Università degli Studi di Padova – Dipartimento di Ingegneria Industriale

Corso di Laurea in Ingegneria Meccanica

Relazione per la prova finale «Analisi cinematica e statica del meccanismo di sollevamento del canestro Sportsystem Hydroplay»

Tutor universitario: Prof. Giulio Rosati

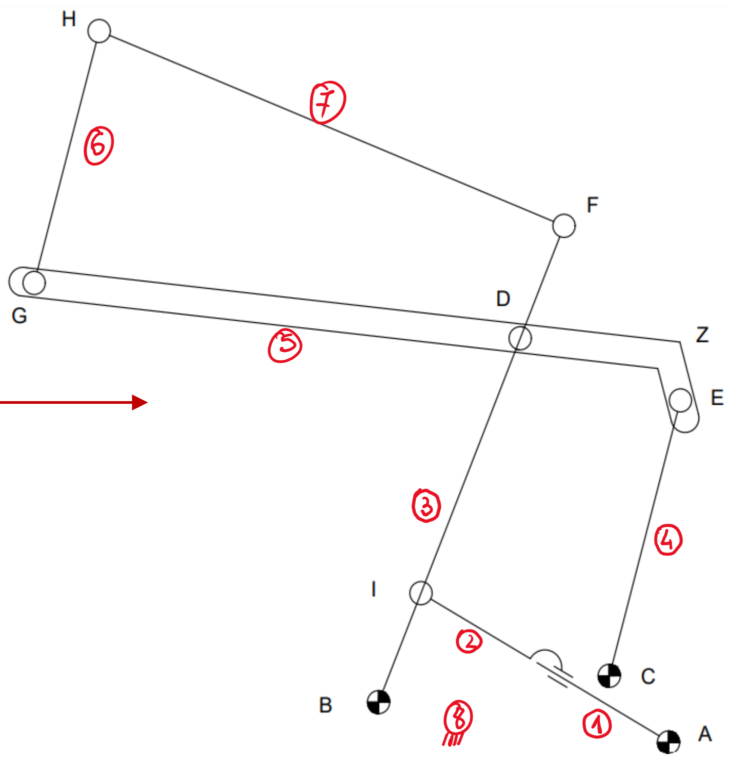
Correlatore: Prof. Bottin Matteo

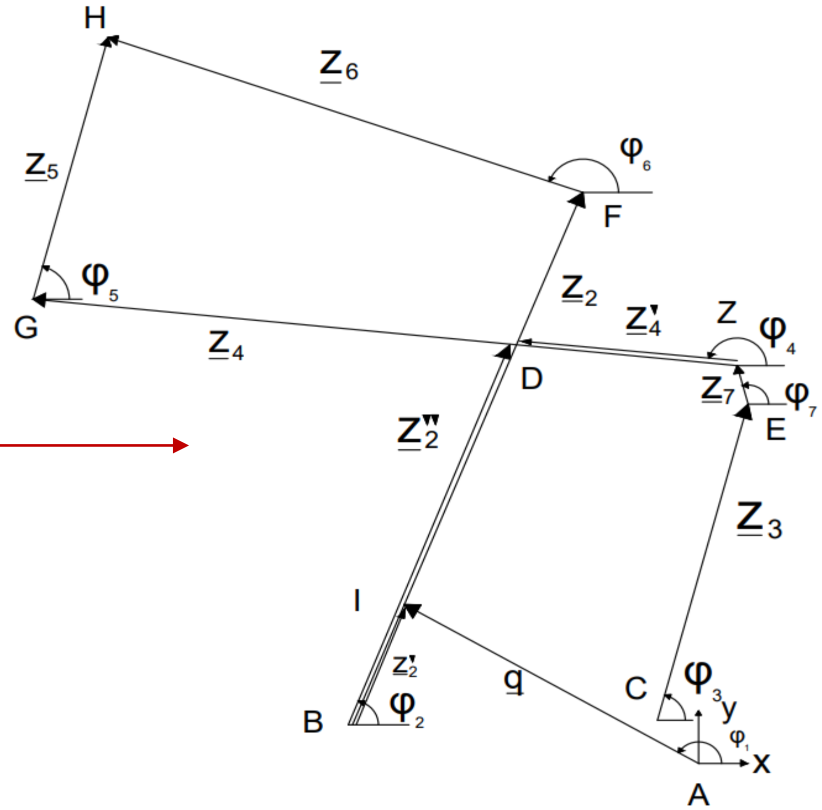
Laureando*: Gasperini Leonardo*

Matricola: 2007918

Padova, 26/09/2023

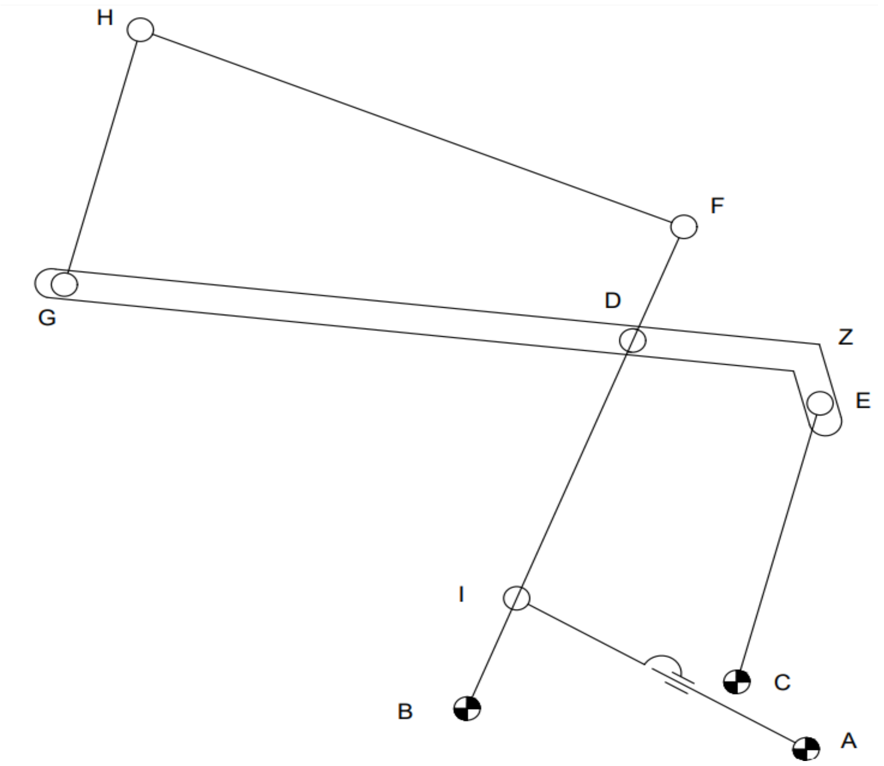
CANESTRO SPORTSYSTEM HYDROPLAY

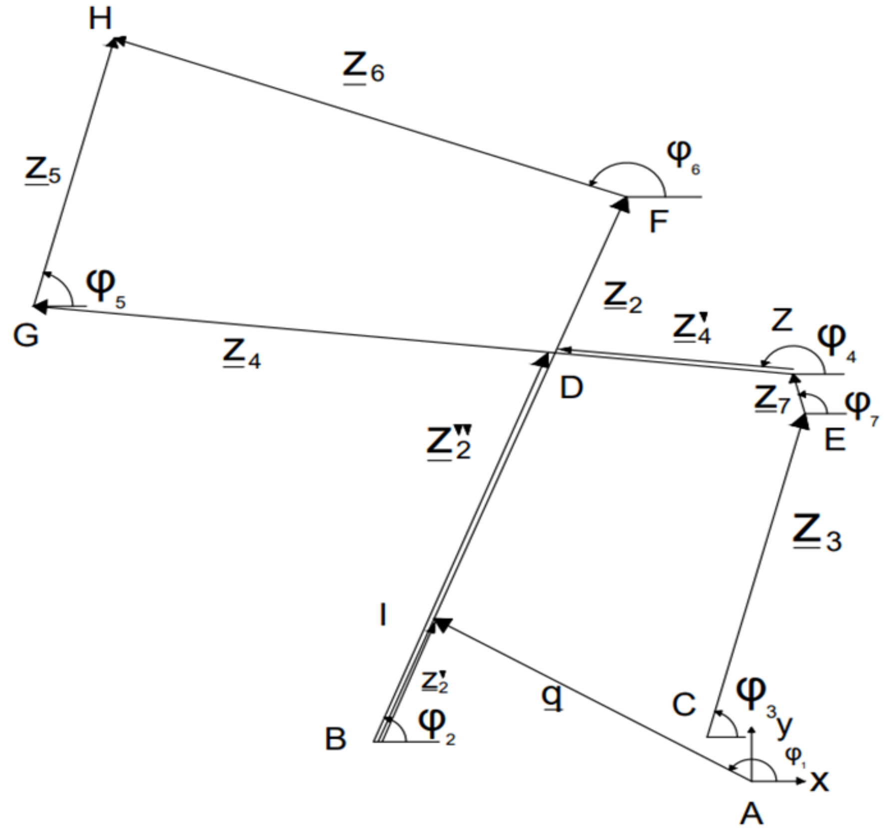




Gradi di Libertà: Equazione di Grubler $3(n-1) - 2R - 2P - 1C = 3(8-1) - 2*9 - 2*1 = 1 \ [gdl]$

ANALISI DEL MECCANISMO





MAGLIE DEL MECCANISMO

1) $\underline{q} - \underline{z'_2} - \begin{Bmatrix} x_B - x_A \\ y_B - y_A \end{Bmatrix} = \underline{0}$

2) $\underline{z''_2} - \underline{z'_4} - \underline{z_7} - \underline{z_3} - \begin{Bmatrix} x_C - x_B \\ y_C - y_B \end{Bmatrix} = \underline{0}$

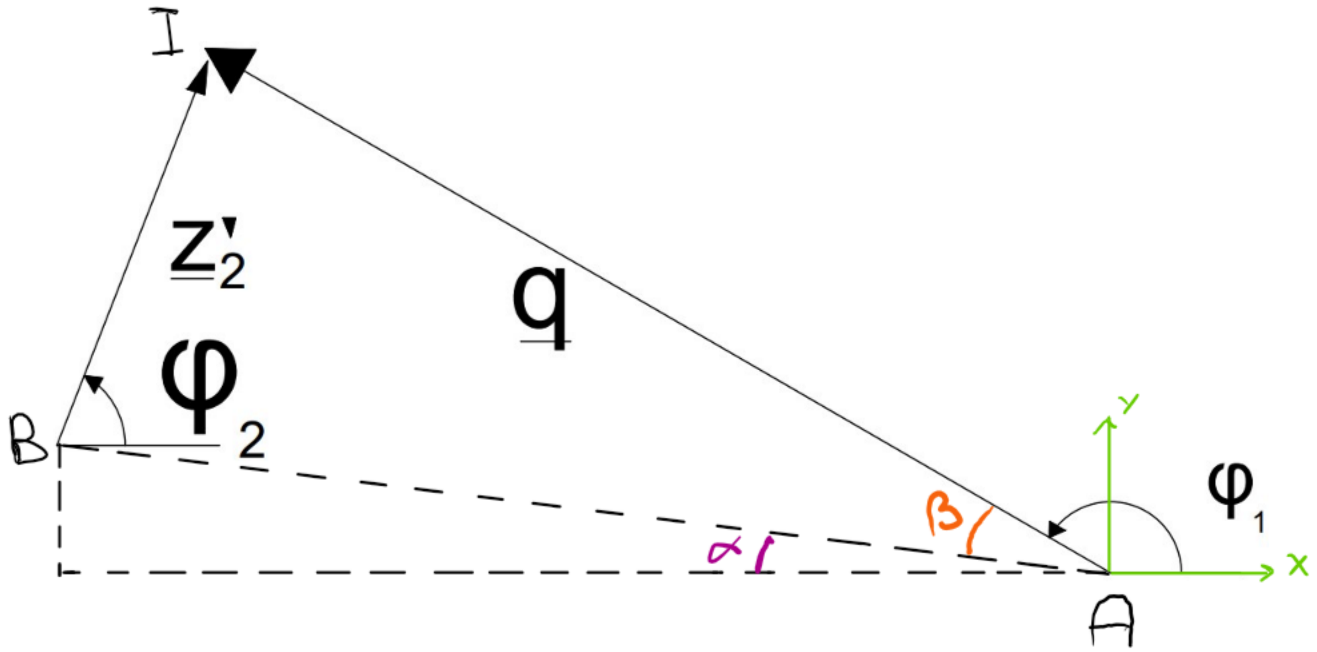
3) $(\underline{z_4} - \underline{z'_4}) + \underline{z_5} - \underline{z_6} - (\underline{z_2} - \underline{z''_2}) = \underline{0}$

COORDINATA LIBERA $\underline{q}$

VARIABILI $\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5, \varphi_6, \varphi_7$

LUNGHEZZE NOTE $a_2, a_2', a_2'', a_3, a_4, a_4', a_5, a_6, a_7, \overline{DE}$

Prima Maglia



$$L_{AB} = \sqrt{{x_B}^2 + y_B^2}\ [m]$$

$$\alpha = \cos^{-1}\left(\frac{x_B^2 + {L_{AB}}^2 - {y_B}^2}{2\, x_B^2\, L_{AB}}\right) [°]$$

$$\beta = \cos^{-1}\left(\frac{q^2 + L_{AB}^2 - a'^2_2}{2\, q\, L_{AB}}\right) [°]$$

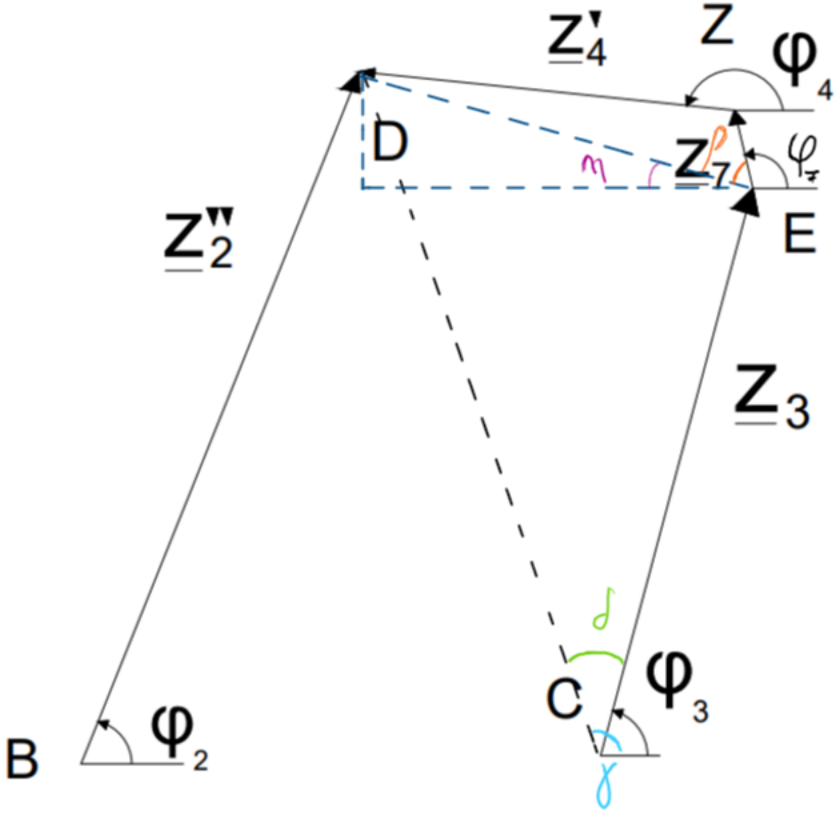
$$\varphi_1 = 180 - (\alpha + \beta)\ [°]$$

$$x_I = q \cos \varphi_1\ [m]$$

$$y_I = q \sin \varphi_1\ [m]$$

$$\varphi_2 = atan2d(y_I - y_B, x_I - x_B)\ [°]$$

Seconda Maglia



$$x_D = a_2'' \cos \varphi_2 \ [m]$$

$$y_D = a_2'' \sin \varphi_2 \ [m]$$

$$L_{CD} = \sqrt{(x_C - x_D)^2 + (y_C - y_D)^2} \ [\mathrm{m}]$$

$$\varphi_3 = atan2d(y_D - y_C, x_D - x_C) - \cos^{-1}\left(\frac{L_{CD}^2 + a_3^2 - L_{DE}^2}{2\ L_{CD}\ a_3}\right) [°]$$

$$x_E = x_C + a_3 cos\varphi_3 [m]$$

$$y_E = y_C + a_3 cos\varphi_3 \ [m]$$

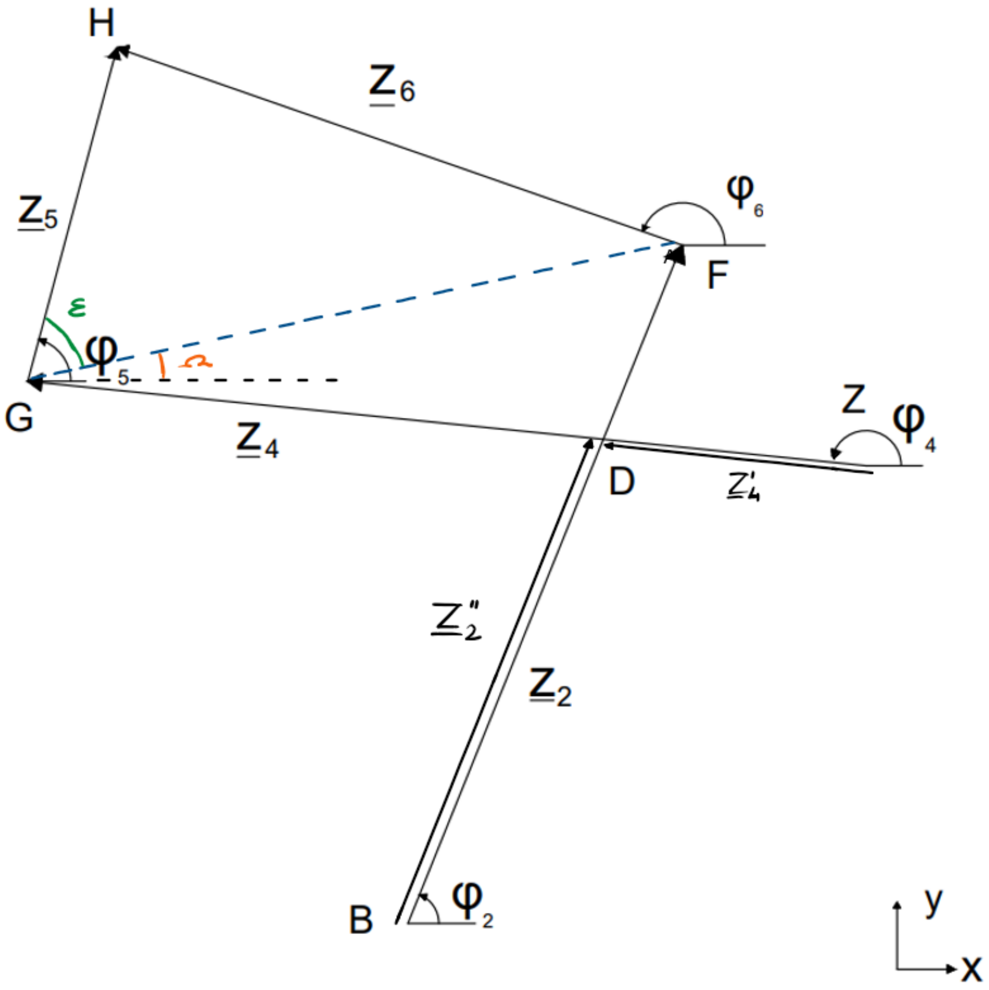
$$\varphi_7 = 180 - \cos^{-1}\left(\frac{L_{DE}^2 + \Delta x_{ED}^2 - \Delta y_{ED}^2}{2\ \Delta x_{ED}\ L_{DE}}\right) - \cos^{-1}\left(\frac{L_{DE}^2 + a_7^2 - a_4'^2}{2\ a_7\ L_{DE}}\right) [°]$$

$$x_Z = x_E + a_7 \cos \varphi_7 \ [m]$$

$$x_Z = x_E + a_7 \cos \varphi_7 \ [m]$$

$$\varphi_4 = atan2d(y_D - y_Z, x_D - x_Z) \ [°]$$

Terza Maglia



$$x_G = x_Z + a_4 cos\varphi_4 \; [m]$$

$$y_G = y_Z + a_4 sin\varphi_4 \; [m]$$

$$x_F = x_C + a_2 cos\varphi_2 \; [m]$$

$$y_F = y_C + a_2 sin\varphi_2 \; [m]$$

$$L_{GF} = \sqrt{(x_G - x_F)^2 + (y_G - y_F)^2} \; [m]$$

$$\Omega = atan2d(y_F - y_G, x_F - x_G)[m]$$

$$\epsilon = \cos^{-1}\left(\frac{L_{GF}^2 + a_5^2 - a_6^2}{2\, L_{GF}\, a_5}\right) \; [°]$$

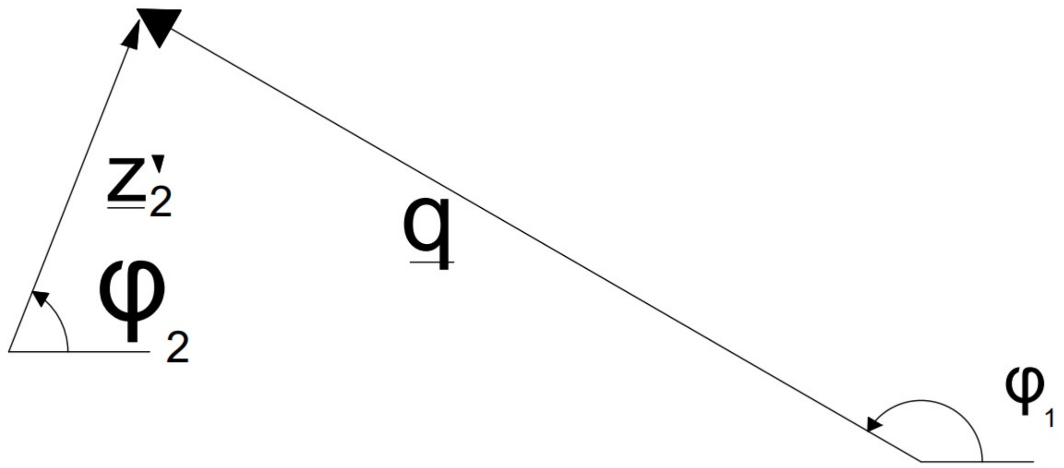
$$\varphi_5 = \epsilon + \Omega \; [°]$$

$$x_H = x_G + a_5 cos\varphi_5 \; [m]$$

$$y_H = y_G + a_5 sin\varphi_5 \; [m]$$

$$\varphi_6 = atan2d(y_H - y_F, x_H - x_F)[°]$$

ANALISI DI VELOCITÀ 1^a MAGLIA



$$\dot{q}\begin{Bmatrix}\cos\varphi_1\\ \sin\varphi_1\end{Bmatrix}+q\begin{Bmatrix}-\sin\varphi_1\\ \cos\varphi_1\end{Bmatrix}\dot{\varphi}_1-a'_2\begin{Bmatrix}-\sin\varphi_2\\ \cos\varphi_2\end{Bmatrix}\dot{\varphi}_2=\begin{Bmatrix}0\\0\end{Bmatrix}$$

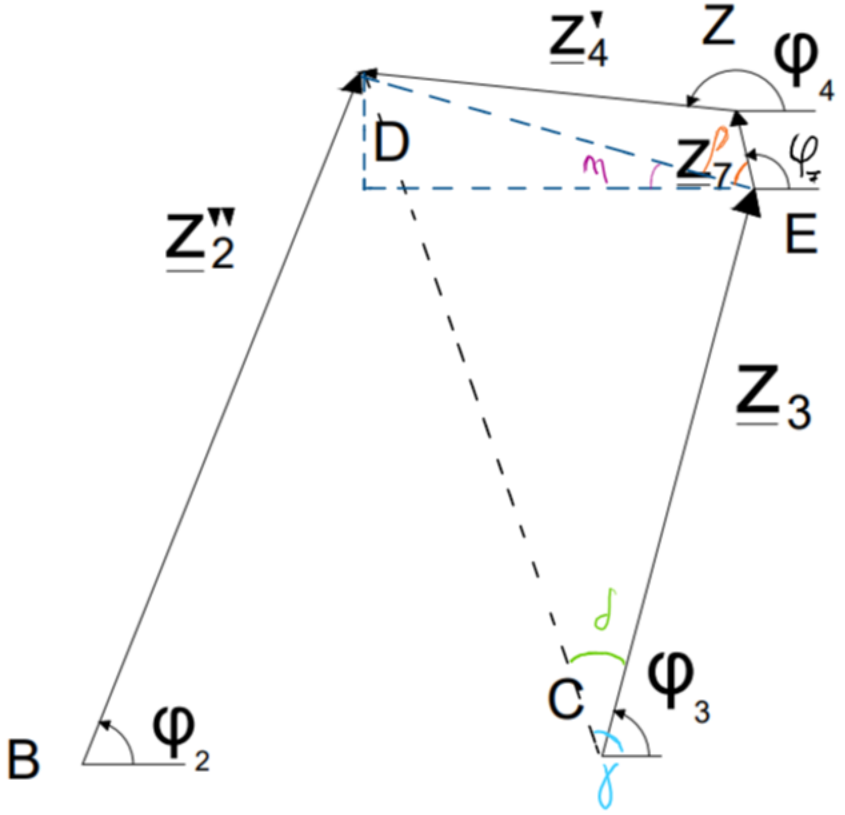
$$\begin{bmatrix}-q sin\varphi_1 & a'_2 sin\varphi_2\\ q cos\varphi_1 & -a'_2 cos\varphi_2\end{bmatrix}\begin{Bmatrix}\dot{\varphi}_1\\ \dot{\varphi}_2\end{Bmatrix}=-\begin{Bmatrix}cos\varphi_1\\ \sin\varphi_1\end{Bmatrix}\dot{q}$$

$$\begin{Bmatrix}\dot{\varphi}_1\\ \dot{\varphi}_2\end{Bmatrix}\frac{1}{\dot{q}}=\frac{1}{a'_2 q sin\,(\varphi_2-\varphi_1)}\begin{bmatrix}a'_2 cos\varphi_2 & a'_2 sin\varphi_2\\ q cos\varphi_1 & q sin\varphi_1\end{bmatrix}\begin{Bmatrix}cos\varphi_1\\ \sin\varphi_1\end{Bmatrix}$$

$$w_{\varphi_1}=\frac{\cos(\varphi_1-\varphi_2)}{q sin(\varphi_1-\varphi_2)}\quad [rad/m]$$

$$w_{\varphi_2}=\frac{1}{a'_2 sin(\varphi_1-\varphi_2)}\quad [rad/m]$$

ANALISI DI VELOCITÀ 2^a MAGLIA



$$a_3\begin{Bmatrix}-\sin\varphi_3\\ \cos\varphi_3\end{Bmatrix}\dot{\varphi}_3 + L_{DE}\begin{Bmatrix}-\sin(\varphi_7+\rho)\\ cos(\varphi_7+\rho)\end{Bmatrix}\dot{\varphi}_7 - a''_2\begin{Bmatrix}-\sin\varphi_2\\ \cos\varphi_2\end{Bmatrix}\dot{\varphi}_2 = \begin{Bmatrix}0\\0\end{Bmatrix}$$

$$\begin{bmatrix}-a_3 sin\varphi_3 & -L_{DE}\sin(\varphi_7+\rho)\\ a_3 cos\varphi_3 & L_{DE}cos(\varphi_7+\rho)\end{bmatrix}\begin{Bmatrix}\dot{\varphi}_3\\ \dot{\varphi}_7\end{Bmatrix} = \begin{Bmatrix}-a''_2 sin\varphi_2\\ a''_2 cos\varphi_2\end{Bmatrix}\dot{\varphi}_2$$

$$\begin{Bmatrix}\dot{\varphi}_3\\ \dot{\varphi}_7\end{Bmatrix}\frac{1}{\dot{q}} = \frac{1}{a_3 L_{DE} sin\,(\varphi_7+\rho-\varphi_3)}\begin{bmatrix}L_{DE}cos(\varphi_7+\rho) & L_{DE}\sin(\varphi_7+\rho)\\ -a_3 cos\varphi_3 & -a_3 sin\varphi_3\end{bmatrix}\begin{Bmatrix}-a''_2 sin\varphi_2\\ a''_2 cos\varphi_2\end{Bmatrix}w_{\varphi 2}$$

$$w_{\varphi_3} = -\frac{a''_2 w_{\varphi_2}\sin(\varphi_7+\rho-\varphi_2)}{a_3 sin(\varphi_7+\rho-\varphi_2)}\quad [rad/m]$$

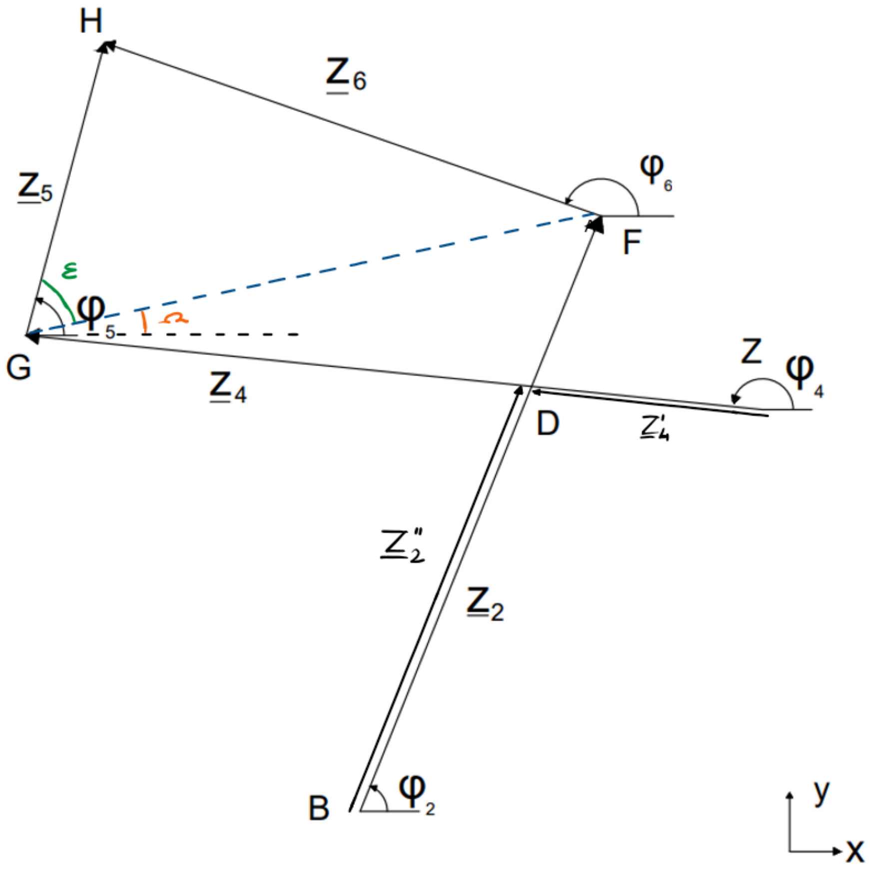
$$w_{\varphi 7} = -\frac{a''_2 a_3 w_{\varphi_2}\sin(\varphi_2-\varphi_3)}{a_4 L_{DE} sin(\varphi_7+\rho-\varphi_3)}[rad/m\,]$$

$$a_3\begin{Bmatrix}-\sin\varphi_3\\ \cos\varphi_3\end{Bmatrix}\dot{\varphi}_3 + a_7\begin{Bmatrix}-\sin\varphi_7\\ cos\varphi_7\end{Bmatrix}\dot{\varphi}_7 + a_4\begin{Bmatrix}-\sin\varphi_4\\ cos 4\end{Bmatrix}\dot{\varphi}_4 - a''_2\begin{Bmatrix}-\sin\varphi_2\\ \cos\varphi_2\end{Bmatrix}\dot{\varphi}_2 = \begin{Bmatrix}0\\0\end{Bmatrix}$$

$$-a_3 sin\varphi_3 w_{\varphi 3} - a_7 sin\varphi_7 w_{\varphi 7} - a'_4 sin\varphi_4 w_{\varphi 4} + a''_2 sin\varphi_2 w_{\varphi 2} = 0$$

$$w_{\varphi 4} = (a''_2 sin\varphi_2 w_{\varphi 2} - a_3 sin\varphi_3 w_{\varphi 3} - a_7 sin\varphi_7 w_{\varphi 7})/(a'_4 sin\varphi_4)\ \ [rad/m]$$

ANALISI DI VELOCITÀ 3^a MAGLIA



$$(a_4 - a_4')\begin{Bmatrix}-\sin\varphi_4\\ \cos\varphi_4\end{Bmatrix}\dot\varphi_4 + a_5\begin{Bmatrix}-\sin\varphi_5\\ \cos\varphi_5\end{Bmatrix}\dot\varphi_5 - a_6\begin{Bmatrix}-\sin\varphi_6\\ \cos\varphi_6\end{Bmatrix}\dot\varphi_6 - (a_2 - a''_2)\begin{Bmatrix}-\sin\varphi_2\\ \cos\varphi_2\end{Bmatrix}\dot\varphi_2 = \begin{Bmatrix}0\\0\end{Bmatrix}$$

$$\begin{bmatrix}-a_5\sin\varphi_5 & a_6\sin\varphi_6\\ a_5\cos\varphi_5 & -a_6\cos\varphi_6\end{bmatrix}\begin{Bmatrix}\dot\varphi_5\\ \dot\varphi_6\end{Bmatrix} = \begin{bmatrix}(a_4 - a_4')\sin\varphi_4 & -(a_2 - a''_2)\sin\varphi_2\\ -(a_4 - a_4')\cos\varphi_5 & (a_2 - a''_2)\cos\varphi_2\end{bmatrix}\begin{Bmatrix}\dot\varphi_4\\ \dot\varphi_2\end{Bmatrix}$$

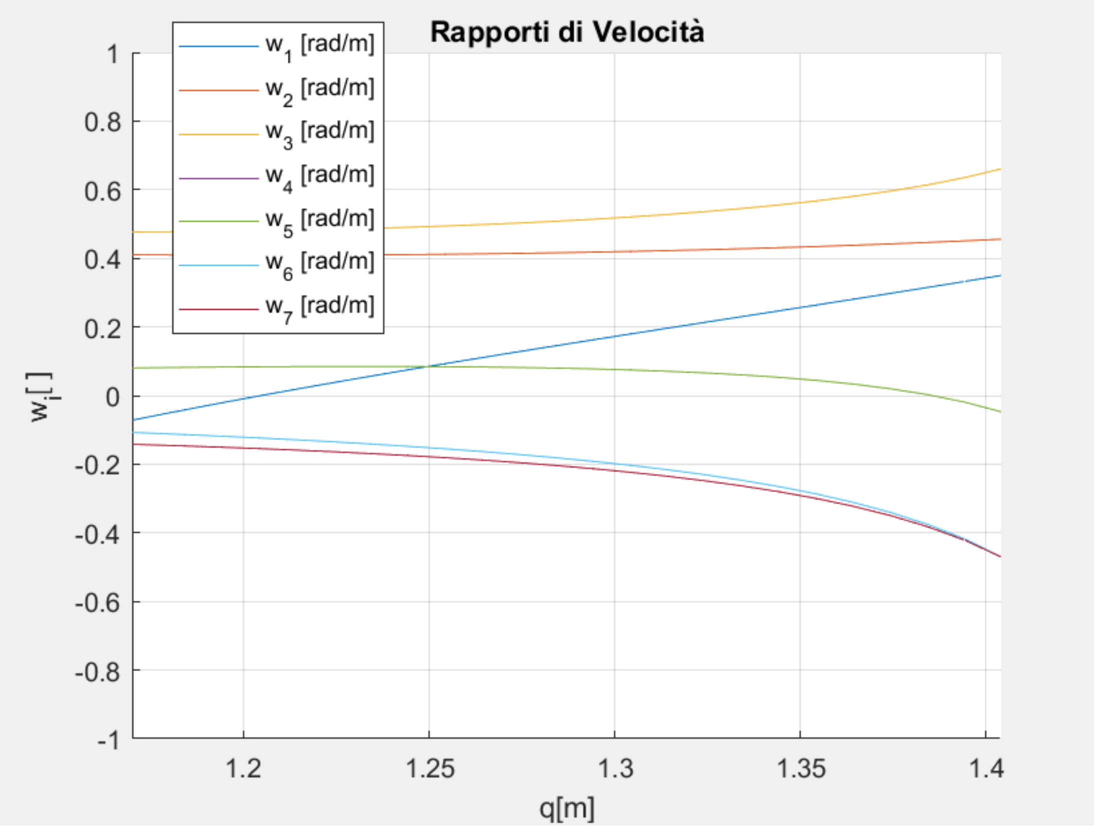
$$\begin{Bmatrix}\dot\varphi_5\\ \dot\varphi_6\end{Bmatrix}\frac{1}{\dot q} = \frac{1}{a_5 a_6\sin(\varphi_5 - \varphi_6)}\begin{bmatrix}(a_4 - a_4')a_6\sin(\varphi_6 - \varphi_4) & (a_2 - a''_2)a_6\sin(\varphi_2 - \varphi_6)\\ (a_4 - a_4')a_5\cos(\varphi_5 - \varphi_4) & (a_2 - a''_2)a_5\cos(\varphi_2 - \varphi_5)\end{bmatrix}\begin{Bmatrix}w_4\\ w_2\end{Bmatrix}$$

$$w_{\varphi_5} = \left(\frac{(a_4 - a_4')\sin(\varphi_6 - \varphi_4)\,w_4 + (a_2 - a''_2)\sin(\varphi_2 - \varphi_6)w_2}{a_5\sin(\varphi_5 - \varphi_6)}\right)\ [rad/m]$$

$$w_{\varphi_6} = \left(\frac{(a_4 - a_4')\sin(\varphi_5 - \varphi_4)\,w_4 + (a_2 - a''_2)\sin(\varphi_2 - \varphi_5)w_2}{a_6\sin(\varphi_5 - \varphi_6)}\right)\ [rad/m]$$

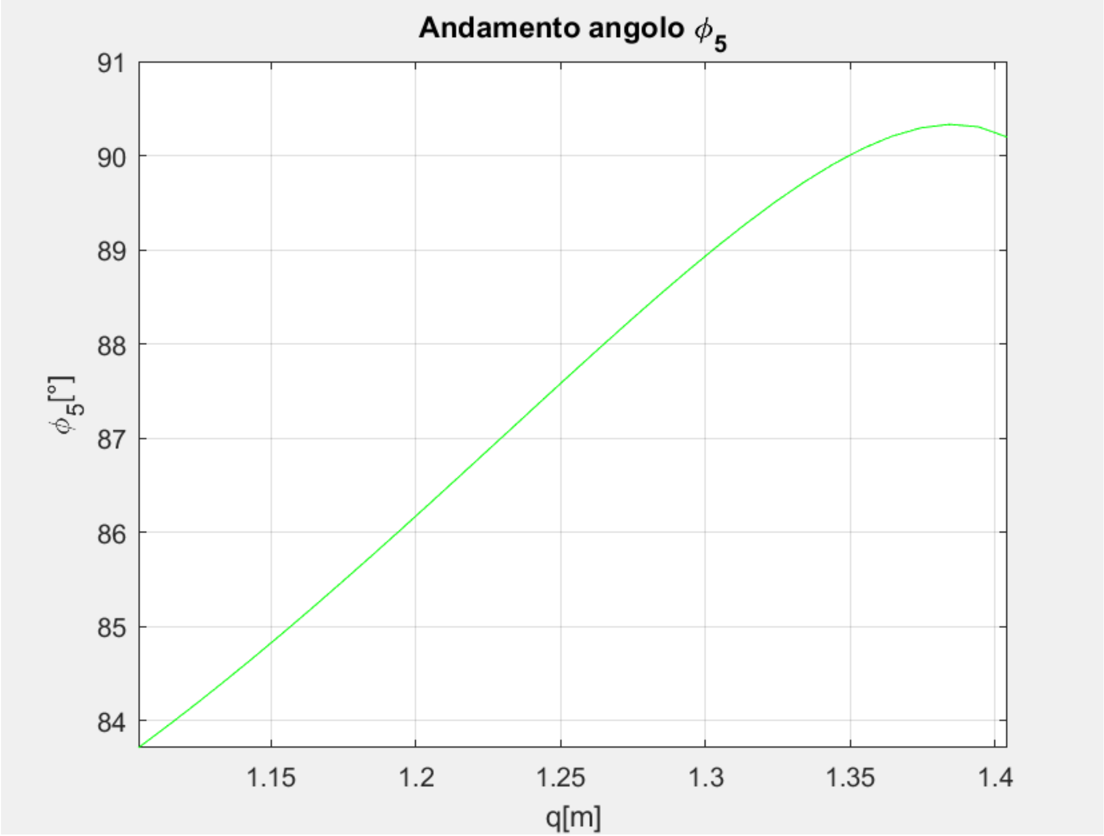
Grafico Andamento Rapporti di Velocità

- $w_{\varphi 4}$ e $w_{\varphi 7}$ sono sovrapposti
- $w_{\varphi 1}$ cambia segno

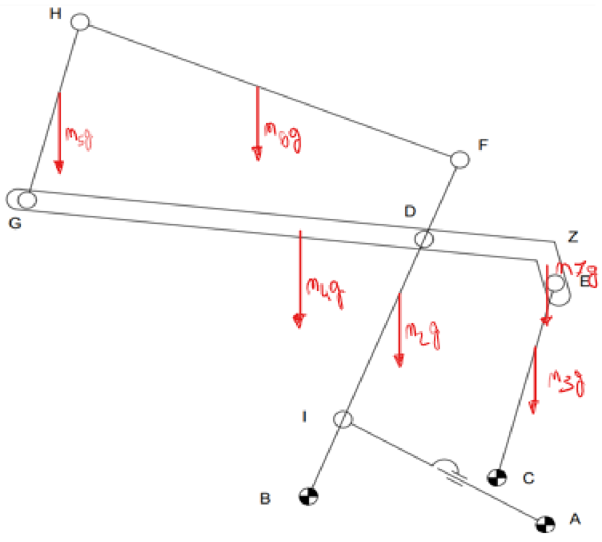


ANALISI ANDAMENTO ANGOLO φ_5

- Resta in un intorno di 90°



ANALISI STATICA



$$\underline{S} = \begin{Bmatrix} F \\ -m_2 g \\ -m_3 g \\ -m_4 g \\ -m_5 g \\ -m_6 g \\ -m_7 g \end{Bmatrix} \qquad \underline{u} = \begin{Bmatrix} q \\ y_{G2} \\ y_{G3} \\ y_{G4} \\ y_{G5} \\ y_{G6} \\ y_{G7} \end{Bmatrix} \qquad \underline{w} = \frac{d\underline{u}}{dq} = \begin{Bmatrix} 1 \\ y'_{G2} \\ y'_{G3} \\ y'_{G4} \\ y'_{G5} \\ y'_{G6} \\ y'_{G7} \end{Bmatrix}$$

$$\underline{u} = \begin{Bmatrix} q \\ y_B + \frac{a_2}{2} sin\varphi_2 \\ y_C + \frac{a_3}{2} \sin\varphi_3 \\ y_C + a_3 sin\varphi_3 + a_7 sin\varphi_7 + \frac{a_4}{2} sin\varphi_4 \\ y_B + a''_2 sin\varphi_2 + (a_4 - a'_4) sin\varphi_4 + \frac{a_5}{2} sin\varphi_5 \\ y_B + a_2 sin\varphi_2 + \frac{a_6}{2} sin\varphi_6 \\ y_C + a_3 sin\varphi_3 + \frac{a_7}{2} sin\varphi_7 \end{Bmatrix}$$

ANALISI STATICA

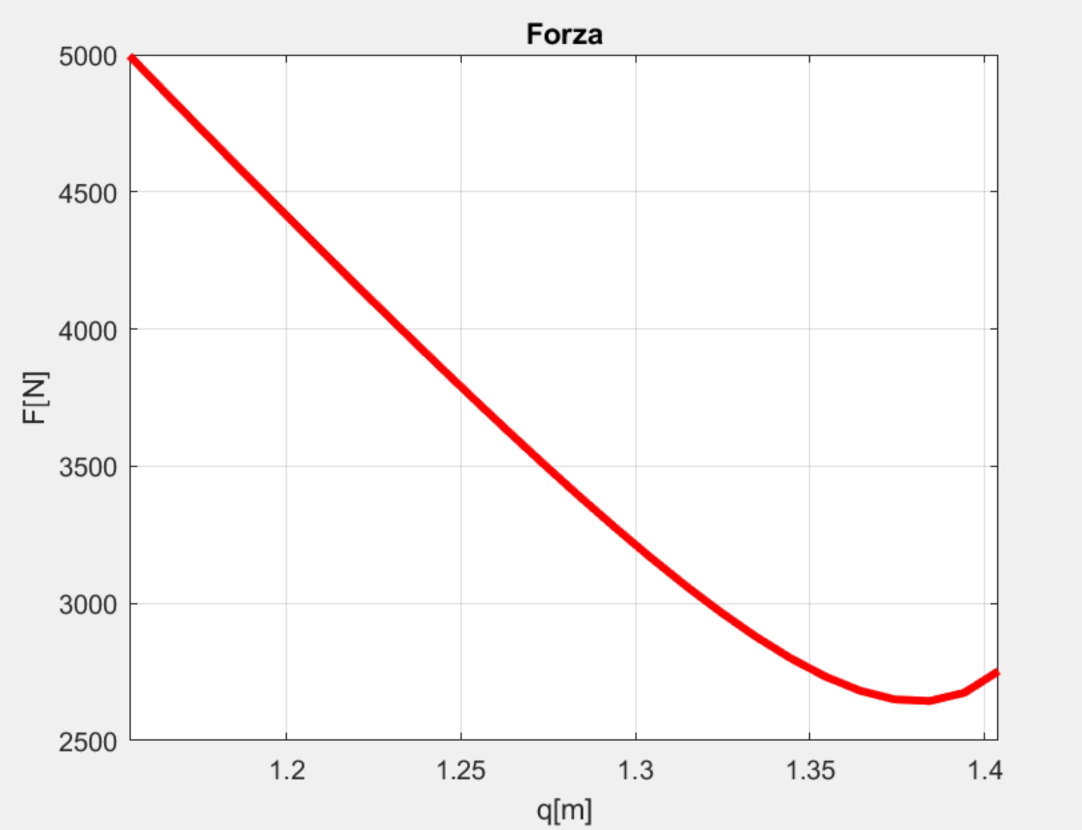
$$\underline{w} = \begin{Bmatrix} 1 \\ \frac{a_2}{2} cos\varphi_2 w_2 \\ \frac{a_3}{2} \cos\varphi_3 w_3 \\ a_3 cos\varphi_3 w_3 + a_7 cos\varphi_7 w_7 + \frac{a_4}{2} cos\varphi_4 w_4 \\ a_2'' cos\varphi_2 w_2 + (a_4 - a_4') cos\varphi_4 w_4 + \frac{a_5}{2} cos\varphi_5 w_5 \\ a_2 cos\varphi_2 w_2 + \frac{a_6}{2} cos\varphi_6 w_6 \\ a_3 sin\varphi_3 w_3 + \frac{a_7}{2} cos\varphi_7 w_7 \end{Bmatrix}$$

$$\underline{S}^T \underline{w} = 0$$

$$F = m_2 g \left(\frac{a_2}{2}\right) cos\varphi_2 w_2 + m_3 g \left(\frac{a_3}{2}\right) cos\varphi_3 w_3 + m_4 g \left(a_3 cos\varphi_3 w_3 + a_7 cos\varphi_7 w_7 + \frac{a_4}{2} cos\varphi_4 w_4\right) + m_5 g \left(a_2'' cos\varphi_2 w_2 + (a_4 - a_4') cos\varphi_4 w_4 + \frac{a_5}{2} cos\varphi_5 w_5\right) + m_6 g \left(a_2 cos\varphi_2 w_2 + \frac{a_6}{2} cos\varphi_6 w_6\right) + m_7 g \left(a_3 cos\varphi_3 w_3 + \frac{a_7}{2} cos\varphi_7 w_7\right)$$

Risultato Analisi Statica

- La forza necessaria diminuisce all’aumentare della corsa del pistone



SIMULAZIONE MECCANISMO