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**"MONETARY POLICY AND THE BUSINESS CYCLE:
DO FINANCIAL CONDITIONS MATTER?"**

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Firma (signature)

To my lovely Monica.

Abstract

This thesis aims to understand if financial conditions are relevant for the identification of a monetary policy shock when considering a structural VAR for the U.S. economy. We model popular proxies for financial conditions - excess bond premium, national financial conditions index, global financial cycle - and a set of standard macroeconomic indicators to appreciate the response of "Main Street" and "Wall Street" to an exogenous hike in the federal funds rate. As an identification strategy, we impose sign restrictions on both the impulse response functions and the systematic components of the monetary policy rule. Using data for the period 1973–2019, we find evidence in favor of a systematic response of the Federal Reserve to financial conditions. Modelling such a response, however, has got only a moderate effect on the estimated business cycle response to the identified monetary policy shock.

Sommario

Questa tesi mira a valutare quanto le condizioni finanziarie siano rilevanti nell'identificare uno shock di politica monetaria nell'economia statunitense quando si considera un VAR strutturale. Per modellare le condizioni finanziarie utilizziamo proxy popolari - excess bond premium, national financial conditions index, global financial cycle - e una serie di indicatori macroeconomici standard. Così facendo, possiamo apprezzare la risposta di "Main Street" e "Wall Street" a un aumento esogeno del tasso di interesse. Come strategia di identificazione, imponiamo restrizioni di segno sia alle funzioni di risposta d'impulso che alle componenti sistematiche della regola di politica monetaria. Utilizzando dati per il periodo 1973 - 2019, troviamo prove a favore di una risposta sistematica della Federal Reserve alle condizioni finanziarie. La modellazione di una tale risposta, tuttavia, ha solo un effetto moderato sulla risposta stimata del ciclo economico allo shock di politica monetaria identificato.

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Introduction

A central question in empirical macroeconomics is how changes in monetary policy affect the response of the economy. To address this question, econometricians employ structural vector autoregressive models and identify the structural shock using a variety of identification schemes. The conventional results they have, by using different length samples and different datasets, is that a monetary policy tightening determines a decline in output and price, and an immediate increase in the policy rate.

In this work, we identify a monetary policy shock in the U.S. economy. Unlike the other identification schemes reported in the literature, our strategy admits simultaneous macro-finance interactions in response to a monetary policy shock and works with an alternative identification scheme made of a combination of sign restriction on the IRFs and sign restriction of the contemporaneous coefficients of the monetary policy rule. To our knowledge, this is the first time that this strategy is implemented.

The main research objectives are to investigate whether the financial variables matter in the identification of the monetary policy shock and if the inclusion of the 2007-2008 financial crisis within the dataset can increase the information set. We have found that the role of the financial variables is moderate when estimating the business cycle response to a monetary policy shock. However, interestingly, financial variables should be accounted for in order to have a more accurate estimation of the monetary policy rule. This leads us to conclude that the central bank deals with financial market swings and, this evidence becomes even stronger after the financial crisis.

This thesis is structured as follows. Chapter 1 provides the state of the art of structural and reduced-form VARs methodologies, as discussed in the literature. Chapter 2 describes the SVAR model we employ, explains our identification strategy, which relies on Uhlig (2005) sign restrictions on the IRFs, and on Arias et al. (2019) sign restrictions on the structural parameters of the monetary policy rule and describes our two datasets, which are referred to as the short datasets (1973-2007) and the extended dataset (1973-2019). The results obtained by implementing the VAR model to the short and extended datasets are discussed in Chapter 3 and Chapter 4, respectively. To prove the robustness of our results, we then report, in Chapter 5, the Forecast Error Variance Decomposition and some robustness checks. Finally, we draw some conclusions and discuss the main outcomes. The Matlab codes required for replicating our results are available in the Appendix.

Chapter 1

State of the art

This chapter introduces the state of the art of structural and reduced-form VARs tools and methods, as discussed in the literature. Furthermore, it provides the foundations of the New Keynesian DSGE model, and it examines how a monetary policy shock impacts the real economy. The last part of the Chapter is devoted to a literature review on the identification procedures that have been developed for the identification of a monetary policy shock in structural VARs.

1.1   The Vector Autoregressive Model

Since the seminal work of Sims (1980), structural vector autoregressive (SVARs) models have become a powerful and reliable tool for data description, forecasting, structural inference and policy analysis (Stock and Watson, 2001). They are used not only in macroeconometric analysis and time series econometrics, but also in many other fields of empirical research such as finance, energy economy and agricultural economics, to name a few (Kilian and Lutkepohl, 2017). The success of this methodological approach is due to its ability in capturing the dynamics of multiple time series with simple equations that are easy to use and implement.

The VAR can be seen as a stochastic process that generalizes univariate autoregressive models by allowing for multivariate time series. Thus, each variable has an equation modelling its evolution over time (Stock and Watson, 2001). The VAR comes in two varieties: the reduced-form and the structural VAR. The reduced-form VAR does not require as much knowledge about the forces influencing a variable as the structural model does. The only prior knowledge required is a list of variables that may affect each other over time. A detailed description of these two forms is given below.

1.1.1   The Structural VAR

The structural VAR, often referred to as SVAR, uses economic theory to clarify the contemporaneous links among the variables (Stock and Watson, 2001). Each variable is expressed by its own past values and both the contemporaneous and lagged values of the remaining n-1 variables.

A structural VAR model with p autoregressive lags is represented by:

$$\boldsymbol{A}_0\boldsymbol{y}_t = \boldsymbol{A}_1\boldsymbol{y}_{t-1} + \ldots + \boldsymbol{A}_p\boldsymbol{y}_{t-p} + \boldsymbol{\varepsilon}_t \text{ for } t \neq s \qquad (1.1)$$

where $\boldsymbol{y}_t$ is a $n \times 1$ vector of time series data, $\boldsymbol{A}_i$ is a $n \times n$ matrix whose elements are autoregressive slope coefficients, $\boldsymbol{A}_0$ is an $n \times n$ matrix representing the instantaneous relationships among the endogenous variables, and $\boldsymbol{\varepsilon}_t$ is a $n \times 1$ vector of structural errors, or shocks. By construction, the shocks are mutually uncorrelated. Furthermore, they are serially uncorrelated, and the error vector has zero mean and variance-covariance matrix $\boldsymbol{\Sigma}_\varepsilon$ of full rank (i.e. the number of shocks is equal to the number of variables). Mathematically, this means that:

$$\mathbb{E}(\boldsymbol{\varepsilon}_t) = \boldsymbol{0} \text{ , } \mathbb{E}(\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t') = \boldsymbol{\Sigma}_\varepsilon = \boldsymbol{I}_n \text{ and } \mathbb{E}(\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_s') = \boldsymbol{0} \text{ for } t \neq s \qquad (1.2)$$

Notice that the structural shocks have not unit of measurement and the error term is assumed to be unconditionally homoskedastic. Furthermore, since the errors are not directly observable, they may be recovered from the corresponding reduced-form VAR (Kilian and Lutkepohl, 2017).

1.1.2   The Reduced-Form VAR

A VAR, o reduced-form VAR, is a linear model composed of n variables in n equations. In a reduced-form VAR, each variable is a linear function of its past value and the past values (i.e. but not the current values) of the other variables composing the system (Stock and Watson, 2001). The reduced-form representation is obtained by multiplying both sides of Eq 1.1 by $\boldsymbol{A}_0^{-1}$:

$$\boldsymbol{A}_0^{-1}\boldsymbol{A}_0\boldsymbol{y}_t = \boldsymbol{A}_0^{-1}\boldsymbol{A}_1\boldsymbol{y}_{t-1} + \ldots + \boldsymbol{A}_0^{-1}\boldsymbol{A}_p\boldsymbol{y}_{t-p} + \boldsymbol{A}_0^{-1}\boldsymbol{\varepsilon}_t \qquad (1.3)$$

If we define $\boldsymbol{B}_i \equiv \boldsymbol{A}_0^{-1}\boldsymbol{A}_i$ for $i = 1, \ldots, p$ and $\boldsymbol{u}_t \equiv \boldsymbol{A}_0^{-1}\boldsymbol{\varepsilon}_t$, we obtain:

$$\boldsymbol{y}_t = \boldsymbol{B}_1\boldsymbol{y}_{t-1} + \ldots + \boldsymbol{B}_p\boldsymbol{y}_{t-p} + \boldsymbol{u}_t \qquad (1.4)$$

where $\boldsymbol{u}_t$ is the reduced-form error terms (or innovations). These errors, which represent the surprise movement in the variables, are white noises (Foroni, 2015) whose elements are serially uncorrelated, and thus:

$$\mathbb{E}(\boldsymbol{u}_t) = \boldsymbol{0} \text{ , } \mathbb{E}(\boldsymbol{u}_t\boldsymbol{u}_t') = \boldsymbol{\Sigma}_u \text{ and } \mathbb{E}(\boldsymbol{u}_t\boldsymbol{u}_s') = \boldsymbol{0} \text{ for } t \neq s \qquad (1.5)$$

Furthermore, they are but mutually correlated.

The reduced-form VAR can be seen as a data generated process (DGP) from the structural VAR (Stock and Watson, 2001). Unlike the SVAR, which provides an interpretation of the data, the reduced-form VAR summarizes the data (Fry and Pagan, 2011). A stable VAR can be also

represented as a Wold moving average process with p lags, MA(p), resulting in:

$$\boldsymbol{y}_t = \mu + \boldsymbol{\Phi}_1 \boldsymbol{u}_{t-1} + \boldsymbol{\Phi}_2 \boldsymbol{u}_{t-2} + \ldots \tag{1.6}$$

where $\mu = \boldsymbol{B}(1)^{-1}\nu$, and $\mathbb{E}(y_t) = \mu$. For more details, refer to Kilian and Lutkepohl (2017).

1.2   Identification Problem

As discussed in Section 1.1, the employment of a VAR model is suitable for many types of analysis. In the macroeconomic environment, the main core of the research focuses on understanding the effect of a shock (e.g. monetary policy shock, technology shock, demand shock...) on the other variables composing the system. Having in mind the structural VAR (see Eq.1.1) and its reduced form (see Eq.1.4), it is clear that for retrieving the structural form from the reduced one is necessary to know the structural impact multiplier matrix $\boldsymbol{A}_0^{-1}$ (Kilian and Lutkepohl, 2017). However, since it is possible to retrieve the same reduced-form from various structural models (Gottschalk, 2001), it is necessary to impose some restrictions (i.e. identifying restrictions) in order to univocally estimate $\boldsymbol{A}_0^{-1}$. In the absence of these assumptions on the data generating process (DGP), no conclusions regarding the structural parameters of the *true model* can be drawn from the data (Gottschalk, 2001).

Estimating structural parameters with economically suitable restrictions is the base of the identification problem. In practice, the identification problem arises because there are more structural parameters than reduced-form parameters. Let us consider the fundamental innovations appearing in Eq. 1.4. This is, by construction:

$$\boldsymbol{u}_t = \boldsymbol{A}_0^{-1} \boldsymbol{\varepsilon}_t \tag{1.7}$$

The variance-covariance matrix of u_t, namely $\boldsymbol{\Sigma}_u$, is given by:

$$\boldsymbol{\Sigma}_u = \mathbb{E}[\boldsymbol{u}_t \boldsymbol{u}_t'] = \boldsymbol{A}_0^{-1} \mathbb{E}[\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t'] \boldsymbol{A}_0^{-1\prime} = \boldsymbol{A}_0^{-1} \boldsymbol{\Sigma}_\varepsilon \boldsymbol{A}_0^{-1\prime} = \boldsymbol{A}_0^{-1} \boldsymbol{A}_0^{-1\prime} \tag{1.8}$$

where $\boldsymbol{\Sigma}_\varepsilon = \boldsymbol{I}_n$. The latter holds because the variance-covariance matrix of the structural error term $\mathbb{E}[\varepsilon_t \varepsilon_t']$ is typically normalized (see Eq. 1.2). If we consider a structural VAR model with (for the sake of simplicity) only 3 endogenous variables, then, Eq. 1.8 becomes, in matrix notation:

$$\boldsymbol{\Sigma}_u = \begin{bmatrix} \sigma_{u,11} & \sigma_{u,12} & \sigma_{u,13} \\ \sigma_{u,21} & \sigma_{u,22} & \sigma_{u,23} \\ \sigma_{u,31} & \sigma_{u,32} & \sigma_{u,33} \end{bmatrix} = \begin{bmatrix} a_{0,11} & a_{0,12} & a_{0,13} \\ a_{0,21} & a_{0,22} & a_{0,23} \\ a_{0,31} & a_{0,32} & a_{0,33} \end{bmatrix}^{-1} \begin{bmatrix} a_{0,11} & a_{0,12} & a_{0,13} \\ a_{0,21} & a_{0,22} & a_{0,23} \\ a_{0,31} & a_{0,32} & a_{0,33} \end{bmatrix}^{-1\prime} \tag{1.9}$$

Eq. 1.9 is a system of non-linear equations for the unknown parameters of $\boldsymbol{A}_0^{-1}$. However, since

Σ_u is symmetric, the system has $n(n+1)/2$ independent equations and n^2 unknowns parameters (Kilian and Lutkepohl, 2017).

When the number of unknown parameters exceeds the number of linear independent equations, additional restrictions are required. Then, to verify that all elements of $\boldsymbol{A}_0^{-1}$ are uniquely identified, rank and order conditions are provided. The order condition states that the maximum number of parameters in $\boldsymbol{A}_0^{-1}$ that can be identified is $n(n+1)/2$. The order condition is necessary for identification, but unfortunately, it is not sufficient. Thus, even if the order condition holds, the set of equations may fail to have a unique solution (Kilian and Lutkepohl, 2017). To ensure identification, the system has to satisfy a further condition, namely the rank condition. The rank condition checks the linear dependency of the equations in the system. Unlike the order condition, the rank condition is a necessary and sufficient condition. This means that the system of equations is really identified when the rank condition is valid, while, if the rank condition does not hold, it is not possible to identify all the structural parameters (Christiano et al., 1999).

Even though in this work we resort to sign restrictions only, for the sick of completeness we briefly introduce some of the possible restrictions that can be implemented in SVAR analysis. According to what is reported in the literature, the most important ones are: short-run restrictions (see Section 1.2.1), long-run restrictions (see Section 1.2.2), sign restrictions (see Section 1.2.3), and statistical identification procedures (see Section 1.2.4).

1.2.1   Identification by Short Run Restrictions

Imposing short-run restrictions is a common practice for solving the identification problem. This method consists in retrieving the structural shock $\boldsymbol{\varepsilon_t}$ from the reduced-form errors $\boldsymbol{u_t}$, by assuming that they are mutually uncorrelated (i.e. orthogonal). One possible solution consists in defining the matrix $\boldsymbol{A}_0^{-1}$ as a lower-triangular matrix. The resulting matrix is called the lower-triangular Cholesky decomposition of $\boldsymbol{\Sigma}_u$ since it holds $\boldsymbol{\Sigma}_u = \boldsymbol{A}_0^{-1}\boldsymbol{A}_0^{-1\prime}$. For this reason, this identification scheme is also called Cholesky identification. According to the Cholesky identification scheme, Eq.1.9 becomes:

$$\boldsymbol{\Sigma}_u = \begin{bmatrix} \sigma_{u,11} & \sigma_{u,12} & \sigma_{u,13} \\ \sigma_{u,21} & \sigma_{u,22} & \sigma_{u,23} \\ \sigma_{u,31} & \sigma_{u,32} & \sigma_{u,33} \end{bmatrix} = \begin{bmatrix} a_{0,11} & 0 & 0 \\ a_{0,21} & a_{0,22} & 0 \\ a_{0,31} & a_{0,32} & a_{0,33} \end{bmatrix}^{-1} \begin{bmatrix} a_{0,11} & 0 & 0 \\ a_{0,21} & a_{0,22} & 0 \\ a_{0,31} & a_{0,32} & a_{0,33} \end{bmatrix}^{-1\prime} \tag{1.10}$$

When $\boldsymbol{A}_0^{-1}$ is restricted as shown above, it has $n(n-1)/2$ zero parameters and, since $n(n+1)/2 > n(n-1)/2$, the order condition is satisfied. At this point, the number of equations is equal to the number of unknown parameters and hence, it is possible, through recursive substitution, to solve the system and retrieve a unique solution for $\boldsymbol{A}_0^{-1}$. For this reason, this technique is also called recursive identification.

The Cholesky decomposition allows the reduced-form errors to be orthogonal. The orthogonalization makes this identify assumption strong. However, the model has a drawback: the

recursive structure must be carefully chosen based on the economic theory. If the recursive order is not properly justified, it may be possible to jeopardize the results and retrieve structural parameters with no economic sense, because imposing zero in some contemporaneous lagged variables, means imposing a well-defined chain in the causality instead of leaving the data to speak about the causal relation (Kilian and Lutkepohl, 2017). To demonstrate this, let us consider Eq. 1.7 and Eq. 1.10. We can obtain the following system of equations:

$$
\begin{bmatrix} u_{1,t} \\ u_{2,t} \\ u_{3,t} \end{bmatrix} = \begin{bmatrix} a_{0,11} & 0 & 0 \\ a_{0,21} & a_{0,22} & 0 \\ a_{0,31} & a_{0,32} & a_{0,33} \end{bmatrix}^{-1} \begin{bmatrix} \varepsilon_{x1,t} \\ \varepsilon_{x2,t} \\ \varepsilon_{x3,t} \end{bmatrix} \tag{1.11}
$$

and the VAR model, in scalar form, reads:

$$
\begin{cases} y_{1,t} = \ldots + a_{0,11} u_{x1,t} \\ y_{2,t} = \ldots + a_{0,21} u_{x1,t} + a_{0,22} u_{x2,t} \\ y_{3,t} = \ldots + a_{0,31} u_{x1,t} + a_{0,32} u_{x2,t} + a_{0,33} u_{x3,t} \end{cases} \tag{1.12}
$$

It is clear that the first shock affects all the variables considered, the second shock affects all the variables considered except the first one, and so on. Furthermore, it is possible to find different solutions A_0^{-1} for different ranks of ordering. For instance, for a VAR model with 6 variables there are $6! = 720$ permutations of ordering (Kilian and Lutkepohl, 2017).

1.2.2   Identification by Long Run Restrictions

An alternative identification approach consists in imposing long-run restrictions on the long-run response of variables to shocks. For imposing long-run restrictions, there exist two frameworks.

The traditional framework has been firstly proposed by Blanchard and Quah (1989) and consists in restricting the cumulative responses in a stationary VAR model. Let us consider the structural VAR model in Eq. 1.1 and the corresponding reduced-form VAR in Eq. 1.4. If the $I(1)$ variables are not cointegrated or the cointegrating relations are known, then all $I(1)$ variables may be transformed to $I(0)$ variables (Kilian and Lutkepohl, 2017). If we call these transformed variables as z_t, the structural and the reduced-form VARs become, respectively:

$$
A_0 z_t = A_1 Z_{t-1} + \ldots + A_p Z_{t-p} + \varepsilon_t \tag{1.13}
$$

$$
z_t = B_1 Z_{t-1} + \ldots + B_p Z_{t-p} + u_t \tag{1.14}
$$

Imposing long-run restrictions means restricting the elements of the $n \times n$ cumulative structural impulse response matrix z_t (Kilian and Lutkepohl, 2017):

$$
z_t = A(1)^{-1} \varepsilon_t = \Theta(L) \varepsilon_t \tag{1.15}
$$

of the structural MA process:

$$\Theta(1) = \sum_{i=0}^{\infty} \Theta_i = A(1)^{-1} = B(1)^{-1} \boldsymbol{A}_0^{-1} \tag{1.16}$$

In this model, by construction, A_0^{-1} is not recursive.

The general framework, instead, involves the vector error correction form (VECM) of the VAR representation:

$$\triangle y_t = \alpha \beta' y_{t-1} + \boldsymbol{\Gamma} \triangle y_{t-1} + \ldots + \boldsymbol{\Gamma}_{p-1} \triangle y_{t-p+1} + u_t \tag{1.17}$$

where α and β are matrices with rank r and dimension $n \times r$. This process has the following Grander representation:

$$y_t = \boldsymbol{\Upsilon} \sum_{i=1}^{t} \varepsilon_i + \boldsymbol{\Xi}^*(L) A_0^{-1} \varepsilon_t + y_0^* \tag{1.18}$$

where $\boldsymbol{\Upsilon}$ is the matrix of long-run multipliers of the VECM:

$$\boldsymbol{\Upsilon} = \boldsymbol{\Xi} \boldsymbol{A}_0^{-1} \tag{1.19}$$

Restricting the long-run effects of the shocks means acting directly on $\boldsymbol{\Upsilon}$. When a shock does not have any long-run effects at all, the corresponding column of $\boldsymbol{\Upsilon}$ is zero (Kilian and Lutkepohl, 2017).

Unlike what happens for the short-run restrictions, most economists agree on the long-run properties of the models. For instance, they agree that, in the long run, demand shocks are neutral while productivity shocks are not (Challe, 2010). Nevertheless, the use of long-run restrictions for structural VAR identification presents several theoretical challenges. First of all, unlike short-run restrictions, long-run restrictions require the presence of exact unit roots. Then, the obtained results may be sensitive to omitted variables; and there is a lack of robustness when the data are at low frequency. Last, long-run restrictions should be coupled with sign restrictions otherwise, the solution is not unique. In addition to these conceptual challenges, there are practical limitations (refer to Kilian and Lutkepohl (2017) for more details). For all these reasons, in this work, we do not even employ long-run restrictions.

1.2.3   Identification by Sign Restrictions

The traditional identification methods based on exclusion restrictions are often difficult to support from an economic perspective. A different identification approach is represented by the sign restrictions, that instead relays on implications of economic theory. Since they allow avoiding the disadvantages of the traditional parametric methods, they have grown in popularity.

Identification by sign restrictions has been developed first by Faust (1998), Canova and

Nicoló (2002), and successively by Uhlig (2005). The concept on the basis of the sign restrictions identification scheme is simple: based on the economic theory, it is possible to recognize the sign in the response between variables and so, imposing the relative sign restriction upon the impulse responses in the presence of a shock. Imposing sign restrictions on the impact response of the variables corresponds to imposing sign restrictions on the structural impact multiplier matrix, $\boldsymbol{A}_0^{-1}$. For this reason, this is also known as static sign restriction.

Consider the structural VAR model in Eq. 1.1 and the (normalized) variance-covariance matrix $\mathbb{E}(\varepsilon_t \varepsilon_t') = \boldsymbol{\Sigma}_\varepsilon = \boldsymbol{I}_n$. The reduced-form VAR innovations are:

$$\boldsymbol{u}_t = \boldsymbol{P}\boldsymbol{\eta}_t \tag{1.20}$$

where $\boldsymbol{\eta}_t$ are mutually uncorrelated shocks with variance equal to 1, and $\boldsymbol{P}$ is the lower triangular matrix of the Cholesky decomposition. Since the solution of the Cholesky decomposition is not unique:

$$\boldsymbol{\Sigma}_u = \boldsymbol{A}_0^{-1}\boldsymbol{A}_0^{-1\prime} = \boldsymbol{A}_0^{-1}\boldsymbol{I}_n\boldsymbol{A}_0^{-1\prime} = \boldsymbol{A}_0^{-1}\boldsymbol{Q}\boldsymbol{Q}'\boldsymbol{A}_0^{-1\prime} = \boldsymbol{P}\boldsymbol{P}' \tag{1.21}$$

there are several solutions for the structural shocks. These candidates are given by:

$$\varepsilon_t^* = \boldsymbol{Q}'\boldsymbol{\eta}_t \tag{1.22}$$

where $\boldsymbol{Q}$ is an $n \times n$ orthogonal matrix such that:

$$\boldsymbol{Q}\boldsymbol{Q}' = \boldsymbol{Q}'\boldsymbol{Q} = \boldsymbol{I}_n \tag{1.23}$$

and where:

$$\boldsymbol{u}_t = \boldsymbol{P}\boldsymbol{Q}\boldsymbol{Q}'\boldsymbol{\eta}_t = \boldsymbol{P}\boldsymbol{Q}\varepsilon_t^* \tag{1.24}$$

The shocks ε_t^* are uncorrelated with unit variance. They are discharged whether the structural impact multiplier matrix, $\boldsymbol{P}\boldsymbol{Q}$, does not satisfy the imposed restrictions on $\boldsymbol{A}_0^{-1}$; or they are retained in the opposite case. It is indeed possible to draw several $\boldsymbol{Q}$ and have as much as $\boldsymbol{P}$ but there is no reason that every $\boldsymbol{P}$ is plausible. Only the matrices $\boldsymbol{P}$ that satisfy sign restrictions on the impulse response function are accepted. This process is repeated several times, to construct the set of all structural models that are consistent with the maintained sign restrictions and the reduced-form parameters (Kilian and Lutkepohl, 2017). Two are the main techniques used to construct the orthogonal matrices $\boldsymbol{Q}$: the Given rotation matrices and the Householder transformation.

Givens rotation matrices

Consider a 3×3 VAR model. Is possible to generate the orthogonal matrix $\boldsymbol{Q}$ by multiplying

the given rotation matrices as follows:

$$\boldsymbol{Q}(\phi_1, \phi_2, \phi_3) = \boldsymbol{Q}_{12}(\phi_1) \times \boldsymbol{Q}_{13}(\phi_2) \times \boldsymbol{Q}_{23}(\phi_3) \tag{1.25}$$

where the matrices are:

$$\boldsymbol{Q}_{12} = \begin{bmatrix} \cos\phi_1 & -\sin\phi_1 & 0 \\ \sin\phi_1 & \cos\phi_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \boldsymbol{Q}_{13} = \begin{bmatrix} \cos\phi_2 & 0 & -\sin\phi_2 \\ 0 & 1 & 0 \\ \sin\phi_2 & 0 & \cos\phi_2 \end{bmatrix} \boldsymbol{Q}_{23} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi_3 & -\sin\phi_3 \\ 0 & \sin\phi_3 & \cos\phi_3 \end{bmatrix} \tag{1.26}$$

and ϕ_i lies between 0 and 2π (Kilian and Lutkepohl, 2017). Since $\cos\phi_i + \sin\phi_i = 1$, the matrix $\boldsymbol{Q}(\phi_1, \phi_2, \phi_3)$ is an orthogonal matrix:

$$\boldsymbol{Q}_{12}\prime\boldsymbol{Q}_{12} = \boldsymbol{Q}_{13}\prime\boldsymbol{Q}_{13} = \boldsymbol{Q}_{23}\prime\boldsymbol{Q}_{23} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{1.27}$$

Even if it is theoretically possible to apply this approach for $n > 3$ dimensional VAR, it is almost never applied in practice because of the high computational effort required.

Householder transformation

The second and most employed technique used for constructing the orthogonal matrices $\boldsymbol{Q}$ has been developed by Rubio-Ramirez et al. (2010) and relies on an algorithm that employs the QR factorization. Following the example of a hypothetical VAR model with 3 endogenous variables, a 3×3 matrix $\boldsymbol{W}$ of random variables with $\mathcal{N}(0, I_n)$ distribution functions can be decomposed with the QR factorization ($\boldsymbol{W} = \boldsymbol{Q}_r\boldsymbol{R}$). The matrix $\boldsymbol{Q}_r$ is an orthogonal matrix ($\boldsymbol{Q}\boldsymbol{Q}\prime = \boldsymbol{I}_n$), while $\boldsymbol{R}$ is an upper-triangular matrix. Clearly ($\boldsymbol{Q}_r = \boldsymbol{I}_n$) corresponds to the matrix $\boldsymbol{Q}$ used in recursive orderings (Fry and Pagan, 2011) and it is known in the literature as the rotation matrix. Thanks to the algorithm provided by Rubio-Ramirez et al. (2010), it is possible to draw several candidate solutions for $\boldsymbol{A}_0^{-1}$. As for the technique of the Givens rotation matrices, also in this case the solutions that are not consistent with the sign restrictions provided are discharged. Once the structural impact multiplier matrix $\boldsymbol{A}_0^{-1}$ is retrieved, the model is identified.

As pointed out by Fry and Pagan (2007), this approach is theoretically equivalent to the Given Rotation Matrices method, however, from a computational point of view, this is much more convenient, especially when the number of the variables in the model increases.

1.2.4   Other Identification Strategies

Identification by heteroskedasticity and identification by non-Gaussianity represent two alternative approaches for achieving identification of structural VAR models. These are purely statisti-

cal identification procedures that exploit properties of the data, such as the volatility of the VAR errors, rather than economic theories. To evaluate if the identified shocks correspond to real economic shocks, these types of identifications are usually coupled with conventional identifying restrictions.

We start with explaining identification by heteroskedasticity. Let us consider the general reduced-form VAR(p) model reported in Eq. 1.4. The reduced-form errors in $\boldsymbol{u}_t$ are serially uncorrelated, mutually correlated, and have zero mean (see Eq. 1.5). Furthermore, they may be subjected to conditional or unconditional heteroskedasticity. They are generally obtained from the following linear transformation $\boldsymbol{u}_t = \boldsymbol{A}_0^{-1}\boldsymbol{\varepsilon}_t$, however, if the conditional variances of the reduced-form shocks change over time, we have that:

$$\mathbb{E}(\boldsymbol{u}_t\boldsymbol{u}_t') = \boldsymbol{\Sigma}_1 \text{ for } t = 1, \ldots, T_1 \text{ and } \mathbb{E}(\boldsymbol{u}_t\boldsymbol{u}_t') = \boldsymbol{\Sigma}_2 \text{ for } t > T_1 \qquad (1.28)$$

with $\boldsymbol{\Sigma}_1 \neq \boldsymbol{\Sigma}_2$. There must, therefore, be a matrix $\boldsymbol{G}$ and a diagonal matrix $\boldsymbol{\Lambda}$ such that:

$$\boldsymbol{\Sigma}_1 = \boldsymbol{G}\boldsymbol{G}' \text{and} \boldsymbol{\Sigma}_2 = \boldsymbol{G}\boldsymbol{\Lambda}\boldsymbol{G}' \qquad (1.29)$$

Given this, the structural shocks become:

$$\boldsymbol{\varepsilon}_t = \boldsymbol{G}^{-1}u_t \qquad (1.30)$$

and have variance:

$$\mathbb{E}(\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t') = \begin{cases} \boldsymbol{I}_K, & t = 1, \ldots, T_1, \\ \boldsymbol{\Lambda}, & t > T_1. \end{cases} \qquad (1.31)$$

Note that, if all the diagonal elements of $\boldsymbol{\Lambda}$ are different, the matrix $\boldsymbol{A}_0^{-1}$ is univocal (apart from changes in sign and permutation). We may obtain unique shocks by simply imposing that the structural shocks are instantaneously uncorrelated. Even if this is a purely statistical approach with no economical meaning, in the last decades, it has been extended in different directions. As an example, Bacchiocchi et al. (2014) consider the possibility of time-varying instantaneous effects of the shocks (Kilian and Lutkepohl, 2017).

Identification by non-Gaussianity exploits the non-gaussian distribution of the reduced-form errors of a VAR model for uniquely and completely identifying a set of structural shocks. If the shocks are stochastically independent, have variance one and at most one has normal distribution, then, the matrix $\boldsymbol{A}_0$ is unique (except for rows sign and permutation) and the shocks are obtained from the following linear transformation (assuming that it exists): $\boldsymbol{\varepsilon}_t = \boldsymbol{A}_0\boldsymbol{u}_t$ (Kilian and Lutkepohl, 2017).

Both these identification approaches are useful for assessing the validity of conventional identifying restrictions (Kilian and Lutkepohl, 2017).

1.3 The New Keynesian DSGE Framework

As Data Generating Process, we use the New Keynesian Dynamic Stochastic General Equilibrium (DSGE) model that employs Bayesian methods for exploring the effect of various shocks (Ramey, 2016).

The New Keynesian paradigm has been employed by different authors all over the world for understanding fluctuations in economic activity and inflation, and for assessing how these fluctuations relate to monetary policies and fiscal policies. Examples of work employing this model include Christiano et al. (2005), Smets and Wouters (2007), Castelnuovo (2013) and Arias et al. (2019). Furthermore, the New Keynesian DSGE model is used by central banks and policy institutions as a tool to assess macroeconomic phenomena (Galí, 2018). Unlike the Real Business Cycle (RBC) model, the New Keynesian framework allows introducing explicitly nominal variables, accounting for positive price mark-up and considering nominal rigidities. This leads to the following two key properties: exogenous changes in monetary policy have a non-trivial effect on real variables, and the shock response depends on the policy rule that is adopted by the central bank (Galí, 2018).

The New Keynesian model is composed of three equations. The first equation is referred to as the Dynamic Investment-Savings (DIS) equation. This equation determines the output gap, $\tilde{y}_t$, given a path for the exogenous natural rate of interest, r_t^n, the actual real rate, r_t, and the expected value of the output gap one period ahead, namely $\mathbb{E}_t\{\tilde{y}_{t+1}\}$. In algebraic terms, the DIS equation reads (Galí, 2015, 2018):

$$\tilde{y}_t = -\frac{1}{\sigma}\left(i_t - \mathbb{E}_t\{\pi_{t+1}\} - r_t^n\right) + \mathbb{E}_t\{\tilde{y}_{t+1}\} \tag{1.32}$$

where σ is the curvature of the utility of consumption (Galí, 2015). In turn, the natural rate of interest is the real interest rate that would prevail in the flexible price equilibrium:

$$r_n^t = \rho - \sigma(1 - \rho_a)\psi_{ya}a_t + (1 - \rho_z)z_t \tag{1.33}$$

while the actual real rate is a function of the nominal rate, i_t, and the expected inflation, π_{t+1}:

$$r_t = i_t - \mathbb{E}_t\{\pi_{t+1}\} \tag{1.34}$$

The latter is the well known Fisher Equation (FE).

The second equation of the New Keynesian model is the AS curve for aggregate supply and says that the inflation depends on the output gap and the expected inflation of one period in the future:

$$\pi_t = \beta \mathbb{E}_t\{\pi_{t+1}\} + \kappa\tilde{y}_t \tag{1.35}$$

where $\kappa \equiv \lambda\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)$. This equation is often called the New Keynesian Phillips curve (NKPC) (it differs from the conventional one because of the introduction of the expectation

term) and represents a key feature of the canonical New Keynesian model.

The last equation is a simple interest rate rule and describes how monetary policy systematically responds to changes in output and prices (Arias et al., 2019):

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \hat{y}_t + \nu_t \tag{1.36}$$

In Eq. 1.36, $\hat{y}_t$ is the deviation of the output from the steady-state ($\hat{y}^n = y^n - y$), the coefficients ϕ_π and ϕ_y are chosen positive by the monetary authority, and ν_t is an autoregressive AR(1) process representing an exogenous monetary policy shock:

$$\nu_t = \rho_\nu \nu_{t-1} + \varepsilon_t^\nu \tag{1.37}$$

When ε_t^ν is positive, the monetary policy shock is contractionary and the nominal interest rate rises. On the other hand, when it is negative, the monetary policy shock is expansionary and the interest rate declines. Note that Eq. 1.36 is consistent with the Taylor rule (Taylor, 1993) that says that:

$$i_t^M = r_t^* + \pi_t + 0.5(\pi_t - \pi^*) + 0.5(y_t - \tau_t^Y) \tag{1.38}$$

In Eq. 1.38, i_t^M is the target nominal interest rate, r_t^* is the equilibrium real interest rate, π^* is the inflation target and τ_t^Y is the (log) of the potential output, which is determined by its linear trend over the sample. This equation well describes the behaviour of the central bank when it attempts to minimize, in each period, its loss function, L. The canonical central bank loss function penalizes deviations from the inflation target and long-run output (Challe, 2010):

$$L(y_t \pi_t) = \lambda(y_t - \bar{y})^2 + (\pi_t - \bar{\pi})^2 \tag{1.39}$$

where λ is a constant. So the interest rate is set by monetary authorities to stabilize the business cycle. More precisely, the nominal interest rate is increased in order to enter a crisis and contrast high inflation and a positive output gap, or nominal interest rate is cut to stimulate investment and consumptions. Every policy rule in which the short-term nominal interest rate appears as dependent variable, function of other variables, takes the name of Taylor rule.

1.4   The Monetary Policy Implementation

Monetary economics represents one of the most prosperous macroeconomic research area. The challenge faced by many researchers is understanding the relationship between monetary policy, inflation and business cycle (Galí, 2015). As previously mentioned, the relation among these variables is well summarized by Eq. 1.36, which describes the systematic response of the central bank to contemporaneous inflation and output gap. As we know, the spending behaviours of private agents are, indeed, determined by real interest rate, r_t, rather than by short-term nominal

interest rate (Challe, 2010). However, the central bank does not control directly r_t, but it sets short nominal interest rates, i_t, that influence longer nominal interest rates and real interest rates, r_t, through the interbank market. Thus, what the central bank tries to do with its monetary policy is to influence r_t that, in turn, depends on an entire path of short-term nominal interest rates. To sum up, the central bank uses the short nominal interest rate, together with the forward guidance, as a tool for implementing the monetary policy. In such a way, thanks to the FE (see Eq. 1.34), the central bank can influence also real interest rates and expected inflation.

Every day, the banks perform a number of operations that determines, at the end of the day, an excess or a deficit of liquidity. This does not count for the soundness of the single bank, but it is physiological to the banking operations. To settle these debts, banks can lend and borrow money to each other overnight at the nominal interest rate, i^M, which is the interest on money exchange between banks, in the very short run. When the interbank market shows no need for extra liquidity, banks in surplus can deposit extra liquidity to the central bank. On the other hand, when the interbank market needs extra liquidity, the accounts in deficit borrow from the central bank. The clearing in the interbank market is also determined by portfolio choices.

The demand for reserve money is sensitive to both the state of the business cycle and the opportunity cost of holding reserves. The first comes from the money demand function:

$$M^d = \$YL(i) \tag{1.40}$$

Since the money demand is a function of real income and interest rate, if the economy is growing, banks will experience more imbalances to be settled. Analogously to the demand for money, the demand for reserves is an increasing function of Y. The second one, i.e. the opportunity cost of holding reserves, is given by the difference between what the bank will gain from lending liquidity to other banks overnight, i^M, and the interest rate on deposits to the central bank overnight, i^R. For any i^R, it must hold $i^M \geq i^R$. If the inequality does not hold, the interbank market would cease to exist, until some banks with liquidity needs, will increase i^M over i^R and restore the canonical inequality. It is possible to sum up the demand for reserves money with:

$$R^d = R^d(\underbrace{i^M - i^R}_{-}, \underbrace{y}_{+}) \tag{1.41}$$

Indeed the central bank has the monopoly on the money supply, that exercises through open market operations. Briefly, the central bank purchases government bonds to inject liquidity into the system and dampen the interest rate, while selling government bonds to limit the liquidity in the market. Thus, the monetary authority completely controls the aggregate quantities of reserves money in the system R^s (Challe, 2010):

$$R^s = R^d(\underbrace{i^M - i^R}_{-}, \underbrace{y}_{+}) \tag{1.42}$$

In equilibrium (i.e. $R^s = R^d$), we obtain:

$$i^M = i^M(\underbrace{i^R}_{+}, \underbrace{y}_{+}, \underbrace{R^0}_{-}) \geq i^R \qquad (1.43)$$

To conclude, the central bank can use two leverage to move (e.g. upward) the interest rate i^M. It can simply move the interest rate i^R that for a given level of R^s adjust i^M accordingly (upward); or it can act on the opportunity cost since changes in the quantity of reserves R^s (reduce) increases the interbank interest rate i^M for a fixed level of i^R. Even if, the target is i^M, what the central bank set during the meeting of the Federal Open Market Committee (FOMC) are both i^R and R^s. It is important to notice that, while i^M is a market rate, i^R is considered to be an exogenous variable since set by the central bank.

1.5   The Monetary Policy Shock in the Literature

As discussed in Section 1.3, the central bank systematically implements the monetary policy to respond to variations in the state of the economy (Christiano et al., 1999). The systematic components of the policy are expressed by the feedback rule that links short-term interest rates managed by the central bank to the rate of inflation or deviation from its target (Armour and Côté, 2000). However, a part of the policy variation can not be explained by the systematic response to the economy. This unexplained variation is formalized by the notion of a monetary policy shock (Christiano et al., 1999). A more compact definition for the monetary policy shock is provided by Miranda-Agrippino and Ricco (2021) who define the monetary shock as an exogenous shift in the policy instrument that surprises market participants, that is unforecastable, and that is not due to the central bank's systematic response to the information set.

Christiano et al. (1999) proposed three economic interpretations for the exogenous variation on the basis of the policy shock. The first interpretation assumes that the shock is caused by a change in the preferences of the central bank and/or a shift in the preferences of the FOMC. The second interpretation, instead, says that the policymakers want to avoid the social costs deriving from disappointing private agents' expectations (Ball, 1995; Chari et al., 1998). The last possible source of exogenous variation is based on technical factors. For example, there may be measurement errors in the indicators used by the FOMC for taking decisions (Bernanke and Mihov, 1998a). Uhlig (2005) agrees with the last hypothesis, indeed he believes that the sources of unexplained variation in the policy are accidental errors made by the central bank, which are quickly reversed.

Structural shocks are in general unobservable (Kilian and Lutkepohl, 2017). In order to analyse them and study their effects, econometricians resort to structural economic models. Among the others, the structural VAR has been widely used to assess the impact of a monetary policy shock on the U.S. economy. The monetary policy shock has been identified by several authors

by considering different sets of endogenous variables and by applying different identification strategies.

The approach commonly used to identify monetary policy shock is to impose alternative sets of recursive zero restrictions on the contemporaneous coefficients (Ramey, 2016). The pioneer of this method, which is also called triangularization, is Sims (1980). The recursiveness assumption considers that the information set at time t does not respond to monetary policy shock realized at time t, but that it responds with a lag. Consider for instance the work of Christiano et al. (1996).

Several authors used the sign restriction method which consists in imposing sign restriction on the response of some variables. This method was pioneered by Faust (1998), who imposes sign restrictions on impact, Canova and Nicoló (2002), who imposes sign restrictions on the cross-correlation of variables in response to shocks, and Uhlig (2005). The key difference from the recursive method is that the model is no longer point identified but set identified.

Other authors tried to recover structural shocks through the narrative approach. This approach considers key historical events when constraining the structural shocks and when performing historical decomposition. Key references for narrative sign restrictions are the works of Romer and Romer (2004) and Antolín-Díaz and Rubio-Ramírez (2018). Romer and Romer (2004) document and collect various historical events from the narrative records of FOMC meetings and internal memos of the Fed about the expected federal funds rate. When a narrative source provides information about the direction but not the magnitude, they resort to quantitative evidence. Then, before estimating the effect of the shock, they remove from the found rate the moments that are results of deliberated decisions. Antolín-Díaz and Rubio-Ramírez (2018) instead, combine the narrative sign restrictions with the traditional sign restrictions. The main novelty of their work is that they use only a single event, or a small number of events instead of an entire time series when imposing narrative information. Moreover, they use a Bayesian method instead of the frequentist approach. They show that narrative sign restrictions can sharpen the inference of structural VARs.

A different identification strategy is proposed by Arias et al. (2019). The main novelty of their work is that they achieve identification by imposing sign and zero restrictions on the systematic components of the monetary policy rule instead of restricting the response on output. In detail, they impose positive sign restrictions on the contemporaneous response of the federal funds rate to an increase in output and prices, zero restrictions on the contemporaneous structural parameters of total reserves and nonborrowed reserves, and they keep unrestricted the response of the federal funds rate to commodity prices. They find that the monetary policy shock determines a decline in output with a high posterior probability. However, they have to add the conventional sign restrictions on the impulse response function (IRF) of prices, nonborrowed reserves and federal funds rate in order to rule out the price and liquidity puzzle.

Even if, there is no consensus about a particular set of assumptions for identifying the effects of monetary policy, there is substantial agreement about the qualitative effects of a shock. The

general consensus says that, after a contractionary monetary policy shock, short term interest rates rise, aggregate output and various monetary aggregates fall, aggregate price level decreases slowly, and various measures of wages fall. In addition, there is agreement that monetary policy shocks account for only a very modest percentage of the volatility of aggregate output and for even less volatility of the movements in the aggregate price level (Christiano et al., 1999).

Only a few authors argue that a contractionary monetary policy shock has a no-clear effect on output. Uhlig (2005) approaches the identification problem with a new agnostic method that let the data decide: he imposes sign restrictions on the impulse response function (IRF) of prices, nonborrowed reserves and federal funds rate while imposing no restrictions on the response of the real GDP. This agnostic identification procedure wants to criticize the conventional iden- tification strategies that impose zero restrictions on the contemporaneous response of output. He finds that a monetary policy tightening produces an unexpected expansion in output. The latter result is achieved also by Ramey (2016) who retrieved the same conclusion relaxing zero restriction on the results of Romer and Romer (2004). In its work, Wolf (2020) says that a pos- sible solution to solve the problem that the output can respond positively to a monetary policy tightening when only sign restrictions on the input response functions are applied, is to add sign restrictions on the Taylor rule (i.e. by adopting the approach proposed by Arias et al. (2019)).

In this work, we aim to identify a monetary policy shock in the U.S. economy. As an iden- tification strategy, we impose sign restrictions on both the IRFs and the systematic components of the monetary policy rule. However, we do not resort to zero restrictions on the coefficient of the monetary policy equation since, as argued by Arias et al. (2019), they have a negligible weight in shaping the effects of a monetary policy shock. In doing so, we consider a VAR that features macro-finance interactions (as financial indicators we use Excess bond premium (EBP) and National Financial Condition Index (NFCI)). Indeed, as demonstrated by Gertler and Karadi (2015), monetary policy shocks exert a significant effect on financial conditions, however, this effect is usually underestimated by recursive VARs (Castelnuovo, 2013). Our results are con- sistent with the economic theory: we have identified the monetary policy shock, ruled out the liquidity and price puzzle, and demonstrated that the central bank deals with financial market swings.

Chapter 2

The Model

In this chapter, we provide detailed insight into the structural VAR model and we derive the systematic components of the monetary policy rule. We then describe our identification strategy and the endogenous variables chosen for the two datasets employed. The first dataset, which ranges from 1973 to 2007, does not include the global financial crisis, while the second dataset, which ranges from 1973 to 2019 does.

2.1 VAR specification

Let us consider n time series. A structural VAR model with ν lags is, in matrix notation, given by:

$$
[y_{1,t} \ \cdots \ y_{n,t}]
\begin{bmatrix}
a_{0,11} & \cdots & a_{0,1n} \\
\vdots & \ddots & \vdots \\
a_{0,n1} & \cdots & a_{0,nn}
\end{bmatrix}
= \sum_{l=1}^{\nu} [y_{1,t-l} \cdots y_{n,t-l}]
\begin{bmatrix}
a_{11,t-l} & \cdots & a_{1n,t-l} \\
\vdots & \ddots & \vdots \\
a_{n1,t-l} & \cdots & a_{nn,t-l}
\end{bmatrix}
\tag{2.1}
$$

$$
+ [c_1 \ \cdots \ c_n] + [\varepsilon_{1,t} \ \cdots \ \varepsilon_{n,t}]
$$

at date $t = 1, \cdots, T$. In compact form representation, Eq. 2.1 becomes:

$$
\boldsymbol{y}_t' \boldsymbol{A}_0 = \sum_{l=1}^{\nu} \boldsymbol{y}_{t-l}' \boldsymbol{A}_l + \boldsymbol{c} + \boldsymbol{\varepsilon}_t' \quad \text{for } 1 \leq t \leq T
\tag{2.2}
$$

where $\boldsymbol{y}_t$ and $\boldsymbol{\varepsilon}_t$ are $n \times 1$ vectors of endogenous variables and structural shocks, respectively, $\boldsymbol{A}_0$ (invertible) and $\boldsymbol{A}_l$ are $n \times n$ matrices of structural parameters, $\boldsymbol{c}$ is a $1 \times n$ vector of parameters, and T is the sample size. Conditional on past information and initial conditions, the vector $\boldsymbol{\varepsilon}_t$ is assumed to be Gaussian with zero mean and covariance matrix $\boldsymbol{I}_n$. Thus, the structural VAR model in Eq. 2.2 can be rewritten as:

$$
\boldsymbol{y}_t' \boldsymbol{A}_0 = \boldsymbol{x}_t' \boldsymbol{A}_+ + \boldsymbol{\varepsilon}_t' \quad \text{for } 1 \leq t \leq T
\tag{2.3}
$$

where $\boldsymbol{A}'_+ = [\boldsymbol{A}'_1 \ \ldots \ \boldsymbol{A}'_\nu \ \boldsymbol{c}']$ and $\boldsymbol{x}'_t = [\boldsymbol{y}'_{t-1} \ \ldots \ \boldsymbol{y}'_{t-\nu} \ 1]$. The matrix $\boldsymbol{A}_+$ has dimensions of $(n\nu + 1) \times n$. In its reduced-form, Eq. 2.3 becomes:

$$\boldsymbol{y}'_t = \boldsymbol{x}'_t \boldsymbol{B} + \boldsymbol{u}'_t \quad \text{for } 1 \leq t \leq T \tag{2.4}$$

where $\boldsymbol{u}'_t$ is the one-step-ahead prediction error with variance-covariance matrix $\boldsymbol{\Sigma}$. The error $\boldsymbol{u}'_t$ is composed of economically meaningful and fundamental innovations. Since people are usually interested in evaluating the impulse response functions to the fundamental innovations, $\boldsymbol{u}'_t$ must be decomposed. How to decompose $\boldsymbol{u}'_t$ is source of disagreement. Following Uhlig (2005), we assume the fundamental innovations to be mutually independent, and thus the matrix $\boldsymbol{A}_0^{-1}$ is such that:

$$\boldsymbol{u}'_t = \boldsymbol{\varepsilon}'_t \boldsymbol{A}_0^{-1} \tag{2.5}$$

Therefore, a consistent estimation of $\boldsymbol{\Sigma}$ is given by:

$$\boldsymbol{\Sigma} = \mathbf{E}[\boldsymbol{u}_t \boldsymbol{u}'_t] = \boldsymbol{A}_0^{-1} \mathbf{E}[\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}'_t] \boldsymbol{A}_0^{-1'} = \boldsymbol{A}_0^{-1} \boldsymbol{I}_n \boldsymbol{A}_0^{-1'} = (\boldsymbol{A}_0 \boldsymbol{A}'_0)^{-1} \tag{2.6}$$

Note that, the matrices $\boldsymbol{B} = \boldsymbol{A}_+ \boldsymbol{A}_0^{-1}$, which appears in Eq. 2.4, and $\boldsymbol{\Sigma}$ are the reduced-form parameters.

The structural VAR in Eq. 2.3 can be written in terms of orthogonal reduced-form parametrization (Arias et al., 2018):

$$\boldsymbol{y}'_t = \boldsymbol{x}'_t \boldsymbol{B} + \boldsymbol{\varepsilon}'_t \boldsymbol{Q}' h(\boldsymbol{\Sigma}) \quad \text{for } 1 \leq t \leq T \tag{2.7}$$

In Eq. 2.7, $\boldsymbol{B}$ and $\boldsymbol{\Sigma}$ are reduced-form parameters, $\boldsymbol{Q}$ is an orthogonal matrix defined from the QR decomposition of $\boldsymbol{X}$ (i.e. $\boldsymbol{X} = \boldsymbol{Q}\boldsymbol{R}$ with $\boldsymbol{R}$ being a diagonal and positive-defined matrix), and $h(\boldsymbol{\Sigma})$ is a $n \times n$ matrix that refers to the Cholesky decomposition of the covariance matrix $\boldsymbol{\Sigma}$. It holds that $h(\boldsymbol{\Sigma})' h(\boldsymbol{\Sigma}) = \boldsymbol{\Sigma}$. As argued by Arias et al. (2018), the orthogonal reduced-form parametrization is convenient for drawing. However, we are interested in making draws from the structural parametrization and thus, we need an invertible function in order to transform $(\boldsymbol{B}, \boldsymbol{\Sigma}, \boldsymbol{Q})$ into $(\boldsymbol{A}_0, \boldsymbol{A}_+)$. This function is defined as:

$$f_h(\boldsymbol{A}_0, \boldsymbol{A}_+) = (\underbrace{\boldsymbol{A}_+ \boldsymbol{A}_0^{-1}}_{\boldsymbol{B}}, \underbrace{(\boldsymbol{A}_0 \boldsymbol{A}'_0)^{-1}}_{\boldsymbol{\Sigma}}, \underbrace{h((\boldsymbol{A}_0 \boldsymbol{A}'_0)^{-1}) \boldsymbol{A}_0}_{\boldsymbol{Q}}). \tag{2.8}$$

while its inverse is given by:

$$f_h^{-1}(\boldsymbol{B}, \boldsymbol{\Sigma}, \boldsymbol{Q}) = (\underbrace{h(\boldsymbol{\Sigma}^{-1} \boldsymbol{Q}}_{\boldsymbol{A}_0}, \underbrace{\boldsymbol{B} h \boldsymbol{\Sigma}^{-1} \boldsymbol{Q}}_{\boldsymbol{A}_+}). \tag{2.9}$$

For any $\boldsymbol{B}$, $\boldsymbol{\Sigma}$ and h, we can see each value of $\boldsymbol{Q} \in \mathcal{O}(n)$ as a specific set of structural parameters (Arias et al., 2018).

To sum up, there are three different ways to express VARs: the structural VAR that is defined by $(\boldsymbol{A}_0, \boldsymbol{A}_+)$, the reduced-form VAR and the orthogonal reduced-form VAR characterized by $(\boldsymbol{B}, \boldsymbol{\Sigma}, \boldsymbol{Q})$.

2.1.1 Conjugate, Priors and Posteriors

Even if the technique developed by Arias et al. (2018) and implemented here theoretically works for any prior distributions, it is more efficient when these prior distributions are conjugate. A family of distribution is conjugate when the posteriors are members of the same probability distribution family as the priors. A family of conjugate distributions for the reduced-form VAR reported in Eq. 2.4 is represented by the normal-inverse-Wishart distribution, whose density is (Arias et al., 2018):

$$\text{NIW}_{(\nu, \boldsymbol{\Phi}, \boldsymbol{\Psi}, \boldsymbol{\Omega})}(\boldsymbol{B}, \boldsymbol{\Omega}) \propto \underbrace{|\det(\boldsymbol{\Sigma})|^{-\frac{\nu+n+1}{2}} e^{-\frac{1}{2}\text{tr}(\boldsymbol{\Phi}\boldsymbol{\Sigma}^{-1})}}_{inverse-Wishart} \cdot$$

$$\underbrace{|\det(\boldsymbol{\Sigma})|^{-\frac{m}{2}} e^{-\frac{1}{2}\text{vec}(\boldsymbol{B}-\boldsymbol{\Psi})'(\boldsymbol{\Sigma}\otimes\boldsymbol{\Omega})'\text{vec}(\boldsymbol{B}-\boldsymbol{\Psi})}}_{conditionally\ normal} \tag{2.10}$$

where $\nu \geq n$ is a scalar, $\boldsymbol{\Phi}$ and $\boldsymbol{\Omega}$ are symmetric and positive definite matrices of dimensions $n \times n$ and $m \times m$ respectively, and $\boldsymbol{\Psi}$ is an $m \times m$ matrix. Let us now define the priors over the reduced form parameter as $\text{NIW}(\bar{\nu}, \bar{\boldsymbol{\Phi}}, \bar{\boldsymbol{\Psi}}, \bar{\boldsymbol{\Omega}})$, and the posterior of the reduced-form parameter as $\text{NIW}(\tilde{\nu}, \tilde{\boldsymbol{\Phi}}, \tilde{\boldsymbol{\Psi}}, \tilde{\boldsymbol{\Omega}})$ where the posteriors parameter are:

$$\tilde{\nu} = T + \bar{\nu} \tag{2.11}$$

$$\tilde{\boldsymbol{\Phi}} = \boldsymbol{Y}'\boldsymbol{Y} + \bar{\boldsymbol{\Phi}} + \bar{\boldsymbol{\Psi}}'\bar{\boldsymbol{\Omega}}^{-1}\bar{\boldsymbol{\Psi}} - \tilde{\boldsymbol{\Psi}}'\tilde{\boldsymbol{\Omega}}^{-1}\tilde{\boldsymbol{\Psi}} \tag{2.12}$$

$$\tilde{\boldsymbol{\Psi}} = \tilde{\boldsymbol{\Omega}}(\boldsymbol{X}'\boldsymbol{Y} + \bar{\boldsymbol{\Omega}}^{-1}\bar{\boldsymbol{\Psi}}) \tag{2.13}$$

$$\tilde{\boldsymbol{\Omega}} = (\boldsymbol{X}'\boldsymbol{X} + \bar{\boldsymbol{\Omega}}^{-1})^{-1} \tag{2.14}$$

for $\boldsymbol{Y} = [\boldsymbol{y}_1, \ldots, \boldsymbol{y}_T]'$ and $\boldsymbol{X} = [\boldsymbol{x}_1, \ldots, \boldsymbol{x}_T]'$. If we take $\pi(\boldsymbol{Q}|\boldsymbol{B}, \boldsymbol{\Sigma})$ as uniform conditional density over $\mathcal{O}(n)$, we have the uniform normal-inverse-Wishart $\text{UNIW}(\nu, \boldsymbol{\Phi}, \boldsymbol{\Psi}, \boldsymbol{\Omega})$. As done in Arias et al. (2018), the prior density parametrization is $\nu = 0$, $\boldsymbol{\Phi} = \boldsymbol{0}_{n\times n}$, $\boldsymbol{\Psi} = \boldsymbol{0}_{m\times n}$ and $\boldsymbol{\Omega}^{-1} = \boldsymbol{0}_{m\times m}$. This parametrization results in prior densities analogues to those obtained by Uhlig (2005). Once the UNIW posteriors are known, we use the invertible function to transform the draws to the structural parametrization $(\boldsymbol{A}_0, \boldsymbol{A}_+)$. This is called a normal-generalized-normal distribution over the structural parametrization $\text{NGN}(\nu, \boldsymbol{\Phi}, \boldsymbol{\Psi}, \boldsymbol{\Omega})$ and has density $\text{NGN}_{(\nu, \boldsymbol{\Phi}, \boldsymbol{\Psi}, \boldsymbol{\Omega})}(\boldsymbol{A}_0, \boldsymbol{A}_+)$. In terms of algorithms for drawing from the uniform distribution over $\mathcal{O}(n)$, there are several examples in the literature. Here we use the algorithm of Rubio-Ramirez et al. (2010).

2.2   The Systematic Component of the Monetary Policy

A large fraction of the variation in monetary policy instruments can be attributed to the systematic reaction of policy authorities to the state of the economy (Leeper et al., 1996; Sims and Zha, 2006). Thus, as argued by Arias et al. (2019), it is necessary to specify the systematic components of the monetary policy in order to identify a monetary policy shock. If we consider the first shock being the monetary policy shock, we obtain from the structural VAR (i.e. from Eq.2.1) the following monetary policy equation:

$$
[y_{1,t} \; \cdots \; y_{n,t}]
\begin{bmatrix} a_{0,11} \\ \vdots \\ a_{0,n1} \end{bmatrix}
= \sum_{l=1}^{\nu} [y_{1,t-l} \cdots y_{n,t-l}]
\begin{bmatrix} a_{11,t-l} \\ \vdots \\ a_{n1,t-l} \end{bmatrix}
+ \varepsilon_{1,t}
\tag{2.15}
$$

that is, in compact notation form, equivalent to:

$$
\boldsymbol{y}_t' \boldsymbol{a}_{0,1} = \sum_{l=1}^{\nu} \boldsymbol{y}_{t-l}' \boldsymbol{a}_{l,1} + \varepsilon_{1,t} \quad \text{for } 1 \le t \le T
\tag{2.16}
$$

where $\varepsilon_{1,t}$ is the structural shock (i.e. first entry of $\boldsymbol{\varepsilon}_t$), and $\boldsymbol{a}_{l,1}$ is the first column of $\boldsymbol{A}_l$ with $a_{l,ij}$ being the systematic component (i.e. the i,j entry of $\boldsymbol{A}_l$). Note that, imposing sign restrictions on these latter components means restricting the systematic component of the monetary policy. The monetary policy rule is then obtained by abstracting Eq. 2.15 from lags:

$$
[y_{1,t} \; \cdots \; y_{n,t}]
\begin{bmatrix} a_{0,11} \\ \vdots \\ a_{0,n1} \end{bmatrix}
= \varepsilon_{1,t}
\tag{2.17}
$$

The general form for the monetary policy rule reads:

$$
y_{n,t} = -\frac{a_{0,11}}{a_{0,n1}} y_{1,t} - \cdots + \frac{\varepsilon_{1,t}}{a_{0,n1}}
\tag{2.18}
$$

where the $\psi_i \equiv -a_{0,i1} a_{0,n1}^{-1}$ are the contemporaneous coefficients.

2.3   Identification Strategy

According to the Taylor-type monetary policy rule, the monetary policy reacts contemporaneously to changes in economic activity and prices (Christiano et al., 1996). Furthermore, unexpected monetary policy shocks strongly affect financial conditions (Castelnuovo, 2013; Rüth, 2017). Thus, to implement our identifications, our reduced-form VAR specification consists of six endogenous variables: output, y_t; prices, p_t; commodity prices, $p_{c,t}$; excess bond premium, e_t; national financial condition index, n_t; and interest rate, r_t. Notably, the interest rate is either

the federal fund rate (see section 2.4) or the policy rate (see section 2.5).

To achieve identification, we have imposed sign restrictions on both the IRFs and the systematic component of the monetary policy equation (i.e. structural parameters). The latter is done by using the Bayesian approach and the techniques developed in Arias et al. (2018) and Arias et al. (2019). Our restrictions can be summarized as follow.

Restriction 1

A monetary policy shock implies a negative response of prices and commodity prices (Uhlig, 2005), and a positive response of EBP, NFCI and interest rate (Gilchrist and Zakrajvsek, 2012). This is achieved by assigning sign restrictions on the IRFs of the above-mentioned variables. However, we do not impose restrictions on the IRF of the output. This agnostic procedure, which let the data decide, is analogues to the identification scheme proposed by Uhlig (2005). For this reason, in this work, we will refer to Restriction 1 also as Uhlig's sign restriction. These restrictions have been applied for five periods after the shock (at all horizons $t = 0, ..., 5$).

Restriction 2

The contemporaneous reaction of the interest rate to output and prices is positive (Arias et al., 2019). This implies assigning sign restrictions on the systematic component of the monetary policy equation. This is consistent with the monetary policy rule of the New Keynesian DSGE model (see Section 1.3). In this case, we assume that the systematic responses of output and price react instantaneously to a tightening in the monetary policy, and thus, the restriction is assigned only at $t = 0$. As explained by Arias et al. (2019), this assumption is reliable since the central bank does not have access to real-time data on output and price, but it does have access to a huge amount of real-time indicators. However, we assume that the central bank can retrieve the data within a month.

Restriction 3.

The monetary shock affects term premia and credit spread by increasing credit costs (Gertler and Karadi, 2015), and exerts a significant effect on the overall financial conditions (Castelnuovo, 2013). Specifically, the contemporaneous reaction of the interest rate to EBP and NFCI is negative (Gilchrist and Zakrajvsek, 2012; Brave and Butters, 2012). Analogously to restriction 2, this is done by restricting the systematic components of the monetary policy rule.

For the dataset under consideration, we obtain from Eq. 2.18 the following monetary policy rule:

$$r_t = -\frac{a_{0,11}}{a_{0,61}}y_t - \frac{a_{0,21}}{a_{0,61}}p_t - \frac{a_{0,31}}{a_{0,61}}pc_t - \frac{a_{0,41}}{a_{0,61}}e_t - \frac{a_{0,51}}{a_{0,61}}n_t + \frac{\varepsilon_{1,t}}{a_{0,61}} \qquad (2.19)$$

that can be rewritten, in terms of contemporaneous structural parameters, as:

$$r_t = \psi_y y_t + \psi_p p_t + \psi_{pc} pc_t + \psi_e e_t + \psi_n n_t + \sigma \varepsilon_{1,t} \tag{2.20}$$

Equipped with this representation of the monetary policy rule, we summarize restrictions 2 as follows: ψ_y, $\psi_p > 0$. Restriction 3, instead, implies that ψ_e, $\psi_n < 0$. ψ_{pc} remains always unconstrained.

Note that we only identify a single structural shock and thus, our structural parameters are partially identified. Furthermore, we impose only inequality constraints (i.e. sign restrictions), and hence, they are set identified (and not point identified).

2.4   Short Dataset (1973 - 2007)

For the benchmark case study, the reduced-form VAR has been applied on monthly based U.S. data from January 1973 to June 2007. This length of time ensures that we do not capture in our predictions the effects of the global financial crisis and unconventional monetary policies. The dataset is composed of 1) real GDP for output, 2) GDP deflator for the price level, 3) a commodity price index, 4) the EBP as a proxy for the credit supply disruption, 5) the NFCI as a comprehensive measure of financial stress in the system, and 6) the federal found rate. Note that, the dataset is analogue to the dataset previously used by Uhlig (2005) and Arias et al. (2019) with two main differences: the introduction of EBP and NFCI instead of total reserves and nonborrowed reserves. The employment of these two variables allows us investigating how central banks react to stress in the credit market or stress in the financial system as a whole.

2.4.1   Real Output

The Gross Domestic Product, or GDP, is the value of all finished goods and services that have been produced in a given economy and in a given time. We can measure the GDP in nominal terms or in real terms. The difference between the two is that the real GDP is adjusted for inflation, while the nominal GDP is not.

The real GDP is usually used as an economic indicator for measuring the output. Here, we employ the GDPC1 retrieved from FRED, which has quarterly frequency and concerns the U. S. economy. Since for our structural VAR model we need monthly data, we interpolate the time series for GDPC1 with a different measure of output that is available monthly: the industrial production index that is referred to as INDPRO.

The Industrial Production Index (INDPRO) is an economic indicator that measures the real output for all the manufacturing, mining, electric, and gas utility industries located in the U.S. (Board of Governors of the Federal Reserve System (US), May 2, 2022b). The index is made of several individual series based on those industries included in the North American Industry

Classification System (NAICS). The INDPRO, as well as the other Industrial Production indexes, has two main sources of data: the outputs measured in physical units and the inputs to the production process, from which the output is inferred (Board of Governors of the Federal Reserve System (US), May 2, 2022b). The data are taken on a monthly basis from FRED.

The interpolation of the real GDP is done accordingly to the method developed by Chow and Lin (1971) for temporal disaggregation (i.e. temporal disaggregation process allows high-frequency data to be retrieved from lower frequency data), and previously employed also by Bernanke and Mihov (1998b) and Mönch and Uhlig (2005). We have chosen this methodology because it has been proven to be the best method for the set of series used (Mönch and Uhlig, 2005). In brief, the Chow and Lin (1971) methodology retrieves unobserved monthly values (y) from quarterly data by taking advantage of a variable (X) available monthly, which, however, contains information about (y):

$$y = X\beta + u \tag{2.21}$$

Assuming u to be an $AR(1)$ with an unknown serial correlation coefficient ρ, the error covariance matrix V takes the standard form $V = \mathbb{E}(uu')$. Eq. 2.21 can be re-written as:

$$Cy = CX\beta + Cu \tag{2.22}$$

where C is a $t \times 3t$ matrix required to convert quarterly data to monthly information. Eq. 2.22 yields:

$$\dot{y} = \dot{X}\beta + \dot{u} \tag{2.23}$$

Since GDP has a quarterly frequency, Eq 2.23 can be consistently estimated by OLS. We can retrieve a consistent estimate of the monthly serial correlation coefficient ρ from the first-order correlation coefficient of the quarterly residuals. Through ρ, it is then possible to build up the covariance matrix of the quarterly residuals, $\hat{V}$, and to re-estimate Eq. 2.23 by GLS. According to Chow-Lin's formula, the estimated monthly values for the component $\hat{y}$ of the GDP are:

$$\hat{y} = X\hat{\beta} + \hat{V}C'(C\hat{V}C')^{-1}\hat{u} \tag{2.24}$$

By applying this formula, the average of the monthly values in each quarter is equal to the corresponding quarterly value (Chow and Lin, 1971).

2.4.2   GDP Deflator

The GDP deflator is an economic indicator that provides a measure of the overall level of prices in an economy. It is computed as the ratio between the nominal GDP and the real GDP, and it shows the effect of movement in prices rather than changes in the volume of the output. For this reason, the GDP deflator is a measure of inflation in a given economy.

The data for the GDP deflator (GDPDEF) are retrieved from FRED. These are quarterly

frequency data that have to be interpolated. The methodology used for the interpolation is the one proposed by Chow and Lin (1971) and already discussed in section 2.4.1. However, in this case, interpolation is done thanks to two monthly series related to the deflator: a consumer price index and a producer price index.

The consumer price index (CPI) is a different method to measure the level of inflation. A CPI is made by creating a hypothetical basket in which selected goods and services are observed over time. The basket reflects the average price level and hence, the decline in the purchasing power of the consumers. Even if both the CPI and the GDP deflator provide the overall level of prices in the economy (i.e. inflation), they have some differences. First, the GDP deflator measures the prices of all goods and services produced, while the CPI measures the price level of the goods and services that are bought by consumers. Another difference regards the fact that the GDP deflator accounts only for goods and services that are produced domestically, whereas the CPI is affected also by their import. Last but not least, the CPI is estimated using a fixed basket of goods and services, instead, the composition of GDP changes over time. In this work, as consumer price index, the CPIAUCSL retrieved from FRED has been employed.

Analogously to the CPI, the producer price index (PPI) measures the level of prices in a basket of goods and services. Nevertheless, the buyers considered in the PPI are not consumers but firms (Mankiv, 2016). In this sense, a PPI provides a measure of inflation based on input cost to producers and returns a metric for the cost of production (of Labor Statistics, May 2, 2022). As producer price index, we resort to the PPIFGS, taken from FRED.

The PPI, as well as the CPI, are published on monthly basis by the Bureau of Labor Statistics (BLS) in the U.S. for the domestic market. For these two indices, data from almost 10,000 products are collected.

2.4.3   Commodity Price Index

The employment of a commodity price index in the dataset is an established practice for the identification of a monetary policy shock, and it comes to solve what is called the liquidity puzzle. The liquidity puzzle is a positive relationship between inflation and federal funds rate (Balke and Emery, 1994). It is defined as a puzzle because it consists of an unexpected increase in the price level after a monetary policy shock, even if a sudden increase in the interest rate should lead to a decrease in the price level (Sims, 1992; Bernanke and Blinder, 1992).

The positive correlation between inflation and interest rate in the U.S. economy has been observed by many authors and different explanations have been provided in the literature (Bernanke and Blinder, 1992; Sims, 1992; Christiano et al., 1994). In their work, Bernanke and Blinder (1992) found that the federal funds rate is informative in forecasting macroeconomic variables and a good indicator of monetary policy actions. In order to understand better this relation, Balke and Emery (1994) broke down the inflation time series into two sub-samples: 1960-79 and 1982-93. As Fig. 2.1 shows, they found evidence of a strong positive relationship between
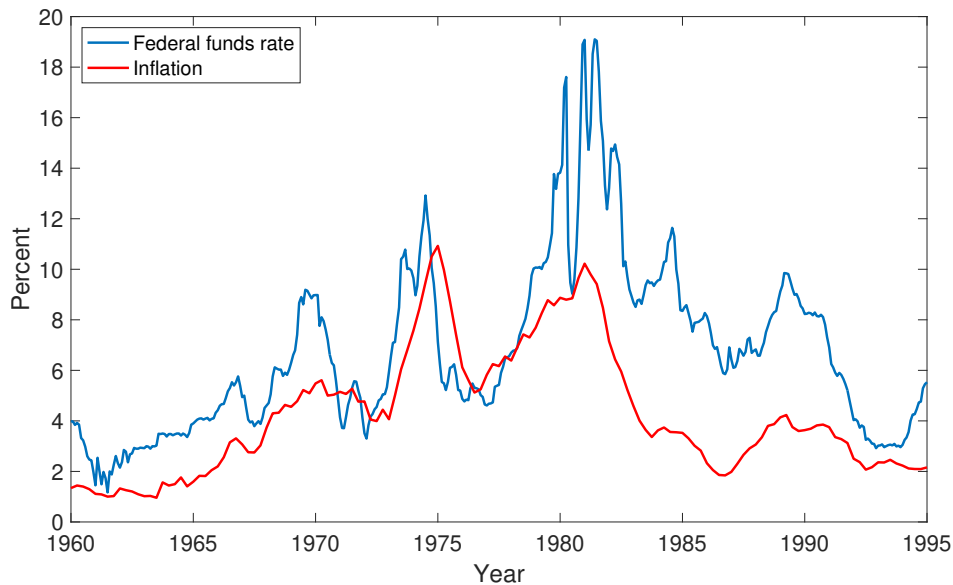
Figure 2.1: Percentage value of federal funds rate (FEDFUNDS) and inflation (GDPDEF_ PC1) from 1960 until 1995. Source: FRED.

the federal funds rate and inflation in the first sample (i.e. the liquidity puzzle is severe), while this positive correlation became weaker, until approaching zero, in the second sub-sample (i.e. but still not negative). The reason why the price puzzle has changed in the early 80s is related to a change in the reaction of the Federal reserves corresponding to the beginning of Paul Volcker's mandate as chairman of the Federal reserves. Regarding the possible explanations for the price puzzle, these are:

- The Federal Reserve increases the federal funds rate when higher future inflation is expected, nevertheless this increase is not enough for preventing inflation to rise. This means that even if the federal reserve is forward-looking, it is not able to prevent upcoming inflation. Moreover, the systematic response to a monetary policy shock is not truly exogenous, since the Federal Reserve owns information about future inflation (Sims, 1992; Balke and Emery, 1994).

- The Federal Reserve responds to the supply shock by raising the federal funds rate. This rise, however, is not enough for extinguishing the inflationary consequences of the supply shock (Balke and Emery, 1994).

As argued by Christiano et al. (1994), in a VAR framework, the price puzzle may be solved with the inclusion of a commodity price index. For the short dataset we chose as commodity price index the Dow Jones Spot Average (Ticker _DJSD) that is commercially available from Global Financial Data. The measure is on monthly basis and is computed as the average of daily data.

2.4.4 Excess Bond Premium

The credit spread proposed by Gilchrist and Zakrajvsek (2012) is a financial indicator that can be split into two different components: the first one reflects the systematic movements in the default risk of individual firms; the second one provides an effective measure of investor sentiment or risk appetite in the corporate bond market. This second component is the so-called Excess Bond Premium.

The EBP is simply obtained as the difference between the GZ credit spread, S_t^{GZ}, and the predicted component of the GZ credit spread, $\hat{S}_t^{GZ}$:

$$EBP_t = S_t^{GZ} - \hat{S}_t^{GZ} \tag{2.25}$$

In turn, S_t^{GZ} is an arithmetic average of credit spreads on outstanding bonds in any given month:

$$S_t^{GZ} = \frac{1}{N_t} \sum_i \sum_k S_{it}[k] \tag{2.26}$$

where N_t is the number of bond/firm observations in month t, and $S_{it}[k]$ is the difference between the corporate bond yield, $y_{it}[k]$, and the synthetic risk-free security yield, $y_t^f[k]$, for bond k of firm i. The synthetic risk-free security replicates the cash flow of a corresponding corporate debt instrument. The cash flow deriving from the coupon payments and the principal has been discounted with the continuously compounded zero-coupon Treasury yields. In this way, it has been possible to compare the two yields without incurring in "duration mismatch" and biases (Gilchrist and Zakrajvsek, 2012).

The predicted component of the GZ credit spread has been obtained in accordance with the work of Berndt et al. (2011). They first used the following linear regression to remove the expected default risk of individual firms from the underlying credit spreads:

$$\ln S_{it}[k] = \beta DFT_{it} + \hat{\gamma}' \mathbf{Z}_{it}[k] + \varepsilon_{it}[k] \tag{2.27}$$

This equation states that the log of $S_{it}[k]$ is linearly related to the firm-specific measure of expected default, DFT_{it}, and a bond-specific characteristic vector, $\mathbf{Z}_{it}[k]$, with a pricing error $\varepsilon_{it}[k]$. Under the assumption of normally distributed disturbances, this yields to:

$$\hat{S}_{it}[k] = \exp\left[\hat{\beta} DFT_{it} + \hat{\gamma}' \mathbf{Z}_{it}[k] + \frac{\hat{\sigma}^2}{2}\right] \tag{2.28}$$

where $\hat{\cdot}$ indicates OLS estimated variables. The predicted component of the GZ credit spread is then averaged across outstanding bonds to the firms at time t analogously to what is done in Eq. 2.26 for $S_{it}[k]$.

The EBP is a robust financial indicator for predicting future economic activity (Gilchrist and Zakrajvsek, 2012). It has been used by the Fed to predict the probability that the U.S. economy

will enter a recession sometime during the next 12 months. Moreover, the results of Brave and Butters (2011) show that the EBP provides a consistent indicator of credit supply conditions: a shortening in the credit supply (an increase in the EBP) causes a contraction in economic activity through the financial accelerator. This is the main reason why it is chosen for our identification strategy. The data are retrieved from the Board of Governors of the Federal Reserves System.

2.4.5 National Financial Conditions Index

In order to have a comprehensive measure of the economic and financial system's health, the Chicago Fed releases, on weekly basis, the National Financial Conditions Index. The NFCI is a high-frequency index that widely involves a measure of risk, a measure of liquidity and a measure of leverage. The former refers to risk premium and volatility of asset prices, the liquidity concerns the willingness to borrow or lend at the prevailing prices, whereas the leverage provides a relative measure for financial debt with respect to equity.

The NFCI is calculated as a weighted average of 105 indicators of financial activity (Brave and Butters, 2011). Each weight is assigned to the indicators through a statistical method called principal component analysis (PCA) according to which the indicators more correlated with their peers receive higher weights. Brave and Butters (2011) have divided the indicators into three categories: 1) money markets, 2) debt and equity markets, and 3) banking system. The money markets category is composed of interest rate spreads and other financial conditions indexes. Moreover, it includes measures of implied volatility and trading volumes of numerous money market financial products. The debt and equity market indicators are mainly composed of stocks and bond market prices, but also residential and commercial real estate prices, municipal and corporate bonds, stock, asset-backed security, and credit derivative market volumes. Note that, the metrics used in this category consider not only the prices but also volatility and risk premiums in their various forms. Then, the banking system category comprises mostly survey-based measures of credit availability, accounting-based measures for commercial banks and so-called shadow banks, but also some interest rate spreads. The first scope of this subindex is to measure liquidity and leverage in the system as well as to catch the risk deriving from the deterioration of credit quality.

The index is expressed relative to its sample standard deviation, thus, a zero value indicates that the financial system is working at the historical average levels of risk, liquidity, and leverage. In the sub-indexes, the risk is accounted for with positive weights, instead, liquidity and leverage have negative weights. An index with positive values implies tighter conditions than on average, instead, negative values mean looser conditions than on average.

Since the NFCI provides a comprehensive measure of U.S. financial conditions, it is chosen as variable for our identification strategy. The data are taken on a monthly basis from FRED.

2.4.6 Federal Funds Rate

The short-term policy instrument for the Fed is the federal funds rate. It is the overnight interest rate at which banks lend to each other (Mankiv, 2016). When banks experience an excess of liquidity, they lend to other depository institutions, which have shortages of cash and need funds as soon as possible. This comes not for free: the bank borrowing will pay interests to the lender. When there is no need for extra liquidity in the interbank market, banks with balance surpluses can deposit at the Fed overnight. The Central Bank will pay interest on the deposit. This interest corresponds to the federal funds rate. The interest paid by the central bank for the deposit must be lower than the interest paid in the interbank market. In this way the Fed uses the federal funds rate, as policy instrument for the implementation of the monetary policy, influencing the other interest rates. The nominal interest rate is set by the Federal Open Market Committee (FOMC), which meets every six weeks and votes on the interest rate target. The interest rate target is set on the market through open market operation, so increasing or decreasing the supply of money in the market. More specifically, when there is the need for expansionary monetary policy, the Fed buys government bonds on the market leading to a decrease in the interest rate. However, for increasing the federal funds rate, the Fed sells bonds and consequently reduces liquidity in the market.

The effective federal funds rate (EFFR) is the most common measure of interest rate for the U.S. economy and is often used in the VAR analysis (Uhlig, 2005; Leeper et al., 1996; Leeper and Zha, 2003; Wu and Xia, 2015; Arias et al., 2018, 2019). Since the rate at which banks borrow and lend money to each other is determined by bilateral negotiations, the effective federal funds rate is computed as the weighted average rate for all of the negotiations (Board of Governors of the Federal Reserve System (US), May 2, 2022a). As explained above, this rate is slightly higher compared to the target rate settled by the FOMC. Although the effective rate is influenced by the target, it tracks the interest rate that clears the interbank market. The data are retrieved from FRED with monthly frequency, and with the ticker FEDFUNDS.

2.5 Extended Dataset (1973 - 2019)

Besides the short dataset, a second case has been studied. In this case, the dataset comprises longer time series, ranging from January 1973 to December 2019. The choice of the time interval is not casual indeed, it includes the global financial crisis previously discharged but it does not account for the Covid-19 pandemic crisis. As discussed in section 2.5.1, indeed, the pandemic crisis introduces problematic outliers in the time series.

The extended dataset has the same composition as the short dataset, except for the federal found rate. The dataset is composed of real GDP for output, GDP deflator for the price level, a commodity price index, the EBP, the NFCI, and the Policy Rate. Nevertheless, due to missing data in some of the previously employed series, we have searched for different proxies. A detailed

description of these indexes is reported below (after Section 2.5.1).

2.5.1 The Pandemic Outlier Problem

Beyond the problem for the real economy and population health, the Covid-19 pandemic cri-sis has introduced troubles also in modelling VARs. This issue arises because of the extreme observations that the pandemic has generated in the key macroeconomics variables. For some variables, indeed, this variation has been measured in two orders of magnitude (Lenza and Prim-iceri, 2020) and represents an unprecedented movement. These outlier observations represent an issue for the estimation of standard time-series models. Understanding how to manage the outlier data, if it is necessary to discard or use these observations as conventional observations if the outlier can or cannot distort the model, are crucial questions that will affect future research, because the outliers deriving from the pandemic will contaminate future works. The solution proposed by Lenza and Primiceri (2020) is to explicitly model the change in shock volatility, to account for the exceptionally large macroeconomic innovations recorded during the period of the pandemic. What makes this volatility shock singular is the perfect knowledge of the moment for increasing the variance.

In their work, Lenza and Primiceri (2020) have first estimated a standard VAR, considering pandemic observations, with monthly U.S. economic variables, and simulated a shock in unem-ployment. The results of their attempt are completely dependent on the finals observations, i.e. from the pandemic time. The Outcome is significantly different with respect to the IRFs obtained by employing only pre-pandemic observations. This leads to the conclusion that the few obser-vations recorded during the pandemic affect the entire sample. For this reason, it is necessary to disaggregate the latest data in the time series before using these types of observations with the ad-hoc procedure of dripping the extreme observations (Lenza and Primiceri, 2020). To analyse the outlier data together with the regular observations, Lenza and Primiceri (2020) first re-scaled the outlier with unknown parameters $\bar{s}_0, \bar{s}_1, \ldots, \bar{s}_j$ estimated by Bayesian approach (Giannone et al., 2015) or likelihood estimation. In doing so, they assumed that the re-scaling is common to all shocks, as previously done also by Carriero et al. (2016). Considering a standard VAR:

$$y_t = C + \boldsymbol{B}_1 y_{t-1} + \ldots + \boldsymbol{B}_p y_{t-} + s_t \varepsilon_t$$

$$\varepsilon_t \sim N(0, \boldsymbol{\Sigma})$$

(2.29)

To re-scale the residual covariance matrix, s_t have been used. Observations with $\bar{s}_0 = \bar{s}_1 = \bar{s}_j = 1$ are treated as regular observations, while observations during the pandemic are pointed out with t^*. Then they have modelled the evolution of the residual variance after the pandemic began with $s_{t^*} = \bar{s}_0, s_{t^*+1} = \bar{s}_1, s_{t^*+2} = \bar{s}_2, \ldots$, assuming that it decays at a constant rate ρ, such as:

$$s_{t^*+j} = 1 + (\bar{s}_2 - 1)\rho^{j-2}$$

(2.30)

then the model is estimated as usual.

In order to verify the model, Lenza and Primiceri (2020) estimated a VAR with a sample containing pandemic observations. They estimate IRFs with a standard VAR and with the VAR they proposed. The median responses are very similar between the two models but significantly different from the benchmark model. These findings illustrate the importance of explicitly modelling the change in shock volatility during the COVID-19 era (Lenza and Primiceri, 2020). The authors' main finding is the necessity of an ad-hoc procedure for the estimation of outliers observation. To this aim, they recommend using their approach, or to drop the data from the pandemic when using standard VAR approaches. Including Outliers in the dataset using standard methods perform poorly in VAR analysis. This approach is more straightforward and allows for more flexibility with respect to the time-varying volatility standard methods.

In light of these considerations, we choose to discard observations deriving from the pandemic by cutting our dataset to December 2019. This allows us to avoid outlier problems and work with standard model VARs.

2.5.2 GDP Deflator

As well as in the short dataset, also in the extended dataset, it has been necessary to interpolate the quarterly data available for the GDPDEF with higher frequency data from proxy, in order to obtain on a monthly measure of the GDP deflator. However, the series for the Producer Price Index previously used, the PPIFGS, ends in 2015. For this reason, we found a good substitute in the Producer Price Index by Commodity (PPIACO). The data are taken on monthly basis from FRED. In Fig. 2.2 the two series, producer price index by commodity for finished goods PPIFGS and producer price index by commodity all commodity PPIACO are reported. As expected, the series show similar co-movements and have a high correlation: $\rho = 0,9538$. The reason for such a high correlation is that the two indexes are different only for a switch in the data aggregation system. In fact, from January 2014, but effectively from January 2016, the BLS transitioned PPIs from a stage of processing aggregation system (SOP) to final demand - intermediate demand aggregation system (FD_ID).

2.5.3 Commodity Price Index

The series we use in the short dataset as commodity price index ceased in 2009. In order to create a dataset made of open-source data, we substituted the index previously used, the Dow Jones spot average _DJSD, with producer price index by commodity all commodity PPIACO, discussed in section 2.5.2. Fig. 2.3 reports the two series. In this case, the series show a correlation of $\rho = 0.6694$.
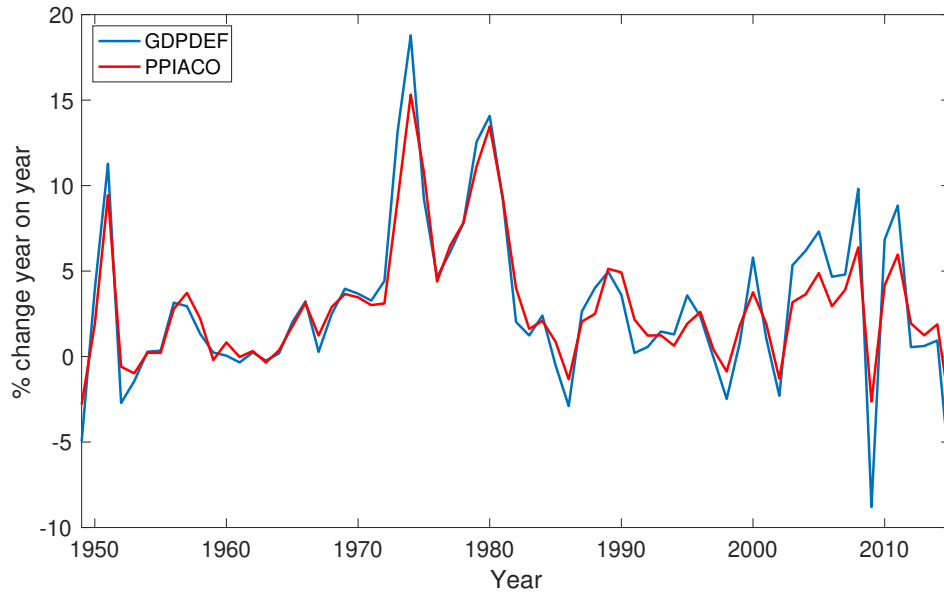
Figure 2.2: Percentage change year on year of PPIFGS and PPIACO from 1949 to 2015. The two time series show $\rho = 0.9538$ correlation. Source: FRED.
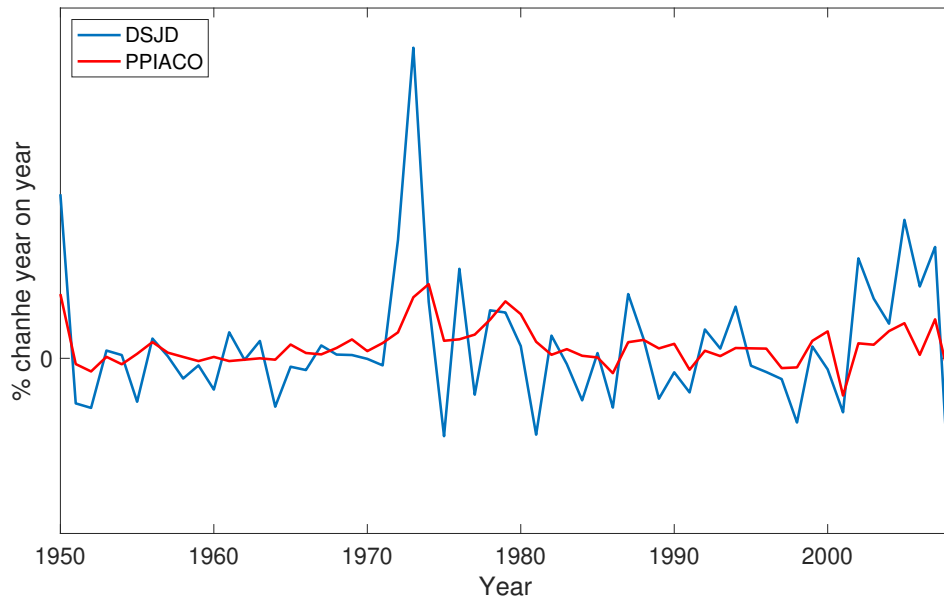
Figure 2.3: Percentage change year on year of DJSD and PPIACO from 1950 to 2009. The two time series show $\rho = 0.6694$ correlation. Source: Global Financial Data and FRED.
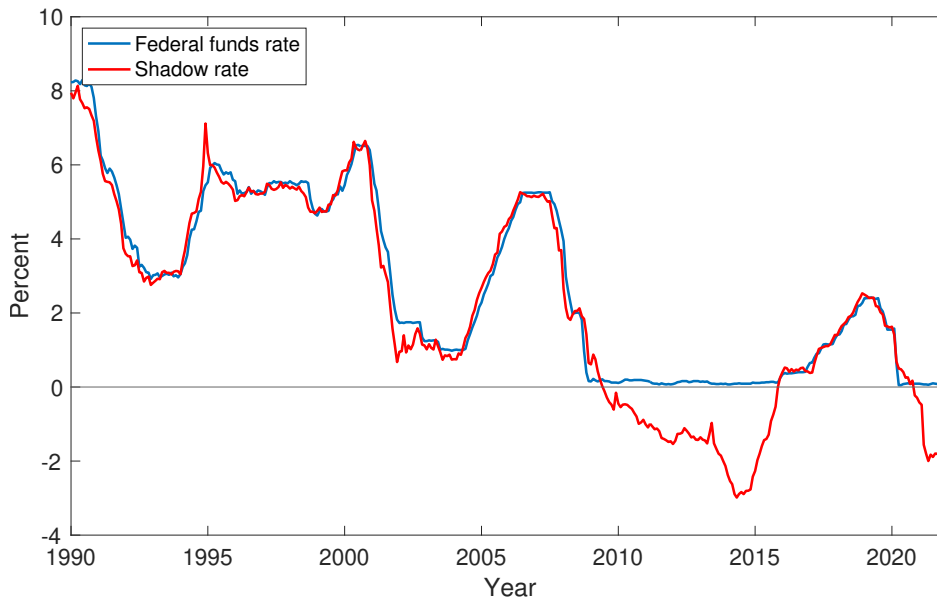
Figure 2.4: The federal funds rate and the shadow rate. Source: FRED and Wu (May 2, 2022).

2.5.4 Policy rate

In response to the global financial crisis that began in 2008, the Fed reacted by lowering the Federal funds rate, as its primary instrument of monetary policy (Wu and Xia, 2015). However, since the end of 2008, the federal funds rate was stably closed to zero and it was impossible for the Fed to further increase the monetary stimulus. Without its first tool, the Fed had to rely on unconventional monetary policy and forward guidance. The first measure consists of open market operations in which the central bank purchases long-term securities in order to lower the yield curve in the long run. The second one, the Forward guidance, concerned the statements released by the Fed aimed to influence the expectations of the economic agents. Both measures have the scope of affecting interest rates and influencing the economy (Wu and Xia, 2015). A scenario in which the interest rate is near zero and the central bank cannot further move it down to stimulate the economy is known as zero lower bound (ZLB). This is the situation the Fed faced after December 2008.

In this particular scenario, the metric of the federal funds rate doesn't convey information anymore, and the VAR model could fail in extracting economically meaningful information from the data (Wu and Xia, 2015). For this reason, for the extended database, we resorted to the shadow federal funds rate as designed by Wu and Xia (2015). It has been proved that the shadow rate is a very powerful tool for summarizing useful information when the ZLB is binding. Furthermore, it provides more information than the effective federal funds rate when the latter is close to zero.

The policy rate we use for the extension of the dataset has only partially replaced the FED-FUNDS. Therefore our policy rate is a combination of the effective federal funds rate and the shadow federal funds rate. More precisely, we have created a series in which we consider the effective federal funds rate when this is higher or equal to 0.25 basis points, and the shadow rate

otherwise. In this way we could use the VAR model, without losing precious information about
the period between the end of 2008 and the middle of 2016, that is when the interest rate was
stuck at the ZLB. In Fig 2.4, it is possible to observe the two series. As pointed out by Wu and
Xia (2015), the difference between the two series is the measurement error that can be scaled
in a few basis points. The data for the shadow rate are retrieved from the personal website of
Professor Wu and have a monthly frequency.

2.6   Conclusions

In this chapter, we have outlined the main characteristic of the structural VAR model. Then,
we have focused both on the identification strategy and the datasets employed. The datasets
are discerned in short and extended datasets. If the former does not include the global financial
crisis, the second does. In both cases, we have considered both economic and financial indicators
as endogenous variables. This allows us to appreciate the response of "Main Street" and "Wall
Street" to a monetary policy shock.

Chapter 3

Results - Short Dataset

In this Chapter, we describe the results of our benchmark model and we estimate the monetary policy equation associated with our identification scheme. Furthermore, we discuss the role of the financial variables in identifying a monetary policy shock.

The dataset is composed of real GDP for output, GDP deflator for the price level, a commodity price index, the EBP, the NFCI, and the Policy Rate. The first three series are entered as log-levels (times 100), EBP and the policy rate are in percentage points, whereas NFCI is expressed in standard deviations from its historical trend. The dataset span the interval from January 1973 to June 2007, and thus, it does not account for the global financial crisis. For more information about the composition of the dataset refer to Section 2.5.

3.1   Results

In this Section, we draw the IRFs obtained by imposing several sets of restrictions, for a time horizon of 60 lags (i.e. 5 years). In the following figures, the solid lines represent the median posterior variance, whereas the confidence intervals correspond to 68% and 95% of the posterior probability. The reduced-form VAR specification uses 12 lags and does not include any deterministic term.

Figure 3.1 shows the IRFs to a contractionary monetary policy shock of one standard deviation when imposing restriction 1, namely when implementing Uhlig's sign restrictions (Uhlig, 2005). Following an exogenous jump in the federal funds rate by around a third of a percentage point, the response of output has a high posterior probability to be positive. The response of prices is negative, with a high posterior probability of persistent decline. Also, commodity price has a negative response but it will probably increase after 18 months. On the EBP side, the impact response is almost always positive at 95% confidence for the first 36 months after the shock. Then, the median response of NFCI is positive for the first 12 months and virtually zero thereafter. The IRFs of prices and commodity prices are in accordance with the theoretical insights of the Taylor rule (Taylor, 1993; Arias et al., 2019). On the other hand, EBP and NFCI feature as described in Castelnuovo (2013) and Gilchrist and Zakrajvsek (2012). The output

Figure 3.1: IRFs to a one standard deviation contractionary monetary policy shock identified using negative sign restrictions on the IRFs of price and commodity price, and positive sign restrictions on the IRFs on EBP, NCFI, and Federal Found rate. These sign restrictions correspond to Uhlig's identification scheme. No restriction on output. This identification scheme corresponds to restriction 1.

Variables	ψ_y	ψ_p	ψ_{pc}	ψ_e	ψ_n
Median	-0.26	1.94	0.10	-1.17	-0.86
68% Prob.	[-1.47,0.43]	[0.88,3.97]	[0.03,0.22]	[-3.09,-0.26]	[-5.26,0.77]
95% Prob.	[-6.09,2.85]	[-3.10,17.97]	[-0.14,0.80]	[-12.31,2.70]	[-27.51,7.78]

Table 3.1: Contemporaneous coefficients in the monetary policy equation identified based on restrictions 1. The 68 and 95% equal-tailed posterior probability intervals are reported in brackets.

behaves as in Uhlig (2005), however, here, the confidence intervals of prices and commodity prices are narrower. But even so, this improvement is not enough to identify a monetary policy shock.

Table 3.1 reports the contemporaneous coefficients in the monetary policy equation identified based on restriction 1. It is interesting to see that the posterior medians of ψ_e and ψ_n are respectively equal to -1.17 and -0.86, meaning that the federal funds rate reacts more than one-to-one to contemporaneous movements in EBP and almost one-to-one to contemporaneous movements in NFCI. Thus, these two variables can be taken as representative indicators of U.S. financial shocks, and their inclusion in our dataset is justified.

Fig.3.2 plots the IRFs to a monetary policy shock of one standard deviation when applying restriction 1 and restriction 2. We can see that imposing additional sign restrictions on the systematic components of the monetary policy rule allows recovering a negative response of the output, with high posterior probability until almost one year. Furthermore, it allows reducing

Figure 3.2: IRFs to a one standard deviation contractionary monetary policy shock identified using negative sign restrictions on the IRFs of price and commodity price, and positive sign restrictions on the IRFs on EBP, NCFI, and Federal Found rate. No restriction on the IRF of the output. In addition, we imposed the following sign restriction on the structural parameters: ψ_y, $\psi_p > 0$. This identification scheme corresponds to restriction 1 plus restriction 2.

Variables	ψ_y	ψ_p	ψ_{pc}	ψ_e	ψ_n
Median	0.35	1.71	0.07	-0.74	-0.31
68% Prob.	[0.10,0.75]	[0.93,2.83]	[0.03,0.14]	[-1.78,-0.10]	[-2.60,0.89]
95% Prob.	[0.02,1.26]	[0.35,4.92]	[0.00,0.32]	[-3.90,0.24]	[-7.92,1.51]

Table 3.2: Contemporaneous coefficients in the monetary policy equation identified based on restrictions 1 plus 2. The 68 and 95% equal-tailed posterior probability intervals are reported in brackets.

the posterior probability of the output while leaving the IRFs of the other variables almost unchanged. The corresponding contemporaneous coefficients are reported in Table 3.2. We can see that the posterior median of the output coefficient becomes positive and equal to $\psi_y = 0.35$. Interestingly, ψ_e and ψ_n are much lower than before. This is the reason why we resorted to a further restriction (i.e. restriction 3) in our identification scheme.

In Fig. 3.3, we explore the implication of combining Uhlig's restrictions on the IRFs with restriction 2 and restriction 3, which instead restrict the systematic component of the monetary policy equation. The responses of our variables are qualitatively similar to those reported in Fig. 3.2. A key implication of our identification scheme can be seen in Table 3.3: the median impact of NFCI on the federal funds rate, namely ψ_n, is -1.39 against -0.31 obtained in Table 3.2. This is in line with the results of Castelnuovo (2013) according to whom an unexpected monetary policy tightening determines a significant reaction to financial conditions. The effect of restricting ψ_e is instead more moderate (see Table 3.3): when unrestricted, ψ_e was equal to

Figure 3.3: IRFs to a one standard deviation contractionary monetary policy shock identified using negative sign restrictions on the IRFs of price and commodity price, and positive sign restrictions on the IRFs on EBP, NCFI, and Federal Found rate. No restriction on the IRF of the output. In addition, we imposed the following sign restriction on the structural parameters: ψ_y, $\psi_p > 0$ and ψ_e, $\psi_n < 0$. This identification scheme corresponds to restrictions 1, 2 and 3.

Variables	ψ_y	ψ_p	ψ_{pc}	ψ_e	ψ_n
Median	0.35	2.08	0.08	-0.95	-1.39
68% Prob.	[0.10,0.75]	[1.30,3.29]	[0.03,0.17]	[-2.19,-0.28]	[-4.04,-0.37]
95% Prob.	[0.02,1.35]	[0.58,5.90]	[0.00,0.34]	[-4.08,-0.02]	[-9.84,-0.06]

Table 3.3: Contemporaneous coefficients in the monetary policy equation identified based on restrictions 1, 2 and 3. The 68 and 95% equal-tailed posterior probability intervals are reported in brackets.

-0.74, while, with restriction, it became equal to -0.95. Furthermore, the confidence interval for the unrestricted case was almost always negative.

Restricting ψ_e and ψ_n has determined an increase in the estimated median coefficients for inflation, EBP and NFCI but, at the same time, has introduced some uncertainty. For this reason, it seemed natural to try an identification strategy that employs the same sign restrictions on the IRFs and on the systematic components as before (i.e. restrictions 1, 2 and 3), but leaving the two financial variables unrestricted one at a time. In Fig. 3.4, we report the IRFs obtained with restrictions 1, 2 and 3, but with ψ_n unrestricted. In the short-run, the response of the output is unchanged, however after 12 months the median posterior output swifts down by a small amount. Again, there is no evidence of price puzzle, and the median response of NFCI is similar to what is reported in Fig. 3.2. However, in this case, it does not reach the peak of +0.1 standard deviation, but it remains stable at +0.05 standard deviation for the first two quarters. In the third quarter after the shock, the median response becomes negative and then stabilizes close to zero.

Figure 3.4: IRFs to a one standard deviation contractionary monetary policy shock identified using negative sign restrictions on the IRFs of price and commodity price, and positive sign restrictions on the IRFs on EBP, NCFI, and Federal Found rate. No restriction on the IRF of the output. In addition, we imposed the following sign restriction on the structural parameters: ψ_y, $\psi_p > 0$ and $\psi_e < 0$.

Variables	ψ_y	ψ_p	ψ_{pc}	ψ_e	ψ_n
Median	0.34	1.70	0.07	-0.83	-0.35
68% Prob	[0.10, 0.731]	[0.94, 2.83]	[0.02, 0.14]	[-1.91,-0.28]	[-2.71, 0.88]
95% Prob	[0.10, 1.18]	[0.38, 5.01]	[0.00, 0.31]	[-4.51,-0.04]	[-8.67, 1.52]

Table 3.4: Contemporaneous coefficients in the monetary policy equation identified based on restrictions 1, 2 and 3 relaxing restriction on ψ_n. The 68 and 95% equal-tailed posterior probability intervals are reported in brackets.

Table 3.4 shows the posterior estimates of the contemporaneous coefficients for the monetary policy equations. Interestingly, the systematic responses are almost identical to what is reported in Table 3.2. However, in Table 3.2 parameters are better estimated but ψ_n has a non-negligible probability to be positive.

Analogously, Fig. 3.5 and Table 3.5 are obtained under restrictions 1, 2 and 3 and leaving ψ_e unrestricted. The results are very similar to those obtained in Fig.3.3 and Table 3.3, respectively. This means that the sign restriction on ψ_e plays a minor role in shaping the effect of a monetary policy shock and it can be left unrestricted. This latter set of restrictions is the most appropriate choice since it requires fewer restrictions but leads to the same results.

According to our identification strategy (see Fig.3.3), a monetary policy shock of one standard deviation corresponds to an unexpected increase in the federal funds rate of around 30 basis points (median), and after the first month, it reaches 40 basis points. The shock leads to a drop in the output, which reaches the minimum (-0.15%) in the third quarter of the first year, for re-
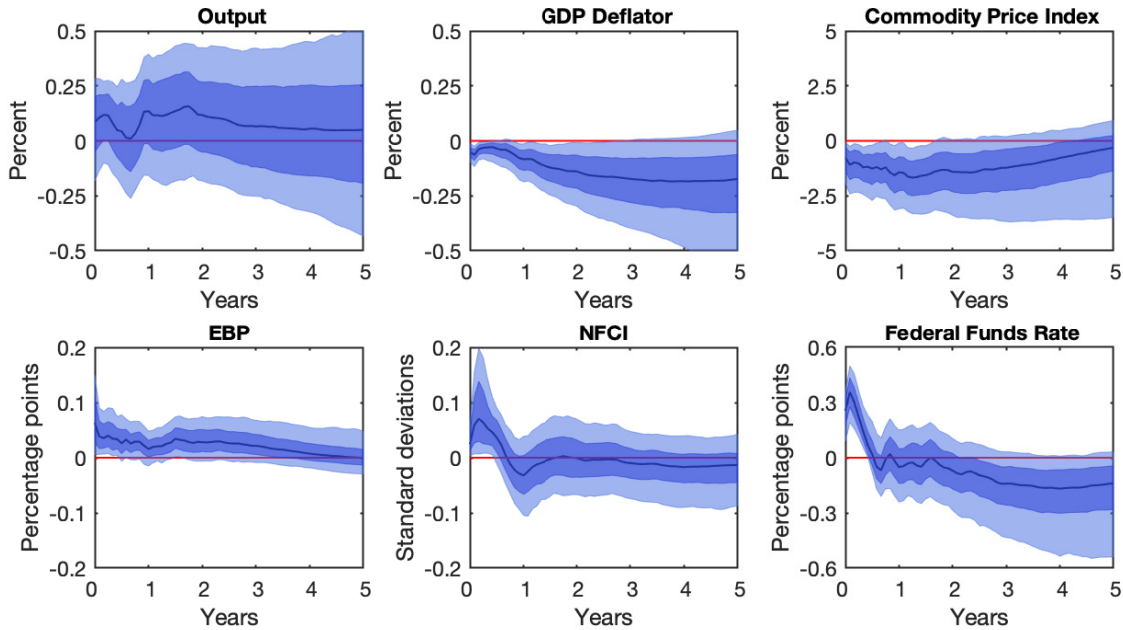
Figure 3.5: IRFs to a one standard deviation contractionary monetary policy shock identified using negative sign restrictions on the IRFs of price and commodity price, and positive sign restrictions on the IRFs on EBP, NCFI, and Federal Found rate. No restriction on the IRF of the output. In addition, we imposed the following sign restriction on the structural parameters: ψ_y, $\psi_p > 0$ and $\psi_n < 0$.

Variables	ψ_y	ψ_p	ψ_{pc}	ψ_e	ψ_n
Median	0.38	2.04	0.08	-0.82	-1.34
68% Prob.	[0.11, 0.76]	[1.28, 3.27]	[0.03,0.17]	[-2.12,-0.12]	[-3.91,-0.33]
95% Prob.	[0.02, 1.36]	[0.70, 5.54]	[0.00, 0.32]	[-4.37, 0.25]	[-9.55, -0.06]

Table 3.5: Contemporaneous coefficients in the monetary policy equation identified based on restrictions 1, 2 and 3 relaxing restriction on ψ_e. The 68 and 95% equal-tailed posterior probability intervals are reported in brackets.

covering quickly to the pre-shock level at the end of the first year, and remaining at around zero thereafter. The decline in price level is protracted in time and precisely estimated, as well as the commodity prices, which, however, recovers the pre-shock level after 48 months. The stress for the credit market is represented by the positive response on EBP that remains stably positive with high posterior probability for many months after the shock. For what concerns NFCI, it shows a positive IRF with a maximum median of 0.1 standard deviations for the first year, and then it goes back to steady-state.

3.2   Conclusions

In this Chapter, we have identified a monetary policy shock in the U.S. economy and we have shown that financial variables do matter. Specifically, we have demonstrated that, although EBP and NFCI do not bring a major contribution in estimating the input response function of the

output, they help achieve a better estimation in the response of the federal funds rate (if compared with the results obtained by Arias et al. (2019)). Regarding the monetary policy rule, it is clear that the first factor influencing the federal fund rate is inflation, followed by financial conditions.

Chapter 4

Results - Extended Dataset

In this chapter, we further analyse whether the contribution of the financial variables matters in the identification of a monetary policy shock. The main difference with respect to the previous chapter is the length of the dataset. We have, indeed, employed data from January 1973 to December 2019. This time period contains both the Great Moderation (i.e. 1980-2007), which is characterized by a reduction in volatility, as well as the global financial crisis (2007-2008), which is characterized by a volatility spillover. However, in building our dataset, we have dropped the extreme observations of the Covid-19 pandemic crisis. According to Lenza and Primiceri (2020), disregarding these outliers is acceptable for the purpose of estimation. The reasons why we have extended the dataset were to find out if the results obtained in the previous chapter lack of stability or if they are sample independent, to investigate if the role of financial markets in impacting the implementation of the monetary policy by the central bank has changed after the crisis and to see if the global financial crisis increases the information set.

The dataset is composed of real GDP for output, GDP deflator for the price level, a commodity price index, the EBP, the NFCI, and the Policy Rate. The first three series are entered as log-levels (times 100), EBP and the policy rate are in percentage points, whereas NFCI is expressed in standard deviations from its historical trend. For more information about the composition of the dataset refer to Section 2.5.

4.1 Results

Following the same path of chapter 3, we draw the IRFs obtained by imposing several sets of restrictions, for a time horizon of 60 lags (i.e. 5 years). In each figure, the solid line corresponds to the median posterior variance, whereas the confidence intervals represent 68% and 95% of the posterior probability.

Fig. 4.1 shows the IRFs to a contractionary monetary policy shock of one standard deviation when applying only restriction 1, namely, sign restriction on IRFs. First, we can see an immediate median increase in the policy rate of about 25 basis points. However, the median impact response becomes virtually zero after 12 months. The median response of output is neg-
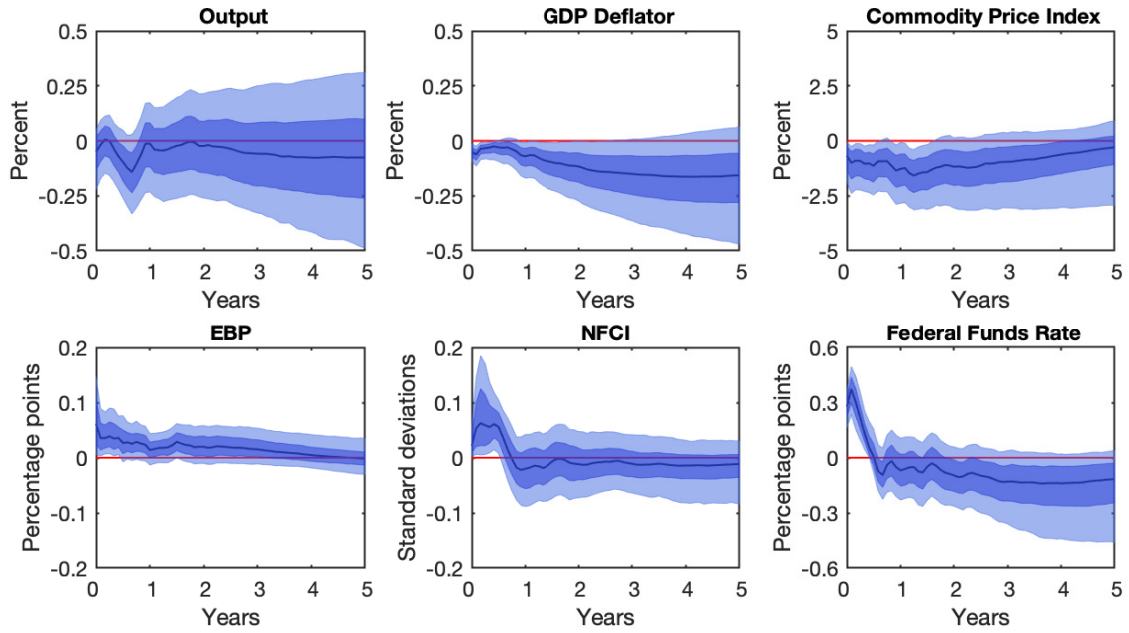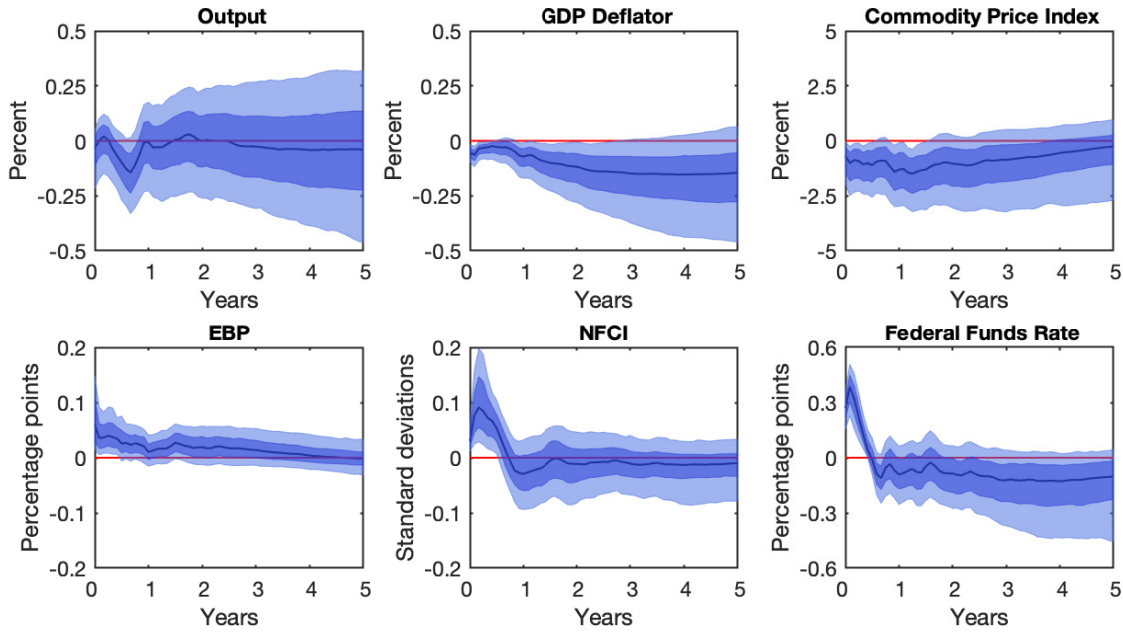
Figure 4.1: IRFs to a one standard deviation contractionary monetary policy shock identified using negative sign restrictions on the IRFs of price and commodity price, and positive sign restrictions on the IRFs on EBP, NCFI, and Federal Found rate. These sign restrictions correspond to Uhlig's identification scheme. No restriction on output. This identification scheme corresponds to restriction 1.

Variables	ψ_y	ψ_p	ψ_{pc}	ψ_e	ψ_n
Median	0.17	2.55	0.35	-1.53	-1.64
68% Prob.	[-1.48,1.72]	[0.24,7.99]	[-0.04,1.11]	[-5.18,0.18]	[-10.42,1.43]
95% Prob.	[-8.91,11.50]	[-31.79,34.02]	[-4.62,5.32]	[-24.81,24.68]	[-66.23,69.26]

Table 4.1: Contemporaneous coefficients in the monetary policy equation identified based on restrictions 1. The 68 and 95% equal-tailed posterior probability intervals are reported in brackets.

ative, and persistent with time, however, it is not accurately estimated. The probability of having positive output is, indeed, not negligible. The reduction in price level is moderate and constant over time; there are only a tiny fraction of posterior draws that imply the price puzzle. For what concerns the commodity price index, it is negative at 68% of probability. The effect of a contractionary monetary policy on the two financial indicators is much better estimated. It leads to an immediate positive median response and a zero response after 12 months. Interestingly, the IRFs of output and interest rate behave differently to those reported in Fig. 3.1 for the short dataset, upon the same restrictions. This is the first evidence that the identification procedure proposed by Uhlig (2005) is sample dependent.

In Table 4.1, there are reported the contemporaneous coefficients of the monetary policy rule, which have been obtained by applying restriction 1. The highest contemporaneous median value coefficient is the one of the price level (i.e. $\psi_p = 2.55$) that says the interest rate reacts more than 2.5 times to contemporaneous price changes. The contemporaneous median coefficients of
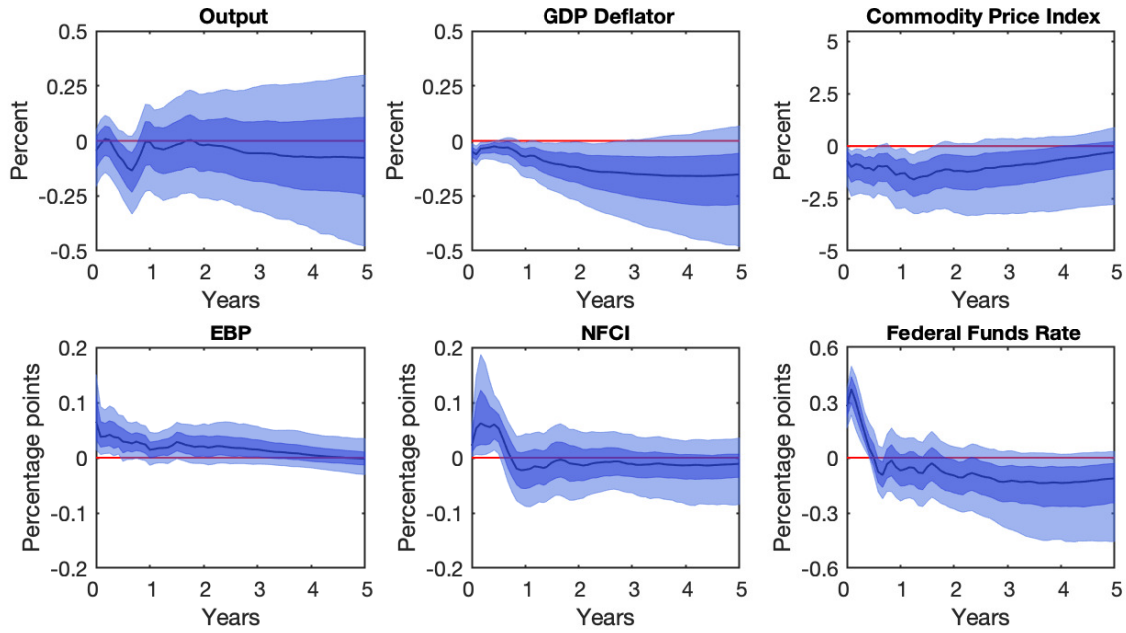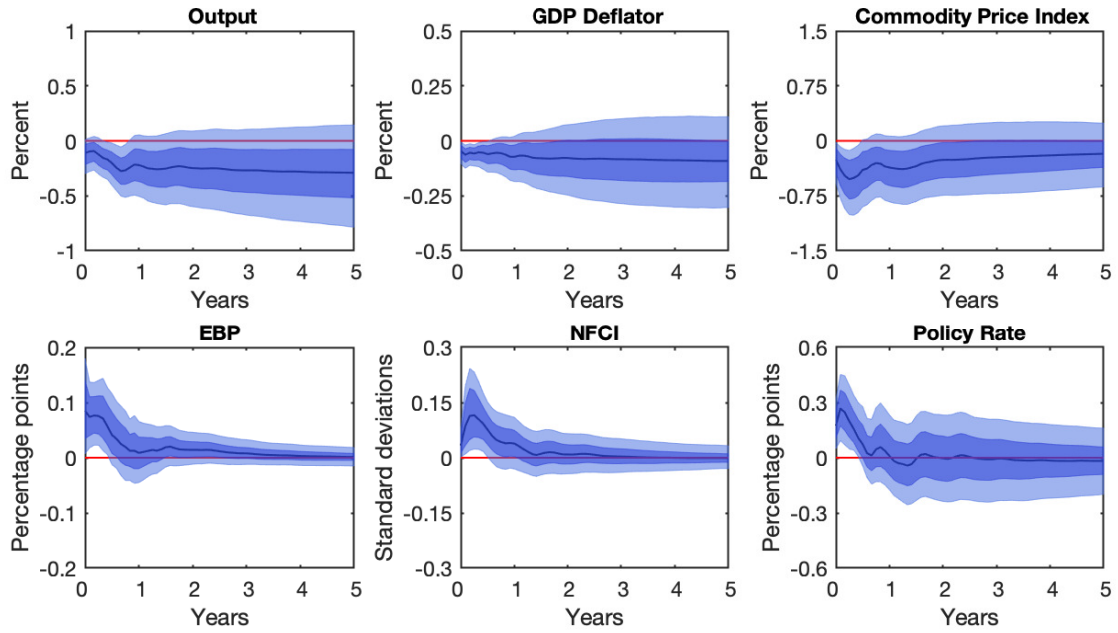
Figure 4.2: IRFs to a one standard deviation contractionary monetary policy shock identified using negative sign restrictions on the IRFs of price and commodity price, and positive sign restrictions on the IRFs on EBP, NCFI, and Federal Found rate. No restriction on the IRF of the output. In addition, we imposed the following sign restriction on the structural parameters: ψ_y, $\psi_p > 0$. This identification scheme corresponds to restriction 1 plus restriction 2.

Variables	ψ_y	ψ_p	ψ_{pc}	ψ_e	ψ_n
Median	0.72	2.62	0.37	-1.25	-1.66
68% Prob.	[0.21 ,1.73]	[0.96, 6.61]	[0.07, 1.00]	[-3.83 ,-0.09]	[-8.88, 0.64]
95% Prob.	[0.03, 5.04]	[0.22, 25.29]	[-0.15, 4.04]	[-13.91 , 1.04]	[-47.19, 1.69]

Table 4.2: Contemporaneous coefficients in the monetary policy equation identified based on restrictions 1 plus 2. The 68 and 95% equal-tailed posterior probability intervals are reported in brackets.

output and commodity price are respectively equal to 0.17 and 0.35, meaning that they have a much lower median effect on the policy rule. For all three cases, however, there is uncertainty about the signs in the probability interval. The median coefficients for ECB and NFCI are around -1.5 and -1.6. This indicates that they have a significant effect on the monetary policy rule, and thus, accordingly to Gilchrist and Zakrajvsek (2012), the central banks take into account the financial markets in developing decisions. Interestingly, these coefficients are higher than the ones obtained by discharging the global financial crisis (see Table 3.1).

We proceed with our investigation by applying both Restriction 1 and Restriction 2 to the model. The corresponding IRFs are plotted in Fig. 4.2 where, it is clear that adding positive sign restrictions on the structural parameters of output and GDP deflator (i.e. ψ_y ,$\psi_p > 0$) leads to a shrinkage of the IRF of the output, and thus to more accurate predictions. With 68% confidence, a contractionary monetary policy shock determines a continuous drop in the output for the first 12 months after the shock, whit permanent effect thereafter. Apart from that, the
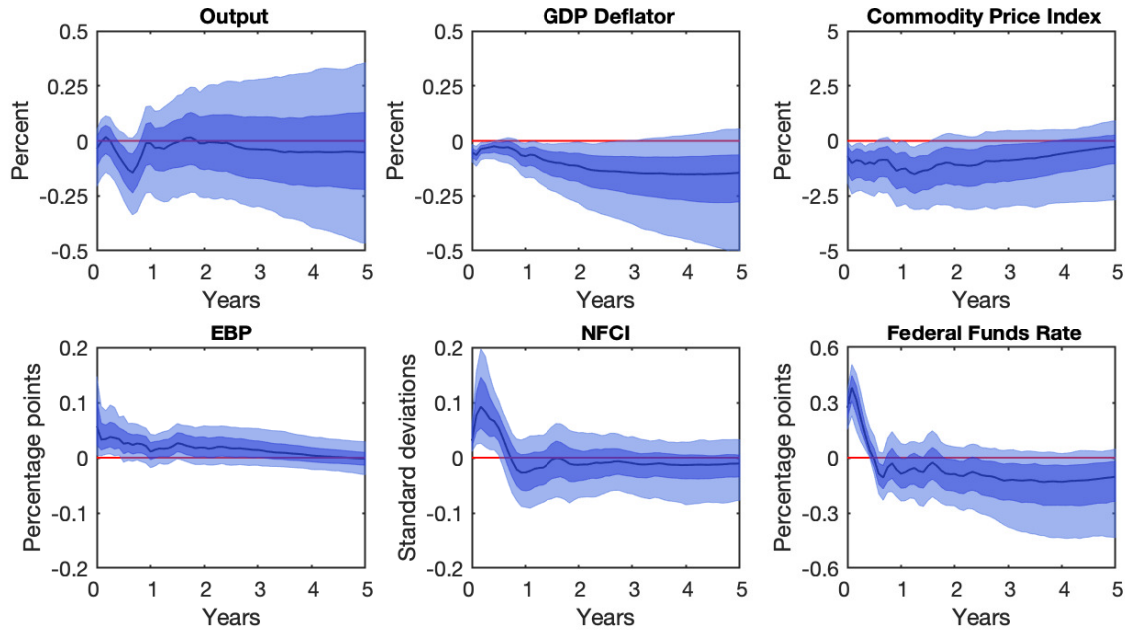
Figure 4.3: IRFs to a one standard deviation contractionary monetary policy shock identified using negative sign restrictions on the IRFs of price and commodity price, and positive sign restrictions on the IRFs on EBP, NCFI, and Federal Found rate. No restriction on the IRF of the output. In addition, we imposed the following sign restriction on the structural parameters: ψ_y, $\psi_p > 0$ and ψ_e, $\psi_n < 0$. This identification scheme corresponds to restrictions 1, 2 and 3.

Variables	ψ_y	ψ_p	ψ_{pc}	ψ_e	ψ_n
Median	0.77	3.20	0.43	-1.73	-3.06
68% Prob.	[0.23 1.97]	[1.32 7.88]	[0.07 1.19]	[-4.82 -0.52]	[-11.58 -0.71]
95% Prob.	[0.04 6.48]	[0.35 31.67]	[-0.22 4.73]	[-21.97 -0.07]	[-64.03 -0.09]

Table 4.3: Contemporaneous coefficients in the monetary policy equation identified based on restrictions 1, 2 and 3. The 68 and 95% equal-tailed posterior probability intervals are reported in brackets.

addition of Restriction 2 does not impact the IRFs of the other endogenous variables (see 4.1).

Table 4.2 reports the contemporaneous coefficients of the monetary rule restricted according to Restriction 1 and Restriction 2. The main improvement with respect to Table 4.1 regards the posterior median and the posterior intervals of ψ_y. The posterior median of ψ_y increases from 0.17 to 0.72, while, by construction, the equal-tailed posterior probability intervals become entirely positive. This means that, consistently with what is reported in the literature, central banks follow monetary policy rules that increase interest rates in response to increases in real GDP. For what concerns the other variables, even if there are no significant differences in the estimation of the posterior medians, there is a contraction in the posterior distributions that allows inferring more accurate conclusions.

We continue our analysis by adding restrictions on the financial indexes and seeing whether this influence or not the result. Fig. 4.3 shows the IRFs under Restriction 1, Restriction 2 and Restriction 3. Overall, restricting the systematic responses of the two financial variables, ECB
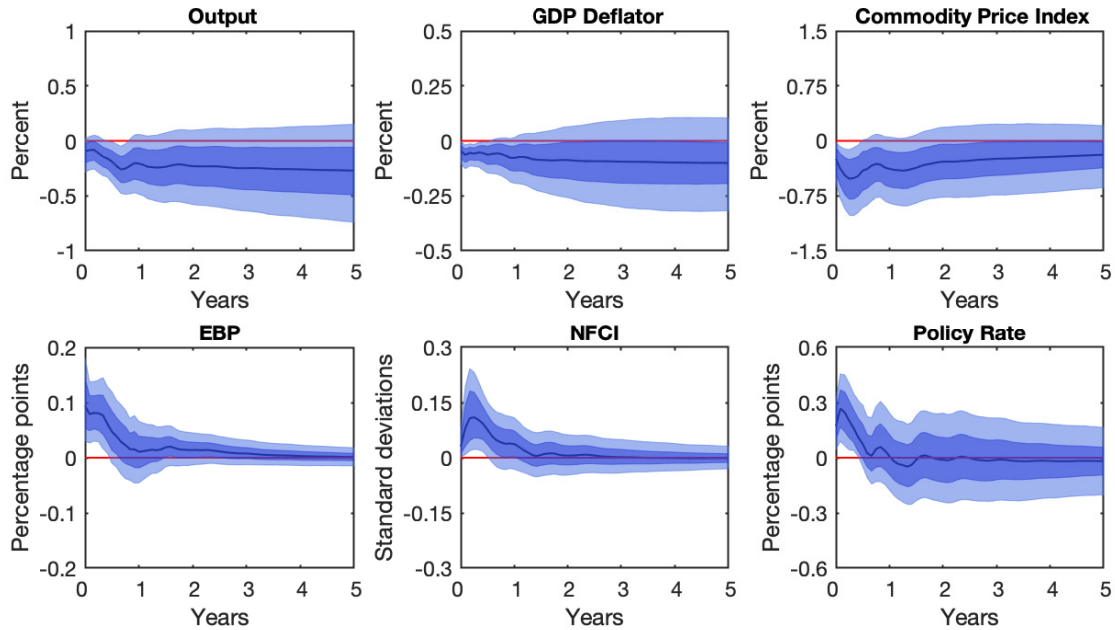
Figure 4.4: IRFs to a one standard deviation contractionary monetary policy shock identified using negative sign restrictions on the IRFs of price and commodity price, and positive sign restrictions on the IRFs on EBP, NCFI, and Federal Found rate. No restriction on the IRF of the output. In addition, we imposed the following sign restriction on the structural parameters: ψ_y, $\psi_p > 0$ and $\psi_e < 0$.

Variables	ψ_y	ψ_p	ψ_{pc}	ψ_e	ψ_n
Median	0.71	2.65	0.35	-1.53	-1.42
68% Prob.	[0.22 1.76]	[1.01, 6.56]	[0.06, 0.94]	[-4.10,-0.49]	[-8.25, 0.70]
95% Prob.	[0.03, 5.24]	[0.24, 23.14]	[-0.19, 3.60]	[-15.06,-0.09]	[-42.81, 1.67]

Table 4.4: Contemporaneous coefficients in the monetary policy equation identified based on restrictions 1, 2 and 3 relaxing restriction on ψ_n. The 68 and 95% equal-tailed posterior probability intervals are reported in brackets.

and NFCI, does not bring any improvements. The IRFs keep displaying weak tracks of the liquidity puzzle, and the response of output is the conventional one. Note that, under the same restrictions, the features of the IRF of the output are different if the global financial crisis is not included in the dataset (see Fig. 3.3)

The contemporaneous coefficients corresponding to the latter identification scheme are reported in Table 4.3. The estimated response of policy rate to real GDP, GDP deflator, and commodity price is quite similar to the one depicted in Table 4.2, without imposing Restriction 3. By contrast, there are huge differences in the estimation of NFCI and EBP: the policy rate reacts three to one to exacerbate financial conditions, and almost two to one to variation in EBP. Nonetheless, the 95% posterior probability intervals present significant uncertainty.

Since restricting the systematic components of the two financial variables leads to very different estimations of the median response of ψ_e ψ_n, and very large probability intervals (especially for the case of ψ_n at 95% confidence), we want to investigate which financial indicator brings a
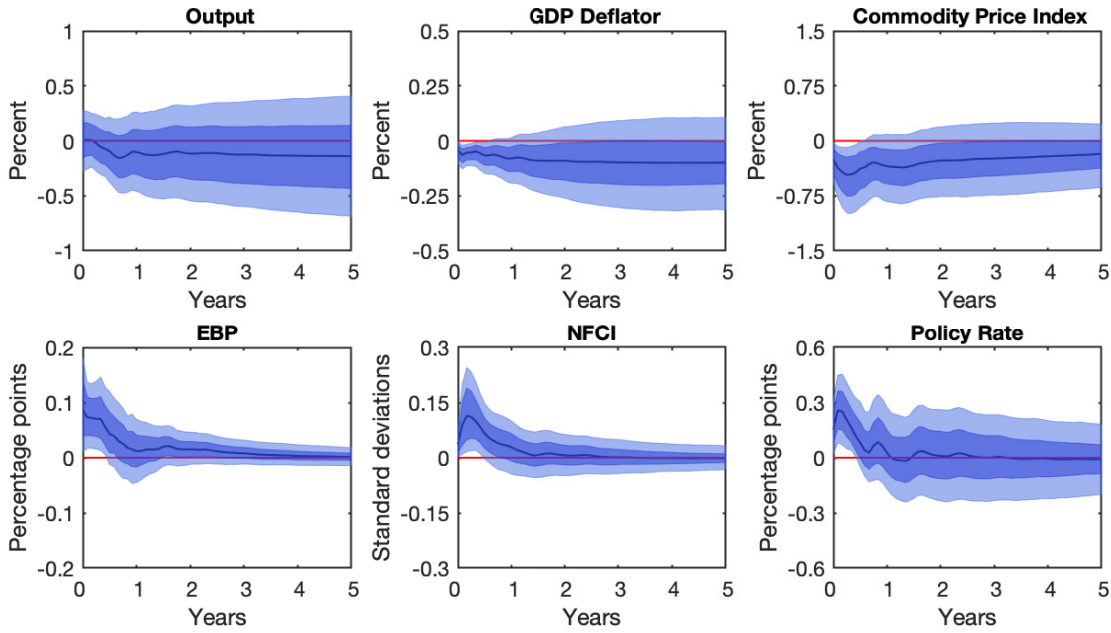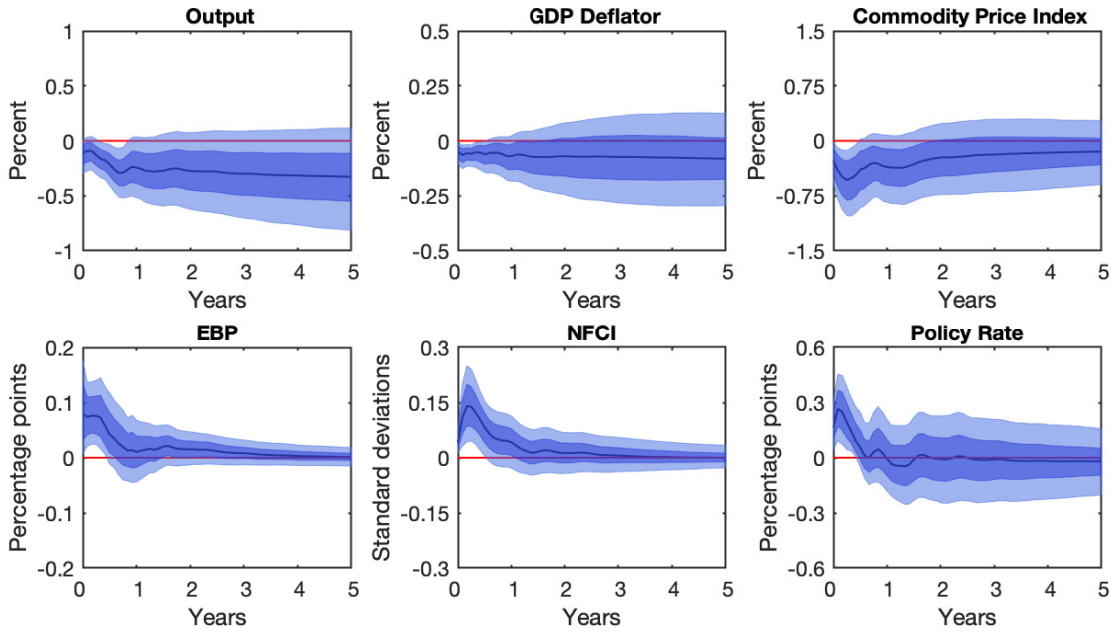
Figure 4.5: IRFs to a one standard deviation contractionary monetary policy shock identified using negative sign restrictions on the IRFs of price and commodity price, and positive sign restrictions on the IRFs on EBP, NCFI, and Federal Found rate. No restriction on the IRF of the output. In addition, we imposed the following sign restriction on the structural parameters: ψ_y, $\psi_p > 0$ and $\psi_n < 0$.

Variables	ψ_y	ψ_p	ψ_{pc}	ψ_e	ψ_n
Median	0.83	3.32	0.46	-1.29	-3.30
68% Prob.	[0.25 ,2.00]	[1.40, 8.01]	[0.09, 1.28]	[-4.21, 0.04]	[-11.99, -0.80]
95% Prob.	[0.04, 7.08]	[0.33, 32.43]	[-0.18, 5.52]	[-17.26, 1.42]	[-67.54 ,-0.11]

Table 4.5: Contemporaneous coefficients in the monetary policy equation identified based on restrictions 1, 2 and 3 relaxing restriction on ψ_e. The 68 and 95% equal-tailed posterior probability intervals are reported in brackets.

major contribution to the identification of the monetary policy shock and to the estimation of the contemporaneous coefficients. To this aim, we proceed analogously to what is done in Chapter 3 for Figures 3.4 and 3.5, and Tables 3.4 and 3.5.

Fig. 4.4 depicts the IRFs for the extended dataset according to restrictions 1 and 2, however, restriction 3 has been partially relaxed, with only ψ_n being restricted. We cannot see any relevant difference with respect to Fig. 4.3. Nevertheless, as shown in Table 4.4, this identification scheme leads to more accurate estimates of the response of policy rate to EPB and NFCI.

Finally, Fig. 4.5 reports the result obtained with the last set of restrictions, namely by imposing Restriction 1, Restriction 2 and part of Restriction 3 (i.e. no restriction on ψ_e, but $\psi_n > 0$). Again, the IRFs seem to be unaffected by any restriction on the financial indicators. On the other hand, these restrictions have a huge influence on the estimation of the monetary policy rule. As Table 4.5 shows, the estimation of the median response of policy rate is higher with respect to the previous sets of restrictions, except for ψ_e which decreases by about 0.5. Furthermore, the

uncertainty in the estimation of ψ_n is recovered.

In general, for all the 5 identification schemes, we have, with the inclusion of the financial crisis within the dataset, a higher uncertainty in the estimation of the monetary policy rule.

4.2   Conclusions

Depending on the length of the time series, and on the historical events covered, the outcomes of VARs models may be different over the same restrictions. In this Chapter, we address this concern by applying the same identification strategies employed in Chapter 3 but considering a longer dataset spanning the global financial crisis. We have found that, in this case, whatever the restrictions applied, a contractionary monetary policy shock always determines a persistent decline in output. Unlike what happens in the short dataset, the impact of the shock on the median response of the interest rate becomes zero after 12 months. Furthermore, the liquidity puzzle is, with high probability, ruled out. Even if changing the restriction does not impact the IRFs, it has a strong effect on estimating the coefficients of the policy rule. Interestingly, the contemporaneous coefficients for the financial indicators are not negligible.

Chapter 5

Instruments for Structural VAR checks

The main results obtained by implementing our VAR model have been presented in Chapters 3 and 4. In this chapter, we first re-estimate the structural VAR model without including the financial indicators (i.e. VAR model with 4 endogenous variables) to understand better to which extent the contribution brought by Wall Street is important. We then provide the forecast error variance decomposition (FEVD) for both the short and the extended datasets, and we perform some robustness checks to assess how the model works with alternative variables. Finally, we evaluate the effect of the sample length on the accuracy of the estimations.

5.1   Four Variables Structural VAR

To see the influence of the two financial variables on the results, we re-estimate the same VAR model with the exclusion of EBP and NFCI. The reduced-form VAR has been applied on monthly based U.S. data from January 1973 to December 2019. The dataset is composed of real GDP for output y_t, GDP deflator for the price level p_t, a commodity price index, pc_t and the policy rate r_t, which is computed as described in Section 2.5.4. The VAR is fitted with 12 lags in logs-levels series except for using the policy rate directly. As usual, we plot the IRFs for 60 lags. The monetary policy equation (see Eq. 2.20) in this case reduces to:

$$r_t = \psi_y y_t + \psi_p p_t + \psi_{p_c} pc_t + \sigma \varepsilon_{1,t} \tag{5.1}$$

In Fig. 5.1, we draw the IRFs to a contractionary monetary policy surprise of one standard deviation resulting from the 4 variables VAR model when applying negative sign restrictions on the IRFs of price and commodity prices, and positive sign restriction on the IRF of policy rate (i.e. Uhlig's sign restrictions). The outcome is qualitatively similar to what was obtained by Uhlig (2005). The median response of the output is positive. After the third quarter, the probability of having positive response of output becomes stronger. Furthermore, we can see that, since most of the mass of GDP deflator is on the negative side, the price puzzle is unlikely. Last, the level of the policy rate remains sustained after the shock with a high probability for the
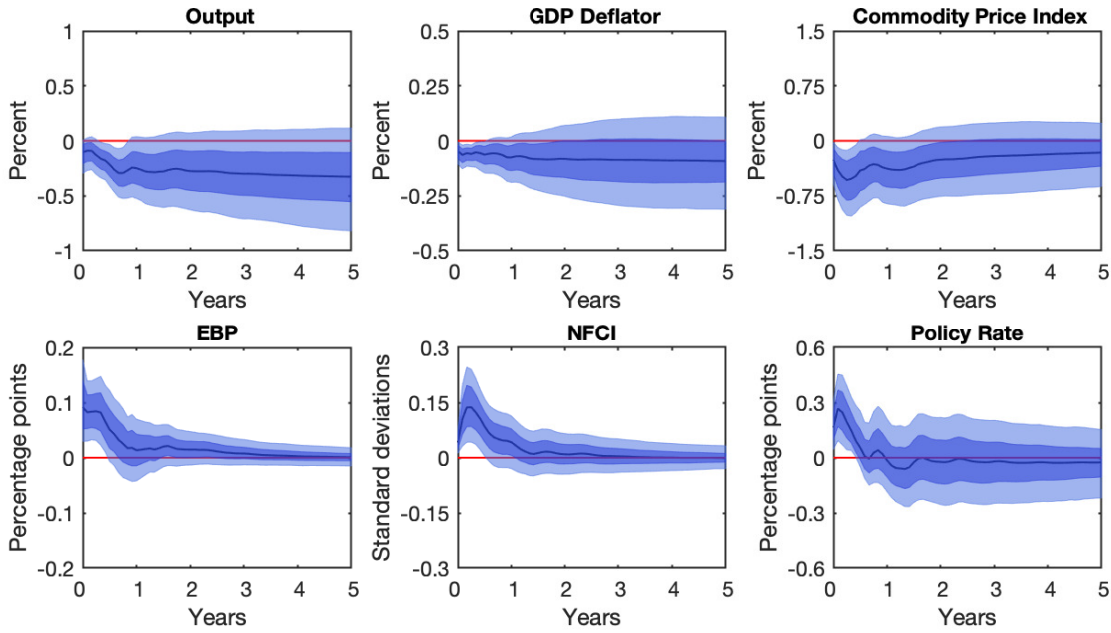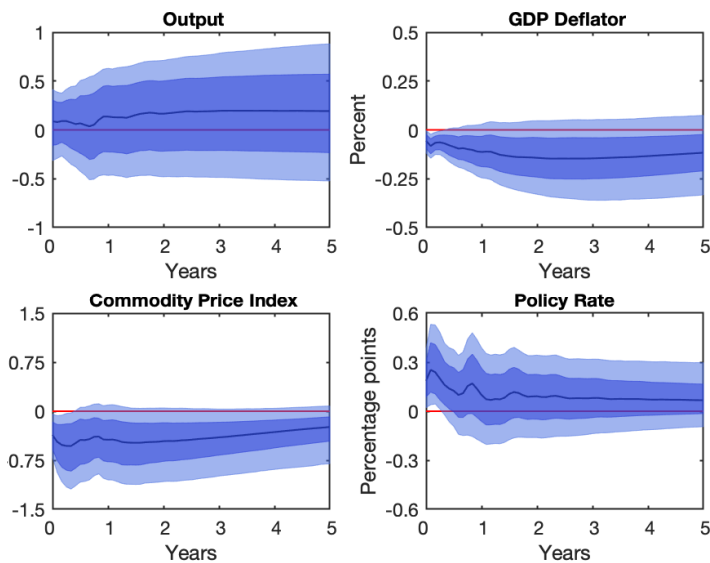
Figure 5.1: IRFs to a one standard deviation contractionary monetary policy shock identified using restriction 1 and the extended dataset, without including the financial variables.

Variables	ψ_y	ψ_p	ψ_{pc}
Median	0.08	2.68	0.47
68% Prob.	[-1.65, 1.05]	[0.64, 7.54]	[0.11, 1.42]
95% Prob.	[-11.61, 9.57]	[-9.26, 26.96]	[-3.17, 6.23]

Table 5.1: Contemporaneous coefficients in the monetary policy equation identified based on restriction 1, considering the extended dataset and no financial indicators. The 68 and 95% equal-tailed posterior probability intervals are reported in brackets.

whole period of analysis. We can conclude that the inclusion of financial indicators allows to recover a contractionary output also when only restriction 1 is applied.

In Table 5.1, the contemporaneous coefficients for the latter case are reported. As well as the IRFs for the output in Figure 5.1, also the contemporaneous coefficient for the real GDP is not well estimated. The median response at $t = 0$ is virtually zero and there is no clear evidence about the sign. On the other hand, the ψ_p is sustained and consistent with the extended dataset. The sign is positive at least at 68 % probability interval.

Following the approach of this work, in addition to imposing sign restrictions on the IRFs, we restrict the systematic components of the monetary policy rule. This set of restrictions is similar to what is imposed in Fig. 3.2 and Fig. 4.2 (notice that, since there are no financial variables in the dataset, we cannot apply restriction 3). To sum up, we impose sign restriction on the IRF of price and commodity price, and positive sign restriction on the IRFs of the Federal found rate, while leaving the output unrestricted. We also impose negative sign restrictions on the systematic response of output and GDP deflator. Fig. 5.2 shows the IRFs of the model according to the last set of restrictions. The main improvement with respect to 5.1 concerns the response of the output: the median response is negative. However, since a great mass is on the positive side, the results are not strong enough to conclude that, four variables are sufficient to
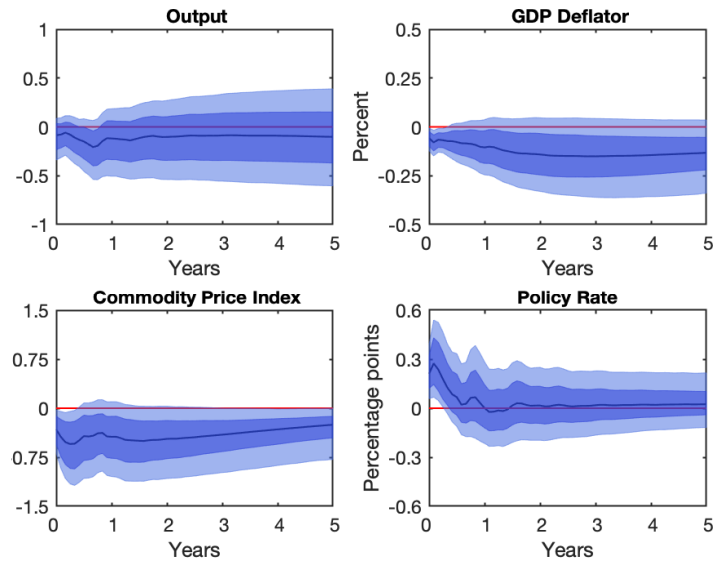
Figure 5.2: IRFs to a one standard deviation contractionary monetary policy shock identified using restriction 1 and restriction 2 to the extended dataset, without including the financial variables.

Variables	ψ_y	ψ_p	ψ_{pc}
Median	0.51	2.51	0.31
68% Prob.	[0.15, 1.18]	[0.76, 6.57]	[0.03, 0.92]
95% Prob.	[0.02, 2.17]	[0.13, 18.08]	[-0.19, 2.29]

Table 5.2: Contemporaneous coefficients in the monetary policy equation identified based on restrictions 1 and 2, considering the extended dataset and no financial indicators. The 68 and 95% equal-tailed posterior probability intervals are reported in brackets.

identify a monetary policy shock. This proves that including the financial variables improves the estimation of the output, even though this does not have a significant role in the estimation of the other variables.

Table 5.2 shows the posterior estimates of the contemporaneous coefficients according to the previous Figure. Not surprising the coefficient for the output recovers a positive sign. Interestingly, the ψ_p holds the value and becomes better estimated.

5.2   Forecast Error Variance Decomposition

This section collects the outcome of the forecast error variance decomposition analysis (FEVD) that allows evaluating the percentage of the error variance made in forecasting a variable due to a specific shock at a given horizon (Stock and Watson, 2001). The FEDV is also known as prediction mean squared error (MSPE). Mathematically, it is possible to compute the MSPE

12 months						
Variables	Output	GDP def.	CP Index	EBP	NFCI	FFR
Median	0.11	0.05	0.10	0.13	0.13	0.15
68% Prob.	[0.03,0.33]	[0.01,0.20]	[0.02,0.33]	[0.04,0.34]	[0.06,0.25]	[0.09,0.22]

24 months						
Variables	Output	GDP def.	CP Index	EBP	NFCI	FFR
Median	0.09	0.07	0.10	0.12	0.13	0.13
68% Prob.	[0.03,0.21]	[0.01,0.25]	[0.02,0.32]	[0.05,0.25]	[0.06,0.22]	[0.07,0.23]

Table 5.3: Forecast error variance decomposition computed by considering a 1-year and 2-year horizon for the short dataset. The terms in brackets refer to 68% equal-tailed probability intervals.
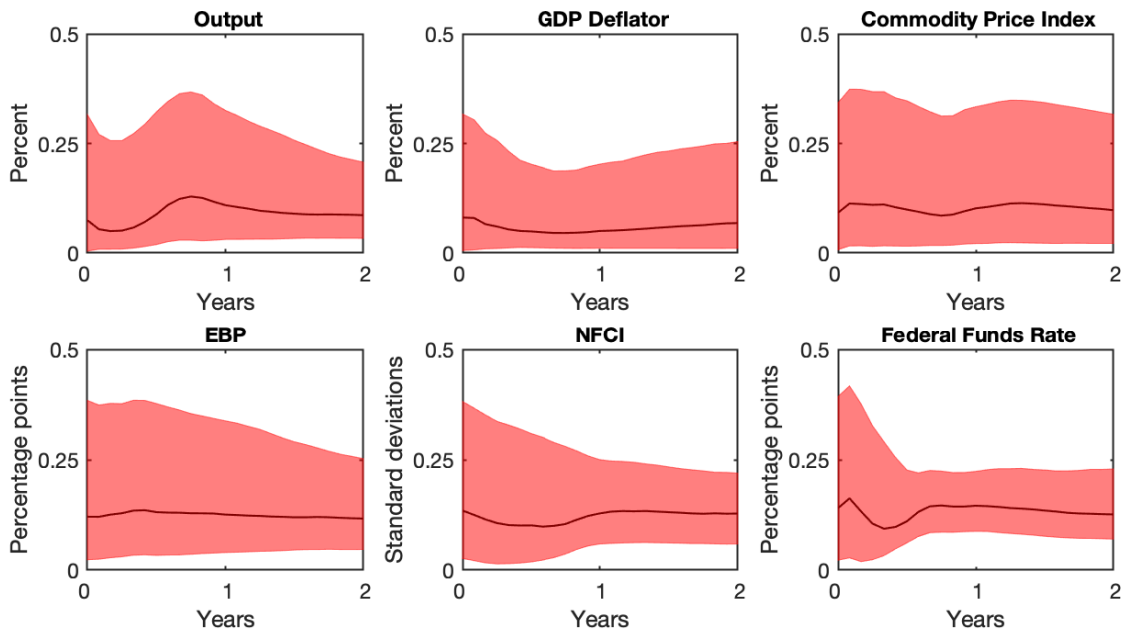
Figure 5.3: Forecast error variance decomposition for the short dataset. The confidence intervals correspond to 68% of probability.

starting from the h-step ahead forecast error:

$$y_{t+h} - y_{t+h|t} = \sum_{i=0}^{h-1} \phi_i \boldsymbol{u}_{t+h-i} = \sum_{i=0}^{h-1} \boldsymbol{\Theta}_i \boldsymbol{\varepsilon}_{t+h-i} \qquad (5.2)$$

since $\boldsymbol{u}_t = \boldsymbol{A}_0^{-1} \boldsymbol{\varepsilon}_t$ is possible to rewrite $\phi_i \boldsymbol{u}_{t+h-i}$ as $\boldsymbol{\Theta}_i \boldsymbol{\varepsilon}_{t+h-i}$ and MSPE at horizon h become

$$MPSE(h) \equiv \mathbb{E}\left[\left(y_{t+h} - y_{t+h|t}\right)\left(y_{t+h} - y_{t+h|t}\right)'\right]$$

$$= \sum_{i=0}^{h-1} \phi_i \Sigma_u \phi_i' = \sum_{i=0}^{h-1} \boldsymbol{\Theta}_i \underbrace{\Sigma_\varepsilon}_{\boldsymbol{I}_K} \boldsymbol{\Theta}_i' = \sum_{i=0}^{h-1} \boldsymbol{\Theta}_i \boldsymbol{\Theta}_i' \qquad (5.3)$$

12 months

Variables	Output	GDP def.	CP Index	EBP	NFCI	FFR
Median	0.20	0.06	0.09	0.13	0.13	0.13
68% Prob.	[0.06,0.45]	[0.01,0.19]	[0.02,0.28]	[0.04,0.34]	[0.05,0.31]	[0.08,0.23]

24 months

Variables	Output	GDP def.	CP Index	EBP	NFCI	FFR
Median	0.16	0.06	0.08	0.12	0.13	0.12
68% Prob.	[0.05,0.37]	[0.01,0.20]	[0.02,0.26]	[0.05,0.28]	[0.06,0.23]	[0.07,0.21]

Table 5.4: Forecast error variance decomposition computed by considering a 1-year and 2-year horizon for the extended dataset. The terms in brackets refer to 68% equal-tailed probability intervals.
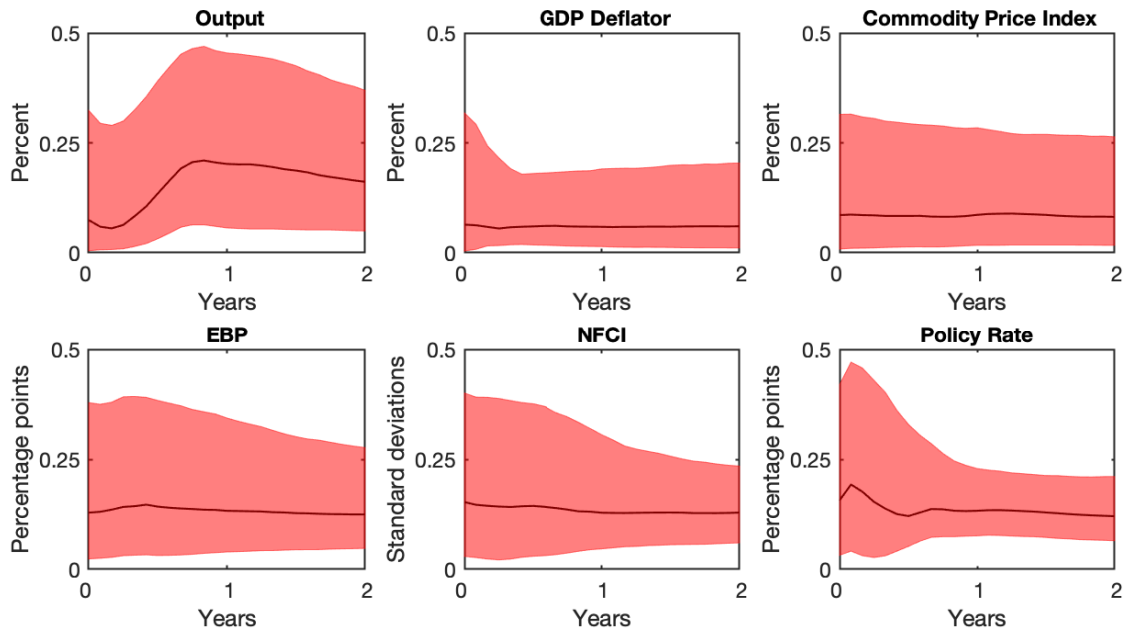
Figure 5.4: Forecast error variance decomposition for the extended dataset. The confidence intervals correspond to 68% of probability.

If we denote with $\boldsymbol{\theta}_{kj,h}$ the elements of $\boldsymbol{\Theta}_h$, we have that the contribution of the shock j to the MSPE of y_{kt} for all horizons is:

$$MPSE_j^k(h) = \boldsymbol{\theta}_{kj,0}^2 + \ldots + \boldsymbol{\theta}_{kj,h-1}^2 \tag{5.4}$$

To perform the FEVD computations, we use a modified version of the code developed by Caggiano et al. (2021), and reported in Appendix.

The first FEVD has been calculated on the short dataset under the three restrictions (see Section 2.3) by considering both 1-year and 2-years ahead horizons (see Table 5.3). After one year, the monetary policy shock is responsible for the 11% of the variation of the output, the 5% of the GDP deflator, the 10% of the consumer price index, and the 13% of EBP and NFCI. The strongest impact of the shock is on the federal funds rate, and the fraction explained is equal to 15%. After two years, the contribution of the shock on the output is found to be more

moderate (only 9% of its volatility is explained by the shock), whereas the impact on other variables remains almost unchanged. Same considerations can be drawn by looking at Fig. 5.3. Overall, except for the GDP deflator, all variables are impacted, in median, for the same amount.

We calculate the FEVD under the same conditions (i.e. restrictions 1, 2 and 3 at 1-year and 2-years ahead horizon) also for the extended dataset. If we look at Table 5.4, we can see that the main novelty regards the output: the shock determines 20% of the volatility of the output after 12 months and 16% after 2 years. The levels of the error variances for the other variables are, instead, not significantly different from the one obtained for the short database, and their median values are stable in time (see also Fig. 5.4). The impact on the policy rate is non-negligible, with monetary surprise accounting for 13% of the volatility of the federal funds rate.

Observing EBP and NFCI together, they account for 26% of the volatility due to the shock. This result suggests that monetary policy affects financial indicators and, in some measure, they have to be considered in the policy decisions.

5.3 Robustness Checks

There are several ways to check the robustness of a VAR model. Here, we employ the two most common approaches that are: assessing how the model performs with alternative variables and analysing how the model works in different sub-samples. In the former case, we employ as alternative variables a measure of term-spread, the index Standard and Poor's 500, and the VIX index. For the latter case, we consider a sample starting from 1985, and thus, we discharge the breaks found by Stock and Watson (2012) in some factor loadings.

5.3.1 Industrial Production and Term Spread

As already discussed in Section 1.4, the central bank set the short-term interest rate with the aim to influence the real interest rate, for the whole length of the yield curve. Since investments in the real economy have usually a medium-long maturity, it is important that the central bank reaches the long-term interest rate when using the monetary policy to stabilize the business cycle. The central bank implements the monetary policy through open market operations, by buying or selling government bonds (i.e. treasury in the U.S.). The difference between the yield on a long-term government bonds and the yield on shorter term public securities, is commonly known as term-spread. Since in normal times longer maturities provide higher yields, the term-spread is positive and the yield curve is upward. However, in the presence of markets on edge, it could happen that a higher yield is required for the shorter maturities with respect to the longer ones. In this case, the difference is negative and the yield curve is downward. For this reason, the term-spread may be used as a predictor of financial instability (i.e. crisis predictor) (Parker and Schularick, 2021).

Here, the term spread is used as an alternative variable for substituting EBP (discussed in

Figure 5.5: IRFs to a one standard deviation contractionary monetary policy shock identified using restrictions 1, 2 and 3. With respect to the original dataset, the EBP is substituted with the term spread while the real GDP is substituted with the industrial production index.

Variables	ψ_y	ψ_p	ψ_{pc}	ψ_{ts}	ψ_n
Median	0.20	0.85	0.19	-1.54	-0.93
68% Prob.	[0.06, 0.39]	[0.32, 1.63]	[0.08, 0.36]	[-1.90 -1,27]	.[-2.31 -0,26]
95% Prob.	[0.01, 0.65]	[0.07, 2.91]	[0.01, 0.58]	[-2.39, -1,05]	[-4.74 -0,04]

Table 5.5: Contemporaneous coefficients from the first robustness check in the monetary policy equation identified based on restrictions 1, 2 and 3, and considering term spread and industrial production index as alternative variables for EBP and real GDP, respectively. The 68 and 95% equal-tailed posterior probability intervals are reported in brackets.

2.4.4). We compute the term spread as the difference between the market yield on U.S. Treasury at 10-years and the market yield on U.S. Treasury at 1-year. The data are retrieved from FRED with monthly frequency. Since the term-spread moves in the same direction as the EBP in response to an increase in the policy rate, we apply the same set of restrictions as before. To stress the test, we also switch the real GDP with the industrial production index (see Section 2.4.1), since it is often used as a proxy for the real GDP (Christiano et al., 1994; Ramey, 2016). The obtained IRFs are reported in Figure 5.5.

The results are consistent with our model: the response of the output is negative at least for 18 months after the shock. The median response remains at around -0.4% for 12 months after the second quarter of the first year and returns to be zero after 60 months. The price puzzle is completely ruled out for the first 12 months and then keeps a large part of its mass with a negative sign. The median response for the GDP deflator is declining for the entire space. Even though the shock in the policy rate is modest, indeed it is estimated to be at around 20 basis points, it is well transmitted to the term spread, whose median response reaches the peak after 18 months. For the

sick of completeness, we report in Table 5.5 the corresponding contemporaneous coefficients.

5.3.2   CPI and S&P500

For the second robustness check, we replace the NFCI (see Section 2.4.5) with the S&P500 index. While the first is a comprehensive measure of the financial stress in the system (Brave and Butters, 2012), the second works as a proxy for the stock market which is indeed sensitive to financial tension (Sum, 2012).

The Standard and Poor's 500 is one of the most popular equity indexes for the U.S. market. It tracks the performance of the 500 largest-cap listed in one of the following: New York stock exchange, American Stock Exchange (AMEX), or Nasdaq. The index is weighted by float-adjusted market capitalization and, according to the provider, it covers approximately 80% of the available market capitalization (Standard & Poor official website, May 2, 2022).

In re-estimating the VAR, we impose restrictions 1, 2 and 3. Furthermore, we impose negative sign restriction on the IRF of the S&P500 index, and positive sign restriction on its systematic component. According to Rigobon and Sack (2004), indeed, an increase in the short-term interest rate have a negative impact on stock prices. Bjørnland and Leitemo (2009) confirm the results and find great interdependence between the interest rate setting and real stock prices, and real stock prices immediately fall after a monetary policy surprise. They also solve the endogeneity problem, due to simultaneity between monetary policy and financial markets, by applying a combination of short-run and long-run restrictions that maintains the qualitative properties of a monetary policy shock (Bjørnland and Leitemo, 2009). For this reason for this check, we do not investigate heteroskedasticity and simultaneity, but we simply assume the results by Rigobon and Sack (2004) and Bjørnland and Leitemo (2009) hold. The S&P500 index is entered in log level in the VAR model (Bjørnland and Leitemo, 2009). To stress the test, we also switch the GDP deflator with the CPI (see Section 2.4.2), since it is often used as a proxy for the deflator (Christiano et al., 1994; Gertler and Karadi, 2015).

Fig. 5.6 reports the IRFs for the second robustness check. Even if the response for the output is not well estimated, the responses are qualitatively similar to our model. We observe a contraction in the output for the first year after the shock, with 68% posterior probability. The liquidity puzzle is absent for the first 12 months, after which there is a small probability to incur in the puzzle. The equity index reacts well to the shock. Its median value registers a decrease up to -2% after 6 months, and a huge part of the mass of its posterior probability remains below the steady-state level for the whole period. Then, exactly as in our model, the median response of the policy rate increases by 25 basis points. The second check confirms even further the robustness of our VAR model.

The systematic responses for the second robustness check are reported in Table 5.6. Interestingly, the estimated coefficient for the S&P500 is virtually zero at 68% posterior probability. The latter result suggests that the central bank does not take into consideration the stock market,
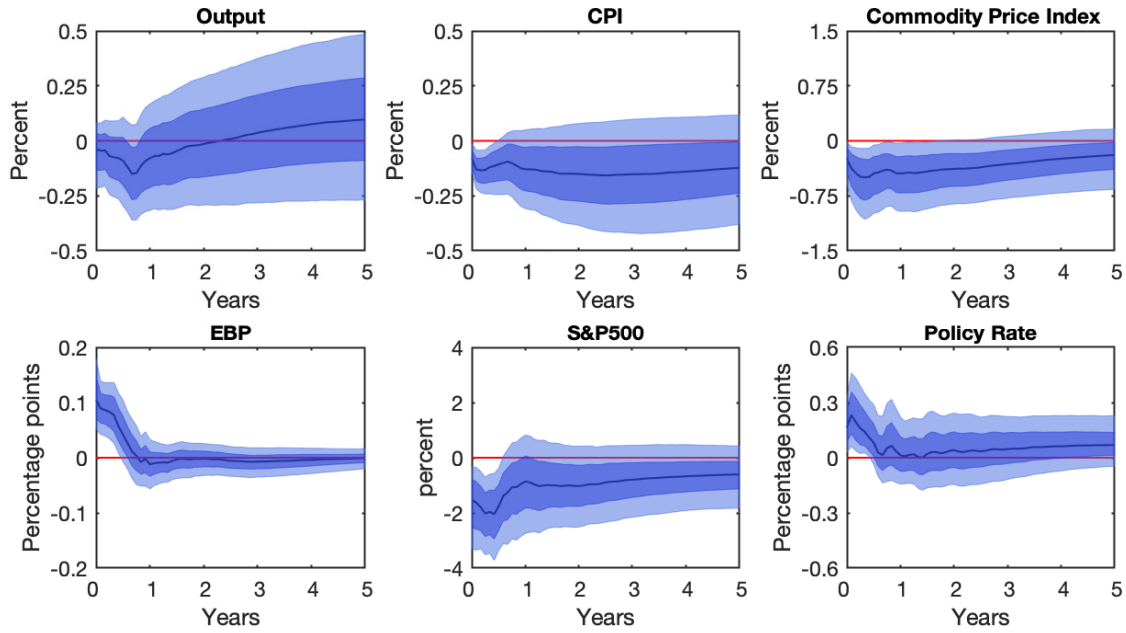
Figure 5.6: IRFs to a one standard deviation contractionary monetary policy shock identified using restrictions 1, 2 and 3. With respect to the original dataset, the EBP is substituted with S&P500 while the GDF deflator is substituted with CPI.

Variables	ψ_y	ψ_p	ψ_{pc}	ψ_e	ψ_{sp}
Median	0.82	2.82	0.02	-1.33	0.07
68% Prob.	[0.27, 1.65]	[0.97, 6.93]	[[-0.50, 0.54]	[-3.64, -0.36]	[0.02, 0.20]
95% Prob.	[0.04, 3.14]	[0.16, 19.13]	[-1.67, 1.67]	[-11.55, -0.06]	[0.00, 0.61]

Table 5.6: Contemporaneous coefficients from the first robustness check in the monetary policy equation identified based on restrictions 1, 2 and 3, and considering S&P500 and CPI as alternative variables for EBP and GDP deflator, respectively. The 68 and 95% equal-tailed posterior probability intervals are reported in brackets.

at least directly.

5.3.3 VIX

In this section, we re-estimate the VAR model by substituting the NFCI, a comprehensive measure of financial stress (Brave and Butters, 2012) (see Section 2.4.5), with an index of expected volatility, the VIX.

The Chicago Board Options Exchange's (CBOE) Volatility Index, or VIX, is a real-time index that represents the market's expectations. It is calculated as mid-quote prices of call and put options with very short maturity on the S&P500 index. For this reason, it generates a 30-days forward projection of volatility. The VIX is also called fear index because it reports the level of stress in the stock market. On a global basis, it is one of the most recognized measures of volatility (CBOE official website). Unfortunately, the VIX has been created only in 1993 by the CBOE, based on the Brenner and Galai's work. For this reason, we reduce the sample interval
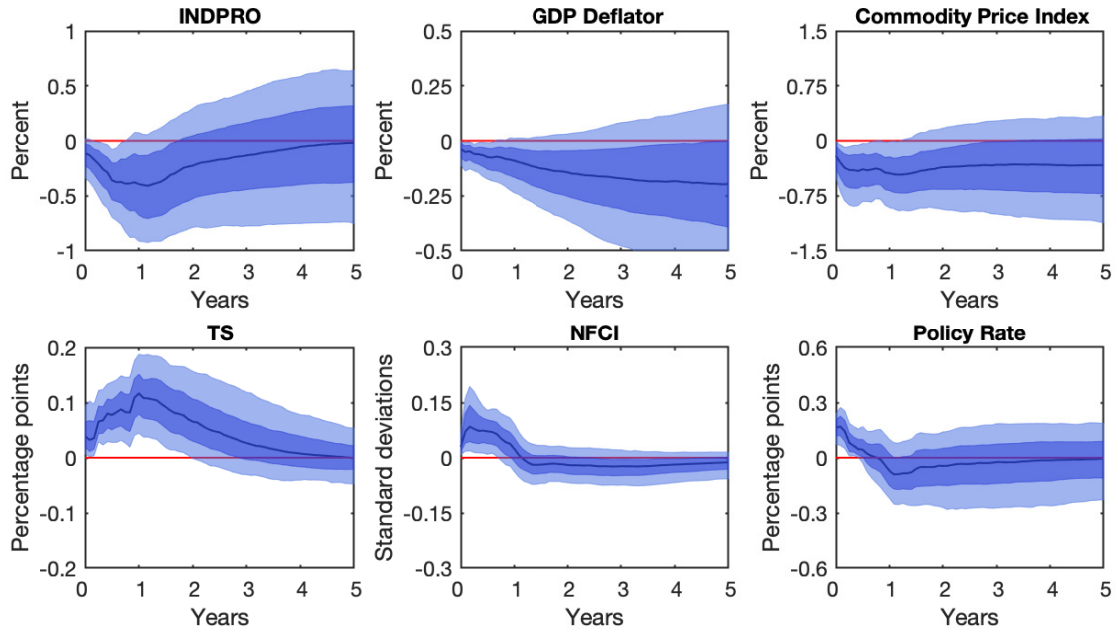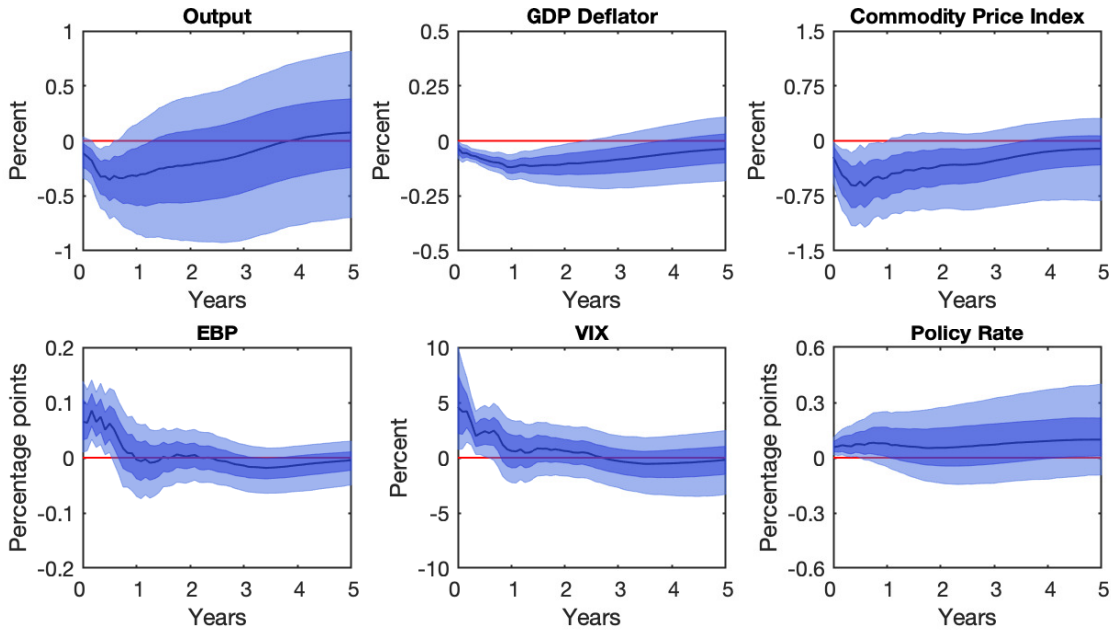
Figure 5.7: IRFs to a one standard deviation contractionary monetary policy shock identified using restrictions 1, 2 and 3. With respect to the original dataset, the NFCI is substituted with VIX.

Variables	ψ_y	ψ_p	ψ_{pc}	ψ_e	ψ_v
Median	0.17	0.93	0.08	-0.48	-0.01
68% Prob.	[0.05, 0.40]	[0.29, 2.27]	[-0.00, 0.23]	[-1.11, -0.16]	[-0.02 -0.00]
95% Prob.	[0.01, 0.86]	[0.06, 5.65]	[-0.06, 0.56]	[-2.26 -0.03]	[-0.03, -0.00]

Table 5.7: Contemporaneous coefficients from the first robustness check in the monetary policy equation identified based on restrictions 1, 2 and 3, and considering VIX as alternative variable for NFCI. The 68 and 95% equal-tailed posterior probability intervals are reported in brackets.

to 1990-2019.

The choice to substitute the NFCI with the VIX can be justified by looking at the literature. Bekaert et al. (2013) show that a cut in interest rate reduces the volatility risk premium in stocks, and that risk aversion is closely connected to monetary policy actions. According to Bruno and Shin (2015), the VIX can therefore be seen as the empirical link between monetary policy and bank leverage decisions, which in turn have macroeconomic implications. They find that a tightening in the monetary policy increases the VIX index, and leads to a decline in banking capital flows and leverage. Furthermore, Passari and Rey (2015) find a relevant degree of co-movements in leverage and credit across countries and a negative correlation with the VIX. For this reason, they consider the VIX as a proxy for the global financial cycle (GFC) in their VAR model and they show that the central bank, with its behaviours, can impact the VIX and that a monetary shock increases the VIX. According to the findings above, we do not need to modify the set of restrictions for this check, however following Passari and Rey (2015) the VIX is entered in log level.

In Fig. 5.7, the obtained IRFs are plotted. Overall, the results are coherent with the main

model. In particular, we can see that there is uncertainty in the posterior of the output, however, it contracts for the first three quarters with over 68% probability. There is no trace of the price puzzle for the first 30 months. The VIX reposts almost a 5% increase after the shock, but the increase in the policy rate is lacking, less than 10%, but persistent. Except for the response of the output after the third quarter, all the responses are well estimated. Overall, the model is robust.

Table 5.7 lists the contemporaneous coefficients of this third robustness check. As well as the coefficient for S&P500, the coefficient for the VIX, ψ_v, is zero. The central bank does not react to volatility in the stock market. The huge difference between the estimation of ψ_n and ψ_v, suggests that the VIX is not a good indicator for the leverage of the banking system, at least in this framework.

5.4   Further Analysis

In their work, Stock and Watson (2012) found evidence that macroeconomic uncertainty is countercyclical. Indeed, the unprecedented events of the 2007-2009 crisis lead to macroeconomic shocks that were the larger version of previous shocks. This means that the economy reacts in a predictable way. Furthermore, they found evidence for a break in the factor loadings in the first quarter of 1984. A break in these coefficients corresponds to a break in the factor dynamics.

To have an idea of how the length of the sample influences our results, and to assess whether the 1984 break found by Stock and Watson (2012) affects the stability of the coefficients, in this section, we re-estimate our VAR model but on a different sub-sample, ranging from January 1985 to December 2019.

The IRFs to a one standard deviation contractionary monetary policy shock under restrictions 1, 2 and 3 obtained from this model are reported in Fig. 5.8. The empirical outcomes are in agreement with our previous results, except that the decline in output is mild, and the monetary policy stance is less tight but more persistent. Furthermore, the responses of output and policy rate are less accurately estimated.

Table 5.8 shows the posterior estimates of the contemporaneous coefficient. The sign and the relative importance of each variable are almost the same as before. This confirms that price, NFCI, EBP are, in this order, the endogenous variables that most influence the monetary policy rule. What change, however, is the magnitude and the accuracy of the estimation. Specifically, the median response of the policy rate to price is estimated to be 1.63, while the posterior median of the NFCI and EBP is -1.53 and -0.47, respectively. Even though these values are in accordance with the contemporaneous coefficient estimated for the short dataset (see Table 3.3), they are roughly half the ones obtained for the extended dataset (see Table 4.3), under the same conditions. Furthermore, starting the sample in January 1985 increases the accuracy of the estimations: the 68% and 95% posterior distributions become narrower by a factor of 8. A similar result has been obtained by Arias et al. (2019).

Overall, the main results remain robust when we consider, in the estimation of the VAR
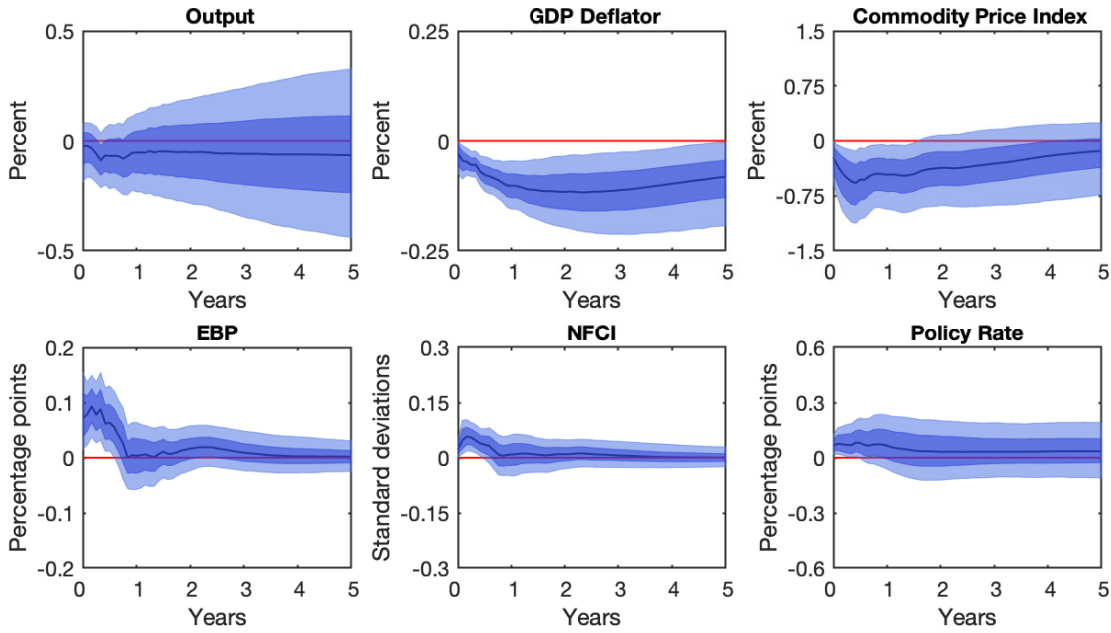
Figure 5.8: IRFs to a one standard deviation contractionary monetary policy shock identified using restrictions 1, 2 and 3. The sample ranges from January 1985 to December 2019.

Variables	ψ_y	ψ_p	ψ_{pc}	ψ_e	ψ_n
Median	0.30	1.63	0.10	-0.47	-1.53
68% Prob.	[0.09, 0.67]	[0.81,3.25]	[0.01, 0.27]	[-1.11, -0.14]	[-3.84, -0.46]
95% Prob.	[0.01, 1.19]	[0.36, 6.74]	[-0.04, 0.64]	[-2.49, -0.02]	[-8.89, -0.08]

Table 5.8: Contemporaneous coefficients from the last robustness check (sub-sample ranging from January 1985 to December 2019) in the monetary policy equation identified based on restrictions 1, 2 and 3. The 68 and 95% equal-tailed posterior probability intervals are reported in brackets.

model, a different sub-sample. This indicates that the economy is predictable. However, discharging the 1984 break in the factor dynamics, allows more accurate estimations to be retrieved.

5.5   Conclusions

In this Chapter, we have provided the results of a 4 variables VAR model without financial indicators, the forecast error variance decomposition analyses, and robustness checks to better interpret the consistency of the results obtained with our VAR model. We have reached three main findings. First, the inclusion of the two financial indicators improves the estimation of the output. Furthermore, if accounted together, the EPB and the NFCI are responsible for 26% of the variability of the monetary policy shock. Second, our identification scheme is appealing because we can retrieve similar results when using alternative variables. Nevertheless, as pointed out by Arias et al. (2019), the set of admissible structural parameters might be very large and include structural parameters with questionable implications. Finally, the results are robust also if we

re-estimate our VAR model with a different sub-sample. In this case, the contemporaneous coefficients may be less or more precisely estimated, accordingly to the amount of volatility included in the sample.

Conclusions

In this work, we have identified a monetary policy shock in the U.S. economy by using set and partially identified structural VAR models. Our reduced-form VAR specification consists of six endogenous variables. Four of them are standard macroeconomic indicators that allow appreciating the effect of the real economy. The other two are proxies for financial conditions. To identify the shock, we have imposed sign restrictions on the impulse response functions and sign restrictions on the contemporaneous coefficient of the monetary policy rule. We have combined these two restrictions because, as demonstrated by Kilian and Murphy (2012), sign restrictions on the IRFs alone are not sufficient to infer the responses of real GDP to the monetary policy shock. Furthermore, according to Wolf (2020), adding sign restriction on the Taylor rule is a possible solution to solve this problem. The model is estimated for two different samples. The first one employs data from January 1973 to December 2019. It spans the Great Moderation and is characterized by a mild volatility. The second dataset ranges from January 1973 to December 2019 and thus, it accounts for both the Great Moderation and the Global Financial Crisis, which is instead characterized by a higher volatility.

With our analysis, we have reached three main conclusions. First, Uhlig's results are sample dependent. Indeed, the response estimated considering the agnostic procedure proposed by Uhlig has an ambiguous effect on the real GDP. When considering the short sample, a contractionary monetary policy shock determines, with a high probability, an expansion of the output (see Fig. 3.1) and this is consistent with Uhlig's findings. However, when considering the extended dataset, the same monetary policy shock induces, with high probability, an output contraction (see Fig. 4.1). The reason why this happens is because in the original sample considered by Uhlig (i.e. January 1965 - December 1996), as well as in our short dataset, there are no events having the high volatility introduced by the global financial crisis. From an econometric perspective, including the 2007-2009 financial crisis within the sample increases the information set.

Another interesting outcome regards the role of financial conditions in the identification of a monetary policy shock and in estimating the policy coefficient. The response of the output estimated by considering EBP and NFCI (see Fig. 4.3) is more accurate than the one retrieved with the 4-variables VAR that does not embed financial indices (see Fig. 5.2). However, the real improvement concerns the estimation of the monetary policy rule: according to our results, the monetary policy rule is mostly influenced by price level, followed by financial conditions. Thus,

financial conditions matter.

Last but not least, the response of output and interest rate estimated considering the short and the extended datasets are qualitatively different. Specifically, when considering the short sample (Fig. 3.3), a monetary policy tightening determines a decline in the median response of output that lasts only 12 months, and a sharp increase followed by a persistent decline in the interest rate. On the other hand, with the full sample (Fig. 4.3), the median decline of output is higher and persistent, while the interest rate becomes negligible after one year. The length of the sample influence also the estimation of the monetary policy coefficients. Not surprisingly, as we increase the length of the sample, the estimated posterior distributions become wider, and the accuracy of the estimation weaker.

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Sitography

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Appendix A

Replication codes

In this Chapter, we provide the Matlab codes necessary to replicate our main results. The Matlab code is divided into four main files that concern, in order, the creation of the dataset from .csv files, the resolution of the VAR model with the desired restrictions, the plot of the IRFs, and the display of tables reporting the estimated contemporaneous coefficient of the monetary policy rule. These replications files are updated from the ones provided by Arias et al. (2019) thus, refer to Arias et al. (2019) to retrieve all the help functions required to run the code. For what concern the Error Variance Decomposition, instead, our replication files are an updated version of the one given by Caggiano et al. (2021).

Replication code for setting up the short dataset

```matlab
%%************************************************************************
% This is a modified version of the code developed by
% Arias et al. (2019) to create the short dataset (1973-2007)
%%************************************************************************

clear all;
clc;
close all;

%-----------------------------------------------------------------------
% % Load useful libraries
%-----------------------------------------------------------------------

addpath('td');    %load Enrique M. Quilis Temporal Disaggregation Library
addpath('csvfiles/');

%-----------------------------------------------------------------------
% % Interpolate real GDP
%-----------------------------------------------------------------------

% read real GDP
```

```matlab
22 raw_data_GDPC1 = readtable('csvfiles/GDPC1.csv');
23 GDPC1          = table2array(raw_data_GDPC1(find(raw_data_GDPC1.DATE=='
      01-Jan-1973'):find(raw_data_GDPC1.DATE=='01-Oct-2007'),2));
24
25 % read industrial production
26 raw_data_IP = readtable('csvfiles/INDPRO.csv');
27 IP           = table2array(raw_data_IP(find(raw_data_IP.DATE=='01-Jan
      -1973'):find(raw_data_IP.DATE=='01-Dec-2007'),2));
28
29 Y  = GDPC1; % Y: Nx1 ---> vector of low frequency data
30 x  = IP;    % x: nxp ---> matrix of high frequency indicators (without
      intercept)
31 ta = 2;      % type of disaggregation ---> average (index)
32 sc = 3;      % quarterly to monthly
33 type = 0;    % estimation method:  (0) weighted least squares   (1)
      maximum likelihood
34 opC  = 1;    % no intercept in hf model
35
36 res = chowlin(Y,x,ta,sc,type,opC,[]);
37
38 monthly_GDP = res.y;
39
40 %-----------------------------------------------------------------------
41 % % Interpolate real GDP deflator
42 %-----------------------------------------------------------------------
43
44 % read GDP Deflator
45 raw_data_GDPDEF= readtable('csvfiles/GDPDEF.csv');
46 GDPDEF          = table2array(raw_data_GDPDEF(find(raw_data_GDPDEF.DATE==
      '01-Jan-1973'):find(raw_data_GDPDEF.DATE=='01-Oct-2007'),2));
47
48 % read CPIAUCSL
49 raw_data_CPIAUCSL= readtable('csvfiles/CPIAUCSL.csv');
50 CPIAUCSL          = table2array(raw_data_CPIAUCSL(find(raw_data_CPIAUCSL.
      DATE=='01-Jan-1973'):find(raw_data_CPIAUCSL.DATE=='01-Dec-2007'),2));
51
52 % read PPIFGS
53 raw_data_PPIFGS= readtable('csvfiles/PPIFGS.csv');
54 PPIFGS          = table2array(raw_data_PPIFGS(find(raw_data_PPIFGS.DATE==
      '01-Jan-1973'):find(raw_data_PPIFGS.DATE=='01-Dec-2007'),2));
55
56 Y  = GDPDEF;                 % Y: Nx1 ---> vector of low frequency data
57 x  = [CPIAUCSL,PPIFGS];      % x: nxp ---> matrix of high frequency
      indicators (without intercept)
58 ta = 2;                      % type of disaggregation ---> average (index)
59 sc = 3;                      % quarterly to monthly
60 type = 0;                    % estimation method:  (0) weighted least
```

```matlab
     squares    (1) maximum likelihood
61  opC   = 1;                      % no intercept in hf model

62

63  res = chowlin(Y,x,ta,sc,type,opC,[]);

64

65  monthly_GDPDEF = res.y;

66

67  %-------------------------------------------------------------------------
68  % % Commodity Price Index: Obtained from Global Financial Data through
69  % % the Board of Governors of the Federal Reserve System
70  %-------------------------------------------------------------------------

71

72  raw_data_CPRINDEX= readtable('csvfiles/DJSD_20170720.csv');
73  CPRINDEX        = table2array(raw_data_CPRINDEX(find(raw_data_CPRINDEX.
        Date=='31-Jan-1973'):find(raw_data_CPRINDEX.Date=='31-Dec-2007'),9));

74

75  %-------------------------------------------------------------------------
76  % % Excess Bond Premium
77  %-------------------------------------------------------------------------

78

79  raw_data_EBP   = readtable('csvfiles/EBP.csv');
80  EBP            = table2array(raw_data_EBP(find(raw_data_EBP.date=='01-
        Jan-1973'):find(raw_data_EBP.date=='01-Dec-2007'),3));

81

82  %-------------------------------------------------------------------------
83  % % National Financial Conditions Index
84  %-------------------------------------------------------------------------

85

86  raw_data_NFCI   = readtable('csvfiles/NFCI.csv');
87  NFCI           = table2array(raw_data_NFCI(find(raw_data_NFCI.DATE=='01-
        Jan-1973'):find(raw_data_NFCI.DATE=='01-Dec-2007'),2));

88

89  %-------------------------------------------------------------------------
90  % % Federal Funds Rate
91  %-------------------------------------------------------------------------

92

93  raw_data_FEDFUNDS= readtable('csvfiles/FEDFUNDS.csv');
94  FEDFUNDS        = table2array(raw_data_FEDFUNDS(find(raw_data_FEDFUNDS.
        DATE=='01-Jan-1973'):find(raw_data_FEDFUNDS.DATE=='01-Dec-2007'),2));

95

96  %-------------------------------------------------------------------------
97  % % Export data
98  %-------------------------------------------------------------------------

99

100 dates = table2array(raw_data_FEDFUNDS(find(raw_data_FEDFUNDS.DATE=='01-
        Jan-1973'):find(raw_data_FEDFUNDS.DATE=='01-Dec-2007'),1));

101
```

```
102 dataset = table(dates,monthly_GDP,monthly_GDPDEF,CPRINDEX,EBP,NFCI,
        FEDFUNDS,IP,CPIAUCSL);

103
104 writetable(dataset,'csvfiles/dataset.csv')

105
106 % Dates
107 dates1 = [dataset.dates];
108 dates = datenum(dates1);

109
110 % Names of variables
111 varNames1={'m_Y'; 'm_P'; 'm_CP'; 'm_EBP'; 'm_NFCI'; 'm_FFR'};
112 varNames=varNames1';

113
114 % Data
115 data = [100*log(dataset.monthly_GDP) 100*log(dataset.monthly_GDPDEF)
        100*log(dataset.CPRINDEX) dataset.EBP dataset.NFCI dataset.FEDFUNDS];

116
117 % Save results
118 save('data_mat.mat','data','dates','varNames');
119 cd ..
```

Replication code for setting up the extended dataset

```matlab
%%***********************************************************************
% This is a modified version of the code developed by
% Arias et al. (2019) to create the extended dataset (1973-2019)
%%***********************************************************************

clear all;
clc;
close all;

%----------------------------------------------------------------------
% % Load useful libraries
%----------------------------------------------------------------------

addpath('td');  % load Enrique M. Quilis Temporal Disaggregation Library
addpath('csvfiles/');

%----------------------------------------------------------------------
% % Interpolate real GDP
%----------------------------------------------------------------------

% read real GDP
raw_data_GDPC1= readtable('csvfiles/GDPC1.csv');
GDPC1          = table2array(raw_data_GDPC1(find(raw_data_GDPC1.DATE=='
    01-Jan-1973'):find(raw_data_GDPC1.DATE=='01-Oct-2019'),2));

% read industrial production
raw_data_IP= readtable('csvfiles/INDPRO.csv');
IP          = table2array(raw_data_IP(find(raw_data_IP.DATE=='01-Jan-1973
    '):find(raw_data_IP.DATE=='01-Dec-2019'),2));

Y  = GDPC1; % Y: Nx1 ---> vector of low frequency data
x  = IP;    % x: nxp ---> matrix of high frequency indicators (without
    intercept)
ta = 2;     % type of disaggregation ---> average (index)
sc = 3;     % quarterly to monthly
type = 0;   % estimation method:  (0) weighted least squares  (1)
    maximum likelihood
opC  = 1;   % no intercept in hf model

res = chowlin(Y,x,ta,sc,type,opC,[]);

monthly_GDP = res.y;

%----------------------------------------------------------------------
% % Interpolate real GDP deflator
```

```matlab
42  %----------------------------------------------------------------------
43
44  % read GDP Deflator
45  raw_data_GDPDEF= readtable('csvfiles/GDPDEF.csv');
46  GDPDEF          = table2array(raw_data_GDPDEF(find(raw_data_GDPDEF.DATE==
       '01-Jan-1973'):find(raw_data_GDPDEF.DATE=='01-Oct-2019'),2));
47
48  % read CPIAUCSL
49  raw_data_CPIAUCSL= readtable('csvfiles/CPIAUCSL.csv');
50  CPIAUCSL         = table2array(raw_data_CPIAUCSL(find(raw_data_CPIAUCSL.
       DATE=='01-Jan-1973'):find(raw_data_CPIAUCSL.DATE=='01-Dec-2019'),2));
51
52  % read PPIFGS
53  raw_data_CONSPI= readtable('csvfiles/PPIACO.csv');
54  CONSPI          = table2array(raw_data_CONSPI(find(raw_data_CONSPI.DATE==
       '01-Jan-1973'):find(raw_data_CONSPI.DATE=='01-Dec-2019'),2));
55
56  Y  = GDPDEF;                % Y: Nx1 ---> vector of low frequency data
57  x  = [CPIAUCSL,CONSPI];     % x: nxp ---> matrix of high frequency
       indicators (without intercept)
58  ta = 2;                     % type of disaggregation ---> average (index)
59  sc = 3;                     % quarterly to monthly
60  type = 0;                   % estimation method:  (0) weighted least
       squares  (1) maximum likelihood
61  opC  = 1;                   % no intercept in hf model
62
63  res = chowlin(Y,x,ta,sc,type,opC,[]);
64
65  monthly_GDPDEF = res.y;
66
67  %----------------------------------------------------------------------
68  % % Commodity Price Index: Obtained from Global Financial Data through
69  % % the Board of Governors of the Federal Reserve System
70  %----------------------------------------------------------------------
71
72  raw_data_CPRINDEX= readtable('csvfiles/PPIACO.csv');
73  CPRINDEX          = table2array(raw_data_CPRINDEX(find(raw_data_CPRINDEX.
       DATE=='01-Jan-1973'):find(raw_data_CPRINDEX.DATE=='01-Dec-2019'),2));
74
75  %----------------------------------------------------------------------
76  % % Excess Bond Premium
77  %----------------------------------------------------------------------
78
79  raw_data_EBP   = readtable('csvfiles/EBP.csv');
80  EBP            = table2array(raw_data_EBP(find(raw_data_EBP.date=='01-
       Jan-1973'):find(raw_data_EBP.date=='01-Dec-2019'),3));
81
```

```matlab
82  %-----------------------------------------------------------------------
83  % % National Financial Conditions Index
84  %-----------------------------------------------------------------------
85
86  raw_data_NFCI  = readtable('csvfiles/NFCI.csv');
87  NFCI           = table2array(raw_data_NFCI(find(raw_data_NFCI.DATE=='01-
       Jan-1973'):find(raw_data_NFCI.DATE=='01-Dec-2019'),2));
88
89  %-----------------------------------------------------------------------
90  % % Policy rate (Federal funds rate and shadow rate)
91  %-----------------------------------------------------------------------
92
93  raw_data_FEDFUNDS       = readtable('csvfiles/FEDFUNDS.csv');
94  raw_data_WuXiaShadowRate = readtable('csvfiles/WuXiaShadowRate.xlsx','
       Sheet','Data');
95  FEDFUNDS                = table2array(raw_data_FEDFUNDS(find(
       raw_data_FEDFUNDS.DATE=='01-Jan-1973'):find(raw_data_FEDFUNDS.DATE=='
       01-Dec-2019'),2)) ;
96  SHADOWR                 = table2array(raw_data_WuXiaShadowRate(find(
       raw_data_WuXiaShadowRate.Var1=='01-Jan-1973'):find(
       raw_data_WuXiaShadowRate.Var1=='01-Dec-2019'),3));
97  POLICYR                 = FEDFUNDS;
98  POLICYR(POLICYR<0.25)   = SHADOWR(POLICYR<0.25);
99
100 %-----------------------------------------------------------------------
101 % % Export data
102 %-----------------------------------------------------------------------
103 dates = table2array(raw_data_FEDFUNDS(find(raw_data_FEDFUNDS.DATE=='01-
       Jan-1973'):find(raw_data_FEDFUNDS.DATE=='01-Dec-2019'),1));
104
105 dataset = table(dates,monthly_GDP,monthly_GDPDEF,CPRINDEX,EBP,NFCI,
       POLICYR,IP,CPIAUCSL);
106
107 writetable(dataset,'csvfiles/dataset.csv')
108
109 % Dates
110 dates1 = [dataset.dates];
111 dates = datenum(dates1);
112
113 % Names of variables
114 varNames1={'m_Y'; 'm_P'; 'm_CP'; 'm_EBP'; 'm_NFCI'; 'm_PR'};
115 varNames=varNames1';
116
117 % Data
118 data = [100*log(dataset.monthly_GDP) 100*log(dataset.monthly_GDPDEF)
       100*log(dataset.CPRINDEX) dataset.EBP dataset.NFCI dataset.POLICYR];
119
```

```matlab
120  % Save results
121  save('data_mat.mat','data','dates','varNames');
122  cd ..
```

Replication code for estimating the IRFs according to restriction 1

```matlab
%%************************************************************************
% This is a modified version of the code developed by
% Arias et al. (2019) to compute the draws when only restriction 1 is
% imposed.
%%************************************************************************

clear variables;
close all;
userpath('clear');
clc;

rng('default'); % reinitialize the random number generator to its
    startup configuration
rng(0);         % set seed

currdir=pwd;
cd ..
get_help_dir_currdir=pwd;
addpath([get_help_dir_currdir,'/helpfunctions']); % set path to helper
    functions
cd(currdir)

%=======================================================================
% load the data and priors
%=======================================================================

data   = readtable([get_help_dir_currdir,'/data/csvfiles/dataset.csv']);
data   = data(find(data.dates=='01-Jan-1973'):find(data.dates=='01-Jun
    -2007'),:);

% data
% all variables are in log times 100 except for the federal funds rate
    that enters the SVAR in annualized percentages
num  = [100*log(data.monthly_GDP) 100*log(data.monthly_GDPDEF) 100*log(
    data.CPRINDEX) data.EBP data.NFCI data.FEDFUNDS];

%=======================================================================
% model setup
%=======================================================================

nlag      = 12; % number of lags
nvar      = 6; % number of endogenous variables
nex       = 0; % set equal to 1 if a constant is included; 0 otherwise
m         = nvar*nlag + nex; % number of exogenous variables
nd        = 4e5; % number of orthogonal-reduced-form (B,Sigma,Q) draws
```

```matlab
41  iter_show = 1e4; % display iteration every iter_show draws
42  horizon   = 60; % maximum horizon for IRFs
43  index     = 40; % define  horizons for the FEVD
44  horizons  = 0:5; % horizons to restrict
45  NS        = 1 + numel(horizons); % number of objects in F(A_{0},A_{+})
        to which we impose sign and zero restrictios: F(THETA)=[A_{0};L_
        {0},...,L_{horizons}]
46  e         = eye(nvar); % create identity matrix
47  maxdraws  = 1e4; % max number of importance sampling draws
48  conjugate = ''; % structural or irfs or empty
49
50  %=========================================================================
51  % identification: declare Ss and Zs matrices
52  %=========================================================================
53  % restrictions on A0  and/or IRFs
54
55  % sign restrictions
56  S = cell(nvar,1);
57  for ii=1:nvar
58      S{ii}=zeros(0,nvar*NS);
59  end
60
61  % negative restrictions on the IRF of price and commodity price.
62  % positive restrictions on the IRF of EBP and NFCI and r.
63  S{1}(1,nvar+2)= -1; S{1}(6,2*nvar+2) = -1; S{1}(11,3*nvar+2) = -1; S
        {1}(16,4*nvar+2)= -1; S{1}(21,5*nvar+2)= -1; S{1}(26,6*nvar+2)= -1;
64  S{1}(2,nvar+3)= -1; S{1}(7,2*nvar+3) = -1; S{1}(12,3*nvar+3) = -1; S
        {1}(17,4*nvar+3)= -1; S{1}(22,5*nvar+3)= -1; S{1}(27,6*nvar+3)= -1;
65  S{1}(3,nvar+4)=  1; S{1}(8,2*nvar+4) =  1; S{1}(13,3*nvar+4) =  1; S
        {1}(18,4*nvar+4)=  1; S{1}(23,5*nvar+4)=  1; S{1}(28,6*nvar+4)=  1;
66  S{1}(4,nvar+5)=  1; S{1}(9,2*nvar+5) =  1; S{1}(14,3*nvar+5) =  1; S
        {1}(19,4*nvar+5)=  1; S{1}(24,5*nvar+5)=  1; S{1}(29,6*nvar+5)=  1;
67  S{1}(5,nvar+6)=  1; S{1}(10,2*nvar+6)=  1; S{1}(15,3*nvar+6) =  1; S
        {1}(20,4*nvar+6)=  1; S{1}(25,5*nvar+6)=  1; S{1}(30,6*nvar+6)=  1;
68
69  % zero restrictions
70  Z=cell(nvar,1);
71  for i=1:nvar
72      Z{i}=zeros(0,nvar*NS);
73  end
74
75  %=========================================================================
76  % Setup info
77  %=========================================================================
78  info=SetupInfo(nvar,m,Z,@(x)chol(x));
79
80  % ZF(A_{0},A_{+})
```

```matlab
81  info.nlag      = nlag;
82  info.horizons = horizons;
83  info.ZF        = @(x,y)ZF(x,y);
84
85  % functions useful to compute the importance sampler weights
86  iw_info = info;
87  fs        = @(x)ff_h(x,iw_info);
88  r         = @(x)ZeroRestrictions(x,iw_info);
89
90  if strcmp(conjugate,'irfs')==1
91      fo                = @(x)f_h(x,iw_info);
92      fo_str2irfs       = @(x)StructuralToIRF(x,iw_info);
93      fo_str2irfs_inv   = @(x)IRFToStructural(x,iw_info);
94      r_irfs            = @(x)IRFRestrictions_more_general(x,iw_info);
95  end
96
97  % function useful to check the sign restrictions
98  fh_S_restrictions  = @(x)SF(x,iw_info,S);
99
100 %=========================================================================
101 % write data in Rubio, Waggoner, and Zha (RES 2010)'s notation
102 %=========================================================================
103
104 % yt(t) A0 = xt(t) Aplus + constant + et(t) for t=1...,T;
105 % yt(t)    = xt(t) B     + ut(t)             for t=1...,T;
106 % x(t)     = [yt(t-1), ... , yt(t-nlag), constant];
107 % matrix notation yt = xt*B + ut;
108 % xt=[yt_{-1} ones(T,1)];
109
110 yt = num(nlag+1:end,:);
111 T  = size(yt,1);
112 xt = zeros(T,nvar*nlag+nex);
113
114 for i=1:nlag
115     xt(:,nvar*(i-1)+1:nvar*i) = num((nlag-(i-1)):end-i,:);
116 end
117
118 if nex>=1
119     xt(:,nvar*nlag+nex)=ones(T,1);
120 end
121
122 % write data in Zellner (1971, pp 224-227) notation
123 Y = yt; % T by nvar matrix of observations
124 X = xt; % T by (nvar*nlag+1) matrix of regressors
125
126 % prior for reduced-form parameters
127 nnuBar             = 0;
```

```matlab
128 OomegaBarInverse     = zeros(m);
129 PpsiBar              = zeros(m,nvar);%forchettone
130 PphiBar              = zeros(nvar);
131
132 % posterior for reduced-form parameters
133 nnuTilde             = T +nnuBar; % tempi - 12
134 OomegaTilde          = (X'*X  + OomegaBarInverse)\eye(m);
135 OomegaTildeInverse   =  X'*X  + OomegaBarInverse;
136 PpsiTilde            = OomegaTilde*(X'*Y + OomegaBarInverse*PpsiBar);
137 PphiTilde            = Y'*Y + PphiBar + PpsiBar'*OomegaBarInverse*PpsiBar
        - PpsiTilde'*OomegaTildeInverse*PpsiTilde;
138 PphiTilde            = (PphiTilde'+PphiTilde)*0.5;
139
140
141 % useful definitions
142 % definitios used to store orthogonal-reduced-form draws, volume
        elements, and unnormalized weights
143 Bdraws          = cell([nd,1]); % reduced-form lag parameters
144 Sigmadraws      = cell([nd,1]); % reduced-form covariance matrices
145 Qdraws          = cell([nd,1]); % orthogonal matrices
146 storevefh       = zeros(nd,1);  % volume element f_{h}
147 storevegfhZ     = zeros(nd,1);  % volume element g o f_{h}|Z
148 uw              = zeros(nd,1);  % unnormalized importance sampler weights
149
150 if strcmp(conjugate,'irfs')==1
151     storevephi      = zeros(nd,1);  % volume element f_{h}
152     storevegphiZ    = zeros(nd,1);  % volume element g o f_{h}|Z
153 end
154
155 % definitions related to IRFs; based on page 12 of Rubio, Waggoner, and
        Zha (RES 2010)
156 J     = [e;repmat(zeros(nvar),nlag-1,1)];
157 A     = cell(nlag,1);
158 extraF = repmat(zeros(nvar),1,nlag-1);
159 F     = zeros(nlag*nvar,nlag*nvar);
160
161 for l=1:nlag-1
162     F((l-1)*nvar+1:l*nvar,nvar+1:nlag*nvar)=[repmat(zeros(nvar),1,l-1) e
        repmat(zeros(nvar),1,nlag-(l+1))];
163 end
164
165 % definition to facilitate the draws from B|Sigma
166 hh               = info.h;
167 cholOomegaTilde = hh(OomegaTilde)'; %this matrix is used to draw B|Sigma
168
169 % initialize counters to track the state of the computations
170 counter = 1;
```

```matlab
171 record   = 1;
172 count    = 0;
173 tStart   = tic;
174
175 while record<=nd
176
177     % step 1 in Algorithm 2
178     Sigmadraw      = iwishrnd(PphiTilde,nnuTilde);
179     cholSigmadraw  = hh(Sigmadraw)';
180     Bdraw          = kron(cholSigmadraw,cholOomegaTilde)*randn(m*nvar,1)
    + reshape(PpsiTilde,nvar*m,1);
181     Bdraw          = reshape(Bdraw,nvar*nlag+nex,nvar);
182
183     % store reduced-form draws
184     Bdraws{record,1}     = Bdraw;
185     Sigmadraws{record,1} = Sigmadraw;
186
187     % steps 2:4 of Algorithm 2
188     w           = DrawW(iw_info);
189     x           = [vec(Bdraw); vec(Sigmadraw); w];
190     structpara  = ff_h_inv(x,iw_info);
191
192     % store the matrix Q associated with step 3
193     Qdraw             = SpheresToQ(w,iw_info,Bdraw,Sigmadraw);
194     Qdraws{record,1} = reshape(Qdraw,nvar,nvar);
195
196     % check if sign restrictions hold
197     signs       = fh_S_restrictions(structpara);
198
199     if (sum(signs{1}*e(:,1)>0))==size(signs{1}*e(:,1),1)
200         count=count+1;
201         % compute importance sampling weights
202
203         switch conjugate
204
205         case 'structural'
206         storevefh(record,1) = (nvar*(nvar+1)/2)*log(2)-(2*nvar+m+1)*
    LogAbsDet(reshape(structpara(1:nvar*nvar),nvar,nvar));
207         storevegfhZ(record,1) = LogVolumeElement(fs,structpara,r);
208         uw(record,1) = exp(storevefh(record,1) - storevegfhZ(record,1));
209
210         case 'irfs'
211         irfpara = fo_str2irfs(structpara);
212         storevephi(record,1) = LogVolumeElement(fo,structpara)   +
    LogVolumeElement(fo_str2irfs_inv,irfpara);
213         storevegphiZ(record,1) = LogVolumeElement(fs,structpara,r) +
    LogVolumeElement(fo_str2irfs_inv,irfpara,r_irfs);
```

```matlab
214        uw(record,1) = exp(storevephi(record,1)-storevegphiZ(record,1));
215
216        otherwise
217        uw(record,1) = 1;
218        end
219
220    else
221        uw(record,1) = 0;
222    end
223
224    if counter==iter_show
225        display(['Number of draws = ',num2str(record)])
226        display(['Remaining draws = ',num2str(nd-(record))])
227        counter =0;
228    end
229
230    counter = counter + 1;
231    record=record+1;
232 end
233
234 tElapsed = toc(tStart);
235 imp_w  = uw/sum(uw);
236 ne = floor(1/sum(imp_w.^2));
237
238 % store draws
239 Ltilde      = zeros(horizon+1,nvar,nvar,ne); % define array to store IRF
240 A0tilde       = zeros(nvar,nvar,ne); % define array to store A0
241 Aplustilde    = zeros(m,nvar,ne); % define array to store Aplus
242 hist_is_draws = zeros(ne,1);   % define array to store draws from
       importance sampler
243
244 for s=1:min(ne,maxdraws)
245
246    % draw: B,Sigma,Q
247    is_draw = randsample(1:size(imp_w,1),1,true,imp_w);
248    hist_is_draws(s,1)=is_draw;
249
250    Bdraw       = Bdraws{is_draw,1};
251    Sigmadraw   = Sigmadraws{is_draw,1};
252    Qdraw       = Qdraws{is_draw,1};
253
254    x=[reshape(Bdraw,m*nvar,1);reshape(Sigmadraw,nvar*nvar,1);Qdraw(:)];
255    structpara = f_h_inv(x,info);
256
257    LIRF =IRF_horizons(structpara, nvar, nlag, m, 0:horizon);
258
259    for h=0:horizon
```

```matlab
            Ltilde(h+1,:,:,s) =  LIRF(1+h*nvar:(h+1)*nvar,:);
    end

    A0tilde(:,:,s)    = reshape(structpara(1:nvar*nvar),nvar,nvar);
    Aplustilde(:,:,s) = reshape(structpara(nvar*nvar+1:end),m,nvar);
end

A0tilde    = A0tilde(:,:,1:s);
Aplustilde = Aplustilde(:,:,1:s);
Ltilde     = Ltilde(:,:,:,1:s);

save('results/figure_1_results_d.mat','Ltilde','A0tilde','Aplustilde','
    imp_w','ne');
cd ..
```

Replication code for estimating the IRFs according to restrictions 1 and 2

```matlab
1  %%*************************************************************************
2  % This is a modified version of the code developed by
3  % Arias et al. (2019) to compute the draws when restrictions 1 and 2
4  % are imposed.
5  %%*************************************************************************
6
7  clear variables;
8  close all;
9  userpath('clear');
10 clc;
11
12 rng('default'); % reinitialize the random number generator to its
      startup configuration
13 rng(0);         % set seed
14
15 currdir=pwd;
16 cd ..
17 get_help_dir_currdir=pwd;
18 addpath([get_help_dir_currdir,'/helpfunctions']); % set path to helper
      functions
19 cd(currdir)
20
21 %========================================================================
22 % load the data and priors
23 %========================================================================
24
25 data   = readtable([get_help_dir_currdir,'/data/csvfiles/dataset.csv']);
26 data   = data(find(data.dates=='01-Jan-1973'):find(data.dates=='01-Jun
      -2007'),:);
27
28 % data
29 % all variables are in log times 100 except for the federal funds rate
      that enters the SVAR in annualized percentages
30 num  = [100*log(data.monthly_GDP) 100*log(data.monthly_GDPDEF) 100*log(
      data.CPRINDEX) data.EBP data.NFCI data.FEDFUNDS];
31
32 %========================================================================
33 % model setup
34 %========================================================================
35
36 nlag       = 12; % number of lags
37 nvar       = 6; % number of endogenous variables
38 nex        = 0; % set equal to 1 if a constant is included; 0 otherwise
39 m          = nvar*nlag + nex; % number of exogenous variables
40 nd         = 2e6; % number of orthogonal-reduced-form (B,Sigma,Q) draws
```

```matlab
iter_show = 1e5; % display iteration every iter_show draws
horizon   = 60; % maximum horizon for IRFs
index     = 40; % define  horizons for the FEVD
horizons  = 0:5; % horizons to restrict
NS        = 1 + numel(horizons); % number of objects in F(A_{0},A_{+})
    to which we impose sign and zero restrictios: F(THETA)=[A_{0};L_
    {0},...,L_{horizons}]
e         = eye(nvar); % create identity matrix
maxdraws  = 1e4; % max number of importance sampling draws
conjugate = 'structural'; % structural or irfs or empty

%========================================================================
% identification: declare Ss and Zs matrices
%========================================================================
% restrictions on A0  and/or IRFs

% sign restrictions
S = cell(nvar,1);
for ii=1:nvar
    S{ii}=zeros(0,nvar*NS);
end

% negative restrictions on the IRF of price and commodity price.
% positive restrictions on the IRF of EBP and NFCI and r.
S{1}(1,nvar+2)= -1; S{1}(6,2*nvar+2) = -1; S{1}(11,3*nvar+2) = -1; S
    {1}(16,4*nvar+2)= -1; S{1}(21,5*nvar+2)= -1; S{1}(26,6*nvar+2)= -1;
S{1}(2,nvar+3)= -1; S{1}(7,2*nvar+3) = -1; S{1}(12,3*nvar+3) = -1; S
    {1}(17,4*nvar+3)= -1; S{1}(22,5*nvar+3)= -1; S{1}(27,6*nvar+3)= -1;
S{1}(3,nvar+4)=  1; S{1}(8,2*nvar+4) =  1; S{1}(13,3*nvar+4) =  1; S
    {1}(18,4*nvar+4)=  1; S{1}(23,5*nvar+4)=  1; S{1}(28,6*nvar+4)=  1;
S{1}(4,nvar+5)=  1; S{1}(9,2*nvar+5) =  1; S{1}(14,3*nvar+5) =  1; S
    {1}(19,4*nvar+5)=  1; S{1}(24,5*nvar+5)=  1; S{1}(29,6*nvar+5)=  1;
S{1}(5,nvar+6)=  1; S{1}(10,2*nvar+6)=  1; S{1}(15,3*nvar+6) =  1; S
    {1}(20,4*nvar+6)=  1; S{1}(25,5*nvar+6)=  1; S{1}(30,6*nvar+6)=  1;

% sign restriction on the contemporaneous coefficients.
% Negative sign restrictions on y and p,
S{1}(31,1)        = -1;
S{1}(32,2)        = -1;
S{1}(33,6)        =  1;

% zero restrictions
Z=cell(nvar,1);
for i=1:nvar
    Z{i}=zeros(0,nvar*NS);
end

```

```matlab
81  %========================================================================
82  % Setup info
83  %========================================================================
84  info=SetupInfo(nvar,m,Z,@(x)chol(x));
85
86  % ZF(A_{0},A_{+})
87  info.nlag     = nlag;
88  info.horizons = horizons;
89  info.ZF       = @(x,y)ZF(x,y);
90
91  % functions useful to compute the importance sampler weights
92  iw_info = info;
93  fs      = @(x)ff_h(x,iw_info);
94  r       = @(x)ZeroRestrictions(x,iw_info);
95
96  if strcmp(conjugate,'irfs')==1
97      fo               = @(x)f_h(x,iw_info);
98      fo_str2irfs      = @(x)StructuralToIRF(x,iw_info);
99      fo_str2irfs_inv  = @(x)IRFToStructural(x,iw_info);
100     r_irfs           = @(x)IRFRestrictions_more_general(x,iw_info);
101 end
102
103 % function useful to check the sign restrictions
104 fh_S_restrictions  = @(x)SF(x,iw_info,S);
105
106 %========================================================================
107 % write data in Rubio, Waggoner, and Zha (RES 2010)'s notation
108 %========================================================================
109 % yt(t) A0 = xt(t) Aplus + constant + et(t) for t=1...,T;
110 % yt(t)    = xt(t) B      + ut(t)            for t=1...,T;
111 % x(t)     = [yt(t-1), ... , yt(t-nlag), constant];
112 % matrix notation yt = xt*B + ut;
113 % xt=[yt_{-1} ones(T,1)];
114
115 yt = num(nlag+1:end,:);
116 T  = size(yt,1);
117 xt = zeros(T,nvar*nlag+nex);
118 for i=1:nlag
119     xt(:,nvar*(i-1)+1:nvar*i) = num((nlag-(i-1)):end-i,:) ;
120 end
121
122 if nex>=1
123     xt(:,nvar*nlag+nex)=ones(T,1);
124 end
125
126 % write data in Zellner (1971, pp 224-227) notation
127 Y = yt; % T by nvar matrix of observations
```

```matlab
128  X = xt; % T by (nvar*nlag+1) matrix of regressors
129
130  % prior for reduced-form parameters
131  nnuBar              = 0;
132  OomegaBarInverse    = zeros(m);
133  PpsiBar             = zeros(m,nvar);
134  PphiBar             = zeros(nvar);
135
136  % posterior for reduced-form parameters
137  nnuTilde            = T +nnuBar;
138  OomegaTilde         = (X'*X  + OomegaBarInverse)\eye(m);
139  OomegaTildeInverse  =  X'*X  + OomegaBarInverse;
140  PpsiTilde           = OomegaTilde*(X'*Y + OomegaBarInverse*PpsiBar);
141  PphiTilde           = Y'*Y + PphiBar + PpsiBar'*OomegaBarInverse*PpsiBar
         - PpsiTilde'*OomegaTildeInverse*PpsiTilde;
142  PphiTilde           = (PphiTilde'+PphiTilde)*0.5;
143
144  % useful definitions
145  % definitios used to store orthogonal-reduced-form draws, volume
         elements, and unnormalized weights
146  Bdraws        = cell([nd,1]); % reduced-form lag parameters
147  Sigmadraws    = cell([nd,1]); % reduced-form covariance matrices
148  Qdraws        = cell([nd,1]); % orthogonal matrices
149  storevefh     = zeros(nd,1);  % volume element f_{h}
150  storevegfhZ   = zeros(nd,1);  % volume element g o f_{h}|Z
151  uw            = zeros(nd,1);  % unnormalized importance sampler weights
152
153  if strcmp(conjugate,'irfs')==1
154      storevephi     = zeros(nd,1);  % volume element f_{h}
155      storevegphiZ   = zeros(nd,1);  % volume element g o f_{h}|Z
156  end
157
158  % definitions related to IRFs; based on page 12 of Rubio, Waggoner, and
         Zha (RES 2010)
159  J     = [e;repmat(zeros(nvar),nlag-1,1)];
160  A     = cell(nlag,1);
161  extraF = repmat(zeros(nvar),1,nlag-1);
162  F     = zeros(nlag*nvar,nlag*nvar);
163  for l=1:nlag-1
164      F((l-1)*nvar+1:l*nvar,nvar+1:nlag*nvar)=[repmat(zeros(nvar),1,l-1) e
         repmat(zeros(nvar),1,nlag-(l+1))];
165  end
166
167  % definition to facilitate the draws from B|Sigma
168  hh              = info.h;
169  cholOomegaTilde = hh(OomegaTilde)'; % this matrix is used to draw B|
         Sigma below
```

```matlab
170
171  % initialize counters to track the state of the computations
172  counter = 1;
173  record  = 1;
174  count   = 0;
175  tStart = tic;
176  while record<=nd
177
178      % step 1 in Algorithm 2
179      Sigmadraw      = iwishrnd(PphiTilde,nnuTilde);
180      cholSigmadraw = hh(Sigmadraw)';
181      Bdraw          = kron(cholSigmadraw,cholOomegaTilde)*randn(m*nvar,1)
     + reshape(PpsiTilde,nvar*m,1);
182      Bdraw          = reshape(Bdraw,nvar*nlag+nex,nvar);
183      % store reduced-form draws
184      Bdraws{record,1}      = Bdraw;
185      Sigmadraws{record,1} = Sigmadraw;
186
187      % steps 2:4 of Algorithm 2
188      w            = DrawW(iw_info);
189      x            = [vec(Bdraw); vec(Sigmadraw); w];
190      structpara  = ff_h_inv(x,iw_info);
191
192      % store the matrix Q associated with step 3
193      Qdraw             = SpheresToQ(w,iw_info,Bdraw,Sigmadraw);
194      Qdraws{record,1} = reshape(Qdraw,nvar,nvar);
195
196      % check if sign restrictions hold
197      signs       = fh_S_restrictions(structpara);
198
199      if (sum(signs{1}*e(:,1)>0))==size(signs{1}*e(:,1),1)
200
201          count=count+1;
202
203          % compute importance sampling weights
204
205          switch conjugate
206
207          case 'structural'
208
209          storevefh(record,1)   = (nvar*(nvar+1)/2)*log(2)-(2*nvar+m+1)*
     LogAbsDet(reshape(structpara(1:nvar*nvar),nvar,nvar));
210          storevegfhZ(record,1) = LogVolumeElement(fs,structpara,r);
211          uw(record,1)            = exp(storevefh(record,1) - storevegfhZ(
     record,1));
212
213          case 'irfs'
```

```matlab
        irfpara                 = fo_str2irfs(structpara);
        storevephi(record,1)   = LogVolumeElement(fo,structpara)   +
    LogVolumeElement(fo_str2irfs_inv,irfpara);
        storevegphiZ(record,1) = LogVolumeElement(fs,structpara,r) +
    LogVolumeElement(fo_str2irfs_inv,irfpara,r_irfs);
        uw(record,1)           = exp(storevephi(record,1) - storevegphiZ
    (record,1));

        otherwise

                uw(record,1) = 1;
        end

    else

        uw(record,1) = 0;
    end

    if counter==iter_show

        display(['Number of draws = ',num2str(record)])
        display(['Remaining draws = ',num2str(nd-(record))])
        counter =0;
    end

    counter = counter + 1;
    record=record+1;
end

tElapsed = toc(tStart);
imp_w  = uw/sum(uw);
ne = floor(1/sum(imp_w.^2));

% store draws
Ltilde     = zeros(horizon+1,nvar,nvar,ne); % define array to store IRF
A0tilde      = zeros(nvar,nvar,ne); % define array to store A0
Aplustilde   = zeros(m,nvar,ne); % define array to store Aplus
hist_is_draws = zeros(ne,1); % define array to store draws from
    importance sampler

for s=1:min(ne,maxdraws)

    % draw: B,Sigma,Q
    is_draw = randsample(1:size(imp_w,1),1,true,imp_w);
    hist_is_draws(s,1)=is_draw;
```

```matlab
257     Bdraw        = Bdraws{is_draw ,1};
258     Sigmadraw    = Sigmadraws{is_draw ,1};
259     Qdraw        = Qdraws{is_draw ,1};
260
261     x=[reshape(Bdraw,m*nvar,1);reshape(Sigmadraw,nvar*nvar,1);Qdraw(:)];
262     structpara = f_h_inv(x,info);
263
264     LIRF =IRF_horizons(structpara, nvar, nlag, m, 0:horizon);
265
266     for h=0:horizon
267         Ltilde(h+1,:,:,s) =  LIRF(1+h*nvar:(h+1)*nvar,:);
268     end
269
270     A0tilde(:,:,s)    = reshape(structpara(1:nvar*nvar),nvar,nvar);
271     Aplustilde(:,:,s) = reshape(structpara(nvar*nvar+1:end),m,nvar);
272 end
273
274 A0tilde     = A0tilde(:,:,1:s);
275 Aplustilde = Aplustilde(:,:,1:s);
276 Ltilde      = Ltilde(:,:,:,1:s);
277
278 % cd results
279 save('results/figure.mat','Ltilde','A0tilde','Aplustilde','imp_w','ne',
        'm','Bdraw','Bdraws');
280 cd ..
```

Replication code for estimating the IRFs according to restrictions 1, 2 and 3

```matlab
%%************************************************************************
% This is a modified version of the code developed by
% Arias et al. (2019) to compute the draws when restrictions 1, 2 and 3
% are imposed.
%%************************************************************************

clear variables;
close all;
userpath('clear');
clc;

rng('default'); % reinitialize the random number generator to its
    startup configuration
rng(0);         % set seed

currdir=pwd;
cd ..
get_help_dir_currdir=pwd;
addpath([get_help_dir_currdir,'/helpfunctions']); % set path to helper
    functions
cd(currdir)

%=======================================================================
% load the data and priors
%=======================================================================

data    = readtable([get_help_dir_currdir,'/data/csvfiles/dataset.csv']);
data    = data(find(data.dates=='01-Jan-1973'):find(data.dates=='01-Jun
    -2007'),:);

% data
% all variables are in log times 100 except for the federal funds rate
    that enters the SVAR in annualized percentages
num  = [100*log(data.monthly_GDP) 100*log(data.monthly_GDPDEF) 100*log(
    data.CPRINDEX) data.EBP data.NFCI data.FEDFUNDS];

%=======================================================================
% model setup
%=======================================================================

nlag      = 12; % number of lags
nvar      = 6; % number of endogenous variables
nex       = 0; % set equal to 1 if a constant is included; 0 otherwise
m         = nvar*nlag + nex; % number of exogenous variables
nd        = 4e5; % number of orthogonal-reduced-form (B,Sigma,Q) draws
```

```matlab
41 iter_show = 1e4; % display iteration every iter_show draws
42 horizon   = 60; % maximum horizon for IRFs
43 index     = 40; % define  horizons for the FEVD
44 horizons  = 0:5; % horizons to restrict
45 NS        = 1 + numel(horizons); % number of objects in F(A_{0},A_{+})
       to which we impose sign and zero restrictios: F(THETA)=[A_{0};L_
       {0},...,L_{horizons}]
46 e         = eye(nvar); % create identity matrix
47 maxdraws  = 1e4; % max number of importance sampling draws
48 conjugate = ''; % structural or irfs or empty
49
50 %=========================================================================
51 % identification: declare Ss and Zs matrices
52 %=========================================================================
53 % restrictions on A0  and/or IRFs
54
55 % sign restrictions
56 S = cell(nvar,1);
57 for ii=1:nvar
58     S{ii}=zeros(0,nvar*NS);
59 end
60
61 % negative restrictions on the IRF of price and commodity price.
62 % positive restrictions on the IRF of EBP and NFCI and r.
63 S{1}(1,nvar+2)= -1; S{1}(6,2*nvar+2) = -1; S{1}(11,3*nvar+2) = -1; S
       {1}(16,4*nvar+2)= -1; S{1}(21,5*nvar+2)= -1; S{1}(26,6*nvar+2)= -1;
64 S{1}(2,nvar+3)= -1; S{1}(7,2*nvar+3) = -1; S{1}(12,3*nvar+3) = -1; S
       {1}(17,4*nvar+3)= -1; S{1}(22,5*nvar+3)= -1; S{1}(27,6*nvar+3)= -1;
65 S{1}(3,nvar+4)=  1; S{1}(8,2*nvar+4) =  1; S{1}(13,3*nvar+4) =  1; S
       {1}(18,4*nvar+4)=  1; S{1}(23,5*nvar+4)=  1; S{1}(28,6*nvar+4)=  1;
66 S{1}(4,nvar+5)=  1; S{1}(9,2*nvar+5) =  1; S{1}(14,3*nvar+5) =  1; S
       {1}(19,4*nvar+5)=  1; S{1}(24,5*nvar+5)=  1; S{1}(29,6*nvar+5)=  1;
67 S{1}(5,nvar+6)=  1; S{1}(10,2*nvar+6)=  1; S{1}(15,3*nvar+6) =  1; S
       {1}(20,4*nvar+6)=  1; S{1}(25,5*nvar+6)=  1; S{1}(30,6*nvar+6)=  1;
68
69 % sign restriction on the contemporaneous coefficients.
70 % Negative sign restrictions on y and p,
71 % Positive sign restrictions on ECB and NFCI.
72 S{1}(31,1)        = -1;
73 S{1}(32,2)        = -1;
74 S{1}(33,4)        =  1;
75 S{1}(34,5)        =  1;
76 S{1}(35,6)        =  1;
77
78 % zero restrictions
79 Z=cell(nvar,1);
80 for i=1:nvar
```

```matlab
81      Z{i}=zeros(0,nvar*NS);
82  end
83
84  %=========================================================================
85  % Setup info
86  %=========================================================================
87  info=SetupInfo(nvar,m,Z,@(x)chol(x));
88
89  % ZF(A_{0},A_{+})
90  info.nlag     = nlag;
91  info.horizons = horizons;
92  info.ZF       = @(x,y)ZF(x,y);
93
94  % functions useful to compute the importance sampler weights
95  iw_info = info;
96  fs      = @(x)ff_h(x,iw_info);
97  r       = @(x)ZeroRestrictions(x,iw_info);
98
99  if strcmp(conjugate,'irfs')==1
100     fo              = @(x)f_h(x,iw_info);
101     fo_str2irfs     = @(x)StructuralToIRF(x,iw_info);
102     fo_str2irfs_inv = @(x)IRFToStructural(x,iw_info);
103     r_irfs          = @(x)IRFRestrictions_more_general(x,iw_info);
104 end
105
106 % function useful to check the sign restrictions
107 fh_S_restrictions  = @(x)SF(x,iw_info,S);
108
109 %=========================================================================
110 % write data in Rubio, Waggoner, and Zha (RES 2010)'s notation
111 %=========================================================================
112
113 % yt(t) A0 = xt(t) Aplus + constant + et(t) for t=1...,T;
114 % yt(t)    = xt(t) B     + ut(t)              for t=1...,T;
115 % x(t)     = [yt(t-1), ... , yt(t-nlag), constant];
116 % matrix notation yt = xt*B + ut;
117 % xt=[yt_{-1} ones(T,1)];
118
119 yt = num(nlag+1:end,:);
120 T  = size(yt,1);
121 xt = zeros(T,nvar*nlag+nex);
122
123 for i=1:nlag
124     xt(:,nvar*(i-1)+1:nvar*i) = num((nlag-(i-1)):end-i,:);
125 end
126
127 if nex>=1
```

```matlab
128      xt(:,nvar*nlag+nex)=ones(T,1);
129 end
130
131 % write data in Zellner (1971, pp 224-227) notation
132 Y = yt; % T by nvar matrix of observations
133 X = xt; % T by (nvar*nlag+1) matrix of regressors
134
135 % prior for reduced-form parameters
136 nnuBar                = 0;
137 OomegaBarInverse      = zeros(m);
138 PpsiBar               = zeros(m,nvar);%forchettone
139 PphiBar               = zeros(nvar);
140
141 % posterior for reduced-form parameters
142 nnuTilde              = T +nnuBar; % tempi - 12
143 OomegaTilde           = (X'*X  + OomegaBarInverse)\eye(m);
144 OomegaTildeInverse    =  X'*X   + OomegaBarInverse;
145 PpsiTilde             = OomegaTilde*(X'*Y + OomegaBarInverse*PpsiBar);
146 PphiTilde             = Y'*Y + PphiBar + PpsiBar'*OomegaBarInverse*PpsiBar
        - PpsiTilde'*OomegaTildeInverse*PpsiTilde;
147 PphiTilde             = (PphiTilde'+PphiTilde)*0.5;
148
149
150 % useful definitions
151 % definitios used to store orthogonal-reduced-form draws, volume
        elements, and unnormalized weights
152 Bdraws        = cell([nd,1]); % reduced-form lag parameters
153 Sigmadraws    = cell([nd,1]); % reduced-form covariance matrices
154 Qdraws        = cell([nd,1]); % orthogonal matrices
155 storevefh     = zeros(nd,1);  % volume element f_{h}
156 storevegfhZ   = zeros(nd,1);  % volume element g o f_{h}|Z
157 uw            = zeros(nd,1);  % unnormalized importance sampler weights
158
159 if strcmp(conjugate,'irfs')==1
160     storevephi     = zeros(nd,1);  % volume element f_{h}
161     storevegphiZ   = zeros(nd,1);  % volume element g o f_{h}|Z
162 end
163
164 % definitions related to IRFs; based on page 12 of Rubio, Waggoner, and
        Zha (RES 2010)
165 J     = [e;repmat(zeros(nvar),nlag-1,1)];
166 A     = cell(nlag,1);
167 extraF = repmat(zeros(nvar),1,nlag-1);
168 F     = zeros(nlag*nvar,nlag*nvar);
169
170 for l=1:nlag-1
171     F((l-1)*nvar+1:l*nvar,nvar+1:nlag*nvar)=[repmat(zeros(nvar),1,l-1) e
```

```matlab
      repmat(zeros(nvar),1,nlag-(l+1))];
end

% definition to facilitate the draws from B|Sigma
hh             = info.h;
cholOomegaTilde = hh(OomegaTilde)'; %this matrix is used to draw B|Sigma

% initialize counters to track the state of the computations
counter = 1;
record  = 1;
count   = 0;
tStart  = tic;

while record<=nd

    % step 1 in Algorithm 2
    Sigmadraw      = iwishrnd(PphiTilde,nnuTilde);
    cholSigmadraw = hh(Sigmadraw)';
    Bdraw          = kron(cholSigmadraw,cholOomegaTilde)*randn(m*nvar,1)
    + reshape(PpsiTilde,nvar*m,1);
    Bdraw          = reshape(Bdraw,nvar*nlag+nex,nvar);

    % store reduced-form draws
    Bdraws{record,1}     = Bdraw;
    Sigmadraws{record,1} = Sigmadraw;

    % steps 2:4 of Algorithm 2
    w          = DrawW(iw_info);
    x          = [vec(Bdraw); vec(Sigmadraw); w];
    structpara = ff_h_inv(x,iw_info);

    % store the matrix Q associated with step 3
    Qdraw           = SpheresToQ(w,iw_info,Bdraw,Sigmadraw);
    Qdraws{record,1} = reshape(Qdraw,nvar,nvar);

    % check if sign restrictions hold
    signs      = fh_S_restrictions(structpara);

    if (sum(signs{1}*e(:,1)>0))==size(signs{1}*e(:,1),1)
        count=count+1;
        % compute importance sampling weights

        switch conjugate

        case 'structural'
        storevefh(record,1) = (nvar*(nvar+1)/2)*log(2)-(2*nvar+m+1)*
    LogAbsDet(reshape(structpara(1:nvar*nvar),nvar,nvar));
```

```matlab
216          storevegfhZ(record,1) = LogVolumeElement(fs,structpara,r);
217          uw(record,1) = exp(storevefh(record,1) - storevegfhZ(record,1));
218
219          case 'irfs'
220          irfpara = fo_str2irfs(structpara);
221          storevephi(record,1) = LogVolumeElement(fo,structpara)   +
    LogVolumeElement(fo_str2irfs_inv,irfpara);
222          storevegphiZ(record,1) = LogVolumeElement(fs,structpara,r) +
    LogVolumeElement(fo_str2irfs_inv,irfpara,r_irfs);
223          uw(record,1) = exp(storevephi(record,1)-storevegphiZ(record,1));
224
225          otherwise
226          uw(record,1) = 1;
227          end
228
229      else
230          uw(record,1) = 0;
231      end
232
233      if counter==iter_show
234          display(['Number of draws = ',num2str(record)])
235          display(['Remaining draws = ',num2str(nd-(record))])
236          counter =0;
237      end
238
239      counter = counter + 1;
240      record=record+1;
241 end
242
243 tElapsed = toc(tStart);
244 imp_w  = uw/sum(uw);
245 ne = floor(1/sum(imp_w.^2));
246
247 % store draws
248 Ltilde      = zeros(horizon+1,nvar,nvar,ne); % define array to store IRF
249 A0tilde      = zeros(nvar,nvar,ne); % define array to store A0
250 Aplustilde   = zeros(m,nvar,ne); % define array to store Aplus
251 hist_is_draws = zeros(ne,1);   % define array to store draws from
    importance sampler
252
253 for s=1:min(ne,maxdraws)
254
255      % draw: B,Sigma,Q
256      is_draw = randsample(1:size(imp_w,1),1,true,imp_w);
257      hist_is_draws(s,1)=is_draw;
258
259      Bdraw       = Bdraws{is_draw,1};
```

```matlab
    Sigmadraw    = Sigmadraws{is_draw,1};
    Qdraw        = Qdraws{is_draw,1};

    x=[reshape(Bdraw,m*nvar,1);reshape(Sigmadraw,nvar*nvar,1);Qdraw(:)];
    structpara = f_h_inv(x,info);

    LIRF =IRF_horizons(structpara, nvar, nlag, m, 0:horizon);

    for h=0:horizon
        Ltilde(h+1,:,:,s) =  LIRF(1+h*nvar:(h+1)*nvar,:);
    end

    A0tilde(:,:,s)    = reshape(structpara(1:nvar*nvar),nvar,nvar);
    Aplustilde(:,:,s) = reshape(structpara(nvar*nvar+1:end),m,nvar)
end

A0tilde    = A0tilde(:,:,1:s);
Aplustilde = Aplustilde(:,:,1:s);
Ltilde     = Ltilde(:,:,:,1:s);

save('results/figure_1_results_d.mat','Ltilde','A0tilde','Aplustilde','
    imp_w','ne');
cd ..
```

Replication code for estimating the IRFs according to restrictions 1, 2 and $\psi_e < 0$

```matlab
%%****************************************************************************
% This is a modified version of the code developed by
% Arias et al. (2019) to compute the draws when restrictions 1, 2 and 3
% are imposed but no restriction is applied on NFCI.
%%****************************************************************************

clear variables;
close all;
userpath('clear');
clc;

rng('default'); % reinitialize the random number generator to its
    startup configuration
rng(0);         % set seed

currdir=pwd;
cd ..
get_help_dir_currdir=pwd;
addpath([get_help_dir_currdir,'/helpfunctions']); % set path to helper
    functions
cd(currdir)

%==========================================================================
% load the data and priors
%==========================================================================

data    = readtable([get_help_dir_currdir,'/data/csvfiles/dataset.csv']);
data    = data(find(data.dates=='01-Jan-1973'):find(data.dates=='01-Jun
    -2007'),:);

% data
% all variables are in log times 100 except for the federal funds rate
    that enters the SVAR in annualized percentages
num  = [100*log(data.monthly_GDP) 100*log(data.monthly_GDPDEF) 100*log(
    data.CPRINDEX) data.EBP data.NFCI data.FEDFUNDS];

%==========================================================================
% model setup
%==========================================================================

nlag       = 12; % number of lags
nvar       = 6; % number of endogenous variables
nex        = 0; % set equal to 1 if a constant is included; 0 otherwise
m          = nvar*nlag + nex; % number of exogenous variables
nd         = 4e5; % number of orthogonal-reduced-form (B,Sigma,Q) draws
```

```matlab
41 iter_show = 1e4; % display iteration every iter_show draws
42 horizon   = 60; % maximum horizon for IRFs
43 index     = 40; % define  horizons for the FEVD
44 horizons  = 0:5; % horizons to restrict
45 NS        = 1 + numel(horizons); % number of objects in F(A_{0},A_{+})
      to which we impose sign and zero restrictios: F(THETA)=[A_{0};L_
      {0},...,L_{horizons}]
46 e         = eye(nvar); % create identity matrix
47 maxdraws  = 1e4; % max number of importance sampling draws
48 conjugate = ''; % structural or irfs or empty
49
50 %========================================================================
51 % identification: declare Ss and Zs matrices
52 %========================================================================
53 % restrictions on A0  and/or IRFs
54
55 % sign restrictions
56 S = cell(nvar,1);
57 for ii=1:nvar
58     S{ii}=zeros(0,nvar*NS);
59 end
60
61 % negative restrictions on the IRF of price and commodity price.
62 % positive restrictions on the IRF of EBP and NFCI and r.
63 S{1}(1,nvar+2)= -1; S{1}(6,2*nvar+2) = -1; S{1}(11,3*nvar+2) = -1; S
      {1}(16,4*nvar+2)= -1; S{1}(21,5*nvar+2)= -1; S{1}(26,6*nvar+2)= -1;
64 S{1}(2,nvar+3)= -1; S{1}(7,2*nvar+3) = -1; S{1}(12,3*nvar+3) = -1; S
      {1}(17,4*nvar+3)= -1; S{1}(22,5*nvar+3)= -1; S{1}(27,6*nvar+3)= -1;
65 S{1}(3,nvar+4)=  1; S{1}(8,2*nvar+4) =  1; S{1}(13,3*nvar+4) =  1; S
      {1}(18,4*nvar+4)=  1; S{1}(23,5*nvar+4)=  1; S{1}(28,6*nvar+4)=  1;
66 S{1}(4,nvar+5)=  1; S{1}(9,2*nvar+5) =  1; S{1}(14,3*nvar+5) =  1; S
      {1}(19,4*nvar+5)=  1; S{1}(24,5*nvar+5)=  1; S{1}(29,6*nvar+5)=  1;
67 S{1}(5,nvar+6)=  1; S{1}(10,2*nvar+6)=  1; S{1}(15,3*nvar+6) =  1; S
      {1}(20,4*nvar+6)=  1; S{1}(25,5*nvar+6)=  1; S{1}(30,6*nvar+6)=  1;
68
69 % sign restriction on the contemporaneous coefficients.
70 % Negative sign restrictions on y and p,
71 % Positive sign restrictions on ECB.
72 S{1}(31,1)        = -1;
73 S{1}(32,2)        = -1;
74 S{1}(33,4)        =  1;
75 S{1}(34,6)        =  1;
76
77 % zero restrictions
78 Z=cell(nvar,1);
79 for i=1:nvar
80     Z{i}=zeros(0,nvar*NS);
```

```matlab
81  end
82
83  %=======================================================================
84  % Setup info
85  %=======================================================================
86  info=SetupInfo(nvar,m,Z,@(x)chol(x));
87
88  % ZF(A_{0},A_{+})
89  info.nlag     = nlag;
90  info.horizons = horizons;
91  info.ZF       = @(x,y)ZF(x,y);
92
93  % functions useful to compute the importance sampler weights
94  iw_info = info;
95  fs      = @(x)ff_h(x,iw_info);
96  r       = @(x)ZeroRestrictions(x,iw_info);
97
98  if strcmp(conjugate,'irfs')==1
99      fo                = @(x)f_h(x,iw_info);
100     fo_str2irfs       = @(x)StructuralToIRF(x,iw_info);
101     fo_str2irfs_inv   = @(x)IRFToStructural(x,iw_info);
102     r_irfs            = @(x)IRFRestrictions_more_general(x,iw_info);
103 end
104
105 % function useful to check the sign restrictions
106 fh_S_restrictions   = @(x)SF(x,iw_info,S);
107
108 %=======================================================================
109 % write data in Rubio, Waggoner, and Zha (RES 2010)'s notation
110 %=======================================================================
111
112 % yt(t) A0 = xt(t) Aplus + constant + et(t) for t=1...,T;
113 % yt(t)    = xt(t) B     + ut(t)              for t=1...,T;
114 % x(t)     = [yt(t-1), ... , yt(t-nlag), constant];
115 % matrix notation yt = xt*B + ut;
116 % xt=[yt_{-1} ones(T,1)];
117
118 yt = num(nlag+1:end,:);
119 T  = size(yt,1);
120 xt = zeros(T,nvar*nlag+nex);
121
122 for i=1:nlag
123     xt(:,nvar*(i-1)+1:nvar*i) = num((nlag-(i-1)):end-i,:);
124 end
125
126 if nex>=1
127     xt(:,nvar*nlag+nex)=ones(T,1);
```

```matlab
128 end
129
130 % write data in Zellner (1971, pp 224-227) notation
131 Y = yt; % T by nvar matrix of observations
132 X = xt; % T by (nvar*nlag+1) matrix of regressors
133
134 % prior for reduced-form parameters
135 nnuBar              = 0;
136 OomegaBarInverse    = zeros(m);
137 PpsiBar             = zeros(m,nvar);%forchettone
138 PphiBar             = zeros(nvar);
139
140 % posterior for reduced-form parameters
141 nnuTilde            = T +nnuBar; % tempi - 12
142 OomegaTilde         = (X'*X  + OomegaBarInverse)\eye(m);
143 OomegaTildeInverse  =  X'*X  + OomegaBarInverse;
144 PpsiTilde           = OomegaTilde*(X'*Y + OomegaBarInverse*PpsiBar);
145 PphiTilde           = Y'*Y + PphiBar + PpsiBar'*OomegaBarInverse*PpsiBar
        - PpsiTilde'*OomegaTildeInverse*PpsiTilde;
146 PphiTilde           = (PphiTilde'+PphiTilde)*0.5;
147
148
149 % useful definitions
150 % definitios used to store orthogonal-reduced-form draws, volume
      elements, and unnormalized weights
151 Bdraws        = cell([nd,1]); % reduced-form lag parameters
152 Sigmadraws    = cell([nd,1]); % reduced-form covariance matrices
153 Qdraws        = cell([nd,1]); % orthogonal matrices
154 storevefh     = zeros(nd,1);  % volume element f_{h}
155 storevegfhZ   = zeros(nd,1);  % volume element g o f_{h}|Z
156 uw            = zeros(nd,1);  % unnormalized importance sampler weights
157
158 if strcmp(conjugate,'irfs')==1
159     storevephi     = zeros(nd,1);  % volume element f_{h}
160     storevegphiZ   = zeros(nd,1);  % volume element g o f_{h}|Z
161 end
162
163 % definitions related to IRFs; based on page 12 of Rubio, Waggoner, and
      Zha (RES 2010)
164 J     = [e;repmat(zeros(nvar),nlag-1,1)];
165 A     = cell(nlag,1);
166 extraF = repmat(zeros(nvar),1,nlag-1);
167 F     = zeros(nlag*nvar,nlag*nvar);
168
169 for l=1:nlag-1
170     F((l-1)*nvar+1:l*nvar,nvar+1:nlag*nvar)=[repmat(zeros(nvar),1,l-1) e
      repmat(zeros(nvar),1,nlag-(l+1))];
```

```matlab
171  end
172
173  % definition to facilitate the draws from B|Sigma
174  hh              = info.h;
175  cholOomegaTilde = hh(OomegaTilde)'; %this matrix is used to draw B|Sigma
176
177  % initialize counters to track the state of the computations
178  counter = 1;
179  record  = 1;
180  count   = 0;
181  tStart  = tic;
182
183  while record<=nd
184
185      % step 1 in Algorithm 2
186      Sigmadraw     = iwishrnd(PphiTilde,nnuTilde);
187      cholSigmadraw = hh(Sigmadraw)';
188      Bdraw         = kron(cholSigmadraw,cholOomegaTilde)*randn(m*nvar,1)
      + reshape(PpsiTilde,nvar*m,1);
189      Bdraw         = reshape(Bdraw,nvar*nlag+nex,nvar);
190
191      % store reduced-form draws
192      Bdraws{record,1}     = Bdraw;
193      Sigmadraws{record,1} = Sigmadraw;
194
195      % steps 2:4 of Algorithm 2
196      w             = DrawW(iw_info);
197      x             = [vec(Bdraw); vec(Sigmadraw); w];
198      structpara    = ff_h_inv(x,iw_info);
199
200      % store the matrix Q associated with step 3
201      Qdraw             = SpheresToQ(w,iw_info,Bdraw,Sigmadraw);
202      Qdraws{record,1} = reshape(Qdraw,nvar,nvar);
203
204      % check if sign restrictions hold
205      signs     = fh_S_restrictions(structpara);
206
207      if (sum(signs{1}*e(:,1)>0))==size(signs{1}*e(:,1),1)
208          count=count+1;
209          % compute importance sampling weights
210
211          switch conjugate
212
213          case 'structural'
214          storevefh(record,1) = (nvar*(nvar+1)/2)*log(2)-(2*nvar+m+1)*
      LogAbsDet(reshape(structpara(1:nvar*nvar),nvar,nvar));
215          storevegfhZ(record,1) = LogVolumeElement(fs,structpara,r);
```

```matlab
216         uw(record,1) = exp(storevefh(record,1) - storevegfhZ(record,1));
217
218         case 'irfs'
219         irfpara = fo_str2irfs(structpara);
220         storevephi(record,1) = LogVolumeElement(fo,structpara)   +
    LogVolumeElement(fo_str2irfs_inv,irfpara);
221         storevegphiZ(record,1) = LogVolumeElement(fs,structpara,r) +
    LogVolumeElement(fo_str2irfs_inv,irfpara,r_irfs);
222         uw(record,1) = exp(storevephi(record,1)-storevegphiZ(record,1));
223
224         otherwise
225         uw(record,1) = 1;
226         end
227
228     else
229         uw(record,1) = 0;
230     end
231
232     if counter==iter_show
233         display(['Number of draws = ',num2str(record)])
234         display(['Remaining draws = ',num2str(nd-(record))])
235         counter =0;
236     end
237
238     counter = counter + 1;
239     record=record+1;
240 end
241
242 tElapsed = toc(tStart);
243 imp_w  = uw/sum(uw);
244 ne = floor(1/sum(imp_w.^2));
245
246 % store draws
247 Ltilde     = zeros(horizon+1,nvar,nvar,ne); % define array to store IRF
248 A0tilde       = zeros(nvar,nvar,ne); % define array to store A0
249 Aplustilde    = zeros(m,nvar,ne); % define array to store Aplus
250 hist_is_draws = zeros(ne,1);   % define array to store draws from
    importance sampler
251
252 for s=1:min(ne,maxdraws)
253
254     % draw: B,Sigma,Q
255     is_draw = randsample(1:size(imp_w,1),1,true,imp_w);
256     hist_is_draws(s,1)=is_draw;
257
258     Bdraw      = Bdraws{is_draw,1};
259     Sigmadraw  = Sigmadraws{is_draw,1};
```

```matlab
260     Qdraw        = Qdraws{is_draw,1};
261
262     x=[reshape(Bdraw,m*nvar,1);reshape(Sigmadraw,nvar*nvar,1);Qdraw(:)];
263     structpara = f_h_inv(x,info);
264
265     LIRF =IRF_horizons(structpara, nvar, nlag, m, 0:horizon);
266
267     for h=0:horizon
268         Ltilde(h+1,:,:,s) =  LIRF(1+h*nvar:(h+1)*nvar,:);
269     end
270
271     A0tilde(:,:,s)    = reshape(structpara(1:nvar*nvar),nvar,nvar);
272     Aplustilde(:,:,s) = reshape(structpara(nvar*nvar+1:end),m,nvar);
273 end
274
275 A0tilde    = A0tilde(:,:,1:s);
276 Aplustilde = Aplustilde(:,:,1:s);
277 Ltilde     = Ltilde(:,:,:,1:s);
278
279 save('results/figure_1_results_d.mat','Ltilde','A0tilde','Aplustilde','
    imp_w','ne');
280 cd ..
```

Replication code for estimating the IRFs according to restrictions 1, 2 and $\psi_n < 0$

```matlab
%%************************************************************************
% This is a modified version of the code developed by
% Arias et al. (2019) to compute the draws when restrictions 1, 2 and 3
% are imposed but no restriction is applied on EBP.
%%************************************************************************

clear variables;
close all;
userpath('clear');
clc;

rng('default'); % reinitialize the random number generator to its
    startup configuration
rng(0);         % set seed

currdir=pwd;
cd ..
get_help_dir_currdir=pwd;
addpath([get_help_dir_currdir,'/helpfunctions']); % set path to helper
    functions
cd(currdir)

%=========================================================================
% load the data and priors
%=========================================================================

data    = readtable([get_help_dir_currdir,'/data/csvfiles/dataset.csv']);
data    = data(find(data.dates=='01-Jan-1973'):find(data.dates=='01-Jun
    -2007'),:);

% data
% all variables are in log times 100 except for the federal funds rate
    that enters the SVAR in annualized percentages
num  = [100*log(data.monthly_GDP) 100*log(data.monthly_GDPDEF) 100*log(
    data.CPRINDEX) data.EBP data.NFCI data.FEDFUNDS];

%=========================================================================
% model setup
%=========================================================================

nlag       = 12; % number of lags
nvar       = 6; % number of endogenous variables
nex        = 0; % set equal to 1 if a constant is included; 0 otherwise
m          = nvar*nlag + nex; % number of exogenous variables
nd         = 4e5; % number of orthogonal-reduced-form (B,Sigma,Q) draws
```

```matlab
41 iter_show = 1e4; % display iteration every iter_show draws
42 horizon   = 60; % maximum horizon for IRFs
43 index     = 40; % define  horizons for the FEVD
44 horizons  = 0:5; % horizons to restrict
45 NS        = 1 + numel(horizons); % number of objects in F(A_{0},A_{+})
      to which we impose sign and zero restrictios: F(THETA)=[A_{0};L_
      {0},...,L_{horizons}]
46 e         = eye(nvar); % create identity matrix
47 maxdraws  = 1e4; % max number of importance sampling draws
48 conjugate = ''; % structural or irfs or empty
49
50 %=========================================================================
51 % identification: declare Ss and Zs matrices
52 %=========================================================================
53 % restrictions on A0  and/or IRFs
54
55 % sign restrictions
56 S = cell(nvar,1);
57 for ii=1:nvar
58     S{ii}=zeros(0,nvar*NS);
59 end
60
61 % negative restrictions on the IRF of price and commodity price.
62 % positive restrictions on the IRF of EBP and NFCI and r.
63 S{1}(1,nvar+2)= -1; S{1}(6,2*nvar+2) = -1; S{1}(11,3*nvar+2) = -1; S
      {1}(16,4*nvar+2)= -1; S{1}(21,5*nvar+2)= -1; S{1}(26,6*nvar+2)= -1;
64 S{1}(2,nvar+3)= -1; S{1}(7,2*nvar+3) = -1; S{1}(12,3*nvar+3) = -1; S
      {1}(17,4*nvar+3)= -1; S{1}(22,5*nvar+3)= -1; S{1}(27,6*nvar+3)= -1;
65 S{1}(3,nvar+4)=  1; S{1}(8,2*nvar+4) =  1; S{1}(13,3*nvar+4) =  1; S
      {1}(18,4*nvar+4)=  1; S{1}(23,5*nvar+4)=  1; S{1}(28,6*nvar+4)=  1;
66 S{1}(4,nvar+5)=  1; S{1}(9,2*nvar+5) =  1; S{1}(14,3*nvar+5) =  1; S
      {1}(19,4*nvar+5)=  1; S{1}(24,5*nvar+5)=  1; S{1}(29,6*nvar+5)=  1;
67 S{1}(5,nvar+6)=  1; S{1}(10,2*nvar+6)=  1; S{1}(15,3*nvar+6) =  1; S
      {1}(20,4*nvar+6)=  1; S{1}(25,5*nvar+6)=  1; S{1}(30,6*nvar+6)=  1;
68
69 % sign restriction on the contemporaneous coefficients.
70 % Negative sign restrictions on y and p,
71 % Positive sign restrictions on NFCI.
72 S{1}(31,1)         = -1;
73 S{1}(32,2)         = -1;
74 S{1}(33,5)         =  1;
75 S{1}(34,6)         =  1;
76
77 % zero restrictions
78 Z=cell(nvar,1);
79 for i=1:nvar
80     Z{i}=zeros(0,nvar*NS);
```

```matlab
81  end
82
83  %=========================================================================
84  % Setup info
85  %=========================================================================
86  info=SetupInfo(nvar,m,Z,@(x)chol(x));
87
88  % ZF(A_{0},A_{+})
89  info.nlag     = nlag;
90  info.horizons = horizons;
91  info.ZF       = @(x,y)ZF(x,y);
92
93  % functions useful to compute the importance sampler weights
94  iw_info = info;
95  fs      = @(x)ff_h(x,iw_info);
96  r       = @(x)ZeroRestrictions(x,iw_info);
97
98  if strcmp(conjugate,'irfs')==1
99      fo               = @(x)f_h(x,iw_info);
100     fo_str2irfs      = @(x)StructuralToIRF(x,iw_info);
101     fo_str2irfs_inv  = @(x)IRFToStructural(x,iw_info);
102     r_irfs           = @(x)IRFRestrictions_more_general(x,iw_info);
103 end
104
105 % function useful to check the sign restrictions
106 fh_S_restrictions  = @(x)SF(x,iw_info,S);
107
108 %=========================================================================
109 % write data in Rubio, Waggoner, and Zha (RES 2010)'s notation
110 %=========================================================================
111
112 % yt(t) A0 = xt(t) Aplus + constant + et(t) for t=1...,T;
113 % yt(t)    = xt(t) B     + ut(t)             for t=1...,T;
114 % x(t)     = [yt(t-1), ... , yt(t-nlag), constant];
115 % matrix notation yt = xt*B + ut;
116 % xt=[yt_{-1} ones(T,1)];
117
118 yt = num(nlag+1:end,:);
119 T  = size(yt,1);
120 xt = zeros(T,nvar*nlag+nex);
121
122 for i=1:nlag
123     xt(:,nvar*(i-1)+1:nvar*i) = num((nlag-(i-1)):end-i,:);
124 end
125
126 if nex>=1
127     xt(:,nvar*nlag+nex)=ones(T,1);
```

```matlab
128  end
129
130  % write data in Zellner (1971, pp 224-227) notation
131  Y = yt; % T by nvar matrix of observations
132  X = xt; % T by (nvar*nlag+1) matrix of regressors
133
134  % prior for reduced-form parameters
135  nnuBar              = 0;
136  OomegaBarInverse    = zeros(m);
137  PpsiBar             = zeros(m,nvar);%forchettone
138  PphiBar             = zeros(nvar);
139
140  % posterior for reduced-form parameters
141  nnuTilde            = T +nnuBar; % tempi - 12
142  OomegaTilde         = (X'*X  + OomegaBarInverse)\eye(m);
143  OomegaTildeInverse  =  X'*X  + OomegaBarInverse;
144  PpsiTilde           = OomegaTilde*(X'*Y + OomegaBarInverse*PpsiBar);
145  PphiTilde           = Y'*Y + PphiBar + PpsiBar'*OomegaBarInverse*PpsiBar
            - PpsiTilde'*OomegaTildeInverse*PpsiTilde;
146  PphiTilde           = (PphiTilde'+PphiTilde)*0.5;
147
148
149  % useful definitions
150  % definitios used to store orthogonal-reduced-form draws, volume
        elements, and unnormalized weights
151  Bdraws          = cell([nd,1]); % reduced-form lag parameters
152  Sigmadraws      = cell([nd,1]); % reduced-form covariance matrices
153  Qdraws          = cell([nd,1]); % orthogonal matrices
154  storevefh       = zeros(nd,1);  % volume element f_{h}
155  storevegfhZ     = zeros(nd,1);  % volume element g o f_{h}|Z
156  uw              = zeros(nd,1);  % unnormalized importance sampler weights
157
158  if strcmp(conjugate,'irfs')==1
159      storevephi      = zeros(nd,1);  % volume element f_{h}
160      storevegphiZ    = zeros(nd,1);  % volume element g o f_{h}|Z
161  end
162
163  % definitions related to IRFs; based on page 12 of Rubio, Waggoner, and
        Zha (RES 2010)
164  J      = [e;repmat(zeros(nvar),nlag-1,1)];
165  A      = cell(nlag,1);
166  extraF = repmat(zeros(nvar),1,nlag-1);
167  F      = zeros(nlag*nvar,nlag*nvar);
168
169  for l=1:nlag-1
170      F((l-1)*nvar+1:l*nvar,nvar+1:nlag*nvar)=[repmat(zeros(nvar),1,l-1) e
        repmat(zeros(nvar),1,nlag-(l+1))];
```

```matlab
171  end
172
173  % definition to facilitate the draws from B|Sigma
174  hh              = info.h;
175  cholOomegaTilde = hh(OomegaTilde)'; %this matrix is used to draw B|Sigma
176
177  % initialize counters to track the state of the computations
178  counter = 1;
179  record  = 1;
180  count   = 0;
181  tStart  = tic;
182
183  while record<=nd
184
185      % step 1 in Algorithm 2
186      Sigmadraw     = iwishrnd(PphiTilde,nnuTilde);
187      cholSigmadraw = hh(Sigmadraw)';
188      Bdraw         = kron(cholSigmadraw,cholOomegaTilde)*randn(m*nvar,1)
189      + reshape(PpsiTilde,nvar*m,1);
189      Bdraw         = reshape(Bdraw,nvar*nlag+nex,nvar);
190
191      % store reduced-form draws
192      Bdraws{record,1}     = Bdraw;
193      Sigmadraws{record,1} = Sigmadraw;
194
195      % steps 2:4 of Algorithm 2
196      w         = DrawW(iw_info);
197      x         = [vec(Bdraw); vec(Sigmadraw); w];
198      structpara = ff_h_inv(x,iw_info);
199
200      % store the matrix Q associated with step 3
201      Qdraw            = SpheresToQ(w,iw_info,Bdraw,Sigmadraw);
202      Qdraws{record,1} = reshape(Qdraw,nvar,nvar);
203
204      % check if sign restrictions hold
205      signs     = fh_S_restrictions(structpara);
206
207      if (sum(signs{1}*e(:,1)>0))==size(signs{1}*e(:,1),1)
208          count=count+1;
209          % compute importance sampling weights
210
211          switch conjugate
212
213          case 'structural'
214          storevefh(record,1) = (nvar*(nvar+1)/2)*log(2)-(2*nvar+m+1)*
      LogAbsDet(reshape(structpara(1:nvar*nvar),nvar,nvar));
215          storevegfhZ(record,1) = LogVolumeElement(fs,structpara,r);
```

```matlab
216        uw(record,1) = exp(storevefh(record,1) - storevegfhZ(record,1));
217
218        case 'irfs'
219        irfpara = fo_str2irfs(structpara);
220        storevephi(record,1) = LogVolumeElement(fo,structpara)   +
   LogVolumeElement(fo_str2irfs_inv,irfpara);
221        storevegphiZ(record,1) = LogVolumeElement(fs,structpara,r) +
   LogVolumeElement(fo_str2irfs_inv,irfpara,r_irfs);
222        uw(record,1) = exp(storevephi(record,1)-storevegphiZ(record,1));
223
224        otherwise
225        uw(record,1) = 1;
226        end
227
228    else
229        uw(record,1) = 0;
230    end
231
232    if counter==iter_show
233        display(['Number of draws = ',num2str(record)])
234        display(['Remaining draws = ',num2str(nd-(record))])
235        counter =0;
236    end
237
238    counter = counter + 1;
239    record=record+1;
240 end
241
242 tElapsed = toc(tStart);
243 imp_w   = uw/sum(uw);
244 ne = floor(1/sum(imp_w.^2));
245
246 % store draws
247 Ltilde       = zeros(horizon+1,nvar,nvar,ne); % define array to store IRF
248 A0tilde        = zeros(nvar,nvar,ne); % define array to store A0
249 Aplustilde    = zeros(m,nvar,ne); % define array to store Aplus
250 hist_is_draws = zeros(ne,1);   % define array to store draws from
       importance sampler
251
252 for s=1:min(ne,maxdraws)
253
254    % draw: B,Sigma,Q
255    is_draw = randsample(1:size(imp_w,1),1,true,imp_w);
256    hist_is_draws(s,1)=is_draw;
257
258    Bdraw       = Bdraws{is_draw,1};
259    Sigmadraw   = Sigmadraws{is_draw,1};
```

```matlab
    Qdraw        = Qdraws{is_draw,1};

    x=[reshape(Bdraw,m*nvar,1);reshape(Sigmadraw,nvar*nvar,1);Qdraw(:)];
    structpara = f_h_inv(x,info);

    LIRF =IRF_horizons(structpara, nvar, nlag, m, 0:horizon);

    for h=0:horizon
        Ltilde(h+1,:,:,s) =  LIRF(1+h*nvar:(h+1)*nvar,:);
    end

    A0tilde(:,:,s)    = reshape(structpara(1:nvar*nvar),nvar,nvar);
    Aplustilde(:,:,s) = reshape(structpara(nvar*nvar+1:end),m,nvar);
end

A0tilde    = A0tilde(:,:,1:s);
Aplustilde = Aplustilde(:,:,1:s);
Ltilde     = Ltilde(:,:,:,1:s);

save('results/figure_1_results_d.mat','Ltilde','A0tilde','Aplustilde','
    imp_w','ne');
cd ..
```

Replication code for plotting the IRFs

```
1 %%*********************************************************************
2 % This is a modified version of the code developed by
3 % Arias et al. (2019) to plot the draws.
4 %%*********************************************************************
5
6 clear variables;
7 close all;
8 clc;
9
10 % Enter the figure
11 load 'figure1.mat';
12
13 Horizon=60;
14
15 % store IRF quantile 50th
16 Ltildeq50=zeros(size(Ltilde,1),size(Ltilde,2),size(Ltilde,3));
17 % store IRF quantile 16th
18 Ltildeq16=zeros(size(Ltilde,1),size(Ltilde,2),size(Ltilde,3));
19 % store IRF quantile 84th
20 Ltildeq84=zeros(size(Ltilde,1),size(Ltilde,2),size(Ltilde,3));
21  % store IRF quantile 025th
22 Ltildeq025=zeros(size(Ltilde,1),size(Ltilde,2),size(Ltilde,3));
23 % store IRF quantile 975th
24 Ltildeq975=zeros(size(Ltilde,1),size(Ltilde,2),size(Ltilde,3));
25
26
27 for ii=1:size(Ltilde,1)
28     for jj=1:size(Ltilde,2)
29         for kk=1:size(Ltilde,3)
30             Ltildeq50(ii,jj,kk) = quantile(Ltilde(ii,jj,kk,:),0.5);
31             Ltildeq16(ii,jj,kk) = quantile(Ltilde(ii,jj,kk,:),0.16);
32             Ltildeq84(ii,jj,kk) = quantile(Ltilde(ii,jj,kk,:),0.84);
33             Ltildeq025(ii,jj,kk) = quantile(Ltilde(ii,jj,kk,:),0.025);
34             Ltildeq975(ii,jj,kk) = quantile(Ltilde(ii,jj,kk,:),0.975);
35         end
36     end
37 end
38
39 ftsizeaxis=11;
40 ftsizexlabel=11;
41 ftsizetitle=11;
42 ftlinewidth = 1.0;
43 medianwidth=1.0;
44
45 H=Horizon;
```

```matlab
46
47  close all
48  hFig = figure (1);
49  set(hFig, 'Position', [20 20 700 350])
50
51  subplot (2 ,3 ,1)
52  plot (0:1:H, squeeze (Ltildeq50 (:,1,1)),'-k','LineWidth', medianwidth)
53  hline (0,'-r')
54  hold on
55  a=(squeeze (Ltildeq16 (:,1,1)))';
56  b=(squeeze (Ltildeq84 (:,1,1)))';
57  a95 =(squeeze (Ltildeq025 (:,1,1)))';
58  b95 =(squeeze (Ltildeq975 (:,1,1)))';
59
60  x = 0:1:H;
61  [~,~]=jbfill(x,a,b,rgb('mediumblue'),rgb('mediumblue'),0,0.5);
62  hold on
63  [~,~]=jbfill(x,a95,b95,rgb('royalblue'),rgb('royalblue'),0,0.5);
64  set(gca ,'XTick',[0;12;24;36;48;60])
65  set(gca ,'XTickLabel',['0 ';' 1';' 2';' 3';' 4';' 5'])
66  set(gca ,'LineWidth',ftlinewidth )
67  set(gca ,'YTick',[-1 -0.5 0 0.5 1])
68  axis([0 60 -1 1])
69  xlabel('Years','FontSize',ftsizexlabel)
70  ylabel('Percent','FontSize',ftsizexlabel)
71  set(gca ,'FontSize',ftsizeaxis)
72  set(gca ,'LineWidth',ftlinewidth)
73  title(' Output','FontSize',ftsizetitle)
74  box on
75
76  subplot (2 ,3 ,2)
77  plot (0:1:H, squeeze (Ltildeq50 (:,2,1)),'-k','LineWidth', medianwidth)
78  hline (0,'-r')
79  hold on
80  a=(squeeze (Ltildeq16 (:,2,1)))';
81  b=(squeeze (Ltildeq84 (:,2,1)))';
82  a95 =(squeeze (Ltildeq025 (:,2,1)))';
83  b95 =(squeeze (Ltildeq975 (:,2,1)))';
84
85  x = 0:1:H;
86  [~,~]=jbfill(x,a,b,rgb('mediumblue'),rgb('mediumblue'),0,0.5);
87  hold on
88  [~,~]=jbfill(x,a95,b95,rgb('royalblue'),rgb('royalblue'),0,0.5);
89  set(gca ,'XTick',[0;12;24;36;48;60])
90  set(gca ,'XTickLabel',['0 ';' 1';' 2';' 3';' 4';' 5'])
91  set(gca ,'LineWidth',ftlinewidth)
92  set(gca ,'YTick',[-0.5 -0.25 0 0.25 0.5])
```

```matlab
93  axis([0 60 -0.5 0.5])
94  xlabel('Years','FontSize',ftsizexlabel)
95  ylabel('Percent','FontSize',ftsizexlabel)
96  set(gca,'FontSize',ftsizeaxis)
97  set(gca,'LineWidth',ftlinewidth)
98  title(' GDP Deflator','FontSize',ftsizetitle)
99  box on
100
101 subplot(2,3,3)
102 plot(0:1:H,squeeze(Ltildeq50(:,3,1)),'-k','LineWidth',medianwidth)
103 hline(0,'-r')
104 hold on
105 a=(squeeze(Ltildeq16(:,3,1)))';
106 b=(squeeze(Ltildeq84(:,3,1)))';
107 a95=(squeeze(Ltildeq025(:,3,1)))';
108 b95=(squeeze(Ltildeq975(:,3,1)))';
109
110 x = 0:1:H;
111 [~,~]=jbfill(x,a,b,rgb('mediumblue'),rgb('mediumblue'),0,0.5);
112 hold on
113 [~,~]=jbfill(x,a95,b95,rgb('royalblue'),rgb('royalblue'),0,0.5);
114 set(gca,'XTick',[0;12;24;36;48;60])
115 set(gca,'XTickLabel',['0 ';' 1';' 2';' 3';' 4';' 5'])
116 set(gca,'LineWidth',ftlinewidth)
117 set(gca,'YTick',[-1.5 -0.75 0 0.75 1.5])
118 axis([0 60 -1.5 1.5])
119 xlabel('Years','FontSize',ftsizexlabel)
120 ylabel('Percent','FontSize',ftsizexlabel)
121 set(gca,'FontSize',ftsizeaxis)
122 set(gca,'LineWidth',ftlinewidth)
123 title(' Commodity Price Index','FontSize',ftsizetitle)
124 box on
125
126
127 subplot(2,3,4)
128 plot(0:1:H,squeeze(Ltildeq50(:,4,1)),'-k','LineWidth',medianwidth)
129 hline(0,'-r')
130 hold on
131 a=(squeeze(Ltildeq16(:,4,1)))';
132 b=(squeeze(Ltildeq84(:,4,1)))';
133 a95=(squeeze(Ltildeq025(:,4,1)))';
134 b95=(squeeze(Ltildeq975(:,4,1)))';
135
136 x = 0:1:H;
137 [~,~]=jbfill(x,a,b,rgb('mediumblue'),rgb('mediumblue'),0,0.5);
138 hold on
139 [~,~]=jbfill(x,a95,b95,rgb('royalblue'),rgb('royalblue'),0,0.5);
```

```matlab
140  set(gca,'XTick',[0;12;24;36;48;60])
141  set(gca,'XTickLabel',['0 ';' 1';' 2';' 3';' 4';' 5'])
142  set(gca,'LineWidth',ftlinewidth)
143  set(gca,'YTick',[-0.2 -0.1 0 0.1 0.2])
144  axis([0 60 -0.2 0.2])
145  xlabel('Years','FontSize',ftsizexlabel)
146  ylabel('Percentage points','FontSize',ftsizexlabel)
147  set(gca,'FontSize',ftsizeaxis)
148  set(gca,'LineWidth',ftlinewidth)
149  title('EBP','FontSize',ftsizetitle)
150  box on
151
152
153  subplot(2,3,5)
154  plot(0:1:H,squeeze(Ltildeq50(:,5,1)),'-k','LineWidth',medianwidth)
155  hline(0,'-r')
156  hold on
157  a=(squeeze(Ltildeq16(:,5,1)))';
158  b=(squeeze(Ltildeq84(:,5,1)))';
159  a95=(squeeze(Ltildeq025(:,5,1)))';
160  b95=(squeeze(Ltildeq975(:,5,1)))';
161
162  x = 0:1:H;
163  [~,~]=jbfill(x,a,b,rgb('mediumblue'),rgb('mediumblue'),0,0.5);
164  hold on
165  [~,~]=jbfill(x,a95,b95,rgb('royalblue'),rgb('royalblue'),0,0.5);
166  set(gca,'XTick',[0;12;24;36;48;60])
167  set(gca,'XTickLabel',['0 ';' 1';' 2';' 3';' 4';' 5'])
168  set(gca,'LineWidth',ftlinewidth )
169  set(gca,'YTick',[-0.3 -0.15 0 0.15 0.3])
170  axis([0 60 -0.3 0.3])
171  xlabel('Years','FontSize',ftsizexlabel)
172  ylabel('Standard deviations','FontSize',ftsizexlabel)
173  set(gca,'FontSize',ftsizeaxis)
174  set(gca,'LineWidth',ftlinewidth)
175  title('NFCI','FontSize',ftsizetitle)
176  box on
177
178  subplot(2,3,6)
179  plot(0:1:H,squeeze(Ltildeq50(:,6,1)),'-k','LineWidth',medianwidth)
180  hline(0,'-r')
181  hold on
182  a=(squeeze(Ltildeq16(:,6,1)))';
183  b=(squeeze(Ltildeq84(:,6,1)))';
184  a95=(squeeze(Ltildeq025(:,6,1)))';
185  b95=(squeeze(Ltildeq975(:,6,1)))';
186
```

```matlab
187  x = 0:1:H;
188  [~,~]=jbfill(x,a,b,rgb('mediumblue'),rgb('mediumblue'),0,0.5);
189  hold on
190  [~,~]=jbfill(x,a95,b95,rgb('royalblue'),rgb('royalblue'),0,0.5);
191  set(gca,'XTick',[0;12;24;36;48;60])
192  set(gca,'XTickLabel',['0 ';' 1';' 2';' 3';' 4';' 5'])
193  set(gca,'LineWidth',ftlinewidth )
194  set(gca,'YTick',[-0.6 -0.3 0 0.3 0.6])
195  axis([0 60 -0.6  0.6])
196  xlabel('Years','FontSize',ftsizexlabel)
197  ylabel('Percentage points','FontSize',ftsizexlabel)
198  set(gca,'FontSize',ftsizeaxis)
199  set(gca,'LineWidth',ftlinewidth)
200  title('Policy Rate','FontSize',ftsizetitle)
201  box on
202
203  set(gcf, 'PaperPositionMode', 'auto');
204
205  saveas(hFig,'../../plots/Figure1','epsc')
206  savefig(hFig,'../../plots/Figure1.fig')
```

Replication code for estimating the contemporaneous coefficients

```
1  %%*************************************************************************
2  % This is a modified version of the code developed by
3  % Arias et al. (2019) to display the contemporaneous coefficents of the
4  % monetary policy rule.
5  %%*************************************************************************
6
7  clear variables;
8  clc
9  close all
10
11 disp('----------------------------------------------------------')
12 disp('**********************************************************')
13 disp('********************** TABLE **************************')
14 disp('**********************************************************')
15 disp('----------------------------------------------------------')
16 cd ..
17 gcd = pwd;
18
19 % Enter the number of the table to display
20 number = 3;
21 eval(['load ',gcd,'/figure',num2str(number),'_TesiMattia/results/figure'
       ,num2str(number),'.mat']);
22 cd table
23
24 A0tilde_11 = squeeze(A0tilde(1,1,:));
25 A0tilde_21 = squeeze(A0tilde(2,1,:));
26 A0tilde_31 = squeeze(A0tilde(3,1,:));
27 A0tilde_41 = squeeze(A0tilde(4,1,:));
28 A0tilde_51 = squeeze(A0tilde(5,1,:));
29 A0tilde_61 = squeeze(A0tilde(6,1,:));
30
31 %response to output
32 ppsiy    = -A0tilde_11./A0tilde_61;
33 %response to prices
34 ppsip    = -A0tilde_21./A0tilde_61;
35 %response to commodity prices
36 ppsipc   = -A0tilde_31./A0tilde_61;
37 %response total reserves
38 ppsiebp  = -A0tilde_41./A0tilde_61;
39 %response non-borrowed reserves
40 ppsinfci = -A0tilde_51./A0tilde_61;
41
42 disp(['*** Relative to Fig ', num2str(number),' ***'])
43
44 mediany          = quantile(ppsiy,0.5);
```

```
45  quarty68          = [quantile(ppsiy,0.16),quantile(ppsiy,0.84)];
46  quarty96          = [quantile(ppsiy,0.025),quantile(ppsiy,0.975)];
47  psiy              = {mediany; quarty68; quarty96};
48
49  medianp           = quantile(ppsip,0.5) ;
50  quartp68          = [quantile(ppsip,0.16),quantile(ppsip,0.84)];
51  quartp96          = [quantile(ppsip,0.025),quantile(ppsip,0.975)];
52  psip              = {medianp; quartp68; quartp96};
53
54  medianpc          = quantile(ppsipc,0.5) ;
55  quartpc68         = [quantile(ppsipc,0.16),quantile(ppsipc,0.84)];
56  quartpc96         = [quantile(ppsipc,0.025),quantile(ppsipc,0.975)];
57  psipc             = {medianpc; quartpc68; quartpc96};
58
59  mediane           = quantile(ppsiebp,0.5) ;
60  quarte68          = [quantile(ppsiebp,0.16),quantile(ppsiebp,0.84)];
61  quarte96          = [quantile(ppsiebp,0.025),quantile(ppsiebp,0.975)];
62  psie              = {mediane; quarte68; quarte96};
63
64  mediann           = quantile(ppsinfci,0.5) ;
65  quartn68          = [quantile(ppsinfci,0.16),quantile(ppsinfci,0.84)];
66  quartn96          = [quantile(ppsinfci,0.025),quantile(ppsinfci,0.975)];
67  psin              = {mediann; quartn68; quartn96};
68
69  Tab=table({'Median';'68\% Prob. Interval';'95\% Prob. Interval'}, ...
70       psiy,psip,psipc,psie,psin, ...
71      'VariableNames',{'Coefficient' '\psi_y' '\psi_p' '\psi_p_c' '\psi_e'
        '\psi_n'});
72  Tab
```

Replication code for estimating the FEVD

```matlab
%%*****************************************************************
% This is a modified version of the code developed by
% Caggiano et al. (2021) to compute the forecast error variance
% decomposition.
%%*****************************************************************

clear
close all
clc

load results_MAIN.mat

addpath('functions')

n = 6 ;                    % number of variables
numSavedNarrative = length(A0_save) ;

hmaxtoplot2 = 24        % 2 yrs ahead
bands = [16,50,84];     % Percentiles for credible sets
p = 12;                 % Maximum lag order of VAR
hmax = 120;             % Set maximum horizon to 10 years

A0_narrative = A0_save;
Beta_narrative = Beta_save;

Draws_FEVDs_narrative = nan(n,n,hmaxtoplot2+1,numSavedNarrative); %
    Initialisation: variable,...,horizon,draw

for draw = 1:numSavedNarrative
    IRFs = getIRFs(Beta_narrative(:,:,draw),A0_narrative(:,:,draw),exog,
    n,p,hmax);
    Draws_FEVDs_narrative(:,:,:,draw) = varianceDecompositionOfVAR(IRFs,
    hmaxtoplot2);
end

% Compute FEVDs of all 7 endogenous variables to FU and EBP shocks over
    a 2-year horizon:
FEVD_percentiles_narrative = zeros(n,2,hmaxtoplot2+1,3); %
    Initialisation: variable,shock,horizon,bands
for jj = 1:2  % Shock
    for ii = 1:n   % Variable
        FEVD_percentiles_narrative(ii,jj,:,:) = prctile(squeeze(
    Draws_FEVDs_narrative(ii,jj,1:end,:)),bands,2);
    end
end
```

```matlab
40
41 % Table Construction for all n endogenous variables to shocks over a 2-
      year horizon
42
43 % 1. Data Arrangements before constructing table:
44
45 % FEVDs for each of the n variables to monetary policy shock over
      selected horizons
46 FEVD_y = squeeze(FEVD_percentiles_narrative(1,:,1:12:end,:));
47 FEVD_p = squeeze(FEVD_percentiles_narrative(2,:,1:12:end,:));
48 FEVD_pc = squeeze(FEVD_percentiles_narrative(3,:,1:12:end,:));
49 FEVD_E = squeeze(FEVD_percentiles_narrative(4,:,1:12:end,:));
50 FEVD_N= squeeze(FEVD_percentiles_narrative(5,:,1:12:end,:));
51 FEVD_r = squeeze(FEVD_percentiles_narrative(6,:,1:12:end,:));
52
53 var = 1;
54 FEVD_y1 = squeeze(FEVD_y(var,:,:));
55 FEVD_p1 = squeeze(FEVD_p(var,:,:));
56 FEVD_pc1 = squeeze(FEVD_pc(var,:,:));
57 FEVD_E1 = squeeze(FEVD_E(var,:,:));
58 FEVD_N1 = squeeze(FEVD_N(var,:,:));
59 FEVD_r1 = squeeze(FEVD_r(var,:,:));
60
61 save('FEVD.mat','FEVD_y1','FEVD_p1','FEVD_pc1','FEVD_E1','FEVD_N1','
      FEVD_r1');
62
63 % Selected horizons labels:
64 Horizon = [0; 12; 24];
65
66 % 2. Table construction:
67
68 Tab1 = table(Horizon,FEVD_y1, 'VariableNames', {'Horizons','16th-50th-84
      th Prctles FEVDs of y to Monetary policy Shock'})
69 Tab2 = table(Horizon,FEVD_p1, 'VariableNames', {'Horizons','16th-50th-84
      th Prctles FEVDs of p to Monetary policy Shock'})
70 Tab3 = table(Horizon,FEVD_pc1,'VariableNames', {'Horizons','16th-50th-84
      th Prctles FEVDs of pc to Monetary policy Shock'})
71 Tab4 = table(Horizon,FEVD_E1, 'VariableNames', {'Horizons','16th-50th-84
      th Prctles FEVDs of EBP to Monetary policy Shock'})
72 Tab5 = table(Horizon,FEVD_N1, 'VariableNames', {'Horizons','16th-50th-84
      th Prctles FEVDs of NFCI to Monetary policy Shock'})
73 Tab6 = table(Horizon,FEVD_r1, 'VariableNames', {'Horizons','16th-50th-84
      th Prctles FEVDs of FFR to Monetary policy Shock'})
```

Replication code for plotting the estimation of FEVD

```matlab
%%******************************************************************
% This is a modified version of the code developed by
% Caggiano et al. (2021) to plot the forecast error variance
% decomposition.
%%******************************************************************

clear variables;
close all;
clc;

load 'FEVD.mat';

Horizon=24;

ftsizeaxis=11;
ftsizexlabel=11;
ftsizetitle=11;
ftlinewidth = 1.0;
medianwidth=1.0;

H=Horizon;

close all
hFig = figure(1);
set(hFig, 'Position', [20 20 700 350])

subplot(2,3,1)
plot(0:1:H,FEVD_y1(:,2),'-k','LineWidth',medianwidth)
hold on
a=(FEVD_y1(:,1))';
b=(FEVD_y1(:,3))';

x = 0:1:H;
[~,~]=jbfill(x,a,b,rgb('red'),rgb('red'),0,0.5);
set(gca,'XTick',[0;12;24])
set(gca,'XTickLabel',['0 ';' 1';' 2';' 3';' 4';' 5'])
set(gca,'LineWidth',ftlinewidth )
set(gca,'YTick',[-0.5 -0.25 0 0.25 0.5])
axis([0 Horizon 0 0.5])
xlabel('Years','FontSize',ftsizexlabel)
ylabel('Percent','FontSize',ftsizexlabel)
set(gca,'FontSize',ftsizeaxis)
set(gca,'LineWidth',ftlinewidth)
title(' Output','FontSize',ftsizetitle)
box on
```

```matlab
46
47
48  subplot(2,3,2)
49  plot(0:1:H,FEVD_p1(:,2),'-k','LineWidth',medianwidth)
50  hold on
51  a=FEVD_p1(:,1)';
52  b=FEVD_p1(:,3)';
53
54  x = 0:1:H;
55  [~,~]=jbfill(x,a,b,rgb('red'),rgb('red'),0,0.5);
56  set(gca,'XTick',[0;12;24])
57  set(gca,'XTickLabel',['0 ';' 1';' 2';' 3';' 4';' 5'])
58  set(gca,'LineWidth',ftlinewidth)
59  set(gca,'YTick',[-0.5 -0.25 0 0.25 0.5])
60  axis([0 Horizon 0 0.5])
61  xlabel('Years','FontSize',ftsizexlabel)
62  ylabel('Percent','FontSize',ftsizexlabel)
63  set(gca,'FontSize',ftsizeaxis)
64  set(gca,'LineWidth',ftlinewidth)
65  title(' GDP Deflator','FontSize',ftsizetitle)
66  box on
67
68  subplot(2,3,3)
69  plot(0:1:H,FEVD_pc1(:,2),'-k','LineWidth',medianwidth)
70  hold on
71  a=FEVD_pc1(:,1)';
72  b=FEVD_pc1(:,3)';
73
74  x = 0:1:H;
75  [~,~]=jbfill(x,a,b,rgb('red'),rgb('red'),0,0.5);
76  set(gca,'XTick',[0;12;24])
77  set(gca,'XTickLabel',['0 ';' 1';' 2';' 3';' 4';' 5'])
78  set(gca,'LineWidth',ftlinewidth)
79  set(gca,'YTick',[-0.5 -0.25 0 0.25 0.5])
80  axis([0 Horizon 0 0.5])
81  xlabel('Years','FontSize',ftsizexlabel)
82  ylabel('Percent','FontSize',ftsizexlabel)
83  set(gca,'FontSize',ftsizeaxis)
84  set(gca,'LineWidth',ftlinewidth)
85  title(' Commodity Price Index','FontSize',ftsizetitle)
86  box on
87
88  subplot(2,3,4)
89  plot(0:1:H,FEVD_E1(:,2),'-k','LineWidth',medianwidth)
90  hold on
91  a=FEVD_E1(:,1)';
92  b=FEVD_E1(:,3)';
```

```matlab
 93
 94 x = 0:1:H;
 95 [~,~]=jbfill(x,a,b,rgb('red'),rgb('red'),0,0.5);
 96 set(gca,'XTick',[0;12;24])
 97 set(gca,'XTickLabel',['0 ';' 1';' 2';' 3';' 4';' 5'])
 98 set(gca,'LineWidth',ftlinewidth)
 99 set(gca,'YTick',[ -0.25 0 0.25 0.5])
100 axis([0 Horizon 0 0.5])
101 xlabel('Years','FontSize',ftsizexlabel)
102 ylabel('Percentage points','FontSize',ftsizexlabel)
103 set(gca,'FontSize',ftsizeaxis)
104 set(gca,'LineWidth',ftlinewidth)
105 title('EBP','FontSize',ftsizetitle)
106 box on
107
108 subplot(2,3,5)
109 plot(0:1:H,FEVD_N1(:,2),'-k','LineWidth',medianwidth)
110 hold on
111 a=FEVD_N1(:,1)';
112 b=FEVD_N1(:,3)';
113
114 x = 0:1:H;
115 [~,~]=jbfill(x,a,b,rgb('red'),rgb('red'),0,0.5);
116 set(gca,'XTick',[0;12;24])
117 set(gca,'XTickLabel',['0 ';' 1';' 2';' 3';' 4';' 5'])
118 set(gca,'LineWidth',ftlinewidth )
119 set(gca,'YTick',[ -0.25 0 0.25 0.5])
120 axis([0 Horizon 0 0.5])
121 xlabel('Years','FontSize',ftsizexlabel)
122 ylabel('Standard deviations','FontSize',ftsizexlabel)
123 set(gca,'FontSize',ftsizeaxis)
124 set(gca,'LineWidth',ftlinewidth)
125 title('NFCI','FontSize',ftsizetitle)
126 box on
127
128 subplot(2,3,6)
129 plot(0:1:H,FEVD_r1(:,2),'-k','LineWidth',medianwidth)
130 hold on
131 a=FEVD_r1(:,1)';
132 b=FEVD_r1(:,3)';
133
134 x = 0:1:H;
135 [~,~]=jbfill(x,a,b,rgb('red'),rgb('red'),0,0.5);
136 set(gca,'XTick',[0;12;24])
137 set(gca,'XTickLabel',['0 ';' 1';' 2';' 3';' 4';' 5'])
138 set(gca,'LineWidth',ftlinewidth )
139 set(gca,'YTick',[-0.6 -0.25 0 0.25 0.5])
```

```matlab
140  axis([0 Horizon 0  0.5])
141  xlabel('Years','FontSize',ftsizexlabel)
142  ylabel('Percentage points','FontSize',ftsizexlabel)
143  set(gca,'FontSize',ftsizeaxis)
144  set(gca,'LineWidth',ftlinewidth)
145  title(' Federal Funds Rate','FontSize',ftsizetitle)
146  box on
147
148  set(gcf, 'PaperPositionMode', 'auto');
149
150  saveas(hFig,'../plots/FigureFEVD','epsc')
151  savefig(hFig,'../plots/FigureFEVD.fig')
```