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"A Multiplicative Error Model approach for trading volumes including company news and announcements"

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Firma dello studente

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To my parents.
Ai miei genitori.

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1 INTRODUCTION

The aim of this thesis is to find whether company news and announcements have any effect in determining trading volumes.

For this purpose, we use an econometric model for high-frequency data, namely a Multiplicative Error Model. This model has been firstly introduced by Engle (2002) and belongs to the Generalized AutoRegressive Conditional Heteroskedasticity (GARCH) family. The motivation of this choice lies in the properties shown by trading volumes series: they take only positive values, they present strong persistence and exhibit clustering. In our framework, trading volumes in the intra-daily interval i of the business day t are then modelled as the product of a deterministic scale factor and an i.i.d. error term. The latter term is supposed to follow a Gamma distribution with shape parameter v and mean equal to one. On the other hand, the deterministic scale factor depends on a parameter vector θ, and is defined in several specifications. For the purpose of this thesis, we are interested in adding some covariates to the baseline specification, such as variables representing the amount of company news and announcements, the corresponding sentiment and the presence of price volatility jumps. All the specifications of our Multiplicative Error Model for trading volumes are then estimated by Maximum Likelihood method, ending up with asymptotically normally distributed parameters. Instead, as regards jumps, we detect them by using the approach of Corsi, Pirino and Renò (2008), based on realized volatility and threshold multipower variation.

The empirical analysis of this thesis uses Apple Inc. intra-daily data, for the period which goes from 4th January 2005 to 31st December 2014. It has been downloaded at one-minute frequency and then aggregated to create five-minute frequency series for all variables included in the dataset: open and close price, low and high price, volumes, amount and sentiment of company news and announcements. As regards company news, the dataset include also categorization by seven topics: all (no filter), 'earnings related', 'litigation', 'M&As', 'newspapers', 'regulatory', 'upgrades and downgrades'. Beyond the general case, we will estimate the same specifications of our Multiplicative Error Model for trading volumes for some of these topics.

Therefore, in Chapter 2 we present some literature about the methodology used in our empirical analysis, that is about high-frequency data properties, realized volatility and jumps detection, and about the Multiplicative Error Model approach, in its baseline and extended versions. Chapter 3 regards the empirical analysis of this thesis. It starts with a detailed description of data about Apple Inc., defines all the specifications of our Multiplicative Error Model for trading volumes, and reports the estimation results. Eventually, Chapter 4 sums the most relevant results and, thus, aims at answering to the initial question of this work.

2 METHODOLOGY

2.1 Notions about high-frequency data

The birth of high-frequency trading in the early 21th century has opened interesting horizons to the study of financial markets activity. In fact, the availability of high-frequency data enables traders, analysts and researchers to precisely study the dynamics of market micro-structure and the impact of traders' behaviour on it. As a consequence, it is possible to construct more efficient estimators and predictors of market variables.
Nowadays, "with the development of computing power and storage capacity, datasets may contain tens of thousands of transactions or posted quotes in a single day time stamped to the nearest second" (Engle, Russell, 2004, p.2). Generally, datasets arising from floor trading contain information on trades and quotes, whereas data from electronic trading often concerns the process of order arrivals as well as – at least partly – of the order book. Typically, "the data is recorded whenever a trade, quote, or – in the informational limiting cases – a limit order occurs. This data is called transaction data, ultra-high frequency data or tick data" (Hautsch, 2012).

It is possible to distinguish four types of datasets according to the level of detail of information included. The first one – called trade data – contains information about individual trades consisting in the time stamp of trades, the price at which a trade was executed and the traded volume in number of shares. The second level – the so-called trade and quote – additionally includes the best ask/bid quote updates, the underlying best ask and bid quotes with the respective market depth, and the trade direction up to identification rules. Apart from what already mentioned, the next level – labelled as fixed level order book data – provides information on limit order activities and on the depth behind the market if the underlying trading system is a fully computerized system. As a consequence, it would be possible to reproduce the limit order book up to a fixed level. Instead, the category belonging to the next level of detail incorporates messages on all limit order activities. It means that time stamps, limit prices, sizes, specific attributes of the submissions, executions, cancellations, amendments, and – if allowed – hidden or iceberg orders are given. Thus, this type of dataset allows to fully recreate the trading flow and the limit order book at any point in time, and to identify if the trade is buyer-initiated or seller-initiated. Lastly, data on order book snap-shots constitutes the whole information from trading activities. They are taken at equidistant time intervals so that it would be useful to study limit order book dynamics. By the way, they are not proper for analysing interactions between the book and trading process, since the matching with the corresponding underlying trading process would be difficult. Although all this information could be available, it is evident that the reconstruction of the order book would require to consider all activities outside trading hours – opening auctions, pre-trading and late-trading periods.

In what follows we expose the characteristics of high-frequency data in order to be aware of how to facilitate the analysis approach. In fact, it could be that some of the following properties may obstruct high-frequency data analysis and econometric modelling. For presenting them we follow the approach in Engle and Russell (2004) replicating – when possible – their examples on the trading volume time series of Apple Inc.

The first feature of tick data is the irregular temporal spacing of observations. Transactions can take few seconds or many minutes one from another to be executed. This stylized fact happens for all types of transaction in financial markets and forces to give particular attention in econometric modelling. To solve this problem, data could be statistically considered as point processes which characterize the random occurrence of single events over time, in dependence of observable characteristics and of the process history. Engle (see Hautsch, 2012, p.2), for the first time, discussed their importance in high-frequency financial econometrics. However, since data used in our empirical analysis has been already aggregated by one-minute and then by five-minute frequency, observations are equally distanced in time and this problem does not appear (see Figure 1). In fact, time aggregation techniques – which will be explained later on – are considered an immediate solution to irregular spacing of observations but neglect retrieving information from timing of trading activity.

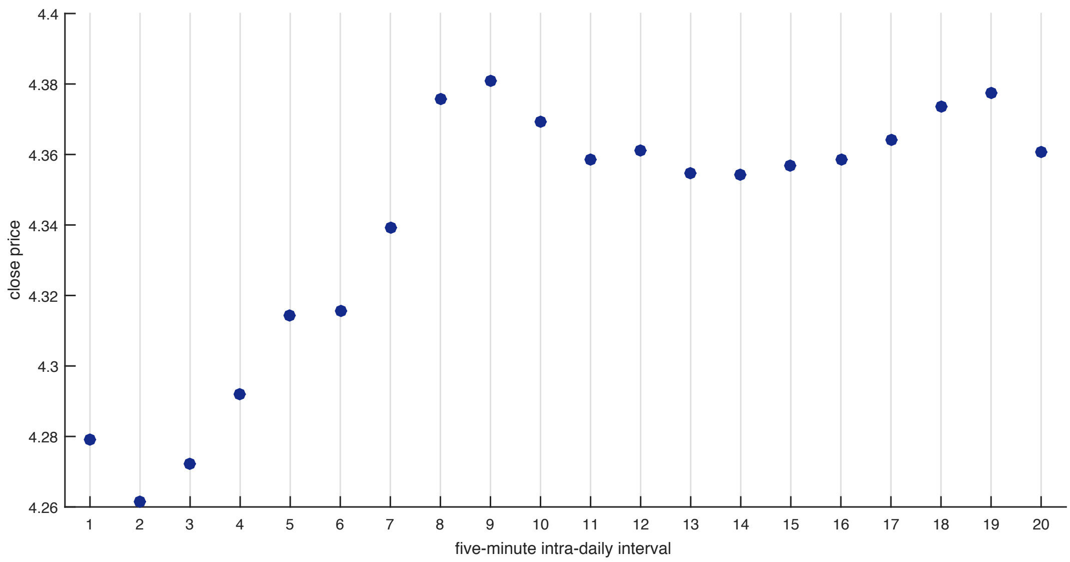

Figure 1. Regular temporal spacing of AAPL close prices.

The second property – embodied in all economic data – is the discreteness. In trading activities, for a transaction to take place, agents are required or imposed to move the price for a pre-specified tick size. For actively traded stocks, it is usual that prices move a small number of ticks from one transaction to another. The resulting price falls on a very small number of possible outcomes. The example in Figure 1 reports the histogram of Apple Inc. close prices of one random trading day present in the dataset. As it can be observed, the majority of close prices lie among very few possible values: in this sample, they moved only few cents around the median value of USD 10.25. This phenomenon has an impact in measuring volatility, dependence and any other characteristic that is small relative to the tick size. Moreover, it implies a high degree of kurtosis (heavy tails) in the data: normal distribution may be no longer appropriate to represent the randomness of the events because tends to underestimate the probability of extreme events.

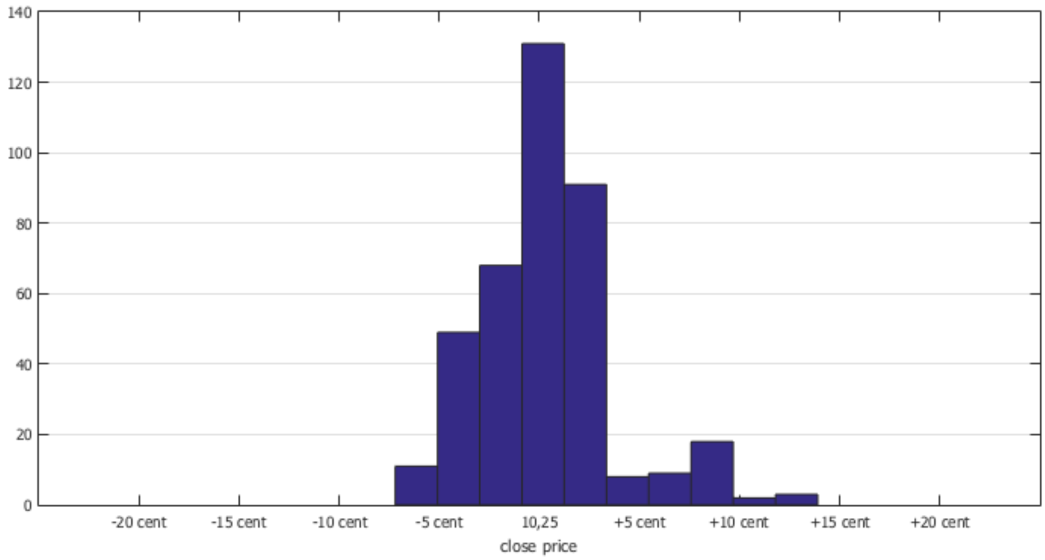

Figure 2. Discreteness of AAPL close prices.

Another property of high-frequency data can be retrieved from the plot of some market variables. A U-shaped pattern over the course of the day defines what is called intra-daily periodicity or seasonality. This phenomenon is associated with high market activity after the opening and before the closure of markets and, conversely, with low market activity around lunch time. This is evident for many market variables: volatility, volumes, spreads are shown to present the same pattern. As an example, In Figure 3 we present the plot of average trading volumes of Apple Inc. over one-minute intra-daily intervals. The U-shaped path is evidently caused by a higher trading volume in the opening and the closing time of the market.

Then, high-frequency financial data present strong persistence. The phenomenon is easily retrievable by looking at autocorrelation functions of the market variable of interest. The example of Apple Inc. trading volumes in Figure 4 shows autocorrelations that go very slow to

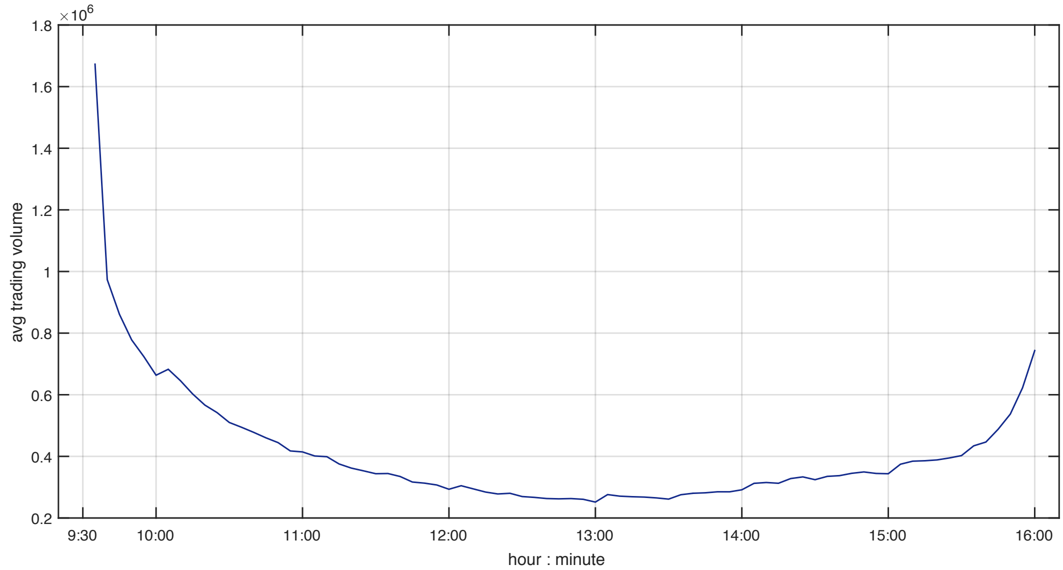

Figure 3. Diurnal pattern for AAPL average trading volumes.

zero and highly significant autocorrelations up to the lag 9. Moreover, clustering of trading volumes is evident: autocorrelation functions show that high volatility periods are likewise followed by periods of high volatility, while low volatility periods are followed by as much periods of low volatility. Nonetheless, this characteristic tends to decrease under temporal aggregation.

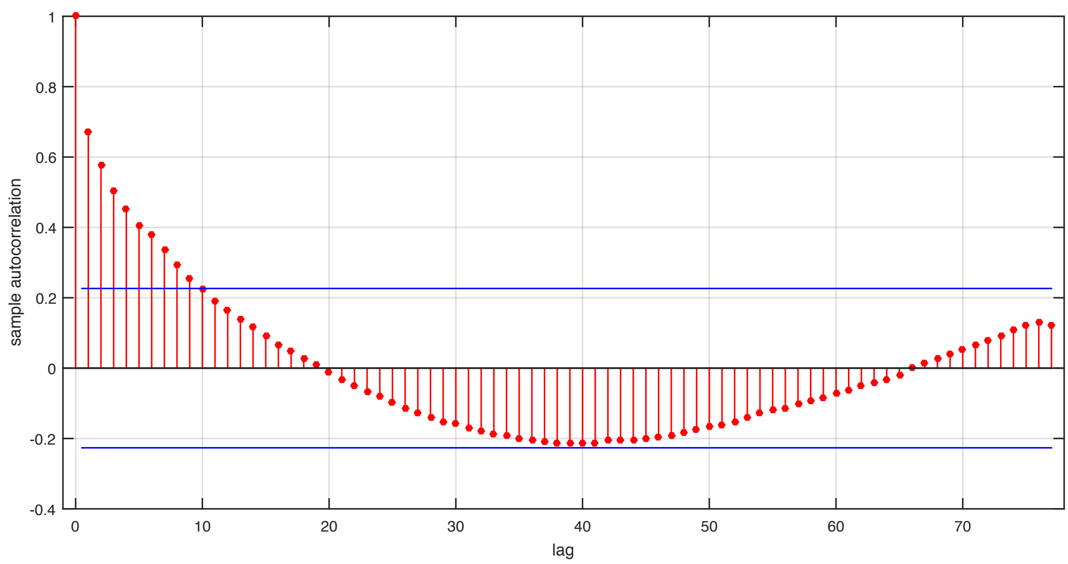

Figure 4. Strong persistence of AAPL trading volumes.

Furthermore, "most high-frequency financial variables take only positive values calling for specific models for positive-valued variables" (Hautsch, 2012, p.62). Therefore, in order to describe the variable of interest, positive-valued random variables – such as Gamma, Weibull, Log-Normal distributions – are required. In some case it would be useful to use a mixture of discrete and continuous component, like for trade size, as a consequence of traders' preference for round numbers.

For the analysis of high-frequency data, it is not uncommon to implement aggregation of data. Even if it causes losses of useful information, it would be appropriate to adopt aggregation schemes. Indeed, aggregation allows to construct economically and practically interesting variables, to reduce the impact of microstructure effects whenever they create noise, and to diminish the amount of data to be analysed if long sample periods or large cross-sections are studied (Hautsch, 2012). There are two possible ways for sampling and aggregating. The first one is the so-called event aggregation that is aggregation of the process according to specific trading events. It is obtained by sampling the process whenever a trade occurs, in this case creating the trade duration associated with the intensity of liquidity demand. The second approach is the time aggregation made in line with calendar time. This means being able to evaluate market activity over equidistant time intervals and to simplify multivariate modelling as well as forecasting over fixed time intervals.

2.2 Modelling high-frequency data

Modelling financial time series has always been a challenge for researchers. In the 80's, the discover that most of asset price series exhibiting volatility clustering could be modelled as an AutoRegressive Conditional Heteroskedasticity (ARCH) or a Generalized AutoRegressive Conditional Heteroskedasticity (GARCH) model brought an innovation in financial econometrics. In this field, the key problem has always been modelling, estimation and forecasting of conditional return volatility and correlation, together with deriving accurate models for pricing, risk management and asset allocation decisions. Nonetheless, the biggest obstacle has always been that regarding volatility, which is unobservable and must be retrieved by indirect modelling. This is the motivation of existence of the above-mentioned parametric models – the ARCH/GARCH family – as well as stochastic volatility and Markov-switching models.

High-frequency data has been a real turning point in modelling and forecasting conditional volatility. A huge number of new models have been created from the combination of multivariate time series models, micro-econometric approaches and statistical techniques to successfully capture the dynamics in the financial data. The literature underlines the role of autoregressive conditional mean models where the conditional mean is treated as an autoregressive process that is updated based on observation driven or parameter driven innovations. Before the availability of high-frequency data, volatility measures – extracted from daily squared returns – had given unsatisfactory results. Standardized returns showed fat tails in their distribution which led to the search for appropriate error distributions. Furthermore, multivariate modelling was difficult to implement for highly dimensioned datasets. Instead, high-frequency data has allowed to built accurate estimates of volatility. Realized volatility – which will be presented in detail in the following paragraph – is a volatility measure based on high-frequency squared returns, easy to be computed.

Together with the models, analysis of stochastic processes, estimators and unknown parameters has started. Moreover, theorems about autocorrelations, stationarity and ergodicity of these processes have been developed. Research in this field is continuously under development.

In the following paragraphs, we firstly present a high-frequency based volatility measure useful for data analysis – the realized volatility – as well as its decomposition in continuous and discontinuous components – the bipower variation and jumps. Later, after a brief presentation of GARCH-type models, the Multiplicative Error Model is introduced in its baseline specification and in its extended version. For concluding, since the focus of this thesis is on trading volumes, some specific literature is presented.

2.2.1 Realized Volatility and jumps detection

In high-frequency data analysis, most econometric models aim at capturing volatility dynamics. For this reason, we now present a volatility measure – called realized volatility – which is directly obtainable from price returns. We then show how it can be decomposed into its continuous component – the integrated volatility – and its discontinuous component – the jumps.

By following the notation of the work of Corsi, Pirino and Renò (2008), suppose we are working in a filtered probability space $(\Omega, (\mathcal{F}_t)_{t\in[0,T]}, \mathcal{F}, \mathcal{P})$. Assume also that $(X_t)_{t\in[0,T]}$ is a real-valued process such that $X_0 \in \mathbb{R}$, measurable with respect to $\mathcal{F}_0$, and

$$dX_t = \mu_t dt + \sigma_t dW_t + dJ_t$$

where μ_t is predictable, σ_t is cádlág, $dJ_t = c_t dN_t$ where N_t is a non-explosive Poisson process whose intensity is an adapted stochastic process λ_t, $\left(\tau_j\right)_{j=1,\dots,N_t}$ denotes the times of the corresponding jumps and c_j are i.i.d. adapted random variables measuring the size of the jump at time τ_j and satisfying $\mathcal{P}\left(\{c_j = 0\}\right) = 0$ for $\forall t \in [0, T]$.

Given a fixed time window T, the quadratic (total) variation of such a process is defined as:

$$[X]_t^{t+T} \coloneqq X_{t+T}^2 - X_t^2 - 2\int_t^{t+T} X_{s-} dX_s$$

where t indexes the interval, typically a day, and it can be decomposed into its continuous and discontinuous component as:

$$[X]_t^{t+T} \coloneqq [X^c]_t^{t+T} + [X^d]_t^{t+T}$$

where $[X^c]_t^{t+T} = \int_t^{t+T} \sigma_s^2 ds$ is the integrated volatility, and $[X^d]_t^{t+T} = \sum_{j=N_t}^{N_{t+T}} c_j^2$ is the discontinuous component in which c_j^2 is the size of the j-th jump at time τ_j. In order to estimate these quantities, the time interval $[t, t+T]$ must be divided into n subintervals of length $\delta = \frac{T}{n}$.

Then, we define the returns as:

$$\Delta_{j,t}X = X_{j\delta+t} - X_{(j-1)\delta+t}$$

for $j = 1, \dots, n$. For simplicity we can write $\Delta_j X$. At this point, finally, an estimator of the quadratic variation is given by the realized volatility defined as:

$$RV_\delta(X)_t = \sum_{j=1}^{n} (\Delta_j X)^2$$

which as $\delta \longrightarrow 0$ converges in probability to $[X]_t^{t+T}$.

In order to separate the continuous quadratic variation from the discontinuous one, Barndorff-Nielsen and Shepard (see Corsi-Pirino-Renò, 2006) introduced the so-called multipower variation defined as:

$$MPV_\delta(X)_t^{[\gamma_1,\dots,\gamma_M]} = \delta^{1-\frac{1}{2}(\gamma_1+\cdots+\gamma_M)} \sum_{j=M}^{[T/\delta]} \prod_{k=1}^{M} |\Delta_{j-k+1}X|^{\gamma_k} .$$

This formula is used for the estimation of $\int_t^{t+T} \sigma_s^2 ds$ and $\int_t^{t+T} \sigma_s^4 ds$. The most important case of multipower variation is the bipower variation obtained by choosing $\gamma_1 = \gamma_2 = 1$:

$$BPV_\delta(X)_t = \mu_1^{-2} MPV_\delta(X)_t^{[1,1]} = \mu_1^{-2} \sum_{j=2}^{[T/\delta]} |\Delta_{j-1}X| \cdot |\Delta_j X|$$

which, as $\delta \longrightarrow 0$, converges in probability to $\int_t^{t+T} \sigma_s^2 ds$. The term μ_1 is given and it is approximately equal to 0.7979. Hence, the previous definition is equal to the more readable version in the work of Caporin, Rossi and Santucci de Magistris (2014):

$$BPV_t = \frac{\pi}{2} \sum_{j=2}^{M} |r_{t,j}| \, |r_{t,j-1}|$$

where $j = 2, \dots, M$ is defining the fixed length intra-daily intervals, $t = 1, \dots, T$ is the time period (day), and $r_{t,j}^2$ is the intraday squared log-return at day t in interval j.

The choice of disentangling the continuous part (bipower variation) from the jumps lies in the fact that volatility jumps are relevant in financial and economic applications. Nonetheless, a lot of past literature had not managed to demonstrate the effect of jumps in determining the future volatility. An example is contained in Andersen et al. (see Corsi-Pirino-Renò, 2006) where jumps are said to bring a negative or a null impact in determining future volatility. On the other hand, if we look at the plot of realized volatility, we easily observe that bursts in volatility are usually initiated by a large and unexpected movement of asset prices. Moreover, it is well known that volatility is associated with dispersion of beliefs and heterogeneous information (Shalen, Wang, Buraschi et al. – see Corsi-Pirino-Renò, 2006). If the occurrence of a jump increases the uncertainty on fundamental values, it is likely to have a positive impact on future volatility. Thus, the affirmation that jumps are not relevant for volatility determination becomes a puzzle. It is due to the fact that bipower variation was used to estimate the continuous integrated volatility and, by difference, the jump contribution to total quadratic variation. By the way, bipower variation is a consistent estimator of integrated volatility as long as the time interval between observations vanishes, but in finite samples it is upper biased in presence of jumps. Therefore, the so-computed bipower variation implies a large underestimation of the jump component.

For this reason, threshold multipower variation has been introduced as alternative estimator of integrated volatility in presence of jumps, firstly by Mancini (see Corsi-Pirino-Renò, 2006), and then by Corsi, Pirino and Renò (2008) as follows. It is considered an ideal candidate to estimate dynamic models of volatility in which we use separately the continuous and discontinuous volatility as explanatory variables.

Let now use a strictly positive threshold function $\vartheta_s: [t, t+T] \to \mathbb{R}^+$ which does not depend on δ. Let the vector $\gamma_1, \dots, \gamma_M$ be positive and define the threshold multipower variation estimator as:

$$TMPV_\delta(X)_t^{[\gamma_1,\dots,\gamma_M]} = \delta^{1-\frac{1}{2}(\gamma_1+\cdots+\gamma_M)} \sum_{j=M}^{[T/\delta]} \prod_{k=1}^{M} |\Delta_{j-k+1}X|^{\gamma_k} I_{\{|\Delta_{j-k+1}X|^2 \le \vartheta_{j-k+1}\}}$$

If our interest is in estimating $\int_t^{t+T} \sigma_s^2 ds$ when δ is finite and large enough to avoid microstructure effects (typically, $\delta = 5$ minutes), the so-computed multipower variation is different from the one previously calculated. As a special case, when $\gamma_1 = \gamma_2 = 1$, integrated variance is estimated by threshold bipower variation as follows:

$$TBPV_\delta(X)_t = \mu_1^{-2} TMPV_\delta(X)_t^{[1,1]} = \mu_1^{-2} \sum_{j=2}^{[T/\delta]} |\Delta_{j-1}X| \cdot |\Delta_j X| \, I_{\{|\Delta_{j-1}X|^2 \le \vartheta_{j-1}\}} I_{\{|\Delta_j X|^2 \le \vartheta_j\}}$$

This estimator is robust to the presence of jumps and unelastic with respect to the choice of the threshold function. Suppose to use a multiple of the local variance σ_t^2 as the threshold function, defined as:

$$\hat{V}_t^Z = \frac{\sum_{i=-L,\ i\neq -1,0,+1}^{L} K\left(\frac{i}{L}\right) (\Delta_{t+i}X)^2 I_{\{(\Delta_{t+i}X)^2 \le c_V^2 \cdot \hat{V}_{t+1}^{Z-1}\}}}{\sum_{i=-L,\ i\neq -1,0,+1}^{L} K\left(\frac{i}{L}\right) I_{\{(\Delta_{t+i}X)^2 \le c_V^2 \cdot \hat{V}_{t+1}^{Z-1}\}}}$$

for $Z = 1,2,\dots$, with the starting value set to $\hat{V}^0 = +\infty$ which corresponds to using all observations in the first step, and $c_V = 3$. At each iteration, jumps are detected by the condition $(\Delta_t X)^2 > c_V^2 \cdot \hat{V}_{t+1}^{Z-1}$ and removed from the time series by means of the indicator function. Each estimate of the variance is multiplied by c_V^2 to get the threshold for the following step. The iterations stop when the removed jumps are the same. On high frequency data, this always happens with $Z = 2,3$ iterations. The bandwidth parameter L determines the number of adjacent returns included in the estimation of the local variance around point t, typically set equal to 25. The choice of the kernel function $K(\cdot)$ is uninfluential; Corsi, Pirino and Renò (2008) use a Gaussian kernel:

$$K(y) = \left(1/\sqrt{2\pi}\right) \exp(-y^2/2) \,.$$

Eventually, the threshold function is set proportional to the local variance $\vartheta_t = c_\vartheta^2 \cdot \hat{V}_t^Z$, where c_ϑ is typically equal to 3.

It can be now introduced a test for jump detection in time series, based on threshold multipower variation. Corsi, Pirino and Renò (2008) use the separation of the quadratic variation in its continuous and discontinuous component to achieve a substantial improvement in volatility forecasting because of the positive impact of past on future volatility. The forecasting power on subsequent volatilities extends for a period of at least one month: this is an important result for risk management and other financial applications involving volatility estimation.
As already mentioned, while threshold multipower variation has the same asymptotic law of multipower variation, we expect it to provide better estimates in small samples. By the way, for finite δ, when $|\Delta X|^2 > \vartheta$ the indicator function zeroes its relative addend. This can be an issue when testing for the presence of jumps bringing to a negative bias under the null hypothesis of no jumps. For this reason, the estimator has been corrected according to the following rule: under the null, when $|\Delta X|^2 > \vartheta$, we replace $|\Delta X|^\gamma$ with its expected value instead of zero. Therefore, the corrected realized threshold multipower estimator is defined as:

$$C - TMPV_\delta(X)_t^{[\gamma_1,\dots,\gamma_M]} = \delta^{1-\frac{1}{2}(\gamma_1+\dots+\gamma_M)} \sum_{j=M}^{[T/\delta]} \prod_{k=1}^{M} Z_{\gamma_k}(\Delta_{j-k+1}X, \vartheta_{j-k+1})$$

where the function $Z_\gamma(x,y)$ takes the following values with $c_\vartheta = 3$. This correction is essential for building test statistics and provides unbiased estimates under the null. Nonetheless, while this version of threshold bipower variation is expected to be unbiased in absence of jumps, it introduces a positive bias if jumps are present in the trajectory of X. Thus, the correction should not be implemented when the test statistics detects a jump in the trajectory.
The test statistics is based on this correction and is defined as:

$$C - Tz = \frac{\delta^{-\frac{1}{2}}(RV_\delta(X)_T \; - \; C - TBPV_\delta(X)_T) \cdot RV_\delta(X)_T^{-1}}{\sqrt{\left(\frac{\pi^2}{4} + \pi - 5\right) \max\left\{1, \frac{C - TTriPV_\delta(X)_T}{(C - TBPV_\delta(X)_T)^2}\right\}}}$$

Since for small δ the correction affects only a finite number of terms, we have that $C - Tz \rightarrow \mathcal{N}(0,1)$ stably in law as $\delta \longrightarrow 0$.

In the empirical analysis of this thesis, this test for jump detection is used in order to determine whether and when jumps in price volatility happened in our sample. Then, a time series is built on the basis of this information and incorporated in the econometric model, together with trading volumes and news arrival series.

2.2.2 *Baseline Multiplicative Error Model*

We now present some of the ARCH and GARCH models which have approached the problem of deviation from normality of random variables. Returns for financial time series are an example, since they depend on their past values and their variance is not constant over time. To mention some of the developed models, Andersen and Bollerslev (see Engle, 2002) used realized volatility to evaluate traditional GARCH specifications, and Andersen et al. (see Engle, 2002) built models based on this measure of volatility.
Nonetheless, GARCH-type models have been introduced for other variables beyond volatility. Engle and Russell (see Engle, 2002) proposed the so-called Autoregressive Conditional Duration (ACD) model in order to capture the dynamics of trade-to-trade durations. Then, Engle (2002) generalized the ACD model into the family of Multiplicative Error Models. It represents a class of time series models for positive-valued random variables which are decomposed into the product of their conditional mean and a positive-valued error term. Moreover – as we describe in the following paragraph - these models can be easily modifiable to capture non-linearities in dynamics, long range dependence, explanatory variables and intraday seasonalities, and are extendable to a multivariate setting.

Following the specification given by Engle and Gallo (2006), in the univariate setting, the baseline Multiplicative Error Model expresses the variable of interest x_t – defined by $\{x_t\}$ that is a discrete time process defined on $[0, +\infty)$, $t \in \mathbb{N}$ – as product of a scale factor μ_t evolving in a conditionally autoregressive way and of an i.i.d. error term with unit mean. Thus, it is defined as:

$$x_t = \mu_t \varepsilon_t \ .$$

where μ_t is a deterministic positive quantity that, conditionally on the available information at $\mathrm{t}-1$, evolves according to a parameter vector θ

$$\mu_t = \mu(\theta, \mathcal{F}_{t-1})$$

and ε_t is a random variable with a probability density function defined over $[0, +\infty)$, with unit mean and unknown constant variance

$$\varepsilon_t \mid \mathcal{F}_{t-1} \sim D^+(1, \sigma^2) \ .$$

As a consequence of these assumptions, whatever specification for $\mu(\cdot)$ and whatever distribution D^+, we have that

$$\mathbb{E}(x_t \mid \mathcal{F}_{t-1}) = \mu_t$$

and

$$Var(x_t \mid \mathcal{F}_{t-1}) = \sigma^2 \mu_t^2 \ . \ .$$

For what concerns the term μ_t, modelled as a GARCH, it can be specified – conditional to the information set $\mathcal{F}_{t-1}$ – as follows:

$$\mu_t = \omega + \beta\mu_{t-1} + \alpha x_{t-1} \,.$$

In more detail, $\beta\mu_{t-1}$ is the inertial component, and αx_{t-1} stands for the contribution of the more recent observation of the variable of interest. Thus, in this case the parameter vector θ to be estimated is composed by (ω, β, α), and may be restricted to ensure positive means for all possible realizations and stationary distributions for x_t. For this purpose, positivity constraints are imposed on all of them – $\omega \geq 0$, $\alpha \geq 0$ and $\beta \geq 0$ – with the coefficient β being additionally restricted to avoid the system to become explosive: $0 \leq \beta \leq 1$.

The conditional distribution of the error term ε_t can be specified by means of any positive-valued probability density function with unit mean and unknown variance. As in the ACD model (Engle and Gallo, 2006), the error term is assumed to follow a Gamma distribution with shape parameter a and scale parameter b, namely $\Gamma(a, b)$ [1]:

$$f(\varepsilon_t \mid \mathcal{F}_{t-1}) \sim \frac{1}{\Gamma(a)b^a}\varepsilon_t^{a-1}\exp\left(-\frac{\varepsilon_t}{b}\right)$$

The property of unit mean [2] implies that $\mathrm{b} = \frac{1}{\mathrm{a}}$ so that the probability density function of the error term ε_t becomes

$$f(\varepsilon_t \mid \mathcal{F}_{t-1}) \sim \frac{1}{\Gamma(a)}a^a\varepsilon_t^{a-1}\exp(-a\varepsilon_t) \,.$$

Hence, the probability density function of the variable of interest x_t is finally defined as

$$f(x_t \mid \mathcal{F}_{t-1}) \sim \frac{1}{\Gamma(a)}a^a x_t^{a-1}\mu_t^{-a}\exp\left(-a\frac{x_t}{\mu_t}\right).$$

It follows that the process x_t has conditional expectation $\mathbb{E}(x_t \mid \mathcal{F}_{t-1}) = \mu_t$ and conditional variance $Var(x_t \mid \mathcal{F}_{t-1}) = \frac{\mu_t^2}{a}$.

Once the distribution of the error term has been specified, the Multiplicative Error Model can be estimated by Maximum Likelihood method [3]. Here we report the estimation procedure as in Cipollini, Engle and Gallo (2006) [4]. As the variable of interest x_t is distributed as $\Gamma(a, \mu_t)$ in shape-mean representation, the likelihood function is given by:

$$\ell_t = \ln L_t = a\ln a - \ln\Gamma(a) + (a - 1)\ln x_t - a\left(\ln\mu_t + \frac{x_t}{\mu_t}\right).$$

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[1] In this context we specify the Gamma distribution as in Engle and Gallo (2006).

[2] $\mathbb{E}(\varepsilon_t) = a \cdot b = 1 \iff b = \frac{1}{a}$.

[3] See Appendix A.

[4] See Appendix B for the transformation for a $\Gamma(a, b)$ in shape-scale representation into a $\Gamma(a, 1)$ in shape-mean representation. Here we have changed Gamma parameters' name with respect to Cipollini, Engle and Gallo (2006).

By maximizing this function with respect to the parameter vector θ and the shape parameter a, the contribution of $\mathrm{x_t}$ to the score is composed by

$$s_{t,\theta} = \nabla_\theta \ell_t = a\nabla_\theta \mu_t \left(\frac{x_t - \mu_t}{\mu_t^2}\right)$$

and

$$s_{t,a} = \nabla_a \ell_t = \ln a + 1 - \psi(a) + \ln\left(\frac{x_t}{\mu_t}\right) - \frac{x_t}{\mu_t}$$

where ∇ denotes the derivatives with respect to the components of θ and with respect to a, while $\psi(a)$ is the digamma function evaluated at a.

The contribution of x_t to the Hessian is $H_t = \begin{pmatrix} H_{t,\theta\theta'} & H_{t,\theta a} \\ H'_{t,\theta a} & H_{t,\phi a} \end{pmatrix}$ with components

$$H_{t,\theta\theta'} = \nabla_{\theta\theta'} \ell_t = a\left(\frac{-2x_t + \mu_t}{\mu_t^3}\nabla_\theta \mu_t \nabla_{\theta'} \mu_t + \frac{x_t - \mu_t}{\mu_t^2}\nabla_{\theta\theta'} \mu_t\right)$$

$$H_{t,\theta a} = \nabla_{\theta a'} \ell_t = \frac{x_t - \mu_t}{\mu_t^2}\nabla_\theta \mu_t$$

$$H_{t,aa} = \nabla_{aa} \ell_t = \frac{1}{a} - \psi'(a)$$

where $\psi'(a)$ is the trigamma function.

From these results the first order conditions for θ and a are respectively:

$$\frac{1}{T}\sum_{t=1}^{T} \nabla_\theta \mu_t \frac{x_t - \mu_t}{\mu_t^2} = 0$$

$$\ln a + 1 - \psi(a) + \frac{1}{T}\sum_{t=1}^{T}\left[\ln\left(\frac{x_t}{\mu_t}\right) - \frac{x_t}{\mu_t}\right] = 0$$

For the second equation to be solved, the estimations for θ are required. As noted by Cipollini, Engle and Gallo (2006), the first order conditions do not depend on a. On the contrary, the second order conditions depend on that parameter, since the Hessian matrix would be proportional to a.

The asymptotic variance-covariance matrix of the Maximum Likelihood estimator is

$$V_\infty = \begin{pmatrix} \phi \lim\limits_{t\to\infty} \sum\limits_{t=1}^{T} \frac{1}{\mu_t^2} \nabla_\theta \mu_t \nabla_{\theta'} \mu_t & 0 \\ 0 & \psi'^{(a)} - \frac{1}{a} \end{pmatrix}^{-1}$$

The estimators for $\hat{\theta}$ and $\hat{a}$ are asymptotically uncorrelated. Finally, inference about (θ, a) must be based on estimates of V_∞ which can be obtained evaluating the average Hessian or the outer product of the gradients evaluated at the estimates $(\hat{\theta}, \hat{a})$. The sandwich estimator

$$V_\infty = \widehat{\bar{H}}_T^{-1}\widehat{\overline{OPG}}_T\widehat{\bar{H}}_T^{-1}$$

with $\widehat{\bar{H}}_T^{-1} = \frac{1}{T}\sum_{t=1}^{T}\widehat{H_t}$ and $\widehat{\overline{OPG}}_T = \frac{1}{T}\sum_{t=1}^{T}\widehat{OPG_t}$ eliminates the dependence of the sub-matrix relative to θ on a altogether.

At the end of maximization of the log-likelihood function with respect to the parameter vector θ and the shape parameter a of the Gamma distribution, we end up with Quasi Maximum Likelihood estimators. As stated in Engle (2002) [5], they are consistent and asymptotically normally distributed. As long as $\mu_t = \mathbb{E}(x_t|\mathcal{F}_{t-1})$, the expected value of the score of θ evaluated at the true parameters is a vector of zeroes even if the distribution of the error term does not belong to the Gamma family. This means that whatever the value of a, the log-likelihood functions can be interpreted as Quasi Maximum Likelihood functions and the corresponding parameters are Quasi Maximum Likelihood estimators. Instead, if we know that a Gamma distribution would be appropriate for the error term ε_t, the same procedure delivers consistency and efficiency for the estimators if a is known. In the case that a is not known, one solution would be to use robust standard errors. Another solution is to introduce a as parameter to be estimated: it would provide information on the shape of the error term distribution and is useful for simulating future values or for making scenario analysis with no impact on the values of the estimates θ since $Cov(\hat{\theta}, \hat{a}) = 0$.

[5] From the corollary to Lee and Hansen (see Engle, 2002):

If

(1) $\mathbb{E}_{t-1}(x_t) \equiv \mu_{0,t} = \omega_0 + \alpha_0 x_{t-1} + \beta_0\mu_{0,t-1}$

(2) $\varepsilon_t \equiv x_t/\mu_{0,t}$ is

- strictly stationary and ergodic
- non-degenerate
- has bounded conditional second moments
- $sup_t\mathbb{E}[\ln(\beta_0 + \alpha_0\varepsilon_t)|\Im_{t-1}] < 0$ a.s.

(3) $\theta_0 \equiv (\omega_0, \alpha_0, \beta_0)$ is the interior of Θ

(4) $L(\theta) = -\sum_{t=1}^{T}(\log(\mu_t) + \frac{x_t}{\mu_t}$ where $\mu_t = \omega + \alpha x_t + \beta\mu_{t-1}$ for $t > 1$, and $\mu_t = \omega/(1-\beta)$

Then

a) The maximizer of L will be consistent and asymptotically normal with a covariance matrix given by the familiar robust standard errors as in Lee and Hansen;

b) The model can be estimated with GARCH software by taking $\sqrt{\mu_t}$ as the dependent variable and setting the mean to zero;

c) The robust standard errors of Bollerslev and Wooldridge (1992) coincide with those in Lee and Hansen.

2.2.3 *Extended Multiplicative Error Model*

Here, we present some literature on possible extensions of the Multiplicative Error Model by defining different conditional mean specifications.
In Engle (2002), we can find a general class of mean functions which include predetermined or weakly exogenous variables into the conditional mean equation. Moreover, it is possible to augment the GARCH baseline specification by adding asymmetric effects about the direction of price movements.
Generally, defining the usual model $x_t = \mu_t \varepsilon_t$, and defining the exogenous variables included in the information set $\mathcal{F}_{t-1}$ in a $k\times 1$ vector z_t, a (p, q) mean specification would be flexibly specified as

$$\mu_t = \omega + \sum_{i=1}^{p} \alpha_i x_{t-i} + \sum_{j=1}^{q} \beta_j \mu_{t-j} + \gamma' z_t$$

where the parameters can be restricted to return positive means for all possible realizations and to ensure stationarity distributions for x.
We can find an example in Engle and Gallo (2006): the Authors built a model where absolute daily returns, daily high-low range and daily realized volatility were jointly considered in order to develop a forecasting model based on their conditional dynamics. Each of these indicators of volatility has been modelled as a GARCH-type process. Specifically, we now list some possible mean specifications of absolute daily returns given by Engle and Gallo (2006) as examples of extensions of a Multiplicative Error Model.
Suppose absolute daily returns are defined as

$$r_t = \log\left(\frac{C_t}{C_{t-1}}\right)$$

where C_t is the daily closing price. The Multiplicative Error Model for absolute daily returns has been built by considering the squared returns r_t^2 as follows:

$$r_t^2 = h_t^r \varepsilon_t$$

$$\varepsilon_t \mid \mathcal{F}_{t-1} \sim D(1, \xi^r)\ .$$

One possible way in which the baseline mean specification of this model can be expanded is considering asymmetric reactions of returns to shocks. In Engle and Gallo (2006), the conditional mean for absolute daily returns is defined as a threshold-type model involving a dummy variable for negative returns:

$$h_t^r = \omega_r + \alpha_r r_t^2 + \beta_r h_{t-1}^r + \gamma_r r_{t-1}^2 d_{t-1}$$

with $d_{t-1} = I(r_t < 0)$, where $I(\cdot)$ denotes the indicator function.

Moreover, it is also possible to add lagged returns r_{t-1} accounting for asymmetry given that they maintain their sign:

$$h_t^r = \omega_r + \alpha_r r_t^2 + \beta_r h_{t-1}^r + \gamma_r r_{t-1}^2 d_{t-1} + \delta_r r_{t-1} \, .$$

Furthermore, given that the other two indicators – the daily high-low range and the daily realized volatility – are included in the information set $\mathcal{F}_{t-1}$, they can enter the conditional mean specification as:

$$h_t^r = \omega_r + \alpha_r r_t^2 + \beta_r h_{t-1}^r + \gamma_r r_{t-1}^2 d_{t-1} + \delta_r r_{t-1} + \varphi_r hl_{t-1}^2 + \vartheta_r hl_{t-1}^2 d_{t-1} + \psi_r v_{t-1}^2 + \lambda_r v_{t-1}^2 d_{t-1}$$

which is called system specification. Here, six exogenous variables appear in the equation. Despite the augmentation of the baseline GARCH-type conditional mean has the advantage to take into consideration other effects, the question is up to which point the Multiplicative Error Model requires the information set to be augmented. For this reason, Engle and Gallo (2006) have estimated several models and then chose the best ones with a general-to-specific strategy and by Schwartz information criteria (BIC).

In the empirical analysis of this thesis, the Multiplicative Error Model for trading volumes is extended by means of the above-explained approach. Exogenous variables are added into the baseline conditional mean specification of the Multiplicative Error Model in order to get information on whether these covariates have an impact on the conditional mean of trading volumes.

2.2.4 *Application to trading volumes*

In Manganelli (2005), trading volumes, price returns and durations are modelled as three different Multiplicative Error Models. The purposes of the Author were to understand whether stocks exhibit trading volume clustering, whether times of greater activity coincide with a higher number of informed traders in the market, and whether the more frequently traded stocks converge more rapidly to their full information equilibrium.
This work is useful to our empirical analysis in two aspects. First, we inspired to the Multiplicative Error Model for trading volumes of our dataset from the one presented in Manganelli (2005). In fact, the so-called Autoregressive Conditional Volume model is defined as an alternative version of the Autoregressive Conditional Duration model of Engle (2002). Second, Manganelli (2005) studies the link between new information arriving in the market and trading activity through the study of trade-to-trade durations. In our empirical analysis, we do not use durations – as our dataset does not convey information about them – but we study the relationship between news arrivals and trading volumes by including a variable expressing the amount of news and announcements into the conditional mean specification of the Multiplicative Error Model.

The so-called Autoregressive Conditional Volume model has been defined as:

$$v_t = \phi_t \eta_t$$

where v_t is a non-negative random variable representing trading volumes, ϕ_t is the conditional mean specified as

$$\phi_t = \omega + \alpha v_{t-1} + \beta \phi_{t-1}$$

and η_t is the error term following a non-negative probability distribution: $\eta_t \sim IID(1, \sigma_\eta^2)$. In order to answer the question about trading volume clustering, this model has been estimated for both frequently and infrequently traded stocks. The evidence highlighted that volumes appear to be a very persistent process for frequently traded stocks. In fact, the values of the autoregressive coefficient (β) has been estimated above 0.9, more specifically above 0.95 for frequently traded stocks. This phenomenon – already observed for durations and volatility – is consistent with the predictions of market micro-structure theories according to which markets are active every time an unexpected piece of information arrives or when there is a clustering of liquidity traders. For the less frequently traded stocks, the persistence is significantly lower: the estimates β take values from 0.7 to 0.8, with some peak of 0.3. Therefore, clustering effect should be stronger for firms with better public reporting and more discretionary liquidity traders. Informational advantages are eliminated by public announcements, and the decision of liquidity traders to postpone their trading until the announcement has been released will make it easier for the market maker to infer informed agent's reasons for trading. For confirming this, it is evident that the frequently traded stocks are also more likely to be characterized by better disclosure of public information, and the higher clustering of trading volumes is consistent. Since private informational advantage is shorter lived for frequently traded stocks, informed traders will try to trade more intensively, thus inducing a greater clustering in trading volumes. The Author has also predicted a stronger autocorrelation in the volume process for the frequently traded stocks.

For what concerns the link between news release and trading volumes, the questions about how long does it take for new information to be incorporated into prices was addressed in Manganelli (2005). New information is incorporated into prices through the interaction between informed and uninformed traders. As a consequence, volumes and durations influence prices because they are correlated with private information about the stock's true value. Since informed trades would trade more frequently and in larger quantities when they have valuable information, a rational market maker will interpret large orders and short durations as evidence of trading by informed agents and will adjust beliefs and prices accordingly. On the other hand, a lack of trade is associated with no new information: in this situation, prices efficiently incorporate all the available information.

Assumed that prices are semi-strong efficient – that is they reflect all public but not private information – market activity and volatility seem to change over time because new information becomes available to traders. When new information is few, there is slow trading, and vice versa. The intuition is simple: trading volume increase as long as there exist traders with superior information who try to transform their informational advantage into new profits. Competition among informed traders performs an important role: as the number of traders increases, the speed with which news and announcements are incorporated into prices increases very sharply. In order to investigate this phenomenon, the Author uses impulse-response functions in calendar time. They show how the system reacts to a perturbation of its long-run equilibrium, and are helpful for understanding the speed with which prices tend to converge to the underlying liquidation value of the asset. Manganelli (2005) affirmed also that price volatility of frequently traded stocks should converge to their long-run equilibrium (after an initial perturbation) more rapidly than the one of infrequently traded stocks. Theory shows that prices do eventually converge to full informational values but not much is said on how much this process takes. Knowing the speed at which new information is incorporated into prices may put some light on the nature of market efficiency, and may give new suggestions for understanding what features are needed for more efficiency in this sense.

3 EMPIRICAL ANALYSIS

3.1 Data description [6]

Data used for the case study in our empirical analysis regards Apple Inc. It is a multinational company which was founded by Steve Jobs, Steve Wozniak and Ronald Wayne on 1st April 1976, and then incorporated on 3rd January 1977 in Cupertino, California. The Company deals with the design, manufacturing and sale of mobile communication and media devices, personal computers, software and digital media content. It is active worldwide and uses both online and retail stores as well as third-party wholesalers and reseller to market its products. Apple's stocks – whose tick is 'AAPL' – are traded in NASDAQ market from 12nd December 1980, when the Corporation went public with an initial offering of USD 22.00 per share. Figure 5 reports a plot of its close prices for the period available in the database, that is from the beginning of 2005 to the end of 2014.

Figure 5. Evolution of AAPL close prices, with minimum and maximum.

As usually happens for high-technology corporations, it can be observed that Apple stock's price has had an irregular evolution through the years. The overall trend is increasing and has become sharper in the the last period of observation. More specifically, the minimum close price – marked in red – has been registered on 6th January 2005 with a value of USD 4.32. On the other hand, Apple's stock has reached its minimum – marked in green – on 28th December 2014 with a close price equal to USD 117.87.

[6] All graphics and computations are obtained in Matlab.

From the fact that close prices have moved in an upward way, it is deducible that this path depends on what happened in the Company, what investors and analysts declared about its activity and what products and services have been offered to customers. When the Company went public in 2005, the top products were the new Mac Mini, the innovative online music shop iTunes Store, and those belonging to the iPod family introduced in the early 21th century. The fact that close price shows a low positive trend until mid-2007 could be a signal of a limited attention paid by investors and general customers for Apple's products. After all, this situation is typical of high-tech companies in the early phases: the line between success and failure of the entire company lies in the acceptance and trust level shown by investors and potential consumers. Large upward price oscillations can be seen in case of positive feedback from the market. This is the case of Apple Inc. when the Mac family products and the revolutionary iPhone have been introduced in 2008. In that period, close prices followed a period of higher volatility with respect the past, and began to increase until a new jump in volatility in the early 2012. That point coincides with the the introduction of a new version of the iPad together with the up-to-date range of personal computers, the MacBook Air and MacBook Pro. Then, although close prices have shown a decrease between the end of 2012 and the half of 2013, they resume a fast increase from about the end of 2013 when iPhone 5 and the new iOS7 have been introduced in the market. Finally, the maximum at the end of 2014 has been probably caused by the launch of iPhone 6. Furthermore, close prices have maintained their average around USD 100 without getting smaller than USD 90 until half of 2016.

For what concerns the empirical analysis of this thesis, we use intra-daily Apple Inc. data, both quantitative and qualitative. For quantitative data it is intended trading volumes and returns – and consequently close prices also – leaving the other available variables (open price, low price and high price) aside. These data have been downloaded from kibot.com - which is a provider of historical intraday data – at one-minute frequency. On the other hand, qualitative data refers to news and announcements about Apple Inc. in each of the one-minute intra-daily interval, retrieved from FactSet StreetAccount database [7]. All data – both quantitative and qualitative – has been arranged in order to build a unique dataset consistent in timing.
The time period covered by the sample observations corresponds to a typical trading day, that is from 9.30 a.m. to 4.00 p.m. The number of days observed is 2516 which coincides with the number of business days between 4th January 2005 and 31st December 2014. Moreover, variables are associated to timing details such as hour, minute, second and number of day for each observation in the dataset.

[7] We thank the PhD candidate Poli F. for StreetAccount data.

Furthermore, it is important to notice that they contain not raw but already filtered data. This means that they are high-frequency but not tick data because price and exact timing of every transaction occurred is not available. Both quantitative and qualitative data measure what happened to Apple’s stocks into every one-minute interval in business days observed. Therefore, data do not present the problem concerning irregular temporal spacing of observations. Moreover, once time aggregation is performed to each of the variables, the first observation for each business day observed is deleted. The reason is that we would not like to take into consideration any micro-structure market effect in the opening of the trading days. In fact, all overnight and pre-trading effects in the first daily interval may add a non-usual behaviour in the series that is better to avoid.

In the following paragraphs, trading volumes, price returns, news and announcements are described in detail.

3.1.1 Trading volumes

Generally speaking, trading volume refers to the total number of shares traded between a buyer and a seller during a transaction. In our case, volume measure the overall number of Apple's stocks traded in a particular one-minute interval in each observed business day. The total size of the sample is then equal to $78 \cdot 2516 = 196248$ positive-valued observations resulting from the product between the number of five-minute intra-daily intervals and the number of business days. We observe a high trading volume when securities are more actively traded, while a low volume when they are less actively traded. Apple Inc. stocks can be considered actively traded since trading volume time series in our dataset does not present any zero values. It means that, in each minute of the overall 2516 business days observed, someone traded at least one Apple's. Below, Figure 6 represents the average daily trading volume throughout all days observed.

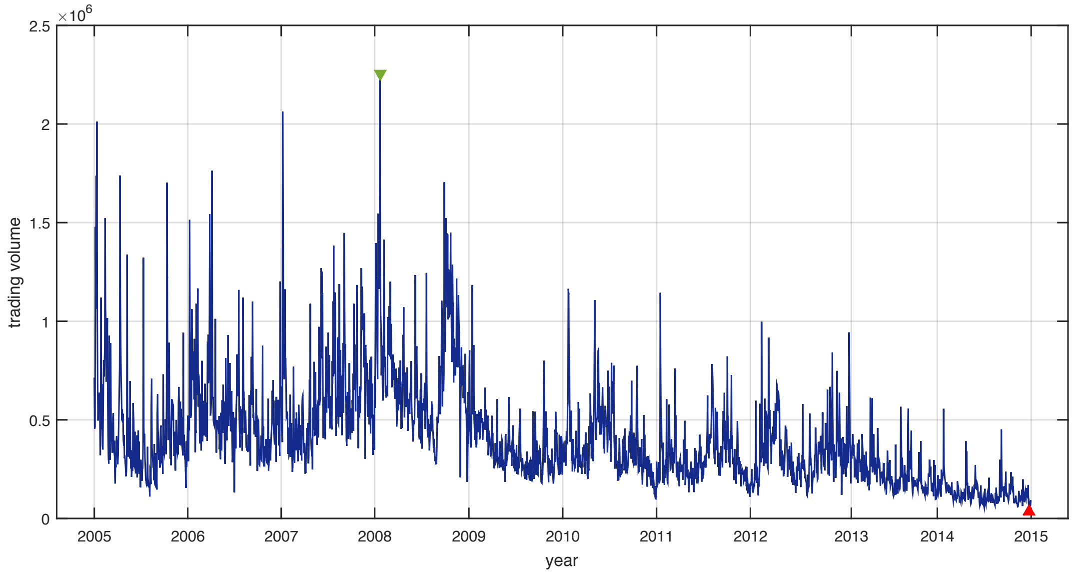


Figure 6. Evolution of AAPL trading volumes, with minimum and maximum.

It is evident that the series presents large oscillations, especially in the first four years of observation. Moreover, periods of high volatility come in succession of periods characterized by high volatility, and vice versa. As we describe in the following, this volatility clustering phenomenon is a consequence of strong persistence of trading volume process over time, and is visible in intra-daily analysis also. It strongly depends on price volatility evolution: when the volatility increases, trading volumes increase as traders trade more frequently. This may be due to the entry of new information in the market and, because of price discovery process, it is exploited by informed traders which translates in higher volumes. Beyond oscillations, Figure 2 shows that the maximum value of average volume is of 2,259,260.59 shares located on 6th February 2008, and the minimum counts 36,019.37 shares traded on 24th December 2014.

Instead, Figure 7 we show the total amount of trading volumes for each year in the dataset.

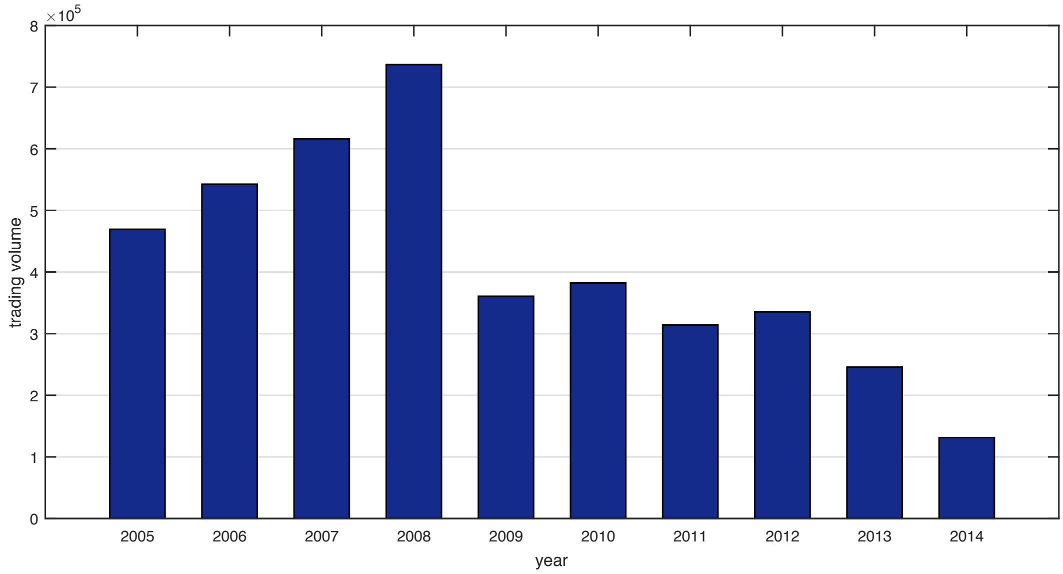


Figure 7. AAPL total amount of trading volumes for each year.

We observe that this histogram follows what shown in the previous plot. We see that there has been a constant increment in the first years until 2008; then, trading volumes have suddenly dropped down and remained constant until 2012. In the last two years, volumes have continued to decrease.

For what concerns intra-daily analysis, we firstly applied time aggregation to the series. This allows – as we said in Chapter 2 – to work with a more smoothed series with respect the starting one, and to notably reduce computation complexity by decreasing the number of observations. Hence, we aggregate the one-minute intra-daily observations and replace them with five-minute intra-daily values. We define trading volume as the positive-valued process $V_{t,n}$ where $t = 1, \dots, 2516$ indicates the business day and $n = 1, \dots, 390$ is the time index for one-minute frequency intra-daily intervals. The original series contained 390 intra-daily observations and has been transformed according to:

$$V_{t,i} = \frac{1}{5}\sum_{k}^{k+4} V_{t,n}$$

with $k = 1, 6, 11, \dots, 386$. The index $i = 1, \dots, 78$ indicate the five-minute intra-daily intervals, and $V_{t,i}$ the new five-minute frequency time series. The aggregated trading volumes has thus been computed as the arithmetic mean of five consecutive trading volume observations.

In Figure 8, average daily trading volumes are represented for both one-minute and five-minute cases.

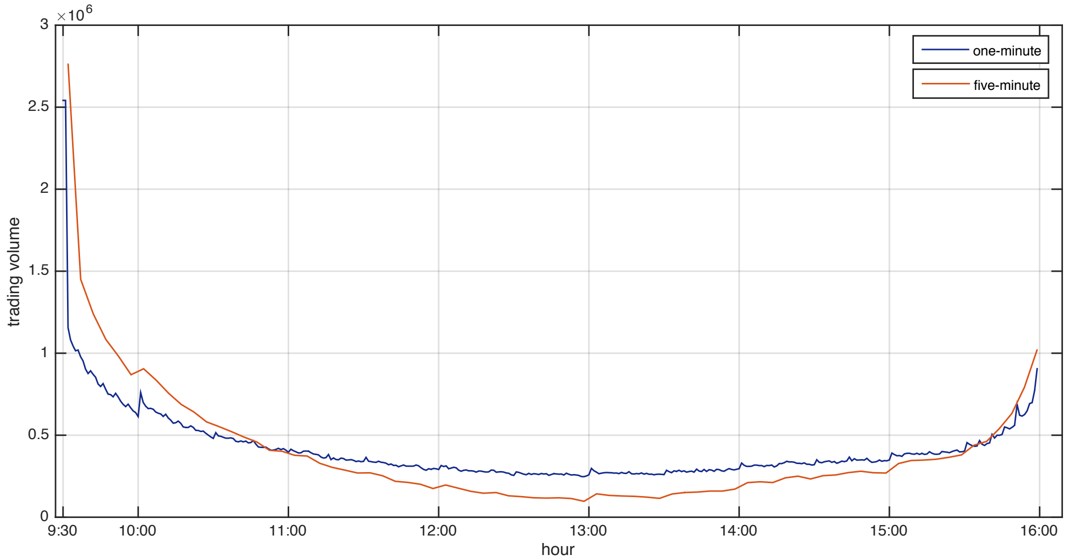


Figure 8. AAPL average daily trading volumes, one-minute and five-minute frequency.

The five-minute case series is immediately distinguishable because – as already said – it is more smoothed. However, in both graphical representations, average daily trading volumes follow a U-shaped pattern which signals for periodicity in the data. This phenomenon is the consequence of a higher market activity in the beginning and in the end of the trading day, and of a lower activity around lunch time. It can be interpreted as a sort of diurnal seasonality: it is a typical path presented by volatility and absolute returns also – as we analyse in the following paragraphs.

In order to use trading volume series in the econometric analysis, it is necessary to remove periodicity from it. Many methods are available for this purpose, but our data calls for harmonic regression. Indeed, as shown in Figure 9, the plot of five-minute trading volumes for three consecutive days present a periodic component that could be approximated by the sum of various sine and cosine curves.

For this purpose, we used Fourier series approach which allows to explain the series entirely as a composition of sinusoidal functions. In order to write the regression equation to remove the periodic component, we begin from the definition of the Multiplicative Error Model for trading volumes which will be used in the empirical analysis. Given $V_{t,i}$ the trading volume at intra-daily interval i in the business day t, the model we use is:

$$V_{t,i} = \mu_{t,i}\varepsilon_{t,i} \, .$$

Regardless of the conditional mean specification of our Multiplicative Error Model – we will specify many of them in the following analysis – we add an intra-daily periodic component S_i into this model:

$$V_{t,i} = S_i\mu_{t,i}\varepsilon_{t,i} \, .$$

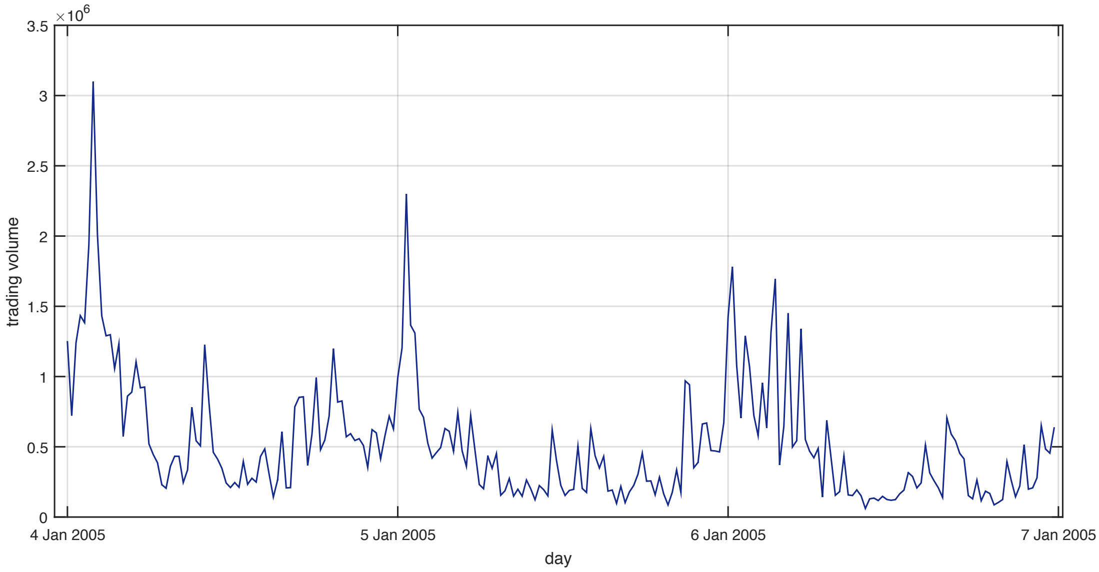


Figure 9. AAPL trading volumes for three consecutive days of the sample.

Then, we specify it as a function containing a constant, a polynomial trend and a combination of four harmonics:

$$S_i = \varphi_0 + \sum_{v=1}^{3} \varphi_v t^v + \sum_{j=1}^{4} \{\alpha_j \cos[2\pi\, jf(t)] + \beta_j \sin[2\pi jf(t)]\}$$

where t^v is a trend that increases very slowly, $f(t)$ is a periodic sequence adapted to the data frequency, and $\varphi_v, \alpha_j, \beta_j$ are the periodic parameters to be estimated. By substituting the periodic component with this expression, the above-mentioned model for trading volumes becomes:

$$V_{t,i} = \left(\varphi_0 + \sum_{v=1}^{3} \varphi_v t^v + \sum_{j=1}^{4} \{\alpha_j \cos[2\pi\, jf(t)] + \beta_j \sin[2\pi jf(t)]\}\right) \mu_{t,i} \varepsilon_{t,i} \,.$$

Since periodic parameters are ideally estimated by the standard least squares approach on the log-transformation, we have to transform our multiplicative model into an additive one:

$$\ln(V_{t,i}) = \ln\left(\varphi_0 + \sum_{v=1}^{3} \varphi_v t^v + \sum_{j=1}^{4} \{\alpha_j \cos[2\pi\, jf(t)] + \beta_j \sin[2\pi jf(t)]\}\right) + error_{t,i}$$

where $error_{t,i} \equiv \ln(\mu_{t,i} \varepsilon_{t,i})$.

Finally, a linear regression of $\ln(V_{t,i})$ is performed on the periodic component, and exponential function is then applied to both fitted and residual values in order to come back to the original multiplicative model. In Figure 10 and Figure 11, we plotted the resulting [8] periodic component and the average intra-daily trading volume series without it.

[8] See Appendix for regression results.

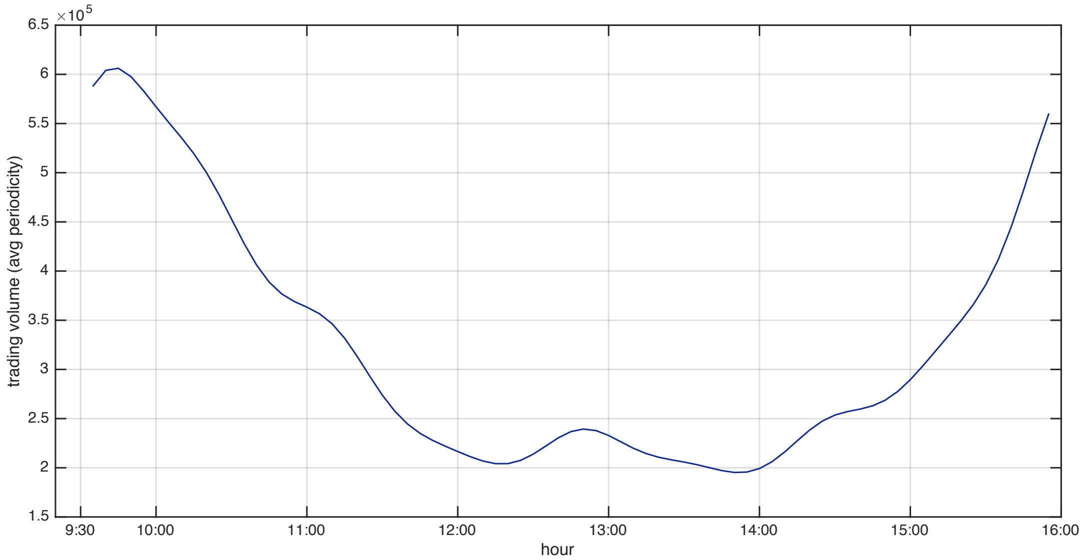


Figure 10. AAPL daily average trading volume periodicity.

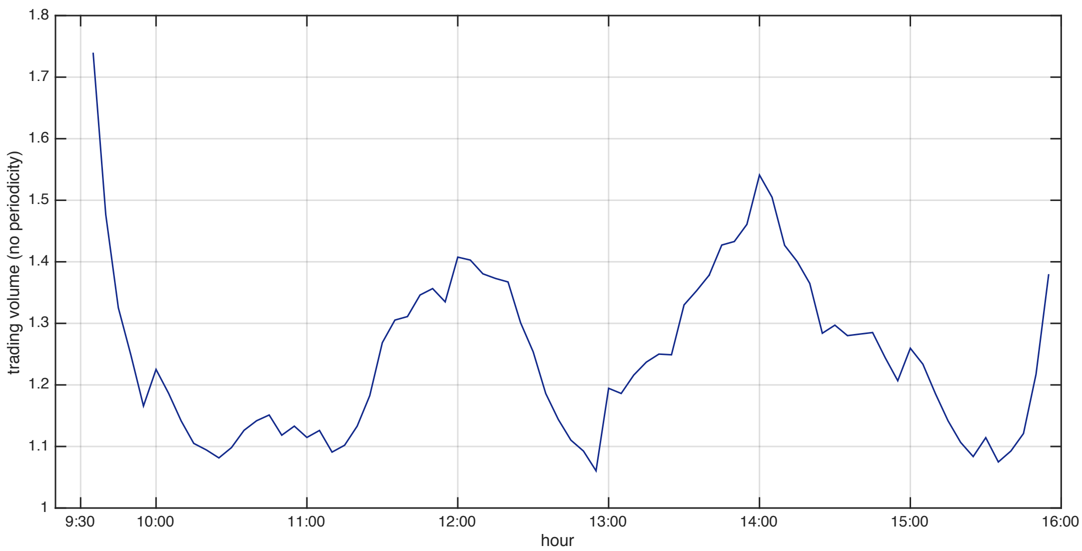


Figure 11. AAPL daily average trading volume without periodicity.

It is evident how the regression has captured the periodicity component in each day: the plot in Figure 6 follows the U-shaped pattern of diurnal periodicity. On the other hand, Figure 7 shows $\tilde{V}_{t,i} \equiv \hat{\mu}_{t,i}\hat{\varepsilon}_{t,i}$ as result of application of exponential function to the last equation of the following ones:

$$\ln(V_{t,i}) = \ln(\hat{S}_i)\, \widehat{error}_{t,i}$$

$$\ln(V_{t,i}) = \ln(\hat{S}_i) + \ln(\hat{\mu}_{t,i}\hat{\varepsilon}_{t,i})$$

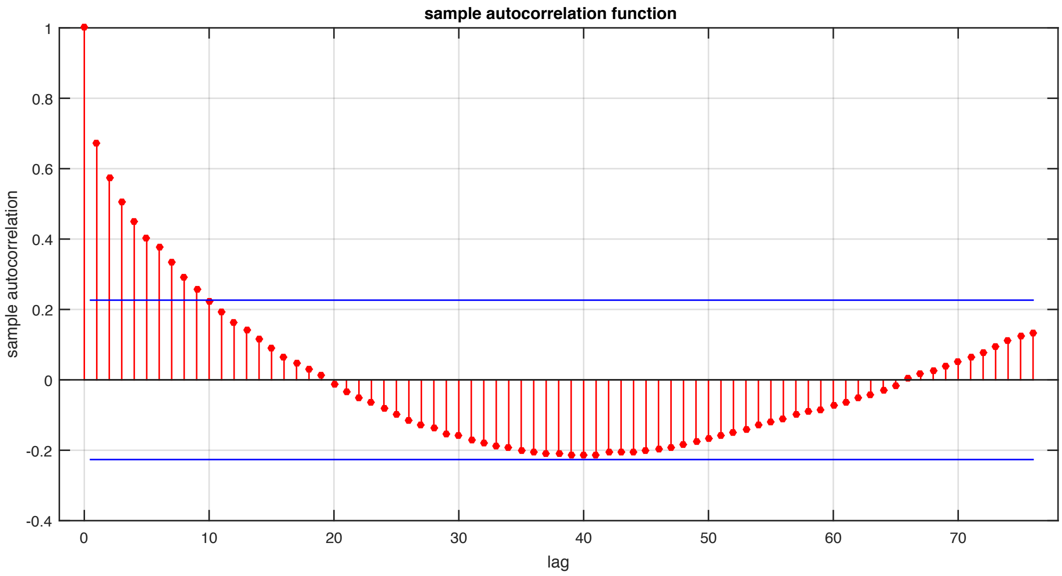
$$\ln(V_{t,i}) - \ln(\hat{S}_i) = \ln(\hat{\mu}_{t,i}\hat{\varepsilon}_{t,i}).$$

Thus, we arrive to define the trading volume time series without periodicity as:

$$\frac{V_{t,i}}{\hat{S}_i} = \tilde{V}_{t,i}\,.$$

In the empirical analysis, we will simply call it $V_{t,i}$ for simplicity's sake.

Another property of trading volume series is strong persistence. It means that there is serial autocorrelation, indicating that observations in the intra-daily time interval i strongly depend on what happens at time $i-1$. For detecting this characteristic, Figure 12 plots the autocorrelation function for the average intra-daily trading volume through all five-minute



intervals.

Figure 12. AAPL average daily trading volumes autocorrelation function (original series).

Autocorrelation function plots the autocorrelations of the process defined as $\rho_j = Corr(V_{t,i}, V_{t,i-1})$ against periods i. As reported in the graph, the sample autocorrelation function of trading volumes goes to zero very slowly, indicating for presence of persistence in the data. Moreover, intra-daily periodicity can be easily retrieved from the evolution of autocorrelations that follow a sinusoidal pattern. Lastly, autocorrelations seem to be dependent one to the other, confirming the presence of clustering.

3.1.2 Returns

Stock returns measure the profit – if positive – or the loss – if negative – from investment made by investors. The return can be in form of profit through trading or in form of dividends given by the company to its shareholders. Generally, at the end of every quarter, the company making profits give a part of them to its shareholders. The most common form of generating stock returns is through trading the stock in the secondary market. There, investors can trade by taking long or short positions according to their risk preferences. Moreover, stock returns are not fixed as bond returns because they depend on market evolution, on investors' preferences and on overall market sentiment. Usually, when a stock is frequently traded, its return changes more frequently and by more amount. This phenomenon is reflected on the stock volatility evolution which tends to increase more likely when some important events happen in the market.

Using Apple Inc. quantitative data, we now define the return $R_{t,i}$ – where $t = 1, \dots, 2516$ indicates the observed day and $i = 1, \dots, 390$ indicates the five-minute intra-daily interval of that day – as the rate of return of two consecutive close prices. Defining the close prices at intra-daily interval i in the business day t as $C_{t,i}$, returns are then computed as:

$$R_{t,i} = \left(\frac{C_{t,i}}{C_{t,i-1}}\right) - 1 \, .$$

Taking into consideration the overall evolution of Apple Inc. stock returns for the entire size of our sample – which comprehends the time period from 4th January 2005 to 31st December 2014 – the resulting pattern is reported in Figure 13.

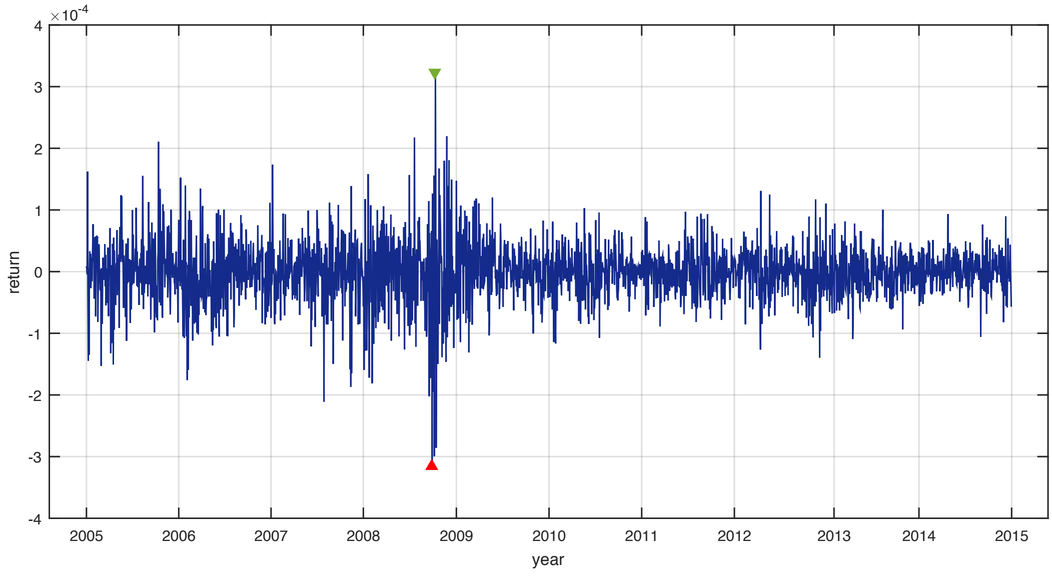


Figure 13. Evolution of AAPL daily average returns, with minimum and maximum.

We can observe that Apple Inc. returns have varied a lot through the years. The mean can be individuated around zero, the average minimum value at -0.0003 (-0.03%) on 29th September

2008 and the average maximum at +0.0003 (+0.03%) on 10th October 2008. It is important to notice that both minimum and maximum are observed in a relatively small time interval: only eight business days have passed from one to the other.

More in detail - as said in Chapter 2 – returns can be divided into a continuous component and a discontinuous part, respectively $[R^c]_t^{t+T}$ and $[R^d]_t^{t+T}$, as follows:

$$[R]_t^{t+T} \coloneqq [R^c]_t^{t+T} + [R^d]_t^{t+T} .$$

The integrated volatility corresponding to the first addend is defined as $\int_t^{t+T} \sigma_s^2 ds$. In intra-daily analysis, this continuous component can be measured by the quadratic variation of the process, namely realized volatility which is equal to:

$$RV_t = \sum_{i=1}^{I} (\Delta_i R)^2$$

where $\Delta_i R = R_{i+1} - R_i$ indicates the return from intra-daily interval $(i-1)$ to i, for $i = 1, \dots, 78$. Below, Figure 14 represents the daily realized volatility for each business day in our dataset.

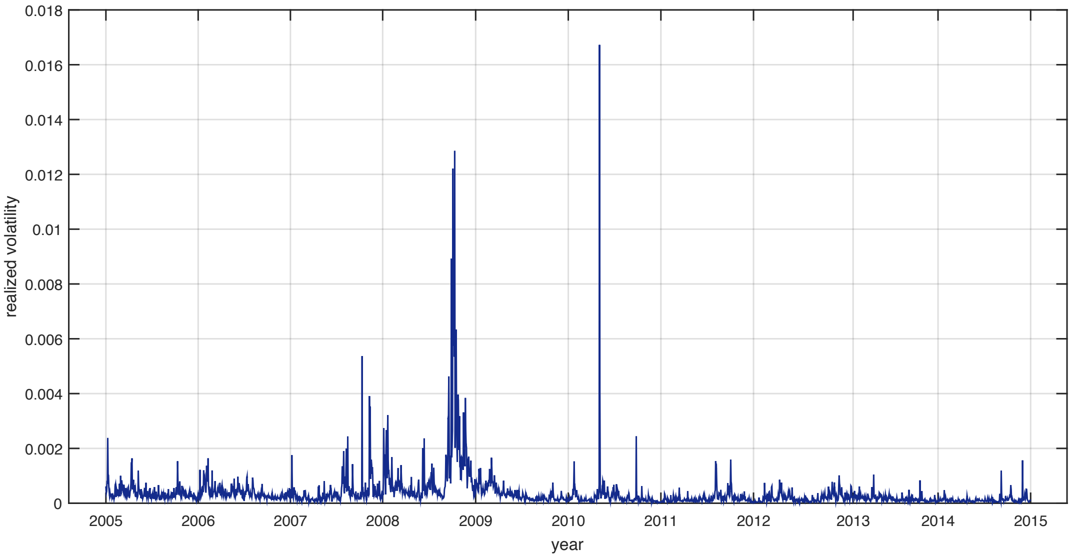


Figure 14. AAPL daily realized volatility.

We observe that there are two larger peaks in Apple Inc. realized volatility: they correspond to observations on 6th May 2010 and on 10th October 2008. The latter happened exactly on the same date of the largest variability in returns. Moreover, as confirmed by Figure 15 which represents average daily returns for each year separately, larger oscillations are thus observed in year 2008 and at the beginning of 2009. From these graphs, we can also retrieve general information on volatility: in particular, the presence of volatility clustering – that is high volatility periods are followed by high volatility days, and vice versa – is proved. In fact, where there are peaks in volatility other large oscillations can be observed in the days around the peak,

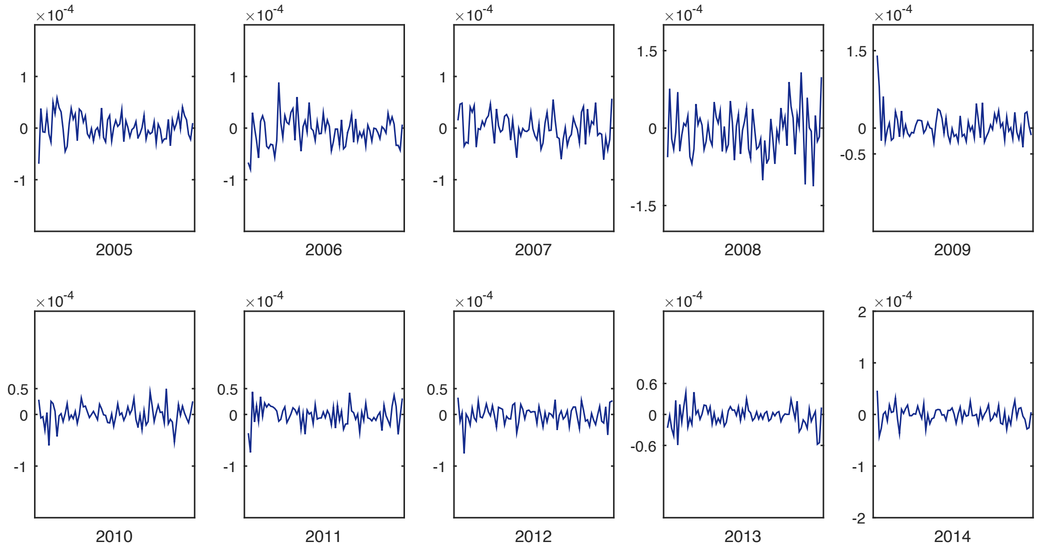


Figure 15. AAPL evolution of returns for each year, separately.

and when volatility has established low, it remains until something happens in the market.
On the other hand, the discontinuous component of returns is defined as $[R^d]_t^{t+T} = \sum_{j=N_t}^{N_{t+T}} c_j^2$ where c_j^2 is the size of the j-th jump at time τ_j, and N_t is a Poisson process indicating for presence of jumps. At this stage, jumps are also individuated in order to be used in the empirical analysis. Their detection has been performed by following the approach of Corsi-Pirino-Renò (2008) that has been explained in Chapter 2.
We now present some graphics in order to inspect some general description of Apple's stocks returns. Figure 16 plots the average yearly evolution of returns through all sample size.

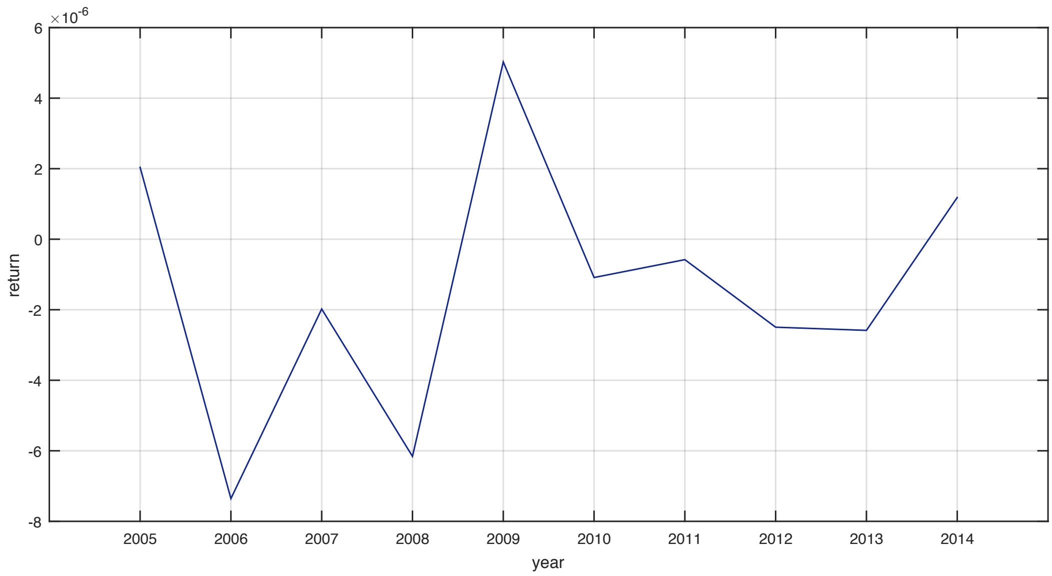


Figure 16. AAPL average yearly return.

It can be immediately recovered that the average return of Apple Inc. stocks in 2005 was positive, while remained negative in the following years until 2009. In the period from 2010 to

2013, returns have remained slightly negative in mean while slightly positive in the last year of observation.

For what concerns intra-daily analysis of returns – as already utilized for trading volumes - we apply time aggregation to the series. We aggregate all the one-minute intra-daily observations in order to get five-minute intra-daily interval observations. Defining returns at one-minute frequency as $R_{t,n}$ where $t = 1, \dots, 2516$ is the business day and $n = 1, \dots, 390$ is the time index for one-minute intra-daily intervals, the original series transforms into the five-minute series $R_{t,i}$ as:

$$R_{t,i} = \frac{1}{5}\sum_{k}^{k+4} R_{t,n}$$

where $k = 1, 6, 11, \dots, 386$, and $i = 1, \dots, 78$ that indicates the so-built five-minute intra-daily intervals. As for trading volumes, the original 390 intra-daily observations become only 78, and the five-minute returns have been simply aggregated by taking the arithmetic mean of five consecutive returns observations.

Figure 17 represents average daily return for both one-minute and five-minute cases.

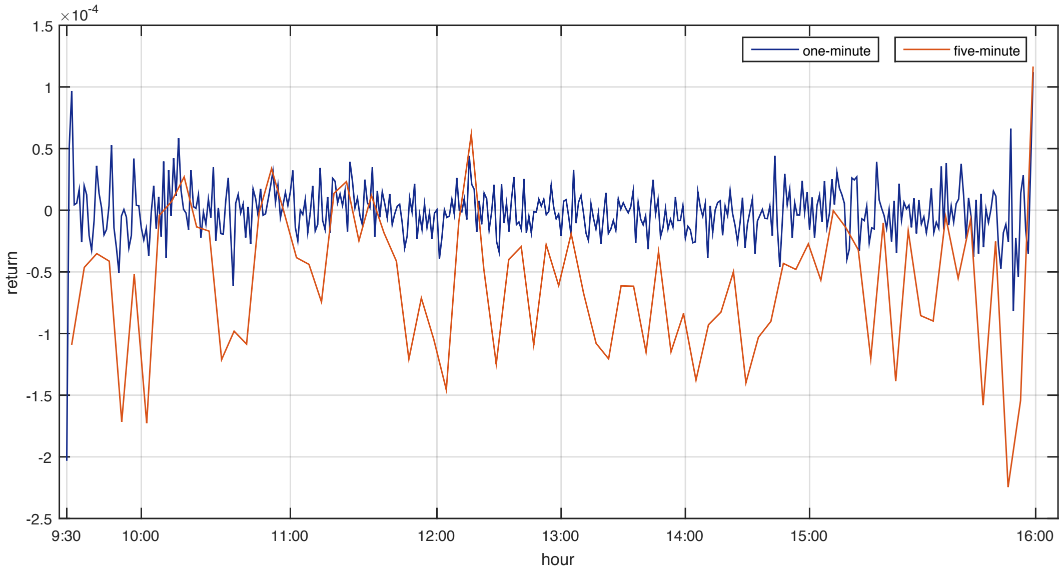


Figure 17. AAPL average daily returns, one-minute and five-minute frequency.

We observe that the five-minute series collects the evolution of returns in a roughly way. This may be due to the different variability that returns follow during the day so that aggregated series smooths the data but loses precision.

By the way, in our empirical analysis we use absolute returns. In Figure 18, average absolute returns of Apple’s stock is represented in both one-minute case and five-minute intra-daily intervals case. Absolute daily returns present diurnal periodicity – as trading volume series – in both cases. This phenomenon presents for the same reason: it is surely linked with a higher activity in the opening and in the closing of the trading day counterbalanced by a lower market activity around lunch time.

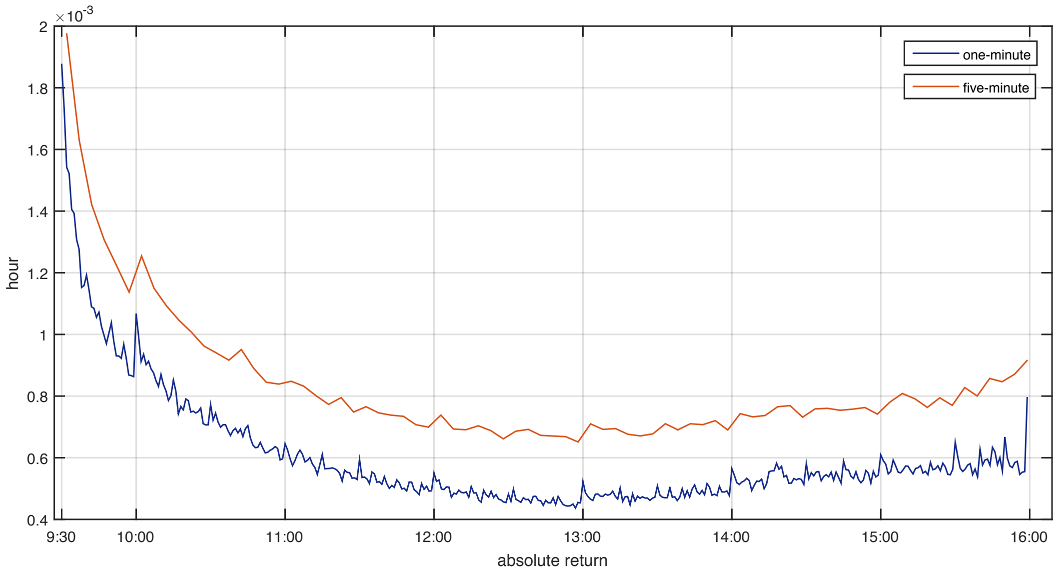


Figure 18. AAPL average daily absolute returns, one-minute and five-minute frequency.

For this reason, also in this case, it is necessary to remove this periodicity from the series before using it into the econometric model. The procedure is the same applied for trading volumes, which means involving Fourier harmonics to estimate the periodic component and then removing it. In fact, as shown in Figure 19, the path followed by absolute returns series in three consecutive days can be easily connected with sinusoidal curves.

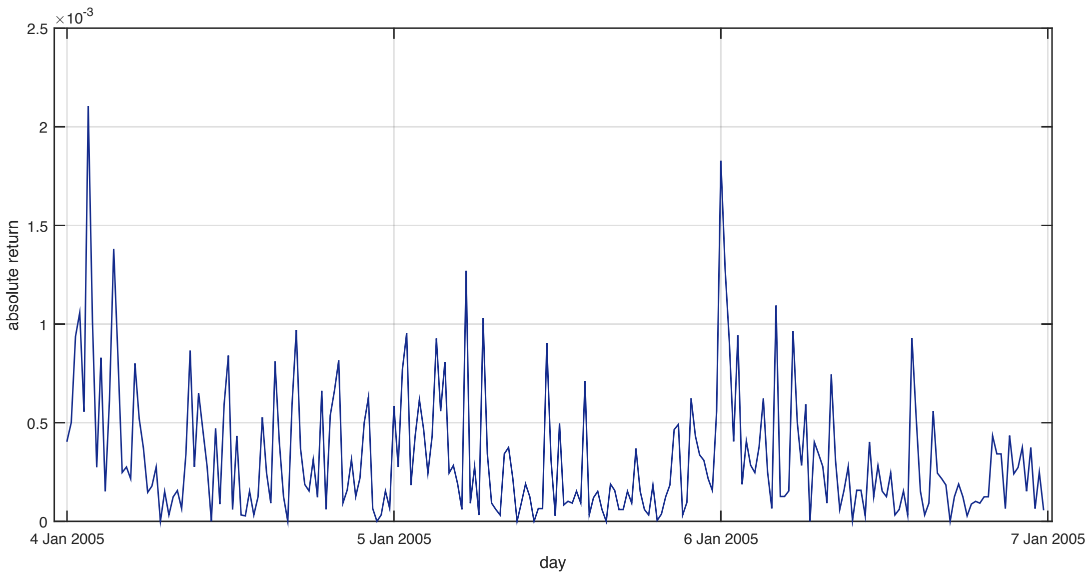


Figure 19. AAPL absolute returns for three consecutive days of the sample.

As in the case of trading volumes, we define absolute returns series as a multiplicative model:

$$Abs\_R_{t,i} = S_i \cdot error_{t,i}$$

where $error_{t,i}$ will be the absolute time series without periodicity. We define S_i as the diurnal component as:

$$S_i = \varphi_0 + \sum_{v=1}^{3} \varphi_v t^v + \sum_{j=1}^{4} \{\alpha_j \cos[2\pi jf(t)] + \beta_j \sin[2\pi jf(t)]\}$$

where φ_0 is a constant, φ_v are the coefficients of the trend component and α_j, β_j the coefficients of the two harmonics included in the periodic component. We remark that t^v increases very slowly and $f(t)$ is a periodic sequence adapted to the data frequency. To remove the periodicity component, it is needed to transform the above-defined multiplicative model for absolute returns into an additive one through logarithms:

$$\ln(Abs\_R_{t,i}) = \ln(S_i) + \ln(error_{t,i})$$

$$\ln(Abs\_R_{t,i}) = \ln\left(\varphi_0 + \sum_{v=1}^{3} \varphi_v t^v + \sum_{j=1}^{4} \{\alpha_j \cos[2\pi jf(t)] + \beta_j \sin[2\pi jf(t)]\}\right) + \ln\left(error_{t,i}\right).$$

Now it is possible to proceed with linear regression, and the resulting absolute returns series without periodicity is given by:

$$\ln(Abs\_R_{t,i}) = \ln(\hat{S}_i) + \ln(\widehat{error}_{t,i})$$

$$\ln(Abs\_R_{t,i}) - \ln(\hat{S}_i) = \ln(\widehat{error}_{t,i})$$

$$\frac{Abs\_R_{t,i-1}}{\hat{S}_i} = \widehat{error}_{t,i}$$

$$\frac{Abs\_R_{t,i-1}}{\hat{S}_i} = \widetilde{Abs\_R}_{t,i}$$

where, finally, $\widetilde{Abs\_R}_{t,i}$ represents the absolute returns time series without periodic component – we simply call it $Abs\_R_{t,i}$ without any marker hereafter. In Figure 20 we plot the estimated daily periodic component – in which it is evident the U-shaped pattern - and in Figure 21 the resulting time series without periodicity.

Figure 22 presents the sample autocorrelation function for returns. We immediately see that the autocorrelation function for returns is different from that of trading volume series. This is explained by the fact that returns are not serially autocorrelated: in fact, autocorrelations are not significant and the process does not show any trend. Returns are thus a so-called white noise process which cannot be predicted since its oscillations are random. As regards absolute returns, however, the situation is not the same. Figure 23, in fact, presents the same evolution of autocorrelation function for trading volumes. Absolute returns, as already said, present an intra-daily U-shaped pattern which can be retrieved by this graph also. Moreover, autocorrelations go very slowly to zero meaning that the process is serially autocorrelated and strong persistent over time.

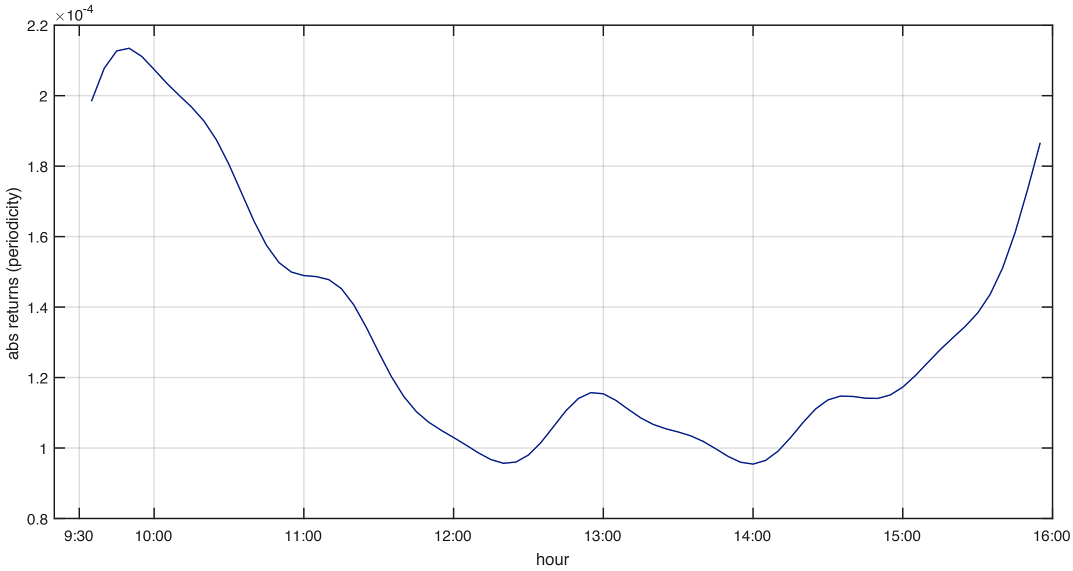


Figure 20. AAPL daily absolute returns periodicity.

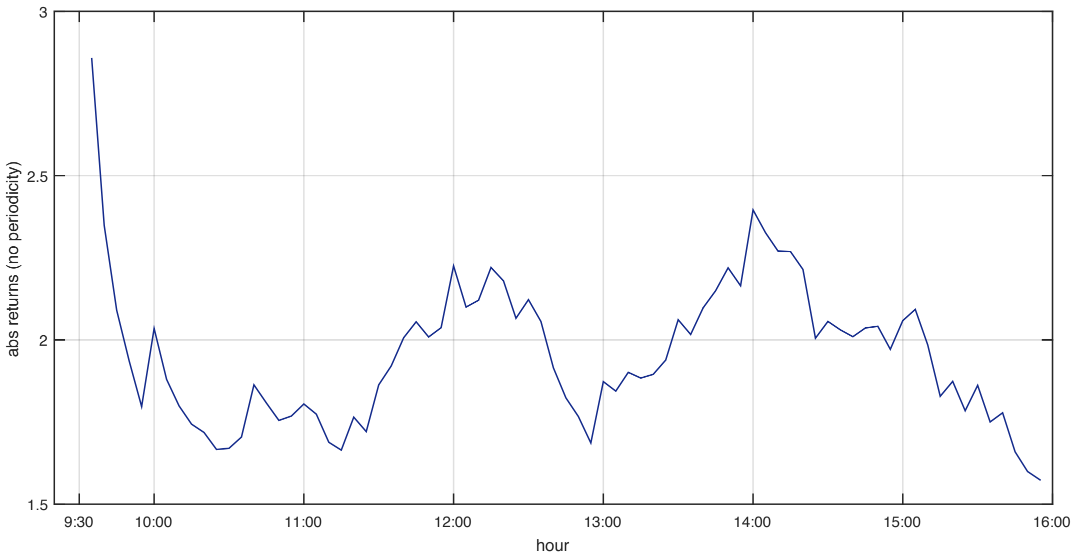


Figure 21. AAPL daily absolute returns without periodicity.

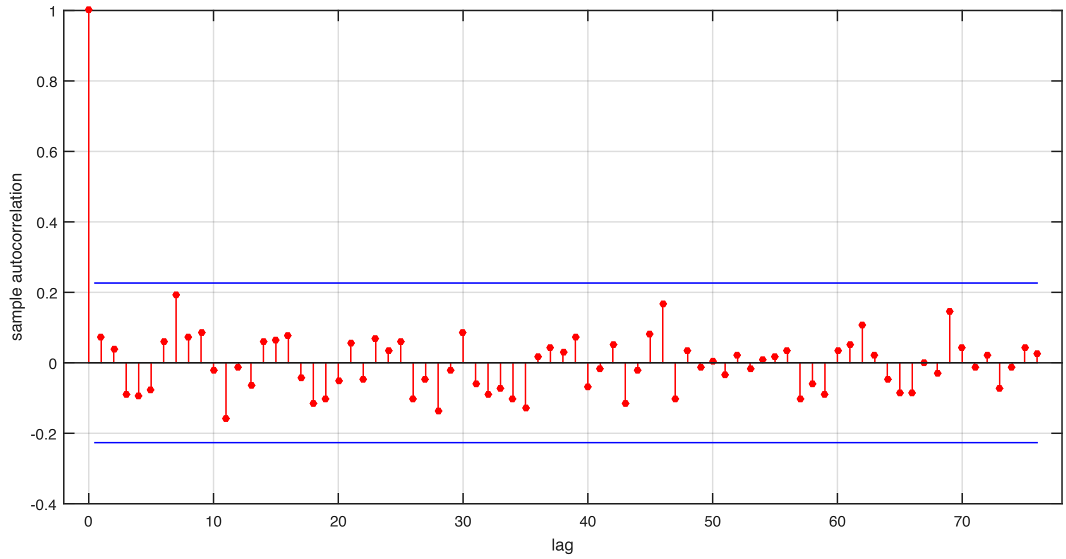


Figure 22. AAPL average intra-daily returns autocorrelation function.

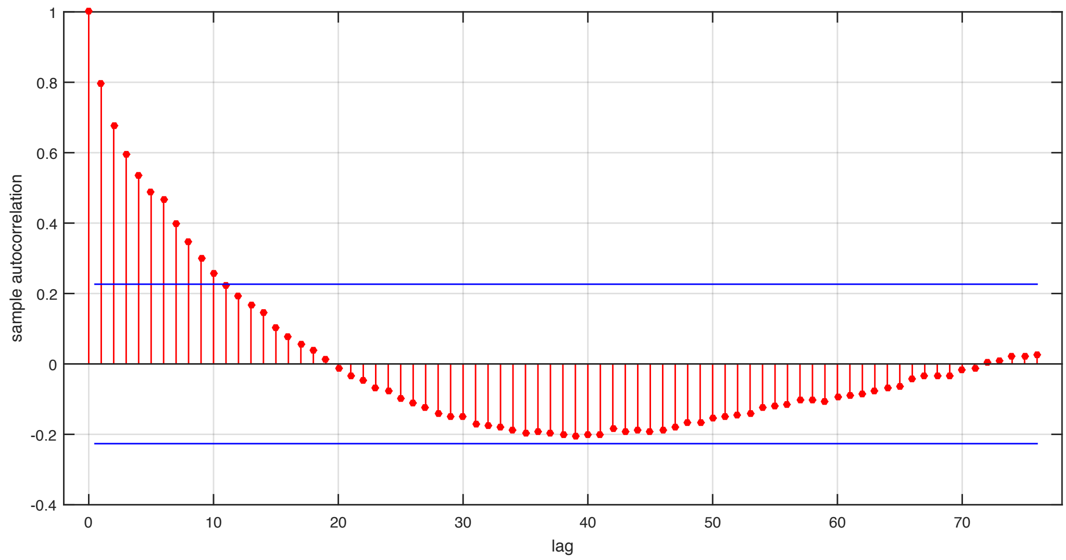


Figure 23. AAPL average intra-daily absolute returns autocorrelation function.

3.1.3 News and announcements

As already mentioned in the introduction of this section, data including news and announcements about Apple Inc. has been retrieved from FactSet StreetAccount database. Raw data appeared in TXT files composed by date, time, headline and text. In order to be comparable with quantitative data, it has been converted into numeric variables which indicate whether or not there are news in each intra-daily interval for every business day together with the corresponding sentiment. Moreover, news and announcements used in our analysis comprehend only intra-daily observations, that is no over-night or pre-trading and after-trading information has been included in the dataset.

The presence of any news or announcement is indicated by a dummy variable which takes the value +1 if there is any available information, otherwise 0. Sentiment information has been retrieved through text analysis. This technique has allowed to distinguish whether the content of a document is good, bad or neutral with respect to the issue it talks about. The sentiment extraction performed in our dataset is based on Loughran and McDonald (2011) [9]. The procedure consists in counting the words belonging to different lists, each of them associated to the category ‘positive’ or ‘negative’. Furthermore, this approach has been robustly modified in order to adjust the sentiment extraction to the way the news is presented and to the possibility of having a negation into the sentence (which could totally change the sentiment). At the end of the analysis, positive words are given a value of 1, negative words -1 and the value is inverted in case of negation. For each news, the values of all words with an associated sentiment are summed to get the sentiment sum:

$$Sent\_Sum = \sum_{i=1}^{n} s_i$$

where s_i is the sentiment of the word indexed by i, and N is the number of words with a sentiment in a text. This quantity has then been divided by the number of words with a sentiment, obtaining a quantity called relative sentiment:

$$Rel\_Sent = \frac{Sent\_Sum}{N}$$

which is comprised between -1 and 1 by construction. If Rel_Sent is bigger or smaller than 0.05 we associate, respectively, a positive (1) or a negative sentiment (-1) to the news, otherwise a neutral sentiment (0) is given:

[9] We thank the PhD candidate Poli F. for the data.

$$Text\_Sent = \begin{cases} -1 & if\ Rel\_Sent < -0.05 \\ 0 & if\ -0.05 < Rel\_Sent < 0.05 \\ +1 & if\ Rel\_Sent > 0.05\,. \end{cases}$$

Turning to the last available type of information in our dataset, when applicable, a numeric variable has been associated with the amount and sentiment of news and announcements according to the topic to which they refer to. FactSet StreetAccount database allows to classify them by eleven topics, but our dataset includes only six of them. Beyond the category called ‘all’ which includes all data with no filter, data has then been divided into: ‘earnings related’, ‘litigation’ (court disputes), ‘mergers and acquisitions’ (M&As), ‘newspapers’, ‘regulatory’, ‘upgrades and downgrades’. In further analysis, this categorization would be useful to identify the most relevant news and announcements to be included as controls in the estimable model. Table 1 presents the total amount of news and announcements for each category together with the corresponding percentage with respect to the total. Graphically, Figure 24 represents the same data in a pie chart. In Table 2, instead, we report one randomly picked news or announcement for each of the six topics as examples of dataset content.

	Amount	%
Earnings related	46	4,42%
Litigation	60	5,76%
M&As	18	1,73%
Newspapers	153	14,7%
Regulatory	11	1,06%
Upgrades & Downgrades	215	20,65%
Not classified	538	51,68%
Total	1041	100%

Table 1. News and announcements, total amount and percentages.

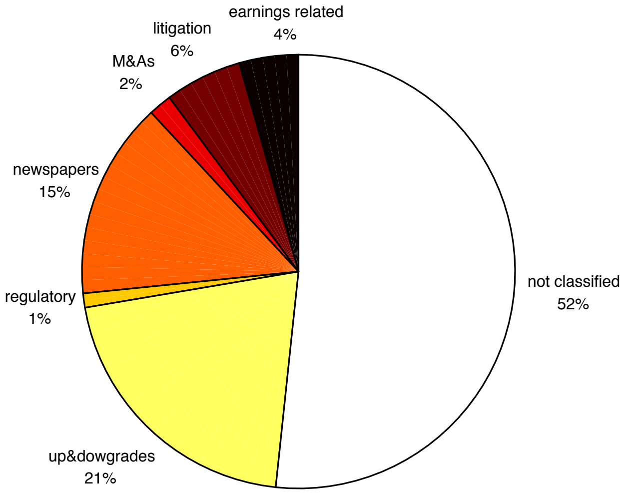


Figure 24. Pie chart of percentages of news and announcements for each topic.

Earnings related	*22-Jan-2013 18:44 (UTC+1)* AAPL StreetAccount Earnings Preview - Apple fiscal Q1 (Dec) - Overview: Apple is scheduled to report fiscal Q1 (Dec) results on 23-Jan, after the close. The company guided EPS to $11.75 on revenue of $52B. It also said that it expects a gross margin of 36%. The FactSet consensus for EPS and revenue is $13.45 and $54.92B, respectively. The StreetAccount consensus for gross margin is 38.4%. There was little discussion in the December quarter previews about the directional risks to consensus EPS and revenue estimates. However, this was not too surprising given the outsized focus on guidance and stock performance. iPhone commentary was mostly upbeat, with analysts highlighting supply chain improvements, the strength in smartphone shipments at Verizon (which reported on Tuesday that it activated 6.2M iPhones in Q4) and AT&T, holiday demand, expanded distribution and price cuts on older models. The StreetAccount consensus for iPhone shipments is 48.3M units. Gross margin commentary also skewed to the positive side, with a number of previews suggesting that recent reports about production cuts fit with thoughts of improving manufacturing yields. While largely overshadowed by all of the attention on the iPhone, there were some mixed thoughts surrounding the iPad. Analysts highlighted the continued strength of the company's tablet platform during the holiday season, though there were also some concerns that supply constraints may have hampered the sell in of the new iPad mini. There was relatively little discussion about Macs. While a few analysts noted the continued cannibalization effects from tablets and smartphones, they also pointed out that Macs have fared much better than the broader PC market. (...)
Litigation	*15-Jan-2014 18:02 (UTC+1)* AAPL Apple decided to provide full consumer refunds of at least $32.5M to settle FTC complaint. It charged for Kids' In-App purchases without parental consent. - Apple has agreed to provide full refunds to consumers, paying a minimum of $32.5M, to settle a FTC complaint that the company billed consumers for millions of dollars of charges incurred by children in kids' mobile apps without their parents' consent. Under the terms of the settlement with the FTC, Apple also will be required to change its billing practices to ensure that it has obtained express, informed consent from consumers before charging them for items sold in mobile apps.
M&As	*17-Aug-2012 16:50 (UTC+1)* AAPL FTC grants antitrust clearance for Apple's purchase of AuthenTec (AUTC) - Transaction was announced on 27-Jul.
Newspapers	*09-Oct-2014 21:20 (UTC+1)* AAPL WSJ's Heard on the Street column says Apple should invest in innovation, not buybacks - The column says that Apple should seek long-term value by directing its cash towards product design, rather than betting on financial engineering.
Regulatory	*02-Aug-2013 20:36 (UTC+1)* AAPL Apple files motion to object the DOJ e-books proposal - Filing of motion by AAPL to object to the proposal (see attached comment) is likely expected.
Upgrades & Downgrades	*05-Dec-2012 19:53 (UTC+1)* AAPL Apple defended at Piper Jaffray - Weakness is a buying opportunity. Reiterate overweight and $900 target. Analyst is Gene Munster.

Table 2. Some examples of AAPL news and announcements.

We observe that news and announcements that have not been classified in any topic count for the majority of the total amount. Then, news about company upgrades and downgrades are the most numerous together with that of the category ‘newspapers’. ‘Litigation’ and ‘earnings related’ – with respective percentages of 6% and 4% - and finally ‘M&As’ and ‘regulatory’ take a secondary and negligible role in the total amount of news and announcements.
As last preliminary operation, time aggregation has been performed to all data. The amount of news and announcements in a given five-minute interval has been computed according to:

$$N_{t,i} = \sum_{k}^{k+4} N_{t,n}$$

where $N_{t,n}$ is the number of news in the one-minute interval $n = 1, \dots ,390$, and $k = 1, 6, 11, \dots , 386$ is the index for building the series, and, eventually, the corresponding five-minute intra-daily number of interval is indicated by $i = 1, \dots ,78$ for each of the business days observed $t = 1, \dots ,2516$. Thus, the amount of news and announcements in five-minute intra-daily intervals in each day is the result of their sum in each of the five-minute interval studied. The same technique has been applied for the corresponding sentiment. The overall sentiment for each five-minute interval is therefore defined as:

$$Sent_{t,i} = \sum_{k}^{k+4} Sent_{t,n}$$

where $Sent_{t,n}$ is the sentiment of the news or announcement in the one-minute interval $n = 1, 5, 10, \dots , 390$ and the corresponding five-minute intra-daily number of interval $i = 1, \dots ,78$ for each of the business days observed $t = 1, \dots ,2516$. In each five-minute intra-daily interval, the overall sentiment is computed as the sum of all sentiments in the interval studied. It is important to notice that there could be three possibilities for which the overall sentiment could be equal to zero: two opposite sentiments in identical amount are summed up (-1 and +1), every sentiment is neutral (0), or a mix of them.
We use now the so-obtained variables about the amount, the sentiment and the topic of news and announcements in our dataset to present some descriptive graphical representations. For this purpose, Figure 25 and Figure 26 consider the amount and the corresponding sentiment divided into the ten years included in the dataset, respectively by entire dataset and by topic.
Firstly, we observe that the quantity of news and announcements is bigger in the first years of observation and then reaches its lowest peak in 2009 before keeping a constant evolution in the last period. Below, the corresponding sentiment is negative for the majority of the years, apart from 2008. This could be interpreted as result of a large amount of negative news over the positive ones, or as result of an overall neutral sentiment which is moved downwards only by

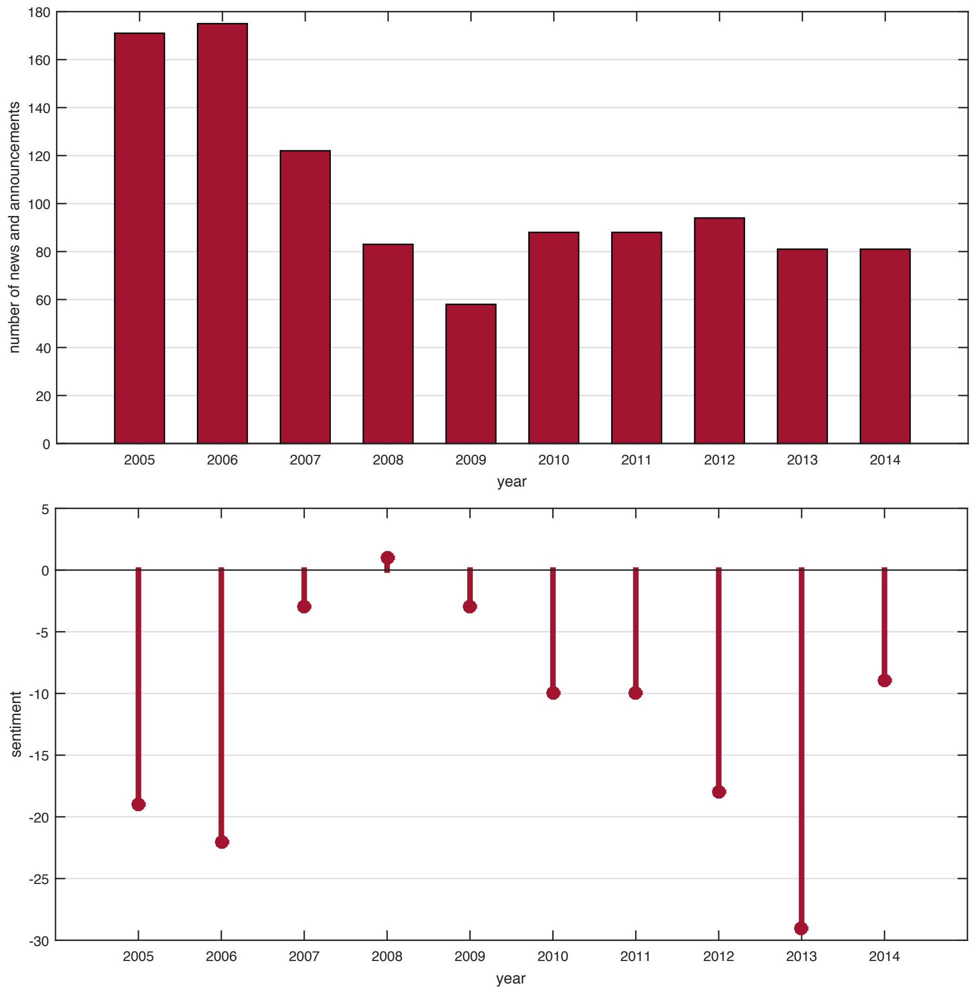


Figure 25. Amount (above) and overall sentiment (below) of news and announcements per year.

an overall negative tendency. As already seen, news and announcements that are not classified in any topic have a pronounced impact on the overall amount and sentiment with respect to the six categories considered, and this is also true for the year-by-year analysis. However, in the following we conduct a description of the data once ‘not classified’ news and announcements have been ruled out.

By analysing the data year by year, we can observe that in 2005 the most relevant topics in terms of quantity of information arrived in the market have been ‘newspapers’, ‘upgrades and downgrades’, ‘earnings related’ and ‘litigation’. News about mergers, acquisitions and regulatory subjects are not present in this period. The overall sentiment in 2005 is negative with a value of -19. This fact is supported by the sentiment of the above-mentioned topics: ‘newspapers’ and ‘earnings related’ support the negative sentiment, while topic ‘upgrades and downgrades’ counterbalances it with a positive sentiment which, however, is not so relevant. ‘Litigation’ sentiment effect is irrelevant since its amount is very small with respect to the other topics. As regards 2006, ‘newspapers’ together with ‘upgrades and downgrades’ take the first place as amount of news and announcements arrived in the market. Newspapers informa-

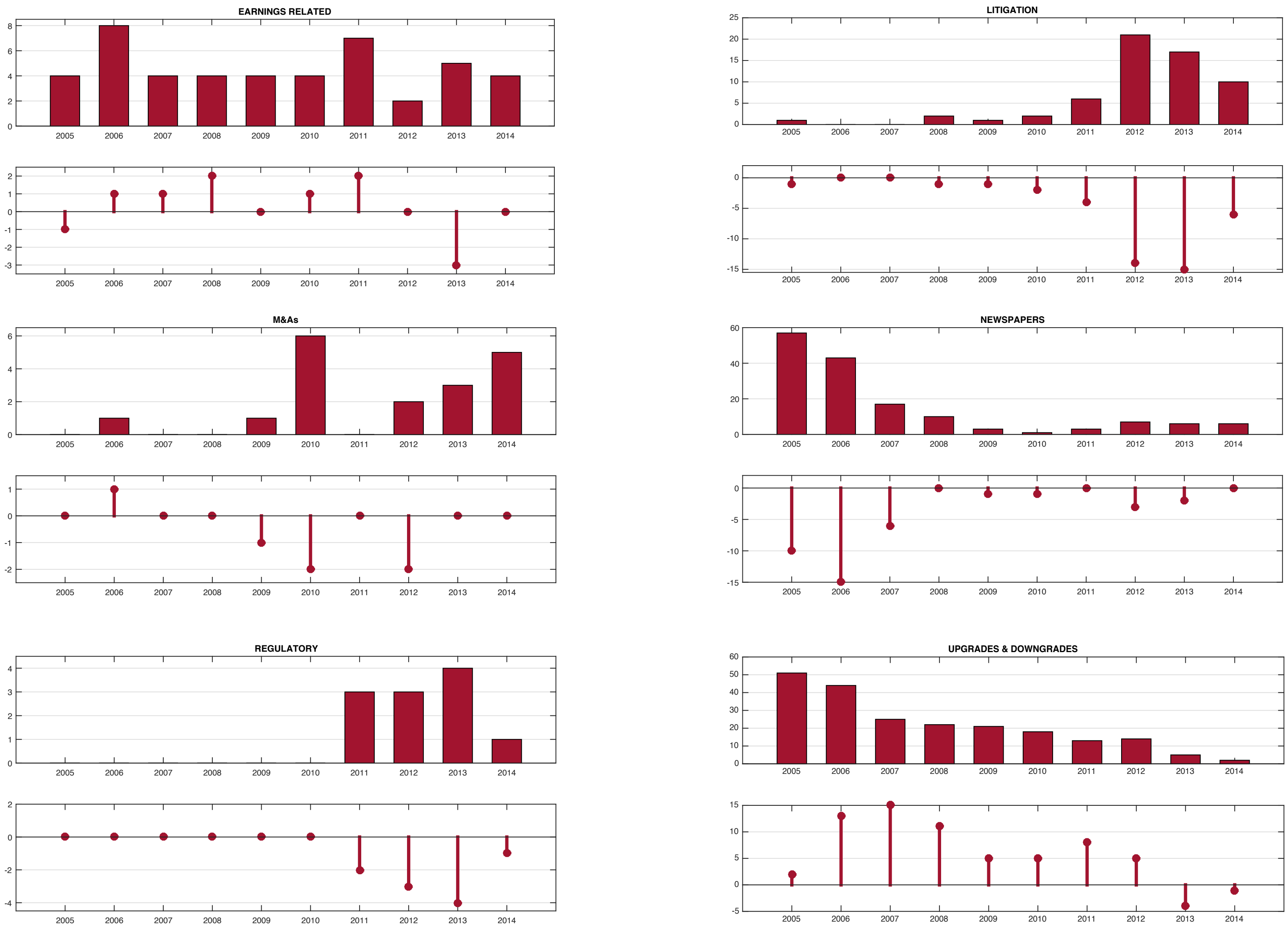


Figure 26. Number and sentiment of news and announcements per year, for each of the six topics.

tion influenced the most the overall negative sentiment, while upgrade news are assessed as positive news. Moreover, ‘earnings related’ and ‘M&As’ news and announcements are relevant in this period but they do not help to explain the negative sentiment in 2006. In this case, the ‘not classified’ topic probably played an important role in the downward tendency of total sentiment which registered a value of -22. In 2007 – as in the following two years – the overall sentiment remains close to the neutrality line. In fact, in 2007 it is slightly negative and it can be explained by a large amount of positive news from ‘upgrades and downgrades’ topic together with ‘earnings related’ news. On the other hand, ‘newspapers’ count for the negative tendency that might be accented by the negative sentiment of the ‘not classified’ news and announcements. Year 2008 shows the only positive overall sentiment over the years of observations. In this period, upgrade information and positive sentiment about earnings marked this slightly positive sentiment of +1. In fact, they counted for the largest amount of news and announcements arrived in that year helped with the fact that ‘newspapers’ overall sentiment has remained neutral. ‘Litigation’ topics has a marginal negative sentiment which, instead, has been counted the most by ‘not classified’ news. In 2009, regulatory and litigation subject have not influenced the most. In contrast, ‘upgrades and downgrades’ news together with those in ‘newspapers’, ‘earnings related’ and ‘M&As’ topics have had a relevant role in terms of quantity of information. As regards the corresponding sentiment, upgrades have had a positive impact, while all the others negatively influenced the overall sentiment apart from ‘earnings related’ and ‘M&As’ whose sentiment remained neutral. For what concerns 2010, we observe that the overall sentiment began to be more negative again from 2006 and has been influenced the most by news coming from litigation subjects, M&As news and newspapers. ‘Earnings related’ and ‘upgrades and downgrades’ topics brought positive values in the overall sentiment, but it has remained negative even for the ‘not classified’ news and announcements. The same negative sentiment – with a total value of -10 – has remained also in 2011 in which ‘litigation’ and ‘regulatory’ topics played an important role together with not classified news and announcements. Positivity brought by ‘upgrades and downgrades’ and ‘earnings related’ topics has not swiped away the overall negative sentiment. In 2012, all topics counted for an amount which goes from 21 of ‘litigation’ to 2 of ‘M&As’ news and announcements. The overall sentiment is negative as in 2005 and it has been influenced the most by litigation, mergers and acquisitions, newspapers and regulatory topics. The effect of ‘earnings related’ is neutral. For 2013 the negativity peak in overall sentiment is registered. It took a value of -20 and this has been caused by negative sentiment about earnings, newspapers titles, litigation and regulatory subjects and – for the first time – also by downgrades information. Lastly, year 2014 saw an increase of general sentiment thanks to few negative weight of 'litigation', 'newspapers', 'M&As' and 'earnings related'.

By concluding this year-by-year analysis, we have seen that the most relevant topics – keeping out the 'not classified' news and announcements – for quantity and sentiment have been: 'newspapers' in the first years of observation, 'earnings related' and 'upgrades and downgrades' for the positive peak in 2008, and 'litigation' and 'regulatory' in determining the overall negative sentiment in the last years of observation.

As regards intra-daily analysis, Figure 27 shows the total amount of news and announcements and the relative sentiment in the dataset for each of the 78 intra-daily intervals.

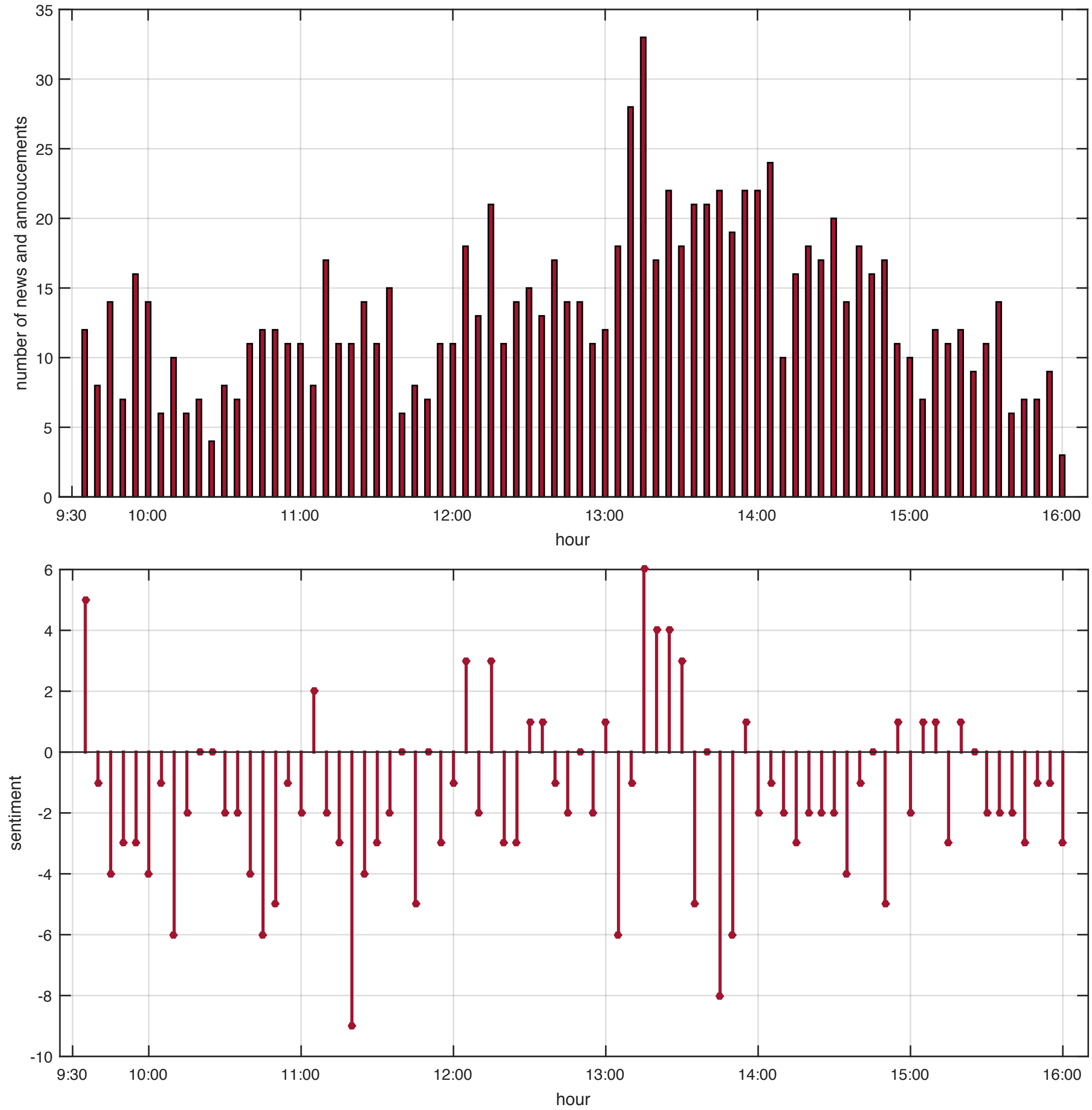


Figure 27. Number of news and announcements per year (above) and the corresponding sentiment (below).

We can observe that the peak is reached around lunch time, while the quantity of news and announcements over the other intra-daily intervals seems to not present any evident pattern. The overall daily sentiment is negative, but nothing can be said about its source. For this reason, Figure 28 represents the intra-daily quantity and corresponding sentiment for each of the six to-

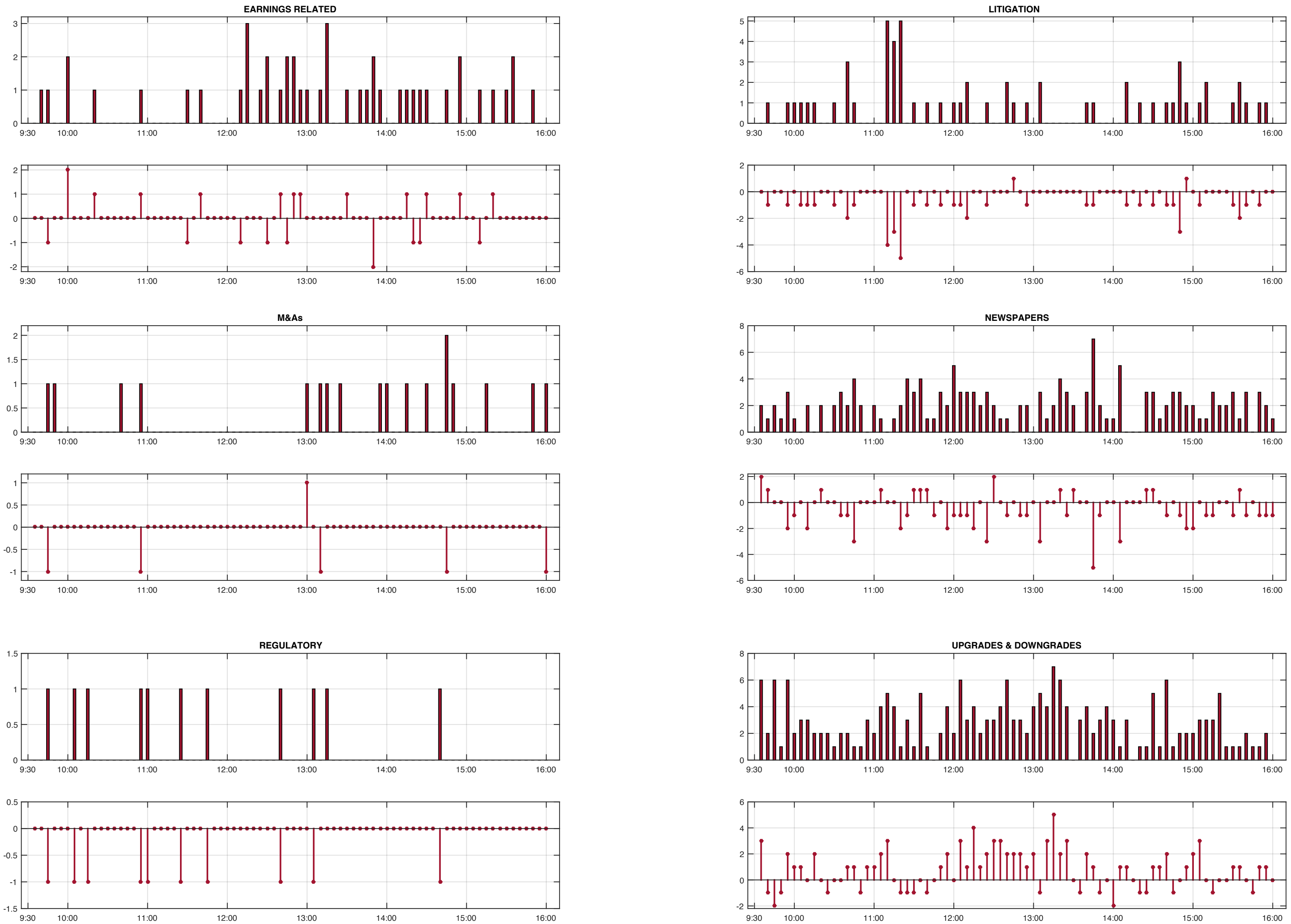


Figure 28. Amount and overall sentiment of news and announcements per year, for each of the six topics.

pics in the dataset. As before, as the ‘not classified’ news and announcements count for the majority, the overall sentiment is influenced by it. However, once its effects have been ruled out, we observe that the most relevant topics are ‘newspapers’, ‘upgrades and downgrades’ and ‘litigation’ topics. They present a relevant and well-distributed amount of news and announcements throughout the five-minute intra-daily intervals observed. In particular, newspapers topics show a constant presence through the trading day. ‘Upgrades and downgrades’ count the most in the opening time, while decreasing in the closing period. By he way, these two categories present opposite overall sentiments. Newspapers subjects are considered as negative or neutral, while ‘upgrades and downgrades’ topic mostly contains positive news and announcements. This phenomenon seems to give an overall neutral sentiment throughout the trading day, but all other categories count for sentiment negativity. ‘Earnings related’ news and announcements brought mostly negative sentiment as ‘M&As’, ‘regulatory’ and ‘litigation’ topics. Their quantity is not so numerous but they count for negativity – as the ‘not classified’ news – throughout the trading day sentiment evolution.

In view of all this, in our empirical analysis we will take into account only some categories for a deepened analysis. We can discard ‘M&As’, ‘regulatory’, ‘litigation’ and ‘earnings related’ topics as their amount is not so large with respect to the other categories. Therefore, it turns out that, in our empirical analysis, we will take into account ‘newspapers’ and ‘upgrades and downgrades’ news and announcements.

3.2 Models Set Up

In the empirical analysis, we model trading volumes of Apple Inc. with the Multiplicative Error Model approach introduced by Engle (2002), as applied by Manganelli (2005). This choice is motivated by the properties embodied in trading volume process. In fact – in the previous Section – it has been shown that volumes they take only positive values, present strong persistence over numerous intra-daily intervals and exhibit non-constant volatility.
Therefore, we firstly define the Multiplicative Error Model for trading volumes with baseline specification of the conditional mean term. Further, we define other specifications – by enriching the baseline one with weakly exogenous variables – in order to understand the effects of company news and announcements with the corresponding sentiment, of price returns and of volatility jumps on trading volumes of Apple Inc. The motivation of this choice is due to the fact that trading volumes are strongly dependent on the arrival of news and announcements. When new information appears in the market, if the market is efficient (Fama, 1991), informed agents trade on the basis on it to bring prices reflect all available information in the market. This phenomenon brings to an increase in trading volumes, either traders take long or short positions. However, we can retrieve information about the direction of the trade from the sign of returns. If news is positive, price returns are likely to be positive, and, on the opposite, if news sentiment is negative, traders sell and returns are likely to be negative. Moreover, price volatility jumps are registered when there are sudden and large – upward or downward – changes in price returns. Once again, since returns are determined by traders' activity in the market, volatility jumps are likely to be caused by sudden events occurred in the market, such as the arrival of company news or announcements. As a consequence, traders react to them and this fact translates into a sudden change in price volatility, inducing clustering in trading volumes.

First of all, we illustrate the econometric model for trading volumes of our dataset. We define them as $V_{t,i}$ where $t = 1, \dots, 2516$ is the business day observed and $i = 1, \dots, 77$ indicates the five-minute intra-daily intervals in each day. As already described, the first intra-daily interval in each of the trading days observed has been excluded from analysis in order to avoid any market micro-structure effect bringing anomalies in the series. Moreover, intra-daily periodicity has been removed in order to eliminate the intra-daily U-shaped path of trading volumes.

Since trading volumes process $V_{t,i}$ is defined over $[0, +\infty)$, they are defined as a Multiplicative Error Model as the product of a deterministic scale factor – i.e. $\mu_{t,i}$ – and an i.i.d. error term with unit mean – i.e. $\varepsilon_{t,i}$:

$$V_{t,i} = \mu_{t,i}\varepsilon_{t,i} \, .$$

The conditional mean term is necessarily a positive quantity that evolves deterministically according to a parameter vector θ:

$$\mu_{t,i-1} = \mu(\theta, \mathcal{F}_{t,i-1})$$

where $\mathcal{F}_{t,i-1}$ indicates the set of available information in the intra-daily interval $i-1$ in each business day t.

On the other hand, the innovation term is a random variable whose support has to be strictly positive. For this reason, $\varepsilon_{t,i}$ is assumed to be Gamma distributed as in Engle and Gallo (2006). More specifically, it is distributed as a $\Gamma\left(\frac{1}{v}, v\right)$ in shape-scale representation. Equivalently, $\varepsilon_{t,i}$ is distributed as a $\Gamma(v, 1)$ in shape-mean Gamma distribution representation, where v is the shape parameter while the second one is the mean that equals 1 because of the unit mean property [10].

Therefore, the density of the Gamma distribution of $\varepsilon_{t,i}$ in shape-mean representation is:

$$\varepsilon_{t,i}|\mathcal{F}_{t,i-1} \sim \frac{1}{\Gamma(v)} v^{v} \varepsilon^{v-1} \exp(-v\varepsilon) \, .$$

The expected value of the error term can be computed as the product between the shape and the scale parameter:

$$\mathbb{E}(\varepsilon_{t,i}|\mathcal{F}_{t,i-1}) = v \cdot \frac{1}{v} = 1$$

which equals to one, as a consequence of the unit mean property imposed above. Instead, the variance of the innovation term is calculated as the product of the shape parameter and the squared scale parameter. The resulting variance is then:

$$Var(\varepsilon_{t,i}|\mathcal{F}_{t,i-1}) = v \cdot \frac{1}{v^2} = \frac{1}{v} .$$

Once the distribution of the error term is defined, it is possible to retrieve the probability distribution of the random variable describing trading volumes. As the process $V_{t,i}$ is the result

[10] For the purpose of this work, as in many other cases, the mean is required to be equal to one. Therefore, the relationship between the shape parameter v and the scale parameter ϑ in a shape-scale representation $\Gamma(\vartheta, v)$ is given by:

$$\mathbb{E}(\varepsilon_{t,i}) = v \cdot \vartheta = 1 \iff \vartheta = \frac{1}{v} .$$

It means to write the Gamma distribution as $\Gamma\left(\frac{1}{v}, v\right)$, which, in turn, is equivalent to the shape-mean representation $\Gamma(v, 1)$.

of the multiplication between the innovation term – whose probability function is known – and a deterministic term $\mu_{t,i}$, it simply follows a Gamma distribution with same shape parameter of the error term and mean term equal to $\mu_{t,i}$:

$$V_{t,i}|\mathcal{F}_{t,i-1} \sim \Gamma(v, \mu_{t,i})$$

in shape-mean representation. It follows that the expected value of trading volume process is equal to the conditional mean term, i.e. $\mathbb{E}(V_{t,i}|\mathcal{F}_{t,i-1}) = \mu_{t,i}$, and its variance corresponds to:

$$Var(V_{t,i}|\mathcal{F}_{t,i-1}) = \frac{\mu_{t,i}}{v}.$$

The baseline specification of the Multiplicative Error Model, introduced by Engle (2002) and then reconsidered by Engle and Gallo (2006), is defined as:

(1) $\mu_{t,i} = \alpha + \beta\mu_{t,i-1} + \delta V_{t,i-1}$

where the indexes t and i respectively indicate the business days and the intra-daily intervals observed. This simple specification replicates a GARCH-model evolution, where $\mu_{t,i-1}$ is the GARCH term and $V_{t,i-1}$ – representing trading volumes – is the ARCH term. This is the conditional mean equation that we define for specification (1). The coefficients α and δ are given positivity constraints ($\alpha > 0, \delta > 0$), and β is also imposed to be less than one ($0 < \beta < 1$) in order to avoid the system to become explosive. As regards the meaning of these parameters, the constant term α can be interpreted as the expected value of $\mu_{t,i}$ when all other variables are set to zero. Then, the coefficient β indicates how much the conditional mean of trading volumes depends on its previous values. If the coefficient is high, it would mean that $\mu_{t,i}$ is strongly dependent on what happens at time $i - 1$, and vice versa. Lastly, the coefficient δ measures the effect of the moving average term $V_{t,i-1}$ on the conditional mean of trading volumes. If it is high, it would mean that the amount of trading volumes at time $i - 1$ greatly affects its mean in the successive intra-daily interval, and vice versa.

Now we expose what are the extensions to this baseline specification of our Multiplicative Error Model for trading volumes. As said before, the motivation of this choice lies in willing to understand whether news and announcements, returns and price volatility jumps have a causal effect on trading volumes.

Firstly, we list questions we would like to answer with estimation of the Multiplicative Error Model for trading volumes with extended specifications for the conditional mean term:

a) Do news and announcements have a causal effect on trading volumes? If yes, does it change according to the sentiment of news and announcements?

b) Do positive and negative price returns have a causal effect on trading volumes? If yes, in both cases, what is the effect of news and announcements according to their sentiment?
c) Do price volatility jumps have a causal effect on trading volumes? If yes, does it change according to the presence or the lack of news and announcements?
d) Given the presence of volatility jumps, what is the effect of news and announcements on trading volumes according to their sentiment?
e) Do positive and negative price returns have a casual effect on trading volumes, given the presence of volatility jumps? If yes, what is the effect of news and announcements – with both positive and negative returns – according to their sentiment, given the presence of volatility jumps?

In order to find the answers to the above-mentioned questions, we now present the extensions of the baseline specification of the conditional mean of our Multiplicative Error Model. To clearly justify the motivation of the choice of particular covariates, we group specifications by relevance with a specific question. Positivity constraints are imposed to all additional variables as established for α and δ in the baseline specification. Moreover, all covariates belong to the information set available at time $i-1$ for each of the days t, that is $\mathcal{F}_{t,i-1}$.

In order to answer to the question a) – which regards the effect of news and announcements on trading volumes – we add a variable expressing the amount of news about Apple Inc. into the baseline specification of our Multiplicative Error Model. We define it as $N_{t,i-1}$, where t indicates the business day and i the intra-daily interval observed. The first extended specification of the conditional mean term of our Multiplicative Error Model is then defined as:

$$(2) \qquad \mu_{t,i} = \alpha + \beta\mu_{t,i-1} + \delta V_{t,i-1} + \gamma_1 N_{t,i-1} \,.$$

Through it, we are able to understand whether company news and announcements have an influence on trading volumes. In fact, the coefficient γ_1 can be interpreted as the effect of a one-unit increase in the amount of news and announcements in the intra-daily interval $i-1$ on the conditional mean of trading volumes at time i, ceteris paribus. If the estimated parameter γ_1 would be high, trading volumes at time i are highly influenced by company news or announcements arrival in the market in the previous intra-daily interval $i-1$.

Supposing that the amount of news and announcements in a particular intra-daily interval affect trading volumes in the subsequent interval, the question becomes whether a particular sentiment – positive, neutral or negative – influences trading activity more than the others.

Specification (3) aims at solving this point by making the news variable enter the equation according to the three sentiment possibilities, thanks to indicator functions:

$$(3) \qquad \mu_{t,i} = \alpha + \beta\mu_{t,i-1} + \delta V_{t,i-1} + \gamma_2 N_{t,i-1} I_{\{Sent_{t,i-1}>0\}} + \gamma_3 N_{t,i-1} I_{\{Sent_{t,i-1}=0\}} + \\ + \gamma_4 N_{t,i-1} I_{\{Sent_{t,i-1}<0\}} \,.$$

The amount of news and announcements arrived in the market in the interval $i-1$ of the business day t is represented by $N_{t,i-1}$, while the corresponding sentiment is indicated by $Sent_{t,i-1}$. Since we use aggregated sentiments for each five-minute intra-daily interval, we divide the cases as follows: $Sent_{t,i-1} > 0$ if sentiment is positive, $Sent_{t,i-1} = 0$ if sentiment is neutral, and $Sent_{t,i-1} < 0$ if sentiment is negative. The interpretation given to the three news coefficients is different according to the sentiment of news and announcements, so that we are able to understand whether trading volumes are affected more by positive, neutral or negative news. The coefficient γ_2 expresses the effect of a one-unit increase in the amount of positive news and announcements in the interval $i-1$ on the conditional mean of trading volumes at time i, keeping all other factors fixed. Similarly, the coefficient γ_3 indicates what happens to trading volumes if news have neutral sentiment. Lastly, the coefficient γ_4 represents the effect of negative news on the mean of trading volume. Guessing what coefficient would be the largest one is not easy. In fact, since investors are generally risk-averse, one can argue that negative news would have more impact on trading volumes with respect to the positive ones. It would mean that investors prefer to sell their securities rather than paying negative consequences with their own capital. On the other hand, when positive news come into the market, traders would prefer to make profits before the others, but only in case of risk inclination. After all, we should expect a higher coefficient γ_4 with respect to γ_2.
Additionally, since traders are more likely to buy securities if news are positive, and, instead, prefer to sell in case of negative news, these effects could contain the effect of trade direction on trading volumes.

To answer to the question b) which is related to the effect of positive or negative returns on trading volumes, together with the effect of news sentiment, we now list two additional specifications for our Multiplicative Error Model for trading volumes. In specification (4), we include a variable for price returns for both positive and negative cases through indicator functions. The conditional mean specification then is defined as:

$$(4) \; \mu_{t,i} = \alpha + \beta\mu_{t,i-1} + \delta V_{t,i-1} + \gamma_5 |R_{t,i-1}| I_{\{R_{t,i-1}>0\}} + \gamma_6 |R_{t,i-1}| I_{\{R_{t,i-1}<0\}}$$

where $R_{t,i-1}$ is the price return in the business day t in the intra-daily interval $i-1$.

Moreover, it is important to notice that price returns enter the equation in absolute terms in order to maintain positivity of the Multiplicative Error Model. The motivation of including the two parameters γ_5 and γ_6 according to the sign of returns in a particular intra-daily interval is the following. We would like to understand the effect of a one-unit increase in absolute returns in the two key cases of positivity and negativity of returns. The coefficient γ_5 represents the effect of a one-unit increase in absolute returns, if they are positive, on the mean of trading volumes in the subsequent intra-daily interval, ceteris paribus. On the other hand, the coefficient γ_6 indicates what happens to trading volumes if absolute returns increase by one, when price returns are negative, ceteris paribus. Hence, coefficients γ_5 and γ_6 indicate whether investors trade more if returns are positive or negative, which may have some link with the sign of sentiment variable. In fact, by the same token of what said in specification (3) about the link between news sentiment and trade direction, positive returns should indicate that traders are buying securities on the market. On the contrary, when returns are negative, traders should be selling securities on the market.

Specification (5) has the purpose of deepen this matter. The above-explained trade direction should be connected with news sentiment also, as said in specification (3). If positive news or announcements arrive in the market, then traders should buy securities, making returns be positive. On the other hand, when negative news arrives into the market, traders should sell securities making returns be negative from one intra-daily interval to another. By this reasoning, the conditional mean specification of our Multiplicative Error Model for trading volumes in specification (5) becomes:

$$
\begin{aligned}
(5)\ \mu_{t,i} = {} & \alpha + \beta\mu_{t,i-1} + \delta V_{t,i-1} + \gamma_7 N_{t,i-1} I_{\{R_{t,i-1}>0\}} I_{\{Sent_{t,i-1}>0\}} + \\
& + \gamma_8 N_{t,i-1} I_{\{R_{t,i-1}>0\}} I_{\{Sent_{t,i-1}=0\}} + \gamma_9 N_{t,i-1} I_{\{R_{t,i-1}>0\}} I_{\{Sent_{t,i-1}<0\}} + \\
& + \gamma_{10} N_{t,i-1} I_{\{R_{t,i-1}<0\}} I_{\{Sent_{t,i-1}>0\}} + \gamma_{11} N_{t,i-1} I_{\{R_{t,i-1}<0\}} I_{\{Sent_{t,i-1}=0\}} + \\
& + \gamma_{12} N_{t,i-1} I_{\{R_{t,i-1}<0\}} I_{\{Sent_{t,i-1}<0\}}
\end{aligned}
$$

where, as usual, $N_{t,i-1}$ indicates the amount of news and announcements arrived in the market in the business day t in the intra-daily interval $i-1$. The coefficient γ_7 indicates the effect of a one-unit increase in news variable at time $i-1$ on the mean of trading volumes in the subsequent interval, when both returns and news sentiment are positive, keeping fixed all other covariates. Similarly, coefficients γ_8 and γ_9 represent the effect of one more news at time $i-1$ in case of positive returns and, respectively, in the cases in which the sentiment is neutral and negative. By the same token, coefficients γ_{10}, γ_{11} and γ_{12} capture the effect of an increase in news variable when returns are negative and in the three different cases of sentiment sign, i.e. positive, neutral or negative. The cases in which we consider neutral sentiment are included as a residual condition. We should expect that positive returns are in line with positive news sentiment, and negative returns in line with negative sentiment. More specifically, we should expect a higher coefficient γ_7 with respect to γ_{10} for the case of positive price returns, and a higher γ_{12} with respect to γ_9 for the case of negative returns.

We now consider price volatility jumps as covariate to add in the conditional mean of the Multiplicative Error Model, to understand – firstly – if they have any effect on trading volumes, and – secondly – if there is a link between jumps and news arrival. This correspond to answering to question c).

We define $J_{t,i}$ as the dummy variable indicating the presence ($J_{t,i} = 1$) or the absence ($J_{t,i} = 0$) of volatility jumps in a particular intra-daily interval i of the observed business day t. The jump variable enters the conditional mean specification (6) of the Multiplicative Error Model as:

$$\text{(6)} \qquad \mu_{t,i} = \alpha + \beta\mu_{t,i-1} + \delta V_{t,i-1} + \psi_1 J_{t,i-1} \,.$$

The coefficient ψ_1 represents the effect of one more volatility jump in the intra-daily interval $i-1$ on the mean of trading volumes in the subsequent interval i. We expect this coefficient to be high, since the presence of volatility jumps signals for sudden and very large changes in price returns, which, in turn, reflect a significant trading activity in the market.

In specification (7), we go more in detail by investigating the effect of volatility jumps in the case of presence or lack of news or announcements in a particular intra-daily interval i. Hence, the conditional mean specification for our Multiplicative Error Model for trading volumes becomes:

$$\text{(7)} \qquad \mu_{t,i} = \alpha + \beta\mu_{t,i-1} + \delta V_{t,i-1} + \psi_2 J_{t,i-1} I_{\{N_{t,i-1}>0\}} + \psi_3 J_{t,i-1} I_{\{N_{t,i-1}=0\}}$$

in which indicator functions are used to divide the two cases. The parameter ψ_2 then represents the ceteris paribus effect of a one-unit increase in jump variable in the interval $i-1$ on the mean of trading volumes at time i, if there the amount of news in such interval is positive. On the other hand, the coefficient ψ_3 measures the same effect but in case of no news or announcements coming in that particular interval, ceteris paribus. Since volatility jumps signal for large changes in price returns and mean for some important event happening in the market (like news and announcements should be), we should expect a higher coefficient ψ_2 with respect to ψ_3.

Supposing that news and announcements are related to volatility jumps, our purpose is to answer to the question d) which refer to the more detailed effect of news and announcements on trading volumes in presence of volatility jumps. For this reason, specification (8) includes news variable for the cases in which sentiment is positive or negative, both with the presence of price volatility jumps. It is then defined as:

$$(8) \qquad \mu_{t,i} = \alpha + \beta\mu_{t,i-1} + \delta V_{t,i-1} + \psi_4\, N_{t,i-1} I_{\{J_{t,i-1}>0\}} I_{\{Sent_{t,i-1}>0\}} + \\ + \psi_5\, N_{t,i-1} I_{\{J_{t,i-1}>0\}} I_{\{Sent_{t,i-1}<0\}}\,.$$

The coefficient ψ_4 then indicated the ceteris effect of a one-unit increase in news or announcements in the intra-daily interval $i-1$ on the mean of trading volumes in the subsequent interval i, given that the news sentiment is positive and given the presence of volatility jump. On the other hand, the coefficient ψ_5 represents the effect of one more news on the mean of trading volumes given volatility jumps, but in case of negative news sentiment, keeping all other factors fixed.

Lastly, in order to answer to question e), specifications (9) and (10) are build for this purpose. The question regards the effect of positive or negative price returns on trading volumes, once the condition about the presence of volatility jumps is verified. Thus, in specification (9), two coefficients enter the baseline specification of the conditional mean for the two different cases of positivity and negativity or returns:

$$(9) \qquad \mu_{t,i} = \alpha + \beta\mu_{t,i-1} + \delta V_{t,i-1} + \psi_6\, |R_{t,i-1}| I_{\{J_{t,i-1}>0\}} I_{\{R_{t,i-1}>0\}} + \\ + \psi_7 |R_{t,i-1}| I_{\{J_{t,i-1}>0\}} I_{\{R_{t,i-1}<0\}}\,.$$

Therefore, the coefficient ψ_6 indicates the ceteris paribus effect of a one-unit increase in absolute returns when returns are positive and there is the presence of volatility jumps. On the other hand, the coefficient ψ_7 expresses the effect of a one-unit increase in absolute returns in case of negative price returns, given the presence of jumps, keeping all other factors fixed. The combination of absolute returns and volatility jumps results in the same interpretation of specification (4) but considering important variations in returns. In fact, basically, price volatility jumps are large discrepancies in returns. It means that, in this case, we consider a subset of observations of those contemplated in specification (4), in which absolute returns are generally considered in the two cases of positivity and negativity of returns.

With specification (10), instead, we go more in detail by examining the effect of news and announcements – with both positive and negative returns – according to their sentiment, given the sign of price returns and the presence of volatility jumps. The conditional mean specification of our Multiplicative Error Model for trading volumes is then defined as:

$$(10) \qquad \mu_{t,i} = \alpha + \beta\mu_{t,i-1} + \delta V_{t,i-1} + \psi_8\, N_{t,i-1} I_{\{J_{t,i-1}>0\}} I_{\{R_{t,i-1}>0\}} I_{\{Sent_{t,i-1}>0\}} + \\ + \psi_9\, N_{t,i-1} I_{\{J_{t,i-1}>0\}} I_{\{R_{t,i-1}>0\}} I_{\{Sent_{t,i-1}<0\}} + \\ + \psi_{10}\, N_{t,i-1} I_{\{J_{t,i-1}>0\}} I_{\{R_{t,i-1}<0\}} I_{\{Sent_{t,i-1}>0\}} + \\ + \psi_{11}\, N_{t,i-1} I_{\{J_{t,i-1}>0\}} I_{\{R_{t,i-1}<0\}} I_{\{Sent_{t,i-1}<0\}}\,.$$

The coefficient ψ_8 represents the ceteris paribus effect of one more news or announcement arriving in the intra-daily interval $i-1$ on the mean of trading volumes in the subsequent interval i, given the presence of volatility jumps, positive returns and positive news sentiment in the same interval. Instead, the coefficient ψ_9 captures the effect of one more news under the same conditions, except for the sign of news sentiment which, in this case, it is considered as negative. By the same token, the coefficients ψ_{10} and ψ_{11} express the ceteris paribus effect of a one-unit increase in news and announcements variable, when there is the presence of volatility jumps, when returns are negative and in the two cases of positivity or negativity of news sentiment. We should expect the coefficients ψ_9 and ψ_{10} to be not significant, since they express the opposite sign of returns and news sentiment. In fact, it should be that traders make price returns be positive when positive news arrives into the market, and negative returns should correspond to negative sentiment about company news or announcements.

For what concerns specifications from (11) to (22) – listed in Table 4 – we replicate the ten above-presented specifications of the conditional mean term for two of the topics of news and announcements about Apple Inc. obtained from FactSet StreetAccount database. Their selection – as explained in the previous Section – is motivated by their relevance in terms of quantity of available information. Thus, we choose the topics about 'newspapers' and 'upgrades and downgrades'. The motivation of including only some of Apple Inc. news and announcements derives from the willing to understand whether they have some specific effect in determining trading volumes. Interpretation of conditional mean estimated parameters follows the same approach described above. Moreover, the amount of news as well the corresponding sentiment are separated into 'newspapers' and 'upgrades and downgrades' categories, while the sign of price returns and the presence of volatility jumps are considered in the general framework, since it is not possible (and useful) divide them by topics.

Table 3. Conditional mean specifications for the Multiplicative Error Model.

(1) $\mu_{t,i} = \alpha + \beta\mu_{t,i-1} + \delta V_{t,i-1}$

(2) $\mu_{t,i} = \alpha + \beta\mu_{t,i-1} + \delta V_{t,i-1} + \gamma_1 N_{t,i-1}$

(3) $\mu_{t,i} = \alpha + \beta\mu_{t,i-1} + \delta V_{t,i-1} + \gamma_2 N_{t,i-1} I_{\{Sent_{t,i-1}>0\}} + \gamma_3 N_{t,i-1} I_{\{Sent_{t,i-1}=0\}} + \gamma_4 N_{t,i-1} I_{\{Sent_{t,i-1}<0\}}$

(4) $\mu_{t,i} = \alpha + \beta\mu_{t,i-1} + \delta V_{t,i-1} + \gamma_5 \left|R_{t,i-1}\right| I_{\{R_{t,i-1}>0\}} + \gamma_6 \left|R_{t,i-1}\right| I_{\{R_{t,i-1}<0\}}$

(5) $\mu_{t,i} = \alpha + \beta\mu_{t,i-1} + \delta V_{t,i-1} + \gamma_7 N_{t,i-1} I_{\{R_{t,i-1}>0\}} I_{\{Sent_{t,i-1}>0\}} + \gamma_8 N_{t,i-1} I_{\{R_{t,i-1}>0\}} I_{\{Sent_{t,i-1}=0\}} + \gamma_9 N_{t,i-1} I_{\{R_{t,i-1}>0\}} I_{\{Sent_{t,i-1}<0\}} +$
$+ \gamma_{10} N_{t,i-1} I_{\{R_{t,i-1}<0\}} I_{\{Sent_{t,i-1}>0\}} + \gamma_{11} N_{t,i-1} I_{\{R_{t,i-1}<0\}} I_{\{Sent_{t,i-1}=0\}} + \gamma_{12} N_{t,i-1} I_{\{R_{t,i-1}<0\}} I_{\{Sent_{t,i-1}<0\}}$

(6) $\mu_{t,i} = \alpha + \beta\mu_{t,i-1} + \delta V_{t,i-1} + \psi_1 J_{t,i-1}$

(7) $\mu_{t,i} = \alpha + \beta\mu_{t,i-1} + \delta V_{t,i-1} + \psi_2 J_{t,i-1} I_{\{N_{t,i-1}>0\}} + \psi_3 J_{t,i-1} I_{\{N_{t,i-1}=0\}}$

(8) $\mu_{t,i} = \alpha + \beta\mu_{t,i-1} + \delta V_{t,i-1} + \psi_4 N_{t,i-1} I_{\{J_{t,i-1}>0\}} I_{\{Sent_{t,i-1}>0\}} + \psi_5 N_{t,i-1} I_{\{J_{t,i-1}>0\}} I_{\{Sent_{t,i-1}<0\}}$

(9) $\mu_{t,i} = \alpha + \beta\mu_{t,i-1} + \delta V_{t,i-1} + \psi_6 |R_{t,i-1}| I_{\{J_{t,i-1}>0\}} I_{\{R_{t,i-1}>0\}} + \psi_7 |R_{t,i-1}| I_{\{J_{t,i-1}>0\}} I_{\{R_{t,i-1}<0\}}$

(10) $\mu_{t,i} = \alpha + \beta\mu_{t,i-1} + \delta V_{t,i-1} + \psi_8 N_{t,i-1} I_{\{J_{t,i-1}>0\}} I_{\{R_{t,i-1}>0\}} I_{\{Sent_{t,i-1}>0\}} + \psi_9 N_{t,i-1} I_{\{J_{t,i-1}>0\}} I_{\{R_{t,i-1}>0\}} I_{\{Sent_{t,i-1}<0\}} +$
$+ \psi_{10} N_{t,i-1} I_{\{J_{t,i-1}>0\}} I_{\{R_{t,i-1}<0\}} I_{\{Sent_{t,i-1}>0\}} + \psi_{11} N_{t,i-1} I_{\{J_{t,i-1}>0\}} I_{\{R_{t,i-1}<0\}} I_{\{Sent_{t,i-1}<0\}}$

Table 4. Conditional mean specifications for the Multiplicative Error Model, 'newspapers' and 'upgrades and downgrades' news cases.

'newspapers' case

(11) $$\mu_{t,i} = \alpha + \beta\mu_{t,i-1} + \delta V_{t,i-1} + \gamma_1 N_{t,i-1}^{nn}$$

(12) $$\mu_{t,i} = \alpha + \beta\mu_{t,i-1} + \delta V_{t,i-1} + \gamma_2 N_{t,i-1}^{nn} I_{\{Sent_{t,i-1}^{nn}>0\}} + \gamma_3 N_{t,i-1}^{nn} I_{\{Sent_{t,i-1}^{nn}=0\}} + \gamma_4 N_{t,i-1}^{nn} I_{\{Sent_{t,i-1}^{nn}<0\}}$$

(13) $$\mu_{t,i} = \alpha + \beta\mu_{t,i-1} + \delta V_{t,i-1} + \gamma_7 N_{t,i-1}^{nn} I_{\{R_{t,i-1}>0\}} I_{\{Sent_{t,i-1}^{nn}>0\}} + \gamma_8 N_{t,i-1}^{nn} I_{\{R_{t,i-1}>0\}} I_{\{Sent_{t,i-1}^{nn}=0\}} + \gamma_9 N_{t,i-1}^{nn} I_{\{R_{t,i-1}>0\}} I_{\{Sent_{t,i-1}^{nn}<0\}} +$$
$$+ \gamma_{10} N_{t,i-1}^{nn} I_{\{R_{t,i-1}<0\}} I_{\{Sent_{t,i-1}^{nn}>0\}} + \gamma_{11} N_{t,i-1}^{nn} I_{\{R_{t,i-1}<0\}} I_{\{Sent_{t,i-1}^{nn}=0\}} + \gamma_{12} N_{t,i-1}^{nn} I_{\{R_{t,i-1}<0\}} I_{\{Sent_{t,i-1}^{nn}<0\}}$$

(14) $$\mu_{t,i} = \alpha + \beta\mu_{t,i-1} + \delta V_{t,i-1} + \psi_2 J_{t,i-1} I_{\{N_{t,i-1}^{nn}>0\}} + \psi_3 J_{t,i-1} I_{\{N_{t,i-1}^{nn}=0\}}$$

(15) $$\mu_{t,i} = \alpha + \beta\mu_{t,i-1} + \delta V_{t,i-1} + \psi_4 N_{t,i-1}^{nn} I_{\{J_{t,i-1}>0\}} I_{\{Sent_{t,i-1}^{nn}>0\}} + \psi_5 N_{t,i-1}^{nn} I_{\{J_{t,i-1}>0\}} I_{\{Sent_{t,i-1}^{nn}<0\}}$$

(16) $$\mu_{t,i} = \alpha + \beta\mu_{t,i-1} + \delta V_{t,i-1} + \psi_8 N_{t,i-1}^{nn} I_{\{J_{t,i-1}>0\}} I_{\{R_{t,i-1}>0\}} I_{\{Sent_{t,i-1}^{nn}>0\}} + \psi_9 N_{t,i-1}^{nn} I_{\{J_{t,i-1}>0\}} I_{\{R_{t,i-1}>0\}} I_{\{Sent_{t,i-1}^{nn}<0\}} +$$
$$+ \psi_{10} N_{t,i-1}^{nn} I_{\{J_{t,i-1}>0\}} I_{\{R_{t,i-1}<0\}} I_{\{Sent_{t,i-1}^{nn}>0\}} + \psi_{11} N_{t,i-1}^{nn} I_{\{J_{t,i-1}>0\}} I_{\{R_{t,i-1}<0\}} I_{\{Sent_{t,i-1}^{nn}<0\}}$$

'upgrades and downgrades' case

(17) $$\mu_{t,i} = \alpha + \beta\mu_{t,i-1} + \delta V_{t,i-1} + \gamma_1 N_{t,i-1}^{ud}$$

(18) $$\mu_{t,i} = \alpha + \beta\mu_{t,i-1} + \delta V_{t,i-1} + \gamma_2 N_{t,i-1}^{ud} I_{\{Sent_{t,i-1}^{ud}>0\}} + \gamma_3 N_{t,i-1}^{ud} I_{\{Sent_{t,i-1}^{ud}=0\}} + \gamma_4 N_{t,i-1}^{ud} I_{\{Sent_{t,i-1}^{ud}<0\}}$$

(19) $$\mu_{t,i} = \alpha + \beta\mu_{t,i-1} + \delta V_{t,i-1} + \gamma_7 N_{t,i-1}^{ud} I_{\{R_{t,i-1}>0\}} I_{\{Sent_{t,i-1}^{ud}>0\}} + \gamma_8 N_{t,i-1}^{ud} I_{\{R_{t,i-1}>0\}} I_{\{Sent_{t,i-1}^{ud}=0\}} + \gamma_9 N_{t,i-1}^{ud} I_{\{R_{t,i-1}>0\}} I_{\{Sent_{t,i-1}^{ud}<0\}} +$$
$$+ \gamma_{10} N_{t,i-1}^{ud} I_{\{R_{t,i-1}<0\}} I_{\{Sent_{t,i-1}^{ud}>0\}} + \gamma_{11} N_{t,i-1}^{ud} I_{\{R_{t,i-1}<0\}} I_{\{Sent_{t,i-1}^{ud}=0\}} + \gamma_{12} N_{t,i-1}^{ud} I_{\{R_{t,i-1}<0\}} I_{\{Sent_{t,i-1}^{ud}<0\}}$$

(20) $$\mu_{t,i} = \alpha + \beta\mu_{t,i-1} + \delta V_{t,i-1} + \psi_2 J_{t,i-1} I_{\{N_{t,i-1}^{nn}>0\}} + \psi_3 J_{t,i-1} I_{\{N_{t,i-1}^{nn}=0\}}$$

(21) $$\mu_{t,i} = \alpha + \beta\mu_{t,i-1} + \delta V_{t,i-1} + \psi_4 N_{t,i-1}^{ud} I_{\{J_{t,i-1}>0\}} I_{\{Sent_{t,i-1}^{ud}>0\}} + \psi_5 N_{t,i-1}^{ud} I_{\{J_{t,i-1}>0\}} I_{\{Sent_{t,i-1}^{ud}<0\}}$$

(22) $$\mu_{t,i} = \alpha + \beta\mu_{t,i-1} + \delta V_{t,i-1} + \psi_8 N_{t,i-1}^{ud} I_{\{J_{t,i-1}>0\}} I_{\{R_{t,i-1}>0\}} I_{\{Sent_{t,i-1}^{ud}>0\}} + \psi_9 N_{t,i-1}^{ud} I_{\{J_{t,i-1}>0\}} I_{\{R_{t,i-1}>0\}} I_{\{Sent_{t,i-1}^{ud}<0\}} +$$
$$+ \psi_{10} N_{t,i-1}^{ud} I_{\{J_{t,i-1}>0\}} I_{\{R_{t,i-1}<0\}} I_{\{Sent_{t,i-1}^{ud}>0\}} + \psi_{11} N_{t,i-1}^{ud} I_{\{J_{t,i-1}>0\}} I_{\{R_{t,i-1}<0\}} I_{\{Sent_{t,i-1}^{ud}<0\}}$$

3.3 Estimation

As regards the estimation of parameters of our Multiplicative Error Model for trading volumes, we use Maximum Likelihood method. In Appendix A, the procedure is explained in detail for the general case, while here we expose the procedure for our particular case.

First of all, probability distribution of the random variable of interest is needed. In our case, it means retrieving the probability distribution of trading volumes $V_{t,i}$. As derived at the beginning of this Section, we know that trading volumes follow a Gamma distribution with shape parameter v and mean parameter equal to $\mu_{t,i}$. Then, it is of the form (Engle and Gallo 2006):

$$f(x_t|\mathcal{F}_{t,i-1}) = \frac{1}{\Gamma(a)} a^a x_t^{a-1} \mu_t^{-a} exp\left(-a\frac{x_t}{\mu_t}\right)$$

from which follows that the process x_t has conditional expectation $\mathbb{E}(x_t|\mathcal{F}_{t,i-1}) = \mu_t$ and conditional variance $Var(x_t|\mathcal{F}_{t,i-1}) = \frac{\mu_t}{a}$. In our case, considering trading volumes modelled as a Multiplicative Error Model means assuming that volumes follow a Gamma distribution $\Gamma(v, 1)$, in shape-mean representation. Thus, the probability distribution for trading volumes $V_{t,i}$ is defined as:

$$f(V_{t,i}|\mathcal{F}_{t,i-1}) = \frac{1}{\Gamma(v)} v^v V_{t,i}^{v-1} \mu_{t,i}^{-v} exp\left(-v\frac{V_{t,i}}{\mu_{t,i}}\right).$$

Consequently, its first two moments are:

$$\mathbb{E}(V_{t,i}|\mathcal{F}_{t,i-1}) = \mu_{t,i}$$

and

$$Var(V_{t,i}|\mathcal{F}_{t,i-1}) = \frac{\mu_{t,i}}{v}.$$

From these results, it is possible to build the log-likelihood function in order to estimate all the parameters which vary according to the conditional mean specification considered. For example, specification (1) is required to estimate – beyond the shape parameter v of the Gamma – the constant α, and the coefficients of the GARCH and ARCH terms, β and δ. The same procedure is applied to all specifications. In order to compute the Maximum Likelihood estimators for these parameters, we set the maximization problem as:

$$\max_{\theta} \sum_{t=1}^{T} \sum_{i=1}^{I} \log(L_{t,i}(\theta))$$

where $t = 1, \dots, 2516$ is the number of the business days observed, i=1,…77 indicates the five-minute intra-daily intervals, and θ the vector including all parameters to be estimated. Since trading volumes follow a Gamma distribution as described above, the problem becomes:

$$\max_{\theta} \sum_{t=1}^{T} \sum_{i=1}^{I} \left(v \ln(v) - \ln\big(\Gamma(v)\big) + (v-1) \ln\big(V_{t,i}\big) - v \ln\big(\mu_{t,i}\big) - v \, \frac{V_{t,i}}{\mu_{t,i}} \right)$$

which turns into:

$$\max_{\theta} \; T \cdot I \cdot v \cdot \ln(v) - T \cdot I \cdot \ln(\Gamma(v)) + (v-1) \cdot$$

$$\cdot \sum_{t=1}^{T} \sum_{i=1}^{I} \ln(V_{t,i}) - v \cdot \sum_{t=1}^{T} \sum_{i=1}^{I} \ln\big(\mu_{t,i}\big) - v \cdot \sum_{t=1}^{T} \sum_{i=1}^{I} \frac{V_{t,i}}{\mu_{t,i}}$$

where $\mu_{t,i}$ changes according to the conditional mean specification considered. For the empirical analysis of this thesis, all computations have been performed in Matlab [11]. The Quasi-Maximum Likelihood estimators – because our time series are non-Gaussian – show some nice asymptotic properties which bring important results for inference. First of all, Maximum Likelihood estimators are consistent, i.e. when the number of observations go to infinity, the absolute value of the difference between the estimate and the true population parameter equals zero. Second, Maximum Likelihood estimators are unbiased, meaning that the expected value of the estimator asymptotically converges to the population parameter. Third, estimators computed by Maximum Likelihood method are efficient: they have the minimum asymptotic variance among all asymptotically unbiased estimators. Four, for large samples their variance is known and it is equal to the inverse of the information matrix, that is the diagonal component of the inverse Hessian matrix. Last, Maximum Likelihood estimators result to be asymptotically normally distributed, so that the building of confidence intervals and hypothesis testing are permitted. After having computed all the parameters, their significance is separately tested through a Wald-statistics, testing for parameters being different from zero. As usual, p-values are also computed by considering the Wald-statistics normally distributed. Moreover, since our Multiplicative Error Model is a dynamic model, multiple restrictions about parameters are not taken into consideration. Together with estimation and statistical significance, results of AIC and BIC are associated for each of the estimated models. These criterions are based on the log-likelihood function evaluated at the Maximum Likelihood estimates $\hat{\theta}$ of the parameters, and are strictly connected each other. AIC (Akaike's information criterion) measures the goodness of estimates in a statistic model tanking into consideration both goodness of fit and model complexity. BIC (Bayesian information criterion or Schwarz criterion) is used for selecting a model among a set of models with different numbers of parameters. By using the results of these criterion, the choice among different models thus depends on the smallest AIC and BIC values.

[11] See Appendix D for codes.

3.3 Results

In this Section we present all the resulting parameters obtained from the estimation of each of the conditional mean specifications (listed in Table 3 and Table 4) of our Multiplicative Error Model for trading volumes. Below, we report the questions to which this thesis aims at answering:

a) Do news and announcements have a causal effect on trading volumes? If yes, does it change according to the sentiment of news and announcements?
b) Do positive and negative price returns have a causal effect on trading volumes? If yes, in both cases, what is the effect of news and announcements according to their sentiment?
c) Do price volatility jumps have a causal effect on trading volumes? If yes, does it change according to the presence or the lack of news and announcements?
d) Given the presence of volatility jumps, what is the effect of news and announcements on trading volumes according to their sentiment?
e) Do positive and negative price returns have a casual effect on trading volumes, given the presence of volatility jumps? If yes, what is the effect of news and announcements – with both positive and negative returns – according to their sentiment, given the presence of volatility jumps?

In the end of this Section, tables of estimation results are reported. Here, we expose them by following the logical order of the above-mentioned questions.

First of all, we begin with the exposition of the resulting parameters of the baseline specification (1) of the Multiplicative Error Model for trading volumes. Below, we present the equation of this specification, the results of the estimated coefficients in Table C, and a graphical representation of them.

(1) $$\mu_{t,i} = \alpha + \beta\mu_{t,i-1} + \delta V_{t,i-1}$$

	(1)
α	0.0568*** (0.0003)
$\mu_{t,i-1}$	0.4485*** (0.0002)
$V_{t,i-1}$	0.4985*** (0.0003)

Table 5. Estimation results for specification (1).

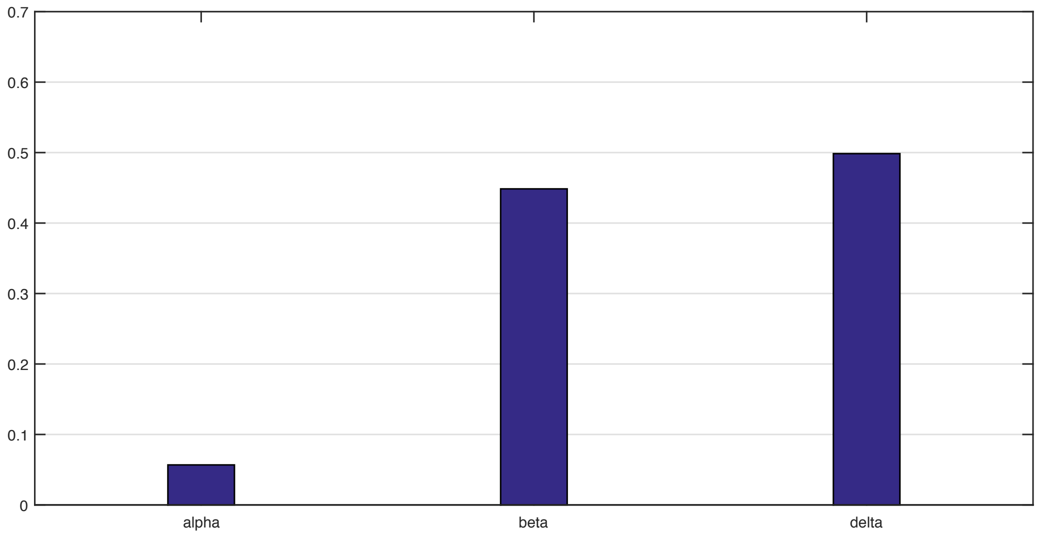


Figure 29. Estimation results for specification (1).

The coefficient α is statistically significant and equal to 0.0568. It means that the conditional mean of trading volumes $\mu_{t,i}$ increases by 0.0568 regardless of what happen to the other variables in the previous intra-daily interval $i-1$. It is a very small value indicating that, when all other variables are set to zero, the expected value of the mean of trading volumes does not influence $\mu_{t,i}$ in a large amount. For what concerns the effects of the GARCH and ARCH terms, the coefficients β and δ are much larger than the constant term. Precisely, the coefficient β is statistically significant and takes a value of 0.4485, meaning that, ceteris paribus, a one-unit increase in the mean of trading volumes in the interval $i-1$ makes increase the mean in the subsequent intra-daily interval by 0.4485. This result indicates that the mean of trading volumes in the intra-daily interval i highly depends on the mean of the previous interval. The strong dependence of trading volume process on its past can be also observed in the value of the coefficient δ. In fact, it is statistically significant and equal to 0.4985, meaning that increasing volumes at time $i-1$ by one would grow the mean of trading volumes by that amount. As a consequence, trading volume seems to be a very strong persistent process, which translates into clustering phenomenon. Indeed, when volumes are high in the intra-daily interval $i-1$, they preserve this characteristic in the subsequent interval i also. On the other hand, when volumes are low, they tend to be low in the following intra-daily interval also.
Estimation results about these three coefficients of the baseline specification are approximately true over all specifications.

We now enter into the interesting exposition of estimation results with the consideration of news and announcements. Then, we begin with answering to the question a) with the resulting coefficients belonging to specifications (2) and (3). As before, we report their equations together with Table D and Figure 30 which give information about estimates.

(2) $$\mu_{t,i} = \alpha + \beta\mu_{t,i-1} + \delta V_{t,i-1} + \gamma_1 N_{t,i-1}$$

(3) $$\mu_{t,i} = \alpha + \beta\mu_{t,i-1} + \delta V_{t,i-1} + \gamma_2 N_{t,i-1} I_{\{Sent_{t,i-1}>0\}} + \gamma_3 N_{t,i-1} I_{\{Sent_{t,i-1}=0\}} + + \gamma_4 N_{t,i-1} I_{\{Sent_{t,i-1}<0\}}$$

	(2)	(3)
$N_{t,i-1}$	0.0498*** (0.0025)	
$N_{t,i-1} I_{\{Sent_{t,i-1>0}\}}$		0.0433** (0.0136)
$N_{t,i-1} I_{\{Sent_{t,i-1=0}\}}$		0.0571*** (0.0121)
$N_{t,i-1} I_{\{Sent_{t,i-1<0}\}}$		0.0467*** (0.0125)

Table 6. Estimation results for specifications (2) and (3).

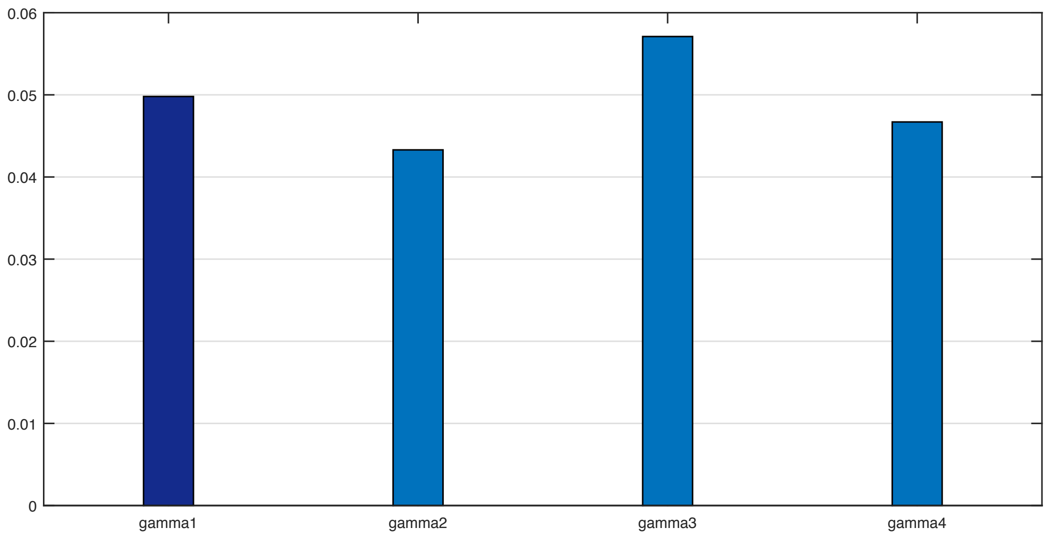


Figure 30. Estimation results for specifications (2) and (3).

The coefficient γ_1 – which is statistically significant and equal to 0.0498 – captures the effect of a one-unit increase in news variable in the interval $i-1$ on the mean of trading volumes in the subsequent interval i. It indicates that, because of company news arrivals, the mean of trading volumes increases by 0.0498, keeping all other factors fixed. The value of this estimate is not very high, but the causal effect of company news and announcements arrival on trading volumes is evident.

As a consequence, it is reasonable to ask whether the effect of news and announcements changes according to the sentiment associated with them. Specification (3) aims at answering to this question and we can see its estimation results in Table D and in Figure 31, graphically. The coefficient γ_2 – statistically significant and with a value of 0.0433 – indicates that the mean of trading volumes in the interval i would increase by that amount when one more positive news or announcement arrives in the market in the interval $i-1$. Instead, the coefficient γ_3 - which also is statistically significant – indicates that the mean of trading volumes in the intra-daily interval i increases by 0.0571 with one more neutral news in the interval i. Eventually, the coefficient γ_4 takes the statistically significant value of 0.0467, meaning that one more news or announcement with negative sentiment arriving in the intra-daily interval $i-1$ affects the amount of traded stocks by increasing it by 0.0467. As for the estimate in specification (2), these are not very high too. By the way, some differences can be seen through the three coefficients γ_2, γ_3 and γ_4. Neutral news and announcements seem to have a bigger impact on trading volumes with respect to the other two cases. Then, negative and positive news affect trading volumes in this order. We then can conclude that news have surely an impact on trading volumes, even if they do not bring any positive or negative sentiment into the market. On the other hand, if they do, negative news have a bigger impact than the positive ones, which may be due to a general risk aversion of traders and investors. In fact, if an agent is risk-averse, he prefers to not bear risk with his own capital. So that, when a negative news or announcement arrives into the market, he is likely to sell its stocks more probably with respect to the situation in which a positive news enters the market. For this reason, trading activity is bigger in case of negative sentiment of news and announcements.

For what concerns the effect of positive or negative returns on trading volumes – before – and according to the sign of news sentiment – after – we now expose the estimation results for specification (4) and (5). Below, we report their equations, the estimates in Table E and their graphical representation.

$$\text{(4)} \qquad \mu_{t,i} = \alpha + \beta\mu_{t,i-1} + \delta V_{t,i-1} + \gamma_5 \left|R_{t,i-1}\right| I_{\{R_{t,i-1}>0\}} + \gamma_6 \left|R_{t,i-1}\right| I_{\{R_{t,i-1}<0\}}$$

$$\begin{aligned}\text{(5)} \qquad \mu_{t,i} = {} & \alpha + \beta\mu_{t,i-1} + \delta V_{t,i-1} + \gamma_7 N_{t,i-1} I_{\{R_{t,i-1}>0\}} I_{\{Sent_{t,i-1}>0\}} + \\ & + \gamma_8 N_{t,i-1} I_{\{R_{t,i-1}>0\}} I_{\{Sent_{t,i-1}=0\}} + \gamma_9 N_{t,i-1} I_{\{R_{t,i-1}>0\}} I_{\{Sent_{t,i-1}<0\}} + \\ & + \gamma_{10} N_{t,i-1} I_{\{R_{t,i-1}<0\}} I_{\{Sent_{t,i-1}>0\}} + \gamma_{11} N_{t,i-1} I_{\{R_{t,i-1}<0\}} I_{\{Sent_{t,i-1}=0\}} + \\ & + \gamma_{12} N_{t,i-1} I_{\{R_{t,i-1}<0\}} I_{\{Sent_{t,i-1}<0\}}\end{aligned}$$

The estimated coefficients γ_5 and γ_6 are both statistically significant and respectively equal to 0.0474 and 0.0159. They mean that a one-unit increase in absolute returns in the intra-daily in-

	(4)	(5)
$\lvert R_{t,i-1}\rvert I_{\{R_{t,i-1>0}\}}$	0.0474*** (0.0013)	
$\lvert R_{t,i-1}\rvert I_{\{R_{t,i-1<0}\}}$	0.0159*** (0.0002)	
$N_{t,i-1}I_{\{R_{t,i-1>0}\}}I_{\{Sent_{t,i-1>0}\}}$		0.0597*** (0.0096)
$N_{t,i-1}I_{\{R_{t,i-1>0}\}}I_{\{Sent_{t,i-1=0}\}}$		0.0783*** (00047)
$N_{t,i-1}I_{\{R_{t,i-1>0}\}}I_{\{Sent_{t,i-1<0}\}}$		0.0713*** (0.0058)
$N_{t,i-1}I_{\{R_{t,i-1<0}\}}I_{\{Sent_{t,i-1>0}\}}$		0.0232*** (0.0085)
$N_{t,i-1}I_{\{R_{t,i-1<0}\}}I_{\{Sent_{t,i-1=0}\}}$		0.1067*** (0.0122)
$N_{t,i-1}I_{\{R_{t,i-1<0}\}}I_{\{Sent_{t,i-1<0}\}}$		0.0393*** (0.0035)

Table 7. Estimation results for specifications (4) and (5).

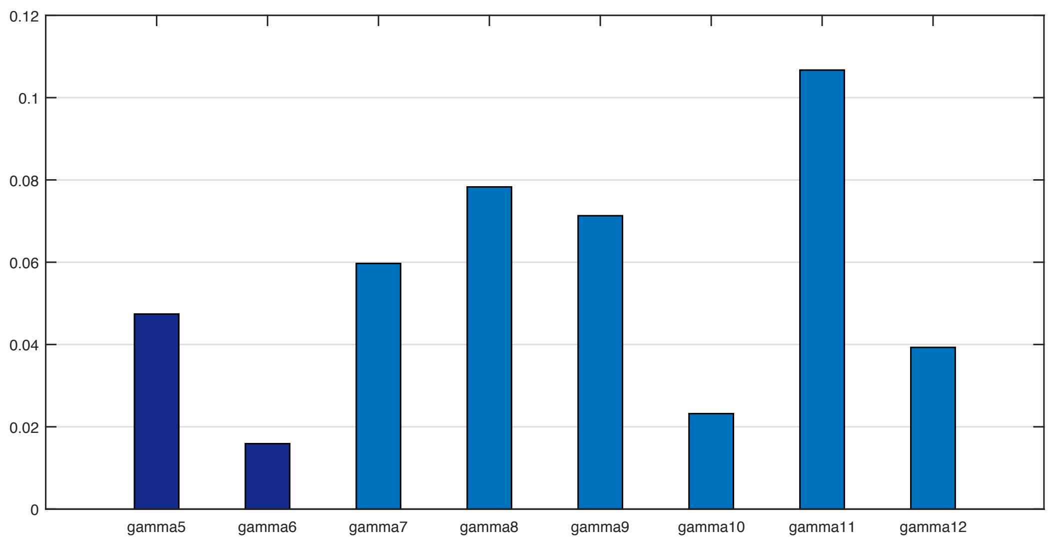


Figure 31. Estimation results for specifications (4) and (5).

terval $i-1$ influences the mean of trading volumes in the subsequent interval i by making them increase by 0.0474 in case of positive returns, while by 0.0159 in case of negative returns. These results then show that changes in returns when they are positive have more effect than the opposite case.

When we take into consideration both signs of price returns and news sentiment, the estimated coefficients confirm what said above for parameters of specification (3). It seems that, in both cases of positive and negative returns, one more neutral news or announcements have a bigger effect on trading volumes with respect the other two situations. Then, negative news sentiment seems to have more relevance than the positive one in determining trading volumes. All these affirmations can be retrieved from the values of the estimated parameters of specification (5). For the case of positive returns, the coefficients γ_7, γ_8 and γ_9 are statistically significant and respectively equal to 0.0597, 0.0783 and 0.0713. Instead, for the case of negative returns, the statistically significant coefficients γ_{10}, γ_{11} and γ_{12} take the value of 0.0232, 0.1067 and 0.0393. Therefore, in order of relevance, we have that a one-unit increase in the amount of neutral news in the intra-daily interval $i-1$ has an increasing effect of 0.0783 – in case of positive returns – and of 0.1067 – in case of negative returns – on the mean of trading volumes in the subsequent intra-daily interval i. Then, we have that negative news makes the mean of trading volumes increase by 0.0713 or by 0.0393, in the case of positive and negative returns respectively. Lastly, the effect of one more positive news or announcement on trading volumes is of 0.0597 in the case of positive returns, or of 0.0393 in the case of negative returns. Once again, it seems that negative news and announcements have a larger influence in determining trading volumes with respect to positive news, regardless of the sign of returns. Moreover, to confirm what resulting from specification (4), positive returns show bigger news sentiment coefficients with respect to the negative case, except for neutral news and announcements. We should also have expected – as said in the previous Section – a higher coefficient γ_7 with respect to γ_{10} for the case of positive returns, and a higher γ_{12} with respect to γ_9 for the case of negative returns. However, it is shown to be true only for the case of positive returns: in fact, 0.0597 is bigger than 0.0232. This would mean that when positive news arrives in the market, the returns follow this sentiment being positive too. On the opposite, it is not true for the case of negative returns: 0.0393 is not bigger than 0.0713. This means that negative news have more impact when price returns are positive. This fact is quite logical: when the overall market sentiment about a stock is positive and a negative news arrives in the market, agents are likely to trade more intensively rather than in the situation of positive news arrival.

As regards the effect of price volatility jumps on trading volumes – addressed in question c) – we now consider the resulting coefficients of specifications (6) and (7) of our Multiplicative Error Model for trading volumes. We illustrate their equations, together with estimates and their graphical representation (Table 8 and Figure 32).

(6) $$\mu_{t,i} = \alpha + \beta\mu_{t,i-1} + \delta V_{t,i-1} + \psi_1 J_{t,i-1}$$

(7) $$\mu_{t,i} = \alpha + \beta\mu_{t,i-1} + \delta V_{t,i-1} + \psi_2 J_{t,i-1} I_{\{N_{t,i-1}>0\}} + \psi_3 J_{t,i-1} I_{\{N_{t,i-1}=0\}}$$

	(6)	(7)
$J_{t,i-1}$	0.0804*** (0.0029)	
$J_{t,i-1}I_{\{N_{t,i-1>0}\}}$		0.7861*** (0.1523)
$J_{t,i-1}I_{\{N_{t,i-1=0}\}}$		0.0798*** (0.0052)

Table 8. Estimation results for specifications (6) and (7).

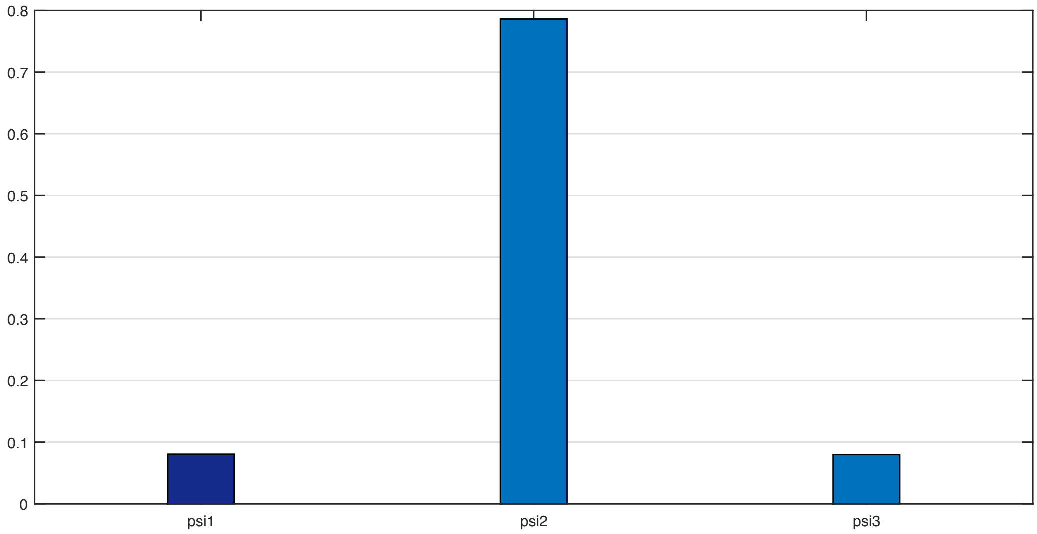


Figure 32. Estimation results for specifications (6) and (7).

In order to understand the overall effect of volatility jumps on trading volumes, we examine the estimate of the coefficient ψ_1. It is statistically significant and equal to the value of 0.0804. It means that the effect of one more volatility jump in the intra-daily interval $i-1$ makes the mean of trading volumes in the subsequent interval i increase by that amount. It is not a high coefficient, but the causal effect of volatility jumps on trading activity is evident. In fact, since jumps correspond to large changes in price returns, they should be moved by a large trading activity of agents in the market.

Differently, in specification (7), we take into consideration the case in which there are news arrivals together with the case of no news arrival. The coefficients ψ_2 and ψ_3 are both statistically significant and equal respectively 0.7861 and 0.0798. It can be observed – from Figure also – that these two coefficients are very different one from the other in terms of relevance. The first one, that is ψ_2, indicates that a one-unit increase in the jump variable in the interval $i-1$, given news arrival in the same interval, makes the the mean of trading volumes in the interval i increase by 0.7861. It is a very high coefficient, signalling for a strong effect of volatility jumps on trading volumes in the case that company news and announcements arrive into the market. Moreover, it indicates that there exists a link between price volatility jumps and company news arrivals. In fact, when news appears in the market, agents trade on the basis of it, making price returns move. On the opposite, with the estimated coefficient ψ_3, we can state that one more volatility jump in the interval $i-1$ makes increase trading volumes in the interval i only by the low amount of 0.0798.

Question d) considers the presence of volatility jumps and ask whether the sentiment of news and announcements affects their effect on trading volumes. For this purpose, specification (8) expresses the conditional mean of trading volumes by adding news effect in the cases of positive and negative news. Its equation, the resulting coefficients and their graphical representations are presented below (Table 9 and Figure 33).

$$\text{(8)} \qquad \mu_{t,i} = \alpha + \beta\mu_{t,i-1} + \delta V_{t,i-1} + \psi_4\, N_{t,i-1} I_{\{J_{t,i-1}>0\}} I_{\{Sent_{t,i-1}>0\}} + \\ + \psi_5\, N_{t,i-1} I_{\{J_{t,i-1}>0\}} I_{\{Sent_{t,i-1}<0\}}$$

	(8)
$N_{t,i-1} I_{\{J_{t,i-1>0}\}} I_{\{Sent_{t,i-1>0}\}}$	0.9894 (2.0077)
$N_{t,i-1} I_{\{J_{t,i-1>0}\}} I_{\{Sent_{t,i-1<0}\}}$	0.6091*** (0.0670)

Table 9. Estimation results for specifications (8).

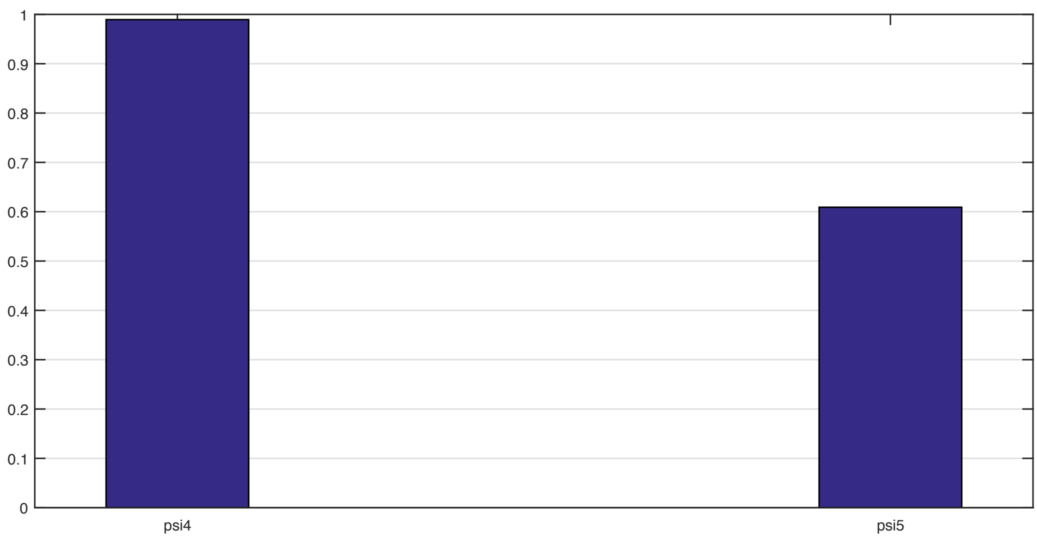


Figure 32. Estimation results for specifications (8).

The coefficient ψ_4 is not statistically significant. On the contrary, the coefficient ψ_5 is statistically significant and is equal to 0.6091. These results mean that news and announcements arrivals seem to not have any causal effect on the mean of trading volumes when news sentiment is positive. On the other hand, when news are negative, a one-unit increase in news in the intra-daily interval $i-1$ makes increase the mean of trading volumes in the subsequent interval i by 0.6091. This result is in line with the resulting coefficients of specification (3), in which, after neutral case, negative news affect trading volumes more than the positive ones. Moreover, the non-significance of coefficient ψ_4 seems to be related to the fact that volatility jumps get more negative direction rather than upward peaks of price returns.

In the last two specifications ((9) and (10)), the aim is answering to the question e): given the presence of jumps, understanding whether positive or negative returns have an effect on trading volumes, and understanding the effect of news in the cases of positive and negative returns, positive and negative sentiment. We report the equations of specifications (9) and (10) with estimation results also.

$$\begin{aligned}(9)\qquad \mu_{t,i} &= \alpha + \beta\mu_{t,i-1} + \delta V_{t,i-1} + \psi_6\,|R_{t,i-1}| I_{\{J_{t,i-1}>0\}} I_{\{R_{t,i-1}>0\}} + \\ &\quad + \psi_7 |R_{t,i-1}| I_{\{J_{t,i-1}>0\}} I_{\{R_{t,i-1}<0\}}\end{aligned}$$

$$\begin{aligned}(10)\qquad \mu_{t,i} &= \alpha + \beta\mu_{t,i-1} + \delta V_{t,i-1} + \psi_8\, N_{t,i-1} I_{\{J_{t,i-1}>0\}} I_{\{R_{t,i-1}>0\}} I_{\{Sent_{t,i-1}>0\}} + \\ &\quad + \psi_9\, N_{t,i-1} I_{\{J_{t,i-1}>0\}} I_{\{R_{t,i-1}>0\}} I_{\{Sent_{t,i-1}<0\}} + \\ &\quad + \psi_{10}\, N_{t,i-1} I_{\{J_{t,i-1}>0\}} I_{\{R_{t,i-1}<0\}} I_{\{Sent_{t,i-1}>0\}} + \\ &\quad + \psi_{11}\, N_{t,i-1} I_{\{J_{t,i-1}>0\}} I_{\{R_{t,i-1}<0\}} I_{\{Sent_{t,i-1}<0\}}\end{aligned}$$

As regards the effect of a one-unit increase in absolute returns, given the presence of volatility jumps, we can observe that both coefficients ψ_6 and ψ_7 are both statistically significant and are equal to 0.0173 and 0.0309, respectively. It is then show that absolute returns make the mean of trading volumes increase more when returns are negative, given the presence of volatility jumps. Once again, jumps correspond to large changes in price returns, there is evidence of the fact that they have downward direction rather than an upward direction. So that we have that, when changes in absolute returns are big and downward (i.e. there is a jump), a one-unit increase in absolute returns in the intra-daily interval $i-1$ makes the mean of trading volumes in the subsequent interval i by 0.0309, keeping all other factors fixed. On the other hand, when changes in absolute returns are upward, the increase in absolute returns by one makes the mean of trading volumes increase by a lower value of 0.0173.

	(9)	(10)
$\lvert R\rvert_{t,i-1} I_{\{J_{t,i-1>0}\}} I_{\{R_{t,i-1>0}\}}$	0.0173*** (0.0015)	
$\lvert R\rvert_{t,i-1} I_{\{J_{t,i-1>0}\}} I_{\{R_{t,i-1<0}\}}$	0.0309*** (0.0015)	
$N_{t,i-1} I_{\{J_{t,i-1>0}\}} I_{\{R_{t,i-1>0}\}} I_{\{Sent_{t,i-1>0}\}}$		0.0150 (0.3259)
$N_{t,i-1} I_{\{J_{t,i-1>0}\}} I_{\{R_{t,i-1>0}\}} I_{\{Sent_{t,i-1<0}\}}$		0.0009 (0.5230)
$N_{t,i-1} I_{\{J_{t,i-1>0}\}} I_{\{R_{t,i-1<0}\}} I_{\{Sent_{t,i-1>0}\}}$		1.9923*** (0.2285)
$N_{t,i-1} I_{\{J_{t,i-1>0}\}} I_{\{R_{t,i-1<0}\}} I_{\{Sent_{t,i-1<0}\}}$		1.4954*** (0.3864)

Table 9. Estimation results for specifications (9) and (10).

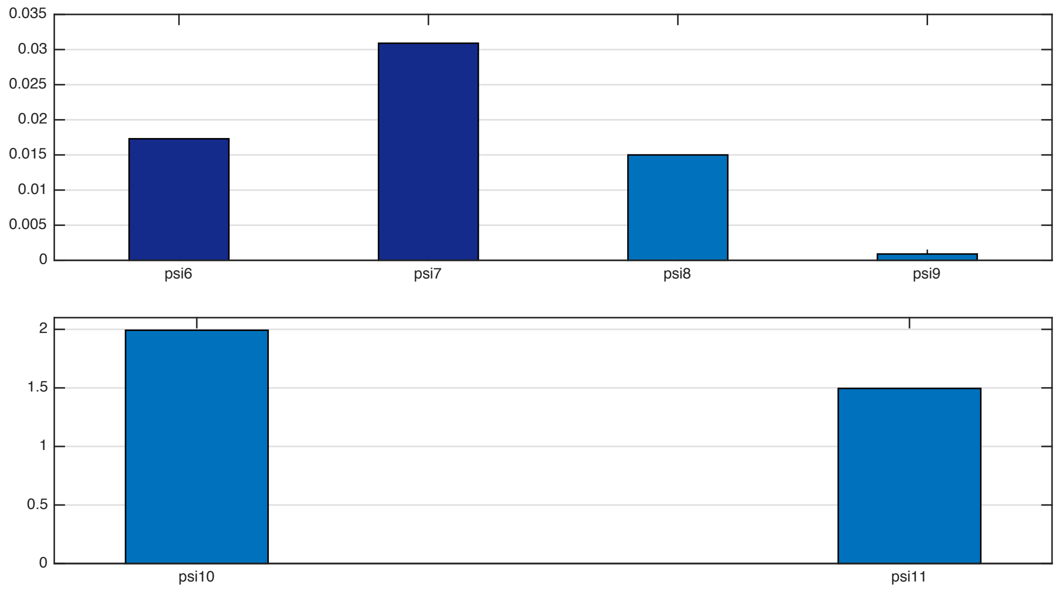


Figure 33. Estimation results for specifications (9) and (10).

Coming to the effect of the amount of news and announcements according to their sentiment in presence of jumps for the cases in which returns are positive or negative, the coefficients in specification (10) aim at capture them. for the case in which positive returns are considered, the coefficients ψ_8 and ψ_9 are not statistically significant. This confirms what already said for coefficient ψ_4: when returns are positive, the effect of one more news in presence of volatility jumps on trading activity is not significant. On the other hand, when negative returns are considered, the coefficients ψ_{10} and ψ_{11} are statistically significant. The coefficient ψ_{10} - which equals 1.9923 – is bigger than the coefficient ψ_{11} – equal to 1.4954 – meaning that, when returns are negative, positive news have more impact on determining the mean of trading volumes. In fact, the arrival of one more positive news in the intra-daily interval $i-1$ in presence of negative jumps makes the mean of trading volumes in the subsequent interval increase by 1.9923. On the other hand, one more negative news arriving in the market makes the mean of trading volumes increase by the lower value of 1.4954.

It is important to notice that the resulting coefficient ψ_{10} goes in the opposite way with respect to the coefficients γ_9 and γ_{12} of specification (5). In fact, in specification (5), we have affirmed that, when returns are positive, negative news affect more trading activity. Here, in the case of volatility jumps, we are observing the opposite: when returns are negative, positive news have more effect on trading volumes. The interpretation of the coefficients γ_9 and γ_{12} was clear: when the sentiment is positive and a negative news arrives in the market, agents are likely to trade more intensively rather than in the situation of positive news arrival. For the case of specification (10), the interpretation is not easy: the result of the coefficient ψ_{10} should derive from the fact that when a negative jump has occurred, positive news arriving in the market help price returns recover. By the way, the difference between ψ_{10} and ψ_{11} is not very high, so that we can generally affirm that, in the case of negative returns and with the presence of volatility jumps, one more news or announcement have a relevant effect on trading volumes.

For what concerns estimation result of our Multiplicative Error Model for trading volumes by considering 'newspapers' and 'upgrades and downgrades' topics of news and announcements, Table 12 and Table 13 sum all the results.

As regards the case of 'newspapers' topic, we can observe that the resulting parameters of the baseline specification of the Multiplicative Error Model are approximately the same of those presented in the general case. Then, in specification (11), we observe a statistically significant coefficient γ_1- equal to 0.0542 – meaning that one more 'newspapers' news arriving in the market in the interval $i-1$ affects trading volumes in the interval i by that amount. By separating this effect by news sentiment in specification (12), however, it can be shown that only the coefficient γ_3 is significant, meaning that 'newspapers' neutral news make trading volumes increase by 0.0748. The coefficients considering positive and negative news are not significant which may be due to the fact of not enough sentiment assignments to the news and announcements of this topic. In specification (13), in which the sign of price returns is also taken into account, we observe only one statistically significant coefficient. We refer to the coefficient γ_{11} which is equal to 0.1967, meaning that a one-unit increase in 'newspapers' news in the case of negative returns and neutral sentiment would affect trading volumes by that amount. The other coefficients of specification (13) are not statistically significant, meaning that the case of positive returns and the case of negative returns with positive and negative sentiment of 'newspapers' news have no causal effect on trading volumes.

For what concerns the inclusion of volatility jumps, we observe – in specification (14) – that the results of the general case are confirmed. In fact, the coefficients ψ_2 and ψ_3 are both statistically significant and signal a bigger effect of volatility jumps when there are news arrivals in the same intra-daily interval. In investigating the effect of one more 'newspapers' news or announcement in the presence of jumps in the cases of positive or negative results, we observe a statistically significant coefficient ψ_5, while a non-statistically significant coefficient ψ_4. These results confirm the general ones, affirming that one more negative 'newspapers' news, given the presence of volatility jumps, has more effect on the mean of trading volumes with respect to the case of positive news. For what concerns specification (16), as in the general case, the coefficients ψ_8 and ψ_9 are not statistically significant. Differently, the coefficients ψ_{10} and ψ_{11} are significant and respectively equal to 171.4940 and 4.3382. Although a very high coefficient ψ_{10}, it can be said that these results also confirm what said for specification (10). It means that, in presence of volatility jumps and in the case of negative returns, a one-unit increase in the 'newspapers' news variable significantly affect trading volumes.

By passing now to the 'upgrades and downgrades' news and announcements, it can be immediately observed that all coefficients of specifications (17), (18) and (19) are not statistically significant. This may be due to the low amount of news belonging to this topics, so that it has not been possible to derive a causal effect on trading volumes. By the way, results of specifications (20) and (21) are significant. In specification (20), the higher coefficient ψ_2 (1.5867) with respect to the coefficient ψ_3 (0.0794) signals, once again, the bigger influence of volatility jumps on trading volumes when there exist news and announcements arrivals in the market. As regards resulting estimates of specification (21), both coefficients ψ_4 and ψ_5 are statistically significant and are respectively equal to 2.9309 and 2.1311. In the case of 'upgrades and downgrades', we observe that, in presence of jumps, one more positive news makes trading volumes increase by 2.9309. Moreover, even when news are negative, a one-unit increase in news makes trading volumes increase by a large amount, that is 2.1311. Eventually, estimation results of specification (22) are not available, since the amount of 'upgrades and downgrades' company news and announcements given the presence of jumps, the separation in negative and positive returns cases together with that into positive and negative sentiment, is equal to zero.

Table 10. Estimation results, specifications from (1) to (5).

	(1)	(2)	(3)	(4)	(5)
α	0.0568*** (0.0003)	0.0568*** (0.0001)	0.0568*** (0.0009)	0.0565*** (0.0001)	0.0568*** (0.0003)
$\mu_{t,i-1}$	0.4485*** (0.0002)	0.4486*** (0.0002)	0.4486*** (0.0016)	0.3887*** (0.0003)	0.4488*** (0.0102)
$V_{t,i-1}$	0.4985*** (0.0003)	0.4981*** (0.0016)	0.4980*** (0.0017)	0.4894*** (0.0001)	0.4978*** (0.0118)
$N_{t,i-1}$		0.0498*** (0.0025)			
$N_{t,i-1}I_{\{Sent_{t,i-1}>0\}}$			0.0433** (0.0136)		
$N_{t,i-1}I_{\{Sent_{t,i-1}=0\}}$			0.0571*** (0.0121)		
$N_{t,i-1}I_{\{Sent_{t,i-1}<0\}}$			0.0467*** (0.0125)		
$\lvert R_{t,i-1}\rvert I_{\{R_{t,i-1}>0\}}$				0.0474*** (0.0013)	
$\lvert R_{t,i-1}\rvert I_{\{R_{t,i-1}<0\}}$				0.0159*** (0.0002)	
$N_{t,i-1}I_{\{R_{t,i-1}>0\}}I_{\{Sent_{t,i-1}>0\}}$					0.0597*** (0.0096)
$N_{t,i-1}I_{\{R_{t,i-1}>0\}}I_{\{Sent_{t,i-1}=0\}}$					0.0783*** (00047)
$N_{t,i-1}I_{\{R_{t,i-1}>0\}}I_{\{Sent_{t,i-1}<0\}}$					0.0713*** (0.0058)
$N_{t,i-1}I_{\{R_{t,i-1}<0\}}I_{\{Sent_{t,i-1}>0\}}$					0.0232*** (0.0085)
$N_{t,i-1}I_{\{R_{t,i-1}<0\}}I_{\{Sent_{t,i-1}=0\}}$					0.1067*** (0.0122)
$N_{t,i-1}I_{\{R_{t,i-1}<0\}}I_{\{Sent_{t,i-1}<0\}}$					0.0393*** (0.0035)
v	5.9688*** (0.0157)	5.9694*** (0.0443)	5.9694*** (0.0196)	6.1268*** (0.0126)	5.9698*** (0.0303)
AIC	-1.2841e+05	-1.2839e+05	-1.2838e+05	-1.2743e+05	-1.2836e+05
BIC	-1.2837e+05	-1.2834e+05	-1.2831e+05	-1.2737e+05	-1.2826e+05
observations	193,732	193,732	193,732	193,732	193,732

Note: standard errors in parenthesis; * p-value < 0.05; ** p-value < 0.01; *** p-value < 0.001

Table 11. Estimation results, specifications from (6) to (10).

	(6)	(7)	(8)	(9)	(10)
α	0.0562*** (0.0008)	0.0563*** (0.0009)	0.0568*** (0.0014)	0.0583*** (0.0007)	0.0568*** (0.0068)
$\mu_{t,i-1}$	0.4482*** (0.0067)	0.4484*** (0.0287)	0.4488*** (0.0238)	0.4457*** (0.0037)	0.4490*** (0.1060)
$V_{t,i-1}$	0.4981*** (0.0160)	0.4975*** (0.0326)	0.4981*** (0.0312)	0.4970*** (0.0007)	0.4979* (0.2157)
$J_{t,i-1}$	0.0804*** (0.0029)				
$J_{t,i-1}I_{\{N_{t,i-1>0}\}}$		0.7861*** (0.1523)			
$J_{t,i-1}I_{\{N_{t,i-1=0}\}}$		0.0798*** (0.0052)			
$N_{t,i-1}I_{\{J_{t,i-1>0}\}}I_{\{Sent_{t,i-1>0}\}}$			0.9894 (2.0077)		
$N_{t,i-1}I_{\{J_{t,i-1>0}\}}I_{\{Sent_{t,i-1<0}\}}$			0.6091*** (0.0670)		
$\lvert R\rvert_{t,i-1}I_{\{J_{t,i-1>0}\}}I_{\{R_{t,i-1>0}\}}$				0.0173*** (0.0015)	
$\lvert R\rvert_{t,i-1}I_{\{J_{t,i-1>0}\}}I_{\{R_{t,i-1<0}\}}$				0.0309*** (0.0015)	
$N_{t,i-1}I_{\{J_{t,i-1>0}\}}I_{\{R_{t,i-1>0}\}}I_{\{Sent_{t,i-1>0}\}}$					0.0150 (0.3259)
$N_{t,i-1}I_{\{J_{t,i-1>0}\}}I_{\{R_{t,i-1>0}\}}I_{\{Sent_{t,i-1<0}\}}$					0.0009 (0.5230)
$N_{t,i-1}I_{\{J_{t,i-1>0}\}}I_{\{R_{t,i-1<0}\}}I_{\{Sent_{t,i-1>0}\}}$					1.9923*** (0.2285)
$N_{t,i-1}I_{\{J_{t,i-1>0}\}}I_{\{R_{t,i-1<0}\}}I_{\{Sent_{t,i-1<0}\}}$					1.4954*** (0.3864)
v	5.9770*** (0.0196)	5.9783*** (0.0168)	5.9698*** (0.0802)	5.9880*** (0.0175)	5.9710*** (0.0153)
AIC	-1.2813e+05	-1.2808e+05	-1.2837e+05	-1.2775e+05	-1.2833e+05
BIC	1.2807e+05	-1.2802e+05	-1.2831e+05	-1.2769e+05	-1.2825e+05
observations	193,732	193,732	193,732	193,732	193,732

Note: standard errors in parenthesis; * p-value < 0.05; ** p-value < 0.01; *** p-value < 0.001

Table 12. Estimation results, specifications from (11) to (16).

	(11)	(12)	(13)
α	0.0568***	0.0568**	0.0568***
	(0.0003)	(0.0188)	(0.0009)
$\mu_{t,i-1}$	0.4485***	0.4485***	0.4486***
	(0.0001)	(0.0087)	(0.0085)
$V_{t,i-1}$	0.4984***	0.4984***	0.4983***
	(0.0005)	(0.0114)	(0.0192)
$N_{t,i-1}$	0.0542***		
	(0.0010)		
$N_{t,i-1}I_{\{Sent_{t,i-1}>0\}}$		0.0102	
		(0.0538)	
$N_{t,i-1}I_{\{Sent_{t,i-1}=0\}}$		0.0748**	
		(0.0264)	
$N_{t,i-1}I_{\{Sent_{t,i-1}<0\}}$		0.0609	
		(0.0312)	
$N_{t,i-1}I_{\{R_{t,i-1}>0\}}I_{\{Sent_{t,i-1}>0\}}$			0.0090
			(0.0869)
$N_{t,i-1}I_{\{R_{t,i-1}>0\}}I_{\{Sent_{t,i-1}=0\}}$			0.0047
			(0.0437)
$N_{t,i-1}I_{\{R_{t,i-1}>0\}}I_{\{Sent_{t,i-1}<0\}}$			0.0826
			(0.0639)
$N_{t,i-1}I_{\{R_{t,i-1}<0\}}I_{\{Sent_{t,i-1}>0\}}$			0.0203
			(0.0377)
$N_{t,i-1}I_{\{R_{t,i-1}<0\}}I_{\{Sent_{t,i-1}=0\}}$			0.1967***
			(0.0564)
$N_{t,i-1}I_{\{R_{t,i-1}<0\}}I_{\{Sent_{t,i-1}<0\}}$			0.0483
			(0.0606)
υ	5.9690***	5.9690***	5.9693***
	(0.0080)	(0.2138)	(0.0192)
AIC	-1.2840e+05	-1.2840e+05	-1.2838e+05
BIC	-1.2835e+05	-1.2832e+05	-1.2828e+05
observations	193,732	193,732	193,732

Note: standard errors in parenthesis; * p-value < 0.05; ** p-value < 0.01; *** p-value < 0.001

	(14)	(15)	(16)
α	0.0562***	0.0568***	0.0568***
	(0.0005)	(0.0005)	(0.0011)
$\mu_{t,i-1}$	0.4483***	0.4485***	0.4487***
	(0.0002)	(0.0023)	(0.0126)
$V_{t,i-1}$	0.4979***	0.4984***	0.4983***
	(0.0004)	(0.0002)	(0.0036)
$J_{t,i-1}I_{\{N_{t,i-1}>0\}}$	0.9359***		
	(0.0061)		
$J_{t,i-1}I_{\{N_{t,i-1}=0\}}$	0.0797***		
	(0.0013)		
$N_{t,i-1}I_{\{J_{t,i-1}>0\}}I_{\{Sent_{t,i-1}>0\}}$		0.0236	
		(0.0085)	
$N_{t,i-1}I_{\{J_{t,i-1}>0\}}I_{\{Sent_{t,i-1}<0\}}$		0.3137***	
		(0.0117)	
$N_{t,i-1}I_{\{J_{t,i-1}>0\}}I_{\{R_{t,i-1}>0\}}I_{\{Sent_{t,i-1}>0\}}$			0.0237
			(0.3301)
$N_{t,i-1}I_{\{J_{t,i-1}>0\}}I_{\{R_{t,i-1}>0\}}I_{\{Sent_{t,i-1}<0\}}$			0.0293
			(0.1389)
$N_{t,i-1}I_{\{J_{t,i-1}>0\}}I_{\{R_{t,i-1}<0\}}I_{\{Sent_{t,i-1}>0\}}$			171.4940***
			(1.3963)
$N_{t,i-1}I_{\{J_{t,i-1}>0\}}I_{\{R_{t,i-1}<0\}}I_{\{Sent_{t,i-1}<0\}}$			4.3382*
			(1.7007)
υ	5.9775***	5.9689***	5.9693***
	(0.0177)	(0.0183)	(0.0180)
AIC	-1.2811e+05	-1.2840e+05	-1.2838e+05
BIC	-1.2805e+05	-1.2834e+05	-1.2830e+05
observations	193,732	193,732	193,732

Note: standard errors in parenthesis; * p-value < 0.05; ** p-value < 0.01; *** p-value < 0.001

Table 13. Estimation results, specifications from (17) to (21).

	(17)	(18)	(19)
α	0.0568***	0.0568	0.0568***
	(0.0033)	(0.0717)	(0.0013)
$\mu_{t,i-1}$	0.4485***	0.4485*	0.4486
	(0.0283)	(0.2101)	(1.0725)
$V_{t,i-1}$	0.4984***	0.4984	0.4983**
	(0.0101)	(0.4427)	(0.1839)
$N_{t,i-1}$	0.0360		
	(0.1047)		
$N_{t,i-1} I_{\{Sent_{t,i-1}>0\}}$		0.0470	
		(0.3365)	
$N_{t,i-1} I_{\{Sent_{t,i-1}=0\}}$		0.0003	
		(0.0673)	
$N_{t,i-1} I_{\{Sent_{t,i-1}<0\}}$		0.0816	
		(0.3732)	
$N_{t,i-1} I_{\{R_{t,i-1}>0\}} I_{\{Sent_{t,i-1}>0\}}$			0.0877
			(0.3769)
$N_{t,i-1} I_{\{R_{t,i-1}>0\}} I_{\{Sent_{t,i-1}=0\}}$			0.0000
			(0.7532)
$N_{t,i-1} I_{\{R_{t,i-1}>0\}} I_{\{Sent_{t,i-1}<0\}}$			0.0000
			(0.9015)
$N_{t,i-1} I_{\{R_{t,i-1}<0\}} I_{\{Sent_{t,i-1}>0\}}$			0.0000
			(0.5883)
$N_{t,i-1} I_{\{R_{t,i-1}<0\}} I_{\{Sent_{t,i-1}=0\}}$			0.0000
			(0.3514)
$N_{t,i-1} I_{\{R_{t,i-1}<0\}} I_{\{Sent_{t,i-1}<0\}}$			0.0000
			(0.5019)
v	5.9689***	5.9689***	5.9691***
	(0.0184)	(0.3363)	(0.2288)
AIC	-1.2840e+05	-1.2840e+05	-1.2839e+05
BIC	-1.2835e+05	-1.2833e+05	-1.2829e+05
observations	193,732	193,732	193,732

Note: standard errors in parenthesis; * p-value < 0.05; ** p-value < 0.01; *** p-value < 0.001

	(20)	(21)
α	0.0562***	0.0568***
	(0.0013)	(0.0002)
$\mu_{t,i-1}$	0.4484***	0.4488***
	(0.0025)	(0.0002)
$V_{t,i-1}$	0.4978***	0.4981***
	(0.0074)	(0.0001)
$J_{t,i-1} I_{\{N_{t,i-1}>0\}}$	1.5867***	
	(0.3914)	
$J_{t,i-1} I_{\{N_{t,i-1}=0\}}$	0.0794***	
	(0.0120)	
$N_{t,i-1} I_{\{J_{t,i-1}>0\}} I_{\{Sent_{t,i-1}>0\}}$		2.9309***
		(0.0148)
$N_{t,i-1} I_{\{J_{t,i-1}>0\}} I_{\{Sent_{t,i-1}<0\}}$		2.1311***
		(0.0060)
v	5.9779***	5.9700***
	(0.1208)	(0.0184)
AIC	-1.2809e+05	-1.2836e+05
BIC	-1.2803e+05	-1.2830e+05
observations	193,732	193,732

Note: standard errors in parenthesis; * p-value < 0.05; ** p-value < 0.01;
*** p-value < 0.001

4 CONCLUSIONS

In the Introduction of this thesis, we posed the question about whether company news and announcements would have any effect on trading volumes. This question has been developed in several aspects according to the different specifications defined for our Multiplicative Error Model for trading volumes. We remind that they have been grouped to answer these different questions:

a) Do news and announcements have a causal effect on trading volumes? If yes, does it change according to the sentiment of news and announcements?
b) Do positive and negative price returns have a causal effect on trading volumes? If yes, in both cases, what is the effect of news and announcements according to their sentiment?
c) Do price volatility jumps have a causal effect on trading volumes? If yes, does it change according to the presence or the lack of news and announcements?
d) Given the presence of volatility jumps, what is the effect of news and announcements on trading volumes according to their sentiment?
e) Do positive and negative price returns have a casual effect on trading volumes, given the presence of volatility jumps? If yes, what is the effect of news and announcements – with both positive and negative returns – according to their sentiment, given the presence of volatility jumps?

The answers to these questions have been already delivered in the previous Section, but here we would like to highlight the most important findings.

First, company news and announcements have a relevant effect on trading volumes. Moreover, it has been observed that this effect is different according to their sentiment. Negative news – after those with neutral sentiment – have a bigger effect on determining trading volumes, signalling for a risk-averse behaviour in agents trading in the market.

Second, taking into account price returns, it is possible to affirm that their changes in absolute terms also have a relevant effect on trading volumes. Moreover, by considering the effect of the amount and the sentiment of news and announcements in the cases returns are positive or negative, we can affirm that, also in this case, negative news – beyond neutral news – have a bigger effect on volumes with respect to the positive ones.

Third, volatility jumps have a significant effect on trading volumes, especially in the case of presence of company news and announcements arrivals. Furthermore, when news sentiment is taken into account, it can be observed that, given the presence of volatility jumps, negative news and announcements have more effect in determining trading volumes with respect to the positive sentiment case.

Fourth – and last – in presence of volatility jumps, changes in price returns in absolute terms affect trading volumes more when returns they are negative. By the way, in presence of jumps and of negative returns, news and announcements have more effect in determining trading volumes when their sentiment is positive rather than negative.

In the light of all these results, it is then evident the link between news arrival and a bigger trading activity. This happens because of the existence of informed traders who trade a certain security with the interest of making profits before the others, making also the price of that security reflect information come into the market. Moreover, it is evident that, generally, agents are risk-averse since they respond more to negative news with respect to the positive ones in both cases of positive and negative overall market sentiment (given by the sign of price returns in the same intra-daily interval). By the way, this is discovered not true in presence of volatility jumps, where it seems that, in case of negative returns, positive news have a bigger impact on trading activity.

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APPENDIXES

A. Maximum Likelihood estimation method

Maximum Likelihood is a method of estimating population characteristics from a sample, by choosing the values of parameters that will maximize the probability of getting the particular sample actually obtained from the population.

The likelihood inference procedure follows these steps.

Step 1. Extract a sample constituted by n i.i.d. random variables X_i with $i = 1, \dots, n$ from a population X which have probability function $f(x, \theta)$.

Step 2. Build the likelihood function $L(x, \theta)$ that is the probability function of the sample assumed that the likelihood function is a function of the parameters θ and that the sample contains fixed realizations x_i of the random variable X_i. The likelihood function is defined as:

$$L(x_1, \dots, x_n; \theta) = \prod_{i=1}^{n} f(x, \theta) .$$

Step 3. Define the log-likelihood function as the logarithm of the likelihood function in order to compute the Maximum Likelihood Estimators $\hat{\theta}$ (hereafter called MLEs):

$$\ell(x, \theta) = \ln L(x, \theta) = \ln \left[\prod_{i=1}^{n} f(x, \theta) \right] = \sum_{i=1}^{n} f(x, \theta) .$$

The log-likelihood function ends up to be the sum of the probability functions over sample observations.

Step 4. Compute MLEs $\hat{\theta}$ as solution of the following maximization problem:

$$\max_{\theta} \ell(x, \theta) .$$

By omitting the argument x for simplicity's sake, the log-likelihood function becomes $\ell(\theta)$. The maximization problem is solved by taking partial derivatives of the log-likelihood function with respect to each of the parameters in the vector θ, and by setting them equal to zero:

$$S(\theta) = 0$$

where $S(\theta)$ is the so-called score or gradient vector which contains all first partial derivatives:

$$S(\theta) \equiv \frac{\partial}{\partial \theta} \ell(\theta) .$$

At the maximum, the second derivative of the log-likelihood is negative.

The matrix of second partial derivatives is called Hessian matrix and is defined as:

$$H(\theta) \equiv \frac{\partial^2}{\partial\theta}\ell(\theta)\,.$$

The curvature at MLEs $\hat{\theta}$ of the log-likelihood is a key quantity in Maximum Likelihood inference. It is called the observed Fisher information matrix and is defined as the negative of the Hessian matrix:

$$I(\hat{\theta}) \equiv -\frac{\partial^2}{\partial\theta}\ell(\theta)\,.$$

Since it is evaluated at $\hat{\theta}$, it contains numbers rather than functions. Low information translates into a high variance of MLEs and confidence intervals will be wide. In contrast, high information translates into a low variance of estimators and into narrow confidence intervals.

Step 5. Compute variance and standard errors of MLEs by using the information matrix $I(\hat{\theta})$. In the normal case, variance is defined as

$$Var(\hat{\theta}) = I^{-1}(\hat{\theta})$$

and, consequently, standard errors are

$$se(\hat{\theta}) = I^{-1/2}(\hat{\theta}).$$

This results are approximately true in non-normal cases also. Therefore, standard errors, calculated as the inverse of the square root of the diagonal elements of the observed Fisher information matrix, are commonly used in supplementing the MLEs.

ML estimates have good properties. In the following some asymptotic properties of MLEs are listed.

a) $\hat{\theta}$ is a consistent estimator of θ, meaning that $|\hat{\theta}-\theta| \to 0$ as $n \to \infty$ where n is the number of observations in the sample.
b) $\hat{\theta}$ is asymptotically unbiased, that is $\lim_{n\to\infty} \mathbb{E}(\hat{\theta}) = \theta$. MLEs can be biased but the bias disappears as the sample size increases.
c) $\hat{\theta}$ is asymptotically efficient which means that, among all asymptotically unbiased estimators, it has the minimum variance asymptotically.
d) The variance of MLEs is known. For large n, it is defined by the inverse of the information matrix: $Var(\hat{\theta}) = I^{-1}(\hat{\theta})$. It is an important result because it allows to compute standard errors of a MLE.
e) $\hat{\theta}$ is asymptotically normally distributed, for large n. This result permits hypothesis testing and the construction of confidence intervals.

With properties b), d) and e) it is possible to conclude that, for large samples, MLE $\hat{\theta}$ of a population parameter θ has an approximate normal distribution with mean θ and variance $I^{-1}(\hat{\theta})$.

Standard errors are used in the so-called Wald test whose null hypothesis is $H_0: \theta = \theta_0$ and alternative hypothesis is $H_1: \theta \neq \theta_0$. After some computations which take into account the curvature of the log-likelihood function, the Wald statistic is defined as:

$$W = \frac{\hat{\theta} - \theta_0}{se(\hat{\theta})}.$$

It is a z-score and, since MLEs are asymptotically normally distributed, it follows that W must follow a standard normal distribution. Thanks to this result, Wald confidence intervals with 5% significance level can be constructed in the following way:

$$\hat{\theta} \pm z_{.0975} se(\hat{\theta})$$

where $z_{.0975}$ is the 0.975 quantile of a standard normal distribution which equals 1.96.

B. Gamma distribution

Gamma distribution is a two-parameter family of continuous probability distributions.
There are three possible different parameterizations. In order to show their equivalence, we start from the one approached in this thesis. In Chapter 2 we saw how the the error term ε_t of the baseline Multiplicative Error Model follows a Gamma distribution, namely $\Gamma(a, b)$, where a and b are respectively the scale and the shape parameter. This representation is called shape-scale parameterization. generally, if a random variable X follows a $\Gamma(a, b)$, its probability density function is defined as:

$$f(X; a, b) \sim \frac{1}{\Gamma(a) b^a} x^{a-1} \exp\left(-\frac{x}{b}\right)$$

where $\Gamma(a)$ is the gamma function evaluated at a. Moreover, the first and the second moments are defined as $\mathbb{E}(X) = a \cdot b$, and $Var(X) = a \cdot b^2$.
For the purpose of this work – and in many other cases – the mean was required to be equal to 1 – the so-called unit mean property – so we find the following relationship between the shape and the scale parameters:

$$\mathbb{E}(X) = a \cdot b = 1 \Leftrightarrow b = \frac{1}{a}.$$

Therefore, the probability density function becomes:

$$f(X; a, b) \sim \frac{1}{\Gamma(a)} a^{a} x^{a-1} \exp(-ax).$$

This property is directly detectable from the second possible representation of the Gamma distribution, called mean-shape parameterization. In fact, the process is said to follow a $\Gamma(a, 1)$, where a is the usual shape parameter and the second parameter is simply the mean μ computed exactly as before.

Lastly, the shape-rate parameterization shows the random variable X to follow a $\Gamma(\alpha, \beta)$. The scale parameter α does not change, i.e. $a = \alpha$, and the second – called rate parameter – is defined as $\beta = \frac{1}{b}$. The probability density function becomes:

$$f(X; \alpha, \beta) \sim \frac{\beta^{\alpha}}{\Gamma(\alpha)} x^{a-1} \exp(-\beta x)$$

that is exactly equivalent to that of the scale-shape representation given the unit mean property. Therefore, in Chapter 2 the $\Gamma(a, b)$ in scale-shape representation is equivalent to the shape-rate parameterization $\Gamma(\phi, \phi)$ in Cipollini, Engle and Gallo (2006).

In the empirical analysis of this thesis, we use a $\Gamma(v, 1)$ in shape-mean representation for the error term of the Multiplicative Error Model. It then becomes a $\Gamma(v, \mu_{t,i})$ where $\mu_{t,i}$ is the mean of trading volumes modelled as $V_{t,i} = \mu_{t,i}\varepsilon_{t,i}$. In the empirical analysis, several conditional mean specifications are discussed together with the form of log-likelihood function for Maximum Likelihood estimation of their parameters.

In Figure A, the Gamma probability density function for different value of shape b and scale a parameters is plotted.

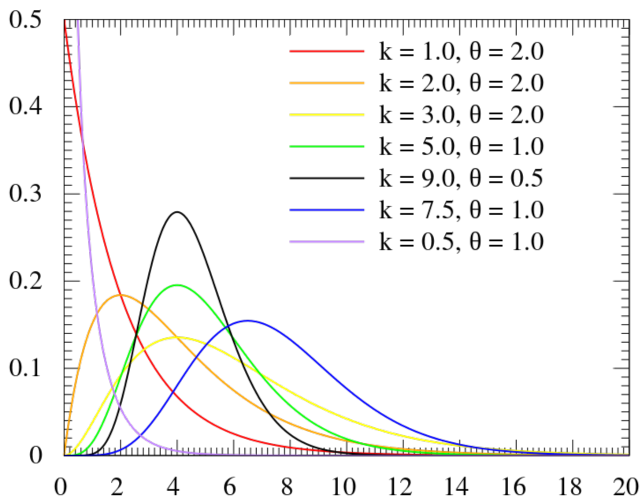

Figure B1. Gamma probability density function. Source: Wikipedia.

C. Regression results for periodicity

	$\log(V_{t,i})$	$\log(\lvert R_{t,i}\rvert)$
intercept	12.6992***	-8.8474***
	(0.0063)	(0.0139)
t	0.1139***	0.1116***
	(0.0028)	(0.0062)
t^2	-0.0128***	-0.0189***
	(0.0003)	(0.0007)
t^3	0.0002***	0.0006***
	(0.0000)	(0.0000)
$cos_1[2\pi jf(t)]$	0.4360***	0.2714***
	(0.0022)	(0.0049)
$sin_1[2\pi jf(t)]$	0.2037***	0.1679***
	(0.0022)	(0.0049)
$cos_2[2\pi jf(t)]$	0.1122***	0.0620***
	(0.0022)	(0.0049)
$sin_2[2\pi jf(t)]$	0.1040***	0.1146***
	(0.0022)	(0.0049)
$cos_3[2\pi jf(t)]$	0.0526***	0.0251***
	(0.0022)	(0.0049)
$sin_3[2\pi jf(t)]$	0.0517***	0.0645***
	(0.0022)	(0.0049)
$cos_4[2\pi jf(t)]$	0.0191***	0.0094
	(0.0022)	(0.0049)
$sin_4[2\pi jf(t)]$	0.0155***	0.0246***
	(0.0022)	(0.0049)
R-squared	0.4255	0.0507
Adj. R-squared	0.4255	0.0507
observations	193,732	193,732
Note: standard errors in parenthesis; * p-value < 0.05; ** p-value < 0.01; *** p-value < 0.001		

D. Matlab codes

Removing periodicity for trading volumes

```
clear;
clc;
load('QUANT_77','volume5');

T=77*2516;
t=((1:T)')/10000; % trend

c01=cos(2*pi*1*(1:T)/(1*77))';     % amplitude 1
s01=sin(2*pi*1*(1:T)/(1*77))';

c02=cos(2*pi*2*(1:T)/(1*77))';     % amplitude 2
s02=sin(2*pi*2*(1:T)/(1*77))';

c03=cos(2*pi*4*(1:T)/(1*77))';     % amplitude 4
s03=sin(2*pi*4*(1:T)/(1*77))';

c04=cos(2*pi*8*(1:T)/(1*77))';     % amplitude 8
s04=sin(2*pi*8*(1:T)/(1*77))';

X=[t t.^2 t.^3 c01 s01 c02 s02 c03 s03 c04 s04];

Y=vec(log(volume5)');

reg=fitlm(X,Y);

fitted=exp(reg.Fitted);
residual=exp(Y)./fitted;     % time series without periodicity

fittedr=(reshape(fitted,77,size(fitted,1)/77))';
figure
plot(mean(fittedr))

volumeD=(reshape(residual,77,size(residual,1)/77))';     % reshape
figure
plot(mean(volumeD))

save('volumeD','volumeD');
```

Removing periodicity for absolute returns

```
clear;
clc;
load('QUANT_77','ret5');

rM=mean(mean(ret5));
r=ret5-rM;
abr=abs(r);

T=77*2516;
t=((1:T)')/10000; % trend

c01=cos(2*pi*1*(1:T)/(1*77))';     % amplitude 1
s01=sin(2*pi*1*(1:T)/(1*77))';

c02=cos(2*pi*2*(1:T)/(1*77))';     % amplitude 2
s02=sin(2*pi*2*(1:T)/(1*77))';

c03=cos(2*pi*4*(1:T)/(1*77))';     % amplitude 4
s03=sin(2*pi*4*(1:T)/(1*77))';

c04=cos(2*pi*8*(1:T)/(1*77))';     % amplitude 8
s04=sin(2*pi*8*(1:T)/(1*77))';

X=[t t.^2 t.^3 c01 s01 c02 s02 c03 s03 c04 s04];
```

```
Y=vec(log(abr)');

reg=fitlm(X,Y);

fitted=exp(reg.Fitted);
residual=exp(Y)./fitted;      % time series without periodicity

fittedr=(reshape(fitted,77,size(fitted,1)/77))';
figure
plot(mean(fittedr))

abrD=(reshape(residual,77,size(residual,1)/77))';    % reshape
figure
plot(mean(abrD))

save('abrD','abrD');
```

Estimating the Multiplicative Error Model

Example: specification (1)

```
% define log-likelihood function
function ll=loglike1(x,data)

v=x(4); % v (density)
a=x(1); % a
b=x(2); % b
d=x(3); % d
T=length(data);

L1=T*v*log(v);
L2=-(T*gammaln(v));
L3=(v-1)*sum(log(data(:,1)));

mu=zeros(T,1);
mu(1)=a;
for i=2:T
    mu(i)=a+b*mu(i-1)+d*data(i-1,1);
end;

L4=-(v*sum(log(mu)))-v*sum(data(:,1)./mu);

ll=-(L1+L2+L3+L4);
end

% load data
clear;
clc;
load('volumeD');

volumeD = vec(volumeD');
mV      = mean(volumeD);
volumeD = volumeD./mV;

% build data matrix
data    = volumeD;

% set constraints for maximization problem
x0      = [0.001;0.001;0.001;0.001];
A       = -(eye(4));
b       = zeros(4,1);
lb      = zeros(4,1);
ub      = [inf;1;inf;inf];

% solve maximization problem
options = optimset('Algorithm','interior-
point','Display','iter','AlwaysHonorConstraints','bounds');
[x,fval,exitflag,output,lambda,grad,hessian] =
fmincon(@loglike1,x0,A,b,[],[],lb,ub,[],options,data);
```

```
% build std errors, aic/bic, p-values
se=sqrt(diag(inv(hessian)));

[aic,bic]=aicbic(fval,4,193732);

z=x./se;
pvalues = 2*(1-normcdf(abs(z)));

% save
save('model1');
```

Example: Specification (5)

```
% define log-likelihood function
function ll=logl4(x,data)

v=x(4); % v (density)
a=x(1); % a
b=x(2); % b
d=x(3); % d
g7=x(5); % g7
g8=x(6); % g8
g9=x(7); % g9
g10=x(8); % g10
g11=x(9); % g11
g12=x(10); % g12
T=length(data);

L1=T*v*log(v);
L2=-(T*gammaln(v));
L3=(v-1)*sum(log(data(:,1)));

mu=zeros(T,1);
mu(1)=a;
for i=2:T
    mu(i)=a+b*mu(i-1)+d*data(i-1,1)+g7*data(i-1,2)+g8*data(i-1,3)+g9*data(i-
1,4)+g10*data(i-1,5)+g11*data(i-1,6)+g12*data(i-1,7);
end;

L4=-(v*sum(log(mu)))-v*sum(data(:,1)./mu);

ll=-(L1+L2+L3+L4);
end

% load data
clear;
clc;
load('volumeD');
load('QUAL_77','all_n5','all_s5');
load('QUANT_77','ret5');
load('abrD');

volumeD = vec(volumeD');
mV      = mean(volumeD);
volumeD = volumeD./mV;
all_n5  = vec(all_n5');
all_s5  = vec(all_s5');
ret5    = vec(ret5');
absret  = vec(abrD');

% build data matrix
data = volumeD;

for i=1:193732;
    if ret5(i,1)>0 && all_s5(i,1)>0;
        data(i,2)=all_n5(i,1);
    else
        data(i,2)=0;
```

```
    end;
end;

for i=1:193732;
    if ret5(i,1)>0 && all_s5(i,1)==0;
        data(i,3)=all_n5(i,1);
    else
        data(i,3)=0;
    end;
end;

for i=1:193732;
    if ret5(i,1)>0 && all_s5(i,1)<0;
        data(i,4)=all_n5(i,1);
    else
        data(i,4)=0;
    end;
end;

for i=1:193732;
    if ret5(i,1)<0 && all_s5(i,1)>0;
        data(i,5)=all_n5(i,1);
    else
        data(i,5)=0;
    end;
end;

for i=1:193732;
    if ret5(i,1)<0 && all_s5(i,1)==0;
        data(i,6)=all_n5(i,1);
    else
        data(i,6)=0;
    end;
end;

for i=1:193732;
    if ret5(i,1)<0 && all_s5(i,1)<0;
        data(i,7)=all_n5(i,1);
    else
        data(i,7)=0;
    end;
end;

% set constraints for maximization problem
x0      = [0.001;0.001;0.001;0.001;0.001;0.001;0.001;0.001;0.001;0.001];
A       = -(eye(10));
b       = zeros(10,1);
lb      = zeros(10,1);
ub      = [inf;1;inf;inf;inf;inf;inf;inf;inf;inf];

% solve maximization problem
options =optimset('Algorithm','interior-
point','Display','iter','AlwaysHonorConstraints','bounds','MaxFunEval', 15000);
[x,fval,exitflag,output,lambda,grad,hessian] =
fmincon(@logl4,x0,A,b,[],[],lb,ub,[],options,data);

% build std errors, aic/bic, p-values
se=sqrt(diag(inv(hessian)));

[aic,bic]=aicbic(fval,10,193732);

z=x./se;
pvalues = 2*(1-normcdf(abs(z)));

% save
save('model5');
```

Example: Specification (6)

```
% define log-likelihood function
function ll=loglike5(x,data)

v=x(4); % v (density)
a=x(1); % a
b=x(2); % b
d=x(3); % d
ps1=x(5); % ps1
T=length(data);

L1=T*v*log(v);
L2=-(T*gammaln(v));
L3=(v-1)*sum(log(data(:,1)));

mu=zeros(T,1);
mu(1)=a;
for i=2:T
    mu(i)=a+b*mu(i-1)+d*data(i-1,1)+ps1*data(i-1,2);
end;

L4=-(v*sum(log(mu)))-v*sum(data(:,1)./mu);

ll=-(L1+L2+L3+L4);
end

% load data
clear;
clc;
load('volumeD');
load('QUAL_77','all_n5','all_s5');
load('jump');

volumeD = vec(volumeD');
mV      = mean(volumeD);
volumeD = volumeD./mV;
jump = vec(jump');

% build data matrix
data    = [volumeD jump];

% set constraints for maximization problem
x0      = [0.001;0.001;0.001;0.001;0.001];
A       = -(eye(5));
b       = zeros(5,1);
lb      = zeros(5,1);
ub      = [inf;1;inf;inf;inf];

% solve maximization problem
options = optimset('Algorithm','interior-
point','Display','iter','AlwaysHonorConstraints','bounds');
[x,fval,exitflag,output,lambda,grad,hessian] =
fmincon(@loglike5,x0,A,b,[],[],lb,ub,[],options,data);

% build std errors, aic/bic, p-values
se=sqrt(diag(inv(hessian)));

[aic,bic]=aicbic(fval,5,193732);

z=x./se;
pvalues = 2*(1-normcdf(abs(z)));

% save
save('model6');
```

Example: Specification (10)

```
% define log-likelihood function
function ll=logl10(x,data)

v=x(4); % v (density)
a=x(1); % a
b=x(2); % b
d=x(3); % d
ps8=x(5); % psi8
ps9=x(6); % psi9
ps10=x(7); % psi10
ps11=x(8); % psi11
T=length(data);

L1=T*v*log(v);
L2=-(T*gammaln(v));
L3=(v-1)*sum(log(data(:,1)));

mu=zeros(T,1);
mu(1)=a;
for i=2:T
    mu(i)=a+b*mu(i-1)+d*data(i-1,1)+ps8*data(i-1,2)+ps9*data(i-1,3)+ps10*data(i-
1,4)+ps11*data(i-1,5);
end;

L4=-(v*sum(log(mu)))-v*sum(data(:,1)./mu);

ll=-(L1+L2+L3+L4);
end

% load data
clear;
clc;
load('volumeD');
load('QUAL_77','all_n5','all_s5');
load('jump');
load('QUANT_77','ret5');

volumeD = vec(volumeD');
mV      = mean(volumeD);
volumeD = volumeD./mV;
all_n5  = vec(all_n5');
all_s5  = vec(all_s5');
ret5    = vec(ret5');
jump = vec(jump');

% build data matrix
data = volumeD;

for i=1:193732;
    if jump(i,1)>0 && ret5(i,1)>0 && all_s5(i,1)>0;
        data(i,2)=all_n5(i,1);
    else
        data(i,2)=0;
    end;
end;

for i=1:193732;
    if jump(i,1)>0 && ret5(i,1)>0 && all_s5(i,1)<0;
        data(i,3)=all_n5(i,1);
    else
        data(i,3)=0;
    end;
end;

for i=1:193732;
    if jump(i,1)>0 && ret5(i,1)<0 && all_s5(i,1)>0;
        data(i,4)=all_n5(i,1);
    else
        data(i,4)=0;
```

```
    end;
end;

for i=1:193732;
    if jump(i,1)>0 && ret5(i,1)<0 && all_s5(i,1)<0;
        data(i,5)=all_n5(i,1);
    else
        data(i,5)=0;
    end;
end;

% set constraints for maximization problem
x0      = [0.001;0.001;0.001;0.001;0.001;0.001;0.001;0.001];
A       = -(eye(8));
b       = zeros(8,1);
lb      = zeros(8,1);
ub      = [inf;1;inf;inf;inf;inf;inf;inf];

% solve maximization problem
options =optimset('Algorithm','interior-
point','Display','iter','AlwaysHonorConstraints','bounds');
[x,fval,exitflag,output,lambda,grad,hessian] =
fmincon(@logl10,x0,A,b,[],[],lb,ub,[],options,data);

% build std errors, aic/bic, p-values
se=sqrt(diag(inv(hessian)));

[aic,bic]=aicbic(fval,8,193732);

z=x./se;
pvalues = 2*(1-normcdf(abs(z)));

% save
save('model10');
```