UNIVERSITA' DEGLI STUDI DI PADOVA

DIPARTIMENTO DI SCIENZE ECONOMICHE ED AZIENDALI "M.FANNO"

CORSO DI LAUREA MAGISTRALE / SPECIALISTICA IN ECONOMICS AND FINACE

TESI DI LAUREA

Alternative Asset Classes: an opportunity for portfolio diversification?

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ANNO ACCADEMICO 2015 – 2016

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ABSTRACT

Il lavoro persegue l'obiettivo generale di indagare sull'efficacia dell'utilizzo di classi di assets alternativi (Alternative Asset Classes) da affiancare ai classici strumenti di investimento, quali Obbligazioni e Azioni, nella scelta di allocazione strategica di portafogli. In finanza non vi è una ben precisa definizione di "investimento alternativo" ma più generalmente esso viene inteso come strumento di investimento che non rientra nell'ordinario portafoglio di un investitore classico. All'interno di questo lavoro vengono utilizzate 7 diverse classi, che fanno riferimento a dei precisi indici, per l'allocazione delle risorse: Hedge Funds, Real Estates, Commodities, Private Equities ed infine Sovereign Bonds, Corporate Bonds ed Equities facenti parte dell'area dei Paesi Emergenti. La Tesi, oltre all'introduzione e alla conclusione, è composta da 5 capitoli. I primi 2 costituiscono la base teorica e fanno riferimento, rispettivamente, alla Portfolio Theory comprendente il modello di Markowitz e l'approccio Risk Budgeting, e agli Indici di Performance, mentre gli ultimi 3 riguardano la descrizione degli indici usati comprendente anche la loro analisi storica, e l'analisi empirica sull'efficacia di diversificazione degli assets alternativi tramite 3 diversi casi e sulla performance tramite l'evoluzione della composizione di 4 portafogli nel tempo.

1 Summary

Introduction

In the last years, the reference interest rates, used by central banks to implement monetary policies, have been brought to extraordinary low levels. The observance of very accommodating monetary conditions was primarily imposed by the will to prevent the risk of an economic recession and to contrast the deflationary movements. At the same time, given the financial perturbations provoked by the generally called “great recession”, the reduction in the interest rates has counterbalanced the excessive growth of the costs associated to refinancing, provoked by the widening of the country *spread.* In addition, the remuneration from risk free financial activities has been very scares, this implies that investors, looking for safe and sound opportunities of investment, have been forced to accept extreme low level of profit. With the fixed income returns estimated to stay low for a long period and the instability on the stock markets, coming from a recovery that struggles to stabilize, the idea of looking at other forms of investment could end up as a winning choice.

Different markets, beyond the American and the European ones, different assets like real estates, private equities, commodities or hedge funds could increase the level of diversification inside the investor’s portfolio and provide with an extra-return beyond those of stocks and bonds.

In my thesis, I will analyse all these classes, generally referred to as Alternative Asset Classes, to understand their potential as a means of portfolio diversification and to understand their abilities to enhance its performance, without overlooking the risk management. The first part of this work will discuss about Markowitz framework as the principal framework to allocate the resources and to build up consistent portfolios; alongside it, I will introduce also the Risk Budgeting theory as an alternative to estimate the fundamental input: the information matrix. An important element of this work is about how to evaluate the results and for this reason I will create a section to illustrate which performance and diversification indices I will employ to assess the portfolios constructed.

Terminated the theoretic part, I will start to analyse what defines an asset class “alternative” and what its characteristics are. The following part will centre on the historical investigation to comprehend how the alternative indices have performed in the past and how the economic cycles have affected them.

To assess the diversification process, I will compare the results obtained by some portfolios, made up of “traditional” assets and constituting the control group, with a series of different portfolios belonging to three different scenarios. In every scenario, I will add two or three different alternative asset classes to the traditional ones, to understand if there are any evident signs of diversification and improvement. In the final part, I will focus only on the performance side of these portfolios to understand more in details the effectiveness of implementing alternative asset classes.

Despite the different methodologies implemented to compute and analyse the indices, the results are quite evident: the Hedge Fund is the index most employed among the possible alternative asset classes. Its level varies from 80 to 95% depending on the specific type of portfolio used. The remaining percentage is distributed among Commodity, Private Equity, Real Estate and EM Equity.
Hedge Fund allows also for the widest diversification within each portfolio when employed, in contrast with the others that permits only a very limited one.
In general, the presence of the alternative asset classes inside the portfolio results quite considerable, more than 30% (with peak of 36%), and this gives the proof of their possible contribution to the investor wealth.

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2 Portfolio theory

The modern state of finance has exponentially increased the investment possibilities, namely the financial instruments available on the market, and has thus widely enhanced the options for the best portfolio allocation. The development of a wide knowledge of new financial theories has opened new ways, improving and simplifying the work carried out by portfolio managers. Nevertheless, the job of an asset manager remains a difficult job, full of risks and uncertainty.

Asset managers are responsible for the asset allocation, a decisional process aimed to invest the clients' wealth over different markets (e.g. Emerging or Frontier Markets), geographical areas (i.e. North America or EMU[1]), sectors (e.g. Utilities or Energy), or asset classes (e.g. Fixed income or Equities), in order to compose a well-diversified portfolio. The process relies on a deep examination by means of an ex-ante analysis of the investor's financial situation (wealth, income statement, balance sheet and fiscal position), risk aversion, investment objectives and not less important investment horizon.

Regarding the composition and the level of management of the portfolio, we can distinguish two asset allocation macro-categories: a passive one and an active one.
The former aims to replicate a reference index, assuming that the market is efficient and thus characterized by assets or asset classes that are not undervalued or overvalued somehow, because all the operators have all the available information (no asymmetric information). The objective is to reproduce as much as possible the return of a particular market, or segment of it, because it is not possible to obtain an extra return apart from the normal one, according to the theory. This approach focuses completely on the concept of diversification, ignoring the market timing of the investment and the stock picking too. Simply put, it creates a portfolio where the weights of different assets or indices reflect those of the reference market or segment of it. By means of the mean-variance criterion, all portfolios that maximise the return, given a certain level of risk, get selected in order to point out the optimal portfolio that suits the investor, provided his own preference/utility profile. The goodness of the passive allocation is measured by estimating the difference (tracking error) between the return from the optimal portfolio and market, or its segment. The investment horizon can be nothing but a medium-long run. It follows that the transaction costs are very low compared to other strategies. "The Exchange

[1] European Monetary Union.

Trade Funds (ETF) are special funds traded on the stock market, which have the only objective to replicate the index (e.g. FTSE MIB, DAX, Nasdaq100, S&P 500) they refer to thought a completely passive management”[2].

The latter is an attempt of portfolio managers to *beat the benchmark* according to the return-risk profile, explicitly aspiring to the realization of an extra-return. The method bases on portfolio adjustments that may produce a better performance than that of the market as benchmark, thanks to the issuer and market analysis. The recall is to the *market timing*, which is the adjustment of the portfolio through the decrease/increase of asset weights, due to positive/negative return forecast, the *stock picking*, namely the analysis and selection of assets based on companies' fundamentals, and the use of *technical-statistical analysis*. Given the technical difficulties and the consequent approximation, this approach applies on a short/medium run and has higher transaction costs due to wide turnover.

These two macro categories have fading boundaries and other types of allocation such as Strategic Allocation, Tactical Allocation, Dynamic Allocation[3], Insured Allocation[4] and Integrated Allocation[5] fall across them.

The Strategic Allocation establishes and adheres to a "base policy mix" - a proportional combination of assets based on expected rates of return for each asset class: generally it implies a “buy-and-hold”[6] strategy, even as the shift in values of assets causes a drift from the initially established policy mix. It is structured on a long run horizon.

The Tactical Allocation can be described as a moderately active strategy. Over the long run, a strategic asset allocation strategy may appear relatively rigid, therefore sometime seems to be necessary to engage in short-term, tactical deviation from the mix to capitalize on unusual or exceptional investment opportunities. This flexibility adds a market-timing component to the portfolio, allowing to participate in economic conditions more favourable for one asset class than for others.

With the Dynamic Allocation, there is a constant adjustment of the mix of assets as markets rise and fall and as economy strengthens and weakens. This method is very active since declining assets are immediately sold and increasing assets are immediately purchased, based on which signs the market produces: consequently the transaction costs are quite high.

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[2] www.borsaitaliana.it

[3] See “Dynamic Asset Allocation”, P. K. Madhogarhia and M. Lam, Journal of Asset Management (2015) 16, 293–302.

[4] See www.economictimes.indiatimes.com

[5] See “Integrated Asset Allocation”, W.F. Sharpe, Financial Analysis Journal, Vol. 43, No. 5 (Sep. - Oct., 1987), pp. 25-32.

[6] Strategy of long-term investment with a low turnover. Its counterpart is Day-Trading.

The main point of an Insured Allocation strategy is to establish a base portfolio value under which the portfolio should not be allowed to drop. As long as the portfolio achieves a return above its base, it is exercised an active management to try to increase the value as much as possible. If, on the contrary, the portfolio drops under the limit value, the weights completely shift to the risk free asset so that the base value becomes fixed. In this case, a re-allocation is the best option, otherwise a change of strategy. Insured Allocation may be suitable for risk-averse investors who desire a certain level of active portfolio management but appreciate the security of establishing a guaranteed floor below which the portfolio is not allowed to decline. With Integrated Allocation, the manager considers both the economic expectations and the risk in establishing an asset mix. While all of the above-mentioned strategies take into account expectations for future market returns, not all of the strategies account for investment risk tolerance. Integrated asset allocation, on the other hand, includes aspects of all strategies, accounting not only for expectations but also for also actual changes in capital markets and risk tolerance.

The portfolio composition occurs to be a complicated problem and requires specific financial, mathematical and statistical skills, in order to ensure continuous long-term returns, and also a certain degree of information, not only from the client (definition of investor's risk aversion and objectives) but from the market too. The problem of portfolio composition has been the centre of a vast literature since the '50s, when Markowitz theorised the Modern Portfolio Theory[7]. In the following years, many studies and researches expanded the work of the Nobel Prize winner[8], such as Tobin [1958][9] and Lintner [1965][10], improving the underlying theory but also the input analysis and computation. This theory has not been exempt from critics or drawbacks. Michaud [1989][11], Black and Litterman [1991][12], Chow et al. [1999][13] and Tutuncu and Koenig [2004][14] are some example.

[7] See Markowitz, H., *Portfolio Selection*, The Journal of Finance, Vol. 7, No. 1. (Mar., 1952), pp. 77-91.
[8] Markowitz won the Nobel Memorial Prize in Economic Sciences in 1990.
[9] See Tobin, J., *Liquidity preference as behaviour towards risk*, The Review of Economic Studies 25, 1958, 65-86.
[10] See Lintner, J., *Portfolios and Capital Budgets*, The Review of Economics and Statistics, Vol. 47, No. 1. (Feb., 1965), pp. 13—37.
[11] See Michaud, R., O., *The Markowitz Optimization Enigma: is 'Optimized' Optimal?.*, Financial Analysts Journal, 1989.
[12] See Black, F. and Litterman, R., *Global Portfolio Optimization*, Financial Analysts Journal; Sep/Oct 1992.
[13] See Chow et al., *Optimal Portfolios in Good Times and Bad,* Financial Analyst Journal, May/June 1999: pp. 65-73.
[14]See Tütüncü, R., H. and Koenig, M., *Robust asset allocation*, Annals of Operations Research, Vol. 132, 2004, pp. 157-187.

An important step in the Portfolio theory has been done thanks to the work of William Sharpe: in 1964 he developed the Capital Asset Pricing Model[15] (CAPM) theory and highlighted the relationship between risk premium of the asset (the difference between the expected return and risk-free rate) and its beta (the systematic risk with respect to the tangency portfolio). The major contribution of Sharpe led to the emergence of index funds and to the increasing development of passive management. Nonetheless, as for the theory of Markowitz, also Sharpe's theory faced many critics (see Black, Jensen and Scholes [1972], Fama and MacBeth [1973], Roll [1977]), especially about the explanatory power of the single risk factor model. Following, different versions of the CAPM saw the light as the Intertemporal CAMP (see Merton [1973]), and the Consumption CAMP as well as Three Factor Model (see Fama and French [1993]), the Multiple Factor Model and the Arbitrage Pricing Theory (APT).

Despite all the critics and drawbacks upon Markowitz's and Sharpe's Model, both are still widely employed in the financial sector. In this work, I use the former's model along with the Risk Budge Approach in order to perform the best strategic asset allocation possible. As to the Risk Budge Approach, it is a methodology that tries to overcome the problems of estimating the expected returns, dealing with only the risk contribution of each asset/asset class. Both of the methods are discussed in the next part.

2.1 The Markowitz model

2.1.1 The efficient frontier

In his paper, Markowitz defined precisely what portfolio selection mean: "*the investor does (or should) consider expected return a desirable thing and variance of return an undesirable thing*". The author explains that an efficient portfolio is the portfolio that maximises the expected return given a certain level of risk (or that minimises the risk given a certain level of expected return). In particular there is not just a single optimal portfolio, but many optimal portfolios that together are called efficient frontier.

First, we need to consider a universe of *n* assets, then we assume:

- $r = (r_1, \dots, r_n)$ is the vector of asset returns where R_i is the return of ith asset,
- $w = (w_1, \dots, w_n)$ is the vector of weights of portfolio,
- The portfolio is completely invested, i.e. $\sum_{i=1}^{n} w_i = \mathbf{1}'w = 1$,

[15] See Sharpe, W., F., *Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk*, The Journal of Finance, Vol. 19, No. 3 (Sep., 1964), pp. 425-442.

In this way, we can compute the return of portfolio as $r(w) = \sum_{i=1}^{n} w_i r_i$ and using the matrix annotation, we obtain $r(w) = w'r$.

Let $\mu(w) = E[r(w)]$ be the vector of expected returns and $\sum = E[(r-\mu)(r-\mu)']$ be the variance-covariance matrix of asset returns. We can now compute the expected return of the portfolio as:

$$\mu(w) = E[r(w)] = E[w'r] = w'E[r] = w'\mu$$

And in the same way the variance of portfolio results as:

$$\sigma^2(w) = E\left[\left(r(w) - \mu(w)\right)\left(r(w) - \mu(w)\right)'\right]$$

$$= E[(w'r - w'\mu)(w'r - w'\mu)]$$

$$= E[w'(r-\mu)(r-\mu)'w]$$

$$= w'E[(r-\mu)(r-\mu)']w$$

$$= w'\sum w$$

The financial investor's problem above mentioned may now be stated in a more formal manner as follows:

- Maximizing the expected return of the portfolio under a volatility constraint (σ-problem):

$$\max \mu(w) \quad u.c. \quad \sigma(w) \leq \sigma^* \tag{1.1}$$

- Minimizing the volatility of the portfolio under a return constraint (μ-problem)

$$\min \sigma(w) \quad u.c. \quad \mu \geq \mu^* \tag{1.2}$$

Example 1. We consider four equity indices: MSCI Equity USA, MSCI Equity JAPAN, MSCI Equity EM[16] and MSCI Equity EMU[17], for the period January 1997 – July 2015. Means, standard deviation and correlation matrix are the following[18]:

Table 1: Computation of mean and standard deviation

	MSCI Equity USA	MSCI Equity JAP	MSCI Equity EM	MSCI Equity EMU
Mean	0.6	0.2	0.55	0.5
St. Dev.	5.26	5.99	7.22	6.91

[16] EM stands for Emerging Market.
[17] EMU stands for European Monetary Union.
[18] Mean, standard deviation, return, volatility and weight are always in percentage.

Table 2: Computation of correlation matrix

	MSCI Equity USA	MSCI Equity JAP	MSCI Equity EM	MSCI Equity EMU
MSCI Equity USA	1	0.53	0.75	0.78
MSCI Equity JAP	0.53	1	0.55	0.49
MSCI Equity EM	0.75	0.55	1	1
MSCI Equity EMU	0.78	0.49	0.76	1

In Figure 1, I run a simulation of a set of portfolios and report their mean and their volatility (orange cycle). Analysing the μ-problem with $\mu^* = 0.53\%$, portfolio A could not be the solution because it is dominated by portfolio C, given the fact that it has lower volatility. On the contrary if we consider the σ-problem with $\sigma^* = 5.015\%$, portfolio D cannot be the solution because dominated by portfolio E, which has a higher mean. From this we can understand that the *efficient frontier* can be defined as the convex curve of all the point (σ(w), μ(w)) of all possible portfolios. In particular, the two optimal portfolios C and E are on the efficient frontier.

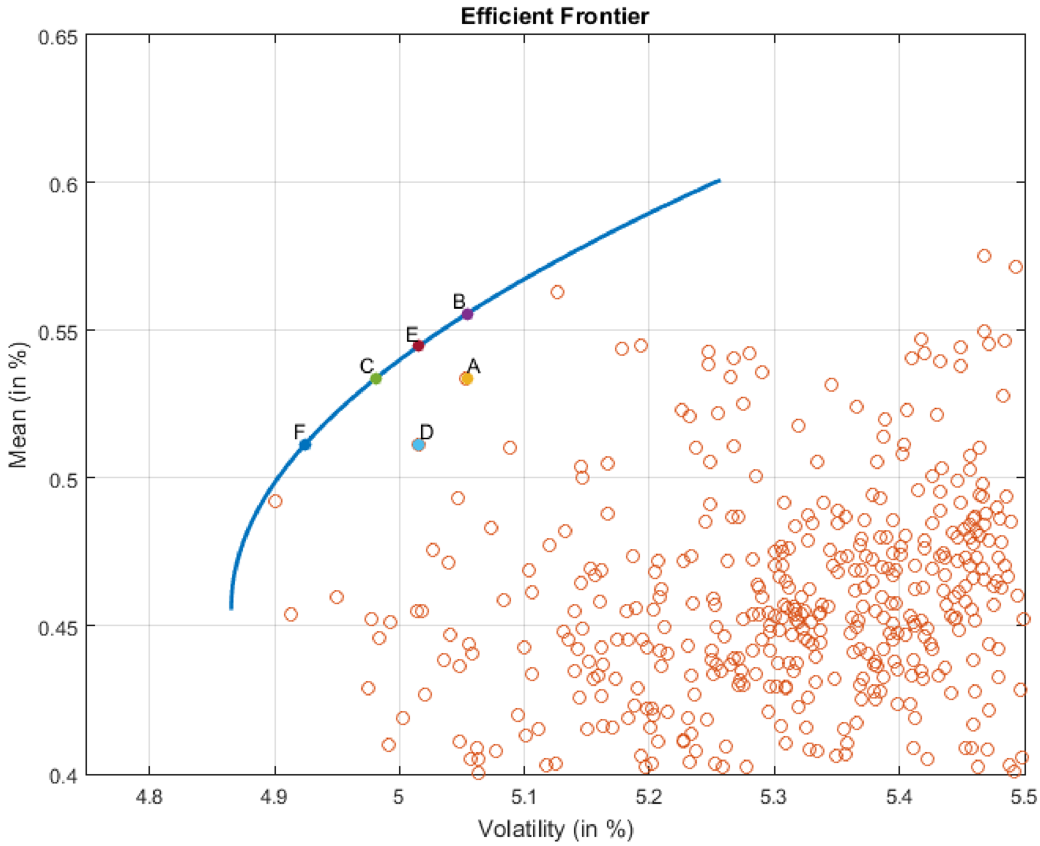

Figure 1: Optimized Markowitz portfolios

If we compute all portfolios belonging to the simplest set define by $\{w \in [0,1]^n: \mathbf{1}'x = 1\}$[19], then we could obtain the expected return and volatility extremes of the portfolios: $\mu^- \leq \mu(w) \leq \mu^+$ and $\sigma^- \leq \sigma(w) \leq \sigma^+$. Then the solution to the maximization problem is simply $\sigma^- \leq \sigma^*$, while the solution for the minimization is $\mu^* \leq \mu^+$. It follows that, if the two conditions are met, the inequality constraints becomes $\sigma(w) = \min(\sigma^*, \sigma^+)$ and $\mu(w) = \max\ (\mu^- \mu^*)$.

2.1.2 The quadratic utility function

A key factor in the portfolio selection is the preference profile of the investor, which is the level of risk that he is willing to bear. Not all the investors are the same, some are more gambler-like and dispose to take more risk for a higher profit, others are more conservative and less prone to suffer great loss. For a very risk-averse investor a suitable portfolio would be the Global Minimum Variance portfolio (GMV), an optimal set of assets that lies on the bottom of the efficient frontier and that possesses the lowest level of volatility. On the contrary a risk-

[19] The simpler case is where Short-selling in not allowed, that is the investor cannot sale a security he does not own or that he has borrowed.

lover mostly would like to hold a Maximum Return (MR) portfolio[20], an optimal set of assets that gives the highest return possible, given the constraint on the portfolio optimization. The risk profile is represented by the utility function, a function, as P. Samuelson [1937] stated, that embodies the desires or what consumers, in our case investors, are willing to pay. In our case the optimal portfolio that the investor is willing to take is the one, following his preference, that is the tangency point between the efficient frontier and the lowest utility curve.

An easier approach to the problem of optimization, other than the original non-linear method, is by means of the use of a quadratic utility function, as stated in Markowitz [1956]. This utility function is the most frequently used in financial economics to describe the investor behaviour, because under the assumption of quadratic utility, mean-variance analysis is optimal[21].

As Roncalli [2012] show, we use the quadratic utility function

$$U(w) = x'w - \frac{\varphi}{2} w'\Sigma w \tag{1.3}$$

to write the optimization problem as

$$\begin{gathered} w^*(\varphi) = \arg\min \ w'\mu - \frac{\varphi}{2} w'\Sigma w \\ \text{u.c. } \mathbf{1}'w = 1 \end{gathered} \tag{1.4}$$

With this formulation we are able to incorporate the investor risk-aversion parameter as φ. In this way if $\varphi = 0$, the optimized portfolio is the one that maximizes the expected return and we have $\mu\big(w^*(0)\big) = \mu^+$. In this case we are facing a risk-lover investor.

On the other hand, if $\varphi = \infty$ the optimization problem is:

$$\begin{gathered} w^*(\infty) = \arg\min \frac{1}{2} w'\Sigma w \\ \text{u.c. } \mathbf{1}'w = 1 \end{gathered} \tag{1.5}$$

This is the optimized portfolio that minimizes the volatility and we have $\sigma\big(w^*(\infty)\big) = \sigma^-$. So this portfolio corresponds to the one that a risk-adverse investor would take: the GMV portfolio. Considering the Example 1, I report in Table 3 the optimal portfolio for different level of φ. We

[20] From a mathematical point this is possible only without short-selling, where the efficient frontier is finite. If allowing for short-selling we could have a theoretically infinite efficient frontier, given the fact that we could go $-\infty$ on one asset and then $+\infty$ on another.

[21] See Appendix A for mathematical demonstration

note that $\mu(w^*(\varphi))$ and $\sigma(w^*(\varphi))$ are both function decreasing with respect to the parameter φ. This mean that the expected return is a function increasing with respect to the volatility.

Table 3: Solving φ-problem

φ	+∞	1	0.1	0.05	0.02	0.01
MSCI Equity USA	0.781	0.796	0.931	1.081	1.53	2.279
MSCI Equity JAP	0.392	0.379	0.258	0.123	-0.281	-0.955
MSCI Equity EM	-0.13	-0.127	-0.097	-0.065	0.033	0.196
MSCI Equity EMU	-0.044	-0.048	-0.091	-0.139	-0.282	-0.52
μ (w*)	0.4548	0.4605	0.5116	0.5685	0.7983	1.023
σ (w*)	4.8150	4.8156	4.8736	5.0455	6.1146	8.9444

In Figure 2, I scatter a set of 5000 portfolios with different level of φ ranging from 0.01 to 0.5099 with intervals of 0.0001 and I plot the efficient frontier using the optimizing portfolio function available in the program *Matlab*, and the result is two curves perfectly overlapping each other. This is the practical demonstration of the theory above mentioned.

2.1.3 Constraints

The optimization problem (1.4) can be modified by the addition of some constraints, these can be linear restrictions or non-linear restrictions. In the latter case we can find some difficulties with the solution, given the fact that the standard quadratic programming algorithm cannot be used to sort out the optimization problem. In this case, can be useful to adopt non-linear optimization algorithms. With the introduction of constraints, we can modify the efficient frontier and this translates into less opportunity arbitrages. This means that, on the mean-volatility plane, the new efficient frontier (constrained) swifts to the right with respect to the old efficient frontier (unconstrained).

One of the most frequently employed constrained is the no short-selling restriction. Usually the institutional investors, including pension funds, mutual funds and in general domestic financial institutions and FIIs[22], are not allowed by law[23] to operate short selling. This is due to the fact that short-selling is considered as financial speculation and thus highly risky,

[22] Foreign Institutional Investors are entities that are not established, registered or incorporated in the country where they invest.
[23] An example is the "Decreto 166" of 2 September 2014 that expressly pronounce about pension funds: "Persist, instead, the prohibition of short-selling or of operations with derivatives equivalent to short selling".

consequently not in line with the fund objectives and risk-profile of the investors. From

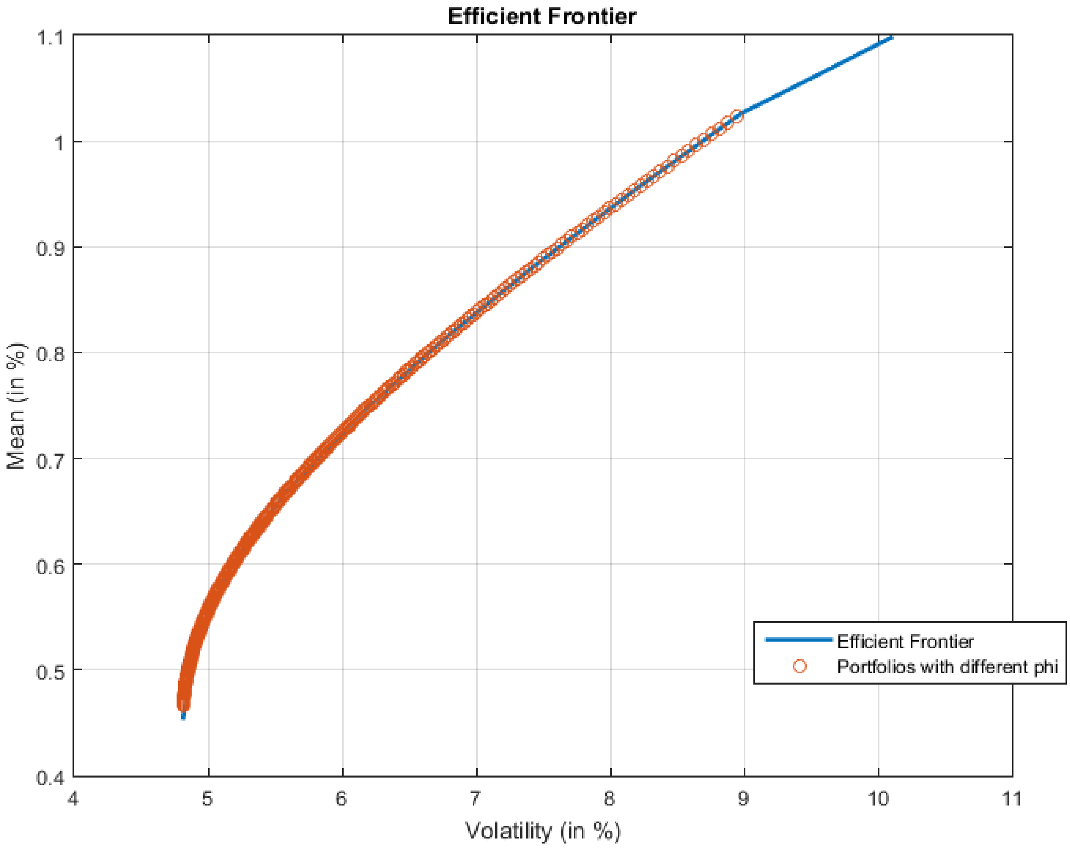

Figure 2: φ-problem optimized efficient frontier

a mathematical approach, we can write the constraint to our optimization problem as $w_i \geq 0$ and $\Omega = [0,1]^n$. The leverage measure of the portfolio w may then be expressed as the sum of the absolute values of the weights:

$$\mathcal{L}(w) = \sum_{i=1}^{n} |w_i|$$

With the no short-selling constraint, the leverage measure is 100% while it is larger than 100% without this restriction. In Figure 3, I have used the data from Example 1 in order to represent three different efficient frontiers. In accordance with the theory, from left to right the unconstraint frontier, with best mean-variance trade-off portfolios and then the others two frontiers with respectfully no short-selling constraint and weight floor and cap for single asset at 0% and 40%.

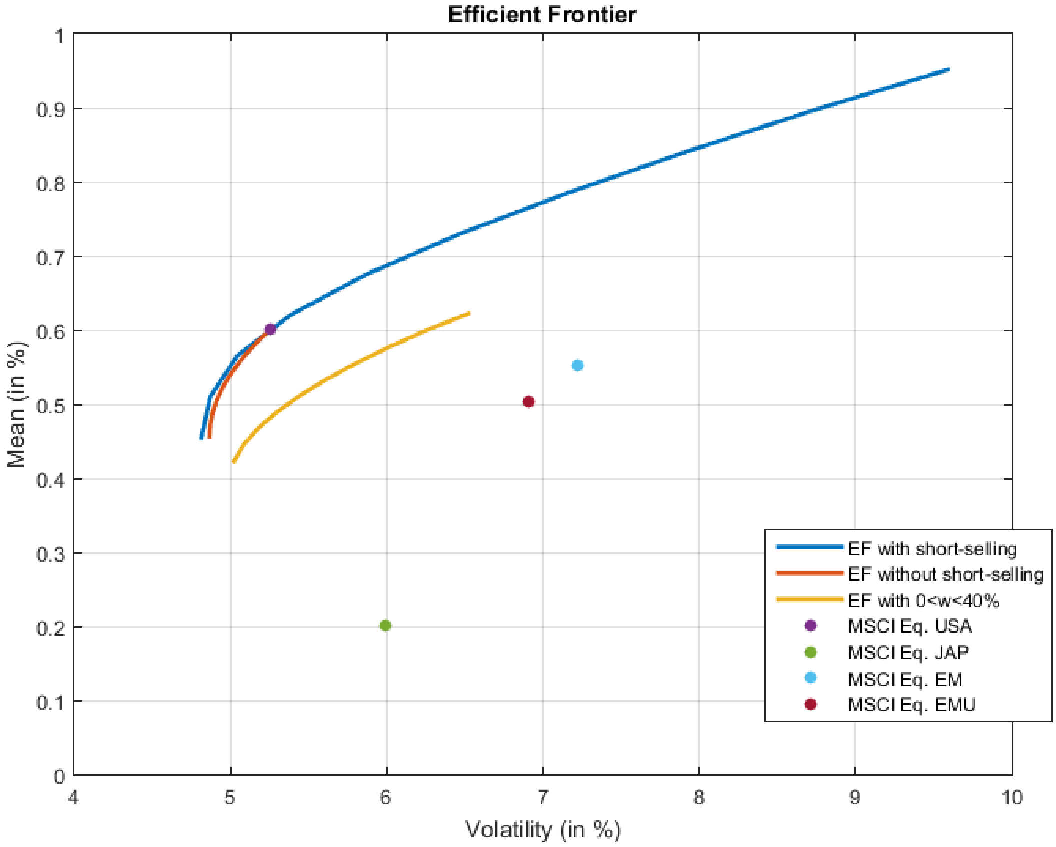

Figure 3: Efficient frontiers with different constraints

2.1.4 The analytical solution

In order to solve the optimization problem (1.4) we recur to the Lagrange function:

$$\mathcal{L}(w;\lambda_0) = w'\mu - \frac{\varphi}{2} w'\Sigma w + \lambda_0(\mathbf{1}'w - 1)$$

where $\lambda_0$is the Lagrange coefficients associated with underlying constraint $\mathbf{1}'w = 1$.

As in Roncalli [2014], the solution x^* verifies the following first-order condition:

$$\begin{cases} \partial_x \mathcal{L}(w;\lambda_0) = \mu - \varphi\Sigma w + \lambda_0 \mathbf{1} = 0 \\ \partial_{\lambda_0} \mathcal{L}(w;\lambda_0) = \mathbf{1}'w - 1 = 0 \end{cases}$$

From the first equation we can obtain $w = \varphi^{-1}\Sigma^{-1}(\mu + \lambda_0 \mathbf{1})$ and then plugging w into the second we have $\mathbf{1}'\varphi^{-1}\Sigma^{-1}\mu + \lambda_0(\mathbf{1}'\varphi^{-1}\Sigma^{-1}\mathbf{1}) = 1$. Isolating λ_0 we obtain:

$$\lambda_0 = \frac{1 - \mathbf{1}'\varphi^{-1}\Sigma^{-1}\mu}{\mathbf{1}'\varphi^{-1}\Sigma^{-1}\mathbf{1}}$$

The analytical solution for the φ-problem is:

$$w^*(\varphi) = \frac{\Sigma^{-1}\mathbf{1}}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} + \frac{1}{\varphi} * \frac{(\mathbf{1}'\Sigma^{-1}\mathbf{1})\Sigma^{-1}\mu - (\mathbf{1}'\Sigma^{-1}\mu)\Sigma^{-1}\mathbf{1}}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} \quad (1.6)$$

With this final equation we are also able to derive the global minimum variance portfolio, imposing as previously φ = 0:

$$w_{GMV} = w^*(\infty) = \frac{\Sigma^{-1}\mathbf{1}}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}$$

We said before that the analytical solution is not possible if we introduce the short-selling restriction and there reason is that if the Lagrange function becomes:

$$\mathcal{L}(w;\lambda_0,\lambda) = w'\mu - \frac{\varphi}{2}w'\Sigma w + \lambda_0(\mathbf{1}'w - 1) + \lambda' w$$

where $\lambda = (\lambda_1, \ldots, \lambda_n)$ is a vector of Lagrange coefficients associated with $w_i \geq 0$. The first-order condition is then $\mu - \varphi\Sigma w + \lambda_0\mathbf{1} + \lambda = 0$. Isolating w we obtain $w = \varphi^{-1}\Sigma^{-1}(\mu + \lambda_0\mathbf{1} + \boldsymbol{\lambda})$. Given the Kuhn-Tucker conditions $\min(\lambda_i w_i) = 0$ for all $i = 1, \ldots, n$ follows that if $w_i > 0$ then $\lambda_i = 0$ or if $w_i = 0$ then $\lambda_i > 0$. We can find also a formula similar to the previous one, but it is endogenous, given the fact that the asset can have only positive weights.

2.1.5 The tangency portfolio

In his Mutual Fund Theorem, Tobin [1958] stated that the portfolio allocation problem can be viewed as a decision to allocate between a riskless asset and a risky portfolio. In the mean-variance framework, cash can serve as a proxy for a riskless asset and an efficient portfolio on the efficient frontier serves as the risky portfolio such that any allocation between cash and this portfolio dominates all the other portfolios on the efficient frontier. This portfolio is called *Tangency Portfolio* because it is located at the point on the efficient frontier where a tangent line that originates at the riskless asset touches the efficient frontier.

To prove this concept, we write the combination of the return from the risk-free asset and a risky portfolio *m* as:

$$r(y) = (1-\alpha)r_f + \alpha r(w) \quad (1.7)$$

where $m = \binom{\alpha w}{1-\alpha}$ is a vector of dimension $(n+1)$[24] and $\alpha \geq 0$ represent the proportion of wealth invested in the risky portfolio. From this we can derive the new portfolio mean $\mu(m)$:

[24] N is the number of asset making up the risky portfolio and 1 is the risk-free asset.

$$\mu(m) = (1-\alpha)r_f + \alpha\mu = r_f + \alpha(-r_f + \mu)$$

and the variance $\sigma(m)^2$:

$$\sigma^2(m) = \alpha^2\sigma^2(w)^{25} \tag{1.8}$$

Plugging the variance equation into the mean equation, we obtain:

$$\mu(m) = r_f + \frac{(\mu(w) - r_f)}{\sigma(w)}\sigma(m)$$

In this form the expected return of portfolio *m* is linearly dependent from the volatility.
In Figure 4, we have the unconstraint efficient frontier from the previous figure, but in this case we also have an orange line that links the risk-free asset ($r_f = 0.1\%$) and the portfolio A, cutting the efficient frontier in two points. Instead the green one links the risk-free asset to the tangency portfolio, staying tangent to the efficient frontier. The first line is defined *suboptimal market line* because it is dominated by the second one, the *optimal market line.* This is due to the fact that all the portfolios from the orange line are suboptimal with respect to the green one. This is true for every combination of risk-free asset and portfolios. Even though we drew as many line as the number of portfolios present on the efficient frontier, they would be all dominated by the optimal market line.

Let's consider the Sharpe ratio of the portfolio *w*:

$$SR(w|r_f) = \frac{\mu(w) - r_f}{\sigma(w)}$$

then thanks to equation (1.7) e (1.8) we can say that

$$\frac{\mu(m) - r_f}{\sigma(m)} = \frac{\mu(w) - r_f}{\sigma(w)} \Leftrightarrow SR\,(m|r_f) \Leftrightarrow SR(w|r_f)$$

The tangency portfolio is then the combination of expected return and volatility that maximizes the angle **k** , which corresponds to the maximum Sharpe ratio. Finally, all portfolios that lie on the optimal market line have the same Sharpe ratio

[25] The variance of risk-free asset is 0 given the fact the return is constant.

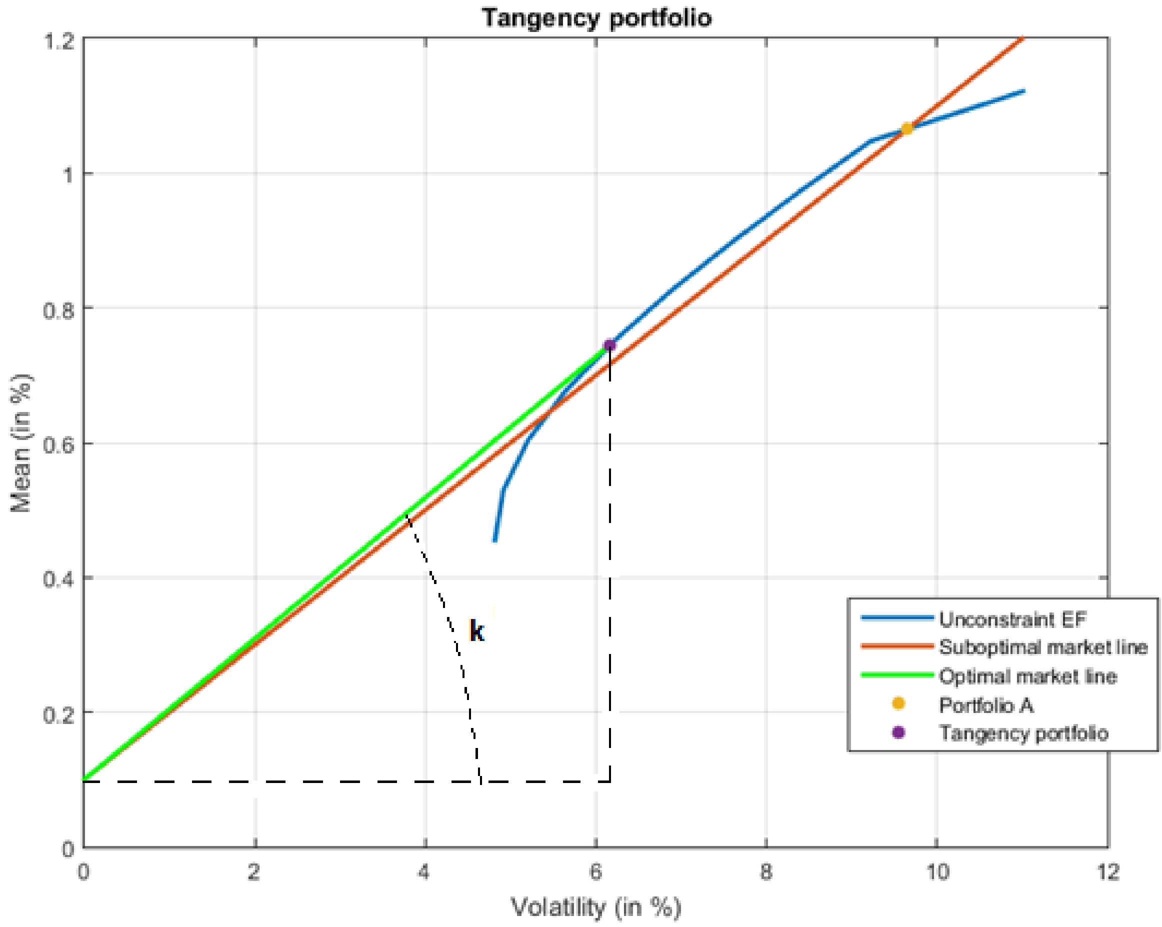

Figure 4: Computing tangency portfolio

2.2 The Risk Budgeting Approach

Markowitz model has been an important framework in finance and more specifically in portfolio investment, but despite its huge impact on the sector, there has been always a gap between theory and reality. The quadratic utility optimization has been the key factor to solve the allocation problem, however this approach has been implemented by replacing the theoretical mean and variance by their historical counterparts, and the estimated mean-variance portfolio, even though computed adopting shrinkage methods or imposing weight constraints, has numerous drawbacks:

- Chopra [1993], and Chopra and Ziemba [1993] show that portfolios are very sensitive to errors in estimating the mean and variance inputs,
- Konno and Hiroaki [1991] highlight that the resolution of a large-scale quadratic optimization problem is not undemanding,
- Green and Hollifield [1992] test that dominant factor in the variance covariance matrix can result in extreme weights in optimal portfolios,

- Portfolios allocations are very erratic over time and that means large transaction costs and high liquidity risk (see, for example, Ledoid and Wolf [2003], DeMiguel *et al.* [2007] and Frahm [2008]).

As Darolles *et al.* [2012] point out, "these difficulties are due mainly to the sensitivity of the mean-variance efficient portfolio allocation to the smallest eigenvalues[26] of the volatility, and to the poor accuracy of the inverse volatility matrix with the standard estimation methods". The literature has proposed different ways to improve the regularization of information matrix and the robustness of optimized portfolios. For the former, some examples are the empirical covariance matrix estimator, the Hayashi-Yoshida estimator (see Hayashi and Yoshida [2005]), the GARCH approach (see Engle [1982]) and factor models, for the latter the resampling techniques (see Jorion [1992]), the variance covariance matrix denoising (see Laloux *et al.* [1999]), Ledoit-Wolf approach and the penalized regression techniques (see Scherer [2007]). Robustness can also be achieved by introducing restrictions in the empirical process of optimization, like the well-known short-selling restriction, gross exposure constraints (Fan *et al.* [2012]a), budget allocation (see Elton and Gruber [1977], Levy [2009], Beleznay *et al.* [2012]) and contribution to total risk (see Maillard *et al.* [2010]). The last constraint is particularly useful and it is the base for the method I provide next: the risk budget approach. It derives its importance from the fact that it requires less discretionary inputs than the Markowitz model does.

2.2.1 Properties of a risk measure

Before discussing about risk allocation and risk budgeting, it is essential to focus the attention on measuring the risk of a portfolio and in general on the risk measures. Volatility of the loss, Value at Risk and Expected Shortfall are all example of risk measures.
In order to be acceptable in terms of risk allocation principle, all the risk measures $R(w)$ should hold some properties. Following Artzner *et al.* [1999], *R* is said to be *coherent* if it satisfies the following properties:

- Subadditivity

$$R(w_1 + w_2) \leq R(w_1) + R(w_2)$$

[26] Principal components analysis (PCA) is a statistical technique used to reduce the dimensionality of data. PCA is unlike traditional factor models such as the CAPM because the factors it creates do not usually have an economic interpretation and hence is entirely statistical in nature. Mathematically, we want to transform the covariance matrix of our original data in such a way so as to maximize the variance of each of these orthogonal factors. We can accomplish this by using an eigenvalue decomposition of the covariance matrix (Jason Hsu).

The total risk of the two portfolios is less or equal to the sum of risks of each individual portfolio.

- Homogeneity

$$R(\lambda w) = \lambda R(w) \qquad \text{if } \lambda \geq 0$$

Leveraging (or deleveraging) the portfolio increases (or decreases) the risk by the same level.

- Monotonicity

$$\text{if } r(w_1) > r(w_2) \text{ then } R(w_1) \geq R(w_2)$$

If the return from portfolio 1 is larger than from portfolio 2 in all scenarios, then the risk measure of portfolio 1 is higher than of portfolio 2

- Translation invariance

$$\text{if } m \in \mathbb{R}, \text{ then } R(w + m) = R(w) - m$$

Adding cash position of amount *m* to the portfolio reduces the risk by *m*.

Follmer and Schied [2002] consider the replacement of the first two conditions with a weak one known as convexity property:

$$R(\lambda w_1 + (1 - \lambda) w_2) \leq \lambda R(w_1) + (1 - \lambda) R(w_2)$$

The total risk of two portfolios is less or equal to the sum of the individual risk of the two portfolios taken separately, that is the diversification does not increase the risk.
By definition, the loss of the portfolio is $L(w) = -r(w)$, we consider then different risk measures:

- Volatility of the loss[27]

$$R(w) = \sigma\big(L(w)\big) = \sigma(w)$$

in matrix form

$$\sigma(w) = \sqrt{w'\Sigma w}$$

The volatility of the loss is the standard deviation of portfolio

[27] Roncalli [2014] point out that the volatility is not a coherent risk measure because it does not verify the translation invariance axiom. This measure is still used since the axiom is based for banking system and not for the portfolio management.

- Standard deviation-based risk measure

$$R(w) = SD_c(w) = E[L(w)] + c * \sigma\big(L(w)\big) = -\mu(w) + c * \sigma(w)$$

in matrix form

$$SD_c(w) = -w'\mu + c * \sqrt{w'\Sigma w}$$

To get this measure we scale the volatility of loss for $c > 0$ and subtract the expected return of portfolio.

- Value at Risk[28]

$$R(w) = VAR_\alpha(w) = \inf\{\ell : \Pr\{L(w) \leq \ell\} \geq \alpha$$

in matrix form

$$VAR_\alpha(w) = -w'\mu + \Phi^{-1}(\alpha)\sqrt{w'\Sigma w}^{29}$$

VaR is defined in terms of α-quantile of the portfolio's loss distribution for a given horizon

- Expected Shortfall

$$R(w) = ES_\alpha = E[L(w)|L(w) \geq VAR_\alpha(w)$$

in matrix form

$$ES_\alpha(w) = -w'\mu + \frac{\sqrt{w'\Sigma w}}{(1-\alpha)}\phi(\Phi^{-1}(1-\alpha)^{30}$$

ES represents the expected loss when the loss is beyond the VaR

As it appears clear, within the Gaussian framework, all the risk measures are based on the volatility of the expected returns. Generally, can be useful to omit the term of expected return, given the fact that if it is larger enough it can seriously affect the risk measurement. In Table 4, I have report the Value at Risk and Expected Shortfall, using data from Example 1.1, for GMV portfolio, Max Sharpe portfolio and Maximum Return portfolio ($w_i \leq 3$) for the α-quantile 90%, 95%, 99% and 99.5%

Table 4: VaR and ES for different α-quantile

Portfolio	*R(w)*	90%	95%	99%	99.5%

[28] VaR does not verify the subadditivity condition in general.

[29] $\Phi^{-1}(\alpha)$ refers to the inverse of the cumulative distribution function of the standardized normal distribution.

[30] Φ refers to the probability density function of the standardized normal distribution.

GMV	VaR	5.72%	7.47%	10.75%	11.95%
	ES	8%	9.48%	12.38%	13.47%
MS	VaR	7.15%	9.39%	13.59%	15.12%
	ES	10.07%	11.96%	15.67%	17.07%
MR	VaR	11.84%	15.51%	22.39%	24.91%
	ES	16.62%	19.73%	25.81%	28.10%

2.2.2 Risk contributions

After measuring the risk, we need to decompose the risk portfolio into contributions from the individual asset. Litterman [1996] defines it as *risk allocation*. These contributions come in hands to:

- clarify which assets are more responsible for portfolio risk
- make decision about rebalancing the portfolio to alter the risk
- construct "risk parity" portfolios, where assets have the same risk contribution

Denault [2001] illustrated many methods, some more efficient than others, to reach our objective, but the most used and valuable is the *Euler Principle*. Based on it, we can define the risk contribution of asset i as:

$$RC_i = w_i \frac{\partial R(w)}{\partial w_i}$$

And the risk measure satisfies the Euler decomposition:

$$R(w) = \sum_{i=1}^{n} w_i \frac{\partial R(w)}{\partial w_i} = \sum_{i=1}^{n} RC_i$$

We are now able to deduce the risk contribution for all the four risk measure:

- Volatility

$$RC_i = w_i \frac{(\Sigma w)_i}{\sqrt{w'\Sigma w}} \tag{1.9}$$

In appendix B, I show this relationship, using the volatility as risk measure, first with the case of $n = 2$ and then with the more general case $n > 2$.

- Standard deviation-based risk

$$RC_i = w_i(-w'\mu + c * \frac{(\Sigma w)_i}{\sqrt{w'\Sigma w}}) \tag{1.10}$$

- Value at Risk

$$RC_i = w_i \left(-w'\mu + \Phi^{-1}(\alpha) \frac{(\Sigma w)_i}{\sqrt{w'\Sigma w}}\right) \tag{1.11}$$

- Expected Shortfall

$$RC_i = w_i[-w'\mu + \frac{(\Sigma w)_i}{(1-\alpha)\sqrt{w'\Sigma w}} \phi\left(\Phi^{-1}(1-\alpha)\right)] \tag{1.12}$$

EXAMPLE 2: We consider four equity indices: BOFA Eur Corp, Inv. Trust Private Equity, WGBI Japan and MSCI Equity USA, for the period 7 January 1997 – 7 July 2015. Means, standard deviation and correlation matrix are the following:

Table 5: Mean and standard deviation

%	Bofa Eur Corp	I.T Priv. Equity	WGBI Japan	MSCI Eq. USA
Mean	0.41	0.68	0.21	0.6
St. dev	0.98	8.76	3.3	5.26

Table 6: Correlation matrix

	Bofa Eur Corp	I.T Priv. Equity	WGBI Japan	MSCI Eq. USA
Bofa Eur Corp	1	0.18	0.2	0.16
I.T Priv. Equity	0.18	1	-0.09	0.68
WGBI Japan	0.2	-0.09	1	-0.05
MSCI Eq. USA	0.16	0.68	-0.05	1

In Table 7, Table 8 and Table 9 I have report the marginal risks (MR_i) as well as risk contributions for volatility, Value at Risk and Expected Shortfall. I have also highlight the total risk as sum of contributions for the two strategies.

Table 7: Risk decomposition of volatility

Strategy	Asset	w_i	MR_i	RC_i	RC_i^*[31]
1	1	25%	0.29	0.07	2.24%
	2	25%	8.02	2	60.23%
	3	25%	0.59	0.15	4.41%
	4	25%	4.41	1.10	33.11%
R(w)				3.33	
2	1	40%	0.42	0.17	8.18%

[31] RC_i^* is the risk contribution expressed in percent of risk measure.

	2	10%	6.54	0.65	31.24%
	3	30%	1.46	0.44	20.94%
	4	20%	4.15	0.83	39.65%
R(w)				2.09	

We note that the asset weights do not reflect the contribution risks, especially with the first strategy: the first asset has a weight of 25% but its risk contribution is only 2.24%. Similarly, the second asset has still a weight of 25% but its risk contribution is over 60%. It is clear that despite the nominal weights are the same, the two assets provide with a very different level of risk.

Table 8: Risk decomposition of VaR with α = 99%

Strategy	Asset	w_i	MR_i	RC_i	RC_i^*
1	1	25%	0.28	0.07	0.96%
	2	25%	17.98	4.49	61.85%
	3	25%	1.15	0.29	3.97%
	4	25%	9.65	2.41	33.22%
R(w)				7.27	
2	1	40%	0.58	0.23	5.22%
	2	10%	14.54	1.45	32.64%
	3	30%	3.19	0.96	21.47%
	4	20%	9.06	1.81	40.66%
R(w)				4.45	

Table 9: Risk decomposition of ES with α = 90%

Strategy	Asset	w_i	MR_i	RC_i	RC_i^*
1	1	25%	0.11	0.02	0.51%
	2	25%	13.39	3.35	62.42%
	3	25%	0.82	0.20	3.81%
	4	25%	7.14	1.78	33.26%
R(w)				5.36	
2	1	40%	0.34	0.13	4.14%
	2	10%	10.80	1.08	33.16%
	3	30%	2.35	0.71	21.66%
	4	20%	6.68	1.34	41.04%
R(w)				3.26	

Litterman [1996] and Garman [1997] propose an interpretation for the risk contribution, using the sensitivity analysis of the risk measure. The marginal risk of asset *i* can specified as:

$$\frac{\partial R(w)}{\partial w_i} = \lim_{h \to 0} \frac{R(w + h\mathbf{e}_i) - R(w)}{h}$$

If h is small enough, we can write:

$$R(w + h\mathbf{e}_i) \cong R(w) + h\frac{\partial R(w)}{\partial w_i} \tag{1.13}$$

This means that if the weight of asset *i* increases by an amount equal to *h*, than the risk increases by *h* times the marginal risk.

In table 10, I report the approximated value of volatility, according to Equation (1.13), following a variation in the asset weights. If the weight of asset 2 (strategy 1) increase of *h* = 10%, the portfolio volatility will be 4.1301%.

Table 10: Marginal analysis of the volatility with respect to factor *h*

Strategy	Asset	σ(w)	+0.01%	+0.1%	+1%	+10%
1	1	3.3282%	3.3282%	3.3285%	3.3312%	3.3580%
	2		3.3290%	3.3362%	3.4084%	4.1301%
	3		3.3282%	3.3288%	3.3341%	3.3869%
	4		3.3286%	3.3326%	3.3723%	3.7690%
2	1	2.0940	2.0941%	2.0944%	2.0983%	2.1368%
	2		2.0947%	2.1006%	2.1594%	2.7481%
	3		2.0942%	2.0955%	2.1086%	2.2402%
	4		2.0944%	2.0982%	2.1355%	2.5091%

2.2.3 Risk budgeting

So far, the risk contribution has been used as a mere instrument to control for the contribution of each asset to the overall portfolio risk: first we have set the asset weight in term of percentage and then we have derived the marginal risk.

In the risk budgeting, the asset weights are computed based on the *budget* allocated for the risk contribution of each asset. Given a set of risk budget $\{B_1,\ldots,B_n\}$ with $\sum_{i=1}^{n} B_i = 1$, the risk budgeting portfolio is then define by the following constraints:

$$\begin{cases} RC_1(w) = B_1 \\ RC_i(w) = B_i \\ RC_n(w) = B_n \end{cases}$$

We can then highlight two main differences between this methodology and the Markowitz model:

- The utility function is not present, therefore we do not need to solve a problem of maximization,
- This model does not depend on the expected return of portfolio but only on the risk dimension

In the previous formulas, the expected return μ was present, but in the practical analysis it is set to zero in order to obtain a conservative risk measure.
In the case of negative risk contributions for some assets, a vast amount of portfolio risk will be held by the other assets, worsening the diversification effect. For this reason, it is better to deal just with long-only portfolios. The risk budget portfolios should then be defined by the following non-linear system:

$$\begin{cases} RC_i(w) = B_i R(w) \\ B_i \geq 0 \\ w_i \geq 0 \end{cases}$$

$$\text{with } \sum_{i=1}^{n} B_i = 1$$

$$\text{and } \sum_{i=1}^{n} w_i = 1$$

2.2.4 Non-normal measures for Value at Risk

All the previous formulas are based on the fact that the expected returns have a Gaussian distribution, but most of the time this is not the case. So for the VaR, there are three the possible alternatives:

- Using the general non-normal Value at Risk:

  $$RC_i = -w_i E[R_i | R(w) = -VaR_\alpha(w)]$$

  The problem with this solution is that it is difficult to retrieve an analytical expression, thus using simulation like the Monte Carlo one is necessary.

- Using historical Value at Risk: "this consists in deriving the quantile of the empirical distribution of losses using a set of historical scenarios, typically the last 260 trading days"[32].

[32] Roncalli [2014]

The advantage of this technique is the fact that we do not need to estimate any parameters like the in Gaussian VaR case.

- Correcting the Gaussian VaR approach by taking into consideration the third and fourth moments. In order to do that, the Cornish-Fisher expansion is needed and the solution for VaR, as proposed by Zangari [1996], becomes:

$$VaR_\alpha(w) = -w'\mu + z\sqrt{w'\Sigma w}$$

where *z* is:

$$z = z_\alpha + \frac{1}{6}(z_\alpha^2 - 1)\gamma_1 + \frac{1}{24}(z_\alpha^3 - 3z_\alpha)(\gamma_2 - 3) - \frac{1}{36}(2z_\alpha^3 - 5z_\alpha)\gamma_1^2$$

With $z_\alpha = \Phi^{-1}$, γ_1 being the skewness and γ_2 the kurtosis. We can then deduce the expression for risk contribution:

$RC_i = w_i(-\mu_i + z\frac{(\Sigma w)_i}{\sqrt{w'\Sigma w}})$ (2.6)

3 VALUATION INDICES

3.1 Performance measure

The performance measure is one of the most important part about the investment assessment. Many are the indices available for the assessment analysis. Most of them are built on the ratio between two elements: a return measure, like the excess-return over the risk-free asset yield, and a risk measure, usually the most used is the standard deviation.

The area around the performance analysis is very active and the current sector studies focus on some features:

- Identify the optimal performance indices base on particular criteria linked to the investor type (e.g. retail, institutional);
- Identify asymmetric performance indices that weight up in different ways profits or losses with respect to the investor utility function;
- Develop the performance measurement based on the most recent findings of the behavioural finance.

I provide with a brief overview about the performance indices[33] that I employ to assess the quality of the portfolios.

3.1.1 Sharpe Index

The Sharpe index (SH) is usually computed as ratio between the mean of the return excess over the risk-free asset yield and the standard deviation of the returns. In my case, to simplify, I impose an $r_f = 0$.

$$SH = \frac{\mu(w) - r_f}{\sigma(w)}$$

It represents therefore a trade-off between return and risk.

From the previous equation, it is quite intuitive that the SH measures the premium for each unit of risk accepted. It is straightforward that the portfolio/asset with the highest value is the one that rewards the most for unit of risk.

Such an index imposes that the standard deviation of returns describes completely the risk. Though, we need to bear in mind that the investors do not prefer negative return and long

[33] For VaR and ES risk measures see previous chapter.

drawdown[34]. On the contrary, as Thaler and Benartzi [1995] prove, they prefer to sacrifice some of their gain in order to avoid larger losses. This asymmetric behaviour is not captured by the index.

As I previously illustrate, the SH is the point on the efficient frontier that, if linked with a line to the risk-free asset, provide with the largest slope. The higher the angular coefficient, the higher the margin return for a single unit of risk.

3.1.2 Sortino Index

The Sortino index (SO) is the ratio between the mean of the return excess over the risk-free asset yield and the *downside* risk, i.e. it focuses only on the negative side of volatility, the negative returns. Again, to simplify, I impose an $r_f = 0$.

$$SO = \frac{\mu(w) - r_f}{DSR}$$

With

$$DSR = \left(\int_{-\infty}^{T} (T - x)^2 f(x) dx\right)^{\frac{1}{2}}$$

Differently from the Sharpe index, SO tries to capture the asymmetry of the return distribution. A high value for the index means the return volatility is mainly concentrated above the minimum acceptable by the investor and vice versa for a low value.

3.1.3 Calmar Ratio

Calmar ratio (Cal) is a performance measurement used especially to evaluate Commodity Trading Advisors and hedge funds. It is the ratio between the portfolio average rate of return and the maximum drawdown.

$$Cal = \frac{\mu(w)}{\max(DD)}$$

Usually the return period is set to 36 months to allow for a better valuation on the recent trends.

[34] Drawdown is the difference between the peak and the trough during a specific period of an investment.

3.1.4 Sterling Ratio

Similar to the Calmar ratio, the Sterling ratio (Ste) is based on the ratio between the portfolio average rate of return and the average largest drawdown.

$$Ste = \frac{\mu(w)}{Avg.\ Largest\ DD}$$

Usually the average largest drawdown is the mean of the maximum drawdown for each year of the sample.

3.1.5 Farinelli-Tibiletti Ratio

The Farinelli-Tibiletti ratio[35] (FT) represents a more generalized measure of Gain-Loss Ratio or Upside-Potential Ratio. This ratio is a comparison between the expected value of return above and below a certain threshold, raised to the power of p e q respectively.

$$FT(p,q,\tau) = \frac{E^{\frac{1}{p}}[\{r-\tau)^{+}\}^{p}]}{E^{\frac{1}{q}}[\{r-\tau)^{+}\}^{q}]}$$

The parameter p, q and τ are some numbers chosen by the investor according to his preference. They determine whether an investor is risk-seeking, risk-neutral or risk-averse above (for p) or below (for q) a reference point or return threshold τ. If p =1 and q=1 the investor is risk-neutral above and below τ. If 0<p<1 the investor is risk-averse above τ. Contrarily, if p>1 the investor is risk-seeking above τ. Similarly, if 0<q<1 the investor is risk-seeking below τ and risk-averse below τ for q>1 (Wiesinger [2010]).
For my purpose p, q and τ are set 1, 2 and 0 respectively.

3.1.6 Information Ratio

The Information ratio (IR) is often used to gauge the skill of managers of mutual funds, hedge funds, etc. It is based on the difference between active return (difference between the return of the portfolio and the return of the reference benchmark) and the tracking error volatility (volatility of the active return).

$$IR = \frac{r_w - r_b}{\sigma(r_w - r_b)}$$

[35] See Farinelli-Tibiletti [2008]

This ratio allows for an assessment of the active management of the portfolio. It shows if the manager is able to obtain an extra-profit over the benchmark return without increasing too much the level of the portfolio risk. The higher the IR, the higher the active return of the portfolio, given the amount of risk taken, and the better the manager.

3.2 Diversification measure

Alongside with the performance indices there are some indices that evaluate the level of diversification of a portfolio, in particular they measure the proximity of the portfolio to the situation of perfect diversification or total concentration. The diversification may be thought in terms of weight budgets or risk budgets.

3.2.1 Diversification Index

The Diversification index (DI) is the ratio between the risk measure of the portfolio and the weighted risk measure of the assets.

$$DI = \frac{R(w)}{\sum_{i=1}^{n} R(w_i)}$$

In my case, I will be using the volatility as risk measure, given the fact that is a coherent risk measure and thus the output range is $0 < DI \leq 1$. Value 1 is obtained if asset returns are perfectly correlated.

3.2.2 Concentration Indices

Another way to measure the diversification if by means of concentration indices, in other words they measure the weight concentration.

The first one is the Gini index (GI), an index based on the Lorenz Curve of inequality:

$$GI = \frac{2\sum_{i=1}^{n} i w_{i:n}}{n\sum_{i=1}^{n} w_{i:n}} - \frac{n+1}{n}$$

With $\{w_{1:n}, \dots, w_{n:n}\}$ the ordered statistic of $\{w_1, \dots, w_n\}$. Output range is $0 < GI \leq 1$ where 1 represents the maximum concentration in terms of *weight budget* in one asset and 0 the maximum diversification.

The second one is the Shannon Entropy (SE):

$$SE = \exp(-\sum_{i=1}^{n} w_i \ln w_i)$$

Output range is $1 < GI \leq n$, where 1 represents the maximum concentration in terms of *risk budget* in one asset and n the maximum diversification.

4 ALTERNATIVE ASSET CLASSES

4.1 The characteristics

Before starting with the cold numbers of my analysis, I think it is necessary to discuss about what alternative asset classes are, what they refer to when we use the adjective *alternative* and their features.

How to define an asset class?

Wilcox and Fabozzi [2013] identify an asset class in term of characteristics that the members of it have in common. These features comprehend:

- The sharing of a common regulatory or legal structure;
- Being characterized by similar return and volatility;
- Being affected by the same major economic factors and, as result, being highly correlated with the returns of the other members included in the asset class.

Kritzman [1999] propose a second way to define an asset class based simply on a group of assets that is treated as an asset class by manager:

> "Some investments take on the status of an asset class simply because the managers of these assets promote them as an asset class. They believe that investors will be more inclined to allocate funds to their products if they are viewed as an asset class rather than merely as an investment strategy".

Part of the difficulty of working with alternative asset classes is defining them. Are they a separate asset class or a subset of an existing asset class? Do they hedge the investment opportunity set or expand it? That is, in terms of Markowitz diversification, do they improve the efficient portfolio for a given level of risk? This means that for a given level of risk, do they allow for a greater expected return than by just investing in traditional asset classes? In most cases, alternative assets are a subset of an existing asset class. This may run contrary to the popular view that alternative assets are separate asset classes. However, I take the view that what many consider separate "classes" are really just different investment strategies within an existing asset class. Usually, they expand the investment opportunity set, rather than hedge it. Finally, alternative assets are generally purchased in the private markets, outside of any exchange. Specifically, most alternative assets derive their value from either the debt or equity markets. For instance, most hedge fund strategies involve the purchase and sale of either equity or debt securities. Additionally, hedge fund managers may invest in derivative instruments

whose value is derived from the equity or debt markets.
In the following subsections, I will review 4 types of the best known alternative asset classes: hedge funds, private equity, commodities and real estate, plus a fifth class that includes three different types of assets – common equities, sovereign bonds and corporate bonds – that belong to the Emerging Market.

4.1.1 Real Estate

Wilson *et al*. [2005] propose five goals for adding real estate to an investment portfolio:

- To achieve absolute returns above the risk-free rate,
- To provide a hedge against inflation,
- As a portfolio diversification tool that provides exposure to a different type of systematic risk and return than stocks and bonds,
- To constitute an investment portfolio that resembles the global investment opportunity set,
- To deliver strong cash flows to the portfolio through lease and rental payments.

Strategies in real estate investing can be classified into the following styles: core, value added, and opportunistic. Core properties are the most liquid, most developed, least leveraged, and most recognizable properties in a real estate portfolio. They tend to be usually held for a long period of time to take full advantage of the lease and rental cash flows that they provide. Value-added properties include hotels, resorts, assisted care living, low-income housing, outlet malls, hospitals, and the like. These properties tend to require a subspecialty within the real estate market to manage well and can involve repositioning, renovation, and redevelopment of existing properties. Relative to core properties, these properties tend to produce less income and rely more on property appreciation to generate the total return. Opportunistic real estate moves away from a core income approach to a capital appreciation approach. Often, opportunistic real estate is accessed through real estate opportunity funds, they invest in real estate with a high risk and return profile, particularly those properties that require extensive development or are turnaround opportunities.

A simple and liquid way to bring real estate into an investor's portfolio is by investing in a real estate investment trust (REIT). REITs are securities listed on major stock exchanges that represent an interest in an underlying pool of real estate properties. They pool investment capital from many small investors and invest the larger collective pool in real estate properties that would not be available for the small investor. The key advantage of REITs is that they provide access to an illiquid asset class for investors who would not otherwise invest in real property. The biggest disadvantage of REITs is being listed on a market stock exchange or traded over-the-counter. Consequently, their prices are affected by the systematic risk coming from the market stock exchange and reduces the diversification benefits. Therefore they are regarded as imperfect substitute or proxy for direct real estate investment.

4.1.2 Hedge Fund

There is no legal definition of what a hedge fund is. Anson [2006] try to define it as "A privately organized investment vehicle that manages a concentrated portfolio of public and private securities and derivative instruments on those securities, that can invest both long and short, and can apply leverage".

Hedge funds employ a wide range of trading strategies and techniques, seeking to generate a high return regardless of the movement of the market, that is, they seek to earn positive absolute returns even in a declining market environment. Managers employ portfolio strategies that typically include leverage, short selling, and the use of derivatives. In contrast to mutual funds, which are publicly traded investment vehicles, hedge funds are private.

The use of derivative strategies requires more sophisticated risk management techniques to control the risks associated by managers due to their larger use of leverage.

Hedge funds do not have any restriction on short positions and, in fact, shorting can be the most important aspect of their investment strategy.

They tend to focus on only one sector of the economy or one segment of the market tailoring their portfolio to extract the most value from their smaller investment sector or segment. Many hedge fund strategies invest in non-public securities, namely securities that have been issued to investors without the support of a prospectus and a public offering. They can be classified in four categories:

- Market directional hedge funds: they employ strategy that involves either retain some systematic risk or follow the movements of the market. Inside this there are also three sub-categories: equity long/short fund, equity market timing fund and short-selling fund,
- Corporate restructuring hedge funds: they profit from significant corporate transactions like merger, acquisition or bankruptcy. These funds usually concentrate on few stocks and have to evaluate not only if the company is under or overvalued but also if the transaction will eventually be accomplished.

- Convergence trading hedge funds: they pursue arbitrage strategy called “risk arbitrage”. It consists on betting that two similar securities that have dissimilar market prices will converge to the same value over some investment horizon. These funds divided in statistical arbitrage, fixed income arbitrage, relative value arbitrage, and convertible arbitrage hedge funds
- Opportunistic hedge funds: they employ strategies designed to take advantages of all the opportunities that present themselves. They are divided usually in global macrohedge funds, global tactical asset allocation hedge funds, and multi-strategy hedge funds.

Finally the *funds of hedge funds* are well diversified investment vehicles made up of a variety of other funds. Some funds of funds invest in hedge funds with a variety of different strategies and a much higher level of diversification, while others, called single-strategy funds, will invest in a variety of funds having the same or similar strategies.

4.1.3 Private Equity

For private equity there is no legal definition. Megginson [2004] define them as “a professionally managed pool of money raised for the sole purpose of making actively managed direct equity investments in private companies and with a well-defined exit strategy (sale or IPO)”. Private companies refer to companies that are not listed on the market stock exchange, and therefore are not allowed to raise capital by means of public stock market. Private equity strategies can be distinguished in 4 categories:

- *Venture capital* refers to equity investments in less mature non-public companies to fund the launch, early development, or expansion of a business,
- *Mezzanine capital* refers to an investment in subordinated debt or preferred stock of a company, without taking voting control of the company. Often these securities have attached warrants or conversion rights into common stock,
- *Growth capital* refers to minority equity investments in mature companies that need capital to expand or restructure operations, finance an acquisition or enter a new market, without a change of control of the company,
- *Leverage buy-out* refers to the purchase of all or most of a company or a business unit by using equity from a small group of investors in combination with a significant amount of debt.

Private equity firms are usually organized as limited partnerships or limited liability company (LLP) that act as holding companies for several private equity funds run by general partners. At the largest private equity firms there may be 20 to 40 general partners. These general partners invest in the fund and also raise money from institutional investors and high-net-worth individuals, who become limited partners in the fund.

Unlike quoted companies, the number of larger shareholders is small, usually they share the same agenda and seat on the board, being operationally involved. The management is very highly incentivised and aligned with the interest of the shareholders. The decision making process is very fast, keeping the costs down and allowing for an optimal strategy. All these characteristics translate into the prospect of outperformance and low correlation with the quoted companies and markets.

4.1.4 Commodity investment

Unlike the majority of alternative asset classes which are no more than alternative investment strategies, like hedge fund or private equity that always trade in stocks and bonds, commodity investment represents a really alternative of investment. This can be achieved through various products. Some investors take passive position in physical commodities and earn the risk premium associated with it. Others actively trade in both physical commodities and commodity derivatives and generate a rate of return that is both a function of the risk premium embedded in this asset class and the trading skills of the manager.

To take economic exposure, the investor could employ several solutions:

- By purchasing directly the underlying commodity, but this solution is not very attractive since with the direct investment cost for the ownership, come also the cost for storage and transportation of physical commodities,
- By owning stocks of a company that derives a significant part of its revenues from the purchase and sale of the physical commodities. An example could be the purchase of some stocks of Eni, which derives three fourths of its revenues from the exploration, refining and sale of petroleum products. Investing in it, could be thought as a play on oil price, however Eni remains a company quoted on the stock market and therefore its value is still might be correlated with it. As a result, the play could be exposed to the firm-specific risk as well as systematic risk of the market as any other share. Moreover, there are other operating risks associated with an investment in any company,

- By acquiring *commodity derivative* contracts such as commodity futures, commodity swaps, and commodity forward contracts. Futures contracts offer several advantages. First, these contracts are traded on an organized exchange. Therefore, they share the same advantages as stock exchanges: a central marketplace, transparent pricing, clearinghouse security, uniform contract size and terms, and daily liquidity. Most importantly, there is minimal counterparty risk. Second, the purchase of a futures contract does not require automatic delivery of the underlying commodity. Commodity swaps and commodity forward contracts perform the same economic function as commodity futures contracts. However, because commodity swaps and forward contracts are custom made for the individual investor, these contracts are less liquid,
- By purchasing a *commodity ETF* that may provide exposure to one commodity or a group of commodities,
- By acquiring a *commodity-linked note* an intermediate-term debt instrument whose value at maturity will be a function of the value of an underlying commodity futures contract or basket of commodity futures contracts.

4.1.5 Emerging Markets

One of the major effects of globalization of capital markets in the last few decades has been the emergence of new capital markets in many countries. The introduction of equity markets in China and Russia in 1990, the opening of Eastern Europe, the founding of markets in Africa and Asia, as well as the general revival and growth of equity markets through the latter part of the twentieth century opened up considerable new opportunities for international investing. The term “emerging markets” was coined by the World Bank to refer to these new exchanges, but there is no single definition of what emerging markets are. Fisher [2010] propose several characteristics which are generally found, with different degree, in this countries:

- Fast-growing economies,
- Low levels of per capital income,
- Relatively immature capital market infrastructure,
- Weak property rights,
- Tenuous adherence to capitalism principles,
- Varying political model,
- Relatively undeveloped institutions,
- Restriction on foreign investors,

- Restriction to foreign exchange and fund repatriation,
- Inherently risky.

In Table 11, 12, and 13 are reported the country breakdown and in Table 14 the sector breakdown according to MSCI Emerging Market Index of 30 September 2015.

Table 11: America quotes

America	Weight (%)
Brazil	6.10
Mexico	4.78
Chile	1.28
Colombia	0.55
Perù	0.38
Total Americas	**13.09%**

Table 12: Asia quotes

Asia	Weight (%)
China	23.44
South Korea	15.53
Taiwan	12.53
India	8.91
Malaysia	3.13
Thailand	2.25
Indonesia	2.14
Philippines	1.47
Total Asia	**69.40%**

Table 13: EMEA quotes

EMEA	Weight (%)
South Africa	7.84
Russia	3.38
Poland	1.55
Turkey	1.42
Qatar	1.14

United Arab Emirates	0.81
Greece	0.25
Hungary	0.25
Egypt	0.22
Czech Republic	0.21
Total EMEA	**17.51%**

Table 14: Sector quotes

Sector Name	Weight %
Financial	28.56
Information Technology	18.08
Telecommunication services	7.37
Materials	6.61
Health Care	2.89
Consumer Discretionary	9.38
Energy	7.49
Industrials	7.48
Consumer Staples	8.79
Utilities	3.35

Studies of emerging markets using IFC data showed that EM indices had high return but also high risk, although the evidence on high return depended to some extent on the time period over which data were measured. Barry *et al.* (1998) show that prior to 1989 their performance was very low and just in the post-1989 some large economies such as Brazil, Russia, India and China (BRICS) assisted to an economic expansion. However, the high performance of the BRICS may be due to an unusual episode in global capital markets rather than being indicative of future higher returns. In addition, some emerging markets had a long history, often interrupted by wars and other adverse events, making them "disappear" and "re-emerging" on the political and economic scenario. It is idea of Goetzman and Jorion [1999] that the growth and integration of re-emerging markets into the world capital markets may therefore be temporary - a result of world market liberalization that is reversed in periods of global distress.

4.2 Index analysis

Before starting with the portfolio analysis, it may be important to define the indices I will use, and perform a deep analysis about them.

In this work, I use 16 different indices divided in 4 categories: equities, sovereign bonds, corporate bonds and alternative assets and I report them in Table 8 along with a simplified name and some features.

Two of them, namely MSCI WORLD and CGBI-WGBI WORLD, are indices used to construct the benchmark in order to run a comparison in the last part of my work. Both of them have a weight of 50% in the benchmark.

Table 15: Classification of the indices

Name	Category	Region	Simplified name
MSCI USA	Equity	United Stated of America	Eq. USA
MSCI EMU	Equity	European Monetary Union	Eq. EMU
MSCI JAP	Equity	Japan	Eq. JAP
MSCI EM	Alternative	Emerging Market	Eq. EM
MSCI WORLD	Equity	World	Eq. WD
CGBI-WGBI US	Government Bond	United Stated of America	SB USA
CGBI-WGBI EU	Government Bond	European Monetary Union	SB EMU
CGBI-WGBI JP	Government Bond	Japan	SB JAP
Barclays EM World Government	Alternative	Emerging Market	SB EM
CGBI-WGBI WORLD	Government Bond	World	CB USA
Barclays US Agg Corporate	Corporate Bond	United Stated of America	CB EMU
BOFA ML EUR Corp	Corporate Bond	European Monetary Union	CB JAP
BOFA ML JP Corp	Corporate Bond	Japan	CB EM
Barclays EM World Corporate	Alternative	Emerging Market	CB WD
S&P GSCI Commodity Total Return	Alternative	World	Commodity
HFRI Fund Weighted Hedge Fund	Alternative	World	Hedge Fund

UK-DS Inv. Trust Private Equity	Alternative	World	Private Equity
MSCI World Real Estate	Alternative	World	Real Estate

The sample for these indices is from January 1997 to December 2014, for a total of 18 years and 216 monthly observations.

One of the major problem in selecting the indices was the time range of the sample, since most of the indices used in the current finance world are born recently and did not exist back in 1997. Consequently, the range of possible choices offered by *Datastream* was quite narrow, in particular for the private equity index.

Moreover, in the case of real estate, there is no common index that can represent all the real estate prices around the world, like for commodities, where there are specific exchange markets and specific derivative instruments.

In addition, real estate prices are affected by many elements such as geographical area, the dimensions and the material used. Thus, for this case I decided to employ an equity index representing the real estate companies as a *proxy*.

4.2.1 Normality

The probability distribution of return is an important aspect when analysis like mine are undergone.

Using a distribution that does not represent the reality of the fact may affect the findings and may have serious consequences, in particular when we employ economic and statistical measures such as the Value at Risk.

In addition, different return distributions can lead to different portfolio optimizations when it comes to risk budgeting allocation.

For these reasons, using the software *Matlab,* I run some tests to verify whether the returns are normally distributed. Table 16 show the results on normality distribution using Jarque-Bera test and Anderson-Darling test with different level of confidence.

Table 16: Normality test

Asset	Jarque-Bera Test			Anderson-Darling Test		
	10%	5%	1%	10%	5%	1%
Eq. USA	x	x	x	x	x	x

Eq. JAP	x	x		x	x	
Eq. EM	x	x		x	x	
Eq. EMU	x	x		x	x	x
Commodities	x	x	x	x	x	
Hedge Fund	x	x	x	x	x	x
Private Equity	x	x	x	x	x	x
Real Estate	x	x	x	x	x	x
SB USA	x	x	x			
SB JAP	x			x		
SB EM	x	x	x	x	x	x
SB EMU						
CB USA	x	x	x	x	x	x
CB JAP						
CB EM	x	x	x	x	x	x
CB EMU	x	x	x	x	x	x
x = reject the null hypothesis of normal distribution						

From Table 16 we can draw some conclusion about the distribution.

Both tests show for all the three levels of confidence that SB EMU and CB JAP have returns that follow a normal distribution while SB JAP reject only at 10%.

Eq. USA, Hedge Fund, Private Equity, Real Estate, CB USA, SB EM, CB EM and CB EMU do not have returns that follow a normal distribution for any level of confidence while Eq. JAP and Eq. EM accept the null hypothesis only at 1%.

Commodity accepts the hypothesis only with Anderson-Darling test at 1% and Eq. EMU only with Jarque-Bera at 1%.

For SB USA we have contrasting results.

Eventually, both methods provide with almost the same result: with Anderson-Darling[36] test 75% of the indices have returns that do not follow a Gaussian distribution whereas with Jarque-Bera it is the 81.25% of them.

[36] At 5% confidence level.

From Figure 5 to 8, I provide with two probability density function estimates for each asset. For the first (green bars) I used the Matlab function *histogram*[37], employing an automatic binning algorithm that returns bins with a uniform width, chosen to cover the range of monthly returns and reveal the underlying shape of the distribution.

For the second I used the Matlab function *ksdensity*, employing an Epanechnikov kernel function[38].

Table 17 shows the skewness and kurtosis for each asset.

Most of the estimates show concordant results with the tests. SB EMU, CB JAP and SB JAP have the lowest levels of kurtosis while the alternative assets such as Hedge Fund, Commodity, Private Equity, and all the corporate bonds (with the exclusion of Japan) show the highest levels of kurtosis. In some cases also the skewness is very high, for instance CB EM, CB USA and Hedge Fund.

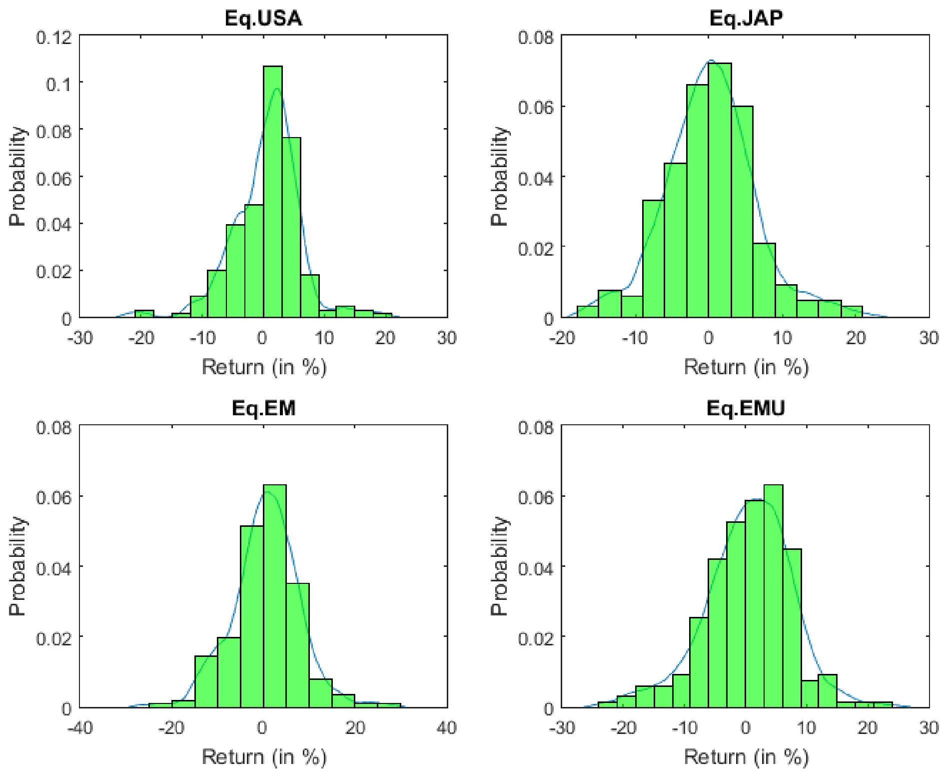

Figure 5: Probability density function estimates for Eq. USA, JAP, EM, EMU

[37] The height of each bar is, (number of observations in the bin) / (total number of observations * width of bin). The area of each bar is the relative number of observations.

[38] The kernel of a probability density function (pdf) is the form of the pdf in which any factors that are not functions of any of the variables in the domain are omitted.

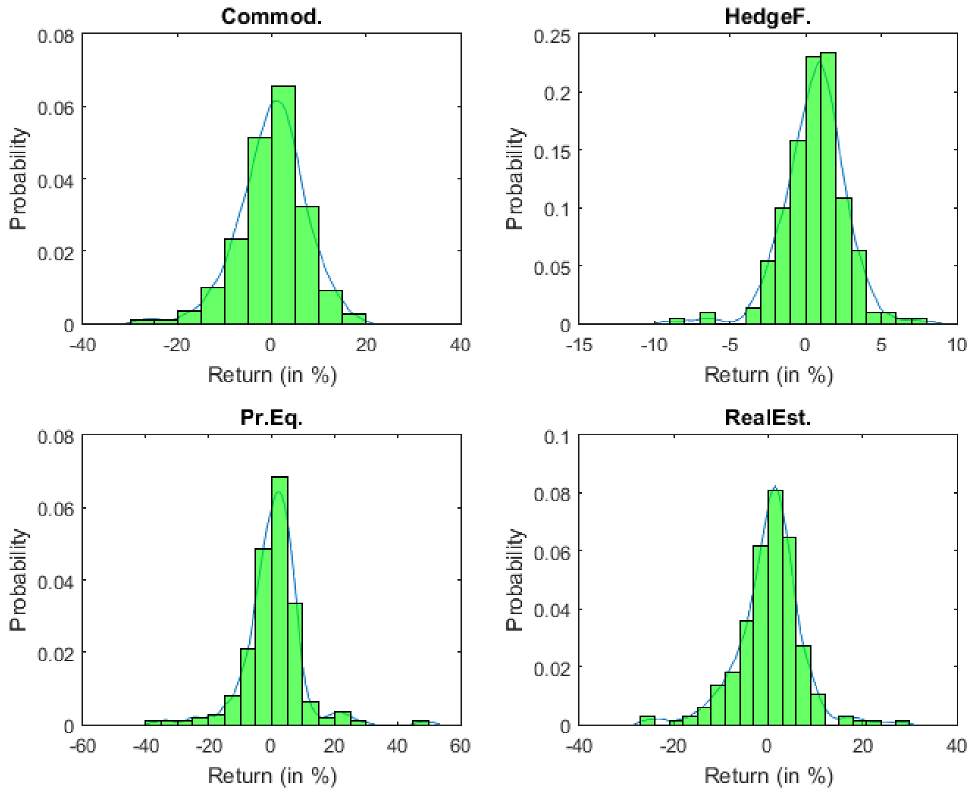

Figure 6: Probability density function estimates for Commodity, Hedge Fund, Private Equity, Real Estate

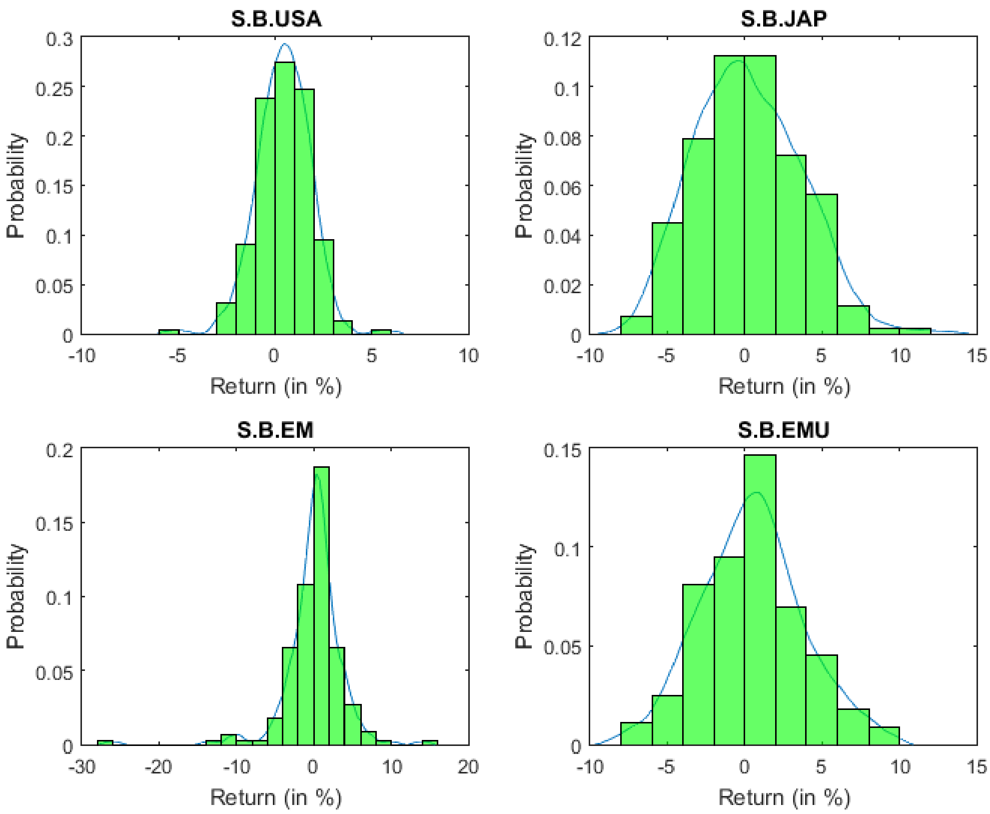

Figure 7: Probability density function estimates for SB USA, JAP, EM, EMU

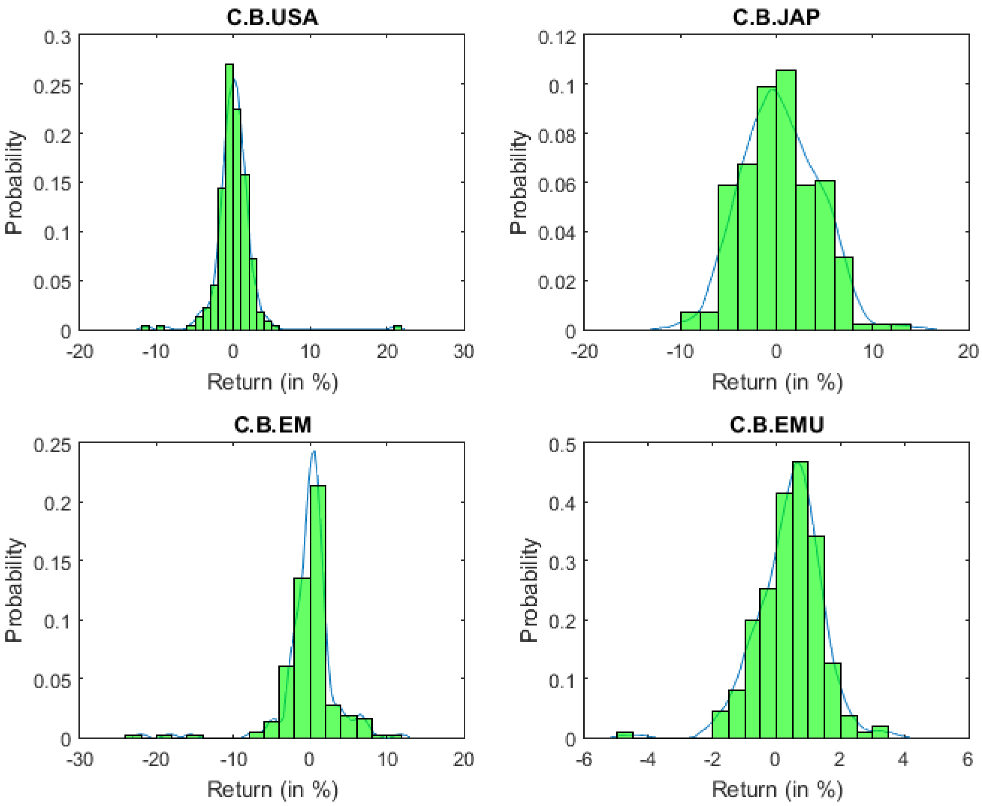

Figure 8: Probability density function estimates for CB USA, JAP, EM, EMU

Table 17: Skewness and kurtosis of all the indices

Asset	Skewness	Kurtosis	Asset	Skewness	Kurtosis
Eq.USA	-0,354	5,560	SB USA	-0,192	4,572
Eq.JAP	0,248	3,860	SB JAP	0,363	2,864
Eq.EM	-0,064	4,057	SB EM	-1,893	16,098
Eq.EMU	-0,410	3,854	SB EMU	0,108	2,912
Commodity	-0,512	4,123	CB USA	2,576	34,991
Hedge Fund	-0,561	5,691	CB JAP	0,208	2,973
Private Equity	0,189	9,205	CB EM	-2,131	16,907
Real Estate	-0,272	5,755	CB EMU	0.179	3,39

4.3 Historic analysis

In this part, I propose an analysis of the evolution of the indices, with the scope to understand the reasons that led their performances during the expansion cycles and the recession periods.

The liberalization of the financial system and the increased globalization of capital market over the past few decades have improved the provision of financial services and the allocation of

resources, but have also enhanced the range of pronounced financial cycles. The cycles have often involved dramatic fluctuations in asset prices that have contributed to the amplification of the business cycles more generally, and occasionally have culminated in both banking and exchange market crisis. While both industrialized and emerging market economies have been affected, emerging markets have tended to occur the heaviest costs.
Typically these financial cycles are generated by a wave of optimism supported by favourable developments in real side of the economy. This optimism contributed to the underestimation of the risks, overextension of credit, excessive asset price fluctuations, over-investment in physical capital, and strong consumer expenditure. Eventually, when expectations realign with fundamentals, the imbalances built up during the boost are corrected suddenly, as excessive optimism turns into excessive pessimism, causing disruption in both the financial system and the real economy.

4.3.1 Asian crisis

In my index sample, the first event that leaves a significant trace is the Asian financial crisis. The crisis, starting in 1997, affected mainly some countries of the Asian South-East belonging to the emerging markets, like Malaysia (3.13%), Indonesia (2.14%) and South Korea (15.53 %) and to the developed countries like Japan.
In Figure 9, I report the cumulated return for Eq. JAP, Eq. EM, SB EM, CB EM and Benchmark for a comparison. There is no doubt that these four indices had been very affected by the recession with cumulative losses of 31%, 47%, 42% and 31% respectively towards the end of 1998.

Born from a multiple financial speculation that caused a strong monetary devaluation and the consequent Asian currency peg break from the Dollar, the crisis was due to the heavier debt load of private sector, which suffered a capital shrinkage from foreign investors and institutions. The crisis showed itself under different aspects: from a speculative attack on the currencies involved, to the slump of the stock market and the real estate sector. Precisely as Mera *et al.* [2000] suppose, the real estate sector was one of the main causes at the base of the Asian South-East boom and of the consequent financial speculations, very similarly to what will occur in 2007-2008. The events of 1998 resulted in reduced levels of production from the emerging markets with the consequence of a consistent drop in raw material demand.

Figure 9: Cumulated returns for Eq. JAP, EM, SB EM, CB EM, Benchmark

Eventually, as a first sign of the concept of *global contagio*, the plunge affected all the commodities markets, in particular those with a strong correlation with the oil price levels, with the Brent index that touched the $9.22 per barrel, the lowest since the '70s oil shortage. In Figure 11, the effects on Real Estate and Commodity that lost, between the end of 1998 and the beginning of the 1999, respectively 45% and 46%.

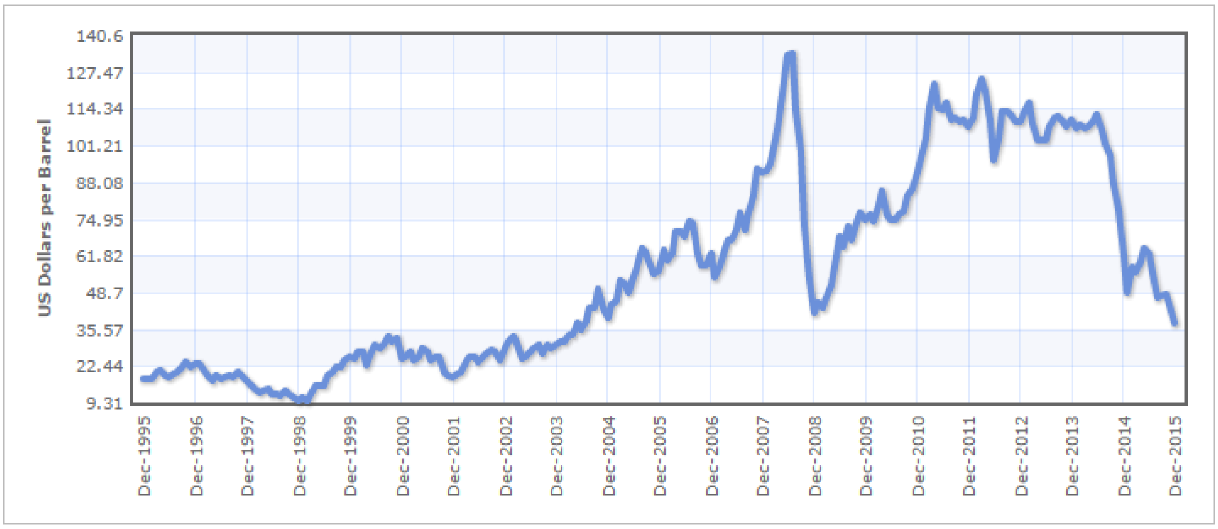

Figure 10: Oil Price from December 1995 to December 2015, Source: Il Sole24Ore

Figure 11: Cumulated returns for Commodity, Real Estate, Benchmark

4.3.2 Dot-com crisis

Eq. USA and Eq. EMU share a very similar path from the beginning of the sample, January 1997, until the middle of 2002, with a general upward and downward trend. The principal reason dated back to the *dot.com* bubble, a speculative bubble developed between 1997 and the first months of 2000. During this period, the capitalization of the most developed countries saw a rapid boost of the value of those companies active in the information technology sector, and along with the growth of it, also the other sectors experienced a rapid rise. Both American and European stock markets benefited from the speculative bubble at the beginning, stimulated by the IT sector with a cumulated return of 111% of Eq. USA (as of April 2000) and 120% for Eq. EMU (as of March 2000) at the momentum peak.

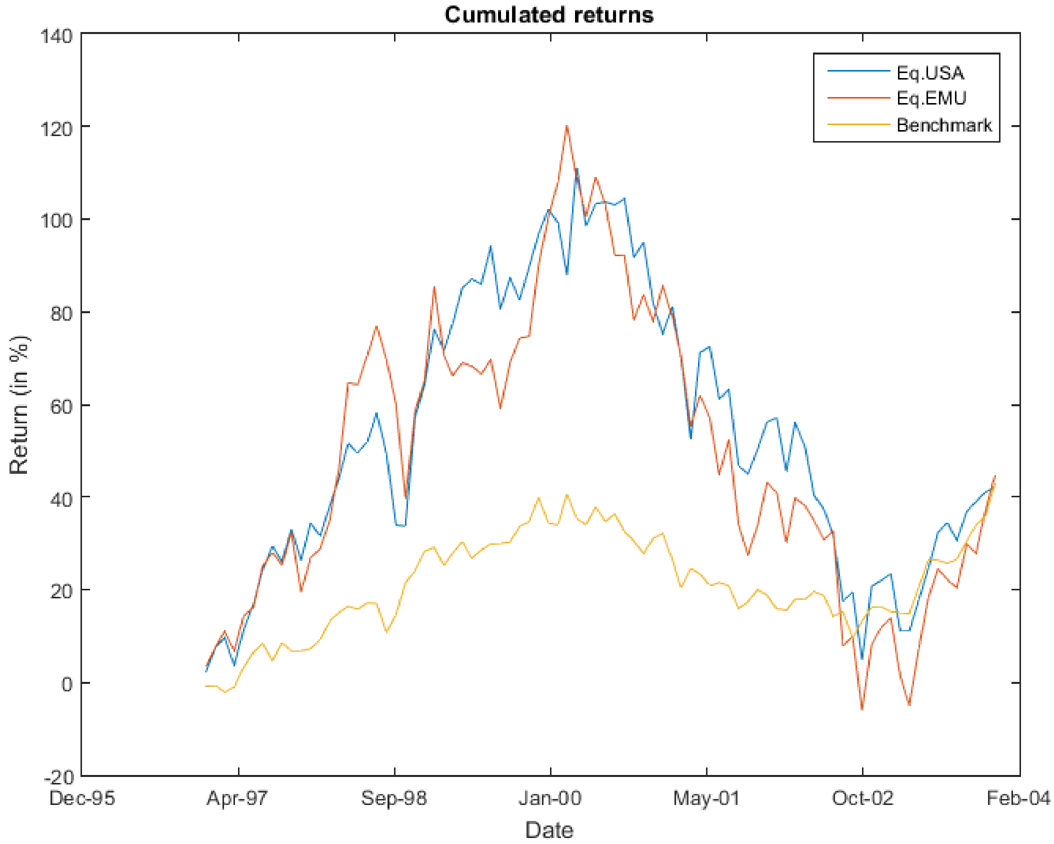

Figure 12: Cumulated returns for Eq. USA, Eq. EMU, Benchmark

The period was marked by the foundation (and consequent bankruptcy) of a high number of firms with the core activities related to the Internet sector usually called Dot-com; they were companies with insufficient capitalization and small dimension, highly exposed in an overrated sector: all typical conditions for the birth of a speculative bubble.

Specifically, speculators and investment companies (in particular private equity firms) saw a large growth of the Dot-com firms that recorded significant stock price rise, and, as result, they switched fast and carelessly great amounts to different investments in the sector, as an attempt to diversify the risk, letting the market choose the best projects.

Thus, the combination of fast increments in stock prices, the market certainty of the firms' capability to provide profits in the future, the speculation on equities and the vast presence of Venture capitals produced an environment in which many investors overlooked the traditional parameters of valuation such as Price/Earnings ratio, blinded by the idea of technological progress.

The collapse of the bubble started between 2000 and 2001. Some companies failed completely, while others lost a large portion of their market capitalization, remaining sound and profitable though: Cysco System stocks lost nearly 86% whereas Amazon almost 94%.

In the slump started in March 2000, the entirely venture capital industry was stricken as the valuation for start-up technology companies collapsed. Many venture firms have been forced to write off large proportion of their investments and many funds saw their values fall below the capital invested.

Figure 13 evidences the plunge of the Private Equity from the peak of 7891$, reached in September 2000, to bottom of 2705$ in October 2002, with a loss of 66%.

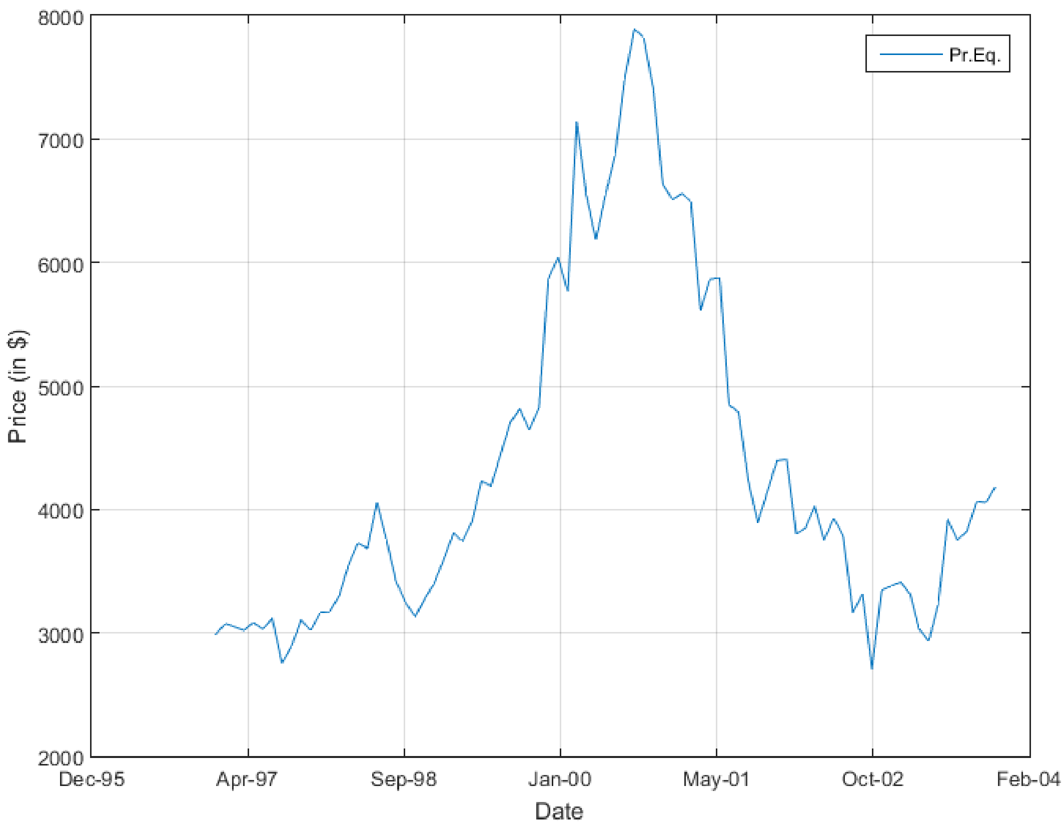

Figure 13: Cumulated return for Private Equity

Just as much as these indices grew, they also greatly slumped, eroding all the gains accumulated in the previous years.

4.3.3 The great crisis of 2008

The great recession was a world economic crisis started in 2007. It developed, first, in the United States following a deep crisis in the real estate sector due to the burst of real estate bubble, and then infected the rest of the economies turning into a world financial recession. Among the causes at its base, we may find the high prices of raw materials (in particular oil price), a world food crisis, the threat of a possible worldwide recession and a credit crunch with the consequent loss of trust in the exchange markets.

Everything began with a real estate bubble developed in USA starting from the first year of the new century: from the 2000 until the middle of 2006, the USA house prices grew very rapidly (15% on the average).

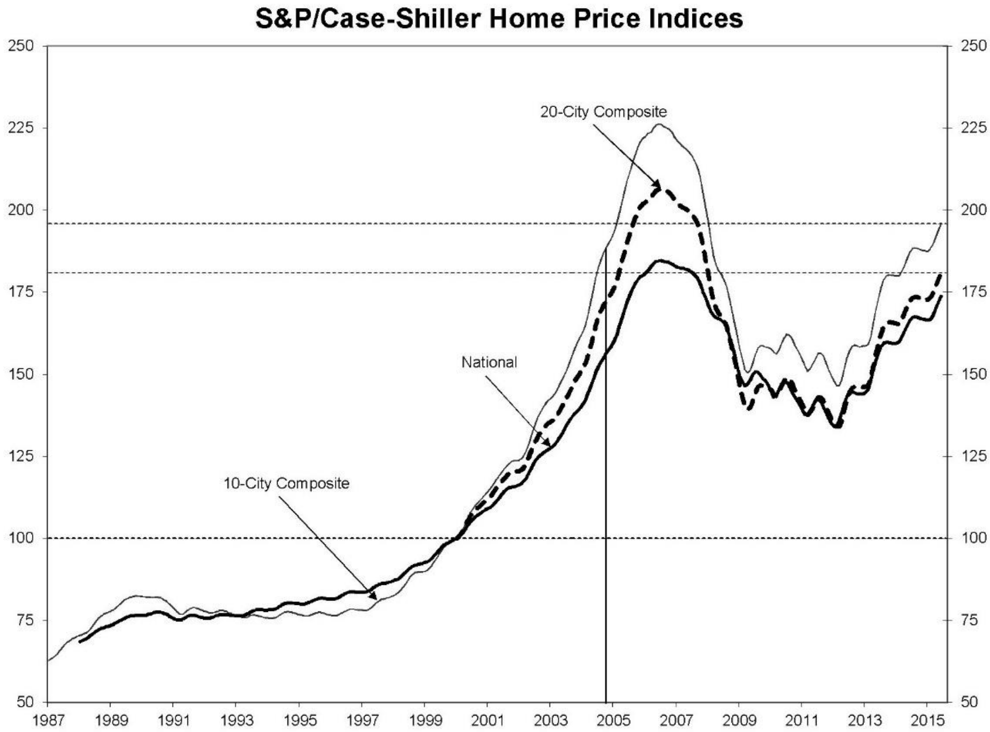

Figure 14: S&P/Case-Shiller Home Price Indices, Source: The Economist

The raising prices, as in Figure 14, were promoted by the institution lending process and by the vast number of mortgages allowed, making the activity apparently safe and sound. Forecasting a continuous increment of house prices, the lenders did not doubt about the incapability of some borrowers to repay the debt, also because, in case of borrower defaults, the credit institutions were always able to resell the houses for a higher price.

Moreover, the mortgage subscriptions with low guarantee (subprime), subscribed also by wealthy people confident in profitable investments, were always more frequent grant even with the awareness of incapacity of repayment by the borrower. The subprime trend grew form $145 billion of 2001 to $635 billion of 2005. This speculation was supported by the low rates of interest, due to the fact that the Federal Reserve was operating economic policies to stimulate the American economy after the 2001 crisis. However between 2005 and 2007, the FED increased the rate of interests from 1.5% to 5.25%, in the attempt to stop the growing speculation and drain the liquidity out of the market.

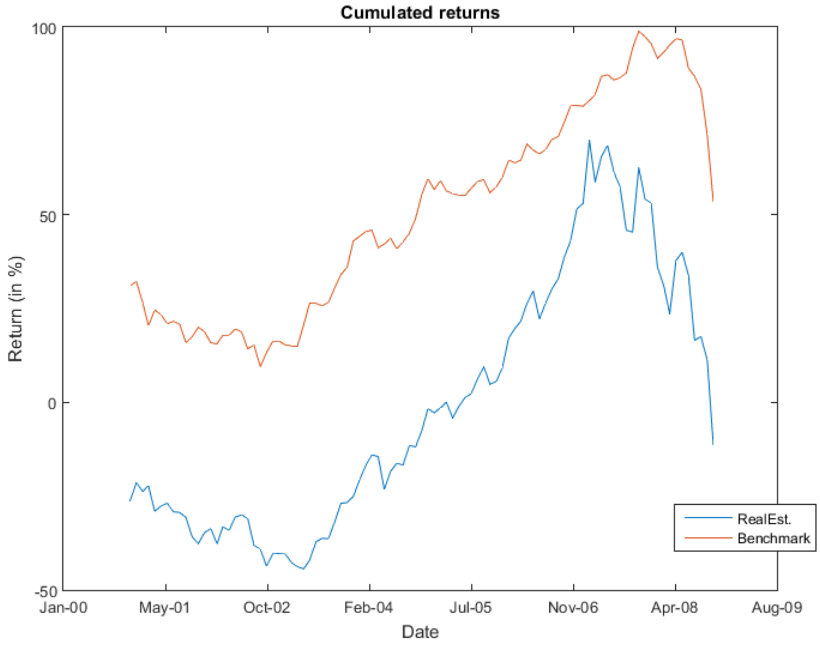

Figure 15: Cumulated return for Real Estate, Benchmark

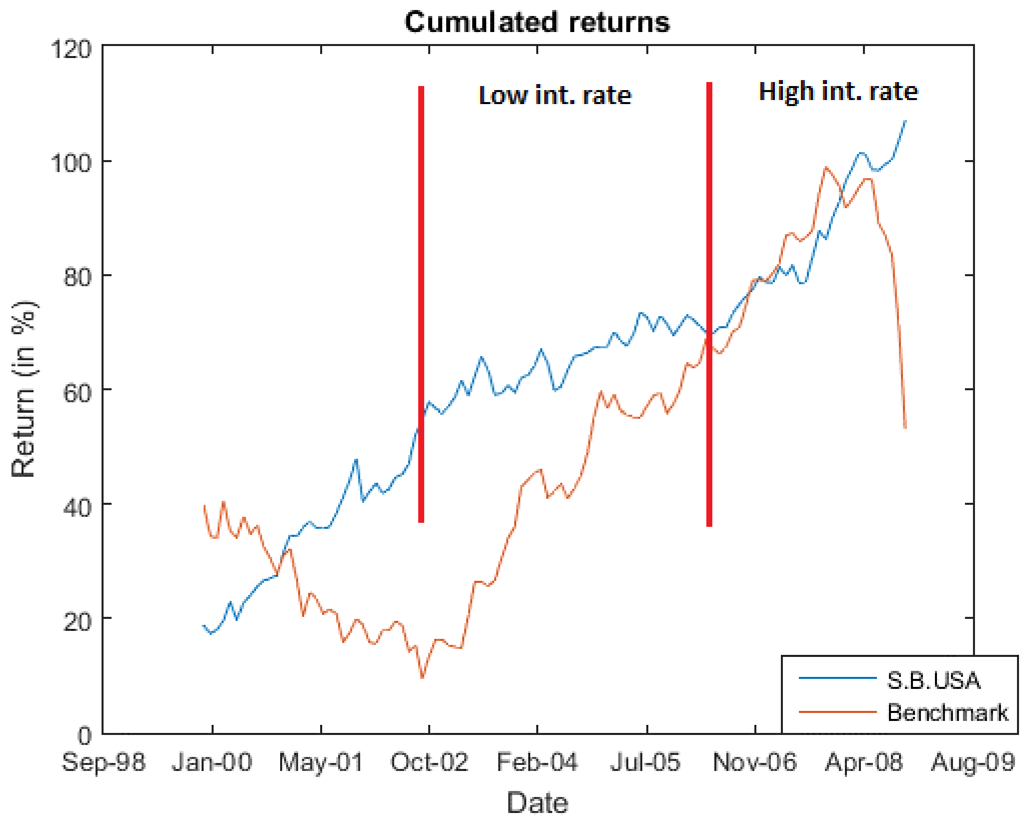

Figure 16: Different trend in SB USA

Immediately in 2006, defaults and insolvencies increased hugely, in particular among the subprime buyers, stricken by the mortgage interest rate boost. When the bubble burst, the house prices sunk, starting a wave of sales and pessimism that ruined many investors and credit institutions, in particular the last ones were overloaded by bridge lending to the private equity firms for their operations of LBO. The result was a complete paralysis of the credit system and the collapse of all the market exchanges, first in USA and then in the rest of the world. Figure 17 shows the drastic loss of value of some indices: after the peak between February and March 2007, in the following two year Eq. USA lost 65%, Eq. JAP 44%, Eq. EM 63%, Eq. EMU 56%, Real Estate 72% and Private Equity 69%.

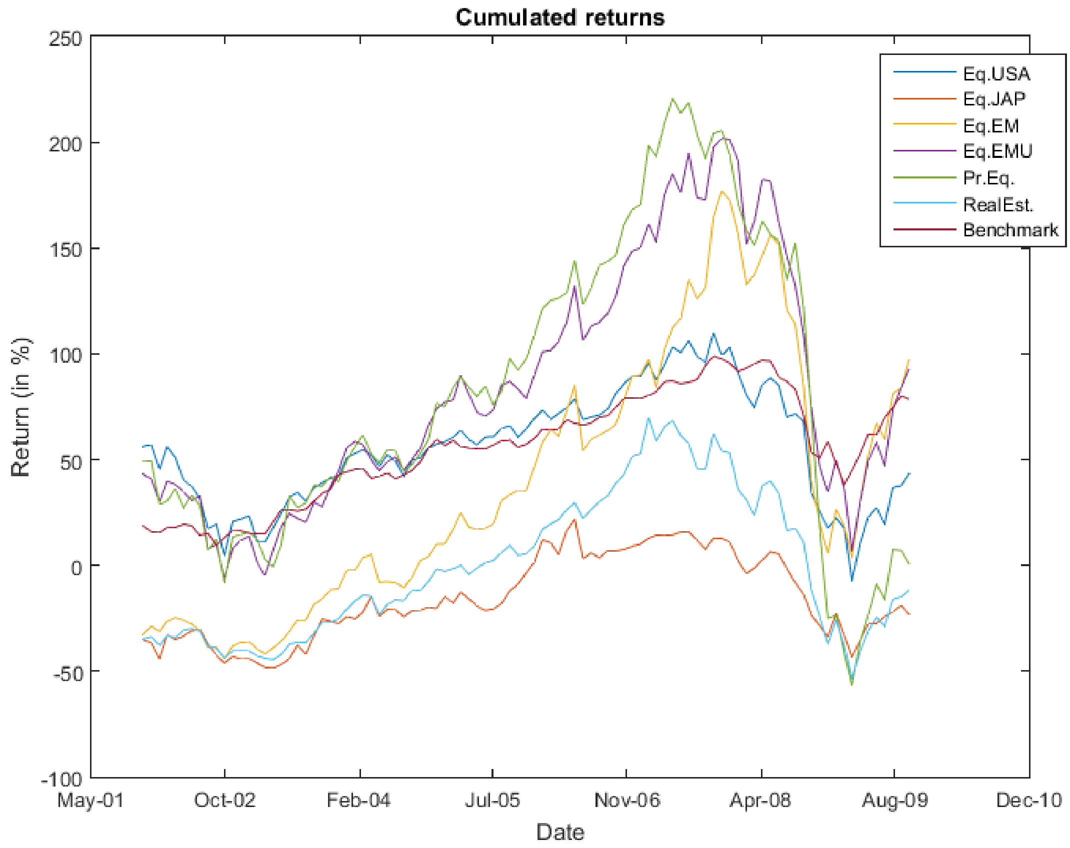

Figure 17: Cumulated returns for Eq. USA, JAP, EM, EMU, private Equity, Real Estate, Benchmark

As show in Figure 10, also the oil price suffered a great collapse during this crisis. Between 2001 and 2008 the oil price increased almost of the 470%, from the initial 20$ to the peak of $147 touched by the Brent. The raise of oil price but also of the other commodities like wheat and soy was due to different reasons:

- A world growing demand, especially from the expansion of the emerging countries,
- The intensified frequency of supply shortages,
- The negative previsions about the future supply,
- The geographical and political instability of certain areas (for instance the Middle East),
- The depreciation of the Dollar,
- The scarcity of technological investment in the extractive industry,
- Speculations on commodity exchanges.

The cost of oil and raw materials was one of the many elements at the base of the crisis, pushing up the stock markets and the financial speculations but when the financial crisis affected also the real economy and the expansionary cycle stopped, the demand of raw materials dropped vertiginously, making the commodity exchange slump too.

From more than 140$ per barrel to less than 40$ in one year. The rally of Commodity is illustrated in figure 16 from $2744 on February 2002 to the peak of $10590 (increase of 285%) on July 2008 to the bottom of $3394 on March 2009 (drop of 212%).

The only exception in this upward and downward trend of the index was along the 2006: it was the result of a wrong bet made by hedge funds and other international investors. Many of them betted on a replay of the severe 2005 hurricane season, stocking oil futures and sending oil prices soaring in the wake of Katrina and Rita, but the 2006 hurricane season was mild, and realized the mistake, they started reducing their exposure with the consequent drop of price.

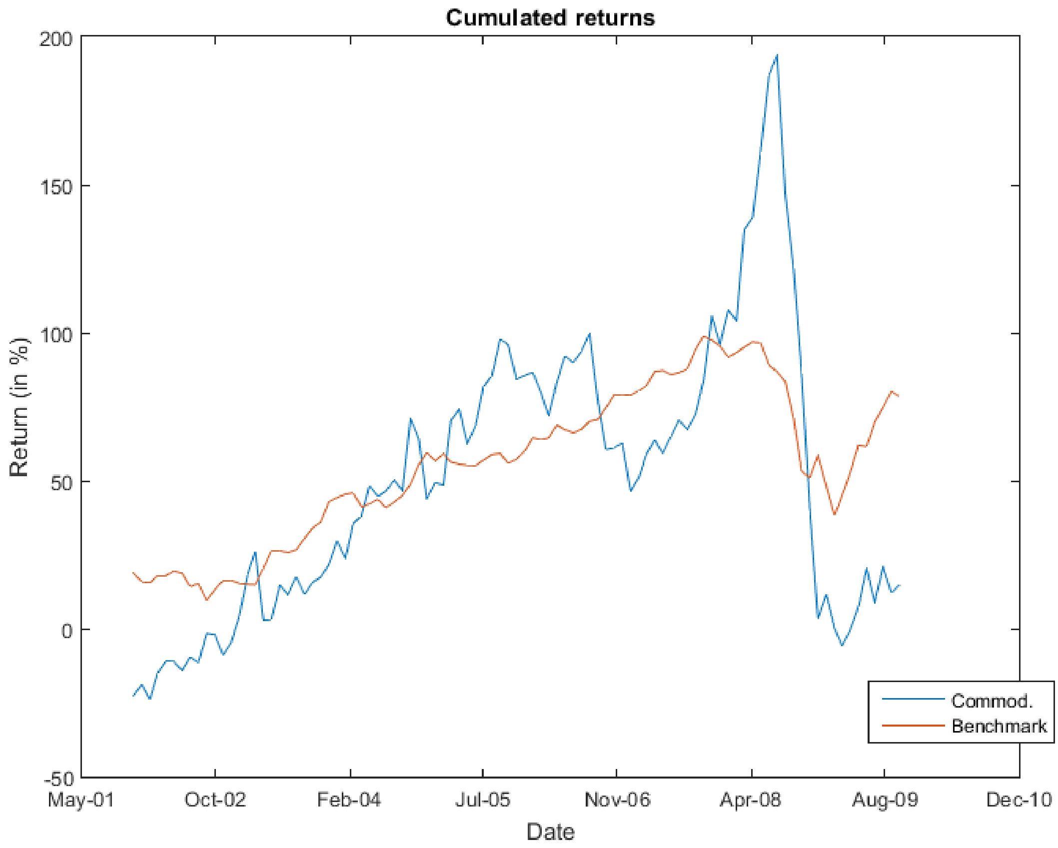

Figure 18: Cumulated returns for Commodity, Benchmark

The Commodity lost about 27%, before bouncing back and starting its new raise.

4.3.4 After the 2008 crash

The sign of a recovery, already in place during the 2009, was marked by a raise in the commodity prices: again Brent surged from $40 per barrel to $70 in June 2009. A more dynamic recovery took place in USA with respect to Europe's, where the insufficient countermeasures against the crisis and a higher work protection imposed high costs on firms, slowing down their plans of reconversion.

However the intervention of the European Central Bank, with its non-conventional operations, relieved the pressure in the interbank market, supporting the banking activities and reducing the credit crunch and the monetary spread.

Stock and bond markets all over the world showed an improving economic expectations since the middle of 2009, consolidating their position at the end of the year.

The 2010 was a positive year for the global economy with a growth of 5%, spread differently based on the geographical area: it was stronger in the emerging markets, with China and India leading with an average 10% and milder in Europe with the only exception of Germany.

This recovery phase lasted till the 2011, where the economies saw sensible reductions of their PILs. This situation worsened when the Europe had to face the sovereign debt crisis. In the first few weeks of 2010, there was renewed anxiety about excessive national debt, with lenders demanding ever-higher interest rates from several countries with higher debt levels, and current account deficits. This made it difficult for four out of eighteen euro-zone governments, namely Greece, Portugal, Cyprus and Ireland, to finance further budget deficits and repay or refinance existing government debt. The states had to be rescued by sovereign bailout programs, which were provided jointly by the International Monetary Fund and the European Commission, with additional support at the technical level from the European Central Bank. Figure 19 shows clearly the movement of the economy of USA, EM and EMU.

Despite sovereign debts have risen in some euro-zone countries, with the three most affected countries Greece, Ireland and Portugal collectively accounting for 6% of the euro-zone's gross domestic product (GDP), other countries benefit from situation, in fact Germany was estimated to have made more than €9 billion out of the crisis since investors flocked to safer but near zero interest rate German federal government bonds.

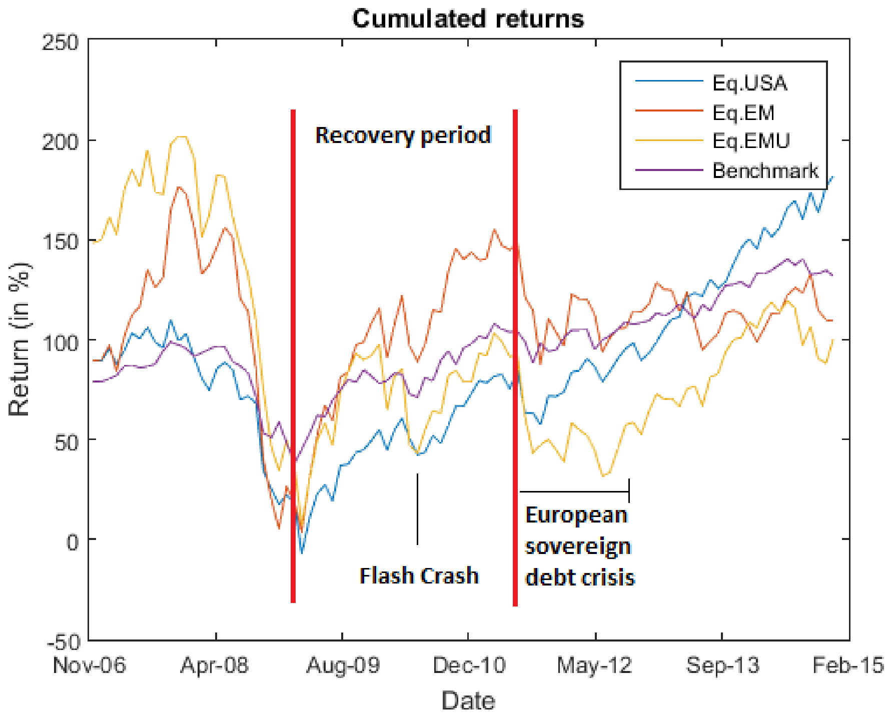

Figure 19: Recovery period

By July 2012 also the Netherlands, Austria, and Finland benefited from zero or negative interest rates. Looking at short-term government bonds with a maturity of less than one year the list of beneficiaries also includes Belgium and France.

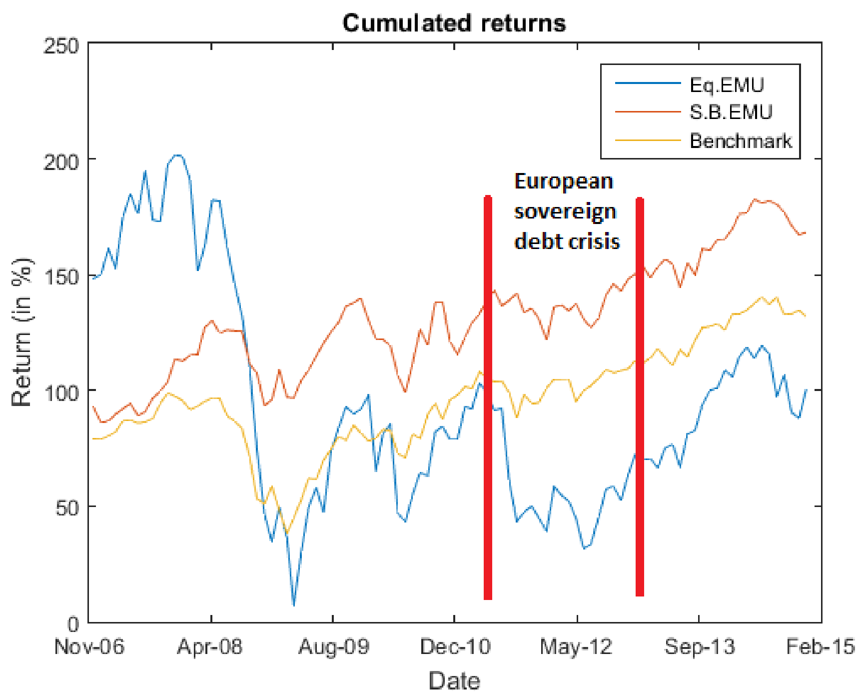

Figure 20: European sovereign debt crisis

As we can see in Figure 20 from the comparison with Eq. EMU, SB EMU was just slightly affected by the sovereign debt crisis.

The significant drop of May 2010 is ascribable to the Flash Crash.
The Flash Crash also known as The Crash of 2:42 was a United States trillion-dollar stock market crash, which started at 2:42 pm and lasted until the 3.07 pm.
Stock indexes, such as the S&P 500, Dow Jones Industrial Average and Nasdaq Composite, collapsed beyond 9% and rebounded very rapidly closing the day with losses around 3%.

5 Portfolio diversification analysis

In this section, I analyse what the impact is and how the alternative asset classes diversify different type of portfolios.

For this purpose I use 216 monthly observations from January 1997 to December 2014, for a total of 18 years.

This analysis will be divided in 4 scenario, in order to better understand the value and the power of the several indices available. In the first I employ only indices referring to the “traditional” asset classes with the aim to construct the first level of comparison and it includes:

- Equity USA
- Equity JAP
- Equity EMU
- SB USA
- SB JAP
- SB EMU
- CB USA
- CB JAP
- CB EM

The second scenario sees the addition of the assets afferent to the emerging market sphere and will be compared against the first case. The reason is that, despite the different geographical regions, these assets have many characteristics in common with the assets in the first group, and includes:

- Scenario No.1 assets
- Equity EM
- SB EM
- CB EM

The third scenario includes the private equities and the hedge funds to widen more the opportunities in term of return and volatility. Despite being evaluated as “alternative”, private equities and hedge funds may be considered as sub-classes of those already existing. In the most of the cases, for instance, hedge funds employ strategies based using fixed income assets and stocks. Even the derivatives used by these funds have most of the time, as underlying, bonds and equities.

The third group is then built up by:

- Scenario No.1 assets
- Hedge Fund
- Private Equity

The last scenario comprehend the assets including commodities and real estates. The last two indices should represent the two “most alternative” classes, given the fact that they refer to assets with different features from the previous and possibly follow different path from the others in response to the economic cycles.
The last group then includes:

- Scenario No.1 assets
- Commodity
- Real Estate

For all the scenarios, the same procedure is implemented.
In the first part, I use the Markowitz model theory to derive the efficient frontier and, to that purpose, I estimate the expected returns and the variance-covariance matrix using the entire sample available. I also plot on the return-risk plane each indices, in order to give a general idea of which asset could provide itself useful for the diversification scope and to understand how optimal they appear with respect to the others.
The correlation matrix is also another valuable instrument to understand furthermore how the indices interact and move with each other.
The following part regards to the portfolio analysis: I provide 9 different portfolios, built up with different technics and constraints, with the objective to diversify them and to comprehend the role played by the different assets.

At the end of each scenario, I also report the following valuation indices in order to extrapolate some information about the performance, the diversification, the risk and the concentration of weight and risk budgets:

- Diversification index
- Sharpe ratio
- VaR
- Shannon entropy
- Gini Index

For an overall portfolio valuation, I create a composite index that is no other than the sum of the ranks, based on the mentioned above indices, multiplied by a specific coefficient, of each portfolio. DI, SH and VaR coefficient equals 1, whereas SE and GI coefficient equals 0.5.

As I stated before, in every scenario I analyse, there are 9 different portfolios constructed with various approaches and constraints. The generic feature is that short-selling is not allowed. The following six portfolio have the same construction for all the four scenarios. The first portfolio is based on Markowitz framework and aims to reduce to the minimum the overall risk, without regards for other factors like the return level. It is called Global Minimum Variance (GMV) portfolio.

This portfolio lies at the bottom left of the efficient frontier and usually is made up by the assets with the lowest risk and correlation.

The second portfolio is the Maximum Return (MR) portfolio, and follows an optimization aimed to maximize the overall return, without bearing in mind any other aspect. This portfolio is composed normally (if no constraint is present) by just one asset with the highest return.

The third portfolio is based on Tobin [1958] expansion[39] of the Markowitz framework and refers to the Sharpe ratio (SH)[40]:

$$MS(w) = \max(\frac{\mu(w) - r_f}{\sigma(w)})$$

In my case the risk free rate is set equal to zero. This portfolio looks for a good compromise between volatility and return.

For the seventh portfolio, I completely changed approach. So far, I relied on the Markowitz framework but for this case and the next one, I switch to the risk budgeting approach[41]. Risk budgeting approaches are based on the risk contributions, which are given by the product of marginal risk for the exposure (weight) to the single index.

Notably, I employ an Equal Risk Contribution (ERC) approach, in which all the assets receive the same budget for the individual risk.

For this portfolio, I use the equation (1.9) for the volatility risk measure to estimate the risk contribution of each index.

In general, the estimation of the risk budgeting portfolio is not feasible from an analytic point of view and even from a computational viewpoint the problem is not simple and requests a

[39] See chapter 1.1.5 for review.

[40] See chapter 3.1 for review.

[41] For the general framework review chapter 2.2.

recursive method to identify the best vector of the index weights.

For portfolio No.8, I use again the risk budgeting approach, yet I do not use the volatility as measure to estimate the risk but the Value at Risk.

Moreover, given the fact that the returns do not follow a normal distribution, I cannot use the classic equation (1.11) but I need to use the Cornish-Fisher expansion to keep in consideration also the skewness and the kurtosis of the curve.

The new equation for risk contributions becomes then:

$$RC_i = w_i(-\mu_i + w'z\frac{(\sum w)_i}{\sqrt{w'\sum w}})$$

where z is the vector of the transformation of the inverse of the cumulative density function of the standardized normal distribution $\Phi^{-1}(\alpha)$ of each index, based on the skewness and kurtosis estimation.

For the allocation of risk budgets, I created two indicators based on the Sortino index and on the Sharpe index.

For the first indicator, I compute the expected return and divide it for its maximum drawdown. Then I divided the values just found for their sum.

For the second indicator, I simply compute the Sharpe ratio and then I divide them by their sum, obtaining in this way a risk allocation based on the performance of the index. To deepen my analysis, when computing the propter allocation for the indices, I use different quantile for the Value at Risk, i.e. $\alpha = 0.99, \alpha = 0.95$ and $\alpha = 0.9$.

The choice of the best allocation is based on the Diversification index. For the analysis of portfolio 8, see Appendix C.

Portfolios No. 4, No.5, No.6 and No.9 have specific constraints so I explain them in each scenario.

5.1 Scenario No.1

Table 18: Mean, median, st. deviation, minimum, maximum, skewness, kurtosis S.1

Asset	Mean (Annual)	Median	St.Deviation (Annual)	Min	Max	Skewness	Kurtosis
Eq. USA	0.625 (7.823)	1.339	5.324 (18,951)	-20.961	19.073	-0.354	5.560
Eq. JAP	0.155 (2.286)	0.094	6.032 (23.293)	-15.226	20.168	0.248	3.860

Eq. EMU	0.566 (7.351)	1.289	6.901 (24.851)	-21.373	21.807	-0.410	3.854
SB USA	0.44 (5.434)	0.512	1.328 (4.416)	-5.098	5.667	-0.192	4.572
SB JAP	0.207 (2.498)	0.141	3.311 (11.775)	-6.801	11.832	0.363	2.864
SB EMU	0.509 (6.301)	0.488	3.211 (12.197)	-7.301	8.459	0.108	2.912
CB USA	0.050 (0.382)	-0.050	2.356 (4.744)	-11.410	21.234	2.576	34.991
CB JAP	0.396 (4,482)	0.219	3.818 (10.572)	-9.944	13.498	0.208	2.973
CB EMU	0.469 (6.001)	0.172	3.191 (13.864)	-8.477	10.415	0.179	3.39

Analysing Table 18, we realize immediately that the indices have different figures and we can make some considerations about them: SB USA has a good trade-off between return and volatility as well as SB EMU and CB EMU, both of them present a minimum value lower, in absolute value, then the maximum.

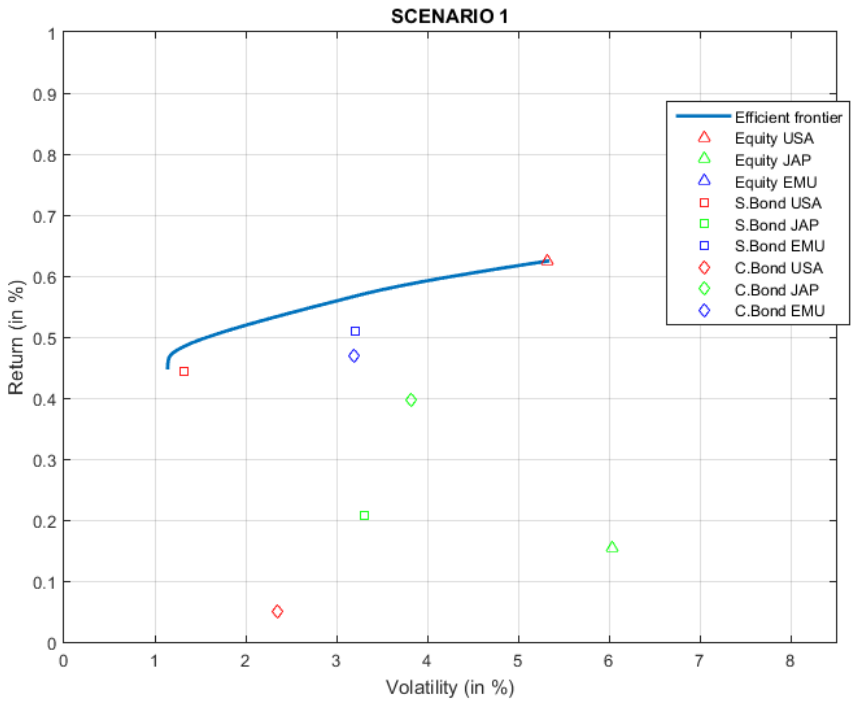

Figure 21: Efficient frontier S.1

Particular is the Equity JAP that has a volatility almost as double as those of SB JAP and CB JAP, but a return 33.5% and 155% lower. The lowest return is for CB USA with only 0.05%, and it is the only asset with a negative median.

In Figure 21, we can note that the best assets are SB USA, with the lowest volatility, SB EMU and Equity USA, with the highest returns. Equity JAP may be defined the worst in the basket, given the fact the all the other indices have practically higher return and lower volatility.

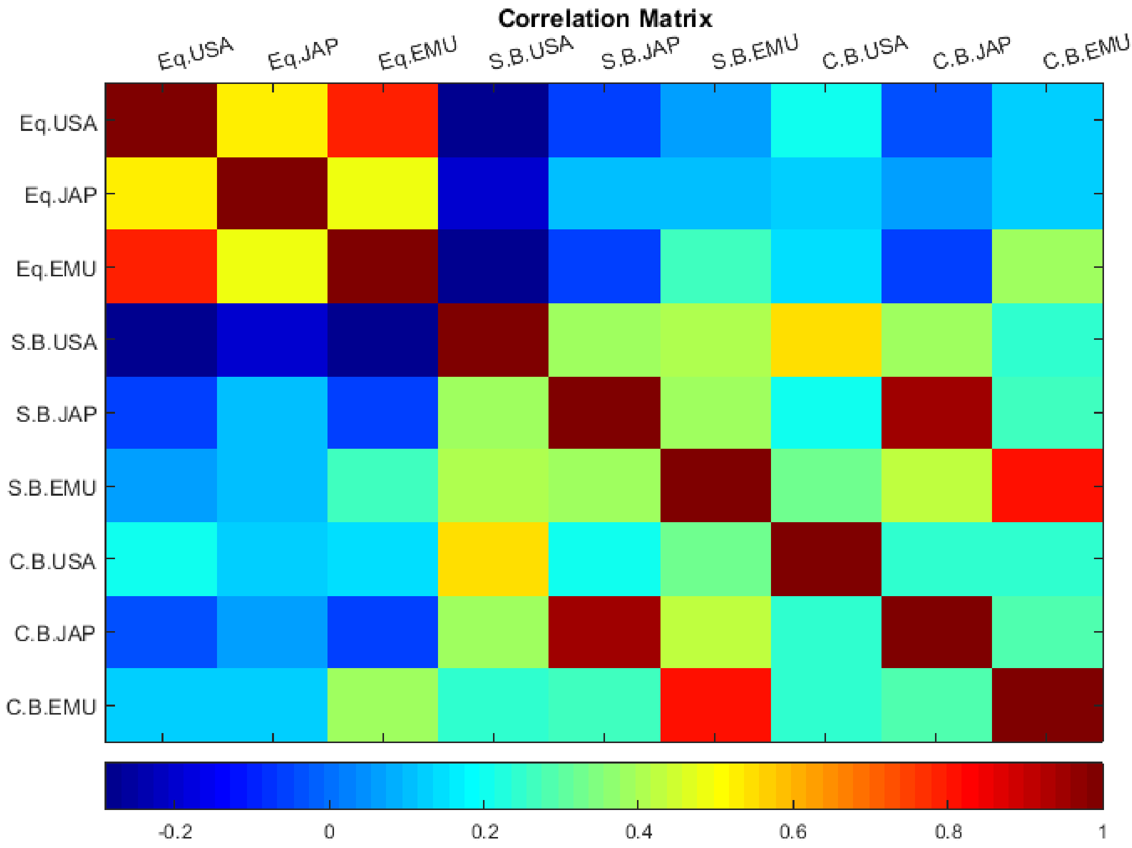

Figure 22: Correlation matrix S.1

In Figure 22, we can observe clearly a high level of positive correlation among equities (close to 0.6), in particular between Eq. USA and Eq. EMU which is nearly 0.8. SB USA holds negative correlation against all equities, symptom that when the stock markets struggle, investors look at USA treasury bonds as a form of defence.

In general, all the fixed income assets hold negative or none correlation against stock markets. Different story when it comes to correlation among bonds; in fact, all the fixed income assets have a positive correlation among each other, with peak logically between corporate bonds and sovereign bonds from the same region (USA 0.55, JAP 0.94, and EMU 0.81).

Portfolio 4 (50/50) is based on a Max Sharpe optimization (see portfolio 3) but with some constraints to improve diversification:

a. All assets must have at least 3% and no more than 30%, $0.03 < w < 0.3$,
b. Bond class must hold 50% of the portfolio, w_{B}[42] $= 0.5$,
c. Equity class must hold 50% of the portfolio, $w_E = 0.5$.

Portfolio 5 (30/30/40) is based on a Global Minimum Variance optimization (see Portfolio 1), namely a more defensive approach that focuses more on the risk side. Some constraints to improve diversification are:

a. All assets must have at least 3% and no more than 30%, $0.03 < w < 0.3$,
b. Sovereign bond class must hold 30% of the portfolio, $w_{SB} = 0.3$,
c. Corporate bond class must hold 40% of the portfolio, $w_{CB} = 0.4$,
d. Equity class must hold 30% of the portfolio, $w_E = 0.3$.

Portfolio 6 (60/20/20) is based on a Max Return optimization (see Portfolio 2) with an aggressive approach, but to avoid the concentration in a single index, I impose again some constraints:

a. All assets must have at least 3% and no more than 30%, $0.03 < w < 0.3$,
b. Sovereign bond class must hold 20% of the portfolio, $w_{SB} = 0.2$,
c. Corporate bond class must hold 20% of the portfolio, $w_{CB} = 0.2$,
d. Equity class must hold 60% of the portfolio, $w_E = 0.6$.

For portfolio 9 (RB with Cap) I decide to use both the Markowitz and risk budgeting approaches. The result is a portfolio with some constraints on the class exposure and some constraints on the specific index:

a. Exposure to equity class = 70%,
b. Exposure to bond class = 30%,
c. All assets must have at least 3% and no more than 30%, $0.03 < w < 0.3$.

Table 19: Portfolio weights S.1

Port.	Eq. USA	Eq. JAP	Eq. EMU	SB USA	SB JAP	SB EMU	CB USA	CB JAP	CB EMU

[42] Bond class includes all sovereign and corporate bonds

1	7.02%	3.15%	1.92%	87.63%	0.00%	0.00%	0.00%	0.00%	0.28%
2	100.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
3	11.48%	0.00%	0.91%	86.02%	0.00%	0.00%	0.00%	0.00%	1.60%
4	30.00%	6.65%	13.35%	30.00%	3.00%	8.00%	3.00%	3.00%	3.00%
5	16.87%	10.13%	3.00%	24.00%	3.00%	3.00%	24.82%	7.80%	7.38%
6	30.00%	3.00%	27.00%	3.00%	3.00%	14.00%	3.00%	3.00%	14.00%
7	7.23%	6.44%	5.11%	32.80%	9.46%	8.37%	13.27%	8.12%	9.20%
8	7.86%	4.26%	4.41%	55.47%	5.91%	7.54%	0.00%	6.17%	8.38%
9	23.33%	12.75%	3.00%	30.00%	10.28%	3.00%	11.64%	3.00%	3.00%

About portfolio 1, we can immediately see that almost the entire portfolio is built up by SB USA (87.63%) while Eq. USA receives only 7.02% despite being the second larger weight. The reason is simply the fact that being the first portfolio based on a GMV optimization, most of the weight goes to the index with the lowest volatility, in this case SB USA with 1.328. More over SB USA has also a low correlation with most of the other indices.

Correlation plays an important role; in fact, equity class receives some weight in spite of their high volatility (high correlation only among equities), while bond class gets no weight.

Portfolio 2 focuses only on return as a meter of estimation and nothing else. This is why all weight is place on Eq. USA that possess the higher return (0.625%).

Portfolio 3 strategy aims at the best mean-variance trade-off and in this case appears very similar to GMV. The weight is concentrated on SB USA (86.02%) and Eq. USA (11.48%) that together amount to almost 100%.

With those constrains in portfolio 4, I try to improve diversification, using a typical portfolio structure which divides the portfolio between fixed income class and stock class, trying to maximize the return from the equities but at the same time minimizing the risk thanks to the bonds. I impose the presence (even in small amount 3%) of all the assets to furtherly diversify, and, to avoid the possibility that one asset of a specific class could get most of the weight, I imposed a cap of 30%. The result is a good diversification, with the portfolio not made up by just one or two indices, but with a good representation of all the classes.

Beside the GMV optimization to focus on the defensive side, in portfolio 5 I decided to increase the total weight dedicated to the bond class. Only SB JAP and SB EMU received the minimum allocation of 3%, all other fix income assets present some good allocation, improving the diversification. The allocation is centred on the USA region with EQ. USA, SB EMU and CB USA receiving respectively 16.87%, 24% and 24.82%.

Differently from the second case, portfolio 6 presents a better diversification with the allocation focused mainly on 4 assets (84% of total): Eq. USA, Eq. EMU, SB EMU and CB EMU. In this case, the presence of the EMU region is quite strong with 55% of the portfolio.

The ERC method provides with a good diversification, all indices are present and have weights above the minimum 3% of the previous cases. Still, there is a strong presence of SB USA but it is less than 33%. The equity class receives a total allocation of 18.78%, while the sovereign bond class and corporate bond class of 50.63% and 30.59% respectively. This proves again that the major contribution to the risk comes from the equity class, given the fact that they get a smaller capital allocation than the other classes do.

Portfolio 8, with its risk allocation based on the VaR, awards SB USA with a weight equal to more than half of the portfolio, and leaves nothing to CB USA. The rest of portfolio weight is equally distributed among the other indices.

Last portfolio offers a well-rounded diversification with a good proportion on Eq. USA and Eq. JAP; in any case, some portfolios just receive the minimum, Eq. EMU, SB EMU, CB JAP and SB EMU, confirming for some indices the same trend seen in the above portfolios.

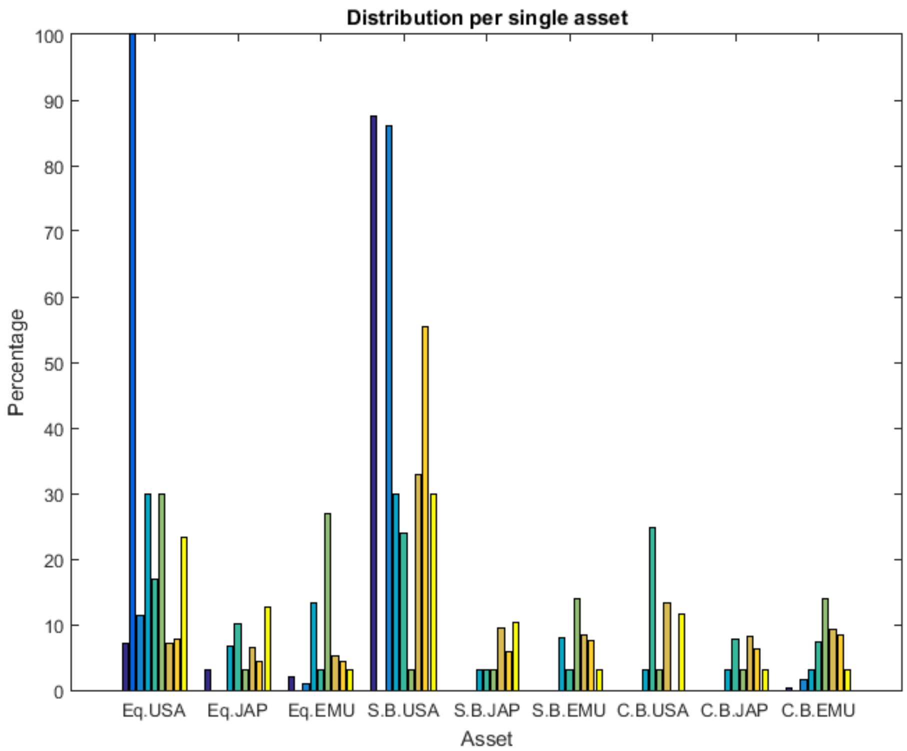

Figure 23: Distribution of single asset S.1

Despite all the portfolios available and the various strategies implemented, it seems clear that some indices just perform too much better than the others and for this reason they receive a way larger percentage of weights as Figure 23 and 24 highlight.

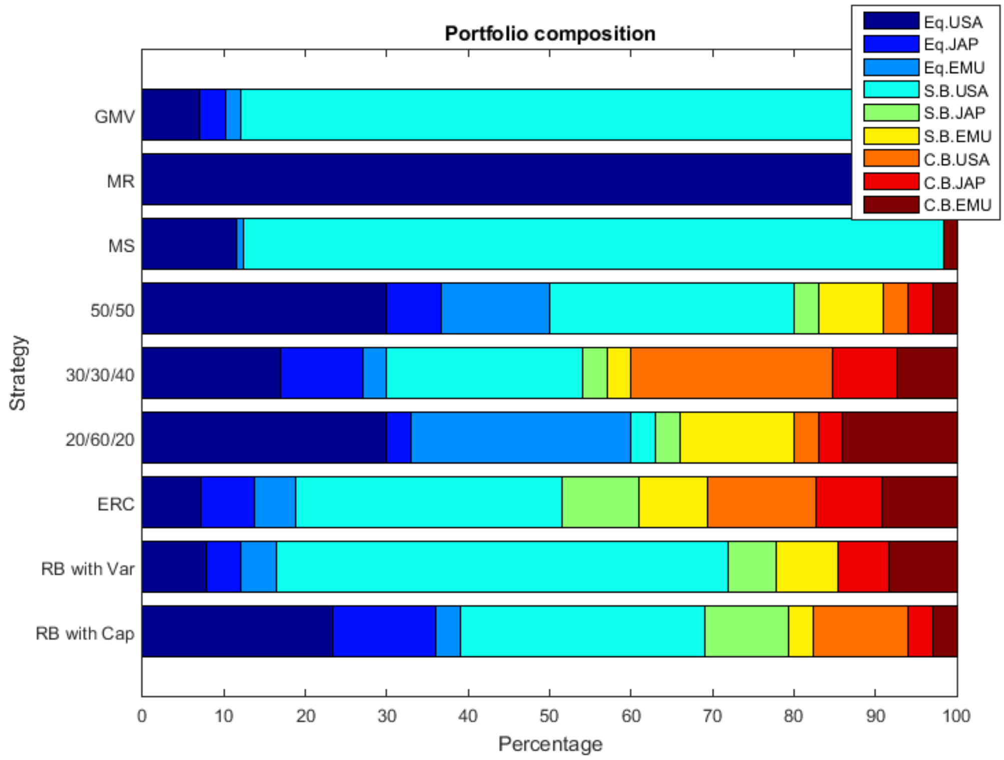

Figure 24: Portfolio composition S.*1*

We can draw already some conclusion: different strategies provide different results, especially in terms of return, where the difference between the best (MR) and the worse (30/30/40) is of 45%, and of volatility, where the difference between GMV and MR is almost 5 folds (366%). In line with Markowitz theory, we find that the portfolio with the lowest risk is the Global Minimum Variance while the one with the best mean-variance trade-off is of course the Max Sharpe.

All portfolio share a negative skewness with a pronounced kurtosis.

Table 20: Mean, standard deviation, skewness, kurtosis for the 10 portfolios S.1

Strategy	Mean	St. Deviation	Skewness	Kurtosis
GMV	0.45	1.14	-0.47	4.54
MR	0.62	5.32	-0.35	5.56
MS	0.47	1.16	-0.34	4.34
50/50	0.48	2.69	-0.36	4.94
30/30/40	0.34	1.95	-0.18	8.75
60/20/20	0.51	3.70	-0.33	4.58
ERC	0.37	1.70	-0.09	3.90
RB with VaR	0.44	1.45	-0.14	3.13
RB with Cap	0.38	2.13	-0.30	5.90

Table 21: Computation of performance, risk, diversification measures S.1

Strategy	DI	SH	VaR	SE	GI
GMV	0.433	0.453	1.657	3.140	0.731
MR	1.000	0.117	13.313	1.000	0.889
MS	0.425	0.461	1.722	3.451	0.720
50/50	0.638	0.188	5.908	2.469	0.449
30/30/40	0.467	0.262	3.293	5.304	0.422
20/60/20	0.708	0.151	7.761	2.429	0.473
ERC	0.416	0.311	2.477	9.000	0.335
RB with VaR'	0.398	0.400	1.896	7.248	0.523
RB + Cap	0.498	0.255	3.604	4.454	0.444

First thing, we can note, is that the portfolio with the lowest VaR is GMV as expected, followed by MS. ERC which gives every index the same risk exposure is only forth after RB with VaR. No surprise to see MR as the worst for risk exposure.

The most diversified portfolio is RB with VaR, very close to ERC, ERC that has the lowest level of concentration with both Shannon entropy and Gini index. The opposite is again MR that is just a one-asset portfolio.

In Table 22, I rank all the portfolios based on each single index: the result is the composite index CI.

Table 22: Computation of composite index S.1

Strategy	CI	DI	SH	VaR	SE	GI
ERC	11	2	4	4	0.5	0.5
RB with VaR	11	1	3	3	1	3
GMV	15.5	5	2	1	4	3.5
MS	16.5	6	1	2	3.5	4
RB with Cap	17.5	4	5	5	2	1.5
30/30/40	19.5	3	7	7	1.5	1
50/50	24.5	7	6	6	3	2.5
20/60/20	28.5	8	8	8	2.5	2
MR	36	9	9	9	4.5	4.5

ERC shows a high degree of diversification and equally balanced weight and risk budgets. The performance in on the average.

For RB with VaR, we have excellent diversification with quite good level of performance and low concentration.

MS and GMV excel on the performance area with a quite good level of concentration. The MR appear to be the worst under all the aspects.

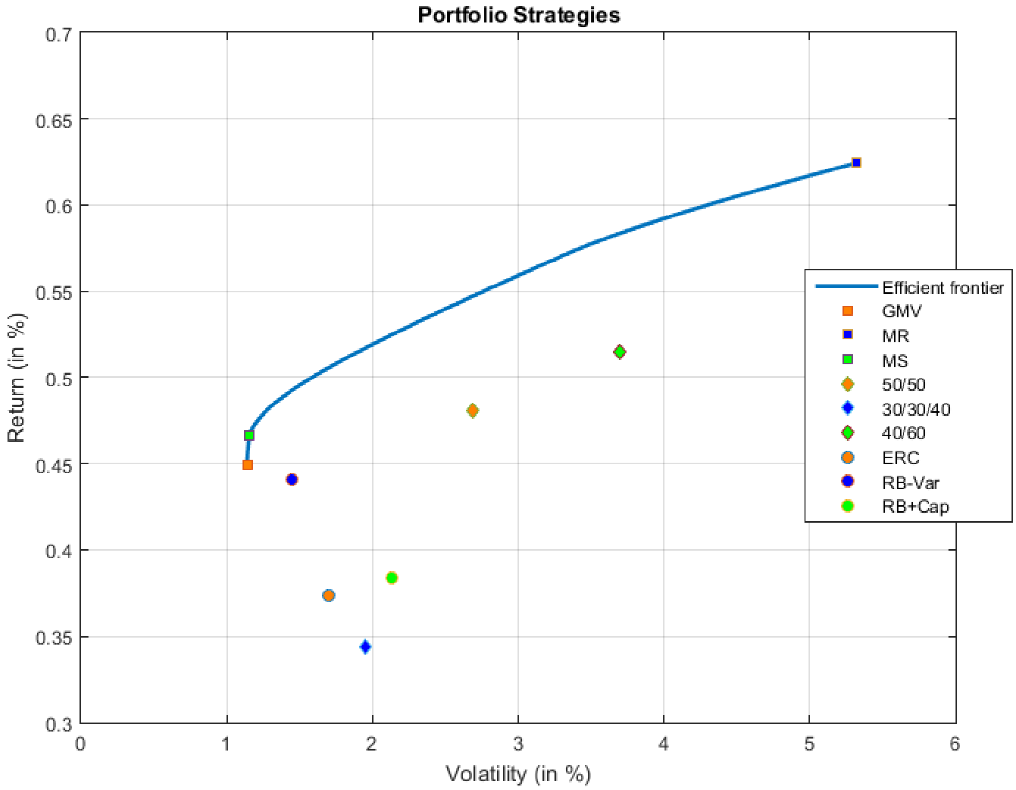

Figure 25: Portfolio strategies S.1

5.2 Scenario No.2

Table 23: Mean, median, st. deviation, minimum, maximum, skewness, kurtosis S.2

Asset	Mean (Annual)	Median	St.Deviation (Annual)	Min	Max	Skewness	Kurtosis
Eq. USA	0.625 (7.823)	1.339	5.324 (18.951)	-20.961	19.073	-0.354	5.560
Eq. JAP	0.155 (2.286)	0.094	6.032 (23.293)	-15.226	20.168	0.248	3.860

Eq. EM	0.609 (10.146)	0.698	7.287 (35.953)	-24.562	26.125	-0.064	4.057
Eq. EMU	0.566 (7.351)	1.289	6.901 (24.851)	-21.373	21.807	-0.410	3.854
SB USA	0.444 (5.434)	0.512	1.328 (4.416)	-5.098	5.667	-0.192	4.572
SB JAP	0.207 (2.498)	0.141	3.311 (11.775)	-6.801	11.832	0.363	2.864
SB EM	0.098 (1.378)	0.261	3.666 (14.621)	-26.108	14.044	-1.893	16.098
SB EMU	0.509 (6.301)	0.488	3.211 (12.197)	-7.301	8.459	0.108	2.912
CB USA	0.050 (0.382)	-0.050	2.356 (4.744)	-11.410	21.234	2.576	34.991
CB JAP	0.396 (4.482)	0.219	3.818 (10.572)	-9.944	13.498	0.208	2.973
CB EM	0.064 (1.531)	0.232	3.364 (18.637)	-22.025	11.782	-2.131	16.907
CB EMU	0.469 (6.001)	0.172	3.191 (13.864)	-8.477	10.415	0.179	3.39

Table 23 sees the addition of the indices regarding the emerging markets. We can note that the returns of SB EM and CB EM are very low, only CB USA has performed worse. At the same time, the standard deviation does not look that small to justify them: SB USA, SB JAP SB EMU and CB EMU possess a smaller volatility. Needs to be considered also the fact the EM indices have the lowest minimum value.

All these aspect do not represent a good sign and there is the risk that the EM may not be able to give a significant contribution to the process of diversification and to improve the portfolio performance.

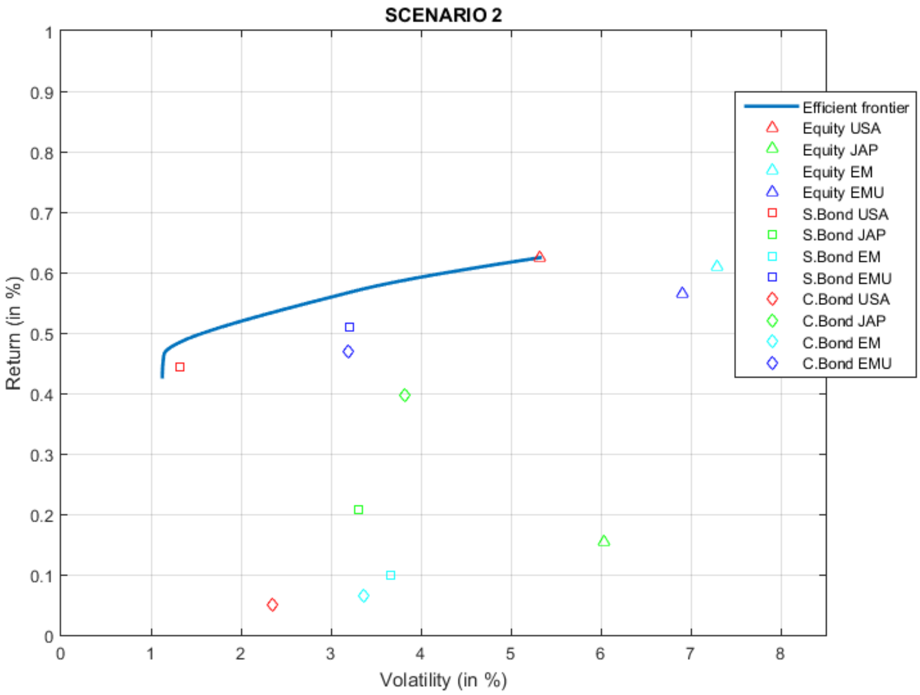

Figure 26: Efficient frontier S.2

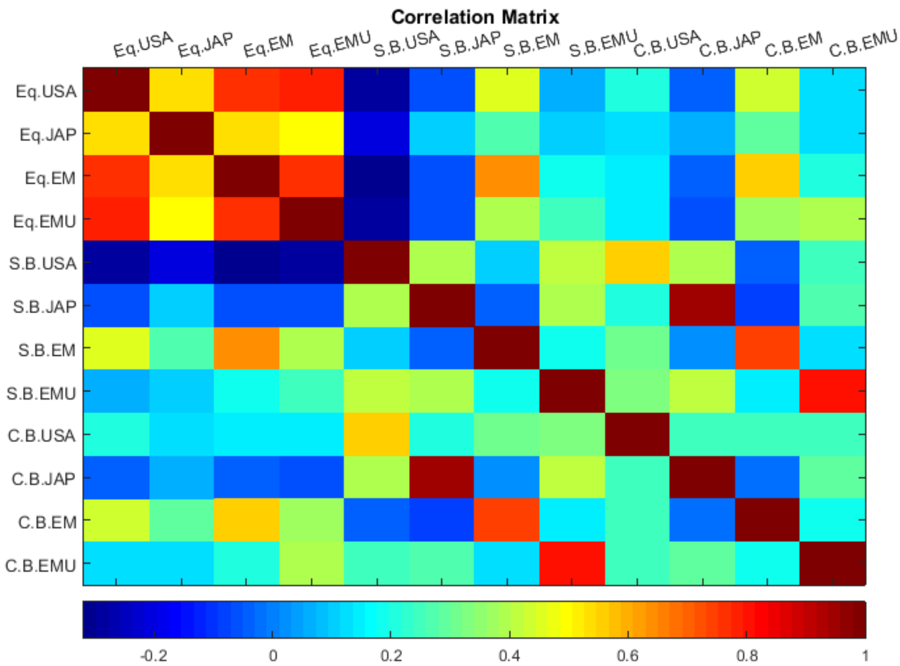

Figure 27: Correlation matrix S.2

Also from a simple graphical prospective, we can understand how poorly the EM indices performed. SB EM and CB EM belongs to those set of classes that have bad performances along with CB USA and Eq. JAP. Even Eq. EM despite its high return cannot be considered a good class given its exposure to risk, the largest among all.

From Figure 27, we can note the quite high level of correlation among the EM classes, around 0.6, with a peak of 0.73 between SB EM and CB EM.

In general, they possess high correlation with equity class, more pronounced with the USA and EMU regions. In one case, Eq. EM shows an effective negative correlation: with SB USA (-0.32).

In six cases, EM classes show a correlation comprised between -0.07 and 0: with SB JAP (-0.06, -0.04 and -0.07), with CB JAP (-0.05 and -0.03) and with SB USA (-0.04). All the others are positive.

Again, this aspect does not pose well in term of possible portfolio diversification.

The strategies for the scenario No.2 are the same of scenario No.1 except for portfolio 6 and 9. Portfolio 6 (25/75) is a different portfolio compared to the one in scenario No.1. In this case, the constraints are placed on the macro-categories: alternative and traditional assets. With this limit, I try to test what happens when more presence is given to the EM indices. The constraints then are:

a. Alternative classes must hold 25% of the portfolio, $w_A = 0.25$,
b. Sovereign bond class must hold 75% of the portfolio, $w_T = 0.75$,
c. All assets must have at least 3% and no more than 30%, $0.03 < w < 0.3$.

Portfolio 9 (RB with Cap) holds slightly different constraints than that of the scenario No.1:

a. Exposure to equity class = 70%,
b. Exposure to bond class = 30%,
c. All assets must have at least 3% and no more than 30%, $0.03 < w < 0.3$,
d. Exposure to tradition asset must be equal or lower than 60%.

Table 24: Portfolio weights S.2 and S.1

No. Port.	Case	Eq. USA	Eq. JAP	Eq. EM	Eq. EMU	SB USA	SB JAP	SB EM	SB EMU	CB USA	CB JAP	CB EM	CB EMU
1	1	7.02%	3.15%		1.92%	87.63%	0.00%		0.00%	0.00%	0.00%		0.28%
	2	5.25%	2.67%	0.70%	1.43%	84.39%	0.00%	0.00%	0.00%	0.00%	0.00%	5.57%	0.00%
2	1	100.00%	0.00%		0.00%	0.00%	0.00%		0.00%	0.00%	0.00%		0.00%
	2	100.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
3	1	11.48%	0.00%		0.91%	86.02%	0.00%		0.00%	0.00%	0.00%		1.60%
	2	9.19%	0.00%	3.10%	0.00%	86.62%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	1.09%
4	1	30.00%	6.65%		13.35%	30.00%	3.00%		8.00%	3.00%	3.00%		3.00%
	2	30.00%	3.00%	8.42%	8.58%	29.00%	3.00%	3.00%	3.00%	3.00%	3.00%	3.00%	3.00%
5	1	16.87%	10.13%		3.00%	24.00%	3.00%		3.00%	24.82%	7.80%		7.38%
	2	14.02%	9.98%	3.00%	3.00%	21.00%	3.00%	3.00%	3.00%	22.73%	8.84%	3.00%	5.43%
6	1	30.00%	3.00%		27.00%	3.00%	3.00%		14.00%	3.00%	3.00%		14.00%
	2	30.00%	3.00%	19.00%	24.00%	3.00%	3.00%	3.00%	3.00%	3.00%	3.00%	3.00%	3.00%
7	1	7.23%	6.44%		5.11%	32.80%	9.46%		8.37%	13.27%	8.12%		9.20%
	2	4.78%	4.73%	3.23%	3.54%	29.35%	8.58%	6.36%	6.82%	10.27%	7.12%	7.70%	7.52%
8	1	7.86%	4.26%		4.41%	55.47%	5.91%		7.54%	0.00%	6.17%		8.38%
	2	5.44%	3.10%	2.99%	3.17%	52.11%	5.36%	3.38%	6.46%	0.00%	5.56%	5.24%	7.20%
9	1	23.33%	12.75%		3.00%	30.00%	10.28%		3.00%	11.64%	3.00%		3.00%
	2	5.96%	10.65%	17.70%	3.00%	25.39%	3.00%	3.00%	3.00%	3.00%	3.00%	19.30%	3.00%

From Table 24, we can understand the effects of the introduction of EM classes. Eq. EM is probably the index that is more useful among the new one. It gets 0% and the minimum requirement only in two cases, in the others it receives some weights. The best performance is with portfolio 6 where it take 19% and 9 where it takes 17.70%. It performs quite well also in portfolio 4. With the Max-Sharpe portfolio, it gains 3.1% and places third, while in portfolio with risk budgeting it is the worst (3.23% and 2.99%).

SB EM is the worst among the new indices, in three cases it receives 0% and in four cases only the minimum requirement. The only two exceptions are portfolio 7 (6.36%) and 8 (3.38%) where it performs only better than Eq. EM.

CB EM get 0% in two cases and 3% in three cases, but it places second in GMV portfolio. Remarkable is also the 19.3% in the last portfolio. In general, it shows low level of risk and low correlation, being able to participate in the first portfolio but also to get a good budget in portfolio 7.

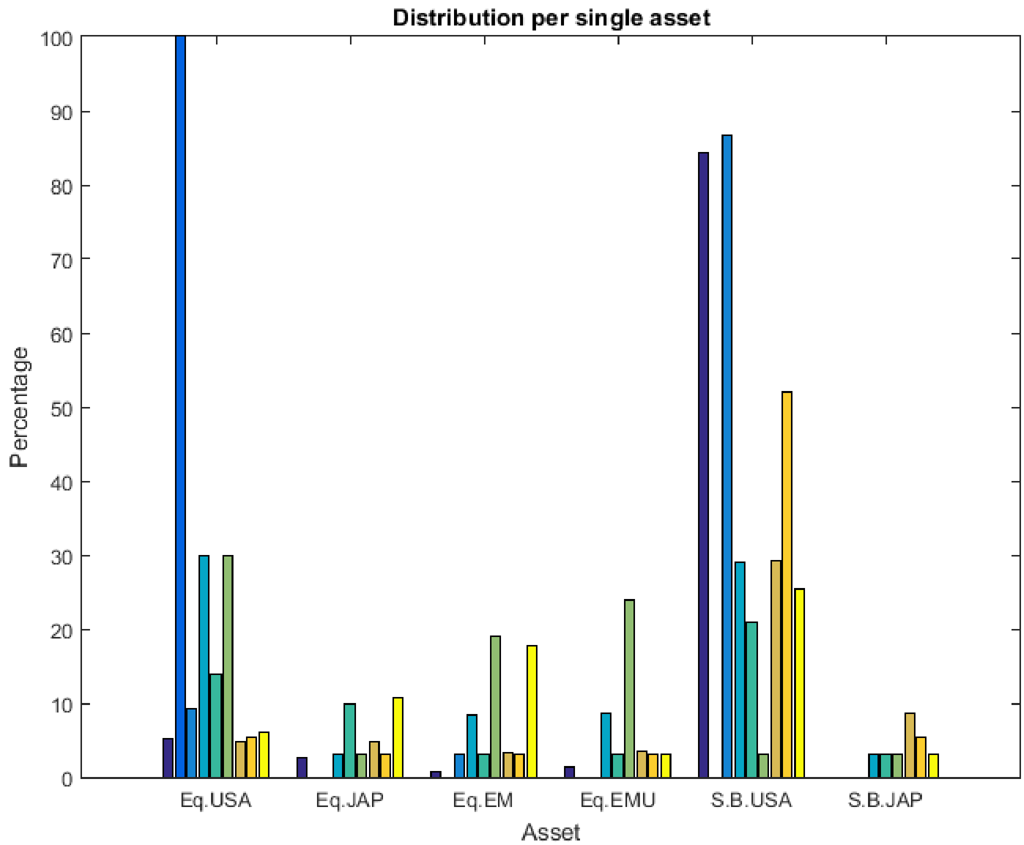

Figure 28: Distribution of single asset S.2a

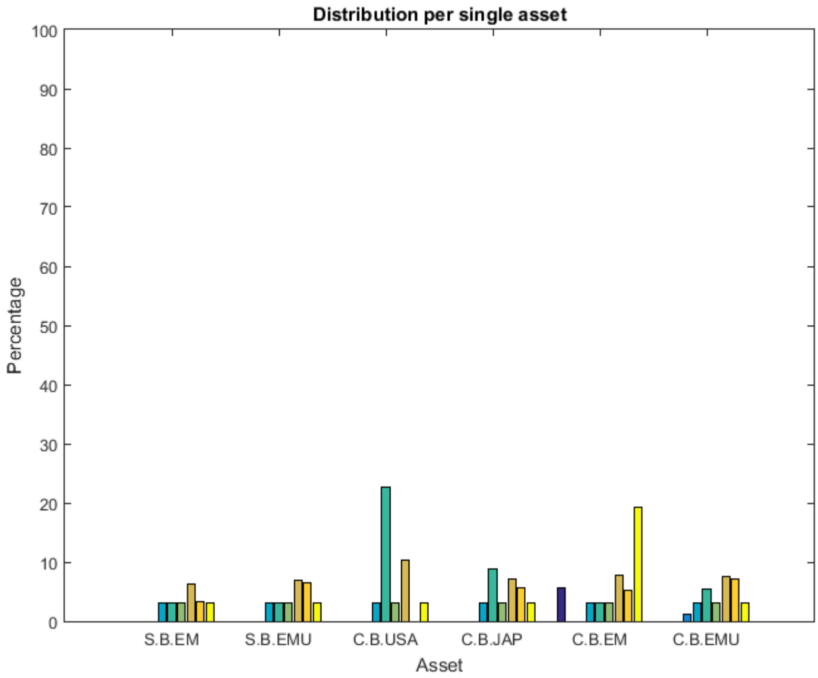

Figure 29: Distribution of single asset S.2b

Also from Figure 28 and 29, Eq. EM appears to perform slightly better than the other EM indices, but without being anything remarkable.

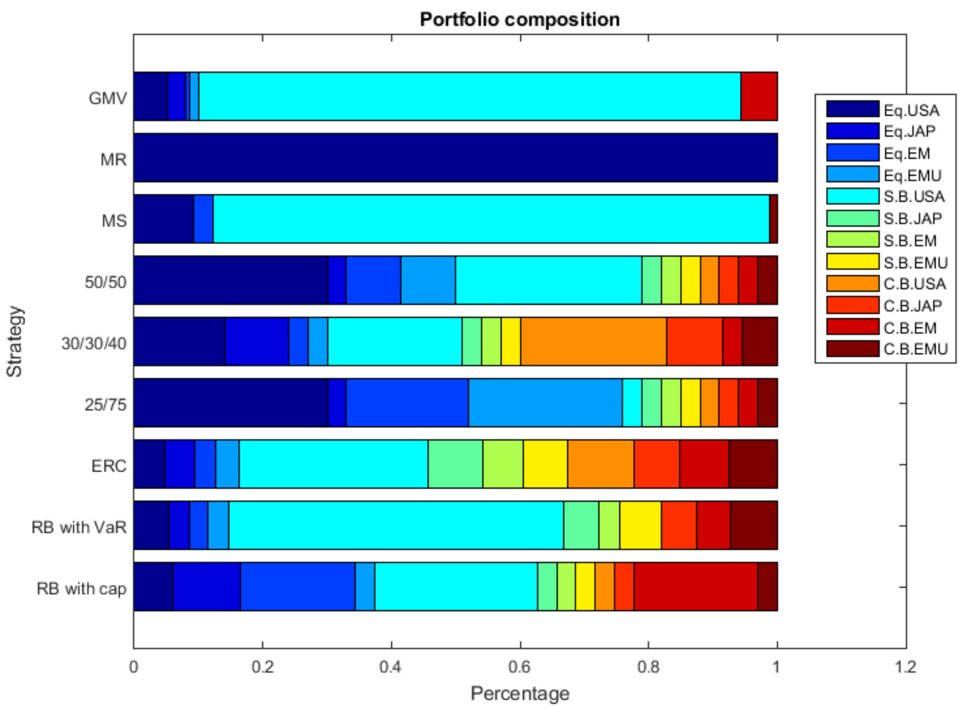

Figure 30: Portfolio composition S.2

Table 25: Mean, standard deviation, skewness, kurtosis for 10 portfolios S.2 and S.1

Strategy	Mean		St.Deviation		Skewness		Kurtosis	
	S.1	S.2	S.1	S.2	S.1	S.2	S.1	S.2
GMV	0.45	0.42	1.14	1.12	-0.47	-0.49	4.54	4.39
MR	0.62	0.62	5.32	5.32	-0.35	-0.35	5.56	5.56
MS	0.47	0.46	1.16	1.14	-0.34	-0.37	4.34	4.21
50/50	0.48	0.47	2.69	2.82	-0.36	-0.33	4.94	5.13
30/30/40	0.34	0.33	1.95	2.02	-0.18	-0.18	8.75	8.03
25/75	0.51	0.51	3.70	4.53	-0.33	-0.28	4.58	4.68
ERC	0.37	0.33	1.70	1.70	-0.09	-0.21	3.90	4.17
RB with VaR	0.44	0.41	1.45	1.43	-0.14	-0.27	3.13	3.42
RB with Cap	0.38	0.35	2.13	2.64	-0.30	-0.40	5.90	5.32

Table 26: Computing performance, risk, diversification measures for S.1 and S.2

Strategy	S.1	S.2	S.1	S.2	S.1	S.2	S.1	S.2	S.1	S.2
	DI		SH		VaR		SE		GI	
GMV	0.611	0.593	0.394	0.379	2.131	2.044	1.655	1.917	0.787	0.844
MR	1.000	1.000	0.117	0.117	13.313	13.313	1.000	1.000	0.916	0.889
MS	0.620	0.604	0.403	0.406	2.247	2.155	1.764	1.815	0.789	0.850
50/50	0.680	0.696	0.179	0.168	6.157	6.695	3.182	3.944	0.464	0.477
30/30/40	0.581	0.577	0.177	0.163	6.513	6.452	6.126	8.098	0.452	0.402
25/75	0.761	0.817	0.139	0.113	8.401	10.734	3.289	3.894	0.498	0.473
ERC	0.556	0.540	0.220	0.199	3.826	3.794	9.000	12.000	0.373	0.304
RB with VaR	0.545	0.527	0.305	0.287	2.812	2.751	7.211	9.688	0.572	0.540
RB with Cap	0.602	0.657	0.180	0.134	5.469	6.298	4.508	5.056	0.436	0.438

From Table 26, we can note that the introduction of the EM indices have contrasting result: generally they reduces the concentration, in fact SE in S.2 are higher than in S.1 and the same goes for GI in the most of the cases.

About the performance, scenario 2 appears worse than scenario 1, given the fact that SH in S.2 are lower than S.1 and vice versa for VaR.

For DI, scenario 2 shows an improvement in 50% of the cases, notably in GMV, MS, ERC and RB with VaR.

Table 27: Computation of composite index S.2

Strategy	CI	DI	SH	VaR	SE	GI
ERC	11	2	4	4	0.5	0.5
RB with VaR	11	1	3	3	1	3
GMV	14	4	2	1	3.5	3.5
MS	16	5	1	2	4	4
30/30/40	17.5	3	6	6	1.5	1
RB with Cap	21.5	6	7	5	2	1.5
50/50	23.5	7	5	7	2.5	2
25/75	30.5	8	9	8	3	2.5
MR	35	9	8	9	4.5	4.5

Table 27 provides with the ranking valuation.

Even in the second scenario, the best portfolio results the ERC followed by RB with VaR and GMV. The worst portfolio is 25/75 with the highest percentage of Eq. EM (19%).

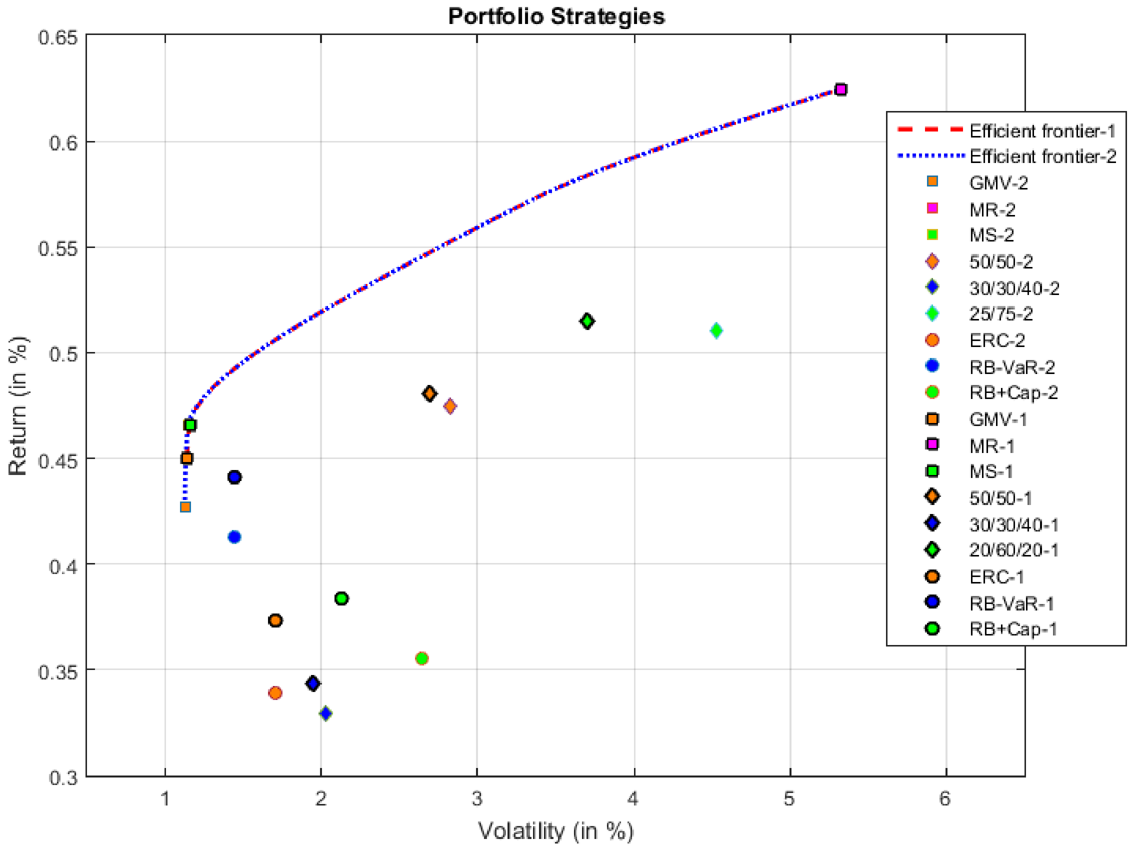

Figure 31: Comparison between S.1 and S.2 strategies

From the comparison in Figure 31, we may draw some conclusions about EM indices. As it was shown in precedent Tables, EM indices do not provide any substantial advantage in terms of return nor volatility.

The two efficient frontiers overlap each other, with a minimal difference only in the bottom: in fact, the efficient frontier-2 is slightly moved more leftward than the efficient frontier-1. This means that there is an improvement in the diversification with the introduction of the new indices.

However, as shown in Table 26, the diversification has improved only slightly: ERC, MS, GMV and RB with VaR have reduced their volatility by a minimal amount (+0.2%, -0.1%, -1.3% and -0.5% respectively) sacrificing a large portion of their return (-10%, -0%, -5% and -7% respectively).

Moreover, for the other portfolios the situation just worsens completely: their returns have diminished and their volatilities have increased.

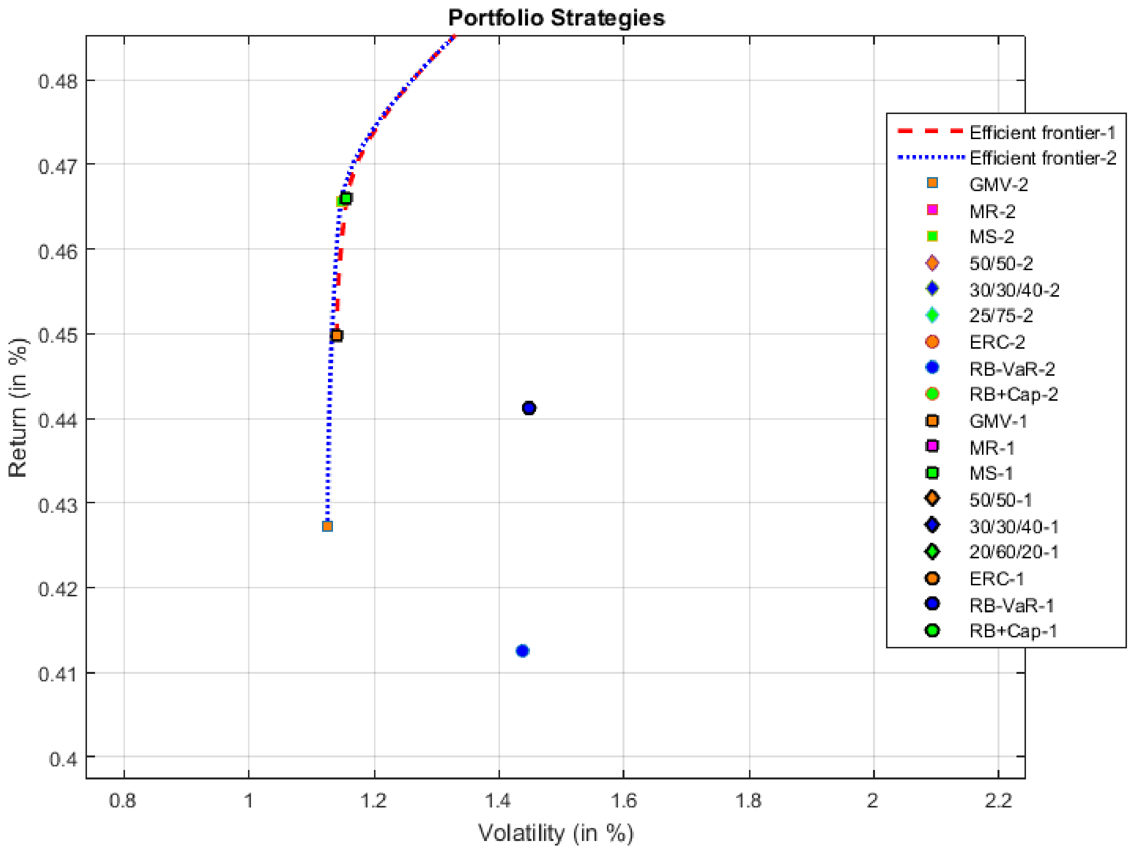

Figure 32: Comparison for GMV and MS S.1 and S.2

5.3 Scenario No.3

Scenario No.3 focuses on Hedge Fund and Private Equity.

Table 28: Mean, median, st. deviation, minimum, maximum, skewness, kurtosis S.3

Asset	Mean (Annual)	Median	St.Deviation (Annual)	Min	Max	Skewness	Kurtosis
Eq. USA	0.625 (7.823)	1.339	5.324 (18.951)	-20.961	19.073	-0.354	5.560
Eq. JAP	0.155 (2.286)	0.094	6.032 (23.293)	-15.226	20.168	0.248	3.860
Eq. EMU	0.566 (7.351)	1.289	6.901 (24.851)	-21.373	21.807	-0.410	3.854
Hedge	0.630 (7.995)	0.767	2.050 (9.685)	-8.700	7.650	-0.561	5.691
Private	0.659 (9.998)	1.135	8.840 (32.717)	-35.584	48.945	0.189	9.205
SB USA	0.444 (5.434)	0.512	1.328 (4.416)	-5.098	5.667	-0.192	4.572
SB JAP	0.207 (2.498)	0.141	3.311 (11.775)	-6.801	11.832	0.363	2.864
SB EMU	0.509 (6.301)	0.488	3.211 (12.197)	-7.301	8.459	0.108	2.912
CB USA	0.050 (0.382)	-0.050	2.356 (4.744)	-11.410	21.234	2.576	34.991
CB JAP	0.396 (4.482)	0.219	3.818 (10.572)	-9.944	13.498	0.208	2.973
CB EMU	0.469 (6.001)	0.172	3.191 (13.864)	-8.477	10.415	0.179	3.390

From Table 28, we can appreciate the figures of the new two classes: Hedge Fund has the second highest expected return (0.63%) and the second lowest volatility (2.05%). In addition, the spread between maximum and minimum value is contained (16.35).

Private Equity instead presents the highest expected return (0.659%) and highest volatility (8.84%) with the widest spread among maximum and minimum value (84.529).

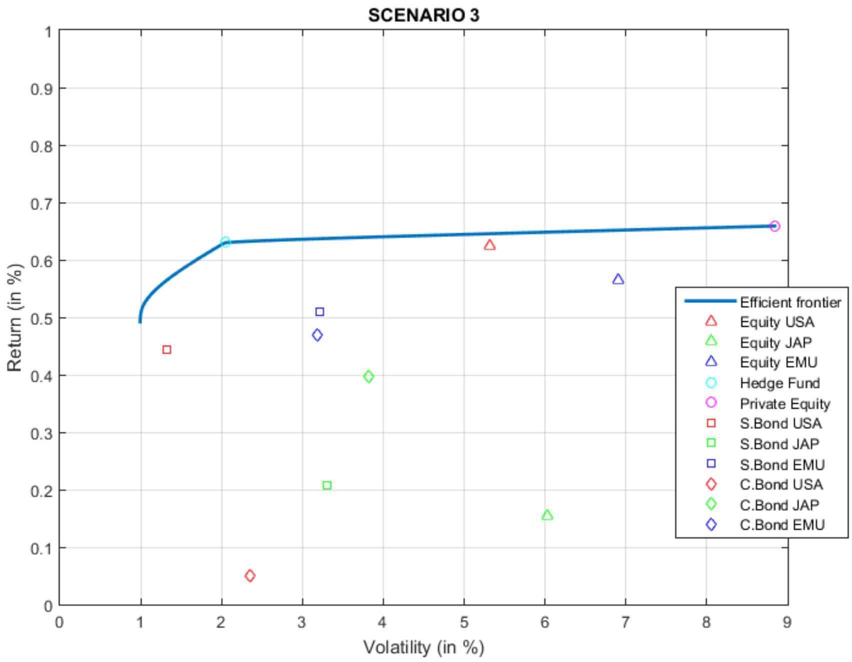

Figure 33: Efficient frontier S.3

From Figure 33, we can observe that both the new indices lie on the efficient frontier, this may be a good sign. They appear to be optimal classes as alternative classes. The portfolio weight analysis should prove this fact.

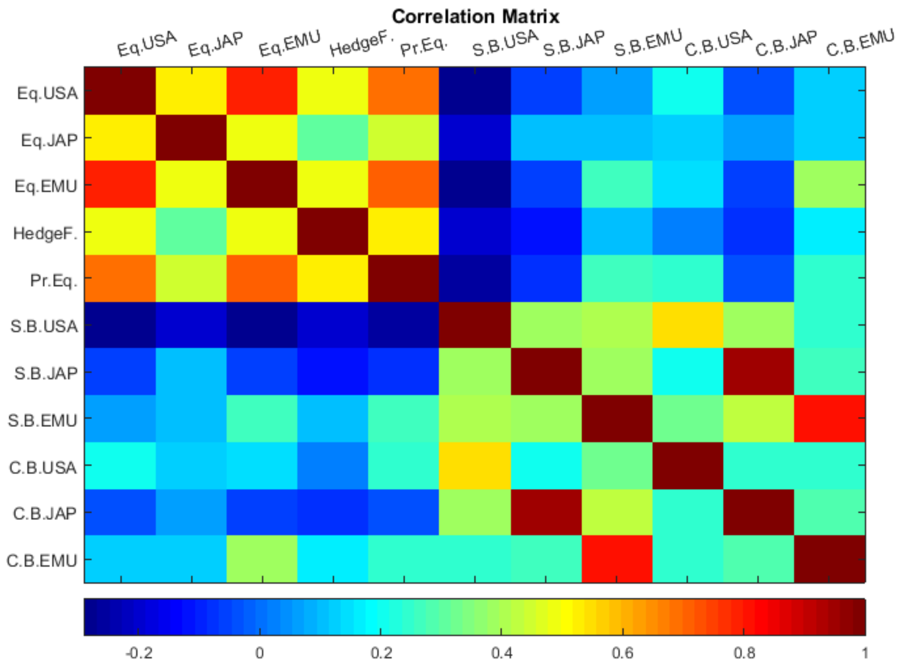

Figure 34: Correlation matrix S.3

From Figure 34, we may see that both indices have a quite positive correlation with equity class, between 0.30 and 0.49 for Hedge Fund, and between 0.44 and 0.71 for Private Equity. Moreover

Hedge Fund has negative or no correlation with the bond classes, the same goes for Private Equity with the exception of SB EMU (0.26) and CB USA (0.24).

There are little changes in the constraints for some portfolio: 50/50 portfolio is slightly different from the scenario No.1. Before, it was represented by the distinction between bond and equity classes, now the distinction is about high risk assets, namely Eq. USA, Eq. JAP, Eq. EMU, Hedge Fund and Private Equity, and low risk assets, i.e. SB USA, SB JAP, SB EMU, CB USA, CB JAP and CB EMU. Minimum and maximum requirements are the same. The process of estimation also is the same.

Portfolio No.5 (30/30/30/30) is a portfolio that gives maximum 30% to each of the classes present: equity, sovereign bond, corporate bond and alternative. This portfolio is a defensive one based on GMV estimation:

a. Alternative classes can hold at the most 30% of the portfolio, $w_A \leq 0.30$,
b. Sovereign bond class can hold at the most 30% of the portfolio, $w_S \leq 0.30$,
c. Corporate bond class can hold at the most 30% of the portfolio, $w_C \leq 0.30$
d. All assets must have at least 3% and no more than 30%, $0.03 < w < 0.3$.

RB with Cap: with this portfolio, I replicate the experiment of portfolio 4. Two different exposure: high risk and low risk, with the same participants.

a. Exposure to high risk assets = 65%
b. Exposure to low risk assets = 35%
c. All assets must have at least 3% and no more than 30%, $0.03 < w < 0.3$.

Table 29: Portfolio weights S.3 and S.1

No. Port	Case	Eq. USA	Eq. JAP	Eq. EMU	Hedge F.	Private E.	SB USA	SB JAP	SB EMU	CB USA	CB JAP	CB EMU
1	1	7.02%	3.15%	1.92%			87.63%	0.00%	0.00%	0.00%	0.00%	0.28%
	3	2.14%	1.97%	0.00%	27.52%	0.00%	67.88%	0.48%	0.00%	0.00%	0.00%	0.00%
2	1	100.00%	0.00%	0.00%			0.00%	0.00%	0,.00%	0.00%	0.00%	0.00%
	3	0.00%	0.00%	0.00%	0.00%	100.00%	0,.00%	0.00%	0.00%	0.00%	0.00%	0.00%
3	1	11.48%	0.00%	0.91%			86.02%	0.00%	0.00%	0.00%	0.00%	1.60%
	3	2.63%	0.00%	0.00%	34.28%	0.00%	63.09%	0.00%	0.00%	0.00%	0.00%	0.00%
4	1	30.00%	6.65%	13.35%			30.00%	3.00%	8.00%	3.00%	3.00%	3.00%
	3	14.00%	3.00%	3.00%	27.00%	3.00%	24.00%	3.00%	3.00%	3.00%	9.24%	7.76%
5	1	16.87%	10.13%	3.00%			24.00%	3.00%	3.00%	24.82%	7.80%	7.38%
	3	3.00%	4.00%	3,.00%	27.00%	3.00%	24.00%	3.00%	3.00%	20.41%	6.59%	3.00%
6	1	30.00%	3.00%	27.00%			3.00%	3.00%	14.00%	3.00%	3.00%	14.00%
	3	30.00%	3.00%	24.00%	3.00%	22.00%	3.00%	3.00%	3.00%	3.00%	3.00%	3.00%
7	1	7.23%	6.44%	5.11%			32.80%	9.46%	8.37%	13.27%	8.12%	9.20%
	3	4.80%	4.71%	3.42%	15.38%	2.74%	29.78%	8.22%	6.51%	10.44%	6.84%	7.17%
8	1	7.86%	4.26%	4.41%			55.47%	5.91%	7.54%	0.00%	6.17%	8.38%
	3	4.26%	2.67%	2.44%	22.46%	1.52%	45.86%	4.89%	5.27%	0.00%	4,.81%	5.81%
9	1	23.33%	12.75%	3.00%			30.00%	10.28%	3.00%	11.64%	3.00%	3.00%
	3	3.00%	3.00%	3.00%	26.09%	3.00%	30.00%	9.17%	3.00%	13.73%	3.00%	3.00%

In general, Hedge Fund takes part in almost every portfolio even as an important part. In the GMV, it gets 27.52%, being second only to SB USA, same goes for MS with 34.28%. In portfolio 4 and 5, it gets 27%, the highest presence. The only exception is portfolio 6 where it obtains only the minimum requirement. In addition, it performs well in risk budget-based portfolios with 15.38%, 22.46% and 26.09%.

On the other side, Private Equity performs the opposite: it is the only asset for portfolio MR, given its highest return, and the one with the most weight in portfolio 6 (based again on MR optimization). In three time it gets only the minimum requirement and in two times nothing. From Figure 35 and 36, we can visualize the weight that each index has in all portfolios. Hedge Fund gains weight in almost all portfolio, together with SB USA. In addition, Private Equity is present but with a very low percentage, except in those portfolios derived with MR estimation.

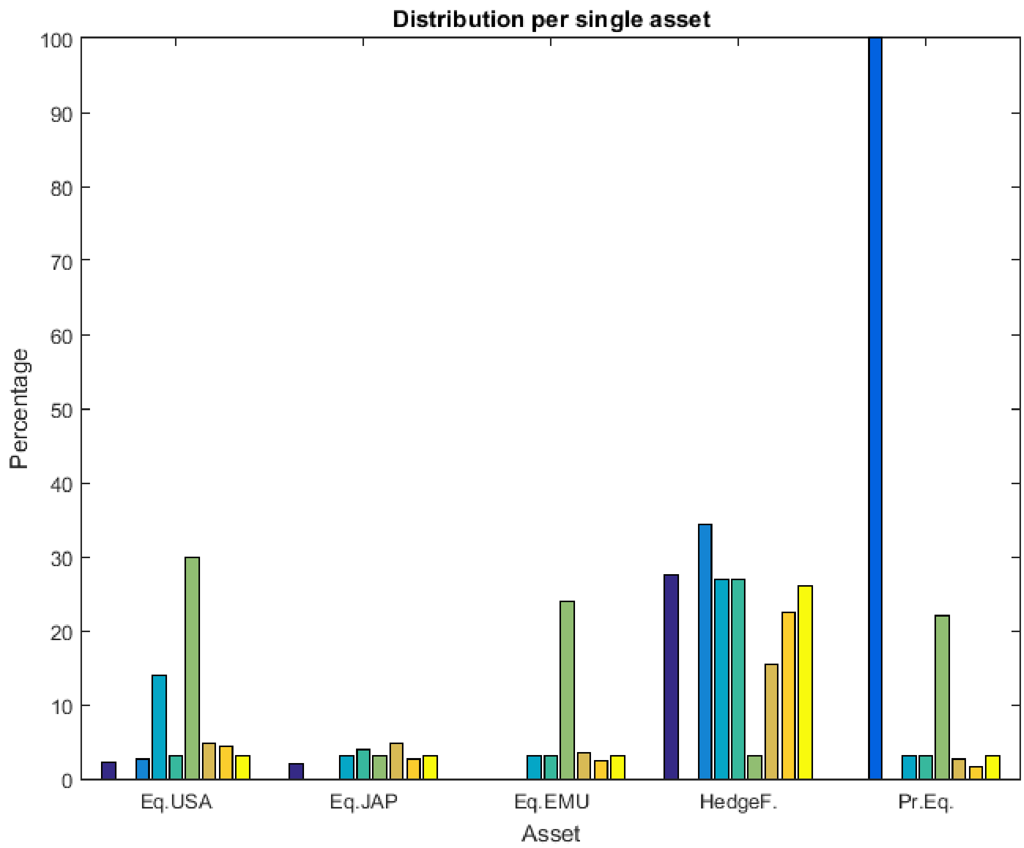

Figure 35: Distribution of single asset S.3a

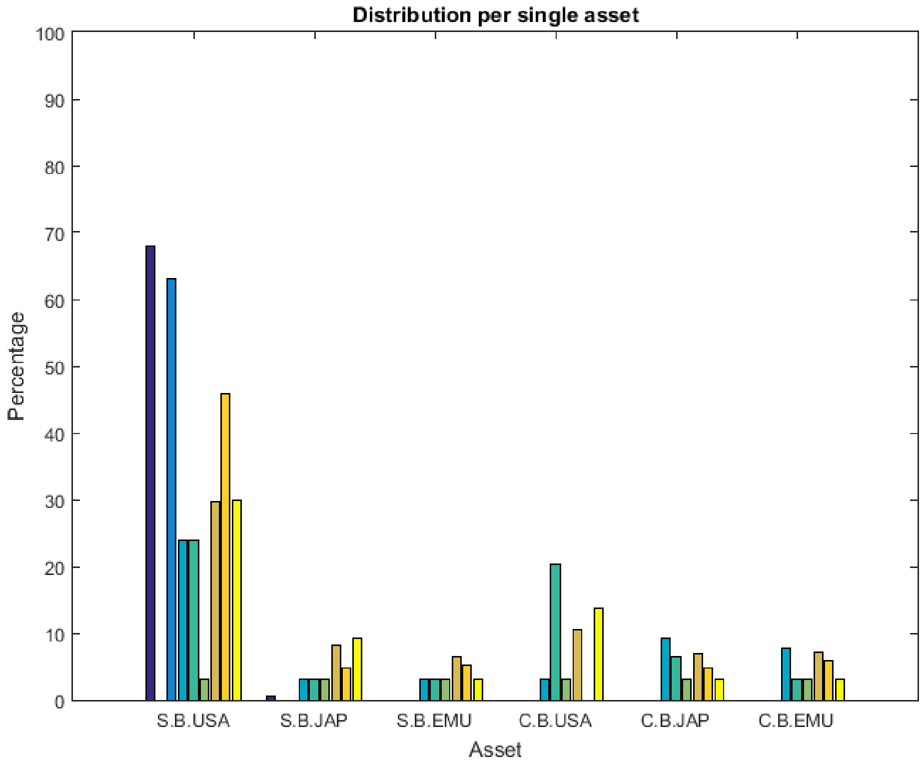

Figure 36: Distribution of single asset S.3b

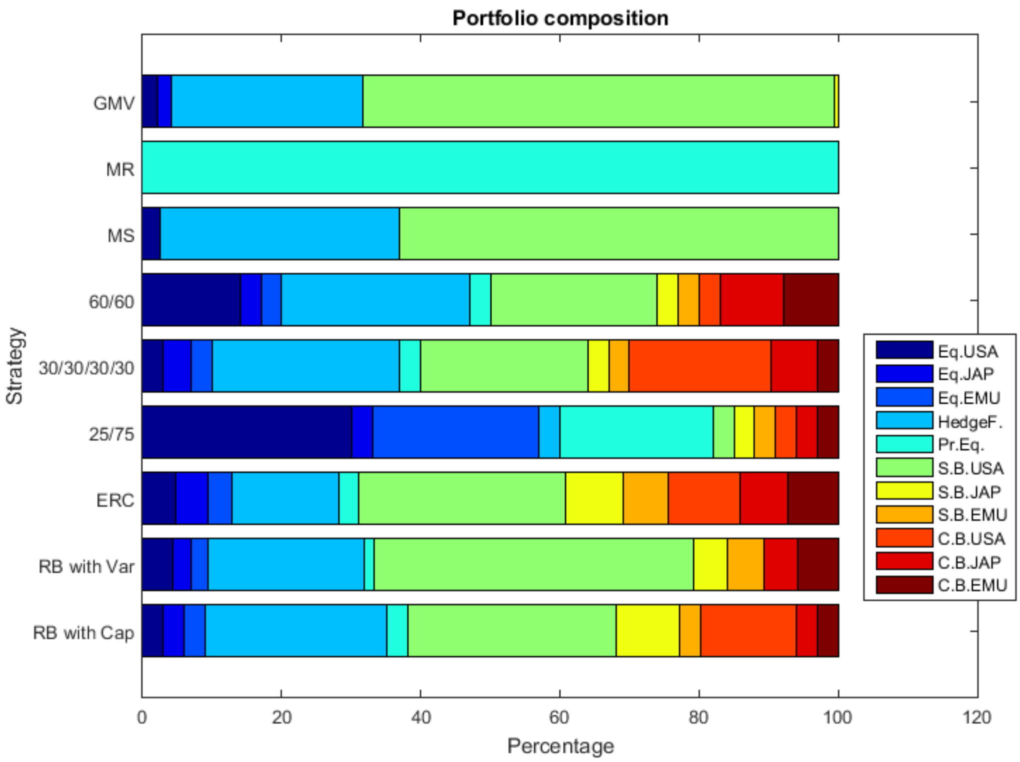

Figure 37: Portfolio composition S.3

What said for Figure 35 and 36 can be said for Figure 37. It is evident that beyond SB USA, Hedge Fund has a large presence in more or less all portfolios. Private Equity exceeds only in two strategy.

Table 30: Mean, st. deviation, skewness, kurtosis for 10 portfolios S.3 and S.2

Strategy	Mean		St.Deviation		Skewness		Kurtosis	
	S.1	S.3	S.1	S.3	S.1	S.3	S.1	S.3
GMV	0.45	0.492	1.14	0.994	-0.47	-0.401	4.54	4.067
MR	0.62	0.659	5.32	8.840	-0.35	0.189	5.56	9.205
MS	0.47	0.512	1.16	1.010	-0.34	-0.366	4.34	3.997
50/50	0.48	0.501	2.69	1.775	-0.36	-0.454	4.94	4.622
30/30/40	0.34	0.410	1.95	1.494	-0.18	-0.170	8.75	7.231
25/75	0.51	0.554	3.70	4.880	-0.33	-0.195	4.58	6.285
ERC	0.37	0.420	1.70	1.557	-0.09	-0.203	3.90	4.256
RB with VaR	0.44	0.483	1.45	1.261	-0.14	-0.315	3.13	3.535
RB with Cap	0.38	0.425	2.13	1.403	-0.30	-0.247	5.90	5.161

Table 31: Computation of performance, risk, diversification measure S.3

Strategy	S.1	S.3	S.1	S.3	S.1	S.3	S.1	S.3	S.1	S.3
	DI		SH		VaR		SE		GI	
GMV	0.611	0.580	0.394	0.495	2.131	1.715	1.655	2.234	0.787	0.837
MR	1.000	1.000	0.117	0.075	13.313	33.840	1.000	1.000	0.916	0.909
MS	0.620	0.601	0.403	0.507	2.247	1.749	1.764	2.236	0.789	0.837
50/50	0.680	0.566	0.179	0.282	6.157	3.570	3.182	7.211	0.464	0.462
30/30/40	0.581	0.540	0.177	0.274	6.513	4.340	6.126	8.833	0.452	0.481
25/75	0.761	0.819	0.139	0.113	8.401	13.778	3.289	3.586	0.498	0.502
ERC	0.556	0.529	0.220	0.270	3.826	3.403	9.000	11.000	0.373	0.378
RB with VaR	0.545	0.512	0.305	0.383	2.812	2.269	7.211	8.713	0.572	0.598
RB with Cap	0.602	0.524	0.180	0.303	5.469	3.262	4.508	9.435	0.436	0,.494

Table 32: Computation the composite index S.3

Strategy	CI	DI	SH	VaR	SE	GI
RB with VaR	12	1	3	3	2	3
RB with Cap	13	2	4	4	1	2
ERC	16	3	7	5	0.5	0.5

GMV	16.5	6	2	1	4	3.5
MS	17.5	7	1	2	3.5	4
50/50	19.5	5	5	6	2.5	1
30/30/30/30	20	4	6	7	1.5	1.5
25/75	29.5	8	8	8	3	2.5
MR	36	9	9	9	4.5	4.5

Table 30, 31 and 32 provide with some results about diversification and performance. RB with VaR results the best choice considering all the possible valuation, also RB with Cap shows good results.

In general, the figures from scenario No.3 result better than scenario No.1, the diversification increases together with the mean-variance trade-off. Both VaR and asset concentration decrease. Only MR and 25/75 hold a behaviour with opposite results: no wonder they rank as the last two.

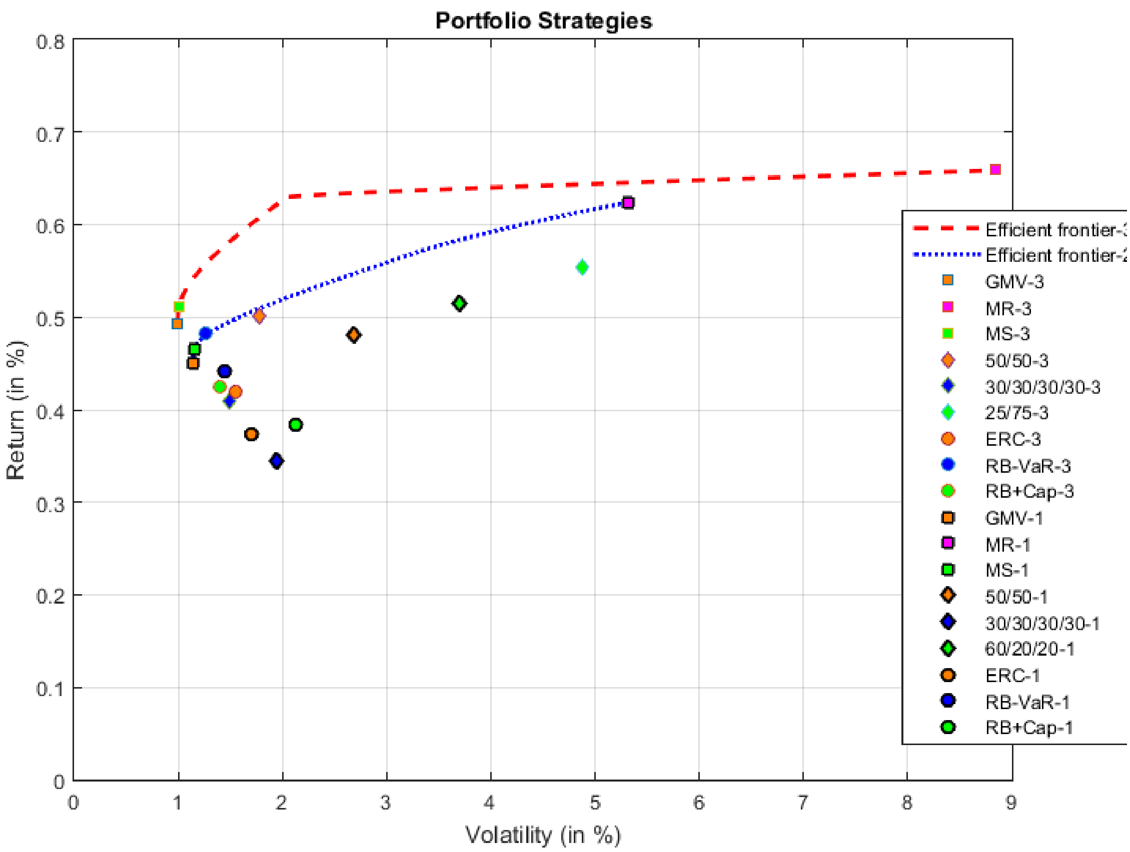

Figure 38: Comparison between S.1 and S.3 strategies

Figure 38 show the effect of introducing the Private Equity and above all the Hedge Fund.

The efficient frontier has moved clearly up and leftward, meaning a true improvement of the portfolio optimization, thanks to a valid diversification provided by the two new indices. MS and GMV from scenario No.3 provide with better performances than their counterparts of scenario No.1 do, with an increase of 8% in return and a reduction of 14% in volatility. Moreover all the other portfolios from scenario No.3 result better or equal (25/75 and MR) to those of scenario No.1 in terms of return and volatility, being placed upper and more to the left. This result may be considered as a first proof of the utility of Hedge Fund as alternative asset class to invest in. Further investigation is necessary about Private Equity, since the worst performing portfolio are those with an elevate participation of it, i.e. MR and 25/75.

5.4 Scenario No.4

In scenario No.4, I introduce the Commodity and Real Estate and analyse their impact on the strategies. This part presents the same portfolios of scenario No.3.

Table 33: Mean, median, st. deviation, minimum, maximum, skewness, kurtosis S.4

Asset	Mean (Annual)	Median	St.Deviation (Annual)	Min	Max	Skewness	Kurtosis
Eq. USA	0.625 (7,823)	1.339	5.324 (18.951)	-20.961	19.073	-0.354	5.560
Eq. JAP	0.155 (2,286)	0.094	6.032 (23.293)	-15.226	20.168	0.248	3.860
Eq. EMU	0.566 (7.351)	1.289	6.901 (24.851)	-21.373	21.807	-0.410	3.854
Commodity	0.255 (3.901)	0.741	6.898 (27.220)	-25.997	16.645	-0,.512	4.123
Real Estate	0.367 (4.944)	0.982	6.750 (24.264)	-24.706	27.204	-0,.272	5.755
SB USA	0.444 (5.434)	0.512	1.328 (4.416)	-5.098	5.667	-0.192	4.572
SB JAP	0,207 (2.498)	0.141	3.311 (11.775)	-6.801	11.832	0.363	2.864
SB EMU	0.509 (6.301)	0.488	3.211 (12.197)	-7.301	8.459	0.108	2.912
CB USA	0,050	-0,050	2,356	-11,410	21,234	2,576	34,991

	(0,382)		(4,744)				
CB JAP	0,396 (4,482)	0,219	3,818 (10,572)	-9,944	13,498	0,208	2,973
CB EMU	0,469 (6,001)	0,172	3,191 (13,864)	-8,477	10,415	0,179	3,390

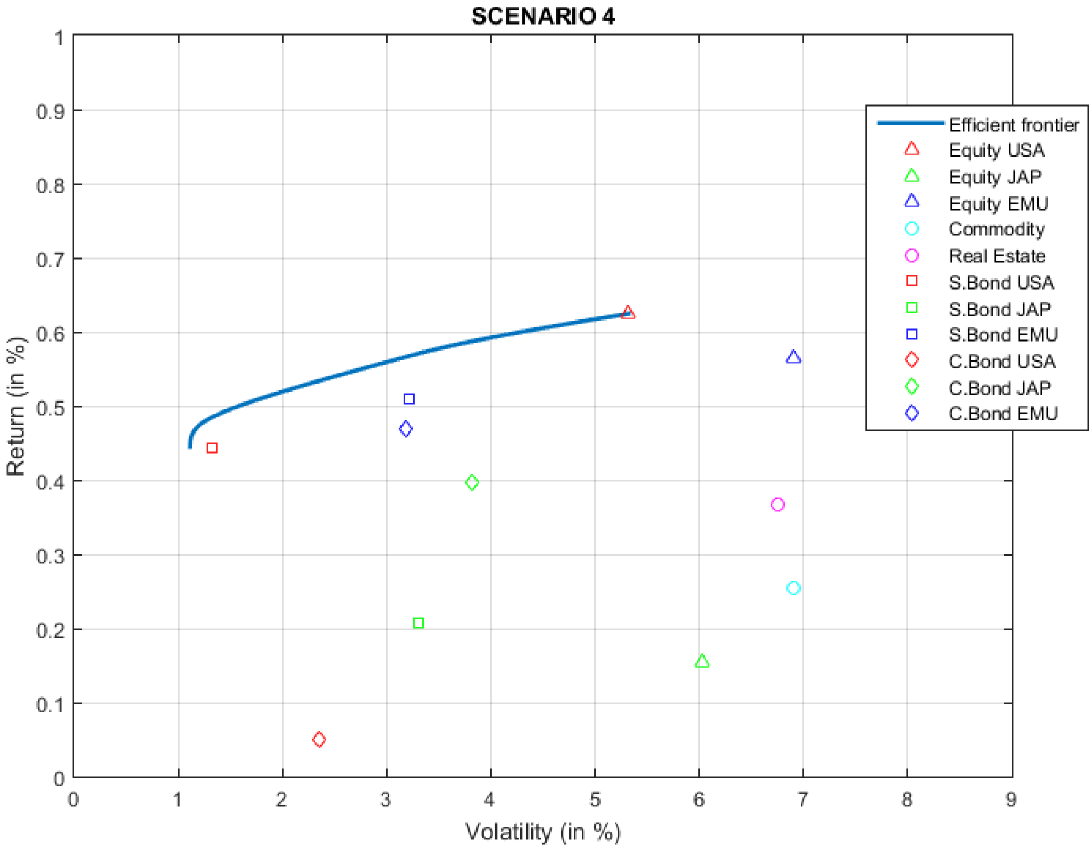

Figure 39: Efficient frontier S.4

Both Commodity and Real Estate are not optimal indices, as Figure 39 shows. They both have a very high risk (6.898 and 6.750), second only to Eq. EMU (6.901) and not a proportional high expected return (0.255 and 0.367). Six indices perform better than they do. As Figure 39 shows, they are placed on the bottom right corner of the risk-return plane, in one of the less optimal position with respect to the other indices.

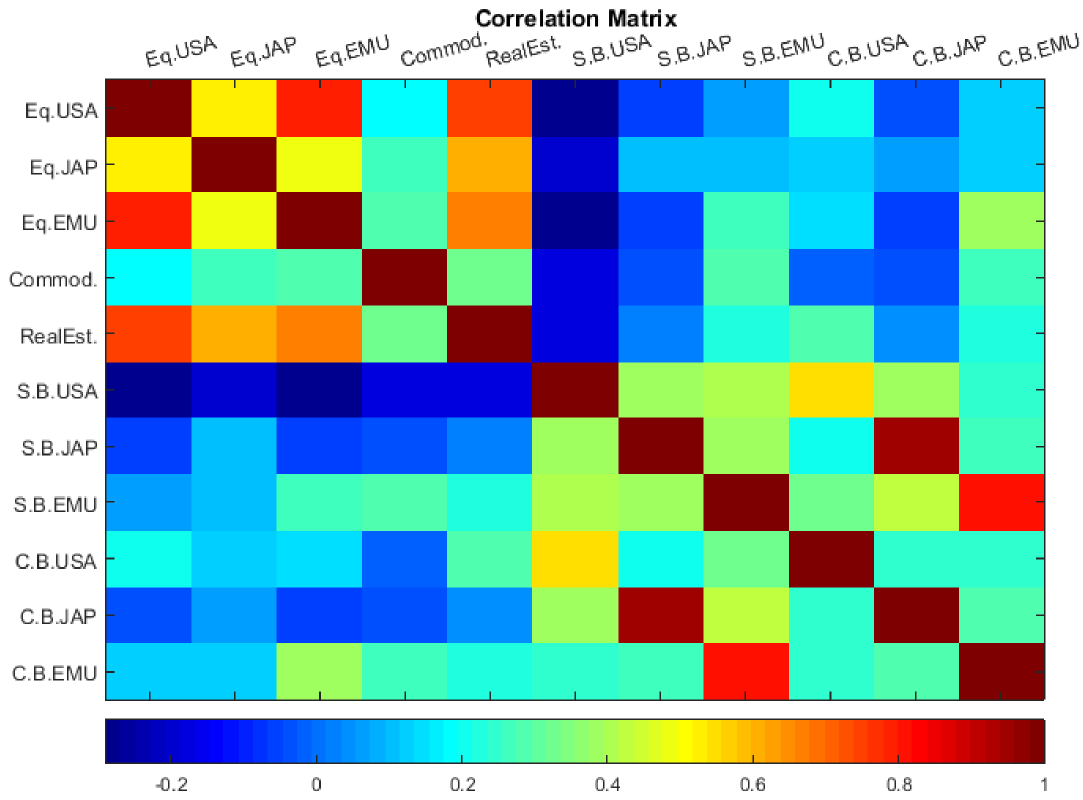

Figure 40: ***Correlation matrix S.4***

From Figure 40, we can note that Commodity has very low positive correlation, also with equity class, practically almost always below 0.3 (0.187, 0.268, 0.293) and in the half of the cases it is negative (-0.179, -0.045, -0.014, -0.028).

Real Estate, instead, shows a strong positive correlation with equity class (between 0.71 and 0.64), while with the other classes it shows low correlation in general (between 0.328 and -0.177).

Table 34: Portfolio weights S.4 and S.1

No. Port	Case	Eq. USA	Eq. JAP	Eq. EMU	Commod.	Real Est.	SB USA	SB JAP	SB EMU	CB USA	CB JAP	CB EMU
1	S.1	7.02%	3.15%	1.92%			87.63%	0.00%	0.00%	0.00%	0.00%	0.28%
	S.4	7.63%	2.16%	0.69%	3.76%	0.00%	85.76%	0.00%	0.00%	0.00%	0.00%	0.00%
2	S.1	100.00%	0.00%	0.00%			0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	S.4	100.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
3	S.1	11.48%	0.00%	0.91%			86.02%	0.00%	0.00%	0.00%	0.00%	1.60%
	S.4	11.12%	0.00%	0.48%	2.69%	0.00%	85.71%	0.00%	0.00%	0.00%	0.00%	0.00%
4	S.1	30.00%	6.65%	13.35%			30.00%	3.00%	8.00%	3.00%	3.00%	3.00%
	S.4	24.00%	3.00%	3.00%	16.79%	3.21%	24.00%	3.00%	3.00%	3.00%	10.80%	6.20%
5	S.1	16.87%	10.13%	3.00%			24.00%	3.00%	3.00%	24.82%	7.80%	7.38%
	S.4	15.46%	6.97%	3.00%	11.57%	3.00%	24.00%	3.00%	3.00%	19.39%	7.61%	3.00%
6	S.1	30.00%	3.00%	27.00%			3.00%	3.00%	14.00%	3.00%	3.00%	14.00%
	S.4	19.00%	3.00%	3.00%	3.00%	22.00%	3.00%	3.00%	24.00%	3.00%	3.00%	14.00%
7	S.1	7.23%	6.44%	5.11%			32.80%	9.46%	8.37%	13.27%	8.12%	9.20%
	S.4	5.75%	5.04%	4.04%	5.57%	3.67%	33,.05%	8.83%	7.01%	11.78%	7.49%	7.76%
8	S.1	7.86%	4.26%	4.41%			55.47%	5.91%	7.54%	0.00%	6.17%	8.38%
	S.4	6.51%	3.25%	3.57%	4.18%	2.43%	55.37%	5.42%	6.41%	0.00%	5.69%	7.18%
9	S.1	23.33%	12.75%	3.00%			30.00%	10.28%	3.00%	11.64%	3.00%	3.00%
	S.4	15.14%	5.96%	3.00%	10.77%	3.00%	30.00%	10.13%	3.00%	13.00%	3.00%	3.00%

Between Commodity and Real Estate, the first has a better impact on the portfolios. This may appears strange, since Real Estate has higher return and lower volatility. The crucial point is the correlation matrix; in fact, Commodity has generally a better correlation than Real Estate has.

Commodity is almost always present, with the only exception in MR portfolio. It represents the third weight in the first, third and fourth portfolio. The worst performances are in the second and sixth portfolio, both with an MR optimization.

For the risk budget-based portfolios, the high volatility penalizes Commodity. Real Estate performs poorly, getting no weight in the first three portfolios and the minimum in the fifth and in the last one.

Poor results also with ERC, RB with VaR and RB with Cap.

The only exception is portfolio 6, where Real Estate gets 22%.

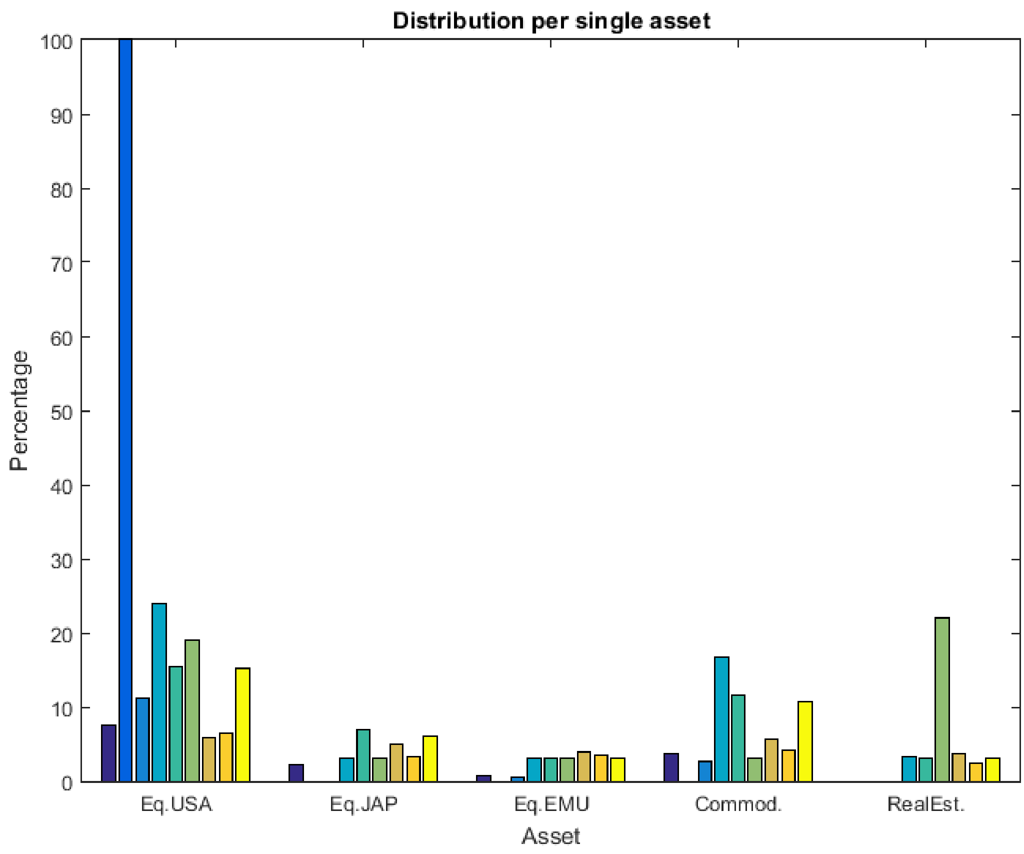

Figure 41: Distribution of single index S.4a

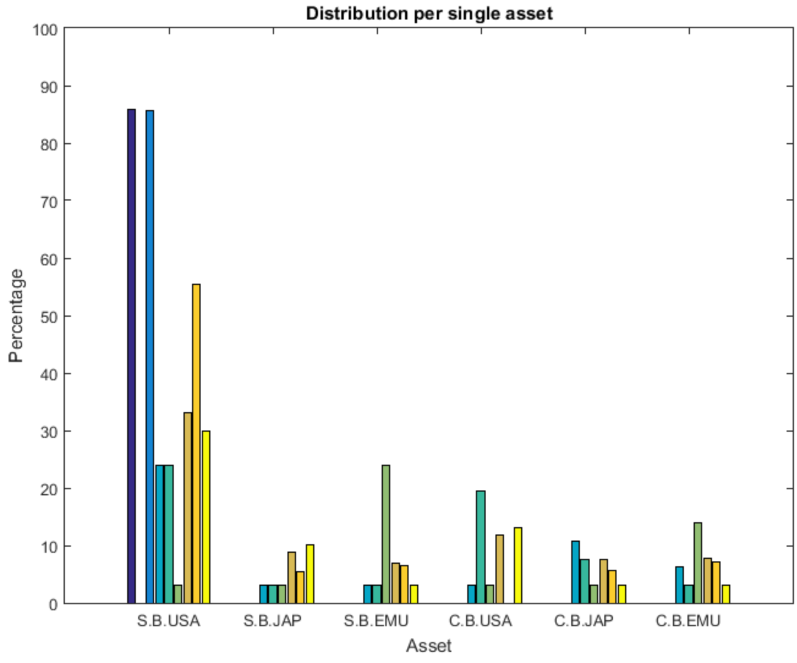

Figure 42: Distribution of single asset S.4b

From Figure 41 and 42, we can see that Eq. USA and SB USA are the indices more employed. Commodity overall performs a bit better that Real Estate on the average. The level of usage of these alternative indices can be appreciated also from Figure 43.

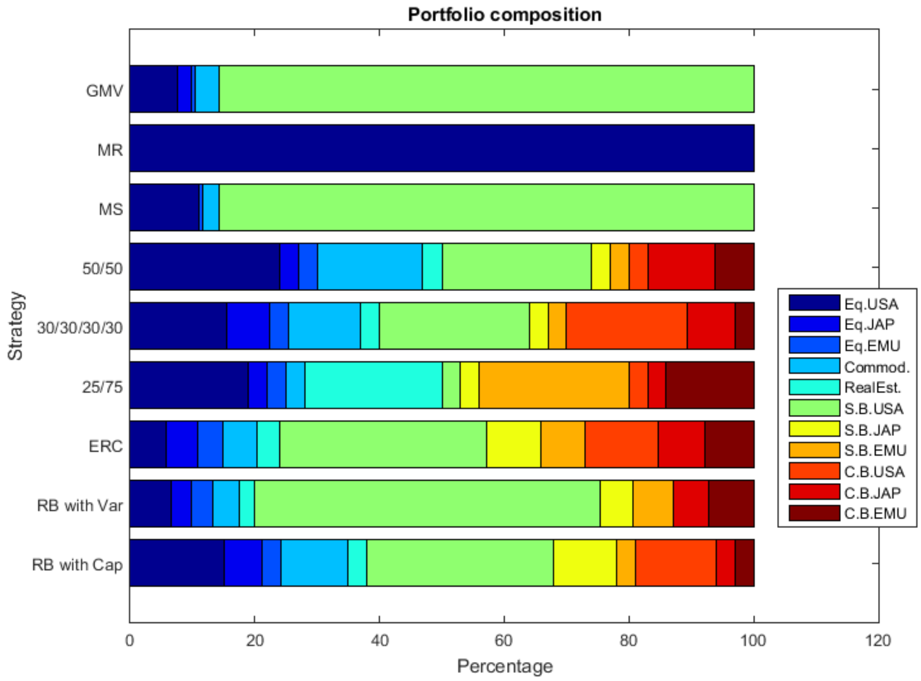

Figure 43: Portfolio composition S.4

Table 35: Mean, st. deviation, skewness, kurtosis S.4 and S.1

Strategy	Mean		St.Deviation		Skewness		Kurtosis	
	S.1	S.4	S.1	S.4	S.1	S.4	S.1	S.4
GMV	0.450	0.445	1.141	1.114	-0.467	-0.544	4.542	4.345
MR	0.625	0.625	5.324	5.324	-0.354	-0.354	5.560	5.560
MS	0.466	0.459	1.157	1.127	-0.340	-0.441	4.336	4.194
50/50	0.481	0.427	2.689	2.437	-0.358	-0.530	4.941	4.702
30/30/30/30	0.344	0.347	1.947	2.090	-0.185	-0.381	8.750	7.122
25/75	0.515	0.449	3.700	3.214	-0.333	-0.222	4.581	4.926
ERC	0.374	0.367	1.701	1.735	-0.088	-0,.220	3.901	4.086
RB with VaR	0.441	0.431	1.447	1.446	-0.137	-0.274	3.127	3.175
RB with Cap	0.384	0.361	2.128	1.960	-0.301	-0.399	5.905	5.705

Table 36: Computation of performance, risk, diversification measures S.4

Strategy	S.1	S.4	S.1	S.4	S.1	S.4	S.1	S.4	S.1	S.4
	DI		SH		VaR		SE		GI	
GMV	0.611	0.562	0.394	0.399	2.131	1.927	1.655	1.766	0.787	0.865
MR	1.000	1.000	0.117	0.117	13.313	13.313	1.000	1.000	0.916	0.909
MS	0.620	0.578	0.403	0.407	2.247	2.030	1.764	1.708	0.789	0.876
50/50	0.680	0.575	0.179	0.175	6.157	5.004	3.182	5.118	0.464	0.453
30/30/30/30	0.581	0.549	0.177	0.166	6.513	5.831	6.126	7.423	0.452	0.418
25/75	0.761	0.694	0.139	0.140	8.401	7.890	3.289	5.175	0.498	0.456
ERC	0.556	0.528	0.220	0.211	3.826	3.799	9.000	11.000	0.373	0.355
RB with VaR	0.545	0.515	0.305	0.298	2.812	2.660	7.211	8.859	0.572	0.569
RB with Cap	0.602	0.535	0.180	0.184	5.469	4.744	4.508	7.772	0.436	0.429

Table 37: Computation of composition index S.4

Strategy	CI	DI	SH	VaR	SE	GI
ERC	11	2	4	4	0.5	0.5
RB with VaR	11	1	3	3	1	3
GMV	15	5	2	1	3.5	3.5
RB with Cap	16	3	5	5	1.5	1.5
MS	18	7	1	2	4	4
30/30/30/30	21	4	7	7	2	1

50/50	23	6	6	6	3	2
25/75	29	8	8	8	2.5	2.5
MR	36	9	9	9	4.5	4.5

From Table 35, we can note that in almost all the portfolios, both the mean and the standard deviation diminish. The only exception is 30/30/30/30 portfolio that has also very high skewness and kurtosis.

From Table 36, we observe that the diversification improves, notably the values decrease. For the Sharpe ratio, the situation in contrasting: for GMV, MS, 25/75 and RB with Cap it increases while for the others decreases. All the VaR figures improve as well as all concentration ones. ERC and RB with VaR result the best portfolios above all.

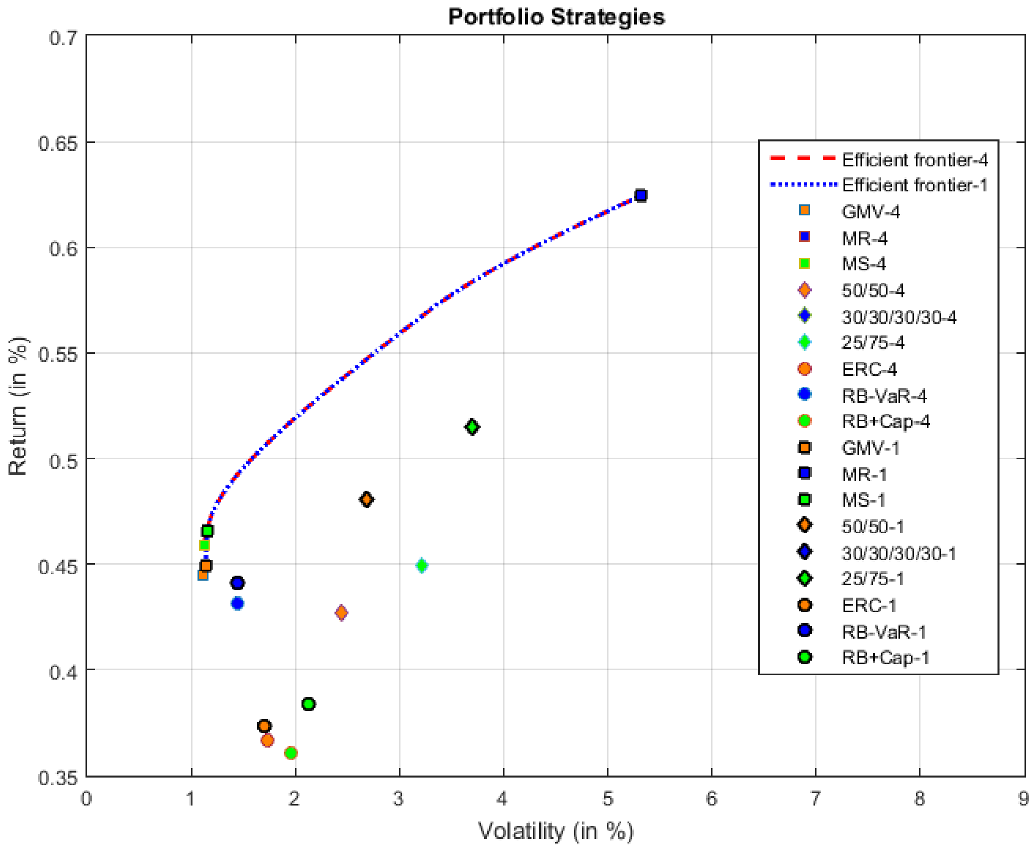

Figure 44: Comparison between S.1 and S.4 strategies

From Figure 44, we comprehend that with Commodity and Real Estate I obtain the same results as I did with the EM classes.

There is no evident change in the two efficient frontiers, except for the lower part. The efficient frontier-4 is slightly more on the left, sign of a diversification but still very little to be considered significant.

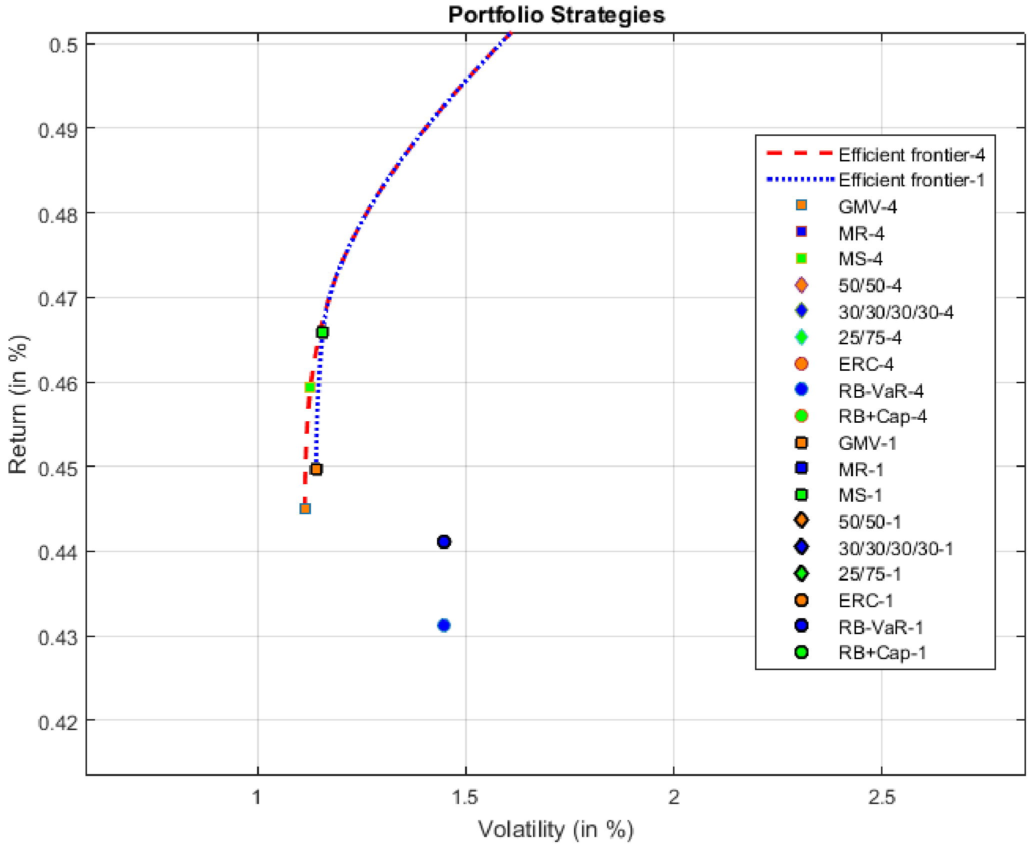

Figure 45: Comparison of GMV, MS and RB with VaR S.1 and S.4

The new portfolios do not result more optimal than those from first scenario do. On the contrary, most of them are worse in terms of mean and risk, being placed lower or more rightward than their counterparts are.

From this first analysis, Commodity and Real Estate do not highlight themselves as alternative classes that may give more advantages than the other classes as equity or sovereign bond or corporate bond do.

I implemented all these four scenarios and the portfolios based on inputs, like mean, variance and correlation, estimated on the entire sample. I compared their 18 years in a lump sum, with the main objective on the diversification process.

However this method cannot be very precise because it does not take into consideration all the characteristics of the index paths, of their cycles, and cannot highlight the portfolio allocations during economic recessions and expansions. A more precise method is needed in order to complete this analysis.

6 Portfolio performance analysis

In this section, I try to increase the precision of my analysis, using a different methodology. The aim now is to maximize the performance and not to focus on the diversification process. For this purpose, I employ one of the most diffused method to estimate the inputs necessary for the Markowitz and Risk Budgeting framework: the rolling method.

This method, also called moving average, is based on a full data set and a fixed subset size, called window of estimation; the first element of the moving average is obtained by taking the average of the initial fixed subset of the data series. Then the subset is modified by shifting forward, namely excluding the first number of the series and including the next number following the original subset in the series. This creates a new subset of numbers, which is averaged.

The same logic used to create a series of means can be also employed to create a series of variance-covariance matrixes.

In this way, instead of having a unique mean and a unique variance-covariance matrix computed on the entire data sample, I have many of them, calibrated on a smaller time section. In this way, I can increase the precision of my analysis.

About the evaluation window, it generally depends on the sample size and on the number of the assets for which I have to estimate the variance-covariance matrix. Sample estimator produces inconsistent results when the number of assets is greater than the elements of the return time series used to estimate the expected returns and information matrix[43]. In the practise, it is often used a 60-month rolling estimation but sometimes it is more convenient to employ a shorter one. Kolusheva [2008] highlight that if there is a parameter instability in the return series, the more distant historical return may no longer carry relevant information and may provide biased information.

On the other side, a shorter estimation window, due to the possible instability in the return series, may provide a larger turnover, increasing the general transaction costs to operate the portfolios.

I think that the best option is to use both a 36 and a 60 month rolling estimation and see what the differences, if any, may be.

[43] Ledoit and Wolf [2003] declared that when the number of stocks is larger than the number of historical returns per stock, the sample covariance matrix is always singular, even if the true covariance matrix is known to be non-singular.

For the weight budget estimations, I use four different optimization: the Max-Sharpe for the first portfolio, the Equal Risk Contribution for the second, the Risk Budgeting derived using the VaR as risk measure for the third and the Global Minimum Variance for the last one. The reason behind this choice is the fact that these portfolios resulted the best in terms of diversification in the previous section. There are no constraints of any kind, given the fact that I focus only on the performance and not on the diversification.

Also, the evaluation indices employed are different, since different is the objective of my analysis: alongside the usual Sharpe ratio and VaR, there are the Sortino index, the Calmar ratio, the Sterling ratio, the Farinelli-Tibiletti ratio and the Information ratio. For the Information ratio but also for the comparison with the different portfolios, I use the Benchmark derived from the combination 50% - 50% of an equity index and a bond index, i.e. MSCI World Index and Citygroup WGBI World All Maturities.

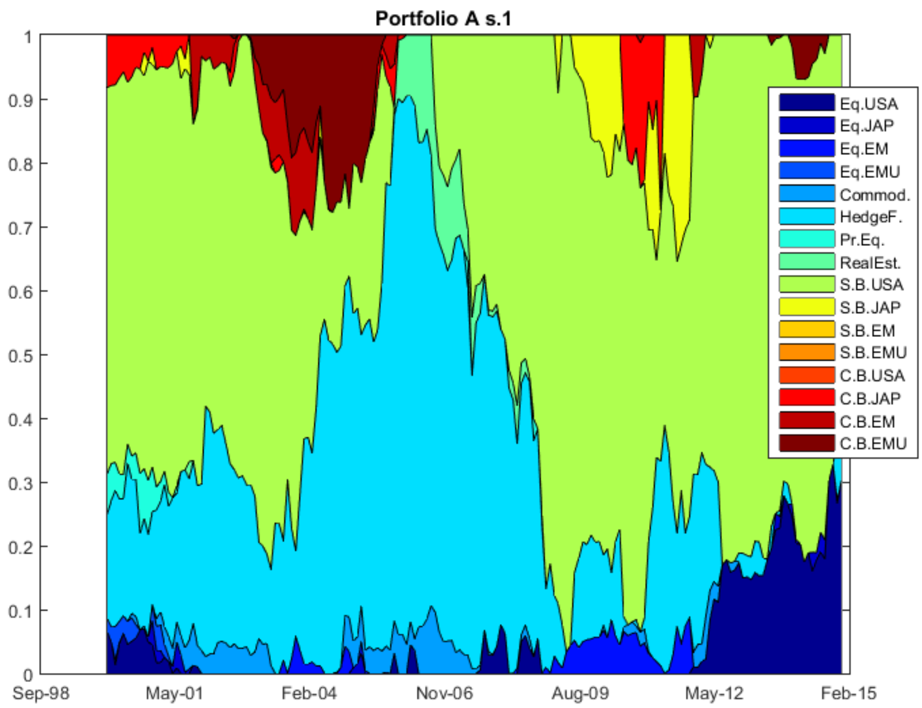

Figure 46: Portfolio A S.1

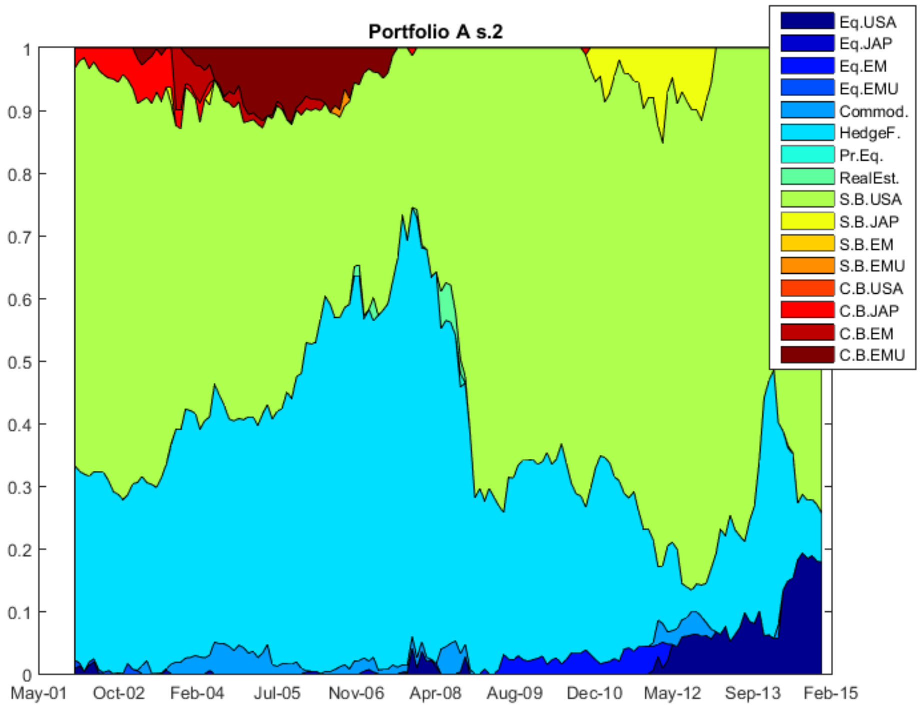

Figure 47: Portfolio A S.2

In Figure 46 and 47 are presented the weight distribution for the portfolio strategy A (Max Sharpe) both with 36 (S.1) and 60 (S.2) month rolling window.

Both distributions tend to have a similar weight evolution, with SB USA and Hedge Fund composing the vast majority of the portfolio.

The main difference lies in the different employment of the indices. In Table 38, we note that moving from a 60 to a 36 rolling window the average usage of SB USA and Hedge Fund diminish while increases the usage of the other indices; in fact a shorter window is more receptive towards the new information and consequently there are more asset allocation variations, while a longer window tends to keep stable the allocations and less receptive towards the new information.

Scenario No. 1 presents a more volatile index usage that scenario No. 2 does but this allows a better use of all the classes available and a better analysis. Effectively, in S.1 Hedge Fund composes the 81.94 % of all the alternative indices while in S.2 the 93.29%.

Hedge Fund is the alternative asset class more used (28.14%), during the expansion period after the dotcom bubble, the same period applies to CB EM (1.45%). Real Estate (1.49%) and Commodity (1.72%) have their largest participation at the peak of the cycle, just before the 2008 great crisis. Eq. EM (1.05%) is present during the recovery in 2009.

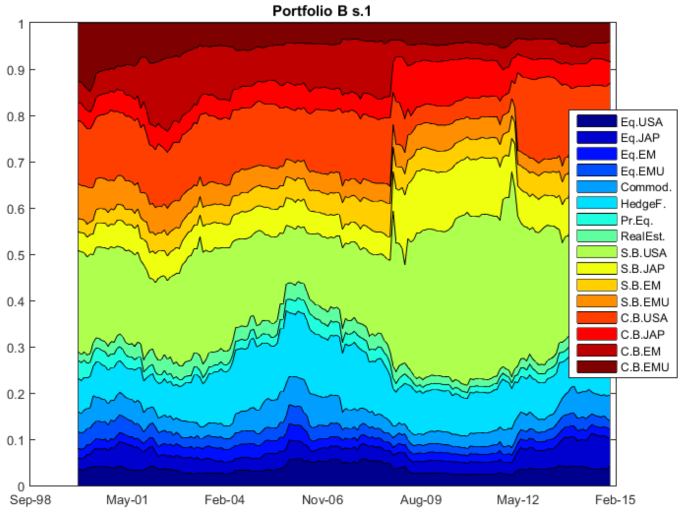

Figure 48: Portfolio B S.1

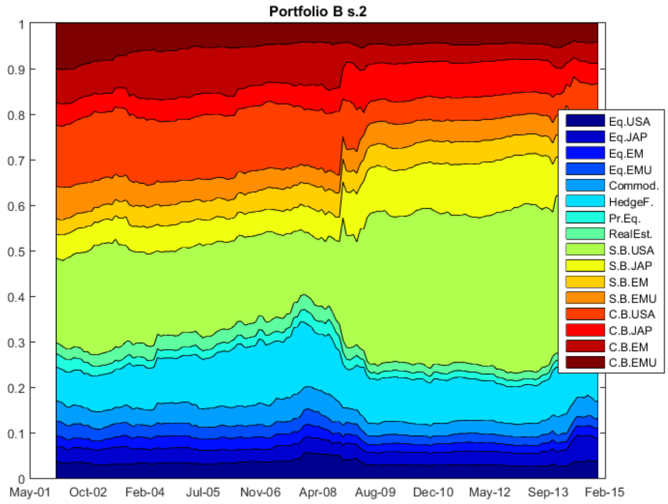

Figure 49: Portfolio B S.2

Also with the ERC strategy, the paths followed by the two portfolios (Figure 48 and 49) appear quite similar, with the S.1 version more "extreme" than the S.2.

This strategy sees the drastic reduction of SB USA (23.33% in S.1, s5.43% in S.2) and Hedge Fund (9.7% in S.1, 10.06% in S.2) weights in change for a more equal distribution of the indices: e.g. Commodity 4.04% in S.1 and 3.77% in S.2, Real estate 2.52% in S.1 and 2.44% in S.2, CB 7.24% in S.1 and 6.72% in S.2, CB USA that receives no weight with strategy A, now has 11.64% in S.1 and 10.32% in S.2.

In general, the distribution is more equal during the expansion period 2001-2007, whereas it is less equal after it at the advantage of SB USA. An example is the strong drop of CB EM after the great recession.

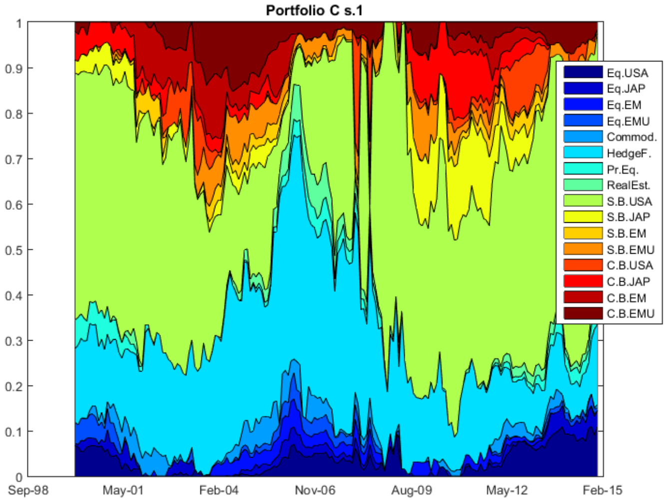

Figure 50: Portfolio C S.1

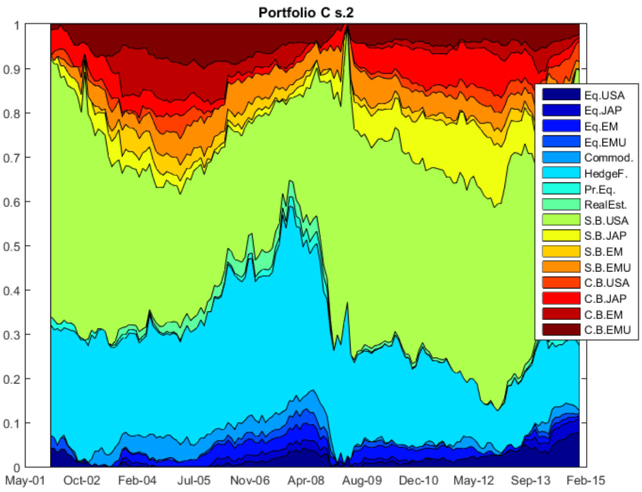

Figure 51: Portfolio C S.2

Figure 50 and 51 represent the weight distribution according to the Risk Budgeting portfolio based on the VaR. This model provides very volatile and "extreme" weights in both the scenario, but in particular in S.1, where a vast turnover is present.

This strategy awards one more time SB USA (41.71% in S.1 and 41.24% in S.2) and Hedge Fund (22.64% in S.1 and 23.47% in S.2).

As in previous B portfolios, also in both C portfolios, Commodity (2.11% in S.1 and 2.08% in S.2), Real Estate (1.70% in S.1 and 1.47% in S.2) and CB EM (3.33% in S.1 and 3.15% in S.2) are present during the expansion cycles, while Private Equity (1.74% in S.1 and 0.73 in S.2) is present only at the beginning of the sample during the boosting dotcom bubble. Eq. EM stands both before and after the 2008 recession (1.47% in S.1 and 1.88% in S.2).

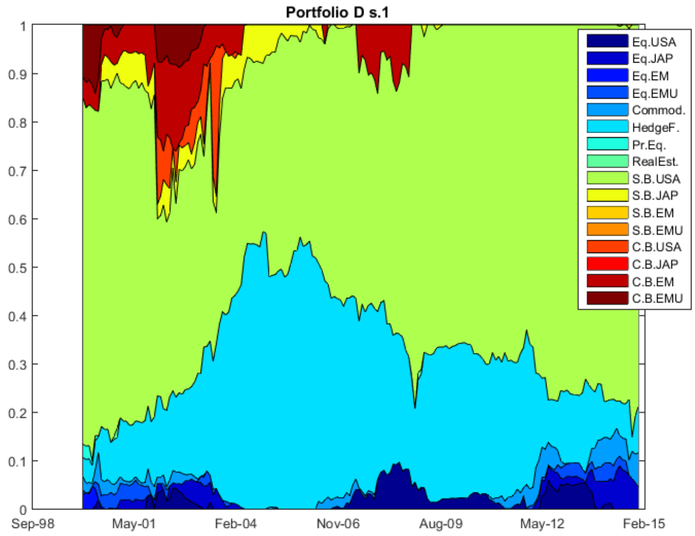

Figure 52: Portfolio D S.1

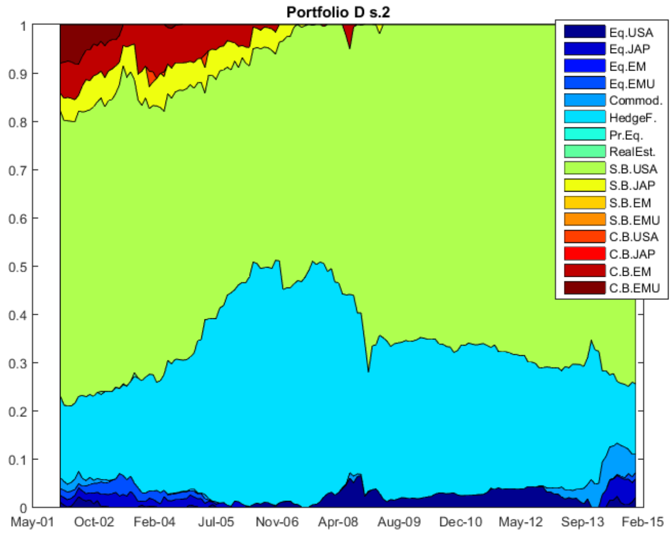

Figure 53: Portfolio D S.2

Portfolio strategy D is based on GMV optimization and consequently on index volatility and on the correlation among them. This causes the portfolio to be mainly composed only by SB USA and Hedge Fund, in an even more extreme scope than the other portfolios. On the average SB USA is present with 60.16% in S.1 and 60.74% in S.2, while Hedge Fund with 27.03% in S.1 and 30.58% in S.2.

Many of alternative classes, such as Eq. EM, Private Equity, Real Estate or SB EM, given their high risk and correlation, receive no weight. The only exceptions are Commodity (1.44% in S.1 and 0.8% in S.2) and CB EM (3.29% in S.1 and 2.29% in S.2).

Table 38: Portfolio weights

Port.	Scenario	Eq. USA	Eq. JAP	Eq. EM	Eq. EMU	Comm.	Hedge F.	Private E.	Real Est.	SB USA
A	S.1	4,33%	0,23%	1,05%	0,21%	1,72%	28,14%	0,49%	1,49%	53,45%
	S.2	2,13%	0,02%	0,64%	0,04%	1,09%	33,95%	0,02%	0,22%	57,30%
B	S.1	3,63%	3,58%	2,16%	2,52%	4,04%	9,70%	2,10%	2,52%	23,33%
	S.2	3,32%	3,22%	2,17%	2,39%	3,77%	10,06%	1,91%	2,44%	25,43%
C	S.1	3,29%	1,44%	1,47%	1,25%	2,11%	22,64%	1,74%	1,70%	41,71%
	S.2	1,75%	0,79%	1,88%	0,81%	2,08%	23,47%	0,73%	1,47%	41,24%
D	S.1	1,78%	1,23%	0,10%	0,74%	1,44%	27,03%	0,06%	0,03%	60,16%
	S.2	1,54%	0,78%	0,00%	0,52%	0,80%	30,58%	0,00%	0,01%	60,74%
Port.	Scenario	SB JAP	SB EM	SB EMU	CB USA	CB JAP	CB EM	CB EMU	Traditional	Alternative
A	S.1	2,62%	0,00%	0,00%	0,00%	1,77%	1,45%	3,05%	65,66%	34,34%
	S.2	1,25%	0,00%	0,04%	0,00%	0,78%	0,46%	2,05%	63,61%	36,39%
B	S.1	7,30%	4,51%	4,68%	11,64%	5,53%	7,24%	5,55%	67,74%	32,26%
	S.2	7,68%	4,50%	4,78%	10,32%	5,90%	6,72%	5,40%	68,45%	31,55%
C	S.1	4,32%	1,60%	3,22%	2,50%	3,48%	3,33%	4,22%	65,42%	34,58%
	S.2	5,01%	1,98%	4,46%	1,64%	4,37%	3,15%	5,18%	65,25%	34,75%
D	S.1	1,94%	0,00%	0,00%	1,07%	0,00%	3,39%	1,04%	67,96%	32,04%
	S.2	2,07%	0,00%	0,00%	0,06%	0,00%	2,29%	0,60%	66,32%	33,68%

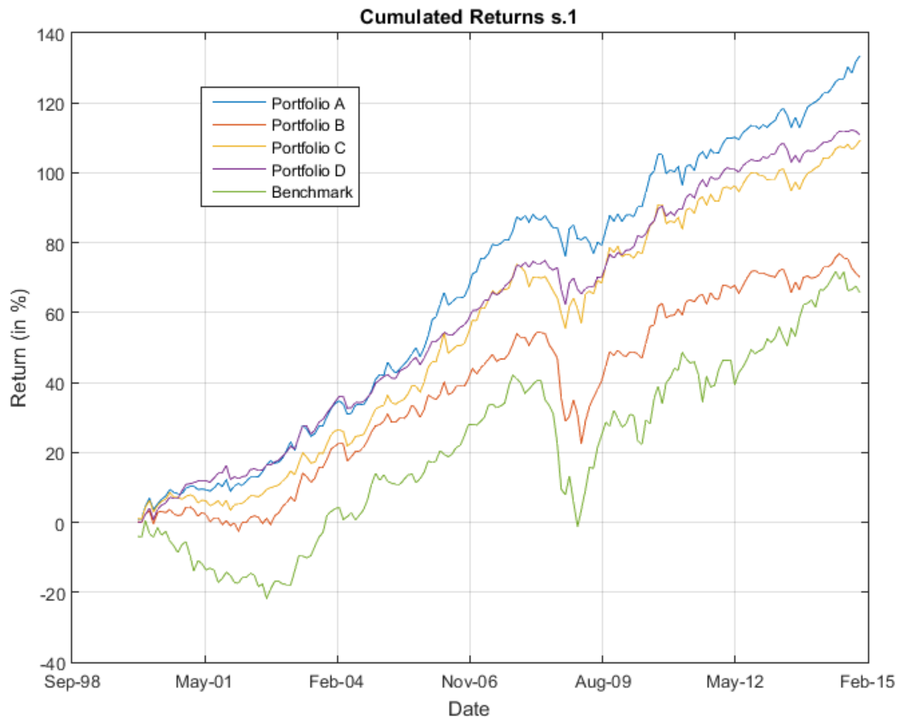

Figure 54: Cumulated return S.1

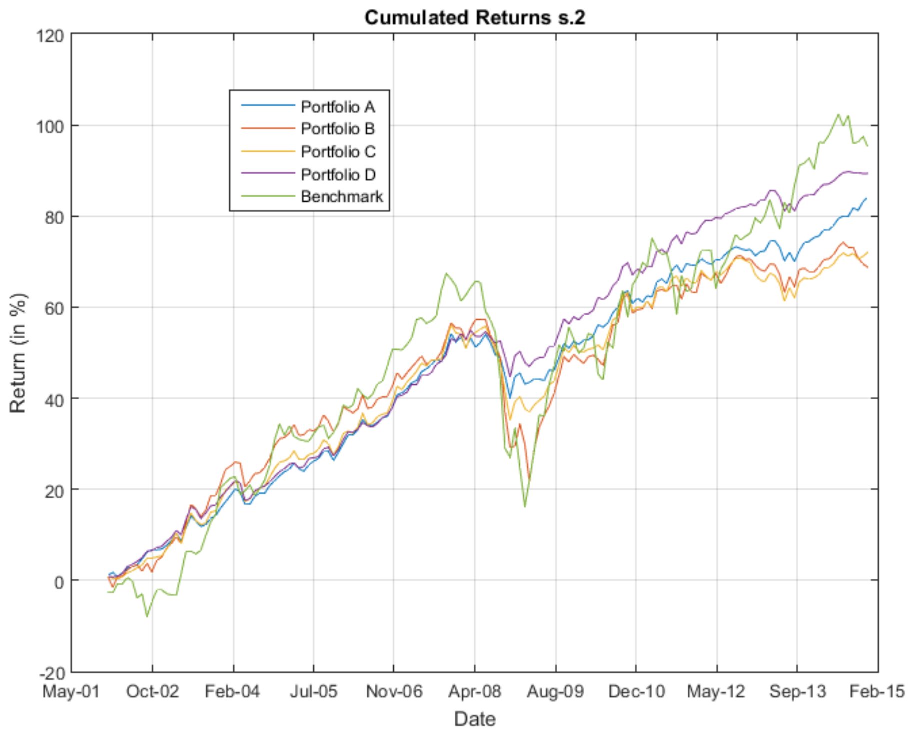

Figure 55: Cumulated return S.2

Table 38 gives us a first result about the utility of the alternative indices: in fact, they have a presence, no matter based on which portfolio or window estimation, that varies from 31.55%

to 36.39%. Of course, I must say that there is a strong different in the allocation for the Hedge Fund and the other classes.

About the returns, using different rolling windows produces very different effects: in Figure 54 (with 36 month window) the benchmark results the worse index while in Figure 55 (with 60 month window) it results the best at the end of the sample.

An important difference is at the beginning of the graphic; in fact in S.1 the benchmark suffers of a great loss, over 20%, until the end of 2002, but in S.2 this loss is very limited. The reason is that in S.2 I use a larger monthly window that includes both the slump but also the recovery of the benchmark, consequently the first moving averages are stationary around the zero. In S.1, the moving averages, given the shorter monthly window, keep in consideration first the slump (during the dotcom crisis) and only after two years, the recovery shows its effect. The portfolio indices suffered less the crisis and gain a substantial advantage from a shorter rolling window.

Another effect is also the different performance of the indices in the two figures: in S.1 strategy A results the best in terms of cumulated returns, followed by portfolio D and C and more distanced portfolio B. In S.2 things are different: portfolio D performs the best, followed by A, while portfolio C and B have same poorer trend.

We can say that a shorter rolling window tends to produce better cumulated returns, while a longer rolling window tends to incorporate more historic information that does not allow the portfolio to vary excessively during different crises and tends to reduce the volatility. This is true in general and a confirmation of these results comes from Table 39, where the average return is higher for S.1 at the cost of an increase in volatility that is lower in S.2. The only exception is portfolio B.

Table 39: Mean, st. deviation, min, max S.1 and S.2

Port.	Scenario	Mean	St.Deviation	Min	Max
A	S.1	0.481	1.303	-3.007	4.474
	S.2	0.398	1.042	-3.292	3.329
B	S.1	0.311	1.762	-7.869	5.098
	S.2	0.353	1.828	-8.075	5.942
C	S.1	0.421	1.443	-3.766	5.278
	S.2	0.357	1.326	-4.377	3.462
D	S.1	0.421	1.077	-3.691	3.755
	S.2	0,415	0,964	-3,209	3,230

Benchmark	S.1	0,316	2,663	-10,542	5,974
Benchmark	S.2	0,464	2,637	-10,542	5,974

However, Figure 55 and 56 show the other side of using a shorter or a longer monthly window.

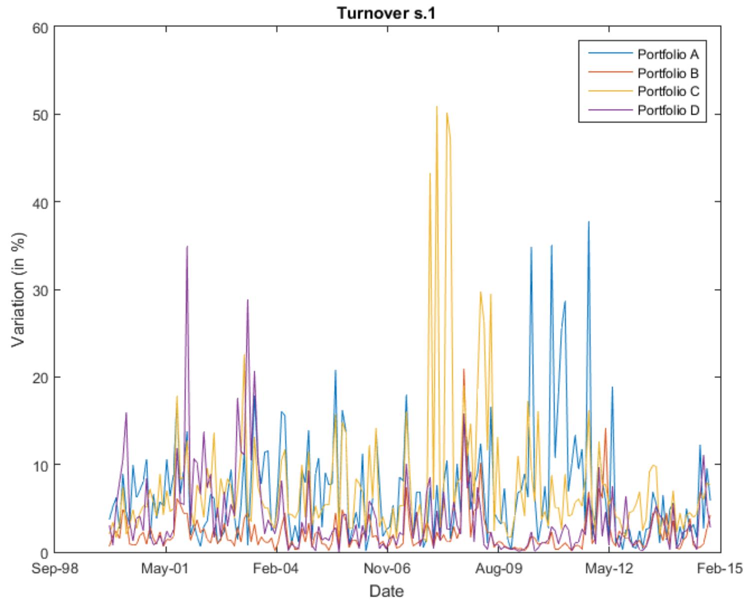

Figure 56: Turnover S.1

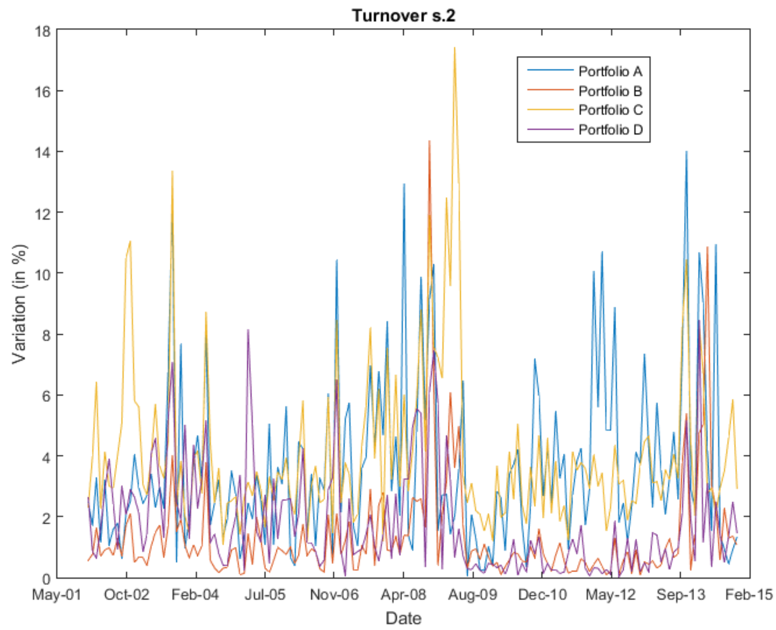

Figure 57: Turnover S.2

In the first scenario, portfolio C reaches a turnover of almost 50% during the 2008 recession while in the second case the highest peak is only 17%.

From Figure 57, all strategies produce high turnover, even if in different period: portfolio D between 2001 and 2004 has two peaks corresponding to 35% and 30% of the allocation and portfolio A between 2010 and 2012 changes often more than 30%.

In Figure 58, the turnover for all portfolios is very limited, usually lower than 10%. A note of mention is for the ERC portfolio that with its well-balanced risk allocation has a turnover lower than 10% in the first scenario and lower than 6% in the second one.

Finally the evaluation of the portfolio through some indices:

Table 40: Computation of performance indices

Port.	Scenario	SH	SO	VaR	Cal	Ste	FT
A	S.1	0.369	0.573	0.239	0.160	0.173	1.102
	S.2	0.381	0.535	0.254	0.120	0.169	1.062
B	S.1	0.177	0.222	0.132	0.039	0.058	0.687
	S.2	0.193	0.229	0.154	0.044	0.064	0.691
C	S.1	0.292	0.448	0.191	0.112	0.134	0.951
	S.2	0.269	0.355	0.184	0.081	0.106	0.846
D	S.1	0.391	0.463	0.286	0.114	0.137	1.012
	S.2	0.430	0.534	0.312	0.129	0.174	1.115
Benchmark	S.1	0.119	0.160	0.070	0.030	0.044	0.638
Benchmark	S.2	0.176	0.223	0.103	0.044	0.064	0.690

Table 41: Computation of composite index

Scenario	Port.	CI	SH	SO	VaR	Cal	Ste	FT
S.1	A	10	2	1	4	1	1	1
	D	14	1	2	5	2	2	2
	C	18	3	3	3	3	3	3
	B	22	4	4	2	4	4	4
	Benchmark	26	5	5	1	5	5	5
S.2	D	11	1	2	5	1	1	1
	A	13	2	1	4	2	2	2
	C	18	3	3	3	3	3	3
	B	24	4	4	2	5	5	4
	Benchmark	24	5	5	1	4	4	5

From Table 41, it is not clear which scenario provides with the best result. Portfolio A and C perform better in scenario 1 while portfolio B and D perform better in scenario 2; effectively, in the first scenario, strategy A is the winner while in the second scenario strategy D is it.

The last valuation is based on the information ratio (IR).

Table 42: Information ratio

Port.	Scenario	TE[44]	TEV[45]	SemiTEV[46]	IR	SemiIR[47]
A	S.1	0.164	8.179	1.730	0.020	0.095
	S.2	-0.067	7.478	1.672	-0.009	-0.040
B	S.1	-0.005	7.767	1.926	-0.001	-0.003
	S.2	-0.112	7.840	2.002	-0.014	-0.056
C	S.1	0.105	7.752	1.753	0.014	0.060
	S.2	-0.107	7.249	1.723	-0.015	-0.062
D	S.1	0.105	7.209	1.579	0.015	0.066
	S.2	-0,.050	7,.281	1.613	-0.007	-0.031

As we can see from Table 42, all the IR and SemiIR from scenario No. 2 result negative while those from scenario No. 1 result positive. This seems to be more favourable for S.1 but we must remember that there is a difference in terms of return for the two benchmark computed with different rolling windows. Thus, even this final valuation cannot be considered completely decisive.

[44] TE stands for tracking error: the difference between portfolio and benchmark return
[45] TEV stands for tracking error volatility: measures the volatility of TE
[46] SemiTEV: measure the volatility of only negative TE
[47] SemiIR: information ration based on SemiTEV

7 Conclusion

The scope of my thesis was an investigation about several alternative asset classes to understand if they could be a good opportunity of diversification and investment.
During this investigation I tried to understand what "alternative" means and how to describe these type of assets. A precise definition is not easy to find. They are called alternative because represent an alternative to the more classical financial instruments like equities or bonds, but maybe they should be considered more as a sub-classes in the most of the cases: after all what is a private equity if not an equity of a company simply not quoted on the stock market?

To conduct my investigation, I focused on several indices linked to these alternative investment such as hedge funds, commodities, private equities, real estates and equities, sovereign bonds and corporate bonds referring to the emerging market countries. I analysed them from an historical point of view, trying to comprehend their development during the cycles of the last 18 years and the way they dealt with the several crises that occurred in that period.
Above all, the hedge fund index was the best, it kept a raising and stable trend, with a low volatility and good rate of return, being lowly affected by the several recessions that occurred. The stability of an index is a key element for being an investment opportunity, in fact in my sample the other best asset in terms of performance and allocation was the American sovereign bond and it, like the hedge fund, had a stable and growing trend.
Private equity was one of the most volatile index, if not the most; it raised strongly during the dotcom bubble and the expansionary period among the 2002 and the 2007, but as fast as it grew, it dropped. Risk-adverse investors should not look in this way to diversify their investment, it may yield a lot if you select it on the upward trend but at the same time you can suffer huge losses in the case you do not know the precise moment to reduce the exposure to it. Analog to the private equity index was also the commodity one. It followed very much the oil price trend, resulting as much volatile as the oil price was and this fact affected it to a large extension.
One of my hope for this index was the fact that it could behave differently from stocks and bonds, that could not be affected by the exchange markets and that could represent a real alternative. However commodity prices are strongly correlated to the real economy, in the ways that a drop in productivity means a drop in commodity demand and this of course affects prices. The 2008 recession is a precise example. The commodity prices grew a lot during the expansion period, pushed high by the increasing demand, but when the crisis stroke and the demand

inverted its trend, the commodity prices slumped.
Highly volatile was also the EM equity index. Emerging markets are markets in expansion, in evolution, that have not a fix structure capable to absorb and mitigate great shocks. Under the expansionist trend large amount of investment flows fast in the companies of these countries but in the same fast way, during period of instability, they flow out, not giving the stability and the necessary risk control to be a consistent index for an institutional investor.
Corporate and sovereign bonds performed very flatly during the 18 years leaving no significant results. Low volatility without a certain level of returns do not represent an interesting opportunity of investment.
About real estate index, I can say that in the first part of the sample data it performed very poorly, significantly below the zero, with the only upward trend during the house bubble that led to the subprime crisis and to well-known consequences. After the slump, it stayed stationary close to zero. Even in this case, the index did not show itself as a valid alternative to the traditional classes.

The conclusions so far made for the single index, thanks to the historical analysis, are the same I made after the numerical portfolio analysis.
I tried several portfolio strategies, with different methodologies and constraints, but the answer was always the same: the only alternative asset class really useful is the hedge fund. Certainly, private equity and commodity as well as EM equity represent a possible good opportunity of investment but only in the measure they are restricted to a small proportion and only in the upward cycles.
The high volatility and the great losses they suffered during the recession, discourage their wide employment.
Both the scenario No.2 and No.4 showed how little their diversification contribution was to the whole allocation process. In addition this result was confirmed by the more extensive analysis using the rolling method of estimation. With little difference from a 36 month window or a 60 month window, the result did not change.
The alternative asset classes, with the exclusion of the hedge fund class, may be employed only in small measure, with the attempt to obtain a high extra-return in a short term period, exploiting their propensity to grow fast during positive economic trend.
On the other side, hedge fund class represents a class where to invest in. In scenario No.3, it showed how it usage could increase the diversification of almost all the portfolio strategies. Also the last analysis with the rolling method pointed in this direction: more than 30% of the

portfolios were composed by alternative indices and of this portion between 80% and 95%, according to the different windows, was made up by the hedge fund index.

The reason of this performance is ascribable to the “independency” from the general economy. Commodity prices are based on the demand and supply relation, private equity and EM equities are based on the companies’ capacity to produce and sell goods or services that consumers want to purchase, real estate prices are based on families’ future economic stability: if the real economy starts to fall apart, commodity demand decreases, companies see their profits reduced and uncertainty about the future takes over the stability, and with them the above mentioned asset classes plunge.
Instead, hedge funds possess different strategies, from short-selling to the use of derivatives and so on, that allow them to make absolute profits also when the economy is on a downward trend or facing a recession.

Concluding, if I were asked to construct my ideal portfolio based on the information I have gathered from my work, I would allocate around 75% to traditional classes as American and European bonds and equities and the remaining one fourth to alternative assets composed at 85% by hedge funds and at 15% by commodities, private equities and EM equities.

Appendix A

The quadratic utility function is greatly used in financial and economic model because we can express the expected utility function in terms of means and variance

Let W be a random variable representing the level of wealth. Then we can write his variance as

$$\sigma_W^2 = E[W - E(W)]^2$$

Squaring the function we obtain

$$\sigma_W^2 = E\{W^2 - 2WE(W) + [E(W)]^2\}$$

The expected value of the sum of random variables is the sum of the expected values, follows that

$$\sigma_W^2 = E(W^2) - E[2WE(W)] + [E(W)]^2$$

Also, the expected value of a constant times a random variable is the constant times the expected value of the random variable, so σ_W^2 can be written as

$$\sigma_W^2 = E(W^2) - 2E[W]E[W] + [E(W)]^2$$

Or

$$\sigma_W^2 = E(W^2) - [E(W)]^2$$

Rearranging for $E(W^2)$

$$E(W^2) = \sigma_W^2 + [E(W)]^2 \quad \text{(A.1)}$$

Using the quadratic utility function

$$U(W) = W - bW^2$$

We take the expected value,

$$E[U(W)] = E[W] - bE[W^2]$$

And then substituting $E[W^2]$ from equation (A.1)

$$E[U(W)] = E[W] - b\{\sigma_W^2 + [E(W)]^2\}$$

Finally, we are able to define the expected utility in terms of means and variance.

Appendix B

First, we write the portfolio volatility for the two asset case

$$\sigma(w) = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho\sigma_1\sigma_2}$$

Then we compute marginal risk of the first asset

$$\frac{\partial\sigma(w)}{\partial w_1} = \frac{w_1\sigma_1^2 + w_2\rho\sigma_1\sigma_2}{\sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho\sigma_1\sigma_2}}$$

And its risk contribution

$$RC_1 = w_1\frac{\partial\sigma(w)}{\partial w_1}$$

$$RC_1 = \frac{w_1^2\sigma_1^2 + w_2\rho\sigma_1\sigma_2}{\sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho\sigma_1\sigma_2}}$$

We can verify its validity by summing up the two risk contributions

$$RC_1 + RC_2 = \frac{w_1^2\sigma_1^2 + w_2\rho\sigma_1\sigma_2}{\sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho\sigma_1\sigma_2}} + \frac{w_2^2\sigma_2^2 + w_2\rho\sigma_1\sigma_2}{\sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho\sigma_1\sigma_2}}$$

$$= \frac{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho\sigma_1\sigma_2}{\sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho\sigma_1\sigma_2}}$$

$$= \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho\sigma_1\sigma_2} = \sigma(w)$$

For the case in which $n > 2$, the volatility in matrix form is

$$\sigma(w) = \sqrt{w'\Sigma w}$$

Then the marginal risk is

$$\frac{\partial\sigma(w)}{\partial w} = \frac{1}{2}(w'\Sigma w)^{-\frac{1}{2}} * (2\Sigma w) = \frac{\Sigma w}{\sqrt{w'\Sigma w}}$$

And the risk contribution for the asset i is

$$RC_i = w_i\frac{(\Sigma w)_i}{\sqrt{w'\Sigma w}}$$

Appendix C

In the following part, I report all the Table regarding the Sortino and Sharpe-based indicator, the different portfolio allocation and the Diversification index for the final choice for the four scenarios.

C.1 Scenario No.1

Table 43: Sortino-based indicator S.1

Asset	SO	Budget
Eq. USA	0.162	9.34%
Eq. JAP	0.042	2.43%
Eq. EMU	0.115	6.67%
SB USA	0.517	29.92%
SB JAP	0.130	7.51%
SB EMU	0.282	16.31%
CB USA	0.031	1.80%
CB JAP	0.195	11.29%
CB EMU	0,.254	14.72%
Total SO =1.73		

Table 44: Sharpe ratio-based indicator S.1

Asset	SH	Budget
Eq. USA	0.117	11.15%
Eq. JAP	0.026	2.44%
Eq. EMU	0.082	7.79%
SB USA	0.334	31.76%
SB JAP	0.062	5.93%
SB EMU	0.159	15.07%
CB USA	0.021	2.00%
CB JAP	0.104	9.87%
CB EMU	0.147	13.98%
Total SR = 1.05		

Table 45: Weights for portfolio 8 with different α-quantiles S.1

Indicator	α	Eq. USA	Eq. JAP	Eq. EMU	SB USA	SB JAP	SB EMU	CB USA	CB JAP	CB EMU
Sortino-based	0.99	7.65%	4.32%	4.32%	54.47%	6.30%	7.87%	0.00%	6.46%	8.60%
	0.95	7.53%	2.74%	3.56%	55.66%	4.96%	8.08%	2.84%	6.08%	8.57%
	0.9	7.85%	1.06%	2.96%	61.30%	4.04%	8,.36%	0.00%	5.73%	8.70%
Sharpe-based	0.99	7.86%	4.26%	4.41%	55.47%	5.91%	7.54%	0.00%	6.17%	8.38%
	0.95	7.88%	2.63%	3.73%	57.36%	4.27%	7.54%	2.83%	5.58%	8.18%
	0.9	8.37%	0.90%	3.23%	63.63%	3.07%	7,.62%	0.00%	5.04%	8.14%

In 4 cases out of 6, the CB USA does not receive any weights, the reason may be the fact that its expected return is only about 0.050, the lowest among the indices. A vast amount of weight is placed on the SB USA, that in the last line almost reach two third of the portfolio. These results are not big surprise because in line with the other portfolios' weights.

Table 46: Diversification index, expected return, risk S.*1*

Indicator	α	Diversification index	Return	Risk
Sortino-based	0.99	0.547	0.440	1.461
	0.95	0.557	0.435	1.418
	0.9	0.573	0.454	1.373
Sharpe-based	0.99	0.545	0.441	1.447
	0.95	0.554	0.438	1.396
	0.9	0.568	0.458	1.344

As we can note, the risk for all the six cases are quite similar, while there are some small differences in the expected return. Particular thing is the fact that the portfolio with the best value for diversification index is the same that has the third lower return and the highest risk, symptom that the most diversified portfolios do not always give the best performance.

C.2 Scenario No.2

Table 47: Sortino-based indicator S.2

Asset	SO		Budget	
	Scenario 1	Scenario 2	Scenario 1	Scenario 2
Eq. USA	0.162	0.162	9.34%	8.51%
Eq. JAP	0.042	0.042	2.43%	2.21%
Eq. EM	NA	0.124	NA	6.53%
Eq. EMU	0.115	0.115	6.67%	6.08%
SB USA	0.517	0.517	29.92%	27.24%
SB JAP	0.130	0.130	7.51%	6.83%
SB EM	NA	0.027	NA	1.44%
SB EMU	0.282	0.282	16.31%	14.85%
CB USA	0.031	0.031	1.80%	1.64%
CB JAP	0.195	0.195	11.29%	10.28%
CB EM	NA	0.019	NA	1.00%
CB EMU	0.254	0.254	14.72%	13.40%
Total SO = 1.8988				

Table 48: Sharpe-based indicator S.2

Asset	SH		Budget	
	S.1	S.2	S.1	S.2
Eq. USA	0.117	0.117	11.15%	9.93%
Eq. JAP	0.026	0.026	2.44%	2.17%
Eq. EM	NA	0.084	NA	7.07%
Eq. EMU	0.082	0.082	7.79%	6.94%
SB USA	0.334	0.334	31.76%	28,.28%
SB JAP	0.062	0.062	5.93%	5.28%
SB EM	NA	0.027	NA	2.27%
SB EMU	0.159	0.159	15.07%	13.42%
CB USA	0.021	0.021	2.00%	1.78%
CB JAP	0.104	0.104	9.87%	8.79%
CB EM	NA	0.019	NA	1.62%
CB EMU	0.147	0.147	13.98%	12.45%

Total SH = 1.1814

In Table 47 and 48, we can visualize the presence of EM indices based on the two indicators. For SB EM and CB EM the presence is still small, while it is quite on the average for Eq. EM.

Table 49: Weights for portfolio 8 with different α-quantiles S.1 and S.2

Indicator	α	Eq. USA	Eq. JAP	Eq. EM	Eq. EMU	SB USA	SB JAP
Sharpe-based	0.99	5.27%	3.15%	2.98%	3.09%	51.23%	5.75%
	0.95	5.53%	1.94%	2.88%	2.67%	54.60%	4.58%
	0.9	6.28%	0.97%	3.14%	2.47%	61.64%	3.97%
Sortino-based	0.99	5.44%	3.10%	2.99%	3.17%	52.11%	5.36%
	0,.95	5.82%	1.84%	2.89%	2.80%	56.08%	3.88%
	0.9	6.69%	0.87%	3.14%	2.68%	63.62%	3.07%
Indicator	α	SB EM	SB EMU	CB USA	CB JAP	CB EM	CB EMU
Sharpe-based	0.99	3.23%	6.81%	0.00%	5.86%	5.19%	7.45%
	0.95	2.08%	7.30%	2.08%	5.68%	2.92%	7.75%
	0.9	0.00%	7.80%	0.00%	5.55%	0.00%	8.17%
Sortino-based	0.99	3.38%	6.46%	0.00%	5.56%	5.24%	7.20%
	0.95	2.32%	6.74%	2.08%	5.17%	3.06%	7.32%
	0.9	0.35%	7.06%	0.00%	4.89%	0.00%	7.61%

From Table 49, we note that the risk measure VaR penalizes SB EM and CB EM for both the indicators at α = 0.9 with empty risk budgets. The level of presence of EM is low, but not as much as CB USA, and it is on the same level of Eq. JAP.

Table 50: Diversification index, expected return, risk S.2

Indicator	α	Diversification index	Expected Return	Risk
Sortino-based	0.99	0.528	0.412	1.449
	0.95	0.539	0.423	1.403
	0.9	0.556	0.456	1.371
Sharpe-based	0.99	0.527	0.413	1.439
	0.95	0.536	0.424	1.385
	0.9	0.552	0.457	1.345

Based on Table 50, I decide to choose the portfolio based on Sharpe-based indicator with a level of confidence of 0.99.

C.3 Scenario No.3

Table 51: Sortino-based indicator S.3

Asset	SO		Budget	
	S.1	S.3	S.1	S.3
Eq. USA	0.162	0.162	9.34%	7.22%
Eq. JAP	0.042	0.042	2.43%	1.88%
Eq. EMU	0.115	0.115	6.67%	5.16%
Hedge	NA	0.411	NA	18.37%
Private	NA	0.097	NA	4.32%
SB USA	0.517	0.517	29.92%	23.13%
SB JAP	0.130	0.130	7.51%	5.80%
SB EMU	0.282	0.282	16.31%	12.61%
CB USA	0.031	0.031	1.80%	1.39%
CB JAP	0.195	0.195	11.29%	8.73%
CB EMU	0.273	0.254	14.72%	11.38%
Total SO = 2.2359				

Table 52: Sortino-based indicator S.3

Asset	SH		Budget	
	S.1	S.3	S.1	S.3
Eq. USA	0.117	0.117	11.15%	8.18%
Eq. JAP	0.026	0.026	2.44%	1.79%
Eq. EMU	0.082	0.082	7.79%	5.72%
Hedge	NA	0.307	NA	21.43%
Private	NA	0.075	NA	5.20%
SB USA	0.334	0.334	31.76%	23.30%
SB JAP	0.062	0.062	5.93%	4.35%

SB EMU	0.159	0.159	15.07%	11.06%
CB USA	0.021	0.021	2.00%	1.47%
CB JAP	0.104	0.104	9.87%	7.24%
CB EMU	0.147	0.147	13.98%	10.26%
Total SH = 1.4338				

Both from Table 51 and 52, we can see that Hedge Fund receives a large weight budget, second only to SB USA. Private Equity, despite its high level of volatility, does not receive the smallest weight budget; in both the tables, Eq. JAP and CB USA perform poorly.

Table 53: Weights for portfolio 8 with different α-quantiles S.1 and S.3

Indicator	α	Eq. USA	Eq. JAP	Eq. EMU	Hedge	Private	
Sortino-based	0.99	4.20%	2.73%	2.42%	21.86%	1.45%	
	0.95	3.78%	1.36%	1.78%	23.96%	1.21%	
	0.9	3.42%	0.03%	1.14%	27.40%	0.47%	
Sharpe-based	0.99	4.26%	2.67%	2,.44%	22.46%	1.52%	
	0.95	3.89%	1.25%	1.83%	24.95%	1.30%	
	0.9	3.57%	0.00%	1.21%	28.44%	0.64%	
Indicator	α	SB USA	SB JAP	SB EMU	CB USA	CB JAP	CB EMU
Sortino-based	0.99	45.36%	5.20%	5.61%	0.00%	5.09%	6.07%
	0.95	46.49%	3.78%	5.62%	1.56%	4.60%	5.86%
	0.9	49.78%	2.68%	5.53%	0.00%	3.93%	5.62%
Sharpe-based	0.99	45.86%	4.89%	5.27%	0.00%	4.81%	5.81%
	0.95	47.51%	3.15%	5.07%	1.53%	4.12%	5.41%
	0.9	51.47%	1.74%	4.75%	0.00%	3.22%	97%

Moving from a 0.99 quintile to 0.9 quintile, only SB USA and Hedge Fund increase their weights while all the other indices generally decrease them.

Table 54: Diversification index, expected return, risk S.3

Indicator	α	Diversification index	Return		Volatility
	0.99	0.513	0.481		1.271

Sortino-based	0.95	0.521	0.484		1.212
	0.9	0.538	0.500		1.143
Sharpe-based	0.99	0.512	0.483		1.261
	0.95	0.520	0.488		1.195
	0.9	0.537	0.505		1.122

Based on Table 54, I decide to choose the portfolio based on Sharpe-based indicator with a level of confidence of 0.99.

C.4 Scenario No. 4

Table 55: Sortino-based indicator S.4

Asset	SO		Budget	
	S.1	S.4	S.1	S.4
Eq. USA	0.162	0.162	9.34%	8.73%
Eq. JAP	0.042	0.042	2.43%	2.27%
Eq. EMU	0.115	0.115	6.67%	6.24%
Comm.	NA	0.051	NA	2.78%
Real Est.	NA	0.070	NA	3.79%
SB USA	0.517	0.517	29.92%	27.96%
SB JAP	0.130	0.130	7.51%	7.01%
SB EMU	0.282	0.282	16.31%	15.24%
CB USA	0.031	0.031	1.80%	1.68%
CB JAP	0.195	0.195	11.29%	10.55%
CB EMU	0.254	0.254	14.72%	13.76%
Total SO = 1.85				

Table 56: Sharpe-based indicator S.4

Asset	SH		Budget	
	S.1	S.4	S.1	S.4
Eq. USA	0.117	0.117	11.15%	10.26%
Eq. JAP	0.026	0.026	2.44%	2.25%
Eq. EMU	0.082	0.082	7.79%	7.17%

Comm.	NA	0.037	NA	3.23%
Real Est.	NA	0.054	NA	4.75%
SB USA	0.334	0.334	31.76%	29.23%
SB JAP	0.062	0.062	5.93%	5.46%
SB EMU	0.159	0.159	15.07%	13.87%
CB USA	0.021	0.021	2.00%	1.84%
CB JAP	0.104	0.104	9.87%	9.08%
CB EMU	0.147	0.147	13.98%	12.86%
Total SH = 1.1433				

Both from Table 55 and 56, we note that both Commodity and Real Estate get small budgets, given their poor result on the two indicators. Again, only CB USA and Eq. JAP perform worse.

Table 57: Weights for portfolio 8 with different α-quantiles S.1 and S.4

Indicator	α	Eq. USA	Eq. JAP	Eq. EMU	Comm.	Real Est.	
Sortino-based	0.99	6.35%	3.31%	3.50%	4.14%	2.33%	
	0.95	6.45%	2.15%	3.00%	2.91%	1.85%	
	0.9	7.05%	0.99%	2.71%	1.52%	1.21%	
Sharpe-based	0.99	6.51%	3.25%	3.57%	4.18%	2.43%	
	0.95	6.71%	2.04%	3.13%	2,.99%	2.02%	
	0.9	7.38%	0.83%	2.89%	1.73%	1.49%	
Indicator	α	SB USA	SB JAP	SB EMU	CB USA	CB JAP	CB EMU
Sortino-based	0.99	54.37%	5.82%	6.75%	0.00%	6.00%	7.43%
	0.95	55.75%	4.68%	7.26%	2.45%	5.76%	7.74%
	0.9	60.97%	3.97%	7.82%	0.00%	5.57%	8.19%
Sharpe-based	0.99	55.37%	5.42%	6.41%	0.00%	5.69%	7.18%
	0.95	57.39%	3.99%	6.70%	2.45%	5.26%	7.31%
	0.9	63.16%	3.04%	7.04%	0.00%	4.88%	7.58%

Table 58: Diversification index, expected return, risk S.4

Indicator	α	Diversification index	Return	Volatility
	0.99	0.517	0.430	1.458

Sortino-based	0.95	0.530	0.429	1.410
	0.9	0.552	0.448	1.367
Sharpe-based	0.99	0.515	0.431	1.446
	0.95	0.527	0.431	1.391
	0.9	0.545	0.451	1.339

Also in this scenario, the portfolio that offers the best diversification is fourth, based on the Sharpe indicator with a confidence level of 0.99.

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