# UNIVERSITÀ DEGLI STUDI DI PADOVA 

Dipartimento di Fisica e Astronomia "Galileo Galilei" Corso di Laurea in Astronomia

Tesi di Laurea

Analysis of fractures in crater floors: the case of Ceres

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Anno Accademico 2018/2019

# Analysis of fractures in crater floors: the case of Ceres 

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#### Abstract

This study is focused on Floor Fractured Craters (FFCs) located on Ceres, a particular category of craters with nets of fractures on their floors. In this work we analyze the orientation and the length distribution of the fractures present on the craters' floors of Dantu and Occator. We find that for each crater the fractures have a preferential orientation and, hence, they are not randomly distributed. In addition, the fractures length frequency distribution analysis reveals that some fractures can penetrate the entire shell of Ceres. This interesting result means that these fractures can serve as conduits for cryovolcanic activity. This analysis provides preliminary results regarding fractures behavior on Ceres surface and will be useful for additional future analyses.


## Sommario

Questo studio è focalizzato su una particolare tipologia di crateri, localizzati su Cerere, denominati Crateri dal Fondo Fratturato. In questo lavoro abbiamo analizzato l'orientazione e la distribuzione delle lunghezze delle fratture presenti sui fondi dei crateri Dantu e Occator. Si è trovato che, per ogni cratere, le fratture non sono casualmente distribuite, ma hanno una direzione di orientamento preferenziale. Inoltre, l'analisi della distribuzione in frequenza delle lunghezze delle fratture, rivela che alcune di queste posso penetrare interamente la crosta di Cerere. Questa conclusione è di estremo interesse, poichè significa che queste fratture possono agire da "condotto" per attività criovulcaniche. Questa analisi ha riportato risultati preliminari riguardanti il comportamento delle fratture su Cerere, e sarà utilizzato per ulteriori analisi che verrano fatte nel prossimo futuro.

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## Chapter 1

## Introduction

On the 6 th of March 2015, after a 14 -month long mission around Vesta, the Dawn spacecraft arrived at Ceres, giving the first opportunity to study from near this unique celestial body.
The probe remained in orbit around Ceres until the 1st of November 2018, when it ended all of its fuel, remaining in uncontrolled orbit around the dwarf planet. Over these 3 years, 7 months and 26 days in orbit around Ceres, Dawn produced more than 100 GB of data, thus reporting the most detailed view that we have ever had of this dwarf planet in more than 200 years from its discovery.

Between the different features observed on the surface of Ceres, we focused our attention on two peculiar impact craters, Dantu and Occator. These two craters are included in a particular category of craters called FFCs (Floor Fractured Craters) that present some peculiar characteristics when compared to normal craters. Indeed, they present wide nets of fractures on their floors.

Specifically, we characterize the fractures pattern deriving: (I) size-frequency distribution of fractures length to understand if fractures can penetrate the entire shell thickness of Ceres and (II) Rose diagram to determine if a preferential orientation of these fractures exists.

## Chapter 2

## Ceres and the Dawn mission

### 2.1 Ceres



Figure 2.1: Ceres as seen by the Dawn spacecraft on the 6 th of May 2015 from $13,600 \mathrm{~km}$, it's visible the bright spot, named "spot 5 " in the Occator crater. Credits: NASA/JPL-Caltech/UCLA/MPS/DLR/IDA.

Ceres, discovered in 1801 by the Italian astronomer Giuseppe Piazzi, is the closest, to Earth, dwarf planet in the Solar System. With a radius of about 476 km , it is the most massive object in the Main Asteroid Belt.
Ceres' sidereal period is about 4.60 years with an orbit that is inclined by 10.593 deg, with respect to the ecliptic. It has an eccentricity of 0.075 , and its average distance from the Sun is 2.8 AU . It rotates around its axis, which is inclined by
4.00 deg , completing one rotation every 9.07 hours.

Before 2006, when it was 'upgraded' to the status of dwarf planet, it was classified as a C-type asteroid meaning that it contains high percentages of Carbon, either in elementary or molecular forms, especially clustered in a type of rock called carbonaceous chondrite. Its surface acceleration of gravity is about 0.28 $\mathrm{m} / \mathrm{s}^{2}$ (Hisinger et al. 2016).
Moreover, Ceres is a unique example of volatile-rich dwarf planet (Fu et al. 2017) and its crust is mechanically strong (the maximum effective viscosity should be around $10^{25} \mathrm{~Pa} \mathrm{~s}$ ). This rheology suggests a crustal composition of carbonates, phyllosilicates and water ice (less than 25 volume percent) in addition to a $30 \%$ volume of low-density, high-strenght materials, probably made by salt and/or clathrate hydrates. The latter are concentrated in a probably 41 km thickness crust with a total average density of $1287 \mathrm{~kg} / m^{3}$ (Fu et al. 2017). Ammonia could also be part of the composition of the crust, indeed, there were found traces of ammonia bounded to the phyllosilicates (Buczkowski et al. 2018). One of Ceres distinct features are the many bright spots, named faculae, that are probably made of various salts as magnesium sulfate and/or sodium carbonate (DeSanctis et al., 2016).
Furthermore, as normal for planetoids in an region like the main belt, Ceres surface is constellated by craters of various sizes and morphologies, but there are not craters larger than 280 km . This is a peculiar aspect of Ceres since it is at least 4.5 billion years old. The most plausible hypothesis for the lack of large impact craters is the presence of somehow geological activity that flattened and, maybe, is flattening the surface, making the bigger craters disappeared over time.
A possible candidate for this kind of geological activity is the viscous relaxation. Indeed, considering that the surface temperature ranges between 120 K (at high latitudes) and 150-160 K (at low latitudes), the ice is not strong enough to retain large-scale topography. Hence, the bigger craters are expected to flatten out on relatively short geological timescales (Bland et al. 2016). Another hypothesis is attributed to cryo-volcanism, due to both the presence of some traces of waterice material in the crust, and flow features on the surface (Krohn et al. 2016). Moreover, the salts observed within the faculae are hypothesized as the result of this cryo-volcanic activities.
Previous works suggest that Ceres crust is quiet heterogeneous, indeed, the features that lead to the presence of water ice-salt mixture, such as ring-mold craters (Krohn et al. 2018), are not evenly distributed on the surface. Thus leading to the presence of complex geological crustal structure (Bland et al., 2016).

From the evidences reported above, it has been suggested that Ceres possibly had a global or partial ocean in the past. The freezing of this ocean could have lead to the crustal salts accumulation that are seen today (Fu et al., 2017).
For this reason, Ceres is thought to be partially differentiated having both a crust and a mantle.
The latter should extend below the crust (nearly 41 km deep) to near the center, for a total of 428.8 km . The mantle is thought to be rich in silicate and is
mechanically weak with maximum viscosity of almost $10^{21} \mathrm{~Pa} s$ in the uppermost part. This suggests the presence of liquid pore fluids in this region (the first 60 km thickness) that avoided igneous differentiation (Fu et al. 2017) and remained brittle and enriched with other substances. From a geomorphological point of view, Ceres is known to be a geological complex dwarf planet with many different features on its surface, such as faculae, many different crater morphologies (Hisinger et al. 2016), chains of pits, as the Samhain Catenae (Scully et al. 2017), and the presence of the Ahuna Mons, whose height is about 4 km . Among the different geomorphological features observed on the surface of Ceres, this work focuses on the so-called Floor Fractured Craters, FFCs, such as Dantu and Occator craters (see Chapter 3).

| Mean radius [km] | 476 |
| :--- | :---: |
| Sidereal period [yr] | 4.60 |
| Orbital inclination [ ${ }^{\circ}$ ] | 10.593 |
| Eccentricity [ ] | 0.075 |
| Average Sun distance [AU] | 2.8 |
| Axis inclination [ ${ }^{\circ}$ ] | 4.00 |
| Rotational period [h] | 9.07 |
| Acceleration of gravity [m/s ${ }^{2}$ ] | 0.28 |

Table 1.1 : Physical parameters of Ceres.

### 2.2 Dawn mission

Dawn is a spacecraft launched on the 27th of September 2007 with the objective of study the asteroid Vesta and the dwarf planet Ceres. It arrived on the dwarf planet on the 5 th of March 2015 and stayed in orbit around it up until the 1st of November 2018, effectively prolonging the mission that was scheduled to end on June 2016. The core of the probe is a graphite composite cylinder, which is surrounded by aluminium panels on which most of the hardware is fixated. On this part of the spacecraft are also mounted the heaters that controls the probe temperature (Dawn at Ceres, press kit NASA).


Figure 2.2: The logo of the Dawn mission. Credits: NASA.

Its main characteristics are:

- 1.64 meters long, 1.27 meters wide and 1.77 meters high spacecraft (solarsystem.NASA.gov).
- An high-gain antenna of 1.52 meters by diameter used for primarly communications with Earth, in addition to three other low-gain antennas used when the principal one is not pointing at Earth (Dawn at Ceres, press kit NASA).
- A solar array 20 meters long ( $36.4 \mathrm{~m}^{2}$ of total surface area), (solarsystem.NASA.gov).
- Weight of 747.1 kg with an addition of 425 kg of xenon propellent and other 45.6 kg of hydrazine fuel, (solarsystem.NASA.gov).
- Two 8.3 x 2.3 meter solar arrays, together providing 1.4 kW at Dawn maximum distance from the Sun, with the energy storage that was a $35 \mathrm{~A} / \mathrm{h}$ rechargeable Ni-H battery, (solarsystem.NASA.gov).
- Three $33 \times 41 \mathrm{~cm}$ cylindrical ion thrusters movable in two axes, providing a thrust of 19 to 91 mN , each unit weights 8.9 kg . Only one thruster at time was used during the mission (Dawn at Ceres, press kit NASA).

This propulsion system accelerated the spacecraft, at full thrust, from 0 to 26.8 $\mathrm{m} / \mathrm{s}$ in 4 days, resulting in the record for the highest variation of velocity in less time for a probe, (solarsystem.NASA.gov). The thrusters work by using an electrical charge to accelerate ions from the Xenon fuel, that was chosen for it's chemical unreactivity and atomic weight, to a speed 7 to 10 times that of conventional chemical engines (Dawn at Ceres, press kit NASA). This propulsion consumed only $3.25 \mathrm{mg} / \mathrm{s}$ of Xenon at full power (Dawn at Ceres, press kit NASA).


Figure 2.3: Schematic structure of the Dawn spacecraft.Credits: NASA.
The payload onboard Dawn is composed by the following instruments:

- A Gamma Ray and Neutron Detector (GRaND), consisting in a total of 21 sensors with a wide field of view capable of detecting the possible emanations of those particles from the surface of the two celestial bodies, allowing to study the composition of the surface up to 1 m in the crust (Dawn at Ceres, press kit NASA).
- A Visual and Infrared Imaging Spectrometer (VIR), this instrument was used to study the surface mineralogy of Vesta and Ceres, each measurement recorded the light intensity at more than 400 wavelenght ranges in each pixel (Dawn at Ceres, press kit NASA).
- Two Framing Cameras (FC), this instrument acquired images of the surface of Ceres that were used for this study, (for more information see section 1.2.1).

The Dawn spacecraft was the first spacecraft orbiting two separate bodies in only one mission and studying both a dwarf planet and an asteroid.

### 2.2.1 Framing Cameras



Figure 2.4: Schematized functioning of the Frame Cameras.
Credits: NASA.

The Framing Camera (FC) is the German contribution to the Dawn mission and allowed the deep study of the physical parameters of Ceres, as well as the reconstruction of its global shape, local topography and surface geomorphology. Moreover, the data provided important informations on the surface composition of Ceres.
The two cameras are perfectly identical, they weight 5.5 kg , have a 19 mm aperture size, a focal length of 150 mm and an Instantaneus Field Of View (IFOV) of $93.3 \mu \mathrm{rad}$. The cameras mounted refractive optics that focus the light on a filter wheel equipped with one clear filter (ranging from 400 to 1100 nm (Hisinger et al. 2016)), and seven band-pass filters that covers the wavelengths from visible to near-IR (438, 555, $653,749,829,917$ and 965 nm (Hisinger at al. 2016), (see table 1.2 for more details). Then, the filtered light is transmitted to a a $1024 \times 1024$ frame transfer CCD (Mastrodemos et al.). To correct and compensate the aberrations were used the following precautions:

- Chromatic aberration was compensated chosing a crown glass, with positive refracting power, for the first lens, followed by two flint glass, with negative refracting powers, then again followed by another flint lens with poitive refracting power (Sierks et al. 2011).
- Spherical aberration was corrected with an aspherical first lens surface (Sierks et al. 2011).
- All the aberrating effect that derives from temperature changings (for example a change in the focal length), were dealt by mounting the two
central lenses in an inner barrel with a different thermal expansion coefficient than the outer one. This allowed to keep the focal plane within a range of $30 \mu \mathrm{~m}$ (Sierks et al. 2011).

The optical system is located in a lens barrel mounted on the camera's head, and all the structure is reinforced with braces from the head to the base of the camera, where the buffle resides (Sierks et al. 2011).
The calibration was done during the flight, (when the cameras were also used for it's navigation, as well during the voyage phase and the orbit phase). Photometric calibrations imaged Vega and various solar analogs, while geometric calibrations used cluster observations to estimate the focal length. The calibration results allowed an astrometric accuracy of $0.09-0.11$ pixels (1- $\sigma$ ) when imaging a few stars at a minimum signal-to-noise (SNR) $>10$ (Mastrodemos et al.).
For estimating the camera's alignment in the body frame were again used the cluster observation, leading to a residual pointing error of $\sim 1$ FC pixel, that is considerably random (Mastrodemos et al.).

The resolution of the Framing Camera was up to 62 m per pixel in low altitude mapping orbit (LAMO), at an angular resolution of 93.7 $\mu \mathrm{rad} / \mathrm{px}$ (Mastrodemos et al.). To protect the optical system the cameras use a resealable front door that, when open, elevates the total height of the cameras to 422 mm (Sierks et al. 2011). The total power consumption in nominal operation was of 17 W (Sierks et al. 2011). At the end of the camera head, there are the filter wheel, the CCD and the electronics for the operations of the CCD. After the head there is the elec-


Figure 2.5: The Framing Camera without the MLI.
Credits: Sierks et al. 2011. tronics box, or E-box, where the Data Processing Unit, the Power Converter Unit and the Mechanism Control Unit are situated, and they are thermally insulated from the camera head (Sierks et al. 2011). All the components are insulated from external sunlight and temperature with a Multi-Layer Insulation, or MLI (Sierks et al. 2011). Thanks to Doppler measurements of the orbiting Dawn and observations of characteristic landmarks, the FCs measured the rotation rate, the orientation of the spin axis, the mass, the gravity field, the shape, the volume and the bulk density of Ceres. Thanks to the huge amount of images acquired, it was possible to obtain mosaics of the entire surface of Ceres, in addition to to a Digital Terrain Model (DTM) produced by the stereophotogrammetry technique with a vertical accuracy of 12 m .

| Item | Specification |
| :--- | ---: |
| Focal Lenght | 150 mm |
| Focal shift | $<20 \mu \mathrm{~m}$ |
| Field of view | $5.5^{\circ} \mathrm{X} 5.5^{\circ}$ |
| IFOV | $93.7 \mu \mathrm{rad}$ |
| Field Curvature | $<10 \mu \mathrm{~m}$ |
| Distortion | $<0.1 \%$ |
| Spectral Range | $400-1050 \mathrm{~nm}$ |
| Spectral Transmission | $>75 \%$ |

Table 2.1: Optical specifications of the FC. Credits: Sierks et al. 2011

| Center $\lambda[\mathbf{n m}]$ | Bandwidth [nm] | Transmission [\%] | Thickness [mm] |
| :---: | :---: | :---: | :---: |
| polychromatic | $450 \pm 10-920 \pm 10$ | 98 | $6.00 \pm 0.05$ |
| $430 \pm 2$ | $40 \pm 5$ | $>75$ | $2.00 \pm 0.05$ |
| $550 \pm 2$ | $40 \pm 5$ | $>75$ | $5.90 \pm 0.05$ |
| $650 \pm 2$ | $40 \pm 5$ | $>75$ | $6.60 \pm 0.05$ |
| $750 \pm 2$ | $40 \pm 5$ | $>75$ | $6.40 \pm 0.05$ |
| $830 \pm 2$ | $40 \pm 5$ | $>75$ | $5.90 \pm 0.05$ |
| $920 \pm 2$ | $40 \pm 5$ | $>75$ | $5.30 \pm 0.05$ |
| $980 \pm 2$ | $80 \pm 5$ | $4.80 \pm 0.05$ |  |

Table 2.2: Tabulated philters specifications. Credits: Sierks et al. 2011

## Chapter 3

## Floor Fractured craters analysis



Figure 3.1: The Humboldt crater on the Moon, a typical example of FFC-I. Credits: NASA, Lunar Recoinnaissance Orbiter (LRO).

Floor Fractured Craters (FFCs) are craters with peculiar nets of fractures on their floor. This type of crater was firstly found on the Moon and then on other bodies, like Mars and Ceres. The fractures systems are present in various form on the craters, they can cut the floor radially, from the center to the outer parts, or can be disposed in concentric patterns around the crater center (Buczkowski et al. 2018).
Another important parameter for characterizing FFCs is the depth to diameter, d/D, ratio. Indeed, lunar FFCs are shallower when compared to normal lunar craters. Cerean FFCs also show shallower craters floor when comparing FFCs to normal craters (Buczkowski et al. 2018).

There are various typologies of FFCs, divided in the following classes:

- Class I: Radial and/ or concentric features, central peak complexes.
- Class II: Uplifted central region, concentric region.
- Class III: Wide moat between wall and interior, radial and/or polygonal fractures.
- Class IV: Characterized by a V-shaped moat, and divided in three categories:
- IVa: Radial or concentric features, convex up floor profile.
- IVb: Pronounced inner ridge on the interior side, subtle fractures, irregular convex up profile.
- IVc: Hummocky interior.
- Class V: Degraded crater walls, radial and/of polygonal fractures.
- Class VI: Mare-flooded interiors, concentric fracture pattern near walls.
(Schultz 1976, Jozwiak et al. 2012, 2015).
This classification is based on the observation of lunar FFCs and can be applied to other bodies. Indeed, on the surface of Ceres there are the FFC Class I and Class IV with all of their subtypes (Buczkowski et al. 2018). This is because the cerean FFCs are relatively young and do not have the heavily degraded walls associated with Class 5 FFCs. Moreover, Cerean FFC have both the radially cutting fractures and the concentric cutting fractures. In addition, they do not have concave-up profiles at their centers or mare-flooded interiors, hence, Class 2 and 6 are excluded on Ceres (Buczkowski et al. 2018). The two craters analyzed in this work, Dantu and Occator, are classified as Class I floor fractured crater. The fractures are an important object of study in planetary sciences because their origin can be related to volcanic, cryo-volcanic or some sort of geological activity that can cause processes like viscous relaxation or floor uplifting (Buczkowski et al. 2018). Indeed, they can serve as fluid conduits bringing the subsurface material to the surface. Therefore, the study of the distribution of fractures on a planetary body can be a tool to infer the thickness of the fractured crust (Lucchetti et al., 2017).
Understanding the origin of fractures can provide information about the Ceres crustal and/or sub-crustal composition, as well as the possible existence of geological acitvity on the body (Buczkowski et al. 2018).

The importance of studying the fractures distribution lies in the fact that fractures following an exponential-kind distribution are thought to not penetrate too deep in the crust, hence, they are not involved in the resurfacing of deepcrust material. On the other hand, those fractures following a powerlaw-kind distribution are thought to be related with fractures that penetrate the entire crust (Mazzarini and D' Orazio, 2003; Mazzarini, 2004; Soliva and Schultz, 2008; Mazzarini and Isola, 2010 ).

From this consideration, we can make a differentiation in the distribution of fractures, leading to two main typologies:

- 1. Localized fractures systems that presents few large fractures that cuts deep in the crust and down to the mechanical discontinuity.
Those presents a distribution of length that follows a powerlaw:

$$
\begin{equation*}
N(>l)=c l^{-\alpha} \tag{3.1}
\end{equation*}
$$

Where $N(>1)$ is the number of fractures with length more than a value 1 , c is a scale factor and $\alpha$ is the exponent of the power law that identify the slope of the log-log graph.

- 2. Distributed fracture systems that are almost evenly spaced. These fractures are confined within a specific layer of the crust and does not grow down to the main discontinuity.
They present a distribution of length that follows an exponential law:

$$
\begin{equation*}
N(>l)=\beta e^{-\lambda \alpha} \tag{3.2}
\end{equation*}
$$

Where $\mathrm{N}(>\mathrm{l})$ is, again, the number of fractures with length more than a value $l, \beta$ and $\lambda$ are parameters for the exponential and $\alpha$ is, again, is the exponent of the exponential that characterize the slope of the plotted line on the log-log graph.

For this reason it is important to study Floor Fractured Craters, especially in case of the dwarf planet Ceres. Indeed, in light of its geological complexity and almost non-uniform surface morphology, the fractures analysis can give unique insights on the origin and evolution of this dwarf planet.

### 3.1 Dantu crater

The Dantu crater, located at coordinates $24.3^{\circ} \mathrm{N} 138.2^{\circ} \mathrm{E}$ in the Vendimia Planitia depression in the northern hemisphere of Ceres, is 120 km in diameter. It shows remnants of a central peak and a wide and flat floor (Kallisch et al. 2017), with a medium depth of 4.4 km (Buczwkowski et al. 2018).
Its age is thought to be $230 \pm 30 \mathrm{Myr}$ that was derived using an ejecta blanket counting model, even if the crater density lowers significantly when reaching the crater floor, indicating a younger surface age (Kallisch et al. 2017). Since there are no indications that melting events occurred in the past, the younger surface is probably caused by debris avalanches and granular flows, that filled up the center of the crater (Kallisch et al. 2017).
Dantu shows a prominent system of fractures, almost all located in the southern portion of the crater (see chapter 2.1 for details), and some of those fractures are associated with bright spots (Kallisch et al. 2017).
Those bright spots are relatively abundant on the crater floor. There are more then 80 with diameters between 100 and 500 m and a few other that are about 1


Figure 3.2: The Dantu crater.
In the image it is well visible the central peak, as well as some of the bright spots in the southern region.
The North is in the upperward direction.
Credits :
km in diameter. Moreover, most of the bright spots are located in the southern part of the crater, and within a $10.5 \times 9 \mathrm{~km}$ area, causing an increment in the local albedo (Kallisch et al. 2017).
This southern area of the crater is on average 1 km higher in elevation compared to the northern crater floor (Kallisch et al. 2017). The Dawn spacecraft performed also a spectral analysis of the Dantu revealing a strong absorption around $2.7 \mu \mathrm{~m}$, associated with structural OH -groups in phyllosilicates, around $3.1 \mu \mathrm{~m}$, likely due to ammoniated phyllosilicates, and around $3.9 \mu \mathrm{~m}$, consistent with carbonate phases (Kallisch et al. 2017). Some other absorption around 3.2-3.5 $\mu \mathrm{m}$ and $4.2 \mu \mathrm{~m}$ were observed, but their origin is not yet understood (Kallisch et al. 2017).
In synthesis, Dantu is a diverse complex crater, showing non-uniformly distributed features like fracture systems, a high number of small bright spots, an asymmetric floor profile, uneven distributed spectral properties and an irregular ejecta distribution (Kallisch et al. 2017).

| Coordinates | Diameter | Depth |
| :---: | :---: | :---: |
| $24^{\circ} 18^{\prime} 00^{\prime} \mathrm{N} ; 138^{\circ} 13^{\prime} 48^{\prime \prime} \mathrm{E}$ | 120 km | 4.4 km |

Table 2.1: Parameters of the Dantu crater (Buczwkoski et al. 2018).

### 3.2 Occator crater



Figure 3.3: The Occator crater.
In the image the Ceralia Facula at the center and the Vinalia Faculae in the eastern part of the crater are shown.
The North is in the upperward direction
Credits :
The Occator crater, located at coordinatesd $19.8^{\circ} \mathrm{N}, 239.3^{\circ}$ E near the Hanami Planum, in the Occator Quadrangle, is 92 km in diameter and 4.3 km in depth (Buczkowski et al. 2018). Its central pit is 9 km wide and almost 1 km deep, with a 2 km wide and 400 m high central dome on its floor (Schenk et al., 2015; Hiesinger et al., 2016; Buczkowski et al., 2016). The floor material on Occator ranges from smooth to knobby, but in the northeast is overlain by lobate flows, thought to be emanated from the center of the crater (Krohn et al. 2016). The floor of Occator is extensively fractured, with type of fractures ranging from linear fractures in the northeastern part to concentric fractures around the central pit and around the crater wall (see chapter 2.1 for details), (Buczkowski
et al., 2018).
Associated with the fractures there are the faculae, or "bright spots". These high albedo surface features are relatively abundant on the cerean surface, but the Occator crater hosts the highest number of them, as well as the biggest one, the Ceralia Facula which is visible in the central part of the crater and it is associated with the dome within the central pit (Buczkowski et al., 2018). Other smaller faculae are the Vinalia Faculae which are located in the northeast part of the crater and associated with the fractures cutting the lobate flows (Buczkowski et al., 2018).
The faculae on Occator are composed almost entirely by sodium carbonates (DeSanctis et al. 2016).

| Coordinates | Diameter | Depth |
| :---: | :---: | :---: |
| $19^{\circ} 49^{\prime} 12^{\prime \prime} \mathrm{N} ; 239^{\circ} 19^{\prime} 48^{\prime} \mathrm{E}$ | 92 km | 4.3 km |

Table 2.2: Parameters of the Occator crater (Buczwkoski et al. 2018).

### 3.3 Dataset and Methods

The dataset consists in the fractures located on the crater floors of Occator and Dantu. The mapping of these fractures was done using the HAMO (High Altitude Mapping Orbit) and LAMO (Low Altitude Mapping Orbit) maps. Those two maps are mosaics made with the images that the FCs cameras acquired during the Dawn mission. We use also the DTM (Digital Terrian model) that provide information about the elevation of the features. The DTM has a vertical accuracy of 12 m , while the LAMO map has a resolution of $20 \mathrm{~m} / \mathrm{px}$.
The DTM was done during the HAMO phase, at an altitude of about 1475 km (astrogeology.usgs.gov), during which the FCs acquired about 2350 images of the surface of Ceres in clear filter (Preusker et al. 2016). To produce the DTM, the images taken at different angles and with similar illumination conditions, were combined using a method called stereophotogrammetry (SPG) (Preusker et al. 2016). We used the DTM to understand the elevation of the different areas we mapped.

The mapping of the fractures was done using the Arcgis software ArcMap. On the LAMO map the fractures were mapped as polylines, modeled as geometrical lines connected by their vertexes. We calculated the length of fractures and the x and y coordinates of the starting point, the ending point and the midpoint of each fracture. These parameters were extrapolated in a txt file, that was then used in Python with the numpy command genfromtxt.
The total amount of fractures is 1462,630 on Dantu and 832 on Occator. (for the complete index of the single fracture lengths see Appendix B).


Figure 3.4: The LAMO mosaic of the surface of Ceres.
The two orange rectangles highlight the two craters under study, Dantu and Occator.
Credits: .


Figure 3.5: The DTM produced using images from the HAMO map acquired by the FCs cameras onboard the Dawn spacecraft.
The two orange rectangles highlight the two craters under study, Dantu and Occator .
Credits: .


Figure 3.6:
a): The Dantu crater in LAMO view with fractures mapped in black.
b): The DTM image of Dantu crater with the fractures mapped in black.


Figure 3.7:
a): The Occator crater in LAMO view with fractures mapped in black.
b): The DTM image of Occator crater with the fractures mapped in black.

### 3.3.1 Rose diagram analysis

The midpoint coordinates were useful for the calculations of the azimuthal angle between the vertical axis of the fracture and the Cerean parallel that passes by the midpoint. This was done to define a preferential angular distribution of the fractures.
This angular analysis was made with an extension of the ArcGis software named Polar Plots (J.Jenness, 2014). This program took the azimuthal angle calculated on the two sides of the intersection of the parallel and the fracture (this precaution was taken because the mapping was "homogeneus", indeed, not all the fractures where mapped from the top to the bottom or from left to right, so it was not correct to use just a single angle). Then, the Polar Plots program weighted every combination of angles with the length of its specific fracture, returning a polar graph, the so-called rose diagram. This reveals if the fracture systems has or not a preferential distribution.

### 3.3.2 Length distribution analysis

We performed an analysis on the length of fractures in Python to understand which distribution fits our dataset. Specifically, we are interested in understanding if our distribution follows a power-law or an exponential trend in order to assess if fractures penetrate or not the entire Ceres shell. Those fractures following a power-law distribution are thought to be involved in the uplifting of subsurface material. After the extrapolation of the data in the txt file, we used the Numpy module of Python to turn it into an array, with the genfromtxt function, then the array was sorted in increasing order with the sort command. After this part we used the Powerlaw module (Jeff Alstott) to do a fit of the data, and this fit was then used to do a comparing between a power-law distribution and an exponential distribution (through the distribution_compare function). Then we derived the $\alpha$ parameter of the fit in power-law, as well as the $\mathrm{x}_{\min }$ parameter to determine the cutoff from which the power-law distribution begins. Using the module Plvar (Jeff Alstott), we then calculated the relative error of the $\alpha$ parameter and the $\mathrm{x}_{\min }$. Using the Plpva module (Jeff Alstott), we calculated the goodness of fitting (GoF) and the p-value associated with the two distribution.
We used for our analysis the statistical method developed by Clauset et al. in 2007.

We used the Matplotlib.pyplot module to do the histograms of the distribution of lengths, in addition to the cumulative distribution. In the latter, we highlighted the $\mathrm{x}_{\text {min }}$ value of the powerlaw distribution with his associated error to assess if there is an effective change in slope around that value (Appendix A).

The Python modules used for the statistical evaluations are based on the methods developed by Clauset et al. in 2007. We report here these methods, remembering that our dataset of lengths are defined as continuous. For estimat-
ing the $\alpha$ parameter of the distributions, the Powerlaw module used the method of maximum likelihood, deriving so a maximum likelihood estimator (MLE). The calculus for this MLE for the alpha parameter is dictated by:

$$
\begin{equation*}
\hat{\alpha}=1+n\left[\sum_{i=1}^{n} \ln \left(\frac{x_{i}}{x_{\min }}\right)\right]^{-1} \tag{3.3}
\end{equation*}
$$

Where $\alpha$ is written as $\hat{\alpha}$ to enlight the fact that this is an estimation of the true value. In the equation $n$ indicates the total number of data in the dataset, $x_{i}$ the i-th data of the dataset and $x_{\text {min }}$ the cutoff value of the power-law (so, for each $\left.\mathrm{i}>0, \mathrm{x}_{\mathrm{i}}>\mathrm{x}_{\text {min }}\right)$.
The standard error associated with $\hat{\alpha}$ is then derived from the width of the likelihood maximum (Clauset et al. 2007), and its calculated from the formula:

$$
\begin{equation*}
\sigma_{\hat{\alpha}}=\frac{\hat{\alpha}-1}{\sqrt{n}}+O\left(\frac{1}{n}\right) \tag{3.4}
\end{equation*}
$$

Where, again n is the total number of data in the dataset.
This method of the maximum likelihood estimators are guaranteed to be unbiased in the asymptotic limit of $n \rightarrow \infty$ (Clauset et al. 2007). So for finite datasets, like ours, the biases are present, but those biases decay as $\mathrm{O}(1 / \mathrm{n})$ for any choice of $\mathrm{x}_{\min }$ (Clauset et al. 2007). So if $\mathrm{n}>50$ the estimates on the parameters are reliable, and $\hat{\alpha} \rightarrow \alpha$ (Clauset et al. 2007). That is our case, indeed the two distribution counted $n=630$ for Dantu and $n=832$ for Occator.

For the estimation of $x_{\min }$ the program choose the value of $\sim x_{\min }$ (the $\sim$ means that this $\mathrm{x}_{\text {min }}$ is an estimation of the true value) that makes the probability distributions of the measured data and the best-fit power-law model as similar as possible above $\sim \mathrm{x}_{\min }$ (Clauset et al. 2007). In general, if the chosen $\sim \mathrm{x}_{\text {min }}$ is higher than the true value $\mathrm{x}_{\text {min }}$, then we are effectively reducing the size of our data set, which will make the probability distributions a poorer match because of statistical fluctuation. Conversely, if the chosen $\sim x_{\text {min }}$ is smaller than the true $\mathrm{x}_{\text {min }}$, the distributions will differ because of the fundamental difference between the data and model by which we are describing it. In between lies the best estimate (Clauset et al. 2007). The measure that the module of Python uses for quantifying the distance between two distributions is the Kolmogorov-Smirnov (KS) statistic defined as follows:

$$
\begin{equation*}
D=\max _{\mathrm{x}>\mathrm{x}_{\text {min }}}|S(x)-P(x)| \tag{3.5}
\end{equation*}
$$

Where $S(x)$ is the Cumulative Distribution Function (CDF) of the data for the observations with value at least $x_{\min }$, and $P(x)$ is the CDF for the powerlaw model that best fits the data in the region $x=x_{\min }$. The estimated value $\sim x_{\min }$ is then the value of $x_{\min }$ that minimizes D (Clauset et al. 2007). To evaluate the error on $\mathrm{x}_{\mathrm{min}}$, given our datasets with n measures, the Powerlaw module generates a synthetic dataset with similar distribution to the original, estimating then, with the KS method, the $\mathrm{x}_{\text {min }}$. Then the program repeated this process 1000 times and calculated the standard deviation of these estimates
over the number of repetition, giving us the uncertainety values that we searched (Clauset et al. 2007).

To strengthen our case for the power-law it was important to rule out the case of a general exponential distribution in each dataset. The comparison between the power-law and exponential distribution is important because only fractures with length distributions that follows a power-law are thought to penetrate deep in the crust (Soliva and Schultz, 2008; Schultz et al., 2010). To do this comparison, we used the distribution_compare function of the Powerlaw module that uses the likelihood ratio test. This test compute the likelihood of the data under two competing distributions, then the one with the higher likelihood is the better fit (Clauset et al. 2007). Then the program took the logarithm of the ratio, this logarithm is then positive or negative depending on which distribution is better, or zero in the event of a tie (Clauset et al. 2007). The function then returned two values, the first is the result of the logarithm of the likelihood ratio, indicating which fit is better, then it spouted another value, the p-value, calculated in the manner developed by Vuong. The p-value tells us whether the observed sign of the log likelihood is statistically significant (Clauset et al. 2007). If this p-value is less than 0.1 then the sign is a reliable indicator of which model is the better fit to the data, viceversa, if this p -value is large, the sign is not reliable.

The next and last step that we did in the statistical analysis part is to test the powerlaw hypothesis, calculating the goodness of fitting (GoF) and the pvalue to quantify the plausibility of the hypothesis. Such tests are based on measurement of the "distance" between the distribution of the empirical data and the hypothesized model (Clauset et al. 2007). The Plpva module (Jeff Alstott) then compared this distance with distance measurements for comparable synthetic datasets drawn from the same model, returning so the p-value defined to be the fraction of the synthetic distances that are larger then the empirical distance (Clauset et al. 2007). If p is large, which means $\sim 1$ because $0<\mathrm{p}<$ 1 , then the those difference between the empirical data and the model can be attributed to statistical fluctuations alone, thus giving credit to our hypothesis. Viceversa, if the p-value is small, the model is not a plausible fit to the data (Clauset et al. 2007).

Once the p-value is calculated, it is important to set a lower boundary of acceptance, we decided to use $\mathrm{p}=0.1$ as this boundary, as defined by Clauset et al. This means that p-values $\leq 0.1$ are associated with hypothesis that are unplausible, and so need to be discarded.

## Chapter 4

## Results and Discussion

The results coming from the analysis previous explained is here reported for Dantu and Occator crater, respectively.

### 4.1 Dantu's results

We mapped 630 fractures located on the floor of the Dantu crater with a length ranging from 150.418 m to 14396.803 m . The histogram of the fracture lengths distribution is shown in Figure 4.1.
The angular analysis made with the Polar Plots (J.Jenness 2014) module of ArcMap, reveal that the fractures of Dantu crater have one preferential direction in their distribution, which is from $150^{\circ}$ to $330^{\circ}$, plus one minor direction from $\sim 75^{\circ}$ to $\sim 255^{\circ}$. Therefore, the Dantu fractures show a NW-SE trend and a NE-SW trend, as shown in Figure 4.2. Thus the fractures are not randomly distributed, but it is clear that they have a preferential orientation.
The Python modules gave us the value of $\alpha$ to be $3.427 \pm 0.306$ and, for the $\mathrm{x}_{\min }$, a value of $2431.403 \pm 341.259 \mathrm{~m}$. Those can be assumed as the real values of the distribution, because the number of datas that have $x \geq x_{\min }\left(n_{\text {tail }}\right)$ is about $170 \pm 41$, so greater than 50 . In Chapter 2 we reported that for Clauset et al. this is the minimum value to accept the approximation in which the "hatted" values tend to be the true values. From the DistributionCompare function, we found that the the powerlaw is the best fit, when compared to the exponential distribution, of the data above $\mathrm{x}_{\min }$. Indeed we obtained a p -value of 0.262 for the comparison, well below the limit value of 0.5 .
Finally, the Plpva module (J.Alstott), gave us a level of confidentiality. Indeed, the p-value found for Dantu is equal to 0.795 , meaning that the powerlaw model for the cumulative distribution of the fractures is a plausible hypothesis. The cumulative distribution is shown in figure 4.3.

| $\mathbf{N}^{\circ}$ of fractures [ ] | 630 |
| :--- | :---: |
| $\alpha[]$ | 3.427 |
| $\sigma_{\alpha}[]$ | 0.306 |
| $\mathbf{x}_{\min }[\mathbf{m}]$ | 2431.403 |
| $\sigma_{\mathbf{x}_{\text {min }}}[\mathbf{m}]$ | 341.259 |
| $\mathbf{n}_{\text {tail }}[]$ | 170.670 |
| $\sigma_{\mathbf{n}_{\text {tail }}}[]$ | 41.018 |
| $\mathbf{p - v a l u e}[]$ | 0.795 |
| GoF [ ] | 0.034 |

Table 3.1: Resulting parameters for the distribution of lengths associated with the Dantu crater.


Figure 4.1: The histogram of the single fracture lengths of Dantu, the x-axis is binned 300 times.


Figure 4.2: The rose plot of the azimuthal angles of the fractures of Dantu; the plot is radially symmetric due to the double counting of the angle.


Figure 4.3: The cumulative histogram of the fractures length distribution of Dantu, the cutoff ( $\mathrm{x}_{\min }$ ) is visible in red with its error dotted, that marks the passage from an exponential-kind of distribution, in the left part, to a power-law distribution, in the right part of the graph. The x -axis is binned 300 times.

### 4.2 Occator's results

We mapped 832 fractures located on the floor of Occator with a length ranging from 110.550 m to 7929.309 m . The histogram of fractures length distribution is shown in Figure 4.4.
The angular analysis made with the Polar Plots (J.Jenness 2014) module of ArcMap reveal that the fractures of Occator crater have just one preferential direction of distribution in the direction, which is from $\sim 50^{\circ}$ to $\sim 230^{\circ}$. Therefore, the Occator fractures show a NE-SW trend, as shown if figure 4.5 meaning that the fractures are not randomly distributed.
The Python modules gave us the value of $\alpha$ to be $3.375 \pm 0.469$ and, for the $\mathrm{x}_{\text {min }}$, a value of $1224.230 \pm 385.216 \mathrm{~m}$. Those can be assumed as the real values of the distribution, because the number of datas that have $x \geq x_{\min }\left(n_{\text {tail }}\right)$ is about $222 \pm 73$, so greater than 50 . For the same reasons explained in the previous section, this is the minimum value to accept the approximation in which the "hatted" values tend to be the true values. From the Distribution_Compare function, we found that the the power-law is the best fit, when compared to the exponential distribution, of the data above $x_{\text {min }}$. Indeed we obtained a p-value of 0.456 for the comparison. This value, in contrary to the distribution of Dantu, is almost at the limit of acceptancy, but we retained correct to infer that the power-law is in any case the best fit. Probably the rise of this p-value is caused by the rise of data avaliable in the Occator dataset with respect to the Dantu dataset.
Finally, the Plpva module (J.Alstott), gave us a level of confidentiality. Indeed, the p-value found for Occator is equal to 0.123 meaning that the our powerlaw model for the cumulative distribution of the fractures is indeed a plausible hypothesis, though not as strong as for the Dantu dataset of lengths. The cumulative distribution is shown in Figure 4.6.

| $\mathbf{N}^{\circ}$ of fractures [ ] | 832 |
| :--- | :---: |
| $\alpha[]$ | 3.375 |
| $\sigma_{\alpha}[]$ | 0.469 |
| $\mathbf{x}_{\min }[\mathbf{m}]$ | 1224.230 |
| $\sigma_{\mathbf{x}_{\min }}[\mathbf{m}]$ | 385.216 |
| $\mathbf{n}_{\text {tail }}[]$ | 222.593 |
| $\sigma_{\mathbf{n}_{\text {tail }}}[]$ | 73.374 |
| $\mathbf{p}$-value [ ] | 0.123 |
| GoF [ ] | 0.044 |

Table 3.2: Resulting parameters for the distribution of lengths associated with the Occator crater.


Figure 4.4: The histogram of the single fractures length of Occator, the x-axis is binned 300 times.


Figure 4.5: The rose plot of the azimuthal angles of the fractures of Occator; the plot is radially symmetric due to the double counting of the angle.


Figure 4.6: The cumulative histogram of the fractures length distribution of Occator; the cutoff (xmin) is visible in red with its error dotted, that marks the passage from an exponential-kind of distribution, in the left part, to a powerlaw distribution, in the right part of the graph. The x-axis is binned 300 times.

## Chapter 5

## Conclusions and future work

In this work, we analyzed the fracture systems located on the crater floors of Dantu and Occator. Specifically, we mapped the fractures deriving their length and orientation. From the analysis reported above, we found that fractures are not randomly distributed, but they have a preferential orientation. In the Dantu case, we obtained that fractures have a NW-SE trend, plus a minor NESW trend, while in the case of Occator, the fractures have NE-SW orientation. In addition, analyzing fractures length distribution, we found that the fractures follow both an exponential and power-law behaviour below and above a certain fracture length. Those fractures following a power-law behavior are the ones that can penetrate below the surface down to the mechanically discontinuity. Hence, we found that there are fractures able to penetrate the entire crust of Ceres in both craters cases. In the next future, it will be interesting to analyze in greater detail these type of fractures in order to quantify the thickness of the Ceres crust giving insights into the internal stratification of the body. Indeed, other technical applications like self-similar clustering (SSC), will allow us to infer how much in depth the fractures penetrate in the crust. This technique was already used on other Solar System bodies, as on Earth (Mazzarini and Isola, 2010), Mars (Pozzobon et al., 2015) and Enceladus (Lucchetti et al., 2017).

In addition, since Ceres hosts a certain number of Class I FFCs, it will be interesting to apply the same analysis also to other craters, such as Ezinu, Guae, Ikapati, Azacca, Haulani and Kupalo. This will enlarge the statistic and characterize entirely the behaviour of this category of peculiar craters, the FFC craters.

## Appendix A

## Codes

```
#import of the modules
import powerlaw as pl
import numpy as np
import matplotlib.pyplot as plt
from numpy import log
import plvar as pv
import plpva as pp
#generating the ordered arrays of fractures lengths
dantu = np.genfromtxt("dantulsp.txt")
d = dantu[::-1]
g = np.genfromtxt("occatorlsp.txt")
occator = np.sort(g)
o = occator [::-1]
#fitting the data to the theorical distributions
z=pl.Fit(dantu)
w=pl.Fit(occator)
#comparing between the powerlaw and the exponential
q=w.distribution_compare('power_law', 'exponential')
p=z.distribution_compare('power_law', 'exponential')
print p
print q
#finding alpha and xmin parameter for the distributions
alpha_d=z.power_law.alpha
xmin_d=z.power_law.xmin
print alpha_d
print xmin_d
```

alpha_o=w. power_law. alpha xmin_o=w. power_law. xmin
print lpha_o
print xmin_o
\#finding the errors on alpha and xmin, also finding ntail with its \#error, the $p$-values and the GoF of the distributions
err_d $=$ pv.plvar (d)
err_o $=$ pv.plvar (o)
p_d $=$ pp.plpva(d, xmin_d)
$\mathrm{p}_{-} \mathrm{o}=\mathrm{pp} \cdot \mathrm{plpva}\left(\mathrm{o}, \quad \mathrm{xmin} \mathrm{A}_{\mathrm{d}}\right)$
\#here we putted the complete list of fractures
\#because, somehow, numpy and plvar or plpva
\#do not get along
print err_d
print err_o
print p_d
print p_o
\#doing the cumulative histograms of the distributions
\#enlightening the cutoff value of xmin
plt.hist (d, bins=300, cumulative= -1 , histtype='step')
plt.xscale('log')
plt.yscale (' $\log$ ')

plt.xlabel ('L(m)')
plt. ylabel ('cumulative $\quad$ frequency')
plt.axvline (x=xmin_d, color='r', label='xmin')
plt.axvline (x=xmin_d+342.0687, color='r', ls='--')
plt.axvline (x=xmin_d-342.0687, color='r', ls='--')
plt.grid (True)
plt.show ()
plt.hist (o, bins=300, cumulative= $=1$, histtype='step')
plt.xscale(' $\log$ ')
plt.yscale (' $\log { }^{\prime}$ )

plt.xlabel('L(m)')
plt. ylabel ('cumulative $f$ frequency')
plt.axvline (x=xmin_o, color='r', label='xmin')
plt.axvline (x=xmin_o $+379.7369, ~ c o l o r=' r ', ~ l s='--')$
plt.axvline (x=xmin_o-379.7369, color='r', ls='--')
plt.grid (True)
plt.show ()

## Appendix B

## Tables

## B. 1 Dantu's fractures lengths [m]

| 150.4177293 |
| :---: |
| 199.8111859 |
| 261.2030634 |
| 296.2375965 |
| 322.4735172 |
| 351.5706298 |
| 381.3313484 |
| 393.7235466 |
| 451.6692713 |
| 472.8667099 |
| 496.0947422 |
| 527.4389194 |
| 545.2822422 |
| 576.853058 |
| 605.3911245 |
| 621.6049303 |
| 642.2950784 |
| 658.3517628 |
| 669.6768615 |
| 682.031305 |
| 705.0617747 |
| 720.7208767 |
| 730.9859581 |
| 767.300329 |
| 790.4719649 |
| 801.8186 |
| 818.4807314 |
| 834.779752 |


| 158.7503175 | 167.3375275 |
| :---: | :---: |
| 221.8211697 | 243.2157564 |
| 270.3119184 | 282.4494311 |
| 299.2192147 | 301.7526035 |
| 329.2767163 | 329.332633 |
| 358.2635969 | 360.4560657 |
| 381.7745501 | 382.6461803 |
| 411.8005808 | 413.4660329 |
| 454.5879306 | 461.8491237 |
| 474.6893013 | 478.0403124 |
| 497.1959527 | 519.4371003 |
| 534.1514809 | 534.1686393 |
| 552.1465462 | 562.2407078 |
| 582.8858018 | 584.8864061 |
| 606.1685005 | 612.2258175 |
| 628.3973306 | 629.019963 |
| 649.5897997 | 654.7849242 |
| 658.777396 | 662.3314071 |
| 670.4803367 | 675.3473937 |
| 689.2256745 | 691.0624092 |
| 707.2361784 | 709.1940301 |
| 722.2187216 | 722.3486318 |
| 732.6854355 | 736.4659407 |
| 771.7145235 | 785.8301099 |
| 790.769417 | 791.8655744 |
| 807.1280775 | 810.1772297 |
| 822.0538039 | 830.4266926 |
| 835.1565585 | 836.3445485 |


| 187.0890434 | 188.5450112 |
| :---: | :---: |
| 246.8427945 | 258.8573158 |
| 283.4887141 | 286.6649733 |
| 320.2448575 | 320.2764109 |
| 340.9085037 | 351.2554812 |
| 365.9014025 | 372.5591615 |
| 386.312822 | 392.7938623 |
| 440.0849719 | 450.4505479 |
| 462.4564586 | 465.4106722 |
| 479.7684818 | 488.1760251 |
| 523.3808081 | 523.4556548 |
| 541.7124357 | 543.9831115 |
| 562.2827782 | 564.8621134 |
| 596.4609526 | 602.4688156 |
| 613.834561 | 613.834561 |
| 638.0306299 | 638.412398 |
| 655.9142246 | 656.0547444 |
| 663.4002293 | 664.7587274 |
| 678.8023265 | 679.8087056 |
| 697.0871602 | 703.7327026 |
| 710.3860751 | 718.5324785 |
| 722.4897903 | 728.3568927 |
| 748.516319 | 755.6289991 |
| 786.678062 | 788.3402256 |
| 796.0552732 | 797.0148745 |
| 811.031408 | 811.2248904 |
| 831.0716723 | 834.3836414 |
| 840.9080515 | 842.8898763 |


| 846.4107357 | 847.2634646 |
| :---: | :---: |
| 858.6245731 | 858.6245731 |
| 879.0036006 | 879.1771174 |
| 897.3473525 | 905.394396 |
| 927.2124922 | 927.2711301 |
| 946.5979275 | 952.2543751 |
| 965.4059822 | 968.2343764 |
| 982.356671 | 1000.85808 |
| 1015.761715 | 1018.110651 |
| 1026.305284 | 1030.860629 |
| 1044.039397 | 1048.93095 |
| 1060.813566 | 1061.321548 |
| 1064.208112 | 1067.943668 |
| 1078.859794 | 1086.191554 |
| 1097.562198 | 1097.687035 |
| 1109.105848 | 1112.250634 |
| 1116.538356 | 1124.711069 |
| 1136.440072 | 1147.060488 |
| 1154.235696 | 1154.863851 |
| 1168.58525 | 1172.145552 |
| 1189.336872 | 1190.954391 |
| 1207.287466 | 1208.723431 |
| 1219.671342 | 1226.702641 |
| 1236.179346 | 1238.321731 |
| 1264.994036 | 1268.141334 |
| 1280.602365 | 1281.049695 |
| 1299.88877 | 1309.118674 |
| 1322.484821 | 1328.009032 |
| 1334.499925 | 1336.760217 |
| 1345.026283 | 1350.348935 |
| 1363.259249 | 1363.591258 |
| 1378.393684 | 1384.73578 |
| 1407.235764 | 1409.93163 |
| 1421.834819 | 1425.397437 |
| 1443.636108 | 1445.582823 |
| 1461.96052 | 1467.051712 |
| 1477.401313 | 1477.460394 |
| 1480.9826 | 1481.151187 |
| 1500.946219 | 1508.045468 |
| 1517.397678 | 1523.883954 |
| 1543.099641 | 1544.707768 |
| 1562.613566 | 1566.272634 |
| 1586.572082 | 1587.381167 |
| 1601.288048 | 1607.105364 |
| 1632.875474 | 1642.507018 |
| 1666.265058 | 1674.142257 |


| 848.9793125 | 850.7451409 | 854.186669 |
| :---: | :---: | :---: |
| 869.4010916 | 871.7810443 | 873.0792917 |
| 886.4300486 | 886.7772475 | 896.3634927 |
| 910.0048749 | 910.8590082 | 911.6342088 |
| 928.3563763 | 941.1545892 | 942.8576675 |
| 957.120068 | 958.7286709 | 959.4212021 |
| 970.5344548 | 971.1632361 | 977.2233693 |
| 1005.31516 | 1008.21201 | 1010.609238 |
| 1020.302496 | 1021.037714 | 1022.632993 |
| 1032.376064 | 1040.406863 | 1041.95283 |
| 1056.945495 | 1057.094486 | 1058.077894 |
| 1061.723961 | 1061.870853 | 1062.957407 |
| 1068.876644 | 1074.07622 | 1074.500442 |
| 1089.562638 | 1090.588886 | 1097.272167 |
| 1103.04266 | 1103.824592 | 1105.745313 |
| 1113.617642 | 1114.048228 | 1114.834277 |
| 1131.742995 | 1132.257699 | 1134.974573 |
| 1148.322912 | 1148.383238 | 1154.192597 |
| 1156.696009 | 1165.152434 | 1167.81934 |
| 1174.984102 | 1178.014203 | 1187.191464 |
| 1201.095497 | 1203.382986 | 1205.33323 |
| 1209.134259 | 1218.846735 | 1218.932479 |
| 1227.701367 | 1229.772259 | 1234.567219 |
| 1243.322398 | 1249.55726 | 1263.695016 |
| 1269.418048 | 1272.293534 | 1274.764829 |
| 1281.851641 | 1288.079106 | 1289.770694 |
| 1309.118851 | 1313.024857 | 1316.926737 |
| 1328.943974 | 1331.879517 | 1333.832023 |
| 1338.217244 | 1339.455804 | 1341.387452 |
| 1355.202011 | 1356.630918 | 1360.993817 |
| 1375.10297 | 1377.424913 | 1377.850024 |
| 1396.461495 | 1397.269579 | 1405.463236 |
| 1413.457576 | 1417.438024 | 1421.253046 |
| 1428.060749 | 1430.7437 | 1439.384031 |
| 1453.045487 | 1454.832533 | 1458.689661 |
| 1469.351843 | 1469.917183 | 1477.0493 |
| 1480.046686 | 1480.240735 | 1480.961073 |
| 1485.924282 | 1489.214309 | 1497.014224 |
| 1511.031623 | 1513.432305 | 1514.075881 |
| 1527.41603 | 1530.10918 | 1534.928934 |
| 1547.991455 | 1551.674429 | 1560.061263 |
| 1568.244163 | 1572.676541 | 1575.047862 |
| 1587.647266 | 1593.230977 | 1595.933423 |
| 1619.874066 | 1625.640093 | 1632.243953 |
| 1642.698916 | 1657.600772 | 1660.699328 |
| 1678.719286 | 1679.110325 | 1695.536127 |
|  |  |  |


| 1699.165381 | 1727.060845 | 1729.388137 |
| :---: | :---: | :---: |
| 1749.375367 | 1769.08117 | 1769.731484 |
| 1771.254255 | 1773.07777 | 1778.277158 |
| 1811.169954 | 1830.414502 | 1835.837839 |
| 1852.478137 | 1865.162848 | 1867.262601 |
| 1882.704764 | 1885.90186 | 1897.501024 |
| 1922.481127 | 1927.040544 | 1931.490857 |
| 1957.774741 | 1971.697083 | 1976.472783 |
| 1990.125633 | 1994.979189 | 1999.341197 |
| 2007.076982 | 2008.456715 | 2013.321316 |
| 2045.75503 | 2046.285503 | 2048.838061 |
| 2066.451042 | 2069.568 | 2081.251501 |
| 2119.381357 | 2124.420202 | 2136.197074 |
| 2148.821276 | 2154.191414 | 2156.365734 |
| 2159.669053 | 2167.310041 | 2183.175593 |
| 2190.991889 | 2196.829687 | 2206.806414 |
| 2236.077417 | 2244.844879 | 2249.830138 |
| 2274.536656 | 2277.79806 | 2278.955939 |
| 2306.041939 | 2314.2941 | 2316.977866 |
| 2338.526258 | 2342.181408 | 2343.988674 |
| 2359.020245 | 2375.98496 | 2377.798767 |
| 2432.876641 | 2433.151071 | 2436.270074 |
| 2448.966848 | 2459.831588 | 2462.161478 |
| 2483.9636 | 2486.04191 | 2492.434577 |
| 2555.387205 | 2570.233644 | 2571.227249 |
| 2614.61052 | 2618.293349 | 2635.402886 |
| 2651.636772 | 2661.713491 | 2662.895686 |
| 2687.326868 | 2689.41035 | 2689.434064 |
| 2719.312691 | 2721.9973 | 2729.982465 |
| 2786.610853 | 2800.05756 | 2805.472992 |
| 2848.616127 | 2854.138124 | 2857.968186 |
| 2875.588561 | 2893.811424 | 2908.702053 |
| 2943.544688 | 2946.925077 | 2983.304779 |
| 3049.601499 | 3054.654782 | 3055.043081 |
| 3105.462303 | 3113.996189 | 3132.788627 |
| 3160.320183 | 3176.743853 | 3176.958818 |
| 3237.707185 | 3240.914837 | 3255.993971 |
| 3304.165555 | 3370.675168 | 3377.27017 |
| 3455.960591 | 3498.616789 | 3516.829645 |
| 3554.291088 | 3564.454883 | 3566.476798 |
| 3620.957871 | 3666.708641 | 3673.418672 |
| 3802.24853 | 3819.286387 | 3831.659652 |
| 3901.026997 | 3940.63738 | 4032.886834 |
| 4246.915845 | 4317.962599 | 4330.649753 |
| 4450.931182 | 4572.592696 | 4576.480393 |
| 4718.036374 | 4721.130227 | 4776.489702 |


| 1734.382392 | 1748.609508 |
| :---: | :---: |
| 1769.974229 | 1770.346911 |
| 1783.283173 | 1800.236247 |
| 1844.950101 | 1851.244119 |
| 1867.452437 | 1869.978054 |
| 1897.703236 | 1913.688481 |
| 1939.529183 | 1956.78099 |
| 1980.700262 | 1988.86626 |
| 1999.440155 | 2003.175922 |
| 2028.825658 | 2032.219513 |
| 2057.318732 | 2061.900635 |
| 2101.727349 | 2118.400267 |
| 2137.577295 | 2143.449978 |
| 2159.497122 | 2159.632025 |
| 2185.47357 | 2190.537887 |
| 2219.650981 | 2233.295322 |
| 2271.686146 | 2273.206352 |
| 2279.951596 | 2285.795092 |
| 2317.765125 | 2320.543126 |
| 2345.536627 | 2347.081387 |
| 2415.803112 | 2431.402743 |
| 2445.12498 | 2445.71068 |
| 2466.49341 | 2482.344896 |
| 2497.638549 | 2549.095913 |
| 2574.995742 | 2605.672779 |
| 2644.119776 | 2648.664074 |
| 2673.553428 | 2677.069687 |
| 2698.883772 | 2701.863148 |
| 2779.818419 | 2784.433324 |
| 2805.688011 | 2806.290128 |
| 2858.461741 | 2875.3174 |
| 2911.213667 | 2915.852414 |
| 3025.045274 | 3026.245238 |
| 3096.742328 | 3098.908163 |
| 3141.170854 | 3147.812671 |
| 3207.400056 | 3219.775717 |
| 3266.141222 | 3271.093086 |
| 3384.997308 | 3443.271081 |
| 3529.341689 | 3538.974368 |
| 3566.97679 | 3608.383917 |
| 3734.054406 | 3789.332343 |
| 3846.064932 | 3872.344393 |
| 4036.356351 | 4151.528644 |
| 4413.570098 | 4444.134952 |
| 4609.804449 | 4617.755102 |
| 4801.22699 | 4838.406961 |
|  |  |


| 4876.307915 | 4881.626825 | 5144.612136 | 5277.563389 | 5361.745358 |
| :---: | :---: | :---: | :---: | :---: |
| 5647.30347 | 5743.863172 | 5793.622935 | 5845.577483 | 5941.258718 |
| 5976.707585 | 6104.940137 | 6163.024522 | 6228.816109 | 6276.755529 |
| 6380.745852 | 6383.733 | 6674.432153 | 6743.045346 | 7005.30858 |
| 7164.522313 | 7343.188358 | 7350.777523 | 7355.803106 | 8740.949562 |
| 10668.97584 | 10856.15485 | 11202.76977 | 13955.35448 | 14396.80306 |

## B. 2 Occator's fractures lengths [m]

| 131 | 143.00144043 | 143.27216154 | 146.698346 | 59 |
| :---: | :---: | :---: | :---: | :---: |
| 153.45694185 | 154.09596752 | 154.8247639 | 162.45535341 | 170.13569183 |
| 174.54515391 | 179.44495227 | 191.12280211 | 195.38262641 | 199.12615024 |
| 201.09671663 | 203.64080279 | 206.70345515 | 209.17190941 | 210.07358862 |
| 210.79625119 | 212.83653034 | 213.89985362 | 214.97438828 | 217.7333254 |
| 217.80719692 | 220.62431314 | 224.89628313 | 225.62659391 | 230.32667305 |
| 231.81614871 | 237.83131249 | 238.49267073 | 238.78693591 | 248.11580458 |
| 251.56346465 | 251.91189313 | 253.17578964 | 253.27179246 | 254.88092835 |
| 254.88092835 | 255.24116001 | 256.47065031 | 259.82827733 | 262.05826167 |
| 262.59198577 | 264.03575928 | 264.91438588 | 264.91438588 | 266.06593504 |
| 270.24924741 | 270.49406355 | 271.97116907 | 272.88645584 | 273.12437348 |
| 274.86440477 | 275.18834643 | 275.2816766 | 275.40606408 | 277.65078683 |
| 278.12452961 | 280.13610754 | 284.49508291 | 289.64970211 | 290.54210459 |
| 291.12426901 | 294.24677018 | 296.63531667 | 298.42845683 | 299.77034609 |
| 300.13276348 | 300.85422892 | 301.34541683 | 302.01414925 | 303.25761172 |
| 303.45662737 | 308.93200861 | 310.0278483 | 311.30953061 | 312.87467757 |
| 312.89347081 | 313.18494592 | 313.71575651 | 314.30959286 | 316.49854959 |
| 317.29249336 | 322.41100159 | 322.81745938 | 323.4043405 | 324.46918541 |
| 327.27216358 | 327.64820417 | 328.3194345 | 331.19516836 | 333.84780361 |
| 335.37931175 | 335.39153129 | 337.46232672 | 340.08435387 | 341.7369253 |
| 343.62129727 | 343.91109627 | 347.28520476 | 352.28953163 | 357.18959241 |
| 358.32314199 | 358.4902809 | 361.79256086 | 363.26121353 | 364.72388223 |
| 364.96025899 | 365.98395074 | 367.90481061 | 369.5054236 | 369.70754832 |
| 371.61142576 | 372.05770946 | 372.82861218 | 373.6538702 | 374.411875 |
| 376.66936183 | 380.13597366 | 380.69639842 | 381.61361357 | 384.54526704 |
| 384.77628594 | 387.96906952 | 389.11650595 | 392.63322215 | 393.68312252 |
| 395.6009096 | 396.54541303 | 396.82121387 | 397.79325094 | 399.56862299 |
| 401.18203373 | 401.34678539 | 402.33773366 | 402.44357107 | 402.7976007 |
| 403.70720408 | 404.29887489 | 404.55693056 | 405.3624979 | 405.39707 |
| 409.20141911 | 409.74100489 | 411.66392216 | 412.00637102 | 413.29320527 |
| 413.41228516 | 415.56428197 | 415.57630202 | 415.94400897 | 417.52088776 |
| 417.96938769 | 419.61753183 | 420.4715111 | 421.0542731 | 421.88475571 |
| 425.18028123 | 425.8314643 | 425.97180166 | 427.40592448 | 427.98853073 |
| 431.69610902 | 432.67874304 | 433.9175345 | 434.22939861 | 436.45130988 |
| 436.95529976 | 441.23138202 | 441.59355781 | 446.30441816 | 447.32673367 |
| 448.34975908 | 448.39137017 | 449.98707115 | 450.44825631 | 451.35837566 |
| 454.25234016 | 457.94260972 | 458.32660965 | 458.57095027 | 460.74831638 |
| 463.61705693 | 466.46036284 | 467.72260842 | 468.66458863 | 469.8326065 |
| 475.32997277 | 476.98534145 | 477.57203951 | 478.69209688 | 479.86244014 |
| 482.62654245 | 483.10743863 | 483.3651591 | 486.35280306 | 487.04072856 |
| 487.16014027 | 489.63859815 | 490.74636856 | 492.02027594 | 495.50897906 |
| 496.97024601 | 497.16271921 | 500.42814643 | 501.42982328 | 505.14078901 |
| 505.57781482 | 506.1091361 | 512.23726144 | 514.7457074 | 516.75014265 |


| 517.20424864 | 518.31848421 | 522.0861455 | 522.85785821 | 523.19853774 |
| :---: | :---: | :---: | :---: | :---: |
| 523.42017749 | 523.92988329 | 524.11966671 | 525.82553045 | 525.96177717 |
| 528.07151856 | 528.70850198 | 528.73010065 | 528.77907809 | 530.23336903 |
| 530.80822031 | 531.64189927 | 534.69141125 | 535.53057516 | 535.73580281 |
| 536.78832734 | 538.64549541 | 540.94546121 | 542.31983435 | 543.83501153 |
| 544.41537495 | 546.51862752 | 548.64737759 | 549.60449137 | 549.61683282 |
| 549.68837428 | 549.69844513 | 551.33647906 | 554.83248104 | 555.39927878 |
| 556.12992063 | 556.5325166 | 561.06328852 | 561.08275475 | 561.64824387 |
| 561.7062144 | 561.82374826 | 562.85688733 | 562.92913122 | 563.15432934 |
| 564.56527618 | 566.19053109 | 568.60415264 | 568.60980074 | 570.12644457 |
| 570.2093007 | 570.25225062 | 571.936332 | 575.10659105 | 575.23720256 |
| 576.82582161 | 582.80799649 | 585.78548199 | 586.1762594 | 587.3057364 |
| 589.01348576 | 593.25993385 | 596.96733977 | 600.13616793 | 601.28560605 |
| 602.50124363 | 603.59615295 | 604.67605452 | 604.84744699 | 606.47537767 |
| 606.72629003 | 608.54288375 | 613.74745626 | 616.74555146 | 617.61313145 |
| 620.43445261 | 623.20801448 | 628.15035561 | 628.6042212 | 629.2564132 |
| 629.48872291 | 629.82883511 | 631.10510439 | 632.88975888 | 633.05331764 |
| 634.29532062 | 637.65988191 | 637.91498144 | 638.23712815 | 638.41239798 |
| 640.76410525 | 640.99160408 | 641.99874043 | 646.10861429 | 646.20241416 |
| 647.4763774 | 648.30639155 | 649.76368118 | 652.046422 | 652.89658728 |
| 656.15578623 | 657.11110439 | 657.23398351 | 658.55371925 | 661.57681828 |
| 665.16996077 | 665.40377271 | 667.11840605 | 667.17642206 | 670.18226818 |
| 670.62872055 | 671.44192113 | 673.54546039 | 675.32118489 | 679.21393377 |
| 680.26872813 | 680.77451197 | 684.32660911 | 685.7876632 | 686.27867527 |
| 687.14339326 | 690.47660022 | 691.40880117 | 692.05783962 | 692.1391106 |
| 692.69190284 | 695.9434714 | 696.1775682 | 696.71511136 | 697.29268363 |
| 700.08659343 | 700.12309726 | 701.21985946 | 704.74112006 | 704.96581506 |
| 706.40674344 | 707.79274085 | 708.14972325 | 708.20067464 | 709.16372736 |
| 711.68655319 | 714.07980173 | 715.64266659 | 716.5162511 | 721.06442031 |
| 722.30838291 | 726.28775252 | 727.72587733 | 728.97733072 | 730.0722521 |
| 730.62076236 | 731.01900864 | 731.06470627 | 741.24336224 | 741.51486138 |
| 742.58090668 | 743.62658414 | 744.1288897 | 745.21248811 | 745.66514446 |
| 747.21457006 | 747.22274798 | 749.92188853 | 750.04805717 | 750.72757009 |
| 750.74709385 | 751.66796107 | 751.88608114 | 754.35433937 | 755.20585534 |
| 755.23812182 | 756.39770227 | 760.25371738 | 760.50334625 | 763.77753711 |
| 765.87536646 | 769.62201705 | 770.95039233 | 775.79420998 | 776.78614871 |
| 776.99376948 | 779.30220345 | 780.75399707 | 782.42933627 | 782.96855082 |
| 785.31057739 | 785.40191077 | 787.34152498 | 787.54033561 | 789.3822902 |
| 791.7773457 | 793.2643259 | 796.55944575 | 800.26076083 | 801.52308826 |
| 802.33408087 | 805.02278429 | 806.50607232 | 806.84736112 | 808.26066603 |
| 808.97665821 | 810.02426205 | 811.48266133 | 811.6637357 | 817.16628655 |
| 817.40841027 | 819.09140139 | 820.93474606 | 826.87530336 | 827.32716333 |
| 829.3850963 | 829.77732514 | 830.77363052 | 833.50087523 | 836.50492533 |
| 837.01953232 | 837.96233207 | 842.78879247 | 843.71984996 | 844.14150462 |
| 847.47646109 | 859.90110703 | 860.34102899 | 860.37609058 | 860.62465108 |
| 863.57594722 | 864.62209966 | 865.4383148 | 866.33828247 | 867.43212026 |


| 918071 | 869.61193112 | 876.03758961 | 876.35117909 | 87 |
| :---: | :---: | :---: | :---: | :---: |
| 1.10781025 | 884.0249398 | 884.61098977 | 886.95923858 | 887.96372842 |
| 889.33310384 | 891.32958821 | 892.49532795 | 896.31959423 | 896.37750369 |
| 899.68240072 | 900.02081219 | 902.17086597 | 903.5417968 | 904.41209743 |
| 909.47522549 | 909.56334171 | 918.66295276 | 920.59978963 | 920.76796247 |
| 927.99681091 | 928.34896121 | 929.5073788 | 930.64460461 | 934.23958956 |
| 935.50481341 | 936.48995145 | 938.91357831 | 940.24581503 | 940.59042337 |
| 941.571176 | 942.01046368 | 944.07353393 | 945.58855136 | 96.90177164 |
| 64157906 | 951.029576 | 959.01705793 | 962.00310212 | 965.82170479 |
| 968.13003483 | 974.77921449 | 976.28829579 | 976.66931771 | 4 |
| 1.3426861 | 981.47170826 | 985.52106731 | 985.61970376 | 985 |
| 986.63792453 | 992.89420842 | 999.05630784 | 1006.02703484 | 1007.98661866 |
| 1009.85634607 | 1010.53025545 | 1013.77352 | 1019.0979074 | 1023.98702533 |
| 1024.31768702 | 1025.72210146 | 1027.26525876 | 1031.53529146 | 1039.86188693 |
| 1042.56500274 | 1045.23155996 | 1045.40752171 | 1049.14323248 | 1050.12989015 |
| 1050.56731344 | 1051.52630322 | 1055.08144702 | 1055.28780458 | 1059.15125708 |
| 1064.21418472 | 1064.33095436 | 1064 | 1066.32220572 | 1066.85237151 |
| 1073.34006083 | 1073.95183607 | 1079.74842396 | 1080.30804219 | 1084.22046021 |
| 1087.00933621 | 1087.41939819 | 1087.86454 | 1096.04477537 | 1099.47356053 |
| 1.3269338 | 1103.19937706 | 1107.7071938 | 1111.24174312 | 1111.45580151 |
| 1111.89435019 | 1114.06525367 | 1114.59924556 | 1123.60003365 | 1128.51634624 |
| 1138.38724998 | 1139.09901914 | 1150.37569608 | 1158.05071419 | 1161.18117411 |
| 1161.51309678 | 1162.68994636 | 1168.692516 | 1170.24976373 | 1170.35459977 |
| 1174.69620628 | 1180.26112571 | 1189.44139623 | 1195.54414193 | 1202.31748227 |
| 1202.89775776 | 1203.80338899 | 1205.29011253 | 1208.25447218 | 1224.23031696 |
| 1227.53247977 | 1228.50897375 | 1230.83050665 | 1232.07743395 | 1235.28839768 |
| 1235.98480038 | 1236.90881869 | 1239.12355994 | 1241.27161913 | 1241.80093265 |
| 1241.8973091 | 1242.97388409 | 1244.25290708 | 1247.73885792 | 1248.07885379 |
| 1250.80831037 | 1252.3882357 | 1270.35455066 | 1271.41476325 | 1272.4418982 |
| 1276.86939883 | 1277.22410165 | 1281.5879993 | 1285.636078 | 1286.69784269 |
| 1300.15302381 | 1300.50066682 | 1305.52650469 | 1307.21882451 | 1307.6203541 |
| 1308.29218672 | 1309.8146821 | 1316.07513031 | 1318.66733668 | 1321.07732614 |
| 1322.94581081 | 1327.63884968 | 1328.09883556 | 1328.34305387 | 1333.59227849 |
| 1335.70570668 | 1336.28826865 | 1337.24139358 | 1339.86405013 | 1339.98735956 |
| 1340.99632142 | 1341.95458455 | 1346.08339276 | 1346.78690006 | 1346.86602004 |
| 48.6131215 | 1352.44412232 | 1352.55933004 | 1353.29969339 | 1356.19185681 |
| 1361.43166246 | 1363.2794977 | 1365.85497885 | 1366.93882807 | 1368.34871066 |
| 1372.27665684 | 1374.34581036 | 1376.85569177 | 1380.6863456 | 1382.95499223 |
| 1392.44804333 | 1402.52006379 | 1403.24975381 | 1403.45514777 | 1403.8781394 |
| 1415.04659203 | 1417.10178153 | 1420.3823844 | 1421.031458 | 1424.28888151 |
| 1425.24216024 | 1429.30706537 | 1431.59310056 | 1432.94946397 | 1435.45522177 |
| 1442.49431617 | 1446.87065136 | 1449.27932912 | 1449.54519297 | 1453.00494044 |
| 1464.72277652 | 1465.31165924 | 1465.47373506 | 1477.30755549 | 1477.57623131 |
| 1481.10987516 | 1481.13192492 | 1483.24238102 | 1485.84215902 | 1485.94015501 |
| 1487.74436968 | 1491.38441038 | 1505.81910523 | 1507.93230309 | 1512.02345625 |
| 1513.32734372 | 1517.75547323 | 1520.61849595 | 1525.39537785 | 1530.35206676 |


| 1531.5326829 | 1532.12598443 | 1545.36999783 | 1546.29453808 | 1546.92595699 |
| :---: | :---: | :---: | :---: | :---: |
| 1564.2845611 | 1573.0345175 | 1578.04545127 | 1588.58292053 | 1591.12768142 |
| 1593.54063793 | 1598.58915371 | 1600.43292033 | 1630.26245711 | 1633.50510927 |
| 1638.37572927 | 1641.42954272 | 1643.20827981 | 1646.40977456 | 1648.00656714 |
| 1654.39276028 | 1668.03473295 | 1670.83156945 | 1676.00325007 | 1676.19490684 |
| 1677.76835189 | 1680.66116809 | 1681.62604411 | 1690.12580396 | 1699.80512809 |
| 1702.93830525 | 1704.08854485 | 1712.03914908 | 1716.75483133 | 1720.48454241 |
| 1733.67786964 | 1746.89351076 | 1756.17559498 | 1756.2912035 | 1780.27335683 |
| 1797.41498561 | 1800.66570159 | 1800.765742 | 1810.23045575 | 1818.96289954 |
| 1823.66180895 | 1844.24277617 | 1847.38566074 | 1852.8595334 | 1853.38996442 |
| 1860.65302474 | 1869.82887132 | 1874.67965294 | 1893.14913043 | 1900.73412763 |
| 1904.4806935 | 1905.26383782 | 1906.53604148 | 1909.39849718 | 1912.96493371 |
| 1922.82946516 | 1929.94695173 | 1949.83642156 | 1953.57991495 | 1957.60519299 |
| 1964.57193661 | 1966.54822285 | 1972.73837211 | 1983.75766207 | 2004.12274113 |
| 2012.67421414 | 2055.48775662 | 2061.98937952 | 2091.23338173 | 2102.46344101 |
| 2114.2258764 | 2116.67297327 | 2127.43083958 | 2131.18823494 | 2141.87763876 |
| 2165.83474729 | 2169.39441643 | 2220.43104896 | 2226.73027202 | 2248.2818945 |
| 2273.49277181 | 2280.49965552 | 2286.30635369 | 2292.83674358 | 2295.93803589 |
| 2300.25710347 | 2300.3407118 | 2307.36716312 | 2334.3428212 | 2341.22356857 |
| 2348.18935143 | 2365.33312696 | 2382.27575906 | 2416.25152884 | 2428.07264289 |
| 2451.71653841 | 2453.57945092 | 2456.82309563 | 2480.22897514 | 2480.75274303 |
| 2513.35720094 | 2527.00351238 | 2557.23503939 | 2577.01244198 | 2577.8625463 |
| 2592.95725602 | 2639.14837059 | 2645.59180744 | 2712.31394138 | 2713.42547786 |
| 2739.32179027 | 2753.55124542 | 2816.50947545 | 2822.94557538 | 2828.94565823 |
| 2865.27388329 | 2870.38721076 | 2915.76028209 | 2924.37684456 | 2931.78970345 |
| 2939.29062342 | 2939.33526798 | 2981.61164417 | 2994.31208679 | 3014.09110519 |
| 3020.05966229 | 3050.04007405 | 3079.46139884 | 3083.98686902 | 3180.3424135 |
| 3180.38572976 | 3205.30553944 | 3221.00133948 | 3307.31779462 | 3509.26407357 |
| 3550.49999606 | 3555.65436155 | 3586.49636017 | 3616.4501463 | 3640.53948605 |
| 3927.08163244 | 4055.83643213 | 4179.36888715 | 4347.50955825 | 4566.88109959 |
| 5323.06087983 | 5906.52425169 | 5915.37018601 | 6314.43040201 | 6787.84425153 |
| 6942.67887095 | 7929.30932145 |  |  |  |
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