



UNIVERSITA' DEGLI STUDI DI PADOVA

DIPARTIMENTO DI SCIENZE ECONOMICHE E AZIENDALI "M. FANNO"

DIPARTIMENTO DI MATEMATICA

CORSO DI LAUREA MAGISTRALE IN BUSINESS ADMINISTRATION

TESI DI LAUREA

**"GHG EMISSION REDUCTION
THROUGH INTERNATIONAL ENVIRONMENTAL TREATIES:
ECONOMIC ANALYSIS AND MATHEMATICAL MODELS"**

RELATORE:

CH.MO PROF. ALESSANDRA BURATTO

LAUREANDO: FILIPPO PAMATO

MATRICOLA N. 1081485

ANNO ACCADEMICO 2015 – 2016

Il candidato dichiara che il presente lavoro è originale e non è già stato sottoposto, in tutto o in parte, per il conseguimento di un titolo accademico in altre Università italiane o straniere.

Il candidato dichiara altresì che tutti i materiali utilizzati durante la preparazione dell'elaborato sono stati indicati nel testo e nella sezione "Riferimenti bibliografici" e che le eventuali citazioni testuali sono individuabili attraverso l'esplicito richiamo alla pubblicazione originale.

Firma dello studente

GHG emission reduction
through international environmental treaties:
economic analysis and mathematical models

Filippo Pamato

Contents

1	Introduction	6
1.1	Classical tools for pollution reduction	7
1.2	GHG as a transboundary pollution problem	8
1.3	Global results against climate change	9
1.4	Fundamentals of game theory	10
1.5	A model for environmental treaties	11
I	Economic and institutional approaches	14
2	Classical tools for pollution reduction	16
2.1	An introduction to economic incentives for pollution reduction	17
2.1.1	A desirable level of pollution	18
2.2	Price rationing	19
2.2.1	Emission taxes	19
2.2.2	Taxes on products	25
2.2.3	Subsidies	26
2.3	Responsibilities rules	30
2.3.1	Sanctions: penalizing non-compliance	30
2.4	Quantity rationing	32
2.4.1	A combination of emission taxes and tradable allowances	35
2.5	Practical conditions for incentives	36
2.6	Evaluation criteria for incentives	36
2.6.1	Effectiveness	37
2.6.2	Efficiency	37
2.6.3	Equity	38

<i>CONTENTS</i>	3
2.6.4 Flexibility	38
3 GHG as a transboundary pollution problem	40
3.1 The role of greenhouse gases	41
3.1.1 The 2°C threshold	42
3.2 The Chinese example	44
4 Global results against climate change	48
4.1 Building the basis for COP21	49
4.1.1 1992: the Rio de Janeiro Earth Summit	50
4.1.2 1997: the Kyoto protocol	53
4.1.3 2010: the road towards the Cancun agreement	57
4.1.4 2012: the Doha amendment to the Kyoto protocol	59
4.2 Achievements in GHG emission reduction	60
4.2.1 Current situation on GHG emissions	60
4.3 COP21 and the <i>Paris agreement</i>	61
4.3.1 The agreement	62
4.3.2 The role of the parties	64
4.3.3 A truly legally-binding agreement?	66
II A game theory approach	68
5 Fundamentals of game theory	70
5.1 An introduction to game theory	70
5.1.1 Cooperative and non-cooperative games	72
5.1.2 Strategic and extensive forms	72
5.1.3 Best response and dominant strategy	74
5.1.4 Nash Equilibrium	75
5.1.5 Some criticisms of game theory	75
5.2 Dynamic games	76
5.3 Differential games	77
5.3.1 Open-loop strategies	78
5.3.2 Markovian strategies	79
5.3.3 Backward induction and subgame perfectness	79
5.4 Optimal control problems	80

<i>CONTENTS</i>	4
5.4.1 The Hamilton - Jacobi - Bellman equation	81
5.4.2 Pontryagin Maximum Principle	83
6 A model for environmental treaties	86
6.1 The non-cooperative scenario	87
6.2 The cooperative solution	91
6.3 Determining α : a regulator's choice	96
6.4 Countries with different bargaining powers	97

Chapter 1

Introduction

The following work focuses on international treaties as a way to reach a sustainable level of anthropogenic pollution. The need for a commonly shared, global answer in the fight against climate change comes from the fact that concentration levels of greenhouse gases in the Earth atmosphere have constantly grown in the last centuries. Starting from the original levels measured just before the Industrial Revolution, the volumes of CO_2 ¹ have steadily increased from year to year, reaching their absolute peak in 2014 and experiencing only in 2015 -for the very first time after more than one and a half century- a slight reduction.

It has been widely accepted and recognised by the vast majority of the scientific community how these gases are the main responsible for climate change, to which is associated a global warming process. If the Earth surface's temperatures will increase beyond a certain cap in the very next years, this would have dramatic consequences for the planet, and would force the human race to modify in a drastic way our lifestyle and probably even the geography of our settlements. Luckily the concerns of the scientific community are more and more being listened and shared by global governments. Even those which in the recent past were more refractory to take into account environmental themes are now changing their behaviors towards climate.

In this precise context numerous international treaties have been drawn up, with a constantly growing number of ratifiers. Countless conferences have been and are being held, with the aim of studying and finding a solution for an issue that is constantly getting worse, with the point of no return for the climate's safety is estimated as belonging to the current century.

¹The main gas among the greenhouse ones, representing the unity of measure on whose basis the others are converted and evaluated.

The first part of this work deals with international environmental treaties from an economic and institutional point of view. In the second part, mathematical models resulting from game theory are formulated in order to depict how treaties work.

1.1 Classical tools for pollution reduction

Economic theories on environmental contamination are the subject of the second chapter, in which we do not directly address the greenhouse gases issue, but the ones of environmental protection instead. We adopt the classical theories of environmental economics, according to which the focus must be set on the environment as a scarce good to be safeguarded. Pollution is therefore defined as a by-product of industrial production (a negative externality), and the aim of the economic literature is to reduce it within acceptable limits.

We see different methods in order to determine the socially optimal emission level, where societies are meant to be constituted by both producers and consumers, as well as citizens who are negatively affected by the contamination problem. From the very beginning we observe that markets alone are not able to guarantee that such a level is going to be reached, and because of this the presence of a *sub-partes* regulator is required. In practice, it may be identified with a national authority exerting a certain degree of powers over its own jurisdiction, or with -and this is our case- a sovranational treaty which can rule the environmental actions of more than one country.

We examine three types of economic incentives that can help reducing (greenhouse gases) emission levels: price rationing measures, including emission taxes, taxes on products and subsidies, whose aim is to modify the producer's behavior by changing his very own cost function; responsibilities rules, and we particularly focus on the sanctions associated with the deviation from the rules; quantity rationing methods, which involve the creation of a market -under the watchful eye of the regulator- for the trade of emission allowances that at the beginning are allocated among all market participants. We also mention an hybrid form, resulting from a combination between the first and third types, in which emission taxes and tradable allowances are gathered in order to create a stronger, more flexible method.

The last two sections of the chapter are dedicated to the practical application of economic incentives. We start by enumerating the practical conditions for their successful establishment, and then we proceed by listing the four criteria required to evaluate their performance: effectiveness, efficiency, equity and flexibility.

1.2 GHG as a transboundary pollution problem

In the third chapter the focus shifts from economic theories to reality. Greenhouse gases are defined as the main and worst example of transboundary pollution, a type of contamination which affects the world as a whole, independently from the place in which they were originated. This is the reason why the drafting of international environmental treaties is so fundamental, as long as the largest possible number of countries is involved. CO_2 emissions are not an exclusive problem regarding just a number of nations or specific locations, even if it is certain that some states suffer or are going to experience graver consequences than others. Having stated that, no country will be completely unharmed by the damages caused by global warming, and therefore it is both obvious and fair that they must all give their contribution in order to reach a common solution.

After having explained what greenhouse gases are and how determinant their role in contributing to climate change is, we analyse what is the ultimate objective set by the experts. As a matter of fact the scientific community has already stated that a certain degree of global warming, intended as some temperature increase with respect to the pre-industrial levels, cannot be avoided anymore. In order to prevent worse damages, a particular threshold was determined, to which important climatic consequences -but neither catastrophic nor irreversible- are associated. We examine how and why such a peculiar value was chosen, as long as some critics on its effectiveness.

Finally, we contextualise this matter by providing an (almost) present-day example. China, currently the world's greatest polluter, has reached the highest pollution levels ever recorded in its already suffering capital in the very same days in which the 21st session of the Conference of the Parties (COP) to the United Nations Framework Convention on Climate Change (UNFCCC) was being held.

Even if this fact probably did not contribute to increase its commitment during the negotiations -since the delegations' works had started a few months earlier- it certainly represented a curious coincidence. During those days the Chinese nation, as well as the world as a whole, was watching its representatives and asking the conference to draw up a document that could really be able to change things.

1.3 Global results against climate change

After having analysed the economic tools and having explained why a greenhouse gases emission reduction is needed, the following chapter deals in a closer way with international environmental treaties.

Anthropogenic contamination is as old as humanity itself. It is believed that the very first forms of pollution appeared together with our civilisation, in the prehistoric caves (SPENGLER *et al.* 1983). About 5000 years ago, when more developed populations grew around big urban settlements, pollution became more important as well, and started to negatively affect nations and geographical regions. However, it took more than 6700 years for it to become a global issue. When coal began to be used on a large scale and when the Industrial Revolution took place, greenhouse gases emissions became the concerning problem we are still facing nowadays.

Despite the longevity of this matter, in order to make our work as current as possible we have chosen to focus on a very small period: the 24 years elapsed between 1992 and 2015. We analyse the path that have brought to the 2015 *Paris agreement*, mentioning both the milestones that have been reached and the failures that have been made.

In the first part of the chapter, our focus is particularly placed on four years: 1992, 1997, 2010 and 2012. In other words, the years in which the UNFCCC, the *Kyoto protocol*, the *Cancun agreement* and the *Doha amendment* were constituted. For each one of these fundamental treaties we underline the elements of innovation, what they have tried to accomplish and in what they have succeeded, and -where possible- the elements of connection with the *Paris agreement*.

In the end, our focus shifts right towards it, the most recent international environmental treaty, after a brief evaluation of the effectiveness of the 1992-2014 international policies against climate change.

As a matter of fact, this work had the luck of being written during an historical moment for the fight against global warming: from November 30 to the very first days of December, the 2015 COP21 was held in Paris, France. Its aim was to find a legally-binding and, for the very first time in history, universal agreement on climate.

At the beginning of the month, the negotiators of each state delegation reunited in Paris and after many sleepless night on December 12 a final draft was presented. On the very morning of that day UN Secretary General KI-MOON announced to the world that:

“we have entered a new era of global cooperation on one of the most complex issues ever to confront humanity. For the first time, every country in the world has pledged to curb emissions, strengthen resilience and join in common cause to take common climate action”².

The text was defined as an ambitious and fair one, and will become legally-binding for its ratifiers in the next few years. Even K. NAIDOO, executive director of Greenpeace International, the non-governmental environmental organisation that in the past has hardly criticised treaties like the *Kyoto protocol*, stated that:

“This deal puts the fossil fuel industry on the wrong side of history [...]. This deal alone won’t dig us out the hole we’re in, but it makes the sides less steep. To pull us free of fossil fuels we are going to need to mobilise in ever greater numbers” (YARDLEY 2015).

In the last section of the fourth chapter we see how this treaties differentiates from the ones that have been written before it, how it works and what it hopes to accomplish.

The chapter does also constitute the end of Part I.

1.4 Fundamentals of game theory

Modern game theory was born in the last century, when J. VON NEUMANN published his book *Theory of Games and Economic Behavior* in 1944: through all these years, this field of study has repeatedly proven to be a valid method in addressing and resolving important matters. Since its birth, it has earned a valid reputation worldwide³, and is now one of the most powerful tools at our disposal to face some of the most important problems of our times.

An extensive scientific literature has used the methodologies provided by game theory for the quest of solutions to the most different environmental issues. During the last century, the more public concerns about global warming as a result of the increasing greenhouse gases emissions grew, the more mathematicians and applied economists dedicated their attention to such matters. Works on the preservation of environmental quality multiplied, reaching their peak in the second half of the nineties, after the drafting of the *Kyoto protocol*.

²Source: <<http://www.newsroom.unfccc.int>>.

³In the last 70 years, eleven economic Nobel prizes were won by game theorist, the latest being Jean Tirole in 2014.

This is precisely the set to which the second part of our work belongs; it is composed by two chapters.

The first one has the objective of providing in a brief way the necessary basis in order to face, comprehend and solve optimal control problems. To do so, we start from the very fundamentals of game theory, by defining what it is, which its main assumptions are and so on, until the formulation of a Nash equilibrium. The first part of the chapter ends with the mention of some of the main critics about game theory, as well as the answers given by its supporters.

From this point the work becomes a little more complex, since we introduce the concept of dynamic games. As a matter of fact these are more realistic but also less simple than the static ones, because they take into account not only the time element, but also the possibility for a player to react in a strategic way with respect to his own previous actions, by utilising some informations that were not previously at his disposal.

We then take into account a particular kind of dynamic games, the differential ones. After having described their functioning and features, we consider two related concepts, the open-loop and Markovian strategies.

The fourth and last part of the chapter is entirely dedicated to optimal control problems, and to their resolution. To this end, we introduce two different methodologies, the Hamilton - Jacobi - Bellman equation and the Pontryagin Maximum Principle, which purpose some sufficient and necessary conditions in order to find potentially optimal solutions for the problem.

1.5 A model for environmental treaties

The last part of our work is dedicated to the explanation -under the form of an optimal control problem- of how international environmental treaties operate. The methods depicted in the previous chapter are used in order to analyse a model built starting from the one created by KAITALA and POHJOLA (1995). We first approach their differential game with an infinite horizon, and then consider a situation characterised by finite time, in which two blocks of nations (representing wealthier and still-developing economies) are facing environmental issues caused by greenhouse gases pollution.

In the beginning we deal with the non-cooperative scenario and find the related solution. We then analyse what happens when the two players reach an agreement, and how their

roles in the process of global warming reduction may be evaluated and more or less rewarded by the regulator. This is exactly what takes place with environmental treaties, since they always tend to recognise a more active role to developed countries in the fight against climate change, while developing ones' efforts are often not required, or just encouraged.

We see that this consideration expressed by the regulator may be represented through the use of a coefficient, α , to which is associated a certain level of discount on the environmental damages produced by both blocks. The more the polluting emissions of the first one are fully considered, the less penalised is the second one, which is able to reach higher contamination levels, that of course are linked with an higher production and therefore with higher profits. Finally, we observe that since in the real world a truly independent and sovranational regulator does not exist, environmental treaties may be seen as the consequence of a process of negotiation performed by the two parties we have previously taken into account.

Starting from this consideration, we consider general solutions to the cases in which parties dispose of different bargaining powers. In order to do such thing, we use the concept of efficient frontier and some different solving methodologies that have been developed after NASH's work (1950).

Part I

Economic and institutional approaches

Chapter 2

Classical tools for pollution reduction

The First Fundamental Welfare Theorem, demonstrated by DEBREU and ARROW (1951-1954) states that under certain, ideal conditions, market equilibria are **Pareto efficient**, meaning that they achieve a state from which it is impossible to make one individual better off without making another one worse off.

Environmental quality may be considered as a scarce resource, since its use or protection involve a certain degree of opportunity costs. According to this theorem, a market for such a particular good should reach an equilibrium point which corresponds also to a socially efficient level of pollution. However, the conditions mentioned by the theorem are very strict and difficult -if not impossible- to be found in the real world. Markets for all possible goods should exist, they should all be in full equilibrium¹, transaction costs should be negligible and all participants should have access to perfect information. Otherwise, we would experience a so-called **market failure**.

Why do markets fail? In the following chapter we introduce some of the main causes (externalities and problems related to the lack of informations), and explain why the presence of a *sub-partes* regulator is required for environmental policies to succeed. We study the general theory about economic incentives introduced in order to reach a desirable level of pollution, and the ways through which they may be used to reduce inefficiencies related to market failures. Are emission taxes and subsidies preferred to a less strict, tradable allowances system? Under which practical conditions do they work? And how can their results be evaluated?

¹A condition under which demand equals supply.

2.1 An introduction to economic incentives for pollution reduction

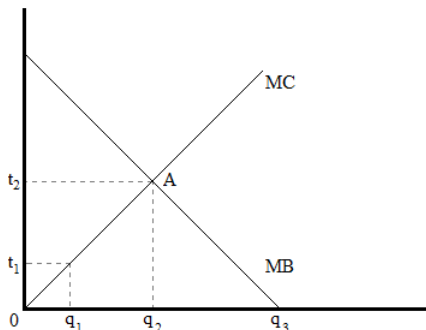
An **externality** is the direct consequence of an economic activity that affects unrelated third parties, who were not aware or able to influence it at the moment of its production. It may be positive or negative, depending on what the third party experiences (either a benefit or a cost). Damages caused by greenhouse gases emissions to the environment represent a perfect example of externalities: solutions found by producers² to these issues will always be different from the optimal social ones, since there is a clear conflict between private and social optimum; under the latter as a matter of fact, a private agent will not maximise his utility function. The problem gets worse when provisions to reduce pollution are taken by economic agents with non cooperational schemes.

A possible solution to the issue requires the introduction of a regulator in the system, someone able to generate incentives that would cause the costs associated with the avoidance of the normative to increase, in a way that agents would not be willing to deviate from his intentions. Incentives are so introduced with the main aim of modifying the producers' pollution strategies. According to HANLEY *et al.* (2007), they should increase the costs related to the deviation from pollution control, under a flexible scenario where agents are willing to minimize their cost function. The economic incentives may be classified into three wide categories:

1. **price rationing**: these incentives try to increase the costs associated with the deviation from the rules by imposing sanctions or subsidies that directly modify the behavior of the polluters;
2. **responsibilities rules**: a set of rules containing schemes of economic incentives like refundable deposits or fines for non-compliance, which try to induce a more responsible behavior from polluters;
3. **quantity rationing**: here the focus is to obtain an acceptable level of pollution by assigning tradable allowances for emissions. This mechanism encourages "less emitting" firms to reduce their pollution functions beyond law requirements, so that they can sell their exceeding allowances to producers who face greater costs in reducing theirs. The social gain deriving from the execution of this instrument is positive.

²From now on also indicated as firms, polluters or private agents.

2.1.1 A desirable level of pollution



Graph 2.1: Pollution control to a socially optimal level

Graph 2.1 shows marginal costs (MC) and marginal benefits (MB) associated with pollution control; t_1 and t_2 represent different values of emission taxes, whose aim is to obtain some level of pollution reduction q . The positive slope of the marginal costs curve implies that control costs³ increase following a growing rate, the more contamination is reduced. This happens because the less the environment is polluted, the more costly it will be to eliminate the last unit of q . We face the highest control costs associated with q_3 .

The negative slope of the marginal benefits curve tells us that benefits related to emission control work in the opposite way: the more control is used, the less marginal benefits for society, until quantity q_3 is reached, where $MB = 0$. The socially optimal level of pollution control is found in point A , where $MB = MC = t_2$; q_2 is associated with the optimal and most efficient level of environmental quality.

Even if this scenario represents a desirable solution for society, it does not hold the same for polluters, who do not maximise their utility functions under it. As a consequence, they will not have incentives to invest in emission control and therefore to reach the socially optimal point, since they would receive more benefits under $q = q_1$, associated with tax t_1 .

These considerations allow us to realise that polluters will pay the taxes imposed by regulators only when these are below the marginal cost of reducing emissions, otherwise they will invest in contamination control. It can be seen how the choice between the two alternatives depends directly on the costs associated with each one of them. If the marginal costs associated with pollution control are greater than the tax for unit of pollution produced, the agents prefer to pay the tax and will not act in a way that reduces their emissions. When

³Costs caused by the reduction of environmental pollution.

the exact contrary happens, they will prefer to invest in pollution reduction, in order not to pay the tax.

The regulator has a great responsibility in deciding which level of taxation to adopt: if the optimal social tax t_2 is greater than the one currently charged (t_1), producers will choose once again to pay and not to reduce their emissions. The optimal level of taxation is to be found where $MC = t$; this type of tax is called **Pigovian**, and tells us that a price is to be assigned to the externality we are facing in order to make pollution costs internal for the firms: by doing so, it is possible to reach a socially optimal solution.

Are these kinds of economic incentives feasible? The answer is not simple, since a regulator trying to implement them would eventually face the lack of information about the total level of emissions produced by an industry, and the impossibility of a punctual esteem of the environmental damages caused by contamination. When a regulator does not hold sufficient (if any) information about polluters, these will always try not to follow environmental policies, in order not to increase their cost functions.

Therefore **information asymmetry** plays an important role in the subject: when drawing a policy of economic incentives against pollution, one of the main factors that can influence its success is the lack of informations available to the regulator, with respect to those at the polluters' disposal. If a producer is able to hide or change some informations regarding his production process to the regulator, he will always have an incentive to evade the costs related to emission control, because such a behavior will always result in additional benefits for him. On the contrary, if the regulator is able to minimise the information asymmetry, his policies will reach a level of pollution reduction very close (if not correspondent) to the socially optimal one.

2.2 Price rationing

2.2.1 Emission taxes

Emission taxes (or taxes for unit of emissions) represent a fiscal imposition that a producer faces everytime he pollutes the environment; they are designed in a way that should lead to pollution reduction, because their aim is to make polluters pay for at least one part of the environmental damages they have caused. Taxes for unit of emissions released in the environment induce producers to reduce their contamination levels to a point in which the incremental costs associated with pollution control equal the emission tax. These contami-

nation control costs are of course different for every producer: the ones who face lower costs will reduce their emissions more than the one who are dealing with higher costs. Then, emissions taxes encourage producers to develop, acquire or modify the technology they use to control pollution in order to decrease their cost functions, which now take into account environmental damages too.

Empirical evidence shows that, mostly in developed realities like the United States or the European Union, emission tax systems have proved to work quite well. However, while they tend to increase the monetary entries of the regulator, they do not actually modify producers' behaviors associated with pollution. HANLEY *et al.* (2007) propose a model which explains how emissions taxes work.

We consider the case of a firm which, by producing some kind of goods x , pollutes the environment. Its profits π to be maximised are

$$\pi = px - c(x)$$

where p is the price at which product x is sold and $c(x)$ is the cost function associated with the production. Such costs increase according to the production at a growing rate

$$\frac{\partial c}{\partial x} > 0 \quad \text{and} \quad \frac{\partial^2 c}{\partial x^2} < 0$$

Thus, the profit maximisation problem for the producer is the following

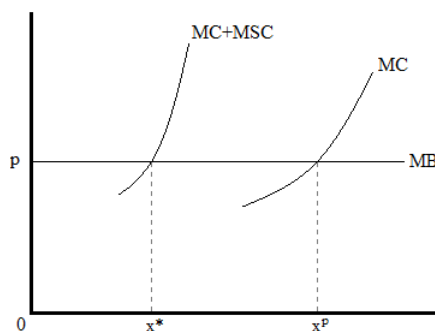
$$\max_x [px - c(x)] \tag{2.1}$$

The producer then selects an optimal level of production, x^p , which maximises his profits: this happens at the point where the marginal benefits of one extra unity of product equal the marginal costs, as well as price⁴

$$MB = MC = p \tag{2.2}$$

The private optimal level of production is represented in **Graph 2.2**: we can see how it is different from the social optimal one, which of course is associated with greater costs ($SMC + MC$).

⁴In a market working under perfect conditions.



Graph 2.2: Social and private optimal levels of production

Recalling that production has a direct consequence over environmental quality; the total level of emissions may be represented through a linear relation $\alpha = \beta x$ where α defines the emissions and β is a coefficient telling us that when we increase production, pollution increases as well: $\beta = \delta\alpha/\delta x > 0$. By $D(\alpha)$ we represent the monetary value of emissions damages to the environment, which increases following the pollution rate

$$\frac{\partial D}{\partial \alpha} > 0 \quad \text{and} \quad \frac{\partial^2 D}{\partial \alpha^2} < 0$$

If the producer can incorporate D , (2.1) must be rewritten as follows

$$\max_x [px - c(x) - D(\beta x)] \quad (2.3)$$

Now the producer will select an optimal level x^* at the point where marginal benefits equal the price and the sum between private and social marginal costs

$$MB = p = \frac{\partial c}{\partial x} + \beta \frac{\partial D}{\partial \alpha} = MC + SMC \quad (2.4)$$

From **Graph 2.2** we can see that level x^* is smaller than x^P (and so are the associated profits), resulting in a loss for the producer with respect to the previous situation.

Now, is it possible to make the social marginal costs fit the producer's own cost function? The answer is positive, but the regulator must be capable of estimating (and making the producer pay) the exact amount of environmental damages associated with the production of x ; in other words a tax

$$t = \beta \frac{\partial D}{\partial \alpha} \quad (2.5)$$

that causes the producer to rewrite at his maximisation problem as follows

$$\max_x [px - c(x) - tx] \quad (2.6)$$

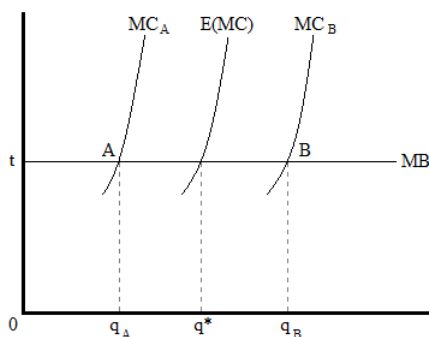
As a result, he will choose the new level of production which still maximises his profits, characterised by

$$p = \frac{\partial c}{\partial x} + t \quad (2.7)$$

If we take the value associated with t from (2.5) and put it into (2.7), we find that (2.7) is identical to (2.4). The producer has internalized the external damages caused by his production, with the result that his private optimum is now equal to the social one.

We have just found an optimal solution for the problem, but some issues may rise when it is time to transfer it to the real world. The main one regards the determination of environmental damages; in fact, informations regarding the physical and economic impacts of pollution are expensive and difficult to be gathered. However, if a regulator perfectly knows what the costs and benefits associated with pollution control are, he may easily reach the optimal social level through emission taxes. We are facing a feasibility problem when this information is not known, nor available.

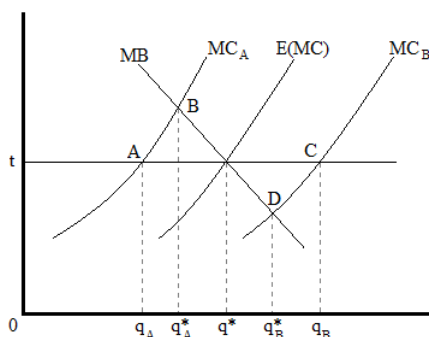
According to WEITZMAN (1974) the effectiveness of a price rationing system through the use of emission taxes depends on the marginal benefits and costs curves associated with the pollution control expected by the regulator. There may be issues of overinvestments or underinvestments when the expected slopes are different from the actual ones. Steeper slopes for marginal benefits and costs associated with emission reduction mean that there will be a greater variation in monetary values when a unit of pollution reduction q changes, while with less steep slopes the exact opposite will happen.



Graph 2.3: Constant emission tax and marginal benefits curve

Graph 2.3 represents a constant marginal benefits curve MB equal to a constant emission tax t , and the marginal cost of controlling the emissions expected by the regulator, $E(MC)$. If these costs were equal to the actual ones, we would obtain a socially optimal level of contamination control q^* . This is not what happens at point A: the actual reduction marginal costs are higher than the expected ones, and the new level is q_A , which is not efficient. We would experience the same inefficiency if the costs were smaller than the expected ones: the level at point B will be q_B .

However, in this case the emission tax is always equal to the marginal benefits, so that for any level of reduction the condition $MC = MB = t$ holds. There are no differences between the optimal private and the optimal social reduction level, but there is only a matter of inefficiency associated with points A and B.

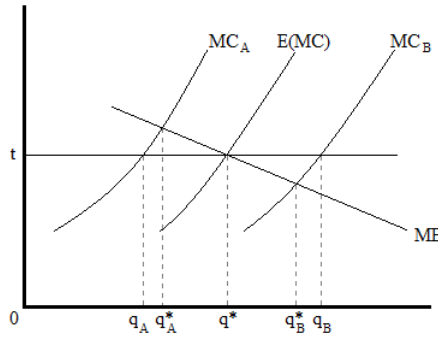


Graph 2.4: Constant emission tax and steep marginal benefits curve

This does not hold when we have a non-constant marginal benefits curve and a constant

tax: **Graph 2.4** shows different levels of optimal reduction. Once again, with actual control costs equal to the expected ones we will have the optimal level q^* . When costs are higher (point A), $t = MC(q_A)$, but the condition $MC = MB$ holds only at point B (q_A^*), meaning that pollution is reduced at an inferior level than the required one. The emission tax is also smaller than the one that should be collected. Producers will not reduce their pollution levels, since the marginal cost of reduction is higher than the emissions tax.

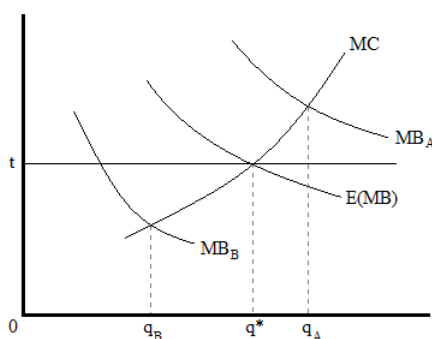
We observe the opposite phenomenon when costs are smaller: at point C marginal costs equal the tax (q_B), but they equal the marginal benefits only at point D (q_B^*). The pollution reduction level is now greater than the one that leads to the social optimum, since the tax is now over the point where $MB = MC$.



Graph 2.5: Constant emission tax and less-steep marginal benefits curve

In **Graph 2.5** we can see what happens when the marginal benefits curve's slope is less steep: the same considerations made for the case depicted in **Graph 2.4** still hold, with the only difference that here a smaller distance between q_A and q_A^* (and between q_B and q_B^* as well) may be noticed, meaning that the negative effects of the non-optimal policies are minor.

The steeper the marginal benefits curve, the bigger the deviation of the policies' impact from the optimal social levels, translating into much higher social costs. It is fundamental for the regulator to have a complete information about the marginal costs associated with pollution reduction, and to esteem efficiently emissions-caused damages, in order to introduce policies that may reach their aim in an efficient and effective way.



Graph 2.6: Constant emission tax and uncertain marginal benefits curve

The last case to be analysed (Graph 2.6) is the one concerning a marginal benefits curve which is not certain⁵: the implementation of a policy of emission taxes under these circumstances will cause either an overestimation (q_A) or an underestimation (q_B) of the pollution reduction levels when actual benefits differ from the expected ones. In all the cases where the reduction is not equal to q^* , the implemented policies are not the optimal ones.

2.2.2 Taxes on products

In theory, emission taxes have demonstrated to be an efficient solution; unfortunately, this does not always hold in practice, mainly because of the already mentioned issues related to information asymmetry, especially those involving moral hazard. Even if it is not directly related to GHG emissions, for the sake of completeness of our discussion on price rationing, it is worth mentioning an alternative solution used by regulators to avoid these problems when they want to establish incentive policies in order to control pollution: a **tax on products**. Here the polluter's behavior is influenced through a tax placed directly on the product whose manufacturing has caused some degree of contamination. In this way, prices associated with materials that are harmful for the environment rise, and consumers will try to find alternative, substitute products more eco-friendly, and not subject to taxation. Through this system the regulator is able to control (and limit) contamination at every single step of a good's productive cycle. Evidence shows that is widely used:

“there is a wide range of environmentally related taxes currently levied in the OECD-countries, among others: water pollution tax, batteries tax, logging

⁵This may happen, for example, when damages caused by pollution are not esteemed correctly.

tax, tyres tax, beverage container tax, toxic waste levy, tax on plastic bags, aircraft noise tax, tax on groundwater extraction, tax on pesticides, and artificial fertilisers, landfill tax, ozone depletion tax...” (SOLLUND 2007, p. 2).

One weakness of this system is that when more and more products are subject to this kind of taxation, administrative costs inflate and affect the achievement of economic efficiency. A possible solution could be the taxation of polluting raw materials only (HANLEY *et al.* 2007). However, the majority of reports about taxes on products have demonstrated that they scarcely affect consumers’ behavior. The reason is to be found in their amount, since -with only very few exceptions- they have been placed at an insufficient level:

“tax revenues raised by countries from environmentally related taxes represent on average about 2-2.5% of GDP in OECD countries, but this figure varies significantly among countries, from more than 3% in Scandinavian countries, the United Kingdom, Turkey and the Netherlands, to less than 1% in the United States. [Furthermore] this figure is not only resulting from the tax mixes but is also influenced by the level of traditional taxes like income tax and VAT in each country” (SOLLUND 2007, p. 3).

2.2.3 Subsidies

Subsidies are a kind of financial aid offered to producers by the regulator, which may be used as an incentive for pollution control. They may consist in a direct money transfer to firms (in order to help them develop infrastructures against contamination), or in the creation of some sort of tax reduction system. Producers may consider subsidies as an opportunity cost: every time they decide to pollute, they are refusing to receive a concrete payment from the regulator.

To quote some examples related to GHG emission reduction, VALSECCHI *et al.* (2009) mention a VAT reduction measure for domestic energy consumption in the UK, fuel tax exemptions for biofuels in Germany or company cars taxation in the Netherlands. To investigate more deeply how subsidies work in order to reduce pollution, we take into account the model proposed by HANLEY *et al.* (2007).

Suppose that a producer receives a subsidy in order to produce a certain amount of product below a level fixed by the regulator (who associates to this production quantity his

desired level of contamination). The subsidy will be equal to

$$s = \gamma(\bar{x} - x)$$

where $\gamma = D'\beta$ represents the social marginal cost associated with an x level of production. If $x = \bar{x}$, the subsidy equals zero and the producer does not receive it. If $x = 0$ (meaning that the producer does not produce anymore), the subsidy will be the maximum one, $s = \gamma\bar{x}$. In a system with subsidies, the producer has to face the following maximisation problem

$$\max_x \pi = [px - c(x) + \gamma(\bar{x} - x)]$$

The optimal condition is found at the point in which the marginal benefits equal the price and the sum between the marginal cost and the opportunity cost of losing the subsidy⁶

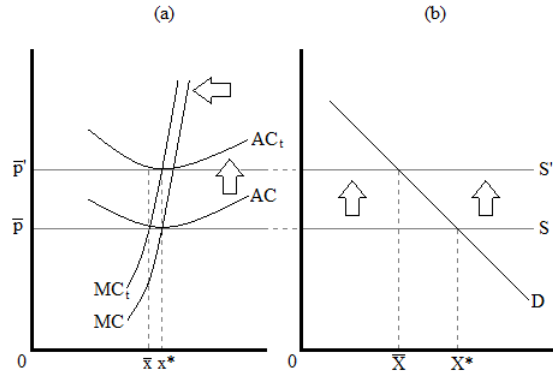
$$\frac{\partial \pi}{\partial x} = p = \frac{\partial c}{\partial x} + \gamma$$

Every unit of product means a loss of one unit of subsidy γ , causing an incentive for the producer to reduce his production until the optimal social level.

This is what happened in the previous case of emissions taxes as well: however, there are some differences in the ways through which taxes and subsidies work if we look at the medium/long term and consider the possibility for producers to entry and exit freely the industry. In the short term, where the industry environment can be considered as close, subsidies and taxes bring the same results. In the medium/long term, taxes reduce the aggregate pollution, while subsidies increase it.

Consider **Graph 2.7**, showing producers entering and exiting from an industry facing emissions taxes.

⁶Which is equal to the social marginal cost of producing one unit of x .



Graph 2.7: Short and medium/long term impacts of emission taxes

Part (a) of the graph represents the behavior of a randomly chosen producer, while part (b) depicts the industry as a whole. The average cost of a product (AC) and the marginal costs are the following

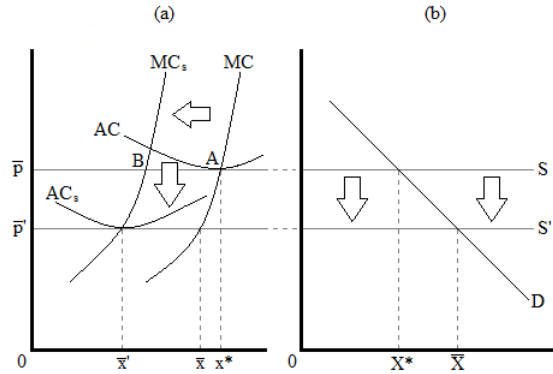
$$AC = \frac{c(x)}{x} \quad \text{and} \quad MC = \frac{\partial c}{\partial x}$$

We suppose a perfectly competitive market: there are no profits (nor losses) for the producer associated with the optimal quantity of product x^* . Perfect competition implies a perfectly elastic aggregate supply curve S , as shown in part (b), associated with a price level \bar{p} . We suppose the aggregate demand curve of the industry as a whole to have a negative slope, so that the aggregate level of production is X^* , the result of the sum of all products x^* of all producers. If a tax like the one described in (2.5) is introduced, the curves above have to be rewritten as

$$AC_t = \frac{c(x)}{x} + t \quad \text{and} \quad MC_t = \frac{\partial c}{\partial x} + t$$

Part (a) of the graph shows that the two new curves are parallel to the previous ones. With the market price still being \bar{p} , the producer will choose the point where his marginal benefits are equal to the price and to the new marginal costs \bar{x} ; at this point, his profits are negative ($\pi < 0$). Because of this fact, some producers will abandon the industry, and the curve [part (b)] of aggregate supply will move upwards from S to S' , in correspondence of the new price level \bar{p}' that makes $\pi = 0$ once again. This causes the level of aggregate products to decrease from X^* to \bar{X} , and the total level of emissions caused by the industry decreases as well.

The producers who remained in business are now producing x^* , but since they are not as many as they used to be, the total level of production (and pollution) has diminished. Taxes have achieved their objective.



Graph 2.8: Short and medium/long term impacts of subsidies

Graph 2.8 depicts a scenario characterised by subsidies; the curves associated with the average cost of a product and the marginal costs are

$$AC_s = \frac{c(x)}{x} + \frac{\varphi \bar{x}}{x} - \gamma \quad \text{and} \quad MC_s = \frac{\partial c}{\partial x} + \gamma$$

with φ being the pollution cap imposed by the regulator. The marginal cost curve is the same that we have analysed in the tax case, since $t = \gamma = D'\beta$, but here the subsidy causes the average cost curve to move to the left, reaching a lower point. Part (a) of Graph 2.8 shows how subsidies influence a producer's cost structure. Initially the producer finds himself in point A, associated with $x = x^*$ and $\pi = 0$; the introduction of a subsidy system will make him move to point B, where the production is reduced ($\bar{x} < x^*$) and profits are greater than zero ($\pi > 0$). The condition $\pi > 0$ will cause new producers to enter the industry, so that the aggregate supply curve will shift to a lower point (from S to S'), resulting in a lower price \bar{p}' and a higher level of aggregate production (from X^* to \bar{X}), as shown in part (b). In this way, each firm is producing a lower quantity of products \bar{x}' (being $\bar{x}' < \bar{x} < x^*$), associated with lower emissions, but there are more producers operating in the industry, so that the total level of pollution tends to increase.

If there are no barriers to entry, subsidies will attract more producers, decreasing emissions at a particular but not at a total level.

2.3 Responsibilities rules

Responsibilities rules are a set of prescriptions that oblige producers to follow some environmental requirements, technological restrictions or an acceptable polluting behavior. There are many specific instruments, including even some forms of bail. We have decided however to focus on the most important and effective one: a sanction scheme where fines are introduced in order to avoid producers' deviance from the regulator's prescriptions. Before starting, it is useful too remind how it is often very difficult to identify the exact contribution of a single producer to the whole emissions affecting the environment: a moral hazard problem must be taken into account.

2.3.1 Sanctions: penalizing non-compliance

Starting from HOLMSTRÖM's (1982) works on producers' behavior in a system with incentives, XEPAPADEAS (1991) has argued that a plausible solution for the free-riding problem⁷ would be to combine subsidies with a scheme based on **random sanctions**: if pollution in a certain area is above the desired level, the regulator punishes at least one randomly chosen producer. The resulting fine should then be splitted in two parts: one to be used by the regulator as a compensation for the damages to the environment, the other to be distributed among the non-sanctioned producers.

This mechanism has some benefits: for example, there are less administrative costs for the regulator to face, since there are less informations to be gathered with respect to those necessary for the taxes/subsidies based systems. Here the only knowledge required concerns the total level of contamination in a given area.

However, HERRIGES *et al.* (1992) demonstrated that this scheme only works when all the producers are risk averse, since one producer's loss constitute an actual gain for the others. Therefore, a producer's choice to comply or not with the law will depend on his expectations about the probability of getting fined against the possibility for his competitors to be sanctioned as well, resulting in a benefit for him. Under these conditions, when the regulator increases the sanction he is also increasing the costs and benefits associated with deviance from the regulation: producers would have to weight the possibility of getting chosen and pay a higher fine versus the one of receiving a larger quota of that sanction. However,

⁷Since pollution is not easily detectable, some producers may keep on contaminating the environment, leaving to others the costs associated with emission reduction.

if they are sufficiently risk averse, they will give more importance to the marginal costs with respect to the possible marginal benefits associated with deviation. The prize associated with compliance (not getting fined) should exceed the one associated with deviation, and this random-fines system would reach its objective.

To better depict this scenario, we could consider a group of producers $i = 1, 2, \dots, N$ where each has to choose a level q_i of pollution reduction; of course the regulator wants the quantity associated with the socially optimum q^* to be selected, but given his inability to control each producer's emissions he decides to introduce a system based on random sanctions. The cap selected as the critical pollution level is represented by $\bar{\varphi}$; if the actual level $\varphi \leq \bar{\varphi}$ every producer will receive a subsidy s_i , expressed as a percentage ρ_i of total social benefits $SB(a(q))$ where $q = (q_1, q_2, \dots, q_N)$. But when $\varphi > \bar{\varphi}$ producer i may be randomly selected and fined (F_i) with probability σ_i , or another producer j ($j \neq i$) may be chosen with probability σ_j , and the other ones would receive the subsidy and a percentage of the sanction (minus a compensation for the environmental damages).

Such a scheme may be summed up as follows

$$S_i(q) = \begin{cases} s_i - \rho_i SB(a(q)) & \varphi \leq \bar{\varphi} \\ -F_i & \varphi > \bar{\varphi} \text{ with probability } \sigma_i \\ s_i + \rho_{ij}[s_j + F_j + \Gamma(a(q))] & \varphi > \bar{\varphi} \text{ with probability } \sigma_j \quad j \neq i \end{cases}$$

where $\rho_{ij} = \rho_i / \sum_{k \neq j} \rho_k$ represents the percentage of producer j 's fine given to i , and $\Gamma(a(q)) = SB(a(q)) - \bar{SB}$ denotes the change in social benefits associated with the pollution level chosen by the regulator, with $\Gamma(a(q)) < 0$ for $\varphi > \bar{\varphi}$. Given this, the risk averse producer selects the emission level that maximises his profits

$$\pi_i = \pi_i^0 - c_i(q_i) + S_i(q)$$

with π_i^0 being the profits earned in the absence of a regulator intervention. The expected utility level of the producer when all the producers (including himself) adopt a compliance strategy is

$$E[U(\pi_i(q_i^*, q_{-i}^*))] = U(\pi_i^0 - c_i(q_i^*) + s_i)$$

where $q_{-i}^* = (q_1^*, q_2^*, \dots, q_{i-1}^*, q_{i+1}^*, \dots, q_N^*)$. If the producer decides to cheat and believes that his competitors will stick to the regulations, the expected utility associated with deviation becomes

$$E[U(\pi_i(q_i, q_{-i}^*))] = \sigma_i U(\pi_i^0 - c_i(q_i) - F_i) + \sum_{j \neq i} \sigma_j U(\pi_i^0 - c_i(q_i) + s_i + \rho_{ij}[s_j + F_j + \Gamma(a(q))])$$

A system which combines incentives, subsidies and fines for non-compliance should bring us to an optimal level of emission reduction if the expected utility associated with cheating is minor than the one associated with following the rules

$$\Omega = E[U(\pi_i(q_i, q_{-i}))] - E[U(\pi_i(q_i^*, q_{-i}^*))] < 0$$

It has been stated by HERRIGES *et al.* (1992) that an increase in the sanction level for all the producers increases the variability of the expected benefits of cheating as well. As a consequence, if all the producers are risk averse, a loss in the expected utility associated with following the rules and being fined exceed the gain associated with cheating, $\Omega_i < 0$.

2.4 Quantity rationing

Quantity rationing introduces the idea of **tradable allowances**, whose *raison d'être* is very simple: the ruler identifies a desired level of emissions for a specific area; the allowances equal the total level of pollution chosen, and are distributed among all the producers in that region, who have the option to trade them. Producers that maintain their level of contamination below the one assigned to them may sell -or even rent- their exceeding allowances. Since the total level of emissions allowed is limited, they may be seen as a scarce good, and this generates an incentive to trade. Of course the regulator must dispose of a sufficient knowledge about the market, including how to establish the validity of the allowances, how to initially assign them in a fair and efficient way, and how to properly monitor the producers. A certain degree of informations must be provided to the producers as well. Moreover, tradable allowances need a clear legal structure describing property rights and how transactions may happen.

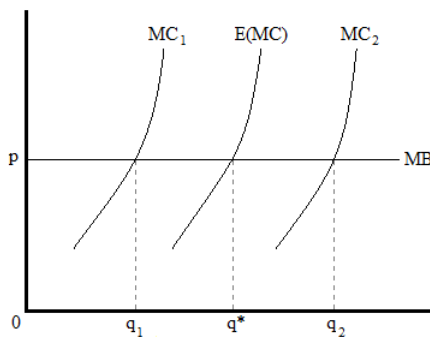
There are many criteria that a tradable allowances system has to satisfy in order to work efficiently (HAHN *et al.* 1990):

1. the number of allowances must be limited and well defined, in order to give them a precisely esteemed value;

2. they must be freely tradable;
3. they may be stored and still maintain their utility⁸;
4. transaction costs must be kept to a minimum;
5. fines associated with the violation of an allowance must be higher than its price, in order to incentive producers to follow the rules;
6. producers must be allowed to keep any profit coming from allowances' trade.

If the regulator perfectly knows the marginal costs and benefits of pollution control, the level of tradable allowances may be imposed in a way that its outcome is a socially optimal emission reduction. This level corresponds to the point where marginal benefits equal marginal costs, like happened in **Graph 2.1**. With allowances traded freely, supply and demand mechanisms should make their price equal to marginal costs and benefits of pollution control, so that $p = MC = MB$. Under complete certainty, this value is also equal to the emission tax: $p = MC = MB = t$ brings to the optimal level of pollution.

But what if the cost function is not known? We analyse three cases by looking at differences in marginal benefits to see if quantity rationing may be preferred to emission taxes.



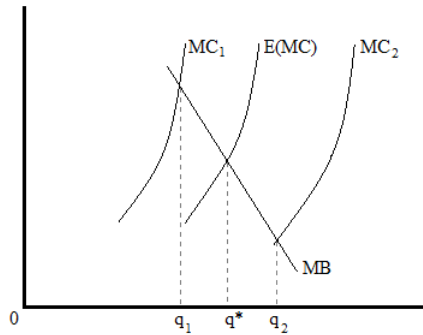
Graph 2.9: Quantity rationing, part 1 of 3

In the first one (**Graph 2.9**), the curve of marginal benefits is a straight line parallel to the x axis: in this situation emission taxes work poorly. The regulator will find that the number of tradable allowances for a desired level of pollution q^* is at the point where marginal benefits MB equals expected marginal costs $E(MC)$. Then, if the real marginal costs are

⁸Still, the possibility for the regulator to indicate an “expiration date” for the allowances is given.

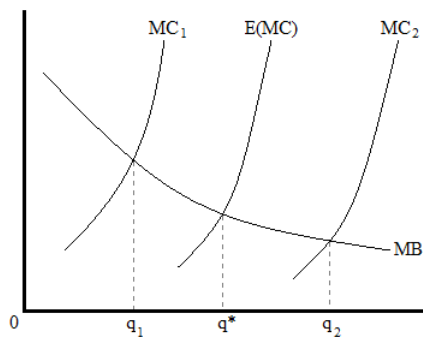
less than the expected ones [$MC_2 < E(MC)$], the outcome is a level of pollution reduction too small, since $q^* < q_2$: the system can not adjust itself to the minor costs if the number of allowances is fixed, resulting in an inefficient, minor level of reduction. On the contrary, if the real costs are higher than the expected ones [$MC_1 > E(MC)$] we are using too much control [$q^* > q_1$]: once again, the allowances quantity does not adjust to the control costs.

In these cases, an emissions tax system is preferable to a tradable allowances one.



Graph 2.10: Quantity rationing, part 2 of 3

Graph 2.10 shows the case of a marginal benefits curve with a very negative slope. Here tradable allowances performance well: if the real control costs are below or under the expected marginal ones, the socially optimal level of pollution is reached, being $q^* \cong q_2 \cong q_1$. In this scenario allowances are preferred with respect to emissions taxes.



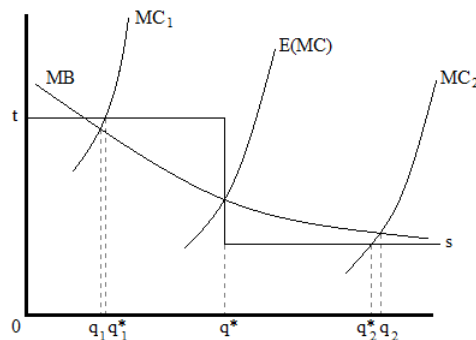
Graph 2.11: Quantity rationing, part 3 of 3

In Graph 2.11 we have the middle case where allowances result in an inefficiency, whose size however is reduced with respect to the marginal benefits curve with constant slope. If

the real costs are higher than the expected ones the result is an excessive reduction level, being $q^* > q_1$, and if they are smaller there is an insufficient level of reduction $q^* < q_2$. It is not clear if here allowances work better than emission taxes: the preferred system depends on the slope of the marginal costs and benefits curves, and on the difference between actual and expected costs.

2.4.1 A combination of emission taxes and tradable allowances

What about a system that combines emission taxes and tradable allowances? Could it be more effective than the two separated? The idea is to combine the strenghts of both methods: allowances protect against the possibility of generating dangerous levels of environmental damages thanks to the existence of an incentive that reduce contamination when actual control costs are greater than the expected ones; taxes provide an incentive to control more pollution than allowances when actual control costs are smaller. By combining these two, we should obtain a producer who is able to respond in a more flexible way to changes in market conditions.



Graph 2.12: A combination of incentives system

Graph 2.12 shows this case: the regulator uses such a system to approach the marginal benefits function by imposing a tax t , a subsidy s and an allowances level q^* . When the real marginal control costs are greater than the expected ones [$MC_1 > E(MC)$] we obtain a contamination control level greater than the optimal one ($q_1^* > q_1$), but not as high as it would have been in a system where allowances are separated from taxes: $q^* > q_1^* > q_1$. Taxes' aim is to reduce the negative impact associated with high deviations from actual to expected marginal costs. If cost allows the compliance of $t > p > s$, the private optimum point equals the social one.

In theory, this combined system should present more and more levels in order for taxes and subsidies to get as close to the marginal benefits curve as possible. This does not hold in the real world: a tax scheme with such a high variance is obviously difficult to achieve.

2.5 Practical conditions for incentives

It is worth noticing that some practical conditions are required for economic incentives to be used in an effective way:

1. a certain level of basic informations;
2. a strong legal structure;
3. a competitive market;
4. policies must be feasible.

With respect to the first point, the basic informations required regard the costs and benefits of different incentive alternatives, the technological and institutional opportunities and to which extent it is possible to control pollution. Incentives are less effective when political objectives are not clear, and when there is not a strong legal structure.

This brings us to the second point: the legislation must specify clearly the hierarchical order of the authorities, how much and to what extent they have power over such matters.

A competitive market is also required: in the hypothesis of its absence, it would be difficult to create a tradable allowances system. Economic incentives work better in markets characterised by a great number of buyers and sellers. These markets should also allow instruments regarding credit, insurance policies and risk management mechanisms, otherwise the bigger producers would have a consistent competitive advantage over the smaller ones.

Of course all these conditions must be translated into feasible policies by the regulator. It is his duty to face the reactions of any winning or losing party from the incentive system, which may include other regulatory entities, producers or individuals that may eventually gather together and (or already have such a power to) influence the regulator's policies.

2.6 Evaluation criteria for incentives

We may evaluate how useful and easy is adopting economic incentives by looking at four criteria: effectiveness, efficiency, equity and flexibility. Putting aside for a moment all the

theoretical components and looking at the situation only through a practical point of view, an incentive scheme will fail if it is not effective in reducing pollution's damages, if it is not efficient in meeting its target, if it violates social rules about equity and if it lacks of flexibility to change along with the environment surrounding it.

2.6.1 Effectiveness

An incentive effectiveness depends on how successfully it reaches the regulator's targets of controlling contamination. According to HANLEY *et al.* (2007), if the aim is to reach a certain level of pollution, a scheme of tradable allowances is the preferred choice. In fact, it establishes a fixed quantity of emissions in a specific region and offers better tools in order to predict and control how much pollution will be, and has been, reduced. Such a mechanism should be adopted also when the risk associated with (small) deviations is perceived as particularly high, since it allows to narrow the potential difference between actual and required emissions.

However, if the objective is to be certain about costs associated with pollution control, a tax system would work better. Through taxes a specific cost for emissions is achieved, but in this case the level of pollution control is uncertain.

2.6.2 Efficiency

Efficiency means that the regulator's objectives are reached at the least possible cost. In theory, the tradable allowances system and the one imposing taxes on emissions are equally efficient, but in practice they may differ significantly, depending on the characteristics and the sources of the contamination. Consider -for example- the costs associated with monitoring and compliance: a tax on emissions requires continuous data on quantities produced, which are likely to be expensive. The regulator must also dispose of the sufficient capacity to use such informations in order to determine an appropriate tax level, and to collect it. On the other hand, if a tradable allowances scheme is chosen, it must have rules regarding the transactions and the market organisation, and it has to monitor the acquisitions.

Tradable allowances should allow for more savings with respect to taxes, but this does not hold with developing economies, where technology and administrative capabilities are not sufficient.

2.6.3 Equity

Economic incentives may influence the distribution of costs and benefits among society members, with obvious effects on equality. It is the regulator's duty to identify the "winners" (who capture the benefits of a cleaner environment) and the "losers" (who face the costs related to it). Should it be recognized to a producer some kind of right to pollute? Or must he face alone the damages and costs caused by his emissions? Some strict measures may have a negative influence on the producer's operations, with bad effects over working places or economic growth. If a tax increases production costs and makes a producer not competitive in the market, he may cease his activity or move to another country.

There might also be equity issues regarding the location of an activity: polluting a specific area may have greater consequences compared to another one, so that an uniform tax on emissions may be perceived as unfair.

If we move our focus, it may be noticed that a regulator concerned with equity should also know in which measure an incentive system may be transferred from producers to consumers (by rising the price of the products), or to workers (by decreasing the salary levels).

2.6.4 Flexibility

The regulator should be able to adapt incentives to changes in market or technology, not to mention social, political and environmental conditions. If we take the example of taxes on emissions, flexibility is crucial in order for the regulator to respond to pollution variations. If modifying a tax requires more levels of authorities, change may be too slow to be efficient. The tax's flexibility should also be considered when dealing with nations characterised by high levels of inflation.

Tradable allowances are concerned with this feature too, since their system assigns a specific value to transactions among producers in the market. These prices should be adjusted in case of economic, technologic and inflation changes as well. Due to this characteristics, tradable allowances may provide more price flexibility than emission taxes, but less flexibility regarding the total level of emissions.

Chapter 3

GHG as a transboundary pollution problem

Transboundary pollution is defined as follows:

“a pollution that originates in one country but, by crossing the border through pathways of water or air, is able to cause damage to the environment in another country” (OECD 1997).

The definition involves the presence of at least two juridically independent entities: since emissions do not respect borders, the contamination caused by a polluter usually have negative effects not only within his home country, but also in the surrounding nations. This of course does not refer only to geographically close nations: as our analysis focuses on negative effects produced by greenhouse gases on the environment, data¹ show that the corresponding process of global warming is influenced more by great polluters like China or the United States, rather than by the state of Kiribati, Oceania. Nevertheless, this little island and its population of 103,000 people are more likely to suffer sooner the resulting dramatic consequences than the two polluters we have mentioned: in 2014 its government was forced to buy some territories from Fiji, planning to move there all of its residents if the oceans' level increase induced by climate change will continue its growing trend.

As a result of situations like this one, it is clear that transboundary pollution deals with themes of fairness and equity: generally air pollution may be transported over hundreds (even thousands) kilometres, meaning that, when it is finally deposited, its negative impacts are experienced in areas far away from their original sources. Moreover, when we refer to

¹From the World Bank organisation, available at <<http://www.worldbank.org>>.

GHG contamination, damages coming from this type of pollution affect directly the Earth's atmosphere as a whole. This means that a country which has made huge investments in renewable energy sources and tries its best to reduce the impact of its emissions over the environment will suffer because of the pollution originated in another state. Since in these kinds of problems more than one independent jurisdiction is involved, the issue gets more complicated with respect to what we have seen in **Chapter 2**. There is currently no such thing as a sovranational institution capable of imposing its will, penalizing the nations that do not respect the treaties signed or rewarding the ones that behave correctly.

In **Chapter 4** we analyse the major and most recent steps taken from a legal point of view in order for countries to reduce their GHG emissions, but we underline how the adherence of states to these kinds of treaties always happens on a voluntary basis: since there are no central entities capable of forcing some kind of environmental-friendly behavior to the world nations, the last word on such policies belong -in the end- to a country's own government.

3.1 The role of greenhouse gases

We have chosen to focus our analysis on the most discussed transboundary pollution problem, one that affects the world as a whole, with no regards of where the original source of pollution is located. We are talking about **greenhouse gases** (GHG), which are defined as:

“those gaseous constituents of the atmosphere, both natural and anthropogenic, that absorb and emit radiation at specific wave-lengths within the spectrum of thermal infrared radiation emitted by the Earth's surface, the atmosphere itself, and by clouds” (BERNSTEIN *et al.* 2008, p. 82).

The most important types that can be found in the atmosphere are water vapor (H_2O), carbon dioxide (CO_2), nitrous oxide (N_2O), methane (CH_4) and ozone (O_3); some, like the halocarbons and other substances containing chlorine and bromine, are entirely human-made. However, it is **carbon dioxide** which has been identified as the principal anthropogenic greenhouse gas causing climate change, as it is a by-product of burning fossil fuels from fossil carbon deposits like gas, oil and coal. It was chosen to be the “reference greenhouse gas”, the one on whose basis the others' negative effects are measured using the so called GWP, global warming potential.

Without entering too much into details about how GHG work, the Sun radiate towards the Earth the energy responsible for its climate at a very short wavelenght (the ultraviolet

section of the spectrum). Almost one third of this solar energy that reaches the top of our atmosphere is sent back through a process of reflection, while the remaining two thirds are absorbed in part by the atmosphere itself, and in a greater part by the Earth's surface. In order for the incoming energy to be balanced, the same amount must be radiated back to space, and this time infrared radiations (characterised by longer wavelengths) are used, because of the great difference in temperature between our planet and the Sun. Greenhouse gases, the ones which absorb the thermal infrared radiation, are then fundamental to ensure life on Earth, since they indirectly regulate the average temperature on its surface. It has been estimated (LE TREUT *et al.* 2007) that without their presence we would experience a 15°C decrease in the average terrestrial temperature.

An increase in their concentration would cause (and is causing) an increased infrared opacity of the atmosphere, along with a rise of the Earth's surface temperature. This is what has happened since the Industrial Revolution (1750): human industrial production based on fossil fuels -and its related negative side-effects, like deforestation- is responsible for a 40% increase of the carbon dioxide concentration in the atmosphere (BERNSTEIN *et al.* 2008).

As a result, it is a proven fact that in the last two centuries surface temperatures have followed an increasing rate in basically every nation, as shown by a variety of heterogeneous studies: from KÖPPEN (1881) to BROHAN *et al.* (2006)². Since Köppen's measurements, the Earth's surface has been warmed of an average of 0.85°C.

3.1.1 The 2°C threshold

To fight climate change, policy makers must first find an answer to a complicated issue: how much temperature rise our planet can tolerate, with respect to the pre-industrial average levels.

A **maximum 2°C increase** by the end of the current century (2100) is the limit that has dominated the prevailing views over such matter in the last twenty years. At a scientific level, it was first mentioned in 1990, when the Intergovernmental Panel on Climate Change (IPCC) produced its first report underlying how “grave damages to ecosystems” and “non linear responses” would have followed the overtaking of such a threshold. Since there, it has become a shared target among policy makers and scientists: it was explicitly mentioned in the *Copenhagen accord* (2009), which was initially signed by 114 countries, and formally incorporated in the *Cancun agreement* (2010): from then it formed the basis of the following

²Both mentioned in LE TREUT *et al.* 2006.

negotiations.

However, even getting close to the 2°C rise without surpassing it would not solve the negative effects related to global warming. It was meant to be the ultimate target not to be crossed in order to avoid catastrophic and unsolvable consequences, but it does not come without serious side-effects:

“Crop yields [could] decline in parts of Africa, Asia and Latin America; sea level rise [could] cause coastal flooding and saline intrusion into freshwater resources; low-lying islands may face a long term existential threat” (COMMITTEE ON CLIMATE CHANGE 2015, p. 21);

just to mention a few, are nowadays unfortunately seen as inevitable. Such a slightly under the cap result would bring enormous losses with respect to global economy and agriculture, but it still represents a somehow desirable objective, which is currently not impossible to achieve. As severe as its collateral damages may seem, they are almost negligible when compared to what would be associated with surpassing the threshold by, for example, two further degrees (reaching a total of 4°C increase):

“Roughly 3-30 million additional people could suffer coastal flooding each year due to 4°C warming, even assuming defences continue to improve with rising population and wealth [...]. Around 50-65% of plant and amphibian species, and around 25-40% of bird and mammal species are expected to lose at least half of their suitable climatic range”. (COMMITTEE ON CLIMATE CHANGE 2015, p. 23).

Critics on the 2°C threshold

While being widely shared, the 2°C cap has also gathered many critics. It has been seen as a somehow arbitrary and overly simplifying measure:

“[This] target has emerged nearly by chance, and it has evolved in a somewhat contradictory fashion: policy makers have treated it as a scientific result, scientists as a political issue” (JAEGER *et al.* 2010, p. 25).

According to TSCHAKERT (2015) it is simply too high: it should be substituted with a more effective 1.5°C, since temperature rises are not equally and uniformly distributed among the world nations; to quote an example, an increase of two degrees at a global level would eventually result in a 3.5°C rising in some parts of Africa.

The 2013 IPCC's *Fifth assessment report* was the first one mentioning the possible inadequacy of limiting emissions-caused global warming to 2°C:

“[recent] studies indicate a threshold of 2°C [...] with respect to pre-industrial levels for near-complete loss of the Greenland ice sheet” (CHURCH *et al.* 2013, p. 1170)

Most of Caribbean and Pacific Island countries would therefore suffer dramatic consequences due to the sea-level rising.

The idea of a 1.5°C limit as a substitute for the 2°C one was expressed for the first time -once again- in the *Copenhagen accord*; in its final part, the decision on the ideal objective was postponed to 2015, on the occasion of COP21:

“We call for an assessment of the implementation of this accord to be completed by 2015, including [...] consideration of strengthening the long term goal [...] including in relation to temperature rises of 1.5 degrees Celsius” (COPENHAGEN ACCORD 2009, art. 12).

3.2 The Chinese example

Data show that global warming caused by greenhouse gases is now more than ever a concrete, almost tangible threat that must heavily addressed as soon as possible. However, despite the fact that the whole world may suffer the catastrophic effects we have enumerated, the lack of a central, sovranational authority we have mentioned in the introduction of this chapter makes it more difficult for countries to find a commonly shared, legally-binding agreement. Is it possible for a whole nation to change its mentality and attitude towards environmental issues?

In this sense, the example of China may be peculiar with respect to our analysis. The country is the **world's worst polluter** since 2006, when it surpassed the United States in this unpleasant ranking. Obviously there is no sovranational entity capable of forcing China to reduce its emissions or to invest in renewable resources, and in the past the country has always been reluctant to take significant steps in this direction. Something must change in its government's attitude in order for China to voluntarily adhere to some sort of international legislation against GHG-caused transboundary pollution.

A curious coincidence happened on December 7, 2015: while all the world nations were attending the COP21 in Paris, negotiating for emission reduction, Beijing reached its highest level of $PM_{2.5}$ (an atmospheric particulate characterised by a fine diameter of 2.5 micrometres or less): 291 micrograms per cubic metre³ (at 07:00 am, local time). It is of course an extremely dangerous quantity, since the World Health Organisation recommends 25 micrograms (a volume almost 12 times lower) as the maximum safe level.

“Traffic thinned and construction sites went silent on Tuesday as the Chinese capital carried out its highest-level pollution alert for the first time, a move experts said marked official acknowledgment of public perception that previous bouts of bad air had been played down. City officials restricted industrial production and urged schools to shut their doors among other three-day emergency measures enacted on Tuesday after the city issued what it calls a red alert over pollution levels” (CHEN 2015).

A measure like this was not easily taken: the red alert was called for the very first time in the history of the nation, provoking huge social costs: the shutdown of outdoor constructions and schools and the heavy traffic limitations (half the vehicles were forced not to circulate) -just to mention a few- are very likely to have important consequences on the country’s economy if these pollution levels will be regularly reached from now on. According to the same data source, from 2008 to 2015 almost 7 out of 10 days in the city registered an “unhealthy, very unhealthy or hazardous” level of air quality, while only 2 out of 100 were considered “good”.

This is one of the reasons why, with the words of MCGRATH, the BBC environment correspondent for the COP21,

“something is changing, as the growing public pressure [in China] is starting to make a difference. A strong agreement in Paris won’t immediately solve China’s air woes, but if it ultimately pushes down the price of renewables even further, it could play a part in solving the issue long term” (MCGRATH 2015).

Historically, the Chinese government had been unwilling to commit to important targets when it came to greenhouse gases reduction, since the country has always relied heavily on coal, which still accounts for more than 60% of its power nowadays. As we mentioned, the country is indeed the worst polluter at a global level, but it has always refused to fully face this fact by claiming that other countries, like the US, have a way larger per capita amount

³Data from the US Embassy’s air pollution monitor in Beijing.

of emissions. However, major investments in renewable energy sources have been made in the last years, and facts like the one that recently happened in Beijing are forcing China to realise how it must cut its dependence on fossil fuels in order to survive or, according to less catastrophic scenarios, remain competitive at a global level.

The country is starting to move its first, important steps towards pollution reduction, and it is also thanks to its previous commitments that in 2015 global emissions of carbon dioxide suffered a decline of 0.6%, as was shown by the scientific journal *Nature climate change* at the COP21. It is the first time in the world history that such a downturn is experienced since the economy as we know it started growing through the Industrial Revolution.

Maybe something is really changing.

Chapter 4

Global results against climate change

One of the main reasons why it is so difficult to reduce emission levels is because pollution sources are often located in other jurisdictions, far from the local regulator. The example we have previously mentioned is the one of carbon dioxide emissions, which negatively affect the planet as a whole, with no regards of where they were originally produced. This kind of polluters' behavior does make sense under an economic point of view, as it is perfectly rational: when benefits associated with production mainly remain inside an area -a jurisdiction-, but part of the resulting costs (pollution) may be exported (by means of wind) to other jurisdictions, it will cause a production of pollution exceeding the socially optimal one. It is the well-known free-riding problem, that we have already mentioned in **Chapter 2**.

The only way to solve such an issue is for regulators to stop looking exclusively at their own jurisdiction: all the parties involved, the polluters and the pollutees, should negotiate the optimal level of pollution at an **interjurisdictional level**. The more countries are involved in signing such a legally-binding agreement, the more effective the outcome will be.

The Coase Theorem, by the namesake, Nobel winner R. COASE, states that:

“If one assumes rationality, no transaction costs, and no legal impediments to bargaining, all misallocations of resources would be fully cured in the market by bargains” (CALABRESI 1968, p. 68)

By applying it to environmental economics, we find that it does not matter which part has an actual right to contaminate. Independently from who is the right-holder, the parties involved will always have an incentive to negotiate until reaching the point where marginal costs and benefits of boths are equal. However, this will not happen in presence of lacks of information, damages diversity, high costs of transactions: the market itself is not currently

able to solutionate this problem. When interjurisdictional externalities like pollution exist, a higher level of jurisdiction is required: a system with powers over the different countries involved.

In **Chapter 5** we start a brief resume of the main features of game theory, and therefore introduce the concept of cooperative games, seeing how two or more players may come to an arrangement and act like a single one, with a unique payoff - to be later shared among them- higher than the one that would have been reached under a non-cooperative situation. Nevertheless, in **Chapter 2** we have already seen that this type of scheme is the one through which it is possible to control environmental damages by reaching an efficient level of pollution: every player, or firm, produces a certain rate of products, leading to a certain profit that he wants to maximise. If we stick to the *polluter pays principle*¹, environmental damages caused by emissions must become a part of each producer's cost function. We have seen that in order to force firms into cooperation, a sovereign entity (the regulator) must be introduced in the game. In the following chapter, we analyse what has been done at a global level to induce nations to cooperate in the fight against climate change caused by GHG emissions. It is a long path, full of (many) failures and (fewer) successes, which however has been eventually able to create the historical, first universal agreement on climate: the *Paris agreement*, presented to the world at the end of the COP21.

4.1 Building the basis for COP21

As the number indicates, COP21 was not the first meeting among the world's leaders to discuss environmental themes and climate change: on the contrary, there is a long history of conferences and treaties, achievements and -more frequently- failures behind it. We have chosen to focus on four years, all mentioned in the COP21 final draft, whose contribution to the agreement reached in Paris on December 2015 was significant: 1992, 1997, 2010 and 2012.

It all started with the Intergovernmental Panel on Climate Change (IPCC), which was instituted in 1988 under the spur of the United Nations Environment Programme (UNEP) and the World Meteorological Organisation (WMO), both belonging to the UN, and which defines itself as the "leading international body for the assessment of climate change"². Two

¹As it happens in the OECD and EU countries.

²Source: <<http://www.ipcc.ch>>

years later, the IPCC published its first assessment report³, depicting future dramatic consequences for the world as a whole, as a result of an indiscriminate GHG emission production by all nations. It also underlined how some serious commitments were to be taken soon, otherwise a *business as usual* situation would have brought to:

“an effective doubling of CO₂ in the atmosphere between now and 2025 to 2050 [...]; a consequent increase of global mean temperature in the range of 1.5°C to 4°-5°C [with] an unequal global distribution [...] and a sea-level rise of about 0.3-0.5 m by 2050 and about 1 m by 2100” (TEGART *et al.* 1990, p.1).

Heavy repercussions would have affected forests, desertification, aquatic ecosystems and human health as well. As a direct result of these scientific prediction, in 1992 the UN held a conference in Rio de Janeiro.

4.1.1 1992: the Rio de Janeiro Earth Summit

From June 3 to 14, 1992 most of world nations reunited in Rio de Janeiro for the United Nations Conference on Environment and Development (UNCED), also known as the **Rio de Janeiro Earth Summit**.

It was not the first time they assembled together to discuss environmental-related issues, but this summit went down in history for its size. The participants' number was truly impressive: 183 nations represented by over 10.000 delegates, a hundred heads of state, over 30.000 people (environmentalists, journalists, experts...) from all over the world. The purpose of the Earth Summit was stated in the previous 44/228 UN resolution (1989); in one of its articles, it specifically mentioned that during the 1992 conference

“[strategies and measures were to be elaborated in order to] halt and reverse the effects of environmental degradation in the context of increased national and international efforts to promote sustainable and environmentally sound development in all countries” (RESOLUTION 44/228 1989, art 3 part I).

Feasible solutions were needed to address in particular climate change and greenhouse gases emission (art. 12, part I); developed countries were also recognized as the main cause of pollution, and therefore identified as the principal responsables for its reduction.

Most of the expected outcomes of the summit were not achieved because of the emerging of opposing positions: the already developed part of the planet was willing to protect the

³Four more would have followed, the last one in 2014.

environment as a whole and to slightly modify its polluting behaviors (with some exceptions), while the less-developed, emerging one did not want to give up its future development based, for convenience reasons, on economical fossil fuels. These different demands were difficult to be mediated; however, it was thanks to the Rio conference that a concept emerged, an idea which as naive as it may seem now was stated there for the first time: the future economic development had to be environmentally **sustainable**.

This position was the basis for the writing of a document, the *Rio declaration on environment and development*, which was not a legally-binding agreement but a simple and quite broad declaration of political intents for a next future. Through its 27 principles, some main, basic statements over environment and development were expressed, reaffirming -as expressed in the preamble- the ones already stated in the previous Stockholm declaration (1972). Among these principles, some are particularly interesting on the basis of our analysis:

“States have [...] the sovereign right to exploit their own resources [but also] the responsibility to ensure that activities within their jurisdiction or control do not cause damage to the environment of other States [principle 2]; States shall cooperate in a spirit of global partnership to conserve, protect and restore the health and integrity of the Earth’s ecosystem, [having] common but differentiated responsibilities [principle 7]” (RIO DECLARATION ON ENVIRONMENT AND DEVELOPMENT 1992)

along with the ones wishing a cooperation among nations with the objective of progressively building up some sort of environmental international right (principles 13 and 27). Another voluntarily-implemented, non legally-binding document resulting from the summit was the *Agenda 21*, which, despite its less preminent juridical impact (with respect to the other papers), still represented a valid action plan on how to achieve a sustainable development. Its objective was to realise the complete integration between environmental care and economic development, having international cooperation among nations as a compass. Its second section (out of a total of four) specifically mentions GHG emission reduction as one of the main objectives to be implemented.

The declaration and the agenda were not the only results of the summit: three legally-binding agreements were opened for signature, collectively known as the *Rio convention*. The first ones -*Convention on biological diversity* and the *United Nations convention to combat desertification*- do not strictly concern our analysis, while the third one -the *United Nations*

framework convention on climate change (UNFCCC)- deserves some attention. It was signed by 154 countries⁴, and was intended as a framework convention, an international agreement whose details are delegated to the single nations ratifying it.

As a matter of fact, this international treaty generally expressed a commitment to initiate a process of greenhouse gas concentrations reduction, with no references to some sort of schedule or to mandatory actions. Its specific aims were to promote the knowledge, in each nation, about all types of emissions and their related damages, to encourage scientific researches on climate change due to GHG emissions, and to stimulate policies against pollution at a national and regional level. However, it included the possibility of negotiating more specific international treaties -protocols- with more strict limits concerning GHG emission reduction. It is worth noticing that there was also a specifically mentioned difference in the results expected from developed countries, since the convention took as inevitable a GHG emission rise in developing countries for the following years, both for their necessity to address social and development issues, both for their dependence on fossil fuels.

The origins of the Paris agreement are to be found in this convention, since after its ratification all the parties involved agreed to meet every year in the so called **Conferences of the Parties** (COP). The first COP (Berlin 1995) was not very effective, while the second one (Geneva 1996) made significant steps in recognizing that climate change was an actual phenomenon, demonstrated by evidences and validated by the scientific community, and that medium-term targets were to be taken and legally enforced. The set was ready for COP 3 (discussed in **Subsection 4.1.2**), which took place in Kyoto in December 1997.

Summing up, the Earth Summit in Rio was welcomed as a possible, concrete solution to problems like climate change, whose real gravity was started to be perceived. It may have disappointed those expectations, but it surely represented a valid starting point for a dialogue over GHG emission reduction: the UNCED eventually promoted the entry of environmental issues and sustainable development (POTTER *et al.* 2003, p.183) in the global agenda.

Parties to the UNFCCC

Another important feature of the convention was the division of world countries into the following groups:

⁴Which are now 196, all member states of the UN, including other entities like the European Union as a whole.

1. **Annex I Parties** are the industrialised, developed countries that were already members of the OECD in 1992, and those nations with economies in transition. They include the Russian federation and several Central and Eastern European states.
2. **Annex II Parties** include the OECD members of the previous group, but not the economies in transition. They are responsible for the financial needs of developing countries in the fight against climate change.
3. **Non-Annex I Parties** represent mostly developing countries; some groups of nations are recognised as “especially vulnerable to the adverse impacts of climate change”, some form part of the LDC sub-classification. These are Least Developed Countries, whose limited capacity to reduce emissions and control pollution levels is recognised and financially helped.

4.1.2 1997: the Kyoto protocol

The main result of COP3 (Kyoto 1997) was the *Kyoto protocol*, which represented for years a milestone with regards to the fight against climate change. The document is an international, legally-binding treaty (for parties that -on a voluntary basis- chose to ratify it) which recognises global warming as an undoubtful, anthropogenic effect of GHG emissions; its parties are therefore obliged to reduce them to bearable limits, through a series of local policies and measures and international **flexibility mechanisms**. In particular, the protocol sets targets concerning the production of six particular greenhouse gases (Annex A): carbon dioxide (CO_2), methane (CH_4), nitrous oxide (N_2O), hydrofluorocarbons ($HFCs$), perfluorocarbons ($PFCs$) and sulphur hexafluoride (SF_6), all translated into CO_2 equivalents in order to determine common reduction levels. Furthermore, a series of secondary activities concerning with accounting, reporting and review was implemented, as well as a Compliance Committee in order to vigilate on the observance by Parties of the determined objectives.

The protocol entered into force on February 16, 2005 -eight year after its drafting- when the condition of being ratified by at least 55 states⁵ (art. 25) was fulfilled. To the present date, 192 parties have signed it -including the European Union as a whole⁶-, one has withdrawn from it (Canada) and one of its signatories has not ratified it yet (USA). Its first commitment period (see Subsection 4.1.4 for the second one) covered the years from 2008 to 2012; by

⁵Including Annex I Parties representing 55% of 1990 levels of CO_2 emissions

⁶Whose states are authorized to work jointly by art. 4.

the end of that year, the objective for total emission reduction was to achieve a 5.2% decrease with respect to the 1990 levels (art. 3). Let us examine this goal more precisely.

Article 10 of the protocol states that it is based on the principle of “common but differentiated responsibilities”, resulting in the fact that developed countries are identified as the responsible for the current GHG levels in the atmosphere: the main abatement measures for their reduction, as well as the financial aids for developing countries in order to assure them an environmentally sustainable development, should come from them. As a matter of fact, only Annex I Parties have chosen to commit themselves to some reduction targets, to be achieved on a national or joint basis. Non-Annex I Parties do not have legally-binding objectives, but can still participate in the fight against global warming: for example, the clean development mechanism, later explained, is one of the measures directly aimed to them. The limitations of GHG emissions are not the same for each country: they were calculated below, at or in some cases even above the base year’s ones. In turn, even the base year is not universally shared: while being 1990 for most countries, there are five exceptions⁷ which have chosen years or averages in the period from 1985 to 1989 as their references. We have selected six among the most important Annex I Parties and shown their respective commitments as a percentage of the base year in the following table:

Annex I Party	Commitment	Base year
Australia	108%	1990
Canada	94%	1990
EU	≈ 95.6%	1990
Japan	94%	1990
Russian federation	100%	1990
USA	93%	1990

Table 4.1: 2008-2012 Kyoto GHG emission targets for some Annex I parties

Two things must be underlined: the EU commitment has been calculated as an average of the EU15 individual targets, with the lowest level (72%) belonging to Luxembourg and the highest one (125%) to Greece; USA are considered among the parties since they did sign the protocol, but it is useful to remind that they have not ratify it yet.

⁷Bulgaria, Hungary, Poland, Romania and Slovenia.

How to achieve Kyoto emission targets

There are two methods for Annex I Parties to achieve such a decrease in GHG emission levels. The first one are general policies and measures to be implemented on a national and regional basis: into this category fall all those specific implementing programs like the improvement of industries' energy efficiency, the promotion of sustainable means of agriculture, the development of renewable energy sources, etc.

The second method is composed by three different flexibility mechanisms: joint implementation (art. 6), clean development mechanism (art. 12) and international emissions trading (art. 17). On the opposite of local policies, all these means act at an international level and allow GHG emission reduction activities implemented by a country out of its jurisdiction to count in determining the reach of its determined quota. We recall from **Chapter 3** that global warming is a transboundary pollution problem: therefore, every measure to reduce GHG-caused pollution is effective without regards of where it is conceived.

The three mechanisms are available for utilisation by all Annex I Parties, at the following conditions:

- they have to ratify the *Kyoto protocol*;
- they have to calculate their assigned quota of CO_2 equivalents (in tonnes);
- they have to implement a system on a national basis to estimate GHG emissions, and transmit such informations to the UNFCCC secretariat every year;
- they have to create a national register to account for GHG emissions quota released, owned, transferred, repaid and erased, and transmit such informations to the UNFCCC secretariat every year;
- they have to demonstrate that the use would only have been an addiction to the courses of actions already undertaken at a national level.

The aim of **joint implementation** is to promote a collaboration between developed countries and economies in transaction, in order for both to reach their own reduction objectives in an efficient way. Annex I Parties may invest into emission reduction projects in the territory of any other such Party; in this way, the global cost of fulfilling Kyoto obligations should be reduced, enabling Parties to abate emissions where is more convenient. The pollution which has been avoided thanks to these projects allows funding countries to achieve emission

credits (called ERU, Emission Reduction Units), that may be used for the compliance with their own Kyoto reduction objectives.

The second international mean to abate contamination levels is a **clean development mechanism**, which works in a similar way. Once again, a process of collaboration is induced: countries (or companies⁸) realising projects based on a clean technology (one that abates GHG emissions) in developing countries (therefore Non-Annex I Parties) receive emission credits equal to the pollution reduction achieved through their intervention, on the basis of a *what would have otherwise been emitted* scenario without the project. These are known as CER, Certified Emission Reductions, and may be sold to the market or stored. This mechanism involves energy efficiency, renewable energy commercialisation and fuel switching, and has been widely used: estimates by World Bank calculate its related pollution abatement to be ten times the one reached through joint implementation.

The first two methods are also known as **project-based** mechanisms, since, despite their differences regarding methods or the recipients of their actions, both are funded on a technological program to be implemented in order to meet Kyoto levels. The third one is deeply different, since it is based on the cap-and-trade system we have mentioned in **Section 2.4**: the **international emissions trading** system allows states (companies) to buy from and sell to other states (companies) a certain amount of emission permits, in order for their GHG emissions to meet their originally assigned quota. The party will sell its tradable allowances on the market with its level of emissions being under the quota, and buy them otherwise. The allowances are called AAU, Assigned Amount Units, while the original quotas were agreed by the Annex I Parties involved in the protocol and are stated in its final part (Annex B). The trading was restricted to the 5 years compliance period from 2008 to 2012, and allowed among Annex I Parties that have agreed to the GHG emission limitation (art. 17).

Emission trading schemes (ETS) were successfully implemented in many Annex I Parties. However, it must be noticed that the one implemented by the EU (the so-called EU ETS) is the only involving more than one jurisdictional entity: all the 28 EU states plus Iceland, Liechtenstein and Norway. A tradable system for GHG emission allowances was constituted by countries like Australia (2003), Canada (2007), Switzerland (2008), the US (2009) or Japan (2010), but it was limited to their territory and saw no other national entities included.

⁸Under the responsibility of their home nations, as determined in the *Marrakech Accords* (2001).

Sanctions

Sanctions for non-compliance with the protocol were decided at the COP11 (Montreal 2005) through the UNFCCC *Decision 27/CMP.1*. The purpose of the decision was to facilitate, promote and strengthen the meeting of the Kyoto targets through a transparent system. There are **no direct economic fines** for countries failing to reach their objective; however, the following sanctions are to be applied:

- 30% surcharge on the emission quantity missed to meet compliance to be added to the post 2012 quota;
- adoption of an action plan for the meeting of the targets;
- Annex I Parties may be prevented from taking part to further emissions trading.

As a result, non-compliance may result costly, in terms of international credibility, increase of the post 2012 obligations and risk of being ousted from emissions trading.

To this point the Canadian case is emblematic: since there are no fines forecasted for parties withdrawing from the protocol (art. 27), the nation decided to abandon the program in 2011, finding itself in the impossibility of respecting the agreement (6% of pollution reduction by the end of 2012, with respect to its 1990 levels). As a matter of fact, what happened between 1990 and 2008 was the exact opposite: Canada experienced an increase of 24.1%⁹ of GHG emissions. By exiting the *Kyoto protocol*, the country experienced only a slight damage (if any at all) in its international reputation, without direct consequences on its economy. On the contrary, heavy penalties would have arisen for the country if it had not withdrawn: it avoided an estimated US\$14 billion in penalties resulting from its failure to miss the Kyoto target (JULL 2012). The sum was calculated by the former Canadian minister of the environment, P. KENT, on the basis of the 30% surcharge for the second period and on one of the protocol's options, suggesting that a non-compliant party has to purchase emission credits from another state to eventually meet its target.

4.1.3 2010: the road towards the Cancun agreement

From 1997 to 2010 thirteen Conferences of the Parties took place¹⁰. Given that none was as remarkable as the Japanese one, no important conclusions were reached either, with few

⁹Data comes from a 2010 UNFCCC report.

¹⁰In 2001 two COP were held, in Germany and Morocco.

exceptions.

COP6 (Bonn 2001) and COP7 (Marrakech 2001) saw the rejection of the Kyoto protocol by the US -which became an observer country- and the acceptance of the three flexible mechanisms we have mentioned in Subsection 4.1.2, with the appropriate compliance and sanctionatory mechanisms. During COP10 (Buenos Aires 2004) the post-Kyoto period was mentioned for the first time, and discussions on how to allocate emission reduction after 2012 began. On February 16, 2005 the Kyoto protocol entered into force and the same-year COP11 (Montreal 2005) was also the first Meeting of the Parties (MOP).

Three years later, COP15 (Copenhagen 2009) was welcomed with high expectations. It was attended by over 100 world leaders, and accomplished the drafting of a document, the *Copenhagen accord*, a non-legally-binding resolution signed by 141 countries¹¹ (accounting for 87.5% of the world GHG emissions) and “taken note of”, but not formally adopted, by the COP itself.

The accord attests the recognition of climate change as a fundamental threat for the world nations, agreeing -for the first time- on the need to limit further temperature rise to **2°C** (recall Subsection 3.1.1). The path to reach a certain quota of GHG emission reduction was set in different ways for Annex I and non-Annex I Parties. The first ones had to commit to the emission targets for 2020 submitted by themselves by the end of the following year to the secretariat (art.4); the objectives for the Annex I Parties that we have previously considered with regards to the *Kyoto protocol* are shown in the table¹²:

Annex I Party	Emission pledge by 2020	Compared to
Australia	5 - 25%	2000
Canada	17%	2005
EU	20 - 30%	1990
Japan	25%	1990
Russian federation	15 - 25%	1990
USA	17%	2005

Table 4.2: Levels of GHG emission pledges by 2020 for some Annex I parties

¹¹As of 2015.

¹²Data from: <<http://www.usclimatenetwork.org>>.

Being developing countries, non-Annex I Parties only had to generally implement mitigation actions, while least developed countries and small island developing states “might” undertake voluntary actions, on the basis of international support (art. 5). Moreover, developing countries were also indicated as the most needy of incentives, in order to reach a sustainable development by following a low emission pathway (art. 7). The task of raising those funds was granted to their developed homologous: US\$30 billion for the period 2010-2012, and US\$100 billion a year by 2020 (art. 8).

The following year COP16 (Cancun 2010) was the natural consequence of Copenhagen. The 2°C threshold was **formally incorporated**, as well as the possibility of its re-evaluation in 2015, officially introducing the idea of a possible, further decrease of 0.5°C. All Annex I Parties submitted their emission targets (Table 4.1), while other countries and emerging economies also contributed by submitting their action plans. All of this happened on a pledge-and-review basis, meaning that nations acted -once again- on a voluntary, non-legally-binding plan. Finally, the *Cancun agreement* called for the creation of a Green Climate Fund to help developing countries to mitigate their GHG emissions, and established an Adaptation Framework to coordinate national and regional adaptation plans.

4.1.4 2012: the Doha amendment to the Kyoto protocol

With respect to what was done in the previous years, during COP17 (Durban 2011) the parties returned to a more rules-based approach. The main result of the conference was to explicitly start the negotiations of a legally-binding document, regarding all countries, that would have flown into the 2015 *Paris agreement*: a rules-based commitment for the post 2020 period.

COP18 (Doha 2012) started with the idea of setting some rules for the post-Kyoto period, and resulted in a series of **amendments** to the original protocol. Its expiration date was postponed from 2012 to 2020; after that year, a “successor” to Kyoto had to produce its effects, and the schedule for constructing such a document was set to 2015. The *Doha amendment to the Kyoto protocol* restricts the trading mechanisms only to the parties that have actually taken part in the second agreement; furthermore, it limits the possibility of access to the emission surplus allowances eventually stored during the Kyoto period only to countries exceeding their allowances in the 2012-2020 timeline. However, many countries

stated their intention not to acquire further allowances, and the vast majority did not ratify the amendment. As a result, the amendment is not currently effective: to enter into force, 144 out of the 192 parties (75%) were requested to sign it, but as of 2015 it has only been ratified by 57 nations (less than 30%).

The following COP19 (Warsaw 2013) and COP20 (Lima 2014) did not accomplish important results, but allowed negotiations among different position to continue, clearing the way for the *Paris agreement*.

4.2 Achievements in GHG emission reduction

There is not an univocal, shared answer to the question over the effectiveness of the *Kyoto protocol*. A study by KUMAZAWA *et al.* (2012) showed that emission reductions differed greatly among developed countries that were bound to reduce their GHG emissions, with the result of 38 industrialised country that were not able to fulfill their obligations by the end of the first Kyoto period. At the same time, others (GRUNEWALD *et al.* 2011) demonstrated a clear CO_2 emissions' decrease trend for the period 1960-2009 that can be brought back to the protocol. The UNFCCC itself (2012), in analysing the period from 1990 to 2010, has affirmed that the total GHG emissions of Annex I Parties experienced a reduction of 8.9%, well beyond the 2012 original target. And it is a fact that 21 countries (mostly European ones) achieved to meet their reduction targets.

However, even these results are disputed, based on the ways on which the original esteems were made, or upon the fact that the most of the “virtuous” nations were not among the main polluters. Latvia, Lithuania, Ukraine and Romania -for example- are the countries that reached the highest levels of CO_2 reduction, with an extraordinary mean positive gap between their percentage target and their actual percentage change of 81.3¹³; nevertheless, if combined together, they still account for less than 1% of the world's share of GHG emissions.

4.2.1 Current situation on GHG emissions

The most recent available data are the ones elaborated by the Climate Action Tracker (CAT), an independent scientific analysis that reunites the results of four different climate research organisations¹⁴ and which has monitored since 2009 GHG emissions of 31 countries. The

¹³UNFCCC data, LULUCF included.

¹⁴Ecofys, Climate analytics, Potsdam Institute for climate impact research (PIK), NewClimate Institute.

sample is composed of the principal, biggest polluters¹⁵¹⁶ and a representation of smaller emitters too, covering about 81.3% of greenhouse gases global emissions and approximately 70% of the world inhabitants. Nations belonging to the sample are **evaluated** on the basis of their GHG emission reduction efforts, considering if their INDCs¹⁷, pledges and current policies are likely to maintain global warming below the 2°C cap, based on their esteemed individual contribution to climate change:

Rating	List of country by category (alphabetical order)
Sufficient	Bhutan, Costa Rica, Ethiopia, Morocco, Gambia
Medium	Brazil, China, EU, India, Kazakhstan, Mexico, Norway, Peru, Philippines, Switzerland, USA
Inadequate	Argentina, Australia, Canada, Chile, Indonesia, Japan, New Zealand, Russian federation, Saudi Arabia, Singapore, South Africa, South Korea,, Turkey, UAE, Ukraine

Table 4.3: List of countries' rating based on their current polluting behavior

In 2015, only 5 out of 31 countries had a *sufficient* behavior, indicating a full consistency with below the 2°C limit. Eleven countries' efforts were judged *medium*, meaning that their current level of GHG emissions is likely to slightly exceed the threshold, while the majority (15) behaved *inadequate*(ly): if all nations followed their conduct, we would experience a 3-4°C temperature rise, widely above the “safety” cap. According to the CAT, no country has adopted a *role model* conduct yet, one that would bring the expected increase below 2°C.

4.3 COP21 and the *Paris agreement*

The 2015 United Nations Climate Change Conference (COP 21) took place from November 30 to December 12 in Paris, France. The conference started in a climate of both protests and great expectations, announcing its ambitious target of producing a legally-binding, commonly shared agreement on climate involving the highest possible number of countries. After many

¹⁵The European Union is considered as one due to its integrated carbon trading scheme (EU ETS).

¹⁶Data and ratings exclude LULUCF, *i.e.* emissions from land use, land use change and forestry.

¹⁷Intended Nationally Determined Contributions, commitments to reduce GHG emissions submitted to the UNFCCC.

sleepless nights and being one day late with respect to the original schedule, the delegations working on the drafts finally reached their aim and presented a document which is expected to start its effects in 2020. For the very first time in history, an universal agreement on climate change was reached and welcomed by consensus by all the 195 countries attending COP21.

4.3.1 The agreement

The final draft of the agreement is a 32-pages document that can be divided into two parts. The first one is a list of 140 actions that are or will be executed by the Conference of the Parties, while the second one (the *Annex*) includes the 29 articles that truly constitute the treaty itself. Through these parts, two ambitious and equally important long term objectives are set.

The first one has 2100 as a target, and its purpose is to keep below the discussed **2°C threshold** the inevitable rise in temperature the world will experience by 2100 as a consequence of GHG emissions, with a parallel attempt to further narrow such increase below the 1.5°C limit. This was introduced for the first time in an official UNFCCC document as a possible objective, being identified as the most desirable -and therefore still feasible- cap in order to prevent the worst global warming-related effects and especially protect the Caribbean and Pacific Island states. By 2018 the IPCC is requested to provide a special report on the impacts of climate change associated with the compliance of such a target. From now on, and differently from what was written in the *Copenhagen accord* (2009), 2°C must be intended as the ultimate, maximum measure, rather than some kind of objective.

The second aim, even more demanding, states the need to reach in the second half of this century a global net zero CO_2 emissions point (art. 4.1). It is the so-called **carbon neutrality**, which is not to be intended as the moment in which GHG-emitting technologies will not be used anymore, but as the situation in which the production of such anthropogenic pollutants will equal the quantity of CO_2 that is naturally absorbed by the environment itself. This will be accomplished by gradually reducing the use of fossil fuels, and it is worth noticing that it is the first international climate agreement to request such thing.

How to achieve Paris emission targets

Five areas are identified as crucial by the agreement in order to determine the course of action to be taken:

- **mitigation** -the most important effort, the one concerned with the actual GHG emission reduction;
- **adaptation** -meaning that countries will strengthen their ways to deal with future climate impacts, which are foreseen as inevitable even with the reach of the 1.5°C target;
- **loss and damage** -along with the previous area, is the enhancement of actions for nations to recover from climate impacts;
- **support** - a series of financial aids necessary to fulfill the agreement's purpose, with a particular focus on less-developed countries;
- **transparency system and global stock-take** -the latter starting in 2023 with the purpose of assessing the results achieved, and to be repeated every five years.

By the end of COP21, 188 countries out of 196 had produced their climate pledges (INDCs) to implement GHG emission reduction for the terms 2025-2030; the UNFCCC published on November 1, 2015 a study evaluating the actions plans that were ready at that time. It showed that, despite the great efforts demonstrated by the parties, the increase in temperature would still be between 2.7°C and 3°C above the settled cap (instead of the 4-5°C resulting from a *business as usual* scenario), even with the full accomplishment of such contributions. Under these results, the Paris Agreement forces its parties to review their pledges on a regular basis (art. 14.2): every five years, starting from 2020, with the explicit prescription for them to be at least as ambitious as the previous ones. This means that the 188 parties of the UNFCCC will not be able to reduce their stated efforts, which now constitute the basis for further, improved and higher emission reductions.

Countries are therefore encouraged to constantly review their contributions and to strengthen their objectives; a country that wants to adjust its pledge by making it stronger can submit its INDC at any time, without expecting the next five-years phase.

The last point of the list refers to the implementation of a process tracking progresses achieved in GHG emission reduction. This will work both at a national level, monitoring the effectiveness of each national contribution, and at global level, verifying the step-by-step compliance with the long term objectives. It has been proved that the previous accounting methods, based on a bottom-up pathway, were in fact insufficient and ineffective in order to reach the global goal.

Therefore, starting from 2018, a global stock-take will regularly monitor each nation's progresses towards that direction; a compliance mechanism will be overseen by a committee of experts, operating in a non-punitive way.

Finally, in the period between now and 2020 (the year when the agreement should enter into force) countries will be required to further implement their current actions towards mitigation and adaptation, and to define the ways through which the required financial aids will be gathered. These five years should constitute a solid foundation for the second period.

Hopes for an international market

One of the main defeats of the Kyoto protocol was its inability to create an **international market** for carbon emission allowances. More accurately, it did achieve the creation of such markets -as we have previously mentioned- but only at a restricted, national level, with the only exception of the EU emission trading scheme, involving 31 sovereign entities. Part of article 6 of the Paris agreement is dedicated to the possibility of the future development of market-based approaches for mitigation:

“[since] some Parties choose to pursue voluntary cooperation in the implementation of their nationally determined contributions [...], the use of internationally transferred mitigation outcomes¹⁸ to achieve [them] shall be voluntary and authorized”.

In the end, and differently from what was prescribed in the Kyoto protocol, the possible future system of tradable ITMOs would have as objective

“the aim to deliver an overall mitigation in global emissions” (art. 6.4, d)

resulting therefore in an actual **net-mitigation impact**, and no longer in some kind of purely compensating mechanism.

4.3.2 The role of the parties

The COP21 agreement will become legally-binding only if ratified by at least 55 countries, representing 55% of the whole GHG emissions in the year between April 22, 2016 and April 21, 2017.

¹⁸ITMOs is the new terminology used by the agreement to indicate allowances.

With regards to its future parties, a clear difference between wealthier and less-developed nations is once again remarked. The first ones are still identified as the main players in the fight against climate change, while the latter are depicted as the ones needing support for mitigation and adaptation to global warming related effects (in the short term) and fundings in order to achieve a cleaner development (art. 9).

Concrete numbers, ways to achieve them or schedules are not included in the legally-binding part of the treaty, but in its first part is mentioned the need to raise by 2020 **US\$100 billion** a year by means of loans and donations. These funds will be used for the finance-ment of adaptation projects and for the transition towards a low GHG emissions future, and collected among industrialised countries that will be obliged in this task after the ratifica-tion of the document. Voluntary contributions from developing countries are expected -and encouraged- as well, since their large, emerging economies have modified their importance on the international stage with respect to the one they had in the very first COPs. The amount is specifically said to be increased: before 2025 a new collective fund, surpassing the previous one, will be defined.

Moreover, mechanisms to allow for a greater degree of financial transparency are provided within the agreement: the global stock-take will also include a review of the contributions provided by the parties. Developed countries will be required to report, on a two years basis, on their projected public climate finance as well as on the supporting measures that they have already granted. In the next COPs, more detailed accounting rules will be established.

Parties of the agreement are not the only ones from whom a course of action is expected: civil society, the private sector, financial institutions, cities and in general subnational au-thorities are recognised as stakeholders in the first part of the agreement, and encouraged and welcomed in their efforts against global warming. A massive contributions will come from these actors as well, under the approval of the COP21; their efforts have now entered the United Nations Non-state Actor Zone for Climate Action (NAZCA) portal, and include the following parties:

- over 7,000 cities from over 100 industrialised and less-developed nations, accounting for a population of 1.25 billion people and for around a third of the global GDP;
- sub-national states and regions occupying 1/5 of total global land area;
- over 5,000 companies, whose combined revenues surpass US\$38 trillion;

- nearly 500 investors, managing assets with a value over US\$25 trillion.

It is hoped that pledges and ambitions formulated by business and subnational actors will eventually increase the determination of their own sovranational governments to strenghten their own ones. All these civil parties are explicitly encouraged in the preamble of the agreement to continue their work of indirect persuasion.

4.3.3 A truly legally-binding agreement?

We have introduced the Paris agreement defining it as the first universal, legally-binding agreement on climate change. However, we have also mentioned the inexistence of a clear sanctionary mechanisms to protect the treaty against non-compliance. How much legal force has been provided to the document? Negotiations on the final draft have resulted in a sort of **hybrid form** for the agreement, with some of its elements being legally-binding, while others, equally important, are not. To quote an example, the two long term objectives and the national reporting requirements are explicitly prescribed as legally-binding, while the national mitigation targets (INDCs) required for the post-2020 period are not (art. 3).

The reason behind this combination of forms may be seen in the fact that in order to constitute an as-large-as-possible basis for the agreement, some compromises have been necessary. These arrangements exposed the document itself to criticisms, the main ones underlining its lack of a sanctionary system, as well as the fact that countries will have an option to choose whether to ratify it or not, therefore adhering to its legally-binding nature on a voluntary basis. During the negotiations, there has been a clear trade-off between the redaction of an universal, commonly shared agreement and the production of a stricter, more demanding treaty.

The first philosophy has clearly prevailed. Nevertheless, the nature of the agreement is the one of an international treaty, giving it a strongly enough framework to produce the desired effects.

Part II

A game theory approach

Chapter 5

Fundamentals of game theory

The purpose of this chapter is to provide with the main tools that will be necessary to address the optimal control problem we provide in the final Chapter 6. To do so, we begin with a brief introduction about what game theory is, starting from its definition and its very first assumptions. Then, among all the variety of types games can assume, we select the most helpful to our case, underlining the importance of the players' choices while distinguishing between cooperative and non-cooperative games, and analysing in which forms games may be depicted. We also introduce the ideas of best response and dominant strategy, and the less strong, yet more fitting to reality, concept of Nash Equilibrium. We mention some of the main criticisms that have arised about game theory, along with the responses to them.

After that, we focus on the time issue, talking about dynamic games and how to solve them using backward induction. From there we move to differential games, a peculiar type of dynamic games, explaining what they are and upon which informations a player determines his strategy: this leads us the the concepts of open-loop and Markovian strategies, that are particularly used in games regarding environmental issues.

Finally, we introduce optimal control problems along with the Hamilton-Jacobi-Bellman equation and the Pontryagin Maximum Principle, two methodologies that are used in order to determine a solution. f

5.1 An introduction to game theory

The *Encyclopædia Britannica* defines the game theory as follows:

“the analysis of a situation involving conflicting interests in terms of gain and

losses among opposing players”.

This is of course a highly broad statement, that shows how game theory can be applied to a multitude of fields, from sociology to politics, from military strategy to philosophy, just to mention a few: all scenarios where people have to interact and make some decisions. Since we are dealing with an economic topic, we have to limit our focus by including some parameters and characteristics that allow us to apply mathematical models to these interactions.

A more punctual definition for our purposes may be the following one:

“Game theory is the study of mathematical models of conflict and cooperation between intelligent rational decision-makers” (MYERSON 1997, p. 1).

The second part of the definition introduces some simplifying assumptions, which are considered necessary in order to build some models depicting the complexity of the real world. In the classic trade-off between solvability and realism, the most common axioms are two:

1. **rationality**;
2. **strategic thinking**.

We define a rational player as one who is able to order his own preferences regarding a number of possible outcomes; these preferences have to satisfy some axioms, depicted by Von Neumann and Morgenstern in 1947¹. Players will choose among the actions at their disposal in order to maximise their utility function², which can be an expression of any quantitative measure. Furthermore, a rational player will know the number of his opponents and the set of all possible strategies, and will be able to develop expectations about any uncertainty about the game. A player will think strategically if:

“when designing his strategy for playing the game, [he] takes into account any knowledge or expectation he may have regarding his opponent’s behavior” (DOCKNER *et al.* 2000, p. 11-12).

There are four basic elements that can be found in any game: (two or more) players, set of strategies available, outcomes and payoffs. These elements are also known as the **rules** of the game, because they constitute its formal description; for this reason, they are not fixed criteria, but change constantly from case to case and are derived from the institutional

¹The “four axioms of rationality” are completeness, transitivity, continuity and independence.

²Measure of preference over some set of goods and services.

environment in which the game is set. It is easy to notice that, since the involved players are always at least two, their final payoffs do not depend only on their own choices: in choosing which strategy among those at their disposal is the best one, they will have to consider also the ones that the other(s) player(s) may implement. To be fully defined, a game must also specify which are the informations and the actions available to each of the participants.

There is a multitude of cases that might be studied, starting from these very few characteristics common to all games: games where players move simultaneously, others where there is a clear sequence for the actions; situations in which there is perfect information, others in which there is not; games that are to be played once, others that will repeat themselves, just to mention the most common ones.

5.1.1 Cooperative and non-cooperative games

A further, peculiar distinction is the one between cooperative and non-cooperative games. **Non-cooperative games** represent all the situations in which players do not cooperate with each other, do not make binding agreements (or are not forced to do so), and are just rivals who choose actions on behalf of their own interest. These kinds of situation are of course characterised by a high degree of uncertainty, since every single player can not know what strategy the others will follow. On the other hand, a game is **cooperative** if the players involved are able to gather themselves into groups, and to coordinate their actions in order to maximise their mutual, expected outcomes. The ability (or will) to make binding agreements largely depends on the institutional environment in which the game takes place. The main focus here is not anymore to decide which action is the best for each player to take, considered the presence of some rivals, but how share the expected earning between all the members of the group, according to the terms of the agreement.

Game theory shows us that to cooperate is always a good decision: the expected outcomes are higher in these games, with respect to the non-cooperative ones.

5.1.2 Strategic and extensive forms

The strategic and the extensive forms are the two types of models used to represent non-cooperative games. In every **strategic form** we find the following elements:

1. a set of players $N = \{1, \dots, N\}$;

2. a set of possible strategies U^i for each player $i \in N$;
3. a real-valued function J^i for each player $i \in N$, such that $J^i(u^1, \dots, u^n)$ represents the payoff of player $i \in N$ if the N players use the strategies $(u^1, \dots, u^N) \in U^1 \times U^2 \times \dots \times U^N$.

Strategy plays a fundamental role in this game form, and it is defined as a function that guides the player in his process of choosing one of the feasible actions at his disposal whenever he has to make a move, taking into account all the possible events that may have occurred until that precise time of the game, known as the history of the game. A strategy guides the player through all possible histories of the game, even those that will never be actually observed, selecting his feasible choices of action. As we have mentioned, strategic forms are used to depict non-cooperative games: this means that a player chooses one particular strategy among his own list of possible ones, without any form of communication or cooperation with his opponents, that have to choose “blindly” their own strategy too. It is from this lack of knowledge that permeates non-cooperative games that rises their typical issue of uncertainty. Strategic forms do not mention explicitly the time element: however, they can still represent in a general way games that go beyond the one-shot option, as they are repeated over time. A player could determine in advance, at the very beginning of the game, the course of actions that he will implement for every possible stage of the game.

An example of a strategic form is given below, where the first player has to decide wheter to play A or B , while the second must choose between C and D .

1 \ 2	C	D
A	(2,1)	(0,0)
B	(-1,1)	(3,2)

Table 5.1: Representation of a strategic form

For a more precise description of games played over time the **extensive form** is used, in the look of a decision tree, that is particularly helpful in describing the sequence in which players will have to move, as well as in highlighting the points in which chance events that can change the course of the game may happen. With regard of the time element, extensive forms are superior to strategic ones: they describe clearly the order of moves available to

the players, which informations are revealed to them (and at what time), and how they may influence the game. On the other hand, a tree may not be easy to be managed and depicted when the complexity of games arise.

The game tree below shows how it is possible to represent the previous example in an extensive form as well.

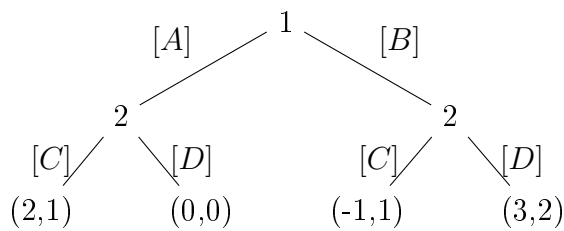


Figure 5.1: Representation of an extensive form

5.1.3 Best response and dominant strategy

We will start by briefly analyzing a non-cooperative, one shot game, where all players move just once, in order to show that even if our randomly chosen player i is not able to predict the moves of his opponents, he can always determine which one should be his best strategy, given a set of possible actions for all the players. Let $u_i(s_i, s_{-i})$ be the payoff (or utility) of player i when the profile of strategies $s = (s_i, s_{-i})$ is played; here s_i represents i 's own strategy, while s_{-i} stands for the strategies of all other players $N - 1$. We will find the definition of best response by considering a game with N players, where i is the generic player whose point of view we are adopting, and $-i$ are all the other players different from i ; the following strategy $s_i^*(s_{-i})$ is a **best response** for player i if it guarantees to him the highest payoff when other players have played s_{-j}

$$u_i(s_i^*, s_{-i}) \geq u_i(s_i, s_{-i}) \quad \forall s_i \in S_i$$

where S_i denotes the set of player i 's strategies.

A (strict) **dominant strategy** s_i^* represents a stronger concept, which is defined as player i 's best reply for all possible strategies played by the other players

$$u_i(s_i^*, s_{-i}) > u_i(s_i, s_{-i})$$

for all s_i and for all s_{-i} . This means that the strategy profile $s = s_1, \dots, s_n$ is an equilibrium in dominant strategy if s_i is a dominant strategy for each $i = 1, \dots, n$. It is important to underline how a dominant strategy represents the best strategy a player may play, and not the one that could give him the maximum payoff available.

5.1.4 Nash Equilibrium

A strategy profile is a **Nash Equilibrium** (NE) of a game if each player is playing a best response to the other players' strategies; in other terms, the strategy profile s^* is a Nash Equilibrium of the n -players game if, for all players $i = 1, \dots, n$,

$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*) \quad \forall s_i \neq s_i^*$$

Every player i will always prefer his s_i^* strategy among all the ones at his disposal, assuming that all the other players play s_j^* ; i will have no unilateral incentive to deviate, unless he wants to worsen his outcome³.

A Nash Equilibrium is of course a weaker concept with respect to the one of the equilibrium in a dominant strategy that we have previously analysed. It can be demonstrated that if a strategy profile is an equilibrium in a dominant strategy, then it is (the unique) Nash Equilibrium as well. The converse, however, does not hold, underlining how this strategy is not the best a player can do in absolute terms, but only conditionally to the others' ones.

5.1.5 Some criticisms of game theory

Game theory has also gathered some criticisms, that we would like to mention briefly in this paragraph, along with the responses given by its supporters, for seek of clarity. Critics to game theory usually belong to very different fields.

There have been some concerns about ethics, starting from the analysis of the assumptions that stand behind game theory. At a first glance, it may seem defective both as a normative theory of action and as a descriptive theory of action. As a matter of fact, if we look at the very first assumptions we have given, players seem to be urged to care only about their own self-interest, while when we focus on the actual depiction of situations, the theory assumes that players move and decide only on the basis of their own self-interest, even when they

³And this is not allowed by the rationality axiom.

apparently do not. These concerns can be easily answered, because they are built on a misunderstanding about the definition of payoff:

“A payoff simply represents a person’s well-ordered preferences, [which may be] altruistic and self-sacrificing. As long as a person’s preferences are well-ordered, even a perfectly altruistic saint [...] is maximising his payoff” (CHUN 2014, p. 29).

Another class of criticisms is concerned with the assumptions behind game theory from a technical point of view, judging them incapable of truly depicting what may happen in the real world. Psychology has largely demonstrated how human beings are not capable of that full rationality which is the first axiom of game theory; they prefer to talk about **bounded rationality**, since the informations we use to understand the environment surrounding us are always incomplete, because of our mind’s inability to process all of them and of the limited amount of time we use for the process. This seems to be the greatest weakness of game theory, since its axioms and assumptions imagine a role for informations that is unbearable for the human mind: are all the informations about the opponents, their strategies, one’s own possible moves always available for the player? Probably not. As we have said in the first paragraphs, there is a clear trade-off between the complexity of a model (that is, its closeness to reality) and its solvability; the “ideal” model, as close to reality as possible, would be too complicated to be predicted. The numbers of variables that it should have to take into account would make it impossible to solve, and therefore useless. But having said that game theory does not truly describe reality, this does not make it less helpful for predicting it:

“We should be aware that models are not supposed to be accurate representations of real-world phenomena, but even very simplified models do not necessarily produce useless predictions. [Game theory] is not [to be applied] in a mechanistic way” (DOCKNER *et al.* 2000, p. 12).

5.2 Dynamic games

Dynamic games are more complex and more correspondent to reality, mostly when we refer to economics and look at firms which can make more than one decision over the game period. The introduction of the time element is not a sufficient condition *per se* in order to differentiate a static game from a dynamic one. **Dynamic games** may be defined as those where at least one player can react strategically to the actions conducted previously, using

informations that were not available when the game started.

The first thing to do when studying a dynamic game is to determine in which order players move, and what informations are available to them when they have to decide their actions. We will focus on games of perfect information, meaning that every player is aware of all the previous actions when he makes his move at time t : his own past actions and his rivals' ones of course, but also every exogenous event that may influence the game. All the players move simultaneously if, at every moment t , they do not know anything about the actions that are being played at the same time by their opponents. We are dealing however with games of perfect information: all past actions until $t - 1$ are known by all the participants; this allows us to introduce the concept of history of actions by time t (denoted by h_t). It corresponds to a sequence u_1, u_2, \dots, u_{t-1} , where any action-profile is a set of N individual actions of the players. The history before the beginning of the game is represented by h_0 , and is an empty set. Payoffs of the players can be defined both as functions of the history h_t , as well as sums of per-period payoffs.

5.3 Differential games

Differential games are dynamic games played in continuous time, with two peculiar characteristics:

1. a set of variables that characterises the state of the dynamical system at any instant of time during the game;
2. the evolution over time of the state variables is described by a set of differential equations.

Differential games, unlike dynamic ones, are usually represented with a strategic form. To deepen our comprehension of differential games, we start by representing the time variable with t , and by supposing that the game will be played during a time interval $[0, T]$, where T , which is the horizon (or planning period), may be infinite or finite, but always greater than zero. The state of the game at each instant t can be described by an n -dimensional vector $x(t) = (x_1(t), x_2(t), \dots, x_N(t)) \in X$, where $X \subseteq \mathbb{R}^n$ is a set containing all possible states, also known as the state space of the game; $x(t)$ is also known as the state vector, and is introduced in order to characterise the current state of a dynamical system. Denote by the variable $u_i(t)$ the action taken at time t by player i , which is referred to as the control of

player i . Furthermore, any action chosen by a player at an instant t must be derived from the player's set of feasible actions, which generally depends on the current time t , the current state $x(t)$ and the set of current actions of the player's opponents.

We have seen that one of the two important features of differential games is that the evolution over time of $x(t)$ is determined by differential equations

$$\begin{aligned}\dot{x}(t) &= f(t, x(t), u_1(t), \dots, u_N(t)) \\ x(0) &= x_0\end{aligned}$$

also known as the state equations, showing that all players have the chance of influencing the rate of change of the state vector through the choice of their current actions. Each player tries to maximise his total payoff over the planning horizon, discounted at the rate $r_i \geq 0$

$$J^i = \int_0^T e^{-r_i t} F^i(t, x(t), u_1(t), \dots, u_N(t)) dt$$

where F^i defines the instantaneous payoff of player i . This equation demonstrates that -in general- every player is able to influence the payoff of player i through the choice of his current actions. The payoff J^i has to be maximised by player i through his choice of the control $u_i(t)$ for every $t \in [0, T]$. The differential game we have just defined is a strategic form game.

5.3.1 Open-loop strategies

Since we are using strategic rather than extensive forms to model a differential game, an issue arises: we need to specify upon which information a player conditions his strategy. The choice here is between a strategy space or an information structure, and the answer comes directly from the institutional environment that frames the game. For example, a player may decide to use a minimum of information and base his strategies only on time, while another may base his strategies on the whole history of actions. These are, of course, two extreme cases: the first one brings us to the concept of **open-loop strategy**, which is conditioned only on current time, meaning that we will have a strategy conditioned on a minimal amount of information. During the game an action is chosen instead of another one only on the basis of the moment in which the move has to be done. By doing so, players leave all informations (except time) out of consideration, either by choice or because they can only observe their own actions (and time, of course). In games regarding renewable resources and environmental

issues, a player using an open-loop strategy cares about the conservation of resources and the preservation of the environment.

5.3.2 Markovian strategies

In a **Markovian strategy** a player is conditioned on current time t and on the state vector $x(t)$ when he has to decide which action is to be chosen. Unlike open-loop strategies, here players' choices are not driven by the game history until time t : only the consequences of the previous moves matter, and they are represented in the current value of the state vector, an effect of the so-called Markov property of memorylessness. Markovian strategies have the important feature of being simple: it can be demonstrated (MASKIN *et al.* 2001) that players use them in a learning context, in which it is expensive -and therefore not convenient- to increase the complexity of a strategy.

In a game that is played through the use of Markovian strategies, a Nash equilibrium is called Markovian Nash equilibrium, or feedback (closed-loop) Nash equilibrium. The resulting equilibrium is perfect in all the subgames of the game, a concept that is developed in the next subsection.

5.3.3 Backward induction and subgame perfectness

As we have already mentioned, the use of the extensive form is much more useful for the analysis of dynamic games with respect to the strategic one; moreover, the principle of **backward induction** is particularly helpful. It is used to solve games with a finite number of time periods $T < \infty$ and with a finite number of strategy sets. The game is solved by first determining the optimal choice in the final moment t and then, working backward, the optimal choice for the previous instant $t - 1$, and so on until the starting point is reached.

Backward induction has a natural extension in the property of **subgame perfectness**, an important concept of dynamic game theory. A subgame may be considered as a subset of any games that includes an initial node -that must be independent from any information set- and all its successor nodes. It is basically a game on its own, a cut version of the whole picture; it may start at time t after a particular history of actions h_t , and we represent it with the symbol $\Gamma(h_t)$. A Nash equilibrium strategy profile for the game as a whole (σ) induces a strategy profile in the subgame $\Gamma(h_t)$, which is the restriction of σ to the subgame $\Gamma(h_t)$. A Nash equilibrium strategy profile σ for the whole game is subgame perfect if, for any history h_t , it holds that the restriction of σ to the subgame $\Gamma(h_t)$ is a Nash equilibrium of $\Gamma(h_t)$. It

is worth noticing that subgame perfectness requires not only that σ is a Nash equilibrium for the whole game, but also that σ 's restrictions are Nash equilibria for every subgame: this means that the Nash equilibrium must exist also in the subgames that are not played.

Subgame perfectness is by definition a stronger equilibrium concept than Nash equilibrium.

5.4 Optimal control problems

We have seen that differential games are situations in which players want to maximise their objective functional subject to some constraints, the most important of which is a differential equation describing the evolution of the state of the game. The concept of Nash equilibrium in such a dynamic situation involving n players is related to the resolution of n **optimal control problems**, one for each player, where the opponents' strategies are considered as parameters. We are then facing an optimisation problem, and the following concepts belong to optimal control theory must be taken into account in order to determine the Nash equilibria.

From Section 5.3, we recall that the differential game spreads over the period $[0, T]$ with $T > 0$, and that every player can make a move at every time $t \in T$, influencing both the evolution of the state of the game and his own and his opponents' objective functionals. We have also introduced the state vector $x(t)$ and we have underlined how the evolution of the state can be described by the following differential equation (which has a specific focus on a single, particular player)

$$\dot{x}(t) = f(x(t), u(t), t) \quad (5.1)$$

which is a description of how the current state $x(t)$ and the player's actions at time t influence the rate of change of the state at time t . It has to satisfy the initial condition

$$x(0) = x_0 \in X \quad (5.2)$$

with X being the state space of the game. In the equation (5.1) $u(t)$ is the abbreviation for $u(t) = (u_1(t), u_2(t), \dots, u_m(t)) \in \mathbb{R}^m$, and represents the vector of actions chosen by our selected player at time t . This allows us to introduce the first constraint that must be obeyed

$$u(t) \in U(x(t), t) \quad (5.3)$$

where $U(x, t) \subseteq \mathbb{R}^m$ represents the set of all feasible actions at time t , if the state of the system is equal to x . These three equations ([5.1], [5.2] and [5.3]) are the constraints of the optimal control problem.

The next step is to introduce the objective functional of our player

$$J(u(\cdot)) = \int_0^T e^{-rt} F(x(t), u(t), t) dt + e^{-rT} S(x(T)) \quad (5.4)$$

recalling that $r \geq 0$ stands for the discount rate. Furthermore, $F(x(t), u(t), t)$ -the utility function- tells us the player's utility when he chooses the control function $u(t)$ at time t , with the current state of the game being $x(t)$, while $S(x(T))$ -the terminal value function- represents the terminal value associated with the state $x(T)$.

A standard optimal control problem consists of maximising the functional J defined in (5.4) over all control paths $u(\cdot)$ which satisfy (5.3), while taking into account that the evolution of the state is determined by the system dynamics (5.1) and the initial condition (5.2). We have just mentioned the concept of control path, whose introduction also brings some problems that have to be addressed in order to continue: solutions to (5.1) and (5.2) may not exist or may not be unique, and the integral in (5.4) may not be defined. To deal with this problems, we have to restrict the set of control paths $u(\cdot)$ in a way that makes the objective functional $J(u(\cdot))$ well defined.

Definition 5.1 A control path $u : [0, T] \mapsto \mathbb{R}^m$ is feasible for the optimal control problem we are considering if the initial value problem (5.1) - (5.2) has a unique absolutely continuous solution $x(\cdot)$ such that the constraints $x(t) \in X$ and $u(t) \in U(x(t), t)$ hold for all t and the integral in (5.4) is well defined. The control path $u(\cdot)$ is optimal if it is feasible and if the inequality $J(u(\cdot)) > J(\tilde{u}(\cdot))$ holds for all possible control paths $\tilde{u}(\cdot)$.

5.4.1 The Hamilton - Jacobi - Bellman equation

We have seen that Markovian strategies may deal with continuous time situations, meaning that a player can decide at any time which decision is to be implemented. Under these assumptions, with the state space and the action space being continuous, a possible approach to the solution of an optimal control problem is the **Hamilton-Jacobi-Bellman** (from now on: HJB) equation.

Through the HJB equation it is possible to find the optimal criterion: when it is solved over the whole state space, it represents a necessary and sufficient condition for an optimum. It is based on two important principles: embedding and recursion. At a first glance, the first

one does not seem very helpful, since it widens exceedingly our field of study: starting from our problem $P(x_0, 0)$ which begins at time $t = 0$ in the initial state x_0 , the principle prescribes not to solve only it, but rather the entire family of problems $\{P(x, t) \mid x \in X, t \in [0, T]\}$ in which our first one is embedded. The new problem $P(x, t)$ begins at time t in the initial state x and can be stated as follows

$$\text{Maximise } \int_t^T e^{-r(s-t)} F(x(s), u(s), s) ds + e^{-r(T-t)} S(x(T)) \quad (5.5)$$

subject to

$$\begin{aligned} x(\dot{s}) &= f(x(s), u(s), s) \\ x(t) &= x \\ u(s) &\in U(x(s), s) \end{aligned}$$

It seems that we now have to solve infinitely many problems instead of one, but here is where the second principle comes in handy and justifies the validity of the HJB equation. Recursion means that we have to start by picking the “smallest” problems of the entire family $-P(x, T), x \in X$ - and work our way backwards to the “largest” ones, which are $P(x, 0), x \in X$. Knowing the solution of all small problems will help to find the solution of any larger one.

We will start by denoting the only feasible (and hence the optimal) value of the objective functional of $P(x, T)$ by $V(x, T)$, which will also denote the optimal value of the objective functional of the problem expressed in (5.5). The optimal value function V satisfies the following equation

$$rV(x, t) - V_t(x, t) = \max\{F(x, u, t) + V_x(x, t)f(x, u, t) \mid u \in U(x, t)\}$$

which is the HJB equation. Is V always differentiable? The answer is no, and that is the reason why no theorem exists stating that the optimal value function V is continuously differentiable and solves the HJB equation. A solution to this issue may be to consider the HJB equation only as a sufficient optimality condition, under the assumption that the optimal value function is continuously differentiable. This would lead us to the formulation of the following theorem.

Theorem 5.1 Let $V : X \times [0, T] \mapsto \mathbb{R}$ be a continuously differentiable function which satisfies the HJB equation

$$rV(x, t) - V_t(x, t) = \max\{F(x, u, t) + V_x(x, t)f(x, u, t) \mid u \in U(x, t)\} \quad (5.6)$$

and the terminal condition

$$V(x, T) = S(x) \quad (5.7)$$

for all $(x, t) \in X \times [0, T]$. Let $\Phi(x, t)$ denote the set of controls $u \in U(x, t)$ maximising the right-hand side of (5.6). If $u(\cdot)$ is a feasible control path with corresponding state trajectory $x(\cdot)$ and if $u(t) \in \Phi(x(t), t)$ holds for almost all $t \in [0, T]$ then $u(\cdot)$ is an optimal control path. Moreover, $V(x, t)$ is the optimal value of problem $P(x, t)$.

The study of perfect Nash equilibria in subgames is based on the HJB equation system: once the solution of the system has been determined, the optimal conditions provide the optimal strategies, at least in an implicit form.

5.4.2 Pontryagin Maximum Principle

Along with the Hamilton-Jacobi-Bellman, another methodology used to approach optimal control problems is the **Pontryagin Maximum Principle**. Its importance is given by the fact that it leads to the formulation of necessary conditions, differently from what happens with the HJB theory that provides sufficient conditions. These necessary conditions -which, by being such, must be satisfied- allow to determine quite easily a solution which is a candidate for optimality. However, a solution satisfying all the necessary conditions is not automatically the optimal one, otherwise those conditions would be sufficient too.

Furthermore, it is useful since there are often dynamic optimisation problems where, because of the extent of the time lapse upon which the system is considered, the utility flows are valued within the objective functional, taking their distribution over time into account (VISCOLANI 2003). This can be obtained by multiplying the function representing the utility flow by an actualisation function which must be continuous, decreasing and with values in $]0, 1]$. Such an actualisation function is embodied by $e^{-\delta t}$, with $\delta > 0$ being a fixed parameter.

Let us consider the following problem

$$\text{Maximise } J(u) = \int_{t_0}^{t_1} e^{-\delta t} f^0(x(t), u(t), t) dt + S(x(t_1)) \quad (5.8)$$

subject to

$$\begin{aligned}
 \dot{x}(t) &= f(x(t), u(t), t) \\
 x(t_0) &= x^0 \\
 x_i(t_1) &= x_i^1 \quad i = 1, \dots, l \\
 x_i(t_1) &\geq x_i^1 \quad i = l + 1, \dots, m \\
 x_i(t_1) &\in \mathbb{R} \\
 u(t) &\in \Omega \subseteq \mathbb{R}^r
 \end{aligned}$$

with $\delta > 0$.

The integrand in the objective functional is the product between the actualisation function $e^{-\delta t}$ and the function $f^0(x(t), u(t), t)$

$$f_0(x(t), u(t), t) = e^{-\delta t} f^0(x(t), u(t), t)$$

Hence, we define H^c as the current value Hamiltonian function

$$H^c(x, u, q, t) = p_0 f^0(x, u, t) + \sum_{i=1}^n q_i f_i(x, u, t)$$

Theorem 5.2 Let $u^*(t)$ be a piecewise continuous optimal control, defined by $[t_0, t_1]$, to which the state function $x^*(t)$ is associated. Then, some constants $p_0, \eta_1, \eta_2, \dots, \eta_n \in \mathbb{R}$ and a piecewise continuous function of class C^1 $q(t)$ ($q : [t_0, t_1] \mapsto \mathbb{R}^n$) exist, such that, for every $t \in [t_0, t_1]$, the following conditions hold:

1. $(p_0, \eta) \neq 0 \in \mathbb{R}^{n+1}$ ($\eta = (\eta_1, \eta_2, \dots, \eta_n)$);
2. $u^*(t)$ maximises $H^c(x^*(t), u, q(t), t)$ for $u \in \Omega$;
3. except for all the t such that $u^*(t)$ is discontinuous,

$$\dot{q}(t) = -\frac{\partial H^c(x^*(t), u^*(t), q(t), t)}{\partial x} + \delta q(t)$$

4. $p_0 \in \{0, 1\}$;

$$5. q_i(t_1) = p_0 e^{\delta t_1} \frac{\partial S(x(t_1))}{\partial x_i} + \eta_i, \quad i = 1, \dots, n$$

$$\eta_i \in \mathbb{R} \quad i = 1, \dots, l$$

$$\eta_i \geq 0 \quad \text{and} \quad \eta_i(x_i(t_1) - x_i^1) = 0 \quad i = l + 1, \dots, m$$

$$\eta_i = 0 \quad i = m + 1, \dots, n$$

Arrow's theorem

It should be highlighted that the Pontryagin Maximum Principle gives the necessary conditions for the optimality. If the principle suggests a certain number of possible solutions, we know that there are no other solutions capable of solving the problem. However, the principle is not able to determine if the solution it has found is an optimal one, and neither if an optimal solution exists. Therefore, some stronger conditions for the concavity of the function are introduced, such that we obtain sufficient conditions for the optimality.

ARROW proposed the following theorem, which represents an alternative condition, yet weaker than the Hamiltonian functions' concavity.

Theorem 5.3 Let $(x^*(t), u^*(t))$ be an admissible pair. If a piecewise continuous function of class C^1 $p(t)$ ($p : [t_0, t_1] \mapsto \mathbb{R}^n$) exists, such that the following conditions (with $p_0 = 1$) are satisfied:

$$1. \dot{p}(t) = -\partial H^*/\partial x, \quad i = 1, \dots, n;$$

$$2. H(x^*(t), u, p(t), t) \leq H(x^*(t), u^*(t), p(t), t) \quad \forall u \in \Omega, \forall t \in [t_0, t_1]$$

3.

$$p_i(t_1) \quad \text{no condition} \quad i = 1, \dots, l$$

$$p_i(t_1) \geq 0 \quad p_i(t_1)(x_i^*(t_1) - x_i^1) = 0, \quad i = l + 1, \dots, m$$

$$p_i(t_1) = 0, \quad i = m + 1, \dots, n$$

$$4. \text{function } \hat{H}(x, p(t), t) = \max_{u \in \Omega} H(x, u, p(t), t)$$

then $(x^*(t), u^*(t))$ is the optimal solution for the problem.

Chapter 6

A model for environmental treaties

Once explained the economic basis of environmental treaties, given evidence of their need for the fight against climate change to succeed and having underlined the serious possible consequences that we are going to face if a commonly shared, global answer will not be soon determined, having summarised the most important steps taken from 1992 to the present date by the international community, and having explained the basis on which optimal control problems are built up, we now describe how international environmental treaties work under the form of mathematical models.

Let us first introduce some simplifying assumptions.

To quote the main one, we do not deal with 192 countries, as happened for the *Kyoto protocol*, or with the 195 states which expressed their consensus for the *Paris agreement*. Even if it is plausible that during the negotiation phases almost each one of these parties had its own position about environmental themes, with serious degrees of differences with the ones hold by the other “players”, we consider only two states, or, more precisely, two blocks of nations. To each one of them a different position on climate change, as well as a different bargaining power (as it is underlined in Section 6.2), can be associated.

The model takes inspiration from the one created by KAITALA and POHJOLA (1995). They proposed a differential game with an infinite horizon where two blocks of countries, suffering in different ways from the environmental damages caused by global warming, find themselves willing to negotiate a commonly shared solution.

In the first section of the chapter we report the model they have built for the scenario characterised by non-cooperation. From that basis, we analyse a slightly different situation, characterised by a finite horizon, with a starting and an ending point, just like environmental treaties consider.

The last part is dedicated to the evaluation of the different bargaining powers associated with nations: who determines the most responsible countries, *i.e.* the ones that will face the highest costs in order to abate GHG emissions? We consider a situation in which such power is attributed to an hypothetical, independent regulator, and then the case in which nations themselves are responsible for the different distribution of responsibilities.

6.1 The non-cooperative scenario

Let us denote the two blocks of nations I_1 and I_2 respectively. They both share a common good -the atmosphere- which is at risk because of greenhouse gases emissions.

In order to formulate the model, we need to determine who the two players are. According to KAITALA and POHJOLA (1995), the division may represent the distinction between countries suffering more or less from climate change. As a matter of fact, we recall from **Chapter 3** that even if global warming is affecting the humanity as a whole, some countries are more likely to experience its negative effects sooner. However, we have chosen to adopt another valid criterion, and to distinguish world countries between developing and industrialised economies. Recalling what we have analysed in **Chapter 4**, that is that Annex II Parties are identified as the responsible of the actual pollution levels, we expect the first ones to be more protected by the treaty than the already developed, second ones. In fact, this is what happened in all the environmental treaties signed in the last decades. In the next section we specify how the different bargaining powers associated with the blocks may influence the cooperative outcome.

Let $Q(t)$ be the deviation at time t from the CO_2 emission levels with respect to the base year, and $e_i(t)$, $i = 1, 2$ the GHG emission level produced by each block at time t . Two different costs are taken into account, the ones due to the emission levels represented by $C_i(e_i(t))$ and also known as emission abatement costs, and the ones dependent on the CO_2 concentration levels in the atmosphere denoted by $D_i(Q(t))$ and also known as damage costs.

In the next section we introduce the finite time element, since a cooperative solution under the cap of an international environmental treaty will take place. However, in the non-cooperative example both I_1 and I_2 are not willing to come to an agreement, and therefore we take into account the infinite horizon where no joint commitment will ever take place. The problem that the two players are facing is to determine their respective emission levels $e_i(t)$ in a way that the cost associated to their chosen level (and therefore the consequences

coming from GHG concentration levels in the atmosphere) is as small as possible.

Furthermore, we suppose that the evolution of $Q(t)$ is represented by the following equation

$$\frac{\partial Q}{\partial t} = \sigma(e_1(t) + e_2(t)) - \beta Q(t) \quad (6.1)$$

where σ and β are two positive environmental parameters. The first one implies the role played by the environment itself in absorbing (part) of all CO_2 emissions, since

“[it has been] estimated that a half of the anthropogenic carbon is removed by the natural sinks, oceans mainly, while the other half remains in the atmosphere” (NORDHAUS 1991, p. 78).

The second one, β , takes into account the life time of greenhouse gases in the atmosphere, before their decay. We assume that the moment at which we observe $t = 0$ corresponds to the year 1990 both for the non-cooperative and for the cooperative situations. In the next section, the levels of GHG emissions associated with 1990 are the basis for the environmental treaty, as it happened for the *Kyoto protocol* and the *Paris agreement*, since they are associated with the known thresholds of 1.5-2°C.

Therefore in 1990 $Q(0) = 0$.

The problem that the two players are facing when deciding to pursue a non-cooperative solution for the issue implies the determination of the emission level minimising at every time the actual value of costs during the period $[0, \infty)$. Or -in other terms- they must find the values of $e_i(t)$ for $i = 1, 2$ responsible of

$$\text{Minimise } \int_0^{\infty} e^{-\rho_i t} [C_i(e_i(t)) + D_i(Q(t))] dt \quad (6.2)$$

subject to

$$\frac{\partial Q}{\partial t} = \sigma(e_1(t) + e_2(t)) - \beta Q(t) \quad (6.3)$$

$$Q(0) = 0 \quad (6.4)$$

for all $t \in [0, \infty)$, $i = 1, 2$ and for all $Q(t)$; ρ_i once again represents the discount rate for each player i ; from now on, we suppose that both of them share the same value ($\rho_1 = \rho_2 = \rho$).

Moreover, we suppose that C_i , $i = 1, 2$ is a decreasing convex function, and that on the contrary D_i , $i = 1, 2$ is an increasing convex one.

To proceed, assume that the cost functions of the two players are both quadratic and equal such that

$$C_i(e_i) = \frac{1}{2}c_i(e_i - e_i^m)^2 \quad (6.5)$$

$$D_i(Q) = \frac{1}{2}d_i(Q)^2 \quad (6.6)$$

where c_i and d_i ($i = 1, 2$) are two positive constants and e_i^m represents the CO_2 emission rate with no reductions carried out.

Through the application of the Hamilton - Jacobi conditions it can be shown that the non-cooperative CO_2 emission policies are given as

$$e_i^* = -\frac{\sigma\epsilon_i^*}{c_i}Q + (e_i^m - \frac{\sigma}{c_i}\gamma_i^*), \quad i = 1, 2 \quad (6.7)$$

where the coefficients ϵ_i^* and γ_i^* represent

$$\epsilon_i^* = \frac{-\left(\rho + 2\beta + 2\frac{\sigma^2\epsilon_j^*}{c_j}\right) \pm \sqrt{\left(\rho + 2\beta + 2\frac{\sigma^2\epsilon_j^*}{c_j}\right)^2 - 4\frac{\sigma^2}{c_i}(-d_i)}}{2\frac{\sigma^2}{c_i}} \quad (6.8)$$

and

$$\gamma_i^* = \frac{\epsilon_i^*\sigma\left(e_i^m + e_j^m - \frac{\sigma}{c_j}\gamma_j^*\right)}{\rho + \beta + \frac{\epsilon_i^*\sigma^2}{c_i} + \frac{\epsilon_j^*\sigma^2}{c_j}} \quad (6.9)$$

Therefore, the value of non-cooperation for each player is

$$V_i^*(Q) = \frac{1}{2}\epsilon_i^*Q^2 + \gamma_i^*Q + \mu_i^* \quad (6.10)$$

with coefficient μ_i^* given by

$$\mu_i^* = \left(-\frac{1}{2}\frac{\sigma^2(\gamma_i^*)^2}{c_i} + \gamma_i^*\sigma\left(e_i^m + e_j^m - \frac{\sigma}{c_j}\gamma_j^*\right)\right) / \rho \quad (6.11)$$

Furthermore, the differential equation for the non-cooperative Q^* becomes

$$\frac{\partial Q^*}{\partial t} = -A^*Q^* + B^* \quad (6.12)$$

where

$$A^* = \sigma^2 \left(\frac{\epsilon_1}{c_1} + \frac{\epsilon_2}{c_2} \right) + \beta \quad (6.13)$$

and

$$B^* = \sigma \left(e_1^m - \frac{\sigma}{c_1} \gamma_1 + e_2^m - \frac{\sigma}{c_2} \gamma_2 \right) \quad (6.14)$$

The solution to (6.12) is given by

$$Q^*(t) = Q^*(0)e^{-A^*t} + \frac{B^*}{A^*}(1 - e^{-A^*t}) \quad (6.15)$$

Thus the state trajectory converges to the value

$$\lim_{t \rightarrow \infty} Q^*(t) = \bar{Q}^* = \frac{B^*}{A^*} \quad (6.16)$$

The solution to the non-cooperative situation is largely inspired by the already mentioned work of KAITALA and POHJOLA (1995). However, we have also introduced some differences with respect to it. The main one concerns the determination of the role of the two players. As a matter of fact, the authors divided the world countries into two groups, with the first one hosting all

“countries vulnerable to the global warming, suffering definite costs from it in the form of physical damages”,

and the second one

“countries that are economically neutral with respect to the global warming, [that] do not suffer from the greenhouse effect” (KAITALA *et al.* 1995, p. 69).

According to this distinction, $D_2(Q) = 0$ for all Q , and since non-cooperative emissions of player 2 are defined as $e_2^*(Q) = e_2^m$ for all Q , his value of the game corresponds to $V_2^*(Q) = 0$.

6.2 The cooperative solution

In the following section we suppose that the parties do come to an arrangement under the form of an environmental treaty, so as to cooperate in the pursuit of a shared solution for their common pollution problem.

This time, I_1 and I_2 want to determine their respective emission levels $e_i(t)$ in the **finite** time $[0, T]$, in a way that once again minimises the cost associated to the chosen level. As in the previous case, the evolution of $Q(t)$ is represented by (6.1).

In order to determine the cooperative solutions, starting from the two objective functions, we obtain the following minimisation problem

$$\text{Min } \alpha \int_0^T e^{-\rho t} [C_1(e_1(t)) + D_1(Q(t))] dt + (1 - \alpha) \int_0^T e^{-\rho t} [C_2(e_2(t)) + D_2(Q(t))] dt \quad (6.17)$$

subject to

$$\frac{\partial Q}{\partial t} = \sigma(e_1(t) + e_2(t)) - \beta Q(t) \quad (6.18)$$

$$Q(0) = 0 \quad (6.19)$$

with $\alpha \in (0, 1)$ showing the different roles attributed by the regulator to the two players in evaluating their reduction of GHG emission levels.

It is useful to remind from **Subsection 6.1** that the treaty we are analysing, as all the ones drafted in the last years, consistently differentiate between developed and developing countries, requiring a much greater effort to the first ones. The regulator may decide to multiply each of the objective functions for a weight w_i , so as to obtain a form of discount for player i 's cost function. The idea is that through this process developing nations can be identified as responsible for just a portion of their whole emissions; the sum discounted corresponds to some kind of inevitable cost for the environment, a price to be paid in order to assure them the development that they legitimately require. Therefore, coefficient α may be seen as the result of the following

$$\alpha = \frac{w_1}{w_1 + w_2}$$

The more w_i gets closer to value 1, the more the environmental damages caused by player i are fully accounted; on the other hand, when $w_i \mapsto 0$ player i 's emission costs tend to zero

as well.

Note that (6.18) and (6.19) equal the conditions of the previous case (6.3) and (6.4).

The two participants in this cooperative game try to find the respective functions $e_{1\alpha}$ and $e_{2\alpha}$ that minimise the objective function. We therefore name $J_1(\alpha)$ and $J_2(\alpha)$ the values of the functionals corresponding to I_1 and I_2 when the emission levels are equal to $e_{1\alpha}$ and $e_{2\alpha}$. The problem must be solved for every α , supposing that the two cost functions are both quadratic

$$C_i(e_{i\alpha}(t)) = \frac{1}{2}c_i(e_{i\alpha}(t) - e_i^m)^2 \quad (6.20)$$

$$D_i(Q_\alpha(t)) = d_i[Q_\alpha(t)]^2 \quad (6.21)$$

where c_i and d_i ($i = 1, 2$) are two positive parameters representing the effects associated to emission levels and to the atmospheric concentration of CO_2 . They are different for each of the two players, since our assumption was that they were not involved in the same way in the global warming issue. The constants e_i^m for $i = 1, 2$ represent the different emission quotas associated with each party. We presume that they were the result of the common agreement from which the cooperative game originated.

The Hamiltonian function of the problem is the following one

$$H(e_{1\alpha}, e_{2\alpha}, \lambda_\alpha, Q_\alpha) = e^{-\rho t} \left[\frac{\alpha}{2} [c_1(e_{1\alpha} - e_1^m)^2 + d_1 Q_\alpha^2] + \frac{1-\alpha}{2} [c_2(e_{2\alpha} - e_2^m)^2 + d_2 Q_\alpha^2] \right] + \lambda_\alpha [\sigma(e_{1\alpha} + e_{2\alpha}) - \beta Q_\alpha] \quad (6.22)$$

Four variables are then introduced: $e_{1\alpha}$ and $e_{2\alpha}$ are the control variables, while $Q_\alpha(t)$ represents the state variable. From **Chapter 5**, we recall that the necessary conditions in order to constitute the Maximum Principle require the introduction of a dummy variable, similar to a Lagrange multiplier. It is also known as costate variable, and it is indicated through $\lambda_\alpha(t)$. The necessary and sufficient conditions for (6.22) are represented below

$$\frac{\partial \lambda_\alpha}{\partial t} = -[e^{-\rho t}(\alpha d_1 Q_\alpha(t) + (1 - \alpha)d_2 Q_\alpha(t)) - \beta \lambda_\alpha(t)] \quad (6.23)$$

$$\frac{\partial H}{\partial e_{1\alpha}} = 0 \quad (6.24)$$

$$\frac{\partial H}{\partial e_{2\alpha}} = 0 \quad (6.25)$$

$$\frac{\partial Q_\alpha}{\partial t} = \sigma(e_{1\alpha}(t) + e_{2\alpha}(t)) - \beta Q_\alpha(t) \quad (6.26)$$

$$Q_\alpha(0) = 0 \quad (6.27)$$

$$\lambda_\alpha(T) = 0 \quad (6.28)$$

From above it can be inferred that the solutions of the problem we are analysing are

$$e_{1\alpha}(t) = e_1^m - \frac{\sigma}{\alpha c_1} \lambda_\alpha(t) e^{\rho t} \quad (6.29)$$

$$e_{2\alpha}(t) = e_2^m - \frac{\sigma}{(1 - \alpha)c_2} \lambda_\alpha(t) e^{\rho t} \quad (6.30)$$

Therefore, the functions determining the evolution of the state variable $Q_\alpha(t)$ and the costate variable $\lambda_\alpha(t)$ must be solutions to the system constituted by the following two linear differential equations

$$\frac{\partial \lambda_\alpha}{\partial t} = -e^{-\rho t}(\alpha d_1 Q_\alpha(t) + (1 - \alpha)d_2 Q_\alpha(t)) + \beta \lambda_\alpha(t) \quad (6.31)$$

$$\frac{\partial Q_\alpha}{\partial t} = \sigma(e_1^m + e_2^m) - \sigma^2 e^{\rho t} \lambda_\alpha(t) \left(\frac{1}{\alpha c_1} + \frac{1}{(1 - \alpha)c_2} \right) - \beta Q_\alpha(t) \quad (6.32)$$

If we set $Z_\alpha(t) = e^{\rho t} \lambda_\alpha(t)$, the previous system may be transformed into one with constant coefficients in the variables Z_α , Q_α , to whom is associated the following matrix

$$A(\alpha) = \begin{pmatrix} \beta + \rho & -(\alpha d_1 + (1 - \alpha)d_2) \\ -\sigma^2 \left(\frac{1}{\alpha c_1} + \frac{1}{(1 - \alpha)c_2} \right) & -\beta \end{pmatrix} \quad (6.33)$$

In order to determine the singular values of the matrix $A(\alpha)$ we must first briefly introduce the **singular value decomposition**, based on the following theorem:

Theorem 6.1 Let $A \in \mathbb{R}^{m \times n}$, therefore a matrix $U \in O(m)$ and a matrix $V \in O(n)$ exist such that

$$U^T A V = \Sigma, \quad i.e. \quad A = U \Sigma V^T$$

where the diagonal element $\Sigma \in \mathbb{R}^{m \times n}$ has the following elements

$$\begin{aligned} \sigma_{ij} &= 0 \quad \text{if } i \neq j \\ \sigma_{ij} &= \sigma_i \quad \text{if } i = j \end{aligned}$$

with

$$\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_r > \sigma_{r+1} = \dots = \sigma_p = 0, \quad p = \min\{m, n\}$$

Columns u_1, \dots, u_m of U are called left-singular vectors of A ; they are eigenvectors of AA^T . Columns v_1, \dots, v_n are called right-singular vectors of A ; they are eigenvectors of $A^T A$. Real numbers $\sigma_1, \dots, \sigma_p$ are known as singular values of A . They correspond to the square roots of the eigenvalues λ_j of $A^T A$

$$\sigma_j = \sqrt{\lambda_j(AA^T)} = \sqrt{\lambda_j(A^T A)}$$

The singular values are univocally determined.

Therefore, the singular values of the matrix $A(\alpha)$ are given by the expression

$$V_i(\alpha) = \frac{1}{2} \left(\rho \pm \sqrt{\rho^2 + 4\sigma^2 \left(\frac{1}{\alpha c_1} + \frac{1}{(1-\alpha)c_2} \right)} \right) (\alpha d_1 + (1-\alpha)d_2) + 4(\beta + \rho)\beta \quad (6.34)$$

By solving it, we obtain two different values: a positive and a negative one. Hence, we can establish that the existing point of equilibrium -represented by (Z_α^*, Q_α^*) - is a saddle point: in other words, here the matrix is undefined.

Let $v_i(\alpha)$ be the singular vector associated to the eigenvalue $V_i(\alpha)$, such that

$$v_i(\alpha) = \left(\frac{\alpha d_1 + (1-\alpha)d_2}{\beta + \rho - V_i(\alpha)} \right) = (v_{1i}(\alpha), v_{2i}(\alpha)) \quad (6.35)$$

The general solution to the system requires, among other things, the determination of Z_α^* and Q_α^* . Recalling our previous considerations about (6.32) and (6.33), we know that

$$Z_\alpha^* = \frac{(\alpha d_1 + (1 - \alpha)d_2)Q_\alpha^*}{\beta + \rho} \quad (6.36)$$

$$Q_\alpha^* = \left(\sigma(e_1^m + e_2^m) - \sigma^2 \left(\frac{1}{\alpha c_1} + \frac{1}{(1 - \alpha)c_2} \right) Z_\alpha^* \right) / \beta \quad (6.37)$$

Once set $A = (\alpha d_1 + (1 - \alpha)d_2)$ and $B = \left(\frac{1}{\alpha c_1} + \frac{1}{(1 - \alpha)c_2} \right)$ we can determine the solution to (6.36) and (6.37)

$$Z_\alpha^* = \frac{A\sigma(e_1^m + e_2^m)}{\beta(\beta + \rho) + AB\sigma^2} \quad (6.38)$$

$$Q_\alpha^* = \left(\sigma(e_1^m + e_2^m) - \sigma^2 B \frac{A\sigma(e_1^m + e_2^m)}{\beta(\beta + \rho) + AB\sigma^2} \right) / \beta \quad (6.39)$$

Once we have established Z_α^* , Q_α^* , the singular values, the singular vectors and the equilibrium, we find the general solution to the system

$$Z_\alpha(t) = Z_\alpha^* + K_1(\alpha)v_{11}(\alpha)e^{V_1(\alpha)t} + K_2(\alpha)v_{12}(\alpha)e^{V_2(\alpha)t} \quad (6.40)$$

$$Q_\alpha(t) = Q_\alpha^* + K_1(\alpha)v_{21}(\alpha)e^{V_1(\alpha)t} + K_2(\alpha)v_{22}(\alpha)e^{V_2(\alpha)t} \quad (6.41)$$

We can determine the values associated to the constants $K_1(\alpha)$ and $K_2(\alpha)$ by recalling that $Q_\alpha(0) = 0$ and that $\lambda_\alpha(T) = 0 = Z_\alpha(T)$. Therefore

$$K_1(\alpha) = \frac{Z_\alpha^* - Q_\alpha^*v_{12}(\alpha)e^{V_2(\alpha)T}}{v_{12}(\alpha)e^{V_2(\alpha)T} - v_{11}(\alpha)e^{V_1(\alpha)T}} \quad (6.42)$$

$$K_2(\alpha) = \frac{Q_\alpha^*v_{11}(\alpha)e^{V_1(\alpha)T} - Z_\alpha^*}{v_{12}(\alpha)e^{V_2(\alpha)T} - v_{11}(\alpha)e^{V_1(\alpha)T}} \quad (6.43)$$

Once we have determined the state and costate functions, we can proceed with the corresponding functionals

$$e_{1\alpha}(t) - e_1^m = \frac{\sigma}{\alpha c_1} \lambda_\alpha(t) e^{\rho t} = \frac{\sigma}{\alpha c_1} Z_\alpha(t) \quad (6.44)$$

$$e_{2\alpha}(t) - e_2^m = \frac{\sigma}{(1-\alpha)c_2} \lambda_\alpha(t) e^{\rho t} = \frac{\sigma}{(1-\alpha)c_2} Z_\alpha(t) \quad (6.45)$$

Finally, we can deduce that

$$J_1(\alpha) = \frac{1}{2} \int_0^T e^{-\rho t} \left[\frac{\sigma^2}{\alpha^2 c_1^2} (Z_\alpha(t))^2 + d_1 (Q_\alpha(t))^2 \right] dt \quad (6.46)$$

$$J_2(\alpha) = \frac{1}{2} \int_0^T e^{-\rho t} \left[\frac{\sigma^2}{(1-\alpha)^2 c_2^2} (Z_\alpha(t))^2 + d_2 (Q_\alpha(t))^2 \right] dt \quad (6.47)$$

Points $(J_1(\alpha), J_2(\alpha))$ with $\alpha \in [0, 1]$ are the ones that constitute the efficient frontier.

6.3 Determining α : a regulator's choice

From the very introduction of the cooperative solution we have underlined how important the role played by coefficient α is. The discounting measures for the environmental costs of each of the two coalitions of countries are associated to the values that it can assume. All the international environmental treaties, from Rio de Janeiro (1992) to Paris (2015), recognise as consistent the efforts required from the already developed countries, while often developing and least developed economies are request to contribute on the basis of their limited possibility, or not to contribute at all. In both cases, there are no mandatory prescriptions for them, according to the idea that while Annex II Parties have already earned their richness (at the environment's expenses), to limitate the chances for the other nations to achieve an equal level of development would not be fair. As a matter of fact, it was never considered as an option by the UNFCCC. On the contrary, wealthier states are obliged to perform in two different ways: they should first reduce their own emission levels, and then provide to the rest of the world the means in order to reach a sustainable economic development.

So, when it is the regulator's duty to determine the level of discount for block I_1 and the correspondent one for block I_2 , a choice must be made by using the values included in the range $[0, 1]$. We start by observing the final extrem; when

$$\lim_{\alpha \rightarrow 1} J_1(\alpha) = 0 \quad (6.48)$$

the second block experiences

$$\lim_{\alpha \rightarrow 1} J_2(\alpha) = +\infty \quad (6.49)$$

This means that if the regulator chooses to punish I_1 (with respect to I_2) by making him fully responsible for his emissions, the only choice for the player is to completely stop his production processes responsible for GHG pollution. On the contrary, since from $\alpha = 1$ all the countries reunited under I_2 experience $(\alpha - 1) = 0$, and the second player will be able to maximise his own production. As a matter of fact, it must be reminded that when $J_1 = 0$ the profits associated with those countries' productions are equal to zero as well ($\pi_1 = 0$).

This implies (and was however predictable) that a possible cost decrease for I_1 can be only achieved at the expenses of I_2 , that would see his costs rising.

We have a similar situation to the one previously described for $\alpha \mapsto 0$, since

$$\lim_{\alpha \rightarrow 0} J_1(\alpha) = +\infty \quad (6.50)$$

and

$$\lim_{\alpha \rightarrow 0} J_2(\alpha) = 0 \quad (6.51)$$

This time, countries belonging to the first block are the ones pardoned by the regulator.

6.4 Countries with different bargaining powers

With respect to what we have previously observed, it should be remarked that international environmental treaties are not drafted by some sort of independent, sovranational committee. In **Chapter 4** we have largely mentioned the voluntary nature of such agreements, as well as the fact that COPs find their origin in the UNFCCC, a convention signed by 196 countries. Therefore, at a global level we lack the presence of a truly stand-alone regulator, and it is more likely that nations themselves negotiate to reach a commonly agreed-upon solution. Considering this, this section deals with the different bargaining powers associated to each of the block of states. Once again we shall consider two players, for the sake of simplicity.

Section 6.2 ended with the identification of two points, $J_1(\alpha)$ and $J_2(\alpha)$, that constitute the efficient frontier. We now describe in general how bargaining powers associated with players can determine the solution to the problem, by considering four different approaches.

The starting point of our analysis is the definition of a negotiation problem: we consider

(X, n) , with $X \subseteq \mathbb{R}^2$ representing the set of achievable agreements and with n being a point of X known as the disagreement point. It depicts what would happen if a scenario in which the players were not able to find a common solution took place.

Supposing that the axiom of rational behavior holds, players want to maximise their expected benefits. Additionally, we suppose that set X is closed and convex. Knowing that a rational player will not accept an agreement whose associated benefits are lower than the ones resulting from the situation represented by the disagreement point, we can consider only the scenarios where $x \geq n$. They are represented by set F .

The first methodology explained could not have been but NASH's (1950), which stated that there exist a unique solution, provided that certain axioms are satisfied:

1. **symmetry** -meaning that the players have the same bargaining power;
2. **strong efficiency** -implying that the solution belongs to the efficient frontier;
3. **individual rationality** -since no player will accept an outcome worse than the one associated with a failure in the agreement;
4. **scale covariance** -stating that the outcome for the parties is independent of the way utility is measured;
5. **independence of irrelevant alternatives** -which signifies that a reasonable outcome that is still achievable after some allocations are removed remains a reasonable outcome. Or, in other terms, let G be the bargaining game with payoff space X and disagreement point n , and let \bar{x} be the solution of the game. If we denote by G^* the game obtained from G by restricting X to $Q \subset X$ such that $n \in Q$ and $\bar{x} \in Q$, then \bar{x} is also the solution of G^* .

From this work, the following methods were developed:

- HARSANYI and SELTEN (1972) criticised Nash for the use of symmetry, the first one of the axioms leading to a Nash solution on our list. They introduced the less strong asymmetric Nash solution, which permits to consider different levels of bargaining power associated to the players of the game, and therefore is more adherent to the empirical evidence.

- KALAI and SMORODINSKY (1975) on the contrary concentrated on the last axiom, independence of irrelevant alternatives, substituting it with a monotonicity condition. According to it, with (U, \underline{u}) being a general Nash bargaining problem and B the set of all bargaining problems, a bargaining solution $\bar{x} \in (\mathbb{R}^n)^B$ is monotone if and only if for all $(U, \underline{u}), (U', \underline{u}') \in B$ such that $\underline{u} = \underline{u}', U \subset U'$ and for some $i = 1, \dots, n$ $\max \pi_i(U) = \max \pi_i(U')$, it holds that for all $j \neq i$

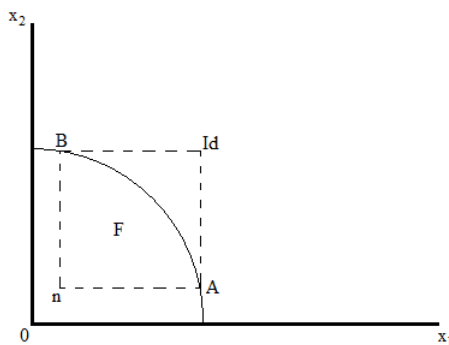
$$x_j(U, \underline{u}) \leq x_j(U', \underline{u}')$$

A few years later, ANBARCI (1993) was responsible for the introduction of the related area monotonic solution.

- Finally, CHUNG (1988) was responsible for the equal-loss solution, whose aim is to seek some sort of equality in bargaining situations. Parties equalise their respective losses with regards to the *best case scenario*, represented by an ideal point I_d placed out of the efficient frontier.

We now suppose to have a parametric representation of the efficient frontier, where the functions that are responsible for such parametrisation are strictly monotonic and continuous.

Hence, let $(x_1(t), x_2(t))$ be a parametrisation of the efficient frontier, with $t \in I$ and I being an interval of R . Recalling that n represents the disagreement point, the achievable agreements' area is circumscribed by the coordinates of such point; this situation is depicted in Graph 6.1.



Graph 6.1: Representation of the achievable agreements

If we look at the efficient frontier, the set of solutions is restricted to the curve's points included between A and B. Set F is represented in **Graph 6.1** as well.

Points A and B can be obtained through two values t_1, t_2 such that $A = (x_1(t_1), n_2)$ and $B = (n_1, x_2(t_2))$. The continuity and the monotonicity of the parametrisation guarantee the existence of these values.

Let us suppose that we are dealing with a scenario where $t_1 < t_2$ and let $J = [t_1, t_2]$. The Nash bargaining solution is represented by the only point belonging to the Pareto frontier which maximises the Nash product $-(x_1 - n_1)(x_2 - n_2)$ - which is the difference between achievable agreements and the value associated with the disagreement.

Therefore, the Nash solution is the solution to the following problem

$$\max_{x \in F} (x_1 - n_1)(x_2 - n_2) \quad (6.52)$$

If a different bargaining power is associated to the players (represented by $b_1, b_2 \in \mathbb{R}$, with $b_1 + b_2 = 1$) we can consider the generalised Nash solution x_N (HARSANYI e SELTEN 1972), that is the point solving the following problem

$$\max_{x \in F} (x_1 - n_1)^{b_1} (x_2 - n_2)^{b_2} \quad (6.53)$$

In other words, the previous solution (6.52) is a particular case of the one (6.53) that we have just found, and it occurs only when $b_1 = b_2$, that is when the bargaining powers are identical.

Since we have considered the efficient frontier through a parametric form, the problem that we have to solve can be set like the following one, concerning the maximisation of a variable

$$\max_{t \in J} (x_1(t) - n_1)^{b_1} (x_2(t) - n_2)^{b_2} \quad (6.54)$$

According to Nash, point n becomes the reference with respect to which the solution has to be calculated. On the contrary, according to KALAI e SMORODINSKY (1975, p. 513-518), we have to take into account two points, the disagreement point n and the ideal point $Id = (Id_1, Id_2)$. The latter is out of reach for both players, but represents the maximum benefits which they can desire. We can find this solution by looking at the point belonging to the efficient frontier and also located on the straight line that unites the disagreement point to the ideal one. It is the same that satisfies the following equation

$$x_2 - n_2 = \frac{n_2 - Id_2}{n_1 - Id_1}(x_1 - d_1) \tag{6.55}$$

If the bargaining powers belonging to each party are not the same, we can determine the generalised Kalai-Smorodinsky solution, and therefore consider the solution resulting from the intersection of the efficient frontier with the straight line

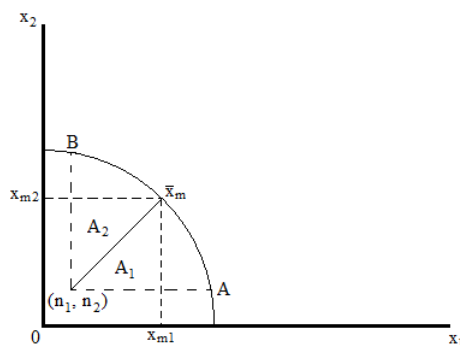
$$x_2 - n_2 = \frac{b_2}{b_1}(x_1 - n_1) \tag{6.56}$$

Differently from what we have previously observed, the new solution does not coincide with the generalised Nash one for the case $b_1 = b_2$. Such a coincidence only exists when the slope of the straight line uniting the ideal point I_d to the disagreement point n equals 1.

We consider exactly this case.

Recalling the parametric representation of the frontier, we are seeking the value of $t \in J$ such that

$$x_2(t) - n_2 = \frac{b_2}{b_1}(x_1(t) - n_1) \tag{6.57}$$



Graph 6.2: Area monotonic solution

Solution $\bar{x}_m = (x_{m1}, x_{m2})$ represents the only point belonging to the efficient frontier such that the closed line segment which is bounded by n and \bar{x}_m divides F in two equal parts, A_1 e A_2 (Graph 6.2). When we have two different bargaining powers (and therefore an asymmetric conflict), we consider the solution thanks to which the relation between the two areas is $\frac{b_1}{b_2}$, that is $A_1 w_1 = A_2 w_2$. Once again, this case coincides with the generalised solution when the bargaining powers are equal, and $\frac{b_1}{b_2} = 1$.

Let S_1 be the area limited by the efficient frontier between x_m and $A = (x_1(t_1), n_2)$, axis x and the straight lines $x_1 = x_{m1}$ and $x_1 = x_1(t_1)$. We will have that

$$A_1 = \frac{1}{2}(x_{m2} - n_2)(x_{m1} - n_1) + S_1 - n_2(x_1(t_1) - x_{m1}) \quad (6.58)$$

Again, let S_2 be the area limited by the efficient frontier between $B = (n_1, x_2(t_2))$ and x_m , axis x and the straight lines $x_1 = n_1$ and $x_1 = x_{m1}$. In this case

$$A_2 = S_2 - \frac{x_{m2} + n_2}{2}(x_{m1} - n_1) \quad (6.59)$$

Finally, we analyse the equal-loss solution (CHUNG 1988), which aims to determine the point belonging to the efficient frontier where the two players experience the same loss in benefits, with regards to their respective ideal scenario (Id_i , $i = 1, 2$). Following the same line of thoughts we have previously expressed, we may consider different bargaining powers for the parties, and deduce that the relation between such powers is reflected in the one between their losses. The equal-loss solution is the point belonging to the efficient frontier such that

$$(x_1 - Id_1)b_1 = (x_2 - Id_2)b_2 \quad (6.60)$$

Bibliography

- [1] ANBARCI N., 1993. *Noncooperative foundations of the area monotonic solution*. Quarterly Journal of Economics, vol. 108.
- [2] BERNSTEIN L., *et al.*, 2008. *IPCC AR4 SYR Appendix Glossary* [online]. Available at: <<http://www.ipcc.ch>>.
- [3] BURATTO A., *et al.*, 2014. *Ottimizzazione dinamica. Modelli economici e gestionali*. 2nd edition. Padova: Edizioni Libreria Progetto.
- [4] CALABRESI G., 1968. *Transaction costs, resource allocation and liability rules: a comment*. 11 Journal of Law and Economics, 67-73.
- [5] CHEN T., 2015. *China's red alert on air pollution puts focus on regulators*. The Wall Street Journal, December, 7th.
- [6] CHUN H., 2014. *Game theory, rational choice theory, and the prisoner's dilemma: some clarifications* [online]. Available at: <<http://www.academia.edu>>.
- [7] CHURCH J., *et al.*, 2013. *Sea level change*. In: Climate change 2013: the physical science basis. Contribution of working group I to the fifth assessment report of the Intergovernmental Panel on Climate Change. Cambridge: University Press.
- [8] COMMITTEE ON CLIMATE CHANGE, 2015. *The scientific and international context for the fifth carbon budget* [online]. Available at: <<http://www.theccc.org.uk>>.
- [9] DIXIT A., *et al.*, 2009. *Games of strategy*. 3rd edition. New York: W. W. Norton & Company.
- [10] DOCKNER E., *et al.*, 2000. *Differential games in economics and management science*. Cambridge: University Press.

- [11] EUROPEAN COMMISSION, 2015. *2015 EU budget at a glance*. Luxembourg: Publications Office of the European Union.
- [12] FAURE M., *et al.*, edited by, 2003. *Climate change and the Kyoto protocol: the role of institutions and instruments to control global change*. Cheltenham: Edward Elgar Publishing.
- [13] GRUNEWALD N., MARTÍNEZ-ZARZOSO I., 2011. *Carbon dioxide emissions, economic growth and the impact of the Kyoto protocol*. In: KIM Y., TANAKA K., MATSUOKA S., 2014. *Environmental and economic consequences of the Kyoto protocol* [online]. Available at: <www.econ.shiga-u.ac.jp>.
- [14] HAHN R., NOLL R., 1990. *Environmental markets in the year 2000* [online]. *Journal of Risk and Uncertainty*, 3 (3). Available at: <<http://www.academia.edu>>.
- [15] HANLEY N., *et al.*, 2007. *Environmental economics in theory and practice*. 2nd edition. London: Palgrave Macmillan.
- [16] HERRERO C., MARCO C., 1992. *A note on the equal-loss principle for bargaining problems* [online]. Available at: <<http://www.ivie.es>>.
- [17] HERRIGES J., *et al.*, 1992. *Budget balancing incentive mechanisms* [online]. Available at: <<http://www.researchgate.net>>.
- [18] HOLMSTRÖM B., 1982. *Moral hazard in teams* [online]. *The Bell Journal of Economics*, vol. 13 (2). Available at: <<http://www.faculty.fuqua.duke.edu>>.
- [19] JAEGER C., JAEGER J., 2010. *Three views of two degrees* [online]. Available at: <<http://www.pik-potsdam.de>>.
- [20] JULI O., 2012. *Canada withdraws from Kyoto protocol to avoid non-compliance penalties* [online]. Ottawa: the Canadian Bar Association newsletter. Available at: <<http://www.cba.org>>.
- [21] KAITALA V., POHJOLA M., 1995. *Sustainable international agreements on greenhouse warming: a game theory study*. In: CARRARO C., FILAR J. A., edited by, 1995. *Control and game-theoretic models of the environment*. Boston: Birkhäuser.
- [22] KALAI E., SMORODINSKY M., 1975. *Other solutions to Nash's bargaining problem* [online]. Available at: <<http://www.jstor.org>>.

- [23] KUMAZAWA R., CALLAGHAN, M., 2012. *The effect of the Kyoto protocol on carbon dioxide emissions*. *Journal of Economics and Finance* , 36 (1), 201–210.
- [24] LE TREUT H., *et al.*, 2007. *Historical overview of climate change*. In: *Climate change 2007: the physical science basis. Contribution of working group I to the fourth assessment report of the intergovernmental panel on climate change*. Cambridge: University Press.
- [25] LEWIS T., SAPPINGTON D., 1993. *Using markets to allocate pollution permits and other scarce resource rights under limited information* [online]. Available at: <<http://www.warrington.ufl.edu>>.
- [26] MCGRATH M., 2015. *China pollution: first ever red alert in effect in Beijing* [online]. BBC news, December, 7th. Available at: <<http://www.bbc.com>>.
- [27] MASKIN E., TIROLE J., 2001. *Markov perfect equilibrium: I. Observable actions*. *Journal of Economic Theory*, 100, 191-219.
- [28] MURRAY B., *et al.*, 2009. *Balancing cost and emissions certainty: an allowance reserve for cap-and-trade*. *Review of Environmental Economics and Policy*, vol. 3 (1). Oxford: University Press.
- [29] MYERSON R., 1997. *Game theory: analysis of conflict*. Reprint edition. Harvard: University Press.
- [30] NORDHAUS W., 1991. *To slow or not to slow: the economics of the greenhouse effect*. *The Economic Journal*, 101, p. 920-937. In: CARRARO C., FILAR J. A., edited by, 1995. *Control and game-theoretic models of the environment*. Boston: Birkhäuser.
- [31] OECD, 1997. *Glossary of environment statistics. Studies in method* [online]. Series F (67). United Nations, New York. Available at: <<http://www.stats.oecd.org>>.
- [32] POTTER R., *et al.*, 2003. *Geographies of development*. 2nd edition. New Jersey: Pearson Prentice Hall.
- [33] ROSSINI L., 2011. *Giochi differenziali con sistemi di incentivi*. Bachelor's degree thesis, Università degli Studi di Padova.
- [34] SOLLUND S., 2007. *Environment taxes* [online]. IFAD Meeting of UN group of experts on domestic resource mobilisation. Rome, 4-5/09/2007. Available at: <<http://www.un.org>>.

- [35] SPENGLER J., SEXTON K., 1983. *Indoor air pollution: a public health perspective* [online]. Science, vol. 221, issue 4605. Available at: <<http://www.ncbi.nlm.nih.gov>>.
- [36] TEGART W., *et al.*, edited by, 1990. *Climate change: the IPCC impacts assessment*. Canberra: Australian Government Publishing Service.
- [37] TSCHAKERT P., 2015. *1.5°C or 2°C: a conduit's view from the science-policy interface at COP20 in Lima, Peru* [online]. Available at: <<http://www.climatechangeresponses.biomedcentral.com>>.
- [38] UNITED NATIONS, 1989. *Resolution 44/228* [online]. Available at: <<http://www.un.org>>.
- [39] UNITED NATIONS, 1992. *Framework convention on climate change* [online]. Available at: <<http://www.unfccc.int>>.
- [40] UNITED NATIONS, 1992. *Rio declaration on environment and development* [online]. Available at: <<http://www.un.org>>.
- [41] UNITED NATIONS, 1998. *Kyoto protocol to the United Nations framework convention on climate change* [online]. Available at: <<http://www.un.org>>.
- [42] UNITED NATIONS, 2005. *Decision 27/CMP.1: procedures and mechanisms relating to compliance under the Kyoto protocol* [online]. Available at: <<http://www.unfccc.int>>.
- [43] UNITED NATIONS, 2009. *Copenhagen accord* [online]. Available at: <<http://www.unfccc.int>>.
- [44] UNITED NATIONS, 2010. *Cancon agreement* [online]. Available at: <<http://www.unfccc.int>>.
- [45] UNITED NATIONS, 2012. *Doha amendment to the Kyoto protocol* [online]. Available at: <<http://www.unfccc.int>>.
- [46] UNITED NATIONS, 2015. *Final draft of the Paris agreement* [online]. Available at: <<http://www.cop21.gouv.fr>>.
- [47] VALSECCHI C., *et al.*, 2009. *Environmentally harmful subsidies: identification and assessment*. Final report for the European Commission's DG Environment.

- [48] VISCOLANI B., 2003. *Controllo ottimo e modelli per l'economia*. Padova: Edizioni Libreria Progetto. In: ROSSINI L., 2011. *Giochi differenziali con sistemi di incentivi*. Bachelor's degree thesis, Università degli Studi di Padova..
- [49] WEITZMAN M., 1974. *Prices vs. quantities* [online]. Available at: <<http://www.scholar.harvard.edu>>.
- [50] XEPAPADEAS A., 1991. *Environmental policy under imperfect information: incentives and moral hazard* [online]. Available at: <<http://www.faculty.ses.wsu.edu>>.
- [51] YARDLEY W., 2015. *Climate talks updates: final draft revealed at United Nations summit* [online]. Los Angeles Times, December, 12nd. Available at: <<http://www.latimes.com>>.