Università degli Studi di Padova

Dipartimento di Ingegneria dell'Informazione

Master Thesis in Control Systems Engineering

Nonlinear Model Predictive Control of an Inductive Power Transfer system

Master Candidate

Valentina Turrato

Student ID 2026925

Supervisor

Prof. Nicola Bianchi

University of Padova

Co-supervisor

Prof. Jon Are Suul

Norwegian University of Science and Technology

Academic Year 2022/2023

27-02-2023

Abstract

Wireless power transfer (WPT) technology is receiving more attention as it provides a way to realise power transmission without a physical contact. Compared to traditional recharging methods, wireless power transfer offers a safer and more flexible way of recharging a battery. One way to realise WPT is with the inductive power transfer (IPT) technology. It is based on the magnetic coupling of coils exchanging power from a stationary primary unit to a secondary system on board the EV. In this work a non-linear model predictive control-based method is used to control the power transmission in a series-series IPT system. Model predictive control is a class of algorithms which use a model to predict the future behaviour of the system. Moreover a comparative analysis between model predictive control (MPC) and the traditional proportional integral (PI) control is developed to show that MPC has a better transient response than the PI controller and ensures more system efficiency when regulating the power transfer

Sommario

Il trasferimento di potenza wireless si sta sviluppando sempre di più negli ultimi anni in quanto permette di trasmettere potenza senza un contatto fisico. In aggiunta, è un metodo di ricarica più sicuro e flessibile dei metodi tradizionali con cavo. Un modo per realizzare il trasferimento wireless di potenza è usando un sistema a trasferimento di potenza induttiva. Tale sistema è basato sull'induzione magnetica tra due bobine, una posta in un'unità fissa primaria e l'altra, secondaria, posta all'interno del veicolo elettrico. In questa tesi, viene presentata una tecnica di controllo del sistema IPT basata sul model predictive control non lineare. Il model predictive control è un insieme di algoritmi che producono l'evoluzione futura di un sistema basandosi su un modello di esso. Viene poi sviluppata un'analisi comparativa tra MPC e il tradizionale controllo Proporzionale Integrativo. L'intento è di dimostrare che MPC ha performance migliori e offre un'efficienza migliore nel trasferimento di potenza.

Contents

List of Figures

1

Introduction

1.0.1 Electric vehicles

Electric vehicles (EV) have become more popular in the recent years for different reasons. Due to global warming, fossil fuel resources have reduced and as the attention on climate change is increasing, people are more conscious and more interested in EV. They are less noisy and less polluting and using them reduces carbon emissions for sure. EVs started being developed following Faraday's work in 1821. The first electric train was built in 1837 and then followed electric trams and trolley cars. By the 1920s thousands of electric vehicles such as cars, trains and buses had been produced. In fact electric vehicles were at that time more reliable than Internal Combustion (IC)) vehicles which smelt and needed to be manually crancked to start. However, due to the limitations of storage batteries and as the oil became widely available, IC road vehicles started to gain more popularity also because they would offer more power and more efficiency. The outlook of electric vehicles shifted in the 1990s when automotive manufacturers started aggressive programs to develop EVs for commercialization. An EV operates by converting electrical energy to kinetic energy via an induction motor for propulsion. On board they are equipped with an energy storage battery unit. The power needed to charge the EV can come from different arrangements of energy sources which also define the type of an EV. For these reasons and thanks to the development of the technology, EV are more accessible for the public. Moreover, EV have many advantages compared to internal combustion engine vehicles (ICEVs), such as low maintenance costs and the travel costs per kilometer is more economical. Now it is possbile to say that EVs are comparable to ICEVs and are even more efficient and less noisy. However, there are some possible limitations and concerns when it comes to EVs. First of all there is a limited driving range and second of all cable recharging is not convenient anymore especially as more and more EVs are put in to the market. As more EV are put on the market it is a necessity to establish battery charging stations and to realize alternative charging methods, such as wireless charging. Wireless charging does not require any physical contact or wired connections: thus it has several advantages such as high safety and reliability and at the same time low maintenance cost and long service life. There are mainly three types of wireless charging sytems: static, quasi-dynamic and dynamic. With static charging the EV is in a stationary position in a parking area. The main limitations of this type of charging are related to the fact that there is a limited overall driving range and usually long charging times. Quasi-dynamic charging provides higher driving ranges because it uses charging pads at bus stops and traffic light stops. As a result the vehicle can be charged not only during parking but also when moving slowly. In dynamic charging the goal is to charge the EV while driving by burying the source of energy into the road. With Wireless power transfer (WPT) there is no need of having a large and space consuming energy storage system onboard resulting in less weight and more efficiency.The charging process is safer, simpler and more comfortable. Besides the three main types of WPT, another distinction can be made according to how power is transferred: it can take place by magnetic coupling, capacitive coupling and microwave propagation. Microwaves are used when the power transfer occurs with low efficiency over long distances. For short distance power transfer, the most popular systems are Inductive and Capacitive Power Systems. The latter is based on a confined electric field distribution between two conductive plates. Inductive power transfer (IPT) systems are the most frequently used and well-known systems. They are unaffected by dirt or chemicals, can be deployed in dynamic charging, offer reliability and are not expensive. [1]

1.0.2 Inductive power transfer systems

In general in a WPT system there are two components: a transmitter and a receiver. In the case of IPT systems, the transmitter and the receiver consists of two coupled but separate coils. On the left hand side of the system, as shown in figure 1.1,the transmitter is composed of a high-frequency DC/AC converter and a primary coil L1, while the receiver consists of a secondary coil L2 and a rectifier. The sending side is then connected to a battery.

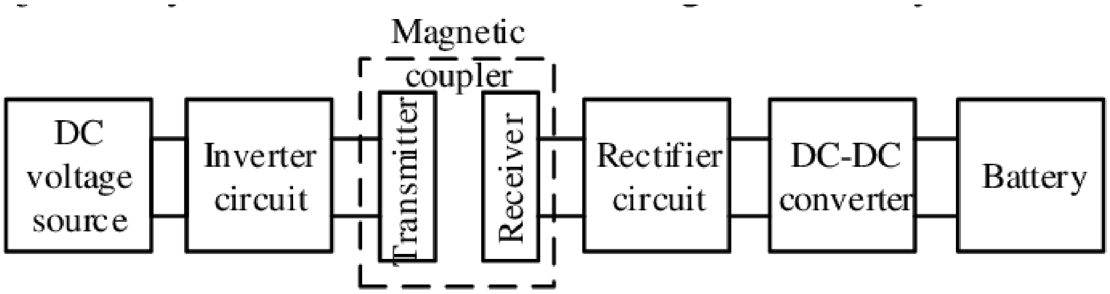

Figure 1.1: Model of an IPT system

The transmitter is fed from an AC power source and thus it produces an alternating current. According to Ampere's law, this alternating current when going through the primary coil generates a magnetic field. This induces a voltage in the primary coil which induces, by Faraday's law, a flux in the secondary coil. The flux induces a voltage in the secondary coil which is then rectified and a current flows through the load connected to the output of the system.

Figure 1.2: Flow chart of an IPT system

One of the coil is installed in the charging terminal (track coil), while the other is embedded in the vehicle (pick-up coil).[2]

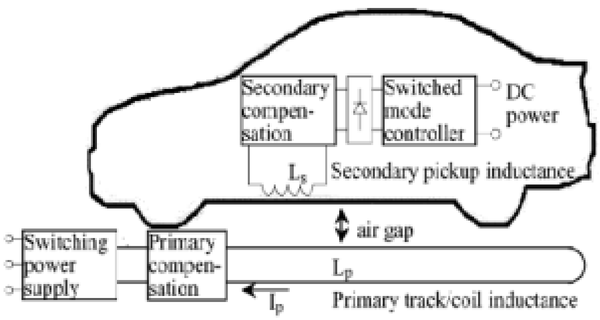

Figure 1.3: Typical inductive power transfer system

When designing an IPT system multiple factors have to be taken into consideration such as the coil design and the distance between the two of them, the operating frequency, compensation topologies and control techniques. The coil design is the most critical one as different types of shapes and sizes of the coil ensure different power transferred values. Depending on the distance between the transmitting and receiving coils, only a fraction of the magnetic flux generated by the transmitter coil penetrates the receiver coil and contributes to the power transmission. The more flux reaches the receiver, the better the coils are coupled. The grade of coupling is expressed by the coupling factor k. The coupling coefficient, a dimensionless parameter, is a value between 0 and 1. It is related to the mutual inductance through this equation

$$M = k \cdot \sqrt{L_1 L_2} \tag{1.1}$$

The coupling factor is determined by the distance between the coils and their relative size. It is further determined by the shape of the coils and the angle between them. If the distance increases beyond a certain limit the coupling decreases and as a consequence the efficiency of the power transmission reduces. IPT systems can be divided into two categories: static and dynamic. Static refers to stationary charging of the EV while dynamic refers to charging the EV while it is in motion. Dynamic charging has to face more challenges than static. The two most challenging ones are the large and possibly varying air gap between the two coils and the subsequent misalignment. These two factors affect and lower the coupling and the efficiency. Therefore it is necessary to design the system in such a way that it is tolerant to mis-alignments. As the mutual inductance and the coupling coefficient play an important role in an IPT system, the gap between the primary and secondary coil needs to be adjusted appropriately. In fact the mutual inductance is affected by the variation in the air gap between the two coils. If the gap is large, a small mutual inductance is produced and at the same time there is a large leakage inductance. This results in the requirement of a large current for transferring a desired amount of power. Moreover this large current induces losses and therefore less efficiency. To prevent this from happening, it is necessary to add a compensating circuit on both sides of the system. It provides high voltages both at the input and at the output for transmitting high power. It is also necessary to reduce the total reactive power which increases when the coupling coefficient is low. Therefore using compensation capacitors helps in doing so and in better controlling the system in general.

There are different compensation topologies based on how the capacitor and the coil are combined. The compensating capacitor can be placed either in parallel on in series. Four combinations are possible:

1. parallel-parallel (PP)
2. parallel-series (PS)
3. series-parallel (SP)
4. series-series. (SS)

Each topology offers different advantages and disadvantages and the type of topology needs to be chosen according to the application. In the SS topology the compensation capacitor values are independent of the resistive load and mutual inductance depending only on resonant frequency and self inductance. For this reason the system stays in resonance and becomes less sensitive in case of misalignment. The main advantage of the SS is that the value of primary capacitance does not depend on the variation of coupling coefficient.

In PS and PP topologies, while the total impedance increases, the current drawn from the source and the load decreases significantly. More complex equations are needed to calculate the primary capacitor, while the capacitor value depends on the mutual inductance and load values. Moreover it as instantaneous voltage changes may happen, a current source is needed at the input.

In order to increase the efficiency of PP and PS structures and to provide an easier inverter current control,it is necessary to add extra inductance to the topology.

1.0.3 Modeling of a Series-series compensated system

In this work a series-series topology is considered. Figure 1.4 shows the system under consideration.

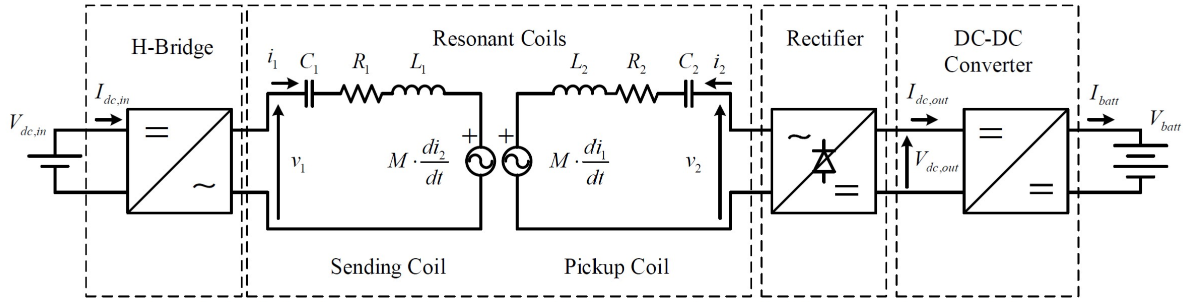

Figure 1.4: SS-compensated IPT-system with Constant Voltage Load(CVL)

On the sending side there is an H-bridge converter and a diode rectifier on the receiving side connected directly to a Constant Voltage Load(CVL) representing a battery. The sending side is energised by the output voltage $v1$ of the H-bridge converter and the diode rectifies the ac signals $v2$ and $i2$ in the receiving coil. Therefore the H-bridge controls directly the frequency and amplitude of the fundamental frequency component $v1$ of the voltage at the sending side. It is desirable for the system to operate in resonance, i.e. the sending and receiving side frequencies must coincide

$$\omega = \frac{1}{\sqrt{C_1 L_1}} = \frac{1}{\sqrt{C_2 L_2}} \tag{1.2}$$

Under the assumption that all currents and voltages in the system are sinusoidal wave-forms, a first harmonic approximation is assumed for the sending side. This means that only the fundamental frequency components of the voltages and currents contribute to the power transfer.

The system dynamics is described by the following equations in the time domain:

$$v_1 = R_1 i_1 + L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} + v_{C1} \tag{1.3}$$

$$v_2 = -R_2 i_2 - L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} - v_{C12} \tag{1.4}$$

$$\frac{dV_{C1}}{dt} = \frac{1}{C_1} i_1 \tag{1.5}$$

$$\frac{dV_{C2}}{dt} = \frac{1}{C_2} i_2 \tag{1.6}$$

1.0.4 Constant voltage and constant resistance load

In a constant voltage (CV) mode the load sets a fixed voltage across the terminals independently from the input current. This configuration allows the load to dynamically change its resistance and maintain the programmed voltage regardless of the current variation of the sourcing device. For this reason the receiving side voltage v_2 will remain constant despite any possible variations. In a constant resistance (CR) mode, the load will change its voltage according to change in the receiving side current due to variations in the operating conditions. This is the opposite behaviour of a CVL. Under this assumption the relationship between the receiving side voltage and current is:

$$v_2 = R_{eq} \cdot i_2 \tag{1.7}$$

where R_{eq} is the equivalent load resistance. In general IPT systems are represented by a CRL. However the CRL model can only be used for steady state conditions and is not suitable for analyzing dynamic characteristics. For this reason the CVL model is adopted. In fact the load voltage v_2 remains constant but with a phase angle determined by the phase of the receiving side current i_2 and amplitude determined by the load voltage $V_{dc,out}$. Thus the CVL mode can be represented by this equation:

$$v_2 = \frac{i_2}{I_2} \cdot \frac{4}{\pi} \cdot V_{dc,out} \tag{1.8}$$

1.0.5 Dq-model representation of the IPT system

The formulation of the system represented by equations (1.3)-(1.6) makes the variable time-periodic when the steady-state is reached. Therefore the system can not be represented with a formulation where the variables will settle at a constant value when evaluated at the equilibrium point. This time-invariant representation can be achieved by representing the variables in a a synchronously rotating dq-reference frame. In this way the phase angle and amplitude of the first frequency armonic components of the currents and voltages in equations (1.3)-(1.6) can be accurately represented. The d- and q-axis state variables are equivalent to the real and imaginary parts of dynamic phasor models.

Under the assumption of constant or slowly varying operating frequency ω

the model defined in Equations (1.3)-(1.6) can be written in state-space form i.e.

$$\dot{x} = f(x, u) \tag{1.9}$$

where the states and input vectors are defined as:

$$\begin{aligned} x &= \begin{bmatrix} i_{1,d} & i_{1,q} & i_{2,d} & v_{C1,d} & v_{C1,q} & v_{C2,d} & v_{C2,q} \end{bmatrix}^T \\ u &= \begin{bmatrix} v_{1,d} & v_{1,q} & \omega & V_{dc,out} \end{bmatrix} \end{aligned} \tag{1.10}$$

Now each AC current and voltage of the IPT system can be represented by two separate state variables for the d- and q- axis components. The CVL representation described by equation (1.8) needs to be considered also in the dq-reference frame. In this case the amplitude I_2 is expressed by the euclidean norm of $i_{2,dq}$, i.e. $|i_{2,dq}| = \sqrt{i_{2,d}^2 + i_{2,q}^2}$. Thus the voltage of the receiving side can be expressed in the dq-components as:

$$v_{2,q} = \frac{i_{2,dq}}{\sqrt{i_{2,d}^2 + i_{2,q}^2}} \cdot \frac{4}{pi} \cdot V_{dc,out} \tag{1.11}$$

Applying the Park's transformation to the system in equations (1.3)-(1.6) to obtain the corresponding synchronous reference frame formulation and taking into consideration the CVL dynamics expressed in (1.8), allows to definition the non-linear state-space model expressed by (1.9) and (1.11).

$$\begin{aligned} \frac{di_{1,d}}{dt} = \omega i_{1,q} - \frac{R_1}{L_{\alpha 1}} i_{1,d} - \frac{MR_2}{L_{\alpha 1} L_2} i_{2,d} - \frac{1}{L_{\alpha 1}} v_{C1,d} - \frac{M}{L_{\alpha 1} L_2} v_{C2,d} \\ + \frac{1}{L_{\alpha 1}} v_{1,d} - \frac{M}{L_{\alpha 1} L_2} \frac{i_{2,d}}{\sqrt{i_{2,d}^2 + i_{2,q}^2} \frac{4}{\pi}} V_{dc,out} \end{aligned}$$

$$\begin{aligned} \frac{di_{1,q}}{dt} = -\omega i_{1,d} - \frac{R_1}{L_{\alpha 1}} i_{1,q} - \frac{MR_2}{L_{\alpha 1} L_2} i_{2,q} - \frac{1}{L_{\alpha 1}} v_{C1,q} - \frac{M}{L_{\alpha 1} L_2} v_{C2,q} \\ + \frac{1}{L_{\alpha 1}} v_{1,q} - \frac{M}{L_{\alpha 1} L_2} \frac{i_{2,q}}{\sqrt{i_{2,d}^2 + i_{2,q}^2} \frac{4}{\pi}} V_{dc,out} \end{aligned}$$

$$\frac{di_{2,d}}{dt} = \omega i_{2,q} - \frac{R_2}{L_{\alpha 2}} i_{2,d} - \frac{MR_1}{L_{\alpha 2}L_1} i_{1,d} - \frac{1}{L_{\alpha 2}} v_{C2,d} - \frac{M}{L_{\alpha 2}L_1} v_{C1,d} + \frac{1}{L_{\alpha 1}} v_{1,q} + \frac{M}{L_{\alpha 2}L_1} v_{1,q} - \frac{1}{L_{\alpha_2}} \frac{i_{2,q}}{\sqrt{i_{2,d}^2 + i_{2,q}^2} \frac{4}{\pi}} V_{dc,out}$$

$$\frac{di_{2,q}}{dt} = \omega i_{2,d} - \frac{R_2}{L_{\alpha 2}} i_{2,q} - \frac{MR_1}{L_{\alpha 2}L_1} i_{1,q} - \frac{1}{L_{\alpha 2}} v_{C2,q} - \frac{M}{L_{\alpha 2}L_1} v_{C1,q} - \frac{1}{L_{\alpha 2}} v_{C2,q} + \frac{M}{L_{\alpha 2}L_1} v_{1,q} - \frac{1}{L_{\alpha_2}} \frac{i_{2,q}}{\sqrt{i_{2,d}^2 + i_{2,q}^2} \frac{4}{\pi}} V_{dc,out}$$

$$\frac{dv_{C1,d}}{dt} = \omega v_{C1,q} + \frac{1}{C_1} i_{1,d}$$

$$\frac{dv_{C1,q}}{dt} = -\omega v_{C1,d} + \frac{1}{C_1} i_{1,q}$$

$$\frac{dv_{C2,d}}{dt} = \omega v_{C2,q} + \frac{1}{C_2} i_{1,d}$$

$$\frac{dv_{C2,q}}{dt} = -\omega v_{C2,d} + \frac{1}{C_2} i_{1,q}$$

where there have been introduced the leakage inductances as

$$L_{\alpha 1} = L_1 - \frac{M^2}{L_2} \qquad L_{\alpha 2} = L_2 - \frac{M^2}{L_1}$$

As the reference frame of the dq-model can be chosen arbitrary, for convenience it is defined to be synchronized with the peak amplitude of v_1. Therefore, the q-axis voltage component at the sending side is zero,$v_{1,q,0} = 0$, and the operating d-axis voltage component is equal to the peak amplitude of v_1, namely $v_{1,d,0} = V_{1,0}$. [3]

In the following it is assumed that there is only one resonance frequency f_0

defined as

$$\omega = \omega_0 = \frac{1}{\sqrt{C_1 L_1}} = \frac{1}{\sqrt{C_2 L_2}} = 2\pi \cdot f_0$$

The nominal parameters of the system are:

Nominal power, P_0	10 kW
Nominal operating frequency, $f0$	85 kHz
Nominal coupling factor, k_{nom}	0.2
Primary coil	
Nominal voltage, $V_{1,nom}$	380 V
Self inductance, L_1	176 μ H
Secondary coil	
Nominal voltage, $V_{2,nom}$	235 V
Self inductance, L_2	41 μ H

1.0.6 Control of an IPT system

Typically it is desired to regulate the battery charging in a wireless power transfer. The control action can take place in the primary side, in the secondary one or in both. The main objective in WPT system is to control the power flow. While controlling the secondary-side is more preferable, primary or dual-side control are more common. It is essential to control the IPT system especially during transients: in dynamic IPT the coupling coefficient varies when the vehicle is moving and also in static IPT fluctuations of the input voltage may happen. This may lead to a decrease in the system efficiency and frequent power oscillations. If the control scheme does not perform well in responding to dynamic variations, long settling times, overshoot or undershoot will occur thus causing power oscillations which will reduce the lifetime of the system. The more common method adopted for controlling the power transfer in an IPT system is the operating frequency shifting. The idea is to change the operating frequency of the system to regulate the system gains and achieve higher efficiency. This is why phase shift modulation is a very spread technique. Proportional Integral (PI) control is for sure the most common control technique. It has low costs, provides with high efficiency but at the same time is not able to handle well changes in the dynamics leading to instability. Sliding mode control (SMC) is another well known control methodology. It provides quick response, robustness to parameters changes and can handle linear and non-linear systems. However the complexity of this controller is higher as the modelling process is more complicated. However, this method fails to achieve a satisfying performance for applications requiring fast dynamic response. In fact it causes large overshoots/undershoots and long settling times.[4] Model predictive control (MPC) is an advanced control method for IPT systems. It has better performances as it shows a faster dynamic response than the PI control and has a simpler mathematical model than SMC. It can predict the future behaviour of the system under different operating conditions. [5]

2

Model predictive control

2.1 Generalities

Model Predictive control (MPC) refers to a vast set of advanced control methods developed from the late '70s. MPC predicts the future behaviour of the controlled system by solving a constrained optimization problem. The control law is computed implicitly thus with MPC it is necessary to investigate more the modeling of the to-be controlled process rather than the design of the controller.

In the automation of systems, feedback controllers (also called closed-loop controllers) are widely used: they compare a reference r with a measured variable y to determine the best value for the manipulated variable u according to the deviation $e = r - y$

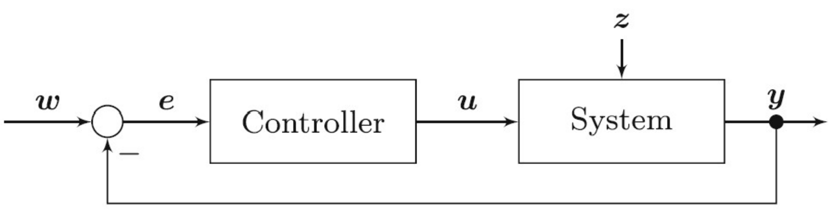

Figure 2.1: Block scheme of a classical feedback control loop

According to the working principle the closed loop controllers can be divided into different categories, i.e classical controllers and predictive controllers. Classical control methods such as PID controllers or bang-bang controllers only focus on the past and current system behaviour. Predictive controllers on the other hand use a system model to predict the future behaviour of the system and computes the optimal control input while minimizing a predefined cost function. The fundamentals of MPC are then:

1. a model of the system to predict the system future behaviour
2. an objective to be minimized to found the optimal control sequence
3. a receding horizon control strategy: at each time instant the control input is evaluated by solving at each time the optimization problem. Then only the first value of the optimal input is used until the next sampling instant. Then the horizon is shifted and the procedure is repeated again.

The prediction horizon, p, is the number of future control intervals the MPC controller must evaluate by prediction when optimizing its MVs at control interval k. If the prediction horizon is too long, it results in increasing sampling time and increasing weights of the cost function. The control horizon, m, is the number of MV moves to be optimized at control interval k. The control horizon falls between 1 and the prediction horizon p. A small control horizon means more stability and fewer variables to compute in the optimization problem, which promotes faster computation.[6] The receding horizon strategy is used because a short-term optimization performs better than a long-term one. It is important to notice that the prediction horizon needs to be long enough to represent the effect of the change.

More formally, consider a generic non-linear system

$$x(k+1) = f(x(k), u(k))$$
$$y(k) = h(x(k))$$

and a generic cost function J where $J(x(k), u(k))$.
At each time instant, considering a prediction horizon of dimension N going from k to $k+N$, the outputs of the systems are predicted. They depend on the values of the states variables and of the output variables at time instant t and on the unknown future control variables.
Then the optimization problem is solved by minimizing the cost function J so that the control inputs sequence

$$u_k = [u(k), u(k+1), ..., u(k+N-1)]$$

is found. Only its first element, u(0), is used while the others are discarded. At the next time instant $k+1$, a new optimization problem is solved on the shifted prediction interval $[k+1, k+N+1]$, based on the available information up to time $k+1$, yielding the control $u(k+1)$. The process is then constantly repeated so that the controller can operate on arbitrarily long time horizons. As the prediction horizon is shifted at each iteration, both the predicted and measured output are evaluated again thus showing different values. The figure shows what explained above.

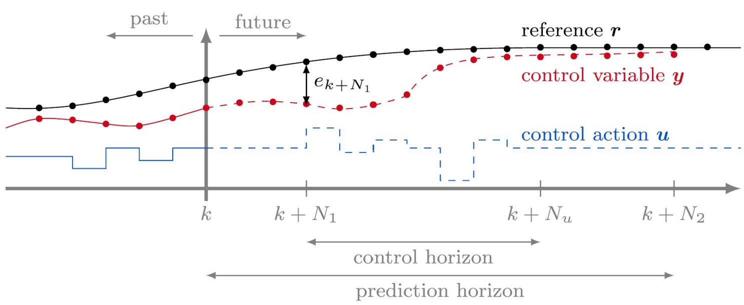

Figure 2.2: Principle of MPC receding horizon

The main adavantages of MPC are

1. the systematic approach to handle constraints and nonlinearites
2. it is applied to multiple input multiple output systems (MIMO)
3. it can be used in a wide spread of applications
4. it offer high control performances.

However, though undoubtedly an interesting technique, MPC presents different disadvantages. Among them:

1. it relies on a model of the system which has to ensure at the same time both accuracy and efficiency
2. the computational cost can be quite challenging
3. the MPC problem has to be solved within the sampling time
4. stability and robustness are not guaranteed
5. the optimization problem might become unfeasible.

In conclusion using optimization and prediction at the same time is what differs MPC from the classical control techniques. In Figure 2.3 a block scheme of the MPC is shown

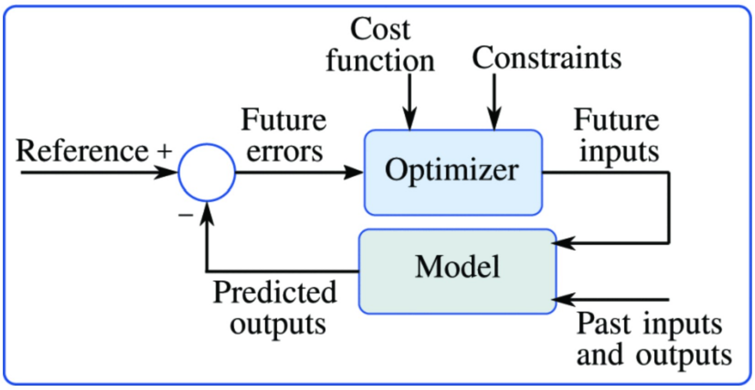

Figure 2.3: Block scheme of the MPC

2.1.1 Unconstrained linear MPC

Consider the n dimensional linear, time-invariant and discrete-time system:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) + Du(k) \end{aligned}$$

Assuming that the state vector can be measured, consider the control sequence

$$u = [u(0), u(1), \dots, u(N-1)]$$

Define a cost function:

$$J(x(0), u) = \frac{1}{2} \sum_{k=0}^{N-1} [x(k)'Qx(k) + u(\hat{k})Ru(k)] + \frac{1}{2} x(N)'P_f x(N) \tag{2.1}$$

with $Q \geq 0, R > 0, P \geq 0$. The control problem is that of finding the control input u minimizing the above cost function. Therefore the control problem can be rewritten as:

$$\begin{aligned} \min_{u} \quad & J(x(0), \boldsymbol{u}) \\ \text{s.t.} \quad & x(k+1) = Ax(k) + Bu(k) \end{aligned} \tag{2.2}$$

To solve the optimal control problem the Dynamical Programming (DP) is used.

Assume the stage cost is $l(x,u) = \frac{1}{2}(x'Qx + u'Ru)$ and the terminal cost is $l_N(x) = \frac{1}{2}x'P_f x$. Then the cost function can be rewritten as:

$$\min_{u(0),x(1),..u(N-2),x(N-1)} \sum_{k=0}^{N-2} l(x(k),u(k)) + \min_{u(N-1),x(N)} \quad l(x_{N-1}, u(N-1) + l_N x(N))$$
$$\text{s.t.} \quad x(k+1) = Ax(k) + Bu(k)$$

Optimizing over $u(N-1)$ and $x(N)$ one obtains:

$$u_0(N-1) = K_N(N-1)x(N-1)$$

with $K_{N-1} = -(B'P_f B + R)^{-}1B'P_f A$. Notice that Π is obtained through the backward Ricatti Iteration. Then the optimal control value, control gain and cost going from k to N are:

$$u_0(k) = K_N(k)x(k)$$
$$K_N(k) = -(B'\Pi(k+1)B + R)^{-}1B'\Pi(k+1)A$$
$$\Pi(k-1) = Q + A'\Pi(k)A - A'\Pi(k)B(B'\Pi(k) + R)^{-}1B'\Pi(k)A$$
$$\Pi(N) = P_f$$

Therefore the unconstrained linear MPC is a standard linear state-feedback law and coincided with the linear quadratic regulator. [7]

2.1.2 Constrained MPC

In the majority of physical systems the manipulated inputs such as voltages, torques etc are bounded. For operability, safety, product quality etc. reasons, constraints can be imposed on the values of the states or the outputs. Moreover, constraints on the rate of change of the input, i.e. $u(k) - u(k-1)$, can be used. There is a distinction between input, states and output constraints: input constraints represent a physical limit. If these constraints are not respected by the controller, the system will enforce them in some ways. On the other hand, states and output constraints are used to prevent the system from operating outside of his safety limits. It might happen that they are not achievable due to disturbances affecting the system. Thus the MPC controller needs to realise in real-time if they can be satisfied or not. For these reasons, constraints can be divided into two categories: $hard$ and $soft$. It is mandatory to satisfy hard constraints and thus they are applied to the input as they represent a physical boundary, as mentioned above. Soft constraints can be slightly violated and they provide flexibility. They are applied to the states variables so that numerical issues which might lead to infeasibility are avoided. In terms of implementation, hard constraints are expressed with an equality sign while soft constraints with an inequality sign. Moreover the state constraints can be softened by introducing a slack variable, ϵ. It is a form of decision variable. The cost penalty is then reformulated so that it can be expressed how much care goes into the state x, the input u and the violation of the state constraint given by ϵ. Thus, infeasibility is avoided in the control problem by choosing a suitable value for ϵ. However, large values of ϵ are undesirable.

$$u_{min} \leq u(k) \leq u_{max}, k = 0, \ldots, N-1$$
$$y_{min} \leq y(k) \leq y_{max}, k = 1, \ldots, N$$
$$y_{min} - \epsilon \leq y(k) \leq y_{max} + \epsilon$$

2.1.3 Stability and Feasibility

Several properties of MPC needs to be adressed:

- feasibility
- stability

Feasibility deals with the formulation of the optimization problem, i.e. with the definition of constraints. As already known, hard input constraints can not be violated as they represent a physical limit. Hard output constraints might render the optimization problem infeasible thus using a slack variable to relax them adds a degree of freedom. Stability deals with the closed-loop and is the property of a system that a bounded input result in a bounded output (BIBO stability). Moreover if the behaviour converges towards an equilibrium point, then the system is asymptotically stable. If the equilibrium is reached for every possible initial state then the system is globally asymptotically stable. Stability can be guaranteed for every system with hard and soft constraints if the optimal control problem can be solved over an infinite prediction horizon. Therefore if the optimization problem is feasible, then the closed-loop trajectories will be feasible at all times. However the optimization problem, for computational restrictions, is often limited to a finite horizon. The receding finite horizon control problem behaviour does not always resemble the one defined over an infinite horizon. In fact at some point the optimal control problem might become infeasible meaning that the control input does not satisfy the constraints.In addition, even if feasibility is always satisfied, asymptotic stability might not be. Therefore stability and feasibility are not always ensured by the receding horizon control law. To achieve stability the cost function is formulated as a $LYAPUNOV$ function and an accurately selected terminal cost is introduced. [7] [8]

2.1.4 Types of Model Predictive Controllers

Summarizing what said so far, the MPC uses the model of the plant, noise and disturbances to estimate the controller states and predicts future plant outputs. With the predicted outputs, the controller solves a quadratic programming optimization problem to determine the optimal control inputs. So far the linear MPC has been investigated. In fact most of the time the to be controlled plant can be approximated by a linear plant around a given operating point. When this approximation is not accurate enough or it is not guaranteed for every operating point, different techniques might be used. In the following some MPC techniques will be presented

Gain scheduling MPC

Gain-schedule MPC is used to control a non-linear plant in a wide range of operating conditions. It uses a predefined set of MPC controllers so that it can switch among them. It is used when the plant dynamics change over time in a predictable way. It requires a lot of design effort even if it is computationally simple to implement. The main idea is to tune a nominal MPC controller for the average conditions. Then simulations are used to determine where the nominal controller looses robustness. A new plant model is then implemented at these operating conditions and a new MPC controller is developed using the nominal one as a starting point. [7][9]

Adaptive MPC

MPC predicts future behaviour of a system using a model of this system. These predictions, though accurate, are never exact. It is important therefore to tune the MPC so that it becomes insensitive to prediction errors. However if the plant is non-linear or it changes a lot over time, the prediction accuracy might become so bad that the controller is not performing well enough anymore. This is why the Adaptive MPC is adopted. The idea behind this technique is that of finding a controller that can address different operating conditions. In fact at each control interval the Adaptive MPC controller updates the plant and the nominal conditions. First it uses a fixed model structure found for the average most likely conditions. As the parameters evolve over time the controller uses a model which is appropriate for the current conditions. To update the controller states, usually a Kalman Filter is used. In Adaptive MPC the kalman gains are adjusted at each time instant so that it is consistent with the updated plant model. Thus the result is a time varying kalman filter. [7][10]

Non-linear MPC

As in linear MPC, the nonlinear MPC (NMPC) computes the control actions using optimization and modeling. The differences between the two techniques are:

- the prediction model can be non linear and show time-varying parameters
- the constraints can be non linear
- the cost function can be non quadratic

Linear MPC is nowadays a successful technique which is used for different applications. NMPC on the other hand has gained more popularity in the last years even though it has always been used in those industry processes dealing with slow systems requiring intensive computational costs. However fast algorithm for NMPC have been developed thus NMPC is being considered also for fast applications. Nevertheless many systems are non linear. The inherent non-linearity, the higher quality specifications and productivity require to use the system in a wide range of operating conditions. Linear models might not be adequate to describe the system dynamics. Using a linear MPC might indeed lead to poor and unsuccesful results as it is not able to satisfy the process requirements. Using a non-linear model would provide better results. At the same time the computational costs would be higher and it would be harder to find a control law and to analise the stability of the system. [11]

2.2 MPC for Inductive power transfer systems

Inductive power transfer systems are sensitive to parameters variations. This affects the efficiency and stability of the system. In this work a non-linear MPC controller for the aformentioned IPT system is developed. Then it is compared to a typical PI controller so that it is possible to develop a comparative analysis of the two techniques.

2.2.1 Implementation of Non-linear MPC

Mathematical formulation

The mathematical formulation of a NMPC problem in continuous-time is here presented. Consider the system described by the following non linear differential equation:

$$x(t)' = f(x(t), u(t)) \tag{2.3}$$

$$x(0) = x_0 \tag{2.4}$$

subject to

$$u(t) \in U, t \geq 0, \tag{2.5}$$

$$x(t) \in X, t \geq 0. \tag{2.6}$$

The system is n dimensional and with m inputs. The input and states set are constraints in this form:

$$U := u \in \boldsymbol{R}^m \mid u_{min} \leq u \leq u_{max} \tag{2.7}$$

$$X := x \in \boldsymbol{R}^n \mid x_{min} \leq x \leq x_{max} \tag{2.8}$$

The optimal control input is found as the solution of the following finite-horizon optimal control problem, solved at every sampling instant:

$$\begin{aligned}
\min_{\bar{u}(\cdot)} \quad & J(x(t), \bar{u}(\cdot)) \\
\text{s.t.} \quad & \bar{x}'(\tau) = f(\bar{x}(\tau), \bar{u}(\tau)), \\
& \bar{x}(t) = x(t), \\
& \bar{u}(\tau) \in U, \tau \in [t, t + T_c], \\
& \bar{x}(\tau) \in X, \tau \in [t, t + T_p]
\end{aligned} \tag{2.9}$$

where

$$J(x(t), \bar{u}(\cdot)) := \int_t^{t+T_p} F(\bar{x}(\tau), \bar{u}(\tau))\, d\tau.$$

T_p and T_c are respectively the prediction horizon and the control horizon and $T_c \leq T_p$.

The cost function is defined in terms of the stage cost F, specifying the performance. In general the cost function is a quadratic one taking the following form:

$$F(x,u) = \left[x - x_{ref}\right]^T Q \left[x - xref\right]^T + \left[u - u_{ref}\right]^T R \left[u - u_{ref}\right]$$

x_{ref} and u_{ref} denote the desired reference trajectory. The positive definite matrices Q, R weight the deviation from the references. As having infinite prediction and control horizon lead to infeasibility in terms of calculations, they are usually finite. However as well as in the linear case, stability problems may arise when considering a finite prediction horizon. To solve this issue most of the time the standard NMPC setup is modified so that stability of the closed-loop can be guaranteed independently of the plant specifications. This is achieved by adding equality or inequality constraints or penalty terms to the standard setup. [8]

Non-linear MPC for the IPT system

The non-linear MPC controller was implemented in $MATLAB$ and $Simulink$ using the $MATLAB$ toolbox for the NMPC. The objective function to be minimized has this structure:

$$J(k) = \frac{1}{N}(\sum_{1}^{N}((P_{in}(k+1)) - P_{ref})^2 + \lambda((V_{1,d})(k) - V_{1,nom})^2) \tag{2.10}$$

The optimal control problem is then formulated in this way:

$$\begin{aligned} \min \quad & J(k) = \frac{1}{N}(\sum_{1}^{N}((P_{in}(k+1)) - P_{ref})^2 + \lambda((V_{1,d})(k) - V_{1,nom})^2) \\ \text{s.t.} \quad & \dot{x}(t) = (1.9) \\ & 0 \leq v_{1,d} \leq \frac{4}{\pi} \cdot V_{dc,out} \end{aligned} \tag{2.11}$$

The prediction horizon is set as $N_p = 4$ and the sampling time is $T_s = \frac{1}{f} = \frac{1}{85000}$. To use the toolbox, a state and output function are to be provided to the NMPC controller. The state function predicts how the states evolve over time, while the output function evaluates the plant outputs given the state and input variables. For this specific application, the operating frequency ω is considered constant i.e. $\omega = 85000$ as well as the output voltage $V_{dc,out} = 184$. The voltage $v_{1,q} = 0$ so that the reactive power is not taken into consideration.
Therefore the non linear system considered in this work becomes:

$$\begin{aligned} \dot{x} &= f(x,u) \\ y &= g(x,u) \end{aligned}$$

where

$$x = \begin{bmatrix} i_{1,d} & i_{1,q} & i_{1,d} & i_{2,q} & v_{C1,q} & v_{C1,d} & v_{C2,d} & v_{C2,q} \end{bmatrix}^T \tag{2.12}$$

$$u = \begin{bmatrix} v_{1,d} \end{bmatrix} \tag{2.13}$$

$$y = \begin{bmatrix} P_{in} \end{bmatrix} = \begin{bmatrix} v_{1,d} i_{1,d} \end{bmatrix} \tag{2.14}$$

These are the states, inputs and output used in the state and output function for the MPC controller. As the non-linear MPC controller is a discrete-time controller, the state function has to be discretized. For this purpose the Forward

Euler discretization method is used, i.e.

$$\dot{x}(t) = \frac{x(k+1) - x(k)}{T_s} \tag{2.15}$$

Again $T_s = 1/85000$ and the discretization is repeated for $N = 4$ steps. Once the discretization has been performed in $MATLAB$, the cost function in (2.9) can be minimized.

Non-linear MPC, as well as linear MPC, solves a constrained optimization problem at each interval. In this case, as the plant is non linear, the optimal control problem is non-linear as well. However the standard cost function for non-linear MPC is a standard quadratic cost function for reference tracking, which is the same for the linear case. Indeed this is the cost function used in this work. The weigths used in the cost functions need to be specified: in this case the ouput variable tuning weight, which prevents the output from deviating from the reference, is set as $1/4$. The manipulated variables tuning weight, used to prevent deviations from the target, is set to 0.4. The tuning weight for the manipulated variable is chosen as small as possible so that the main focus is on the output.

The optimization problem is carried out by the optimization toolbox using the SQP algorithm. The optimal control value

$$U \star (k) = argminJ(k) = v_{1d,opt}(k), v_{1d,opt}(k+1), ..$$

is then found.

Here, the main file in *MATLAB* is presented: after specifying the dimension of the model, i.e. the number of input, output and states, the main file calls the functions needed to specify the model.

```
/*Create a nonlinear MPC controller with a prediction model that
has 8 states, 1 output, and 1 input.
nlobj = nlmpc(nx,ny,nu) creates a nonlinear MPC object.
The inputs are positive integers that define prediction model
dimensions: nx: number of states,
ny: number of outputs, nu: number of inputs  */
nlobj = nlmpc(8,1,1);

/* Specify the controller sample time, prediction horizon
 and control horizon.  */
Ts=1/85000;
nlobj.Ts = Ts;
nlobj.PredictionHorizon = 4;
nlobj.ControlHorizon = 1;

/*Specify the state and output function */
nlobj.Model.StateFcn = myStateFunctionCT;
nlobj.Model.OutputFcn = myOutputFunction;

/*Specify the tuning weights for the cost function */
nlobj.Weights.OutputVariables = 1/4;
nlobj.Weights.ManipulatedVariablesRate = 0.4;

/*Simulation of the model*/
sim(mdl)
```

In *SIMULINK* the NMPC controller is implemented in an open-loop way as shown in the figures below:

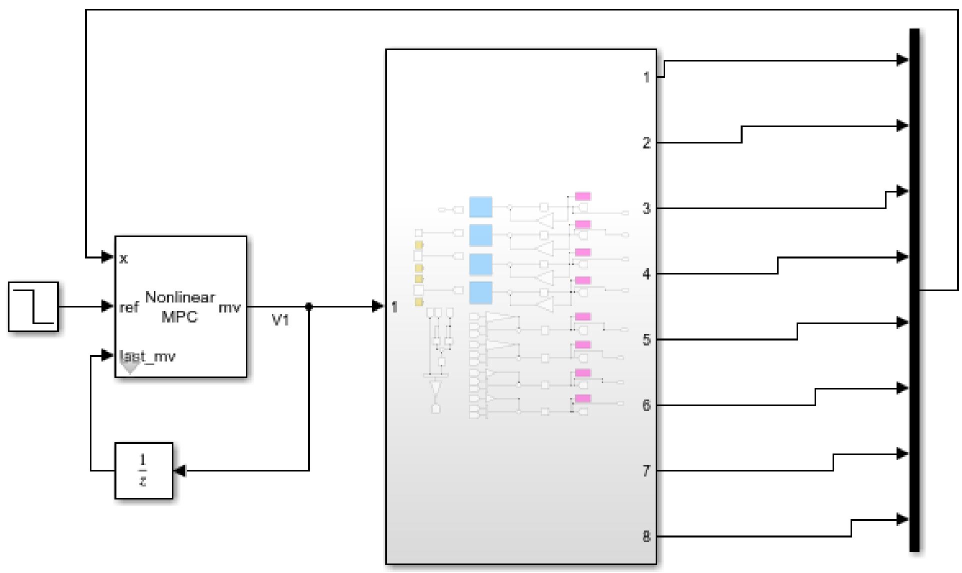

Figure 2.4: Block scheme of the MPC

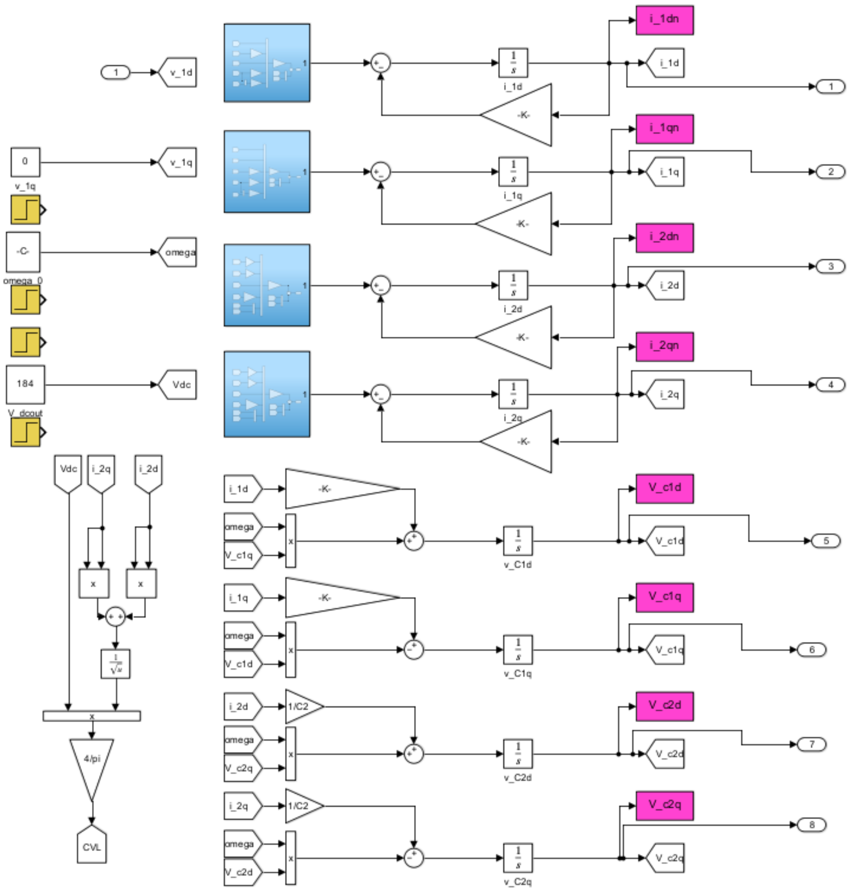

Figure 2.5: Dq-model of the IPT system

3 Modulation

3.1 Phase shift and pulse density modulation

As mentioned above the IPT system consists of an H-bridge in the sending side. An H-bridge is an electronic circuit that allows the inversion of the polarity of a voltage and thus the current applied to a load. Usually H-bridges are applied to DC motors to make them run forward or backwards. The H-bridge consists of four switching elements, as shown in figure, which can be either diodes or mosfets. When switches S1 and S4 are on and thus closed a positive voltage will be applied to the circuit. On the other hand when switches S2 and S3 are open, this voltage is reversed. Notice that switches S1 and S2 should never be open at the same time otherwise there would be a short circuit. The same reasoning is applied to switches S3 and S4. The H-bridge is connected to a DC voltage source as input. The output of the H-bridge is a voltage fed to a high frequency transformer. This voltage is then rectified by a diode rectifier to get the required DC output voltage. By adjusting the phase shift between S1 and S2, it is possible to change the width of the transformer voltage. If the width increases then the output voltage increases while if the width decreases the output voltage decreases as well.

The *Simulink* implementation of the IPT system under consideration is here shown:

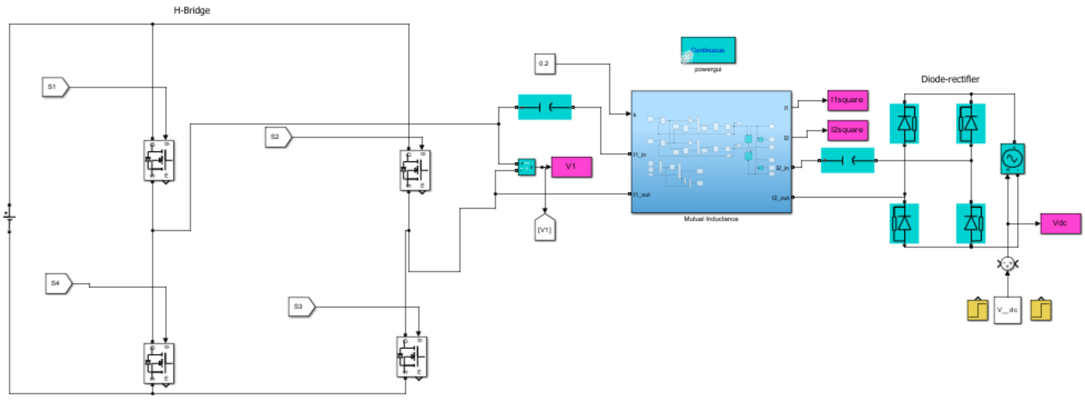

Figure 3.1: Simulink implementation of the IPT system

In electronics modulation is the process of adding information on a radio-frequency carrier wave by changing one or more characteristics of the wave according to the modulation signal which contains information to be transmitted. The modulation signal may be an audio, video or digital signal. In general the carrier signal has a higher frequency. There are different forms of modulation according to what characteristic of the signal needs to be altered. Modulation schemes can be analog or digital. Analog techniques work by modulating a continuous carrier wave, rather than a signal encoded in binary digits as with digital techniques. Digital modulation is used to transmit binary signals. In order to transmit digitized information over a communications channel, an analog carrier wave can be modulated to reflect the binary nature of the digital baseband signal.
In this work phase shifting modulation and pulse density modulation are presented.

Phase shift modulation (PSM) is an analog modulation technique. The modulation is applied continuously in response to the analog information signal.

With PSM, the phase of the carrier signal is varied to reflect possible changes in the frequency of the data. The frequency is unchanged though. Is it important to notice that the changing rate of the phase is directly proportional to the frequency of the modulation signal. Pulse density modulation (PDM) is a modulation technique used to represent an analog signal in the digital domain. The relative density of the pulses corresponding to the analog signal amplitude is encoded into codewords of pulses with different weights. In PDM, 1 corresponds to a pulse while 0 corresponds to the absence of a pulse. PDM is realised through the process of sigma-delta modulation. It generates a pulse stream in which the frequency of the pulses is proportional to the analog voltage input. [12] [13]

A WPT system needs to provide the required power to the load and work with the highest possible efficiency. Therefore a control technique is implemented combining first a NMPC controller and then a Proportional Integral(PI) controller with the aformentioned modulation techniques.
As the goal is to regulate the power transfer provided to the load, in theory it would be necessary to use the output power as the feedback signal in the control loop. However this would require a wireless feedback across the air gap of the system. For this reason and for simplicity it is better to use feedback signals coming from the sending side to regulate the power transfer.
First phase shift modulation will be analyzed. The phase shift angle, α, is a function of the voltage of the sending coil and of the input voltage and is evaluated through the formula:

$$\alpha = arcsin(\frac{\pi}{4}\frac{V_1}{V_{in}}) \tag{3.1}$$

It is then fed to a phase shift modulator for full bridge converters which takes as inputs the frequency, which is 85000 Hz, and the angle in degrees.

In figure 3.2, the implementation of the phase shift modulation with NMPC is shown.

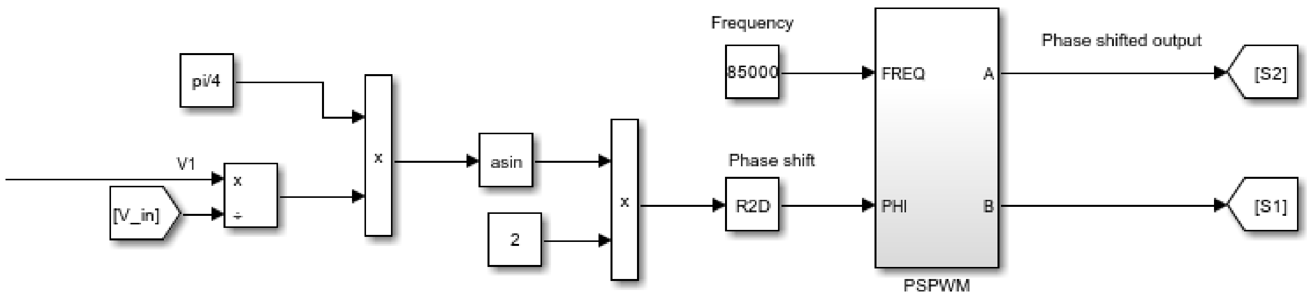

Figure 3.2: Block scheme of the Phase shift modulation with MPC

In figure 3.3 the control scheme with a PI controller is shown. The input power is used as feedback for the reason mentioned above. In this case as well, the output of the PI controller is the control variable $V_{1,d}$ of the sending coil. The optimal value of $V_{1,d}$ was obtained by tuning the PI controller with a trial and error technique. The optimal gains of the controller are $K_I = 200$ and $K_P = 0.4$.

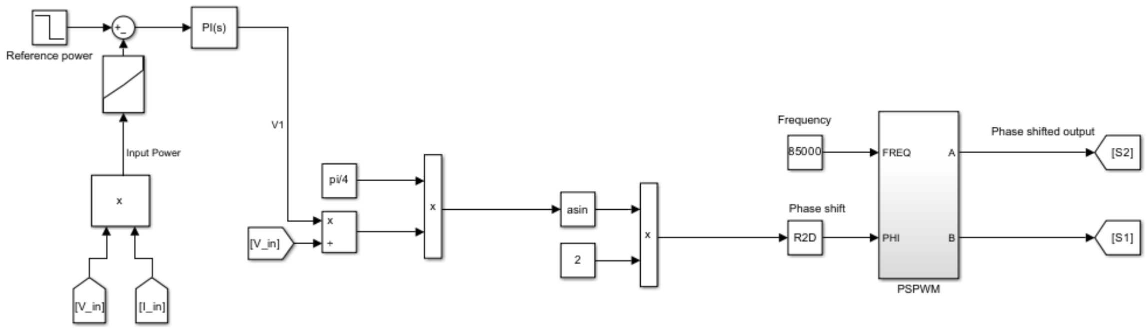

Figure 3.3: Block scheme of the phase shift modulation

Pulse density modulation is a more attractive method compared to phase shift modulation because it ensures soft switching and maintains high efficiency even when dealing with large voltage ranges. The main idea behind PDM is that it adjust the pulse density by skipping pulses. Some of the pulses are removed in a switching cycle and hard switching is avoided. The pulse density D is defined as the ratio of the number of remaining pulses to the number of switching cycles. Therefore by adjusting the pulse density D the output power of IPT systems with PDM can be constrained as

$$V_1 = 2 \cdot \sqrt{2} \cdot D \cdot \frac{V_{in}}{\pi} \tag{3.2}$$

The implementation of PDM with MPC is shown in the figure below. Again at each time instant the NMPC computes the optimal value for $V_{1,d}$ and it is used to compute the optimal value for the pulse density.

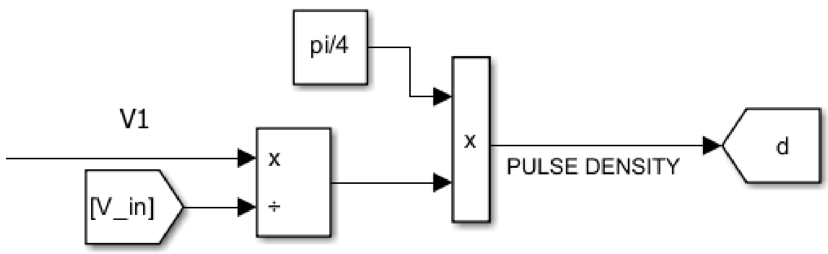

Figure 3.4: Block scheme of the pulse density modulation with NMPC

The pulse density is then passed to the pulse density modulation circuit. The implementation of PDM is carried out through delta-sigma modulation. There is an accumulator which is triggered by the rising edges of the input pulses and accumulates the difference between the pulse density D and the output of a comparator. The latter is then combined with the delayed input pulses by an AND gate whose output is the modulated output pulses. [14]

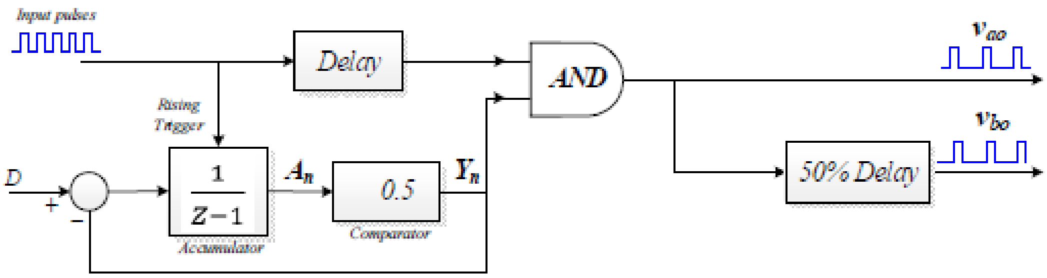

Figure 3.5: Block scheme of the Pulse density modulation

The control scheme with PI and PDM is shown in the following picture. Again the PI controller uses as reference the difference between the nominal power the input power to compute the control variable. The tuning of the PI controller has been performed with a trial and error approach also in this case, leading to the following optimal gains: $K_I = 100$ and $K_P = 0.01$.

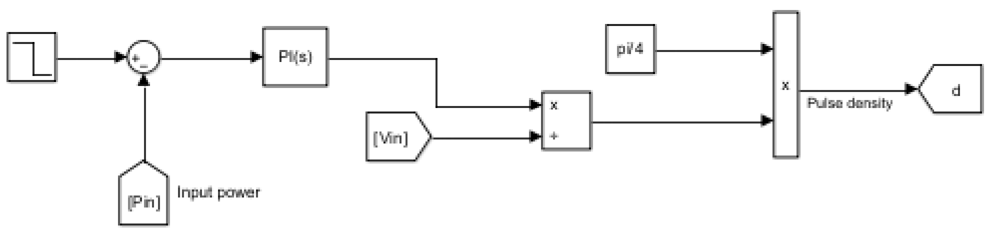

Figure 3.6: Control scheme

4
Simulations

4.0.1 Modulation with PI controller

Phase shift modulation

The simulation time is set as 0.1 s and as input of the control loop there is a reference power represented as a step. With a step time of 0.05 s, it goes from 5000 to 4000 W.

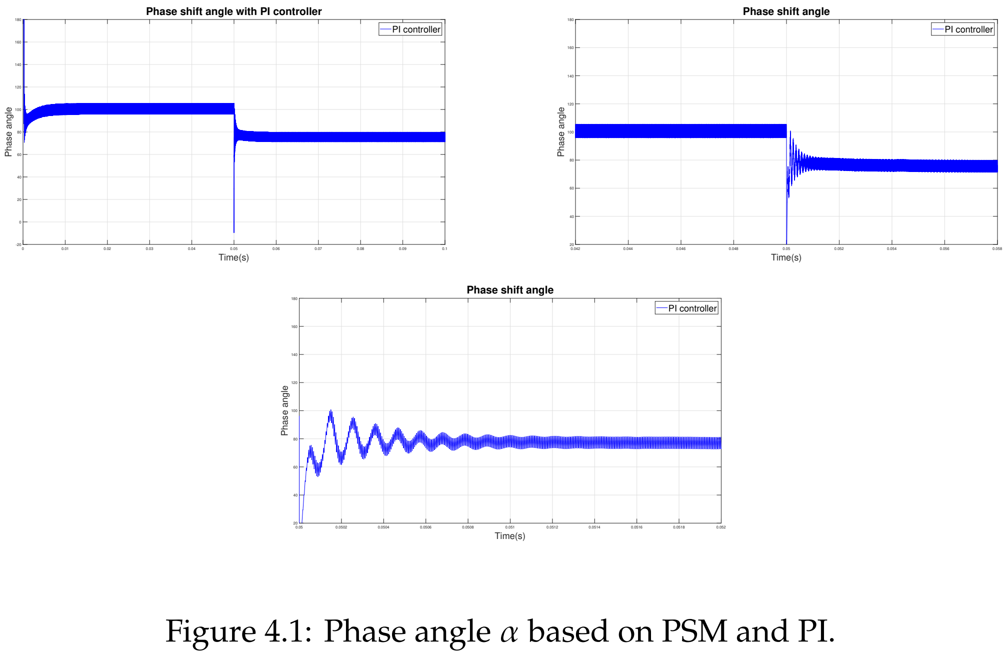

Figure 4.1: Phase angle α based on PSM and PI.

In Figure 4.1 the phase angle behaviour is shown. When the step reference changes, there are some oscillations but then it gets stable.
The voltage v_1 is shown in figure 4.2.

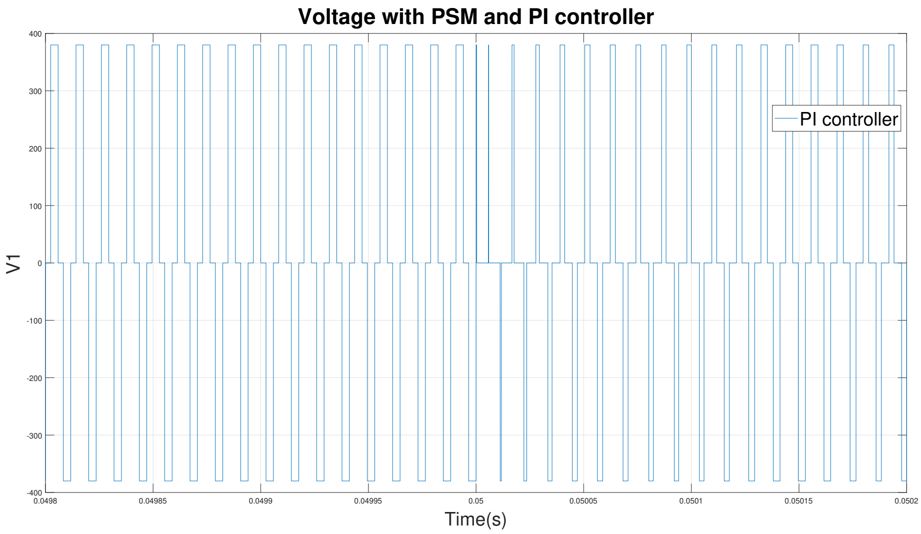

Figure 4.2: Voltage V_1 based on PSM and PI.

The behaviour of the currents of the sending and receiving sides is shown below in figure 4.3. It is typical of phase shift modulation.

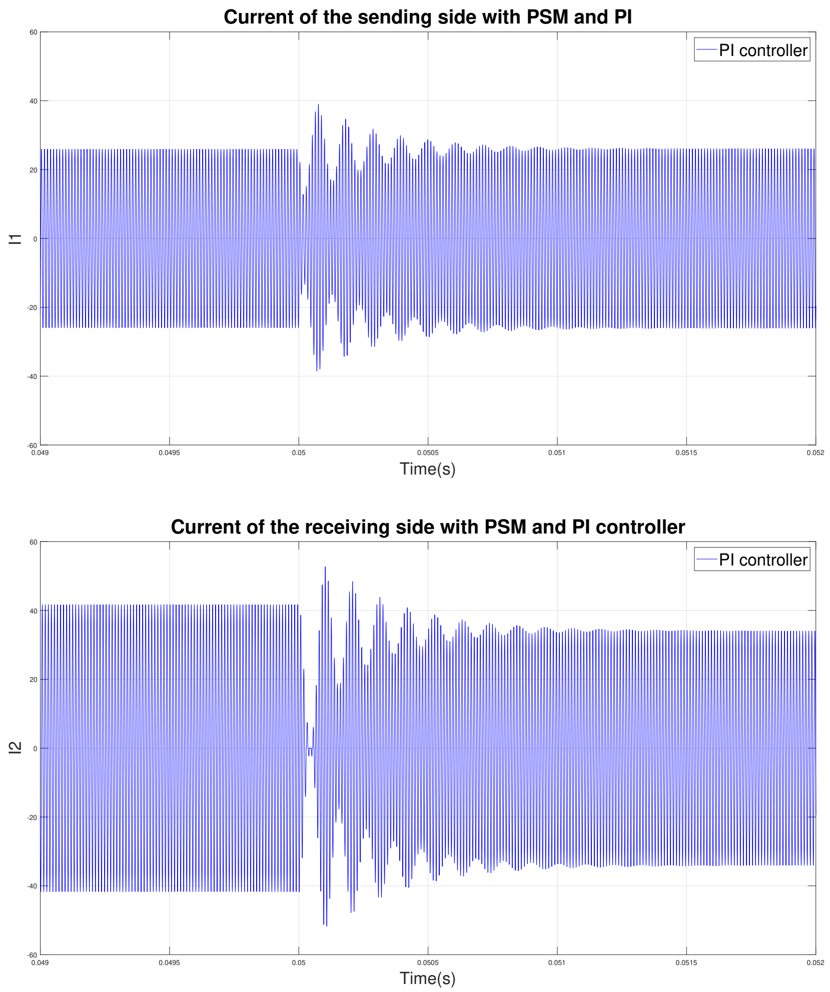

Figure 4.3: Sending and receiving currents based on PSM and PI.

At 0.05, when the reference changes the currents adopt this behaviour:

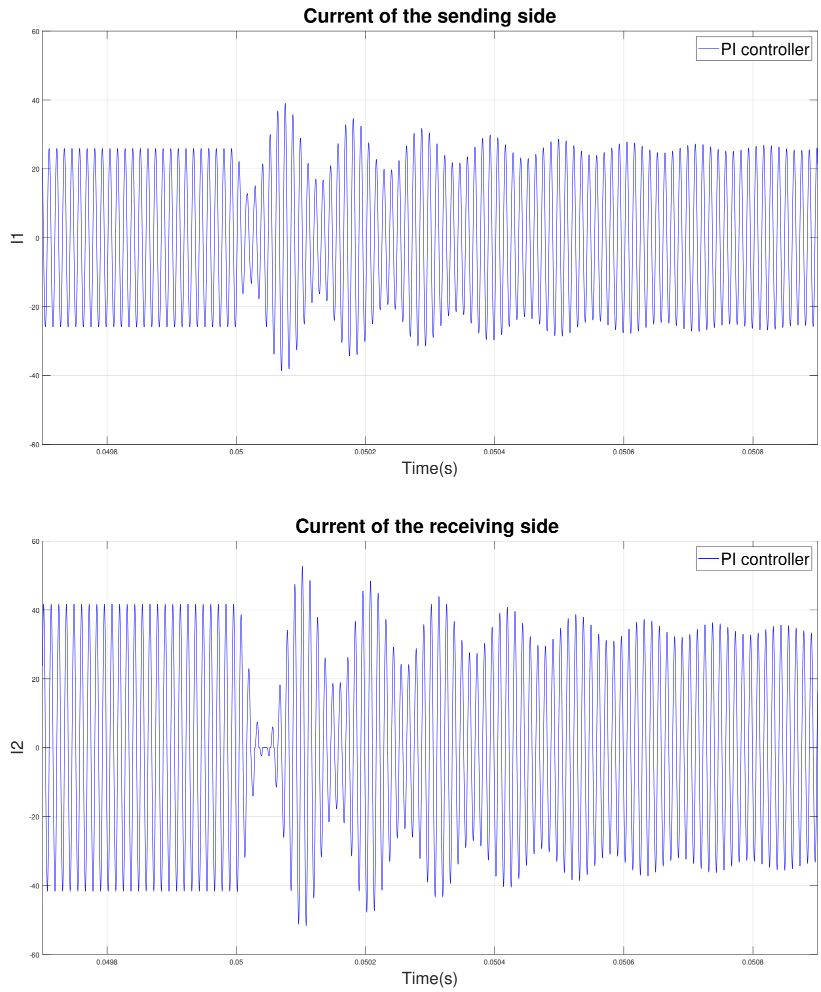

Figure 4.4: Highlight of the changes in the currents.

As the main goal is to control the power transfer, the input power and the output power are compared.

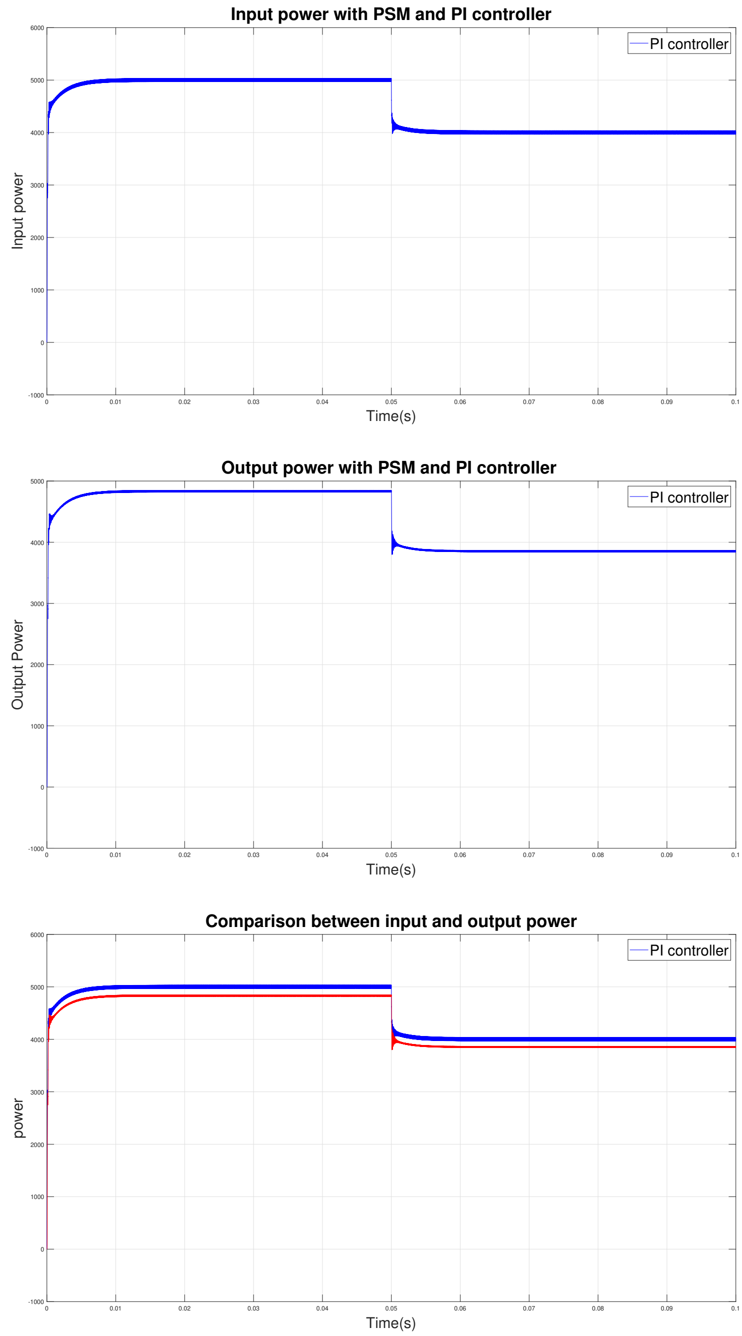

Figure 4.5: Input and output power based on PSM and PI.

The above results show that the PI controller is able to quite accurately follow the constant power frequency trajectory. The efficiency of the system is around the 94%. There are still power losses due to the hard switching caused by phase shift modulation itself.

Pulse density modulation

In figure 4.6 the behaviour of the pulse density is shown. After applying the input to the system,it can be seen the output takes a bit more time before reaching the steady state.

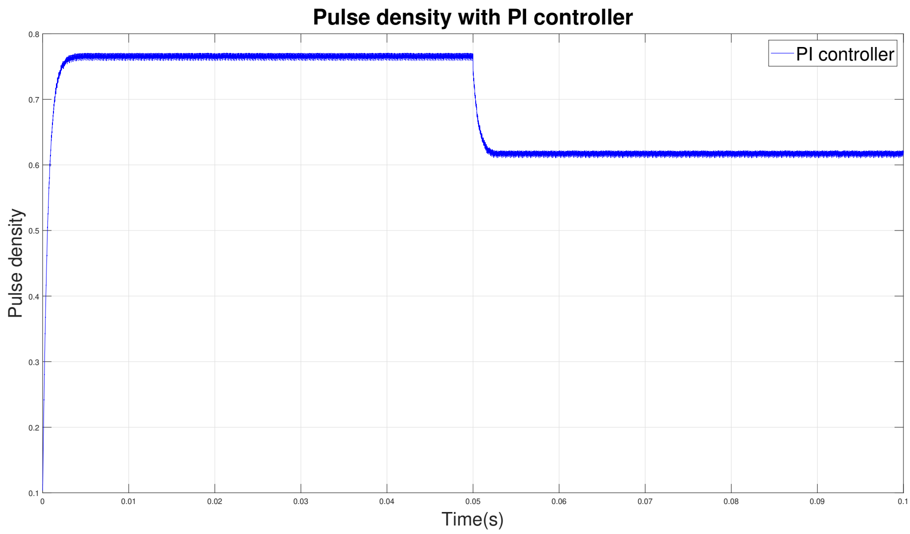

Figure 4.6: Pulse Density

The voltage v_1 shows this behaviour: a pulse is skipped and the voltage is zero.

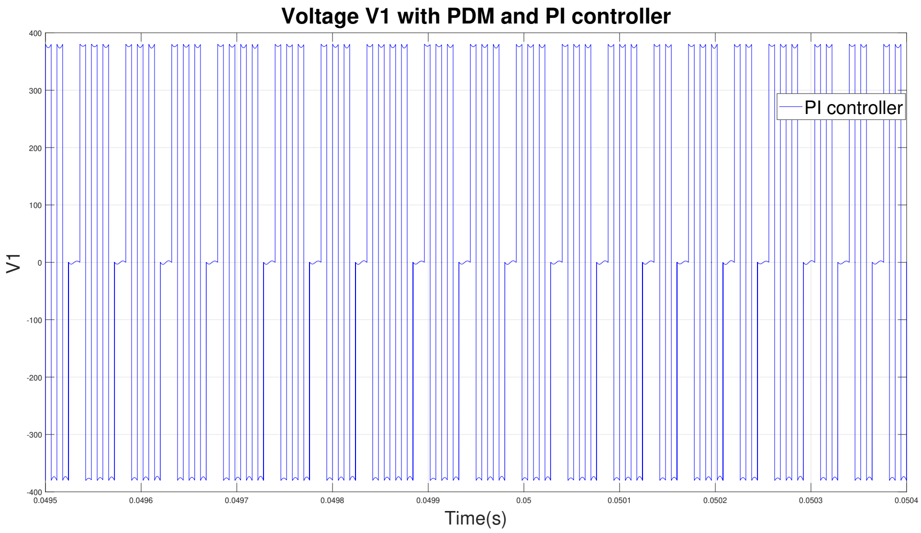

Figure 4.7: Voltage V_1 based on PDM and PI

The currents adopt this behaviour.

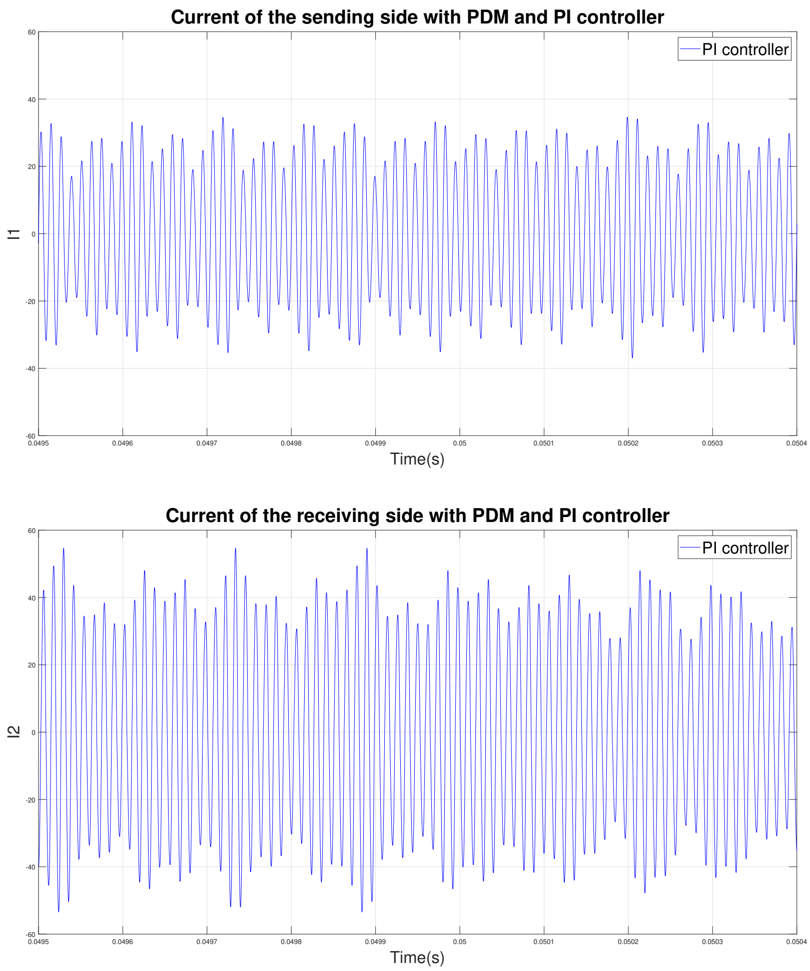

Figure 4.8: Sending and receiving currents based on PDM and PI

As it can be seen, when one pulse is skipped the current is excited and there is a ripple. When the voltage is zero, the current goes up and down. This causes a frequency oscillation.

The input and output power are:

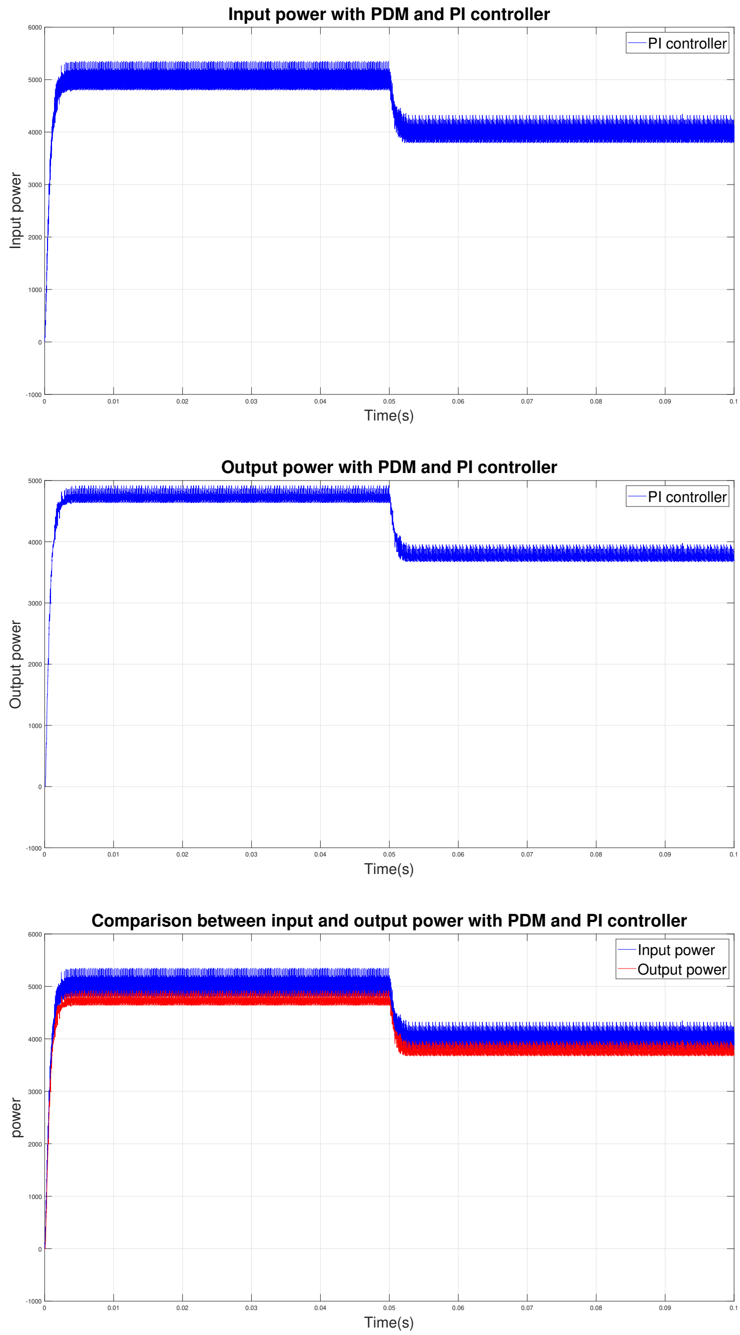

Figure 4.9: Input and output power based on PDM and PI.

The PI controller is able in this case as well to track the input power in a quite good way. The efficiency of the controller is equal to 93.3% . There are a lot of oscillations due to the modulation itself. In conclusion the performance is acceptable.

4.0.2 Modulation with MPC controller

Phase shift modulation

The simulation time is $t_s = 0.1$. The reference signal goes from 12000 to 9000 and is represented as a step with step time, $t_{step} = 0.05$. As it can be seen the controller is able to find the optimal phase angle without a lot of oscillations before reaching the steady state.

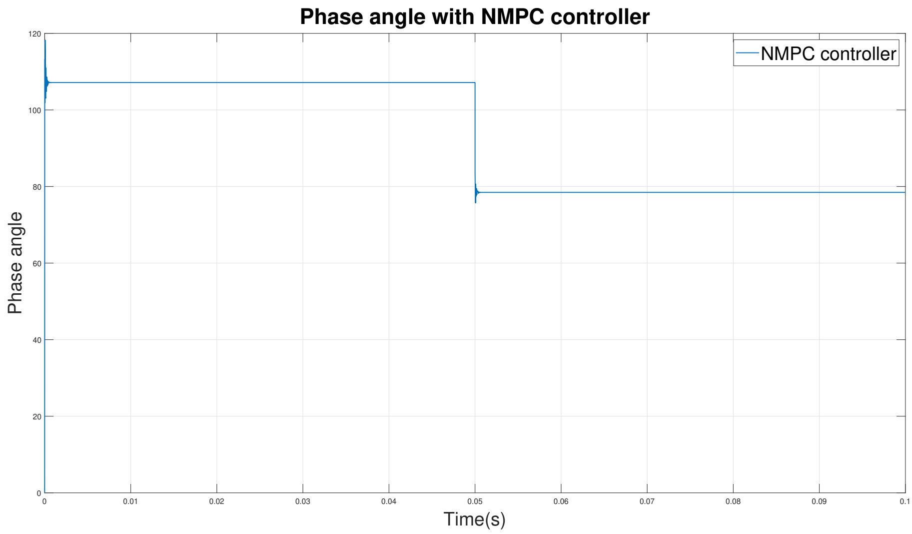

Figure 4.10: Phase angle based on PSM and NMPC

The voltage V_1 is shown in picture 4.11.

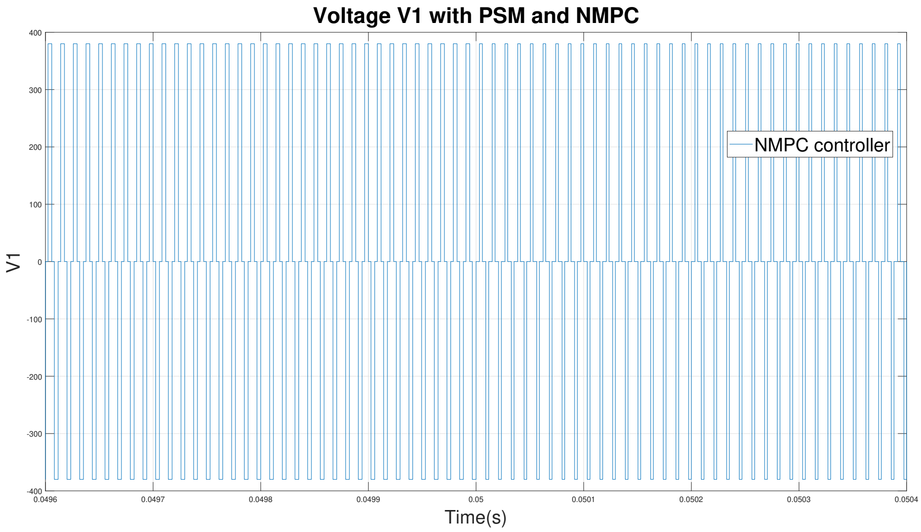

Figure 4.11: Voltage V_1 based on PSM and NMPC

In figure 4.10 the general behaviour of the current of the sending and receiving side is shown.

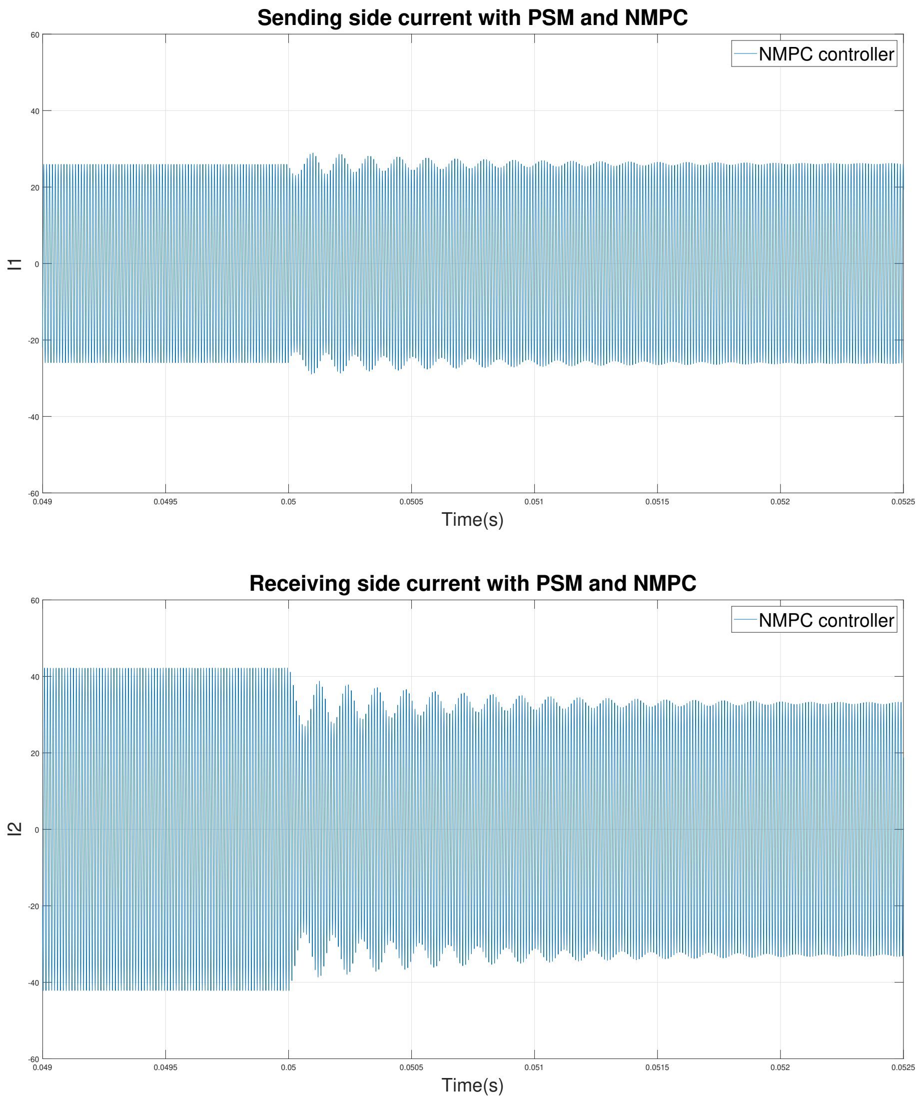

Figure 4.12: Sending and receiving currents based on PSM and PDM

The input and output power are shown in figures 4.13.

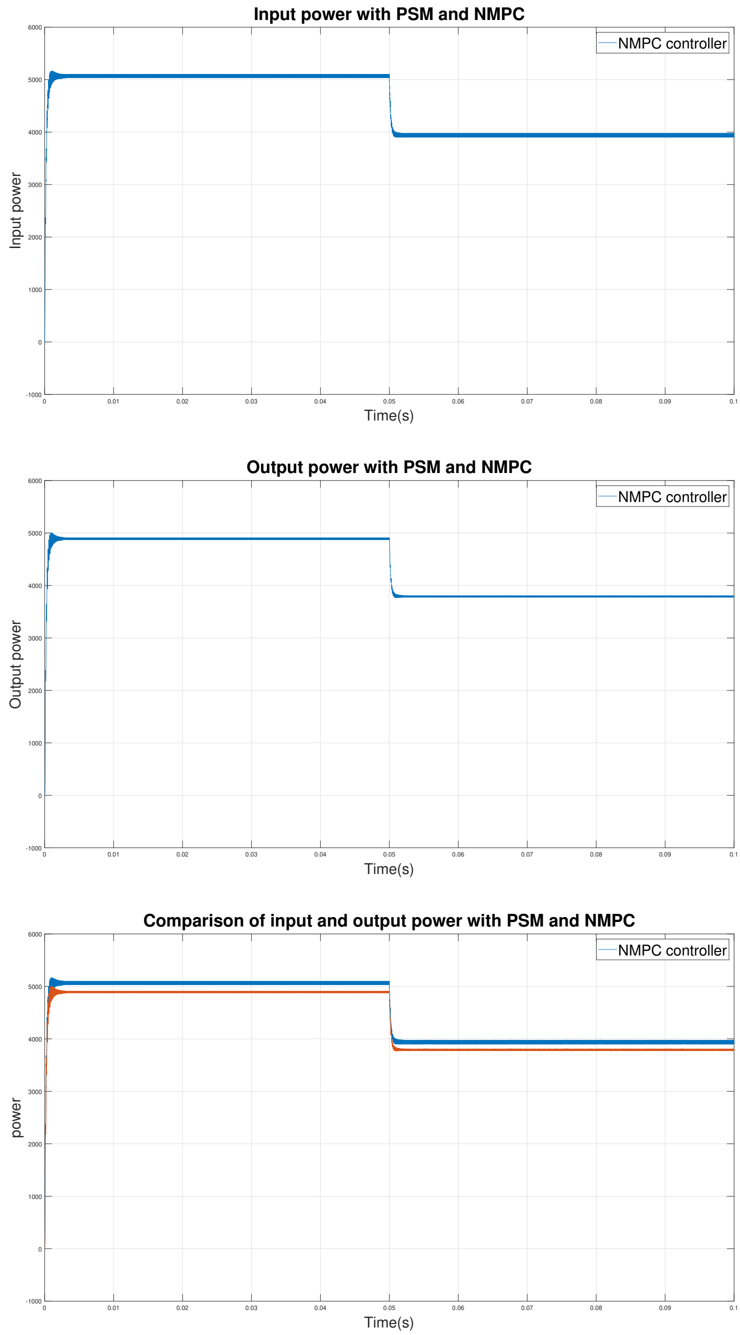

Figure 4.13: Input and output power based on PSM and NMPC.

As shown in figure 4.14 the NMPC controller with PSM tracks the input power but it is still not able to perfectly track it. The efficiency is around the 95%.

Pulse density modulation

The simulation time is $t_s = 0.009$. It is lower than the other simulations for feasibility reasons. In figure below the pulse density is shown. As it can be seen the MPC controller is able to find the optimal value quite fast and without oscillations.

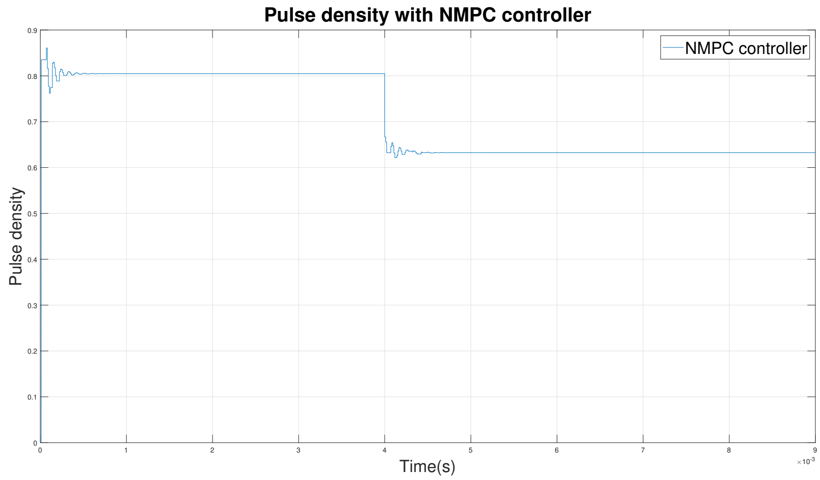

Figure 4.14: Pulse density based on PDM and NMPC

The voltage v_1 is shown in picture 4.15.

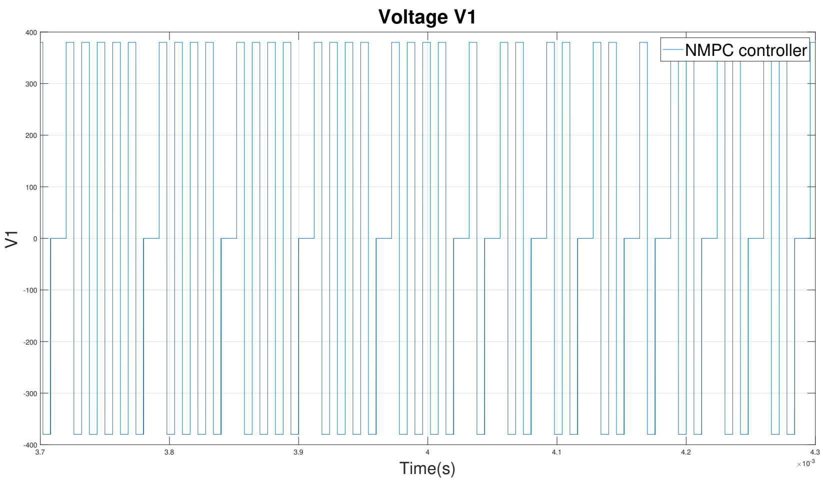

Figure 4.15: Voltage V_1 based on PDM and NMPC

The receiving and sending currents are shown.

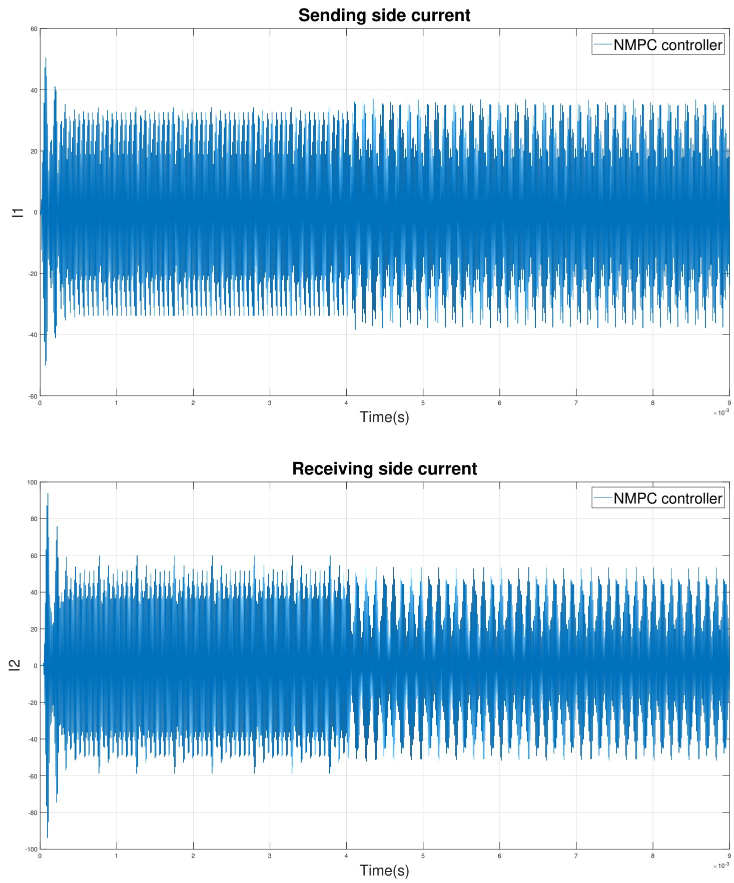

Figure 4.16: Sending and receiving currents based on PDM and NMPC

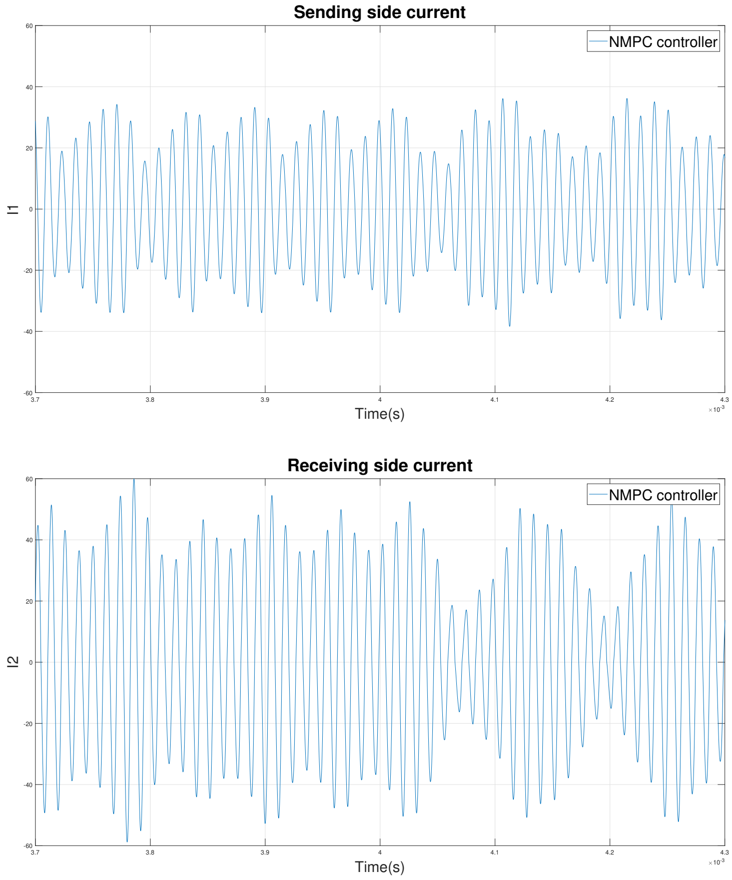

Figure 4.17: Sending and receiving currents based on PDM and NMPC

Pulse density modulation even with the NMPC controller cause large oscillations in the currents. Again as the voltage is zero, the currents show a ripple. Comparing the currents on the sending side shown in pictures 4.8 and 4.17 one can see that there are not significant differences. Thus both the PI and the MPC show the same performances when it comes to control the ripple.

In the figures below the input and output powers are compared. As it can be seen the controller is able to track the input power and there are not significant losses in the output power. The efficiency of the system is in fact equal to 98.7%

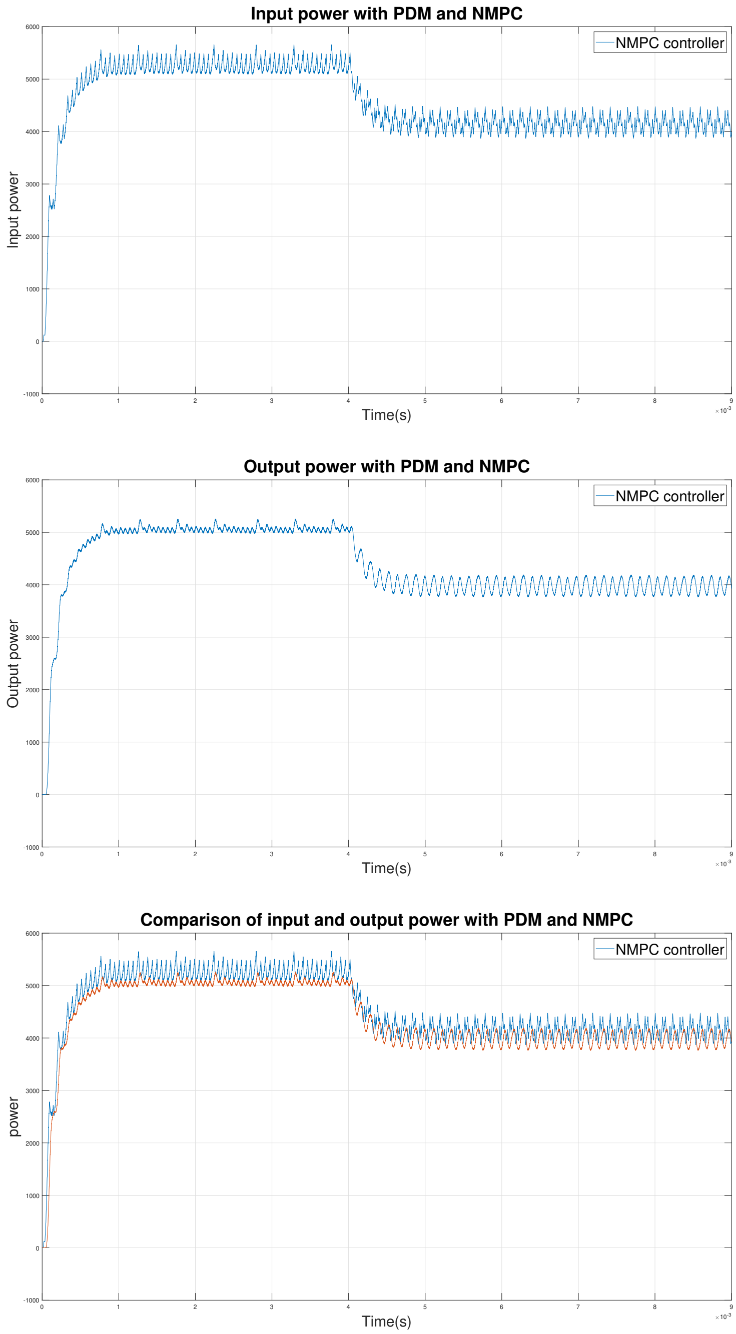

Figure 4.18: Input and output power based on PDM and NMPC.

4.0.3 Comparisons between NMPC and PI

Phase shift modulation

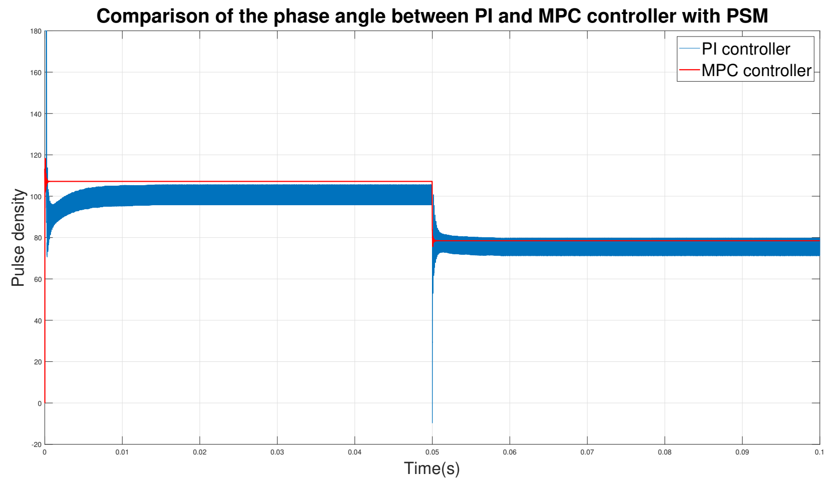

Figure 4.19: Comparison of the phase angle based on PSM between NMPC and PI

Regarding PSM, as shown in the pictures above, the NMPC controller shows a better transient response than the PI controller. The phase angle in fact, with NMPC reaches the optimal value quite immediately and with very little oscillations also when the step reference changes. On the other hand with the PI controller it takes more time to reach the steady state value and the transient state is longer.

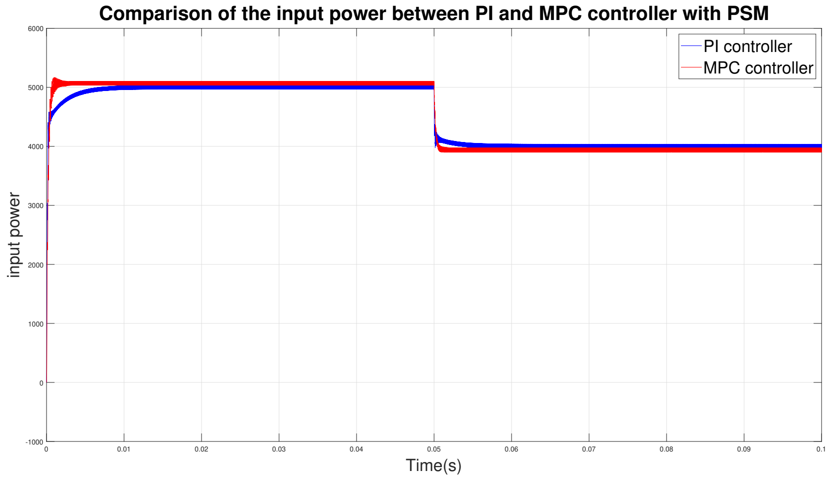

Figure 4.20: Comparison of the input power, based on PSM, between NMPC and PI

The system, under the NMPC controller, reaches immediately the required input power and does the same when the step reference change. With the PI controller there are some more oscillations and the input power value is not reached instantely.

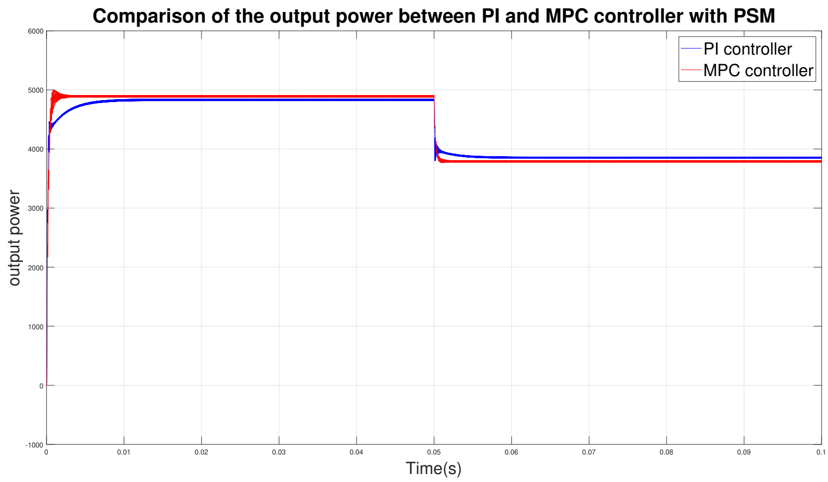

Figure 4.21: Comparison of the output power, based on PSM, between NMPC and PI

When it comes to the output power, the NMPC is again performing better than the PI control. The PI controller takes more time and more oscillation to settle than the NMPC. However PSM ensures power losses even with the NMPC.

Pulse density modulation

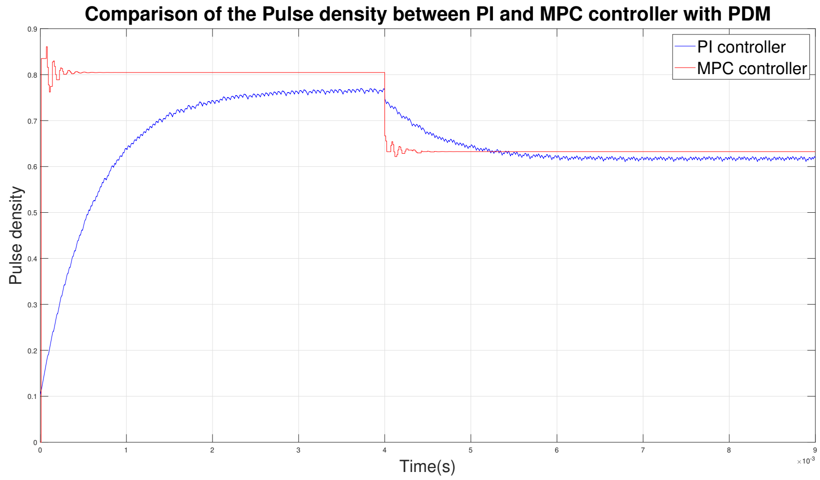

Figure 4.22: Comparison of the pulse density, based on PDM, between NMPC and PI

In the figure above it is clear that with NMPC the optimal pulse density is found quite immediately with very little oscillations while with the PI control the curve is more damped and the transient response is clearly longer than with the NMPC.

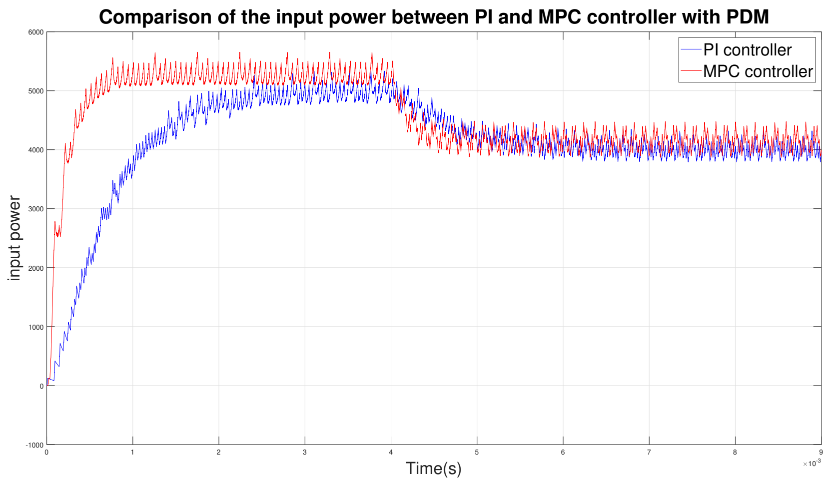

Figure 4.23: Comparison of the input power,based on PDM, between the NMPC and PI

With NMPC the input power reference value is reached sooner that with the PI controller. Again the transient state is longer under the PI controller.

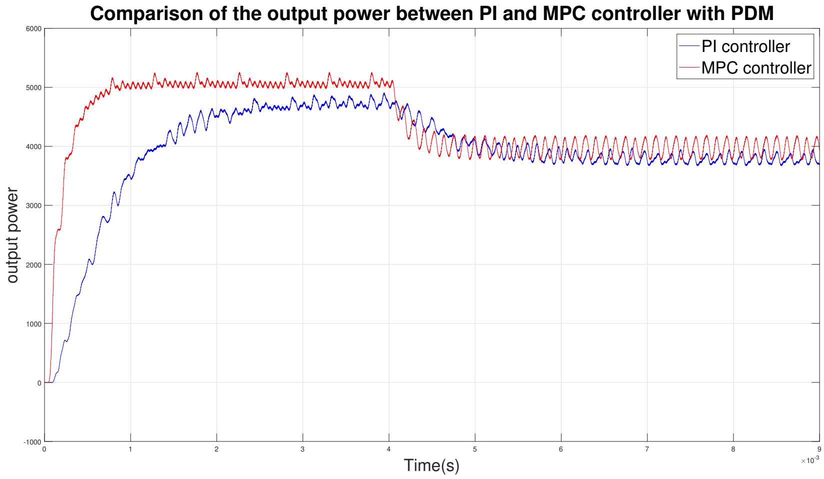

Figure 4.24: Comparison of the output power, based on PDM, between the NMPC and PI

The output power with NMPC reaches the values imposed by the input power thus there are not significant losses which on the other hand is what happens with the PI controller. In fact the efficiency of the system with the NMPC controller is almost 100 thus confirming the superiority of the MPC control.

5

Conclusion

In this work a non-linear model predictive control for an IPT system has been presented. It is shown how NMPC control-based methods, though bringing more computational effort, perform better than traditional PI controller-based ones. In fact NMPC has a better and faster transient response and reaches the steady state values sooner. Moreover the efficiency of the system under the NMPC control is higher than with the PI control. All these considerations were drawn with a constant coupling coefficient. However in an IPT for dynamic WPT the coupling coefficient is varying over time. Therefore a possible improvement would be to take this factor into consideration. Moreover, the implementation of the NMPC was achieved in an open loop way, without applying the system directly to the controller. Doing this would be bring the implementation closer to a real system.

In general still a lot of work has to be put into the development of a control scheme for an IPT system.

References

1. Aydin, E., Aydemir, M., Aksöz, A., El Baghdadi, M. & Hegazy, O. *Inductive Power Transfer for Electric Vehicle Charging Applications: A Comprehensive Review* in. **15** (July 2022), 4962.
2. Salem, M., Jusoh, A., Idris, N. R. N. & Alhamrouni, I. *A Review of an Inductive Power Transfer System for EV Battery Charger* in. **134** (Aug. 2015), 41–56.
3. Guidi, G. & Suul, J. *Modelling techniques for designing high-performance on-road dynamic charging systems for electric vehicles* in (Sept. 2018).
4. Cao, L., Sun, W., he, J. & Loo, K. *A Computationally Efficient Hybrid Optimization-Based Model Predictive Control for Inductive Power Transfer Systems* in. **Early Access** (June 2022).
5. Liu, S. *et al. Dynamic Improvement of Inductive Power Transfer Systems With Maximum Energy Efficiency Tracking Using Model Predictive Control: Analysis and Experimental Verification* in. **35** (2020), 12752–12764.
6. *Prediction horizon and control horizon* `https://it.mathworks.com/help/mpc/ug/choosing-sample-time-and-horizons.`.
7. Beghi, A. *Lectures notes on adative and model predictive control* in (2021).
8. Schwenzer, M., Ay, M., Bergs, T. & Abel, D. *Review on model predictive control: an engineering perspective* in. **117** (Nov. 2021), 1–23.
9. *Gain-Scheduled MPC* `https://it.mathworks.com/help/mpc/ug/gain-scheduling-mpc.html.`.
10. *Adaptive MPC* `https://it.mathworks.com/help/mpc/ug/adaptive-mpc.html`.
11. Findeisen, R. & Allgöwer, F. *An Introduction to Nonlinear Model Predictive Control* in (Jan. 2002).
12. *Modulation* `https://en.wikipedia.org/wiki/Modulation`.

13. *Modulation* `https://www.britannica.com/technology/loading-communications`.

14. Zhou, J., Guidi, G., Ljøkelsøy, K. & Suul, J. A. *Analysis and Mitigation of Oscillations in Inductive Power Transfer Systems with Constant Voltage Load and Pulse Density Modulation* in *2021 IEEE Energy Conversion Congress and Exposition (ECCE)* (2021), 1565–1572.