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Master's Degree in Control Systems Engineering
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CONTROL ARCHITECTURES FOR ENERGY COMMUNITIES

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Contents

In questa tesi proponiamo diverse soluzioni ad un problema di ottimizzazione distribuita per la gestione efficiente delle comunità energetiche (CEs). In particolare ci concentriamo sul caso italiano in cui gli agenti delle CEs possono ricevere un incentivo monetario basato sulla quantità di “energia condivisa”.
In una comunità l’energia rinnovabile prodotta dai membri dotati di fonti energetiche rinnovabili (RESs) può essere “condivisa” anche con membri non fisicamente collegati e che quindi possono consumare energia ad un costo ridotto grazie agli incentivi statali specificati secondo norme specifiche.
C’è inoltre la possibilità di equipaggiare i membri con sistemi di accumulo dell’energia ovvero delle batterie (BESSs).
Nel modello si presuppone che ciascun membro sia dotato di nessuno, solo uno o entrambi questi sistemi di produzione e stoccaggio di energia rinnovabile.
Vengono proposti modelli di ottimizzazione distribuita cooperativa che costituiscono la base per il controllo predittivo distribuito di una rete di BESSs che modifica la quantità oraria di energia condivisa consumata dalla comunità. In questo modo è possibile preservare la riservatezza dei dati di consumo degli utenti.
Confrontando ciascun approccio analizziamo se ci sono benefici riguardanti il costo dell’energia per la comunità e se la cooperazione tra i BESSs sia vantaggiosa.
Valutiamo inoltre quanto la realizzazione di comunità energetiche consenta di ridurre le emissioni di CO2 nell’atmosfera.

Abstract

In this thesis we propose different solutions to a distributed optimization problem for the efficient management of energy communities (ECs). In particular we focus on the Italian case where agents of ECs can receive a monetary incentive based on the amount of "shared energy".

In a community the renewable energy produced by members equipped with renewable energy sources (RESs) can be "shared" also with not physically connected members which can consume energy at a reduced cost due to state incentives according to specific rules.

There is also the possibility to equip members with battery energy storage systems (BESSs). The model assumes that each member has none, only one or both of these renewable energy production and storage systems.

Cooperative distributed optimization models are proposed and they are the basis for distributed predictive control of a network of BESSs that changes the hourly amount of shared energy consumed in the community. In this way it is possible to preserve privacy of user consumption data.

By comparing each approach we analyze if there are benefits regarding the cost of energy for a community and if the cooperation among BESSs owned by the members is beneficial.

We also evaluate how much the creation of energy communities allows us to reduce CO2 emissions into the atmosphere.

Chapter 1

Energy Communities

Nowadays it is impossible to remain indifferent to climate changes that are occurring on our planet. Catastrophic events as intense droughts, water shortages, fires, floods and strong storms have been increasing in recent years. Atmospheric temperatures are rising uncontrollably causing global warming which is responsible for melting polar ice and consequently for rising sea levels. All of this not only causes many problems for humans, just think of the continuous hardships in the agricultural sector, but it also causes a reduction in biodiversity.
One example is the strong reduction in the number of polar bears that are forced to migrate in other regions due to food shortage. This travel in most cases consists in long swims that, often, could be fatal to them. An other proof of these climate changes is the persevering drought that is afflicting some African countries in recent years causing a decimation of local wildlife.
Not to mention the continuous increase of CO2 in the atmosphere. This phenomena induces also ocean acidification and, together with the warming of the oceans, it is responsible for disasters for aquatic flora and fauna: large swaths of tropical coral reefs in the three major ocean basins are under extreme stress causing coral bleaching. An other example could be the continuous change in the migratory routes of the whales, sharks and sea turtles that are not able to adapt themselves as quickly as changes occur.
Fortunately all these types of problems have alarmed world governments who have therefore begun to introduce measures to try to limit damages at least in long term. During the United Nations Framework Convention on Climate Change (COP21)

held in Paris in 2015 almost all country on the world signed up to a plan to fight global warming. COP21 suggested that global warming can be reduced applying incisive energy policies as increasing the fossil flues costs and pointing to encourage low carbon technologies. After COP21 some benefits have been obtained but it is still not enough. As described in the Intergovernmental Panel on Climate Change (IPCC) report [7] of 2023, objectives previously set will not be reached if other improvements are not quickly realized. As analyzed it is obvious that only if we intervene immediately reducing the CO2 emissions in the next years we will be able to maintain the mean rising global temperature below $2°C$ compared to pre-industrial era. Furthermore the report suggests that one possibility to reduce global warming and CO2 emissions is to immediately implement an energy transition. In fact, as reported in the data published by the World Resources Institute (WRI), it is possible to observe that the energy sector is the one that emits more CO2 than the others. It is responsible for approximately 73% of CO2 emissions in the world. In fig. 1.1 taken from [9] it is represented global greenhouse gas emissions by sector in 2016 but nowadays it is not much different. To limit this phenomena there are many possibilities. Firstly, in order to reduce the dependency from the the fossil flues, it is important to increase the energy produced by: photovoltaic panels, hydroelectric turbines, wind turbines and all the other elements that consent to produce energy in low carbon mode obtaining the so-called "green energy". Then it is important that this produced energy is well distributed avoiding losses and making it accessible to all those who need it. In this approach it is also convenient to consider the economic aspect since green energy is cheaper than the one produced using fossil flues. These strategies in fact paved the way for the realization of smart-grids and energy communities.

1.1 What are Energy Communities?

Although they were not yet called energy communities, aggregations based on the concept of shared energy were created for the first time in Denmark in the 1970s with the installation of some wind farms managed by cooperatives of citizens interested in promoting renewable energy. In the following years some other realizations of this type took place in Europe but the real energy communities, as we know them

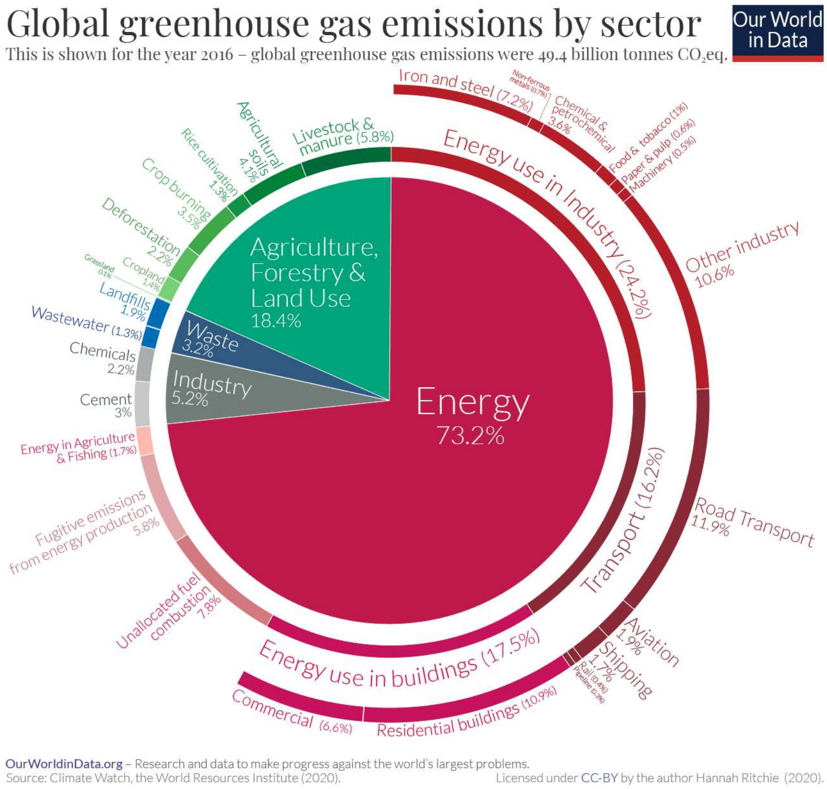

Figure 1.1: Global greenhouse gas emissions by sector.

today, grew up and developed in the 2000s when the liberalization of the energy market and continuous technological innovation allowed rapid growth of Renewable Energy Communities (RECs). In fig. 1.2[1] it is represented an example of energy community.
Nowadays in Italy the Ministry of the Environment and Energy Security, publishing the CER decree of 7 December 2023, is allocating considerable funds to achieve an energy transition and encourage the creation of energy communities.

But roughly speaking... what are energy communities?

By definition energy communities are non-profit legal entities whose members share in the production, distribution and use of renewable energy at a local level. Through

[1] Taken from `https://www.comune.assisi.pg.it/le-comunita-energetiche-rinnovabili/`

Figure 1.2: Example of energy community.

certain incentives, their members can save on energy expenditures and hence obtaining a reduction in bill costs. There are no restrictions on the type of building that can make up this community and renewable generators can be installed in public and private buildings indiscriminately, they just need to be built close to its consumer-members. Components of an energy community can also be equipped with a battery to store energy to be used when it will be necessary. The presence of a storage unit is very convenient for the community since it is possible to obtain greater exploitation and better management of energy produced from renewable sources, obtaining greater self-consumption. In addiction it consents to reduce peaks in power sold to the grid and imbalances due to renewable sources which do not guarantee a continuous supply of energy.
Members equipped with renewable generators and storage units are also called prosumers. Prosumer is an abuse of notation and it was created combining the two words: consumer and producer. This is due to fact that this type of members do not have the passive mode only, typical of a consumer, that consents to only buy energy but they can also produce it becoming in this way prosumers.
So a prosumer is an active protagonist in the management of energy flows. The

two main benefits to be a prosumer are the relative autonomy and the economic return.

How do energy communities work?

The general idea behind these communities is that each prosumer uses the generated energy to satisfy its energy needs and the surplus is stored in a battery, if it is present, or shared among the community. If the total amount of shared energy is grater than the community consumption the surplus is sold to the main grid and the grid operator will reward the community which determines how to share revenues among its members. This procedure is typically managed by optimization algorithms that are able to obtain high efficiency considering all the possible changes that could happen during the operation time of the community. This mechanism consents the members to enjoy energy at a lower cost than the one they would purchase directly from the main grid operator.

What are the main benefits of energy communities?

Bill costs reduction is not the only benefit regarding these aggregations. In fact the main advantage that can be obtained from energy communities concerns the ecological field since they are able to significantly reduce the CO2 emission in the atmosphere encouraging renewable energy. Furthermore they consent to reduce energy losses in the main grid since energy is shared locally and it has not to travel long distances to reach the final destination. In addiction citizens that do not have the possibility to install renewable generators can be part of the community too and they can also enjoy the profits of being a member of the community.

... So what at the end?

To obtain maximum results from energy communities it is important to make them easy to access for anyone who wants to be part of them. Government has to raise awareness about the importance of this new type of energy management and it must encourage citizens to be part of it. In addition government has to support anyone who need an help with government incentives for installation of renewable energy generators in order to obtain bigger aggregations with the obvious consequence of strong increasing all the benefits listed above. Each citizen must be assisted in the economic and bureaucratic fields otherwise the risk is that this technology becomes a monopoly of big private energy companies only helping themselves to increase their revenues.

In conclusion, the creation of energy communities in a correct way surely represents a precious weapon against global warming. It also raises awareness among future generations and citizens around the world on how important it is to take care of our planet by intelligently using the numerous technological innovations of recent years.

Chapter 2

An Energy Community Model

In this section we introduce the developed model of an energy community on which all the control architectures described in the following chapters will be implemented. For all details regarding used notation, please refer to the dedicated chapter "Notations".

2.1 Model of the Energy Community

In order to implement a model of an energy community we consider that a prosumer, as mentioned before, can satisfy its energy needs producing power thanks to a Renewable Generator (RG). In addition it is equipped with a Battery Energy Storage System (BESS) that has the role of storing the excess energy produced. We exam an energy community composed by $n \in \mathbb{N}$ interconnected prosumers. It is possible to consider this aggregation as an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ where nodes ($\mathcal{V}$) represent the agents that make up the energy community and the edges ($\mathcal{E}$) represent the interconnections among them. So at this point it is important to define three quantities regarding each prosumer $i \in \mathcal{V}$, with $i = 1, ..., n$:

- $c_i(t) \in \mathbb{R}_{\geq 0}$ represents the power consumption [kW]
- $g_i(t) \in \mathbb{R}_{\geq 0}$ represents the power generation [kW]
- $e_i(t) \in \mathbb{R}_{\geq 0}$ represents the energy storage [kWh]

In the following these three elements are analyzed in detail.

2.1.1 Power Consumption

The consumption is the power that a prosumer needs in order to satisfy the energy requests of all loads embedded in it as lights, appliances or industrial machinery, depending on the type of building that we are considering.
Loads can be divided in two main categories:

Controllable: This type of loads consent user to select a time window, indicating a starting and finishing time, in which they can be turned on at any time. The only important thing is that a load has to finish its work strictly inside the selected time window.

Non-Controllable: Unlike previous ones the time in which loads are turned on is fixed and hence it can not be modify by a user.

So these considerations suggest that the power consumption $c_i(t) \in \mathbb{R}_{\geq 0}$ can be split into two components: $c_i^c(t)$ and $c_i^n(t)$, representing the consumption of controllable and non-controllable loads respectively as reported in equation (2.1).

$$c_i(t) = c_i^c(t) + c_i^n(t) \tag{2.1}$$

2.1.2 Power Generation

The generated power $g_i(t) \in \mathbb{R}_{\geq 0}$ is produced by RGs. Reminding that it can be used to satisfy the energy requests of the prosumer or it can be sold to the grid if there is a surplus, it is possible to divide the power generation in two components obtaining equation (2.2).

$$g_i(t) = g_i^c(t) + g_i^s(t) \tag{2.2}$$

Where $g_i^c(t)$ and $g_i^s(t)$ represent the part of the generated power consumed by the user and sold to the grid respectively.
It is important to observe that these two quantities are complementary and consequently it is possible to obtain the following relation (2.3):

$$\begin{cases} {g_i}^c(t) = \alpha_g(t) g_i(t), \\ {g_i}^s(t) = (1 - \alpha_g(t)) g_i(t) \end{cases} \tag{2.3}$$

Where $\alpha_g(t) \in [0,1]$ is a variable useful to determine the amount of $g_i^c(t)$ and $g_i^s(t)$.

2.1.3 Energy Storage

We consider the case in which each component of the community has its own BESS. Referring to Section III-A of [10] it is possible to model Battery Energy Storage Systems as:

$$e_i^{MAX} \frac{d}{dt} \varepsilon_i(t) = \eta_i r_i(t) - d_i(t) \tag{2.4}$$

This model is supposed to be valid for every i-th BESS of the community, with $i = 1, ..., n$.
Let's now analyze the meaning of each element that composes equation (2.4):

- $e_i^{MAX} \in \mathbb{R}_{\geq 0}$ represents the maximum energy storage capacity of the BESS;
- $\varepsilon_i(t) \in [0,1]$ represents the State of Charge of the BESS. It could be written as $\varepsilon_i(t) = e_i(t)/e_i^{MAX}$ and it can take values belonging to the interval [0,1];
- $r_i(t) \in \mathbb{R}_{\geq 0}$ represents the recharging power of the BESS;
- $d_i(t) \in \mathbb{R}_{\geq 0}$ represents the discharging power of the BESS;
- $\eta_i \in [0,1]$ represents the ratio between the energy supplied to the storage system and the energy retrieved from it. It is called *Round Trip Efficiency* (RTE).

In this model we suppose that the battery can not be recharged and discharged at the same time and as a consequence we must have:

$$\begin{aligned} r_i(t) > 0 &\implies d_i(t) = 0 \\ d_i(t) > 0 &\implies r_i(t) = 0 \end{aligned} \tag{2.5}$$

In order to optimize the benefits for the prosumers, we consider that the battery could be discharged not only to satisfy the consumption but also to sell energy to the main grid when it is convenient. Due to this consideration it is therefore possible to split $d_i(t)$ in two components as reported in equation (2.6).

$$d_i(t) = d_i^c(t) + d_i^s(t) \tag{2.6}$$

where $d_i^c(t)$ and $d_i^s(t)$ represent the discharged power consumed by the prosumer and the discharged power sold to the grid respectively.

In addition it is possible to quantify the amount of these two quantities introducing a variable $\alpha_d \in [0,1]$ obtaining:

$$\begin{cases} d_i^c(t) = \alpha_d(t) d_i(t) \\ d_i^s(t) = (1 - \alpha_d(t)) d_i(t) \end{cases} \tag{2.7}$$

2.1.4 Interaction with Utility Grid

We can model the interaction with the utility grid of the i-th prosumer, $i = 1, ..., n$, using the following two equations:

$$\begin{aligned} b_i(t) &= r_i(t) + c_i(t) - d_i^c(t) - g_i^c(t), \\ s_i(t) &= d_i^s(t) + g_i^s(t) \end{aligned} \tag{2.8}$$

where $b_i(t) \in \mathbb{R}$ is the total energy bought from the main grid while $s_i(t) \in \mathbb{R}$ is the total energy sold to the utility grid.

The following picture 2.1 illustrates how energy is exchanged between the various components of a prosumer and the energy community.

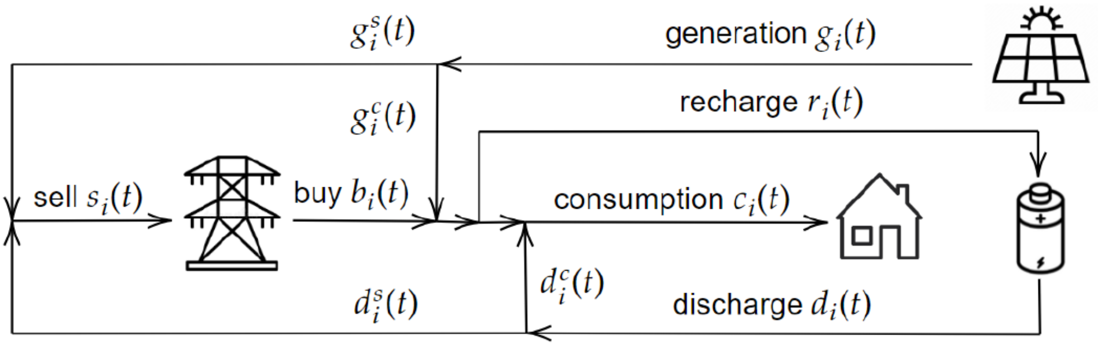

Figure 2.1: Energy exchanged between prosumer and energy community.

Chapter 3

Basic Heuristic Approaches

To compare the results of the optimization approaches described in the following chapters it is important to develop some heuristics scenarios. In this chapter we describe two of the three heuristic approaches that are used in this thesis.
The two methods are implemented following a basic if-else approach and they are tested using an energy community composed by $n = 4$ agents. The objective is to quickly verify if the community takes advantages from the installation of a BESS.

3.1 First Basic Heuristic Scenario

In this section we exam an energy community in which only one prosumer is present. It is equipped by an RG but without the storage system. The other three members are considered as passive components which means that they can only buy energy from the main grid since energy can not be produced or stored due to the absence of BESSs and RGs. Furthermore it is considered that all the surplus of the generated power arising from the prosumer is directly sold to the main grid.
In image 3.1 the flowchart used to implement this approach is reported. This process is repeated for each i-th member at each time instant $t \in \mathbb{N}_+$ with $i = 1, ..., n$ and $t = 1, ..., T$. The value $T \in \mathbb{N}_+$ is the total number of samples obtained sampling the desired continuous-time signal in the time horizon H. Considering a sampling time $\Delta \in \mathbb{R}_{\geq 0}$ the relation between H and T is $H = T\Delta$. Obviously if a member of the community is not equipped with RG, *Gens* and *Sold* values are set to zero.

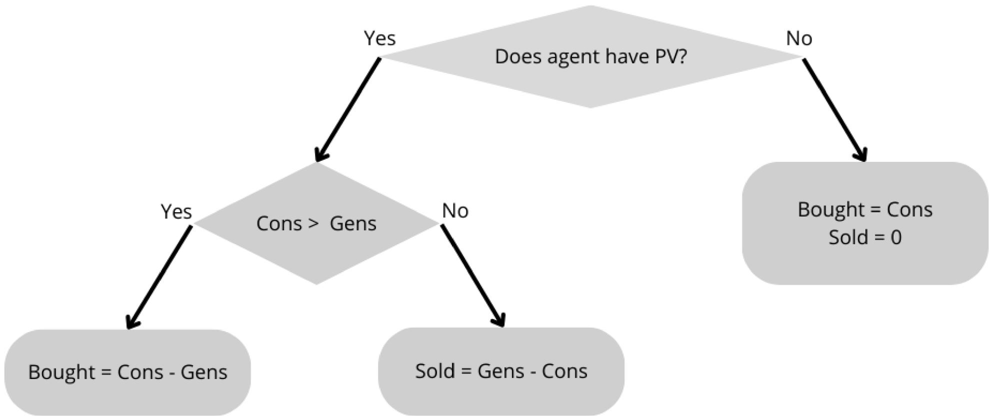

Figure 3.1: Flowchart of first basic heuristic approach.

3.2 Second Basic Heuristic Scenario

As before we exam an energy community in which only one prosumer is present. It is equipped by an RG and BESS. The other components are considered to be in passive mode as previously described. The storage system of the prosumer is charged if there is a surplus of generated power and it is discharged when the prosumer's consumption can not be satisfied by the produced energy. In figure 3.2 the flowchart used to implement this second approach is reported. In this case we suppose to obtain a reduction of bought energy since otherwise the use of BESS does not produce improvements in the community.
The legend of the words used in images 3.1 and 3.2 is:

- *Cons* refers to the agent power consumption
- *Gens* refers to the agent generated power
- *Bought* refers to the agent energy bought from the main grid
- *Sold* refers to the agent energy sold to the grid operator
- *Charging* refers to the agent amount of energy stored in the BESS
- *eb* refers to the maximum capacity of the BESS

Note: At each step of the flowchart we are considering the time instant t. When we refer to a different time instant it is specified in round brackets as it is visible observing fig. 3.2.

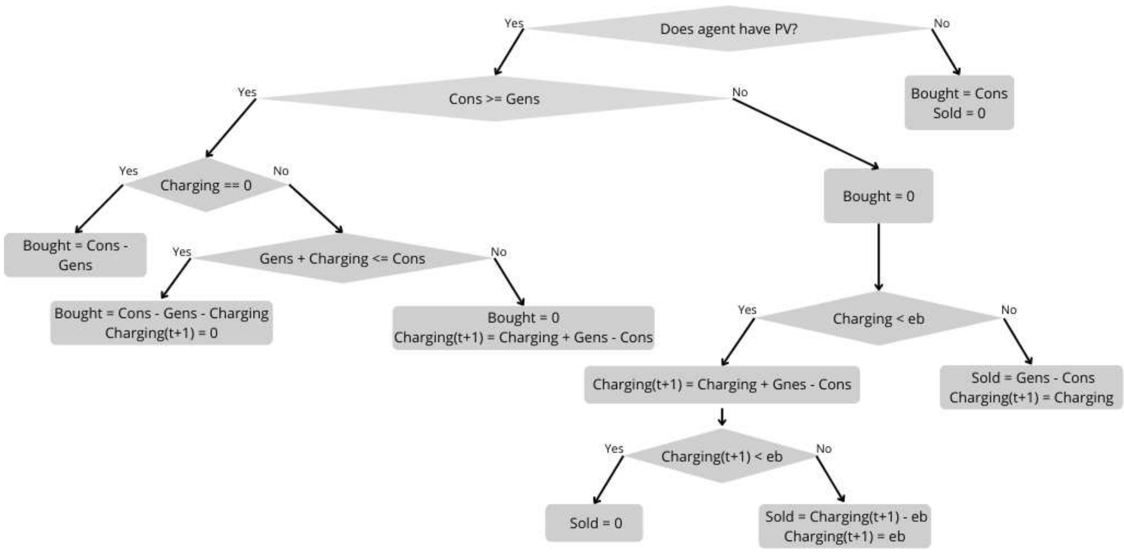

Figure 3.2: Flowchart of the second basic heuristic approach

3.3 Comparison of the two approaches

In this section we report the results obtained implementing these two approaches. Due to absence of interactions among elements of the community it is sufficient to compare the results regarding the total amount of energy bought by the community. In image 3.3 on the left we can observe the amount of the energy bought by the community implementing the first basic heuristic approach while on the right the same quantity is reported but using the second basic heuristic approach.
At first view it seems that there are no benefits installing a BESS but if we overlap the two graphs as depicted in the bottom of fig. 3.4 it is possible to observe that during the discharging of the battery in the second approach there is a reduction of the energy bought from the main grid by the community.
Considering a mean cost of 0.2 €/kWh to buy energy from the grid and of 0.02 €/kWh to sell energy to the grid we obtain:

- Energy community total expenditure in the first case: 217.46 €
- Energy community total expenditure in the second case: 210.17 €

Using a BESS embedded into the prosumer we have a daily cost saving of 7.29 €. So it is possible to conclude that the installation of BESSs is convenient for the community since it consents to obtain benefits also in a very basic heuristic case.

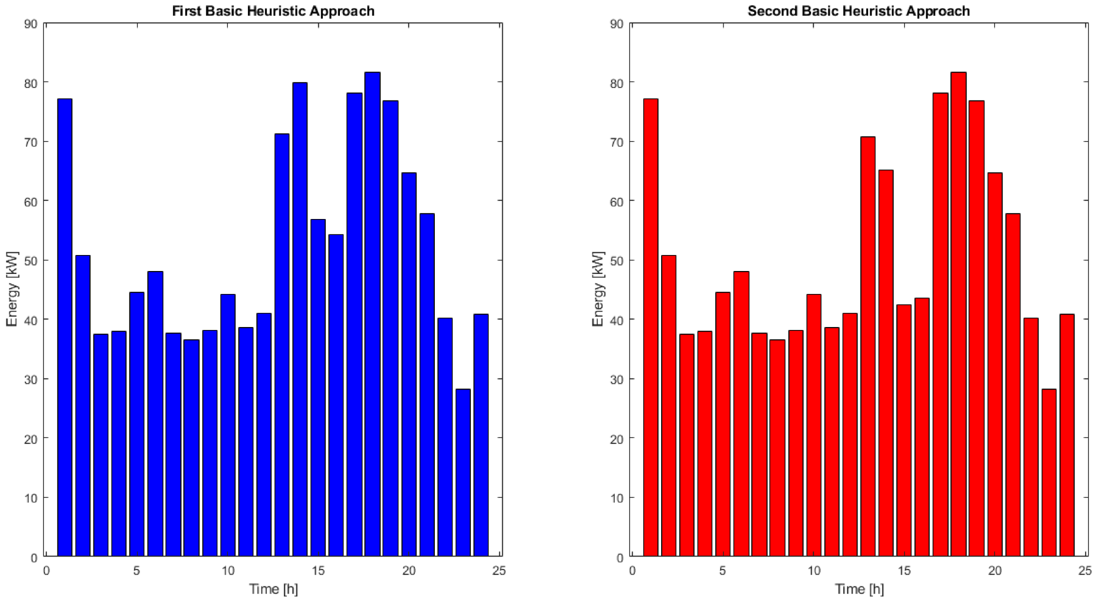

Figure 3.3: Daily total bought energy by the community using first approach (on the left) and second approach (on the right).

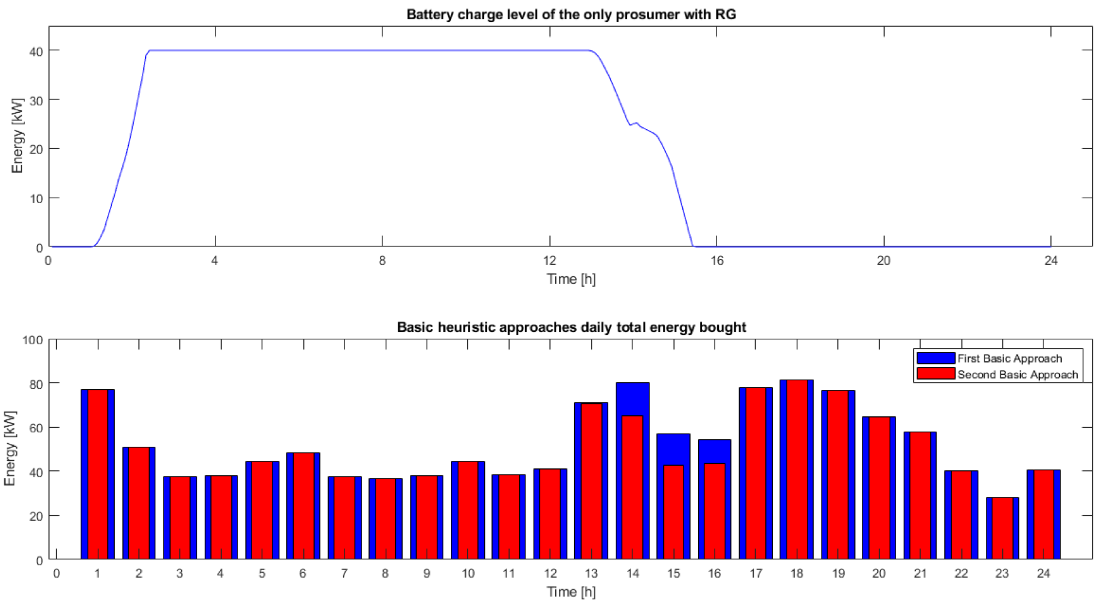

Figure 3.4: Daily prosumer battery charge level (on the top) and overlapping of the daily total energy bought by the community in the two basic heuristic approaches (on the bottom).

Chapter 4

Decision Variables and Local Constraints

In this section we introduce the concept of optimization problem and we provide an explanation of all decisions variables and local constraints that are essential in order to formulate the various control architectures developed in the following chapters.

4.1 What is an Optimization Problem?

As reported by the National Institute of Standards and Technology (NIST) an optimization problem is:

> “A computational problem in which the object is to find the best of all possible solutions. More formally, find a solution in the feasible region which has the minimum (or maximum) value of the objective function.”

Roughly speaking an optimization problem consists on maximizing or minimizing a function, known as the objective function, subject to certain constraints. The solution to an optimization problem provides the most cost-effective outcome based on the given criteria. There are many types of optimization problem but the the main categories are linear/nonlinear, integer, constrained/unconstrained and combinatorial. So an optimization problem is composed by three main ingredients: objective function, decision variables and local constraints.

What is an Objective Function?

An objective function is a mathematical expression that needs to be maximized or minimized and it defines the goal of an optimization problem. It consents to represent in a mathematical form real world problems in order to find the best solution respecting some specific constraints.

What are Decision Variables?

A decision variable is an input to the objective function and constraints. It represents a choice or decision that the optimization problem can make in order to find the best solution since its value affects the result of the objective function solution.

What are Local Constraints?

Local constraints are conditions or restrictions applied to decision variables as equalities or inequalities. They define the feasible region of an optimization problem within which the solution must lie and ensure that the decision variables meet some specific criteria at a local level.

Let's now analyze what are the decision variables and local constraints that are used in our model and how they are derived.

4.2 Decision Variables

We consider as decision variables the following quantities that are valid for each member i, with $i = 1, \ldots, n$, of the community:

- The recharging power of the battery $\mathbf{r}_i \in \mathbb{R}^{\mathrm{T}}$
- The discharging power of the battery $\mathbf{d}_i \in \mathbb{R}^{\mathrm{T}}$
- The discharging power of the battery consumed by the agent $\mathbf{d}_i^c \in \mathbb{R}^{\mathrm{T}}$
- The generated power consumed by the agent $\mathbf{g}_i^c \in \mathbb{R}^{\mathrm{T}}$

We do not consider the consumption $\mathbf{c}_i \in \mathbb{R}^{\mathrm{T}}$ and the generated power $\mathbf{g}_i \in \mathbb{R}^{\mathrm{T}}$ as decision variables. We decide to use these quantities as fixed parameters representing the expected power consumption and power generation of the agent in the optimization horizon. These quantities are supposed to be known and they could be, for example, the result of a machine learning technique that, analyzing a collection of historical data, generates the most possible accurate previsions for these parameters.

4.3 Local Constraints

The solution of the optimization problem has to satisfy some local constraints in order to model the energy community in the most realistic way. Regarding decision variables they have to respect the inequalities that are reported in equation (4.1).

$$\begin{array}{rcccc} \mathbf{0} & \leq & \mathbf{r}_i & \leq & r_i^{MAX}\mathbf{1}, \\ \mathbf{0} & \leq & \mathbf{d}_i & \leq & d_i^{MAX}\mathbf{1}, \\ \mathbf{0} & \leq & \mathbf{d}_i^c & \leq & \mathbf{d}_i, \\ \mathbf{0} & \leq & \mathbf{g}_i^c & \leq & \mathbf{g}_i, \\ \mathbf{0} & \leq & \mathbf{d}_i/d_i^{MAX} + \mathbf{r}_i/r_i^{MAX} & \leq & \mathbf{1}. \end{array} \tag{4.1}$$

As communicated by the manufactures, to increase battery life, BESSs have to be recharged and to be discharged respecting specific power ranges. These restrictions are described in the first two lines of (4.1) where $r_i^{MAX} \in \mathbb{R}_{\geq 0}$ and $d_i^{MAX} \in \mathbb{R}_{\geq 0}$ denotes the maximum allowable recharge and discharge power of each BESS, respectively. The third and fourth lines describe the obvious relationships that the discharged and generated power consumed by an agent must be less or equal to the total amount of generated and discharged power. The last line is a direct consequence of equation (2.5). It ensures that batteries can not be charged and discharged at the same time.
We have to take in consideration that the amount of energy retrieved and injected into the main grid in most cases is limited due to physical installed infrastructures.

This consideration is took in consideration imposing the following constraints reported in (4.2).

$$\begin{aligned} \mathbf{0} \;\leq\;& \mathbf{r}_i + \mathbf{c}_i - \mathbf{d}_i^c - \mathbf{g}_i^c &\leq\; b_i^{MAX}\mathbf{1}, \\ \mathbf{0} \;\leq\;& \mathbf{d}_i^s + \mathbf{g}_i^s &\leq\; s_i^{MAX}\mathbf{1}. \end{aligned} \tag{4.2}$$

Recalling equation (2.8), the first and second lines of (4.2) denote the bounds that the total bought and sold energy of each member i of the community have to respect. We denote by $b_i^{MAX} \in \mathbb{R}_{\geq 0}$ and $s_i^{MAX} \in \mathbb{R}_{\geq 0}$ the maximum allowable power transfer from and to the grid respectively.
Battery manufacturers impose in their products that the SoC has to remain within specific bounds. This condition is stated in the following equation (4.3).

$$\varepsilon_i^{MIN}\mathbf{1} \leq \varepsilon_i \leq \varepsilon_i^{MAX}\mathbf{1}, \tag{4.3}$$

where $\varepsilon_i^{MIN} \in [0,0.5]$ and $\varepsilon_i^{MAX} \in [0.5,1]$ represent the minimum and maximum allowable SoC. Regarding BESS it is necessary to include in the local constraint also the relation described in equation (2.4). To do so it is useful to introduce two quantities: $\mathbf{e}_i \in \{0,1\}^{\mathrm{T}}$ and $D \in \mathbf{R}^{\mathrm{T}\times\mathrm{T}}$, which are reported hereafter.

$$\mathbf{e}_i = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \qquad D = \begin{bmatrix} 1 & 0 & \dots & \dots & 0 \\ -1 & 1 & 0 & \dots & 0 \\ 0 & \ddots & \ddots & \ddots & \vdots \\ \vdots & \dots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & -1 & 1 \end{bmatrix}$$

Thanks to these two elements it is possible to transform the continuous-time equation (2.4) in a more useful discrete-time one as it is reported in equation (4.4).

$$\frac{e_i^{MAX}}{\Delta}(D\varepsilon_i - \mathbf{e}_i\varepsilon_i(t_{k-1})) = \eta_i\mathbf{r}_i - \mathbf{d}_i \tag{4.4}$$

This procedure consents to write the the final constraint (4.5) that it is obtained combining (4.3) and (4.4).

$$\varepsilon_i^{MIN}\mathbf{1} \leq D^{-1}\left[\frac{e_i^{MAX}}{\Delta}(\eta_i\mathbf{r}_i - \mathbf{d}_i) + \mathbf{e}_1\varepsilon_i(t_{k-1})\right] \leq \varepsilon_i^{MAX}\mathbf{1}. \tag{4.5}$$

4.4 Shared Energy

To implement our control architectures we have to introduce the important concept of "shared energy" that is denoted in the following as E_{sh}.

Defining pricing windows as $W = \Upsilon\Delta$ of length $\Upsilon \in \mathbb{N}$, the shared energy is defined as the minimum between the energy sold to the grid and the energy bought from the main grid within that windows. In addition it is considered that the optimization horizon $H = \Delta T$ can contain only $h \in \mathbb{N}_+$ positive integer number of pricing windows W with the consequence that the following relation has to hold:

$$\lfloor T/\Upsilon \rfloor = h \geq 1.$$

In order to develop a mathematical formulation of the shared energy concept it is necessary to introduce a function that sums the samples of vector $\mathbf{x} = [x]_k^{\mathrm{T}}$ within windows of length Υ.

These function is defined as follows:

$$g(\mathbf{x}, \Upsilon) = \Delta \begin{bmatrix} \mathbf{1}^\top [x]_k^{\Upsilon - mod(k,\Upsilon)} \\ I_{h-1} \otimes \mathbf{1}_\Upsilon^\top [x]_{\lceil (k+1)/\Upsilon \rceil \Upsilon}^{(h-1)\Upsilon} \\ \mathbf{1}^\top [x]_{(\lceil k/\Upsilon \rceil + h - 1)\Upsilon}^{mod(k,\Upsilon} \end{bmatrix}.$$

Thanks to this expression it is so possible to define the shared energy E_{sh} as:

$$E_{sh}(\mathbf{b}, \mathbf{s}, \Upsilon) = \min \left\{ \sum_{i \in \mathcal{V}} g(\mathbf{b}_i, \Upsilon), \sum_{i \in \mathcal{V}} g(\mathbf{s}_i, \Upsilon) \right\} \in \mathbb{R}^{\lceil k, \Upsilon \rceil}, \tag{4.6}$$

where $\mathbf{b} = [\mathbf{b}_1^\top, \ldots, \mathbf{b}_n^\top]^\top$ and $\mathbf{s} = [\mathbf{s}_1^\top, \ldots, \mathbf{s}_n^\top]^\top$ are vectors with n components.

Note: E_{sh} has dimension $\lceil k, \Upsilon \rceil$ since it can either contains h or $h+1$ elements. This is a direct consequence of the $mod(a, b)$ operator which returns the remainder of a division between its first (a) and second (b) component. In our case if $mod(k, \Upsilon) = 0$ the last element of the matrix is equal to an empty vector otherwise it contains at least one element since $mod(k, \Upsilon) \in [1, \Upsilon - 1]$.

4.4.1 A variant for Shared Energy

There is the posibility to express the shared energy E_{sh} in a simpler way. Considering the generated power $\mathbf{g}_i$, it can be decompose as reported in the following equation (4.7).

$$\mathbf{g}_i = \mathbf{g}_i^c + \mathbf{g}_i^{sc} + g_i^{sg}, \tag{4.7}$$

where two new variables are introduced:

- $\mathbf{g}_i^{sc}$ that represents the generated power sold to the community,
- $\mathbf{g}_i^{sg}$ that represents the generated power sold to the main grid.

From the definition of $\mathbf{g}_i^{sc}$ it is possible to state that:

$$\sum_{i\in\mathcal{V}} \mathbf{g}_i^{sc} \quad \leq \quad \sum_{i\in\mathcal{V}} \mathbf{b}_i \quad \implies \quad \sum_{i\in\mathcal{V}} g(\mathbf{g}_i^{sc}, \boldsymbol{\Upsilon}) \quad \leq \quad \sum_{i\in\mathcal{V}} g(\mathbf{b}_i, \boldsymbol{\Upsilon})$$

and as a consequence it is possible to define the shared energy E_{sh} as:

$$E_{sh}(\mathbf{b}, \mathbf{s}, \boldsymbol{\Upsilon}) = \sum_{i\in\mathcal{V}} g(\mathbf{g}_i^{sc}, \boldsymbol{\Upsilon}).$$

The two formulations are equivalent and they lead to same results, hence it is the same to choose one rather than the other[1].

[1] The proof of this equivalence is reported in Appendix A.

Chapter 5

Centralized and Distributed LP Problem Formulation

In this section we introduce the concept of centralized and distributed control architecture and we analyze the main benefits and drawbacks of these two approaches. Then we develop an LP formulation for our problem with the aim of realising an optimization problem that is able to minimize the costs of the whole energy community. Considering Italian policy a community is proportionally rewarded based on the amount of energy that is shared among the community members. The amount of the reward is established by the Italian GME (Gestore dei Mercati Elettrici).

5.1 Centralized vs Distributed Approach

A centralized approach in energy communities refers to a model where all decisions regarding energy flows are controlled by a single central entity. This approach is useful only in a small scale community since, increasing the number of the members, the amount of data that a single component has to manage grows exponentially and this can leads to some problems due to limited capabilities of physical controllers. In addition there could be privacy issues from the moment that each member has to communicate all its private information to the global manager with the risk of introducing a monopolistic behavior in the community.
All these limitations and drawbacks of the centralized case lead to the implementation of a distributed approach. In this last case privacy problems are solved

since each member has its own power manager and only a limited amount of their own information are shared with the community. The amount of data managed by the controller is reduced and as a consequence decision making capabilities are increased leading to higher efficiency. In addition a new agent can be more easily integrated into the community since the global structure of the control architecture does not require major modifications. As a consequence the number of members composing the community can grow with no restrictions, except for physical limitations, taking advantages in the amount of shared energy.
In conclusion in many cases, if there is the possibility, it is better to prefer a distributed approach with respect to a centralized one due to numerous advantages and benefits that derive from it.

5.2 Centralized LP Problem Formulation

In order to implement a satisfying objective function that describes our problem we need to introduce the following two quantities:

- $p_e \in \mathbb{R}_{\geq 0}$ describes the price of the bought energy,
- $p_{sh} \in \mathbb{R}_{\geq 0}$ describes the economic reward for the shared energy.

In addition we group the decision variables previously described into a single vector obtaining:

$$\mathbf{v}_i = [\mathbf{r}_i^\top, \mathbf{d}_i^\top, \mathbf{d}_i^{c\top}, \mathbf{g}_i^{c\top}]^\top$$

and consequently we have:

$$\mathbf{v} = [\mathbf{v}_1^\top, \dots, \mathbf{v}_n^\top]^\top.$$

Now that all the necessary components are available it is possible to define the objective function $f(\mathbf{v})$ as reported in (5.1).

$$f(\mathbf{v}) = p_e^\top \sum_{i=1}^{n} g(\mathbf{b}_i, \boldsymbol{\Upsilon}) - p_{sh} E_{sh}(\mathbf{b}, \mathbf{s}, \boldsymbol{\Upsilon}). \tag{5.1}$$

Notwithstanding $f(\mathbf{v})$ perfectly represents a mathematical representation of our problem it is non-linear, due to the minimum operation embedded in E_{sh}, hence it can not be used as objective function in linear programming optimization.

To solve this drawback we can consider that the minimum between two quantities is always less or equal to both values. So it is possible to replace the shared energy with a new variable $\vartheta \in \mathbb{R}^{\lceil k, \Upsilon \rceil}$ subject to the following constraints:

$$\vartheta = \min \left\{ \sum_{i \in \mathcal{V}} g(\mathbf{b}_i, \Upsilon), \sum_{i \in \mathcal{V}} g(\mathbf{s}_i, \Upsilon) \right\} \implies \begin{cases} \vartheta \leq \sum_{i \in \mathcal{V}} g(\mathbf{b}_i, \Upsilon), \\ \vartheta \leq \sum_{i \in \mathcal{V}} g(\mathbf{s}_i, \Upsilon). \end{cases} \tag{5.2}$$

Thanks to this fictitious variable it is possible to rewrite the objective function as:

$$f(\mathbf{v}, \vartheta) = p_e^\top \sum_{i=1}^{n} g(\mathbf{b}_i, \Upsilon) - p_{sh}^\top \vartheta. \tag{5.3}$$

Now the objective function is linear and hence it is possible to realize a centralized LP formulation of our problem which is reported in (5.4) where, in addition to the decoupled constraints, there are two additional coupled ones which are needed due to the presence of the variable ϑ into the model.

$$\begin{aligned} \min_{\mathbf{v}, \vartheta} \quad & f(\mathbf{v}, \vartheta) \\ \text{subject to:} \quad & (4.1), (4.2), (4.5) \qquad \forall i \in \mathcal{V}, \\ & \vartheta - \sum_{i \in \mathcal{V}} g(\mathbf{b}_i, \Upsilon) \leq \mathbf{0}, \\ & \vartheta - \sum_{i \in \mathcal{V}} g(\mathbf{s}_i, \Upsilon) \leq \mathbf{0}. \end{aligned} \tag{5.4}$$

5.3 Distributed LP Problem Formulation

Knowing the advantages deriving from a decentralized approach in this section we derive a distributed formulation for our problem starting from the previous one.
To reach our goal we can note that the first term of the objective function

$$\sum_{i=1}^{n} g(\mathbf{b}_i, \Upsilon),$$

is already in a distributed form since it is the sum over the agents of the bought energy. The second term can be transform in a distributed one introducing a local variable ϑ_i which, summed over the agents, consents to obtain the original variable ϑ as described in the following relation:

$$\vartheta = \sum_{i \in \mathcal{V}} \vartheta_i.$$

Replacing these quantities in (5.3) we obtain a fully decoupled objective function as described in (5.5).

$$f(\mathbf{v}, \vartheta) = \sum_{i \in \mathcal{V}} \left(p_e^\top g(\mathbf{b}_i, \Upsilon) - p_{sh}^\top \vartheta_i \right) \tag{5.5}$$

At the end we obtain an objective function that is linear in the decision variables and simply convex. This last characteristic is a limitation in the problem solution since it could lead to the duality-gap problem. This condition is verified when, solving a LP problem, the solution of the primal has different value with respect to the dual[1]. To avoid this problem we have to reformulate the objective function in (5.5) in order to make it strictly convex.
Considering:

$$f(\mathbf{v}, \vartheta) = \sum_{i \in \mathcal{V}} f_i(\mathbf{v}_i, \vartheta_i), \tag{5.6}$$

with

$$f_i(\mathbf{v}_i, \vartheta_i) = p_e^\top g(\mathbf{b}_i, \Upsilon) - p_{sh}^\top \vartheta_i,$$

and observing that $\mathbf{v}_i$ satisfies the constraint:

$$\mathbf{v}_i^{MIN} \leq \mathbf{v}_i \leq \mathbf{v}_i^{MAX},$$

it is possible to make $f_i(\mathbf{v}_i, \vartheta_i)$ strictly convex by adding two regularization terms as done in (5.7).

$$\tilde{f}_i(\mathbf{v}_i, \vartheta_i) = f_i(\mathbf{v}_i, \theta_i) + \sigma ||\mathbf{v}_i - \bar{\mathbf{v}}_i||_2^2 + \zeta ||\vartheta_i||_2^2 \ , \tag{5.7}$$

where $\bar{\mathbf{v}}_i$ is equal to:

$$\bar{\mathbf{v}}_i = \frac{1}{2}(\mathbf{v}_i^{MIN} + \mathbf{v}_i^{MAX}),$$

and $\sigma \in \mathbb{R}_{\geq 0}$ and $\zeta \in \mathbb{R}_{\geq 0}$ are two positive numbers weighting the effects of the regularization terms.
In addition we use two local variables:

$$\alpha_i \geq 0 \quad \text{and} \quad \beta_i \geq 0, \quad \forall i \in \mathcal{V} \tag{5.8}$$

[1] For more details regarding primal and dual of an LP problem see Appendix B.

that are valid in order to transform the coupled inequality constraints of (5.4) into equality constraints.
At the end we obtain the following distributed LP formulation for our problem:

$$\begin{aligned} \min_{\mathbf{v}_i,\vartheta_i,\alpha_i,\beta_i} \quad & \sum_{i\in\mathcal{V}} \tilde{f}(\mathbf{v}_i,\vartheta_i) \\ \text{subject to:} \quad & (4.1),(4.2),(4.5),(5.8) \qquad \forall i\in\mathcal{V}, \\ & \sum_{i\in\mathcal{V}} (\vartheta_i - g(\mathbf{b}_i,\Upsilon) + \alpha_i) = \mathbf{0}, \\ & \sum_{i\in\mathcal{V}} (\vartheta_i - g(\mathbf{s}_i,\Upsilon) + \beta_i) = \mathbf{0}. \end{aligned} \tag{5.9}$$

In (5.9) the last two constraints refer to the coupled equality constraints while the others refer to local inequalities constraints.
In this work we decide to solve the problem (5.9) using the Alternating Direction Method of Multipliers (DC-ADMM) algorithm developed in [4] due to its robustness and efficiency in solving optimization problems as ours. The implemented algorithm (Algorithm 1) is reported below.
Regarding this procedure it is important to make some considerations:

- each node $i \in \mathcal{V}$ stores in memory auxiliary variables $\mathbf{y}_i$ and $\mathbf{p}_i$ in addition to $\mathbf{v}_i, \vartheta_i, \alpha_i, \beta_i$;
- The only information that each node $i \in \mathcal{V}$ transmits to all its neighbors $j \in \mathcal{N}$ is the variable $\mathbf{y}_i$ and as a consequence privacy is enhanced;
- Algorithm 1 consists in three main steps:
  1. The agent updates variables $\mathbf{v}_i, \vartheta_i, \alpha_i, \beta_i, \mathbf{y}_i$;
  2. The agent transmits the variable $\mathbf{y}_i$;
  3. The agent updates the variable $\mathbf{p}_i$.
- the real number $\rho \geq 0$ is a free parameter and it influences the convergence rate of the algorithm. Algorithm 1 converges to an optimal solution for any choice of $\rho \geq 0$ (cfr. [4],[1]);
- if $\sigma \to 0$ and $\zeta \to 0$ it means that regularization terms are neglected and hence the found optimal solution coincides with an optimal solution of the problem solved using the objective function in (5.5).

Here the realization of Algorithm 1 used to solve the distributed scenario is reported.

Algorithm 1 DC-ADMM algorithm used to solve problem (5.9)

Require: Initial values $\mathbf{v}_i(0)$, $\theta_i(0)$, $\alpha_i(0)$, $\beta_i(0) \in \mathbb{R}^{2\lceil k,\Upsilon\rceil}$ and $\mathbf{p}_i(0) = 0 \in \mathbb{R}^{2\lceil k,\Upsilon\rceil}$ for $i \in \mathcal{V}$ and parameter $\rho > 0$

1: **for** $k = 1, 2, 3, \dots$ (until a stopping criterion is satisfied) **do**

2: **for** each prosumer $i \in \mathcal{V}$ **do (in parallel) do**

$$\begin{bmatrix} \mathbf{v}_i(k) \\ \vartheta_i(k) \\ \alpha_i(k) \\ \beta_i(k) \end{bmatrix} = \arg\min_{\mathbf{v}_i,\vartheta_i,\alpha_i,\beta_i} \{*\}$$

where $(*)$ is:

$$\tilde{f}_i(\mathbf{v}_i, \vartheta_i) + \frac{\rho}{4|\mathcal{N}_i|} \left\| \begin{bmatrix} (\vartheta_i - g(b_i, \Upsilon) + \alpha_i)/\rho \\ (\vartheta_i - g(s_i, \Upsilon) + \beta_i)/\rho \end{bmatrix} - \frac{1}{\rho}\mathbf{p}_i(k-1) + \sum_{j\in\mathcal{N}_i} (\mathbf{y}_i(k-1) + \mathbf{y}_j(k-1)) \right\|_2^2$$

subject to: (4.1),(4.2),(4.5),(5.8) $\quad \forall i \in \mathcal{V}$,

$$y_i(k) = \frac{1}{2|\mathcal{N}_i|} \left(\begin{bmatrix} (\vartheta_i(k) - g(b_i(k), \mathcal{Y}) + \alpha_i(k)/\rho) \\ (\vartheta_i(k) - g(s_i(k), \mathcal{Y}) + \beta_i(k)/\rho) \end{bmatrix} - \frac{1}{\rho}\mathbf{p}_i(k-1) + \sum_{j\in\mathcal{N}_i} (\mathbf{y}_i(k-1) + \mathbf{y}_j(k-1)) \right)$$

$\mathbf{p}_i(k) = \mathbf{p}_i(k-1) + \rho \sum_{j\in\mathcal{N}_i} (y_i(k) - y_j(k))$

3: **end for**

4: **end for**

5.4 Optimized Heuristic Scenario

In order to test the effectiveness of introducing optimization in the management of an energy community we develop the third heuristic scenario. As for the previous, we consider an energy community composed by $n = 4$ agents in which only one of them is equipped with RG and BESS. Instead of using an if-else approach we use the optimization problem formulation (5.7) to solve the energy flow problem of the community. In this case we are not considering shared energy represented by the variable ϑ_i and hence the community is not cooperating and each agent optimizes

only its own behaviour. To observe possible benefits we compare the results obtained using this approach with the second basic heuristic scenario. In fig. 5.1, on the bottom plot, the daily total amount of energy bought from the main grid using these two methods is represented.
It is astonishing to observe that using an optimization approach the total energy bought from the main grid is drastically reduced. This benefits derive from the fact that if a prosumer is equipped with RG and BESS the energy stored in the battery is used more efficiently as represented in fig. 5.2. In fact optimization approach consents to preserve the life of the BESS since it is not fully charged, as instead happen using an if-else algorithm. In addition it consents to use stored energy in a more clever way since the BESS is discharged slowly with the result of making the prosumer more independent from the main grid as it is show on the top plot of fig. 5.1 where it is reported the daily total energy bought from the agent equipped with both RG and BESS.
Considering a mean cost of 0.2 €/kWh for bought energy from the utility grid and a reward of 0.02 €/kWh to sold energy to the main grid we obtain a total expenditure for energy purchase of about 193.27 €. Comparing this cost with the one obtained from the second basic approach that was 210.17 €, optimization consents to save about 16.90 €, that is a reduction of about 8.04%.
At the end it is possible to conclude that an optimization approach is preferable since it consents to obtain better results. In addiction it consents to develop a more robust control architecture. Opting for an optimization approach it is possible to avoid some possible mistakes that could derive from the realization of an if-else algorithm where there are a lot of loops and variable updates. Not to mention that if an error occurs in a big size if-else approach it is very hard to solve.

5.5 Centralized and Distributed Results

In this section we compare the results obtained using the LP optimization formulation of the centralized and distributed approach. We consider an energy community composed by $n = 4$ members in which only one of them is equipped with an RG. All agents have their own storage system and they are able to charge their battery

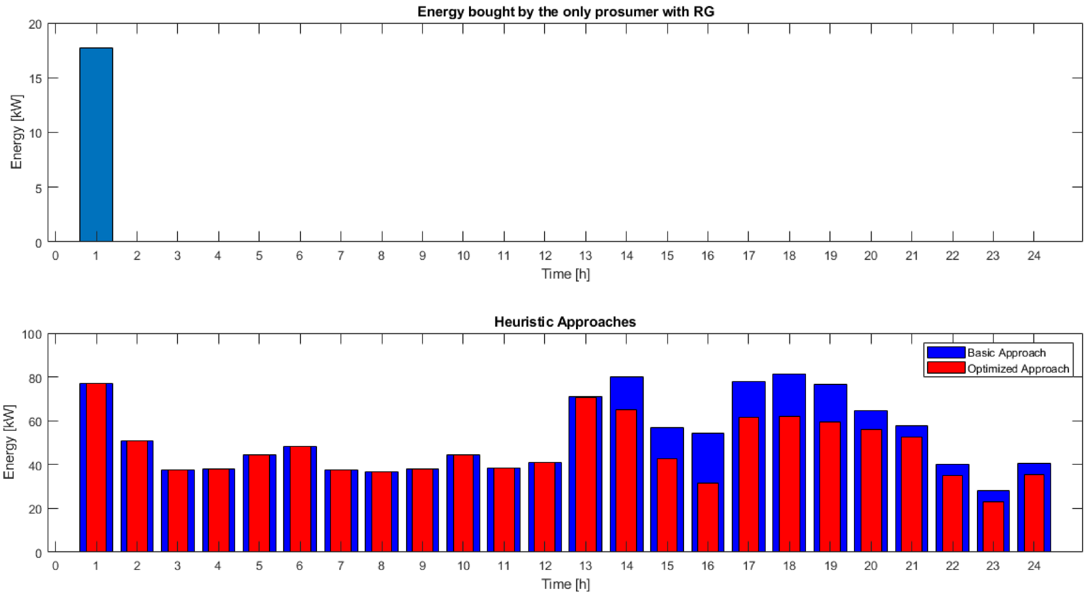

Figure 5.1: Daily energy bought from the prosumer equipped with RG (on the top) and comparison of daily total energy bought by the community(on the bottom).

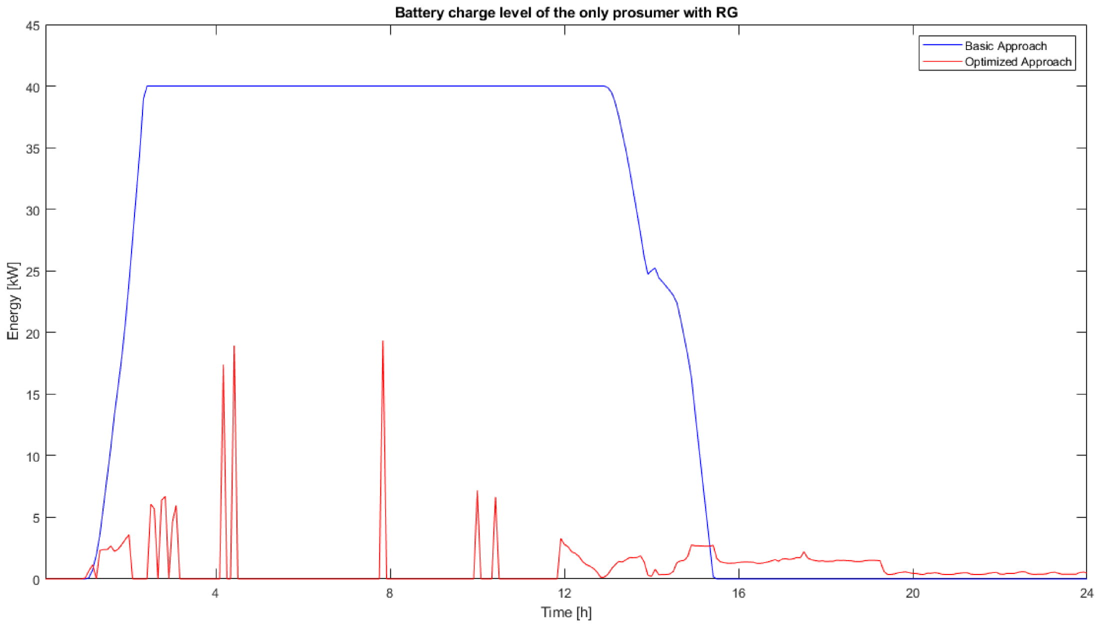

Figure 5.2: Prosumer with RG and BESS daily battery charge level in basic heuristic scenario and in the optimized heuristic scenario.

thanks to the concept of shared energy. First of all it is useful to take a look to the main quantities of each agent as: $\mathbf{r}_i(t)$, $\mathbf{d}_i(t)$, $\mathbf{g}_i(t)$, $\mathbf{c}_i(t)$, $\mathbf{b}_i(t)$ and $\mathbf{s}_i(t)$. These results are reported in fig. 5.3 for the centralized (on the left) and distributed (on

the right) case. In fig. 5.4 the comparisons between the total bought, sold and shared energy of the two approaches is instead reported.
Observing the figures it is possible to make some important considerations. Agent 1 is the only one having produced energy and consequently it is the only member equipped with RG. Agents charge their batteries only when there is power generation. In the centralized solution the energy generated is fully used to charge the batteries of each member of the community consenting them to fulfil their future energy expected consumption. In the distributed case this is not completely true, here agents are forced to buy energy from the grid also after the 12-th hour consenting the community to maximize the shared energy. This is verified by fig. 5.4 where it is evident that the amount of shared energy during production time in the distributed approach is higher than the one of the centralized approach. In both cases batteries life is safeguarded since they are never fully charged during the day. Considering again a mean cost of 0.2 €/kWh for bought energy from the utility grid, a reward of 0.02 €/kWh to sold energy to the main grid and a reward of 0.05 €/kWh for the shared energy it is possible to obtain in the centralized approach:

- Community total expenditure: 146.67 €;
- Community total shared energy: 1052.11 kWh;

while in the distributed approach:

- Community total expenditure: 147.70 €;
- Community total shared energy: 1051.66 kWh;

The two solutions are very closed to each other, as expected, since the two approaches are solving the same problem.
In addiction it is useful to observe that these approaches consent to obtain a reduction about the:

- 30.06% with respect to the best basic heuristic approach,
- 23.94% with respect to the optimized heuristic approach.

These results consent to state that an optimization approach that takes in consideration the concept of shared energy produces consistent benefits for the community members.

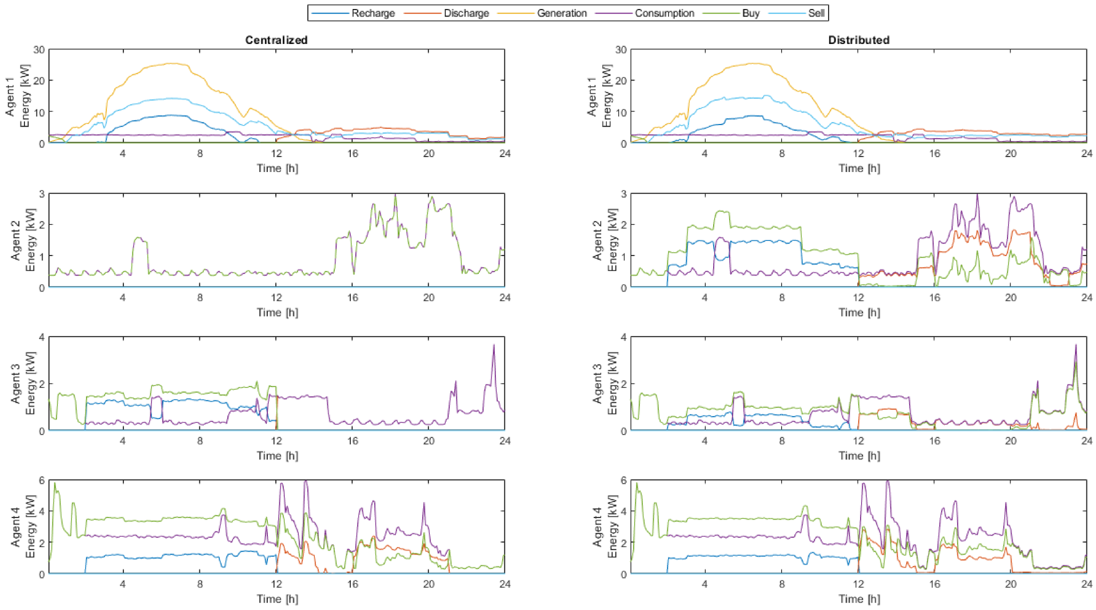

Figure 5.3: Comparison between the temporal evolution of recharge $\mathbf{r}_i(t)$, discharge $\mathbf{d}_i(t)$, generation $\mathbf{g}_i(t)$, consumption $\mathbf{c}_i(t)$, and the corresponding energy bought from the grid $\mathbf{b}_i(t)$ and sold to the grid $\mathbf{s}_i(t)$ obtained by solving the centralized problem (left) and the distributed regularized problem (right).

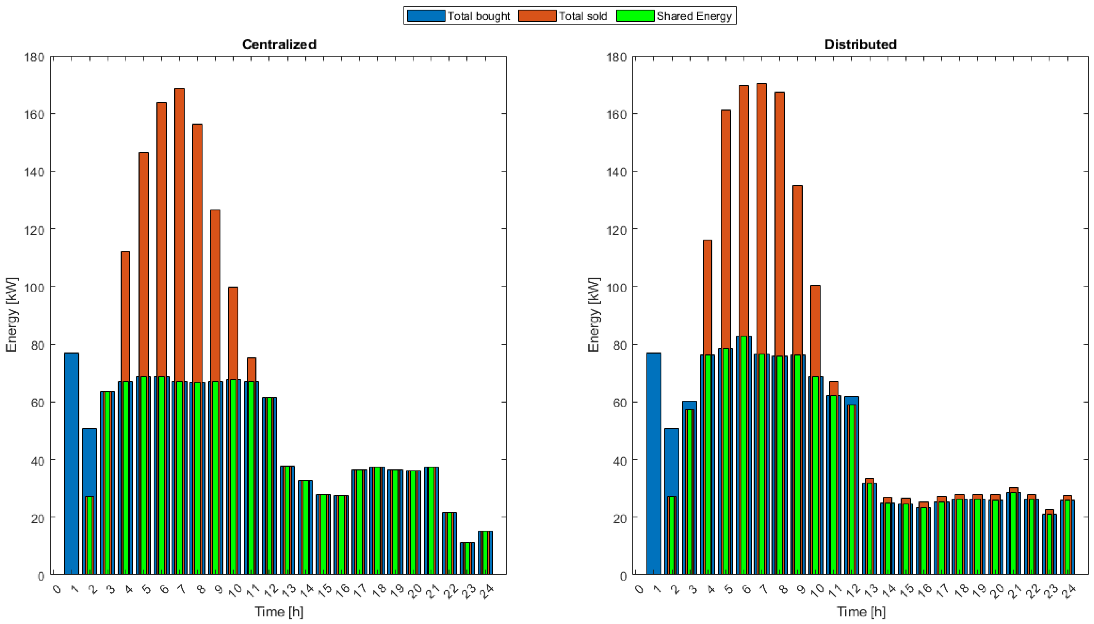

Figure 5.4: Daily total energy bought, total energy sold and total shared energy by the community in the centralized (on the left) and distributed (on the right) approach.

Chapter 6

MILP Problem Formulation

In this chapter we describe the concept of Mixed Integer Linear Programming (MILP) and we analyze the differences that distinguish it from the previous analyzed LP method. Then we provide a MILP formulation of our problem and we discuss the results obtained in this scenario.

6.1 What is MILP?

Mixed Integer Linear Programming (MILP) is an extension of linear programming (LP) where some of the decision variables are required to take on integer values. A problem can be solved as a MILP if it can be reduced to the form described in equation (6.1).

$$\begin{aligned} \min_{\mathbf{x}} \quad & c^\top \mathbf{x} \\ \text{subject to:} \quad & A\mathbf{x} \leq b \\ & \mathbf{x} \geq 0 \\ & \mathbf{x}_j \in \mathbb{Z}, \qquad \forall j \in \mathcal{I}. \end{aligned} \tag{6.1}$$

Where $\mathbf{x}$ is the unknown vector to be determined, $\mathbf{c}$ and $\mathbf{b}$ are given known vectors, A is a given matrix, $\mathbf{c}^\top\mathbf{x}$ is a linear objective function, $A\mathbf{x} \geq b$ and $\mathbf{x} \geq 0$ are linear constraints and $\mathcal{I}$ is a set containing all indices of variables that must be integer. If all variables need to be integer, MILP is called a (pure) integer linear program. Imposing to have integer variables it allows MILP to model a broader range of real-world problems that involve discrete decisions alongside continuous

variables. Despite the possibility of describing real world problems more precisely, solving MILP is generally more complex than solving a standard LP due to the fact that MILP has to respect the integer constraints.
To solve MILP problems there are three main approaches:

- Branch and Bound: It explores branches of a tree composed by subsets of feasible solutions and it uses bounds to find the best optimal solution among all the other;
- Branch and Cut: It works similarly to Branch and Bound but it embeds in the procedure also cutting planes techniques in order to restrict solutions feasible regions speeding up the resolution process;
- Heuristic and Metaheuristic Methods: They are used to find solutions of complex problems in a reduced amount of time but however they can lead to a solution that can be the non-optimal one.

The capabilities of MILPs to represent real world problems in a more realistic way lead us to convert our LP approach to a MILP one even if it is more complex to solve.

6.2 MILP Modelling

Since Mixed Integer Linear Programming is a extension of LP, MILP formulation of our problem is very similar to the previous one but with some important differences. Considerations made for consumption $\mathbf{c}_i \in \mathbb{R}^T$ and generation $\mathbf{g}_i \in \mathbb{R}^T$ remain valid as well as for the bounds described in (4.5) regarding BESSs. So also in MILP formulation we have to include the following constraint referring to the energy storage system:

$$\varepsilon_i^{MIN}\mathbf{1} \leq D^{-1}\left[\frac{e_i^{MAX}}{\Delta}(\eta_i\mathbf{r}_i - \mathbf{d}_i) + \mathbf{e}_1\varepsilon_i(t_{k-1})\right] \leq \varepsilon_i^{MAX}\mathbf{1}. \tag{6.2}$$

First change regards the model of the bought and sold energy. Differently from (2.8) these quantities are modeled as follow:

$$\begin{aligned} b_i(t) &= r_i(t) + \beta_i^l(t)c_i(t) - d_i^c(t) - g_i^c(t), \\ s_i(t) &= d_i^s(t) + g_i^s(t). \end{aligned} \tag{6.3}$$

The quantity $\beta_i^l(t)$ is introduced to show how much the required load is satisfied. Remembering that the amount of energy retrieved and injected into the main grid can be limited due to physical installed infrastructures, the following constraints reported in (6.4) have to be taken in consideration.

$$\begin{aligned} \mathbf{0} \leq \; & \mathbf{r}_i + \beta_i^l \mathbf{c}_i - \mathbf{d}_i^c - \mathbf{g}_i^c && \leq (b_i^{MAX} + \varepsilon)\delta_i^g - \varepsilon \mathbf{1}, \\ \mathbf{0} \leq \; & \mathbf{d}_i^s + \mathbf{g}_i^s && \leq -\delta_i^g (s_i^{MAX} + \varepsilon) + \mathbf{s}_i^{MAX} \mathbf{1}, \\ & \delta_i^g \in \{0,1\}. \end{aligned} \tag{6.4}$$

Variable $\delta_i^g \in \{0,1\}$ introduced in (6.4) has the following meaning:

$$\begin{cases} \delta_i^g = 1 & \iff \quad 0 \leq b_i(k), \\ \delta_i^g = 0 & \iff \quad 0 \leq s_i(k). \end{cases}$$

Regarding constraints related to the decision variables described in eq. (4.1) they are replaced by:

$$\begin{aligned} \mathbf{0} \leq \; & \mathbf{r}_i && \leq (r_i^{MAX} + \varepsilon)\delta_i^b - \epsilon \mathbf{1}, \\ \mathbf{0} \leq \; & \mathbf{d}_i && \leq -\delta_i^b (d_i^{MAX} + \varepsilon) + d_i^{MAX} \mathbf{1}, \\ \mathbf{0} \leq \; & \mathbf{d}_i^c && \leq \mathbf{d}_i, \\ \mathbf{0} \leq \; & \mathbf{g}_i^c && \leq \mathbf{g}_i, \\ \mathbf{0} \leq \; & \beta_i^l && \leq \mathbf{1}, \\ & \delta_i^b \in \{0,1\}. \end{aligned} \tag{6.5}$$

In this case last constraint of (4.1) is replaced by the variable $\delta_i^b \in \{0,1\}$ which has the following behaviour:

$$\delta_i^b = \begin{cases} 1 & \text{if the battery } i \text{ is charging,} \\ 0 & \text{otherwise.} \end{cases}$$

This binary variable consents to insert in the constraints the fact that batteries can not be charged and discharged simultaneously.
So at the end the considered decision variables in this scenario are:

- The recharging power of the battery $\mathbf{r}_i \in \mathbb{R}^{\mathrm{T}}$;
- The discharging power of the battery $\mathbf{d}_i \in \mathbb{R}^{\mathrm{T}}$;

- The discharging power of the battery consumed by the agent $\mathbf{d}_i^c \in \mathbb{R}^{\mathrm{T}}$;
- The generated power consumed by the agent $\mathbf{g}_i^c \in \mathbb{R}^{\mathrm{T}}$;
- The two binary variables $\delta_i^b \in \{0{,}1\}$ and $\delta_i^g \in \{0{,}1\}$.

Regarding shared energy E_{sh} it does not undergo changes and hence it has the same formulation that is:

$$E_{sh}(\mathbf{b}, \mathbf{s}, \Upsilon) = \min \left\{ \sum_{i \in \mathcal{V}} g(\mathbf{b}_i, \Upsilon), \sum_{i \in \mathcal{V}} g(\mathbf{s}_i, \Upsilon) \right\} \in \mathbb{R}^{\lceil k, \Upsilon \rceil} \tag{6.6}$$

where $\mathbf{b} = [\mathbf{b}_1^\top, \dots, \mathbf{b}_n^\top]^\top$ and $\mathbf{s} = [\mathbf{s}_1^\top, \dots, \mathbf{s}_n^\top]^\top$ are vectors with n components. Considerations regarding its dimension are still valid.

6.3 Centralized MILP Problem Formulation

To implement a MILP formulation of our problem it is possible to follow similarly steps to what we did for the LP formulation.
First of all we stack all decision variables into a single vector:

$$\mathbf{v}_i = [\mathbf{r}_i^\top, \mathbf{d}_i^\top, \mathbf{d}_i^{c\top}, \mathbf{g}_i^{c\top}, \delta_i^{b\top}, \delta_i^{g\top}]^\top$$

and consequently we have:

$$\mathbf{v} = [\mathbf{v}_1^\top, \dots, \mathbf{v}_n^\top]^\top.$$

And again denoting by:

- $p_e \in \mathbb{R}_{\geq 0}$ the price of the bought energy,
- $p_{sh} \in \mathbb{R}_{\geq 0}$ the economic reward for the shared energy,

it is possible to obtain the objective function that is reported in (6.7).

$$f(\mathbf{v}) = p_e^\top \sum_{i=1}^{n} g(\mathbf{b}_i, \Upsilon) - p_{sh} E_{sh}(\mathbf{b}, \mathbf{s}, \boldsymbol{\Upsilon}). \tag{6.7}$$

Since also in this case $f(\mathbf{v})$ is not linear due to the minimum operation necessary to obtain the shared energy, we need to introduce a variable $\vartheta \in \mathbb{R}^{\lceil k, \Upsilon \rceil}$ having the same properties of the one introduced in the LP formulation. This consents to obtain the new objective function (6.8).

$$f(\mathbf{v}, \vartheta) = p_e^\top \sum_{i=1}^{n} g(\mathbf{b}_i, \Upsilon) - p_{sh}^\top \vartheta. \tag{6.8}$$

So now we have all the elements to formulate our MILP problem formulation that is reported in (6.9).

$$\begin{aligned} \min_{\mathbf{v},\vartheta} \quad & f(\mathbf{v}, \vartheta) \\ \text{subject to:} \quad & (6.2), (6.4), (6.5) \qquad \forall i \in \mathcal{V}, \\ & \vartheta - \sum_{i \in \mathcal{V}} g(\mathbf{b}_i, \Upsilon) \leq \mathbf{0}, \\ & \vartheta - \sum_{i \in \mathcal{V}} g(\mathbf{s}_i, \Upsilon) \leq \mathbf{0}. \end{aligned} \tag{6.9}$$

6.4 Distributed MILP Problem Formulation

The objective function of the centralized MILP problem formulation is the same as the one described in section (5.2) and hence all previous made considerations are still valid also in this case. A distributed objective function can be built following the same steps as described in section (5.3) that we recall hereafter.
So it is possible to introduce a local variable ϑ_i which, summed over the agents, consents to obtain the original variable ϑ as described in the following relation:

$$\vartheta = \sum_{i \in \mathcal{V}} \vartheta_i.$$

Replacing these quantities in (6.8) we obtain a fully decoupled objective function as described in eq. (6.10).

$$f(\mathbf{v}, \vartheta) = \sum_{i \in \mathcal{V}} f_i(\mathbf{v_i}, \vartheta_i) = \sum_{i \in \mathcal{V}} \left(p_e^\top g(\mathbf{b}_i, \Upsilon) - p_{sh}^\top \vartheta_i \right). \tag{6.10}$$

We recall that it is linear in the decision variables and simply convex. Remembering that it is preferable to have strictly convex objective function in order to avoid duality gap problems we introduce two regularization terms as follows.

$$\tilde{f}_i(\mathbf{v}_i, \vartheta_i) = f_i(\mathbf{v}_i, \theta_i) + \sigma ||\mathbf{v}_i - \bar{\mathbf{v}}_i||_2^2 + \zeta ||\vartheta_i||_2^2 \tag{6.11}$$

Where $\bar{\mathbf{v}}_i$ is equal to:

$$\bar{\mathbf{v}}_i = \frac{1}{2}(\mathbf{v}_i^{MIN} + \mathbf{v}_i^{MAX}),$$

and $\sigma \in \mathbb{R}_{\geq 0}$ and $\zeta \in \mathbb{R}_{\geq 0}$ are two positive numbers weighting the effects of the regularization terms.
Then adding two local variables:

$$\alpha_i \geq 0 \quad \text{and} \quad \beta_i \geq 0, \quad \forall i \in \mathcal{V}, \tag{6.12}$$

in the coupled inequalities constraints of problem (6.9) we have the following distributed LP formulation for our problem:

$$\begin{aligned} \min_{\mathbf{v}_i, \vartheta_i, \alpha_i, \beta_i} \quad & \sum_{i \in \mathcal{V}} \tilde{f}(\mathbf{v}_i, \vartheta_i) \\ \text{subject to:} \quad & (4.1), (4.2), (4.5), (5.8) \qquad \forall i \in \mathcal{V}, \\ & \sum_{i \in \mathcal{V}} (\vartheta_i - g(\mathbf{b}_i, \Upsilon) + \alpha_i) = \mathbf{0}, \\ & \sum_{i \in \mathcal{V}} (\vartheta_i - g(\mathbf{s}_i, \Upsilon) + \beta_i) = \mathbf{0}. \end{aligned} \tag{6.13}$$

We obtain a distributed MILP problem formulation with a strictly convex objective function. Differently from the distribuited LP formulation we decide to solve this problem using dual decomposition procedure that is described in Section IV-A of [2].

6.5 Centralized and Distributed MILP Results

In this section we exam the results obtained using the centralized and distributed MILP formulation of our problem. Considering again a mean cost of 0.2 €/kWh for energy bought from the utility grid, a reward of 0.02 €/kWh to energy sold to the main grid and a reward of 0.05 €/kWh for the shared energy, what we obtain in the centralized approach is:

- Community total expenditure: 151.05 €;
- Community total shared energy: 1085.29 kWh;

while in the distributed approach:

- Community total expenditure: 151.05 €;
- Community total shared energy: 1085.29 kWh;

They are equal as expected since they are solutions of the same problem. It is worth to notice that the costs and the shared energy are very similar to the LP formulation. This is a good result since, as said before, MILP is a more similar model to the real world scenario and it is more constrained. This consents to conclude that this optimization approach is a valid implementation to manage a real energy community. Differently from the LP formulation, in which small differences occurs, we obtain the same results for the centralized and distributed scenario. This is a direct consequence of the fact that MILP has more constraints than LP leading to a restriction of solutions feasible region for both scenarios. It is also important to observe that the agents that do not generated power buy needed energy only when there is some power generation into the community. This happens since in that period the shared energy is very high and hence the community try to use the green energy produced by it-self avoiding to buy non-renewable energy. Observing fig. 6.1 it is possible to state that agent 1, in distributed case, sells more energy than in the centralized case. This is due to the fact that in a distributed scenario each agent has its own power manager and hence agent 1 is able to consume less generated energy with the direct consequence of increasing the amount of sold energy.

At the end we can conclude that MILP formulation consents to obtain more useful results for a realistic scenario and in addition it is preferable with respect to a centralized scenario for the reason that it leads to the same result and moreover it enjoys all benefits listed above.

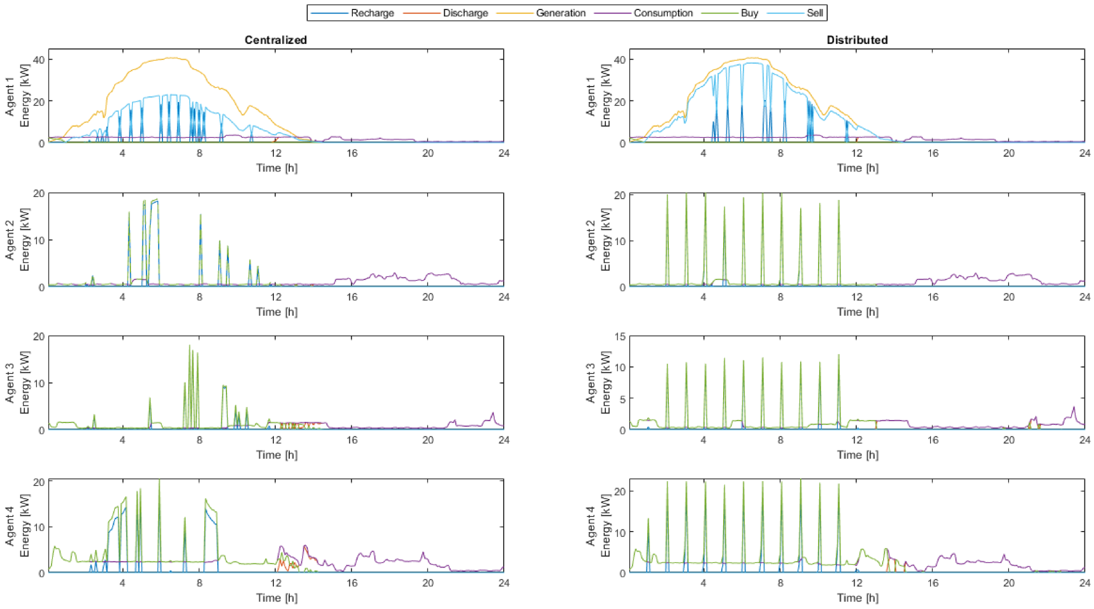

Figure 6.1: Comparison between the temporal evolution of recharge $\mathbf{r}_i(t)$, discharge $\mathbf{d}_i(t)$, generation $\mathbf{g}_i(t)$, consumption $\mathbf{c}_i(t)$, and the corresponding energy bought from the grid $\mathbf{b}_i(t)$ and sold to the grid $\mathbf{s}_i(t)$ obtained by solving the centralized problem (left) and the distributed regularized problem (right).

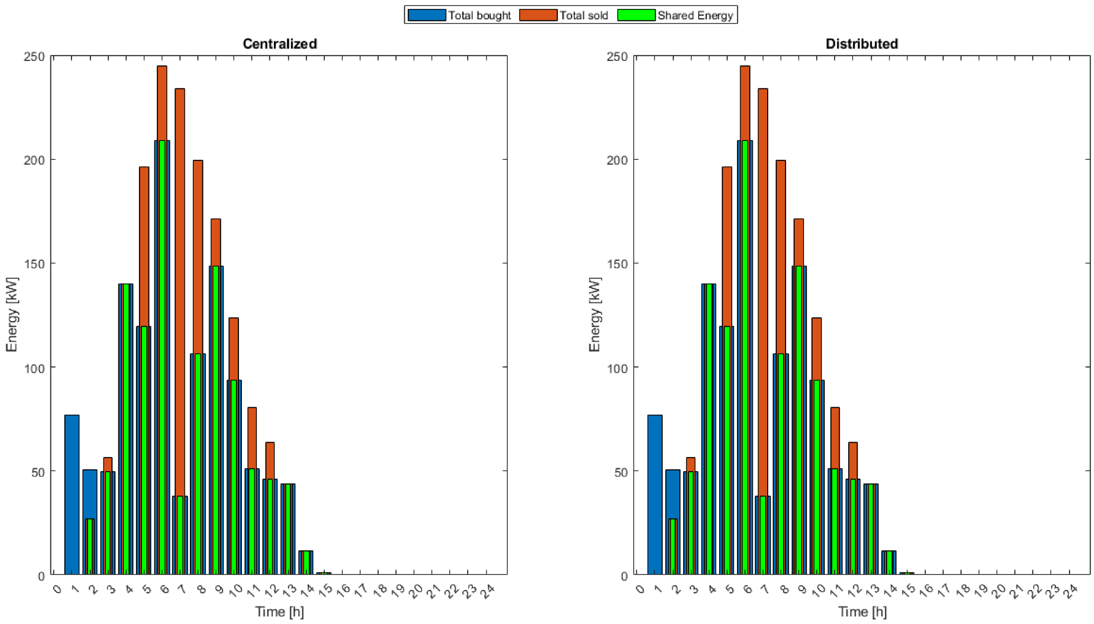

Figure 6.2: Daily total energy bought, total energy sold and total shared energy by the community in the centralized (on the left) and distributed (on the right) approach.

Chapter 7

CO2 emissions

In this section we analyze the amount of CO2 emitted by the community considering all scenarios previously described.
First of all it is important to understand how this quantity is calculated. It is know that in an energy community each member has two important quantities:

- $\mathbf{b}_i \longrightarrow$ Total energy bought from the main grid by agent i, $i \in \mathcal{V}$;
- $\mathbf{s}_i \longrightarrow$ Total energy sold to the main grid by agent i, $i \in \mathcal{V}$.

Selling energy to the grid is a fundamental factor in reducing CO2 emissions. Energy flowing into the main grid is chiefly composed by two quantities:

- Green energy produced using RGs;
- Not renewable energy produced using fossil flues.

The presence of a non renewable part is necessary since nowadays RGs are not able to guarantee energy continuity. As reported by the Italian Ministry of Environment to produce $1kWh$ of energy that has to be injected into the grid $531g$ of CO2 are generated.
When a prosumer sells its energy it consents to increase the quantity of green energy available in the main grid reducing the CO2 pollution. So in order to calculate the CO2 emissions of the community we have to take in account this characteristic.

7.1 How to calculate CO2 emissions

To calculate the community CO2 emissions we first have to find the total amount of bought and sold energy. To do so we can use the following two equations that are reported in (7.1) and (7.2).

$$\mathbf{b}^{TOT} = \sum_{i \in \mathcal{V}} \mathbf{b}_i, \tag{7.1}$$

where $\mathbf{b}^{TOT} \in \mathbb{R}^T$ represents the community total energy bought from the main grid.

$$\mathbf{s}^{TOT} = \sum_{i \in \mathcal{V}} \mathbf{s}_i, \tag{7.2}$$

where $\mathbf{s}^{TOT} \in \mathbb{R}^T$ represents the community total energy sold to the main grid.
Once these two quantities are obtained it is possible to calculate the community CO2 emission using equation (7.3).

$$\mathbf{CO2} = c^{CO2}(\mathbf{b}^{TOT} - \mathbf{s}^{TOT}) \tag{7.3}$$

where $c^{CO2} = 0.531\ kg/kWh$. $\mathbf{CO2} \in \mathbb{R}^T$ is a vector that can have negative components. If this happen it means that the community are not producing CO2 since it is selling green energy. So we have to take in account only the positive values of this vector and to obtain the desired result we have to sum up its components.
In the following we analyze the results obtained in previous different scenarios.

7.2 Basic vs Optimized Heuristic Scenarios

Following the procedure described above in this section we analyze the community CO2 emissions obtained solving our problem in the three heuristic scenarios.
We obtain:

- First basic heuristic approach CO2 emissions: 433.45 kg
- Second basic heuristic approach CO2 emissions: 426.82 kg
- Optimized heuristic approach CO2 emissions: 383.69 kg

It is possible to observe that only making little improvements in the management of the energy flow it is possible to reduce the CO2 emission of about 11.48%. The presence of the battery and its efficient use allows the community to become more sustainable. This can be confirmed also by fig. 7.1 in which the reduction of the CO2 emissions can be noticed when the BESS is present.

7.3 Centralized vs Distributed LP formulation

In this section we analyze the community CO2 emissions obtained solving our problem using the centralized and distributed LP approaches.
We obtain:

- Centralized approach CO2 emissions: 79.20 kg
- Distributed approach CO2 emissions: 79.20 kg

As expected the amount of CO2 emitted in the atmosphere is equal in the centralized and distributed approach. Comparing the results it is possible to observe that applicating an optimization approach that uses the concept of shared energy it is possible to obtain a reduction of about 79.36% of the CO2 emissions with respect to scenarios where there are no cooperation among the community members. This result can be proved observing fig. 7.2 and comparing it with fig.7.1 in which it is evident that the CO2 pollution is drastically reduced using an optimization architecture. Also BESSs play an important rule in this aspect since they consent community to become more self-sufficient. Observing fig. 7.2 it can be noticed that after the power generation period (≈13-th hour) a drastically reduction of the CO2 emissions is obtained since the community starts to utilize energy stored into BESSs.

7.4 Centralized vs Distributed MILP formulation

In this section we analyze the community CO2 emissions obtained solving our problem using the centralized and distributed MILP approaches.
We obtain:

- Centralized approach CO2 emissions: 53.46 kg

- Distributed approach CO2 emissions: 53.46 kg

As expected the amount of CO2 emitted in the atmosphere is equal in the centralized and distributed approach. Comparing the results obtained using the LP formulation and the MILP one it is possible to observe that in the last case we obtain a reduction of about 32.50%. This benefit is the consequence of the fact that MILP is more constrained than LP and it consents to achieve a more efficient behaviour of the BESSs and of the shared energy. Consequently the community become self-sufficient after the period in which there is power generation causing a reduction of the bought energy and hence of the CO2 emissions. These considerations can be observed comparing the right parts of the plots reported in fig. 7.2 and fig. 7.3. In addition it is important to observe that during the power generation in all cases there are no CO2 emissions and this is the consequence of the fact that the community do not buy energy from the main grid thanks to the presence of the renewable generator. This is an important proof of the fact that the installation of RGs can be a precious helper to fight the increasing of CO2 pollution in our planet.

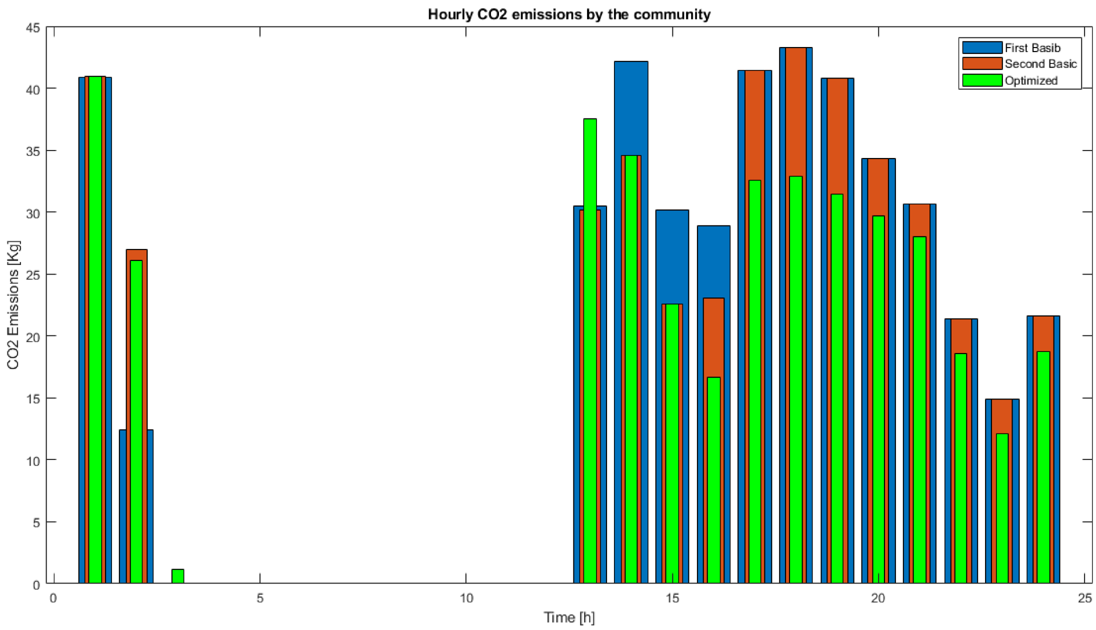

Figure 7.1: Hourly CO2 emissions by the community in the three heuristic approaches.

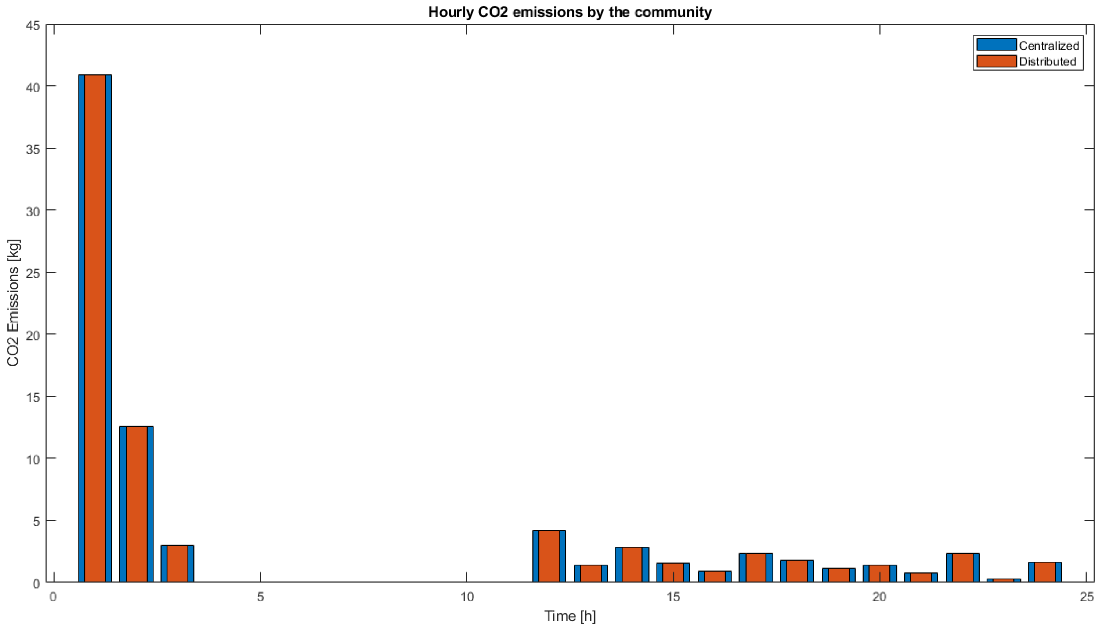

Figure 7.2: Hourly CO2 emissions by the community in the centralized and distributed LP approach.

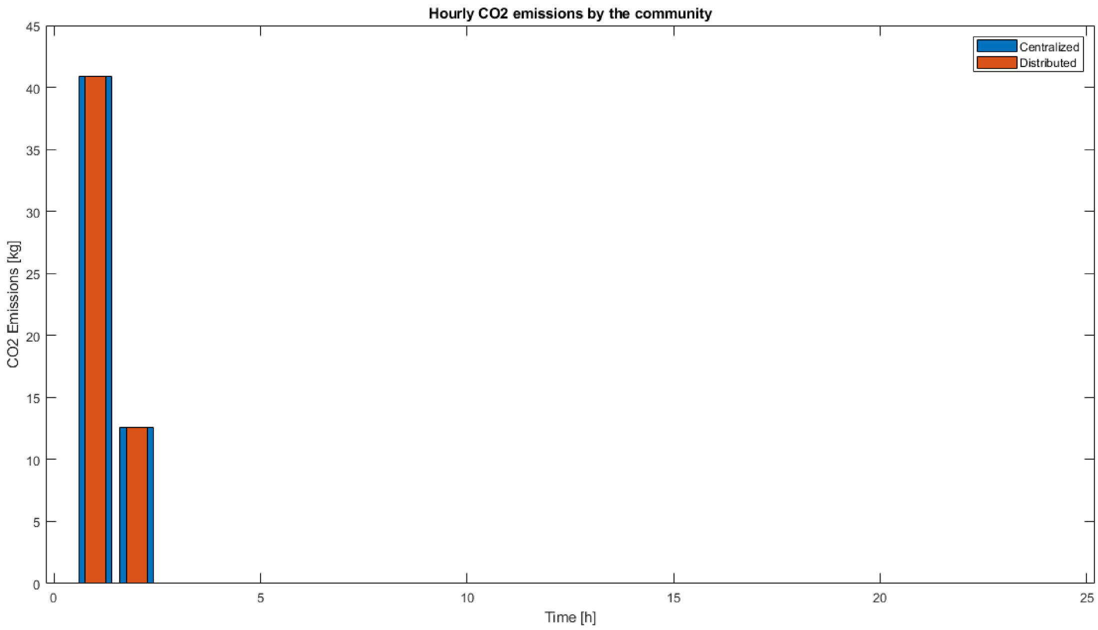

Figure 7.3: Hourly CO2 emissions by the community in the centralized and distributed MILP approach.

Chapter 8

Electric Vehicles

The number of people showing interest into the energy transition topic is continuously increasing. This fact has a big impact also in the transport sector. All the largest companies in the automotive world are investing large capital in the development of electric vehicles due to the fact that the use of electric propulsion engines allows CO2 emissions reduction in the atmosphere. In recent years in fact we have seen the creation of Formula E, a global car racing competition with electrically powered vehicles which is gathering many fans. Furthermore to make commercial transport more sustainable, prototypes of electric trucks and boats have been developed by numerous companies and should be on the market in a few years. Attracted by numerous state incentives and continuous innovation that is leading to greater autonomy and reliability of electric vehicles, many citizens are purchasing an electric vehicle. Due to these reasons in this section we add to the community previously examined a modification. We want to consider a scenario in which each component of the energy community is equipped with an electric vehicle.

8.1 LP Problem Formulation

Our aim is to bouild a model able to fully charge the EV battery if the vehicle is plugged in. To do so we have to consider some constrains that have to be added to the previous LP model. Firstly we have to consider that each EV battery has a maximum and a minimum possible amount of charge. These two quantities are described by the constants $E_{ev}^{MAX} \in \mathbb{R}_{\geq 0}$ and $E_{ev}^{MIN} \in \mathbb{R}_{\geq 0}$. We consider that the power consumption of each agent can be written as in equation (8.1).

$$\mathbf{c}_i = \mathbf{c}_i^l + \mathbf{c}_i^{ev} \quad , \qquad \forall i \in \mathcal{V} \tag{8.1}$$

where:

- $\mathbf{c}_i^l \in \mathbb{R}^T$ is the power consumption that i-th agent needs to satisfy its loads;
- $\mathbf{c}_i^{ev} \in \mathbb{R}^T$ is the EV charging power of the i-th agent.

Furthermore we consider that the battery of the EV, once plugged in, can only be charged and hence it can not be shared with any other components of the energy community and this can be modeled using the inequality reported in (8.2).

$$\mathbf{c}_i^{ev} \geq 0 \quad \Longrightarrow \quad \mathbf{c}_i^{ev} \in \mathbb{R}_{\geq 0}^T. \tag{8.2}$$

At the end we have to model the fact that the battery of the EV has to be fully charged during the day and hence we have to respect the following equality (8.3).

$$\sum_{k \in H} \mathbf{c}_i^{ev}(k) = E_{ev}^{MAX}. \tag{8.3}$$

Electric vehicles manufacturers impose in their products that the SoC of batteries embedded in in their machines has to remain within specific bounds. This condition is stated in the following equation (8.4).

$$\varepsilon_{i,ev}^{MIN} \mathbf{1} \leq \varepsilon_{i,ev} \leq \varepsilon_{i,ev}^{MAX} \mathbf{1}, \tag{8.4}$$

where $\varepsilon_{i,ev}^{MIN} \in [0,0.5]$ and $\varepsilon_{i,ev}^{MAX} \in [0.5,1]$.
Recalling (2.4) it is possible to model EV Battery Energy Storage Systems as:

$$e_{i,ev}^{MAX} \frac{d}{dt} \varepsilon_{i,ev}(t) = \eta_i r_{i,ev}(t) \tag{8.5}$$

In the above equation the discharging power $d_i(t)$ is not present since the battery, as said before, can not be discharged when the EV is connected with the prosumer. At the end, following the same considerations done in section 4.3, we can obtain the final constraint reported in (8.6).

$$\varepsilon_{i,ev}^{MIN} \mathbf{1} \leq D^{-1} \left[\frac{e_{i,ev}^{MAX}}{\Delta} (\eta_i \mathbf{r}_{i,ev}) + \mathbf{e}_1 \varepsilon_{i,ev}(t_{k-1}) \right] \leq \varepsilon_{i,ev}^{MAX} \mathbf{1}. \tag{8.6}$$

Now, considering the problem formulation in (5.4) and adding the constraints just mentioned, it is possible to obtain an LP problem formulation as in (8.7) that takes into account also the part regarding EVs.

$$\begin{aligned} \min_{\mathbf{v},\vartheta} \quad & f(\mathbf{v},\vartheta) \\ \text{subject to:} \quad & (4.1),(4.2),(4.5),(8.1),(8.2),(8.3),(8.6) \qquad \forall i \in \mathcal{V}, \\ & \vartheta - \sum_{i\in\mathcal{V}} g(\mathbf{b}_i,\Upsilon) \leq \mathbf{0}, \\ & \vartheta - \sum_{i\in\mathcal{V}} g(\mathbf{s}_i,\Upsilon) \leq \mathbf{0}, \end{aligned} \tag{8.7}$$

in which we have to add into the decision variables also the consumption regarding the electric vehicles as reported in the following.

$$\mathbf{v}_i = [\mathbf{r}_i^\top, \mathbf{d}_i^\top, \mathbf{d}_i^{c\top}, \mathbf{g}_i^{c\top}, \mathbf{c}_i^{ev\top}]^\top$$

and consequently we have:

$$\mathbf{v} = [\mathbf{v}_1^\top, \ldots, \mathbf{v}_n^\top]^\top.$$

8.2 Results

In this case we obtain a community total expenditure of about: 156.26 €.
This value is very similar to one obtained in the previous scenarios. It means that the community is able to satisfy also small additional loads, that in this case are represented by EV batteries, without resorting to much to the energy grid. In figure 8.1 it is possible to observe that the EV battery is charged when there is power generation. This consents the community to avoid buying energy from the main grid an hence to reduce costs and CO2 emissions. In addition we observe that batteries are fully recharged at the end of their charging period since their state of charge are all equal to the value one as it is visible in figure 8.2 and hence the initial objective is achieved.

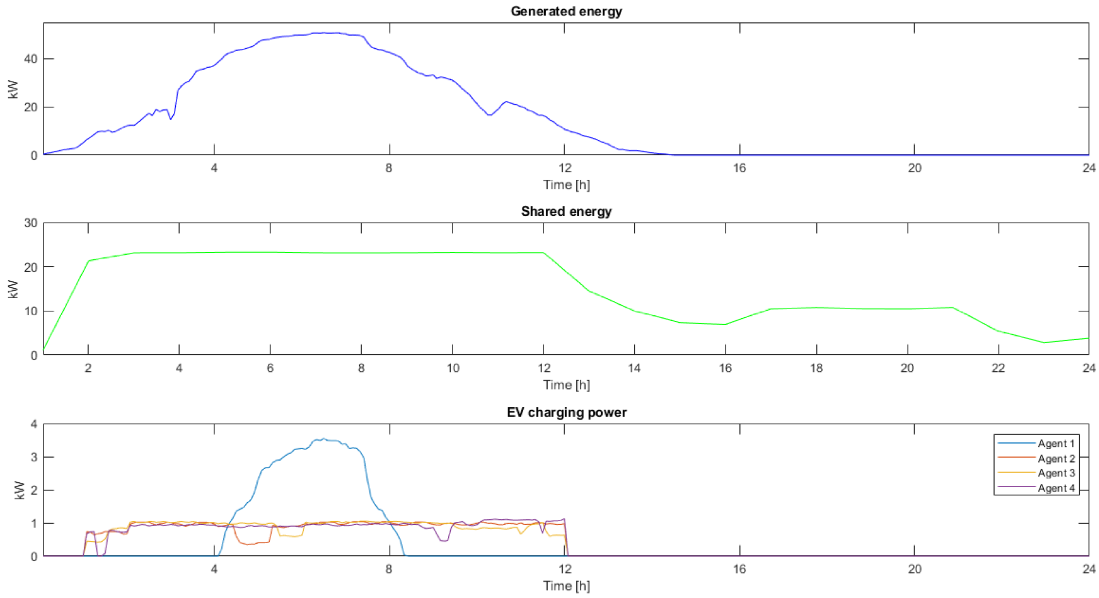

Figure 8.1: Total amount of generated power (on the top), total amount of shared energy (in the middle), charging power of each EV battery (on the bottom).

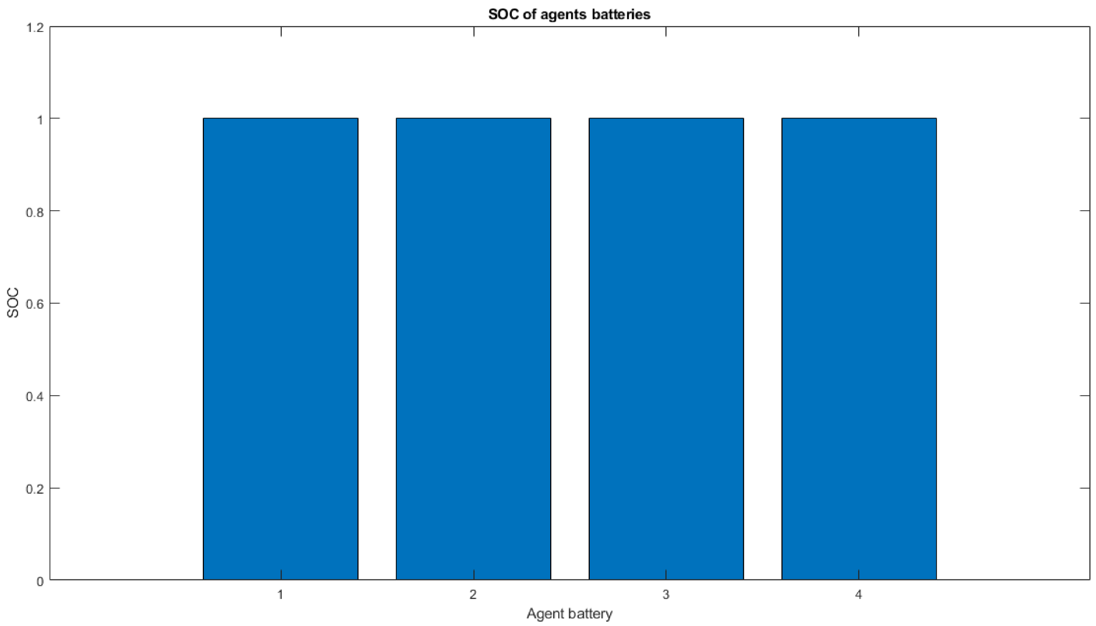

Figure 8.2: Final SOC of each battery that is present the community.

Chapter 9

Shiftable Loads

In this section we introduce the concept of shiftable loads on our energy community. They are loads in which their starting time can be shifted along a specific time interval in order to obtain more benefits from the optimization algorithm.

9.1 LP Problem Formulation

In this case the power consumption $\mathbf{c}_i \in \mathbb{R}^T$ can be separated into two components as reported in (9.1).

$$\mathbf{c}_i = \mathbf{c}_i^n + \mathbf{c}_i^s, \tag{9.1}$$

where $\mathbf{c}_i^n \in \mathbb{R}^T$ and $\mathbf{c}_i^s \in \mathbb{R}^T$ represent the non-controllable and shiftable loads respectively.
We want to model the scenario in which a shiftable load is active or not in a specific time instant k. To do so the introduction of a binary variable is needed. We denote this binary variable by $\delta_i^s(k) \in \{0,1\}$ and its behaviour is described in (9.2).

$$\delta_i^s(k) = \begin{cases} 1 & \text{if shiftable load of i-th agent is ON at k} \\ 0 & \text{otherwise.} \end{cases} \tag{9.2}$$

We consider that the total amount of time needed by the shiftable load to complete all its functions is known in advance. We call this quantity *Total ON-time* and we describe it with the constant T_i^a. Furthermore it is obvious that the sum of the the time instants in which the i-th variable load is ON has to be equal to T_i^a. This consideration is described in equation (9.3).

$$T_a = \sum_{k \in H} \delta_i^s(k), \tag{9.3}$$

where H represents the whole time horizon.

In this work we consider that shiftable loads are uninterruptible. It means that once turned on they must complete their works with no interruptions before being shut down. To implement this characteristic we have to introduce into the model the inequality constraints reported in (9.4).

$$-1 < \frac{-\sum_{k \in H} \delta_i^s(k)}{T_a} \leq \delta_i^s(k) - \delta_i^s(k-1). \tag{9.4}$$

Variables $\delta_i^s(k)$ are stored inside an unique vector that in what follows we call $\delta_i^s \in \mathbb{R}^T$.

In order to take in consideration shiftable loads into the problem formulation we have to modify the model of the bought energy $\mathbf{b}_i$ as reported in equation (9.5).

$$\mathbf{b_i} = \mathbf{r}_i + \mathbf{c}_i + \delta_i^s \bar{\mathbf{c}}_i^s - \mathbf{d}_i^c - \mathbf{g}_i^c. \tag{9.5}$$

In the above equation the term $\bar{\mathbf{c}}_i^s \in \mathbb{R}^T$ represents the mean power consumption of the i-th shiftable load. It is useful to use these quantity instead of the original one in order to simplify the problem by reducing the number of variables and constraints. Instead of optimizing the power usage for every time step, it is possible to use an average value, which makes the problem less complex and faster to solve.

Once the best starting time is found it is so possible to consider the original signal for the shiftable loads in the bought energy $\mathbf{b}_i \in \mathbb{R}^T$ to obtain the final result as reported in equation (9.6).

$$\mathbf{b_i} = \mathbf{r}_i + \mathbf{c}_i + \delta_i^s \mathbf{c}_i^s - \mathbf{d}_i^c - \mathbf{g}_i^c. \tag{9.6}$$

It is possible to solve this problem using the centralized LP problem formulation reported in (9.7).

$$\begin{aligned}
\min_{\mathbf{v},\vartheta} \quad & f(\mathbf{v}, \vartheta) \\
\text{subject to:} \quad & (4.1), (4.2), (4.5), (9.2), (9.3), (9.4) \qquad \forall i \in \mathcal{V}, \\
& \vartheta - \sum_{i \in \mathcal{V}} g(\mathbf{b}_i, \Upsilon) \leq \mathbf{0}, \\
& \vartheta - \sum_{i \in \mathcal{V}} g(\mathbf{s}_i, \Upsilon) \leq \mathbf{0},
\end{aligned} \tag{9.7}$$

in which we have to add into the decision variables also the binary variable that decides the starting time of the shiftable load as represented in the following equation.

$$\mathbf{v}_i = [\mathbf{r}_i^\top, \mathbf{d}_i^\top, \mathbf{d}_i^{c\top}, \mathbf{g}_i^{c\top}, \delta_i^{s\top}]^\top,$$

and consequently we have:

$$\mathbf{v} = [\mathbf{v}_1^\top, \dots, \mathbf{v}_n^\top]^\top.$$

9.2 Results

Shiftable loads that we use in this section are reported in fig. 9.1. Their new ON-time obtained by solving the problem is reported on the top of fig. 9.2. It is possible to note that the optimization problem decides to place shiftable loads starting time where there is power generation and hence more shared energy. In fact we obtain:

- New starting time for agent 1 variable load = 195-th minute;
- New starting time for agent 2 variable load = 99-th minute;
- New starting time for agent 3 variable load = 135-th minute;
- New starting time for agent 4 variable load = 120-th minute.

So at the end shiftable loads are placed according to their new ON-time as reported on the bottom of fig.9.2.
To test the effectiveness of this approach we have to compare the community total expenditure in the case in which shiftable load is placed in a wrong time period and the scenario in which shiftable loads are placed at the optimum time period. To do so we consider the original position of shiftable load to test the first case.
We obtain:

- Community total expenditure in the first case: 335.58 €
- Community total expenditure in the second case: 303.22 €

So this approach consents to obtain a daily cost saving of about 32.36 € that is a reduction of about 9.64%.

Obviously the community total expenditure is increased with respect to the scenarios previously analyzed but it is the direct consequence of the fact that the energy needed by the community has undergone a large increase.

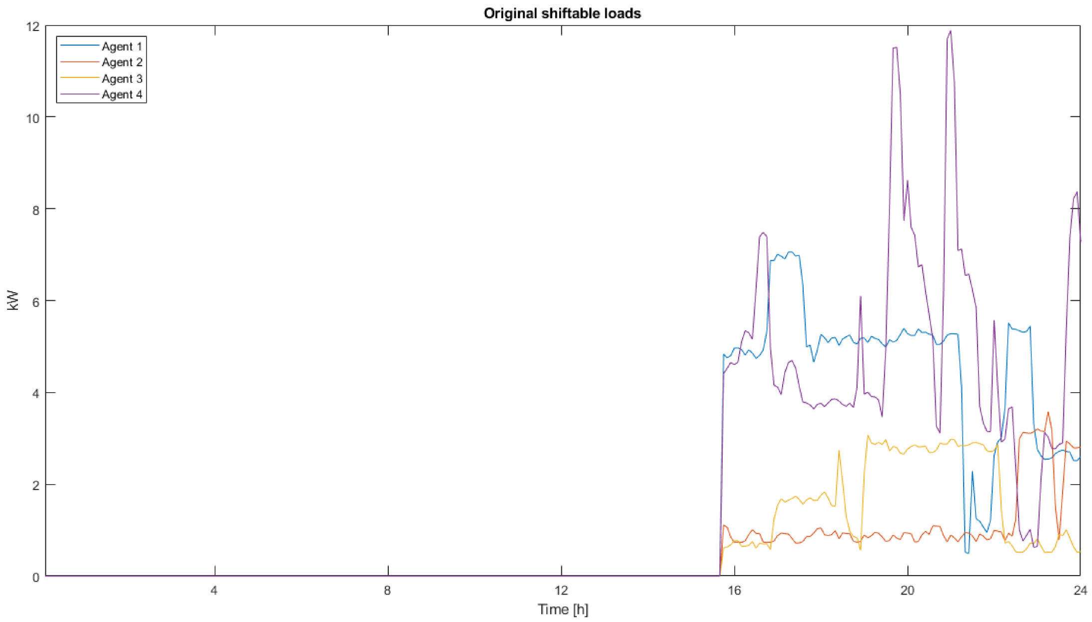

Figure 9.1: Initial representation of shiftable loads.

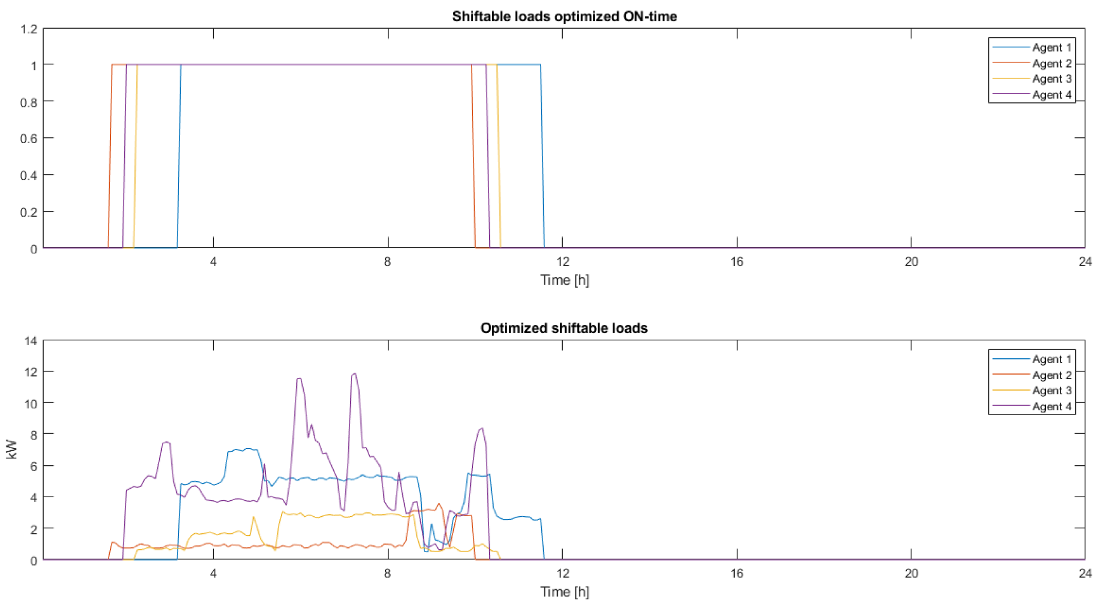

Figure 9.2: New shiftable loads ON-time (on the top) and optimized position for shiftable loads (on the bottom).

Chapter 10

Conclusions

In this thesis we have presented a model for energy communities that takes into account the concept of shared energy. We have proposed an LP and a MILP problem formulations in centralized and distributed approach to solve our problem. We have therefore concluded that optimization approaches consent to obtain considerable benefits regarding costs savings and CO2 emissions reduction with respect to any heuristic approach. We have observed that a MILP formulation consents to model in a more realistic way the energy community studied and in addition it consents to obtain a reduction of the CO2 emissions with respect to the LP formulation.
In the following we report a summary of the results obtained:

Best heuristic approach

- Community total expenditure: 193.27 €;
- Community total shared energy: 0 kWh;
- CO2 emissions: 383.69 kg.

LP problem formulation
Centralized approach:

- Community total expenditure: 146.67 €;
- Community total shared energy: 1052.11 kWh;

- CO_2 emissions: 79.20 kg.

Distributed approach:

- Community total expenditure: 147.70 €;
- Community total shared energy: 1051.66 kWh;
- CO_2 emissions: 79.20 kg.

MILP problem formulation
Centralized approach:

- Community total expenditure: 151.05 €;
- Community total shared energy: 1085.29 kWh;
- CO_2 emissions: 53.46 kg.

Distributed approach:

- Community total expenditure: 151.05 €;
- Community total shared energy: 1085.29 kWh;
- CO_2 emissions: 53.46 kg.

At the end we have modified the model inserting first electric vehicles and secondly shiftable loads. It has been noticed that our model can be easily changed in order to take in consideration these modifications. The obtained results show that it is possible to completely charge the EV batteries during a day with a reduced cost for the community thanks to the concept of shared energy. In addition we have observed that our model is able to correctly find the best starting time if shiftable loads are present in the community members.
We firmly believe that, based on the results obtained, the implementation of energy community represents a significant opportunity to reduce the CO_2 pollution of the planet. It consents to become less dependent from non-renewable resources and it consents to reduce cost for energy purchase making the community members more self-sufficient. These results are the proof that an energy transition is possible and government should support this technology and invest in its development since it consents to obtain an huge amount of benefits for citizens and for the planet.

Future Works

Future works can be based on the implementation of a distributed scenario regarding also the case in which EVs and shiftable loads are considered. In addition it would be possible to try to implement a single problem in which these two last cases are both embedded together. It would also be curious to understand how much a community can grow before observing consistent differences between the centralized and distributed scenario.
An other interesting scenario would be to try to introduce different types of renewable generators as wind or hydroelectric turbines since they are able to produce energy also in cloudy days or during the night. In this way it may be possible to obtain energy continuity and observe if this approach can lead to consistent benefits. Moreover it would be curious to try to adapt this model also for the management of heating systems in order to obtain a more sustainable community.

In this last section we have reported only a limited number of possible future works that could be developed based on the studied model since its quality is that it can be adapted to solve many different real world problems.

Notations

- $\mathbb{R}$, $\mathbb{Z}$: sets of real and integer numbers
- $\mathbb{R}_{\geq 0}$, $\mathbb{N}$, $\mathbb{R}_{+}$, $\mathbb{N}_{+}$: non negative and positive entries of $\mathbb{R}$ and $\mathbb{Z}$
- Matrices are denoted by uppercase letters whose components are denoted by non bold letters (i.e. $\mathrm{M} = \{m_{ij}\}$ [1])
- Vectors are denoted by bold lowercase letters whose components are denoted by non bold letters (i.e. $\mathbf{x} = [x_1, ..., x_n]$ [2])
- I_n, $\mathbf{1}_n$: identity matrix and vector of ones of dimension $n \in \mathbb{N}_+$

Signals and Sampling

Considering a continuous time signal $x(t) \in \mathbb{R}$ with $t \in \mathbb{R}$ and a sampling time $\Delta \in \mathbb{N}_+$, we denote by $t_k = \Delta k$ $(k \in \mathbb{N})$ the discrete time at which the signal is sampled obtaining the discrete time signal $x(t_k) \in \mathbf{R}$. If we refer to a collection of $T \in \mathbf{N}$ samples of $x(t)$ starting from t_k we use the notation:

$$[x]_k^T = [x(t_k), ..., x(t_{k+T-1})]^\top \in \mathbb{R}^{\mathrm{T}}$$

When it is clear from the context we use the slender notation: $\mathbf{x} := [x]_k^T$.

Networks and Graphs

Networks are considered composed by $n \in \mathbb{N}_+$ interconnected agents. The pattern of interactions is described by the graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. The set of nodes modeling

[1] In this case M represent a square matrix of dimension $n \in \mathbb{N}_+$ with entries $m_{ij} \in \mathbb{R}$ with $i, j = 1, ..., n$.

[2] In this case $\mathbf{x}$ denotes a vector of $n \in \mathbb{N}_+$ entries $x_i \in \mathbb{R}$ with $i = 1, ..., n$.

the agents is represented by $\mathcal{V}$ instead the set of edges modelling the point-to-point interaction is described by $\mathcal{E} \subseteq (\mathcal{V} \times \mathcal{V})$. In this work undirected graph is considered, it means that if $(i, j) \in \mathcal{E}$ then $(j, i) \in \mathcal{E}$, as a consequence interactions among agents of the community is bidirectional. It is also considered graphs without self-loops (i.e. $i \notin \mathcal{N}_i$).
Some other useful definitions regarding graphs are reported hereafter:

- Nodes $i, j \in \mathcal{V}$ are *neighbors* if an edge connecting them exists (i.e. $(i, j) \in \mathcal{E}$)
- The set that collect all neighbors of the i-th node node is represented by $\mathcal{N}_i$ with $\mathcal{N}_i = \{j \in \mathcal{V} : (i, j) \in \mathcal{E}\}$

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Appendix A

Two Formulations of Shared Energy: Proof

The objective is to prove that the quantity

$$E_{sh}(\mathbf{b}, \mathbf{s}, \Upsilon) = \min \left\{ \sum_{i \in \mathcal{V}} g(\mathbf{b}_i, \Upsilon), \sum_{i \in \mathcal{V}} g(\mathbf{s}_i, \Upsilon) \right\}$$

is equal to

$$E_{sh}(\mathbf{b}, \mathbf{s}, \Upsilon) = \sum_{i \in \mathcal{V}} g(\mathbf{g}_i^{sc}, \Upsilon).$$

Proof:

Since the total energy sold $\mathbf{s}_i$ can be expressed as:

$$\mathbf{s}_i = \mathbf{g}_i^{sc} + \mathbf{g}_i^{sg},$$

it is straightforward that the shared energy E_{sh} can be written as:

$$E_{sh}(\mathbf{b}, \mathbf{s}, \Upsilon) = \min \left\{ \sum_{i \in \mathcal{V}} g(\mathbf{b}_i, \Upsilon), \sum_{i \in \mathcal{V}} \left(g(\mathbf{g}_i^{sc}, \Upsilon) + g(\mathbf{g}_i^{sg}, \Upsilon) \right) \right\}.$$

To reach our goal it is necessary to consider two cases:

- Case 1 -

We analyze the scenario:

$$\sum_{i\in\mathcal{V}} g(\mathbf{b}_i,\Upsilon) \quad = \quad \sum_{i\in\mathcal{V}} g(\mathbf{g}_i^{sc},\Upsilon).$$

If this equality holds it is straightforward to have:

$$\sum_{i\in\mathcal{V}} g(\mathbf{s}_i,\Upsilon) \quad \geq \quad \sum_{i\in\mathcal{V}} g(\mathbf{b}_i,\Upsilon).$$

The direct consequence is that the shared energy E_{sh} takes value:

$$E_{sh}(\mathbf{b},\mathbf{s},\Upsilon) = \sum_{i\in\mathcal{V}} g(\mathbf{b}_i,\Upsilon) = \sum_{i\in\mathcal{V}} g(\mathbf{g}_i^{sc},\Upsilon)$$

- Case 2 -

We analyze the scenario:

$$\sum_{i\in\mathcal{V}} g(\mathbf{b}_i,\Upsilon) \quad \geq \quad \sum_{i\in\mathcal{V}} g(\mathbf{g}_i^{sc},\Upsilon).$$

If this inequality is verified this means that:

$$\sum_{i\in\mathcal{V}} g(\mathbf{g}_i^{sc},\Upsilon) = 0$$

and consequently:

$$\sum_{i\in\mathcal{V}} g(\mathbf{s}_i,\Upsilon) \quad \leq \quad \sum_{i\in\mathcal{V}} g(\mathbf{b}_i,\Upsilon).$$

So at the end the shared energy E_{sh} takes value:

$$\begin{aligned} E_{sh}(\mathbf{b},\mathbf{s},\Upsilon) &= \sum_{i\in\mathcal{V}} g(\mathbf{s}_i,\Upsilon) \\ &= \sum_{i\in\mathcal{V}} g(\mathbf{b}_i,\Upsilon), \sum_{i\in\mathcal{V}} \big(g(\mathbf{g}_i^{sc},\Upsilon) + g(\mathbf{g}_i^{sg},\Upsilon)\big) \\ &= \sum_{i\in\mathcal{V}} g(\mathbf{g}_i^{sc},\Upsilon) \end{aligned}$$

We can therefore conclude that the two quantities are equal, QED[1].

□

[1] Quod Erat Demonstrandum

Appendix B

Linear Programming (LP)

Linear programming (LP) or Linear Optimization provides a powerful framework for making optimal decisions in complex scenarios with limited resources. It is a mathematical technique used for optimizing an objective function, subject to a set of constraints. The peculiarity of the LP is to have a linear objective function and linear equalities or inequalities as constraints. An LP could be represented in two equivalent formulations: Canonical form (on the left) and Standard form (on the right).

$$\begin{aligned} \min_{\mathbf{x}} \quad & \mathbf{c}^\top \mathbf{x} \\ \text{s.t.:} \quad & A\mathbf{x} \geq b \\ & \mathbf{x} \geq 0 \end{aligned} \quad , \quad \begin{aligned} \min_{\mathbf{x}} \quad & \mathbf{c}^\top \mathbf{x} \\ \text{s.t.:} \quad & A\mathbf{x} = b \\ & \mathbf{x} \geq 0 \end{aligned}$$

where:

- $\mathbf{x}$ is the unknown vector to be determined,
- $\mathbf{c}$ and $\mathbf{b}$ are given known vectors,
- A is a given matrix,
- $\mathbf{c}^\top \mathbf{x}$ is the linear objective function,
- $A\mathbf{x} \geq b$ and $\mathbf{x} \geq 0$ are the linear constraints.

The two formulations are equivalent, yet the conversion from one to the other may implicate the variation of the number of constraints and variables of the problem.

The main methods used to solve LP problems are: Simplex Method, Interior-Point Method, Dual Simplex Method, Revised Simplex Method and Cutting-Plane.
Once solved the problem it is useful to quickly check if the obtained solution is feasible or not. To do so it is useful to introduce the LP dual. The dual problem (D) allows to obtain an underestimate of the starting primal problem (P).
Primal and the Dual are two different representations of the same problem, they are defined by the same vectors $\mathbf{c}^\top$, $\mathbf{b}$ and matrix A but in the dual these quantities play different roles, in particular:

- The objective vector $\mathbf{c}^\top$ becomes the righter side vector;
- The righter side vector $\mathbf{b}$ becomes the objective vector;
- Matrix A becomes $A^\top$.

Primal (on the right) and Dual (on the left) are reported hereafter:

$$\begin{aligned} \min_{\mathbf{x}} \quad & \mathbf{c}^\top \mathbf{x} \\ \text{s.t.:} \quad & A\mathbf{x} = \mathbf{b} \\ & \mathbf{x} \geq 0 \end{aligned} \quad , \quad \begin{aligned} \max_{\mathbf{b}} \quad & \mathbf{u}^\top \mathbf{b} \\ \text{s.t.:} \quad & \mathbf{u}A^\top \leq \mathbf{c} \end{aligned}$$

So the primal and the dual solutions $\mathbf{x}^P$,$\mathbf{x}^D$ are an overestimate and an underestimate of the optimal solution $\mathbf{x}^*$ of the problem respectively. So the following inequalities are obtained:

$$\mathbf{x}^D \leq \mathbf{x}^* \leq \mathbf{x}^P$$

If the primal and dual solutions coincide this means that we have obtained an optimal solution for our problem, while if they do not coincides this means that our problem suffers of Duality Gap and hence the solution is not optimal.
More precise details regarding Duality Gap problem can be found in [3] and [12].

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"It always seems impossible until it's done."

– Nelson Mandela –