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Offshoring and Reshoring: a Model Based on Continuous-Time Markov Chain

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Abstract

Offshoring, intended as the relocation of a business's productive structure to a country different from the origin one, is a widespread phenomenon, along with reshoring, its inverse: causes are varied, ranging from economies of localisation, convenience in input prices, government incentives. This thesis analyses a two-country model (North and South) in which the location of production is represented by a controlled Markov chain: firms (with homogeneous characteristics and identified by a representative agent) can costly increase the rate of probability of transitioning from one country to the other in a finite time interval. The optimal choice of firms is derived from a backward differential equation that represents at each instant the value attached to offshoring. The defining characteristic of the countries is that input prices in the South are lower, while the North can give monetary incentives to firms at the end of the programming interval, defining the terminal value of a Cauchy's problem. The effects of different parameters on the offshoring and reshoring times are analysed, along with the cost for each country of reacting to the modification of some parameters by the other.

Abstract in Italiano

L'offshoring - cioè la rilocalizzazione della struttura produttiva di un'azienda in un paese diverso da quello d'origine - è un fenomeno diffuso, assieme al suo inverso, il reshoring: le cause sono varie, dalle economie di localizzazione alla convenienza nei prezzi degli input produttivi agli incentivi statali. Questa tesi analizza un modello a due paesi (Nord e Sud) in cui la localizzazione della produzione è rappresentata da una catena di Markov controllata: le aziende (con caratteristiche omogenee e individuate da un agente rappresentativo) possono aumentare pagando un prezzo il tasso di probabilità di passare da un paese all'altro. La scelta ottima delle aziende è derivata da un'equazione differenziale *backward* che rappresenta in ogni istante il valore attribuito all'offshoring. La caratteristica distintiva dei due stati è che i prezzi degli input sono inferiori (per il Sud), mentre il Nord può offrire degli incentivi monetari alle aziende alla fine dell'intervallo di programmazione, definendo il valore finale del problema di Cauchy. Gli effetti dei diversi parametri sul tempo di offshoring e reshoring sono analizzati, assieme al costo per ogni paese di reagire alla modifica del sistema da parte dell'altro.

Contents

1 Offshoring

While international trade has been the object of economic analysis for a long time, the efforts of the bright minds that have tackled its intellectual challenges has brought about no shortage of material for further examination by the curious economist, both for its complexity and its ever-changing aspects. One such challenge is presented by the recently rising phenomena of offshoring and reshoring. Offshoring is the relocation of part or all of the production of a firm in another country, either by moving production plants or by externalisation, while reshoring is its inverse. This section shall be devoted to setting them in the economic and business analysis.

1.1 What is Offshoring?

Classical models of international trade - such as the Ricardo, the Heckscher-Ohlin and the Specific Factors models described in [8] by Krugman et al. (2022) - are usually concerned with the effect of commercial opening to foreign countries: they focus on specialisation of production and how it impacts the exchange of goods and services. The interplay of consumer preferences, production functions, market structure and endowments yields different equilibrium outputs and divergent welfare allocations, based on the assumptions that try to capture different aspects of the economic reality. The core concept is *comparative advantage*: replicating the famous Smithian concept of division of labour, if countries can produce a set of goods with different efficiency, it is convenient in order to maximise profits and increase total output that each serves the market with the good it can produce better. The main differentiating characteristic of the models lies if the definition of the cause of the comparative advantage: it may be different relative productivity (Ricardo) or different endowments (H-O), or more recently, internal or external economies of scale (this is the argument of the so called ”New Trade Theory”). An absolute advantage, that is, a country that is better at providing all goods in a market, on the other hand, will prevent international trade. One of the key features of this framework is that agents are identified with their country of origin: firms are fixed, acting as mere aggregators of inputs and maximising profits of commerce in goods - or, equivalently, production factors incorporated in those goods. The focus is on import and export.

It reality, though, one observes that firms' productive choices include much more than that. Firms do decide where they establish their production, which markets they should serve and from which market they should source inputs.

Tunisini et al. (2020) in [14] include *economies of location* among the *instruments of growth* that allow a firm to lower its average total costs.

A thorough causal explanation for economies of location is elusive, because of the plethora of possible determining factors: hypotheses may be advanced about different taxation regimes, internal or external economies of scale, infrastructure quality or many others (see for example the overview in [3] by Chang 2012). The political environment seems a particularly strong element among these: subsidies in the form of tax benefits or monetary incentives are a commonly used instrument by States to steer decision processes (see [15] by Wang et al. 2023), particularly to incentivise firms to reshore (as highlighted by Yang et al. 2021 in [16]). Economies of location, as any other growth factor, may become negative too, due for example to congestion or high transaction costs.

It is no wonder, therefore, that production structures across countries are an ever-evolving web of connections and follow no easily intelligible trajectory: relocation has been a stable trend for a long time (documented for the European case in [13] by Schwörer 2013), but in more recent years a reversion of the phenomenon has arisen, with producers coming back to their countries of origin (some aspects that will later come up are discussed in [4] by Chen and Hu 2017 and [15] by Wang et al. 2023). The vexed *make or buy* dilemma emerges too, as often production of intermediate inputs is delegated to foreign firms.

Recent research has not been silent on the matter, and progress has been made in trying to incorporate explanatory features in the setting of the models mentioned above: examples may be paper [10] by Rodríguez-Clare (2010) or [3] by Chang (2012).

1.2 Types and Trade-offs of Internationalisation

Given that the international productive strategies of firms are multifaceted and complex, a much needed categorisation can follow Chapter 8 in *International Economics* [8] by Krugman et al. (2022). A distinction is made between:

- **FDI**, foreign direct investments
  - horizontal: the replication of production processes in their entirety in foreign countries
  - vertical: the fragmentation of a production line and relocation of its components in other countries
- **outsourcing**, the externalisation of the production of some inputs, whose output the firm at home acquires as intermediate goods

"Offshoring" includes both outsourcing and vertical FDIs.

Choosing the optimal strategy among these involves a number of trade-offs: one of these is the *proximity/concentration trade-off*, underlying an IDE decision.
Building a new productive plant has fixed costs F and a marginal profit made by costs savings ξ in input prices (for vertical IDEs) or transportation costs (for horizontal IDEs). There is a certain threshold quantity, therefore, after which the fixed cost is sufficiently covered by the δ units of good sold, making profits positive. Mathematically, for the investment to be profitable, $F \leq \xi\delta$, or equivalently, the average fixed cost F/δ must be lower that the marginal (and average) profit ξ (Figure 1).

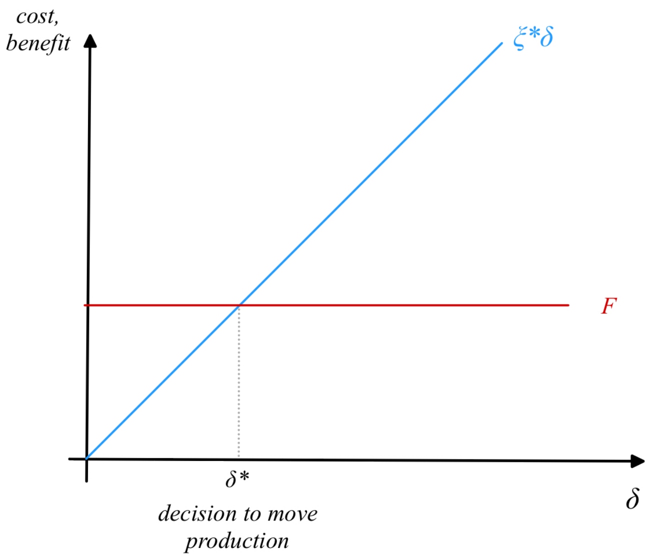


Figure 1: Costs-Profit trade-off for an IDE

1.3 Modelling Offshoring, Environment and Business strategy

The scope of this thesis shall be to better understand some theoretical instruments which may help to describe this kind of underlying trade-offs.
As pointed out in the first paragraph, assumptions and modelling serve as magnifying glasses, that idealise and abstract the traits of the real world one wants to study. The framework will be stochastic dynamic optimisation, which has great applicability to socio-economic sciences (for some of the applications, see [6] by Gomes et al. 2014). Tractability imposes a loss of generality: offshoring is going to be from now on intended as total relocation of production[1] and assumed as always convenient for the firm (a country has an absolute productive advantage), but elements of uncertainty and intertemporal optimisation will be introduced, along with exogenous monetary incentives, and this will produce different results from those one would expect: even in a situation of absolute advantage, the more productive country shall not always be preferred.

The business study framework provides a well-fitting parallel for the interpretation and application of the seemingly more abstract elements of the model. These studies have analysed in great depth firms' productive and strategic choices, from most varied perspectives. Complementary approaches may in facts provide insight that, though differing on many levels in their conclusions, help to highlight some salient features of economic behaviour.

A first point is that firms operate and interact with their environments. Many theories try to define such relation, such as the *objectivist* paradigm (as Costa et al. 2021 explain in [5], Chapter 3). According to it, the environment is a set of resources (physical or non-physical) that are causally linked to an organisation's structure and actions. Such relation is not unidirectional,

[1] since the value chain has a single link, this may be though of as a horizontal IDE

but rather circular, as the firm itself can exert a changing force while being the object of external influence. This aspect is later integrated in the *feedback* component of the model, as a firm's action takes into account the current state of the system.
Costa et al. (2021) in [5] describe the environment as made up of *markets*, *technology* and *institutions*. The first may be *input* or *output markets*. Each of these components bears impact on the strategic choices of a firm with different intensity: in [9] by Kudrenko 2024, for example, the interaction of supply-chain uncertainty and trade policy is evaluated. Paper [12] by Schmeisser (2013, p.390) further highlights that "[...] offshoring can be understood as a specific manifestation of firm internationalization that is primarily concerned with the internationalization of the firm's input-market side rather than with the internationalization of sales on the output-market side of the value chain". These elements are taken too into account, as the demand of goods shall be assumed fixed in one country, while the input market will be the main source of cost saving.

Lastly, two factors the model captures are uncertainty and dynamism. The system presented in section 3 falls into the category of *stochastic processes*, which are mathematical representation of a non-deterministic environment: the economic agents shall be bound to choose while not sure of the positive outcome of their actions.

2 Markov Chains

A sequence of non deterministic events in intertemporal setting may be represented by a stochastic process. Intuitively, this is a function whose output is a realisation of an underlying distribution of probability. More formally, a stochastic process is a family of time-indexed random variables such that

$$X_t(\omega) : T \times S \to \mathbb{R}$$

$$(t, \omega) \mapsto x_t(\omega)$$

The state space S is the set of all possible states of the system, while T is the time set. Both of these may be discrete or continuous, finite or infinite. This overview will consider stochastic processes with a finite state space and over a finite time.

A particular family of stochastic processes is called *Markov chains*: the main features these present is the so-called *memorylessness*: the probability of transition of the system from one state to another at time $t+1$ (or $t+s$ in continuous time) is not conditioned by its past history, but rather only by the state of the system at time t. This will be particularly useful in modelling the transition probability from the offshoring to reshoring state, as presented in Section 3.

2.1 Discrete time Markov Chains

A discrete time Markov chain (MC) is a stochastic process whose dynamic is described by the Markov property:

$$P\big(X(t_{n+1}) = j \big| X(t_n) = i, X(t_{n-1}) = i_{n-1}, \ldots, X(t_0) = i_0\big) =$$

$$= P\big(X(t_{n+1}) = j \big| X(t_n) = i\big)$$

For example, suppose the state space is $S = \{O, N\}$, where O stands for “offshoring” and N for “no offshoring”. At each time t, the distribution of probability for x at $t+1$ may be described by the graph in Figure 2 or the following matrix, also named *(one-step) probability matrix*:

$$P = \begin{bmatrix} p_{O,O} & p_{O,N} \\ p_{N,O} & p_{N,N} \end{bmatrix}$$

2.2 Continuous Time Markov Chains

When time is a continuous variable, the memorylessness property of a Continuous Time Markov Chain (CTMC) becomes:

$$P\big(X_{t+s} = j \big| X(s) = i, X_{s_n} = i_n, \ldots, X_{s_0} = i_0\big) = P\big(X_t = j \big| X_0 = i\big)$$

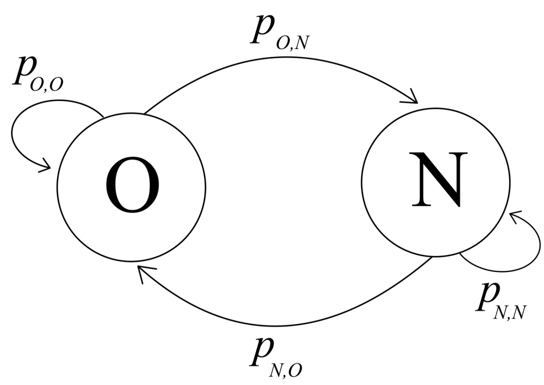


Figure 2: A binary-state Markov Chain in the form of a graph

where $s_n \geq s_{n-1} \geq s_{n-2} \geq ... \geq s_0$. In this case, the inter-transition time T_i -meaning the time after which the process changes state ("jumps"), is itself is a random variable with exponential distribution of parameter υ_i ($T_i \sim$ exp $\{\upsilon_i\}$). This is because the exponential distribution is the unique absolutely continuous random variable that satisfies the memoryless property.

Describing the dynamics of a CTMC is a more challenging task, since state transitions do not follow a commonly determined time structure. The one step probability transition matrix is therefore no longer useful: its place is taken by a *transition rate matrix*. Rates are the incremental ratios of change in jump probability with respect to an infinitesimal amount of time. As such, they need not be between 0 and 1, but can take any real value.
Each non-diagonal entry of the matrix, that is, every transition rate is

$$q(i,j) = \lim_{h \to 0} \frac{p_h(i,j)}{h}$$

while the diagnal entries are equal to

$$q(i,i) = -\sum_{i \neq j} q_{i,j}$$

so as to make each row sum to 0, coherently with the fact that the probability of remaining in one state is constantly decreasing the longer a jump doesn't occur, and that the total change in a row must be 0, since the probability that a jump will either happen or not happen must remain 1.

Assuming the same state space $S = \{O, N\}$ as before, a binary transition rate matrix will have the following structure:

$$Q = P' = \begin{bmatrix} -q_{O,N} & q_{O,N} \\ q_{N,O} & -q_{N,O} \end{bmatrix}$$

The transition rate matrix Q is strictly connected to the *infinitesimal generator* of the CTMC.

2.3 Optimal Control and Markov Chains with Binary State and Finite Time

2.3.1 Stocahstic control problem and Dynamic Programming

In a programming interval $[0, T]$, with $T \in (0, +\infty)$, a stochastic optimal control problem is the optimisation of an objective functional of the kind:

$$J[u] = \mathbb{E}^u_{t,x} \left\{ \int_0^T g(X_t, U_t, t) dt + G(X_T) \right\}$$

$$s.t \begin{cases} dX_t = f(X_t, U_t, t) + \sigma(t, X_t, U_t) dW_t \\ x(0) = x_0 \end{cases}$$

where g is a profit function, $u : [0, T] \mapsto \mathbb{R}$ is a control function, $G(X_T)$ is a scrap value function and $t \in [0, T]$. The first constraint represents the motion of the stochastic process, the second the initial state.

In economic analysis this is a good way of modelling an agent's optimal decision along time: X_t and dX_t represent the state of the system and its evolution, g and G are costs or profits (in the case of firms), respectively achieved at each t and at the end of the programming interval. U_t is the *control variable*, and can be thought of as the action chosen in a feasible set U. Its optimisation brings the system's dynamics to follow an *optimal path*. If U_t is a feedback control, the state of the system X_t is accounted for in the optimal policy, hence $U(X_t, t)$.

The solution to an optimal control problem can be found through the technique of dynamic programming (DP): given a *value function* $V(t, x^*)$, representing the optimal value of the functional $J[u]$ at each t, the following Hamilton-Jacobi-Bellman (HJB) function's solution yields x^* and consequently u^*:

$$\begin{cases} -\partial_t V(t, x^*) = \max_w \left\{ g(x^*, w, t) + f(x^*, w, t) \cdot \partial_x V(t, x^*) + \frac{1}{2}\sigma^2(x, w, t) \cdot \partial^2_{xx} V(t, x) \right\} \\ V(T, x^*) = G(x^*) \end{cases}$$

2.3.2 DP in a Markov Process with binary state

To favour economic abstraction, it would be useful to reduce the state space to a discrete one. Following [2], the state space is defined as $S := \{+1, -1\}$. The system's dynamics is set as the following Markov property:

$$\mathbb{P}(X_{t+h} = -x | X_t = x, U_t = u) = \ell(x, u)h + o(h)$$

which describes a CTMC. The transition rate from one state to another in infinitesimal time is dependant on u, the control variable. The control set is $[0, v]$, with $v > 0$: this means an agent

may decide to either increase the rate or not to change it at all. To find the optimal $u^*(t)$, a modified version of the HJB is needed.
The Λ_t^V operator is defined[2]:

$$\Lambda_t^u V(x) := \ell(x, u(t,x)) \underbrace{(V(-x) - V(x))}_{\nabla_x f(x)}$$

Such operator (which is the infinitesimal generator of the CTMC) shall serve as the equivalent of $f \cdot \partial_x V + \frac{1}{2}\sigma^2 \cdot \partial^2_{xx} V$ in the ordinary HJB, representing the discrete change of the value function with respect to x and the transition rate form x to $-x$, since

$$\lim_{h \to 0} \frac{\mathbb{P}(X_{t+h} = -x | X_t = x, U_t = u)}{h} = \lim_{h \to 0} \frac{\ell(x,u)h + o(h)}{h} = \ell(x,u)$$

As a derivative with respect to x of V is not possible, the HJB presented above shall assume the following form:

$$\begin{cases} \partial_t V(t,x) + \max_{w \in [0,v]} \{g(t,x,w) + \Lambda_t^w V(t,x)\} = 0 \\ V(T,x) = G(x) \end{cases}$$

2.3.3 Solution with backward ODE

This result allows a solution through a backward differential equation equation: since the derivative with respect to x is not defined, the evolution of the difference of the value functions $\nabla_x f(x)$ is evaluated at all t, describing its optimal evolution backwards, and consequently defining the optimal path of the control function. This strategy shall be applied in Section 3, after having defined a functional form for running profits and cost. Figure 3 provides a graphical intuition for this.

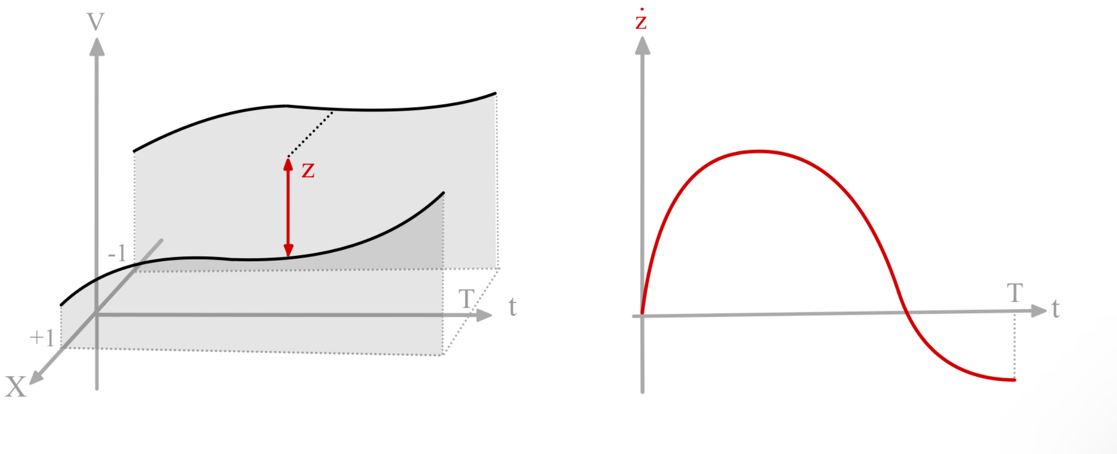


Figure 3: The algebraic evaluation of the evolution of the value function's discrete gradient allows for the construction of a backward differential equation

[2]The notation is altered to keep consistency with the rest of the Section

3 A Markov-chain Model of Offshoring

Having individually defined the elements of the model, it is now the time to put them toghether: this section of the thesis will review paper [1] written by Brambilla et al. (2024), remarking on the economic intuitions the model presents.

The basic framework is that of a North-South model inspired by that of Saito (2018) in [11], with an infinite number of firms[3]. In a programming interval $[0, T]$ the state of the firm can take two values, $+1$ for location the North and -1 for location in the South: it is therefore a binary state variable $X_t \in \{-1, +1\}$.
The production cost in the South is ρ, lower than in the North where it is $\rho + \xi$ with $\xi > 0$. Companies only sell in the North market, at constant price $\pi > 0$ and demand $\delta > 0$ at each instant of time. A market-clearing condition is set, so all produced goods are immediately sold. Each company cannot directly control its state, but only the probability rate of transition from their current one to the other at each instant t of the programming interval: this allows for a model with binary state Markov Chains, more precisely a *controlled Markov Chain* (a thorough overview can be found in paper [7] of Guo and Hernández-Lerma 2009), whose dynamic is

$$\mathbb{P}(X_{t+h} = -X_t | X_s, s \leq t) = U_t \cdot h + o(h) \tag{1}$$

where $U_t = \phi(t, X_t) \geq 0$ is the feedback control. The activation of such control is costly, and the cost has a quadratic functional shape of the kind $\frac{\kappa}{2}u^2 + \theta u$. This means that the instantaneous marginal cost of transition is $\kappa u + \theta$: a higher probability of transition implies a linearly increasing cost, but also an "activation cost" θ that must be sustained even if $u \to 0^+$.
At the end of the programming interval, the North country sets a monetary incentive for companies to reshore, γ, which is awarded to companies in the state $+1$. There is perfect information about the state of all companies. The functional each firm faces is therefore

$$J[U_s] = \mathbb{E}\left\{\int_0^T \left[\delta\left(\pi - \rho - \frac{\xi}{2}(1 + X_t)\right) - \frac{\kappa}{2}U_t^2 - \theta U_t\right] dt + \frac{\gamma}{2}(1 + X_T)\right\} \tag{2}$$

Inside the integral are the instantaneous profits, while $\frac{\gamma}{2}(1 + X_T)$ is the scrap value function at T.
Notice that when $X_t = -1$, meaning the company is in the South where the production cost is lower,

$$\frac{\xi}{2}(1 - 1) = 0$$

and the production cost is just ρ. When, on the other hand, $X_t = +1$,

$$\frac{\xi}{2}(1 + 1) = \xi$$

[3] Theoretically, the number N of firms can be lower, making the computations quite harder: the model can be though of as a limit form game, even though it does not strictly fit the formal definition of a game given the lack of strategic interaction

and the production cost is $\rho + \xi$. The same mechanism applies to the final incentive γ.

From the perspective of the policymaker, the representative company's state shall be of interest, which is the average state of the companies at each time, m_t. Since $m \in [-1, +1]$ by definition, at $m = +1$ all companies' productions are located in the North country, at 0 they are evenly split, at -1 they are all in the South.

3.1 HJB and optimal control

Equations (1) and (2) and a starting condition X_0 characterise a stochastic optimal control problem that can be solved using the technique described by Brambilla et al. (2023) in [2], as outlined in Section 2.

The associated value function is

$$V(t,x) = \sup_{U_s \geq 0} \mathbb{E}\left\{ \int_t^T \left[\delta\left(\pi - \rho - \frac{\xi}{2}(1 + X_s) \right) - \frac{\kappa}{2} U_s^2 - \theta U_s \right] ds + \frac{\gamma}{2}(1 + X_T) \big| X_t = x \right\}$$

with the relative HJB equation

$$\begin{cases} \partial_t V(t,x) + \max_{u \geq 0} \left\{ u \nabla_x V(t,x) + \delta\big(\pi - \rho - \frac{\xi}{2}(1+x)\big) - \frac{\kappa}{2} u^2 - \theta u \right\} = 0 \\ V(T,x) = \frac{\gamma}{2}(1+x) \end{cases} \tag{3}$$

where $\nabla_x V(t,x) := V(t,-x) - V(t,x)$ is the discrete gradient of the value function defined in Section 2. Second order conditions are satisfied, as $u \mapsto u \nabla_x V(t,x) + \delta\big(\pi - \rho - \frac{\xi}{2}(1+x)\big) - \frac{\kappa}{2} u^2 - \theta u$ is strictly concave with respect to u.
One can solve for the zero of the first derivative $u \mapsto \nabla_x V(t,x) - \kappa u - \theta$, which is the global maximum point in $[0, +\infty)$. Therefore

$$u^* = \frac{[\nabla_x V(t,x) - \theta]^+}{\kappa} \tag{4}$$

where the notation $[a]^+$ represents the positive part of a, that is $\max\{0, a\}$, coming from the positivity condition on u.

Following the strategy defined in the last paragraph of Section 2, a backward ODE can be constructed. The auxiliary variable

$$z_t := \nabla_x V(t,1) = V(t,-1) - V(t,1)$$

represents the difference of the value functions of state -1 and state 1 at each t, which is the value attributed to going from North to South. If $z_t > 0$ and the starting state x is 1, the transition will make profits positive.

The last passage to obtain the backward ODE is the substitution of (4) in (3) and evaluate both states:

$$\partial_t V(t,1) = -\frac{[\nabla_x V(t,1)-\theta]^+}{\kappa}\nabla_x V(t,1) - \delta(\pi-\rho-\xi) + \\ + \frac{\kappa}{2}\left(\frac{[\nabla_x V(t,1)-\theta]^+}{\kappa}\right)^2 + \theta\frac{[\nabla_x V(t,1)-\theta]^+}{\kappa} \quad (5)$$

and

$$\partial_t V(t,-1) = -\frac{[\nabla_x V(t,-1)-\theta]^+}{\kappa}\nabla_x V(t,-1) - \delta(\pi-\rho-\xi) + \\ + \frac{\kappa}{2}\left(\frac{[\nabla_x V(t,-1)-\theta]^+}{\kappa}\right)^2 + \theta\frac{[\nabla_x V(t,-1)-\theta]^+}{\kappa} \quad (6)$$

Since $\nabla_x V(t,-1) = -z_t$, to obtain $\dot{z}$ one can simply subtract (6) from (5). Intuitively, the variation of the difference between $V(t,1)$ and $V(t,-1)$ w.r.t time is the difference of the two variations w.r.t. time:

$$\begin{aligned}\dot{z}_t &= \partial_t V(t,-1) - \partial_t V(t,1) \\ &= \frac{z_t}{\kappa}\left[[-z_t-\theta]^+ + [z_t-\theta]^+\right] + \frac{1}{2\kappa}\left[\left([-z_t-\theta]^+\right)^2 - \left([z_t-\theta]^+\right)^2\right] \\ &\quad + \frac{\theta}{\kappa}\left[[-z_t-\theta]^+ - [z_t-\theta]^+\right] - \delta\xi \quad (7)\end{aligned}$$

Given that $[-a]^+ = [a]^-$, $|a| = [a]^+ + [a]^-$ and $a = [a]^+ - [a]^-$, the final differential equation becomes

$$\dot{z}_t = \frac{z_t+\theta}{\kappa}[z_t+\theta]^- + \frac{(z_t-\theta)}{\kappa}[z_t-\theta]^+ + \frac{1}{2\kappa}\left[([z_t+\theta]^-)^2 - ([z_t-\theta]^+)^2\right] - \delta\xi$$

There are three different cases, depending on the sign of $(z_t-\theta)$ and $(z_t+\theta)$:

$$\dot{z}_t = \begin{cases} -\frac{(z_t+\theta)^2}{2\kappa} - \delta\xi & z_t < -\theta \\ -\delta\xi & |z_t| \le \theta \\ \frac{(z_t-\theta)^2}{2\kappa} - \delta\xi & z_t > \theta \end{cases} \quad (8)$$

The optimal feedback control from (4) is

$$\phi^*(t,x) = \frac{[\nabla_x V(t,x)-\theta]^+}{\kappa} = \frac{[z_t\cdot x-\theta]^+}{\kappa}$$

as the $\nabla_x V(t,x)$ is z_t if $x=1$ and $-z_t$ if $x=-1$. Notice how the control is activated only if $z_t\cdot x-\theta > 0$, meaning that $z_t\cdot x > \theta$: firms only try to increase the probability rate if the value obtained by transitioning is greater than the activation cost.
The final condition is $z_T = -\gamma$: at T, being in South bears the opportunity cost of the foregone monetary incentive.

3.2 Offshoring without incentives

Without incentives, the final condition for the backward ODE (8) is $z_T = 0$. There are now two possible cases, depicted in the following fugure:

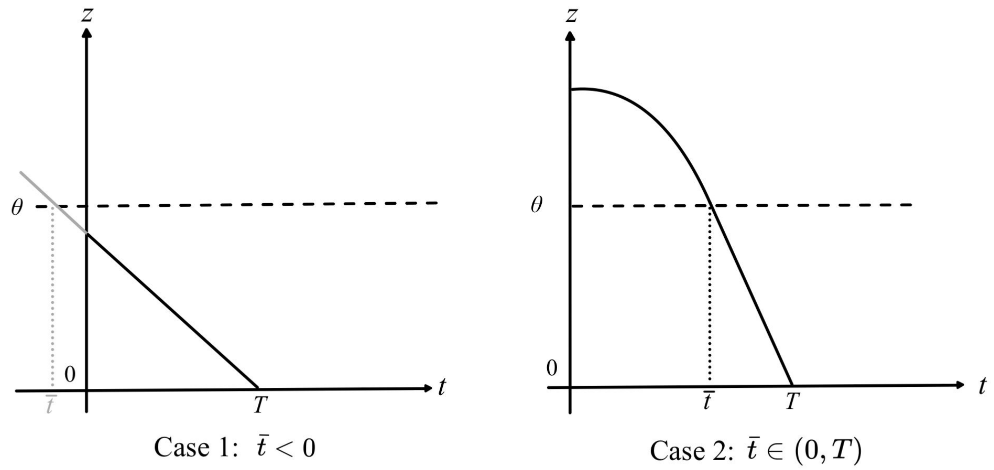


Figure 4: Two cases for$z_T = \gamma = 0$

- $z_t < \theta \; \forall \, t \in [0, T]$
- $z_t > \theta \; \forall \, t \in [0, \bar{t}]$ and $z_t < \theta \; \forall \, t \in (\bar{t}, T]$

where $\bar{t}$ marks the intervals in which $z_t > \theta$ and $z_t \leq \theta$. Notice that, if $\bar{t} < 0$, the solution to the second problem defines entirely z_t.
If on the other hand $\bar{t} > 0$ and $t \in [0, \bar{t}]$, firms activate their control from state 1 to state -1, because the increase in value attached to going South is higher that its cost.
Now, one can solve two different Cauchy problems to describe the optimal z, one for the interval of time in which $z_t > \theta$, with final condition $z_{\bar{t}} = \theta$, and the other for $z_t < \theta$.

$$\begin{cases} \dot{z}_t = \frac{(z-\theta)^2}{2\kappa} \\ z_{\bar{t}} = \theta \end{cases} \qquad \begin{cases} \dot{z}_t = -\delta\xi \\ z_T = 0 \end{cases} \tag{9}$$

Therefore,

- if $\bar{t} > 0$

$$\begin{cases} z_t = \theta + \sqrt{2\delta\xi\kappa} \tanh\left(\sqrt{\frac{\delta\xi}{2\kappa}}(\bar{t} - t)\right) & t \in [0, \bar{t}] \\ z_t = \delta\xi(T - t) & t \in [\bar{t}, T] \end{cases}$$

To find $\bar{t}$, one can set z_t from to second equation equal to θ, and obtain

$$\bar{t} = T - \frac{\theta}{\delta\xi}$$

The control shall be active in the interval $[0, \bar{t}]$ from 1 to -1:

$$\phi^*(t,x) = \begin{cases} \sqrt{\frac{2\delta\xi}{\kappa}} \tanh\left(\frac{\delta\xi}{2\kappa}(\bar{t}-t)\right) & x=1 \\ 0 & x=-1 \end{cases}$$

Not only does a higher $\bar{t}$ increase the offshoring interval, but it makes the control - the derivative of the probability of transitioning to the South - higher, as it increases $(\bar{t}-t)$.

- if $\bar{t} < 0$

$$z_t = \delta\xi(T-t)$$

and the control is never activated.

It may be useful to see graphically how $\bar{t}$ changes with each parameter, *ceteris paribus*. Assuming starting parameters of $T = 2$, $\delta = 0.5$, $\xi = 0.5$, $\theta = 0.2$, $\kappa = 0.5$ the single effects are the following [4] .

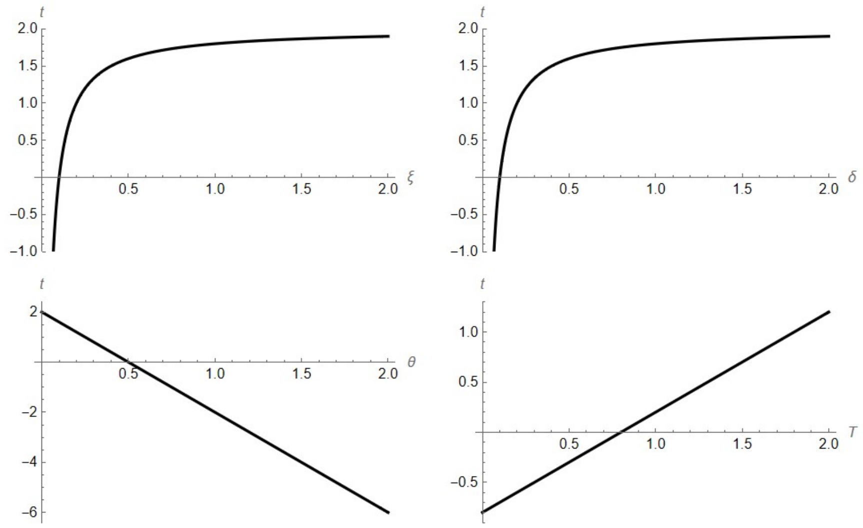


Figure 5: Effects of parameters on $\bar{t}$

While the effects of additional profits - whether by higher demand or lower production costs - is marginally decreasing, the ones of transition costs and end of the programming interval is linear; this is because the first two are conditioned by the probability of transition and by the state of firms: as $\bar{t} \to T^-$, there is less time and a lower chance to enjoy extra profits, while the activation cost remains the same. As ξ or $\delta \to T^-$, $\bar{t}$ asymptotically approaches T because there must be by continuity an interval of time in which $z_t < \theta$ (given that $\theta > 0$).

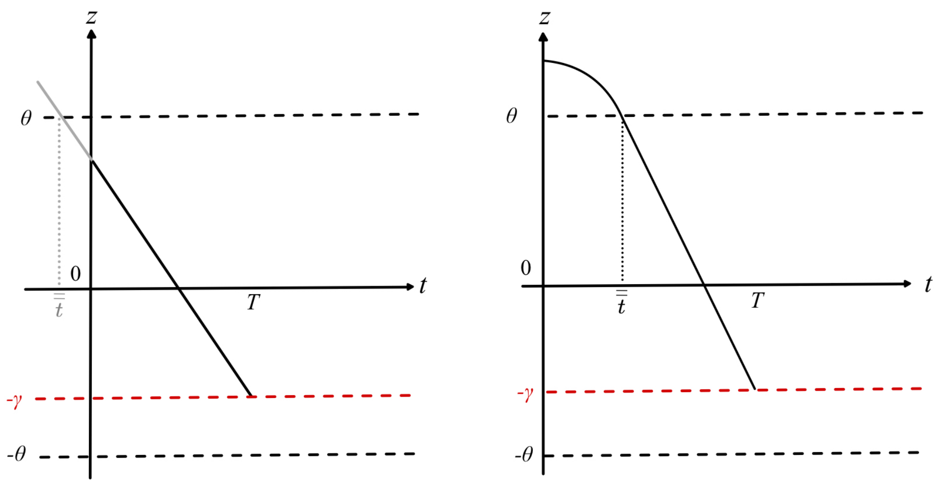
[4]All the following numerical simulations are performed in Wolfram Mathematica 14 (www.wolfram.com/mathematica), running on Windows 11 Pro and processor Intel(R) Core(TM) i5-1035G1 CPU @ 1.00GHz 1.19 GHz

3.3 Offshoring with incentives

The terminal value γ poses a further challenge: one must evaluate differently the cases of (8), depending on whether $z_T = \gamma$ is lower or higher than θ .

3.3.1 Low incentives, $0 < \gamma \leq \theta$

The solution is similar to the one carried in section 3.2: the final condition is here set to $-\gamma$. There are again two cases, based on $\bar{\bar{t}}$, the time at which $z_t = \theta$.



Case 1: $\bar{\bar{t}} < 0$

Case 2: $\bar{\bar{t}} \in (0, T)$

Figure 6: z_t with $\gamma < \theta$

- $\bar{\bar{t}} \leq 0$ From (8),

$$\begin{cases} \dot{z}_t = -\delta\xi \\ z_T = -\gamma \end{cases}$$

  The solution is

$$z_t = \delta\xi(T - t) - \gamma$$

  while setting $z_t = \theta$ one obtains:

$$\bar{\bar{t}} = T - \frac{\theta + \gamma}{\delta\xi}$$

- $\bar{\bar{t}} > 0$ There are now two different solutions for z_t:

$$\begin{cases} z_t = \delta\xi(T - t) - \gamma & z_t > \bar{\bar{t}} \\ z_t = \theta + \sqrt{2\delta\xi\kappa}\tanh\left(\sqrt{\frac{\delta\xi}{2\kappa}}(\bar{\bar{t}} - t)\right) & z_t \leq \bar{\bar{t}}. \end{cases}$$

In the interval where $\bar{\bar{t}} > 0$ and $z_t > \theta$, the control is active from state 1 (North) to state -1 (South) in the form:

$$\phi^*(t,x) = \begin{cases} \sqrt{\frac{2\delta\xi}{\kappa}} \tanh\left(\frac{\delta\xi}{2\kappa}(\bar{\bar{t}} - t)\right) & x = 1 \\ 0 & x = -1 \end{cases}$$

The effects of parameters on the de-activation time are again similar (notice how θ is kept higher than γ). A relevant point for policymakers is that while incentives are not able to make firms not activate their control, they delay the time at which they do, thereby lowering their chances to successfully relocate, as ϕ depends positively on $\bar{\bar{t}}$, which in turn depends negatively on incentives.

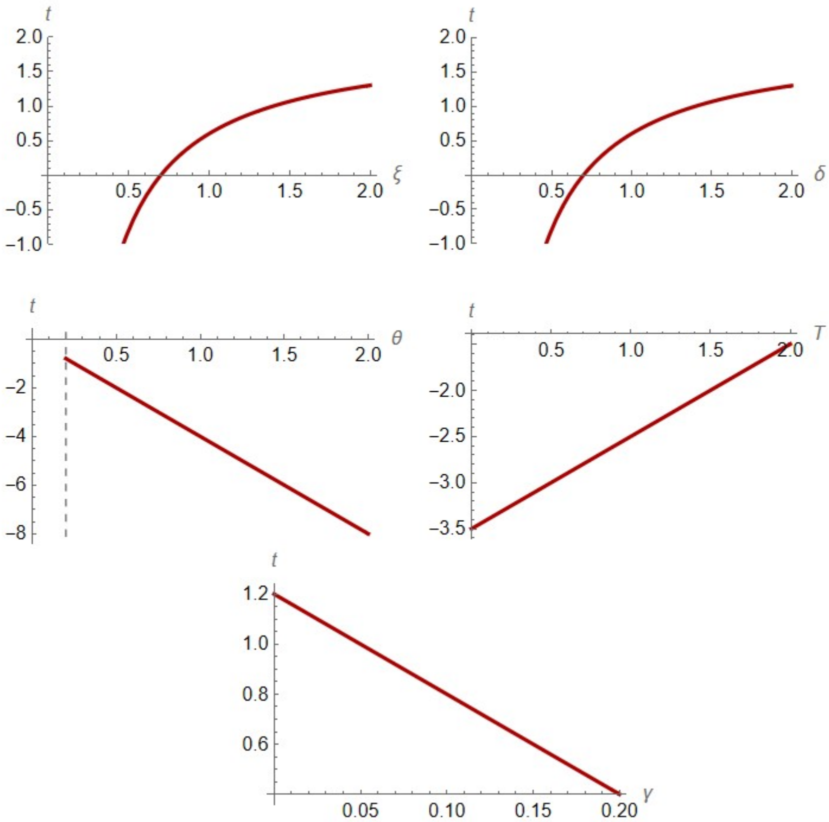


Figure 7: Effects of single parametres on $\bar{\bar{t}}$

3.3.2 High incentives, $\gamma > \theta$

This case is the most complex. Three different situations can be distinguished, based on the time $\bar{\bar{t}}$ at which z_t is higher than θ and the time $\tilde{t}$ at which z_t lower than $-\theta$. The first defines the time of offshoring, as before. The second that of reshoring, as the opportunity cost of going North is higher than the saved cost θ.

There must exist an interval in which $z_t < -\theta$ by continuity; the final condition $z_t = -\gamma$

completes the Cauchy problem for the last part of the interval

$$\dot{z} = \begin{cases} -\frac{(z+\theta)^2}{2\kappa} - \delta\xi \\ z_T = -\gamma \end{cases}$$

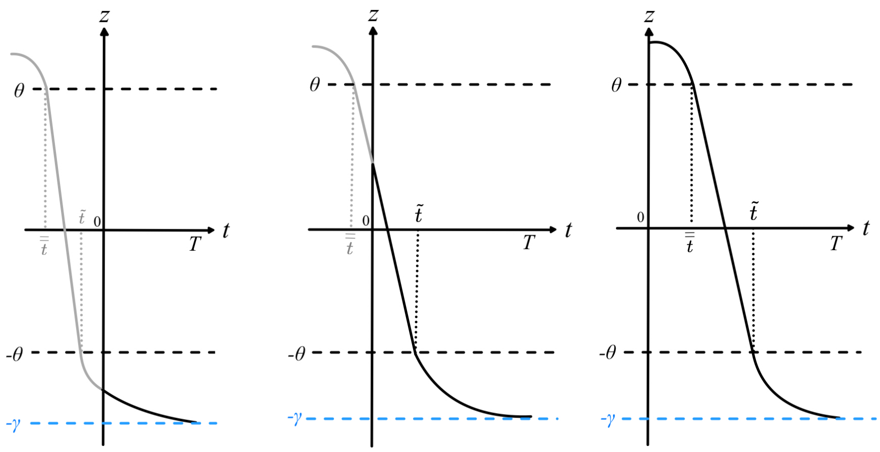
So,

$$z_t = -\theta + \sqrt{2\delta\xi\kappa}\tanh\left[\sqrt{\frac{\delta\xi}{2\kappa}}(T-t) + \arctan\left(\frac{\theta-\gamma}{\sqrt{2\delta\xi\kappa}}\right)\right] \tag{10}$$

for $z_t < -\theta$. One can now find

$$\tilde{t} = T - \sqrt{\frac{2\kappa}{\delta\xi}}\arctan\left(\frac{\gamma-\theta}{\sqrt{2\delta\xi\kappa}}\right)$$

to distinguish the following cases:



Case 1: $\bar{\bar{t}}, \tilde{t} \leq 0$ Case 2: $\bar{\bar{t}} \leq 0, \tilde{t} \in (0, T]$ Case 3: $\bar{\bar{t}}, \tilde{t} \in (0, T]$

Figure 8: z_t with $\gamma > \theta$

- $\tilde{t} < 0$

  The solution is given by (10) for the whole programming interval

- $\bar{\bar{t}} < 0, \tilde{t} > 0$

  In this instance there is again only reshoring, but from $\tilde{t}$ to T. In the second part of the interval, the solution is again (10). Solving the Cauchy problem for the first, one easily finds that $z_t = \delta\xi(\tilde{t} - t) - \theta$ for $t \in [0, \tilde{t})$ and that

$$\bar{\bar{t}} = \tilde{t} - \frac{2\theta}{\delta\xi}$$

- $\bar{\bar{t}}, \tilde{t} > 0$

  This is the most informative case: there are in facts both offshoring in $t \in [0, \bar{\bar{t}}]$ and reshoring in $t \in [\tilde{t}, T]$. Here it is possible to piece together the two solutions above with the one in Paragraph 3.2 to obtain:

$$z_t = \begin{cases} \theta + \sqrt{2\delta\xi\kappa}\tanh\left(\sqrt{\frac{\delta\xi}{2\kappa}}(\bar{\bar{t}} - t)\right) & t \in [0, \bar{\bar{t}}] \\ \delta\xi(\tilde{t} - t) - \theta & t \in [\bar{\bar{t}}, \tilde{t}] \\ -\theta + \sqrt{2\delta\xi\kappa}\tanh\left[\sqrt{\frac{\delta\xi}{2\kappa}}(T - t) + \arctan\left(\frac{\theta-\gamma}{\sqrt{2\delta\xi\kappa}}\right)\right] & t \in (\tilde{t}, T] \end{cases}$$

The control is therefore active for offshoring in $t \in [0, \bar{\bar{t}}]$ if $\bar{\bar{t}} > 0$ as above

$$\phi^*(t,x) = \begin{cases} \sqrt{\frac{2\delta\xi}{\kappa}}\tanh\left(\frac{\delta\xi}{2\kappa}(\bar{\bar{t}} - t)\right) & x = 1 \\ 0 & x = -1 \end{cases}$$

and for reshoring in $t \in (\tilde{t}, T]$ it is defined by

$$\phi^*(t,x) = \begin{cases} 0 & x = 1 \\ -\sqrt{\frac{2\delta\xi}{\kappa}}\tanh\left[\sqrt{\frac{\delta\xi}{2\kappa}}(T - t) + \arctan\left(\frac{\theta-\gamma}{\sqrt{2\delta\xi\kappa}}\right)\right] & x = -1 \end{cases}$$

by putting together (4) and (10)

What changes most significantly in this last instance is the effect of θ: while raising it previously impacted negatively only on the South, here it prevents offshoring on the one hand, but makes reshoring more difficult too. A more accurate description of these trade-offs is carried in the next Section. κ has now a strongly negative effect as well.
Standard parameters are as in the last paragraphs, γ is set at 0.5. The red line is $\bar{\bar{t}}$, being always lower than $\tilde{t}$.

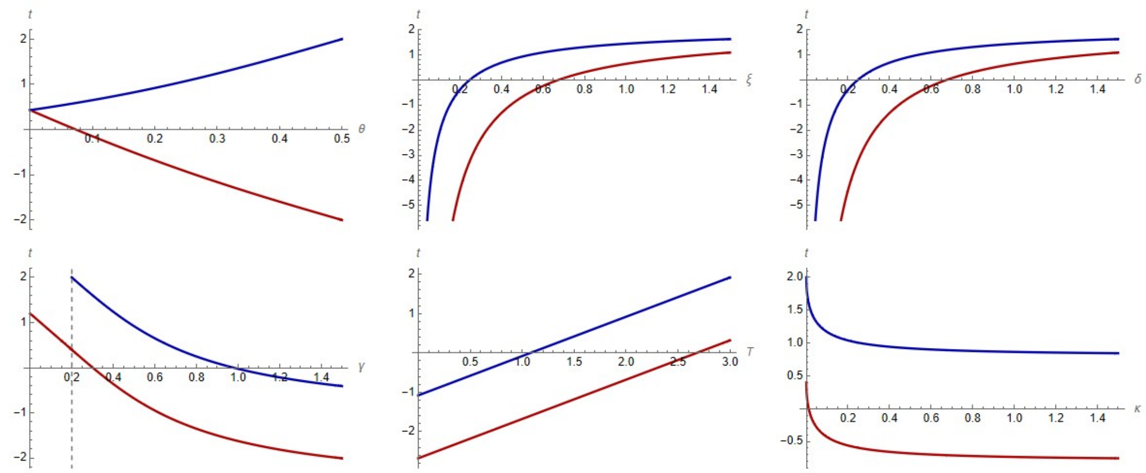


Figure 9: Effects of single parameters on $\bar{\bar{t}}$ and $\tilde{t}$, all else equal

4 Economics and trade-offs of offshoring

While Section 3 focused on effects of single parameters on the critical points in time at which the activation of the control started or was interrupted, this Section shall be devoted to analyse the trade-offs between such parameters.

The two countries have opposing interests in the actions of firms: each would like to either encourage offshoring and contrast reshoring (the South) or the contrary (the North). From their perspective, the cost of such modification of the system should be taken into account.
To do so, the approach shall be to graphically observe the necessary reaction to the other government's modification on the system in order to keep the status quo.
This method bears resemblance to the evaluation of trade-offs performed in microeconomics, where the utility of an agent is kept constant and the sacrifice of an unit of good is compared with the necessary compensation in another.

Each of the cases of Section 3 shall now be considered. The standard parameters are $\delta = 0.5$, $\kappa = 0.5$ and $T = 2$, while ξ and θ are the main object of interest; γ is set at 0.15 when lower than θ and at 0.5 when it is higher.

4.1 No incentives trade-offs, $\gamma = 0$

As Brambilla et al. (2024) in [1] highlight in Proposition 1, $\bar{t}$ is positive if

$$T\delta\xi - \theta > 0$$

which means that $\bar{t}$ is positive when the additional profit obtained by locating in the South throughout the while programming interval ($T\delta\xi$) are greater than the activation cost. This implies that the two countries can contrast or incentivise offshoring by acting upon ξ or θ. The North can in facts decrease the marginal cost of production in its territory or impose duties to lower the difference in profits [5]. The South can try to decrease the marginal cost of transition, which in this case is the only parameter that negatively affects $\bar{t}$.
It would be particularly insightful therefore, to see what the trade-off between the parameters ξ and θ is, by comparing how much one of them needs to be changed to keep $\bar{t}$ fixed.

[5] It would be more unrealistic to expect an influence on the programming interval or on δ. A change in the latter would have in any case the same effect as a change in ξ, as Figure 5 depicts

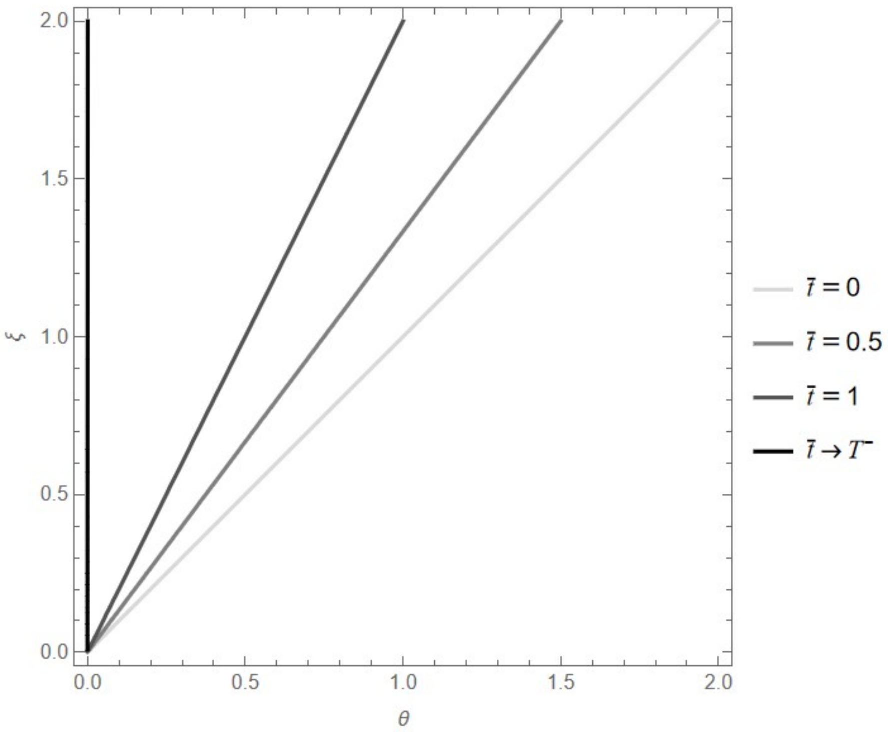


Figure 10: Trade-off between ξ and θ to keep $\bar{t}$ constant

Recalling that $\bar{t} = T - \frac{\theta}{\delta\xi}$, it is easy to see how the reaction to counteract the shift in one parameter for is linear and depending on $\bar{t}$, T and δ. This can be shown analytically by fixing $\bar{t}$ and making explicit ξ:

$$\xi = \frac{\theta}{(T - \bar{t}^{*})\delta}$$

This is displayed in the first figure of the paragraph: the slope of the line can be interpreted as the necessary compensation in offshoring profit to keep firms from reducing relocation to the South when its cost rises, or alternatively as the necessary "counter-action" by one nation to the change of parameters of the other. Take, for example, $\theta = 1, \xi = 1$, with $\bar{t} = 0$: if the South decides to bring $\bar{t}$ to 1 by lowering θ to 0.5, the North will need to sacrifice 0.5 ξ, bringing it down to 0.5, to prevent relocation.
One can therefore conclude that:

- the trade-off between ξ and θ is linear, given $\bar{t}$, δ and T . As $\bar{t} \to T$, such trade-off becomes more and more steep, up to infinity at $\bar{t} = T$, because the effect of ξ is positive but marginally decreasing: it takes an increasingly stronger incentive to make firms activate the control and try to switch state because the interval time for which marginal savings ξ can be enjoyed thins out;

- the trade-off's steepness is negatively affected by δ and T: this is because the higher the demand, the higher the convenience of offshoring, since firms can gain more profits from each unit of produced good. Similarly, firms can take advantage of offshoring for a longer time if T is increased: a rise in either of these two parameters will make the compensation needed not to switch more less high, as depicted in the next figure.

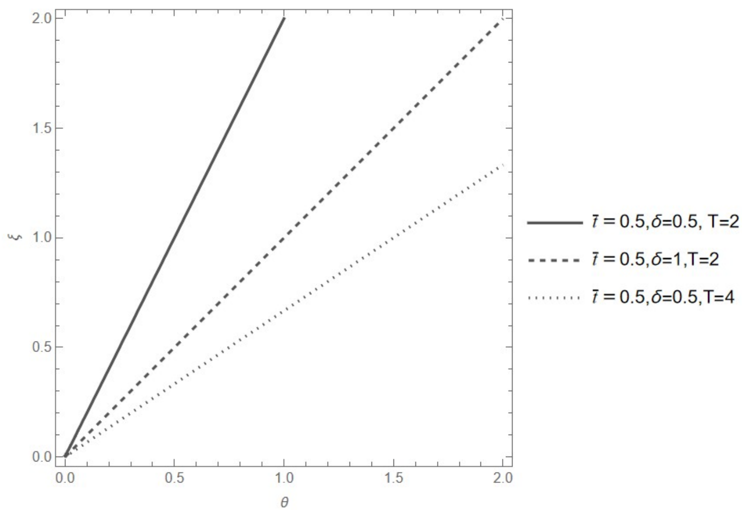


Figure 11: Effects of δ and T on the trade-off between ξ and θ, all else equal

4.2 Low incentives trade-offs, $0 < \gamma \leq \theta$

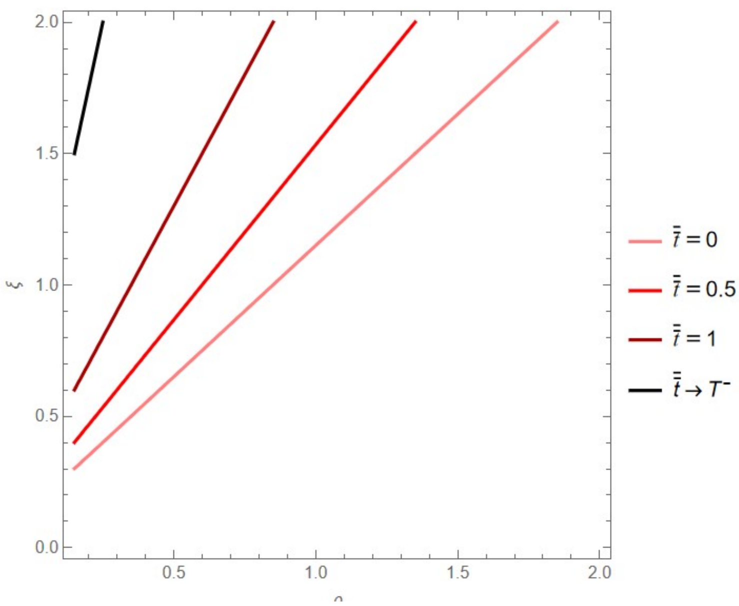


Figure 12: Trade-off between ξ and θ to keep $\bar{\bar{t}}$ constant

The analysis is in this case rather similar to the last one, with only one main difference: $\gamma = 0.15$ shift the locus of points in the parametric space that keep the deactivation time constant towards the left by an amount equal to its magnitude.

This means that for each vector (θ, ξ) in the parametric space, $\bar{\bar{t}} < \bar{t}$: incentives, though not high enough as to warrant reshoring, can still have a counteracting effect. This is shown in the figure below with different levels of γ for $\bar{\bar{t}} = 1$. The intensity of the trade-off between the considered parameters is not modified and the effects of δ and T are not different from those of the last section.

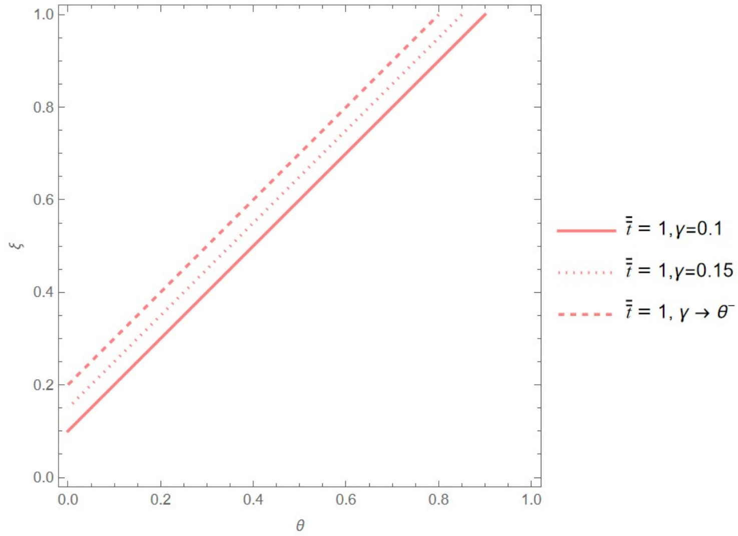


Figure 13: Effects of γ on the trade-off between ξ and θ to keep $\bar{\bar{t}}$ constant, all else equal

4.3 High incentives trade-offs, $\gamma > \theta$

The trade-offs between cost θ and profits ξ is more complex in the case of high incentives. While $\bar{t}$ and $\bar{\bar{t}}$ represented in the last paragraphs the total amount of time firms were offshoring - and consequently had a direct relationship with the proportion of the programming interval in which the control was activated to go South - one must now take into account that at $\tilde{t}$ the effect is counteracted. The opposite effect of θ on both the activation times poses a further challenge.
A possible strategy is to analyse the *net* effect of transition cost and profits on the activation times, which is

$$n = \bar{\bar{t}} - (T - \tilde{t})$$

n is the time a firm offshores more than reshores: if it is positive, the first phenomenon shall be prevalent in the time interval and vice versa.
Figure 14 illustrates a much "flatter" trade-off: this is in line with the observation of the double effect of θ. Moreover, the trade-off is non-linear here. Raising θ is less and less effective (as the figure 9 of paragraph 3.3.2 depicts as well), and needs therefore a decreasing amount of ξ for its effect to be offset.

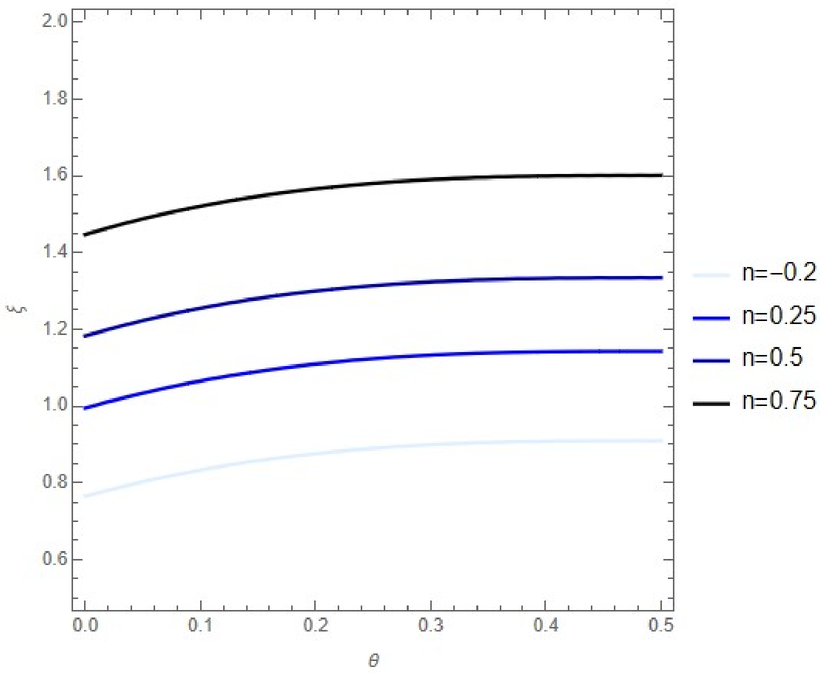


Figure 14: Trade-off between ξ and θ to keep net offshoring n constant

T's effect on this trade-off is similar to that seen before. δ ad κ, on the other hand, have a visible effect on the linearity of the trade-off, acting as "amplifiers" of θ and ξ: as they increase, both make the trade-off flatter, the first downwards and the second upwards.

Increasing δ makes the net offshoring much stronger, boosting ξ and requiring a very high θ to be offset. On the contrary, κ's increase has a relatively small impact, lowering n at each point, and slightly decreasing the counteraction needed even when ξ is lower and its effect is stronger.

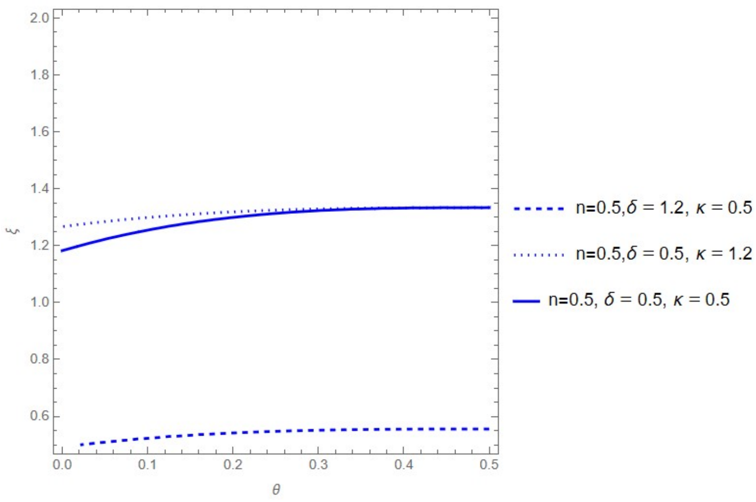


Figure 15: Effects of δ and κ on the trade-off between ξ and θ to keep $\tilde{t}$ constant, all else equal

Lastly, the effect of γ should be taken into consideration, since this is the configuration of the system where incentives are most important: they again shift the curve left, and make the trade-off considerably sharper.

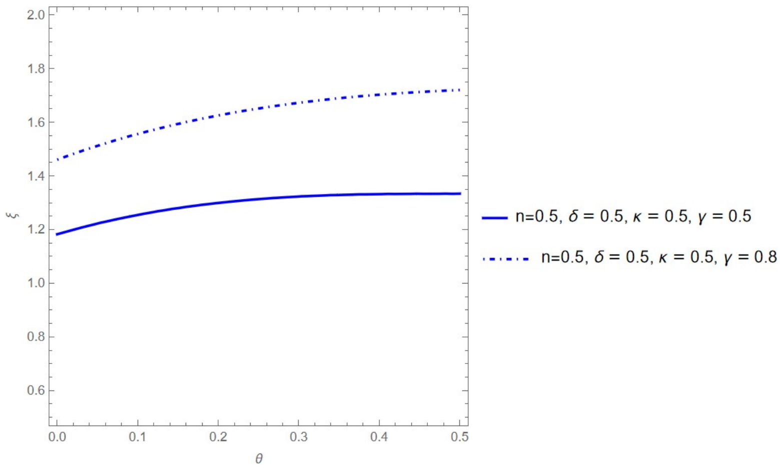


Figure 16: Effects of γ on the trade-off between ξ and θ to keep n constant, all else equal

5 Conclusions

This thesis' objective has been to overview and expand on the work in paper [1] by Brambilla et al. (2024), in which a stochastic optimisation problem describes the behaviour of the representative agent of an infinite number of firms in a two-country model where the location is a controlled Markov Chain in finite time. The solution is given by a backward differential equation that describes the evolution of the value function increment from one state to the other. The main parameters identified were the transitioning cost θ, the lower production cost in the South ξ, and the incentives offered by the North at the end of the programming interval γ. The trade-off between the first two were analysed, to display what are the main implications for policy-makers, and the cost necessary to respond to the actions of the other country by fixing the time of activation of the control, and plotting in the parametric space of ξ and θ the curves along which the activation time remained the same, toghether with the distortions in the trade-off that all the other parameters could cause.
There is great room for extension, both of the original model (making it a mean field game, in which parameters depend on the concentration of firms on one country on the line of oligopolistic competition models, for example) and of the analysis technique (in trying to identify for all combinations of parameters the least costly policy).

Number of words: n. 5487

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