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"ANALYSIS OF INVESTMENTS TIMING IN VERTICAL RELATIONSHIPS: A REAL OPTION APPROACH"

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Abstract

In the analysis of investment opportunities in the corporate finance literature a commonly used standard framework is the real option approach. This examines the timing and the value of investment projects relying on the basic idea that a financial option on a real asset as underling can be thought as the opportunity (option) to invest in a project that can return a positive payoff. So in an environment characterized by irreversibility and uncertainty, an agent considering an investment opportunity needs to account for the fact that, at the time of the investment, he forgoes the option to postpone the investment decision in the future when the uncertainty will be, naturally, partly resolved. With this as starting point this thesis will focus on the timing and the value of an irreversible investment made in different vertical relationships settings and finally on the possibility to introduce the intervention of a policy maker to see if this can be useful in improving the timing and enhancing the value for the different agents involved.

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Introduction

As reported by McGrath and Nerkar (2004) one of the most important features for a firm is the ability to innovate, this in fact can be thought as one of the main reasons that allows a company to succeed and dominate the market. Therefore, one of the major problems that an entrepreneur face has to do with the decision to make new investment in technology, product or service. But how to account for these investments? One of the most widespread technique is represented by the net present value rule (NVP), that represent the basis of the neoclassical theory of investment. However, the NVP approach sometimes may not be the best way to evaluate some kind of investments, in fact often it assumes that the investment can be reversed, so that in some way, once a project is undertaken, this can be undone or that the expenditure sustained can be recovered, and also in the cases in which the irreversibility of a project is taken into account often this is thought as a “do it now or never” opportunity, so that the choice of the firm are reduced to invest in the present or forgo the chance. For these reasons, and given that often a significant part of the market value of a firm consists in asset that are not yet in place (Miller and Modigliani, 1961) that will be the results of important irreversible investment decisions, another approach to evaluating a project can fit better the needs. In fact, these types of assets can be viewed as financial option, (Myers, 1977) in the sense that one agent may have the right but not the obligation to invest, consequently in the moment in which an agent decides to undertake an irreversible investment, it exercises its option to invest, and so it gives up the opportunity to wait for new information that could affect the timing or the eligibility of the project. Given this, usually these opportunities to acquire real assets through an irreversible expense are called in the literature “Real options”. My work consists in presenting different cases based on the application of the standard real option framework that consider investment opportunities as options on real assets, providing a way to apply option pricing methods to investment decision problems. In facts, from the above considerations, the classical net present value method is not always the best instrument given that it does not takes into accounts the value of the possibility (option) to delay an irreversible investment decision characterized by uncertainty. Given the large applicability of the subject I choose to focus on the study of vertical relationships taking as reference the basic theory of irreversible investment under uncertainty as in Dixit and Pindyck (1994) and the classic presentation of vertical relationships as described by Tirole (1988). I will consider a framework where at the center there is a potential risk neutral investor (that it may be think as an innovative start-up) interested in entering in a new high profitable growing market by undertaking a project for which there is the need to make an irreversible investment. Starting at the beginning of Chapter 1 from the simplest case in which it is assumed the investor will have both the financial resources and the infrastructure needed to get the input, I will progressively expand the problem taking into consideration different scenarios. Indeed, I will assume that the completion of the investment can be conditional at one or both conditions: the participation of an investment partner (willing to bear part of the costs in exchange of part of the profits deriving by the project) or/and the purchase of a discrete input from a supplier with market power. In all the cases examined the focus will be on the timing and on the value of the investment, and in which way the introduction of different agents will affects the results. Strictly linked to Chapter 1, in the Chapter 2 of my thesis I will run a simulation in which I will examine the effect that the variation of different parameters may have on both the investment timing and on the values of the option to invest for the potential investor, providing numerical examples for the different cases examined in Chapter 1. In Chapter 3 I will consider a more innovative framework where more than three agents are asked to interact together in order to undertake the project. This section is based by the fact that usually when a new technology or a new investment opportunity is available on the market, there may some competition provided by the presence of other agents. So, by assuming this, I will show the effects that a Bertrand competition and a Collusion agreement can have on the value and on the timing of the investment, considering first the case in which two upstream supplier produce the same input and are looking for a client and second the case in which two downstream potential investors interact to gain the furniture of the input they need to undertake the same project. Finally, in Chapter 4, I will assume that the realization of the investment is of public interest, so that the project is will produce a positive social benefit. In this new framework I will consider the introduction of a policy maker that it will have an incentive to provide a subsidy with the aim to facilitate the investment and to smooth the friction between the different subjects involved.

Literature review

Starting by the pioneers of the formal theory of the valuation of options Fischer Black, Myron Scholes and Robert C. Merton, the term "Real Options", was coined for the first time in the seventies by Myers[1] and from there has been a frequent subject of study. One of major references in this field is given by Dixit and Pindyck (1994), that provides a treatment of the investment decision under the uncertainty in the environment in which these are taken, highlighting the analogy of the classical financial option theory with the values of the option to wait for new information when one is considering an investment opportunity. Moreover, on this, other important contributes are given by Baldwin and Meyer (1979) that discuss irreversibility in the case mutually exclusive investment opportunities are faced stochastically over time; Bernanke (1983) that exploit a model in which is shown that waiting before undertake an investment can be optimal pending the resolution of a situation of significant uncertainty; and by McDonald and Siegel (1986) that studied the optimal time in irreversible investment decisions, considering both the benefit and the costs of the project to follow a continuous-time stochastic process.[2]

Overall, the analysis regarding the real option has been used for many applications. In development of new technologies and R&D, between the many (e.g. McGrath, 1997, Folta, 1998), interesting is the work of McGrath and Nerkar, (2004) that examined the outcome of the investment decisions taken by firms in the pharmaceutical sector over a long period of time with the aim to see whether a real option reasoning can explain for some of the differences between the actual managerial investment behavior and theorized investment behavior; specifically, they confirmed the use by the decision maker of a real option approach and they found that the decisions to pursue an option on a new technological area are influenced by the scope of the technological opportunity, the competition in the area, and a firm's past investment behavior. In the dynamic of outsourcing agreement between the many (e.g. Moretto and Rossini, 2012; Di Corato et al., 2017), Alvarez and Stenbacka, (2007) found that the optimal threshold for the establishment of partial outsourcing is an increasing function of the underlying market uncertainty, showing also that an increasing in market's uncertainty induces a higher optimal proportion of outsourced production once the threshold is reached. In the venture

[1] See Myers S.C. (1977).

[2] See also Brealey & Myers (1981).

capital investments[3] Lukas et al., (2016) presents a dynamic model of entrepreneurial venture financing under uncertainty based on option exercise games between an entrepreneur and a venture capitalist (VC), specifically they analyze the impact of multi-staged financing and both economic and technological uncertainty on optimal contracting in the context of VC-financing, showing that both sources of uncertainty positively impact the VC-investor's optimal equity share; moreover, by combining compound option pricing with sequential non-cooperative contracting in determining whether renegotiation improve the probability of coming to an agreement and proceed with the venture, they show that higher uncertainty leads to a larger stake in the venture, and renegotiation may result in a dramatic shift of control rights in the venture, while with low volatility, situations might occur where the VC-investor loses his first-mover advantage. In joint ventures arrangements, interesting is the work of Cvitanić and Šikić (2011)[4] that, by analyzing three contract designs: the risk-sharing, the timing-incentive and the asymmetric contract decisions design, they studied the optimal time for entering in a joint venture by two firms, and the optimal linear contract for sharing the profits. They found that that if the firms are risk-neutral and if the cash payments are allowed, all three designs are equivalent, while if at least one of the two firms is risk averse, the optimal contract parameters may vary significantly across the three designs and across varying levels of risk aversion. Finally, regarding the M&A operations, between the many (e.g. Benson and Ziedonis, 2009; Tong and Li, 2011) Lambrecht (2004) analyzes the timing of mergers motivated by economies of scale, showing as subsist an incentive to merge in periods of economic expansion, and that, by relaxing the assumption that firms are price takers, the market power strengthens the firms' incentive to merge and speeds up merger activity; moreover, by comparing mergers with hostile takeovers, they show that the way merger synergies are divided not only influences the acquirer's and the acquiree's returns from merging, but also the timing of the restructuring.

Given the large applicability of the subject, my work is thought as a connection between the standard real option theory (ROT) as presented in Dixit and Pindyck (1994) with the classical framework of vertical relationships, as treated by Tirole (1988) and more recently by others, between which: Chevalier-Roignant (2011) that combines the ROT with game theory in study the interactions among firms as competitors or potential collaborators, and Azevedo and Paxson (2014) that shows as an investment decision in a competitive market can be thought as a "game" in which firms are able to implicitly take into account the reactions of other firms in their own investment decision process. Among the many, the most closely related works I have identified

[3] See also Vrande and Vanhaverbeke, (2013).

[4] See also Li J, Dhanaraj C, Shockley (2008); and Banerjee S, Güçbilmez I.U., Pawlina G. (2014).

are the publications of Zormpas D. (2017), Chan (2012), Lukas and Welling (2014) and de Villemeur Billette et al. (2013). The work of Zormpas D. (2017) consider a potential investor who contemplates entering an uncertain new market under the conditions to purchase a discrete input from an upstream firm and to interact with an investment partner who is willing to bear some of the investment costs. By using the ROA (real option approach), the paper shows as the involvement of any of the two alien agents causes the postponement of the investment and examines the synchronous effect of outsourcing and external funding both in a non-cooperative and in a cooperative (Nash bargaining solution) game-theoretic setting, highlighting as the endogeneity of the sunk investment cost affects the timing and the value of the option to invest. Chan (2012) explore a model regarding the supply chain management, acknowledging the fact that volatile market conditions cause the future cash flows along the supply chain more difficult to anticipate, so that the usual NVP approach may be hard to apply; for this reason they propose a two-stages dynamic optimization model by using a real option approach in which the supplier and the retailer cooperatively determine the optimal entry time, and, performing a sensitivity analyses, they investigate the impact that some critical factors (growth rate, volatility of demand shock, sunk cost, and relating operational costs) may have on both the option value and the investment threshold. Lukas and Welling (2014) by defining a model for the optimal timing in "climate-friendly" investments, enlarge the contribution of Chen (2012) by adopting a non-cooperative real option game setting according to which the optimal timing is not decided jointly, but by one of the participating firms, moreover they provide a further extension of the previous model by allowing for the participation of more than two agents and showing that the supply chain becomes less efficient with every additional link as the timing distortion builds up. Finally, de Villemeur Billette et al. (2013), show as the standard analysis of vertical relationships transposes directly to investment dynamics, highlighting the fact that an investment not always is made in-house, but often may be conditioned to the furniture of a discrete input produced by an upstream supplier with market power, so that the cost of the investment is endogenous since it is determined by the vertical relationship between the potential investor and the external supplier.

I my thesis I will analyze different cases in which a potential investor (that can be though as an innovative start-up) wants to enter in a new high growing market by investing in a project. To do this face the problem to decide when it is optimal to undertake the irreversible investment under a situation of uncertainty. I will use the real options approach in order to examine the interaction among: the firm who is contemplating entering in a new market (downstream firm), a firm who acts like an investment partner helping in financing the project in exchange of a share of the final payoff gained by the investment and an upstream supplier with market power who is responsible for the provision of an input that is necessary for the investment to take place. I will start providing different cases in which, progressively, more agents will be involved considering different type of interactions and comparing different game setting. Particularly, I will examine as, in the case in which there is the participation of an outside investment partner, the timing and the value for the potential investor may be affected depending on the role that the two takes with respect to each other. Moreover, I will provide also some innovative extensions. First, by taking into accounts the presence of competition between downstream firms or upstream firms, I will examine both a competitive and a collusive setting. Second, starting by the assumption that the investment produces positive social benefits, I will consider as the intervention of the policy maker can affect both the timing and the value of the potential investor's option to invest.

Chapter 1: Analysis of different investment scenarios.

This first chapter of my thesis is focused on providing different case study in which at the center there is a potential investor willing to undertake an irreversible investment under uncertainty for which the participation of an investment partner helping by financing part of the project and/or the interaction with an upstream supplier providing a discrete input necessary to start the production, may be needed. Specifically, in section 1.1 I will examine the benchmark case under which the potential investor is able to produce the input in-house and has the resources to finance the project. In section 1.2 I will assume the discrete input will be provide by an upstream supplier but still the downstream firm has the finance resources to finance the investment. In section 1.3 I return to the case in which the input is produced in house, but with the difference that the start-up firm does not have the financial resources and so the participation of an investment partner is needed. In section 1.4 I examine how the involvement of the two alien agents can affects the investment timing and how the observed timing discrepancies are reflected in the value of the opportunity to invest. Finally, in section 1.5, I will make a comparison between cases comparing the different investment threshold and the value of the options to invest from the investor prospective.

1.1. Benchmark/integrated case: Internal founding and internal input production.

In the framework of a start-up firm willing to undertake a project, an investment in a discrete input is needed to operate on the final market. In this first section I will assume this input will be produced internally by the firm (integration case) and that the firm owns the financial resources to realize the project.

Now, in line with the framework presented by Billette de Villlemeur (2013) I will provide some variable and assumptions that will be useful throughout the paper:

- I denote the sunk cost of the investment and is assumed to be positive.
- $\pi_M Y_t$ represents the profit flows deriving from the investment, given that $\pi_M > 0$ is the monopolistic profit per unit of Y_t.
- Y_t is a positive scale parameter assumed to follow a geometric Brownian Motion with drift: $dY_t = \alpha Y_t dt + \sigma Y_t dW_t$, where the parameters σ and α are both positive and represent respectively the volatility and the expected growth rate of the market and W_t is

the standard increment of a Winer process uncorrelated over time satisfying $E[dW_t] = 0$ and $E[W_t^2] = dt$.

- The discount rate r is assumed to be larger than the expected growth rate of the market $(r > \alpha)$ [5].
- A lower case $y = Y_0$ is used to indicate the current level of the state variable and it is assumed the initial size of the market to be positive and with a value sufficiently small so that it is preferable to wait rather than to invest today.

These assumptions are in line with the expression of the option value from McDonald and Siegel (1986) where the optimal investment strategy is thought as a trigger strategy in the sense that one should invest as soon as the value of a project is greater than a threshold, the value of which increases with uncertainty[6]. About this O'Brien et al. (2013) find empirical evidence about the fact that entrepreneurs take into account the value of the option to delay the investment when they are deciding to enter in a new market, so the potential investor will invest at the time in which the expected payoff derived from the project exceeds the investment cost by the option value of further postponing the investment in the future. With this background the startup firm is also assumed to be able to observe the current market size and so to decide when to invest and gain the subsequent profit flows.

Given the previous assumption we have, following the standard real option framework by Dixit and Pindyck (1994), that the value of the investment opportunity (so the value of the option to invest) for the firm is represented by the maximization of the expected present value of the payoff $(F_T - I)$ that it will be obtained at time T (time in which the investment occurs) [7]:

$$V_I(y) = max_T E[(F_T - I)e^{-rT}] = max_T E[e^{-rT}](F_T - I)$$
$$= max_{y^*}\left(\frac{\pi_M}{r-\alpha}y^* - I\right)\left(\frac{y}{y^*}\right)^{\beta} \tag{1}$$

Where $F_t = E_t[\int_t^\infty Y_s \pi_M e^{-r(s-t)}ds] = \frac{\pi_M}{r-\alpha}Y_t$ constitutes the value of the project and $T = \inf\{t \geq 0 | Y_t = y^*\}$ represent the random first time in which Y_t hits the barrier y^* that trigger the investment. In words, y^* represent the time in which the market size reaches the width

[5] Otherwise, for $r \leq \alpha$ the firm may have no incentive to undertake the project and delay indefinitely the investment.

[6] Dixit and Pindyck (1994).

[7] Appendix 1.1.a. shows the proof of the relation in the (1).

needed to make the firm willing to exercise its option to invest in the project. While β is a function of parameters[8]:

$$\beta(\alpha,\sigma,r) = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} \tag{2}$$

In Appendix 1.1b I show the sensitivity analysis of $\beta(\alpha, \sigma, r)$, what it is important to observe here is that the function of parameters β is > 1 and it is increasing in *r* and decreasing both in σ and α. Note moreover that the assumption made regarding $\alpha < r$ it guarantees convergence. With this background, by solving the maximization problem (1) it is possible to retrieve the value-maximizing investment trigger[9]:

$$y^* = \frac{r-\alpha}{\pi_M}\frac{\beta}{\beta-1}I \tag{3}$$

Note that y^* is increasing in the sunk investment cost *I* and in the volatility σ (though the function of parameters β) but it is decreasing in the present value of the profit flow $\frac{\pi_M}{r-\alpha}$. In practice this means that it does worth more to hold an investment option with high strike price (represented by *I*), high underlying asset volatility (σ) and small return $\left(\frac{\pi_M}{r-\alpha}\right)$, rather than exercise it. By substitution, we have that the value of the option to invest at the value maximizing threshold y^* is equal to:

$$V_I(y) = \frac{I}{\beta-1}\left(\frac{y}{y^*}\right)^{\beta} \tag{4}$$

Note that, in this case, because we assumed the firm is able to undertake the project on its own, and the investment it is undertaken in a completely new market this value corresponds also to the value of the industry, as such in the integrate case we have only one player.

1.2. Separated case: Internal founding and external input production.

In this section I will examine a case in which the production of the discrete input needed in order to undertake the investment and the decision about the investment timing are made by

[8] See Dixit and Pindyck (1994) Ch. 5 or Chevalier-Roignant and Trigeorgis (2010), Ch. 11-12.

[9] See Appendix 1.1c. for calculations.

two different firms[10]. For simplicity I will call A the downstream firm that make the investment decision and B the upstream firm that will provide the input. It is important to point out that, as input producer, B do not observe the magnitude of the downstream market size and its only choice consists on the determination of the input price (p_1) that, once chosen, is taken as constant. Moreover, as in Tirole (1988) is the supplier that chooses the contract, so A is assumed to be the price taker.

Now, by moving backwards, taking into consideration the price (p_1), A observes the current market size and decides the optimal investment threshold y_1. Then B, accounting for the timing chosen by A, sets the p_1. So, I start analyzing the optimization problem of A, and considering that now the costs of the investment are represented by p_1, we have:

$$V_{A1}(y) = max_{y_1(p_1)} \left(\frac{\pi_M}{r - \alpha} y_1(p_1) - p_1\right) \left(\frac{y}{y_1(p_1)}\right)^{\beta} \tag{5}$$

Because the (5) is very similar to the (1) following the same procedure of Appendix 1.1c it is possible to retrieve the optimal investment threshold, that is given by:

$$y_1(p_1) = \frac{r - \alpha}{\pi_M} \frac{\beta}{\beta - 1} p_1 \tag{6}$$

Note that the only difference between the investment threshold in the integrated case and the one in the above expression, consists in the fact that in the latter the costs sustained by A are no more equal to the pure investment cost (*I*), but instead they are defined by the price set by the supplier. Moreover, note that $y_1(p_1)$ is increasing in p_1[11], so that for a higher price payed the potential investor will delay the project's realization.

By proceeding now with the decision problem of B, the supplier will provide the discrete input needed to the investment and in exchange will receive p_1 from A. So that the maximization problem of B is simply reduced to the following:

$$V_{B1}(y) = max_{p_1} (p_1 - I) \left(\frac{y}{y_1(p_1)}\right)^{\beta} \tag{7}$$

By solving for p_1 yelds[12]:

[10] As in Billette de Villemeur et al. (2013).

[11] Note that when $p_1 = I$, B charges the input cost and the firm will invest at the same investment trigger of the integrated case in Section 1.1.

[12] Calculations are shown in Appendix 1.2.a.

$$p_1 = \frac{\beta}{\beta - 1} I \tag{8}$$

Note that p_1 it is always greater than *I*, given that by construction $\beta > 1$. This makes sense, in fact in order to gain a positive payoff, B has to charge for the furniture of the input a price greater than the costs sustained to produce it. Now, substituting the (8) back into the (6) the optimal investment threshold becomes:

$$y_1(p_1) = \frac{\beta}{\beta - 1} y^* \tag{9}$$

From which it is trivial note that $y_1(p_1) > y^*$. So, in the case in which there is the presence of an upstream firm providing the input, the optimal investment trigger will be higher so that the investment will be delayed with respect the benchmark case in Section 1.1. It is interesting also to note that both y_1 and p_1 are decreasing in β and so are increasing in the volatility (σ), this means that A will tend to give up very risky projects or delay them as much as possible. The same effect holds also for α, with the intuition that if A expects higher market growth rate in the future then it will be inclined to wait more before to enter in the market. From a mathematical point of view this can be seen by the fact that the $\lim_{\beta \to 1} y_1(p_1) \to \infty$.

Now, substituting back the (9) and the (8) in the expression for the value of the option to invest for the A, we obtain:[13]:

$$V_{A1}(y) = V_I(y) \left(\frac{\beta}{\beta - 1}\right)^{1-\beta} \tag{10}$$

The above expression tells us that the option to invest in the integrated case has higher value with respect to the case under separation, in fact $V_{A1}(y) < V_I(y)$ given that $\left(\frac{\beta}{\beta-1}\right)^{1-\beta}$ is in the interval (0;1). So, the presence of a supplier providing the input has the effect to both delay the investment and to reduce the value of the option of A. Now, regarding B, by substituting p_1 into the expression for $V_{B1}(y)$, the value for the supplier is given by:

$$V_{B1}(y) = V_I(y) \left(\frac{\beta}{\beta - 1}\right)^{-\beta} \tag{11}$$

From which it is easy to see that the value of the option to invest for A is higher than the value for B, in particular $V_I(y) > V_{A1}(y) > V_{B1}(y)$, with $\frac{V_{A1}(y)}{V_{B1}(y)} = \frac{\beta}{\beta-1}$. So, $V_{A1}(y)$ is $\frac{\beta}{\beta-1}$ times

[13] See Appendix 1.2.b for the calculations.

bigger than $V_{B1}(y)$, and this difference depends to the function of parameters β (so from the magnitude of volatility, expected market growth and discount rate). Indeed, as β increases the distance between the two decreases[14].

Now, by taking the sum of the values of the two agents it is possible to analyze the situation from an industry prospective:

$$V_1(y) = V_{A1}(y) + V_{B1}(y) = V_I(y)\left(\frac{\beta-1}{\beta}\right)^{\beta}\frac{2\beta-1}{\beta-1} \tag{12}$$

Now, by comparing the (12) with the value of the industry in the benchmark case, and given that the function $f(\beta) = \left(\frac{\beta-1}{\beta}\right)^{\beta}\frac{2\beta-1}{\beta-1}$ is positive but < 1, we have that the introduction of a supplier in the industry leads to a lower value from an industry prospective. In fact, the presence of B increases the optimal investment threshold for the firm A though the effect of p_1 and at the same time it reduces its value of the option to invest. Moreover, the value added from B to the industry is not enough to compensate for the reduction in the value of A, so that the industry value in the integrated case results to be higher.

1.3. External funding case: external founding and internal input production.

In this section I start by the case presented in section 1.1 (vertical integration), with the difference that here I will assume that the start-up firm does not have the financial resources to undertake the project. So, as in Chesbrough and Schwartz (2007), that reports as the timely investments demands in many cases too much resources for a single, the participation of an investment partner willing to bear part of the costs in exchange of a share of the returns is usually requested. Now, referring to the start-up firm as A and to the outside investor as C, I assume two different scenarios in which the two subjects enter in a leader-follower game. In line with Robert and Berry (1985), the frameworks that I will develop can be thought as a corporate VC that takes a minority equity stake in a relatively new start up, with the aim to value early-stage projects. So, in section 1.3.1 I will assume C is the game leader that submit

[14] $\lim_{\beta\to\infty}\frac{\beta}{\beta-1} \to 1$

the compensation offer and A is the game follower that decides the timing for the investment[15]. This framework it's more appropriate in cases in which the project is highly innovative or very desirable, so that the investment partner makes the first step declaring his interest to invest. While in subsection 1.3.2 I will treat a case similar to the one described by Lukas and Welling (2014), in which C takes the role of game follower (deciding the timing) and A is the game leader deciding the compensation share.

1.3.1. Investment partner as game-leader and potential investor as game-follower.

In the following A and C are assumed to negotiate on the equity share that the former has to give to the latter. Specifically, C is assumed to make an offer at time zero for helping in financing the project in exchange of a participation share $\gamma_F \in (0,1)$ while A has the option to accept and immediately undertake the project or to wait. In other word C will decide the compensation offer and A will choose the optimal timing for the investment taking in consideration the share γ_F.

Going backwards, I start analyzing the choice of A, that sets the optimal investment threshold by solving the following maximization problem:

$$V_{A2}^{F}(y) = max\ _{y_F(\gamma_F)}\left((1-\gamma_F)\frac{y_F(\gamma_F)\pi_M}{r-\alpha} - (1-\theta)I\right)\left(\frac{y}{y_F(\gamma_F)}\right)^{\beta} \qquad (13)$$

With $\theta \in (0,1)$ and representing the exogenously given share of investment cost payed by C in order to participate in a share γ_F of the future profits deriving from the project. From the solution of the (13) we get[16]:

$$y_F(\gamma_F) = \left(\frac{1-\theta}{1-\gamma_F}\right)y^{*} \qquad (14)$$

That represents the optimal investment threshold that once reached from below it triggers the investment. As one can expect, $y_F(\gamma_F)$ is increasing in γ_F and decreasing in θ[17]. In words this

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[15] Similar framework is presented in Cvitanić J, Radas S, Šikić H. (2011).

[16] See appendix 1.3.1.a. for the calculation.

[17] Taking the derivative with respect to γ_F and θ gives: $\frac{d\,y_F(\gamma_F)}{d\gamma_F} = \frac{1-\theta}{(1-\gamma_F)^2}y^{*} > 0$ and : $\frac{d\,y_F(\gamma_F)}{d\theta} = -\frac{y^{*}}{1-\gamma_F} < 0$

means that in order to anticipate the investment it is necessary that C participates in a higher share of the costs, while the investment will be delayed as the compensation share increases. C is facing a dilemma, in fact a high compensation leads to higher return but the cash-in will be delayed, whereas low compensation implies low returns but with shorter waiting period. Given that the investment partner takes in consideration the timing chosen by A, the compensation offer will be derived by the solution of the maximization problem:

$$V_{C2}^{F}(y) = max_{\gamma_F}\left(\gamma_F \frac{y_F(\gamma_F)\pi_M}{r-\alpha} - \theta I\right)\left(\frac{y}{y_F(\gamma_F)}\right)^{\beta} \tag{15}$$

From which we have that the optimal compensation share required by C in order to participate in a share θ of the costs, is given by[18]:

$$\gamma_F = \frac{1 - 2\theta + \theta\beta}{\beta - \theta} \tag{16}$$

As ones can expect γ_F is increasing in θ and decreasing in β [19], this means that a higher level of the cost share payed by C, will correspond to a higher compensation required by the investment partner. Moreover, through the effect of β, an increase in the expected volatility will increase the compensation share, while an increase in the discount factor and in the expected market growth will reduce γ_F. This makes sense, in facts for riskier project an investor will demand more return, while for higher expected growth in the market a lower share can be enough for the investment partner in order to participate.

Now, by substituting back the (16) in the (14) we can write the optimal threshold as:

$$y_F(\gamma_F) = \left(\frac{\beta - \theta}{\beta - 1}\right) y^* \tag{17}$$

Note that the terms $\left(\frac{\beta-\theta}{\beta-1}\right)$ is >1 given that by construction $\theta \in (0,1)$, so the investment will be delayed with respect to the benchmark case presented in Section 1.1. Moreover, interesting enough $y_F(\gamma_F)$ is decreasing both in β and in θ. So here there are two effects regarding θ: the first one is given by the fact that $\frac{d\, y_F(\gamma_F)}{d\theta} < 0$ so increasing the share of the costs payed by C will have a direct effect on $y_F(\gamma_F)$ that will lead to an anticipation of the investment by A, while the second effect is the one previously seen, for which an increasing in the share of the

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[18] See Appendix 1.3.1.b. for calculations.

[19]Taking the derivative with respect to θ and β gives: $\frac{d\gamma_F}{d\theta} = \frac{(\beta-1)^2}{(\beta-\theta)^2} > 0$ and $\frac{d\gamma_F}{d\beta} = -\frac{(1-\theta)^2}{(\beta-\theta)^2} < 0$

costs covered by C leads to an higher compensation (γ_F) that increases $y_F(\gamma_F)$ and so delay the project. These two have opposite effects, however given that $y_F(\gamma_F) > y^*$, overall we can say that the introduction of an investment partner in the framework analyzed induce to a postponing in the project with respect the benchmark case.

Now, regarding the value of the option to invest for the two agents involved, by substituting the (16) and (17) into the equation for $V_{A2}^F(y)$ and $V_{C2}^F(y)$ we obtain:

$$V_{A2}^F(y) = (1-\theta)\left(\frac{\beta-1}{\beta-\theta}\right)^{\beta} V_I(y) \tag{18}$$

And:

$$V_{C2}^F(y) = \left(\frac{\beta-1}{\beta-\theta}\right)^{\beta-1} V_I(y) \tag{19}$$

Note that, because $\left(\frac{\beta-1}{\beta-\theta}\right) < 1$ (given that the investment partner is assumed to cover only a share of the costs, so that $\theta < 1$), none of the above expression are greater than the value of the option to invest in the benchmark case. So in the case where the completion of the project depends on the participation of an agent providing funding, the investment not only will be delayed ($y_F(\gamma_F) > y^*$) but the potential investor's option to invest will be less valuable with respect to the integrated benchmark case: $V_{A2}^F(y) < V_I(y)$.

Now, regarding the industry prospective, by adding the value of the two player we have:

$$V_2^F(y) = V_{A2}^F(y) + V_{C2}^F(y) = (\beta-1)^{\beta-1}\left[\frac{2\beta-1-\theta\beta}{(\beta-\theta)^{\beta}}\right] V_I(y) \tag{20}$$

From which the industry value results to be lower than the benchmark case, in fact being $(\beta-1)^{\beta-1}\left[\frac{2\beta-1-\theta\beta}{(\beta-\theta)^{\beta}}\right]$ positive but less than one, we have that when the presence of an outside investor is needed the industry value is lower than in the integrated case. This has to do with the fact that the delay of the project caused by the interaction between A and C, leads to a reduction in the value of A: $V_{A2}^F(y) < V_I(y)$ and at the same time the value of C is not enough to compensate for this loss of value: $V_{C2}^F(y) < V_I(y) - V_{A2}^F(y)$.

1.3.2. Potential investor as game-leader and investment partner as game-follower.

In this second sub-section, similarly to the previous one, I will assume that the two players engage in a leader-following game, but here the roles will be reversed: A takes the role of game leader and at time zero makes a compensation offer represented by the share $\gamma_L \in (0,1)$ to C, while C as game follower, taking into account the proposal of A, has the option to accept immediately the offer and to pay straightaway an exogenously given share of the costs needed to the project $\theta \in (0,1)$, or it can wait and postpone the contribution. This framework is in line with the one of Lukas and Welling (2014) in which an investment partner is assumed to be willing to participate in a given share of the sunk costs $\theta \in (0,1)$ in exchange of receiving a fraction of the project $\gamma_L \in (0,1)$.

By going backwards, I now start analyzing the problem of C, that taking into consideration the offer of A, decides the optimal timing of the investment by solving the following maximization problem:

$$V_{C2}^{L}(y) = max_{\ y_L(\gamma_L)} \left(\gamma_L \frac{y_L(\gamma_L)\pi_M}{r-\alpha} - \theta I\right)\left(\frac{y}{y_L(\gamma_L)}\right)^{\beta} \qquad (21)$$

From the solution of the above expression[20] we obtain:

$$y_L(\gamma_L) = \frac{\theta}{\gamma_L} y^* \qquad (22)$$

As one can see from the above equation $y_L(\gamma_L)$ is decreasing in the compensation share[21] offered by A, while is increasing in the in the exogenous share of costs that is required to C in order to participate in the investment[22]. This is intuitively right, in fact higher are costs that C will have to pay, and more the investment will be delayed. While, on the other hands, for a higher compensation offered by A, the investment partner will be willing to anticipate the investment. Note in fact from the (22) that for $\theta = \gamma_L$ the investment will be undertaken at the same time than in the integrated benchmark case: $y_L(\gamma_L) = y^*$. Consequently it is easy to see as A is facing a dilemma: on one hand it can offer a lower share of the profits so that to retrain a greater compensation for himself, but this will delay the implementation of the project and so also the generation of the returns, while on the other hands it can offer a higher compensation

[20] See Appendix 1.3.2.a for the calculations.

[21]Taking the first derivative with respect to γ_L: $\frac{d\, y_L(\gamma_L)}{d\gamma_L} = -\frac{\theta}{(\gamma_L)^2} y^* < 0$

[22] Taking the first derivative with respect to θ: $\frac{d\, y_L(\gamma_L)}{d\theta} = \frac{1}{\gamma_L} y^* > 0$

to the investment partner that so shortens the waiting period, but at the cost of give up a higher share of profits.

The above considerations indeed are taken into account by the potential investor, that so sets the compensation offer by solving the following maximization problem:

$$V_{A2}^{L}(y) = max_{\ \gamma_L} \left((1-\gamma_L)\frac{y_L(\gamma_L)\pi_M}{r-\alpha} - (1-\theta)I \right)\left(\frac{y}{y_L(\gamma_L)}\right)^{\beta} \tag{23}$$

From which yields[23]:

$$\gamma_L = \frac{\theta(\beta-1)}{\theta+\beta-1} \tag{24}$$

Note that the optimal compensation that A is willing to offer to C depends both on θ and β. More precisely a higher share of the costs paid by C leads the potential investor to offer a higher compensation in order to anticipate the investment, but this has a limit, in facts also for higher value of θ it is never optimal for A to offer a share $\gamma_L > \theta$. Regarding the effect of β, we have that γ_L is increasing in β, this is important because it means that an increasing in the volatility (σ) or in the drift of the stochastic scale parameter (α) will lead to a decrease in the optimal share offered by A, while at the contrary an increase in the interest rate (*r*) increases the γ_L.

Now, by substituting the (24) inside the (22) we obtain:

$$y_L(\gamma_L) = \frac{\theta+\beta-1}{\beta-1} y^* \tag{25}$$

From which we can see as the optimal investment threshold decided by C is decreasing in β[24] and is always higher than the threshold set in the integrated case $y_L(\gamma_L) > y^*$, given $\theta > 0$ by construction. Note moreover that, and as we seen before, for a higher θ we have a double effect on the timing: through γ_L we can expect an increasing in the compensation offer by A and so an anticipation of the investment while if we examine the effect directly on $y_L(\gamma_L)$ we have that an higher cost share leads to a longer waiting period. The reason is that while γ_L is a concave function of θ, the optimal investment threshold is a linear increasing function of the cost

[23] See appendix 1.3.2.b to the calculations.

[24] Taking the first derivative with respect to β: $\frac{d\, y_L(\gamma_L)}{d\beta} = -\frac{\theta}{(1-\beta\,)^2} < 0$.

share[25], so also if the compensation offer increase, in real terms it worsens, in fact what A is giving up by offering a higher γ_L it is not remunerated by a shorter waiting period.

Now, substituting the (25) and (24) into the equation for the value of the option to invest for the potential investor and the investment partner we have respectively:

$$V_{A2}^{L}(y) = V_I(y)\left(\frac{\beta - 1}{\theta + \beta - 1}\right)^{\beta-1} \tag{26}$$

And:

$$V_{C2}^{L}(y) = V_I(y)\left(\frac{\beta - 1}{\theta + \beta - 1}\right)^{\beta} \theta \tag{27}$$

As we can see, in line with the fact that the investment is postponed the value for the potential investor is lower than in the integrated case: $V_{A2}^{L}(y) < V_I(y)$, recalling that $\theta > 0$ and $\beta > 1$. So, similarly to the previous section we have that the introduction of an investment partner has the effect not only to delay the realization of the project, but also to lower the value of the option to invest for the potential investor, and this is independent from the role chosen by the two agent in the leader-following game examined.

Now, analyzing the situation from an industry prospective, we have that the overall value of the industry is given by:

$$V_2^{L}(y) = V_{A2}^{L}(y) + V_{C2}^{L}(y) = V_I(y)\left(\frac{\beta - 1}{\theta + \beta - 1}\right)^{\beta}\left(\frac{\theta\beta + \beta - 1}{\beta - 1}\right) \tag{28}$$

Note that for $\theta = 0$, so basically in the case there is no participation by the investment partner in the costs of the project, the value is reduced to the one analyzed in the benchmark case. While for higher values of θ, we have that the industry value is decrease, and this means that the value added by C to the industry is not enough to compensate for the loss of value that A has on its option with respect to the integrated case.

1.4. Three-agent case: external founding and external input production.

[25] Given that $\frac{d\, y_L(\gamma_L)}{d\theta} = \frac{1}{1-\beta} > 0$.

In this section I put together the situations examined in sections 1.2 and 1.3. So, I will consider the case in which the potential investor (A) is not capable to produce the input required for the investment and at the same time does not have enough financial resources at his disposal. As consequence of this A will have to interact both with an outside supplier (B) and with an investment partner (C) in order to undertake the project. Now, providing that B will be always considered as game-leader, also here I will analyze different scenarios. In section 1.4.1. I will assume A as time deciding agents and that C submit the compensation offer to participate in a share of the costs of the investment. While in section 1.4.2. the roles will be reverted: A will decide the compensation share to submit to C that in turn will set the investment timing. In both cases, keeping in mind that both A and C are assumed to be able to observe continuously the magnitude of Y_t (while the knowledge of B is limited to the structural parameters of the stochastic process), I will show as the presence of the three agents will affect the timing and the value of the option to invest from the point of view of the potential investor.

1.4.1. Three-agent case with the potential investor as time deciding agent.

Starting with the situation in which the potential investor takes the role of time deciding agent, the three players are assumed to follow these steps:

I. Player B chooses the optimal price (p_3^F) that is willing to take in order to provide the furniture of the discrete input.
II. Given the choice of B, C decides optimally the compensation share (γ_3^F) that is willing to take in order to participate in an exogenously given share (θ) of the investment costs.
III. Given both the choice of B and C, A determine the optimal investment threshold $\left(y_3^F(\gamma_3^F, p_3^F)\right)$, and so the time at which the investment will occur.

Now, by proceeding backwards, I will start with examine first the optimization problem of player A and after the choices of the other two subjects.

Starting with the potential investor, the optimal investment threshold is given by the solution of the following maximization problem:

$$V_{A3}^F(y) = max_{y_3^F(\gamma_3^F, p_3^F)} \left((1-\gamma_3^F) \frac{y_3^F(\gamma_3^F, p_3^F)\pi_M}{r-\alpha} - (1-\theta)p_3^F \right) \left(\frac{y}{y_3^F(\gamma_3^F, p_3^F)} \right)^\beta \qquad (29)$$

Note that the above expression is similar to the (13) in section 1.3.1, the only difference is that now the investment costs are represented by the price p_3^F necessary to acquire the furniture of the input. So, we have that[26]:

$$y_3^F(\gamma_3^F, p_3^F) = \left(\frac{1-\theta}{1-\gamma_3^F}\right)\frac{p_3^F}{I}y^* \tag{30}$$

So, by taking into account the compensation set by the investment partner and the price of the input as given, the potential investor will undertake the project when the optimal threshold $y_3^F(\gamma_3^F, p_3^F)$ is reached from below. Note that the expression for $y_3^F(\gamma_3^F, p_3^F)$ is similar to the one of $y_F(\gamma_F)$ in section 1.3.1., in fact if we consider the case in which $p_3^F = I$ we obtain exactly the (14). Given this, the same considerations made in Section 1.3.1 holds also for this case, with the difference that now the total costs for investing in the project are represented by p_3^F.

Proceeding backward the optimal compensation offer made by C is derived by the solution of the following maximization problem:

$$V_{C3}^F(y) = max_{\gamma_3^F}\left(\gamma_3^F\frac{y_3^F(\gamma_3^F, p_3^F)\pi_M}{r-\alpha} - \theta p_3^F\right)\left(\frac{y}{y_3^F(\gamma_3^F, p_3^F)}\right)^\beta \tag{31}$$

Solving we get[27]:

$$\gamma_3^F = \frac{1-2\theta+\theta\beta}{\beta-\theta} \tag{32}$$

Note that holds the same result as in section 1.3.1, in fact the investment partner do not care about the presence of B, and requires a compensation share that it's independent from the cost of the input. Recall that, as previously seen, the compensation share is increasing in θ and decreasing in β.

Finally, the upstream supplier anticipates the choice of the other two players, and it sets the optimal price for the furniture of the discrete input by solving the following maximization problem:

$$V_{B3}^F(y) = max_{p_3^F}(p_3^F - I)\left(\frac{y}{y_3^F(\gamma_3^F, p_3^F)}\right)^\beta \tag{33}$$

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[26] Note that the procedure to retrieve $y_3^F(\gamma_3^F, p_3^F)$ is the same as the one used in section 1.3.1. to get $y_F(\gamma_F)$. See Appendix 1.3.1.a.

[27] The procedure to obtain γ_3^F follows the same calculation made in Appendix 1.3.1.b.

Note that, the value of the option to invest of B it is independent from the presence of an investment partner and the solution of the above expression is given by:

$$p_3^F = \frac{\beta}{\beta - 1} I \tag{34}$$

That it's equal to the optimal price retrieved in Section 1.2. The intuition here is simple, the outside supplier does not care the way in which the project is financed as long as the price for the furniture of the input is payed. Note moreover that, though the effect of β, higher volatility (σ) lead to higher price, and the same holds also for the expected growth in the market (α)[28].

By combining the (30), (32) and (34) the explicit form for the optimal investment threshold is given by:

$$y_3^F(\gamma_3^F, p_3^F) = \frac{\beta(\beta - \theta)}{(\beta - 1)^2} y^* \tag{35}$$

Note that the terms $\frac{\beta(\beta-\theta)}{(\beta-1)^2}$ is greater than one given that $\beta > 1$ and $\theta < 1$ by construction, so as one can expect, the introduction of two agents leads to a postponement of the project with respect to the benchmark case examined in Section 1.1. Moreover note that $y_3^F(\gamma_3^F, p_3^F)$ is decreasing in θ[29], this is intuitive, in fact more the investment partner participates in the costs of the project and earlier this will be undertaken by A.

Now, substituting the optimal investment threshold obtained in the (35) into the equations for the value of the option to invest of the three agents, we obtain:

$$V_{A3}^F(y) = (1 - \theta)\left(\frac{\beta - 1}{\beta - \theta}\right)^{\beta}\left(\frac{\beta - 1}{\beta}\right)^{\beta - 1} V_I(y) \tag{36}$$

$$V_{B3}^F(y) = \left(\frac{\beta - 1}{\beta - \theta}\right)^{\beta}\left(\frac{\beta - 1}{\beta}\right)^{\beta} V_I(y) \tag{37}$$

$$V_{C3}^F(y) = \left(\frac{\beta - 1}{\beta - \theta}\right)^{\beta - 1}\left(\frac{\beta - 1}{\beta}\right)^{\beta - 1} V_I(y) \tag{38}$$

From these last equations we can observe that in the case in which the project's completion is subordinated to the presence of both an outside supplier (B) producing the input and an

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[28] As reported in de Villemeur Billette et al. (2013) the drift (α) of the stochastic scale parameter Y_t can be thought as the market's expected growth rate.

[29] Given that $\frac{dy_3^F(\gamma_3^F, p_3^F)}{d\theta} = -\frac{\beta}{(\beta-1)^2}$.

investment partner (C) providing external founding, this has the effect to delay the timing of the investment ($y_3^F(\gamma_3^F, p_3^F) > y^*$) and at the same time to reduce the value of the option to invest for the potential investor (A). Note in fact that $V_{A3}^F(y) < V_I(y)$ given $\beta > 1$ and $\theta < 1$.

Now, analyzing the situation from an industry prospective, by adding the (36), (37) and (38) we get:

$$V_3^F(y) = V_{A3}^F(y) + V_{B3}^F(y) + V_{C3}^F(y) = \\ = \left(\frac{\beta - 1}{\beta}\right)^\beta \left(\frac{\beta - 1}{\beta - \theta}\right)^\beta \left[\frac{\beta^2(3-\theta) - 3\beta + 1}{(\beta - 1)^2}\right] V_I(y) \tag{39}$$

Note that, the (39) is always lower than the value of the industry in the benchmark case, in fact the component resulting by taking $\frac{V_3^F(y)}{V_I(y)} = \left(\frac{\beta-1}{\beta}\right)^\beta \left(\frac{\beta-1}{\beta-\theta}\right)^\beta \left[\frac{\beta^2(3-\theta)-3\beta+1}{(\beta-1)^2}\right]$ is always lower than 1. This means that the additional value given by the introduction of two more agents in the industry is not sufficient to compensate for the decreasing in value in the option to invest for A: $V_{B3}^F(y) + V_{C3}^F(y) < V_I(y) - V_{A3}^F(y)$.

1.4.2. Three-agent case with the investment partner as time deciding agent.

In this second sub-section I will invert the roles of the investment partner (C) and the potential investor (A). I will so expand the framework analyzed in sub-section 1.3.2 by considering now that in order to undertake the project A will have to acquire the input from an upstream supplier (B). So, by considering C as time deciding agent, the three players are assumed to interact following these steps:

I. Player B chooses the optimal price (p_3^L) that is willing to receive in order to provide the furniture of the discrete input.

II. Given the choice of B, A decides optimally the compensation share (γ_3^L) that is willing to offer to C in order to have him participating in an exogenously given share (θ) of the investment costs.

III. Finally, given both the choice of B and A, C will choose the optimal time to accept the offer and to provide the funds needed to the acquisition of the input, determining so the optimal investment threshold $\left(y_3^L(\gamma_3^L, p_3^L)\right)$.

By starting from the behavior of the investment partner, the optimal investment threshold is given by the solution of the following maximization problem:

$$V_{C3}^{L}(y) = max_{y_3^L(\gamma_3^L, p_3^L)}\left(\gamma_3^L \frac{y_3^L(\gamma_3^L, p_3^L)\pi_M}{r-\alpha} - \theta p_3^L\right)\left(\frac{y}{y_3^L(\gamma_3^L, p_3^L)}\right)^{\beta} \tag{40}$$

Note that the above expression is similar to the (21) in section 1.3.2, the only difference is that now the costs payed by C are represented by θp_3^L, and solving we have that[30]:

$$y_3^L(\gamma_3^L, p_3^L) = \frac{\theta}{\gamma_3^L}\frac{p_3^L}{I} y^* \tag{41}$$

So, by taking into account the compensation offer made by the potential investor and the price for the furniture of the input as given, the investment partner will provide the funds needed to the project when the optimal threshold $y_3^L(\gamma_3^L, p_3^L)$ is reached from below. Here it is easy to see as if the compensation share (γ_3^L) and the share of the costs payed by C (θ) are equal, the investment will be anyway postponed with respect to the benchmark case in Section 1.1 proportionally with the magnitude of the price p_3^L necessary to pay the furniture of the input. This highlight the impact of presence of the outside supplier (B) that, as we had seen in section 1.2, in order to have positive profit will charge a price for the furniture greater than the pure costs I, leading so to a further delay in the project. Note moreover that, similarly to the (22) the investment threshold is increasing in θ and decreasing in γ_3^L, this makes sense in fact higher are the costs to sustain for C, and more time usually it takes to provide the funds, while on the other hands higher is the compensation share and more the investment partner is impatient to undertake the investment.

Proceeding backward the potential investor, anticipates the choice of C about the timing and set the compensation offer as the solution of the following maximization problem:

$$V_{A3}^{L}(y) = max_{\gamma_3^L}\left((1-\gamma_3^L)\frac{y_3^L(\gamma_3^L, p_3^L)\pi_M}{r-\alpha} - (1-\theta)p_3^L\right)\left(\frac{y}{y_3^L(\gamma_3^L, p_3^L)}\right)^{\beta} \tag{42}$$

From which we get:

$$\gamma_3^L = \frac{\theta(\beta-1)}{\theta+\beta-1} \tag{43}$$

[30] Note that the procedure to retrieve $y_3^L(\gamma_3^L, p_3^L)$ is the same as the one used in section 1.3.2. to get $y_L(\gamma_L)$. See Appendix 1.3.2.a.

Note that holds the same result as the (24) in section 1.3.2, in fact given the exogeneity of the share costs (θ), the potential investor do not care about the presence of B, and so it requires a compensation share that it's independent from the cost of the input.

Finally, the upstream supplier anticipates the choice of the other two agents and set the optimal price for the furniture of the discrete input by solving the following maximization problem:

$$V_{C3}^{L}(y) = max_{p_3^L}(p_3^L - I)\left(\frac{y}{y_3^L(\gamma_3^L, p_3^L)}\right)^{\beta} \tag{44}$$

Note that, the value of the option to invest for B it is independent from the presence of an investment partner, and the solution of the above expression is given by:

$$p_3^L = \frac{\beta}{\beta - 1}I \tag{45}$$

That it's equal to the optimal price retrieved in Section 1.2. where C was missing. In fact, the outside supplier does not care the way in which the project is financed as long as the price for the furniture of the input is totally payed. Note moreover that, though the effect of β[31], higher volatility (σ) and higher expected growth in the market (α) lead both to a higher price.

Now, by substituting the (43), (45) into the (41) we have that the optimal investment threshold can be written as:

$$y_3^L(\gamma_3^L, p_3^L) = \frac{\beta(\theta + \beta - 1)}{(\beta - 1)^2}y^* \tag{46}$$

From the above expression it is possible to note as the terms $\frac{\beta(\theta+\beta-1)}{(\beta-1)^2} > 1$, given that $\theta > 0$ and $\beta > 1$ by construction. So, similarly to the previous section where the roles of A and C were reverted, also in this framework the introduction of two more agents leads to a delay in the project with respect to the integrated case (section 1.1.).

Interesting enough is the effect of θ on the timing of the investment: on one hand, unlike $y_3^F(\gamma_3^F, p_3^F)$ in the previous section, the optimal investment threshold is increasing in θ[32], on the other hands an higher costs share (θ) leads also to an higher compensation offer by A that should result in a shorter waiting period. The reason lies on the fact that while γ_3^L is a concave function of θ, $y_3^L(\gamma_3^L, p_3^L)$ is a linear function of θ, so in real terms an increase in the cost share

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[31] Taking the derivative w.r.t. β we have: $\frac{dp_3^L}{d\beta} = -\frac{I}{(\beta-1)^2}$.

[32] Given that $\frac{dy_3^L(\gamma_3^L, p_3^L)}{d\theta} = \frac{\beta}{(\beta-1)^2}$

worsen the situation: what A sacrifice giving up a higher share of the future profits is not compensated by sufficient acceleration of the investment. This is the same reasoning made in section 1.3.2 where the input was internally produced, with the only difference that here, because of the presence of B, the effect on the timing is exacerbated. Note in fact that by comparing the slopes of $y_3^L(\gamma_3^L, p_3^L)$ and $y_L(\gamma_L)$ with respect to θ, we have that the former is $\frac{\beta}{1-\beta}$ times greater than the latter[33] and that the terms $\frac{\beta}{1-\beta}$ represent what the supplier charges on the price to provide the furniture of the input.

Now, by substituting the (43), (45) and (46) into the equations of the values of the option to invest for the three agents, we obtain:

$$V_{A3}^L(y) = \left(\frac{\beta-1}{\beta-1+\theta}\right)^{\beta-1} \left(\frac{\beta-1}{\beta}\right)^{\beta-1} V_I(y) \tag{47}$$

$$V_{B3}^L(y) = \left(\frac{\beta-1}{\beta-1+\theta}\right)^{\beta} \left(\frac{\beta-1}{\beta}\right)^{\beta} V_I(y) \tag{48}$$

$$V_{C3}^L(y) = \theta\left(\frac{\beta-1}{\beta-1+\theta}\right)^{\beta} \left(\frac{\beta-1}{\beta}\right)^{\beta-1} V_I(y) \tag{49}$$

From the above expressions it is easy to observe that when the realization of the project is subordinated by the presence of both an investment partner (C) helping in financing the project and a supplier (B) providing the furniture of the input, this leads to a delay in the investment with respect to the benchmark case in section 1.1 ($y_3^L(\gamma_3^L, p_3^L) > y^*$) and at the same times it reduces the value of the option to invest for the potential investor (A)[34].

Finally, analyzing the situation from an industry prospective, summing up the (47), (48) and (49) we get:

$$\begin{aligned} V_3^L(y) &= V_{A3}^L(y) + V_{B3}^L(y) + V_{C3}^L(y) = \\ &= \left(\frac{\beta-1}{\beta}\right)^{\beta-1} \left(\frac{\beta-1}{\beta-1+\theta}\right)^{\beta} \left[\frac{\beta^2(2+\theta)-3\beta+1}{\beta(\beta-1)}\right] V_I(y) \end{aligned} \tag{50}$$

The above expression represents the value generated in the industry by the project. Note as this value is always lower than the value of the industry in the benchmark case: $\frac{V_3^L(y)}{V_I(y)} < 1$. In words

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[33] Comparing the slopes: $\frac{\frac{dy_3^L(\gamma_3^L, p_3^L)}{d\theta}}{\frac{dy_L(\gamma_L)}{d\theta}} = \frac{\beta}{1-\beta}$

[34] Note that $\left(\frac{\beta-1}{\beta}\right)^{\beta-1} \left(\frac{\beta-1}{\beta-1+\theta}\right)^{\beta-1} < 1$.

this means that the additional value given by the introduction of two more agents in the industry is not sufficient to compensate for the decreasing in value in the option to invest for A: $V_{B3}^{L}(y)+V_{C3}^{L}(y)<V_{I}(y)-V_{A3}^{L}(y)$.

Chapter 2: Comparison between cases.

In this second chapter of my thesis I will sum up the main results obtained from the previous sections of Chapter 1 and I will compare the different cases both from a timing and a value prospective. The sections are organized as follows: in Section 2.1 I will recall the main results about the timing of the investment, making some consideration between the different situations analyzed, in Section 2.2 I will sum up the finding about the value of the option to invest both from a potential investor (A) and from an industry prospective, finally in Section 2.3, I will run a simulation using the MATLAB compiler, in order to give a better sense of the magnitude of the changing between the different cases.

2.1. Analysis on the investment timing.

In this first section I will proceed by analyzing the results about the optimal investment threshold obtained in the previous chapter. Recall that this threshold is the one that, ones reached from below trigger the investment in the project, so a higher threshold implies a longer waiting period, while for lower values we will have an anticipation of the project. The following table sum up the major results in Chapter 1:

Benchmark/Integrated Case:	$y^* = \frac{r-\alpha}{\pi_M}\frac{\beta}{\beta-1}I$
Separated Case:	$y_1(p_1) = \frac{\beta}{\beta-1}y^*$
External Funding Case: (Potential investor as time deciding agent)	$y_F(\gamma_F) = \frac{\beta-\theta}{\beta-1}y^*$
External Funding Case: (Investment partner as time deciding agent)	$y_L(\gamma_L) = \frac{\beta-1+\theta}{\beta-1}y^*$
Three-agent Case: (Potential investor as time deciding agent)	$y_3^F(\gamma_3^F, p_3^F) = \frac{\beta(\beta-\theta)}{(\beta-1)^2}y^*$
Three-agent Case: (Investment partner as time deciding agent)	$y_3^L(\gamma_3^L, p_3^L) = \frac{\beta(\beta-1+\theta)}{(\beta-1)^2}y^*$

Recalling that $\beta > 1$, we can easily observe as an increase in the number of agents involved leads to a delay of the project with respect to the optimal threshold in the Integrated/Benchmark case in Section 1.1. of Chapter 1. So, generally, we can report the following ranking:

$$y^* < \left(y_F(\gamma_F) \lesseqqgtr y_L(\gamma_L)\right) < y_1(p_1) < \left(y_3^F(\gamma_3^F, p_3^F) \lesseqqgtr y_3^L(\gamma_3^L, p_3^L)\right)$$

As we can see, more interesting is to analyze the timing in the cases in which the numbers of players involved are the same, note in fact that in both the "External Funding Case" and the "Three-gent Case" we have analyzed two different framework. First, in Sections 1.3.1 and 1.4.1, I have assumed the investment partner (C) submit the compensation offer to the potential investor (A) that decides the timing of the investment. Second, in Sections 1.3.2 and 1.4.2, I have reverted the roles of the two players, so considered A as the agent setting the compensation offer and C deciding the optimal timing for the disbursement of the funds needed to undertake the investment. As one can note in what reported on the table above, the effect that the exogenously given share of the costs (θ) has on the ranking of the timing depends on the framework analyzed. In fact, recalling that we assumed $0 < \theta < 1$, by comparing the two situations in both the "External Funding Case" and in the "Three-gent Case" we have that:

$$\frac{y_F(\gamma_F)}{y_L(\gamma_L)} = \frac{y_3^F(\gamma_3^F, p_3^F)}{y_3^L(\gamma_3^L, p_3^L)} = \frac{\beta - \theta}{\beta - 1 + \theta} \lesseqqgtr 1$$

So, from a timing prospective, we have that for $0 < \theta < \frac{1}{2}$ it is more convenient for the potential investor to set the compensation offer and to let the investment partner decide the timing. In this case the ranking becomes:

$$y^* < y_L(\gamma_L) < y_F(\gamma_F) < y_1(p_1) < y_3^L(\gamma_3^L, p_3^L) < y_3^F(\gamma_3^F, p_3^F)$$

While for $\frac{1}{2} < \theta < 1$ the potential investor accelerates the realization of the project by taking the role of time deciding agent. In this case the ranking becomes:

$$y^* < y_F(\gamma_F) < y_L(\gamma_L) < y_1(p_1) < y_3^F(\gamma_3^F, p_3^F) < y_3^L(\gamma_3^L, p_3^L)$$

Note however that for $\theta = \frac{1}{2}$ we will have that the potential investor will be indifferent in the role to pick up, in fact in this case we will have that the investment threshold will be the same, no matter which of the two framework are considered, so $y_F(\gamma_F) = y_L(\gamma_L)$ and $y_3^F(\gamma_3^F, p_3^F) = y_3^L(\gamma_3^L, p_3^L)$.

Another useful observation about the timing can be sought by noting that where there are two agents, in order to anticipate the investment, it is always preferable for the potential investor interact with C, in fact the presence of the supplier (B) has always the effect to delay the investment threshold by the same coefficient $\frac{\beta}{\beta-1}$. This has to do with the fact that first, as already seen, B do not care in which way the project is financed as long as A pays the price for the furniture, second this is the amount that the upstream firm charges on the "pure costs" (I) given that it has the monopoly on the discrete input. As we will see in the next Chapter things may change in the case there is a competitor that is also able to provide the furniture to A.

2.2. Analysis on the value of the option to invest.

In this section I will procced by analyzing the result obtained in Chapter 1, providing some consideration both from an individual prospective and from an industry prospective.

By starting to analyze the situation form the single agents prospective, we have that, by studying the values of the options to invest derived in Chapter 1, for the different players involved it is possible to establish the following ranking:

Agent A	$V_I(\mathrm{y}) > V_{A2}^L(y) > V_{A1}(y) > V_{A3}^L(y) > V_{A2}^F(y) > V_{A3}^F(y)$
Agent B	$V_{B1}(y) > \left(V_{B3}^F(y) \lesseqgtr V_{B3}^L(y)\right)$
Agent C	$V_{C2}^F(y) > V_{C3}^F(y) > V_{C2}^L > V_{C3}^L(y)$

As we can see from the above table, some interesting considerations can be made.

Regarding the potential investor (A), first note that an increase in the number of agents involved leads to a delay in the project (as reported in the previous section), but not always leads to a lower value of the option to invests. In facts, also if the integrated case is the most favorable and the separated case is always better than the cases in which all three players are involved, we can see as, from a value prospective, the situation in which A takes the role of leader in the relation with the investment partner, is always preferable than the case where A is the game follower, no matter if there is or not the additional presence of B. In other word, we have that by considering A as time deciding agent (as in sections 1.3.1 and 1.4.1) the value of the option to invest for the potential investor is always lower than the case in which A set the compensation offer (as in sections 1.3.2 and 1.4.2) even if in the former scenario we take the situation in which only two players are involved, versus the latter case in which all three agents are considered. Moreover, note that these consideration holds independently from the exogenously given share of costs covered by the investment partner.

Regarding the outside supplier (B), we can see how the increasing in the number of agents involved (so as the additional presence of the investment partner) has the effect to both delay the investment and to reduce the value of the option to invest. Anyway, when we consider the presence of all three players, the ranking of the value of B depends indirectly also on the type of relation between the investment partner and the potential investor. Indeed, on one hand, for a given share of the investment costs $\left(0 < \theta < \frac{1}{2}\right)$ covered by C, the value of the option of B is higher in the case in which A takes the role of game leader in interacting with C $\left(V_{B3}^{L}(y) > V_{B3}^{F}(y)\right)$; while on the other hands, for a given share of the investment costs $\left(\frac{1}{2} < \theta < 1\right)$, the value of the option of B is higher in the case in which A takes the role of game follower $\left(V_{B3}^{F}(y) > V_{B3}^{L}(y)\right)$. This is coherent with the ranking of the timing made in the previous section, in fact a delay of the investment for B means a longer waiting period before start collecting the profits from the selling of the discrete input to the potential investor.

Regarding the investment partner (C), we can analyze the situation by two different points of view. First, if we consider that the type of interaction between A and C does not change, the addition of an outside supplier providing the furniture of the input leads generally to a delay of the investment and so to a lower value of the option to invest for the investment partner. Second, if we compare the type of interaction between the investment partner and the potential investor, we can see that it is always preferable for C to set the optimal compensation offer for participate in a given share of the investment costs, letting so A deciding the timing of the investment, all this independently from the presence of B.

Analyzing now the situation from an industry prospective, the following table sum up the major results obtained in Chapter 1:

Benchmark/Integrated Case:	$V_I(\mathrm{y}) = \left(\frac{\mathrm{y}}{y^*}\right)^{\beta} \frac{I}{\beta - 1}$
Separated Case:	$V_1(y) = \left(\frac{\beta - 1}{\beta}\right)^{\beta} \frac{2\beta - 1}{\beta - 1} V_I(y)$
External Funding Case: (Potential investor as time deciding agent)	$V_2^F(y) = (\beta - 1)^{\beta - 1} \left[\frac{2\beta - 1 - \theta\beta}{(\beta - \theta)^{\beta}}\right] V_I(y)$
External Funding Case: (Investment partner as time deciding agent)	$V_2^L(y) = \left(\frac{\beta - 1}{\theta + \beta - 1}\right)^{\beta} \left[\frac{\theta\beta + (\beta - 1)}{\beta - 1}\right] V_I(y)$

Three-agent Case: (Potential investor as time deciding agent)	$V_3^F(y) = \left(\frac{\beta-1}{\beta}\right)^{\beta}\left(\frac{\beta-1}{\beta-\theta}\right)^{\beta}\left[\frac{\beta^2(3-\theta)-3\beta+1}{(\beta-1)^2}\right]V_I(y)$
Three-agent Case: (Investment partner as time deciding agent)	$V_3^L(y) = \left(\frac{\beta-1}{\beta}\right)^{\beta-1}\left(\frac{\beta-1}{\beta-1+\theta}\right)^{\beta}\left[\frac{\beta^2(2+\theta)-3\beta+1}{\beta(\beta-1)}\right]V_I(y)$

By comparing the value of the industry in the different cases on table above, it is possible to obtain the following general ranking:

$$V_I(\mathrm{y}) > \left(V_2^F(y) \lesseqgtr V_2^L(y)\right) > V_1(y) > \left(V_3^F(y) \lesseqgtr V_3^L(y)\right)$$

In line with what we have seen in the comparison between the investment thresholds, generally an increase in the number of players involved in the realization of the project leads to a delay of the investment and, as we can see from above, to a lower industry value. Again, the most favorable scenario is represented by the benchmark case analyzed in section 1.1 of Chapter 1. Moreover, when two players are considered, we can observe that it is always preferable to interact with an investment partner rather than with an outside supplier. Interesting is the comparison of the situations presented in Section 1.3.1 and Section 1.4.1 (where the potential investor takes the role of game follower in the interaction with the investment partner) with those evaluated respectively in Section 1.3.2 and Section 1.4.2 (where the role of the two agents are reverted). Here the analysis is similar to the one made for the investment timing: for values of the exogenously given share of the costs (θ) covered by the investment partner between zero and one-half we have that the realization of the project is anticipated if A takes the role of time deciding agent (as in Section 1.3.1 and Section 1.4.1), consequently in this case the above ranking will be:

$$V_I(\mathrm{y}) > V_2^F(y) > V_2^L(y) > V_1(y) > V_3^F(y) > V_3^L(y)$$

While for values of θ between one-half and one, the things are reverted and the anticipation of the investment will occurs in the cases in which the potential investor sets the compensation offer (as in Section 1.3.2 and 1.4.2), so the ranking will be:

$$V_I(\mathrm{y}) > V_2^L(y) > V_2^F(y) > V_1(y) > V_3^L(y) > V_3^F(y)$$

Note moreover that for $\theta = \frac{1}{2}$ similarly to what we have seen for the investment threshold in the previous section, we have that $V_2^L(y) = V_2^F(y)$ and $V_3^L(y) = V_3^F(y)$.

2.3. Numerical simulation.

In order to give a sense of the magnitude of the results obtained in Chapter 1, and to better understand the effect that a delay in the project can have on the value of the option to invests both from an individual prospective and from an industry prospective, I will first provide a simulation by assuming as references the following values and afterwards I will go to see how a variation in the parameters of β can affects the results. Starting by assuming:

- Initial level of the stochastic parameter $Y_0 = y$ and instantaneous monopoly profit per unit of Y_t, π_M are both assumed to be unitary ($Y_0 = \pi_M = 1$).
- Sunk costs of the investment are assumed to be: $I = 100$.
- Expected growth rate of the market: $\alpha = 0.02$.
- Volatility: $\sigma = 15\%$;
- Interest rate: $r = 3\%$.
- Share of the investment costs financed by the investment partner (when is considered): $\theta = 40\%$.

The following table shows what has been obtained[35]:

TABLE 1

Cases	Threshold	VA	VB	VC	Industry_Value
'Benchmark'	4.451	50.302	0	0	50.302
'Outsourcing'	19.811	32.635	7.332	0	39.967
'External_funding_A as time agent'	13.667	7.1011	0	36.341	43.442
'External_funding_C as time agent'	10.595	39.124	0	6.5743	45.698
'Three_agents_A as time agent'	60.833	4.6071	1.7251	23.577	29.91
'Three_agents_C as time agent'	47.159	25.383	2.3957	4.2653	32.044

As one can observe from Table 1, the benchmark situation, that represents the integrated case in section 1.1. of Chapter 1, is always the best alternative both in terms of timing and value. Moreover, it is possible to remark that because we assumed an exogenously given share of the costs $0 < \theta < 50\%$ we have that, a confirmation of the analysis reported in the section above, it is better for the potential investor to take the role of time deciding agent in the relation with the investment partner, consequently on the fact that in this case the project is anticipated and so the value of his option is higher. Note now that the introduction of one more agents with

[35] The simulations are made by using a program in MATLAB, check Appendix 1.5.

respect to the benchmark case leads to a significant delay of the investment: in the case in which the production of the input is entrusted to an external supplier (outsourcing case in the table) the threshold for the investment is more than fourfold, while, depending on the framework, it is more than doubled or tripled in the cases where there is the introduction of an investment partner. However, the worse effect from a timing prospective is obtained when the interaction of all three agents is required in order to undertake the project: the investment threshold is about ten time higher in the case in which the investment partner takes the role of time deciding agent in the relation with the potential investor and even higher if the roles between the two are reversed.

Regarding the value of the option to invest, the same considerations made in the previous section holds. More precisely, as we can see from the table, taking as reference the agent A, the effect of the introduction of an outside supplier leads to a reduction of more 30% of the value with respect to the benchmark case, while if we add instead the presence of an investment partner, considering A as time deciding agent (A as follower) we have a reduction in the value of more than 85%, while by considering C as time deciding agent (A as leader) the reduction of value is around 20% with respect to the benchmark case. Note moreover that if keep as reference the same framework between the potential investor and the investment partner, we have that in both cases the additional presence of a supplier leads to the same percentage reduction in the value of the option of A[36], that with the above values is roughly 35%.

Finally, for what concerns the value of the industry, as we have seen analytically, generally an increase in the number of the agents involved leads to a decrease in the value of the option to invest. However here the effects are less dramatic, in fact the industry value is the sum of the option to invest for the different agents involved and we can see as, in the cases where A has an important reduction of value, this reduction is partially compensated by the presence of one or more agents.

Now, keeping as reference the results in Table 1, I will provide an analysis of how the investment threshold and the value of the option to invest for the potential investors can change if, ceteris paribus, some parameters vary.

2.3.1 Effects of a change in the drift.

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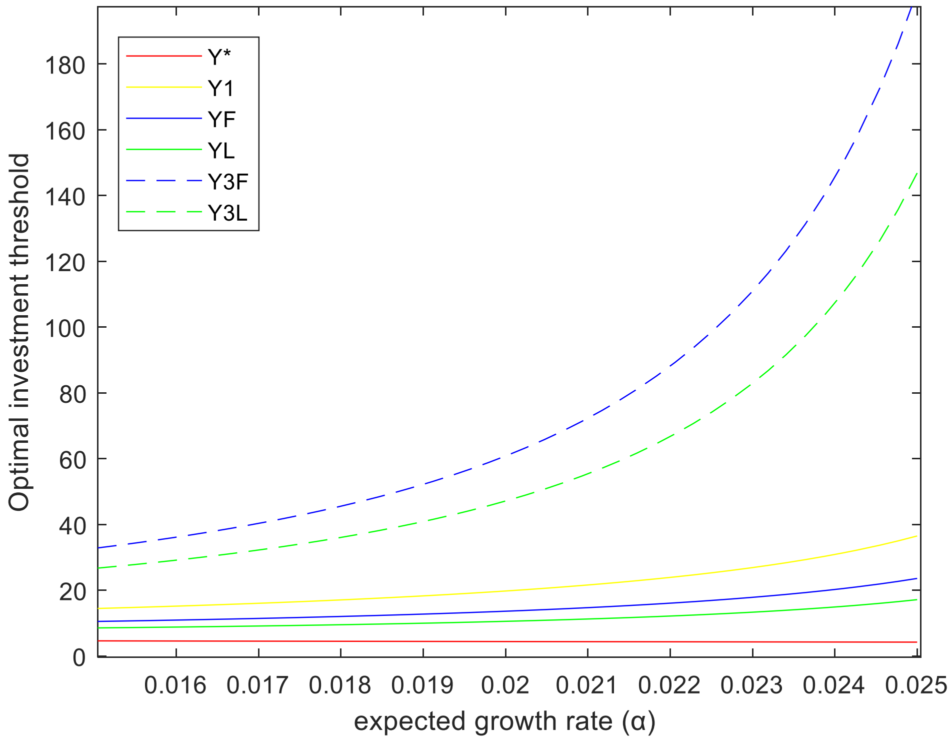
[36] In fact, taking the usual notation it easy to observe: $\frac{V_2^L(y)}{V_3^L(y)} = \frac{V_2^F(y)}{V_3^F(y)} = \left(\frac{\beta-1}{\beta}\right)^{\beta-1}$

By starting from a variation in the drift α, that can be thought as a parameter indicating the expected growth in the market[37], the following two figures show respectively the variation in the investment threshold (Figure 1) and in the value of the option to invest for the potential investor (Figure 2) for α in the interval [0.015 , 0.025].

FIGURE 1 – Effect of a change in α on the investment threshold



From the above figure we can observe as generally an increase in the drift α leads to an increase in the investment threshold and to a delay in the project in almost all the cases examined. Interesting in fact is the integrated case (represented by the red line), for which an increasing in α has the effect to slightly anticipate the realization of the investment. The reason has to do with the interaction of two different forces. By recalling the (3) in Section 1.1 of chapter 1, we can rewrite the expression of y^* as:

$$\frac{\pi_M}{r-\alpha} y^* = \frac{\beta}{\beta-1} I$$

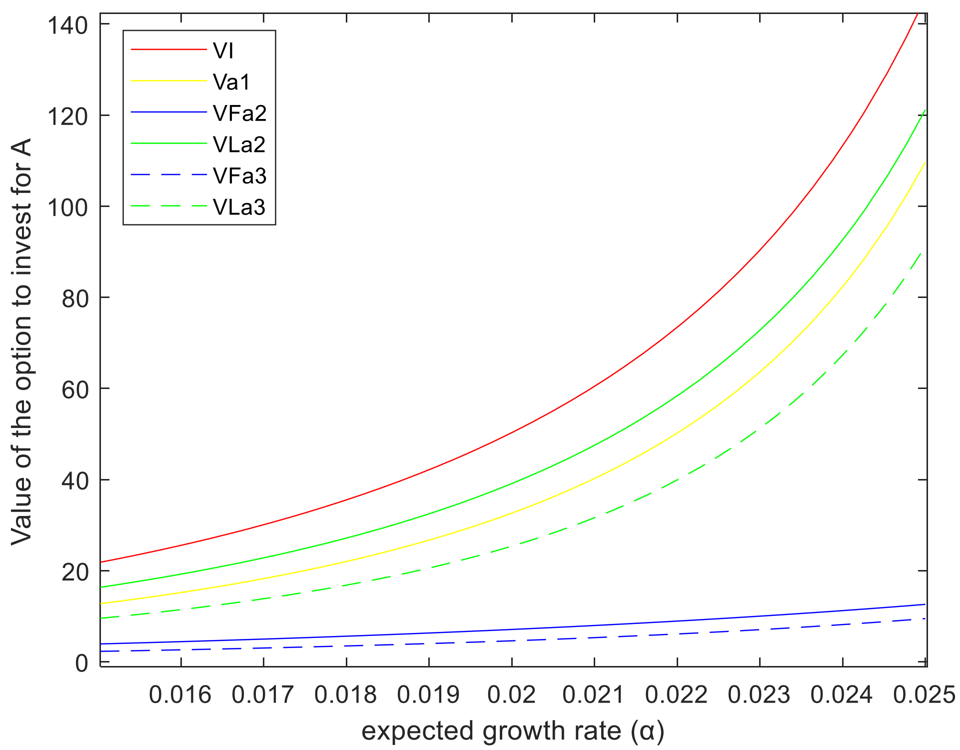
Where the right hands side represents the profits of A and the left hands side represents the costs of the investment. Now, note that an increase in α on one hand imply an increase in the terms $\frac{\beta}{\beta-1}$ that so would lead to higher costs and a delay in the project, while on the other hand

[37] As reported in Billette de Villemeur et al. (2013).

it imply also an higher present value of the profit flows $\frac{\pi_M}{r-\alpha}$ that favors the anticipation of the investment. As it is possible to observe by the simulation, the second effect seems to prevail. Note moreover that these two forces are present in all the threshold examined, but more agents are introduced and more the effect generating though β seem to be strong. As consequence of this, increasing α and the numbers of player involved leads to an exponential increasing in the investment threshold and so to a longer waiting period for the project's realization.

Now, the following figure analyze the effect that α has on the value of the option to invest for the potential investor (A).

FIGURE 2 – Effect of a change in α on the value of the option to invest for A.



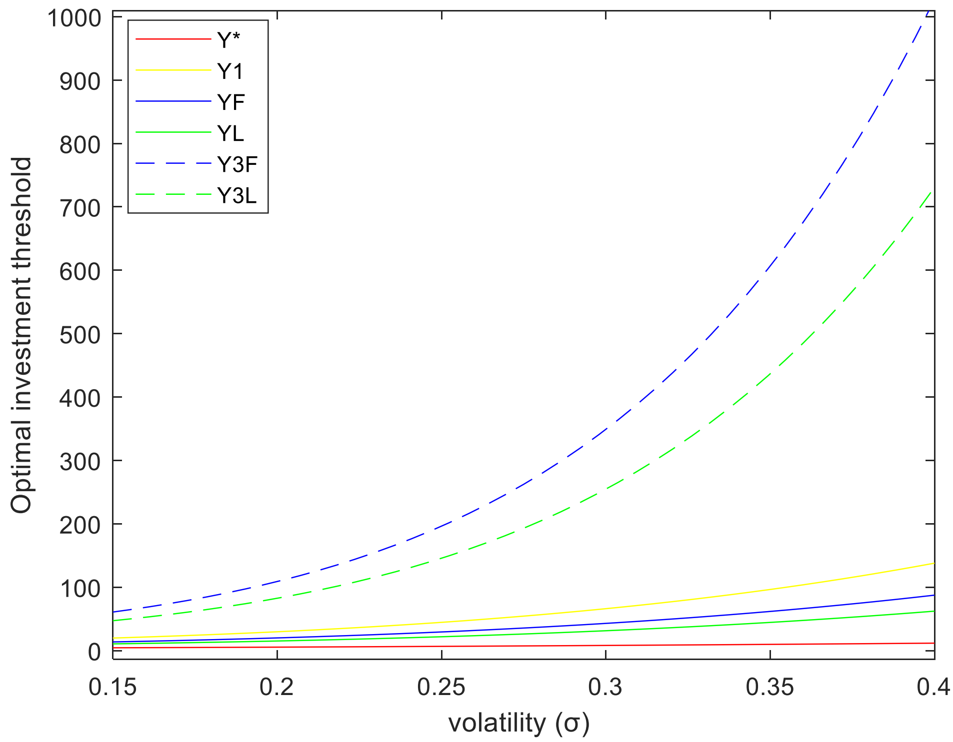
From the above figure at first it is evident that the ranking of the option to invest is the same to the one reported in section 2.2 of this chapter, one established this, we can see as, overall, an increasing in α leads, through its effect on the investment threshold, to a higher value of the option to invest. This result however is different in its intensity, in fact we can see as in the cases in which the potential investor takes the role of time deciding agents in the relation with the investment partner the effect of a variation in the parameter α in the interval considered does not affect so much the value of the option to invest, while in the other cases we have an important increase in value as the expected market growth rate increases. This because in the cases in which is the potential investor set the timing of the investment (two blue lines in the graph), the investment partner (C) decides the compensation share to submit to A, so that because C anticipate that the project will be delayed it set an higher compensation share that so reduces drastically the remain profit for the potential investor. Note however that this last effect is counterbalanced by the fact that, generally, an increase in the investment threshold makes the value of the option to be more valuable. Moreover, in the opposite scenario, where the investment timing is set by the investment partner (two green lines in the graph), the value of the compensation share is derived by A, that in the same way, because anticipates that the investment will be delayed, sets a compensation share that is lower as α, (and so the waiting period), increases so that to retain an higher share of the profits.

2.3.2 Effects of a change in the volatility.

Now, I will analyze the variation that a change in volatility can have on both the investment threshold (Figure 3) and on the value of the option to invest (Figure 4), by assuming σ takes the interval of values [0.15, 0.4].

Starting with the analysis of the timing of the investment the following figure represent the different levels of the optimal investment threshold if, ceteris paribus, we have a variation of the volatility parameter (σ).

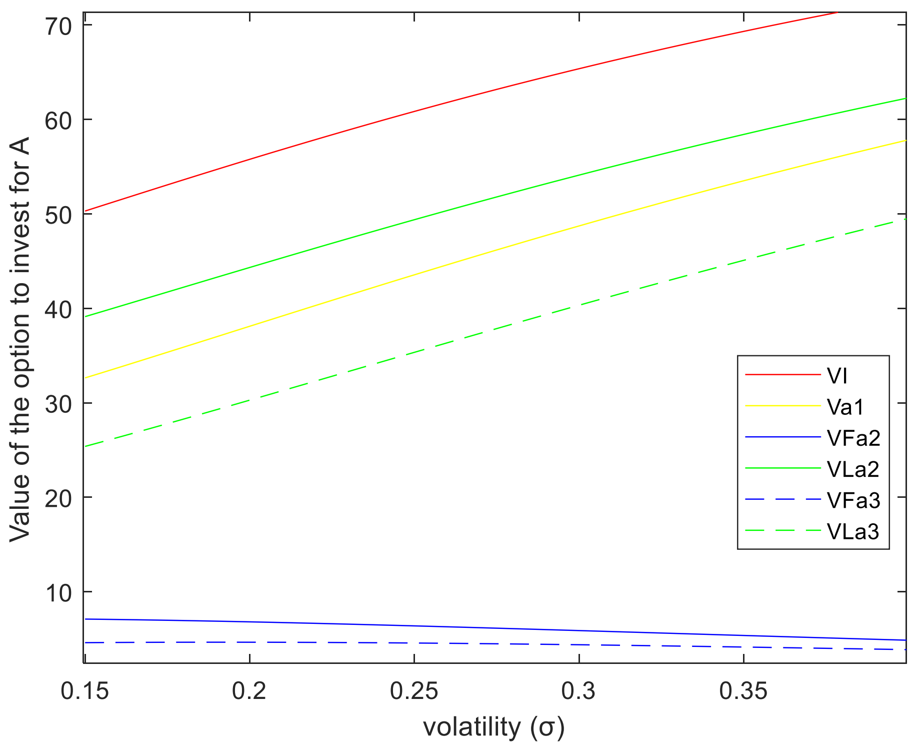
FIGURE 3 – Effect of a change in σ on the investment threshold



From the figure above we can see that, overall, an increasing in the level of volatility leads to an increase in the investment threshold and so to a delay in the project. This is intuitively right and in line with the classic real option literature, given that more uncertainty related with the investment makes the investor to be more prudent and to wait more. Note that unlike the previous case (where we examined the variation in the parameter α), here also the timing in the integrated case (red line) positively vary with σ, this has to do with the fact that there is no more the presence of two different forces, but the variation in the threshold is due only from the effect that σ has on the function of parameters β. Note also that even in this case it is possible to appreciate as more agents are involved, more sensitive the investment threshold is to an increase in the volatility, moreover note that, in accordance to what already seen in section 2.1 of this chapter, from a timing prospective it is always preferable for the potential investor to interact with an outside supplier instead of with an investment partner, for every level of volatility considered.

Keeping in mind the above discussion about the timing, the following figure analyze the effect that, ceteris paribus, a changing in the volatility parameter (σ) has on the value of the option to invest for the potential investor (A).

FIGURE 4 – Effect of a change in σ on the value of the option to invest for A

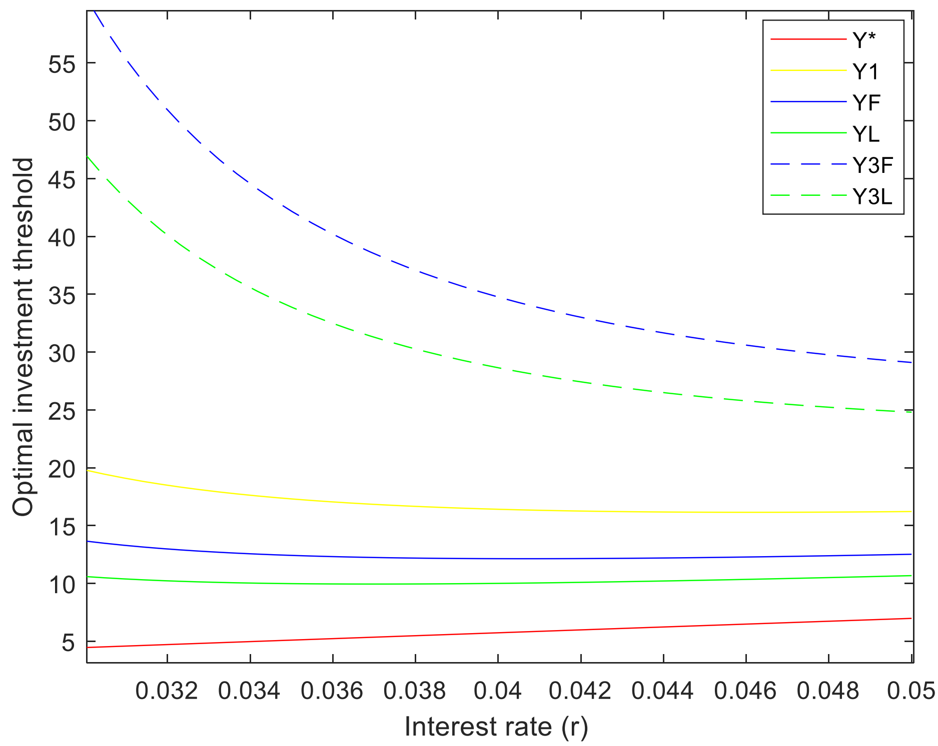


As we can see from the above figure an increasing in the volatility leads to an increase of the value of the option to invest both in the benchmark case (red line) and in the case of outsourcing the production of the input (yellow line), this is coherent with the fact that for a longer waiting period there is an increase in value of the option to wait for undertake an investment characterized by irreversibility and uncertainty. Now, given this, interesting are the cases where the participation of an investment partner is taken into accounts. Here we can easily see that there is a huge difference in value between the frameworks analyzed in Section 1.3.1 and 1.4.1 of Chapter 1 (where the potential investor took the role of time deciding agent), represented by the blue lines, and the frameworks analyzed in Section 1.3.2 and 1.4.2 of Chapter 1 (where the potential investor sets the compensation share to offer at the investment partner), represented by the green lines. Not only, but also the effects of the volatility changes if we compare the two frameworks, in fact we have that an increasing in σ will lead to an higher value of the option to invest for the potential investor if the investment partner takes the role of time deciding agent, while it will lead to a lower value if it is the potential investor to set the timing. These differences are due to two opposite forces: on one hand, the effect of higher uncertainty leads to a delay of the project that makes the option more valuable, on the other hands, in the cases in which the investment partner sets its compensation, an higher level of volatility calls for a higher compensation share demanded by C, that so reduces the value of the option for the potential investor, while in the opposite cases where the investment partner takes the role of time deciding agent, a higher level of volatility implies a lower compensation share offered by the potential investor, that so have the effect to boost the value of his option. To sum up, in the cases in which A is the time deciding agent (blue lines) we observe that there is a decreasing in its option to invest, and this is due to the fact that C sets a compensation for participating in a given share of the costs that is so high that reduce the increment in value that normally the potential investor would have given the longer waiting period caused by an increase in the volatility.

2.3.3 Effects of a change in the discount rate.

Finally, I will now provide an analysis about the effect that a variation in the level of the interest rate r can have on both the investment threshold (Figure 5) and on the value of the option to invest (Figure 6), by assuming, ceteris paribus, that r takes values in the interval [0.03, 0.05].

FIGURE 5 – Effect of a change in r on the investment threshold



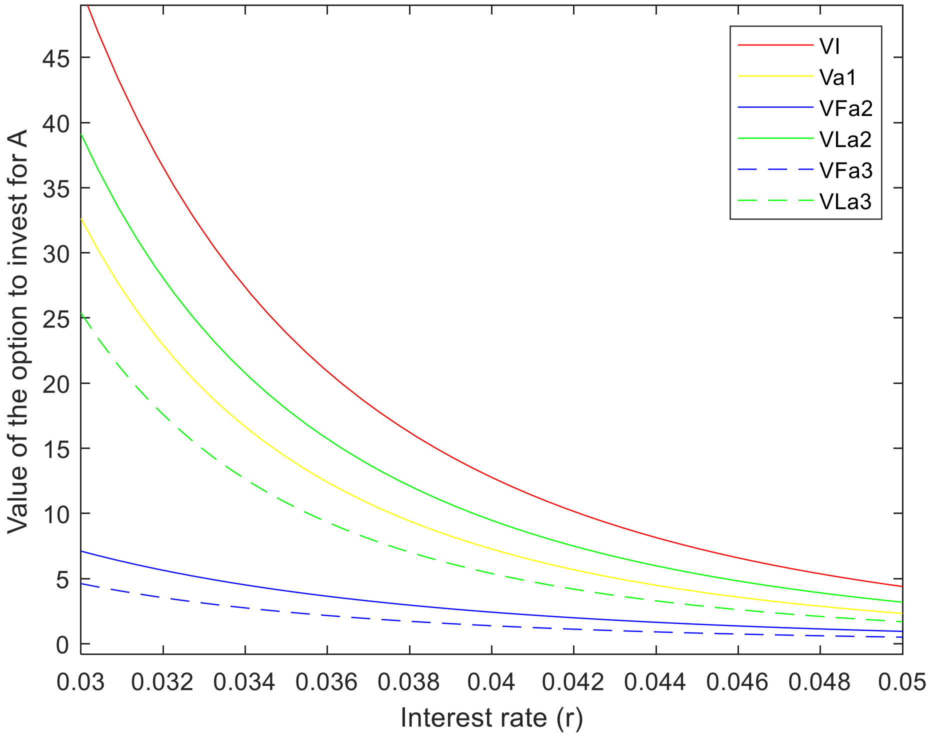
From the figure above it is possible to observe as the effect of a change in the interest rate on the optimal investment threshold is not the same across all the cases examined. Starting with the benchmark case (red line) we can see that, overall, an increase in r leads to a higher investment threshold, however this is the result of two opposite forces: on one hand, for a higher interest rate we have that the present becomes more valuable with respect to the future so that the potential investor would anticipate the investment by setting a lower threshold, while on the other hands, the present value of the future profit flows generating by the project is indirectly affected by the interest rate and this makes the potential investor willing to postpone the investment. As one can observe from the figure above the latter force prevails in the benchmark case. Note however that these two forces are acting in all the cases studied, but by increasing the number of agents involved we have that the second effect loses power. Specifically, if we first consider the two agent cases in which there is the presence of an investment partner (blue and green solid lines) we can see as the effect of an increase in the interest rate it is almost null. This because in these cases the investment threshold is also affected by the compensation share that the potential investor has to pay to the investment partner, so on one hand, if this share is set by the A (so that C is the time deciding agent), given its impatient, it will be willing to offer an higher compensation share to the investment partner in order to try to shorten the waiting period[38], on the other hands, if the compensation share is set by C (so that A is the time deciding agent), he will anticipate the intentions of A to postpone the investment (as in the integrated case), and because of this he will set a lower compensation offer in the attempt to reduce the investment threshold[39]. Now, keeping in mind these effects we have that by adding the presence of an outside supplier, the investment threshold is importantly reduced as the interest rate increase, this effect is clear if we look at the three agent cases (dashed lines). The reason behind this is that, while for both the potential investor and the investment partner the investment generate a profit flow that must be discounted, for the supplier the profits are represented by a lump sum payment equal to the price for the furniture of the input. So, the effect of the interest rate on the option to invest for the supplier goes only though the level of the investment threshold, and this makes B very impatient as r increases so that it's willing to lower the price of the input in order anticipate the investment.

Finally, keeping in mind the above considerations on the investment timing, I will now provide a brief discussion about the effect that a change in the interest rate (r) has on the value of the option to invest for the potential investor (A).

FIGURE 6 – Effect of a change in r on the value of the option to invest for A

[38] Recall that, in the cases examined in sections 1.3.2 and 1.4.2 of Chapter 1, the optimal compensation share was: $\gamma_L = \gamma_3^L = \frac{\theta(\beta-1)}{\theta+\beta-1}$, that, though the effect on β it is increasing in r, while the optimal investment threshold was decreasing in the compensation offer.

[39] Recall that, in the cases examined in sections 1.3.1 and 1.4.1 of Chapter 1, the optimal compensation share was: $\gamma_F = \gamma_3^F = \frac{1-2\theta+\theta\beta}{\beta-\theta}$, that, though the effect on β it is decreasing in r, while the optimal investment threshold was increasing in the compensation offer.



As we can easily observe for the figure above, an increase in the interest rate leads to a generally lower value in the option to invest for the potential investor. The reason of this is twofold: on one hand there is the effect on the investment timing examined before, and overall from the standard real option literature we know that generally a lower waiting period makes the value of the option to be less valuable. On the other hands there is the effect of the interest rate on the discount factor of the profit flows generating by the project, in fact, a higher r, ceteris paribus, has the effect to increase the discount factor and this it leads to a lower present value of the future profit flows and consequently to a lower value in the option to invest.

Chapter 3: Allowing for competition between agents.

In this Chapter of my thesis I will enlarge the framework seen in Chapter 1. Specifically, by following the general idea that the potential investor (A) willing to undertake the project is thought as a start-up and so do not have the financial resources needed, I will always consider the presence of an investment partner (C) providing part of the funds needed to finance the investment. Given this, in the following sections I will analyze first the case in which there is the presence of two upstream firms that can compete or coordinate each other to provide the input needed to the project. Second I will analyze the case in which the outside supplier is only one, but there are two start-up firms willing to undertake the same project that need to obtain the input and so again they can go under a cooperative setting or can compete against each other. In order to simplify the analysis, I will take as reference the framework seen in Section 1.4.1, so by always considering as time deciding agent the potential investor. Anyway, this do not affect the major results given that, as previously shown in section 1.4 of Chapter 1, the price for the acquisition of the input from an outside supplier it is independent by the type of interaction that the investment partner has with the potential investor.

3.1. Competition between suppliers.

In this first subsection I will examine the case in which there is the presence of two outside suppliers that need the furniture of the discrete input and one potential investor willing to undertake the project under the condition of receiving part of the funds needed by an investment partner. So, while the relationship with the external investor is similar to the one seen in section 1.4.1, here the difference will lies on the determination of the input price that will be the result of the interaction between the two outside suppliers (called S1 and S2). In the following subsections two situations are examined: first the two upstream firms are assumed to engage in a Bertrand competition over price and second the suppliers decide to collude. In order to proceed some assumption are needed:

- Two upstream firm will have constant and equal production costs: I.
- They are both able to deliver the same homogeneous discrete input (so that the downstream firm will buy from who makes the lowest price).
- S1 and S2 will set their prices simultaneously (I assume that once they make the price offer for the production of the input, the potential investor has to choose and sign the contract for the future furniture, so that the price will remain the same).

- If the same price is set, the potential investor will choose randomly the supplier of the input.

3.1.1 Bertrand competition case.

Given the above assumptions, in this sub-section I will consider the case in which the two suppliers will engage in a Bertrand competition over price, so each firm will have an incentive to offer for the furniture of the input a price lower than the other. Given this, the interaction between B1 and B2 it is reduced to a simple game in which the unique Nash Equilibrium is represented by the fact that both suppliers will set a price near to their marginal costs (represented by *I*) such that:

$$p_{S1} = p_{S2} = p_S = \omega I \tag{51}$$

Where ω is a constant greater than, but near to one, such that ωI represents the minimum price with which the suppliers realize some profit, so that they will have an incentive to provide the discrete input. Note that each upstream firm would always like to undercut the other because doing so it will be able to capture the entire market and to become the only monopolistic supplier. Following this we will have that the profits, and so the value of the option to invest for the two suppliers will be positive although near to zero and so that the potential investor will be able to buy the discrete input at a price similar to the one that he could have by produce it by himself. Given this background, the four players are assumed to follow these steps:

I. The two suppliers engage in a competition over price that will end up in fixing the same input price: $p_{S1} = p_{S2} = p_S = \omega I$.
II. The investment partner (C) decides optimally the compensation share (γ_4) that is willing to take in order to participate in an exogenously given share (θ) of the investment costs.
III. Given the price and the compensation chosen in the previous steps, the potential investor (A) determines the timing.

Proceeding backwards, following the same procedure of section 1.4.1, the optimal investment threshold is given by the solution of the following maximization problem:

$$V_{A4}(y) = max_{y_4(\gamma_4, p_S)} \left((1-\gamma_4) \frac{y_4(\gamma_4, p_S)\pi_M}{r-\alpha} - (1-\theta)p_S \right) \left(\frac{y}{y_4(\gamma_4, p_S)} \right)^{\beta} \tag{52}$$

From which we retrieve:

$$y_4(\gamma_4, p_S) = \left(\frac{1-\theta}{1-\gamma} \right) \frac{p_S}{I} y^* \tag{53}$$

Note that, as previously seen, the decision about the compensation share made by C is independent from the price at which the discrete input is bought, and so we have that, similarly to section 1.4.1:

$$\gamma_4 = \gamma_3^F = \frac{1-2\theta+\theta\beta}{\beta-\theta} \tag{54}$$

Now, proceeding backward, the suppliers engage in a Bertrand competition over price that ends up in fixing the same price[40], such that no one has the incentive to deviate (to change his own price) and:

$$p_{S1} = p_{S2} = p_S = \omega I \tag{55}$$

Given the above expression, on one hand, if a firm try to gain by increase its price it will sell nothing, and its profits will be zero because A will buy the input by the one that fix the lower price. On the other hands if a firm try to gain by decrease its price, it will capture the market entirely, but it will sell at a loss or without making profits. From these considerations, it is clear that the above price represents the unique Nash equilibrium of the game. Now, substituting the (54) and (55) into the (53) we have:

$$y_4(\gamma_4, p_S) = \left(\frac{\beta-\theta}{\beta-1} \right) \omega y^* \tag{56}$$

The above expression represents the optimal investment threshold from the potential investor prospective. Note that in this case the project will be delayed with respect to the integrated case , while it will be anticipated with respect to the case in section 1.4.1, in fact $y_3^F(\gamma_3^F, p_3^F) > y_4(\gamma_4, p_S) > y^*$.

[40] See Osborne M.J. (2009).

Now, by substituting the (56) into the equations for the value of the option to invest of the different agents, we can retrieve[41]:

$$V_{S2}(y) = V_{S1}(y) = \frac{1}{2}\frac{(\omega - 1)}{\omega^{\beta}}\frac{(\beta - 1)^{\beta+1}}{(\beta - \theta)^{\beta}}\, V_I(y) \tag{57}$$

$$V_{A4}(y) = \omega^{1-\beta}(1-\theta)\left(\frac{\beta - 1}{\beta - \theta}\right)^{\beta}\, V_I(y) \tag{58}$$

$$V_{C4}(y) = \omega^{1-\beta}\left(\frac{\beta - 1}{\beta - \theta}\right)^{\beta-1}\, V_I(y) \tag{59}$$

Note that the (57) represent the expected value of the option to invest for both suppliers, in fact following the assumptions, we will have that in the case in which the suppliers sets the same price for producing the input the potential investor will choose randomly between the two, so that the expected value of the option to invest for the upstream firm is:

$$E[V_S(y)] = \frac{1}{2}\left[\frac{(\omega - 1)}{\omega^{\beta}}\frac{(\beta - 1)^{\beta+1}}{(\beta - \theta)^{\beta}}\, V_I(y)\right] + \frac{1}{2}[0] = V_{S2} = V_{S1} \tag{60}$$

Note moreover that in the case in which the suppliers are willing to gain zero profits in order to win the competition (ω=1) the minimum price that they are willing to accept become exactly equal to their marginal costs (represented by I) and the value of their option to invest goes to zero. In this scenario we will have that the presence of an outside firm providing the discrete input would have the same effect of the case in which the potential investor produce the input by itself, and so both the level of the optimal investment threshold and value of the option to invest for A and C would be equal to the case examined in section 1.3.1. of Chapter 1.

3.1.2 Collusion case.

In this subsection I will examine the case in which the two upstream firms will decide to collude instead of to compete over price. Recalling the previous assumptions, the four players are assumed to follow these steps:

I. The two suppliers collude and decide together to fix the same price[42]: $p_{S1}^C = p_{S2}^C = p_S^C$.

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[41] See Appendix 3.1.1 to the calculation.

[42] The apex "C" over the notation of this section stands for "Collusion case".

II. The investment partner (C) decides optimally the compensation that he is willing to take in order to participate in the investment.

III. Given the price and the compensation chosen in the previous steps, the potential investor (A) determine the optimal investment threshold.

Note that the procedure is similar to the one analyzed in the previous section, with the only difference that now the price of the input is set by the collusion of the two upstream suppliers. So, by proceeding backwards, the optimal time for the investment is given by the following expression:

$$y_4^C(\gamma_4^C, p_S^C) = \left(\frac{1-\theta}{1-\gamma_4^C}\right)\frac{p_S^C}{I}y^* \tag{61}$$

That is equivalent to the threshold in section 3.1.1. and to the one retrieved in section 1.4.1, so similarly, by proceeding backwards, the investment partner will ask for a compensation share given by:

$$\gamma_4^C = \gamma_3^F = \frac{1-2\theta+\theta\beta}{\beta-\theta} \tag{62}$$

Note that, as before, the compensation share asked by the investment partner is independent from the price that will be set by the interaction of the two upstream firms, so also in this case it will be determined in the same way of section 1.4.1. Now, regarding the choice of the two suppliers, in case of collusion, the price set will be equal to the monopoly price[43], that in our case coincides with the price that maximize the following value function:

$$V_{SC1}(y) = V_{SC2}(y) = max_{p_S^C}(p_S^C - I)\left(\frac{y}{y_4^C(\gamma_4^C, p_S^C)}\right)^{\beta} \tag{63}$$

Note that the above maximization problem is the same of the one analyzed in sections 1.2 and 1.4, where there was the presence of only one supplier, in fact in choosing the price to set S1 and S2 will behave as if they were the only firm able to produce the input. So, the monopolistic price for the furniture is equal to[44] :

$$p_{S1}^C = p_{S2}^C = p_S^C = \frac{\beta}{\beta-1}I \tag{64}$$

[43]See Osborne, M.J. (2009).

[44] See Appendix 1.2.a. for the calculations.

Now, by substituting the (62) and the (64) into the (61), the expression of the optimal investment threshold is given by:

$$y_4^C(\gamma_4^C, p_S^C) = \frac{\beta(\beta-\theta)}{(\beta-1)^2} y^* \tag{65}$$

That it is equivalent to the one retrieved in Section 1.4.1. So, given the above analysis, we have that in case of collusion the values of the options to invest for both the potential investor and the investment partner are the same to the one derived in Section 1.4.1, and given that, we can write:

$$V_{A4}^C(y) = (1-\theta)\left(\frac{\beta-1}{\beta}\right)^{\beta-1}\left(\frac{\beta-1}{\beta-\theta}\right)^{\beta} V_I(y) \tag{66}$$

$$V_{C4}^C(y) = \left(\frac{\beta-1}{\beta}\right)^{\beta-1}\left(\frac{\beta-1}{\beta-\theta}\right)^{\beta-1} V_I(y) \tag{67}$$

While for what regard the value of the options of the two suppliers, we have that:

$$V_{S1}^C(y) = V_{S2}^C(y) = \frac{1}{2}\left(\frac{\beta-1}{\beta}\right)^{\beta}\left(\frac{\beta-1}{\beta-\theta}\right)^{\beta} V_I(y) \tag{68}$$

So, given that the two firms collude, they set the same monopoly price for the furniture of the input and the value of their option to invest will be equal to half the value that the supplier gets in the case examined in section 1.4.1 where there is only one to provide the input. Indeed, recalling the general assumption made at the beginning of section 3.1, the potential investor will choose randomly who to rely on the production of the input, so that the (68) represents the expected value of the option to invest for each supplier, in fact we have:

$$E[V_S^C(y)] = \frac{1}{2}\left[\left(\frac{\beta-1}{\beta}\right)^{\beta}\left(\frac{\beta-1}{\beta-\theta}\right)^{\beta} V_I(y)\right] + \frac{1}{2}[0] = V_{S1}^C(y) = V_{S2}^C(y) \tag{69}$$

Note that we would have had the same result also assuming that in case of equal price the investor decides to split the furniture, again the case it would have been the same as in section 1.4.1 with the difference that the two suppliers would have shared the market equally and so the value of their option to invest would have been halved.

Most important to note is the fact that this is not a Nash equilibrium, in fact both suppliers have an incentive to deviate and undercut each other. This last observation is crucial if we analyze the situation in an "one-shot game" prospective, where we will have a price competition game equivalent to the prisoner's dilemma[45], as it is possible to observe from the following representation:

S1\S2	COMPETE	COLLUDE
COMPETE	$V_{S1}(y)$; $V_{S2}(y)$	$V_{S1}^M(y)$; 0
COLLUDE	0 ; $V_{S2}^M(y)$	$V_{S1}^C(y)$; $V_{S2}^C(y)$

Where $V_{S1}^M(y)$ and $V_{S2}^M(y)$ represent the values of the options to invest if, respectively, S1 decides to deviate from collusion while S2 collude and vice versa. In this case we have that the supplier that decides to deviate will offer a price just lower the monopolistic one that the two had decided to propose in collusion so that to be able to capture the entire market and to receive a value near to the monopolistic one. So, by offering a price $p_{S'}^C = p_S^C(2-\omega)$ [46] the supplier that deviates will get a value:

$$V_{S1}^M(y) = V_{S2}^M(y) = \frac{\beta(1-\omega)+1}{(2-\omega)^\beta} V_{B3}^F(y) \tag{70}$$

Where $\frac{\beta(1-\omega)+1}{(2-\omega)^\beta}$ represent the loss of value in the option to invest given by the fact that the supplier that compete, in order to win the entire market (or to be sure to be chosen by the potential investor) has to fix a price just lower than the monopolistic one, so that the value of his option to invest will be just lower than the value under the monopoly situation examined in section 1.4.1 of chapter 1, where the unique producer of the input where able to gain entirely the value $V_{B3}^F(y)$.

Now, given that each upstream firm is rational and has an incentive to reduce its price slightly below the other firm's price, as we can see in the game representation above, the two agents will end up playing the only Nash equilibrium: (COMPETE, COMPETE) nevertheless there is an alternative (COLLUDE, COLLUDE) that is Pareto-superior but not sustainable.[47]

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[45] See Dixit A.K, Skeath S, and Reiley D.H. (2009).

[46] Note that because ω is greater than, but close to one, so $(2-\omega)$ will be lower than but close to one.

[47] Note in fact that: $\left(V_{S1}^M(y) = V_{S2}^M(y)\right) > \left(V_{S1}^C(y) = V_{S2}^C(y)\right) > \left(V_{S1}(y) = V_{S2}(y)\right)$.

3.2. Competition between potential investors.

In this subsection I will come back on the case exposed in section 1.4.1 of chapter 1, where I considered only one supplier providing the discrete input and one outside investor helping in financing the investment. Now, as previously anticipated at the beginning of this chapter, I will assume that there are two potential investors (called here D1 and D2) with similar ideas that are willing to undertake the same project. So, while the relationship with the external investor (C) is analogue to what we have seen in section 1.4.1, here we will have a difference input price, that will be the result of the interaction between the two start-up firms with the outside supplier (B).

In the following subsections two cases are examined: first the firms engage in a Bertrand competition and second the two try to collude. In order to do this some assumption are needed in order to simplify the problem:

- The two potential investors will offer a price simultaneously. I assume that once they make the price offers for the acquisition of the input, the supplier must choose and sign the contract for the future furniture, so that the price will remain the same.
- The two start-up firms are assumed to have similar infrastructure, so that they can gain the same profit stream from realization of the project.
- The investor that will place the highest bid will be chosen, if the same price is set, the supplier will choose randomly the client for the input.
- The outside investment partner (C) is assumed to be willing to finance only the investor that obtains the exclusive furniture from the supplier.

3.2.1 Bertrand competition case.

Given the above assumptions, in this sub-section I will consider the case in which the two downstream firms (D1 and D2) engage in a Bertrand competition over price, so that each one has an incentive to offer the highest possible price for the acquisition of the input. Note that, by doing so, intuitively we can expect that the net present value of the two potential investors will be brought to zero, while the value for the supplier will be increased with respect to the case without competition examined in section 1.4.1 of Chapter 1.

Given this background, the four players are assumed to follow these steps:

I. The two potential investors (D1 and D2) mutually discover they have a competitor for the acquisition of the input, so that they engage in a competition over price, and the upstream supplier (B) will choses to who provide the furniture.

II. The investment partner (C) decides optimally the compensation share (γ_4) that is willing to take in order to participate in an exogenously given share (θ) of the investment costs.

III. The two potential investors determine the optimal investment threshold at which they would like to undertake the investment.

So, by proceeding backward, starting with the problem of D1 and D2, the optimal investment threshold at which the two would like to undertake the investment is given by the solution of the following maximization problem:

$$V_{D1}(y) = V_{D2}(y) = max_{y_5(\gamma_5,p_D)}\left((1-\gamma_5)\frac{y_5(\gamma_5,p_D)\pi_M}{r-\alpha} - (1-\theta)p_D\right)\left(\frac{y}{y_5(\gamma_5,p_D)}\right)^{\beta} \quad (71)$$

Note that the above expression is equivalent to the one examined in section 1.4.1 of Chapter 1, so that we obtain:

$$y_5(\gamma_5,p_D) = \frac{(1-\theta)}{(1-\gamma_5)}\frac{p_D}{I}y^* \quad (72)$$

Given this, by proceeding backwards, the investment partner will set the compensation offer by solving the following problem:

$$V_{C5}(y) = \left(\gamma_5\frac{y_5(\gamma_5,p_D)\pi_M}{r-\alpha} - \theta p_D\right)\left(\frac{y}{y_5(\gamma_5,p_D)}\right)^{\beta} \quad (73)$$

From which it yields:

$$\gamma_5 = \frac{1-2\theta+\theta\beta}{\beta-\theta} \quad (74)$$

In fact, similarly to the situation in section 1.4.1 of Chapter 1, we have that the investment partner do not care about the presence of a supplier, and so requires a compensation that it is independent from the cost of the input. In fact, γ_5 will only depends directly on the amount of the exogenously given share of the costs (θ) and indirectly from the parameters contained in the function β.

Now, proceeding backwards the D1 and D2 engage in a Bertrand competition over prices, that will push up their offers to the limit for which the net present value of the project for the two potential investors will be brought to zero, so the maximum price set by the two competitors will be the one that makes the following equation equal to zero:

$$V_{D1}(y) = V_{D2}(y) = \left((1-\gamma_5)\frac{y_5(\gamma_5, p_D)\pi_M}{r-\alpha} - (1-\theta)p_D\right) = 0 \tag{75}$$

From which:

$$p_D = p_{D1} = p_{D2} = \frac{\pi_M}{r-\alpha} y_5(\gamma_5, p_D) \frac{(1-\gamma_5)}{(1-\theta)} (2-\omega) \tag{76}$$

Where ω is a constant greater than but near to one, so that the price in the above expression reduces the net present value to the minimal value for which the two potential investors are willing to undertake the project. Note in fact that, for higher prices, the value of the two potential investors undertaking the project will be completely null or negative, so that they do not have an incentive to make the investment. While, on the other hand, if one of the two offers a lower price, then it will be sure that it will lose the opportunity to obtain the furniture of the input from the supplier, that it will choose the other potential investor that offered a slightly higher price.

Now, by receiving the same offer, the unique supplier will pick out randomly to which player provide the furniture of the input needed to the project. But by doing so, given that the price offered depends also from the magnitude of the optimal investment threshold, we have that B has an incentive to wait the achievement of a given threshold before start providing the furniture. Specifically, B will have to solve the following problem:

$$V_{B5}(y) = max_{y_5(\gamma_5, p_D)}(p_D - I)\left(\frac{y}{y_5(\gamma_5, p_D)}\right)^{\beta} \tag{77}$$

In other words, the competition between D1 and D2 makes B in the condition to dry up the expected profit flows deriving from the project, that otherwise (in absence of competition) would go to the potential investor. So, by the solution of the above expression we have:

$$y_5(\gamma_5, p_D) = \frac{(1-\theta)}{(1-\gamma_5)} y^* \frac{1}{(2-\omega)} \tag{78}$$

That is practically equal to the investment threshold at which the potential investor would undertake the investment in the case in which there is no competition and it is able to produce internally the input needed for the project[48]. So, by substituting the (78) in the equation of the price offered by the two competitor we will have:

$$p_D = \frac{\beta}{\beta - 1} I \tag{79}$$

That it represents also the optimal price that the supplier asks for the production of the input in the case in which there is no competition. Now, provided this, by substituting the (74) into the equation (78) of the optimal investment threshold determined by the supplier, we can write:

$$y_5(\gamma_5, p_D) = \frac{(\beta - \theta)}{(\beta - 1)} y^* \frac{1}{(2 - \omega)} \cong \frac{(\beta - \theta)}{(\beta - 1)} y^* \tag{80}$$

So, note that the supplier put in place another constraint, such that the investment in the end will take place at the above threshold. Given this, by substituting the (79) and the approximation in the (80) into the values of the options to invest for the different agents involved we have:

$$V_{D1}(y) = V_{D2}(y) \cong 0 \tag{81}$$

$$V_{B5}(y) \cong V_I(y) \left(\frac{\beta - 1}{\beta - \theta}\right)^{\beta} \tag{82}$$

$$V_{C5}(y) \cong V_I(y) \left(\frac{\beta - 1}{\beta - \theta}\right)^{\beta} \left(\frac{\beta}{\beta - 1}\right) (1 - \theta) \tag{83}$$

So, as result of the competition, the two potential investors will get nothing, while the supplier it is able to obtain its optimal price and to anticipate the investment with respect the no competition case examined in section 1.4.1 of chapter 1. As consequence of this, we will also have that the value of the option to invest for both the supplier and the investment partner will be higher than in the three-agent case in section 1.4.1 of chapter 1.

3.2.2. Collusion case.

[48] Note in fact that being ω a constant greater than but near to one, the terms $\frac{1}{(2-\omega)}$ will be so close to one that can be negligible.

In this subsection I will examine the case in which the two downstream firms D1 and D2 decide to collude instead of competing each other over price. I will assume, as in section 3.1.1 that the supplier is willing to grant the furniture only in the case of positive profits, so that the minimum price that he is willing to accept is just greater than his costs *I*. Given this the downstream firms, that have an incentive to offer the lower price possible to boost their value, will coordinate to offer a price[49]:

$$p_{D1}^C = p_{D2}^C = p_D^C = \omega I \tag{84}$$

Where ω is a constant greater than, but near to one, such that ωI represents the minimum price with which the supplier makes some profit and so has an incentive to provide the discrete input. Now, recalling the previous assumptions, the four players are assumed to follow these steps:

I. The two potential investors, by colluding decide together to fix the same price: $p_{D1}^C = p_{D2}^C = p_D^C$.
II. The investment partner (C) decides optimally the compensation share that he is willing to take in order to participate in an exogenously given share (θ) of the costs of the investment.
III. Given the price and the compensation chosen in the previous phases, the two potential investors determine the optimal investment threshold.

With this premises, by proceeding backwards, we will have that, the optimal investment threshold is given by the following maximization problem:

$$V_{D1}^C(y) = V_{D2}^C(y) = max_{y_5^C(\gamma_5^C, p_D^C)} \frac{1}{2}\left(\left(1-\gamma_5^C\right)\frac{y_5^C\left(\gamma_5^C, p_D^C\right)\pi_M}{r-\alpha} - (1-\theta)p_D^C\right)\left(\frac{y}{y_5^C\left(\gamma_5^C, p_D^C\right)}\right)^\beta \tag{85}$$

From which we get:

$$y_5^C\left(\gamma_5^C, p_D^C\right) = \left(\frac{1-\theta}{1-\gamma_5^C}\right)\frac{p_D^C}{I}y^* \tag{86}$$

Now, recalling that the decision made by C regarding the optimal compensation share (γ_5^C) it is independent from the price at which the potential investor buy the input from the suppliers and by noting that the above expression of the optimal investment threshold it is similar to the one

[49] The apex "C" over the notation for this section stands for "Collusion case".

already examined in section 1.4.1 of Chapter 1, we have that the compensation share is given by the following expression:

$$\gamma_5^C = \gamma_3^F = \frac{1 - 2\theta + \theta\beta}{\beta - \theta} \tag{87}$$

Now, as previously anticipated, the two potential investors deciding to collude set the minimum price necessary to make the unique supplier willing to provide the input:

$$p_{D1}^C = p_{D2}^C = p_D^C = \omega I \tag{88}$$

Note however that this does not constitute a Nash equilibrium, in fact each potential investor has always the incentive to offer a higher price in order to be sure to obtain the furniture of the input. Now, by substituting the (87) and the (88) into the equation for the optimal investment threshold we get:

$$y_5^C(\gamma_5^C, p_D^C) = \left(\frac{\beta - \theta}{\beta - 1}\right)\omega y^* \tag{89}$$

Note that the above investment threshold it is similar to the one obtained in section 1.3.1 of Chapter 1, where the supplier was not taken into consideration. This because by colluding the two potential investors set a price for acquiring the input near to the investment costs that they would pay if they were able to produce it, so given this, the values of the option to invest for the supplier and the investment partner are respectively equal to:

$$V_{B5}^C(y) = (p_D^C - I)\left(\frac{y}{y_5^C(\gamma_5^C, p_D^C)}\right)^\beta = \frac{(\omega - 1)}{\omega^\beta}\frac{(\beta - 1)^{\beta+1}}{(\beta - \theta)^\beta}\, V_I(y) \tag{90}$$

$$V_{C5}^C(y) = \left(\gamma_5^C \frac{y_5^C(\gamma_5^C, p_D^C)\pi_M}{r - \alpha} - \theta p_D^C\right)\left(\frac{y}{y_5^C(\gamma_5^C, p_D^C)}\right)^\beta = \omega^{1-\beta}\left(\frac{\beta - 1}{\beta - \theta}\right)^{\beta - 1} V_I(y) \tag{91}$$

Note that that if $\omega = 1$, so the price offered by the two potential investors in collusion it is perfectly equal to the pure costs *I*, and in this case we would have that the value of the option to invest for the supplier (B) goes to zero, while the value for the investment partner (C) will be exactly the same of the one retrieved in section 1.3.1 of Chapter 1.

Now, the value of the option to invest for the two potential investors will be given by:

$$V_{D1}^C(y) = V_{D2}^C(y) = \frac{1}{2}\left[\omega^{1-\beta}(1 - \theta)\left(\frac{\beta - 1}{\beta - \theta}\right)^\beta\right] V_I(y) \tag{92}$$

So, given that the two potential investors collude, they agree to set the same price for acquire the furniture of the input from the supplier, that so it will choose randomly to who provide the input. For this reason, the above expression represents the expected value of the option to invest for each potential investor:

$$E[V_D^C(y)] = \frac{1}{2}\left[\omega^{1-\beta}(1-\theta)\left(\frac{\beta-1}{\beta-\theta}\right)^{\beta} V_I(y)\right] + \frac{1}{2}[0] = V_{D1}^C(y) = V_{D2}^C(y) \tag{93}$$

Note that we would have had the same result also assuming that in case of equal price the unique supplier decides to split the furniture, so the two potential investor would have shared the market equally and so the value of their option to invest would have been halved.

Finally, it is important to highlight that this is not a Nash equilibrium, in fact each potential investor has an incentive to deviate and offer a price just higher than the other competitor, this because by doing so they will have the certainty to be chosen by the supplier and so to become monopolists. In fact, similarly to the situation presented in Section 3.1.2 of this chapter, we will have that this situation is equivalent to the prisoner's dilemma[50], as illustrated in the following representation:

D1\D2	COMPETE	COLLUDE
COMPETE	$V_{D1}(y)$; $V_{D2}(y)$	$V_{D1}^M(y)$; 0
COLLUDE	0 ; $V_{D2}^M(y)$	$V_{D1}^C(y)$; $V_{D2}^C(y)$

Where $V_{D1}^M(y)$ and $V_{D2}^M(y)$ represent respectively the values of the options to invest if D1 decides to deviate from collusion while D2 collude and vice versa. In this case we have that one potential investor deviate and offer a price just higher the minimum agreed with the competitor, so that to obtain the exclusive furniture and so to receive a value closer to the one that it would have had if it had been able to produce the input internally[51]. So, by offering a price $p_{D'}^C = p_D^C\omega = I\omega^2$ the player that deviates will get a value:

$$V_{D1}^M(y) = V_{D2}^M(y) = \omega^{2(1-\beta)} V_{A2}^F(y) \tag{94}$$

[50] See Dixit A.K, Skeath S, and Reiley D.H. (2009).

[51] That is the value of the option to invest examined in section 1.3.1 of Chapter 1.

Where, recalling that ω is a constant greater than, but close to one, and $\beta > 1$, we will have that the terms $\omega^{2(1-\beta)}$ is lower than, but close to one. Now, given that each potential investor is assumed to be rational, both D1 and D2 will have an incentive to slightly increase the price with respect to the one set by the competitor, so that, similarly to what we have seen in section 3.1.2 of this section, the two agents will end up playing the only Nash equilibrium: (COMPETE, COMPETE) given that the alternative (COLLUDE, COLLUDE), as shown, it is not sustainable.

3.3. Discussion of results.

In this chapter I have expanded the situation presented in the section 1.4.1 of chapter 1, where the three agents were assumed to interact without the presence of competitors. First, I have considered, in section 3.1, the situation in which the presence of another upstream firm is added to the framework, so that the two suppliers are first assumed to compete over prices (Bertrand competition) and then to collude. Second, in section 3.2, I have analyzed the case in which the outside supplier is only one, but there are two potential investors willing to undertake the project that, in order to obtain the furniture of the input needed can again decide to cooperate or to compete over prices. From the analysis is evinced that in both cases the situation in which the two competitors decide to collude is the more profitable, but, given that this does not constitute a Nash equilibrium the two players will inevitably end up competing.

Specifically, in the framework of section 3.1, the two upstream suppliers will engage in a Bertrand competition that will lead basically to the same situation of section 1.3.1 of chapter 1 in which was assumed that the potential investor was able to produce internally the input needed to undertake the project. In fact, the competition will lead the two suppliers to lower the price at a level such that the values of their option to participate in the project will be reduced approximately to zero, while the optimal investment threshold and the values of the option to invest for the potential investor and the investment partner will be roughly the same to the one retrieved in section 1.3.1 of chapter 1.

While, more interesting is the situation examined in section 3.2. of this chapter. Also in this case the two competitors will end up engaging in a Bertrand competition over prices that will reduce the values of the option to invest for the two potential investors roughly to zero, but as result of the fact that the price set by the two potential investor is a function of the optimal investment threshold here we will have that the supplier will set a threshold so that to obtain its optimal price and to anticipate the investment with respect to the three agent case examined in section 1.4.1 of chapter 1. Specifically, the investment will be undertaken at the same threshold of the one retrieved in the case of section 1.3.1 of chapter 1 $\left(y_5(\gamma_5, p_D) = y_F(\gamma_F)\right)$, and, as result of this, the value of the option to invest for the supplier will be even higher than the one in the separated case examined in section 1.2 of chapter 1. While, regarding the value of the option to invest for the investment partner, this will be lower than the one in the external funding case in section 1.3.1 of chapter 1, given that, for the same investment threshold, in this situation the costs for undertake the investment will be higher $(p_D > I)$.

Chapter 4: Introducing the policy maker in the external funding case.

In this section I will provide an extension of the framework discussed in section 1.3 of Chapter 1. Starting by the assumption that the potential investor's project has a positive impact on the society, it is plausible to consider a government involvement. In fact, if we think the potential investor investing in a project as a high-tech entrepreneurial firm (usually referred in the literature as New Technology-Based Firms or NTBFs[52]) we have that, given its contribution to the social welfare, it represent an important policy target that the government should pursue[53]. So, in this chapter I will study the effect that the introduction of a policy maker can have on both the timing and the value of the investment. Note that here I will assume for simplicity that the potential investor is able to produce internally the input needed so that there the presence of an outside supplier is not taken into account[54].This Chapter, is divided in three sections: in section 4.1 I will consider the potential investor (A) as the time deciding agents, in Section 4.2 I will assume that the optimal time of the investment will be chosen by the private investment partner (C) and finally in section 4.3 I will sum up the major results obtained.

4.1. Potential investor as time deciding agent.

In this section I will expand the framework presented in Section 1.3.1 of Chapter 1, where the financial resources needed to undertake the project were provided by an external investment partner acting as leader in the relation with the potential investor (A). Now, given the assumption that the project's realization generates annual constant social benefits (G_B), I will add the presence of the policy maker that, for high enough level of G_B, it will have an incentive to intervene in order to try to facilitate the investment. Given this, the potential investor will receive on one hand a subsidy (G_C^F) by the government (G) and on the other hand part of the financial resources needed by a private investment partner (C), that can be thought as a venture capitalist. Now, I will assume that the timing of the game will be the following:

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[52] Referring to the standard definition proposed by Arthur D. Little (1977) that identifies an NTBF as an independent firm that is less than 25 years old and is active in high-technology industries.
[53] See Audretsch, (1995) and Stam and Garnsey, (2008).

[54] I already shown in Chapter 1 the effect of the additional presence of an outside supplier and it is possible to prove that for the aim of this Chapter its absence does not affect the major results.

I. The government (G), given a high enough level of the annual social benefit (G_B), offers to finance together with the C the project.

II. The private investment partner (C) decides optimally the compensation share (γ_G^F) that it is willing to take to cover a given share of the costs of the investment (θ).

III. The potential investor (A) decides if accept the offer made by C and it set the timing of the investment.

Proceeding backward, I start from the problem of the potential investor that, taking into consideration the actions of the other two agents involved, decides the optimal investment threshold that trigger the investment by solving the following:

$$V_{AG}^F(y) = max_{y_G^F(\gamma_G^F, G_C^F)} \left[(1-\gamma_G^F)\frac{\pi_M}{r-\alpha} y_G^F(\gamma_G^F) - (1-\theta)I + G_C^F\right]\left(\frac{y}{y_G^F(\gamma_G^F, G_C^F)}\right)^{\beta} \quad (95)$$

From the solution of the above problem, optimal investment threshold is given by[55]:

$$y_G^F(\gamma_G^F, G_C^F) = y^* \frac{I(1-\theta) - G_C^F}{(1-\gamma_G^F)I} = y^* \frac{\Phi}{(1-\gamma_G^F)} \quad (96)$$

Where $\Phi = \frac{I(1-\theta)-G_C^F}{I}$ represents the percentage of total costs sustained by A in order to invest in the project. In fact, recall that from the pure investment costs (*I*) the potential investor has to subtract the costs θI payed by the private investment partner (C) and the amount of the subsidy given by the government (G_C^F) so that, in percentage, Φ represent the effective contribution that A has to pay. Now, as expected, an increase in Φ (so a decreasing in θ or a lower G_C^F) has the effect to delay the investment, while a decreasing in the compensation share (γ_G^F) that A has to pay to C accelerate the realization of the project.

Now, proceeding backward, the private investment partner (C), takes into accounts the investment threshold chosen by A and decides the optimal compensation offer (γ_G^F) by maximizing the value of its option to invest:

$$V_{CG}^F(y) = max_{\gamma_G^F} \left[\gamma_G^F \frac{\pi_M}{r-\alpha} y_G^F(\gamma_G^F, G_C^F) - \theta I\right]\left(\frac{y}{y_G^F(\gamma_G^F, G_C^F)}\right)^{\beta} \quad (97)$$

Note that the value of the option to invest for C it is not directly dependent from the subsidy offered by the government, and so from the above expression it yields[56]:

[55] See Appendix 4.1.a for calculations.

[56] See Appendix 4.1.b for calculations.

$$\gamma_G^F = \frac{\Phi + \theta(\beta - 1)}{\Phi\beta + \theta(\beta - 1)} \tag{98}$$

From which it is possible to observe that, even if the share of costs (θ) paid by the private investment partner are near to zero, the compensation share required by the (C) in order to enter in the contract is still significantly positive and it depends from β[57].

Now, by substituting γ_G^F into the equation of the optimal investment threshold we obtain:

$$y_G^F(\gamma_G^F, G_C^F) = \frac{y^*}{\beta - 1}[\beta\Phi + \theta(\beta - 1)] \tag{99}$$

Finally, the government sets the amount of the subsidy (G_C^F) so that to maximize the value of its option to participate:

$$V_G^F(y) = max_{G_C^F}\left[\frac{\mathrm{G_B}}{\mathrm{r} - \alpha} - G_C^F\right]\left(\frac{\mathrm{y}}{y_G^F(\gamma_G^F, G_C^F)}\right)^{\beta} \tag{100}$$

from which we get[58]:

$$G_C^F = \frac{\mathrm{I}}{\beta - 1}\left[\beta\frac{\mathrm{G_B}}{(\mathrm{r} - \alpha)\mathrm{I}} - \frac{(\beta - \theta)}{\beta}\right] \tag{101}$$

This represent the optimal amount of subsidy that the government should offer to the start-up in order to maximize its revenue. Note that, in order to have an incentive to helping the potential investor in financing the project, the value of the annual social benefit ($\mathrm{G_B}$) must be sufficiently high. Specifically, in order to provide a positive level of subsidy must be that:

$$\mathrm{G_B} > \frac{(\beta - \theta)}{\beta^2}(\mathrm{r} - \alpha)\mathrm{I} \tag{102}$$

That represent the minimum level of social benefit that trigger the government intervention. Given this, by substituting the (101) into the equation for the optimal investment threshold, we can rewrite the (99) as:

$$y_G^F(\gamma_G^F, G_C^F) = y^*\left(\frac{\beta}{\beta - 1}\right)^2\left[\frac{\beta - \theta}{\beta} - \frac{\mathrm{G_B}}{I(\mathrm{r} - \alpha)}\right] \tag{103}$$

Note that, the above equation is decreasing in ($\mathrm{G_B}$), this is intuitively right, in fact for a higher level of social benefit generating by the project, the government will be willing to offer a higher amount of subsidy that in turn will lower the costs of the potential investor that so will anticipate

[57] In facts, for $\theta = 0$ we would have $\gamma_G^F = \frac{1}{\beta}$ and $\frac{d\gamma_G^F}{d\beta} = -\frac{1}{\beta^2}$.

[58] See Appendix 4.1.c for calculation.

the investment. Now, by substitution the (98), (101) and (103) into the values of the options to invest for the different subjects we get:

$$V_{AG}^{F}(y) = V_I(y)\left(\frac{\beta-1}{\beta}\right)^{2\beta-1}\left[\frac{\beta-\theta}{\beta}-\frac{G_B}{(r-\alpha)I}\right]^{-\beta}\left[\left(\frac{\beta-\theta}{\beta}-\frac{G_B}{I(r-\alpha)}\right)-\theta\left(\frac{\beta-1}{\beta}\right)^{2}\right] \quad (104)$$

$$V_{CG}^{F}(y) = V_I(y)\left(\frac{\beta-1}{\beta}\right)^{2(\beta-1)}\left[\frac{\beta-\theta}{\beta}-\frac{G_B}{(r-\alpha)I}\right]^{1-\beta} \quad (105)$$

$$V_{G}^{F}(y) = V_I(y)\left(\frac{\beta-1}{\beta}\right)^{2\beta}\left[\frac{\beta-\theta}{\beta}-\frac{G_B}{(r-\alpha)I}\right]^{1-\beta} \quad (106)$$

As we can observe the values of the options to invests are all dependent from the amount of social benefits generated by the project. For this, in section 4.3 of this chapter I will provide an analysis about how the different values may vary for different level of G_B, comparing also this case with the one examined in section 1.3.1 of Chapter 1, in which the presence of the government was not taken into account.

4.2. Investment partner as time deciding agent.

Starting from a framework similar to the one in Section 3.1.2, I consider the case in which the potential investor (A) is able to produce internally the input but it does not have the financial resources needed to undertake the project, so that the presence of a private investment partner, (C) is needed. Indeed, in this section I will assume that A at time zero makes a compensation offer to C represented by the share $\gamma_G^L \in (0,1)$, while C, taking into account the proposal of A, has the option to accept immediately the offer and to pay straightaway an exogenously given share of the costs $\theta \in (0,1)$ or it can wait and postpone the contribution. Now, in this framework, given that we assumed the investment would produce positive externalities, the policy maker has an incentive to intervene by providing part of the funds needed in order to have in return an annual social benefit (G_B). So, the government, for a high enough level of (G_B), commits himself to pay a subsidy G_C^L at the moments in which the investment will be undertaken. Given this, I will assume that the timing of the game will be the following:

I. The government (G), given a high enough level of the annual social benefit (G_B), offers to finance together with C the project.
II. The potential investor, taking into accounts the offer made by the policy maker, determines the optimal compensation share (γ_G^L) to offer to the private investment partner (C).

III. The investment partner decides if accept the compensation offer made by A and set the timing of the investment.

Now, I start analyzing the problem of the private investment partner that, taking into accounts the choice of the other two agents, derives the optimal investment threshold by solving the following maximization problem:

$$V_{CG}^{L}(y) = max_{y_G^L(\gamma_G^L)} \left[\gamma_G^L \frac{\pi_M}{r - \alpha} y_G^L(\gamma_G^L) - \theta I\right] \left(\frac{y}{y_G^L(\gamma_G^L)}\right)^{\beta} \tag{107}$$

From which, similarly to section 1.3.2 of chapter 1, we get:

$$y_G^L(\gamma_G^L) = y^* \frac{\theta}{\gamma_G^L} \tag{108}$$

So, from the above expression, it easy to see as the optimal investment threshold it is increasing in the share of the investment costs (θ) payed by the investment partner while it is decreasing in the optimal compensation offer (γ_G^L) determined by A. This is intuitively right, in fact, ceteris paribus, for higher costs is plausible that C would like to postpone the disbursement of the contribution and so the realization of the investment, while for an higher compensation it would like to anticipate the realization of the project. Now, proceeding backwards the potential investor, taking into accounts the subsidy offered by the Government, determine the optimal compensation offer (γ_G^L) that maximize the value of its option to invest:

$$V_{AG}^{L}(y) = max_{\gamma_G^L} \left[(1-\gamma_G^L) \frac{\pi_M}{r - \alpha} y_G^L(\gamma_G^L) - (1-\theta)I + G_C^L\right] \left(\frac{y}{y_G^L(\gamma_G^L)}\right)^{\beta} \tag{109}$$

From which it yields[59]:

$$\gamma_G^L = \frac{I\theta(\beta - 1)}{I\theta + (I - G_C^L)(\beta - 1)} \tag{110}$$

Note that the compensation share that A is willing to offer to C it depends on the subsidy that the potential investor receives from the government and, more precisely, a higher G_C^L allows A to offer an higher compensation share to the private investment partner.

Now, by substituting the (110) into the (108) we have that it is possible to write the optimal investment threshold as:

$$y_G^L(\gamma_G^L) = y^* \left[\frac{\theta + (\beta - 1)}{\beta - 1} - \frac{G_C^L}{I}\right] \tag{111}$$

[59] See appendix 4.2.a for calculations.

From which we can observe as the timing of the investment depends both on the share of the costs payed by the private investment partner (θ) and on the subsidy offered by the government for the project's realization. Specifically, a higher contribution (G_C^L) by the policy maker, leads to a lower investment threshold and so to an anticipation of the project, while a higher cost share (θ) imply a higher cost for the time deciding agent (C) and so a delay in the investment.

By proceeding backward, we have that the government chooses the subsidy G_C^L so that to maximize its value:

$$V_G^L(y) = max_{G_C^L}\left[\frac{\mathrm{G_B}}{\mathrm{r}-\ \alpha} - G_C^L\right]\left(\frac{\mathrm{y}}{y_G^L(\gamma_G^L)}\right)^{\beta} \tag{112}$$

Recalling that $\mathrm{G_B}$ represent the annual constant social benefit gain by the society in the case in which the project is realized, by the solution of the above maximization problem we have[60]:

$$G_C^L = \frac{\mathrm{I}}{(\beta - 1)}\left[\beta\frac{\mathrm{G_B}}{(\mathrm{r}-\alpha)\mathrm{I}} - \frac{\theta + (\beta - 1)}{(\beta - 1)}\right] \tag{113}$$

Note that the government decides the amount of subsidy also by taking into accounts the share of the costs (θ) payed by the private investment partner, so that a higher (θ) leads to lower contribution by the policy maker[61]. Moreover, it is important to underline that the policy maker, in order to have an incentive to help the potential investor in financing the project, must have in return a sufficiently high value of the social benefit $(\mathrm{G_B})$, in particular, in order to provide a positive level of subsidy must be that:

$$\mathrm{G_B} > \frac{\theta + (\beta - 1)}{\beta(\beta - 1)}(\mathrm{r}-\alpha)\mathrm{I} \tag{114}$$

That represent the minimum level of social benefit that trigger the government intervention. Given this, by substituting the (113) into the equation for the optimal investment threshold, we can rewrite the (111) as:

$$y_G^F(\gamma_G^F, G_C^F) = y^*(\frac{\beta}{\beta - 1})\left[\frac{\theta + (\beta - 1)}{(\beta - 1)} - \frac{\mathrm{G_B}}{I(\mathrm{r}-\alpha)}\right] \tag{115}$$

Note that, the above equation is decreasing in $(\mathrm{G_B})$, this is intuitively right, in fact for a higher level of social benefit generating by the project, the government will be willing to offer a higher amount of subsidy that in turn will lower the costs for the potential investor that so will be able

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[60] See appendix 4.2.b for calculations.

[61] As one can check: $\frac{dG_C^L}{d\theta} = -\frac{I}{(\beta-1)^2} < 0$.

to offer a higher compensation to the private investment partner that anticipates the investment. Finally, by substituting the (110), (113) and (115) into the equation for the values of the option to invest for the different agents involved, we have that:

$$V_{AG}^{L}(y) = V_I(y)\left(\frac{\beta}{\beta-1}\right)^{1-\beta}\left[\frac{\theta+(\beta-1)}{\beta-1} - \frac{\mathrm{G_B}}{(\mathrm{r}-\alpha)\mathrm{I}}\right]^{1-\beta} \tag{116}$$

$$V_{CG}^{L}(y) = V_I(y)\left(\frac{\beta}{\beta-1}\right)^{-\beta}\theta\left[\frac{\theta+(\beta-1)}{\beta-1} - \frac{\mathrm{G_B}}{(\mathrm{r}-\alpha)\mathrm{I}}\right]^{-\beta} \tag{117}$$

$$V_{G}^{L}(y) = V_I(y)\left(\frac{\beta}{\beta-1}\right)^{-\beta}\left[\frac{\theta+(\beta-1)}{\beta-1} - \frac{\mathrm{G_B}}{(\mathrm{r}-\alpha)\mathrm{I}}\right]^{1-\beta} \tag{118}$$

As one can easily observe the values of the options to invests are all function of the amount of social benefits generated by the project. Now, in the following section I will provide an analysis about how the different values may vary for different level of $\mathrm{G_B}$, comparing also the finding with the case examined in section 1.3.2 of Chapter 1, in which the presence of the government was not taken into account.

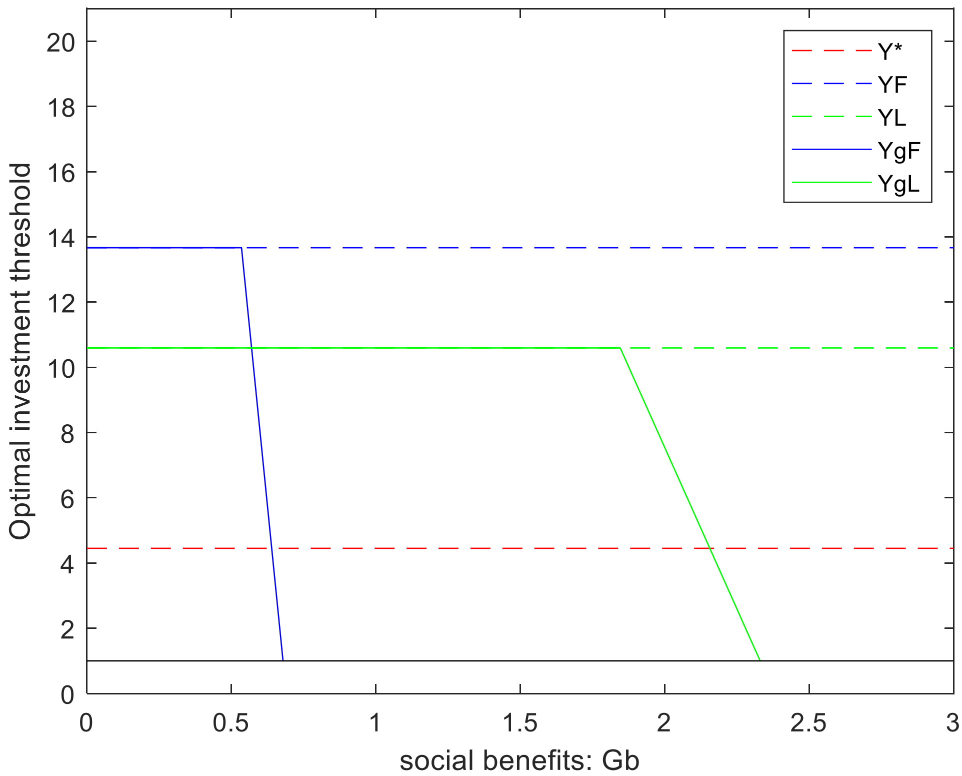
4.3. Discussion of results.

In this third section of the chapter I will provide some consideration about the results obtained in the two previous sections. So, I will show how both the investment threshold and the value of the option to invest for the potential investor change in relation to a different value of the social benefit ($\mathrm{G_B}$) generating by the project. Moreover, I will compare this analysis with the cases examined in section 1.3 of Chapter 1 in which the government was not taken into accounts, so to see the effects of the intervention of the policy maker. Note that in the following, to facilitate the comparison, I will consider the same data used in the numerical simulation provided in section 1.5 of Chapter 1[62].

Starting with the optimal investment threshold, the following figure shows the effect that the added presence of the policy maker can have on the timing of the investment for a $\mathrm{G_B}$ in the interval [0 ; 3].

FIGURE 7 – Effect of $\mathrm{G_B}$ on the optimal investment threshold

[62] Appendix 4.3 reports the code used to run the simulation.



The first thing to note about the above figure is that it shows a comparison between some of the values for the optimal investment thresholds calculated through the simulation in section 1.5 of Chapter 1 (solid lines), with the values of the optimal threshold in the two cases examined in this chapter (dashed lines). Now, recalling that the government has an incentive to intervene only for a sufficiently high level of the social benefit (G_B), from the above figure it is easy to see as if the policy maker do not intervene, the optimal investment thresholds are the same to the ones retrieved in section 1.3 of Chapter 1, while for values of G_B sufficiently high, the government intervention is triggered and we can appreciate a rapid acceleration of the project.

Specifically, starting with the case in which the potential investor is the time deciding agent (blue lines), the government will be willing to pay a positive amount of subsidy for G_B higher than $\frac{(\beta-\theta)}{\beta^2}(r-\alpha)I$, that in the figure correspond to the point in which we see the deviation of the blue solid line from the dashed one. Now, for higher value of G_B, the policy maker is willing to provide a higher subsidy to the potential investor, that so will anticipate the investment up to a point in which it will be willing to undertake the project immediately. Note in fact that the intersection of the blue line with the black one represents the point for which $\frac{y}{y_G^F(\gamma_G^F)} = 1$, so that the potential investor has no incentive to postpone the investment. This, happen for a value of social benefit at least equal to:

$$G_B = \frac{\beta - \theta}{\beta}(r - \alpha)I - \left(\frac{\beta - 1}{\beta}\right)^3 y\pi_M \quad (119)$$

Recalling that y represent the current level of the state variable. So, also if for higher level of G_B the government would be willing to offer a higher subsidy in order to further anticipate the investment, we have that the maximum level of subsidy that the government will offer will be for:

$$G_C^F = I\left[\frac{\beta - \theta}{\beta} - \frac{y}{y^*}\left(\frac{\beta - 1}{\beta}\right)\right] \quad (120)$$

That represent the value of G_C^F such that the investment will be immediately undertaken. Note that, also if for higher level of G_B the government would like to offer an higher subsidy, because it is not possible to anticipate the investment in the past, the government will have no incentive to do so, because what he would pay above the value in the (120) will be not compensate by a further project's acceleration. Note moreover that, from the (120) it is also easy to see the level of G_C^F for which the investment will occurs at the same threshold of the benchmark case (y^*), represented by the red dashed line in the above figure[63].

Now, a similar reasoning can be made also for the situation in which the time deciding agent is represented by the private investment partner (green lines). Here, the government will intervene for a level of the social benefit (G_B) higher than $\frac{\theta+(\beta-1)}{\beta(\beta-1)}(r - \alpha)I$, represented in the figure by the point in which there is the the deviation of the green solid line from the dashed one. So, the policy maker will provide a level of subsidy G_C^L that is increasing in G_B, and this will lead to an anticipation of the investment, up to the point in which the project will be immediately undertaken, that is represented by the intersection of the green line with the black one. In fact, for $\frac{y}{y_G^L(\gamma_G^L)} = 1$, the private investment partner has no incentive to postpone the investment. Now, this point is reached for a value of the social benefit at least equal to:

$$G_B = \frac{\theta + \beta - 1}{\beta - 1}(r - \alpha)I - \left(\frac{\beta - 1}{\beta}\right)^2 y\pi_M \quad (121)$$

Recalling also here that y represent the current level of the state variable. Now, the maximum level of subsidy that the government will offer will be for:

[63] In fact, by setting $\frac{y}{y^*} = 1$ we will have that the amount of subsidy necessary to make the potential investor willing undertake the project at the benchmark level will be $G_C^F = I\left[\frac{\beta-\theta}{\beta} - \left(\frac{\beta-1}{\beta}\right)\right]$.

$$G_C^L = I\left[\frac{\theta+\beta-1}{\beta-1} - \frac{y}{y^*}\right] \quad (122)$$

That represent the value of G_C^L such that the investment will be immediately undertaken. So, also here, the government will have no incentive to offer a higher subsidy, because what he would pay above the value in the (122) will be not compensate by a further acceleration of the project. Note moreover that, from the (122) it is also easy to see the level of G_C^L for which the investment will occurs at the same threshold of the benchmark case (y^*), represented by the red dashed line in the above figure[64].

Finally, from the Figure 7, it is possible to appreciate as the policy maker will intervene earlier if the potential investor takes the role of time deciding agent (blue line), while it will wait for higher level of the social benefit in the case in which is the investment threshold is decided by the private investment partner (green line). However, note that when the government will intervene, it will be more expensive for him to make the private investment partner willing to immediately undertake the project than make the potential investor be willing to anticipate the investment in the present. In fact, by looking at the slope of the two investment thresholds we have that $y_G^F(\gamma_G^F, G_C^F)$ is more sensitive to an increasing in the subsidy than $y_G^L(\gamma_G^L, G_C^L)$[65], and given that the variation of the subsidy with respect to the amount of social benefit generated by the project is the same, independently by which is the time deciding agent: $\frac{dG_C^L}{\mathrm{G_B}} = \frac{dG_C^F}{\mathrm{G_B}} = \frac{\beta}{\beta-1}$,

this means that it will take lower financial resources to the policy maker to make the potential investor anticipate the investment.

Now, going to the analysis of the value of the option to invest for the potential investor (A), the following figure will show the effect that the added presence of the policy maker can have for a $\mathrm{G_B}$ in the interval [0 ; 3].

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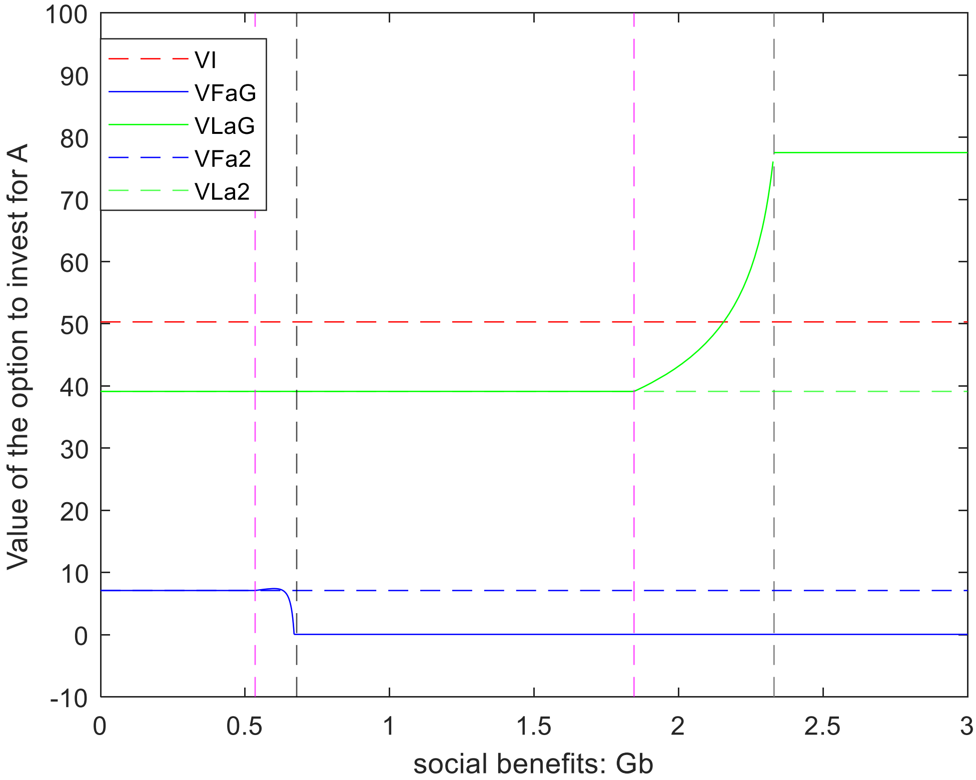
[64] In fact, by setting $\frac{y}{y^*} = 1$ we will have that the amount of subsidy necessary to make the private investment partner willing to undertake the project at the benchmark level will be $G_C^L = \frac{\theta+\beta-1}{\beta} I$.

[65] Specifically we have $\frac{dy_G^F(\gamma_G^F, G_C^F)}{dG_C^F} = -\frac{y^*}{\mathrm{I}}\frac{\beta}{(\beta-1)}$ and $\frac{dy_G^L(\gamma_G^L, G_C^L)}{dG_C^L} = -\frac{y^*}{\mathrm{I}}$

FIGURE 8 – Effect of G_B on the value of the option to invest for A



The first thing to say about the above figure is that the two vertical purple dashed lines represent the levels of the social benefit that trigger the government intervention in the two cases examined, while the two vertical black lines are the thresholds for which the subsidy offered by the policy maker is so high that the investment is immediately undertaken.

As we can observe before the policy maker intervention, the values of the option to invest for the potential investor are the same to those retrieved in Section 1.3 of Chapter 1. Now, once the level of social benefit G_B is sufficiently high the government is willing to provide a subsidy in order to anticipate the realization of the project, and as we can see from the above figure this has a different effect on the value of the option to invest depending on which role the potential investor takes in the relation with the private investment partner.

Specifically, starting by analyzing the situation in which the potential investor (A) is the time deciding agent (blue lines), we have that before the government intervention the value of its option to invest it is equal to the one retrieved in section 1.3.1 of chapter 1. Subsequently, for higher level of G_B the government intervene providing a subsidy in order to anticipate the investment, and the value of the option to invest at first is slightly increasing but right after decreases up to a point in which it goes approximately to zero. This is the result of different forces interacting. First, a higher level of G_C^F leads both to a reduction of the investment costs (that should increase the value of the option to invest) and to an anticipation of the investment (that has the effect to reduce the level of the optimal investment threshold and so to reduce the overall value of the option to invest). Second, by setting the timing the potential investor leaves the power to the private investment partner (C) to set its compensation share. So that C, by taking into account the rapid acceleration of the project derived from an increasing level of G_C^F, would like to have an increasing compensation share (γ_G^F) that it will reduce the value of the option to invest for A. As we can see from Figure 8, this second effect prevails, and the value of the potential investor is brought approximately to zero. About this note moreover that for: $G_C^F > I\left[\frac{\beta-\theta}{\beta} - \theta\left(\frac{\beta-1}{\beta}\right)\right]$, the private investment partner would like to set a compensation share that is higher than one, but provided the fact that in this scenario A will get a negative value (so it is likely that will not undertake the investment), C will limit his compensation share to the maximum he can set provided that the value of A will not be negative, (so that it will have an incentive to realize the project). So, the private investment partner will set, progressively with the project acceleration, a higher compensation up to $\gamma_G^F \cong 1$[66] and this will lead the value for A to be approximately zero. In other words C, aware of the fact that A will receive a subsidy from the government and that this will anticipate its investment decision, wants to set a higher compensation share to be remunerated by the fact that it will have to provide its share of the investment costs earlier, and this will drain out all the profit from A.

Analyzing now the situation in which the role of time deciding agent is taken by the private investment partner (green lines) we can easily see from the above figure as the things drastically change for the potential investor. In fact, while we have that before the government intervention the value of its option to invest it is equal to the one retrieved in section 1.3.2 of chapter 1, after the intervention of the policy maker, the value of the option to invest for A rapidly increases up to overcome the level of the option to invest retrieved in the benchmark case (red dashed line), reaching so an even higher value. The reason of this has to do with the interaction of different forces. First note that a higher level of G_C^L leads both to a reduction of the investment costs (that should increase the value of the option to invest) and to an anticipation of the investment (that has the effect to reduce the level of the optimal investment threshold and so to reduce the overall value of the option to invest). Second, more importantly, given that for the government is more expensive to anticipate the investment in this case, it will have to provide a high level of subsidy to make the investment partner willing to anticipate the investment, so that the

[66] Note in fact that for a level of the compensation share exactly equal to one ($\gamma_G^F = 1$), the potential investor could be refrained in undertaking the project, given that it will not get anything in return.

potential investor will get a level of G_C^L greater than the one needed to completely offset the costs of the investment and in this way it can provide a higher compensation share to the investment partner that so will be willing to accelerate the project's realization. Note in fact that, if the policy maker wants to anticipate the investment at the present $\left(\frac{y}{y_G^L(\gamma_G^L)} = 1\right)$ it will have to provide a subsidy equal to $G_C^L = I\left[\frac{\theta+\beta-1}{\beta-1} - \frac{y}{y^*}\right]$ and so, the potential investor will get a value equal to:

$$V_{AG}^L(y) = V_I(y)\left(\frac{y^*}{y}\right)^{\beta-1} \tag{123}$$

Note that because $\left(\frac{y^*}{y}\right)^{\beta-1} > 1$ given the assumption used though the paper, we have that in the above expression $V_{AG}^L(y) > V_I(y)$. Moreover, from the (123) it is also possible to check that in the case in which the investment it is undertaken at the benchmark level, we will have $V_{AG}^L(y) = V_I(y)$, as reported in the above figure by the intersection of the dashed green line with the red one.

So, given the above analysis it is obvious that, from the potential investor prospective, it is always better to be the game leader in the relation with the private investment partner, deciding so the optimal compensation share to offer to C and leave the investment partner set the optimal investment threshold. On the other hand, if we look the situation from a government prospective, the policy maker will always prefer to intervene in a situation in which the role of time deciding agent is taken by the potential investor, because in this scenario it will be able to anticipate the investment by offering a much lower level of subsidy.

Conclusions

In my thesis I have presented different cases base on the application of the standard real option framework that consider investment opportunities as options on real assets providing a way to apply option pricing methods to investment decision problems. Given the large applicability of the subject my focus has been on the study of vertical relationships. So, by considering the situation in which a risk neutral potential investor is contemplating to enter in a new high profitable growing market characterized by uncertainty, it will have to face two problems: first, it will have to obtain a discrete input needed to the project's competition (by producing it by himself or by relying on the furniture provided by an upstream supplier); second, in the case in which it does not have enough financial resources, it will have to interact with an investment partner that, in return of a share of the profits generating by the project, it will provide an exogenously given share of the sunk investment costs. In order to study the interaction between these agents, I used a stochastic dynamic programming model based on the standard real option approach, and I have examined different cases.

First in chapter 1 I chose to consider as benchmark case the situation in which the potential investor has both the financial resources and the infrastructure needed to produce the input required to undertake the investment. Subsequently I have expanded the basic framework allowing for the introduction of different agents interacting with the potential investor in order to see the effect that the introduction of these can have on both the investment threshold and the value of the option to invest. I found that the best situation with respect to the timing of the investment and in relation to the value of the option to invest is obtained in the case in which the potential investor acts independently. While the additional presence of one more agent causes the postponement of the project and lower the value of the option to invest. Given this, I also showed that regarding the time of the investment, it is always preferable for the potential investor to interact with the investment partner instead of the supplier, while the worse scenario is represented by the simultaneous involvement in the project of the upstream supplier and the investment partner. Instead, different is the situation if we look at the value of the option to invest. In fact, here we have that the ranking depends on the type of the interaction that the potential investor has with the investment partner, with the worse scenario represented by the cases in which the potential investor takes the role of time deciding agent, leaving so to the investment partner the decision about the compensation share that it will receive for participating in an exogenously given share of the investment cost. While in the opposite case (investment partner as time deciding agent) we have that, by comparing situations with the same number of agents involved, it is always better for the potential investor to interact with the investment partner[67].

Second, in chapter 3, by taking as reference the three-agents case in which the potential investor takes the role of time deciding agent[68], I have expanded the framework allowing first for the competition between suppliers and second allowing the presence of two potential investors. From the analysis is evinced that in both cases two competitors will inevitably end up competing over prices (Bertrand competition), and this will lead to an anticipation of the investment with respect the three-agents case. Specifically, I found that this happen at approximately the same threshold retrieved in the external funding case (where only the private investment partner and the potential investor are involved). Different instead is the situation from a value prospective. In both cases analyzed the two agents that compete over price will end up reducing the values of their option to participate in the project approximately to zero, while, the thigs change for the other two players involved. In particular, when the two upstream suppliers compete, they lower so much the input price that the costs of the investment are reduced roughly to the same level that was assumed in the case in which the potential investor was able to produce internally the input needed to undertake the project, so that the values of the option to participate in the project for the investment partner and the potential investor are approximately equal to the one obtained in the external funding case. On the other hand, when the two potential investors compete, the supplier is able both to obtain its optimal price and to anticipate the investment with respect to the three-agents case, so that its value of the option to invest is even higher than the one in the separated case[69], while, as consequence of this, the value for the investment partner will be lower than the one retrieved in the external-funding case, given that, for the same investment threshold, the total investment costs will be higher.

Finally, in Chapter 4, starting from the external funding case[70], I further expanded the framework by assuming that the project, once realized, has positive impact on the society, so that the involvement of the policy maker is plausible in order to try to facilitate the investment. Here I found that the government intervention is conditioned by the generation of a minimum level of social benefit achieved by the project, and that this threshold is higher in the case in which the investment partner takes the role of time deciding agent in the relation with the potential investor. Now, given the achievement of the minimum level of social benefit, the

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[67] See the analysis on the value of the option to invest provided in section 2.2 of chapter 2.

[68] See the case in section 1.4.1 of chapter 1.

[69] See section 1.2 of chapter 1.

[70] See section 1.3 of chapter 1.

policy maker will offer a subsidy to the potential investor that, from a timing prospective, will have the effect to rapidly accelerate the project's realization in both cases examined. While, from a value prospective, I found that the effect of the subsidy provided by the government change in relation to which player takes the role of time deciding agent. Specifically, in the case in which the optimal investment threshold is set by investment partner, the value of the option to invest for the potential investor is increasing in the amount of subsidy and, for a given levels of social benefit, also overcome the value of the option to invest retrieved in the integrated case (in which the potential investor is assumed to act independently). While different is the situation in which the potential investor set the timing of the investment, here I fund that the value provided by higher subsidy offered by the government goes to the investment partner that, by setting its own compensation share, will end up asking for almost the totality of the profits generating by the investment so that the value of the option to invest for the potential investor will be approximately null. Concluding, it is obvious that, from the potential investor prospective, it is always better to be the game leader in the relation with the private investment partner (deciding so the optimal compensation share to offer to the investment partner), while from a government prospective, the policy maker will always prefer to intervene in a situation in which the role of time deciding agent is taken by the potential investor, because in this scenario it will be able to anticipate the investment by offering a much lower level of subsidy[71].

[71] See section 4.3 of chapter 4.

Appendix

Appendix 1.1.a – Proof of the relation: $\boldsymbol{V_I(y) = max_{y^*}\left(\frac{\pi_M}{r-\alpha}y^* - I\right)\left(\frac{y}{y^*}\right)^{\beta}}$

In order to verify that:

$$V_I(y) = max_T E[(F_T - I)e^{-rT}] = max_T E[e^{-rT}](F_T - I) = max_{y^*}\left(\frac{\pi_M}{r-\ \alpha}y^* - I\right)\left(\frac{y}{y^*}\right)^{\beta}$$

I will divide the demonstration in two parts:

I) I will show that:

$$F_t = \ E_t[\int_t^{\infty} Y_s \pi_M e^{-r(s-t)} ds] = \frac{\pi_M}{r-\ \alpha} Y_t$$

By considering for simplicity that the profit flows generating upon the investment in the project are denoted by $\varepsilon_t = Y_t \pi_M$ we have that, their present value at $t \geq 0$ can be represented by:

$$F(\varepsilon_t) = \ E_t[\int_t^{\infty} \varepsilon_s e^{-r(s-t)} ds]$$

Where, obviously, $\varepsilon_t = \pi_M Y_t = \alpha \pi_M Y_t \ dt + \sigma \pi_M Y_t \ dW_t = \alpha \varepsilon_t \ dt + \sigma \varepsilon_t \ dW_t$.

Now, for small dt it is possible to decompose $F(\varepsilon_t)$ as:

$$\begin{aligned} F(\varepsilon_t) = \ E_t[\int_t^{\infty} \varepsilon_s e^{-r(s-t)} ds] &= E_t[\int_t^{t+dt} \varepsilon_s e^{-r(s-t)} ds] + E_t[\int_{t+dt}^{\infty} \varepsilon_s e^{-r(s-t)} ds] \\ &= \varepsilon_t \ dt + e^{-rdt} E_t[\int_{t+dt}^{\infty} \varepsilon_s e^{rdt} e^{-r(s-t)} ds] \\ &= \varepsilon_t \ dt + e^{-rdt} E_t[\int_{t+dt}^{\infty} \varepsilon_s e^{-r[s-(t-dt)]} ds] = \varepsilon_t \ dt + e^{-rdt} E_t[F(\varepsilon_{t+dt})] \end{aligned}$$

Now, since $\varepsilon_{t+dt} = \varepsilon_t + d\varepsilon_t$, we have the following Bellman Equation:

$$F(\varepsilon_t) = \varepsilon_t \ dt + e^{-rdt} E_t[F(\varepsilon_t + d\varepsilon_t)]$$

By dropping for convenience, the time representation and recalling that by using Taylor expansion $e^{-rdt} \cong (1 - rdt)$ we can rewrite the above expression as:

$$F(\varepsilon) = \varepsilon dt + (1 - rdt) E[F(\varepsilon + d\varepsilon)]$$

And expanding through the Ito Lemma we get:

$$F(\varepsilon) = F(\varepsilon) + \left[\frac{1}{2}\sigma^2 \varepsilon^2 F_{\varepsilon\varepsilon}(\varepsilon) + \alpha \varepsilon F_{\varepsilon}(\varepsilon) - rF(\varepsilon) + \varepsilon\right] dt$$

So that, rearranging it yields:

$$\frac{1}{2}\sigma^2\varepsilon^2F_{\varepsilon\varepsilon}(\varepsilon) + \alpha\varepsilon F_{\varepsilon}(\varepsilon) - rF(\varepsilon) + \varepsilon = 0$$

That represents a second order non-homogeneous differential equation, for which the solution will be represented by a homogenous part and a particular solution. Starting from the homogenous part, we guess a solution: $F(\varepsilon) = L\varepsilon^{\beta}$. So that the first and the second partial derivative w.r.t. ε will be respectively $F_{\varepsilon}(\varepsilon) = L\beta\varepsilon^{\beta-1}$ and $F_{\varepsilon\varepsilon}(\varepsilon) = L\beta(\beta-1)\varepsilon^{\beta-2}$. Substituting in the homogenous part we get:

$$\frac{1}{2}\sigma^2\varepsilon^2L\beta(\beta-1)\varepsilon^{\beta-2} + \alpha\varepsilon L\beta\varepsilon^{\beta-1} - rL\varepsilon^{\beta} = 0 \rightarrow L\varepsilon^{\beta}\left[\frac{1}{2}\sigma^2\beta(\beta-1) + \alpha\beta - r\right] = 0$$

So that solving for the quadratic equation: $Z(\beta) = \frac{1}{2}\sigma^2\beta(\beta-1) + \alpha\beta - r = 0$ the two solutions ($\beta_1 > 1;\ \beta_2 < 0$) are given by:

$$\beta_1, \beta_2 = \frac{1}{2} - \frac{\alpha}{\sigma^2} \pm \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}}$$

Then the solution of the homogenous part is:

$$F(\varepsilon) = L_1\varepsilon^{\beta_1} + L_2\varepsilon^{\beta_2}$$

Where $L_1$and $L_2$are two constants.

Now, proceeding with the particular solution, by guessing a solution of the type: $F(\varepsilon) = K\varepsilon$, the first and the second partial derivative w.r.t. ε will be respectively $F_{\varepsilon}(\varepsilon) = K$ and $F_{\varepsilon\varepsilon}(\varepsilon) = 0$ so that, by substituting into the Ito Lemma's expansion we have:

$$\alpha\varepsilon K - rL\varepsilon + \varepsilon = 0 \rightarrow \varepsilon[\alpha K - rL + 1] = 0$$

That is solved for:

$$K = \frac{1}{r-\alpha}$$

From which the particular solution is given by:

$$F(\varepsilon) = K\varepsilon = \frac{\varepsilon}{r-\alpha}$$

Finally, by putting together the two parts we have that the general solution is:

$$F(\varepsilon) = \left[L_1\varepsilon^{\beta_1} + L_2\varepsilon^{\beta_1}\right] + \frac{\varepsilon}{r-\alpha}$$

Now, in order to determine the value of $L_1$and L_2 we have to impose two terminal conditions: because the process is free to move in the positive interval $(0, \infty)$, on one hand if $\varepsilon \rightarrow 0$ also the value of the project should go to zero, so that $L_1 = 0$, on the other hand if $\varepsilon \to \infty$ the value of the project cannot be higher than the fundamental, so that $L_2 = 0$. Given the two conditions we have:

$$F(\varepsilon) = \frac{\varepsilon}{r - \alpha}$$

and recalling the initial assumption for which $\varepsilon_t = Y_t \pi_M$ we have shown:

$$F(\varepsilon_t) = E_t[\int_t^{\infty} \varepsilon_s e^{-r(s-t)} ds] = E_t[\int_t^{\infty} Y_s \pi_M e^{-r(s-t)} ds] = \frac{\pi_M}{r - \alpha} Y_t = F(Y_t)$$

II) I will show that

$$max_T E[(F_T - I)e^{-rT}] = max_{F^*}(F^* - I)\left(\frac{F}{F^*}\right)^{\beta}$$

By dropping the time representation for simplicity, and by assuming that the payoff deriving by a project is $(F - I)$, I will consider that F evolves accordingly to the following Geometric Brownian Motion:

$$dF = \alpha F dt + \sigma F\, dW$$

Now, considering $V(F)$ be the value of the option to invest in the project and F^* the trigger of the optimal investment we can write:

$$V(F) = max_T E[(F_T - I)e^{-rT}]$$

So, by maximizing with respect to T we get, over the continuation region $(F \leq F^*)$:

$$V(F) = \frac{E[V(F + dF)]}{1 + rdt} \to V(F) r dt = E[dV(F)]$$

Now, by assuming $r \leq \alpha$, and by expanding $dV(F)$ by using the Ito Lemma we can rewrite the above problem as the following second order homogenous differential equation:

$$\frac{1}{2}\sigma^2 F^2 V_{FF}(F) + \alpha F V_F(F) - rV(F) = 0$$

Where $V_F(F)$ and $V_{FF}(F)$ represent respectively the first and the second partial derivative w.r.t. F. Now, by guessing a solution of the form $V(F) = AF^{\beta}$ we have that: $V_F(F) = A\beta F^{\beta-1}$ and $V_{FF}(F) = A\beta(\beta - 1)F^{\beta-2}$. So, by substitution in the above equation we get:

$$Z(\beta) = \frac{1}{2}\sigma^2 \beta(\beta - 1) + \alpha\beta - r = 0$$

For which the two solutions $(\beta_1 > 1;\ \beta_2 < 0)$ are given by:

$$\beta_1, \beta_2 = \frac{1}{2} - \frac{\alpha}{\sigma^2} \pm \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}}$$

Then the general solution will be given by:

$$V(F) = A_1 F^{\beta_1} + A_2 F^{\beta_1}$$

Where $A_1$and $A_2$are two constants. Now, given that $V(F)$ can go to zero but not to infinity (provided that it exist a point that it trigger the investment), we can get rid of the second term by setting $A_2 = 0$. So, the above expression must be solved accordingly to following boundary conditions:

a) $V(0) = 0$

b) $V(F^*) = F^* - I$

c) $V'(F^*) = 1$

Where the b) is called the "value-matching" condition, that states that at the moment in which the option is exercised its net payoff is given by $(F^* - I)$, and the c) is the "smooth-pasting" or condition, and ensures that the exercise trigger is chosen to maximize the value of the option.

So, by solving the system of equations we get:

$$F^* = \frac{\beta}{\beta - 1} I$$

And:

$$A = \frac{F^* - I}{(F^*)^\beta}$$

Finally, by substituting these in the $V(F)$ we have:

$$V(F) = \begin{cases} (F^* - I)\left(\frac{F}{F^*}\right)^\beta & for \quad F \leq F^* \\ (F - I) & for \quad F > F^* \end{cases}$$

And in conclusion we can say that:

$$V(F) = max_T E[(F_T - I)e^{-rT}] = max_T E[e^{-rT}](F_T - I) = max_{F^*}\left(\frac{F}{F^*}\right)^\beta (F^* - I)$$

Now, recalling from the first part that of the demonstration that:

$$F(\varepsilon_t) = E_t[\int_t^\infty \varepsilon_s e^{-r(s-t)} ds] = E_t[\int_t^\infty Y_s \pi_M e^{-r(s-t)} ds] = \frac{\pi_M}{r - \alpha} Y_t = F_t$$

By putting the things together, we will have:

$$max_T E[(F_T - I)e^{-rT}] = max_T E[e^{-rT}](F_T - I) = max_{y^*}\left(\frac{\pi_M}{r-\alpha}y^* - I\right)\left(\frac{y}{y^*}\right)^{\beta}$$

Appendix 1.1.b – Sensitivity analysis of the function of parameter $\boldsymbol{\beta}$.

$\beta(\alpha, \sigma, r) > 1$ is the positive root of the characteristic equation $\frac{1}{2}\sigma^2\beta(\beta-1) + \alpha\beta - r = 0$

$$\beta(\alpha, \sigma, r) = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}}$$

The derivatives with respect to parameters α (expected growth of the market), σ (volatility of the market) and r (discount rate) are:

$$\frac{d\beta}{d\alpha} = \frac{-\beta}{s} < 0 \quad ; \quad \frac{d\beta}{d\sigma} = \frac{-2(r - \beta\alpha)}{\sigma s} \leq 0 \quad ; \quad \frac{d\beta}{dr} = \frac{1}{s} > 0$$

Defining: $s = \sigma^2\left(\beta + \left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)\right) \geq 0$

Note that $\frac{d\beta}{d\sigma} = 0$ only if $\sigma = 0$ (no volatility case) and that $r > \beta\alpha$.

Appendix 1.1.c – Investment threshold of potential investor in the integrated (benchmark) case.

Going through the maximization problem of the firm we can write $V_I(y)$ as:

$$V_I(y) = \left(\frac{\pi_M}{r-\alpha}y^* - I\right)\left(\frac{y}{y^*}\right)^{\beta} = \frac{\pi_M}{r-\alpha}y^*\left(\frac{y}{y^*}\right)^{\beta} - I\left(\frac{y}{y^*}\right)^{\beta}$$

$$= \frac{\pi_M}{r-\alpha}y^{\beta}(y^*)^{1-\beta} - Iy^{\beta}(y^*)^{1-\beta}$$

Solving the derivative with respect to y^*we have:

$$\frac{dV(y)}{dy^*} = \frac{\pi_M}{r-\alpha}y^{\beta}(1-\beta)(y^*)^{-\beta} + \beta I y^{\beta}(y^*)^{-\beta-1}$$

Now by setting the derivative to zero and dividing for $\left(\frac{y}{y^*}\right)^{\beta}$ we get:

$$\frac{\pi_M}{r-\alpha}(1-\beta) + \beta I(y^*)^{-1} = 0$$

Rearranging and solving for y^* we get:

$$y^* = \frac{r-\alpha}{\pi_M}\frac{\beta}{\beta-1}I$$

Remember we assumed that initially the market size is sufficiently small, so that it is convenient to delay the investment, we can say now more precisely that $Y_0 < y^*$.

Appendix 1.2.a. – Optimal price sets by the Supplier in the separated case.

By going through the maximization problem of the upstream firm, substituting $y_1(p_1)$ into the equation for $V_{B1}(y)$ we can write:

$$V_{B1}(y) = (p_1 - I)\left(\frac{y}{y_1(p_1)}\right)^{\beta} = (p_1 - I)y^{\beta}\left(\frac{r-\alpha}{\pi_M}\frac{\beta}{\beta-1}\right)^{-\beta} p_1^{-\beta}$$

Defining for simplicity $a = \left(\frac{r-\alpha}{\pi_M}\frac{\beta}{\beta-1}\right)^{-\beta}$and rewriting we get:

$$V_{B1}(y) = p_1^{1-\beta}y^{\beta}a - Iay^{\beta}p_1^{-\beta}$$

Solving the derivative with respect to p_S :

$$\frac{dV_{B1}(y)}{dp_1} = (1-\beta)p_1^{-\beta}y^{\beta}a + \beta aIy^{\beta}p_1^{-\beta-1} = \ ay^{\beta}p_1^{-\beta}(1-\beta+I\beta p_1^{-1})$$

By setting the derivative to zero, the equation is solved for:

$$(1-\beta) + I\beta p_1^{-1} = 0 \rightarrow (\beta-1)p_1 = I\beta \rightarrow p_1 = \frac{\beta}{\beta-1}I$$

That represents the optimal price set by the supplier of the input in the separated case.

Appendix 1.2.b. – Value of the option to invest for the potential investor in the separated case.

The value of the option to invest for the firm A is:

$$V_{A1}(y) = \left(\frac{\pi_M}{r-\alpha}y_1(p_1) - p_1\right)\left(\frac{y}{y_1(p_1)}\right)^{\beta}$$

Substituting for the value of $y_1(p_1)$ and p_1 we get:

$$V_{A1}(y) = \left(\frac{\pi_M}{r-\alpha}\frac{r-\alpha}{\pi_M}\left(\frac{\beta}{\beta-1}\right)^2 - \frac{\beta}{\beta-1}I\right)\left(\frac{y}{y_1(p_1)}\right)^{\beta} = I\frac{\beta}{(\beta-1)^2}\left(\frac{y}{y_1(p_1)}\right)^{\beta}$$

Finally substituting forward for $y_1(p_1) = \frac{\beta}{\beta-1}\ y^*$:

$$V_{A1}(y) = I\frac{\beta}{(\beta-1)^2}y^{\beta}\frac{(\beta-1)^{\beta}}{(\beta y^*)^{\beta}} = I\frac{\beta}{(\beta-1)^2}\frac{(\beta-1)^{\beta}}{\beta^{\beta}}\left(\frac{y}{y^*}\right)^{\beta}$$
$$= \left[I\frac{\beta}{\beta-1}\left(\frac{y}{y^*}\right)^{\beta}\right]\frac{\beta}{\beta-1}\frac{(\beta-1)^{\beta}}{\beta^{\beta}} = V_I(y)\left(\frac{\beta}{\beta-1}\right)^{1-\beta}$$

Appendix 1.3.1.a. - Investment threshold set by the potential investor in the case the investment is partially externally funded.

In order to retrieve the optimal investment threshold, the potential investor maximizes:

$$V_{A2}^F(y) = max_{y_F(\gamma_F)}\left((1-\gamma_F)\frac{y_F(\gamma_F)\pi_M}{r-\alpha} - (1-\theta)I\right)\left(\frac{y}{y_F(\gamma_F)}\right)^{\beta}$$
$$= max_{y_F(\gamma_F)}\left((1-\gamma_F)\frac{\pi_M}{r-\alpha}y_F(\gamma_F)\left(\frac{1}{y_F(\gamma_F)}\right)^{\beta}y^{\beta} - (1-\theta)I\ \left(\frac{y}{y_F(\gamma_F)}\right)^{\beta}\right)$$

Taking the derivative with respect to $y_F(\gamma_F)$ we get:

$$\frac{dV_{A2}^F(y)}{dy_F(\gamma_F)} = (1-\gamma_F)(1-\beta)\frac{\pi_M}{r-\alpha}\left(\frac{y}{y_F(\gamma_F)}\right)^{\beta} + \beta(1-\theta)I\ \left(\frac{y}{y_F(\gamma_F)}\right)^{\beta}\frac{1}{y_F(\gamma_F)}$$
$$= \left(\frac{y}{y_F(\gamma_F)}\right)^{\beta}\left((1-\gamma_F)(1-\beta)\frac{\pi_M}{r-\alpha} - \beta(1-\theta)I\ \frac{1}{y_F(\gamma_F)}\right)$$

Setting to zero and solving for $y_F(\gamma_F)$ we obtain:

$$y_F(\gamma_F) = \frac{(1-\theta)}{(1-\gamma_F)}\frac{r-\alpha}{\pi_M}I\frac{\beta}{\beta-1} = \frac{(1-\theta)}{(1-\gamma_F)}y^*$$

Appendix 1.3.1.b. – Optimal compensation offer made by the investment partner in the case the investment is partially externally funded.

The investment partner anticipating the investment timing that will be set by A, in order to retrieve the optimal compensation offer, maximizes:

$$V_{C2}^{L}(y) = max_{\gamma_F}\left(\gamma_F \frac{y_F(\gamma_F)\pi_M}{r-\alpha} - \theta I\right)\left(\frac{y}{y_F(\gamma_F)}\right)^{\beta}$$

$$= max_{\gamma_F}\left[\frac{(1-\theta)}{(1-\gamma_F)} y^{*}\gamma_F \frac{\pi_M}{r-\alpha} - \theta I\right] y^{\beta}\left(\frac{(1-\theta)}{(1-\gamma_F)} y^{*}\right)^{-\beta}$$

$$= max_{\gamma_F}\left[\frac{(1-\theta)}{(1-\gamma_F)} \frac{r-\alpha}{\pi_M} \frac{\beta}{\beta-1} I\gamma_F \frac{\pi_M}{r-\alpha} - \theta I\right] y^{\beta}\left(\frac{(1-\theta)}{(1-\gamma_F)} y^{*}\right)^{-\beta}$$

$$= max_{\gamma_F}\left[\frac{(1-\theta)}{(1-\gamma_F)} \frac{\beta}{\beta-1}\gamma_F - \theta\right](1-\gamma_F)^{\beta} I\left(\frac{y}{y^{*}}\right)^{\beta}(1-\theta)^{\beta}$$

From which, it is equivalent maximize:

$$max_{\gamma_F}\left[\frac{(1-\theta)}{(1-\gamma_F)} \frac{\beta}{\beta-1}\gamma_F - \theta\right](1-\gamma_F)^{\beta} = max_{\gamma_F}[\gamma_F(1-\gamma_F)^{\beta-1}\frac{\beta}{\beta-1}(1-\theta) - \theta(1-\gamma_F)^{\beta}]$$

Taking the derivative with respect to γ_F and setting to zero we get:

$$(1-\gamma_F)^{\beta-1}\frac{\beta}{\beta-1}(1-\theta) - \gamma_F(\beta-1)(1-\gamma_F)^{\beta-2}\frac{\beta}{\beta-1}(1-\theta) + \theta\beta(1-\gamma_F)^{\beta-1}$$

$$= 0 \rightarrow (1-\gamma_F)^{\beta-1}\left[\frac{(1-\theta)}{(\beta-1)} - \frac{\gamma_F}{(1-\gamma_F)}(1-\theta) + \theta\right] = 0$$

$$\rightarrow \frac{(1-\theta)}{(\beta-1)}(1-\gamma_F) + \theta(1-\gamma_F) = \gamma_F(1-\theta)$$

$$\rightarrow \frac{(1-\theta)}{(\beta-1)} - \gamma_F\frac{(1-\theta)}{(\beta-1)} + \theta - \theta\gamma_F = \gamma_F - \gamma_F\theta \rightarrow \gamma_F + \gamma_F\frac{(1-\theta)}{(\beta-1)}$$

$$= \frac{1-\theta}{\beta-1} + \theta \rightarrow \gamma_F\left(1 + \frac{(1-\theta)}{(\beta-1)}\right) = \frac{1-2\theta+\theta\beta}{\beta-1}$$

From which we can solve for γ_F:

$$\gamma_F = \frac{1-2\theta+\theta\beta}{\beta-\theta}$$

Appendix 1.3.2.a. - Investment threshold set by the investment partner in the case the investment is partially externally funded.

In order to retrieve the optimal time to provide the financial resources to undertake the investment, the investment partner maximizes:

$$V_{C2}^{F}(y) = max_{y_L(\gamma_L)}\left(\gamma_L \frac{y_L(\gamma_L)\pi_M}{r-\alpha} - \theta I\right)\left(\frac{y}{y_L(\gamma_L)}\right)^{\beta}$$

$$= max_{y_L(\gamma_L)}\left(\gamma_L \frac{\pi_M}{r-\alpha} y_L(\gamma_L)\left(\frac{1}{y_L(\gamma_L)}\right)^{\beta} y^{\beta} - \theta I \left(\frac{y}{y_L(\gamma_L)}\right)^{\beta}\right)$$

Taking the derivative with respect to $y_L(\gamma_L)$ we get:

$$\frac{dV_{C2}^F(y)}{dy_L(\gamma_L)} = \gamma_L(1-\beta)\frac{\pi_M}{r-\alpha}\left(\frac{y}{y_L(\gamma_L)}\right)^\beta + \beta\theta I\ \left(\frac{y}{y_L(\gamma_L)}\right)^\beta \frac{1}{y_L(\gamma_L)}$$
$$= \left(\frac{y}{y_L(\gamma_L)}\right)^\beta \left(\gamma_L(1-\beta)\frac{\pi_M}{r-\alpha} - \beta\theta I\ \ \frac{1}{y_L(\gamma_L)}\right)$$

Setting to zero and solving for $y_L(\gamma_L)$ we obtain:

$$y_L(\gamma_L) = \frac{\theta}{\gamma_L}\frac{r-\alpha}{\pi_M} I \frac{\beta}{\beta-1} = \frac{\theta}{\gamma_L} y^*$$

Appendix 1.3.2.b. – Optimal compensation offer made by the potential investor in the case the investment is partially externally funded.

The potential investor, anticipating the reactions of the investment partner, sets the optimal compensation offer in order to maximize:

$$V_{A2}^L(y) = max\ _{\gamma_L}\left((1-\gamma_L)\frac{y_L(\gamma_L)\pi_M}{r-\alpha} - (1-\theta)I\right)\left(\frac{y}{y_L(\gamma_L)}\right)^\beta$$
$$= max_{\gamma_L}\left[\frac{\theta}{\gamma_L}y^*\frac{\pi_M}{r-\alpha} - \frac{\theta}{\gamma_L}y^*\gamma_L\frac{\pi_M}{r-\alpha} - (1-\theta)I\right]\left(\frac{y}{y^*}\right)^\beta\left(\frac{\gamma_L}{\theta}\right)^\beta$$
$$= max_{\gamma_L}\left[\frac{\theta}{\gamma_L}\frac{\beta}{\beta-1}I - \frac{\beta}{\beta-1}I\theta - (1-\theta)I\right]\left(\frac{y}{y^*}\right)^\beta\left(\frac{\gamma_L}{\theta}\right)^\beta$$
$$= max_{\gamma_L}\left[\frac{\beta}{\beta-1}I\theta^{1-\beta}\gamma_L^{\beta-1} - \frac{\beta}{\beta-1}I\theta^{1-\beta}\gamma_L^\beta - (1-\theta)I\theta^{-\beta}\gamma_L^\beta\right]\left(\frac{y}{y^*}\right)^\beta$$

From which, it is equivalent maximize the term inside the square brackets:

$$max_{\gamma_L}\left[\frac{\beta}{\beta-1}I\theta^{1-\beta}\gamma_L^{\beta-1} - \frac{\beta}{\beta-1}I\theta^{1-\beta}\gamma_L^\beta - (1-\theta)I\theta^{-\beta}\gamma_L^\beta\right]$$

Taking the derivative with respect to γ_L and setting to zero we get:

$$\beta I\theta^{1-\beta}\gamma_L^{\beta-2} - \frac{\beta^2}{\beta-1}I\theta^{1-\beta}\gamma_L^{\beta-1} - \beta(1-\theta)I\theta^{-\beta}\gamma_L^{\beta-1} = 0$$
$$\rightarrow \left(\beta I\theta^{-\beta}\right)\gamma_L^{\beta-1}\left[\frac{\theta}{\gamma_L} - \frac{\beta}{\beta-1}\theta - 1 + \theta\right] = 0$$

Given that the terms outside the square brackets is by construction always positive, we can solve for:

$$\frac{\theta}{\gamma_L} - \frac{\beta}{\beta-1}\theta - 1 + \theta = 0 \rightarrow \gamma_L\left(\frac{\beta}{\beta-1}\theta + 1 - \theta\right) = \theta \rightarrow \gamma_L\left(\frac{\beta\theta + \beta - 1 - \theta\beta + \theta}{\beta-1}\right) = \theta$$

$$\rightarrow \gamma_L = \frac{\theta(\beta - 1)}{\theta + \beta - 1}$$

Appendix 2 – MATLAB code used to run the simulations.

The following code has been used to make the simulations relatively to Table 1 and to retrieve the graphs utilized for the analysis in Section 2.2 of Chapter 2.

```
clc
clear
%%%%%%%%%%%%%%%%%%%%%%%%_____DEFINE VARIABLES_____%%%%%%%%%%%%%%%%%%%%%%%%

I=100;                  %sunk costs of the investment
r=0.03;                 %interest rate
Y=1;                    %initial level of the stochastic parameter Y0
Pie=1;                  %instantaneous monopoly profit per unit of Yt
alpha=0.02;             %expected growth rate of the market
st=0.15;                %volatility
theta=0.4;              %share of the investment costs financed by C
%syms r real
%int=[0.03 0.05];
beta= 1/2 - alpha/(st^2) + ((alpha/(st^2)-1/2)^2 +(2*r)/(st^2))^(1/2);


%%%%%%%%%%%%%%%%%%%%%%%%%%%_____CHAPTER 1_____%%%%%%%%%%%%%%%%%%%%%%%%%%%

%CASE 1.1: BENCHMARK/INTEGRATED CASE

%optimal investment threshold
Yop=((r-alpha)/Pie)*(beta/(beta-1))*I;
%The value of the option to invest for A
VI=(I/(beta-1))*((Y/Yop)^beta);

%CASE 1.2: SEPARATED CASE

%optimal investment threshold
Y1=Yop*(beta/(beta-1));
%The value of the option to invest for A
Va1=VI*((beta/(beta-1))^(1-beta));
%The value of the option to invest for B
Vb1= VI*((beta/(beta-1))^(-beta));
%Value of the industry
V1=Va1+Vb1;

%CASE 1.3.1: EXTERNAL FUNDING CASE - (A as time deciding agent)

%The optimal investment threshold
YF=Yop*((beta-theta)/(beta-1));
%The value of the option to invest for A
VFa2=VI*(1-theta)*(((beta-1)/(beta-theta))^beta);
%The value of the option to invest for C
VFc2=VI*(((beta-1)/(beta-theta))^(beta-1));
%Value of the industry
VF2=VFa2+VFc2;

%CASE 1.3.2: EXTERNAL FUNDING CASE - (C as time deciding agent)
```

```
%The optimal investment threshold
YL=Yop*((beta-1+theta)/(beta-1));
%The value of the option to invest for A
VLa2=VI*(((beta-1)/(beta-1+theta))^(beta-1));
%The value of the option to invest for C
VLc2=VI*(theta)*(((beta-1)/(beta-1+theta))^(beta));
%Value of the industry
VL2=VLa2+VLc2;

%CASE 1.4.1: THREE AGENTS CASE - (A as time deciding agent)

%The optimal investment threshold is
Y3F=Yop*(beta*(beta-theta)/((beta-1)^2));
%The value of the option to invest for A
VFa3=VI*(1-theta)*(((beta-1)/(beta))^(beta-1))*(((beta-1)/(beta-
theta))^beta);
%The value of the option to invest for B
VFb3= VI*(((beta-1)/(beta))^beta)*(((beta-1)/(beta-theta))^beta);
%The value of the option to invest for C
VFc3= VI*(((beta-1)/(beta))^(beta-1))*(((beta-1)/(beta-theta))^(beta-1));
%Value of the industry
VF3=VFa3+VFb3+VFc3;

%CASE 1.4.1: THREE AGENTS CASE - (C as time deciding agent)

%The optimal investment threshold is
Y3L=Yop*(beta*(beta-1+theta)/((beta-1)^2));
%The value of the option to invest for A
VLa3=VI*(((beta-1)/(beta))^(beta-1))*(((beta-1)/(beta-1+theta))^(beta-1));
%The value of the option to invest for B
VLb3=VI*(((beta-1)/(beta))^(beta))*(((beta-1)/(beta-1+theta))^(beta));
%The value of the option to invest for C
VLc3=VI*(theta)*(((beta-1)/(beta))^(beta-1))*(((beta-1)/(beta-
1+theta))^beta);
%Value of the industry
VL3=VLa3+VLb3+VLc3;

%%%%%%%%%%%%%%%%%%%%_____TABLE OF THE SIMULATION_____%%%%%%%%%%%%%%%%%%%%

Cases={'Benchmark'; 'Outsourcing'; 'External_funding_A as time agent';...
    'External_funding_C as time agent';'Three_agents_A as time
agent';'Three_agents_C as time agent'};
Threshold=[Yop; Y1; YF; YL; Y3F; Y3L];
VA=[VI; Va1; VFa2; VLa2; VFa3; VLa3];
VB=[0; Vb1; 0; 0; VFb3; VLb3];
VC=[0; 0; VFc2; VLc2; VFc3; VLc3];
Industry_Value=[VI; V1; VF2; VL2; VF3; VL3];
Tab1=table(Cases,Threshold,VA,VB,VC,Industry_Value)


%%%%%%%%%%%%%%%%%%%%%%%_____PLOT OF THE GRAPHS_____%%%%%%%%%%%%%%%%%%%%%%%

%Investment thresholds
F0=figure;
fplot(Yop,int,'r')
hold on
fplot(Y1,int,'y')
hold on
fplot(YF,int,'b')
hold on
fplot(YL,int,'g')
hold on
```

```
fplot(Y3F,int,'b--')
hold on
fplot(Y3L,int,'g--')
hold off
legend('Y*','Y1','YF','YL','Y3F','Y3L')
xlabel('Interest rate (r) ');
ylabel('Optimal investment threshold');

%Values of the option to invest for A
F1=figure;
fplot(VI,int,'r')
hold on
fplot(Va1,int,'y')
hold on
fplot(VFa2,int,'b')
hold on
fplot(VLa2,int,'g')
hold on
fplot(VFa3,int,'b--')
hold on
fplot(VLa3,int,'g--')
hold off
legend('VI','Va1','VFa2','VLa2','VFa3','VLa3')
xlabel('Interest rate (r)');
ylabel('Value of the option to invest for A');
```

Appendix 3.1.1. – Values of the option to invest for the agents involved in the case in which suppliers engage in a Bertrand competition over prices.

The value of the option to invest for the two upstream suppliers is:

$$V_S(y) = (p_S - I)\left(\frac{y}{y_4(\gamma_4, p_S)}\right)^{\beta} = (\omega - 1)I\left(\frac{y}{y_4(\gamma_4, p_S)}\right)^{\beta} = (\omega - 1)I\left(\frac{y}{y^*}\right)^{\beta}\left(\frac{\beta - 1}{\beta - \theta}\right)^{\beta}\omega^{-\beta}$$

$$= (\omega - 1)I\left(\frac{y}{y^*}\right)^{\beta}\left(\frac{\beta - 1}{\beta - \theta}\right)^{\beta}\omega^{-\beta}\frac{\beta - 1}{\beta - 1} = \frac{(\omega - 1)}{\omega^{\beta}}V_I(y)\frac{(\beta - 1)^{\beta+1}}{(\beta - \theta)^{\beta}}$$

The value of the option to invest for the potential investor is:

$$V_{A4}(y) = \left[(1-\gamma_4)\frac{\pi_M\, y_4(\gamma_4, p_S)}{r-\alpha} - (1-\theta)p_S\right]\left(\frac{y}{y_4(\gamma_4, p_S)}\right)^{\beta}$$

$$= \left[\left(1 - \frac{1-2\theta+\theta\beta}{\beta-\theta}\right)\frac{\beta-\theta}{\beta-1} y^* \omega \frac{\pi_M}{r-\alpha} - (1-\theta)I\omega\right]\left(\frac{y}{y_4(\gamma_4, p_S)}\right)^{\beta}$$

$$= \left[\frac{(\beta-1)(1-\theta)}{(\beta-\theta)}\frac{\beta-\theta}{\beta-1} y^* \omega \frac{\pi_M}{r-\alpha} - (1-\theta)I\omega\right]\left(\frac{y}{y^*}\right)^{\beta}\left(\frac{\beta-1}{\beta-\theta}\right)^{\beta}\omega^{-\beta}$$

$$= \left[y^* \frac{\pi_M}{r-\alpha} - I\right](1-\theta)\omega^{1-\beta}\left(\frac{\beta-1}{\beta-\theta}\right)^{\beta}\left(\frac{y}{y^*}\right)^{\beta}$$

$$= \left[\frac{\beta}{\beta-1} - 1\right] I(1-\theta)\omega^{1-\beta}\left(\frac{\beta-1}{\beta-\theta}\right)^{\beta}\left(\frac{y}{y^*}\right)^{\beta}$$

$$= \frac{I}{\beta-1}\left(\frac{y}{y^*}\right)^{\beta}(1-\theta)\omega^{1-\beta}\left(\frac{\beta-1}{\beta-\theta}\right)^{\beta} = (1-\theta)\omega^{1-\beta}\left(\frac{\beta-1}{\beta-\theta}\right)^{\beta} V_I(y)$$

The value of the option to invest for the investment partner is:

$$V_{C4}(y) = \left(\gamma_4 \frac{y_4(\gamma_4, p_S)\pi_M}{r-\alpha} - \theta p_S\right)\left(\frac{y}{y_4(\gamma_4, p_S)}\right)^{\beta}$$

$$= \left[\frac{(1-2\theta+\theta\beta)}{(\beta-\theta)}\frac{(\beta-\theta)}{(\beta-1)} y^* \omega \frac{\pi_M}{r-\alpha} - \theta I\omega\right]\left(\frac{y}{y_4(\gamma_4, p_S)}\right)^{\beta}$$

$$= \left[\frac{(1-2\theta+\theta\beta)}{(\beta-1)}\frac{\beta}{\beta-1} I\omega - \theta I\omega\right]\left(\frac{y}{y_4(\gamma_4, p_S)}\right)^{\beta}$$

$$= \left[\frac{(1-2\theta+\theta\beta)\beta - \theta(\beta-1)^2}{(\beta-1)^2}\right] I\omega\left(\frac{y}{y_4(\gamma_4, p_S)}\right)^{\beta}$$

$$= \left[\frac{\beta-\theta}{\beta-1}\right]\frac{I}{\beta-1}\left(\frac{y}{y^*}\right)^{\beta}\left(\frac{\beta-1}{\beta-\theta}\right)^{\beta}\omega^{1-\beta} = V_I\left(\frac{\beta-1}{\beta-\theta}\right)^{\beta-1}\omega^{1-\beta}$$

Appendix 4.1.a – Optimal investment threshold set by the potential investor in the case in which the government co-finance the project.

Proceeding with the maximization problem of agent A in order to retrieve the optimal investment threshold, we solve:

$$V_{AG}^F(y) = max_{y_G^F(\gamma_G^F, G_C^F)}\left[(1-\gamma_G^F)\frac{\pi_M}{r-\alpha} y_G^F(\gamma_G^F, G_C^F) - (1-\theta)I + G_C^F\right]\left(\frac{y}{y_G^F(\gamma_G^F, G_C^F)}\right)^{\beta}$$

Taking the derivative with respect to $y_G^F(\gamma_G^F, G_C^F)$ and setting to zero we have:

$$(1-\gamma_G^F)(1-\beta)\frac{\pi_M}{r-\alpha}\left(\frac{y}{y_G^F(\gamma_G^F,G_C^F)}\right)^{\beta}+\beta(1-\theta)I\left(\frac{y}{y_G^F(\gamma_G^F,G_C^F)}\right)^{\beta}\frac{1}{y_G^F(\gamma_G^F,G_C^F)}$$

$$-\beta G_C^F\left(\frac{y}{y_G^F(\gamma_G^F,G_C^F)}\right)^{\beta}\frac{1}{y_G^F(\gamma_G^F,G_C^F)}=0\rightarrow$$

$$\left(\frac{y}{y_G^F(\gamma_G^F,G_C^F)}\right)^{\beta}\left[-(1-\gamma_G^F)(\beta-1)\frac{\pi_M}{r-\alpha}+\beta(1-\theta)I\frac{1}{y_G^F(\gamma_G^F,G_C^F)}-\beta G_C^F\frac{1}{y_G^F(\gamma_G^F,G_C^F)}\right]=0$$

Given that $\left(\frac{y}{y_G^F(\gamma_G^F,G_C^F)}\right)^{\beta}>0$ for assumption, the equation is verified for:

$$\begin{aligned}
y_G^F(\gamma_G^F,G_C^F)&=\frac{r-\alpha}{\pi_M}\frac{1}{(1-\gamma_G^F)}\frac{1}{(\beta-1)}\beta[(1-\theta)I-G_C^F]\\
&=\frac{r-\alpha}{\pi_M}\frac{\beta}{(\beta-1)}I\frac{(1-\theta)}{(1-\gamma_G^F)}-G_C^F\frac{r-\alpha}{\pi_M}\frac{1}{(1-\gamma_G^F)}\frac{\beta}{(\beta-1)}\frac{I}{I}\\
&=\frac{(1-\theta)}{(1-\gamma_G^F)}y^*-\frac{r-\alpha}{\pi_M}\frac{\beta}{(\beta-1)}I\frac{G_C^F}{I(1-\gamma_G^F)}=y^*\frac{(1-\theta)}{(1-\gamma_G^F)}-y^*\frac{G_C^F}{I(1-\gamma_G^F)}\\
&=\frac{y^*}{(1-\gamma_G^F)}\left[(1-\theta)-\frac{G_C^F}{I}\right]=\frac{y^*}{(1-\gamma_G^F)}\left[\frac{I(1-\theta)-G_C^F}{I}\right]=\frac{y^*}{(1-\gamma_G^F)}\Phi
\end{aligned}$$

With $\Phi=\frac{I(1-\theta)-G_C^F}{I}$.

Appendix 4.1.b - Optimal compensation share set by the private investment partner in the case in which the government co-finance the project.

Going through the maximization problem of agent C, we have that the optimal compensation offer is derived from the solution of the following expression:

$$\begin{aligned}
V_{CG}^F(y)&=max_{\gamma_G^F}\left[\gamma_G^F\frac{\pi_M}{r-\alpha}y_G^F(\gamma_G^F,G_C^F)-\theta I\right]\left(\frac{y}{y_G^F(\gamma_G^F,G_C^F)}\right)^{\beta}\\
&=max_{\gamma_G^F}\left[\gamma_G^F\frac{\pi_M}{r-\alpha}\left(\frac{r-\alpha}{\pi_M}\frac{\beta}{\beta-1}I\right)\frac{\Phi}{(1-\gamma_G^F)}-I\theta\right]\left(\frac{y}{y_G^F(\gamma_G^F,G_C^F)}\right)^{\beta}\\
&=max_{\gamma_G^F}\left[\frac{\gamma_G^F}{(1-\gamma_G^F)}\frac{\beta}{\beta-1}I\Phi-I\theta\right]\left(\frac{y}{y_G^F(\gamma_G^F,G_C^F)}\right)^{\beta}\\
&=max_{\gamma_G^F}\left[\gamma_G^F(1-\gamma_G^F)^{\beta-1}\frac{\beta}{\beta-1}I\Phi^{1-\beta}\left(\frac{y}{y^*}\right)^{\beta}-I\theta\left(\frac{y}{y^*}\right)^{\beta}(1-\gamma_G^F)^{\beta}\Phi^{-\beta}\right]\\
&=max_{\gamma_G^F}\left[\gamma_G^F(1-\gamma_G^F)^{\beta-1}\frac{\beta}{\beta-1}I\Phi^{1-\beta}-I\theta(1-\gamma_G^F)^{\beta}\Phi^{-\beta}\right]\left(\frac{y}{y^*}\right)^{\beta}
\end{aligned}$$

Given that $\left(\frac{y}{y^*}\right)^{\beta}$ does not contain γ_G^F we can proceed by taking the derivative w.r.t. γ_G^F of the argument inside the square brackets and set it to zero:

$$1(1-\gamma_G^F)^{\beta-1}\frac{\beta}{\beta-1}\mathrm{I}\Phi^{1-\beta}-\gamma_G^F(\beta-1)(1-\gamma_G^F)^{\beta-2}\frac{\beta}{\beta-1}\mathrm{I}\Phi^{1-\beta}+\beta(1-\gamma_G^F)^{\beta-1}\Phi^{-\beta}\mathrm{I}\theta$$
$$=0\rightarrow(1-\gamma_G^F)^{\beta-1}\frac{\Phi^{1-\beta}}{\beta-1}-\gamma_G^F(1-\gamma_G^F)^{\beta-2}\Phi^{1-\beta}+(1-\gamma_G^F)^{\beta-1}\Phi^{-\beta}\theta=0$$
$$\rightarrow(1-\gamma_G^F)^{\beta-1}\frac{1}{\beta-1}-\gamma_G^F(1-\gamma_G^F)^{\beta-2}+(1-\gamma_G^F)^{\beta-1}\Phi^{-1}\theta=0$$
$$\rightarrow(1-\gamma_G^F)^{\beta-1}\left[\frac{1}{\beta-1}-\gamma_G^F(1-\gamma_G^F)^{-1}+\Phi^{-1}\theta\right]=0\rightarrow$$

Given that by assumption $0<\gamma_G^F<1$ and so $(1-\gamma_G^F)^{\beta-1}>0$ we have that the solution of the above equation is given by:

$$\frac{1}{\beta-1}-\gamma_G^F(1-\gamma_G^F)^{-1}+\Phi^{-1}\theta=0\rightarrow\frac{\gamma_G^F}{(1-\gamma_G^F)}=\frac{1}{\beta-1}+\Phi^{-1}\theta\rightarrow\gamma_G^F$$
$$=\frac{1}{\beta-1}-\frac{\gamma_G^F}{\beta-1}+\frac{\theta}{\Phi}-\frac{\theta}{\Phi}\gamma_G^F\rightarrow\gamma_G^F\left(1+\frac{1}{\beta-1}+\frac{\theta}{\Phi}\right)=\frac{1}{\beta-1}+\frac{\theta}{\Phi}$$
$$\rightarrow\gamma_G^F\left(\frac{\Phi(\beta-1)+\Phi+(\beta-1)\theta}{(\beta-1)\Phi}\right)=\frac{\Phi+\theta(\beta-1)}{(\beta-1)\Phi}\rightarrow\gamma_G^F$$
$$=\frac{\Phi+\theta(\beta-1)}{\Phi(\beta-1)+\Phi+(\beta-1)\theta}=\frac{\Phi+\theta(\beta-1)}{\beta\Phi+\theta(\beta-1)}$$

Appendix 4.1.c – Value of the subsidy set by the Government in the case the potential investor is the time deciding agent.

By substituting γ_G^F into the expression for $y_G^F(\gamma_G^F,G_C^F)$ we get that the optimal investment threshold becomes:

$$y_G^F(\gamma_G^F,G_C^F)=\frac{y^*}{(1-\gamma_G^F\ )}\Phi=y^*\Phi\left[\frac{\beta\Phi+\theta(\beta-1)}{\Phi(\beta-1)}\right]=\frac{y^*}{(\beta-1)}[\beta\Phi+\theta(\beta-1)]$$
$$=y^*\frac{\beta}{\beta-1}\left[1-\theta-\frac{G_C^F}{I}\right]+y^*\theta=y^*\frac{\beta}{\beta-1}-y^*\frac{\beta}{\beta-1}\theta-y^*\frac{\beta}{\beta-1}\frac{G_C^F}{I}+y^*\theta$$
$$=y^*\frac{\beta}{\beta-1}\left(1-\frac{G_C^F}{I}\right)+y^*\theta\left(1-\frac{\beta}{\beta-1}\right)=y^*\frac{\beta}{\beta-1}\left(\frac{I-G_C^F}{I}\right)-\frac{y^*}{\beta-1}\theta$$

Now, given that $\frac{dy_G^F(\gamma_G^F,G_C^F)}{dG_C^F} = -y^* \frac{\beta}{(\beta-1)I}$ and that the government want to set G_C^F so that to maximize the value of its option to participate, we have that G_C^F is derived by the solution of the following problem:

$$V_G^F(y) = max_{G_C^F}\left[\frac{G_B}{r-\alpha} - G_C^F\right]\left(\frac{y}{y_G^F(\gamma_G^F, G_C^F)}\right)^{\beta}$$

So, by taking the derivative with respect to G_C^F :

$$\frac{dV_G^F(y)}{dG_C^F} = -\beta\left(\frac{y}{y_G^F(\gamma_G^F, G_C^F)}\right)^{\beta-1}\left(\frac{dy_G^F(\gamma_G^F, G_C^F)}{dG_C^F}\right)\left(\frac{y}{(\,y_G^F(\gamma_G^F, G_C^F)\,)^2}\right)\left[\frac{G_B}{r-\alpha} - G_C^F\right]$$
$$-\left(\frac{y}{y_G^F(\gamma_G^F, G_C^F)}\right)^{\beta}$$

Now, by setting the derivative to zero and solving for G_C^F:

$$\left(\frac{y}{y_G^F(\gamma_G^F, G_C^F)}\right)^{\beta}\left[-\beta\left(\frac{y}{y_G^F(\gamma_G^F, G_C^F)}\right)^{-1}\left(-y^*\frac{\beta}{(\beta-1)I}\right)\left(\frac{y}{(y_G^F(\gamma_G^F, G_C^F))^2}\right)\left(\frac{G_B}{r-\alpha} - G_C^F\right) - 1\right]$$
$$= 0 \rightarrow \beta\left(y^*\frac{\beta}{(\beta-1)I}\right)\left(\frac{(\beta-1)}{y^*[\beta\Phi + \theta(\beta-1)]}\right)\left(\frac{G_B}{r-\alpha} - G_C^F\right) = 1$$
$$\rightarrow \frac{\beta^2}{I[\beta\Phi + \theta(\beta-1)]}\left(\frac{G_B}{r-\alpha} - G_C^F\right) = 1 \rightarrow \frac{\beta^2}{I}\left(\frac{G_B}{r-\alpha} - G_C^F\right)$$
$$= [\beta\Phi + \theta(\beta-1)] \rightarrow \frac{\beta^2}{I}\left(\frac{G_B}{r-\alpha} - G_C^F\right) = \beta\left[\frac{I(1-\theta) - G_C^F}{I}\right] + \theta(\beta-1)$$
$$\rightarrow \frac{G_B}{r-\alpha}\beta - \beta G_C^F = I(1-\theta) - G_C^F + \frac{(\beta-1)}{\beta}\theta I \rightarrow G_C^F(\beta-1)$$
$$= \frac{G_B}{r-\alpha}\beta - I(1-\theta) - \frac{(\beta-1)}{\beta}\theta I \rightarrow G_C^F = \frac{\beta}{\beta-1}\frac{G_B}{r-\alpha} - \frac{I(1-\theta)}{\beta-1} - \frac{\theta I}{\beta}$$
$$\rightarrow G_C^F = \frac{\beta}{\beta-1}\frac{G_B}{r-\alpha} - I\left[\frac{(\beta-\theta)}{\beta(\beta-1)}\right] = \frac{1}{\beta-1}\left[\beta\frac{G_B}{r-\alpha} - \frac{I(\beta-\theta)}{\beta}\right]$$
$$= \frac{1}{\beta-1}\left[\beta\frac{G_B}{r-\alpha} - I + \frac{I\theta}{\beta}\right]$$

Appendix 4.2.a – Optimal compensation offer set by the potential investor in the case in which the project is financed both by private VC and by the government.

The potential investor determines the optimal compensation share by solving the following maximization problem:

$$V_{AG}^{L}(y) = max_{\gamma_G^L}\left[(1-\gamma_G^L)\frac{\pi_M}{r-\ \alpha}y_G^L(\gamma_G^L) - (1-\theta)I + G_C^L\right]\left(\frac{y}{y_G^L(\gamma_G^L)}\right)^{\beta}$$

$$= max_{\gamma_G^L}\left[\frac{(1-\gamma_G^L)}{\gamma_G^L}\frac{\beta}{\beta-1}\theta I - I + \theta I + G_C^L\right]\left(\frac{y}{y^*}\right)^{\beta}\left(\frac{\gamma_G^L}{\theta}\right)^{\beta}$$

$$= max_{\gamma_G^L}\left[(1-\gamma_G^L)(\gamma_G^L)^{\beta-1}\frac{\beta}{(\beta-1)}\frac{\theta I}{\theta^{\beta}} - I\left(\frac{\gamma_G^L}{\theta}\right)^{\beta} + (\gamma_G^L)^{\beta}\frac{\theta I}{\theta^{\beta}}\right.$$

$$\left.+\left(\frac{\gamma_G^L}{\theta}\right)^{\beta}G_C^L\right]\left(\frac{y}{y^*}\right)^{\beta}$$

Note that, because $\left(\frac{y}{y^*}\right)^{\beta}$ does not contains γ_G^L we maximize the argument inside the square brackets. So, by taking the derivative w.r.t γ_G^L and set it to zero we have:

$$-\frac{\theta I}{\theta^{\beta}}\frac{\beta}{(\beta-1)}(\gamma_G^L)^{\beta-1} + \beta(1-\gamma_G^L)(\gamma_G^L)^{\beta-2}\frac{\theta I}{\theta^{\beta}} - \beta(\gamma_G^L)^{\beta-1}I\frac{(1-\theta)}{\theta^{\beta}} + \beta G_C^L\frac{(\gamma_G^L)^{\beta-1}}{\theta^{\beta}} = 0$$

$$\rightarrow -\frac{\theta I}{(\beta-1)} + \theta I\frac{(1-\gamma_G^L)}{\gamma_G^L} - I(1-\theta) + G_C^L = 0$$

$$\rightarrow \gamma_G^L\left[-\frac{\theta I}{(\beta-1)} - I(1-\theta) + G_C^L - I\theta\right] + \theta I = 0$$

$$\rightarrow \gamma_G^L\left[\frac{\theta I}{(\beta-1)} + I(1-\theta) - G_C^L + I\theta\right] = \theta I \rightarrow \gamma_G^L$$

$$= \frac{I\theta(\beta-1)}{I\theta + I(\beta-1) - G_C^L(\beta-1)} = \frac{I\theta(\beta-1)}{I\theta + (I-G_C^L)(\beta-1)}$$

Appendix 4.2.b – Value of the subsidy set by the Government in the case in which the private investment partner is the time deciding agent.

Before proceeding note that $\frac{dy_G^L(\gamma_G^L,G_C^L)}{dG_C^L} = -\frac{r-\alpha}{\pi_M}\frac{\beta}{(\beta-1)}$ and that the government want to set G_C^L to maximize the value of its option to participate. So, we have that G_C^L is derived by the solution of the following problem:

$$V_G^L(y) = max_{G_C^L}\left[\frac{G_B}{r-\ \alpha} - G_C^L\right]\left(\frac{y}{y_G^L(\gamma_G^L, G_C^L)}\right)^{\beta}$$

By taking the derivative w.r.t G_C^L and set it to zero we have:

$$-\beta\left(\frac{y}{y_G^L(\gamma_G^L,G_C^L)}\right)^{\beta-1}\left(\frac{dy_G^L(\gamma_G^L,G_C^L)}{dG_C^L}\right)\left(\frac{y}{(y_G^L(\gamma_G^L,G_C^L)\,)^2}\right)\left[\frac{G_B}{r-\alpha}-G_C^L\right]-\left(\frac{y}{y_G^L(\gamma_G^L,G_C^L)}\right)^{\beta}=0$$

$$\rightarrow\left[-\beta\left(\frac{1}{(y_G^L(\gamma_G^L,G_C^L)}\right)\left(\frac{dy_G^L(\gamma_G^L,G_C^L)}{dG_C^L}\right)\left(\frac{G_B}{r-\alpha}-G_C^L\right)-1\right]\left(\frac{y}{y_G^L(\gamma_G^L,G_C^L)}\right)^{\beta}=0$$

$$\rightarrow\left[\beta\left(\frac{1}{(y_G^L(\gamma_G^L,G_C^L)}\right)\left(\frac{r-\alpha}{\pi_M}\frac{\beta}{(\beta-1)}\right)\left(\frac{G_B}{r-\alpha}-G_C^L\right)-1\right]\left(\frac{y}{y_G^L(\gamma_G^L,G_C^L)}\right)^{\beta}=0$$

Now, given that, by construction $\left(\frac{y}{y_G^L(\gamma_G^L,G_C^L)}\right)^{\beta}>0$ the above expression is verified for:

$$\beta\left(\frac{1}{(y_G^L(\gamma_G^L,G_C^L)}\right)\left(\frac{r-\alpha}{\pi_M}\frac{\beta}{(\beta-1)}\right)\left(\frac{G_B}{r-\alpha}-G_C^L\right)-1=0$$

$$\rightarrow\left(\frac{\pi_M(\beta-1)^2}{(r-\alpha)[I\theta+(I-G_C^L)(\beta-1)]}\right)\left(\frac{r-\alpha}{\pi_M}\frac{\beta}{(\beta-1)}\right)\left(\frac{G_B}{r-\alpha}-G_C^L\right)-1=0$$

$$\rightarrow\left(\frac{\beta(\beta-1)}{[I\theta+(I-G_C^L)(\beta-1)]}\right)\left(\frac{G_B}{r-\alpha}-G_C^L\right)-1=0$$

$$\rightarrow\beta(\beta-1)\frac{G_B}{r-\alpha}-G_C^L\beta(\beta-1)=I\theta+(I-G_C^L)(\beta-1)$$

$$\rightarrow G_C^L(\beta-1)=\beta\frac{G_B}{r-\alpha}-\frac{I\theta}{(\beta-1)}-I\rightarrow G_C^L$$

$$=\frac{\beta}{(\beta-1)}\frac{G_B}{r-\alpha}-\frac{I\theta}{(\beta-1)^2}-\frac{I}{(\beta-1)}$$

$$=\frac{\beta}{(\beta-1)}\frac{G_B}{r-\alpha}-\frac{I}{(\beta-1)}\left[\frac{\theta+(\beta-1)}{(\beta-1)}\right]$$

Appendix 4.3 – MATLAB code used to run the simulations.

The following code is the continuation of the one that has been used to make the simulations relatively to Table 1 in section 1.5 of Chapter 1.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%_____CHAPTER 4_____%%%%%%%%%%%%%%%%%%%%%%%%%%%


GbLimFd=((beta-theta)/(beta^2))*(r-alpha)*I;
GbLimF1=((beta-theta)/(beta))*(r-alpha)*I-(Y*((beta-1)/(beta))^3)*Pie;
GbLimFu=((beta-theta)/(beta))*(r-alpha)*I;
GbLimLd=((theta+beta-1)/((beta-1)*beta))*(r-alpha)*I;
GbLimLu=((theta+beta-1)/(beta-1))*(r-alpha)*I;
GbLimL1=((beta-1+theta)/(beta-1))*(r-alpha)*I-(Y*((beta-1)/(beta))^2)*Pie;
GbLimGF=(((beta-theta)/(beta))-(((((beta-1)/beta)^2))*theta))*(r-
alpha)*I*0.9999;
XlimF=I*(r-alpha)*((beta-theta)/(beta));
XlimL=I*(r-alpha)*((theta+beta-1)/(beta-1));
```

```matlab
syms Gb real
int=[0 3];
int1=[0 GbLimF1]
int2=[0 GbLimL1]
int3=[GbLimF1 3]
int4=[GbLimL1 3]
int5=[0 GbLimGF]


%CASE 4.1 - A as time deciding agent

%subsidy payed by the government
GcFF=(1/(beta-1))*(((Gb*beta)/(r-alpha))-I*((beta-theta)/beta));
GcFu=(1/(beta-1))*(((GbLimGF*beta)/(r-alpha))-I*((beta-theta)/beta));
GcF=subplus(GcFF);
%constant phi
phi=1-theta-(GcF/I);
phiu=1-theta-(GcFu/I);
%optimal compensation share
gammaGF=(phi+(theta*(beta-1)))/((phi*beta)+(theta*(beta-1)));
gammaGFu=(phiu+(theta*(beta-1)))/((phiu*beta)+(theta*(beta-1)));
%optimal investment threshold
YgF=Yop*phi/(1-gammaGF);
YgFu=Yop*phiu/(1-gammaGFu);
%Value of the option to invest for A
VFaG=((1-gammaGF)*(YgF*Pie/(r-alpha))-(1-theta)*I+GcF)*((Y/YgF)^beta);
VFaGu=((1-gammaGFu)*(YgFu*Pie/(r-alpha))-(1-theta)*I+GcFu)*((Y/YgFu)^beta);
%The value of the option to invest for C
VFcG=((gammaGF)*(YgF*Pie/(r-alpha))-(theta*I))*((Y/YgF)^beta);
%The value of the option to invest for G
VFg=((Gb/(r-alpha))-GcF)*((Y/YgF)^beta);


%CASE 4.1 - A as time deciding agent

%subsidy payed by the government
GcLL=(I/(beta-1))*(((beta*Gb)/((r-alpha)*I))-(theta/(beta-1))-1);
GcLu=(I/(beta-1))*(((beta*GbLimL1)/((r-alpha)*I))-(theta/(beta-1))-1);
GcL=subplus(GcLL);
%optimal compensation share
gammaGL=(I*theta*(beta-1))/(I*theta+((I-GcL)*(beta-1)));
gammaGLu=(I*theta*(beta-1))/(I*theta+((I-GcLu)*(beta-1)));
%optimal investment threshold
YgL=Yop*(theta/gammaGL);
YgLu=Yop*(theta/gammaGLu);
%Value of the option to invest for A
VLaG=((1-gammaGL)*(YgL*Pie/(r-alpha))-(1-theta)*I+GcL)*((Y/YgL)^beta);
VLaGu=((1-gammaGLu)*(YgLu*Pie/(r-alpha))-(1-theta)*I+GcLu)*((Y/YgLu)^beta);
%The value of the option to invest for C
VLcG=((gammaGL)*(YgL*Pie/(r-alpha))-(theta*I))*((Y/YgL)^beta);
%The value of the option to invest for G
VLg=((Gb/(r-alpha))-GcL)*((Y/YgL)^beta);


%%%%%%%%%%%%%%%%%%%%%%%_____PLOT OF THE GRAPHS_____%%%%%%%%%%%%%%%%%%%%%%%

%Investment thresholds
F0=figure;
fplot(Yop,int,'r--')
hold on
fplot(YF,int,'b--')
hold on
```

```
fplot(YL,int,'g--')
hold on
fplot(YgF,int1,'b')
hold on
fplot(YgL,int2,'g')
hold on
fplot(Y,int,'black')
hold off
legend('Y*','YF','YL','YgF','YgL')
xlabel('social benefits: Gb');
ylabel('Optimal investment threshold');
limits=[0 21];
ylim(limits);


%Values of the option to invest for A
F1=figure;
fplot(VI,int,'r--')
hold on
fplot(VFaG,int5,'b')
hold on
fplot(VLaG,int2,'g')
hold on
fplot(VFa2,int,'b--')
hold on
yline(VLa2,'g--');
hold on
fplot(VFaGu,int3,'b')
hold on
fplot(VLaGu,int4,'g')
hold on
xline(GbLimF1, 'black--')
hold on
xline(GbLimFd, 'm--')
hold on
xline(GbLimLd, 'm--')
hold off
legend('VI','VFaG','VLaG','VFa2','VLa2')
xlabel('social benefits: Gb');
ylabel('Value of the option to invest for A');
limits1=[-10 100];
ylim(limits1);
```

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