UNIVERSITA' DEGLI STUDI DI PADOVA

DIPARTIMENTO DI SCIENZE ECONOMICHE ED AZIENDALI "M.FANNO"

DIPARTIMENTO DI SCIENZE STATISTICHE

CORSO DI LAUREA MAGISTRALE IN ECONOMICS AND FINANCE

TESI DI LAUREA

"ASSET & LIABILITY MANAGEMENT IN PENSION FUND"

RELATORE:

CH.MO PROF. MASSIMILIANO CAPORIN

LAUREANDO/A: MANOU K. MWELWA

MATRICOLA N. 1156461

ANNO ACCADEMICO 2018 – 2019

Il candidato dichiara che il presente lavoro è originale e non è già stato sottoposto, in tutto o in parte, per il conseguimento di un titolo accademico in altre Università italiane o straniere.
Il candidato dichiara altresì che tutti i materiali utilizzati durante la preparazione dell'elaborato sono stati indicati nel testo e nella sezione "Riferimenti bibliografici" e che le eventuali citazioni testuali sono individuabili attraverso l'esplicito richiamo alla pubblicazione originale.

The candidate declares that the present work is original and has not already been submitted, totally or in part, for the purposes of attaining an academic degree in other Italian or foreign universities. The candidate also declares that all the materials used during the preparation of the thesis have been explicitly indicated in the text and in the section "Bibliographical references" and that any textual citations can be identified through an explicit reference to the original publication.

Firma dello studente

INDEX

I.   INTRODUCTION

Before we start this work, we think that it is useful to spend some words on what to expect from this thesis.

The work is part of the thematic of quantitative financial investment. In particular, it is an attempt to apply several tools acquired from the field of Economics, Finance, Mathematics and Statistics to the Pension fund financial investment decision process.

The pension funds, unlike several other financial institutions, are concerned, while making investment decision, by the problem of taking into account the commitment to repay, at some future date, a determined amount of money. The integrated consideration of the commitment to repay in the investment policy is a shade of what is known as Asset and Liability Management-ALM.

ALM has received an increasing interest from practitioners and scientists and literature is full of models and opinions on this topic, most of them complex. The model we are proposing (inspired from Barberis, 2000) tries to apply the usual quantitative investment tools to ALM.

The Problem we are searching a solution for is the one of a representative worker. He is currently aged 45. He should decide on how to best invest his wealth in order to have a decent income when he will have to retire.

Therefore, over a 45 years period, we will conduct analyses and, according to the case and considering a set of constraints, we will propose an optimum solution to be intendent as a portfolio allocation allowing to approach the most a target value while satisfying the constraints.

This analysis is conducted both from the perspective of the worker and the pension manager.

 The results have, according to us, a practical relevance and the implications from the various approach give interesting cues for policy making.

To achieve this objective, we started this work by a general overview on pension systems, their pillars, the process that caused the creation of pension fund in Italy etc.

In the third chapter, we focused on deepening the world of pension fund intended as institutions. Then we presented several approaches to asset and liability management proposed by the literature.

The fourth chapter focuses instead on setting up the problem. It explains in detail what are the activities to be cared in order to reach our objectives.

In the fifth chapter we formalized the work, presented the data used (S&P US data), the methodology and all relevant assumptions made.

The following sixth chapter presents all the results in detail from the cases dealt with.

The last point contains a summary of all the results and comments on them followed by a final conclusion.

II. PENSION SYSTEMS -THE ITALIAN SOCIAL SECURITY

Pension systems are part of the broader world of social security.

In simple terms, social security refers to **any plan that provides monetary assistance to people with an inadequate or no income.** It may also refer to the actions, programs of an organization intended to promote the welfare of the population through assistance measures guaranteeing access to sufficient resources for food and shelter and to promote health and well-being for the population at large and potentially vulnerable segments such as children, elderly, sick and the unemployed.

Historically, the first social security interventions happened in mutuals, cooperatives, associations and other foundations organized and managed by workers and categories of workers in order to receive **health insurance and get access to medical and hospital cares**.

Those primitive forms of assistance were financed by contributions of the same workers and their employers.

The next step in development of social security is linked to industrialization.

Industrialization caused an immense expansion of the working class. As a result, social security had been considered as a public good and progressively shifted from self-organized management of workers to a broader compulsory system, managed by public institutions. The latter provided benefit for various purposes such as medical care, unemployment, old age, maternity, invalidity benefits etc.

II.1 Pension Systems in the Economic Theory

A pension (intended as a scheme, a plan, a system) is a "fund" into which a sum of money is progressively added during the entire employee's working period, and from which payments are drawn to support the person's inactivity in the form of periodic payments.

The pension systems are characterized by the fact that they have a compulsory feature and a relevant number of participants into it.

Once the worker stopes working, the pension (intended as payment) can take the following form (According to the reason that brings him to stop working):

1. old-age pensions;
2. retirement pensions;
3. disability pensions.

The above-mentioned list is not exhaustive. Indeed, there are also survivors' pensions (*Per Superstiti*) and social pensions but, unlike the old-age and the retirement pensions that fulfill an "insurance" function, they have a mere assistance nature.

The old-age and retirement pension forms often lead to confusion even if they have 2 distinct definitions.

The old-age pension is the main form of public insurance. It consists of an economic benefit, represented by a monthly payment of an amount of money to a worker **who has reached a certain age.**

Pursuant to some regulations, this form of pension may require a minimum number of contributions years.

On the other hand, the retirement pension consists of a **benefit given to the employee, in case certain contribution requirements are satisfied.**

In presence of those conditions the eligible employee is allowed to **anticipate the time of retirement regardless of his age.**

In some formulations, the retirement pension cannot be obtained before a given age.

If we have to give a classification of pension systems, we have to point out that there are various ways to classify them:

The first option is to classify the pension systems according to the model issued by World Bank. It clusters them into the following 3 pillars:

1. **Public mandatory system** financed by taxes with public management and differs from one country to another;
2. **Compulsory system privately managed**;
3. **Voluntary system privately managed**.

Pension systems falling in these 3 pillars can be classified further according to the **methods of financing** and to the **criteria for defining the amount of benefits**.

A. **Financing Method:** Regarding the financing methods, we can identify **pay-as-you-go** and **capitalization (funded)** systems.

In the first case, workers and employers pay the contributions each year. The contributions are then used to finance the pension benefit paid to retirees in the contemporaneous period. Therefore, **the working generation pays the benefit to the generation that has ceased working**.

As we will see later, this system is characterized by several risks mainly due to the consequences linked to a slowing economic growth, ageing of the population, unemployment or any other factor that could cause the current contributions to be insufficient to cover the contemporaneous benefit payments.

In the **capitalization system** instead, the contributions paid are invested in the capital market and, at the time of retirement, the pension is equal to the paid contributions increased by the rate of return obtained from the investment. The main risk, in this case, is the financial risk

B. **Criteria for Defining the Amount of Benefit:** As to the criteria for defining the amount of benefits, we have **the defined benefit system and the defined contribution.** In the first one, benefits are based on the salary of the worker (that is, of the last period or an average). This can be justified as an attempt to guarantee a consumption standard similar to the one enjoyed in the working period. They are sometimes called final salary or career average pensions.

In the second case, it is a kind of forced saving in view of the period of inactivity. The rate of remuneration in this case is defined by the law *a priori*. At retirement, one will get what he has paid as contribution increased by a specific rate previously defined.

II.2. Evolution of the Italian Pension System

In Italy, the old-age pensions and disability pensions was introduced in 1864.

Initially, this kind of pensions was reserved to public sector employees.

Private workers had to wait until 1919 to be included into public pension system.

Twenty years after, in 1939, it has been recognized to private workers also the so-called survivors' pensions.

This progressive inclusion process of private workers went on over the years and, during the '50s-'60s, the compulsory pension system has been extended to include also other categories of workers (artisans, merchants etc.).

The '70s instead were characterized by the oil crises. Being a country with very few natural resources, Italy is strongly dependent on oil imports. The two crises had a severe impact on the economy and public finance.

As a result, the country experienced a stage of stagflation—weak economic growth combined with high unemployment and a high inflation rate—and, consequently, a worsening of its public finance.

The '80s therefore inherited of this situation. A recovery plan was necessary. Italy had to intervene in order to stabilize its public finance.

As suggested by the figure 1, intervention consisted in reforms able to scale back the current expenses.
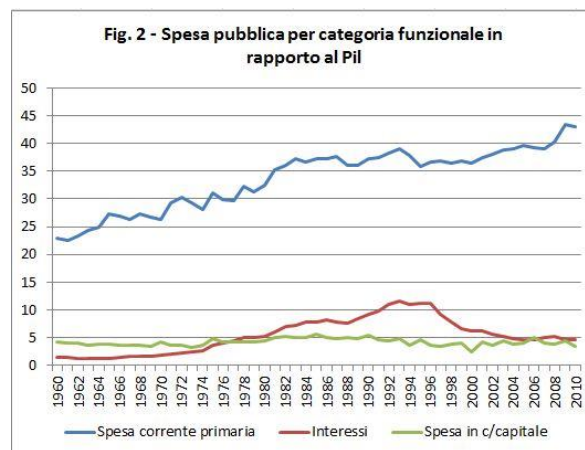

Fig. 1: **Italian component of the Italian Public expenditure.**

We should point out that current expenses also include pension expenses.

The choice of reducing current expenses is coherent with what is highlighted in figure 2. Social and pension interventions have persistently composed the bigger part of public expenditure.

According to Giancarlo Morcaldo, ex central director of *Banca d'Italia*, the growth of pension expenditures was the result, on one hand, of the progressive formation of a pension and welfare system extended to the entire resident population and, on the other hand, is the result of gradual rising of pension's quality (they were becoming more and more generous).

Concerning the pension quality, it is crucial to note that, until then, a worker enrolled to INPS (Italian National Institute for Social Security) was receiving a pension whose amount was linked to wage of the last working period, with a revaluation of 2% each year of contribution. There was also a revaluation of wage to consider inflation. This implied systematic increase of wages over time in presence of inflation.

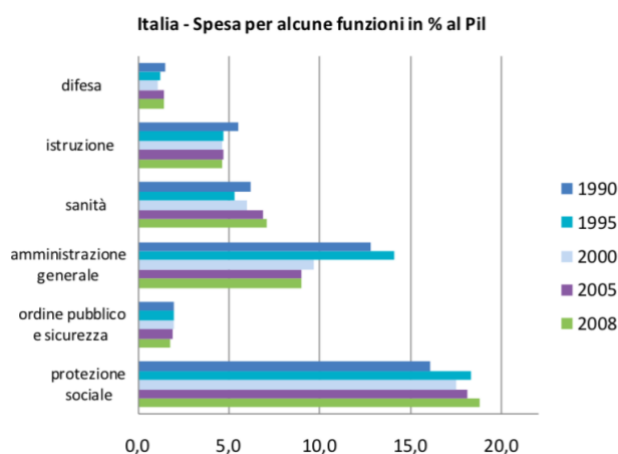Consequently, the base on which to compute the amount of pension benefit resulted systematically high.

Italia - Spesa per alcune funzioni in % al Pil

Fig. 2. **Functional split of the Italian public expenses**
Source: dipartimento della ragionieria generale dello stato

Furthermore, from the '70s, early retirement became a common practice.

The figure 3 reports the evolution of the effective age of retirement. This, combined with the ageing population, implied that a higher amount of pension benefit would have to be paid for longer periods.

We should consider also the fact that the welfare system had been extended to the entire resident population. Has anticipated before, the public social security has been extended to all workers during the '50s. This situation was combined with an improper use of invalidity pensions.

In fact, this instrument was used to support and maintain workers in difficult situation. As one could figure out, this generated some perverse effects during the oil crises.

Finally, the ageing of the population was also an issue. During the extension phase of the Italian pension system, there has been a gradual shift from a capitalization system (funded) to a Pay-as-you-go system.

The adoption of a pay-as-you-go system and the observed ageing of the population had an influence on the increase of expenditures experienced in Italy after the second world war.
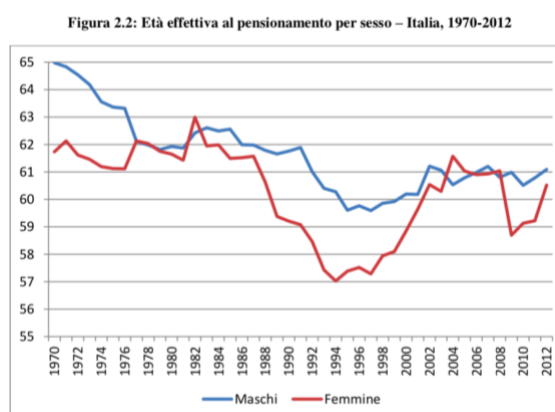

Figura 2.2: Età effettiva al pensionamento per sesso – Italia, 1970-2012

Fig. 3 **Italian effective retirement age** Source: Il Bilancio del Sistema Previdenziale Italiano. Rapporto No.3 2016

To understand this point, we introduce the concept of **elderly dependency ratio**.

It is a measure showing the ratio of the number of individuals over the age of 65 to the total population aged from 15 to 64. This indicator gives insight into the amount of people in retired age compared to the number of those in working age.

A high dependency ratio means individuals in working age, and the overall economy, face a greater burden in supporting the ageing population.
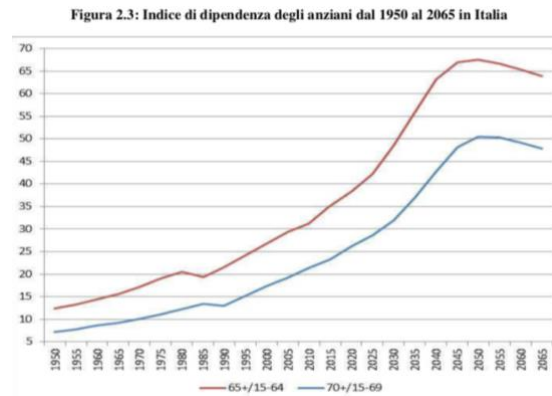
Figura 2.3: Indice di dipendenza degli anziani dal 1950 al 2065 in Italia

Fig. 4 **Italian Dependency Ratio** Source: Il Bilancio del Sistema
Previdenziale Italiano. Rapporto No.3 2016

From this figure, we can see the increase that occurred in the post war period. The trend has always been positive. The situation should be interpreted as the result of a mix of extraordinary progress in longevity and an intense drop of births.

If we take as reference point the year 1965 where the ratio is equal to15 percent (7 individuals in working age against 1 retired), in only 15 years, the ratio experienced about a 40 percent growth meaning that in 1980 there were only 5 individuals in working age for each retired individual. This, unless corrective measures are taken, will be reflected either as an increase of the burden for individuals in working age and/or an increase of public expenditures.

In front of the evolution of pension expenses and the threat to the sustainability of the Italian public finance, several reforms had to be implemented.

In fact, a mix of intervention aimed at reducing the amount of social security expenses and at increasing the volume of introit was needed.

The reduction of social security expenses could be achieved through a reduction of the amount of pension benefit and the length of the period the pension will be provided, that is, delaying the age of retirement.

Increase the volume of introit could be achieved through a scale up of the contribution rate.

The first of these reforms is the *Riforma Amato* in 1992 (*Decreto lgs. 503 del 1992*). The main aim of this reform was to **contain the expansion of the pension expenses**.

In particular, the reform wanted to **stabilize the incidence of the pension expenditures on the Gross Domestic Product**.

The reforms introduced, among other provisions, a gradual increase of the retirement age for old-age pension (from 60 to 65 for male and 55 to 60 for female), the progressive extension of

the period used to determine the pension base (from last 5 working years to last 10 years for those with more than 15 years of contributions).

Furthermore, with this reform, the mechanism of revaluation of salary indexed to inflation had been definitively removed.

The result was in line with the expectation of the legislator. It emerged a huge decrease of the pension benefit received compared to the salary.

It is crucial to recall that, up to this intervention, the supplementary social security was only part of the banking and private firms' activities and was limited exclusively to their employees.

The low level of benefit resulting from the reform has generated a greater interest toward supplementary forms of social security.

The only compulsory public benefit was no longer sufficient to guarantee the maintenance of the standard of living.

Therefore, it was necessary to establish rules for regulating supplementary pension provision.

Specifically, pension funds have been set up for collective and open membership.

II.3 Pillars of the Italian Pension System

In light of the above, the Italian pension system, as a result of the multiple reforms that have taken place over the years, is based on three fundamental pillars:

1. The first pillar consists of **compulsory public insurance**, financed by workers and employers throughout the working life. It is the traditional system resulting from the inclusion process. It has been profoundly modified with the reforms of the 1990s. These changes have meant that, for the new pensions, the relationship between the benefit and the last salary received was lower than that of the individuals already retired This is the main reason why a second pillar was added to compulsory social security;

2. The second pillar (complementary pension) is implemented through pension funds to which **workers join collectively and contribute by allocating their own severance pay (*TFR*)**. Pension funds are managed according to the system of capitalization and the contributions collected are invested in order to generate an amount to be converted

into annuity at the time of retirement, through management that no longer passes through the State but through managers specifically selected by the funds.

3. The third pillar includes individual supplementary pensions, **which anyone can make, at his discretion**, by means of individual savings schemes, with the aim of integrating both public and collective pension provision, to keep their standard of living unchanged once ceased work

The social security form of the second pillar is the most important for our purposes.

Adhering to a complementary pension scheme means regularly setting aside a portion of your savings during your working life to get a pension that is added to that received from compulsory pension provision. The management of the amount set aside is the responsibility of pension funds (seen as an entity).

In the next chapter, we will analyze the various forms of pension funds provided in the Italian regulations and their functioning.

III.   PENSION FUND – ASSETS LIABILITIES MANAGEMENT

Pension funds collect wealth from workers, invest them in profitable activities and, at the moment of retirement, provide pension benefits to the participants.

According to the Italian regulations, **the amount of pension benefit should depend on the total amount paid, on the length of the contribution period and on the returns obtained from the financial investment.**

It is therefore a defined contribution scheme where the contribution is fixed at the beginning and the final benefit depends on the performance and financial management of the contributions paid.

This means that both parties know from the moment of registration what the amount of periodical contributions to the fund will be. It is not possible instead to predict the level of income that will be perceptible at the time of retirement, as it depends on the results of the management of the capital set aside.

Even if the final benefit is not predetermined in the case of defined contributions, the common practice is to provide a minimum guaranteed benefit, or, in any case, to protect the invested capital.

Exceptionally, the Italian regulation allows for defined benefit scheme. The latter scheme is in fact reserved only to self-employed workers and freelancers.

In this kind of scheme, the value of the final benefit is initially set. What varies is the contribution of the member. From the time of accession to the fund, the amount of the benefit to be obtained is established, while you will have a contribution that will be adjusted over time to achieve the goal set.

It is important to point out that, currently, it is not anymore possible to create and commercialize new defined benefit pension funds. Those already existing instead are allowed to function.

III.1 Types of pension funds

According to the materials available on the website of the Italian Supervisory Board for Pension Funds, *Covip* (*Commissione di vigilanza sui fondi pensione*)[1], **supplementary social security can have 4 different forms**: Contractual pension funds (*Fondi Pensione Negoziali*), Open pension funds (*Fondi Pensione Aperti*), Individual insurance plans (*Piani Individuali di Tipo Assicurativo -PIP*), Pre-existing pension funds (*Fondi Pensione Preesistenti*).

 a. Contractual pension funds are supplementary pension schemes set up as part of national or industrial collective bargaining (contractual and non-coercive origin).

They are designed for specific categories of workers such as employees of the private sector belonging to the same contractual category, to the same enterprise or group of companies or territory. It is also design for the public-sector employees, cooperative members, self-employed and free-lancers, even when organized by professional and territorial areas.

They are established based on collective agreements, including corporate agreements, signed by representatives of employers and workers. In the absence of a genuine agreement, they may be established through company internal regulations. For cooperatives, they are established based on agreements between worker members. For the other categories, there are agreements between self-employed workers, free-lancers promoted by their regional or national trade unions or associations.

Membership is voluntary and takes place on the basis of the collective agreements stipulated between the parties (representatives of workers and employers) who have set up the fund.

From an organizational point of view, the assembly is usually composed of representatives of workers and companies. As a rule, it appoints the members of the Board of Directors and the Board of Statutory Auditors, approves the Bylaws (including any subsequent amendments) and the financial statements, and resolves upon the possible dissolution of the pension fund.

---

[1] http://www.covip.it/wp-content/uploads/guidaintroduttivaallaprevidenzacomplementare.pdf

The Board of Directors is made up equally by representatives of workers and companies. The members must be in possession of the requisites of professionalism and integrity required by the law. The board administers the pension fund and decides its investment policy; select the manager, the depositary, the insurance company for the provision of the annuities; elect the Chairman, who also acts as legal representative of the pension fund itself and the General Manager with executive functions of the Board of Directors' addresses; appoints the Head of the fund.

The Board of Statutory Auditors is made up equally by representatives of workers and companies. The members must be in possession of the requisites of professionalism and integrity required by the law. The Board supervises compliance with the law, the Bylaws, compliance with the principles of correct administration, the adequacy of the organizational, administrative and accounting structure adopted by the fund and its actual operation. It also performs an accounting audit if it is not entrusted to an external auditor.

the person in charge of the pension fund appointed by the Board of Directors must be in possession of specific requisites of integrity and professionalism required by law. The position of Manager may also be conferred on the General Manager or one of the fund's directors. The Manager verifies that the management of the pension form is carried out in the exclusive interest of the members, in compliance with the regulations, also secondary, issued by the COVIP, and the contractual provisions;. He monitors the compliance with investment limits, the transactions in conflict of interest, the adoption of suitable operational practices to better protect members; carries out its activity autonomously and independently; provides for sending data and news on the fund's activity to the COVIP.

b. **Open pension funds:** they are complementary pension schemes set up by banks, insurance companies, asset management companies and securities brokerage firms to which all those who intend to form a supplementary pension can join regardless of the working situation (private or public sector employee, self-employed, free-lancer, other). Open pension funds can collect membership on an individual and collective basis.

They are constituted as separated and autonomous assets with respect to that of the parent company since they are exclusively intended for payment of benefits to members. Therefore, they cannot be used to repay the parent company's creditors.

Membership is voluntary and, as underlined, independent of the working condition.

From an organizational point of view, the person in charge of the open pension fund, appointed by the company, must be in possession of specific requisites of integrity and professionalism required by law. The Manager verifies that the management of the pension form is carried out in the exclusive interest of the members, in compliance with the regulations (including secondary legislation issued by the COVIP) and the provisions contained in the Regulations; monitors compliance with investment limits, transactions in conflict of interest, the adoption of suitable operational practices to better protect members. He carries out his activity autonomously and independently, reporting directly to the administrative body of the company regarding the results of the activity carried out; provides for sending data and news on the fund's activity to the COVIP.

The Supervisory Body is envisaged in the case of an open pension fund with collective membership. The Body represents the interests of the members and verifies that the administration and management of the fund take place in their exclusive interest.

Individual insurance plans: they are supplementary pension schemes set up by insurance companies. PIPs can only collect membership on an individual basis regardless of the job status.

They are also constituted as separated and autonomous assets compared to that of the company that establishes them.

PIPs are implemented through *ramo I e III* or mixed life insurance contracts. In the first case, the revaluation of the individual position is linked to one or more separate internal procedures, while in the second the revaluation is linked to the value of the units of one or more internal funds held by the insurance company.

Membership is voluntary, takes place on an individual basis and regardless of the job status.

From an organizational point of view, the Manager of the PIP, appointed by the insurance company, must be in possession of specific requisites of integrity and professionalism required by law. The Manager verifies that the management of the PIP is carried out in the exclusive interest of the members, in compliance with the regulations (even secondary ones issued by the COVIP); monitors compliance with investment limits, transactions in conflict of interest, the adoption of suitable operational practices to better protect members.

He carries out his activity autonomously and independently, reporting directly to the administrative body of the insurance company regarding the results of the activity carried out; provides for sending data and news on the activity of the PIP to the COVIP.

c.   Pre-existing pension funds: they are complementary pension forms so called because they were already established before the *Decreto legislativo No. 124 del 1993* which introduced for the first time an organic discipline of the sector.

The *Decreto Legislativo 252/2005* (replacing the *Decreto Legislativo 124/1993*) set new rules for the supplementary social security system, also envisaging a gradual adjustment to the new regulation of pre-existing pension funds.

They are divided into pre-existing internal funds and autonomous pension funds. The latter are endowed with juridical subjectivity. The former are within a company as a separate asset.

Membership of a pre-existing pension fund is usually voluntary and takes place through collective membership. Each pre-existing pension fund is aimed at certain categories of workers (of a given company or group of companies or of specific professional categories, for example managers of a particular company, etc.).

From an organizational point of view, pre-existing pension funds are equal to those of contractual pension funds.

III.2 Asset - liabilities management in the pension fund industry

Pension funds, from a purely technical point of view, are the tools that allow workers to receive a complementary pension, to be added to what would be provided by the compulsory social security institutions.

Through a pension fund the worker then sets aside a portion of his earnings earned during his working life in order to guarantee additional pension benefits.

The resources collected by the pension fund are invested in the financial markets in order to produce a return that is added to the periodical contribution paid in individual positions and will be used to pay pension benefit at the moment of retirement of the member.

In this regard, Ian Tonks (in *Pension Fund Management and Investment Performance*, 2006) points out that during the collection phase the fund tends to grow mainly for 2 reasons: the contributions of the members and the return generated by the assets of the fund.

He continues by illustrating how minimum changes in asset returns can have substantial impacts on the value of the fund and, consequently, on payable pensions.

The returns of the pension fund will therefore vary over the years introducing a risk component on the amount of benefits.

Since the principal and investment returns on assets are used to satisfy future liabilities, an integrated portfolio management and a joint evaluation of risks and benefits for assets and liabilities are therefore crucial. It could allow to take into consideration the risk and return of invested income, as well as the corresponding pension liabilities. This leads us the concept of Asset & Liabilities management (ALM).

Investopedia defines the ALM as "the process of managing the use of assets and cash flows to meet a company's obligations in order to reduce the firm's risk of loss from not paying a liability on time". It is typically used for banks' loan portfolios and pension plans[2].

It is a mechanism to address the risk faced by a financial institution due to a mismatch between assets and liabilities due to liquidity (institution's ability to meet its liabilities either by borrowing or converting assets).

Banks in particular, apart from liquidity, may also have a mismatch due to changes in interest rates as banks typically tend to borrow short term and lend long term.

The concept of ALM focuses therefore on the timing of cash flows because company managers need to know when liabilities must be paid.

---

[2]https://www.investopedia.com/terms/a/asset-liabilitymanagement.asp#ixzz5VJo3XSB4 (access 29 November 2018)

It involves the management of assets in such a way to earn an adequate return while maintaining a comfortable surplus of assets over existing and future liabilities (*Gulpinar and Pachamanova, 2013*).

In other words, ALM is concerned with solvency (availability of assets to pay the liabilities as they come due). The solvency is usually measured through the Funding Ratio, the ratio of assets over liabilities.

Basically, in the absence of ALM, a Pension fund manager could decide to operate either on the return maximization principle or the surplus maximization principle.

In this regard, according to Sharpe and Tint (1990), pension fund managers had an all-or-nothing approach to consideration of liabilities in the sense that under the return maximization principle, a fund manager focuses only on the asset side of the balance sheet and, therefore, care about finding the optimal asset allocation strategy. Under the surplus maximization instead, he had to include all the liabilities and focus on maximizing the surplus for the fund's investors.

ALM could allow for partial consideration of liabilities. Managers could decide which liabilities will be included and hedged in the decision process and which will be excluded. In this way, managers could adopt a middle way approach between asset only and full surplus optimization.

ALM has increasingly become over the years the standard for pension management.

In an article by IPE[3] from 2011, we could read that almost two-thirds of pension plans around the world are using a liability-driven investment (LDI) approach. At this number, we must add the other users of this instrument. We can think about Banks and insurance companies for instance.

III.3 Approaches to ALM in Pension Fund Industry

The traditional and most conservative approach to ALM is the **cash flow matching**, where assets are invested in fixed income securities for which the coupon and principal payments

---

[3] https://www.ipe.com/two-thirds-of-pension-funds-now-use-ldi-finds-sei-strategy/www.ipe.com/two-thirds-of-pension-funds-now-use-ldi-finds-sei-strategy/43358.fullarticle (access 29 November 2018)

match as closely as possible the liabilities both in terms of timing and magnitude, thereby eliminating most if not all risk.

In larger liability portfolios where liability cash flow matching is difficult, traditional ALM matches the risk profile, specifically interest rate risk and liquidity risk, of the liabilities.

Technically, there are many criteria to classify the various approaches to ALM. We are going to present some of them.

For instance, Van der Meer R. and Smink M. (in *strategies and Techniques for Asset-Liability Management, 1993*) suggest 3 distinct groups of ALM techniques and strategies: **Static, dynamic Value driven and dynamic return driven**. According to this criteria, the example above was a clear case a static approach.

Static models do not make optimal use of the opportunity to react to future short term circumstances. They do not allow, as suggested by the name, for recursive decisions. Therefore, decisions would not reflect a correct tradeoff between short term effects and longer term effects.

The only advantage linked to static approach is the low level of computational effort.

Static models include cash flow matching, Gap analysis, segmentation and cash flow payment calendars. They can be extended to multi-scenario analysis.

Dynamic models instead can be employed to compute policies that consist of actions to be taken now, and sequences of reactions to future developments.

They have the privilege of providing more optimal results.

According to Platankis E. and Sutcliffe C. (*in Asset Liability Modelling and Pension Schemes: The Application of Robust Optimization to USS*), there are a variety of techniques for deriving such optimal ALM strategies for pension funds. They fall into four categories: **Stochastic Programming, Dynamic Programming, Portfolio Theory and Stochastic simulation** with stochastic programming being the most popular technique.

Stochastic programming can be used for supporting decision making under uncertainty, **while considering the probability distributions of uncertain parameters**.

It usually focuses on finding optimal investment rules over a set of scenarios for the future returns on the assets and the liabilities of the company.

The main drawback of such a method is, According to Gulpinar and Pachamanova, the difficulty to apply them in practice. The reasons are many as they suggested.

First, ALM is a multi-period problem, and the number of scenarios needed to represent reality satisfactorily increases exponentially with the number of time periods under consideration. Thus, the dimension of the optimization problem, and correspondingly its computational difficulty, increases.

Second, the scenario generation itself requires sophisticated statistical techniques, which is a deterrent to practitioners who need to make decisions in a short amount of time.

Finally, often little is known about the specific distributions of future uncertainties in the ALM problem, and little data are available for estimating the probability distributions of these uncertainties.

For this reason, the starting point of our work will be the Portfolio Theory.

Modern Portfolio Theory started with MARKOWITZ' famous article (1952). The Markowitz approach is based on the creation of a set of efficient portfolios which can be represented as combinations of two reference portfolios ("two funds separation").

From this model has been derived the Capital Asset Pricing Model (Sharpe and Lintner).

In the Markowitz portfolio selection model, the "expected return" on a portfolio is measured by the mean of the random portfolio return, and the associated "risk" is quantified by the variance of the portfolio return.

Markowitz showed that, given either a maximum level of risk that the investor is willing to take or a minimum level of return the investor is willing to accept, the optimal portfolio can be obtained by solving a convex quadratic programming problem.

His approach therefore gives, for a given desired level of risk, the portfolio with the higher expected return. Conversely, at a given level of desired expected return, the Markowitz approach gives the portfolio with the lowest risk. The latter portfolio is to be intended as a combination of n Assets $S_1...., S_n$. Each one of these assets enters in the portfolio with a percentage weight $x_1,....,x_n$ such that the sum of these weights equals one.

If the expected return of the ith asset is given μ_i, the portfolio's return, μ_p, is given by the weighted sum of each security's expected return:

$$\mu_p = \sum_{i=1}^{n} x_i \, \mu_i$$

The risk measure of each single asset is given by the variance of that asset. The variance of the portfolio instead is given by:

$$\sigma^2 = \sum_{i=1}^{n} x_i^2 \, \sigma_i^2 + \sum_{i=1}^{n} \sum_{j<1} x_i \, x_j \, \mathrm{Cov}(i,j)$$

It is assumed in this model that the random returns of securities are normally distributed.

Furthermore, it is assumed that investors prefer greater return and less risk. Any portfolio can be represented in the σ-μ plane.

The optimization problem takes the following form:

$$\text{Minimize } \tfrac{1}{2}\mathbf{x}'\mathbf{V}\,\mathbf{x} = \tfrac{1}{2}\mathrm{Var}(P)$$

$$\text{Such that: } \mathbf{x}'\mu = \mu_p = E(P)$$

$$\mathbf{x}'\mathbf{1} = 1$$

where:

- $\mathbf{x} = [x_1, x_2\ldots., x_n]$' is a column vector of portfolio weights for each security
- $\boldsymbol{\mu} = [\mu_1,..., \mu_n]$, the vector of expected return
- $\mathbf{V}$ is the covariance matrix of returns
- $\mathbf{1} = [1, 1,\ldots,1]$'
- and μ_P is the desired level of expected return for the portfolio.

From the optimal solution of the minimization problem, one can determine the entire set of efficient portfolios by plugging the optimal solution (function of μ_P) in the expression for the portfolio variance.

The result is the so called **Efficient Frontier** i.e. the set of optimal portfolios that offers the highest expected return for a defined level of risk or the lowest risk for a given level of expected return. A rational should investor should therefore choose a portfolio that lays on the Efficient Frontier.

The extension of the Markowitz model to asset and liability management has been proposed for the first time by Sharpe and Tint (1990).

They put the ALM problem in a close relationship to Markowitz' mean variance portfolio theory providing a procedure that allows the proper incorporation of coexisting asset or liabilities into the assets mix considerations for the portfolio allocation.

Their model starts with the definition of the Surplus – the value of assets less the value of liabilities. This value is known in the present but not in the future. a pension fund is therefore concerned with the future value of the surplus.

$$S_1 = A_1 - kL_1$$

where S_1 is the value of the surplus in the next period, A_1 the value asset in the next period, L_1 the Value of liabilities. k stands for an indicator variable taking 0 in case on asset only optimization or 1 in case of full surplus optimization (remember *all or nothing approach*).

By expressing the surplus in relation to the current value of assets and multiplying and dividing the second part of the previous expression by L_0 (current liabilities), we obtain:

$$\frac{A_1}{A_0} - k\frac{L_0}{A_0}\frac{L_1}{L_0}$$

Which can be rewritten

$$1 + R_A - k\frac{L_0}{A_0}(1 + R_L) \text{ or } \left[1 - k\frac{L_0}{A_0}\right] + \left[R_A - k\frac{L_0}{A_0}R_L\right]$$

With R_A rate of return of assets and R_L rate of growth of liabilities.

The first bracketed expression does not involve uncertainty and, therefore, is not relevant for asset allocation decisions. Their model focuses on the second element.

This element shows that the incidence of liability returns is directly related to the magnitude of the liabilities vis-à-vis the asset value suggesting that funds with better funding ratio (lower $\frac{L_0}{A_0}$) should have smaller effect of liabilities on asset allocation. They also suggest that only a fund without surplus should simply subtract the liability return from the asset return.

The maximization take the following form[4]:

---

[4] Making abstraction from constraints.

$$\text{Max U}\left[\text{Expected (Z)} - \frac{\text{Variance(Z)}}{t}\right]$$

Where

t is the investor's risk tolerance coefficient (inverse of the risk aversion coefficient);

$$Z = R_A - k\frac{L_0}{A_0}R_L$$

$$\text{Expected (Z)} = \text{Expected}(R_A) - k\frac{L_0}{A_0}\text{Expected } R_L;$$

$$\text{Variance(Z)} = \text{Variance }(R_A) - 2k\frac{L_0}{A_0}\text{Covariance}(R_A, R_L) + \frac{k^2 L_0^2}{A_0^2}\text{Variance }(R_1);$$

The last term includes only constants and the second term in the expression of Expected (Z) is not affected by asset allocation.

The problem becomes:

$$\text{Max U}\left[\text{Expected}(R_A) - \frac{\text{Variance}(R_A)}{t} + 2\frac{k}{t}\frac{L_0}{A_0}\text{ Covariance }(R_A, R_L)\right]$$

the first two terms are those used in the traditional asset only optimization (Risk-adjusted expected return) while the last term have been called by the authors **Liability Hedging Credit.**

If a portfolio has a Liability Hedging Credit equal to 3% for instance, an investor is indifferent between this asset mix and another with 3% more expected return but with no ability to serve as a hedge against fluctuations in liabilities values.

It depends on the tolerance to risk (the higher the tolerance, the smaller the LHC), the relative value of liabilities with respect to assets and the covariance between the asset mix and the liabilities. If the latter is equal to zero, it means that the assets provide no hedging against liability increases. If instead the covariance is positive, asset returns tend to be high when liability growth increases providing some hedging. Finally, if the covariance is negative, there will be an exacerbation of adverse movement of liabilities. If they increase, the return on the asset mix will decrease not mitigating the adverse movement.

In the previous equations, the expected return of asset has the same definition as in the Markowitz model, i.e. it is equal to the weighted sum of the assets returns.

Throughout the time, the model by Sharpe & Tint has had many followers, but was also challenged by skeptics at academic and financial institutions alike.

In fact, being a model based on the Markowitz framework, it is concerned by the same criticisms addressed to the Markowitz approach[5].

There are several reasons for the lack of acceptance of MPT among practitioners.

The most significant is the argument that "optimal" portfolios obtained through the mean-variance approach are often "counterintuitive", "inexplicable" and "overly sensitivity input the parameters".

The real world is made of situations that make the sole historical mean and variance of returns unable to give consistent results.

Michaud (1998) for instance argues that mean-variance optimization overweighs those assets with a large estimated return to estimated variance ratio (under weighs those with a low ratio) and that these are precisely the assets likely to have large estimation errors (error maximization). He quoted: "*Although Markowitz efficiency is a convenient and useful theoretical framework for portfolio optimality, in practice it is an error-prone procedure that often results in error maximized and investment-irrelevant portfolios*".

Some correction attempts have been made to create better and more stable mean-variance optimal portfolios by utilizing expected return estimators that have a better behavior when used in the context of the mean-variance framework.

A common technique, according to Ceria S. and Stubbs R. (2006)[6], is the use of the James-stein estimators that shrink the expected returns towards the average expected return based on the volatility of the asset and the distance of its expected return from the average.

Black and Litterman (1990) instead have developed an approach for producing stable expected return estimates that combines equilibrium expected returns and investor's views on specific assets or weighted groups of assets.

In their model, one should compute the implied market returns derived from the CAPM. Then he can either use the equilibrium returns so obtained or adjust them in order to take into

---

[5] Cornelius Ludovicus Dert (1995) for instance suggest that ALM can be seen as an investment decision process that has to fit a given set of liabilities. "*It is a variant of mean-variance investment problem where the return is equal the return from the investment less the growth of liabilities. The variance instead has the consider also the covariance between asset and liabilities items*". We are therefore comfortable to extrapolate the criticism of the Markowitz model to the Sharpe and Tint model.

[6] In *Incorporating estimation errors into portfolio selection: Robust Portfolio construction.*

consideration his view (either express in relative or absolute terms) and the associated confidence level.

The expected return, E(R), is given by

$$E(R) = [(\tau\Sigma)^{-1} + P^T\Omega]^{-1}[\ (\tau\Sigma)^{-1}\Pi + P^T\Omega Q]$$

Where

- τ Scalar number indicating the uncertainty of the CAPM distribution;
- P Matrix with investors views. Each row is equivalent to a specific view of the market and each entry of the row represents the weights of each assets;
- Q Expected returns of the portfolios from the views described in matrix P;
- Ω Diagonal covariance matrix with entries of the uncertainties within each view. It embodies the confidence level component;
- Σ Covariance matrix of assets;
- Π Implied returns from the CAPM.

With the computed returns, we can compute the covariance matrix and implement the asset allocation.

But, since they are using estimates, there will always be room for estimation errors (even though they are sensibly lower with respect to the standard Markowitz approach).

Another development that has received much attention is the portfolio resampling methodology of Michaud (1999).

He introduced a statistical resampling technique that indirectly considers estimation error by averaging the individual optimal portfolios that result from optimizing with respect to many randomly generated expected-return and risk estimates.

Ben-Tal and Nemirovski from their part (1998, 1999) proposed the introduction of Robust Optimization which considers **uncertainty in unknown parameters directly and explicitly in the optimization problem**. The perturbations in the market parameters are modeled as unknown, but bounded.

This approach is generally concerned with ensuring that decisions are "adequate" even if estimates of the input parameters are incorrect.

It is acknowledged that robust optimization can be used to address the same type of problems as dynamic programming and stochastic programming do, but, it takes a worst-case approach

to optimization formulations in the sense that, among all possible values of the uncertain parameters, one is to choose the worst one while conducting his analysis.

At this regard, Gulpinar and Pachamanova stressed that this is not as restrictive as it sounds at first.

The robust optimization approach solves an optimization problem assuming that the uncertain input data belong to an uncertainty set, and finds the optimal solution if the uncertainties take their worst-case values within that uncertainty set. One is free to allow for different level of "conservatism" by varying the width of the bounded interval for unknown parameters.

Given the fact that ALM is concerned with ensuring a level of minimum guaranteed performance to meet future liabilities, robust-optimization-based strategies that place special emphasis on the worst-case realizations of uncertainties seem to be particularly appealing in the ALM context.

The next pages will focus on the modeling, estimation and presentation of results

IV. PROBLEM SET UP AND ISSUES

In light of the above, our work will consist in attempting to implement an integrated asset allocation considering both the asset and liability side items of a balance-sheet in such a way that all our constraints are satisfied.

One issue in making the aforementioned integrated allocation is the estimation of the covariance between assets and liabilities returns.

In fact, we are concerned that, by using historical data, the resulting estimated covariance could be biased. If a given pension fund has always used ALM as a standard policy, it could be that the historical structure of the asset is a result of adaptation to fluctuation of liabilities.

Hence, we could have high non-random correlation between assets returns and liabilities returns.

For this reason, we feel more comfortable if we treat assets and liabilities separately by making projection of asset returns on one side and of liabilities on the other side. We will then either include a non-inferiority constraint.

At this regard, we deem useful to take a step back and consider the concept of ALM.

In practice, there are various cases of dealing with risk in pension schemes. These cases are different among them (mainly due to the particular risk considered) and require different approaches and methodologies each of which can be seen as a ALM process.

We have spotted 3 different situations relevant to our purposes:

- **Superannuation Funds** (*Casse Previdenza*): Under the Italian legislation, they are part of the compulsory pension scheme for freelancers. They are entitled to carry the two sides of the scheme's process i.e. the accumulation/investing side and the pension benefits payment. in this case, the ALM will consist in investing the contributions in order to meet the future obligations of paying the benefits for the residual life of the retired worker. They deal with both the financial risk and the demographic risk since the residual life of the worker is an uncertain variable;

- **Pension Funds** (voluntary): Unlike the superannuation funds, pension funds carry only the accumulation/investing phase of the process. The benefit payment activity is

delegated to an insurance company. In this particular case, the ALM activity is an asset-only process aimed at finding the optimal asset allocation in the risk-return framework;

- **Insurance Companies**: It is the opposite side of the coin of the previous point. These company manage the payment of the benefits from the moment of retirement of the worker to the end of his life carrying in this way a demographic risk consisting in the probability that the retired worker will receive the benefits for period of time longer than forecasted.

In light of the above, and given the fact that we don't have adequate actuarial instruments for dealing with the demographic risk, our work will consist in finding the optimal asset allocation decisions of an investment manager of a non-mature defined benefit pension fund under an assets and liabilities management prospective. Over a 45 years' time horizon and in compliance with the relevant regulatory constraints, we have to implement an ALM in order to: maximize (reach the pre-defined) retirement benefit, minimize the periodical contributions.

The time horizon is to be split into an initial 25 years period representing the accumulation/investing phase and a residual 20 years period for the payment of benefits. This because we implicitly assumed that the average member of the fund is aged 45 years, will retire at 70 and will have a residual life of 20 years.

The payment of the benefit is delegated to an outside firm. Therefore intermediate money outflows are ignored[7]

IV.1 projection of assets.

There are various tools and method for achieving the projection of the items on the asset-side of the balance sheet.

The projection typically consists on the estimation and forecast of the future assets return.

The common practice is to forecast returns through stochastic processes and simulations.

---

[7] In the model by Sharpe and Tint, this is equivalent to assume k=1 and no initial surplus.

Ruud Kleynen for instance followed a Vector Autoregressive approach in "Asset liability Management for Pension Funds, A case study". The same methodology has been used by Binsbergen and Brant (2007).

Others widespread approach are the Bootstrap and Montecarlo simulations.

The implementation of the simulations is quite straightforward. What is more critical instead is the setting-up of an adequate portfolio whose future returns are to be forecasted.

As already introduced, Markowitz has proposed for first (1952) a mathematical model for deriving optimal portfolio.

However, despite being the most important contribution to modern portfolio theory, Markowitz approach have many drawbacks and assumptions that may not properly represent reality, as we have already introduced.

The general criticism addressed to Markowitz is about the input parameters used in the model. Using the sample mean and variance as measure of expected return and risk is not as optimal as it seems. We can recall Michaud's error maximization as an example critic (1999). According to him, *the important contributor to the error-maximizing character of Markowitz optimization when using historical data is that the usual estimation procedure-which replaces the expected returns with their sample means-is not optimal.* Alternatives have been discussed in the chapter 3.

For our purposes, we will test several remedies proposed in the literature and retain the one that fit better in our problem.

IV.2 Projection of liabilities

The forecast of the path of liabilities is the task of the actuary.

Many studies suggest considering the present actuarial value of liabilities in each year to construct their evolution over time.

Liabilities are mainly made of future benefit payments. Being a defined benefit pension plan, the way in which the level of those payments will be computed is already known. The actual level instead is uncertain. It is subject to the development of the characteristics of the participants.

These characteristics are determined by future career paths, life, death[8]. It depends also on inflation in cases where the benefits level is indexed to inflation.

Therefore, given the characteristics of the current participants in the fund, the expected rate of inflation, the demographic specificities and the expected development of the characteristics, we can estimate the amount of the expected future benefit in each period.

But as we have introduced before, we do not have the adequate actuarial tools for achieving the estimation of these variables. We will instead provide assumptions on the way they are set to find their values.

IV.3 Regulatory constraints

There are various sources of regulations concerning the pension fund industry.

The most important is the **EU DIRECTIVE 2016/2341** of the European Parliament and of the Council of 14 December 2016 on the activities and supervision of Institutions for Occupational Retirement Provision (IORPs). It is an upgrade of the Directive 2003/41/CE on supervision on IORPs.

The directive provides general rules on the operation of IORPs (authorization, activities, operating requirements etc.)., we will only focus however on quantitative requirements.

It is important to stress that **the regulation does not give precise and exact (numerical) rules**.

It relies heavily on words like **"adequacy", "prudence" "sufficiency"** instead of numerical values or benchmarks.

Nevertheless, regarding the investment rules, the directive provides the following rules (article 19, 1):

- the assets shall **be invested in the best long-term interests of members and beneficiaries**. In the case of a potential conflict of interest, an IORP, or the entity which manages its portfolio, shall ensure that the investment is made in the sole interest of members and beneficiaries;

---

[8] By Cornelius Ludovicus Dert (1995)

- the assets shall **be invested in such a manner as to ensure the security, quality, liquidity and profitability of the portfolio** as a whole;

- investment in **derivative** instruments **shall be possible insofar as such instruments contribute to a reduction in investment risks or facilitate efficient portfolio management…**. IORPs shall also avoid excessive risk exposure to a single counterparty and to other derivative operations;

- the assets shall be properly **diversified in such a way as to avoid excessive reliance on any particular asset, issuer or group of undertakings and accumulations of risk in the portfolio** as a whole.

Under the point 3 of article 19, it is made **prohibited to borrow or act as a guarantor on behalf of third parties**. However, it is left to Member States the freedom to authorize IORPs to carry out some borrowing but only for liquidity purposes and on a temporary basis.

As introduced before, the European regulation does not provide precise values.

We find more detailed rules in the Italian *Decreto legislativo n. 252 del 5 dicembre 2005 (Disciplina delle Forme Pensionistiche Complementari)*.

Among other rules, the point 2 of article 11 set that **the right to retirement benefits can be acquired only when the requirements set under the compulsory scheme to which the individual belongs are met,** with at least five years of participation in the supplementary pension schemes.

The point 5-bis of article 6 instead states that **the competent authorities should identify the assets in which pension funds are allowed to invest** considering the pursuit of the interest of the members, **possibly setting maximum investment limits** if they are justified from a prudential point of view.

Point 13 of the same article **forbids to take or grant loans and to provide guarantees in favor of third parties.**

The *Decreto Ministeriale n.166* of september, the 2nd 2015, which enacts the point 5-bis of article 6 of *Decreto n. 252 del 5 dicembre 2005* provides more interesting rules on investments. What we must keep in mind from this Decree is:

- Pension funds must pursue the interests of members and beneficiaries and comply with the following criteria: (art 3)

- o **Optimization of the income-risk combination of the portfolio as a whole**, by choosing the best instruments in relation to quality, liquidity, yield and risk level, in line with the investment policy adopted;
  - o Adequate diversification of the portfolio aimed at limiting the concentration of risk and the dependence of the result of the management on certain issuers, groups of businesses, sectors of activity and geographical areas;
  - o efficient management aimed at optimizing results, containing transaction, management and operating costs in relation to the size and complexity and characteristics of the portfolio.
- Regarding the prohibition of taking and conceding loans and guarantees, pension funds are allowed, under some circumstances[9], to enter in repurchase agreement and loans of securities for the purposes of the efficient management of the portfolio. Furthermore, they are allowed to used derivatives exclusively for the purpose of reducing the investment risk or efficient management of the portfolio (art 4, 2 a-c).

Finally, the *Decreto n.259 del 7 dicembre 2017* of the *Minestero dell'Economia e delle Finanze* which enacts the article 7-bis point 2 of the d*ecreto legislativo 5 dicembre 2005, n. 252* provides useful rules for our purposes.

On point 3,*i* of its article 4, it is stated that the interest rate to be used to compute the reserve cannot be higher than the interest rate used on the prevision of the medium long term public debt.

Some of these provisions are not relevant for our purposes and concern the tactical asset allocation (e.g. diversification constraints). The remaining provisions will be translated in constraints to be included in our forecasting and optimization problems.

The constraints will mainly concern risk contributions, short-sale constraints and maximum bound on weights and choice of input of the model.

---

[9] They are allowed if realized within a standardized system organized by a recognized compensation and guarantee body or if concluded with reliable, solid and reputed counterparts subject to supervision of public authority.

V.   ASSUMPTIONS & DATA

V.1 Formalization of the Problem & Main Assumptions

Our problem covers 45 years' time. The time horizon will be split into an initial 25 years of accumulation/investing and 20 years of payment.

The Accumulation phase is mainly a financial problem. It will consist on finding the best combination of returns (Portfolio Allocation), in order to satisfy the constraints we have set.

The payment phase instead can be seen as an issue concerning insurance company. It cares about being able to meet the commitment toward the investor/retired and avoid situation of insolvency and illiquidity.

Formally, the work will consist in finding, under various hypothesis, the best combinations of assets (weights in the portfolio), based on their expected return. The word Best is to be intended as "giving the result closest to our goals while satisfying our constraints". As we will see later on, finding the optimality is all about dealing with trade-off between reaching the desired value and satisfying the constraints,

We will adopt the following notation:

- **K_i** Asset Classes
- **W** Optimal Weights Vector
- **R_t** Monthly assets' returns (t for time period)
- **C_t** Monthly contributions to the fund
- **P_t** Monthly Pension Payment received from the fund
- **A_t** Aggregate pension value at time t

At any given period of time **during the accumulation phase**, the aggregate pension value is given by its value on the previous period, increased (decreased) by the positive (negative) return from financial investment and by the monthly contribution of the participant. Formally, we have:

$$A_t = A_{t-1}(1 + R_t'w) + C_t \quad \text{with } A_0 = 0$$

$$\text{For } t = 1 \dots 300$$

During the **payment phase** instead, the aggregate pension value is given by:

$$A_t = A_{t-1}(1 + R'_t W) - P_t$$
$$\text{for } t = 301 \dots 540$$

That is, in each period of time, the residual value of the fund is equal to the value one period before, increased by the return obtained from investment and corrected by the contemporaneous pension payed to the participant. The said participant is one average/representative individual with several characteristic that we will expose latter on.

The contributions are assumed to follow the process:

$$C_t = C_{t-1}(1 + \pi_1)$$
$$\text{for } t = 1 \dots 300$$
$$C_0 = \rho \check{S}_0$$

$-$ Where π_1 captures increases of contributions linked to inflation and career development.

It is assumed to be $\pi_1 = \pi_0 + \delta$

π_0 Stands for the monthly inflation rate

δ captures the carrer development (initially set to zero)

$- \check{S}_0$ is the monthly wage at $t = 0$ of our considered average member

$- \rho$ Is the share of salary brought as provision to the fund

The monthly pension payment is defined as:

$$P_t = P_{t-1}(1 + \pi_0)$$
$$\text{For } t = 302 \dots 540$$
$$P_{301} = \gamma \check{S}_{300}$$

We initially assume ρ=10%, γ= 40%, δ=0, π_0=2%/12 $\check{S}_1$=1800. We will then see latter on how the result are sensitive to changes of these values.

We set the rate of contribution to the fund based on the data used in practice. In fact, the contribution to a complementary pension scheme by a worker is equal to the severance pay (TFR[10]) increased by 1 to 2.05% of the salary paid by the employer and 0.5 to 1% paid by the

---

[10] In Italy, the amount of the severance indemnity is determined by allocating for each year of work a portion equal to 6.91% of the gross remuneration, adding for each year of service (or

worker. These values (excepted the TFR) vary based on various factors such as policy of the company managing the fund, the age of the worker, the year the worker started to work, the industrial sector etc.

For the rate of conversion wage to pension, we didn't find a reliable value to be used as proxy. While searching for this value, we noticed that the value is not set in an exogenous way by pension fund. This means that there is not a predetermined value to be used in all the cases (for all the participants) but it is tailored to fit each particular case.

Generali Italia S.p.A, for instance, makes projections on the future paths of the wage of the participant, estimates the last period salary and the corresponding pension amount receivable from the public pension system and computes the **Pension Gap** which is the difference between the last wage and the pension from the public mandatory system. The amount of benefit is then based on this pension gap, which is in line with the integrative feature of the pension funds.

Being our case based on the complementary pension, we thought it would not be wrong if use the aforementioned procedure as a proxy. In particular, using the data from our average participant, we found that the ratio between the pension gap and the last salary, assuming a moderate-low career development, can be fairly approximated at 40%.

The remaining part of the work will focus on analyzing optimal portfolio choices adopting both a static Buy-and-hold strategy and a dynamic rebalancing strategy. Formally, the work will search for optimal vector **W** under different assumptions.

V.2 Data Used

The investment universe we adopted is made of 4 asset classes:

- US Equity

---

part of a year) a portion equal to the amount of remuneration due for the year itself, divided by 13.5 (in the case of fraction of a year the share is proportionately reduced, counting fractions of a month equal to or greater than 15 days as a whole month). The quotas thus set aside are revalued annually, on a compound basis, with the application of a rate consisting of 1.5% in a fixed measure and 75% of the increase in the consumer price index calculated by the ISTAT with respect to the last year. ***Source: Temi.camera.it (05/06/2019)***

- US Long Term Government Bonds
- US Short Term Government Bonds
- US Investment Grade Corporate Bonds

These asset classes are represented by indexes rather than single securities. They are completely taken from the US market.

The decision of using US data rather than Italian or European data is justified by pure convenience linked to availability of data and length of time series.

- Equity

The equity asset class is represented by the Standard & Poor 500 index. It is a market-capitalization-weighted index of the 500 largest U.S. publicly traded companies. The index is widely regarded as the best gauge of large-cap U.S. equities.

The companies included in the S&P 500 are selected so they are representative of the industries in the United States economy.

The index includes also non-U.S. companies, both formerly U.S.-incorporated companies that have re-incorporated outside the United States, as well as firms that have never been incorporated in the United States.
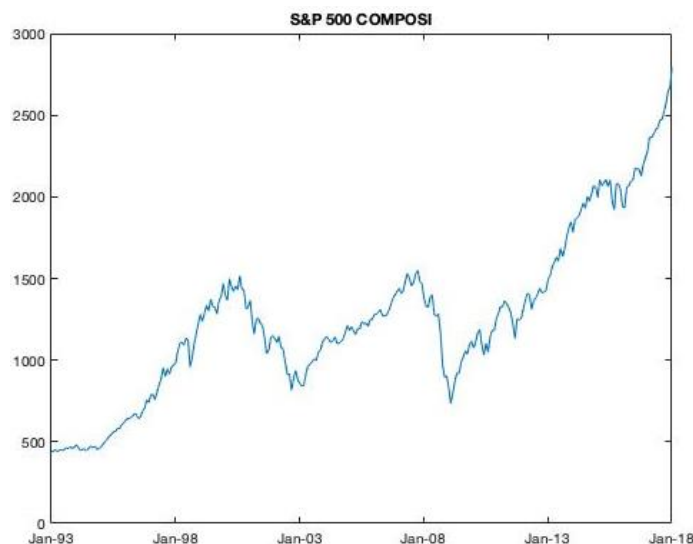
Fig. 6. S&P 500 Composite Price Index. Source: Own Elaboration.
EIKON Data

The initial part of the graph shows an uptrend with some fluctuations up to the 2000's. in fact, on March 2000, the index reached a high around 1,550, at the peak of the dot-com bubble; a

high not to be exceeded for the following seven years. On May 30, 2007, the S&P 500 closed at around 1,530, to set its first all-time closing high in more than seven years.

In mid-2007, the subprime mortgage crisis spread to the wider U.S. financial sector. The resulting situation became acute in September 2008, ushering in a period of unusual market volatility.
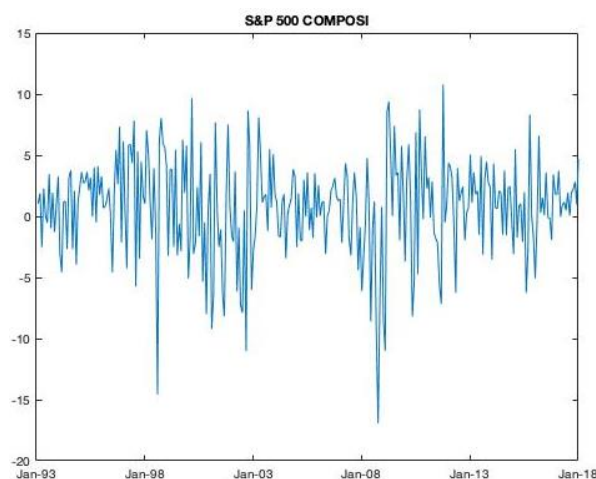
Fig. 7. S&P 500 Composite Return. Source: Own Elaboration.
EIKON Data

On November 2008, the index closed at 750, its lowest since early 1997. The market continued to decline in early 2009, surrounding the financial crisis of 2008

On April 29, 2011, the index closed at 1,300, but it had a sharp drop in August and briefly broke 1,100 in October. Gains continued despite significant volatility amid electoral and fiscal uncertainty, and the 2012 close of the S&P 500 following QE3 was its third-highest ever, at 1,426.22 points.

- Short Term Government Bonds

The short-term index used in our work is represented by the Standard and poor S&P U.S. Treasury Bond 1-5 Year. This index is designed to track the performance of U.S. dollar denominated domestic market sovereign debt issued by the U.S. government, with remaining term to final maturity of at least one year and at most five years.
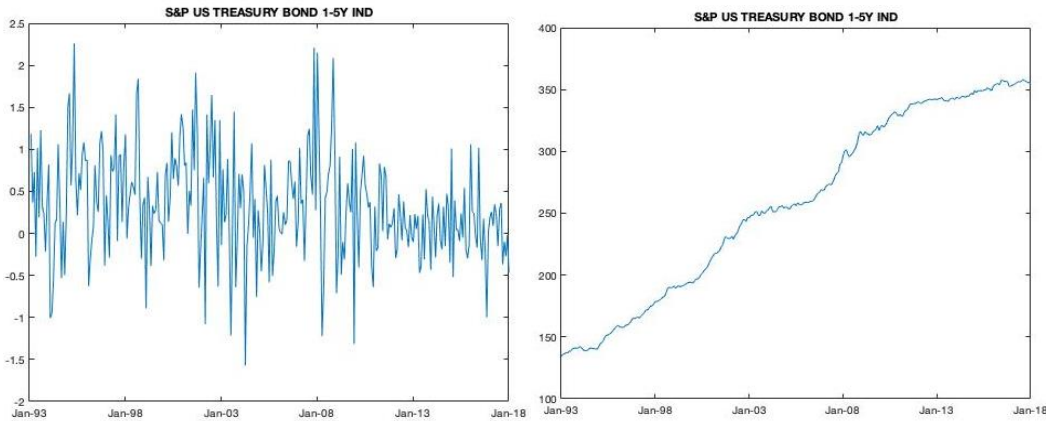
**Fig. 8 & 9. S&P US Treasury Bond 1-5 Years Total Return Index
and Daily Return**
Source: Own Elaboration. EIKON Data

The index exhibits a sustained positive trend over time with some slight downturns.

Unlike the Equity asset class, During the periods following the 2-relevant crises (2000,2008), the trend in this case tends to take even a sharper slope. This should be related to interest rate policies adopted to contrast the effects of the crises.

In fact, implementing an expansionary monetary policy and reduction of the interest to stimulate the economy is likely to cause an appreciation of the price of bonds, Ceteris Paribus.

- Long Term Government Bonds

We used the Standard and Poor S&P U.S. Treasury Bond 10+ Year Index. The S&P U.S. Treasury Bond 10+ Year Index is designed to track the performance of U.S. dollar denominated domestic market sovereign debt issued by the U.S. government, with remaining term to final maturity of 10 years or more. As in the previous case, we used the total return index.
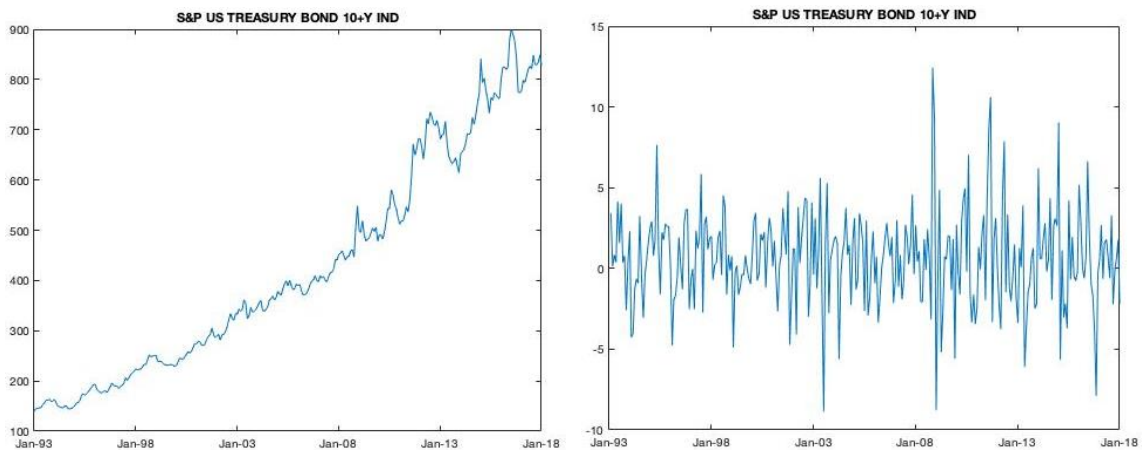
**Fig. 10 &11. S&P US Treasury Bond 10+ Years Total Return and
Daily Return Index.** Source: Own Elaboration. EIKON Data

Unlike the short-term treasury bond index, the long-term bond index exhibit more fluctuations and heavier downturns mostly in the post 2008 period and from 2013 on. In line with financial theory which states that government bonds tend to perform better than equity for instance during financial crisis, also the long term shows quite a good behavior during the relevant crises suggesting an allocation privileging this asset class in distressed market.

- Corporate Bonds

In this case, we used the S&P 500 Investment Grade Corporate Bond Index. It seeks to measure the performance of U.S. corporate debt issued by constituents in the S&P 500 with an investment-grade rating. The S&P 500 Bond Index is designed to be a corporate-bond counterpart to the S&P 500, which is widely regarded as the best single gauge of large-cap U.S. equities.

It is crucial to highlight how this index is also sensitive to financial turbulence as shown in the graph below in late 2008 and early 2013.
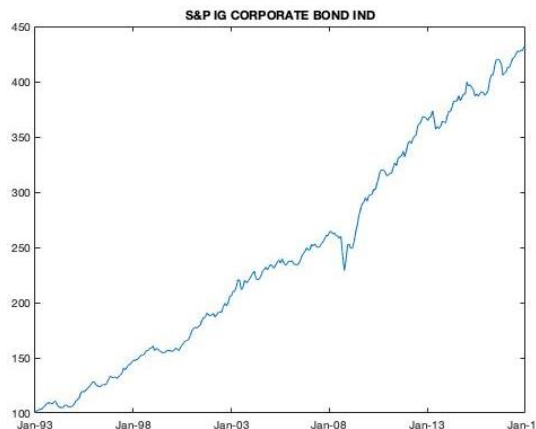
Fig. 12. S&P US Investment Grade Corporate Bonds Total Return Index. Source: Own Elaboration. EIKON Data

The Table below reports the summary statistics.

Mean	Median	StDev	Min	Max	Skew	Kurt
0,68	1,11	4,11	-16,94	10,77	-0,69	4,34
0,32	0,28	0,61	-1,57	2,26	0,24	3,65
0,61	0,66	2,90	-8,87	12,44	0,22	4,63
0,48	0,49	1,33	-6,57	6,15	-0,58	7,42

Tab 1. Summary Statistics
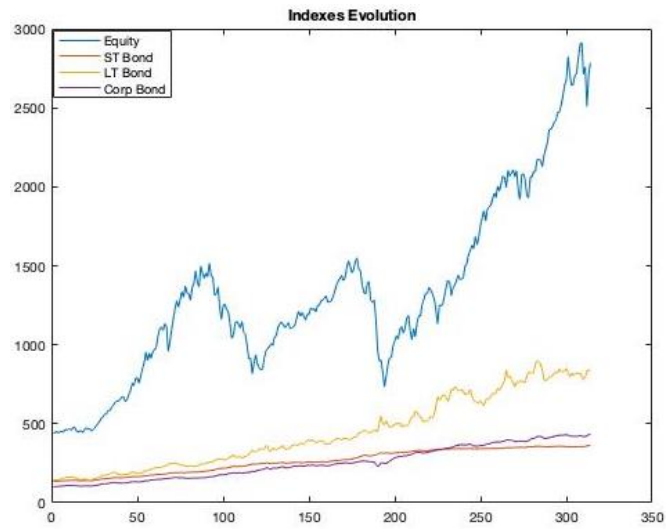
Fig. 13. Indexes Evolution. *S*ource: Own Elaboration. EIKON Data

VI.   METHODOLOGY & RESULTS

The aim of the work was finding a good asset allocation for a representative worker/investor who is 45 years old and want to set aside wealth during the following 25 working years and expected to live 20 years from retirement.

We had then to find a benchmark both under a static-buy-and-hold and a dynamic optimal rebalancing strategy. This benchmark should have the characteristic of being chosen today and achieve our objective function today and in the future.

Since the future values of all relevant variables of our problem are unknown and uncertain, we had to find a procedure of estimating them rather than assuming perfect knowledge of value or inefficiently used the sample mean.

To do so, we had to choose between the Bayesian Learning approach as suggested by Nicholas Barberis (2000) and a bootstrap based resampling approach. In this regard, our choice was guided by the work by Campbell R. Harvey, John C. Liechty and Merril W. Liechty (2008). They conducted a comparison based on an investment game where one investor uses a Bayesian approach and the other a resampling one. Based on their performance measure[11], they found that the resampling approach has practical merit **when the future returns are not consistent with the historical returns** (e.g., when the underlying statistical model has been misspecified or the data is drawn from a distribution other than the predictive distribution) **or when the investor has a very long investment horizon**.

It follows that we adopt Bootstrap approach.

Briefly, the procedure we adopted can be summarized as follow:

1. Generate N simulations of the future returns of assets and get a 540x10000 matrix

$$\begin{bmatrix} N_{1,1} & ... & N_{1,10000} \\ \vdots & \vdots & \vdots \\ N_{540,1} & \vdots & N_{540,10000} \end{bmatrix}$$

---

[11] They priory proposed a "true" value of parameter generating history and compared which approach were more in line with that value.

Each row represents one period of time (month). Therefore 540 represent the investment horizon (25 years + 20 years). Each column represents a particular simulation of future asset returns.
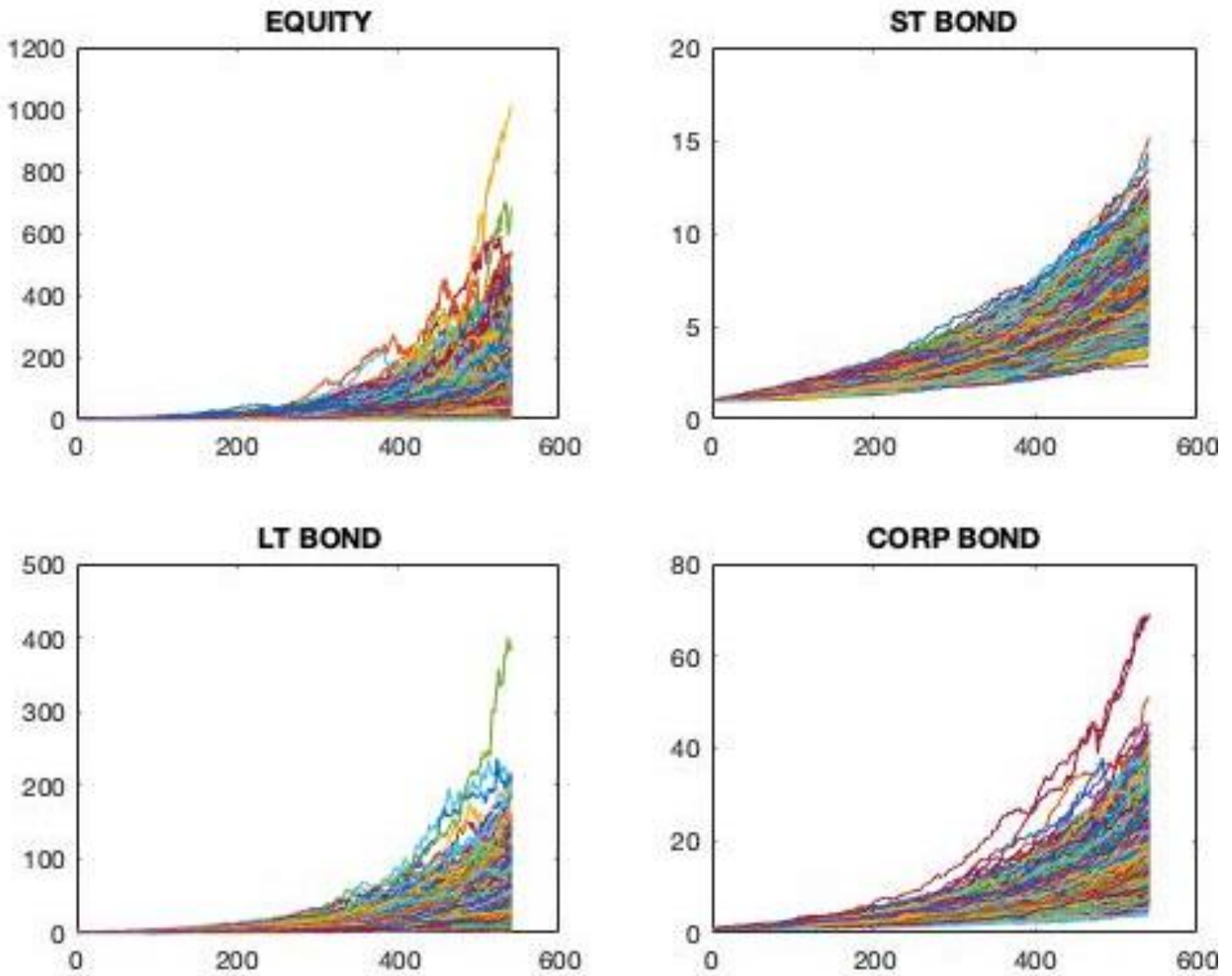

Fig. 14. Simulations of Future Asset cumulated returns

The graph above is the representation of the 10000 simulations of 540 future time period of each asset class. To have a better insight, it is useful to plot some quantiles of these simulations as reported in the graph below which reports, for each asset class, the 50% quantile in red and the 5 and 95% quantiles in dotted black.
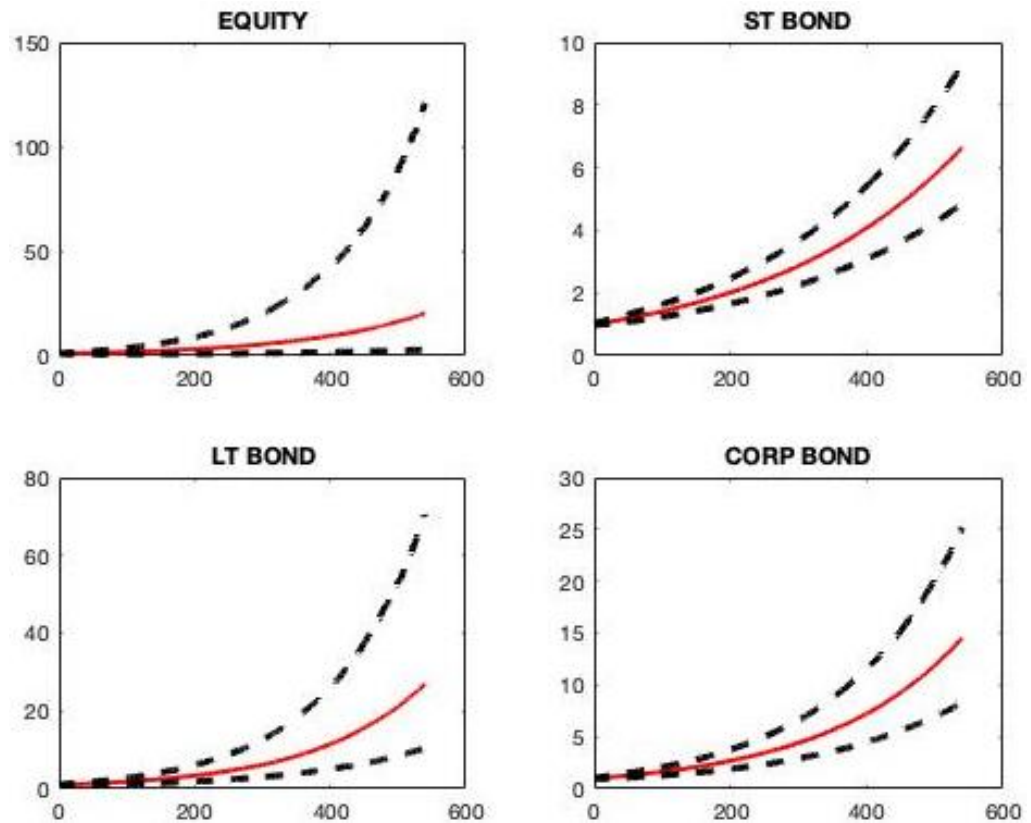
Fig. 14. Quantile Representation of Simulated Cumulated future Returns

2. Create a matrix containing all possible combinations of weights.

In order to insure a minimum level of diversification and avoid corner-solution type allocation, we decided to set a lower bound on weight equal to 10% and an upper bound equal to 60%.

From these extremities, we generate series of number starting from 10% to 60% with a step of 1 (i.e. 10,11,12…58,59,60). But the resulting matrix had enormous dimensions and practically impossible to manipulate in an efficient way. The time of execution of simple command was unthinkable.

For this reason, we decided to switch from a step of 1 to a step 2,5 (i.e. 10,12.5, 15…55, 57.5, 60) in order to gain in efficiency. The result is a 4x2845 matrix. By doing so, we definitely lose something from the accuracy/precision of results point of view. Nevertheless, we believe that the qualitative implications that can be retrieve from the results are unchanged.

3. Use these simulations of future returns and matrix of weights to evaluate all possible evolutions of the pension fund taking into consideration all the other variables (contributions, payment, inflation etc.). We kept trace of the few quantiles useful to our analysis and get a 540x2845x3 array.)
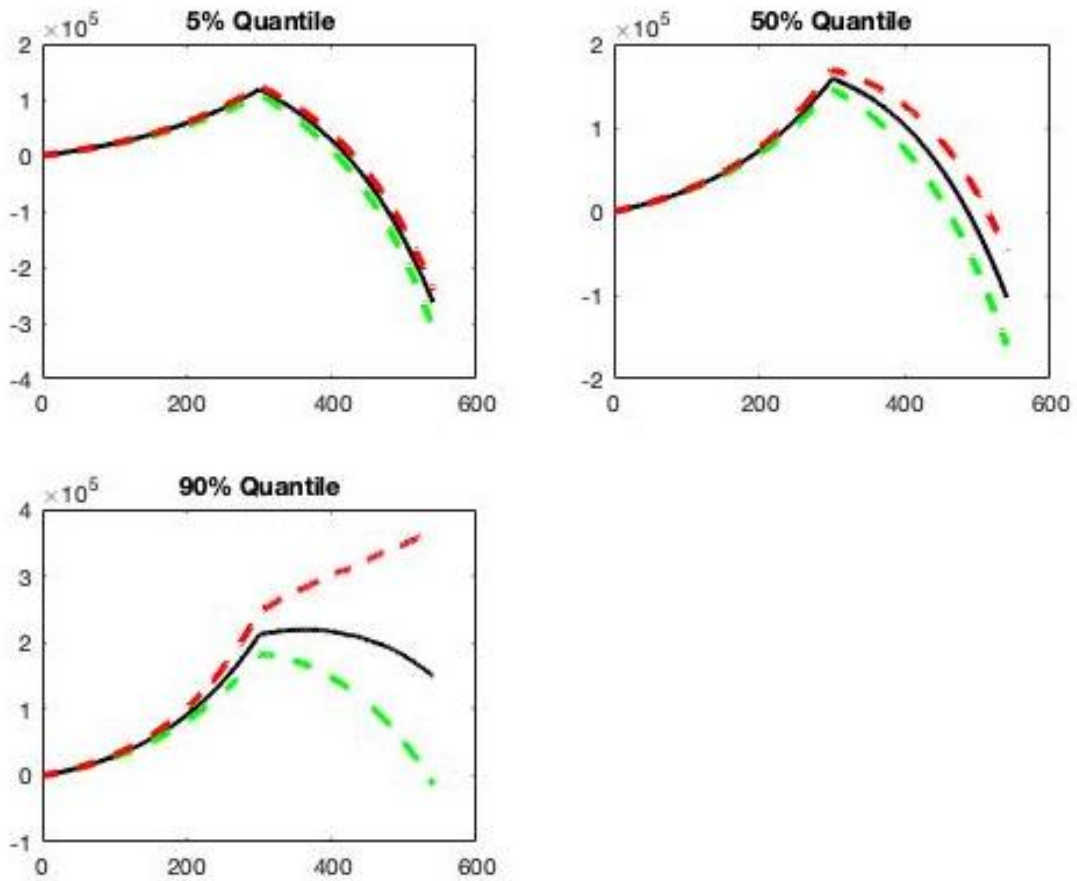

Fig. 15. Representation of Quantiles Taken Over Portfolios
Each Subplot represents one Quantile over the number of simulations. Within each Subplot, 3 trajectories
for 5% portfolio quantile (Green), 50% (Black), 90% (Red)

The graph above shows, for the 3 quantiles made over the 10000 simulations, the quantiles made over the combination of weights i.e. it reports how the value of the pension fund changes switching from an allocation on one extremity to one on the other extremity passing through a near equally weighted one. This for each quantile made over the simulations. In other words, they report the evolution of the pension fund provided by a bad performing portfolio, an average and a good performing.

The graphs in Figure 16 gives an insight about the portfolio composition of each case.

The 3 subplots on the top left corner (5% quantile) for instance tell us that, in case of distressed market, the equity asset class is the worst performer and the long term government bonds tend to be the best performer.

With the result from point 3, we will try to find which particular combination of weight solve our problem.
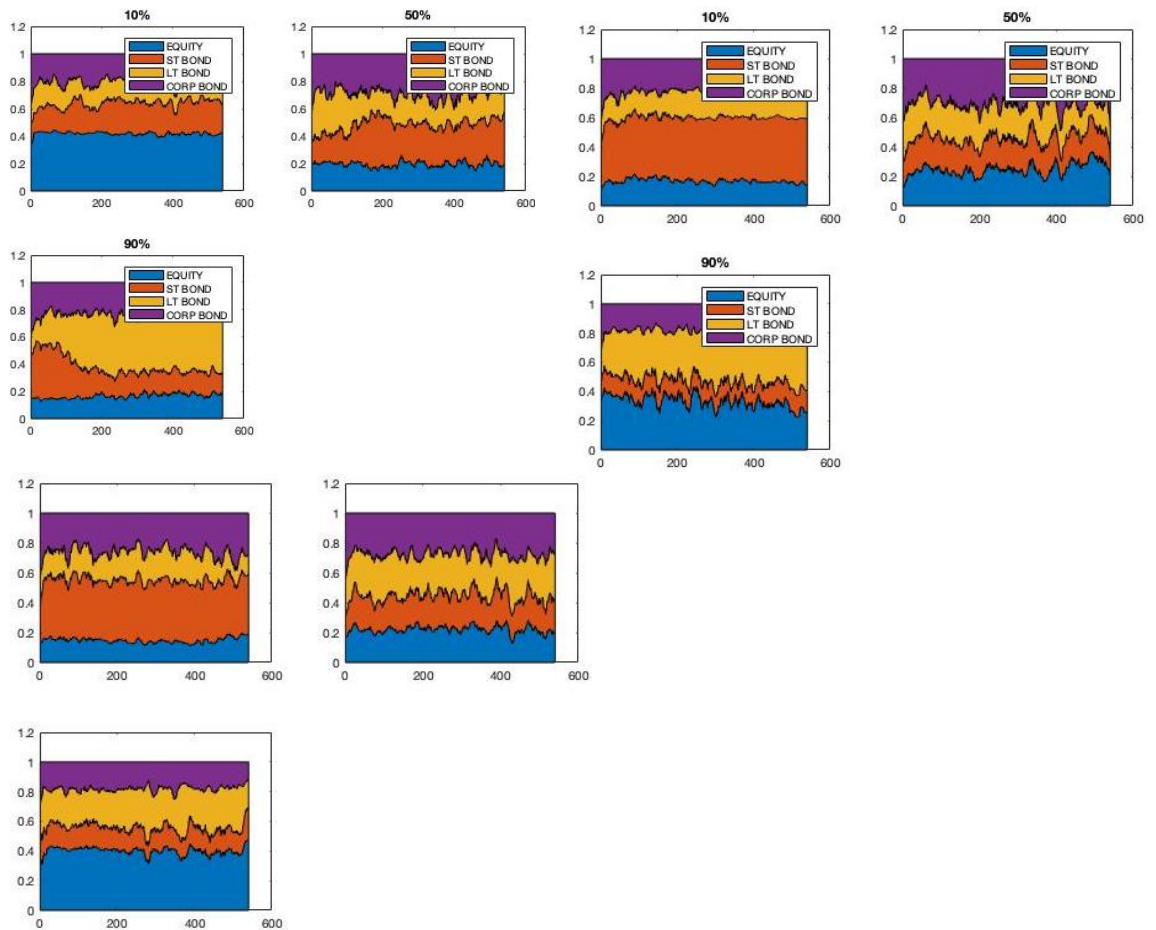

Fig. 16. Portfolios Corresponding to Quantiles Made Over Portfolios Compositions
The 3 subplots on the top left corner are referred to the 10% Subplot in Fig 15. The 3 in the top right are referred to 50% quantile in Fig 15 and the remaining 3 in the bottom left are referred to the 90% quantile.

VI.1 Base Case Solution

Having all the elements presented above, the procedure for finding the benchmark was quite straightforward.

We evaluated all the possible values (10000x2845) of the fund at the investment horizon and searched for the combination of weights that makes this value positive and close to zero in

expectation. Formally, starting from all the A_{540}, we wanted to find which combination of weights has on average a positive value close to zero.

At this stage of the work, it is crucial to explain the reason leading us to the choice of this criteria rather than another such as maximizing the final value of the fund.

Two main reasons have been considered:

1. The work as a whole is adopting the prospective of the investor/worker. It tries the give a decision tool to the investor who is to inject his wealth in the fund. If we had adopt the prospective of the fund pension managing board, it would have make more sense to set the final value of the fund as its maximum.

   But from the prospective of the investor, it is optimal to have back from the fund everything we can, on expectation, and therefore putting the terminal value of the fund positive but as close as possible to zero.

2. The second motivation has to do with concepts of risk and diversification. All along this work, we have not make consideration about risk, covariance and diversification. Beside setting upper and lower bounds, we believed that adding this condition on the terminal value, rather than maximizing it, could have helped us to get rid of corner solutions, concentrated allocation and excessive risk exposure toward one particular asset class. In this regard, it is a common belief for practitioners that, beside all the advantages of diversification such as reduction of risk, diversification has got also several drawbacks. One of them is the limitation of the upside gains. The fact that underperforming assets are compensated with well performing asset creates an ex-post loss of gain opportunity. In fact, if one could have invested only on the well performing asset, he would have ended up better off than if he had diversified. In other words, diversification allows to get rid of the downside movement of portfolio but limits also to upside movement. By setting the final value of the fund close to zero, we give up to the upside movement and, we believe, this is implicitly requiring avoiding non-diversified portfolio and therefore implying some level of diversification.

We implemented this procedure using Matlab Software and the result can be summarized by the graph below.
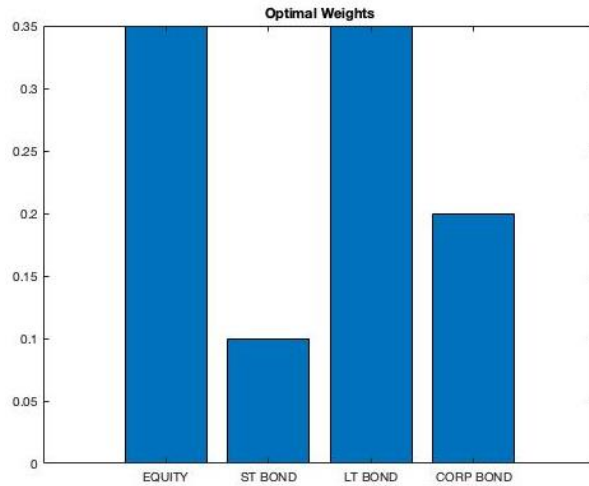
Fig. 17. Benchmark Portfolio. ρ=10%, γ=40%.

As highlighted in the graph, the procedure we adopted suggests us an allocation made of 35% on Equity and Long-term Government Bonds, 10% on Short-Term Bonds and 20% on Corporate Bonds.

Given the fact that is diversified and in line with our constraints, we retained it as the benchmark.

It can also be seen as the "optimal" portfolio for a buy-and-hold strategy.

The resulting evolution of the pension fund value is reported in the graph in Figure 18 below.
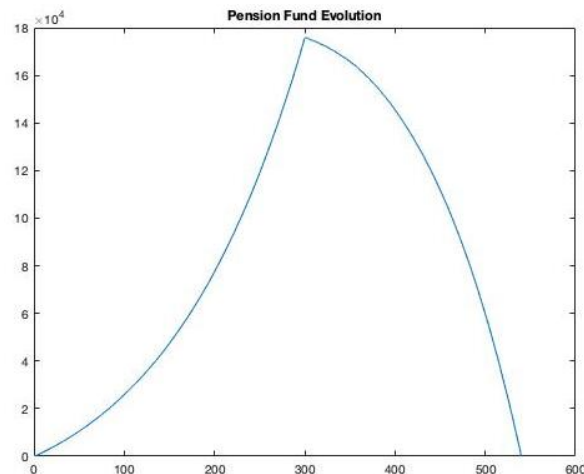
Fig. 18. Evolution of the Pension Fund under the Benchmark
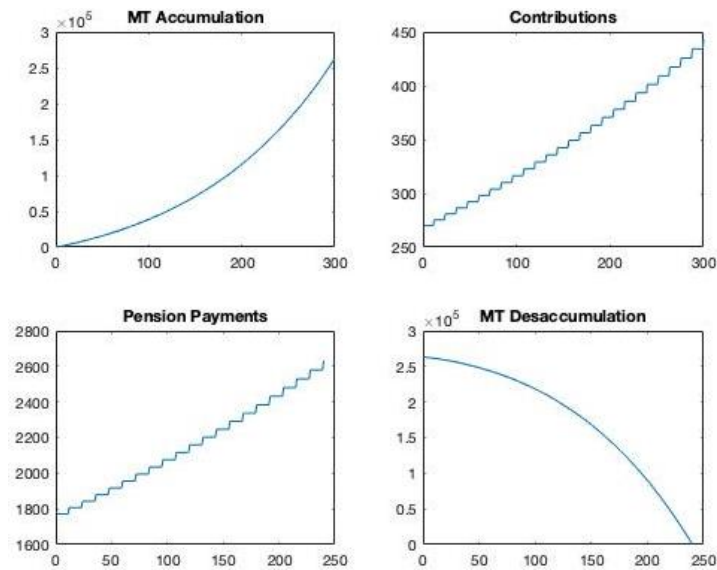
Fig. 19. Evolution of Relevant Variables

It is interesting to see also how the results change moving along the columns of the weight combinations matrix.

As shown in the graphs below, we get improvement of the value of the pension by progressively sacrificing the Short-Term Bond asset class in favor of Equity and Long-Term Bond
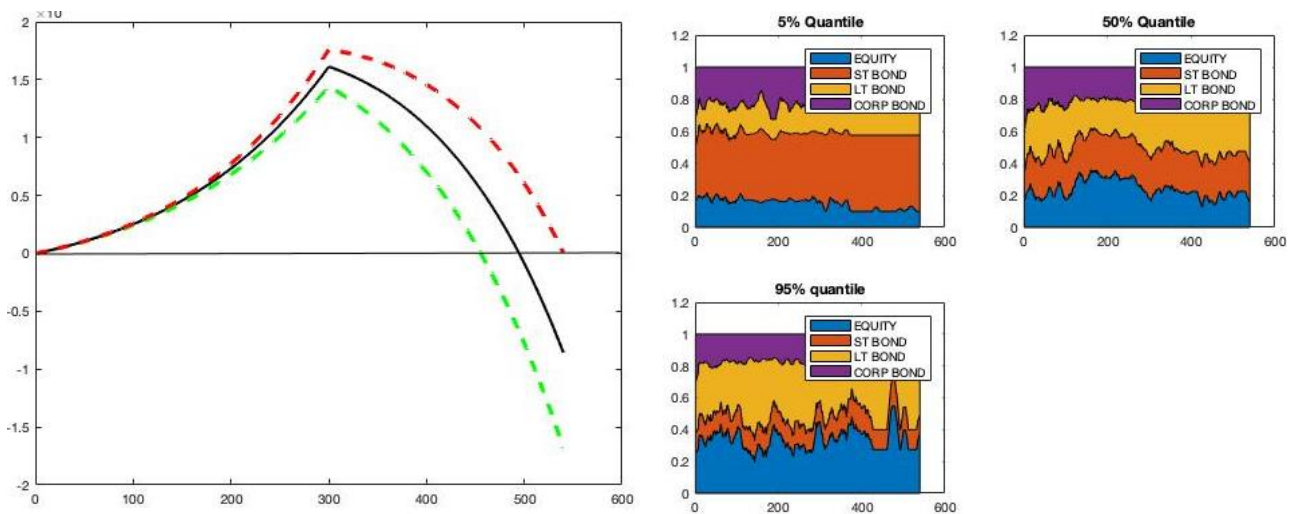

Fig. 20. Quantile Representation Of Expected Pension Value over Portfolio Composition
The graph on the left represents the quantiles, the one on right, the portfolio composition of each quantile

By confronting the graph on the left (mean) with the one in figure 16, especially the 50% quantile (median), we found that there is a slight difference among them. In fact, in graph 16, at t=540, all the pension fund values are negative. In graph 20 instead they are some positive

values. By checking how many portfolios in the matrix are able to give, on average, positive terminal value of the pension fund, our calculations tell that only 5% of portfolio can satisfy this requirement versus 0% if we consider the median. This data will be useful after in this work.

The only reason justifying, according to us, the fact that in graph 16 have no positive terminal values unlike the graph 20 is the shape of the distribution. In fact, the skewness index of the distribution over weights, at t=540, is slightly positive (0.0738). This could justify the difference between mean and median. The reason we found is that we have considered many (too much) portfolio compositions, most of them irrelevant and under-performing. By implementing these quantiles, which are based on percentage, we include also these under-performing portfolios and the result is that only 5% of them can give a positive final value.
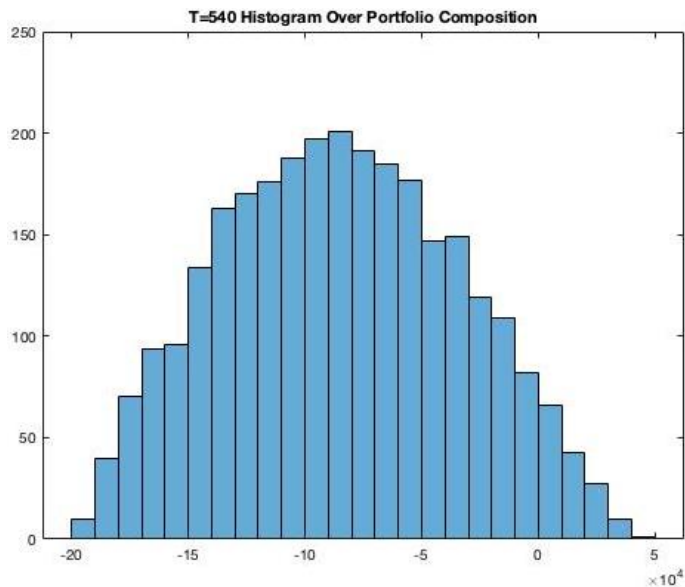

Fig. 21. Histogram of Terminal Values over Portfolio compositions

This suggest that, even if we have only 5% of weight giving positive final value, we are taking extremely risky allocation, even if the portfolio is located on the tail and not around the mode. Once again, the reason is the fact that we have considered many portfolio combinations.

After finding the benchmark, the next step was focus on finding optimal short term deviations from the benchmark. In fact, as we have mentioned before, the benchmark was a decision of

the type: "find today what is optimal from now to the end period". What if we have to take advantage of short-term opportunity in the market? We expose it on the next point.

VI.2 Short Term Deviation from The Benchmark

The benchmark tells us that given the data and the objectives set, the first best solution to our problem can be achieved by investing as suggested.

However, the use of this static approach does not allow us to take advantage of short-term market movements in order to avoid loss or make additional gain. It is therefore interesting seeing what could be the per-period optimal allocation, seeing in which measure we could deviate from the benchmark to respond in an effective way to market movement.

We conduct this analysis using the 3 quantiles of returns set before and assumed they represent 3 market conditions: distressed (5%), quite (50%), bullish (95%).

We therefore looked for the portfolio that maximizes the value of the fund in each period, for each market condition. In other words, we are assuming to act as a pension fund manager that decides today what is the best portfolio (that maximize the pension value) for next period, then for 2 period ahead, for 3 period ahead and so on.
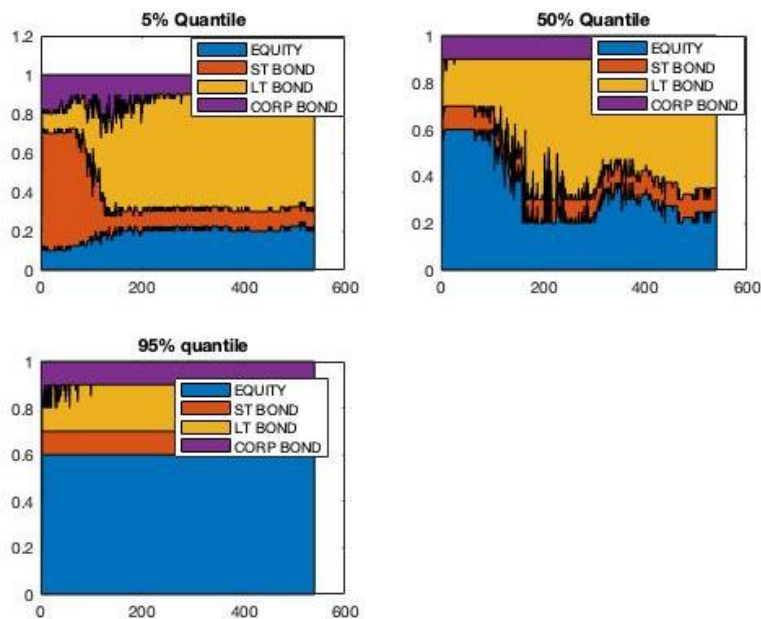
The results are reported on the next graph.

Fig. 22. Short-term Deviation from the Benchmark
For each of the 3 quantiles of return, it reports the portfolio allocation that maximizes the pension value in each period

As one could have expected, the allocation is going toward equity in case of bullish market, long term bond during distressed market, mixed equity-long-term bond during quite market situation.

Note that this procedure only says that the allocation allow gives the highest value of the fund in each period. This means that from one period to another, several allocations reported in the graph could have a worse performance than other not reported. To take into consideration this effect, we should consider the maximization of the percentage difference from on period to the other.

This has been done but the results were not stable giving irrelevant and non-feasible allocation due to excessive rebalancing. It will also cause issues if we must consider transaction costs.

Even by weighting the percentage difference by a risk measure, the results were still unstable.

We recall that these results are based on several inputs assumed before. What if we use other variables? How will the results change? These questions will be answered in the next chapter.

VII. SCENARIO ANALYSIS

The following chapter is focused on redoing the same analysis made at the previous but assuming other input variables.

The aim is to see how the optimal portfolio found on the previous point reacts and how sensible it is to input parameter.

We consider it interesting to see what happen to the if:

1. We change the value of the rate of contribution to the fund
2. We change the rate of conversion of wage to pension amount
3. We allow to withdraw a given portion of the fund, several periods retiring
4. We adopt the perspective of the insurance company with the task of paying the pensions

We are interested in seeing how these changes will affect:

1. The investment strategy
2. The length of time during which the pension is paid

One can think about this analysis as a study of the implications of more competitive commercial policy adopted by fund managing company.

VII. 1 Rate of Contribution to the Fund

In the base case of the work, we used a rate of contribution to the fund equal to 10%. The investor has then to pour to the fund, each period of time, 10% of his salary.

The choice of this value was partly arbitrary. In fact, the Italian regulation provides that the contribution that the worker has to pay to fund is mainly made of the severance pay. We used this as a proxy.

In practice, the amount of severance pay is the result of the sum of 6.91% of the salary i.e. the wage divided 13.5.

We have also to take into consideration the contribution to be paid by the employer and the worker. As introduced before, they vary from 0.5% to 2.05. For professional offices for instance, the worker should pay 0.55% and the employer 1.55. A more adequate rate of contribution would be 11%. But we arbitrary set 10.

To check how sensitive are our result, we used other value for this parameter. In particular, we wanted to see what happen if we use the more adequate rate of 11% on one side, and on the other side one commercially competitive rate (8%). The results are reported in the next point.

A. Benchmark

For the case of $\rho=11$ the consequence is an increase of wealth available to the fund, more wealth to invest and more wealth also available for the payment of pension. The logic expectation on the allocation is having the same allocation as before or an allocation less exposed to equity. The next graph represents the benchmark of this case. The benchmark has been find in the same way as in the base case.
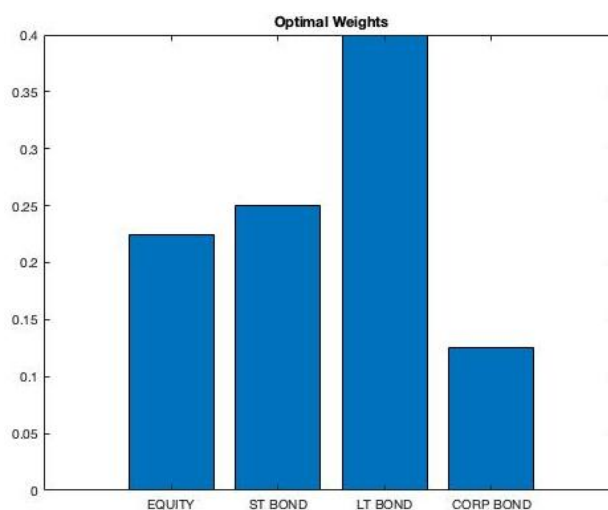

Fig. 23. ρ=11 Benchmark

As anticipated and as one could have imagine, the benchmark in this case is allocating more to Long-Term bonds and Short-Term Bonds. The economic intuition behind this result is, we believe, the one given in the previous paragraph. The fact that the fund has more wealth at its

disposal makes it non-necessary to allocate more than that to equity which has the characteristic of given higher returns.

This allocation seems not satisfying the diversification criteria at his best. It seems to be too much concentrated to one asset class. In fact, it allocates more than half of the wealth to Long-Term Government bonds. However, while valuating this allocation, one should also take into consideration the fact that Long-Term Bonds have a low level of risk and, in normal circumstances, can be considered as risk free investment. Therefore, the concentration on one asset class resulting from our analysis is not necessarily an issue from the riskiness of the portfolio point of view.

By looking at risk of the portfolio using the sample standard deviation, the benchmark has a risk of 1.8040 against 1.5560 from this case, which confirms our previous affirmation.

At this point, a question worth asking is to see what would happen if we change Rho but keep the allocation found in the base case. the only variable that can be affected in this environment is the time during which the pension is paid. To check it, we will look at the value of the fund at the end date (t=540) and see how much is still available on the fund.

The result of this exercise is a surplus of 72600€. If we take as proxy the pension payment made in the last period (around 1800€), this implies the ability to pay pension for further 3 years (40 months around).
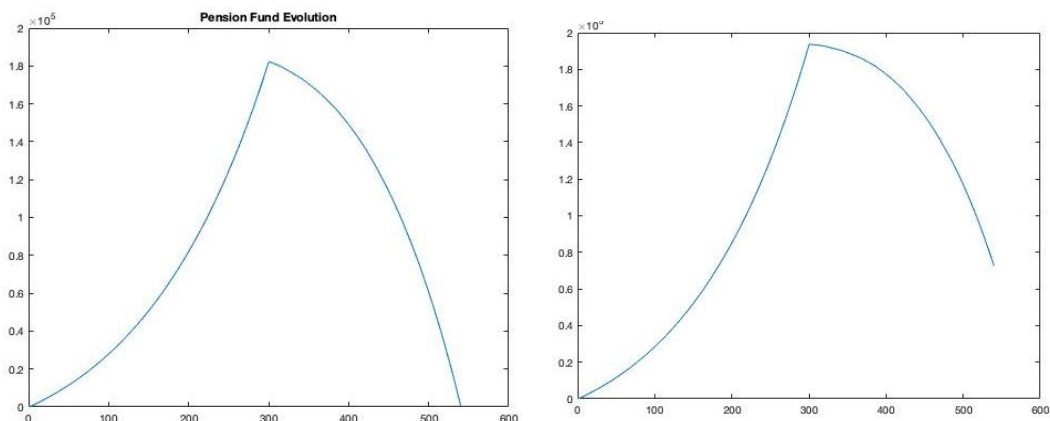

Fig. 24. Pension Fund Evolution Comparison between Benchmarks. ρ=11
Left graph for the benchmark assuming ρ=11, right graph, benchmark assuming ρ=10

Finally, by using a rate of contribution of 11%, we found that, in expectation, **around 35% of the possible portfolios can give positive terminal value of the fund.**

The second case we wanted to investigate is the case in which ρ=8 that is a lower burden for the investor and a kind of commercial competitive strategy by the fund manager.

The fact that this policy implies less wealth available to the fund, the prediction is an allocation toward the equity asset class. This, in order to compensate the reduced amount of wealth available to the fund by the eventual higher returns that the equity asset class can offer.
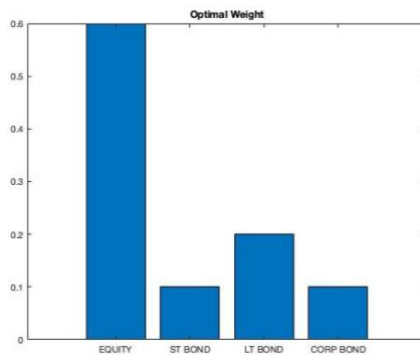

Fig. 25. ρ=8 Benchmark

The graph above reports the result. As anticipated, the allocation is toward equity. it concentrates 60% of the wealth in equity and the remaining goes to long-term bonds mainly.

It is a kind of corner solution. It allocates whatever it can to equity. in this case 60% because of the upper bound. Then it tries to allocate whatever it can to long-term bonds (20) and satisfies the lower bounds set at 10%.

Another relevant aspect of this case is that not only the allocation is affected, but the time length of payment of pension is also affected. In fact, we have not highlight it but the results reported above are not derived in the same way as in the base case or in the case with ρ=10. Here, we find the benchmark in some period preceding the end period initially set in the work. The reason is that if we look at the end period, all the portfolios provide negative value of the pension fund.
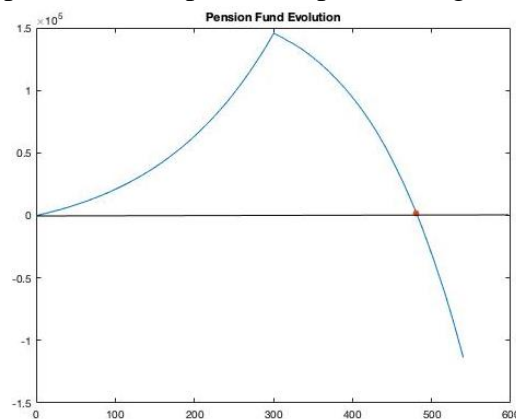

Fig. 26. Pension Fund Evolution. ρ=8

By doing a backward check, with find the first positive value at the 481st month from the beginning. This positive value is the one related to the allocation reported above.

The consequence is that, ceteris paribus, the length of period of pension payment is reduced by 5 years around. Given the assumptions of our model, the worker would retire at 70, perceive the pension up to 85 years old instead of 90. Or in alternative, the worker should accept to receive reduced amount of pension allowance.

B. Short-Term Deviation from the benchmark

The result for the short-term deviation from the benchmark are identical as in the base case. The reason behind this similarity is the fact that the short-term deviation consists on finding the portfolio that **maximize** the fund value in each period. This seems not being influenced by a linear increase of the contributions.

VII. 2 Rate of conversion of wage to pension

After checking how the results changes in reaction to change of the rate of contribution, we now want to see how the results change when is the rate of conversion of wage to pension to be changed, i.e. how will the result change if we promise to the worker/investor to pay a higher percentage his final wage as pension.

In the base case, we used a rate equal to 40%. We want to see what happen if we use a rate of 48% instead.

As in the case of the contribution rate, this can be seen as a competitive strategy. Promise to pay more to the investors that will choose to invest in our fund.

- Benchmark

The expectation on the result that one could have is once again an allocation "aggressive" toward equity. once again, promise to pay more as pension imply a higher cost for the fund,

fewer money available to it during the pension payment phase. As in the previous case, the switch toward equity is an attempt to compensate this reduced availability of wealth by the higher return typical of the equity asset class.

In line with our expectations, the portfolio composition is made of a 60% exposure toward equity. Long term bond is the second class in order of magnitude. This result is similar to the one found on the previous point.
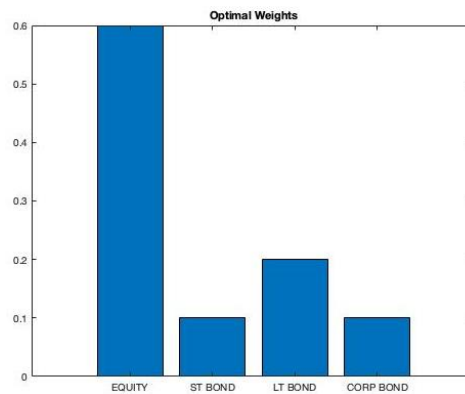

Fig. 27. Benchmark. γ=48%

Another similarity with the previous point is the fact that this allocation does not allow to cover all time length of pension payment. in fact, even in this case, none of the portfolios is able to give a non-negative value of the pension fund at the end period (t=540). Implementing the check to find the first non-negative value, we found that this allocation allow to pay the pension benefit up to the 491st month from the beginning.
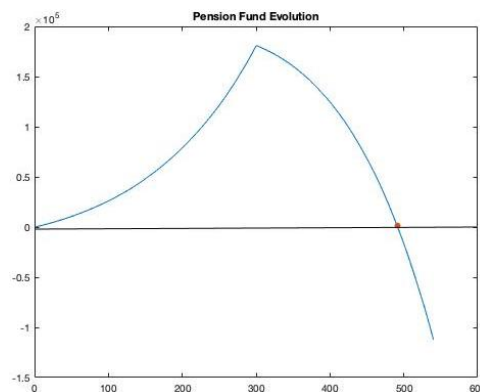

Fig. 28. Pension Fund Evolution γ=48%

As a consequence, the best solution that can be offer the investor, given this rate of conversion, implies that he will receive the pension for 4 years less than in the base case i.e. he will retire

at 70 and perceive the pension (higher pension!) up to 86. Or, in alternative, reduce the amount of the pension allowance

- Short term deviation from the benchmark

As in the previous case, since the deviation from the benchmark is a maximization of the pension fund in each period, it is not sensitive to linear deterministic change of the payment policy.

Therefore, the result is the same as in all the previous cases.

VII. 3 Possibility of withdrawing money from the fund.

This section of the work want to answer to the following question: what happen if we allow the participant to fund to withdraw a given amount of money at a given time? i.e. given the contribution policy, given the portfolio allocation found in the base, what will happen to the pension payment phase if we allow the investor to pick up 10% of the fund value, say, 10 years before retiring.

The ratio behind the inclusion of this hypothesis is the fact, under the current rules, it is allowed to the investor to withdraw money from the fund under several circumstances. However, most of the time, participants to the fund do not know what would be the consequence of picking up money,

About the circumstances under which it is allowed to withdraw, we have found that there are 3 cases giving the right to the anticipated payment[12]:

- To cope with health costs (up to 75% of the individual position);
- For the first home purchase or renovation;
- Other needs.

---

[12] https://www.propensione.it/approfondimenti/anticipazioni-sulle-pensioni-integrative-la-flessibilita-della-previdenza-26532/

For other needs, after 8 years of participation in the fund it is possible to request advances also for generic reasons, without any need to justify them. In this case the amount anticipated cannot exceed 30% of the accrued individual position.

The 8 years' requirement is also present in case of purchase and renovation of first home and the limit is 75% of the personal position.

We investigated the implications of the policy on the monetary amount of pension payable to the investor and on the length of the pension payment period.
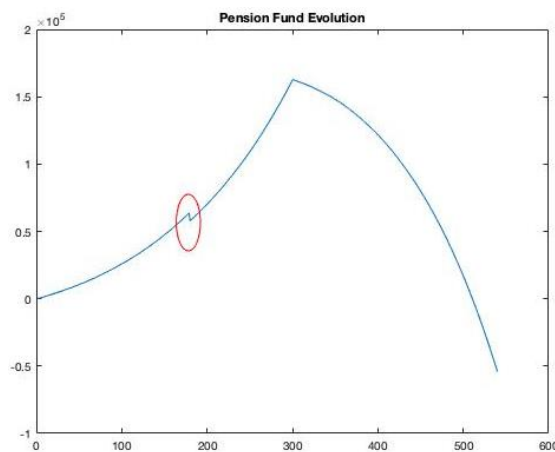

Fig. 29. Pension Fund Evolution. Withdrawal allowed

The result reported on the graph above tells that the consequence of an anticipated withdrawal is a reduction of the length of the period of payment. In particular, withdrawing 10% of the fund (6500€) implies that the participant will receive the pension for 29 months lesser than in the base case.

A more interesting case is seeing what happen if it is allowed to withdraw and, instead of reducing the length of the period of payment, we reduce the amount of the pension allowance.

In this particular case, we found that, in the face of a 10% anticipated withdrawal, the overall amount of pension payable, intended as the sum of pensions payed from the moment of retirement to the extinction of the fund, is reduced by 3.07% with respect to the base case. Each periodical pension amount is reduced by an amount of 44€.
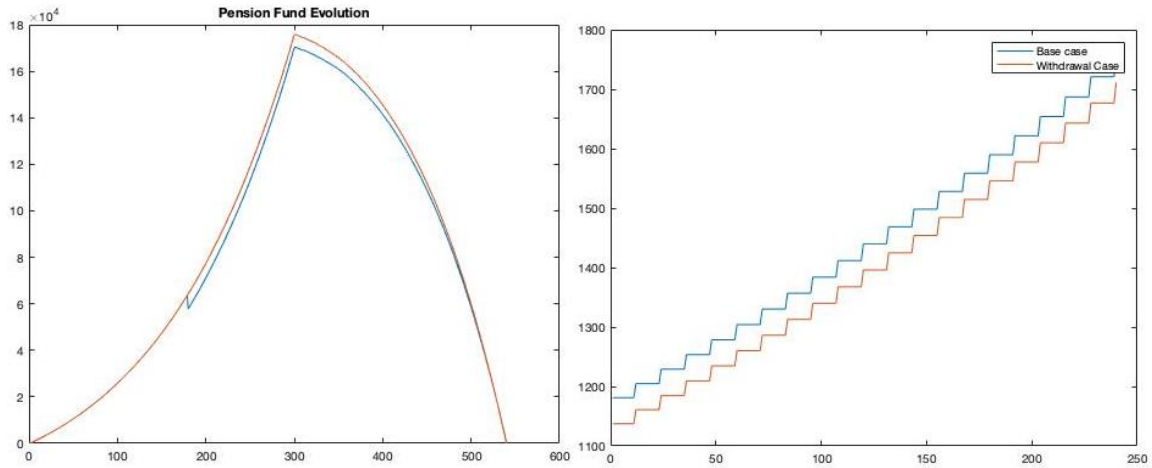
Fig. 30. Comparison between Withdrawal case and Base Case
Evolution of pension fund and pension payments.

VII. 4 ALM, An Insurance Problem

Finally. we were curious to see how would the results change if we use the perspective of an insurance company entitled to meet all the payments of pension. To see what would be the best allocation under a prudential approach that requires to meet all the payment, not on average, but in way such that, at T=540, the probability of being solvent is higher than the one of being insolvent.

That is, instead of putting the mean close to zero, we want to see what happens if we work we a lower quantile, say, 30%. In this way, the worst cases (on the left of the chosen) are fewer than the better cases.
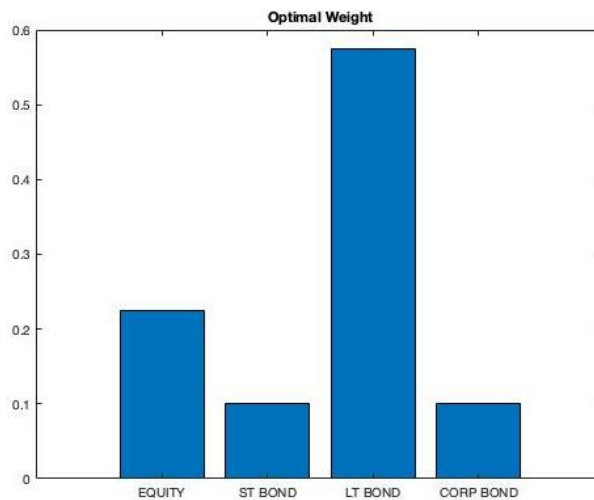

Fig. 31. Insurance case allocation.

The aim is to find the best allocation for the insurance company which has the task of paying the pensions to the retired participant under a prudential perspective. The result is reported on the graph in figure 31.

This portfolio is concentrated on long-term bonds. The concentration in this case is even more important than in the previous case (Fig. 23). The portfolio as standard deviation of 1.8924 evidencing the lack of diversification.

Another particularity of this allocation, the same of many of the previous cases, is the fact that it does not allow to meet all the payments up to the extinction of the fund. In fact, it guarantees to pay up to the 493rd month leaving the participant "Naked" for around 4 years.

In alternative, we could reduce the amount of pension benefit to ensure the payment up to the end. It is therefore crucial to combine this allocation with a reduction of the amount of pension allowances.

Our calculation let us that by renouncing at 27€ on each pension allowance, the payments could be met in each period up to the extinction of the fund. The graph below plot the evolution of the pension fund.
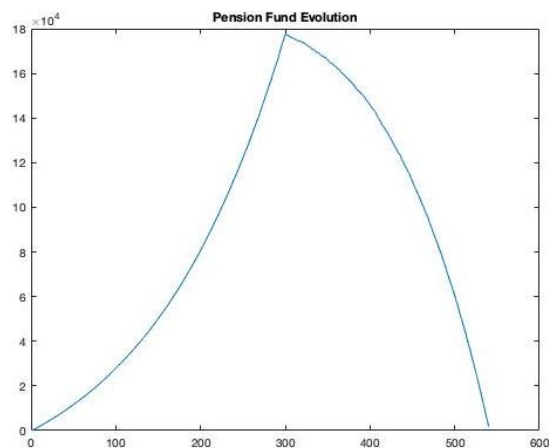

Fig. 32. Insurance case Pension Fund Evolution

VIII. SUMMARY CONCLUSION

After exploring all the cases exposed above, we think it useful to make a brief summary of what we have done.

The study was centered on finding the best allocation of the wealth of an average individual considered as investor/participant in the pension fund. This individual is currently aged 45, has a current wage equal to 1800€.

He desires to set aside a portion of his salary for the next 25 working years in order to avoid the risk of consuming less when he will retire and, therefore, will not perceive a salary. lacking adequate tools to infer on the life expectancy, and dealing with an average individual, we assume it equal to 90 years.

The main variables of the model were defined as follow:

$$A_t = A_{t-1}(1 + R'_t w) + C_t \quad \text{with } A_0 = 0$$
$$\text{For } t = 1 \dots 300$$

$$A_t = A_{t-1}(1 + R'_t W) - P_t$$
$$\text{for } t = 301 \dots 540$$

For the aggregate pension value

The contributions instead are assumed to follow the process:

$$C_t = C_{t-1}(1 + \pi_1)$$
$$\text{for } t = 1 \dots 300$$
$$C_0 = \rho \check{S}_0$$

$-$ Where π_1 captures increases of contributions linked to inflation and career development.

It is assumed to be $\pi_1 = \pi_0 + \delta$

π_0 Stands for the monthly inflation rate

δ captures the carrer development (initially set to zero)

$-$ $\check{S}_0$ is the monthly wage at $t = 0$ of our considered average member

$-$ ρ Is the share of salary as provision to the fund

The monthly pension payment is defined as:

$$P_t = P_{t-1}(1 + \pi_0)$$

$$\text{For } t = 302 \ldots 540$$

$$P_{301} = \gamma \check{S}_{300}$$

We followed various approaches and the results are reported on the table below.

	Cases						
	1 (Base case)	2	3	4	5	6	7
Assumptions							
ρ	10	**11%**	8	10	10	10	10
γ	40	40	40	48	40	40*	40**
Withdraw all allowed?	No	No	No	No	Yes	Yes	No
Results							
Allocation	Diversified	Aggressive on LT bond	Aggressive on Equity	Aggressive on Equity	Aggressive on Equity	Diversified	Agressive on LT bond
Risk of Portfolio [13]	1.8040	1.5560	2.4866	2.4866	2.4866	1.8040	1.8924
Last payment	T=540	T=540	T=481	T=491	T= 511	T=540	T=540
Amount of pension	Base	Base	Base	Higher	Base	45€ lower	27 Lower

Tab 2: Summary Table

The base case gave us a portfolio with 37,5 to Equity and to LT bond, 15% on short term bonds and 10 on corporate bond. For the fact that it is complying with our constraints (diversification and risk taking), we considered it as our benchmark.

A competitive, but technically identic, is the solution on case 6. Using the allocation found in the case 1, it allows for withdrawing an amount equal to 10% of the fund value. This reduction will be compensated by a lowering of the pension allowance of 65€ and an overall reduction of the pension received in percentage term equal to 3.1%.

---

[13] Measure by the sample Standard Deviation
*Rate based on 40% but in fact it is 45€ less

In conclusion, this work allowed us deepening how the quantitative tools in our possession could be used in an environment different from the familiar Markowitz framework. In particular, we were able to apply them in the ALM problem for pension fund.

We started by an overview on the pension systems, their birth, evolution, particularities, characteristic, how it switched from private to public etc.

We then focused on the Italian case where the pension expenditure by the state is one of the biggest issues linked to Italian public finance. In fact, many reforms have been introduced to improve the situation.

Among the results of those reforms, there is the scaling back of the current expenses, within which we find the pension expenditure.

The pension funds were created to help removing the disequilibria generated by those reforms.

Finding a model for quantitative investment for pension fund in line with the tools was not an easy task. For this reason, we rely on the work by Barberis (2000) that gave us a huge input in our work.

Despite the fact that the results are enough satisfying, we have to note that all the work is based on many assumptions. They are useful simplifications but they also reduce the accuracy of the results. One of them is about the average investor/participant. It could be far more interesting to evaluate and make inference on his characteristic (mainly demographic). This however requires actuarial tools that we were not able to use.

Nevertheless, as said before, the results are enough satisfying for our level. We believe that the result could have practical use as the portfolios we obtained are satisfying the usual constraints and, based on our data, allow to meet, on average, the commitment toward the participants, which is one of the greats preoccupations of ALM.

Beside the practical use, the work allowed us to notice several interesting things. One of them is the sensibility to input parameter. Apparently irrelevant change in input parameters have huge impact in term of portfolio allocation and surplus available to fund. This is the case of switching from a contribution rate equal to 10% to one equal to 11%. The result was a less risky portfolio and an increase of the percentage of portfolios giving positive final value of the fund. Another one is the tradeoff emerging from the various cases we performed. Briefly, one is to

choose between level of pension allowance, length of pension payment period, rate of conversion of wage to pension. If one prefer higher pension allowance, he must either accept to receive it for reduced period of time or accept to bear a riskier investment strategy. The same for an individual who needs to receive the pension for a longer period of time.

This has also some policy implications. Given a desired risk budget from investment, given the demographic characteristic of the population, the policy maker should implement a pension system taking into account this tradeoff.

Because of the limitations mentioned above, we leave room for eventual future improvements. In particular, we have not exploited several cases. It could have been interesting to see what could happen in case 2 3 and 4 if instead of changing the allocation, we keep the first best and reduce the amount of pension payment for instance and meet all the commitment up to extinction of the fund (T=540).

IX. BIBLIOGRAPHY

Barberis Nicholas, "Investing for the Long Run when Returns Are Predictable", The Journal of Finance, 2000

Ben-Tal A., Nemirovski A., (Robust Solutions of Uncertain Linear Programs), Operations Research Letters, 1999

Gulpinar Nalan, Pachamanova Dessislava, "A Robust Optimization Approach to Asset-Liability Management under time Varying Investment Opportunities", journal of Banking and Finance, 2013

Goldfarb D., Iyengar G., "Robust Portfolio Selection Problems", Mathematics of Operations Research, 2003

H. van Binsbergen Jules, Brandt Michael W., "Optimal Asset Allocation in Asset Liability Management", 2007

Jahera John S., "Bank Asset-Liability Management Theory Revisited", Journal of Banking Finance Management, 2018

Kane Edward J., Malkiel Burton G., "Bank Portfolio Allocation, Deposit Variability, and the Availability Doctrine", The Quarterly Journal Of Economics, 1965.

Keel Alex, Muller Heinz H., "Efficient Portfolios in the Asset Liability Context", Astin Bulletin, 1995.

Kusy M. I., Ziemba W. T., "A Bank Asset and Liability Management Model", 1993, available on ResearchGate from March 2016.

Martellini Lionel, Mlhau Vincent, "An Integrated Approach to Asset-Liability Management: Capital Structure Choices, Pension Fund Allocation Decisions and the Rational Pricing of Liability Streams", EDHEC-Risk Institute, 2011

Michaud Richard O., "The Markowitz Optimization Enigma: is "Optimized" Optimal?", Financial Analysts Journal, 1989 .

Platanakis Emmanouil, Sutcliffe Charles, "Asset Liability Modelling and Pension Schemes: The Application of Robust Optimization to USS", Henley Business School Discussion Paper, 2014

Sharpe William F., Tint Lawrence G., "Liabilities, a New Approach", Journal of Portfolio Management, 1990.

Sze Michael, "The Process of Pension Forecasting", Journal of Actuarial Practice, 1993

Tonks Ian, "Pension Fund Management and Investment Performance", Oxford University Press, 2005.

Van der Meer Robert, Smink Meye, "Strtegies and Techniques for Asset-Liability Management: An Overview", The Geneva Papers on Risk Insurance, 1993

OTHER RESEARCHS AND STUDIES

"Il Bilancio del Sistema Previdenziale Italiano, Rapporto 3 (2016), 4 (2017)

Istat, "Pensioni e Pensionati, Percorsi di Analisi", 2013

Ministero dell'Economia e delle Finanze, "La Spesa dello Stato dall'Unità d'Italia", 2011

Ministero del Lavoro e delle Politiche Sociali, "Gli Andamenti del Sistema Pensionistico Obbligatorio", 2002

Morcaldo Giancarlo, "Pensioni, Neccessità di una Riforma".

OECD Guidelines on Pension Fund Asset Management, 2006

OECD, "Pensions at a Glance 2017: OECD and G20 Indicators", OECD Publishing, Paris. 2017 *http://dx.doi.org/10.1787/pension_glance-2017-en*

"Risk Management and Asset and Liability Management in Banks", Asian Development Bank Technical Assistance Consultant's Reports, 2008

Squeglia Michele, "La gestione Finanziaria nei Fondi Pensione Negoziali tra Vincolo Previdenziale, Massimizzazione dei Rendimenti e Modelli Alternativi di Gestione", in Massimario di Giurisprudenza del Lavoro (Il Sole 24 ore), May 2011.

SITOGRAPHY

http://www.assogestioni.it/index.cfm/1,413,0,49,html/il-sistema-previdenziale 24 October 2018

https://www.ecnews.it/tre-pilastri-della-previdenza/ 24 October 2018

https://www.covip.it/?cat=319 25 October 2018

https://www.investopedia.com/terms/a/asset-liabilitymanagement.asp 29 October 2018

https://www.borsaitaliana.it/notizie/sotto-la-lente/asset-and-liability-management-banche167.htm 30 October 2018

https://www.focus-economics.com/countries/italy 5 November 2019

http://goofynomics.blogspot.com/2012/05/la-spesa-pubblica-al-bar-dello-sport.html 5 November 2018

https://www.borsaitaliana.it/notizie/speciali/pensioni/normativa/le-riforme-da-amato-ad-oggi/le-riforme-da-amato-ad-oggi.htm 6 November 2018

https://www.cambridgeassociates.com/research/liability-hedging-handbook-a-guide-to-best-practices-for-us-pension-plans/ 19 November 2018

http://www.pensionanalysis.com/CM/LearnAboutPensions/How-to-do-a-Pension-Valuation.html 29 November 2018

https://www.finanzaeinvestimenti.it/rubriche/pensione-integrativa/01519-previdenza-complementare-fondo-a-contribuzione-definita-e-fondo-a-prestazione-definita.html 3 December 2018