Università degli Studi di Padova – Dipartimento di Ingegneria Industriale

Corso di Laurea in Ingegneria Aerospaziale

Relazione per la prova finale
«Il modello geo-potenziale terrestre»

Trattazione teorica, implementazione e applicativi in MATLAB per calcolo dello station keeping in orbita GEO

Tutor universitario:

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Laureando:

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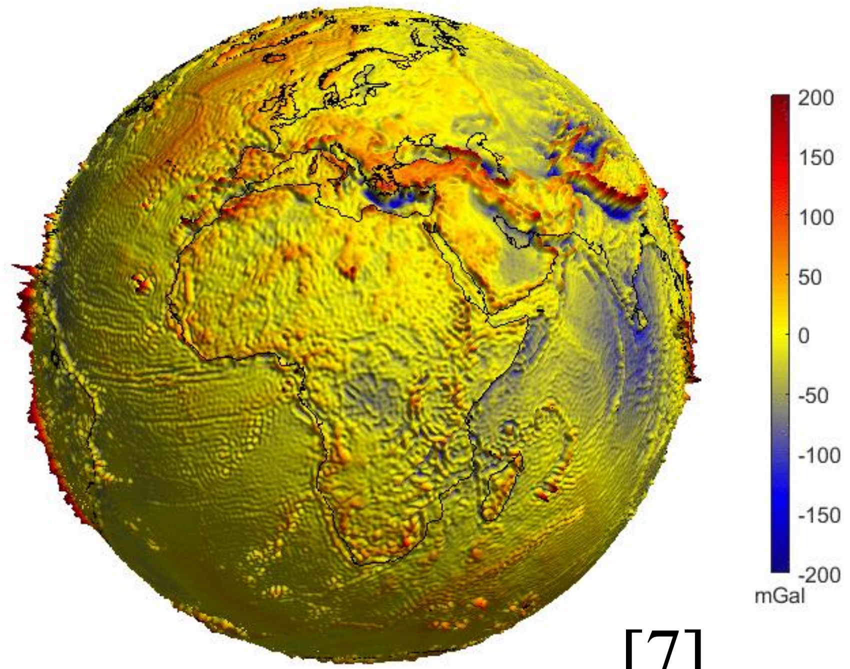
Padova, 11/03/2022

INTRODUZIONE E OBIETTIVI DEL LAVORO

1) Che «forma» ha il pianeta Terra? Come si determina la sua vera forma?

2) La costante g è davvero costante? Perché il moto dei satelliti non è stazionario?

3) Come si ricavano matematicamente i coefficienti Jn studiati nei vari corsi?



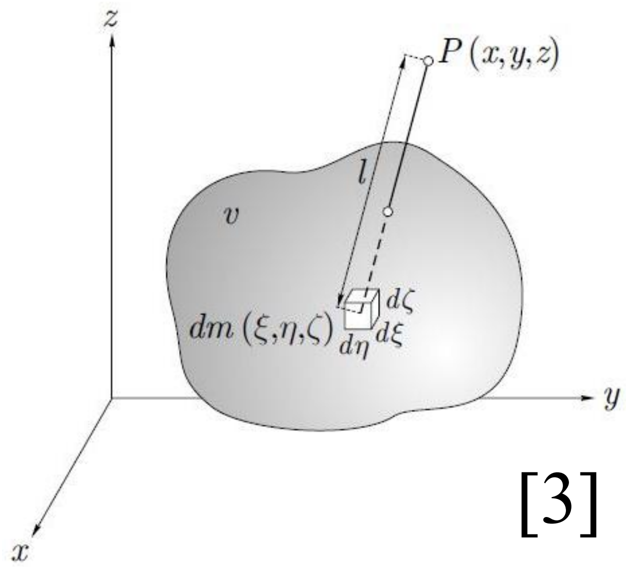
[7]

Gravity disturbance (EGM2008, nmax=1000)

FORZA GRAVITAZIONALE ED ESPANSIONE DEL POTENZIALE

$$\mathbf{F} = G\frac{m_1 m_2}{l^2} = G\frac{m}{l^2} = [X, Y, Z] = grad\, V$$

OBIETTIVO ⟶ $V = G\iiint_V \frac{dm}{l} = G\iiint_V \frac{\rho}{l} dv$



[3]

$$\Delta V = -4\pi G\rho$$ **Sulla superficie**

$$\boxed{\Delta V = 0}$$ **Fuori dalla distribuzione di massa**

«Eq. di Laplace»

$$\Delta V = 0 \; (*) \longrightarrow$$ **Soluzioni: <u>FUNZIONI ARMONICHE</u> (da ricercare)**

$$\Delta V = G\Delta\left[\iiint_V \frac{\rho}{l} dv\right] = G\iiint_V \rho\Delta\left(\frac{1}{l}\right) dv = 0$$

1) Trasformare il Laplaciano di V in coordinate sferiche

$$\Delta V = \frac{\partial^2 V}{\partial r^2} + \frac{2}{r}\frac{\partial V}{\partial r} + \frac{1}{r^2}\frac{\partial^2 V}{\partial \theta^2} + \frac{\cot\theta}{r^2}\frac{\partial V}{\partial \theta} + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial \lambda^2} = 0$$

2) Trovare le soluzioni di (*)

- Funzioni armoniche sferiche solide $\longrightarrow$ $V = \sum_{n=0}^{\infty} r^n Y_n(\theta,\lambda)$ *e* $V = \sum_{n=0}^{\infty} \frac{Y_n(\theta,\lambda)}{r^{n+1}}$

- Funzione armonica sferica superficiale $\longrightarrow$ $Y_n(\theta,\lambda) = \sum_{m=0}^{n}[a_{nm}P_{nm}(\cos\theta)\cos m\lambda + b_{nm}P_{nm}(\cos\theta)\sin m\lambda]$

3) Trovare la soluzione completa come C.L. delle due

$$\begin{cases} V_i(r,\theta,\lambda) = \sum_{n=0}^{\infty} r^n \sum_{m=0}^{n}[a_{nm}P_{nm}(\cos\theta)\cos m\lambda + b_{nm}P_{nm}(\cos\theta)\sin m\lambda] \\ V_e(r,\theta,\lambda) = \sum_{n=0}^{\infty} \frac{1}{r^{n+1}} \sum_{m=0}^{n}[a_{nm}P_{nm}(\cos\theta)\cos m\lambda + b_{nm}P_{nm}(\cos\theta)\sin m\lambda] \end{cases}$$

FORZA GRAVITAZIONALE ED ESPANSIONE DEL POTENZIALE

Una funzione generica del tipo $f(\theta,\lambda)$ appartenente alla superficie di una sfera puo' essere espansa attraverso una serie di armoniche sferiche superficiali

$$f(\theta,\lambda)=\sum_{n=0}^{\infty} Y_n(\theta,\lambda)=\sum_{n=0}^{\infty}\sum_{m=0}^{n} a_{nm}\mathcal{R}_{nm}(\theta,\lambda)+b_{nm}\mathcal{S}_{nm}(\theta,\lambda)$$

$$\mathcal{R}_{nm}(\theta,\lambda)=P_{nm}(\cos\theta)\cos m\lambda \qquad \mathcal{S}_{nm}(\theta,\lambda)=P_{nm}(\cos\theta)\sin m\lambda$$

a_{nm} e b_{nm}?

4) Semplificare tramite «relazioni di ortogonalità» e trovare un'espressione utile per i coefficienti della C.L.

$$\begin{cases}\iint_\sigma \mathcal{R}_{nm}(\theta,\lambda)\,\mathcal{R}_{sr}(\theta,\lambda)d\sigma=0\\ \iint_\sigma \mathcal{S}_{nm}(\theta,\lambda)\,\mathcal{S}_{sr}(\theta,\lambda)d\sigma=0\end{cases}\quad \text{se s} \neq \text{n oppure r} \neq \text{m o entrambi}$$

$$\iint_\sigma \mathcal{R}_{nm}(\theta,\lambda)\,\mathcal{S}_{sr}(\theta,\lambda)=0 \quad \text{sempre}$$

$$\iint_\sigma \left[\mathcal{R}_{nm}(\theta,\lambda)\right]^2 d\sigma=\iint_\sigma \left[\mathcal{S}_{nm}(\theta,\lambda)\right]^2 d\sigma=\frac{2\pi}{2n+1}\frac{(n+m)!}{(n-m)!}$$

$\longrightarrow$

$$\begin{cases}a_{nm}=\frac{2n+1}{2\pi}\frac{(n-m)!}{(n+m)!}\iint_\sigma f(\theta,\lambda)\mathcal{R}_{nm}(\theta,\lambda)d\sigma\\ b_{nm}=\frac{2n+1}{2\pi}\frac{(n-m)!}{(n+m)!}\iint_\sigma f(\theta,\lambda)\mathcal{S}_{nm}(\theta,\lambda)d\sigma\end{cases}\quad \text{se m}\neq 0$$

L'espansione necessita di differenziare casi in cui $m=0$ da casi in cui $m\neq 0$

5) «Normalizzare» le armoniche

$$\begin{cases}\bar{\mathcal{R}}_{nm}(\theta,\lambda)=\sqrt{2(2n+1)\frac{(n-m)!}{(n+m)!}}\mathcal{R}_{nm}(\theta,\lambda)\\ \bar{\mathcal{S}}_{nm}(\theta,\lambda)=\sqrt{2(2n+1)\frac{(n-m)!}{(n+m)!}}\mathcal{S}_{nm}(\theta,\lambda)\end{cases}\quad \text{se m}\neq 0$$

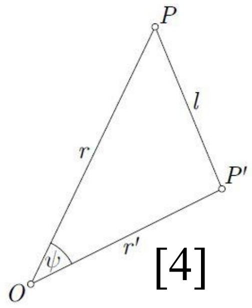
$$\bar{a}_{nm}=\frac{1}{4\pi}\iint_\sigma f(\theta,\lambda)\bar{\mathcal{R}}_{nm}(\theta,\lambda)d\sigma$$

$$\bar{b}_{nm}=\frac{1}{4\pi}\iint_\sigma f(\theta,\lambda)\bar{\mathcal{S}}_{nm}(\theta,\lambda)d\sigma$$

$$f(\theta,\lambda)=\sum_{n=0}^{\infty}\sum_{m=0}^{n}\left[\bar{a}_{nm}\bar{\mathcal{R}}_{nm}(\theta,\lambda)+\bar{b}_{nm}\bar{\mathcal{S}}_{nm}(\theta,\lambda)\right]$$

I coefficienti dell'espansione non sono che la media integrale del prodotto della funzione stessa per le corrispondenti funzioni armoniche normalizzate

6) Espandere il reciproco della distanza «l» tra i punti di interesse



[4]

$$\frac{1}{l}=\frac{1}{\sqrt{r^2-2rr'\cos\psi+r'^2}}=\frac{1}{r\sqrt{1-2\alpha u+\alpha^2}}=\sum_{n=0}^{\infty}\alpha^n P_n(u)=P_0(u)+\alpha P_1(u)+\alpha^2 P_2(u)+\dots$$

$$\frac{1}{l}=\sum_{n=0}^{\infty}\frac{r'^n}{r^{n+1}}P_n(\cos\psi)$$ *

7) Esprimere la funzione di Legendre in funzione delle coordinate sferiche

Si vuole esprimere $P_n(\cos\psi)$ in funzione delle coordinate sferiche $\theta,\lambda,\theta',\lambda'$ Formula di decomposizione

$$P_n(\cos\psi)=P_n(\cos\theta)P_n(\cos\theta')+2\sum_{m=1}^{n}\frac{(n-m)!}{(n+m)!}[\mathcal{R}_{nm}(\theta,\lambda)\mathcal{R}_{nm}(\theta',\lambda')+\mathcal{S}_{nm}(\theta,\lambda)\mathcal{S}_{nm}(\theta',\lambda')]$$

↓ *

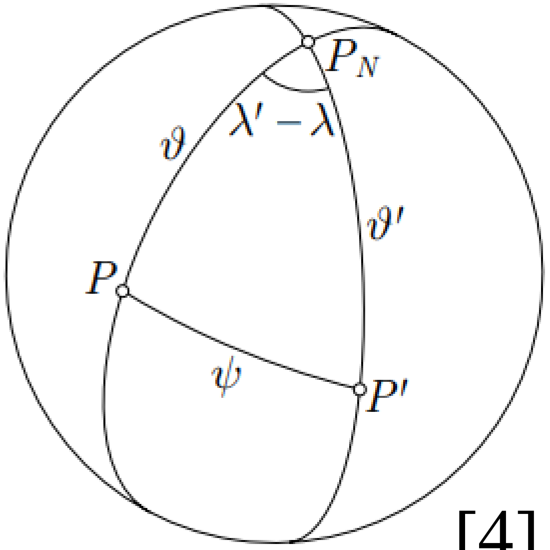
$$\frac{1}{l}=\sum_{n=0}^{\infty}\left\{\frac{P_n(\cos\theta)}{r^{n+1}}r'^nP_n(\cos\theta')+2\sum_{m=1}^{n}\frac{(n-m)!}{(n+m)!}\left[\frac{\mathcal{R}_{nm}(\theta,\lambda)}{r^{n+1}}r'^n\mathcal{R}_{nm}(\theta',\lambda')+\frac{\mathcal{S}_{nm}(\theta,\lambda)}{r^{n+1}}r'^n\mathcal{S}_{nm}(\theta',\lambda')\right]\right\}$$

↓

$$P_n(\cos\psi)=\frac{1}{2n+1}\sum_{m=0}^{n}[\bar{\mathcal{R}}_{nm}(\theta,\lambda)\bar{\mathcal{R}}_{nm}(\theta',\lambda')+\bar{\mathcal{S}}_{nm}(\theta,\lambda)\bar{\mathcal{S}}_{nm}(\theta',\lambda')]$$

↓

$$\frac{1}{l}=\sum_{n=0}^{\infty}\sum_{m=0}^{n}\frac{1}{2n+1}\left[\frac{\bar{\mathcal{R}}_{nm}(\theta,\lambda)}{r^{n+1}}r'^n\bar{\mathcal{R}}_{nm}(\theta',\lambda')+\frac{\bar{\mathcal{S}}_{nm}(\theta,\lambda)}{r^{n+1}}r'^n\bar{\mathcal{S}}_{nm}(\theta',\lambda')\right]$$



[4]

$$V=G\iiint_V\frac{\rho}{l}dv$$

$$V=G\iiint_{earth}\left\{\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{1}{2n+1}\left[\frac{\bar{\mathcal{R}}_{nm}(\theta,\lambda)}{r^{n+1}}r'^n\bar{\mathcal{R}}_{nm}(\theta',\lambda')+\frac{\bar{\mathcal{S}}_{nm}(\theta,\lambda)}{r^{n+1}}r'^n\bar{\mathcal{S}}_{nm}(\theta',\lambda')\right]\right\}dM$$

FORZA GRAVITAZIONALE ED ESPANSIONE DEL POTENZIALE

$$\begin{cases} V = \sum_{n=0}^{\infty}\sum_{m=0}^{n} \frac{\bar{\mathcal{R}}_{nm}(\theta,\lambda)}{r^{n+1}} \iiint_{earth} G\frac{1}{2n+1} r'^{n} \bar{\mathcal{R}}_{nm}(\theta',\lambda') dM + \sum_{n=0}^{\infty}\sum_{m=0}^{n} \frac{\bar{\mathcal{S}}_{nm}(\theta,\lambda)}{r^{n+1}} \iiint_{earth} G\frac{1}{2n+1} r'^{n} \bar{\mathcal{S}}_{nm}(\theta',\lambda') dM \\ \bar{\mathcal{A}}_{nm} = \frac{G}{2n+1} \iiint_{earth} r'^{n} \bar{\mathcal{R}}_{nm}(\theta',\lambda') dM \\ \bar{\mathcal{B}}_{nm} = \frac{G}{2n+1} \iiint_{earth} r'^{n} \bar{\mathcal{S}}_{nm}(\theta',\lambda') dM \end{cases}$$

$$V = \sum_{n=0}^{\infty}\sum_{m=0}^{n} \left[\bar{\mathcal{A}}_{nm} \frac{\bar{\mathcal{R}}_{nm}(\theta,\lambda)}{r^{n+1}} + \bar{\mathcal{B}}_{nm} \frac{\bar{\mathcal{S}}_{nm}(\theta,\lambda)}{r^{n+1}} \right]$$

funzione $\rho(r',\theta',\lambda')$ non nota: **Dirichlet's boundary-value problem**

8) Riorganizzare l'espressione di V esprimendo le armoniche con nuovi coefficienti

$$V = \sum_{n=0}^{\infty}\sum_{m=0}^{n} \left[\mathcal{A}_{nm} \frac{\mathcal{R}_{nm}(\theta,\lambda)}{r^{n+1}} + \mathcal{B}_{nm} \frac{\mathcal{S}_{nm}(\theta,\lambda)}{r^{n+1}} \right]$$

$$\begin{cases} A_{nm} = GMa^n C_{nm} \\ B_{nm} = GMa^n S_{nm} \end{cases} \quad n\neq 0$$

$$V = \frac{GM}{r}\left\{ 1 + \sum_{n=1}^{\infty}\sum_{m=0}^{n} \left(\frac{a}{n}\right)^n \left[C_{nm}\mathcal{R}_{nm}(\theta,\lambda) + S_{nm}\mathcal{S}_{nm}(\theta,\lambda) \right] \right\}$$

9) Estrarre le armoniche di grado n e ordine 0 (armoniche zonali)

$$V = \frac{GM}{r}\left\{ 1 - \sum_{n=2}^{\infty} \left(\frac{a}{r}\right)^n J_n P_n(\cos\theta) + \sum_{n=2}^{\infty}\sum_{m=1}^{n} \left(\frac{a}{r}\right)^n \left[C_{nm}\cos m\lambda + S_{nm}\sin m\lambda \right] P_{nm}(\cos\theta) \right\}$$

$$V = \frac{GM}{r}\left\{1 - \sum_{n=2}^{\infty}\left(\frac{a}{r}\right)^n J_n P_n(\cos\theta) + \sum_{n=2}^{\infty}\sum_{m=1}^{n}\left(\frac{a}{r}\right)^n [C_{nm}\cos m\lambda + S_{nm}\sin m\lambda] P_{nm}(\cos\theta)\right\}$$

Come determiniamo il valore delle armoniche (i coefficienti)?

Satellite tracking

Misura delle perturbazioni osservate nelle orbite dei satelliti stessi

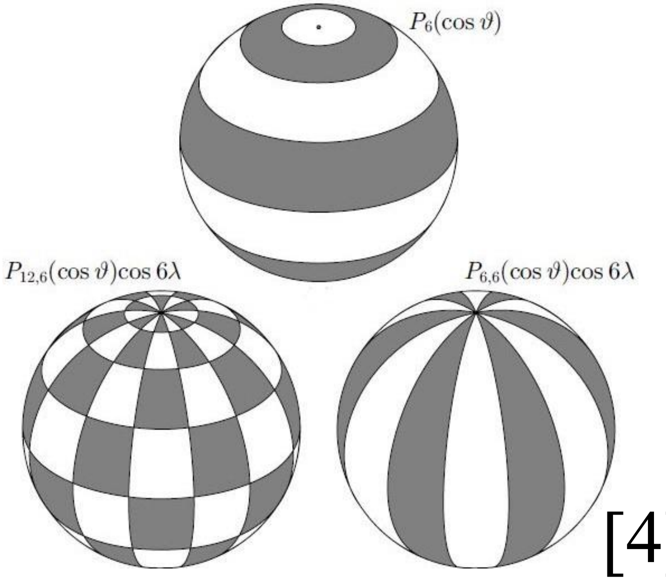
Gravimetria in superficie

Utilizzo di apparati ultra-sensibili e delicati (gravimetri) che misurano l'accelerazione gravitazionale locale

Dati altimetrici

Utilizzo di altimetri che misurano l'altezza dei satelliti sopra il livello del mare determinando il livello medio della superficie

Rappresentazione fisica
delle armoniche zonali, tesserali e settoriali



[4]

Espansione alternativa passando per armoniche ellittiche/ellissoidali

8.1) Trasformare V in coordinate ellittiche, distinguere potenziale «gravitazionale» da potenziale della «gravità», ri-convertire in coordinate sferiche

$$V(u,\beta) = \sum_{n=0}^{\infty} \frac{Q_n(i\frac{u}{E})}{Q_n(i\frac{b}{E})} A_n P_n(\sin\beta)$$

$$\Phi(u,\beta) = \frac{1}{2}\omega^2(u^2+E^2)\cos^2\beta$$

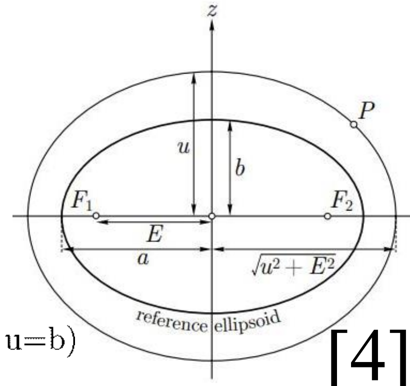
$$U(u,\beta) = \sum_{n=0}^{\infty} \frac{Q_n(i\frac{u}{E})}{Q_n(i\frac{b}{E})} A_n P_n(\sin\beta) + \frac{1}{2}\omega^2(u^2+E^2)\cos^2\beta$$

e ponendoci sull'ellissoide S_0, $u=b$, $U=U_0$:

$$\sum_{n=0}^{\infty} A_n P_n(\sin\beta) + \frac{1}{2}\omega^2(u^2+E^2)\cos^2\beta = U_0$$

* $E=\sqrt{a^2-b^2}=\sqrt{a^2-u^2}$ (su S_0 u=b)

$$\cos^2\beta = \frac{2}{3}[1-P_2(\sin\beta)]$$



[4]

*
$$\sum_{n=0}^{\infty} A_n P_n(\sin\beta) + \frac{1}{3}\omega^2a^2 + \frac{1}{3}\omega^2a^2P_2(\sin\beta) - U_0 = 0$$

↓ esplicitando i primi tre termini della serie e raccogliendo:

$$(A_0 + \frac{1}{3}\omega^2a^2 - U_0)P_0(\sin\beta) + A_1P_1(\sin\beta)$$

$$+ (A_2 - \frac{1}{3}\omega^2a^2)P_2(\sin\beta) + \sum_{n=3}^{\infty} A_n P_n(\sin\beta) \overset{*}{=} 0$$

* L'espressione sara' pari a 0 solo se tutti i coefficienti dei P_n si annullano

$$\begin{cases} V(u,\beta) = \sum_{n=0}^{\infty} \frac{Q_n(i\frac{u}{E})}{Q_n(i\frac{b}{E})} A_n P_n(\sin\beta) \\ A_0 = U_0 - \frac{1}{3}\omega^2a^2 \\ A_1 = 0 \\ A_2 = \frac{1}{3}\omega^2a^2 \\ A_3 = A_4 = \cdots = 0 \end{cases}$$

↓

$$V(u,\beta) = (U_0 - \frac{1}{3}\omega^2a^2)\frac{Q_0(i\frac{u}{E})}{Q_0(i\frac{b}{E})} + \frac{1}{3}\omega^2a^2\frac{Q_2(i\frac{u}{E})}{Q_2(i\frac{b}{E})}P_2(\sin\beta)$$

$$\begin{cases} Q_0(i\frac{u}{E}) = -i\tan^{-1}\frac{E}{u} \\ Q_2(i\frac{u}{E}) = \frac{i}{2}\left[\left(1+3\frac{u^2}{E^2}\tan^{-1}\frac{E}{u} - 3\frac{u}{E}\right)\right] = q \\ Q_0(i\frac{b}{E}) = \frac{i}{2}\left[\left(1+3\frac{b^2}{E^2}\tan^{-1}\frac{E}{b} - 3\frac{b}{E}\right)\right] = q_0 \end{cases}$$

↓

$$V(u,\beta) = (U_0 - \frac{1}{3}\omega^2a^2)\frac{\tan^{-1}\frac{E}{u}}{\tan^{-1}\frac{E}{b}} + \frac{1}{3}\omega^2a^2\frac{q}{q_0}P_2(\sin\beta)$$

FORZA GRAVITAZIONALE ED ESPANSIONE DEL POTENZIALE

$$\begin{cases} \tan^{-1}\frac{E}{u} = \frac{E}{u} + O(1/u^3) \\ \frac{1}{u} = \frac{1}{r} + O(1/r^3) \end{cases} \rightarrow \quad \tan^{-1}\frac{E}{u} = \frac{E}{r} + O(1/r^3)$$

$$\downarrow$$

$$\begin{cases} V = (U_0 - \frac{1}{3}\omega^2 a^2)\frac{E}{\tan^{-1}\frac{E}{b}}\frac{1}{r} + O(1/r^3) \\ V = \frac{GM}{r} + O(1/r^3) \quad \text{(Noto dalla meccanica celeste)} \end{cases}$$

$$\downarrow$$

$$\frac{GM}{r} = (U_0 - \frac{1}{3}\omega^2 a^2)\frac{E}{\tan^{-1}\frac{E}{b}}\frac{1}{r} + O(1/r^3)$$

Moltiplicando per r e mandando $r \to 0$:

$$GM = (U_0 - \frac{1}{3}\omega^2 a^2)\frac{E}{\tan^{-1}\frac{E}{b}}$$

$$\downarrow$$

$$U_0 = \frac{GM}{E}\tan^{-1}\frac{E}{b} + \frac{1}{3}w^2 a^2$$

$$V(u,\beta) = (U_0 - \frac{1}{3}\omega^2 a^2)\frac{\tan^{-1}\frac{E}{u}}{\tan^{-1}\frac{E}{b}} + \frac{1}{3}\omega^2 a^2 \frac{q}{q_0} P_2(\sin\beta)$$

$$V = \frac{GM}{E}\tan^{-1}\frac{E}{u} + \frac{1}{3}w^2 a^2 \frac{q}{q_0} P_2 \sin\beta$$

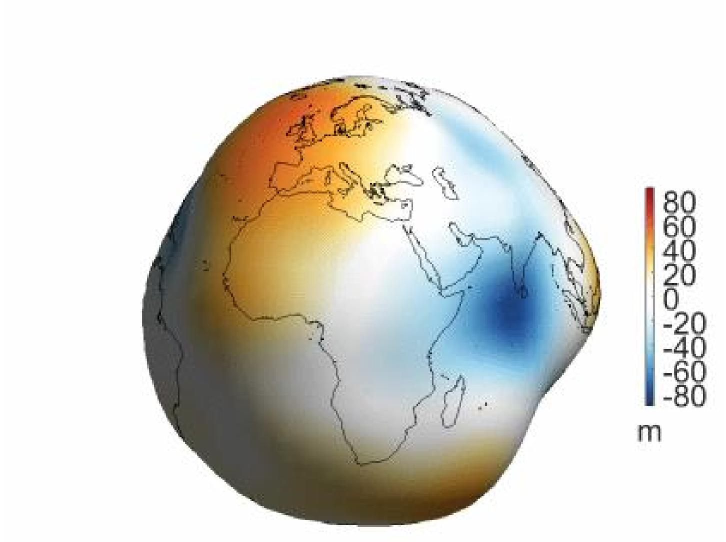
Conversione in coordinate sferiche (armoniche restano ellittiche)

$$V = \frac{GM}{r} + A_2\frac{P_2(\cos\theta)}{r^3} A_4\frac{P_4(\cos\theta)}{r^5} + \cdots = \frac{GM}{r} + \sum_{n=1}^{\infty} A_{2n}\frac{1}{r^{2n+1}}$$

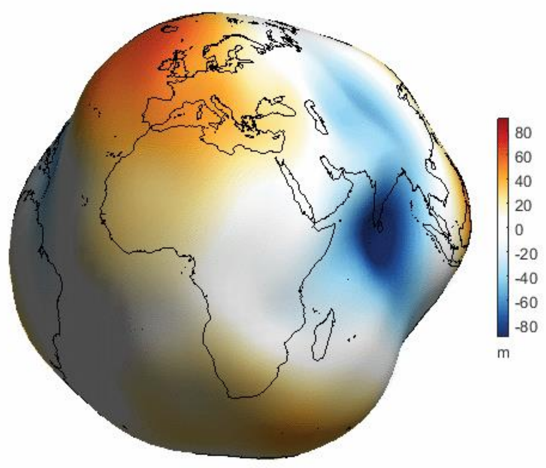
$$\begin{cases} V = \frac{GM}{r} + \sum_{n=1}^{\infty} A_{2n}\frac{1}{r^{2n+1}} \\ A_{2n} = (-1)^n \frac{GME^{2n}}{2n+1}\left(1 - \frac{2n}{2n+3}\frac{me'}{3q_0}\right) \end{cases}$$

$$\begin{cases} V = \frac{GM}{r}\left[1 + \sum_{n=1}^{\infty} C_{2n}\left(\frac{a}{r}\right)^{2n} P_{2n}(\cos\theta)\right] \\ C_{2n} = -J_{2n} = (-1)^n \frac{3e^{2n}}{(2n+1)(2n+3)}\left(1 - n + 5n\frac{C-A}{ME^2}\right) \\ C_{20} = -\frac{C-A}{Ma^2} \rightarrow J_2 = \frac{C-A}{Ma^2} \end{cases}$$

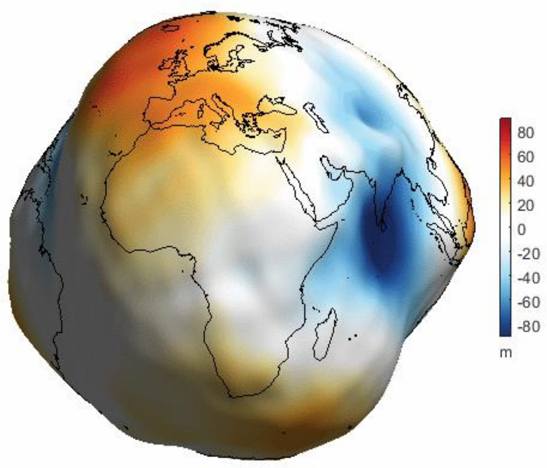
MODELLI GEOPOTENZIALI E LORO IMPLEMENTAZIONE



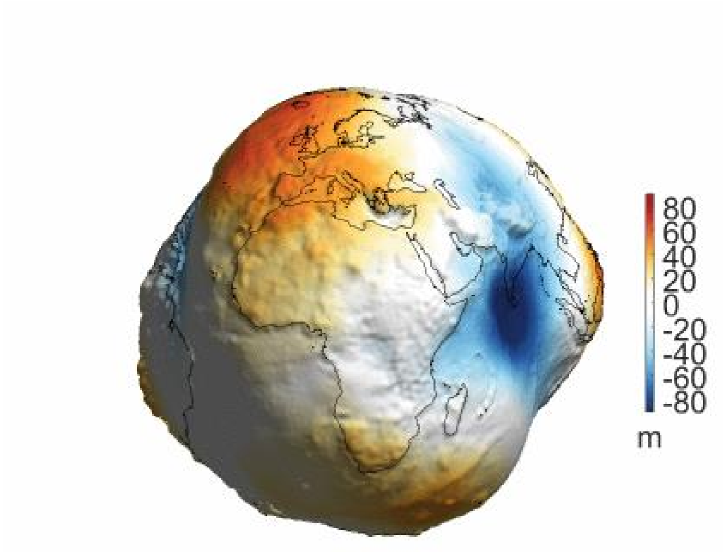
Geoid height (SE1, nmax=15)



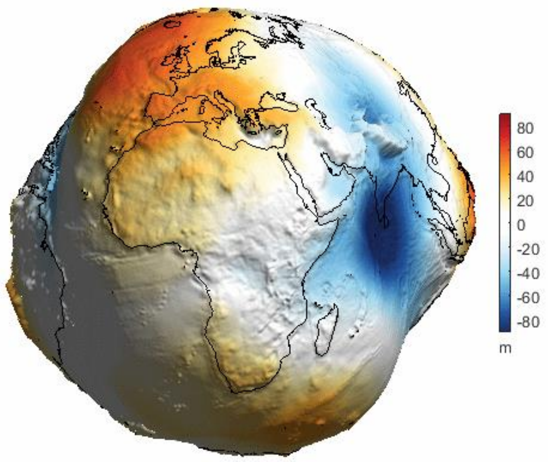
Geoid height (GEM1, nmax=24)



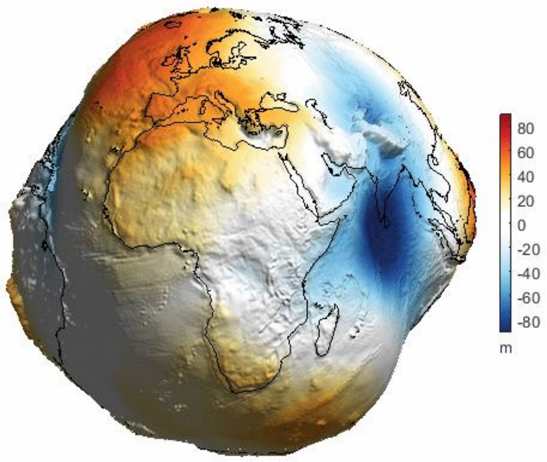
Geoid height (JGM1, nmax=60)



Geoid height (OSU81, nmax=180)

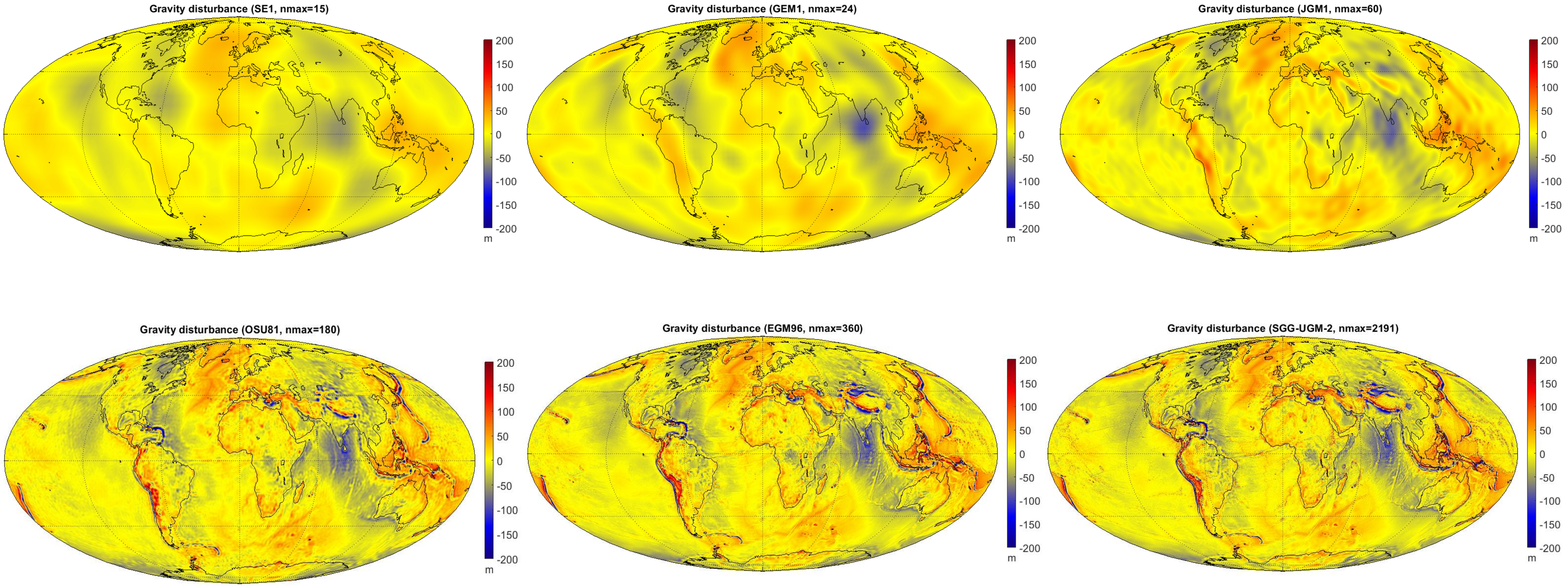


Geoid height (EGM96, nmax=360)



Geoid height (EGM2008, nmax=1000)

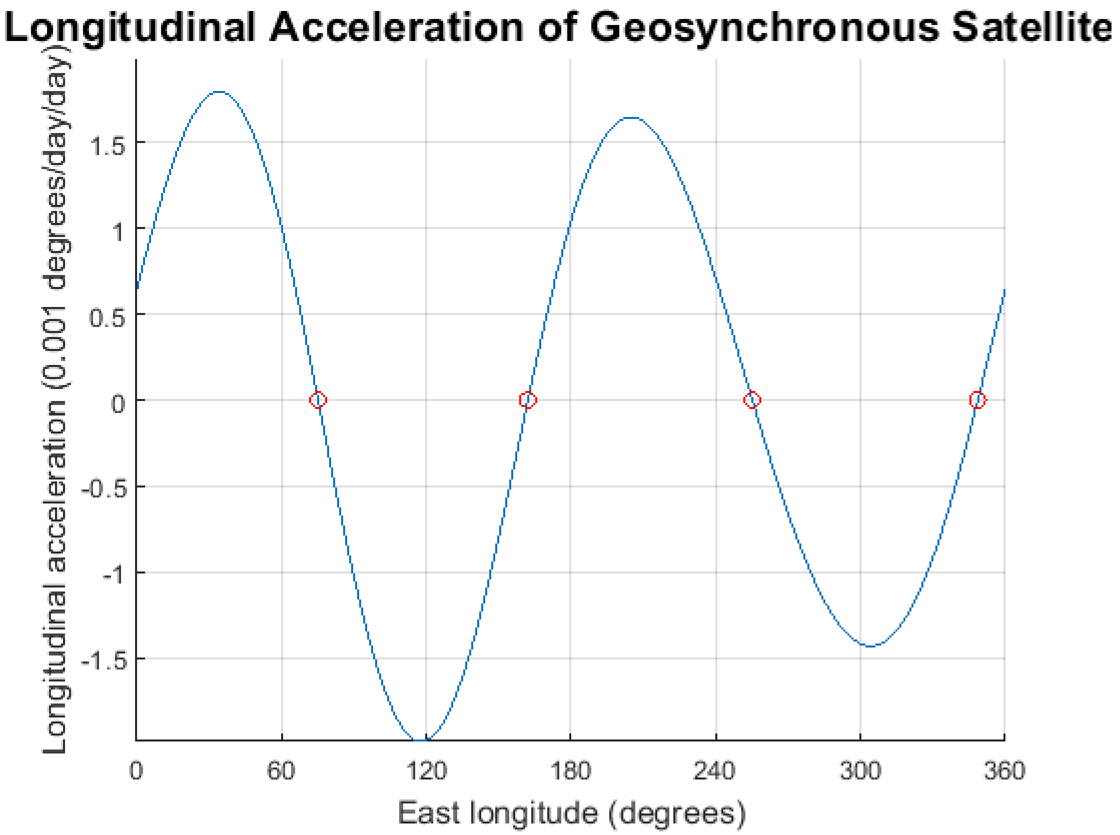
MODELLI GEOPOTENZIALI E LORO IMPLEMENTAZIONE



Ales Bezdek e Josef Sebera: "Matlab script for 3D visualizing geodata on a rotating globe"
International Centre for Global Earth Models (ICGEM) [2]

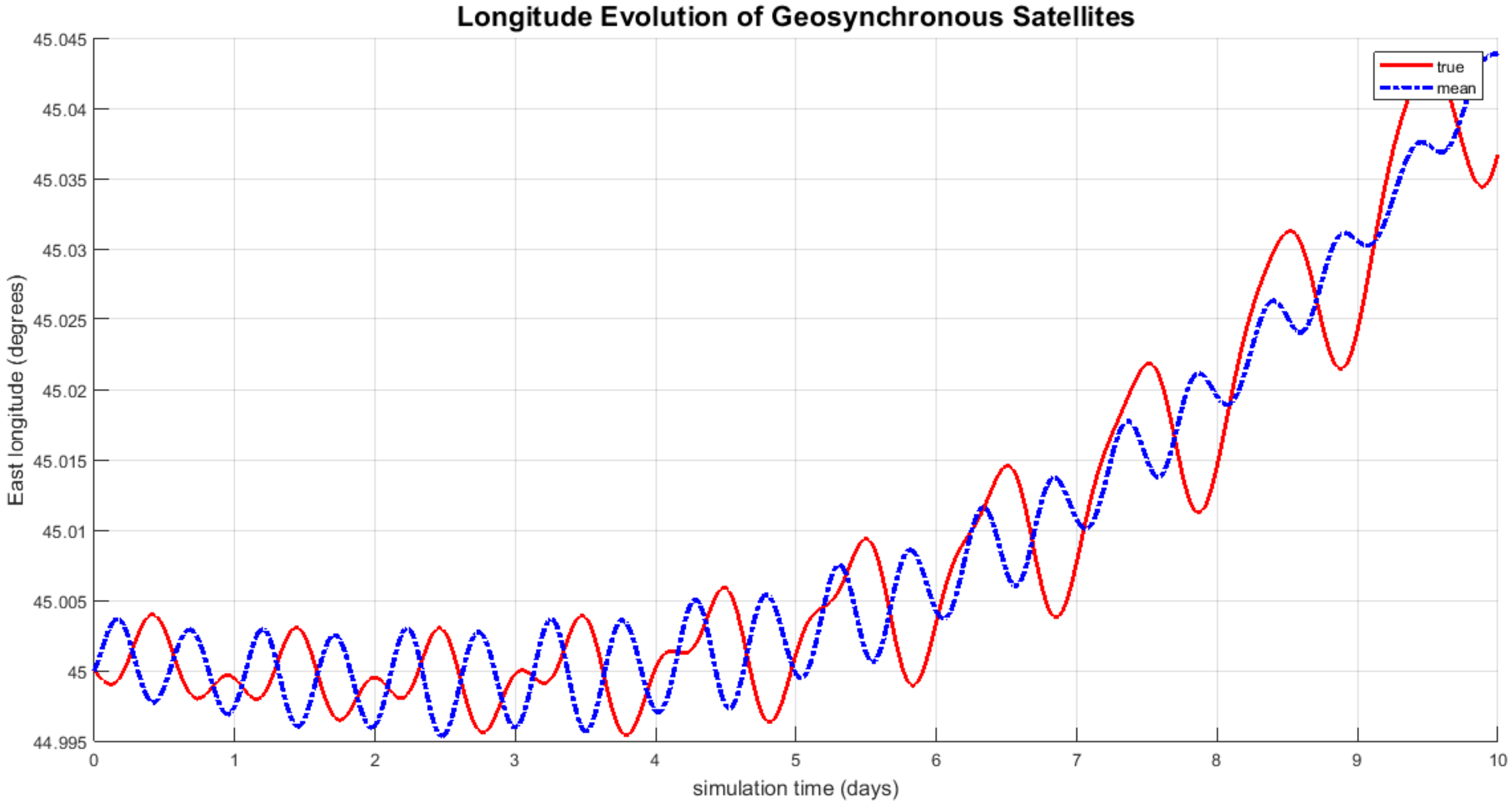
GEOSYNC1

```
                    program geosync1

     < equilibrium longitudes, radii and acceleration >

location       east longitude        radius          drift acceleration
                  (degrees)        (kilometers)       (degrees/day^2)

   1               75.0602          42166.2409        -2.0741305128e-18

   2              162.0816          42166.2847        -2.1909590887e-17

   3              255.0880          42166.2411         9.3970810989e-19

   4              348.5962          42166.2811         4.5376896117e-18

drift cycle period at longitude   75.0602 degrees =  2339.2602 days

drift cycle period at longitude  255.0880 degrees =  2883.5951 days

   < please press any key to continue >
```

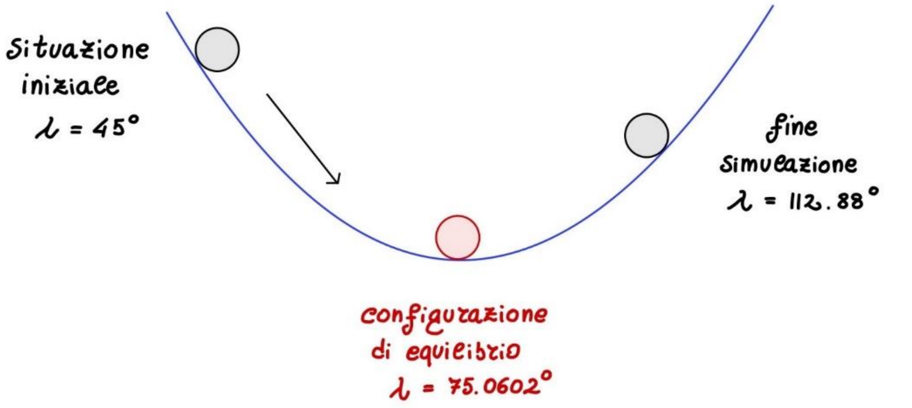


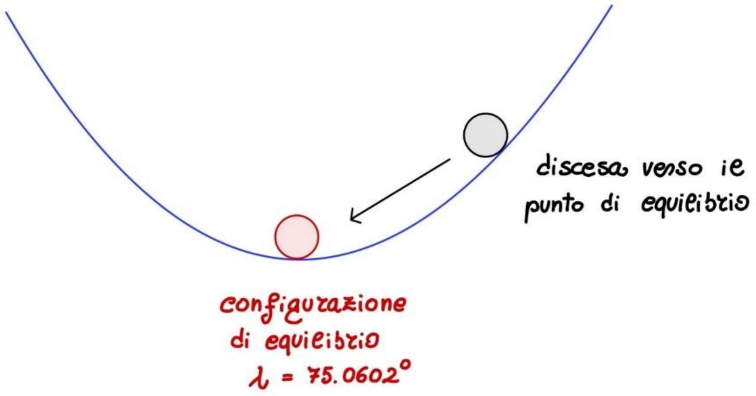
GEOSYNC2

```
                    program geosync2

    < geosynchronous osculating semimajor axis >

initial calendar date           27-Feb-2022
initial universal time          00:00:00.000

initial mean east longitude      45.000000  degrees
final mean east longitude       112.879992  degrees

initial geodetic latitude         0.000000  degrees
final geodetic latitude           0.836662  degrees

reflectivity constant             1.850000

cross-sectional area             10.000000  sqr meters

spacecraft mass                2000.000000  kilograms

degree of gravity model          15.0
order of gravity model           15.0

number of orbits modeled         50.0
```



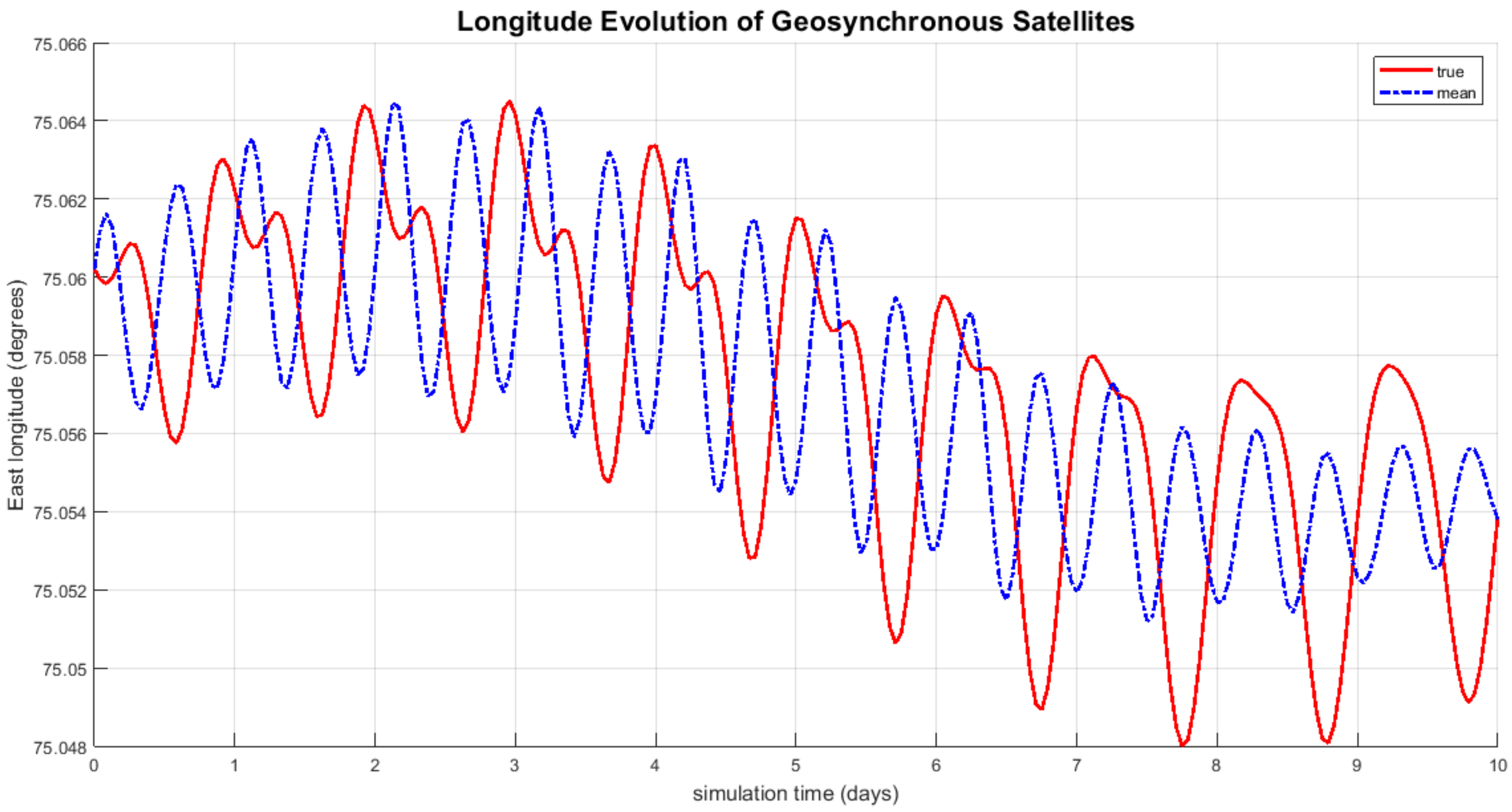
GEOSYNC2


Situazione iniziale
λ = 45°
fine simulazione
λ = 112.88°
configurazione di equilibrio
λ = 75.0602°


discesa verso il punto di equilibrio
configurazione di equilibrio
λ = 75.0602°

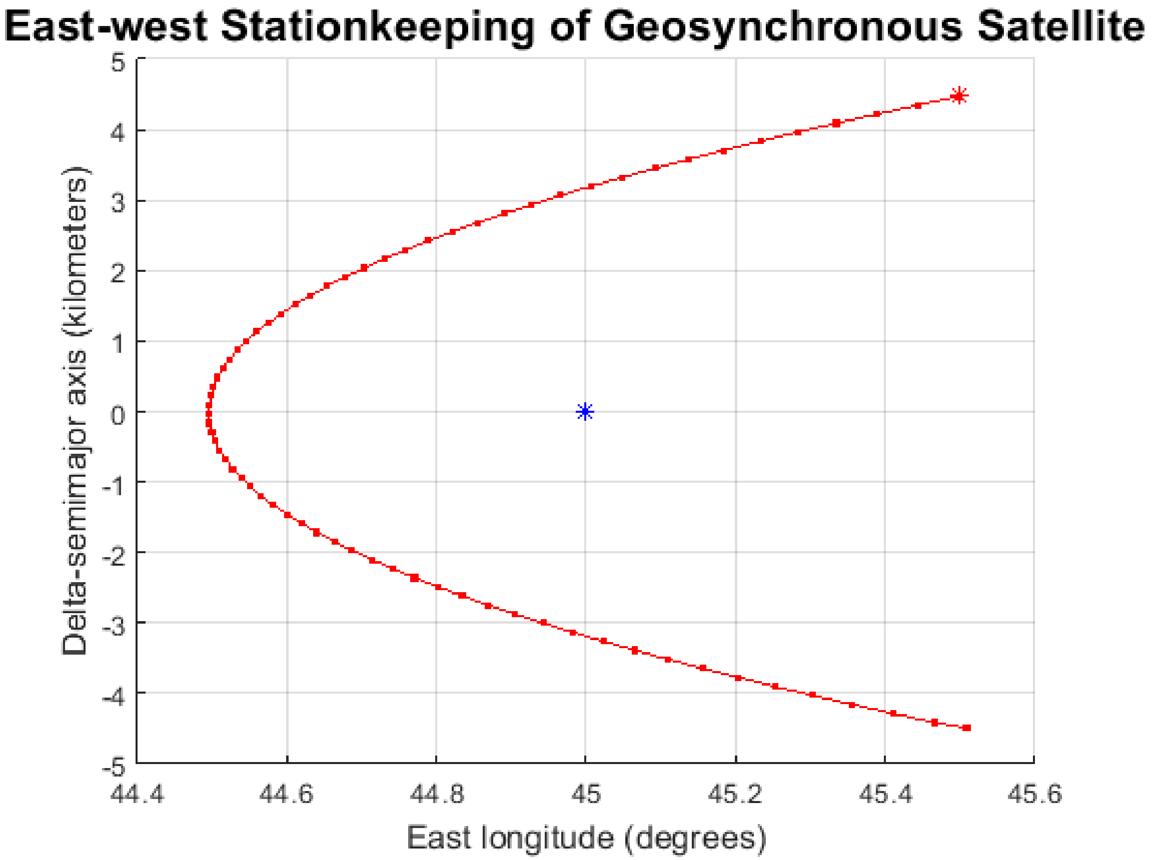
GEOSYNC2

```
program geosync2

< geosynchronous osculating semimajor axis >

initial calendar date          27-Feb-2022
initial universal time         00:00:00.000

initial mean east longitude     75.060200   degrees
final mean east longitude       75.060200   degrees

initial geodetic latitude        0.000000   degrees
final geodetic latitude          2.882883   degrees

reflectivity constant            1.850000

cross-sectional area            10.000000   sqr meters

spacecraft mass               2000.000000   kilograms

degree of gravity model         15.0
order of gravity model          15.0

number of orbits modeled        30.0
```

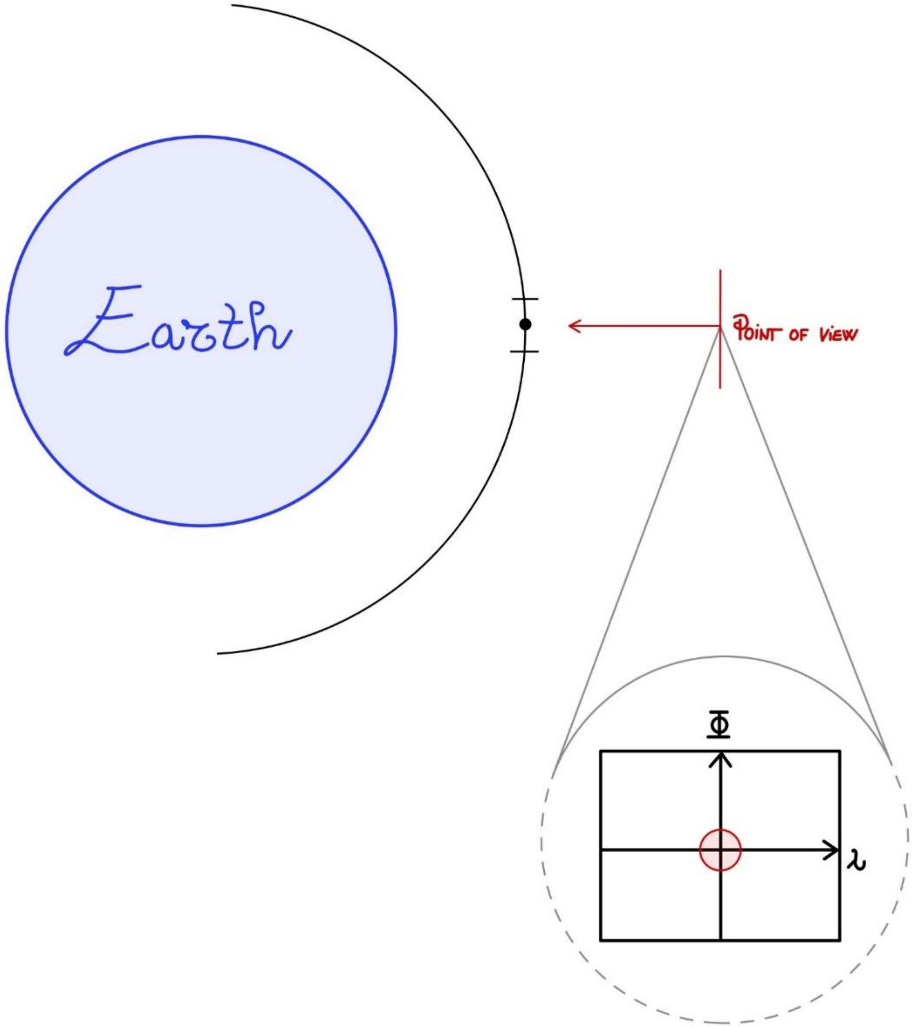


GEOSYNC4

```
                  program geosync4

< east-west stationkeeping of geosynchronous satellites >

satellite east longitude             45.0000 degrees

total longitude deadband              1.0000 degrees

initial drift rate                    0.0575 degrees/day

single maneuver delta-v               0.3262 meters/second

total annual delta-v                  1.7113 meters/second/year

drift cycle period                   69.6248 days


geosynchronous semimajor axis     42166.2534 kilometers
drift cycle semimajor axis        42170.7272 kilometers
delta semimajor axis                  4.4738 kilometers
```



STATION KEEPING IN ORBITA GEO



Ahmed Kamel, Donald Ekman e Richard Tibbitts: "East-west stationkeeping requirements of nearly synchronous satellites due to Earth's triaxiality and luni-solar effects" [5]

Thomas J. Kelly, Lisa K. White e Donald W. Gamble: "Stationkeeping of geostationary satellites with simultaneous eccentricity and longitude control" [7]

6.1 *Bibliografia essenziale*

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6.2 *Letture d'approfondimento*

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GRAZIE PER L'ATTENZIONE