UNIVERSITY OF PADUA

Department of Industrial Engineering DII

Degree in Mechanical Engineering

Thesis for the final test

ANALYTICAL METHODS FOR FATIGUE DESIGN WITH NOTCH EFFECTS

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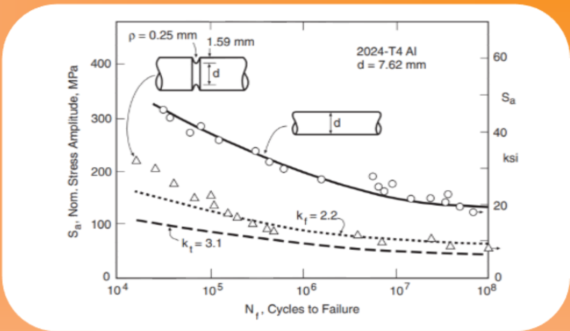
- ## Reasons:

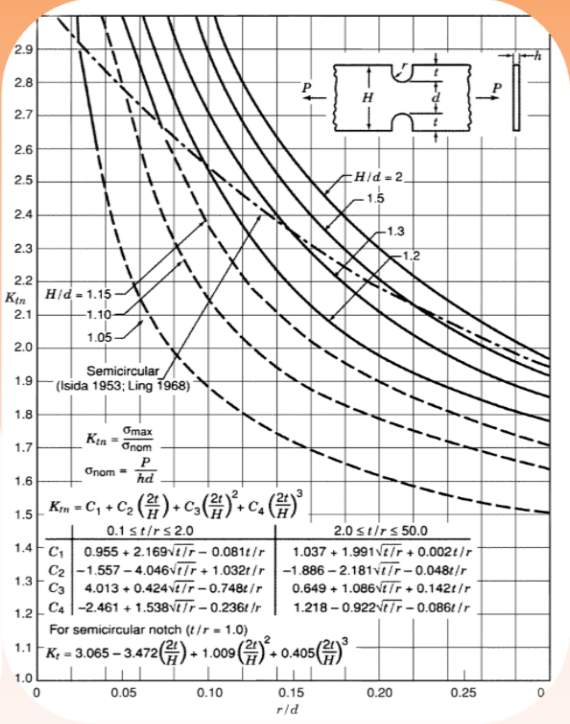
- *For small notch radii, the formulas for K_f and for the fatigue limit seen in Machine Design course's lectures, based on the Peterson studies, differed from the empirical data available in literature.*

- *As K_{tn} increases, the real fatigue limit continues to decrease, but not as quickly as expected, then it tends to a plateau that the Peterson formula does not describe well. As the notch radius decreases, the notch acts as a crack of equal depth at some point.*



- ## Aim:

- *I therefore decided to address the topic of descriptions of alternative methods that can be found in the literature that solve, at least in part, this problem, which was later studied in MatLab environment. The fidelity between the parameters K_{tn} taken from the book Peterson's Stress Concentration Factors by Walter D. Pilkey and its possible approximations of common use for some geometries has been verified in this relation.*



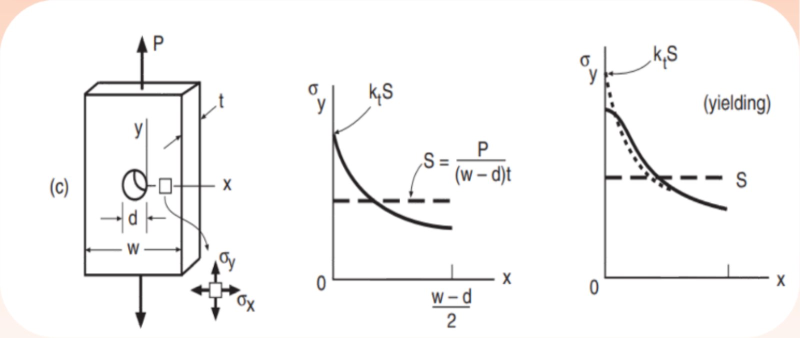
- ## Introduction:

- *This article deals with the fatigue study for notched components, mainly through a stress-control approach. The article investigates the tension of specimens with stress raisers.*

- *Slope of the notched specimen shifted down from the one of the unnotched piece. Factor that characterize the severity of the notch as a stress raiser: $K_t = \frac{\sigma}{S}$.*

- *Example of values of K_t (Peterson's stress concentration factors).*

- *Assumed linear elastic behaviour of the piece.*



Phases

- *The work is divided in two phases.*
- *1) the concept of fatigue of materials, parameters and related factors that influence their resistance are introduced. Then different methods for the fatigue study of notched components are presented.*
- *2) Finally, the results from the graphs of the book Peterson's Stress Concentration Factors will be compared with graphs produced in MatLab environment through approximated formulas.*

Index

Matlab code for an isotropic two-dimensional drilled plate, under uniaxial tension

- $K_{tg} = \frac{2+(1-\frac{d}{W})^3}{1-\frac{d}{W}}$, $K_{tn} = 2 + (1-\frac{d}{W})^3$ (Heywood (1952)).
- Further formulae of theoretical stress concentration factors are obtained from the Peterson manual.
- Matlab script used to study how, as the ratio $\frac{d}{W}$ varies, vary:

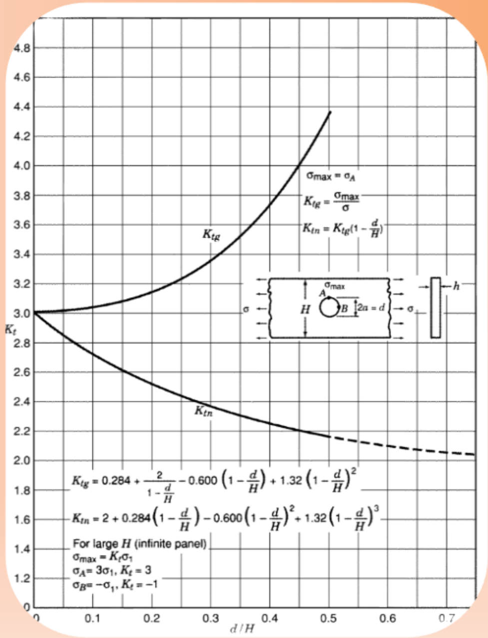
A) K_{tg} and K_{tn} for Peterson formulae and the approximate ones from Heywood

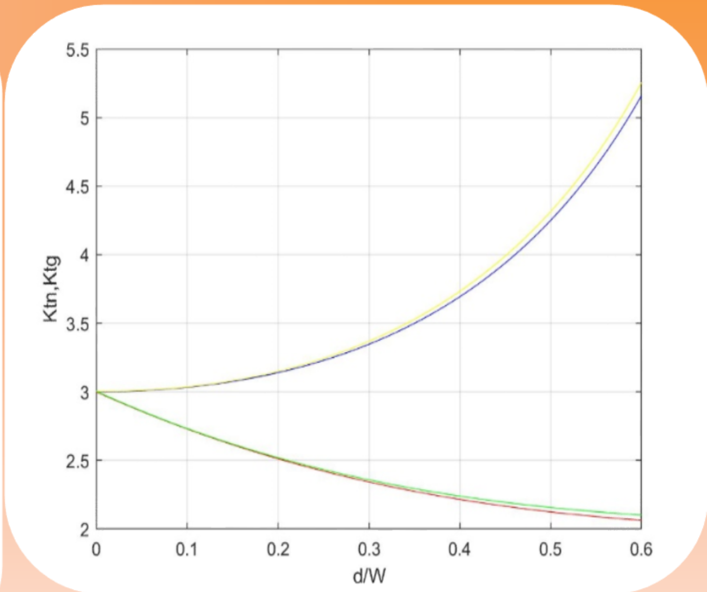
B) the notch sensitivity index q

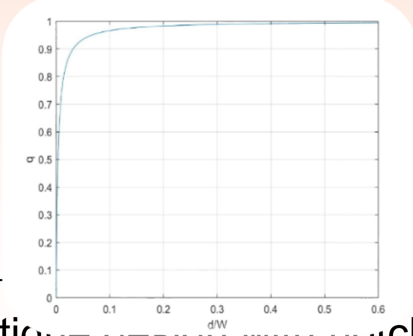
C) the fatigue life reduction factor K_f

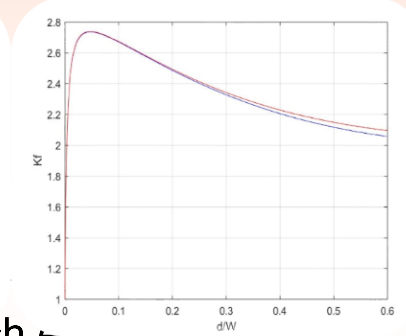
D) the fatigue limit for a notched specimen

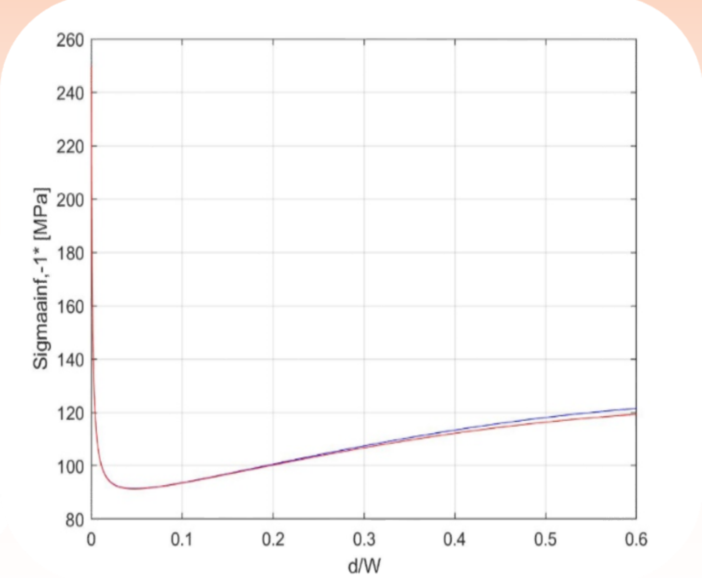
- In blue the theoretical stress concentration factor referred to the gross tension (Heywood formula);in yellow the same factor (Peterson's formula); in red the theoretical stress formula); in red the theoretical stress concentration factor referred to the net tension (Heywood formula); in green the same factor through (Peterson's formula).
- Notch sensitivity index →(Peterson) $q \triangleq \frac{1}{1+\frac{a}{r}}$.
- Radius ↔ $\frac{d}{W}$. (Material with tensile strength of 500 MPa → a=0.265 from table IX of the UNI 7670).
- In blue the trend of the fatigue life reduction factor for a notched specimen (Heywood); in red the "more correct formula" to calculate K_{tn}.
- R_f=0.5 + $\sigma_R = 500\ MPa \rightarrow \sigma_{a\infty,-1} = 250\ MPa.$
- **syms x y**
- **dersig1= diff(realtension1,x);**
- **dersig11=matlabFunction(dersig1);**
- **dersig2= diff(realtension2,x);**
- **dersig22=matlabFunction(dersig2);**
- **criticalpoint1=fzero(dersig11,0.2)**
- **criticalpoint2=fzero(dersig22,0.2)**
- **criticalpoint1 = 0.0475**
- **criticalpoint2 = 0.0474**
- Critical at a value of $\frac{d}{W} \cong 0.047$, which corresponds to a diameter of the hole of $d \cong 7\ mm$ for a plate's width of $W = 150\ mm.$





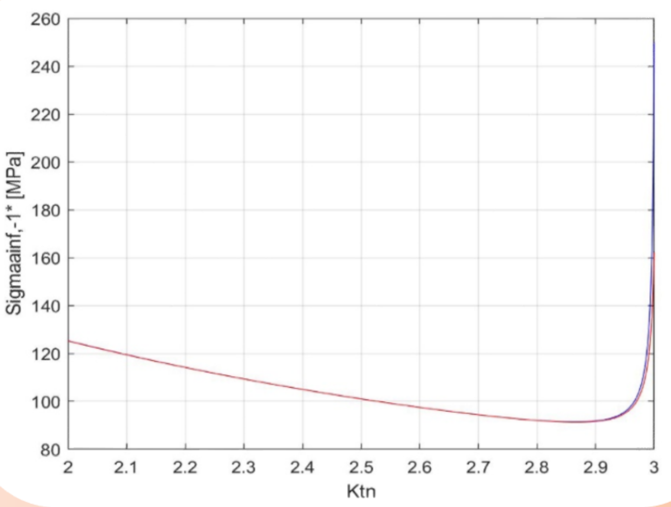


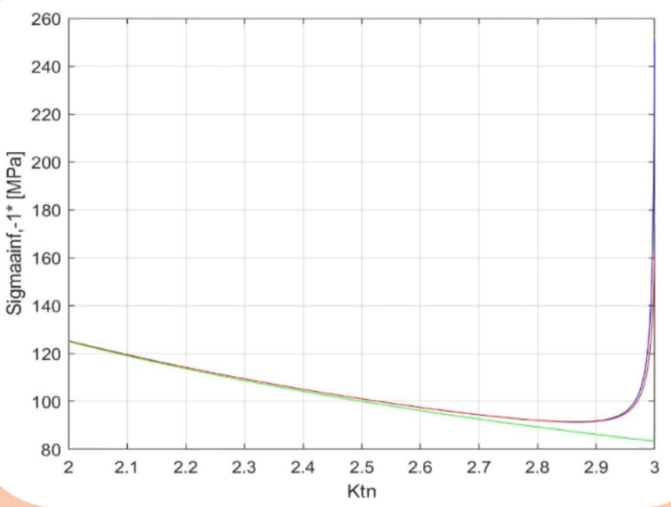


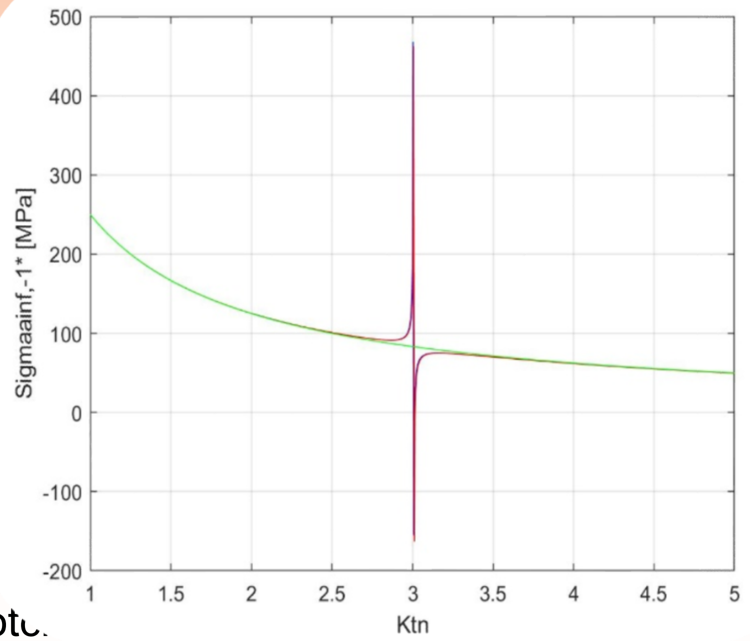


Matlab code for an isotropic two-dimensional drilled plate, under uniaxial tension

- Define the fatigue limit of the notched specimen vs the K_{tn} factor: $\sigma *_{a\infty,-1} = f(K_{tn})$, and thus also $q = f(K_{tn}) \rightarrow \frac{d}{W} = f(K_{tn})$.
- **u=linspace(2,3,1000);**
- **S=zeros(1,length(u));**
- **for i=1:length(S)**
- **p=u(i);**
- **syms x**
- **S_i=vpasolve(2+0.284*(1-x)-0.6*(1-x).^2+1.32*(1-x).^3-p);**
- **S(1,i)=S_i(1);**
- Vpasolve → three roots: a real one and two complexes conjugated.
- **end**
- **%we do likewise for the Heywood formula**
- **A=zeros(1,length(u));**
- **for i=1:length(A)**
- **p=u(i);**
- **syms x**
- **A_i=vpasolve(2+(1-x).^3-p);**
- **A(1,i)=A_i(1);**
- **end**
- **%I can thus express fatigue limits as functions of the Ktn factor**
- **realtension1=250./((1./(1+(0.265./((150.*A)./2)))).*(u-1)+1);**
- **realtension2=250./((1./(1+(0.265./((150.*S)./2)))).*(u-1)+1);**
- **figure(5)**
- **plot(u,realtension1,'b'); grid**
- **hold on**
- **plot(u,realtension2,'r')**
- Incorrect result after $K_{tn} \cong 2.9$, $(\frac{d}{W} \cong 0.047)$.
- The curves with q as a variable and with q as a unit-value diverge more and more as $K_{tn} \rightarrow 3$.
- K_{tn} between 2 and 3. Different study interval (between 1 and 5) → essential discontinuity (vertical asymptote) , in the case of a variable notch sensitivity index.

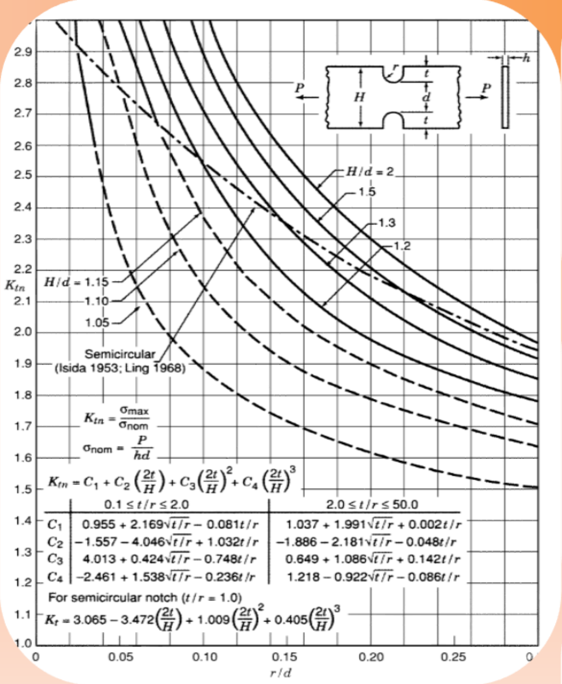




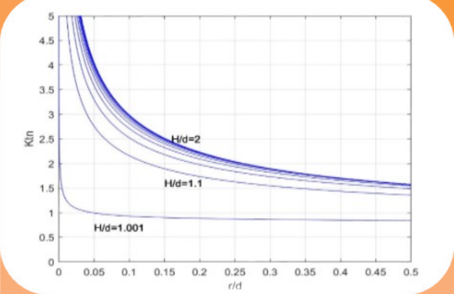


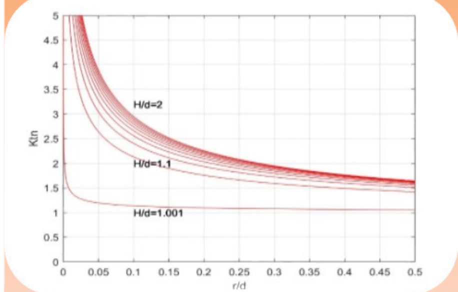
Matlab code for an isotropic two-dimensional bar with opposite u-shaped notches, under uniaxial tension

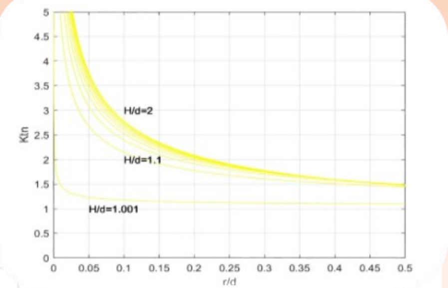
- Formulas applied only to very thin flat members, with two-dimensional state of stress (plane stress).
- For an infinite or semi-infinite plate, the stress concentration factor for an elliptical hole: $K_{tn} = 1 + 2\sqrt{\frac{t}{r}}$ (same equation in literature for a crack of length 2t); also applied to U-shaped notches, especially for $\frac{t}{r} \to 1$ (Isida, 1955).
- F.I. Barrata and D.M. Neal (1970) → photoelastic tests → $K_{tn} = \left(0.780 + 2.243\sqrt{\frac{t}{r}}\right)\left[0.993 + 0.180\left(\frac{2t}{H}\right) - 1.060\left(\frac{2t}{H}\right)^2 + 1.710\left(\frac{2t}{H}\right)^3\right]\left(1 - \frac{2t}{H}\right)$
- Instead, R.B. Heywood (1952) gave: $K_{tn} = 1 + \left(\frac{\frac{t}{r}}{1.55\left(\frac{H}{d}\right) - 1.30}\right)^n$, with $n = \frac{\frac{H}{d} - 1 + 0.5\sqrt{\frac{t}{r}}}{\frac{H}{d} - 1 + \sqrt{\frac{t}{r}}}$ and $t = \frac{H-d}{2}$
- Other formulas from Peterson's manual (Kikukawa (1962), Flynn and Roll (1966), Appl and Koerner (1969))
- MatLab script, same goals as before (K_{tn} for Heywood, Kikukawa and Barrata, index q, life reduction factor K_f, fatigue limit)
- *Note that Barrata's formula will be displayed in Blue, Heywood's one in red, Kukawa's formula (in case of 2<(t/r)<50) in yellow, Kukawa's formula (in case of 0.1<(t/r)<2) in green.*
- Demonstration of the relations and express them according as a function of $\frac{r}{d}$: $x = \frac{r}{d}$; $a = \frac{H}{d}$; $\frac{1}{r} = \frac{d}{r} * \frac{1}{d} = \frac{1}{x*d}$; $\frac{t}{r} = \frac{H-d}{2*r} = \frac{a*d-d}{2*r} = \frac{d}{r} * \frac{a-1}{2} = \frac{a-1}{2*x}$; $\frac{2*t}{H} = \frac{H-d}{H} = 1 - \frac{d}{H} = 1 - \frac{1}{a} = \frac{a-1}{a}$
- $\frac{H}{d} = 1$ would implies no notch (t=r=0), so $K_{tn} = 1$.

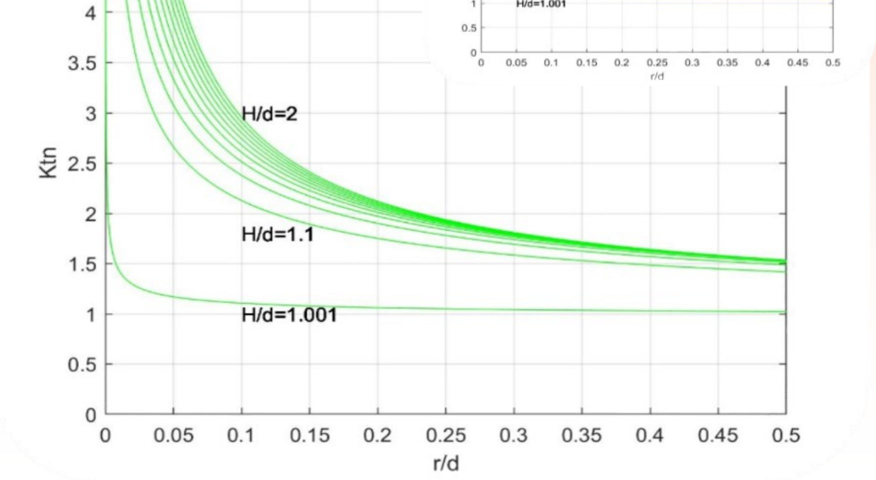


	$0.1 \le t/r \le 2.0$	$2.0 \le t/r \le 50.0$
C_1	$0.955 + 2.169\sqrt{t/r} - 0.081t/r$	$1.037 + 1.991\sqrt{t/r} + 0.002t/r$
C_2	$-1.557 - 4.046\sqrt{t/r} + 1.032t/r$	$-1.886 - 2.181\sqrt{t/r} - 0.048t/r$
C_3	$4.013 + 0.424\sqrt{t/r} - 0.748t/r$	$0.649 + 1.086\sqrt{t/r} + 0.142t/r$
C_4	$-2.461 + 1.538\sqrt{t/r} - 0.236t/r$	$1.218 - 0.922\sqrt{t/r} - 0.086t/r$

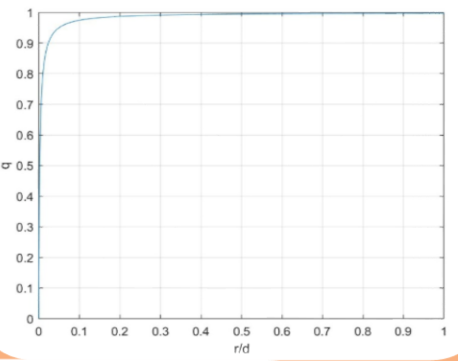




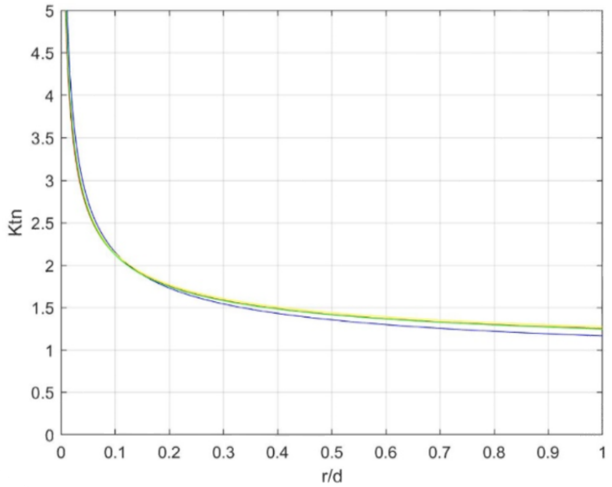


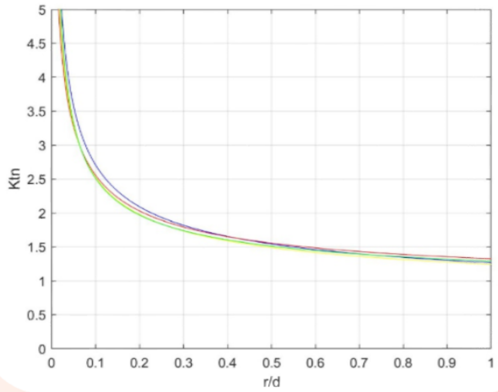


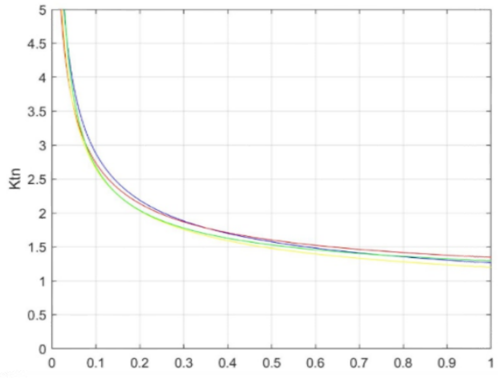
Matlab code for an isotropic two-dimensional bar with opposite u-shaped notches, under uniaxial tension

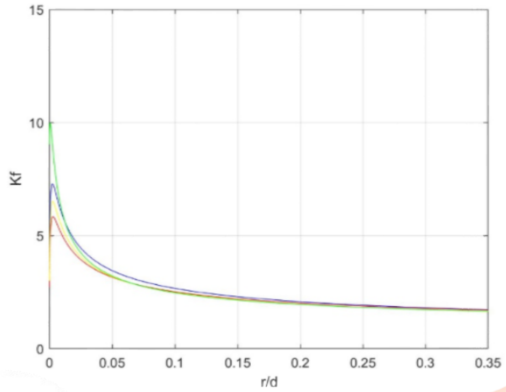


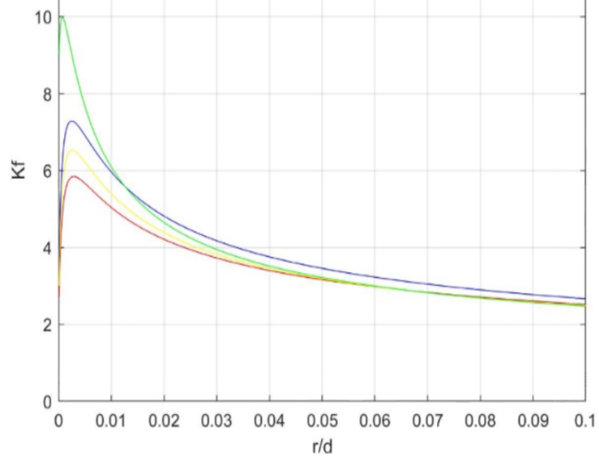
- Specific case: H/d=1.1, H/d=1.2, H/d=1.5
- Material with 500 MPa tensile strength, (a=0.265 from table IX UNI7670), a=H/d=1.3 and d=100 mm → H=130 mm and t=(H-d)/2=15 mm
- Rf=0.5 → Fatigue limit = 250 MPa
- *At very low ratios → limit of applicability (critical points* 0.0025, 0.0029, 0.0026, 7.2166e-04).
- Fatigue limit vs Ktn (four different matrices filled with the real roots of the problem).

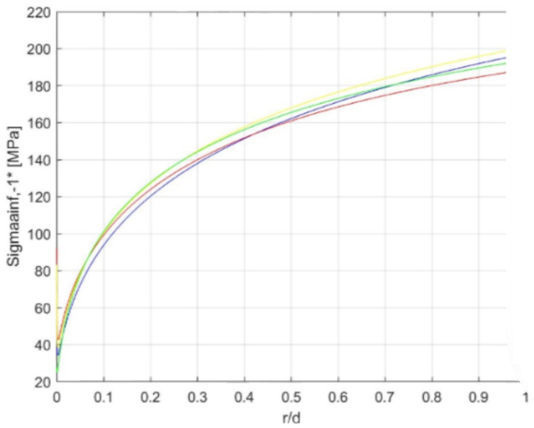


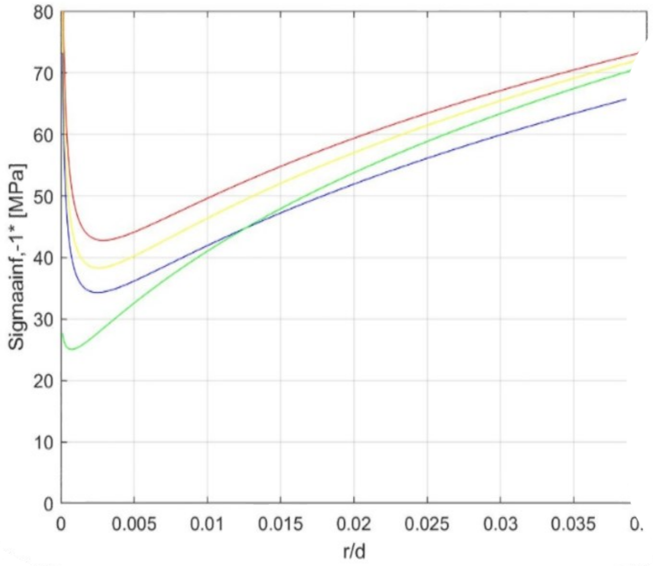


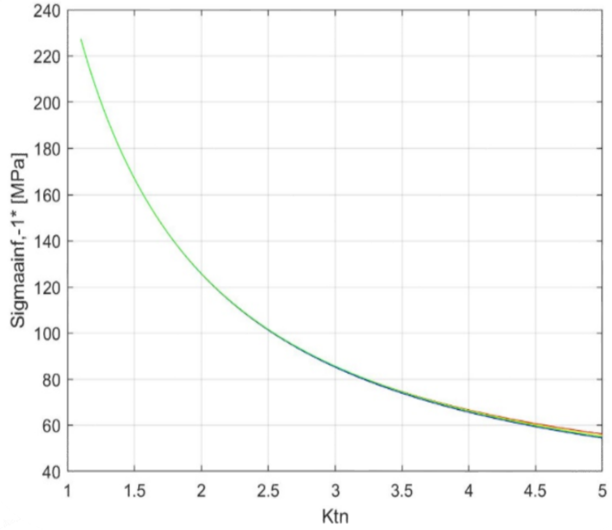




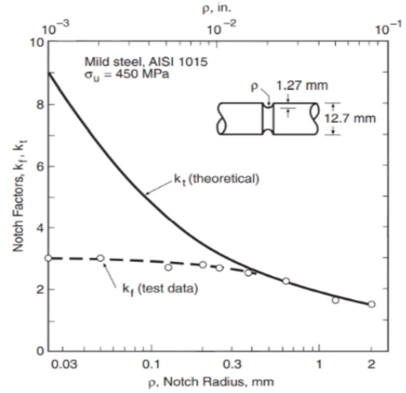


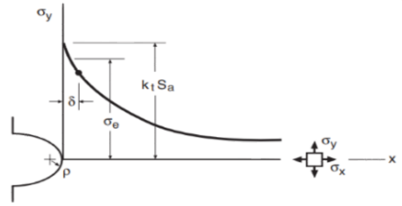


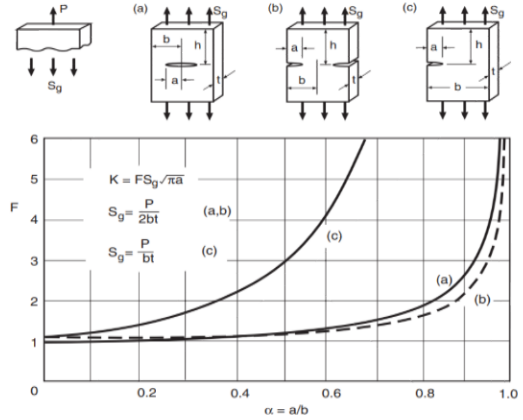




The fatigue notch factor, fracture mechanics, notches sensitivity and empirical estimates to know the value of K_f



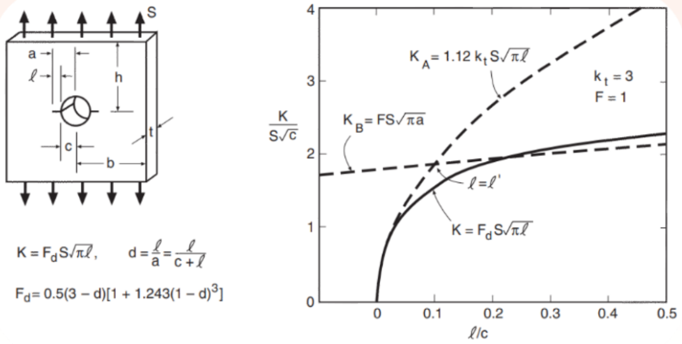




Values for small a/b and limits for 10% accuracy:

(a) $K = S_g\sqrt{\pi a}$ $(a/b \le 0.4)$ (b) $K = 1.12S_g\sqrt{\pi a}$ $(a/b \le 0.6)$ (c) $K = 1.12S_g\sqrt{\pi a}$ $(a/b \le 0.13)$

- The real reduction factor, the fatigue notch factor $K_f = \frac{\sigma_{ar}}{S_{ar}}$. $K_f \neq K_t$ explained by:
- (1) Local stress value decreases while moving away from the notch (decreasing of the stress gradient $\frac{d\sigma}{dx}$), the material sensitive to average stress in the process zone. $K_f \triangleq \frac{The\ average\ \sigma\ in\ the\ process\ zone}{S_a} = \frac{\sigma_e}{S_a} < K_t$; crystal grains vs. the equalization of the stress on small dimensions
- (2) Weakest-link effects
- (3) Analysis of cracks in the fracture mechanics theory (non-propagating cracks). Stress intensity factor $K = \lim_{r,\theta \to 0} (\sigma_y\sqrt{2\pi r}),\ K = FS\sqrt{\pi a}$. K (the continuous curve fourth fig.) → two different curves K_A and K_B. "l'": crack length where $K_A = K_B, l' = \frac{c}{\left(1.12\frac{k_t g}{F}\right)^2 - 1}$.
  $K_s = 1.12\sigma_s\sqrt{\pi l}$ of the unnotched member similar to the K_a for the notched member. After l', K in the notched always lower than the value of K_s. Drop of K_b as compared to K_s → cracks to grow slower on the carved piece → longer lives. Interesting situation : smaller l' value for sharper notches (fatigue limit).
- (4) Reverse yielding, at low life cycles for high stress amplitudes (strain-approach).
- Notch sensitivity: $q \triangleq \frac{K_f - 1}{K_t - 1}$. Completely sensitive piece to notches: $q = 1$ and $K_f = K_t$; no sensitiveness: $q = 0$ and $K_f = 1\ \forall\ K_t$.
- Peterson's formula (1959): $q = \frac{1}{1+\frac{\alpha}{\rho}}$. α tabulated or, for steels, α as a function of the ultimate stress in axial or bending loading conditions: $log\alpha = 2.654 * 10^{-7}\sigma_u^2 - 1.309 * 10^{-3}\sigma_u + 0.01103$. $K_f = 1 + \frac{K_t - 1}{1+\frac{\alpha}{\rho}}$.
- Kuhn and Hardrath (1952) based on H. Neuber relations: $q = \frac{1}{1+\sqrt{\frac{\beta}{\rho}}}$. β as a function of the ultimate stress too (Neuber) (for steels and for heated-treated aluminum).
- These equations for estimating K_f not used for acute notches.



Summary

- The equations used in MatLab codes for estimating K_f should not be used for acute notches.
- Peterson's formula increasingly incorrect for more and more ductile materials at short life, owing to the yielding of the specimen → need to define another factor $K_f{}'$ that varies according to the life of the piece: $1 \leq K_f{}' \leq K_f$. The limit on the left is achieved at short life, the one on the right at long life.
- For low stresses applied for many times (the long-life fatigue strength over 10^6 cycles), other factors are used (for the type of load, for the actual size and surface finish etc.,) to obtain the true fatigue limit.
- Moreover, if the stress exceeds the yield strength or if we encounter sharp notches or some great complexities, the methods checked before become useless or at least definitely wrong → the need to find another method, such as the "estimated S-N curves". Advantage of including geometric and manufacturing details usually very difficult to evaluate. Juvinall or Budynas: very imprecise with damages due to occasional serious cycles or due to corrosion.
- Effect of the mean stress for notched members → the Goodman or Gerber equations. Cons: complications of these formulas by local yielding. In these cases, it is better to use the Smith, Watson and Topper (SWT) equation or the Walker equation.
- If available in other manuals, an S-N curve from test data. Not every possible geometry can be found on manuals → data of similar components or notched members. Matching by notch radius or a similar length parameter l'.
- Problem with the sequence effect, that characterize a loading history with small fraction numbers of high-stress cycles, leading to local failure at the notches by yielding, subsequently causing a variation of the mean stress due to residual stresses. Do not use S-N curves and Palmgren-Miner rule.
- To overcome this problem:

A) Using the rule Palmgren-Miner formula in a relative form: $B_f\left(\sum_{1\,repetition}\frac{N_j}{N_{failure,j}}\right) = D \neq 1$ where D can be different from unity, as written;

B) Corten-Dolan cumulative damage procedure (1968);

C) Strain-based approach, especially in local yielding cases;

D) Fracture mechanics theory based on a crack-growth approach.

- In the second part of the report, I explained how, for some specific geometric examples, the formulas concerning the fatigue life of a specimen that can be found in the literature have practical limits: after decreasing the value of notch radius by a lot, they find a limit of applicability (different limits for different geometries and formulas).
- I created a code where, once the input data are determined, a theoretical fatigue life limit is calculated, as a function of the K_t and for the different possible formulations given by the scientific literature.