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TESI DI LAUREA

Optimal Investment with Incentives for Renewable Energy Communities: a Stochastic Approach

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Chapter 1

Abstract

After the European Union (EU) has introduced a regulation for Renewable Energy Communities (REC) in 2018, the attention on them is growing quickly. Renewable Energy Communities can be composed by citizens, small and medium-sized companies, and local administrations with the goal of self-producting and self-consuming of energy from renewable sources and, at the same time, is a way to increase the efficiency of the energy system and to reduce the environmental pollution.

In this thesis we discuss about a stochastic model for optimizing investment in Renewable Energy Communities. We start from the paper [1] and we focus on a particular type of REC composed by an household and a biogas producer. In this case, the potential demand of the community is given by the household's demand, while both members produce renewable energy. In this type of REC, the biogas producer converts biogas into electricity and sells it in the electricity market, while the biogas that is not transformed into energy can be sold on the gas market. Meanwhile, the household invests in photovoltaic panels to reduce the energy purchased from the market in order to cover its own power demand, and can also sell the energy which is not self-consumed.

The advantage of entering into a REC for both players is that they will be rewarded with a governmental incentive, in particular we use the incentive approved by the italian government by the "Decreto CER" [3].
We set the problem as a leader-follower problem, where the leader decides how to share the incentive, while the followers decide their own optimal installation strategy. Our goal is to find an optimal way to balance investments in renewable energies.

Chapter 2

Introduction

2.1 Environmental issues

Environmental issues are disruptions in the usual function of ecosystems. Further, these issues can be caused by humans (human impact on the environment) or they can be natural. These issues are considered serious when the ecosystem cannot recover in the present situation, and catastrophic if the ecosystem is projected to certainly collapse.
Major current environmental issues may include:

- climate change
- pollution
- biodiversity loss.

2.1.1 Climate change

Climate change refers to long-term shifts in temperatures and weather patterns. Such shifts can be natural, due to changes in the sun's activity or large volcanic eruptions. But since the 1800s, human activities have been the main driver of climate change, primarily due to the burning of fossil fuels like coal, oil and gas. Burning fossil fuels generates greenhouse gas emissions that "chokes" the Earth and makes it suffer by raising temperatures. The main greenhouse gases that are causing climate change include carbon dioxide and methane. These come, for example, from using gasoline for driving a car or coal for heating a building, clearing land and cutting down forests can also release carbon dioxide. Agriculture, oil and gas operations are major sources of methane emissions. Energy, industry, transport, buildings, agriculture and land use are among the main sectors causing greenhouse gases [9].

2.1.2 Pollution

Pollution in our world affects two essential aspects of our planet: air and water. Although their pollutants are emitted in completely different ways, they both harm living organisms.
Air pollution is predominately emitted through the exhaust of motor vehicles and the combustion of fossil fuels, but also animals and vegetation emit some substances which are not naturally part of the atmosphere; whereas water pollution is mostly caused by human involvement. It is the result of industrial waste and environmental accidents. There are main areas of polluting substances that cause disruption or change in the chemical make

up of the world's waters, and effect the aquatic environment. Some basic pollutants include: wastes, radioactive material, sediments, inorganic chemicals, oil, synthetic organic compounds (i.e. pesticides) and toxic metals (i.e. mercury) [10].

2.1.3 Biodiversity loss

Biodiversity loss refers to the decline or disappearance of biological diversity, understood as the variety of living things that inhabit the planet, its different levels of biological organization and their respective genetic variability, as well as the natural patterns present in ecosystems.
The causes of biodiversity loss are multiple: climate change, that impacts biodiversity at various levels like species distribution, population dynamics, community structure and the functioning of the ecosystem; destruction of habitats, that can be the consequence of soil pollution and of changes due to activities such as deforestation; invasive alien species, that are the second biggest cause of loss of biodiversity in the world, and finally overexploitation of natural resources, that is the consumption of natural resources at a speed greater than that of their natural regeneration.
Biodiversity loss has many consequences, not only for the environment, but also for human beings at the economic and health level. Some of these consequence are: extinction of species, proliferation of pests and increase in CO_2 emissions [11].

2.2 Renewable Energies

A large chunk of the greenhouse gases are generated through energy production, by burning fossil fuels to generate electricity and heat. Fossil fuels, such as coal, oil and gas, are by far the largest contributor to global climate change.
About 80% of the global population lives in countries that are net-importers of fossil fuels, that's about 6 billion people who are dependent on fossil fuels from other countries, which makes them vulnerable to geopolitical shocks and crises.

Science says that, to avoid the worst impacts of climate change, emissions need to be reduced by almost half by 2030 and reach net-zero by 2050.
To achieve this, human reliance on fossil fuels should end, so to start to and invest in alternative sources of energy that are clean, accessible, affordable, sustainable, and reliable. Renewable energy sources, provided by the sun, wind, water, waste, and heat from the Earth, are replenished by nature and emit no greenhouse gases or pollutants into the air.
Unlike fossil fuels, which are present in small areas of the world, renewable energy sources are available in all countries, and their potential is yet to be fully harnessed. The International Renewable Energy Agency (IRENA) estimates that 90% of the world's electricity can and should come from renewable energy by 2050.
Renewables offer a way out of import dependency, allowing countries to diversify their economies and protect them from the unpredictable price swings of fossil fuels.[12]

The change of the electricity operator towards renewable sources therefore becomes a fundamental step in the transition towards a more sustainable energy system, actively involving consumers and promoting the adoption of clean energy sources. Diversification of energy supply, together with appropriate government policies and incentives, can accelerate the adoption of renewable energy, creating jobs and stimulating sustainable economic development. In conclusion, the importance of renewable energy in combating climate problems is undeniable and requires a global commitment to guarantee a better future for our planet and future generations.

2.3 Renewable Energy Communities

A possible solution, born in recent years to encourage and spread the use of renewable energies, are Renewable Energy Communities (RECs).

2.3.1 About Renewable Energy Communities

Renewable Energy Communities are a recent concept in the world of sustainable energy and environmentalism. Energy communities enable collective and citizen-driven energy actions to support the clean energy transition. An Energy Community is an association that produces and shares renewable energy, generating and managing cost-effective green energy autonomously, reducing CO2 emissions and energy waste.

The community may be composed of local citizens, businesses, public administrations, small and medium-sized enterprises, etc. basically, any public or private entity that wants to form a Renewable Energy Community, for example, people who live in the same neighborhood and want to develop a REC may do so. They can contribute to increasing public acceptance of renewable energy projects and make it easier to attract private investments in the clean energy transition. Energy communities can be an effective means of re-structuring our energy systems, by empowering citizens to drive the energy transition locally and directly benefit from better energy efficiency, lower bills, reduced energy poverty and more local green job opportunities. Acting as a single entity means energy communities can access all suitable energy markets on a level-playing field with other market actors.

Through the Clean energy for all Europeans package, adopted in 2019, the EU introduced the concept of energy communities in its legislation, notably as citizen energy communities and renewable energy communities. More specifically, the Directive on common rules for the internal electricity market (EU/2019/944) aims to support the uptake of energy communities. It introduced new rules to enable active consumer participation, individually or through citizen energy communities, in all markets, by generating, consuming, sharing or selling electricity, or by providing flexibility services through demand-response and storage. EU countries should enable this through available support schemes, ensuring energy communities can participate on equal footing with larger participants.[13].

2.3.2 Different types of Renewable Energy Communities

There are two tipes of energy communities:

- Renewable Energy Community
- Citizen Energy Community.

The first is based on open and voluntary participation, is autonomous, and is effectively controlled by shareholders or members that are located in the proximity of the renewable energy projects that are owned and developed by that legal entity. It is a restricted membership capable of remaining autonomous from individual members or other traditional market actors that participate in the community as members or shareholders, natural persons and local authorities, who's membership/participation is not their primary economic activity.

The second is based on voluntary and open participation and is effectively controlled by members or shareholders that are natural persons, local authorities, including municipalities, or small enterprises. Electricity directive does not bind energy communities to immediate vicinity. Any actor can participate, but stakeholders involved in large-scale commercial activity where energy is the primary economic activity cannot make decisions: in fact, decision-making powers should be limited to those members or shareholders that are not engaged in large-scale commercial activity in the energy sector [14].

2.4 Renewable Energy Communities in Italy

The new Italian regulations concerning Renewable Energy Communities give a significant boost to distributed generation, encouraging the development of ‘zero-mile’ local energy production and smart grids.
Through the "Decreto CER" [3], the italian goverment introduced an incentive for RECs in order to encourage the use of renewable energies.

In this thesis we propose a simplified version of REC which is composed by a biogas producer and a representative household. The biogas producer contributes with power production, while the household (which could be interpreted as a whole building whose decision is taken by the building administrator) contributes with demand and power production.
In Chapter 3 we introduce a mathematical model based on a REC composed by a biogas producer and an household and we end up with an analytical expression of the incentive.
In Chapter 4 we show the numerical implementation to calculate the incentive and both the biogas producer’s and household’s profit with different market price and demand, and the results obtained from the implementation.
In Chapter 5 it is possible to find the conclusions of the entire work.

Chapter 3

Mathematical model

In this Chapter we describe the mathematical model and the problem formulation of a simplified version of Renewable Energy Communities.
Let us consider two members who together form a simplified REC: one is a biogas producer, the other is an household. The goal of this work is to analyze the possible profit of both members with and without the incentives present in [3], approved by the Italian government and effective from 24^{th} January 2024.

3.1 Introduction to the model and profits without the incentive

3.1.1 Introduction

Consider a complete filtered probability space $(\Omega, \mathcal{F}, \mathcal{F}_t, \mathbb{P})$ satisfying the usual conditions, define W_c, W_v, W_p, W four Brownian motions and define the correlation $\rho_c = Corr(W, W_c)$.

As told before, we consider two members of a REC, a biogas producer and an household. The biogas producer has a total gas production capacity K_g, whose hourly output is equal to $bKg \cdot 1\text{h}$ ($b > 0$ is the conversion factor MW/m^3), which is sold in the market at the gas spot price $(P^p(t))_{t\geq 0}$, that evolves according to a geometric Brownian motion with initial value p as

$$P^p(s) = pe^{\mu_p s + \sigma_p W_p(s)},$$

with $\mu_p \in \mathbb{R}$ and $\sigma_p > 0$.
The biogas producer can transform the gas into electricity through a Gas-to-Power turbine, and sell it at the spot sale electricity price $(X_v^{x_v}(t))_{t\geq 0}$, which can be defined as

$$X_v^{x_v}(s) = x_v e^{\mu_v s + \sigma_v W_v(s)},$$

with $\mu_p \in \mathbb{R}$ and $\sigma_p > 0$.

3.1.2 Profit of both biogas producer and household without the incentives

The turbine that transforms gas into electricity has capacity $y_b \leq \theta_b$, where θ_b is the maximum allowed installation for the biogas producer. Since the biogas producer has a total gas production capacity K_g, the turbine reduces the output of gas that can be sold to the market, then the residual gas output is $(bK_g - y_b)$. So the biogas producer's profit functional is

$$\begin{aligned} J_b^0(x_v, x_c, p, d, y_b, y_h) = \mathbb{E}\left[\int_0^\infty e^{-rs} X_v^{x_v}(s) y_b ds\right] + \\ + \mathbb{E}\left[\int_0^\infty e^{-rs} P^p(s)(bK_g - y_b) ds\right] - c_b y_b, \qquad (3.1) \end{aligned}$$

where c_b is the cost (€/MW) of installing a turbine with capacity y_b, and $r > 0$ is a discount rate.

The household buys all the energy at the purchase electricity price, which follows a geometric Brownian motion

$$X_c^{x_c}(s) = x_c e^{\mu_c s + \sigma_c W_c(s)},$$

with $\mu_c \in \mathbb{R}$ and $\sigma_c > 0$.

We assume that the demand of the community is equal to the household's power demand $(D^d(t))_{t \geq 0}$, defined as

$$D^d(s) = d e^{\mu_d s + \sigma_d W(s)},$$

with $\mu_d \in \mathbb{R}$ and $\sigma_d > 0$.

Moreover, the household can install new photovoltaic panels of capacity $y_h \leq \theta_h$, so the profit of the household is

$$\begin{aligned} J_h^0(x_v, x_c, p, d, y_b, y_h) = \mathbb{E}\left[\int_0^\infty e^{-rs} X_v^{x_v}(s) y_h ds\right] + \\ - \mathbb{E}\left[\int_0^\infty e^{-rs} X_c^{x_c}(s) D^d(s) ds\right] - c_h y_h, \end{aligned} \tag{3.2}$$

where c_h is the installation cost of photovoltaic panels per MW.

3.1.3 Calculation of both profits J_b^0 and J_h^0

Now, we find the optimal installation for both members in absence of incentives. First assume that:

$$r_v = r - \mu_v - \frac{1}{2}\sigma_v^2, \qquad r_p = r - \mu_p - \frac{1}{2}\sigma_p^2,$$
$$r_c = r - \mu_c - \frac{1}{2}\sigma_c^2, \qquad r_d = r - \mu_d - \frac{1}{2}\sigma_d^2,$$
$$r_{cd} = r_c + r_d - r - \rho_c\sigma_c\sigma_d,$$

are strictly positive.

From these, we can calculate

$$\mathbb{E}\left[e^{-rs}X_v^{x_v}(s)\right], \qquad \mathbb{E}\left[e^{-rs}P^p(s)\right], \qquad \mathbb{E}\left[e^{-rs}X_c^{x_c}(s)D^d(s)\right].$$

The first term becomes:

$$\mathbb{E}\left[e^{-rs}X_v^{x_v}(s)\right] = e^{-rs}\mathbb{E}\left[X_v^{x_v}(s)\right] = e^{-rs}\mathbb{E}\left[x_v e^{\mu_v s + \sigma_v W_v(s)}\right] =$$
$$= x_v e^{-rs+\mu_v s}\mathbb{E}\left[e^{\sigma_v W_v(s)}\right] = x_v e^{-rs+\mu_v s+\frac{1}{2}\sigma_v^2 s} = x_v e^{-r_v s}.$$

The second term's computation is similar to the first, so:

$$\mathbb{E}\left[e^{-rs}P^p(s)\right] = e^{-rs}\mathbb{E}\left[pe^{\mu_p s+\sigma_p W_p(s)}\right] =$$
$$= pe^{-rs+\mu_p+\frac{1}{2}\sigma_p^2 s} = pe^{-r_p s}.$$

The last has two correlated terms, in fact:

$$\mathbb{E}\left[e^{-rs}X_c^{x_c}(s)D^d(s)\right] = e^{-rs}\mathbb{E}\left[x_c e^{\mu_c s+\sigma_c W_c(s)} d e^{\mu_d s+\sigma_d W(s)}\right] =$$
$$= dx_c e^{-rs+\mu_c s+\mu_d s}\mathbb{E}\left[e^{\sigma_c W_c(s)+\sigma_d W(s)}\right] =$$
$$= dx_c e^{-rs+\mu_c s+\mu_d s+\frac{1}{2}\sigma_c^2 s+\frac{1}{2}\sigma_d^2 s+\sigma_c\sigma_d\rho_c s} = dx_c e^{-r_{cd}s}.$$

From the previous calculations, we obtain the following proposition.

Proposition 3.3. *The payoffs for the household and for the biogas producer without the incentives are:*

$$J_b^0(x_v, x_c, p, d, y_b, y_h) = y_b g_b + \frac{pbK_g}{r_p},$$
$$J_h^0(x_v, x_c, p, d, y_b, y_h) = y_h g_h - \frac{x_c d}{r_{cd}},$$

where $g_b = \frac{x_v}{r_v} - \frac{p}{r_p} - c_b$ *and* $g_h = \frac{x_v}{r_v} - c_h$.

Proof.

From (3.1), knowing that

$$\mathbb{E}\left[e^{-rs}X_v^{x_v}(s)\right] = x_v e^{-r_v s} \quad \text{and} \quad \mathbb{E}\left[e^{-rs}P^p(s)\right] = pe^{-r_p s},$$

we have

$$J_b^0(x_v, x_c, p, d, y_b, y_h) = \int_0^\infty y_b x_v e^{-r_v s} ds + \int_0^\infty (bK_g - y_b) pe^{-r_p s} ds - c_b y_b =$$
$$= \frac{x_v y_b}{r_v} + \frac{(bK_g - y_b)p}{r_p} - c_b y_b = y_b g_b + \frac{pbK_g}{r_p},$$

and from (3.2), knowing that

$$\mathbb{E}\left[e^{-rs}X_v^{x_v}(s)\right] = x_v e^{-r_v s} \quad \text{and} \quad \mathbb{E}\left[e^{-rs}X_c^{x_c}(s)D^d(s)\right] = dx_c e^{-r_{cd} s},$$

we have

$$J_h^0(x_v, x_c, p, d, y_b, y_h) = \int_0^\infty y_b x_v e^{-r_v s} ds + \int_0^\infty dx_c e^{-r_{cd} s} ds - c_h y_h =$$
$$= \frac{x_v y_b}{r_v} - \frac{x_c d}{r_{cd}} - c_h y_h = y_h g_h - \frac{x_c d}{r_{cd}}.$$

□

3.2 Profits of the biogas producer and of the household with incentives

We now assume that, by creating a REC, the community receives an incentive. Both members contribute to the total power produced by the community: the household provide power y_h by installing solar panels, while the biogas producer contributes with power y_b by installing turbines to transform the gas into electricity. Hence the energy shared and self-consumed by the community at time t can be expressed as

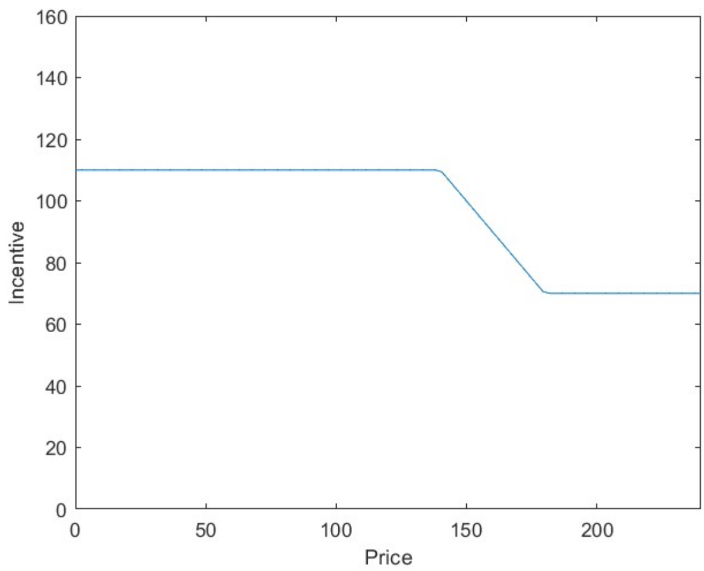
$$min\{D^d(t), y_h + y_b\}.$$

3.2.1 Introduction of the incentives

As we already anticipated, the billing system is organized in a way to offer an incentive to REC, moreover, this incentive is proportional to the self consumed energy of the REC.
As explained in [3], this incentive depends on the power of the system and on the geographical location, we assume to have a system of power > 600 kW and to be in Veneto, so we have a bonus of +10 €/MWh. The incentive provides a reward, dependent on the current market electricity price defined as

$$Z(X_v^{x_v}(s)) = 60 + \max\{0, 180 - X_v^{x_v}(s)\} + 10, \tag{3.4}$$

where $X_v^{x_v}(s)$ is the current market electricity price. This function has a maximum value of 110€/MWh, reached when the price is ≤ 140 and a minimum value of 70 €/MWh, when the price is ≥ 180 as showed in the following graph:



3.2.2 Profits of the biogas producer and the household

Now, we can define the profit of the biogas producer and the household. The profit of the biogas producer is defined as

$$J_b(x_v, x_c, p, d, y_b, y_h, \beta) = J_b^0(x_v, x_c, p, d, y_b, y_h) + (1-\beta)\omega(y_h, y_b, d, x_v), \tag{3.5}$$

where β is the household's proportion of the incentive and $1-\beta$ is that of the biogas producer. Furthermore, J_b^0 is the biogas producer's profit without the incentives (3.1) and $\omega(y_h, y_b, d, x_v)$ is the total revenue from the incentives based on self-consumption

$$\omega(y_h, y_b, d, x_v) = \mathbb{E}\left[\int_0^\infty e^{-rs} Z(X_v^{x_v}(s)) \min\{D^d(s), y_h + y_b\} ds\right]. \tag{3.6}$$

Whereas, the profit of the household is

$$J_h(x_v, x_c, p, d, y_b, y_h, \beta) = J_h^0(x_v, x_c, p, d, y_b, y_h) + \beta\omega(y_h, y_b, d, x_v). \quad (3.7)$$

Now we want to calculate the function $\omega(y_h, y_b, d, x_v)$.
First of all we notice that ω depends on $y_b, y_h \geq 0$ only as parameters, so we can consider ω as a function of d and x_v. Then we have

$$\begin{aligned}\omega(d, x_v) &= \mathbb{E}\left[\int_0^\infty e^{-rs} Z(X_v^{x_v}(s)) \min\left\{D^d(s), y_h + y_b\right\} ds\right] = \\ &= \int_0^\infty e^{-rs} \mathbb{E}\left[Z(X_v^{x_v}(s)) \min\left\{D^d(s), y_h + y_b\right\}\right] ds.\end{aligned}$$

Assuming that $X_v^{x_v}(t)$ and $D^d(t)$ are independent, we obtain

$$\omega(d, x_v) = \int_0^\infty e^{-rs} \mathbb{E}\left[Z(X_v^{x_v}(s))\right] \mathbb{E}\left[\min\left\{D^d(s), y_h + y_b\right\}\right] ds. \quad (3.8)$$

In the following Section, we introduce a method that will help us to calcolate (3.8).

3.3 Fourier methods

In this section we introduce Fourier methods in order to determine the expression of a call option (we follow the method explained in [8]). We present our approach for analytically determining the Fourier transform of the option value and of the time value in terms of the characteristic function of the risk-neutral density.

Consider the problem of evaluating a European call of maturity T, written on the terminal spot price

$$S_T = S_0 e^{\mu T + \sigma W_T}$$

of some underlying asset (where μ is the drift of S_T under the risk-neutral measure $\mathcal{Q}$). The logarithmic price is

$$s_T = \ln(S_T) = \ln(S_0) + \mu T + \sigma W_T$$

So the characteristic function of s_T is defined by:

$$\phi_T(u) = \mathbb{E}\left[e^{ius_T}\right]$$

Now suppose that K is a strike and $k = \ln(K)$, $C_T(k)$ is a call option and $q_T(s)$ is the risk-neutral density of the logarithmic price s_T. Then, the price of a discounted call option with strike $K = e^k$ and maturity T can be expressed as

$$C_T(k) = \mathbb{E}\left[(S_T - K)^+\right] = \int_k^\infty (e^s - e^k) q_T(s) ds,$$

and the characteristic function of the density $q_T(s)$ is

$$\phi_T(u) = \int_{-\infty}^{\infty} e^{ius} q_T(s) ds.$$

Now let

$$c_T(k) = e^{\alpha k} C_T(k) = e^{\alpha k} \int_k^\infty (e^s - e^k) q_T(s) ds \tag{3.9}$$

with $\alpha > 0$, we can define the Fourier transform of $c_T(k)$

$$\psi_T(v) = \int_{-\infty}^{\infty} e^{ivk} c_T(k) dk. \tag{3.10}$$

Substituting (3.9) in (3.10), we obtain

$$\begin{aligned}
\psi_T(v) &= \int_{-\infty}^{\infty} e^{ivk} \int_{k}^{\infty} e^{\alpha k}(e^s - e^k) q_T(s) ds dk = \\
&= \int_{-\infty}^{\infty} q_T(s) \int_{-\infty}^{s} (e^{s+\alpha k} - e^{(\alpha+1)k}) e^{ivk} dk ds = \\
&= \int_{-\infty}^{\infty} q_T(s) \left[\frac{e^{(\alpha+1+iv)s}}{\alpha + iv} - \frac{e^{(\alpha+1+iv)s}}{\alpha + 1 + iv} \right] ds = \\
&= \int_{-\infty}^{\infty} q_T(s) \frac{e^{(\alpha+1+iv)s}}{(\alpha + iv)(\alpha + 1 + iv)} ds = \\
&= \frac{1}{(\alpha + iv)(\alpha + 1 + iv)} \int_{-\infty}^{\infty} e^{i(v-(\alpha+1)i)s} q_T(s) ds,
\end{aligned}$$

and writing the previous expression in terms of the characteristic function ϕ_T, it becomes

$$\psi_T(v) = \frac{\phi_T(v - (\alpha + 1)i)}{(\alpha + iv)(\alpha + 1 + iv)}. \tag{3.11}$$

Now we use the inverse Fourier transform and obtain:

$$\begin{aligned}
C_T(k) &= \frac{e^{-\alpha k}}{2\pi} \int_{-\infty}^{\infty} e^{-ivk} \psi_T(v) dv = \\
&= \frac{e^{-\alpha k}}{2\pi} \int_{-\infty}^{\infty} e^{-ivk} \frac{\phi_T(v - (\alpha + 1)i)}{(\alpha + iv)(\alpha + 1 + iv)} dv.
\end{aligned}$$

Since $\phi_T(v-(\alpha+1)i)$ is the characteristic function of the Gaussian distribution of s_T, i.e. $s_T \sim \mathcal{N}(\ln(S_0)+\mu T, \sigma^2 T)$ (remember that the characteristic function of a Gaussian distribution with mean μ and variance σ^2 is $\phi_x(t) = e^{i\mu t - \frac{1}{2}\sigma^2 t^2}$), we have that

$$\phi_T(v-(\alpha+1)i) = e^{i(\ln(S_0)+\mu T)(v-(\alpha+1)i)-\frac{1}{2}\sigma^2 T(v-(\alpha+1)i)^2}.$$

Finally, we obtain that a discounted call option can be expressed as

$$C_T(k) = \frac{e^{-\alpha k}}{2\pi}\int_{-\infty}^{\infty} e^{-ivk}\frac{e^{i(\ln(S_0)+\mu T)(v-(\alpha+1)i)-\frac{1}{2}\sigma^2 T(v-(\alpha+1)i)^2}}{(\alpha+iv)(\alpha+1+iv)}dv. \tag{3.12}$$

3.4 Calculation of the incentive

Now, we return to equation (3.8) and we write the two means inside the integral in terms of call options.

3.4.1 Computation of $\omega(d, x_v)$

First of all, we can assume that the first term $\mathbb{E}\left[Z(X_v^{x_v}(s))\right]$ is equal to a constant term g, which is the starting value of the incentive, minus a call option plus another call option. In our case, g is the value of the incentive at the initial price $x_v \leq 140$, so $g = 110$. For the expressions of the call options, we use (3.12), with two logarithmic strikes $k_1 = \ln 140$ and $k_2 = \ln 180$, obtained from (3.4). So we have that

$$\mathbb{E}\left[Z(X_v^{x_v}(s))\right] = g - C_s^v(k_1) + C_s^v(k_2),$$

where

$$C_s^v(k_j) = \frac{e^{-\alpha k_j}}{2\pi}\int_{-\infty}^{\infty} e^{-ivk_j}\frac{e^{i(\ln x_v+\mu_v s)(v-(\alpha+1)i)-\frac{1}{2}\sigma_v^2 s(v-(\alpha+1)i)^2}}{(\alpha+iv)(\alpha+1+iv)}dv$$

with $j=1,2$.

Meanwhile the second term $\mathbb{E}\left[\min\left\{D^d(s), y_h+y_b\right\}\right]$ is equal to a function (which we call $D(s)$) minus a call option with strike $k_3=\ln(y_h+y_b)$, so

$$\begin{aligned}\mathbb{E}\left[\min\left\{D^d(s), y_h+y_b\right\}\right] &= \mathbb{E}\left[D^d(s) - C_s^d(k_3)\right] = \\ &= \mathbb{E}\left[de^{\mu_d s+\sigma_d W(s)}\right] - C_s^d(k_3) = \\ &= de^{(\mu_d+\frac{1}{2}\sigma_d^2)s} - C_s^d(k_3) = \\ &= D(s) - C_s^d(k_3), \end{aligned} \tag{3.13}$$

where we defined

$$\begin{aligned} D(s) &= de^{(\mu_d+\frac{1}{2}\sigma_d^2)s}, \\ C_s^d(k_3) &= \frac{e^{-\alpha k_3}}{2\pi}\int_{-\infty}^{\infty} e^{-iwk_3}\frac{e^{i(\ln d+\mu_d s)(w-(\alpha+1)i)-\frac{1}{2}\sigma_d^2 s(w-(\alpha+1)i)^2}}{(\alpha+iw)(\alpha+1+iw)}dw. \end{aligned}$$

Now, since we want to calculate (3.8), we have to calculate the product

$$\begin{aligned} &\mathbb{E}\left[Z(X_v^{x_v}(s))\right]\mathbb{E}\left[\min\left\{D^d(s), y_h+y_b\right\}\right] = \\ &\qquad = (g - C_s^v(k_1) + C_s^v(k_2))(D(s) - C_s^d(k_3)) = \\ &\qquad = gD(s) - gC_s^d(k_3) - C_s^v(k_1)D(s) + \\ &\qquad + C_s^v(k_1)C_s^d(k_3) + C_s^v(k_2)D(s) - C_s^v(k_2)C_s^d(k_3). \end{aligned}$$

Finally, defining

$$
\begin{aligned}
A^{x_v}(v,k_j) &= -ivk_j + (iv+\alpha+1)\ln x_v \qquad j=1,2,\\
A^d(w,k_3) &= -iwk_3 + (iw+\alpha+1)\ln d,\\
B(w) &= (\alpha+iw)(\alpha+1+iw),\\
C^v(v) &= (iv+\alpha+1)\mu_v - \frac{1}{2}\sigma_v^2(v-(\alpha+1)i)^2,\\
C^d(w) &= (iw+\alpha+1)\mu_d - \frac{1}{2}\sigma_d^2(w-(\alpha+1)i)^2,
\end{aligned}
\tag{3.14}
$$

we obtain that (3.8) can be rewritten as

$$
\begin{aligned}
\omega(d,x_v) &= \int_0^\infty e^{-rs}\mathbb{E}\left[Z(X_v^{x_v}(s))\right]\mathbb{E}\left[\min\left\{D^d(s), y_h+y_b\right\}\right]ds =\\
&= gd\int_0^\infty e^{(-r+\mu_d+\frac{1}{2}\sigma_d^2)s}ds - g\frac{e^{-\alpha k_3}}{2\pi}\int_{-\infty}^\infty \frac{e^{A^d(w,k_3)}}{B(w)}\left[\int_0^\infty e^{(-r+C^d(w))s}ds\right]dw+\\
&- d\frac{e^{-\alpha k_1}}{2\pi}\int_{-\infty}^\infty \frac{e^{A^{x_v}(v,k_1)}}{B(v)}\left[\int_0^\infty e^{(-r+C^v(v)+\mu_d+\frac{1}{2}\sigma_d^2)s}ds\right]dv+\\
&+ \frac{e^{-\alpha k_1-\alpha k_3}}{4\pi^2}\int_{-\infty}^\infty\int_{-\infty}^\infty \frac{e^{A^d(w,k_3)+A^{x_v}(v,k_1)}}{B(w)B(v)}\left[\int_0^\infty e^{(-r+C^d(w)+C^v(v))s}ds\right]dvdw+\\
&+ d\frac{e^{-\alpha k_2}}{2\pi}\int_{-\infty}^\infty \frac{e^{A^{x_v}(v,k_2)}}{B(v)}\left[\int_0^\infty e^{(-r+C^v(v)+\mu_d+\frac{1}{2}\sigma_d^2)s}ds\right]dv+\\
&- \frac{e^{-\alpha k_2-\alpha k_3}}{4\pi^2}\int_{-\infty}^\infty\int_{-\infty}^\infty \frac{e^{A^d(w,k_3)+A^{x_v}(v,k_2)}}{B(w)B(v)}\left[\int_0^\infty e^{(-r+C^d(w)+C^v(v))s}ds\right]dvdw.
\end{aligned}
\tag{3.15}
$$

3.4.2 Conditions for α

In order to have finite integrals in (3.15) respect to the variable s, we have to impose some condition on r.

For all $v, w \in \mathbb{R}$ and $\alpha > 0$, we suppose that

$$
\begin{aligned}
& r > \mu_d + \frac{1}{2}\sigma_d^2, \\
& r > Re\{C^d(w)\}, \\
& r > Re\{C^v(v)\} + \mu_d + \frac{1}{2}\sigma_d^2, \\
& r > Re\{C^d(w)\} + Re\{C^v(v)\},
\end{aligned}
$$

where

$$
\begin{aligned}
Re\{C^v(v)\} &= (\alpha+1)\mu_v - \frac{1}{2}\sigma_v^2(v^2 - \alpha^2 - 2\alpha - 1), \\
Re\{C^d(w)\} &= (\alpha+1)\mu_d - \frac{1}{2}\sigma_d^2(w^2 - \alpha^2 - 2\alpha - 1).
\end{aligned}
$$

Since the conditions on r need to be valid for all $v, w \in \mathbb{R}$ and $\alpha > 0$, this means that r have to be grater then the supremum in v and w. So first we calculate the supremum of $Re\{C^d(w)\}$ and $Re\{C^v(v)\}$:

$$
\begin{aligned}
\sup_{w\in\mathbb{R}} \left[Re\{C^d(w)\}\right] &= \sup_{w\in\mathbb{R}} \left[(\alpha+1)\mu_d - \frac{1}{2}\sigma_d^2(w^2 - \alpha^2 - 2\alpha - 1)\right] \\
&= (\alpha+1)\mu_d + \frac{1}{2}\sigma_d^2(\alpha^2 + 2\alpha + 1 + \sup_{w\in\mathbb{R}}\left[-w^2\right]) = \\
&= (\alpha+1)\mu_d + \frac{1}{2}\sigma_d^2(\alpha+1)^2,
\end{aligned}
$$

$$\sup_{v\in\mathbb{R}}\left[Re\{C^v(v)\}\right] = \sup_{v\in\mathbb{R}}\left[(\alpha+1)\mu_v - \frac{1}{2}\sigma_v^2(v^2-\alpha^2-2\alpha-1)\right]$$

$$= (\alpha+1)\mu_v + \frac{1}{2}\sigma_v^2(\alpha^2+2\alpha+1+\sup_{v\in\mathbb{R}}\left[-v^2\right]) =$$

$$= (\alpha+1)\mu_v + \frac{1}{2}\sigma_v^2(\alpha+1)^2.$$

The inequalities for r become

$$r > \sup_{w\in\mathbb{R}}\left[Re\{C^d(w)\}\right] = (\alpha+1)\mu_d + \frac{1}{2}\sigma_d^2(\alpha+1)^2,$$

$$r > \sup_{v\in\mathbb{R}}\left[Re\{C^v(v)\}\right] + \mu_d + \frac{1}{2}\sigma_d^2 =$$

$$= (\alpha+1)\mu_v + \frac{1}{2}\sigma_v^2(\alpha+1)^2 + \mu_d + \frac{1}{2}\sigma_d^2,$$

$$r > \sup_{v,w\in\mathbb{R}}\left[Re\{C^d(w)\} + Re\{C^v(v)\}\right] =$$

$$= \sup_{w\in\mathbb{R}}\left[Re\{C^d(w)\}\right] + \sup_{v\in\mathbb{R}}\left[Re\{C^v(v)\}\right] =$$

$$= (\alpha+1)\mu_d + \frac{1}{2}\sigma_d^2(\alpha+1)^2 + (\alpha+1)\mu_v + \frac{1}{2}\sigma_v^2(\alpha+1)^2 =$$

$$= (\mu_d+\mu_v)(\alpha+1) + \frac{1}{2}(\sigma_d^2+\sigma_d^2)(\alpha+1)^2.$$

These inequalities give us an upper bound for α. In fact:

- from the first inequality we obtain

$$\begin{cases} (-\sqrt{\mu_d^2+2\sigma_d^2 r} - \mu_d - \sigma_d^2)\frac{1}{\sigma_d^2} < \alpha \\ \alpha < (\sqrt{\mu_d^2+2\sigma_d^2 r} - \mu_d - \sigma_d^2)\frac{1}{\sigma_d^2}, \end{cases}$$

- from the second we have

$$\begin{cases}(-\sqrt{\mu_v^2 - 2\sigma_v^2(\mu_d + \frac{1}{2}\sigma_d^2 - r)} - \mu_v - \sigma_v^2)\frac{1}{\sigma_v^2} < \alpha \\ \alpha < (\sqrt{\mu_v^2 - 2\sigma_v^2(\mu_d + \frac{1}{2}\sigma_d^2 - r)} - \mu_v - \sigma_v^2)\frac{1}{\sigma_v^2},\end{cases}$$

- from the third

$$\begin{cases}(-\sqrt{(\mu_d + \mu_v)^2 + 2(\sigma_d^2 + \sigma_v^2)r} - (\mu_d + \mu_v) - (\sigma_d^2 + \sigma_v^2))\frac{1}{\sigma_d^2 + \sigma_v^2} < \alpha \\ \alpha < (\sqrt{(\mu_d + \mu_v)^2 + 2(\sigma_d^2 + \sigma_v^2)r} - (\mu_d + \mu_v) - (\sigma_d^2 + \sigma_v^2))\frac{1}{\sigma_d^2 + \sigma_v^2}.\end{cases}$$

From these inequalities, knowing that $\alpha > 0$, we obtain these conditions for α:

$$\begin{cases}\alpha > 0 \\ \alpha < (\sqrt{\mu_d^2 + 2\sigma_d^2 r} - \mu_d - \sigma_d^2)\frac{1}{\sigma_d^2} \\ \alpha < (\sqrt{\mu_v^2 - 2\sigma_v^2(\mu_d + \frac{1}{2}\sigma_d^2 - r)} - \mu_v - \sigma_v^2)\frac{1}{\sigma_v^2} \\ \alpha < (\sqrt{(\mu_d + \mu_v)^2 + 2(\sigma_d^2 + \sigma_v^2)r} - (\mu_d + \mu_v) - (\sigma_d^2 + \sigma_v^2))\frac{1}{\sigma_d^2 + \sigma_v^2}.\end{cases} \tag{3.16}$$

3.4.3 Condition for r

Now, note that, since $\alpha > 0$, we need that all the three upper bounds for α in (3.16) are grater then zero, this implies the following inequalities:

$$\begin{cases} (\sqrt{\mu_d^2 + 2\sigma_d^2 r} - \mu_d - \sigma_d^2)\frac{1}{\sigma_d^2} > 0 \\ (\sqrt{\mu_v^2 - 2\sigma_v^2(\mu_d + \frac{1}{2}\sigma_d^2 - r)} - \mu_v - \sigma_v^2)\frac{1}{\sigma_v^2} > 0 \\ (\sqrt{(\mu_d + \mu_v)^2 + 2(\sigma_d^2 + \sigma_v^2)r} - (\mu_d + \mu_v) - (\sigma_d^2 + \sigma_v^2))\frac{1}{\sigma_d^2 + \sigma_v^2} > 0. \end{cases}$$

- the first inequalitiy gives us

$$\begin{aligned} &\sqrt{\mu_d^2 + 2\sigma_d^2 r} - \mu_d - \sigma_d^2 > 0 \\ \Leftrightarrow &\sqrt{\mu_d^2 + 2\sigma_d^2 r} > \mu_d + \sigma_d^2 \\ \Leftrightarrow &\mu_d^2 + 2\sigma_d^2 r > \mu_d^2 + 2\mu_d\sigma_d^2 + \sigma_d^4 \\ \Leftrightarrow &r > \mu_d + \frac{1}{2}\sigma_d^2, \end{aligned}$$

  which is the same condition that we found assuming that $r_d > 0$,

- from the second inequality we obtain

$$\begin{aligned} &\sqrt{\mu_v^2 - 2\sigma_v^2(\mu_d + \frac{1}{2}\sigma_d^2 - r)} - \mu_v - \sigma_v^2 > 0 \\ \Leftrightarrow &\sqrt{\mu_v^2 - 2\sigma_v^2(\mu_d + \frac{1}{2}\sigma_d^2 - r)} > \mu_v + \sigma_v^2 \\ \Leftrightarrow &\mu_v^2 - 2\sigma_v^2\mu_d - \sigma_v^2\sigma_d^2 + 2\sigma_v^2 r > \mu_v^2 + 2\mu_v\sigma_v^2 + \sigma_v^4 \\ \Leftrightarrow &r > \mu_d + \frac{1}{2}\sigma_d^2 + \mu_v + \frac{1}{2}\sigma_v^2, \end{aligned}$$

so we find a new condition for r,

- finally, the third inequality gives us

$$
\begin{aligned}
&\sqrt{(\mu_d + \mu_v)^2 + 2(\sigma_d^2 + \sigma_v^2)r} - (\mu_d + \mu_v) - (\sigma_d^2 + \sigma_v^2) > 0 \\
\Leftrightarrow &\sqrt{(\mu_d + \mu_v)^2 + 2(\sigma_d^2 + \sigma_v^2)r} > \mu_d + \mu_v + \sigma_d^2 + \sigma_v^2 \\
\Leftrightarrow &\ \mu_d^2 + 2\mu_d\mu_v + \mu_v^2 + 2\sigma_d^2 r + 2\sigma_v^2 r > (\mu_d + \mu_v + \sigma_d^2 + \sigma_v^2)^2 \\
\Leftrightarrow &\ r > \mu_d + \frac{1}{2}\sigma_d^2 + \mu_v + \frac{1}{2}\sigma_v^2,
\end{aligned}
$$

which is the same condition found in the second inequality.

So we obtain two conditions for r:

$$
\begin{cases}
r > \mu_d + \frac{1}{2}\sigma_d^2 \\
r > \mu_d + \frac{1}{2}\sigma_d^2 + \mu_v + \frac{1}{2}\sigma_v^2,
\end{cases}
\tag{3.17}
$$

where we remember that $\mu_d, \mu_v \in \mathbb{R}$ and $\sigma_d, \sigma_v > 0$.
Notice that the first lower bound is equal to suppose that $r_d > 0$, whereas the second lower bound is a new condition to satisfy in order to have finite integrals in (3.15) when we integrate in the variable s.

3.4.4 Computation of the integrals of $\omega(d, x_v)$ in the variable s

Finally, we are able to calculate the integrals in (3.15) in the variable s, with r satisfying (3.17), and obtain

$$\begin{aligned}\omega(d, x_v) = {} & \frac{gd}{r - \mu_d - \frac{1}{2}\sigma_d^2} - g\frac{e^{-\alpha k_3}}{2\pi}\int_{-\infty}^{\infty}\frac{e^{A^d(w,k_3)}}{B(v)(r - C^d(w))}dw + \\ & - d\frac{e^{-\alpha k_1}}{2\pi}\int_{-\infty}^{\infty}\frac{e^{A^{xv}(v,k_1)}}{B(v)(r - C^v(v) - \mu_d - \frac{1}{2}\sigma_d^2)}dv + \\ & + \frac{e^{-\alpha k_1 - \alpha k_3}}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{e^{A^d(w,k_3)+A^{xv}(v,k_1)}}{B(w)B(v)(r - C^d(w) - C^v(v))}dvdw + \\ & + d\frac{e^{-\alpha k_2}}{2\pi}\int_{-\infty}^{\infty}\frac{e^{A^{xv}(v,k_2)}}{B(v)(r - C^v(v) - \mu_d - \frac{1}{2}\sigma_d^2)}dv \\ & - \frac{e^{-\alpha k_2 - \alpha k_3}}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{e^{A^d(w,k_3)+A^{xv}(v,k_2)}}{B(w)B(v)(r - C^d(w) - C^v(v))}dvdw.\end{aligned} \tag{3.18}$$

From this expression we will calculate the values of $\omega(d, x_v)$ numerically using Matlab.

Chapter 4

Model implementation

4.1 Introduction to the Matlab implementation

In this chapter we want to solve (3.18) for one particular case and to do this, we will use MATLAB. MATLAB (MATrix LABoratory) is a proprietary multi-paradigm programming language and numeric computing environment which allows matrix manipulations, plotting of functions and data, implementation of algorithms, creation of user interfaces, and interfacing with programs written in other languages.

4.1.1 Initial data

In our case we suppose to be in Veneto, so we have the function of the incentive (3.4), and we set the following initial data:

- the maximum allowed installation for the biogas producer is $\theta_b = 0.32$

MW,

- the maximum photovoltaic panels capacity for the household is $\theta_h = 0.2$ MW,
- the total gas production capacity is $K_g = 18.9394$ m^3,
- the conversion factor is $b = 0.01056$ MW/m^3,
- the installation cost of photovoltaic panels is $c_h = 900000$ €/MW,
- the cost of installing a turbine of capacity y_h is $c_b = 2500000$ €/MW,
- the cost of the energy is $x_c = 65$ €/MWh,
- the correlation factor is $\rho_c = 0.01$,
- the gas spot price is $p = 74.7$ €/MWh.

Moreover, we set

$$\mu_d = -7.269 \cdot 10^{-4}, \qquad \sigma_d = 0.03812835,$$

$$\mu_v = -0.0043076, \qquad \sigma_v = 0.09281802,$$

$$\mu_p = -0.35, \qquad \sigma_p = 0.8371437,$$

$$\mu_c = -2.14 \cdot 10^{-6}, \qquad \sigma_c = 0.00128.$$

Now, from (3.17), we calculate the conditions for r (the variables `LB_r1` and `LB_r2`), we obtain that: `LB_r1` $= -1.4463 \cdot 10^{-8}$ and `LB_r2` $= -2.2045 \cdot 10^{-8}$, so we set

$$r = 3.4247 \cdot 10^{-6}.$$

We do the same with α, first we compute the upper bounds from (3.16), which we call `UB_alpha1`, `UB_alpha2` and `UB_alpha3`, obtaining: `UB_alpha1` $= 0.0047$, `UB_alpha2` $= 7.9952 \cdot 10^{-4}$ and `UB_alpha3` $= 6.8416 \cdot 10^{-4}$, then we set

$$\alpha = 1 \cdot 10^{-4}.$$

Finally, we suppose that y_h equal to the maximum photovoltaic panels capacity and y_b equal to the maximum allowed installation, so:

$$y_h = \theta_h, \qquad y_b = \theta_b.$$

4.1.2 Computation of the profits and of the incentives

Then we start two cicles to compute the incentive and the profits with and without the incentive:

- one with the variables x_v varying from 60 to 200 with step 10,
- one with d varying from 0.1 to 2 with step 0.1.

For each of these values, first we compute the profits without the incentive (called `Jh0` and `Jb0`) from Proposition 3.3; then we compute the incentive from (3.18); and finally, from (3.5) and (3.7), we compute the biogas producer and the household profit (called `Jb` and `Jh`) with the incentive, assuming that $\beta = 0.5$.

To calculate the incentive, we compute the integrals in (3.18) one by one, using the MATLAB functions:

- `integral1`, which numerically integrates the function from `xmin` to `xmax`, using global adaptive quadrature and default error tolerances,

- `integral2`, which approximates the integral of the function over the planar region xmin $\leq x \leq$ xmax and ymin $\leq y \leq$ ymax,

where we set xmin = ymin = − M, xmax = ymax = M and M = 10000.
Finally, we plot five surfaces representing the incentive and the profits with and without the incentive.

4.2 Code

In this Section we show the MATLAB code used for the implementation.

4.2.1 Data setting of the problem

```
% Master's thesis in Mathematical Engineering, Ivano Severino

% Fixed data:

% -thetah is the maximum allowed installation for
% the household (MW)
% -thetab is the maximum allowed installation for
% the biogas producer (MW)
% -Kg is the total gas production capacity (m3)
% -b>0 is the conversion factor (MWh/m3)
% -ch is the installation cost of photovoltaic panels
% per MW (€/MW)
% -cb is a cost coefficient of installing a turbine (€/MW)
```

```
% -xc initial value of the purchase electricity price (€/MWh)
% -roc is the correlation factor between cost and demand
% of the electricity
% -p is the initial value of the gas spot price (€/MWh)
% -r is the discount rate (1/h)
% -beta is the share factor of the incentive
% (we assume that is 0.5)

thetah = 0.32;
thetab = 0.2;
Kg = 18.9394;
b = 0.01056;
ch = 2500000;
cb = 900000;
xc = 65;
roc = 0.01;
p = 74.7;
beta = 0.5;

mud = -7.269*10^(-4);
sigmad = 0.03812835;
muv = -0.0043076;
sigmav = 0.09281802;
mup = -0.35;
sigmap = 0.8371437;
muc = -2.14*10^(-6);
```

```
sigmac = 0.00128;

% LB_r1 and LB_r2 are the lower bounds for r,
% we need to have r grater than these values

LB_r1 = mud+sigmad^2*0.5;
LB_r2 = mud+sigmad^2*0.5+muv+sigmav^2*0.5;

r = 3.4247*10^(-6);
rv = r-muv-sigmav^2*0.5;
rp = r-mup-sigmap^2*0.5;
rd = r-mud-sigmad^2*0.5;
rc = r-muc-sigmac^2*0.5;
rcd = rc+rd-r-roc*sigmac*sigmad;

% UB_alpha1, UB_alpha2, UB_alpha3 are the three upper bounds
% for alpha

UB_alpha1 = (sqrt(mud^2+2*sigmad^2*r)-mud-sigmad^2)/sigmad^2;
UB_alpha2 = (sqrt(muv^2-2*sigmav^2*(mud+0.5*sigmad^2-r))-...
    muv-sigmav^2)/sigmav^2;
UB_alpha3 = (sqrt((mud+muv)^2+2*(sigmad^2+sigmav^2)*r)-...
    (mud+muv)-(sigmad^2+sigmav^2))/(sigmad^2+sigmav^2);

% -M is the value that we use in the definite integrals
% yh if the capacity of the photovoltaic panels
```

```
% yb is the capacity if a gas turbine

M = 10000;
yh = thetah;
yb = thetab;

% k1, k2, k3 are three strike values
% g is the maximum value of the incentive

xmin = -M;
xmax = M;
ymin = -M;
ymax = M;
k1 = log(140);
k2 = log(180);
k3 = log(yh+yb);
g = 110;
alpha = 1e-4;
```

4.2.2 Computation of the incentive and the profits

```
% I use "contatored" and "contatorexv" as indices to create
% matrices

contatored = 0;
```

```
% Loop on d and xv to obtain different values of the
% incentive

for d = 0.1:0.1:2
contatored = contatored+1;
contatorexv = 0;
for xv = 60:10:200
contatorexv = contatorexv+1;

gb = xv/rv-p/rp-cb;
gh = xv/rv-ch;

% -Jb0 is the profit of the biogas producer without the
% incentive
% -Jbh is the profit of the household without the incentive

Jh0(contatored,contatorexv) = yh*gh-xc*d/rcd;
Jb0(contatored,contatorexv) = yb*gb+p*b*Kg/rp;

% I define a function handle in order to calculate
% the integrals using the matlab function "integral"
% for single integrals and the function
% "integral2" for double integrals.
% For double integrals i used iterated method with
% tollerance = 10^-8
```

```
int1 = @(w) exp(-i*w*k3+(i*w+alpha+1)*log(d))/((alpha+i*w)*...
    (alpha+1+i*w)*(r-((alpha+1+i*w)*mud-...
    (w^2-2*i*alpha*w-2*i*w-alpha^2-2*alpha-1)*sigmad^2*0.5)));

omega1 = integral(int1,xmin,xmax,'ArrayValued',true);



int2 = @(v) exp(-i*v*k1+(i*v+alpha+1)*log(xv))/((alpha+i*v)*...
    (alpha+1+i*v)*(r-((alpha+1+i*v)*muv-(v^2-2*i*alpha*v-...
    2*i*v-alpha^2-2*alpha-1)*sigmav^2*0.5)-mud-sigmad^2*0.5));

omega2 = integral(int2,xmin,xmax,'ArrayValued',true);



int3 = @(w,v) exp(-i.*w*k3+(i.*w+alpha+1)*log(d)-i.*v*k1+...
    (i.*v+alpha+1)*log(xv))./((alpha+i.*w).*(alpha+1+...
    i.*w).*(alpha+i.*v).*(alpha+1+i.*v).*(r-((alpha+1+...
    i.*w)*mud-(w.^2-2*i*alpha.*w-2*i.*w-alpha^2-2*alpha-...
    1)*sigmad^2*0.5)-((alpha+1+i.*v)*muv-(v.^2-...
    2*i*alpha.*v-2*i.*v-alpha^2-2*alpha-1)*sigmav^2*0.5)));

omega3 = integral2(int3,xmin,xmax,ymin,ymax,'Method',...
    'iterated','AbsTol',1e-5,'RelTol',1e-6);



int4 = @(v) exp(-i*v*k2+(i*v+alpha+1)*log(xv))/...
```

```
    ((alpha+i*v)*(alpha+1+i*v)*(r-((alpha+1+i*v)*muv-...
    (v^2-2*i*alpha*v-2*i*v-alpha^2-2*alpha-1)*sigmav^2*...
    0.5)-mud-sigmad^2*0.5));

omega4 = integral(int4,xmin,xmax,'ArrayValued',true);



int5 = @(w,v) exp(-i.*w*k3+(i.*w+alpha+1)*log(d)-i.*v*k2+...
    (i.*v+alpha+1)*log(xv))./((alpha+i.*w).*(alpha+1+...
    i.*w).*(alpha+i.*v).*(alpha+1+i.*v).*(r-((alpha+1+...
    i.*w)*mud-(w.^2-2*i*alpha.*w-2*i.*w-alpha^2-2*alpha-...
    1)*sigmad^2*0.5)-((alpha+1+i.*v)*muv-(v.^2-...
    2*i*alpha.*v-2*i.*v-alpha^2-2*alpha-1)*sigmav^2*0.5)));

omega5 = integral2(int5,xmin,xmax,ymin,ymax,'Method',...
    'iterated','AbsTol',1e-5,'RelTol',1e-6);

% omega is the function of the incentive

omega = g*d/(r-mud-0.5*sigmad^2)-g*exp(-alpha*k3)/(2*pi)*...
    omega1-d*exp(-alpha*k1)/(2*pi)*omega2+exp(-alpha*k3-...
    alpha*k1)/(4*pi^2)*omega3+d*exp(-alpha*k2)/(2*pi)*...
    omega4-exp(-alpha*k3-alpha*k2)/(4*pi^2)*omega5;

asse_d(contatored) = d;
asse_xv(contatorexv) = xv;
```

```
% Real_omega is the real part of the incentive

Real_omega(contatored,contatorexv) = real(omega);

end
end

% Now I calculate both the profit of the biogas producer Jb
% and the household Jh with the incentive.

Jb = Jb0+(1-beta)*Real_omega;
Jh = Jh0+beta*Real_omega;
```

4.2.3 Graphic outputs

```
% I plot the graphs for the incentive, and for both the
% profits.

figure1 = figure;
surf(asse_xv,asse_d,Jb0);
xlabel('price')
ylabel('demand')
zlabel('BiogasProducer profit no incentive')
colorbar
```

```
figure2 = figure;
surf(asse_xv,asse_d,Jh0);
xlabel('price')
ylabel('demand')
zlabel('Household profit no incentive')
colorbar

figure3 = figure;
surf(asse_xv,asse_d,Real_omega);
xlabel('price')
ylabel('demand')
zlabel('incentive')
colorbar

figure4 = figure;
surf(asse_xv,asse_d,Jb);
xlabel('price')
ylabel('demand')
zlabel('BiogasProducer profit')
colorbar

figure5 = figure;
surf(asse_xv,asse_d,Jh);
xlabel('price')
ylabel('demand')
zlabel('Household profit')
```

```
colorbar
```

4.3 Graphics and results

In this section, we explain the results obtaining from the implementation with the set of data showed in subsection 4.1.1.

4.3.1 Outputs for the household's and the biogas producer's profit without the incentive

First we show two graphs referred to the household and the biogas producer profit without the presence of the incentive (i.e. J_h^0 and J_b^0):

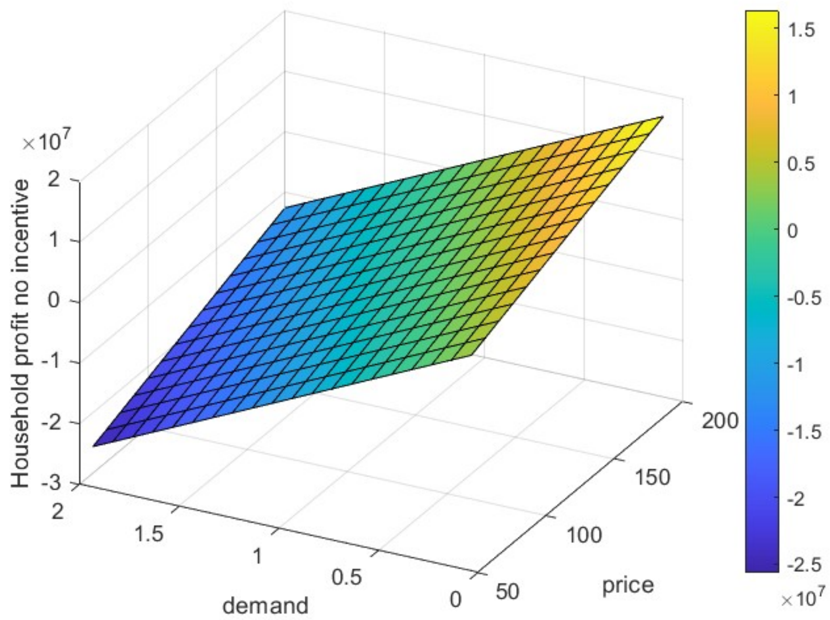


Figure 4.1: graph of the household's profit without the incentive

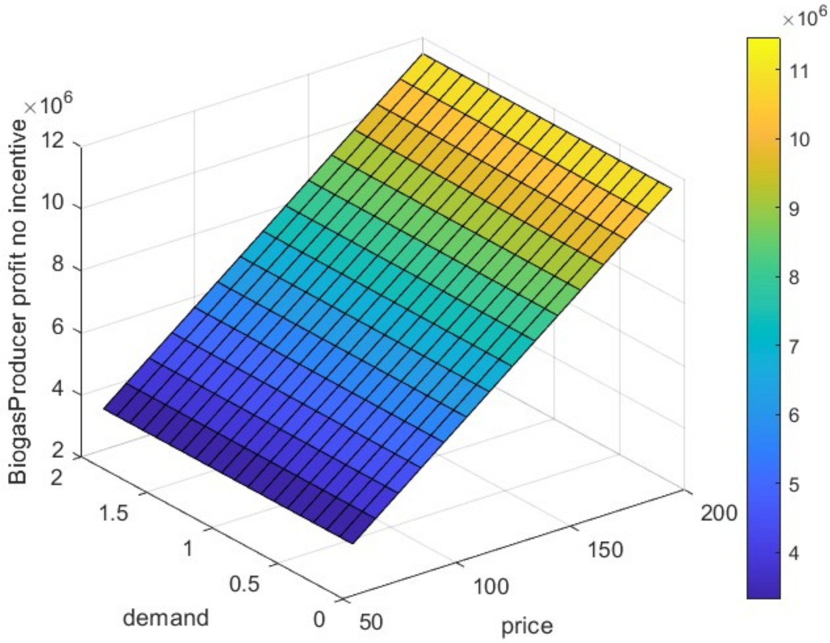


Figure 4.2: graph of the biogas producer's profit without the incentive

4.3.2 Outputs for the incentive

Now, we show the results of the computation of the incentive.

First let us consider the function

$$\bar{\omega}(d) = \int_0^\infty e^{-rs}\mathbb{E}\left[\min\{D^d(s), y_h + y_b\}\right] ds,$$

which is the total expected unitary revenue from the incentives based on self-consumption defined in [1]. From (3.13), we know that

$$\mathbb{E}\left[\min\{D^d(s), y_h + y_b\}\right] = D(s) - C_s^d(k_3) =$$
$$= de^{(\mu_d + \frac{1}{2}\sigma_d^2)s} - \frac{e^{-\alpha k_3}}{2\pi}\int_{-\infty}^{\infty} \frac{e^{A^d(w,k_3)+C^d(w)s}}{B(w)}dw,$$

(using the notation (3.14)) and this implies that

$$\bar{\omega}(d) = \int_0^{\infty} e^{-rs} \left[d e^{(\mu_d + \frac{1}{2}\sigma_d^2)s} - \frac{e^{-\alpha k_3}}{2\pi} \int_{-\infty}^{\infty} \frac{e^{A^d(w,k_3)+C^d(w)s}}{B(w)} dw \right] ds =$$

$$= d \int_0^{\infty} e^{(\mu_d + \frac{1}{2}\sigma_d^2 - r)s} ds - \frac{e^{-\alpha k_3}}{2\pi} \int_{-\infty}^{\infty} \frac{e^{A^d(w,k_3)}}{B(w)} \int_0^{\infty} e^{(C^d(w)-r)s} ds dw =$$

$$= \frac{d}{r - \mu_d - \frac{1}{2}\sigma_d^2} - \frac{e^{-\alpha k_3}}{2\pi} \int_{-\infty}^{\infty} \frac{e^{A^d(w,k_3)}}{B(w)(r - C^d(w))} dw.$$

Defining $Z_1 = 70$ and $Z_2 = 110$ and implementing numerically $Z_1 \cdot \bar{\omega}(d)$ and $Z_2 \cdot \bar{\omega}(d)$ with d varying from 0.1 to 2, we obtain two curves, which are respectively a lower and an upper bound of the incentive function $\omega(d, x_v)$ defined in (3.8), as showed in the following graph:

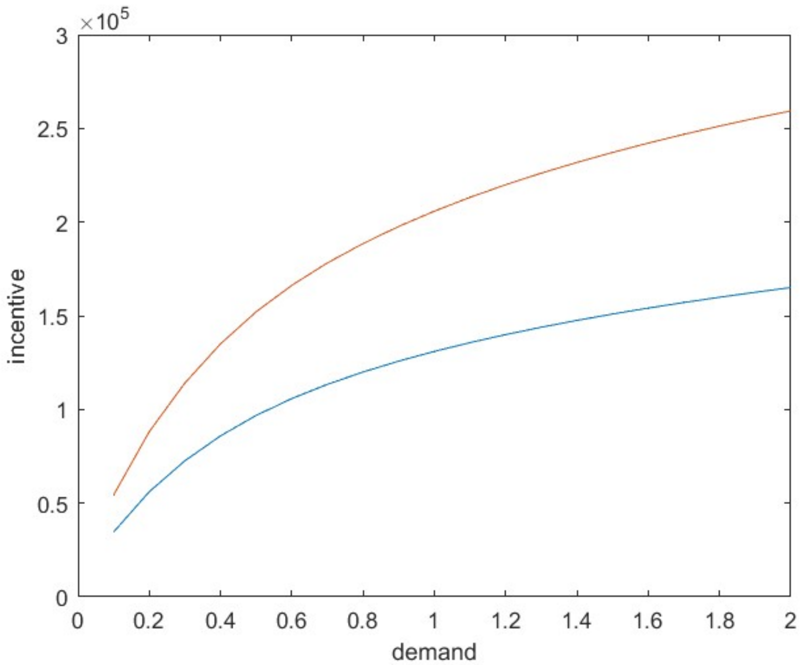


Figure 4.3: maximum and minimum value of the incentive

We can finally show the surface of the incentive with d varying from 0.1 to 2 with step 0.1, and x_v varying from 60 to 200 with step 10:

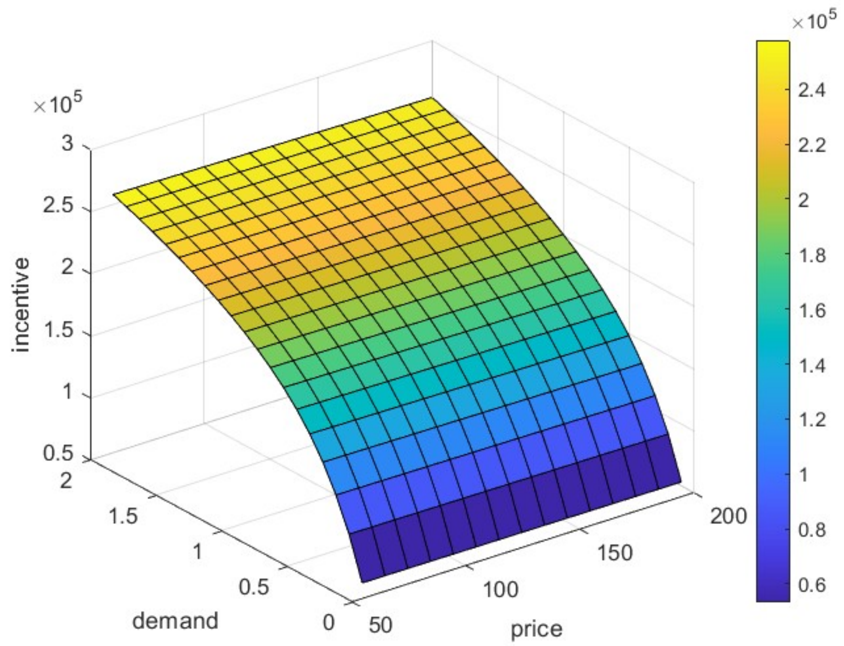


Figure 4.4: graph of the incentive with respect to d and x_v, note that it increases as the demand increases

4.3.3 Outputs for the household's and for the biogas producer's profit with the incentive

Finally, we show the profits of the household and of the biogas producer with the presence of the incentive.

In figure 4.5 we have the surface of the household's profit:

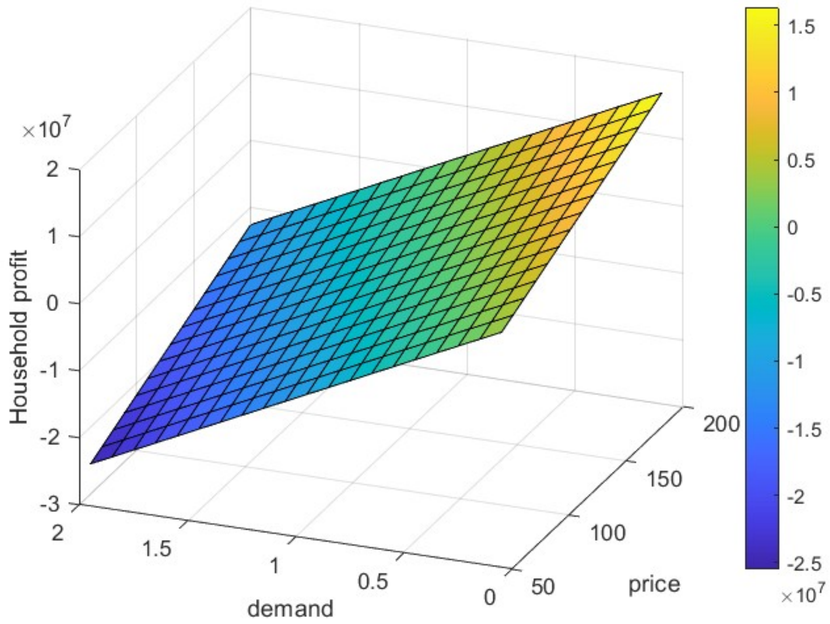


Figure 4.5: graph of the household's profit with the incentive

Notice that the more the initial price raises, the more the household's profit increases, while the profit decreases as the demand increases. This is due to the definition of J_h^0 (3.2), where the demand affects negatively the profit of the household.

Then, in figure 4.6, we have the surfaces of the biogas producer profit with the incentive:

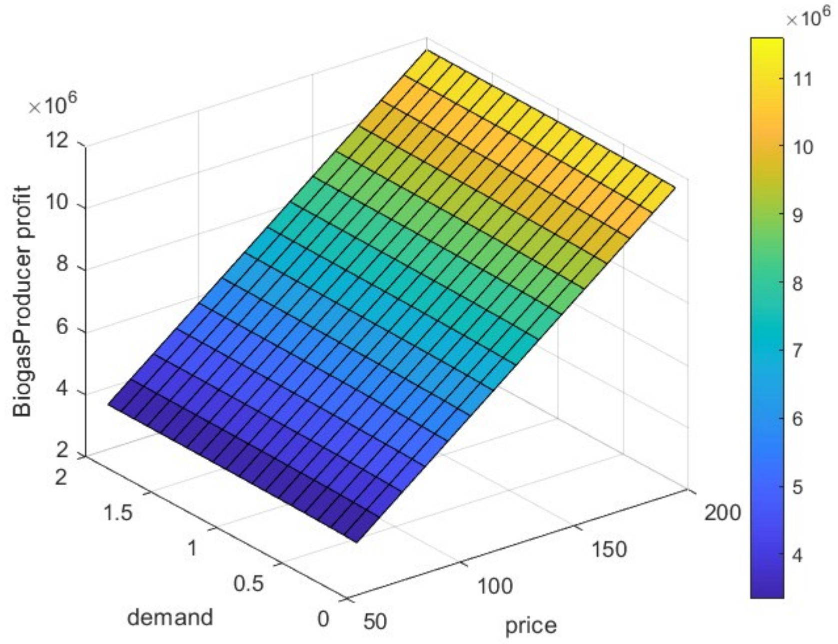


Figure 4.6: graph of the biogas producer's profit without the incentive

Here we can notice that the biogas producer's profit increases as the demand and the initial price increase.

Chapter 5

Conclusions

In approaching the conclusion of this discussion, we summarize the key points and the results of this work.

We start from the environmental issues and the importance of renewable energies, then we introduce Renewable Energy Communities and finally we explain the goal of this work.

In this thesis we begin from the model introduced in [1], which examines a particular type of REC composed of a "representative" household and a biogas producer, where the potential demand of the community is given by the household's demand, while both members produce renewable energy. The biogas producer invests in technology to convert biogas into electricity and sell it in the electricity market at the spot price. However the biogas that is not transformed into energy can be sold on the market at the gas spot price. The household invests in photovoltaic panels to reduce the energy purchased from the market in order to cover its own power demand, moreover the household has the possibility to sell the excess of energy not used for self-consumption. The advantage of entering into a REC for both players is that their joint self-consumption is rewarded with a governmental incentive.

In this work we analyze the total expected revenue from the incentive based on self-consumption and the profits of this simplified REC with the Italian government incentive taken from [3]. We begin defining the gas and electricity spot prices and the household's power demand, afterwards we define the biogas producer's and household's profit functionals without the presence of the incentive. Then we introduce the total expected revenue from the incentive based on self-consumption and we define the profit of the biogas producer and the household with the incentive.

Now we start to compute the function of the incentive, first with an analytical approach, where we end up with a particular exspression of the incentive, then with a numerical approach, where we compute numerically the function of the incentive in particular case. Finally, we show the obtained results through graphs of the incentive and of the profits of both members with and without the incentive.

We conclude with the hope that this work can help and speed up the transition to the use of renewable energies with the help of Italian and European government incentives. Only through collective effort and shared will can we effectively address climate problems and build a better future for generations to come.

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