

# UNIVERSITÀ DEGLI STUDI DI PADOVA

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## THE STRING LANDSCAPE AND THE SWAMPLAND. THE WEAK GRAVITY CONJECTURE AND SCALAR FIELDS.

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# 1 Introduction

Our Universe is governed by few fundamental forces: the strong interaction, the weak interaction, the electromagnetic interaction, the Higgs-mediated interaction and the gravitational interaction.

The first four forces are described by a Quantum Field Theory called the Standard Model (SM) and extensions of it.

A Quantum Field Theory (QFT) lagrangian is determined by the invariance group  $G$  of the theory; by the spectrum of spin-0, spin- $\frac{1}{2}$  and spin-1 particles composing the matter content of the model with their representations with respect to  $G$  and by the interactions respecting the symmetry  $G$ . In particular [1], the Standard Model is based on the symmetry group

$$G = SU(3)_{color} \times SU(2)_{weak} \times U(1)_{hypercharge},$$

where the factor  $SU(3)_{color}$  refers to the strong interaction and the sector  $SU(2)_{weak} \times U(1)_{hypercharge}$  takes into account the unified weak and electromagnetic interactions. The matter content of the theory is composed by the Higgs scalar field ( $\Phi$ ); by three similarly organized generations of fermionic particles (quarks and leptons), which are

$$q_1 = \begin{pmatrix} u \\ d \end{pmatrix} \quad l_1 = \begin{pmatrix} e \\ \nu_e \end{pmatrix}; \quad q_2 = \begin{pmatrix} c \\ s \end{pmatrix} \quad l_2 = \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix}; \quad q_3 = \begin{pmatrix} t \\ b \end{pmatrix} \quad l_3 = \begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix}$$

and by the gluons, the  $W^\pm$  and the  $Z^0$  bosons and the photon. By means of the Higgs mechanism, the Higgs field lets (almost all) these matter particles to gain a mass. The Standard Model is requested to be a renormalizable theory: its lagrangian density  $\mathcal{L}_{SM}$ , whose schematic structure is

$$\mathcal{L}_{SM} = \sum_i c_i O_i$$

(for the operators  $\{O_i\}_i$  with associated coefficients  $\{c_i\}_i$ ), has to be such that the mass dimensions of  $O_i$  and  $c_i$  (for any  $i$ ) have to satisfy

$$[O_i] \leq 4; \quad [c_i] \geq 0.$$

Once all these ingredients are taken under consideration, the lagrangian (or the action) of the Standard Model is entirely determined and its study can be pushed forward.

The gravitational interaction is described by General Relativity, instead. The crucial idea General Relativity is based upon is that gravity is a manifestation of the spacetime geometry. As a consequence, it does not influence the motion of a body as all the other interactions do: a body that feels the gravitational interaction moves freely in a deformed spacetime. This brilliant intuition that Einstein was able to make evident substantiates in the so called *Equivalence Principle*.

The geometric nature of gravity and its relation with the substance constituents of our Universe is then expressed by the action

$$S_{GR} = \int d^4x \sqrt{-g} \left[ -\frac{M_P^2}{2} R + \mathcal{L}_{rest\ of\ the\ world} \right],$$

where  $M_P$  is the (reduced) Planck mass;  $g_{\mu\nu}$  is the spacetime metric;  $R$  is the corresponding Ricci scalar and  $\mathcal{L}_{rest\ of\ the\ world}$  is the lagrangian density grouping the contributions from all the components of Universe but the gravitational field. By making advantage of  $S_{GR}$  the famous Einstein's equations can be deduced.

General Relativity is amazingly well tested by experiments [2–5]. For instance, Eat–Wash torsion balance experiment is able to test the Weak Equivalence Principle with a precision of  $\mathcal{O}(10^{13})$ . Some experiments involving an appropriately modified version of Dirac equation (to take into account of possible violations of local Lorentz invariance) and based on measures of nuclear energy levels allow to confirm the validity of the Einstein Equivalence Principle with  $\mathcal{O}(10^{29})$  of precision; some measurements of the gravitational red-shift effect performed with gravitational clocks test the Einstein Equivalence Principle with a precision of  $\mathcal{O}(10^4)$ . By studying the system Earth-Moon in the gravitational field of the Sun the Laser Lunar Ranging Experiment has managed to confirm the Strong Equivalence Principle with a precision level of  $\mathcal{O}(10^{13})$ .

General Relativity is a classical (in the sense of non-quantum) theory. However, being convinced that gravity has to be a fundamental force governing our Universe, for consistency with the other fundamental interactions, gravity has to be quantic too and the way to quantize it has to be found.

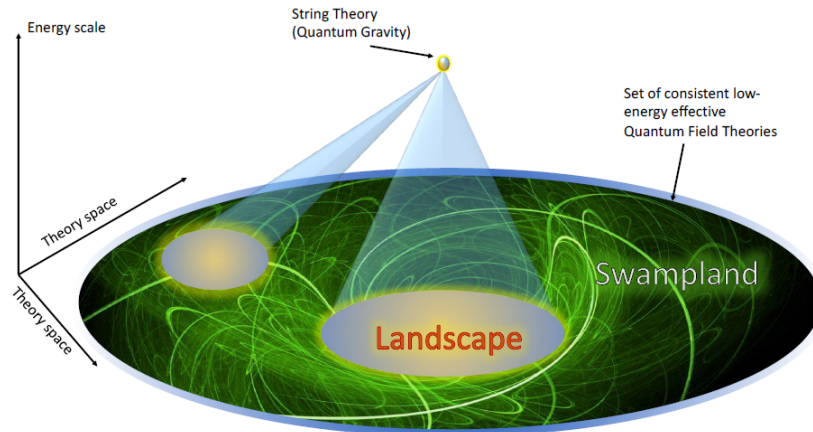
Nowadays, String Theory is the only consistent model we can refer to and we can make calculations and formulate predictions with in the attempt of getting some information on the construction of a theory of quantum gravity.

One of the main ideas String Theory is based on is that the fundamental entities composing our Universe are extended (rather than point-like) objects. When supersymmetry (which is basically a symmetry that exchanges bosonic and fermionic degrees of freedom) enters the game, in order to preserve Lorentz invariance, String Theory becomes consistent in ten (or eleven) dimensions. Since we are sensitive to four dimensions only, there emerge six (or seven) extra dimensions. They are associated to a manifold (denoted as internal manifold) and have to be "small" enough so that they do not apparently affect any experimental result. This is in substance the idea lying behind the compactification procedure. Because of the enormously rich variety of internal manifolds allowed by String Theory the extra dimensions can be referred to and the influence that the properties of the internal manifold actually exert on the four observable dimensions, an infinite number of potential 4-dimensional universes is obtained. Once fluxes are turned on in the compactification, this number reduces to  $\mathcal{O}(10^{600})$ .

Animated by the belief that String Theory should be predictive regardless the previous estimate, the question on how our Universe can be recognized and selected in this "jungle" of potential universes spontaneously arises. Thinking that there is a way Nature has made such a choice by and being interested in uncovering this mechanism, C. Vafa has introduced the distinction between the so called *Landscape* and *Swampland* in String Theory [6].

The string Landscape can be defined as the set of those effective QFTs that admit a high-energy completion in String Theory. Still, because of the richness of choices for the geometry of the internal space, studying the Landscape by means of the compactification technique is hard. One can then be led to the construction of consistent-looking 4-dimensional theories and to the deduction of the relevant 4-dimensional physics they give rise to without caring about their possible origin from a

compactification procedure. From this perspective String Theory would become useless. However, the majority of seemingly consistent 4-dimensional theories can not be deduced as descendants of String Theory. All those effective QFTs that appear consistent but are not completable in String Theory at high energies are said to belong to the string Swampland.



**Figure 1.1** [7]:

The figure schematically shows the set of apparently self-consistent effective QFTs. The subset that can arise from String Theory is called the string Landscape; all the other theories are said to belong to the string Swampland.

The concepts of string Landscape and Swampland can be extrapolated to Quantum Gravity (QG). In this respect the quantum gravitational Landscape is made by all consistent-looking effective QFTs that descend from Quantum Gravity and the quantum gravitational Swampland is composed by all those seemingly consistent effective QFTs that do not admit a completion in Quantum Gravity. Since the proper characteristics of the quantum theory of gravity relevant for our Universe are not known and String Theory is not necessarily such a theory, the QG Landscape and Swampland and the string Landscape and Swampland do not in principle coincide. We do not know how to circumscribe the QG Landscape from the QG Swampland or, either said, we do not know what are the (additional) properties that define a theory consistently accounting for the quantization of gravity and are absent (instead) when gravity doesn't play any role.

The attempt of getting (at least) some line-guide principle to uncover the characteristics of a consistent theory of quantum gravity and the absence of an evident alternative way of proceeding lead to identify, as far as practical purposes only are concerned, the QG Landscape and Swampland with the string Landscape and Swampland. In this framework evidence for some criteria distilling out the Landscape from the Swampland can be gained. These criteria are formulated as conjectural statements and are motivated by examples coming from String Theory (as it could be easily guessed) and by arguments arising from black hole physics.

Among the various Swampland conjectures the present work deals in particular with the so called "Weak Gravity Conjecture" (WGC). It can be phrased as the claim that gravity acts as the weakest force in any circumstance.

In its best known and understood version the WGC states that [7]

A theory coupled to gravity with a  $U(1)$  gauge field should have a state with mass  $m$  and charge  $q$  satisfying

$$m \lesssim gqM_P, \tag{1.1}$$

where  $g$  is the gauge coupling associated to the gauge field in the theory.

The previous inequality guarantees that the gravitational interaction between two identical  $(m, q)$  particles set at a mutual distance  $r$  is beaten in strength by the electromagnetic force that is acting between the two,

$$\frac{m^2}{r^2} \lesssim \frac{q^2}{r^2}$$

(in appropriate units). Coherently with what mentioned above, the WGC (1.1) expresses the weakness of gravity with respect (for instance) to the electromagnetic interaction.

Since in our Universe two scalar fields (the Higgs field and the inflaton) seem to play a crucial role and many scalar fields appear in theories going beyond the Standard Model or characterize supergravity theories (traditionally thought as low-energy descendants of Superstring Theory), finding out a formulation of the WGC when scalar fields are present is a dutiful interesting and challenging task.

There are several versions of the Scalar WGC (SWGC) in the literature (e.g [7, 8]). In April E. Gonzalo and L. Ibàñez published an article [9] where they presented a Strong version of the SWGC (SSWGC), that depends on and constrains the scalar self-interactions described by the scalar potential  $V(\phi)$  of a theory with one scalar field.

The SSWG proposed in [9] is

*The potential  $V(\phi)$  of a canonically normalized real scalar field  $\phi$  in the theory (under consideration) must satisfy for any value of the field the constraint*

$$2 (V''')^2 - V''V'''' \geq \frac{(V'')^2}{M_P^2} \tag{1.2}$$

(with the “ ’ ” denoting the derivative with respect to  $\phi$ ).

Even recognizing that (1.2) could be an incredibly powerful tool potentially constraining the Standard Model and many inflationary models, when studying Gonzalo and Ibàñez’s article, we have noticed that (1.2) appears critical under various points of view. The first criticism consists in the fact that, despite of pretending to be a general statement, Gonzalo and Ibàñez’s bound seems to be valid around a state that minimizes the potential of the theory (or, either said, in the vicinity of a vacuum state). If it wasn’t so, there would be no reason why terms like  $VV''''''$  and  $V'V''''''$  should not appear. Another critical aspect of Gonzalo and Ibàñez’s claim is concerned with the coefficients multiplying the terms appearing in (1.2): the Feynman graph derivation of the conjecture that



the same authors briefly suggest in their article do not justify them. On top of that, even though Gonzalo and Ibàñez’s conjecture should be inspired by the physical principle according to which gravity has to act as the weakest force in any circumstance, in the attempt of deducing (1.2) in an appropriate Quantum Field Theory context we have been able to conclude that the terms Gonzalo and Ibàñez made correspondent to the scalar self-interactions are always beaten in strength by the gravitational interaction term.

The main problem of such a proposal is its generality: the fact that the scalar field is arbitrary and so massive (in principle) obliges the scalar force to decay with the distance (in the configuration space) more rapidly than the gravitational force does.

Being conscious of the criticisms Gonzalo and Ibàñez’s statement exhibits, we would like to get to a general and consistent statement that can replace (1.2). In order to do this we have considered a model including gravity and two scalar fields one of which is strictly massless. The idea is then to compare the strength of the gravitational interaction mediated by the (massless) graviton and the strength of the scalar interaction associated to the massless scalar by regarding the massive scalar field as a probe. The request that gravity has to be the weakest force translates into a bound on the parameters of the theory that we have studied: this constraint is coherent (as we will motivate in detail) with the SWGC that E. Palti presented in [8] and gives support to Palti’s approach to the study of scalar version of the WGC.

More specifically, the content of the present thesis work is organized as follows.

In **Section 2** the notions of Landscape and Swampland are defined by referring with particular care to String Theory and Quantum Gravity; however, it is emphasized there that a Landscape and a Swampland can be introduced also when dealing with any Effective Field Theory (EFT) and its completion in a Quantum Field Theory (QFT) at higher energies.

The abstract concepts of Landscape and Swampland acquire consistency when it is possible to circumscribe the Landscape within the Swampland: **Section 3** is devoted to the description and the analysis of some Swampland criteria. Among the various Swampland conjectures we have decided to present the “No Global Symmetry Conjecture” (stating that any global symmetry characterizing a low-energy theory that is completable in String Theory or Quantum Gravity at high energies has to be gauged or broken); the “Weak Gravity Conjecture” (WGC) (that claims the weakness of gravity with respect to all the other interactions); the “Swampland Distance Conjecture” (SDC) (dealing with the finiteness of the fields’ range in the fields’ space) and the “de Sitter Conjecture” (dSC) (that is concerned with the late-time acceleration phase that our Universe is undergoing and the possible absence of de Sitter vacua in String Theory).

By making advantage of such an overview of some relevant Swampland criteria, in **Section 4** a deepening on the Weak Gravity Conjecture is made. Following E. Palti [8], we first describe a version of the WGC in the presence of multiple gauge and scalar fields and we present a proposal for the WGC in the presence of gravity and scalar fields only. Then, by referring to the latter conjectural statement, we emphasize the relation between the WGC and the SDC: this underlines that the Swampland criteria are interconnected and somehow suggests the research of a general criterion all the others are descendant of. After that, with the attempt of generalizing Palti’s SWGC we discuss

the claim that E. Gonzalo and L. Ibàñez have made in [9]. Finally, we try to propose an alternative to Gonzalo and Ibàñez's statement by studying a theory including gravity and one massive and one strictly massless scalar field. The former plays the role of a probe in testing the strength of the scalar interaction mediated by the latter with respect to the strength of the gravitational interaction. In **Section 5** we present a way (in a particular case) to give scalar charge to a classical particle. It is based on a specific modification (coherent with the models that have been studied in **Section 4**) of the Polyakov action but different with respect to what the Liénard–Wiechert mechanism usually prescribes. Once again, by requiring that gravity acts more weakly than the scalar force, we get a bound supporting Palti's approach to the study of the WGC.

## 2 The Swampland and the Landscape

As already emphasized in the **Introduction**, String Theory is the consistent framework we can refer to to try to get information on the construction of a consistent theory of quantum gravity.

The central idea behind String Theory is that the fundamental entities composing our Universe are extended objects rather than point-like particles. When supersymmetry is taken into account, String Theory works in ten (or eleven) dimensions. As a consequence, in substituting a theory of 4-dimensional point-like objects with a theory of extended objects, geometry and topology enter the game as crucial protagonists: the extra dimensions String Theory requires (for consistency) are associated to a rich variety of complicated manifolds. According to their structure, various string configurations can emerge and, more importantly, depending on the choice of the internal geometry, “different universes” are obtained. However, since we are sensitive to four dimensions only, all extra dimensions have to effectively disappear in the sense that they can be considered so “small” that they do not (apparently) influence experimental results.

The previous observations introduce the great problem of how String Theory is connected to experiments and of how the string vacuum (if there is one) corresponding to the observed Universe can be selected.

In this context a natural question is concerned with the attempt of understanding what kind of effective Quantum Field Theories (QFTs) can be obtained by String Theory.

If any seemingly consistent QFT was coherent with String Theory, then constructing string vacua (by referring to complicated geometries of the internal manifolds) would become quite non-relevant or (at least) it would be postponed until so high energies are reached that the effective quantum field description under consideration breaks down and the question on how gravity can be quantized arises.

Despite this, there is evidence that not all consistent-looking QFTs can be completed in the  $UV$  in String Theory. A distinction among the vast set of apparently good QFTs can indeed be made [6, 7]. Any QFT that is constructed about one of the vacua of the rich vacuum structure of String Theory and is consistent with String Theory is said to belong to the string Landscape. The string Landscape is then defined as the large spectrum of effective QFTs that are consistent-looking and can be completed in String Theory at high energies.

All around the stringy landscape of vacua, there is an even more vast region that is called the string Swampland. The string Swampland is made by all those seemingly consistent QFTs that are actually inconsistent, meaning that they can't be  $UV$  completed within String Theory.

The distinction between a Landscape and a Swampland can be actually framed more generally than we have just done. If in discussing the relation between String Theory and QFTs we have introduced a string Landscape and a Swampland, we can proceed similarly in studying the link among Effective Field Theories (EFTs) and QFTs. Indeed, it can be shown that not all apparently consistent EFTs are properly  $UV$  completable in a QFT. Those consistent-looking theories admitting a  $UV$  completion in a QFT are points in the so called QFT Landscape; instead, all EFTs that are seemingly consistent but don't admit a  $UV$  completion in a QFT are said to be in the QFT

Swampland<sup>1</sup>.

For clarity, let us mention a simple example.

Consider a theory with a  $U(1)$  global symmetry that is spontaneously broken. As a consequence of the spontaneous symmetry breaking, a Goldstone boson  $\pi$  appears in the spectrum of the theory. The particle  $\pi$  is characterized by the shift symmetry

$$\pi \longrightarrow \pi + c \quad (2.1)$$

( $c$  being a constant) and so

$$\partial\pi \longrightarrow \partial\pi. \quad (2.2)$$

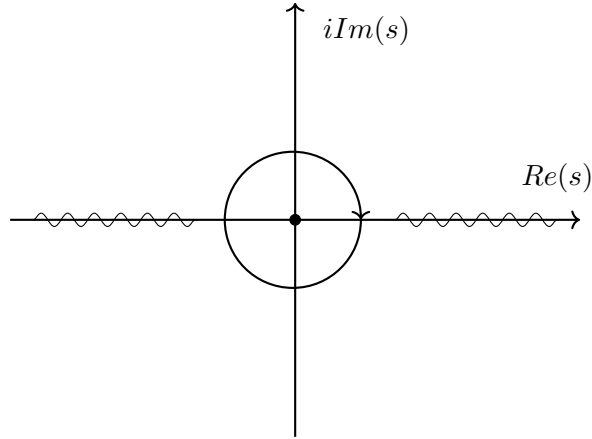
The lagrangian density of the weakly coupled theory (of  $\pi$ ) at low energy is

$$\mathcal{L} = \frac{1}{2}(\partial\pi)^2 + \frac{a}{\Lambda^4}(\partial\pi)^4 + \dots, \quad (2.3)$$

where  $a$  is a non-running parameter and  $\Lambda$  is the cut-off energy scale of the effective theory.

Adopting the perspective of the EFT, any value of the parameter  $a$  ( $a > 0$ ,  $a = 0$ ,  $a < 0$ ) is consistent with the symmetries of the theory itself and is therefore allowed. However, only when  $a > 0$ , the EFT (2.3) can be completed in the  $UV$  to a QFT.

To show this, let us adopt an approach based on interaction theory and amplitudes' positivity [10]. Let us consider the scattering process  $\pi\pi \longrightarrow \pi\pi$  and denote as  $s$  and  $t$  two of the Maldestam variables. By referring to the complexified  $s$ -plane (with  $t = 0$ ) and assuming there is a mass gap



$a$  can be expressed as

$$a = \frac{1}{2\pi} \oint \frac{M(s, t = 0)}{s^3} ds, \quad (2.4)$$

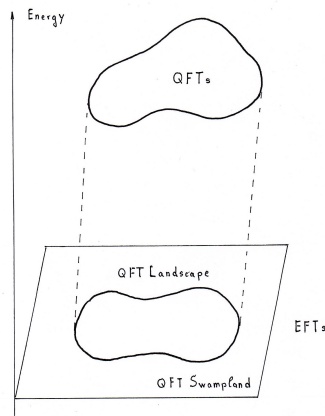
$M(s, t = 0)$  being the scattering amplitude. The integral in (2.4) (taken over the deformable circle in the previous graph) can be evaluated as the sum of (twice) the integral of  $M(s, t = 0)$  over the

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<sup>1</sup>The concept of *consistency* we are referring to is connected to  $UV$  dynamics; we require unitarity and causality, Lorentz invariance and a notion of locality.

discontinuity and the integral of  $M(s, t = 0)$  over the Wick circle. Since the former is positive definite and the latter vanishes when the Wick circle is sent to infinity,  $a$  turns out to be positive [11]. We conclude that only those EFTs having  $a > 0$  can be completed consistently in a QFT. The circumstances  $a = 0$  (corresponding to the free theory) and  $a < 0$ , even respecting the EFT's symmetries, do not result from a  $UV$  flow originating from some QFT.

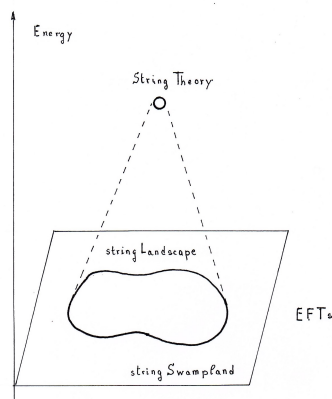
Coming back to a general treatment, we can schematically depict what happens in relating QFTs and EFTs as far the Swampland debate is concerned as



**Figure 2.1:**

The figure illustrates the notions of QFT Swampland and Landscape. Only some consistent-looking EFTs are selected as  $IR$  descendants of higher-energy defined QFTs.

and, including the discussion (we are really interested in) about the  $UV$  completion of an effective QFT in String Theory (or, more generally, in Quantum Gravity (QG)), the relevant reference picture becomes



**Figure 2.2:**

The figure shows how the  $UV$  flow originating from String Theory isolates the string Landscape, distilling it out from the string Swampland.

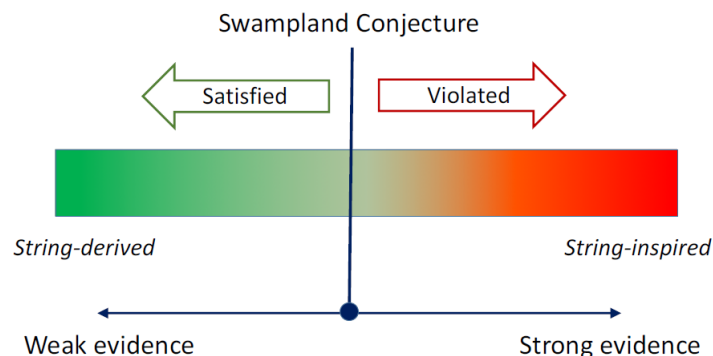
As briefly mentioned above, the notions of string Landscape and Swampland can be extrapolated to Quantum Gravity. The abstract concept of the QG Swampland has no meaning unless we understand how to distill out those effective theories that belong to the QG Landscape from those in

the QG Swampland. But, because we don't know the quantum theory of gravity and its characteristic properties, we don't actually know how to circumscribe the QG Landscape from the QG Swampland.

In the absence of other alternative evident ways of proceeding, we can try to get some information on the construction of quantum gravity theories by formulating some conjectural criteria distinguishing the QG Landscape from the QG Swampland as if they were the string Landscape and the string Swampland. This identification (usually performed in the literature when dealing with the Swampland program) is quite illegitimate, because it is not clear if String Theory can be a candidate theory of quantum gravity relevant for our Universe. On the other hand, the adoption of this (hoping temporary) line of thinking has allowed and allows to formulate various Swampland conjectures that (with different levels of strength) impose constraints on the effective QFTs, which may be experimentally tested.

As it can be easily guessed from the previous observations, String Theory is extensively used to gain evidence for the formulation of Swampland statements. String vacua can be distinguished in the so called *string-derived* vacua and the *string-inspired* ones. The first class is composed by the best understood string vacua: they are characterized by a full string world-sheet description and by relatively simple geometries for the extra dimensions and are usually supersymmetric. The second category of vacua is instead composed by those vacua of String Theory that are better thought of as constructions in a Quantum Field Theory framework and are motivated by the usual type of structures that one finds in String Theory.

A Swampland criterion would be typically referred to some region in the string-derived and string-inspired spectrum of vacua and proposed to be valid for all vacua with increasing rigour (as far as their construction with respect to String Theory is concerned). The question of whether a Swampland statement holds in String Theory or not hasn't a sharp answer. If one insists in accounting for the string-derived vacua only, then any conjecture may be satisfied by all the known string vacua; however, there could be many other string vacua that can't be rigorously constructed violating the conjecture. Being conscious of this, such a debate can be faced by saying that String Theory offers evidence for a given Swampland criterion with varying levels of strength.



**Figure 2.3 [7]:**

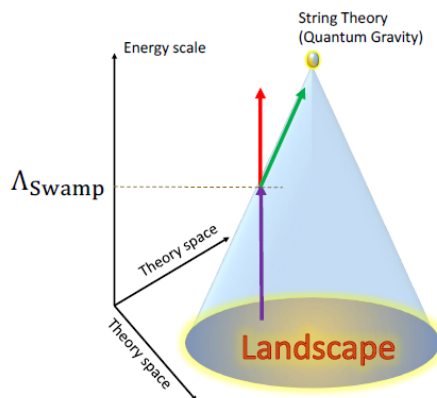
The figure schematically illustrates the spectrum of vacuum constructions in String Theory. The most rigorously understood vacua, the string-derived ones, are presented on the left hand side, whereas the most loosely connected vacua, the string-inspired ones, are on the right hand side. A Swampland conjecture can be placed on the spectrum such that all known vacua of increasing rigour satisfy it. A conjecture placed to the left hand side of the spectrum is said to have weaker evidence with respect to one that is placed on the right hand side, because (in principle) less string vacua may satisfy it.

Beside using String Theory as a reference setup, another approach to contribute to the Swampland program is based on the use (justified by the expectation that some low-energy aspects of quantum gravity are universal) of quantum gravity arguments directly in the low-energy effective theory. In this framework the study of black hole physics places itself and motivates some Swampland statements. Despite of having the advantage of broader generality with respect to referring to String Theory constructions, this Swampland-testing proposal often lacks of details and concreteness (which are typical of String Theory founded proposals).

The ideal approach to gain evidence for the Swampland criteria would be a direct derivation from microscopic physics. Even though any physical model can't be formulated (by definition) with no assumptions, it may be that one could uncover some  $UV$  principle of String Theory (for instance), which we have missed so far or hasn't been appropriately appreciated, leading directly to the Swampland conjectures. Finding out this underlying microscopic physics can be regarded as the final actual purpose of the Swampland program and String Theory constructions and general quantum gravity arguments can be considered as the experimental data supplied to try to develop such a physical theory.

There are some signs that the Swampland program is pointing towards the right direction: its approaches seem to be in tune and the Swampland criteria they give rise to appear to be interconnected.

When dealing with a Swampland conjecture, one considers a set of effective QFTs that are consistent up to a cut-off scale  $\Lambda_{QFT}$ . After these theories have been coupled to gravity<sup>2</sup>, a new energy scale  $\Lambda_{Swampland}$  enters the game. It can be intended as the scale at which the coupled-to-gravity QFT has to plan some (mild or more substantial) modifications so that it can be consistently completed in a theory of quantum gravity at higher energies. In other words, the theory would become inconsistent (after the coupling to gravity has been performed), if it wasn't modified above  $\Lambda_{Swampland}$ .



**Figure 2.4** [7]:

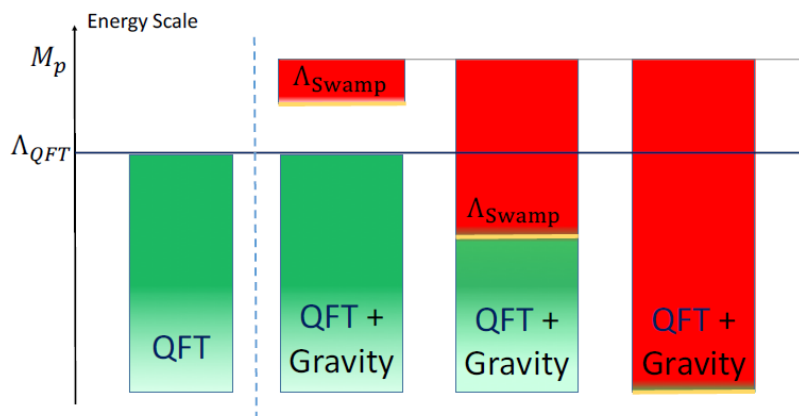
The figure presents a schematic interpretation of the energy scale  $\Lambda_{Swampland}$  at which an effective QFT has to be modified in order for it to be able to be completed into QG in the  $UV$ .

<sup>2</sup>The coupling of a given QFT to gravity is associated with a finite gravitational strength or, equivalently, with a finite value of the Planck mass  $M_P$ .

Depending on the values of the parameters of the QFT under consideration, the scale  $\Lambda_{Swampland}$  can be above or below  $\Lambda_{QFT}$ .

If  $\Lambda_{Swampland} > \Lambda_{QFT}$ , then the changes required at  $\Lambda_{Swampland}$  do not affect the effective QFT. If, on the contrary,  $\Lambda_{Swampland} < \Lambda_{QFT}$ , then the effective QFT with the original cut-off becomes inconsistent because of the coupling to gravity.

Since  $\Lambda_{Swampland}$  emerges as a consequence of the coupling to gravity, it usually diverges as  $M_P$  is sent  $+\infty$ . However, it can happen that  $\Lambda_{Swampland} \ll \Lambda_{QFT} \ll M_P$  and the extremal case where  $\Lambda_{Swampland}$  sets itself below any non-trivial energy scale in the effective theory may occur: these are of course the more interesting circumstances.



**Figure 2.5** [7]:

The figure shows various cut-off scales on effective QFTs. The first case (on the left hand side) corresponds to a pure QFT with a cut-off  $\Lambda_{QFT}$ . The second circumstance represents a QFT, coupled to gravity, that is characterized by parameter values such that the new Swampland cut-off scale  $\Lambda_{Swampland}$  lies above  $\Lambda_{QFT}$ . Varying the parameters one may reach the third case where  $\Lambda_{Swampland} \ll \Lambda_{QFT}$ : this leads to a strong constraint on the effective QFT due to the presence of gravity. Finally, the fourth case (on the right hand side) illustrates that, for certain parameter values' ranges, it may happen that the QFT is inconsistent already at the lowest non-trivial energy scale in the theory.

Having these general observations in mind, we will now turn to the analysis of some Swampland criteria. Despite of the fact that we will usually provide evidence for them by making reference to String Theory, which practically leads (together with black hole physics and aspects concerning the holographic nature of gravity) the Swampland research field, the Swampland constraints should be in principle understood as not necessarily and strictly related to String Theory. This way (and in the temporary absence of strong evident alternatives) the Swampland program points towards the attempt of getting information on the microscopic physics underlying all the Swampland statements and definitely on the construction of the theory of quantum gravity relevant for our Universe.



### 3 An overview on some Swampland criteria

As just presented, to give sense to the abstract definition of the Swampland one has to formulate sensible criteria allowing its distinction from the Landscape. In this chapter we will describe some Swampland conjectures focusing our attention on those that will play a relevant role in the following. More precisely, we will discuss the absence of global symmetries in a quantum gravity theory (the “No Global Symmetry Conjecture”), the weakness of gravity (the “Weak Gravity Conjecture”), the finiteness of the fields’ range of the theory (the “Swampland Distance Conjecture”) and we will describe the “de Sitter Conjecture”. These criteria will be motivated by low-energy quantum gravity properties (and black hole physics based arguments, in particular) and by referring to examples in String Theory. For a more detailed treatment and a review we refer the reader to [7].

#### 3.1 The No Global Symmetry Conjecture

The absence of global symmetries in a theory of quantum gravity can be considered as a prototype of a network of criteria distilling out the Landscape from the Swampland.

The **No Global Symmetry Conjecture** [12, 13] is stated as follows:

*A theory, coupled to gravity, does not have exact global symmetries.*

A strong motivation for this criterion comes from black hole physics.

Black holes are solutions of the Einstein’s equations of General Relativity. Under fairly general conditions, once the mass  $M$ , the (gauge) charge  $Q$  and the angular momentum  $J$  of a black hole have been fixed, the No Hair Theorem states that the solution of Einstein’s equations is uniquely determined [14].

Let us suppose that global symmetries are admitted and consider the  $U(1)$  exact global symmetry *number of particle-antiparticle pairs*. We can imagine to throw particles, which are charged under this  $U(1)$  symmetry, in the black hole. Since the charge is global and there is no gauge field emanating from it, the black hole’s horizon is insensitive to this global charge. As it can be shown in a semiclassical framework, a black hole evaporates thanks to Hawking radiation [15, 16]; however, there is no preference for the emission of the global charge in the sense that there is nothing which breaks the symmetry between positive and negative charges at the horizon. The number of antiparticles that the black hole should emit does not correspond to the given number of particles that have been thrown in: the number of particle-antiparticle pairs is characterized by an infinite uncertainty. This indeterminateness unavoidably affects the measures of the black hole’s global charge that an observer sitting outside the horizon is trying to make: its measures are infinitely uncertain and the conserved  $U(1)$  global charge the black hole has been endowed with is violated.

Moreover, following the semiclassical derivation made by Bekenstein and Hawking, we can associate to a black hole an entropy, which is related to its horizon’s area. Since the latter is finite, the black hole’s entropy is finite. In spite of this, in the presence of a global charge, the fact that there is an infinite uncertainty characterizing the determination of the black hole’s global charge is a signal for the possibility to construct an infinite number of black hole states with a given mass and differing (only) for their arbitrary global charge. The impossibility to measure the black hole’s global charge

corresponds to an infinite entropy and at the end to an inconsistency. By requiring consistency we have to conclude the absence of global symmetries in a quantum theory of gravity.

Following [17], we would like to show now that the incompatibility that we have just described of ordinary global symmetries with quantum gravity applies also to the so defined generalized global symmetries.

Ordinary global symmetries are described by a parameter  $\lambda$ ; it generates a one-parameter family of transformations on the (operatorial) fields of the theory, preserving the expectation values.

Nöther's Theorem associates to this continuous global symmetry a conserved current by means of which the charge (operator) can be defined. The charged objects are particles and the charged operators are local 0-dimensional operators.

Generalized global symmetries are described by an arbitrary closed  $p$ -form symmetry parameter. The Nöther conserved current corresponding to such a symmetry is a  $(p+1)$ -form. Charged objects are higher-dimensional branes and charged observables are higher-dimensional objects defined on  $p$ -cycles (rather than inserted at a point, which is a 0-cycle).

In order to simply appreciate the concept of generalized global symmetries let us consider pure Einstein–Maxwell theory with a  $U(1)$  gauge field  $A_\mu$  with gauge coupling  $g$ ,

$$S = \int d^4x \sqrt{-g} \left[ \frac{M_P^2}{2} R - \frac{1}{4g^2} F^2 \right]. \quad (3.1)$$

The theory is invariant under local gauge transformations acting as

$$A_\mu \longrightarrow A_\mu + \partial_\mu \lambda, \quad (3.2)$$

where  $\lambda(x)$  is a scalar parameter. However, we can also consider the transformation

$$A_\mu \longrightarrow A_\mu + \Lambda_\mu \quad (3.3)$$

with

$$\partial_{[\nu} \Lambda_{\mu]} = 0. \quad (3.4)$$

If locally in spacetime these two transformations are indistinguishable, the difference is made by taking into account global aspects of the space. For instance, by considering the spacetime to be topologically  $\mathbb{R}^{1,2} \times S^1$ , the Wilson loops  $e^{\int_{S^1} A_\mu dx^\mu}$  transform under (3.3) but not under (3.2).

The transformation (3.3) represents an additional symmetry of the theory: it is an example of 1-form generalized global symmetry, whose parameter is the closed 1-form  $\Lambda$ .

Generalizing, let us consider a theory with a  $p$ -form gauge field  $A_p$ . In the absence of charged objects the theory is characterized by the  $p$ -form global symmetry

$$A_p \longrightarrow A_p + \Lambda_p \quad (3.5)$$

(with  $\Lambda_p$  a closed  $p$ -form,  $d\Lambda_p = 0$ ) such that

$$dF_{p+1} = 0, \quad (3.6)$$

$F_{p+1}$  being the field strength  $F_{p+1} = dA_p$ .

The conserved Nöther current is  $j = F_{p+1}$  and the charged operators are given by the exponential of the generalized Wilson lines of the gauge potential on non-trivial  $p$ -cycles.

At infinitesimal level the symmetry acts on an operator  $O_{C_p}$  defined on the  $p$ -cycle  $C_p$  as

$$\delta O_{C_p} = \left( \int_{C_p} \Lambda_p \right) O_{C_p} \quad (3.7)$$

and the correlators are invariant under such infinitesimal shifts.

As we will better detail, the existence of generalized global symmetries is expected to be obstructed within a theory of quantum gravity. This incompatibility does not amount to the absence of generalized global symmetries in an effective theory coming (for instance) from String Theory, but it rather means that they have to be either gauged or broken in the  $UV$  theory the effective theory is descendant of.

The gauging of a global symmetry consists on dropping the requirement that the parameter  $\Lambda_p$  must be a closed form while maintaining the invariance property (3.5).

In the case of a 0-form (i.e ordinary) global symmetry this recipe amounts to promote the parameter  $\lambda$  to a function of the spacetime coordinates  $\lambda(x)$  and to introduce an additional 1-form potential, transforming with the exterior differential of  $\lambda(x)$ . As far as generalized  $p$ -form global symmetries are then concerned, their gauging prescribes the promotion of the closed  $p$ -form  $\Lambda_p$  (characteristic for the transformation property of the  $p$ -form gauge field  $A_p$ ) to an arbitrary  $p$ -form and the introduction of a  $(p+1)$ -form potential (say,  $A_{p+1}$ ), modifying the kinetic term to

$$\frac{1}{2} |dA_p - gA_{p+1}|^2 \quad (3.8)$$

(where  $g$  is the gauge coupling) and making

$$\begin{cases} A_p \longrightarrow A_p + \Lambda_p \\ A_{p+1} \longrightarrow A_{p+1} + d\Lambda_p \end{cases} \quad (3.9)$$

the gauge invariance properties of the theory <sup>3</sup>.

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<sup>3</sup>It is interesting to note that the gauging of a generalized global symmetry has consequences on the objects (higher-dimensional branes, in general) that are charged under it. A charged  $(p-1)$ -brane would couple electrically to  $A_p$  by means of its  $p$ -dimensional world-volume  $\mathcal{W}_p$  as

$$\int_{\mathcal{W}_p} A_p.$$

This world-volume coupling is not invariant under (3.7). However, if  $\mathcal{W}_p = \partial\mathcal{W}_{p+1}$ , the operator

$$\int_{\mathcal{W}_p} A_p - g \int_{\mathcal{W}_p} A_{p+1}$$

is gauge invariant. So,  $(p-1)$ -branes exist as boundary of a  $p$ -branes which are electrically coupled to  $A_{p+1}$ .

Since generalized global symmetries can't often be gauged [18], their breaking mechanisms have to be studied.

As for ordinary global symmetries, (continuous and discrete) higher-form global symmetries can be spontaneously or explicitly broken.

Following the Coleman, Mermin and Wagner's analysis about the possibility of spontaneous symmetry breaking in  $d$  dimensions, it is possible to show that  $p$ -form global symmetries (with  $p > 0$ ) are always spontaneously unbroken in  $d \leq p + 2$  dimensions if continuous and in  $d \leq p + 1$  dimensions if discrete [18]. These arguments are usually regarded as the realization of the so called Coleman–Mermin–Wagner (CMW) Theorem, preventing spontaneous symmetry breaking in the vacuum and in any thermal state in sufficiently low dimensions.

As it can be easily guessed, spontaneous symmetry breaking is not the general mechanism for global symmetries to be broken and other breaking procedures have to be studied [17].

Rather than a generic treatment we will now consider a specific example and in its respect we will try to analyze some breaking techniques. The following discussion will be also interesting as far as the exploration of the role of generalized global symmetries within a theory of quantum gravity is concerned: by making use of black hole physics arguments we will discover that generalized global symmetries can be placed on the same footing as the ordinary ones.

Let us now consider a 4-dimensional theory including gravity and with a 2-form field  $B_2$ . The field  $B_2$  might come from dimensional reduction of a higher  $p$ -form field (with  $p > 2$ ) or it can be interpreted as the dual potential of an axion  $\phi$  in four dimensions. If a potential dependence of the kinetic term coefficient on other fields is ignored, the relevant part of the action is

$$\int \frac{1}{2} |H_3|^2, \quad (3.10)$$

where  $H_3 = dB_2$ . The system is characterized by the 2-form global symmetry

$$B_2 \longrightarrow B_2 + \Lambda_2, \quad (3.11)$$

$\Lambda_2$  being an arbitrary closed 2-form. If the spacetime  $X_4$  has a non-contractible 2-cycle  $\Sigma$ , the periods of  $B_2$ , which are a higher-dimensional generalization of Wilson lines, can be defined,

$$\int_{\Sigma} B_2. \quad (3.12)$$

By taking into account the identifications provided by gauge transformations involving the closed form  $\Lambda_2$  the periods of  $B_2$  take values in the quotient of cohomology groups<sup>4</sup>

$$\frac{H_2(X_4, \mathbb{R})}{H_2(X_4, \mathbb{Z})} \quad (3.13)$$

and the 2-form symmetry translates into a continuous shift symmetry of the periods of  $B_2$ .

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<sup>4</sup>The cohomology group  $H_p(X)$  is defined as the set of equivalence classes of closed  $p$ -forms that differ only by exact forms on the topological space  $X$ .

Even though the presence of a non-trivial 2-cycle is not so manifest, let us consider Schwarzschild spacetime.

Using the  $S^2$  volume-form  $d\Omega$ , we can introduce a 2-form  $B$ -field background

$$B = Qd\Omega. \quad (3.14)$$

By writing then  $\Lambda_2$  as

$$\Lambda_2 = \lambda d\Omega \quad (3.15)$$

the symmetry characterizing the periods of  $B$  shifts  $Q$  by  $\lambda$  over the horizon  $S^2$ .

For different values of the  $B$ -field a one-parameter family of Schwarzschild solutions is defined. Since  $H_3 = dB = 0$  and there is no back-reaction, the metric is independent on the  $B$ -field value. This setup is known as the Bowick–Giddings–Harvey–Horowitz–Strominger (BGHHS) black hole.

BGHHS black holes are indeed legitimate states of the theory and they can be put in evidence directly through the wiggling and settling down of a string electrically coupled to the field  $B$  near a Schwarzschild black hole; or by an Euclidean instanton producing a pair of black holes whose horizons are connected by a 3-chain on the boundary of which a  $B$ -field (with  $H = 0$  everywhere) can be turned on.

In the literature (e.g [19, 20]) the  $B$ -field was proposed as an example of global quantum hair for a black hole. Despite of the No Hair Theorem, because the  $B$ -field is not a local observable (and in any contractible region it can be simply gauged away) and being its curvature (at least) outside the horizon  $H = 0$ , turning on the  $B$ -field does not in principle affect the properties of the black hole such as its mass or the Hawking evaporation process it is subjected to. This means that, at least classically, we can build black holes with a given mass and an arbitrary value of  $Q = \int_{S^2} B$ .

By adopting an Euclidean perspective the black hole geometry is described by the metric

$$ds^2 = \left(1 - \frac{2M_{BH}}{r}\right) d\tau^2 + \frac{1}{1 - \frac{2M_{BH}}{r}} dr^2 + r^2(\sin^2 \theta d\theta^2 + d\phi^2) \quad (3.16)$$

and the topology of the solution is  $\mathbb{R}^2 \times S^2$ , where  $\mathbb{R}^2$  is parameterized by the coordinates  $(\tau, r)$  ( $\tau$  being the periodic Euclidean time) and  $S^2$  is described by the angular variables  $(\theta, \phi)$ .

By compactifying on  $S^2$ , we get a 2-dimensional theory on  $\mathbb{R}^2$  in which the 2-form symmetry of  $B_2$  becomes a continuous shift symmetry for the 2-dimensional axion  $\phi = \int_{S^2} B_2$ .

Classically, the BGHHS charge  $Q$  corresponds to the vacuum expectation value of  $\phi$ . It seems that we have a continuous shift symmetry which is spontaneously broken. However, the CMW Theorem prevents spontaneous symmetry breaking for our setup: an Euclidean-Schwarzschild black hole (and so a thermal state at Hawking temperature) in  $d = 2$  ( $\leq 4$ ) dimensions. The CMW Theorem sets the semiclassical charge  $Q$  to zero in the quantum theory such that for an observer outside the black hole its charge  $Q$  becomes completely unobservable. As in the case of ordinary global symmetries, the problems concerned with the black hole entropy and the measurability of the black hole's global charge arise once again.

Having in mind the formulation of a consistent theory of quantum gravity, the conclusion we have just reached encourages further investigation for an efficient mechanism by means of which generalized global symmetries can be broken.

In the context of the 2-dimensional effective theory on  $\mathbb{R}^2$  we have just dealt with we can try to break the shift symmetry of the axion  $\phi$  by introducing strings: they are the electrically charged objects under the 2-form symmetry involving the field  $B_2$  in the 4-dimensional theory. However, in two dimensions there are some *IR* effects that obstruct the symmetry violation. The idea is that the electric field sourced by the strings decays so slowly that the effective action diverges unless the total charge vanishes.

More precisely, let us suppose a string couples to the field  $B$  through the usual world-volume coupling,  $\int_{W_2} B_2$ . When we compactify on  $S^2$ , the dimensional reduction of the action (3.10) gives a canonical kinetic term for  $\phi$  and the lower-dimensional version of Gauss's law is

$$\int *d\phi = Q, \quad (3.17)$$

where  $Q$  is the net string charge wrapped on the 2-cycle. In two dimensions the theory suffers from *IR* divergence:

$$\int \frac{1}{2} |d\phi|^2 \sim Q^2 \int \frac{dr}{r^2} \rightarrow +\infty. \quad (3.18)$$

The selection rule  $Q = 0$  is imposed and (part of) the 2-form global symmetry of the original 4-dimensional theory remains unbroken.

This argument can be generalized and the pattern we have just described can be regarded as a generic feature of any continuous shift symmetry in two dimensions and it is typical of any theory of gravity or compactification of String Theory in two dimensions; and, imagining the theory is sensitive upon compactifications, these considerations apply to global symmetries in higher dimensions too.

Since the introduction of electrically (or magnetically) charged objects under a  $p$ -form global symmetry ( $(p-1)$ -branes, for example) does not generically guarantee the breaking of the symmetry, another approach consists in an explicit breaking procedure by coupling a  $(d-p-1)$ -form to the  $p$ -form gauge potential by its field strength.

In this respect and still considering the previous setup, the violation of the global shift symmetry of the 2-dimensional axion  $\phi$  may be achieved by means of explicit breaking terms in the lagrangian (preserving the discrete axion periodicity) as

$$\int \phi X_2, \quad (3.19)$$

where  $X_2$  is (for instance)  $X_2 = NF_2$  with  $N \in \mathbb{Z}$  and  $F_2 = dA_1$  ( $A_1$  being an ordinary 1-form gauge potential). By uplifting to four dimensions, the axion can be seen as coming from the field  $B_2$  and the coupling can be regarded as descendant of a 4-dimensional Stueckelberg coupling  $BF$ . Due to this coupling the 2-dimensional axion  $\phi$  acquires a potential or the 2-form  $B_2$  and the  $U(1)$  gauge boson are massive. This breaks the axionic continuous symmetry and indeed the 2-form symmetry. This breaking procedure is explicit and valid at any point in spacetime.

However, there are many String Theory compactification examples where the quadratic (Stueckelberg) coupling terms  $BF$  are absent. If no alternative breaking mechanism could be found, String Theory would apparently suffer from global symmetry problems. But in [17] a more flexible and efficient seemingly generic mechanism of generalized global symmetry breaking has been proposed. It is based on the existence of ubiquitous cubic Chern–Simons terms: they explicitly break the symmetry and, even when the fields’ vacuum expectations render them apparently harmless, the symmetry breaking occurs via localized bubble configurations within which the symmetry is broken. By referring to the usual 2-dimensional setup with a 0-form symmetry for an axion, the leading observation is that we can break the axionic shift symmetry by slightly modifying the  $BF$ -coupling procedure thanks to Chern–Simons terms

$$\int G_0 \phi F_2, \tag{3.20}$$

where  $G_0$  is a new non-dynamical field strength (which can be intended as the 2-dimensional dual of another gauge field strength  $G_2$ ). The values of  $G_0$  are quantized and the theory contains membranes under which  $G_0$  shifts by the integer  $N$  [12]. The theory contains different phases that are identified by the coupling  $N\phi F_2$  for different integer values of  $N$  and the electrically charged particles coupled to  $G_0$  (domain walls) separate these phases. The current  $j = d\phi$  associated to the axionic shift symmetry is now such that

$$d * j = G_0 F_2. \tag{3.21}$$

If  $G_0 = N \neq 0$ , the symmetry is broken. However, when  $G_0 = N = 0$ , the Chern–Simons terms don’t seem to play any role, leaving the symmetry unbroken. But it is possible to show that taking into account Chern–Simons terms amounts to the inclusion in the path integral of configurations with a localized bubble where  $G_0 = N \neq 0$ . Within the bubble the  $U(1)$  gauge field is in a Higgs branch eating  $\phi$  through the coupling  $N\phi F_2$ . In the BGHHS system the triple term

$$\int G_0 B_2 \wedge F_2 \tag{3.22}$$

in the action allows the nucleation of bubbles within which the gauge field is Higgsed. Inside the bubbles the value of the field  $B$  is quantized and it is related to the black hole charge modulo  $G_0$ . Whereas outside the bubble  $B$  could freely fluctuate, in the bubble it acquires a definite value and this breaks the symmetry.

Extrapolating, Chern–Simons terms (giving rise to symmetry breaking bubble configurations) provide an efficient (without leading to an  $IR$  divergent action, for instance) and seemingly generic mechanism accounting for generalized global symmetry violation [17].

Convinced that for any global symmetry in String Theory it is always possible to find phases in which the symmetry is gauged or broken and there are always domain walls connecting these phases, M. Montero, A. Uranga and I. Valenzuela have recently proposed the following conjecture [17]

*Consistent theories of quantum gravity suffer from a Chern–Simons pandemic, id est every consistent theory weakly coupled to gravity with higher  $p$ -form potentials should have the appropriate Chern–Simons terms, so that, independently on the compactification procedure, no global symmetries survive in two dimensions.*

This rule is supported by lots of examples in String Theory.

*Exempli causa*, it is interesting to consider the case of Kaluza–Klein (KK) photons. They are protagonists of  $U(1)$  effective theories coming from a compactification and emerge from continuous isometries of the internal manifold. These gauge bosons give rise to 1-form global symmetries and the presence of the breaking Chern–Simons terms is not *a priori* guaranteed. However, it can be shown that KK photons always exhibit the required Chern–Simons terms. In the context of type IIA theories [7], let us concentrate on a KK compactification on  $M_5 \times S^1$  (where  $M_5$  is a 5-dimensional manifold) to four dimensions and on a KK photon arising by moving from five to four dimensions on  $S^1$ . By compactifying on  $M_5$  we get a scalar  $\phi$  defined as the period of the Ramond–Ramond field  $C_5$  on  $M_5$ ; and then, by requiring that a general axion compactified on a circle (say, with radius  $R$ ) with a periodic coordinate (say,  $z$ ) admits periodic boundary conditions  $\phi(z + 2\pi R) = \phi(z) + 2\pi N$ <sup>5</sup>, the compactification on  $S^1$  gives a Stuckelberg lagrangian characterized by the coupling

$$Nd\phi \wedge *A \tag{3.23}$$

(with  $N = \int_{S^1} d\phi$ ). Integration by parts allows to recognize a  $BF$  coupling between the KK photon  $A$  and the Ramond–Ramond field  $C_2$ , dual of  $\phi$  in four dimensions. Moving to the T-dual frame, the Stuckelberg coupling is induced by a cubic Chern–Simons term that has the structure  $B_2 \wedge F_3 \wedge F_5$  in the original 10-dimensional action and leading to  $G_0 B_2 \wedge F_3$ , where  $G_0 = \int_{M_5} F_5$  in five dimensions. This is indeed a coupling between the field  $B_2$ , which usually accompanies gravity in string compactification, and the dual field to the axion.

The general idea is that (in the dual frame) the Chern–Simons terms in the 10-dimensional action involving the field  $B$  (which gives rise to the KK photon  $A$  by dimensional reduction) provide Chern–Simons terms involving  $A$  in the 4-dimensional effective theory. In the original frame, the appropriate Chern–Simons term (manifesting itself in the form of a Stuckelberg coupling) comes from Scherk–Schwarz compactification of some Ramond–Ramond field.

The Montero, Uranga and Valenzuela’s rule is strongly motivated by the leading principle according to which Chern–Simons terms are ubiquitous in String Theory; so, they may really represent a generic characteristic of consistent quantum theories of gravity.

As a consequence, those theories that do not contain the suitable Chern–Simons terms belong to the Swampland.

So, if the conjecture is true, it can be regarded as a new criterion by which Landscape effective theories are distinguished by the Swampland ones.

A remarkable theory which is claimed to be in the Swampland by [17] is pure Einstein gravity in  $d \geq 4$  dimensions.

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<sup>5</sup>This is the so called Scherk–Schwarz ansatz; for a motivation we refer the reader to [21].



Since the conclusion could be easily generalized to  $d > 4$ , let us focus on 4-dimensional pure gravity. By compactifying on a torus  $T^2$  a 2-dimensional axion  $\phi_{KK}$  appears: it is the Wilson line associated to the  $U(1)$  gauge boson  $A_{KK}$  that emerges in the compactification to three dimensions on a circle  $S^1$ , when the compactification to two dimensions on the other  $S^1$  is performed. The 0-form symmetry for the scalar  $\phi_{KK}$  would require some Chern–Simons term for its violation, but it is absent in the theory, making it inconsistent (according to [17]).

Anyway this inconsistency can be cured by modifying the theory with the inclusion of a 4-dimensional axion  $\phi$ . This allows to perform compactifications assuming the Scherk–Schwarz ansatz  $\phi(z + 2\pi) = \phi(z) + 2\pi n$  so that  $e^{i\phi}$  has charge  $n$  under the KK photon and the gauge invariant quantity in the dimensionally reduced theory is  $d\phi - nA_{KK}$ . This provides a mass for  $A_{KK}$  and, after further compactification to two dimensions, to the axion  $\phi_{KK}$  too. The axionic shift symmetry is so broken. This argument can be reformulated in terms of the Chern–Simons rule. By introducing the 2-dimensional dual  $\eta$  to the axion  $\phi$  there is a cubic coupling  $n\eta F_{KK}$  (with  $F_{KK} = dA_{KK}$ ), where  $n$  has to be regarded as a geometric background flux. Even when  $n = 0$ , bubbles circumscribing regions with  $n \neq 0$  break the global symmetry.

According to Montero, Uranga and Valenzuela’s criterion also  $N = 8$  Supergravity is proposed to be in the Swampland, the reason being that the theory contains a 2-form gauge field giving rise to a 2-form generalized global symmetry without the Chern–Simons terms required to break it.

This argument supports what M. B. Green, H. Ooguri and J. H. Schwarz stated in [22], claiming  $N = 8$  Supergravity can’t be reached as a suitable decoupling limit of toroidally compactified String Theory.

However, Montero, Uranga and Valenzuela’s argument has to be better understood and contextualized. To propose some hints in order to do this, let us briefly comment on the structure of  $N = 8$  Supergravity (ignoring fluxes, at first).

In *four* dimensions  $N = 8$  Supergravity is characterized, as far as its field content is concerned, by a spin-2 graviton  $g_{\mu\nu}$ , which carries 2 on-shell bosonic degrees of freedom; eight gravitini  $\psi_\mu^{\alpha[A]}$  (with  $A$  running from 1 to 8), which have spin- $\frac{3}{2}$  and correspond to  $16 = 8 \times 2$  on-shell fermionic degrees of freedom; twenty-eight spin-1 vector fields  $A_\mu^{[AB]}$ , which contribute with  $56 = 28 \times 2$  units to the on-shell bosonic degrees of freedom of the theory; fifty-six spinors  $\chi^{[ABC]}$ , which have spin- $\frac{1}{2}$  and participate to the on-shell fermionic degrees of freedom of the model with  $112 = 56 \times 2$  units and by seventy spin-0 scalar fields  $\Phi^{[ABCD]}$ , which carry 70 on-shell bosonic degrees of freedom.

The symmetry group of  $N = 8$  Supergravity is the exceptional group  $E(7)$  whose dimension is 133. It has various non-compact realizations and that for which the group generators are organized in 63 compact and 70 non-compact generators allows to retrieve the representation associated to the scalar fields  $\Phi^{[ABCD]}$ , whose coset manifold is  $E(7)/SU(8)$  [23, 24].

$N = 8$  Supergravity in *eleven* dimensions is based instead on the fields  $G_{MN}$ ,  $C_{MNP}$  and  $\Psi_M^\alpha$  (with  $\alpha$  denoting 32 spinorial components).  $G_{MN}$  carries 44 on-shell bosonic degrees of freedom<sup>6</sup>; the bosonic on-shell degrees of freedom corresponding to the 3-form  $C_{MNP}$  are given by the combinatorial factor  $\binom{9(=11-2)}{3} = 84$  and  $\Psi_M$  is responsible for  $128 = (9 - 1) \times 16$  on-shell fermionic degrees of freedom

<sup>6</sup>The calculation is performed by using the vielbeins  $e_M^N$ , where, in  $D$  dimensions,  $M$  and  $N$  assume (respectively)  $D - 2$  values (because of gauge transformations) and  $D$  values. By taking into account the local Lorentz transformations the degrees of freedom associated to  $e_M^N$  (and  $G_{MN}$ ) are then  $(D - 2)D - \frac{D(D-1)}{2}$  and they are 44 in  $D = 11$  dimensions.

(by accounting for  $\gamma$ -matrices and the 32 spinorial components).

Even though the field content of  $N = 8$  Supergravity in eleven dimensions is different from that of  $N = 8$  Supergravity in four dimensions, the counting of the on-shell degrees of freedom results in both cases in 128 bosonic and 128 fermionic freedoms.

Having introduced the fields participating to the definition of  $N = 8$  supergravity theories in four and eleven dimensions, we can try to reduce the 11-dimensional  $N = 8$  Supergravity to four dimensions by compactifying it on a 7-dimensional torus  $(S^1)^7$ , playing the role of (flat) internal manifold.

The field  $G_{MN}$  produces  $g_{\mu\nu}$ ,  $g_{\mu I}$  and  $g_{IJ}$ ; the field  $C_{MNP}$  gives rise to  $C_{\mu\nu I}$ ,  $C_{\mu IJ}$  and  $C_{IJK}$  and the field  $\Psi_M^\alpha$  gives rise to  $\psi_\mu^{\hat{\alpha}, A}$  and  $\psi_I^{\hat{\alpha}, A}$ , where  $\mu, \nu = 0, \dots, 3$ ;  $I, J, K = 1, \dots, 7$ ;  $\hat{\alpha} = 1, \dots, 4$  and  $A = 1, \dots, 8$ . In four dimensions we then retrieve a graviton ( $g_{\mu\nu}$ ), eight gravitini ( $\psi_\mu^{\hat{\alpha}, A}$ ), twenty-eight vectors ( $g_{\mu I}$  and  $C_{\mu IJ}$ ) and fifty-six spinors ( $\psi_I^{\hat{\alpha}, A}$ ); but the scalar field content of the dimensionally reduced 11-dimensional  $N = 8$  Supergravity (made by the sixty-three scalar fields  $g_{IJ}$  and  $C_{IJK}$ ) doesn't seem to reproduce the scalar field content of the 4-dimensional  $N = 8$  Supergravity. However, when led to four dimensions by compactification, in the spectrum of the 11-dimensional  $N = 8$  Supergravity seven tensor modes ( $C_{\mu\nu I}$ ) appear. By exploiting duality symmetries and the consequent fact that any massless tensor is equivalent to a scalar<sup>7</sup>, the spectra of the 4-dimensional  $N = 8$  Supergravity and of the 11-dimensional theory can be identified.

Since in the 4-dimensional  $N = 8$  Supergravity there are no 2-forms, Montero, Uranga and Valenzuela's statement can't be strictly referred to such a theory. It can be raised (at most) in considering the dimensional reduction to four dimensions on the torus  $(S^1)^7$  of the 11-dimensional  $N = 8$  Supergravity (where seven 2-forms appear) and a particular choice of duality frame: that in which the 2-forms are dualized to scalar fields. The 2-forms of the 11-dimensional  $N = 8$  Supergravity are indeed involved in cubic couplings through their strength tensors but they do not define the coupling structure required in [17] (with  $B_2$  and the strength tensors of other fields). This poses the question whether the absence of appropriate Chern–Simons terms makes (pure)  $N = 8$  Supergravity (intended, as precised before) fall in the Swampland<sup>8</sup>. Reasonably, a deep and comprehensive study of this problem can't prescind from the analysis of generic duality frames and of the gaugings  $N = 8$  Supergravity can be subjected to [25]. It is possible that turning on fluxes (which we have ignored so far), referring to a generic duality frame and gauging some isometries of the 11-dimensional  $N = 8$  Supergravity may deform the theory so that the appropriate Chern–Simons terms are obtained and break the non-desired generalized global symmetries.

<sup>7</sup>Having in mind the compactification to four dimensions, the lagrangian density of the 11-dimensional  $N = 8$  Supergravity can be schematically written as

$$F_4 \wedge *F_4 + F_4 \wedge F_4 \wedge C_3$$

in terms of the 4-form  $F_4$  and the 3-form  $C_3$  and denoting with “ $*$ ” the Hodge star operation. We can decide to confine the seven internal indices to the coupling term  $F_4 \wedge F_4 \wedge C_3$  in such a way that one of them is associated to the first  $F_4$ , three of them are made correspondent to the second  $F_4$  and the other three internal indices are associated to the form  $C_3$ . We then obtain a cubic coupling whose structure is  $\epsilon^{\mu\nu\rho\sigma} \epsilon^{I_1 I_2 I_3 I_4 I_5 I_6 I_7} F_{\mu\nu\rho I_1} F_{\sigma I_2 I_3 I_4} C_{I_5 I_6 I_7}$ . Let us write  $F_{\sigma I_2 I_3 I_4}$  as  $F_{\sigma I_2 I_3 I_4} = \partial_\sigma C_{I_2 I_3 I_4}$  and  $F_{\mu\nu\rho I_1}$  in the form  $F_{\mu\nu\rho I_1} = \partial_\mu B_{\nu\rho I_1}$ . Since there are no fluxes turned on, the 2-form  $B_2$  always appears accompanied by a derivative. After having introduced  $H_3 = dB_2$ , we can modify the reference lagrangian density by adding the topological term  $H_3 \wedge d\Phi$  (with  $\Phi$  a 0-form), which is a boundary term. By passing through the equations of motion of  $H_3$ ,  $\Phi$  can be made correspondent to  $H_3$ . This correspondence founds the duality relation between a massless tensor and a scalar.

<sup>8</sup>Of course, nothing forbids to refer to compactifications on internal manifolds that are not  $(S^1)^7$ . In such cases the situation may be different and Montero, Uranga and Valenzuela's argument may be overcome.

The question on the possible belonging of  $N = 8$  Supergravity to the Swampland appears to us still open. Further work is required to really get a definite and conclusive statement.

### 3.2 The Weak Gravity Conjecture

Another relevant and challenging criterion to try to chart the Swampland is the **Weak Gravity Conjecture** (WGC).

In its most familiar and best understood formulation involving a  $U(1)$  gauge field it can be stated as follows [26]:

*Consider a theory, coupled to gravity, with a  $U(1)$  gauge symmetry (whose gauge coupling is  $g$ ) in four dimensions*

$$S = \int d^4x \sqrt{-g} \left[ \frac{M_P^2}{2} R - \frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu} + \dots \right]. \quad (3.24)$$

*Electric WGC. There exists a particle in the theory with mass  $m$  and charge  $q$  satisfying the inequality*

$$m \leq \sqrt{2} g q M_P. \quad (3.25)$$

*Magnetic WGC. The cutoff scale  $\Lambda$  of the effective theory is bounded from above approximately by the gauge coupling as*

$$\Lambda \lesssim g M_P. \quad (3.26)$$

By making reference to black hole physics (in particular) we would like to motivate this statement. Let us consider a black hole with mass  $M$  and charge  $Q$  under a  $U(1)$  gauge symmetry (in four dimensions). The black hole  $(M, Q)$  is meant to be the solution of the Einstein's equations expressed by

$$ds^2 = -f(r) dt^2 + f(r)^{-1} dr^2 + r^2 (\sin^2 \theta d\theta^2 + d\phi^2) \quad (3.27)$$

with

$$f(r) = 1 - \frac{2M_{BH}}{r} + \frac{2g^2 Q^2}{r^2}, \quad (3.28)$$

where  $M_{BH}$  is  $M_{BH} = G_N M$  ( $G_N$  being the Newton's constant);  $g$  is the gauge coupling constant and coordinates  $(t, r, \theta, \phi)$  adapted to an observer at infinity are used.

Since  $f(r)$  is quadratic, there are two horizons located at

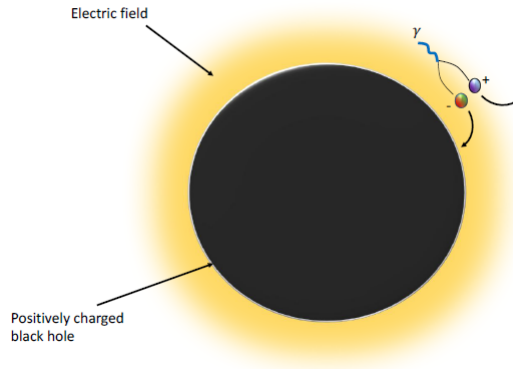
$$r_{\pm} = M_{BH} \pm \sqrt{M_{BH}^2 - 2g^2 Q^2}. \quad (3.29)$$

To make the previous solution a black hole, the extremality bound

$$M_{BH}^2 \geq 2g^2 Q^2 \quad (3.30)$$

has to be satisfied. When the bound is saturated, the black hole is said to be *extremal*, meaning it has the minimal mass to admit a horizon, once its charge  $Q$  has been fixed. A violation of the extremality bound leads to a naked singularity; but naked singularities are thought not to be there according to the Cosmic Censorship [27].

Let us suppose that the black hole  $(M, Q)$  is extremal; indeed,  $M = Q$  in appropriate units (where the reduced Planck mass  $M_P$  has been set to 1). This black hole can loose mass thanks to Hawking radiation and discharge through an analogous process made possible by the field around the horizon that the black hole's charge induces [15, 16]. The two main discharging processes are the thermal one, occurring when the black hole's Hawking temperature is greater than the mass of the particles the black hole is discharging in; and the Schwinger pair production one which is relevant for extremal or near extremal black holes. While evaporating, the black hole emits particles with mass and charge  $(m_i, q_i)$ .



**Figure 3.1** [7]:

The figure shows a black hole's evaporation and discharge processes. A pair of charged particle and antiparticle are produced in the electric field outside the black hole; for instance, the antiparticle crosses the black hole's horizon and the particle escapes.

For the black hole to remain a black hole while evaporating, step by step in the emission process the mass of the black hole should be greater or equal to its charge. Moreover, the decay of the charged black hole is constrained by energy and charge conservation ( $M \geq \sum_i m_i$  and  $Q = \sum_i q_i$ ) such that

$$\frac{M}{Q} \geq \frac{\sum_i m_i}{Q} = \frac{1}{Q} \sum_i \frac{m_i}{q_i} q_i \geq \frac{m}{q} \Big|_{min}. \quad (3.31)$$

As a consequence of the relation (3.31), we can argue the existence of at least a particle whose charge-to-mass ratio is greater or equal than that of the black hole. By exploiting then the extremality condition (hence  $M = Q$ ) we constrain further these particles to be such that gravity acts as the weakest force on them (since  $m \leq q$ ).

The weakness of gravity with respect to the other interactions is really the physical principle behind the WGC.

To further motivate the conjecture as a Swampland criterion, let us try to understand what happens if we set the electromagnetic force to be weaker than the gravitational one for the particle(s) with the largest charge-to-mass ratio in the theory.

Attracting rather than repelling, such two WGC particles would form a bound state. Because of energy and charge conservation, the energy of the bound state would be smaller than  $2m$  and its charge would be exactly  $2q$ . Having a charge-to-mass ratio larger than the charge-to-mass ratio of the particle(s) with the largest charge-to-mass ratio in the theory, the bound state just formed couldn't discharge emitting particles: it would be stable. By adding more and more particles, since they attract each other, it would be possible to produce stable bound states with arbitrary charge. These  $(m, q)$  particles' bound states can be weakly coupled and are stable due to their charge. Even though the comprehension of what goes wrong with them microscopically is still an open question, it is sensible not to expect the existence of such bound states.

These observations provide sensible evidence for the Electric WGC.

Let us now come back to black holes and try to distinguish the cases in which they are charged under a global or gauged  $U(1)$  symmetry.

As already mentioned, in the presence of a  $U(1)$  global symmetry we can in principle create an infinite number of black hole states with an arbitrary global charge and the same finite mass. Instead, as far as a  $U(1)$  gauge symmetry is concerned, the number of states below a given energy scale is finite due to the extremality bound (which implies that any charge increase corresponds to a mass increase for an otherwise naked singularity to be shielded). Once a mass scale  $\Lambda$  has been fixed, the number of possible black holes  $N_{BH}$  is

$$N_{BH} = \frac{\Lambda}{gM_P}. \quad (3.32)$$

The entropy based argument presented in **Subsection 3.1** against global symmetries no longer works for gauge interactions: in fact, at least theoretically, it is possible to measure the black hole's charge thanks to the flux of the gauge field.

However, the relation (3.32) gives interesting constraints in the limit  $g \rightarrow 0$ . In this case,  $N_{BH}$  diverges and it becomes impossible to determine the black hole's charge, because there is no more flux emanating from it. In other words, when the gauge coupling of a gauge symmetry is sent to zero, the circumstance where a global symmetry is in the game is retrieved. This naturally inspires the elaboration of a statement expressing how Quantum Gravity (QG) opposes to the continuous flow (in the couplings' space) towards the forbidden global symmetry limit. If we agree with the argument against global symmetries in QG, we have to accept the black hole argument against the vanishing gauge coupling limit of a gauge symmetry<sup>9</sup>.

Thanks to the above considerations the Magnetic WGC arises.

As it can be perceived by intuition from what has been presented so far, the Electric WGC suggests the study of black holes' discharge processes.

It is an open question whether the black holes' discharge can be considered a good condition to chart the Swampland and so if charged black holes must be able to decay or not. In this respect, showing that stable charged black holes at a given energy scale (which can be, for instance, the scale

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<sup>9</sup>Let us notice that making this argument quantitative is rather difficult. The infinite amount of time requested to measure precisely the black hole's charge (the sphere measuring the flux is at infinity) is an obstacle to the desired quantification. For small gauge couplings the uncertainty on the black hole's charge becomes larger and larger and so the Bekenstein–Hawking entropy may be violated.

that the Magnetic WGC fixes [7]) carry an intrinsic inconsistency would amount to a proof of the Electric WGC.

When dealing with charged black holes, it is interesting to note that they may have a self-instability and therefore no charged particle is requested in order for them to decay. In other words, it is possible that a charged black hole discharges in smaller charged black holes.

For instance, a charged black hole  $(M, Q)$  with horizon area  $A$  can bifurcate in two charged black holes  $(M_1, Q_1)$  and  $(M_2, Q_2)$  with horizon areas  $A_1$  and  $A_2$  (respectively) if

$$M_1 = M_2 = \frac{M}{2}; \quad Q_1 = Q_2 = \frac{Q}{2} \quad (3.33)$$

and (consequently)  $A = A_1 + A_2$ , saturating the constraints  $A \geq A_1 + A_2$ ,  $M \geq M_1 + M_2$  and  $Q = Q_1 + Q_2$  and representing (in this particular case) a decay without the emission of gravitational waves.

Such a cascade of a charged black hole's bifurcations can be followed down towards the Planck scale and problems such as those related to black hole entropy bounds arise.

Considering Einstein–Maxwell theory and including some other massive structure, the low-energy effective theory receives corrections from higher derivative terms, which come out of the integration on the massive structure itself. In this context, there are examples in which extremal black hole solutions do not saturate any more the inequality (3.30) (or (3.25)). The possible success of the efforts in showing that the higher derivative terms increase the charge-to-mass ratio of black holes would amount to prove a formulation of the WGC where the state is a black hole and that can't be valid indeed in the regime in which the state is a particle. In some cases, the structure of the higher derivative terms has been found to be coherent with the idea that the charge-to-mass ratio of extremal black holes is raised above one. This has been recently shown by using arguments of scattering amplitudes' positivity in [10], where also a  $S$ -matrix proof of (a weak version of) the WGC has been provided. Anyway, further work on this line of research is still needed.

These last observations (and [10] too) suggest one of the subtleties of the conjecture that we have proposed at the beginning of this chapter. We have formulated it using the word *particle* meaning a state whose mass is below the Planck scale; in spite of this, the conjecture may be referred to states which are much heavier than  $M_P$ ; they can be regarded as extended objects such as black holes<sup>10</sup>. Another criticism of the statement under analysis is then that the action  $S$  ((3.24)) doesn't fix the gauge coupling by itself; the  $g$  normalization is given by choosing the gauge field normalization to have canonical coupling to matter currents.

Finally, the meaning of the cutoff scale  $\Lambda$  (in (3.26)) is not really made clear by the proposed criterion; as we will explain in the following dealing with the Swampland Distance Conjecture,  $\Lambda$  can be interpreted as the mass scale of an infinite tower of states.

However, before doing this, let us skip to study a refinement of the WGC that will be useful for later purposes.

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<sup>10</sup>This is coherent with the fact that we are motivating a weak version of the WGC: rather than referring to the lightest object in the theory, we are claiming that the conjecture is satisfied by those states with the smallest mass-to-charge ratio. As it seems natural, more evidence favouring the weak version of the WGC rather than the strong one can be provided [10, 26]

### 3.2.1 A refinement of the Weak Gravity Conjecture

Let us now study a Swampland criterion that is profoundly inspired by the Weak Gravity Conjecture and can be considered a refinement of it; it is a stronger condition than the WGC itself and requires additional assumptions.

#### The Weak Gravity Conjecture for dyons with $\theta$ -angle

Dyons are objects that carry both electric and magnetic charge; they can be considered fundamental particles of a given theory if the mutual locality condition is satisfied<sup>11</sup>; otherwise, they must be described like an electric particle and a monopole soliton.

Let's consider the action

$$S = \int d^4x \mathcal{L} = \int d^4x \sqrt{-g} \left[ \frac{M_P^2}{2} R + I_{IJ} F_{\mu\nu}^I F^{J,\mu\nu} + R_{IJ} F_{\mu\nu}^I * F^{J,\mu\nu} \right], \quad (3.34)$$

where the index  $I$  (as well as  $J$ ) runs over the ( $N$ )  $U(1)$  gauge fields with electric field strengths  $F_{\mu\nu}^I$ ; the “ $*$ ” denotes the Hodge star operation ( $*F_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$ ) and  $I_{IJ}$  and  $R_{IJ}$  are the (symmetric) gauge kinetic matrix and the (symmetric)  $CP$ -violating  $\theta$ -angle matrix.

If  $R_{IJ} \neq 0$ , the magnetic field strengths can be introduced as [7]

$$G_{I,\mu\nu} = R_{IJ} F_{\mu\nu}^J - I_{IJ} * F_{\mu\nu}^J. \quad (3.35)$$

The electric and magnetic charges of the particles in the theory are defined (by using a differential form notation) in terms of fluxes passing through a sphere at infinity as

$$Q^I = \frac{1}{4\pi} \int_{S^\infty} *F^I; \quad P^I = \frac{1}{4\pi} \int_{S^\infty} F^I \quad (3.36)$$

and can be grouped in the vector

$$Q = \begin{pmatrix} P^I \\ Q^I \end{pmatrix}.$$

In terms of the quantized charges

$$Q_Z = \begin{pmatrix} p^I \\ q_I \end{pmatrix} \in \mathbb{Z} \times \mathbb{Z}$$

$Q$  is given by

$$Q = \begin{pmatrix} P^I \\ Q^I \end{pmatrix} = \begin{pmatrix} p^I \\ (I^{-1}Rp)^I - (I^{-1}q)^I \end{pmatrix}. \quad (3.37)$$

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<sup>11</sup>For instance, two dyons  $(q,p)$  and  $(q',p')$ , identified by their electric and magnetic charges, are said to be mutually local if  $qp' - q'p = 0$

The charge vector  $Q_Z$  transforms under the symplectic group  $Sp(2N, \mathbb{Z})$  such that [28]

$$Q_Z \longrightarrow Q'_Z = \mathcal{S}Q_Z \quad (3.38)$$

where  $\mathcal{S} \in Sp(2N, \mathbb{Z})$ .

In order to construct invariant quantities under symplectic transformations a matrix  $U$  expressed in terms of the gauge kinetic matrix and the  $CP$ -violating  $\theta$ -angle matrix can be introduced [28]: it is defined as

$$U = \begin{pmatrix} I + RI^{-1}R & -RI^{-1} \\ -I^{-1}R & I^{-1} \end{pmatrix}.$$

The combination

$$Q^2 = -\frac{1}{2}Q_Z^T U Q_Z \quad (3.39)$$

is then invariant under the action of the symplectic group  $Sp(2N, \mathbb{Z})$ .

By making use of  $Q^2$  the **Weak Gravity Conjecture** for a theory whose particles are **dyons** can be formulated as follows [8]:

*A theory, coupled to gravity, with multiple  $U(1)$ s, with gauge kinetic matrix  $I_{IJ}$  and  $CP$ -violating  $\theta$ -angle matrix  $R_{IJ}$  (so with an action as (3.34)) should have a particle with mass  $m$  satisfying the inequality*

$$Q^2 M_P^2 \geq m^2. \quad (3.40)$$

A first check of this statement comes from the observation that for an electrically charged particle in the presence of a single  $U(1)$  gauge symmetry  $Q^2 = 2g^2 q^2$  and the (4-dimensional) bound (3.25) is restored.

Moreover, by interpreting (as it seems sensible to do)  $Q^2$  as the strength of the repulsion between two dyonic particles and  $m^2$  as their attraction, the inequality (3.40) coherently expresses the weakness of gravity. It is also interesting to note that dyonic black hole solutions referred to the action (3.34) satisfy the extremality bound  $M_{BH}^2 \geq Q^2 M_P^2$ ; but, characterizing the particles such black holes decay in is a complicated task.

### 3.3 The Swampland Distance Conjecture

Beside the Weak Gravity Conjecture, another relevant criterion the Landscape can be distilled out by the Swampland through is the **Swampland Distance Conjecture** (SDC).

It can be formulated in the following way [29]:

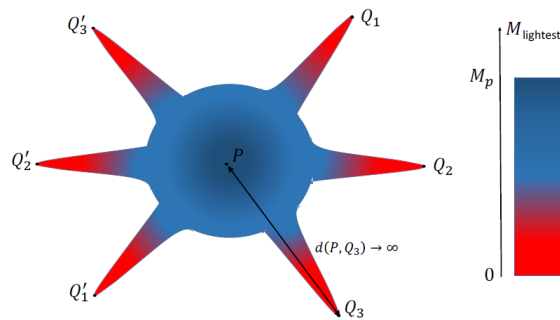


Consider a theory, coupled to gravity, with a moduli space  $\mathcal{M}_\phi$  which is parametrized by the expectation values of some fields  $\{\phi^i\}_i$  which have no potential. Starting from any point  $P \in \mathcal{M}_\phi$  there exists another point  $Q \in \mathcal{M}_\phi$  such that the geodesic distance between  $P$  and  $Q$ , denoted as  $d(P, Q)$ , is infinite. Moreover, there exists an infinite tower of states with an associated mass scale  $M$  such that

$$M(Q) \sim M(P)e^{-\alpha d(P, Q)}, \quad (3.41)$$

where  $\alpha$  is some positive constant.

The structure of the moduli space  $\mathcal{M}_\phi$  appearing in the conjecture can be depicted as **Figure 3.2** does.



**Figure 3.2** [7]:

This figure illustrates schematically the structure of a string moduli space. The distance from any point  $P$  in the bulk to any point  $Q_i$  or  $Q'_i$  is infinite.  $M_{lightest}$  denotes the mass scale of the lightest tower of states in the theory: it goes to zero at any point  $Q$ . The points  $Q_i$  and  $Q'_i$  are related by duality and the light towers of states are interchanged between them.

As it is explained below, support to this statement is given by referring to String Theory and String Theory compactification.

It is known that the characterization of the geodesic motion of a point-like particle in an arbitrary frame within a spacetime which is not necessarily flat is given by the Polyakov action

$$S_P = -\frac{1}{2} \int_\gamma d\tau \sqrt{-\gamma_{\tau\tau}} \left[ \gamma^{\tau\tau} \frac{dX^\mu}{d\tau} g_{\mu\nu}(X) \frac{dX^\nu}{d\tau} + m^2 \right], \quad (3.42)$$

where  $X^\mu$  are the spacetime coordinates;  $g_{\mu\nu}(X)$  is the spacetime metric;  $\gamma$  is the  $\tau$  parameterized world-line of the particle (of mass  $m$ , including  $m = 0$ ) we are studying the motion of and  $\gamma_{\tau\tau}$  is the metric on its world-line.

The action (3.42) can be easily generalized to the description of the geodesic motion of extended objects such as strings. A string sweeps out a world-sheet  $\Sigma$  which is parameterized by two coordinates  $(\sigma, \tau)$ . Indeed,

$$\Sigma : (\sigma, \tau) \longrightarrow X^\mu(\sigma, \tau) \in \mathbb{R}^{1, D-1}, \quad (3.43)$$

being  $0 \leq \sigma \leq 2\pi$  with  $\sigma = \sigma + 2\pi$  (as far as closed strings are concerned) and  $\tau \in \mathbb{R}$ .

By denoting by  $(\xi^a)_{a=0,1} = (\sigma, \tau)$  the string world-sheet coordinates, the string dynamics is described by the Polyakov action

$$S_P = -\frac{T}{2} \int_{\Sigma} d^2\xi \sqrt{-\det h} h^{ab}(\xi) \partial_a X^\mu \partial_b X^\nu g_{\mu\nu}, \quad (3.44)$$

where  $h_{ab}$  is the metric on the string world-sheet and  $T$  is the string tension,  $T = \frac{1}{2\pi\alpha'}$  (with  $\alpha'$  the square of the string length).

The action (3.44) defines a two-dimensional theory with scalar fields  $X^\mu(\xi)$  parameterizing the spacetime (called target-space) in which the string propagates.

By making use of (3.44)'s invariance under local diffeomorphisms  $\xi^a \rightarrow \tilde{\xi}^a(\xi)$  and Weyl transformations  $\delta X^\mu = 0$  and  $h_{ab} \rightarrow \tilde{h}_{ab} = e^{2\Lambda(\xi)} h_{ab}$  and so moving to the flat gauge; once the light-cone coordinates ( $\xi^\pm = \tau \pm \sigma$ ) have been introduced, the quantization of the string can be performed quite straightforwardly. The string spectrum can be characterized too: it consists of a tachyonic mode; the massless symmetric tensor  $g_{ij}$ , called graviton; the Kalb–Ramond field  $B_{ij}$ , which is an antisymmetric traceless tensor; a massless scalar (called dilaton)  $\Phi$  and massive oscillator string modes. By requiring then the conservation of the target-space Lorentz invariance at quantum level the quantum bosonic string appears to be consistent only in  $D = 26$  dimensions ([7]).

When supersymmetry enters the game, the tachyonic mode of the bosonic string disappears and the number of dimensions a superstring is consistent upon is reduced to  $D = 10$  ([7]).

The fact that the bosonic string and the superstring live in  $D = 26$  and in  $D = 10$  dimensions (respectively) is inconsistent with the observed Universe. To solve this contradiction it can be thought that the additional (or extra) dimensions are compact and small such that they can't be perceived. This is the basic principle behind string compactifications.

Let us consider a  $(D = d + 1)$ -dimensional spacetime and a compactification on a circle: the spatial direction  $X^d$  is taken to be compact on the shape of a circle. By working in Planck units  $M_P^d = 1$  (and so by making masses and lengths adimensional), the  $X^d$  periodicity can be fixed by

$$X^d \simeq X^d + 1. \quad (3.45)$$

We are interested in studying the effective theory in the  $d$  non-compact dimensions. The metric on the  $D$ -dimensional spacetime can be written as a product metric as

$$ds^2 = G_{MN} dX^M dX^N = e^{2\alpha\phi} g_{\mu\nu} dX^\mu dX^\nu + e^{2\beta\phi} (dX^d)^2 \quad (3.46)$$

with  $M, N = 0, \dots, d$  and  $\mu, \nu = 0, \dots, d-1$ . The metric contains a parameter  $\phi$  which can be regarded as a  $d$ -dimensional scalar field. The constants  $\alpha$  and  $\beta$  are such that

$$\beta = -(d-2)\alpha; \quad \alpha = \sqrt{2} \left( \frac{d-1}{d-2} \right)^{\frac{1}{2}} \quad (3.47)$$

to make (the Ricci sector of) the  $D$ -dimensional action

$$\int d^D X \sqrt{-G} e^{-2\Phi} R^D, \quad (3.48)$$

in which  $\Phi$  is the dilaton and  $R^D$  is the Ricci scalar in  $D$  dimensions, to (dimensionally) reduce to

$$\int d^d X \sqrt{-g} \left[ R^d - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi \right], \quad (3.49)$$

where  $\phi$  is canonically normalized. The circumference of the circle  $X^d$  is constricted on is given by

$$2\pi R = \int_0^1 \sqrt{G_{dd}} dX^d = e^{\beta\phi}. \quad (3.50)$$

It turns out that the radius  $R$  is a dynamical field in  $d$  dimensions. It is therefore interesting to study the behaviour of the  $d$ -dimensional theory under variations of the size of the compactification circle, which amount to variations of the field  $\phi$ 's expectation value.

Let us consider now a massless  $D$ -dimensional scalar field  $\Psi$  and decompose it as

$$\Psi(X^M) = \sum_{n=-\infty}^{+\infty} \psi_n(X^\mu) e^{2\pi i n X^d}, \quad (3.51)$$

by exploiting the fact that the  $X^d$  periodicity translates into the periodicity of  $\Psi$  along  $X^d$ . The mode  $\psi_0$  is called zero-mode, whereas the  $\{\psi_n\}_n$  are named as the Kaluza–Klein (KK) modes.

By assuming (for simplicity)  $g_{\mu\nu} = \eta_{\mu\nu}$ <sup>12</sup>; by noting the quantization of the momentum along  $X^d$

$$-i \frac{\partial}{\partial X^d} \Psi = 2\pi n \Psi \quad (3.52)$$

(with  $n \in \mathbb{Z}$ ) and by passing through the equation of motion for  $\Psi$

$$\partial^M \partial_M \Psi = 0, \quad (3.53)$$

the mass of the KK modes can be extracted

$$M_n^2 = \left(\frac{n}{R}\right)^2 \left(\frac{1}{2\pi R}\right)^{\frac{2}{d-2}}. \quad (3.54)$$

The  $d$ -dimensional theory has a massive tower of states with increasing mass, called the KK tower. If gaining evidence on the KK tower of states has involved field theory considerations, it is interesting to move to String Theory and to the analysis of strings on a circle of radius  $R$ . Having in mind the string quantization and the adoption of light-cone coordinates, the spacetime coordinates  $X^M$  can be decomposed in right and left movers along the string

$$X^M = X_L^M(\xi^+) + X_R^M(\xi^-) \quad (3.55)$$

with (in principle) independent momenta  $p_L^M$  and  $p_R^M$ .

In the string frame, where part of the reference action is of the kind of (3.48) with a common function of the dilaton multiplying the Ricci sector (and the matter one too), we then impose

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<sup>12</sup> $\eta_{\mu\nu}$  is the flat metric; by convention the signature  $\eta_{\mu\nu} = \text{diag}(-1, +1, \dots, +1)$  is adopted in this chapter.

$$X_{(s)}^\mu(\sigma + 2\pi, \tau) = X_{(s)}^\mu(\sigma, \tau) \quad (3.56)$$

for the non-compact directions and

$$X_{(s)}^d(\sigma + 2\pi, \tau) = X_{(s)}^d(\sigma, \tau) + \omega 2\pi R \quad (3.57)$$

(with  $\omega \in \mathbb{Z}$ ) for the circle a winding string is wrapping around.

By exploiting the relations

$$p_L^\mu = p_R^\mu; \quad \frac{\alpha'}{2}(p_L^d - p_R^d) = \omega R \quad (3.58)$$

the spectrum of the string in such a background can be built. In the Einstein frame, where the metric is scaled such that there is no dilaton function multiplying the Ricci scalar (and the matter action) in a reference action starting as (3.48), the spectrum is

$$M_{n,\omega}^2 = \left(\frac{1}{2\pi R}\right)^{\frac{2}{d-2}} \left(\frac{n}{R}\right)^2 + (2\pi R)^{\frac{2}{d-2}} \left(\frac{\omega R}{\alpha'}\right)^2 \quad (3.59)$$

with  $M_P^d = 1$ . There, not only the KK tower can be recognized but also another tower of states manifests itself: this is the winding modes' tower.

Having characterized the spectrum of states of the  $d$ -dimensional theory, we would like to study how it changes under variations of the expectation value of the field  $\phi$ .

The expectation values of  $\phi$  sweep out a space  $\mathcal{M}_\phi$  which, in the special example we are treating, has an infinite real dimension.

Energy scales can be associated to the two towers of states that the theory exhibits, the tower of the Kaluza–Klein modes and the winding modes' tower. In particular, we have

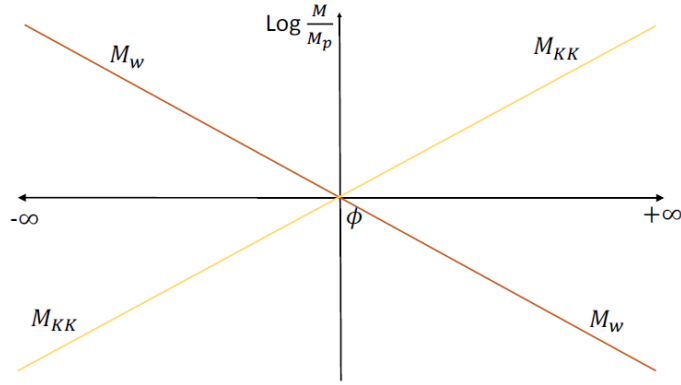
$$M_{KK} \sim e^{\alpha\phi}; \quad M_\omega \sim e^{-\alpha\phi}, \quad (3.60)$$

$\alpha$  being a  $\mathcal{O}(1)$  parameter.

After having introduced  $\Delta\phi = \phi_f - \phi_i$ , we can state that for any  $\Delta\phi$  there exists an infinite tower of states with mass scale  $M$  that becomes light at an exponential rate in  $\Delta\phi$ :

$$M(\phi_i + \Delta\phi) = M(\phi_i)e^{-\alpha|\Delta\phi|}. \quad (3.61)$$

In the attempt of understanding such behaviour as deeply connected to String Theory it is worth noting that the KK tower and the winding modes' one are strictly related. There is a  $\mathbb{Z}_2$  symmetry, called T-duality, which interchanges them. This is manifest in the string frame when substituting  $R$  with  $\frac{\alpha'}{R}$  and viceversa. The dual KK tower and the winding modes' tower of states are such that the product of their mass scales is independent on  $\phi$  and, whatever being the sign of  $\Delta\phi$ , one of the two towers becomes lighter and lighter proceeding towards infinite distances in the parameter space  $\mathcal{M}_\phi$ . When  $|\Delta\phi| \rightarrow +\infty$ , an infinite number of states becomes massless and therefore the  $d$ -dimensional effective description of such a locus of  $\mathcal{M}_\phi$  breaks down.



**Figure 3.3** [7]:

The figure shows in a log-plot the mass scales for the towers of Kaluza–Klein and winding modes as functions of the field  $\phi$ 's expectation value. Because of T-duality the figure is symmetric.

Extrapolating, within the study of the spectrum of an effective theory coming from String Theory, it is quite natural to expect many towers of states which are mutually dual. Because of such a duality, when moving in the moduli space of the theory, the product of the mass scales of two dual towers is constant and one of them should become light in any direction. This implies that an Effective Field Theory with a cutoff scale  $\Lambda$  below the mass scale of an infinite tower of states could have only a finite range of validity in the parameter space.

These arguments can be well regarded as a motivation to the formulation of the SDC we have proposed at the beginning of the chapter.

However, it is worth noting that statement is strongly influenced by String Theory and, as a consequence, it may not be applied in more general circumstances: for instance, the existence of points at infinite distance in the parameter space is not as immediate as it could seem.

We would like to reformulate the previous version of the SDC ((3.41)) trying to generalizing it. In order for this purpose to be achieved the starting point is a more detailed discussion concerning the moduli space  $\mathcal{M}_\phi$ .

Let us consider the action

$$S = \int d^d X \sqrt{-g} \left[ R^d - g_{ij} \partial \phi^i \partial \phi^j + \dots \right], \quad (3.62)$$

where the expectation values of the fields  $\{\phi^i\}_{i=1, \dots, \dim_{\mathbb{R}} \mathcal{M}_\phi}$  (with no potential) are coordinates of the moduli space  $\mathcal{M}_\phi$ , whose metric is  $g_{ij}$ .

A point  $P \in \mathcal{M}_\phi$  is fixed by specifying the expectation vales of the fields  $\{\phi^i\}_i$ .

The geodesic distance between two such points  $P$  and  $Q$  is defined as

$$d(P, Q) = \int_\gamma ds \left( \frac{\partial \phi^i}{\partial s} g_{ij} \frac{\partial \phi^j}{\partial s} \right)^{\frac{1}{2}}, \quad (3.63)$$

being  $\gamma$  the shortest geodesic connecting  $P$  and  $Q$  and  $ds$  the line element along  $\gamma$ .

In the first formulation of the SDC that we have given it is stated that for any point  $P \in \mathcal{M}_\phi$  there exists a point  $Q \in \mathcal{M}_\phi$  at infinite distance. This is the case for a moduli space such as  $\mathcal{M}_\phi = \mathbb{R}$  (as already observed); but if  $\mathcal{M}_\phi = S^1$  (for example) the conjecture does not apply; and this occurs even though periodic moduli are allowed in Quantum Gravity. To solve this apparent contradiction the idea is that periodic scalar fields have to be thought of as part of a larger parameter space: they are not the only fields defining the moduli space of the theory.

Having in mind String Theory compactification on a circle, we have been led to (3.61). However this relation can't be expected to hold generically in the moduli space of a theory of quantum gravity. For instance, if we consider a periodic direction  $\hat{\phi}$  in the moduli space, the mass scale  $M$  is a periodic function of  $\hat{\phi}$ ,  $M(\hat{\phi} + 2\pi) = M(\hat{\phi})$ ; so, for any point  $P$  there is a point  $Q$  such that  $d(P, Q) > 0$  and  $M(Q) \geq M(P)$ .

In spite of the complicated geometry that can characterize the moduli space, its basic structure can be depicted as **Figure 3.2** schematically does.

On such a background the SDC has to be intended as a statement about what happens in the asymptotic regions of the parameter space  $\mathcal{M}_\phi$ . The construction of the conjecture is therefore animated by the question on what the magnitude of  $d(P, Q)$  should be in order for the exponential behaviour of a tower of states' mass scale to be a good approximation for any starting point  $P$ . In principle, this would also lead to a bound on the value of  $\alpha$ : if  $\alpha \ll 1$ , it is possible to have  $d(P, Q) \gg 1$  (in Planck units) without a relevant change in the tower mass scale. Although, no precise statements which constrain  $\alpha$  have been formulated yet and  $\alpha$  is considered to be of  $\mathcal{O}(1)$ .

Trying to summarize all these refining arguments in a single statement we can propose the following **refined** version of the **Swampland Distance Conjecture** [30, 31]:

*Consider a theory, coupled to gravity, with a moduli space  $\mathcal{M}_\phi$  which is parametrized by the expectation values of some fields that have no potential. Let the geodesic distance between any point  $P \in \mathcal{M}_\phi$  and another point  $Q \in \mathcal{M}_\phi$  be denoted by  $d(P, Q)$ . There exists an infinite tower of states with mass scale  $M$  such that*

$$M(Q) < M(P)e^{\frac{-\alpha d(P, Q)}{M_P}} \quad (3.64)$$

*if  $d(P, Q) \gtrsim M_P$ . Moreover, the previous statement holds not only for moduli but also for fields with a potential, where the moduli space is replaced by the field space of the effective theory.*

Before concluding this section, let us note that the first part of the conjecture is related to the original one but different because of the sharp sign characterizing (3.64). With respect to (3.61) the mass scale  $M$  is allowed to decay faster than exponentially in  $d(P, Q)$ . Another distinction is made by the tight condition on  $d(P, Q)$ : the exponential behaviour sets in at  $1-2M_P$ , whereas it is not really a good approximation at distances of (say)  $10M_P$ . As a consequence, before the exponential decrease the behaviour has to be such that the mass scale doesn't increase too much. As an aside remark, let us observe that this actually poses an implicit constraint on  $\alpha$ .

The second part of the conjecture is instead different from its original formulation because it accounts for the possibility that the fields are subjected to a potential. The construction of a field distance conjecture for fields with a potential has as a crucial prerequisite the formulation of a precise statement on finite distances. In fact, it is not clear if an asymptotic point at infinite distance exists within the effective theory and it can be shown that the cutoff  $\Lambda$  of the effective theory bounds the fields  $\{\phi^i\}_i$  to be such that  $V(\phi)^{\frac{1}{4}} < \Lambda$ .

### 3.4 The de Sitter Conjecture

At the beginning of 20<sup>th</sup> century the first steps for the construction of a scientific theory describing the Universe and its properties were accomplished. Since there were no well-structured empirical basis to found these theories upon, some leading principles were adopted.

Having in mind that it is possible to reduce the degrees of freedom of a system by exploiting symmetries, the *Cosmological Principle* was formulated to model the Universe, its kinematics and its dynamics. It states that

*Any comoving observer observes the Universe around itself at (cosmic) time fixed (in its reference frame) to be isotropic and homogeneous on average.*

An observer is said to be *comoving* if it moves integrally with the source of the geometry of the Universe. Practically, a comoving observer is one that measures the Cosmic Microwave Background (CMB) to be isotropic at per million level (and so apart from the intrinsic anisotropies)<sup>13</sup>. The *cosmic time* is the proper time of comoving observers. The properties of average *isotropy* and *homogeneity* are referred to the mass-energy distribution on great scales, when observing the Universe with small spatial resolution. The hypothesis of isotropy is confirmed (at an appropriate precision level) by experiments revealing CMB or the abundance of elements such as Helium or measuring the isotropy in the statistic properties in the scattering of galaxies. On the contrary, because of our limited ability in the direct exploration of the Universe, the hypothesis of homogeneity can't be tested experimentally on large scales and has to be posed. To understand the hypothesis of homogeneity in the part of the Universe we have in principle access to a principle of General Relativity can be used: it claims that isotropy around any (comoving) observer at time fixed is equivalent to homogeneity.

The Cosmological Principle is an abstract statement that is not actually realistic, but it is really helpful in writing down the equations governing the dynamics of the Universe itself.

The Universe is composed by a four-dimensional spacetime with a maximally symmetric three-dimensional space. Spatial rotations and translations surviving as invariance properties, the cosmological spacetime symmetry group has six generators. With respect to Minkowski spacetime, because

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<sup>13</sup>Around the end of the '60s and the beginning of the '70s a dipole anisotropy of CMB (whose mean temperature is 2.725K) was measured: CMB is "hotter" along a direction and "colder" in the opposite one, at per mill level. The Earth is not a comoving reference frame with respect to the average mass-energy distribution of the Universe; and even taking into account the motion of the Earth around the Sun, of the Sun referred to the center of mass of our Galaxy and of the Milky Way with respect to the Local Group of Galaxies, a residual dipole anisotropy remains: it can be interpreted as the result of the Doppler Effect due to the velocity of the Local Group relative to an observer moving with CMB. This velocity is estimated to be 600Km/s.

the Universe is expanding and there is a privileged reference frame (that of comoving observers) due to the presence of cosmic matter and energy, time translation invariance and Lorentz boost invariance are lost as symmetries.

Coherently with the Cosmological Principle the geometric properties of the Universe are described thanks to the so called Robertson–Walker’s metric that can be expressed as

$$ds^2 = c^2 dt^2 - a(t)^2 \left[ \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right], \quad (3.65)$$

whit  $d\Omega^2 = \sin^2 \theta d\theta^2 + d\phi^2$ ,  $(t, r, \theta, \phi)$  being the coordinates adapted to a comoving observer. The coordinate  $r$  is adimensional and  $k$  is an adimensional constant that can take three values:  $-1$ ,  $0$  or  $+1$ . They correspond to the three equivalence classes of (would be) geometries of the Universe:  $k = -1$  stands for the infinite set of open and negative curvature spaces;  $k = 0$  denotes the case of a spatially flat universe and  $k = +1$  groups the infinite class of close and positive curvature spaces. The factor  $a(t)$  (which has the dimensions of a length) allows to describe the expansion or the contraction of the Universe and is named *scale factor*.

After having chosen (3.65) as spacetime metric, Einstein’s equations

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu} \quad (3.66)$$

(where  $g_{\mu\nu}$  is the metric,  $R_{\mu\nu}$  is the Ricci tensor,  $R$  is the Ricci scalar and  $T_{\mu\nu}$  is strength energy tensor) result in the so called Friedman’s equations

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3} \rho - \frac{kc^2}{a^2} \quad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left( \rho + 3\frac{P}{c^2} \right) \quad \dot{\rho} = -3\frac{\dot{a}}{a} \left( \rho + \frac{P}{c^2} \right) \quad (3.67)$$

(denoting with the “ $\cdot$ ” the derivative with respect to the cosmic time)<sup>14</sup>.

In (3.67)  $\rho$  and  $P$  are the energy density and the isotropic pressure of the constituents of substance of the Universe. They can be modelled as perfect fluids characterized by the equation of state

$$P = w\rho c^2, \quad (3.68)$$

$w$  being a constant depending on the constituent.

By evaluating the first of the equations in (3.67) ignoring the spatial curvature term, a critical energy density

$$\rho_c(t) = \frac{3H(t)^2}{8\pi G} \quad (3.69)$$

(where  $H(t) = \frac{\dot{a}}{a}$  is the Hubble parameter) can be defined. With  $\rho_c$  the measurable quantity

$$\Omega(t) = \frac{\rho(t)}{\rho_c(t)} \quad (3.70)$$

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<sup>14</sup> $a$  is different from 0 at any time after the Big Bang, if the Big Bang occurred.



can be introduced:  $\Omega(t)$  is the density parameter at cosmic time  $t$ . The Planck Mission managed to estimate the deviation from 1 of the total density parameter of the Universe “today” (at  $t_0$ ). It is

$$\Omega_{tot}(t_0) - 1 = -0.001 \pm 0.002. \quad (3.71)$$

When Friedman’s equations were written down first, scientists thought that the Universe was made of ordinary matter. Then, after the surprising observation and analysis of the rotational curves of spiral galaxies (at the beginning of the '70s), the existence of another constituent, called dark matter (DM), was proposed (and confirmed later on by solid evidence coming, for example, from the study of nucleosynthesis processes and the formation of clusters of galaxies).

By consistency between theory and experiments,  $\Omega_{matter}(t_0)$  can be fixed to be  $\Omega_{matter}(t_0) \sim 0.05$ <sup>15</sup> and  $\Omega_{DM}(t_0)$  can be set to be  $\Omega_{DM}(t_0) \sim 0.25$ . If one accounts for these components of substance only, the trustful experimental result (3.71) can’t be reproduced.

The inclusion of radiation (CMB) and massive neutrinos which contribute with  $\Omega_{radiation}(t_0) \sim 10^{-5}$  and  $\Omega_{neutrinos}(t_0) \sim 10^{-4}$  (respectively) to the evaluation of the energetic budget of the Universe doesn’t solve the problem.

The analysis of the anisotropies of CMB seem then to suggest the existence of another constituent of the Universe: it is named dark energy (DE).

As for dark matter, we don’t know what dark energy really is. A way to interpret dark energy was unwillingly given by Einstein.

At the beginning of the 20<sup>th</sup> century the scientific community was debating on the staticity of the Universe: the majority of scientists (and Einstein too) thought that the Universe was static and only a few were convinced that the Universe had to be dynamic.

If the Universe is composed by matter (as it was originally believed), a static universe can’t be regarded as a solution of Einstein’s equations. This can be easily seen by requiring  $P = 0$  (for matter) and  $\dot{a} = \ddot{a} = 0$  in (3.67).

Having noticed that and afraid of the fact the static universe couldn’t be a solution of his equations, Einstein decided to modify them. He proposed

$$G_{\mu\nu} - \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}, \quad (3.72)$$

where  $\Lambda$  is the so called *cosmological constant*.

Einstein introduced the cosmological constant as a modification of the Universe spacetime geometry. By moving  $\Lambda g_{\mu\nu}$  to the right hand side of (3.72), the cosmological constant term can be intended (*a posteriori*) as an ingredient participating to the definition of the substance content of the Universe. In this respect, the original strength energy tensor  $T_{\mu\nu} = \text{diag}(\rho, -P, -P, -P)$  has to be substituted with  $\tilde{T}_{\mu\nu} = \text{diag}(\tilde{\rho}, -\tilde{P}, -\tilde{P}, -\tilde{P})$ <sup>16</sup>, where

$$\tilde{\rho} = \rho + \frac{\Lambda c^2}{8\pi G}; \quad \tilde{P} = P - \frac{\Lambda c^4}{8\pi G}. \quad (3.73)$$

<sup>15</sup>In order for the abundance of elements (such as  $He^4$ ,  $Li^3$ ,  $H^3$  or  $H^2$ ) in the Universe to be as observations state, the theory of nucleosynthesis imposes that  $0.011 < \Omega_{matter}(t_0) h^2 < 0.025$ , where  $h$  is a constant giving  $H(t_0)$  as  $H(t_0) = 100 h(\text{km/s})/\text{Mpc}$ .

<sup>16</sup>This, in the convention  $\eta_{\mu\nu} = \text{diag}(+1, -1, -1, -1)$ .

If one repeats the calculation that has led to (3.67) from Einstein's equations with  $T_{\mu\nu}$  for the modified strength energy tensor  $\tilde{T}_{\mu\nu}$ , Friedman's equations

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\tilde{\rho} - \frac{kc^2}{a^2} \quad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\tilde{\rho} + 3\frac{\tilde{P}}{c^2}\right) \quad \dot{\tilde{\rho}} = -3\frac{\dot{a}}{a}\left(\tilde{\rho} + \frac{\tilde{P}}{c^2}\right) \quad (3.74)$$

are obtained. They are the analogue of (3.67) but with the replacements  $\rho \rightarrow \tilde{\rho}$  and  $P \rightarrow \tilde{P}$ . Convinced that a static universe should exist, Einstein required  $P = 0$  (for matter) and  $\dot{\rho} = \dot{a} = \ddot{a} = 0$  and found the desired static solution corresponding to a closed universe with the cosmological constant given in terms of the scale factor as  $\Lambda = \frac{1}{a^2}$ . But Friedman noticed soon that this solution was unstable and Einstein claimed that the introduction of the cosmological constant was the greatest mistake of his life.

As already mentioned, even though Einstein's idea of the cosmological constant was profoundly wrong, the cosmological constant can be regarded as a constituent of the Universe. More precisely, dark energy can be described as a cosmological constant participating to the energy budget of the Universe today with  $\Omega_{DE}(t_0) \sim 0.70$  (as the study of the CMB's anisotropies and the analysis of how galaxies group together indirectly suggest).

The recent cosmological observation of the CMB and the experimental data relative to the spectra of Supernovae of Type IA allow to conclude that our Universe is entering a phase of accelerated expansion [32–34]<sup>17</sup>. Since an ordinary matter or dark matter distributions give rise to an attractive gravitational field, in order to have

$$\ddot{a} > 0 \quad (3.75)$$

the second Friedman's equation requires an exotic substance, whose isotropic pressure is (sufficiently) negative

$$P < -\frac{1}{3}\rho c^2. \quad (3.76)$$

Dark energy in the form of the cosmological constant plays this role: in fact, it satisfies

$$P_{DE} = w_{DE}\rho_{DE}c^2 = -\rho_{DE}c^2 \quad (3.77)$$

(as it can be deduced from (3.73)).

In the presence of the cosmological constant only and so imposing  $P = \rho = 0$ , the relevant Friedman's equations (3.74) become

$$\frac{\dot{a}^2}{a^2} = \frac{\Lambda c^2}{3} - \frac{kc^2}{a^2} \quad \frac{\ddot{a}}{a} = -\frac{\Lambda c^2}{3}. \quad (3.78)$$

If, for simplicity, the spatial curvature term is ignored, one obtains

$$a(t) = e^{Ht}, \quad (3.79)$$

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<sup>17</sup>To have successful nucleosynthesis in the radiation-dominated era and an appropriate ambience for the formation of cosmic structures during the matter-dominated epoch, the present acceleration of the Universe has started during a recent past.

where

$$H = \left( \frac{\Lambda c^2}{3} \right)^{\frac{1}{2}} = \text{constant}. \quad (3.80)$$

This is the de Sitter solution of Einstein's equations when the cosmological constant dominates and the spatial curvature is negligible.

When DE is regarded as vacuum energy<sup>18</sup>, a great problem emerges.

As precised above, experimental observations set the DE energy density to be roughly

$$\rho_{DE}^{exp} \sim 0.7\rho_0^c \sim 0.7(3 \times 10^{-3} \text{eV})^4 \sim 10^{-11} \text{eV}^4, \quad (3.81)$$

where  $\rho_0^c$  is the critical energy density today.

As the estimate (3.81) suggests,  $\rho_{DE}$  is tiny with respect to the typical energy scales of Particle Physics (ignoring neutrinos' mass scales).

By introducing then a cut-off scale at  $M_P$  for instance (so that quantum gravity effects modifying the behaviour of the theory in the  $UV$  can be ignored) the theoretical expectation for the DE energy density is

$$\rho_{DE}^{th} \sim M_P^4 \sim (10^{19} \text{GeV})^4 = 10^{112} \text{eV}^4. \quad (3.82)$$

It can be easily seen that  $\rho_{DE}^{th}$  is more less 123 orders of magnitude greater  $\rho_{DE}^{exp}$ . This incredible discrepancy between what theoretical predictions and experimental results suggest is known as the *Cosmological Constant Problem* [33, 37, 38].

Even accounting for supersymmetry and its breaking in the real world, a discrepancy of between 50 and 60 orders of magnitude is left.

The Cosmological Constant Problem remains (in a milder form) also when exploring the possibility DE is not due to vacuum energy.

As already observed in the **Introduction**, String Theory predicts (once fluxes are turned on)  $\mathcal{O}(10^{600})$  vacua. In order to face the apparently lack of predictive value String Theory seems to be characterized by, three approaches are viable.

One possibility is to not caring that there are  $\mathcal{O}(10^{600})$  vacua: independently on how we have reached it, we are in a vacuum and we can simply try to describe and understand what happens in its own vicinity.

Another approach consists in thinking that there is actually a mechanism that operates a selection among the  $\mathcal{O}(10^{600})$  universes String Theory gives rise to: by studying the Swampland program we can endeavour to gain comprehension on how such a mechanism works and on how our Universe has been selected.

The third possibility founds itself on the observation that not all the stringy  $\mathcal{O}(10^{600})$  vacua are compatible with "life" in the form we know: there are some conditions that have to be satisfied in

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<sup>18</sup>It is worth noting that, (also) because dark energy can't be observed directly, its actual composition is still unknown. Despite of being considered as a cosmological constant, there are other possible DE candidates [35, 36]. They are all characterized by negative pressure and are able to drive the accelerated expansion of the Universe.

order for observers to exist and only a few universes respect such constraints. This approach is based on the so called *Anthropic Principle*: among the “jungle” of possible vacua originating from String Theory, the only ones we should care of are those that we can in principle inhabit [37,38].

An attempt to deal with the Cosmological Constant Problem consists in making reference to the Anthropic Principle as it has just been stated. The Landscape selected by the Anthropic Principle provides a very large (but discrete!) number of vacua where the cosmological constant can take a value that is as small as anthropic arguments tell us it should be. More precisely, the idea behind the anthropic selection solution to the Cosmological Constant Problem is that a scalar field “sitting” on the profile of a scalar potential (induced, for instance, by a compactification of the underlying higher dimensional theory) gives rise to an expanding universe with a certain value of the cosmological constant; because of its fluctuations, the scalar field may be subjected to a phase transition that brings it to a new local minimum configuration (where the potential’s value is less than it was previously): this determines other subuniverses (vacuum bubbles) that are characterized by a different value of the cosmological constant (and so on). By waiting sufficiently long, the majority of the regions of the Universe (at least those we are in causal contact with) are characterized by the present value of the cosmological constant. In this framework a de Sitter vacuum of String Theory is meant to be a vacuum that is a local minimum of an appropriate scalar potential whose value at the minimum itself is positive.

The present acceleration epoch that our Universe is undergoing may be due to a positive cosmological constant.

It seems very difficult to construct de Sitter vacua in String Theory and this may be due to the fact that the starting theory is supersymmetric whereas de Sitter space is not or because de Sitter vacua require the stabilization of all the moduli in the theory but there are no well-understood mechanisms to do so [7].

The attitude in facing these difficulties might be to consider them as just technical problems or as a substantial obstruction to the construction of de Sitter vacua in String Theory<sup>19</sup>.

In the first case, before the technical difficulties would be overcome, only reasonable proposals on how de Sitter vacua can be constructed within String Theory (as the KKLT ones [7]) can be formulated. If (instead) the second circumstance realizes, de Sitter vacua fall in the Swampland.

The possibility that String Theory doesn’t admit de Sitter vacua seems to be in contrast with the experimental results that show that the Universe is entering a late-time acceleration phase. However, as inflation, which was a primordial phase of accelerated expansion that our Universe has passed through, is likely led by a scalar field rolling down a potential [39]<sup>20</sup>, it is reasonable to think that such a mechanism may allow to describe also the expansion of the Universe “today”. This scenario

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<sup>19</sup>As aside comment, it is important to notice that there is no evidence for de Sitter spacetime in the physical Universe. Then, if a de Sitter spacetime condition would ever be reached in the future because of the decaying with increasing  $a$  of the energy density of all the substance components of the Universe but DE (as vacuum energy) whose energy density is constant, this could happen only asymptotically. So, the fact that de Sitter vacua are or are not admitted by String Theory may be actually considered a relatively crucial problem.

<sup>20</sup>The New Inflation model proposed by A. Guth was characterized by the so called “graceful exit” problem, according to which the phase transition to the true vacuum was never complete in a sizeable part of the actual volume of the Universe [42]. To get out of this puzzle A. D. Linde introduces an inflationary model where a scalar field slowly rolls down its potential: this ensures that there is sufficient time available for the phase transition throughout the actual volume of the Universe.

is known as Dynamical Dark Energy (DDE) or quintessence [40, 41]. By adopting the perspective of DDE or quintessence models to explain the present cosmological acceleration epoch, de Sitter vacua may fall in the Swampland and no contradiction with cosmological observations can arise.

Having in mind a DDE scenario and recovering the anthropic selection solution to the Cosmological Constant Problem (which is not lost, if de Sitter vacua are in the Swampland), we can then think that String Theory may allow for a landscape of potentials which have flat enough regions to lead to the accelerated expansion of the Universe and that anthropic arguments can limit the magnitude of the potential in those regions.

The idea that String Theory does not allow for de Sitter vacua has recently gained impetus thanks to a proposal for a constraint that potentials that are in the Landscape have to obey. Animated by examples coming from String Theory, the **de Sitter Conjecture** (dSC) states that [43]

*The scalar potential of a theory coupled to gravity must satisfy a bound on its derivative with respect to the scalar fields*

$$|\nabla V| \geq \frac{C}{M_P} V, \quad (3.83)$$

where  $|\nabla V|$  is the norm of the vector of derivatives of  $V$  with respect to the scalar fields in the theory and  $C$  is a constant of  $\mathcal{O}(1)$ .

Even though the conjecture doesn't fix the value of the constant  $C$ , the experimental data concerned with the present acceleration of the Universe pose  $C < 0.6$ .

The de Sitter Conjecture in the form that we have just proposed is incoherent with the Standard Model. As [44] shows, the top of the Higgs potential would violate (3.83):

$$\frac{|\nabla V|}{V} \sim \frac{10^{-55}}{M_P}. \quad (3.84)$$

In order to avoid the possible counter-examples coming from the Standard Model and extensions of it [45] a refinement of (3.83) has to be found. The **Refined de Sitter Conjecture** is [43]:

*The scalar potential of a theory coupled to quantum gravity satisfy either*

$$|\nabla V| \geq \frac{C}{M_P} V \quad (3.85)$$

or

$$\min(\nabla_i \nabla_j V) \leq -\frac{C'}{M_P^2} V, \quad (3.86)$$

where  $C$  and  $C'$  are positive constants of  $\mathcal{O}(1)$  and  $\min(\nabla_i \nabla_j V)$  is the minimum eigenvalue of the Hessian of  $V$  (in an orthonormal frame).

Regardless the violation of (3.83), the top of the Higgs potential satisfies the Refined de Sitter Conjecture and (3.86). In particular:

$$\frac{\min(\nabla_i \nabla_j V)}{V} \sim -\frac{10^{35}}{M_P^2}. \quad (3.87)$$

Similarly, for QCD axions and QCD phase transitions there could be violations of the original de Sitter Conjecture [46, 47]; as before, they are prevented thanks to the Refined de Sitter Conjecture.

It is also worth noting that for an axion-like particle, whose potential has as leading contribution

$$V \sim -\cos \frac{\phi}{f} \quad (3.88)$$

with

$$\frac{\min(\nabla_i \nabla_j V)}{V} \leq -\frac{1}{f^2}, \quad (3.89)$$

the Refined de Sitter Conjecture is satisfied whenever  $f \leq M_P$ . This result is coherent with what the WGC for axions prescribes [7, 48, 49].

Having fixed the ground for the conjecture, let us study some properties of de Sitter space and explore the connection between the de Sitter Conjecture and the distance criteria.

A  $d$ -dimensional de Sitter space ( $dS_d$ ) can be described as a hypersurface of a  $(d+1)$ -dimensional Minkowski space ( $M_{d+1}$ )<sup>21</sup>

$$-X_0^2 + X_1^2 + \dots + X_d^2 = R^2, \quad (3.90)$$

where  $R$  is the radius of de Sitter space. The radius  $R$  is related to the cosmological constant  $\Lambda$  as

$$\Lambda = \frac{(d-2)(d-1)}{2R^2}. \quad (3.91)$$

In global coordinates the line element of  $dS_d$  is

$$ds^2 = -dt^2 + R^2 \cosh^2 \frac{t}{R} d\Omega_{d-1}^2 \quad (3.92)$$

( $\Omega_{d-1}$  representing the angular coordinates of a unit Euclidean  $(d-1)$ -sphere). De Sitter space can be thought of as a sphere whose radius evolves in time starting from infinite size and then becoming infinitely large again after having reached the size  $R$ .

In static coordinates (when the metric doesn't depend on time and the mixed time and space metric elements are zero)  $dS_d$ 's line element is

$$ds^2 = -\left(1 - \frac{r^2}{R^2}\right) dt^2 + \left(1 - \frac{r^2}{R^2}\right)^{-1} dr^2 + r^2 d\Omega_{d-2}^2 : \quad (3.93)$$

---

<sup>21</sup>This, in the convention  $\eta_{\mu\nu} = \text{diag}(-1, +1, \dots, +1)$ .

de Sitter space has a horizon at radius  $R$ . It is the casual patch available to a given observer so that it can't test distances greater than  $R$ . To this horizon one can associate a temperature<sup>22</sup>,

$$T_{dS} = \frac{1}{2\pi R} \quad (3.94)$$

and, consequently, an entropy  $S_{dS}$ , which is

$$S_{dS} = \frac{2M_P^2}{T_{dS}^2} = 8\pi^2 R^2 M_P^2 = 4\pi^2 \frac{(d-1)(d-2)}{\Lambda} M_P^2 \quad (3.95)$$

for  $d = 4$  [50].

Even though we are not dealing with black holes, by asymptotically defining de Sitter microstates as those reproducing de Sitter space in the future, de Sitter entropy can be interpreted as the logarithm of the dimension of the Hilbert space  $\mathcal{H}$  associated to those microstates [50–54]

$$S_{dS} = \log \dim \mathcal{H}. \quad (3.96)$$

Actually, it is not clear how to define this Hilbert space: there are many aspects of de Sitter space that are difficult to make precise because of the absence of a spatial infinity to define asymptotics. For instance, there is no  $S$ -matrix on de Sitter space and defining String Theory on such a space is not an easy task. There is no a notion of energy conservation and de Sitter space is not supersymmetric.

Coherently with the (would be) definition of de Sitter vacua in String Theory as minima of a scalar potential derived from a compactification procedure, de Sitter vacua in a quantum gravitational context are at least meta-stable. Moreover, it can be argued that, rather than being meta-stable, de Sitter space is actually unstable. This instability leads to a justification of the Refined de Sitter Conjecture [55–58].

The Refined de Sitter Conjecture is connected to the SDC and follows from an interpretation of (3.64) in terms of duality at parametrically large distances in field space [45].

---

<sup>22</sup>By passing through the partition function for a quantum statistical mechanical system with infinite degrees of freedom

$$Z = \text{Tr} e^{-\beta H} = \int \mathcal{D}q e^{-\int_0^\beta L(q) d\tau}$$

(with  $L(q) = \frac{1}{2}\dot{q}^2 + V(q)$  and  $q(0) = q(\beta)$ ,  $\beta$  being related to the temperature  $T$  as  $\beta = \frac{1}{k_B T}$ ), that is

$$Z = \int \mathcal{D}\phi e^{-\int d\tau \int d^3x \mathcal{L}(\phi)},$$

by imposing appropriate boundary conditions and by interpreting  $\tau$  as a time coordinate,  $Z$  can be written as a path integral

$$Z = \int \mathcal{D}\phi e^{-\frac{i}{\hbar} S(\phi)},$$

where  $S(\phi)$  is the action  $S(\phi) = \int d^4x \mathcal{L}(\phi)$ . By moving to Euclidean spacetime it is possible to state the following correspondence: a Quantum Field Theory in Euclidean spacetime in the presence of a temperature corresponds to a statistical system where the temperature is related to the periodicity of time. The Euclidean time coordinate is associated to a singular circle and a conical singularity manifests. The cyclicity of the singular time coordinate can be made correspondent to a temperature.

The idea is that, at large distances in field space, towers of states become light, so increasing the number of states in the theory and the dimension of the Hilbert space. The corresponding interpretation of (3.96) suggests that the entropy should increase with the field distance. More precisely, at parametrically large distances in field space the exponentially large number of light states dominates the Hilbert space and determines a monotone increase in the entropy. Since the de Sitter entropy is inversely proportional to the cosmological constant ((3.95)), the latter decreases moving to large distances in field space. This reasoning suggests that the cosmological constant is not actually a constant but a scalar potential that has a non-vanishing derivative. The exponential nature of the mass scale of the towers of states protagonists of the distance criteria can be mapped in a property of the potential according to which its derivative should be proportional to itself. Relying on a semiclassical notion of entropy for de Sitter space, this reproduces the first condition of the Refined de Sitter Conjecture.

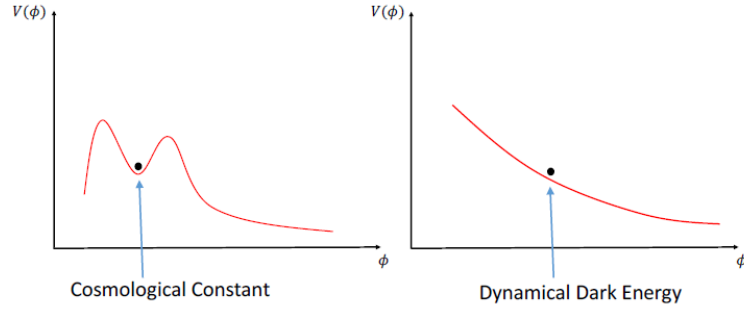
After having assigned an entropy to a field rolling down a potential, it can be shown that a finite de Sitter temperature induces a positive mass (of the order of the potential) to the scalar field, at horizon scales. The second derivative of the potential may be negative and still no instability would manifest, unless its magnitude is greater than the potential. In this case an instability enters the game at horizon scales and spoils the entropic interpretation of the horizon. This motivates the second condition of the Refined de Sitter Conjecture.

The relation between the Swampland distance conjectures and the de Sitter Conjecture is explicit at parametrically large distances in field space. Since in String Theory all the coupling constants are field dependent and the weak coupling condition is at large distance in field space, the argument presented above is valid in any parametrically controlled regime in String Theory.

Besides of being connected with the distance criteria, another important aspect of the de Sitter conjecture is that it has the purpose of linking microscopic and quantum aspects of de Sitter space with properties of scalar potentials of effective field theories arising from String Theory. Even though String Theory may not admit de Sitter vacua, this forms a framework where the attempt of constructing de Sitter vacua in String Theory may find a basis of developing. In this line of research the KKLT proposal sets itself. It is based on the idea of uplifting a KKLT vacuum to a de Sitter vacuum by using the positive energy due to  $D$ -terms and (commonly) anti- $D3$  branes [59, 60]. However, the question whether the KKLT scenario leads to true de Sitter vacua of String Theory remains nowadays subjective and controversial.

So far we have presented the de Sitter Conjecture and some motivations for it; to conclude this section, it is interesting to briefly analyze the cosmological implications of the conjecture itself. The observation that our Universe is entering a phase of late-time acceleration suggests that the scalar potential of the Universe should have a positive value,  $V > 0$ . The de Sitter Conjecture implies that it can't be at a minimum (where  $|\nabla V| = 0$ ). So, the Universe is rolling down a potential slowly enough that the potential energy dominates over the kinetic one and the accelerated expansion can take place. As already mentioned, this scenario is called DDE or quintessence and it is illustrated in the following figure.





**Figure 3.4** [7]:

The figure shows the scalar potential of the Universe along a particular scalar direction denoted by  $\phi$ . The current state of the Universe is indicated with the black dot. The possibility on the left hand side corresponds to a cosmological constant driving the present day accelerated expansion; it violates the de Sitter Conjecture. The potential on the right hand side represents a DDE scenario where the accelerated expansion is driven by a rolling scalar field; it is compatible with the de Sitter Conjecture.

A prediction of the quintessence models is that DE equation of state has to vary in time. If DE is described as a fluid with

$$P_{DE} = w_{DE}\rho_{DE}c^2, \quad (3.97)$$

for a scalar field rolling down a potential

$$w_{DE} = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)}. \quad (3.98)$$

The cosmological constant scenario takes pace in the limit  $w_{DE} \rightarrow -1$ . Current observations of DE equation of state parameter  $w_{DE}$  bound its possible deviation from a cosmological constant. These bounds constrain the constant  $C$  in the de Sitter Conjecture to be  $C < 0.6$  [61].

The de Sitter Conjecture interacts also with inflationary models, because the parameter  $C$  is strictly related to the slow-roll parameter during inflation. The constraints from inflation (and from the non observation of tensor modes, in particular) set  $C < 0.09$  [61]. This is somehow in tension with the conjecture, but this depends on how sharply the condition  $C \sim \mathcal{O}(1)$  is interpreted.

The de Sitter Conjecture implies that the Universe “today” must correspond to a scalar field that is rolling (down) to larger and larger expectation values. It is possible that this would lead to an effective negative cosmological constant causing a phase transition in the Universe or to an expectation value of the scalar field that is so large that the light states of the SDC start to affect the Universe with a possible consequent phase transition.

In exploring the cosmological implications of the de sitter Conjecture it is also worth mentioning the study of its relation with the distance criteria in favouring multi-field inflation [62] and the analysis of the interactions of the de Sitter Conjecture with tensor modes in inflation [63, 64], with warm inflation [65–69] and with eternal inflation [70, 71].

As one can guess, this is a rich and florid field of research where still a lot has to be done and understood.



## 4 The Weak Gravity Conjecture in the presence of scalar fields

In **Section 3** we have presented the WGC in its best known version, that is for a theory coupled to gravity with a  $U(1)$  gauge symmetry.

In many theories Beyond Standard Model (BSM) various scalar fields appear and Supergravity Theories, traditionally regarded as effective descendants of Superstring Theory, are characterized by a lot of scalar fields too. In any effective theory that descends from Great Unification Theories or String Theory scalar fields play a relevant role: the parameters appearing in the effective lagrangians are not actually parameters but vacuum expectation values of some scalar field defined at (very) high energies; its massive fluctuations can't be excited at (sufficiently) low energy and this fixes the field at its vacuum expectation value.

It is interesting and challenging to find the version of the WGC when (also) scalar fields are present. This will be the topic of the paragraph we are undertaking the discussion of.

We will first describe a proposal for the scalar WGC stated by E. Palti in [8]. Then, we will analyze the strong version of the WGC proposed by E. Gonzalo and L. Ibáñez in [9], putting in evidence its inconsistency with the physical principle underlying the WGC itself (and some other criticisms). We will finally consider a theory including gravity and (at least) two scalar fields, one of which is strictly massless. We will do this with the purpose of finding (if there is one) a general formulation for the scalar WGC. For the model under investigation, we will get a constraint on the parameters of the scalar potential that is coherent (as sufficient condition) with Palti's conjecture and seems not to be altered by quantum 1-loop corrections.

### 4.1 The Weak Gravity Conjecture with scalar and gauge fields

So far, we have formulated the WGC by referring to theories including gravity and gauge fields. We can generalize our analysis (modifying, for instance, (3.34)) by adding scalar fields.

Since the WGC can be tied to black hole physics, we can try to gain intuition on the desired generalization by exploiting the properties of  $N = 2$  supergravity theories and considering black hole solutions in this context.

Let us consider the model with action

$$S = \int d^4x \sqrt{-g} \left[ \frac{M_P^2}{2} R - g_{i\bar{j}}(z) \partial^\mu z^i \partial_\mu \bar{z}^{\bar{j}} + I_{IJ}(z) F_{\mu\nu}^I F^{J,\mu\nu} + R_{IJ}(z) F_{\mu\nu}^I * F^{J,\mu\nu} \right], \quad (4.1)$$

where  $\{z^i\}_i$  (with  $i = 1, \dots, n_V$ ,  $n_V$  being the number of vector multiplets) are complex scalar (or pseudo-scalar) fields,  $z^i = b^i + it^i$ , and  $g_{ij}$  is the field space metric. The metric  $g_{ij}$ , the kinetic function  $I_{IJ}$  and the  $CP$ -violating matrix  $R_{IJ}$  are allowed to depend on the fields  $\{z^i\}_i$ .

The geometric structure of the field space is determined by the periods  $\{X^I, F_I\}$  which are related by a symplectic matrix  $N = (N_{IJ})_{IJ}$  such as  $F_I = N_{IJ} X^J$ . The matrix  $N$  defines also the matrices  $I$  and  $R$ :  $I_{IJ} = \text{Im}(N_{IJ})$  and  $R_{IJ} = \text{Re}(N_{IJ})$ .

The Kähler potential for the scalar field space metric can be written in terms of the periods  $\{X^I, F_I\}$  and takes the form

$$K = -\ln i \left( \bar{X}^I F_I - X^I \bar{F}_I \right). \quad (4.2)$$

As we did in discussing the WGC for dyons with a  $\theta$ -angle, it is useful to introduce the matrix

$$U = \begin{pmatrix} I + RI^{-1}R & -RI^{-1} \\ -I^{-1}R & I^{-1} \end{pmatrix}$$

and the relations

$$Q^2 = -\frac{1}{2}Q^T U Q \quad (4.3)$$

and

$$(Q, Q') = -\frac{1}{2}Q^T U Q', \quad (4.4)$$

$Q = \begin{pmatrix} p^I \\ q_I \end{pmatrix}$  and  $Q' = \begin{pmatrix} p'^i \\ q'_i \end{pmatrix}$  representing two vectors of symplectic quantized charges (which are arbitrary constants).

The central charge can be introduced as the symplectic product between the vectors  $V = \begin{pmatrix} e^{K/2} X^I \\ e^{K/2} F_I \end{pmatrix}$  and  $Q = \begin{pmatrix} p^I \\ q_I \end{pmatrix}$  as

$$Z = \langle V, Q \rangle = V^T \Omega Q = e^{K/2} (q_I X^I - p^I F_I), \quad (4.5)$$

$\Omega$  being the symplectic matrix

$$\Omega = \begin{pmatrix} 0_N & 1_N \\ -1_N & 0_N \end{pmatrix} \in Sp(2N, \mathbb{R})$$

(where  $N$  is the number of gauge fields in the theory) [72].

In [72] an identity involving  $Q^2$  and the central charge  $Z$  was pointed out. It is

$$Q^2 M_P^2 = |Z|^2 + g^{i\bar{j}} D_i Z \bar{D}_{\bar{j}} \bar{Z} M_P^2, \quad (4.6)$$

whit  $D_i \psi^j = \partial_i \psi^j + \Gamma_{ik}^j \psi^k + \frac{p}{2} (\partial_i K) \psi^j$ ,  $\psi$  being an object with Kähler weight  $p$  ( $Z$  has Kähler weight 1).

It is worth noting that, after having introduced  $V_i = D_i V$  and expressed (4.6) in terms of symplectic products as

$$Q^2 M_P^2 = |\langle V, Q \rangle|^2 + g^{i\bar{j}} \langle V_i, Q \rangle \langle \bar{V}_{\bar{j}}, Q \rangle M_P^2,$$

the invariance of  $Q^2$  under symplectic transformations is manifest.

As we will appreciate soon, the relation (4.6) will play an important role in our discussion.

We are interested in the black hole solutions of (4.1).

Black holes in  $N = 2$  supersymmetric theories (in the presence of gravity, gauge fields and scalar fields) exhibit a phenomenon that is called the *Attractor Mechanism* [73]. We expect that the total number of microstates corresponding to an extremal black hole is determined by the quantized charges that the black hole carries and so it does not vary continuously. If the counting of microstates

agrees with Bekenstein–Hawking interpretation of black holes’ entropy, regarding it as the horizon area, the total number of microstates is expressed by the charges alone. This suggests that the scalar moduli fields that determine the horizon area have to take fixed values at the horizon and such values depend only on the charges and are independent of the asymptotic values for the moduli<sup>23</sup>.

Let us consider a  $N = 2$  supersymmetric extremal black hole solution with quantized charges  $(q_I, p^J)$  (related to their non-quantized partners as in (3.37)). Its mass  $M_{BH}$  is expressed in terms of the central charge when the scalar fields’ values are taken at spatial infinity,

$$M_{BH} = |Z|_\infty. \quad (4.7)$$

By substituting this definition into (4.6) we obtain

$$Q^2 M_P^2 = M_{BH}^2 + g^{ij} D_i Z \bar{D}_j \bar{Z}. \quad (4.8)$$

The Attractor Mechanism fixes the values of the scalar fields on the horizon of an extremal black hole in terms of the black hole’s charges as  $D_i Z = 0$ . Therefore, there are two types of extremal black holes, those for which the scalar field values at infinity differ from the values on the horizon so that there is a scalar field spatial gradient and those for which the values at infinity are equal to the values at the horizon so that there is a constant spatial profile. In these two circumstances the extremal black hole mass is maximized with respect to its charge at infinity.

The identity (4.8) can be rewritten as

$$Q^2 M_P^2 = M_{BH}^2 + 4g^{ij} \partial_i M_{BH} \bar{\partial}_j M_{BH} M_P^2 \quad (4.9)$$

(where  $M_{BH}$  is thought of as a function of the fields  $\{z^i\}_i$ , rather than in the vacuum) in the form of an extremality condition (valid at infinity)<sup>24</sup>.

To generalize (3.25) or (3.40) we can follow the logic of the existence of at least a particle such that the black hole is able to decay.

Since  $N=2$  extremal black holes are BPS states, they can only decay to other BPS states. The last term in (4.8) is positive definite. So, in order for these black holes to decay we can impose the existence of a particle with mass  $m$  such that  $Q^2 M_P^2 \geq m^2$ . If the last term in (4.8) is non-vanishing, the previous inequality becomes a strict relation. Furthermore, because the particle the black hole decays in has to be a BPS state itself, its mass is given by the central charge and

$$Q^2 M_P^2 = m^2 + 4g^{ij} \partial_i m \bar{\partial}_j m M_P^2 \quad (4.10)$$

holds.

---

<sup>23</sup>It is worth noting that this intuitive argument for the Attractor Mechanism doesn’t rely on supersymmetry and has pushed forward the search for a non-supersymmetric version of the Attractor Mechanism itself.

<sup>24</sup>Since (as just mentioned) the scalar fields may have a spatially varying profile, we need to specify that the extremality condition (4.8) holds at infinity and, indeed, if on the extremal horizon the scalar fields are fixed to their attractor values, they are solutions of  $\partial_i M_{BH} = 0$ .

By extrapolating (4.10) for more general (and possibly non-supersymmetric) cases we can state [7, 8]

A theory with scalar fields  $\{z^i\}_i$ , which have no potential, with general action (4.1) should have a particle with mass  $m(z)$  (depending on the scalar fields, in general) satisfying the bound

$$Q^2 M_P^2 \geq m^2 + 4g^{i\bar{j}} \partial_{z^i} m \bar{\partial}_{z^{\bar{j}}} m M_P^2 \quad (4.11)$$

with  $Q^2 = -\frac{1}{2} Q^T U Q$  (as already defined).

The last term in the previous inequality represents the scalar force mediated by the scalar fields  $\{z^i\}_i$ , which is induced by a cubic coupling of these fields with two WGC states.

The physics underlying (4.11) can be phrased as the statement that the (repulsive)  $U(1)$  gauge force between two WGC states<sup>25</sup> acts at least as strongly as the (attractive) gravitational and scalar forces combined. As already observed, when dealing with BPS states, the inequality (4.11) becomes an equality expressing a no force condition.

A suggestive intuition on the possibility of extending this result to a non-supersymmetric context is given by the fact that for any extremal black hole it is possible to define a black hole scalar potential as  $V_{BH} = Q^2$ . If this potential can be written as [74]

$$V_{BH} = Q^2 = \mathcal{W}^2 + 4g^{i\bar{j}} \partial_i \mathcal{W} \bar{\partial}_{\bar{j}} \mathcal{W} \quad (4.12)$$

(where  $\mathcal{W}$  is a real function of the complex scalar fields in the theory, named “fake superpotential”), then the black hole mass is

$$M_{BH} = |\mathcal{W}|_{\infty} \quad (4.13)$$

and on the horizon the fields solve  $\partial_i \mathcal{W} = 0$ .

This observation suggests that (4.8) is tied to extremality rather than to supersymmetry.

The relation (4.9) could generally hold but, because of the difficulties in describing the black hole discharge in terms of the particles the black hole decays in (the dependence of  $M_{BH}$  on the scalar fields may be different from that of  $m$ ), it does not imply (4.11). Despite this, the analogy of (4.9) with (4.10) suggests the existence of a relation between them.

More precisely, the requirement for the black hole to decay can be stated as the existence of a particle  $(m, q)$  with charge-to-mass ratio greater than that of the black hole. So, from (4.9),

$$\frac{Q^2 M_P^2}{M_{BH}^2} = 1 + 4g^{i\bar{j}} \partial_i \ln M_{BH} \bar{\partial}_{\bar{j}} \ln M_{BH} M_P^2 \leq \frac{q^2 M_P^2}{m^2} \quad (4.14)$$

---

<sup>25</sup>The gauge force contribution given by  $Q^2$  is more complicated than in (3.25), the reason being it is a general expression that is valid for dyonic objects in the presence of a non-vanishing  $\theta$ -angle matrix  $R_{IJ}$ .

and the particle is required to be super-extremal.

The expression (4.11) would ask for a replacement of  $M_{BH}$  with  $m$  in (4.14). It is natural that the charge-to-mass relation for the particle  $(m, q)$  depends on  $m$  rather than on  $M_{BH}$ . Moreover, by changing the values of the scalar fields at infinity  $M_{BH}$  changes and, in order for the black hole to maintain a decay channel,  $m$  has to change too. Even though in a non-supersymmetric context it is difficult to formulate a precise general motivation for (4.11) (rather than (4.14)) using black hole decay only, it seems quite reasonable to think that (4.11) and (4.14) are indeed related.

To further support (4.11), it is worth noting that (4.11) is almost uniquely fixed by requiring really basic principles: invariance under scalar fields reparameterizations and electromagnetic duality.

Before proceeding, in order to better clarify the principle underlying (4.11) and its structure, we can consider a (toy) model in which a canonically normalized massless scalar field  $\phi$  interacts with a WGC scalar field  $H$ , whose mass is  $m_0$ , through

$$\mathcal{L} \supset (2m_0\mu\phi + m_0^2)|H|^2 = m^2|H|^2, \quad (4.15)$$

where  $\mu$  is the (adimensional) interaction strength between  $H$  and  $\phi$ ; the expectation value for  $\phi$  has been set to zero and  $m$  defines the (general or effective) mass term for  $H$ .

Then,

$$\partial_\phi m^2 = 2m\partial_\phi m = 2m_0\mu \quad (4.16)$$

and by evaluating this relation at the vacuum we recognize  $\mu$  to be the derivative of  $m$  with respect to  $\phi$  in the vacuum<sup>26</sup>

$$\mu = \langle \partial_\phi m \rangle. \quad (4.17)$$

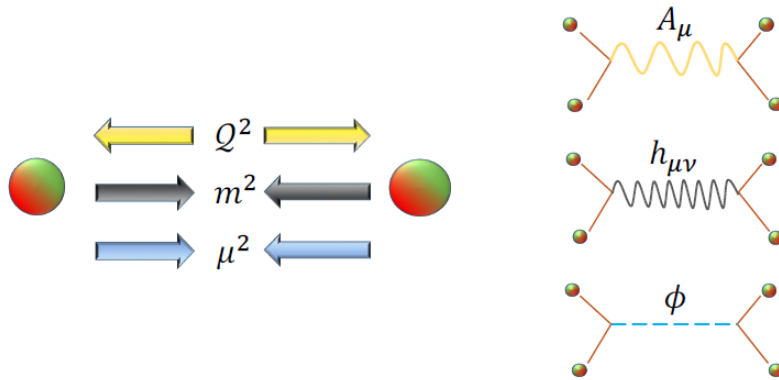
The three-point coupling  $\phi|H|^2$  gives rise to a long-range Coulomb attractive force mediated by  $\phi$  (which is a spin-0 field) acting on two  $H$  states. In modulus, this force is given by

$$F_{scalar} = \frac{\mu^2}{4\pi r^2} \quad (4.18)$$

and is involved, together with the electromagnetic and gravitational force, in expressing the interactions between two WGC particles.

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<sup>26</sup>The same result can be achieved in the cases in which  $H$  is a fermionic field or  $\phi$  is a pseudo-scalar [75].



**Figure 4.1** [7]:

This figure illustrates the long-range forces acting on a pair of WGC particles and the corresponding Feynman diagrams. The repulsive electromagnetic force is mediated by the gauge field  $A_\mu$  and its strength is given by  $Q^2$ ; the attractive gravitational force is mediated by the exchange of the graviton  $h_{\mu\nu}$  and it acts with strength  $m^2$  and the attractive scalar force is mediated by  $\phi$  and its strength is expressed by  $\mu^2$ . The conjecture (4.11) requires the repulsive force to be stronger or (at most) equal in strength than the total attractive force.

Coming back to the general discussion, let us observe that the WGC can be understood (as we have mainly emphasized so far) by referring to black hole decay processes or as the statement that gravity is the weakest force, forbidding gravitationally bound states of the WGC states to exist.

The potential presence of stable WGC bound states would be problematic, as it was argued in [26, 76]. Since the scalar forces act attractively between equal charged particles and the field space metric  $g_{i\bar{j}}$  is positive definite, the sum over all scalar forces contributes positively on the side of gravity. Asking for the absence of WGC bound states amounts to require that the gauge field repulsion overcomes the gravitational and scalar attractions.

In the light of the previous observations the WGC when gauge and scalar fields are present can be stated more generally as

*Consider a theory coupled to gravity with gauge kinetic matrix  $I_{IJ}$ ,  $\theta$ -angle mixing matrix  $R_{IJ}$  and massless scalar fields  $\{z^i\}_i$  (with field space metric  $g_{i\bar{j}}$ ). Then, there must exist a particle with mass  $m$  satisfying*

$$Q^2 M_P^2 \geq m^2 + g^{i\bar{j}} \mu_i \mu_{\bar{j}} M_P^2 \quad (4.19)$$

*where  $\mu_i$  is the non-relativistic coupling of the WGC state with  $z_i$ . If the mass  $m$  is regarded as a function of the scalars  $\{z^i\}_i$ , then  $\mu_i = \partial_i m$ .*

An interesting question is whether (4.19) holds over all the scalar field space or only on certain regions.



In a  $N = 2$  setting, the BPS nature of the states an extremal black hole decays in seems to suggest that the saturated inequality (4.19) is valid at all loci of the scalar field space. A quite natural guess is that (4.19) holds over all field space, but this does not mean that a decay channel for an extremal black hole is available at all points in field space.

By thinking of gravitationally bound states one can say that the particle with the largest charge-to-mass ratio should not form any bound state, because it is stable. As one moves around field space, it could happen that a different particle becomes that with the largest charge-to-mass ratio in the theory. Since the original particle can decay to the new one, gravitationally bound states of the former are allowed. Therefore, we can conclude that at any point in field space there is one state that satisfies (4.19) but it may not be the same state over all field space.

In our following analysis we will assume that (4.19) holds for at least one state at any locus in the scalar field space and we will be conscious of the fact that, having in mind black holes' decay, it may be valid only at specific loci of field space.

The analysis we have proposed so far applies to massless gauge fields and massless scalar fields. It would be interesting to understand how (4.19) should be modified in the presence of massive mediators. Nowadays we don't have any conclusive and precise statement on the way to do this and we can only propose some qualitative and intuitive observations trying to shed light on such research [8].

The classical long-range analysis we are founding our discussion upon relies (at best) on taking the mass of the force mediators much smaller than that of the WGC state. So, as long as the mediators' mass is sufficiently smaller than the mass of the WGC state, it should not modify the mass of the WGC state itself or its coupling to scalar fields. Despite of being expected (4.19) to hold even when the force mediators are massive with a mass that is well below that of WGC state, it is unclear if, for instance, the analysis of bound states supports this conclusion or not.

To better detail this discussion let us first consider the case of massive gauge fields. At sufficiently large distances the intensity of the gravitational interaction would always overcome that associated to the massive force carrier and, as a consequence, bound states would form. The length scale of these states will be like the inverse mass of the mediator. By giving a small mass to the force carrier and by assuming (4.19) to be satisfied, bound states would appear but at very large distances<sup>27</sup> and they may be less problematic from a Quantum Gravity perspective.

If then the scalar fields gain a mass, we could imagine to violate (4.19) without forming bound states at length scales that are larger than the inverse mass of the scalar mediators. But, bound states at arbitrary small distances could form. These bound states are only classically bound (through a barrier) and it is uncertain if they are problematic or not.

As already observed, it is a difficult task to have a conclusive statement on if and how (4.19) gets modified when the gauge and scalar fields have a mass. It could be interesting to investigate this further (also with the attempt of understanding the  $IR$  aspects of (4.19)).

Another interesting question is whether gravity acts more weakly than the scalar forces themselves. This will be the topic of the next subsection.

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<sup>27</sup>When discussing the violation of (4.19) (or (4.11)), we have referred to bound states whose length scale can be arbitrary small.

## 4.2 The Weak Gravity Conjecture with scalar fields

Having in mind the previous paragraph, we would like now to study the relative magnitude of the scalar and gravitational forces. We will start establishing some relations among the forces acting on the WGC states in the context of  $N = 2$  Supergravity; then, trying to uncover the relevant physics, we will propose some generalizations.

The relation (4.6) ( $Q^2 M_P^2 = |Z|^2 + g^{i\bar{j}} D_i Z \bar{D}_{\bar{j}} \bar{Z} M_P^2$ ) can be regarded as an equation capturing the self-interactions of a WGC state. When dealing with different states, one can replace (4.6) with

$$(Q, Q') \operatorname{Re} \left( \frac{Z \bar{Z}'}{|Z \bar{Z}'|} \right) - \frac{1}{2} (q_I p'^I - q'_I p^I) \operatorname{Im} \left( \frac{Z \bar{Z}'}{|Z \bar{Z}'|} \right) = |Z| |\bar{Z}'| + \operatorname{Re} (4g^{i\bar{j}} \partial_i |Z'| \bar{\partial}_{\bar{j}} |Z|), \quad (4.20)$$

where the contributions of the gauge forces and of the non-mutual locality of the states appear in the left hand side and the gravitational and scalar force terms define the right hand side of (4.20) [8].

Let us consider mutually local states<sup>28</sup>.

If the gauge force between such states vanishes and so  $(Q, Q') = 0$ , then the scalar force cancels the gravitational one. In other words, for states with vanishing vector interactions the scalar forces act repulsively.

As, led by the obstruction in the existence of gravitationally bound states, we have generalized the  $N = 2$  results, we can apply the same logic to investigate the relative magnitude of the scalar and gravitational forces when the gauge interactions are absent.

The absence of stable gravitationally bound states requires that the scalar forces act more strongly than gravity.

If we consider the theory (4.1) and two WGC states of masses  $m$  and  $m'$  that are mutually local and have vanishing gauge interactions, then the scalar forces must act repulsively and at least as strongly as gravity [8]

$$-g^{i\bar{j}} \partial_{z_i} m \bar{\partial}_{\bar{z}_j} m' M_P^2 \geq mm'. \quad (4.21)$$

In other words, by making reference (at first) to a  $N = 2$  context and by taking mutually local states (because the  $N = 2$  formalism makes sense in this case); when the gauge forces are not exerted<sup>29</sup>, (4.20) gives rise to (4.21).

Let us remark that this generalization is less clear than that leading to (4.19) from (4.6). One important difference is that we are requiring the presence of (at least) a scalar field rather than study the implications of the (possible) presence of massless scalar fields. Since the scalar fields can be massive, we would have to be able to know how the mass of the scalar field affects the setup. The relation (4.21) seems to guarantee that if a bound state exists than its typical length scale is fixed by the inverse mass of the scalar field. This reproduces the discussion on the implications for the WGC when the gauge fields have a mass.

<sup>28</sup>Two states  $(q_I, p^I)$  and  $(q'_I, p'^I)$ , identified by their electric and magnetic charges, are said to be mutually local if  $q_I p'^I - q'_I p^I = 0$ .

<sup>29</sup>If one purely electric and one purely magnetic state are considered, the gauge force between them vanishes.

Having stated (4.21) as a possible (and not univocal [8]) generalization of (4.20), it is interesting to deepen the relation between the magnitudes of the scalar and gravitational forces for the interaction of a WGC state with itself.

The relevant  $N = 2$  relation is [72]

$$Q^2(F) = |Z|^2 - g^{i\bar{j}} D_i Z \bar{D}_{\bar{j}} \bar{Z}, \quad (4.22)$$

where  $Q^2(F) = Q^2(N_{IJ} \rightarrow F_{IJ})$  with  $F_{IJ} = \partial_I F_J$  and  $F_I = \partial_I F$ ,  $F$  being a prepotential.

If (4.6) can be interpreted as a bound on the sum of the scalar and gravitational forces, (4.22) gives information on their relative magnitude.

The matrix  $I_{IJ}(F) = I_{IJ}(N_{IJ} \rightarrow F_{IJ})$  has  $n_V$  strictly positive eigenvalues and one strictly negative eigenvalue. There is a basis where  $n_V$  WGC states have the scalar force acting more strongly than gravity and one WGC state on which gravity acts more strongly than the scalar force (because the graviphoton has no scalar superpartners).

We can then state that [8]

*For the theory (4.1), for each scalar field, there is a WGC state with mass  $m$  (depending on the scalar fields) on which gravity acts as the weakest force and so satisfying the bound*

$$g^{i\bar{j}} \partial_{z_i} m \bar{\partial}_{\bar{z}_j} m M_P^2 > m^2. \quad (4.23)$$

In the  $N = 2$  case the spectrum of states is such that all the states satisfy (4.23) but an electric and a magnetic one. The idea is that there is at most one state that can violate a non-strict inequality version of (4.23) and, if it does, (4.23) becomes a strict inequality for all the other states.

The generality of (4.23) away from a  $N = 2$  framework can't be directly deduced by thinking about the absence of bound states: the scalar and gravitational forces act both attractively. According to E. Palti [8], (4.23) may be deduced from (4.21), but it is not clear how to do so.

From now on we will refer to (4.21) or more generally to (4.23) as the **Scalar Weak Gravity Conjecture** (SWGC). It should hold as a statement about the scalar interaction of the WGC states associated to gauge fields (Gauge SWGC) or it should hold completely generally even in the absence of gauge fields (General SWGC). In the latter case the SWGC configures itself as the more general statement that gravity is really the weakest force and it can be phrased as the following claim:

*For each scalar field there is a state on which gravity acts more weakly than the other interactions.*

As already emphasized, the justification of this conjecture by obstructing the existence of gravitationally bound states is rather uncertain. Since there is no gauge symmetry to make reference to, it is not clear what could give stability to such states. One possible solution could be that of associating a charge (at least approximately conserved) to scalar fields. Anyway, in the absence of a solid argument for the (in)stability of bound states coupled to scalar fields only, the evidence for the General SWGC remains weak.

Before proceeding with the analysis of (4.23), it may be worth noting another identity relative to a  $N = 2$  setting (and generalizable perhaps in a non-supersymmetric context)<sup>30</sup> [8]

$$g^{i\bar{j}} D_i \bar{D}_{\bar{j}} |Z|^2 = n_V |Z|^2 + g^{i\bar{j}} D_i Z \bar{D}_{\bar{j}} \bar{Z}. \quad (4.24)$$

This equation can be interpreted as a relation between the four-point coupling, the mass and the three-point coupling of the WGC states interacting with scalar fields. Following the usual way of proceeding, it would be interesting to find a plausible generalization of (4.24) (as we have done for (4.6) and (4.20)).

Turning back to the conjecture (4.23), a stronger version of this statement can be proposed by demanding it should hold for any scalar field. In an appropriate modified version, this request implies that the state satisfying the bound (4.23) needs not to be a particle: it can be an extended object. If it is the case, the derivative of the mass in expressing the strength of the scalar force is then substituted with a coupling strength related to the tension.

In this respect, it may be useful to mention a version of the WGC for a  $p$ -form which has been formulated in [26].

Let  $C_{\mu_1, \dots, \mu_p}^{(p)}$  be an antisymmetric tensor of rank  $p$ , transforming as  $\delta C_{\mu_1, \dots, \mu_p}^{(p)} = \partial_{\mu_1} \lambda_{\mu_2, \dots, \mu_p}^{(p-1)}$ . A  $d$ -dimensional theory with a  $p$ -form field, whose kinetic term is  $\frac{1}{2g_p^2} |F^{p+1}|^2$  (where  $g_p$  is the analogue of the gauge coupling for a  $p$ -form;  $F_{\mu_1, \dots, \mu_{p+1}}^{(p+1)} = \partial_{\mu_1} C_{\mu_2, \dots, \mu_{p+1}}^{(p)}$ ; and  $|F^{p+1}|^2 = \frac{1}{(p+1)!} F_{\mu_1, \dots, \mu_{p+1}} F^{\mu_1, \dots, \mu_{p+1}}$ ), should have a  $(p-1)$ -dimensional object (the  $p$ -form field is eventually coupled to by an integration over the world-volume of the object) with (quantized) charge  $q_p$  and tension  $T_p$  satisfying

$$\frac{p(d-p-2)}{d-2} T_p^2 \leq q_p^2 g_p^2 \left( M_P^d \right)^{d-2}. \quad (4.25)$$

The previous statement applies to  $p$ -form fields and  $(p-1)$ -dimensional objects charged under them when  $p > 0$ . If, instead,  $p = 0$  the conjecture fails and an alternative way to treat this circumstance has to be found: the  $p = 0$  case enters the game in the models of natural inflation where periodic axions, which are 0-form fields, are counted as inflaton candidates [48, 49].

As far as axions are concerned, instantons are the charged objects and the role of the gauge coupling is played by the inverse of the axion decay constant. Following [48], the instantons correct the scalar potential of an axion  $\phi$  by terms of the form

$$V(\phi) \sim e^{-S_E} \left[ 1 - \cos \left( \frac{\phi}{f} \right) \right], \quad (4.26)$$

where the suppression of the correction is controlled by the Euclidean action  $S_E$ . It is the analogue of the mass in (3.25) and so determines whether the charged particle is heavy enough to be integrated

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<sup>30</sup>The factor  $n_V$  appears because the WGC states only couple to one combination of gauge fields but to all the moduli:  $n_V$  comes with the coupling to the moduli  $\{t^i\}_i$ . This can be seen if one restricts to electric charges so that the WGC state couples to one linear combination of axions and by calculating the relation for the axions  $b^i$  and the moduli  $t^i$  separately.

out in the low-energy effective theory. A natural guess for a generalization of the WGC to axions is indeed

$$S_E \lesssim \frac{M_P}{f}; \tag{4.27}$$

for an axion with decay constant  $f$  at least one instanton should exist satisfying the previous bound. When the axion decay constant is taken to be larger than the Planck mass, the instantons satisfying the bound (4.27) give consistent corrections to the scalar potential (4.26); as a consequence, by requiring (also) perturbative control, the inflaton is generically prevented to perform trans-Planckian trips in its field space. This represents an obstacle in constructing large-field inflationary models with a single axion [77]. Since the bound (4.27) seems to be a well-motivated generalization of the WGC (as it is shown in [49] by an explicit calculation involving gravitational instantons), by trying to support large-field inflationary models, various possible loopholes (usually referred to multiple axion models) can be advocated. For instance, one proposal consists in thinking that the instantons satisfying the WGC bound might not be those with the smallest action; these instantons would be suppressed and other instantons that do not satisfy the WGC bound would give the dominant contribution allowing for super-Planckian field variations [78]. Another possibility is that of exploiting a discrete gauge symmetry which in the presence of  $N$  axions prevents the existence of some instantons so that the true bound on  $S_E$  is larger than (4.27) [49]. A further potential solution (seemingly better realizable in a consistent String Theory framework) proposes that, in certain models, instantonic corrections may be accompanied by extra suppression factors such that they can be suppressed in the scalar potential even though their Euclidean action is small; in such models, the WGC might be satisfied and super-Planckian field ranges can be obtained [79]. It is not clear whether super-Planckian field trips are allowed by the WGC or not; a deeper theoretical understanding is needed and future cosmological observations leading to a measure of the tensor-to-scalar ratio of primordial fluctuations would certainly be crucial.

Coming now back to the Palti's strong version of the WGC (and to the statement on 0-forms, in particular) and anticipating something we will better precise, it is worth noting that the possibility that the state the scalar fields couple to is an extended object matches nicely with what the Swampland Distance Conjecture (SDC) may prescribe. The SDC seems to be related to (4.23) and this can be interpreted as a first hint for a relation between the former and (more generally) the Weak Gravity Conjecture.

## **A proposal for accounting quantum corrections to the WGC in the presence of scalar fields**

Before deepening the relation between the WGC and the SDC, a relevant observation on the former has to be mentioned.

The arguments we have described so far in presenting (4.19) and (4.23) are somehow classical in nature: we have dealt with scalar fields that are not subjected to any potential. However, since the scalar fields couple (at least) with the WGC particle, once quantum corrections are added they will in general acquire a potential (as the Coleman–Weinberg mechanism prescribes). In other words,

any statement involving scalar fields with no potential has to be considered as a tree-level argument which has to be corrected at loop-level.

The exception to this rule is given by circumstances where  $N = 2$  supersymmetry is present: the scalar fields' expectation values form a true moduli space; if there is no potential at classical level, no potential arises at loops.

As just mentioned, in the absence of extended supersymmetry, it is expected that all scalar fields acquire a potential; but, unfortunately, it is not clear how to generalize (4.19), for instance. If we apply (4.19) to the case of one real scalar field  $\phi$  with gauge coupling  $\mu = \langle \partial_\phi m \rangle$ , indeed

$$Q^2 M_P^2 \geq m^2 + \mu^2 M_P^2; \quad (4.28)$$

by giving the scalar a mass  $m_\phi$  ( $\ll m$ ) and by placing two WGC particles (whose interaction is mediated by  $\phi$ ) at a distance set by  $m^{-1}$  (where a classical force analysis can be more less performed), the replacement

$$\mu^2 \longrightarrow \mu^2 + \mathcal{O}\left(\frac{\mu^2 m_\phi}{m}\right) \quad (4.29)$$

is suggested. This proposal is motivated by thinking of a Yukawa-type force for the scalar (which becomes irrelevant at distances much larger of the inverse mass of the scalar itself) as means to quantify the magnitude of the quantum corrections to (4.19) (and (4.23) too) [75].

Anyway, a general recipe hasn't been found yet.

### 4.3 Relations between the Weak Gravity Conjecture and the Swampland Distance Conjecture

Having studied the SDC and the WGC in the various forms it can present itself, it is remarkable to evidence the existing relations between the two.

For instance, the scalar version of the WGC is quite straightforwardly related to the SDC [30]. Applying (4.23) to the case of a single real scalar field  $\phi$  which is canonically normalized, the bound

$$|\partial_\phi m| > m \quad (4.30)$$

is found.

If we ask for varying  $\phi$  but maintaining (4.30), any power-law behaviour  $m \sim \phi^n$  wouldn't be coherent with the inequality (4.30) for sufficiently large  $\phi$ ; whereas the exponential scenario  $m \sim e^{-\alpha\phi}$  would be consistent. We therefore recover the exponential behaviour that is typical of the distance conjectures. Reversing the reasoning is also allowed.

In words, we can say that the *sine qua non condition* for gravity to be the weakest force acting on a particle is that the latter must have a mass that decreases exponentially even for large scalar expectation values.

It is interesting to note that the Magnetic WGC is strictly related to the distance conjectures too [30, 80, 81].

The Magnetic WGC mass scale  $\Lambda \sim gM_P$  can be associated to an infinite tower of states and this tower can be identified with the tower of states of the distance conjectures. The previous identification suggests that the gauge couplings are functions of the scalar fields: the gauge coupling  $g$  depends exponentially on the canonically normalized field  $\phi$  as

$$g \sim e^{-\phi}; \tag{4.31}$$

this is precisely what happens in String Theory, where special extended objects are protagonists.

To further motivate the relations between the WGC and the SDC let us consider a situation where a scalar field has an expectation value that spatially changes. If we restrict the spatial variations to a region of size  $R$  and consider the variation  $\Delta\phi$  from one side of the region to the other, when keeping  $R$  fixed and trying to increase  $\Delta\phi$ , an obstruction is eventually reached: the energy in the scalar spatial gradient will have an associated Schwarzschild radius larger than  $R$  and the system undergoes gravitational collapse. This argument leads to a gravitational censorship for large field variations which can be related to the SDC.

To make this link more explicit the argument can be refined. Let us consider gravity in a Newtonian regime. For a general potential it can be shown that a scalar field may undergo super-Planckian spatial variations within the Newtonian regime, but the maximum variation  $\Delta\phi$  is obtained from a logarithmic spatial profile  $\phi \sim \log r$ , where  $r$  is the radial coordinate. Since  $\phi \sim \log r$  and therefore  $\partial_r\phi \sim \frac{1}{r} \sim e^{-\phi}$ , the energy density  $\rho$  stored in the field varies at least exponentially.

Localized sources that are charged under a  $U(1)$  gauge symmetry induce a potential for the scalar field; in the case the gauge coupling depends on the scalar field  $\phi$ , a scalar spatial gradient which can support super-Planckian variations is developed. Since, away from the source, the gauge coupling has similar magnitude of the scalar field energy density coming from the kinetic terms, if the latter increases exponentially, the cutoff scale of the theory, set by the Magnetic WGC to be  $\Lambda \sim gM_P$ , has to be subjected to an exponential increase too and has to stay above the energy density. This implies the relation (4.31), motivating the SDC from the WGC again [7].

#### 4.4 Gonzalo and Ibàñez's conjecture

In **Section 3** we have presented the most studied and understood version of the WGC: that involving a  $U(1)$  gauge boson coupled to gravity. The conjecture states the existence of at least a particle with mass  $m$  and charge  $q$  such that  $m \lesssim gqM_P$  in the theory. As we have seen, this Swampland criterion is motivated by black hole physics based arguments or by explicit examples in String Theory.

In the attempt of understanding which is the physical principle *deus ex machina* of the WGC (as we know) two options manifest: either the WGC is strictly related to black holes and their stability or it is a statement concerning the weakness of gravity with respect to the other interactions.

By insisting in considering gravity as the weakest force in any circumstance, E. Gonzalo e L. Ibàñez have formulated a version of the Scalar Weak Gravity Conjecture (SWGC) corresponding to the requirement that the self-interactions of a scalar must be stronger than the gravitational interactions it feels. This statement is proposed to be true for any scalar in the theory (and not only for states playing the role of WGC particles) and for all the values of the scalar itself.

Following [9], let us consider a real (for simplicity) particle  $H$  with (effective) mass  $m$  that is coupled to a light real scalar field  $\phi$  thanks to a trilinear coupling whose strength is proportional to  $\mu = \partial_\phi m$ . In the limit  $m_\phi \rightarrow 0$ , Palti's SWGC (4.23) is rewritten in [9] as

$$(\partial_\phi m)^2 > \frac{m^2}{M_P^2}. \quad (4.32)$$

Since it will be convenient to motivate their new conjecture, in Gonzalo and Ibàñez's perspective let us set the mass  $m^2(\phi)$  of the WGC particle  $H$  to be the second derivative with respect to the field  $H$  of the potential  $V(\phi, H) = \frac{1}{2}(m_0^2 + 2m_0\mu\phi)H^2$ ,  $m^2(\phi) = V''$ . With this identification (4.32) can be written as [9]<sup>31</sup>

$$(V''')^2 > \frac{4(V'')^2}{M_P^2}. \quad (4.33)$$

In a similar philosophy to the WGC for  $U(1)$  gauge interactions, the particle  $H$  plays the role of the WGC particle and it is there to guarantee that there is at least a state with scalar self-interactions stronger than gravity. Palti's bound doesn't apply to any scalar field, but only to WGC scalars interacting with a scalar  $\phi$  and whose mass depends on  $\phi$ . The conjecture doesn't apply to the field  $\phi$  itself.

By insisting in finding a constraint that is valid for any scalar field, Gonzalo and Ibàñez have noted that Palti's claim in the form of (4.33) is in tension with (4.27). Once it is applied to an axion  $\phi$  subjected to the potential  $V \sim -\cos \frac{\phi}{f}$  (where  $f$  is, as usual, the axion decay constant), (4.33) results in  $|f| < \frac{M_P}{2} \left| \tan \frac{\phi}{f} \right|$  and, since this inequality has to be satisfied for any value of  $\phi$  and so also for  $\phi = 0$ , the just noted tension between (4.33) and (4.27) emerges.

With the attempt of generalizing Palti's results and (also) correcting the previous inconsistency, Gonzalo and Ibàñez formulate a **Strong** version of the **Scalar Weak Gravity Conjecture** (SSWGC). It can be stated as follows [9]:

*The potential  $V(\phi)$  of a canonically normalized real scalar field  $\phi$  in the theory (under consideration) must satisfy for any value of the field the constraint*

$$2(V''')^2 - V''V'''' \geq \frac{(V'')^2}{M_P^2} \quad (4.34)$$

*(with the " ' " denoting the derivative with respect to  $\phi$ ).*

Coherently with the physical principle the conjecture is animated by, the inequality (4.34) requires that the strength of the scalar force must always be stronger than that of the gravitational interaction.

Gonzalo and Ibàñez's claim appears really attractive because of its generality: it is proposed to be true for any scalar field and any value of the scalar field.

We can, for instance, apply the bound to axions and to their periodic potential (appearing in

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<sup>31</sup>In [9] the factor 4 lacks: or it was simply forgotten or confusion in intending the derivatives with respect to  $H$  and  $\phi$  was made.



String Theory whenever an axion-like scalar couples to a non-Abelian gauge group) and see if the inconsistency of (4.33) (appropriately intended) with (4.27) is solved.

By subjecting to (4.34) the leading contribution to the axion potential instantons give rise to, we get

$$\frac{1}{f^6} \left[ 2 \sin^2 \frac{\phi}{f} + \cos^2 \frac{\phi}{f} \right] \geq \frac{1}{M_P^2 f^4} \cos^2 \frac{\phi}{f} \quad (4.35)$$

and by dividing both sides of the inequality by  $\cos^2 \frac{\phi}{f}$ , we then obtain

$$f^2 \leq \left[ 1 + 2 \tan^2 \frac{\phi}{f} \right] M_P^2. \quad (4.36)$$

Since we require the conjecture to be valid for any value of the field  $\phi$ , we deduce

$$f \leq M_P. \quad (4.37)$$

Thanks to the addition of the quartic coupling term in (4.34) with respect to (4.33), we find that Gonzalo and Ibàñez's conjecture agrees with (4.27) (when perturbative control is required,  $S_E \geq 1$ ), differently from what happens for (4.33) itself. This can be regarded as a test for Gonzalo and Ibàñez's bound.

If Gonzalo and Ibàñez's SSWG conjecture would be true, it would have relevant implications for Cosmology and Particle Physics.

As clarifying examples, let us consider how (4.34) constrains some inflationary models and the Standard Model.

## Inflation

Among polynomial potentials, the linear case is the only one that allows for trans-Planckian excursions according to (4.34). In the models of chaotic inflation [82] those based on linear potentials are singled out as the class that can lead to sufficient inflation: correspondingly to the tensor-to-scalar ratio  $r \sim 0.07$ , they give  $50 \div 60$   $e$ -folds of inflation. Instead of considering purely linear potentials, one can deal with potentials that behave linearly for  $|\phi| > M_P$ . Such potentials appear in String Theory when studying monodromy inflation [83–87]. One type of potential in this category is

$$V_{mi}(b) = A \left[ 1 + B \left( \frac{b}{M_P} \right)^2 \right]^{\frac{1}{2}}, \quad (4.38)$$

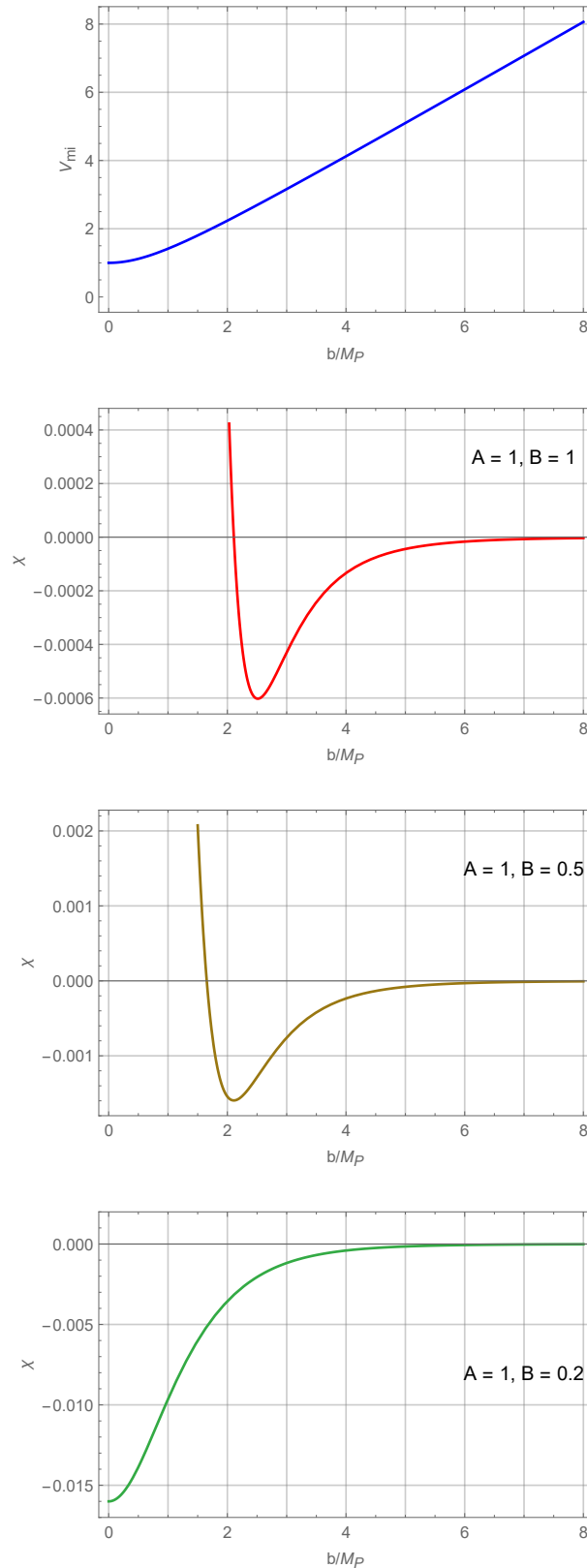
where  $b$  is a type IIB monodromic axion field.

To check Gonzalo and Ibàñez's conjecture we can introduce the combination

$$\chi = 2 (V''')^2 - V'' V'''' - \left( \frac{V''}{M_P} \right)^2 : \quad (4.39)$$

(4.34) is satisfied whenever  $\chi \geq 0$ .

In **Figure 4.2** the behaviour of  $V_{mi}(b)$  and  $\chi$  with respect to  $\frac{b}{M_P}$  is represented for  $A = 1$  and some values of  $B$  (in appropriate units).



**Figure 4.2:**

This figure illustrates the behaviour of  $V_{mi}(b)$  and that of  $\chi$  with respect to  $\frac{b}{M_P}$  (for  $b \geq 0$ ) in the cases  $A = 1$  and  $B = 1, 0.5, 0.2$ . Gonzalo and Ibáñez's conjecture is satisfied if  $\chi \geq 0$ .

When dealing with inflationary models two parameters, usually denoted as  $\epsilon$  and  $\eta$ , can be introduced: they are defined in terms of the inflationary potential  $V$  and its derivatives as

$$\epsilon = \left( \frac{V'}{V} \right)^2 M_P^2$$

and

$$\eta = \frac{V''}{V} M_P^2.$$

The condition for “getting out” of inflation realizes when  $\epsilon$  and  $\eta$  are of  $\mathcal{O}(1)$ .

Testing the agreement of an inflationary model with Gonzalo and Ibàñez’s conjecture amounts to compare the value of the inflaton corresponding to the exit from inflation and its value at which the combination  $\chi$  becomes negative. Going from larger to smaller values of  $\phi$ , if one gets out of inflation before  $\chi$  starts to be negative, no tension between the inflationary model that one is considering and Gonzalo and Ibàñez’s bound arises; if, instead, the getting out of inflation follows the becoming negative of  $\chi$ , by assuming that Gonzalo and Ibàñez’s inequality is a true Swampland criterion, the inflationary model has to be modified.

In the case of  $V_{mi}(b)$  (for  $b \geq 0$ ), when (for instance)  $A = 1$  and  $B = 1$ ,  $\epsilon_{mi}^{(A=1,B=1)} = \eta_{mi}^{(A=1,B=1)} = 1$  (in Planck units) for  $b = 0$ . The corresponding  $\chi_{mi}^{(A=1,B=1)}$  is negative for  $\frac{b}{M_P} \gtrsim 2.11$ , as **Figure 4.2** shows. So, the inflationary model referred to the potential (4.38) (with  $A = B = 1$ ) is in tension with Gonzalo and Ibàñez’s bound. A similar analysis can be made when  $A$  and  $B$  are chosen to be  $A = 1$  and  $B = 0.5$  or  $B = 0.2$ .

We conclude that the potential (4.38) violates Gonzalo and Ibàñez’s conjecture at the per-mil level above  $b \simeq 2M_P$ . But, because we do not have control on the theory at such precision, we can state that the monodromic inflationary model based on  $V_{mi}(b)$  passes the test.

We can then consider monomial potentials of the form

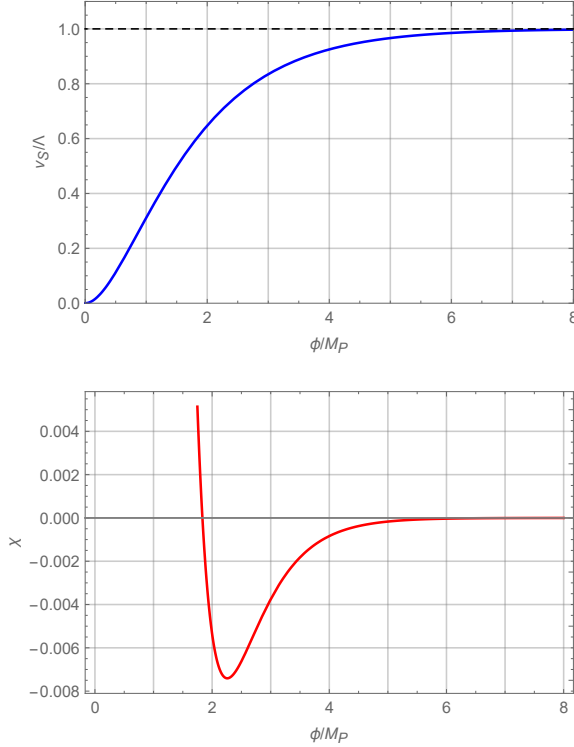
$$V_m(\phi) = \phi^a \tag{4.40}$$

for  $a \geq 0$ . The condition  $\chi \geq 0$  translates into  $(a-2)(a-1)M_P^2 - \phi^2 \geq 0$  for  $a \neq 0, 1$ . When  $0 < a < 1$  the potential is characterized by tiny violations of (4.34) in the region  $\phi < \sqrt{(a-1)(a-2)}M_P$ . For  $a > 2$  there are still violations of the bound and they are trans-Planckian when  $a > 2.7$ . For  $1 < a \leq 2$  (4.34) is violated at all points in field space. The cases  $a = 0$  and  $a = 1$  are those for which Gonzalo and Ibàñez’s conjecture is satisfied at any locus in field space.

The recent cosmological observations support the idea that, after having fixed the initial condition as chaotic inflation prescribes, the Starobinsky inflationary model is that better reproducing the experimental data. It is based on the potential

$$V_S(\phi) = \Lambda \left( 1 - e^{-\sqrt{\frac{2}{3}} \frac{\phi}{M_P}} \right)^2, \tag{4.41}$$

where  $\phi$  is the inflaton and  $\Lambda$  is the typical energy scale of inflation [88, 89].



**Figure 4.3:**

This figure represents the behaviour of  $V_S(\phi)$  and that of  $\chi$  with respect to  $\frac{\phi}{M_P}$  (for  $\phi \geq 0$ ). Gonzalo and Ibáñez's bound is satisfied if  $\chi \geq 0$ .

By imposing  $\epsilon_S = \eta_S = 1$  (in Planck units) for the Starobinsky model (4.41), we get that the getting out of inflation occurs around  $\frac{\phi}{M_P} = 1$ . As **Figure 4.3** shows, the combination  $\chi_S$  is negative for  $\frac{\phi}{M_P} \gtrsim 1.83$ . Therefore,  $V_S(\phi)$  is not consistent with the constraint (4.34).

To gain consistency with Gonzalo and Ibáñez's conjecture the Starobinsky's inflationary model in its original formulation needs to be modified at trans-Planckian distances. The possible addition of a perturbation may render it consistent.

### The Standard Model and its 3-dimensional compactification

Let us consider the SM and compactify it on a circle of radius  $R$ , which is given by  $R = e^{\frac{\phi}{\sqrt{2}M_P^3d}}$  in terms of the quantum fluctuation field  $\phi$  (with canonically normalized kinetic term).

If we concentrate in the deep infrared region where  $R \gg \frac{1}{m_{e^-}}$ , the 3-dimensional 1-loop effective potential is given (in terms of  $R$ ) by [90]

$$V(R) = \frac{2\pi r^3 \Lambda_4}{R^2} - 4 \left( \frac{r^3}{720\pi R^6} \right) + \sum_{\nu_e, \nu_\mu, \nu_\tau} r^3 V_C(R, m_{\nu_i}), \quad (4.42)$$

where  $r$  is a given reference scale to measure  $R$  (it can be fixed to 1 GeV) and

$$V_C(R, m_{\nu_i}) = \frac{n_{\nu_i} m_{\nu_i}^2}{8\pi^4 R^4} \sum_{j=0}^{\infty} \frac{K_2(2\pi m_{\nu_i} j R)}{j^2}, \quad (4.43)$$

being  $n_{\nu_i}$  the helicities for each neutrino ( $n_{\nu_i} = 2$  for Majorana neutrinos and  $n_{\nu_i} = 4$  for Dirac neutrinos) and  $K_{2,j}$  the modified Bessel functions of the second kind. The first term of (4.42) comes from the 4-dimensional cosmological constant term  $\Lambda_4$  after dimensional reduction and going to the 3-dimensional Einstein frame. The second term corresponds to the 1-loop Casimir energy of the massless photon and graviton; since they are characterized by two helicity degrees of freedom each, a factor 4 appears. The last term represents the contribution to the Casimir energy of the three neutrinos compactified with periodic boundary conditions. All other possible contributions from higher thresholds are exponentially suppressed by factors of the order  $e^{-\frac{m}{m_\nu}}$ .

If the neutrinos are of Majorana type, the potential (4.42) exhibits AdS local minima. In fact, the lightest neutrino contributes positively to the potential with two degrees of freedom; this is not sufficient to compensate the negative terms associated to the photon and the graviton, corresponding to four degrees of freedom. If, instead, the lightest neutrino is a Dirac neutrino, it participates with four degrees of freedom to the value of the potential and this is enough to compensate the contributions from the four massless degrees of freedom of the photon and the graviton. In this last circumstance the potential monotonously decreases with  $R$  and no AdS minima develop. By imposing that AdS non-supersymmetric minima are in the Swampland [91] we can obtain constraints on the neutrino masses and on the 4-dimensional cosmological constant [92].

In particular, we deduce that the lightest neutrino is a Dirac neutrino and its mass is  $m_{\nu_1} \leq 7.7 \times 10^{-3}$  eV in normal hierarchy and  $m_{\nu_3} \leq 2.5 \times 10^{-3}$  eV for the inverted hierarchy.

As we have discussed above, quantum gravity may ensure the absence of bound states that are protected from decay by their charge or by the particle being the lightest in the theory. In the SM neutrinos are the lightest massive particles and they could form a tower of stable bound states. At long distances the only force neutrinos feel is gravity and, therefore, in empty space they would form bound states. However, this might not be the case when a cosmological constant is present. In the weak gravity regime the cosmological constant can be modelled as a repulsive linear force so that the gravitational interaction between two neutrinos at distance  $d$  is

$$F_{gr}^{(\nu)} = m_\nu \left( -\frac{m_\nu}{d^2} + \frac{\Lambda_4 d}{3} \right) \quad (4.44)$$

( $m_\nu$  being the mass of the lightest neutrino) up to the scale  $d \sim m_\nu^{-1}$ . For neutrinos not to form stable bound states we require [75]

$$\Lambda_4 > m_\nu^4. \quad (4.45)$$

The bound  $\Lambda_4 > m_\nu^4$  can be regarded as a condition on how small the cosmological constant should be or as a constraint on how heavy neutrinos could be. Following the latter interpretation, it can be translated into a bound on the electroweak scale and, as a consequence, on the Higgs v.e.v. . An upper bound on the neutrino masses implies an upper bound on the Higgs v.e.v. .

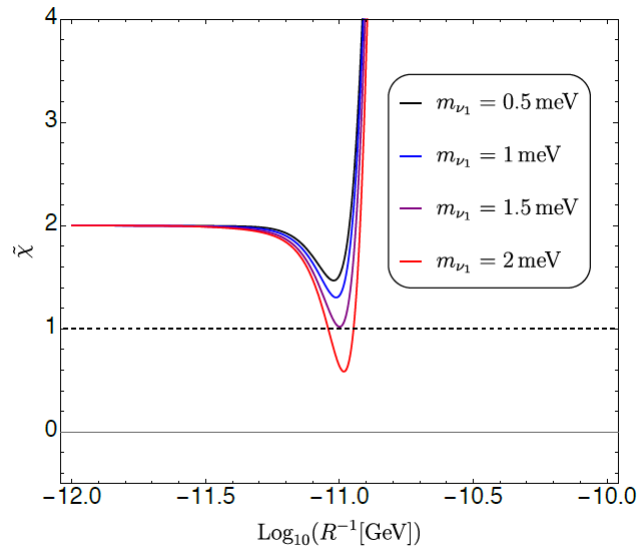
Gonzalo and Ibàñez's conjecture (extended to three dimensions) allows to get similar (and not identical) constraints on the SM. Gonzalo and Ibàñez's perspective is attractive, because the AdS Swampland condition can be applied when the AdS minima are absolutely stable and this is always difficult to prove.

To verify if the effective potential (4.42) of the SM compactified on a circle satisfies (4.34) it is convenient to define

$$\frac{\tilde{\chi}}{M_P^2} = 2 \left( \frac{V''''}{V''} \right)^2 - \frac{V''''}{V''} \quad (4.46)$$

and test the condition  $\frac{\tilde{\chi}}{M_P^2} \geq 1$ .

By taking the derivatives with respect to  $\phi$  and calculating them analytically by using the standard formulas for the Bessel functions  $K_{2,j}$ , we get that (4.34) is violated unless the lightest Dirac neutrino mass is  $m_{\nu_1} < 1.5 \times 10^{-3} \text{eV}$  in normal neutrino hierarchy or the lightest Dirac neutrino mass satisfies  $m_{\nu_3} > 1.6 \times 10^{-3} \text{eV}$  for the inverted hierarchy [9].



**Figure 4.4** [9]:

This figure represents the bounds on the lightest neutrino mass for normal hierarchy when the SM is compactified on a circle of radius  $R$  and its 3-dimensional 1-loop effective potential is tested with Gonzalo and Ibàñez’s conjecture.

By combining the results in [92,93], we conclude that, if the AdS Swampland condition and Gonzalo and Ibàñez’s conjecture are true, then the SM with inverted hierarchy would be in the Swampland. Normal hierarchy is another non-trivial prediction that Gonzalo and Ibàñez’s bound allows to make.

The just proposed discussion suggests Gonzalo and Ibàñez’s SSWG C can be in principle an incredibly powerful tool. It gives constraints on models (such as inflationary models or the Standard Model) our Universe is described by and so it really seems to point towards the formulation of a consistent theory of quantum gravity that would be able to select, among a “jungle” of possible vacua, the state corresponding to our Universe.

In spite of being so, we will now evidence various criticisms of Gonzalo and Ibàñez’s conjecture (4.34) that make us think that their statement is not appropriately formulated.

When looking at (4.34), one poses the natural question on how the coefficients of the terms appearing in the inequality have been derived.

Following the diagrammatic approach Gonzalo and Ibàñez briefly suggest in their article, the coefficients that they propose can't be found.

As the authors stated during the *String Phenomenology Conference* that took place at *CERN* in June (L. Ibàñez, *On Towers and Scalars*, String Phenomenology 2019 (June 24<sup>th</sup>-28<sup>th</sup> 2019, CERN), <http://indico.cern.ch/event>), the particular choice of the numerical coefficients appearing in the conjecture is justified by saturating the inequality (4.34). More precisely, let us consider the extremal condition of (4.34) for a single scalar. The scalar interactions equal the gravitational one and the constraint may be written as a differential equation on the field dependent mass  $m^2(\phi)$ ,

$$2((m^2)')^2 - m^2(m^2)'' - \frac{m^4}{M_P^2} = 0. \quad (4.47)$$

The extremal solution for  $m^2$  (by expressing  $\phi$  in units of  $M_P$ ) is

$$m^2(\phi) = \frac{Ae^\phi}{Be^{2\phi} + 1}, \quad (4.48)$$

where  $B \geq 0$  (and  $A > 0$ , as a choice). By defining  $R = e^{\frac{\phi}{2}}$  (with kinetic metric  $2(dR/R)^2$ ) (4.48) can be more suggestively rewritten as

$$m^2(\phi) = \frac{m_0^2}{1/(NR)^2 + (R/M)^2} = m_0^2 \frac{(NM)^2}{M_{M,N}^2}, \quad (4.49)$$

being

$$M_{M,N}^2 = N^2 R^2 + \frac{M^2}{R^2}. \quad (4.50)$$

The quantity  $M_{M,N}$  looks like the spectrum of a string compactification on a circle with the duality invariance

$$R \longleftrightarrow \frac{1}{R}; \quad M \longleftrightarrow N. \quad (4.51)$$

For large and small  $R$  one obtains that

$$m_{\phi \rightarrow +\infty}^2 \longrightarrow m_0^2 M^2 e^{-\phi}; \quad m_{\phi \rightarrow -\infty}^2 \longrightarrow m_0^2 N^2 e^\phi. \quad (4.52)$$

If  $N$  and  $M$  are integers, this is the structure of towers of winding and momenta modes becoming light as  $|\phi| \rightarrow +\infty$ . Gonzalo and Ibàñez's interpretation of this result is that these towers are the WGC scalars that are required so that gravity keeps on being the weakest force when  $|\phi|$  goes to infinity. If  $\phi$  is identified with a modulus, the statement of the distance conjectures is precisely retrieved.

Even though Gonzalo and Ibàñez's choice of the coefficients allows coherence with the SDC and points towards the weaving of a network of Swampland criteria, this *a posteriori* justification for them seems a rather weak motivation.

In spite of pretending to be a general statement, another criticism (actually preventing such generality) appears at first sight when considering Gonzalo and Ibàñez's conjecture.

In the form (4.34) is written down, we would expect the appearance of at least two other terms

that are lacking though. In principle, there is no reason why a general statement should not include terms such as  $V'V''''$  and  $VV''''''$ .

The absence of these two further contributions to the inequality (4.34) can be understood by thinking that the conjecture applies in the vacuum for, say,  $\langle\phi\rangle = 0$  (where  $V' = 0$  and one can rid of the cosmological "constant term" including  $V$ ) and in its vicinity.

Extending the validity of the conjecture "outside" the vacuum is an absolutely non-trivial task. Because it is not clear to us how to formulate a statement "outside" the vacuum, in the following (when referring to (4.34) or to our attempts in correcting it) we will set the discussion around the vacuum.

Having evidenced two first-sight criticisms of Gonzalo and Ibàñez's bound, we will dedicate the remain part of this paragraph to try to investigate its meaning and content. In order to do so, as briefly suggested in [9], we will use a Quantum Field Theory approach.

Let us consider the theory of a real scalar field  $\phi$  subjected to an arbitrary potential  $V(\phi)$  and including gravity; its action is

$$S = \int d^4x \sqrt{-g} \left[ -\frac{M_P^2}{2} R + \frac{1}{2} g_{\mu\nu} \partial^\mu \phi \partial^\nu \phi - V(\phi) \right]. \quad (4.53)$$

In the weak gravitational field limit and having in mind Minkowski spacetime as background, the spacetime metric  $g_{\mu\nu}$  can be expanded as [94]

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{2}{M_P} h_{\mu\nu}, \quad (4.54)$$

where  $\eta_{\mu\nu} = \text{diag}(+1, -1, -1, -1)$  is the Minkowski metric and  $h_{\mu\nu}$  represents the quantum fluctuation of the gravitational field.

Since we would like to perform interaction theory, it is convenient to appropriately express the  $\sqrt{-g}$  factor, involving the determinant of the metric  $g_{\mu\nu}$ .

By factorizing  $\eta_{\mu\lambda}$  in expressing  $g_{\mu\nu}$ , we have that  $\sqrt{-g} = \sqrt{-\det g_{\mu\nu}}$  is equal to

$$\begin{aligned} \sqrt{-g} &= \left[ -\det(\eta_{\mu\lambda}) \det\left(\delta_\nu^\lambda + \frac{2}{M_P} h_\nu^\lambda\right) \right]^{\frac{1}{2}} = \left[ e^{\text{tr}\left(\ln\left(\delta_\nu^\lambda + \frac{2}{M_P} h_\nu^\lambda\right)\right)} \right]^{\frac{1}{2}} = \\ &= \sum_{i=0}^{+\infty} \frac{1}{i!} \left[ \frac{1}{2} \sum_{j=1}^{+\infty} \frac{(-1)^{j+1}}{j} \left( \frac{2}{M_P} h_\nu^\lambda + \dots \right)^j \right]^i. \end{aligned} \quad (4.55)$$

By moving to the momentum space, the Feynman rules for the scalar and the graviton's propagators and for the interaction vertices can be deduced [94].

The propagator for the scalar field  $\phi$  is given by

$$\begin{array}{c} \text{p} \\ \xrightarrow{\hspace{1.5cm}} \\ \text{-----} \end{array} = \frac{i}{p^2 - V''|_{\phi=0}}, \quad (4.56)$$

where  $V''|_{\phi=0} = m^2 (> 0)$ .

The propagator for the graviton  $g_{\mu\nu}$  is



$$\begin{array}{c}
\text{q} \\
\longrightarrow \\
\text{---} \\
\mu\nu \qquad \qquad \rho\sigma
\end{array} = \frac{i}{q^2} \frac{1}{2} [\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \eta_{\mu\nu}\eta_{\rho\sigma}]. \quad (4.57)$$

Being interested in a tree-level analysis, the interaction vertices that are relevant for our purposes are<sup>32</sup>

$$\begin{array}{c}
\text{---} \\
p_1 \qquad \qquad p_3 \\
\swarrow \qquad \searrow \\
\bullet \\
\swarrow \qquad \searrow \\
p_2 \qquad \qquad \text{---}
\end{array} = -iV'''|_{\phi=0} \frac{1}{3!}; \quad (4.58)$$

$$\begin{array}{c}
\text{---} \\
p_1 \qquad \qquad p_3 \\
\swarrow \qquad \searrow \\
\blacksquare \\
\swarrow \qquad \searrow \\
p_2 \qquad \qquad p_4
\end{array} = -iV''''|_{\phi=0} \frac{1}{4!} \quad (4.59)$$

and

$$\begin{array}{c}
\text{---} \\
p_1 \qquad \qquad q \\
\swarrow \qquad \searrow \\
\bullet \text{---} \\
\mu\nu \\
\swarrow \qquad \searrow \\
p_2 \qquad \qquad \text{---}
\end{array} = \frac{i}{M_P} [p_{1,\mu}p_{2,\nu} + p_{1,\nu}p_{2,\mu} - \eta_{\mu\nu}(p_1p_2 - V''|_{\phi=0})] \frac{1}{2!}. \quad (4.60)$$

In writing the Feynman rules for the vertices we have referred to the conventions of [95, 96]. For instance, each vertex has an associated symmetry factor which prevents the over-counting of the possibilities a diagram can be constructed by when external lines are going to be attached to the fixed vertices' legs.

Let us now study the tree-level interactions that have the field  $\phi$  as protagonist. When two  $\phi$  fields in the initial state interact giving rise to two  $\phi$  particles in the final state thanks to a  $\phi$  mediator, three diagrams (corresponding to the channels  $s$ ,  $t$  and  $u$ ) have to be considered.

<sup>32</sup>It is worth noting that, apart from the omitted  $\delta$ -functions expressing the conservation of the momenta at the vertices, the Feynman rules involving only the scalar field  $\phi$  are independent of the momenta themselves.

They are

$$\mathcal{M}_s^{(1)} = \text{Diagram (4.61)} \quad (4.61)$$

$$\mathcal{M}_t^{(1)} = \text{Diagram (4.62)} \quad (4.62)$$

and

$$\mathcal{M}_u^{(1)} = \text{Diagram (4.63)} \quad (4.63)$$

By taking into account the way the external fields on both sides of the diagram can be glued to the legs of each corresponding vertex, Feynman rules give

$$i\mathcal{M}_s^{(1)} = \left( \frac{-iV'''|_{\phi=0}}{3!} \right)^2 \frac{i}{(p_1 + p_2)^2 - V''|_{\phi=0}} \times (3!)^2 = -i \frac{(V'''|_{\phi=0})^2}{(p_1 + p_2)^2 - V''|_{\phi=0}}; \quad (4.64)$$

$$i\mathcal{M}_t^{(1)} = -i \frac{\left(V'''|_{\phi=0}\right)^2}{(p_1 - p_3)^2 - V''|_{\phi=0}} = -i \frac{\left(V'''|_{\phi=0}\right)^2}{q^2 - V''|_{\phi=0}} \quad (4.65)$$

and

$$i\mathcal{M}_u^{(1)} = -i \frac{\left(V'''|_{\phi=0}\right)^2}{(p_1 - p_4)^2 - V''|_{\phi=0}}. \quad (4.66)$$

Let us refer to the non-relativistic limit (where  $p_i^0 \simeq m + \frac{\vec{p}_i^2}{2m}$  for  $i = 1, \dots, 4$ ) and use the static approximation (for which the energy  $q^0$  exchanged during the scattering is taken to be 0).

Furthermore, let us make reference to the rest frame (" ' ") of the system (1,2). There,  $\vec{p}_1^{\vec{j}} + \vec{p}_2^{\vec{j}} = \vec{0}$  and (as a consequence)  $p_1'^0 = p_2'^0$ ; moreover, because of four-momentum conservation,  $\vec{p}_3^{\vec{j}} + \vec{p}_4^{\vec{j}} = \vec{0}$  ( $= \vec{p}_1^{\vec{j}} + \vec{p}_2^{\vec{j}}$ ) and so  $p_3'^0 = p_4'^0 = p_1'^0 = p_2'^0$ .

By exploiting (also) external particles' on-shellness, the amplitudes  $\mathcal{M}_s^{(1)}$ ,  $\mathcal{M}_t^{(1)}$  and  $\mathcal{M}_u^{(1)}$  become

$$\mathcal{M}_s^{(1)} = - \frac{\left(V'''|_{\phi=0}\right)^2}{4m^2 \left(1 + \frac{\vec{p}_1^{\vec{j}2}}{m^2}\right) - m^2} \simeq - \frac{\left(V'''|_{\phi=0}\right)^2}{3m^2} = \text{constant}; \quad (4.67)$$

$$\mathcal{M}_t^{(1)} = + \frac{\left(V'''|_{\phi=0}\right)^2}{\vec{q}'^2 + m^2} \quad (4.68)$$

and

$$\mathcal{M}_u^{(1)} = - \frac{\left(V'''|_{\phi=0}\right)^2}{\vec{q}'^2 - 4p_1'^2 - m^2} = + \frac{\left(V'''|_{\phi=0}\right)^2}{2p_1'^2 (1 + \cos \theta) + m^2} \quad (4.69)$$

(either by using the Maldestam relation  $s + t + u = 4m^2 \simeq (4m^2 + 4p_1^{\vec{j}2}) - \vec{q}'^2 + u$  or by simply making use of the rest frame relations among three-momenta and denoting as  $\theta$  the angle between  $\vec{p}_1^{\vec{j}}$  and  $\vec{p}_3^{\vec{j}}$ ).

Another contribution to the scalar interactions is represented by the self-interaction diagram

$$\mathcal{M}^{(2)} = \quad \begin{array}{c} \diagup \text{---} \text{---} \text{---} \text{---} \diagdown \\ p_1 \quad \quad \quad p_3 \\ \quad \quad \quad \blacksquare \\ \diagdown \text{---} \text{---} \text{---} \text{---} \diagup \\ p_2 \quad \quad \quad p_4 \end{array} \quad (4.70)$$

and thanks to Feynman rules the amplitude

$$\mathcal{M}^{(2)} = \frac{-V''''|_{\phi=0}}{4!} \times 4! = -V''''|_{\phi=0} = \text{constant} \quad (4.71)$$

is obtained. The factor  $4!$  is justified (as usual) by counting the possibilities in representing the diagram by connecting the external fields with the vertex legs.

The gravitational interaction between two  $\phi$  states is described by diagrams where a graviton acts as mediator. By distinguishing the three possible channels these graphs are

$$\mathcal{M}_s^{(3)} = \begin{array}{c} \begin{array}{c} \text{---} p_1 \text{---} \\ \text{---} p_2 \text{---} \\ \text{---} p_3 \text{---} \\ \text{---} p_4 \text{---} \end{array} \\ \begin{array}{c} \leftarrow -q \quad \rightarrow q \\ \mu\nu \quad \rho\sigma \end{array} \end{array} \quad (4.72)$$

$$\mathcal{M}_t^{(3)} = \begin{array}{c} \begin{array}{c} \text{---} p_2 \text{---} \\ \text{---} p_4 \text{---} \\ \text{---} p_1 \text{---} \\ \text{---} p_3 \text{---} \end{array} \\ \begin{array}{c} q \uparrow \quad \rho\sigma \\ -q \downarrow \quad \mu\nu \end{array} \end{array} \quad (4.73)$$

and

$$\mathcal{M}_u^{(3)} = \begin{array}{c} \begin{array}{c} \text{---} p_2 \text{---} \\ \text{---} p_3 \text{---} \\ \text{---} p_1 \text{---} \\ \text{---} p_4 \text{---} \end{array} \\ \begin{array}{c} q \uparrow \quad \rho\sigma \\ -q \downarrow \quad \mu\nu \end{array} \end{array} \quad (4.74)$$

According to Feynman rules, we find

$$i\mathcal{M}_s^{(3)} = -i \frac{2}{M_P^2 (p_1 + p_2)^2} [(p_1 p_3)(p_2 p_4) + (p_1 p_4)(p_2 p_3) - (p_1 p_2)(p_3 p_4) - m^2(p_1 p_2) + \\ -m^2(p_3 p_4) - 2m^4]; \quad (4.75)$$

$$i\mathcal{M}_t^{(3)} = -i \frac{2}{M_P^2 (p_1 - p_3)^2} [(p_1 p_2)(p_3 p_4) + (p_1 p_4)(p_2 p_3) - (p_1 p_3)(p_2 p_4) + m^2(p_1 p_3) + m^2(p_2 p_4) - 2m^4] \quad (4.76)$$

(with  $p_1 - p_3 = q$ ) and

$$i\mathcal{M}_u^{(3)} = -i \frac{2}{M_P^2 (p_1 - p_4)^2} [(p_1 p_2)(p_3 p_4) + (p_1 p_3)(p_2 p_4) - (p_1 p_4)(p_2 p_3) + m^2(p_1 p_4) + m^2(p_2 p_3) - 2m^4]. \quad (4.77)$$

In the non-relativistic limit; in the static approximation and by referring (as before) to the (1,2) rest frame, the amplitudes take the leading order forms

$$\mathcal{M}_s^{(3)} = + \frac{3m^2}{2M_P^2 \left(1 + \frac{p_1^2}{m^2}\right)} \simeq + \frac{3m^2}{2M_P^2} = \text{constant}; \quad (4.78)$$

$$\mathcal{M}_t^{(3)} = + \frac{2m^4}{M_P^2 q^2} = + \frac{m^4}{M_P^2 p_1^2 (1 - \cos \theta)} \quad (4.79)$$

and

$$\mathcal{M}_u^{(3)} = - \frac{2m^4}{M_P^2 (q^2 - 4p_1^2)} = + \frac{m^4}{M_P^2 p_1^2 (1 + \cos \theta)} \quad (4.80)$$

(either by using the Maldestam relation or by simply making use of the rest frame relations among three-momenta and denoting as  $\theta$  the angle between  $\vec{p}_1$  and  $\vec{p}_3$ ).

Let us focus on the limit<sup>33</sup>

$$|\vec{q}'| \longrightarrow 0, \quad (4.81)$$

that is let us consider a scattering condition characterized by small exchanged energies and large interaction distances.

In this circumstance, only the  $t$ -channels  $\mathcal{M}_t^{(1)}$  and  $\mathcal{M}_t^{(3)}$  may diverge,  $p_1^2$  being some given non-vanishing number. But, because the field  $\phi$  is massive ( $m > 0$ ),  $\mathcal{M}_t^{(1)}$  doesn't diverge either in the interesting limit  $|\vec{q}'| \longrightarrow 0$ .

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<sup>33</sup>This limit can be also regarded as a non-deflection limit: in the center of mass frame,  $\theta \longrightarrow 0$  and the scattering is of  $s$ -wave type.

More precisely, by moving from the momentum space to the configuration space (where the interaction distance is called  $|\vec{x}|$ ) through Fourier transform operations (denoted by a “ $\tilde{\phantom{x}}$ ”) we find that

$$\mathcal{M}_s^{(1)} \propto \delta^{(3)}(\vec{x}); \quad \mathcal{M}^{(2)} \propto \delta^{(3)}(\vec{x}); \quad \mathcal{M}_s^{(3)} \propto \delta^{(3)}(\vec{x}), \quad (4.82)$$

because the Fourier transform of a constant is proportional to the Dirac  $\delta$ -function; then

$$\mathcal{M}_t^{(1)} \propto \frac{e^{-m|\vec{x}|}}{|\vec{x}|}; \quad (4.83)$$

and

$$\mathcal{M}_u^{(1)} \propto \frac{e^{i\sqrt{4p_1^2 + m^2}|\vec{x}|}}{|\vec{x}|}; \quad \mathcal{M}_u^{(3)} \propto \frac{e^{2i|p_1||\vec{x}|}}{|\vec{x}|}, \quad (4.84)$$

the Fourier transform of  $\frac{1}{q^2 \pm a^2}$  (with  $a$  a real number) being the Yukawa-type term  $\frac{e^{-a|\vec{x}|}}{|\vec{x}|}$  and  $\frac{e^{ia|\vec{x}|}}{|\vec{x}|}$  respectively; and finally

$$\mathcal{M}_t^{(3)} \propto \frac{1}{|\vec{x}|}, \quad (4.85)$$

since the Fourier transform of  $\frac{1}{q^2}$  has the Coulomb-type behaviour  $\frac{1}{|\vec{x}|}$ .

The relations (4.82), (4.83), (4.84) and (4.85) allow to conclude that, because all the scalar scattering terms are either contact interaction terms or terms with a decaying behaviour bounded between (say)  $-\frac{1}{|\vec{x}|}$  and  $+\frac{1}{|\vec{x}|}$  or exponentially suppressed Yukawa-like interactions, they can't beat the gravitational force in the large distance regime: in its Coulomb-type  $t$ -channel gravity decays much less rapidly than the scalar force does.

Being so, we claim that Gonzalo and Ibàñez's conjecture requires a condition that is in contrast with its Quantum Field Theory derivation (which the same authors suggest in their article).

Even though Gonzalo and Ibàñez's bound is inspired by the physical principle that gravity has to be the weakest force in any circumstance, we have found that, in the well-understood regime of large distances, gravity acts more strongly than the scalar force. The reason behind this result is the freedom in choosing an arbitrary field  $\phi$ : because in principle it is massive, the scalar  $t$ -channel (in particular) can't ever compete with the gravitational  $t$ -channel as it could do if the scalar force mediator was massless.

To overcome the tension we have just pointed out we can try to study models with many scalar fields among which one is strictly massless and see what are the constraints that the request that gravity has to be the weakest force imposes on the parameters of such theories. This will be the topic of the next paragraph.

## 4.5 Some models to try to get a general statement

As we have just emphasized, the fact that the field  $\phi$  is arbitrary and so (*a priori*) massive makes Gonzalo and Ibáñez's conjecture "contradictory". We would like to amend (4.34) and try to get (if there is one) a general statement.

In the same spirit of Palti's proposals for a scalar version of the WGC ([8]), we will consider a model including gravity and involving two scalar fields, namely  $\phi$  and  $H$ , with  $\phi$  that is taken to be strictly massless. The idea behind the analysis we are approaching to is to compare the interaction strengths of the force mediated by the massless scalar  $\phi$  and the gravitational force, taking  $H$  as a testing particle. By imposing the scalar force has always to beat gravity we will deduce some constraints on the parameters of the theory.

Let us consider the action

$$S = \int d^4x \sqrt{-g} \mathcal{L} = \int d^4x \sqrt{-g} \left[ -\frac{M_P^2}{2} R + \frac{1}{2} g_{\mu\nu} \partial^\mu \phi \partial^\nu \phi + \frac{1}{2} g_{\mu\nu} \partial^\mu H \partial^\nu H - V(\phi, H) \right], \quad (4.86)$$

where

$$V(\phi, H) = \frac{1}{2} m^2 H^2 + \frac{1}{2} \mu \phi H^2 + \frac{1}{4} \lambda \phi^2 H^2. \quad (4.87)$$

We will study the theory (4.86) in a weak gravitational regime, where the spacetime metric  $g_{\mu\nu}$  can be expanded as

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{2}{M_P} h_{\mu\nu} \quad (4.88)$$

( $\eta_{\mu\nu} = \text{diag}(+1, -1, -1, -1)$  being the Minkowski metric); and around a (supposed) minimum of the potential  $V(\phi, H)$ , for which  $\partial_\phi V = 0 = \partial_H V$  and, say,  $\langle \phi \rangle = 0$  and  $\langle H \rangle = 0$ .

Since we want the scalar field  $\phi$  to be strictly massless, we have to prevent it from acquiring even an effective mass (when, in case, integrating over  $H$ ). This request translates into the conditions  $\partial_\phi^2 V = 0$  and  $\partial_\phi \partial_H V = 0$  and so determines the form of the potential (4.87)<sup>34</sup>.

By moving to the momentum space the relevant Feynman rules for the scalars and the graviton's propagators and for the interaction vertices can be deduced [94–96].

From  $\frac{1}{2} \eta_{\mu\nu} \partial^\mu \phi \partial^\nu \phi$ , the propagator for the scalar field  $\phi$  is derived

$$\begin{array}{c} \text{P} \\ \xrightarrow{\hspace{1.5cm}} \\ \text{-----} \end{array} = \frac{i}{p^2}; \quad (4.89)$$

from  $\frac{1}{2} \eta_{\mu\nu} \partial^\mu H \partial^\nu H - \frac{1}{2} m^2 H^2$ , the propagator for the scalar  $H$  results in

$$\begin{array}{c} \text{P} \\ \xrightarrow{\hspace{1.5cm}} \\ \text{-----} \end{array} = \frac{i}{p^2 - m^2} \quad (4.90)$$

---

<sup>34</sup>From now on, with a small abuse of notation, we will denote as  $\phi$  and  $H$  the quantum fluctuations of the same named fields around their vacuum expectation values.

and, from  $\frac{1}{2}\partial_\mu h_{\nu\lambda}\partial^\mu h^{\nu\lambda} - \frac{1}{4}\partial_\mu h_\nu^\nu\partial^\mu h_\lambda^\lambda$ , the graviton propagator is

$$\begin{array}{c} \text{q} \\ \xrightarrow{\hspace{2cm}} \\ \bullet \text{---} \text{wavy line} \text{---} \bullet \\ \mu\nu \qquad \qquad \rho\sigma \end{array} = \frac{i}{q^2} \frac{1}{2} [\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \eta_{\mu\nu}\eta_{\rho\sigma}]. \quad (4.91)$$

The interaction vertices that will be relevant for the following discussion are

$$\begin{array}{c} \diagup \\ \text{---} \\ k_1 \\ \diagdown \\ \text{---} \\ k_2 \end{array} \begin{array}{c} \text{---} \\ p \\ \text{---} \end{array} = -i\mu \frac{1}{2}, \quad (4.92)$$

from  $-\frac{1}{2}\mu\phi H^2$ ;

$$\begin{array}{c} \text{---} \\ p_1 \\ \text{---} \\ \text{---} \\ p_2 \\ \text{---} \end{array} \begin{array}{c} \text{---} \\ k_3 \\ \text{---} \\ \text{---} \\ k_4 \\ \text{---} \end{array} = -i\lambda \frac{1}{4}, \quad (4.93)$$

from  $-\frac{1}{4}\lambda\phi^2 H^2$ <sup>35</sup> and

$$\begin{array}{c} \diagup \\ \text{---} \\ k_1 \\ \diagdown \\ \text{---} \\ k_2 \end{array} \begin{array}{c} \text{---} \\ q \\ \text{---} \\ \text{---} \\ \mu\nu \end{array} = \frac{i}{M_P} [k_{1,\mu}k_{2,\nu} + k_{1,\nu}k_{2,\mu} - \eta_{\mu\nu}(k_1k_2 - m^2)] \frac{1}{2}, \quad (4.94)$$

from  $\frac{1}{M_P} [h^{\mu\nu}\partial_\mu H\partial_\nu H - \frac{1}{2}h_\mu^\mu\partial_\nu H\partial^\nu H + \frac{1}{2}h_\mu^\mu m^2 H^2]$ .

Let us start analyzing the tree-level interactions that have the scalar field  $H$  as protagonist. When two  $H$  particles interact (in the initial state) and give rise to other two  $H$  particles (in the final state) thanks to the mediator  $\phi$ , the following diagrams have to be considered:

<sup>35</sup>The previous scalar Feynman rules are independent of the momenta of the scalar fields, because the couplings that determine them don't involve derivatives.



$$\mathcal{M}_s^{(1)} = \begin{array}{c} \begin{array}{ccc} \diagup & & \diagdown \\ \text{---} & \xrightarrow{q} & \text{---} \\ \text{---} & & \text{---} \\ \diagdown & & \diagup \end{array} \\ \begin{array}{ccc} p_1 & & p_3 \\ \uparrow & & \uparrow \\ p_2 & & p_4 \\ \uparrow & & \uparrow \end{array} \end{array} \quad (4.95)$$

$$\mathcal{M}_t^{(1)} = \begin{array}{c} \begin{array}{ccc} \diagdown & & \diagup \\ \text{---} & & \text{---} \\ \text{---} & & \text{---} \\ \diagup & & \diagdown \end{array} \\ \begin{array}{ccc} p_2 & & p_4 \\ \uparrow & & \uparrow \\ -q & & \downarrow \\ \text{---} \\ \begin{array}{ccc} \diagup & & \diagdown \\ \text{---} & & \text{---} \\ \text{---} & & \text{---} \\ \diagdown & & \diagup \end{array} \\ \begin{array}{ccc} p_1 & & p_3 \\ \uparrow & & \uparrow \end{array} \end{array} \end{array} \quad (4.96)$$

and

$$\mathcal{M}_u^{(1)} = \begin{array}{c} \begin{array}{ccc} \diagdown & & \diagup \\ \text{---} & & \text{---} \\ \text{---} & & \text{---} \\ \diagup & & \diagdown \end{array} \\ \begin{array}{ccc} p_2 & & p_3 \\ \uparrow & & \uparrow \\ -q & & \downarrow \\ \text{---} \\ \begin{array}{ccc} \diagup & & \diagdown \\ \text{---} & & \text{---} \\ \text{---} & & \text{---} \\ \diagdown & & \diagup \end{array} \\ \begin{array}{ccc} p_1 & & p_4 \\ \uparrow & & \uparrow \end{array} \end{array} \end{array} \quad (4.97)$$

Feynman rules give

$$i\mathcal{M}_s^{(1)} = \left(\frac{-i\mu}{2}\right)^2 \frac{i}{(p_1 + p_2)^2} \times (2)^2 = -i\frac{\mu^2}{(p_1 + p_2)^2}; \quad (4.98)$$

$$i\mathcal{M}_t^{(1)} = -i\frac{\mu^2}{(p_1 - p_3)^2} \quad (4.99)$$

(with  $p_1 - p_3 = q$ ) and

$$i\mathcal{M}_u^{(1)} = -i\frac{\mu^2}{(p_1 - p_4)^2} \quad (4.100)$$

(by taking into account the ways the external legs can be attached to the vertex lines).

By considering the non-relativistic limit (where  $p_i^0 \simeq m + \frac{\vec{p}_i^2}{2m}$  for  $i = 1, \dots, 4$ ) and the static approximation (for which the energy  $q^0$  exchanged during the scattering is negligible and is taken to be 0) and by choosing as reference frame the center of mass frame (" ' ") of the system (1,2), the scattering amplitudes  $\mathcal{M}_s^{(1)}$ ,  $\mathcal{M}_t^{(1)}$  and  $\mathcal{M}_u^{(1)}$  for the on-shell  $H$  particles become

$$\mathcal{M}_s^{(1)} = -\frac{\mu^2}{4m^2 \left(1 + \frac{\vec{p}_1^2}{m^2}\right)} \simeq -\frac{\mu^2}{4m^2} = \text{constant}; \quad (4.101)$$

$$\mathcal{M}_t^{(1)} = +\frac{\mu^2}{q^{\vec{2}}} = +\frac{\mu^2}{2p_1^{\vec{2}} (1 - \cos \theta)} \quad (4.102)$$

and

$$\mathcal{M}_u^{(1)} = -\frac{\mu^2}{q^{\vec{2}} - 4p_1^{\vec{2}}} = +\frac{\mu^2}{2p_1^{\vec{2}} (1 + \cos \theta)} \quad (4.103)$$

(either by using the Maldestam relation  $s + t + u = 4m^2 \simeq (4m^2 + 4p_1^{\vec{2}}) - q^{\vec{2}} + u$  or by simply making use of the rest frame relations among three-momenta and denoting as  $\theta$  the angle between  $\vec{p}_1$  and  $\vec{p}_3$ ).

The gravitational interaction between two  $H$  states is described by diagrams where a graviton is exchanged.

The  $s$ -channel diagram is

$$\mathcal{M}_s^{(3)} = \begin{array}{c} \begin{array}{c} \begin{array}{c} \nearrow p_1 \\ \searrow p_2 \end{array} \\ \bullet \\ \begin{array}{c} \leftarrow -q \\ \rightarrow q \end{array} \\ \bullet \\ \begin{array}{c} \nearrow p_3 \\ \searrow p_4 \end{array} \end{array} \\ \text{wavy line} \\ \begin{array}{c} \mu\nu \\ \rho\sigma \end{array} \end{array} \quad (4.104)$$

the  $t$ -channel graph can be depicted as

$$\mathcal{M}_t^{(3)} = \begin{array}{c} \begin{array}{c} \nearrow p_2 \\ \searrow p_4 \end{array} \\ \bullet \\ \begin{array}{c} \uparrow q \\ \downarrow -q \end{array} \\ \bullet \\ \begin{array}{c} \nearrow p_1 \\ \searrow p_3 \end{array} \end{array} \\ \text{wavy line} \\ \begin{array}{c} \rho\sigma \\ \mu\nu \end{array} \end{array} \quad (4.105)$$

and the  $u$ -channel diagram is

$$\mathcal{M}_u^{(3)} = \begin{array}{c} \begin{array}{c} \nearrow p_2 \\ \searrow p_3 \end{array} \\ \bullet \\ \begin{array}{c} \uparrow q \\ \downarrow -q \end{array} \\ \begin{array}{c} \rho\sigma \\ \mu\nu \end{array} \\ \bullet \\ \begin{array}{c} \nearrow p_1 \\ \searrow p_4 \end{array} \end{array} \quad (4.106)$$

Feynman rules give  $i\mathcal{M}_s^{(3)}$ ,  $i\mathcal{M}_t^{(3)}$  and  $i\mathcal{M}_u^{(3)}$ . Their expressions are the same as in (4.75), (4.76) and (4.77) (with the only *caveat* that here not  $\phi$  but  $H$  particles are involved). For completeness, let us repeat the results when adopting the non-relativistic limit and the static approximation and making reference to the center of mass frame of the particles 1 and 2:

$$\mathcal{M}_s^{(3)} = + \frac{3m^2}{2M_P^2 \left(1 + \frac{p_1^2}{m^2}\right)} \simeq \frac{3m^2}{2M_P^2} = \text{constant}; \quad (4.107)$$

$$\mathcal{M}_t^{(3)} = + \frac{2m^4}{M_P^2 q^2} = + \frac{m^4}{M_P^2 p_1^2 (1 - \cos\theta)}; \quad (4.108)$$

$$\mathcal{M}_u^{(3)} = - \frac{2m^4}{M_P^2 (q^2 - 4p_1^2)} = + \frac{m^4}{M_P^2 p_1^2 (1 + \cos\theta)} \quad (4.109)$$

(either by using the Maldestam relation or by simply making use of the rest frame relations among three-momenta and denoting as  $\theta$  the angle between  $\vec{p}_1$  and  $\vec{p}_3$ ).

In the interesting limit

$$|\vec{q}'| \longrightarrow 0 \quad (4.110)$$

the only diagrams that (diverging) can give relevant contributions are  $\mathcal{M}_t^{(1)}$  and  $\mathcal{M}_t^{(3)}$ .

Since both graphs behave as  $\frac{1}{q'^2}$  (or as  $\frac{1}{|\vec{x}|}$  in the configuration space,  $|\vec{x}|$  being the interaction distance), the comparison between the scalar and the gravitational interaction strengths is given by looking at the coefficients  $\mathcal{M}_t^{(1)}$  and  $\mathcal{M}_t^{(3)}$  are expressed by.

Animated by the physical principle gravity has to be the weakest force in any circumstance, we deduce a constraint on the parameters of the potential  $V(\phi, H)$ . It is

$$\left(\partial_\phi \partial_H^2 V|_{\phi=0=H}\right)^2 \geq \frac{2 \left(\partial_H^2 V|_{H=0}\right)^2}{M_P^2}, \quad (4.111)$$

that is

$$\frac{\mu^2}{m^2} \geq \frac{2m^4}{M_P^2}. \quad (4.112)$$

By recovering Palti's notation [7] (and so by substituting  $m_0^2$  to  $m^2$  and  $\mu^2 m_0^2$ , where  $\mu$  is now adimensional, to  $\mu^2$ ), we find

$$\mu^2 \geq \frac{2m_0^2}{M_P^2}. \quad (4.113)$$

This is a more strict but coherent result (as sufficient condition) with respect to that proposed by Palti in [7, 8].

The analysis we have dealt with so far ignores loop effects. We will now enrich the discussion by taking into account the 1-loop scalar contribution to the scattering of two  $H$  particles, dictated by the lagrangian term  $-\frac{1}{4}\lambda\phi^2 H^2$ . In considering the possible quantum corrections to the statement (4.113) we won't include the loop gravitational contributions: because they would appear with a suppression factor at least of  $\mathcal{O}(\frac{1}{M_P^4})$ , they can be reasonably regarded (at first approximation) as negligible terms.

Using the Feynman rules (4.89) and (4.93), let us calculate the diagrams

$$\mathcal{M}_s^{(1),1-loop} = \text{Diagram (4.114)} \quad (4.114)$$

$$\mathcal{M}_t^{(1),1-loop} = \text{Diagram (4.115)} \quad (4.115)$$

and

$$\mathcal{M}_u^{(1),1-loop} = k - p_1 + p_4 \quad (4.116)$$

As an example of calculation let us consider the  $t$ -channel graph  $\mathcal{M}_t^{(1),1-loop}$ .

According to Feynman rules and by using dimensional regularization in  $D = 4 - \epsilon$  dimensions,  $\mathcal{M}_t^{(1),1-loop}$  can be expressed as<sup>36</sup>

$$i\mathcal{M}_t^{(1),1-loop} = \frac{1}{2} (-i\lambda)^2 \int \frac{d^D k}{(2\pi)^D} \frac{i}{k^2} \frac{i}{(k - p_1 + p_3)^2} = \frac{\lambda^2}{2} \int \frac{d^D k}{(2\pi)^D} \frac{1}{k^2 (k - q)^2}; \quad (4.117)$$

by exploiting now Feynman parameterization technique, the previous integral becomes

$$\begin{aligned} i\mathcal{M}_t^{(1),1-loop} &= \frac{\lambda^2}{2} \int_0^1 dx \int \frac{d^D k}{(2\pi)^D} \frac{1}{[k^2(1-x) + (k-q)^2x]^2} = \\ &= \frac{\lambda^2}{2} \int_0^1 dx \int \frac{d^D k}{(2\pi)^D} \frac{1}{[(k-xq)^2 + q^2x(1-x)]^2} \end{aligned} \quad (4.118)$$

and, by implementing the substitution  $k \rightarrow k + xq$ , it results to be

$$i\mathcal{M}_t^{(1),1-loop} = \frac{\lambda^2}{2} \int_0^1 dx \int \frac{d^D k}{(2\pi)^D} \frac{1}{[k^2 + q^2x(1-x)]^2}. \quad (4.119)$$

By moving then to the Euclidean space we find

$$i\mathcal{M}_t^{(1),1-loop} = \frac{i\lambda^2}{2} \int_0^1 dx \int \frac{d^D k_E}{(2\pi)^D} \frac{1}{[k_E^2 + q_E^2x(1-x)]^2}. \quad (4.120)$$

So [97]

$$\mathcal{M}_t^{(1),1-loop} = \frac{1}{2} \left( \frac{\lambda}{4\pi} \right)^2 \left( \frac{4\pi\rho^2}{q_E^2} \right)^{\frac{\epsilon}{2}} \Gamma \left[ \frac{\epsilon}{2} \right] \int_0^1 dx \frac{1}{(x(1-x))^{\frac{\epsilon}{2}}}, \quad (4.121)$$

where  $\rho$  is the dimensional regulator for  $\lambda$  and  $\Gamma[z]$  is the Euler  $\Gamma$ -function.

<sup>36</sup>The factor  $\frac{1}{2}$  at the beginning of the (4.117) takes into account the identity of internal lines.

Being

$$\int_0^1 dx \frac{1}{(x(1-x))^{\frac{\epsilon}{2}}} = \frac{\Gamma[1 - \frac{\epsilon}{2}]^2}{\Gamma[2 - \epsilon]}; \quad (4.122)$$

by defining  $\psi(n) = \frac{\Gamma'[n]}{\Gamma[n]}$  (for  $n \in \mathbb{Z}$ ) and expanding the Euler  $\Gamma$ -functions  $\Gamma[\frac{\epsilon}{2}]$ ,  $\Gamma[1 - \frac{\epsilon}{2}]^2$  and  $\Gamma[2 - \epsilon]$  in powers of  $\epsilon$ ,  $\mathcal{M}_t^{(1),1-loop}$  turns out to be

$$\mathcal{M}_t^{(1),1-loop} = \left(\frac{\lambda}{4\pi}\right)^2 \left(\frac{4\pi\rho^2}{q_E^2}\right)^{\frac{\epsilon}{2}} \left[\frac{1}{\epsilon} + \frac{1}{2}\gamma_E + \psi(2) + O(\epsilon)\right] \quad (4.123)$$

(with  $\gamma_E = -\psi(1) = 0.5772\dots$  and  $\psi(2) = 1 - \gamma_E = 0.4228\dots$ ). In the interesting limit  $\epsilon \rightarrow 0$  (4.123) results in

$$\mathcal{M}_t^{(1),1-loop} = \left(\frac{\lambda}{4\pi}\right)^2 \left[\frac{1}{\epsilon} + \frac{1}{2}\gamma_E + \psi(2)\right] \quad (4.124)$$

and exhibits a divergent part going as  $\frac{1}{\epsilon}$  and a finite part.

The amplitudes  $\mathcal{M}_s^{(1),1-loop}$  and  $\mathcal{M}_t^{(1),1-loop}$  can be evaluated by following exactly the same procedure and their final expressions match (4.124).

By summing over the three channel contributions, at the end we obtain

$$\begin{aligned} \mathcal{M}^{(1),1-loop} &= \mathcal{M}_s^{(1),1-loop} + \mathcal{M}_t^{(1),1-loop} + \mathcal{M}_u^{(1),1-loop} = \left(\frac{\lambda}{4\pi}\right)^2 \left[\frac{3}{\epsilon} + 3\left(\frac{1}{2}\gamma_E + \psi(2)\right)\right] = \\ &= \left(\frac{\lambda}{4\pi}\right)^2 \left[\frac{3}{\epsilon} + 3 - \frac{3}{2}\gamma_E\right]. \end{aligned} \quad (4.125)$$

Since  $\mathcal{M}^{(1),1-loop}$  has a divergent part

$$\mathcal{M}^{(1),1-loop}|_{DIV} = \left(\frac{\lambda}{4\pi}\right)^2 \frac{3}{\epsilon} \quad (4.126)$$

to get rid of this divergence a renormalization procedure is required.

Let us consider the action

$$\begin{aligned} S = \int d^4x \sqrt{-g} \left[ -\frac{M_P^2}{2} R + \frac{1}{2} g_{\mu\nu} \partial^\mu \phi \partial^\nu \phi + \frac{1}{2} g_{\mu\nu} \partial^\mu H \partial^\nu H - \frac{1}{2} m^2 H^2 - \frac{1}{2} \mu \phi H^2 + \right. \\ \left. -\frac{1}{4} \lambda \phi^2 H^2 \right] \end{aligned} \quad (4.127)$$

and study its renormalization by adopting as regularization procedure the dimensional one (and so moving to  $D = 4 - \epsilon$  dimensions). In doing so, we will ignore all the gravitational contributions to the renormalization itself: this approximation is legitimate by the fact that the loop gravitational amplitudes are suppressed by the inverse and (at least) fourth power of the Planck mass  $M_P$ .

The (relevant) renormalized lagrangian density  $\mathcal{L}_R$  can be written as

$$\mathcal{L}_R = -\frac{1}{2}\phi_0\Box\phi_0 - \frac{1}{2}H_0\Box H_0 - \frac{1}{2}m_0^2 H_0^2 - \frac{1}{2}\mu_0\phi_0 H_0^2 - \frac{1}{4}\lambda_0\phi_0^2 H_0^2 \quad (4.128)$$

and it satisfies  $[\int d^D x \mathcal{L}_R] = 0$ . The renormalized quantities  $\mathcal{L}_R$  depends on are defined in terms of the original ones in the following way:

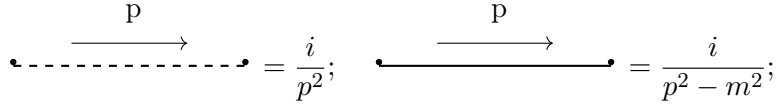
$$\phi_0 = Z_\phi^{\frac{1}{2}}\phi; \quad H_0 = Z_H^{\frac{1}{2}}H \quad (4.129)$$

and

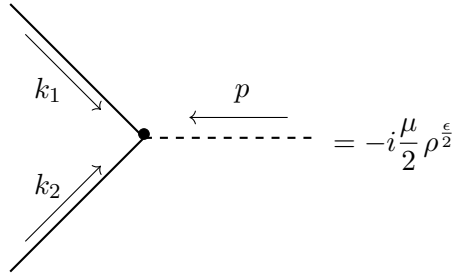
$$m_0^2 = m^2 + \delta m^2; \quad \mu_0 = Z_\mu Z_\phi^{-\frac{1}{2}} Z_H^{-1} \rho^{\frac{\epsilon}{2}} \mu; \quad \lambda_0 = Z_\lambda Z_\phi^{-1} Z_H^{-1} \sigma^\epsilon \lambda, \quad (4.130)$$

where  $\rho$  and  $\sigma$  are two dimensional regulators allowing to correctly express the mass dimensions of the parameters  $\mu_0$  and  $\lambda_0$ . With appropriately defined  $Z_\phi$ ,  $Z_H$ ,  $\delta m^2$ ,  $Z_\mu$  and  $Z_\lambda$ , the correlation functions referred to  $\mathcal{L}_R$  (rather than  $\mathcal{L}$ ) are finite.

Before proceeding, let us recall the relevant Feynman rules:

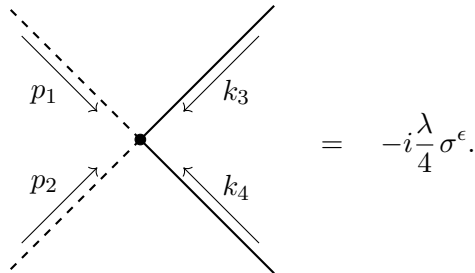


$$\begin{array}{c} \text{---} \xrightarrow{p} \text{---} \\ \text{---} \end{array} = \frac{i}{p^2}; \quad \begin{array}{c} \text{---} \xrightarrow{p} \text{---} \\ \text{---} \end{array} = \frac{i}{p^2 - m^2};$$



$$\begin{array}{c} \text{---} \xrightarrow{k_1} \\ \text{---} \xrightarrow{k_2} \\ \text{---} \xleftarrow{p} \end{array} = -i \frac{\mu}{2} \rho^{\frac{\epsilon}{2}}$$

and



$$\begin{array}{c} \text{---} \xrightarrow{p_1} \\ \text{---} \xrightarrow{p_2} \\ \text{---} \xleftarrow{k_3} \\ \text{---} \xleftarrow{k_4} \end{array} = -i \frac{\lambda}{4} \sigma^\epsilon.$$

Let us analyze first the two-point functions for  $\phi$  and  $H$  and then the three-point and four-point vertex functions.

At order 0, from the quadratic  $\phi$ -sector of the renormalized action, the two-point function for  $\phi$  is

$$\Gamma_\phi^{(2),0} = p^2 + (Z_\phi - 1) p^2 = S_\phi^{(2),0} + \Delta S_\phi^{(2),0}. \quad (4.131)$$

At 1-loop, the topologically non-equivalent diagrams that contribute (once the external legs have been fixed) are

$$\Gamma_{\phi,A}^{(2),1-loop} = \text{Diagram (4.132)} \quad (4.132)$$

and

$$\Gamma_{\phi,B}^{(2),1-loop} = \text{Diagram (4.133)} \quad (4.133)$$

The first graph turns out to be

$$\begin{aligned} i\Gamma_{\phi,A}^{(2),1-loop} &= -i\lambda\sigma^\epsilon \int \frac{d^D k}{(2\pi)^D} \frac{i}{k^2 - m^2} = -i\lambda\sigma^\epsilon \int \frac{d^D k_E}{(2\pi)^D} \frac{1}{k_E^2 + m^2} = \\ &= -i \frac{\lambda m^2}{(4\pi)^2} \Gamma \left[ -1 + \frac{\epsilon}{2} \right] \left( \frac{4\pi\sigma^2}{m^2} \right)^{\frac{\epsilon}{2}}; \end{aligned} \quad (4.134)$$

in the limit  $\epsilon \rightarrow 0$ , one finds

$$\Gamma_{\phi,A}^{(2),1-loop} = -\frac{\lambda m^2}{(4\pi)^2} \Gamma \left[ -1 + \frac{\epsilon}{2} \right] \quad (4.135)$$

and isolates

$$\Gamma_{\phi,A}^{(2),1-loop} \Big|_{DIV} = \frac{\lambda m^2}{8\pi^2 \epsilon}. \quad (4.136)$$

For the second 1-loop contribution to the two-point function for  $\phi$ , Feynman rules give<sup>37</sup>

$$i\Gamma_{\phi,B}^{(2),1-loop} = \frac{1}{2} (-i\mu)^2 \rho^\epsilon \int \frac{d^D k}{(2\pi)^D} \frac{i}{(k-p)^2 - m^2} \frac{i}{k^2 - m^2}. \quad (4.137)$$

By using Feynman parameterization (similarly to what we did in (4.118)); by operating the shift  $k \rightarrow k + xp$  and moving to the Euclidean space, one gets

<sup>37</sup>The factor  $\frac{1}{2}$  takes into account the identity of the internal loop lines.



$$i\Gamma_{\phi,B}^{(2),1-loop} = i\frac{\mu^2}{2}\rho^\epsilon \int_0^1 dx \int \frac{d^D k_E}{(2\pi)^D} \frac{1}{[k_E^2 + M^2]^2} \quad (4.138)$$

(having called  $p_E^2 x(1-x) + m^2 = M^2$ ). In the limit  $\epsilon \rightarrow 0$  one deduces

$$i\Gamma_{\phi,B}^{(2),1-loop} = \frac{i}{2} \left(\frac{\mu}{4\pi}\right)^2 \Gamma\left[\frac{\epsilon}{2}\right] \int_0^1 dx \left(\frac{4\pi\rho^2}{M^2}\right)^{\frac{\epsilon}{2}} \longrightarrow \frac{i}{2} \left(\frac{\mu}{4\pi}\right)^2 \Gamma\left[\frac{\epsilon}{2}\right] \quad (4.139)$$

and can extract  $\Gamma_{\phi,B}^{(2),1-loop}$ 's divergent part: it is

$$\Gamma_{\phi,B}^{(2),1-loop}|_{DIV} = \frac{\mu^2}{16\pi^2\epsilon}. \quad (4.140)$$

In the minimal subtraction scheme the requirement is then

$$i\Delta S_\phi^{(2),0} + i\Gamma_{\phi,A}^{(2),1-loop}|_{DIV} + i\Gamma_{\phi,B}^{(2),1-loop}|_{DIV} = 0, \quad (4.141)$$

that is

$$(Z_\phi - 1)p^2 + \frac{\lambda m^2}{8\pi^2\epsilon} + \frac{\mu^2}{16\pi^2\epsilon} = 0. \quad (4.142)$$

This renormalization condition suggests that the field  $\phi$  acquires a mass at loop-level. We can add a mass term for  $\phi$  in  $\mathcal{L}$  and introduce the corresponding counter-term  $\delta m_\phi^2$ . After having modified the definition of  $\Delta S_\phi^{(2),0}$  in (4.131) as

$$\Delta S_\phi^{(2),0} = (Z_\phi - 1)(p^2 - m_\phi^2) - Z_\phi \delta m_\phi^2, \quad (4.143)$$

(4.141) results in

$$(Z_\phi - 1)(p^2 - m_\phi^2) - Z_\phi \delta m_\phi^2 + \frac{\lambda m^2}{8\pi^2\epsilon} + \frac{\mu^2}{16\pi^2\epsilon} = 0. \quad (4.144)$$

By replacing the dimensional coupling  $\mu$  with  $\mu^* m$  (where, as  $\mu$  in (4.113),  $\mu^*$  is adimensional and  $m$  is the typical mass scale for the theory (4.127)) (4.144) gives

$$Z_\phi = 1 \quad (4.145)$$

and

$$\delta m_\phi^2 = \frac{2\lambda + \mu^{*2}}{16\pi^2\epsilon} m^2. \quad (4.146)$$

Since the field  $\phi$  has to be massless, its physical mass has to be zero: in symbols,

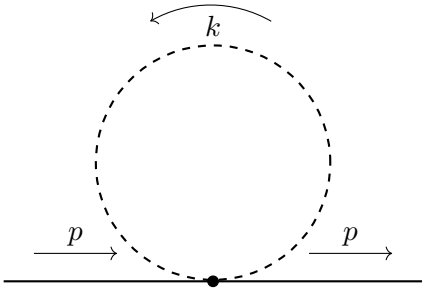
$$m_{0,\phi}^2 = m_\phi^2 + \delta m_\phi^2 = 0 \quad (4.147)$$

(or, equivalently,  $m_\phi^2 = -\frac{2\lambda + \mu^{*2}}{16\pi^2\epsilon} m^2$ ).

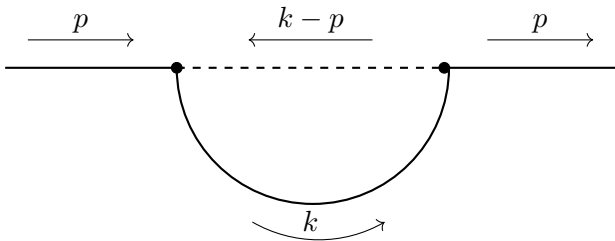
At order 0, the  $H$ -quadratic terms of the renormalized action determine the two-point function for the field  $H$ ; it is

$$\Gamma_H^{(2),0} = (p^2 - m^2) + (Z_H - 1)(p^2 - m^2) - Z_H \delta m^2 = S_H^{(2),0} + \Delta S_H^{(2),0}. \quad (4.148)$$

At 1-loop, the topologically non-equivalent diagrams that correct  $\Gamma_H^{(2),0}$  (once the external legs have been fixed) are

$$\Gamma_{H,A}^{(2),1-loop} = \text{Diagram (4.149)} \quad (4.149)$$


and

$$\Gamma_{H,B}^{(2),1-loop} = \text{Diagram (4.150)} \quad (4.150)$$


When evaluated by taking into account the introduction of a bare mass term for  $\phi$ , the Feynman graph (4.149) is

$$i\Gamma_{H,A}^{(2),1-loop} = -i\lambda\sigma^\epsilon \int \frac{d^D k}{(2\pi)^D} \frac{i}{k^2 - m_\phi^2}; \quad (4.151)$$

and by following exactly the same procedure used for (4.132) one obtains that

$$\Gamma_{H,A}^{(2),1-loop} \Big|_{DIV} = \frac{\lambda m_\phi^2}{8\pi^2 \epsilon}. \quad (4.152)$$

The contribution (4.150) to the  $H$  two-point function

$$i\Gamma_{H,B}^{(2),1-loop} = (-i\mu)^2 \rho^\epsilon \int \frac{d^D k}{(2\pi)^D} \frac{i}{(k-p)^2 - m_\phi^2} \frac{i}{k^2 - m^2} \quad (4.153)$$

can be calculated as (4.133) (intending now  $M^2$  as  $M^2 = p_E^2 x(1-x) + m^2(1-x) + m_\phi^2 x$ ). Since we are referring to four dimensions, its divergent part is

$$\Gamma_{H,B}^{(2),1-loop} \Big|_{DIV} = \frac{\mu^2}{8\pi^2 \epsilon}. \quad (4.154)$$

In the minimal subtraction scheme the requirement to get rid of divergences consists in

$$i\Delta S_H^{(2),0} + i\Gamma_{H,A}^{(2),1-loop}|_{DIV} + i\Gamma_{H,B}^{(2),1-loop}|_{DIV} = 0, \quad (4.155)$$

that is

$$(Z_H - 1)(p^2 - m^2) - Z_H \delta m^2 + \frac{\lambda m_\phi^2}{8\pi^2 \epsilon} + \frac{\mu^2}{8\pi^2 \epsilon} = 0. \quad (4.156)$$

Therefore,

$$Z_H = 1 \quad (4.157)$$

and, by exploiting (4.147) to express  $m_\phi^2$  and keeping only the  $\mathcal{O}(\frac{1}{\epsilon})$  term<sup>38</sup>,

$$\delta m^2 \simeq \frac{\mu^2}{8\pi^2 \epsilon} = \frac{\mu^{*2} m^2}{8\pi^2 \epsilon}. \quad (4.158)$$

Having considered the two-point functions for the scalar fields  $\phi$  and  $H$ , let us now move to the analysis of the three-point and four-point vertex functions.

The 1-loop correction to the 0-order three-point function

$$\Gamma_\mu^{(3),0} = -\mu\rho^{\frac{\epsilon}{2}} - (Z_\mu - 1)\mu\rho^{\frac{\epsilon}{2}} \quad (4.159)$$

is expressed thanks to the diagram

$$\Gamma_\mu^{(3),1-loop} = \text{Diagram} \quad (4.160)$$

that is

$$i\Gamma_\mu^{(3),1-loop} = (-i\mu\rho^{\frac{\epsilon}{2}})^3 \int \frac{d^D k}{(2\pi)^D} \frac{i}{k^2 - m_\phi^2} \frac{i}{(k-p_2)^2 - m^2} \frac{i}{(k+p_1)^2 - m^2}. \quad (4.161)$$

The graph (4.160) is superficially convergent in four dimensions and, being a 1-loop diagram, it does not have sub-divergences. Standing then to Weinberg's theorem ([95, 96]),  $\Gamma_\mu^{(3),1-loop}$  is finite.

Therefore, the renormalization condition results in

$$Z_\mu = 1. \quad (4.162)$$

<sup>38</sup>The terms of  $\mathcal{O}(\frac{1}{\epsilon^2})$  behave as typically 2-loop divergent terms do.

Finally, the 0-order four-point function (deduced from the quartic sector of (4.127) in the fields)

$$\Gamma^{(4),0} = -\lambda\sigma^\epsilon - (Z_\lambda - 1)\lambda\sigma^\epsilon \quad (4.163)$$

is corrected at 1-loop by

$$\Gamma_A^{(4),1-loop} = \text{Diagram (4.164)} \quad (4.164)$$

and

$$\Gamma_B^{(4),1-loop} = \text{Diagram (4.165)} \quad (4.165)$$

For the diagrams (4.164) and (4.165) Feynman rules give

$$i\Gamma_A^{(4),1-loop} = (-i\mu\rho^{\frac{\epsilon}{2}})^4 \int \frac{d^D k}{(2\pi)^D} \frac{i}{k^2 - m_\phi^2} \frac{i}{(k-p_2)^2 - m^2} \frac{i}{(k+p_1+p_4)^2 - m^2} \cdot \frac{i}{(k+p_1)^2 - m^2}. \quad (4.166)$$

and

$$i\Gamma_B^{(4),1-loop} = (-i\mu\rho^{\frac{\epsilon}{2}})^2 (-i\lambda\sigma^\epsilon) \int \frac{d^D k}{(2\pi)^D} \frac{i}{k^2 - m_\phi^2} \frac{i}{(k-p_2)^2 - m^2} \frac{i}{(k+p_1)^2 - m^2}, \quad (4.167)$$

respectively. Weimberg's theorem ensures that (4.164) and (4.165) are finite, because they superficially converge in four dimensions and are 1-loop graphs.

The renormalization condition turns out to be equivalent to

$$Z_\lambda = 1. \quad (4.168)$$

Summarizing, the renormalization procedure we have just dealt with gives

$$Z_\phi = Z_H = Z_\mu = Z_\lambda = 1; \quad \delta m^2 \simeq \frac{\mu^2}{8\pi^2\epsilon}, \quad (4.169)$$

implying that the couplings  $\mu$  and  $\lambda$  do not run with an appropriately defined energy scale as, instead,  $m$  (and  $\mu^*$ ) does.

The running of  $m$  and  $\mu^*$  is described by their  $\beta$ -functions.

Having in mind  $m = \frac{\mu}{\mu^*}$ , the  $\beta$ -function for  $m$  can be defined as

$$\beta_m = \rho \frac{\partial}{\partial \rho} m^2(m_0^2, \rho, \epsilon) \Big|_{m_0^2(m^2, \rho, \epsilon)} \quad \epsilon \rightarrow 0 \quad (4.170)$$

and explicitly is<sup>39</sup>

$$\beta_m = \frac{\mu^2}{8\pi^2} > 0. \quad (4.171)$$

By introducing

$$\mu_0^* = \frac{\mu_0}{m_0} \simeq \mu^* \rho^{\frac{\epsilon}{2}} \left[ 1 - \frac{\mu^{*2}}{16\pi^2\epsilon} \right], \quad (4.172)$$

the  $\beta$ -function for  $\mu^*$  is

$$\beta_{\mu^*} = \rho \frac{\partial}{\partial \rho} \mu^*(\mu_0^*, \rho, \epsilon) \Big|_{\mu_0^*(\mu^*, \rho, \epsilon)} \quad \epsilon \rightarrow 0 \quad (4.173)$$

and it turns out to be

$$\beta_{\mu^*} = -\frac{\mu^{*3}}{16\pi^2} < 0. \quad (4.174)$$

Let us now come back to the diagrams (4.114), (4.115) and (4.116) (whose divergent parts can be properly taken into account by the renormalization procedure). These graphs give a 1-loop contribution to the scalar interaction between two  $H$  particles that sums up to the classical term, corresponding to the diagram (4.96). Since (4.102) and the finite part of (4.125) agree as far their sign is concerned, the 1-loop term increases the scalar force strength. But, the mass being running with the interaction energy ( $m^2 = m^2(E)$ ), one may ask whether there could exist an energy scale at which the gravitational interaction equals in strength the scalar force and so if there could be solutions of

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<sup>39</sup>To a mass parameter  $m$  the anomalous dimension  $\gamma_m$  is usually associated (rather than the  $\beta$ -function): it is defined as

$$\gamma_m = \frac{1}{2} \rho \frac{\partial}{\partial \rho} \ln m^2(m_0^2, \rho, \epsilon) \Big|_{m_0^2(m^2, \rho, \epsilon)} \quad \epsilon \rightarrow 0$$

and, for the theory we are considering, it is

$$\gamma_m = \frac{\mu^2}{16\pi^2 m^2} > 0$$

$$\frac{\mu(E_0)^2}{E^2} + \hbar C \frac{\lambda(E_0)}{16\pi^2} = \frac{2m^2(E)}{M_P^2 E^2}, \quad (4.175)$$

where  $C$  is a numerical positive constant of  $\mathcal{O}(1)$  and  $E_0$  is a reference energy scale.

Since (as it is possible to deduce from (4.171))  $m^2$  grows logarithmically (and so tremendously slowly) with the energy  $E$ , (4.175) can't be (reasonably) satisfied.

Requiring that the scalar force is stronger than the gravitational one at tree-level is indeed enough: quantum (1-loop) corrections don't seem to alter the significant result (4.111) (or (4.113)).

## 5 Giving scalar charge to a classical particle

We would like to understand how (if there is a practicable way to do so) a classical particle can acquire a scalar charge.

As it is known, the geodesic motion of a point-like massive or massless particle in a generic spacetime is described by means of the isomorphism and reparameterization invariant Polyakov action

$$S_P = -\frac{1}{2} \int d\tau \sqrt{-\gamma_{\tau\tau}} \left[ \gamma^{\tau\tau} \frac{dx^\mu(\tau)}{d\tau} g_{\mu\nu}(x) \frac{dx^\nu(\tau)}{d\tau} + m^2 \right], \quad (5.1)$$

where  $\{x^\mu\}_{\mu=0,\dots,3}$  are the (adimensional) coordinates chosen to parameterize the spacetime (whose metric is  $g_{\mu\nu}(x)$ ) the particle lives in;  $\tau$  is the variable that parameterizes the world-line  $\gamma = (x^0(\tau), \vec{x}(\tau))$  of the particle ( $\gamma_{\tau\tau}$  being the metric along it) and  $m$  is the mass of the particle.

The action  $S_P$  expresses the interaction of the free particle with the spacetime metric and such a coupling explicitly manifests in the world-line  $\gamma$ , whose parameter is tied to the line-element  $ds^2$  as  $ds^2 = dx^\mu g_{\mu\nu} dx^\nu = \gamma_{\tau\tau} d\tau^2$ .

The action  $S_P$  can be modified by adding other terms that account for the interaction of the particle with something else. For instance, the particle is made interact with an external  $U(1)$  gauge field  $A_\mu(x)$  through the action

$$S_A = q_A \int dx^\mu A_\mu(x), \quad (5.2)$$

where  $q_A$  is the charge of the particle with respect to  $A_\mu$ . The action  $S_P + S_A$  describes the motion in a curved spacetime of a particle, charged (say) under the electromagnetic field  $A_\mu(x)$ .

Because of the role it can play as far as the main results of our thesis are concerned, we would like to couple the particle to a scalar field (rather than to a gauge field).

Following the same logic we have just presented, the only possible extension of  $S_P$  we could imagine is given by the term

$$S_\phi = q_\phi \int dx^\mu \partial_\mu \phi(x), \quad (5.3)$$

$\phi$  being the scalar field of interest. However, because the integration is meant to be taken over all spacetime (in the absence of a boundary),  $S_\phi$  is identically 0 and it seems that the particle can't get any scalar charge.

Despite this, an alternative procedure to make the particle interact with the scalar field  $\phi$  appears to us to be reasonably viable.

The particle's mass  $m$  in  $S_P$  is usually regarded as an intrinsic and fixed property of the particle itself, but it could be interpreted as an effective mass depending on  $\phi$ . The idea we are going to develop now is that the particle interacts with  $\phi$  (and so gets charged under it) thanks to the dependence of its mass on the scalar field.

Let us consider the action

$$S = -\frac{1}{2} \int d\tau \sqrt{-\gamma_{\tau\tau}} \left[ \gamma^{\tau\tau} \frac{dx^\mu(\tau)}{d\tau} g_{\mu\nu}(x) \frac{dx^\nu(\tau)}{d\tau} + m^2(\phi) \right] + \int d^4x \sqrt{-g} \frac{1}{2} \partial^\mu \phi(x) g_{\mu\nu}(x) \partial^\nu \phi(x), \quad (5.4)$$

where, inspired by Palti's model (4.15), we can set  $m^2(\phi) = m_0^2 + 2\mu\phi(x(\tau))$ .  $S$  describes the motion of a particle along the world-line  $\gamma = (x^0(\tau), \vec{x}(\tau))$  in a curved spacetime (with metric  $g_{\mu\nu}$ ) in the presence of a scalar field  $\phi(x)$ .

We choose to refer to Minkowski spacetime,

$$g_{\mu\nu} = \eta_{\mu\nu} = \text{diag}(-1, +1, +1, +1) \quad (5.5)$$

and consider the non-relativistic limit.

By varying the action (5.4) with respect to the field  $\phi$  we can get (thanks to its equation of motion) the field  $\phi$  the particle moving along  $\gamma$  gives rise to.

Being the reference frame we observe the motion of the particle from that we reveal the field  $\phi$  through, we set  $x^0 = x^0(\tau)$ . The equation of motion of  $\phi$  is then

$$\square \phi \delta(x^0 - x^0(\tau)) = -e_\tau \mu \delta^{(4)}(x - x(\tau)) \quad (5.6)$$

with  $e_\tau = \sqrt{-\gamma_{\tau\tau}}$ .

In order to solve the previous equation we move from configuration space to momentum space by adopting the convention

$$\hat{F}(k) = \frac{1}{(2\pi)^2} \int d^4k e^{-ikx} F(x) = \frac{1}{(2\pi)^{\frac{1}{2}}} \int dk^0 e^{+ik^0 x^0} \frac{1}{(2\pi)^{\frac{3}{2}}} \int d\vec{k} e^{-i\vec{k}\cdot\vec{x}} F(x^0, \vec{x}). \quad (5.7)$$

Coherently with the non-relativistic limit we are referring to, we require the static approximation too and so impose  $k^0 = 0$ . Then, for  $\vec{k}^2 \neq 0$ , we have to deal with

$$\hat{\phi} = \frac{e_\tau \mu}{(2\pi)^{\frac{3}{2}}} \frac{e^{-i\vec{k}\cdot\vec{x}(\tau)}}{\vec{k}^2}. \quad (5.8)$$

The anti-Fourier transform of  $\hat{\phi}$  is given by

$$\phi = \frac{1}{(2\pi)^{\frac{3}{2}}} \int d\vec{k} e^{+i\vec{k}\cdot\vec{x}} \frac{e_\tau \mu}{(2\pi)^{\frac{3}{2}}} \frac{e^{-i\vec{k}\cdot\vec{x}(\tau)}}{\vec{k}^2}. \quad (5.9)$$

By passing to spherical coordinates ( $d\vec{k} = \rho^2 d\rho d\cos\theta d\alpha$ ),  $\phi$  is expressed as

$$\phi = \frac{e_\tau \mu}{(2\pi)^3} \int_0^{2\pi} d\alpha \int_0^{+\infty} d\rho \rho^2 \int_{-1}^{+1} d\cos\theta \frac{e^{i\rho|\vec{x}-\vec{x}(\tau)|\cos\theta}}{\rho^2} = \frac{e_\tau \mu}{2\pi^2} \int_0^{+\infty} d\rho \frac{\sin\rho|\vec{x}-\vec{x}(\tau)|}{\rho|\vec{x}-\vec{x}(\tau)|} \quad (5.10)$$

and



$$\phi = \frac{e_\tau \mu}{2\pi^2} \frac{1}{|\vec{x} - \vec{x}(\tau)|} \int_0^{+\infty} dy \frac{\sin y}{y} \quad (5.11)$$

(once the substitution  $y = \rho|\vec{x} - \vec{x}(\tau)|$  has been performed). Since  $\int_0^{+\infty} dy \frac{\sin y}{y} = \frac{\pi}{2}$ ,  $\phi$  turns out to be

$$\phi = \frac{\sqrt{-\gamma_{\tau\tau}} \mu}{4\pi|\vec{x} - \vec{x}(\tau)|} = \phi(|\vec{x} - \vec{x}(\tau)|). \quad (5.12)$$

In the non-relativistic limit and in the static approximation a particle in  $\vec{x}(\tau)$  at  $x^0$  and whose mass depends linearly on a scalar field  $\phi$  gives rise to a Coulomb-type behaving  $\phi$  in  $\vec{x}$ .

Having found the explicit  $\phi$ -field solution, let us study the way the motion of another identical particle with associated world-line  $\gamma' = (x^0(\lambda), \vec{x}(\lambda))$  is influenced by the field  $\phi(|\vec{x}(\lambda) - \vec{x}(\tau)|)$  (for  $\vec{x}(\tau)$  fixed).

Let us consider the action

$$S' = -\frac{1}{2} \int d\lambda \left[ -e_\lambda^{-1} \frac{dx^\mu(\lambda)}{d\lambda} \eta_{\mu\nu} \frac{dx^\nu(\lambda)}{d\lambda} + e_\lambda m^2(\phi) \right] + \int d^4x \frac{1}{2} \partial^\mu \phi(x) \eta_{\mu\nu} \partial^\nu \phi(x), \quad (5.13)$$

where  $m^2(\phi) = m_0^2 + 2\mu\phi(x(\lambda))$  and  $e_\lambda = \sqrt{-\gamma_{\lambda\lambda}}$ .

By varying the action  $S'$  with respect to the spacetime coordinates  $x^\mu(\lambda)$  we get the equations of motion for the particle in Minkowski spacetime and in the presence of the scalar field  $\phi(|\vec{x}(\lambda) - \vec{x}(\tau)|)$ .

The variation of (5.13) is

$$\delta S' = -\frac{1}{2} \int d\lambda \left[ 2e_\lambda^{-1} \frac{d^2 x_\mu}{d\lambda^2} + 2e_{\lambda\mu} \partial_\mu \phi - \frac{\delta e_\lambda^{-1}}{\delta \lambda} \frac{\delta \lambda}{\delta x^\mu} \frac{dx^\nu}{d\lambda} \frac{dx_\nu}{d\lambda} + \frac{\delta e_\lambda}{\delta \lambda} \frac{\delta \lambda}{\delta x^\mu} (m_0^2 + 2\mu\phi) \right] \delta x^\mu \quad (5.14)$$

(after having made advantage of an integration by parts and ignored the boundary term). By imposing

$$\delta S' = 0 \quad (5.15)$$

we deduce the equations

$$2e_\lambda^{-1} \frac{d^2 x_\mu}{d\lambda^2} + 2e_{\lambda\mu} \partial_\mu \phi - \frac{de_\lambda^{-1}}{d\lambda} \frac{d\lambda}{dx^\mu} \frac{dx^\nu}{d\lambda} \frac{dx_\nu}{d\lambda} + \frac{de_\lambda}{d\lambda} \frac{d\lambda}{dx^\mu} (m_0^2 + 2\mu\phi) = 0 \quad (5.16)$$

that, once multiplied by  $e_\lambda$ , become

$$2 \frac{d^2 x_\mu}{d\lambda^2} + 2e_{\lambda\mu}^2 \partial_\mu \phi + e_\lambda^{-1} \frac{de_\lambda}{d\lambda} \frac{d\lambda}{dx^\mu} \frac{dx^\nu}{d\lambda} \frac{dx_\nu}{d\lambda} + \frac{1}{2} \frac{de_\lambda^2}{d\lambda} \frac{d\lambda}{dx^\mu} (m_0^2 + 2\mu\phi) = 0 \quad (5.17)$$

or

$$2 \frac{d^2 x_\mu}{d\lambda^2} - 2\gamma_{\lambda\lambda} \mu \partial_\mu \phi + \frac{1}{\sqrt{-\gamma_{\lambda\lambda}}} \frac{d\sqrt{-\gamma_{\lambda\lambda}}}{d\lambda} \frac{d\lambda}{dx^\mu} \frac{dx^\nu}{d\lambda} \frac{dx_\nu}{d\lambda} - \frac{1}{2} \frac{d\gamma_{\lambda\lambda}}{d\lambda} \frac{d\lambda}{dx^\mu} (m_0^2 + 2\mu\phi) = 0. \quad (5.18)$$

In the possible case  $\gamma_{\lambda\lambda}$  doesn't depend on  $\lambda$  (that is if  $\lambda$  is an affine parameter) the particle's equations of motion reduce to

$$\frac{d^2 x_\mu}{d\lambda^2} - \gamma_{\lambda\lambda} \mu \partial_\mu \phi = 0. \quad (5.19)$$

By distinguishing the time and the spatial components and by making use of  $\eta_{\mu\nu}$  we have

$$\begin{cases} \frac{d^2 x^0}{d\lambda^2} = 0 \\ \frac{d^2 x^i}{d\lambda^2} - \gamma_{\lambda\lambda} \mu \partial_i \phi = 0; \end{cases}$$

from which

$$\begin{cases} x^0(\lambda) = a\lambda + b \\ \frac{d^2 x^i}{d\lambda^2} + \frac{\gamma_{\lambda\lambda} \sqrt{-\gamma_{\tau\tau}} \mu^2}{4\pi} \frac{x^i(\lambda) - x^i(\tau)}{|\vec{x}(\lambda) - \vec{x}(\tau)|^3} = 0 \end{cases}$$

(with  $a$  and  $b$  some constant parameters); or

$$\begin{cases} x^0(\lambda) = a\lambda + b \\ r^3 \frac{d^2 r^i}{d\lambda^2} + \frac{\gamma_{\lambda\lambda} \sqrt{-\gamma_{\tau\tau}} \mu^2}{4\pi} r^i = 0, \end{cases}$$

where

$$r^i(\lambda, \tau) = x^i(\lambda) - x^i(\tau)$$

and

$$r = |\vec{x}(\lambda) - \vec{x}(\tau)|.$$

Let us consider two classical particles such as those participating to the previous discussion and set them at a distance  $r = |\vec{x}(\lambda) - \vec{x}(\tau)|$  (with  $\vec{x}(\lambda)$  and  $\vec{x}(\tau)$  fixed). These particles interact through the exchange of the scalar field  $\phi$  and the graviton. Differently from what we have done so far, by regarding the spacetime coordinates as dimensional, the scalar and the gravitational forces are expressed (as far as their strengths are concerned) by

$$F_{scalar} = \frac{\sqrt{-\gamma_{\tau\tau}} \mu^2}{4\pi m_0^2 r^2} \quad (5.20)$$

and

$$F_{gr} = \frac{G_N m_0^2}{r^2} \quad (5.21)$$

( $G_N$  being the Newton's constant). Once  $G_N$  has been written in terms of the (reduced) Planck mass as

$$G_N = \frac{1}{8\pi M_P^2}$$

(in natural units), the request that the scalar force acts more strongly than gravity translates into the bound

$$\frac{\mu^2}{m_0^2} \geq \frac{m_0^2}{2\sqrt{-\gamma_{\tau\tau}}M_P^2}. \quad (5.22)$$

By recovering Palti's notation [7] (and so substituting  $\mu$  with  $\mu m_0$ , where  $\mu$  is now adimensional) (5.22) becomes

$$\mu^2 \geq \frac{m_0^2}{2\sqrt{-\gamma_{\tau\tau}}M_P^2}. \quad (5.23)$$

The inequality (5.23) mimics the structure of Palti's SWGC (4.23) (when one scalar modulus only is present). In particular, if the parameter  $\tau$  can be considered as proper time, we get the constraint

$$\mu^2 \geq \frac{m_0^2}{2M_P^2}, \quad (5.24)$$

that is coherent (as necessary condition) with Palti's bound.



## 6 Conclusions

Animated by the desire of finding the way quantum gravity constrains physics and convinced that String Theory is predictive, we have presented the Swampland program. Even though it is actually an abuse, in order to try to get some information on the properties of a consistent theory of quantum gravity, we have identified the QG Landscape and Swampland with the string Landscape and Swampland. The abstract notion of Swampland becomes relevant if it is possible to distill it out from that of Landscape: String Theory examples and black hole physics arguments are used as a sort of experimental tools to gain conjectural evidence for the Swampland criteria and construct a network among them. In this framework we have presented some of the main Swampland conjectures such as the the “No Global Symmetry Conjecture”, the “Weak Gravity Conjecture” (WGC), the “Swampland Distance Conjecture” (SDC) and the “de Sitter Conjecture” (dSC).

We have concentrated our attention on the WGC and, after having presented its best understood version (in the presence of a single  $U(1)$  gauge field), we have moved to the analysis of its refinements and extensions.

Following E. Palti [8], we have tried to extend the WGC in the presence of scalar fields. We have placed ourselves in a  $N = 2$  supersymmetric context and we have made use of the structure of  $N = 2$  black holes to generalize the WGC when multiple gauge fields (with arbitrary gauge kinetic functions) and scalar fields (with an arbitrary scalar field space metric) are in the game. By taking as an assumption that the WGC particles should not form a tower of stable gravitationally bound states and by requiring the decay of extremal black holes (exhibiting certain properties that are typical in the  $N = 2$  supersymmetric framework), the desired generalization emerges: it can be phrased as the statement that there should be at least a state on which the gauge force acts more strongly than the gravitational and the scalar forces combined.

Having described such an extension of the WGC, we have posed the question whether the scalar force acts more strongly than the gravitational force by itself. By exploiting a crucial  $N = 2$  identity (which can be intended as a bound on the relative magnitude of the scalar and the gravitational interactions), we have given evidence to a Scalar version of the WGC (SWGC). In its general form it can be regarded as the claim that for every scalar field there must exist a state on which gravity acts more weakly than the scalar interaction. This amounts to impose that gravity is the weakest force. The General SWGC is motivated by forbidding gravitationally bound states, but in the absence of a gauge symmetry it is not clear how their stability would be ensured. To gain a more solid motivation for the conjecture further work has to be done.

In [9] E. Gonzalo and L. Ibáñez claimed that Palti’s SWGC is not compatible with the WGC for axions. In the attempt of correcting and generalizing Palti’s statement they proposed a new bound. Because of its general structure (Gonzalo and Ibáñez’s conjecture is stated to be valid for any scalar field and any value it assumes) the Strong Scalar WGC (SSWGC) presented in [9] is really attractive, giving (for instance) interesting constraints on the Standard Model and on inflationary models. Despite this, our study of Gonzalo and Ibáñez’s proposal has made evident its various criticisms. Even though claimed as a general statement, Gonzalo and Ibáñez’s bound is valid around a vacuum state: its validity far away from such a vacuum would require additional terms that are absent. The

investigation of the extension of Gonzalo and Ibàñez’s conjecture away from the vacuum asks for further work. Moreover, the derivation of the SSWG by making use of a Quantum Field Theory approach does not reproduce the coefficients the various terms are accompanied by in the conjecture and, on top of that, it allows to conclude that the scalar force is actually weaker in strength than the gravitational interaction. In spite of being apparently animated by the physical principle according to which gravity is the weakest force, Gonzalo and Ibàñez’s claim seems to be in tension with such a principle. The motivation behind this incoherence is the fact that the scalar field taking part to the conjecture is arbitrary and so *a priori* massive.

In the same spirit of Palti, we have tried to overcome the criticisms characterizing the SSWG and possibly get to a general statement. In order to do this we have considered a model including gravity and two scalar fields and we have required that one of these scalars is strictly massless (in the sense that it does not have neither its own mass nor acquire an effective mass). The comparison between the gravitational force (whose carrier is the graviton) and the scalar interaction (whose mediator is the massless scalar) by taking the massive scalar as a probe and the request that gravity acts more weakly than the scalar force lead to a constraint on the parameters of the theory that is coherent (as sufficient condition) with Palti’s SWGC.

Having in mind the Polyakov action and the Lienard–Wiechert mechanism, we have also investigated a proposal to couple a classical particle to a scalar field. In a case that mimics what Palti did in a toy model to motivate his SWGC [7], we have promoted the mass parameter appearing in the Polyakov action (usually intended as a fixed property of the particle) to an effective function of the scalar field the particle couples to. The scalar field solution we have deduced in the non-relativistic limit and in the static approximation and the request that the scalar force that a classical particle feels is stronger than the gravitational one allow to retrieve a condition that is coherent (as necessary condition) with Palti’s bound.

Having not succeeded in finding a general statement for the scalar WGC and somehow feeling that Palti’s approach to the study of the WGC in the presence of scalar fields points towards the right direction, we would like (hopefully soon) to proceed forward.

In [8] an intriguing  $N = 2$  relation is proposed:

$$g^{i\bar{j}}D_i\bar{D}_{\bar{j}}|Z|^2 = n_V|Z|^2 + g^{i\bar{j}}D_iZ\bar{D}_{\bar{j}}\bar{Z}. \quad (6.1)$$

This equation can be interpreted as a relation between the four-point coupling, the mass and the three-point coupling of the WGC states interacting with scalar fields. By specifying the previous relation to black holes and to the analysis of their decay and by making the hypothesis that there should be a state satisfying a corresponding (in)equality, we think that a constraint involving four-point couplings (and overcoming Gonzalo and Ibàñez’s proposal) can be obtained. In the end, we think that a generalization of the scalar versions of the WGC that have been described in this thesis may be evidenced by considering (6.1) as a starting point.

As final comment, let us notice that this thesis has briefly suggested another topic it would be interesting to develop further: according to [17],  $N = 8$  Supergravity should belong to the Swampland.

However, the claim in [17] depends on the choice of a specific duality frame in which all 2-forms have been dualized to scalar fields.

We think that a more accurate investigation should be done for generic duality frames and also considering the gauging procedure, which can affect the result.

For the moment we stop here and we leave a deeper study on all these lines of research for a hoping future work.

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## Errata Corrige

From (4.123) that is

$$\mathcal{M}_t^{(1),1-loop} = \left(\frac{\lambda}{4\pi}\right)^2 \left(\frac{4\pi\rho^2}{q_E^2}\right)^{\frac{\epsilon}{2}} \left[\frac{1}{\epsilon} + \frac{1}{2}\gamma_E + \psi(2) + O(\epsilon)\right],$$

we actually obtain

$$\mathcal{M}_t^{(1),1-loop} = \left(\frac{\lambda}{4\pi}\right)^2 \left[\frac{1}{\epsilon} + \frac{1}{2}\gamma_E + \psi(2) + \frac{1}{2}\ln\left(\frac{4\pi\rho^2}{q_E^2}\right)\right],$$

instead of (4.124).

Then, because the amplitudes  $\mathcal{M}_s^{(1),1-loop}$  and  $\mathcal{M}_u^{(1),1-loop}$  can be evaluated by following the same procedure adopted for  $\mathcal{M}_t^{(1),1-loop}$ , we find

$$\begin{aligned} \mathcal{M}^{(1),1-loop} &= \left(\frac{\lambda}{4\pi}\right)^2 \left[\frac{3}{\epsilon} + 3\left(\frac{1}{2}\gamma_E + \psi(2) + \frac{1}{2}\ln\left(\frac{4\pi\rho^2}{q_E^2}\right)\right)\right] = \\ &= \left(\frac{\lambda}{4\pi}\right)^2 \left[\frac{3}{\epsilon} + 3 - \frac{3}{2}\gamma_E + \frac{3}{2}\ln\left(\frac{4\pi\rho^2}{q_E^2}\right)\right], \end{aligned}$$

instead of (4.125).

After having renormalized the theory and by considering (sufficiently) low energies (coherently with the non-relativistic limit and the static approximation we are referring to), the finite part of  $\mathcal{M}^{(1),1-loop}$  gives a positive contribution to the scalar interaction between two  $H$  particles, that sums up to the positive-signed classical term (4.102). But, the mass being running with the interaction energy ( $m^2 = m^2(E)$ ), we may ask whether there could exist an energy scale at which the gravitational interaction equals in strength the scalar force and so if there could be solutions of

$$\frac{\mu(E_0)^2}{E^2} + \hbar \frac{\lambda(E_0)^2}{16\pi^2} \left(3 - \frac{3}{2}\gamma_E + \frac{3}{2}\ln\left(\frac{4\pi\rho^2}{E^2}\right)\right) = \frac{2m^4(E)}{M_P^2 E^2},$$

instead of (4.175). Since (as it is possible to deduce from (4.171))  $m^2$  changes logarithmically with the energy  $E$ , it seems that the previous corrected equation can't be (reasonably) satisfied.