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“ASSESSING AVERAGE INFLATION TARGETING FOR THE EURO AREA: AN EMPIRICAL INVESTIGATION”

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Abstract

Major central banks in the world, notably the Fed and the ECB, have decided to revised their monetary policy strategies, due to the economic environment of the last decade, characterized by low level of inflation and by the zero lower bound. In this thesis, I estimated a DSGE model for the euro area and I study the effect of the adoption of the average inflation targeting regime based on a welfare loss function. The benefits of the average inflation target arise depending on the design of the monetary policy rule. Specifically, the central bank must answer aggressively to the deviation of the average inflation to its target.

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Chapter 1

Introduction

In the last decade, for most of the industrialized countries in the world, the economy has been characterized by a low level of inflation rate, slow growth, and monetary policy has been constrained by the zero lower bound (ZLB), straining the effectiveness of traditional inflation targeting policies. Many economists tried to rationalize these features, Koester et al. (2021) highlighted the main causes of low inflation in the euro area over the period 2013 - 2019, arguing that, the underestimation of the economic slack, the inflation expectations being less well anchored to the ECB's inflation aim, structural trends like globalisation, digitization and demographic change, have resulted in a persistent shortfall in inflation, enhanced by the fact that monetary policy was constrained by the effective lower bound (ELB); Caldara et al. (2020) emphasized that the large negative Global Financial Crisis (GFC) shock sharply raised unemployment and the inflation has persistently undershot the Federal Open Market Committee's (FOMC) target rate. Generally speaking, policymakers and market participants expected a faster return of inflation to 2 percent and a stronger economic activity than realized, moreover, together with structural transformations that were difficult to ascertain in real time, this has lead to forecast error, who have delayed the implementation of fast and effective monetary responses.
As a result, many central banks, notably the U.S. Federal Reserve and the European Central Bank, decided to review their monetary policy strategy. The ECB, in July 2021, moved its inflation target (IT) from the ceiling of the 2 percent to a symmetric target over the medium term, which means that the Governing Council will consider a negative and a positive deviations from this target as equally undesirable and with the aim of stabilize the change in prices at 2 percent. Overseas, at the 2020 Jackson Hole Economic Policy Symposium, Federal Reserve Chair Jerome Powell announced a revision to the Fed's long-run monetary policy framework, replacing the flexible inflation target with average inflation target (AIT) to achieve its dual mandate of price stability and maximum sustainable employment, see Powell (2020).

Under an AIT regime, a period of below-target inflation should be followed by a period in which inflation is systematically above the target. Standard inflation targeting, instead, implies that inflation should move to its target regardless of its deviations in the previous period. AIT would benefit the economy by anchoring longer-term inflation expectations at a level consistent with the central bank's target, thereby avoiding the downward bias in inflation expectations that can arise under IT when the zero lower bound potentially constrains the policy rate, maintaining policy space for stabilization policy. Moreover, the promise of an above-target future inflation under AIT during low level of inflation should raise near and medium-term inflation expectations, thereby reducing ex-ante real rates and stimulating the economy as households increase their consumption.

Although AIT has gained a lot of consensus among policy makers, it has still many open questions. In a recent article on The Financial Times written by William Buiter, see Buiter(2021), AIT is strongly criticized, defining it ”illiterate”. He argues that the new monetary strategy of the Fed did not clearly expressed the average window of the target, which could lead to unnecessary period of above target inflation. He also condemn the make-up feature of this policy: past inflation can influence expectations of future inflation which can in turn drive actual current and future inflation, but many other drivers guide expected future inflation such as past, present, and anticipated future money growth, forward guidance, expected and unanticipated changes in the monetary and fiscal policy regime, supply side developments, with no strong evidences that past failure of inflation determine future deliberate failures.

In this work, I try to address some of the open questions. I contribute to the extant literature along three dimensions. First, I assess the effect of the average inflation targeting in an estimated model for the euro area. Thus, I estimated the Smets and Wouters (2007) model over the period 1995Q1 - 2019Q4 using macroeconomic observable variables of the euro area. This allows me, to understand how the model fit the data and how policy related question can be interpret. Second, I study the effect of an average inflation target over different time horizons. I include the average of realized past inflation or the average of the future expected inflation. Finally, I evaluate the welfare properties of the Average Inflation Targeting rules on the basis of a central bank's loss function.

I find that the design of the monetary policy rule is crucial to reap the benefits of the average inflation targeting strategies. Especially, only under an aggressive response to average inflation deviation from its target, the central can improve its objective function.

The thesis proceeds as follows. Chapter 1 describe the state of the literature, reporting the main contributions on the evaluation of the AIT and highlighting the weakness and the open questions. Chapter 3 presents the log-linearized Smets and Wouters (2007) model. Chapter 4 reports the results of the Bayesian estimation, and Chapter 5 evaluate the different monetary policies on the basis of a central bank's loss function.

Chapter 2

Literature

AIT has gained increased attention among policymakers and academics in recent years. Nessén and Vestin (2005) introduced AIT into the central bank's optimization problem. Under perfect communication and rational expectation, when the Philips curve has forward-looking components, adopting an average inflation targeting strategy will result in lower welfare loss than a one-period inflation target objective. The history dependence of the AIT interacts favourably with expectations of future inflation. Targeting two-period average inflation, a positive shock to inflation in one period will lead to lower expectations in the following period. This is crucial under a forward-looking Philips curve, because, this change in expectations, will improve the short-run trade-off faced by the central bank and lead to lower social loss. Similarly, Nakata et al. (2020) studied the implications of average inflation targeting in a New Keynesian model with a lower bound on nominal interest rates. They have analyzed the optimization problem of a central bank that takes the assigned objective function as given and sets the short-term nominal interest rate under discretion. They have considered two variants of the model. Under rational expectations, AIT improves macroeconomic outcomes and increases people's welfare when compared to standard inflation targeting. Under bounded-rational expectations, as long as cognitive limitations remain small, the results remain the same. However, if cognitive limitations are sufficiently strong, the optimal averaging window is finite, and the welfare improvement from abandoning standard inflation targeting in favour of average inflation targeting can be small.

Similarly, but focusing on an interest rate targeting rule, instead of optimal policy, Mertens and Williams (2019) show that AIT can mitigate the effects of the zero lower bound by raising inflation expectations when inflation is low. They use a standard New Keynesian model augmented with a lower bound on interest rates and they investigate three classes of monetary frameworks with their policy rules: inflation targeting, average inflation targeting and price-level targeting. Then, they compare with the benchmark case of optimal policy under discretion. They affirm that all of these policies work through affecting expectations. However, among all of these policies, AIT appears to be the best in raising inflation expectations during a period of low inflation, resulting in an overall lower social loss. Also Svensson (2020a), which studied monetary policy strategy of "forecast targeting" within the Fed mandate, affirms that AIT, if well receipted by the public, would anchor inflation expectation towards the target.

The paper of Arias et al. (2020) is the closet to this thesis. Indeed, their analysis largely focuses on a class of makeup strategies in which policymakers seek to stabilize average inflation around the inflation target over some horizon. They compare a set of monetary policy rules, under the same estimated New Keynesian model, specifically the FRB/US model, and they evaluate the different rules based on a welfare loss function. They find that makeup strategies generally improve macroeconomic stability compared with a standard inflation target approach, however, the size of these gains is moderate across the strategies considered, with longer makeup windows yielding somewhat larger gains. Companion of this paper is the work of Hebden et al. (2020). They examine how AIT performs under the FRB/US macroeconomic model, but modify the assumptions about expectations formation. They find that, as long as financial market participants understand the AIT strategies and believe in the central bank's commitment to implement them, the make-up rule can counteract the real effects of adverse economic shocks. To be credible, the central bank will initially answer to a period of low with an aggressive monetary policy accommodation. This will lead to overheating the economy and bring a sustained period of inflation above the target. Moreover, they show that in a period of low inflation and when the central bank is constrained by the ELB, the make-up strategies are more credible since the public would see a strong commitment by the policymakers to mitigate the economy's issues. They also report that the adoption of a makeup strategy could inadvertently unanchor long-run inflation expectations. Furthermore, AIT works better than IT also under uncertainty about the natural rate of unemployment, because makeup strategies implicitly correct policy errors induced by misconception of slack in the economy.

As well stressed before, the effectiveness of AIT hinges on the extent to which people understand how these strategies are known and understood by households and firms. Coibion et al. (2020) studied this issue using a daily survey of U.S. households run before and after Powell's speech of the adoption of the new monetary policy regime, and then followed up by another survey around one year after the announcement. First of all, the authors show that the announcement did not significantly affect the general public's perception of monetary policy, and neither being exposed to news about monetary policy changed households' perceptions of how the Federal Reserve would act. Moreover, only a few people were able to recognize AIT as the Fed new monetary policy strategy. In addition, they also control for the fact that could take time to understand the new strategy, however, even after a year the results remain the same. Finally, they studied the case when information about AIT was presented directly and concisely to individuals: they wanted to understand if households' beliefs could change in a manner consistent with the theory. In detail, they provided some individuals with information about AIT, others with information about IT. However, they find no significant differences in expectations between individuals who are provided information about AIT compared to IT. Overall, these findings suggest that AIT is unlikely to provide many of the economic benefits that theory often attributes to it.

Most papers have seen before either impose rational expectations or assume that agents are bounded-rational, but have perfect knowledge about the policy structure. Honkapohja and McClung (2021) analyzed the performance of AIT when agents have imperfect knowledge about the policy structure and learn to forecast over time. They show that the Central Bank can fail to anchor expectations around the target steady-state if prices are flexible or the speed of learning is very slow. Furthermore, an opaque AIT policy will typically fail to move from a liquidity trap.

Another fundamental issue to take into account in implementing a policy of average-inflation-targeting is the size of the time window used to calculate the average inflation rate. Related to this topic Amano et al. (2020) examine how much history dependence should be embedded in the AIT rule for the policy to be effective. They find that the optimal length for AIT is less than two years. In contrast, the optimal window length declines substantially to about two quarters when the economy is only subject to cost-push shocks and firms have adaptive expectations. Related to the miscommunication of the horizon over which the Fed should target the average inflation at 2 percent is the work of Jia and Wu (2021). They show that Fed ambiguous communication about the time horizon of AIT is intentional and welfare improving. According to this work, under uncertainty about economic fundamentals, the central bank has to convince the private sector about its commitment to implement AIT. In this way, the central bank can improve the trade-off between inflation and real activity. Only ex-post, the central bank has the incentive to deviate from its communication and implement IT instead. This strategy is welfare improving, but time-inconsistent. The central bank must be able to convince private agents about its communication, but then it will be different from its actions.

The literature on average inflation targeting policies is still young compared to Price-level Targeting and nominal GDP targeting, and many questions are still open. In this work, I especially try to address the importance of the design of the AIT rule. However, before jumping into the monetary policy rule analysis, I estimated a DSGE model for the euro area, for which I present the result in the next chapter.

Chapter 3

The model

For a better and consistent analysis, I estimated the Smets and Wouters (2007) (SW, hereafter) medium-scale DSGE model, a well-known and recognized framework among central banks and academics. It is particularly suited for evaluating the effect of different monetary policy rules. Indeed, this model can be viewed as the foundation of frameworks used for policy analysis at many central banks and policy institutions. Moreover, the SW model, being empirically consistent with optimizing behavior, should be less prone to the Lucas (1976) critique, compare to other studies based on backward-looking models. Largely based on the work of Christiano et al. (2005), the SW model contains many shocks and frictions. The economy is populated by a representative household, which receives, net of taxes, income from wages, from financial asset, from renting capital to firms, and the interest rate determines the inter-temporal time pattern of consumption. Moreover, they introduced habit formation in consumption, adjustment costs for investments, utilisation costs, that make variables more sluggish and give random shocks a more long-lasting effect. Furthermore, the model features price and wage rigidities. In addition, the stochastic dynamics are driven by seven orthogonal structural shocks: total factor productivity shocks, risk premium shocks, investment-specific technology shocks, wage and price mark-up shocks, exogenous spending and monetary policy shocks.

This section develops the sticky-price and flexible-price economies of the log-linearized version[1].

[1] All the variables in lowercase letters represent log deviation from the non-stochastic steady-state.

3.1 Sticky-price economy

3.1.1 The goods market equilibrium condition

The aggregate resource constraint is given by

$$y_t = c_y c_t + i_y i_t + z_y z_t + \epsilon_t^g. \tag{3.1}$$

Output y_t is equal to consumption c_t, investment i_t and capital utilization rate z_t. c_y is the steady-state share of consumption in output (equals to $1 - g_y - i_y$), where g_y and i_y are respectively the steady-state exogenous spending-output ratio and investment-output ratio; $i_y = \gamma - 1 + \delta$, where γ is the steady-state growth rate, while δ is the depreciation rate of capital. Finally, $z_y = R_*^k k_y$, where R_*^k (equals to $\beta^{-1}\gamma^{\sigma_c} - 1 - \delta$) is the steady-state rental rate of capital. ϵ^g is the government spending shock.

3.1.2 Consumption

Consumption is defined as

$$\begin{aligned} c_t = {} & \frac{h}{1+h} c_{t-1} + \frac{1}{1+h} E_t[c_{t+1}] - \frac{1-h}{\sigma_c(1+h)}(r_t - E_t[\pi_{t+1}] + \epsilon_t^b) \\ & + \frac{(\sigma_c - 1)}{\sigma_c(1+h)} \frac{W_*^h L_*}{C_*}(l_t - E_t[l_{t+1}]) \end{aligned} \tag{3.2}$$

where c_t represent the aggregate private consumption, r_t the central bank nominal interest rate, π_t the inflation rate and l_t the household's hours worked. Consumption at time t depends on a weighted average of past and expected future consumption, on expected growth in hours worked and the ex ante real interest rate. W_*^h is the steady-state aggregate nominal wage rate, L_* the steady-state composite labour input and C_* the steady-state household's consumption. The parameter h, the modified household's consumption habits parameter, is equal to $\lambda \,/\, \gamma$, where λ is the the external habit formation. σ_c is the inverse of the elasticity of inter-temporal substitution. Finally, ϵ^b is the risk premium shock and represent a wedge between the interest rate controlled by the central bank and the return on assets held by the households.

3.1.3 Investment

The investment equation is given by

$$i_t = \frac{1}{1+\beta\gamma^{1-\sigma_c}} i_{t-1} + \frac{\beta\gamma^{1-\sigma_c}}{1+\beta\gamma^{1-\sigma_c}} E_t[i_{t+1}] + \frac{1}{\varphi\gamma^2(1+\beta\gamma^{1-\sigma_c})} q_t + \epsilon_t^i \tag{3.3}$$

where i_t is the aggregate investment and q_t the real value of the existing capital stock. β is the discount factor, φ the steady-state elasticity of the capital adjustment cost function and ϵ^i represents a disturbance to the investment-specific technology process.

3.1.4 Capital

The value of capital equation is

$$q_t = \frac{1-\delta}{1-\delta+R_*^k} E_t[q_{t+1}] + \frac{R_*^k}{1-\delta+R_*^k} E_t[r_{t+1}^k] - (r_t - E_t[\pi_{t+1}] + \epsilon_t^b) \tag{3.4}$$

where q_t is the value of capital stock at time t, which depends positively on the its expected future value and the expected rental rate on capital r_t^k, while negatively on the ex ante real interest rate and the risk premium shock.
The capital accumulation equation is

$$k_t = \frac{1-\delta}{\gamma} k_{t-1} + \frac{\gamma+\delta-1}{\gamma} i_t + (\gamma - 1 + \delta)\left(1+\beta\gamma^{(1-\sigma_c)}\right)\gamma\varphi\epsilon_t^i \tag{3.5}$$

k_t is the installed capital that is a function of the flow of investment and the relative efficiency of these investment expenditures captured by the investment specific technology shock.

3.1.5 Production

Turning to the supply side, the aggregate production function is defined as

$$y_t = \phi_p \left(\alpha k_t^s + (1-\alpha) l_t + \epsilon_t^a\right) \tag{3.6}$$

where output y_t is produced using capital k_t^s and labor services l_t (hours worked). Parameters ϕ_p and α represent respectively the presence of fixed cost in production and the share of capital in production.

Capital services used in production

$$k_t^s = k_{t-1} + z_t \tag{3.7}$$

are a function of capital installed in the previous period and the degree of capital utilization z_t, which is a positive function of the elasticity of the capital utilization adjustment cost function with respect to utilization normalized to be between zero and one

$$z_t = \frac{1-\psi}{\psi} r_t^k. \tag{3.8}$$

and r_t^k is the rental rate of capital equals to

$$r_t^k = -(k_t - l_t) + w_t \tag{3.9}$$

that is negatively related to the capital-labor ratio and positively to real wage.

3.1.6 Wages

The real wage equation is given by:

$$\begin{aligned} w_t =& \frac{\beta\gamma^{1-\sigma_c}}{1+\beta\gamma^{1-\sigma_c}} E_t\left[w_{t+1}\right] + \frac{1}{1+\beta\gamma^{1-\sigma_c}} w_{t-1} \\ & \frac{\beta\gamma^{1-\sigma_c}}{1+\beta\gamma^{1-\sigma_c}} E_t\left[\pi_{t+1}\right] - \frac{\beta\iota_w\gamma^{1-\sigma_c}}{1+\beta\gamma^{1-\sigma_c}} \pi_t + \frac{\iota_w}{1+\beta\gamma^{1-\sigma_c}} \pi_{t-1} \\ & - \frac{1}{1+\beta\gamma^{1-\sigma_c}} \frac{\left(1-\beta\gamma^{1-\sigma_c}\xi_w\right)(1-\xi_w)}{\xi_w\left(1+(\phi-1)\epsilon_w\right)} \mu_t^w + \epsilon_t^w \end{aligned} \tag{3.10}$$

where w_t is a function of expected and past wages, expected and past inflation, wage mark-up shock ϵ_t^w and wage mark-up μ_t^w, which, in the monopolistically competitive labor market, is given by the equation

$$\mu_t^w = w_t - \left(\sigma_l l_t + \frac{1}{1-h}(c_t - hc_{t-1})\right) \tag{3.11}$$

that is the difference between the real wage and the marginal rate of substitution between working and consuming, where σ_l is the elasticity of labor supply with respect to real wage and h is the habit parameter in consumption.

In general, real wages do not depend on lagged inflation if wage indexation is zero ($\iota_w = 0$) and the speed of adjustment to the desired wage mark-up depends on the degree of wage stickiness ξ_w, the curvature of the Kimball labor market aggregator ϵ_w and the steady state labor mark-up $(\phi_w - 1)$.

3.1.7 Prices

Under monopolistic competitive goods market, cost minimization by firms implies that the price mark-up (μ_t^p) is equal to the difference between the marginal product of labor and the real wage:

$$\mu_t^p = mpl_t - w_t = \alpha\left(k_t{}^s - l_t\right) - w_t + \epsilon_t^a \tag{3.12}$$

Due to price stickiness, as in Calvo (1983), and partial indexation to lagged inflation of those prices that can not be reoptimized, prices adjust only sluggishly to their desired mark-up. Profit maximization by price-setting firms gives rise to the following New-Keynesian Phillips curve:

$$\begin{aligned}\pi_t = & \frac{\beta\gamma^{1-\sigma_c}}{1+\beta\iota_p\gamma^{1-\sigma_c}} E_t\left[\pi_{t+1}\right] + \frac{\iota_p}{1+\beta\iota_p\gamma^{1-\sigma_c}}\pi_{t-1} \\ & - \frac{1}{1+\beta\iota_p\gamma^{1-\sigma_c}} \frac{\left(1-\beta\gamma^{1-\sigma_c}\xi_p\right)\left(1-\xi_p\right)}{\xi_p\left(1+(\phi-1)\epsilon_p\right)}\mu_t^p + \epsilon_t^p\end{aligned} \tag{3.13}$$

Therefore, inflation depends positively on past and expected future inflation, price mark-up shock ϵ_t^p, while negatively on the price mark-up. As for wages, when the degree of indexation to past inflation, $\iota_p = 0$, the inflation equation reverts to a standard, purely forward-looking Phillips curve, so, assuming that all prices are indexed to lagged inflation ensures that the Phillips curve is vertical in the long run. The speed of adjustment to the desired mark-up depends on the degree of price stickiness (ξ_p), the curvature of the Kimball goods market aggregator (ϵ_p), and the steady-state mark-up ($\phi_p - 1$).

3.1.8 Monetary policy rule

The monetary authority adjusts r_t following a generalized taylor rule given by the equation:

$$r_t = \rho r_{t-1} + (1-\rho)[r_\pi\pi_t + r_y(y_t - y_t^p)] + r_{\Delta y}[(y_t - y_t^p) - (y_{t-1} - y_{t-1}^p)] + \epsilon_t^r. \tag{3.14}$$

which depends on inflation and on output gap, represented by the difference between actual and potential output (y_t^p) defined as the level of output that would prevail under flexible prices and wages in the absence of the two "mark-up" shocks. In addition, it also depend on a short-run feedback from the change in the output gap. The parameter ρ captures the degree of interest rate smoothing. Finally, the monetary policy shock is given by ϵ_t^r.

3.2 Flexible-price economy

The model is expanded with a flexible-price-wage version of the previous 14 equations in order to calculate the model-consistent output gap.

3.2.1 Flexible-price equilibrium

The market equilibrium under flexible price is given by:

$$y_t^p = c_y c_t^p + i_y i_t^p + z_y z_t^p + \epsilon_t^g \tag{3.15}$$

3.2.2 Flexible-price consumption

The flexible-price consumption is defined as

$$\begin{aligned} c_t^p = & \frac{h}{1+h} c_{t-1}^p + \frac{1}{1+h} E_t\left[c_{t+1}^p\right] - \frac{1-h}{\sigma_c(1+h)}(r_t^p + \epsilon_t^b) \\ & + \frac{(\sigma_c - 1)}{\sigma_c(1+h)} \frac{W_*^h L_*}{C_*}(l_t^p - E_t[l_{t+1}^p]) \end{aligned} \tag{3.16}$$

where c_t^p is flexible-price consumption, r_t^p the natural interest rate and l_t^p flexible-price worked hours.

3.2.3 Flexible-price investment

Investment under flexible price is equal to

$$i_t^p = \frac{1}{1+\beta\gamma^{1-\sigma_c}} i_{t-1}^p + \frac{\beta\gamma^{1-\sigma_c}}{1+\beta\gamma^{1-\sigma_c}} E_t[i_{t+1}^p] + \frac{1}{\varphi\gamma^2(1+\beta\gamma^{1-\sigma_c})} q_t^p + \epsilon_t^i \tag{3.17}$$

where i_t^p is flexible-price investment and q_t^p the flexible-price real value of existing capital stock.

3.2.4 Flexible-price capital

Flexible-price capital stock is given by

$$q_t^p = \frac{1-\delta}{1-\delta+R_*^k} E_t[q_{t+1}^p] + \frac{R_*^k}{1-\delta+R_*^k} E_t[r_{t+1}^{kp}] - (r_t + \epsilon_t^b) \tag{3.18}$$

where r_t^{kp} is the flexible-price rental rate on capital equal to

$$r_t^{kp} = -(k_t^p - l_t^p) + w_t^p \tag{3.19}$$

and the flexible-price capital accumulation equation is given by

$$k_t^p = \frac{1-\delta}{\gamma} k_{t-1}^p + \frac{\gamma+\delta-1}{\gamma} i_t^p + (\gamma - 1 + \delta)\left(1 + \beta\gamma^{(1-\sigma_c)}\right)\gamma\varphi\epsilon_t^i \tag{3.20}$$

3.2.5 Flexible-price production

The flexible-price aggregate production function is defined as

$$y_t^p = \phi_p \left(\alpha k_t^{sp} + (1-\alpha)\, l_t^p + \epsilon_t^a\right) \tag{3.21}$$

where y_t^p represents the pontential output that is a function of flexible-price labor services l_t^p and flexible-price capital services k_t^{sp}, that is equal to

$$k_t^{sp} = k_{t-1}^p + z_t^p \tag{3.22}$$

that is a function of flexible-price capital installed in the previous period (k_{t-1}^p) and the degree of flexible-price capital utilization (z_t^p)

$$z_t = \frac{1-\psi}{\psi} r_t^k. \tag{3.23}$$

3.2.6 Flexible-price wages

Real wages under flexible-price are equal to

$$w_t^p = \sigma_l l_t^p + \frac{1}{1-h}\left(c_t^p - hc_{t-1}^p\right) \tag{3.24}$$

3.2.7 Flexible-price marginal cost

Under flexible prices there is no marginal cost by definition. This leads to

$$\epsilon_t^\alpha = \alpha r_t^{kp} + (1-\alpha) w_t^p \tag{3.25}$$

3.3 The stochastic structure

3.3.1 AR(1) shocks

The total factor of productivity shock ϵ_t^a, the risk premium shock ϵ_t^b, the investment-specific technology shock ϵ_t^i and the monetary policy shock ϵ_t^r follow a first-order autoregressive functional form as

$$\epsilon_t^k = \rho_k \epsilon_{t-1}^k + \eta_t^k \tag{3.26}$$

$\forall k \in \{a, b, i, r\}$, where $\rho_k \in [0, 1[$ is the first-order autoregressive parameter of the shock k, and η_t^k an i.i.d-normal error term.

3.3.2 Government spending shock

In addition, the government spending shock ϵ_t^g follows a first-order autoregressive process impacted by technology shocks such as

$$\epsilon_t^g = \rho_g \epsilon_{t-1}^g + \eta_t^g + \rho_{ga} \eta_t^a \tag{3.27}$$

where $\rho_g \in [0, 1[$ is the first-order autoregressive parameter and η_t^g is an i.i.d normal error term. The inclusion of the technology shock is empirically motivated by the fact that, in estimation, the exogenous spending also includes net exports, which may be affected by domestic productivity developments (ρ_{ga}).

3.3.3 ARMA(1,1) shocks

Finally, the price and the wage markup shock follows an ARMA(1,1) functional form such as

$$\epsilon_t^s = \rho_s \epsilon_{t-1}^s + \eta_t^s - \mu_s \eta_{t-1}^s \tag{3.28}$$

$\forall s \in \{p, w\}$, where $\rho_s \in [0, 1[$ is the first-order autoregressive parameter of the shocks s, μ_s is the moving average term and η_t^s an i.i.d normal error term. The inclusion of the MA term is designed to capture the high-frequency fluctuations in inflation and wages.

Summing up, the model determine 14 endogenous variables: y_t, c_t, i_t, q_t, k_t^s, k_t, z_t, r_t^k, μ_t^w, w_t, μ_t^p, π_t, l_t and r_t. The stochastic behaviour of the system of linear rational expectations equations is driven by all the seven exogenous disturbances reported above.

Chapter 4

Model estimation

4.1 Bayesian estimation

The model is estimated using Bayesian techniques as in the works of Smets and Wouters (2003, 2007). Prominent academics, such as Geweke (1999), Landon-lane et al. (2000), Lubik and Schorfheide (2003), An and Schorfheide (2007), and in more recent works of Christiano et al. (2018) and Fernández-Villaverde and Guerrón-Quintana (2021), demonstrated how the Bayesian approach has become an excellent tool to estimate DSGE models. First, Bayesian estimation fits the complete, solved DSGE model, likewise, estimation is based on the likelihood generated by the DSGE system. Moreover, it uses the information about the prior distributions coming from micro or macro-econometric studies, thereby linking them with the previous calibration-based literature. The inclusion of priors also helps identifying parameters. Furthermore, Bayesian estimation explicitly addresses model misspecification by including shocks, that are observation errors, in the structural equations. Finally, the resulted posterior distribution can be used to evaluate how the model fits the data, and how it can be useful for policy analysis.

The posterior distribution estimation combines the prior assumption of the distribution of the parameters with a likelihood function, which describe the data, under the Bayesian's law:

$$p(\theta|Y) = \frac{p(Y|\theta)p(\theta)}{p(Y)} \tag{4.1}$$

First, $p(\theta)$ is the prior distribution of the parameter vector θ before observing the sample Y. It stands for a probability density function such as a normal, gamma, inverse gamma, beta, or uniform distribution.

The likelihood function, $p(Y|\theta)$, describes the density of the observed data, given the model and its parameters.

The marginal density, $p(Y)$, of the data conditional to the model is equal to the equation:

$$p(Y) = \int p(Y|\theta)p(\theta)\,d\theta \tag{4.2}$$

and normalizes the posterior density such that it integrates to one. Therefore, the posterior kernel correspond to the numerator of the posterior density, such that:

$$p(\theta|Y) \propto p(Y|\theta)p(\theta) \equiv \mathcal{K}(\theta|Y) \tag{4.3}$$

and this equation is fundamental to rebuild all the posterior moments of interest. Estimating the likelihood function with the use of the Kalman filter, it is possible to derive the log-likelihood and rewrite the above equation as the log posterior kernel:

$$\ln \mathcal{K}(\theta|Y) = \ln \mathcal{L}p(Y|\theta) + \ln p(\theta) \tag{4.4}$$

where the first term on the right hand side is now known after carrying out the Kalman filter recursion.

Maximizing the log posterior kernel with respect to θ, it is possible to find the mode of the posterior distribution. Finally, trough the use of the Metropolis-Hasting algorithm, it is possible to simulate the posterior distribution of the parameters. The MH algorithm is a "rejection sampling algorithm" used to generate a sequence of sample, known as "Markoc Chain" from a distribution that is unknown at the outset. First of all, it starts from a point of θ, which is typically the posterior mode and it run in a loop. It draws a propose, that is θ^* from a jumping ditribution of the form

$$J(\theta^*|\theta^{t-1}) = \mathcal{N}(\theta^{t-1}, c\Sigma_m) \tag{4.5}$$

where Σ_m is the inverse of the Hessian computed at the posterior mode. Then it calculate the acceptance ratio, such as

$$r = \frac{\mathcal{K}(\theta^*|Y)}{\mathcal{K}(\theta^{t-1}|Y)} \tag{4.6}$$

and finally accept or discard the proposal, and update, if necessary, the jumping distribution:

$$\theta^t = \begin{cases} \theta^*, & \text{with probability min(r,1)} \\ \theta^{t-1}, & \text{otherwise} \end{cases} \tag{4.7}$$

The acceptance ratio and the scale factor are very important in this procedure. If the scale factor is too small, the acceptance rate will be too high and the Markov Chain chain is likely to get "stuck" around a local maximum. On the other hand, if the scale factor is too large, the acceptance rate will be very low and the chain will spend too much time in the tails of the posterior distribution.

Once the algorithm has draw the distribution, this will produce the mean and the standard deviation of each parameters and the DSGE model is estimated. For a more detailed and completed analysis I remaind to the book of Herbst and Schorfheide (2015)

The next section report the Bayesian estimation of the SW DSGE model on the Euro area. Section 4.2 will present the data set and its transformation, section 4.3 will describe the prior distribution, section 4.4 will comment the posterior distribution and finally, section 4.5 will analysis the results.

4.2 Data

The data needed for the estimation are mainly sourced from the ECB statistical warehouse, except for the interest rate and the weekly working hours time series. I used data over the period 1995Q1-2019Q4 for the Euro Area 19. These quarterly time series include real GDP, GDP deflator, private final consumption, gross fixed capital formation, population level, employment and compensation per hours. To identify the policy interest rate, I combined the short-term rate obtained from the Area Wide Model database, Fagan et al. (2005), with the *shadow interest rate* developed by Wu and Xia (2017) for the euro area. This strategy allows to account for unconventional monetary policy implemented when the economy is stuck at the zero lower bound, experienced in the Euro zone after the global financial crisis, capturing important information otherwise absent in the ECB deposit rate. Similarly, data for average weekly working hours are collected from the work of Botelho et al. (2021) in which they derived the series from the data available in the European Union Labour Force Survey (EU-LFS).

One of the characteristic of this model is that it features a deterministic growth rate driven by labor-augmenting technological progress, so that the data do not need to be detrended before estimation. As a result, to obtain the seven key macroeconomic variable needed for the estimation, I transformed the observed time series according to the following equations:

$$GDP_t = 100ln\left(\frac{RGDP_t}{POPLEV_t}\right), \tag{4.8}$$

$$INF_t = 100ln\left(\frac{RGDPDEF_t}{RGDPDEF_{t-1}}\right), \tag{4.9}$$

$$RATE_t = 100ln\left(\frac{INTRATE_t}{4}\right), \tag{4.10}$$

$$CONS_t = 100ln\left(\frac{PFC_t}{RGDPDEF_t * POPLEV_t}\right) \tag{4.11}$$

$$INV_t = 100ln\left(\frac{GFCF_t}{RGDPDEF_t * POPLEV_t}\right) \tag{4.12}$$

$$WAGE_t = 100ln\left(\frac{COMPHOUR_t}{RGDPDEF_t}\right) \tag{4.13}$$

$$HOURS_t = 100ln\left(HOURWORKED_t\frac{EMPL_t}{POPLEV_t}\right) \tag{4.14}$$

where $RGDP_t$ is the real gdp, $RGDPDEF_t$ is the real gdp deflator, PFC_t is the private final consumption, $GFCF_t$ is gross fixed capital formation, $INTRATE_t$ is the time series for the interest rate constructed as descried above, $COMPHOUR_t$ is the average compensation per hour, $HOURWORKED_t$ is average weekly hours worked, whereas $POPLEV_t$ and $EMPL_t$ stand for total level of population and the employment level respectively, and they are both transformed in indexes of the same base. Finally, the transformed observed time series enter in the model through the following measurements equations:

$$GDP_t = y_t - y_{t-1} + \overline{\gamma} \tag{4.15}$$

$$CONS_t = c_t - c_{t-1} + \overline{\gamma} \tag{4.16}$$

$$INV_t = i_t - i_{t-1} + \overline{\gamma} \tag{4.17}$$

$$WAGE_t = w_t - w_{t-1} + \overline{\gamma} \tag{4.18}$$

where GDP_t, $CONS_t$, INV_t and $WAGE_t$ are respectively the data series of the real output, the real consumption, the real investment and the wage inflation described above, on the other hand $\overline{\gamma} = 100(\gamma - 1)$ is the common quarterly trend growth rate to real GDP, consumption, investment and wages.

The inflation, interest rate and worked hours data are measured by:

$$INF_t = \pi_t + \overline{\pi} \tag{4.19}$$

$$RATE_t = r_t + \overline{r} \tag{4.20}$$

$$HOURS_t = l_t - l_{t-1} + \bar{l} \tag{4.21}$$

where $\bar{\pi} = 100(\Pi_* - 1)$ is the quarterly steady state inflation rate, $\bar{r} = 100(\beta^{-1}\gamma^{\sigma_c}\Pi_* - 1)$ is the steady state nominal interest rate, which will be determined by estimating the discount rate, and $\bar{l}$ is steady-state hours worked, normalized to be equal to zero.

4.3 Calibration and prior distributions of the parameters

All the prior distributions are specified as in Smets and Wouters (2007). First of all, some of the parameters were kept fixed in the estimation. These include the depreciation rate δ, which is set to be equal to 0.025, the steady state exogenous spending share g_y is fixed at 0.18, the steady-state mark-up in the labor market λ_w is set at 1.5 and the curveture parameters of the Kimball aggregators in the goods and labor market (ϵ_p and ϵ_w) are both set at 10. Other two parameters were kept fixed and not clearly identify: the persistence productivity and spending shock (ρ_a and ρ_g) both fixed at 0.98.
For what concerns the prior distributions, the parameters of the utility functions are set as follows: the mean of the inter-temporal elasticity of substitution, σ_c, is 1.5 and the standard error is 0.375 under Normal distribution; the mean of the elasticity of labor supply, σ_l, which also follows a Normal distribution, is 2 with a standard error of 0.75; the habit formation parameter, h, has a Beta distribution with mean of 0.7 and a standard error of 0.1. The adjustment cost parameter for investment, φ, is assume to be Normal around a mean of 4 with a standard error of 1.5. The share of fixed cost in production, ϕ_p, follows a Normal distribution with prior mean of 1.25 and a standard error of 0.125 and the Beta distribution of the capacity utilization elasticity parameter, ψ, is set with a mean of 0.5 and a standard deviation of 0.15. The parameters describing the price and the wage setting all follows a Beta distribution. The Calvo probabilities (ξ_w and ξ_p) are assumed to fluctuate around a mean of 0.50 and a standard deviation of 0.10, while the prior mean of the degree of indicization to past inflation is set to be 0.5 with a standard error of 0.15.

The parameter of the monetary policy rule, that are the long-run reaction on inflation (π_r), the long-run reaction on output gap (π_y) and the short-run reaction coefficient to the change in the output gap are described by a Normal distribution with mean around 1.5, 0.125 and 0.125 and standard errors of 0.125, 0.05 and 0.05, respectively. The coefficient on the lagged interest rate, ρ follows a Beta distribution with mean 0.75 and standard deviation 0.10.

Finally, the standard errors of the shocks σ_k, $\forall k \in \{a, b, g, i, r, p, w\}$, follow an inverse gamma

Table 4.1: Prior and posterior distribution of the structural parameters

	Prior			Posterior			
	Dist.	Mean	Std. dev.	Mode	Mean	HPD inf	HPD sup
φ	Normal	4.00	1.50	4.89	5.89	3.80	7.95
σ_c	Normal	1.50	0.37	1.23	1.38	1.05	1.69
λ	Beta	0.70	0.10	0.70	0.67	0.60	0.80
ξ_w	Beta	0.50	0.10	0.90	0.89	0.84	0.94
σ_l	Normal	2.00	0.75	2.09	2.10	1.07	3.12
ξ_p	Beta	0.50	0.10	0.95	0.93	0.90	0.95
ι_w	Beta	0.50	0.15	0.19	0.20	0.07	0.32
ι_p	Beta	0.50	0.15	0.31	0.34	0.14	0.53
ψ	Beta	0.50	0.15	0.73	0.71	0.55	0.88
ϕ_p	Normal	1.25	0.12	1.69	1.72	1.57	1.86
r_π	Normal	1.50	0.25	1.47	1.41	1.01	1.73
ρ	Beta	0.75	0.10	0.93	0.91	0.87	0.95
r_y	Normal	0.12	0.05	0.15	0.17	0.11	0.23
$r_{\Delta y}$	Normal	0.12	0.05	0.23	0.22	0.15	0.29
$\bar{\pi}$	Gamma	0.62	0.10	0.54	0.54	0.43	0.64
$100(\beta^{-1}-1)$	Gamma	0.25	0.10	0.31	0.27	0.09	0.44
$\bar{\gamma}$	Normal	0.40	0.10	0.24	0.20	0.10	0.28
ρ_{ga}	Normal	0.50	0.25	1.31	1.33	1.14	1.52
α	Normal	0.30	0.05	0.07	0.06	0.03	0.09

Notes: The posterior distribution are derived from the estimation of a Euro Area 19 sample data over the period 1995Q1 - 2019Q4. The results are obtained using the Metropolis-Hastings algorithm with 10 millions draws and two parallel chains, neglecting 2 millions draws and resulting in an acceptance ratio of 32%. The convergence diagnostic test is based on Brooks and Gelman (1998) and compares between and within moments of multiple chains. HPDi (Highest Posterior Density interval) is the shortest interval among all of the Bayesian credible intervals: any point within the interval has a higher density than any other point outside. All the estimation is implemented in Dynare.

distribution with a mean of 0.10 and 2 degrees of freedom. The persistence parameters of the AR(1) ρ_s, $\forall s \in \{b, i, r, p, w\}$, and MA ($\mu_p$ and μ_w) processes follow a beta distribution with mean 0.5 and a standard deviation of 0.2.

4.4 Posterior distribution of the parameters

Tables 4.1 and 4.2 give the mode, the mean and the highest posterior density interval of the parameters obtained by the Matropolis-Hasting algorithm with 10 millions draws and two parallel chains, and an acceptance ratio of 32 percent. The figure representing the multivariate Brooks and Gelman (1998) convergence test is reported in the appendix B.1.

The posterior mean of the steady-state inflation rate is about 2 percent and the mean of the discount rate is about 1 both on annual basis. The posterior mean of the trend growth rate is equal to 0.24, which is similar to the average quarterly growth rate of output per capita over the

Table 4.2: Prior and posterior distribution of the structural parameters

	Prior			Posterior			
	Dist.	Mean	Std. dev.	Mode	Mean	HPD inf	HPD sup
η^a	Invgamma	0.10	2.00	0.32	0.34	0.27	0.40
η^b	Invgamma	0.10	2.00	0.03	0.04	0.02	0.05
η^g	Invgamma	0.10	2.00	0.39	0.39	0.33	0.45
η^i	Invgamma	0.10	2.00	1.25	1.23	1.07	1.39
η^m	Invgamma	0.10	2.00	0.13	0.13	0.11	0.15
η^p	Invgamma	0.10	2.00	0.19	0.18	0.13	0.22
η^w	Invgamma	0.10	2.00	0.11	0.11	0.08	0.13
ρ_b	Beta	0.50	0.20	0.97	0.94	0.90	0.98
ρ_i	Beta	0.50	0.20	0.06	0.08	0.01	0.15
ρ_r	Beta	0.50	0.20	0.25	0.31	0.15	0.47
ρ_p	Beta	0.50	0.20	0.66	0.64	0.38	0.91
ρ_w	Beta	0.50	0.20	0.81	0.73	0.51	0.91
μ_p	Beta	0.50	0.20	0.86	0.80	0.53	0.96
μ_w	Beta	0.50	0.20	0.76	0.62	0.39	0.85

Notes: The posterior distribution are derived from the estimation of a Euro Area 19 sample data over the period 1995Q1 - 2019Q4. The results are obtained using the Metropolis-Hastings algorithm with 10 millions draws and two parallel chains, neglecting 2 millions draws and resulting in an acceptance ratio of 32%. The convergence diagnostic test is based on Brooks and Gelman (1998) and compares between and within moments of multiple chains. HPDi (Highest Posterior Density interval) is the shortest interval among all of the Bayesian credible intervals: any point within the interval has a higher density than any other point outside. All the estimation is implemented in Dynare.

sample data which is around 0.28. The implied mean steady-state real interest rate is to 4 percent on an annual basis. For what concerns the posterior distribution of the main parameter, many of them are worth to mention. The mean of the steady-state elasticity of the capital adjustment cost function, φ, is higher then its prior, suggesting a slower response of investment to changes in value of capital. The Calvo probabilities (ξ_w and ξ_p) turn out to be much higher than assumed a priori, suggesting an contract duration of more than 2 years. On the other hand, the mean of the degree of wage and price indexation is less than 0.50, where the degree of wage stickiness is lower ($\xi_w = 0.20$) than price stickiness ($\xi_p = 0.34$). Furthermore, the estimated mean of the capacity utilization cost ($\psi = 0.71$) and of the fixed cost share ($\phi_p = 1{,}72$) is much higher than assumed in assumed prior distribution, while the share of capital cost in production α much lower. The rest of the estimated behavioural parameters are very similar to their prior.

Turning to the monetary policy reaction function parameter, there is a very high degree of interest rate smoothing, with a mean equal to 0.91. The long run coefficient is a bit smaller than its prior ($r_\pi = 1.41$), and the policy respond strongly to both output gap level ($r_y = 0.17$) and to change in the output gap ($r_{\Delta y} = 0.22$) in the short run.

A worth discussion is needed for the estimated processes of the exogenous shock variable.

Especially, the persistence of the risk premium shock turns out to be very high. However, the absence of financial frictions in model induce the risk-premium shock to capture the output and the consumption drop of the global financial crisis. When there is the presence of this breaks in the data, the AR(1) process tend to move as a random walk. Also the persistence parameters of the price and wage mark-up shocks are very high. On the contrary, the persistence of monetary policy shock is relatively low, while for the investment-specific technology process is extremely low.

Overall, the parameters seem to be very informative. For a more detailed anyalsis I reported the figures of the prior and the posterior distributions in the Appendix B.2.

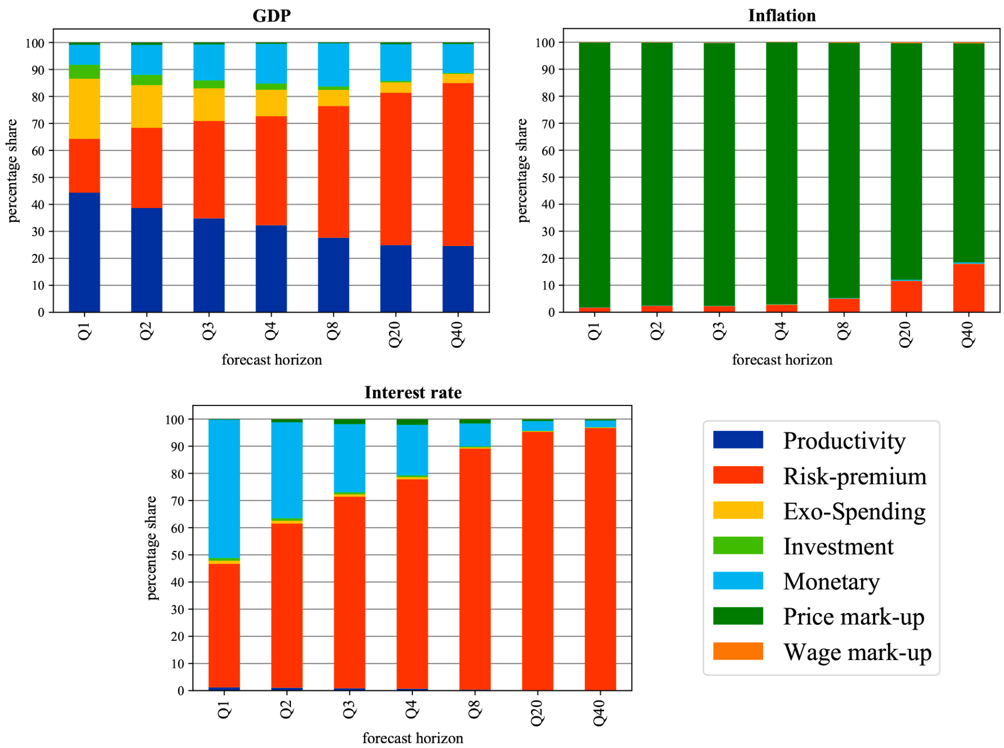
4.5 FEVD, IRFs and Historical Variance Decomposition

Now I use the estimated model to analyse what guides output, inflation and interest rate dynamics, how the economy reacted to the Global Financial Crisis and how monetary policy responded to it.

Figure 4.1 reports the forecast error variance decomposition based on the mode of the model's posterior distribution. The forecast horizon includes 6 period: the first four quarters and quarter 8 (short run), and five and ten years (the medium-long run). Movements in GDP are clearly driven by demand shocks, even in the long run, dominated by the strong persistence of the risk premium shock, as anticipated in section 4.3. In the first quarter the exogenous spending shock accounts for the 50 percent of the total demand shock. It vanishes in the long run when the risk-premium shock dominates the others. Indeed, the latter accounts for the 60 percent of the total movement of the GDP after 10 years. Large part of forecast error variance is also explained by the productivity shock, which represents the 45 percents of the movement in the first quarter and it decreases over time. However, it is still persistent after 10 year explaining 25 percent of the movements. On the other hand, price and wage mark-up shocks do not have an impact over the forecast horizon. The impact of monetary policy shocks on output is in line with the literature (see Christiano et al. (1999), which in long run do not strongly determines the output movements (around 10 percent).

The second and the third graphs of figure 4.1 show show that, the forecast error variance decomposition of the interest rate, after one quarter, is explained by 50 percent by the monetary policy shock and 50 percent by the risk premium shock. The latter, in the long run, explains almost the total movement of the interest rate dynamics. On the other hand, forecast error variance for inflation is totally driven by price mark-up shocks: prices explain idiosyncratically movements

Figure 4.1: Forecast Error Variance Decomposition



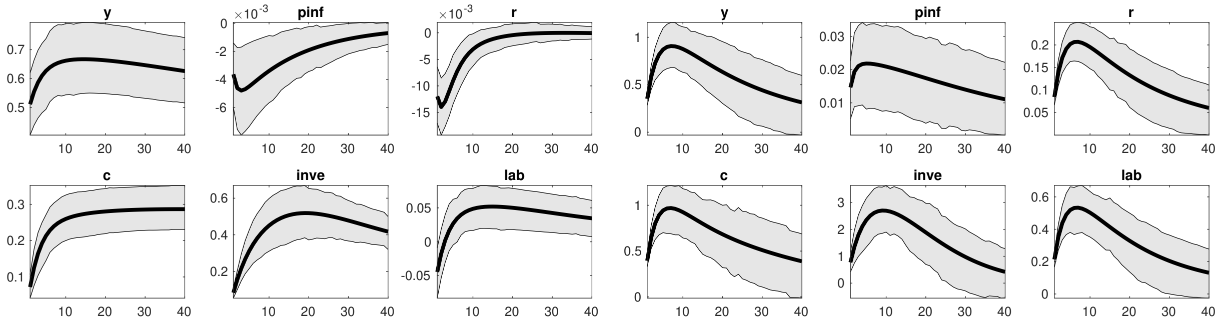
Note: The FEVD is computed at the mode of the posterior distribution.

in inflation. Notably, wage mark-up shocks does not influence the inflation's dynamics. The strong impact of risk premium on the real economics variables are also evident in the figure 4.2, where I reported the estimated impulse response functions. The risk-premium shock explains much of the variation in the short run of output, consumption and investments compare to the other shocks, even though a big part of investment movements in the short-run is explained by the investments shock itself. As also expected, a tightening monetary policy action by the central bank would reduce output, consumption and investments in the short run, but very little effects in the long-run. Furthermore, it negatively impacts the inflation and positively the interest rate.

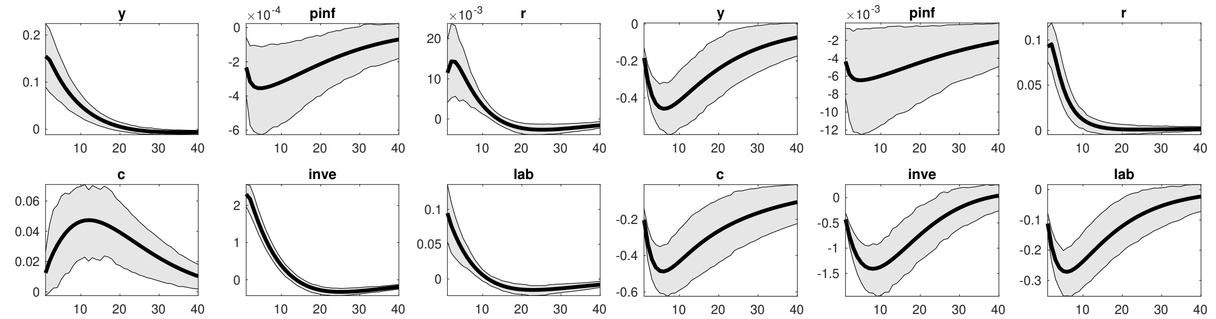
As in Gali (1999), Francis and Ramey (2005) and Smets and Wouters (2007) the positive technology shock increases the output, consumption, and investments, but immediately reduces hours worked, which returns positive after one here.

The estimated model allowed also to analyse how the economy responded to the Global Financial Crisis. Figure 4.3 shows the historical variance decomposition of real GDP per capita growth and real consumption per capita growth. In both series, the drop caused by the Global Financial Crisis is driven by the risk-premium shock. Given the absence of financial frictions

Figure 4.2: The Estimated Bayesian Impulse Response Functions



(a) Estimated IRFs to productivity shock (b) Estimated IRFs to risk-premium shock



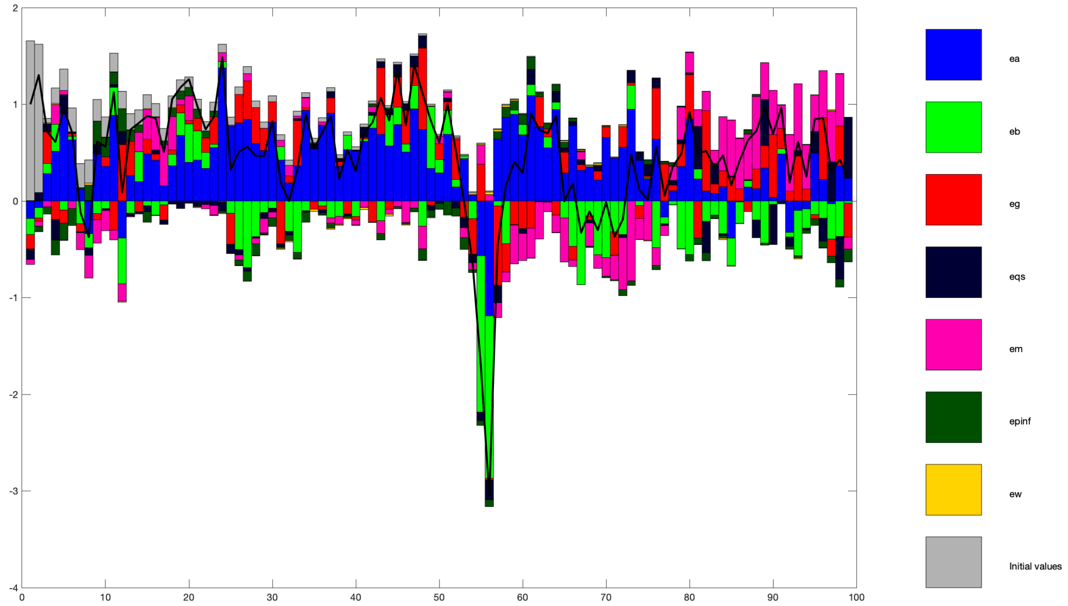
(c) Estimated IRFs to investment shock (d) Estimated IRFs to monetary policy shock

Note: *y* represents the output, *pinf* the inflation rate, *r* the nominal interest rate, *c* the consumption, *inve* the investments, and *lab* the hours worked.

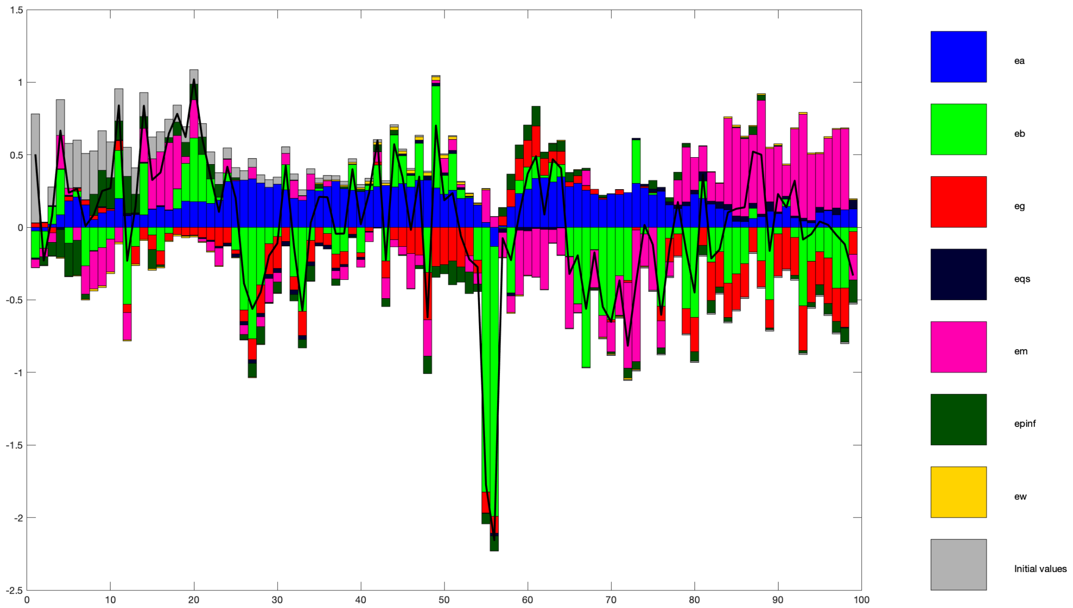
in the model, the risk-premium shock captures the movements of a financial shock during the recession. High risk premium rates lowered the demand, causing the consumption, investment and output drop. The central bank tried to answer to the shock of risk premium lowering the interest rates, aiming to the negative shock in demand cushioning the crisis. This is evident in the figure 4.4b where the interest rate is driven down by the risk-premium shock. Finally, the figure 4.4a shows the historical variance decomposition of inflation. The common shifts in inflation are mainly driven by price mark-up shocks. Moreover, the graph shows that the negative demand shocks contributed to low inflation, particularly after the GFC.

These results suggests the necessity to add financial frictions to the model. As pointed out by Christiano et al. (2018), DSGE model prior the Global Financial Crisis were not equipped by financial frictions because economic crisis did not seem closely tied to disturbances in financial markets. Moreover, the financial frictions that were included in dynamic stochastic general equilibrium models did not seem to have very big effects. However, after the GFC many researcher integrated financial frictions in DSGE models (e.g. Gertler and Kiyotaki (2015) and Christiano et al. (2014). However, the results of the estimation represents quite well the dynamic of the euro area economy, which allow me to analyse the effect of the introduction of an average inflation targeting rule in the next chapter.

Figure 4.3: Historical Variance Decomposition



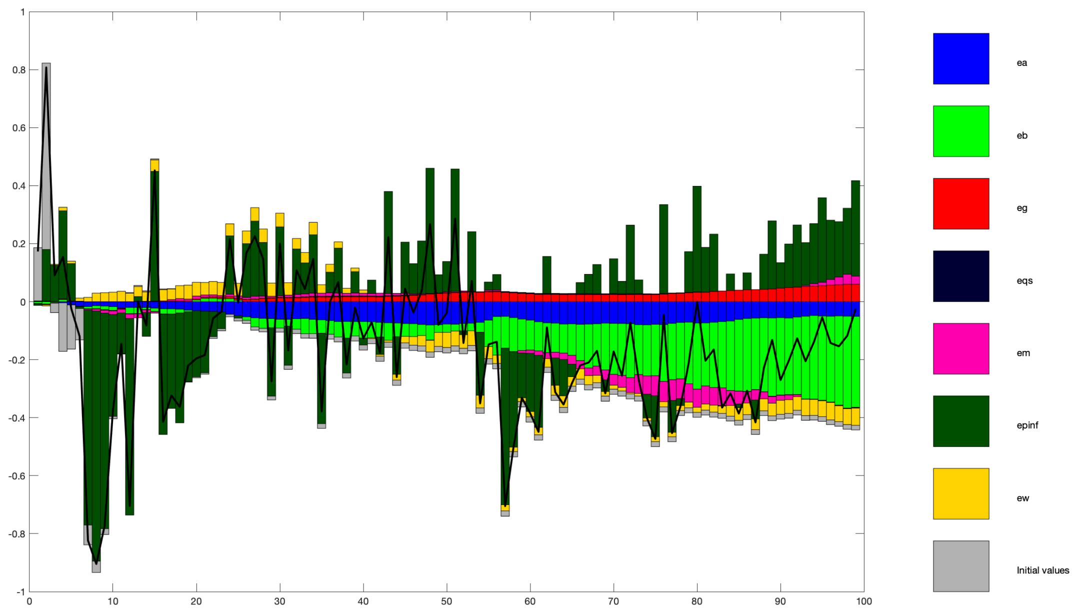
(a) Historical variance decomposition of real GDP growth



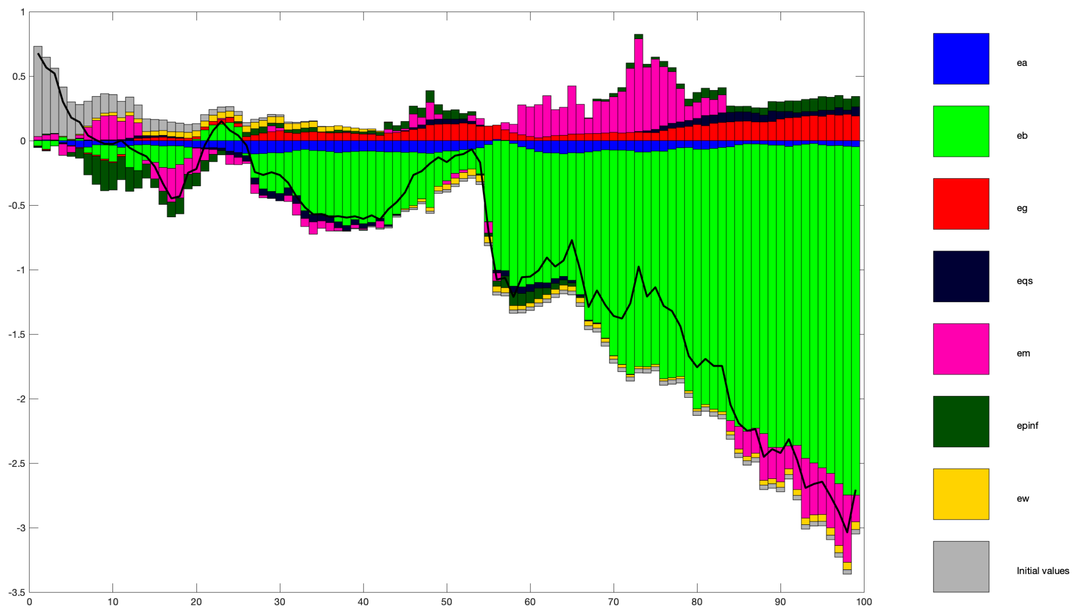
(b) Historical variance decomposition of real consumption growth

Note: Historical data over the period 1995Q1 - 2019Q4. The big drop describe the GFC. Each color represents a different shock where, *ea* (blue) is the productivity shock, *eb* (light green) is the risk-premium shock, *eg* (red) the exogenous government spending shock, *eqs* (balck) is investment-specific technology shock, *em* (pink) is the monetary policy shock, *epinf* (dark green) is the price mark-up shock, and *ew* (yellow) is the wage mark-up shock. Trend per-capita growth is estimated at 0.96 percent.

Figure 4.4: Historical Variance Decomposition



(a) Historical variance decomposition of inflation rate



(b) Historical variance decomposition of the shadow interest rate

Note: Historical data over the period 1995Q1 - 2019Q4. The big drop describe the GFC. Each color represents a different shock where, *ea* (blue) is the productivity shock, *eb* (light green) is the risk-premium shock, *eg* (red) the exogenous government spending shock, *eqs* (balck) is investment-specific technology shock, *em* (pink) is the monetary policy shock, *epinf* (dark green) is the price mark-up shock, and *ew* (yellow) is the wage mark-up shock. Mean inflation is estimated at 2.16

Chapter 5

Evaluating monetary policy rules

Having estimated the model parameters, I can now run a series of exercises to evaluate several monetary policy rules and assess the effect of the average inflation targeting on the euro area. In this section, I first present each rule, I discuss the result of the stochastic simulation and finally, I compare a welfare function under different policy regimes to evaluate which strategy works best.

5.1 Moneteary policy rules

5.1.1 Flexible Inflation Targeting rules

For standard inflation targeting rules, I refers to flexible inflation targeting rules or Taylor-type reaction functions, see Svensson (1999). The term ”flexible” indicates that the central bank is also concerned to stabilize the real economy, for instance, the output variability. This is different to a ”strictly” inflation targeting rule, which aims to only stabilize the prices movements. Historically, this strategy has been interpreted as an implicit adherence to a “bygones be bygones” approach. Policymakers react to their best estimate of current economic conditions and the medium-term outlook. They do not take into account the history of inflation. Even if there has been a long period of inflation undershooting there is no attempt to later overshoot the target for some time. This type of policies have been structured *à la* Taylor (1993). The first flexible IT rule I am going to consider is the Taylor (1999)’s inertial feedback rule:

$$r_t = \rho r_{t-1} + (1 - \rho)[r_\pi \pi_t + r_y(y_t - y_t^p)] + \varepsilon_t^r \tag{5.1}$$

”Inertial” because the coefficient ρ denotes the degree of policy rate smoothing and such inertia serve to introduce some history dependence in the policy. In this equation, the interest rate gradually responds to the deviation of inflation from its target, which is normalized to zero, and the output gap, that is the difference between actual output (under sticky prices) and potential output, when the prices are flexible (see Chapter 3). Note that, in the original Taylor rule, the natural interest rate is constant. However, log-linearization of the model around the steady-state eliminates this natural interest rate from the rule.

The second flexible inflation targeting rule is the policy rule introduced in the Smets and Wouters (2007) model:

$$r_t = \rho r_{t-1} + (1-\rho)[r_\pi \pi_t + r_y(y_t - y_t^p)] + r_{\Delta y}(\Delta y_t - \Delta y_t^p) + \varepsilon_t^r \tag{5.2}$$

This rule gradually responds to deviation in inflation from its inflation objective, which is normalized to zero, to the output gap, defined as the difference between actual output (under sticky prices) and potential output, when the prices are flexible, and, in addition to the previous rule, to the deviations of the output gap from the previous periods. Also, this equation present the inertial parameter and the natural interest rate has been eliminated due to the log-linearization around the steady-state.
Although they are both Taylor-type rules, hereafter, I will call the first monetary policy Taylor rule and the second one SW rule.

5.1.2 Average Inflation Targeting rules

Average inflation targeting rules enter in that class of rules called "Make-up Strategies" or "forecast targeting rules", see Svensson (2020b). Under these strategies, the policymakers try to stabilize average inflation around its target over some horizon. Differently, from price-level targeting rules, AIT seeks to undo past deviations of inflation from its medium-long-run target goal over a window of fixed length. As discussed previously, the optimal horizon window is still unclear among researchers. In this exercise I propose some interpretations of the AIT rule based on the works of Arias et al. (2020), Svensson (2020a), Coenen et al. (2021). Average inflation is calculated over lengths of 4 or 8 years of the average past inflation and 4 or 8 years of the future expected inflation. Thus, π_t^{AIT} is equal to

$$\pi_t^{AIT} = \frac{\pi_t + \pi_{t-1} + ... + \pi_{t-n}}{n}, n = \{16, 32\} \tag{5.3}$$

or

$$\pi_t^{AIT} = \frac{\pi_t + \pi_{t+1} + ... + \pi_{t+n}}{n}, n = \{16, 32\} \tag{5.4}$$

where n = 16 indicates 16 quarters, or 4 year, therefore n = 32 represents 8 years.
The average inflation then enters in the Taylor-type rules presented above in the form of:
inertial Taylor (1999) rule:

$$r_t = \rho r_{t-1} + (1-\rho)[r_\pi \pi_t^{AIT} + r_y(y_t - y_t^p)] + \varepsilon_t^r \tag{5.5}$$

and Smets and Wouters (2007) rule:

$$r_t = \rho r_{t-1} + (1-\rho)[r_\pi \pi_t^{AIT} + r_y(y_t - y_t^p)] + r_{\Delta y}(\Delta y_t - \Delta y_t^p) + \varepsilon_t^r \tag{5.6}$$

As it can be noticed, the AIT and IT are structured in the same way. If $\pi_t^{AIT} = \pi_t$, then the central bank will react as under a flexible inflation target regime. In the end, considering all the time windows and the two base forms, there are 4 AIT rules which are built on the line of the SW rule, and another 4 AIT rules which are based on the structure of the Taylor rule. The only difference is in how inflation is identified: for AIT rules, inflation enters as an average variable over a defined time horizon 4 or 8 years of average future inflation, and 4 or 8 years of past realized inflation. Summing up, I will analyze 10 monetary policy rules: 2 flexible IT rules, and 8 AIT rules.

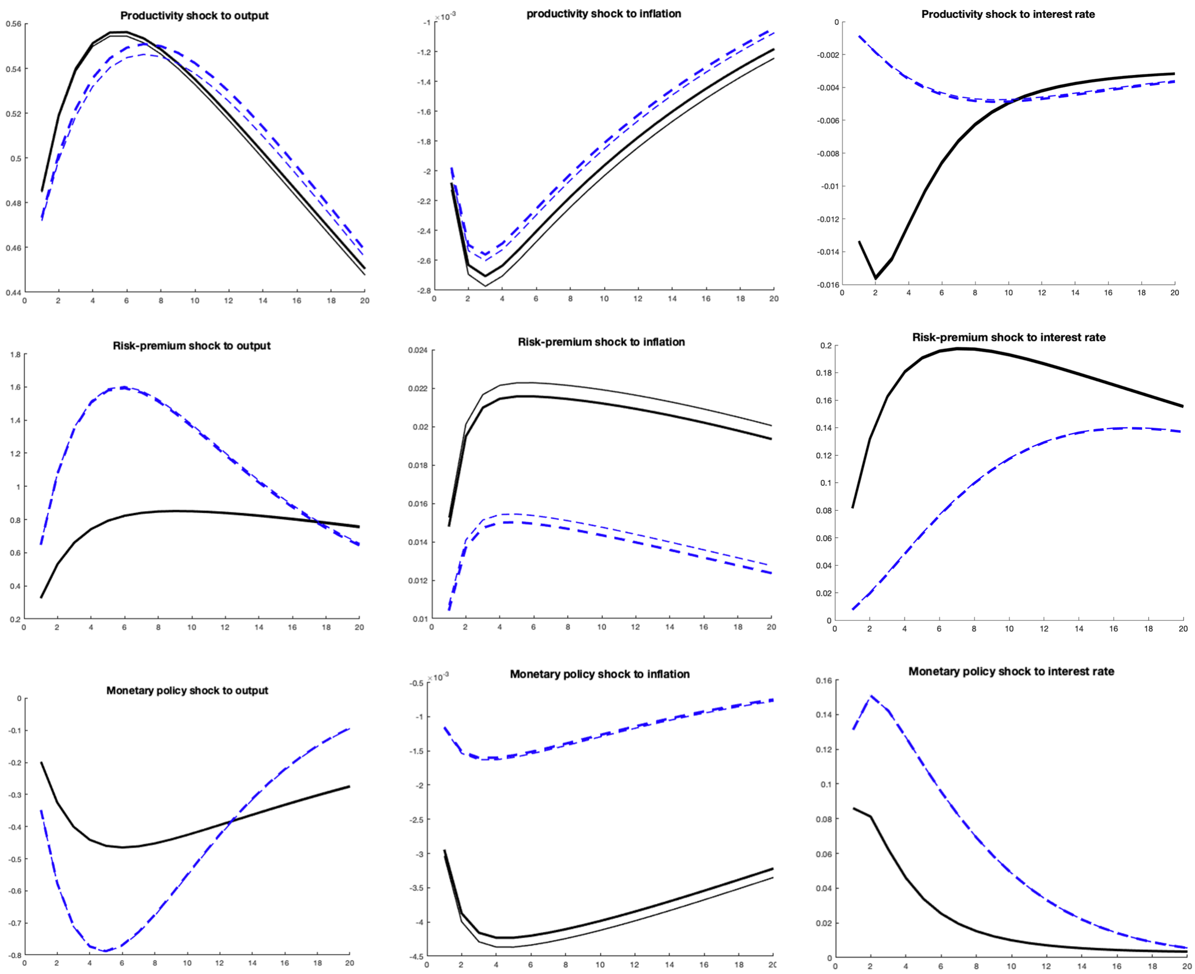
5.1.3 Models results

To study the effects of all the rules presented above, I run 10 stochastic simulations of the same Smets and Wouters (2007) model presented in chapter 2, but, each time, accounting for the ten 10 different monetary policy rules. The parameters of the models were calibrated with the estimated mode of the posterior distributions. In this section, I report the impulse response functions and the variances of variables that I will use to compute the welfare loss function later.

Figure 5.1 displays the IRFs of the shock of (from the top to the bottom) productivity, risk premium and monetary policy to three key macroeconomic variables (from left to the right): output, inflation and interest rate. To make the graphs easier to read, I describe the behaviour of only four monetary policy rules: two for IT and two for AIT. These are the SW and the Taylor rules and their respective AIT counterpart which I consider as the average time horizon of 4 years of expected future inflation.

Overall, there is a significant difference between the SW rules and the Taylor rules, but not

Figure 5.1: IRFs of simulated models under different monetary policy rules

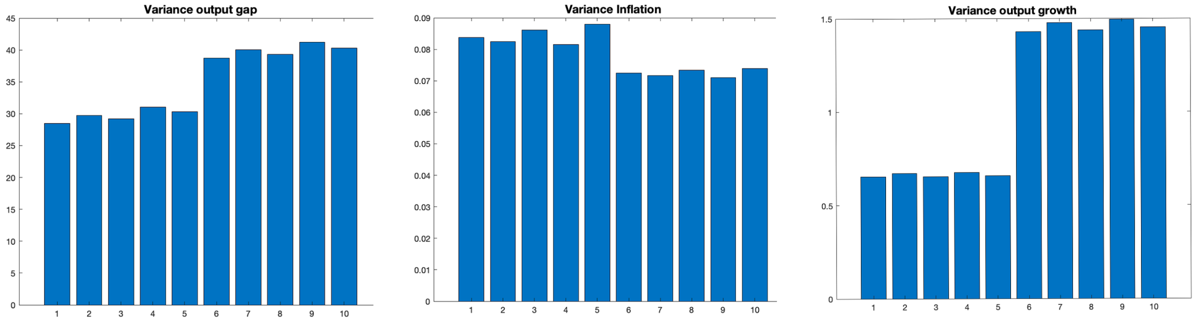


Note: Bold solid line: SW rule; thin solid line: AIT SW rule - average of 4 years future expected inflation, bold dashed line: Taylor rule; thin dashed line: AIT Taylor rule - average of 4 years future expected inflation.

much difference between the IT rules and their AIT counterparts. In detail, a productivity shock leads to an increase in output and a drop in the inflation rate. The interest rate also decreases, however, in the short term, the impact is stronger under the SW rule, while it has almost no effect under the Taylor rule. A risk-premium shock results in an increase in output, which is stronger under the Taylor rule, and in inflation. This shock increases the interest rate by 0.1 percent under both rules, even though the curve in the short term is steeper under the SW rule. The monetary policy shock leads to a decrease in output, which in the short term is stronger under the Taylor rule, however, it reaches the steady-state equilibrium faster than the SW rule. The monetary policy shock lowers also inflation and rises the interest rate under both policy rules.

Figure 5.2 reports the results of the variances of the output gap, inflation, and output growth computed through the stochastic simulation over the ten models. For each figure, the first 5

Figure 5.2: Variance results of the simulated models



Note: the number 1 to 10 on the x-axes represent each of the ten models analyzed. 1 is the SW rule, 6 is the Taylor rule. From 2 to 5 are, in order, the AIT-SW rule that consider as average inflation period 4 years past realized inflation (2), 4 years expected future inflation (3), 8 years past realized inflation (4), 8 years expected future inflation (5). In parallel , from columns 7 to 10, the AIT-Taylor rule that consider as average inflation period 4 years past realized inflation (7), 4 years expected future inflation (8), 8 years past realized inflation (9), 8 years expected future inflation (10).

columns belong to the family of the SW rule, while the last 5 columns to the Taylor rule. Overall, SW rules better stabilize the output gap e and output growth, instead, the Taylor rules are more efficient to stabilize inflation. The output variances of the AIT rules are higher than the IT rules (which are described by columns 1 and 6). Notably, on the other hand, the AIT rules that consider an average time window of 4 or 8 years of the realized past inflation (columns 2, 4, 7, and 9) show a lower variance, meaning that the make-up strategies better stabilize the inflation rate.

5.2 Central Bank Losses

The preferences of the central bank are represented by a loss function that seeks to minimize. This function also represent the objective of the society, therefore an indication of the welfare. It is traditional to assume that the loss function it based on the historical variances of the variables of interest of the central bank, see Galí (2015). The welfare loss function equation is defined as

$$L_t = var(\pi_t) + \lambda_a var(x_t) \tag{5.7}$$

where x_t takes the form of output gap or output growth and λ_a is the weight that the central bank gives to the output. Literature has largely stated that, see Woodford (2003), the central bank should assign only a small weight to the measures of economic activity in its objective function. Also, Blanchard and Galí (2007) established that stabilizing inflation allows the central bank to simultaneously stabilize welfare-relevant measures of economic activity, therefore λ_a should be

Table 5.1: Central Bank Loss Functions
x_t = outputgap

Monetary rule	Time horizon	$\lambda_a = 0.048$	$\lambda_a = 0.25$	$\lambda_a = 1.042$
SW (2007)	flexible inflation targeting	1	1	1
	Average of 4 years future expected π	1.026	1.026	1.026
	Average of 4 years past realized π	1.039	1.042	1.043
	Average of 8 years future expected π	1.064	1.065	1.065
	Average of 8 years past realized π	1.082	1.089	1.089
Taylor (1993)	flexible inflation targeting	1.329	1.355	1.359
	Average of 4 years future expected π	1.347	1.374	1.379
	Average of 4 years past realized π	1.369	1.399	1.404
	Average of 8 years future expected π	1.379	1.408	1.413
	Average of 8 years past realized π	1.408	1.439	1.446

Notes: Welfare loss functions obtained by the sum of the variance of the inflation and output gap computed with stochastic simulations of the Smets and Wouters model using 10 different monetary policy rules and calibrated at the mode of the posterior distribution estimated in chapter 4.

close to zero. Recently, Debortoli et al. (2019) showed that assigning a high weight on standard measures of economic activity could be strongly beneficial. For example, an output measure could stand for the welfare-relevant variables which are not included in the simple mandate. As a result, to test the results of the different simulations as the relative importance assigned to real volatility in the loss function varies, I decided to consider three values of λ_a. As a proxy of strict inflation target, $\lambda_a = 0.048$ as in Woodford (2003), as a proxy of dual mandate, $\lambda_a = 0.25$. Yellen (2012) defined the dual mandate as a loss function that assigns equal weights for inflation and the unemployment gap. The weight of the unemployment gap equals one converts into a coefficient equals 0.25 of the output gap. Finally, $\lambda_a = 1.042$, the optimal parameter found in Debortoli et al. (2019). With the results of the variances obtained above, I can compute the loss function for each of the ten models. The results reported in table 5.1 are calculated from the variance of inflation and output gap. This table shows the results of the central bank objective function of each model relative to the outcome of the SW rule which is fixed at 1. First of all, it can be noticed that the results are consistent over the λ values. Moreover, the SW rule dominates all the others, which rank. Particularly, it reaches a stronger result than the standard Taylor rule. The latter is 30 percent higher than the benchmark case. Differently from the Taylor rule, is that the central bank, when acting according to a SW rule, set the interest reacting to the deviations of the output gap, but also to the deviations of the output gap from the previous periods. This leads to a smaller value of the variance of the output gap, resulting in a lower loss function. Overall, AIT does not seem to behave as expected. It performs worse than the IT rules. Moreover, as more as the time horizon increases, the value of the loss function rises. Additionally, since it

Table 5.2: Central Bank Loss Functions
x_t = output growth

Monetary rule	Time horizon	$\lambda_a = 0.048$	$\lambda_a = 0.25$	$\lambda_a = 1.042$
SW (2007)	flexible inflation targeting	1	1	1
	Average of 4 years future expected π	1.022	1.012	1.005
	Average of 4 years past realized π	1.038	1.023	1.012
	Average of 8 years future expected π	0.999	1.013	1.022
	Average of 8 years past realized π	0.995	1.014	1.027
Taylor (1993)	flexible inflation targeting	1.201	1.689	2.019
	Average of 4 years future expected π	1.207	1.698	2.030
	Average of 4 years past realized π	1.216	1.714	2.050
	Average of 8 years future expected π	1.207	1.727	2.078
	Average of 8 years past realized π	1.208	1.742	2.102

Notes: Welfare loss functions obtained by by the sum of the variance of the inflation and output gap computed by the stochastic simulation of the SW model calibrated at the mode of the posterior distribution estimated in chapter 4.

is difficult for the central bank to observe the dynamics of the potential output, I also compute the value of a welfare loss function which include in its preferences the output growth, which is easier to observe. Table 5.2 reports the results of the loss functions considering the variance of the output growth. When λ_a is equal to 0.25 and 1.042 the SW rule dominates the other as in the case of the output gap. Instead, in a ”strict” inflation target regime, the AIT Smets and Wouters rules, which count for an average inflation target over a window of eight years best perform on the others. However, the outcome is very similar: 0.1 and 0.5 percent better than its flexible inflation target counterpart. On the contrary, AIT Taylor rules evidence a greater loss function compared to the standard IT-Taylor rule. Overall, the results are consistent with the previous table: this exercise evidence that the SW rule better fulling the central bank objective function, leading to the best result in terms of welfare improvement. To understand how the welfare functions respond to the economic shocks, I also investigate how the rules behave when only one shock per time is active in the model. The results are reported in table 5.3. Column three, ”Total loss”, represent the results of the loss function where the variance of inflation is summed up with the variance of the output gap, and the latter is multiplied by $\lambda_a = 1.042$. This benchmark case coincides with column 5 of table 5.1, but they are presented in levels instead of relative shares. Columns 4 - 6 show the contribution to the loss function of every single shock. I decided to report only the shocks with the highest share. Risk-premium determines most of the variance of the loss function, followed by the monetary policy shock and by productivity shock.

Table 5.3: Central Bank Loss Functions
shock analysis

			Single shock active per time		
Monetary rule	Time horizon	Total loss	Productivity	Risk-premium	Monetary
SW (2007)	IT	29.12	0.06	23.81	5.01
	AIT 4y fut. exp. π	30.20	0.08	24.76	5.16
	AIT 4y past π	30.60	0.08	25.03	5.21
	AIT 8y fut. exp. π	31.56	0.11	25.92	5.32
	AIT 8y past π	32.04	0.10	26.27	5.38
Taylor (1993)	IT	35.89	0.08	29.28	6.26
	AIT 4y fut. exp. π	36.71	0.09	30.04	6.36
	AIT 4y past π	36.83	0.10	30.05	6.35
	AIT 8y fut. exp. π	37.54	0.12	30.75	6.44
	AIT 8y past π	37.70	0.11	30.82	6.44

Notes: Welfare loss functions obtained by the stochastic simulation of the SW model calibrated at the mode of the posterior distribution estimated in chapter 4. The simulations were run for only one shock active per time. This means that the standard deviation of 6 shocks out of 7 were set to zero.

Moreover, this table confirms the dominance of the SW rule on the other policy: it figures in a lower loss over all the single shocks.

To check the validity of these results, I try to understand if the dominance of the SW rule is guided by that that I calibrated the models with the mode of the posterior distribution estimated using the SW rule itself. Therefore, I re-estimated the model but considering the Taylor rule as the monetary policy function. The results of the estimation are in Appendix C. Overall, the

Table 5.4: Central Bank Loss Functions
estimated model with Taylor rule

Monetary rule	Time horizon	$\lambda_a = 0.048$	$\lambda_a = 0.25$	$\lambda_a = 1.042$
SW (2007)	flexible inflation targeting	1	1	1
	Average of 4 years future expected π	1.000	0.999	0.998
	Average of 4 years past realized π	0.983	0.999	1.008
	Average of 8 years future expected π	1.006	1.004	1.003
	Average of 8 years past realized π	0.978	0.999	1.009
Taylor (1993)	flexible inflation targeting	1.024	1.080	1.109
	Average of 4 years future expected π	1.017	1.075	1.106
	Average of 4 years past realized π	1.008	1.080	1.117
	Average of 8 years future expected π	1.021	1.079	1.110
	Average of 8 years past realized π	1.003	1.079	1.118

Notes: Welfare loss functions obtained by the stochastic simulation of the SW model calibrated at the mean of the posterior distribution reported in Appendix C.3.

mean of the posterior distribution is similar to the benchmark model: the risk-premium shock is persistent, there is a high degree of price indicization, and the policy responds strongly to the deviation in the output gap.

Then, I run the same exercise to compute the loss functions for the 10 models. Table 5.4 reports the results calculated over a loss function that sums the variance of inflation and output growth. Overall, the loss functions are very close to each other, and even the SW rule does not dominate as in the previous table, still performs as the best rule. Taking into account the average period of past and future expected value of inflation under the Taylor rule leads to a similar outcome of the SW inflation target rule. The results of AIT are not so stronger to justify a change in the monetary policy regime. However, as in table 5.2, when λ_a is equal to 0.048 or when the central bank does not strongly weigh the output in its loss function, make-up strategies improve the welfare and help the central bank to reach its objectives.

5.3 The importance of the design of the rule

In the previous section, I assumed that the central bank gives the same importance to inflation independently of the policy chosen. The estimated Taylor-principle parameter was around 1.5, for every monetary policy rule, I have considered so far. However, Arias et al. (2020) and Coenen et al. (2021) showed that when the central bank decides to adopt an AIT rule, it put more weight on the inflation parameter. In their works, the AIT rule takes this form:

$$R_t = 0.85R_{t-1} + 0.15\left(r^* + \bar{\pi}_t^{(4)} + y_t^{gap} + T\left(\bar{\pi}_t^{(4T)} - \pi^*\right)\right) \tag{5.8}$$

where T is the length of the make-up window, which they set at 4 or 8 years and the numbers in brackets indicate the respective number of quarters. Since T appears also as the coefficient of the average inflation to its long-run target, this means that ”as the length of the window increases, the rule puts more weight on deviations of average inflation from the long-run objective” Arias et al. (2020).

Therefore, I decided to translate these features into my model and run the same exercise as before to evaluate the resulted loss functions. I compare three rules structured *à la* SW rule.

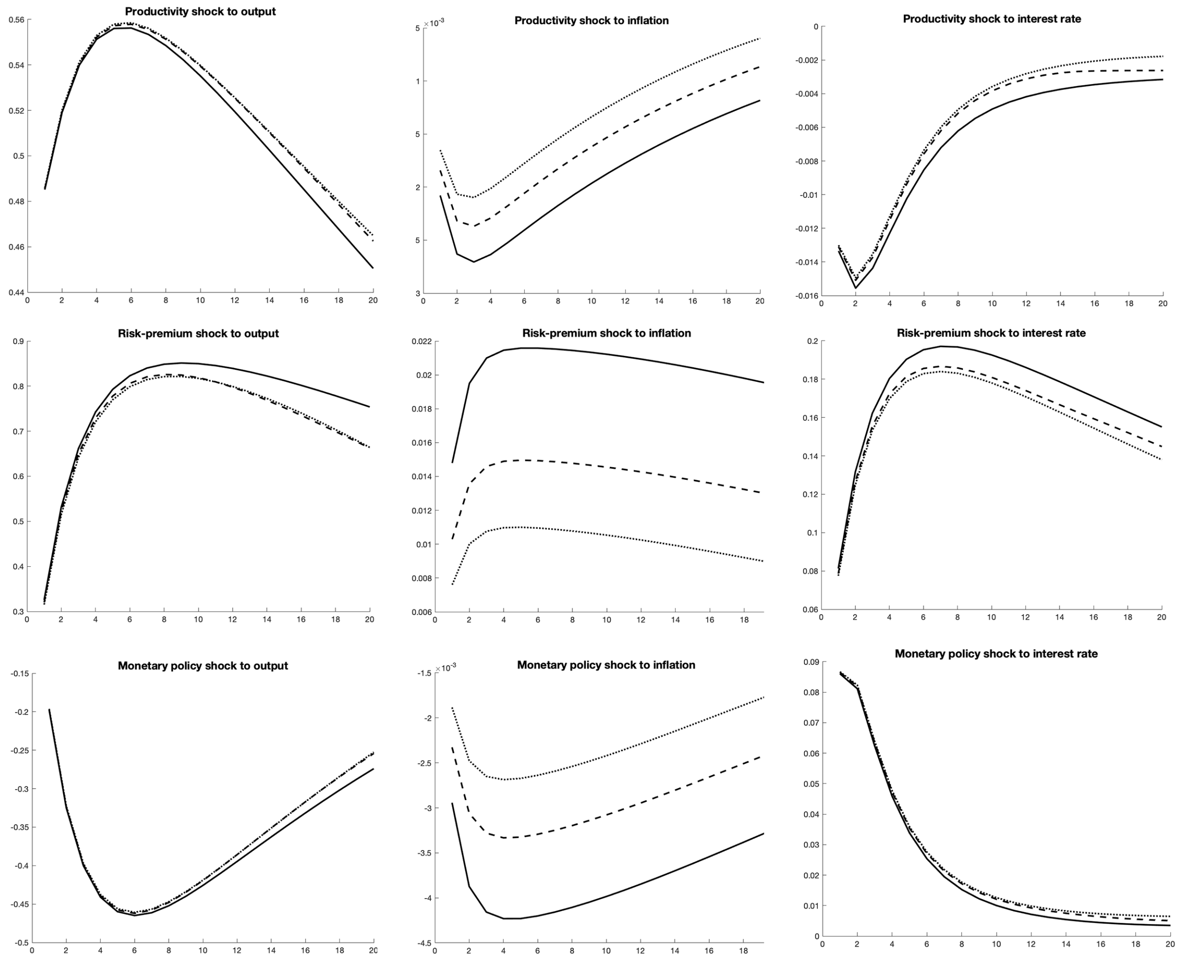
$$r_t = \rho r_{t-1} + (1-\rho)[1.47\pi_t + r_y(y_t - y_t^p)] + r_{\Delta y}(\Delta y_t - \Delta y_t^p) + \varepsilon_t^r \tag{5.9}$$

$$r_t = \rho r_{t-1} + (1-\rho)[4\bar{\pi}_t^{16} + r_y(y_t - y_t^p)] + r_{\Delta y}(\Delta y_t - \Delta y_t^p) + \varepsilon_t^r \tag{5.10}$$

$$r_t = \rho r_{t-1} + (1-\rho)[8\bar{\pi}_t^{32} + r_y(y_t - y_t^p)] + r_{\Delta y}(\Delta y_t - \Delta y_t^p) + \varepsilon_t^r \tag{5.11}$$

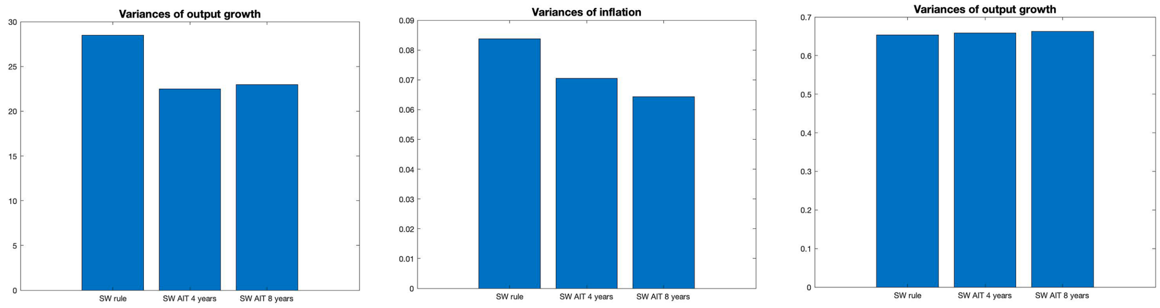
Equation 5.9 is the benchmark case, a flexible inflation target rule, in which I set the inflation parameter equal to 1.47 as a result of the estimation presented in chapter 4. Equations 5.10 and 5.11 stand for an AIT rule calculated on the average of 4 and 8 years of realized past inflation. Under a 4 (8) year make-up window, the central bank responds to the deviation of inflation with a coefficient equal to 4 (8). The inflation objective is normalized to zero and the monetary authority answer also to deviation in the output gap, and output gap from the previous period. Figure 5.3 presents the impulse response functions obtained by the stochastic simulations of the SW model with the above policy rules. The IRFs show the shock (from the top to the bottom) of productivity, risk premium and monetary policy to three key macroeconomic variables (from left to the right): output, inflation and interest rate. The solid line is the benchmark SW rule, the dashed line the 4 years AIT SW rule and the dotted line is the 8 years AIT SW rule. Overall, the

Figure 5.3: IRFs of simulated models under different monetary policy rules



Note: The solid line is the benchmark SW rule, the dashed line the 4 years AIT SW rule and the dotted line the 8 years AIT SW rule.

Figure 5.4: IRFs of simulated models under different monetary policy rules



Note: The solid line is the benchmark SW rule, the dashed line the 4 years AIT SW rule and the dotted line the 8 years AIT SW rule.

effect of shocks on output and interest are similar among the three rules. A productivity shock rises output and decreases interest rate in the short. A risk premium shock leads to an increase in output and interest rate, which is higher under the IT rule. Generally, AIT rules cushion the effects of a risk premium shock over all the variables. The impact of a monetary policy shock of the AIT rules on output and interest rate is much the same as the IT rule. On the other hand, inflation responds differently to the shocks among the three rules, where the AIT rules better stabilize inflation. Overall the AIT rules better respond to the economic shocks.

With the variances reported in Figure 5.4, I computed the loss functions using equation 5.7 and the results are displayed in table 5.5. Assuming a strong reaction to inflation deviations by the central bank under AIT rules results in lower loss functions. The outcomes are consistent among the different values of λ_a and if either output gap (A) or output growth (B) are in the loss equation. When the loss functions are computing summing the output gap, the AIT rules improve the welfare by 20 percent compared to a flexible IT rule. These results are in line with the works of Arias et al. (2020) and Coenen et al. (2021).

Table 5.5: Central Bank Loss Functions - aggressive answer to inlation

Monetary rule	Time horizon	$\lambda_a = 0.048$	$\lambda_a = 0.25$	$\lambda_a = 1.042$
(A) SW (2007)	Flexible inflation targeting	1	1	1
	Average of 4 years past realized π	0.79	0.79	0.79
	Average of 8 years past realized π	0.80	0.80	0.80
(B) SW (2007)	Flexible inflation targeting	1	1	1
	Average of 4 years past realized π	0.89	0.95	0.98
	Average of 8 years past realized π	0.85	0.93	0.98

Notes: (A) results of the sum of the variance of inflation and output gap; (B) results of the sum of the variance of inflation and output growth. Welfare loss functions obtained by the stochastic simulation of the SW model calibrated at the mode of the posterior distribution estimated in chapter 4.

Conclusions

The economic environment of the last decade has led the major central banks to review their monetary policy strategies bringing into attention the make-up strategies rule. In this thesis I try to assess the effects of such policies in an estimated model for the euro area. First of all, the model works well and fit the data. However, since I estimated the model over the period of GFC, the introduction of financial friction could help to better identify the behaviour of some parameters, see Christiano et al. (2018). Moreover, introducing an AIT rule in the estimated model, there are no significant improvements on the basis of a welfare loss functions. However, I showed that the benefits of AIT strategies came up only when there is a strong and aggressive response to inflation deviation by the central bank, as assumed in Arias et al. (2020). Under this conditions, AIT rule better cushions the shocks to the inflation rate and significantly reduce the variability of output gap and inflation, resulting in a lower welfare loss function, comparing it with a standard flexible inflation targeting rule.

Many questions are not addressed in this thesis. Practical challenges regarding the implementations of the AIT strategy are not treated in this model analysis. For example, how people understand the new policy regime, or how the central bank is coherent and transparent in their communication. This is crucial for the analysis of expectation formations and how they interact with the new policy. Introducing information frictions in a DSGE model would help to understand if the benefits of the AIT strategies would be the same as under the assumption of rational expectations. This would be interesting as a future research work.

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Appendix A

Legend - variables and parameters

Table A.1: Endogenous

Variable	LaTeX	Description
robs	$RATE$	Observed interest rate
pinfobs	INF	Inflation
dy	GDP	Output growth rate
dc	$CONS$	consumption growth rate
dinve	INV	Investment growth rate
dw	$WAGE$	Wage growth rate
labobs	$HOURS$	log hours worked
yf	y^p	Output flex price economy
wf	w^p	real wage flex price economy
mc	μ_p	gross price markup
zcap	z	Capital utilization rate
rk	r^k	rental rate of capital
k	k^s	Capital services
pk	q	real value of existing capital stock
c	c	Consumption
inve	i	Investment
y	y	Output
lab	l	hours worked
pinf	π	Inflation
w	w	real wage
r	r	nominal interest rate
kp	k	Capital stock

Table A.2: Exogenous

Variable	LaTeX	Description
ea	η^a	productivity shock
eb	η^b	risk premium shock
eg	η^g	Spending shock
eqs	η^i	Investment-specific technology shock
em	η^m	Monetary policy shock
epinf	η^p	Price markup shock
ew	η^w	Wage markup shock

Table A.3: Parameters

Variable	LaTeX	Description
crhob	ρ_b	persistence risk premium shock
crhoqs	ρ_i	persistence risk premium shock
crhoms	ρ_r	persistence monetary policy shock
crhopinf	ρ_p	persistence price markup shock
crhow	ρ_w	persistence wage markup shock
cmap	μ_p	coefficient on MA term price markup
cmaw	μ_w	coefficient on MA term wage markup
curvw	ε_w	Curvature Kimball aggregator wages
curvp	ε_p	Curvature Kimball aggregator prices
csadjcost	φ	investment adjustment cost
csigma	σ_c	risk aversion
chabb	λ	external habit degree
cprobw	ξ_w	Calvo parameter wages
csigl	σ_l	Frisch elasticity
cprobp	ξ_p	Calvo parameter prices
cindw	ι_w	Indexation to past wages
cindp	ι_p	Indexation to past prices
czcap	ψ	capacity utilization cost
cfc	ϕ_p	fixed cost share
crpi	r_π	Taylor rule inflation feedback
crr	ρ	interest rate persistence
cry	r_y	Taylor rule output level feedback
crdy	$r_{\Delta y}$	Taylor rule output growth feedback
constepinf	$\bar{\pi}$	steady state inflation rate
constelab	$\bar{l}$	steady state hours

Table A.3 – Continued

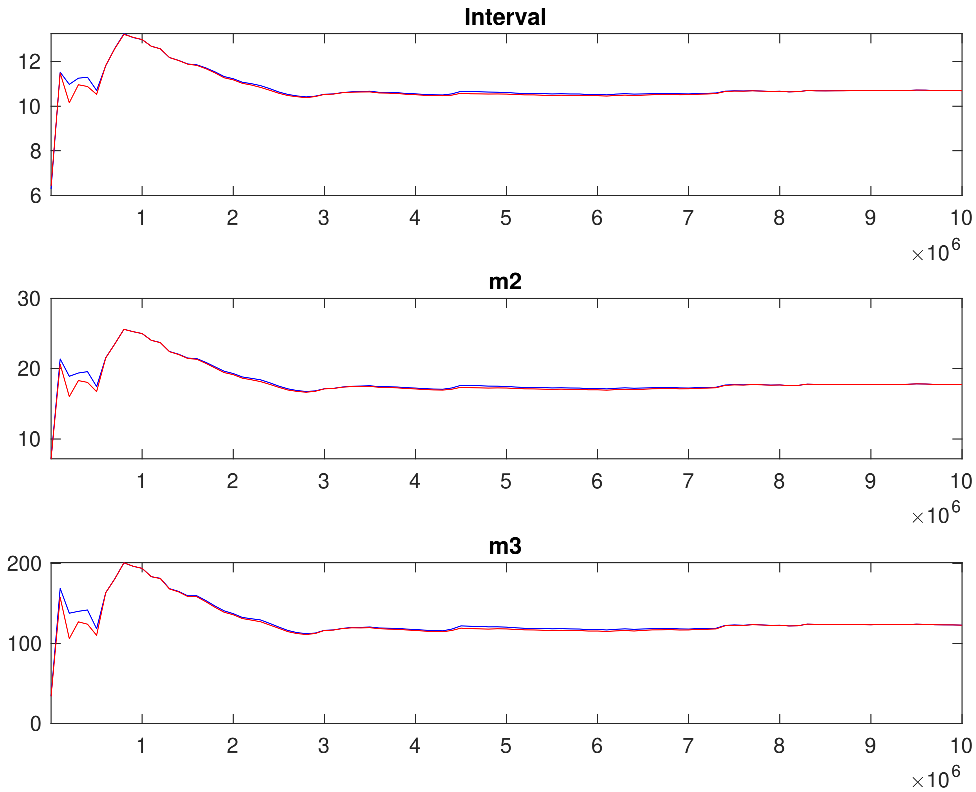
Variable	LaTeX	Description
`constebeta`	$100(\beta^{-1} - 1)$	time preference rate in percent
`ctrend`	$\bar{\gamma}$	net growth rate in percent
`cgy`	ρ_{ga}	Feedback technology on exogenous spending
`calfa`	α	capital share
`cg`	$\frac{\bar{g}}{\bar{y}}$	steady state exogenous spending share

Appendix B

Results estimation SW model with the SW rule

B.1 Multivariate convergence diagnostic test

Figure B.1: Multivariate convergence diagnostics for the Metropolis-Hastings. The first, second and third rows are respectively the criteria based on the eighty percent interval, the second and third moments. The different parameters are aggregated using the posterior kernel.



B.2 Prior and Posterior distributions

Figure B.2: Priors and posteriors.

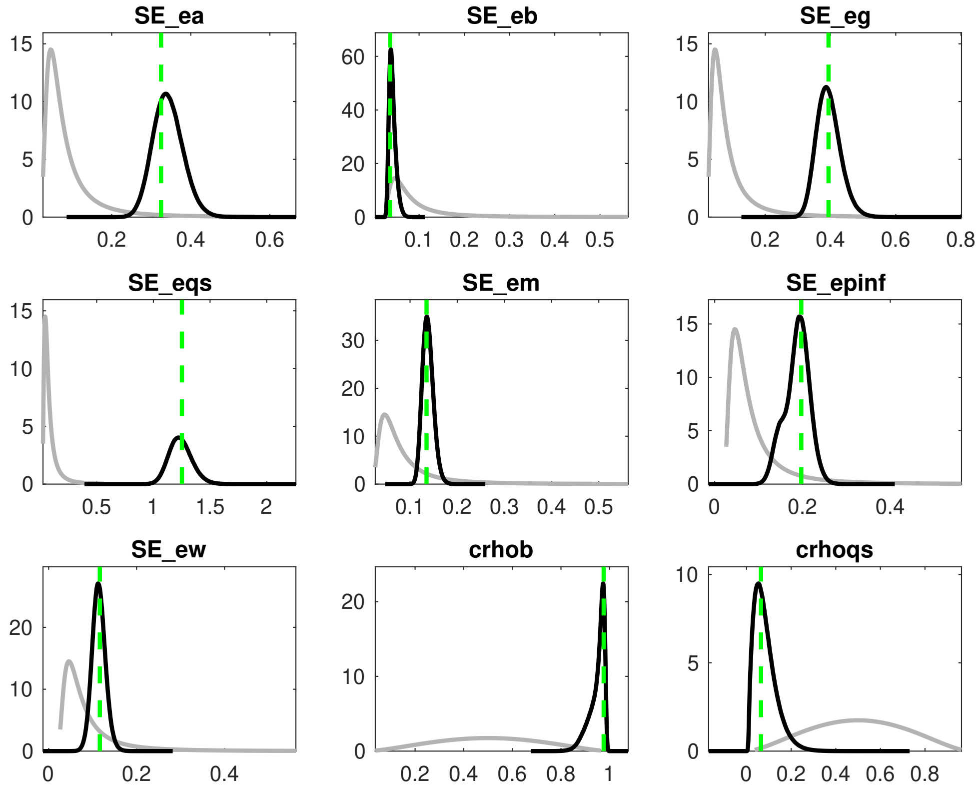


Figure B.3: Priors and posteriors.

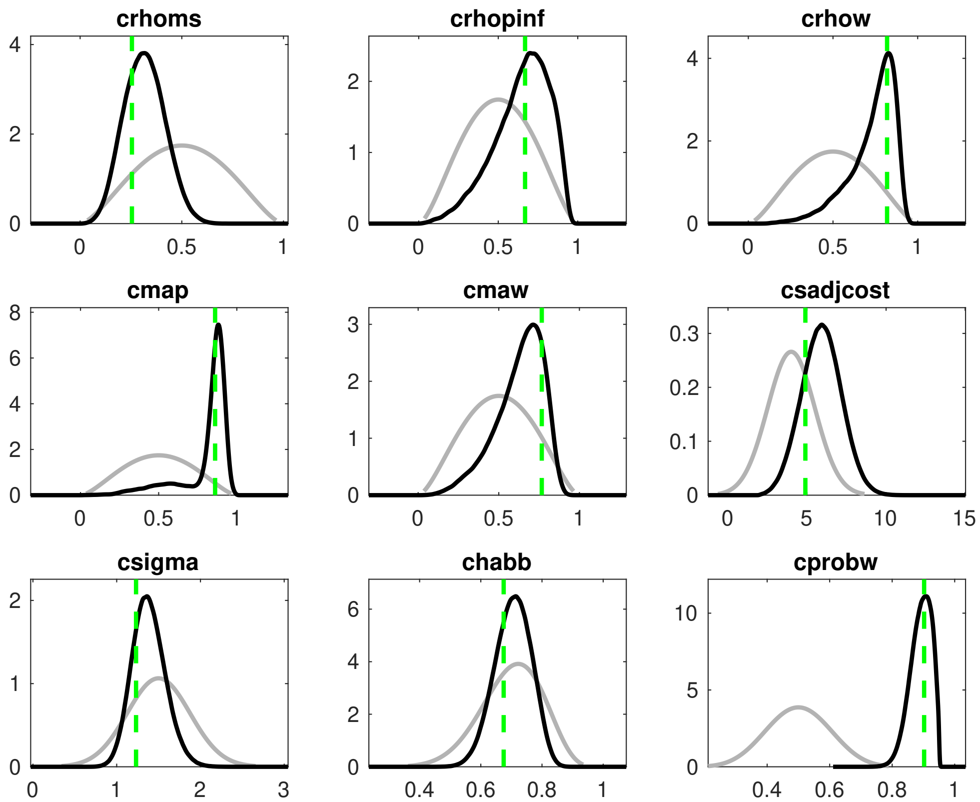


Figure B.4: Priors and posteriors.

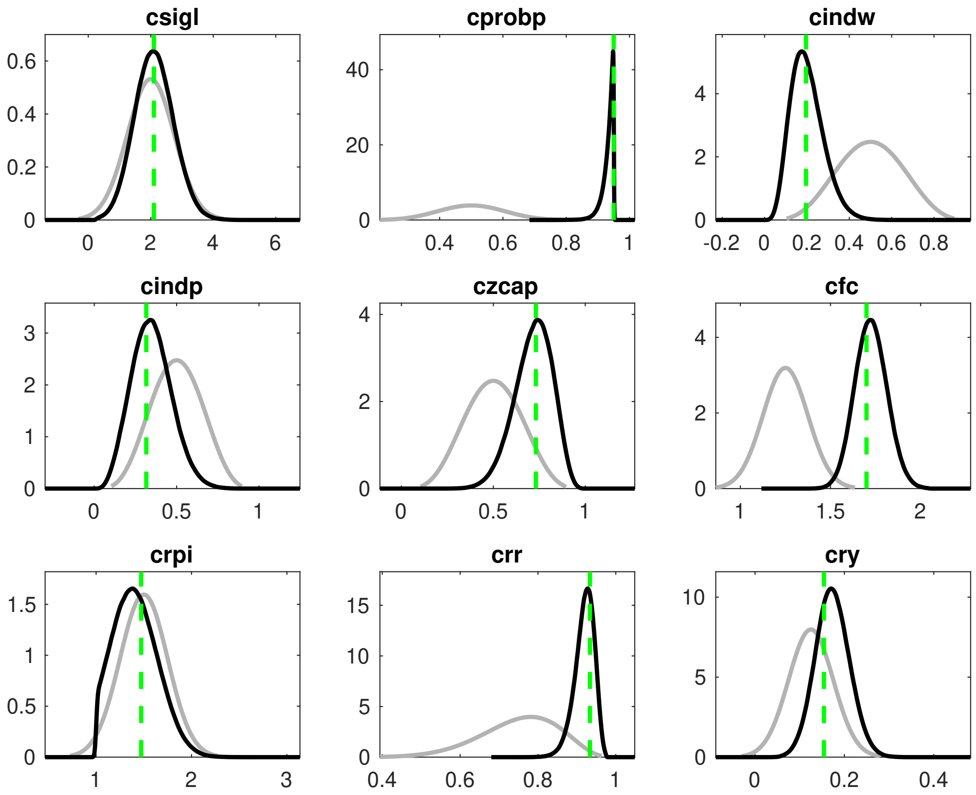
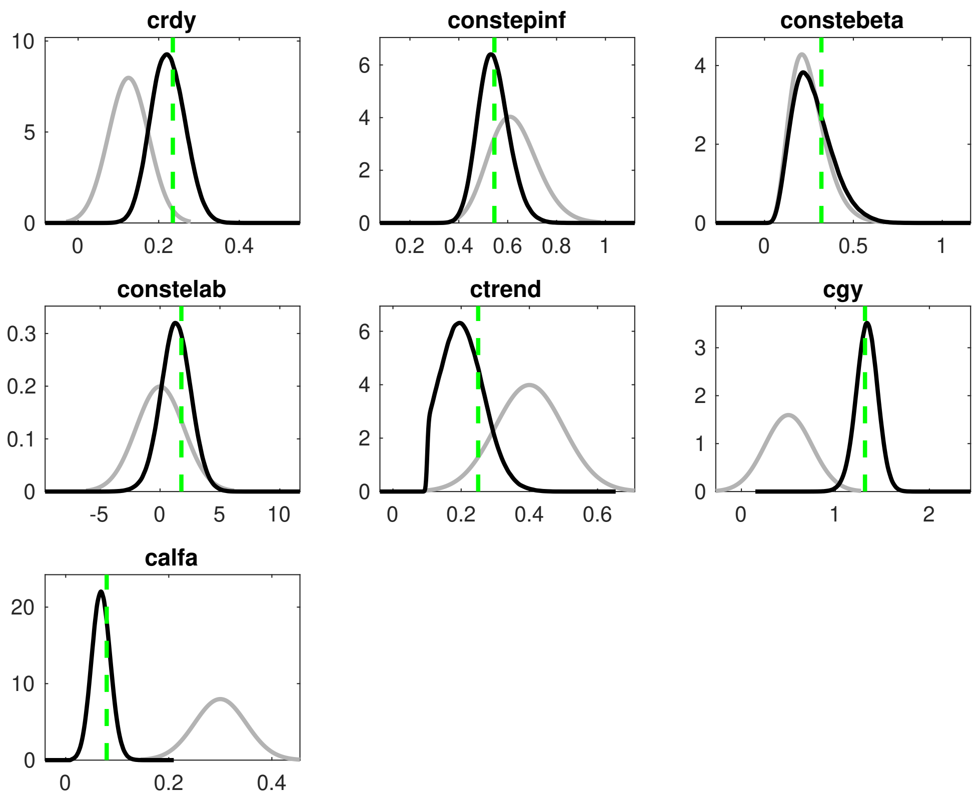


Figure B.5: Priors and posteriors.



B.3 Bayesian IRFs

Figure B.6: Bayesian IRF: Orthogonalized shock to η^a.

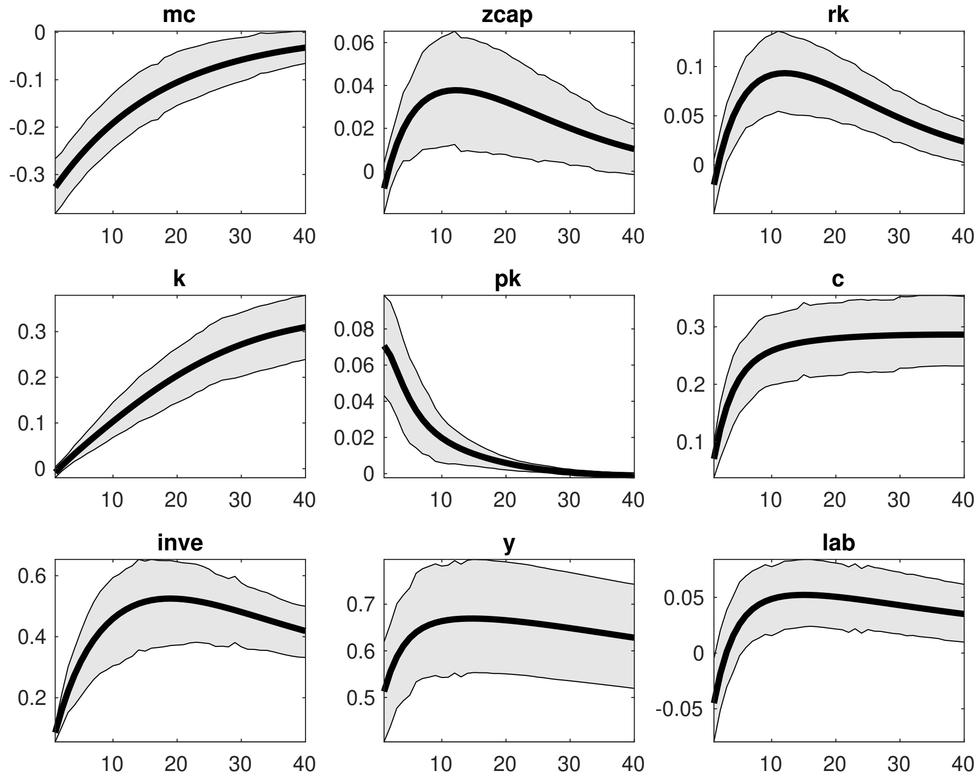


Figure B.7: Bayesian IRF: Orthogonalized shock to η^a.

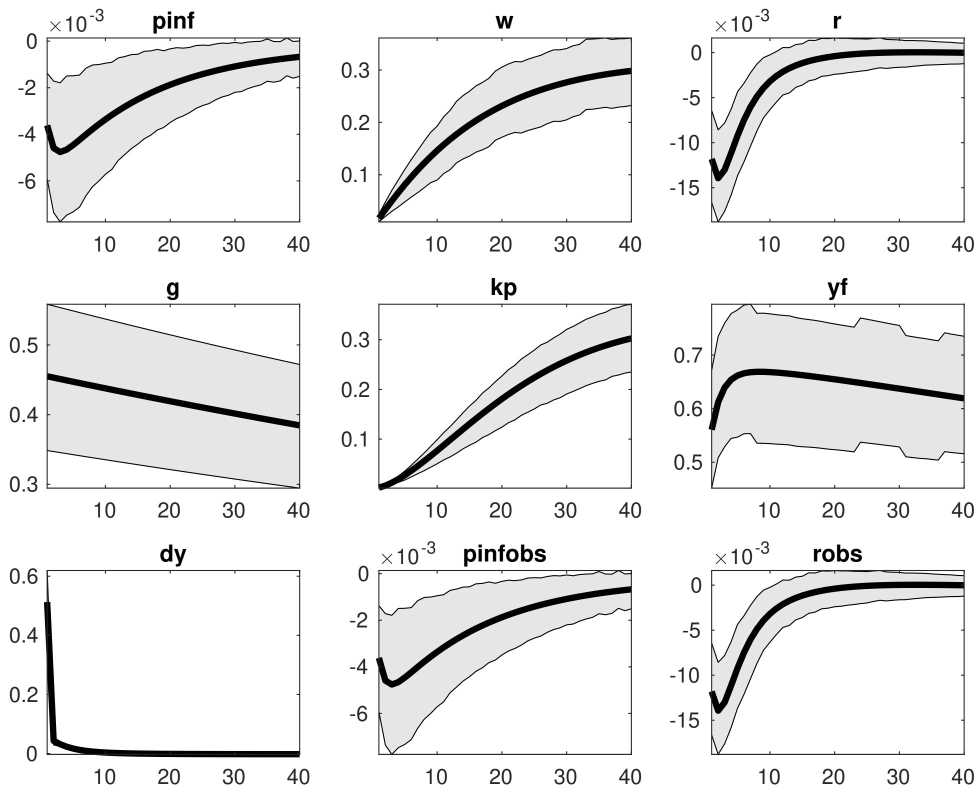


Figure B.8: Bayesian IRF: Orthogonalized shock to η^a.

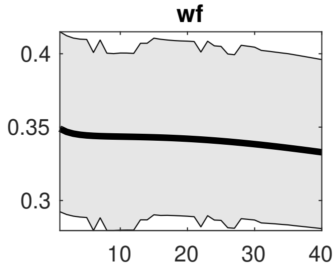


Figure B.9: Bayesian IRF: Orthogonalized shock to η^b.

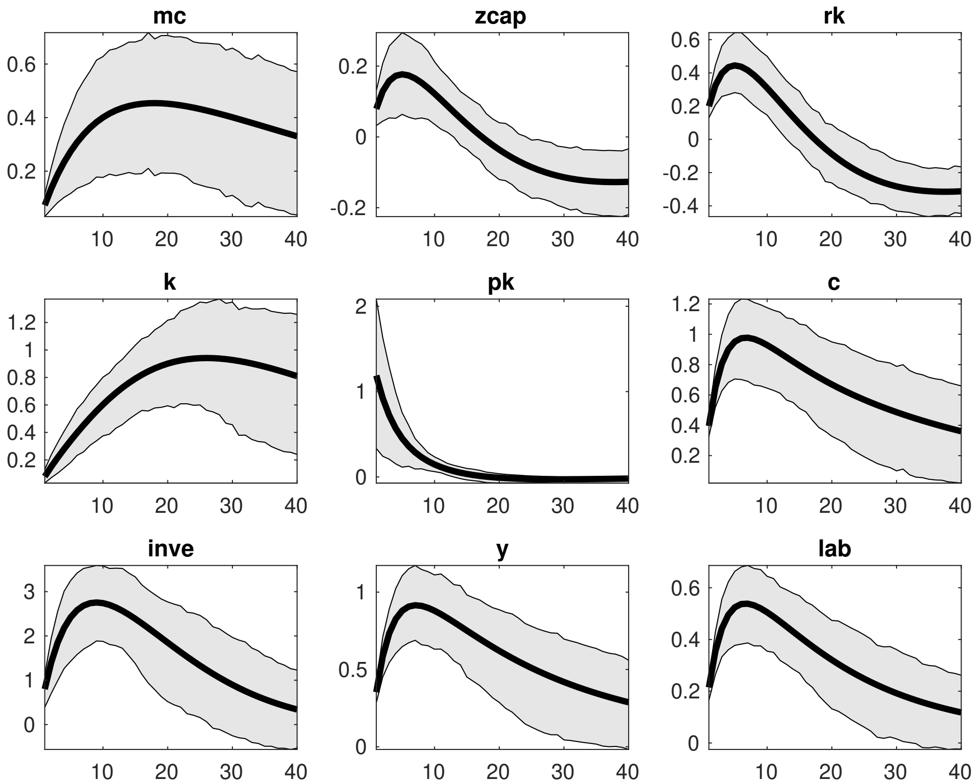


Figure B.10: Bayesian IRF: Orthogonalized shock to η^b.

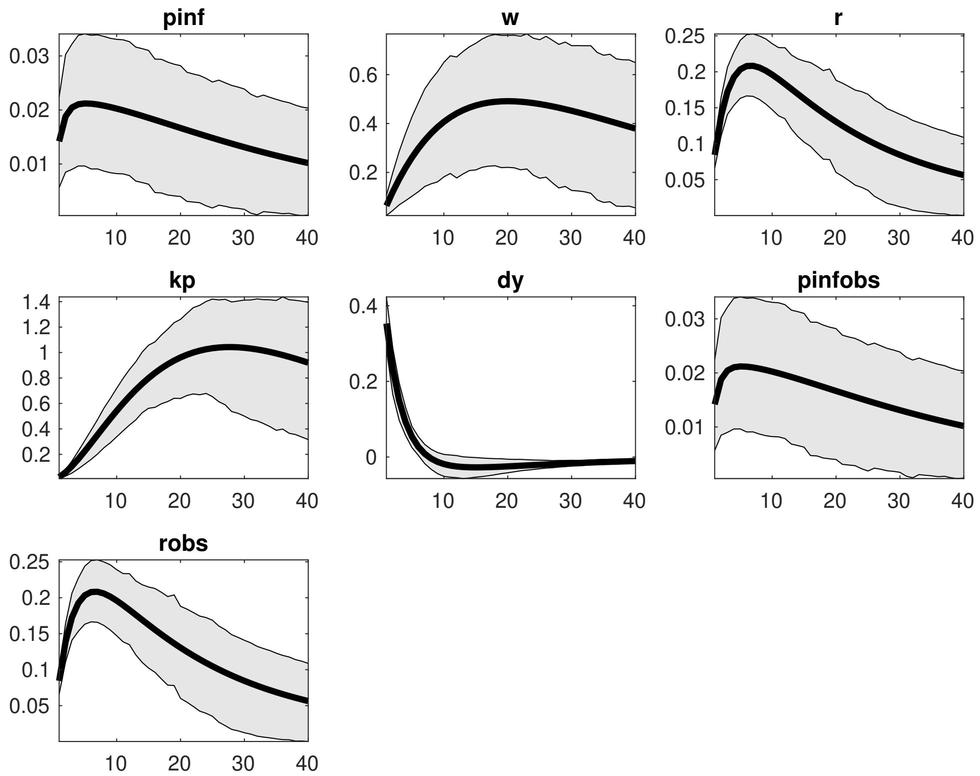


Figure B.11: Bayesian IRF: Orthogonalized shock to η^g.

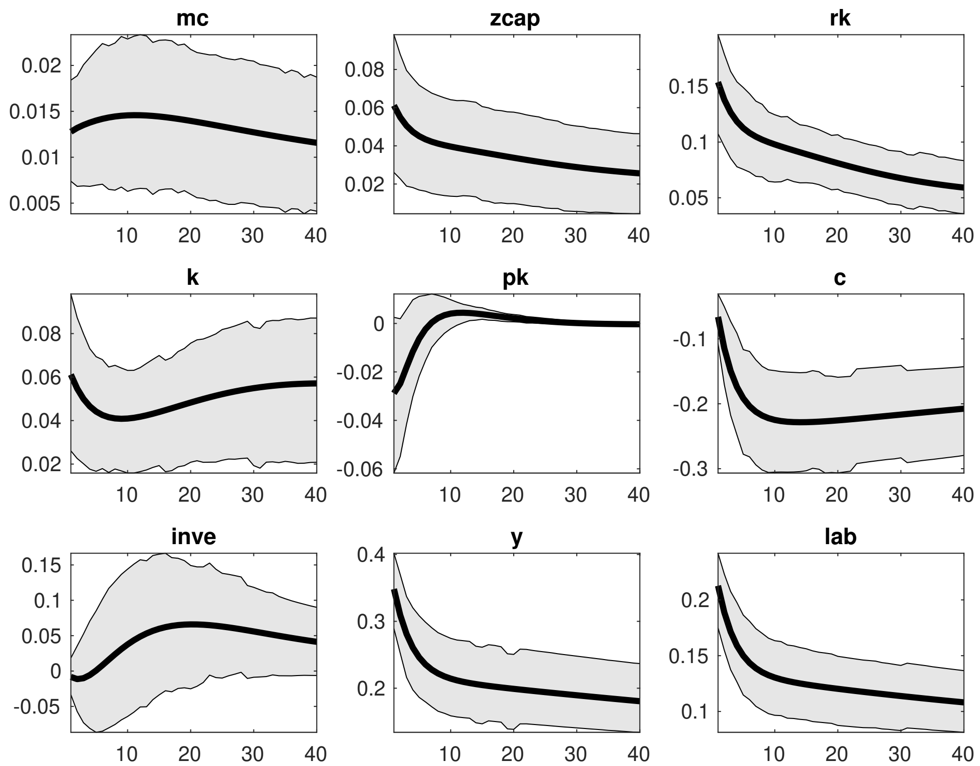


Figure B.12: Bayesian IRF: Orthogonalized shock to η^g.

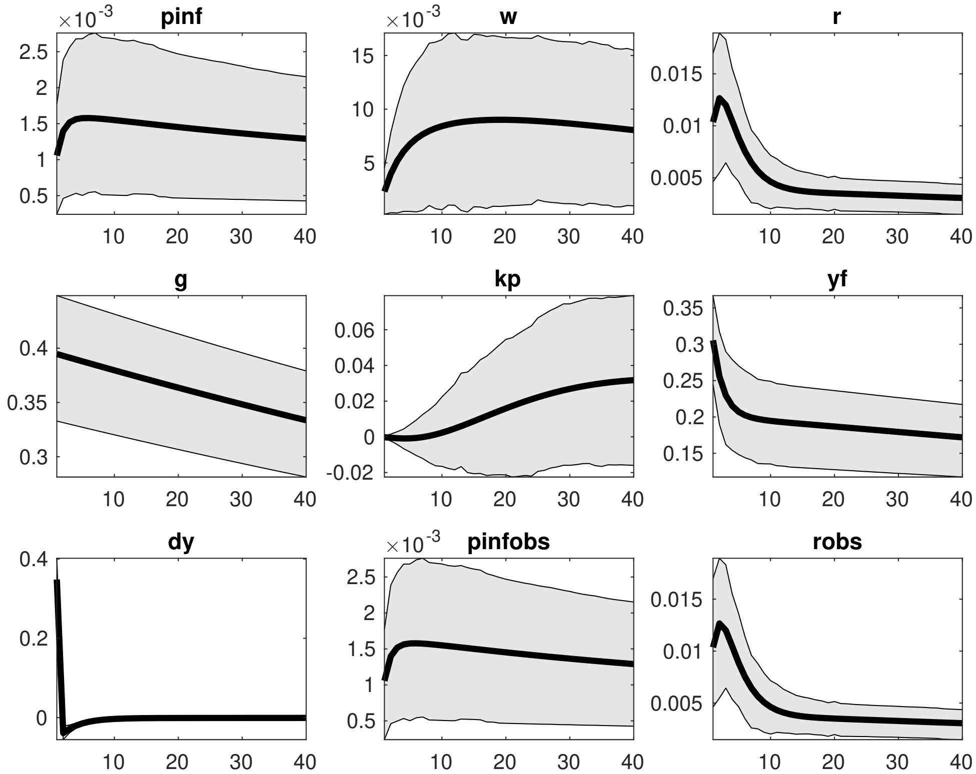


Figure B.13: Bayesian IRF: Orthogonalized shock to η^g.

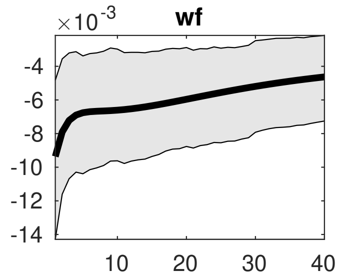


Figure B.14: Bayesian IRF: Orthogonalized shock to η^i.

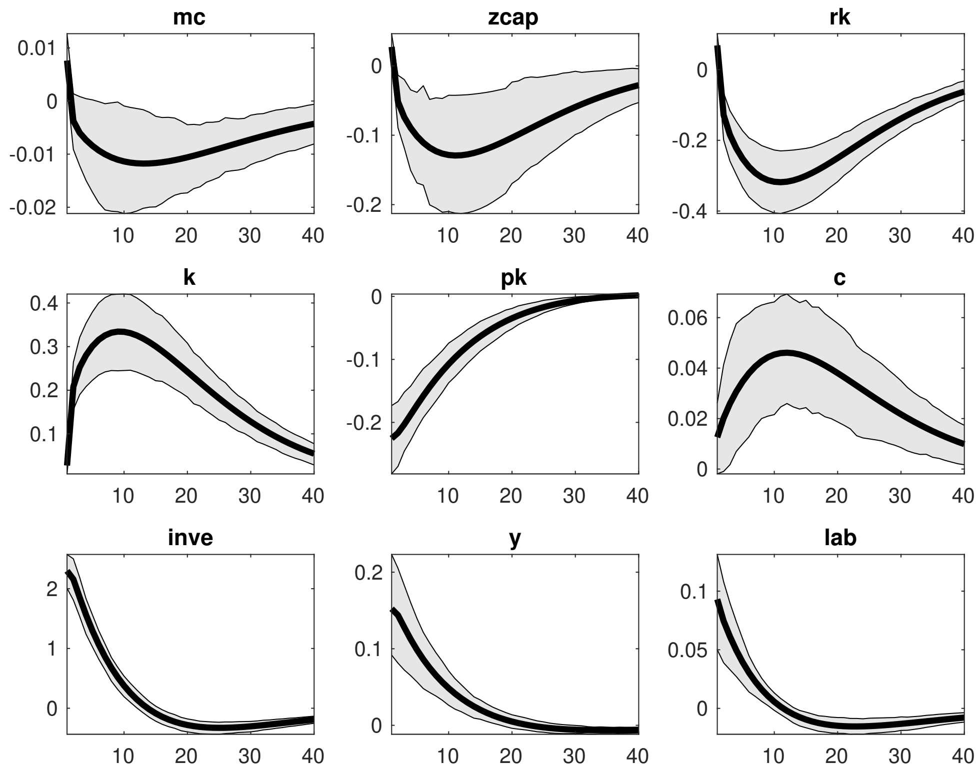


Figure B.15: Bayesian IRF: Orthogonalized shock to η^i.

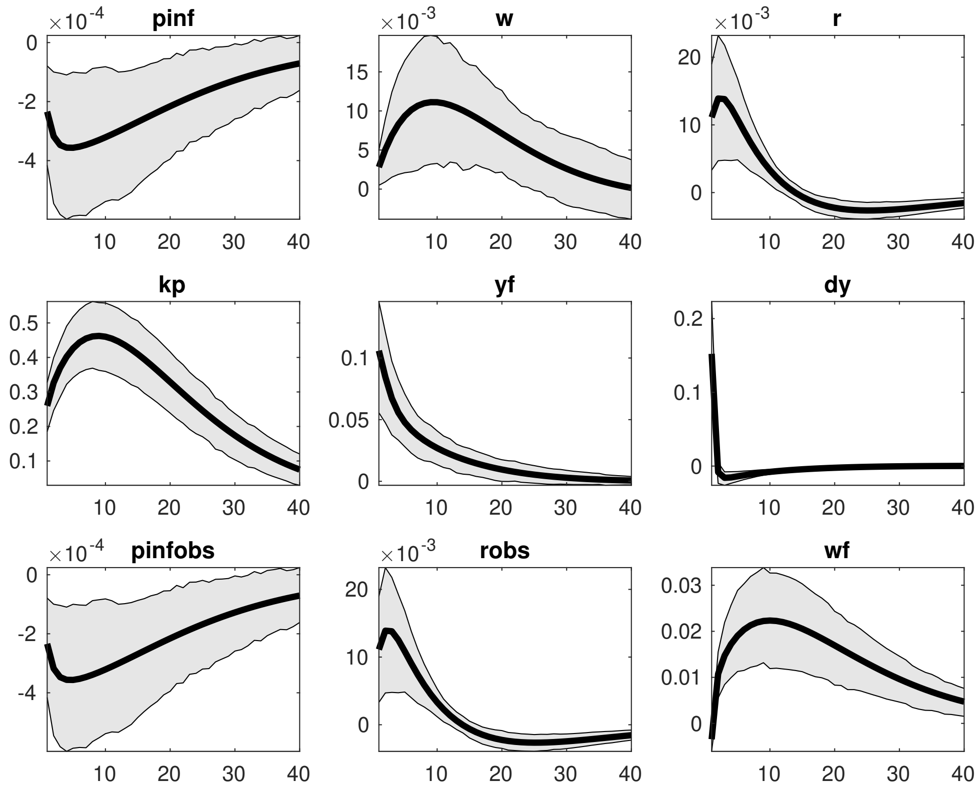


Figure B.16: Bayesian IRF: Orthogonalized shock to η^m.

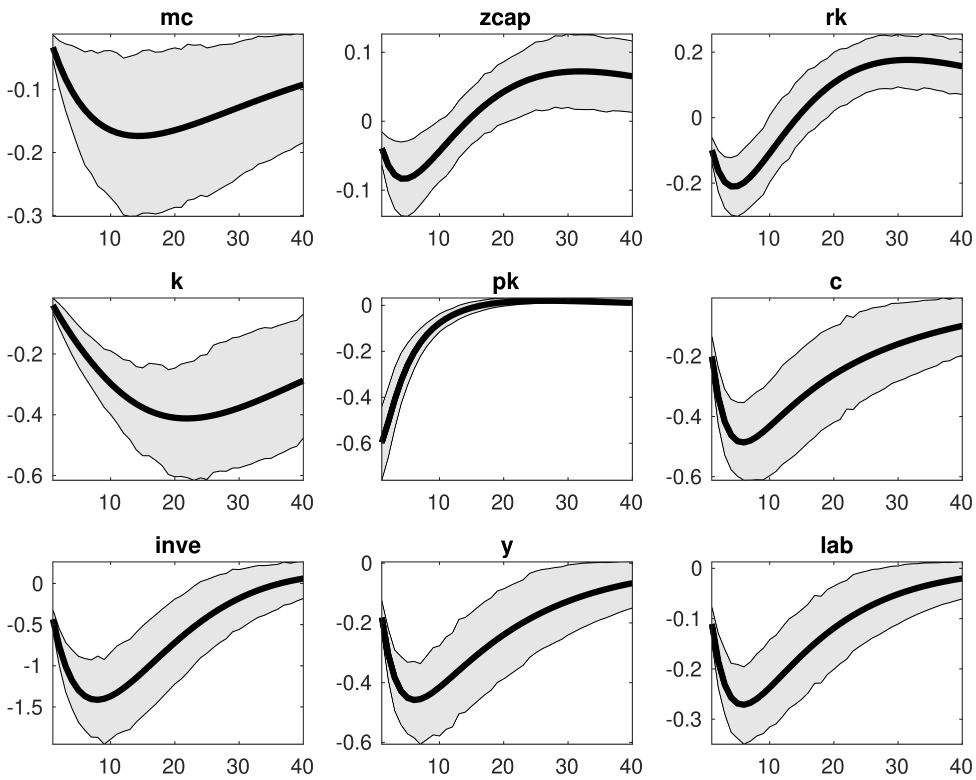


Figure B.17: Bayesian IRF: Orthogonalized shock to η^m.

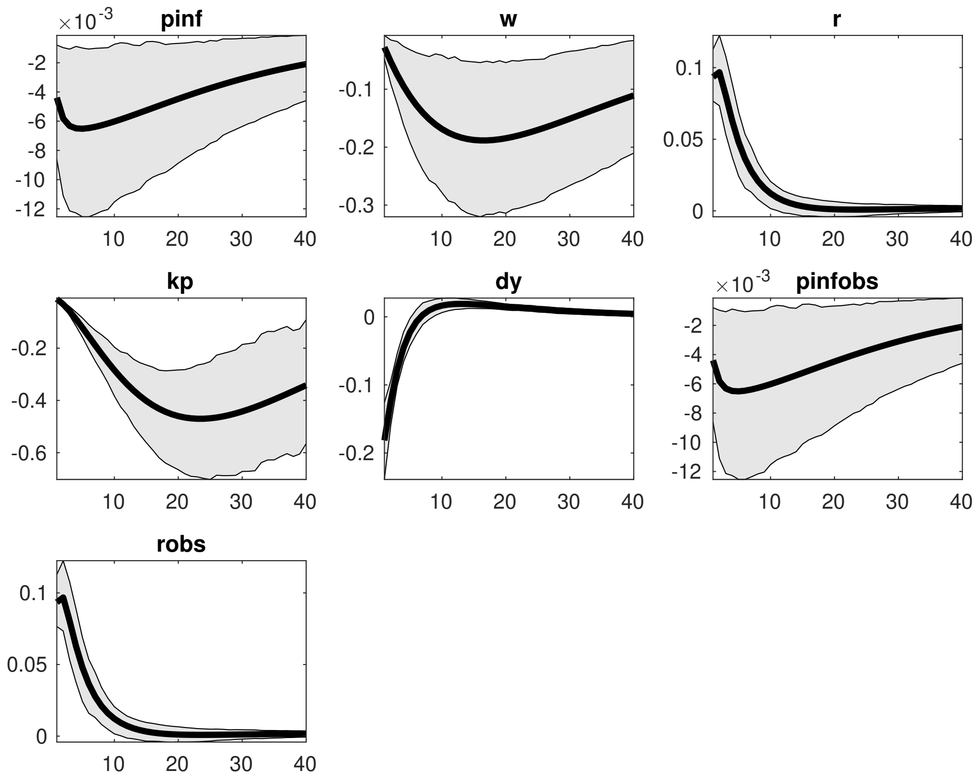


Figure B.18: Bayesian IRF: Orthogonalized shock to η^p.

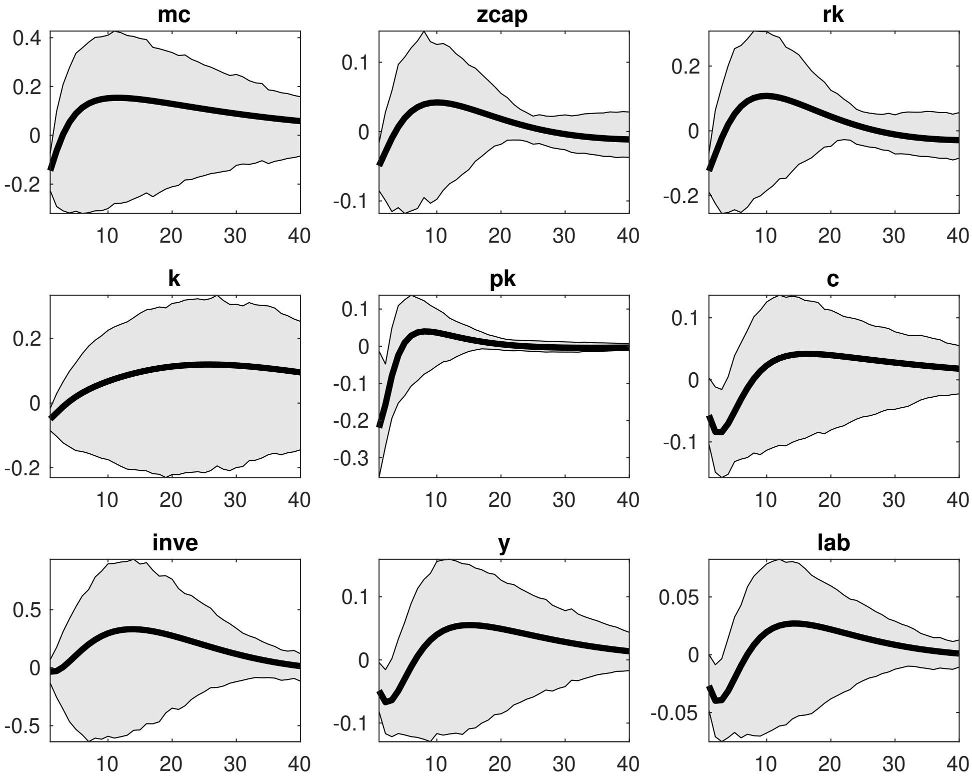


Figure B.19: Bayesian IRF: Orthogonalized shock to η^p.

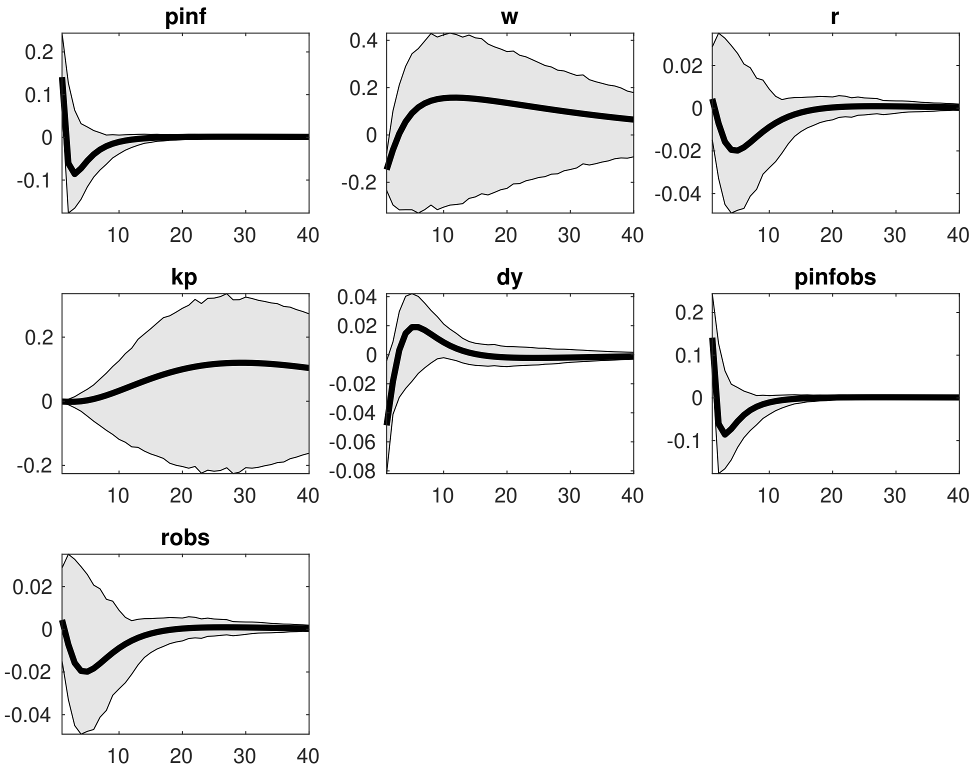


Figure B.20: Bayesian IRF: Orthogonalized shock to η^w.

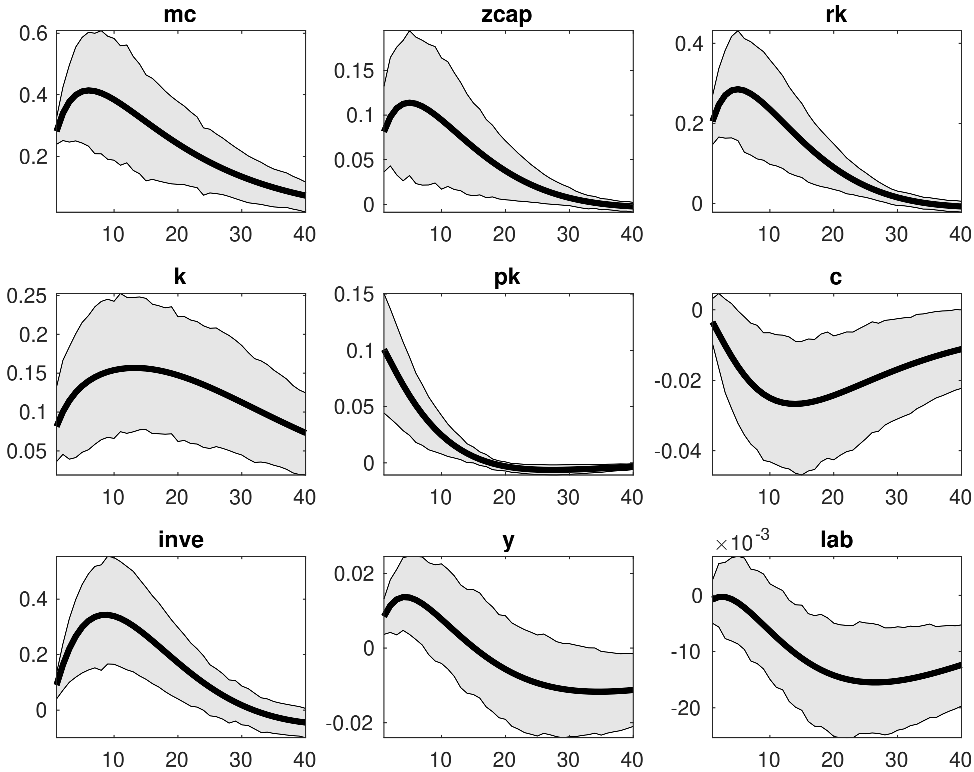
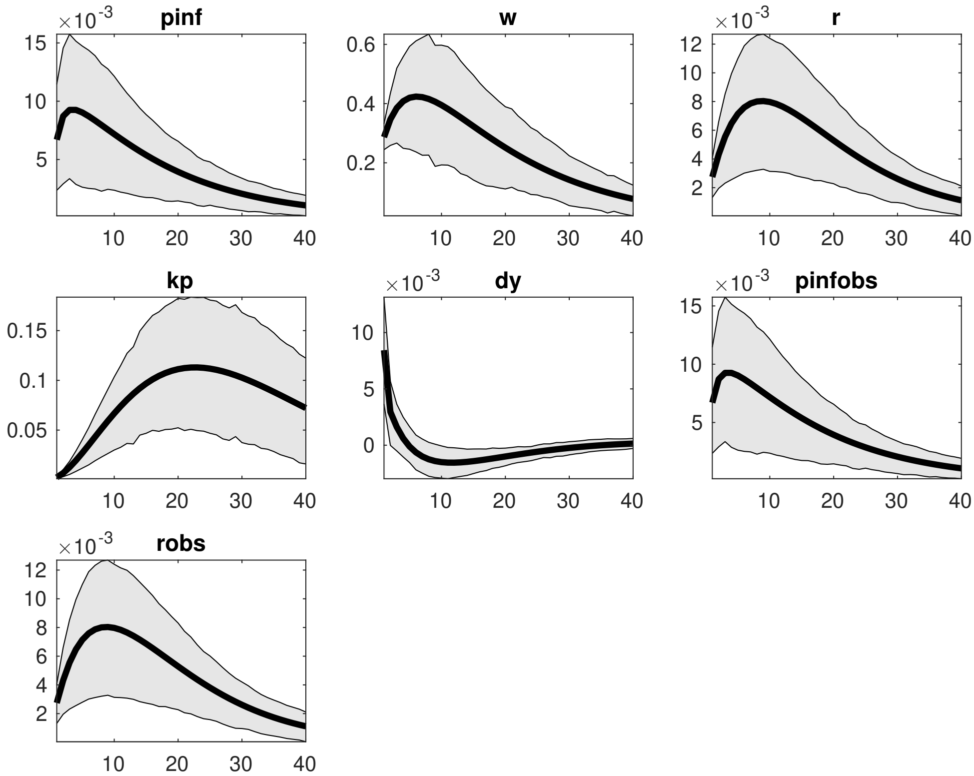
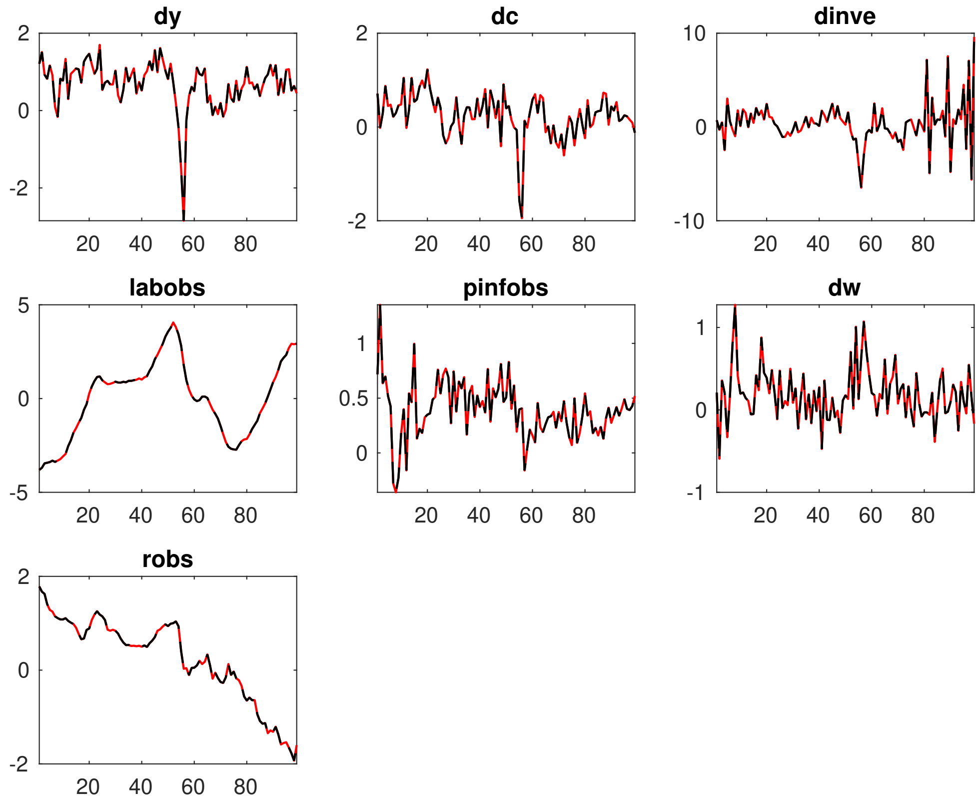


Figure B.21: Bayesian IRF: Orthogonalized shock to η^w.



B.4 Historical and smoothed variables

Figure B.22: Historical and smoothed variables.

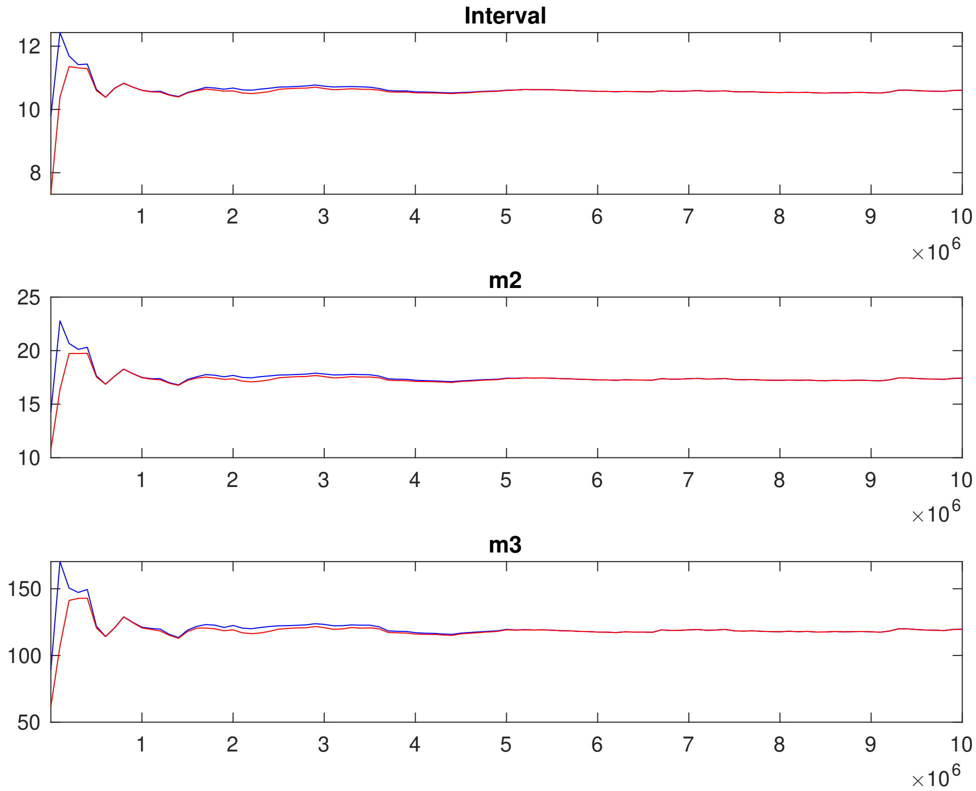


Appendix C

Results estimation SW with Taylor rule

C.1 Multivariate convergence diagnostic test - Taylor rule

Figure C.1: Multivariate convergence diagnostics for the Metropolis-Hastings. The first, second and third rows are respectively the criteria based on the eighty percent interval, the second and third moments. The different parameters are aggregated using the posterior kernel.



C.2 Prior and Posterior distributions - Taylor rule

Figure C.2: Priors and posteriors.

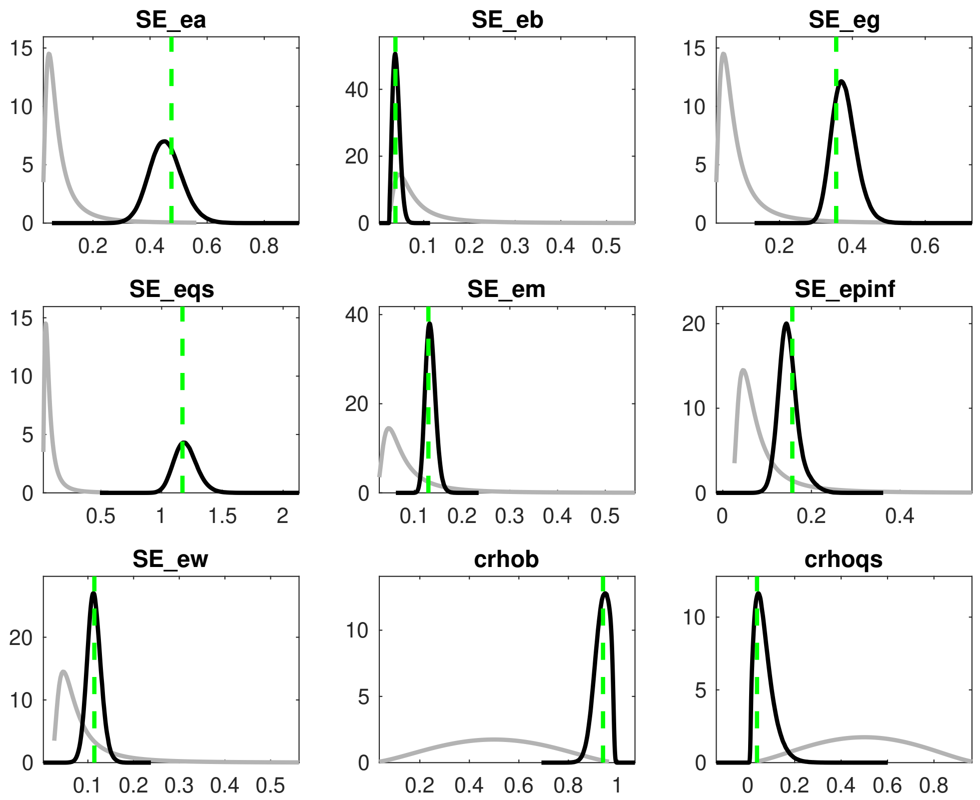


Figure C.3: Priors and posteriors.

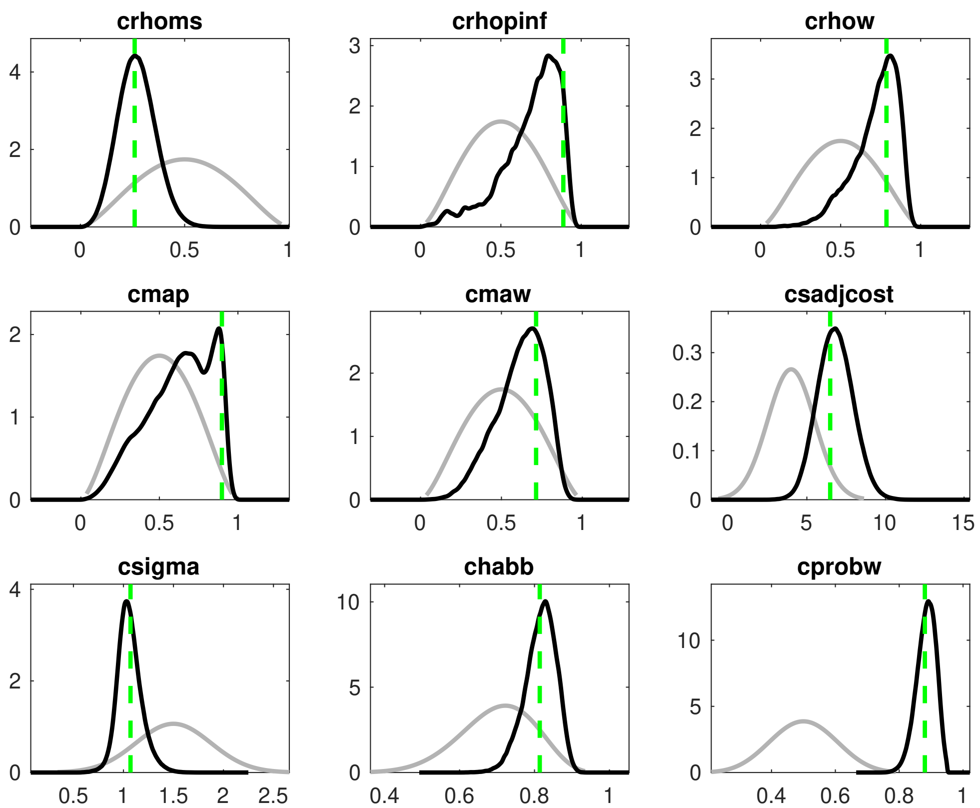


Figure C.4: Priors and posteriors.

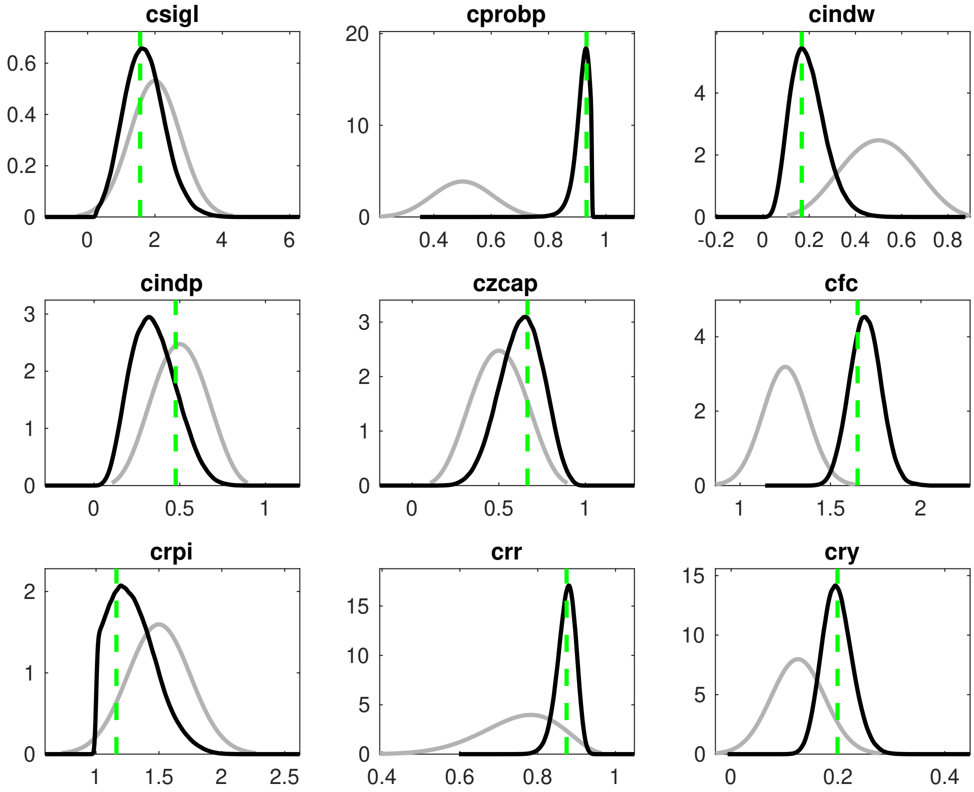
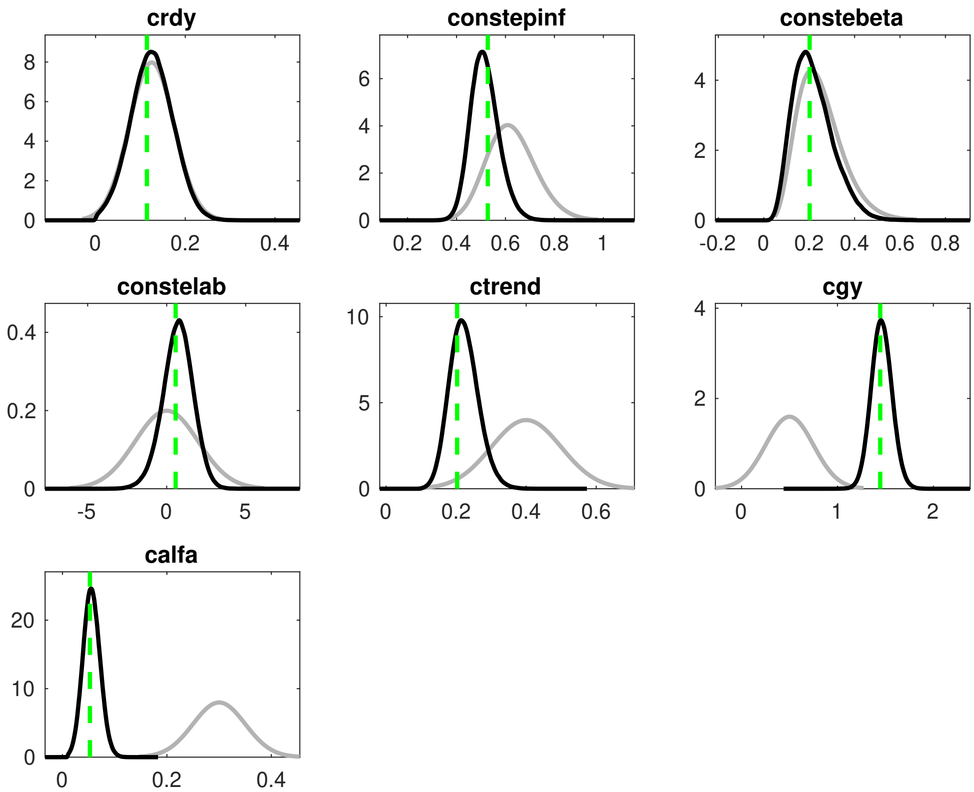


Figure C.5: Priors and posteriors.



C.3 Posterior mean - Taylor rule

Table C.1: Results from Metropolis-Hastings (parameters)

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
ρ_b	beta	0.500	0.2000	0.935	0.0289	0.8928	0.9814
ρ_i	beta	0.500	0.2000	0.069	0.0412	0.0100	0.1244
ρ_r	beta	0.500	0.2000	0.269	0.0889	0.1218	0.4136
ρ_p	beta	0.500	0.2000	0.685	0.1794	0.4243	0.9356
ρ_w	beta	0.500	0.2000	0.717	0.1386	0.5003	0.9188
μ_p	beta	0.500	0.2000	0.625	0.2029	0.3106	0.9311
μ_w	beta	0.500	0.2000	0.613	0.1497	0.3752	0.8517
φ	norm	4.000	1.5000	6.792	1.1282	4.9266	8.6259
σ_c	norm	1.500	0.3750	1.060	0.1211	0.8695	1.2593
λ	beta	0.700	0.1000	0.815	0.0415	0.7489	0.8828
ξ_w	beta	0.500	0.1000	0.884	0.0304	0.8357	0.9336
σ_l	norm	2.000	0.7500	1.684	0.5991	0.6800	2.6351
ξ_p	beta	0.500	0.1000	0.914	0.0277	0.8800	0.9500
ι_w	beta	0.500	0.1500	0.196	0.0756	0.0737	0.3129
ι_p	beta	0.500	0.1500	0.349	0.1297	0.1370	0.5576
ψ	beta	0.500	0.1500	0.628	0.1236	0.4276	0.8330
ϕ_p	norm	1.250	0.1250	1.692	0.0862	1.5491	1.8322
r_π	norm	1.500	0.2500	1.293	0.1854	1.0000	1.5480
ρ	beta	0.750	0.1000	0.873	0.0243	0.8347	0.9126
r_y	norm	0.125	0.0500	0.198	0.0283	0.1514	0.2443
$r_{\Delta y}$	norm	0.125	0.0500	0.125	0.0464	0.0482	0.2020
$\bar{\pi}$	gamm	0.625	0.1000	0.515	0.0576	0.4209	0.6081
$100(\beta^{-1}-1)$	gamm	0.250	0.1000	0.217	0.0869	0.0799	0.3540
l	norm	0.000	2.0000	0.689	0.9593	-0.8780	2.2708
$\bar{\gamma}$	norm	0.400	0.1000	0.222	0.0413	0.1538	0.2886
ρ_{ga}	norm	0.500	0.2500	1.458	0.1074	1.2823	1.6348
	norm	0.300	0.0500	0.056	0.0162	0.0288	0.0820

Table C.2: Results from Metropolis-Hastings (standard deviation of structural shocks)

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
η^a	invg	0.100	2.0000	0.455	0.0577	0.3598	0.5485
η^b	invg	0.100	2.0000	0.040	0.0078	0.0273	0.0519
η^g	invg	0.100	2.0000	0.377	0.0340	0.3214	0.4312
η^i	invg	0.100	2.0000	1.196	0.0950	1.0392	1.3473
η^m	invg	0.100	2.0000	0.134	0.0107	0.1162	0.1507
η^p	invg	0.100	2.0000	0.148	0.0230	0.1102	0.1837
η^w	invg	0.100	2.0000	0.113	0.0153	0.0881	0.1383

C.4 Bayesian IRFs - Taylor rule

Figure C.6: Bayesian IRF: Orthogonalized shock to *ea*.

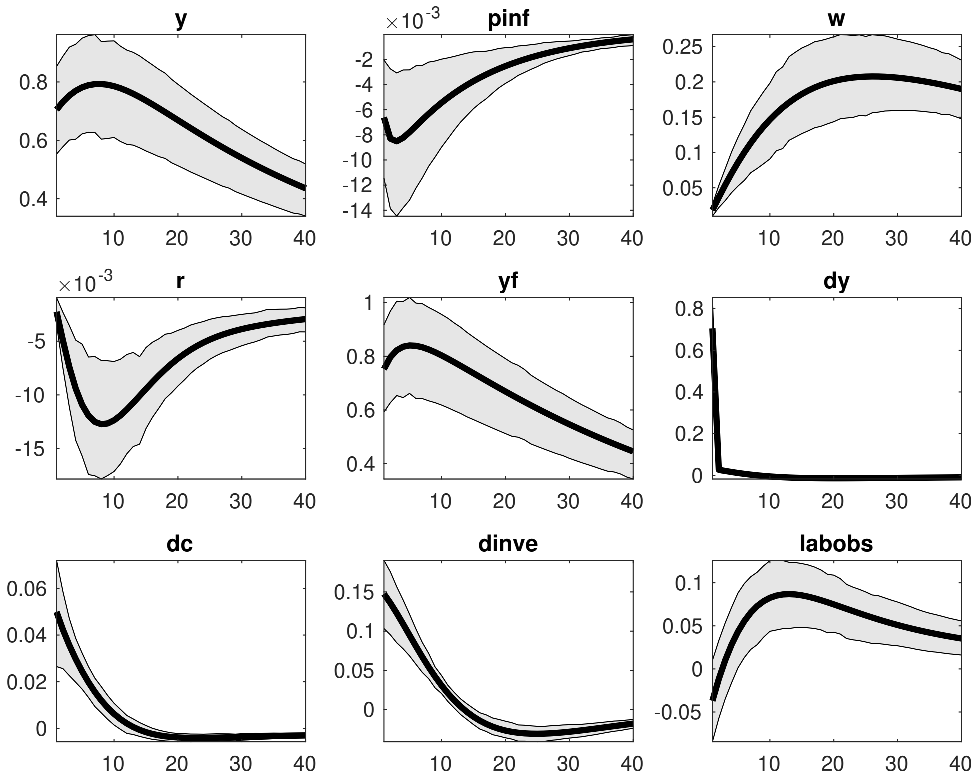


Figure C.7: Bayesian IRF: Orthogonalized shock to *ea*.

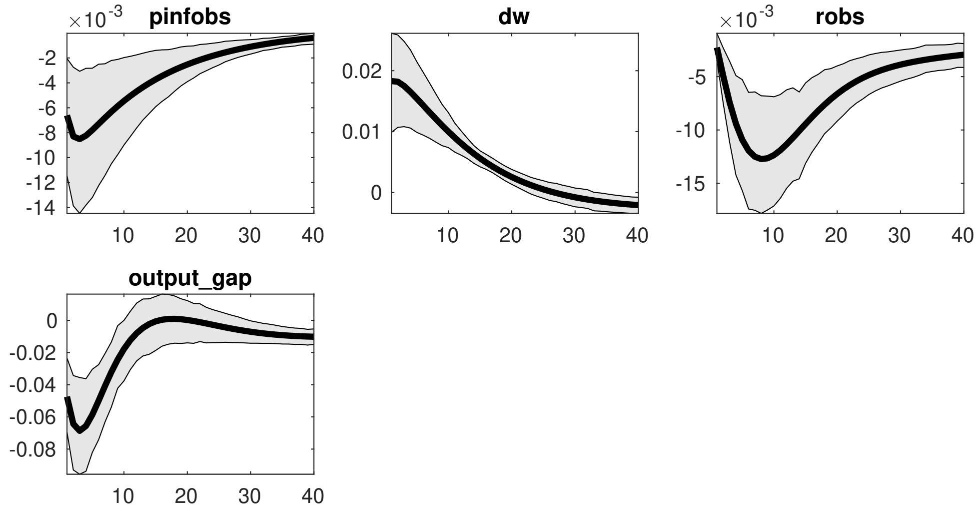


Figure C.8: Bayesian IRF: Orthogonalized shock to *eb*.

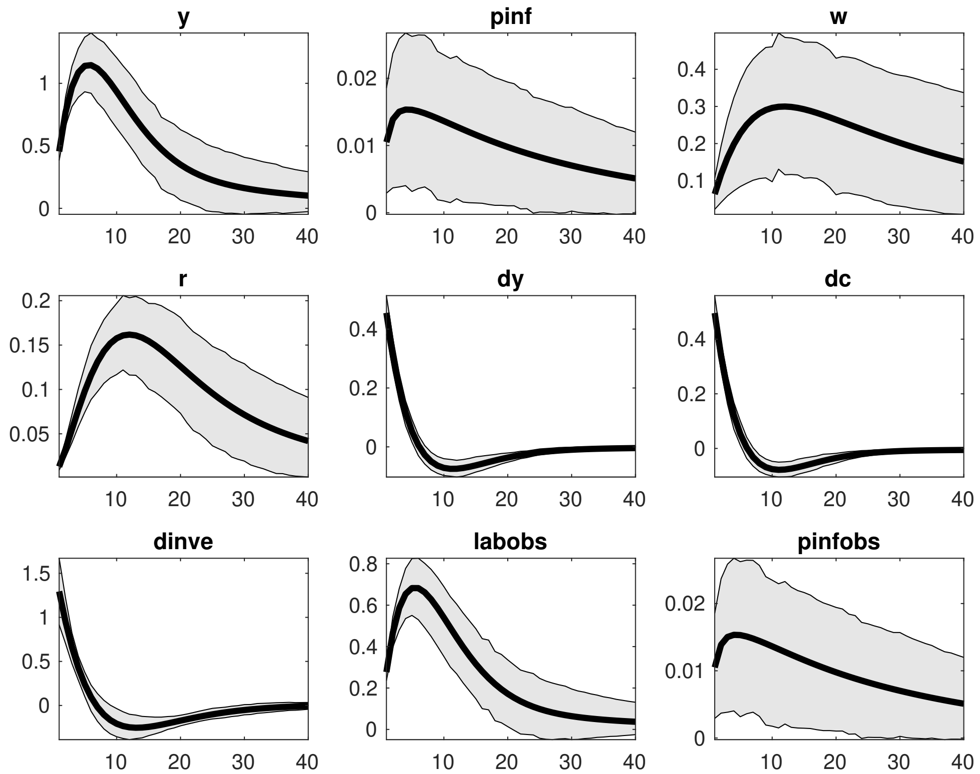


Figure C.9: Bayesian IRF: Orthogonalized shock to *eb*.

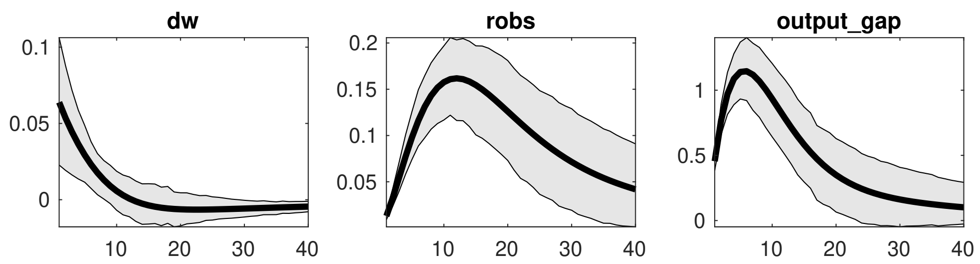


Figure C.10: Bayesian IRF: Orthogonalized shock to *eg*.

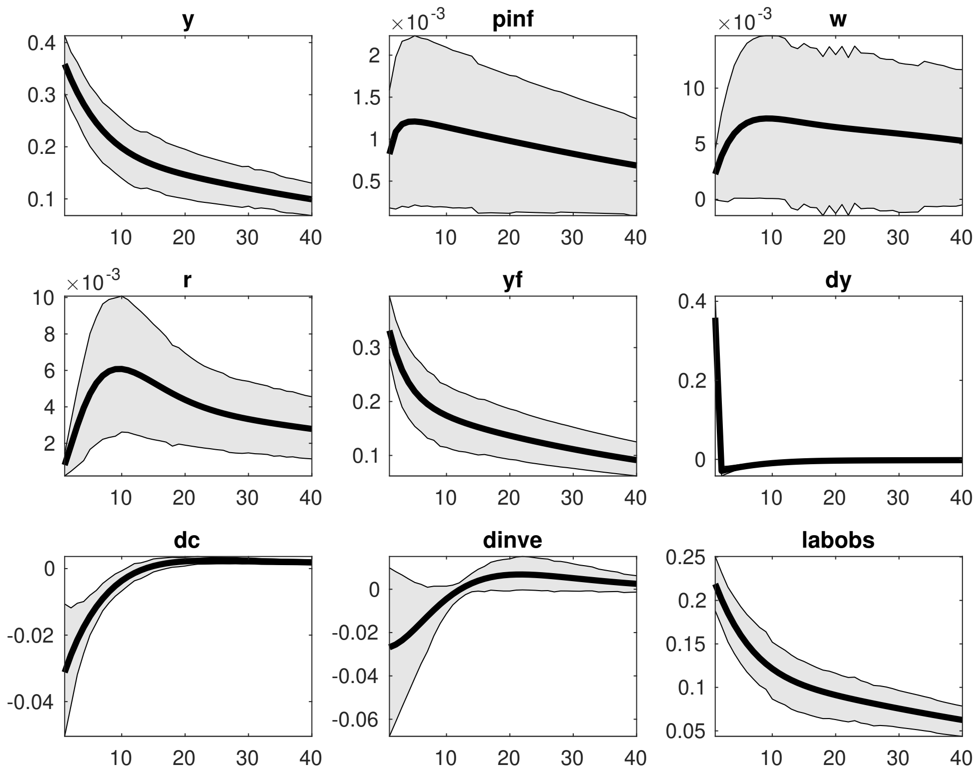


Figure C.11: Bayesian IRF: Orthogonalized shock to *eg*.

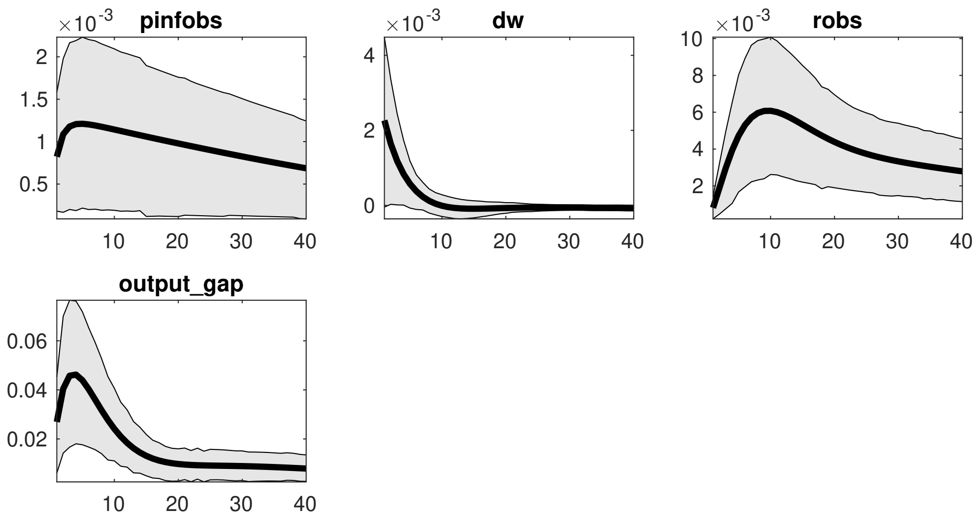


Figure C.12: Bayesian IRF: Orthogonalized shock to *eqs*.

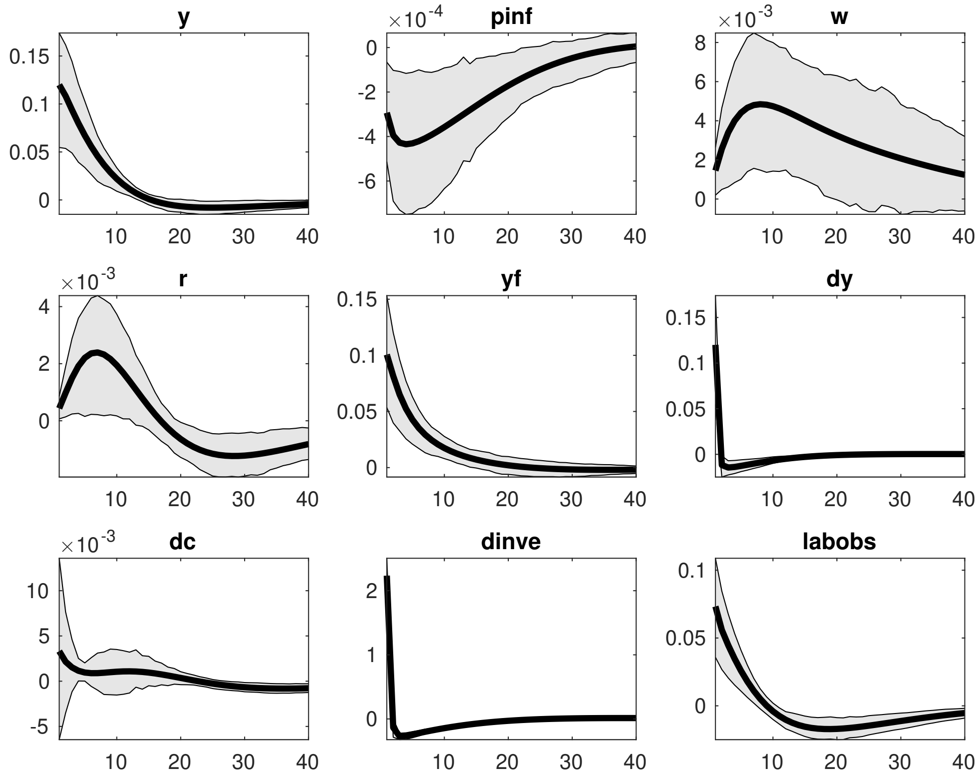


Figure C.13: Bayesian IRF: Orthogonalized shock to *eqs*.

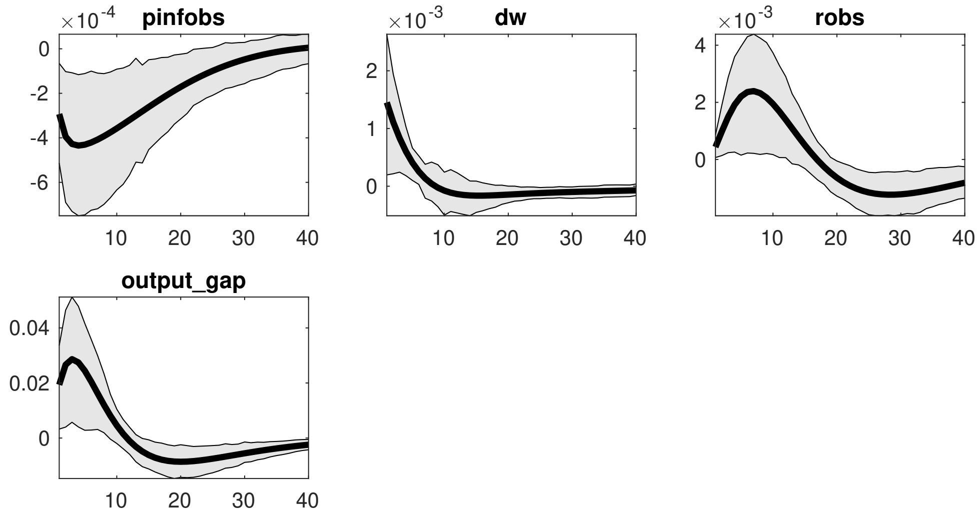


Figure C.14: Bayesian IRF: Orthogonalized shock to *em*.

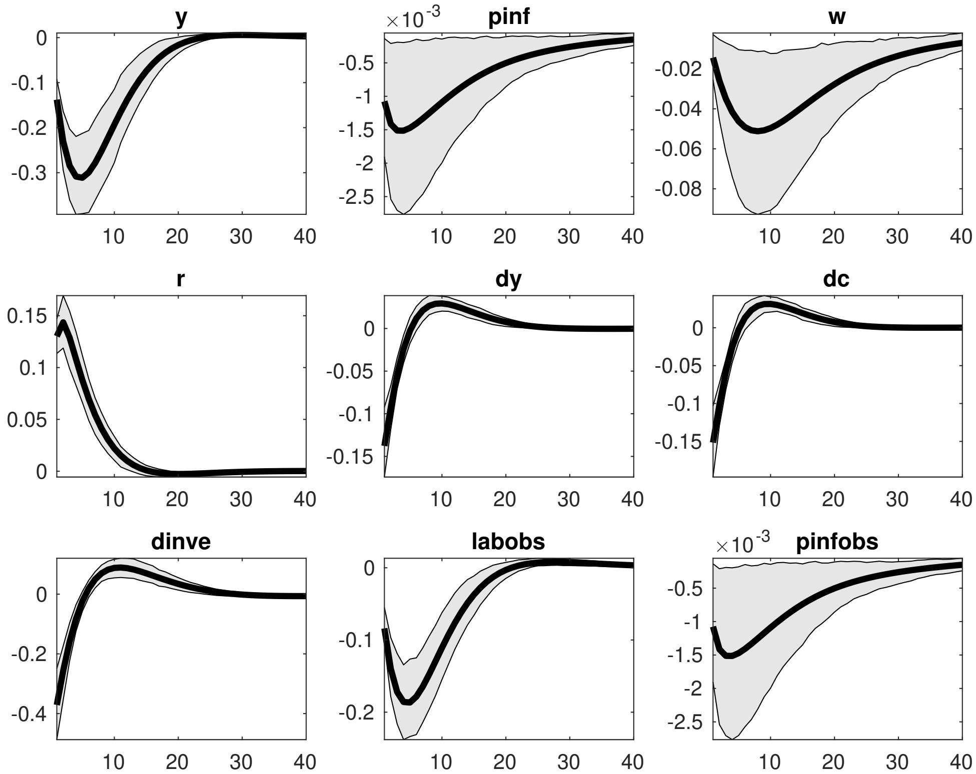


Figure C.15: Bayesian IRF: Orthogonalized shock to *em*.

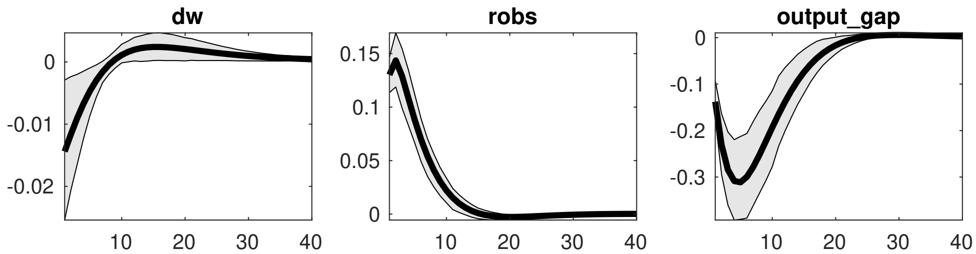


Figure C.16: Bayesian IRF: Orthogonalized shock to $epinf$.

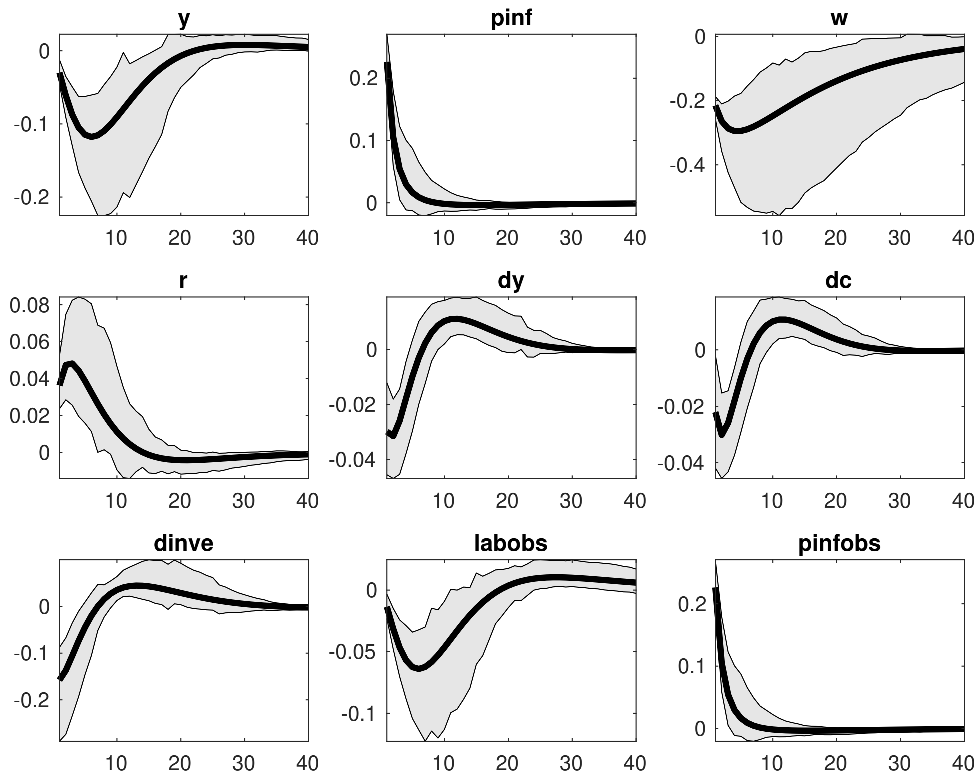


Figure C.17: Bayesian IRF: Orthogonalized shock to $epinf$.

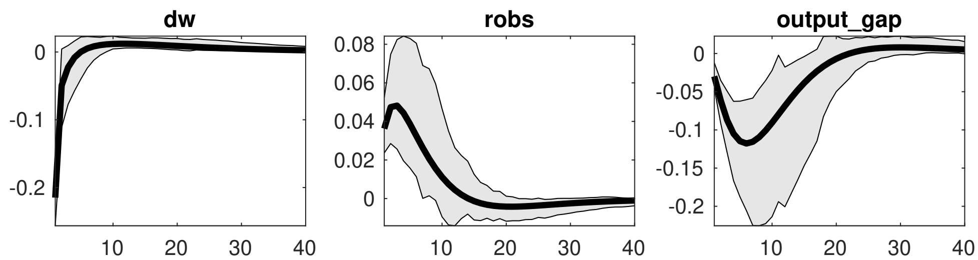


Figure C.18: Bayesian IRF: Orthogonalized shock to *ew*.

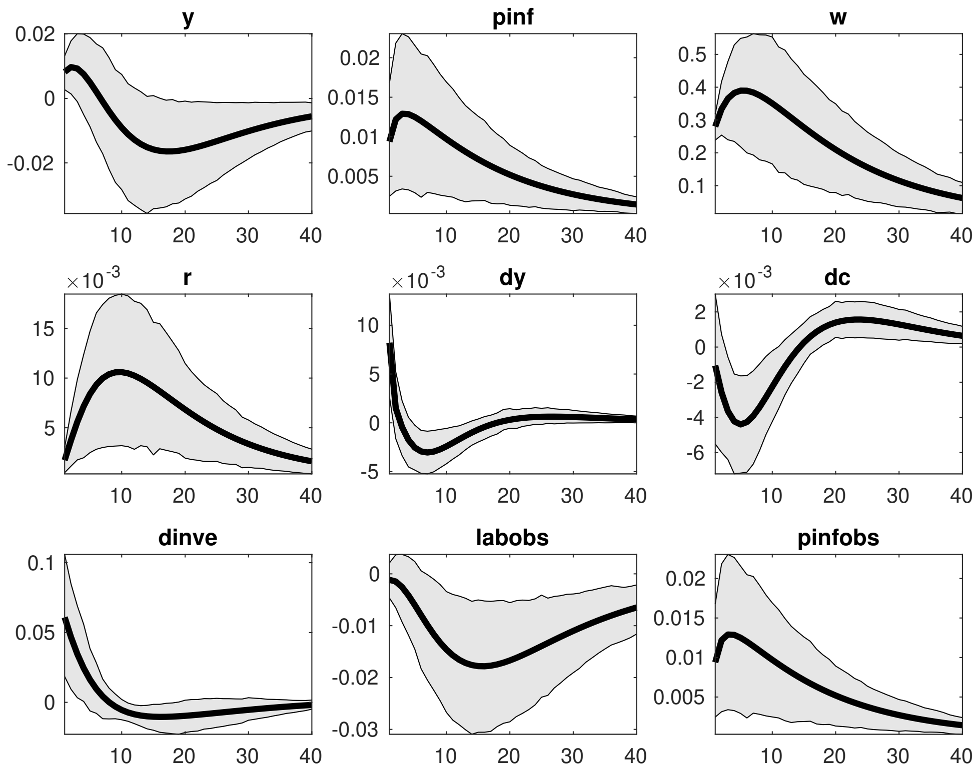
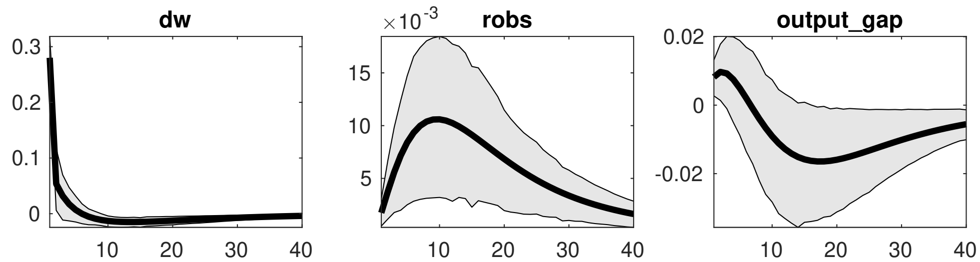


Figure C.19: Bayesian IRF: Orthogonalized shock to *ew*.



Appendix D

Codes

D.1 Dynare code

Here I report the Dynare code that I used to estimate the model and to run the stochastic simulations. The code was developed by Johannes Pfeifer and it is available at his GitHub's page.

```
var labobs robs pinfobs dy dc dinve dw ewma epinfma zcapf rkf kf pkf cf
    invef yf labf wf rrf mc zcap rk k pk c inve y lab pinf w r a b g qs ms
    spinf sw kpf kp;


varexo ea eb eg eqs em epinf ew;

parameters curvw cgy curvp constelab constepinf constebeta cmaw cmap calfa
    czcap csadjcost ctou csigma chabb ccs cinvs cfc
    cindw cprobw cindp cprobp csigl clandaw
    crdpi crpi crdy cry crr
    crhoa crhoas crhob crhog crhols crhoqs crhoms crhopinf crhow
    ctrend cg;


// fixed parameters
ctou=.025;
clandaw=1.5;
cg=0.18;
curvp=10;
curvw=10;

// estimated parameters initialisation
calfa=.24;
cbeta=.9995;
csigma=1.5;
```

```
cfc=1.5;
cgy=0.51;

csadjcost= 6.0144;
chabb=   0.6361;
cprobw=  0.8087;
csigl=   1.9423;
cprobp=  0.6;
cindw=   0.3243;
cindp=   0.47;
czcap=   0.2696;
crpi=    1.488;
crr=     0.8762;
cry=     0.0593;
crdy=    0.2347;

crhoa=   0.98;
crhob=   0.5799;
crhog=   0.98;
crhols=  0.9928;
crhoqs=  0.7165;
crhoas=1;
crhoms=0;
crhopinf=0;
crhow=0;
cmap = 0;
cmaw = 0;

//constelab=0;

model(linear);
//deal with parameter dependencies; taken from usmodel_stst.mod
#cpie=1+constepinf/100;
#cgamma=1+ctrend/100 ;
#cbeta=1/(1+constebeta/100);

#clandap=cfc;
#cbetabar=cbeta*cgamma^(-csigma);
#cr=cpie/(cbeta*cgamma^(-csigma));
#crk=(cbeta^(-1))*(cgamma^csigma) - (1-ctou);
#cw = (calfa^calfa*(1-calfa)^(1-calfa)/(clandap*crk^calfa))^(1/(1-calfa));
//cw = (calfa^calfa*(1-calfa)^(1-calfa)/(clandap*((cbeta^(-1))*(cgamma^csigma)
    - (1-ctou))^calfa))^(1/(1-calfa));
```

```
#cikbar=(1-(1-ctou)/cgamma);
#cik=(1-(1-ctou)/cgamma)*cgamma;
#clk=((1-calfa)/calfa)*(crk/cw);
#cky=cfc*(clk)^(calfa-1);
#ciy=cik*cky;
#ccy=1-cg-cik*cky;
#crkky=crk*cky;
#cwhlc=(1/clandaw)*(1-calfa)/calfa*crk*cky/ccy;
#cwly=1-crk*cky;

#conster=(cr-1)*100;

// flexible economy

        0*(1-calfa)*a + 1*a = calfa*rkf+(1-calfa)*(wf) ;
        zcapf = (1/(czcap/(1-czcap)))* rkf ;
        rkf = (wf)+labf-kf ;
        kf = kpf(-1)+zcapf ;
        invef = (1/(1+cbetabar*cgamma))* ( invef(-1) +
            cbetabar*cgamma*invef(1)+(1/(cgamma^2*csadjcost))*pkf ) +qs ;
         pkf = -rrf-0*b+(1/((1-chabb/cgamma)/(csigma*(1+chabb/cgamma))))*b
             +(crk/(crk+(1-ctou)))*rkf(1) + ((1-ctou)/(crk+(1-ctou)))*pkf(1) ;
        cf = (chabb/cgamma)/(1+chabb/cgamma)*cf(-1) +
            (1/(1+chabb/cgamma))*cf(+1)
            +((csigma-1)*cwhlc/(csigma*(1+chabb/cgamma)))*(labf-labf(+1)) -
            (1-chabb/cgamma)/(csigma*(1+chabb/cgamma))*(rrf+0*b) + b ;
        yf = ccy*cf+ciy*invef+g + crkky*zcapf ;
        yf = cfc*( calfa*kf+(1-calfa)*labf +a );
        wf = csigl*labf  +(1/(1-chabb/cgamma))*cf -
            (chabb/cgamma)/(1-chabb/cgamma)*cf(-1) ;
        kpf = (1-cikbar)*kpf(-1)+(cikbar)*invef +
            (cikbar)*(cgamma^2*csadjcost)*qs ;

// sticky price - wage economy

        mc = calfa*rk+(1-calfa)*(w) - 1*a - 0*(1-calfa)*a ;
        zcap = (1/(czcap/(1-czcap)))* rk ;
        rk = w+lab-k ;
        k = kp(-1)+zcap ;
        inve = (1/(1+cbetabar*cgamma))* ( inve(-1) +
            cbetabar*cgamma*inve(1)+(1/(cgamma^2*csadjcost))*pk ) +qs ;
         pk = -r+pinf(1)-0*b
             +(1/((1-chabb/cgamma)/(csigma*(1+chabb/cgamma))))*b +
```

```
     (crk/(crk+(1-ctou)))*rk(1) + ((1-ctou)/(crk+(1-ctou)))*pk(1) ;
c = (chabb/cgamma)/(1+chabb/cgamma)*c(-1) + (1/(1+chabb/cgamma))*c(+1)
    +((csigma-1)*cwhlc/(csigma*(1+chabb/cgamma)))*(lab-lab(+1)) -
    (1-chabb/cgamma)/(csigma*(1+chabb/cgamma))*(r-pinf(+1) + 0*b) +b ;
y = ccy*c+ciy*inve+g + 1*crkky*zcap ;
y = cfc*( calfa*k+(1-calfa)*lab +a );
pinf = (1/(1+cbetabar*cgamma*cindp)) * ( cbetabar*cgamma*pinf(1)
    +cindp*pinf(-1)
      +((1-cprobp)*(1-cbetabar*cgamma*cprobp)/cprobp)/((cfc-1)*curvp+1)*(mc)
          ) + spinf ;
w =  (1/(1+cbetabar*cgamma))*w(-1)
      +(cbetabar*cgamma/(1+cbetabar*cgamma))*w(1)
      +(cindw/(1+cbetabar*cgamma))*pinf(-1)
      -(1+cbetabar*cgamma*cindw)/(1+cbetabar*cgamma)*pinf
      +(cbetabar*cgamma)/(1+cbetabar*cgamma)*pinf(1)
      +(1-cprobw)*(1-cbetabar*cgamma*cprobw)/((1+cbetabar*cgamma)*cprobw)*(1/((cla
      (csigl*lab + (1/(1-chabb/cgamma))*c -
          ((chabb/cgamma)/(1-chabb/cgamma))*c(-1) -w)
      + 1*sw ;
//Inflation average - 4 year future expected inflation
pinfait = 1/16*(pinf + pinf(+1) + pinf(+2) + pinf(+3) + pinf(+4) +
    pinf(+5) + pinf(+6) + pinf(+7) + pinf(+8) + pinf(+9) + pinf(+10) +
    pinf(+11) + pinf(+12) + pinf(+13) + pinf(+14) + pinf(+15));
 //Inflation average - 4 year past realized inflation
pinfait = 1/16*(pinf + pinf(-1) + pinf(-2) + pinf(-3) + pinf(-4) +
    pinf(-5) + pinf(-6) + pinf(-7) + pinf(-8) + pinf(-9) + pinf(-10) +
    pinf(-11) + pinf(-12) + pinf(-13) + pinf(-14) + pinf(-15));
//Inflation average - 8 year future expected inflation
pinfait = 1/32*(pinf + pinf(+1) + pinf(+2) + pinf(+3) + pinf(+4) +
    pinf(+5) + pinf(+6) + pinf(+7) + pinf(+8) + pinf(+9) + pinf(+10) +
    pinf(+11) + pinf(+12) + pinf(+13) + pinf(+14) + pinf(+15)+
    pinf(+16) + pinf(+17) + pinf(+18) + pinf(+19) + pinf(+20) +
    pinf(+21) + pinf(+22) + pinf(+23) + pinf(+24) + pinf(+25) +
    pinf(+26) + pinf(+27) + pinf(+28) + pinf(+29) + pinf(+30) +
    pinf(+31));
//Inflation average - 8 year past realized inflation
pinfait = 1/32*(pinf + pinf(-1) + pinf(-2) + pinf(-3) + pinf(-4) +
    pinf(-5) + pinf(-6) + pinf(-7) + pinf(-8) + pinf(-9) + pinf(-10) +
    pinf(-11) + pinf(-12) + pinf(-13) + pinf(-14) + pinf(-15) +
    pinf(-16) + pinf(-17) + pinf(-18) + pinf(-19) + pinf(-20) +
    pinf(-21) + pinf(-22) + pinf(-23) + pinf(-24) + pinf(-25) +
    pinf(-26) + pinf(-27) + pinf(-28) + pinf(-29) + pinf(-30) +
    pinf(-31));
```

```
       //SW rule
        r = crpi*(1-crr)*pinf
              +cry*(1-crr)*(y-yf)
              +crdy*(y-yf-y(-1)+yf(-1))
              +crr*r(-1)
              +ms ;
      //AIT SW rule
      r = crpi*(1-crr)*pinfait
             +cry*(1-crr)*(y-yf)
             +crdy*(y-yf-y(-1)+yf(-1))
             +crr*r(-1)
             +ms ;
      //Taylor rule
       r   = (1 - crr)*(crpi*pinf + cry*(y - yf))
           + crr*r(-1) + ms;
      //AIT Taylor rule
       r   = (1 - crr)*(crpi*pinfait + cry*(y - yf))
           + crr*r(-1) + ms;


        a = crhoa*a(-1) + ea;
        b = crhob*b(-1) + eb;
        g = crhog*(g(-1)) + eg + cgy*ea;
        qs = crhoqs*qs(-1) + eqs;
        ms = crhoms*ms(-1) + em;
        spinf = crhopinf*spinf(-1) + epinfma - cmap*epinfma(-1);
            epinfma=epinf;
        sw = crhow*sw(-1) + ewma - cmaw*ewma(-1) ;
            ewma=ew;
        kp = (1-cikbar)*kp(-1)+cikbar*inve + cikbar*cgamma^2*csadjcost*qs ;

// measurment equations

dy=y-y(-1)+ctrend;
dc=c-c(-1)+ctrend;
dinve=inve-inve(-1)+ctrend;
dw=w-w(-1)+ctrend;
pinfobs = 1*(pinf) + constepinf;
robs =   1*(r) + conster;
labobs = lab + constelab;

end;
```

```
steady_state_model;
dy=ctrend;
dc=ctrend;
dinve=ctrend;
dw=ctrend;
pinfobs = constepinf;
robs =
    (((1+constepinf/100)/((1/(1+constebeta/100))*(1+ctrend/100)^(-csigma)))-1)*100;
labobs = constelab;
end;

shocks;
var ea;
stderr 0.4618;
var eb;
stderr 1.8513;
var eg;
stderr 0.6090;
var eqs;
stderr 0.6017;
var em;
stderr 0.2397;
var epinf;
stderr 0.1455;
var ew;
stderr 0.2089;
end;



estimated_params;
// PARAM NAME, INITVAL, LB, UB, PRIOR_SHAPE, PRIOR_P1, PRIOR_P2, PRIOR_P3,
    PRIOR_P4, JSCALE
// PRIOR_SHAPE: BETA_PDF, GAMMA_PDF, NORMAL_PDF, INV_GAMMA_PDF
stderr ea,0.4618,0.01,3,INV_GAMMA_PDF,0.1,2;
stderr eb,0.1818513,0.025,5,INV_GAMMA_PDF,0.1,2;
stderr eg,0.6090,0.01,3,INV_GAMMA_PDF,0.1,2;
stderr eqs,0.46017,0.01,3,INV_GAMMA_PDF,0.1,2;
stderr em,0.2397,0.01,3,INV_GAMMA_PDF,0.1,2;
stderr epinf,0.1455,0.01,3,INV_GAMMA_PDF,0.1,2;
stderr ew,0.2089,0.01,3,INV_GAMMA_PDF,0.1,2;
crhob,.2703,.01,.9999,BETA_PDF,0.5,0.20;
```

```
crhoqs,.5724,.01,.9999,BETA_PDF,0.5,0.20;
crhoms,.3,.01,.9999,BETA_PDF,0.5,0.20;
crhopinf,.8692,.01,.9999,BETA_PDF,0.5,0.20;
crhow,.9546,.001,.9999,BETA_PDF,0.5,0.20;
cmap,.7652,0.01,.9999,BETA_PDF,0.5,0.2;
cmaw,.8936,0.01,.9999,BETA_PDF,0.5,0.2;
csadjcost,6.3325,2,15,NORMAL_PDF,4,1.5;
csigma,1.2312,0.25,3,NORMAL_PDF,1.50,0.375;
chabb,0.7205,0.001,0.99,BETA_PDF,0.7,0.1;
cprobw,0.7937,0.3,0.95,BETA_PDF,0.5,0.1;
csigl,2.8401,0.25,10,NORMAL_PDF,2,0.75;
cprobp,0.7813,0.5,0.95,BETA_PDF,0.5,0.10;
cindw,0.4425,0.01,0.99,BETA_PDF,0.5,0.15;
cindp,0.3291,0.01,0.99,BETA_PDF,0.5,0.15;
czcap,0.2648,0.01,1,BETA_PDF,0.5,0.15;
cfc,1.4672,1.0,3,NORMAL_PDF,1.25,0.125;
crpi,1.7985,1.0,3,NORMAL_PDF,1.5,0.25;
crr,0.8258,0.5,0.975,BETA_PDF,0.75,0.10;
cry,0.0893,0.001,0.5,NORMAL_PDF,0.125,0.05;
crdy,0.2239,0.001,0.5,NORMAL_PDF,0.125,0.05;
constepinf,0.7,0.1,2.0,GAMMA_PDF,0.625,0.1;//20;
constebeta,0.7420,0.01,2.0,GAMMA_PDF,0.25,0.1;//0.20;
constelab,1.2918,-10.0,10.0,NORMAL_PDF,0.0,2.0;
ctrend,0.3982,0.1,0.8,NORMAL_PDF,0.4,0.10;
cgy,0.05,0.01,2.0,NORMAL_PDF,0.5,0.25;
calfa,0.24,0.01,1.0,NORMAL_PDF,0.3,0.05;
end;

varobs dy dc dinve labobs pinfobs dw robs;
write_latex_dynamic_model;
write_latex_parameter_table;
write_latex_definitions;
options_.TeX=1;
//for the estimation
estimation(mode_check,order=1,mode_compute=6,datafile=eudata01,first_obs=1,
   bayesian_irf,
   presample=4,lik_init=2,prefilter=0,mh_replic=10000000,mh_nblocks=2,mh_jscale=0.20,mh_drop=0
   mc zcap rk k pk c inve y lab pinf w r g kp yf dy pinfobs robs wf;
shock_decomposition y dy dc dinve labobs pinfobs dw robs;

//for the stochastic simulation
stoch_simul(periods=0,order=1,nodisplay,irf=20,conditional_variance_decomposition=[1:8,20,40,1
   c inve pinf r drate y dy yf output_gap lab w;
```

```
collect_latex_files;
```

D.2 Matlab codes

To run the dynare files for stochastich simulations and save the variances

```
clear all
addpath /Applications/Dynare/4.6.1/matlab

*%path/0-smets-and-wouters-rule
dynare stoch_simul_smets_and_wouters;

%getting moments variances
var = diag(oo_.var);
str = ["c"; "inve" ; "pinf"; "r"; "drate"; "y"; "dy"; "yf"; "output_gap";
    "lab"; "w"];
columns = ["variable", "variance"];
X=[str,var];
T=[columns;X];
cd path/xlsx_var_output
writematrix(T,'***.xlsx');
```

For the IRFs charts

```
clear all
load("***/stoch_simul_smets_and_wouters_results.mat");

y_ea = oo_.irfs.y_ea
pinf_ea = oo_.irfs.pinf_ea
r_ea = oo_.irfs.r_ea
c_ea = oo_.irfs.c_ea
inve_ea = oo_.irfs.inve_ea
lab_ea = oo_.irfs.lab_ea
w_ea = oo_.irfs.w_ea

y_ea = y_ea.'
pinf_ea = pinf_ea.'
r_ea = r_ea.'
c_ea = c_ea.'
inve_ea = inve_ea.'
lab_ea = lab_ea.'
w_ea = w_ea.'
```

```
T_a = table (y_ea, pinf_ea, r_ea, c_ea, inve_ea, lab_ea, w_ea)
filename = "***/irfs_SW.xlsx"
writetable(T_a,filename,"Sheet",1)
```

D.3 Python code

To compute the loss functions

```
#Model 1, Loss function file.
#Using calibrating model of Smets and Wouters work of 2007
#Monetary policy rule: Smets and Wouter (2007)

import pandas as pd
import numpy as np
import glob

path = r'***'
all_files_eu01 = glob.glob(path + "/*.xlsx")

#loading results of the 01 model conditional on schocks
df_01=pd.read_excel(path + "/var_SM.xlsx")
df_aitSW4FL=pd.read_excel(path + "/var_aitSW4FL.xlsx")
df_aitSW4BL=pd.read_excel(path + "/var_aitSM4BL.xlsx")
df_aitSW8FL=pd.read_excel(path + "/var_aitSM8FL.xlsx")
df_aitSW8BL=pd.read_excel(path + "/var_aitSM8BL.xlsx")
df_TR=pd.read_excel(path + "/var_TR.xlsx")
df_aitTR4FL=pd.read_excel(path + "/var_aitTR4FL.xlsx")
df_aitTR4BL=pd.read_excel(path + "/var_aitTR4BL.xlsx")
df_aitTR8FL=pd.read_excel(path + "/var_aitTR8FL.xlsx")
df_aitTR8BL=pd.read_excel(path + "/var_aitTR8BL.xlsx")
df_PLT=pd.read_excel(path + "/var_PLT.xlsx")
df_GR=pd.read_excel(path + "/var_garin_rule.xlsx")

#Compiting the variance of the 01 model conditional on schocks
variances = ['pinf', 'output_gap', 'drate', 'dy']

var_SW=[]
for i in variances:
    var_SW.append(df_01.loc[df_01['variable'] == i, 'variance'].iloc[0])

var_aitSW4FL=[]
```

```
for i in variances:
    var_aitSW4FL.append(df_aitSW4FL.loc[df_aitSW4FL['variable'] == i,
        'variance'].iloc[0])

var_aitSW4BL=[]
for i in variances:
    var_aitSW4BL.append(df_aitSW4BL.loc[df_aitSW4BL['variable'] == i,
        'variance'].iloc[0])

var_aitSW8FL=[]
for i in variances:
    var_aitSW8FL.append(df_aitSW8FL.loc[df_aitSW8FL['variable'] == i,
        'variance'].iloc[0])

var_aitSW8BL=[]
for i in variances:
    var_aitSW8BL.append(df_aitSW8BL.loc[df_aitSW8FL['variable'] == i,
        'variance'].iloc[0])

var_TR=[]
for i in variances:
    var_TR.append(df_TR.loc[df_TR['variable'] == i, 'variance'].iloc[0])

var_aitTR4FL=[]
for i in variances:
    var_aitTR4FL.append(df_aitTR4FL.loc[df_aitTR4FL['variable'] == i,
        'variance'].iloc[0])

var_aitTR4BL=[]
for i in variances:
    var_aitTR4BL.append(df_aitTR4BL.loc[df_aitTR4BL['variable'] == i,
        'variance'].iloc[0])

var_aitTR8FL=[]
for i in variances:
    var_aitTR8FL.append(df_aitTR8FL.loc[df_aitTR8FL['variable'] == i,
        'variance'].iloc[0])

var_aitTR8BL=[]
for i in variances:
    var_aitTR8BL.append(df_aitTR8BL.loc[df_aitTR8FL['variable'] == i,
        'variance'].iloc[0])
```

```
var_PLT=[]
for i in variances:
    var_PLT.append(df_PLT.loc[df_PLT['variable'] == i, 'variance'].iloc[0])

var_GR=[]
for i in variances:
    var_GR.append(df_GR.loc[df_GR['variable'] == i, 'variance'].iloc[0])

#computing the loss function for different lambda (weight of the output_gap)
lambdag = (0.048, 0.25, 1.042)
var=[var_SW, var_aitSW4FL, var_aitSW4BL, var_aitSW8FL, var_aitSW8BL, var_TR,
    var_aitTR4FL, var_aitTR4BL, var_aitTR8FL, var_aitTR8BL, var_PLT, var_GR]
loss_matrix=[]
for v in var:
    for x in lambdag:
        l = v[0] + x*v[1] + v[2]
        loss_matrix.append(l)
loss_matrix=pd.DataFrame(loss_matrix, columns=['model_1'])

lamba=[]
schock=['SW_', 'aitSW4FL_', 'aitSWfBL_', 'aitSW8FL_', 'aitSW8BL_', 'TR_',
    'aitTR4FL_', 'aitTR4BL_', 'aitTR8FL_', 'aitTR8BL_', 'PLT_', 'GR_']
value=['0.048', '0.25', '1.042']
for s in range(len(schock)):
    for v in range(len(value)):
        str= schock[s] + value[v]
        lamba.append(str)
lamba=pd.DataFrame(lamba, columns=['rule_lamba'])
loss_functions_eu01=loss_matrix.join(lamba)
display(loss_functions_eu01)

#prepare the final dataset
numbers = [0.048,0.25,1.042]
loss_functions_eu01['lambda_type'] = np.tile(numbers,
    len(loss_functions_eu01)//len(numbers) + 1)[:len(loss_functions_eu01)]
schocks = pd.Series(['SW', 'aitSW4FL', 'aitSWfBL', 'aitSW8FL', 'aitSW8BL',
    'TR', 'aitTR4FL', 'aitTR4BL', 'aitTR8FL', 'aitTR8BL', 'PLT', 'GR'],
    name="rule").to_frame()
schocks_df = pd.DataFrame(np.repeat(schocks.values,3,axis=0),
    columns=schocks.columns)
loss_functions_eu01 = pd.concat([loss_functions_eu01,schocks_df],axis=1)
```

```
loss_functions_eu01 =loss_functions_eu01.drop(['rule_lamba'], axis=1)

loss_functions_eu01 = loss_functions_eu01.pivot(index="rule",
    columns="lambda_type", values="model_1")
#Final dataset to excel
loss_functions_eu01.to_excel("loss_functions_SW_estimated_with_mode.xlsx")
```