nu}^M$ under (2.62). The new field strengths $\mathcal{H}_{\mu\nu}^M$ must appear into the Lagrangian in place of $\mathcal{F}_{\mu\nu}^M$, together with some topological Chern–Simons-like term depending on the vector fields and on the two-forms (see [16] for details on this). All terms containing two-forms eventually combine to give rise to a first-order field equation, meaning that they are the on-shell duals of the already present scalar fields, and thus they do not provide any additional degree of freedom.

The terms we introduced so far are proportional to the gauge coupling g and produce new unwanted variations (proportional to g) upon application of the supersymmetry transformations (2.36). As we said, these variations can be canceled if we introduce into the Lagrangian feasible masslike terms and define new supersymmetry variations for the fermions. Indeed, these modifications generate (among other terms) precisely the type of variations that may cancel the extra unwanted contributions. The masslike terms to be inserted into (2.35) are written as

$$e^{-1}\mathcal{L}_{\text{ferm-mass}} = g\left(\frac{\sqrt{2}}{2}A_{1ij}\bar{\psi}_\mu{}^i\gamma^{\mu\nu}\psi_\nu{}^j + \frac{1}{6}A_{2i}{}^{jkl}\bar{\psi}_\mu{}^i\gamma^\mu\chi_{ijk} + A_3^{ijk,lmn}\bar{\chi}_{jkl}\chi_{lmn}\right) + \text{h.c.},\tag{2.63}$$

where

$$A_3^{ijk,lmn} = \frac{\sqrt{2}}{144}\varepsilon^{ijkpqr}[lm A_2{}^n]\tag{2.64}$$

and the new fermion variations to be added to (2.36) are

$$\begin{aligned}\delta_g\psi_\mu{}^i &= \sqrt{2}gA_1{}^{ij}\gamma_\mu\varepsilon_j, \\ \delta_g\chi^{ijk} &= -2gA_{2l}{}^{ijk}\varepsilon^l.\end{aligned}\tag{2.65}$$

Here, A_1 and A_2 are functions of the scalar fields transforming in proper representations of $\text{SU}(8)$ and can be systematically constructed by exploiting the so-called T-tensor formalism [16]. Although these modifications are sufficient to cancel the unwanted terms to the linear order in g , the application of (2.65) on (2.63) produces terms of order g^2 , which are fully compensated by adding to the Lagrangian the scalar potential

$$\begin{aligned}
V &= g^2 \left(\frac{1}{24} |A_{2i}{}^{jkl}|^2 - \frac{3}{4} |A_1^{ij}|^2 \right) \\
&= \frac{1}{336} g^2 \mathcal{M}^{MN} \left(8 \mathcal{P}_M{}^{ijkl} \mathcal{P}_{Mijkl} + 9 \mathcal{Q}_{Mi}{}^j \mathcal{Q}_{Nj}{}^i \right) \\
&= \frac{1}{672} g^2 \left(X_{MN}{}^R X_{PQ}{}^S \mathcal{M}^{MP} \mathcal{M}^{NQ} \mathcal{M}_{RS} + 7 X_{MN}{}^Q X_{PQ}{}^N \mathcal{M}^{MP} \right),
\end{aligned} \tag{2.66}$$

where we have introduced the real symmetric (and positive definite) field-dependent 56×56 matrix $\mathcal{M}^{MN} \equiv \mathcal{V}_M{}^{ij} \mathcal{V}_{Nij} + \mathcal{V}_{Mij} \mathcal{V}_N{}^{ij}$ and its inverse $\mathcal{M}^{MN} = \Omega^{MP} \Omega^{NQ} \mathcal{M}_{PQ}$. It can be shown that the above-mentioned modifications are eventually enough to close the new supersymmetry algebra also at the order g^2 keeping the local gauge invariance (2.62).

2.5 Eleven-dimensional supergravity

In this last part we present a brief overview of 11-dimensional supergravity [3, 4, 5], which will be the starting point for the dimensional reductions leading to our 4-dimensional model. In the first subsection, we provide a few information about its low-energy origin, in particular explaining why the 11-dimensional action can be constructed without any recourse to M-theory. In the second subsection, instead, we will describe field content of the theory and write explicitly the Lagrangian, which turns out to be uniquely determined by local supersymmetry invariance.

2.5.1 The low-energy origin

The particle spectrum of a generic string theory is made of a finite number of massless states and an infinite tower of massive excitations, whose mass must be at a scale determined by a fundamental parameter, the string tension, approximately of the order of the Planck mass (10^{19} GeV) in order for the graviton to interact with the usual Newtonian strength. If we are interested in a low-energy phenomenological description of the theory, however, it should not be necessary to describe the explicit behaviour of the massive states and, therefore, it is natural to formulate an effective action entirely based on the very light degrees of freedom. Unfortunately, due to the rich structure of nonlocality typical of string theory, this *exact* effective action is extremely complicated and, obviously, nonlocal.

Anyway, since each derivative corresponds to a suppression of a factor E/M where M is the characteristic mass scale of the string theory, an expansion in the number of derivatives (truncated to the first terms) represents an excellent low-energy approximation of the effective action, despite the inevitable loss of the nice UV properties of the exact string theory. Furthermore, in the extreme low-energy limit, the leading terms of the effective action can be even constructed just from invariance principles, i.e. gauge invariance and local supersymmetry, and hence the action can be determined with a relatively little effort. The theories arising from such an expansion are indeed supergravity theories and represent the low-energy description of the lightest degrees of freedom of a string theory. To be precise, since

string theories exist only in ten space-time dimensions, their low-energy limits are 10-dimensional supergravities. Instead, the theory we are going to face, supergravity in eleven dimensions, derives from M-theory, which is the analogue of string theories in eleven dimensions and is widely considered to be the master theory that contains the various 10-dimensional string theories (and can be actually obtained by IIA supergravity in the strong-coupling limit).

Let us examine how the above expansion can be done in practice. Given that every derivative has the dimension of an inverse length, we would like to expand the effective action in powers of $(\text{length})^{-1}$. If we denote by N_∂ and N_f , respectively, the number of derivatives and the number of fermions in a given term of the Lagrangian, it turns out [4] that the integer number

$$n = N_\partial + \frac{1}{2}N_f \quad (2.67)$$

precisely counts the powers of lengths in that term. Hence, terms of larger and larger n are less and less relevant at high wavelengths. Moreover, it can be shown that the $n = 0$ and $n = 1$ contributions are incompatible with supersymmetry in eleven dimensions, and therefore the long wavelength behaviour is governed by terms with $n = 2$, which can be determined just from supersymmetry invariance without any recourse to M-theory. Besides, the addition of higher-order contributions (certainly present in M-theory) implies the impossibility to restrict the Lagrangian to definite values of n . In fact, if we introduce some term with $n > 2$, attempting to close the supersymmetry algebra forces the inclusion of terms with higher and higher n . This is the reason why, in the following, we will present the 11-dimensional supergravity Lagrangian restricted just to the $n = 2$ contributions.

It should be noticed that, since the link to string theory was unknown when $D = 11$ supergravity was constructed for the first time, its evident lack of renormalizability led to the belief that it does not approximate a consistent quantum field theory. Nowadays, however, viewed as a low-energy limit of M-theory, $D = 11$ supergravity actually possesses a well-defined quantum interpretation.

2.5.2 Field content and Lagrangian

Compared at least to the massless spectrum of the superstring theories in ten dimensions, the field content of 11-dimensional supergravity is actually very simple [3].

- First, due to the presence of gravity, 11-dimensional supergravity obviously contains the **graviton**. Since in D dimensions the graviton is a symmetric traceless tensor of $\text{SO}(D - 2)$, which is the little group for a massless particle, the 11-dimensional graviton g_{MN} has

$$\frac{(D - 2)((D - 2) + 1)}{2} \Big|_{D=11} - 1 = 44 \quad (2.68)$$

independent physical degrees of freedom. As we will see, supergravity in eleven dimensions includes also spinors and, therefore, it is necessary to use the vielbein (*elfbein*, in eleven dimensions) formalism and represent the graviton

by the field E_M^A . Let us note that M, N, \dots and A, B, \dots are respectively the curved and flat indices in eleven dimensions and transform nontrivially, respectively, under general coordinate and local Lorentz transformations.

- The gauge field indispensable for local supersymmetry is the **gravitino** field Ψ_M , which must be part of the field content. Apart from the explicit index M , the gravitino has an implicit spinor index α which makes Ψ_M a 32-components Majorana spinor for every fixed value of M .

Once spinors are included, the little group in 11 dimensions becomes Spin(9) (the covering group of SO(9)), whose spinor and vector representations have respectively dimensions 16 and 9. Therefore an 11-dimensional vector-spinor transforms in the product $\mathbf{9} \times \mathbf{16} = \mathbf{128} + \mathbf{16}$ (which, in four dimensions, would have led to a spin 1/2 plus a spin 3/2 representation). However, the local gauge invariance $\delta\Psi_M = \partial_M\varepsilon$ of the free gravitino action

$$S_\Psi \sim \int \bar{\Psi}_M \Gamma^{MNP} \partial_N \Psi_P d^{11}x \quad (2.69)$$

ensures, as in four dimensions, that the gravitino physical degrees of freedom correspond only to the **128** representation and are pure spin 3/2. So, in eleven dimensions the gravitino has 128 independent physical polarization states.

- The missing $128 - 44 = 84$ bosonic degrees of freedom required by an honest supergravity model are obtained by inserting a rank-3 antisymmetric tensor A_{MNP} , associated with the **three-form** A_3 . This field must appear in the action with the usual invariance under the gauge transformations

$$A_3 \rightarrow A_3 + d\Lambda_2, \quad (2.70)$$

where Λ_2 is a generic two-form. In fact, this invariance implies that the independent polarizations are just transverse (as in the case of the electromagnetic field) and their number is exactly

$$\left. \frac{(D-2)(D-3)(D-4)}{3!} \right|_{D=11} = 84. \quad (2.71)$$

This value indeed matches the number of propagating degrees of freedom of the gravitino, which is the only Fermi field in the theory.

In order to construct the classical action describing the dynamics of the above fields, we have to take into account the requirements of general coordinate invariance and local Lorentz invariance, together with the gauge invariance for the three-form A_3 . Moreover, as we said in the previous section, we should restrict just to the terms with $n = 2$, which can contain two derivatives and no fermionic fields, one derivative and two fermionic fields or no derivatives and four fermionic fields. Together, the mentioned constraints uniquely fix the Lagrangian up to some numeric coefficients, which are fully determined by the request of local supersymmetry invariance. Hence, the 11-dimensional supergravity action does not depend on any modifiable parameter apart from an overall constant κ , however fixed by the gravitational strength.

Without entering in details about the derivation, we now present the full 11-dimensional Lagrangian (see for instance [12, 19]), which can be cast in the form

$$\mathcal{L}(E_M^A, \Psi_M, A_3) = \mathcal{L}_b(E_M^A, A_3) + \mathcal{L}_{\text{bf}}(E_M^A, \Psi_M, A_3) , \quad (2.72)$$

where \mathcal{L}_b depends only on the bosonic fields (and contains just terms with two derivatives), while \mathcal{L}_{bf} includes also gravitino terms. The bosonic part is

$$e^{-1}\mathcal{L}_b = \frac{1}{2\kappa^2} \left[R - \frac{1}{4!} |F|^2 - \frac{2\sqrt{2}}{(144)^2} e^{-1} \epsilon^{M_1 \dots M_{11}} F_{M_1 \dots M_4} F_{M_5 \dots M_8} A_{M_9 M_{10} M_{11}} \right] \quad (2.73)$$

where

- F is the field strength of A_3 , defined by $F = dA_3$ or, in terms of explicit components, $F_{MNPQ} = 4\partial_{[M}A_{NPQ]}$. Thanks to the fact that the ordinary derivatives commute, F is invariant under gauge transformations and therefore the second term, which includes $|F|^2 \equiv F_{MNPQ}F^{MNPQ}$, is automatically gauge invariant
- the last term, called Chern–Simons term, is independent of the metric and is gauge invariant as well, despite the explicit appearance of A_{MNP} . In fact, the action related to this term can be written as

$$S_{\text{C-S}} = -\frac{\sqrt{2}}{6\kappa^2} \int F \wedge F \wedge A_3 , \quad (2.74)$$

and by taking the gauge variation $\delta A_3 = d\theta$, using $F = dA_3$ as well as the Bianchi identity $dF = 0$ and integrating by parts, we easily obtain $\delta S_{\text{C-S}} = 0$.

The part including also the gravitino is

$$e^{-1}\mathcal{L}_{\text{bf}} = -\frac{1}{2\kappa^2} \left[\bar{\Psi}_M \Gamma^{MNP} D_N \left(\frac{1}{2}(\omega + \hat{\omega}) \right) \Psi_P - \frac{\sqrt{2}}{192} \bar{\Psi}_M (\Gamma^{MNPQRS} + 12\Gamma^{NP} G^{MQ} G^{RS}) \Psi_S (F_{NPQR} + \hat{F}_{NPQR}) \right] \quad (2.75)$$

where $D_M(\omega)$ is the covariant derivative with connection ω and we defined

$$\begin{aligned} \omega_{MAB} &= \omega_{MAB}(e) + K_{MAB} , \\ \hat{\omega}_{MAB} &= \omega_{MAB}(e) - \frac{1}{4} (\bar{\Psi}_M \gamma_B \Psi_A - \bar{\Psi}_A \gamma_M \Psi_B + \bar{\Psi}_B \gamma_A \Psi_M) , \\ K_{MAB} &= -\frac{1}{4} (\bar{\Psi}_M \gamma_B \Psi_A - \bar{\Psi}_A \gamma_M \Psi_B + \bar{\Psi}_B \gamma_A \Psi_M) + \frac{1}{8} \bar{\Psi}_N \Gamma^{NR}{}_{MAB} \Psi_R , \\ \hat{F}_{MNR S} &= F_{MNR S} + \frac{3}{2} \sqrt{2} \bar{\Psi}_{[M} \Gamma_{NR} \Psi_{S]} . \end{aligned} \quad (2.76)$$

The full action (2.72) is invariant under certain local supersymmetry transformations, which reduce to the original gauge symmetry $\delta\Psi_M = \partial_M \varepsilon$ of the free gravitino action when the graviton and the three-form are set to zero. In particular, under this local supersymmetry, the fields transform according to

$$\begin{aligned} \delta E_M^A &= \frac{1}{2} \bar{\varepsilon} \Gamma^A \Psi_M , \\ \delta \Psi_M &= D_M(\hat{\omega}) \varepsilon + \frac{\sqrt{2}}{288} \left(\Gamma^{ABCD}{}_M - 8\Gamma^{BCD} \delta_M^A \right) \hat{F}_{ABCD} \varepsilon , \\ \delta A_{MNP} &= -\frac{3\sqrt{2}}{4} \bar{\varepsilon} \Gamma_{[MN} \Psi_{P]} . \end{aligned} \quad (2.77)$$

It is especially relevant that, in the interacting theory, local supersymmetry is related to a set supercharges that form a 32-component Majorana-spinor. This is the minimum amount of supersymmetry a theory can enjoy in eleven dimensions and so there cannot be less supersymmetry than that. Besides, it can be shown that one cannot introduce more supersymmetry charges without inserting spin-higher-than-2 particles. For this reason, the above Lagrangian contains the only allowed amount of supersymmetry for an 11-dimensional theory describing particles with spin at most equal to 2. Even more interestingly, we cannot construct any supersymmetric theory with spin-higher-than-2 particles for $D > 11$.

3 M-theory reductions to four dimensions

As we saw in section 2.5.1, the low-energy formulation of string theories and M-theory leads naturally to supergravity models in a space-time with extra dimensions. Out of these theories, 11-dimensional supergravity is of prominent importance for our purposes, because it gives rise to the maximal $D = 4$ supergravities we have described in the previous chapter as well as the isotropic models we will eventually examine. Furthermore, the fact that the maximum dimension in which one can consistently formulate a supersymmetric field theory is indeed eleven makes 11-dimensional supergravity even more interesting.

In order to get back the four ordinary dimensions in an extra-dimensional context such as 11-dimensional supergravity, we have to require the extra dimensions to be somehow hidden with respect to the ordinary energy and distance scales at which we usually base our experience. This issue can be correctly addressed through the concept of *compactification*, namely the assumption that the extra dimensions are “compactified” in a geometric structure with a finite (and very small) volume. In particular, a 4-dimensional compactified theory and its field content are strictly related not only to initial higher-dimensional Lagrangian, but also to the chosen geometry for the extra-dimensional (or internal) space. Thus, different kinds of compactification give naturally rise to different 4-dimensional actions, which explains the huge variety of lower-dimensional theories that can be derived starting from the same higher-dimensional one.

This chapter is dedicated to the compactifications of 11-dimensional supergravity leading to our 4-dimensional models. In the first section we therefore describe the Kaluza–Klein dimensional reduction, which is the general mechanism for the compactification of all theories with extra dimensions. In the second section we instead specify the discussion to 11-dimensional supergravity, explicitly showing that its structure naturally admits a compactification to four dimensions, in which the internal space curvature may be either flat or positive. The last section finally describes how our STU-models arise from the 11-dimensional theory upon orbifold reductions.

3.1 Kaluza–Klein reductions

The Kaluza–Klein reduction [20, 21, 22] is a general method that allows to construct a reduced theory in 4 space-time dimensions starting from a $4 + d = D$ dimensional theory. In view of the compactification, we conveniently denote by $z^M = (x^\mu, y^m)$, $M = 0, \dots, D - 1$, the $4 + d$ space-time variables, where, as usual, x^μ stand for the ordinary 4-dimensional space-time variables, while y^m (with $m = 1, \dots, d$) represent the additional ones. In general, the D -dimensional model depends on the set of fields $\{\Phi_i(x, y)\}$ (plus the metric $g_{MN}(x, y)$, if gravity is present) and is described by the D -dimensional action

$$S_D = \int d^4x d^d y \mathcal{L}_D(\{\Phi_i(x, y)\}, g_{MN}(x, y)) . \quad (3.1)$$

The Kaluza–Klein reduction of the D -dimensional theory is based on the following two assumptions.

- (i) First, we require that the D -dimensional space-time on which the integral (3.1) is defined can be written as the direct³ product $M_4 \times M_d$, where M_4 is a 4-dimensional manifold with coordinates x^μ and usual signature $(-+++)$, while M_d is a d -dimensional compact space with coordinates y^m and Euclidean signature $(+\cdots+)$. If gravity is present, this request means that the combined field equations of $\{\Phi_i(x, y)\}$ and $g_{MN}(x, y)$ must admit a so-called “ground-state” solution in which the metric displays the diagonal form

$$\langle g_{MN}(x, y) \rangle = \begin{pmatrix} \hat{g}_{\mu\nu}(x) & 0 \\ 0 & \tilde{g}_{mn}(y) \end{pmatrix}. \quad (3.2)$$

In this case, the theory is said to exhibit “spontaneous compactification”, in the sense that the structure of the theory itself spontaneously leads to a product space $M_4 \times M_d$, where M_d is ready to be compactified. Usually we further require that the 4-dimensional space-time M_4 possesses maximal symmetry, which implies that it has a constant curvature $\hat{R}_{\mu\nu\rho\sigma} = \frac{1}{3}\Lambda(\hat{g}_{\mu\rho}\hat{g}_{\nu\sigma} - \hat{g}_{\mu\sigma}\hat{g}_{\nu\rho})$, and hence that it is an Einstein space with Ricci tensor $\hat{R}_{\mu\nu} = \Lambda\hat{g}_{\mu\nu}$, being Λ its cosmological constant. Instead, no requirement is in general posed on the compact manifold M_d , which can be Einstein, homogeneous or neither of the two.

- (ii) Secondly, we assume that the x and y dependence of each field $\Phi_i(x, y)$ and of the metric $g_{MN}(x, y)$ can be factorized according to

$$\Phi_i(x, y) = \sum_n \phi_i^{(n)}(x)Y^n(y), \quad g_{MN}(x, y) = \sum_n g_{MN}^{(n)}(x)Y^n(y), \quad (3.3)$$

where the functions $Y^n(y)$ depend only on the internal coordinates and are the same in the expansion of each field. As we will see soon, consistency of the theory imposes that the functions $Y^n(y)$ are not arbitrary and are determined by the structure of the internal manifold M_d .

With these two assumption, the 4-dimensional reduced theory can be derived substituting equations (3.3) in the D -dimensional the action (3.1) and performing the integration over the variables y^m which are constrained to the domain M_d . In this way we obtain the 4-dimensional action

$$S_4 = \int_{M_4} d^4x \int_{M_d} d^d y \mathcal{L}_D \left(\left\{ \sum_n \phi_i^{(n)}(x)Y^n(y) \right\}, \sum_n g_{MN}^{(n)}(x)Y^n(y) \right), \quad (3.4)$$

whose Lagrangian reads

$$\mathcal{L}_4(x) = \int_{M_d} d^d y \mathcal{L}_D \left(\left\{ \sum_n \phi_i^{(n)}(x)Y^n(y) \right\}, \sum_n g_{MN}^{(n)}(x)Y^n(y) \right) \quad (3.5)$$

and therefore describes in general an infinite number of degrees of freedom, associated to the new 4-dimensional fields $\phi_i^{(n)}(x)$ and $g_{MN}^{(n)}(x)$. However, in order for

³In general the product is sufficient, but direct products will be considered for simplicity.

the 4-dimensional theory to be meaningful, we have to further demand that the equations of motion of the reduced Lagrangian \mathcal{L}_4 , obtained from the D -dimensional Lagrangian \mathcal{L}_D by means of (3.3), consistently describe a 4-dimensional theory, and this imposes nontrivial constraints on the functions $Y^n(y)$. As we now prove in a simple example, it turns out that these constraints are equivalent to the fact that $Y^n(y)$ satisfies an eigenvalue equation of the form

$$\Delta Y^n = m_n^2 Y^n , \quad (3.6)$$

where Δ is the Laplace mass operator. This means that $Y^n(y)$ are harmonic functions on the compact manifold M_d . Hence, in general, the Kaluza–Klein reduction directly proceeds through the expansion (3.3) in harmonics on M_d , which automatically implies the consistency of the 4-dimensional theory.

Example. As an example, let us apply the preceding construction to a very simple case and discuss the consequences on the particle spectrum of the 4-dimensional theory. We consider the 5-dimensional action of a free complex scalar field $\Phi(x, y)$

$$\begin{aligned} S_5 &= \int d^4x \int dy \left[- \left(\partial^M \Phi \right)^* \left(\partial_M \Phi \right) - M_0^2 \Phi^* \Phi \right] \\ &= \int d^4x \int dy \left[- \left(\partial^\mu \Phi \right)^* \left(\partial_\mu \Phi \right) - \left(\partial_y \Phi \right)^* \left(\partial_y \Phi \right) - M_0^2 \Phi^* \Phi \right] , \end{aligned} \quad (3.7)$$

where M_0 is the 5-dimensional mass and there is no gravitational background, namely the metric $g_{MN} = \eta_{MN}$ is flat. Obviously the first Kaluza–Klein assumption is verified, since for all solutions of the scalar field equations the metric g_{MN} displays the diagonal form (3.2), and the theory exhibits the spontaneous compactification $M_4 \times M_1$. The simplest possibility is to compactify the fifth dimension on a circle S^1 of radius R , which is defined as the quotient space \mathbb{R}/τ , where $\tau : \mathbb{R} \rightarrow \mathbb{R}$ is the translation map $y \rightarrow y + 2\pi R$. So, S^1 can be essentially considered as the interval $[-\pi R, \pi R]$ with the ends identified. The structure of quotient space gives naturally rise to periodic boundary conditions for the variable y , and thus our choice $M_1 = S^1$ implies $\Phi(x, -\pi R) = \Phi(x, \pi R)$ for any x^μ . Following the step (ii), we then have to expand the 5-dimensional scalar field as

$$\Phi(x, y) = \sum_n \phi^{(n)}(x) Y^n(y) , \quad (3.8)$$

where $\{\phi^{(n)}(x)\}$ is an infinite set of 4-dimensional scalar fields and periodicity implies $Y^n(-\pi R) = Y^n(\pi R)$. Substituting (3.8) in (3.7) we find the reduced Lagrangian in four dimensions

$$\begin{aligned} \mathcal{L}_4 &= - \sum_{m,n} \partial^\mu \phi^{(m)*} \partial_\mu \phi^{(n)} \int_{-\pi R}^{\pi R} dy Y^{m*} Y^n \\ &\quad - \sum_{m,n} \phi^{(m)*} \phi^{(n)} \int_{-\pi R}^{\pi R} dy \left[\partial_y Y^{m*} \partial_y Y^n + M_0^2 Y^{m*} Y^n \right] , \end{aligned} \quad (3.9)$$

whose equations of motion are consistent with a 4-dimensional theory describing an infinite set of complex scalar fields $\phi^{(n)}$ with masses m_n if and only if

$$\begin{cases} \int_{-\pi R}^{\pi R} dy Y^{m*} Y^n = \delta_{mn} \\ \int_{-\pi R}^{\pi R} dy \left[\partial_y Y^{m*} \partial_y Y^n + M_0^2 Y^{m*} Y^n \right] = m_n^2 \delta_{mn} . \end{cases} \quad (3.10)$$

By means of the former equation, which is an orthonormality condition for the functions Y^n , the latter becomes $\int_{-\pi R}^{\pi R} dy [\partial_y Y^{m*} \partial_y Y^n - (m_n^2 - M_0^2) Y^{m*} Y^n] = 0$. Integrating by part the first term and using $Y^n(\pi R) - Y^n(-\pi R) = 0$ as well as the fact that $Y^n(y)$ does not vanish identically, we find that $Y^n(y)$ must satisfy the eigenvalue equation

$$-\partial_y^2 Y^n(y) = (m_n^2 - M_0^2) Y^n(y) , \quad (3.11)$$

which, as anticipated, is exactly (3.6) with $\Delta = -\partial_y^2$. The solutions of this equation are $Y^n(y) = A e^{i\sqrt{m_n^2 - M_0^2}y}$ and taking account also of the periodicity condition $Y^n(-\pi R) = Y^n(\pi R)$ we conclude that m_n is quantized according to

$$m_n^2 = M_0^2 + \frac{n^2}{R^2} , \quad n \in \mathbb{Z} . \quad (3.12)$$

Due to the orthonormality constraint, the solutions $Y^n(y)$ of (3.10) assume the form

$$Y^n(y) = \frac{1}{2\pi R} e^{i\frac{ny}{R}} , \quad (3.13)$$

which are the very well-known harmonics on the circle. So, the Kaluza–Klein compactification of a free 5-dimensional scalar field of mass M_0 on a circle simply led to the 4-dimensional effective Lagrangian (3.9) describing an infinite but discrete set (the so-called *tower*) of free scalar fields of square masses $m_n^2 = M_0^2 + n^2/R^2$. These modes comprehend a 4-dimensional scalar with the original mass M_0 (corresponding to $n = 0$) and an infinite number of additional scalars, the so-called Kaluza–Klein modes, whose masses are bigger than M_0 and are quantized in units of the fundamental mass R^{-1} , determined by the “size” of the internal manifold and therefore called *compactification scale*. In particular, the lightest degree of freedom comes from the zero-eigenvalue mode of the equation (3.11), which is just a constant function for such a simple compactification. Instead, the infinite nonzero Kaluza–Klein modes start with mass $\sim R^{-1}$ and grow with the excitation order, and do not conflict with the everyday sensation of inhabiting in a world with a finite number of particles provided R is small with respect to M_0^{-1} , so that m_n is much bigger than M_0 and cannot be observed at low energies. In the simplest higher-dimensional phenomenological model, the general idea is that each Standard Model particle can be identified with the zero-mode of the Kaluza–Klein tower and all heavier modes are new particles beyond the Standard Model, which appear as copies of the lightest particle with bigger and bigger mass.

Although many features of the compactification on a circle strongly depend on the choice of this particularly simple manifold, the qualitative structure of the spectrum we sketched in the above example is general and derives from constraining the extra dimensions to a compactified structure with periodic boundary conditions. In fact, this constraint selects a discrete number of eigenfunctions $Y^n(y)$ of (3.6) corresponding to a quantized number of 4-dimensional modes, of which a finite set comes from the zero-harmonics and an infinite (massive) tower from the nonzeros. However, when the manifold is more complicated, the precise structure of

the spectrum is not so trivial, because, for instance, also the zero-harmonics may depend on y . When the manifold has a treatable structure such as a group coset, anyway, the harmonic analysis can be done group theoretically and the spectrum can be found analytically [20, 22].

Example. Let us explore another example in which we compactify the 5-dimensional scalar Lagrangian (3.7) on a slightly more complicated manifold, namely an orbifold, showing the mentioned difference on the particle spectrum. In general, “orbifolding” is the procedure of reducing a fundamental domain by a set of identifications, such as, for example, the \mathbb{Z}_2 transformation $\sigma : \mathbb{R} \rightarrow \mathbb{R}$ acting on the line \mathbb{R} as $y \rightarrow -y$. The compact space we focus on is the S^1/\mathbb{Z}_2 orbifold, which is defined as the quotient space S^1/σ and can be viewed as the above circle $[-\pi R, \pi R]$ in which the points y and $-y$ are further identified. Essentially, this identification leads to the half-circle $[0, \pi R]$, where the end points $y = 0, \pi R$ are included but not identified with each other. If we choose to compactify (3.7) on $M_1 = S^1/\mathbb{Z}_2$, the procedure we did for S^1 equally holds, except for the fact that we need to also require that $\Phi(x, y) = \Phi(x, -y)$, and so $Y^n(y) = Y^n(-y)$. Noting that $e^{i\frac{ny}{R}} = \cos \frac{ny}{R} + i \sin \frac{ny}{R}$, this requirement implies that the solutions of (3.10) are just

$$Y^n(y) = \frac{1}{2\pi R} \cos \frac{ny}{R}, \quad n = 0, 1, 2, \dots, \quad (3.14)$$

and that the masses m_n of the 4-dimensional modes are still determined by (3.12) but with $n = 0, 1, 2, \dots$. Therefore, the compactification on the orbifold S^1/\mathbb{Z}_2 actually slightly modified the spectrum, since the Kaluza–Klein modes are reduced by a factor 2 (however, the zero-mode with mass M_0 remains).

Example. As a concrete, although somewhat unrealistic, example of the Kaluza–Klein procedure on a gravitational background [20], we consider pure gravity in five dimensions, described by the action

$$S_5 = \frac{1}{2\pi\kappa^2} \int d^4x dy \sqrt{-\hat{g}} \hat{\mathcal{R}}, \quad (3.15)$$

being $\hat{\mathcal{R}}$ the Ricci scalar of the 5-dimensional metric $\hat{g}_{MN}(x, y)$ and \hat{g} its determinant. The field equations are the usual Einstein equations $\hat{R}_{MN} - \frac{1}{2}\hat{g}_{MN}\hat{\mathcal{R}} = 0$, which obviously admit the ground state solution $\langle \hat{g}_{MN} \rangle = \eta_{MN}$, and therefore the theory exhibits (among the others) the spontaneous compactification $M_4 \times M_1$. For the sake of simplicity, we choose to compactify the internal space M_1 on the circle S_1 of radius R and we consider the change of variables

$$\hat{g}_{MN} = \phi^{-1/3} \begin{pmatrix} g_{\mu\nu} + \kappa^2 A_\mu A_\nu & \kappa\phi A_\mu \\ \kappa\phi A_\nu & \phi \end{pmatrix}, \quad (3.16)$$

which is simply a different (but convenient) way of defining the components of $\hat{g}_{MN}(x, y)$ in terms of the fields $g_{\mu\nu}(x, y)$, $A_\mu(x, y)$ and $\phi(x, y)$. Following the

outlined general procedure, we have to assume the expansions

$$\begin{aligned}
g_{\mu\nu}(x, y) &= \sum_{n=-\infty}^{\infty} g_{\mu\nu}^{(n)}(x) Y^n(y) , \\
A_\mu(x, y) &= \sum_{n=-\infty}^{\infty} A_\mu^{(n)}(x) Y^n(y) , \\
\phi(x, y) &= \sum_{n=-\infty}^{\infty} \phi^{(n)}(x) Y^n(y) ,
\end{aligned} \tag{3.17}$$

where, for consistency, $Y^n(y)$ are the harmonics on the circle given by (3.13) and the reality of $g_{\mu\nu}^{(n)}$, $A_\mu^{(n)}$ and $\phi^{(n)}$ imposes $g_{\mu\nu}^{(n)*} = g_{\mu\nu}^{(-n)}$, $A_\mu^{(n)*} = A_\mu^{(-n)}$ and $\phi^{(n)*} = \phi^{(-n)}$. The 4-dimensional Lagrangian \mathcal{L}_4 is obtainable substituting (3.17) into (3.15) and integrating over the variable y constrained to S^1 . In principle \mathcal{L}_4 depends on the infinite set of fields $g_{\mu\nu}^{(n)}(x)$, $A_\mu^{(n)}(x)$ and $\phi^{(n)}(x)$, but we choose to set all fields to zero retaining just the $n = 0$ ones, which, as we said in general, are the lightest and the only observable provided R is small. Since on the circle the zero-harmonic $Y^0(y)$ is actually independent of the internal coordinate, this truncation is exactly equivalent to rewriting the initial action eliminating the dependence of $g_{\mu\nu}(x, y)$, $A_\mu(x, y)$ and $\phi(x, y)$ on the internal coordinates. After having computed the Christoffel symbols in terms of $g_{\mu\nu}^{(0)}(x)$, $A_\mu^{(0)}(x)$ and $\phi^{(0)}(x)$, plugged their expression into (3.15) and performed the integral over y , we find the 4-dimensional action

$$S_4 = \int d^4x \sqrt{-g^{(0)}} \left[\frac{1}{\mathcal{K}^2} R^{(0)} - \frac{1}{4} \phi^{(0)} F_{\mu\nu}^{(0)} F^{(0)\mu\nu} - \frac{1}{6\mathcal{K}^2 \phi^{(0)2}} \partial^\mu \phi^{(0)} \partial_\mu \phi^{(0)} \right]. \tag{3.18}$$

Here, $R^{(0)}$ and $g^{(0)}$ are respectively the Ricci scalar and the determinant of $g_{\mu\nu}^{(0)}$, we have defined the field strength $F_{\mu\nu}^{(n)} = 2\partial_{[\mu} A_{\nu]}^{(n)}$, the indices are raised and lowered by $g_{\mu\nu}^{(0)}$ and $\mathcal{K}^2 = \kappa^2/R$ has the meaning of 4-dimensional Newton constant and is linked to κ by the volume of the internal space. As is clear looking at (3.18), the equations of motion of the reduced (and truncated) theory consistently describe a massless spin 2 field, $g_{\mu\nu}^{(0)}$, a massless vector, $A_\mu^{(0)}$, and a massless scalar, $\phi^{(0)}$, as a consequence of the compactification.

We now take advantage of this simple model to discuss two general characteristics of Kaluza–Klein reductions (that are indeed present here), i.e. the connection between 4- and 5-dimensional symmetries and the consistency of the truncation we have done.

- (i) For what concerns the symmetries, we note that the reduced action (3.18) is invariant under general coordinate transformations with parameter $\xi_{(0)}^\mu(x)$, which read $\delta g_{\mu\nu}^{(0)} = 2\partial_{(\mu} \xi_{(0)\rho}^\rho g_{\nu)\rho}^{(0)} + \xi_{(0)}^\rho \partial_\rho g_{\mu\nu}^{(0)}$, $\delta A_\mu^{(0)} = \partial_\mu \xi_{(0)}^\rho A_\rho^{(0)} + \xi_{(0)}^\rho \partial_\rho A_\mu^{(0)}$ and $\delta \phi^{(0)} = \xi_{(0)}^\rho \partial_\rho \phi^{(0)}$. Besides, thanks to the masslessness of $A_\mu^{(0)}$, (3.18) is also invariant under local U(1) gauge transformations of $A_\mu^{(0)}$ with parameter $\kappa^{-1} \xi_{(0)}^5(x)$, namely $\delta A_\mu^{(0)} = \kappa^{-1} \partial_\mu \xi_{(0)}^5$. Even the global scale transformation with parameter λ , acting as $\delta A_\mu^{(0)} = \lambda A_\mu^{(0)}$, $\delta \phi^{(0)} = -2\lambda \phi^{(0)}$, is a further symmetry of

the 4-dimensional action. However, this global symmetry is not preserved by the vacuum (in which, as we said, $\langle g_{\mu\nu}^{(0)} \rangle = \eta_{\mu\nu}$, $\langle A_\mu^{(0)} \rangle = 0$ and $\langle \phi^{(0)} \rangle = 1$) and so it is spontaneously broken. Thanks to the Goldstone theorem, this spontaneous symmetry breakdown provides one Goldstone boson, which is exactly $\phi^{(0)}$.

The mentioned symmetries are not accidental at all, because, as we now see, they are the 4-dimensional manifestation of the obvious invariance of the 5-dimensional action (3.15) under general coordinate transformations, which act on the 5-dimensional metric as

$$\delta \hat{g}_{MN} = 2\partial_{(M}\xi^R \hat{g}_{N)R} + \xi^R \partial_R \hat{g}_{MN} , \quad (3.19)$$

for any infinitesimal parameter $\xi^M(x, y)$. In fact, consistency between the metric expansion (3.17) and the transformation (3.19), together with the assumed topology of the ground state, requires $\xi^M(x, y)$ to be expandable in terms of the harmonics on the circle $Y^n(y)$, in such a way that

$$\xi^\mu(x, y) = \sum_{n=-\infty}^{\infty} \xi_{(n)}^\mu(x) Y^n(y) , \quad \xi^5(x, y) = \sum_{n=-\infty}^{\infty} \xi_{(n)}^5(x) Y^n(y) . \quad (3.20)$$

It is therefore clear that the general covariance of (3.18) derives from the original invariance under (3.19) upon choosing $\xi_{(n)}^\mu = 0$ for $n \neq 0$ and $\xi_{(n)}^5 = 0$ for any n . Moreover, the local U(1) gauge invariance of (3.18) under $\delta A_\mu^{(0)} = \kappa^{-1} \partial_\mu \xi_{(0)}^5$ corresponds to the initial general covariance with $\xi_{(n)}^5 = 0$ for $n \neq 0$ and $\xi_{(n)}^\mu = 0$ for any n .

This simple example explicitly shows a general feature of Kaluza–Klein reductions, namely that what we perceive to be *internal* symmetries of the reduced theory in four dimensions are really *space-time* symmetries in the extra dimensions. Besides, the fact that the gauge group (U(1) in this case) coincides with isometry group of the internal manifold (the circle) is not even accidental. Indeed, it turns out that, in general, the $n = 0$ states of a theory compactified on a manifold with isometry group G include Yang-Mills fields with gauge group G . Hence, for instance, the search of phenomenologically suitable compactifications of 11-dimensional supergravity should be restricted to internal manifolds which contain $SU(3) \times SU(2) \times U(1)$ as a subgroup of the isometry group, in order to take into account at least of the Standard Model gauge group.

- (ii) The theory described by the Lagrangian (3.18) is *not* the full reduced theory, since it is obtained by just considering the zero-modes. The actual theory that results from retaining the $n \neq 0$ fields describes, in addition to the above massless states, an infinite tower of charged, massive, purely spin-2 particles with charges $q_n = n\mathcal{K}/R$ and masses $m_n = |n|/R$.

However, in general, we are not entitled to truncate a theory by setting to zero some of its degrees of freedom $\hat{\Phi}_i$ (as we have done in this example for the massive states), because nothing ensures that this truncation is consistent, namely that the solutions of the field equations of the truncated theory plus $\hat{\Phi}_i = 0$ are also solutions of the equations of the original one. The compactification on the circle (and, in general, on the n -tours T^n defined as the product of n circles) is particularly fortunate, because, as we now prove, in this case truncating the

theory to the $n = 0$ modes (which consists in eliminating the dependence on the internal coordinates) is automatically consistent. In fact, let us schematically denote by $\square\Phi_{\text{ext}} = f_{\text{ext}}(\Phi_{\text{int}}, \Phi_{\text{ext}})$ and $\square\Phi_{\text{int}} = f_{\text{int}}(\Phi_{\text{int}}, \Phi_{\text{ext}})$ the full equations of motion for the fields $\Phi_{\text{ext}}(x)$ (which depend just on the external coordinates) and $\Phi_{\text{int}}(x, y)$ (which depend also on the internal coordinates). The fields Φ_{ext} can enter in $f_{\text{int}}(\Phi_{\text{int}}, \Phi_{\text{ext}})$ only if each term of $f_{\text{int}}(\Phi_{\text{int}}, \Phi_{\text{ext}})$ contains a field Φ_{int} , and this implies $f_{\text{int}}(0, \Phi_{\text{ext}}) = 0$. On the other hand, if $f_{\text{ext}}(\Phi_{\text{int}}, \Phi_{\text{ext}})$ includes some term with Φ_{int} , the dependence on Φ_{int} should be quadratic (i.e. $|\Phi_{\text{int}}|^2$), in such a way that, thanks to the properties of $Y^n(y)$, the dependence on y vanishes. Hence, the solutions of $\square\Phi_{\text{ext}} = f_{\text{ext}}(0, \Phi_{\text{ext}})$ together with $\Phi_{\text{int}} = 0$, are also solutions of the initial equations, given that the initial equations for Φ_{int} are trivially satisfied (thanks to $f_{\text{int}}(0, \Phi_{\text{ext}}) = 0$), while the ones for Φ_{ext} exactly reduce to $\square\Phi_{\text{ext}} = f_{\text{ext}}(0, \Phi_{\text{ext}})$. Therefore, the truncation to the $n = 0$ modes (independent of y on the circle) indeed makes sense, and so the Lagrangian (3.18) correctly describes the behaviour of the massless fields. Likewise, in the case of the 5-dimensional free scalar action (3.7) compactified on the circle, the truncation to the $n = 0$ modes would have been meaningful, and this is obvious since the full reduced action describes an infinite set of noninteracting scalars.

Even though in the previous example a consistent truncation of the reduced theory has been obtained taking fields to be independent of the extra coordinates, in presence of more complicated compactifications (such as, for instance, S^7) obtaining a consistent low-energy truncated theory is not so trivial, because the $n = 0$ modes are not independent of y^m . However, at least for 11-dimensional supergravity, consistent truncations with a *finite* number of fields actually exist, and are representable by the ordinary supergravity theories in four dimensions, as we will discuss in the next section.

3.2 M-theory on T^7 and S^7

Not all higher-dimensional theories exhibit spontaneous compactification. In $D = 11$ supergravity, however, spontaneous compactification not only works, but the dimension four of the ordinary space-time is an output rather than an input [20]. Moreover, this theory allows the possibility that extra dimensions are either topologically flat or seven-spheres, and that, at the same, the ordinary 4-dimensional space-time has maximal symmetry.

We now justify these statements by looking for solutions of the equations of motion of 11-dimensional supergravity which might be candidates for a ground state of the form (3.2), described by the direct product $M_4 \times M_7$ with coordinates (x^μ, y^m) . Since we are interested in obtaining a 4-dimensional theory which admits maximal space-time symmetry, the vacuum solutions $\langle g_{MN} \rangle$, $\langle F_{MNPQ} \rangle$ and $\langle \Psi_M \rangle$ should be invariant under $\text{SO}(1, 4)$, Poincaré or $\text{SO}(2, 3)$ as the cosmological constant of M_4 is positive, zero or negative.

- First, the requirement of maximal symmetry implies that the vacuum expectation value of any fermion should vanish, and so we focus on solutions in which

$\langle \Psi_M \rangle = 0$. This entails a considerable simplification, because the gravitino field appears just in the piece \mathcal{L}_{bf} of the total Lagrangian (2.72) and only in pairs: this means that the equations of motion of the total Lagrangian plus the condition $\Psi_M = 0$ coincide with the equations of motions of the sole bosonic piece \mathcal{L}_{b} . Varying (2.73) with respect to g_{MN} and A_{MNP} , these equations read

$$\left\{ \begin{array}{l} R_{MN} - \frac{1}{2}g_{MN}R = \frac{1}{6} \left[F_{MPQR}F_N{}^{PQR} - \frac{1}{8}g_{MN}F_{PQRS}F^{PQRS} \right] \end{array} \right. \quad (3.21)$$

$$\left\{ \begin{array}{l} D_M F^{MNPQ} = -\frac{\sqrt{2}}{72} \varepsilon^{M_1 \dots M_8 N P Q} F_{M_1 \dots M_4} F_{M_5 \dots M_8} . \end{array} \right. \quad (3.22)$$

- Furthermore, maximal symmetry requires that all tensors in the solution have to be invariant under the infinitesimal general coordinate transformation $\delta_i x^\mu = k_i^\mu$, generated by the Killing vectors k_i^μ of one of the mentioned maximal isometry groups. In other terms, we should restrict to the solutions of (3.21-3.22) in which the Lie derivative of g_{MN} and F_{MNPQ} vanishes, namely

$$\left\{ \begin{array}{l} \mathcal{L}_k g_{MN}(x, y) = k_i^\mu \partial_\mu g_{MN}(x, y) + (\partial_M k_i^\mu) g_{\mu N} + (\partial_N k_i^\mu) g_{\mu M} = 0 \\ \mathcal{L}_k F_{MNPQ}(x, y) = k_i^\mu \partial_\mu F_{MNPQ}(x, y) - 4(\partial_{[M} k_i^\mu) F_{NPQ]\mu} = 0 . \end{array} \right. \quad (3.23)$$

The solutions of (3.23), combined with the requirement of a ground state of the form (3.2), are

$$\left\{ \begin{array}{l} g_{\mu\nu}(x, y) = g_{\mu\nu}(x) \\ g_{mn}(x, y) = g_{mn}(y) \\ g_{\mu n} = 0 \end{array} \right. , \quad \left\{ \begin{array}{l} F_{\mu\nu\rho\sigma}(x, y) = F_{\mu\nu\rho\sigma}(x) = m(x)\varepsilon_{\mu\nu\rho\sigma} \\ F_{mnpq}(x, y) = F_{mnpq}(y) \\ F_{\mu\nu\rho q} = F_{\mu\nu pq} = F_{\mu npq} = 0 . \end{array} \right. \quad (3.24)$$

In these formulas, the y -independence of $g_{\mu\nu}$ and the x -independence of g_{mn} derive from requiring that g_{MN} is a product metric, whereas the y -independence of $F_{\mu\nu\rho\sigma}$ and the x -independence of F_{mnpq} are consequences of the Bianchi identity $\partial_{[M} F_{NPQR]} = 0$ and of $F_{\mu\nu\rho q} = F_{\mu\nu pq} = F_{\mu npq} = 0$. Actually, it is important to note that the maximal space-time symmetry requirement (3.23) *alone* would not rule out the so-called ‘‘warped-product’’ solution $g_{\mu\nu}(x, y) = f(y)g_{\mu\nu}(x)$. However, in the following we shall confine our attention to $f(y) = 1$, also because no solution with warp factor leads to manifolds M_7 which are group cosets. Moreover, the relation $F_{\mu\nu\rho\sigma}(x) = m(x)\varepsilon_{\mu\nu\rho\sigma}$ descends from the fact that, up to multiplicative constants, there is only one rank-4 antisymmetric tensor in four dimensions.

Substituting (3.24) into (3.22), we find that our ground state solution must satisfy

$$\begin{aligned} D_\mu F^{\mu\nu\rho\sigma} &= 0 , \\ D_m F^{mnpq} &= -\frac{\sqrt{2}m}{72} \varepsilon^{\mu\nu\rho\sigma npqabcd} \varepsilon_{\mu\nu\rho\sigma} F_{abcd} = \frac{\sqrt{2}m}{3} \varepsilon^{npqabcd} F_{abcd} , \end{aligned} \quad (3.25)$$

which are trivially verified if we assume the Freund–Rubin [23] ansatz

$$\left\{ \begin{array}{l} F_{\mu\nu\rho\sigma} = m\varepsilon_{\mu\nu\rho\sigma} \\ F_{mnpq} = 0 , \end{array} \right. \quad (3.26)$$

i.e. that the internal components of F_{MNPQ} vanish identically and the external ones do *not* depend on x (in (3.26), in fact, m does not depend on x). Let us analyze the consequences of this ansatz on the metric field equation (3.21). Contracting (3.21) with g_{MN} we obtain $R = \frac{1}{72} F^{MNPQ} F_{MNPQ}$, and therefore we can rewrite (3.21) in the equivalent form

$$R_{MN} = \frac{1}{6} \left[F_{MPQR} F_N{}^{PQR} - \frac{1}{12} g_{MN} F^{PQRS} F_{PQRS} \right]. \quad (3.27)$$

Plugging the ansatz (3.26) into this equation, we find that the metric field equation (3.21) is satisfied if and only if the components of the Ricci tensor are

$$\begin{aligned} R_{\mu\nu} &= \frac{1}{6} \left[m^2 \varepsilon_{\mu\alpha\beta\gamma} \varepsilon_\nu{}^{\alpha\beta\gamma} + \frac{1}{12} 24 m^2 g_{\mu\nu} \right] = -\frac{2}{3} m^2 g_{\mu\nu}, \\ R_{mn} &= -\frac{1}{72} g_{mn} m^2 (-24) = \frac{1}{3} m^2 g_{mn}, \\ R_{\mu n} &= 0. \end{aligned} \quad (3.28)$$

Therefore, any value of g_{MN} and F_{MNPQ} that fulfills equations (3.26) and (3.28) determines a ground state solution $M_4 \times M_7$ in which M_4 has maximal symmetry. However, M_4 and M_7 defined in this way are not arbitrary, since further constraints on both M_4 and M_7 derive from equation (3.28).

- First of all, also M_7 should be an Einstein space, with signature $(++++++)$.
- Secondly, M_4 cannot be an arbitrary maximally symmetric space, but either AdS with cosmological constant $-2m^2/3$ (if $m \neq 0$) or Minkowski (if $m = 0$). Correspondingly, the 7-dimensional Einstein space M_7 exhibits either positive or flat curvature.

The important point is that complete Einstein spaces of positive curvature and Euclidean signature are automatically compact [24] and hence spontaneous compactification to four dimensions has indeed been achieved. Besides, the choice of dimension four for the ordinary space-time is not ad hoc, but a consequence of the field equations, since in (3.24) maximal symmetry singles out exactly the *four-dimensional* Levi-Civita symbol. This, in turn, is dictated by the fact that the F tensor has rank four, which is a result of $D = 11$ supersymmetry. Actually, we could have alternatively requested that F_{mnpq} had rank four, and in this case we would have obtained an acceptable ground state solution $M_7 \times M_4$ with F_{mnpq} proportional to ε_{mnpq} and $F_{\mu\nu\rho\sigma} = 0$. Thereby, 11-dimensional supergravity *naturally* admits spontaneous compactification to both four and seven dimensions (in contrast with the examples of section 3.1, in which the dimension four has been chosen by hand).

There are infinitely many 7-dimensional Einstein spaces with flat or positive curvature. For instance, a possible flat choice is the seven-torus T^7 , whose (abelian) isometry group is $[U(1)]^7$. Particularly simple and treatable manifolds with positive curvature are homogeneous spaces corresponding to group cosets G/H , such as S^7 , M^{pqr} and N^{010} (see [22, 20] for more details). Remarkably, the 4-dimensional theories arising from such compactifications (and upon suitable truncations of the massive modes) are nothing but the 4-dimensional gauged supergravities discussed in the previous chapter [15].

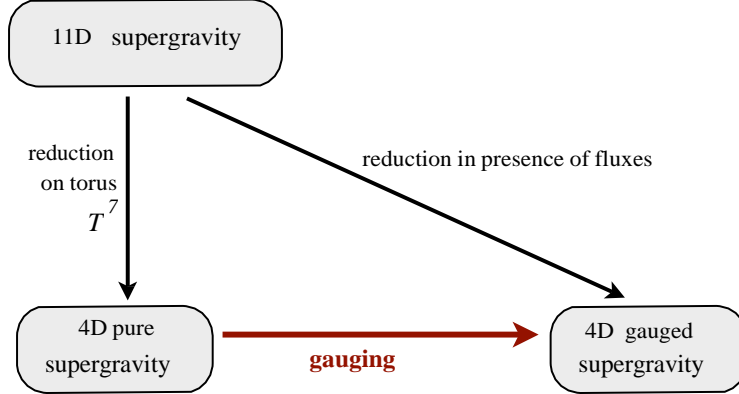


Figure 1: Gauged supergravities from dimensional reductions.

- For example, if we compactify 11-dimensional supergravity on $M_7 = T^7$ keeping only the dependence on the internal coordinates (i.e. making the consistent truncation to the zero-modes and discarding the massive states, as discussed in section 3.1), the result is exactly ungauged maximal supergravity in four dimensions. This was explicitly shown by Cremmer and Julia in [19], where further duality and other field transformations are performed in order to cast the reduced Lagrangian into its manifestly $E_{7(7)} \times SU(8)$ invariant form. Such a compactification corresponds to the vertical arrow of the diagram in Figure 1.
- On the other hand, one may consider more complicated compactifications (the diagonal arrow in Figure 1) where the torus is replaced by manifolds with more structure (such as S^7). In this situation, higher-dimensional p -form field strengths $\mathcal{F}^{(p)}$ may acquire nontrivial background fluxes $C_\Sigma = \int_\Sigma \mathcal{F}^{(p)}$ along nontrivial circles Σ of the internal manifold. All these compactifications, together with suitable truncations of the massive modes, lead to more complicated effective theories in four dimensions, which typically come with non-abelian gauge symmetries and which turn out to be gauged supergravities. As we said, the most systematic approach for constructing these theories is the gauging procedure described in section 2.4 and represented by the horizontal arrow in Figure 1. Fortunately, all different gaugings are encoded in the embedding tensor, which, from the point of view of flux compactifications, can be seen as a very compact tool to group all different possible flux (or deformation) parameters C_Σ , which are indeed connected to the components of Θ_M^α . Therefore, the gauging of a supergravity theory corresponds to reducing 11-dimensional supergravity in presence of nontrivial background fluxes, codified by the embedding tensor.

An example of such a construction is the standard $G = SO(8)$ gauged theory, proposed for the first time in [26], which derives from the S^7 compactification of 11-dimensional supergravity, with $SO(8)$ properly embedded into the global $E_{7(7)}$ symmetry group. However, other 4-dimensional $SO(8)$ gauged supergravities have been recently discovered [27, 28] and it has been proven that actually none of them descends from the 11-dimensional theory [29].

3.3 STU-models from orbifold reductions

Apart from the SO(8) gauged supergravity arising from the compactification on the seven-sphere, our interest is also triggered by the so-called STU-models [30], which derive from M-theory compactifications as well, and, as we will see, consist in a generalization of the SO(8) gauged theory properly truncated.

An STU-model is an $N = 1$ supergravity theory with no vector multiplet and seven chiral multiplets $\{\phi_i\}$, conventionally named $S, T_1, T_2, T_3, U_1, U_2, U_3$.

- The Kähler potential for these models is completely determined by the kind of compactification they descend from, and can be written as the sum of seven equal contributions, one for each supermultiplet (note that $M_P = 1$):

$$K(\phi_i, \phi_{\bar{j}}) = -\log(S + \bar{S}) - \sum_{i=1}^3 \log(T_i + \bar{T}_i) - \sum_{i=1}^3 \log(U_i + \bar{U}_i). \quad (3.29)$$

As we said in section 2.2.2, K defines the structure of the scalar manifold \mathcal{M}_{sc} , which has to be the direct product of seven identical factors $\hat{\mathcal{M}}$, since each term in (3.29) has the same structure but depends on a different multiplet. We can calculate at least the isometry group \hat{G} of $\hat{\mathcal{M}}$, associated to the Kähler potential $K_0 = -\log(\phi + \bar{\phi})$, by explicitly deriving the (homomorphic) Killing vectors $\xi_I(\phi)$ of $\hat{\mathcal{M}}$ and their algebra. First, the Killing equation $\mathcal{L}_{\xi} g_{\phi\bar{\phi}} = 0$, with $g_{\phi\bar{\phi}} = \partial_{\phi} \partial_{\bar{\phi}} K_0$, is solved by the three independent Killing vectors $\xi_0(\phi) = i$, $\xi_1(\phi) = \phi$ and $\xi_2(\phi) = \frac{i}{2}\phi^2$, and therefore $\hat{\mathcal{M}}$ has dimension equal to three. Defining as $\xi_I f \equiv \xi_I \partial_{\phi} f + \bar{\xi}_I \partial_{\bar{\phi}} f$ the action of a Killing vector on a generic function $f(\phi, \bar{\phi})$ on the internal manifold, the Killing algebra reads $[\xi_I, \xi_J]f = f_{IJ}{}^K \xi_K f$, where the only nonvanishing structure constants are $f_{01}{}^0 = f_{12}{}^2 = f_{20}{}^1 = 1$. Hence, since the real linear combinations $\xi'_0 = \xi_0 - \xi_1$, $\xi'_1 = \xi_0 - \xi_1 - \xi_2$, $\xi'_2 = \xi_1 + \xi_2$ satisfy the algebra $f'_{01}{}^0 = f'_{12}{}^0 = -f'_{20}{}^1 = 1$, the isometry group \hat{G} coincides with SU(1, 1). Actually, it turns out that the factor $\hat{\mathcal{M}}$ is the coset manifold SU(1, 1)/U(1) and therefore the scalars parametrize the full manifold

$$\mathcal{M}_{\text{sc}} = \underbrace{\left(\frac{\text{SU}(1,1)}{\text{U}(1)} \right) \times \cdots \times \left(\frac{\text{SU}(1,1)}{\text{U}(1)} \right)}_{\times 7}. \quad (3.30)$$

- On the contrary, the superpotential $W(\phi_i)$ is not fully determined by the compactification and its structure can be either polynomial or not [42, 38]. In particular, a polynomial structure (up to the seventh degree) arises naturally from perturbative effects in the compactification in presence of fluxes, whereas a nonperturbative contribution would contain, for instance, exponential terms. In the following we will consider just polynomial superpotentials, whose coefficients are directly connected with generalized flux parameters in the compactification of the higher-dimensional theory [31]. However, as we will see when talking about the uplift, only for particular choices of the coefficients the uplift of the $N = 1$ vacua to eleven dimensions is actually clear at the present time.

These STU-models derive from 11-dimensional supergravity upon compactifying M_7 on the orbifold $T^7/(\mathbb{Z}_2 \times \mathbb{Z}'_2 \times \mathbb{Z}''_2)$, where \mathbb{Z}_2 , \mathbb{Z}'_2 and \mathbb{Z}''_2 are parity transformations we now specify, and upon truncating the result to the zero-modes (namely, imposing that the fields do not depend on the internal coordinates). Each \mathbb{Z}_2 transformation acts only on the internal coordinates (z^4, \dots, z^{10}) , which parameterize the seven-torus, as $z^M \rightarrow P^M z^M$, where P^M can be either 1 or -1 according to the table:

	z^4	z^5	z^6	z^7	z^8	z^9	z^{10}
\mathbb{Z}_2	+	-	-	-	-	+	+
\mathbb{Z}'_2	+	-	-	+	+	-	-
\mathbb{Z}''_2	-	-	+	-	+	-	+

For example, the first \mathbb{Z}_2 transformation sends $(z^4, z^5, z^6, z^7, z^8, z^9, z^{10})$ into $(z^4, -z^5, -z^6, -z^7, -z^8, z^9, z^{10})$, and similarly for the others. As explained in the example of section 3.1, if $P^M = -1$, the orbifolding procedure induces (for each \mathbb{Z}_2 factor) the identification between z^M and $-z^M$, in such a way that, at the end, the actual manifold $T^7/(\mathbb{Z}_2 \times \mathbb{Z}'_2 \times \mathbb{Z}''_2)$ is just a part of the full seven-torus T^7 , obtained indeed by the projection of T^7 under $\mathbb{Z}_2 \times \mathbb{Z}'_2 \times \mathbb{Z}''_2$.

Let us analyze how the (bosonic) field content of the STU-models can be recovered by reducing 11-dimensional supergravity on $T^7/(\mathbb{Z}_2 \times \mathbb{Z}'_2 \times \mathbb{Z}''_2)$ and why considering an orbifold in place of the full seven-torus is essential.

- If we compactify 11-dimensional supergravity on the full T^7 discarding the massive modes, the constitutive elements of the 11-dimensional supermultiplet give rise to a huge (but finite) number of 4-dimensional fields, which can be listed treating the indices of the internal coordinates as fixed. For example, in the bosonic sector, the 11-dimensional graviton g_{MN} produces the 4-dimensional graviton $g_{\mu\nu}$, 7 vectors $g_{\mu n}$ and 28 scalars g_{mn} , whereas the three-form A_{MNP} generates a 4-dimensional three-form $A_{\mu\nu\rho}$, 7 two-forms $A_{\mu\nu p}$, 21 vectors $A_{\mu np}$ and 35 scalars A_{mnp} . The three-form $A_{\mu\nu\rho}$ just provides nondynamical degrees of freedom and the 7 two-forms $A_{\mu\nu p}$ are the on-shell duals of 7 real scalar fields. Therefore, the field content of the reduced theory includes 70 real scalars, 28 vectors and one graviton, which, together with the fermions, precisely constitute the massless $N = 8$ supermultiplet of maximal supergravity (not by chance, 11-dimensional supergravity reduced on T^7 is exactly ungauged maximal supergravity, as we said).
- Due to the projection $\mathbb{Z}_2 \times \mathbb{Z}'_2 \times \mathbb{Z}''_2$, however, the compactification on $T^7/(\mathbb{Z}_2 \times \mathbb{Z}'_2 \times \mathbb{Z}''_2)$ produces a lower number of fields. In fact, out of the above ones, remain only the components which are invariant under \mathbb{Z}_2 , \mathbb{Z}'_2 and \mathbb{Z}''_2 , since the others are identified with their opposite and thus vanish. For instance, the component $A_{\mu 67}$ is left invariant by \mathbb{Z}_2 but it is identified with its opposite by both \mathbb{Z}'_2 and \mathbb{Z}''_2 , and therefore disappears. Applying this method to all components, one easily realizes that the requirement of invariance under $\mathbb{Z}_2 \times \mathbb{Z}'_2$ imposes that only survive (in addition to $g_{\mu\nu}$)
 - (a) 1 of the 7 vectors $g_{\mu n}$
 - (b) 10 of the 28 scalars g_{mn}
 - (c) 1 of the 7 two-forms $A_{\mu\nu n}$

- (d) 3 of the 21 vectors $A_{\mu np}$
(e) 10 of the 28 scalars A_{mnp}

In the following table are displayed the explicit components invariant under $\mathbb{Z}_2 \times \mathbb{Z}'_2$, and only the ones which are not further invariant under \mathbb{Z}''_2 are canceled.

Scalars	Vectors	Two-forms
g₅₆ , g₇₈ , g₉₁₀ , g ₄₄ , g ₅₅ , g ₆₆ g ₇₇ , g ₈₈ , g ₉₉ , g ₁₀₁₀ A ₅₆₄ , A ₇₈₄ , A ₉₁₀₄ , A₅₇₉ A ₅₇₁₀ , A ₅₈₉ , A₅₈₁₀ , A ₆₇₉ , A₆₇₁₀ , A₆₈₉ , A ₆₈₁₀	g_{μ4} A_{μ56} , A_{μ78} , A_{μ910}	A_{μν4}

Hence, the reduction on $T^7/(\mathbb{Z}_2 \times \mathbb{Z}'_2 \times \mathbb{Z}''_2)$ eventually provides a bosonic spectrum made of the metric, 14 real scalars and no vector, which indeed constitute the bosonic part of the graviton multiplet and of the 7 chiral multiplets of the STU-models.

We can easily prove that also the fermionic counterparts indeed match the expected amount of fields. For this purpose, we note that all 4-dimensional Fermi fields must derive from the 11-dimensional gravitino Ψ_M^α , which is a 32-component Majorana vector-spinor and the only Fermi field of the higher-dimensional theory.

- Upon compactifying on T^7 and truncating to the $n = 0$ modes, the field $\Psi_M^\alpha(x, y)$ turns out to be decomposed as $\Psi_M^\alpha(x, y) = \psi_M^{\hat{\alpha}i}(x) \otimes \eta_i^A(y)$, where $\hat{\alpha} = 1, \dots, 4$ denotes the spinorial index of a 4-component Majorana spinor, the index $i = 1, \dots, 8$ spans the Killing spinors $\eta_i^A(y)$ (8 for T^7 and S^7) and indicates the transformation properties under the SU(8) R -symmetry group and $A = 1, \dots, 8$ is the spinorial index of SO(7). For the torus and the sphere it is always possible to choose a basis in which $\eta_i^A(y)$ is constant and furthermore $\eta_i^A(y) = \delta_i^A$. Thus, the truncated 4-dimensional fields are 8 gravitinos $\psi_\mu^{\hat{\alpha}i}$ (which should be indeed 4-components Majorana vector-spinors) and $7 \times 8 = 56$ spinors $\psi_M^{\hat{\alpha}i}$: these fields together correctly compose the fermionic field content of maximal supergravity.
- On the other hand, if we instead compactify on $T^7/(\mathbb{Z}_2 \times \mathbb{Z}'_2 \times \mathbb{Z}''_2)$, just the fields further invariant under $\mathbb{Z}_2 \times \mathbb{Z}'_2 \times \mathbb{Z}''_2$ survive. In order to list the invariant components, we note that the parity transformations act on both space-time and SU(8) indices, and the two transformation rules $z^M \rightarrow U^M_N z^N$ and $t^i \rightarrow V^i_j t^j$ (where, as we said, $U^M_N = \delta^M_N P^M$) are linked by the simple relation $V^i_j = U_{MN}(\gamma^{MN})^i_j$. It is therefore straightforward to see that also $V^i_j = \delta^i_j P'^i$ (so that $t^i \rightarrow P'^i t^i$), where P'^i is either 1 or -1 according to the table:

	1	2	3	4	5	6	7	8
\mathbb{Z}_2	+	+	+	+	-	-	-	-
\mathbb{Z}'_2	+	+	-	-	+	+	-	-
\mathbb{Z}''_2	+	-	+	-	+	-	+	-

As a consequence, all gravitinos $\psi_\mu^{\hat{\alpha}i}$ except $\psi_\mu^{\hat{\alpha}1}$ transform under at least one parity, and so just $\psi_\mu^{\hat{\alpha}1}$ remains upon compactifying on $T^7/(\mathbb{Z}_2 \times \mathbb{Z}'_2 \times \mathbb{Z}''_2)$. Moreover, among the spinors $\psi_M^{\hat{\alpha}i}$, the only invariant are those for which the transformations

of the indices M and i coincide for all parities, and so, comparing the two above tables, we deduce that just the 7 components

$$\psi_4^{\hat{\alpha}2}, \quad \psi_{10}^{\hat{\alpha}3}, \quad \psi_9^{\hat{\alpha}4}, \quad \psi_8^{\hat{\alpha}5}, \quad \psi_7^{\hat{\alpha}6}, \quad \psi_6^{\hat{\alpha}7}, \quad \psi_5^{\hat{\alpha}8} \quad (3.31)$$

last. Hence, the field content includes 1 gravitino and 7 spinors, which, as predicted, constitute the fermionic part of the graviton multiplet and of the 7 chiral multiplets.

Therefore, this kind of compactification ensures (at least) the correct field content: however, although we do not give further details, this reduction gives actually rise to an $N = 1$ Lagrangian with Kähler potential of the form (3.29) and superpotential determined by flux parameters [31].

4 STU-truncations of SO(8) supergravity

The discussion of chapters 2 and 3 has provided all fundamental ingredients for analyzing the existence and properties of stable dS vacua in minimal and maximal supergravities. As a first step, in this chapter, we begin the vacua analysis focusing on the SO(8) gauged supergravity, which is the simplest possible deformation of maximal supergravity and for which many results are already known by the old and recent literature [25, 32, 33, 34, 35, 36], both analytically and numerically. Actually, our investigation will concern not only the theories arising from the S^7 compactification, but also the new SO(8) gauged supergravities that do not descend from M-theory.

As we have seen in section 2.4.3, however, in all gauged supergravities the scalar potential (2.66) depends quadratically on the embedding tensor and nonlinearly on the 70 real scalars which parameterize the $E_{7(7)}$ -valued matrix \mathcal{V}_M^{ij} . Therefore, even though the embedding tensor components Θ_M^α are fixed at the proper values for the SO(8) gauging, the analytic search of stationary points is anything but straightforward, especially due to the huge number of scalars. For this reason, in the following we do not consider the full SO(8) gauged theory, but, rather, we concentrate on a consistent truncation of it leading to a *particular* STU-model, for which the search is obviously much more manageable. Fortunately, the consistency of the truncation ensures that all vacua of the truncated model are also vacua of the full theory, although many other solutions of the full theory are necessarily cut off. So, in the first section we give some details about the $N = 1$ model deriving from such a truncation, while in the second we present our explicit analytic results, also searching a correspondence with the present literature.

4.1 The $N = 1$ model

In analogy with the orbifolding procedure, our $N = 1$ truncation of SO(8) gauged supergravity is based on the elimination of the field components which are *not* invariant under a certain set of three parity transformation. Actually, these transformations have been already introduced in section 3.3 for the fermionic sector and, as we have seen, if interpreted as action on the extra dimensions, they are exactly the projections that induce the orbifold $T^7/(\mathbb{Z}_2 \times \mathbb{Z}'_2 \times \mathbb{Z}''_2)$.

To explain how these transformations act on a generic field of maximal supergravity, we remind that its field content can be represented by

$$\left(\phi^{ijkl}, \chi^{ijk}, A_\mu^{ij}, \psi_\mu^i, g_{\mu\nu} \right), \quad (4.1)$$

where the indices i, j, \dots assume 8 values and describe the transformation properties under the SU(8) R -symmetry group. The three parity transformations, denoted by $\mathbb{Z}_2, \mathbb{Z}'_2$ and \mathbb{Z}''_2 , act on each field φ in (4.1) as $\varphi \rightarrow P_\varphi \varphi$, where P_φ is either 1 or -1 depending on the structure of indices in φ . In particular, each \mathbb{Z}_2 parity transforms the indices i, j, \dots according to the table on page 49 and thus, for instance, ψ_μ^1 is left invariant by all parity transformations, whereas ψ_μ^5 is transformed into its opposite just by \mathbb{Z}_2 . Likewise, A_μ^{34} is left invariant by both \mathbb{Z}_2 and \mathbb{Z}'_2 , but sent into $-A_\mu^{34}$ by \mathbb{Z}''_2 , since only the index 4 changed.

The $N = 1$ truncation is obtained from the initial $\text{SO}(8)$ gauged theory by identifying each field φ with its transformed $P_\varphi\varphi$, for every parity transformation. Therefore, if a component $\tilde{\varphi}$ is odd under at least one among $\mathbb{Z}_2, \mathbb{Z}'_2$ and \mathbb{Z}''_2 , in the truncated theory $\tilde{\varphi}$ is imposed to be equal to $-\tilde{\varphi}$ and vanishes. Hence, as anticipated at the beginning, the $N = 1$ model displays the sole field components which are invariant under all three parity transformations.

Let us now analyze explicitly which components actually vanish in order to provide the right STU-spectrum. Since the graviton $g_{\mu\nu}$ does not transform under parity, it remains unchanged in the truncated model. Moreover, a quick inspection of the above table tells us that only the gravitino component ψ_μ^1 survives: together with the graviton, ψ_μ^1 therefore constitutes the only graviton multiplet of the STU-model. On the contrary, the structure of $\mathbb{Z}_2, \mathbb{Z}'_2$ and \mathbb{Z}''_2 implies that just the vectors of the form A_μ^{ii} are left invariant by all parities, and so, given the antisymmetry of the indices i, j, \dots , no vector remains (this is indeed coherent with the expected field content). Out of the 35 spinors χ^{ijk} , the only invariant under \mathbb{Z}_2 have the structure $\chi^{\tilde{i}\tilde{j}\tilde{k}}$ or $\chi^{\hat{i}\hat{j}\hat{k}}$ (where $\tilde{i} = 1, 2, 3, 4$ and $\hat{i} = 5, 6, 7, 8$), and are $4 + 24 = 28$ in total: of these, just 7 survive also under \mathbb{Z}'_2 and \mathbb{Z}''_2 . Similarly, the (real) scalars invariant under \mathbb{Z}_2 are $\phi^{\tilde{i}\tilde{j}\tilde{k}\tilde{l}}, \phi^{\tilde{i}\tilde{j}\hat{k}\hat{l}}$ and $\phi^{\hat{i}\hat{j}\hat{k}\hat{l}}$ ($1 + 36 + 1 = 38$ in total), and 14 of them eventually last. The following table summarizes the explicit components of ϕ^{ijkl} and χ^{ijk} left, which properly combine into the 7 chiral multiplets (ϕ^i, χ^i) of the STU-model.

Scalars	Spinors
$\phi^{1234}, \phi^{1256}, \phi^{1278},$ $\phi^{3456}, \phi^{3478}, \phi^{5678},$ $\phi^{1357}, \phi^{1368}, \phi^{1458}, \phi^{1467},$ $\phi^{2358}, \phi^{2367}, \phi^{2457}, \phi^{2468}$	$\chi^{234}, \chi^{256}, \chi^{278},$ $\chi^{357}, \chi^{368}, \chi^{458}, \chi^{478}$

Remarkably, a generic truncation dictated by the invariance under parity transformations \mathbb{Z}_2 is automatically consistent. In fact, let us collectively call Φ_e and Φ_o the fields which are respectively even and odd under \mathbb{Z}_2 . The equations of motions of the full theory are schematically $\square\Phi_e = f_e(\Phi_e, \Phi_o)$ and $\square\Phi_o = f_o(\Phi_e, \Phi_o)$, where Φ_e can enter in $f_o(\Phi_e, \Phi_o)$ only if every term in $f_o(\Phi_e, \Phi_o)$ contains Φ_o (in order for $f_o(\Phi_e, \Phi_o)$ to be odd), and so $f_o(\Phi_e, 0) = 0$. On the other hand, Φ_o can appear into $f_e(\Phi_e, \Phi_o)$ only quadratically, so that $f_e(\Phi_e, \Phi_o)$ is even. Thereby, setting to zero the odd fields, the equation $\square\Phi_o = f_o(\Phi_e, \Phi_o)$ is trivially satisfied, while $\square\Phi_e = f_e(\Phi_e, \Phi_o)$ reduces to $\square\Phi_e = f_e(\Phi_e, 0)$, which is the equation of motion of the truncated theory. As a consequence, imposing $\Phi_o = 0$ is indeed consistent, and the above $\mathbb{Z} \times \mathbb{Z}'_2 \times \mathbb{Z}''_2$ truncation of $\text{SO}(8)$ gauged supergravity is actually meaningful.

Once the $N = 1$ truncation has been performed, the $\text{SO}(8)$ gauged supergravity reduces to a particular STU-model, whose Kähler potential is fixed by (3.29) and whose field content is made of $(g_{\mu\nu}, \psi_\mu^1)$ and of the 7 chiral multiplets (ϕ^i, χ^i) . In particular, the conditions on the embedding tensor which determine the $\text{SO}(8)$ gauging translate into suitable conditions on the coefficients of the superpotential $W(\phi_i)$

of the truncated theory. For instance, the $\text{SO}(8)$ gauging in absence of magnetic duals (i.e. $\Theta^{\Sigma\alpha} = 0$ in equation (2.61)) produces the polynomial superpotential

$$W_1(\phi_i) = 1 + ST_1T_2T_3 + U_1U_2T_1T_2 + U_1U_3T_1T_3 + U_2U_3T_2T_3 \\ + ST_1U_2U_3 + ST_2U_1U_3 + ST_3U_1U_2, \quad (4.2)$$

made of the sum between 1 and a polynomial of the fourth degree in ϕ_i , in which each coefficient is equal to 1. In contrast, the inclusion of the magnetic duals introduces into the right-hand side of (4.2) a new contribution $W_2(\phi_i)$, given by

$$W_2(\phi_i) = i(ST_1T_2T_3U_1U_2U_3 + U_1U_2U_3 + SU_3T_3 + SU_2T_2 + SU_1T_1 \\ + T_2T_3U_1 + T_1T_3U_2 + T_1T_2U_3), \quad (4.3)$$

where each term is obtained “dualizing” the corresponding term in (4.2), namely inserting the missing fields in that term and removing the ones originally present (and multiplying by i). For instance, the second term in $W_2(\phi_i)$, $iU_1U_2U_3$, is the dual of $ST_1T_2T_3$ because the only missing fields in $ST_1T_2T_3$ are exactly U_1, U_2, U_3 . As a consequence, $W_2(\phi_i)$ is a polynomial with imaginary coefficients made of a seventh-degree term, $iST_1T_2T_3U_1U_2U_3$, dual to 1, and of a third-degree contribution, dual to the remainder of $W_1(\phi_i)$. While the STU-model with superpotential $W_1(\phi_i)$ is the truncation of the standard $\text{SO}(8)$ gauged supergravity, in presence of $W(\phi_i) = W_1(\phi_i) + W_2(\phi_i)$ the full $\text{SO}(8)$ theory does not descend from S^7 compactification.

Interestingly enough, non-compact gauge versions of $\text{SO}(8)$ supergravity, in which $\text{SO}(8)$ is replaced by $\text{SO}(p, q)$ with $p + q = 8$, admit similar consistent truncations to STU-models. Moreover, the superpotential arising from such truncations is obtainable by slightly modifying equations (4.2-4.3). In fact, if no magnetic dual is added, the $\text{SO}(p, q)$ gauged theory leads to a superpotential $W_1(\phi_i)$ which differs from (4.2) by the only fact that p of the 8 coefficients (arbitrarily chosen) are -1 in place of 1. Analogously, in presence of magnetic duals, the superpotential takes also a contribution $W_2(\phi_i)$, equal to (4.3) except for the fact that the duals of the p terms in $W_1(\phi_i)$ with coefficient -1 have now coefficient $-i$. Remarkably, also these non-compact gaugings turn out to be upliftable to eleven dimensions [37], when $W(\phi_i) = W_1(\phi_i)$.

The above specification of the superpotential in presence and absence of magnetic duals completely determines the $\text{SO}(p, q)$ reduced theories. Although the STU-models descending from such truncations are very particular, in the following chapter we will however consider generalizations of them, in which the coefficients can be also different from ± 1 or $\pm i$ and other polynomials (up to the seventh degree) are allowed into both (4.2) and (4.3).

4.2 Vacua from the truncations of the $N = 8$ models

Now that we have described all necessary ingredients for our analysis, we can present the results obtained for the truncation of a generic $G = \text{SO}(p, q)$ gauging.

Some preliminary observations facilitated our work. First, since the superpotential appears uniquely in pairs into the scalar potential V of (2.10), an overall sign in front of the superpotential produces the same V . By the above discussion it is

therefore obvious that $\text{SO}(7, 1)$ and $\text{SO}(1, 7)$, $\text{SO}(6, 2)$ and $\text{SO}(2, 6)$, $\text{SO}(5, 3)$ and $\text{SO}(3, 5)$ have respectively the same scalar potential, and thus the same vacua. So, even though we should in principle perform the vacua analysis for each $p = 1, \dots, 8$ in order to study all non-compact gaugings, we can restrict to $p = 4, 5, 6, 7, 8$. Moreover, for each p fixed, it is not necessary to consider all combinations of p negative coefficients in (4.2) and (4.3), because most of them can be obtained starting from a certain combination and redefining the fields ϕ_i . Finally, given that the Kähler potential (3.29) depends just on $\phi_i + \bar{\phi}_i$, it is particularly convenient to utilize as independent variables the real and imaginary parts of S , T_i and U_i , defined by

$$S = s + i\sigma, \quad T_i = t_i + i\tau_i, \quad U_i = u_i + i\nu_i. \quad (4.4)$$

Let us first discuss the vacua in presence of magnetic duals, namely setting $W(\phi_i) = W_1(\phi_i) + W_2(\phi_i)$ with the appropriate number of -1 coefficients. The general expression for the extrema of the scalar potential cannot be found analytically, and thus the best initial approach is to focus on the existence of critical points such that $\sigma = \tau_i = \nu_i = 0$ and $t_1 = t_2 = t_3 \equiv t$, $u_1 = u_2 = u_3 \equiv u$ (which are isotropic solutions). This is indeed sufficient to reproduce many known vacua of the $\text{SO}(p, q)$ theories, reported in Table 1.

#	Gauge group	G_{res}	SUSY	Λ	$m^2_{(\text{multiplicity})}$
i	$\text{SO}(8)$	$\text{SO}(8)$	Yes	-3	$-\frac{2}{3}_{(14)}$
ii	$\text{SO}(8)$	$\text{SO}(7)$	No	$-\frac{25\sqrt{5}}{16}$	$2_{(1)}, -\frac{4}{5}_{(6)}, -\frac{2}{5}_{(7)}$
iii	$\text{SO}(7, 1)$	$\text{SO}(7)$	No	$-\frac{5}{4}$	
iv	$\text{SO}(8)$	$\text{SO}(6)$	No	-4	$2_{(2)}, -1_{(5)}, -\frac{1}{4}_{(4)}, 0_{(3)}$
v	$\text{SO}(7, 1)$	$\text{SO}(6)$	No	$-1.38\dots$	
vi	$\text{SO}(6, 2)$	$\text{SO}(2)^4$	Yes	0	$\frac{(1+u^2)^3}{2u^3}, \frac{(1+u^2)^3}{8u^3}_{(4)}, 0_{(9)}$
vii	$\text{SO}(5, 3)$	$\text{SO}(3) \times \text{SO}(5)$	No	$\frac{3}{4}$	$4_{(2)}, -2_{(2)}, 2_{(5)}, \frac{4}{3}_{(4)}, -\frac{2}{3}$
viii	$\text{SO}(4, 4)$	$\text{SO}(4) \times \text{SO}(4)$	No	1	$-2, 2_{(9)}, 1_{(4)}$

Table 1: Vacua of the truncated theory with magnetic duals satisfying $\sigma = \tau_i = \nu_i = 0$ and $t_1 = t_2 = t_3, u_1 = u_2 = u_3$. When $\Lambda \neq 0$, the scalar masses are expressed in units of the cosmological constant.

The Table shows the scalar mass spectrum (given by the eigenvalues of $g^{-1}\partial^2V$) and the cosmological constant generated around the vacua, which either preserve or break supersymmetry as specified in the fourth column. All these vacua have been already classified, for example in [35, 36], and for each of them has also been calculated the residual symmetry group G_{res} (made of the gauge symmetries not spontaneously broken at the vacuum), reported for completeness in the third column.

- The solutions i, iii, vii, viii are located at the origin $s = t = u = 1$, and, in particular, i is the well-known maximally supersymmetric AdS vacuum of the $\text{SO}(8)$ theory. Since in supergravity all AdS vacua satisfying the so-called Breitenlohner–Freedman bound $|m^2/\Lambda| < 3/4$ possess real vacuum energy (although $m^2 < 0$) [11, 12], vacuum i is of course stable. On the contrary, vacua ii

and iii are AdS with residual symmetry group $\text{SO}(7)$, but for the 6 directions with mass eigenvalues $m^2 = -4/5$ the Breitenlohner–Freedman bound is not respected, and so they are unstable. Similarly, iv and v are not stable due to the presence of 5 mass eigenvalues $m^2 = -1$.

- The solution vi is Minkowski (and supersymmetric) and actually refers to an infinite set of vacua for which $s = t = 1$ and parametrized by the values of u . In order to match the masses with the values $m^2 = (2, 1/2_{(4)}, 0_{(9)})$ displayed in the references, we need to select particular values of u , in such a way that $(1+u^2)^3/u^3 = 4$. Moreover, as predicted by the general discussion of section 2.2.3, these supersymmetric vacua are automatically stable.
- Finally, vii and viii are the only dS vacua and break spontaneously supersymmetry (as they were expected to do): however, in spite of our desires, they are unstable.

As we said at the beginning, these vacua from the truncated theory are also vacua of the $N = 8$ models, thanks to the consistency of the truncation we have done. Anyway, the spectrum in Table 1 indicates only the masses of the 14 scalars left upon the truncation, and therefore, for a full description of the $N = 8$ solutions, we still need to understand the completion of the mass spectrum (also because additional unstable directions might appear).

As a second step, we may look for critical points with nonvanishing imaginary part. The simplest possible vacuum solutions (actually traceable with the minimum effort) are those satisfying $s = t = u$ and $\sigma = \tau_i \equiv \tau = \nu_i \equiv \nu \neq 0$: such vacua are indeed present and summarized in Table 2. Besides, we are also forced to assume $s = t = u \neq 0$, since the Kähler potential (3.29) is defined just for strictly positive real parts.

#	G	G_{res}	SUSY	Λ	$m^2_{(\text{multiplicity})}$
ix	$\text{SO}(8)$	$\text{SO}(7)$	No	$-5^{3/4}$	$2_{(1)}, -\frac{4}{5}_{(6)}, -\frac{2}{5}_{(7)}$
x	$\text{SO}(8)$	G_2	Yes	$-\frac{108}{25} \sqrt{\frac{2}{5}} \sqrt[4]{3}$	$\frac{4 \pm \sqrt{6}}{3}, -\frac{11 + \sqrt{6}}{18}_{(6)}, -\frac{-11 + \sqrt{6}}{18}_{(6)}$
xi	$\text{SO}(7, 1)$			$-\frac{12}{25} \sqrt{\frac{1}{5}} (62\sqrt{6} - 117)$	
xii	$\text{SO}(7, 1)$	$\text{SO}(7)$	No	$-2\sqrt{2\sqrt{3} - 3}$	$2_{(2)}, -\frac{1}{3}_{(12)}$

Table 2: Vacua of the truncated theory with magnetic duals satisfying $s = t = u$ and $\sigma = \tau = \nu \neq 0$. When $\Lambda \neq 0$, the scalar masses are expressed in units of the cosmological constant.

All new vacua are AdS and the ones preserving supersymmetry have, as residual symmetry group, a G_2 subgroup of $\text{SO}(7)$. It is interesting to note that vacuum ix has the same features as ii (even the same mass spectrum), but thanks to the different cosmological constant we can certainly conclude that ii and ix do not coincide. Just ix does not respect the Breitenlohner–Freedman bound (as ii, after all) and hence, at least from the $N = 1$ point of view, x, xi, xii are stable.

Despite its simplicity, this analysis has reproduced most of all simplest vacua for the $N = 8$ landscape, even though the mass spectrum we provided is only partial.

Let us now turn to the $SO(p, q)$ gauged theories without magnetic duals, setting therefore $W(\phi_i) = W_1(\phi_i)$. Repeating the two previous steps also with this superpotential, we obtain respectively Tables 3 and 4.

#	G	G_{res}	SUSY	Λ	$m^2_{(\text{multiplicity})}$
i' \leftrightarrow i	SO(8)	SO(8)	Yes	$-\frac{3}{2}$	$-\frac{2}{3}_{(14)}$
ii \leftrightarrow ix	SO(8)	SO(7)	No	$-\frac{5^{3/4}}{2}$	$2_{(1)}, -\frac{4}{5}_{(6)}, -\frac{2}{5}_{(7)}$
iii' \leftrightarrow vii	SO(5, 3)	SO(3) \times SO(5)	No	$\frac{4\sqrt{3}}{2}$	$4_{(2)}, -2_{(2)}, 2_{(5)}, \frac{4}{3}_{(4)}, -\frac{2}{3}$
iv' \leftrightarrow viii	SO(4, 4)	SO(4) \times SO(4)	No	$\frac{1}{2}$	$-2, 2_{(9)}, 1_{(4)}$

Table 3: Vacua of the truncated theory without magnetic duals satisfying $\sigma = \tau_i = \nu_i = 0$ and $t_1 = t_2 = t_3, u_1 = u_2 = u_3$.

#	G	G_{res}	SUSY	Λ	$m^2_{(\text{multiplicity})}$
v' \leftrightarrow x	SO(8)	G(2)	Yes	$-\frac{54}{25}\sqrt{\frac{2}{5}}\sqrt[4]{3}$	$\frac{4\pm\sqrt{6}}{3}, -\frac{11+\sqrt{6}}{18}_{(6)}, -\frac{-11+\sqrt{6}}{18}_{(6)}$
vi' \leftrightarrow ii	SO(8)	SO(7)	No	$-\frac{25\sqrt{5}}{32}$	$2_{(1)}, -\frac{4}{5}_{(6)}, -\frac{2}{5}_{(7)}$

Table 4: Vacua of the truncated theory without magnetic duals satisfying $s = t = u$ and $\sigma = \tau = \nu \neq 0$.

Interestingly, in absence of magnetic duals the number of vacua is drastically reduced. In contrast with the previous case, for example, there is no vacuum satisfying $\sigma = \tau_i = \nu_i = 0$ or $(s = t = u$ and $\sigma = \tau = \nu)$ for the SO(7, 1) and SO(6, 2) gaugings. Moreover, also the original SO(8) vacuum iv with residual symmetry group SO(6) disappears. All other vacua instead remain, included the maximally supersymmetric ground state of the SO(8) theory. In particular, by simply comparing the scalar square masses and the values of the cosmological constant, we can deduce the correspondence between the vacua in presence and absence of magnetic duals, reported in the first column of Tables 3 and 4.

For all the vacua found in absence of magnetic duals (Tables 3 and 4) the uplift to eleven dimensions has been recently determined: for instance, the uplift of the SO(5, 3) and SO(4, 4) solutions is described in [37], where explicit ansatzs for the 11-dimensional graviton and the three-form are provided. However, as we have seen, none of such vacua is stable and exhibits a positive cosmological constant at the same time. This difficulty of reproducing stable dS vacua derives from the particular superpotential (4.2-4.3) we analyzed, and can be actually solved considering generalizations of the superpotential where the coefficients are no longer ± 1 , but arbitrary. We will study such more general STU-models in the following chapter, trying to not ruin, on the other hand, the vacua upliftability.

5 Searching Minkowski and de Sitter vacua

The analysis in the previous chapter revealed that stable dS vacua are hardly accessible from the STU-truncations of SO(8) gauged supergravity. A possible remedy for this issue is to forget about the SO(8) origin of the STU-models and to allow for completely generic polynomial superpotentials, hoping that, for some values of the coefficients, dS stable vacua indeed arise. Although this search might seem rather straightforward from the $N = 1$ point of view, vacua of too generic STU-models actually do not have, at the present time, an interpretation in terms of maximal supergravity and of M-theory, and therefore, despite their presence in the $N = 1$ model, cannot be uplifted. Hence, the research of a compromise between stability and upliftability is needed and will be the actual scope of this last chapter.

In particular, we first study the stability of dS vacua in a general STU-model, without taking care about the possible uplift to eleven dimensions. In this context, inspired by the line of investigation opened in [38, 39, 42], we provide a systematic method for constructing stable dS solutions close to Minkowski vacua and show in explicit examples that wide analytic classes of such vacua indeed exist. In contrast with [42], we entirely base our search on polynomial superpotentials induced by perturbative effects, proving therefore that nonperturbative contributions do not bring anything new. We then constrain the superpotential to a very special class of complex polynomials (whose uplift is actually clear nowadays) and repeat the same analysis, looking for explicit realizations of stable vacua close to Minkowski solutions. We will see firsthand that, in this case, stability is much less likely to emerge and furthermore stable dS vacua seem to be completely absent.

5.1 General strategy

In this section we outline our general procedure for identifying the critical points of the scalar potential (2.10) in an STU-model with a generic polynomial superpotential (up to the seventh degree). Solving the extremality condition for arbitrary coefficients is in fact absolutely impracticable: as we will see, however, we can combine two crucial observations that transform the extremality conditions into a system of quadratic equations in *some* of the superpotential couplings. Fortunately, these considerations together drastically simplify the search of stable vacua.

- The first fact we use holds for any supergravity model in which the scalars span an homogeneous space, just as $(\text{SU}(1,1)/\text{U}(1))^7$, and has been proposed for the first time in [40] and applied in [41] and in [35, 43] for the $N = 4$ and $N = 8$ landscapes respectively. In the maximal gauged theory, the underlying idea is based on the fact that the embedding tensor Θ_M^α determines the scalar potential, which depends nonlinearly on the 70 scalar fields, but only quadratically on Θ_M^α itself. Besides, since the scalar manifold is the coset space $\text{E}_{7(7)}/\text{SU}(8)$, any point can be mapped to a chosen base point by an $\text{E}_{7(7)}$ duality transformation. Therefore, by a proper duality transformation acting on both the scalar fields and the embedding tensor, we can map any critical point to the base point of the scalar manifold, however changing the explicit form of the embedding tensor at the same time (in such a way that the scalar potential is eventually left invariant). This means that to find *all* vacua of the theory we can just solve

the critical point conditions at the base point in terms of the embedding tensor values allowed for the chosen gauging. The advantage of this procedure is that we reduced the problem to a system of quadratic equations in the constants Θ_M^α , much simpler than minimizing a complicated nonlinear function of 70 real variables.

Thanks to the structure of coset space of the scalar manifold, this formulation is equally valid for the STU-models, with the only difference that the duality transformations belong to the $SU(1,1)^7$ duality group and that the role of Θ_M^α is played by the coefficients of the superpotential, which, analogously, enter in the scalar potential quadratically. Therefore, in order to find all vacua of a certain STU-model, we can just restrict to the vacuum at the origin $S = T_i = U_i = 1$ keeping the superpotential coefficients as general as possible. In fact, all other vacua are obtainable starting from the vacuum at the origin of STU-models with different (and typically more complicated) coefficients, and then performing suitable $SU(1,1)^7$ duality transformations, which take the coefficients to the original values and, at the same time, the vacua far from the origin. Again, this method converts the extremality condition into a system of quadratic equations in the superpotential couplings, whose solution is much more accessible. However, for nonperturbatively induced superpotentials such a search of solutions usually implies a loss of generality.

- The second relevant fact we make use of is that not all the superpotential couplings enter in the determination of the vacuum at the origin and of its properties. In fact, let us define the fields $\Phi^\alpha = (S - 1, T_i - 1, U_i - 1)$, which at the origin obviously assume the value $\Phi^\alpha = 0$. The superpotential can be conveniently expanded in terms of Φ^α around the origin as

$$W(\Phi^\alpha) = \tilde{W}_0 + \tilde{W}_\alpha \Phi^\alpha + \frac{1}{2!} \tilde{W}_{\alpha\beta} \Phi^\alpha \Phi^\beta + \frac{1}{3!} \tilde{W}_{\alpha\beta\gamma} \Phi^\alpha \Phi^\beta \Phi^\gamma + \dots, \quad (5.1)$$

where $\tilde{W}_0, \tilde{W}_\alpha, \tilde{W}_{\alpha\beta}$ and $\tilde{W}_{\alpha\beta\gamma}$ are symmetric complex coefficients and the dots represent additional contributions of the fourth, fifth, sixth and seventh degree in Φ^α . Since in the scalar potential (2.10) at most the first derivatives of $W(\Phi^\alpha)$ appear, the critical point condition at the origin (determined by the first derivatives $\partial_\alpha V|_{\Phi=0}$ and $\partial_{\bar{\alpha}} V|_{\Phi=0}$) depends at most on the second derivatives of $W(\Phi^\alpha)$ calculated for $\Phi^\alpha = 0$. Hence, $\tilde{W}_0, \tilde{W}_\alpha$ and $\tilde{W}_{\alpha\beta}$ are the only parameters that establish the stationarity of the origin. Analogously, its stability, determined by the second derivatives $\partial_\alpha \partial_\beta V|_{\Phi=0}$ and $\partial_\alpha \partial_{\bar{\beta}} V|_{\Phi=0}$, depends at most on the third derivatives of $W(\Phi^\alpha)$ calculated for $\Phi^\alpha = 0$, and so just on $\tilde{W}_0, \tilde{W}_\alpha, \tilde{W}_{\alpha\beta}$ and $\tilde{W}_{\alpha\beta\gamma}$. As a consequence, all higher-degree terms in the dots affect neither the stationarity nor the stability of the origin (nor the cosmological constant $\Lambda = V|_{\Phi=0}$, fixed by \tilde{W}_0 and \tilde{W}_α): therefore, for our purposes, they can be set to zero. This entails a considerable simplification, since we reduced the huge number of original free parameters to the complex coefficients of a cubic polynomial. However, the superpotential $W(\Phi^\alpha)$ is a generalization of the superpotentials (4.2-4.3) as long as every term in $W(\Phi^\alpha)$ includes at most one power of each field Φ^α : this constraint is essential to have any hope of finding a correspondence with the $N = 8$ theory. So, it must be also $\tilde{W}_{\alpha\alpha} = \tilde{W}_{\alpha\alpha\beta} = 0$.

Thanks to the two above observations, the research of stable dS vacua in an STU-model exactly coincides with the determination of feasible values for the parameters $\tilde{W}_0, \tilde{W}_\alpha, \tilde{W}_{\alpha\beta}$ and $\tilde{W}_{\alpha\beta\gamma}$ which make the origin a stable critical point with $V|_{\Phi=0} > 0$. The number of free parameters is anyhow too large to proceed immediately with an analytic search, and therefore we subdivide our study into two stages.

- (i) As a first step, we restrict the research to the isotropic sector, namely we constrain the 14-dimensional space (S, T_i, U_i) on the subspace $T_i = T$ and $U_i = U$, in such a way that the resulting theory is an $N = 1$ model with 3 chiral multiplets for which the Kähler potential (3.29) takes the form

$$K(\phi^\alpha) = -\log(S + \bar{S}) - 3\log(T + \bar{T}) - 3\log(U + \bar{U}) . \quad (5.2)$$

The superpotential deriving from such a constraint can be obtained setting $T_i = T$ and $U_i = U$ in (5.1), and therefore can be written in terms of the shifted fields $\Phi^\alpha = (S - 1, T - 1, U - 1)$ as

$$W(\Phi^\alpha) = W_0 + W_\alpha \Phi^\alpha + \frac{1}{2!} W_{\alpha\beta} \Phi^\alpha \Phi^\beta + \frac{1}{3!} W_{\alpha\beta\gamma} \Phi^\alpha \Phi^\beta \Phi^\gamma + \dots , \quad (5.3)$$

where, again, the terms into the dots do not affect the determination of the vacua and can be put to zero. The new coefficients $W_0, W_\alpha, W_{\alpha\beta}$ and $W_{\alpha\beta\gamma}$ of the isotropic superpotential are of course connected to the *isotropic* versions of the old coefficients $\tilde{W}_0, \tilde{W}_\alpha, \tilde{W}_{\alpha\beta}$ and $\tilde{W}_{\alpha\beta\gamma}$. In fact, by expanding (5.1) with the isotropic constraint, we easily figure out that

$$W_0 = \tilde{W}_0 , \quad W_\alpha = n_\alpha \tilde{W}_\alpha , \quad W_{\alpha\beta} = n_\alpha n_\beta \tilde{W}_{\alpha\beta} , \quad W_{\alpha\beta\gamma} = n_\alpha n_\beta n_\gamma \tilde{W}_{\alpha\beta\gamma} , \quad (5.4)$$

where $n_S = 1$ and $n_T = n_U = 3$. Moreover, the requirement $\tilde{W}_{\alpha\alpha} = \tilde{W}_{\alpha\alpha\beta} = 0$ translates into $W_{SS} = W_{SS\alpha} = 0$, but since there are three T -type and U -type fields no additional restriction is requested for $W_{TT}, W_{TT\alpha}$ and $W_{UU}, W_{UU\alpha}$. The advantage of first studying the isotropic model descending from such identifications relies on the relatively small number of free parameters. In fact, W_0 and W_α lead to $1 + 3$ complex coefficients, while, thanks to the symmetry of the indices, $W_{\alpha\beta}$ and $W_{\alpha\beta\gamma}$ include respectively just $\frac{3 \cdot 4}{2!} - 1 = 5$ and $\frac{3 \cdot 4 \cdot 5}{3!} - 3 = 7$ independent parameters (1 and 3 are subtracted given that W_{SS} and $W_{SS\alpha}$ must vanish). So, as long as the isotropic model is concerned, the vacuum at the origin is determined by 16 complex parameters in total, and hence the analytic search is indeed accessible.

- (ii) However, due to the truncation we have done, in general not all the values of $W_0, W_\alpha, W_{\alpha\beta}$ and $W_{\alpha\beta\gamma}$ that make the origin a stable vacuum in the isotropic model are related by (5.4) to values of $\tilde{W}_0, \tilde{W}_\alpha, \tilde{W}_{\alpha\beta}$ and $\tilde{W}_{\alpha\beta\gamma}$ that produce a stable origin in the full model (instead, the inverse obviously holds). For this reason, once we have found proper stability conditions for the isotropic model, as a second step we need to check whether at least a subset of them actually persist also in the full one. In any case, this analysis eventually determines only *isotropic* values of $\tilde{W}_0, \tilde{W}_\alpha, \tilde{W}_{\alpha\beta}$ and $\tilde{W}_{\alpha\beta\gamma}$, and so more general solutions for which, for instance, $\tilde{W}_{T_i} \neq \tilde{W}_{T_j}$ for $i \neq j$, are automatically ruled out, although they probably exist.

Inspired by the line of research of [38, 42], in the next section we apply the above steps according to the following general strategy. First of all, we find the most general expression for Minkowski supersymmetric and Minkowski no-scale vacua in terms of the parameters $W_0, W_\alpha, W_{\alpha\beta}$ and $W_{\alpha\beta\gamma}$ of the isotropic model. We then deform these conditions introducing two small perturbation parameters ϵ and λ , representing respectively the supersymmetry breaking scale and the cosmological constant, and look for regions in the plane (ϵ, λ) in which the perturbed coefficients $W_0(\epsilon, \lambda), W_\alpha(\epsilon, \lambda), W_{\alpha\beta}(\epsilon, \lambda)$ and $W_{\alpha\beta\gamma}(\epsilon, \lambda)$ generate a stable dS origin. Finally, we check whether, for any subregion, the corresponding parameters $\tilde{W}_0(\epsilon, \lambda), \tilde{W}_\alpha(\epsilon, \lambda), \tilde{W}_{\alpha\beta}(\epsilon, \lambda)$ and $\tilde{W}_{\alpha\beta\gamma}(\epsilon, \lambda)$ defined by (5.4) induce a stable dS origin also in the full model.

This strategy is applied in two cases. In the following section we consider a general superpotential of the form (5.3) with the only necessary constraint $W_{SS} = W_{SS\alpha} = 0$, depending therefore on 16 complex parameters. Instead, in the next section we further demand that the superpotential belongs to a very special class of complex polynomials, which ensures that the possible vacua have an interpretation in terms of maximal supergravity and of M-theory.

5.2 A systematic procedure for building dS vacua

In this section we present in detail our general method for establishing the conditions for stable dS vacua in our STU-models. As anticipated, we start from the isotropic truncation defined by the Kähler potential (5.2) and the superpotential (5.3), depending in total on 16 free complex parameters.

In place of requiring the stationarity condition and the positiveness of Λ and randomly choosing the other parameters hoping that stable solutions arise, a good first step towards our goal may be the one of determining the values of $W_0, W_\alpha, W_{\alpha\beta}$ and $W_{\alpha\beta\gamma}$ that produce Minkowski vacua. In fact, supersymmetric Minkowski vacua are relatively easy to find and, since the Kähler potential fulfills the no-scale requirement (2.19), also no-scale vacua are potentially present. Moreover, as we saw, both supersymmetric and no-scale Minkowski vacua are automatically minima of the scalar potential, and so are (marginally) stable: it is therefore reasonable to expect that, sufficiently closely to these vacua, stable dS solutions indeed emerge. In particular, no-scale vacua especially trigger our interest since they already break supersymmetry, as any dS vacuum should do.

5.2.1 General Minkowski solutions

Let us describe the general conditions for a Minkowski vacuum in terms of W derivatives, both in the supersymmetric and in the no-scale case.

- (i) A supersymmetric Minkowski origin has to satisfy (2.15), and therefore is completely identified by the choice

$$W_0 = W_\alpha = 0, \tag{5.5}$$

whereas all the other 12 complex higher derivatives $W_{\alpha\beta}, W_{\alpha\beta\gamma}$ stay completely arbitrary. The above choice already implies that $\partial_\alpha V|_{\Phi=0}$ and $\partial_{\bar{\alpha}} V|_{\Phi=0}$ vanish,

and so the origin is automatically a stationary point for any value of the additional parameters. Moreover, as we proved, the condition (5.5) guarantees the semi-positiveness of the Hessian matrix and, hence, marginal stability.

- (ii) On the other hand, a no-scale origin enjoys (2.21) with $\phi^T = T$, which implies $W_0 \neq 0$, $W_S = \frac{1}{2}W_0$, $W_T = 0$, $W_U = \frac{3}{2}W_0$. In contrast with the supersymmetric case, however, these conditions do *not* automatically ensure stationarity. Thus, we need to explicitly impose the constraint $\partial_\alpha V|_{\Phi=0} = \partial_{\bar{\alpha}} V|_{\Phi=0} = 0$, which, taking into account the above no-scale condition, translates into the additional 6 real conditions $W_{ST} = W_{TU} = W_{TT} = 0$. As a consequence, a no-scale vacuum at the origin is determined by

$$\begin{cases} W_0 \neq 0, & W_S = \frac{1}{2}W_0, & W_U = \frac{3}{2}W_0 \\ W_T = W_{ST} = W_{TU} = W_{TT} = 0, \end{cases} \quad (5.6)$$

and, as we said, is always (marginally) stable for any choice of the remaining 9 complex parameters. Although we have selected T as the supersymmetry breaking field, also U could have been a good candidate, thanks to the structure of the Kähler potential. Instead, the no-scale requirement (5.5) for the field S is incompatible with the stationarity condition, and thus no Minkowski solution breaking supersymmetry just along S exists (this is essentially due to the absence of the factor 3 in front of $\log(S + \bar{S})$ in (5.2), which prevents the possibility that $\partial_\alpha V|_{\Phi=0} = 0$ unless $W_0 = 0$).

5.2.2 Perturbing Minkowski vacua

In order to find stable dS vacua, we now need to construct a consistent ansatz to move away from the supersymmetric and no-scale Minkowski solutions (5.5-5.6). We therefore introduce two real perturbation parameters, ϵ and λ : the former is supposed to represent the scale of supersymmetry breaking (dictated by the size of the F-terms), while the latter can be used to regulate the value cosmological constant once the former is fixed. We then have to properly define perturbed coefficients $W_0(\epsilon, \lambda)$, $W_\alpha(\epsilon, \lambda)$, $W_{\alpha\beta}(\epsilon, \lambda)$, $W_{\alpha\beta\gamma}(\epsilon, \lambda)$ that reduce to (5.5) or (5.6) in the limit $\epsilon, \lambda \rightarrow 0$, and that hopefully determine a stable dS origin for some values of ϵ and λ . Such coefficients can be systematically constructed as follows.

- (a) First, we arbitrarily fix the functions $W_0(\epsilon, \lambda)$ and $W_\alpha(\epsilon, \lambda)$, with the only requirement that, in the limit $\epsilon, \lambda \rightarrow 0$, they reproduce either supersymmetric or no-scale Minkowski solutions. Alternatively, in place of directly fixing $W_\alpha(\epsilon, \lambda)$, we can also define perturbed F-terms $F_\alpha|_{\Phi=0}(\epsilon, \lambda) = W_\alpha(\epsilon, \lambda) + K_\alpha|_{\Phi=0}W_0(\epsilon, \lambda)$, which must tend either to the Minkowski supersymmetric or to the no-scale F-terms when $\epsilon, \lambda \rightarrow 0$. By solving the above equation, we can indeed determine $W_\alpha(\epsilon, \lambda)$, which automatically reduces to (5.5) or (5.6) in the Minkowski limit. This first step completely establishes 4 complex parameters.
- (b) In order to fulfill the critical point condition, we then impose the 3 complex equations $V_\alpha \equiv \partial_\alpha V|_{\Phi=0} = 0$, which, by hermitian conjugation, immediately imply $\partial_{\bar{\alpha}} V|_{\Phi=0} = 0$. Given that the 5 complex parameters $W_{\alpha\beta}$ enter linearly in V_α , these equations are easy to solve by fixing 3 of the $W_{\alpha\beta}$, for example

W_{ST}, W_{TT} and W_{TU} , in terms of the other 2 and of ϵ and λ . For consistency, in particular, in the no-scale case we should recover, for $\epsilon, \lambda \rightarrow 0$, the conditions on W_{ST}, W_{TT} and W_{TU} in (5.6), actually deriving from stationarity as well. On the contrary, the parameters W_{SU} and W_{UU} stay completely arbitrary.

- (c) In order to significantly simplify the problem of getting all positive eigenvalues for the scalar mass matrix

$$m^2 = g^{-1} \partial^2 V = \begin{pmatrix} K^{\alpha\bar{\gamma}} \partial_{\bar{\gamma}} \partial_{\beta} V & K^{\alpha\bar{\gamma}} \partial_{\bar{\gamma}} \partial_{\bar{\beta}} V \\ K^{\bar{\alpha}\gamma} \partial_{\gamma} \partial_{\beta} V & K^{\bar{\alpha}\gamma} \partial_{\gamma} \partial_{\bar{\beta}} V \end{pmatrix} \quad (5.7)$$

and hence construct proper minima of the potential, it might be useful to observe that m^2 becomes block diagonal at the origin upon imposing $V_{\alpha\beta} \equiv \partial_{\alpha} \partial_{\beta} V|_{\Phi=0} = 0$, which consist in 5 independent complex conditions, given that V_{SS} automatically vanishes due to $W_{SS} = W_{SS\alpha} = 0$. Since in the second derivatives of V the 7 coefficients $W_{\alpha\beta\gamma}$ appear linearly, it is straightforward to solve $V_{\alpha\beta} = 0$ in terms of 5 of the $W_{\alpha\beta\gamma}$, for example $W_{STT}, W_{STU}, W_{TTT}, W_{TTU}, W_{TUU}$, in such a way that they can be expressed in terms of ϵ and λ and of $W_{SU}, W_{UU}, W_{SUU}, W_{UUU}$. Therefore, with the assumption that (5.7) is block diagonal, the whole superpotential is fixed in terms of the perturbation parameters and of $W_{SU}, W_{UU}, W_{SUU}, W_{UUU}$, which are the only coefficients left.

However, in general, it is not necessary to enforce the block diagonality condition, because stable dS vacua actually happen to arise even if $V_{\alpha\beta}$ is different from zero. Nevertheless, stability is much more likely to emerge if the off-diagonal blocks vanish, and this can be easily understood taking into account that the off-diagonal blocks are one the hermitian conjugate of the other, and thus they provide a negative definite contribution to the determinant of m^2 . Moreover, it is obvious that such an assumption implies a pairwise organization of the mass spectrum, which will possess at most 3 distinct eigenvalues.

- (d) Steps (a), (b) and (c) determined a stationary deformation of the initial Minkowski origin, which can be either dS or AdS (and either stable or unstable) depending on the values remaining parameters. As a final step, we plot the positiveness of the eigenvalues of (5.7) as well as the positiveness of $\Lambda = V|_{\Phi=0}$ in the plane (ϵ, λ) , and we look for feasible values of $W_{SU}, W_{UU}, W_{SUU}, W_{UUU}$ that produce, in the plane (ϵ, λ) , regions where an overlap between stability and positiveness of Λ indeed arises. For such values of $W_{SU}, W_{UU}, W_{SUU}, W_{UUU}$ the superpotential is completely fixed in terms of ϵ and λ and the theory admits a class of stable dS vacua.

Once proper values for the parameters $W_0(\epsilon, \lambda), W_{\alpha}(\epsilon, \lambda), W_{\alpha\beta}(\epsilon, \lambda), W_{\alpha\beta\gamma}(\epsilon, \lambda)$ have been selected, we then must check whether the corresponding parameters $\tilde{W}_0(\epsilon, \lambda), \tilde{W}_{\alpha}(\epsilon, \lambda), \tilde{W}_{\alpha\beta}(\epsilon, \lambda), \tilde{W}_{\alpha\beta\gamma}(\epsilon, \lambda)$ defined by (5.4) produce a (marginally) stable dS origin also in the full STU-model. Actually this is not obvious at all, since 8 of the 14 direction have been truncated and some of them may display negative eigenvalues. In fact in many explicit computations where the block-diagonal condition was either fulfilled or not, we found that, although dS vacua were present in the isotropic model, none of them remained stable also in the full one.

5.2.3 Three relevant examples

We now present two concrete analytic examples of stable dS solutions obtained by utilizing the technical machinery outlined in the previous subsection: the former will concern stable vacua close to Minkowski supersymmetric solutions, while the latter to no-scale. In order to facilitate the production of stable vacua, both examples display a block diagonal mass matrix. We then propose a third explicit realization of stable dS vacua without enforcing block diagonality: however, the off-diagonal blocks will be required to be smaller than the diagonal ones.

- (1) In order to construct a simple instance of vacua close to Minkowski supersymmetric solutions, we select the coefficients $W_0(\epsilon, \lambda)$ and $W_\alpha(\epsilon, \lambda)$ according to the trivial prescription

$$W_0 = 2i\epsilon, \quad (F_S, F_T, F_U)|_{\Phi=0} = (\epsilon, \lambda, 0), \quad (5.8)$$

which leads to $(W_S, W_T, W_U) = ((1+i)\epsilon, \lambda + 3i\epsilon, 3i\epsilon)$. Obviously, this choice is consistent with a Minkowski supersymmetric vacuum (5.5) in the limit $\epsilon, \lambda \rightarrow 0$ and induces the cosmological constant

$$\Lambda = V|_{\Phi=0} = \frac{1}{96}(-6\epsilon^2 + \lambda^2), \quad (5.9)$$

which is positive for $|\lambda| > \sqrt{6}|\epsilon|$. Following the step (b), we then have to impose the 3 complex stationarity conditions $V_\alpha = 0$, which can be solved by fixing the 3 parameters

$$\begin{aligned} W_{ST} &= \frac{\lambda^2 - 6i\epsilon^2 + (3+3i)\lambda\epsilon}{2\lambda}, & W_{TT} &= 2\lambda + \frac{9i\epsilon^3}{\lambda^2}, \\ W_{TU} &= \frac{3(\lambda^2 - 2W_{SU}\epsilon + (3+3i)\epsilon^2 + 3i\lambda\epsilon)}{2\lambda}, \end{aligned} \quad (5.10)$$

as functions of ϵ, λ and W_{SU}, W_{UU} . Furthermore, the block diagonality of the mass matrix, namely $V_{\alpha\beta} = 0$, determines $W_{STT}, W_{STU}, W_{TTT}, W_{TTU}, W_{TUU}$ in terms of ϵ, λ and $W_{SU}, W_{UU}, W_{SUU}, W_{UUU}$, and their explicit expressions are

$$\begin{aligned} W_{STT} &= \lambda - \frac{9(1-i)\epsilon^3}{2\lambda^2} - \frac{6i\epsilon^2}{\lambda} + \frac{3\epsilon}{2}, \\ W_{STU} &= \frac{3(\lambda^2 + 2W_{SU}(\lambda - (1+i)\epsilon))}{4\lambda}, \\ W_{TTT} &= \frac{3(\lambda^4 + 18\epsilon^4 + 9i\lambda\epsilon^3 - 3\lambda^2\epsilon^2 - 2i\lambda^3\epsilon)}{2\lambda^3}, \\ W_{TTU} &= \frac{3(2\lambda^3 + 2W_{SU}\epsilon(-2\lambda + 3i\epsilon) + 9\epsilon^3 + 6(1+i)\lambda\epsilon^2)}{2\lambda^2}, \\ W_{TUU} &= \frac{3(\lambda^2 - 2W_{SUU}\epsilon + W_{UU}(\lambda + (1-i)\epsilon))}{2\lambda}. \end{aligned} \quad (5.11)$$

Let us now study the stability of the origin. Interestingly, thanks to the particularly simple choice (5.8), the positiveness of the mass matrix turns out to be *independent* of the residual parameters $W_{SU}, W_{UU}, W_{SUU}, W_{UUU}$, which therefore can be just set to zero for simplicity. However, this independence does not occur in general,

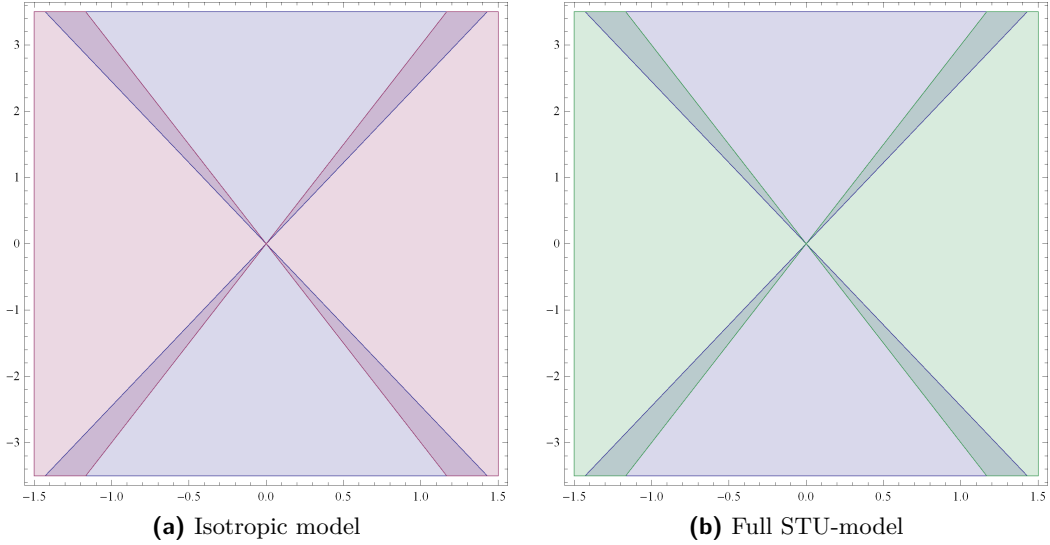


Figure 2: Stability of the origin as a function of the parameters ϵ and λ for the nearly-Minkowski supersymmetric vacuum. Blue represents $\Lambda > 0$, purple represents stability for the isotropic sector, while green represents (marginal) stability in the full model.

and is actually strictly related to the form (5.8) of the F-terms. Since the Hessian matrix is block diagonal and the upper-left block is the conjugate of the down-right, we can analyze just the former, $\partial_\alpha \partial_{\bar{\beta}} V$, which reads

$$\begin{pmatrix} \frac{36\epsilon^4 + 6\lambda^2\epsilon^2 + \lambda^4}{384\lambda^2} & -\frac{\epsilon(36\epsilon^4 + 6\lambda^2\epsilon^2 + \lambda^4)}{128\lambda^3} & -\frac{3(1+i)\epsilon^2(3\epsilon^2 + i\lambda^2)}{64\lambda^2} \\ -\frac{\epsilon(36\epsilon^4 + 6\lambda^2\epsilon^2 + \lambda^4)}{128\lambda^3} & \frac{324\epsilon^6 + 54\lambda^2\epsilon^4 + 27\lambda^4\epsilon^2 - 2\lambda^6}{384\lambda^4} & \frac{9(1+i)\epsilon^3(3\epsilon^2 + i\lambda^2)}{64\lambda^3} \\ \frac{3(1+i)\epsilon^2(3i\epsilon^2 + \lambda^2)}{64\lambda^2} & \frac{9(1-i)\epsilon^3(3\epsilon^2 - i\lambda^2)}{64\lambda^3} & \frac{54\epsilon^4 + 27\lambda^2\epsilon^2 + \lambda^4}{128\lambda^2} \end{pmatrix}. \quad (5.12)$$

and whose positiveness, when $\epsilon, \lambda \neq 0$, depends only on the ratio λ/ϵ , given that $\frac{1}{\epsilon^2} \partial_\alpha \partial_{\bar{\beta}} V$ is a function of just λ/ϵ . In effect, by explicitly calculating the eigenvalues (or, alternatively, by utilizing the Sylvester's criterion) we deduce that (5.12) is positive definite if and only if $|\lambda| < 3|\epsilon|$, which is therefore the stability condition for the isotropic sector.

Combining the two conditions above, we conclude that strictly stable dS solutions are indeed present in the region $\sqrt{6}|\epsilon| < |\lambda| < 3|\epsilon|$ at least for the isotropic model, as represented in Figure 2(a). In particular, the plot can be seen as a polar plot, in which the origin corresponds to a supersymmetric Minkowski point and the polar angle $\arctan(\lambda/\epsilon)$ identifies solutions with the same cosmological constant and mass spectrum up to a multiplicative factor. For instance, along the direction defined by $\lambda/\epsilon = 5/2$ the cosmological constant is $\Lambda = \frac{1}{348}\epsilon^2$, while the approximate (halved) mass spectrum, given by the eigenvalues of $K^{\alpha\bar{\gamma}} \partial_{\bar{\gamma}} \partial_{\beta} V$, reads

$$(0.580 \epsilon^2, 0.140 \epsilon^2, 0.0124 \epsilon^2). \quad (5.13)$$

However, let us note that, although we can adjust ϵ to make the cosmological constant small at will, the ratio between Λ and the gravitino mass $e^K|_{\Phi=0}|W_0|^2 =$

$2^{-7}(2\epsilon)^2 = \epsilon^2/32$ is fixed and is only of the order of 10^{-1} , and therefore this example is not suitable for a realistic phenomenology, in which the gravitino mass should be much bigger than Λ .

Looking at the mass spectrum (5.13), and also at the Hessian matrix (5.12), it is straightforward to realize that the Minkowski origin obtained in the limit $\epsilon, \lambda \rightarrow 0$ has 6 flat directions. This is quite obvious since we have set $W_{SU}, W_{UU}, W_{SUU}, W_{UUU}$ to zero, and hence the superpotential vanishes identically in the limit $\epsilon, \lambda \rightarrow 0$. Of course, the vanishing of the eigenvalues does not occur in general. In fact, if we had chosen, for example, $W_{SU} = 1$ and $W_{UU} = W_{SUU} = W_{UUU} = 0$, the mass spectrum along the direction $\lambda/\epsilon = 5/2$ would have been a more complicated function of ϵ , which in the limit $\epsilon \rightarrow 0$ would have assumed the approximate form

$$\begin{aligned} & (0.0617 - 0.0551 \epsilon + 0.165 \epsilon^2 + \mathcal{O}(\epsilon^3)) , \\ & 0.0617 - 0.315 \epsilon + 5.827 \epsilon^2 + \mathcal{O}(\epsilon^3) , \\ & 0.0129 \epsilon^2 + \mathcal{O}(\epsilon^3) . \end{aligned} \tag{5.14}$$

In this case the superpotential does not vanish identically for $\epsilon, \lambda \rightarrow 0$, and actually the Minkowski origin has just 2 flat directions, corresponding to the 2 copies of the eigenvalue $0.0129\epsilon^2 + \mathcal{O}(\epsilon^3)$. This result is in accordance with a general no-go theorem, proven in [38], establishing that one can obtain stable vacua perturbing a supersymmetric Minkowski vacuum only if the original Minkowski vacuum exhibits at least 2 massless direction. Thus, also scanning all the values of $W_{SU}, W_{UU}, W_{SUU}, W_{UUU}$, there is no hope of finding strictly stable Minkowski vacua for $\epsilon, \lambda \rightarrow 0$.

Eventually, it is interesting to point out that the origin $(\epsilon, \lambda) = 0$ in the plot represents a different supersymmetric Minkowski solution for each direction along which one approaches it, namely the Minkowski point depends on the ratio λ/ϵ : in fact, choosing $\lambda/\epsilon \neq 5/2$, we would have gotten different values for the mass spectrum (5.14), which would not have approached to $(0.0617, 0.0617, 0)$ for $\epsilon \rightarrow 0$.

As a final step, we should check whether the found stability persists in the STU-model with parameters $\tilde{W}_0(\epsilon, \lambda)$, $\tilde{W}_\alpha(\epsilon, \lambda)$, $\tilde{W}_{\alpha\beta}(\epsilon, \lambda)$, $\tilde{W}_{\alpha\beta\gamma}(\epsilon, \lambda)$ defined by (5.4). First, it is convenient to note that, in place of calculating all these coefficients using (5.4), we can equivalently reconstruct the full superpotential (5.1) by just substituting $T \rightarrow \frac{1}{3}(T_1 + T_2 + T_3)$ and $U \rightarrow \frac{1}{3}(U_1 + U_2 + U_3)$ in the isotropic superpotential (5.3). Anyway, it can be shown that this isotropic choice of the coefficients in the full superpotential implies that the full Hessian matrix has *automatically* rank 10 in place of 14, and so 4 eigenvalues are zero and strict stability is ruled out. As a consequence, our method only allows for marginal stability and actually excluded strictly stable vacua from the beginning.

The determination of the full Hessian is straightforward, but it turns out that the off-diagonal blocks do not vanish, although the condition on $W_{\alpha\beta}(\epsilon, \lambda)$ made the isotropic Hessian block diagonal. Nonetheless, the computation of the positiveness can be done analytically (also making use of the Sylvester's criterion) and, remarkably, at least in this example the remaining 10 eigenvalues are positive in the same region as the isotropic truncation, namely if and only if $|\lambda| < 3|\epsilon|$, as depicted in Figure 2(b). Our procedure therefore led to a class of marginally stable solutions.

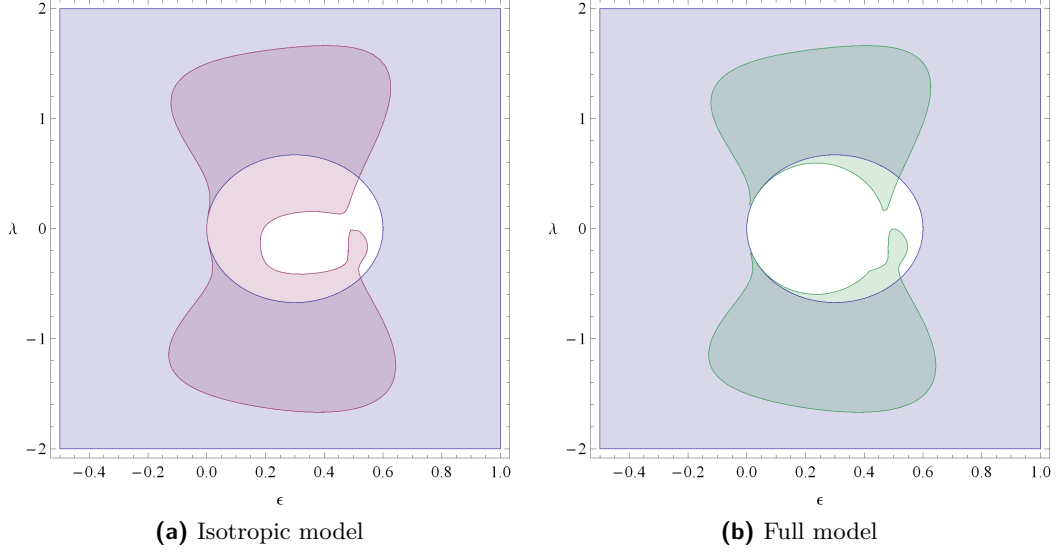


Figure 3: Stability of the origin as a function of the parameters ϵ and λ for a nearly-no-scale model. Blue represents $\Lambda > 0$, purple represents stability for the isotropic sector, while green represents (marginal) stability in the full model.

- (2) To show in a simple example that stable dS solutions can emerge also around no-scale vacua, it is sufficient to choose the following values for W_0 and F_α :

$$W_0 = 1, \quad (F_S, F_T, F_U)|_{\Phi=0} = (\epsilon, K_T|_{\Phi=0}W_0 + (1+i)\epsilon, \lambda). \quad (5.15)$$

The corresponding parameters, $(W_S, W_T, W_U) = (\epsilon + \frac{1}{2}, (1+i)\epsilon, \lambda + \frac{3}{2})$, are indeed coherent with the no-scale requirement (5.6) when $\epsilon, \lambda \rightarrow 0$, and induce the cosmological constant

$$\Lambda = V|_{\Phi=0} = \frac{1}{96} (\epsilon(-3 + 5\epsilon) + \lambda^2), \quad (5.16)$$

which is positive outside the ellipse $\epsilon(-3 + 5\epsilon) + \lambda^2 = 0$ in the plane (ϵ, λ) . Let us note that both the expressions (5.9) and (5.16) of Λ are quadratic in ϵ and λ , and not by chance: in fact $V|_{\Phi=0}$ depends at most quadratically on W_0 and W_α , which were chosen to be linear in ϵ and λ .

By imposing stationarity and block diagonality, we can find explicit expressions for the coefficients W_{ST}, W_{TT}, W_{TU} and $W_{STT}, W_{STU}, W_{TTT}, W_{TTU}, W_{TUU}$ in terms of the perturbation parameters and of $W_{SU}, W_{UU}, W_{SUU}, W_{UUU}$. However, due to the more complicated form (5.15) of the F-terms, we cannot repeat the same analytic study as before, and thus we are forced to investigate stability only numerically, i.e. plotting the positiveness of m^2 as a function of ϵ and λ for suitable choices of $W_{SU}, W_{UU}, W_{SUU}, W_{UUU}$. In particular, the values

$$W_{SU} = \epsilon + \frac{3}{4}, \quad W_{UU} = \lambda + \frac{3}{2}, \quad W_{SUU} = W_{UUU} = 0, \quad (5.17)$$

are compatible with stability and produce the pattern shown in Figure 3(a), for

which the superposition of stable regions (purple) and dS regions (blue) is clear. This means that stable dS vacua are of course present in the isotropic sector.

In [38, 41] similar explicit realizations of stable dS vacua close to a no-scale points were provided, but only either in presence of exponential contributions to the superpotential or of a Polonyi field. Anyhow, this example, and many other examples that can be constructed likewise, prove that generating nonperturbatively the superpotential (as well as introducing Polonyi fields) is unnecessary for producing stable dS vacua in the isotropic sector.

As we saw in the previous example, as long as the full STU-model is concerned, strict stability is no longer guaranteed, since the full mass matrix (which is not block diagonal anymore) acquires 4 vanishing eigenvalues deriving from the isotropic choice of the coefficients. In any case, as represented in Figure 3(b), the additional 10 eigenvalues are simultaneously positive in a subregion of the original stability region, meaning that this model admits at least marginally stable dS vacua. Interestingly, even though the width of the stability area decreases, it looks like that just AdS vacua are infected by instability⁴: this phenomenon systematically occurred in many explicit computations, but we cannot exclude that it is related to the particular choices of the parameters we have done.

- (3) As we anticipated at the beginning, block diagonality is not mandatory (though very useful), since we can easily construct also examples of stable dS vacua in which $V_{\alpha\beta} \neq 0$ for some α, β . This possibility was explicitly shown in [38, 41], but, again, only in presence of nonperturbative superpotentials or Polonyi fields. However, as we said, the off-diagonal block $V_{\alpha\beta}$ always takes a negative contribution to the determinant of the full matrix, and therefore, if $|V_{\alpha\beta}| > |V_{\bar{\alpha}\beta}|$, a positive determinant can arise only for some miraculous conspiracy. As a consequence, stable vacua are quite likely to emerge just for configurations in which the off-diagonal blocks are small with respect to the diagonal ones.

A simple example of stable dS vacua where block diagonality is not enforced can be obtained utilizing the same coefficients as in (2), except the ones that determined $V_{\alpha\beta} = 0$, namely $W_{STT}, W_{STU}, W_{TTT}, W_{TTU}, W_{TUU}$. These parameters can be fixed demanding instead that the off-diagonal block $V_{\alpha\beta}$ has a precise structure that makes $|V_{\alpha\beta}| \lesssim |V_{\bar{\alpha}\beta}|$ near the origin $(\epsilon, \lambda) = 0$, such as, for instance, the diagonal structure

$$V_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \epsilon^j & 0 \\ 0 & 0 & \epsilon^k \end{pmatrix}, \quad (5.18)$$

where the exponents j, k should be sufficiently big (note that, as we said, V_{SS} must automatically vanish as a direct consequence of $W_{SS} = W_{SS\alpha} = 0$). With this choice, it turns out that, if at least one between j and k is 1, no stable solution arises: this is highly reasonable, given that the diagonal blocks are of order ϵ^2 , exactly as in (5.12), and thus are smaller than $V_{\alpha\beta}$ near the origin. On the contrary, if $j, k \geq 2$, stable vacua begin to emerge and, as depicted in Figure 4 for $j = k = 2, 4, 6$, some of them also persist in the full STU-model up the usual 4 flat directions. In

⁴Here, the discussion of the stability of AdS vacua is linked to the positiveness of m^2 , and we did not check whether some of them however respect the Breitenlohner–Freedman bound.

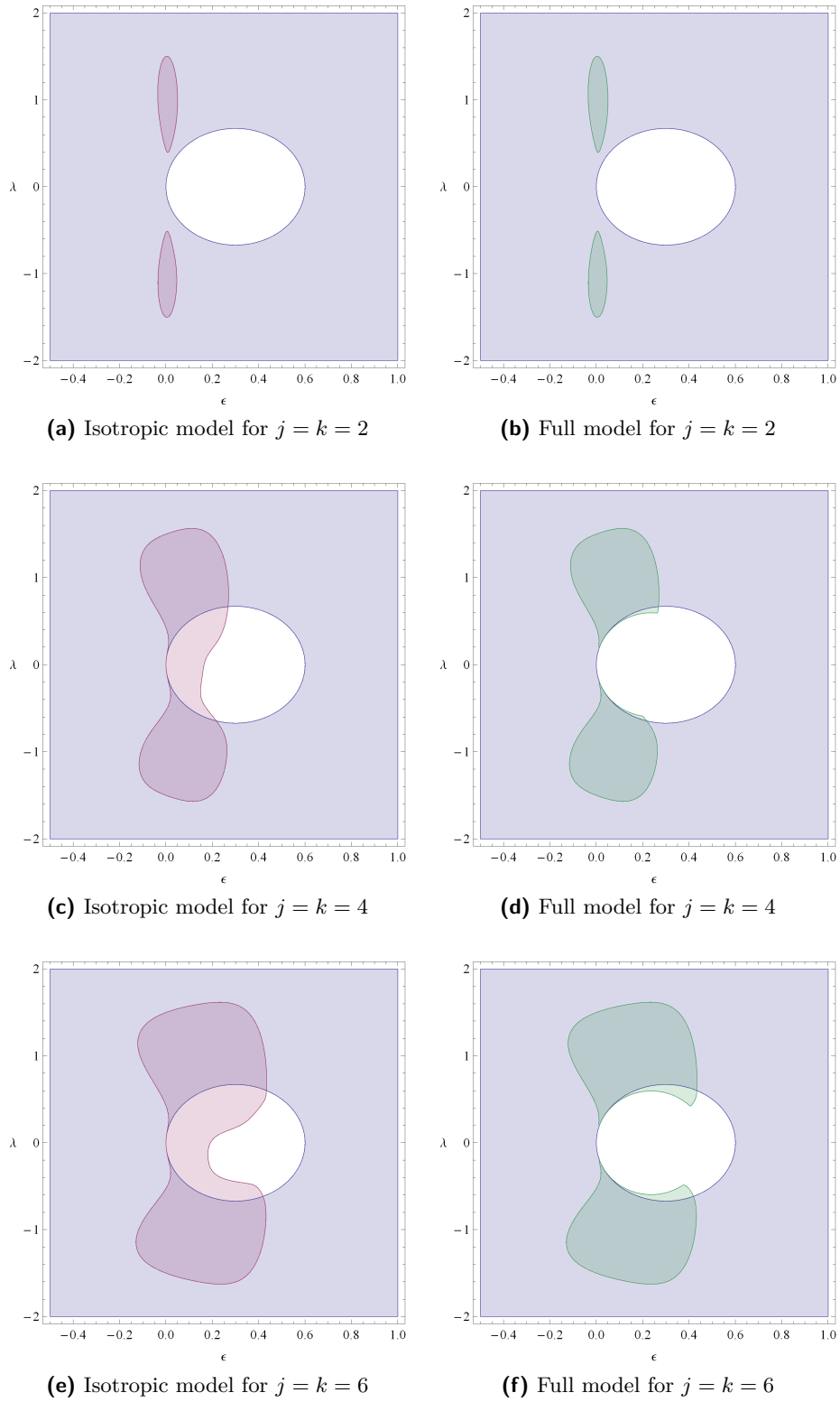


Figure 4: Stability of the origin as a function of the parameters ϵ and λ for a nearly-no-scale model with off-diagonal contributions. Blue represents $\Lambda > 0$, purple represents stability for the isotropic sector, while green represents (marginal) stability in the full model.

particular, the larger are j and k , the smaller are the off-diagonal contributions and, therefore, the more the stability region approaches to the one of previous example. Remarkably, also in this case just AdS vacua become unstable.

This strategy for constructing stable dS vacua in presence of off-diagonal contributions equally holds for much more complicated structures of $V_{\alpha\beta}$ and provides stable solutions as long as $|V_{\alpha\beta}|$ is at most of the order of the diagonal terms. Actually, the outlined method provides stable vacua much more easily (and systematically) than randomly choosing the values of $W_{STT}, W_{STU}, W_{TTT}, W_{TTU}, W_{TUU}$.

It is important to point out that off-diagonal contributions are interesting because they allow for a completely general mass spectrum, not necessarily pairwise organized. Moreover, we can even adjust the values of $V_{\alpha\beta}$ in order to regulate the splitting of those eigenvalues that were originally degenerate: however, too large splittings, dictated by too big off diagonal terms, eventually lead to instability.

The above examples have shown that many realizations of (marginally) stable dS vacua can be constructed with the minimum effort in general STU-models. However, whether it is actually possible to adjust the parameters in order to obtain phenomenologically viable examples with the desired hierarchy $\Lambda \ll m_{\text{grav}}^2 \ll m_{\text{scalars}}^2$ still remains to be seen. Besides, unfortunately, all the previous superpotentials are too generic to have a clear $N = 8$ interpretation and therefore none of the vacua we have found is upliftable.

5.3 Stable vacua from upliftable superpotentials

We now particularize the preceding discussion to superpotentials belonging to a very special class of complex polynomials, whose coefficients actually have an interpretation in terms of maximal supergravity as well as of M-theory flux compactifications. In the isotropic sector, these superpotentials display the form

$$\begin{aligned}
W(\phi^\alpha) = & W'_0 \\
& + i(W'_S S + W'_T T + W'_U U) \\
& + W'_{ST} ST + W'_{SU} SU + W'_{TT} T^2 + W'_{TU} TU + W'_{UU} U^2 \\
& + i(W'_{STT} ST^2 + W'_{STU} STU + W'_{SUU} SU^2 + W'_{TTT} T^3 \\
& \quad + W'_{TTU} T^2 U + W'_{TUU} TU^2 + W'_{UUU} U^3) ,
\end{aligned} \tag{5.19}$$

where the parameters $W'_0, W'_\alpha, W'_{\alpha\beta}, W'_{\alpha\beta\gamma}$ are no longer complex, but only *real*. As can be easily noted, $W(\phi^\alpha)$ is the most general polynomial of the third degree, with the restriction $W_{SS} = W_{SS\alpha} = 0$ and with real or purely imaginary coefficients for even- or odd-degree terms respectively. The constraint on the coefficients clearly halves the number of independent parameters, and in fact $W'_0, W'_\alpha, W'_{\alpha\beta}, W'_{\alpha\beta\gamma}$ are 16 real numbers in place of the 32 of the general superpotential (5.3). This restricted freedom on the superpotential makes the research of stable dS vacua extremely problematic, also because the general method described in the previous section (based on 32 free real parameters) cannot be applied anymore and must be properly adapted.

The first step for matching our general strategy and the particular superpotential (5.19) is translating the reality of $W'_0, W'_\alpha, W'_{\alpha\beta}, W'_{\alpha\beta\gamma}$ into 16 real conditions on the real and imaginary parts of the 16 complex parameters $W_0, W_\alpha, W_{\alpha\beta}, W_{\alpha\beta\gamma}$ of the general superpotential (5.3). This can be done by expanding the explicit expression of (5.3) in terms of S, T, U and requiring that the coefficients in front of even-degree terms are real (leading to 6 conditions) and that the ones in front of odd-degree terms are imaginary (leading to other 10 conditions). After proper manipulations, the end result can be expressed as

$$\left\{ \begin{array}{l} \text{Re } W_{SU} = \text{Re } W_S - \text{Re } W_{ST} \\ \text{Re } W_{TU} = -\text{Re } W_{ST} + \text{Re } W_T - \text{Re } W_{TT} \\ \text{Re } W_{UU} = -\text{Re } W_S + 2\text{Re } W_{ST} - \text{Re } W_T + \text{Re } W_{TT} + \text{Re } W_U \\ \text{Im } W_{UU} = -3\text{Im } W_0 + 3\text{Im } W_S - 2\text{Im } W_{ST} - 2\text{Im } W_{SU} \\ \quad + 3\text{Im } W_T - \text{Im } W_{TT} - 2\text{Im } W_{TU} + 3\text{Im } W_U \\ \\ \text{Re } W_{STT} = \text{Re } W_{STU} = \text{Re } W_{SUU} = 0 \\ \text{Re } W_{TTT} = \text{Re } W_{TTU} = \text{Re } W_{TUU} = \text{Re } W_{UUU} = 0 \\ \\ \text{Im } W_{STU} = \text{Im } W_{ST} - \text{Im } W_{STT} \\ \text{Im } W_{SUU} = -\text{Im } W_{ST} + \text{Im } W_{STT} + \text{Im } W_{SU} \\ \text{Im } W_{TTU} = -\text{Im } W_{STT} + \text{Im } W_{TT} - \text{Im } W_{TTT} \\ \text{Im } W_{TUU} = -\text{Im } W_{ST} + 2\text{Im } W_{STT} - \text{Im } W_{TT} + \text{Im } W_{TTT} + \text{Im } W_{TU} \\ \text{Im } W_{UUU} = -3\text{Im } W_0 + 3\text{Im } W_S - 3\text{Im } W_{STT} - 3\text{Im } W_{SU} + 3\text{Im } W_T \\ \quad - \text{Im } W_{TTT} - 3\text{Im } W_{TU} + 3\text{Im } W_U . \end{array} \right. \quad (5.20)$$

As one can immediately notice, the conditions fix all the coefficients appearing in the left hand sides of (5.20) as functions of the remaining ones. In particular, the constraints are subdivided into three blocks: the first gives 4 conditions for $W_{\alpha\beta}$, the second shows that all the 7 real parts of $W_{\alpha\beta\gamma}$ must vanish, while the third fixes the imaginary parts of 5 of $W_{\alpha\beta\gamma}$. Among these conditions, the fact that $\text{Re } W_{\alpha\beta\gamma}$ is zero can be easily understood taking into account that, since the maximum degree of (5.3) is 3, the only contributions to the imaginary parts of $W'_{\alpha\beta\gamma}$ derive from the real parts of $W_{\alpha\beta\gamma}$, which therefore must vanish.

Equations (5.20) correctly select only 16 free real parameters and, fortunately, are just linear in $W_0, W_\alpha, W_{\alpha\beta}, W_{\alpha\beta\gamma}$. From now on we will assume that (5.20) is imposed, in such a way that the superpotential automatically exhibits the upliftable form and depends just on the following independent coefficients:

Real parts	Imaginary parts
W_0	W_0
$W_S \quad W_T \quad W_U$	$W_S \quad W_T \quad W_U$
$W_{ST} \quad \cancel{W_{SU}} \quad W_{TT} \quad \cancel{W_{TU}} \quad \cancel{W_{UU}}$	$W_{ST} \quad W_{SU} \quad W_{TT} \quad W_{TU} \quad \cancel{W_{UU}}$
$\cancel{W_{STT}} \quad \cancel{W_{STU}} \quad \cancel{W_{SUU}} \quad \cancel{W_{TTT}} \quad \cancel{W_{TTU}} \quad \cancel{W_{TUU}} \quad \cancel{W_{UUU}}$	$W_{STT} \quad \cancel{W_{STU}} \quad \cancel{W_{SUU}} \quad W_{TTT} \quad \cancel{W_{TTU}} \quad \cancel{W_{TUU}} \quad \cancel{W_{UUU}}$

Our general strategy demands, as a second step, the characterization of Minkowski supersymmetric and no-scale vacua in presence of the new superpotential: these

vacua will be then properly perturbed to hopefully extract stable dS solutions.

5.3.1 Upliftable Minkowski solutions

Both Minkowski supersymmetric and no-scale upliftable solutions can be obtained considering simultaneously the general Minkowski conditions (5.5-5.6) and the upliftability requirement (5.20).

- (i) For example, Minkowski supersymmetric vacua satisfy $W_0 = W_\alpha = 0$ and therefore, by just looking at the preceding table, it is clear that the only free parameters on which they depend are $\text{Re } W_{ST}, \text{Re } W_{TT}$ and $\text{Im } W_{ST}, \text{Im } W_{SU}, \text{Im } W_{TT}, \text{Im } W_{TU}, \text{Im } W_{STT}, \text{Im } W_{TTT}$ (8 in total). The 16 additional parameters are functions of these and can be determined substituting $W_0 = W_\alpha = 0$ into (5.20).

Once the substitutions have been performed and the parameters have been plugged in (5.3), the most general superpotential leading to a Minkowski supersymmetric origin can be eventually cast in the upliftable form (5.19) and the above requirements translate into the 8 constraints

$$\begin{cases} W'_0 = 0 \\ W'_U = -W'_S - W'_T \\ W'_{SU} = -W'_{ST} \\ W'_{TU} = -W'_{ST} - 2W'_{TT} \\ W'_{UU} = W'_{ST} + W'_{TT} \\ W'_{SUU} = -W'_S - W'_{STT} - W'_{STU} \\ W'_{TUU} = -2W'_{STT} - W'_{STU} - W'_T - 3W'_{TTT} - 2W'_{TTU} \\ W'_{UUU} = W'_S + 2W'_{STT} + W'_{STU} + W'_T + 2W'_{TTT} + W'_{TTU} \end{cases} \quad (5.21)$$

which completely determine $W'_0, W'_U, W'_{SU}, W'_{TU}, W'_{UU}, W'_{SUU}, W'_{TUU}, W'_{UUU}$ in terms of the 8 remaining coefficients. Correctly, the dependent parameters are 8, since $W_0 = W_\alpha = 0$ exactly gives 8 real conditions. Moreover, as we said, the (marginal) stability of the origin in this case is automatically assured.

- (ii) No-scale vacua can be extracted similarly, with the only difference that the no-scale requirement (5.6) is more restrictive, because it also contains $W_{ST} = W_{TU} = W_{TT} = 0$ (which must be imposed to guarantee stationarity). However, upliftability already provides a condition relating W_T, W_{ST}, W_{TU} and W_{TT} , i.e. the second equation in (5.20), which may be in contrast with $W_{ST} = W_{TU} = W_{TT} = 0$. Fortunately, since also $W_T = 0$ for a no-scale vacuum, the second equation in (5.20) is automatically satisfied and so no-scale vacua are indeed compatible with the upliftable form of the superpotential. As a consequence, for a no-scale vacuum upliftability leads to just 7 additional independent constraints (in place of 8), and by means of the above table we can easily deduce that the only free parameters left are $W_0 \neq 0, \text{Im } W_{SU}, \text{Im } W_{STT}, \text{Im } W_{TTT}$, the others being completely fixed by (5.6) and (5.20).

By substituting equations (5.6) and (5.20) into the general form (5.3), the most general superpotential with a no-scale origin assumes the upliftable struc-

ture (5.19) with the constraints

$$\left\{ \begin{array}{l} W'_0 = W'_T = 0 \\ W'_S \neq -W'_{SU} + W'_{STT} - W'_{SUU} \\ W'_U = -W'_{STT} + W'_{SUU} \\ W'_{ST} = W'_{TT} = W'_{TU} = 0 \\ W'_{UU} = W'_{SU} \\ W'_{STU} = -2W'_{STT} \\ W'_{TTU} = -W'_{STT} - 3W'_{TTT} \\ W'_{TUU} = 2W'_{STT} + 3W'_{TTT} \\ W'_{UUU} = W'_S - W'_{STT} - W'_{TTT} , \end{array} \right. \quad (5.22)$$

which completely determine $12 - 1 = 11$ dependent parameters, correctly deriving from the 12 real no-scale conditions $W_S = \frac{1}{2}W_0$, $W_U = \frac{3}{2}W_0$ and $W_T = W_{ST} = W_{TU} = W_{TT} = 0$, one of which, as we said, is automatically satisfied by the upliftable structure. Moreover, the second equation in (5.22) is the counterpart of the remaining condition $W_0 \neq 0$. Also in this case, the (marginal) stability of the origin is automatic.

5.3.2 Searching stable dS vacua

We are now ready to perturb Minkowski solutions in order to reach (A)dS vacua, maintaining at the same time the upliftable form of the superpotential. First, a very naive count of the free coefficients in above table shows that, if the ansatz to move away from Minkowski solutions is arbitrary, there is *not* a sufficient number of parameters to simultaneously fulfill the stationarity condition (leading in general to 6 independent real constraints) and the block diagonality of the mass matrix (fixing in general other $5 \times 2 = 10$ real parameters). Therefore, we will be necessarily forced to analyze general solutions where the off-diagonal pieces $V_{\alpha\beta}$ are indeed present, which drastically reduces the probability of getting stable vacua. However, as we have seen, the fact $V_{\alpha\beta} \neq 0$ does not prevent the arising positive eigenvalues as long as the off diagonal blocks are at most of the order of the diagonal ones.

Since no-scale vacua already break supersymmetry (as any dS vacuum should do), in the following our search will focus on deformations of no-scale solutions. There are infinitely many consistent ansatzs to move away from a no-scale vacuum, among which the simplest is introducing just one deformation parameter ϵ , to be added (either multiplied by i or not) to the right hand side some of the equations $W_S = \frac{1}{2}W_0$, $W_U = \frac{3}{2}W_0$ and $W_T = 0$ that define a no-scale origin. Let us analyze in three explicit examples whether this kind of deformations can lead to stable dS vacua for some values of ϵ .

- (1) As a first try, we consider the ansatz

$$W_0 \neq 0 , \quad W_S = \frac{1}{2}W_0 , \quad W_T = i\epsilon , \quad W_U = \frac{3}{2}W_0 , \quad (5.23)$$

in which we just added an imaginary contribution to W_T and which completely fixes W_S, W_T and W_U . Equation (5.23) also sets the cosmological constant

$$\Lambda = \frac{1}{96}\epsilon(\epsilon - 3 \operatorname{Im} W_0) , \quad (5.24)$$

correctly vanishing in the limit $\epsilon \rightarrow 0$. We have now to solve the stationarity condition $V_\alpha = 0$ by fixing some of the parameters that are *not* yet determined by the upliftability requirements (5.20), summarized in the Table on page 70. The explicit computation of V_α shows that the only possibility that leads to a non-negative definite cosmological is to select the 6 conditions

$$\begin{aligned} \operatorname{Re} W_0 = \operatorname{Re} W_{ST} = \operatorname{Re} W_{TT} = 0 , \\ \operatorname{Im} W_{ST} = \frac{\epsilon}{2} , \quad \operatorname{Im} W_{TT} = 2\epsilon , \quad \operatorname{Im} W_{TU} = \frac{3\epsilon}{2} , \end{aligned} \quad (5.25)$$

which, consistently, reduce to the no-scale stationarity $W_{ST} = W_{TU} = W_{TT} = 0$ in the limit $\epsilon \rightarrow 0$. Therefore, looking at the Table, we immediately deduce that the only 4 parameters left are $\operatorname{Im} W_0 \neq 0$, $\operatorname{Im} W_{SU}$, $\operatorname{Im} W_{STT}$, $\operatorname{Im} W_{TTT}$, which, as we said, are not sufficient to eliminate the off-diagonal blocks of the Hessian matrix. Anyhow, since there is still a chance of obtaining positive eigenvalues, we analyze the positive definiteness of the mass matrix as a function of $\operatorname{Im} W_0$, $\operatorname{Im} W_{SU}$, $\operatorname{Im} W_{STT}$, $\operatorname{Im} W_{TTT}$, searching for values that produce all positive eigenvalues in the region where $\Lambda > 0$. For this purpose, we conveniently plot the positiveness of Λ and the simultaneous positiveness of all eigenvalues as a function of ϵ and $\operatorname{Im} W_0$, manipulating then $\operatorname{Im} W_{SU}$, $\operatorname{Im} W_{STT}$, $\operatorname{Im} W_{TTT}$, as depicted in Figure 5 for some fixed values of $\operatorname{Im} W_{SU}$, $\operatorname{Im} W_{STT}$, $\operatorname{Im} W_{TTT}$.

As can be immediately seen, there exist values of $\operatorname{Im} W_0$, $\operatorname{Im} W_{SU}$, $\operatorname{Im} W_{STT}$, $\operatorname{Im} W_{TTT}$ that make the origin stable for continuous sets of ϵ , but, oddly, in all cases these stable solutions systematically emerge in the region where $\Lambda \leq 0$. We have checked many other values of the parameters, but it looks like that our initial ansatz is actually always incompatible with stable dS vacua. However, this difficulty of obtaining stable dS solutions may be due to the particularly simple deformation we have assumed, and stable dS vacua might still appear properly modifying (5.23).

- (2) Therefore, as a second attempt, we study the consequences the more complicated ansatz

$$W_0 \neq 0 , \quad W_S = \frac{1}{2}W_0 + i\epsilon , \quad W_T = i\epsilon , \quad W_U = \frac{3}{2}W_0 + i\epsilon , \quad (5.26)$$

which is very similar to the one in the second example of section 5.2.3, provided we set $\lambda = \epsilon$. In fact, as can be immediately verified, also this ansatz easily produces stable dS vacua if upliftability is not enforced. The cosmological constant fixed by equation (5.26) reads

$$\Lambda = \frac{1}{96}\epsilon(5\epsilon - 3 \operatorname{Im} W_0) , \quad (5.27)$$

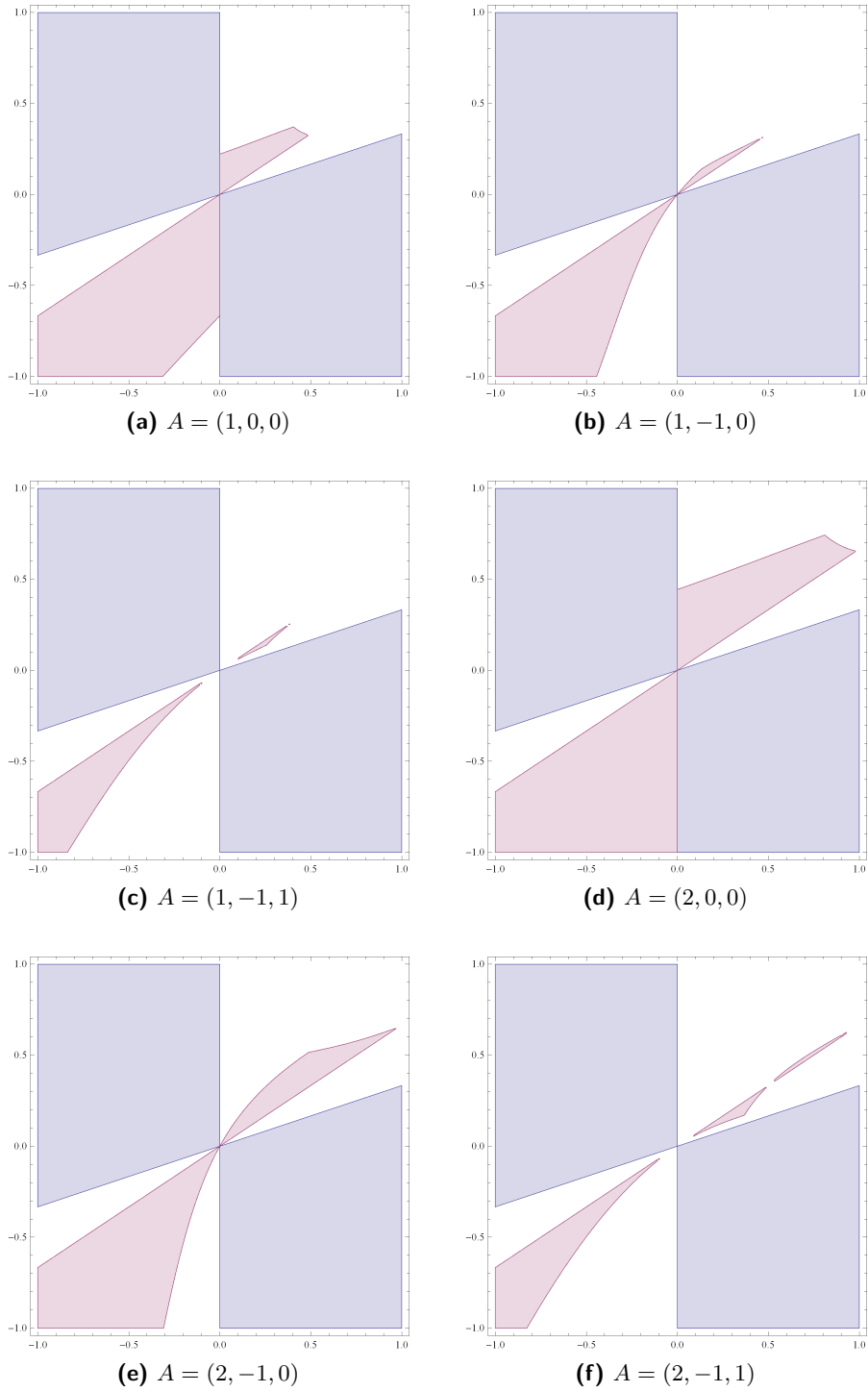


Figure 5: Stability of the origin as a function of the parameters ϵ (x-axis) and $\text{Im } W_0$ (y-axis) for fixed values of $A = (\text{Im } W_{SU}, \text{Im } W_{STT}, \text{Im } W_{TTT})$ in a nearly-no-scale model with ansatz (5.23). Blue represents $\Lambda > 0$, while purple represents stability.

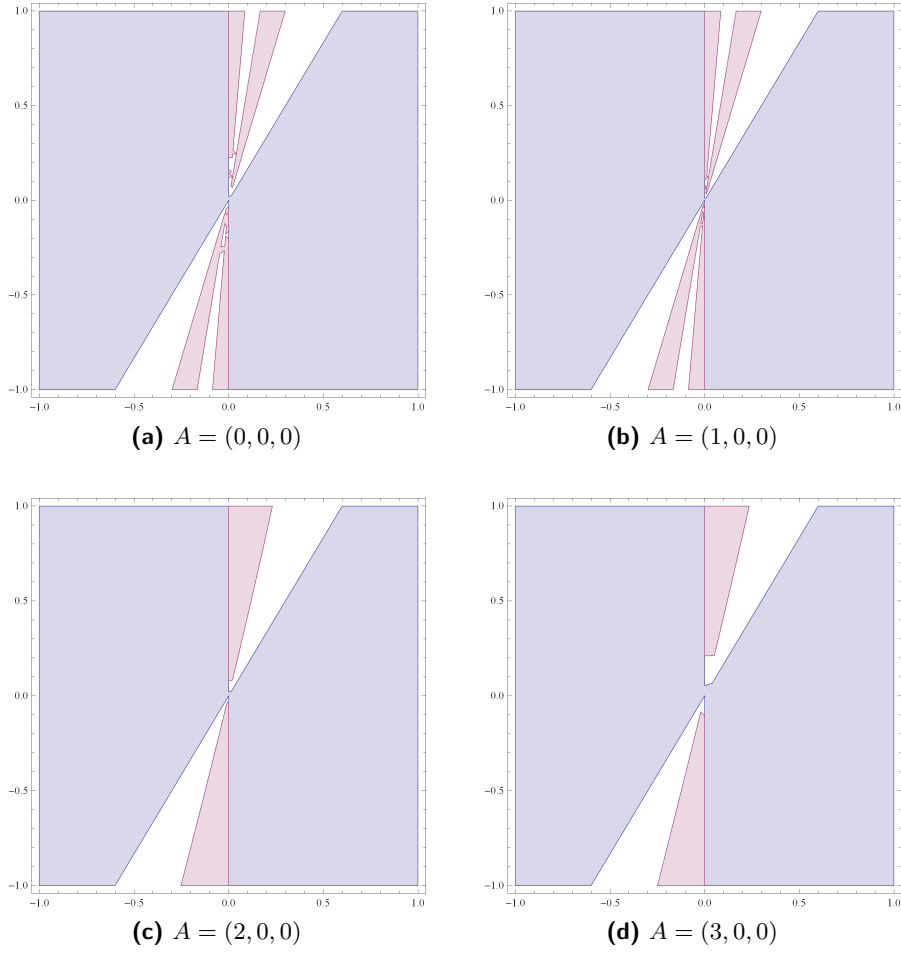


Figure 6: Stability of the origin as a function of the parameters ϵ (x-axis) and $\text{Im } W_0$ (y-axis) for fixed values of $A = (\text{Im } W_{ST}, \text{Im } W_{STT}, \text{Im } W_{TTT})$ in a nearly-no-scale model with ansatz (5.26). Blue represents $\Lambda > 0$, while purple represents stability.

and stationarity can be guaranteed by setting

$$\begin{aligned}
\text{Re } W_0 &= \text{Re } W_{ST} = \text{Re } W_{TT} = 0, \\
\text{Im } W_{SU} &= \frac{3}{4} \text{Im } W_0 \left(\frac{2 \text{Im } W_{ST}}{\epsilon} - 1 \right) - \text{Im } W_{ST} + 4, \\
\text{Im } W_{TT} &= \frac{2\epsilon (3 \text{Im } W_0^2 + 4 \text{Im } W_0 (\text{Im } W_{ST} - 2\epsilon) - 6\epsilon^2)}{3 \text{Im } W_0^2}, \\
\text{Im } W_{TU} &= \frac{3 \text{Im } W_0^2 (\text{Im } W_{ST} + \epsilon) - 2 \text{Im } W_0 \epsilon (4 \text{Im } W_{ST} + \epsilon) + 12\epsilon^3}{3 \text{Im } W_0^2}.
\end{aligned} \tag{5.28}$$

As a consequence, the remaining free parameters to be scanned in order to find stable dS vacua are $\text{Im } W_0 \neq 0$, $\text{Im } W_{ST}$, $\text{Im } W_{STT}$, $\text{Im } W_{TTT}$. In analogy with the preceding example, we represent the positiveness of Λ and of the mass matrix as a function of ϵ and $\text{Im } W_0$, varying $\text{Im } W_{ST}$, $\text{Im } W_{STT}$, $\text{Im } W_{TTT}$, as illustrated in Figure 6. By the above diagrams and by many other instances analyzed, we figure

out that, in the few cases in which stable vacua arise, they regularly emerge where $\Lambda \leq 0$. Moreover, interestingly, we did not find any stable vacuum for nonvanishing values of $\text{Im } W_{STT}$, $\text{Im } W_{TTT}$, because they give positive contributions to the off-diagonal block of the mass matrix. Therefore, also this ansatz seems to completely rule out upliftable stable dS solutions, although they are certainly present when the superpotential is not constrained by (5.20).

- (3) One way to force dS vacua to appear is to select the perturbation ansatz and/or the stationarity condition in such a way that the cosmological constant is positive for *any* choice of the remaining parameters. As a third try, hence, let us consider

$$W_0 \neq 0, \quad W_S = \frac{1}{2}W_0 + i\epsilon, \quad W_T = 0, \quad W_U = \frac{3}{2}W_0 + i\epsilon, \quad (5.29)$$

which induces the cosmological constant $\Lambda = \frac{1}{24}\epsilon^2$, always positive for $\epsilon \neq 0$. There are two possibilities to solve the stationarity condition $V_\alpha = 0$ fixing 6 of the free parameters, and the simplest one is to select

$$\begin{aligned} \text{Re } W_0 &= \text{Re } W_{ST} = \text{Re } W_{TT} = 0, \\ \text{Im } W_{SU} &= \frac{3 \text{Im } W_0 \text{Im } W_{ST}}{2\epsilon} + 2\epsilon, \\ \text{Im } W_{TT} &= \frac{4\epsilon(\text{Im } W_0(6 \text{Im } W_{ST} - 9\epsilon) + 4\epsilon(\text{Im } W_{ST} - 3\epsilon))}{(3 \text{Im } W_0 + 2\epsilon)^2}, \\ \text{Im } W_{TU} &= \frac{3(3 \text{Im } W_0^2 \text{Im } W_{ST} - 4 \text{Im } W_0 \text{Im } W_{ST}\epsilon - 4\epsilon^2(\text{Im } W_{ST} - 2\epsilon))}{(3 \text{Im } W_0 + 2\epsilon)^2}. \end{aligned} \quad (5.30)$$

As a consequence, like in the previous example, the remaining parameters are $\text{Im } W_0 \neq 0$, $\text{Im } W_{ST}$, $\text{Im } W_{STT}$, $\text{Im } W_{TTT}$. Given that the initial ansatz already implies $\Lambda > 0$ as long as $\epsilon \neq 0$, in order to find stable dS solutions we just need to investigate the simultaneous positiveness of Hessian eigenvalues outside the line $\epsilon = 0$. A convenient strategy is to plot the simultaneous positiveness of the first 5 eigenvalues and of the sixth one separately as a function of ϵ and $\text{Im } W_0$, manipulating then the remaining parameters $\text{Im } W_{ST}$, $\text{Im } W_{STT}$, $\text{Im } W_{TTT}$ to find an overlap between the two regions. Anyway, as can be immediately seen in Figure 7, for some mysterious conspiracy the latter region seems to always emerge within the complement of the former, and therefore the two never intersect. We thus conclude that the ansatz (5.29) is not suitable for stable dS vacua as well.

All the previous attempts, as well as many other instances we have analyzed but not reported, explicitly showed that obtaining stable dS vacua is highly nontrivial in presence of upliftable superpotentials. Besides, we did not even study the full STU-model, since the difficulty of getting stable dS solutions already turned up while examining its isotropic truncation.

We should however stress that, even though block diagonality was not enforced, stable upliftable vacua did arise (and also very easily). Therefore, the upliftability condition (5.20) actually does not preclude stability itself: on the contrary, for some miracle, it systematically prevents the simultaneous appearance of a positive cosmological constant and all positive eigenvalues. The many explicit examples we investigated, together with the fact that at the present time no upliftable stable

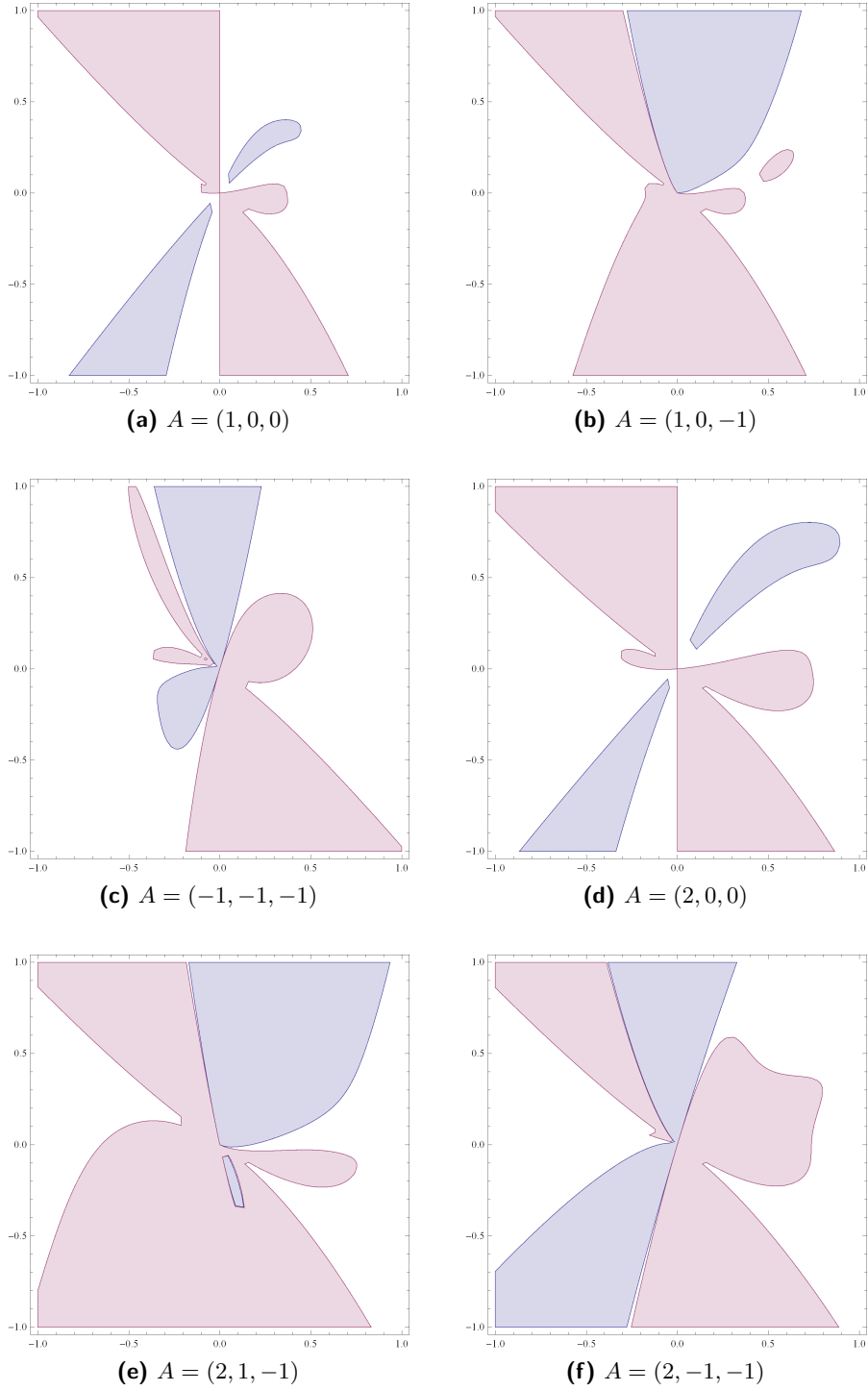


Figure 7: Stability of the origin as a function of the parameters ϵ (x-axis) and $\text{Im } W_0$ (y-axis) for fixed values of $A = (\text{Im } W_{ST}, \text{Im } W_{STT}, \text{Im } W_{TTT})$ in a nearly-no-scale model with ansatz (5.29). Blue represents the positiveness of the first 5 eigenvalues, while purple represents the positiveness of the sixth one.

dS vacuum is known for these models, actually lead to believe that this systematic absence is not random, but it derives from a more general result (not yet proved) that totally excludes, in these STU-models, the existence of stable dS vacua upliftable to eleven dimensions.

6 Conclusions and outlook

Motivated by the need of understanding the origin of stable dS vacua in String Theory, in this thesis we studied the conditions for the occurrence of stable dS solutions in particular $N = 1$ supergravity models, the STU-models. We first restricted ourselves to the theories deriving from the truncations of the $SO(8)$ gauged supergravity and reproduced most of the results in the literature, none of which, unfortunately, includes stable vacua with a positive cosmological constant. We then extended the analysis to generic STU-models, searching for stable dS vacua close to Minkowski supersymmetric and no-scale solutions. However, although many (marginally) stable dS vacua have been found without requiring the upliftable form of the superpotential, for all explicit examples with a clear higher-dimensional origin stability and positiveness of the cosmological constant seemed to be completely incompatible. This is a further evidence of the difficulty of reproducing stable dS vacua in String Theory, but it does not necessarily mean that such upliftable solutions are absent. Indeed, we can extend our analysis in various directions that might hopefully lead to the desired result.

First of all, in the general STU-models we considered just isotropic conditions on the parameters $\tilde{W}_0(\epsilon, \lambda)$, $\tilde{W}_\alpha(\epsilon, \lambda)$, $\tilde{W}_{\alpha\beta}(\epsilon, \lambda)$ and $\tilde{W}_{\alpha\beta\gamma}(\epsilon, \lambda)$, which not only ruled out many possible solutions, but also automatically implied marginal stability. In fact, if the coefficients are isotropic, the 14×14 Hessian matrix (2.16) immediately acquires rank 10 in place of 14, because obviously just 2 of the 6 rows corresponding to T_1, T_2, T_3 and U_1, U_2, U_3 are linearly independent (and likewise for the conjugates). A possible idea to obtain strictly stable vacua is to start from fixed isotropic coefficients that lead to marginally stable vacua and to add to them further perturbative contributions depending on ϵ and λ in such a way that isotropy is canceled. In particular, it should be sufficient to insert different terms into W_{T_2} , W_{T_3} and W_{U_2} , W_{U_3} : if these terms are small around the origin (i.e. at most of the order ϵ^2 and λ^2), it is reasonable to expect that the 4 previously vanishing eigenvalues become small and maybe positive.

Secondly, we have chosen the perturbation ansatzs quite randomly, and most of the times we were forced to proceed with numerical analyses. However, it could be interesting to understand whether a systematic study of all ansatzs can be performed (at least of the linear ones) and whether some of them allow to carry out an analytic analysis, which may give useful information about the relation between stability and positiveness of Λ .

Furthermore, in nearly all the examples the size of the cosmological constant has not been compared with the gravitino and the scalar square masses, and therefore we do not know whether the desired inequality $\Lambda \ll m_{\text{grav}}^2 \ll m_{\text{scalars}}^2$ can be respected. In this context, constructing an explicit example which exhibits this phenomenologically viable hierarchy can be of great interest.

Finally, let us note that we have actually studied just isotropic truncations of upliftable STU-models, and so the absence of stable dS vacua concerns just upliftable superpotentials with isotropic coefficients. It is possible that, for full STU-models with non-isotropic coefficients, upliftable stable dS vacua arise.

If also these further investigations prove to be insufficient, however, the evidence of the absence of upliftable dS vacua will be even stronger.

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