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Aging and Debt: An empirical analysis of U.S. trends

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Marieline Wehbe

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I hereby declare that I have read and understood the "Anti-plagiarism rules" approved by the Council of the Department of Economics and Management and I am aware of the consequences of making false statements. I declare that this thesis has not been previously submitted – either fully or partially – for fulfilling the requirements of an academic degree, whether in Italy or abroad. Furthermore, I declare that the references used for this work – including the digital materials – have been appropriately cited in the text and in the "References" section.

Student signature

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Abstract

In the last decade, public debt in the United States and many other countries, has been on the rise, posing a growing challenge to fiscal stability. While many economists recognize population aging as a significant economic concern, much of the focus has been on its effects on the broader macro economy. However, a closer look reveals that population aging may have serious implications for fiscal health, particularly through its influence on public debt. In this study, we examine how population aging affects public debt in the U.S. by analyzing key macroeconomic variables such as interest rates, economic growth, and the primary surplus. Our findings indicate that the trend component of the dependency ratio, representing the long-term aging of the population, is a primary driver of increasing public debt. In contrast, the cycle component of the dependency ratio, representing the short term fluctuations, did not have any noticeable effect on debt, as all of its responses were statistically insignificant. Policymakers must account for this demographic shift when developing strategies to manage debt and consider reforms that can address this growing fiscal pressure. Additionally, we discuss how migration and other policy reforms could play a crucial role in reducing the fiscal strain caused by population aging and maintaining debt sustainability. Further research is definitely needed on these and related topics to clarify the conclusions reached here. Nevertheless, we remain confident that demographic change presents a threat for fiscal stability, but may also open opportunities if addressed with the right policies at the right time.

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Chapter 1

Introduction

Over the last decade, federal debt has surged in the United States and is now higher compared to gross domestic product (GDP) at any point in the U.S. history, expect for the period following World War II. The U.S government is spending more money than it is taking in, so it borrows the difference to finance its spending. According to a report published by the Congressional Budget Office (CBO) in 2024, it is projected that Debt-to-GDP ratio will continue to climb, reaching 166 percent of GDP in 2054 (Figure 1.1). Understanding how the federal budget reached this point requires examining the key factors that have contributed to the significant rise in federal debt over the past decade. The government makes money in one key way: taxes. It then spends those money on healthcare and security systems, but it doesn't have enough money to spend it on those programs at once, so it borrows money. There is a huge debate between economists on how much is this debt important for the economy. Some economists generally think that it is okay for debt to increase when the economy needs a boost, noting that interest rates on long-term federal debt are extremely low, which justifies additional federal borrowing. The problem is that debt is set to keep on increasing to its highest level since 1946 even though the economy is now in a much better shape. Other observers argue that high debt means that the government has less fiscal space to address recessions or other unforeseen events, threatening the country's economic future. (Elmendorf and Sheiner (2017)) The

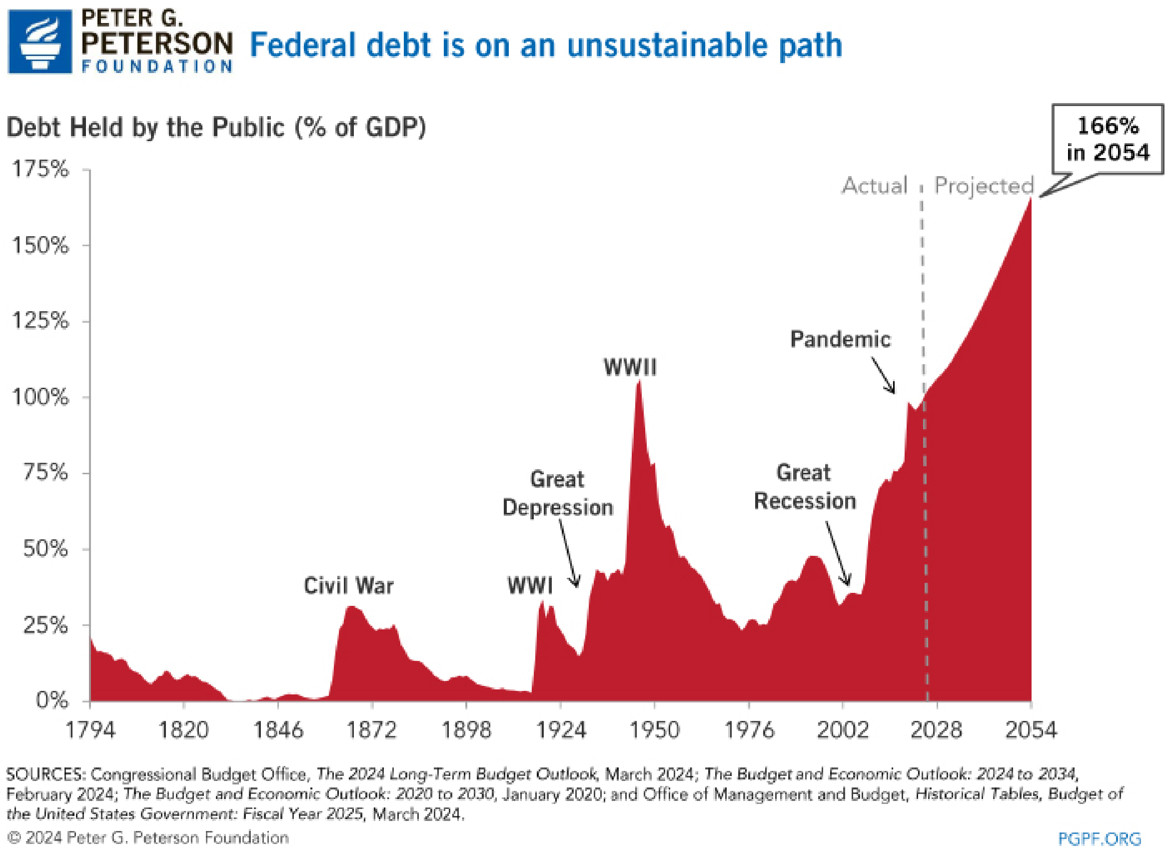


Figure 1.1: Federal debt held by the public

important question is: what is driving this public debt?

Many factors contribute to the increasing U.S. public debt, such as interest rates, productivity, primary deficit, social security and Medicare. One of the main issues that may lead to the increase in debt is demographics. The United States, along with other developed nations, is undergoing a significant demographic transition due to the unfolding of the post-war baby boom. Fertility rates began to decline, with mortality rates declining because of the advancements in medicine, technological changes, and extended knowledge. Therefore, the average life expectancy in the United States has increased and the population has aged. According to Bowens and Withrow (2024) projections, there will be 96.9 million seniors by 2070, an increase from 40.5 million seniors in 2010. (Figure 1.2)

The American population is getting older, meaning that the government have to pay more for programs for senior citizens. For example, spending will increase more on medicare, pensions, and social security. The baby boomer generation is getting older; they are retiring and they will definitely live longer, which will have a huge impact on the budget and debt. The report of Bowens and Withrow (2024) shows that the U.S. spends, on the average healthcare cost, $ 12,555 per person. According to the CBO's projections, federal spending on Medicare will rise from 3.2 percent of GDP in 2024 to 5.4 percent by 2054, with federal revenues not keeping pace with the increasing spending. Individual income taxes, which make up more than half of federal revenues, are expected to drop slightly in the near future, decreasing from 8.8 percent of GDP in 2024 to 8.6 percent the following year. After 2025, they are projected to rise again because of the planned expiration of certain parts of the 2017 Tax Cuts and Jobs Act. Even though America is known as one of the safest places to put your money in, economists have warn that a debt crisis is coming, where the U.S. wont be able to pay its bills. The US director of national intelligence Dan coats called the growing debt a "dire threat to our economic and national security".

According to Gagnon et al. (2021), the United States economy has entered a historical phase known as the "new normal" and characterized by low growth and low interest rates. This low-interest-rate environment has significant implications for public debt: it decreases the cost of borrowing for the government, leading to lower debt services, but it potentially leads the government to borrow more and substantially increases public debt levels over time. On the other hand, interest rates are expected to increase in the upcoming 30 years. Since March 2022, the Federal reserves has increased the fund rates 11 times while trying to fight inflation. The average interest rate on federal debt held by the public increased from 2.5 percent in 2023 to 3.1 percent in 2024. Interest rates on public debt were on average of 1.9 percent from 2014 to 2023, but this average is likely to increase to 3.5 percent from 2025 to 2054, meaning that some of the existing debt will be refinanced at higher costs. As mentioned in Peter G. Peterson Foundation (2024),

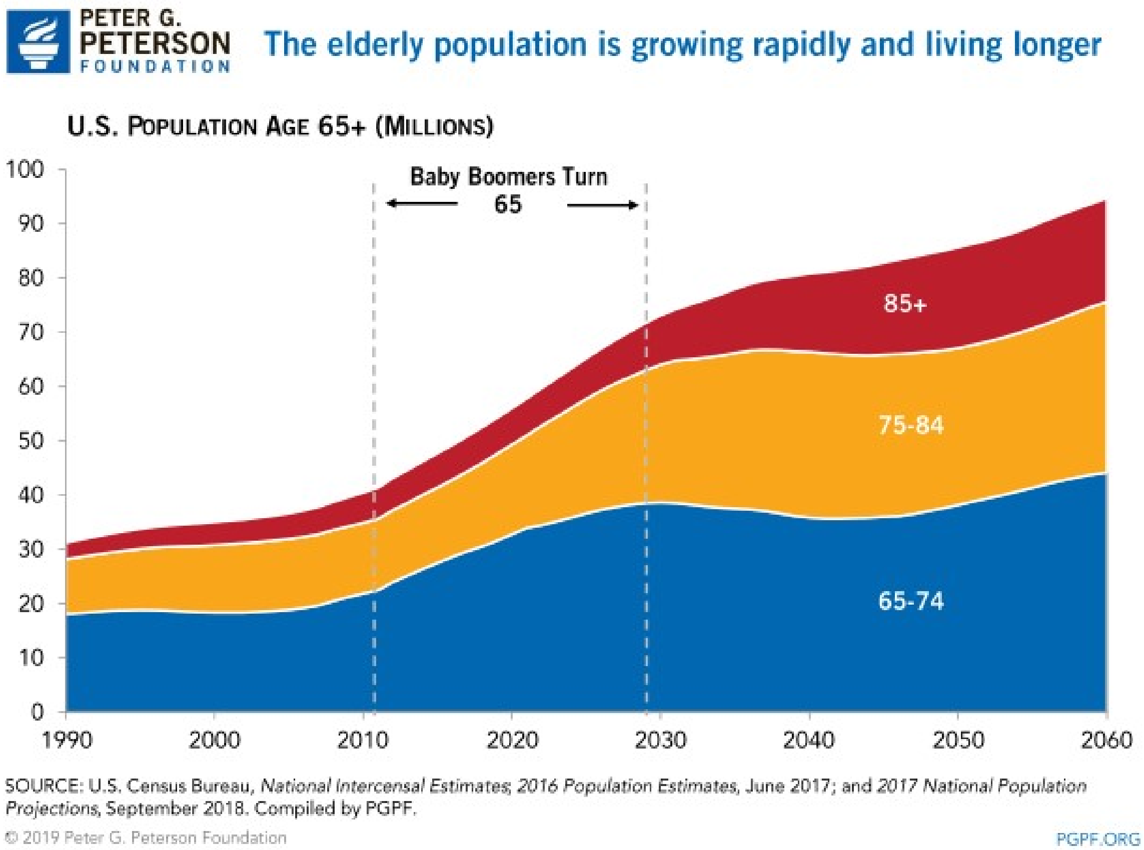


Figure 1.2: Projected senior population in the U.S. (65+)

interest costs on the federal debt will increase to 6.3 percent of GDP by 2054, where they would account for 34 percent of federal revenues.

This paper aims to understand how much of the increase in the United states public debt, as well as other macroeconomic variables, can be explained by demographic factors. Therefore we investigate the extent to which demographic changes can explain the high debt levels. In order to test our hypothesis, we decided to decompose the dependency ratio into two components: trend and cycle. The trend component captures the long term structural demographics shift that persist over time like the aging population coming from a decrease in birth rates and higher life expectancy. As for the cycle component, it captures the short term fluctuations that deviate from the long term trend in the dependency ratio. They are permanent deviations that do not impact the overall trend, such as economic booms or recessions, or health crisis.

Hence, we propose two hypothesis regarding the effects of demographic shifts on the public debt:

Hypothesis 1: A positive shock in the trend component of the dependency ratio will lead to a significant increase in public debt in the US.

A positive shock in this trend indicates an increase in the proportion of elderly individuals (known as dependents) relative to the working-age population. This demographic shift can strain public finances due to higher expenditures on healthcare, pensions, and social security, coupled with a relatively smaller tax base. As a result, the government may need to borrow more, leading to an increase in public deb.

Hypothesis 2: A positive shock in the cyclical component of the dependency ratio will lead to a slight and temporary increase in public debt in the US.

A positive shock in this cycle indicates a short-term increase in the dependency ratio, which may lead to higher public spending on social welfare programs or unemployment benefits during economic downturns. However, as these are temporary changes, the impact on public debt might be moderate and could be even close to zero as the economic conditions improve and the dependency ratio returns to its long-term trend.

In the face of global aging, understanding the implications of demographic shifts on public finances has never been more critical. It is crucial for policy makers to study the impact of demographic transitions on public debt, as well as on various economic indicators. A higher dependency ratio can lead to lower interest rates, reduced investments in productive assets, pressure on productivity and the overall economic output, and threats to the social security systems. Therefore, studying these effects comprehensively will help policy makers in enhancing fiscal stability and fostering sustainable economic growth. They can take many preventive measures to reduce the effects of an aging population, such as implementing fiscal reforms like increasing the retirement age based on life expectancy to make pension systems more resilient to demographic changes, encouraging higher labor force participation by including flexible working hours and support for working parents, and by implementing fiscal policy decisions that include taxation and government spending.

Chapter 2

Literature Review

To understand the global economy, it is important to understand the demographic changes and their challenges, especially when it comes to fiscal and monetary policy-makers. Population aging in the United States is a result of two main factors: the drop in fertility after the "baby boom" that followed World War II and the persistent increase in longevity. According to the United Nations, the U.S. fertility rate is 1.78 births per woman, which is less than the 2.1 replacement rate set by the UN to keep population stable. As mentioned in Mester (2024), this is due to the shifting preferences for smaller families because of the high costs of raising children and educating them. There was also a shift from rural to urban areas, which declined the need for more labors in the farms and therefore less children. Moreover, there was a decline in the mortality rates because of the technological advancements in medicine and public health. Therefore, the average life expectancy in the United States has increased and the population has aged. Demographic shifts can affect the growth rate of the economy through different channels: structural productivity growth, living standards, savings rates, consumption, and investment. It can also influence the unemployment rate and equilibrium interest rate, housing market trends, and the demand for financial assets (Mester (2024)). In this paper, we will discuss the demographic transition, and their influence on different macroeconomic variables, specifically the high level of US public debt.

2.1 Demography and its impacts on macroeconomic variables

2.1.1 Aging and the interest rates

The high and rising debt in the United States have threatened the country's economic future, reducing the government's fiscal space to respond to recessions or unexpected surprises, so economists wants to reduce the federal borrowing by asking the government for changes in taxes or spending policies. According to Elmendorf and Sheiner (2017), the two main factors that play a significant role in the US debt-to-gdp ratio are the aging of the US population and interest rates on US government debt. Since the Great Recession, real interest rates in the U.S. have remained at historically low levels. To help the economic recovery, the Federal Reserve kept the federal funds rate near zero for over seven years and purchased substantial amounts of long-term securities. There have been widely discussions about the causes and the macroeconomics consequences of this decline in interest rates, even some observers reviving the specter of secular stagnation, a prolonged period of slow economic growth and low interest rates (Hansen (1939); Summers (2014)). The decline in interest rates not only poses difficulties to monetary policy, but it also challenges fiscal policy. These low levels of interest rates have attracted significant attention from economists and policymakers, with various explanations being proposed as they seek to understand the driving factors behind this decline. Laubach and Williams (2016) argues that a key reason of the decline in interest rates is the fall in the growth rate of trend output. According to them, as the economy's potential growth slows, the demand for investment declines, leading to lower equilibrium interest rates. This is based on the idea that slower economic growth reduces returns on investments, pushing down interest rates over the long run. Alternatively, Hamilton et al. (2016) used a cross-country data going back to the 19th century, they found a weak correlation between the natural rate of interest and trend output growth, suggesting that other factors beyond growth may play important roles in determining interest rates. For the United

States, this relationship is a bit complicated as real interest rates were high during the 1970s and 1980s, even though productivity growth was relatively low during this period. Moreover, interest rates started declining in the 1990s, even as productivity growth accelerated. This means that factors other than productivity may influence the long-term interest rates.

Other explanations for the low levels of interest rates can be attributed to factors that can influence the desired levels of saving and investment. The most distinguished one is the ongoing demographic transition. In their paper, Gagnon et al. (2021) investigate the extent to which demographic shifts, especially those related to the baby boom, can explain the currently low levels of real interest rates and GDP growth in the U.S. They built an overlapping-generations (OG) model, which predicted that GDP growth and interest rates will remain low by historical standards, consistent with a "new normal" for the U.S. economy. They tried to understand how much of the new normal can be explained by demographic factors in the United States, and they found that demographic factors alone account for a little more than a 1 percentage point decline in the equilibrium real interest rate in the model since the 1980s. When they kept fertility, mortality, and employment rates all fixed at their 1960 values, Gagnon et al. (2021) saw that there would have been a slight increase in the equilibrium real interest rate since the 1980s. Differently speaking, the entirety of the decline in the equilibrium real interest rate that the model found for the recent decades is a direct consequence of the demographic changes that happened from 1960 onward. The baby-boom generation's transition from working age to retirement has contributed to slower labor supply growth and higher savings, both of which have led to lower real interest rates. In the 2000s, the baby boomers generation started to get out of the labor force, decreasing the labor supply as they move to retirement. This shift reduced GDP growth. Lower fertility rates and increased life expectancy led to a larger capital-to-labor ratio. With fewer new workers entering the labor force and more people saving for longer retirements, the supply of capital increased relative to labor, putting a pressure on the return on capital and, consequently, pushing down real interest rates. Increased life expectancy has led households to save more for retirement, further increasing the supply of capital and reducing real interest rates. The demographic changes have led to an abundance of capital relative to the supply of labor, which Gagnon et al. (2021) say has contributed to the persistent decline in real interest rates since the 1980s. Furthermore, Lunsford and West (2019) find that safe real interest rates are correlated as expected with demographic measures, with the long-run correlation between these real rates and labor force hours growth being positive (which is consistent with overlapping generations models), and the long-run correlation with the proportion of 40 to 64 year-old in the population being negative. These findings are in line with the standard theory (as well as with my results as shown in 4.1), which states that middle-aged workers are high savers who drive down real interest rates. They also show that there exists a positive long-run correlation between the safe rate and the dependency ratio. In conclusion, many economists argue that an increase in life expectancy leads workers to save more expecting a longer retirement period, therefore decreasing the interest rates.

2.1.2 Aging and the labor force

Life expectancy is on a rising path in the United States, adding years of life to the current working ages of 20 to 65, with projections that people will spend 24 percent of total expected life years in retirement in the year 2050, increasing from 19 percent in 2010. (Board of Trustees et al. (2011) and Lee (2014)). National Research Council (2012) projected life expectancy to rise to 84.5 by 2050, as well as several other studies. The future macroeconomic impacts of an aging population will partly depend on how long people decide to remain in the workforce. The shares of workers aged 55+ in the labor force will rise form 12 percent in 1990 to 27 percent in 2050, while the share of workers aged 25-54 decreasing, indicating that the population and the labor force will be aging (National Research Council (2012)). Demographics show that the labor force growth will be slower than it has been in the previous decades, which will put downward pressure on the long run economic growth. In addition, most of the literature suggest that the aging of the population may have a negative effect on structural productivity growth. To what extent is this true?

As mentioned in Mester (2024), there is historical evidence about the presence of a hump-shaped relationship between age and productivity, meaning that productivity increases when a person enters the workforce, stabilizes, and then decreases toward the end of a person's work life. Additional research shows that a person's s innovative activity and scientific output peak between the ages of 30 and 40, although that age profile has been increasing over time. Even though a lot of economists show that the aging of the population will negatively affect growth, some theories show otherwise. According to Mester (2024), the magnitude of the negative effect of the aging workforce on productivity growth appears to be quite small. To better determine the economy's long run growth, we should measure how effectively the economy combines its labor and capital inputs to create output. Looking at labor productivity, which measures the output per hour worked, gives us an insight of this growth. Mester (2024) argued that labor productivity has grown at an annual rate of only about a half of a percent, attributing part of the slowdown to cyclical, persistent effects of the Great recession. Older workers tend to stay longer in their jobs than younger workers, allowing them to gain deeper experience, which can be positive for productivity growth. On the other hand, this may force workers to remain in jobs that are not the best match to their skill sets, which will be negative for productivity growth. The problem is that we cannot know the exact magnitude and timing of these effects, as they depend on complicated dynamics and the behavior of consumers and businesses. Several research has been conducted to check whether individual performance in different domains varies with age, but it had no relevance for productivity. According to the projections of the National Research Council (2012), age distributions of the labor force between the years 2010 and 2030 has negligible effects on productivity. They also argued that other small literature on this relationship is very fragile, concluding that any effects appear to be very minimal. The only way by which population aging and postponed retirement can negatively affect innovation is by constricting the resources available to younger scientists, other than that there is no reason for population aging to decline production growth (Mester (2024)). These interpretations align with our results of the labor productivity response to the shock in the trend component of the dependency ratio (4.1), but it contradicts a huge literature which argues that an aging population will slow economic growth. For example, Maestas et al. (2023) found that a 10 percent increase in the population aged 60+ will reduce the per-capita GDP by 5.5 percent, estimating that the aging population decreased the growth rate in GDP per-capita by 0.3 percentage points per year between the years 1980-2010. On the other hand, Cutler et al. (1990) found that diminished fertility represents an opportunity rather than a problem; in their model, stating that with a low labor growth, labor becomes scarce, which may induce more rapid technical change. Based on their model, there is evidence that countries with slower labor force growth tend to experience faster productivity growth, which may offset the full consequences of increased dependency. Nevertheless, this result is uncertain, but a more definitive finding is the absence of empirical support for the pessimistic view that aging societies experience reduced productivity growth.

Furthermore, the anticipation of the demographic transition, more particularly the aging population, can give two opportunities for more rapid economic growth. As argued in Mason and Lee (2004), the first dividend occurs when we have a larger proportion of producers relative to consumers (dependents), leading to an economic boost, but this effect is transitory. Whereas the second dividend occurs when the aging population starts to save more in anticipation, leading to capital deepening and higher productivity if these savings are well invested in productive assets. However, this effect will only be realized if early reforms and appropriate policies are taken, promoting capital accumulation rather than the reliance on family transfers. The authors suggest several policies that are necessary to achieve the second demographic dividend. One of the most important policies suggested is the pension reforms, where people should shift from reliance on intergenerational transfers such as PAYGO pension systems, to policies that promote private savings and investments. This will encourage people to save for their own retirement instead of relying only on public pensions. The paper also promoted the need for financial market reforms, which strengthen financial institutions to ensure they can manage and grow retirement savings effectively, converting them into productive investments. Investments in infrastructure, technology, and businesses are crucial to ensure that savings lead to growth. They also emphasize on the importance of investing in human capital through lifelong learning programs and health investments, to ensure that workers are up to date with technological advancements, and highly productive even as they age. In addition, they emphasized on promoting policies like increasing the retirement age and promoting part time work for the elderly to encourage longer labor force participation, allowing older workers to stay productive for longer periods. Finally, Mason and Lee (2004) concluded that population aging, if managed through effective policies that tackle the second demographic dividend, can stimulate capital accumulation and potentially lead to sustained economic growth.

While this analysis focuses on demographic factors, particularly aging population, it is important not to forget other significant factors that influence productivity as well. One important factor is the increase in women's labor force participation, driven by declining birth rates, the rising costs of children, and higher levels of women's education, which allowed more women to join the labor force. Lee (2003) has estimated that women went from spending 70 percent of their adult lives bearing and rearing young children before the demographic transition, to spending only about 14 percent more recently. All of these factors facilitated women's entry into a wide range of occupations, leading to a higher labor force participation. With higher investments in education, women and men tend to enter the workplace, resulting in a more productive labor force (Bloom et al. (2003)). Another main factor is migration, especially the migration of young, productive labor to aging countries which can have a significant positive impact on productivity and economic growth. By providing essential labor and skills, immigration helps reduce labor shortages, while remittances and the return of migrants with enhanced human capital further contribute to growth in both sending and receiving countries (Hatton and Williamson (1998, 2006)). Lastly, technological advancements have the potential to significantly boost productivity by improving the efficiency of labor. Research shows that innovations in technology can lead to sustained growth in labor productivity, as demonstrated by Lunsford and West (2019), who highlight the positive impact of technological progress on economic output.

2.2 Demography and Fiscal policy

2.2.1 Public debt drivers

There are a lot of factors that are contributing to the increasing trend of the public debt. We can refer to the law of motion of debt to better analyse those factors.

$$\frac{B_{t+1}}{Y_{t+1}} = \left(\frac{1+r}{1+g}\right)\frac{B_t}{Y_t} - \frac{S_t}{Y_t}$$

Where:

- B_t is the public debt at time t,
- Y_t is the GDP at time t,
- r is the real interest rate,
- g is the real growth rate of the economy,
- S_t is the primary surplus at time t (or primary deficit if negative),
- $\frac{B_t}{Y_t}$ is the debt-to-GDP ratio at time t.

Looking at this formula, we can see three direct factors that contribute to the increasing public debt: interest rates, growth rate or productivity, and the primary deficit.

Higher interest rates directly raises the cost of servicing existing debt. As the government pays higher interest on its outstanding obligations, more resources are redirected toward debt repayment, leading to an overall rise in public debt unless offset by higher revenues or lower spending.

The primary deficit, defined as the gap between government spending and revenues (excluding interest payments), also contributes to rising debt. When the government increases its expenditures without increasing its taxes or its revenues, it is accumulating more deficit. A higher primary deficit implies that the government is borrowing to finance not only its interest obligations but also its regular expenditures. As borrowing increases to cover these deficits, the public debt grows.

The growth rate, on the other hand, plays a crucial role as well. A higher growth rate increases the denominator (GDP) in the debt-to-GDP ratio, which can help reduce debt relative to the size of the economy, and vice versa. However, the U.S had experienced a period of low productivity. This low productivity can lead to slow growth, which in turn will reduce the economy's capacity to generate sufficient output and tax revenues. This slower growth increases the debt burden by shrinking the government's fiscal space to manage or reduce debt sustainably. Low productivity further restrains economic expansion, making it harder to offset rising debt through economic growth alone.

The fourth driver that is not equally talked about by macroeconomists is demographics. It shows implicitly in the law of motion of debt, so let's develop the formula we saw above to get:

$$b_{t+1} = \frac{(1+r_t)}{(1+g_y+g_n)}b_t - (T_t - G_t)$$

Where:

- $g = g_y + g_n$
- $PS = T - G$

We can divide the growth rate (g) into two components: Per capita GDP growth g_y which reflects productivity, and population growth g_n which reflects demographic changes, so the overall growth rate can be represented as $g = g_y + g_n$. An aging population can lead to lower population growth g_n, meaning fewer working-age people, which might slow the overall economic growth, as the number of contributors to the economy decreases. This in turn puts upward pressure on the debt-to-GDP ratio because the economy grows more slowly, making it harder to offset debt through growth alone. Moreover, productivity affects per capita GDP growth g_y. If productivity is low, even with a stable working-age population, economic growth remains slow. Therefore, a low growth rate, driven by both demographics and productivity, leads to a higher debt burden.

Demographics also implicitly appear their effect on the public debt in the formula through the primary surplus which is taxes minus government expenditures. An aging population leads to higher government spending on healthcare and pensions, we will discuss this in the following section in details. This leads to a reduction in the primary surplus or an increase in the primary deficit, especially if these costs increase faster than revenues from a shrinking workforce. Therefore, this highlights how demographics play an important role as public debt drivers.

2.2.2 Aging and Public debt

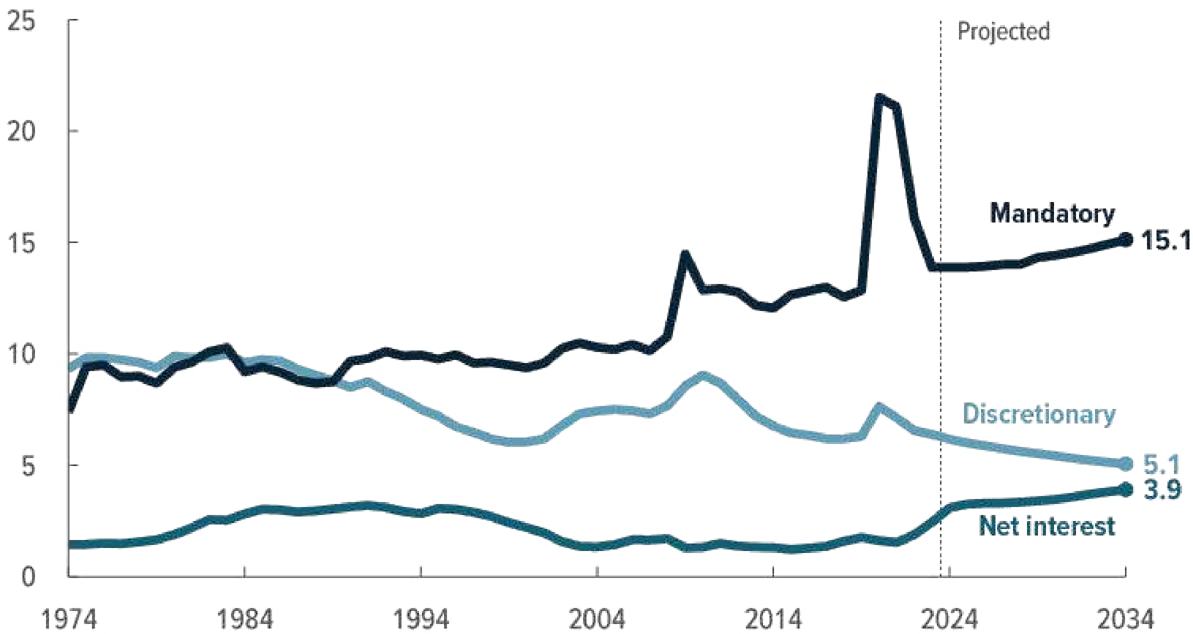
The rapid growth in the share of Americans aged 65 and above has sharply increased spending for Social Security, Medicare, and certain other federal programs relative to GDP, which will increase the per-capita healthcare spending. In addition, population aging will lower the share of the population in the labor force, which will reduce the consumption relative to what it would have been otherwise. Therefore, Elmendorf and Sheiner (2017) show that the optimal social response to population aging would be an increase in national savings in about 1 percent of GDP over the coming decade. In the projections of the Congressional Budget Office (CBO) (2016), federal deficits increase to approximately 5 percent of GDP by 2026, and federal debt held by the public is projected to reach 141 percent of GDP by 2046, and to be on a rising trajectory. According to

Elmendorf and Sheiner (2017), the main reason behind this increase in projected deficits and debt is the growth in federal spending for older Americans and for health care that is not fully offset by cutting in other spending or rises in revenues. Population aging and a projected increase in per-capita healthcare spending now explain more than all of the projected growth in noninterest federal spending over the next few decades (Elmendorf and Sheiner (2017)). Population aging also lowers the share of the population in the labor force, which diminishes the consumption relative to what it would be otherwise, knowing that the normal response to aging population would be higher savings. The key factor leading to the projected increase in deficits and debt is growth in federal spending for older Americans and for health care that is not fully offset by reductions in other spending or increases in revenues (Cutler et al. (1990)). According to the Congressional Budget Office (CBO) (2024), mandatory spending, which is defined by the CBO as government expenditures required by law for programs such as Social Security and Medicare, is projected to increase substantially. As population ages and life expectancy continues to rise, more people will retire and the number of individuals eligible for these programs increase, leading to higher expenditures. As shown in 2.1, mandatory spending is expected to rise to 15.1% of GDP in 2034, up from a level of 13.9% in 2023, showing the pressure exerted by demographic shifts. Spending on Social Security and other healthcare programs like Medicare will mostly contribute to this rise. The primary deficit is projected to worsen from -3.8% of GDP in 2024 to -2.8% by 2034, as outlays exceed revenues (as shown in Fig. 2.1). This increasing gap between revenues and expenditures is due to the increasing mandatory spending on aging-related programs, which will force the government to borrow more, contributing directly to growing public debt because, as the government is forced to borrow more to cover its deficit, the debt held by the public will increase substantially (as we can see the increase from the year 2024 to 2034). The increase in mandatory spending compared to discretionary spending and net interest in fig. 2.2, shows that much of this fiscal strain will come from aging-related programs. Cutler et al. (1990) stated that most of the government expenditures comes from spending on individuals aged 65 and above, with average outlays in 1986 of $6,138 per person, on social security only. The government expenditures on the three programs (social security, healthcare, and education) combined, for the elderly, is more than double that of any other group. The rising cost of net interest payments due to higher debt levels worsens this issue. By 2034, net interest is projected to rise to 3.2% of GDP, up from 2.4% in 2024. As these interest payments rise, they further strain the federal budget, leading to a place where more debt is issued to cover the deficit and interest payments. Rising interest rates are an important driver of public debt as they increase the cost of servicing existing debt, leading to higher net interest payments. These increasing payments will lead the government to borrow more in order to cover both the interest obligations and other expenditures, further adding to the overall public debt. Therefore, we can see how an aging population can have long-term fiscal consequences by increasing mandatory spending and driving up both the primary deficit and the overall federal debt.

		Percentage of GDP				Billions of dollars			
	Average, 1974–2023	Actual, 2023	2024	2025	2034	Actual, 2023	2024	2025	2034
Revenues	**17.3**	**16.5**	**17.5**	**17.1**	**17.9**	**4,439**	**4,935**	**4,996**	**7,474**
Individual income taxes	8.0	8.1	8.8	8.6	9.5	2,176	2,469	2,520	3,973
Payroll taxes	6.0	6.0	5.9	5.9	5.9	1,614	1,663	1,734	2,466
Corporate income taxes	1.8	1.6	2.0	1.7	1.3	420	569	494	551
Other	1.5	0.8	0.8	0.8	1.2	229	234	247	485
Outlays	**21.0**	**22.7**	**23.1**	**23.1**	**24.1**	**6,123**	**6,517**	**6,768**	**10,032**
Mandatory	11.0	13.9	13.9	13.9	15.1	3,742	3,908	4,061	6,298
Social Security	4.4	5.0	5.2	5.3	5.9	1,348	1,453	1,545	2,471
Major health care programs	3.4	5.8	5.6	5.5	6.7	1,556	1,574	1,619	2,781
Medicare	2.1	3.1	3.2	3.2	4.2	832	896	940	1,740
Medicaid, CHIP, and marketplace subsidies	1.3	2.7	2.4	2.3	2.5	724	678	679	1,042
Other mandatory	3.2	3.1	3.1	3.1	2.5	838	881	897	1,046
Discretionary	8.0	6.4	6.2	6.0	5.1	1,722	1,739	1,756	2,106
Defense	4.2	3.0	2.9	2.9	2.5	805	822	845	1,034
Nondefense	3.7	3.4	3.3	3.1	2.6	917	917	911	1,071
Net interest	2.1	2.4	3.1	3.2	3.9	659	870	951	1,628
Total deficit (-)	**-3.7**	**-6.2**	**-5.6**	**-6.1**	**-6.1**	**-1,684**	**-1,582**	**-1,772**	**-2,557**
Primary deficit (-)	-1.6	-3.8	-2.5	-2.8	-2.2	-1,025	-712	-821	-929
Debt held by the public at the end of each period	48.3	97.3	99.0	101.7	116.0	26,240	27,897	29,749	48,300

Figure 2.1: The Budget outlook, by fiscal year, for the U.S.

Notes: Source: Congretional Buget Office (CBO), February 2024. *The Budget and Economic Outlook: 2024 to 2034* (Congressional Budget Office (CBO) (2024)). *P.S: Deficits and outlays have been adjusted to exclude the effects of shifts that occur in the timing of certain payments when the fiscal year begins on a weekend. Without those adjustments, the deficit projected for 2024 is $1.5 trillion (or 5.3 percent of GDP).*



(Congressional Budget Office (CBO) (2024)).

Figure 2.2: Outlays, by category, for the United States

Notes: Source: Congretional Buget Office (CBO), February 2024. *The Budget and Economic Outlook: 2024 to 2034*

In this paper, we aim to investigate the impact of demographic changes, particularly an aging population, on the U.S. public debt. Our study is motivated by the increasing burden of mandatory spending on programs such as Social Security and Medicare, driven by an aging population. We focus on the old-age dependency ratio (OADR) as a key indicator of demographic shifts. To capture the effects of these changes, we estimate a model that analyzes the response of the debt-to-GDP ratio to a shock in the trend component of the OADR, including other macroeconomic variables. By focusing on the trend, we aim to understand the structural long-term effects of an aging population on public debt. In the the next chapter, we will see a detailed discussion of the data used in our analysis, explain the estimation methodology, and present the results of our model.

Chapter 3

Data and Model specification

3.1 Macroeconomic Time Series

This paper uses yearly data from 1966 to 2019, gathered from the Federal Reserve Economic Data (FRED), as shown in figure 3.1. Both labor productivity and NRI are gathered from the paper of Lunsford and West (2019). To measure demographics, we used the old age dependency ratio, which measures the proportion of elderly people (people older than 65) to the working-age population (ages 15-64). FRED calculates this ratio by dividing the number of the population over 65 by the population aged 15-64, then multiplying by 100 to express it as a percentage. As for measuring public debt, we took the Debt-to-GDP ratio, which compares the country's public debt to its gross domestic product (GDP), indicating the size of the debt related to the country's economic output. It is expressed as a percentage of the total public debt by the GDP of this country at the same year. We used the Natural Rate of Interest (NRI), which is the real interest rate consistent with the economy operating at full capacity while maintaining stable inflation. We also used Labor Productivity, which measures the amount of economic output that is generated per unit of labor input. It is usually calculated by dividing the total output (GDP) by the number of hours worked. In our model, we transformed this variable using $100 \times \log(\text{LaborProductivity})$ to linearize its relationship with other variables in the model. We also included the primary surplus, which is the difference between a government's current revenue (excluding debt interest payments) and its current expenditures (it is expressed as a percentage of GDP). Lastly, we included hours index, which represents the total number of hours worked by all employees in the economy. It is also transformed by using 100xlog(hoursindex).

The selection of these variables for my model is based on the law of motion for public debt. This formula captures the relationships between debt, interest rates, economic growth, and fiscal policy, providing a theoretical basis for the inclusion of key macroeconomic variables:

$$B_t = (1 + r - g)B_{t-1} + PD_t$$

In my model, I have chosen to use labor productivity and hours worked instead of GDP growth as a measure of economic performance. As Lunsford and West (2019) argued in their paper, GDP growth can be influenced by short-term fluctuations and does not always reflect the underlying drivers of economic output. They also mentioned that total factor productivity (TFP), which represents improvements in efficiency, including technological advancements, can be volatile and harder to measure consistently over time. Labor productivity, which captures the efficiency with which labor is used, and hours worked, which reflects the total labor input, offer a more direct understanding of the factors contributing to long-term growth. By focusing on these variables, I aim to capture the structural components of economic performance that are crucial in understanding the impacts of demographic changes on growth. Moreover, we checked the correlation between both TFP and labor productivity, getting 0.988, as well as the correlation between both RDGP and labor productivity, getting as well a correlation of 0.994 (check Appendix for the scatter plots). The high correlation between labor productivity and both variables indicates that labor productivity is a strong proxy for output growth, making it a suitable variable to use in place of real GDP in our model. This high correlation indicates that labor productivity effectively captures the same underlying economic dynamics as real GDP or TFP.

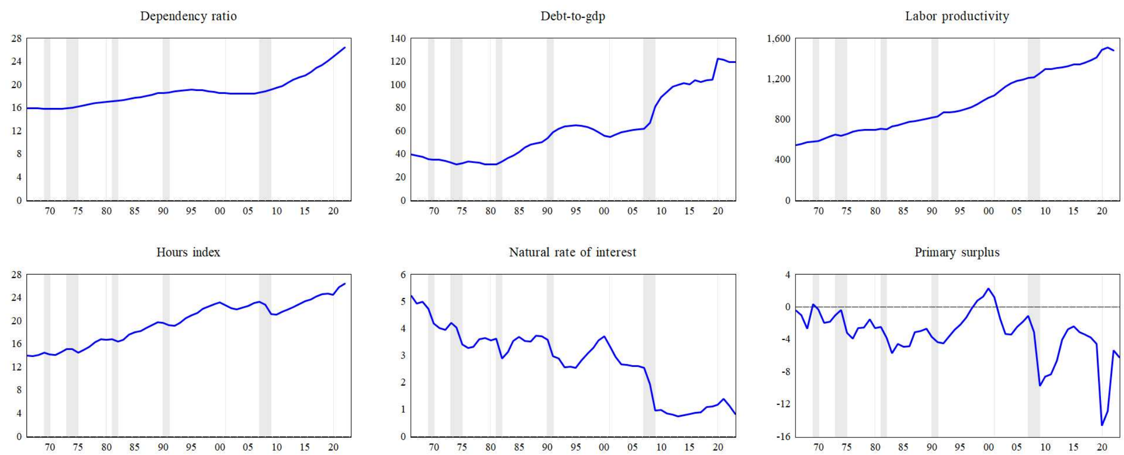


Figure 3.1: Macroeconomic time series

3.1.1 Decomposition of the dependency ratio

We then decomposed the dependency ratio into trend and cycle (as observed in figure 3.2), using the Hodrick-Prescott (HP) filter on Eviews, because of the persistency of the data, thus we wanted to differentiate between the long-term structural components and the cyclical, temporary fluctuations.

The HP filter is used to decompose a time series into a trend and a cyclical component. As Hodrick and Prescott (1997) explained in their article, the filter solves the following optimization problem:

$$\min_{\tau} \left\{ \sum_{t=1}^{T} (y_t - \tau_t)^2 + \lambda \sum_{t=2}^{T-1} \left[(\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1}) \right]^2 \right\}$$

where:

- y_t is the actual data at time t,
- τ_t is the trend component of the data at time t,
- λ is the parameter that controls the smoothness of the trend component.

In our case, y_t represents the dependency ratio at time t, and the objective is to separate τ_t (trend component) from c_t (cycle component), though not explicitly mentioned in the initial equation, where:

$$c_t = y_t - \tau_t$$

Here our objective is to minimize the fluctuations around the trend to isolate τ_t that represents the long-term demographic shifts impacting the dependency ratio, such as aging population, or changes in birth rates.

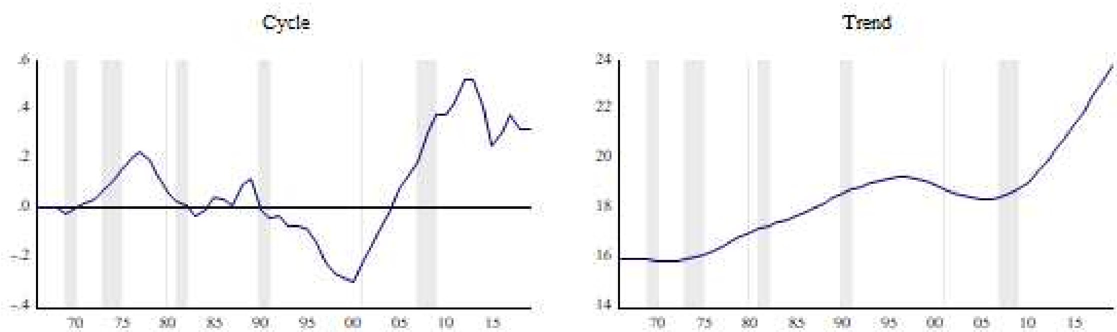


Figure 3.2: The trend and the cycle components of the dependency ratio.

3.2 The Model

Let y_t be 7-dimensional vector of seven endogenous variables: both the trend and the cycle of the dependency ratio, debt-to-gdp, labor productivity, hours index, primary surplus, and the natural rate of interest. The dynamics of these variables are captured by a structural vector autoregressive model of order p, SVAR(p):

$$B_0 y_t = A_0 + \sum_{i=1}^{p} A_i y_{t-i} + \epsilon_t$$

where ϵ_t is the vector of innovations at time t, denoting a mean zero serially uncorrelated error term, also referred to as a structural innovation or structural shock. The non-singular matrix B_0 is the matrix of contemporaneous coefficients, and A_i represents the coefficient matrices for lags $i = 1, \dots, p$. The error term is assumed to be unconditionally homoskedastic, unless otherwise noted. Consequently, the variance-covariance matrix of ϵ_t is an identity matrix:

$$E(\epsilon_t \epsilon_t') = I_n \quad \text{and} \quad E[\epsilon_t \epsilon_t' | s \neq t] = 0 \quad \forall s \neq t.$$

By dividing the structural VAR by B_0, we get the reduced form version of the SVAR model as follows:

$$y_t = \mathcal{A}_0 + \mathcal{A}_1 y_{t-1} + \cdots + \mathcal{A}_p y_{t-p} + u_t$$

where

$$\mathcal{A}_0 = B_0^{-1} A_0 \quad \text{and} \quad \mathcal{A}_i = B_0^{-1} A_i$$

and the reduced-form VAR innovations are linear combinations of the structural shocks:

$$u_t = B_0^{-1} \epsilon_t$$

.

Furthermore, the reduced-form error variance-covariance matrix is:

$$E(u_t u_t') \equiv \Sigma_u = B_0^{-1} B_0'^{-1}$$

Formally, B_0^{-1} collects the impact coefficients. Due to the symmetry of Σ_u, the last equation represents a system of $\frac{n(n+1)}{2}$ independent equations. This system can be solved for the unknown parameters in B_0^{-1} using numerical methods, as long as the number of unknown parameters in B_0^{-1} does not exceed the number of independent equations in the equation. In order to get there, additional restrictions on B_0^{-1} need to be imposed.

3.3 Identification of the structural shocks

According to Cesa-Bianci, the identification problem consists in finding a mapping from the reduced form VAR to its structural counterpart:

$$u_t = B \epsilon_t$$

. This means that the contemporaneous structure of the model is typically underdetermined —there are more potential relationships among variables than the data can uniquely resolve. This means that without additional constraints, the coefficients of the VAR model cannot be uniquely determined from the observed data alone. In VAR models, a common technique that is used often to resolve this problem is the Cholesky decomposition, or the zero contemporaneous restrictions. It involves decomposing the variance-covariance matrix of the innovations into a lower triangular matrix L and its transpose:

$$\Sigma = LL'$$

This decomposition implies a specific ordering of the variables, assuming that variables earlier in the order can contemporaneously affect those later in the order, but not vice versa. This assumption leads to a unique solution for the structural shocks.The Cholesky decomposition also implies that the upper triangular part of the matrix B_0^{-1} is zero. It implies that there are no instantaneous effects of later variables on earlier ones in the ordering, facilitating clear identification of the model. These zero constraints are very important in order to achieve identification in the structural model.

$$\begin{bmatrix} \text{Trend}_t \\ \text{Cycle}_t \\ \text{LABPROD}_t \\ \text{HRSINDX}_t \\ \text{NRI}_t \\ \text{DEBT}_t \\ \text{SURP}_t \end{bmatrix} = A_0 + \sum_{i=1}^{p} A_i y_{t-i} + \begin{bmatrix} b_{11} & \cdots & b_{17} \\ \vdots & \ddots & \vdots \\ b_{71} & \cdots & b_{77} \end{bmatrix} \begin{bmatrix} \epsilon_t^{\text{TREND}} \\ \epsilon_t^{\text{CYCLE}} \\ \epsilon_t^{\text{LABPROD}} \\ \epsilon_t^{\text{HRSINDX}} \\ \epsilon_t^{\text{NRI}} \\ \epsilon_t^{\text{DEBT}} \\ \epsilon_t^{\text{SURP}} \end{bmatrix}$$

3.3.1 Imposing zero restrictions

We then impose zeros for the upper triangle, here is the transmission matrix:

$$\begin{bmatrix} b_{11} & 0 & 0 & 0 & 0 & 0 & 0 \\ b_{21} & b_{22} & 0 & 0 & 0 & 0 & 0 \\ b_{31} & b_{32} & b_{33} & 0 & 0 & 0 & 0 \\ b_{41} & b_{42} & b_{43} & b_{44} & 0 & 0 & 0 \\ b_{51} & b_{52} & b_{53} & b_{54} & b_{55} & 0 & 0 \\ b_{61} & b_{62} & b_{63} & b_{64} & b_{65} & b_{66} & 0 \\ b_{71} & b_{72} & b_{73} & b_{74} & b_{75} & b_{76} & b_{77} \end{bmatrix}$$

The order of the variables determines how contemporaneous shocks to one variable can affect others within the same period. Here's the proposed order for our variables, along with the rationale for each choice:

- 1. Trend Component of Dependency Ratio: it represents long-term demographic shifts, which are persistent changes affecting all other macroeconomic variables over extended periods, which justifies its placement first

- 2. Cycle Component of Dependency Ratio: Placed after the trend because it is also a demographic measure, but it captures shorter-term fluctuations compared to the overarching trends

- 3. Labor Productivity: influences and is influenced by labor market dynamics, usually follows demographic shifts.

- 4. Hours Index: closely related to labor productivity

- 5. Natural Rate of Interest (NRI): A key financial indicator that responds to changes in labor markets and productivity, as well as demographic changes.

- 6. Debt-to-GDP Ratio: Reflects fiscal health and is influenced by the overall economic conditions as well as demographic shifts.

- 7. Primary Surplus: Positioned last, it is directly influenced by the debt levels and the broader economic conditions.

By placing the trend component of the dependency ratio first, we impose the restriction that long-term demographic trends can contemporaneously affect all other variables in our system, but no other variables can contemporaneously affect this trend. This reflects the idea that demographic transitions are exogenous to short-term economic conditions. Putting the cycle secondly comes from the economic theory which states that while short-term factors can respond to long-term trends, they do not alter those trends in the immediate term. The placement of the labor productivity after that comes from the theory that productivity levels, while crucial for economic output, are themselves affected by underlying demographic structures, such as the working-age population, and are not immediate drivers of demographic changes. The hours worked in the economy are placed after productivity, because of the idea that labor market adjustments, such as changes in hours worked, are a response to existing economic conditions rather than a cause of them. The placement of the NRI aligns with the theory that the natural rate is a resultant condition of the economy, shaped by deeper structural forces rather than a direct influence of those forces in the short term. The debt-to-GDP ratio is placed after because we assumed that it is influenced by the demographics, productivity, and interest rates, but does not have a contemporaneous effect on them. Lastly, the primary surplus, which reflects fiscal policy decisions, is assumed to be influenced contemporaneously by all other variables but does not immediately influence them. This comes from the theory that fiscal policy responds to economic conditions rather than driving them contemporaneously.

Chapter 4

Structural Dynamic Analysis

4.1 Structural Impulse Responses

Our interest usually is not in the structural shocks ϵ_t themselves, but in the responses of each element in the vector y_t to a one time impulse in $\epsilon_t^{\text{TREND}}$ or to a one-time impulse in $\epsilon_t^{\text{CYCLE}}$. (Kilian and Lütkepohl (2017))

$$\frac{\partial y_{j,t+i}}{\partial \epsilon_t^{\text{TREND}}} = \theta_{j,i}^{\text{TREND}}$$

for $j = 1, \ldots, 7$ and $i = 0, \ldots, H$.

In the upcoming part, we will represent the dynamic responses of our endogenous variables to a trend shock as well as to a cycle shock.

4.1.1 Impulses Responses to a trend shock

To test **Hypothesis 1** which states that a positive shock in the trend component of the dependency ratio will lead to an increase in public debt in the US, we used a VAR model with 2 lags. It identifies a trend shock in the dependency ratio using the zero contemporaneous restrictions, as we explained earlier. We will see the response of all of our variables to a shock in the trend component, but our primary aim is to see whether the debt-to-GDP ratio, representing the public debt, will increase after a trend shock.

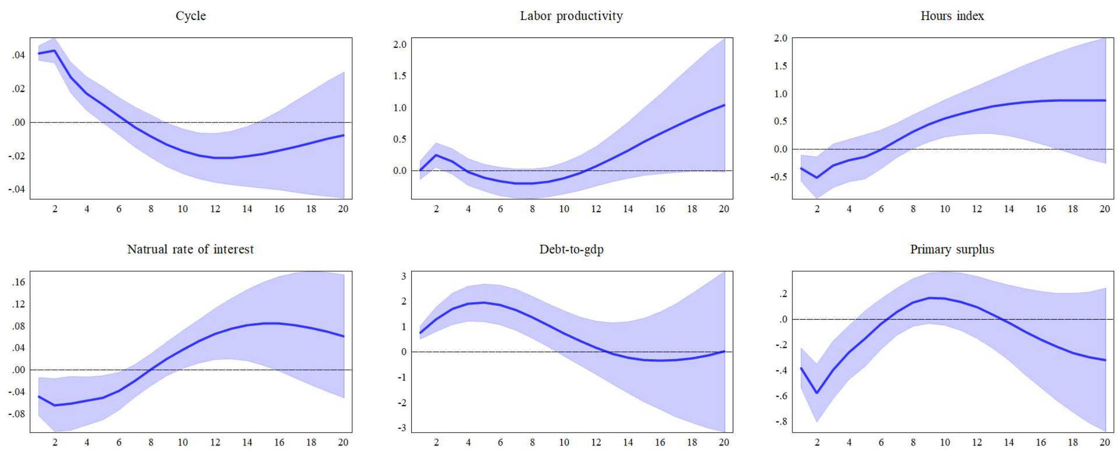


Figure 4.1: The impulse response function (IRF) of a shock in the trend component of the dependency ratio

Notes: Impulse responses of the endogenous variables to a one standard deviation shock in the trend component of the dependency ratio. Sample record: 1966-2019. The blue solid lines depict the impulse responses, while the shaded areas represent the 68% Analytic asymptotic confidence intervals.

As we can see in Figure 4.1, Debt-to-GDP ratio increases when the trend component of the dependency ratio increases, indicating that a higher dependency ratio leads to an increase in public debt relative to GDP. As mentioned before, trend component represents long-term structural changes in the dependency ratio like an aging population or a change in the birth rates. The positive response of the debt here can be due to a rise in government spending on pensions, healthcare, and other services for the aging population, combined with slower economic growth, which reduces the denominator (GDP) in this ratio. This positive response of the debt-to-GDP ratio to a shock in the trend component of the dependency ratio aligns with my hypothesis, proving our theoretical expectation that an increase in the aging population (coming from the rise in the dependency ratio), would increase the public debt. An aging population means that the government has to increase its spending on pensions and healthcare, leading to higher fiscal pressures. The response function of the debt-to-GDP ratio supports our view that demographic shifts significantly contribute to the accumulation of public debt over time.

Looking at the response of the Natural rate of interest (NRI), we can observe a decrease in the NRI following the trend shock. This is consistent with the hypothesis that demographic shifts decrease the demand for investment and slow economic growth, leading to a lower equilibrium interest rate. As mentioned in Gagnon et al. (2021), demographic changes lead to a higher capital-to-labor ratio, increasing savings and reducing the return on capital, which in turn will cause downward pressure on interest rates. This suggests that a higher proportion of dependents in the population, initially lowers the natural rate of interest. In addition, older people increase the demand on safe assets with low-risk, like government bonds, which drives the price of the bonds up, leading to lower rates due to the inverse relationship between prices of bonds and the interest on them.

The primary surplus responds negatively to the trend shock, implying that the government runs larger deficits in response to an increase in the dependency ratio. This could be due to higher spending obligations that come from an aging population, accompanied maybe by lower taxes revenue that are caused by a lower working age population. This will force the government to borrow more in order to finance its expenditures, which aligns with the response of public debt to the trend shock.

Looking at the Labor productivity response to the trend shock, we can see a small increase at period 2, but the effect becomes insignificant later. The small positive effect that we saw at the beginning, shows that labor productivity will increase right after a trend shock, then this increase starts to fade, only to become insignificant in the long run. The increase at first can be due to adaptive improvements in technology or efficiency as the economy adjusts to the demographic change. We can refer to Mester (2024), in which it was mentioned that with an aging population, older workers tend to stay longer in their jobs than younger workers, who are more likely to change jobs and employers. This allows older workers to gain deeper experience, which can be positive for productivity growth, and might explain the positive response of labor productivity in our analysis. Our results here align with National Research Council (2012), who showed that aging has a negligible effect on productivity, it is actually very minimal. In addition, as discussed in the previous sections, Mason and Lee (2004) argued that a second demographic dividend can arise if the aging populations increase the incentive to save and accumulate capital in preparation for retirement. This capital deepening can lead to sustained growth in output per worker. In other words, we will have a higher capital-labor ratio, which might increase the productivity. As mentioned above, our results puzzling regarding the response of the labor productivity, because they are not really clear and they do not really show a clear path for productivity in response for population aging.

As for the hours index, they show a negative response on impact, meaning that fewer workers are available as a response to the increase in the dependency ratio. This will necessitate longer hours of work for the remaining workers to maintain economic output, which explains the later increase in the hours index. The remaining working population may be compensating for the demographic shift by working more hours, which increases the hours index in the long run, or there could also be policy responses like higher retirement age, that push people to stay in the labor market longer.

4.1.2 Impulse responses to a cycle shock

To test **Hypothesis 2,** which states that a positive shock in the cycle component of the dependency ratio will barely affect the public debt, having a slight and short term impact on it, we also used a VAR model with 2 lags.

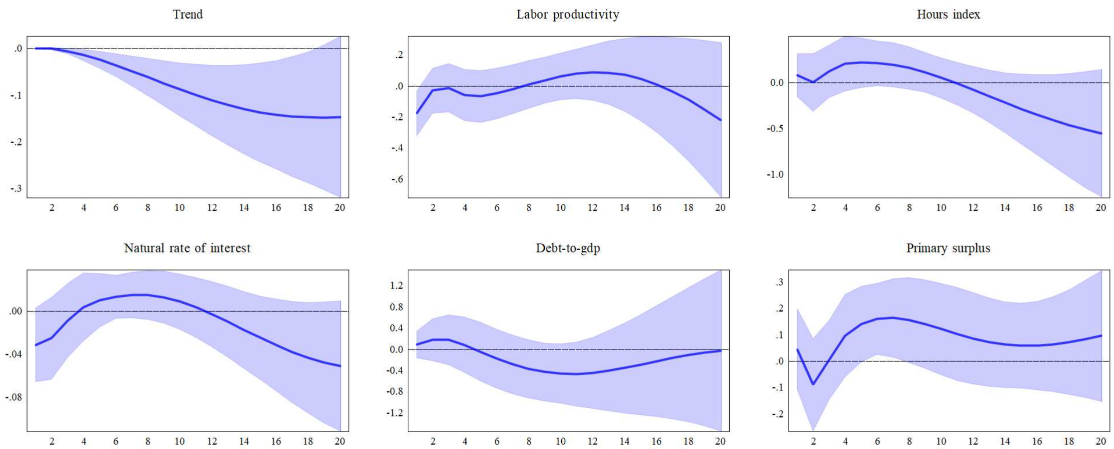


Figure 4.2: The impulse response function (IRF) of a shock in the cycle component of the dependency ratio

Notes: Impulse responses of the endogenous variables to a one standard deviation shock in the cycle component of the dependency ratio. Sample record: 1966-2019. The blue solid lines depict the impulse responses, while the shaded areas represent the 68% Analytic asymptotic confidence intervals.

As we can see from Fig. 4.2, the confidence interval consistently includes the zero in all of the responses, showing that short-term fluctuations in the dependency ratio do not have a significant immediate impact on all of our variables, including the debt-to-gdp ratio.

The results from these IRFs show that the shock to the cycle component of the dependency ratio has statistically insignificant effects on all the key macroeconomic variables considered in our analysis. This indicates that short-term fluctuations in the dependency ratio do not have substantial or lasting impacts on the broader economy, as opposed to more persistent, long-term trends, which proves our hypothesis that the trend component of the dependency ratio is what drives the public debt.

4.2 Forecast Error Variance Decomposition

A second important question that a structural VAR model can answer is how much of the forecast error variance or prediction mean squared error (MSPE) of $y_t + h$ at horizon for $h = 0, 1, \ldots, H$ is accounted for by each structural shock ϵ_t (Kilian and Lütkepohl (2017)). For a VAR process like ours, the h-step head forecast error is:

$$y_{t+h} - y_{t+h|t} = \sum_{i=0}^{h-1} \Phi_i u_{t+h-i} = \sum_{i=0}^{h-1} \Theta_i \epsilon_{t+h-i}$$

where $u_t = B_0^{-1} \epsilon_t$ allows us to replace $\Phi_i u_{t+h-i}$ by $\Theta_i \epsilon_{t+h-i}$. Hence, the mean squared prediction error (MSPE) at horizon h is

$$\text{MSPE}(h) = E\left\{(y_{t+h} - y_{t+h|t})(y_{t+h} - y_{t+h|t})'\right\} = \sum_{i=0}^{h-1} \Phi_i \Sigma_u \Phi_i'$$

$$= \sum_{i=0}^{h-1} \Theta_i \Sigma_\epsilon \Theta_i' = \sum_{i=0}^{h-1} \Theta_i \Theta_i'$$

Let $\theta_{nj,h}$ be the nj^{th} element of Θ_h. Then the contribution of shock j to the MSPE of y_{nt}, $n = 1, \ldots, N$, at horizon h is Let $\theta_{nj,h}$ be the nj^{th} element of Θ_h. Then the contribution of shock j to the MSPE of y_{nt}, $n = 1, \ldots, N$, at horizon h is

$$\text{MSPE}_j^n(h) = \theta_{nj,0}^2 + \cdots + \theta_{nj,h-1}^2$$

and the total MSPE of y_{nt}, $n = 1, \ldots, N$, at horizon h is:

$$\text{MSPE}^n(h) = \sum_{j=1}^{N} \text{MSPE}_j^n(h) = \sum_{j=1}^{N} \left(\theta_{nj,0}^2 + \cdots + \theta_{nj,h-1}^2\right)$$

Dividing $\text{MSPE}_j^n(h)$ by $\text{MSPE}^n(h)$ yields the following decomposition for given h and n:

$$1 = \frac{\text{MSPE}_1^n(h)}{\text{MSPE}^n(h)} + \frac{\text{MSPE}_2^n(h)}{\text{MSPE}^n(h)} + \cdots + \frac{\text{MSPE}_N^n(h)}{\text{MSPE}^n(h)}$$

where each ratio gives the fraction of the contribution of shock j to the MSPE of variable k for $j = 1, \ldots, N$. In other words, $\text{MSPE}_j^n(h)$ represents the fraction of the contribution of shock j to the forecast error variance of y_{nt}. By multiplying the fractions by 100, we obtain percentages.

4.2.1 The Forecast Error Variance Decomposition of the debt

The Forecast Error Variance Decomposition (FEVD) is a tool used in VAR models to determine the contribution of each structural shock to the forecast error variance of an endogenous variable over time, that variable being the debt-to-gdp in our case. Essentially, it helps identify which shocks are most influential in driving the fluctuations of debt, providing insight into the relative importance of different sources of variation within the model.

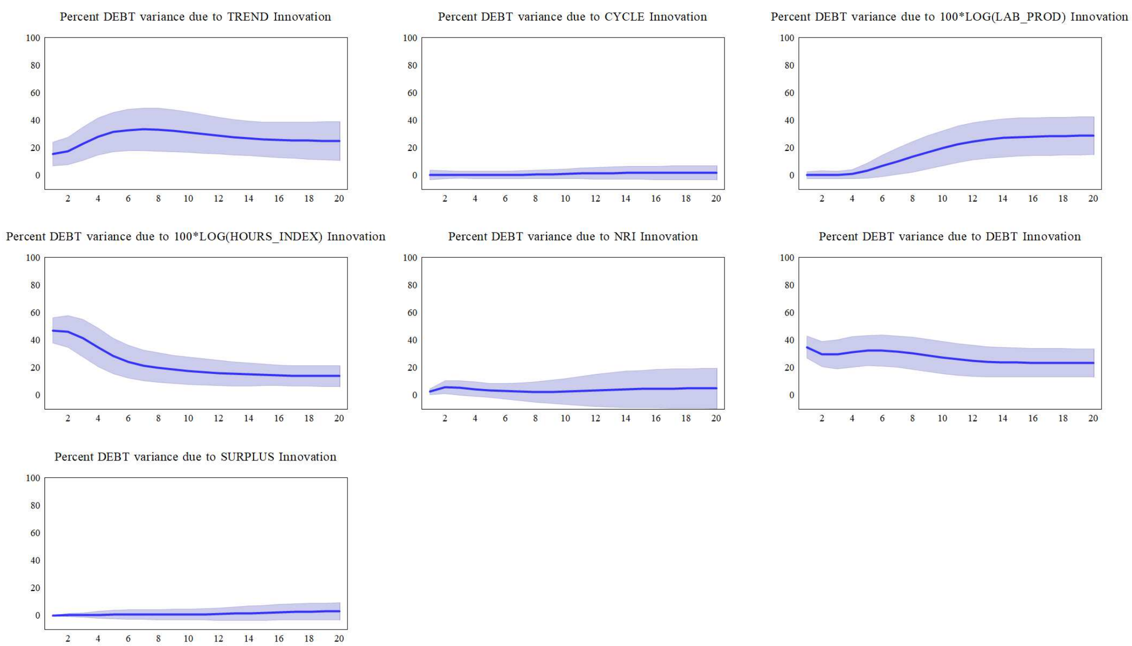


Figure 4.3: The forecast error variance decomposition (FEVD) for the debt

Notes: Variance decomposition using Cholesky factors, and 68% Confidence interval, using Mnte Carlo S.E. with 100 replications.

According to our results shown in Fig. 4.3, we can see that the FEVD shows that a substantial portion of the forecast error variance in the debt-to-GDP ratio is explained by shocks to the trend component of the dependency ratio, specially that this contribution increases over time to reach around 35%. This huge impact shows that long-term demographic trends, such as an aging population, have a considerable influence on the trajectory of public debt. In contrast, the FEVD shows that the cycle component of the dependency ratio contributes minimally to the variance in the debt-to-GDP ratio, with its impact remaining close to zero throughout the forecast horizon. This indicates that short-term fluctuations in the dependency ratio, which might reflect temporary economic cycles, do not significantly influence public debt. These results strongly support our hypothesis, showing that the trend component of the dependency ratio is a major determinant of public debt, whereas short-term demographic fluctuations are not a primary driver of public debt. This finding emphasizes the importance of long-term demographic trends over short-term cyclical changes.

Looking at our results, we can also see that the NRI contributes significantly to the variance in public debt, accounting for about 30-40% of the variance initially. This influence decreases gradually over time but remains significant. This might indicates that interest rates, influenced by broader economic and demographic factors, play a crucial role in determining public debt levels.

Furthermore, the impact of both labor productivity and hours index is relatively smaller compared to the trend component of the dependency ratio. However, the impact of labor productivity increases gradually over time, suggesting its growing importance in the long run.

These results highlight the importance of focusing on the aging population and other long-term demographic changes when analyzing public debt dynamics and formulating fiscal policy.

4.3 Robustness check

Robustness in econometric analysis refers to the stability and reliability of our results when subjected to various tests or modifications in the model specification. A robust model produces consistent results even when certain assumptions are changed or when additional variables are included in the analysis (Stock and Watson (2001)). In our paper, a robustness check was conducted to verify the stability of the results obtained from the original model specification. This process ensures that the findings regarding the effects of demographic changes on public debt are not sensitive to the inclusion of additional variables or changes in the model structure. To perform the robustness check, we added the "inflation rate" as an additional variable in the model. Inflation is a critical macroeconomic factor that has significant influence on public debt and fiscal dynamics, by including it in our model we take into consideration the inflationary pressures that could affect both the economy and fiscal sustainability. Inflation can affect the debt by reducing the real burden of repayment for governments. Inflation is also strongly linked to interest rate policies, where central banks raise or decrease rates to control inflation, which has a direct impact on borrowing costs and public debt levels. By including inflation in the he law of motion for public debt, the nominal interest rate i_t replaces the real rate r_t, and inflation is explicitly represented in the real return on debt. This is given by:

$$\frac{B_t}{Y_t} = \frac{1 + i_t - \pi_t}{1 + g_t} \cdot \frac{B_{t-1}}{Y_{t-1}} - \frac{PS_t}{Y_t}$$

Where:

- B_t is the nominal public debt at time t,
- Y_t is the nominal GDP at time t,
- i_t is the nominal interest rate at time t,
- π_t is the inflation rate at time t,
- g_t is the real GDP growth rate at time t,
- PS_t is the primary surplus at time t.

By including inflation, the aim is to test whether the original findings remain consistent when accounting for the potential effects of price level changes on the dependent variables. We used the variable CPI from FRED, and we got it as a percent change from year ago (as shown in fig. 4.5).

4.3.1 Aging and inflation

In their article, Katagiri et al. (2019) argue that there is a negative correlation between aging and deflation in the developed countries. In our model, we have a correlation of -0.55 between inflation rate and the OADR (as we can see in the scatter plot in fig. 4.4). An aging population can be caused by two main elements: rising life expectancy or lower birth rates. According to the authors, aging is deflationary when caused by an increase in longevity but inflationary when caused by a decline in birth rates. They take the example of Japan, which in the last two decades, experienced both aging and deflation. However, the connection between population aging and deflation is puzzling because population aging is a factor that is expected to increase future fiscal deficits due to higher social security expenditures and declining tax revenues, which eventually generates an inflationary pressure rather than the low rate of inflation observed recently in Japan and some other developed nations.

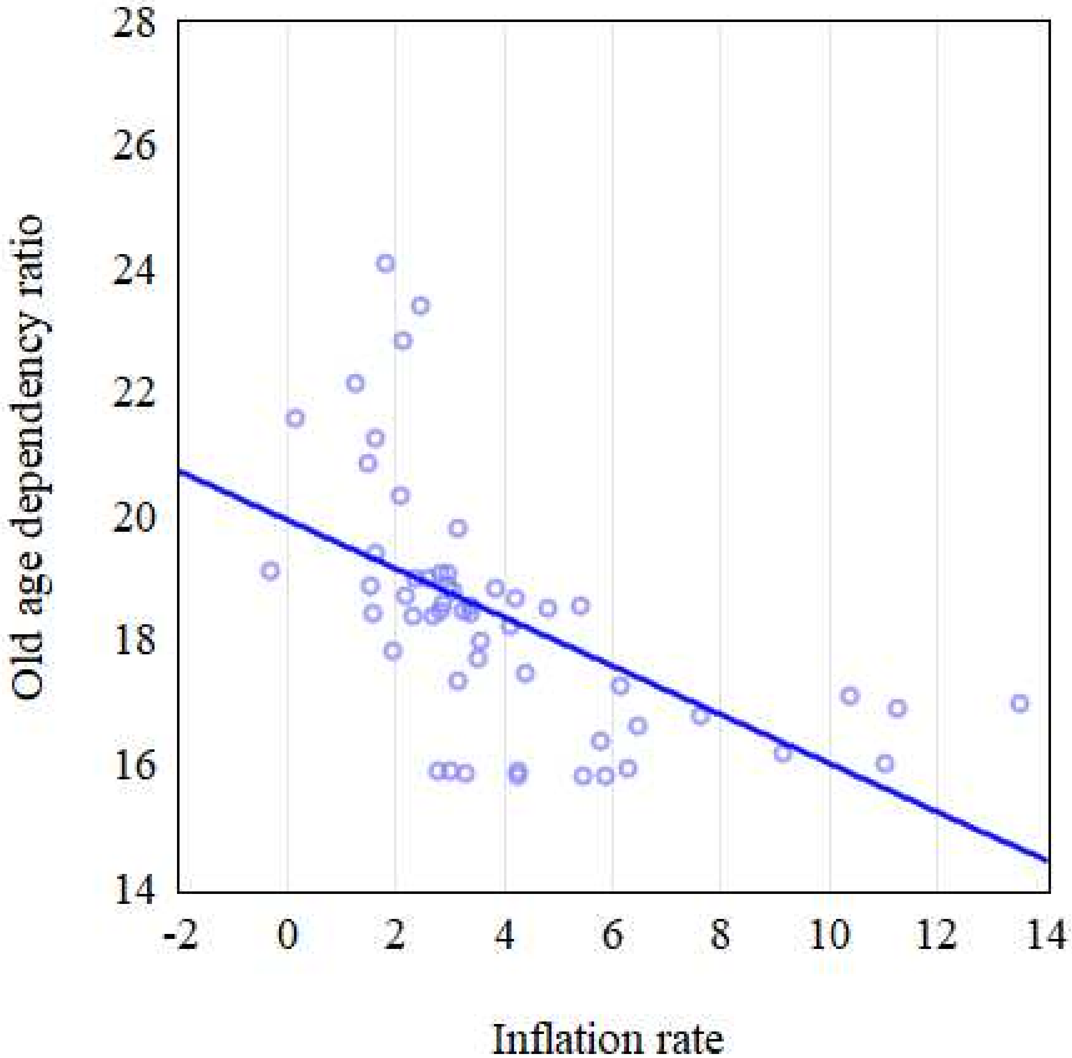


Figure 4.4: Aging and inflation in the U.S.

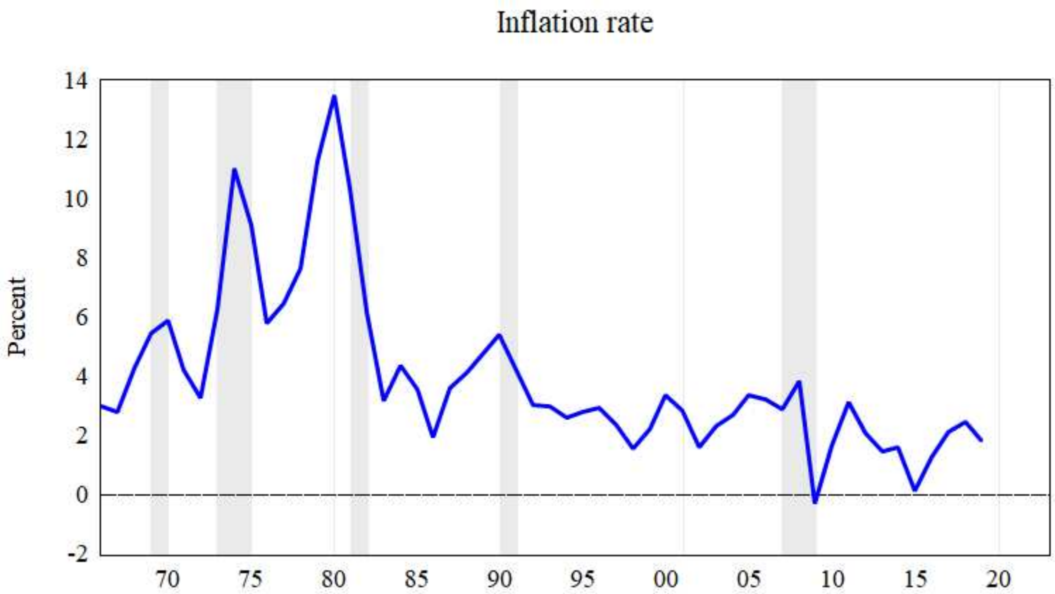


Figure 4.5: Inflation rate series

Notes: Inflation rate time series, sample period: 1966-2019. Source: FRED

The authors found that population aging affects the price levels depending on its causes, with two counteracting impacts, one is economic and the other is political. The economic impact of aging is inflationary when aging is driven by lower birth rates, due to the shrinking tax base and increased fiscal deficits. On the other hand, aging has deflationary effects when driven by increased life expectancy, which will give more political power to the older generation as they tend to vote more actively than younger people. Since elderly people hold a significant portion of nominal assets, like government bonds, they benefit from deflation, which increases the real value of their nominal assets, meaning that the money they hold becomes more valuable. Therefore, the government is incentivized to keep inflation low because it is easier than increasing pensions.

4.3.2 The results with inflation

After including inflation in the model, the results remained consistent with the original analysis. The impulse response functions (shown in fig. 4.6 and fig. 4.7) and variance decomposition (fig. 4.8) indicated that the trend component of the dependency ratio continues to be the primary driver of public debt, while the cycle component remains insignificant. The persistence of these results, even with the addition of inflation, reinforces the robustness of the conclusions drawn from the initial model.

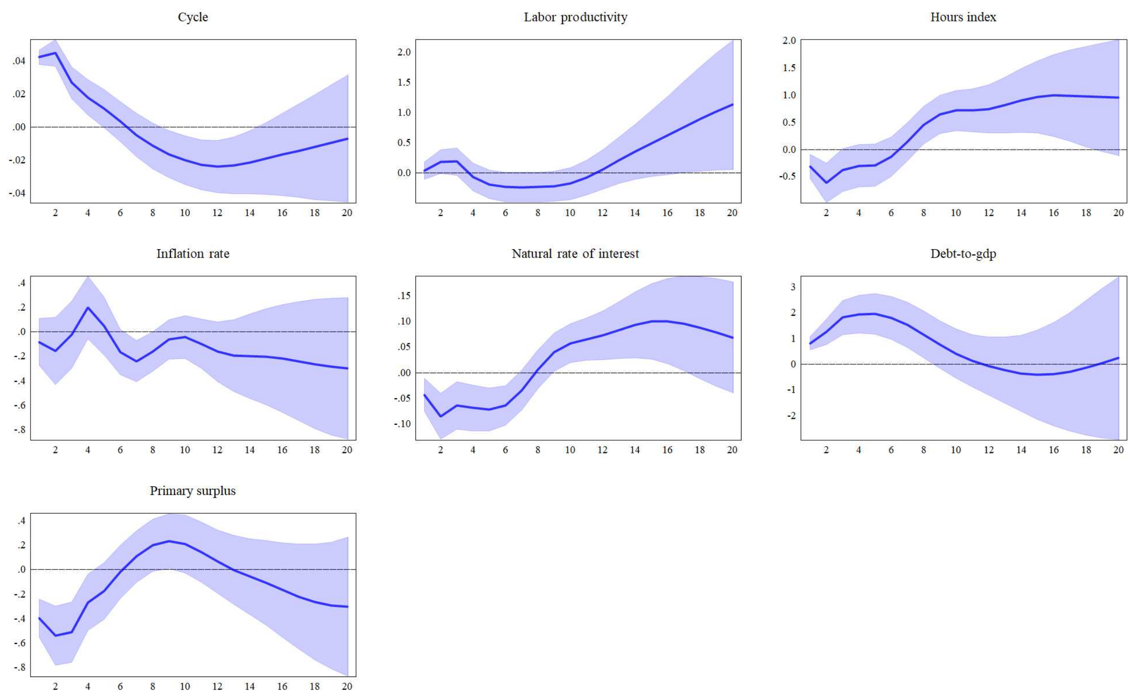


Figure 4.6: The Impulse response function (IRFs) of a trend shock including inflation rate

Notes: This graph is done using the data from our model.

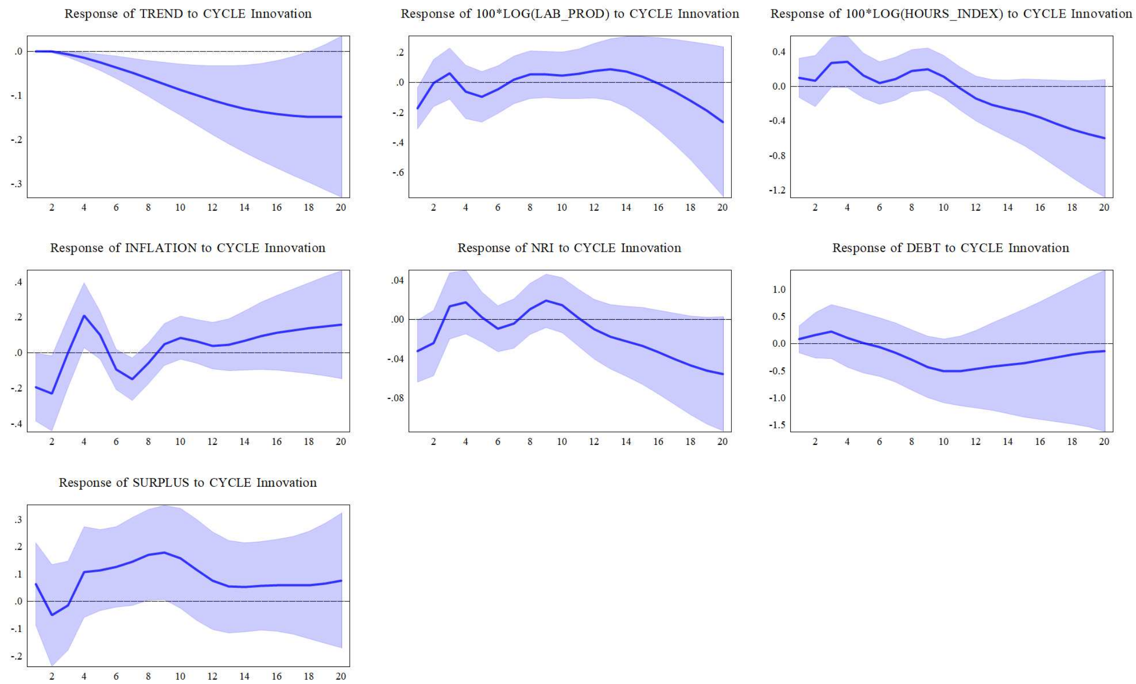


Figure 4.7: The Impulse response function (IRFs) of a cycle shock including inflation rate

The robustness check confirms that the conclusions regarding the impact of demographic trends on public debt are not only reliable but also resilient to changes in the model specification. The inclusion of inflation did not change the fundamental dynamics captured in our original analysis, thereby enhancing the confidence in the results presented in this study.

Figure 4.8: The Forecast Error Variance Decomposition (FEVD) with inflation rate included

4.4 Interpretation

Looking at the dependency ratio graph (fig. 3.1), we can see a small drop in the graph in the year 2000. In 2000, the Baby Boomer generation aged between 36 and 54, were still in the working age population, but they were nearing the retirement age. Several researchers tried to understand the reason behind this fall. there are different stories and scenarios regarding the drop of the dependency ratio in the 2000. Some observers argued that it might be caused by the health issues of the soldiers who came alive from the world war 2. Others said it might be the result of the increased use of opioids by elderly people, which are commonly prescribed for managing chronic pain, and can lead to serious health risks in older adults, including overdose, falls, fractures, and respiratory depression. Therefore they argued that it might have led to an increase in mortality, leading to a decline in the dependency ratio.

However, if we look at Figure 4.9, we can see the two graphs combined: dependency ratio and hours level worked. As shown in this picture, the hours level worked increased in the 2000, meaning that we had a higher labor participation. This can be due to a several reasons like more female participation, and increased immigration.

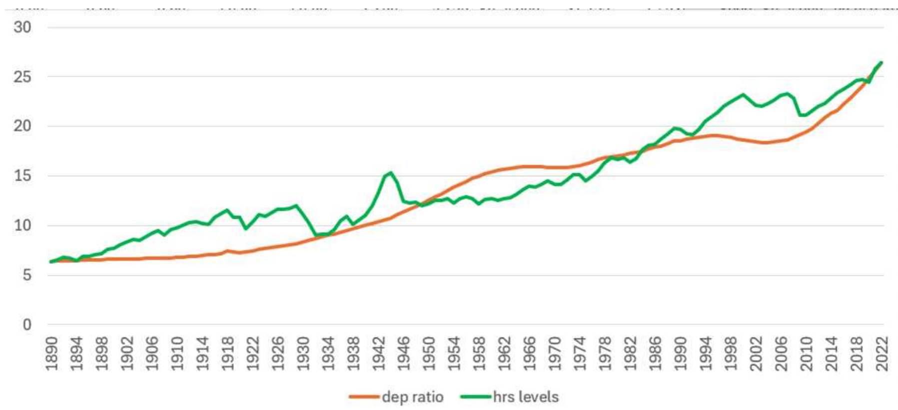


Figure 4.9: Dependency ratio and Hours levels worked

If we dig deeper, we see that the drop in the dependency ratio was a drop in the trend component of the dependency ratio (as we can see in fig. 3.2), which indicates that this can be due to the increased migration in the United States in that year.

Migration can be classified as part of the trend component of the dependency ratio when viewed over the long term, due to the structural shifts in the population that significantly influence demographic patterns. In 2000, the U.S. experienced a huge increase in immigration (as shown in fig. 4.10), which contributed to a growing working-age population. This inflow of working age individuals led to a decrease in the dependency ratio, explaining the decline observed during that period. Unlike short-term economic fluctuations, this peak in migration reflects a sustained demographic shift with lasting effects on the workforce and economic structure.

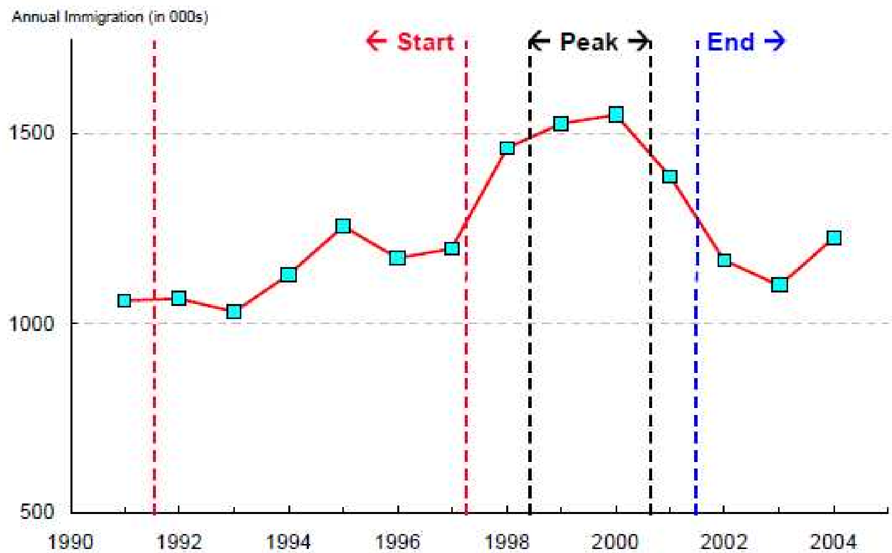


Figure 4.10: Immigration in the United States

Notes: Annual immigration to the U.S. peaked in 1999-2000, based on Census 2000, ACS and CPS data: 1991-2004.

According to Cutler et al. (1990), high immigration to the United States would reduce the dependency burdens as immigrants on average are younger than non-immigrants. In their article, George Borjas stated that only 3.1 percent of those who immigrated to the U.S. between 1975 and 1979 were older than 65 in 1980, compared with 10.6 percent of the non-immigrants. As mentioned in Cutler et al. (1990), immigrants arrive as young people and start working and paying taxes immediately, to support the elderly. They may increase economic welfare of the preexisting population, even if they are ultimately eligible for transfer payments in old age. According to Lee (2014), increasing net immigration by almost one million per year until 2050, can reduce the old age dependency ratio by 10 percent. However, Goodhart and Pradhan (2017) stated that immigration of labors from nations with fast-growing population like India and Africa, to the advanced economies like the United States, can offset the demographic transitions. Nevertheless, optimism about immigration has died because of politics mainly, but if labour cannot be imported into ageing economies, why not export capital instead to economies with growing populations, and produce and import finished goods from there? Some of this will naturally happen, but exporting capital to economies where the labour force is younger is not quite easy. Recently, the largest growth in working population will come mostly from India or Africa, and the developed countries should decide whether to take advantage from this or not, by allowing more immigration.

In addition, we can see a drop in the debt-to-gdp ratio (fig. 3.1) in the year 2000 as well. We can conclude here, that migration can offset the increasing dependency ratio by adding more young people to the labor force, increasing therefore the working-age population and lowering the dependency ratio. In turn, this decline in the dependency ratio can reduce the debt-to-gdp ratio, and lower the fiscal burdens.

Chapter 5

Conclusion

The findings of this paper highlight the complex relationship between demographic shifts, particularly the aging population, and public debt. The analysis of dependency ratio shocks on the US public debt suggests that an aging population does lead to higher levels of public debt. This outcome aligns with broader concerns about the sustainability of current fiscal policies, as the ratio of public debt to GDP is likely to continue increasing unless significant changes are made to spending programs, the tax code, or both. The urgency for fiscal reforms is enhanced by the historically high federal debt relative to GDP. Restoring fiscal space would not only enable the government to respond more effectively to unforeseen events but also address the long-term challenges posed by an aging population.

As more people enter the retirement age, public expenditures on pensions and healthcare are expected to rise, further straining the fiscal balance. While immigration can reduce some of these pressures by restoring the workforce and reducing the dependency ratio, its effect has not been substantial enough to reverse the aging trend. Nevertheless, policies aimed at increasing immigration levels could reduce some of the demographic forces driving higher public debt. Additionally, the concept of the ”second demographic dividend” discussed in Mason and Lee (2004), offers an optimistic outlook, suggesting that increased life-cycle savings in low fertility and low mortality environments could lead to a larger capital-labor ratio, which could, to some extent, offset the burden of an aging population.

Moreover, the response of labor productivity to an aging population was unusual. When most of the literature states that population aging leads to slower economic growth, our model showed otherwise. One way in which productivity might not have been affected by the aging population can be through innovation and more educated labor force. Cutler et al. (1990) argues that a reduction in the labor growth tends to increase capital intensity, which leads to an increase in labor productivity. Another way in which productivity can be positively affected is through the higher participation of women in the workforce. The historical rise in educational attainment, driven in part by parental investments in their children's education, has contributed to a more skilled labor force. This increase in educational levels, coupled with the societal shift that removed children from the workplace, has led to long-term productivity gains (Reher (2011)). As the economy absorbed large numbers of female and immigrant workers without causing significant unemployment, it is reasonable to argue that policies supporting higher labor force participation among older workers would not necessarily displace younger workers. In fact, such policies could provide a barrier against the economic effects of population aging.

To address these challenges, several policy reforms should be considered. More flexible work hours and gradual retirement options would make it easier for older workers to remain employed. Retraining and continuing education programs for older workers may also be beneficial, although the effectiveness of such programs remains questionable. Encouraging later retirement could help workers prepare for shorter retirement periods while contributing to higher labor supply, which will also reduce the pressure on government expenditures (Lee (2014)). Primarily, research has shown no clear evidence that increasing the employment of older workers would reduce opportunities for younger workers.

The aging population's impact on public debt underscores the need for a macroeconomic adjustment, which will likely require a combination of lower consumption (either through increased savings or higher taxes) and an expanded labor supply, potentially through delayed retirement. Population aging affects programs such as Social Security and Medicare, further worsening the fiscal pressure. Bongaarts (2004) argues that encouraging higher fertility or permitting more migration could help counteract some of these demographic trends, but it is clear that significant structural reforms will be necessary to achieve long-term fiscal sustainability.

While the overall trend points toward an increase in public debt due to an aging population, our study also reveals that demographic shifts do not necessarily lead to slower productivity growth. In fact, the ”second demographic dividend” could drive higher rates of economic growth, especially if policies are designed to promote savings and capital accumulation. Early and effective exploitation of this dividend could lead to longer, maybe temporary, periods of higher growth.

However, further research is needed to fully understand the long-term implications of demographic shifts on macroeconomic variables. For instance, clarifying estimates by incorporating data spanning longer periods could provide a clearer picture of how these changes affect key variables such as labor productivity, capital formation, and output growth.

In conclusion, while demographic shifts are likely to negatively affect public debt levels, ongoing research is crucial to clarify our understanding of how these changes interact with macroeconomic outcomes. Nevertheless, our primary conclusion remains clear: demographic changes, particularly population aging, will continue to play a significant role in shaping public debt trajectories, and careful policy planning is essential to mitigate these effects.

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Appendix A

Appendix

A.1 The 68% Confidence interval

In our paper, we used the 68% confidence intervals (CI) when interpreting the results of the VAR model. While the conventional literature often relies on 90% or 95% confidence intervals, there is no universally agreed-upon significance level, and recent literature has begun to adopt the 68% interval. This shift is particularly relevant when dealing with models that have many coefficients, like the one used here, where the complexity of the model can increase the uncertainty in the estimates. Given that we have a moderate sample size of 54 yearly observations, demanding a 90% or 95% confidence interval may impose overly demanding requirements on the model (Lunsford and West (2019)). These higher confidence levels can produce excessively wide intervals, making it difficult to draw meaningful conclusions. Using a 68% confidence interval allows for a more balanced approach, providing a reasonable range for interpretation while acknowledging the uncertainty without overly inflating the confidence bands.

A.2 Vector autoregression estimates

The table below presents the estimated coefficients for the VAR model used in this paper, including 52 yearly observations from 1968 to 2019 (adjusted). The model includes several macroeconomic variables, with the coefficients, standard errors, and t-statistics provided for each variable and its respective lags.

Vector Autoregression Estimates
Date: 09/15/24 Time: 20:15
Sample (adjusted): 1968 2019
Included observations: 52 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	TREND	CYCLE	100*LOG(LA	100*LOG(H	NRI	DEBT	SURPLUS
TREND(-1)	2.189372	0.407293	-7.538259	-0.739652	-0.765565	8.175979	-1.072867
	(0.18422)	(0.30065)	(7.47338)	(12.2440)	(1.79499)	(13.9986)	(8.29284)
	[11.8846]	[1.35470]	[-1.00868]	[-0.06041]	[-0.42650]	[0.58406]	[-0.12937]
TREND(-2)	-1.197131	-0.440253	9.014306	0.657558	0.864295	-9.435309	-0.026530
	(0.21294)	(0.34753)	(8.63854)	(14.1529)	(2.07485)	(16.1811)	(9.58576)
	[-5.62190]	[-1.26682]	[1.04350]	[0.04646]	[0.41656]	[-0.58311]	[-0.00277]
CYCLE(-1)	-0.031534	0.660294	6.422984	-3.237448	-0.053550	-0.872139	-4.105773
	(0.14816)	(0.24180)	(6.01053)	(9.84734)	(1.44364)	(11.2585)	(6.66959)
	[-0.21283]	[2.73073]	[1.06862]	[-0.32876]	[-0.03709]	[-0.07746]	[-0.61560]
CYCLE(-2)	-0.213321	-0.373561	-3.496922	-1.646968	-0.053537	2.617234	1.686573
	(0.09306)	(0.15188)	(3.77524)	(6.18515)	(0.90675)	(7.07152)	(4.18919)
	[-2.29230]	[-2.45964]	[-0.92628]	[-0.26628]	[-0.05904]	[0.37011]	[0.40260]
100*LOG(LAB_PROD(-1	-0.001536	-0.003527	0.842876	0.601564	0.014548	0.048782	0.199165
	(0.00434)	(0.00708)	(0.17598)	(0.28831)	(0.04227)	(0.32963)	(0.19527)
	[-0.35398]	[-0.49819]	[4.78970]	[2.08651]	[0.34418]	[0.14799]	[1.01993]
100*LOG(LAB_PROD(-2	0.008985	0.016887	0.103981	-0.694590	-0.032432	-0.016285	-0.275040
	(0.00447)	(0.00729)	(0.18130)	(0.29704)	(0.04355)	(0.33961)	(0.20118)
	[2.01038]	[2.31530]	[0.57351]	[-2.33837]	[-0.74477]	[-0.04795]	[-1.36710]
100*LOG(HOURS_IND	-0.003026	-0.004398	-0.142977	1.398652	0.050499	0.183274	0.197122
	(0.00420)	(0.00685)	(0.17032)	(0.27904)	(0.04091)	(0.31903)	(0.18899)
	[-0.72078]	[-0.64194]	[-0.83947]	[5.01235]	[1.23444]	[0.57447]	[1.04301]
100*LOG(HOURS_IND	-0.005000	-0.008814	0.100421	-0.402398	-0.048032	0.013504	-0.107582
	(0.00403)	(0.00658)	(0.16352)	(0.26791)	(0.03928)	(0.30630)	(0.18145)
	[-1.24037]	[-1.33978]	[0.61411]	[-1.50201]	[-1.22294]	[0.04409]	[-0.59289]
NRI(-1)	0.024698	0.061132	-1.296568	-2.962097	0.689181	-2.936508	0.497708
	(0.03170)	(0.05174)	(1.28600)	(2.10691)	(0.30888)	(2.40884)	(1.42700)
	[0.77911]	[1.18163]	[-1.00822]	[-1.40590]	[2.23125]	[-1.21906]	[0.34878]
NRI(-2)	0.006556	0.008441	0.516911	0.487221	-0.091008	4.382084	-0.878631
	(0.03348)	(0.05463)	(1.35803)	(2.22493)	(0.32618)	(2.54377)	(1.50694)
	[0.19583]	[0.15450]	[0.38063]	[0.21898]	[-0.27901]	[1.72267]	[-0.58306]
DEBT(-1)	0.008614	0.011229	0.158307	-0.362649	-0.018903	1.175445	-0.247117
	(0.00424)	(0.00691)	(0.17183)	(0.28151)	(0.04127)	(0.32185)	(0.19067)
	[2.03382]	[1.62444]	[0.92132]	[-1.28822]	[-0.45803]	[3.65212]	[-1.29607]
DEBT(-2)	-0.009579	-0.011968	-0.179217	0.389390	0.019001	-0.232098	0.346907
	(0.00443)	(0.00722)	(0.17959)	(0.29422)	(0.04313)	(0.33639)	(0.19928)
	[-2.16384]	[-1.65654]	[-0.99795]	[1.32345]	[0.44052]	[-0.68997]	[1.74082]
SURPLUS(-1)	0.010085	0.005907	0.169962	-0.662917	-0.061903	-0.311161	0.252745
	(0.00686)	(0.01119)	(0.27815)	(0.45570)	(0.06681)	(0.52100)	(0.30864)
	[1.47089]	[0.52790]	[0.61105]	[-1.45473]	[-0.92661]	[-0.59723]	[0.81889]
SURPLUS(-2)	-0.002689	0.001113	0.131809	0.029570	0.036731	-0.004002	0.063674
	(0.00465)	(0.00759)	(0.18875)	(0.30924)	(0.04534)	(0.35356)	(0.20945)
	[-0.57789]	[0.14652]	[0.69832]	[0.09562]	[0.81020]	[-0.01132]	[0.30401]
C	-2.559874	-4.706504	28.82121	69.95808	10.67465	-59.82817	38.63303
	(0.62712)	(1.02348)	(25.4410)	(41.6812)	(6.11054)	(47.6544)	(28.2306)
	[-4.08194]	[-4.59852]	[1.13286]	[1.67841]	[1.74692]	[-1.25546]	[1.36848]
R-squared	0.999860	0.968231	0.998981	0.992999	0.964812	0.995033	0.816891
Adj. R-squared	0.999807	0.956210	0.998595	0.990350	0.951498	0.993154	0.747606
Sum sq. resids	0.025162	0.067018	41.40963	111.1509	2.388868	145.2908	50.98864
S.E. equation	0.026078	0.042559	1.057913	1.733228	0.254094	1.981610	1.173913
F-statistic	18843.79	80.54646	2591.052	374.8616	72.46373	529.4673	11.79037
Log likelihood	124.6909	99.22007	-67.86382	-93.53558	6.306227	-100.4996	-73.27414
Akaike AIC	-4.218883	-3.239233	3.187070	4.174446	0.334376	4.442294	3.395159
Schwarz SC	-3.656024	-2.676375	3.749929	4.737304	0.897235	5.005152	3.958018
Mean dependent	18.41116	0.082300	680.7736	296.3686	2.830310	58.45311	-2.891852
S.D. dependent	1.875691	0.203380	28.22852	17.64394	1.153754	23.94958	2.336665

A.3 Scatter plots

As mentioned before, the choice of using labor productivity as a variable measuring growth, we got that it is highly correlated with real GDP as well as with total factor productivity. Here are the scatter plots proving this.

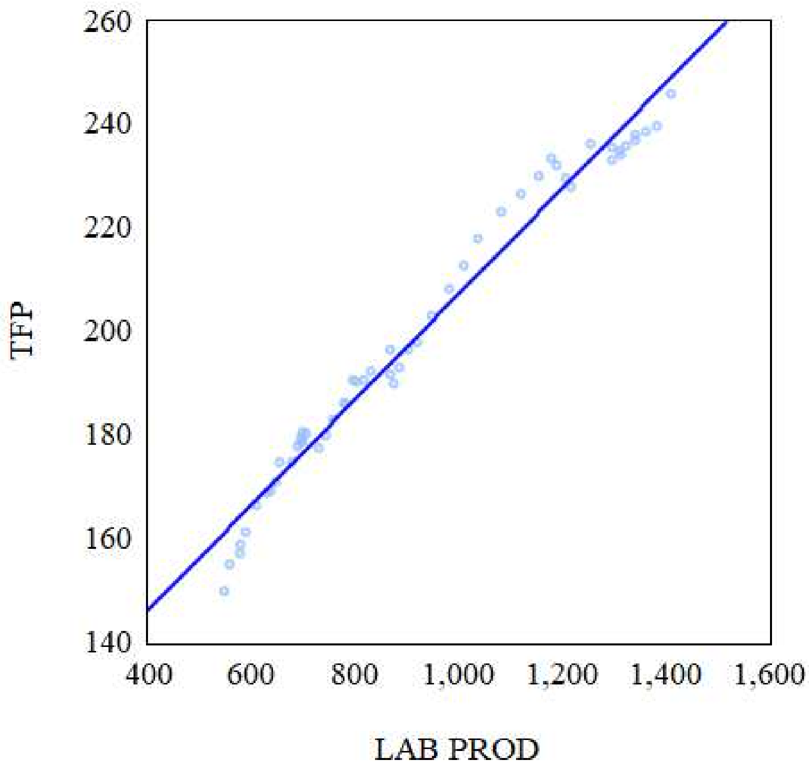

Figure A.2: The correlation between labor productivity and total factor productivity

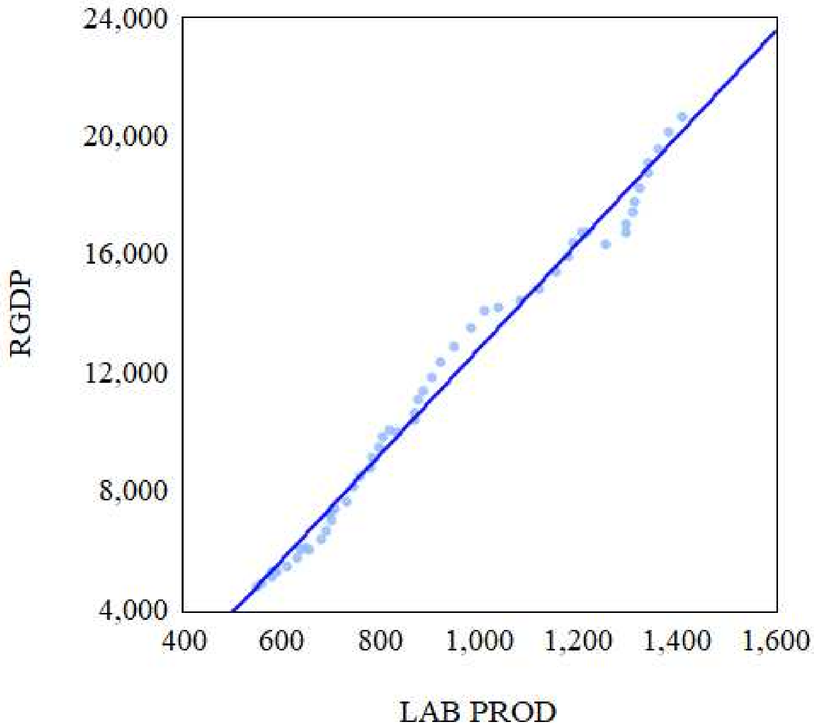

Figure A.3: The correlation between real GDP and labor productivity

A.4 Table of correlations

Here is a correlation table that presents the relationships between each of the variables used in our analysis.

	TREND	CYCLE	LAB_PROD	HOURS_IND	NRI	DEBT	SURPLUS
TREND	1.0	0.345469	0.865484	0.881798	-0.82131	0.916438	-0.240683
CYCLE	0.345469	1.0	0.508882	0.174598	-0.646926	0.543351	-0.627675
LAB_PROD	0.865484	0.508882	1.0	0.912008	-0.907682	0.924297	-0.336908
HOURS_IND	0.881798	0.174598	0.912008	1.0	-0.75127	0.822863	-0.102
NRI	-0.82131	-0.646926	-0.907682	-0.75127	1.0	-0.933357	0.555331
DEBT	0.916438	0.543351	0.924297	0.822863	-0.933357	1.0	-0.367212
SURPLUS	-0.240683	-0.627675	-0.336908	-0.102	0.555331	-0.367212	1.0

Figure A.4: Correlation table

A.5 Table of abbreviations

Abbreviation	Full Term
VAR	Vector Autoregression
CI	Confidence Interval
FRED	Federal Reserve Economic Data
NRI	Natural Rate of Interest
CBO	Congressional Budget Office
OADR	Old-Age Dependency Ratio
HP	Hodrick-Prescott
IRF	Impulse Response Function
FEVD	Forecast Error Variance Decomposition

Figure A.5: The list of abbreviations

A.6 A second robustness check

We chose to do another robustness check to be more confident with our results. We decided to exclude one variable from our model: the natural rate of interest.

Below we can see the IRF to both shocks in the trend and cycle components of the dependency ratio, as well as the FEVD of the debt. We can see that the overall responses are similar to our results, with the magnitudes of the response changing a bit, but the directions of the variables is still the same. We can still see that the variable driving the public debt is the trend component of the dependency ratio, and not the cycle, proving our hypothesis that long term structural changes in the dependency ratio are driving public debt up.

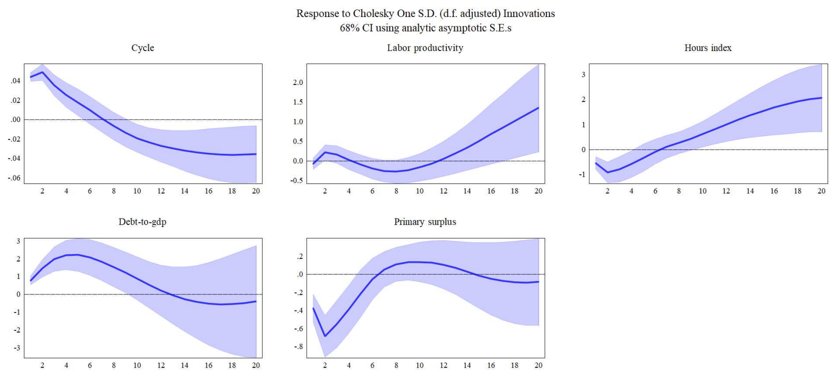

Figure A.6: The Impulse response function (IRF) to a shock in the trend component of the dependency ratio.

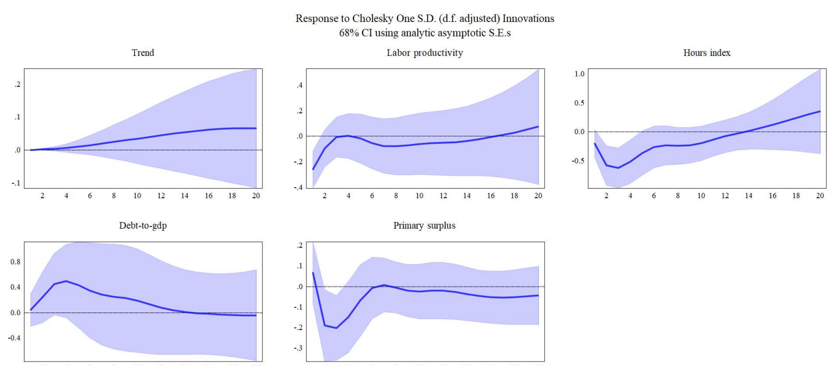

Figure A.7: The Impulse response function (IRF) to a shock in the cycle component of the dependency ratio

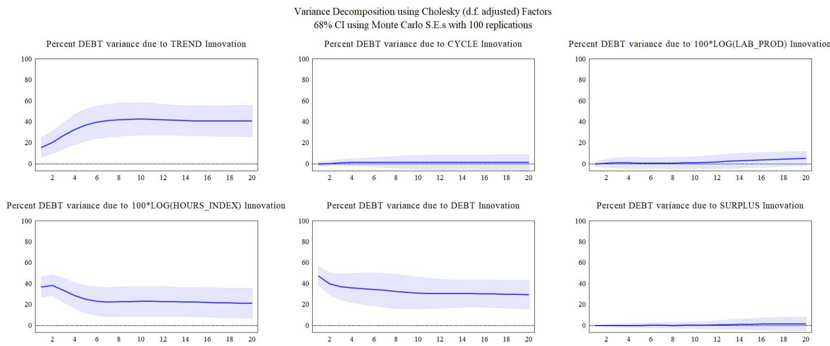

Figure A.8: The forecast error variance decomposition (FEVD) of the debt

A.7 The table of variables

Here is a table showing the source of each of our variables, and if we used the 100*log or not.

Variable	Source	100*Log Used
OADR	FRED	No
Debt-to-GDP	FRED	No
Hours Index	FRED	Yes
Primary Surplus to GDP	FRED	No
NRI	Lunsford and West (2019)	No
Labor Productivity	Lunsford and West (2019)	Yes

Figure A.9: Table of the variables and sources

A.8 The Lag-Order Selection Procedure

In our VAR model, we used two lags. We used yearly data in our model, and Wooldridge (1999) mentioned in his book that for annual data, the number of lags is typically small, with 1 or 2 lags commonly used. For quarterly data, it is common to use 1 to 4 lags, and sometimes more depending on the dynamics of the system, and for monthly data, it is more common to use 12 or even 24 lags when data points allow, to capture the seasonal or monthly variations adequately. Some economists mentioned that yearly data typically requires fewer lags due to longer observation intervals, while quarterly and monthly data require more lags to capture finer fluctuations in time series data.

An alternative to sequential testing procedures is the use of information criteria for lag-order selection (Kilian and Lütkepohl (2017)). The three most commonly used information criteria for VAR models are known as the AIC, HQC, and SIC: Akaike Information Criterion, Hannan-Quinn Criterion, and Schwarz Information Criterion. Both the HQC and SIC showed that we should be using 2 lags in our model (as shown in fig. A.10).

VAR Lag Order Selection Criteria
Endogenous variables: TREND CYCLE 100*LOG(LAB PROD) 100*LOG(HOURS
Exogenous variables: C
Date: 09/29/24 Time: 20:29
Sample: 1966 2019
Included observations: 51

Lag	LogL	LR	FPE	AIC	SC	HQ
0	-696.3026	NA	2242.600	27.58050	27.84565	27.68182
1	-50.70564	1088.654	1.58e-07	4.184535	6.305755	4.995116
2	81.71064	186.9406*	6.66e-09*	0.913308	4.890596*	2.433148*
3	132.0668	57.26778	8.51e-09	0.860126*	6.693482	3.089224

* indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

Figure A.10: The information criteria tests