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Project Valuation Using Real Options Analysis

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Sommario

In questo lavoro verrà introdotta la moderna tecnica *Real Options Analysis* per la valutazione di progetti. Verranno descritti ed analizzati i metodi di valutazione correntemente più affermati (*Discounted Cash Flow, Monte Carlo Simulation, Decision Tree Analysis*) e verranno messe in mostra le loro limitazioni; si vedrà come la Real Options Analysis possa sopperire a queste limitazioni. Infine, verra spiegato come si effettua la valutazione di un progetto con un procedimento a sei passi ed utilizzando la tecnica di risoluzione binomiale. Seguono esempi di applicazioni e conclusioni.

Abstract

In this work we are going to introduce the *Real Options Analysis* method for project valuation. The currently most used valuation methods (*Discounted Cash Flow, Monte Carlo Simulation, Decision Tree Analysis*) will be described and analyzed; we will show how Real Options Analysis can overcome their limitations. A six steps framework for the resolution of a valuation problem using the binomial technique will be introduced. Examples of application of the method will precede the conclusions.

Un ringraziamento speciale a mio papà Lorenzo, a mia mamma Paola ed alle mie sorelle Silvia ed Elisa, ovvero la famiglia che ha sempre creduto in me e mi ha sempre sostenuto in questi quasi ventitre anni.
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Padova, 29 Marzo 2011

Francesco

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1 INTRODUCTION

The fundamental aspect that permits the realization of a project is its *value*. By the term value we intend not only the mere dollar-money value, but also *strategic* value, such as knowledge on a certain subject, or position on a certain market. This makes clear that estimating the correct value of a project falls well ahead of its very own launch: the rule of thumb says that it is useless to initiate a project if it doesn't show to have any kind of intrinsic value.
Nowadays organizations put a lot of effort and resources into project management; having a large pool, or *portfolio*, of projects under development requires an efficient and coordinated supervision. A team of experts is often in charge of the conduction on the projects; they arrange time and resources and plan for every running project in the portfolio before reporting to upper management. Needless to say, the essential information they want to know about is *how much* it is going to yield. To get the project up and running this number must be evaluated ahead, using the information available at the moment.
A series of methods can be applied to estimate the value of a project; in this work, we are going to examine some of the traditional methods that have been used for decades, which are solid and reliable although somewhat limited, and the modern *Real Options Analysis* method, which is based on financial markets theory and compensate the other methods' limitations.

1.1 FINANCIAL AND REAL OPTIONS

Financial Options definition and terminology

In finance, an option is a *derivative*, a financial instrument whose value depends on the values of other, more basic, underlying variables. A stock option, for example, is a derivative whose value is dependent on the price of a stock. The option itself is a right - not an obligation - to either buy or sell the stock, the underlying asset, at a predetermined cost on or before a predetermined date. An option to buy is called a *call option*; the sell option is called a *put option*. The price paid to acquire the option is called *option price* or *premium*, while the price at which the option is exercised is called the *exercise price* or *strike price*. The *expiration date* is the date when the option expires or *matures*; European options have fixed maturity dates, whereas American options can be exercised on or any time before the options' expiration date.

Call Option value

The call option value (C) at expiration is the maximum of two values: (1) zero and (2) the difference between the underlying asset value (S) at the time when the asset is bought at maturity and the exercise price (X) at maturity:

$$C = max[0, S - X]$$

Put Option Value

while the put option value (P) at expiration is the maximum of two values: (1) zero and (2) the difference between the exercise price (X) at maturity and the the underlying asset value (S) when the asset is sold at maturity:

$$P = max[0, X - S]$$

A call option is in the money (shows a profit) if $S - X > 0$, at the money (shows no profit but also no losses) if $S - X = 0$, and out of the money if $S - X < 0$. Vice versa, a put option is in the money if $S - X < 0$, at the money if $S - X = 0$, and out of the money if $S - X > 0$.

Figures 1.1 and 1.2 are called payoff diagrams and show the cash payoff of a call and put option, respectively, at expiration. With a call option, if the underlying asset value is less than the strike price at the time of option expiration, the option is considered to be "out of the money" and, rationally speaking, will not be exercised. Thus, your net payoff in this case is negative and equal to the option price, also called the call price. If the asset value exceeds the strike price, the option is "in the money" and, rationally speaking, will be exercised and your gross payoff will be positive. Your net payoff, however, may be positive or negative depending on the call price. When the asset value is exactly equal to the strike price, the option is considered to be "at the money." At this point, your gross profit is zero, but the net profit is negative and is equal to the call price.

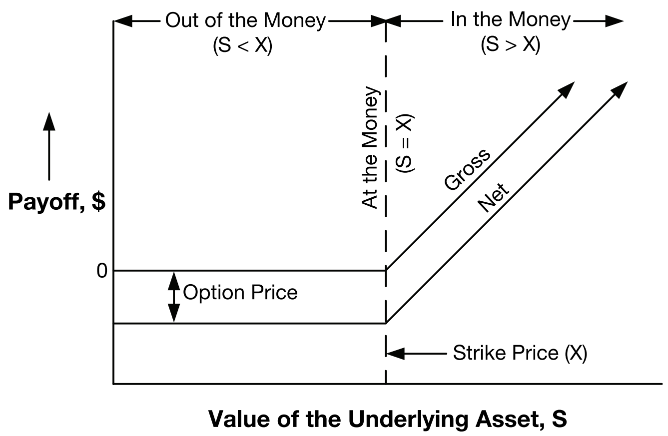

Figure 1.1: Call Option Payoff Diagram

As shown in Figure 1.2, the net payoff of a put option remains negative and equivalent to the put price (price paid to buy the option), as long as the underlying asset value at the expiration time remains above the strike price or the option is out of the money. In other words, you lose what you paid for the put. If the asset value is less than the strike price (that is, the option is in the money), the gross payoff is equal to the difference between the strike price and the value of the underlying asset. The net payoff will be negative until the put price is recovered and from that point goes into the positive territory.

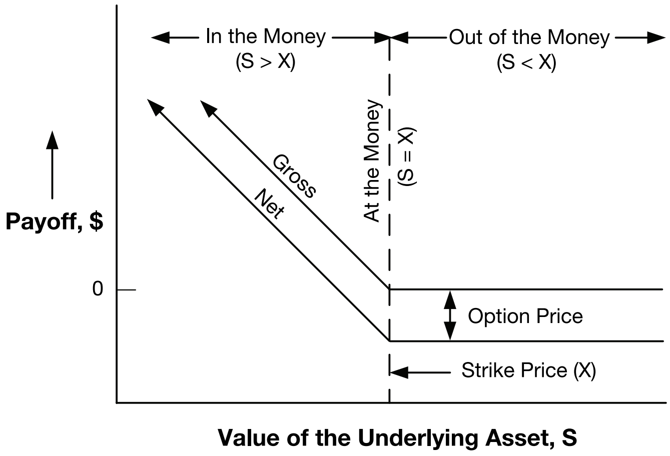

Figure 1.2: Put Option Payoff Diagram

Financial options operate on assets that are primarily stocks and bonds traded in financial markets. The options for most of these assets are listed on exchanges such as the Chicago Board Options Exchange and the American Stock Exchange. What exactly are Real Options? The basic concept is that you can reserve yourself the option to take a decision on an action to perform at any time in the future. The asset on which the decision is taken usually is a real estate property, a project, an intellectual property...which are not traditional traded assets. So a real option is a right - not an obligation - to take an action on an underlying nonfinancial, real asset. The action may involve, for example, abandoning, expanding, or contracting a project or even deferring the decision until a later time, whilst with financial options you usually "buy" or "sell". The real options can be either American, which can be exercised on or before a predetermined expiration date, or European, which can be exercised on a fixed date only. Even if real options have a wider application range, they share the same characteristics as the financial options and, therefore, the same terminology is used. Table 1.1 offers a comparison of financial and real options.

Table 1.1: Financial versus Real Options

	Financial Options	Real Options
Option Price	Price paid to acquire the option, which is fixed by the financial markets	Price paid to acquire or create the option, keep it alive, and clear the uncertainty (for instance, price paid to acquire a patent, maintain it, and conduct market research to identify its potential). The option price is not fixed (for example, the price to buy a patent is negotiable).
Exercise price	Price paid to buy/sell the underlying stock; a fixed value defined in the option contract	Cost of buying/selling the underlying real asset (e.g., the cost of commercializing a new technology is a call option exercise price, the underlying asset being the profits from the commercialization; the selling price of abandoned manufacturing assets is a put option exercise price, the underlying asset being the manufacturing assets).
Expiration time	Defined in the options contract and is clearly known	Clearly known in some cases (e.g., leases may be signed on oil fields involving options on drilling) and not so in others (e.g., for technology projects, it depends on the market conditions and competition).
Timing of payoff	Immediately after the option are exercised; basically instantaneous	Often not until some time after the option has been exercised. May be spread over a long period of time. For example, after a decision has been made to commercialize a new technology, the commercialization itself takes months, and the profits from the sales are spread over a period of many years.
Option holder's control on its value over the option's life	None	Proper management action can increase the option value while limiting the downside potential. For example, the holder of a new, novel technology option can invest in developing other complementary technologies, increasing the value of the original option.
Option value as a function of option life	Larger for longer life of a given option	Larger for longer life of a given option, especially related to patents and property with exclusive rights. But with many options, the asset value may be diminished because of entry of competition, thereby bringing the option value down.
Option value as a function of the underlying asset's volatility	Increases	Increases.

2 THE TRADITIONAL APPROACHES: DISCOUNTED CASH FLOW, MONTE CARLO SIMULATION, DECISION TREE ANALYSIS

There are many different methods developed by financial analysts for determine the value of a project. Simpler techniques may be used when you need just a rough estimation, while complex, more sophisticated methods are used in a project pre-launch analysis to guarantee maximum precision; in any case, the valuation starts with estimation of development and production phase costs and net revenues (which are called *free cash flows*) over the project life. Because of the time value of money, each cash flow from the future is converted into today's dollars, using the formula

Present Value calculation

$$PV = \frac{FV}{(1+r)^n}$$

where FV = future value, PV = present value, r = discount rate per time period and n = number of the time period.
The traditional approach to project valuation is based on the calculation of the *net present value* of a project over its entire life cycle. Investment costs and production phase free cash flows are accounted using a discount factor that best represents the risk associated with the project, thus giving their present value calculated using the above formula. You can obtain one or multiple Net Present Values by using different methods and varying input variables.
The resulting average Net Present Value of the project gives an immediate indicator of its "dollar value", which is basically the net difference between the project revenues and costs over its entire life cycle. If the net revenues during the production phase are higher than the investments cost, the project is considered worthy of investment.

2.1 DISCOUNTED CASH FLOW METHOD

The Discounted Cash Flow method involves the use of only one set of variables, making it a deterministic method.
The project Net Present Value is the summation of the Present Values of all the cash inflows and cash outflows from the development and production phases of a project:

Project NPV = PV of cash flows in production phase - PV of investment costs

The Net Present Value Calculation gives an immediate evaluation of the project value

If the project Net Present Value based on the Discounted Cash Flow analysis is greater than zero, the project is considered financially attractive. In other words, if the total Present Value of expected free cash flows (the project payoff) is greater than the total Present Value of the investment costs, the project is deemed worthy of investment.
Large business firms usually have a "portfolio" which contains all the projects currently under development, and all the descriptions of the projects that may or may not be launched; to facilitate "go/no-go decision", the simplest approach is to compare a candidate project's Net Present Value with those of other projects. All the projects are then ordered by descending Net Present Value to provide a simple prioritization. This means that even if a given project is attractive based on its Net Present Value, it may not be selected for investment if there are other competing projects that are even more attractive.

2.2 MONTE CARLO SIMULATIONS

The Monte Carlo method involves simulation of thousands of possible project scenarios, calculation of the project Net Present Value for each scenario using the Discounted Cash Flow method, and analyzing the probability distribution of the Net Present Value results. The probability distribution of the resulting Net Present Value is then returned with its average value and variance as indicators of the project value.

First Monte Carlo Simulations approach: one random variable for each one of the input parameters

In the most common approach each project scenario is created by taking a random value for each one of the input parameters of the Discounted Cash Flow method and solving for the Net Present Value. These random values are described by their probability distribution, identified by its average value and standard deviation, usually obtained from historical data, or calculated from optimistic/pessimistic estimates based on management's judgement with the use of standard normal frequency tables or professional software.
The Net Present Value of the scenario is then calculated using Discounted Cash Flow method using one value for every input parameter taken from within its distribution.

Second Monte Carlo Simulations approach: starting from the Net Present Value distribution, conduct sensitivity analysis on the most influential parameters

In another approach based on the Monte Carlo Simulations, you may start with an expected average project Net Present Value calculated by the Discounted Cash Flow method using one set of input values and conduct thousands of simulations around it. The probability distribution of the average Net Present Value is required, with its expected variance (which can be calculated again using standard normal frequency tables or professional software from best/worst case scenarios - corresponding to 1% and 99% confidence levels - estimated by management to represent the lowest and highest ends of the Net Present Value distribution).
Since the estimation of the distributions of the input variables is a difficult task, it is common practice to conduct sensitivity

analysis on the parameters to individuate two or three variables that have the highest impact on the Net Present Value calculated with the deterministic Discounted Cash Flow method, and then focus on simulations conducted on those variables. The uncertainty associated with random input variables yield additional information in the estimation of the Net Present Value of the project.

2.3 DECISION TREE ANALYSIS

Decision trees incorporate the effects of management decisions on uncertain situations with Discounted Cash Flow calculation, giving a "road map" which depicts costs, possible outcomes (with their payoffs) and probability of those decisions. The project Net Present Value is obtained using the *expected value* approach. The Expected Value of an event is simply the product of its probability of occurrence and its outcome, commonly expressed in terms of its cash flow value. The probabilities used in Decision Tree Analysis are one of the most important inputs in the valuation process.
Let's use a simple example to illustrate Decision Tree Analysis.

EXAMPLE: PRODUCT DEVELOPMENT A start-up company wants to introduce on the market a new patent-pending product. The technical effectiveness of the product first has to be proved through development effort, which is expected to cost \$ 1.5 million and take one year. Successful development will be followed by commercialization of the technology, which is estimated to take one additional year and cost \$3 million at that time. Discounted Cash Flow analysis shows a project payoff of \$30 million (year 2 dollars) over the project horizon. Although this payoff is attractive compared to the investment costs, the company is not certain about the technical and commercial success of the project, because the respective success probabilities are estimated to be 0.55 and 0.65. Therefore, the company decides to use Decision Tree Analysis to facilitate "go/no-go" decisions for the two phases of the investment.
Let's take a look a the calculation of the project Net Present Value (a 8% discount rate is assumed):

1. Starting at the far right of the decision tree (figure 2.1), calculate the Expected Value of the payoff at year 2 considering the mutually exclusive outcomes related to product launch:
   For the success outcome, $EV = 0.65\$ * (\$30million) = \$19.5million$
   For the failure outcome, $EV = 0.35 * (\$0) = \$0$

2. Add the Expected Values of the two outcomes:
   Total EV at the end of year 2: $+\$19.5million + \$0 = \$19.5million$

3. Calculate the Present Value of the payoff at year 1 by discounting the total Expected Value of year 2 using a discount rate of 8%:
   Payoff PV at year 1 $= (\$19.5\text{million})/(1+0.08)^1 = \$18.06\text{million}$

4. Calculate the Net Present Value of the project at product launch at year 1 by subtracting the launch cost from the payoff Present Value:
   $\$18.06\text{million} - \$3\text{million} = \$15.06\text{million}$

5. Calculate the Expected Value of the payoff at year 1 considering the mutually exclusive outcomes related to the product development:
   For the success outcome, $EV = 0.55 * (\$15.06\text{million}) = \$8.28\text{million}$
   For the failure outcome, $EV = 0.45 * (\$0) = \$0$

6. Add the Expected Values of the two outcomes:
   Total EV at the end of year 1 $+\$8.28\text{million} + \$0 = \$8.28\text{million}$

7. Calculate the Present Value of the project payoff at year 0 by discounting the total Expected Value of year 1 using a discount rate of 8%:
   Payoff PV at year 0 $= (\$8.28\text{million})/(1+0.08)^1 = \$7.67\text{million}$

8. Calculate the Net Present Value of the project at year 0 by subtracting the development cost from the payoff Present Value:
   $\$7.67\text{million} - \$1.5\text{million} = \$6.17\text{million}$

Additional information provided by Decision Tree Analysis: best/worst case scenarios

Decision Tree Analysis also gives a perspective on the relative upside and downside to the project, in form of scenario analysis. Best case represents the scenario where only the best outcomes are experienced and worst case represents the scenario with only the worst outcomes by following the rational decision shown by the Decision Tree. In our example, the best case is where

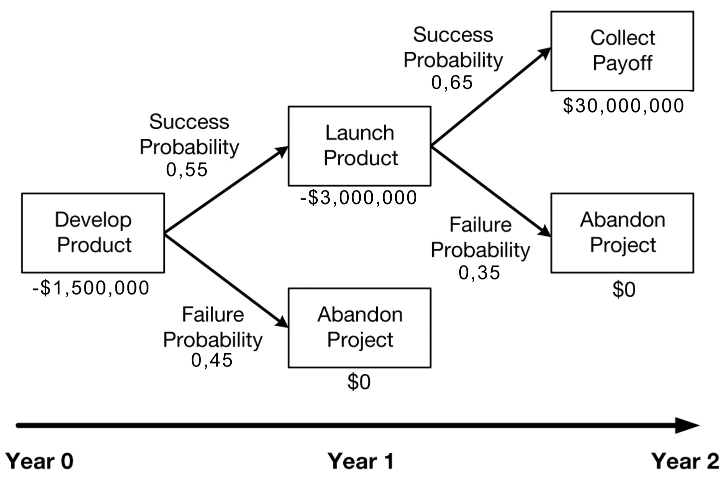

Figure 2.1: Decision Tree for the above example

both the product development and commercialization efforts are successful, while the worst case involves successful development followed by commercial failure. The most likely case is represented by the project's expected Net Present Value today, which corresponds to decision at time = 0. To be financially attractive, a project most likely case Net Present Value should be close to the best case one. Scenario analysis is also a simple way to show the project development based on the decision tree to upper management.

Decision Tree Analysis is not an alternative to but an extension of the Discounted Cash Flow method: the payoffs for different outcomes used in the Decision Tree Analysis are based on the Discounted Cash Flow analysis, but you gain additional information into the project decision process because of the presence of nodes that represent future contingent decisions. This is where Decision Tree Analysis adds value, because Discounted Cash Flow assumes a fixed path and does not account for management's contingent decisions.

2.4 RISKS

Both Discounted Cash Flow and Decision Tree Analysis calculations use discount rates to account for the risks related to the project under scrutiny. The difficulty in the determination of accurate success/failure probabilities and discount rates is a major drawback for these methods, since the outcome probabilities are subjective and should be estimated by subject matter experts. Sensitivity analysis can be applied to calculate the input variables of decision trees, including the discount rate, which depends on the level of risk the project is exposed to during its lifetime.

Higher risks imply higher outcomes

From a business point of view, risk includes both positive and negative outcomes: the common notion is that the higher the risk one is willing to take, the higher the (not guaranteed) results. A simple statistical definition of risk relies on the variance of real outcomes around the expected outcome: the greater this variance, the higher the risk is perceived to be.

Risk in investments is always perceived through the eyes of the investor: he presumably looks at the risk from the standpoint of what the market is willing to bear. This principle dictates what discount rate should be using in calculating the future cash flows in project valuation using Discounted Cash Flow/Decision Tree Analysis.

In the finance world, risks are broadly classified as *market risks* and *private risks*.

Market risks versus private risks

The market risk of a project can be considered due to the volatility of its expected future payoff (the net cash flow) driven by market forces, such as market demand, competition, and so on. The private risks are related to the efficiency of an organization in completing the project as well as the effectiveness of the technology related to the project. Since financial option pricing models are based on the premise that the market risks of an underlying asset is captured by a *traded security*, the validity of the application of these models to real options valuation may be questionable if there is no traded security corresponding to the uncertainty of the underlying asset: for this reason, and for the fact that investors are willing to pay a risk premium for the market driven risks but not for the private risks, it is important to recognize and differentiate private risks from market risks and correctly evaluate them.

First of all, let's examine what type of risk influence the two broad cash flows streams that are common to most projects, *investment costs* and *production phase free cash flows*. The former consist of development cost (Research & Development, design) and production phase capital cost (building a manufacturing plant, marketing campaign): organizations are expected to estimate these costs with very little uncertainty, since they depend on the efficiency of the organization itself and on the effectiveness of the technology, which are not subject to market forces. Therefore, these costs are influenced by private risk only. The latter, instead, are considered to be influenced by market risk only, because the uncertainty of the cash flows is primarily dictated by the market forces. Generally cash outflows are discounted at a risk-free rate, or a rate slightly higher than that, while cash inflows are discounted in accordance with their risk level by adding a risk premium.

Investment costs: development costs and production phase capital costs

Production phase free cash flows: net revenues calculated from the expected future revenues and costs associated with the product (or service offering) in its production phase

Now we are going to analyze what discount rate should be applied to a given cash flow stream if it is dictated by private risk, market risk or neither of them, by addressing these three questions:

1. **If there is non uncertainty - that is, no risk - associated with a cash flow stream, what is the appropriate discount rate?**

   Such cash flows should be discounted using a *risk-free interest rate*, which corresponds to a riskless investment. This can be earned by assets that are considered entirely creditworthy during the life of the investment, such as Government Bonds, which are reckoned to be risk free and hence used as the benchmark risk-free rate.

   No uncertainty implies the use of a risk-free interest rate

2. **If the cash flow stream is dictated by private risk, what is the appropriate discount rate?**

   Theoretically, a risk-free should be used, but in the real world two major consideration are affecting this decision: first, it may be difficult to completely differentiate the private risk from the market risk, and second, any project investment requires capital which has a cost of acquisition. Therefore, for discounting cash flows that are subject to private risk, a rate slightly higher than the risk-free rate or a rate that is commensurate with the organization's weighted average cost of capital is used.

   Cost of capital represents the cost of financing organization's activities. The Weighted Average Cost of Capital of different cost components of issuing debts, preferred stock, and common equity is:

Weighted Average Cost of Capital calculation

$$WACC = W_d C_d (1 - t) + W_p C_p + W_e C_e$$

where W represents the respective weights; C is the cost corresponding to debt (d), preferred stock (p), and common equity (e); and t is the effective corporate tax rate.
Although the WACC characterizes the cost of capital at the organizational level, it can be used as a proxy to represent the private risks related to project investment costs.

3. **If a cash flow stream is influenced by market risk, what is the appropriate discount rate?**
In account for higher market risks a proportional risk premium should be added to the risk-free rate. Because investors expect higher returns for taking higher risks, it is only reasonable to discount the market-driven project cash flows at a rate that is defined by the risk level of the project. The idea is that if you can find a *proxy* (a project, a portfolio of projects, a security, an organization...) that has the same (or multiple of) cash flows expected from the project under evaluation, you can use the proxy's annual returns as the discount rate to discount the project cash flows. To determine this rate one can use a direct method, such as Capital Asset Pricing Model, or run a calculation on the Weighted Average Cost of Capital to obtain a Weighted Average Cost of Capital-Based Discount Rate, or take benchmark Hurdle Rates, or evaluate Opportunity Cost.
These rates work well for both Discounted Cash Flow and Monte Carlo Simulations methods. A few considerations need to be examined for Decision Tree Analysis. First of all, inside the decision tree, contingent decisions are related to cash flows driven by private risk, that should consequently be discounted by a single risk-free rate. Many academics, however, believe that the Weighted Average Cost of Capital is a better representative of the risks inside the decision tree because the outcome probabilities do not truly account for all the risks, and it is difficult to completely separate private risks from market risks associated with investment cash flow streams inside the decision tree. There can also be uncertainty associated with investment-related cash outflows, due to market forces.

Decision Tree Analysis needs an intermediate discount rate

Using the Weighted Average Cost of Capital as the only discount factor across the entire decision tree has some limitations though, since as you move from the left to the right the success probability of the project increases and the risks decreases. For this reason the recommended discount factor should be slightly higher than the risk-free rate and lower than the Weighted Average Cost of Capital, for the entire decision tree.

2.5 LIMITATIONS OF THE TRADITIONAL APPROACHES

While Discounted Cash Flow, Monte Carlo Simulations and Decision Tree Analysis are well-established techniques that have been successfully used for several decades in valuation of projects (as well as organization as a whole), they do fall short if you want to have a dynamic, non-deterministic view on the project which can give you an evaluation of the value closer to reality.

Why Discounted Cash Flow and Monte Carlo Simulations methods are limited: they are deterministic approaches and only the downside of risk is accounted

Discounted Cash Flow, for example, is a solid method and it is effective in many scenarios that are applicable to project investment decisions often faced by upper management; however, it takes a deterministic approach based only on a single set of input values, not accounting the uncertainty of cash flows. Adding sensitivity analysis to the method through studying different scenarios may give more insight about the uncertainty, but still each scenario is based on a fixed path outcome, which in any case does not take into account management's flexibility to change the course of the project. As a result, the additional value created because of these contingent decisions is not captured in Discounted Cash Flow. Another flaw is that to account for the risk associated with the project payoff, Discounted Cash Flow discounts the cash flows at a higher rate by adding a risk premium to the risk-free rate; the higher the risk, the higher the risk premium added. This means only the downside of the risk is accounted for, causing the rejection of potentially high successful projects just because of their high uncertainty.

Monte Carlo Simulations, as an extension of the Discounted Cash Flow method, has the same drawbacks: it does not take into account the contingent decisions and their impact on the project valuation.

Why Decision Tree Analysis method is limited: difficult probabilities estimation

Decision Tree Analysis is a more sophisticated tool than Discounted Cash Flow and offers value when a project is multistage and contingent decisions are involved. It differs from Discounted Cash Flow in the sense that, to account for market risks, it uses probabilities of outcomes rather than risk-adjusted discount rates. Although Decision Tree Analysis accounts for contingent decisions, it faces its own limitations: it is difficult to estimate correctly the probabilities for decision outcomes, since they may include both private and market risks, and there is no consensus in the entire finance community on what is the most appropriate discount rate to discount the cash flows inside the decision tree.

3 REAL OPTIONS ANALYSIS

Whereas Discounted Cash Flow is a deterministic model, Real Options Analysis accounts for the change in the underlying asset value due to uncertainty over the life of a project. The fundamental notion is that the option value increases with uncertainty. There can be a range of possible outcomes over the life of a project, with the uncertainty increasing as a function of time. As a result, the range of asset values would take the shape of a curve, called the "cone of uncertainty" (Figure 3.1).

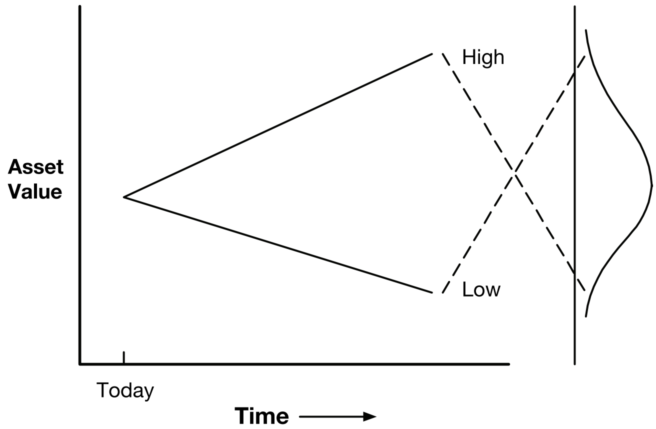

Figure 3.1: Cone of uncertainty

Real Options Analysis calculates the *Real Option Value* of a project: assuming that the decision makers will always take the value-maximizing decision at each decision point in the project life cycle, the value of the project increases as only the outcomes that are favorable (i.e. options are exercised) are considered while those that are not (letting the options expire) are ignored. Projects with very high or very low Net Present Values represent an easy "go/no go" approach so Real Options Analysis does not provide much value. On the other hand, Real Options Analysis is most valuable when there is high uncertainty with the underlying asset value and management has significant flexibility to change the course of the project in a favorable direction and is willing to exercise the option, keeping in consideration the long-term strategic value of the investments.

Real Options Analysis is most valuable when there is high uncertainty

When a project is related to high market uncertainty, its option value is maximized. This means that the risks involved in the project are elevated, but the project also has an high upside potential. The key action to perform before committing to a project is to clear the uncertainty associated to it, so that positive outcomes become more probable. You can do that by making small investments (e.g. market surveys, product test limited rollout) or

Active and passive learning help in resolving uncertainty just by waiting for the uncertainty to clear by itself during the course of time (since the market is subject to sequential changes). If you invest to gain information and clear uncertainty you are performing *active learning*, while if you are waiting for the uncertainty to clear itself you are performing *passive learning*. You can decide to move forward with the project (scale up, move to the next phase...) if the newly obtained information dictate favorable results, taking advantage of the upside potential of the project, or scale down (or even abandon) the project in case of unfavorable results.

Resolution of market uncertainty is necessary but not sufficient to capture the true options value, if the private uncertainty related to the technology effectiveness is high. Resolution of such private uncertainty requires active upfront investment. For example, a prototype may need to be built to demonstrate that the technology works, or a drug must pass clinical trials to prove its effectiveness.

How Real Options Analysis overcomes traditional methods' limitations

As said before, Real Options Analysis is not a substitute for but rather an extension of the Discounted Cash Flow method. Laying out a "map" that outlines contingent decisions, this technique helps the analyst in delineating possible paths of development of the project, and suggests at any time the correct decision to maximize the project payoff. It is a lot more flexible than the deterministic Discounted Cash Flow approach, which provide only one path for investment decisions.

The additional value provided by Real Options Analysis is captured by the risk premium added to the discount rate to account for uncertainty, adjusted taking into consideration the effectiveness of contingent decisions, especially those related to private and market uncertainty. When the payoff uncertainty is high, this additional value represent a significative percentage of the Net Present Value calculated with the standard Discounted Cash Flow method; therefore, you can gain valuable information for "go/no-go" decisions and project prioritization in a portfolio, as less attractive projects ranked lower based on Discounted Cash Flow value alone can move higher on the ranking scale and receive approval for investment, bumping other projects.

When both market risks and private risks exist as well as opportunities for contingent decisions to change the future course of the project, Real Options Analysis in combination with Decision Tree Analysis can provide better valuation.

Again: Real Options Analysis is not a substitute for either Discounted Cash Flow or Decision Tree Analysis. It supplements and integrates the traditional tools into a more sophisticated valuation technique.

3.1 TYPES OF OPTIONS

Options can be grouped into two basic categories: *simple options* and *compound options*. Option to expand, contract, defer and

abandon are examples of simple options; we will examine them and many others later. Most of them are considered to be American options, as they can be exercised on or at any time before the expiration date; call-type options give you the right to invest in the project, while put-type options give you the right to sell your project assets.
Compound options are common in many multiphase projects, where the initiation of one phase depends on the successful completion of the preceding phase. At the end of each phase you have the option to continue to the next phase, abandon the project, or defer it to a later time. Each phase becomes an option itself; the value of a compound option depends on the value of another option rather than the underlying asset value.
A compound option can be called a *learning option* if it involves resolution of either private or market uncertainty; an example of this might be an initial market test performed to clear the market uncertainty.
A *chooser option* is an option that gives you the right to choose from a variety of options, while a *switching option* gives you the right to switch between to modes of operation, if the project you are working on has this capability.
Finally, there is another group of options called *rainbow options*, for which multiple sources of uncertainty exist: they may be either simple or compound.

3.1.1 Simple Options

OPTION TO ABANDON The option to abandon is embedded in virtually every project, and has the characteristics of a put option. The contingent decision in this option is to abandon the project if the expected payoff, the underlying asset value, falls below the project *salvage value*, the strike price. This option is especially valuable where the net present value is marginal but there is a great potential for losses. As the uncertainty surrounding the payoff clears and if the payoff is not attractive, you can abandon the project early on without incurring significant losses. The losses can be minimized by selling off the project assets either on the spot or preferably by prearranged contracts.

OPTION TO EXPAND The option to expand provides particularly significant value to long-term projects. A project with high uncertainty may have marginal or even negative initial net present value, but you may accept that in the short term because of the high potential for growth in the future. The option to expand is common in high-growth companies, especially during economic booms. Lately this has been the case for a lot of internet start-ups. Investment for expansion is the strike price that will be incurred as a result of exercising the option. The option would be exercised if the expected payoff is greater than the strike price, thereby making it a call option.

OPTION TO CONTRACT The option to contract is significant in today's competitive marketplace, where companies need to downsize or outsource swiftly as external conditions change. Organizations can hedge themselves through strategically created options to contract. The option to contract has the same characteristics as a put option, because the option value increases as the value of the underlying asset decreases.

OPTION TO CHOOSE The option to choose consists of multiple options combined as a single option. The multiple options are abandonment, expansion, and contraction. The reason it is called a chooser option is that you can choose to keep the option open and continue with the project or choose to exercise any one of the options to expand, contract, or abandon. The main advantage with this option is the choice. This is a unique option in the sense that, depending upon the choice to be made, it can be considered a put (abandonment or contraction) or call (expansion) option.

OPTION TO WAIT The option to wait, also called the option to defer, is also embedded in virtually every project. It basically portrays the passive learning attitude - an organization may want to wait to invest in a project because it currently shows either negative or marginal net present value but has high uncertainty, which when cleared may tip the project into the high-net present value territory. The project investment corresponds to the strike price of the option: if and when the payoff is expected to be greater than the investment, the decision would be to make the investment at that time or else no investment would be made. These characteristics make the option to wait a call option. Deferral options are most valuable on assets where the owners have proprietary technology or exclusive ownership rights, but they don't want to invest because of high entry barriers. Because of the exclusivity of the project realization, they are not losing revenues to the competition by waiting.

BARRIER OPTION A barrier option is an option where your decision to exercise it depends not only on the strike and asset prices but also on a predefined "barrier" price. This type of option can be either a call or a put option, such as an option to wait or an option to abandon, respectively. A traditional call option is in the money when the asset value is above the strike price, whereas a barrier call option is in the money when the asset value is above the barrier price, which is predefined at a value higher than the strike price. As a rational investor, you exercise the barrier call only when the asset value is above the barrier price, irrespective of the strike price. Similarly, you exercise a barrier put when the asset value is below the barrier price, which is set at a value lower than the strike price. Usually, a barrier put option implies the option to abandon the project, while a barrier call implies the option to wait.
A particular case of barrier option is called *exit option*: the barrier

price, smaller then the effective strike price, is a critical asset value below which the project is abandoned. This option is effective when for some reason (usually political or psychological) management does not want to give up on a project, because it thinks that the reduction in the asset value may be temporary and the project may still be worth of investment.

3.1.2 Compound Options

Many project initiatives (research and development, capacity expansion, launching of new services, etc.) are multistage project investments where management can decide to expand, scale back, maintain the status quo, or abandon the project after gaining new information to resolve uncertainty.

These are compound options where every decision taken implies the prosecution of the project on a specific path. Exercising one option generates another, thereby making the value of one option contingent upon the value of another option, not the underlying asset. Every investment creates the right but not the obligation to make another investment, or to abandon, contract, or scale up the project.

Compound Options value is contingent upon the value of other options

Depending on the layout of the project realization, a compound option can either be *sequential* or *parallel* (also known as *simultaneous*). Sequential options must be exercised in a predetermined order, while parallel options may be exercised simultaneously. Given these priorities, you get that the life of a compound option is longer than or equal to the option it depends on. For both sequential and compound options, valuation calculations are essentially the same except for minor differences.

3.1.3 Rainbow Options

Most options are associated to only one uncertainty-related input variable, the volatility factor, which is typically calculated as an aggregate parameter built from many of the uncertainties associated with the project. If one of the sources of uncertainty has a significant impact on the options value compared to the others or if management decisions are to be tied to a particular source of uncertainty, you may want to keep the uncertainties separate in the options calculations: the use of different volatility factors makes the option a Rainbow Option. The options solution method is basically the same as for a single volatility factor, but it usually yields to more complex payoff calculations.

3.1.4 Other Options

SWITCHING OPTION A switching option refers to the flexibility in a project to switch from one mode of operation to another. Simple real life example: a "bi-fuel" car offers the option of switching from gasoline to natural gas and vice versa, giving you at any time the possibility to choose the less expensive fuel, or the one which gives more power. This flexibility has value and accounts for the price premium for "bi-fuel" cars compared to "single-fuel" cars, which can use only one kind of fuel, regardless of the fact that is less or more expensive than the other. Stressing this example, a "tri-fuel" car would be more valuable than a "bi-fuel" car because of the extra choice of fuel, resulting in an additional option value.

COMPLEX COMPOUND OPTIONS The sequential compound option basically translates to a series of simple options: exercising the first option creates the second one, exercising the second creates the third, and so on. In more complex compound options, the exercise of an option may create more than one option, opening new possibilities of development for the project. In valuing such projects, it is important to take into account all the resulting options, underlining the possible paths and outcomes. In this case, the most effective but also more difficult approach is the use of Decision Tree Analysis combined with Real Options Analysis.

3.2 REAL OPTIONS ANALYSIS CALCULATIONS

The traditional methods are based on a simple theoretical framework, and the math involved is easily acquirable. The solving technique is straightforward and does not involve heavy calculation, except for simulations, which are usually run with special software.

Real Options Analysis: complex theoretical framework, easy calculations

Real Options Analysis is far more complex compared to these traditional tools and requires a higher degree of mathematical understanding, unless you practice a "black-box" approach. The calculation involved in Real Options Analysis solution, however, are relatively simple, and can be done easily with a handheld calculator once you frame the problem and set up the right equations.

The first thing to evaluate is the uncertainty related to the project: if there is no uncertainty, management can make a decision today and there is no option value, while higher uncertainty creates future management decision opportunities that are reflected in an higher option value. Once you established that the Real Options approach can add value to the project, you can perform a Net Present Value calculation using the traditional Discounted Cash Flow method (a risk-adjusted discount rate is used). Then you can choose between a variety of techniques to incorporate the investment cost (which becomes the strike price of the option) and the value created by the uncertainty of the asset value and flexibility due to the contingent decision.

Black, Merton and Scholes won the Nobel Memorial Prize in Economic Sciences in 1997

Real options solutions are based on models developed for pricing financial options. The Nobel Prize-winning breakthrough by three MIT economists - Fischer Black, Robert Merton, and Myron Scholes - paved the way to simple and elegant solution of financial option problems, which in turn became the foundation for real options applications. Several methods are available to calculate option values, and within each method there are many alternative computational techniques to deal with the mathematics. The choice depends on simplicity desired, available input data, and the validity of the method for a given application.

BLACK-SCHOLES-MERTON MODEL This complex method for calculating the call/put option value was developed in 1973 by professors Fischer Black and Myron Scholes and was based on the results of a preceding study by Robert Merton and Paul Samuelson. The model mathematic background is complex; the derivation of the final partial differential equations involves solving a partial differential equation with specified boundary conditions (type of option, option values at known points and extremes, etc.) that describe the change in option value with respect to measurable changes of certain variables in the market.
In a closed form analytical solution to the partial differential equation, the option value is given by only one equation, named Black-Scholes equation, which for a call option is:

$$C = N(d_1)S_0 - N(d_2)X^{(-rT)}$$

where C = value of the call option, S_0 = current value of the underlying asset, X = cost of investment or strike price, r = risk-free rate of return, T = time to expiration, $d_1 = [\ln(S_0/X) + (r + 0.5\sigma^2)T]/\sigma\sqrt{T}$, $d_2 = d_1 - \sigma\sqrt{T}$, σ = annual volatility of future cash flows of the underlying asset, and $N(d_1)$ and $N(d_2)$ are the values of the standard normal distribution at d_1 and d_2 (we will discuss all of these input parameters later on), while for a put option is:

$$P = N(-d_2)X^{(-rT)} - N(-d_1)S_0$$

where P =value of the put option, and all the other parameters are the same as above.

The Black-Scholes equation was developed for financial options, therefore it is not so valid for Real Options Analysis

The Black-Scholes equation is easy to use and widely employed in financial options valuation; its application in Real Options Analysis is limited though. First of all, Black and Scholes developed their model for European financial options, which means that the option is exercised only on a fixed date and no dividends are paid during the option life; the inconsistency with the real options model is obvious, since real options can be exercised at any time during their life and there can be *leakages*, which are equivalent to the dividends of a financial security. There are adjustments that can be made to account for those leakages, but only if they are uniform, and in any case this takes to complication in the formula's usage.
Black-Scholes assumes a lognormal distribution of the underlying asset value, which may not be true with the cash flows related to real assets; it also assumes that the increase in the underlying asset value is continuous as dictated by its volatility and does not account for any drastic ups and downs. It allows only one strike price for the option, which can change for a real option during

its life.
While some of these limitations can be overcome by making adjustments to the Black-Scholes approach, the already complex model becomes even more complex; this promotes a "black-box" approach, where the intuition behind the application is lost, thereby making it difficult to get management buy-in.

SIMULATIONS The simulation method for solving real options problems is similar to the Monte Carlo technique for Discounted Cash Flow analysis. It involves simulation of thousands of paths the underlying asset value may take during the option life given the boundaries of the cone of uncertainty (Figure 3.2) as defined by the volatility of the asset value.

Monte Carlo Simulations apply to Real Options Analysis in a similar way as they apply to the Discounted Cash Flow method

In the simulation method, the option life is divided into a selected number of time steps, and thousands of simulations are made to identify the asset value at each step of each simulation. At time = 0, for every simulation, you start with the expected underlying asset value (S_0). In the next step, the asset value, which may go up or down, is calculated by using the following equation with a random variable function:

$$S_t = S_{t-1} + S_{t-1}(r * \delta t + \sigma \epsilon \sqrt{\delta t})$$

where S_t and S_{t-1} are the underlying asset value at time t and $t-1$ respectively: σ is the volatility of the underlying asset value and ϵ is the simulated value from a standard normal distribution, usually taken with mean zero and a variance of 1.0. Asset values are calculated for each time step until the end of the option life. Each resulting project value is then discounted back to today using a risk-free rate; the average of these values from all the simulations is the option value for the project.
Simulations are more easily applicable to European options, where there is a fixed exercise date. The computations, however, become tedious when simulating all the possible option exercise dates for an American option. It becomes an even bigger challenge when dealing with sequential options, because each decision leads to a new path. This can involve millions of simulations, which can be an enormous computational task even with today's fast computers.

LATTICES Lattices look like decision trees and basically lay out, in the form of a branching tree, the evolution of possible values of the underlying asset during the life of the option. The approach is similar to the Decision Tree Analysis one: once you calculate all the possible outcomes, optimize the future decisions exercising the option where appropriate and then recursively combine the results climbing back to the tree root to obtain the option value.
The most commonly used lattices are binomial trees. The binomial model can be represented by the binomial tree shown in figure 3.3. S_0 is the initial value of the asset. In the first time increment, it either goes up or down and from there continues to

go either up or down in the following time increments. The up and down movements are represented by u and d factors, where u is > 1 and d is < 1 and we assume $u = 1/d$. The magnitude of these factors depends on the volatility of the underlying asset.

The first time step of the binomial tree has two nodes, showing the possible asset values (S_0u, S_0d) at the end of that time period. The second time step results in three nodes and asset values (S_0u^2, S_0ud, S_0d^2), the third time step in four (S_0u^3, S_0u^2d, S_0ud^2, S_0d^3), and so on. The last nodes at the end of the binomial tree represent the range of possible asset values at the end of the option life. These asset values can be represented in the form of a frequency histogram. Each histogram signifies a single asset value outcome, and the height of the histogram is a function of the number of times that outcome will result through all possible paths on the binomial tree (Figure 3.4).

The total time length of the lattice is the option life and can be represented by as many time steps as you want. While the range (minimum and maximum) of outcome values at the end of the lattice may not change significantly with an increase in the number of time steps, the number of possible outcomes increases

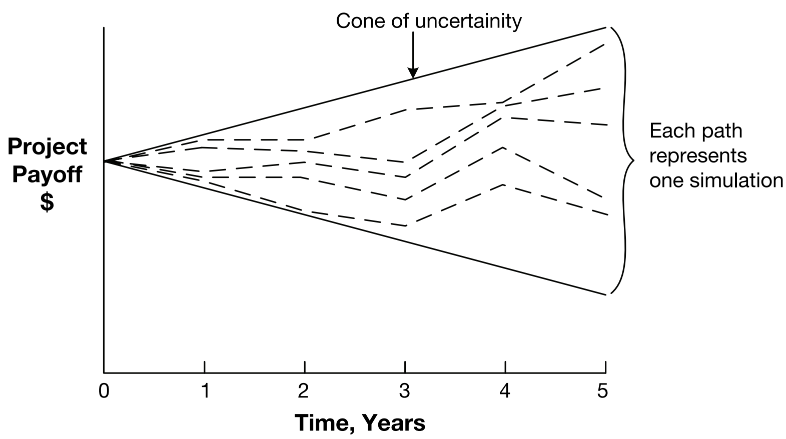

Figure 3.2: Cone of uncertainty and Monte Carlo Simulations

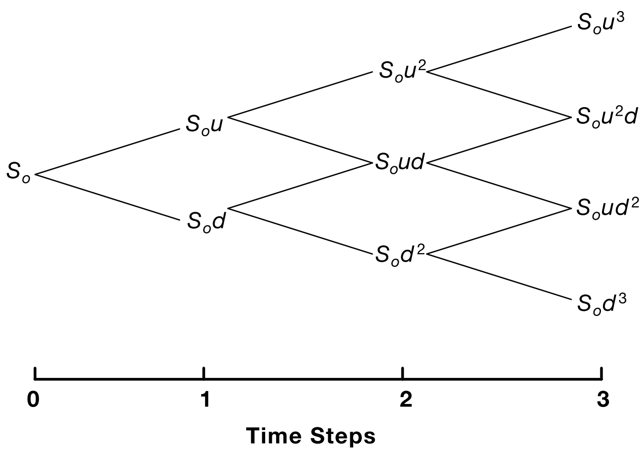

Figure 3.3: Recombining Binomial Tree

exponentially and their frequency distribution curve will become smoother. The higher the number of time steps, the higher the level of accuracy of option valuation.

Risk-neutral Probabilities method for binomial lattices solution

A simple approach for the solution of binomial lattices is the *risk-neutral probabilities* method, which assumes a risk-free rate for discounted cash flows throughout the lattice. This method applies to every kind of option and the calculations involved are easy once you determine the problem parameters; results are significative for the most common cases and quickly obtainable, making this an efficient method for Real Options Analysis solutions.

The up and down factors, u and d, are a function of the volatility of the underlying asset and can be described as follows:

$$u = e^{(\sigma\sqrt{\delta t})}$$

$$d = e^{(-\sigma\sqrt{\delta t})} = \frac{1}{u}$$

where σ is the volatility (%) represented by the standard deviation of the natural logarithm of the underlying free cash flow returns, and δt is the time associated with each time step of the binomial tree.

The risk-neutral probability, p, is defined as follows:

$$p = \frac{e^{r\delta t} - d}{u - d}$$

where r is the risk-free rate. The risk-neutral probabilities are not the same as objective probabilities: they are just mathematical intermediates that will enable you to discount the cash flows using a risk-free interest rate. Once you calculate all these three parameters, you can start laying out the binomial tree starting from the root and expanding its upward and downward branches. At the final time step you obtain the the asset value for the terminal nodes, and you can start the evaluation of the option value,

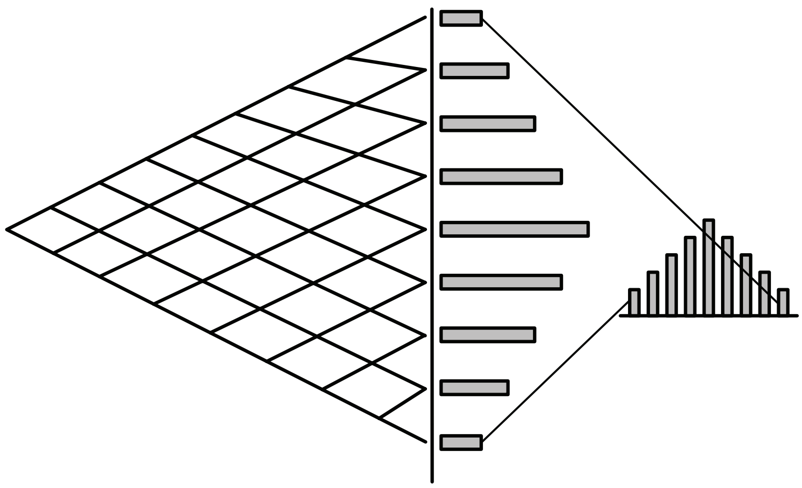

Figure 3.4: Distribution of outcomes for a binomial recombining lattice

comparing the asset value with the strike price and deciding to exercise the option or not. Then you calculate the expected asset value at second-last time step using the neutral probability using this formula:

Expected Asset Value ad node = [$p *$ (Option Value at next upward node) + $(1 - p)$(Option Value at next downward node)] $* e^{(-r\delta t)}$

Iterate this calculation recursively until you get to the root of the tree, obtaining the final Expected Asset Value which you should compare with the project Net Present Value previously calculated using the Discounted Cash Flow method to find the additional Option Value of the project.

Non-recombining binomial lattices

Binomial recombining lattices are the most commonly used; however, other kind of trees can be used in particular cases. Non-recombining lattices such as the one in figure 3.5 have center nodes (for example at level 2, S_0ud* and S_0du*) that are different for the predecessor nodes' downward (S_0u) and upward (S_0d) movements, whilst in a recombining lattice a center node (again, at level 2 S_0ud) is the same for its upper predecessor's (S_0u) downward movement and its lower predecessor's (S_0d) upward movement. More computationally complex trinomial and quadrinomial lattices also can be used to solve real options problem; for example, a quadrinomial lattice involves two sets of upward and downward factors that are applied to the asset values.

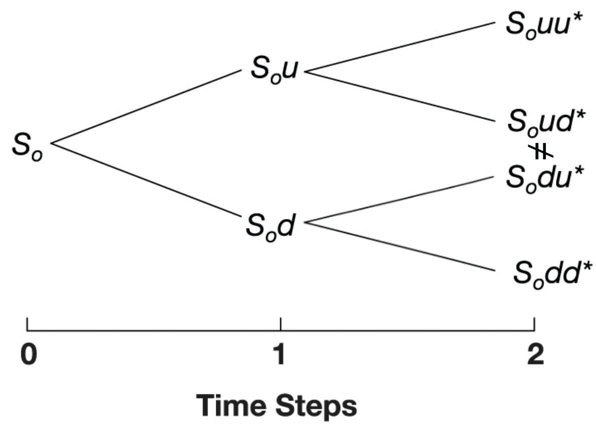

Figure 3.5: Nonrecombining Binomial Lattice

The binomial method is the preferred method for most analysts because its advantages far outweigh the drawbacks. It doesn't give the most accurate option value that you could obtain with Black-Scholes equation, but, since since the underlying mathematical framework is the same, you get an approximation of the result which is really good for practical purposes and can be obtained with few time steps in the binomial tree. Input parameters such as strike price and volatility can be changed easily over the option life; jumps and leakages also can be accommodated without any complex changes; all of these factors, plus the transparency in the underlying framework, make the results easy to explain to upper management for buy-in and approval.

3.3 INPUT VARIABLES

UNDERLYING ASSET VALUE (S_0) The value of the underlying security at time zero represents the underlying asset value and is easily known for a financial option, because it is a traded security. With real options, however, the asset value is estimated from the cash flows the asset is expected to generate over the production phase of the project life cycle. **The present value of the expected free cash flows based on the DCF calculation is considered the value of the underlying asset**. As part of this standard calculation, the analyst should forecast the revenues based on assumed market share, number of units expected to be sold, price per unit, the cost associated with these sales, and so on. Most likely, there will be great uncertainty associated with these estimates, and therefore you will use an appropriate risk adjusted discount rate in the Discounted Cash Flow calculation.

Jumps and leakages

A few problems show up in the calculation of the underlying asset value if you work with long-term options: there may be *jumps*, sharp up and down turns, in the value changing process, usually due to an unexpected turn of events totally out of control. It is difficult to account for such jumps with the Black-Scholes method, but the binomial method allows easy corrections.

The basic Black-Scholes model assumes that there are no dividends (negative cash flows) given out during the option life, and the underlying asset value changes as dictated by the volatility factor only. With real options, both dividend equivalent negative cash flows as well as positive cash flows can affect the underlying asset value, and these "leaks" must be adjusted for accordingly. These changes are not related to the volatility of the asset value dictated by the market conditions or to the jump processes mentioned above. Examples of positive cash flow leaks include royalty income and interest, and cash outflows can be royalty fees, storage costs, etc. The leakage can be constant or varied during the option's life. Adjustments for leakage can be incorporated into either the Black-Scholes or the binomial model, although the latter offers more flexibility and makes the results more transparent.

VOLATILITY OF THE UNDERLYING ASSET VALUE AND VOLATILITY FACTOR (σ) Volatility is an important input variable that can have a significant impact on the option value and is probably the most difficult variable to estimate for real options problems. It represents a measure of the variability of the total value of the underlying asset over its lifetime, as the uncertainty associated with the cash flows that comprise the underlying asset value. The volatility factor σ used in the option models, however, is the volatility of the *rates of return*, which is measured as the standard deviation of the natural logarithm of cash flows *returns*, which are the ratios of a certain time period cash flow to the preceding one. In any options model, the volatility factor used should be consistent with the time step used in the corresponding equations. For instance, if the time steps are annual, the volatility factor should be annualized. The volatility factor based on one time frame can be converted to another using the following equation:

$$\sigma(T_2) = \sigma(T_1) * \sqrt{T_2/T_1}$$

where $\sigma(T_2)$ and $\sigma(T_1)$ are the volatility factors based on time steps T_2 and T_1, respectively.

Volatility depends on several uncertainties

Volatility is a function of uncertainties related to several variables that control the cash flows: unit price, quantity expected to be sold, margins, etc. You can keep the variances separated if you believe the controlling variables are independent of each other, evolve differently over time, and impact the asset value in different directions. If these sources of uncertainty are uncorrelated, keeping the variances separate and utilize a rainbow option value makes it a better estimate and gives you more useful insight into the problem. When there are more than two such independent sources of uncertainty, you can conduct a preliminary analysis to determine which two of the variables have the greatest impact on the asset value and focus on them. You can alternatively decide to combine all the uncertainties into one aggregate value and use it in solving the options model.

Since historical information is usually not available on real options, estimate the volatility is a difficult challenge. Here are some of the commonly used methods:

Logarithmic Cash Flow Returns Method provides a volatility factor that is based on the variability of the same cash flow estimates that are used in calculating the underlying asset value itself; therefore, it is most representative of the volatility of the asset value. The steps involved are as follows:

1. Forecast project cash flows during the production phase of the project at regular time intervals (for example, years);

2. Calculate the relative returns for each time interval, starting with the second time interval, by dividing the current cash flow value by the preceding one;

3. Take the natural logarithm of each relative return;

4. Calculate the standard deviation of the natural logarithms of the relative returns from the previous step, which becomes the volatility factor (σ) for the underlying asset value. This factor is commonly expressed as a percentage and is specific to the time period.

This method is simple to use, mathematically valid and consistent with the assumed variability of the very cash flows that are used to calculate the asset value, but has a major disadvantage: when a cash flow is negative, the return associated with it will also be a negative number, for which a natural logarithm does not exist. This may produce erroneous results. Furthermore, some of the mathematical models (e.g. time series, constant growth rate) used to forecast the cash flows may also result in erroneous data for volatility estimation. Therefore, caution should be exercised in using this method.

Monte Carlo Simulation provides numerous cash profiles that are simulated over the project life; the volatility factor is computed for each profile using the logarithmic cash flow returns method presented in the preceding section. This method thus produces as many volatility factors as the number of simulations, thereby providing a distribution of these factors rather than just one.
The input data (expected average and variance of the input variables such as revenues, costs and discount factor) can be generated based on historical information or management estimates. Although many input variables contribute to the asset value, usually only a few have the most impact. Practitioners typically identify such variables by performing an initial sensitivity analysis and focus on them in simulations to calculate the volatility factor. This method offers the most insightful information on the volatility of the underlying asset value, since it offers the distribution of the volatility factor, which can be used in evaluating the sensitivity of the real options value of the project under scrutiny to the volatility factor.

Project Proxy Approach is an indirect approach to estimate the volatility factor of the underlying asset. It uses as a proxy the data from a historical project which is assumed to have market performance and a cash flow profile similar to the project being considered. This translates to using the volatility factor of a previous project that has real world market information.

Market Proxy Approach is similar to the project proxy approach except that instead of using cash flow information from a similar historic project, the closing stock price of a publicly traded company that has a cash flow profile and risks comparable to the project under consideration is used.

Management Assumptions Approach use optimistic (S_{opt}), pessimistic (S_{pes}), and average (S_0) expected payoffs estimated by management for a given project lifetime (t). An optimistic estimate of $100 million means that there is a 98% probability that the payoff will not exceed $100 million. Similarly, a pessimistic estimate of $20 million means that there is only a 2% probability that the payoff will be less than $20 million. The average estimate corresponds to 50% probability. Assuming that the payoff follows lognormal distribution, by knowing any two of the three estimates mentioned above, you can calculate the volatility of the underlying asset value using one of the following equations:

$$\sigma = \frac{\ln(\frac{S_{opt}}{S_0})}{2\sqrt{t}}$$

$$\sigma = \frac{\ln(\frac{S_0}{S_{pes}})}{2\sqrt{t}}$$

$$\sigma = \frac{\ln(\frac{S_{opt}}{S_{pes}})}{4\sqrt{t}}$$

Option pricing theory assumes that the volatility of the underlying asset value remains constant over the option life, which is reasonable for short-term options on traded stocks. Real options, however, typically have longer lives, and the volatility can change over time. The change may be due to a shift in general economic conditions, sudden market fluctuations, unexpected global events, and so on. Although many such events are unpredictable, you may want to assume more probable changes in estimating volatility variations. Both the Black-Scholes and binomial models allow you to incorporate volatility changes, although the latter offers more flexibility and transparency.

EXERCISE OR STRIKE PRICE While exercise is instantaneous with financial options, in the real options world exercising an option typically involves actions such as development of a product or launching a large marketing campaign which does not happen in an instant but in fact takes a long time. This shortens the true life of the option compared to the stated life. For example, the true life of an option to turn a patent into a marketable product is less than the stated life because of the long product development and commercialization time.
The strike price or the investment cost directly impacts the option value, the sensitivity of which must be evaluated to gain better insight into the option value. It is possible that the strike price may change during the option life, and therefore, the option valuation equations must be adjusted accordingly.

OPTION LIFE Again, while the time to maturity is clearly known for a financial option, in most cases that is not true for a real option. Often, you do not know exactly how long the opportunity will exist to exercise the option, and usually there is no deadline by which the decision must be made. The option life has to be long enough for the uncertainty to clear, but not so long that the option value becomes meaningless because of entry of competitors in the meantime. In the case of a financial option, the value of the option increases with time to maturity, because the range of possible payoff values increases with long time frames, thereby boosting the upside potential. With real options, this relationship is not so direct, except when dealing with proprietary

Option life is a relevant parameter in Real Options Analysis or patented assets. Issues related to loss of market share due to late market entry, loss of first mover advantage, competitive threats, and so on can reduce the option value even when the time to maturity increases.

RISK-FREE INTEREST RATE The risk-free annual interest rate used in real options models is usually determined on the basis of the U.S. Treasury spot rate of return, with its maturity equivalent to the option's time to maturity. The Black-Scholes and binomial models use continuously compounded discount rates as opposed to discretely compounded rates. The continuous rate can be calculated from the discretely compounded rate as follows:

$$r_f = \ln(1 + r_d)$$

where r_f and r_d are the continuously and discretely compounded risk-free rates, respectively.

3.4 APPLICATION OF REAL OPTIONS SOLUTIONS

When approaching the analysis of the value of a new project you should think about the most appropriate tool to use by framing the application, map out the structure and the sequence of the project steps and evaluate strategic decision checkpoints over its life time. A simple Discounted Cash Flow analysis will usually suffice in providing the right information for project with low associated uncertainty, or low flexibility with contingent decisions. On the other hand, for projects with high uncertainty (only if it related to market risk though) Real Options Analysis should be the right tool to use.
The standard approach is to use both the binomial and Black-Scholes methods, the former for flexibility, transparency, and easy communication and the latter to verify and gain better insight into the binomial results.
A six-step process can be used for the binomial method:

1. **Frame the application**
   Framing a real option is more difficult than framing a financial option. It involves describing the problem in simple words and pictures, identifying the option, and stating clearly the contingent decision and the decision rule. Some applications involve more than one decision or option. For example, chooser options may include abandon, defer, expand, contract, and other options. Compound options involve options on options, which may be parallel or sequential. You must identify these dependencies very clearly. Keeping the problem simple and making it more intuitive will help you communicate the results more effectively to get upper management's buy-in.

2. **Identify the input parameters**
   The basic input parameters for the binomial method to

value any type of option include the underlying asset value, strike price, option life, volatility factor, risk- free interest rate, and time increments to be used in the binomial tree. These parameters can be calculated using the approaches suggested earlier. Additional information is required for some of the options, such as expansion and contraction options.

3. **Calculate the option parameters**
   The option parameters are intermediates to the final option value calculations and are calculated from the input variables. These are the up (u) and down (d) factors and the risk-neutral probability (p) required for the binomial solution.

4. **Build the binomial tree and calculate the asset values at each node of the tree**
   The binomial tree is built based on the number of time increments selected. The underlying asset value at each node of the tree is calculated starting with So at time zero at the left end of the tree and moving toward the right by using the up and down factors.

5. **Calculate the option values at each node of the tree by backward induction**
   Starting at the far right side of the binomial tree, the decision rule is applied at each node and the optimum decision selected. The option value is identified as the asset value that reflects the optimum decision. Moving toward the left of the tree, the option values at each node are calculated by folding back the option values from the successor nodes by discounting them by a risk-free rate and using the risk-neutral probability factor. This process is continued until you reach the far left end of the tree, which reflects the option value of the project. Whereas asset valuation (step 4) shows the value of the underlying asset at each node without accounting for management decision, the option valuation step identifies the asset value that reflects management's optimal decision at that node.

6. **Analyze the results**
   After the option value has been calculated, the appropriate first step is to compare the net present value derived from the Discounted Cash Flow method versus Real Options Analysis and evaluate the value added as a result of the flexibility created by the option(s). In order to get a better perspective on the option solution, several analyses can be performed on the sensitivity of the option value to input parameters variations, or to different management decisions. To gain more information, option value changes are estimated in particular situations, such as presence of jumps or leaks, private risk, multiple sources of uncertainty, staged options chains and so on.

It is particularly important to compare the option value based on the binomial method with the solution of Black-Scholes formula to verify the presence of errors in the calculation.

3.4.1 Example: Option to Abandon

We are now going to examine an application of the six-steps process for a simple problem.

1. **Frame the application**
   A television station wants to fully value the real options linked to a new drama series that is on the point of commissioning. The total cost for the production of one episode of the series is estimated to be \$65000. The station will produce three seasons of the show and broadcast one season per year, evaluating at the end of each year the share ratings for the show which translate in advertising revenues for advertising slots during the episodes. A forecast on the share ratings based on historical experience at the channel suggests that such formats would generate a rating of around 20 points in prime-time, implying an advertising revenue per episode of \$70000.
   At the end of any of the seasons, based on the results, the station can either decide to broadcast the next season of the show or take it off the air freeing space for more remunerative shows. The episodes that will not make the air will then be distributed on alternative channels (DVD sells, Internet streaming) generating an estimate revenue of \$45500. The annual volatility of the logarithmic returns of the future cash flows is calculated to be 35%, and the continuous annual riskless interest rate over the next three years is 5%. What is the value of the abandonment option?

2. **Identify the input parameters**
   We suppose that every season of the show gets the same number of episodes, and that the ratings estimation is the same for each episode of the season, so we can perform the analysis on a single episode. We get:

$$S_0 = \$70000$$

$$X = \$45500$$

$$T = 3 \; years$$

$$\sigma = 35\%$$

$$r = 5\%$$

$$\delta t = 1 \; year$$

3. **Calculate the option parameters**

$$u = e^{\sigma\sqrt{\delta t}} = e^{0,35*1} = 1.419$$

$$d = \frac{1}{u} = \frac{1}{1.419} = 0.705$$

$$p = \frac{e^{r\delta t} - d}{u - d} = \frac{e^{0.05*1} - 0.705}{1.419 - 0.705} = 0.485$$

4. **Build the binomial tree and calculate the asset values at each node of the tree**
   Build a binomial tree, as shown in figure 3.6, using one-year time intervals for three years and calculate the asset values over the life of the option. Start with S_0 at the very first node on the left and multiply it by the up factor and down factor to obtain S_0u (\$70000 $*$ 1.419 = \$93330) and S_0d (\$70000 $*$ 0.705 = \$49350), respectively, for the first time step. Moving to the right, continue in a similar fashion for every node of the binomial tree until the last time step. In figure 3.6, the top value at each node represents the asset value at that node.

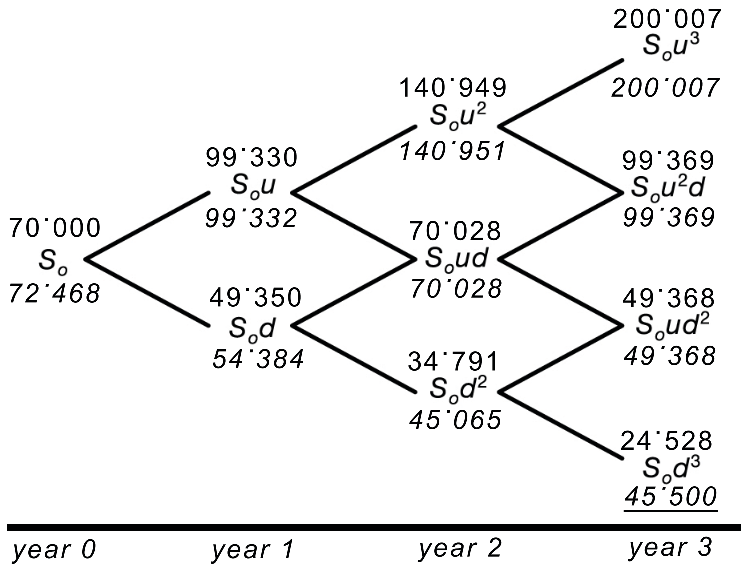

Figure 3.6: Binomial Lattice for the example

5. **Calculate the option values at each node of the tree by backward induction**
   Figure 3.6 shows the option values (bottom italicized numbers) at each node of the binomial tree by backward induction. Each node represents the value maximization of abandonment versus continuation. At every node, you have an option to either abandon the project for a salvage value of \$45500 or continue keeping the option open until it expires.
   A. Start with the terminal nodes that represent the last time step. At node S_0u^3, the expected asset value

is $200007, compared to the salvage value of $45500. Since you want to maximize your return, you would continue rather than abandon the project. Thus the option value at this node is $200007. The same goes for nodes S_0u^2d and S_0ud^2.

B. At node S_0d^3, the expected asset value is $24528, compared to the salvage value of $45500; therefore, it makes sense to sell off the asset and abandon the project, which makes the option value at this node $45500 (bottom underlined number in the figure).

C. Next, move on to the intermediate nodes, one step away from the last time step. Starting at the top, at node S_0u^2, calculate the expected asset value for keeping the option open. This is simply the discounted (at the risk-free rate) weighted average of potential future option values using the risk-neutral probability as weights:

$$[p(S_0u^3) + (1-p)(S_0u^2d)] * e^{-r\delta t} =$$

$$= [0.485 * \$200007 + (1-0.485) * \$99369] * e^{-0.05*1} =$$

$$= \$140951$$

Since this value is larger than the salvage value of $45500, you would keep the option open and continue; therefore, the option value at S_0u^2 is $140951.

D. Complete the option valuation binomial tree all the way to time = 0 using the approach outlined above.

6. **Analyze the results**
The payoff of the project based on the Discounted Cash Flow method without flexibility is $70000, but the cost to produce every episode is $65000, leaving a relatively small Net Present Value of $5000. Real Options Analysis, however, shows a total project value of $72468, yielding an additional $2468 for episode due to the abandonment option. Using the Black-Scholes equation for this put option you can verify the validity of the result. Thus, the project Net Present Value improves by 50% because of the abandonment option.
Additional information on the probability of the survival of the show also can be of use to the station for renewal decision. That information can be obtained easily from the binomial lattice used in solving the options problem. An examination of figure 3.6 reveals four possible asset values at the end nodes of the three-time-step binomial lattice. Of the four, only one node is below the strike price and hence will trigger the option exercise. At the surface, it may seem, therefore, that the probability of exercising the option - that is, not renewing the show - at the end of the option life is 1/4 or 0.25. This in fact is wrong.
Since the binomial tree in this example is recombining, there

are many different paths leading to each node. The number of paths contributing to each node must be calculated first before estimating the probability of exercising the option at the end of the option life. It is easy to verify that the the total number of paths is given by $2^{total\ number\ of\ time-steps}$, in this example $2^3 = 8$. Therefore, the probability that the show will not be renewed at the end of the option life is $1/8 = 12.5\%$. Conversely, the probability that the station will broadcast all three seasons of the show is 87.5%. This kind of information on the success of a project can be helpful in making decisions in a project portfolio context when projects with similar Net Present Values and even similar option values are compared.

4 REAL OPTIONS ANALYSIS IN THE REAL WORLD

Whereas business operators have been making capital investment decisions for centuries, the term "real option" is relatively new. Several works on the "options" available to a business owner were produced in the XX century but the description of such opportunities as "real options", however, came after the development of analytical techniques for financial options, such as Black-Scholes in 1973. As such, the term "real option" itself is closely tied to these new methods; in fact, it was officially coined by Professor Stewart Myers at the MIT Sloan School of Management in 1977.
Real options are today an active field of academic research. An academic conference on real options is organized yearly (Annual International Conference on Real Options).
This popularization is such that Real Options Analysis is now a standard offering in postgraduate finance degrees, and often, even in Master of Business Administration curricula at many Business Schools.
As for the commercial use of the method, several authors have documented potential applications of real options in various industries, including aerospace, automotive, banking, chemicals, consumer goods, electronics, insurance, medical products, oil and gas, pharmaceuticals, technology, telecommunications, transportation, and all kind of IT-based ventures (e-commerce, e-business, internet start-ups etc.).

Real Options Analysis applications are unlimited and can serve a wide variety of business areas

Today's corporate world, in general, seems to take either a "love it or hate it" attitude toward Real Options Analysis. Those who truly understand the principles behind real options appreciate the value Real Options Analysis brings to project valuation and have embraced it. The energy, oil and gas, and pharmaceutical sectors are the leaders in successfully adopting the real options framework. They recognized the value of the real options approach and adopted it quickly. Being the early pioneers, they have been able to accumulate the historical input data that are so vital to the successful application of real options tools.
Despite its potential for broad-based application, the real options framework has been applied and accepted to a limited extent only. Potential first-time users have shown resistance because of the real or perceived limited success of real options in the real world. There are several reasons why the real options framework has received so little support in the finance community.

Real Options Analysis is still not very popular

First, Real Options Analysis is a relatively new tool compared to the traditional methods such as Discounted Cash Flow analysis that have been around for decades. Even Discounted Cash Flow took decades to become a staple in the financial world. As with any new tool, it may take a few years before Real Options Analysis gains wider acceptance. It is a more sophisticated and complex technique compared to the traditional tools and requires a higher level of understanding. Some organizations seem to shy away from this new technique because of the higher level of mathematics involved in solving the real options problems, as well as lack of a clear understanding of the principles and benefits of the method. This is why many in-house analysts as well as outside consultants have promoted a "black box" approach to Real Options Analysis, instead of opening the black box to demystify it and explain what is inside in simple terms easily understood by upper management and decision makers. Unfortunately, Black- Scholes, the most well-known options model, although it is not the most appropriate for many real options problems, easily lends itself to the black box approach because of its outward simplicity (the solution involves just one equation) and the enormous theoretical complexity behind it.

Right now it is also difficult to master the method if you are not familiar with the theoretical framework behind it - a business venture who would like to incorporate Real Options Analysis into its Project Management workflow would have to either hire an external consultant or train its current employees. Of course simpler approaches to the technique are going to be developed and efficient tools are more likely to appear in the future as more organizations embrace the new technique.

Real options proponents believe that it is just a matter of time before Real Options Analysis becomes widely accepted and integrated into the standard project valuation framework of the finance world.

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