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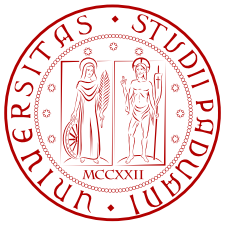

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Algorithms for two-dimensional guillotine packing problems

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Abstract

Packing problems are a class of optimization problems that require to pack items into containers. The Guillotine Two-Dimensional Packing Problems are a sub-class of these problems, where both items and containers are rectangles and with the constraint that every packed item should be possibly retrieved with a series of vertical and horizontal cuts that divide the container into two parts without cutting items. Two exact and two heuristic algorithms have been developed in this thesis, to solve respectively the Guillotine Two-Dimensional Knapsack Problem and the Guillotine Two-Dimensional Bin Packing Problem. The first problem has the goal to pack some of the items into one bin maximizing the total profit. The second one, on the contrary, has the goal to pack all the items using the lowest number of bin as possible.

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Chapter 1

Introduction

The following thesis is focused on the packing problems. The main purpose of this work is to study a way to solve these kind of problems, which are computationally very hard to solve.
These problems are not only a mere theoretical study, but they have many practical application. Usually the main fields where these techniques are used are packaging, transportation and storage issues. A simple example could be the problem of fill in the best way possible a truck to minimize the empty space inside and so to minimize the number of times it has to bring everything to the destination.
Usually very clever techniques are needed to solve packing problems, in particular enumeration techniques that must be thought very carefully, otherwise the risk is to have too poor performance, that means a too high computation time that runs the risk of not completing.

Hereafter there is a quick overview on what next chapters will deal with:

- **Chapter 2: Problem Description**: In this chapter all the typologies of problems and algorithms touched by this work will be explained in more detail. So there will be first a general description on what Packing problems are; the attention will be then focused on the problems actually treated, that are Knapsack problem and Bin-Packing problem, considering, in particular, the cases with shelf divisions, of which problems with guillotine cuts are a particular instance. Eventually there will be an explanation on what NP-Hard problems are and on the difference between exact and heuristic algorithms.

- **Chapter 3: Algorithms Description**: In this section the algorithms will be very particularly explained from a theoretical point of view. In particular the reasons of certain decisions will be explained.

- **Chapter 4: Used Tools**: Here it will be discussed more deeply about the tools that have been necessary to implement the desired algorithms. So there will be a presentation of the C language used to implement the algorithms, a brief description of the integrated development environment (IDE) Eclipse and an illustration of the two debugging tools used, Valgrind, a very useful framework that helps a lot find memory leak and referencing problems and Gnuplot, a command-line program that allows you to visualize graphically the output of the algorithm and that has been useful for logical debugging.

- **Chapter 5: Implementation**: Here it will be discussed about the implementation of all the algorithms. In particular the reasons of certain choices will be pointed out, explaining

also which other choices had been thought or tried and which kind of problems they had brought.

- **Chapter 6: Tests**: In this chapter the computational experiments are described. First the benchmark of instances from the literature are presented, then the characteristics of machines where tests have been performed are illustrated and finally results of tests will be presented and discussed.
- **Chapter 7: Conclusions**: In this section the conclusions on the performed work will be drawn and suggestions for a possible future continuation of the work will be indicated.

Chapter 2

Problem Description

2.1 Packing Problems

Packing problems are a class of optimization problems that require to pack objects into containers. The goal is to either pack a single container as densely as possible by selecting a subset of items from a given set or pack all objects using as few containers as possible.
In packing problems usually the following objects are given:

- Containers, that usually are or mono-dimensional, so they only impose an upper bound on the maximum weight of the selected items, or a two- or three-dimensional convex region, or, more rarely, they are an infinite space. In the case they are a finite region, depending from the type of problem, they can be just one, a finite number or an infinite number.
- A set of items, some or all of which must be packed into one or more containers. The set may contain different items with given sizes (or just weights in the mono-dimensional case), or a single item of a fixed dimension that can be used repeatedly.

Two-dimensional Case

Usually the packing must be without overlaps between items and other items or the container walls. In some variants, the aim is to find the configuration that packs a single container with the maximal density. More commonly, the aim is to pack all the objects into as few containers as possible. In some variants the overlapping (of objects with each other and/or with the boundary of the container) is allowed but should be minimized.
When the container size is increased in all directions, the problem becomes equivalent to the problem of packing objects as densely as possible in infinite Euclidean space (an infinite number of objects, all with the same shape). This has brought to the study of the most different shapes, starting from the regular ones, arriving to the curved ones.

2.2 Knapsack Problem

2.2.1 One-dimensional Knapsack Problem

The Knapsack Problem (KP) [4] is probably the most studied and known packing problem.
Given a set of items {1,...,n}, to each of those a profit p_j and a weight w_j are associated and given

a knapsack of capacity W, the Knapsack Problem (KP) consists in choosing a subset of object with maximum profit to insert into the knapsack.
This problem has a great importance in all those context where the goal is to load in an optimal way a bin (that could be a camion, a container, a knapsack, etc.) and in finance, where, given a fixed budget, it is necessary to select which things to buy. In addition, KP arises as sub-problem in many complex problems; for instance, it turns out to be the pricing problem when column generation techniques are used to solve the Bin Packing Problem.
Without loss of generality in the model it is possible to suppose that p_j, w_j and W are positive integers, with $w_j < W$ for each $j = 1,...,n$ and that $\sum_{j=1}^{n} w_j > W$

Introducing the following decisional variables:

$$x_j = \begin{cases} 1, & \text{if object } j\text{-th is selected} \\ 0, & \text{otherwise} \end{cases}$$

it is possible to set an integer linear programming (ILP) model:

$$z^* := \max \sum_{j=1}^{n} p_j x_j$$

$$\sum_{j=1}^{n} w_j x_j \leq W$$

$$0 \leq x_j \leq 1 \text{ integer}, j \in \{1, ..., n\}.$$

The objective function maximize the total profit of the selected items. The capacity constraint imposes that the sum of the weights of the selected items does not exceed the container capacity; finally the x_j variables are integer variables with value 0 or 1.
It is correct to specify, anyway, that although this model is the most used to represent a Knapsack Problem, this is not the only one. However in this work we will refer to this model.
Usually this model is solved with the branch-and-bound technique or, if W is a not too big integer, with the dynamic programming. This limitation is due to the fact that using the dynamic programming the complexity is $O(nW)$, which means that Knapsack Problem is solvable pseudopolynomial time, because W can be exponential in the input size so the overall running would become not polynomial.

2.2.2 Two-dimensional Knapsack Problem

The problem considered in this work is not exactly the knapsack problem described by the model mentioned above. Each item $j \in N=\{1,...,n\}$ is a rectangle with height h_j, width w_j and profit p_j often equal to that item's area and the container is a sheet with height H and width W.
Without loss of generality in the model it is possible to suppose that p_j, w_j, h_j, W and H are positive integers, with $w_j < W$ and $h_j < H$ for each $j = 1,...,n$.
Each edge of the items must be packed parallel to an edge of the container. Thus, considering the case with $h_j = H$ for all $j \in N$ (or, in the same way, $w_j = W$ for all $j \in N$), the problem can be

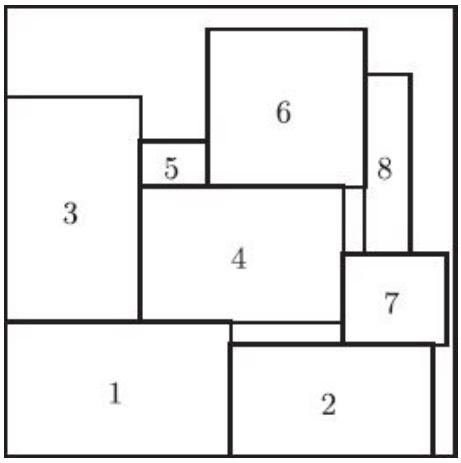

Figure 2.1. Example of solution of Two-dimensional Knapsack Problem

reduced to the mono-dimensional case.

Real-world applications of Two-dimensional knapsack problem (2KP) arise in loading, transportation, resource allocation, just to mention a few.
A possible variant of the problem is the possibility of rotate or not the shapes of items by 90°. This can be useful in some applications, because it can allow less waste of material, while in other it is not possible to use it, because of features of the material or of the desired items. An example that allows rotation is the creation of plywood panels, because it makes no difference which way you cut them; an example that does not allow rotation, on the other hand, is the marble cutting, because the grains must be followed.
In this work only the so-called fixed-orientation (with no rotation) case has been developed.

2.2.3 2-Staged Knapsack Problem

A classical variant on the 2KP is the 2-staged 2KP, in which the maximum number of cuts allowed to obtain each item is fixed to 2 [6]. A cut must be parallel to the edges of the sheet and must go from a part to the other of the portion of surface remained after the previous cuts. It also must not pass through other items.
If a third stage of cutting is allowed only to separate an item from a waste area, this is called the non-exact case of 2-staged 2KP or 2-staged 2KP with trimming. Otherwise, we have the exact case of 2-staged 2KP, or 2-staged 2KP without trimming. Because of the particular disposition that must have the items in the solution, this type of problem is also called 2KP with shelf divisions. In figure 2.2 it is possible to see examples of not-exact and exact 2-staged 2KP.

In recent years, most of the literature on two-dimensional packing referred to general k-staged packing. This constraint (and in particular the constraint with k=2) can make the problem much easier than pure 2KP from a computational viewpoint, but it increases a lot the amount of wasted space in the solution. However, if automatic machines are used for unloading/cutting items and the sheet is not significantly expensive, it could preferable to look for solutions of this type.

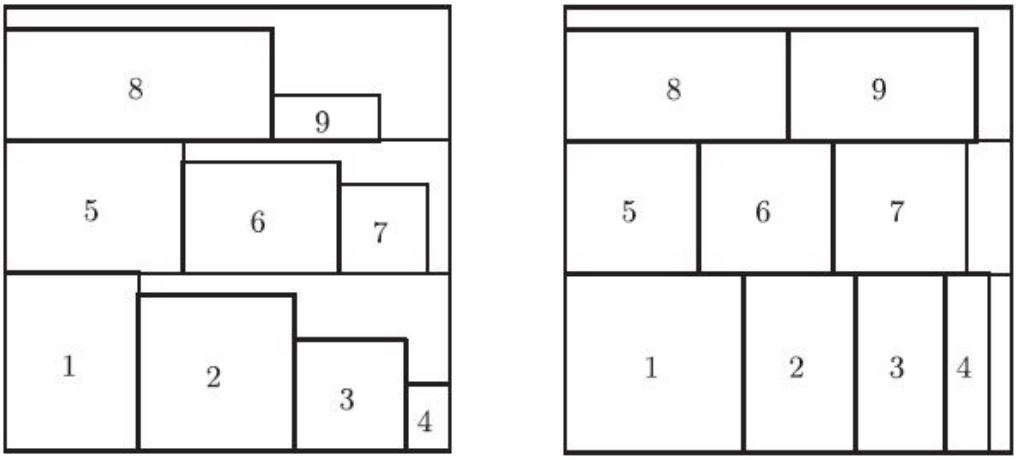

Figure 2.2. Example of 2-staged patterns: a)Not exact and b)exact

2.2.4 k-Staged Knapsack Problem

A more general case is that where the restriction to the maximum number of cuts allowed to unload each item is fixed to a given threshold k. In this case the problem is called k-Staged 2KP [3]. Because k can assume any value it is possible to consider in this section also the case when no restriction on the number of cuts is present. This is the general case of the two-dimensional guillotine knapsack problem (G2KP), which is exactly the case that will be considered in this thesis.

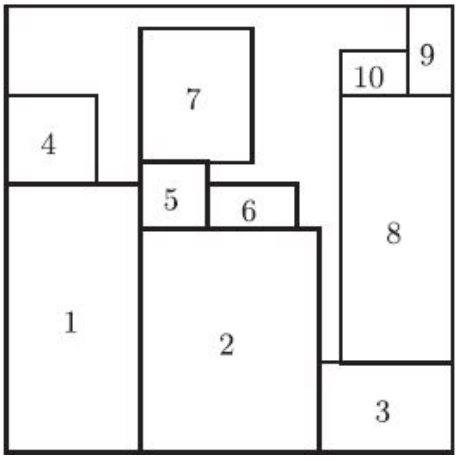

Figure 2.3. Example of solution of Two-dimensional Guillotine Knapsack Problem

This problem can be considered a middle ground between the classic 2KP and the 2-staged 2KP. The real-world application where it is useful is, like the 2-staged 2KP, that of the automatic machines. This kind of problem is not more difficult than pure 2KP, but it finds a solution that can be used by the automatic machine, meanwhile it is harder than 2-staged 2KP from a computational

point of view, but decreases significantly the wasted space in the solution.
Hence this problem is considered a very good compromise and this is one of the reasons that led to focus on this kind of problems.

2.3 Bin Packing Problem

Given a set $N = \{1,...,n\}$ of items, the k-th with weight $w_k > 0$ and a set $M = \{1,...,m\}$ of containers (bin), each one with capacity W, the Bin Packing Problem (BPP) requires to insert all the items into containers observing these constraints:

- each item must be inserted in one bin;
- the total weight of objects inserted in a bin must not be greater that W;
- the number of bins used must be minimized.

A necessary condition for a feasible solutions is that, for each $k \in N$, $w_k \leq W$. Under this condition it is always possible to find a feasible solution in the case that $m \geq n$. If $m < n$ the problem of verify if a feasible solution exists is NP-Hard. It will be considered only the case with $m \geq n$ (supposing m as big as necessary).

One-dimensional bin packing (1BPP) is a classic problem with many practical applications related to minimization of space. Two possible examples can be placing computer files with specified sizes into memory blocks of fixed size, or the recording of all of a composer's music, where the length of the pieces to be recorded are the weights and the bin capacity is the amount of time that can be stored on an audio CD.

Again, we consider a generalization of 1BPP that arises when more than one dimension is present and additional constraints are necessary. This case is the two-dimensional guillotine bin packing problem (G2BPP), that can be easily derived from the previous case of the knapsack problem. Also for the BPP, in fact, the features of the two-dimensional bin packing problem (2BPP) are the same of those of 2KP, with the difference of the goal of the problem that always is to minimize the number of bins (sheets) to insert all the items (with rectangular shape). The same approach can be used for 2-staged 2BPP and k-staged 2BPP.

2.4 NP-Hard problems

In computer science over the years it was necessary to introduce a discipline that determines if a certain problem was solvable or not and how easily it could be solved. The theory of computation is this: it is the branch of computer science that discuss whether and how efficiently problems can be solved on a model of computation, using an algorithm. This field is divided into three major branches: automata theory, computability theory and computational complexity theory.
In particular the computational complexity theory studies which are the minimal necessary resources (considering above all computational time and memory) needed by a problem to be solved. Thus, problems are classified in different complexity classes, according to the most efficient algorithm that is known to solve each problem.
Therefore a complexity class is a class that includes a number of different problems having the

same resource-based complexity. Usually different classes are defined in this way:
set of problems that can be solved by an abstract machine M using O(*f(n)*) of resource R, where n is the size of the input.
An abstract machine is a theoretical model of a computer. Here the abstract machine that has been decided to be used is the Turing machine.
Two of these classes are particularly important: NP and P. NP is the class of decision problems whose solutions can be determined by a non-deterministic Turing machine in polynomial time; P, otherwise, is the class of decision problems whose solutions can be determined by a deterministic Turing machine in polynomial time.
P then is, trivially, just a subset of NP. But NP has another subset of problems, the so-called NP-complete problems, for which no polynomial-time algorithms are known for solving them (although they can be verified in polynomial time). This takes to biggest open question of the complexity theory, the P = NP problem, which asks whether polynomial time algorithms actually exist for NP-complete and, accordingly, all NP problems.
Finally there are the so-called NP-hard problems. This class of problem is not properly a subset of NP, because its problems don't necessarily belong to NP; the name derives from the fact that problems belonging to this class are at least as hard as the hardest problem in NP. So a problem H is NP-hard if and only if there is a NP-complete problem that can be reduced to H by a deterministic Turing Machine in polynomial time.
Knapsack Problem and Bin Packing Problem are two examples of NP-hard problems.

2.5 Exact and Heuristic Algorithms

In this work an exact algorithm to solve the G2KP has been implemented. An exact algorithm is an algorithm that solves a problem to optimality [12]. For NP-hard problems, anyway, probably no polynomial time algorithms exist, but it doesn't mean that it is not possible to find an exact algorithm.
Two examples of techniques that lead to exact algorithms are Branch and Bound and Dynamic Programming.

Branch And Bound

Branch and bound is a general technique for finding optimal solutions of various combinatorial problems. A branch and bound algorithm consists of a systematic enumeration of all candidate solutions, from which some of possible candidates are discarded using upper and lower estimated bounds of the quantity being optimized.
This procedure is usually used to maximize or minimize a function. Suppose that the goal is to find the maximum value for a function *f(x)*, where x ranges over some set S of admissible or candidate solutions. Branch and bound is a recursive algorithm that performs 3 steps each time it is called:

1. **Branching**: splitting procedure that returns two or more subset S_1,..., S_n whose union it is the initial set S. All these new subset are children nodes of a tree where their father node is S;

2. **Bounding**: procedure that computes upper and lower bounds for the maximum value of *f(x)* within a given subset of S;

3. **Pruning**: if the upper bound for some tree node (set of candidates) A is lower than the lower bound for some other node B, then A may be safely discarded from the search.

The recursion stops when the current candidate set S is reduced to a single element, or when the upper bound for set S matches the lower bound. Either way, any element of S will be a maximum of the function within S.

Dynamic Programming

Dynamic programming is a method for solving complex problems by breaking them down into simpler sub-problems. It is applicable to problems exhibiting the properties of overlapping sub-problems (it can be reduced into sub-problems which are reused several times) and optimal sub-structure (an optimal solution can be constructed efficiently from optimal solutions of its sub-problems).
The idea behind dynamic programming is quite simple. In general, to solve a given problem, it is necessary to solve different parts of the problem (sub-problems), then combine the solutions of the sub-problems to reach an overall solution. Often when using a more naive method, many of the sub-problems are generated and solved many times. The dynamic programming approach seeks to solve each sub-problem only once, thus reducing the number of computations: once the solution to a given sub-problem has been computed, it is stored. So the next time the same solution is needed, it is simply looked up. This approach is especially useful when the number of repeating sub-problems grows exponentially as a function of the input size.
Examples of algorithm that can be implemented using dynamic programming are Dijkstra's algorithm for the shortest path problem, Fibonacci sequence, tower of Hanoi, matrix chain multiplication.

Heuristics

Often, however, the determination of the optimal solution of a NP-hard problem may be too costly in terms of computation time and in some of these cases even the technological development can not significantly reduce this time. But sometimes these problems are required to be implemented for a practical application, so it is not possible to wait for dozen years to have the exact result. Moreover often some parameters are just estimates, so it has no sense to try to find the exact solution of a problem that comes from uncertain values.
Therefore the need arises to solve the difficult problems in reasonable computation times sacrificing the optimality of the solution.
It is due to these reasons that heuristic algorithms are important. Literally they are any method that finds a solution of the problem. Ideally one would like a heuristic algorithm to be able to always determine the optimal solution of a problem. But for most of the problems this is not possible. So generally heuristic algorithms are meant to be algorithms that are likely to find reasonably good solutions in a short time [11].
Unfortunately some problems exist for which even heuristic algorithms can not guarantee to find a solution. In these cases heuristic algorithms are able to find an upper bound (if the problem is a minimization problem) or a lower bound (if the problem is a maximization problem).
In this thesis some heuristic algorithm have been implemented. They have been used above all when the optimal solution was not needed, because it was enough knowing if a solution existed or having just a good solution and not necessary the optimal one.

Chapter 3

Algorithms Description

In this section the implemented algorithms will be described.
Because in this thesis we are dealing with two-dimensional problems, sheets of width W and height H will be considered. In addition to this a set N=$\{1,...,n\}$ of types of rectangles (called items) is given. The j-th type of items contains a_j items, each having a width w_j, a height h_j and a profit p_j.

3.1 Procedure Recursive

The goal of this algorithm is to solve the Guillotine Two-dimensional Knapsack Problem (G2KP) [3].
It must receive in input a parameter z_0, which is a lower bound on the profit of desired solution. This solution will be a subset of items that can be put into the sheet satisfying the constraints of the problem. In the solution not only the list of these items will be returned, but also their position in the sheet, by means of the coordinates of the bottom-left vertex of each selected item, assuming dimensions be labelled from 0 to W and from 0 to H respectively.
In general we denote each assignment of a subset of items to the sheet as a feasible packing, so the solution is a feasible packing with its own profit that is greater than z_0. A feasible packing can be usually represented as a vector with length n, in which each element of the vector defines the number of items of that type that are present in the packing: $f = [f_1, \ldots, f_n]$.
Procedure Recursive is called with this name because it implicitly enumerates all feasible packings by recursively dividing the sheet into two parts, so that each of these divisions represents a (either horizontal or vertical) guillotine cut.

As observed by Christofides and Whitlock [1], for any two-dimensional packing problem an optimal solution corresponding to a normal pattern exists, i.e. a solution in which any item is packed with its left edge adjacent either to the right edge of another item or to the left edge of the sheet. Thanks to this result it is possible to consider just vertical cuts that are in correspondence of linear combinations of widths of the items. The same observation can be done also with horizontal cuts. So these 2 sets are obtained as follows:

$$\mathcal{W} = \{x : x = \sum_{i=1}^{n} w_i \alpha_i \ , \ 1 \leq x \leq W \ , \ 0 \leq \alpha_i \leq a_i \ , \ i = 1, ..., n\}.$$

$$\mathcal{H} = \{y : y = \sum_{i=1}^{n} h_i\alpha_i \ , \ 1 \leq y \leq H \ , \ 0 \leq \alpha_i \leq a_i \ , \ i = 1, ..., n\}.$$

We assume that both $\mathcal{W}$ and $\mathcal{H}$ are sorted in ascending order and let $t = |\mathcal{W}|$ and $s = |\mathcal{H}|$.
Given $x \in \mathcal{W}$ and $y \in \mathcal{H}$ and the lower bound solution value z_0, let $F(x, y, z_0)$ be the set of all feasible packings of the given items into a sheet of size $x \times y$ that can produce a profit larger than or equal to z_0 considering their profit plus the profit that could come from a possible packing of some of the remained items into the residual area of the sheet.
Moreover, given two sets of feasible packings $F^1 = \{f^1, \ldots, f^k\}$ and $F^2 = \{\bar{f}^1, \ldots, \bar{f}^m\}$, it is possible to denote by $F^1 \oplus F^2$ the so-called pairwise sum of the packings in those two sets:

$$F^1 \oplus F^2 := \{f^i + \bar{f}^j : i = 1, \ldots, k, \ \ j = 1, \ldots, m\}.$$

Where the sum between two packings is defined as follows: given two feasible packing $f = [f_1, \ldots, f_n]$ and $\bar{f} = [\bar{f}_1, \ldots, \bar{f}_n]$ a new packing $\hat{f} = f + \bar{f}$ is defined int this way: $\hat{f}_i := \min\{f + \bar{f}_i, a_i\}$ $(i = 1, \ldots, n)$.
Hence, $F^1 \oplus F^2$ is the set of packings that can be obtained combining any packing $f^i \in F^1$ with any packing $\bar{f}^j \in F^2$.
So, thanks to the fact that $\mathcal{W}$ and $\mathcal{H}$ are ordered, once set $F(x_1, y_1, z_0)$ has been found, also sets $F(x_1, y_2, z_0)$, $F(x_1, y_3, z_0)$, $\ldots$, $F(x_1, y_s, z_0)$ can be computed and, in a similar way, also $F(x_2, y_1, z_0)$, $F(x_3, y_1, z_0)$, $\ldots$, $F(x_t, y_1, z_0)$. Therefore it is possible to notice that each packing $f \in F(x_j, y_i, z_0)$ can be computed as the sum of two feasible packings defined for smaller sizes of the bin. Thus, knowing $F(x_i, H, z_0)$ and $F(W, y_j, z_0)$ for every $i = 1, ..., t-1$ and $j = 1, ..., s-1$, $F(H, W, z_0)$ can be easily found.

For each packing f that is computed for a certain sheet with width x and height y, an upper bound *U(f)* is computed on the profit obtainable from the knapsack instance with capacity $WH - xy$, n types of items, the j-th available in $a_j - f_j$ copies, each with profit p_j and weight $w_j h_j$. The upper bound is computed according to a procedure proposed by Martello and Toth [9] that will be described later (see section 3.2).
This calculation is fundamental to determine whether each found packing is really feasible for the sheet $x \times y$ or not and this can be easily verified through this simple inequality:

$$p(f) + U(f) \geq z_0$$

with *p(f)* that is given by $\sum_{i=1}^{n} p_i f_i$, where with f_i the number of items of type i that are in the packing is meant.
If the aforementioned inequality is satisfied the packing is kept, otherwise it is discarded.
In addition to this, another control is introduced, that is the control if a packing is maximal or not and allows to discard all the not maximal packings. We say that a feasible packing f is *maximal* if no further items can be packed into the sheet, i.e. if, for example, a packing f = [0,2,1,0] was feasible, then packings like f = [0,2,0,0], f = [0,1,1,0] and f = [0,1,0,0] would not be maximal, because at least one item can be added still obtaining a feasible packing.

Procedure *Recursive* (z_0)

1. compute $\mathcal{W} = \{x_1, x_2, \ldots, x_t\}$
   compute $\mathcal{H} = \{y_1, y_2, \ldots, y_s\}$
   $F(x_0, y_i, z_0) := \emptyset,\ i = 1, \ldots, s$
2. for $i = 1$ to s do
   for $j = 1$ to t do
   $S := F(x_{j-1}, y_i, z_0)$
   $q = 1$
   while $x_q \leq \frac{x_j}{2}$ do
   if there is a $\bar{q}$ such that $x_q + x_{\bar{q}} = x_j$ then
   $S := S \cup (F(x_q, y_i, z_0) \oplus F(x_{\bar{q}}, y_i, z_0))$
   $q := q + 1$
   $S := S \cup F(x_j, y_{i-1}, z_0)$
   $q = 1$
   while $y_q \leq \frac{y_i}{2}$ do
   if there is a $\bar{q}$ such that $y_q + y_{\bar{q}} = y_i$ then
   $S := S \cup (F(x_j, y_q, z_0) \oplus F(x_j, y_{\bar{q}}, z_0))$
   $q := q + 1$
   if there is an item with dimension $y_i \times x_j$ add it to S
   $F(x_j, y_i, z_0) := \{f \in S : p(f) + U(f) \geq z_0,\ f \text{ maximal}\}$
   endfor
   endfor
3. $S := F(x_t, y_s, z_0)$

Figure 3.1. Procedure Recursive

Let us take now a closer look on this procedure presented in figure 3.1.
First of all $\mathcal{W}$ and $\mathcal{H}$ are computed like discussed above and $F(x_0, y_i, z_0)$ is initialised to $\oslash$ for each i from 1 to s.
Then we enter in a double loop where all the possible dimensions of the smaller rectangles are considered. Here set S stores the set of all the possible packings that could be feasible for a rectangle $x_j \times y_i$.
After that we enter in an inner loop, which is very important because here the constraint on guillotine cuts is implemented. In this cycle, some x_q that satisfies two features are considered: they must be lower than half of the current x_j (because in the second half solutions would be repeated) and a $x_{\bar{q}}$ so that $x_q + x_{\bar{q}} = x_j$ must exist. These x_q are possible positions where vertical cuts can be placed. All the possible packings found from the pairwise sum between the sets of feasible packings in rectangles with the same height y_i and width x_q and $x_{\bar{q}}$ are then added to S through the union, so that there are no repeated packings.
Then the exactly same process is performed for possible horizontal cuts.
At the end of these two phases, if there is an item with dimension that is exactly $x_j \times y_i$, it is added to S.
Finally on each packing in S the two controls mentioned above are performed: the control on the

reachability of the minimum desired profit z_0 and that on the maximality of the packing.
At the end of the algorithm the solution of the problem can be found in $F(x_t, y_s, z_0)$.

To find the coordinates of the bottom-left corner of each item in the solution track of the characteristics of each cut performed is kept; in this way, a backward search like in a tree structure allows to find out the desired coordinates from the leaves of the tree, that are the sets of feasible packings where each item in the solution is inserted into a packing for the first time (that is when a rectangle with the same dimensions of the item is considered).

3.2 UB Martello and Toth

Before describing the algorithm created by Martello and Toth to compute an upper bound (UB) of the knapsack problem, it is necessary to present the algorithm found by Dantzig [2].
In Dantzig algorithm first of all it is necessary to order all the items by profit over weight (p/w) ratio. Then the critical object s is searched, that is the first object of the list that can not be inserted into the container. Formally: $s = min\{t : \sum_{j=1}^{t} w_j > c\}$. After this computation the residual capacity is defined as the capacity of the container that is still free before inserting items: $\bar{c} = c - \sum_{j=1}^{s-1} w_j$. Dantzig UB is then the sum of the profits of the items before the critical object plus part of the profit the critical object, this amount being proportional to the part that could be inserted in the residual capacity. Formally:

$$UB(Dantzig) = \sum_{j=1}^{s-1} p_j + \bar{c}\frac{p_s}{w_s}$$

The algorithm of Martello and Toth [9] starts in the same way of that of Dantzig. First there is the ordering of items, then there are computations of critical object and residual capacity. At this point there are two possibilities: either not choosing the critical object or choosing it.
If the critical object is not chosen, the UB is computed like the Dantzig UB but using the item after the critical object in the ordered list in its stead:

$$UB_0 = UB(x_s = 0) = \sum_{j=1}^{s-1} p_j + \bar{c}\frac{p_{s+1}}{w_{s+1}}$$

If the critical object is chosen, then the item before it is just partially chosen:

$$UB_1 = UB(x_s = 1) = \sum_{j=1}^{s} p_j + (\bar{c} - w_s)\frac{p_{s-1}}{w_{s-1}}$$

Once these two UBs have been computed the algorithm of Martello and Toth states that the maximum between them is the searched UB:

$$UB(Martello - Toth) = max(UB_0, UB_1)$$

3.3 Exact Algorithm for G2KP

The previous algorithm is able to find an exact solution for G2KP, but it requires to have a lower bound on this solution as input, which could assume the most different values and is impossible to know in advance.
To overcome this problem this algorithm has been introduced. We iteratively execute procedure Recursive with different threshold values in order to find the optimal input z_0 to have the best possible packing.

First of all a heuristic algorithm (that will be described in detail in the next section) is used to find a reasonable solution z_H that is used as lower bound then an upper bound U is also computed using the exact algorithm by Martello and Toth for 1KP.
After these two preliminary computations, we enter in a loop where the procedure Recursive is iteratively called. The lower bound z_0 passed as input is updated at the end of each iteration of the cycle. For the first time this value z_0 coincides with U but then it assumes always values that are new.
At this point there are two possible situations: either the procedure finds (at least) a feasible packing or it does not and returns NULL. In this case, we know that the value of z_0 is an upper bound on the optimal solution value; thus U is updated. Otherwise, if a feasible packing is found, the procedure Recursive is used again, but this time with threshold $z_0 + 1$ to understand if z_0 was the optimal value of the solution or it can be improved. If the algorithm returns NULL it means that the optimal solution has been found at it is possible to exit the loop. Otherwise it means that it is possible to find a better solution, therefore z_0 is a lower bound and so it is possible to update z_H with this value. Moreover z_0 is set again equal to U because to update the value of z_0 is necessary to start from the upper bound and decrease it.
At the end of the cycle z_0 is decreased of a value $(U - z_H)/50$. The value 50 has been computed empirically as trade off between the necessity of a fast improvement of the value and the necessity of not finding lower bounds for Recursive that are too small, because in that case a too large number of packings is found during the execution of the algorithm and this slow down the performance a lot, both considering computational time and memory used. Hence we have preferred to use a not too big decrease of U, so that the case where the lower bound must be updated rarely happens and, when it happens, it is with a value that will not be much lower than the optimal solution value. The only contraindication is that in this manner a bigger number of cycles is performed, but, as they have almost all a value of z_0 that is an upper bound on the solution, less packings are found during the execution of Recursive so it computes faster.

3.4 Heuristic algorithms for G2KP and G2BPP

These algorithms try to find the best possible packing, which will not be necessarily the optimal one; they return respectively the profit of the best packing found and the minimum number of bins necessary to pack items.

3.4.1 Heuristic G2KP

In this algorithm items are no more divided by type, here each one is considered separately from the others. So first of all, just for the first time, they are all ordered by profit over weight.

Then we enter in a loop that is performed a fixed number of times and can not be terminated before. In this cycle there is another loop inside, where the first k items are considered, such that the sum of their profit is better than the actual best profit and that their total area is smaller than the area of the container. If the desired list of items does not respect one of the two constraint the cycle is performed again, for maximum a fixed (and low) number of times.
When exited from the inner loop, if no set of items is returned no computations are made. Otherwise these items are tried to be inserted in the container with a strategy that has been called "MixedStrategy". Two versions of this have been created. These differ from each other just because one begins with a vertical shelf while the other with a horizontal one. This has been made because some times items can be packed in a way and not in the other. So in the heuristic algorithm first the vertical strategy is tried and then, if it does not work, also the horizontal one is tested. If a packing is found the variable containing the best profit is updated and the cycle starts again with the items in the same order (because it could be possible to add another item and obtain again a feasible packing), otherwise all the items are ordered randomly.

The MixedStrategy has been called in this way because it tries to pack items with an algorithm based on shelves, but with the difference that these are not all horizontal or vertical, but they are alternated. So in the case of the so-called "MixedStrategyV_G2KP" the first shelf that is tried to be filled is vertical, the second one is horizontal, the third vertical again and so on. For the "MixedStrategyH_G2KP" the algorithm works exactly in the same way starting from the horizontal one. Each item of the list is tried to be inserted in the first possible shelf of the order. If in none of them it is possible to insert the item, another shelf is opened. If also it is not possible to open another shelf then it means that a packing for those elements with this strategy does not exist.

3.4.2 Heuristic G2BPP

Like for the previous algorithm, also in this one items are no more divided by type, but each one is considered separately from the others. Initially, however, they are all ordered not by profit over weight, but by decreasing size of area.
After that we enter in a loop where two strategies are performed to compute the total number of bins that are necessary to pack all the items. Also here these strategies are based on the MixedStrategy and differ from each other because of the different direction of the first shelf (vertical against horizontal).
Here the cycle is repeated until a strategy finds that all the items can be packed in only one bin or until the maximum number of times that the loop must be performed is reached (fixed before the beginning of the algorithm).

The MixedStrategy works exactly in the same way of the previous for G2KP and the same is also true for "MixedStrategyV_G2BPP" and "MixedStrategyH_G2BPP". The differences are only those connected to the different goals of the problems. So the main difference is that here it is not possible to have a situation where the packing is not feasible: in the worst case, if it is not possible to open a new shelf in a bin, a shelf in a new bin is opened.
For this kind of problem, anyway, it is possible to terminate the computation before the end of the algorithm: this strategy receives in input the best solution that has been found until that moment and so, if during the execution of the algorithm the i-th bin is open when i is the best solution, the algorithm can be terminated because there will not be an improvement on the solution.

3.5 Exact Algorithm for G2BPP

The exact algorithm to solve G2BPP is based on the branch and bound technique described above (see Martello and Vigo [10]).
First of all also here all items are considered separately and no more divided by type and, as for the Heuristic G2BPP, they are ordered by decreasing size of area.
Then we enter in the recursive algorithm that uses branch and bound. It can be considered as a tree structure where each level represents an item and, for each level, the nodes of the upper level have as many children as the number of bins where it is possible to pack the item that corresponds to this level. This number of children is the current depth in the tree, because it is assumed that each item can be inserted in all the possibly previous opened bins plus one (so the total number of nodes for each level is the number of nodes of the upper level times the depth of each level). To check all the possible combination we begin from the root and then we go down through nodes with a depth first search. In the meanwhile, to avoid visiting all possible nodes, many nodes can be pruned or, if the best solution has been found, the search can be stopped. Each time that the search reaches a leaf (that will be certainly on the last level) it means that a new better solution is found.

To check if the solution found is the best solution and stop the search, the so-called lower bound 2 (LB2) has been used. This can be found in literature and gives a lower bound on the minimum number of bins that are necessary to pack all the items. Hence if a solution that is equal to the value of LB2 is found, it must be the optimal solution and the algorithm can terminate.
To prune some nodes two techniques are used:

1. if the current node should be inserted into a new bin i and the best solution so far is i as well, that node and all the branches that start from it are pruned, because there would not be an improvement of the solution if they were visited;

2. if the current item is the same type of item of the previous one (they are ordered by size, so equal items are close to each other), supposing that the previous item has been inserted in the j-th bin, all nodes that would represent the insertion into bin from 1 to j-1 are pruned, because if it was not possible to insert the previous item there, so it will not be possible to insert the current one as well (symmetry breaking).

For each node that is not pruned we try to insert the corresponding item into each bin starting from the first possible. If the insertion succeeds the algorithm moves down recursively to its first child (obviously checking the pruning conditions), otherwise it moves to its left sibling until it is able to insert the element in a bin. If it is not possible to insert this item in any already opened bin a new bin is opened.

To try to insert the current item in a bin four steps are performed, where the first three guarantees a faster execution of the algorithm:

1. the total area of the items already inserted in the bin summed with the area of the current item must be lower than (or equal to) the area of the bin, otherwise it is not possible the insertion;

2. if LB2, computed on the items already in the bin plus the current one, is greater than 1 it means that only one bin is not sufficient to pack all these items and so it is not possible the insertion;

3. the heuristic for G2BPP aforementioned is used. If it returns 1 it means that all items can be packed into just one bin and so the insertion is performed, otherwise nothing can be said about it;

4. if the heuristic returns a value greater than 1 the procedure recursive is used. It tries to solve the G2KP where all items have profit equal to 1 and the desired total profit is the total number of items that are wanted to be packed. If it returns NULL no packing is possible for these elements in one bin, otherwise the insertion is performed.

Chapter 4

Used Tools

4.1 C Programming Language

Figure 4.1. C Logo

C is a general-purpose programming language [5]. Initially it was associated with the UNIX system, within which it was developed, because both the system and many of its programs were written with C. Anyway this language is not constrained to any machine or operative system, in fact just as effectively it has been used to create primary utilities in different fields.
Many basic ideas of C come from BCPL language, but its influence has been mitigated by the language B, created by Ken Thompson in 1970.
An important difference, anyway, is that BPCL and B are languages with the absence of types, while C owns a wide range of them. The fundamental types (primitive types) are characters, integer numbers and decimal numbers. Also some derived types exist, created with pointers, vectors, structures and unions. Pointers give rise to an arithmetic of addresses that is independent from the machine.
C provides the basic constructs to regulate the flow of control, constructs required for the development of well-structured programs: instructions grouping, the implementation of decisions (*if-else*), the choice of one among the possible cases (*switch*), the use of a cycle with a stopping condition at the beginning (*while, for*) or at the end (*do*), the early exit from a cycle (*break*).
Values returned from functions can be both primitive or derived types. Any function can be called recursively. Local variables are typically "automatic" or created from scratch at each call.

Functions of a same program can be divided in different files, called "source files", which must be compiled separately. Variables can be internal to a function, external but visible only within a given source file, or visible in the entire program (global variables).
Before the compilation, another phase called "preprocessing" exists. Here macros, which are conventional abbreviations, are substituted with their effective values in the program and other source files can be included.

C is considered a relatively "low level" language. This definition means that C uses the same kind of objects of many computers, like characters, sets, lists, vectors, that can be combined through arithmetical and logical operators used by real machines.
C does not provide commands to directly deal with complex objects (like strings, sets, lists, vectors). It does not offer specific tools to allocate memory, with the exception of static definitions and the local stack of functions. Also the garbage collection is not present. This language does not even offer input/output functionalities, it has no read or write operations and no incorporated methods to access file as well. All these "high level" mechanisms must be performed with apposite functions, so many implementations of the language provide a reasonably uniform collection of these functions.
Despite the absence of some features could seem a big lack, the reduced dimension of the language brings some benefits, like the fact that C can be easily learned and that a programmer can expect to know and use the whole language.

C was initially developed by Dennis Ritchie between 1969 and 1973. For many years the so-called "Reference Manual" (a first edition of a book about C) represented the definition of C. In 1983, the American National Standards Institute (ANSI) founded a committee in order to arrive to a modern and exhaustive definition of C. At the end of 1988 the result was the so-called standard "ANSI C". It bases on the original reference manual, because one of the goals of the committee was to assure that most of the already existing programs would remain valid.
The biggest change was that the declaration of a function could then contain a description of its arguments and the syntax of the definition evolved consequently. This change has been very useful, because helps compilers to find errors due to arguments that are incoherent with the expected types of the function.
Other minor changes has come, like, for example, floating point computation can now be also executed with single precision.
Anyway The standard gave a second relevant contribution that was the definition of a library of functions to support C. It included functions to access the operative system (for example to read and write files), to format input and output data, to allocate memory, to manipulate strings and so on. Some so-called headers allow uniform access to functions declarations and to data types.

C is not a "strongly typed" language, in the sense that it does not adopt an absolutely rigid policy of consistency of data types; in its evolution, however, it has strengthened control over the types. For example the original version of C tolerated the interchange between pointers and integer numbers, feature that with the new standard has been removed. Now own declarations and explicit conversions are needed. In spite of this, C keeps its originally philosophy, that programmers know what they are doing, it only asks that their intentions are clearly expressed.

4.2 Eclipse

Figure 4.2. Eclipse Foundation Logo

Eclipse [13] is a multi-language Integrated Development Environment (IDE) comprising a base workspace and an extensible plug-in system for customizing the environment. It is written mostly in Java. It can be used to develop applications in Java and, by means of various plug-ins, other programming languages including C thanks to the so-called "C Development Tools" (CDT).

The Eclipse Platform uses plug-ins to provide all functionality within and on top of the runtime system. A plug-in in Eclipse is a component that provides a certain type of service within the context of the Eclipse workbench. Under this point of view Eclipse can also be considered as an extensible platform for building IDEs. The basic mechanism of extensibility in Eclipse is that new plug-ins can add new processing elements to existing plug-ins.
This plug-in mechanism is used for the most different branches of computer science. In addition to allow the Eclipse Platform to be extended using other programming languages such as C and Python, the plug-in framework permits the Eclipse Platform to work with typesetting languages like LaTeX, networking applications such as telnet and database management systems. The plug-in architecture supports writing any desired extension to the environment, such as for configuration management. Java and CVS support is provided in the Eclipse SDK, with support for other version control systems provided by third-party plug-ins.

Brief History

Industry leaders Borland, IBM, MERANT, QNX Software Systems, Rational Software, Red Hat, SuSE, TogetherSoft and Webgain formed the initial eclipse.org Board of Stewards in November 2001. By the end of 2003, this initial consortium had grown to over 80 members.
On Feb 2, 2004 the Eclipse Board of Stewards announced Eclipse's reorganization into a not-for-profit corporation. Originally a consortium that formed when IBM released the Eclipse Platform into Open Source, Eclipse became an independent body that will drive the platform's evolution to benefit the providers of software development offerings and end-users. All technology and source code provided to and developed by this fast-growing community is made available royalty-free via the Eclipse Public License.

Releases

Since 2006, the Foundation has coordinated an annual Simultaneous Release. Each release includes the Eclipse Platform as well as a number of other Eclipse projects (like, for example, new versions

of some plug-ins).
So far, each Simultaneous Release has occurred on the fourth Wednesday of June.

At the time of the work the latest released version of the platform was Eclipse 4.2 (Juno project), but on 26 June 2013 the version 4.3 (Kepler project) has been released. The version used, anyway, was Eclipse 3.8, which stands between the 3.7 and 4.2 versions and provides bugfixes for Indigo (3.7 version) and adds Java 7 support. The CDT version used, however, is the 8.1.1, which was the latest release at the time.

4.3 Valgrind

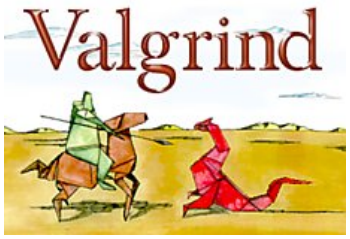

Figure 4.3. Valgrind Logo

Valgrind [16] is a GPL'd system for debugging and profiling Linux programs. With Valgrind's tool suite the user can automatically detect many memory management and threading bugs, avoiding hours of frustrating bug-hunting, making his programs more stable. He can also perform detailed profiling to help speeding up his programs.
The Valgrind distribution currently includes six production-quality tools: a memory error detector, two thread error detectors, a cache and branch-prediction profiler, a call-graph generating cache and branch-prediction profiler, and a heap profiler. It also includes three experimental tools: a heap/stack/global array overrun detector, a second heap profiler that examines how heap blocks are used, and a SimPoint basic block vector generator. It is also possible to use Valgrind to build new tools.

Features

- Valgrind is Open Source / Free Software, and is freely available under the GNU General Public License, version 2;
- Valgrind works with all major Linux distributions and since version 3.5 also on Mac OS X. The latest release is the version 3.8.1 that is the one used for this work.
- It is possible to automatically detect many memory management and threading bugs. This gives the user confidence that programs will be free of many common bugs.
- Profiling can help find bottlenecks in programs and so speed up them.
- Valgrind has been used for very complex programs (with up to 25 milion lines of code), for any type of software (from games to scientific programs, to business software) and all over the world (in over 30 countries)

- Valgrind works with programs written in any language, but most of all it is aimed at programs written in C and C++, because programs written in these languages tend to have the most bugs. However it can, for example, be also used to debug and profile systems written in a mixture of languages like Java, Perl, Python, Fortran and many others.
- Valgrind gives 100% coverage of user-space code, even within system libraries. You can even use Valgrind on programs for which you don't have the source code.

Tool Memcheck

The default and most used tool is Memcheck. It is also the tool used for the debugging of this work.
Memcheck inserts extra instrumentation code around almost all instructions, which keeps track of the validity (all unallocated memory starts as invalid or "undefined", until it is initialized into a deterministic state, possibly from other memory) and addressability (whether the memory address in question points to an allocated, non-freed memory block), stored in the so-called V bits and A bits, respectively. As data is moved around or manipulated, the instrumentation code keeps track of the A and V bits so they are always correct on a single-bit level.
In addition, Memcheck replaces the standard C memory allocator with its own implementation, which also includes memory guards around all allocated blocks (with the A bits set to "invalid"). This feature enables Memcheck to detect off-by-one errors where a program reads or writes outside an allocated block by a small amount. The problems Memcheck can detect and warn about include the following:

- Use of uninitialized memory;
- Reading/writing memory after it has been freed;
- Reading/writing off the end of malloc'd blocks;
- Memory leaks.

The price of this is lost performance. Programs running under Memcheck usually run much slower than running outside Valgrind and use more memory (there is a memory penalty per-allocation). Thus, few developers run their code under Memcheck (or any other Valgrind tool) all the time. They most commonly use such tools either to trace down some specific bug, or to verify there are no latent bugs (of the kind Memcheck can detect) in the code.

The command used to launch Valgrind was the following:

```
valgrind --tool=memcheck --leak-check=yes --max-stackframe=26118672 ./a.out
```

where –tool=memcheck indicates which tool of Valgrind it is going to be used; the –leak-check option turns on the detailed memory leak detector; the –max-stackframe increases the size of the stack to the desired value; the last parameter is the name of the executable program to analyse.

Here below there is an example of what this tool of Valgrind can return:

In figure 4.4 it is possible to see a kind of leak of memory. This happens because after having allocated dynamic memory in the C code, it has not been deallocated. And this forgetfulness has happened two times: for the allocation made in the method recursive and for that made in readInput. Valgrind also informs of how many bytes of memory has been lost.

```
==3033==
==3033== HEAP SUMMARY:
==3033==     in use at exit: 472 bytes in 2 blocks
==3033==   total heap usage: 9,697,113 allocs, 9,697,111 frees, 1,980,010,702 by
tes allocated
==3033==
==3033== 72 bytes in 1 blocks are definitely lost in loss record 1 of 2
==3033==    at 0x4C2C73C: malloc (vg_replace_malloc.c:270)
==3033==    by 0x445130: recursive8 (main.c:12510)
==3033==    by 0x4012B0: main (main.c:527)
==3033==
==3033== 400 bytes in 1 blocks are definitely lost in loss record 2 of 2
==3033==    at 0x4C2C73C: malloc (vg_replace_malloc.c:270)
==3033==    by 0x4464A1: readInput (readInput.c:42)
==3033==    by 0x400C4A: main (main.c:226)
==3033==
==3033== LEAK SUMMARY:
==3033==    definitely lost: 472 bytes in 2 blocks
==3033==    indirectly lost: 0 bytes in 0 blocks
==3033==      possibly lost: 0 bytes in 0 blocks
==3033==    still reachable: 0 bytes in 0 blocks
==3033==         suppressed: 0 bytes in 0 blocks
==3033==
==3033== For counts of detected and suppressed errors, rerun with: -v
==3033== ERROR SUMMARY: 2 errors from 2 contexts (suppressed: 0 from 0)
```

Figure 4.4. Example of final report after the use of memcheck with memory leaks

```
==2847==
==2847== HEAP SUMMARY:
==2847==     in use at exit: 0 bytes in 0 blocks
==2847==   total heap usage: 10,078,716 allocs, 10,078,716 frees, 1,980,042,955
bytes allocated
==2847==
==2847== All heap blocks were freed -- no leaks are possible
==2847==
==2847== For counts of detected and suppressed errors, rerun with: -v
==2847== ERROR SUMMARY: 0 errors from 0 contexts (suppressed: 0 from 0)
```

Figure 4.5. Example of final report after the use of memcheck with no memory leaks

In figure 4.5, otherwise, it is possible to see what happens if all these forgetfulness have been corrected. The program informs that all heap blocks have been freed and so no memory leaks are possible.

4.4 Gnuplot

Gnuplot [14] is a portable command-line driven graphing utility for Linux, OS/2, MS Windows, OSX, VMS, and many other platforms. The source code is copyrighted but freely distributed. It was originally created to allow scientists and students to visualize mathematical functions and data interactively, but has grown to support many non-interactive uses such as web scripting. It is also used as a plotting engine by third-party applications like Octave. Gnuplot has been supported and

under active development since 1986.

Gnuplot supports many types of plots in either 2D and 3D. It can draw using lines, points, boxes, contours, vector fields, surfaces, and various associated text. It also supports various specialized plot types.

Gnuplot supports many different types of output: interactive screen terminals (with mouse and hotkey input), direct output to pen plotters or modern printers, and output to many file formats (eps, emf, fig, jpeg, LaTeX, pdf, png, postscript, ...). Gnuplot is easily extensible to include new output modes. Recent additions include interactive terminals based on wxWidgets (usable on multiple platforms), and Qt. Mouseable plots embedded in web pages can be generated using the svg or HTML5 canvas terminal drivers.

The used version is the 4.6.1, which was the latest one at the beginning of the work. At the moment versions till 4.6.3 have been released.

The importance of Gnuplot in this work has been that of "logical debugging". It has been used to check whether the output of the algorithms were correct or not, printing on the screen the plot representing the resulting packing of rectangular items. To have a better debugging it has been possible to set the length of the axes that must be visible in a single window in order to represent a single bin.
To represent items in the bin, objects Rectangles of the program has been used. To plot them it was enough to set the coordinates of the bottom left and of the upper right corners. They also have been coloured with the command "fillcolor" to avoid confusing them with the empty spaces in the bin, thing that could have happened if the rectangles would have been filled with white as the background of the bin.
Here below there is an example of a file that creates a plot with Gnuplot.

```
set xrange [0:537]
set yrange [0:244]
set object rect from 504,178 to 534,244 fillcolor rgb "#87cefa"
set object rect from 458,0 to 537,89 fillcolor rgb "#87cefa"
set object rect from 458,89 to 537,178 fillcolor rgb "#87cefa"
set object rect from 474,178 to 504,244 fillcolor rgb "#87cefa"
set object rect from 400,90 to 458,178 fillcolor rgb "#87cefa"
set object rect from 444,178 to 474,244 fillcolor rgb "#87cefa"
set object rect from 347,0 to 458,45 fillcolor rgb "#87cefa"
set object rect from 347,45 to 458,90 fillcolor rgb "#87cefa"
set object rect from 414,178 to 444,244 fillcolor rgb "#87cefa"
set object rect from 120,0 to 236,90 fillcolor rgb "#87cefa"
set object rect from 236,0 to 347,45 fillcolor rgb "#87cefa"
set object rect from 236,45 to 347,90 fillcolor rgb "#87cefa"
set object rect from 212,90 to 400,134 fillcolor rgb "#87cefa"
set object rect from 212,134 to 400,178 fillcolor rgb "#87cefa"
set object rect from 384,178 to 414,244 fillcolor rgb "#87cefa"
set object rect from 0,66 to 119,90 fillcolor rgb "#87cefa"
set object rect from 159,90 to 212,147 fillcolor rgb "#87cefa"
set object rect from 168,147 to 210,178 fillcolor rgb "#87cefa"
set object rect from 256,178 to 384,244 fillcolor rgb "#87cefa"
set object rect from 90,0 to 120,66 fillcolor rgb "#87cefa"
set object rect from 106,90 to 159,147 fillcolor rgb "#87cefa"
set object rect from 126,147 to 168,178 fillcolor rgb "#87cefa"
set object rect from 0,178 to 128,244 fillcolor rgb "#87cefa"
```

```
set object rect from 128,178 to 256,244 fillcolor rgb "#87cefa"
set object rect from 60,0 to 90,66 fillcolor rgb "#87cefa"
set object rect from 0,90 to 53,147 fillcolor rgb "#87cefa"
set object rect from 53,90 to 106,147 fillcolor rgb "#87cefa"
set object rect from 84,147 to 126,178 fillcolor rgb "#87cefa"
set object rect from 0,0 to 30,66 fillcolor rgb "#87cefa"
set object rect from 30,0 to 60,66 fillcolor rgb "#87cefa"
set object rect from 0,147 to 42,178 fillcolor rgb "#87cefa"
set object rect from 42,147 to 84,178 fillcolor rgb "#87cefa"
```

Code 4.1. Gnuplot input file to plot a packing

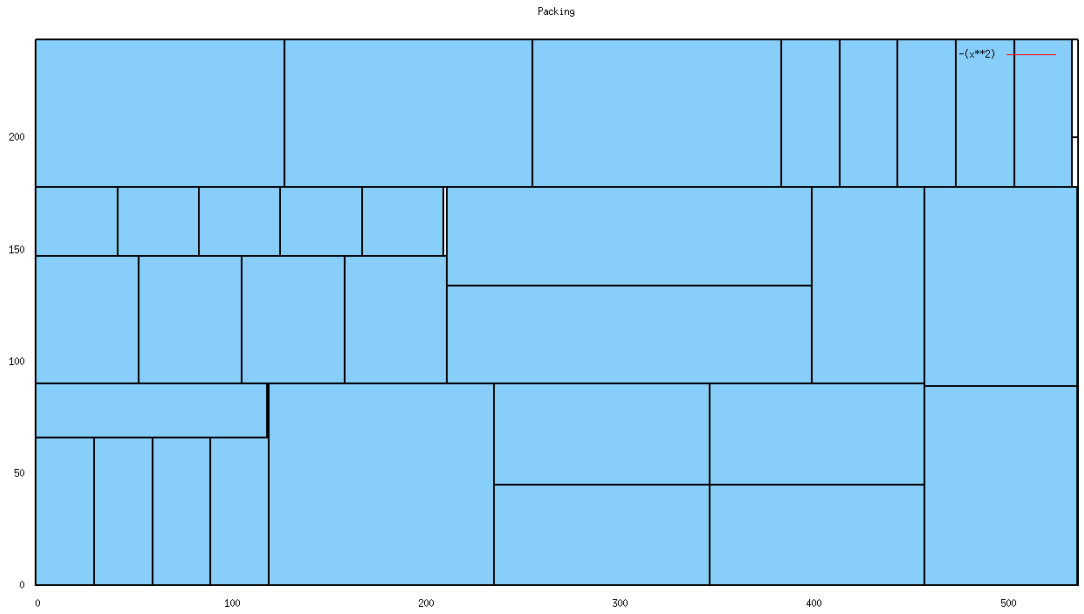

Figure 4.6. The resulting plot of the packing

Chapter 5

Implementation

5.1 Input File

For the implementation of the problem it has been decided to use as input the same kind of files that are used in literature. So they have a very well defined structure, which will be described here.

```
20
40 70
31 43 4 500
30 41 2 480
29 39 4 460
28 38 4 440
27 37 3 420
26 36 4 410
25 35 3 400
24 34 4 380
33 23 4 360
22 32 3 340
31 21 3 320
29 18 3 300
17 27 2 280
15 24 2 240
16 25 4 260
15 24 1 240
23 14 4 220
21 12 3 180
19 11 4 160
 9 17 1 140
```

Code 5.1. Example of an input file

The structure of the file must be the following:

- on the first row there must be the number of the types of rectangles that are listed in the following lines;
- on the second row there must be the values of the dimensions of the bin. The first number represents the width and the second one stands for the height;
- all the next lines have the same structure and they are exactly the quantity present in the first line. For each of these rows the first two numbers represent respectively width and

height of the rectangle, the third number stands for the number of rectangles that have that shape and the fourth is the profit that each rectangle with that shape has.

In the example, moreover, rectangles are ordered by profit, but this is not the rule: to have a valid input file it is enough to follow the three points described here above, with no limitations on the order of rectangles.

5.2 Organization of Files

To make the implementation as readable as possible, the work has been structured following the idea of modularity. So different files have been created for different algorithms and also in the same file, functions have been programmed using sub-functions to increase readability and because, if a sub-function is used more than one time in the main function and there is the desire to modify it, in this manner it is possible to modify only one time the code to change it everywhere.
The files that have been implemented are the following:

- **structures.h**: here all the structs that are needed to represent objects present in this kind of problem are implemented: for example some of them are Sheet, Rectangle, Packing, Container;
- **auxiliaryFunctions.h**: header file where all the functions present in auxiliaryFunctions.c are listed;
- **auxiliaryFunctions.c**: here all the functions of its header file are implemented. These functions are not sub-functions of algorithms to solve G2KP and G2BPP, but they are external. Here there are, for example, the method to parse the input and a method to order rectangles by profit over weight ratio;
- **functionsG2KP.h**: header file where all the functions present in functionsG2KP.c are listed;
- **functionsG2KP.c**: here all the functions of its header file are implemented. These are all the functions related to G2KP, so those necessary to compute the exact solution of G2KP and the procedure recursive, plus all the related sub-functions;
- **functionsG2BPP.h**: header file where all the functions present in functionsG2BPP.c are listed. Here also some struct and some types necessary to use LB2 are defined;
- **functionsG2BPP.c**: here all the functions of its header file are implemented. These are all the functions related to G2BPP, therefore that used to compute the exact solution of G2BPP and all its sub-functions, like heuristic functions and branch and bound method;
- **main.c**: in this file just the main of the program has been implemented where can be decided to perform G2KP or G2BPP.

In addition to these files other two file called 3dbpp.h and 3dbpp.c [8], which have been implemented by David Pisinger [15], are used. These are necessary, because the implementation of LB2 is contained within them.

5.3 Structures

In these section all the implemented structures are described.
To understand why some data types are used instead of others it is important to briefly explain what is a struct. A definition of a struct is the same of defining a variable, because both are defining a limited memory space. The difference is that a struct has inside some fields. These fields are not necessarily all of the same size (in terms of memory space). If all the fields have the same size, then the space used by the structure is the sum of them, otherwise it is a bit different. The size of a struct must be a multiple of the biggest of its fields, so the sum of all of them is computed and, if the result is not perfectly a multiple of the biggest element, a padding is used to reach the next multiple.
Hereafter the implemented structures are presented:

- **Sheet**: it represents the bin where items must be packed. It has only two fields of integer, which represent the dimensions, width and height, respectively.
- **Rectangle**: it represents each item that must be packed. As it is possible to see from the code, it has five fields that represent respectively width and height of the rectangle, the total number of rectangles that have that shape, the profit of that kind of rectangles and the index, that is the position that it has in the input file from where it has been retrieved.

```
/**
 *      Structure that represents rectangles
 */
typedef struct rectangle {

        int width;      //width of the rectangle
        int height;     //height of the rectangle
        int number;     //number of rectangles with this size
        int profit;     //profit of this kind of rectangles
        int index;      //index of the rectangle

} Rectangle;
```

Code 5.2. Example of code for a structure

- **Shelf**: it represents a shelf in the heuristic algorithms. It can be used either as horizontal or vertical shelf thanks to its four fields: the first two determines the width or the height of the shelf, used respectively in the case if the shelf is vertical or horizontal. In the case of vertical shelf, then, to determine if any other packing can still be packed the residualHeight field is used, while for the horizontal there is the residualWidth field. All these fields are integer.
- **Bin**: it represents the bin in the implementation with shelves inside in the heuristic algorithms. It has four fields: a pointer to an array of shelves, where they are saved, an integer that gives the number of shelves (the length of the array) and other two integers that represent residual height and residual width in the bin.
- **Packing**: it represents each possible packing of items in the bin. This is the most critical structure in terms of memory space used, because the number of packings that might be saved at the same time could be very high. So, above all in this case, it has been tried to minimize the space necessary for each field. Moreover, for each packing has been necessary to save some information that are used at the end of the algorithm to retrieve the coordinates of the position of items in the packing. So six fields have been implemented:

  - unsigned char* pack: pointer to an array that represents the packing: each number represents the total number of rectangles with that shape that are in the packing.
  - unsigned short k: index of the array $\mathcal{W}$ or $\mathcal{H}$ where to cut, which is needed to retrieve the position of the first of the two packing, between which the cut has been performed, in the matrix where all packings are saved. If it is a "base case" (which will be better defined later), so there is only field element in the packing and can not be cut, it is equal to 0.
  - unsigned short q: index of the other packing that brings to the feasible Packing through the cut obtained through the pairwise sum; like the previous one if we are in a base case, it assumes the value 0.
  - char direction: it can assume three values: 'W' if there cut is vertical, 'H' if the cut is horizontal, '0' if no cut can be performed
  - unsigned char positionK: index of the position of the packing in the cell of the matrix found thanks to the field k.
  - unsigned char positionQ: index of the position of the packing in the cell of the matrix found thanks to the field q.

  Hence, the total memory space occupied by this struct is the following: 8 bytes for the pointer, if the architecture of the computer is 64-bit, 1 byte for each of the three chars and 2 bytes for each of the two shorts. So the sum is 15 bytes, but, because of the pointer, the struct occupies 16 bytes, with 1 byte of padding.

- **Container**: it represents a set of packings. It is used in the matrix where all packings are saved to represent each cell of the matrix. It has only two fields, which are a pointer to the array of packing and the length of this array.
- **Node**: it represent a node of the tree used to retrieve the positions of items in the feasible packing, starting from the root and descending to the leaves. There are seven fields. The first is a pointer to the next packing from where another cut (and consequently 2 sub-packings) can be retrieved. Then w and h are the indexes of the matrix where this packing is saved. x and y are the searched coordinates of the bottom-left vertex. hDx and wDx are the indexes used to set the right child. They are necessary to determine position taking into account also cuts previously made.
- **RectPosition**: it represents the position of each item in the feasible packing. It has three fields: the index of the item, the coordinate x of the position and the coordinate y of the position, all of them are integers.

5.4 Auxiliary Functions

In the file of the auxiliary functions just three methods are present (two if we consider that mergeSort and merge are used together). We will focus only on the method readInput, because mergeSort has the classic recursive implementation.

```
int readInput(char* inputFileName, Sheet* sheet, Rectangle** rects, int* numRects);
void mergeSortRectangles(Rectangle** rects, int first, int last);
void mergeRectangles(Rectangle** a, int first, int center, int last);
```

Code 5.3. Functions present in auxiliaryFunctions.h

readInput, first of all, tries to open the file just in reading mode with the function "fopen" and, if there is any problem, it terminates with an error. Then with some "scanf" information are retrieved from the input file and saved into the correct fields of a Rectangle and each of these is placed into an array.
It has been noticed, however, that some of the input files, even if they were from literature, had some of the types of rectangles that were equal to each other, despite being written in different rows. So in this function a control has been introduced to check if this happens and, in case, to merge the two types into one.
This method returns 0 if the reading was successful, 1 otherwise, and returns also the array of Rectangles and the length of this array, thanks to the fact that it received their addresses as input.

5.5 Functions For G2KP

The list of functions present in functionsG2KP.h has been divided in two parts: in the first part there are all those related to the function exactG2KP, while in the second there are all those related to the procedure recursive.

For the first part first of all we must consider the function exactG2KP. It has been described in detail above and the implementation is really simple, thanks to the modularity of the code. The only fact that is particularly interesting to notice is that this function returns not only the list of the coordinates of the items in the feasible packing, but also its length and the optimal solution value, thanks to the fact that the input variables optimalSolution and numPackedRects are addressed and so these values are updated directly there and they do not need a return statement.

```
RectPosition* exactG2KP (int maxWidth, int maxHeight, int numRects, Rectangle* rects,
                         int* optimalSolution, int* numPackedRects);

int heuristicG2KP(int maxHeight, int maxWidth, int numRects, Rectangle* rects);
int mixedStrategyV_G2KP(Rectangle* randomOrder, int numElem, int maxHeight, int maxWidth);
int mixedStrategyH_G2KP(Rectangle* randomOrder, int numElem, int maxHeight, int maxWidth);
void findRandomOrder(Rectangle** rectList, int totRects);
void mergeSortRectanglesByNumber(Rectangle** rects, int first, int last);
void mergeRectanglesByNumber(Rectangle** a, int first, int center, int last);
```

Code 5.4. Functions present in functionsG2KP.h part 1

Also the heuristicG2KP implementation is quite simple. An auxiliary array of Rectangles *randomOrder* has been introduced where all the randomizations of the items are saved. For the first time, where, as mentioned before, all elements should be ordered by profit over weight ratio, it is enough to copy elements from array rects, because they should be already ordered in this way. Then, considering the inner loop, has been decided to proceed in the way described in chapter 3, because in this manner the 2 strategies in the outer cycle are computed only for set of items that potentially can improve the best solution value saved at the time.
To randomize the order of items has been used the function srand that seeds the random number generator with the input number. The input number is the current time, so it gives a good randomization. Each random number that is generated is saved in the field "number" of the Rectangles, because they are no more divided by type, so each Rectangle would have value 1 in that field. Then items are ordered with the mergesort by decreasing order of field number.
Let us consider now the mixedStrategy functions. We will discuss only the vertical one, because the horizontal one is the complementary.
First of all this function is characterized by a big external loop that is performed on each Rectangle

to find the correct Shelf where to pack it. This search is performed starting from the first Shelf and then trying the following. Shelves with even index are considered as vertical (respectively: horizontal), those with odd index are considered as horizontal (vertical) (the first index is zero, so it will start with a vertical one). To check if an item can be packed into a vertical (horizontal) shelf two controls are necessary: its width (height) should be lower than the width (height) of the shelf and its height (width) should be lower than the residualHeight (residualWidth) of the shelf. If these two controls are passed, the residualHeight (residualWidth) is updated decreasing it by the height of the item. If we arrive to consider the index of a shelf that has not been opened yet, we try to open it (vertical or horizontal, depending from the index). The control to perform to verify if it is possible is to check if height and width of the current item are lower than the residual height and residual width of the bin. These two variables have been initialised at the beginning of this function with the values of the dimensions of the bin and each time a new shelf is opened (passing the last control mentioned) they are updated. If the shelf is vertical the residual width is decreased by the item width, otherwise the residual height is decreased by the item height. Moreover, when a new vertical (horizontal) shelf is created, also the width (height) and the residualHeight (residualWidth) of the shelf are set as respectively the width (height) of the current item and the difference between the residual height (width) of the bin and the height (width) of the current item. The method returns 1 if at the end a packing for each item is found, otherwise it returns 0 if it happens that the conditions for opening a new shelf can not be met.

Now the procedure recursive and all its sub-functions will be considered.

```
RectPosition* recursive (int z0, int maxWidth, int maxHeight, int numRects,
                         Rectangle* rects, int* numPackedRects);

void saveDataAndQuantities(int* value, int* number, int** arrayData, int** arrayQuantity,
                           int* size, int* counter);
int* linearCombination(int maximum, int** arrayData, int** arrayQuantities,
                       int counterInput, int* counterOutput);
Container pairwiseSum(Container a, Container b, int numRects, Rectangle* rects, int q,
                      int cut, char direction);
Container unionPackingsUB(Container a, Container b, int numRects, Rectangle* rects,
                          int maxWidth, int maxHeight, int width, int height, int isLastW,
                          int isLastH, int z0);
void mergeSortPackings(Container *pack, int first, int last, int numRects);
void mergeSort(Packing** a, int first, int last, int numRects);
void mergePackings(Packing** a, int first, int center, int last, int numRects);
int* resizeArrayInt(int* array, int* oldSize, int newSize);
Packing* resizeArrayPacking(Packing* array, int* oldSize, int newSize);
Node* resizeArrayNode(Node* array, int* oldSize, int newSize);
RectPosition* resizeArrayRectPos(RectPosition* array, int* oldSize, int newSize);
int minVal(int a, int b);
int maxVal(int a, int b);
int findCriticalObject(int profit_j, int area_j, int maxNum_j, int* residualCapacity,
                       int previousUpperBound, int* criticalObjectFound);
int comparePackings(unsigned char** a, unsigned char** b, int size);
```

Code 5.5. Functions present in functionsG2KP.h part 2

It is possible to consider the procedure recursive as divided into three parts: a first part where $\mathcal{W}$ and $\mathcal{H}$ are computed, a second part where all possible packings are generated and at the end the feasible packing, if exists, is found and a third part where the positions of the items in the feasible packing are retrieved.
In the first part the computation of $\mathcal{W}$ and $\mathcal{H}$ is pretty simple in the method recursive, because all the work is performed by the two auxiliary functions saveDataAndQuantities and linearCombination.
To compute $\mathcal{W}$ saveDataAndQuantities saves into two different arrays all the possible different

widths of the items and the number of items that have each width. Then it does the same with heights to compute $\mathcal{H}$. To implement this, because of the small size of input, for each element a simple exhaustive search has been used to find the position where to insert it.
LinearCombination uses the two arrays computed with saveDataAndQuantities to find $\mathcal{W}$ (or $\mathcal{H}$ respectively). This algorithm will be explained only for the widths, because for the heights it is the complementary. First of all an array A is created with length equal to the width of the sheet plus one and it is initialised with all zeros except in the position of index 0 where it is initialised with 1. This happens because each cell of the array is considered as a possible dimension and in each cell is saved the value of the iteration when that dimension is found or zero if it has still not been found at that moment. For each element in the input arrays (j) each element of A (k) is checked and considered only if it has been computed on the previous iteration. In this case, considering the quantity (i) that corresponds to the current element, the new sizes are computed as the linear combination between the current size k and the dimension corresponding to the index j weighted by the quantities i. To explain it more clearly, it has been computed with the following formula:

```
//Iterate on quantities
for (i = 0; i <= (*arrayQuantities)[j-1]; i++) {

        //Compute the value
        int value = k + i*(*arrayData)[j-1];

}
```

Code 5.6. Formula for linear combination

After this computation the found value is checked to be lower than the length of A and, if this is the case, the number of the computation is saved into the cell corresponding to the computed size. At the end of the algorithm all the found sizes are saved into another array that is returned with its size.

Going back to procedure recursive, starting the second part we have $\mathcal{W}$, $\mathcal{H}$ and their respective sizes t and s. The second part is characterized by a double loop on all values of $\mathcal{W}$ and $\mathcal{H}$. At the end of each iteration a set of Packings is saved into the correct cell of a matrix $t \times s$ of Containers, called F.
For the implementation of this part of the algorithm, because the biggest problems were coming from the memory space needed to save all the possible packing, we have decided to change a little the algorithm to reduce the number of packings that were saved into the matrix. To obtain this result the control on the UB of the profit for each packing is performed each time a union must be executed.
In this part some other variables of type Container have been introduced, because each of them represents one set of packing, in the pseudo-code represented with $F(x, y, z_0)$. Moreover, when is computed the Container *temp* as solution of the pairwise sum, it is ordered with a mergeSort by lexicographical order before performing the union with the Container S containing the packing that could be insert in the current cell of the matrix, so both S and *temp* are ordered and it is much easier to perform the union.
Considering the pairwise sum, to call this function from the recursive procedure, it is necessary to pass as its input the values of the cut, so the index q found, his complementary index called k (that is the $\bar{q}$ of the pseudo-code), which is the real position where the cut is performed, and the direction of the cut, that could be horizontal ('H') or vertical ('W'). In this method then a double cycle on all the elements of the two Containers passed as input is performed to find the pairwise sum and, when a packing belonging to it is computed, all the features of the cut are saved as well. Here there is a fragment of code where it can be seen:

```
//Cycle on all the elements of a
for (l = 0; l < a.size; l++) {

        //Cycle on all the elements of b
        for (k = 0; k < b.size; k++) {


                //Cycle on all the components of the feasable packings
                for (i = 0; i < numRects; i++) {

                        int value = a.f[l].pack[i] + b.f[k].pack[i];
                        sum.f[l*b.size + k].pack[i] = minVal(value,rects[i].number);
                        sum.f[l*b.size + k].positionK = k;
                        sum.f[l*b.size + k].positionQ = l;
                        sum.f[l*b.size + k].direction = direction;
                        sum.f[l*b.size + k].q = q;
                        sum.f[l*b.size + k].k = cut;
                }

        }

}
```

Code 5.7. Fragment of the function to compute the pairwise sum

In unionPackingsUB (this is the name of the function were both union and control of the UB on the profit of packings are performed) the packings of the two Containers to merge are lexicographically compared. The first container is always S and it is considered already checked and ordered. If they are equal the pointer to the packings of the second Container is moved to the next packing and nothing else is performed. Otherwise if they are different the smallest is considered. If it belongs to S it can be directly inserted into the new Container, otherwise it must be checked if it satisfies the control on the UB and only after this it is inserted. Then if all the elements of one of the Containers have been checked, the other are tried to be inserted into the new Container. If the remaining Container is the first one they can directly be inserted, otherwise the control of UB must be performed also in this case, after having checked that the current element was not just inserted into the Container (it could happen because the second Container, although it is ordered, may contain duplicates).
Now let us take a closer look on the implementation of the UB by Martello and Toth. This implementation follows the general case described above, but we have had the necessity to manage some other particular cases:

- if the critical object is not found it means that all the items can be packed and so the UB is simply the sum of their profits;
- the standard situations occurs when the critical item is neither the first nor the last item in this list;
- if the critical object is the last element of the array, the UB is computed as the maximum between Dantzig UB and the UB computed by using the entire critical object and a part of the object before;
- if the critical object is the first position of the array, the UB is computed as the maximum between the Dantzig UB and the UB computed by using part of the object after the critical object instead of the critical one.

Considering again the procedure recursive, let us give now a deeper look on the moment when an item perfectly matches the dimension of the current sub-rectangle of the sheet, which has been

called "base case". It is called in this manner because no other cuts are performed to insert it. When it is inserted, all the informations about cuts are set to 0, including also the direction where the char '0' is inserted. This is important so, when we are trying to retrieve the position of items, we know when to stop, because we have found a packing of a single item.
Going forward with the procedure recursive, there is the last part of the algorithm, that is checking if a packing is maximal or not. To implement this a simple observation is done: a packing is not maximal if there is at least another packing that has all the components greater or equal compared to the packing that is being checked. So in the implementation each packing is compared with all the others until a component of the second one is smaller than that of the first one. If for a packing this can not be found, this means that the packing that is being checked is not maximal and can be discarded.
After this last control, memory is freed and S is resized and saved in F[i][j], that is the cell of the matrix considered in that iteration. In F[t-1][s-1], if exists, a feasible packing is present, otherwise the algorithm returns NULL.

Now we enter in the third part of the algorithm, that to retrieve all the positions of the elements in the packing. The binary tree used to retrieve all the elements is implemented using an array where all the left children are in the positions with indexes $(2 \times i) + 1$, where i is the index of the father, and all the right children in the positions with indexes $(2 \times i) + 2$. The root is initialised with all data recovered from the feasible packing in F[t-1][s-1], as it can be seen in the listing here below.

```
//Initialize root of the tree
all[0].w = t-1;
all[0].h = s-1;
all[0].x = 0;
all[0].y = 0;
all[0].wDx = 0;
all[0].hDx = 0;
all[0].p = (Packing*) malloc(1*sizeof(Packing));
all[0].p->direction = F[t-1][s-1].f[0].direction;
all[0].p->k = F[t-1][s-1].f[0].k;
all[0].p->q = F[t-1][s-1].f[0].q;
all[0].p->positionK = F[t-1][s-1].f[0].positionK;
all[0].p->positionQ = F[t-1][s-1].f[0].positionQ;
all[0].p->pack = (unsigned char*) malloc(numRects*sizeof(unsigned char));
for (j = 0; j < numRects; j++) {
        all[0].p->pack[j] = F[t-1][s-1].f[0].pack[j];
}
```

Code 5.8. Initialisation of the root of the binary tree

Some other variables are initialised at the beginning of this part, like the total number *totRects* of Rectangles present in the solution and the number of leaves *numLeaves* found in the solution. There is also a system to manage the size of the array all by taking trace of his size and increasing it threefold each time that it is necessary. This kind of management is crucial because, as the tree is not balanced, in the worst case it could need a very long array even for representing just few nodes. To recognize each cell that does not belong to the tree (empty nodes), their field x has been set to -1.
The algorithm then iterates on each element of the array of Nodes *all* until *numLeaves* is equal to *totRects*. For each iteration of the cycle four possibilities can happen:

1. The node is empty. Its left and right children are set as empty nodes as well and then the iteration continues on the next node.

2. The node is a leaf, because the direction is '0'. Here in fields x and y the true coordinates are found and the index of the item is recovered by the position of the item in the packing.

Then the fields x of its two children are set to empty and the variable *numLeaves* is increased by 1.

3. The cut direction of the node is vertical. Its left and right children must be updated. Considering first the left child, its fields x, y, wDx, hDx, and h assume the same value of the father, while w assumes the value all[i].p->k. The values of the packing are, in the general case, recovered from the matrix F and more precisely from the cell with indexes all[i].p->k and all[i].h in the position all[i].p->positionK of its array of packings.
   In the right child things are a little more complicated: field w assumes the value all[i].p->q, while h the same of the father. Also y assumes the same of the father, while x assumes the value $\mathcal{W}$[all[i].p->k] + $\mathcal{W}$[all[i].wDx] if all[i].wDx > 0, otherwise just the value $\mathcal{W}$[all[i].p->k]. Finally hDx assumes the same value of the father and wDx assumes the value all[i].p->k. The values of the packing are, in general, recovered from F[all[i].p->q][all[i].h] and they can be found in the position all[i].p->positionQ of its array of packings.

4. The cut direction of the node is horizontal. It is very similar to the previous case, here are the few differences: in the left child x assumes the value of the father, while y becomes all[i].p->k. The packing is now retrieved from F[all[i].w][all[i].p->k] always in the position all[i].p->positionK. In the right child w, x and wDx assume the same values of the father, h assume the value all[i].p->q, hDx the value all[i].p->k and y the value $\mathcal{H}$[all[i].p->k] + $\mathcal{H}$[all[i].hDx] if all[i].hDx > 0, otherwise just the value $\mathcal{H}$[all[i].p->k]. The packing is recovered from F[all[i].w][all[i].p->q], always in the position all[i].p->positionQ.

The algorithm terminates returning the array of RectPositions where the coordinates have been saved and his size and freeing the binary tree *all* and the matrix F.

5.6 Functions For G2BPP

Eventually the functions developed in functionsG2BPP.c are considered. To use these functions, in particular to use the function exactG2BPP, it is necessary to include the functions for G2KP as well, because of the use of the procedure recursive.

```
RectPosition* exactG2BPP(int maxHeight, int maxWidth, int numRects, Rectangle* rects,
                         unsigned char* optSolution, int* numPackedItems);

void mergeSortRectanglesByArea(Rectangle** rects, int first, int last);
void mergeRectanglesByArea(Rectangle** a, int first, int center, int last);


unsigned char branchAndBound(int currElem, unsigned char currZ, short* currSol,
                             Rectangle* rects, int totRects, int binHeight, int binWidth,
                             boolean* terminate, short** solution);

int insertCurrent(int bin, int currElem, short* currSol, Rectangle* rects, int binHeight,
                  int binWidth);
int heuristicG2BPP(int maxHeight, int maxWidth, int numRects, Rectangle* rects);
int mixedStrategyV_G2BPP(Rectangle* randomOrder, int numElem, int maxHeight, int maxWidth,
                         int maxBins);
int mixedStrategyH_G2BPP(Rectangle* randomOrder, int numElem, int maxHeight, int maxWidth,
                         int maxBins);
```

Code 5.9. Functions present in functionsG2BPP.h

Initially, in exactG2BPP, before the branch and bound, only the creation of an array of all different items, no more divided by type of Rectangles, and the initialization of an array where to save the solution of the branch and bound (the index of the bin where each element is packed) are performed. After the execution of branch and bound, for each bin all the items that must be packed there, are inserted into an array, divided by type of Rectangles and with profit set to 1; this array is then passed as input of a recursive procedure that finds their exact packing into one bin.

For branch and bound, being a recursive procedure, to stop the search when the optimal solution is found, it has been necessary to use a boolean variable that pointed out if the optimal solution was reached or not. To be sure that its value would not change calling the method recursively its address has been used as input parameter. The same has been done with the array representing the solution, while the best solution found at the moment is saved as global variable.
The implementation is simple: first there are two controls that makes the function return, which are the control on the boolean flag described above and the control if a leaf of the tree is reached, finding then a better solution. If these controls fail, the current item is tried to be inserted in each bin that has already been opened. If it succeed branch and bound is called on the next item, otherwise, another bin is opened if the solution could be still potentially improved and then the branch and bound on the next item is performed.

The method for the insertion of an item in a bin follows exactly what has been described in section 3.5 and so does the heuristic method with what is described in 3.4.2, so we will consider now the mixed strategy.
The mixed strategy is exactly the same algorithm used for G2KP, but with one big difference: here packings are always possible, because if it is not possible to insert all the items into one bin, another bin can be opened. So the differences in the algorithm are that for each item first there is a cycle that checks each opened bin and then another cycle that checks all the opened shelves in that bin. It is possible to notice that inside the cycle that checks the bins, the algorithm is almost the same of that for G2KP. The only difference is that there is a flag that allows to break also the loop on the bins when a shelf is found. At the end if it is not possible to insert an item in any bin, another bin is opened and the item is inserted there. Anyway, to avoid useless iteration, a control when a new bin is opened is performed: if with the opened bin the number of bin opened reaches a value *maxBin* passed as input parameter, containing the value of the best solution found at the moment in the heuristic function, it means that there will not be improvement in the solution of the heuristic method and so the function returns.

5.7 Outputs

In Code 5.10 it is possible to see how an output of a G2BPP is. For each item its type (which is the position in the input file), index (which increases for each element of the same type), profit, dimension, the bin where it is packed and the coordinates of the bottom-left corner are specified. At the end we also report the value of the optimal solution and the time needed to perform the computation.
For a G2KP, on the other hand, the output is the same, with the differences that the number of the bin is not present, because there is only one bin, and the optimal solution is no more the number of bins used, but the best profit found, for which there is a feasible packing.

```
Type: 10, Index: 0, Profit: 759, Dimension: 23 x 33, Bin: 0, Position: 43 x 0
Type: 18, Index: 0, Profit: 1333, Dimension: 43 x 31, Bin: 0, Position: 0 x 0
Type: 0, Index: 0, Profit: 153, Dimension: 17 x 9, Bin: 0, Position: 0 x 31
Type: 17, Index: 0, Profit: 1230, Dimension: 41 x 30, Bin: 1, Position: 0 x 0
Type: 10, Index: 1, Profit: 759, Dimension: 23 x 33, Bin: 1, Position: 41 x 0
Type: 16, Index: 0, Profit: 1131, Dimension: 39 x 29, Bin: 2, Position: 0 x 0
Type: 10, Index: 2, Profit: 759, Dimension: 23 x 33, Bin: 2, Position: 39 x 0
Type: 15, Index: 0, Profit: 1064, Dimension: 38 x 28, Bin: 3, Position: 0 x 0
Type: 9, Index: 0, Profit: 704, Dimension: 32 x 22, Bin: 3, Position: 38 x 0
Type: 6, Index: 0, Profit: 459, Dimension: 27 x 17, Bin: 3, Position: 38 x 22
Type: 14, Index: 0, Profit: 999, Dimension: 37 x 27, Bin: 4, Position: 0 x 0
Type: 9, Index: 1, Profit: 704, Dimension: 32 x 22, Bin: 4, Position: 37 x 0
Type: 6, Index: 1, Profit: 459, Dimension: 27 x 17, Bin: 4, Position: 37 x 22
Type: 13, Index: 0, Profit: 936, Dimension: 36 x 26, Bin: 5, Position: 0 x 0
Type: 11, Index: 0, Profit: 816, Dimension: 34 x 24, Bin: 5, Position: 36 x 0
Type: 5, Index: 0, Profit: 400, Dimension: 25 x 16, Bin: 5, Position: 36 x 24
Type: 12, Index: 0, Profit: 875, Dimension: 35 x 25, Bin: 6, Position: 0 x 0
Type: 4, Index: 0, Profit: 360, Dimension: 24 x 15, Bin: 6, Position: 0 x 25
Type: 12, Index: 1, Profit: 875, Dimension: 35 x 25, Bin: 6, Position: 35 x 0
Type: 4, Index: 1, Profit: 360, Dimension: 24 x 15, Bin: 6, Position: 35 x 25
Type: 8, Index: 0, Profit: 651, Dimension: 21 x 31, Bin: 7, Position: 49 x 0
Type: 11, Index: 1, Profit: 816, Dimension: 34 x 24, Bin: 7, Position: 0 x 0
Type: 3, Index: 0, Profit: 322, Dimension: 14 x 23, Bin: 7, Position: 34 x 0
Type: 5, Index: 1, Profit: 400, Dimension: 25 x 16, Bin: 7, Position: 0 x 24
Type: 4, Index: 2, Profit: 360, Dimension: 24 x 15, Bin: 7, Position: 25 x 24
Type: 7, Index: 0, Profit: 522, Dimension: 18 x 29, Bin: 8, Position: 42 x 0
Type: 8, Index: 1, Profit: 651, Dimension: 21 x 31, Bin: 8, Position: 0 x 0
Type: 8, Index: 2, Profit: 651, Dimension: 21 x 31, Bin: 8, Position: 21 x 0
Type: 7, Index: 1, Profit: 522, Dimension: 18 x 29, Bin: 9, Position: 46 x 0
Type: 7, Index: 2, Profit: 522, Dimension: 18 x 29, Bin: 9, Position: 28 x 0
Type: 5, Index: 2, Profit: 400, Dimension: 25 x 16, Bin: 9, Position: 0 x 23
Type: 3, Index: 1, Profit: 322, Dimension: 14 x 23, Bin: 9, Position: 0 x 0
Type: 3, Index: 2, Profit: 322, Dimension: 14 x 23, Bin: 9, Position: 14 x 0
Type: 1, Index: 0, Profit: 209, Dimension: 11 x 19, Bin: 10, Position: 50 x 0
Type: 1, Index: 1, Profit: 209, Dimension: 11 x 19, Bin: 10, Position: 50 x 19
Type: 2, Index: 0, Profit: 252, Dimension: 12 x 21, Bin: 10, Position: 38 x 0
Type: 1, Index: 2, Profit: 209, Dimension: 11 x 19, Bin: 10, Position: 38 x 21
Type: 5, Index: 3, Profit: 400, Dimension: 25 x 16, Bin: 10, Position: 0 x 23
Type: 2, Index: 1, Profit: 252, Dimension: 12 x 21, Bin: 10, Position: 26 x 0
Type: 1, Index: 3, Profit: 209, Dimension: 11 x 19, Bin: 10, Position: 26 x 21
Type: 3, Index: 3, Profit: 322, Dimension: 14 x 23, Bin: 10, Position: 0 x 0
Type: 2, Index: 2, Profit: 252, Dimension: 12 x 21, Bin: 10, Position: 14 x 0
Optimal solution = 11
Time required (seconds) = 0.3400
```

Code 5.10. Example of output of G2BPP

Chapter 6

Computational Experiments

In this section all the tests that have been performed are presented, not only those concerning the performance of the algorithms, but also those made to study possible improvements of the algorithms, even if they did not bring the desired benefits.

Besides the numeric values in the tables containing results that are presented in this section, also three abbreviations are present: NA = Not Applicable, T.L. = Time Limit, M.L. = Memory Limit.

6.1 Instances

The instances used as input for the computational experiments come from literature. These instances are the following: WANG20, from OKP01 to OKP05, from CGCUT01 to CGCUT03, from GCUT01 to GCUT13 and from ATP30 to ATP49.
GCUT instances are characterised by not having two items with the same shape, by having item profits equal to item areas and by having containers with a squared shape that goes from 250×250 to 3000×3000.
CGCUT instances and WANG20 are very small and easy to solve, WANG20 has item profits equal to item areas.
OKP instances do not have two items with the same shape and their containers have a squared shape 100×100.
ATP instances are divided in two groups: ATPs from 30 to 39 that have item profits equal to item areas and that are a little easier to solve than ATPs from 40 to 49 that do not have this feature. Anyway all the ATPs are very difficult to solve, because of the large number of items present for each instance.
In table 6.1 the features of every instance are shown. From the values of the last two columns it is possible to understand why ATPs are more difficult: the average number of items that can packed in the solution is much higher than for the other instances, so a much greater number of packings is generated at every iteration.

	#SHAPES	#ITEMS	W	H	AVERAGE w	AVERAGE h	AVERAGE #ITEMS HORIZ	AVERAGE #ITEMS VERT
WANG20	19	42	40	70	23,83	23,21	1,68	3,02
CGCUT01	7	16	10	15	4,50	3,25	2,22	4,62
CGCUT02	10	23	70	40	14,17	15,13	4,94	2,64
CGCUT03	19	62	70	40	24,66	28,10	2,84	1,42
OKP01	15	50	100	100	37,52	43,40	2,67	2,30
OKP02	30	30	100	100	31,73	52,50	3,15	1,90
OKP03	30	30	100	100	30,03	57,60	3,33	1,74
OKP04	33	61	100	100	35,16	52,48	2,84	1,91
OKP05	29	97	100	100	29,56	48,47	3,38	2,06
GCUT01	10	10	250	250	108,60	146,00	2,30	1,71
GCUT02	20	20	250	250	118,35	117,50	2,11	2,13
GCUT03	30	30	250	250	110,57	123,97	2,26	2,02
GCUT04	50	50	250	250	122,50	117,10	2,04	2,13
GCUT05	10	10	500	500	262,80	206,40	1,90	2,42
GCUT06	20	20	500	500	259,80	241,90	1,92	2,07
GCUT07	30	30	500	500	243,27	268,03	2,06	1,87
GCUT08	50	50	500	500	245,00	228,64	2,04	2,19
GCUT09	10	10	1000	1000	405,00	497,70	2,47	2,01
GCUT10	20	20	1000	1000	543,40	499,15	1,84	2,00
GCUT11	30	30	1000	1000	478,43	460,53	2,09	2,17
GCUT12	50	50	1000	1000	496,74	505,18	2,01	1,98
GCUT13	32	32	3000	3000	573,28	771,56	5,23	3,89
ATP30	38	192	152	927	214,26	26,68	0,71	34,75
ATP31	51	258	964	856	186,55	217,17	5,17	3,94
ATP32	55	249	124	307	69,20	25,05	1,79	12,26
ATP33	44	224	983	241	57,27	216,87	17,16	1,11
ATP34	27	130	456	795	165,77	78,23	2,75	10,16
ATP35	29	153	649	960	212,62	124,95	3,05	7,68
ATP36	28	153	244	537	107,08	57,31	2,28	9,37
ATP37	43	222	881	440	99,32	193,71	8,87	2,27
ATP38	40	202	358	731	174,39	70,23	2,05	10,41
ATP39	33	163	501	538	137,08	126,67	3,65	4,25
ATP40	56	290	138	683	146,64	32,45	0,94	21,05
ATP41	36	177	367	837	205,57	89,64	1,79	9,34
ATP42	59	325	291	167	32,42	66,21	8,98	2,52
ATP43	49	259	917	362	79,41	195,76	11,55	1,85
ATP44	39	196	496	223	45,34	108,29	10,94	2,06
ATP45	33	156	578	188	38,41	134,83	15,05	1,39
ATP46	42	197	514	416	89,79	124,57	5,72	3,34
ATP47	43	204	554	393	89,50	139,94	6,19	2,81
ATP48	34	167	254	931	195,15	57,78	1,30	16,11
ATP49	25	119	449	759	150,97	93,03	2,97	8,16

Table 6.1. Features of instances

6.2 Packed Area Percentage

This test has been made in the algorithm to solve G2BPP to understand if it was possible to add another control to speed up the algorithm even more. The idea of this kind of test came from a simple observation: if the items that we want to pack occupy just a little portion of the total area of the bin, it should be very likely to be able to pack them into the bin, despite the guillotine constraints. So from this observation we wanted to check if this was just a feeling or if there was really a bound on the percentage of area occupied by the items under that the packing was certainly possible or over that it was certainly impossible.
To perform this test we have computed the percentage of area occupied by items that have been tried to be packed into a bin in the function insertCurrent used to solve G2BPP. Once this has been computed, this value has been saved into a file together with the result of the insertion. At that point the list of values have been ordered and the four maximum values and the four minimum values for both insertions successful and non- have been saved into a table.
The input files that have been used for this test were some files from literature, called CGCUT01, CGCUT02 and CGCUT03 and all GCUTs from GCUT01 to GCUT13. GCUT08 has reached memory limit, because it is a very difficult problem for G2BPP and too much data were returned, so we did not have enough computational power to analyse them.

CGCUT01			
MIN AREA NO	MAX AREA NO	MIN AREA SI	MAX AREA SI
0.780000	0.973333	0.213333	0.933333
0.913333	0.953333	0.293333	0.893333
0.946667	0.946667	0.373333	0.833333
0.953333	0.913333	0.426667	0.753333

CGCUT02			
MIN AREA NO	MAX AREA NO	MIN AREA SI	MAX AREA SI
0.917857	0.952857	0.150000	0.912143
0.927857	0.946429	0.201071	0.877857
0.928929	0.944643	0.252143	0.800714
0.944643	0.928929	0.303214	0.723571

CGCUT03			
MIN AREA NO	MAX AREA NO	MIN AREA SI	MAX AREA SI
0.617143	0.998571	0.387143	0.801786
0.632500	0.979643	0.427500	0.795357
0.646786	0.959286	0.471786	0.774286
0.648214	0.952143	0.485357	0.755000

GCUT01			
MIN AREA NO	MAX AREA NO	MIN AREA SI	MAX AREA SI
0.595040	0.897792	0.522432	0.773888
0.602416	0.871456	0.576080	0.677568
0.621376	0.859728	0.588304	0.673808
0.678720	0.833856	0.592064	0.592064

GCUT02			
MIN AREA NO	MAX AREA NO	MIN AREA SI	MAX AREA SI
0.542416	0.956640	0.435696	0.854368
0.545328	0.944128	0.524288	0.833440
0.550048	0.925984	0.531776	0.783968
0.561984	0.923200	0.535744	0.776864

GCUT03			
MIN AREA NO	MAX AREA NO	MIN AREA SI	MAX AREA SI
0.493728	0.993888	0.395472	0.935344
0.516704	0.992800	0.418592	0.904144
0.520256	0.990928	0.444960	0.879184
0.530592	0.990640	0.445504	0.860608

GCUT04			
MIN AREA NO	MAX AREA NO	MIN AREA SI	MAX AREA SI
0.537504	0.997680	0.457712	0.957008
0.539984	0.996032	0.502864	0.943856
0.568064	0.994640	0.548944	0.939728
0.578432	0.993568	0.553936	0.938432

GCUT05			
MIN AREA NO	MAX AREA NO	MIN AREA SI	MAX AREA SI
0.605728	0.887864	0.410728	0.771628
0.608136	0.886324	0.565568	0.727928
0.737708	0.883916	0.570664	0.681644
0.745184	0.839692	0.679756	0.679756

GCUT06			
MIN AREA NO	MAX AREA NO	MIN AREA SI	MAX AREA SI
0.522972	0.997388	0.356272	0.897596
0.527268	0.994652	0.461100	0.844496
0.530400	0.993856	0.463100	0.796872
0.544800	0.993832	0.465396	0.792576

GCUT07			
MIN AREA NO	MAX AREA NO	MIN AREA SI	MAX AREA SI
0.575216	0.998856	0.428496	0.843764
0.581896	0.995672	0.555044	0.820596
0.586600	0.994264	0.572440	0.810352
0.586812	0.985108	0.583540	0.780784

GCUT08			
MIN AREA NO	MAX AREA NO	MIN AREA SI	MAX AREA SI
M.L	M.L.	M.L.	M.L.
M.L	M.L.	M.L.	M.L.
M.L	M.L.	M.L.	M.L.
M.L	M.L.	M.L.	M.L.

GCUT09			
MIN AREA NO	MAX AREA NO	MIN AREA SI	MAX AREA SI
0.578313	0.899784	0.395576	0.904212
0.593577	0.883892	0.477442	0.772152
0.614664	0.854902	0.524804	0.722042
0.839005	0.854102	0.639292	0.639292

GCUT10			
MIN AREA NO	MAX AREA NO	MIN AREA SI	MAX AREA SI
0.506205	0.998262	0.442379	0.890585
0.542570	0.997037	0.450789	0.888976
0.542885	0.996722	0.450913	0.877079
0.551554	0.992736	0.455090	0.876265

GCUT11			
MIN AREA NO	MAX AREA NO	MIN AREA SI	MAX AREA SI
0.518808	0.999407	0.319910	0.935397
0.536141	0.998233	0.457402	0.874532
0.536499	0.998105	0.457475	0.831853
0.547330	0.997795	0.462518	0.826283

GCUT12			
MIN AREA NO	MAX AREA NO	MIN AREA SI	MAX AREA SI
0.551838	0.999794	0.408240	0.970744
0.555042	0.999767	0.531306	0.957526
0.555828	0.999483	0.531525	0.945437
0.560130	0.997492	0.535702	0.941884

GCUT13			
MIN AREA NO	MAX AREA NO	MIN AREA SI	MAX AREA SI
0.638182	0.956931	0.145100	0.912187
0.913518	0.956430	0.195001	0.906529
0.914032	0.953452	0.234922	0.854664
0.914045	0.953087	0.241924	0.792697

Table 6.2. Table of results of area percentages

The final results (see table 6.2) have not been too far from the expectations, in fact an upper bound on the percentage of area needed to pack items can not be found, as it was easy to forecast, because feasible packings can exists also occupying the whole area.
A bit more surprising were the values obtained from the lower bound of percentage of area of items that can not be packed. It was expected to be a little higher, although not too much. But then, retrieving the set of items with the lowest percentage of area that it was not possible to pack, it has been understood that it is not possible to determine a lower bound on the percentage of area of the set of items to be packed as well, because there can be always particular cases where items can not be packed even if they occupy a very little portion of the total area. A simple example is the following: if we have a bin with dimensions 100×100 and two items with dimensions 1×100 and 100×1 they can not be packed, even if their total area is $(100+100)/(100*100) = 200/10000 = 2\%$ of the area of the bin. And following this case many other examples can be built, also obtaining a lower percentage on the area of the bin.

6.3 Combo Upper Bound

Trying to improve the performance of G2KP, we have tried to use the so-called Combo UB [7]. This UB is stronger than Martello and Toth UB, because Martello and Toth solves a continuous relaxation, while Combo determines an optimal solution solving an instance of KP. So our goal was to decrease consistently the number of packings saved into the matrix F of the procedure recursive eliminating them during the test that verifies if the minimum required profit can be reached.
Unfortunately tests have demonstrated that this was not convenient: as it can be seen in table 6.3, in only two of the tested instances of input there was an improvement of the performance. Most of the times, in fact, and above all when the instances were more difficult like the ATP, the percentage of packings that were cut in this way was meaningless and did not justify the greater computational time needed to compute Combo UB.

	#PACKING MT	TIME (s) MT	#PACKING COMBO	TIME (s) COMBO	DIFF #PACKING (%)	DIFF TIME (%)
WANG20	740	0,00	710	0,05	-4,05%	NA
CGCUT01	736	0,00	575	0,01	-21,88%	NA
CGCUT02	303	0,00	258	0,01	-14,85%	NA
CGCUT03	5266	0,10	4251	0,17	-19,27%	+70,00%
OKP01	11472	1,05	9368	1,24	-18,34%	+18,10%
OKP02	88234	99,33	60344	48,73	-31,61%	-50,94%
OKP03	66829	50,85	47164	29,42	-29,43%	-42,14%
OKP04	3961	0,27	2486	0,32	-37,24%	+18,52%
OKP05	5461	0,30	3715	0,34	-31,97%	+13,33%
GCUT01	4156	0,00	3722	0,06	-10,44%	NA
GCUT02	8160	0,05	7191	0,48	-11,88%	+860,00%
GCUT03	12069	0,12	10464	2,30	-13,30%	+1816,67%
GCUT04	25140	0,53	23128	9,71	-8,00%	+1732,08%
GCUT05	11988	0,03	8971	0,19	-25,17%	+533,33%
GCUT06	5376	0,02	4559	0,19	-15,20%	+850,00%
GCUT07	13636	0,10	10889	1,38	-20,15%	+1280,00%
GCUT08	12479	0,38	11619	15,00	-6,89%	+3847,37%
GCUT09	1981	0,00	1692	0,03	-14,59%	NA
GCUT10	25264	0,09	19196	0,58	-24,02%	+544,44%
GCUT11	125783	1,21	100651	16,98	-19,98%	+1303,31%
GCUT12	107604	1,45	86477	59,65	-19,63%	+4013,79%
ATP30	5727	5,40	5662	12,07	-1,13%	+123,52%
ATP31	319307	126,35	318989	2162,79	-0,10%	+1611,75%
ATP32	145046751	587468,20	T.L.	T.L.	T.L.	T.L.
ATP33	46603	13,34	46261	80,94	-0,73%	+506,75%
ATP34	3032892	2789,39	3032036	11486,22	-0,03%	+311,78%
ATP35	823650	141,25	822741	6586,46	-0,11%	+4562,98%
ATP36	35987	3,99	35744	39,47	-0,68%	+889,22%
ATP37	11109	14,93	11096	53,48	-0,12%	+258,20%
ATP38	42312	9,09	42254	90,53	-0,14%	+895,93%
ATP39	27970	8,04	27902	79,63	-0,24%	+890,42%

Table 6.3. Table that shows differences between number of packings and times using Combo or Martello and Toth

Hence, it has been tried to understand if it was possible that the eliminations of packings happen only when the number of items passed as input in the Combo function is varying between two bounds, defined as percentage on the total number of items of the original problem. If this idea was correct, it could have been possible to use Combo UB only for the instances between these bound, reducing in this way the number of times it is called and speeding up the overall computation.
The results, shown in table 6.4, are very different, but for almost all the tested instances it can be noticed that those bounds varies between 50% and 100%, with the exception of two very small problems. At first look it could seem a very good result, because it seems to halve the number of instances where it should be used, but actually it is not so good. The dimension of the instances passed as input of Combo function depend, in this case, by the number of item present in the packing that is being analysed. So if the total number of items of the problem is n and the packing has k items, Combo will receive $n - k$ items in input. For medium and big problems it is very difficult that a packing will contain more than half of the total number of items, so the dimension of the instances passed as input to the Combo function is almost always included between the two found bounds and so the result can not considered positive.

	Dim Initial KP	n_max(1)	n_min (2)	% on the total (1)	% on the total (2)
WANG20	42	39	33	92,86%	78,57%
CGCUT01	16	15	6	93,75%	37,50%
CGCUT02	23	22	10	95,65%	43,48%
CGCUT03	62	61	53	98,39%	85,48%
OKP01	50	49	33	98,00%	66,00%
OKP02	30	29	19	96,67%	63,33%
OKP03	30	29	19	96,67%	63,33%
OKP04	61	60	50	98,36%	81,97%
OKP05	97	96	84	98,97%	86,60%
GCUT01	10	9	6	90,00%	60,00%
GCUT02	20	19	14	95,00%	70,00%
GCUT03	30	29	23	96,67%	76,67%
GCUT04	50	48	42	96,00%	84,00%
GCUT05	10	9	5	90,00%	50,00%
GCUT06	20	19	15	95,00%	75,00%
GCUT07	30	29	25	96,67%	83,33%
GCUT08	50	48	43	96,00%	86,00%
GCUT09	10	9	5	90,00%	50,00%
GCUT10	20	19	15	95,00%	75,00%
GCUT11	30	29	23	96,67%	76,67%
GCUT12	50	49	44	98,00%	88,00%
ATP30	192	164	145	85,42%	75,52%
ATP31	258	241	208	93,41%	80,62%
ATP32	249	T.L.	T.L.	T.L.	T.L.
ATP33	224	213	184	95,09%	82,14%
ATP34	130	110	66	84,62%	50,77%
ATP35	153	135	107	88,24%	69,93%
ATP36	153	137	115	89,54%	75,16%
ATP37	222	208	186	93,69%	83,78%
ATP38	202	182	157	90,10%	77,72%
ATP39	163	150	133	92,02%	81,60%

Table 6.4. Table that shows maximal and minimal percentage of the total elements for which using Combo at least one packing is cut

6.4 Performance of G2KP

First of all the dimension of loops in heuristicG2KP has been defined. For the inner loop has been seen empirically that it is better to avoid to perform it too many times, because if a very good heuristic solution is found early in the execution, it is very difficult to find values that are better and that at the same time are also coming from a set of possibly feasible items. So it has been decided to set the variable *NUMBER_OF_INNER_LOOPS* to the value 50. On the other hand, for the outer cycle, it has been necessary to execute it a sufficient number of times to increase the possibility of finding a good heuristic solution, to decrease the chances of being obliged to perform the procedure recursive with a too small lower bound, as described before. So it has been decided after some tests, that a good compromise between execution speed and goodness of the solution is to set the variable *NUMBER_OF_LOOPS* to the value 2000000.

Let us now discuss about the performance of the algorithm for G2KP.
Normally, we should describe performances of the entire algorithm exactG2KP. But, as these

performances can vary a lot depending on the result of the heuristic algorithm, which is very randomized, we have decided to focus here more on performances of the procedure recursive, which is the real bottleneck of the algorithm and that is not affected by random values. The computation of procedure recursive has been performed with the optimal lower bound passed as input of the function to determine computational time and maximum memory needed for the execution of this function. The values computed for exactG2KP must be considered only as indicative because of the already mentioned random component.
Tests have been executed on a blade server used for cluster computing. On the cluster any 32- or 64-bit application can be run that might normally be run on a linux pc or any program that has been recompiled. The cluster is composed by 14 computing resources, each one with:

- 2 quad core processors Intel Xeon E5450 (12M Cache, 3.00 GHz, 1333 MHz FSB);
- 16 GB RAM;
- 2 hard disk of 72 GB in configuration RAID-1 (mirror).

The various requests are managed by the server management program, queued according to a priority system and executed by the most appropriate computing resource at that time. The results of computation that normally would appear on the screen are saved in an output file. The management program adopted is Sun Grid Engine (SGE). To launch a program is necessary to create a job file containing the required computing resources, the program to run, the files to use as input and the file where to write as output. This job can be then used to queue the request.
With SGE on the cluster different groups have been defined, each one with more or less privileges. We belonged to the group called "Medium" so the maximum available memory (i.e. memory limit M.L.) for our test was 10 GB. The maximal computational time (i.e. time limit T.L.) has been assumed to be 15 days.

In the following table (Table 6.5), the obtained results are presented.

	#SHAPES	#ITEMS	OPT SOL	TIME RECURS (s)	MEMORY RECURS (MB)	TIME EXACT (s)	MEMORY EXACT (MB)
WANG20	19	42	2721	0,00	NA	10,01	125,879
CGCUT01	7	16	244	0,00	NA	7,46	125,879
CGCUT02	10	23	2892	0,00	NA	4,66	125,879
CGCUT03	19	62	1860	0,08	NA	311,88	125,879
OKP01	15	50	27589	0,91	126,383	36,62	126,508
OKP02	30	30	22502	64,48	132,988	3161,06	133,652
OKP03	30	30	24019	23,73	132,945	1240,51	133,621
OKP04	33	61	32893	0,21	NA	306,78	126,164
OKP05	29	97	27923	0,23	NA	42,88	126,160
GCUT01	10	10	48368	0,00	NA	8,02	125,875
GCUT02	20	20	59307	0,04	NA	14,74	126,152
GCUT03	30	30	60241	0,10	NA	10,61	126,754
GCUT04	50	50	60942	0,40	NA	32,09	127,738
GCUT05	10	10	195582	0,03	NA	6,59	126,375
GCUT06	20	20	236305	0,01	NA	12,04	125,887
GCUT07	30	30	238974	0,08	NA	21,71	126,973
GCUT08	50	50	245758	0,20	NA	26,24	127,668
GCUT09	10	10	919476	0,00	NA	4,75	125,879
GCUT10	20	20	903435	0,07	NA	13,46	126,918
GCUT11	30	30	955389	0,66	134,848	30,13	135,549
GCUT12	50	50	970744	0,79	NA	60,38	137,758
GCUT13	32	32	8532720	T.L.	-	T.L.	-
ATP30	38	192	140904	2,77	127,633	45,75	127,777
ATP31	51	258	823976	48,52	162,066	68927,06	2503,000
ATP32	55	249	38068	587468,20	9189,000	587513,00	9189,000
ATP33	44	224	236611	7,27	131,918	309,26	142,836
ATP34	27	130	361398	1645,50	328,297	79863,59	NA
ATP35	29	153	621021	81,34	186,473	6241,58	371,211
ATP36	28	153	130744	2,22	128,570	108,17	135,051
ATP37	43	222	387276	12,69	131,137	551,75	146,109
ATP38	40	202	261395	5,92	131,746	369,20	179,875
ATP39	33	163	268750	5,00	134,766	219,81	138,270
ATP40	56	290	67154	255923,83	1071,000	T.L.	-
ATP41	36	177	206542	T.L.	-	T.L.	-
ATP42	59	325	33503	T.L.	-	T.L.	-
ATP43	49	259	214651	T.L.	-	T.L.	-
ATP44	39	196	73868	T.L.	-	T.L.	-
ATP45	33	156	74691	T.L.	-	T.L.	-
ATP46	42	197	149911	23949,42	464,301	T.L.	-
ATP47	43	204	150234	T.L.	-	T.L.	-
ATP48	34	167	167660	T.L.	-	T.L.	-
ATP49	25	119	219354	T.L.	-	T.L.	-

Table 6.5. Table that shows performances of procedure Recursive and of exactG2KP

From the table it is possible to see that for the greater part of the input instances has been possible to find a solution. For those for that it has not been possible because of the time limit, during their execution we were checking the maximum memory usage and it was quite low (few gigabytes), so we can say that probably with no limitations of time they would have terminated. But anyway, we have must remember that this is not the whole algorithm to compute G2KP, but just the recursive part, which is repeated many times. So it will be important in a possible future work trying to speed up the algorithm to be able to end up also all the ATP input files in reasonable times. The computed times, anyway, must be considered as indicative, because they depend on the load of the machine.

Table 6.6 contains the values obtained by the heuristic algorithm executing it five times, so the seed that generates random values changes and different values are obtained. It also contains the average of these values and the optimal solution. In the last column is present which percentage of the optimal value is the average of the values found. This gives us a performance indicator of the algorithm used and, because all the average values are above 70% of the optimal value and almost all also above 80%, we can say that the heuristic algorithm implemented can be considered very good.

	OPT SOL	TEST #1	TEST #2	TEST #3	TEST #4	TEST #5	AVERAGE	% OF THE OPT
WANG20	2721	2479	2559	2702	2307	2613	2532	93,05%
CGCUT01	244	230	244	244	229	240	237	97,30%
CGCUT02	2892	2360	2323	2614	2324	2343	2393	82,74%
CGCUT03	1860	1600	1580	1660	1600	1560	1600	86,02%
OKP01	27589	21471	21628	20906	23853	20967	21765	78,89%
OKP02	22502	15744	15988	15597	16502	17591	16284	72,37%
OKP03	24019	18871	18773	22130	19781	19957	19902	82,86%
OKP04	32893	26214	24867	26474	26391	27360	26261	79,84%
OKP05	27923	20155	21864	23321	23997	25417	22951	82,19%
GCUT01	48368	48368	42025	41378	42348	48368	44497	92,00%
GCUT02	59307	48288	48664	52148	55111	53323	51507	86,85%
GCUT03	60241	57462	55424	58396	58714	55804	57160	94,89%
GCUT04	60942	57822	60522	60195	59117	60073	59546	97,71%
GCUT05	195582	177617	192907	187987	192907	189856	188255	96,25%
GCUT06	236305	228442	216644	228675	216481	224166	222882	94,32%
GCUT07	238974	226069	221511	232723	228130	223253	226337	94,71%
GCUT08	245758	220262	232064	225736	233058	233963	229017	93,19%
GCUT09	919476	786653	858697	851794	835406	851794	836869	91,02%
GCUT10	903435	839394	877079	805607	827223	847308	839322	92,90%
GCUT11	955389	903187	918825	864879	879182	897941	892803	93,45%
GCUT12	970744	914253	941884	943625	933988	970744	940899	96,93%
GCUT13	8532720	7594031	7740992	7354480	7408240	7514303	7522409	88,16%
ATP30	140904	128918	126349	126992	125459	129960	127536	90,51%
ATP31	823976	761800	744577	748714	747602	759090	752357	91,31%
ATP32	38068	34395	34736	34608	34795	33910	34489	90,60%
ATP33	236611	212807	212897	211738	215630	209572	212529	89,82%
ATP34	361398	325023	327239	329322	328881	326658	327425	90,60%
ATP35	621021	562979	554089	545209	562001	573636	559583	90,11%
ATP36	130744	115754	119316	117264	116164	119913	117682	90,01%
ATP37	387276	342280	343677	348652	339340	343596	343509	88,70%
ATP38	261395	234816	235670	236071	234105	236440	235420	90,06%
ATP39	268750	247479	246807	255880	246119	248941	249045	92,67%
ATP40	67154	53495	50839	56444	52615	55423	53763	80,06%
ATP41	206542	172873	180189	181823	175524	175617	177205	85,80%
ATP42	33503	27062	26880	27356	26992	28645	27387	81,74%
ATP43	214651	177980	167411	167800	169983	167415	170118	79,25%
ATP44	73868	61166	64091	58657	58136	61032	60616	82,06%
ATP45	74691	65784	66431	65358	65137	65350	65612	87,84%
ATP46	149911	123669	126195	124818	127287	126526	125699	83,85%
ATP47	150234	123171	123809	122932	121504	124345	123152	81,97%
ATP48	167660	133433	139095	132705	138814	135845	135978	81,10%
ATP49	219354	176047	169359	170733	172859	170209	171841	78,34%

Table 6.6. Table that shows average values found by the heuristic algorithm for G2KP

6.5 Performance of G2BPP

Like the case of G2KP, also for G2BPP first of all the dimension of the loop in heuristicG2BPP has been defined. The problem here is different for two reasons: the first one is that here it is possible to exit the loop before its end and the second one is that this loop is performed many times during the execution of the algorithm, not just one time at the beginning like G2KP.
Starting from these considerations the value searched should not have been a too high value, but also it was not necessary that it was too small. Eventually, after some tests, it has been seen that a good value for this variable $NUMBER\_OF\_LOOPS$ is 1000.

To test the efficiency of the algorithm that solves G2BPP the same machine used for G2KP has been adopted. Considering the performance of G2BPP in terms of computational time and memory used, the result that we have obtained can be considered very good. For this kind of

problem the input instances ATPs have not been considered, because we have based on the same kind of instances used by Martello and Vigo in [10]. Moreover, with this algorithm, we also want to find the coordinates that will have the items in the packings and not only the number of bins used to pack all of them. In table 6.7 the time needed and the memory used by the instances are represented.

	#ITEMS	OPT SOL	TIME (s)	MEMORY (MB)	2BPP TIME (s)
WANG20	42	11	0,27	NA	NA
CGCUT01	16	2	0,02	NA	0,01
CGCUT02	23	2	17,75	128,633	0,01
CGCUT03	62	23	0,61	NA	0,04
OKP01	50	4	16,80	126,492	NA
OKP02	30	4	0,61	NA	NA
OKP03	30	4	385,41	126,715	NA
OKP04	61	7	359,86	126,613	NA
OKP05	97	-	T.L.	-	NA
GCUT01	10	5	0,02	NA	0,02
GCUT02	20	6	0,14	NA	0,02
GCUT03	30	8	0,10	NA	0,01
GCUT04	50	14	3,67	125,875	3,73
GCUT05	10	3	0,01	NA	0,01
GCUT06	20	7	1,91	125,875	0,90
GCUT07	30	11	0,17	NA	0,28
GCUT08	50	-	T.L.	-	T.L.
GCUT09	10	3	0,01	NA	0,02
GCUT10	20	8	3,73	125,875	1,22
GCUT11	30	9	1264,46	125,875	909,70
GCUT12	50	16	0,52	125,875	0,17
GCUT13	32	2	34,09	*	0,01

Table 6.7. Table that shows performances of exactG2BPP

Comparing results with results of Martello and Vigo present in the last column, it is possible to see that even with the addition of Guillotine constraints the optimal solution remains the same for all instances with the exception of GCUT10. Moreover, also computational times are all very close to those presented in the paper and sometimes even better. The only two instances that has a big performances decay are CGCUT02 that passes from 0.01 seconds needed for the 2BPP to the 17.75 seconds needed here for G2BPP and GCUT13 that passes from 0.17 to 34.09. GCUT13 has a * on the memory column because that is the time needed to compute only the best solution, to compute also the optimal packing much more time is needed.
OKPs and WANG20 were not used by Martello and Vigo for their tests, but they have been used here because their size is similar to the other used instances. OKP05 is the only of these instances that reaches time limit, but it is not surprising because its size is almost two times bigger than all the other considered instances.
Considering the memory usage, on the other hand, results obtained are great, because the memory used is very limited, so no problem connected with a big memory usage should be present.

Chapter 7

Conclusions

To conclude, therefore, a function that computes an exact packing of a Guillotine Two-dimensional Knapsack Problem with a minimum profit z_0 has been implemented. This function is also able to return the correct position where the various items that are in the solution should have in the container to obtain the correct packing.
Thanks to this function it has been possible to compute first of all the exact solution for a G2KP even without knowing a lower bound and secondly also the exact solution for a G2BPP, returning for each item the bin where it should be packed and the correct coordinates into that bin.
Moreover, to obtain these results, also two heuristic algorithms have been developed and implemented to speed up the global computation.

Considering the memory performances, that was the main goal, the created algorithms can be considered very positively, because even with big input instances it never reaches the memory limit.
Considering, on the other hand, the time performances, the procedure recursive should be improved, because to be able to guarantee the absence of memory problems, the speed of the algorithm has been sacrificed and so for many of the ATP4x instances (which are very difficult) the algorithm is not able to end up before the time limit. Anyway this has not been a problem for G2BPP, because the instances where it is computed are not very big, because all items must be packed, and so the procedure recursive works well, in line with the times obtained by the exact algorithm for 2BPP implemented by Martello and Vigo.

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