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Currency hedging strategies in international portfolios

RELATORE:

CH.MO PROF. MASSIMILIANO CAPORIN

LAUREANDO: ANDREA MILANI

MATRICOLA N. 1058495

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Abstract

Il lavoro persegue l'obiettivo di analizzare l'efficacia di diverse strategie di currency hedging, anche basate su modelli dinamici per l'individuazione dell'optimal hedging ratio (rapporto di copertura ottimale), applicate a differenti portafogli di indici azionari/obbligazionari, government e corporate, in particolare con composizioni del tipo full bond, full equity e bilanciato.

L'analisi è condotta dal punto di vista di un investitore europeo che opera investimenti nei mercati EMU e US, con conseguente esposizione al tasso di cambio EUR/US dollar.

Le strategie di copertura sono implementate grazie all'utilizzo dello strumento dei contratti di currency future, sempre attraverso l'uso di pertinenti indici.

L'analisi tiene conto sia degli aspetti di performance, che di riduzione del rischio all'interno dei portafogli per il confronto con il portafoglio non hedged per il quale non sono stati introdotti strumenti di copertura del rischio sul tasso di cambio.

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1 Introduction

Globally diversified portfolios have historically dominated domestic-only ones on the efficient frontier. Starting from the pioneering study by Solnik (1974) that found optimal portfolio is internationally diversified thanks to the consequently risk reduction, numerous studies have established the benefits of global diversification even if markets are becoming increasingly integrated around the world.

In the last years, European investors benefited from international diversification thanks to lower risk than domestic-only investments and exploitation of countries' different level of growth and market opportunities.

International bond and stock portfolios' distinguishing feature is their exposure to currency risk. Exposure to exchange risk would increase the volatility of international portfolios without a commensurate increase in expected returns. Such exposure could reduce or even eliminate the benefit from international diversification if not managed properly.

The optimal strategy could be hedging exchange rate risk through currency forwards or futures. Indeed, many international investors choose to hedge all or part of their currency risk exposure. Others simply choose to ignore it.

As stated by De Roon et al. (2001) benefits from hedging international asset portfolios is investor specific. Investors with a mean–variance utility function may find hedging benefits but oppositely it could not be necessarily the same for investors with other utility functions.

In their study on international diversifying portfolio, De Roon et al. (2011) show that, while the volatility reduction benefits of currency hedging are always statistically significant, the economic benefits are at best weak or insignificant. They find that in-sample, the gains from speculative currency investing are highly statistically significant and economically meaningful. Schmittmann (2010) points out that, even if currency hedging reduces the volatility of portfolio returns, hedging will be beneficial only if this risk reduction is not accompanied by an offsetting decrease in returns.

In addition to these aspects of significance of benefits from implementing a hedging strategy, there is no consensus over how much exposure should be hedged, i.e. the optimal hedging ratio that should be applied, nor on the way to perform hedging.

A large literature studies strategies and optimal hedging ratios cross different models, portfolios and investors' points of view, for example Black (1990), Campbell et al. (2010), Schmittmann (2010), Brown et al. (2012) and many others.

Starting from static or unconditional hedging strategies that imply the use of hedging ratios estimated on historical basis, minimizing the variance of the hedged portfolio returns, studies moved to dynamic models.

Static models provide hedging ratio constant over time, considering a time-invariant covariance matrix. On the opposite, dynamic models take into account the dynamic evolution of returns and associated risk, regarding both investment assets and currency.

Caporin et al. (2013) consider a set of hedging decisions based on the use of future currency contracts and run different dynamic models, derived from the time-varying nature of financial returns distribution and using risk minimization approach of multivariate GARCH models.

These kind of approaches have been explored before by different papers by Kroner and Sultan (1993), Chakraborty and Barkoulas (1999), Brown et al. (2012), Chang et al. (2013).

These works show how dynamic hedging strategies outperform static strategies in terms of volatility reduction and hedging effectiveness in most cases.

In this work I start from the analysis by Caporin et al. (2013) to investigate currency hedging benefits.

I take a Euro-based investor's point of view who choose to diversify the hold portfolio of indices with small/medium percentage of US Dollar based indices.

I set three portfolios: Bond Portfolio, Equity Portfolio and Balanced Portfolio. In Bond portfolio, I consider an investment on Bond indices, both government and corporate bonds, related to EMU and US markets. Equity portfolio is constituted of two Indices: the first Index captures large cap representation across the 10 Developed Markets countries in the EMU, the second one measures the performance of the large and mid-cap segments of the US market. Balanced Portfolio represent both previously described portfolio of Indices.

I use a monthly allocation strategy, setting the chosen investment percentage in each Indices of the portfolio the first day of the month. At the same point, I fix the level of hedging indicated by the model considered and implemented with the use of a long or short position on a Future Index related to Euro-Dollar exchange rate. Every month the investor redefines the level of each Index in the portfolio, coming back to the originally set allocation, and decide the level of hedging.

In my analysis, hedging level is defined by a minimum variance problem related to the variance-covariance matrix of the unhedged portfolio returns and currency future returns.

Four different dynamic models are considered in the estimation of the variance-covariance matrix: the first based on a rolling mean, the second on the Exponential weighted moving average, the third on Glosten-Jagannathan-Runkle GARCH (GJR) model and the fourth on another GARCH model which exogenous variables.

Out-of-sample analysis run on the entire period considered (2002-2015) is completed with a benchmark given by the OLS static hedging model that comes from the already described minimum variance problem.

I analyze the impact of different hedging strategies comparing returns, volatility and other performance measure of the five portfolios described and showing the differences with the Non hedged portfolio statistics.

The implementation of hedging strategies based on currency future investment, in particular with the definition of Optimal Hedging Ratios through dynamic models such as GJR or EWMA, gives some evidence of improvement of Portfolios' performance in terms of risk-return profile.

Even if the entire period analysis comes with these results, Sharpe ratios robust tests give no evidence of statistical significance of the improvements registered by hedging strategies with respect to the unhedged portfolio.

The thesis is organized as follows. Section 2 present the method used to define the optimal level of hedging with currency future contracts and the consequent different hedging strategies implemented. In Section 3 I present the data and in particular I describe the main characteristics of the Indices chosen to construct the three portfolios, currency and Future Index used for this empirical study. Section 4 focuses on the basic characteristics of the portfolios and in Section 5 I comment the results obtained, showing tests on significance of the differences highlighted and other consideration. Section 6 draws some conclusions.

2 Hedging strategies

In order to investigate hedging strategies, I consider the use of future contracts instead of the issue of direct investment in currencies. This is a relatively inexpensive and reliable strategy for hedging foreign exchange risk. Investor commonly short a certain amount of futures contracts as they take a long position in international portfolios.

The amount of futures contracts that should be held for each unit of the underlying portfolio can be determined by minimizing the variance of the hedged portfolio returns. This is defined as the optimal hedging ratio (OHR) and it is obtained using the time series of portfolio and currency futures returns from which conditional and unconditional covariance matrices can be estimated with different approaches. Considering static or unconditional hedging, hedge ratios are estimated on a historical basis without taking into account the dynamic evolution of the returns both the ones of the portfolio assets and the currency or of their risk.

Dynamic model, instead provide different level of OHR that are derived within a risk-minimization framework, thus making use of econometric models belonging to the multivariate GARCH class or in a simpler way by using rolling mean or exponential weighted moving average of the portfolio returns.

2.1 Futures contracts

Foreign exchange derivatives are financial contracts like forwards, futures, swaps, options that can be used to hedge currency risk by “locking in” exchange rates. In particular, both forwards and futures require delivery of a specific quantity of currency on a specific date at a specified exchange rate.

I choose to cover currency risk in the portfolio with future contracts instead of forward contracts because, first, they are much more liquid. Then with a forward contract it is necessary to find a counterparty and it makes them more difficult to sell and offset. Furthermore, forward contracts expose investors to counterparty risk.

I have to solve a minimization problem in order to find the level of hedging of the considered international portfolio. According to different input and models used, the solution of the problem comes with different optimal hedging ratios that determine future contracts percentage in the portfolios.

With the purpose of using future contracts to solve the optimal problem described before and to evaluate the impact of hedging strategies on portfolio returns I use a future total return time series.

Futures Continuous Series used in this study is a Thomson Reuters calculated time series available on the Datastream product.

AEX-EURO/DOLLAR CONTINUOUS - SETT. PRICE is a perpetual series of futures prices derived from individual futures contracts. The series starts at the nearest available contract month, which forms the first values for the continuous series until the first business day of the notional contract month. At this point, a volume weighting calculation between the near and second nearest contract months is applied to the prices until the near contract reaches its expiry date. No adjustment for price differentials is made.

Unlike individual futures contracts, continuous series do not expire until the actual future contract ceases to exist.

2.2 Hedging models

In the following paragraphs, I describe the methodology I implement with the aim of hedging various international portfolios I set and which I describe in details in section 4.

2.2.1 Unhedged portfolio

The analysis on currency hedging necessarily starts from the definition of the generic portfolio to hedge.

I define the generic nominal unhedged portfolio return at time t , r_t^{UH} , as:

$$r_t^{UH} = w^{EMU} (w^{EMU,GB} r_t^{EMU,GB} + w^{EMU,CB} r_t^{EMU,CB} + w^{EMU,E} r_t^{EMU,E}) + w^{US} (w^{US,GB} r_t^{US,GB} + w^{US,CB} r_t^{US,CB} + w^{US,E} r_t^{US,E}) \quad (1)$$

where $r_t^{EMU,GB}$ and $r_t^{EMU,CB}$ are the returns in Euro for a European investor holding respectively EMU government and corporate bonds; $r_t^{US,GB}$ and $r_t^{US,CB}$ are the returns in Euro for an European investor investing in US government and corporate bonds. $r_t^{EMU,E}$ and $r_t^{US,E}$ are the

returns in Euro of the investment in EMU and US equity; $w^{country,asset}$ is the weight of each instrument of the portfolio at EMU or US level. Note that $w^{country,GB} + w^{country,CB} + w^{country,E} = 1$ and $w^{EMU} + w^{US} = 1$.

2.2.2 Full Hedged portfolio

In order to reach a theoretical Full hedging strategy it is possible to consider the returns of the international portfolio without converting them to the investor's country currency.

Exchange rate risk is completely avoided and the result is comparable to a hedging strategy that provides coverage for an amount equal to the volume of foreign currency in the Portfolio in the continuous time.

Such a strategy would imply a constant calculation of the percentage of foreign currency in the portfolio and a continuous adjustment of the level of futures contracts in order to hedge the exposure.

This particular situation is considered to evaluate the extreme case of full hedging with respect to other hedging strategies implemented as well as the absence of a hedging strategy.

2.2.3 Optimal Hedging Ratio

The currency exposure to US dollar exchange rate changes of the unhedged portfolio defined in (1) could be hedged by selling an appropriate number of future currency contracts denominated in USD.

As already discussed, there is no consensus on the hedging level the investor should maintain. One possibility could be to hedge completely the exposure, taking a future position for an amount equal to the total USD exposure. A large literature shows that this approach could not be the optimal one. A reduction of the percentage of the portfolio to be covered could be reached exploiting the correlation between currency returns and the portfolio composition.

Investor needs to minimize the variance of a hedged portfolio return given by:

$$r_t^H = r_t^{UH} - \beta r_t^{Fut} \quad (2)$$

where r_t^{UH} is the portfolio return in equation (1), and r_t^{Fut} is the changes in USD futures price. The optimal number of futures contracts in USD that the investor should sell for each

Euro invested in the international portfolio is β , the optimal hedging ratios (OHR). A positive value of β implies that the USD future tends to appreciate against the Euro when the unhedged returns, denominated in Euros, increase. In this case the investor could reduce the volatility of the unhedged portfolio by holding a short position on future contracts. A negative value, on the opposite, suggests that future currency would tend to depreciate as the unhedged returns increased and the investor should hold a long position on futures contracts. To identify the optimal values of β , the standard practice considers hedged portfolio's variance.

Considering the vector of unhedged portfolio and future index monthly returns, the vector β is the vector that minimizes the variance of the hedged portfolio returns. The minimum problem to solve is:

$$\min_{\beta} = \text{Var}[r_t^H] \quad (3)$$

The hedged portfolio variance is defined as a function of the variance–covariance matrix of the unhedged portfolio returns and currency future returns. The variance-covariance matrix is presented below.

$$\text{Var} \begin{bmatrix} r_t^{UH} \\ r_t^{Fut} \end{bmatrix} = \begin{bmatrix} \sigma_{UH}^2 & \sigma_{UH,Fut} \\ \sigma_{UH,Fut} & \sigma_{Fut}^2 \end{bmatrix} \quad (4)$$

where σ_{UH}^2 and σ_{Fut}^2 are respectively the variance of the unhedged portfolio returns and of the currency future returns; $\sigma_{UH,Fut}$ represents the covariance between the currency futures and unhedged portfolio returns.

Given the matrix showed above, the hedged portfolio variance is:

$$\text{Var} [r_t^H] = [1 - \beta] \begin{bmatrix} \sigma_{UH}^2 & \sigma_{UH,Fut} \\ \sigma_{UH,Fut} & \sigma_{Fut}^2 \end{bmatrix} \begin{bmatrix} 1 \\ -\beta \end{bmatrix} = \sigma_{UH}^2 + \beta^2 \sigma_{Fut}^2 - 2\beta \sigma_{UH,Fut} \quad (5)$$

The solution of the minimum problem comes by equating the first-order conditions to zero

$$\min_{\beta} (\sigma_{UH}^2 + \beta^2 \sigma_{Fut}^2 - 2\beta \sigma_{UH,Fut}) \quad (6)$$

$$\frac{\partial}{\partial \beta} (\sigma_{UH}^2 + \beta^2 \sigma_{Fut}^2 - 2\beta \sigma_{UH,Fut}) = 2\beta \sigma_{Fut}^2 - 2\sigma_{UH,Fut} = 0 \quad (7)$$

$$\hat{\beta} = 2\beta \sigma_{Fut}^2 - 2\sigma_{UH,Fut} = 0 \quad (8)$$

$$\hat{\beta} = \frac{\sigma_{UH,Fut}}{\sigma_{Fut}^2} \quad (9)$$

The following models are based on this approach and they elaborate the variance-covariance matrix in order to introduce dynamic in the model. They will define different vector of betas

that will delineate the strategy the investor should use, regarding to the level of currency future contracts to hold in the international portfolio considered.

2.2.4 Static Beta

The first is the simplest approach, where I used ordinary least square to estimate a static optimal hedging ratio, β_1 of the following equation.

$$r_t^{UH} = \beta_0 + \beta_1 r_t^{Fut} + u_t \quad (10)$$

The relationship between a continuous response variable, in the case at hand r_t^{UH} , and a continuous explanatory variable, r_t^{Fut} , may be represented using a line of best-fit, where r_t^{UH} is predicted, at least to some extent, by r_t^{Fut} . If this relationship is linear, it may be appropriately represented mathematically using the straight line equation $r_t^{UH} = \beta_0 + \beta_1 r_t^{Fut}$.

The relationship between the two variables is described using the equation of the line of best fit with β_0 indicating the value of intercept and β_1 indicating the slope of the line, the regression coefficient. The regression coefficient β_1 describes the change in r_t^{UH} that is associated with a unit change in r_t^{Fut} .

u_t is a vector of errors with mean zero that makes equation (10) true.

A positive β_1 is interpreted as the number of future contracts the European investor should sell since they tend to appreciate against the Euro when the unhedged returns increase.

A negative β_1 is interpreted as the number of future contract the European investor should buy since they tend to depreciate against the Euro when the unhedged returns increase.

2.2.5 Rolling mean Beta

A simple way to introduce “dynamic component” in the model could be exploit one of the most diffused method: the rolling method.

This method, also called moving average, is based on a full data set and a fixed subset size, called window of estimation; the first element of the moving average is obtained by taking the average of the initial fixed subset of the data series. Then the subset is modified by shifting forward, namely excluding the first number of the series and including the next number following the original subset in the series.

The only relevant element of the methodology is the size of valuation window adopted. It generally depends on the sample size and on the number of the assets.

For this work, I use this approach in order to estimate OHR β through ordinary least square.

The logic is the same described in the previous paragraph “Static beta”, but in this case, I apply the regression to each different subset of returns time series found thanks to the rolling method. It results in a vector of betas, which I name “rolling mean beta” (referring to the approach used to calculate it).

In the practice, it is often used 60-month rolling window of estimation, but sometimes it is more convenient to employ a shorter one. The aim of the OLS β , applied to the rolling subset of returns (unhedged portfolio and currency futures returns), is the definition of the amount of futures contracts to sell (or buy) for each Euro invested in the international portfolio. β is defined through the relation between unhedged portfolio and currency futures returns. It should be useful to consider a period for the rolling window short enough to do not lose relevant information that a window too wide could not carry on. For this reason, I use a window of 36 months that seems appropriate also for the time series considered of about 15 years.

The vector of betas is a series of optimal hedging ratios (see paragraph 2.2.2) that defines the allocation for each month of the period that I analyze. To implement the rolling method first 36 months of the time series returns of indices and futures from which derives the composition of the international portfolio are used to set the first subset, so the analysis start 36 months after the first available monthly return of the portfolio, theoretically constructible. Further details about the asset allocation will be given in chapter 3.

2.2.6 Exponential weighted moving average Beta

Next models introduce dynamic also in the approach adopted to identify variance-covariance matrix related to unhedged portfolio and currency futures returns series. This is at the base of the minimum problem that determine the amount of future contracts to buy or sell. The hedge ratios will be dynamic and will be based on the estimated conditional variances and covariances/correlations patterns.

To give more weights to recent observations I adopt a weighted moving average of returns, where the weights are decreasing over time, higher for recent observations and small for

observations far in the past. The EWMA is characterized by two elements: the size of the initialization window and the factor lambda (λ , i.e., the smoothing parameter). Also in this case I choose a period of 36 months as estimation window.

About the smoothing parameter, JP Morgan suggests a lambda equal to 0.97, which is also quite common in financial market practice. In

I start from the vector or returns $r_t^H = r_t^{UH}, r_t^{Fut}$ with a conditional mean, zero, and a conditional covariance matrix, H_t :

$$r_t = \varepsilon_t = \Sigma_t^{1/2} \eta_t \quad (11)$$

where η_t is i.i.d. with $E(\eta_t) = 0$ and $Var(\eta_t) = I_n$, and I consider the class of conditional covariance matrices that are the weighted sum of the cross products of past returns and the elements of the variance-covariance matrix:

$$\Sigma_t = (1 - \lambda)r_{t-j}r'_{t-j} + \lambda\Sigma_{t-1} \quad (12)$$

With this approach I obtain time-varying conditional variances, covariances, and indirectly, time-varying conditional correlations. The EWMA one-month volatility estimate changes every month as it incorporates new information, discarding older observations. Recent data is weighted more heavily and the previous ones decay exponentially. The choice of λ , is made also because factors lower than 0,95 tend to weight recent data more heavily.

The vector of betas comes from the minimization of variance for each 36-month's window and it will give the position of future contracts the investor should held, according to what I explained in paragraph 2.2.3.

EWMA estimations adapt to the shocks quickly than rolling estimations. However, they can produce large estimation errors if this shock is isolated and it does not represent a change in the trend.

2.2.7 GJR Beta

This model and the following are designed to capture asymmetry, i.e. negative shocks impact on conditional variances is higher than that of positive shocks.

Since the development of the Autoregressive Conditional Heteroskedasticity (ARCH) model by Engle (1982) and the extension to generalized ARCH (GARCH) model by Bollerslev (1986) many models of this family have been developed in order to improve the models and to overcome some shortcomings.

The Glosten, Jagannathan, and Runkle¹ (GJR) model is a generalization of the GARCH model that is appropriate for modelling asymmetric volatility clustering. The model posits that the current conditional variance is the sum of these linear processes, with coefficients:

- Past conditional variances (the GARCH component or polynomial).
- Past squared innovations (the ARCH component or polynomial).
- Past squared, negative innovations (the leverage component or polynomial).

More specifically, considering the vector or returns as described above with a conditional mean, zero,

$$r_t = \varepsilon_t \quad (13)$$

where ε_t is a zero-mean white noise. Despite of being serially uncorrelated, the series ε_t does not need to be serially independent. For instance, it can present conditional heteroskedasticity. The Glosten-Jagannathan-Runkle GARCH (GJR-GARCH) model assumes a specific parametric form for this conditional heteroskedasticity. More specifically, $\varepsilon_t \sim \text{GJR-GARCH}$ if it is possible to write $\varepsilon_t = \sigma_t z_t$, where z_t is standard Gaussian and:

$$\sigma_t^2 = \omega + (\alpha + \gamma I_{t-1})\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \quad (14)$$

where

$$I_{t-1} = \begin{cases} 0, & \text{if } r_{t-1} \geq 0 \\ 1, & \text{if } r_{t-1} < 0 \end{cases}$$

I start from this definition of the variance to solve the minimum problem and find the vector of betas of the optimal hedging ratio, that every month define the level of currency future contracts of the international portfolio.

¹ See Lawrence R. Glosten, Ravi Jagannathan and David E. Runkle, 1993. On the Relation between the Expected Value and the Volatility of the Nominal Excess Return on Stocks. *The Journal of Finance*. Vol. 48, No. 5, 1993, pp. 1779–1801.

2.2.8 GARCH model with exogenous variables Beta

Another generalization of the GARCH model is represented by the inclusion of exogenous variables. The idea behind this procedure for financial applications is that additional sources of information help to better understand the market's behaviour and hence to improve the prediction of the market's reactions. Thus, the investor is able to apply early solution given the possibility of future risky situations. This is the case for example in Ashok et al. (2011) who improve the GARCH model by introducing stock's volume as a proxy for information flow and company specific announcements in the volatility equation. Sharma et al. (1996) extended a GARCH (1,1) model through volume of traded stock and Engle and Patton (2001) introduce interest rate levels in many GARCH models.

The general model implemented, which includes GARCH (1,1) model, defines:

$$\sigma_t^2 = \omega + c(\varepsilon_{t-1})\sigma_{t-1}^2 + u(x_{t-1}) + v(y_{t-1}) + \dots \quad (15)$$

where x_t , y_t represents the exogenous process used for the improvement of the modeling behaviour and c , u , v are real-valued nonnegative continuous functions.

Including exogenous variable in the GARCH model allows considering a new variance-covariance matrix for the definition of the vector of optimal hedging ratios, betas.

The exogenous variables that I include in the model have the aim of introduce factors that could influence returns and volatility of the international portfolio's assets.

In particular, I consider:

- Crude oil price, commodity that represents a crucial element in the global economy.
- VIXCLS, a Chicago Board Options Exchange indicator that measures market expectation of near term volatility conveyed by stock index option prices.

The VIX is a measure of the uncertainty and risk that investors see over the near future (specifically, the next 30 days). Constructed from options on S&P500 index futures, the VIX is technically a gauge of what is called implied volatility.

- STLFSI, St. Louis Fed Financial Stress Index measures the degree of financial stress in the markets and is constructed from 18 weekly data series: seven interest

rate series, six yield spreads and five other indicators. Each of these variables captures some aspect of financial stress. Accordingly, as the level of financial stress in the economy changes, the data series are likely to move together.

- NFCI, Chicago Fed's National Financial Conditions Index provides a comprehensive weekly update on US financial conditions in money markets, debt and equity markets and the traditional and "shadow" banking systems.
- European EPU, Economic Policy Uncertainty Index for Europe, an index based on newspaper articles regarding policy uncertainty. It is constructed drawing on two newspapers per country: Le Monde and Le Figaro for France, Handelsblatt and Frankfurter Allgemeine Zeitung for Germany, Corriere Della Sera and La Repubblica for Italy, El Mundo and El Pais for Spain, and The Times of London and Financial Times for the United Kingdom. It is considered the number of newspaper articles containing the terms uncertain or uncertainty, economic or economy, and one or more policy-relevant terms.
- US EPU, Economic Policy Uncertainty Index for United States constructed from three types of underlying components. One component quantifies newspaper coverage of policy-related economic uncertainty. A second component reflects the number of federal tax code provisions set to expire in future years. Temporary tax measures are a source of uncertainty for businesses and households because Congress often extends them at the last minute, undermining stability in and certainty about the tax code. The third component uses disagreement among economic forecasters as a proxy for uncertainty, drawing on the Federal Reserve Bank of Philadelphia's Survey of Professional Forecasters. This quarterly survey covers a wide range of macroeconomic variables and for the definition of the index are utilized the individual-level data for three of the forecast variables, the consumer price index (CPI), purchase of goods and services by state and local governments, and purchases of goods and services by the federal government.

3 Investment Features

In this section, I present the main features of the investment universe, which includes four Bond Indices and two Equity Indices, covering the following geographical area: European Monetary Union (EMU) and United States (US).

These indices are the base for the construction of three different portfolios (Bond Portfolio, Equity Portfolio and Balanced Portfolio) used to proxy the investment choice of a European investor. This international investment strategy is exposed to currency risk and it represents the base to study the potential benefit of hedging strategy. In order to provide a simplified framework, I consider investments in two currencies: Euro (EUR) and US Dollar (USD).

Indices quotations, spot exchange rate and futures continuous series were downloaded from Dastastream with daily frequency, from January 1999 until November 2015, covering more than 16 years.

In portfolio construction process, I take a European investor perspective. Thus, I convert quotations to monthly frequency and in Euro. The conversion in domestic currency is performed to test the benefit of hedging strategy. In fact, through the conversion in domestic currency, currency risk and performance are internalized by monthly returns.

In the following paragraphs, I present the characteristics of these indices, of the spot and future exchange rate between EUR and USD. In particular I analyze the characteristics of the returns distribution (mean, volatility, skewness, kurtosis, etc.), both in local currency (for USD based indices) and in Euro.

3.1 Bond Indices

The Bond Indices are representative of both Government and Corporate Bonds of the EMU and US area. All the indices are in local currency and I convert them in Euro. I select the following widely used indices²:

- JPM UNITED STATES GOVT. BOND (US\$): index representative of US Government Bonds of all maturities;
- JPM EMU GOVERNMENT ALL MATS. (E): index representative of EMU countries Government Bonds of all maturities;
- BOFA ML US CORP MSTR (\$): index representative of US Corporate Bonds of all maturities;
- BOFA ML EUR CORP (E): index representative of Euro Area Corporate Bonds of all maturities.

3.1.2 Monthly and cumulated returns time series

For each bond index belonging to the investment universe, I computed some basics characteristics of monthly returns on the full sample period, both in Local currency and in Euro.

Figure 1 and Figure 2 provide the graphs of monthly returns and cumulated returns time series, respectively. Table 1 provides average, median, volatility, minimum and maximum, skewness and kurtosis of monthly returns in Local currency and in Euro.

² About the Corporate Bond Indices, I select two indices provided by Bank of America – Merryll Lynch (BofA-ML) because they were the only indices available in Datastream comparable in terms of Data Provider (and thus methodology about the composition) and time series length.

Figure 1 – Bond Indices, monthly returns

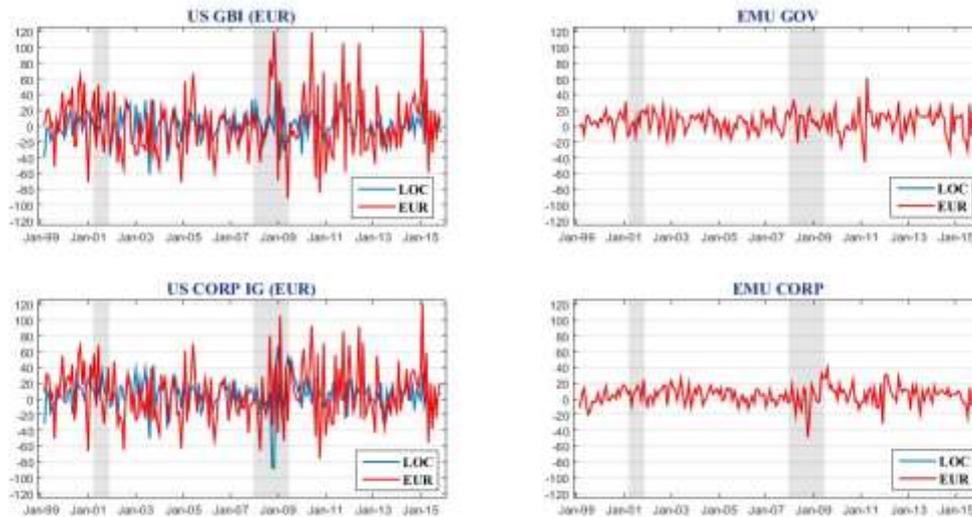


Figure 2 – Bond Indices, cumulated returns

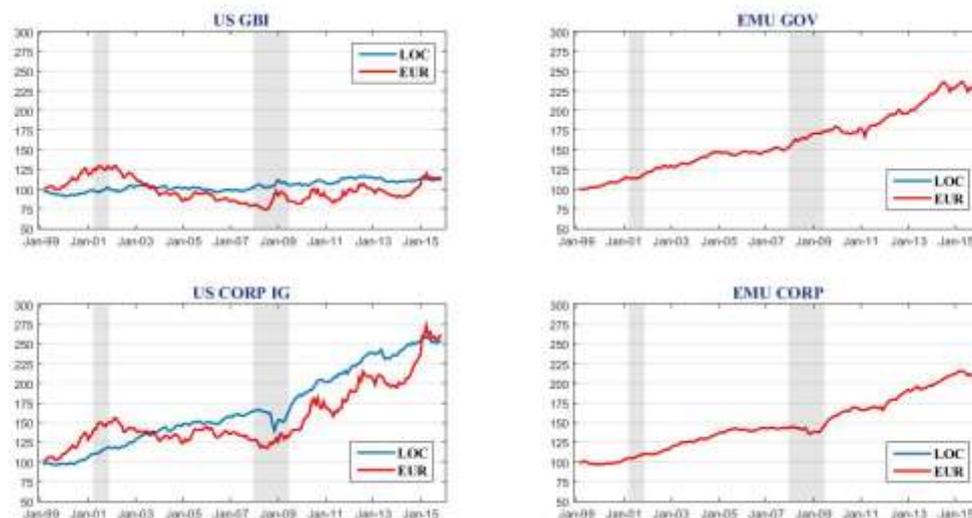


Table 1 – Bond Indices’ statistics

Index	Currency	Mean	Median	Std Dev	Min	Max	Skeweness	Kurtosis
US GBI	USD	0.78	1.39	4.77	-61.39	64.23	-0.16	4.49
US GBI	EUR	1.41	-0.70	10.87	-92.15	121.88	0.54	3.76
EMU GOV	EUR	5.12	7.31	4.03	-46.04	61.23	-0.18	4.28
US CORP IG	USD	5.72	6.42	5.45	-88.58	67.14	-1.04	8.55
US CORP IG	EUR	6.24	3.70	10.04	-76.25	122.02	0.39	3.27
EMU CORP	EUR	4.54	5.49	3.40	-49.13	38.92	-0.52	4.89

Monthly returns of US Government and Corporate indices show higher volatility than EMU indices. This feature can be easily observed from Figure 1. In particular, if we observe monthly returns in Euro the volatility is more than twice with respect to EMU indices (10.87 vs 4.03 per cent for Government indices and 10.04 vs 3.40 per cent for Corporate indices). The conversion in Euro increased the volatility of US indices' monthly returns between 5.00 and 6.00 percentage point.

Average monthly return is higher for EMU Government index and US Corporate index than for their peers (5.12 vs 1.41 per cent and 6.24 vs 4.54 per cent, respectively). Currency has a positive contribution to average returns. The conversion from USD to EUR increase average monthly returns of more than 50 bps (6.00 percentage point on annual basis).

All the indices' monthly returns present skewness different from zero, implying asymmetry. US indices exhibit negative skewness when considered in local currency and positive skewness when converted in Euro. The conversion in Euro also sensibly reduces the kurtosis, bringing it close to normal distribution (3.76 for Government and 3.27 for Corporate). After the conversion, US indices exhibit a kurtosis lower than EMU indices. However, the distribution of all the indices' monthly returns cannot be considered normal.

3.2 Equity Indices

The Equity Indices are representative of EMU and US stock markets, both mid and large cap firms. All the indices are in local currency and I convert them in Euro. I select the following widely used indices:

- MSCI USA (US\$): index representative of the most important mid and large cap quoted in the US stock markets;
- MSCI EURO (E): index representative of the most important large cap from thirteen countries quoted in EMU stock markets.

I used geographical indices because they provide the representation of entire stock markets, covering different sectors (IT, Energy, Materials, Utilities, Consumer Staples and Discretionary, Industrials, Health Care, Communications), and they are characterized by a lower propensity to large movements typical of individual stocks. Exchange rate movements can have strong impacts on some sectors or individual stocks, reducing the robustness of the analysis.

3.2.2 Monthly and cumulated returns time series

For each equity index belonging to the investment universe, I computed some basic characteristics of monthly returns on the full sample period, both in Local currency and in Euro.

Figure 3 and Figure 4 provide the graphs of monthly returns and cumulated returns time series, respectively. Table 2 provides average, median, volatility, minimum and maximum, skewness and kurtosis of monthly returns in Local currency and in Euro.

Figure 3 – Equity Indices, monthly returns

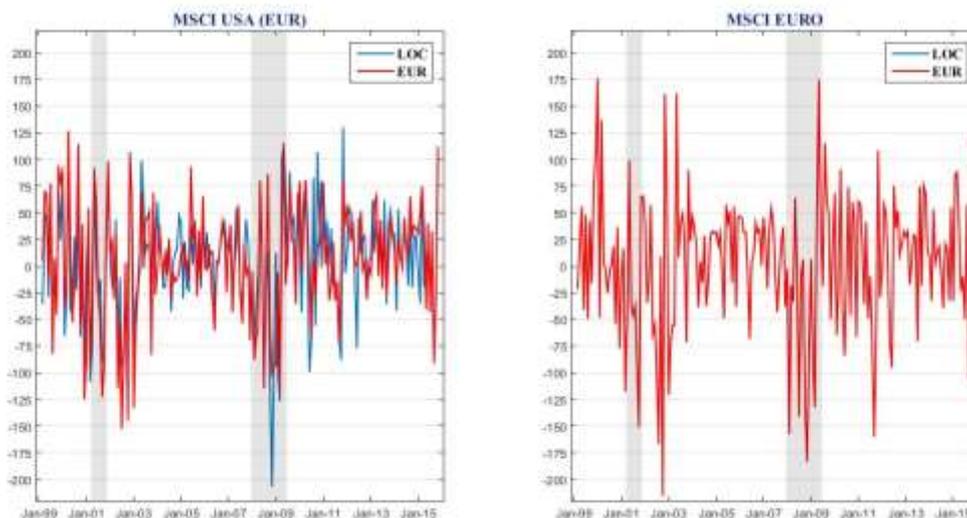


Figure 4 - Equity Indices, cumulated returns

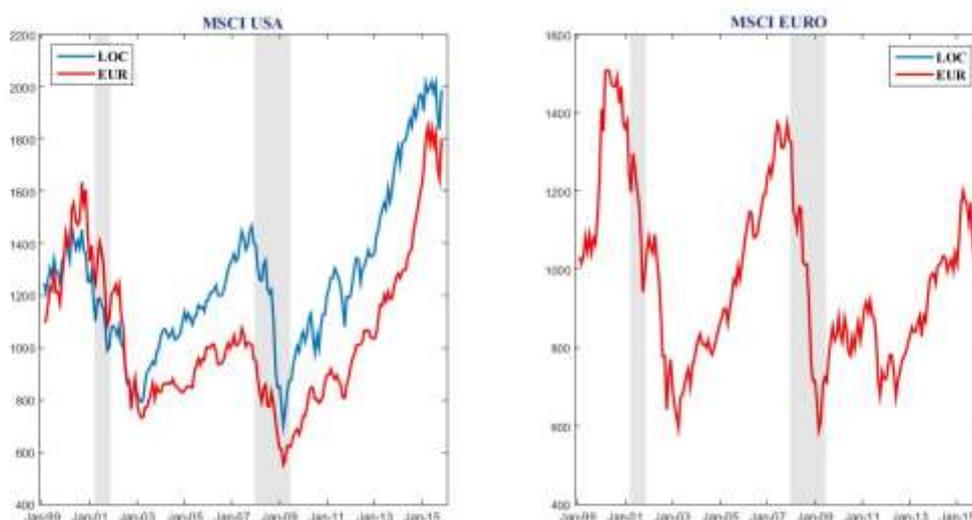


Table 2 - Equity Indices' statistics

Index	Currency	Mean	Median	Std Dev	Min	Max	Skeweness	Kurtosis
MSCI USA	USD	3.94	9.70	15.20	-206,97	129,99	-0.51	3.88
MSCI USA	EUR	4.17	5.69	15.61	-152,70	126,73	-0.45	3.18
MSCI EURO	EUR	2.24	8.05	18.69	-215,48	176,42	-0.38	3.87

Differently from monthly returns of bond indices, US Equity index exhibit lower volatility than EMU Equity index (15.61 vs 18.69 per cent). Even if the volatility is very close between the two of them, this feature can be observed from Figure 3. The conversion in Euro increased, also in this case, the volatility of US indices' monthly returns. However, this increase is lower (0.40 vs more than 5.00 percentage points) in equity indices than in bond indices, confirming the expectation the equity returns partially internalize the currency risk.

Average monthly return is higher for US Equity index than for EMU Equity index (4.17 vs 2.24 per cent). Currency has again a positive contribution to average returns, but the magnitude is much smaller than in bond indices (25 vs 50 bps), confirming again the expectation that equity returns internalize exchange rate movements.

All the indices' monthly returns present skewness different from zero, implying asymmetry. Both US and EMU indices exhibit negative skewness, independently from currency denomination. The conversion in Euro reduces the kurtosis, but again the magnitude is smaller than in bond indices. In general, currency conversion does not affect equity indices' monthly returns characteristics. Even in this case, the distribution of all the indices' monthly returns cannot be considered normal.

3.3 Currencies

In this paragraph, I analyze the characteristics of spot and future exchange rate between Euro and US Dollar. As I will show, spot and future exchange rate present similar basic characteristics and correlations with the other indices.

I will also present some tests about the validity of Futures index prices as a predictor of exchange rate and about Uncovered Interest Parity validity, in order to verify if the relation between domestic and foreign interest rates could be a good predictor of exchange rate movements in the absence of Futures index prices.

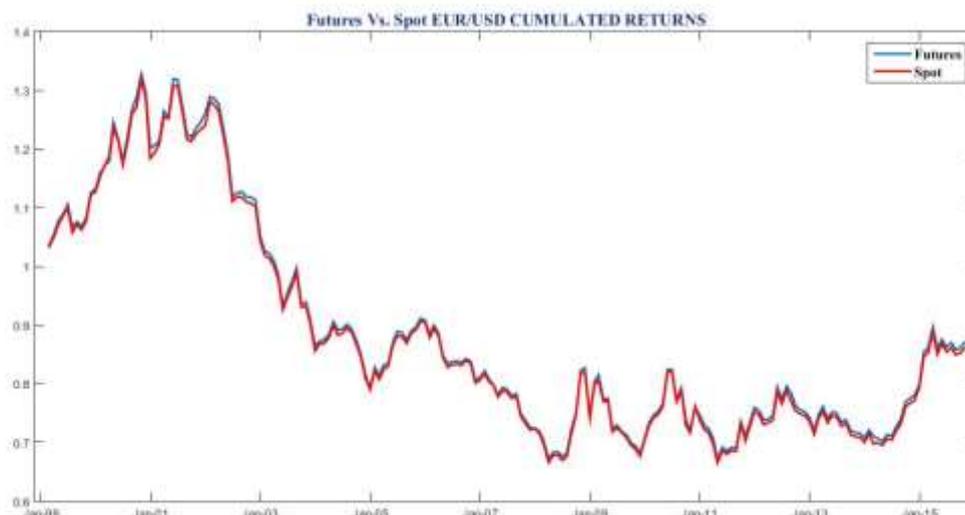
3.3.1 Spot Exchange Rate (EUR/USD) and Futures Index

Figure 5 and 6 show the time series of spot exchange rate and Future index prices and the respective cumulated returns.

Figure 5 – Spot EUR/USD exchange rate and Futures index prices



Figure 6 – Spot EUR/USD exchange rate and Futures index, cumulated returns



Both price and cumulated returns time series are very similar between spot exchange rate and Futures index price. From price time series, we can observe that the starting and the final point are almost the same (1.12 USD vs 1.10 USD per Euro). However, the prolonged period of Euro appreciation from 2002 till 2009 lead to negative cumulated returns (almost 12.00 percentage points in 17 years).

Table 3 and 4 show basic characteristics of spot exchange rate and Futures index monthly returns and the correlations with the bond and equity indices' monthly returns.

Table 3 - Spot exchange rate EUR/USD and Futures Index, monthly returns

	Mean	Median	Std Dev	Min	Max	Skeweness	Kurtosis
Spot	-0.36	1.42	10.31	-114.57	116.70	0.04	3.82
Futures	-0.31	0.49	10.22	-120.19	117.90	0.06	3.96

Monthly returns distribution are very similar between spot exchange rate and Future index. In particular, both exhibit a negative average monthly return (close to -0.3 per cent) and a volatility about 10 per cent. The skewness is close to 0.0, suggesting that the distribution is almost symmetric. Kurtosis is close to 4.00 for both distributions.

Table 4 – Correlation Currency/Indices and Futures/Indices

	US GBI	EMU GOV	US CORP IG	EMU CORP	MSCI USA	MSCI EURO
Currency correlation	-0.13681	-0.06037	-0.33429	-0.16681	-0.29118	-0.10645
Futures correlation	-0.14299	-0.0547	-0.33024	-0.15703	-0.28504	-0.10518

About the correlations between Euro based indices' monthly returns and the exchange rate are always negative, as expected, even if they are never below -0.30. In general, currency contribution to returns is partially offset by an opposite movement in asset prices. Positive performance of US financial assets lead to capital inflows in US financial market with a consequent USD appreciation. For a European investor that holds US financial asset denominated in Euro, this adjustment determine a negative contribution of the currency to returns on financial assets.

4 Portfolio Analysis

With the aim of studying optimal hedging ratio calculated and applied to different Indices' Portfolios, I define the composition of three portfolios: Bond Portfolio, Equity Portfolio and Balanced Portfolio.

I use a monthly allocation strategy, setting the chosen investment percentage in each Indices of the portfolio the first day of the month. Every month the investor redefines the level of each Index in the portfolio, coming back to the originally set allocation.

In Bond portfolio, I consider an investment on Bond indices that follows the structure listed in the table below.

Table 5 – Structure of the Bond Portfolio

Index	Description	Percentage of the Investment
JPM UNITED STATES GOVT. BOND (US\$)	index representative of US Government Bonds of all maturities	15%
JPM EMU GOVERNMENT ALL MATS. (E)	index representative of EMU countries Government Bonds of all maturities	70%
BOFA ML US CORP MSTR (\$)	index representative of US Corporate Bonds of all maturities	5%
BOFA ML EUR CORP (E)	index representative of Euro Area Corporate Bonds of all maturities	10%

The exposure to US Government and Corporate Bonds, and consequently to US-dollar, is limited at 20% of the overall Portfolio.

Equity portfolio is constituted of two Indices: the first Index captures large cap representation across the 10 Developed Markets countries in the EMU, the second one measures the performance of the large and mid-cap segments of the US market.

I choose the following allocation of the Equity Portfolio.

Table 6 – Structure of Equity Portfolio

Index	Description	Percentage of the Investment
MSCI USA (US\$)	index representative of the most important mid and large cap quoted in the US stock markets	20%
MSCI EURO (E)	index representative of the most important large cap from thirteen countries quoted in EMU stock markets	80%

The exposure to US Index, and consequently to US-dollar, is limited at 20% of the overall Portfolio.

Balanced Portfolio represent both previously described portfolio of Indices. The composition of it follow the structure below.

Table 7 – Structure of Balanced Portfolio

Index	Description	Percentage of the Investment
JPM UNITED STATES GOVT. BOND (US\$)	index representative of US Government Bonds of all maturities	10%
JPM EMU GOVERNMENT ALL MATS. (E)	index representative of EMU countries Government Bonds of all maturities	40%
BOFA ML US CORP MSTR (\$)	index representative of US Corporate Bonds of all maturities	5%
BOFA ML EUR CORP (E)	index representative of Euro Area Corporate Bonds of all maturities	15%
MSCI USA (US\$)	index representative of the most important mid and large cap quoted in the US stock markets	5%
MSCI EURO (E)	index representative of the most important large cap from thirteen countries quoted in EMU stock markets	25%

The overall exposure to US-dollar, because of the investments in US Indices, is again limited at 20%. In terms of nature of the Portfolio, it is composed by 70% of Bond Indices and 30% Equity Indices.

Clearly, during each month the percentage of US dollar denominated investment could become larger or smaller due to the effect of the underlying Indices returns (see par. 4.5).

In the following section, I analyze the main statistics of the three Portfolio considering the different cases of study:

- **Non Hedged Portfolio**, with no hedging techniques implemented;
- **Full Hedged Portfolio**, the theoretical completely covered Portfolio that comes from the returns of the Indices allocation without the conversion of US dollar to Euro;
- **Hedged Portfolio**, with different hedging strategies implemented, in particular I call:
 - **Static Beta**, the Hedged Portfolio with the fix OHR obtained as explained in par. 2.2.3;
 - **Rolling Beta**, the Hedged Portfolio with an OHR given by the series of rolling mean beta described in par. 2.2.4;
 - **EWMA Beta**, the Hedged Portfolio with an OHR given by the vector of EWMA betas calculated as described in par. 2.2.5;
 - **GJR Beta**, the Hedged Portfolio with an OHR that is calculated according to the GJR model defined in par. 2.2.6;
 - **GARCH with exogenous variables Beta (or GARCH w/ex Beta)**, the Hedged Portfolio that implement GARCH model with the inclusion of exogenous variables, described in par. 2.2.7, to determine OHR.

4.1 Statistics of returns

Table 8 provides average, median, volatility, minimum and maximum, skewness and kurtosis of monthly returns.

Table 8 - Portfolios' statistics of returns (annualized)

Strategy	Mean	Median	Std Dev	Min	Max	Skewness	Kurtosis	
Bond Portfolio	Non Hedged Portfolio	4,02	5,07	3,67	-33,25	38,89	-0,07	3,41
	Full Hedged Portfolio	4.29	5.38	3.25	-31.48	41.86	-0.26	3.67
	Hedged Portfolio (Static Beta)	4.40	6.19	3.23	-31.69	39.99	-0.29	3.65
	Hedged Portfolio (Rolling Beta)	4.44	5.77	3.27	-31.62	40.48	-0.25	3.52
	Hedged Portfolio (EWMA Beta)	4.31	5.94	3.24	-31.45	40.82	-0.22	3.58
	Hedged Portfolio (GJR Beta)	4.54	6.04	3.27	-31.09	43.05	-0.19	3.72
	Hedged Portfolio (GARCH w/ex Beta)	4.33	6.12	3.24	-31.84	39.73	-0.28	3.64
Equity Portfolio	Non Hedged Portfolio	2.48	10.31	17.06	-201.43	162.89	-0.58	4.12
	Full Hedged Portfolio	2.86	11.09	17.40	-199.80	162.38	-0.60	4.10
	Hedged Portfolio (Static Beta)	2.39	10.17	17.03	-201.82	163.02	-0.58	4.12
	Hedged Portfolio (Rolling Beta)	4.94	5.72	16.61	-194.39	164.23	-0.57	4.98
	Hedged Portfolio (EWMA Beta)	5.31	9.07	16.69	-205.40	171.59	-0.62	5.16
	Hedged Portfolio (GJR Beta)	2.76	11.58	17.16	-199.68	162.59	-0.59	4.16
	Hedged Portfolio (GARCH w/ex Beta)	2.59	10.66	17.10	-201.07	162.79	-0.59	4.13
Balanced Portfolio	Non Hedged Portfolio	3.56	5.25	5.34	-56.05	54.07	-0.39	3.80
	Full Hedged Portfolio	3.86	6.15	5.49	-55.92	53.59	-0.47	3.73
	Hedged Portfolio (Static Beta)	3.79	5.92	5.30	-55.10	53.76	-0.44	3.75
	Hedged Portfolio (Rolling Beta)	4.57	5.96	5.21	-52.98	57.37	-0.34	4.44
	Hedged Portfolio (EWMA Beta)	4.60	6.20	5.20	-53.02	57.40	-0.38	4.46
	Hedged Portfolio (GJR Beta)	3.97	5.99	5.34	-53.34	54.14	-0.42	3.85
	Hedged Portfolio (GARCH w/ex Beta)	3.95	5.68	5.31	-54.65	53.70	-0.45	3.81

Out of sample analysis is ran on the entire period (2002 -2015) that considers an estimation window necessary to implement dynamic models. According to Diebold and Rudebusch (1991), this kind of analysis is more effective in reflecting information available to the forecaster in each period.

Considering the overall set of Portfolios analyzed the one that reaches the best combination of average return and volatility is Hedged Bond Portfolio (GJR Beta) with an average annualized monthly return of 4.54 per cent and a standard deviation of 3.27.

Hedged Equity Portfolio (Rolling Beta) and (EWMA Beta) present higher monthly returns, respectively of 0.40 and 0.77 percentage point in a year, but the small improvement in returns is accompanied by more than 13.00 percentage points of standard deviation.

It is possible to notice how in Bond Portfolio the implementation of any hedging strategy partly reduce volatility (in the order of about 0.40 percentage points for all the Hedged Portfolio) and this is accompanied by an increase in returns, at least in the order of 0.29 per cent (Hedged Bond Portfolio EWMA Beta). Differently for the results according to Schmittmann (2010) analysis, in the case at hand Full Hedging is not the dominant strategy for Bond Portfolio considering the risk-return profile.

Half of the observation of the Hedged Bond Portfolio - that is to say the monthly return - are higher than 5.77 per cent. The median of the Full Hedged Portfolio is instead 5.38, only 0.21 percentage point higher than Non Hedged Portfolio.

The distribution of each Bond Portfolio analyzed shows Kurtosis higher than 3, but in general close to the Normal distribution, even if they are characterized by a certain asymmetry with longer tail in the left side as the negative Skewness indicates.

The effect of the different hedging strategies applied on Equity Portfolio is not clearly defined. The introduction of hedging has a controversial effect. Volatility is not reduce by the use of future currency contracts following Static Beta, GJR Beta and GARCH with exogenous variables Beta Strategy, moreover for the last two mentioned volatility slightly increases. Static Beta even shows a worse mean of monthly returns. Also the theoretical Full Hedged Portfolio has a higher standard deviation with respect to Non Hedged Equity Portfolio.

Only Rolling Beta and EWMA Beta strategies has partly centered the objective of volatility reduction and returns improvements: the two methods that consider the moving average to determine OHR double the monthly returns with respect to Non Hedged portfolio, exploiting the use of future contracts. Reduction in terms of standard deviation is about 0.40 per cent, as registered for all hedging strategies applied on Bond Portfolio.

Considering the characteristics of the distribution of the different strategies adopted the series show Kurtosis higher than 3, in particular Rolling Beta and EWMA Beta Hedged Portfolios show the highest values. Skewness is always negative implying asymmetry in the distribution.

The introduction of Equity investment in the initial analyzed Bond Portfolio does not show a positive effect on returns improvement: the overall period of the investment, as shown in preceding paragraphs, is characterized by lower average returns for Equity indices with respect to Bond Indices used in this analysis.

Another time volatility is not reduce by the implementation of even strategies and Full Hedged portfolio even shows an increase in standard deviation with respect to Non Hedged Equity Portfolio. Moreover, full hedging, in comparison with other strategies, is the one that register lower monthly returns in mean. Again, Rolling Beta and EWMA Beta Hedged Portfolios have the best returns, improving Non Hedged Portfolio by more than 1.00 percentage point. As already anticipated, this is not accompanied by a reduction in volatility: standard deviation is reduce only by 0.10 per cent.

Kurtosis of these two portfolios is higher than 3 and also higher than Non Hedged Portfolio Kurtosis. For the other Portfolios it is close to the normal distribution. Considering Skewness of the various series, it is always negative.

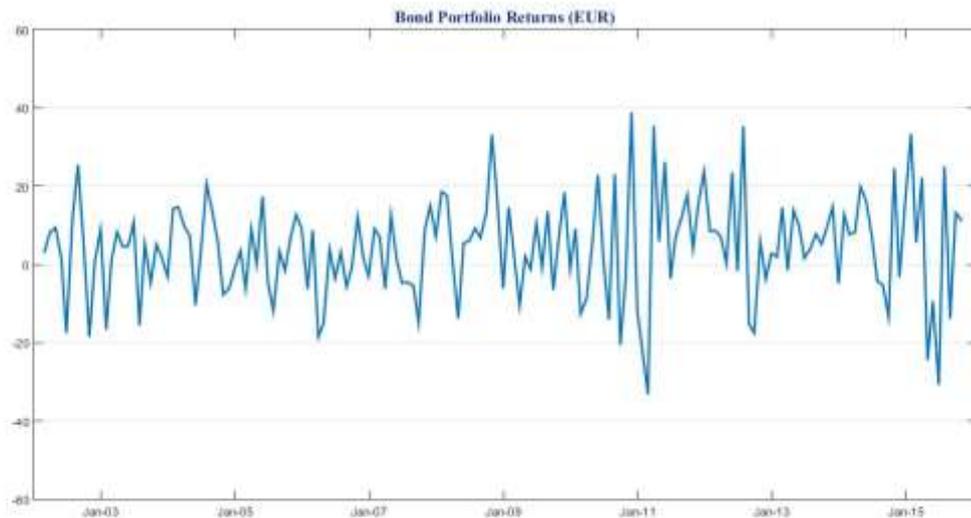
4.2 Monthly returns

In the following paragraphs, I present the different graphs related to Monthly annualized returns of the different Portfolio and strategies analyzed.

4.2.1 Bond Portfolio

Figure 7 shows monthly returns of Bond Portfolio in which the Indices that compose the investment are converted into investor's country currency, Euro. This imply that the movements of EUR-US dollar exchange rate influence returns, amplifying or reducing the performance of the underlying US dollar denominated Indices.

Figure 7 – Non Hedged Bond Portfolio



Non Hedged Bond Portfolio monthly returns graph is compared with the different Hedged Portfolios in which various strategies are implemented in order to cover currency exposure in Bond Portfolio.

Figure 8 - Full Hedged and Non Hedged Bond Portfolio

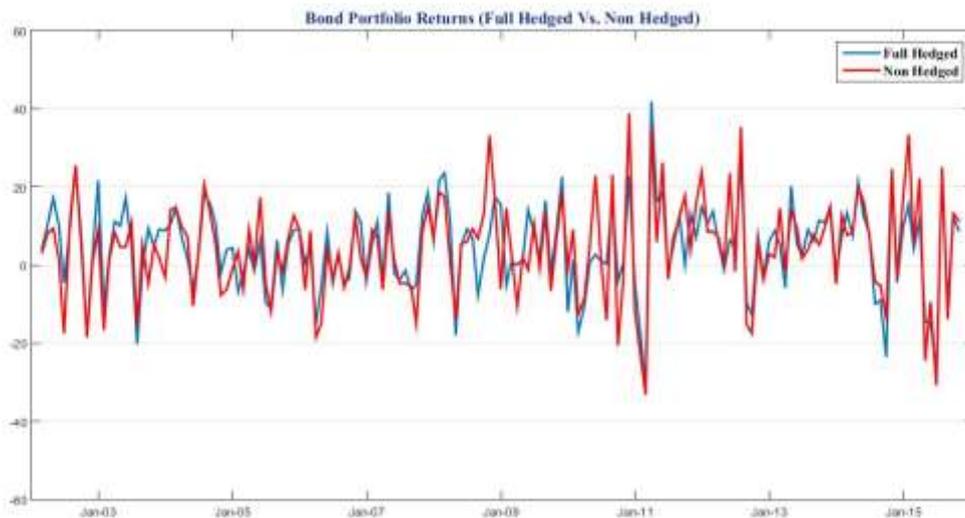
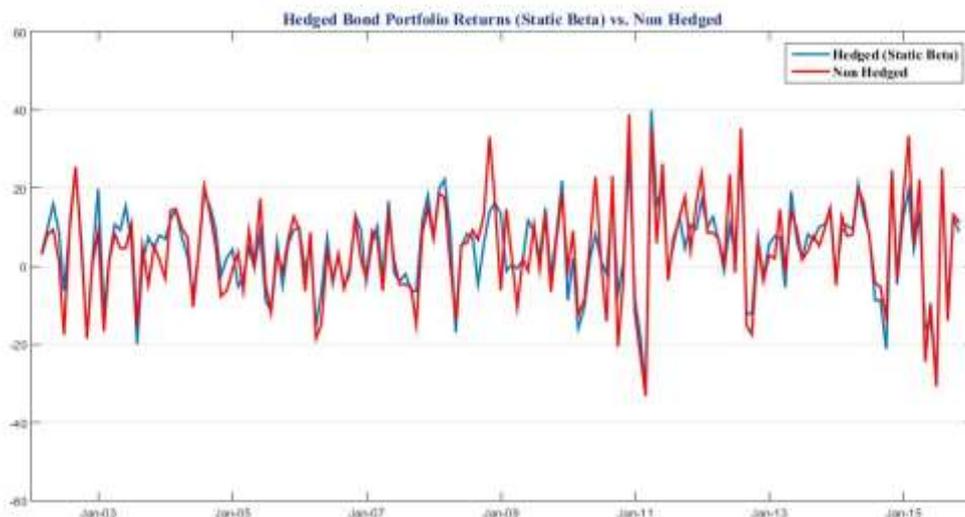


Figure 8 highlights some period in which Full Hedged Portfolio outperforms Non Hedged one, in particular when Currency Futures' trend is positive, like for example in 2003-2004 and before the financial crisis in 2008. On the other hand, when Euro depreciates against US-dollar, full hedging strategy is not the best solution to adopt, see for example end-2008 and end-2014.

Figure 9 - Hedged Bond Portfolio (Static Beta)



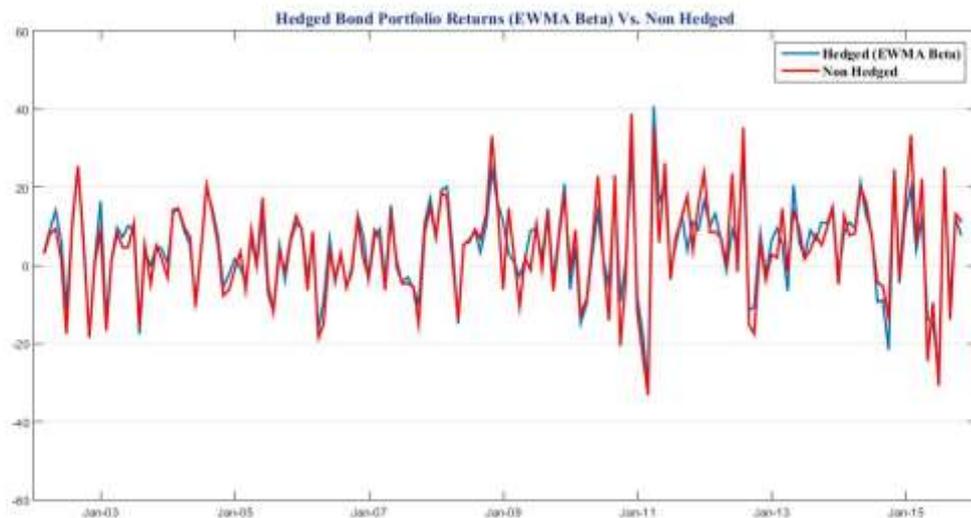
In Figure 9 it is possible to notice the evidence already underlined for Full Hedged Portfolio, with the difference of better returns in periods of US-dollar appreciation with respect to that strategy.

Figure 10 - Hedged Bond Portfolio (Rolling Beta)



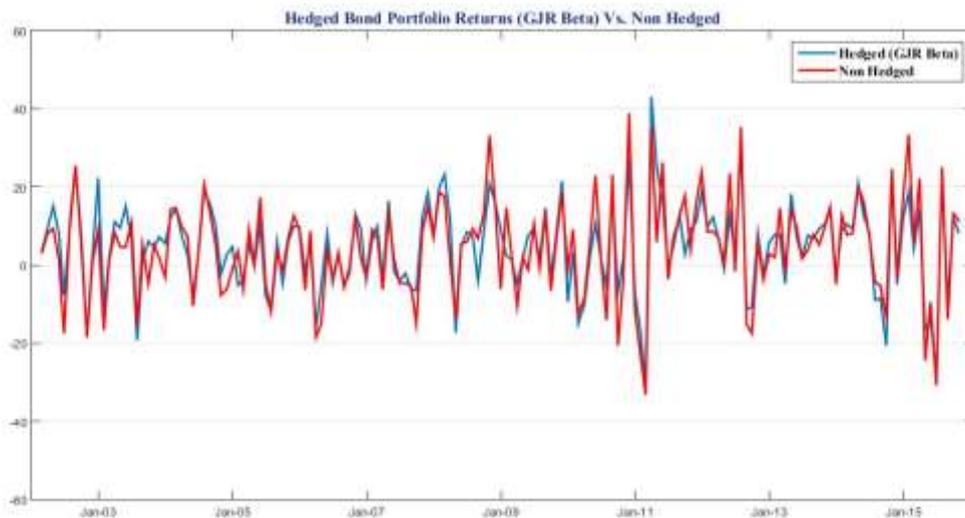
Rolling Beta Hedged Portfolio registers again slightly better performance in returns during appreciation period of the currency and improves them with respect to the other strategies described in previous graphs in depreciation periods.

Figure 11 - Hedged Bond Portfolio (EWMA Beta)



EWMA Beta Hedged Portfolio's returns seem to be in line with the Non Hedged Portfolio in period of negative trend of the currency futures at the cost of worse returns in positive trends' periods.

Figure 12 - Hedged Bond Portfolio (GJR Beta)



GJR Beta Hedged Portfolio reaches the maximum peak in monthly returns among the different Bond Portfolios analyzed but not always gets to outperform other hedging strategies, in particular in currency depreciation period.

Figure 13 - Hedged Bond Portfolio (GARCH with exogenous variables Beta)

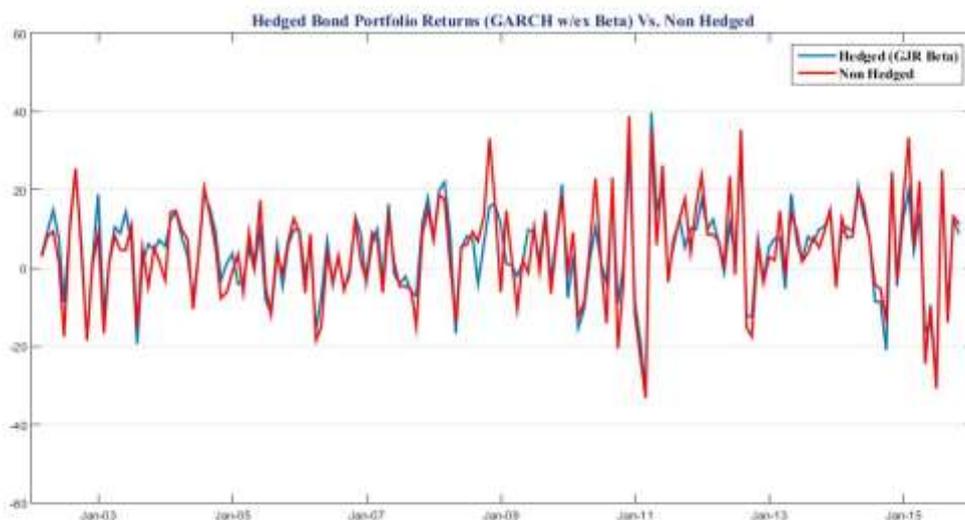
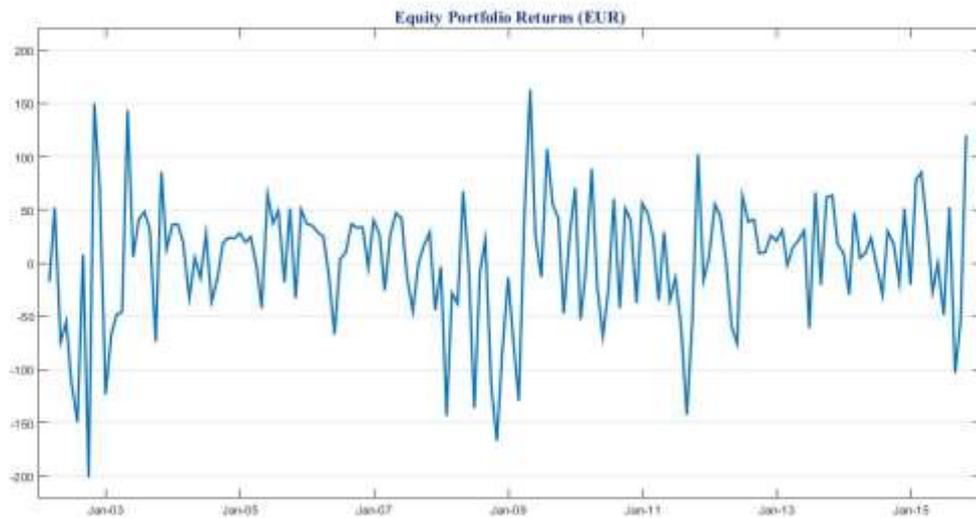


Figure 13 highlights no particular improvements in returns with respect to other strategies implemented. GARCH with exogenous variables method in identifying OHR does not seem to outperform for example naïve Static Beta strategy for the period of analysis considered.

4.2.2 Equity Portfolio

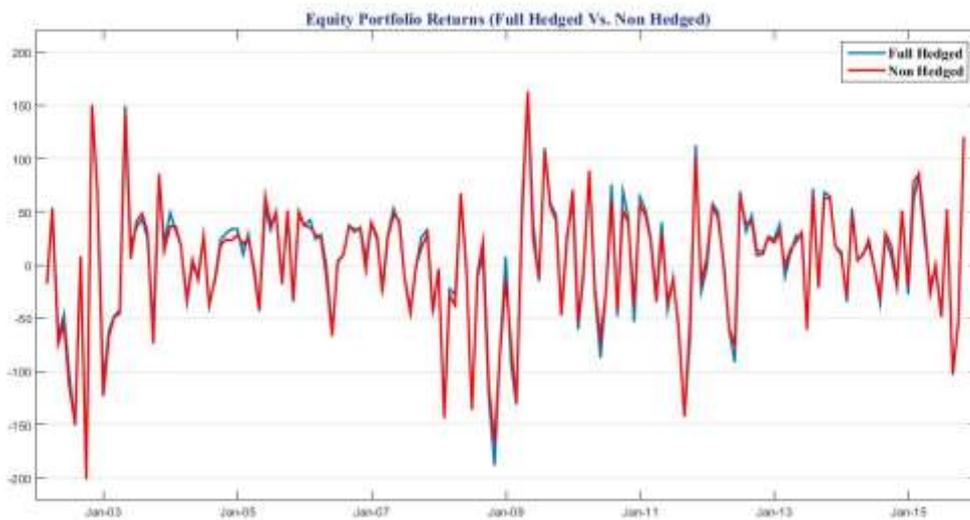
Figure 14 shows monthly returns of Equity Portfolio in which the US stock market Index is converted into investor's country currency, Euro. The graph shows monthly-annualized returns of this Non Hedged Equity Portfolio.

Figure 14 – Non Hedged Equity Portfolio



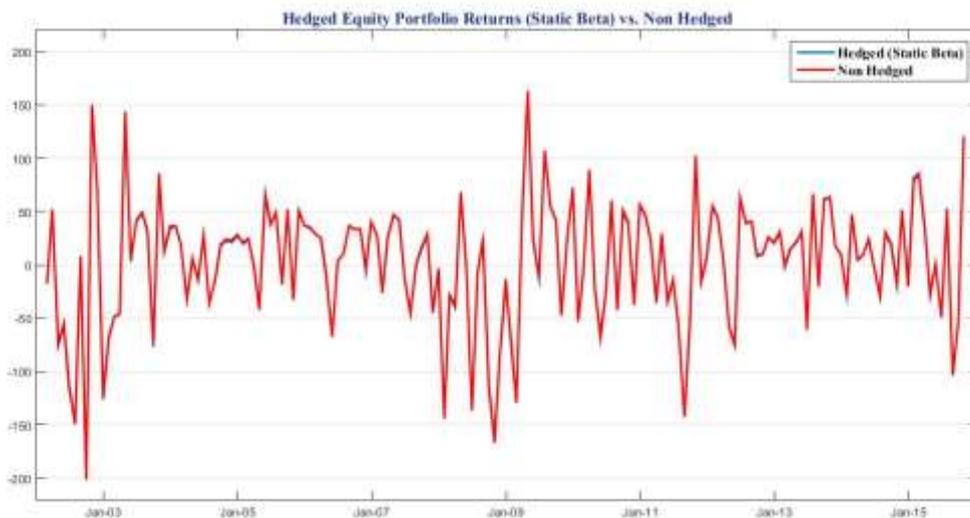
Equity Portfolio, even if is characterized by the same macro trend already seen in Bond Portfolio, except for 2009's larger recovery, is much more volatile and the difference between maximum and minimum return is much more larger. In the following figures, I compare the unhedged portfolio with the portfolios derived from the implementation of the various strategies.

Figure 15 – Full Hedged and Non Hedged Equity Portfolio



Equity Portfolio, even if is characterized by the same macro trend already seen in Bond Portfolio, is much more volatile and the difference between maximum and minimum return is much more larger.

Figure 16 - Hedged Equity Portfolio (Static Beta)



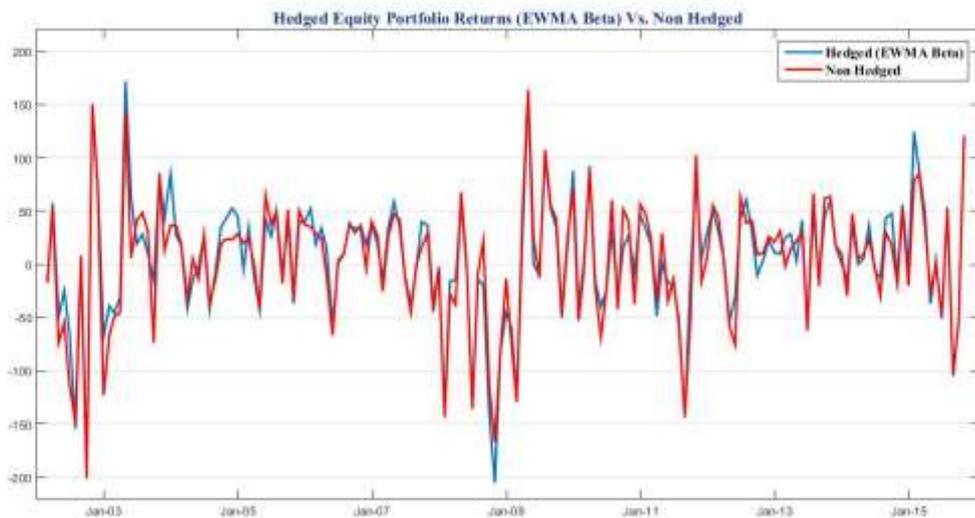
The naïve Static Beta model applied to Equity Portfolio does not give any value added in hedging of currency risk, almost replicating Non Hedged Equity Portfolio returns.

Figure 17 - Hedged Equity Portfolio (Rolling Beta)



Figure 17 highlights Rolling Beta Equity Portfolio succeeds in outperforming Non Hedged Portfolio largely in various periods, early and end-2004 and in 2014 the most evident cases. The 36-months' time window for the rolling mean implementation allows to exploit the hedging in order to get better returns in the portfolio.

Figure 18 - Hedged Equity Portfolio (EWMA Beta)



EWMA Beta Equity Portfolio, like Rolling Beta one, outperforms unhedged portfolio reaching even higher peak in returns (see half-2003). On the other hand the minimum value of returns, touched in end-2008, is lower than the minimum of Rolling Beta Portfolio.

Figure 19 - Hedged Equity Portfolio (GJR Beta)

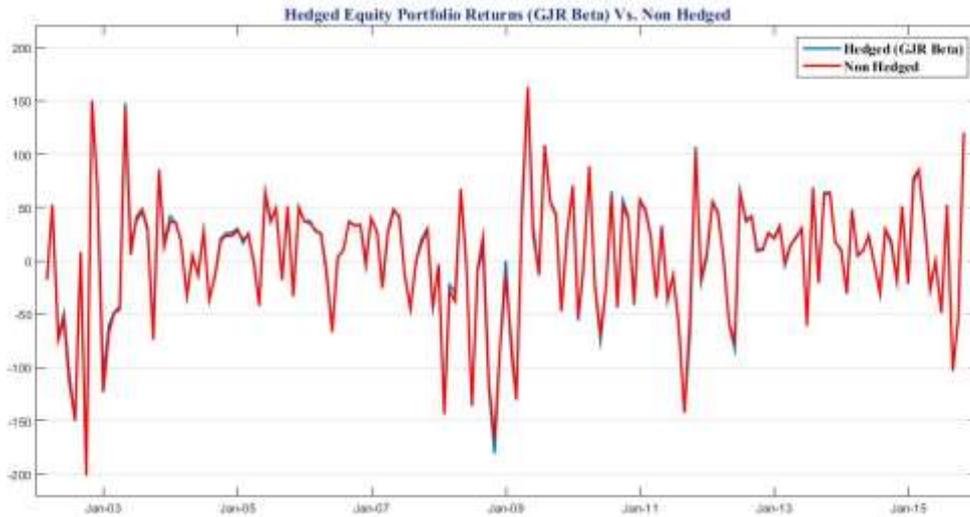
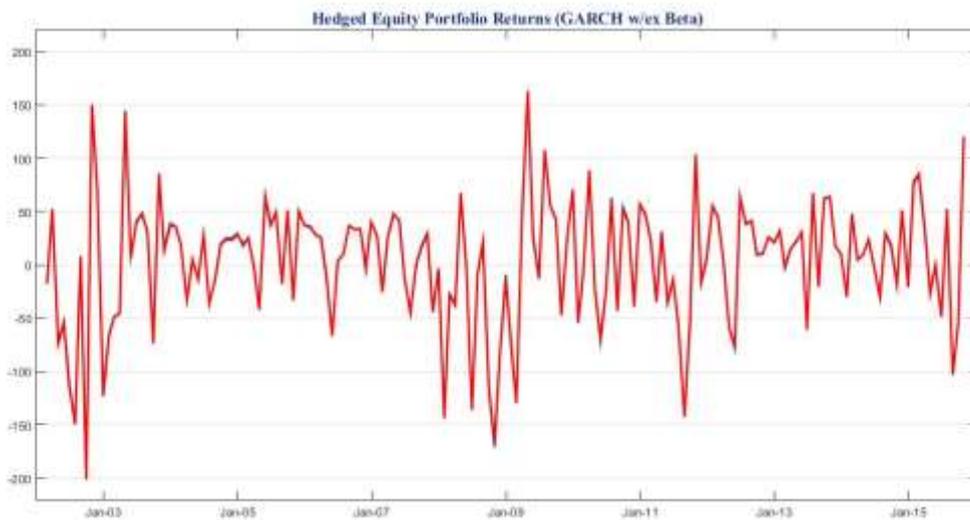


Figure 20 - Hedged Equity Portfolio (GARCH with exogenous variables Beta)

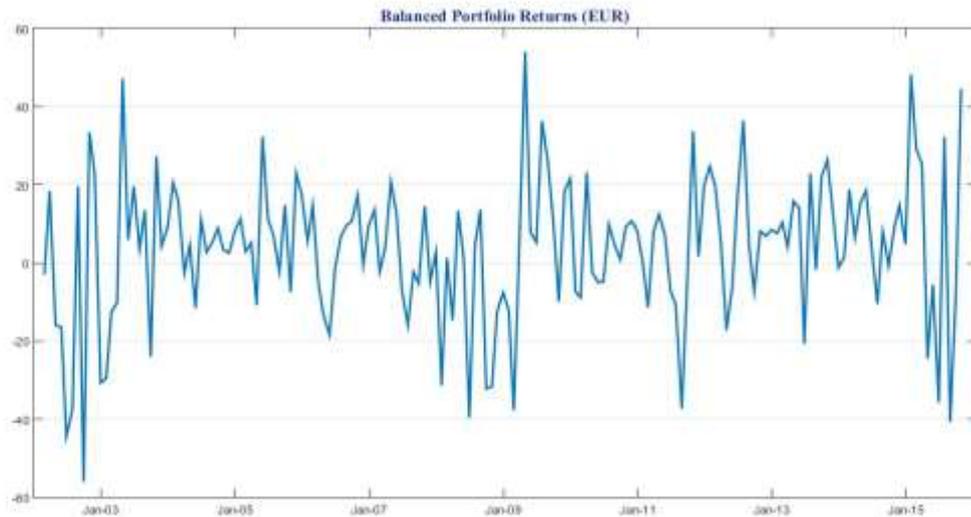


Both GJR and GARCH with exogenous variables models are quite ineffective in hedging Equity Portfolio, replicating returns registered by the Non Hedged Portfolio.

4.2.3 Balanced Portfolio

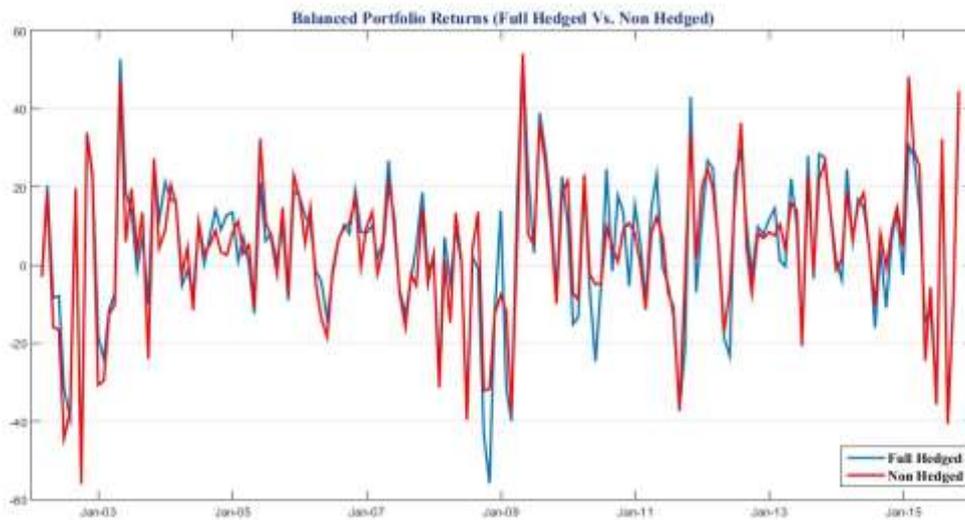
Monthly annualized returns of Non Hedged Balanced Portfolio are represented in Figure 21. As described in the beginning of this Section, Balanced Portfolio is composed for 70 per cent of a mix of the Bond Indices (Government and Corporate) and for 30 per cent of Equity Indices. The large presence of Bond Indices in the Portfolio sensibly reduces volatility with respect to Equity Portfolio.

Figure 21 – Non Hedged Balanced Portfolio



Balanced Portfolio incorporates characteristics of both Bond and Equity Portfolio. Following figures show the comparison between the various strategies implemented in hedging Balanced Portfolio and the monthly returns of the unhedged portfolio.

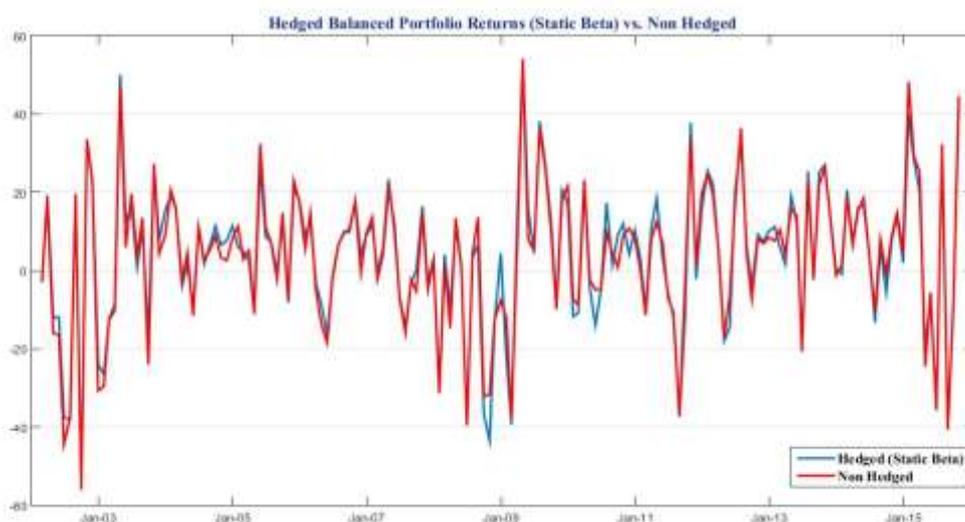
Figure 22 – Full Hedged and Non Hedged Balanced Portfolio



Theoretical full hedging strategy outperforms un-hedging one in various moment during the overall period of the analysis, in particular when US-dollar tends to depreciate against the Euro.

On the other hand, trying to cover the overall exposure to foreign currency is a strategy that amplifies negative movement of financial assets, during period in which currency futures trend is negative.

Figure 23 - Hedged Balanced Portfolio (Static Beta)



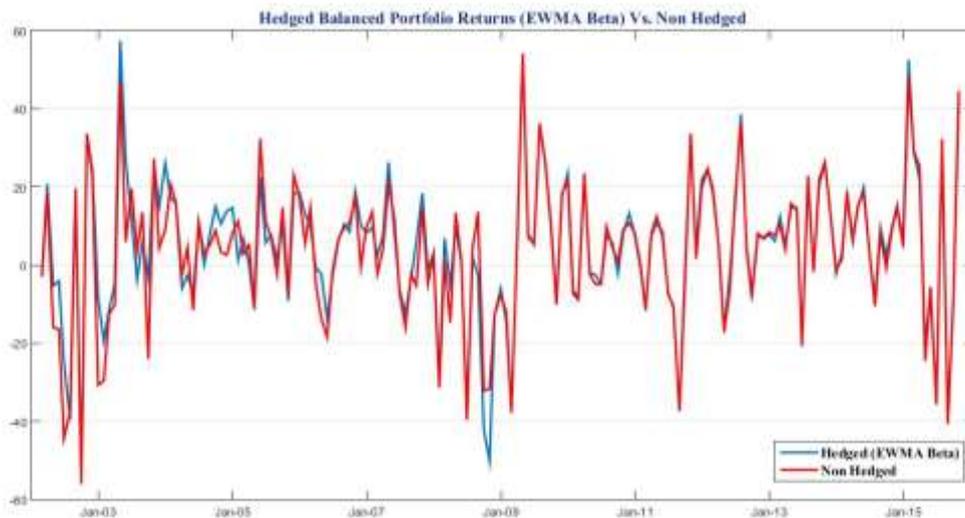
Static Beta Portfolio, with respect to Full Hedged Portfolio and other portfolios' monthly returns reported in the following figures, succeeds less frequently to outperform un-hedging strategy and when it happens, the magnitude is smaller comparing to them..

Figure 24 - Hedged Balanced Portfolio (Rolling Beta)



Figure 24 highlights how adopting a rolling mean beta strategy slightly improves performance of the Balanced Portfolio. In particular, in end-2004, considering the positive trend experienced by currency futures since 2002 with few slight declines, monthly returns exceed Non Hedged strategy. However, in end-2008 returns get worse implementing this strategy. The particular period in which this result is registered may not be representative of the actual power of the hedging strategy.

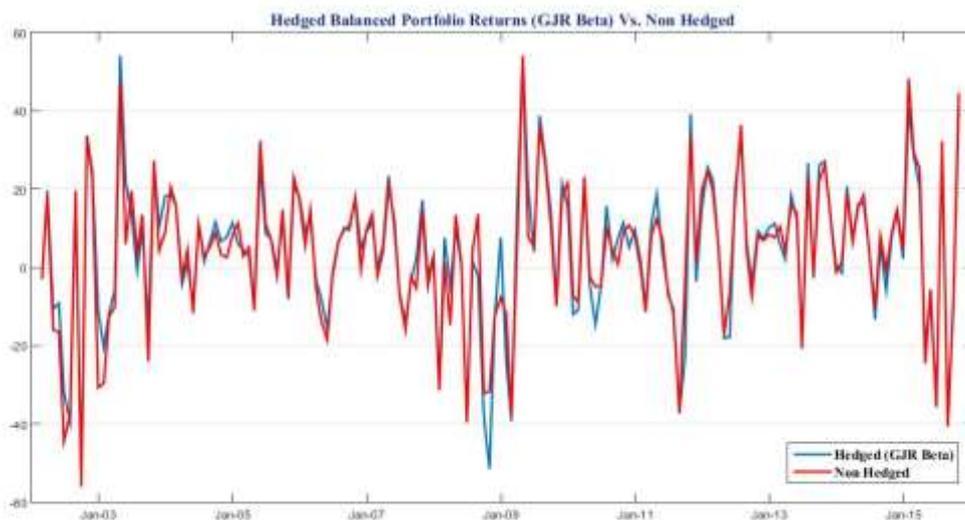
Figure 25 - Hedged Balanced Portfolio (EWMA Beta)



In Figure 25 it is possible to notice the same evidence of Rolling Beta Hedged Portfolio' monthly returns. Among the various strategies implemented, moving average used both in the simpler rolling method and in exponential weighted form gives the best results in terms of monthly returns during the period of the analysis.

Overall, however, the benefits achieved compared to the Non Hedged Portfolio do not appear to be of such great importance.

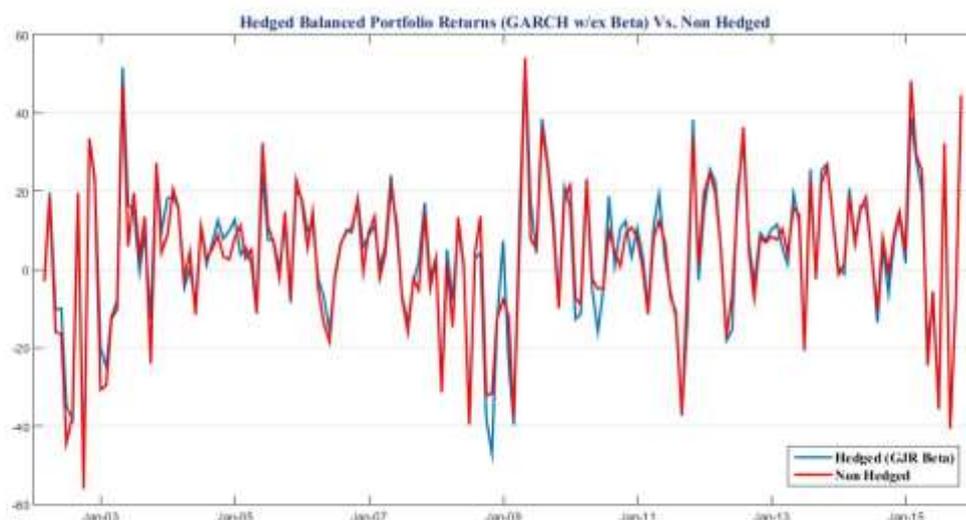
Figure 26 - Hedged Balanced Portfolio (GJR Beta)



In general, the results of implementing the GJR Beta strategy reflect in worse monthly returns with respect to the two previous showed moving average strategies.

However, it could be interesting to notice how in 2009 and in 2011, in two periods of positive trend for currency Futures, GARCH models succeed in outperform un-hedging strategy, when moving average models fail.

Figure 27 - Hedged Balanced Portfolio (GARCH with exogenous variables Beta)



In figure 27 it is possible to find the same evidence described for GJR Beta strategy.

In some cases, GARCH with the exogenous variables shows higher monthly returns with respect to GJR Beta Hedged Portfolio, for example in 2010 and 2011.

Monthly annualized returns, presented through the previous figures, give indication about how different strategies implemented respond to the objective of improving performance of the Portfolio.

In the following paragraph, I analyzed the results of the implementation of hedging strategies for the overall period of the analysis, in order to understand the significance of the possibly positive gap registered in cumulated returns.

4.3 Cumulated returns

In this paragraph, I present returns of portfolio realized during the entire period analyzed. The cumulated total return is what the investments realize from the date of first implementation to the last day of the analysis, in other words what the investor hypothetically gained from the investment in each portfolio and implementing the various hedging strategies already described.

4.3.1 Bond Portfolio

Figure 28 – Bond Portfolios Cumulated Returns

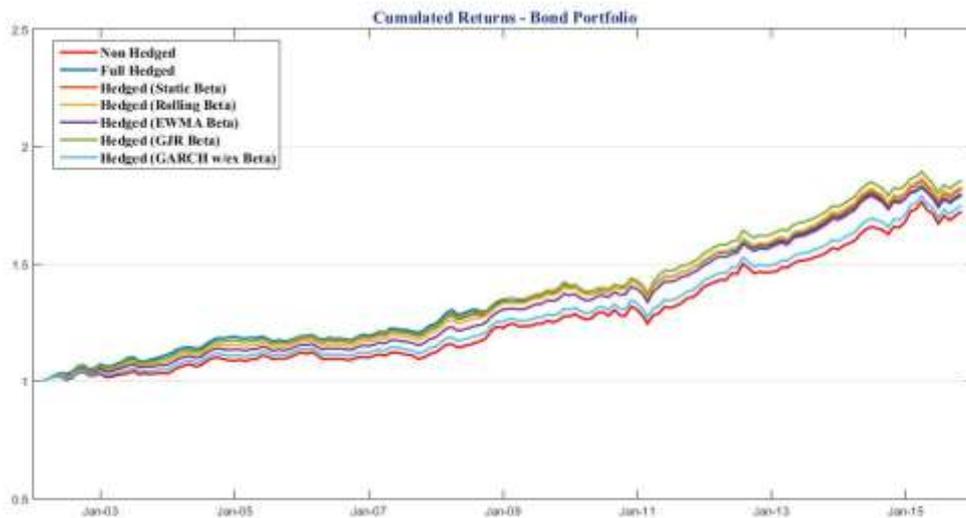


Figure 28 shows cumulated returns of the Bond Portfolio analyzed.

During the entire period, the kind of investor considered, who chooses the composition of her portfolio based on the criteria described at the beginning of this section for Bond Portfolio, obtains the results represented in the figure.

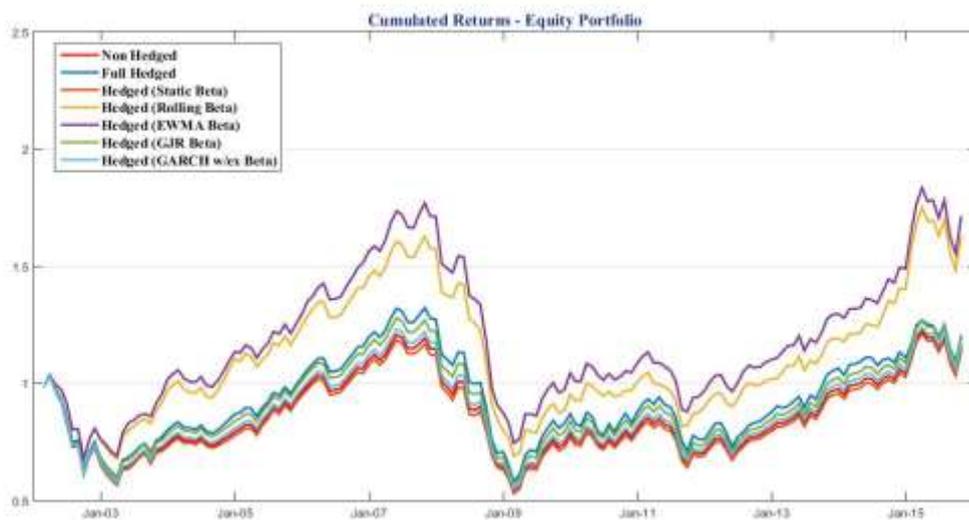
Every hedging strategy is characterized by higher cumulated returns with respect to what realized by Non Hedged Portfolio, 71.89 per cent in almost 14 years, considering a period of the analysis starting in January 2002 that ends in November 2015.

Consistently with the results already showed, the higher cumulated returns is registered by the GJR Beta Hedged Portfolio, that at the end of the analysis has gained 85.17 percentage point.

A remarkable result are the cumulated returns that characterized Static Beta Hedged Portfolio, over 80 per cent in 14 years. In fact, excluded GARCH with exogenous variables Beta Hedged Portfolio, other hedging strategies does not guarantee much higher cumulated returns during the considered period.

4.3.2 Equity Portfolio

Figure 29 – Equity Portfolios Cumulated Returns



Cumulated returns of the Equity Portfolio, reported in Figure 29, are in general characterized by period of decline in correspondence of the various crises that have hit the financial markets in the years analysed.

Again, it is clear that hedging strategies beat un-hedging one, but, in this case, the positive gap in cumulated returns that characterize moving average strategies, Rolling Beta and EWMA Beta, is more evident with respect to what registered for Bond Portfolio.

Rolling mean method, in about 14 years, reaches a cumulative result of 62.58 percentage points and EWMA method guarantees even better returns, with 70.78 per cent. Cumulated returns of the Non Hedged Portfolio stops at 14.82 at the end of the period analysed.

Other Portfolio in which the remaining hedging strategies are implemented does not show the same magnitude of improvement, for example GJR Beta Portfolio has a cumulated return of 18.95 per cent, a little bit more than 400 bps.

Moving average strategies seem to outperform all the other hedging strategies during each period of the analysis by a relevant measure. The positive gap thins only in correspondence of general negative trends in the graph.

4.3.3 Balanced Portfolio

Figure 30 – Balanced Portfolios Cumulated Returns

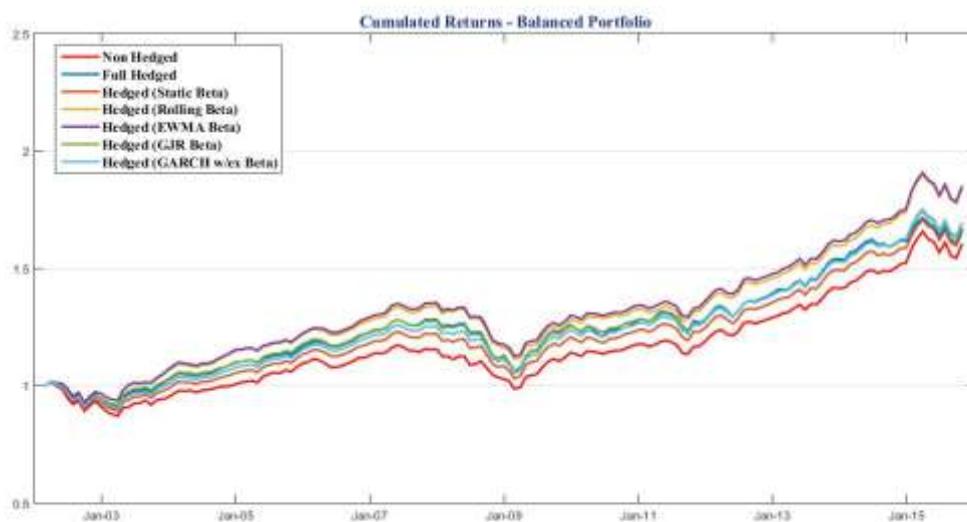


Figure 30 represents cumulated returns of the various Balanced Portfolios implemented.

The larger presence of Bond Indices in the composition of the Balanced Portfolio smooth the trend showed by Equity Portfolio.

Also in this case hedging strategies succeed in the objective of guaranteeing better performance in returns.

In about 14 years, Rolling Beta and EWMA Beta Hedged Portfolios gain respectively 83.81 and 84.47 percentage point. The improvement with respect to not implementing any hedging strategies is about 240 bps. The two strategies maintain the gap throughout the overall investment period.

Other strategies reach the level of the benchmark Full Hedged Portfolio, between 65.14 and 69.04 per cent in 14 years.

Considering the overall period of investment, including Equity in the original only Bond Portfolio in general leads to worse performance with regard to cumulated returns. On the other hand, the improvement with respect to Non Hedged Portfolio has a larger impact on the hedged portfolios cumulated returns.

4.4 Performance Measures

In order to evaluate the performance of the various portfolios analyzed and, consequently, the impact of diverse hedging strategies adopted I calculate some of the indices of performance. Most of them are built on the ratio between two elements: a return measure, like the average portfolio return, and a risk measure, usually the most used is the standard deviation.

In the following paragraphs, I propose an overview about the performance indices that I use to assess the quality of the portfolios.

4.4.1 Sharpe ratio

Sharpe ratio (SH) is usually computed as ratio between the mean of returns and their standard deviation.

$$SH = \frac{r_H}{\sigma_H} \quad (16)$$

The indicator represents the trade-off between return and risk: it measures the premium for each unit of risk accepted. It is straightforward that the portfolio with the highest value is the one that rewards the most for unit of risk. Such an index imposes that the standard deviation of returns describes completely the risk. Usually investors are not prone to suffer negative returns and long *drawdown*, i.e. the difference between the peak and the trough during a specific period of an investment. On the contrary, as Thaler and Benartzi (1995) prove, they prefer to sacrifice some of their gain in order to avoid larger losses. This asymmetric behaviour is not captured by the index.

4.4.2 Sortino ratio

Sortino ratio (SO) is the ratio between the mean of the return excess over the risk-free asset yield and the *downside* risk, i.e. it focuses only on the negative side of volatility, the negative returns volatility.

$$SO = \frac{r_H}{V[r_H I(r_H < 0)]} \quad (17)$$

where V represents standard deviation of the returns below zero. Sortino ratio's aim is to capture the asymmetry of the return distribution. Higher the value of the indicator, more the volatility is mainly concentrated on positive returns, instead of negative ones, as investor should prefer.

4.4.3 Value-at-Risk

Value-at-risk is a statistical method that quantifies the risk level associated with a portfolio. The VaR measures the maximum amount of loss over a horizon equivalent to the returns frequency, and at a given confidence level $(1 - \alpha)$. With a probability α (I fix $\alpha = 0.05$) the loss will be larger than the Value-at-Risk. The VaR is a quantile of the returns density that satisfies:

$$\int_{-\infty}^{VaR(\alpha)} r_t^H f(r_t^H) dr_t^H = \alpha \quad (18)$$

4.4.4 Expected shortfall

The Expected Shortfall was propose to overcome a limit of the VaR, the lack of the so-called sub-additivity property of a risk measures. The VaR of a portfolio should be smaller than the combination of the VaR of the underlying assets.

The Expected Shortfall is a conditional expectation and, simplifying, it equals the mean of returns below the VaR.

Analytically, the Expected Shortfall is equal to

$$ES(r_t^H, \alpha) = \mathbb{E}[r_t^H | r_t^H < VaR(\alpha)] \quad (19)$$

4.4.5 Calmar ratio

The Calmar ratio (Cal) is a performance measurement given by the ratio between the portfolio average rate of return and the maximum *drawdown*, i.e., as defined above, the difference between the peak and the trough of the investment.

$$Cal = \frac{r_H}{Max(DD)} \quad (20)$$

4.4.6 Sterling ratio

The Sterling ratio (Ste), similarly the Calmar one, is defined as the ratio between the portfolio average rate of return and the average of the k largest *drawdowns* (in absolute value).

I set $k = 5$.

$$\text{Ste} = \frac{r_H}{\text{Avg. Largest DD}} \quad (21)$$

4.4.7 Results

In the following table are reported the values of the performance measures described above for each portfolios of this analysis. It shows also the results of the currency (i.e. the US dollar / Euro exchange rate trend) performance and the future index used for hedging purpose.

Table 9 – Performance measures of the Portfolios

Portfolio/ Index	Strategy	Sharpe ratio	Sortino ratio	VaR	Expected Shortfall ratio	Calmar ratio	Sterling ratio
Currency		-0.010	-0.016	-0.006	-0.005	-0.003	-0.004
Futures		-0.009	-0.013	-0.005	-0.004	-0.003	-0.003
Bond Portfolio	Non Hedged Portfolio	0.316	0.528	0.229	0.172	0.120	0.151
	Full Hedged Portfolio	0.381	0.603	0.276	0.198	0.135	0.177
	Hedged Portfolio (Static Beta)	0.392	0.626	0.274	0.206	0.138	0.183
	Hedged Portfolio (Rolling Beta)	0.392	0.646	0.292	0.209	0.140	0.187
	Hedged Portfolio (EWMA Beta)	0.384	0.636	0.289	0.208	0.136	0.184
	Hedged Portfolio (GJR Beta)	0.401	0.671	0.283	0.216	0.145	0.193
	Hedged Portfolio (Ex Beta)	0.386	0.625	0.271	0.203	0.135	0.181
Equity Portfolio	Non Hedged Portfolio	0.042	0.055	0.021	0.017	0.012	0.015
	Full Hedged Portfolio	0.047	0.062	0.025	0.019	0.014	0.017
	Hedged Portfolio (Static Beta)	0.041	0.053	0.020	0.016	0.012	0.015
	Hedged Portfolio (Rolling Beta)	0.086	0.109	0.046	0.032	0.025	0.030
	Hedged Portfolio (EWMA Beta)	0.092	0.115	0.048	0.034	0.026	0.032
	Hedged Portfolio (GJR Beta)	0.046	0.061	0.025	0.018	0.014	0.017
	Hedged Portfolio (Ex Beta)	0.044	0.057	0.022	0.017	0.013	0.016
Balanced Portfolio	Non Hedged Portfolio	0.193	0.266	0.108	0.087	0.064	0.081
	Full Hedged Portfolio	0.203	0.281	0.117	0.090	0.069	0.083
	Hedged Portfolio (Static Beta)	0.207	0.287	0.103	0.092	0.069	0.088
	Hedged Portfolio (Rolling Beta)	0.253	0.342	0.136	0.110	0.086	0.102
	Hedged Portfolio (EWMA Beta)	0.255	0.339	0.133	0.110	0.087	0.102
	Hedged Portfolio (GJR Beta)	0.214	0.299	0.116	0.095	0.074	0.089
	Hedged Portfolio (Ex Beta)	0.215	0.299	0.112	0.095	0.072	0.090

Currency and Futures show very similar results: as already found above, Futures Index is a good proxy for the exchange rate tendency and effectively performs its hedging function.

For each portfolio of the analysis (Bond, Equity and Balanced), I compare the performance measure related to the various strategies adopted.

Among the Bond Portfolios, we notice how each Hedged Portfolio outperforms the Non Hedged Portfolio. The best performance on all measures are obtain by GJR Beta Strategy, outperformed by EWMA and Rolling Beta Strategies only for less than 0.01. In comparison with the unhedged Portfolio, GJR Beta Strategy shows an improvement in Sharpe ratio of almost 0.10 and in Sortino even of 0.15. On the other side, it comes with a worsening, of only less than 0.06, in risk measures, VaR and Expected Shortfall.

Considering Equity Portfolio, it is immediately clear how during the analyzed period, risk premium is very low, close to zero. EWMA Beta Portfolio shows slightly better performance measure, but they are accompanied by a little worse risk measures in the order of 0.02 of value.

With Balanced Portfolio the best improvements with respect to Non Hedged Portfolio is showed again by EWMA Beta Portfolio, and by the Rolling Beta Portfolio. However, also in these cases, it is possible to notice a slightly worsening of risk measures, in comparison to the unhedged portfolio.

The Equity component in the investment seems to not be touched by any risk reduction given the hedging strategy adopted. On the contrary, some interesting results come from Bond Portfolio, which potentially could benefit by the introduction of hedging.

In section 5 I present the results about the Sharpe ratio robust test, for the statistically significance of the outperformance of hedging portfolios with respect to the unhedged ones.

4.5 Beta Analysis

In the following table, I present some characteristics of the different vector of Betas, the optimal hedging ratios calculated through the four approaches.

Table 10 – Betas’ statistics

	Beta	Mean	Median	Std Dev	Min	Max	Skeweness	Kurtosis
Bond Portfolio	Rolling Beta	0.155	0.145	0.049	0.058	0.269	0.626	2.608
	EWMA Beta	0.132	0.117	0.051	0.044	0.248	0.366	2.091
	GJR Beta	0.163	0.155	0.035	0.095	0.322	1.169	5.394
	GARCH Ex Beta	0.148	0.149	0.008	0.128	0.168	-0.007	2.137
Equity Portfolio	Rolling Beta	-0.020	0.139	0.598	-0.862	0.911	0.018	1.319
	EWMA Beta	0.056	-0.080	0.547	-0.652	0.993	0.052	1.328
	GJR Beta	0.083	0.067	0.042	0.042	0.292	2.359	8.975
	GARCH Ex Beta	0.036	0.036	0.003	0.032	0.044	0.535	2.366
Balanced Portfolio	Rolling Beta	0.102	0.146	0.164	-0.125	0.342	0.003	1.244
	EWMA Beta	0.108	0.152	0.141	-0.074	0.362	0.059	1.352
	GJR Beta	0.136	0.117	0.059	0.066	0.404	2.086	7.487
	GARCH Ex Beta	0.128	0.125	0.015	0.109	0.161	0.378	1.925

Mean column represents the average position on currency future contracts a European investor should take in order to minimize the risk of the international portfolio, given the application of the different strategies.

For example, based on the EWMA approach, for each Euro invested in Bond Portfolio, to minimize the volatility of the portfolio of a European investor should hold, on average, 0.132 Euro short positions in USD futures, respectively.

In just one case, optimal hedging strategy for Equity Portfolio, lead to an average long position in future contracts. Actually, vector of Betas for Equity Portfolio is slightly different from zero.

Standard deviation measures the dispersion of the values of Beta, in the following paragraph I will show Beta’s time series. The table highlights how Rolling Beta and EWMA Beta values calculated for the Equity Portfolio and partially for the Balanced Portfolio are more spread out with respect to other methods and related portfolios.

Finally, GJR Beta of each Portfolio are characterized by significant higher kurtosis, which suggest fat tails of the distribution.

With the purpose of comparing the behavior of the different optimal hedging ratios, I propose again figure 32, Future/Currency time series.

4.5.1 Bond Portfolio

The following table shows the level of Bond Portfolio's part denominated in US dollar, i.e. the daily percentage of foreign currency indices that compose the portfolio.

Figure 31 – Foreign currency percentage in Bond Portfolio

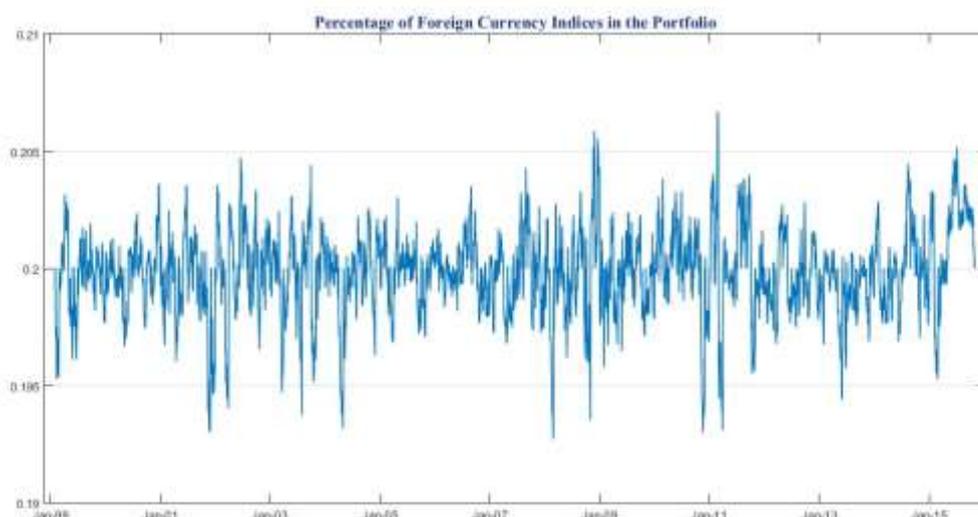
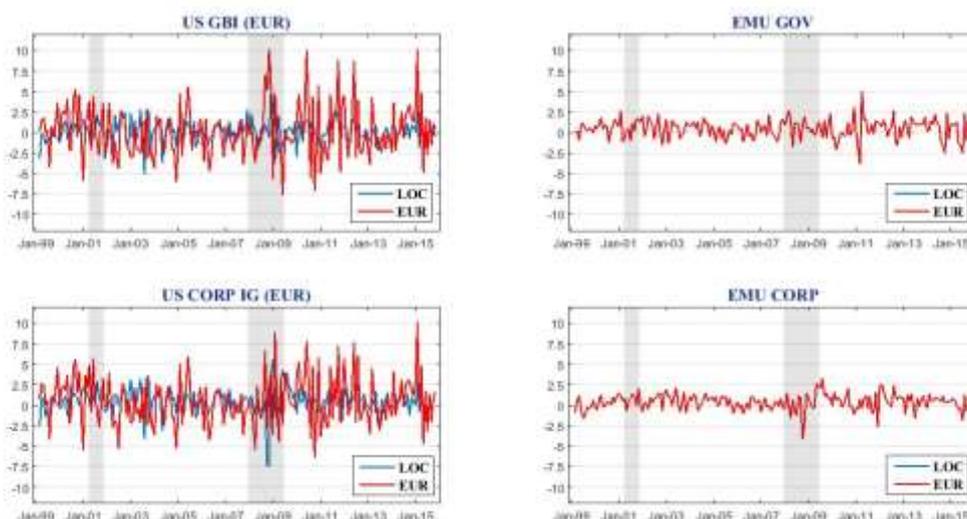


Figure 32 - Bond Indices, monthly returns



The daily percentage of US dollar denominated investment compared to the initial composition, which is re-set every first day of the month, increases or decreases in US currency component, due to the effect of the underlying Indices.

It is possible to notice some volatility clusters in particular in correspondence to the same volatility cluster of the related indices, as shown by Figure 31 and Figure 32. They are referred to specific moment of Indices' history, in which their returns affect more deeply the exposition to the foreign currency.

Figure 33 – Betas for Bond Portfolio's hedge

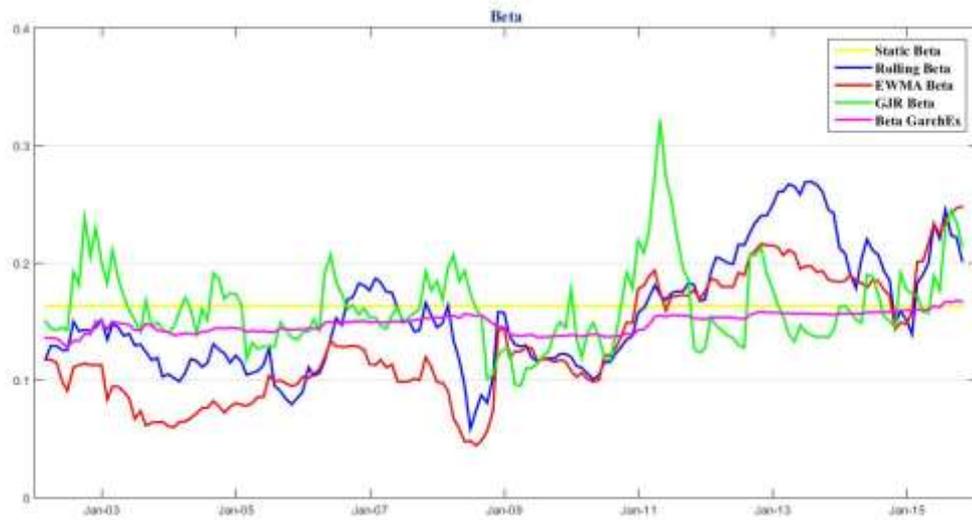


Figure 34 - Spot EUR/USD exchange rate and Futures index prices



Figure 33 represents the various Beta vectors for Bond Portfolio. They show a certain correlation with the Currency Spot/Future time series.

Considering for example Rolling Beta and EWMA Beta, the graph highlights Beta time series and Spot/future time series are negative correlated, taking into account the models' delay given by the time window necessary to the implementation.

GJR Beta embodies early changes in currency framework and at the same time presents more accentuated peaks with respect to the other strategies' Betas.

Finally, GARCH with exogenous variables seems to be more stable and to be in line with the naïve Static Beta.

4.5.2 Equity Portfolio

The following table shows the daily percentage of MSCI USA index in the Equity Portfolio.

Figure 35 - Foreign currency percentage in Equity Portfolio

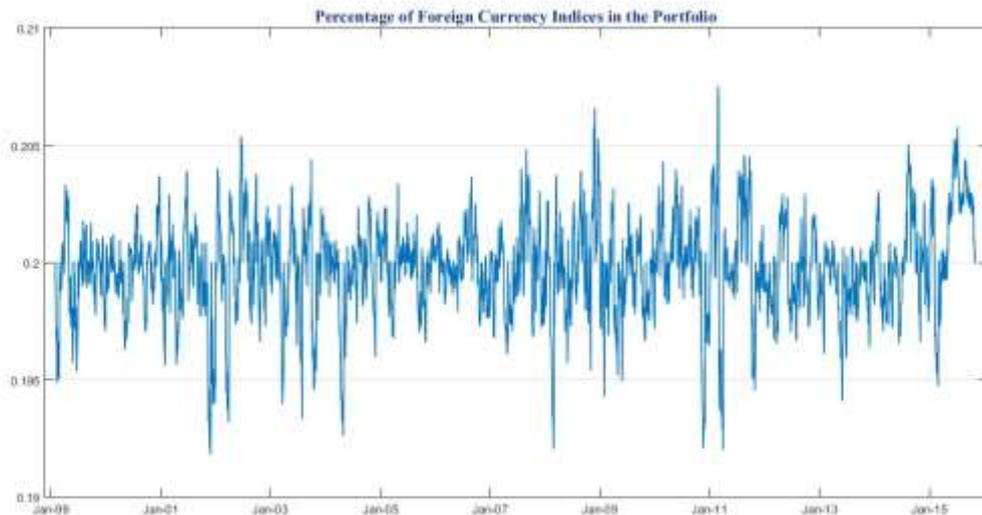
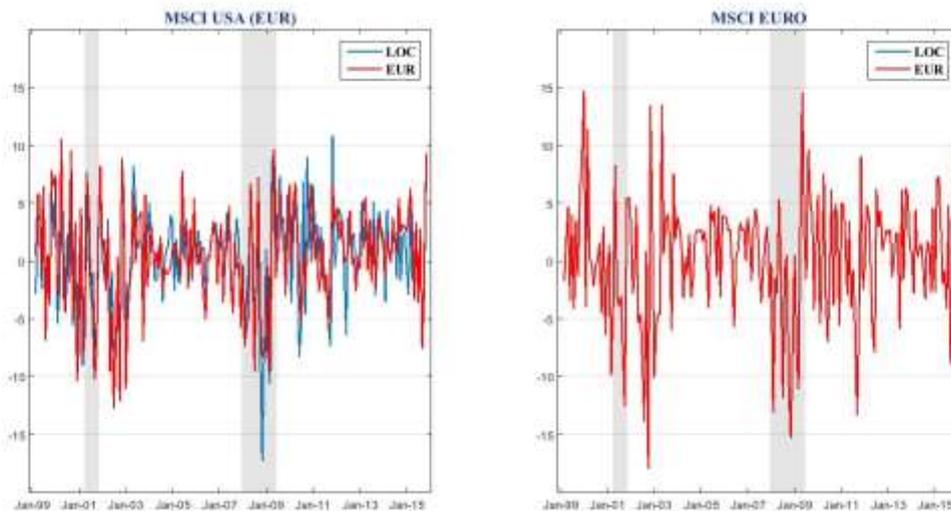


Figure 36 – Equity Indices, monthly returns



The daily percentage of US denominated investment increases when MSCI EURO Index falls in returns, for example during Euro Sovereign debt crisis, or decreases when MSCI USA Index returns are very low, as in 2008 crisis.

Volatility clusters in the graph correspond to particular moments of volatility in returns of the two Indices at the basis of the composition of this portfolio.

Figure 37 - Betas for Equity Portfolio's hedge

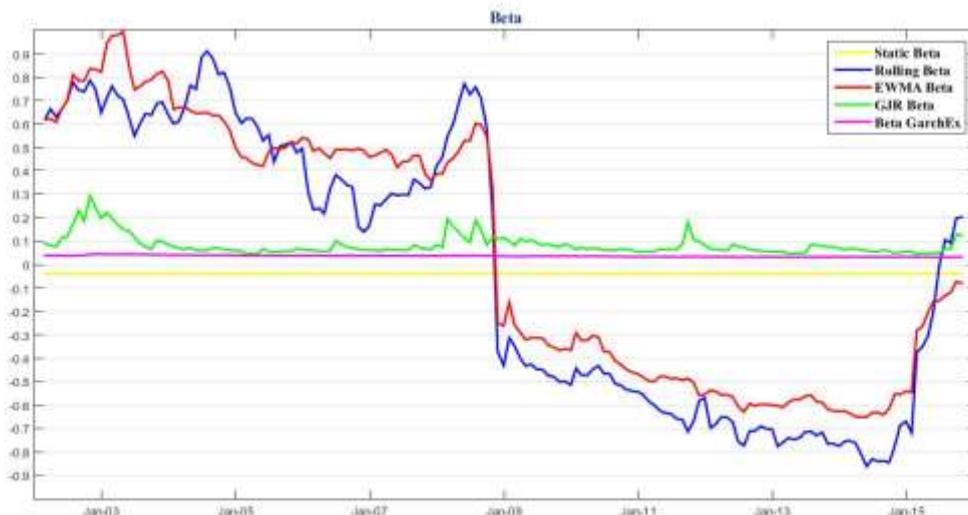


Figure 38 - Spot EUR/USD exchange rate and Futures index prices



Figure 37 represents the various Beta vectors for Equity Portfolio.

GARCH with exogenous variables Beta stays approximately on the zero level, as the naïve Static Beta. They suggest to not cover at all the Equity Portfolio: during the overall time window positive Betas compensate the negative ones.

GJR Beta seems to be more sensitive to falls in Future Index that suggests to held a short position on Currency Futures.

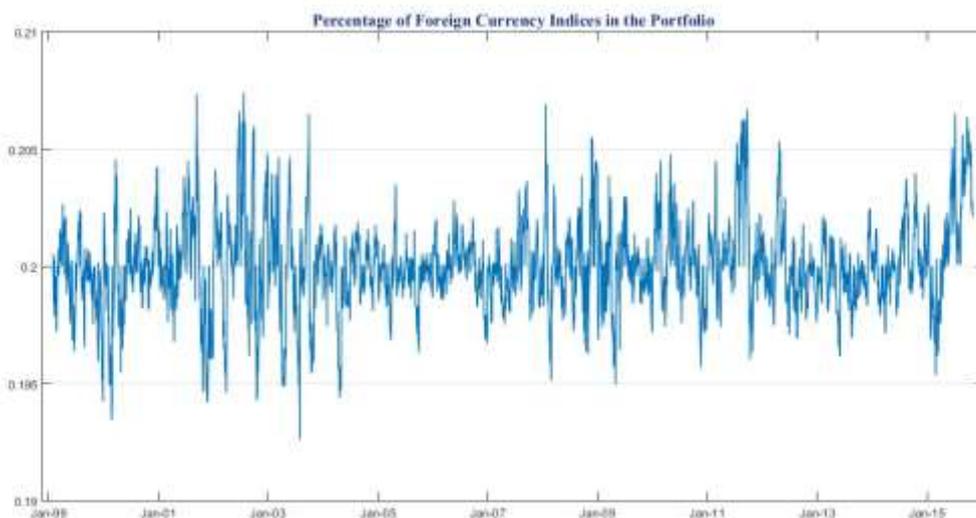
Considering Rolling Beta and EWMA Beta, the graph highlights Beta time series and Spot/future time series are negative correlated: it is clear considering how the positive period pre-2008 crisis period influences the falls in Beta levels, suggesting the investor to buy Future contracts.

The depreciation of US-dollar against Euro in the last period of the analysis drives Rolling Beta and EWMA Beta to a level near zero and in line with the other Betas calculated for the Equity Portfolio.

4.5.3 Balanced Portfolio

The following table shows the level of Balanced Portfolio's part denominated in US dollar, i.e. the daily percentage of foreign currency indices that compose the portfolio.

Figure 39 - Foreign currency percentage in Balanced Portfolio



In the figure above it is possible to notice some volatility clusters already identify in the graph of each Indices underlying the composition of the Balanced Portfolio.

During the last period of the analysis, the daily percentage of US denominated investment increases due to the higher returns of the US Indices and the depreciation of the US dollar.

Figure 40 - Betas for Balanced Portfolio's hedge

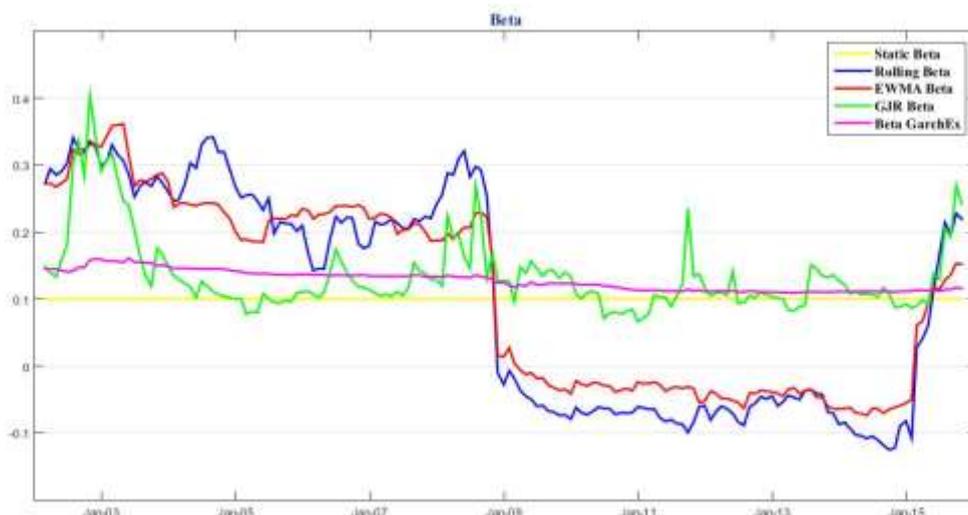


Figure 41 - Spot EUR/USD exchange rate and Futures index prices

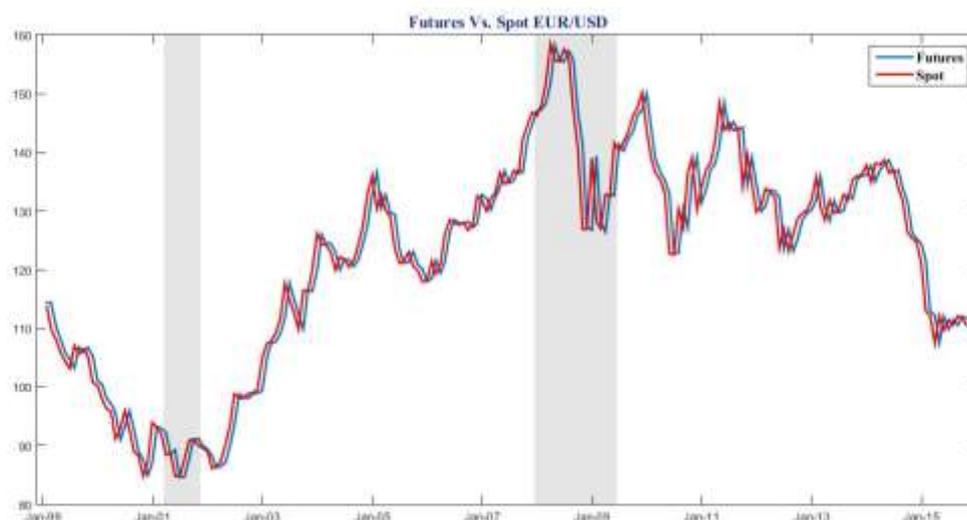


Figure 40 represents the various Beta vectors for Balanced Portfolio.

GARCH with exogenous variables Beta is the more stable series that stays approximately at the same level of the Static Beta.

GJR Beta catches the movements of the Futures Index series with a certain sensitivity in particular in relation to US dollar depreciation, maintaining an overall stability.

Rolling Beta and EWMA Beta, the graph highlights Beta time series and Spot/future time series are negative correlated. The main movements of the Future Indices are caught, with the delay of the two model's initial time window, by the two Betas. The US dollar depreciation periods drive to a lower level of future contracts that the investor should short and, in particular, Beta becomes negative, i.e. the investor should buy Future contracts. The two Beta models reduce the volatility of the Future/spot index at the basis of their construction thanks to the "set up" time window of 36 months. An overall vision permits to understand their behavior connected to Currency trends.

5 Results

5.1 Hedging effectiveness

In order to compare the performance of the optimal hedging ratios obtained from the different models considered I evaluate the so-called hedging effectiveness. It is commonly measured by the variance reduction, more specifically the variance reduction for any hedged portfolio compared with the unhedged portfolio.

Thus, a hedging effective index (HE) is given as:

$$HE = \left[\frac{\sigma_{UH}^2 - \sigma_H^2}{\sigma_{UH}^2} \right] \quad (22)$$

Where σ_H^2 denotes the variances of the hedged portfolio returns r_t^H . The value of HE indicator should be interpreted in relation to its size: higher the hedging effectiveness and larger the risk reduction.

According to Ku et al. (2007) a more accurate model of conditional volatility should also be superior in terms of hedging effectiveness. Among the different hedging strategies proposed, the one with a higher HE indicator is considered the superior hedging strategy.

The following table shows results about hedging effectiveness divided by hedging strategy for each portfolio analyzed.

Table 11 – Hedging effectiveness

	HE (Static Beta)	HE (Rolling Beta)	HE (EWMA Beta)	HE (GJR Beta)	HE (Ex Beta)
Bond Portfolio	22.5637%	20.8018%	22.3235%	20.8364%	22.0930%
Equity Portfolio	0.4116%	5.1943%	4.3046%	-1.1357%	-0.3875%
Balanced Portfolio	1.3126%	4.7967%	5.0585%	-0.0517%	1.1353%

Hedging effectiveness is higher for Bond Portfolio assuming a value over 20% in all hedging strategies implemented. Among the non-naïve methods, EWMA Beta hedge is regard as the superior with HE equal to 22.32 %. Considering Static Beta strategy, that use a unique Beta calculated on the basis of the unhedged Portfolio on the entire period of study, it has the largest

value of HE. This could derive from the particular period chosen. A further analysis could explore the correspondent results of different time windows to investigate how Static Beta Strategy's HE varies.

It is immediately clear how HE largely decreases in Equity Portfolio. This phenomenon is related to the well-known fact that Equity markets are much more volatile than Bond ones. In some cases we even reach negative values of HE, which means hedging strategies adopted failed in their aim of reduce risk. GJR and GARCH with exogenous variables Beta hedged portfolio display larger variance than the related unhedged Portfolio. About Equity Portfolio, the best method seems the one which uses Rolling mean to find the vector of betas, which presents 5.2% HE.

As I have just stated, when equity indices enter the portfolio, hedging effectiveness decreases. This is confirmed looking at the results of Balanced Portfolio. HE related to the different strategies are the most heterogeneous. GJR Beta method gives near-zero HE and also GARCH with exogenous variables method does not seem to be much efficient. On the other hand Rolling mean Beta and EWMA Beta methods show HE near to 5%, the last one is regarded as the superior hedging strategy.

5.2 Sharpe ratio robust test

Risk reduction is not the only driver for the evaluation of hedging strategies applied to the different portfolios.

The package RobustSharpe is a Matlab implementation of the robust Sharpe ratio testing based on Ledoit and Wolf (2008). They suggest constructing a studentized time series bootstrap confidence interval for the difference of the Sharpe ratios of two different strategies.

They propose to test this difference, $H_0: \Delta = 0$, by inverting a bootstrap confidence interval. That is, one constructs a two-sided bootstrap confidence interval with nominal level $(1 - \alpha)$ for Δ . If this interval does not contain zero, then H_0 is rejected at nominal level α .

The advantage of this “indirect” approach is that one can simply resample from the observed data. If one wanted to carry out a “direct” bootstrap test, one would have to resample from a probability distribution that satisfied the constraint of the null hypothesis, that is, from some modified data where the two empirical Sharpe ratios were exactly equal.

I employ this particular method in order to test the null hypothesis " H_0 : Difference of Sharpe ratios is zero" about each Hedged Portfolio versus the Non Hedged Portfolio.

The results of this test gave always the same outcome: I accept the null hypothesis H_0 , that is to say I did not find statistically significant differences in Sharpe ratios of the Hedged strategies with respect to the Sharpe ratio related to the unhedged portfolio.

This is an important point for my analysis proving that, for the period of study and the particular composition of the various portfolios evaluated, the hedging strategies implemented do not reach does not reach a level of improvement such as to justify the efforts for the same implementation of such strategies.

5.3 Mark-to-market

Another important point I do not consider in this work is the mark-to-market component in future contract's price. It is reflected on the possible request of depositing a "margin" when an investor enters a future contract.

In future contract case the broker who propose the contract could require the investor to deposit funds in a margin account. The amount that must be deposited at the time the contract is entered into is known as the initial margin. At the end of each trading day, the margin account is adjusted to reflect the investor's gain or loss. This practice is referred to as daily settlement or marking to market.

The daily settlement is not merely an arrangement between broker and client. When there is a decrease in the futures price so that the margin account of an investor with a long position is reduced by a certain amount, the investor's broker has to pay the exchange and the exchange passes the money on to the broker of an investor with a short position. Similarly, when there is an increase in the futures price, brokers for parties with short positions pay money to the exchange and brokers for parties with long positions receive money from the exchange.

The investor is entitled to withdraw any balance in the margin account in excess of the initial margin. To ensure that the balance in the margin account never becomes negative a maintenance margin, which is somewhat lower than the initial margin, is set. If the balance in the margin account falls below the maintenance margin, the investor receives a margin call and she is expected to top up the margin account to the initial margin level by the end of the next day. The

extra funds deposited are known as a variation margin. If the investor does not provide the variation margin, the broker closes out the position.

A futures contract is in effect closed out and rewritten at a new price each day. Minimum levels for initial and maintenance margins are set by the exchange.

Individual brokers may require greater margins from their clients than those specified by the exchange. However, they cannot require lower margins than those specified by the exchange. Margin levels are determined by the variability of the price of the underlying asset. The higher this variability, the higher the margin levels. The maintenance margin is usually about 75% of the initial margin.

Note that margin requirements are the same on short futures positions as they are on long futures positions. It is just as easy to take a short futures position as it is to take a long one. The spot market does not have this symmetry.

This particular requirement represent first an opportunity cost for the amount that has to be deposit to fill up the margin and it could also come with the erosion of the returns of the portfolio in which hedging strategy is employ.

6 Conclusions

This work investigate the effect of hedging strategies in different Indices Portfolio that consider an investment on Bond indices, on Equity indices and on a mix of the two. The indices are denominated in both US dollar and Euro implying the exposure to US dollar/EUR exchange rate.

As reported in the previous section, Portfolio Analysis and Results, the implementation of hedging strategies based on currency future investment, in particular with the definition of Optimal Hedging Ratios through dynamic models such as GJR or EWMA, gives some evidence of improvement of Portfolios' performance in terms of risk-return profile.

However, as shown by robust tests, the impact of such strategies does not get statistically significant differences in Sharpe ratios of the Hedged Portfolio strategies with respect to the Sharpe ratio related to the unhedged portfolio. This implies non-significance of the benefit in terms of risk reduction and returns improvements.

To this first consideration should be added the costs related to the hedging, both as regards the management of the position in currency futures contracts and the possible margin required to be deposited.

This partly contradicts other studies on hedging in international portfolios that exploit similar approach to determine the part of portfolio to be covered by the use of future or forward contracts that found evidence of significant improvements in international portfolios' performance.

Some clarifications should be provide.

The out-of-sample analysis shown in this work is computed on a time window of about 13 years, which includes period of crisis, recession and recovery. A further analysis could investigate how the different models applied in order to identify the OHR respond in financial turmoil period rather than stable or expansion period.

This could possibly lead the investor to modify the hedging strategy adopted as she manages to correct interpret the characteristics of the period from time to time considered.

Obviously, it is related also to the different Optimal Hedging Ratios that comes from the changing correlation between the unhedged portfolio and currency futures movements from a period to another. Other insights on this field could be conduct to investigate the single effect of currency futures trend on the definition of the level of hedging.

Considering allocation strategy, in this work I assume a monthly allocation: the first day of the month investor sets composition of the Portfolio and the hedging ratio to maintain. Other works in literature, for example Caporin et al. (2013), consider daily updates of the hedging level, with all the impacts that could derive from such a techniques.

A more frequent update of the hedging from one point of view could be more efficient in capturing trend of exchange rate, bond or equity markets or other macroeconomic variables on the basis of the different model adopted for OHR identification.

On the other side, changing currency future allocation involves higher management costs that could also erode any returns' improvement possibly reached.

Eventually, the importance of implementing any hedging strategy is related to the level of exposure on foreign currency denominated investments. In this work, I consider three portfolios that are exposed for 20 % on US- dollar investments: it could be significant studying different international Portfolios with other volume of exposure to foreign currency, and consequently to exchange rate risk.

Investors more often differentiate their portfolios by investing in a number of markets higher than two, as I consider in this work. Thus, another element to consider should be the correlation between different currencies present in the portfolio.

Therefore, a number of consideration could be taken into account in currency hedging decisions. As partly this work and official literature prove there are some evidence of risk reduction and performance improvements for hedged portfolio, but this cannot be the only driver on the choice of whether adopt, and possibly in which measure, a hedging strategy.

In this work I assume passive management of the portfolio where the unique active element is currency risk management. Dynamic models applied on the identification of OHR lead to better performance in risk-return profile than unhedged passive portfolio, hedged portfolio where static models are implemented in the same portfolio.

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