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Firma dello studente

Laura Paoletti

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Introduction

Since the end of the 20th century, European and US electricity markets experienced a liberalisation process, triggered by the expectations of greater efficiency arising from a more competitive industry. This wave of liberalisation affected the Italian power market as well, driving the gradual evolution from the vertically integrated market managed by ENEL, to the current more liberalised power exchange.

The subsequent restructuring of the markets' design, brought electricity markets closer to financial markets. However, unlike any other financial asset, electricity presents some distinctive peculiarities that make the electricity market unique. One of the most remarkable features is that existing power systems do not allow to store electricity extensively, at reasonable costs. Hence, the balance between demand and supply needs to be ensured on a continuous basis. The so-called non-storability of electricity results in the exceptionally high volatility of spot prices, coupled with sudden price spikes. As these unique features of electricity strongly impact prices' behaviour, integrating them in the specifications of a mathematical model for electricity prices is crucial.

The academic literature dealing with the modelling of electricity prices has increasingly grown since the liberalisation of the sector, providing a rich variety of electricity pricing models. This dissertation focuses on the reduced-form approach that models economic factors through stochastic processes to capture the peculiar features of electricity prices.

One of the earliest and most notable electricity pricing models is the mean-reverting diffusion model proposed by Lucia and Schwartz (2002). This model sets the basis from which more complex and refined models were developed.

This paper attempts to assess whether the specification of the mean-reverting diffusion model of Lucia and Schwartz (2002) can adequately fit the Italian electricity prices. Therefore, the study applies such model to the Italian power market, adjusting it to better capture the characteristics that emerged from the analysis of the observed time series of prices.

More specifically, this dissertation is structured as follows;

Chapter 1 describes the fundamental features of electricity markets to be considered in order to provide a correct mathematical representation of electricity prices. Accordingly, the chapter outlines the most remarkable characteristics of the electricity supply chain, the system's design, and time structure, with a specific reference to the Italian electricity market. In addition, we identify some key stylized facts of electricity prices resulting from such features.

Chapter 2 deals with the main electricity derivatives contracts that are nowadays traded in electricity markets. The rise of a liberalised power sector fostered, indeed, the development of new risk management tools to control risk through properly designed hedging instruments. This chapter presents an overview of the most standard electricity derivatives contract, as well as some exotic ones, with a focus on the Italian derivatives exchange.

Chapter 3 provides a survey of the most relevant classes of electricity pricing models that have been proposed by the academic literature. We present an overview of the most remarkable modelling approaches, giving particular relevance to the spot-based approach.

Chapter 4 examines in depth the mean-reverting diffusion model of Lucia and Schwartz (2002), analysing separately the two components it consists of, namely the deterministic and the stochastic one. To empirically illustrate the features of these components, the chapter presents some simulations of the original model as well as of some of its possible variants.

The simulations conducted in this chapter and the computations performed in the following chapter have been carried out through the use of the statistical software R. The Appendices at the end of this dissertation report the codes implemented in these chapters.

Chapter 5 presents an application of the model proposed by Lucia and Schwartz (2002) to the Italian power market. After an initial description of the dataset under analysis, the chapter outlines the estimation process of the model, explaining in detail how the non-linear least squares method has been implemented. The analysis further proceeds by adapting the model to the specificities of the observed time series. Therefore, the seasonal function has been adjusted to cope with the strong weekly seasonality that emerged from the analysis of the data and a jump component, estimated by maximum likelihood method, has been integrated to model the price spikes.

Finally, the last chapter summarises the main results that emerged from the study and suggests possible further developments of this dissertation.

Chapter 1

Electricity Markets: Basic Design and General Features

1.1 The Electricity Supply Chain

Electricity is a form of energy that denotes the ability to perform a work through an ordered flow of electrons.

The process that makes it available to final consumers in a useful form has developed into a complex network composed of generators, transmission and distribution wires, and loads, as outlined in the so-called Electricity Supply Chain (hereafter ESC). From the production phase, up to the dispatching one, the ESC identifies and defines the five main tasks that all agents fulfill in the electricity system. More specifically, production, transmission, distribution, retailing, and dispatching are the five stages it consists of.

The production phase is the one in which power plants generate electricity through the transformation of primary energy sources. In this phase, the range of processes and the technologies that plants can implement is rather wide, as well as the spectrum of sources of energy generation. The latter can vary from fossil fuels to nuclear, including also the more recent-developed class of renewable energy sources, such as geothermal energy, solar energy, wind energy, and biomass energy.

The transmission is the phase in which the authorised entities, namely the System Operators (SOs), move electricity from the plants where it was generated to sub-stations, whose purpose is to convert high voltage electricity into lower voltage electricity, to subsequently feed it into the distribution network. The purpose of SOs is to operate within the network to ensure order and avoid system imbalances. If both the ownership of the transmission networks and their control and usage belong to the SOs, then they are defined as Transmission System Operators (TSOs). On the contrary, if a separate entity owns the network, System Operators are called Independent System Operators, better known as ISO.

The distribution phase is managed by the Distribution System Operators (DSOs) who transfer medium-low voltage electricity from sub-stations to final consumers. Such entities are also in charge to carry out a control on the power quality, to ensure that voltage and current do not cross the predetermined acceptance thresholds and, consequently, guarantee the constant balance of the net.

In the retail phase, retailer companies sell electricity to end-customers, hence connect electricity markets to electricity users. They stipulate agreements with power plants for the provision of electricity, based on forecasts for both short-term and middle-term demand. The retailing task can be performed, also, by entities involved in other functions along the ESC. Often retailers coincide, indeed, with the DSOs.

Lastly, the dispatching phase consists in the optimal selection of plants and loads, defined by Cretí and Fontini (2019) as the points within an electricity system where electrons meet a resistance and, therefore, the current is used to perform a work. This phase differs from the transmission one since it does not focus only on moving electricity towards loads but considers, also, the optimality conditions, such as the cost minimisation or the profit maximisation, that support the flow of electricity and that allow SOs to rank plants and loads.

1.2 Classification of Electricity System Designs

The gradual evolution of the structure of electricity markets and the recent process of liberalisation, that took place in many countries worldwide, have led to the unbundling of the traditional vertically integrated supply chain and have implemented the separation of the roles through the ESC. Through such development, the degree of vertical integration of the entities operating along the ESC has started to diversify. Consequently, different market structures have risen, evolving from integrated monopolies towards more competitive markets.

Before the liberalisation of electricity markets, vertically integrated power industries prevailed. Monopoly has been, indeed, the most common industrial organisational structure worldwide and, nowadays, represents the current status of those industries that have not experienced a process of restructuring. According to this design, a unique firm, or a set of companies behaving like a single entity, carries out all the activity of the ESC. Such entity is usually governmental and serves a given area, exploiting the vertically integrated structure to minimise risks and transaction costs.

The recent wave of electricity markets liberalisation, that occurred in Europe and in the USA by the end of the 20^{th} century, induced the restructuring of the aforementioned electricity markets' design. After the market opening, a separation among the tasks of the ESC occurred. The unbundling of the production phase from the rest of the supply chain represents a common first step towards market liberalisation. This scenario involves some independent producers, owning and managing power plants, to supply electricity to a unique buyer that performs all the other tasks.

Liberalisation introduced the principle of matching supply and demand for determining the price of electricity. Consequently, the prices have become more competitive and more volatile, in accordance with the free price competition, typical of financial markets. If such deregulating process was fully accomplished, it would result in a market structure analogue to a wholesale and retail market model. The resulting supply chain, established on this competitive model, would be characterised by the total disaggregation of its tasks among different agents. From the production phase to the delivery phase, the functions of the ESC would be split and performed by separated entities, competing among them to generate electricity, transmit it and retail it to final consumers.

The fully liberalised market model is the antithesis of the vertically integrated one. As discussed by Pollitt (2008), nowadays, there is, however, no significant example of electricity markets that achieved it totally. Even though there is ample freedom among countries to choose the degree of liberalisation, the electricity deregulating process is, eventually, constrained by national regulations. A common setting for markets that experienced some restructuring tends, indeed, to involve distinct entities generating power and supplying end-consumers. On the contrary, the transmission and distribution phases are likely to reflect the monopoly model, remaining governmental and handled by a unique agent.

1.3 Electricity Market Time Structure

In electricity markets, contractual arrangements occur before the physical delivery of electricity. Wholesale markets can be, therefore, classified according to their specific time-structure, considering the time lag between the arrangement among agents and the real-time delivery. Each country presents its own peculiarities, which may differ from market to market. However, the underlying microstructure exhibits some recurrent features that allow a general categorisation based on different temporal-horizon and tasks. More specifically, as Aïd (2015) (Market Microstructure In *Electricity Derivatives* (pp. 9-21) claimed, such categorisation provides that wholesale markets can be divided into three sub-markets:

- intraday market and balancing mechanism
- day-ahead market
- forward market

A key feature of electricity is that, unlike other energy commodities such as natural gas, oil, and coal, it cannot be stored at reasonable costs. Therefore, electricity must be consumed the moment it is exchanged. Consequently, the balance between demand and supply is very fragile, since it must be ensured on a continuous basis. The retailers that are not able to preserve the balance between the amount of electricity demanded by their clients and the amount of electricity supplied are charged a fee to restore the necessary equilibrium. Hence, their aim is to preserve the balance of the portfolio they own, adjusting it through intraday transactions, just few hours before the gate closure, i.e. the moment after which only the SOs are allowed to operate, in order to assure the equilibrium and the safety of the system.

Similarly, the objective of the SOs is to guarantee the overall balance of the electricity system in which they operate, by balancing production and consumption of electricity.

To pursue this purpose of equilibrium, the electricity markets established two coexisting systems: an intraday market, in which electricity market agents can manage to reduce the possible imbalances between demand and supply of their customers, through market transactions among themselves, and a balancing mechanism, used by the SOs to maintain the whole system balanced. The latter is carried out concurrently to the balancing transactions of the intraday market but continues also after the gate closure. The main purpose of the balancing mechanism is to guarantee the security of the grid, avoiding drops in voltage and consequent harmful imbalance issues, such as *brownouts*. By performing their task, namely, by adjusting the system upward or downward, according to whether the grid is experiencing a lack or an excess of supply, the SOs establish a market for these adjustments and ensure the transparency of the prices the agents need to pay to cover such imbalances.

In the day-ahead market, as suggested by the name itself, customers can buy or sell electricity each day for a specific hour, or time interval, of the following day. This process is handled through an auction in which electricity producers submit their offers, defining the volume and the price of electricity they are willing to accept for each time period of the following day. The offers are matched, through a price-merit criterion, with the bids that constitute the demand for electricity of the market participants. As a result, this process outlines the total volume and the corresponding price of electricity expected to be traded the next day.
A unique characteristic of the day-ahead electricity market is that prices, submitted by market participants to buy and sell, can be negative. Negative prices imply that electricity might be, in some circumstances, a waste product. This means, according to De Jong and Sewalt (2003), that its destruction might be less harmful than its production. Hence, agents might be willing to pay to sell electricity or to be remunerated to buy it.
As mentioned above, in electricity markets, the balance between demand and supply must be continuously maintained. If an imbalance occurs, for instance during the night, for an excess of supply, submitting negative prices might imply lower opportunity costs than those required to reduce the production or to shut a plant down. Therefore, the lack of flexibility of power plants to adapt to disequilibrium is the reason for the existence of negative prices in the electricity day-ahead market.

The forward market refers to the electricity market in which participants can trade electricity, in a future delivery period, at a pre-determined price. As liberalisation spread across electricity markets, the forward market gained much relevance. Long-term contracts, such as futures and forwards, allow, indeed, to cope with price risk due to short-term high volatility, typical of electricity spot prices. In every electricity market, organised exchanges submit standardised future contracts and manage to reduce the counterparty's risk. However, most electricity long-term contracts are traded OTC with the signing of a master agreement that sets the terms of the trades and mitigates the risk for the parties. Electricity forward contracts differ from other energy commodity contracts because of a peculiar feature of electricity, the non-storability. Therefore, market participants that want to hedge the risk of short-term volatility through a long-term contract would be required to negotiate a contract for each hour of the validity period of the contract. Of course, this would not be an efficient solution. It follows that forward and futures contracts in the electricity market have to ensure the delivery of electricity over a pre-determined period, not at a specific hour, as commonly occurs.

The forward, the day-ahead, the intraday, and the balancing markets are organised according to a temporal sequence which allows breaking down electricity markets into different sub-markets with subsequent time horizons. Starting from the forward market, which can precede the moment of the physical delivery of electricity for a period up to six years, the subsequent sub-market is the day-ahead, which is temporally located much closer to the real-time setting, more precisely, one day before the delivery period. The intraday market follows, opening just a few hours before the physical delivery. The sequence ends with the balancing market which is the closest to the real-time delivery. Such structure is common among electricity markets. However, the precise opening and closing time of the markets are specific to each country.

1.4 The Italian Electricity Market

1.4.1 An Overview of the Current Electricity System

The Italian electricity market, better known as Italian Power Exchange (IPEX), is a centralised marketplace managed by a market operator named GME, acronym for the Italian *Gestore del Mercato Elettrico*. The objective of the GME is to reach short-run and long-run efficiency within the electricity system, following the principles of free competition and transparency, and in accordance with the provisions established by the regulatory authority for energy and environment (ARERA). GME runs the four different sub-markets that IPEX consists of, namely the forward market, the day-ahead market, the intraday market, and the market where daily products are traded on a continuous basis, under the guidance of the Ministry of Economic Development.

IPEX is a wholesale power market with a non-mandatory nature. Hence, eligible purchasers and wholesalers submit bids and offers voluntarily. The day-ahead market hosts most of the electricity trading transactions through an hourly auction mechanism to which agents participate by submitting offers indicating the volume and the limit price they are willing to accept. The market session opens at 8.00 am on the ninth day preceding the day of delivery and closes at 12.00 pm on the day before the day of delivery. Once the market session is over, the offers are examined on the basis of an economic criterion and in compliance with the transmission constraints existing between the zones that divide the Italian electricity system. The accepted demand and supply offers result in a system marginal price for each hour and for each electrical zone. The average of the marginal prices, weighted for the amount of electricity traded, determines, in turn, a unique national price (PUN). In case of impossibility of compliance with the programs defined in the day-ahead market, agents are allowed to modify their offers through further trades during the seven sections that make up the Italian intraday market. The offers are assessed with the same criterion applied in the day-ahead market, but the final price is the zonal one rather than the average national price.

Currently, the Italian electricity market is organised as a zonal system that splits the transmission network into six different regions. The entity responsible for the management of each market zone is a TSO named Terna which identifies the zones as the geographical areas within which no internal transmission constraint is set. The division of the net into these zones is functional to the efficient management of the transit of high voltage electricity across the national territory, despite the limited interconnection capacities of the Italian transmission network.

A critical issue for the Italian electricity industry derives from the kind of sources employed for electricity production, as outlined by Cavallo and Termini (2007). The most commonly exploited energy source is, indeed, natural gas. This is reflected in the relatively high cost of production implied by the fossil-fuels and oil-related inputs. The relevance of natural gas within the Italian electricity scene has not been affected by the recent incentives of the European Emission Trading System which have led the industry to resort more to renewable sources but have left, indeed, the share of natural gas employed in the total production rather constant. On the contrary, because of the shift to renewables occurring in the last years, the share of oil has been decreasing, inducing a slight reduction of the prices, which has been partially offset by a general increase in the demand.

1.4.2 The Italian Liberalisation Process

The structure of the Italian electricity market has not always been as above-described, but rather it has been evolving over time, following a process of liberalisation that involved electricity markets worldwide. Before this liberalisation process was implemented, the Italian electricity market was vertically integrated. In 1962, as a result of the foundation of the governmental entity ENEL, the electricity sector was nationalised. ENEL exercised a monopoly on the market, aiming to exploit the benefits deriving from the organisation of the industry as a natural monopoly, in which the costs are subadditive. It was, indeed, a common belief that achieving large economies of scale was the most efficient way to deal with the increase in electricity demand due to the rapid industrialisation of the 20$^{\text{th}}$ century. However, such belief was soon reverted when a new trend of thought supporting the privatisation of the electricity sector spread.

Starting from 1999, the Italian electricity system has progressively enacted its deregulation, following the steps of the global pioneer, namely the UK. Indeed, the liberalisation of the Italian electricity sector entered into force that year, in April, when the Legislative Decree 79/1999, known as *Bersani Decree*, was first implemented. This decree, which was the national transposition of the European Directive 96/92/EC, outlined the crucial steps for a gradual shift towards a liberalised market, calling for the unbundling of the ESC phases. Yet the implementation of the decree showed, in the beginning, only slight results as, for instance, the production, transmission, and dispatching phases were still largely dominated by ENEL.

A greater degree of liberalisation was reached in 2003 with the European Directive 2007/54/EC which anticipated the launch of IPEX and the consequent opening of the market to all commercial consumers. The private sector had to wait for the free competition to be enforced until July 2007, when the Decree 73/2007 was implemented.

As concerns the transmission and dispatching activities, in 1999, in response to the Bersani Decree, a new entity operating the transmission network as an ISO was established. Such entity, named GRTN, managed the grid under the direction of the Ministry of Production Activity, while the new-born Terna, at that time held by ENEL, owned the line. In 2005 Terna acquired GRTN, merging in a unique transmission company that both owned and managed the transmission network. To guarantee non-discriminatory access to the line, Terna set a limit of 20% on the property of its shares.

1.5 Key Characteristics of Electricity Markets

Liberalised electricity markets share some distinctive features with financial markets. They are both competitive markets and they both stand on the principle of matching demand and supply of participants. However, unlike any other financial asset, electricity presents some unique features that make the electricity market one of its kind.

The particular way in which electricity flows through the ESC, from power plants to end-consumers, the different cost of production that generators bear, as well as the distinct geographical, productive, and social features specific for every country worldwide, are all reflected on the spot and forward prices of electricity wholesale markets. Consequently, each market presents its own peculiarities.

Nevertheless, the vast literature that analyzed this topic has highlighted some *stylized facts* of electricity prices, recurrent among markets, regardless of the aforementioned specificities. Since the impact of these common features on prices is significant, they need to be considered for a correct specification of mathematical models aiming to accurately describe spot and forward prices.

The following subsections describe the main characteristics that distinguish the electricity markets and that have to be incorporated for an efficient formulation of models for risk management of electricity prices.

1.5.1 Non-storability

One of the most remarkable features of electricity markets is that electricity cannot be stored at reasonable costs. Unlike other common energy commodities, electricity can be treated as a *flow commodity*, since it cannot be easily stored and transported.

Currently, existing batteries are very expensive and can hold only a limited amount of electricity. The most efficient mean of storage of electric energy potential on a large scale is pumped storage, which exploits the energy potential of hydroelectric reservoirs for balancing energy demand variations. However, such storage solution is not totally reliable and extensively enforceable as it relies on the availability of a sufficient reserve of water and on the specific hydroelectric potential of each geographical area.

The general absence of reserves requires that plants and loads are always able to meet the demand for electricity, maintaining the system balanced on a second-by-second basis. It follows that electricity prices are very volatile since they are easily affected by any exogenous shock in demand and supply.

The consequence of this key peculiarity is that it implies the failure of the common spot-forward relationship. As highlighted by Geman and Roncoroni (2006), the literature concerning the pricing of commodities traded in financial markets, typically, employ the concept of *convenience yield*, i.e. the benefit arising from holding a physical asset rather than a forward contract written on that same asset, to link spot prices and forward prices. The convenience yield captures, indeed, the expectations of the market about the future availability of the commodity it refers to. Hence, it allows performing forecasts about future demand and availability of an underlying asset, affecting the associated forward prices more significantly than the spot prices.

However, in the electricity market, the concept of the convenience yield and its forecasting properties break down. As electricity is not efficiently storable at reasonable costs, the benefit deriving from holding it cannot exist.

Therefore, as a direct consequence of the non-storability of electricity, spot and forward price processes have to be sufficiently complete to summarise the intrinsic properties of electricity, without recurring to the above-mentioned yield.

1.5.2 Probability Distribution of the Prices

Traditional financial models commonly rely on the assumption of log-normality of the prices. Even though the distributions of financial data typically tend to exhibit some excess of kurtosis with respect to a Gaussian distribution, the hypothesis of Normality adapts quite well to this kind of data. Consequently, commodity pricing literature has widely exploited it to model this kind of historical series. The most classic example of the implementation of the log-Normality assumption is surely the Black-Scholes-Merton pricing model.

The log-Normal distribution assumption, however, does not fit the electricity spot prices. There is significant evidence in the literature of a non-negligible deviation of the logarithmic returns of the electricity spot price from Normal distribution. Lucia and Schwartz (2002) show, for example, that the series of the log-prices of the Nordic Power Exchange, during the sample period 1993-1999, revealed an estimated positive skewness and an estimated kurtosis of 4.5. The unsuitability of the Gaussian assumption was confirmed also by Geman and Roncoroni (2006), that worked on the daily average prices between 1997 and 1999 of three of the major US power markets, finding an even more leptokurtic series (Figure 1.1).

Electricity markets are, indeed, subject to large price fluctuations across different seasons, days of the week or hours of the day. This is reflected in the fat tails of the log-returns of the spot prices, which suggest a leptokurtic distribution. Moreover, the tails tend to be thicker on the right side of the distribution. Electricity spot prices show, indeed, a positive skewness, indicating that spot prices are more likely to assume high extreme values rather than low ones. Moreover, the possible presence in the day-ahead market of negative spot prices, discussed above, is a further argument against the applicability of the common log-Normal price processes to the electricity sector.

Unlike spot prices, forward prices are more related to those of other commodity prices. It is indeed crucial when approaching the forward market, to consider the theory, originally developed by Longstaff and Wang (2002), about electricity forward prices being significantly less volatile than expected spot ones. As further underlined by De Jong and Sewalt (2003), the large fluctuations due to the non-storability of electricity, typical of the spot market, are reduced in the long-run. The longer the validity period of the

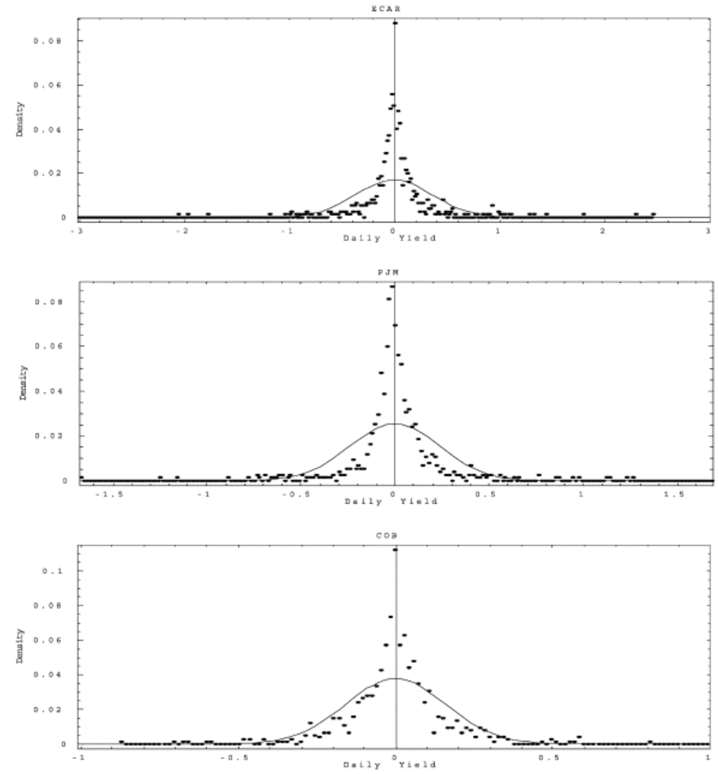

Figure 1.1: Empirical price returns distributions of ECAB, PJM, and COB vs. Normal distributions with equal means and variances.
Source: Geman, H., & Roncoroni, A. (2006). Understanding the Fine Structure of Electricity Prices. The Journal of Business, 79(3), pp.1225-1261.

forward contract, the more the volatility decreases. Hence, both skewness and kurtosis do not significantly depart from the Gaussian values and the assumption on log-Normal distribution might fit the data.

1.5.3 Volatility Clusters and Spikes

The non-storability of electricity and the consequent need to ensure a constant balance between demand and supply results in an exceptionally high volatility of spot prices. The standard deviation of the logarithmic returns of the electricity spot price is, indeed, significantly more relevant than that observed for other energy commodities. Furthermore, such volatility is also not constant, since it is subject to clustering. Accordingly, volatility conditioned at time t is correlated with volatility conditioned at time $t-1$, outlining periods in which price variations are high and others in which they are lower. To handle this stochastic variance, the literature developed different classes of stochastic volatility models to efficiently capture, in continous time, the phenomenon of the volatility clustering. The Heston model, assuming an arbitrary volatility, the CEV model, which represents both the volatility and the leverage effect of equities and commodities, and the SABR model, a widely diffused volatility smile model for derivatives market are just a few examples of those models which allow treating the random distribution of the volatility term. However, most of the studies concerning electricity pricing models have resorted to ARCH-type discrete-time heteroskedastic models, in which the volatility at time t conditioned to previous values is deterministic. These include the ARCH models as well as the GARCH models, which are able to represent volatility terms following, respectively, an autoregressive process and an autoregressive moving average process.

In addition to the phenomenon of volatility clustering, electricity spot price processes exhibit also sharp *spikes*, namely sudden and extreme jumps which run out in a very short time-frame. Typically, spikes occur close to lower-intensity jumps, setting up clusters of high volatility. (Figure 1.2)

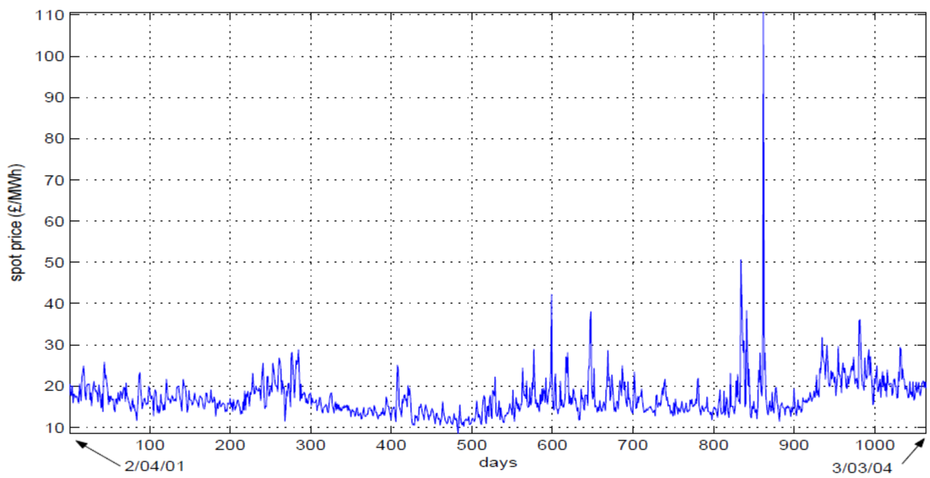

Figure 1.2: Averaged daily prices in England and Wales from 2/04/01 to 3/03/04.
Source: Cartea, A., & Figueroa, M. G. (2005). Pricing in Electricity Markets: A Mean Reverting Jump Diffusion Model with Seasonality. Applied Mathematical Finance, 12(4).

This is due to the limits on the storage of electricity and to the lack of demand elasticity. As stated by Cretí and Fontini (2019), the price jumps tend, indeed, to gather during the *peak hours*, when the generation capacity of the plants is fully exploited to meet the maximum level of the demand, for example, during the central hours of a working day. To capture this peculiar behaviour, a strand of the literature claims that the model representing the spot prices should include a jump component, as originally proposed by

Merton (1976) with his jump-diffusion model for equity prices. Another strand, supported by Schmidt (2008), proposes the shot noise class of models to account more flexibly for extreme spikes, as well as for seasonality and mean reversion which, as subsequently discussed, are key features of electricity prices.

The relevance of the influence of the spikes on electricity price processes cannot be neglected, as the spikes are reflected in the above-mentioned high volatility and, in turn, in the non-Gaussian distribution of the returns. The accuracy of such causal relationship has been proved by Mayer, Schmid, and Weber (2011) who extracted the spikes from an initially non-Normal series of prices of electricity traded in the European Energy Exchange, obtaining empirical returns predicted by a Gaussian distribution.

1.5.4 Seasonality

Another common assumption of traditional financial models is requiring the returns to be independently distributed. Such condition is verifiable conducting an autocorrelation test on the logarithmic returns which allows evaluating the existing time-dependency among different values within a series. A resulting correlation coefficient close to zero would imply an independent distribution.

Electricity markets, however, typically exhibit a strong level of autocorrelation. The literature has shown that spot prices often present predictable patterns, attributable to some form of seasonality. Such fluctuations are recorded within different time-periods, including the year, the month, the week, up to the single day. An intra-day seasonality, for instance, is observed in every electricity market and it is represented by peaks of prices. Accordingly, the analysis of the electricity historical series distinguishes the daily prices among the base-load prices, namely, the daily average of all hourly prices recorded during the day, the peak prices, namely, the daily average of the hourly prices related to the hours of greater energy demand, and the off-peak prices, which are the average prices once the peak prices are extracted. Seasonality can occur also within the week. Energy consumption and, in turn, energy prices regularly decline during the weekend, when industrial and commercial activities reduce.

In the electricity sector, besides the aforementioned patterns which are common among all markets, it is important to consider also the specific weather and seasonal conditions of a given geographical area. The reason is trivial; a market serving a cold Nordic region will face a demand peak during the winter season, while a market serving a warmer area is more likely to have an intensified demand for electricity during the hottest period of the year.

1.5.5 Mean Reversion

Electricity spot prices, similarly to the other energy commodities, fluctuate around an average value, in line with the assumption of mean reversion. When the price of a commodity is low, its supply is likely to decline and its demand to expand, lifting the level of prices. While, when the spot price is particularly high, the supply tends to increase and the demand to decrease, pushing the level of prices down. Prices tend, indeed, to converge to the marginal cost of production, that is the value around which the market expects the prices will stabilise in the long run. Therefore, models describing the behaviour of electricity prices should include mean reversion components. However, including only one mean-reversion coefficient, hence considering only one mean-reversion rate, could still lead to biased results. More specifically, it might result in an insufficient removal of extreme price variations and, conversely, in an excessive detrendisation of the smoother periods which do not experience any intense movement. This is due to the spikes that the electricity price processes commonly exhibit. Since spikes are, by definition, high jumps that run out very fast, they adjust back to the average value quickly and, consequently, enlarge the mean-reversion-rate. To overcome this problem, some studies proposed models that included more than one mean-reversion rate, distinguishing between periods of extreme movements and "normal" periods.

Chapter 2

Electricity Derivatives Contracts

The wave of liberalisation that worldwide electricity markets experienced in the last decades, triggered by the common expectations of greater efficiency arising from a more competitive industry, resulted in a sharp increase in price volatility. Consequently, the need for new risk management tools and techniques became essential. Hence, along with the establishment of a competitive environment, electricity markets implemented also different financial derivatives, aiming to control risk through properly designed hedging strategies.

This chapter presents an overview of the financial derivatives contracts that electricity markets enforced the most in order to hedge risk.

2.1 Electricity Futures, Forward, and Swap Contracts

Electricity futures and forwards are the most conventional form of electricity derivatives. Fixing future power prices, they help to reduce the commonly large degree of uncertainty affecting electricity markets due to the high volatility of spot prices. Futures and forwards represent, indeed, the obligation of the holder to buy, at some future date, a pre-determined amount of electricity, at a pre-determined price and, at the same time, the obligation of the seller to supply such quantity. Alike regular financial contracts, electricity futures are highly standardised contracts traded on organised exchanges. However, the largest share of transactions takes place OTC, where parties meet to set bilateral agreements. As both electricity futures and forwards belong to the class of the commodity contracts, they can be distinguished between physical and purely financial contracts. The former implies the physical delivery of the underlying electricity, while the latter only tracks the movements of the market price indexes referred to the underlying electricity.

The underlying electricity spot prices are subject to considerable fluctuations which commonly result in significant price differentials based on the period in which the delivery of electricity occurs, namely whether it takes place during peak-hours or not. In line with such phenomenon, electricity markets commonly classify futures and forward power contracts based on the nature of the period of the day during which the delivery happens. Accordingly, on-peak, base-load, and off-peak electricity contracts are distinguished. Such categorisation is particularly relevant since, as underlined by Deng and Oren (2006), it applies to most of the electricity derivatives contracts.

A further peculiarity of futures and forward power contracts, already mentioned in Section 1.3, concerns the fact that these contracts ensure the delivery of the underlying electricity over a pre-determined period rather than at a specific point in time. This characteristic implies, on one hand, that the electricity market is incomplete and, consequently, the hedge of risk cannot be totally precise, on the other hand, that, as outlined by Benth and Koekebakker (2008), futures and forwards can be seen as swap contracts. The literature, indeed, frequently regard them as swaps, given that they entail a series of future payments, rather than a single one, as expected from standard futures and forward contracts.

The role played in the academic literature by these types of derivatives is crucial since the empirical distribution of futures and forward prices fits well to models implying a log-Normal distribution. Their volatility tends, indeed, to be restricted, as well as their skewness and kurtosis. This allows for this kind of data to adapt generally well to the assumption of log-Normality which, in turn, empowers for futures and forward electricity contracts to be employed as the underlying of more complex derivatives contracts.

2.2 Electricity Options

Along with the futures and forward derivatives contracts, electricity options provide the proper hedging tools to implement in the risk management sector of both the power production and distribution segment. Since the liberalisation of the industry, electricity markets developed a variety of option contracts, including both plain vanilla options as well as more exotic ones, considering a wide range of electricity attributes such as the type of primary energy source employed for the generation of the electricity carrier, the volume of electricity exchanged, the delivery location and the timing of the delivery.

Call and Put Options

Electricity call and put options, alike standard plain call and put options, are non-linear derivatives which give the holder the right, but not the obligation, to buy or sell a pre-determined amount of the underlying asset, at a fixed strike price, by the maturity of the contract. Their design is very similar to the standard financial options one, resuming both the payoff structure and the fundamental distinction between European and American type of contract. What distinguishes the electricity options contracts is that the underlying can be the physical electricity exchanged or electricity futures contracts. Despite their simple structure, electricity calls and puts are, currently, among the most effective tools for hedging risk deriving from price volatility, exploited by both power producers and retailers.

Spread Options

A relevant family of more exotic electricity options, which often serve as a building block of more elaborate derivatives, is made up of spread options, analyzed in detail by Deng, Johnson, and Sogomonian (2001). The principal categories of spread options that electricity markets employ as risk management tools are the fuel-spread options and the locational-spread options. The fuel-spread options play a significant role in different energy markets as they allow to compare the value of a final energy carrier with the costs of the inputs needed for the production process, representing, in turn, the profitability of the process and the efficiency of the primary energy source. This category, which includes both the *spark spread options*, when the electricity carrier is produced with natural gas, and the *dark spread options*, when the primary energy source is coal, pays the difference between the price of electricity and the price of the fuel employed in the generation phase. Accordingly, the holder, for instance, of a European fuel-spread call option has the right, but not the obligation, to buy at expiration a pre-determined amount of electricity, paying a fixed rate of the value at maturity of the fuel used to generate such amount. It follows that these contracts are crucial for assessing the efficiency of the primary energy sources as well as for hedging the electricity price risk arising from the uncertainty due to the fluctuation of the value of the fuels. As concerns the locational-spreads, their applicability in risk management issues remains unchanged but the risk they hedge derives from the transmission costs and constraints affecting different locations which may give rise to significant differences in electricity prices. The locational-spread options, indeed, pay out the price differential existing between two different locations, due to the electricity non-storability and transmission problems discussed in the previous chapter.

Swing Options

The swing options are among the most popular daily options that allow the holder to repeatedly exercise, during the validity period of the contract, the right to receive a certain amount of the underlying electricity. More specifically, as outlined by Keppo (2004), the owner of a swing option has the right to get an amount of electricity settled by some pre-determined bounds, during a given number of days within the lifetime of the option, at a pre-fixed price, provided that the total amount received complies with the given periodic quantity constraints. Such a structure grants the owner of the option with some flexibility in the amount of underlying traded and, concurrently, provide protection against price and demand jumps during peak days. These features, as remarked by Jaillet, Ronn, and Tompaidis (2004), make the swing options an efficient risk-hedging tool for electricity markets. Electricity markets participants experience, indeed, the need to hedge against both the price volatility and the electricity demand-spiking tendency, attributable to the non-storability of electricity and, in turn, to the lack of demand elasticity.

2.3 Exotic Electricity Derivatives Contracts

With the advent of liberalised, competitive markets, the electricity risk management sector was significantly restructured too, developing a brand new electricity derivatives market. The unique features of electricity did require the enforcing of both traditional risk-management tools, addressed in the previous sections, and more exotic, custom-tailored contracts. The purpose of the implementation of the latter was to cope with any kind of risk implied by the peculiarities of such energy commodity; from the exposure suffered by power plants, loads, and retailers due to the significant price volatility, to the risk experienced by the dispatchers because of the electricity transmission constraints. It is not the aim of this dissertation to deal with these exotic derivatives contracts in depth. Therefore, the following is just a brief overview of some of the principal contracts that the literature examined the most.

A significant class of derivatives deemed proper of the electricity sector consists of *tolling contracts*. Such derivatives have been embraced by the wholesale electricity market in virtue of their potential as a risk-transfer mechanism. The currently existing tolling contracts exhibit different designs but, in general terms, their structure resumes that of a rental agreement between a generating plant and a power marketer. More specifically, a tolling is a contract that gives the buyer the right to set the power generation scheduling of the counterpart plant or to receive the underlying electricity within some pre-determined time span, under some pre-specified constraints, against the payment of a premium to the plant (Aïd, R. (2015). Real Derivatives In *Electricity Derivatives* (pp. 21-26)). Therefore, this contract enables the owner of a plant to cope with the uncertainty associated with the market price of electricity or the fuel needed to produce it.

The electricity derivatives market developed addressing not only the risk arising in wholesale markets but involving also the retail segment and its specific risk-management needs. From a consumer perspective, the risk derives from the volatility of the prices combined with the fluctuations in the amount consumed. Accordingly, a wide range of tailored and structured derivatives emerged in response to the demand for a supply contract for variable consumption needs. Among all, it is worth mentioning the *full-requirement power contracts* which, as highlighted by Deng and Oren (2006), enable the buyer to pay a fixed rate for each unit of electricity, whatever the total amount consumed.

Running the transmission network efficiently is crucial for the well-functioning of both the wholesale and retail segments of the electricity market. Therefore, the electricity derivatives market adapted to include also financial instruments aimed at improving the performance of the transmission net. The resulting products designed for hedging the transmission risk are different and vary depending on the specificities of each transmission network. Financial and physical *transmission rights* are relevant examples of contracts belonging to this class of derivatives. The former entitles the holder to receive the price difference across transmission borders, while the latter enables to transmit electricity across borders at a fixed price. Likewise, the *contracts for differences* constitute another remarkable example. This type of contract, described by Cretí and Fontini (2019), relieves the subscribing importing node from the price uncertainty, transferring the risk to the counterparty, the exporting node. The buyer agrees, indeed, to pay a fixed premium to receive in exchange the difference between the strike price of the derivative and the spot price of the underlying electricity.

2.4 The Italian Electricity Derivatives Exchange

Borsa Italiana S.p.A., part of the London Stock Exchange Group, launched in 2008 the Italian Derivatives Energy Exchange (IDEX), on the line of already-founded exchanges, NordPool and EEX.

IDEX is a regulated market, developed as a new segment of the Italian Derivatives Market, where investors can trade electricity financial derivatives. Currently, it allows the exchange of base-load power futures, which provide for the delivery of the underlying electricity during any hour of any day of the week, and on-peak futures contracts, which imply the delivery of the underlying electricity only during the peak period, namely from 8 a.m. to 8 p.m. of the weekdays. The delivery period of the futures listed on IDEX can be monthly, quarterly, or yearly. They are organised according to a cascading structure, which entails that when a monthly future extinguishes, the quarterly ones are split into three monthly futures, while the yearly ones are divided into three quarterly and three monthly futures, so that, eventually, only the contracts with a monthly delivery period are settled. The great majority of these derivatives are purely financial contracts, offering only a cash settlement, paid by *Cassa di Compensazione e Garanzia*.

The settlement price differs between base-load and on-peak futures. For the former, it corresponds to the average of the unique national prices computed during the delivery month in the day-ahead market, while for the latter, it coincides with the average of the unique national price computed only on the above-mentioned peak period. However, IDEX allows the customers that request it and that GME approves to trade also physical futures contracts, which provide the physical delivery of the underlying electricity. In this case, the cash settlement does not take place. On the contrary, the physical power futures settled on IDEX are executed on a separate platform (CDE), where the transactions are recorded.

Chapter 3

Electricity Pricing Models

The academic literature concerning the modelling of electricity spot and futures prices is very rich and has increasingly grown since the liberalisation of the sector. Consequently, the variety of different approaches and classes of models currently available is considerably vast. Such wide range of models is not intended to provide spot and futures price forecasts, but rather, it aims to develop the processes of derivatives assessment and risk management. The pricing of electricity derivatives relies, indeed, on the identification of a realistic process for the underlying electricity spot or futures prices. Due to the peculiarities of electricity, outlined in Section 1.5, the pricing models significantly differ from traditional financial models. The most notable consequence of these distinctive features is that the Geometric Brownian Motion (GBM), commonly employed for the assessment of financial products and exploited by Black and Scholes (1973) for option pricing purposes, is not appropriate for the modelling of electricity prices.

This chapter reviews some of the most relevant classes of electricity pricing models with the purpose of providing the reader with a survey of different possible approaches.

3.1 An Overview of the Modelling Approaches

Along with the development of various electricity pricing models, two main distinct modelling approaches emerged, namely, a *structural approach* and a *technical approach*. As presented by Carmona and Coulot (2014), the former relies on market information and economic theory to describe the processes running electricity markets which in turn provide realistic models for prices. This branch of literature includes different classes of models. Starting from one of the earliest-developed models based on the demand-supply equilibrium, suggested by Barlow (2002), some studies (e.g. Kanamura, & Ohashi, 2007) extended to more sophisticated models, considering also optimisation procedures minimising production costs. The structural approach provides particularly realistic models since it exploits forward-looking information about expected future market changes not already reflected in prices such as, according to Carmona and Coulot (2014), upcoming changes in the power generation mix or in the regulatory framework of a country. Such additional information can be used to identify the sources of volatility that the other approach is not able to observe. However, since the structural approach is not specifically designed to support derivatives pricing, it tends to be computationally prohibitive when applied to the risk management sector (Deng & Oren, 2006). Therefore, hereafter, we will focus on the technical approach.

The technical approach, as highlighted by Carmona and Coulon (2012), aims to model electricity market prices through suitable stochastic processes, exploiting historical data and statistical analysis. The resulting so-called *reduced form processes*, indeed, adjust common financial parametric processes to the peculiarities of the electricity prices, to capture the dynamics of power prices in simple functions from which derivatives prices can be computed. Accordingly, reduced-form models tend to be more computationally tractable than structural models. However, the main shortcoming of this approach is their inability to capture those fundamental sources of randomness which, on the contrary, the structural approach deals with.

The variety of existing reduced-form models for spot and futures electricity prices suggested in the literature is wide. According to Cartea and Figueroa (2005), the approaches implemented in electricity pricing models can be further grouped into two broad classes, namely, a spot- based one and a futures-based one. The former initially models spot prices from which futures prices are retrieved. The latter, conversely, starts from the modelling of the observed futures prices and, eventually, derives spot prices. Models following the Heath–Jarrow–Morton (HJM) approach belong to this class.

Initially introduced by Heath, Jarrow, and Morton (1992) to model the term structure of interest rates, this approach proposes to model forward rates directly. Bjerksund, Rasmussen, and Stensland (2000) extended the approach to power markets to model the dynamics of electricity futures contracts. Since this first appearance in the literature concerning power markets, the HJM approach has been extensively adopted by various studies. The advantages provided by the futures-based approach are, indeed, several. As outlined by Benth and Koekebakker (2008), directly modelling futures prices allows to avoid having to deal with the complex electricity spot-futures relationship, to use quoted futures prices directly in model fitting, and simplifies the computation of risk management measures like the Greeks or the VaR. However, most of the existing literature about electricity markets rely on the spot-based approach, implementing stochastic models to fit the dynamics of spot prices.

In line with this strand of literature, the following sections review some of the most commonly used classes of stochastic models for the time evolution of the electricity spot prices.

3.2 A first proposal: a two-factor mean-reverting model

The inability of the traditionally-exploited Geometric Brownian Motion to capture the periodic seasonal behavior of electricity prices and the mean-reversion phenomena, according to Hikspoors and Jaimungal (2007), fostered the development of new models, able to capture these specific characteristics. An early attempt to account for mean-reversion was presented by Gibson and Schwartz (1990) and Cortazar and Schwartz (1994). Their one-factor models, however, required further improvements as they were not able to capture the dynamics of futures prices. To this end, Pilipovic (1997) proposed a two-factor mean-reverting spot price model. The model is structured as follows:

$$S_t = S_t^{Und} + f(t) \tag{3.1}$$

where S_t^{Und} is the stochastic component of the model, representing the underlying spot prices that the author defines as the seasonally adjusted electricity prices, while $f(t)$ is the deterministic component modelling the seasonality. The latter is defined as a sinusoidal function, more precisely, a cosine function, reflecting the annual and semiannual seasonal pattern of the price time series. On the other hand, the stochastic component S_t^{Und} is described by the following mean-reverting process with two dependent variables:

$$dS_t^{Und} = \beta(\theta_t - S_t^{Und})dt + \sigma_S S_t^{Und} dW_t^{(1)} \tag{3.2}$$

$$d\theta_t = \alpha\theta_t dt + \sigma_\theta \theta_t dW_t^{(2)} \tag{3.3}$$

where θ_t represents the stochastic long-run mean that spot prices S_t revert to, σ_S and σ_θ are the constant volatilities of S_t and θ_t respectively, β is the coefficient indicating the speed with which the price S_t reverts to the stochastic mean θ_t, and, lastly, $W_t^{(1)}$ and $W_t^{(2)}$ are the two correlated Brownian risk factors: $d[W^{(1)}, W^{(2)}]_t = \rho dt$. The stochastic differential equation (3.2) adapts the GBM to the mean-reversion hypothesis, while equation (3.3) identifies the GBM describing the long-run dynamics of θ_t. The stochastic long-run mean, θ_t, indeed, varies over time, following a log-Normal probability distribution.

This model, as stated by Hikspoors and Jaimungal (2007), results in log-Normally distributed spot prices which, on one hand, simplifies the calibration process, on the other hand, contrasts with the real, leptokurtic, and skewed distribution of the spot prices.

3.3 Further developments of continuous-time diffusion models

3.3.1 Mean-reverting pure diffusion models

A diffusion process is a continuous-time Markov process with continuous sample paths. Relevant examples of processes belonging to this class are the Brownian Motion, the GBM, and the mean-reverting Ornstein-Uhlenbeck (OU) process. Mean-reverting pure diffusion models, neglecting the complexity deriving from volatility clustering and spikes, requires the estimation of only three parameters, namely the long-run mean, the speed of mean reversion, and the volatility of the process. Such a limited number of parameters makes these models a simple tool to describe continuous-time stochastic processes. This simplicity fostered the widespread use of mean-reverting diffusion models within power markets. The earliest and most notable applications of these models to electricity spot prices are provided by Lucia and Schwartz (2002), and Barlow (2002). Such models represented the starting point from which several authors further developed spot price models, integrating additional sources of complexity. Accordingly, the approaches proposed, for instance, by Escribano et al. (2002), Villaplana (2003), and Geman and Roncoroni (2006) relied on- and extended- the work proposed by Lucia and Schwartz (2002), adding to the pure diffusion model a jump component describing the spike phenomenon, typical of electricity spot prices.

Similarly to the model of Pilipovic(1997) defined in equation (3.1), the diffusion model proposed by Lucia and Schwartz (2002) identifies a predictable deterministic component, which captures the regularities that affect the evolution of spot prices, and a stochastic component, which is the only source of uncertainty. That is

$$P_t = f(t) + X_t \tag{3.4}$$

representing the spot price P_t as the sum of the deterministic component $f(t)$ and the stochastic component, X_t. The former can be defined as a sinusoidal function or as the sum of a constant and two dummy variables which distinguish between working and non-working days, and reproduce the seasonal evolution of prices over the year. Therefore, this model not only captures the annual and semiannual seasonality, as the model proposed by Pilipovic (1997), but also models the changes in the price level during the weekends or the holiday seasons. On the other hand, the latter is a mean-reverting OU process with a zero long-run mean defined by the stochastic differential equation (SDE)

$$dX_t = -kX_t dt + \sigma dZ_t \tag{3.5}$$

where k is the intensity of the mean-reversion, σ is the volatility of the process. It follows that the process describing the behaviour of spot prices can be defined as

$$dP_t = k(\alpha(t) - P_t)dt + \sigma dZ_t \tag{3.6}$$

where $\alpha(t)$ is defined as $\alpha \equiv \frac{1}{k}\frac{df}{dt}(t) + f(t)$.

Hence, unlike the approach proposed by Pilipovic (1997), this one-factor model suggests that spot prices in the long-run tend to a mean value which is a function of the deterministic seasonal component.

Despite the different specifications of the seasonal component and of the long-run mean, both the model proposed by Pilipovic (1997) and the one developed by Lucia and Schwartz (2002) tend to be analytically very tractable. However, they lack some realism as they do not allow to account for the possible spikes of the prices.

3.3.2 Mean-reverting jump-diffusion models

For a more accurate representation of the behaviour of electricity prices, the academic literature introduced a new class of models integrating a continuous-time stochastic process with a component describing the typical spikes of prices. Jump-diffusion models combine, indeed, a diffusion process and a jump process which is a stochastic process presenting discrete movements, namely the jumps. This class of models was first introduced by Merton (1976) to model equity dynamics and was later extended in order to be implemented in power markets by several authors, including Eydeland and Wolyniec (2003), Cartea and Figueroa (2005), and Geman and Roncoroni (2006).

A commonly-implemented jump-diffusion model represents the jumps through a Poisson process, a stochastic process presenting stationary and independent increments following the Poisson distribution. Accordingly, in every time interval of length Δ, prices may experience a jump with probability proportional to Δ.

The structure of the jump-diffusion model proposed by Cartea and Figueroa (2005) is recurring in several other models belonging to the same class. Therefore, the general design of this model is briefly presented below

$$\ln S_t = g(t) + Y_t \tag{3.7}$$

where $\ln S_t$ is the natural logarithm of the spot price S_t, $g(t)$ is the deterministic log-seasonality function, and Y_t is a zero level mean-reverting jump-diffusion process for the electricity spot price S_t whose dynamics are described by the SDE

$$dY_t = -\alpha Y_t d_t + \sigma(t) dZ_t + \ln J dq_t \tag{3.8}$$

in which α is the speed of mean reversion, $\sigma(t)$ is the time-dependent volatility, J is the proportional random jump-size such that $\ln J \sim \mathcal{N}(\mu_J, \sigma_J^2)$, Z_t is a standard Brownian Motion, and q_t is a Poisson process such that

$$dq_t = \begin{cases} 1 & \text{with probability } l\Delta \\ 0 & \text{with probability } 1 - l\Delta \end{cases} \tag{3.9}$$

where the parameter l is the rate or intensity of the process defining the probability with which an event, i.e. a jump, occurs within a time interval Δ.

This class of model enhances the pure diffusion models discussed in section 3.3 enabling to capture some of the most important characteristics of electricity spot prices namely seasonality, mean reversion, and jumps. By combining a jump component with a mean-reverting process, that brings the prices back to original levels, it allows, indeed, to compensate for the inability of pure diffusion models to represent the typical sudden power price spikes. However, this class of model is less parsimonious as it entails a high number of parameters, which, if coupled with an insufficient amount of data, greatly complicates the parameters estimation process.

3.4 Regime-switching models

The need for realistic models of power price dynamics motivated the development of another class of reduced-form models, namely the regime-switching models. As outlined by Hamilton (2008), such models describe the dynamics of a variable by identifying two or more states, namely the regimes, within which the variable can follow different processes. Although the literature presents some applications of multi-regime-switching models, two-regime-switching models have been more widely applied in the electricity sector (Möst & Keles, 2010). These models are, indeed, exploited by the literature concerning the electricity sector to represent the spikes. The previously discussed jump-diffusion models assume that, once the jump occurs, prices return to the average level at the same speed described by the intensity of the mean-reverting process. However, the spikes actually observed in power markets run out much faster. Allowing to distinguish between a base regime, i.e. the mean-reverting regime, and a regime for price peaks, regime-switching models overcome the jump-diffusion models' shortcomings and more realistically represent the price dynamics.

This class of models requires to define a law of probability that regulates the transition from one regime to the other one. The specification of these models applied to power markets generally assumes that the probability of a change of regime depends only on the value of the most recent regime. It follows that, as outlined by Fanelli (2019), the probability of switches between regimes is defined by the following transition matrix

$$\pi = \begin{pmatrix} 1-\gamma dt & \eta dt \\ \gamma dt & 1-\eta dt \end{pmatrix} \tag{3.10}$$

with γ and η assumed constant, γdt representing the probability of transition from the base regime to the spike one in an infinitesimal time interval Δ, and ηdt defining the probability for the opposite transition.

A time-invariant Markov chain adapts well to this assumption as it represents a memoryless stochastic process describing a finite number of states whose probability to take place depends only on the most recently-occurred state. Under this specification, a regime-switching model can be defined as a Markov regime-switching model.

A simple application of this class of models to power markets is the one proposed by Weron (2009). The author modelled deseasonalized log-prices, $Y_t = \log(X_t)$, with a two-regime switching model, in which the base regime dynamics are described by a mean-reverting OU process

$$dY_{t,1} = (c_1 - \beta Y_{t,1})dt + \sigma_1 dW_t \tag{3.11}$$

where c_1 is the long-run mean, β is the speed of mean reversion, σ_1 is the volatility of prices in the mean-reversion regime, and W is Brownian motion. On the other hand, the spike regime dynamics follow a log-Normal or a Pareto distribution, namely:

$$\log(Y_{t,2}) \sim \mathcal{N}(c_2, {\sigma_2}^2) \tag{3.12}$$

or

$$\log(Y_{t,2}) \sim F_{Pareto}(c_2, {\sigma_2}^2) = 1 - \left(\frac{{\sigma_2}^2}{x}\right)^{c_2} \tag{3.13}$$

where c_2 and σ_2 are, respectively, the long-run mean and the volatility in the spike regime.

3.5 Discrete-time time series models

The literature concerning electricity pricing also considers discrete-time models, exploiting statistical models for time series to capture the seasonality, the mean-reversion tendency, and the spikes.

As discussed by Möst and Keles (2010), the equivalent in discrete time of the mean-reverting OU process, previously mentioned in Section 3.3.1, is the AutoRegressive (AR) process of order 1. To account for price seasonality and ensure the stationarity of the data, a Seasonally Integrated component can be included in such process, resulting in a Seasonal Autoregressive Integrated Moving Average (SARIMA) model. However, the high level of volatility affecting electricity prices which induces the phenomena of spikes and volatility clustering requires models that, unlike SARIMA models, capture heteroskedasticity. Accordingly, a frequently-implemented discrete-time time series model is the Generalised Autoregressive Conditional Heteroskedasticity (GARCH) model. As stated by Hua et al. (2005), GARCH models are a widely applied tool for financial risk-analysis and derivative pricing, able to capture the dynamics of the error term of a time series. The general specification of the conditional variance of a GARCH(p, q) model, provided by Hua et al. (2005), is the following

$$\sigma_t = \omega + \sum_{j=1}^{n} \beta_j \sigma_{t-j} + \sum_{i=1}^{q} \alpha_i e_{t-i}^2 \tag{3.14}$$

where $\omega > 0$ is the drift, $\alpha_i \geq 0$ and $\beta_j \geq 0$ are the parameters associated with the square of the past error term, e_{t-i}^2, and the past values of the conditional variance, σ_{t-j}, respectively. Lastly, p represents the number of lags of the autoregressive part included in the model, while q indicates the number of lags of the moving average component.

Further extensions of such model have been considered. One of the most widely accepted extensions is the Exponential GARCH (EGARCH) model, as some studies (e.g. Bowden & Payne, 2008) proved its ability to better capture the behavior of spot electricity price with greater accuracy.

Chapter 4

The Model of Lucia & Schwartz (2002): Presentation and Simulations

The mean-reverting diffusion model applied to electricity spot prices proposed by Lucia and Schwartz (2002) is one of the most popular and widely accepted in electricity markets. The basic one-factor model developed by the two authors describes the spot price P_t at time t as the combination of a deterministic and a stochastic component, namely

$$P_t = f(t) + X_t, t \in [0, \infty) \tag{4.1}$$

as already defined by equation (3.4). Such simple specification has been the starting point for several subsequent studies that, as already mentioned in Section 3.3, extended the model integrating a jump component.

This chapter aims to analyse in depth the mean-reverting diffusion model of Lucia and Schwartz (2002) to apply it, in the following chapter, to Italian power spot prices.

4.1 The deterministic component

The deterministic component of spot prices, $f(t)$, aims to capture those predictable phenomena deriving from a regular behaviour of electricity prices and characterising their evolution over time. Lucia and Schwartz (2002) proposed different functions to account for the specificities of a variety of datasets.

The simplest approach suggested by the authors defines the deterministic component as a constant. This specification neglects any regularities in the evolution of power prices. Therefore, it implies that spot prices follow a mean-reverting process without deterministic seasonality effects. The implementation of a constant deterministic component is adequate to model time series in which the deviations from the "normal" level implied by seasonal phenomena are so frequent that cannot be predicted.

A more refined approach, which enables to reflect the periodic pattern of prices implied by seasonality, involves the use of a sinusoidal function such as the cosine function. In this regard, the authors refer to the specification proposed by Pilopovic (1998). As electricity demand commonly exhibits periodic patterns which are mainly driven by atmospheric conditions, power prices are likely to present strong seasonality as well. Typically, throughout the year two price peaks occur, one in the coldest and one in the warmest season, due to the use, respectively, of heating and air conditioning. Conversely, the spring and fall seasons commonly exhibit the lower level of demand and price. Such seasonality can be represented by the following specification

$$f(t) = \gamma_A \cos(2\pi(t - t_A)) + \gamma_{sA} \cos(4\pi(t - t_{sA})) \tag{4.2}$$

where the first cosine function describes the annual seasonal pattern just discussed, with γ_A representing the magnitude of the seasonal contribution, namely the amplitude of the fluctuations, and t_A the time of the year defining the center of the seasonal peak. Accordingly, during its period of cycle whose length is 1, such function presents, at time t_A, a peak whose amplitude is defined by γ_A, capturing the winter or summer price peaks. Similarly, the second cosine function, whose cycle is defined over the period $\frac{2\pi}{|4\pi|} = \frac{1}{2}$, defines the semi-annual seasonal component, with magnitude γ_{sA} and center t_{sA}.

Such seasonal pattern is represented in Figure 4.1 which plots a standardised version of function (4.2) in a time span of 2 years, with $\gamma_A, \gamma_{sA} = 1$ and $t_A, t_{sA} = 0$.

A further solution developed to account for seasonality implies the integration of a piecewise function in the price model. Following the approach proposed by Jaillet et al. (1998), and Manoliu and Tompaidis (2002), the deterministic function $f(t)$ can include seasonal dummy variables. Accordingly, Lucia and Schwartz (2002) suggested two different piecewise functions as possible versions of the deterministic component of the model.

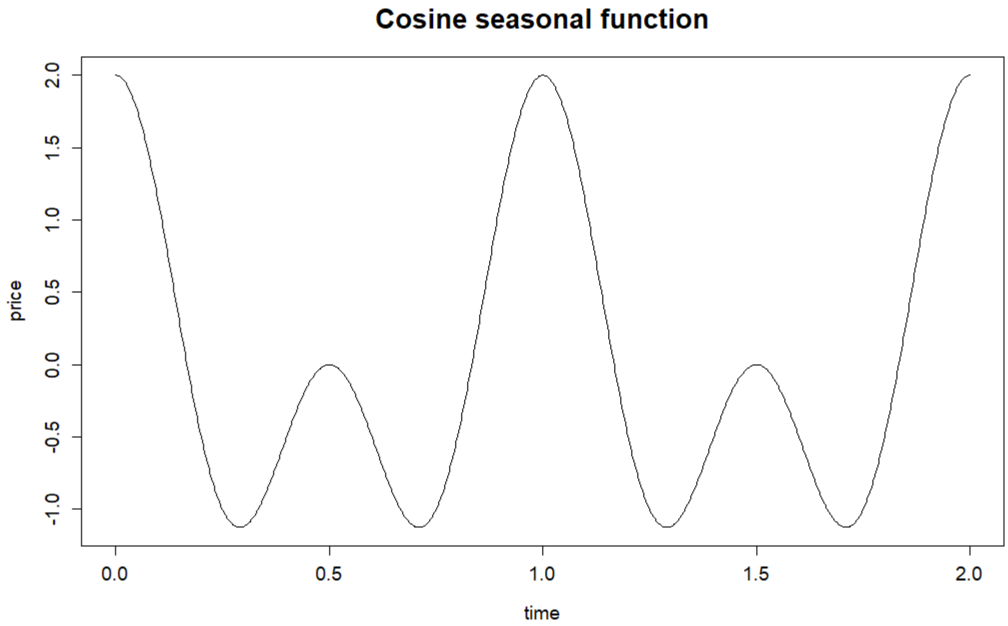

Figure 4.1: Shape of the seasonal function (4.2) for a time span of 2 years.

These functions have been constructed to capture both the fluctuations of prices between working and non-working days, as well as the variation of the level of prices throughout the year. One version is described by the following equation

$$f_1(t) = \alpha + \beta D_t + \sum_{i=2}^{12} \beta_i M_{it} \tag{4.3}$$

in which α is the constant term, β is the parameter characterising the changes in prices occurring during the holidays or the weekends, and β_i, for $i = 2, \ldots, 12$, are the parameters capturing the fluctuations of prices occurring during the different months of the year. D_t and M_{it} are the two dummy variables such that

$$D_t = \begin{cases} 1 & \text{if } t \text{ corresponds to a holiday or a weekend} \\ 0 & \text{otherwise} \end{cases},$$

$$M_{it} = \begin{cases} 1 & \text{if } t \text{ belongs to the i - th calendar month} \\ 0 & \text{otherwise} \end{cases}, \text{ for } i = 2, \ldots, 12$$

The resulting seasonal pattern is illustrated in Figure 4.2 which plots equation (4.3) using the parameters estimated by Lucia and Shwartz (2002). More precisely, we defined $\alpha =$

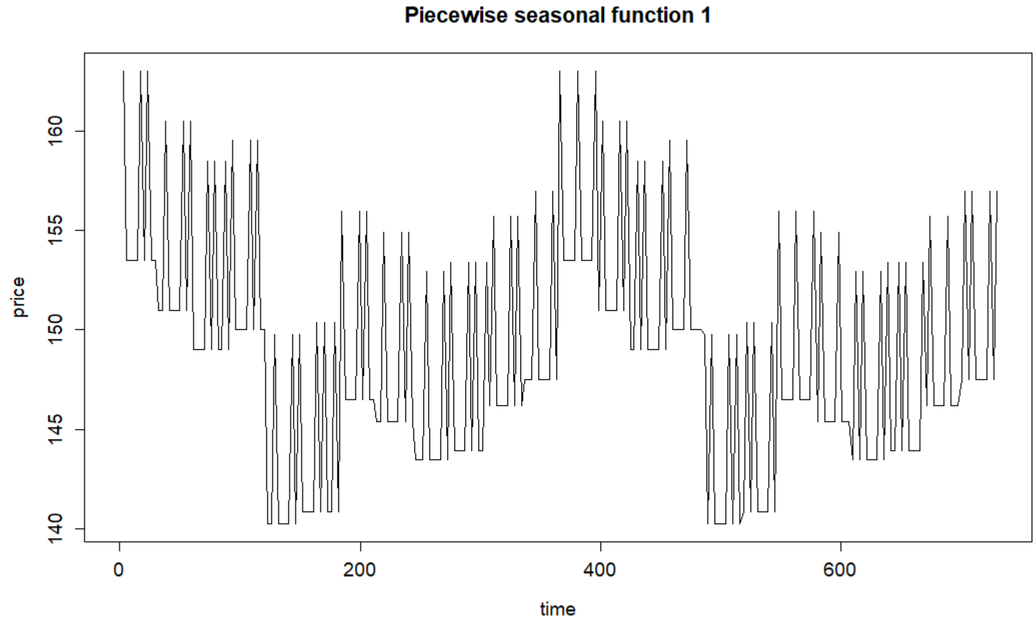

Figure 4.2: Seasonal function (4.3) for a time span of 2 years.

153.51, $\beta = 9.514$, and $\beta_i = (-2.527, -4.511, -3.484, -13.248, -12.656, -7.038, -8.109, -10.061, -9.597, -7.304, -6.019)^\intercal$. This definition of the values of the parameters implies that the range of the values taken by the spot prices in Figure 4.2 is much higher than the range plotted in Figure 4.1. Defining the dummy variable M_{it}, we omitted January to avoid multicollinearity. The graph represents the seasonal behaviour of prices for two years, namely 730 days. Comparing it to Figure 4.2, it presents a less smooth pattern, as it is made up of a sequence of small jumps representing the steps of the function, but it still outlines the periodic peaks that reflect the above-mentioned weather-related drivers, similarly to the cosine function.

The second version of the seasonal function still represents the variations of prices between working and non-working days with a dummy variable. However, it describes the seasonal pattern in the evolution of prices throughout the year with a cosine function as follows

$$f_2(t) = \alpha + \beta D_t + \gamma \cos\left((t + \tau)\frac{2\pi}{365}\right) \tag{4.4}$$

where α, β, γ are the constant parameters defined above, τ is the phase displacement

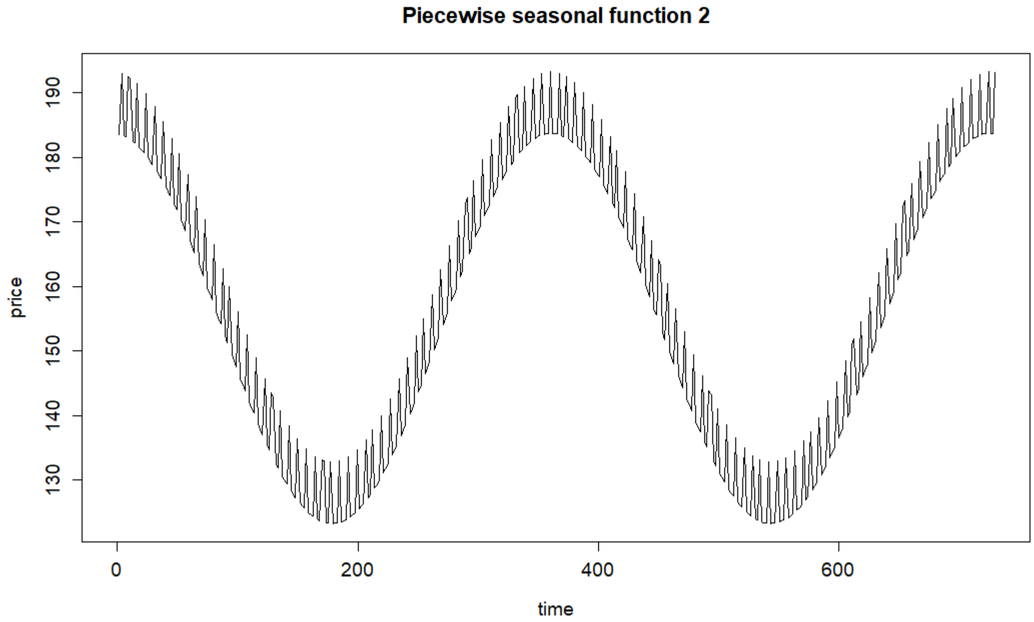

Figure 4.3: Seasonal function (4.4) for a time span of 2 years.

that shifts the seasonal peaks in time, and D_t is the following dummy variable

$$D_t = \begin{cases} 1 & \text{if } t \text{ corresponds to a holiday or a weekend} \\ 0 & \text{otherwise} \end{cases}$$

indicating whether the price refers to a working or to a non-working day.

Figure 4.3 plots the path of this function, using the value of the parameters estimated by Lucia and Schwartz (2002) for a time span of 2 years. The function we implemented assumes $\alpha = 151.08$, $\beta = -10.24$, $\gamma = 30.27$, and $\tau = 3.96$. Due to the integration of a sinusoidal function in this specification, the graph presents a more clear seasonal pattern than Figure 4.3. However, some irregularities, given by the dummy variable, can be noticed.

4.2 The stochastic component

The only source of uncertainty involved in the model (3.4) is provided by the variable X_t. X_t is defined, indeed, as a continuous-time diffusion process that models the stochastic component driving spot prices. As already mentioned in Section 3.3, Lucia and Schwartz (2002) assume that X_t follows a zero-mean-reverting Ornstein-Uhlenbeck process whose dynamics can be expressed by the following stochastic differential equation

$$dX_t = -kX_t dt + \sigma dZ_t \tag{4.5}$$

where $X(0) = x_0$, $k > 0$ represents the speed of mean-reversion, and σ indicates the volatility of the process. Given the probability space $(\Omega, F, \mathbb{P})$ with a filtration $\mathbb{F}$, Z_t represents a standard Brownian Motion starting from $Z_0 = 0$ and adapted to $\mathbb{F}$.

Simulating the solutions to the stochastic differential equation (3.4), assuming the values of the parameters k and σ are, respectively, -0.0014 and 2.36, as estimated by Lucia and Schwartz (2002), we obtained the seasonal-adjusted trajectory of prices, plotted in Figure 4.4.

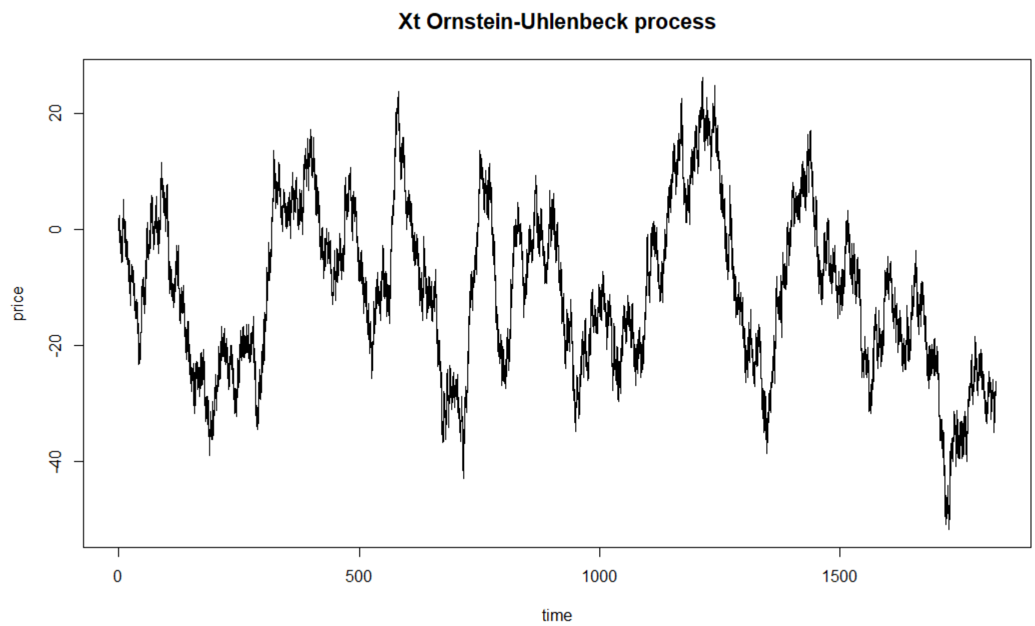

Figure 4.4: Simulated solutions to the stochastic differential equation (4.5) for a time horizon of 5 years.

Knowing that $X_t = P_t - f(t)$, and provided that the deterministic function $f(t)$ meets the required regularity conditions, equation (4.5) can be rewritten as

$$d(P_t - f(t)) = k(f(t) - P_t)dt + \sigma dZ_t \tag{4.6}$$

It follows that the dynamics of P_t can be decribed by the stochastic differential equation

$$dP_t = k(\alpha(t) - P_t)dt + \sigma dZ_t \tag{4.7}$$

with $\alpha \equiv \frac{1}{k}\frac{df}{dt}(t) + f(t)$. It should be noted that this expression of α requires the seasonal function $f(t)$ to be differentiable. Hence, it only applies to the specification (4.2).

By applying Itô's formula, we can derive an explicit solution for equation (4.5), namely:

$$X_t = e^{-kt}X_0 + \sigma \int_0^t e^{-k(t-s)}dZ_s, \qquad \forall t \in [0, \infty) \tag{4.8}$$

which, together with equation (3.4), results in the following equation

$$P_t = f(t) + e^{-kt}X_0 + \sigma \int_0^t e^{-k(t-s)}dZ_s \tag{4.9}$$

A basic property of the Brownian Motion provides that the increment of a Brownian Motion follows a Normal distribution, such that $Z_{t+\Delta} - Z_t \sim \mathcal{N}(0, \Delta)$. Exploiting this property, we can compute the conditional distribution of the spot price P_t.

Defining $X_0 = P_0 - f(0)$ and applying the Brownian Motion property according to which $\mathbb{E}[Z_t] = 0$, the conditional mean of P_t is given by

$$\begin{aligned} \mathbb{E}_s[P_t] &\equiv \mathbb{E}[P_t|X_s] \\ &= f(t) + e^{-kt}X_0 + \sigma\mathbb{E}\Big[\int_0^t e^{-k(t-s)}dZ_s\Big] \\ &= f(t) + e^{-kt}P_0 - f(0), \qquad t < s \end{aligned} \tag{4.10}$$

Implementing a similar process which, again, exploits the null expected value of a Brownian Motion and applying the Itô isometry we can compute the conditional variance of P_t as follows

$$
\begin{aligned}
Var_s[P_t] &\equiv Var[P_t|X_s] \\
&= \mathbb{E}[P_t^2|X_s] - (\mathbb{E}[P_t|X_s])^2 \\
&= \mathbb{E}\big[\big(f(t) + e^{-kt}X_0 + \sigma \int_0^t e^{-k(t-s)}dZ_s\big)^2|X_s\big] - (f(t) + e^{-kt}X_0))^2 \\
&= (f(t) + e^{-kt}X_0)^2 + 2\sigma(f(t) + e^{-kt}X_0)\mathbb{E}\big[\int_0^t e^{-k(t-s)}dZ_s|X_s\big] \\
&\qquad + \sigma^2\mathbb{E}\big[\big(\int_0^t e^{-k(t-s)}dZ_s|X_s\big)^2\big] - (f(t) + e^{-kt}X_0))^2 \\
&= \sigma^2\mathbb{E}\big[\int_0^t e^{-2k(t-s)}ds|X_s\big] \\
&= \sigma^2 \int_0^t e^{-2k(t-s)}ds \\
&= \frac{\sigma^2}{2k}(1 - e^{-2kt}), \qquad k > 0, \qquad t < s
\end{aligned}
\tag{4.11}
$$

It follows that

$$
P_t \sim \mathcal{N}\Big(f(t) + e^{-kt}X_0; \tfrac{\sigma^2}{2k}(1 - e^{-2kt})\Big), \qquad t \in [0, +\infty) \tag{4.12}
$$

Consequently, P_t follows a Gaussian mean-reverting processes, reverting towards the value $\alpha(t)$ at a rate k, as outlined by (4.7). This result highlights that, given its initial value P_0, the realisations of P_t concentrate around a function of $f(t)$. Moreover, the variance proves to be a decreasing function of time t. It decreases as t increases, reaching its limit $\frac{\sigma^2}{2k}$ as t tends to $+\infty$.

This presentation of the model further proceds by graphically representing the simulated trajectories of the prices resulting from the specifications just described. In line with the analysis performed by Lucia and Schwartz (2002) we used the specification (4.3) as deterministic component of the model of spot prices and the previously simulated solutions to the stochastic differential equation (4.5) as stochastic component to plot the simulated trajectory of P_t (Figure 4.5). The R code used to implement these simulations is provided in Appendix A. The stochastic behaviour of prices represented in the graph shows the typical features of spot power prices discussed in Section 1.5, namely, high volatility, spikes, strong seasonality, and a tendency to revert to the mean value.

To illustrate how the different shapes of the deterministic seasonal function affect

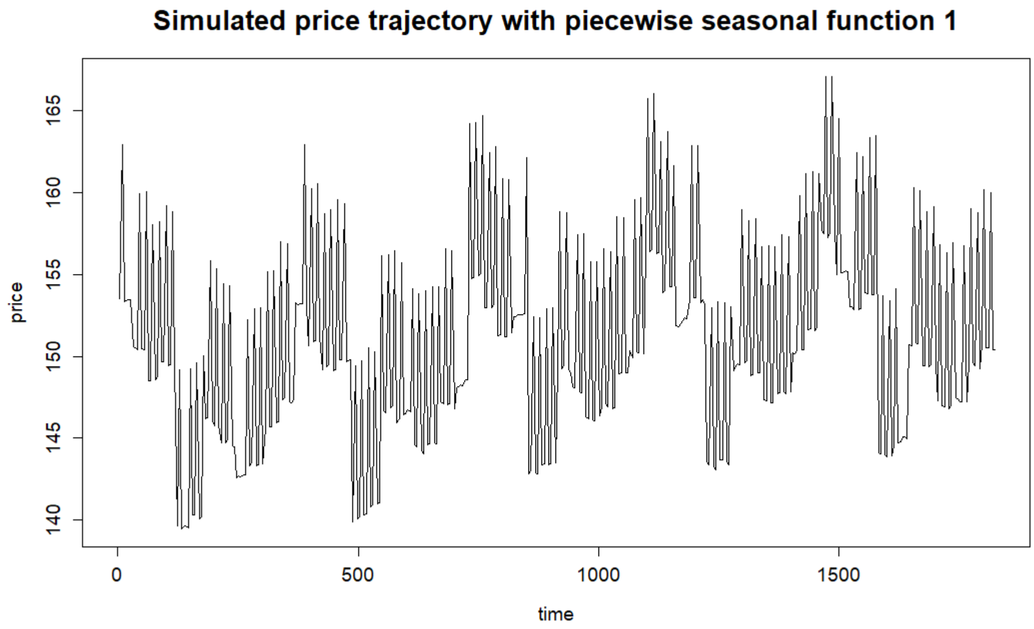

Figure 4.5: Simulated trajectory of the process of spot prices for a time horizon of 5 years.

the overall process followed by prices, we also plotted the simulated trajectories of spot prices, using as deterministic function the equations (4.2) and (4.4) (Figure 4.6). Since the estimates of Lucia and Schwartz (2002) for the parameters of the specification (4.4) were missing, and since the purpose of Figure 4.6 is to compare the possible different shapes of the price trajectories, we put them on the same scale, implementing the standardised versions of the deterministic functions. Hence, we assumed for the specification (4.2) that $\alpha = 0$, $\beta = 1$, and β_i is a (11 x 1) vector of ones. Similarly, in the specification (4.4) we set $\alpha = 0$, $\beta = 1$, $\gamma = 1$, and $\tau = 0$. This explains the large discrepancy between the values of prices illustrated in this graph and in Figure 4.5.

The two processes plotted in Figure 4.6 present similar paths. They both seem to evolve around their mean value, exhibit frequent spikes and present a sinusoidal shape. Such a shape, which is given by the cosine functions present in both equation (4.2) and (4.4), is reflected in the periodic increase in prices that seems to be concentrated towards the end of each year. Similarly to the process plotted in Figure 4.5, the simulated trajectory obtained using the piecewise seasonal function (4.4) exhibit, although less evidently, a sequence of small steps which plots the dummy variable D_t.

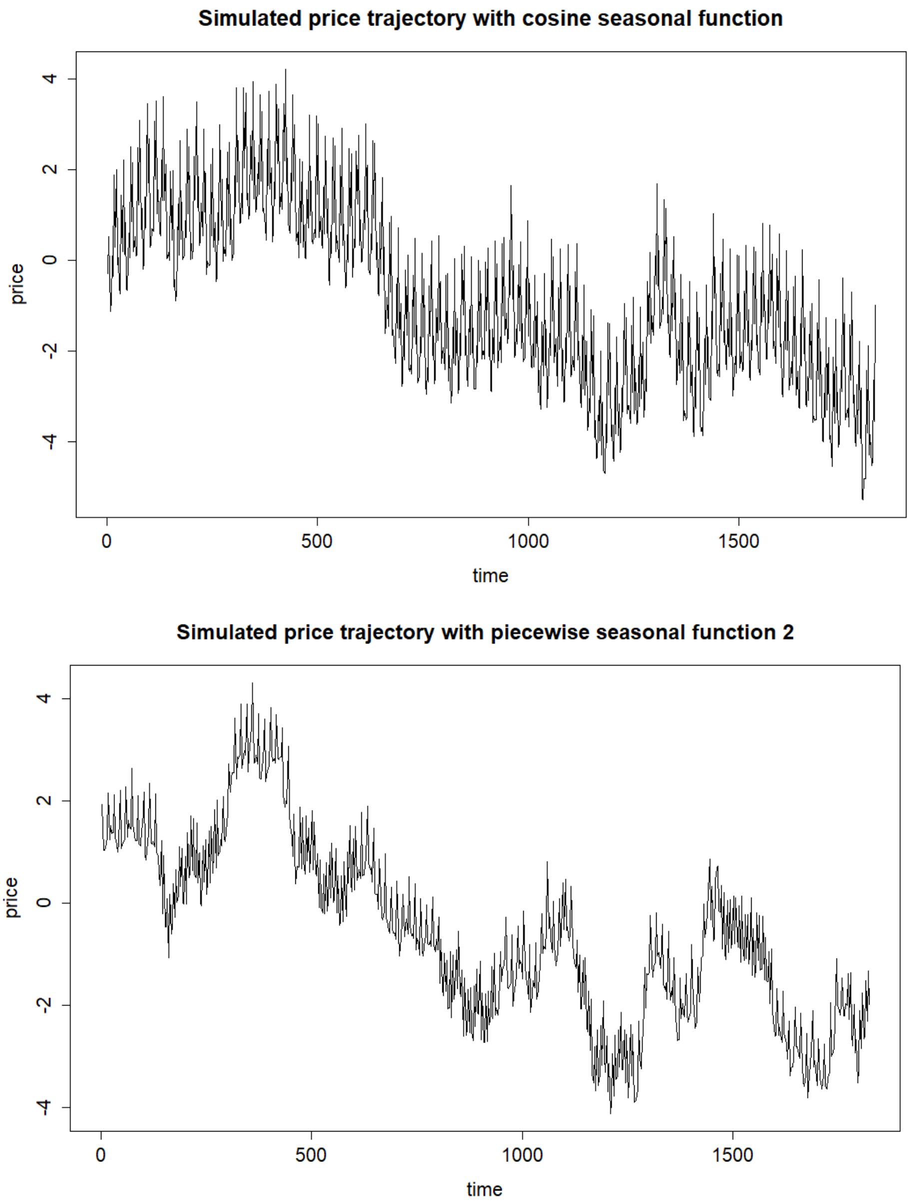

Figure 4.6: Simulated trajectory of the process of spot prices for a time horizon of 5 years using the standardised version of the cosine function (4.2) (above) and the standardised version of the piecewise function (4.4) (bottom)

4.3 Variants of the model

Lucia and Schwartz (2002) focused their analysis on the one-factor model just described, calibrating it using data from the NordPool. However, the authors did not only present this specification of the model but also developed different variants which are briefly presented in this section.

4.3.1 One-factor model of log spot prices

A first variant resumes the structure of the model (3.4) but is formulated with respect to the natural logarithm of spot prices. The logarithmic transformation allows, indeed, to stabilise the variance and to deal with extreme skewness (Gentle, 2020). Accordingly, the process $\ln P_t$ can be described by the following equation

$$\ln P_t = f(t) + Y_t, \qquad t \in [0, \infty) \tag{4.13}$$

where $f(t)$ is the deterministic function detailed in Section 4.1, while Y_t is the stochastic component following the zero-mean-reverting process defined by the stochastic differential equation (4.5). Similarly as above, this specification, together with equation (4.13), implies that $\ln P_t$ follows a Normal distribution.

Applying the reasoning explained in the previous section and assuming that $f(t)$ is differentiable, we obtain the following equation for the logarithm of prices

$$\ln(P_t) = f(t) + e^{-kt}Y_0 + \sigma \int_0^t e^{-k(t-s)} dZ_s, \qquad \forall t \in [0, \infty) \tag{4.14}$$

In order to compare the shape of the trajectory resulting from this version of the model with the one plotted in Figure 4.5, we simulated the trajectory of the process of the log prices for a time horizon of 5 years, using as deterministic component the specification (4.3) (Figure 4.7). To perform this simulation we exploited the estimates of the parameters computed by Lucia and Schwartz (2002). Hence, we assumed $\alpha = 4.938$, $\beta = 0.090$ and $\beta_i = (-0.027, -0.041, -0.041, -0.185, -0.097, -0.062, -0.101, -0.094, -0.067, -0.052, -0.057)^\intercal$. As expected, Figure 4.7 shows a much smoother trajectory than Figure 4.5. The logarithm of prices fluctuates in a smaller range and the steps given by the piecewise function are less visible. It can be still noticed some seasonality and some spikes but the volatility is reduced.

4.3.2 Two-factor model of spot prices

A further version aims at improving the realism of the specifications just discussed, enriching the model with an additional stochastic component, ε_t. The resulting model accounts

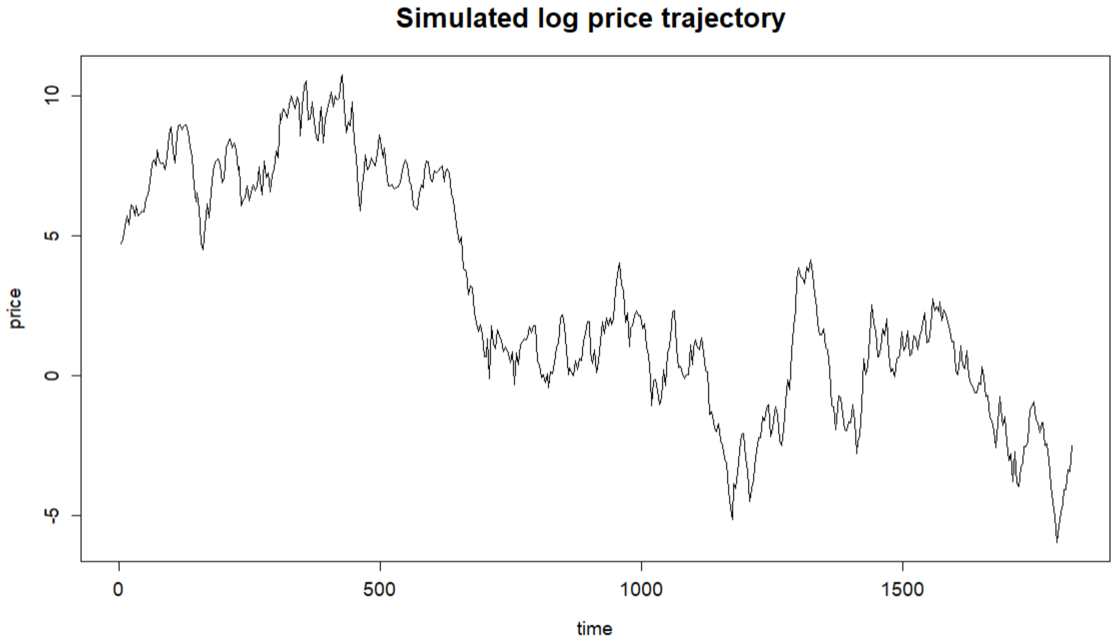

Figure 4.7: Simulated trajectory of the process of the log prices for a time horizon of 5 years.

for both a short-term mean-reverting stochastic component and a long-term stochastic component capturing the equilibrium level that prices tend to reach in the long-run. This version of the model takes the following form

$$P_t = f(t) + X_t + \varepsilon_t \tag{4.15}$$

where $f(t)$ is the deterministic seasonal component of the spot price P_t, while X_t and ε_t are the short-term and long-term stochastic component respectively.

The dynamics of the two stochastic components are specified by the following stochastic differential equations

$$dX_t = -kX_t dt + \sigma_X dZ_X \tag{4.16}$$

$$d\varepsilon_t = \mu_\varepsilon dt + \sigma_\varepsilon dZ_\varepsilon \tag{4.17}$$

where the two Brownian Motion processes, Z_X and Z_ε have correlation coefficient ρ. X_t follows the same mean-reverting process described by equation (4.5), capturing short-term changes in prices that are not expected to persist but rather are assumed to revert towards zero at a rate k. On the other hand, ε_t, which follows an arithmetic Brownian Motion with drift μ_ε, volatility σ_ε, and initial value ε_0, describes persistent fundamental changes in the level of spot prices.

4.3.3 Two-factor model of log spot prices

Resuming the approach provided in Section 4.3.1, the authors specified the structure of model (4.15) also for the natural logarithm of spot prices, $\ln P_t$, as follows

$$\ln P_t = f(t) + X_t + \varepsilon_t \tag{4.18}$$

where X_t and ε_t are the stochastic components following the processes described respectively by equation (4.16) and (4.17) in which the parameters k, σ_X, μ_ε, and σ_ε take a different value because of the logarithmic transformation.

Chapter 5

Application to the Italian Power Market

The previous chapter presented the model under study following a theoretical approach. This chapter aims to extend the analysis integrating the study with an empirical approach. It provides an application of the model to the Italian power market, illustrating the characteristics of the time series of the unique national price (PUN) and estimating the model on the basis of historical data.

5.1 Dataset presentation

The analysis presented in this chapter is based on a dataset that consists of a daily time-series covering a five year period, from January 1, 2015 to December 31, 2019. The data were collected on the website of the Italian electricity market operator, the GME, which records the unique national hourly prices expressed in euro per Megawatt-hour. The R code used for the following analysis is provided in Appendix B.

As the estimation of the model requires the use of daily prices, we generated a time series including only the closing price, for each day. This time series is plotted in Figure 5.1, which highlights a highly volatile and spiky behaviour typical of spot power prices. To describe more in depth the time series we computed the descriptive statistics reported in Table 5.1. These statistics confirm a high degree of variability of prices as the sample variance is particularly high, and the range of the dataset reveals a large dispersion of

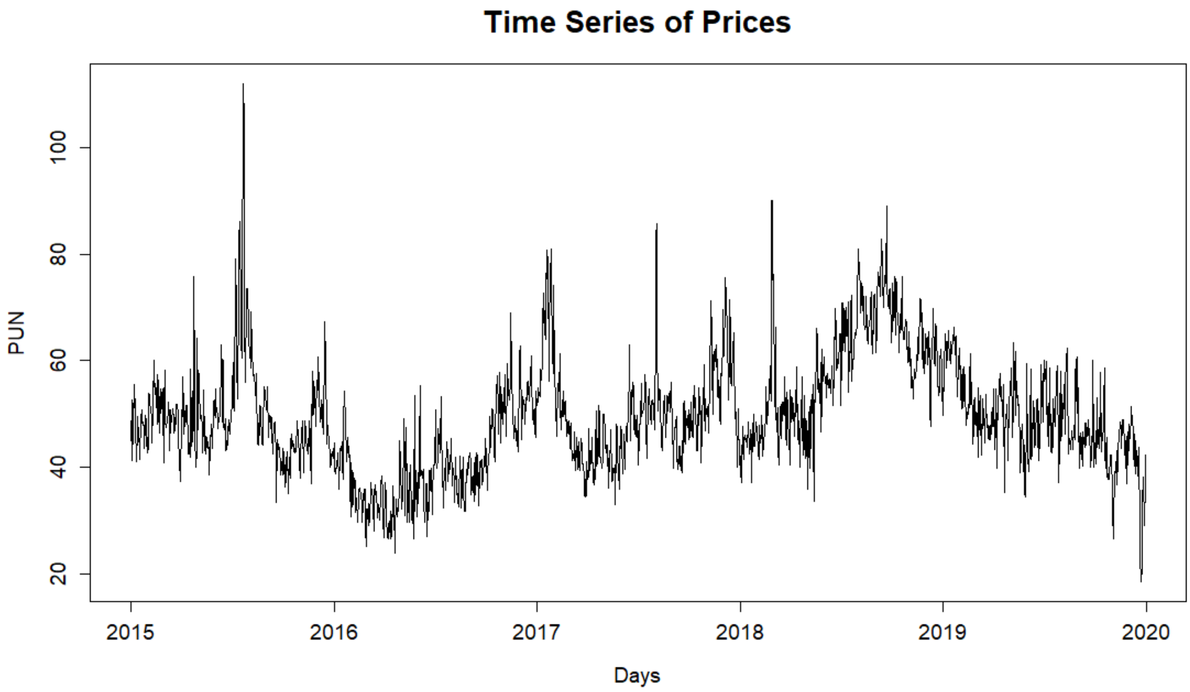

Figure 5.1: Daily time series of the PUN for the time interval 2015-2019.

prices. It is, however, important to underline that these results may be biased because of the presence of large spikes that significantly increase variability and expand the range of prices. The sample skewness is positive, indicating a distribution skewed to the right, presenting a longer right tail. Unlike the typical distribution of power spot prices, which is commonly leptokurtic, the sample kurtosis is lower than 3, implying a light-tailed distribution. This may be motivated by the fact that, as illustrated in Figure 5.1, the time

Statistics	**PUN**
N.Obs	1826.00
Minimum	18.58
Maximum	111.93
1. Quartile	42.63
3. Quartile	55.14
Mean	49.62
Median	48.47
Variance	116.50
Stdev	10.79
Skewness	0.73
Kurtosis	1.45

Table 5.1: Descriptive statistics of the time series of the PUN for the time interval 2015-2019.

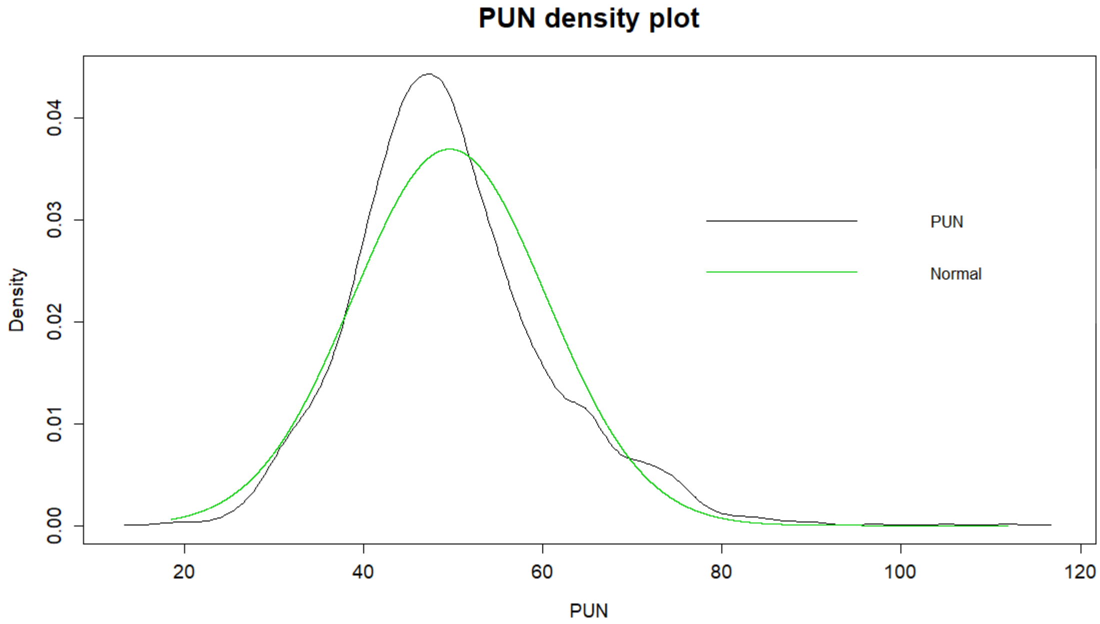

Figure 5.2: Density function of the PUN for the time interval 2015-2019.

series under study include only a few, mostly upward spikes, while great negative deviations from the mean are almost absent. This lack of large negative spikes could have significantly reduced the value of the sample kurtosis.

Figure 5.2 plots the empirical density function of the PUN and compares it with the probability density function of a Normal distribution having the mean and standard deviation equals to the sample ones. The graph confirms the moderate positive skewness of the sample distribution as the curve is left-leaning. It also shows that the right tail is slightly heavier than the Normal one, while the left tail is lighter, supporting the motivation for the low sample kurtosis provided above. In general, Figure 5.2 highlights a clear deviation of the distribution of prices from the Normal distribution. To verify this graphical evidence we conducted a series of Normality tests, including the Shapiro-Wilk test, the Kolmogorov-Smirnov Normality test, and the Jarque-Bera test, that rejected the null hypothesis of the sample distribution matching the Normal one (Table 5.2).

To analyse the time series more in depth, we performed some non-stationarity tests (Table 5.3). With a p-value of 0.01218, the augmented Dickey-Fuller test for unit-root rejected the hypothesis of presence of a unit root at a 5% significance level, revealing the non-stationarity of the series. Using the Ljung-Box test, which strongly rejected the null hypothesis of independently distributed data, the presence of serial correlation among

Test	Result	p value	Pr(>|t|)
Shapiro-Wilk	0.97172	< 2.2e-16	***
Kolmogorov-Smirnov	0.067171	1.409e-07	***
Jarque-Bera	322.2118	< 2.2e-16	***

Table 5.2: Normality tests of the time series of the PUN. 'Pr(>|t|)' probability of observing more extreme test results than the results actually observed, assuming the null hypothesis is correct. '***' significance level 0.1%.

data was confirmed. In this regard, we computed the autocorrelation coefficients until lag 35 and reported them in Table 5.4

As outlined by the coefficients of Table 5.4 and clearly illustrated by the sample autocorrelation function (ACF) and partial autocorrelation function (PACF) graphs in Figure 5.3, power prices seem to significantly depend on past prices. Moreover, this serial correlation significantly persists for several lags. It can be noticed that the correlations tend to be slightly more intense at lags multiple of seven. The ACF plotted in Figure 5.3 shows, indeed, recurrent peaks at the seventh, fourteenth, twenty-first, ... lags, outlining a periodic pattern. Such a regular pattern is even more evident in the ACF graph of the first differences in the level of prices (Figure 5.4). As depicted by the ACF plot, the autocorrelation coefficients that exceed the 95% confidence interval, hence, that are significantly different from 0, recur at lags multiple of seven. This recurrent pattern can suggest the presence of a weekly seasonality. Accordingly, the level of prices in the day t, as well as the daily increment of prices, are significantly influenced by the level of prices, or by the daily increment in the level of prices, of several weeks before.

Test	Result	p value	Pr(>|t|)
Augmented Dickey-Fuller	-3.9323	0.01218	*
Ljung-Box	1390.6	< 2.2e-16	***

Table 5.3: Stationary tests of the time series of the PUN. 'Pr(>|t|)' probability of observing more extreme test results than the results actually observed, assuming the null hypothesis is correct. '*' significance level 5%, '***' significance level 0.1%.

	Autocorrelation Coefficient of lag									
	1	2	3	4	5	7	14	21	28	35
PUN	0.872	0.795	0.756	0.736	0.731	0.758	0.657	0.590	0.542	0.510
$\mathbf{\Delta P_t}$	-0.198	-0.151	-0.077	-0.052	-0.115	0.166	0.166	0.137	0.124	0.107

Table 5.4: Autocorrelation coefficients of the PUN time series(first row) and of the first difference of prices (second row) computed till lag 35.

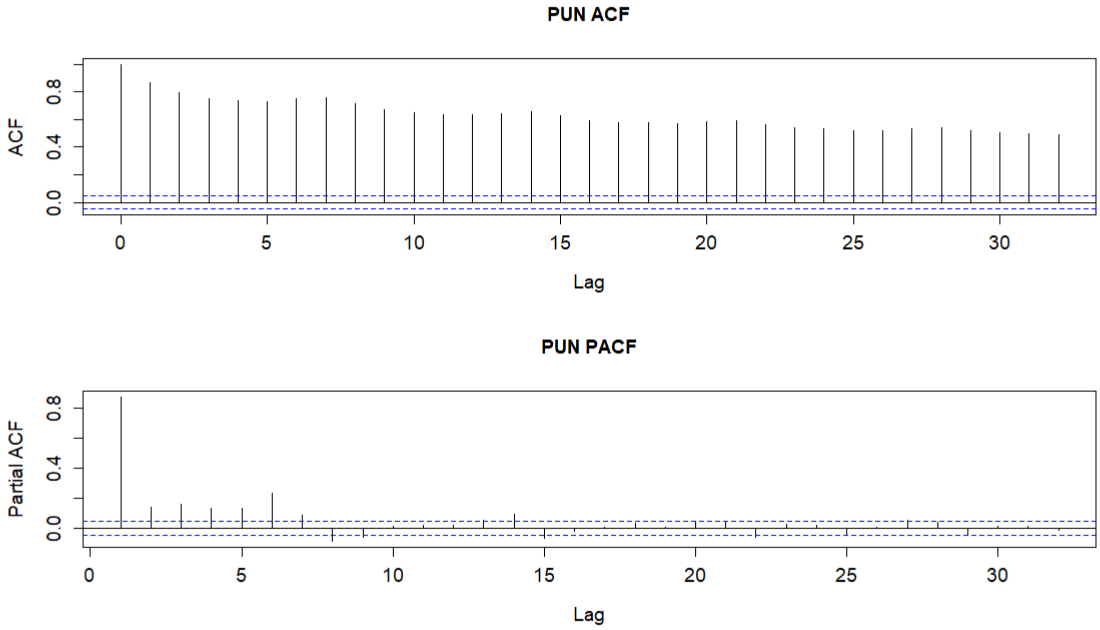

Figure 5.3: PUN autocorrelation function (above) and partial autocorrelation function (below).

5.2 Model estimation

The next step of this analysis is the estimation of the stochastic model. To implement it in the software R and adapt it to the PUN time series, a necessary requirement is to carry out a discretisation process. The model under study described in Chapter 4 is, indeed, a continuous-time model. It needs to be transformed into a discrete form to be able to use the available daily data in the empirical estimation process. Therefore, we express the Ornstein-Uhlenbeck process involved in the stochastic component of the model described in equation (4.5) as the following AR(1) process

$$X_j = (1-k)X_{j-1} + u_j, \qquad j = 0, 1, 2, \ldots, N \tag{5.1}$$

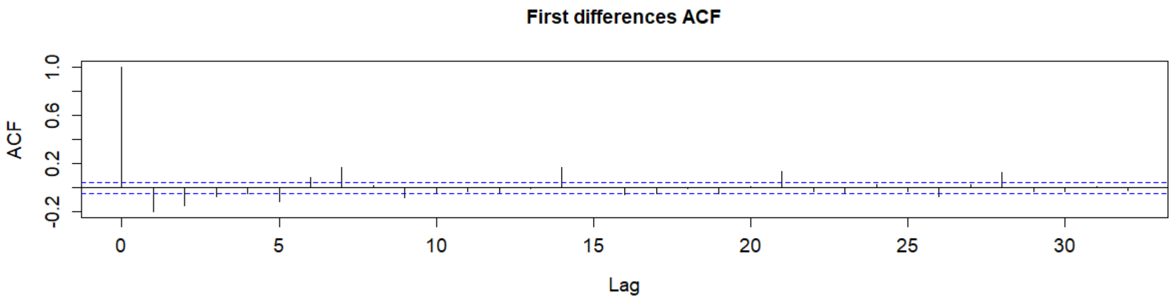

Figure 5.4: Autocorrelation function of the first differences in the level of prices.

where j is the time index representing a day, and $u_j \sim i.i.d.\mathcal{N}(0, \sigma^2_\Delta)$, with σ^2_Δ corresponding to $\sigma^2\Delta$, where Δ is the annualised discrete time interval and σ^2 is the parameter defined in (4.5).

Using the piecewise function (4.2), $f_1(j) = \alpha + \beta D_j + \sum_{i=2}^{12} \beta_i M_{ij}$, defined in the previous chapter, as deterministic component, it follows that the discrete form of the model is given by

$$\begin{aligned} P_j &= \alpha + \beta D_j + \sum_{i=2}^{12} \beta_i M_{ij} + X_j \\ X_j &= \phi X_{j-1} + u_j \end{aligned} \tag{5.2}$$

with $\phi = (1 - k) < 1$, and $\sigma_\Delta > 0$ representing the standard deviation of the error term u_j, the discrete-time analogue of the Brownian motion Z_t defined in equation (4.5).

The model (5.2) can be rewritten as

$$\begin{aligned} P_j &= f_1(j) + \phi X_{j-1} + u_j \\ &= f_1(j) - \phi f_1(j-1) + \phi P_{j-1} + u_j \end{aligned} \tag{5.3}$$

This representation was exploited to estimate the parameters by non-linear least squares method which, as outlined by Scales (1985), allows to fit the data with the non-linear model, minimising the sum of squared residuals. In order to specify appropriate initial guesses of the parameters, necessary for the estimation process, we computed a first estimate of the parameters by applying the least squares method to the deterministic component. Subsequently, to model the stochastic component, we extracted the residuals and adapted them to an AR(1) model, obtaining an estimate also of the parameters of the stochastic component. By using the results of this two-step procedure as initial guesses of the non-linear least squares estimation process, we obtained the estimated values reported in Table 5.5.

Parameters:	Estimate	Std. Error	t value	Pr(>|t|)	
alpha	4.83698	0.43769	11.051	< 2e-16	***
beta	-1.38316	0.18163	-7.615	4.21e-14	***
beta2	-0.35534	0.32932	-1.079	0.28073	
beta3	-0.88074	0.32479	-2.712	0.00676	**
beta4	-0.51546	0.32753	-1.574	0.11571	
beta5	-0.61603	0.32372	-1.903	0.05720	
beta6	-0.38459	0.32415	-1.186	0.23560	
beta7	0.34350	0.32105	1.070	0.28479	
beta8	-0.09136	0.32033	-0.285	0.77552	
beta9	-0.33435	0.32338	-1.034	0.30130	
beta10	-0.21968	0.32055	-0.685	0.49322	
beta11	0.04400	0.32297	0.136	0.89166	
beta12	-0.21665	0.32025	-0.676	0.49882	
phi	0.84501	0.01184	71.389	< 2e-16	***
sigma_delta	26.79083	0.88664	30.216	< 2e-16	***

Table 5.5: Estimates of the parameters of the model of prices (5.2). 'sigma_delta' represents the adjusted sample variance of the residuals. '***' significance level 0.1%, '**' significance level 1%.

The constant term α results being significantly different from zero, as well as the coefficient of the dummy variable D, i.e. β. This suggests that distinguishing between working and non-working days significantly contributes to capturing the seasonal pattern of prices. Also ϕ, the coefficient characterising the lagged stochastic component is significant. On the contrary, most of the coefficients β_i, referred to the monthly dummy variable M, are not significantly different from zero, except for β_2, the coefficient that multiplies the dummy for the month of March, which is significant at a 1% significance level. Hence, it seems that considering the month a price belongs to is not so relevant, as if prices were not greatly influenced by a monthly seasonality.

As the parameter σ^2_Δ is not directly included in the model, we estimated it separately.

The estimated $\hat{\sigma^2_\Delta}$ reported in Table 5.5 is the adjusted sample variance of the residuals of the model, namely $\hat{\sigma^2_\Delta}\frac{n}{n-1}$, where n indicates the sample size. It can be proved that $(n-1)\left(\hat{\sigma^2_\Delta}\frac{n}{n-1}/\sigma^2_\Delta\right) \sim \chi^2(n-1)$, from which the distribution of $\hat{\sigma^2_\Delta}\frac{n}{n-1}$ can be indirectly derived. Since the distribution of this estimator is known, we were able to compute the estimated standard deviation as $\sqrt{\widehat{Var(S^2)}} = 2\widehat{\sigma^4_\Delta}/(n-1)$ (Pace, & Salvan (2001)), which in turn allows to compute the relative t-test and p-value.

Aiming to visually assess the goodness of fit of the model, the graph in Figure 5.5 overlays the graph of the PUN with the values predicted by the model.

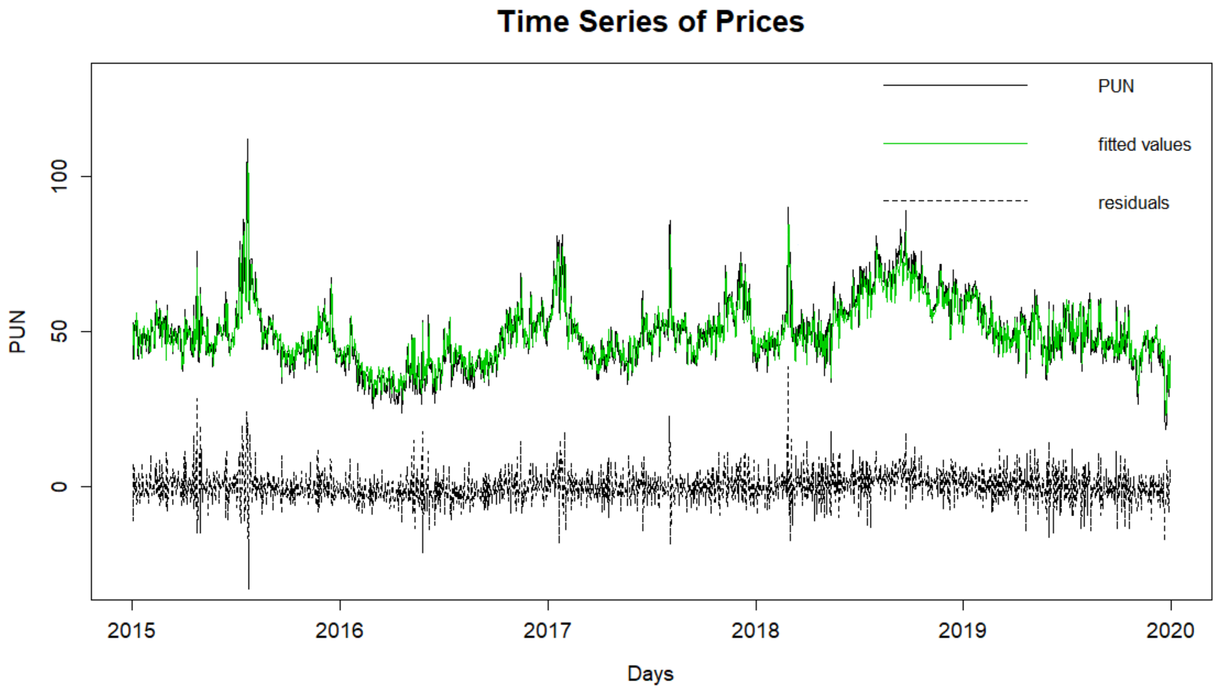

Figure 5.5: PUN time series, fitted values from model (5.2), and residuals.

In general, the model seems to adapt well to the data. However, it does not predict the most extreme values. The green line in Figure 5.5 depicts, indeed, spikes which are lower than the actual ones. The fitted values appear, therefore, less volatile and the resulting graph more smooth than the graph of the PUN. Such inability to properly capture the spiky behaviour of prices is due to the fact that the model does not include any jump component.

To provide a more precise evaluation of the estimated model, we analysed the resulting residuals, plotted in Figure 5.5. As outlined by Johnson and Straume (1992), the residuals emerging from the implementation of an appropriate model should exhibit a mean value close to zero, should be homoskedastic and not serially correlated. From the computations performed on the model of power prices, the residuals result fluctuating around a mean value of 0, with a standard deviation of 5.175. They also present a slightly positive skewness and a kurtosis of 4.199 that reject the Normality hypothesis. The greatest concern comes from the autocorrelation function of the residuals which reveals significant correlation coefficients at lags multiple of seven. This suggests that the model is not able to capture all the weekly seasonality of the data.

Applying this model to the logarithm of prices we obtain similar results. More precisely, the model under study becomes

$$\begin{aligned} \ln(P_j) &= \alpha + \beta D_j + \sum_{i=2}^{12} \beta_i M_{ij} + Y_j \\ Y_j &= \phi Y_{j-1} + u_j \end{aligned} \tag{5.4}$$

The resulting estimates are reported in Table 5.6.

Analogously to the estimates reported in Table 5.5, also in this specification, the only significant parameters are the constant term, α, the coefficient of the dummy variable D, β, the coefficient referred to the stochastic component, ϕ, and the parameter σ_Δ. The parameters of the monthly dummy variable are non-significantly different from zero, with the exception of β_2. The analysis of the residuals revealed similar results too. The residuals present, indeed, a positive skewness, even if lighter than the previous model. Also the tails of the distribution of the residuals are lighter, as the kurtosis results equal to 1.956. Accordingly, a Normality test rejects the hypothesis of Normal distribution. The residuals exhibit serial correlation but the coefficients are slightly lower than the ones resulting from the residuals of the previous model. This analysis of the residuals could suggest that, although still not perfect, the logarithmic variant of the model improves the fit to the observed data.

The analysis further proceeds by applying to the series of the PUN the models that integrate, as deterministic component, the following specifications, discussed in Chapter 4

$$f(j) = \gamma_A \cos(2\pi(j - t_A)) + \gamma_{sA} \cos(4\pi(j - t_{sA})) \tag{5.5}$$

$$f_2(j) = \alpha + \beta D_j + \gamma \cos\left((j + \tau)\tfrac{2\pi}{365}\right) \tag{5.6}$$

The model with the seasonal function (5.5) provide more extreme values for the estimates than model (5.2). Therefore, it seems to adapt better to the positive jumps of prices. The positive values of the residuals are, indeed, lower than the residuals resulting from model (5.2). However, its estimates of the negative peaks tend to be more extreme than the actually observed ones. The residuals extracted from this model are not independently distributed and they reject the Normality hypothesis as their distribution, despite being approximately symmetrical, exhibits heavy tails.

Lastly, the application of the model that integrates the deterministic function (5.6) provides results analogous to model (5.2). The model does not seem able to predict the most extreme values, the resulting residuals exhibit serial correlation and have a skewed and leptokurtic distribution.

To provide a quantitative measure of the goodness of fit of the estimated models of PUN, Table 5.7 presents the root-mean-square error (RMSE) of the three specifications

Parameters:	**Estimate**	**Std. Error**	**t value**	**Pr(>|t|)**	
alpha	0.342182	0.028014	12.215	$<$ 2e-16	***
beta	-0.028863	0.003651	-7.905	4.61e-15	***
beta2	-0.007088	0.006625	-1.070	0.28481	
beta3	-0.017676	0.006537	-2.704	0.00691	**
beta4	-0.010245	0.006596	-1.553	0.12052	
beta5	-0.012135	0.006512	-1.863	0.06256	
beta6	-0.007152	0.006518	-1.097	0.27272	
beta7	0.005782	0.006453	0.896	0.37040	
beta8	-0.001649	0.006443	-0.256	0.79806	
beta9	-0.007039	0.006508	-1.082	0.27961	
beta10	-0.003883	0.006447	-0.602	0.54703	
beta11	0.001541	0.006496	0.237	0.81253	
beta12	-0.004216	0.006442	-0.654	0.51293	
phi	0.843921	0.011871	71.093	$<$ 2e-16	***
sigma_delta	0.010828	3.385569e-04	28.083	$<$ 2e-16	***

Table 5.6: Estimates of the parameters of the model of the logarithm of prices (5.4). 'sigma_delta' represents the adjusted sample variance of the residuals. '***' significance level 0.1%, '**' significance level 1%.

of the model of PUN, measuring the deviations of the predicted values from the actual values of the series.

Model	RMSE
5.3	5.17315
5.5	5.448324
5.6	5.191277

Table 5.7: Root-mean-square error (RMSE) for the three specifications of the model of PUN

The RMSE can be interpreted as a measure of the accuracy of a model's fit, as it measures the deviations of the estimated values from the actual values of the series. Accordingly, a lower value of the RMSE suggest a better ability of the model to adapt to the data. The results reported in Table 5.7 indicate that the model with the seasonal function (5.5) provide the worst fit to the data among the estimated models, while the specifications (5.3) and (5.6) result being virtually equivalent.

To further evaluate and compare the models of PUN estimated in this chapter, we computed the Akaike Information Criterion (AIC) reported in Table 5.8.

Model	AIC	BIC
5.3	11207.83	11290.47
5.5	11379.05	11412.06
5.6	11202.65	11235.66

Table 5.8: Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) for the models of PUN

As the logarithmic trasformation of the dataset does not allow to compare the information criteria of model (5.4) with the ones computed on the original dataset, we reported the values obtained for the model of the log of PUN in Table 5.10 below.

Focusing on the the three specifications of the model of PUN, the results suggest that, to minimise the expected information loss, the model should apply equation (5.3) or (5.6) as deterministic component. On the contrary, the application of the cosine function (5.5) as seasonal component of the model seems to provide the least goodness of fit among the three models.

To consider that model (5.3) is less parsimonious than the others, we computed the Bayesian Information Criterion (BIC) as well (Table 5.8). As outlined by Dziak et al. (2020), with BIC the penalty for the number of parameters is, indeed, larger than with AIC. More precisely, it is $\ln(n)k$, where n is the sample size and k the number of parameters. Hence, as n increases, the criterion becomes more conservative, preferring the model with the fewest parameters. As expected, BIC is higher than AIC for model (5.3). However, the final ranking of the three specifications of the model of PUN does not change, since, as suggested by AIC, the models with the higher quality results being the ones whose deterministic component is represented by equation (5.3) or (5.6).

So far we modelled the data under study using the specifications presented by Lucia and Schwartz (2002). However, the results that emerged from fitting these models to the data highlighted the inability of these specifications to capture the strong weekly seasonality of the series of the Italian prices. To improve the goodness of fit of the model, we adapted the specification (5.4), the most appropriate one according to the analysis of the residuals, to this feature. The resulting model is defined as follows

$$\begin{aligned} \ln(P_j) &= \alpha + \beta D_j + \sum_{i=2}^{7} \gamma_i W_{ij} + Y_j \\ Y_j &= \phi Y_{j-1} + u_j \end{aligned} \tag{5.7}$$

where α is the constant term, D is the dummy variable for the working days defined in equation (5.4), β is the parameter capturing the changes in prices occurring between the working and non-working days, and $\phi = 1 - k$. To cope with the weekly seasonality, we defined γ_i, for $i = 2, \ldots, 7$ as the parameters that represent the fluctuations of the prices during the days of the week, and W_{ij} as a dummy variable such that

$$W_{ij} = \begin{cases} 1 & \text{if } j \text{ belongs to the i - th day of the week} \\ 0 & \text{otherwise} \end{cases}, \qquad \text{for } i = 2, \ldots, 7$$

Using the estimation approach previously described we obtained the results summarised in Table 5.9

In this specification the coefficients γ_2,γ_4, and γ_6, referred to the weekly dummy vari-

Parameters:	Estimate	Std. Error	t value	Pr(>|t|)	
alpha	0.278146	0.026542	10.480	< 2e-16	***
beta	0.018090	0.027528	0.657	0.511	
gamma2	-0.070318	0.015644	-4.495	7.40e-06	***
gamma3	-0.010725	0.009125	-1.175	0.240	
gamma4	-0.063357	0.012738	-4.974	7.18e-07	***
gamma5	-0.008149	0.012783	-0.637	0.5246	
gamma6	-0.144466	0.028757	-5.024	5.57e-07	***
gamma7	-0.034895	0.031537	-1.106	0.269	
phi	0.885366	0.010944	80.901	< 2e-16	***
sigma_delta	0.009873	0.811348	0.012	0.496	

Table 5.9: Estimates of the parameters of the model (5.7). 'sigma_delta' represents the adjusted sample variance of the residuals. '***' significance level 0.1%, '**' significance level 1%.

able W, are significant, confirming the relevance of distinguishing among the different days of the week to capture the pattern of prices fluctuations. On the other hand, the integration of the dummy variable for the day of the week reduces the significance of distinguishing between working and non-working days. Accordingly, the coefficient β is not significantly different from zero.

The graph in Figure 5.6 aims to visually illustrate this version of the model, showing the fitted values and the resulting residuals.

As represented by the graph, the residuals of this model fluctuate around a mean value of 0 with a standard deviation of 0.099391, lower than the one resulting from the residuals of model (5.4), which was 0.104030. However, the residuals still do not satisfy the hypothesis of Normality as they present a skewness of -0.067531 and a kurtosis of 2.062680. The autocorrelation function shows an improvement in the goodness of fit. As plotted in Figure 5.7 , by differentiating among the days of the week, the serial correlation of the residuals significantly reduces and becomes neglegible within the two-weeks lag.

Such improvement is confirmed by the reduction with respect to model (5.4) of the RMSE which indicates a greater accuracy of the model's fit (Table 5.10).

In line with these results, the Akaike and the Bayesian information criteria turned out lower for the model (5.7). Hence, both the values of the AIC and BIC reveal an increase

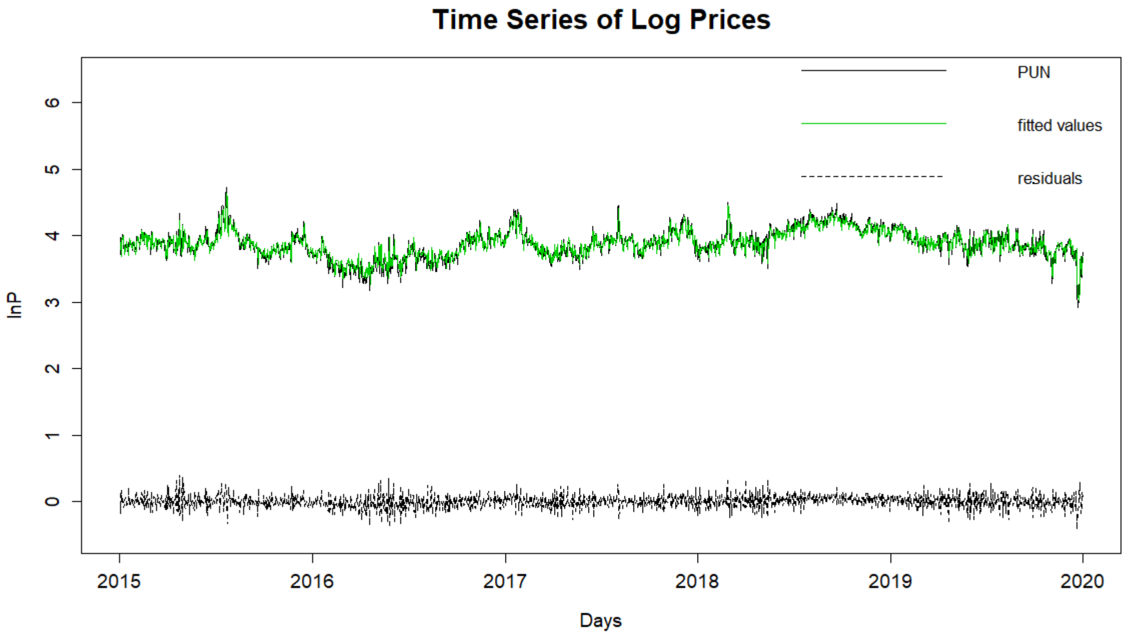

Figure 5.6: Log of PUN time series, fitted values from model (5.7), and residuals.

in the quality of the model that considers the weekly seasonality compared to model (5.4) that does not take it into account.

Model	AIC	BIC	RMSE
without weekly seasonality	-3051.853	-2969.479	0.104001
without weekly seasonality	-3228.503	-3173.531	0.099363

Table 5.10: Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and Root-Mean-Square Error for the models without weekly seasonality (5.4) and with weekly seasonality (5.7)

In light of the outcomes of this analysis, it could be concluded that, to model the price series object of study, a seasonal function considering a weekly seasonality is more adequate than a seasonal function modelling a monthly seasonality.

5.3 Jump component integration

One of the limits that emerged from the application of the diffusion model presented in the previous section is the inability to capture the price spikes of the observed series. Hence, we added to the model of the logarithm of prices (5.7), the one that in the previous

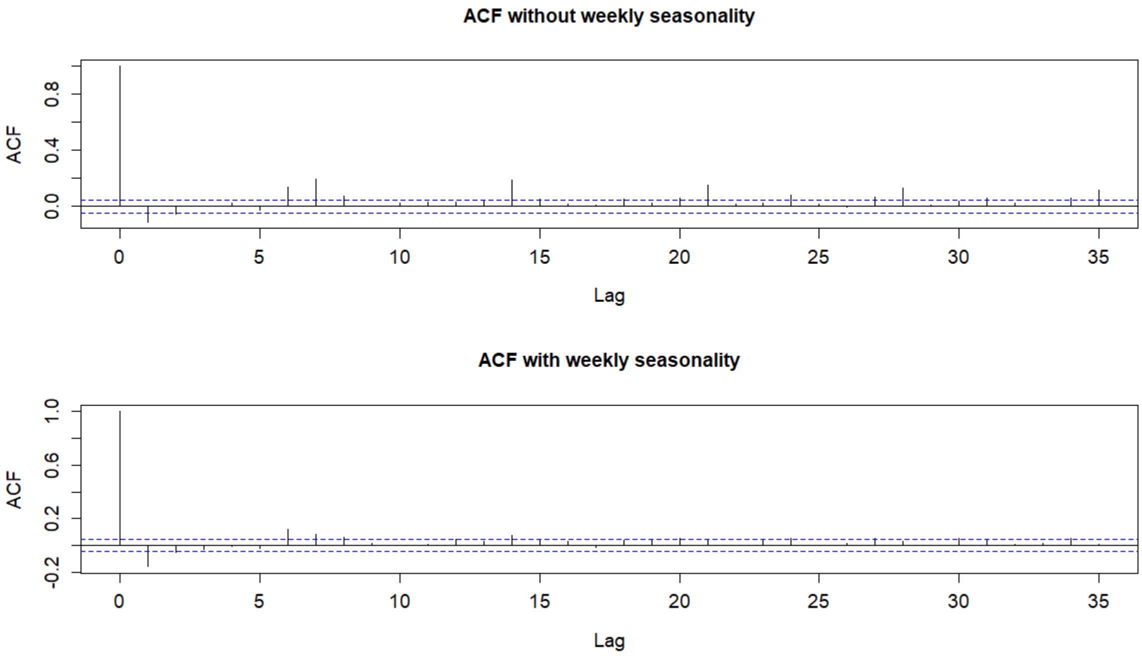

Figure 5.7: Autocorrelation functions (ACF) of the residuals resulting from model (5.2)(above) and (5.7)(below).

section seemed to better fit the observed data, a jump component. We followed the approach proposed by Cartea and Figueroa (2005) (see Chapter 3) adapting it to the time series under study. The integration of a jump component with jumps following a Normal distribution, as presented by the two authors, did not significantly improve the results obtained with the model with no jump component. Therefore, we tested a different variant of this jump diffusion model. Since the time series of the logarithm of PUN mainly exhibits positive spikes, we assumed that the jumps follow an exponential distribution.

The dynamics of the stochastic component of the resulting model is, therefore, described by the following equation

$$dY_t = -kY_t dt + \sigma dZ_t + J dq_t \tag{5.8}$$

where $J \sim Exp(\lambda)$ represents the jump size following an exponential distribution, with $\lambda > 0$ the rate parameter, and q_t is a Poisson process with constant intensity l, defining the probability with which a jump occurs.

The estimation process of this model has been divided into two steps, implementing an approach similar to the one used in the previous section to initialise the parameters for the estimation process. The first step consists of calibrating the seasonal function using the least squares method and removing the estimated seasonality from the series of prices. The function we used in this step to represent the seasonality is the piecewise function defined in equation (5.7).

In the second step, the remaining stochastic component Y_t is estimated. This estimation process requires Y_t to be discretised. Following the approach proposed by Cartea and Figueroa (2005), that approximated the Poisson process with a Bernoulli process, the discretised model of Y_t can be defined as

$$Y_j = \begin{cases} \phi Y_{j-1} + \sigma_\Delta u_j + v_j & \text{with probability } l\Delta \\ \phi Y_{j-1} + \sigma_\Delta u_j & \text{with probability } (1 - l\Delta) \end{cases} \tag{5.9}$$

where $\phi = 1 - k$, σ_Δ is the standard deviation of the error term when no jump occurs, corresponding to $\sigma^2\Delta$ where Δ is the annualised discrete time interval and σ^2 is defined in (4.5), and u_j and v_j are independent random variables following a Normal and an exponantial distribution, respectively.

Therefore, Y_j can be expressed as the mixture of two random variables, with weights $l\Delta$ and $(1-l\Delta)$, following, respectively, a Normal distribution, with mean ϕY_{j-1} and variance σ_Δ^2, and an exponentially modified Gaussian distribution, with parameters ϕY_{j-1}, σ_Δ^2, and λ, where λ is the rate parameter characterising the exponential distribution of the term v_j.

Hence, the density function of Y_j, conditional on Y_{j-1}, is known and can be defined as follows

$$f(Y_j|Y_{j-1}) = l\Delta g_1(Y_j|Y_{j-1}) + (1 - l\Delta)g_2(Y_j|Y_{j-1}) \tag{5.10}$$

with

$$g_1(Y_j|Y_{j-1}) = \frac{\lambda}{2}\exp\left(\frac{\lambda}{2}(2\phi Y_{j-1} + \lambda\sigma_\Delta^2 - 2Y_j)\right) + \operatorname{erfc}\left(\frac{\phi Y_{j-1} + \lambda\sigma_\Delta^2 - Y_j}{\sqrt{2}\sigma_\Delta}\right),$$

$$g_2(Y_j|Y_{j-1}) = \frac{1}{\sqrt{2\pi\sigma_\Delta^2}}\exp\frac{-(Y_j - \phi Y_{j-1})^2}{2\sigma_\Delta^2}$$

where erfc is the complementary error function defined as

$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-t^2} dt$$

Equation (5.10) allows to estimate the parameter $\theta = (\phi, \sigma_\Delta, \lambda, l)$ by minimising the negative log-likelihood function of Y_t, namely

$$\min_\theta - \sum_{j=1}^{N} \log(f(Y_j|Y_{j-1})) \tag{5.11}$$

This estimation process is subject to the constraints $\phi < 1$, which ensures that Y_j reverts back to its long-term mean, $\sigma_\Delta > 0$, $\forall j = 1, \ldots, N$, $\lambda > 0$, and $0 < l\Delta < 1$ as it represents a probability. Moreover, the computation assumes $\Delta = \frac{1}{365}$.

The estimates obtained are summarised in Table 5.11. The statistics of the estimated parameters reported in the Table were computed by exploiting the asymptotic Normality of maximum likelihood estimators. As explained by Greene (2003), under some basic assumptions, that, for the sake of simplicity, we assume to be satisfied, a maximum likelihood estimator is asymptotically Normally distributed. More precisely, $\hat{\theta} \overset{a}{\sim} \mathcal{N}(\theta_0, I(\theta_0)^{-1})$, where θ_0 is the true value of the parameter, while $I(\theta_0)$ is the Fisher information matrix computed in θ_0.

Parameter	**Estimate**	**Std. Error**	**t value**	**Pr(>|t|)**	
ϕ	0.8519	0.025962	32.813	<2e-16	***
σ	0.0552	0.003602	15.326	<2e-16	***
λ	0.0081	0.077166	0.105	0.440	
l	1e-5	0.002317	0.004	0.496	

Table 5.11: Estimates of the parameters of the jump component (5.9). '***' significance level 0.1%.

From these results it follows that the estimated daily mean reversion rate, $\hat{k}$, equals 0.1481, indicating that prices revert back to their mean level in 6.75 days. Similarly to the specifications estimated in the previous section, this model strongly relies on the lagged stochastic component, characterised by the parameter ϕ, to explain prices behaviour. As expected, the estimated volatility is low, due to the logarithmic transformation of prices that stabilised the variance. The estimate of λ, the rate parameter characterising the jump component, resulted quite low as well. This implies that the model expects the jumps to be quite high and volatile. However, the probability that a jump occurs during a day is almost null and both the parameters λ and l are non-significative. This result clearly suggests that the data under analysis do not allow to estimate the jump component with sufficient statistical significance.

As outlined by Cartea and Figueroa (2005), this estimation approach may imply some issues. As the model includes a large number of parameters, it suffers from the risk of overparametrisation, affecting the reliability of the estimates. Accordingly, the estimated probability of the event of a jump close to 0 could suggest that the data may not contain enough information to estimate the parameters reliably.

The resulting fitted values are plotted in Figure 5.8, along with the actual values and the resulting residuals

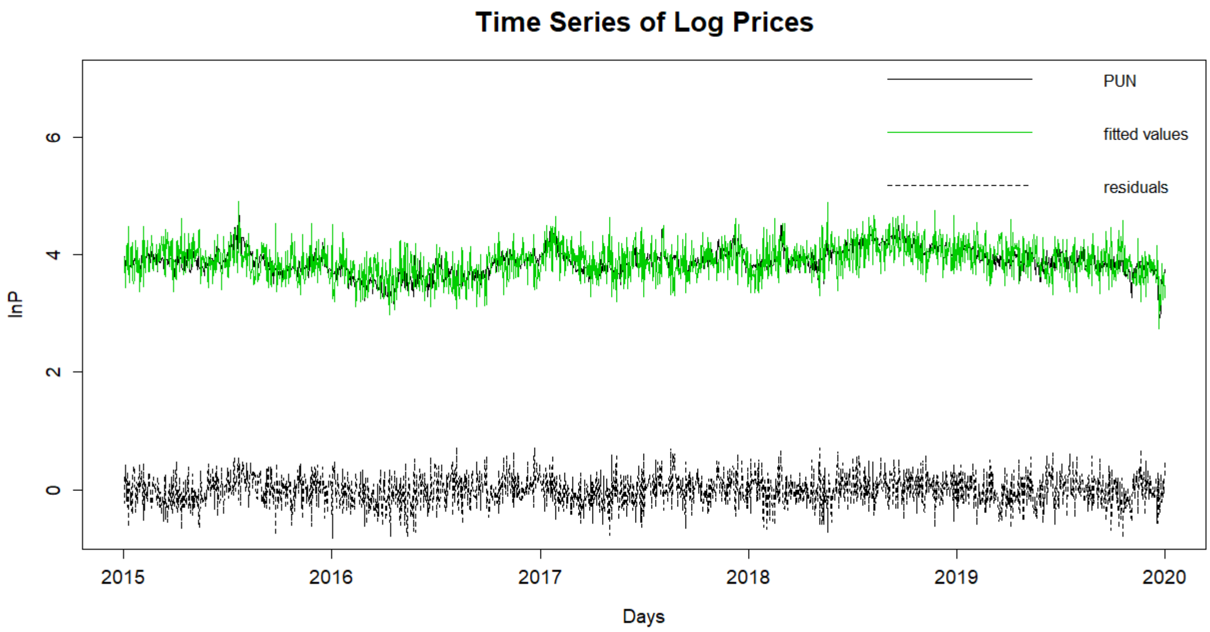

Figure 5.8: log PUN time series, fitted values, and residuals from model (5.8).

From Figure 5.8, it can be noticed that, after the integration of the jump component, the model does not fit the actual data precisely, as it over-estimates the spikes. The resulting residuals fluctuate around a mean value of -0.003946, with a standard deviation of 0.265847, which is higher than the volatility of the residuals of the model of the logarithm of prices without the jump component, which was 0.099391. Since they exhibit a negative skewness and a low kurtosis, the residuals reject the hypothesis of Normal distribution. Moreover, as illustrated by the autocorrelation and partial autocorrelation functions plotted in Figure 5.9, the residuals present some serial correlation even though it becomes non-significant after the two-weeks lag.

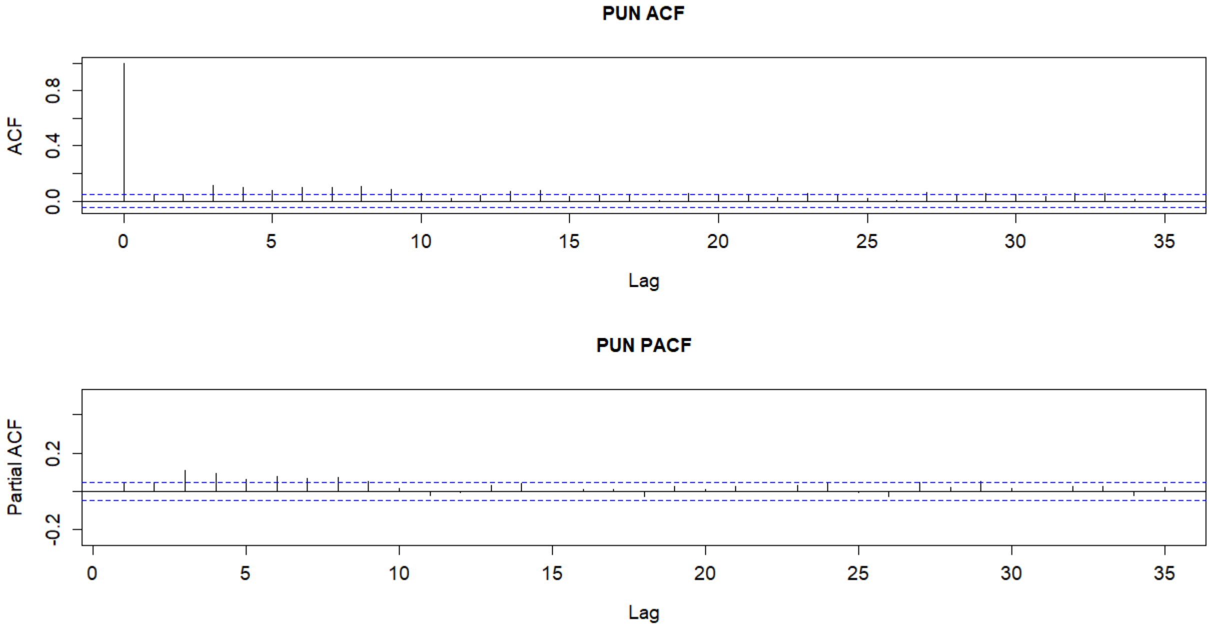

Figure 5.9: Autocorrelation functions (ACF) and partial autocorrelation function (PACF) of the residuals resulting from model (5.8).

To compare the model that includes the jump component with the model of the logarithm of prices without any jump component, we computed the root-mean-square errors reported in Table 5.12.

In line with what Figure 5.8 illustrates, the results reported in Table 5.12 clearly indicate that the models with the jump component provide a worst accuracy level than model (5.7).

Such a poorer goodness of fit is confirmed by the Information Criteria reported in Table 5.12 as well. The values of both the Akaike Information Criterion and Bayesian Information Criterion of model (5.9) are, indeed, much higher than those of model (5.7) , suggesting that including the jump component reduced the ability of the model to fit the actual data, minimising the information loss.

Model	AIC	BIC	RMSE
without jump component	-3228.503	-3173.53105	0.099363
with jump component	370.8799	440.5013	0.265803

Table 5.12: Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and Root-mean-square error (RMSE) for the models of the log of PUN, with and without the jump component

These results seem to suggest that a jump diffusion model with exponentially distributed jumps cannot improve the ability of a diffusion model without any jump component, as the one described by equation (5.7), to fit the series of prices under study.

Conclusions

The worldwide liberalisation of electricity markets fostered an increasing interest in the study of the behaviour of electricity prices.

The electricity market presents, indeed, unique peculiarities, and electricity differs from any other financial asset. Accordingly, the academic literature extensively proved that the non-storability of electricity strongly impacts the development of electricity prices, demonstrating the existence of some stylized facts recurrent among markets. One direct consequence of the non-storability is the need to ensure a constant balance between demand and supply, which in turn results in large price fluctuations. Such high volatility tends to be subject to clustering, showing periods in which price variations are high and others in which they are lower. Furthermore, spot prices typically exhibit sudden and sharp jumps which revert back to previous levels in a very short time-frame, reflected in a leptokurtic distribution of the prices. The literature has further shown that spot electricity prices present some seasonality and a mean-reversion tendency.

The analysis performed in this dissertation on the time series of Italian electricity prices, from January 1, 2015, to December 31, 2019, confirmed the presence of the above-mentioned features, revealing particularly high volatility, a distribution with a heavy right tail, and a strong weekly seasonality. In light of these peculiarities, this dissertation aims to model the time series under analysis adapting the mean-reverting diffusion model developed by Lucia and Schwartz (2002) to the key features that emerged from the data.

The original specification of the electricity pricing model proposed by Lucia and Schwartz (2002) is one of the most popular in electricity markets and has been a key starting point for more complex developments. To summarise, the model consists of a deterministic component, capturing the seasonality, and a stochastic component following a mean-reverting Ornstein-Uhlenbeck process. We tested the fit of the model to the

Italian electricity prices, estimating it using non-linear least squares. To specify appropriate initial guesses of the parameters, necessary for the estimation process, a two-step procedure was implemented. The first step consisted of estimating the parameters of the deterministic component by least squares method, while, in the second step, we extracted the residuals and adapted them to an AR(1) model, obtaining initial guesses for the parameters of the stochastic component as well.

The results that emerged from this approach suggest that the model adapts fairly well to the data, even though it does not seem able to accurately predict the most extreme values. The analysis of the residuals further revealed the inability of the model to entirely capture the weekly seasonality of the prices.

The subsequent analysis, performed on other variants of the model of Lucia and Schwartz (2002), proved that applying a logarithmic transformation to the original model improves the goodness of fit to the observed time series, though still not capturing the strong weekly seasonality of the Italian electricity prices. To cope with such inability, we proposed a variant of the piecewise seasonal function presented by the two authors, replacing the monthly dummy variable with a weekly dummy variable. The analysis of the residuals of the model, as well as the computation of the root-mean-square error and the Akaike and Bayesian information criteria, indicated that differentiating among the different days of the week increased the accuracy of the model's fit and its quality. Therefore, it could be concluded that a seasonal function modelling a weekly seasonality adapts better to our dataset than a seasonal function modelling a monthly seasonality.

One of the limits of these models is the lack of an adequate representation of the price spikes. To address this shortcoming, we introduced in the model a jump component, in the spirit of the approach proposed by Cartea and Figueroa (2005). The integration of a jump component with jumps following a Normal distribution did not significantly improve the results obtained with the model with no jump component. Therefore, we tested another variant of this jump-diffusion model, by assuming exponentially distributed jumps aiming to capture the predominantly positive spikes of the prices. The estimation process of this model exploits an approach similar to the two-step procedure previously described. We initially estimated the seasonal function by least squares method and, later, we removed it from the data to estimate the stochastic component by maximum likelihood methods.

The outcome of this approach clearly suggests that the data under analysis do not allow to estimate the jump component with sufficient statistical significance. Moreover, the model seems to provide a poorer goodness of fit than the previous specifications, suggesting that a jump-diffusion model with exponential jumps cannot improve the ability of a diffusion model to fit the series of the Italian electricity prices.

In conlusion, this dissertation presents an overview of the most popular approaches to electricity modelling, providing a complete application of one of these approaches to the Italian market. This study can, therefore, offer opportunities for further analysis concerning the pricing of electricity derivatives. Accordingly, due to the unique features of electricity, the pricing of these contracts relies on the identification of an adequate process for the underlying spot electricity prices. Moreover, this study may be a useful starting point for the implementation of different electricity price estimation methods, such as a jump recursive filter estimation procedure, on an updated time series.

Appendix A

```r
## R CODE OF THE SIMULATIONS PERFORMED IN CHAPTER 4
rm(list=ls())
#### Deterministic component ####
# define the cosine seasonal function
f1 <- function(t,gammaA,tA,gammaSA,tSA){
        gammaA*cos(2*pi*(t-tA))+gammaSA*cos(4*pi*(t-tA))
}
# plot the function f1, assuming gammaA, gammaSA=1 and tA, tSA=0
curve(f1(x,1,0,1,0),0,2,400,xlab="time",ylab="price",
        main="Cosine␣seasonal␣function",
        cex.main="1.7", cex.lab="1.2", cex.axis="1.2")

## Piecewise function 1
# generate a sequence of days for two years
days <- seq(as.Date("2021/1/1"), as.Date("2022/12/31"), by = "day")
date <- as.POSIXlt(days,format="%Y-%m-%d")
# load the package timeDate
library(timeDate)
# define the holidays
easter<-c(as.Date(Easter(2021:2022, 0)))
christmas<-c(as.Date(ChristmasDay(2021:2022)))
newyear<-c(as.Date(NewYearsDay(2021:2022)))
holidays <- c(as.Date(easter),as.Date(christmas),as.Date(newyear))
# generate the dummy for the weekends and holiday
D <- ifelse(weekdays(date)=="sabato" | weekdays(date)=="domenica"
        | days %in% holidays,1,0)
```

```
# generate the dummy for the i-th calendar month
Mfeb <- ifelse(months(date)=="febbraio",1,0)
Mmar <- ifelse(months(date)=="marzo",1,0)
Mapr <- ifelse(months(date)=="aprile",1,0)
Mmag <- ifelse(months(date)=="maggio",1,0)
Mgiu <- ifelse(months(date)=="giugno",1,0)
Mlug <- ifelse(months(date)=="luglio",1,0)
Mago <- ifelse(months(date)=="agosto",1,0)
Mset <- ifelse(months(date)=="settembre",1,0)
Mott <- ifelse(months(date)=="ottobre",1,0)
Mnov <- ifelse(months(date)=="novembre",1,0)
Mdic <- ifelse(months(date)=="dicembre",1,0)

M <- cbind(Mfeb,Mmar,Mapr,Mmag,Mgiu,Mlug,Mago,Mset,Mott,Mnov,Mdic)

# define the parameters
betai <- c(-2.527, -4.511, -3.484, -13.248, -12.656, -7.038, -8.109,
         -10.061, -9.597, -7.304, -6.019)
alpha2 <- 153.51
beta2 <- 9.514

# define the first piecewise function
f2 <- function(t,alpha2,beta2,betai){
alpha2+beta2*D[t]+sum(betai*M[t,])
}
f2.Vec <- Vectorize(f2,"t")
# plot the function using the above-defined parameters
curve(f2.Vec(x,alpha2,beta2,betai),0,365*2,400,xlab="time",
        ylab="price", main="Piecewise␣seasonal␣function␣1",
        cex.main="1.7", cex.lab="1.2", cex.axis="1.2")

## Piecewise function 2
# define the parameters
```

```r
alpha3 <- 151.08
beta3 <- -10.24
gamma <- 30.27
tau <- 3.96

# define the second piecewise function
f3 <- function(t,alpha3,beta3,gamma,tau){
alpha3+beta3*D[t]+gamma*cos((t+tau)*(2*pi)/365)
}
f3.Vec <- Vectorize(f3,"t")
# plot function, assuming the parameters take the values reported above
curve(f3.Vec(x,alpha3,beta3,gamma,tau),0,365*2,400,xlab="time",
        ylab="price", main="Piecewise␣seasonal␣function␣2",
        cex.main="1.7", cex.lab="1.2", cex.axis="1.2")

#### Stochastic component ####
# load the package sde
library(sde)
# define the parameters and set an initial seed
set.seed(1)
k <- expression(-0.0014 * x)
s <- expression(2.36)
# simulate the solutions to the stochastic differential equation
x <- sde.sim(t0=0,T=1,X0=0,N=1825*5,delta=1/365,drift=k,sigma=s,
        method="euler")
# plot the trajectory of the simulated solutions
plot(x, xlab="time", ylab="price",
        main="Xt␣Ornstein-Uhlenbeck␣process",
        cex.main="1.7", cex.lab="1.2", cex.axis="1.2")

#### Model for spot prices ####
# define the model of prices
Pt <- function(t){
```

```
f2(t,alpha2,beta2,betai)+x[t]
}
Pt_V <- Vectorize(Pt)
# plot the trajectory of prices
curve(Pt_V,0,5*365,400,xlab="time",ylab="price",
   main="Simulated␣price␣trajectory␣with␣piecewise
␣␣␣seasonal␣function␣1",
   cex.main="1.7", cex.lab="1.2", cex.axis="1.2")

# define the standardised model of prices
        # using the deterministic function f1 and f3
Pt_1 <- function(t){
f1(t,1,0,1,0)+x[t]
}
Pt_1V <- Vectorize(Pt_1)

Pt_3 <- function(t){
f3(t,0,1,1,0)+x[t]
}
Pt_3V <- Vectorize(Pt_3)
# plot the trajectories
op=par(mfrow=c(2,1))
curve(Pt_1V,0,5*365,500,xlab="time",ylab="price",
        main="Simulated␣price␣trajectory␣with␣cosine␣seasonal␣function",
        cex.main="1.2", cex.lab="1.2", cex.axis="1.2")
curve(Pt_3V,0,5*365,500,xlab="time",ylab="price",
        main="Simulated␣price␣trajectory␣with␣piecewise
␣␣␣␣␣␣␣␣seasonal␣function␣2",
        cex.main="1.2", cex.lab="1.2", cex.axis="1.2")
par(op)
# define the parameters of the logarithm model
alphalog <- 4.938
betalog <- -0.090
```

```
betailog <- c(-0.027, -0.041, -0.041, -0.185, -0.097, -0.062, -0.101,
         -0.094, -0.067, -0.052, -0.057)
# define the deterministic component
f2log <- function(t,alphalog,betalog,betailog){
        alphalog+betalog*D[t]+sum(betailog*M[t,])
}
# simulate the stochastic component
k_two <- expression(-0.0077 * x)
s_two <- expression(5.77)
set.seed(1)
x_two <- sde.sim(t0=0,T=1,X0=0,N=(365*2)*365,
        delta=1/365,drift=k_two,sigma=s_two,method="euler")
# define and plot the trajectory
Pt_log <- function(t){
        f2log(t,alphalog,betalog,betailog)+x_two[t]
}
Pt_logV <- Vectorize(Pt_log)
curve(Pt_logV,0,5*365, 500,xlab="time",ylab="price", main="Simulated␣log
␣␣␣␣␣␣␣␣␣price␣trajectory␣", cex.main="1.7", cex.lab="1.2", cex.axis="1.2")
```

Appendix B

```
## R CODE FOR THE ESTIMATION OF THE MODELS PRESENTED IN CHAPTER 5
# load the dataset
dati <- read.table(file.choose(), sep=";")
head(dati)
# rename the columns
names(dati)[1] <- "Data"
names(dati)[2] <- "PUN"
# extract the closing prices
close_data <- (dati[seq(24, nrow(dati), 24),])
attach(close_data)
# load the package lubridate
library(lubridate)
# change the format of the Data column in date
Data <- as.Date(Data, format="%Y-%m-%d")
# change the format of PUN into time series
PUN <- ts(PUN,freq = 1)
# plot the closing prices
plot.ts(PUN, main="Time␣Series␣of␣Prices", xlab="Days",
         cex.main="1.7", cex.lab="1.2", cex.axis="1.2")

##### Descriptive Statistics ####
# basic descriptive statistics
library(fBasics)
basicStats(PUN)

## Normality tests
```

```
# empirical density
plot(density(PUN), main="PUN␣density␣plot", xlab="PUN",
cex.main="1.7", cex.lab="1.2", cex.axis="1.2")
m=mean(PUN)
sd=stdev(PUN)
x=seq(min(PUN),max(PUN), 0.001)
lines(x, dnorm(x,m,sd),col=3,lwd=1.5)
legend(70, 0.035, legend=c("PUN", "Normal"),
        col=c(1,3), lty=1:1, box.lty=0, cex = 1.05)
# qqplot
qqnorm(PUN)
qqline(PUN, col=3, lty=2, lwd=2)
# Shapiro test
shapiro.test(PUN)
# Kolmogorov-Smirnov
ks.test(PUN, pnorm, mean(PUN), sd(PUN))
# Jarque Bera test
jarqueberaTest(PUN)

## augmented Dickey-Fuller test
library(tseries)
adf.test(PUN)

## autocorrelation of PUN
op=par(mfrow=c(2,1))
acf(PUN,main="PUN␣ACF",cex.main="1.7",cex.lab="1.2",cex.axis="1.2")
pacf(PUN,main="PUN␣PACF",cex.main="1.7",cex.lab="1.2",cex.axis="1.2")
par(op)
acf(PUN, plot=FALSE)
# Ljung-Box test
Box.test((PUN), type = "Ljung-Box")
# compute the first difference of the prices
P_t1 <- diff(PUN)
```

```
# autocorrelation of the first differences
acf(P_t1, main="First␣differences␣ACF", cex.main="1.7",
        cex.lab="1.2", cex.axis="1.2")
acf(P_t1, plot=FALSE)

#### Application of the model ####

## Deterministic component
# load the package data.table
library(data.table)
# define a lagged series of data
data1 <- shift(Data, 1)
head(data1)
library(purrr)
# remove the missing value
data1 <- discard(data1, is.na)
# remove the first value of Data so that the vectors DATA and data1 have
#       the same length
Data <- Data[!Data %in% Data[1]]

#define D and M
date <- as.POSIXlt(Data,format="%Y-%m-%d")
library(timeDate)
easter<-c(as.Date(Easter(2015:2019, 0)))
christmas<-c(as.Date(ChristmasDay(2015:2019)))
newyear<-c(as.Date(NewYearsDay(2015:2019)))

holidays <- c(as.Date(easter),as.Date(christmas),as.Date(newyear))
D <- ifelse(weekdays(date)=="sabato" | weekdays(date)=="domenica"
        | Data %in% holidays,1,0)
Mfeb <- ifelse(months(date)=="febbraio",1,0)
Mmar <- ifelse(months(date)=="marzo",1,0)
Mapr <- ifelse(months(date)=="aprile",1,0)
Mmag <- ifelse(months(date)=="maggio",1,0)
```

```
Mgiu <- ifelse(months(date)=="giugno",1,0)
Mlug <- ifelse(months(date)=="luglio",1,0)
Mago <- ifelse(months(date)=="agosto",1,0)
Mset <- ifelse(months(date)=="settembre",1,0)
Mott <- ifelse(months(date)=="ottobre",1,0)
Mnov <- ifelse(months(date)=="novembre",1,0)
Mdic <- ifelse(months(date)=="dicembre",1,0)
M <- cbind(Mfeb,Mmar,Mapr,Mmag,Mgiu,Mlug,Mago,Mset,Mott,Mnov,Mdic)

# define the lagged D and M
date1 <- as.POSIXlt(data1,format="%Y-%m-%d")

D1 <- ifelse(weekdays(date1)=="sabato" | weekdays(date1)=="domenica"
        | data1 %in% holidays,1,0)
Mfeb1 <- ifelse(months(date1)=="febbraio",1,0)
Mmar1 <- ifelse(months(date1)=="marzo",1,0)
Mapr1 <- ifelse(months(date1)=="aprile",1,0)
Mmag1 <- ifelse(months(date1)=="maggio",1,0)
Mgiu1 <- ifelse(months(date1)=="giugno",1,0)
Mlug1 <- ifelse(months(date1)=="luglio",1,0)
Mago1 <- ifelse(months(date1)=="agosto",1,0)
Mset1 <- ifelse(months(date1)=="settembre",1,0)
Mott1 <- ifelse(months(date1)=="ottobre",1,0)
Mnov1 <- ifelse(months(date1)=="novembre",1,0)
Mdic1 <- ifelse(months(date1)=="dicembre",1,0)
M1 <- cbind(Mfeb,Mmar,Mapr,Mmag,Mgiu,Mlug,Mago,Mset,Mott,Mnov,Mdic)

## A first estimate of the parameters to define the initial guesses
# estimate the deterministic model with least mean squares method
model <- lm(PUN ~ D + M, data = close_data)
summary(model)
# extract the residuals
res <- residuals(model)
```

```
# transform the residuals in time series format to adapt them to the AR
rests <- ts(res,freq=1)
# estimate the AR(1) model for the stochastic component
library(tseries)
AR <- arma(rests, order = c(1,0))
print(AR)

## non-linear least squares method
# define the deterministic functions
f2 <- function(alpha2,beta2,betai){
alpha2+beta2*D+(M%*%betai)
}
f2lag <- function(alpha2,beta2,betai){
alpha2+beta2*D1+(M1%*%betai)
}

# define the lagged series of prices
library(data.table)
P_t1 <- shift(PUN, 1)
P_t1 <- ts(P_t1, freq=1)
# remove the missing value
library(purrr)
P_t1 <- discard(P_t1, is.na)
# adjust the dimesion of the vector PUN to the length of vector P_t1
PUN <- as.numeric(PUN)
PUN <- PUN[2:1826]

# define the vector of parameters betai
betai= c(-2.527,-4.511,-3.484,-13.248,-12.656,-7.038,-8.109,-10.061,
        -9.597,-7.304,-6.019)
# transpose the horizontal vector betai
betai<-matrix(betai)
```

```
# define the model
model <- function(alpha2,beta2,betai,phi){
        phi*P_t1 + f2(alpha2,beta2,betai) +
        phi*f2lag(alpha2,beta2,betai)
}
# estimate the model
modelnls2 <- nls(PUN ~ model(alpha2,beta2,betai,phi),
        data =list(close_data),
        start=list(alpha2= 53.2313, beta2= -3.4269, betai= betai,
        phi=0.86791))

summary(modelnls2)
# extract the residuals
RESID <- residuals(modelnls2)
# plot the fitted values and the residuals

PUN <- as.numeric(PUN)
plot(date,PUN, type="l", main="Time␣Series␣of␣Prices", xlab="Days",
         cex.main="1.7",  cex.lab="1.2", cex.axis="1.2")
lines(date, predict(modelnls2), type="l", col=3)
lines(date, RESID, type="l", lty=2)
legend("topright",legend=c("PUN", "fitted␣values", "residuals),
␣␣␣␣␣␣␣␣col=c(1,3,1),␣lty=c(1,1,2)␣,␣bg␣=␣NULL,␣box.lty=0,␣cex␣=␣1.05)

#␣compute␣some␣descriptive␣statistics␣of␣the␣residuals
basicStats(RESID)
#␣plot␣the␣autocorrelation␣function␣of␣the␣residuals
acf(RESID)
#␣test␣the␣Normality
shapiro.test(RESID)

#␣compute␣some␣information␣criteria
library(AICcmodavg)
```

```
# Akaike's Second-Order Corrected Information Criterion
AIC(modelnls2)
# Bayesian information criterion
BIC(modelnls2)
# compute the adjusted sample variance and its std deviation
sigma_2 <- var(RESID)
S_2 <- (length(PUN)/(length(PUN)-1))*sigma_2
dev_std_S <- sqrt(2*sigma_2^2/length(PUN)-1)
test_t_S <- S_2/dev_std_S
#### Application of the model to the log of prices ####
# generate the time series of the logarithm of PUN
lnP <- log(PUN)
lnP <- ts(lnP, freq=1)

# define the linear regression model to compute the starting values
modelln <- lm(lnP ~ D + M, data = close_data)
summary(modelln)
# extract the residuals
resln <- residuals(modelln)
# transform the residuals in time series format to model them
restsln <- ts(resln,freq=1)
# define the AR(1) model for the stochastic component
AR <- arima(restsln, order = c(1,0,0))
print(AR)

## non-linear least squares method
# define the lagged value of the log of prices
lnP_t1 <- log(P_t1)
lnP_t1 <- ts(lnP_t1, freq=1)

# define the deterministic functions
f2ln <- function(alpha2ln,beta2ln,betailn){
        alpha2ln+beta2ln*D+(M%*%betailn)
```

```
}
f2lagln␣<-␣function(alpha2ln,beta2ln,betailn){
␣␣␣␣␣␣␣␣alpha2ln+beta2ln*D1+(M1%*%betailn)
}

#␣define␣the␣model
model_ln␣<-␣function(alpha2ln,beta2ln,betailn,philn){
␣␣␣␣␣␣␣␣philn*lnP_t1␣+␣f2ln(alpha2ln,beta2ln,betailn)␣+
␣␣␣␣␣␣␣␣philn*f2lagln(alpha2ln,beta2ln,betailn)
}
#␣define␣and␣transpose␣the␣vector␣of␣parameters␣beta_i
betailn␣<-␣c(-0.081021,-0.161178,-0.159283,-0.135217,-0.076919,
␣0.056489,␣0.006497,␣-0.058708,␣-0.036086,␣-0.004507,␣-0.010554)
betailn␣<-␣matrix(betailn)
#␣estimate␣the␣parameters
modelnls2_ln␣<-␣nls(lnP␣~␣model_ln(alpha2ln,beta2ln,betailn,philn),
␣␣␣␣␣␣␣␣data␣=list(close_data),
␣␣␣␣␣␣␣␣start=list(alpha2ln=␣3.955179,␣beta2ln=-0.066830,
␣␣␣␣␣␣␣␣betailn=␣betailn,␣philn=0.8664))

summary(modelnls2_ln)
#␣plot␣the␣series␣of␣log␣prices␣along␣with␣the␣fitted␣values
plot(date,lnP,␣type="l")
lines(date,␣predict(modelnls2_ln),␣type="l",␣col=3)
#␣extract␣the␣residuals
RESID_ln␣<-␣residuals(modelnls2_ln)
#␣analyse␣the␣residuals
plot(date,␣RESID_ln,␣type="l")
basicStats(RESID_ln)
acf(RESID_ln)
shapiro.test(RESID_ln)
#␣compute␣the␣adjuste␣sample␣variance␣and␣its␣std␣deviation
sigma_2ln␣<-␣var(RESID_ln)
```

```
S_2ln␣<-␣(length(lnP)/(length(lnP)-1)*sigma_2ln
dev_std_ln␣<-␣sqrt(2*sigma_2ln^2/(length(lnP)-1)
test_t_ln␣<-␣S_2ln/dev_std_ln

#### Application of the model with the cosine seasonal function ####
# define the sequence of days for the sample time horizon
t0 <-seq(0,(1-(1/(5*365))), by=1/(5*365))
t1 <- seq((0+(1/(5*365))), 1, by=1/(5*365))

# define the deterministic functions
f1 <- function(gammaA,tA,gammaSA,tSA){
        gammaA*cos(2*pi*(t0-tA))+gammaSA*cos(4*pi*(t0-tSA))
}
f1lag <- function(gammaA,tA,gammaSA,tSA){
        gammaA*cos(2*pi*(t1-tA))+gammaSA*cos(4*pi*(t1-tSA))
}
# define the model
modelcos <- function(gammaA,tA,gammaSA,tSA,phi){
        phi*P_t1 + f1(gammaA,tA,gammaSA,tSA) +
        phi*f1lag(gammaA,tA,gammaSA,tSA)
}
# estimate the model
model.nls.cos <- nls(PUN~modelcos(gammaA,tA,gammaSA,tSA,phi),
        data =list(close_data),
        start=list(gammaA=runif(1,40,80),tA=runif(1),
        gammaSA=runif(1,25,55), tSA=runif(1),phi=runif(1)))

summary(model.nls.cos)

# extract the residuals, plot them, and compute the descriptive statistics
RESID_cos <- residuals(model.nls.cos)
plot(date, RESID_cos, type="l")
basicStats(RESID_cos)
shapiro.test(RESID_cos)
```

```
acf(RESID_cos)

# compute some information criteria, Akaike and Bayesian
AIC(model.nls.cos)
BIC(model.nls.cos)

#### Application of the model with the second piecewise function ####
# define the sequence of days for the sample time horizon
t <-seq(2,1826)
# define the deterministic functions
f3 <- function(alpha3,beta3,gamma3,tau){
        alpha3+beta3*D+gamma3*cos((t+(tau*365))*(2*pi)/365)
}
f3lag <- function(alpha3,beta3,gamma3,tau){
        alpha3+beta3*D1+gamma3*cos(((t-1)+(tau*365))*(2*pi)/365)
}
# define the model
model3 <- function(alpha3,beta3,gamma3,tau,phi3){
        phi3*P_t1 + f3(alpha3,beta3,gamma3,tau) +
        phi3*f3lag(alpha3,beta3,gamma3,tau)
}
# estimate the model
model.nls.3 <- nls(PUN~model3(alpha3,beta3,gamma3,tau,phi3),
        data =list(close_data),
        start=list(alpha3=runif(1,5,35),beta3=runif(1,0,25),gamma3=4,
        tau=runif(1),phi3=runif(1)))

summary(model.nls.3)

# analyse the residuals
RESID_3 <- residuals(model.nls.3)
plot(date, RESID_3, type="l")
basicStats(RESID_3)
```

```
shapiro.test(RESID_3)
acf(RESID_3)

# Information Criteria
AIC(model.nls.3)
BIC(model.nls.3)

## Goodness of fit, compute the RMSE for the three models of PUN
library(Metrics)
prediction1 <- predict(modelnls2)
prediction2 <- predict(model.nls.cos)
prediction3 <- predict(model.nls.3)
rmse(prediction1,PUN)
rmse(prediction2,PUN)
rmse(prediction3,PUN)

## Change the seasonal function to consider the weekly seasonality
# create the dummy variable W for the week days
Wmar <- ifelse(weekdays(date)=="martedi",1,0)
Wmer <- ifelse(weekdays(date)=="mercoledi",1,0)
Wgio <- ifelse(weekdays(date)=="giovedi",1,0)
Wven <- ifelse(weekdays(date)=="venerdi",1,0)
Wsab <- ifelse(weekdays(date)=="sabato",1,0)
Wdom <- ifelse(weekdays(date)=="domenica",1,0)
W <- cbind(Wmar,Wmer,Wgio,Wven,Wsab,Wdom)
# lag the dummy variable W
Wmar1 <- ifelse(weekdays(date1)=="martedi",1,0)
Wmer1 <- ifelse(weekdays(date1)=="mercoledi",1,0)
Wgio1 <- ifelse(weekdays(date1)=="giovedi",1,0)
Wven1 <- ifelse(weekdays(date1)=="venerdi",1,0)
Wsab1 <- ifelse(weekdays(date1)=="sabato",1,0)
Wdom1 <- ifelse(weekdays(date1)=="domenica",1,0)
W1 <- cbind(Wmar1,Wmer1,Wgio1,Wven1,Wsab1,Wdom1)
# define the seasonal function
```

```
f2w <- function(alpha2w,beta2w,betaiw){
        alpha2w+beta2w*D+(W%*%betaiw)
}
f2lagw <- function(alpha2w,beta2w,betaiw){
        alpha2w+beta2w*D1+(W1%*%betaiw)
}
# define the model
modelw <- function(alpha2w,beta2w,betaiw,phiw){
        phiw*lnP_t1 + f2w(alpha2w,beta2w,betaiw) +
        phiw*f2lagw(alpha2w,beta2w,betaiw)
}
# estimate the model
# the initial guesses were computed with the two-step procedure
betaiw <- c(0.009046, 0.014407, 0.018593, 0.031931, 0.083420, 0.036820)
betaiw <- matrix(betaiw)

modelnls2w <- nls(lnP ~ modelw(alpha2w,beta2w,betaiw,phiw),
        data =list(close_data),
        start=list(alpha2w= 3.885432, beta2w= 0.110361,
        betaiw= betaiw, phiw=0.885035))
# analyse the residuals
RESIDW <- residuals(modelnls2w)
basicStats(RESIDW)
op=par(mfrow=c(2,1))
acf(RESID_ln,main="ACF without weekly seasonality", lag.max=35,
        cex.main="1.7",  cex.lab="1.2", cex.axis="1.2")
acf(RESIDW,main="ACF with weekly seasonality", lag.max=35,
        cex.main="1.7",  cex.lab="1.2", cex.axis="1.2")
par(op)
predictionw <- predict(modelnls2w)
rmse(PUN, predictionw)
# compute the information criteria
AICc(modelnls2)
```

```
BIC(modelnls2)
# compute the adjusted samle variance and its std deviation
sigma_2W <- var(RESIDW)
S_2W <- (length(PUN)/(length(PUN)-1))*sigma_2W
dev_std_SW <- sqrt(2*sigma_2W^2/(length(PUN)-1))
test_t_SW <- S_2W/dev_std_SW

### Integration of the jump component ###
# compute the deseasonalized prices
modelln <- lm(lnP ~ D + W, data = close_data)
Y <- residuals(modelln)
modelln1 <- lm(lnP1 ~ D1 + W1, data = close_data)
Y1 <- residuals(modelln1)

## Test the model with Normal jumps
#  define the probability distribution function of Y
pdf <- function(phi,  sigmaSq, sigmaSq_J,lambda){
        (lambda)*exp((-(Y-phi*Y1-1)^2)/(2*(sigmaSq+sigmaSq_J)))*
        (1/sqrt(2*pi*(sigmaSq+sigmaSq_J)))
        + (1-lambda)*exp((-(Y-phi*Y1)^2)/(2*sigmaSq))*
        (1/sqrt(2*pi*sigmaSq))
}
# define the delta
delta <- 1/365
#  define the negative log-likelihood function of Y
norm_nllik <- function(theta){
        phi <- theta[1]
        sigmaSq <- theta[2]
        sigmaSq_J <- theta[3]
        l <- theta[4]
        -sum(log((l)*exp((-(Y-phi*Y1-1)^2)/
        (2*(sigmaSq+sigmaSq_J)))*
        (1/sqrt(2*pi*(sigmaSq+sigmaSq_J)))
```

```
        + (1-l)*exp((-(Y-phi*Y1)^2)/(2*sigmaSq))*
        (1/sqrt(2*pi*sigmaSq)))))
}
# estimate the parameters with negative minimum likelihood
norm_smv <- nlminb(start=c(1e-5,var(Y),1e-5,0.1),
        objective= norm_nllik, hessian=T,
        lower=c(-Inf,1e-5,1e-5,1e-5), upper=c(1-1e-5,Inf,Inf,1-1e-5))

# the fit of the model was tested using the same procedure
# reported for the model with exponential jumps below

## Test the model with exponential jumps
# define the delta
delta <- 1/365
#  define the negative log-likelihood function of Y
nllik <- function(theta){
        phi <- theta[1]
        sigma1 <- theta[2]
        lambda <- theta[3]
        l <- theta[4]
        -sum(log((1-l*delta)*exp((-(Y-phi*Y1)^2)/(2*sigma1))*
        (1/sqrt(2*pi*sigma1)) +
        (l*delta)*((lambda)/2)*exp(((lambda)/2)*((2*phi*Y1)+
        ((lambda)*sigma1^2)-(2*Y)))*
        erfc(((phi*Y1+(lambda)*(sigma1^2)-Y)/sqrt(2*sigma1^2)))))
}
# estimate the parameters with negative minimum likelihood,
# with lower and upper bounds
smv <- nlminb(start=c(1e-5,var(Y),1e-5,0.1), objective= nllik,
        hessian=T, lower=c(-Inf,1e-5,1e-5,1e-5),
        upper=c(1-1e-5,Inf,Inf,1-1e-5))
# extract the estimates of the parameters
smv$par
```

```
# define the parameters of the model
phi <- smv$par[1]
l <- smv$par[4]
k <- (1-smv$par[1])
prob <- smv$par[4]*delta
lambda <- smv$par[3]
sigma1 <- sqrt(smv$par[2])
eps <- rnorm(1825,0, 1)
eps_J <- rexp(1825, rate= lambda)
mu_J <- 1/lambda
sigma_J <- sqrt(1/lambda^2)

# compute the fitted values of the stochastic component
fit <- (1-k)*Y1 + sigma1*eps + prob*(eps_J)
plot(date, fit, type="l")

# compute the fitted values of the prices
prices <-  alpha2w + beta2w*D + (W%*%betaiw) + fit
# extract the residuals
res <- lnP - prices
# plot the log of prices, fitted values, and residuals
plot(date,lnP,type="l",main="Time Series of Log Prices",
        xlab="Days", cex.main="1.7",  cex.lab="1.2", cex.axis="1.2")
lines(date, prices, col=3)
lines(date, res, lty=2)
legend("topright",legend=c("log(PUN)", "fitted values"),
        col=c(1,3), lty=1:1 , bg = NULL, box.lty=0, cex = 1.05)
# analyse the residuals
op=par(mfrow=c(2,1))
acf(res,main="PUN ACF", lag.max=35, cex.main="1.7",
         cex.lab="1.2",cex.axis="1.2")
pacf(res,main="PUN PACF",lag.max=35, cex.main="1.7",
         cex.lab="1.2", cex.axis="1.2")
```

```
par(op)
acf(res, lag.max=62, plot=FALSE)

library(fBasics)
basicStats(res)
shapiro.test(res)
# compute the RMSE of the model with and without the jump component
library(Metrics)
rmse(lnP, prezzi)
predictionln <- predict(modelnls2w)
rmse(lnP,predictionln)

# compute the information criteria of the two models
AIC_jump <- dim(lnP)*(log(2*pi)+1+log((sum(res^2)/dim(lnP))))
        + ((13+1)*2)
AIC_ln <- dim(lnP)*(log(2*pi)+1+log((sum(RESIDW^2)/dim(lnP))))
         + ((13+1)*2)
bic_jump <- dim(lnP)*(log(2*pi)+1+log((sum(res^2)/dim(lnP)))) +
         14*log(dim(lnP))
bicln <- dim(lnP)*(log(2*pi)+1+log((sum(RESIDW^2)/dim(lnP)))) +
         14*log(dim(lnP))

# compute the iverse of the Fisher information matrix
library(numDeriv)
theta=c(phi, sigma1, lambda, l)
fish <- hessian(nllik,theta)
I <- solve(fish)
# compute the statistics for the parameters
Std_dev <- sqrt(diag(I))
T_par <- smv$par/Std_dev
```

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