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## "THE REPRESENTATIVENESS HEURISTIC AS A DETERMINANT OF HERD BEHAVIOR"

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## INTRODUCTION:

Making decisions is an essential part of behavior in each person's everyday life.
Decision-making can be defined as the process by which individuals choose a specific course of action to respond to opportunities and problems they face.

Sometimes individuals engage in programmed decisions to respond to issue that happened in the past and are recurring in the same way in the present.
In order to make the best choice, they exploit performance programs, a standard sequence of behaviors that they follow routinely whenever they encounter that specific problem or opportunity.

In other situations, individuals have to face new problems that involve non-programmed decisions.

It means that people must decide their actions in a scenario of uncertainty.
"Uncertainty is the fact of life and business. Probability is the guide for a good life and successful business. The concept of probability occupies an important place in the decision making process, whether the problem is one faced in business, in government, in the social sciences, or just in one's own everyday personal life. In very few decision-making situations perfect information are available. Most decisions are made in the face of uncertainty. Probability enters into the process by playing the role of a substitute for complete knowledge. Decision makers often face a severe lack of information. Probability assessment quantifies the information gap between what is known, and what needs to be known for an optimal decision. The probabilistic models are used for protection against adverse uncertainty, and exploitation of propitious uncertainty. Difficulty in probability assessment arises from information that is scarce, vague, inconsistent or incomplete" ${ }^{1}$.

In order to make rational choices involving uncertainty, individuals should try to revise their prior information through the acquisition of new information later obtained that help them to assess more precisely the probability of an event to occur. In the language of probability theory, they should estimate conditional probabilities following the Bayes theorem.
In the real economy, it often happens that consumers have to choose between different products or services without being aware of which is the best.
In these situations, they may try to evaluate the quality of the products/services on the basis of the purchasing behavior of other individuals that have already experimented them. In fact, the purchasing decisions of previous consumers may convey additional information on the quality

[^0]of the products that the individual could use to update his/her prior beliefs.
However, the process of passing from prior to posterior beliefs is not necessarily obvious and there can be some consumers who are not able to perform it. In particular, it is possible that individuals do not weigh correctly the new information acquired and their beliefs, ending up with distorted posterior beliefs.
To see this, suppose that an individual has to choose between two alternatives, A and B, without knowing which of them is the best. Suppose that he observes how many consumers chose either product in the past. To get a correct assessment of the posterior probability that the product A is better than B , he/she should use Bayes Theory, giving correct weights to the information conveyed by other people's choices and to his/her own prior beliefs. If, instead, the individual overweighs this new information (i.e. under-weighs the value of the prior probability), relying too much on what other people chose before, he/she will typically tend to imitate the decisions made by the majority of consumers. This effect is called "Herd Behavior" and it occurs when persons copy actions of the largest group. This behavior can be perfectly rational, but in other cases may be due to some bias in the beliefs updating process.

The situation described above is the subject of this work. In particular, our aim is to analyze how consumers use the information provided by previous consumers' choices in order to assess the quality of a product/service of (a priori) uncertain quality and to show how distortions in the updating process may lead to herd behavior.
In the first chapter, we introduce the concept of herd behavior through a review of some contributions on this topic provided by the literature.

The second chapter presents a three-period model in which consumers are called to choose between two products of unknown quality. The model shows how rational consumers should use Bayes theorem to exploit the information conveyed by other consumers' choices in order to turn their prior beliefs into posterior probabilities. This updating process allows consumers to make the "correct" consumption decision. We will show that, in this context, a rational herd behavior may emerge but that it is unlikely that this herd behavior will determine an inefficient outcome in which the inferior product prevail in the market. The last part is dedicated to comments on possible market outcomes.

The third and last chapter introduces a possible bias in the way people update their beliefs: the representativeness heuristic. In the first part, there is a description of this heuristic and an explanation of the main factors that may cause it.

Then, the chapter goes on with the illustration of the same model now under the effect of the representativeness heuristic; we will show that, when the representativeness heuristic leads
individuals to put too much weight on the new information relative to their prior beliefs, the result will be an increase in the probability of an inefficient herd behavior (i.e., one in which the inferior product prevails).

The last chapter concludes.

## 1. AN INTRODUCTION TO HERD BEHAVIOR

Herd behavior is defined as a phenomenon in which a large number of persons act in the same way. It usually occurs when someone takes decisions basing on what other individuals around him or her are doing. For instance, in everyday life, some people seem to decide in which store they want to purchase, in which restaurant they want to have dinner, what school they want to attend on the basis of how popular they seem to be. The same process occurs also in the politic field: voters are often influenced by opinion polls to vote in the direction that the poll predicts will win. These situations display the so-called herd behavior: everyone does whatever others are doing, even when their private information suggests doing something quite different.

Abhijit Banerjee, who studied social interactions, introduces a common example that could take place in the real life:
"Consider a situation where there is a population of 100 people who are all facing such a choice. There are two restaurants A and B that are next to each other, and it is known that the prior probabilities are 51 percent for restaurant A being the better and 49 percent for restaurant B being better. People arrive at the restaurants in sequence, observe the choices made by the people before them, and decide on one or the other of the restaurants. Apart from knowing the prior probabilities, each of these people also got a signal which says either that A is better or B is better (of course the signal could be wrong). It is also assumed that each person's signal is of the same quality.

Suppose that of the 100 people, 99 have received signals that B is better but the one person whose signal favors A gets to choose first. Clearly, the first person will go to A. The second person will now know that the first person had a signal that favored A , while her own signal favors B. Since the signals are of equal quality, they effectively cancel out, and the rational choice is to go by the prior probabilities and go to A .

The second person thus chooses A regardless of her signal. Her choice therefore provides no new information to the next person in line: the third person's situation is thus exactly the same and so on. Everyone ends up at restaurant A even if, given the aggregate information, it is practically certain that B is better" ${ }^{2}$.

When an agent starts herding, learning stops altogether, as her action do not provide any additional information, leaving her successor in the same position where the evidence of the history outweighs any private signals.

[^1]The conformal behavior of all subsequent agents is called information cascade.
An information cascade has the potential to occur when people make decisions sequentially, with later people watching the actions of earlier people, and from these actions inferring something about what the earlier people know.
In the example of the restaurant, when the first diners to arrive chose restaurant $B$, they conveyed information to later diners about what they know. A cascade then develops when people abandon their own information in favor of interferences based on earlier people's actions.

Here we can see that the crucial moment occurs when the second person has to choose between restaurant A and B . If she had followed her own belief, her decision would have been to go to the restaurant B. This choice would have conditioned all the subsequent individuals and, from that point to the end, all individuals would have chosen restaurant B , in contrast to the example.

The decision of the second person to ignore its own information generates a negative externality on the rest of the individuals involved in the decision-making process.

The bad equilibrium arises from that "herd externality" of imitating others and thereby concealing one's own information. That people do indeed try seriously to infer information by observing the actions of others ${ }^{3}$.
If there are more than just two possible action alternatives, informational cascades can still result. However, as the set of alternatives becomes larger and richer, cascades tend to take longer to form and aggregate more information.

If the set of action alternatives is continuous (for example, all points on the interval $[0,1]$ ), then even an individual late in the sequence will still adjust his action at least slightly based on the private signal (Lee, 1993). Consequently, private signals can be perfectly inferred from actions, information aggregates efficiently, and cascades do not form. However, if individuals cannot distinguish between nearby actions taken by their predecessors, cascades do arise ${ }^{4}$.

[^2]Anderson and Holt conducted an experiment in order to analyze when an information cascade emerges and how it evolves over time.

In order to be useful, the experiment must respect some hypotheses:

- There must exist a decision to be made.
- People make decisions sequentially.
- Each people can observe the choices made by others.
- Each person has some private information that could influence the decision.
- Individuals do not know other people's information; they can only infer them from the choices they make.

We imagine the experiment taking place in a classroom with a large group of students as participants.
There exist two different urns; one of them contains two red marbles and one blue marble, the other one contains two blue marbles and one red marble. The two urns are called respectively "majority red" and "majority blue".

The experimenter puts one of the two urns at the front of the room and the choice of the urn is taken randomly with $50 \%$ probability each.

Now, one by one, each student comes to the front of the room and draws a marble from the urn; he looks at the color and then places it back in the urn without showing it to the rest of the class. The student must guess which of the two urns, "majority red" (MR) and "majority blue" (MB), was used to conduct the experiment, making the estimation public. This is a crucial point because the following students can't see the previous draw; they can only make judgment on the basis of the guesses made before.

We also assume that, at the very end of the experiment, each student who has guessed correctly receives a monetary reward in order to be encouraged to give the right answer.

This example is strictly linked to the first one of the two restaurants: one-bye-one, each diner needs to guess which is the better restaurant, and while they can't see the reviews read by previous diners, they make their estimations on the restaurant chosen by others.

Let's now consider what we should expect to happen when this experiment is performed.

The first student. The first student makes his estimation following a simple decision-making rule: if he sees a red marble, he guesses the "majority-red" urn; otherwise he guesses the "majority-blue" one. This process is rational and clear: since two of the three red marbles are
in the "major-red" urn, the posterior probability of the event (MR) after having seen a red marble is $2 / 3$. On the other hand if the first student draws a blue marble, the posterior probability of the event (MR) becomes $1 / 3$ despite the posterior probability of the event (MB) 2/3.

At the beginning of the experiment the prior probabilities that the urn used is the majority red or the majority blue are both equal to 0,5 ; so we can conclude that the first student chooses the urn that shows, given the draw, a posterior probability higher than the prior one. From a mathematical point of view he must evaluate this equation:

## $\operatorname{Prob}(M B$ or $M R \mid$ colour obeserved $)>\frac{1}{2}$

The second student. Here two different cases could occur. If the second student sees the same color that the first student drew, again the choice is quite simple: he should guess the same urn as well. If, instead, the second student observes the opposite color, since he knows exactly what the first student has previously guessed, he remains indifferent between the choice "majority-red" and "majority-blue" urns. Given the fact that he must take a decision, he breaks the tie by choosing the color seen from the draw.

So we can conclude that, independently from the color observed, also his guess conveys perfect information about what he has seen.

The third student. Here things complicate again. If the first two students have guessed different urns, then the third student should choose the color he has observed.

The problem arises when he observes one color (for example red) and the first two students have both guessed the opposite color (blue). The third student perceives that the sequence of draws is: blue, blue, red and, consequently he estimates:

```
\(\operatorname{Prob}(M B \mid\) blue, blue, red \()=\)
\(=\frac{\operatorname{Prob}(\text { blue }, \text { blue }, \text { red } \mid M B) * \operatorname{Prob}(M B)}{\operatorname{Prob}(b l u e, b l u e, r e d \mid M B) * \operatorname{Prob}(M B)+\operatorname{Prob}(\text { blue }, \text { blue }, \operatorname{red} \mid M R) * \operatorname{Prob}(M R)}=\)
```

$=\frac{\frac{2}{3} * \frac{2}{3} * \frac{1}{3} * \frac{1}{2}}{\frac{2}{3} * \frac{2}{3} * \frac{1}{3} * \frac{1}{2}+\frac{1}{3} * \frac{1}{3} * \frac{2}{3} * \frac{1}{2}}=\frac{\frac{2}{27}}{\frac{1}{9}}=\frac{2}{3}$

Since we have supposed that the first two subjects convey perfect information,as a consequence, he should guess that the urn is "majority blue", ignoring his own private information that is the red marble observed.
"More generally, the point is that when the first two guesses are the same, the third student should guess this color as well, regardless of which color he draws from the urn. And the rest of the class will only hear his guess; they don't get to see which color he has drawn.
In this case, an information cascade has begun. The third student makes the same guess as the first two, regardless of which color he draws from the urn, and hence regardless of his own private information" ${ }^{5}$.

The fourth student and onward. Consider the case in which the first two subjects guessed the blue color and the third student guessed the red color regardless of what he has observed from the draw. The situation faced by the fourth individual is depicted by three "majority-blue" in a row. He knows that the first two guesses conveyed right information about the draws observed, but he also knows that the third student decided independently from the color of his draw, so he didn't add any useful information.

As a result, the fourth student finds himself in the same position as the third student: whatever color he observes, the decisions of the first two individuals outweighs his personal information. So he guesses "Majority-blue" independently from his draw.

This process will continue affecting all the subsequent students: they all guess the "majorityblue".

The way in which a cascade takes place teaches us some principles about it.
First, a cascade could occur easily, given the right structural condition. It also shows that a situation in which all people take the same decision could occur even if individuals are behaving rationally.
Second, an information cascade shows that it could lead to non-optimal outcomes. If, for instance, we consider a "majority-red" marble, it exists a probability of $\frac{1}{3}$ that the first student draws a blue marble, and one more time, a probability of $\frac{1}{3}$ that the second student observes another blue marble. Since these events are independent there is a $\frac{1}{9}$ probability that both first and second students guess the "majority-blue" urn and, consequently, all the following students in terms of information cascade.

[^3]This source of error, that occurs with probability $\frac{1}{9}$, is not limited by considering a larger sample of students because each individual, after having observed that two of them made the same choice before, is persuaded to follow it without taking into account his private information, no matter how copious is the sample.
Third, the experiment illustrates that cascades can also be fragile.
Suppose 100 students form the sample and the first two of them guess the "majority-blue" urn and so, also the following individuals forming information cascade. If students 50 and 51 both draw red marbles and they act in a different way, for example cheating by showing directly their marbles to the rest of the class. In this case, the cascade loses its continuity because the $52^{\text {nd }}$ student perceives new useful information: he observes that people 1 and 2 drew blue marbles and people 50 and 51 drew red marbles. So he should take his choice on the basis of his private information, that is the color of his marble drawn.
The point is that everyone knew the initial run of 49 "blue" guesses had very little information supporting it, and so it was easy for a fresh infusion of new information to overturn it. This is the essential fragility of information cascades: even after they have persisted for a long time, they can be overturned with comparatively little effort ${ }^{6}$.

If there are more than just two possible action alternatives, informational cascades can still result. However, as the set of alternatives becomes larger and richer, cascades tend to take longer to form and aggregate more information. If the set of action alternatives is continuous (for example, all points on the interval [ 0,1$]$ ), then even an individual late in the sequence will still adjust his action at least slightly based on the private signal. Consequently, private signals can be perfectly inferred from actions, information aggregate efficiently, and cascades do not form. However, if individuals cannot distinguish between nearby actions taken by their predecessors, cascades do arise.
More generally, one of the main lessons to be learned from cascades is that people must be careful when they have to make decisions basing them on the behavior of others because, as we have just observed, the crowd can be wrong even if every individual is rational. These observations suggest that information cascades can affect the behavior of individuals in some real economical scenarios.

For example, marketers exploit the idea of herding in order to generate a buying cascade for a specific product. If they are able to induce an initial set of people to buy a new product, then other individuals that want to complete the purchase later may adopt the same product even if

[^4]it isn't the best one in the market. This process becomes incredibly effective if customers are able to check choices of previous people but they don't know anything about the level of satisfaction perceived. This situation could be prevented, avoiding a herding effect, if the payoffs of each customer are available.
In the following chapter, an example of a decision-making process is presented. The aim of that work is to focus the attention first on how people make decisions under uncertainty under the assumption of rationality.

## 2. A MODEL OF RATIONAL UPDATING

### 2.1. The Model

The model presented in this chapter helps analyzehow people make decisions in a scenario of uncertainty.

First of all it's fundamental to describe the hypotheses underlyingthe model.
The model is based on a timeline divided in three periods. In each of these, two people are drawn randomly from a population of ten potential customers. These subjects must choose between two products, denoted A and B.

Let $q_{A}$ and $q_{B}$ be the true value of quality of A and B ; we assume that the quality can be either high or low, and we normalize high quality to 1 and low quality to 0 . In symbols, $q_{A}, q_{B} \in\{0,1\}$.

At the beginning of period one, nobody knows the true quality of the products. The idea is that they have just been launched in the market, so, as a consequence, it doesn't exist the possibility to seek and exploit information inferred by other people's choices and the characteristics of the two products remain unknown. The only knowledge shared by everybody is that both products are equally likely to be of high or low quality.

It means that, for each consumer, the prior probability about the quality of product A is equal to the prior probability of the quality of product B and both are equal to 0.5 .

$$
\operatorname{Prob}\left(q_{A}=1\right)=\operatorname{Prob}\left(q_{B}=1\right)=0,5 .
$$

Then we denote by " $\alpha_{t}$ " the number of people that chooses the product A in each period, and the number of periods in our model varies from 1 to 3 .

Given the fact that, from the second period onwards, people drawn could be first customers or individuals that have already had the possibility to try the product(s), we calculate in each period the probability that subjects drawn are going to decide which product to buy for the first, second or third time.

What we are interested in is to realize how consumers evaluate the probability that one product is better than the other one over time, and consequently how people make their decisions and if herd behavior emerges over time.

For this purpose we start analyzing the first period in which the first two people are randomly selected to choose between product A and B; then we go on through the second period in
which we have to highlight the differences in terms of decision-making process between people who buy one of the two products for the first time in the second period and people who buy for the second time in the second period.

Finally we take into account the third period in which the scenario complicates again because consumers who buy for the first time in the third period should take into account previous consumers' decisions.

### 2.2 Customer's behavior

## Period 1

As we have already said, in the first period two competing products (A and B) are launched in the market.

Two people drawn from a population of ten potential customers must choose one of the two alternatives.

Given the fact that the priors are 0.5 for both products, they will choose the product randomly because they don't know the effective quality and there is no possibility to learn from others. The aim of the first period is to check the related probability of the three possible cases: customers buy the product A , no one buys A or one of the two customers buys A .
From a mathematical point of view the answer is given by the use of the Binomial distribution.

The Binomial distribution is a discrete distribution that gives the probability of having $x$ "successes" (in our case successes are equal to the number of people that choose the product A in the first period, so we can consider $x=\alpha_{1}$ ) in $n$ independent trials, where the probability of a success in each trial is $p$.
In symbols:

$$
\operatorname{Bin}_{n, p}(x)=\binom{n}{x} p^{x}(1-p)^{n-x}
$$

Applying the Binomial distribution to our model and denoting by $x$ the number of people who chose product A we have that:
$\operatorname{Prob}\left(\alpha_{1}=0\right)=\operatorname{Bin}_{2,0.5}(0)=\binom{2}{0}(0.5)^{0}(0.5)^{2}=0,25=25 \%$
$\operatorname{Prob}\left(\alpha_{1}=1\right)=\operatorname{Bin}_{2,0.5}(1)=\binom{2}{1}(0.5)^{1}(0.5)^{1}=0.50=50 \%$
$\operatorname{Prob}\left(\alpha_{1}=2\right)=\operatorname{Bin}_{2,0.5}(2)=\binom{2}{2}(0.5)^{2}(0.5)^{0}=0.25=25 \%$.

## Period 2

In the second period two people are drawn again from the same population of potential customers, but the scenario changes a little bit.
Here, among all the possible results of the extraction, two kinds of purchasers can be selected: someone who has already bought the product in the previous period and someone who has not.

In order to understand the number of subjects that will decide to buy A or B it's first of all fundamental to calculate the probability of three different events: both customers drawn in the second period have already been drawn in the first period so they both purchase the product for the second time in the second period: we denote this occurrence by[(2,2)]; only one customer is drawn two times in a row so one individual purchases one product for the first time in the second period, and the other one does it for the second time [in symbols $(2,1)$ ]; both customers are drawn for the first time in the second period so they both choose between A and B for the first time in the second period [in symbols $(1,1)]$.

$$
\begin{aligned}
& \operatorname{Prob}(2,2)=\frac{2}{10} \times \frac{1}{9}=0,0 \overline{2} \\
& \operatorname{Prob}(2,1)=\frac{2}{10} \times \frac{8}{9}=0,3 \overline{5} \\
& \operatorname{Prob}(1,1)=\frac{8}{10} \times \frac{7}{9}=0,6 \overline{2}
\end{aligned}
$$

Once we have in mind the real probabilities that one of the three events occurs, the second step is to understand the beliefs of the two possible customers and the motivations behind the purchase of product A/B.
If the individual drawn was already extracted in the previous period, he will decide to buy either A or B basing the choice on the quality of his past experience:he will purchase A if he chose A in the pastand he was satisfied about the quality of it, or if he chose the product B before, but he was not satisfied with it. This can be summarized in an equation:
$\operatorname{Prob}\left(\alpha_{2} \neq 0\right)=\operatorname{Prob}\left(\alpha_{1} \neq 0\right) * \operatorname{Prob}\left(q_{A}=1\right)+\operatorname{Prob}\left(\alpha_{1}=0\right) * \operatorname{Prob}\left(q_{B}=0\right)$

The left part of the equation stands for the probability that at least one subject in the second period will choose the product A.

The term " $\operatorname{Prob}\left(\alpha_{1} \neq 0\right)$ "means the probability that in the previous period at least one individual chose A ; $" \operatorname{Prob}\left(q_{A}=1\right)$ " indicates the probability that the product A really performs well.
The term " $\operatorname{Prob}\left(\alpha_{1}=0\right)$ " means the probability that no one chose A in the previous period and so as a consequence B was chosen by both consumers; " $\operatorname{Prob}\left(q_{B}=0\right)$ " indicates the probability that the product B is a bad product.

If instead the individual is drawn for the first time in the second period, the way by which he takes his decision differs.

In fact, assuming that customers are rational, they know that those individuals, who tried the products in the first period, chose randomly because they did not have any information about the quality of the two products and they weren't able to receive any advice collected by others. So it becomes clear that people drawn for the first time in the second period will behave in the same way as people in the first period: at random.
Now it's possible to go on with the model calculating the probability that in the second period no one chooses product A , one of the two people drawn chooses A , both individuals choose A.

In order to keep the model clear the letter " $\alpha$ " is introduced and it stands for the number of individuals that, at each time, choose the product A.

$$
\begin{aligned}
\operatorname{Prob}\left(\alpha_{2}=0\right) & =\operatorname{Prob}\left(\alpha_{2}=0 \mid(1,1)\right) * \operatorname{Prob}(1,1)+\operatorname{Prob}\left(\alpha_{2}=0 \mid(2,1)\right) * \operatorname{Prob}(2,1) \\
& +\operatorname{Prob}\left(\alpha_{2}=0 \mid(2,2)\right) * \operatorname{Prob}(2,2)
\end{aligned}
$$

$$
\operatorname{Prob}\left(\alpha_{2}=1\right)=\operatorname{Prob}\left(\alpha_{2}=1 \mid(1,1)\right) * \operatorname{Prob}(1,1)+\operatorname{Prob}\left(\alpha_{2}=1 \mid(2,1)\right) * \operatorname{Prob}(2,1)
$$

$$
+\operatorname{Prob}\left(\alpha_{2}=1 \mid(2,2)\right) * \operatorname{Prob}(2,2)
$$

$$
\operatorname{Prob}\left(\alpha_{2}=2\right)=\operatorname{Prob}\left(\alpha_{2}=2 \mid(1,1)\right) * \operatorname{Prob}(1,1)+\operatorname{Prob}\left(\alpha_{2}=2 \mid(2,1)\right) * \operatorname{Prob}(2,1)
$$

$$
+\operatorname{Prob}\left(\alpha_{2}=2 \mid(2,2)\right) * \operatorname{Prob}(2,2)
$$

Starting from the first equation and substituting with the related numbers we obtain:

- $\operatorname{Prob}\left(\alpha_{2}=0 \mid(1,1)\right) * \operatorname{Prob}(1,1)=0,25 * 0,6 \overline{2}=0,1 \overline{5}$
- $\operatorname{Prob}\left(\alpha_{2}=0 \mid(2,1)\right) * \operatorname{Prob}(2,1)=\left[1-\left(0,5 q_{A}+0,5\left(1-q_{B}\right)\right)\right] * 0,5 * 0,3 \overline{5}$
- $\operatorname{Prob}\left(\alpha_{2}=0 \mid(2,2)\right) * \operatorname{Prob}(2,2)=\left[1-\left(0,5 q_{A}+0,5\left(1-q_{B}\right)\right)\right]^{2} * 0,0 \overline{2}$

Summing up all the three equations above we can calculate the probability that, in the second period, no one chooses the product A , which is:

$$
\begin{aligned}
& \operatorname{Prob}\left(\alpha_{2}=0\right)= \\
& \quad=0,1 \overline{5}+\left[1-\left(0,5 q_{a}+0,5\left(1-q_{b}\right)\right)\right] * 0,5 * 0,3 \overline{5}+\left[1-\left(0,5 q_{a}+0,5\left(1-q_{b}\right)\right)\right]^{2} * 0,0 \overline{2}
\end{aligned}
$$

Going on with the second group of equation of the model we obtain:

- $\operatorname{Prob}\left(\alpha_{2}=1 \mid(1,1)\right) * \operatorname{Prob}(1,1)=0,5 * 0,6 \overline{2}=0,3 \overline{1}$
- $\operatorname{Prob}\left(\alpha_{2}=1 \mid(2,1)\right) * \operatorname{Prob}(2,1)=\left(0,5 q_{A}+0,5\left(1-q_{\square}\right)\right) * 0,5 * 0,3 \overline{5}+$ $\left[1-\left(0,5 q_{A}+0,5\left(1-q_{B}\right)\right)\right] * 0,5 * 0,3 \overline{5}=0,5 * 0,3 \overline{5}$
- $\operatorname{Prob}\left(\alpha_{2}=1 \mid(2,2)\right) * \operatorname{Prob}(2,2)=\left(0,5 q_{A}+0,5\left(1-q_{B}\right)\right) *\left[1-\left(0,5 q_{A}+\right.\right.$ $0,51-q B * 0,02$

Summing up,it's possible to calculate the probability that, in the second period, one of the two customers chooses the product A , which is:

$$
\begin{aligned}
& \operatorname{Prob}\left(\alpha_{2}=1\right)= \\
& \quad=0,3 \overline{1}+0,5 * 0,3 \overline{5}+\left(0,5 q_{a}+0,5\left(1-q_{b}\right)\right) *\left[1-\left(0,5 q_{a}+0,5\left(1-q_{b}\right)\right)\right] * 0,0 \overline{2}
\end{aligned}
$$

Finally we take into account the third case:

- $\operatorname{Prob}\left(\alpha_{2}=2 \mid(1,1)\right) * \operatorname{Prob}(1,1)=0,25 * 0,6 \overline{2}=0,1 \overline{5}$
- $\operatorname{Prob}\left(\alpha_{2}=2 \mid(2,1)\right) * \operatorname{Prob}(2,1)=\left(0,5 q_{A}+0,5\left(1-q_{B}\right)\right) * 0,5+0,3 \overline{5}$
- $\operatorname{Prob}\left(\alpha_{2}=2 \mid(2,2)\right) * \operatorname{Prob}(2,2)=\left(0,5 q_{A}+0,5\left(1-q_{B}\right)\right)^{2} * 0,0 \overline{2}$

Summing up, it's possible to calculate the probability that, in the second period, both consumers choose the product A , which is:

```
\(\operatorname{Prob}\left(\alpha_{2}=2\right)=\)
\(=0,1 \overline{5}+\left(0,5 q_{A}+0,5\left(1-q_{B}\right)\right) * 0,5+0,3 \overline{5}+\left(0,5 q_{A}+0,5\left(1-q_{B}\right)\right)^{2} * 0,0 \overline{2}\)
```

Once the three possible events are clear, it's possible to go on with the third period.

## Period 3

The third period starts in the same way as the previous ones. Two individuals are drawn a third time from the same population of ten "possible customers" that was considered in the first and second period.
The aim is always to identify and analyze the purchasing decision made by the two subjects, but here the scenario complicates again.

The types of possible individuals drawn in this timeline become three: there is a (low) probability that a customer drawn in the third period has been already drawn in both previous periods; some probability that a consumer drawn in the third periodhas been previously drawn once; and some probability that he is drawn for the first time.
In order to be able to understand with which probability each event occurs, we have to pay attention to the behavior of the three possible customers described in the previous paragraph.

First we start analyzing individuals that are drawn for the second time in the third period.They are called out to choose product A or B for the second time.
It's clear that they have already tried one of the two products, so they know exactly its characteristics, but they don't know anything about the other one.
This scenario is perfectly represented by the one of the second period in which the customer is drawn two times in a row.

He chooses the product A if he has already bought that product before and felt satisfied; on the other hand, he also chooses A if he has previously bought the product B but he was not satisfied with it.

The second case concerns individuals that could have been drawn in both previous periods. They are called out to choose between product A or B for the third time.

Here, they look at their past experiences and they base their decision on the level of satisfaction.

If they have bought the product A in the first period and they thought that was good, they continue buying that product also in the second and third period.
They choose the product A in the third period also in another two cases: if they have bought the product B in the first period but the quality perceived was bad, in the second period they opt for the product A tasting its quality. If A performs well, they obviously continue buying
that product also in the third period; if also A performs badly, we assume that they prefer to stick with A rather than to go back to B (the idea is that there are some small switching costs). Clearly, customers choose the product B if it presents good characteristics.

The third case concerns individuals that must choose between the two products for the first time, so they have no personal experiences about the quality of each of them.

Here, however, those customers take their decision in a different way with respect to the "new entries" of the second period.

From the beginning of the model that corresponds to the launch of the two products, two periods are passed by and some information about the true quality of the products have been spread out through previous customers' choices.

Hence, period-3 new customers do not choose to buy A or B randomly, but instead they evaluate the probability that the product A is good $\left(q_{A}=1\right)$ on the basis of the number of persons that bought it in the previous period.

In order to evaluate carefully that probability, those customers must consider how the second period evolved. Thus they take into account three different probabilities diversified by the number of people who have chosen A in $T_{2}$.

- $\operatorname{Prob}\left(q_{A}=1 \mid \alpha_{2}=2\right)$
- $\operatorname{Prob}\left(q_{A}=1 \mid \alpha_{2}=1\right)$
- $\operatorname{Prob}\left(q_{A}=1 \mid \alpha_{2}=0\right)$

To compute these probabilities we apply the Bayes rule.
Bayes Rule is a theorem in probability theory that answers the question, "When you encounter new information, how much should it change your confidence in a belief?" It's essentially about how probabilities should be updated or revised when new evidence emerges.

Bayes' theorem is summarized by the following equation:
$\mathrm{P}(\mathrm{A} \mid \mathrm{B})=\frac{\mathrm{P}(\mathrm{B} \mid \mathrm{A}) * \mathrm{P}(\mathrm{A})}{\mathrm{P}(\mathrm{B})}$
Where $A$ and $B$ are events and $P(B) \neq 0$.
$P(A)$ and $P(B)$ are the probabilities of observing $A$ and $B$ without regard to each other.
$\mathrm{P}(\mathrm{A} \mid \mathrm{B})$ is the probability of observing event A given that B is true.
$P(B \mid A)$ is the probability of observing event $B$ given that $A$ is true.

Coming back to our model, we can rewrite the first equation as follows:
$\operatorname{Prob}\left(q_{A}=1 \mid \alpha_{2}=2\right)=\frac{\operatorname{Prob}\left(\alpha_{2}=2 \mid q_{A}=1\right) * \operatorname{Prob}\left(q_{A}=1\right)}{\operatorname{Prob}\left(\alpha_{2}=2\right)}$

Bayes rule tells the probability that the product A is good, given that both customers chose A in the past period, is equal to the probability that two customers chose A in the second period, given that the quality of the product is good, multiplied by the prior probability that the quality is really good, all divided for the real probability that both customers bought A in the previous period.

In order to solve that equation the first step is the evaluation of the term $\operatorname{Prob}\left(\alpha_{2}=2 \mid q_{A}=1\right)$.

As it was already said in the initial part of the chapter, potential customers are different on the basis of how many times they were drawn.
So we have to make that equation written above contingent upon the different kinds of customers.

$$
\begin{aligned}
\operatorname{Prob}\left(\alpha_{2}=2 \mid q_{A}=1\right)= & \\
& =\operatorname{Prob}\left(\alpha_{2}=2 \mid(1,1), q_{A}=1\right) * \operatorname{Prob}(1,1) \\
& +\operatorname{Prob}\left(\alpha_{2}=2 \mid(2,1), q_{A}=1\right) * \operatorname{Prob}(2,1) \\
& +\operatorname{Prob}\left(\alpha_{2}=2 \mid(2,2), q_{A}=1\right) * \operatorname{Prob}(2,2)
\end{aligned}
$$

$$
\begin{aligned}
& \operatorname{Prob}\left(\alpha_{2}=2 \mid q_{A}=1\right) \\
&=\left[0,25 q_{A}+(1-0,25)\left(1-q_{B}\right)\right] * 0,0 \overline{2}+\left[0,5 q_{A}+0,5\left(1-q_{B}\right)\right] * 0,5 \\
& * 0,3 \overline{5}+0,25 * 0,6 \overline{2}
\end{aligned}
$$

The denominator, instead, can be computed through the total probability rule. It refers to the general probability that both consumers choose the product A in the second period, without conditioning it to the quality of the product; so it can be calculated in this way:

$$
\begin{aligned}
& \operatorname{Prob}\left(\alpha_{2}=2\right)=\operatorname{Prob}\left(\alpha_{2}=2 \mid q_{A}=1\right) * \operatorname{Prob}\left(q_{A}=1\right) \\
&+\operatorname{Prob}\left(\alpha_{2}=2 \mid q_{A}=0\right) * \operatorname{Prob}\left(q_{A}=0\right)
\end{aligned}
$$

The second term $\operatorname{Prob}\left(\alpha_{2}=2 \mid q_{A}=0\right)$ stands for the probability that both consumers choose the product A conditional on the low quality of the product. This event occurs only if the consumers taken into account are "new entries" of the third period and they both randomly choose A. So:
$\operatorname{Prob}\left(\alpha_{2}=2 \mid q_{A}=0\right)=0,25 * 0,6 \overline{2}$

The probability that A is good conditional on the fact that both customers chose A in the second period is finally calculated as follows:

$$
\operatorname{Prob}\left(q_{A}=1 \mid \alpha_{2}=2\right)=
$$

$$
=\frac{\operatorname{Prob}\left(\alpha_{2}=2 \mid q_{A}=1\right) * \operatorname{Prob}\left(q_{A}=1\right)}{\operatorname{Prob}\left(\alpha_{2}=2 \mid q_{A}=1\right) * \operatorname{Prob}\left(q_{A}=1\right)+\operatorname{Prob}\left(\alpha_{2}=2 \mid q_{A}=0\right) * \operatorname{Prob}\left(q_{A}=0\right)}
$$

The only passage needed to reach the numerical value is to substitute the number of the numerator and the denominator, but this calculation is computed in the next paragraph where results are showed.

If, instead, the individual drawn for the first time in the third period finds that only one customer chose the product A before, he takes his decision considering the probability that A is good conditional on the connected number of individual that chose A in the previous period.

He, thus, evaluates the following probability:

$$
\begin{aligned}
& \operatorname{Prob}\left(q_{A}=1 \mid \alpha_{2}=1\right)= \\
& \qquad=\frac{\operatorname{Prob}\left(\alpha_{2}=1 \mid q_{A}=1\right) * \operatorname{Prob}\left(q_{A}=1\right)}{\operatorname{Prob}\left(\alpha_{2}=1 \mid q_{A}=1\right) * \operatorname{Prob}\left(q_{A}=1\right)+\operatorname{Prob}\left(\alpha_{2}=1 \mid q_{A}=0\right) * \operatorname{Prob}\left(q_{A}=0\right)}
\end{aligned}
$$

The left part of the numerator can be conditioned also to the different kinds of customers that are called out to choose between A and B.

We can rewrite it as follows:

$$
\begin{aligned}
& \operatorname{Prob}\left(\alpha_{2}=1 \mid q_{A}=1\right)= \\
&=\operatorname{Prob}\left(\alpha_{2}=1 \mid(1,1), q_{A}=1\right) * \operatorname{Prob}(1,1) \\
&+\operatorname{Prob}\left(\alpha_{2}=1 \mid(2,1), q_{A}=1\right) * \operatorname{Prob}(2,1) \\
&+\operatorname{Prob}\left(\alpha_{2}=1 \mid(2,2), q_{A}=1\right) * \operatorname{Prob}(2,2) \\
& \operatorname{Prob}\left(\alpha_{2}=1 \mid q_{A}=1\right)= \\
&=0,5 q_{B} * 0,0 \overline{2}+\left\{0,5 * 0,5 q_{B}+0,5 *\left[0,5 q_{A}+0,5\left(1-q_{B}\right)\right]\right\} * 0,3 \overline{5}+0,5 * 0,6 \overline{2}
\end{aligned}
$$

The right part of denominator, instead, is calculated as follows:

$$
\begin{aligned}
& \operatorname{Prob}\left(\alpha_{2}=1 \mid q_{A}=0\right)= \\
& \quad=\left\{0,5 *\left[0,5\left(1-q_{a}\right)+0,5 q_{b}\right]+0,5 * 0,5\left(1-q_{b}\right)\right\} * 0,3 \overline{5}+0,5 * 0,6 \overline{2}
\end{aligned}
$$

We can conclude that, in order to calculate the probability that the product A is good, observing that only one over the two consumers chose A in the previous period, we have to substitute respectively the value of the numerator and denominator in the earlier equation. The results are commented in the next paragraph.

If, finally, the "new customer" in the third period observes that no one has previously chosen A , he estimates the following probability:
$\operatorname{Prob}\left(q_{A}=1 \mid \alpha_{2}=0\right)=$

$$
=\frac{\operatorname{Prob}\left(\alpha_{2}=0 \mid q_{A}=1\right) * \operatorname{Prob}\left(q_{A}=1\right)}{\operatorname{Prob}\left(\alpha_{2}=0 \mid q_{A}=1\right) * \operatorname{Prob}\left(q_{A}=1\right)+\operatorname{Prob}\left(\alpha_{2}=0 \mid q_{A}=0\right) * \operatorname{Prob}\left(q_{A}=0\right)}
$$

Starting, as always, from the analysis of the left part of the numerator, he must consider the probability that the previous customers have already bought one of the two products or not yet.

So, that probability, from a mathematical point of view, is estimated as follows:

$$
\begin{aligned}
\operatorname{Prob}\left(\alpha_{2}=0 \mid q_{A}=1\right)= & \\
& =\operatorname{Prob}\left(\alpha_{2}=0 \mid(1,1), q_{A}=1\right) * \operatorname{Prob}(1,1) \\
& +\operatorname{Prob}\left(\alpha_{2}=0 \mid(2,1), q_{A}=1\right) * \operatorname{Prob}(2,1) \\
& +\operatorname{Prob}\left(\alpha_{2}=0 \mid(2,2), q_{A}=1\right) * \operatorname{Prob}(2,2)
\end{aligned}
$$

$$
\operatorname{Prob}\left(\alpha_{2}=0 \mid q_{A}=1\right)=0,25 q_{B} * 0,0 \overline{2}+0,5 * 0,5 q_{B} * 0,3 \overline{5}+0,25 * 0,6 \overline{2}
$$

Then, he must analyze the right part of the denominator:

$$
\begin{aligned}
\operatorname{Prob}\left(\alpha_{2}=0 \mid\right. & \left.q_{A}=0\right) \\
& =\left(0,25\left(1-q_{A}\right)+0,5 q_{B}\right)^{2} * 0,0 \overline{2}+0,5 *\left(0,25\left(1-q_{A}\right)+0,5 q_{B}\right) * 0,3 \overline{5} \\
& +0,25 * 0,6 \overline{2}
\end{aligned}
$$

$\operatorname{Prob}\left(\alpha=0_{t 2} \mid q_{a}=0\right)=0,0 \overline{2}+0,5 * 0,3 \overline{5}+0,25 * 0,6 \overline{2}$.

Again, substituting the computed numerator and denominator within the main equation, the final numerical result of that probability is reached.

Now we can go on with a numerical example that highlights the results of the estimation of these probabilities in a case in which one product is good and the other one is bad.

### 2.3. Market Outcome

The aim of the example concerns the analysis of a specific case in which two different products A and B are launched in the market. The value of the quality of A is equal to 1 . The value of the quality of $B$ is equal to 0 .
Given the fact that the prior probabilities related to the quality of products A and B are both equal to 0.5 , as we have already discussed in the theory of the model, each "first customer" in each period will choose randomly because they, initially, have no preferences.

We can focus the attention on the results of the third period.
Starting from the three final equations reached in the previous chapter, and substituting for the product $\mathrm{A}: q_{A}=1$ and for the product $\mathrm{B}: q_{B}=0$, we obtain these results:

- $\operatorname{Prob}\left(q_{A}=1 \mid \alpha_{2}=2\right)=$

$$
=\frac{(0,0 \overline{2}+0,5 * 0,3 \overline{5}+0,25 * 0,6 \overline{2}) * 0,5}{(0,0 \overline{2}+0,0 \overline{5} * 0,3 \overline{5}+0,25 * 0,6 \overline{2}) * 0,5+0,25 * 0,6 \overline{2} * 0,5}=0,69565
$$

The posterior probability that the quality of the product A is high, given the fact that both customers chose A before, is equal to $69,6 \%$.

It means that a rational customer will choose the product A due to the fact that the posterior probability referred to its quality is higher than the prior probability.

- $\operatorname{Prob}\left(q_{A}=1 \mid \alpha_{2}=1\right)=$

$$
=\frac{(0,5 * 0,3 \overline{5}+0,5 * 0,6 \overline{2}) * 0,5}{(0,5 * 0,3 \overline{5}+0,5 * 0,6 \overline{2}) * 0,5+(0,5 * 0,3 \overline{5}+0,5 * 0,6 \overline{2}) * 0,5}=0,5
$$

The posterior probability that the quality of the product A is high, given the fact that only one over the two customers chose A before, is equal to $50 \%$.

In this case the new customer isn't able to add new information from the analysis of other customers' decision. It is highlighted by the fact that the value of the posterior probability is equal to the value of the prior one. As a consequence, he must choose the purchase randomly also in the third period.

- $\operatorname{Prob}\left(q_{A}=1 \mid \alpha_{2}=0\right)=$

$$
=\frac{0,1 \overline{5} * 0,5}{0,1 \overline{5} * 0,5+(0,0 \overline{2}+0,5 * 0,3 \overline{5}+0,25 * 0,6 \overline{2}) * 0,5}=0,30434
$$

The posterior probability that the quality of the product A is good, given the fact that no one chose A before, is equal to $30,4 \%$.

It means that the posterior probability is lower than the prior probability " $q_{A}$ ", the customer will choose the product $B$, and we can conclude that it is a rational behaviour.

That case captures our attention because it answers to another question: "is it possible that customers in the market choose the worst product?"
The answer is yes. If an individual with no information has to choose between two unknown products, he looks at the past capturing signals from other customers.

If he observes that no one chose the product A in the past, he assigns around $70 \%$ probability that A is bad. The customer reaches this percentage because he is rational and he considers in the right way that the previous individuals drawn in the second period could have been people who had already bought one of the two product also in the first period or people without any experience.
So there is a probability that, even if the quality of A is good, in the third period no one choses that product.

Given the fact that A is better than B , If individuals drawn for the first time in the third period observes that in the previous period both customers bought the product B , they follow the same choice, but that could be the case in which those people drawn in the second period were in turn individuals who bought B randomly without knowing that the choice was wrong. In order to give an estimation to the probability that this event occurs, we have to start from the case in which two individuals are drawn for the first time in the second period and they both buy the product B randomly:

$$
\operatorname{Prob}\left(\alpha_{2}=0 \mid(1,1)\right) * \operatorname{Prob}(1,1)=0,25 * 0,6 \overline{2}=0,1 \overline{5}
$$

Then the following step concerns the probability that two new customers are drawn in the third period and they both choose the product B basing their decision by looking at the previous period in the timeline.

$$
\begin{aligned}
& \operatorname{Prob}\left(\alpha_{3}=0 \mid(1,1) * \operatorname{Prob}(1,1)_{3}=\right. \\
& \qquad=1 *\left[\operatorname{Prob}(2,2)_{2} * \frac{8}{10} * \frac{7}{9}+\operatorname{Prob}(2,1)_{2} * \frac{7}{10} * \frac{6}{9}+\operatorname{Prob}(1,1)_{2} * \frac{6}{10} * \frac{5}{9}\right] \\
& =0,0 \overline{2} * \frac{8}{10} * \frac{7}{9}+0,3 \overline{5} * \frac{7}{10} * \frac{6}{9}+0,6 \overline{2} * \frac{6}{10} * \frac{5}{9}=0,38716049 .
\end{aligned}
$$

The probability that both individuals choose the product B in the third period, conditioned to the fact that they are new customers, in our example is equal to 1because it's known that they analyze the decisions made by the others discovering that $\operatorname{Prob}\left(q_{A}=1 \mid \alpha_{2}=0\right)=$ $30,4 \%$ (Less than the prior probability 0,5 ).

Finally, these two probabilities must be multiplied in order to complete the equation:

$$
\begin{aligned}
\operatorname{Prob}\left(\alpha_{3}=0 \mid(1,1), q_{A}=1\right) & =\operatorname{Prob}\left(\alpha_{2}=0 \mid(1,1)_{2}\right) * \operatorname{Prob}(1,1)_{2} * \operatorname{Prob}(1,1)_{3}= \\
& =0,1 \overline{5} * 0,38716049=0,060224965 .
\end{aligned}
$$

$\operatorname{Prob}\left(\alpha_{3}=0 \mid(1,1), q_{A}=1\right)=6 \%$.

We can define the result as a positive restrained probability that, even if the quality of the product A is high, both customers choose the other product. It means that the worst product wins on the favorite one.

Clearly, if we consider a longer time horizon, then, sooner or later, all consumers will be drawn for two times: at that point, they will be able to make the correct choice because they won't have any doubt about which of the two products is the best one.

Following that concept it seems it doesn't matter the order of choice accomplished by individuals, how the estimation of probability evolves over time from priors to posterior ones. There is no space for errors and biases of the human mind; the whole model is only focused on the amount of time needed to present a sample of customers that have already tried both products once.

As we have already said, the model has the aim to simplify a potential scenario of real market and it is conditioned to initial hypotheses.

In the real world, given the fact that the periods flow, also the population modifies itself through times. Let's think about replacement of generations: new customers could believe that those products are worthless. Considering, instead, the same customers, they could switch their priors and preferences.

The two products could also generate new needs and desires perceived by individuals not present in the market yet. This scenario could lead to an expansion of the sample through new uninformed customers that, once drawn, will define their preferences following the same set of thought presented in the third period of the model.

Coming back to the question: "is it possible that customers in the market choose the worst product?" if we consider only the model and a set of periods tending to infinity the first answer is no. There will be a moment in which all customers have already tried at least one product and perhaps both.

It means they will follow their preferences based on the level of satisfaction of the choice previously made, and nobody will need to analyze the decisions of other individuals.

That is true for sure, but another time, in order to give a rational explanation to the possible consequences of the model, we have to pay attention to other elements that, in a real market, affects the conclusions.

In fact, in the short-term period, the model states that there is a positive possibility that the worst product (in our example B) becomes the more sold. As a consequence, the firm that produces it, always in the short-term period, earns more. Its structure could improve; it could obtain a better financing offer from banks or private institutions; managers could develop a more effective marketing campaign and so on. On the other hand, the firm that produces the best product in the market suffers a loss in sales and profits. It is forced to decrease the quality of the product or to increase prices in order to overtake the break-even point, establishing a ripple effect that leads to a business crisis.

It follows that the answer is more complex than it appears. If the rare but real probability that in the third period all customers chose the product B occurs, even if the quality is lower, from that point the economical scenario of the two firms changes and the producer of the outcome $B$ could gain from that disparity in terms ofsales.
We can conclude saying that the firm which produces the worst product in the market could benefit from a negative herd behavior due to that $6 \%$ probability and due to all the economical consequences that follow; and, in the long-term view, it could reach the best position in the market.

## 3. THE HEURISTIC REPRESENTATIVENESS AND ITS EFFECTS ON THE MARKET

### 3.1. Heuristics

In the previous chapter we have analyzed the behavior of rational individuals called out to choose between two different alternatives in a situation of uncertainty and its market outcome. The underlying hypothesis was the fact that people are perfectly able to infer correctly the information owned by other individuals by observing their actions. In other words, we have supposed complete rationality.

Economists, in the past, have typically described this behavior of humans as strictly logical, centered on a clearly defined goal and free from the unsteady influences of emotion or irrationality instead of focusing on the uncertain, error-prone groping with which most of us are familiar. Of course, some human behavior does fit the rational pattern so beloved of those economists.

In the recent years, they, instead, are focusing on the analysis of the irrationality. It is becoming increasingly fashionable for economists, especially the younger, more ambitious ones, to borrow insights from psychologists to try to explain some types of behaviors that seem to defy rationality.

Many decisions are based on beliefs concerning the likelihood of uncertain events such as the outcome of an election, the guilt of a defendant, or the future value of the dollar. These beliefs are usually expressed in statement such as "I think that...", "chances are...", "it is unlikely that..." and so forth. Occasionally, beliefs concerning uncertain events are expressed in numerical form as odds or subjective probabilities.

Given that decision makers can never acquire or process all the information they need to make the best or optimal decision, it is not surprising that as they try to select the best alternative several sources of error may hurt their decision making.

Some of these sources of error are pervasive and recurring, and frequently decision makers succumb to these errors and so make less than satisfactory decisions.

One of the major sources of error arises from the rules of thumb or heuristics that people use to make decisions.

Amos Tversky and Daniel Kahnemanwere among the first to deeply analyze the decision process followed by individuals in the presence of uncertainty without assuming ex-ante full rationality. They show that people rely on a limited number of heuristic principles that reduce
the complex tasks of assessing probabilities and predicting values to simpler judgmental operations.

In general, these heuristics are quite useful, but sometimes they lead to severe and systematic errors.

The subjective assessment of probability resembles the subjective assessment of physical quantities such as distance or size. These judgments are all based on data of limited validity, which are processed according to heuristic rules. For example, the apparent distance of an object is determined in part by clarity. The more sharply the object is seen, the closer it appears to be. This rule has some validity, because in any given scene the more distant objects are seen less sharply than nearer objects. However, the reliance on this rule leads to systematic errors in the estimation of distance. Specifically, distances are often overestimated when visibility is poor because the contours of objects are blurred. On the other hand, distances are often underestimated when visibility is good because the objects are seen sharply. Thus, the reliance on clarity as an indication of distance leads to common biases. Such biases are also found in the intuitive judgment of probability. ${ }^{7}$

Amos Tversky and Daniel Kahneman described three different kinds of heuristics that can occur when predicting values, so in this chapter we are going to introduce a narrow description of them and then we focus our attention on the representativeness heuristic, the one that will be analyzed also in our model.

The first one is called availability heuristic.
When making decisions, individuals often have to judge the frequency with which different events will occur and their likely causes. Theavailability heuristic reflects the tendency to determine the frequency of an event and its causesby how easy these events and causes are to remember.

If people have no difficulties to remember a circumstance, they overestimate the probability that it occurs. The same process arises when an event comes in mind more quickly.
The availability heuristic can help individuals when making decisions because usually frequent events become easier to remind, but sometimes it can cause errors of estimations that cause wrong decisions.

One such bias is the overestimation of the frequency of extremeevents and their causes

[^5]because these kinds of event are memorable.

Another one is the overestimationof the frequency of recent events and their causes because they are simply the freshest in memory.

Another kind of heuristic to mention is the anchoring heuristic.
The anchoring and adjustment heuristic is the tendency to make decisions based on adjustments or estimates from some initial amount or quantity.

For example salaries increase by choosing a percentage executed on an employee's current salary.

Budget decisions are often plannedstarting from the ongoing financing conditions and then deciding whether to increase or decrease it.

Decisions about the degree to which costs must be cut are often based on the current level of costs.

In situations like these, ifthe initial amounts are reasonable, then the anchoring and adjustment heuristic might be a goodshortcut for the decision-making.

A decision maker, in this way, could choose the final value rapidly saving time otherwise spent to count and estimate all possible variables starting from zero.

But if the original amount from which a decision or adjustment is made is not reasonable, the anchoring and adjustment heuristic will lead to biased decision making.

For example, if employees'current salary levels are low in comparison to what they could be earning in similar kinds of jobsand companies, even a relatively large percentage increase, say $20 \%$, may still leave themunderpaid.

The last type of heuristic is called representativeness heuristic.
The representativeness heuristic is the tendency to predict the likelihood of an event occurring in the future basing its estimation on how it is similar or representative of events that have occurred in the past.

The representativeness heuristic sometimes is useful because it allows reaching an evaluation of the probability rapidly through the help of the analysis of similar events or past events that act as good predictors.

On the other hand, it can also lead decision makers to disregard essential information about the frequency of such events; if they overestimate or underestimate it, as a consequence biases on the evaluation of the likelihood of events birth.

In the next paragraph we analyze the representativeness heuristic and we discover which are
the main factors that lead to this kind of bias in the estimation of probability.

### 3.2. The Representativeness Heuristic

Usually people ask themselves questions that belong to one of the following types: "what is the probability that an event A originates from process B?" " What is the probability that process $B$ will generate the event $A$ ?"
In those cases, individuals rely on the representativeness heuristic. They evaluate that likelihood on the basis of the degree to which A is representative of B and consequently A resembles B.

For an illustration of judgment by representativeness consider an individual who has been described by a former neighbor as follows: "Steve is very shy and withdrawn, invariably helpful, but with little interest in people, or in the world reality. A meek and tidy soul, he has a need for order and structure, and a passion for detail." How do people assess the probability that Steve is engaged in a particular occupation from a list of possibilities (for example, farmer, salesman, airline pilot, librarian, of physician)? How do people order these occupations from most to least likely? In the representativeness heuristic the probability that Steve is a librarian for example, is assessed by the degree to which he is representative of, or similar to, the stereotype of a librarian. Indeed research with problems of this type has shown that people order the occupation by probability and by similarity in exactly the same way. This approach to the judgment of probability leads to serious errors because similarity or representativeness is not influenced by several factors that should affect judgments of probability ${ }^{8}$.

Tversky and Kahneman reported another example that underlines the effect of the heuristic representativeness on the estimation of probability.
"A stimulus sample or description, D, may describe a person named Linda as ' 31 years old, single, outspoken, and very bright, with a major in philosophy; has concerns about discrimination and social justice; and was involved in anti-nuclear demonstrations while a university student'. This person is judged to be more likely to belong to category ' $\mathrm{A} \& \mathrm{~B}$ ' (Women who are active in the feminist movement and bank tellers) than to category ' B ' (Women who are bank tellers). But this often- replicated finding violates the conjunction rule, which says that the conjunction of two events (active feminist and bank teller) cannot be more probable than any of the two events alone (bank teller): $\mathrm{P}(\mathrm{A} \& \mathrm{~B}) \leq \mathrm{P}(\mathrm{B})$. Yet, according to the representativeness heuristic, the description of Linda (D) is more representative of the

[^6]conjunction (A \& B) than of the conjunct (B). ${ }^{9,}$
One of the first causes that lead to the heuristic representativeness is the insensitivity to prior probability of outcomes or base-rate frequency of outcomes. It is a cognitive error whereby too little weight is placed on the base (original) rate of possibility.
In the previous example of Steve, reported by Tversky and Kahneman, even though he has the characteristics of a librarian compared to a farmer, the fact that there are many more farmers than librarians in his population needs to be taken into account when assessing the likelihood of him having one occupation over the other. If in Steve's population there are a lot of farmers because of the rich soil in his area, the base-rate frequency suggests that Steve is more likely to be a farmer than a librarian.

Considerations of the base-rate frequency, however, do not affect the similarity of Steve to any of the stereotypes of workers. If someone evaluates the probability that he is a farmer or a librarian basing the choice relying on representativeness, those prior probabilities will not be considered.

In 1973, Kahneman and Tversky conducted an experiment to show how often people overlook base-rate frequency when assessing probability outcomes.
Participants were shown short personality descriptions of workers sampled from one hundred people, lawyers and engineers. The aim of that experiment consisted in assessing which people were likely to belong to one of the two types of jobs.
In one experimental condition, participants were given the following base rate, 70 engineers and 30 lawyers. In another condition, the base rate was 30 engineers and 70 lawyers.

Given those different prior conditions, the odds that any descriptions belong to an engineer should be higher in the first case where the majority of workers practices engineering; on the other hand the probability that any descriptions belong to a lawyer should be higher in the second scenario where the prior information tell us that $70 \%$ of the whole sample are lawyers. Despite all these rational assumptions, people produced the same probability judgments. Apparently, subjects estimated the likelihood that the descriptions belonged to one of the two kind of job basing it on the degree to which those descriptions were representative of a lawyer or an engineer without paying attention to the prior probability of the sample they were analyzing.

If the same individuals that had to complete the experiment didn't have any information about the personality of the workers, they would have used correctly the prior probabilities. In the absence of the descriptions, they would have estimated that an unknown individual were

[^7]supposed to be an engineer with $70 \%$ probability in the first scenario and $30 \%$ in the second one.

However, when in the experiment descriptions are introduced, individuals tend to ignore the prior probabilities also in the case in which information are useless.

Edmund Fantino reported a famous example and gave an explanation that highlights the failure of considering prior probabilities:
"A cab was involved in a hit-and-run accident at night. Two cab companies, the Green and the Blue, operate in the city. You are given the following data: a) $85 \%$ of the cabs in the city are Green and $15 \%$ are Blue; and b) a witness identified the cab as Blue. The court tested the reliability of the witness under the same circumstances that existed on the night of the accident and concluded that the witness correctly identified each one of the two colors $80 \%$ of the time and failed $20 \%$ of the time. What is the probability that the cab involved in the accident was blue rather than green?"

In this problem, the molar probability is that $85 \%$ of all cabs in the city are green and $15 \%$ are blue. The case cue is the witness's claim (which is $80 \%$ reliable) that the cab is blue. On average, subjects report that the probability that the cab is blue is $80 \%$, which corresponds to the witness's accuracy; whereas simple mathematics shows that there is actually only a $41 \%$ chance that the cab is really blue ${ }^{10}$.

The second factor that causes heuristic representativeness is called insensitivity to sample size.

If probabilities are assessed by representativeness, people evaluate the likelihood of a sample result by the similarity of that result to the corresponding parameter. The similarity of sample statistic to a population parameter does not depend on size; therefore, the judged probability of a sample statistic, in this way, is independent of sample size. Following this hypothesis, if individuals have to estimate an average value from samples of different sizes, they produce identical distributions.

Again, Kahneman and Tversky reported a clear example of insensitivity to sample size:
"A certain town is served by two hospitals. In the larger hospital about 45 babies are born each day, and in the smaller hospital about 15 babies are born each day. As you know about $50 \%$ of all babies are boys. However, the exact percentage varies from day to day. Sometimes it may be higher than $50 \%$ sometimes lower. For a period of one year, each hospital recorded the days on which more than $60 \%$ of the babies born were boys". Which hospital recorded

[^8]more such days?
Most people answered to this question assessing the same probability that more than $60 \%$ of babies born were boys for both large and small hospitals. The cause of this incongruence is due to the fact that they didn't perceive any differences in term of descriptions; so they thought that both scenarios were equally representative of the general population.
In contrast, the sampling theory states that the assessments of those probabilities must be different. The expected number of days on which more than $60 \%$ of babies born are boys is greater in the small hospital because a narrow sample can stray from the mean $50 \%$ more easily. Clearly, on the other hand, increasing the sample size, implicates a greater probability that the value estimated remains closer to the mean.

Another factor that explains the birth of heuristic representativeness is the misconception of chances.

People expect that a randomly generated sequence of events will represent the essential characteristics of that process even when the sequence is short. In other words, if we consider tosses of a coin for heads or tails, people think about the sequence H-T-H-T-T-H to be more probable than, for example, the sequence H-H-H-T-T-T, which does not appear random. This phenomenon arises from the fact that people expect that the essential characteristics of the process will be represented, not only globally in the entire sequence but also locally in each of its parts.

Rationally speaking T or H has to be assessed as an individual probability event. In other words, in trail one, you have a $50 \%$ chance of getting a T or an H . In the second trail, the results of the first have no impact; therefore, you again have a $50 \%$ chance to get either letter. Another cognitive bias due to representativeness heuristic is the gambler's fallacy.

Tversky and Kahneman proposed that: "after observing a long run of red on the roulette wheel, for example, most people erroneously believe that black will result in a more representative sequence than the occurrence of an additional red ${ }^{11}$. The justification of this behavior relies on the fact that people interpret chance as a self-correcting process in which a deviation from the main direction implicates another deviation in the opposite direction that restores the previous equilibrium.

Another element that Tversky and Kahneman took into account is the insensitivity to predictability. It occurs when people make predictions that do not correspond to the predictability. For example, if someone gives a description of a firm to a person asking him to predict if it will perform well, the answer depends on the information of the company. It

[^9]means that if the company's sketch is favorable, an high profit will be appear more representative of that description. Though, Tversky and Kahneman underlines that: "the degree to which the description is favorable is unaffected by the reliability of that description or by the degree to which it permits accurate prediction. Hence if people predict solely in terms of the favorableness of the description, their predictions will be insensitive to the reliability of the evidence and to the expected accuracy of the prediction" ${ }^{12}$.

Illusion of validity is another cognitive bias referred to the heuristic representativeness, in which a person overestimates its ability to interpret and predict accurately the outcome when analyzing a set of data, in particular when the data show a very consistent pattern, that is, when the data "tell" a coherent story. The confidence they have in their prediction stands on how it is representative and less on how the prediction is accurate.

Coming back to a previous case proposed by Tversky and Kahneman about the description of Steve that could have worked as a librarian or something else; people overestimate the probability that he is a librarian if they find that his description matches the crucial characteristics of that kind of work even if the description is unreliable. That illusion persists even when people know that there are some factors that limit the accuracy of their predictions. For example, people express more confidence in estimating a grade-point average of a student who collected in the whole year only B's than of another student who collected many A's and C's. The reason why this event occurs stands on the fact that people tend to have great confidence with redundant input variables.

The last factor that contributes to the rise of the heuristic representativeness, proposed by the two psychologists, is the misconceptions of regression.
"Suppose a large group of children has been examined on two equivalent versions of an aptitude test. If one selects ten children from among those who did best on one of the two versions, he will usually find their performance on the second version to be somewhat disappointing. Conversely, if one selects ten children from among those who did worst, they will be found, on average, to do somewhat better on the other version. More generally, consider two variables X and Y that have the same distribution. If one selects individuals whose average X score deviates from the mean of X by K units, then the average of their Y scores will usually deviate from the mean of $Y$ by less than $K$ units".

In the normal course of life people encounter many example of misconception of regression: comparison of height of father and son, difference in terms of intelligence, or performance of

[^10]an individual along a timeline. Usually, in those cases, they do not develop correct intuition about the regression towards the mean; first because people tend to stray from the idea that the regression is a possible, or probable, phenomenon. Consequently, in the rare cases in which people recognize the regression, they often originate spurious casual explanations for it.

In a discussion of a flight training, experienced instructors noted that praise for an exceptionally smooth landing is typically followed by a poorer landing on the next try, while harsh criticism after a rough landing is usually followed by an improvement on the next try. The instructors concluded that verbal rewards are detrimental to learning while verbal punishments are beneficial ${ }^{13}$.
These conclusions are unwarranted. Considering the regression towards the mean, the reach of a good result usually leads to a consequent poorer performance; on the other hand a bad performance leads to a better one in the future. If the individual that has to judge the series of flight does not take into account the effect of the regression, he overestimates the effects of a punishment and underestimates the outcomes of a reward.

Modern research on judgment and decision making, cognitive psychology, and in social cognitive psychology still relies heavily on Kahneman and Tversky's work, referring to the representativeness when explaining stereotypes and casual attributions, to availability when explaining the overestimation of salient risks, and egocentric overestimations of one's own contributions, and to anchoring when explaining biased cost estimations or courtroom decisions. The notion of heuristics has become common sense shared by students of many other disciplines, medical scientists, journalists, practitioners, and even politicians.

Nevertheless, their studies were not accepted by everyone and, consequently, some other psychologist criticized their works. One of the most important scholars who disapproved the theories proposed by Kahneman and Tversky was GerdGigerenzer, a German psychologist who has studied the use of bounded rationality and heuristics in decision-making.
In one of his articles he claims:
"The heuristics in the heuristics-and-biases program are too vague to count as explanations. They are labels with the virtue of Rorschach inkblots: A researcher can read into them what he or she wishes. The reluctance to specify precise and falsifiable process models, to clarify the antecedent conditions that elicit various heuristics, and to work out the relationship

[^11]between heuristics has been repeatedly pointed out" ${ }^{14}$.
This critique comprises two main arguments. The first one concerns the fact that, by focusing mainly on coherence standards, the theory ignores the environment. It means that too less importance is given to all facts, events, phenomenon that modifies the context in which the judgment rises. The second debate instead concerns the fact that, for Gigerenzer, all concepts like anchoring, representativeness and availability are too vague and cannot justify the processes that act as the base of the judgment of individual's behavior.

Even if some psychologists followed these critiques, we can conclude that the last forty years of studies on heuristics changed the way economists have faced the analysis of the market, the decision-making process of individuals, and bounded rationality.

In the next paragraph we are now going to discover the effects of the heuristic representativeness in our model.

[^12]
### 3.3. Consumer's behavior under the Heuristic Representativeness

Coming back to our model proposed in the previous chapters, we can recall how people, over periods, follow their decisional process and then we introduce the concept of representativeness heuristic and discover if it produces some effects on the decision making process of individuals.
We have seen that, if people called out to choose between A or B in the third period are first customers, they cannot rely on their past experience.

From that time onwards they however analyze the posterior probability that the quality of a product (for example A) is high conditioned to the choices made by other individuals before. So we have argued that, if those posterior probabilities that the product analyzed is good are higher than the prior probabilities, the decision-maker chooses that product; otherwise he chooses the alternative.

Even if this process of decision-making could be defined as rational, it could generate a negative effect in the market. In fact we have seen that, with a small but positive probability, a scenario in which all customers choose the worst product can occur. This because, in the third period, individual called out to chooseone of the two products for the first time will rely on choices made by other individuals in the previous period; and it's possible that these individuals chose the worst product.
The question here is: "what happens if people are not fully rational?"
The model showed that individuals, in order to take the decision accurately, must evaluate carefully all the probabilities involved in the decision making process. It means people, before purchasing the product A (or the alternative B), should discover all possible events that could occur and, consequently, they should keep in mind them calculating each related probability of fulfillment. Clearly, in the real life the majority of scenarios present a set of linked events that increase the complexity of the final decision of the customer. Thus, usually he is not able to follow step by step the rational process that should lead to the optimal solution. What could be the consequences and in what way the model changes?

In the previous section we have introduced and described the concept of heuristic representativeness and, especially, one of its causal factor: the base rate neglect.

The base rate neglect occurs when people ignore prior probability information (that is the "base rate") in favor of more recent or descriptive information when making a probability judgment. It is the tendency for people to erroneously judge the likelihood of a situation by not taking into account all relevant data and focusing more heavily on new information without acknowledging how the new information impacts the original assumption.

This can be explained in terms of Bayesian statistics where an individual forms probabilities or base rates. When he receives new information, a rational individual updates those probabilities to reflect the new information. When subjects fall into the base rate neglect, however too little weight is placed on the base rates (prior probabilities) and too much for the new information. The opposite effect also is a common cognitive error known as the conservatism bias, whereby the base rate is overly weighted.
In the model presented in the previous chapter "new customers" drawn in the third period rely on decisions taken by other customers of past periods in order to evaluate the probability that the quality of a specific product (A) is high.

The equation that expresses numerically this concept is:
$\operatorname{Prob}\left(q_{A}=1 \mid \alpha_{2}=n\right)=$
$=\frac{\operatorname{Prob}\left(\alpha_{2}=n \mid q_{A}=1\right) * \operatorname{Prob}\left(q_{A}=1\right)}{\operatorname{Prob}\left(\alpha_{2}=n \mid q_{A}=1\right) * \operatorname{Prob}\left(q_{A}=1\right)+\operatorname{Prob}\left(\alpha_{2}=n \mid q_{A}=0\right) * \operatorname{Prob}\left(q_{A}=0\right)}$

Where $" \operatorname{Prob}\left(q_{A}=1 \mid \alpha_{2}=n\right)$ is the probability that the quality of the product is high $\left(q_{a}=1\right)$ conditioned to the fact that a certain number of individuals chose the product A in the previous period $t_{2}\left(\alpha_{2}=n\right.$, with $\left.0 \leq n \leq 2\right)$
This is defined as a posterior probability and it adds new information that should help individuals to decide to buy either A or B.
In order to be useful, it must be estimated calculating the real probability that each event, constructing the equation, occurs.
Recalling the analysis of the numerator made in the second chapter:

$$
\begin{aligned}
\operatorname{Prob}\left(\alpha_{2}=n \mid q_{A}=1\right)= & \\
& =\operatorname{Prob}\left(\alpha_{2}=n \mid(1,1), q_{A}=1\right) * \operatorname{Prob}(1,1) \\
& +\operatorname{Prob}\left(\alpha_{2}=n \mid(2,1), q_{A}=1\right) * \operatorname{Prob}(2,1) \\
& +\operatorname{Prob}\left(\alpha_{2}=n \mid(2,2), q_{A}=1\right) * \operatorname{Prob}(2,2)
\end{aligned}
$$

In order to estimate it, we conditioned the probability that a certain number of individuals have previously chosen A to the fact that the quality of the product A is good and also to the fact that customers drawn in $t_{2}$ could have been "first customers" or "old customers".
Only through this way customers in the third period could weigh precisely the probability that each event occurs, reaching an accurate estimation of the quality of the product.

What happens if people involved in measuring these probabilities do not weigh events accurately? One of the main sources of error is represented by the case in which individuals neglect the real probability that an individual could be drawn two times in a raw despite the probability that in each period all "first customers" are extracted.
For what concerns our model, people may tend to overestimate the probability that in the previous periods people take their decisions being aware of the quality of the product chosen. This scenario may occur because some consumers may ignore (or may have forgotten) the moment in which products were launched in the market. In particular, if they erroneously think that the products were launched a long time ago, they will tend to believe that current consumers are well informed because they already tried the products over and over. This is an example of overestimation of probability (or over-inference) because it is more representative than the case in which people choose randomly because they haven't tried products yet.
To illustrate the potential effect of this, let's consider three different scenarios with different parameters in which individuals perceive $\operatorname{Prob}(2,2) \operatorname{Prob}(2,1)$, and $\operatorname{Prob}(1,1)$ erroneously.

### 3.4. Market outcome

If people perceive erroneous chances that previous consumers have already tried the product, they base their decisions on misleading probabilities about the quality of the two products A and B.

With the help of the model presented In the second chapter, the aim here is to analyze through three numerical examples what values these probabilities reach and what are the effects that occur as a consequence of the heuristic representativeness in the consumer's decision-making process.

## First Example:

People, looking at the past period, may not be able to distinguish the probability that previous customers have already been drawn or not. In our model this error in the estimation leads people to evaluate equally the likelihood that previous customers have already been drawn or not.

$$
\operatorname{Prob}(2,2)=\operatorname{Prob}(2,1)=\operatorname{Prob}(1,1)=0,3 \overline{3} .
$$

Let's see in what way this belief distorts the evaluation of all the probabilities involved, starting from the results concerning the rational decision-making process:

- $\operatorname{Prob}\left(q_{A}=1 \mid \alpha_{2}=2\right)=$

$$
=\frac{(1 * 0,0 \overline{2}+0,5 * 0,3 \overline{5}+0,25 * 0,6 \overline{2}) * 0,5}{(1 * 0,0 \overline{2}+0,0 \overline{5} * 0,3 \overline{5}+0,25 * 0,6 \overline{2}) * 0,5+0,25 * 0,6 \overline{2} * 0,5}=0,6959
$$

- $\operatorname{Prob}\left(q_{A}=1 \mid \alpha_{2}=1\right)=$

$$
=\frac{(0,5 * 0,3 \overline{5}+0,5 * 0,6 \overline{2}) * 0,5}{(0,5 * 0,3 \overline{5}+0,5 * 0,6 \overline{2}) * 0,5+(0,5 * 0,3 \overline{5}+0,5 * 0,6 \overline{2}) * 0,5}=0,5
$$

- $\operatorname{Prob}\left(q_{A}=1 \mid \alpha_{2}=0\right)=$

$$
=\frac{0,1 \overline{5} * 0,5}{0,1 \overline{5} * 0,5+(0,0 \overline{2}+0,5 * 0,3 \overline{5}+0,25 * 0,6 \overline{2}) * 0,5}=0,30434
$$

These equations, under the base rate neglect become as follows:

- $\operatorname{Prob}\left(q_{A}=1 \mid \alpha_{2}=2\right)=$

$$
=\frac{(1 * \mathbf{0}, \mathbf{3} \overline{\mathbf{3}}+0,5 * \mathbf{0}, \mathbf{3} \overline{\mathbf{3}}+0,25 * \mathbf{0}, \mathbf{3} \overline{\mathbf{3}}) * 0,5}{(1 * \mathbf{0}, \mathbf{3} \overline{\mathbf{3}}+0,5 * \mathbf{0}, \mathbf{3} \overline{\mathbf{3}}+0,25 * \mathbf{0}, \mathbf{3} \overline{\mathbf{3}}) * 0,5+(0,25 * \mathbf{0}, \mathbf{3} \overline{\mathbf{3}}) * 0,5}=0,875
$$

Following the evaluation of $\operatorname{Prob}\left(q_{A}=1 \mid \alpha_{2}=2\right)$ " under heuristic representativeness the obtained result is wrong. It means that people, observing both individuals purchasing the product A in the previous period, consider the product A as good with probability around $87 \%$. In the previous chapter we have discovered that a rational decision-making process leads people to estimate the same probability with a value around $69 \%$, so people, in this case, tend to overestimate the probability.

The second probability, under heuristic representativeness, is estimated as follows:

$$
\begin{aligned}
& \operatorname{Prob}\left(\boldsymbol{q}_{A}=\mathbf{1} \mid \boldsymbol{\alpha}_{\mathbf{2}}=\mathbf{1}\right)= \\
& \quad=\frac{(0,5 * \mathbf{0}, \mathbf{3} \overline{\mathbf{3}}+0,5 * \mathbf{0}, \mathbf{3} \overline{\mathbf{3}}) * 0,5}{(0,5 * \mathbf{0}, \mathbf{3} \overline{\mathbf{3}}+0,5 * \mathbf{0}, \mathbf{3} \mathbf{3}) * 0,5+(0,5 * \mathbf{0}, \mathbf{3} \overline{\mathbf{3}}+0,5 * \mathbf{0}, \mathbf{3} \overline{\mathbf{3}}) * 0,5}=0,5
\end{aligned}
$$

Here we can see that the result do not differ from the one obtained on the basis of the rational decision making process. The reason concerns the fact that, in both cases, if people discover that the number of customers who bought the product A in the previous period is half of the total, they do not reach any useful information which modify the prior probability.

The last evaluation, under heuristic representativeness, is reached through the equation:

$$
\begin{aligned}
\operatorname{Prob} & \left(\boldsymbol{q}_{A}=\mathbf{1} \mid \boldsymbol{\alpha}_{2}=\mathbf{2}\right)= \\
& =\frac{(0,25 * \mathbf{0}, \mathbf{3} \overline{\mathbf{3}}) * 0,5}{(0,25 * \mathbf{0}, \mathbf{3} \overline{\mathbf{3}}) * 0,5+(1 * \mathbf{0}, \mathbf{3} \overline{\mathbf{3}}+0,5 * \mathbf{0}, \mathbf{3} \overline{\mathbf{3}}+0,25 * \mathbf{0}, \mathbf{3} \overline{\mathbf{3}}) * 0,5}=0,125
\end{aligned}
$$

Again the result is misleading. It shows that, if a person in the third period observes that no one has bought the product A before, heestimatesabout $12,5 \%$ probability that the quality of A is high. Following the rational decision-making process, he reaches a value about $30,4 \%$, so in this case the "new customer" underestimates the probability that, even if no one has bought that product, the quality of A is high.

## Second Example:

People, looking at the previous period, overestimate erroneously the probability that the choices of other individuals are taken by customers who have already tried the product.

So, following this heuristic behavior, the probability that both subjects have bought the product A in the previous period goes up to $70 \%$, the probability that one over the two individuals is a "first customer" and the other an "old customer" decreases a little going down to $20 \%$, and finally the residual $10 \%$ concerns the probability that the other customers have chosen randomly.

Reassuming:

- $\operatorname{Prob}(2,2)=0,7$
- $\operatorname{Prob}(2,1)=0,2$
- $\operatorname{Prob}(1,1)=0,1$

Again, starting from the general equations already cited in the first case, the process continues as follows:

$$
\begin{aligned}
& \operatorname{Prob}\left(\boldsymbol{q}_{A}=\mathbf{1} \mid \boldsymbol{\alpha}_{\mathbf{2}}=\mathbf{2}\right)= \\
& \quad=\frac{(1 * \mathbf{0}, 7+0,5 * \mathbf{0}, \mathbf{2}+0,25 * \mathbf{0}, \mathbf{1}) * 0,5}{(1 * \mathbf{0}, 7+0,5 * \mathbf{0}, \mathbf{2}+0,25 * \mathbf{0}, \mathbf{1}) * 0,5+(0,25 * \mathbf{0}, \mathbf{1}) * 0,5}=0,9705
\end{aligned}
$$

An overestimation of the probability that the product A is good is the direct consequence of a misleading perception due to the effect of the representativeness heuristic.

If people observe that only one of the two customers has purchased $A$ in the previous period They try to estimate: $\operatorname{Prob}\left(q_{a}=1 \mid \alpha=1_{t 2}\right)$.

The value of that probability, contaminated by the effect of the heuristic representativeness, is reached as follows:

$$
\begin{aligned}
\operatorname{Prob}\left(\boldsymbol{q}_{A}\right. & \left.=\mathbf{1} \mid \boldsymbol{\alpha}_{\mathbf{2}}=\mathbf{2}\right)= \\
& =\frac{(0,5 * \mathbf{0}, \mathbf{2}+0,5 * \mathbf{0}, \mathbf{1}) * 0,5}{(0,5 * \mathbf{0}, \mathbf{2}+0,5 * \mathbf{0}, \mathbf{1}) * 0,5+(0,5 * \mathbf{0}, \mathbf{2}+0,5 * \mathbf{0}, \mathbf{1}) * 0,5}=0,5
\end{aligned}
$$

As we have already said for the first case, here it doesn't matter how the heuristic representativeness modifies the perception about the probability that an individual can be drawn two times in a raw. If customers discover that half individuals have chosen the product A and the other half the product B, they do not take into account this information because it becomes useless. They continue buying randomly and the faith rules.

The last equation concerns the probability that the quality of A is high given that no one has chosen that product before.
That value is altered by the heuristic representativeness as follows:

$$
\begin{aligned}
\operatorname{Prob}\left(\boldsymbol{q}_{A}\right. & \left.=\mathbf{1} \mid \boldsymbol{\alpha}_{2}=\mathbf{2}\right)= \\
& =\frac{(0,25 * \mathbf{0}, \mathbf{1}) * 0,5}{(0,25 * \mathbf{0}, \mathbf{1}) * 0,5+(1 * \mathbf{0}, 7+0,5 * \mathbf{0}, \mathbf{2}+0,25 * \mathbf{0}, \mathbf{1}) * 0,5}=0,029
\end{aligned}
$$

Again, people perceive a misleading probability. The value decreases from about $30 \%$ to $3 \%$. That means they leave very little room for uncertainty that, rationally speaking, should be considered.

The situation where people overestimate the probability that an individual has already been drawn could finally end up in the extreme case in which they think the only existing customers are those who have already tried the product. Here, $\operatorname{Prob}\left(q_{a}=1 \mid \alpha=2_{t 2}\right)$ reaches $100 \%, \operatorname{Prob}\left(q_{a}=1 \mid \alpha=1_{t 2}\right)$ remains $50 \%$, and finally $\operatorname{Prob}\left(q_{a}=1 \mid \alpha=0_{t 2}\right)$ decrease to $0 \%$.

## Third Example:

The opposite effect of the base-rate neglect also is a common cognitive error known as the conservatism bias, whereby the base rate is overly weighted.

People, despite all we have already said in the two cases before, could believe that the probability of analyzingchoices made by individuals who have experienced the product before is very low.

Those beliefsare modified as reported below:

- $\operatorname{Prob}(2,2)=0$
- $\operatorname{Prob}(2,1)=0$
- $\operatorname{Prob}(1,1)=1$
$\operatorname{Prob}\left(\boldsymbol{q}_{A}=\mathbf{1} \mid \boldsymbol{\alpha}_{\mathbf{2}}=\mathbf{2}\right)=\frac{(1 * \mathbf{0}+0,5 * \mathbf{0}+0,25 * \mathbf{1}) * 0,5}{(1 * \mathbf{0}+0,5 * \mathbf{0}+0,25 * \mathbf{1}) * 0,5+(0,25 * \mathbf{1}) * 0,5}=0,5$
$\operatorname{Prob}\left(\boldsymbol{q}_{A}=\mathbf{1} \mid \boldsymbol{\alpha}_{\mathbf{2}}=\mathbf{1}\right)=\frac{(0,5 * \mathbf{0}+0,5 * \mathbf{1}) * 0,5}{(0,5 * \mathbf{0}+0,5 * \mathbf{1}) * 0,5+(0,5 * \mathbf{0}+0,5 * \mathbf{1}) * 0,5}=0,5$
$\operatorname{Prob}\left(\boldsymbol{q}_{\boldsymbol{A}}=\mathbf{1} \mid \boldsymbol{\alpha}_{\mathbf{2}}=\mathbf{0}\right)=\frac{(0,25 * \mathbf{1}) * 0,5}{(0,25 * \mathbf{1}) * 0,5+(1 * \mathbf{0}+0,5 * \mathbf{0}+0,25 * \mathbf{1}) * 0,5}=0,5$

All these probabilities lead to the same result: 0,5 . Clearly, if people estimate erroneously that each individual in every period chooses one of the two products for the first time, they consequently think all the past purchases have been done casually.

It follows that also customers belonging to the third period choose between A or B randomly, and the effects on the market could be heavy.

In fact, in a rational making process, the product that performs well is supposed to gain market shares over the bad one due also to herd behavior. If, instead, people perceive these
misleading beliefs, they continue buying randomly and preferences fluctuates between A and B over periods casually.

We have seen that it is rational for individuals to look at other individuals' choices in order to get new information that might be used when they themselves have to make decision. However, this process of inferring information form others' behavior, require to correctly update beliefs, to get from prior to posterior probabilities. Some individuals may not be able to perform this process of Bayesian updating or may willingly decide to avoid it, recurring instead to simplifying heuristics, like representativeness.

When this decision-making shortcut eases too much the process of updating probabilities, it may lead to misleading estimations.
In fact, from the comparison between the results of the two models (rational and heuristic), except the case concerning half people making a choice and the other half the opposite, we discover in all other cases differences in terms of final values.

Under the heuristic representativeness people think that, two individuals who have previously chosen one product should be translated in a probability of around $97 \%$ that the same product performs well.
Considering a perfectly rational decision-making process the same probability reaches only a value of around $69 \%$. It means that people, under heuristic representativeness, overestimates the probability that the product is good.

We have already said that, due to the hypotheses of the model, people are forced to choose one of the two alternatives and, as a consequence, if the rational posterior probability is greater than 0,5 (from the model: 0,69 ), it is enough to lead eachconsumer to choose that product generating herd behavior.
However, the real world is more complex than the model and people could exploit the option of waiting. It means that, if they are not sure about which is the best product between two alternatives, they could to wait and observe how markets and the decision of others evolve over periods.

For example, people could think that a posterior probability that reaches a value of $69 \%$ is not enough to convince completely themselves about the purchase. They wait one period or more in order to acquire additional information and to discover if the posterior probability is accurate or misleading.
If, instead, after having observed that both individuals have previously bought the same product, the probability comes up to $97 \%$, people have no doubt on which is the product that they should purchase.

The overestimation of the posterior information reduces too much the uncertainty that could keep the market in equilibrium in the early stages of the timeline.

If the heuristic representativeness generates those misleading beliefs, it's sufficient that two individuals have purchased the wrong product randomly in order to provoke powerful herd behavior affecting all the following customers' decisions.
The model is created in order to simplify the real economy, so it presents some limitations. Following the model it seems that, when the whole population has already tried at least one of the two alternatives, customers have clear in mind the product that they should buy, but in a more real scenario, customers changes their desires and needs, so the population is in a continuous evolution and in each period there will always be some incoming individuals who haven't experienced any product yet.
This fact explains the potential power and, at the same time, danger of herd behavior: if in each period the new incoming customers copy other individuals' choices relying on misleading beliefs, the firm that produces the worst product continue gaining market share against the other one, and consequently it is able to exploit all the advantages that the surplus of sales brings to it.

## CONCLUSION

The original studies conducted by Kahneman and Tversky on the representativeness heuristic generated a new vision of the decision-making process of individuals in a scenario of uncertainty.
With the help of the model proposed in this work, we have discovered which can be the effects of the representativeness heuristic on the decision-making process of potential consumers and in what way it can lead to misleading posterior probabilities assessments.

We have seenthe representativeness heuristic can enhance the perceived probability that previous consumers have already experienced the product, due to the fact that this situation becomes more representative and, consequently, easier to comprehend.

The difference between values computed rationally and values distorted by biases in estimations defines the factor that enhances herd behavior. People in that way tend to copy other people's decisions increasing the probability that the worst product could establish itself in the market, at least for some periods, despite the best one.

In view of that,it becomes crucial for firms to gain rapidly market shares, especially when they introduce new low quality products, because it enhances the probability that first consumers, making some biases in estimations, lead all following purchasers to copy their actions establishing a powerful herd behavior which benefits the worst product despite the best one.

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