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Cosmological measurements with unequal time galaxy correlations

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Abstract

Current and forthcoming cosmological surveys will provide measurements of galaxy clustering with unprecedented precision. This means that data will need to be compared to theoretical predictions of comparable accuracy.

Currently, the statistical modeling used to analyze data makes use of several simplifying assumptions, aimed at reducing computational complexity, but at the risk of reducing the accuracy.

In this thesis we investigate the impact of unequal time measurements of 2point galaxy correlations and how this could affect cosmological model testing for future galaxy surveys.

INTRODUCTION

Cosmology is the science that studies the origin and development of the universe. It relies on understanding the distribution and behaviour of matter and energy across the entire cosmic history.

In this thesis, we examine the matter power spectrum for different tracers of the large-scale structure, focusing on the impact of unequal-time corrections. Unequal-time correlations represent one new step in the refinement of our cosmological models and improve the precision of upcoming cosmological surveys, offering new insights into the fundamental properties of the universe. By exploring these corrections, we aim to improve our understanding of the cosmic matter distribution at linear scales. We will consider the theoretical framework that governs these unequal-time correlations and evaluate their impact on the power spectrum across different tracers. This in turn informs our understanding of fundamental cosmological parameters, including the Hubble constant, the matter density and the nature of dark energy.

We will focus on understanding the matter power spectrum for different tracers of large-scale structure. Tracers are observables that map the underlying matter distribution; some common tracers are galaxies, quasars and the Cosmic Microwave Background. Since these tracers may not directly follow the distribution of matter due to the complexities of biasing, it is essential to model their relationship with the dark matter field carefully. The study of these tracers offers valuable insights into both the distribution of matter and the processes that have shaped the Large-Scale Structure (or “LLS”).

One of the key challenges in analysing the matter power spectrum is accounting for the intricate dynamics that govern the evolution of the universe. A significant area of interest has been the study of unequal-time correlations. These corrections arise when considering the evolution of the matter power spectrum across different cosmic epochs. The standard cosmological model assumes a linear approximation of these correlations, such as those stemming from unequal-time correlations, are considered. These corrections are important for more accurate predictions of LSS surveys, including galaxy redshift surveys and future missions like the Euclid and LSST surveys.

Our central goal is studying how unequal-time corrections impact the constrains on the most important cosmological parameters assuming a low level of Primordial non-Gaussianity.

THEORY

In the minimal cosmological scenario, the universe features several species with spatial fluctuations, described with different equations because of their distinct properties. Cold dark matter (CDM) is non-relativistic and collisionless, neutrinos are ultra-relativistic and collisionless at the times of interest, baryons are non-relativistic and smoothly interpolating from a strongly coupled to decoupled regime and finally photons are ultra-relativistic and interpolating between the same two regimes.

BACKGROUND

The standard cosmological model assumes as valid the cosmological principle, which states that the mass distribution in the Universe is statistically homogeneous and isotropic on large scales (beyond hundreds of Mpc). This postulate allows to write the line element in an universe maximally symmetric by using Friedmann-Lemaıtre-Robertson-Walker (FLRW) metric expressed in spherical polar coordinates

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2\right]$$

The constant k depends on the curvature of the universe: it is k = 0 for the case of flat space, k > 0 for a closed space and k < 0 for an open space, while t is the proper time.

The presence of a dark energy component in the energy density of the Universe modifies the gravitational growth of large scale structures. We can trace Large-scale structure by considering galaxies as tracers; the rate at which structure grows from small perturbations offers a key discriminant between cosmological models, as different models predict measurable differences in the growth rate of large-scale structure with cosmic time. The distribution of galaxies contributes to grow overdense regions and empty underdense regions.

In linear perturbation theory, it is possible to describe the growth of a generic small amplitude density fluctuation through a second-order differential equation, when the linearised continuity, Euler and Poisson equations are combined, the result being a second order linear differential equation for the overdensity δ_m:

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G\rho\delta_m = 0$$

The physical interpretation of this equation is that the perturbations grow according to a source term involving the amount of matter able to cluster and are diluted by a term arising from the expansion of the universe. When further combined with the unperturbed solution for the evolution a(τ) of the cosmic scale factor, it leads to growing and decaying solutions:

$$\delta_m(r,a) \sim D(a)$$

which evolve in time without change of shape. The interesting solution is the unstable, growing solution, and *D(a)* is the linear growth factor of the growing mode:

$$\frac{Haf}{H_0 a_0}\delta_m + \nabla \cdot \boldsymbol{v}_m = 0$$

where f is the dimensionless linear growth rate of the growing mode. It measures how rapidly structures in the Universe are growing as a function of cosmic time or redshift and it is given by the logarithmic derivative of the linear growth rate with respect to the scale factor:

$$f \equiv \frac{H_0 a_0}{Ha}\frac{d\ln D}{d\tau} = \frac{d\ln D}{d\ln a}$$

And *D* is the linear growth factor of the growing mode.

Now we need to introduce the important concept of bias. This is a factor that accounts for the fact that galaxies form in dark matter halos and don't trace perfectly the underlying matter distribution; thus, when we observe a galaxy distribution we need a model to relate that distribution of visible matter to the distribution of the mass. The simplest model of bias postulates that the galaxy overdensity δ is linearly biased by a constant factor, the linear bias factor b, relative to the underlying mass density δm, so that:

$$\delta = b\delta_m$$

The linearised continuity equation for the matter, evaluated at the present time, together with the linear bias model, yield the linearised continuity equation for galaxies

$$\beta\delta + \boldsymbol{\nabla}\cdot\boldsymbol{v} = 0$$

where the dimensionless quantity β is related to the value of the linear growth rate f, Equation and the bias factor b, by:

$$\beta = \frac{f}{b}$$

In redshift surveys we typically measure this parameter but if we assume the linear bias model, to obtain f we only have to rescale the measured value of β by the constant b. For this reason in what follows we express the redshift space correlation function, and all the quantities related, in function of the growth factor f, that is more useful for our purpose to study models of gravity.

POWER SPECTRUM

The statistical properties of the overdensity field are completely determined by its irreducible moments, or n-point correlation functions. If fluctuations generated in the early Universe were the result of a superposition of many independent random processes, as for example quantum fluctuations during inflation, then the Central Limit Theorem guarantees that the fluctuations will form a multivariate Gaussian field. In the case of a multivariate Gaussian random field by definition all correlation functions of order higher than the third must vanish, and so the first two moments completely specify the statistical properties of the field. Fluctuations on large, linear scales today would then also be Gaussian. The first two irreducible moments are the mean, a constant in space, and the covariance, or 2 -point correlation function, that is a function of separation. For this reason, the most basic and interesting statistic that can be constructed from the overdensity field is its variance, its second irreducible moment, or the correlation function $\xi(r_{12})$, defined as:

$$\xi(r_{12}) \equiv \langle\delta(\boldsymbol{r}_1)\delta(\boldsymbol{r}_2)\rangle$$

This equation states that the expectation value of the product of overdensities at a pair of randomly positioned points separated by r_{12} in the Universe is $\xi(r_{12})$.

The correlation function is the Fourier transform of the power spectrum, that by definition is the covariance of Fourier modes. Since the correlation function $\xi(r_{12})$ is a function only of separation, we can write for the power spectrum:

$$P(k) \equiv \int e^{i\boldsymbol{k}\cdot\boldsymbol{r}}\xi(r)d^3r$$

The fundamental advantage of working in Fourier space (and so with the unredshifted power spectrum) is that the Fourier modes of a statistically homogeneous random field are uncorrelated: their covariance matrix is a diagonal matrix.

MATTER POWER SPECTRUM

We can start by looking at the component of the total energy perturbation in the universe:

$$\delta\rho_{tot} = \delta\rho_b + \delta\rho_{cdm} + \delta\rho_\gamma + \delta\rho_\nu + \delta\rho_{tot} + \delta\rho_{DE} + \cdots$$

While the dots account for extra relics, possible Dark Energy (DE) perturbations are represented by the last term; but in the minimal ΛCDM model, only the first four components are present.

Many Large-Scale Structure (LSS) observables are related to the power spectrum $\delta\rho_{tot}$, at different wavenumbers and redshifts. All these observations probe the power spectrum during matter or dark energy domination, when photons are subdominant and $\delta\rho_\gamma$ can be neglected. If neutrinos are still ultra-relativistic today, $\delta\rho_\nu$ can also be neglected. So we are left only with two dominant components: baryons and cold dark matter. Only in models with large dark energy perturbations or modifications of Einstein gravity, it might be important to consider also the other components of the total energy perturbation.

For Gaussian initial conditions and as long as perturbations are linear, the power spectrum at a given time can be written as the product of the primordial spectrum by the square of the relevant transfer function. So, assuming a power-law primordial spectrum, we have:

$$P(z,k) = \frac{2\pi^2}{k^3}A_s\left(\frac{k}{k_*}\right)^{n_s-1}\delta_m^2(z,k)$$

Today we know that the matter balance of our universe is prevalently constituted by CDM, and secondary by baryons; so we can first assume to live in a ΛCDM universe with a negligible amount of baryons:

$$\Omega_{cdm} \gg \Omega_b \ \ and \ \ \delta_m \simeq \delta_{cdm}$$

Then, by combining the continuity and Euler equation of CDM perturbations, we can obtain an equation of motion of this form:

$$\delta''_{cdm} + \frac{a'}{a}\delta'_{cdm} = -k^2\psi + 3\phi'' + 3\frac{a'}{a}\phi'$$

The second term on the LHS is often called "Hubble friction term"; the name refers to the effect of the Hubble expansion on the clustering rate: expansion increases distances, weakens gravitational forces, and slows down clustering processes. On the RHS, the first term represents gravitational forces, and the last two terms account for dilation effects (namely for local distortions of the scale factor with respect to *a*).

If we assume to stay inside the Hubble radius, we can neglect dilation terms. Using this approximation and replacing the gravitational potential term, we obtain the Meszaros equation:

$$\delta''_{cdm} + \frac{a'}{a}\delta'_{cdm} - \frac{3}{2}\left(\frac{a'}{a}\right)^2 \Omega_{cdm}(a)\,\delta_{cdm} = 0$$

It turns out that, under our assumption on the ratio ${}^{\Omega_{cdm}}/_{\Omega_b}$, the above equation is a very good approximation for the CDM equation of evolution in all regimes; it applies even during radiation domination, when photon fluctuations are large.

By solving the Meszaros equation at different epochs, we can look at the evolution of the matter density perturbation during the entire history of the universe. In summary, during radiation domination, $\delta_{cdm}(\eta, k)$ is constant on super-Hubble scales and grows logarithmically on sub-Hubble scales. A more precise calculation would show that up to a numerical factor of order one, δ_{cdm} is given on sub-Hubble scales by $\delta_{cdm}(\eta, k) = \log(\eta k)$. During matter domination, it is still constant on super-Hubble scales, and grows like η^2 on sub-Hubble scales. Finally, during Λ domination, it grows more slowly. These different behaviours are reported in the next figure.

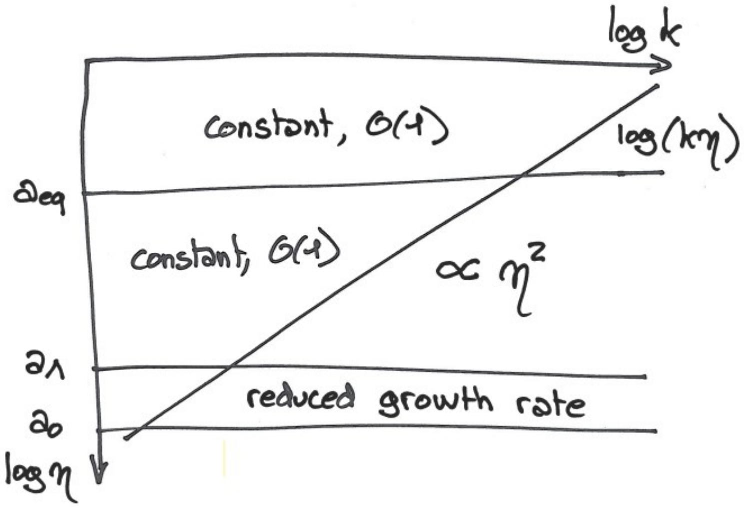

Figure 1 – Refers from: Lesgourgues J., Cosmological perturbations

Now it's easy from our results to infer the shape of the matter power spectrum at different times.

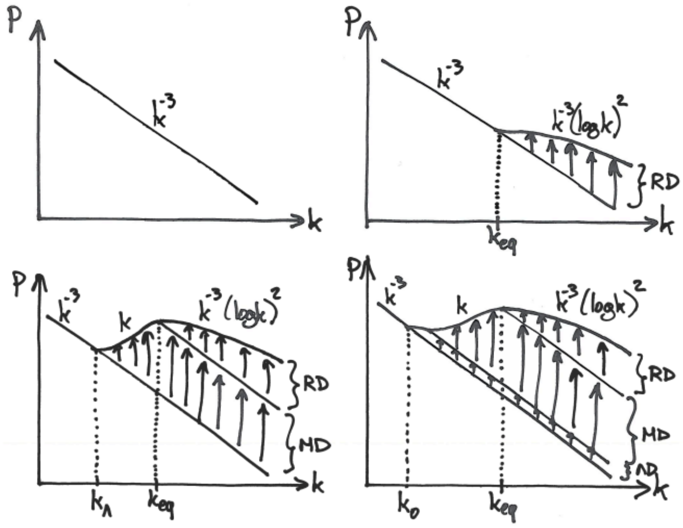

Figure 2 – Refers from: Lesgourgues J., Cosmological perturbations

All this discussion was carried under the assumption of a scale-invariant primordial spectrum. If the spectral index differs from 1 (which is the spectral index of the "Harrison-Zel'dovich spectrum"), the above shape should simply be rescaled by a factor $k^{n_s - 1}$, and the three branches of the power spectrum are given by:

$P(k) \sim k^{n_s - 4}$ if $k < k_0$

$P(k) \sim k^{n_s}$ if $k_\Lambda < k < k_{eq}$

$P(k) \sim k^{n_s - 4} (\log k)^2$ if $k > k_{eq}$

The matter power spectrum quantifies the distribution of matter fluctuations in the universe, providing insight into the underlying physics that governs cosmic growth, from the early universe to the present day. This discussion was carried under the assumption of a scale-invariant primordial spectrum and in the limit of negligible Ω_b. Now we want to add the so called "baryon corrections" by considering a small but non-negligible baryon fraction.

BARYON ACOUSTIC OSCILLATIONS

Before photon decoupling, cosmological perturbations excite sound waves in the photon-baryon fluid; after recombination, these oscillations became frozen into the distribution of matter in the Universe. The smoking gun in the recent universe of this process are called Baryon Acoustic Oscillations (or "BAO"). This imprint can be seen in the power spectrum of matter at large *k* (so on scales much smaller than the radius of the current Hubble radius).

The BAO pattern can be obtained by perturbing the matter power spectrum well after the baryon drag time, when baryons and CDM perturbations have reached gravitational equilibrium: if Ω_b is smaller than Ω_{cdm} but not negligible (as it is the case in our universe), the power spectrum departs slightly from a pure CDM one, with a smooth step-like suppression plus the well-known small oscillatory patterns.

At the time when photon-baryon oscillations were frozen, a preferred scale was selected: the sound horizon. This defines a standard ruler whose length is the distance sound can travel between the Big Bang and recombination (~150 Mpc commoving). The underlying physics which sets this scale is well understood and involves only linear perturbations in the early Universe. Since photons decouple at η_{dec} the scale of oscillations on the last scattering surface is set by the scale (comoving) $rs(\eta_{dec}) = \frac{ds(\eta_{dec})}{a(\eta_{dec})}$. In the same way, since baryons decoupled at η_{drag} (or "baryon drag time"), the scale of BAOs depends on $rs(\eta_{drag})$. Now, we know that the sound horizon at a given time depends on ω_b and ω_m. In the case of $ds(\eta_{drag})$ the dependence on ω_b is even stronger because the time of baryon drag itself depends strongly on the baryon abundance.

The BAO are directly observed in the CMB angular power spectrum. The angular size of the oscillations in the CMB revealed that the Universe is close to flat. These observations scale in a statistical distribution of galaxies over a large range of redshift are a powerful measurement tool to probe the expansion of the Universe, which in turns is a crucial handle to constrain the nature of dark energy. An unprecedented experimental effort is undergoing to obtain galaxy surveys that are deep, larger and accurate enough to trace the BAO feature as a function of redshft. Before these surveys can even be designed it is crucial to know how well a survey with given characteristic will do.

LINEAR REDSHIFT SPACE DISTORTIONS

Hubble's law (1929) states that the recession velocity of a galaxy is proportional to its distance with constant of proportionality equal to the Hubble constant (its present day value). The recession velocity of a galaxy can be measured from the redshift of its spectrum; this has been a primary motivation for redshift surveys, which map the universe in 3-dimensions using the recession velocity of each galaxy as a measure of its distance. Hubble's law, however, is not perfect: galaxies have peculiar velocities v relative to the general Hubble expansion. So it is necessary in general to distinguish between a galaxy's redshift distance $s = cz$ and its true distance $r = H_0 d$, so that we have $s = r + v$.

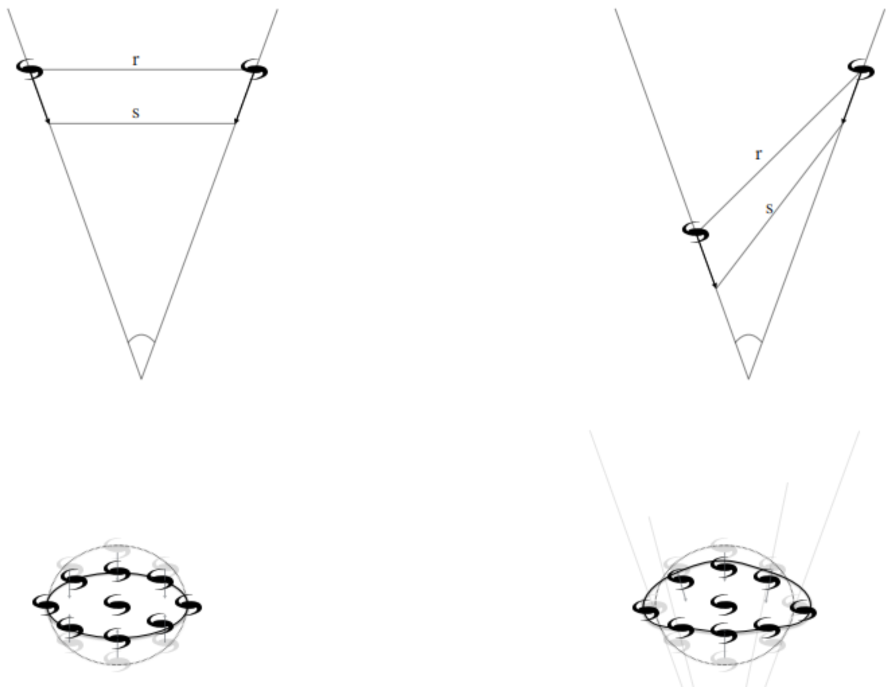

Figure 3 – Refers from: Raccanelli A. et al., Doppler term in the galaxy two-point correlation function: wide-angle, velocity, Doppler lensing and cosmic acceleration effects

The peculiar velocities of galaxies cause them to appear displaced along the line of sight in redshift space. These displacements lead to redshift distortions in the pattern of clustering of galaxies in redshift space. Redshift space distortions arise due to the fact that we observe galaxies not in their true, real-space positions, but rather in the redshift space, where their radial distance is inferred from their redshift. This distortion affects the apparent positions of galaxies along the line of sight, especially in large-scale structure surveys.

Although such distortions complicate the interpretation of redshift maps as positional maps, they have the important advantage of bearing information about the dynamics of galaxies. In particular, the amplitude of distortions on large scales yields a measure of the linear redshift distortion parameter β, which is related to the cosmological density Ω_0 (the present day ratio of the matter density of the Universe to the critical density required to close it) by:

$$\beta = b\, f(\Omega_0) \simeq b\, \Omega_0^{\gamma}$$

In which b is the light-to-mass bias and we assume a power law for the growth rate f with and index γ (fiducial at 0.55). The goal of much of the current work on redshift distortions is to measure this distortion parameter and to determine the cosmological density itself.

Linear RSD refers to the first-order (linear) effects of these distortions, which primarily arise from the peculiar velocities of galaxies, i.e. the velocities of galaxies relative to the Hubble flow, caused by gravitational infall or motion due to large-scale structure growth. The most famous linear RSD effect is Fingers of God (FoG): a squashing of structures along the line of sight due to the random motions (peculiar velocities) of galaxies within large-scale structures (like clusters).

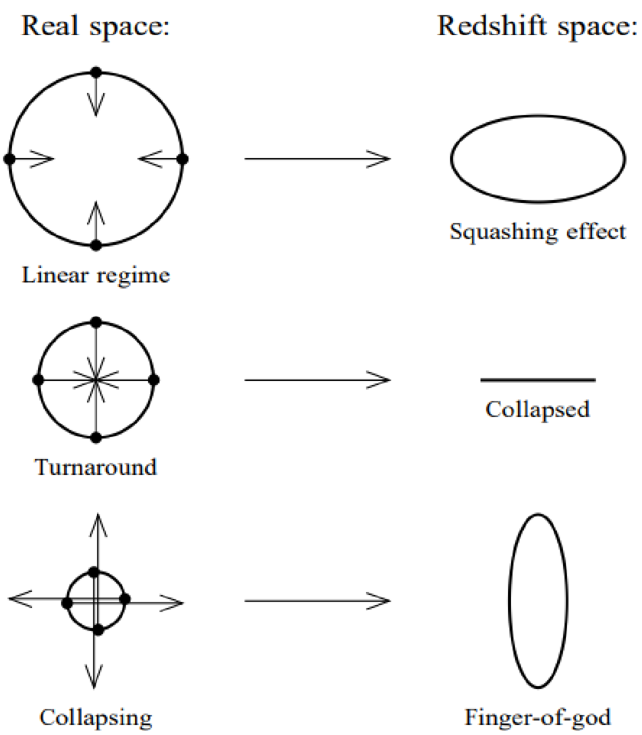

Figure 3 – Refers from: Hamilton A. J. S., Linear redshift distrotions: a review

The relation between redshift and radial comoving distance, and between angle and transverse comoving distance, is different for different cosmological models. The differences produce a cosmological redshift distortion that is zero at zero redshift, but that becomes more marked at higher redshift. The idea of using cosmological redshift distortions to measure cosmological parameters, notably the cosmological constant was firstly proposed by Alcock & Paczynski. So there is a geometric distortion caused by our peculiar motion through the universe, which can also affect the measurement of distances in redshift space. This cosmological distortion (which is not easy to distinguish from redshift distortions caused by peculiar velocities) constitutes a squashing in the radial direction that occurs if there is a large cosmological constant.

The linear RSD model accounts for how the peculiar velocities of galaxies distort their observed positions and is usually described by a differential equation that includes the growth rate of cosmic structures (how structures grow due to gravitational instability), with the key parameter being the redshift-space distortion parameter. This relies on the assumption that the growth of large-scale structure can be described in a linear regime where we can apply a linear bias model.

Redshift-space distortions by peculiar velocities were first computed by Kaiser leading to the well-known formula for the matter density fluctuation:

$$\delta_g(\boldsymbol{k}, z) \simeq D(z)\,[b(z) + f(z)\mu^2]\,\delta_m(\boldsymbol{k})$$

This formula is approximate in a variety of ways, which must be improved as precision of measurements improves. The so-called Doppler term arises from the combination of two effects: a geometrical modification to the RSD operator and the Doppler lensing. The Doppler lensing terms account for apparent modifications of angular size and magnitude of a galaxy, due to the motion of the source galaxy with respect to the observer. This effect originates from the aberration effect in special relativity. Galaxies with the same (apparent) redshift space position can be, in real space, more distant (if they are moving toward us) than galaxies with no peculiar velocity, or closer to us (if they are moving away from us). This generates two opposite effects on the observed angular size of the galaxy: a galaxy that is more distant in real space is observed under a smaller solid angle, but its photons were emitted at an earlier time, when the value of the scale factor of the universe was smaller. Hence, those photons experienced a larger stretch in their path toward us, which increases the observed angular size of the galaxy. The Doppler term is represented by:

$$\alpha(b_e, Q, z) = -\frac{H(z)\,\chi(z)}{(1+z)}\left[b_e(z) - 1 - 2Q(z) + \frac{3}{2}\,\Omega_m(z) - \frac{2\,(1 - Q(z))\,(1+z)}{H(z)\,\chi(z)}\right]$$

Finally, on large scales there are relativistic effects. Their extra contribution to the standard galaxy power spectrum is suppressed by k_c^{-2}, where $k_c = \frac{ck}{aH}$, and is thus effectively limited to the few largest-scale modes and very difficult to detect; however, a correlation (k^{-1}) is generated between the real and imaginary parts of the Fourier space density fields of two different types of galaxy, which would otherwise be zero, so that the cross-power spectrum has an imaginary part. The pure relativistic term is represented by:

$$A(b_e, Q, z) = \left(\frac{H(z)}{(1+z)}\right)^2 \frac{3}{2}\,\Omega_m(z)\,B(b_e, Q, z)$$

$$B(b_e, Q, z) = b_e(z)\left(1 - \frac{2f(z)}{3\,\Omega_m(z)}\right) + 1 + \frac{2f(z)}{\Omega_m(z)} + \frac{3}{2}\,\Omega_m(z) - \frac{2\,(1 - Q(z))\,(1+z)}{H(z)\,\chi(z)} - 4Q(z) - f(z)$$

In conclusion, we need to remember that linear redshift space distortions are a first-order approximation that assumes linear growth of cosmic structures and using peculiar velocities to model galaxy positions in redshift space.

WHAT DO WE OBSERVE?

The crucial point is the difference between the usual theoretical power spectrum (defined as the ensemble average of two galaxy overdensity fields):

$$\langle \delta(\boldsymbol{k}, t), \delta(\boldsymbol{k}', t') \rangle = (2\pi)\,\delta^3(\boldsymbol{k} - \boldsymbol{k}')\,P(\boldsymbol{k}, t, t')$$

and the observed power spectrum (which is a reconstruction of the modes along the line of sight):

$$P_{obs}\left(k_n, \frac{\boldsymbol{l}}{\bar{\chi}}, \bar{\chi}\right) = \frac{1}{\bar{\chi}^2}\int e^{-i\delta\chi k_n}\,C^s(\boldsymbol{l}, \bar{\chi}, \delta\chi)\,d\delta\chi$$

The first one (unequal-time, or "U-T") isn't observable because it is properly defined only in a fully 4D space and could be accessible only by what we call a "meta-observer" outside of the system (see reference "Raccanelli A. et al., *The observed power spectrum & frequency-angular power spectrum"*).

The second one (equal-time, or "E-T") is what we measure and it folds the unequal-time information around the average position into the wave modes along the line of sight. We want to unpack this power spectrum to recover the important piece of information which is folded inside the radial (which is also the temporal) direction; this is done by studying the inclusion of unequal-time correlations to the angular power spectrum. In particular we derive an expansion around the equal time case. Relying on linear theory calculations, we estimate that unequal-time corrections, while generally small, can amount to a few percent on large scales and high redshifts (where also Doppler terms and local general relativistic corrections become non-negligible). Interestingly, such corrections depend on the bias of the tracers, the growth rate, but also their time derivatives, opening up the possibility of new tests of cosmological models. These radial mode effects also introduce anisotropies in the observed power spectrum, in addition to the ones arising from redshifts-pace distortions, generating non-vanishing odd multiples and imaginary contribution.

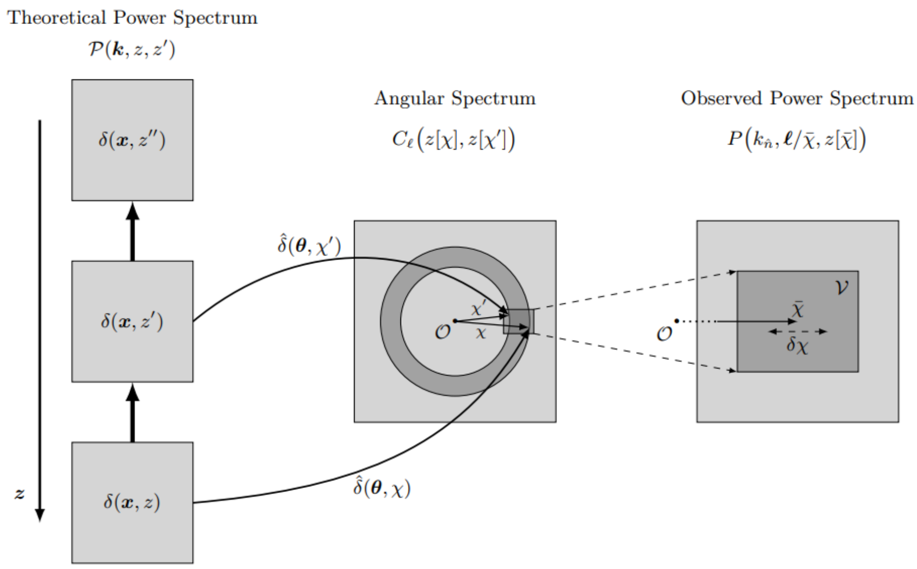

Figure 4 – Refers from: Raccanelli A. et al., The observed power spectrum & frequency-angular power spectrum

The fact that we only observe only our background lightcone and that we do not truly know the distance of the observed galaxy but only its redshift is not only an additional difficulty, but even more a new opportunity for future galaxy surveys. We have to map our light-cone observables into the underlying physical densities, velocities, and other relevant quantities; relating the physical densities of sources to underlying cosmological parameters is then a separate further problem requiring detailed understanding of the source population and the bias.

Using an appropriate comparison (see reference "Raccanelli A. et al., *Power spectrum in the cave"*), the situation is similar to Plato's "allegory of the cave": as the prisoners can only see shadows (projections) of the real world, we can't observe directly our targets so we have to extract all possible information from what we measure. More precisely, when we observe correlators of the Large-Scale Structure of the Universe from our fixed point of view, we need to take into account for these projection effects, linking them to the real motion of galaxies.

WHICH GAUGE DO WE CHOOSE?

The perturbation of any quantity in a given point is the difference between the true and the average quantity in this point; for example, for the total energy density ρ we have:

$$\rho(\vec{x},t) = \rho_0(t) + \delta\rho(\vec{x},t)$$

While $\rho(\vec{x},t)$ is a locally, unambiguously defined quantity, $\rho_0(t)$ depends on the choice of equal-time hypersurface going through the point (t, ~x). With a different choice, $\rho(\vec{x},t)$ would be compared to the average performed on a different sheet and would take a different value. Hence $\delta\rho(\vec{x},t)$ also depends on the choice of time slicing. A gauge is a choice of time slicing. Gauge transformations are induced by coordinate transformations xμ 7→ xμ+μ mapping the points of one time slicing to those of another time slicing. Not all coordinate transformations induce a valid gauge transformation: the condition that perturbations must still be linear after the transformation restricts μ to be small in every point.

In an idealised FL universe, there is only one time slicing compatible with the assumption of homogeneity. Instead, in a perturbed universe, there are infinites choices of time slicing compatible with the theory. Up to now galaxy surveys has not to deal too much with the so called cosmological gauge problem since, for observations well within our Hubble volume today, gauge effects are small (at large wavenumbers), and cosmic variance is large. In reality when comparing observations of the matter power spectrum to theory, we should ask ourselves: in which gauge should the perturbations be calculated?

The density fluctuation δ(x, t) which we calculate in a given Friedman background isn't gauge invariant. We derive gauge invariant expressions which are correct to first order in perturbation theory and which are straightforward to compare with observations. This is an important first step for this problem as the gauge issue is mainly relevant on very large scales, where perturbations are small so that first order perturbation theory is justified. In our treatment, we will use the synchronous commoving gauge.

UNEQUAL-TIME CORRECTIONS

Unequal-time corrections (UTCs) arise when we consider the evolution of matter and tracers over different time slices in the universe. In standard cosmological models, one typically assumes that the power spectrum of the matter density field is taken at a single time slice, or that the correlation between two points in space is calculated at the same time. However, in reality, the universe evolves over time, and this evolution means that the correlations between two points should be evaluated consistently; for this reason, in the context of unequal-time corrections, the power spectrum becomes a function of two different times. These corrections are derived from the fact that the matter at a given redshift may have evolved from perturbations at earlier times, and this evolution influences the observed clustering, including in redshift space.

In UT correlators, we account for the fact that, in a growing structure, the density fluctuations at two points in space evolve differently at different times. UT correction's inclusion modifies the interpretation of observational data, particularly for large-scale structure surveys like galaxy redshift surveys. In the standard linear RSD model, only first-order peculiar velocities and the linear growth rate are considered. These corrections can change the apparent strength of the redshift-space distortions and lead to a more precise determination of cosmological parameters. Incorporating unequal-time corrections into the redshift-space power spectrum allows for a more accurate and comprehensive model of the evolution of large-scale structure.

Now we are going to build the observed power spectrum in two steps:

- projecting the two-point correlator on the sky to obtain the angular power spectrum
- folding the unequal-time information into the Fourier modes along the line of sight

Let's start by the definition of the observed galaxy overdensity in the synchronous gauge (or "time-orthogonal gauge"):

$$\delta_g(\boldsymbol{k}, z) = D(z)\left[b(z) + f(z)\mu^2 - \frac{i\,\mu\, f(z)\,\alpha(b_e, Q, z)}{k\,\chi(z)} + \frac{A(b_e, Q, z)}{k^2}\right]\delta_m(\boldsymbol{k})$$

The third term is the Doppler term or velocity term, which, including general relativistic effects, while the last term is a purely relativistic contribution.

For given value of b_e and Q, A and B are only functions of redshift, and incorporate the relativistic bias as well as the volume distortion and magnification effects. On small scales ($k \gg 1$), we recover the usual Fourier space galaxy overdensity of the Newtonian theory that we write in EQ.

In order to build the observed angular power spectrum, we write down the projected density field:

$$\hat{\delta}^s(\boldsymbol{l}, z) = \int \frac{d\chi}{\chi^2}\, W(\chi) \int \frac{dk_n}{2\pi}\, \delta\left(k_n \boldsymbol{n}, \frac{\boldsymbol{l}}{\chi}, z\right) e^{ik_n\chi} \left[b(z) + f(z)\mu^2 - \frac{i\,\mu\, f(z)\,\alpha(b_e, Q, z)}{k\,\chi(z)} + \frac{A(b_e, Q, z)}{k^2}\right]_{k=\sqrt{k_n^2 + (\boldsymbol{l}/\chi)^2}}$$

Now we can compute the two-point correlator of this object (we remember that this is directly linked to the theoretical power spectrum) as:

$$\langle \hat{\delta}^s(\boldsymbol{l}, z), \hat{\delta}^{s*}(\boldsymbol{l}, z)\rangle = \int \frac{d\chi}{\chi^2}\frac{d\chi'}{\chi'^2}\, W(\chi)W'(\chi') \int \frac{dk_n}{2\pi}\, (2\pi^2)\, \delta_D^{(2)}\left(\frac{\boldsymbol{l}}{\chi} - \frac{\boldsymbol{l}'}{\chi'}\right) P(k, z, z') \left[b_A(\chi) + f(\chi)\mu^2 - \frac{i\,\mu\, f(\chi)\ \alpha_A(\chi)}{k\,\chi} + \frac{A_A(\chi)}{k^2}\right]\left[b_B(\chi') + f(\chi')\mu^2 - \frac{i\,\mu\, f(\chi')\ \alpha_B(\chi')}{k\,\chi} + \frac{A_B(\chi')}{k^2}\right] e^{ik_n\chi}$$

Then we recognize in the RHS the flat-sky unequal-time angular power spectrum Cs:

$$C^s(l, \bar{\chi}, \delta\chi) = \frac{1}{\chi\chi'} \int \frac{dk_n}{2\pi}\, P(k_n\boldsymbol{n}, \boldsymbol{k}_\perp, \bar{\chi}, \delta\chi) \left[b_A(\chi) + f(\chi)\mu^2 - \frac{i\,\mu\, f(\chi)\ \alpha_A(\chi)}{k\,\chi} + \frac{A_A(\chi)}{k^2}\right]\left[b_B(\chi') + f(\chi')\mu^2 - \frac{i\,\mu\, f(\chi')\ \alpha_B(\chi')}{k\,\chi} + \frac{A_B(\chi')}{k^2}\right] e^{ik_n\chi}$$

This represent the "flat-sky version" of the full angular power spectrum.

Now remembering the definition of the ET observed power spectrum and expanding the matter power spectrum and the kernels around the equal-time case, we obtain the link between the observed power spectrum in redshift space and the theoretical power spectrum:

$$P_{obs}\left(k_n, \frac{l}{\bar{\chi}}, \bar{\chi}\right) = D(\bar{\chi})^2 \sum_{n=0}^{\infty} \left(i\mu H(\bar{\chi}) \frac{d}{dk}\right)^n \left(c_n(k, \mu, \bar{\chi})\, P_0(k)\right)$$

In which $P_0(k)$ is the power spectrum calculated at redshift equal to 0.

This is the fundamental equation describing the observed power spectrum including UT corrections. The coefficient c_0 contains the equal-time kernels, while the other c_n are for UT corrections.

When considering this power spectrum (as any observed quantity), we should always think at the effect of the introduction of an observer: it breaks isotropy. In this case the observer can avoid its position in space affecting its observations, so that the radial direction is degenerate with the time's one.

The last equation is a reconstruction of the modes along the line of sight; this is done by folding the UT information contained in the angular power spectrum. Note that the obtained power spectrum by definition assumes the plane-parallel approximation, which allows us to ignore the problem of dealing with the redshift distortion produced (or rather removed) by the motion of the Local Group; so all k-modes will be projected along the same line of sight. In the plane-parallel approximation redshift distortions do preserve statistical homogeneity and structures are sufficiently far away that line-of-sight peculiar velocities are effectively plane-parallel; we could say it assumes the flatness of the sky. It is similar but not strictly the same of the distant observer limit. In future, there will be an extension of this formalism that including wide-angle corrections in Fourier space.

What we have done is to add to the ET power spectrum the first order correction in $\delta\chi$; this can be found from the last equation and has the following form (from now on we refer to the mean redshift $\bar{z}$ as z):

$$D(z)^{\,2}\left(i\mu H(z)\frac{d}{dk}\right)\left(c_1(k,\mu,z)\, P_0(k)\right)$$

In this expression the coefficient $c_1(k, \mu, z)$ is constituted by a sum of nine $c_{1\,xy}(z)$ coefficients that depend on the tracers and that can be divided in two groups; the *x* and *y* subscripts represent respectively the power of μ and the power of $\frac{H}{k}$ that multiplies that coefficient in the sum. We have chosen the arithmetic mean $\bar{\chi} = \frac{1}{2}(\chi_A + \chi_B)$ so that we have $\delta\chi = (\chi_A - \chi_B)$ and $\frac{d\chi_{A,B}}{d\delta\chi} = \pm\frac{1}{2}$: with this choice of mean distance, the coefficient c140 vanishes due to equivalence principle, since it only receives contributions from the $f\mu^2$ terms of the kernels.

In the absence of Doppler and general relativistic terms, this first order unequal-time correction would vanish in the single-tracer case, while in the multi-tracer case it would be purely imaginary, and sourced by the first set of coefficients. When including Doppler terms and general relativistic corrections, instead, the second set of coefficients is non-vanishing, even in the single-tracer case, giving rise to a real part as well.

PRIMORDIAL NON-GAUSSIANITY

The Gaussian distribution is the maximum entropy distribution for a given mean and variance. In the absence of additional information about the initial conditions of the universe, assuming a Gaussian distribution is the "most unbiased" choice. This is a crucial assumption in modern cosmological models and it has several motivations rooted in both theoretical and observational considerations. However, not all models of the early universe predict purely Gaussian fluctuations and certain aspects of inflationary models suggest that the primordial fluctuations could display non-Gaussian characteristics: these are called Primordial non-Gaussianities (or "PNGs").

As we said in the introduction, UT corrections are important on large scales; at these scales, we can also look for the imprint of local-type Primordial non-Gaussianity. Usually Φ is equal to the Newtonian gravitational potential ϕ in the Poisson Gauge (or "orthogonal zero-shear gauge"), but deviation from the Gaussian hypothesis can be introduced modifying the expression of the Bardeen's gauge-invariant potential through a new dimensionless parameter f_{NL}:

$$\Phi = \phi + f_{NL}(\phi^2 - \langle\phi^2\rangle)$$

In this new context, Φ coincides with ϕ only on sub-horizon scales, so that the Newtonian gravitational potential turns out to be the Gaussian part of the Bardeen's gauge-invariant potential.

The parameter f_{NL} (fiducial at 0.1) enters also in the expression of the halo bias, making it scale-dependent:

$$b(k,z) = b_G(z) + b_{NG}(k,z) = b_G(z) + f_{NL}\, b_\phi(z)\, \frac{3\Omega_{m,0}H_0^2}{2k^2T(k)D(z)}$$

What we have done here is to rename the previous linear bias $b(z)$ as $b_G(z)$ (we could call it "Gaussian linear bias"), while the effect of PnG have been encapsulated in the new "non-Gaussian bias" $b_{NG}(k,z)$. Differences between the general-relativistic and Newtonian predictions are in most cases below cosmic variance, though potentially a source of bias in cross-correlation studies. For the study of scale-dependent halo bias (as a probe of primordial non-Gaussianity) using an analysis of multiple biased tracers, small corrections may be more important since there is only one underlying cosmological perturbation, and hence in principle no cosmic variance on the difference between source counts.

For $b_\phi(z)$, we assume the universality relation:

$$b_\phi(z) = 2\delta_c(b_G(z) - 1)$$

In which δ_c is the critical value of the matter overdensity for spherical collapse in an Einstein-de Sitter universe (equal to 1.686).

The non-Gaussian bias leads to some additional UT corrections through the coefficient $c_{1,NG}(k,\mu,z)$; again this is constituted by a sum of $c_{1\ xy,NG}(z)$ coefficients that depend on the tracers, with the same meaning of the subscripts as before.

The scale of inflation is a most uncertain parameter and can range across orders of magnitude without contradicting current observations. If inflation takes place at the highest energies, significant efforts in trying to detect primordial gravitational waves will triumphantly determine this scale. However, if inflation takes place at lower energies, Primordial non-Gaussianity will be our unique source of information as, unlike gravitational waves, their amplitude does not diminish with energy (deviations from Gaussianity directly translate into signatures of the dynamics and the field content driving inflation). Hence, by complementing gravitational wave searches, the study of non-Gaussianity will provide profound new information about the early Universe by directly probing inflationary dynamics and field content at energy scales far beyond those accessible through laboratory experiments.

FISHER MATRIX FORMALISM

How much information do the data x provide about the parameter theta? Moreover, how accurately one can measure model parameters from a given data set without simulating the data set? These questions have a clear answer in the context of Fisher information, which is exactly a measure of the amount of information about a parameter, provided by a random variable or a sample.

We start by introducing the concept of Likelihood: given a set of parameters, it's the probability of that this data set could have occurred. It is represented by the term $P(D|H)$ in the famous Bayes theorem:

$$P(H|D) = \frac{P(D|H)\,P(H)}{P(D)}$$

In which $P(H|D)$ is called "posterior" and represents the probability of the occurrence of *H* given the data *D*, $P(H)$ and is called "prior" and represents the independent probability of the occurrence of the hypothesis.

The magnitude of the derivative of the likelihood (or log-likelihood) is related to the amount of information provided by the data. The Fisher information is large if on average the log-likelihood is very sensitive to a change in the parameter theta; this appends when the likelihood is highly curve. Then, Fisher matrix approach generalizes the above procedure to the case of many parameter. Given a data set of real numbers arranged in a vector (they can be for example power spectrums of galaxies) and considering this vector as a random variable with probability distribution which depends in some way on the vector of model parameters α, the Fisher information matrix is defined as:

$$F_{ij} = \left\langle \frac{\partial^2 ln\mathbb{L}}{\partial_{\alpha_i}\partial_{\alpha_j}} \right\rangle$$

In which $\mathbb{L}$ is the Likelihood.

Note that in many applications the likelihood for the data is assumed to be Gaussian and the data-covariance is assumed not to depend on the parameters. The covariance is a measure of the relationship or dependence between two random variables and indicates the degree to which two variables change together: if it is zero, then there is no linear relationship between the two, if positive they are directly related and if negative they are inversely related.

An estimate of the covariance for the parameters is given by:

$$\sigma^2_{\alpha_i\alpha_j} \geq (F^{-1})_{ij}$$

It means that the standard deviation is given by the reciprocal of the square root of the diagonal element ii of the Fisher matrix.

In practice, you will choose a fiducial model and compute the above at the fiducial model. Then we can expand $\mathbb{L}$ in Taylor series around its maximum (or the fiducial model).

We now assume that the likelihood function for the band-powers *P(k)* is Gaussian thus we can approximate the Fisher matrix by:

$$F_{ij} = \int_{-1}^{1}\int_{k_{min}}^{k_{max}} \frac{dlnP(k,\mu)}{d\theta_i}\frac{dlnP(k,\mu)}{d\theta_j}V_{eff}(k,\mu)\frac{k^2dkd\mu}{2(2\pi)^2}$$

In which V_{eff} denotes the effective survey volume and it is given by:

$$V_{eff}(k,\mu) = \left[\frac{nP(k,\mu)}{nP(k,\mu)+1}\right]^2 V$$

In the equation of F_{ij} the derivatives should be evaluated at the fiducial model. Also we have assumed that over the entire survey extension the line-of-sight direction does not change: in other words we made the flat sky approximation.

We can summary this part by saying that the Fisher information matrix is the covariance matrix of the log-likelihood gradient with respect to the parameters when we evaluate that gradient at the true parameter vector; the randomness comes from this last vector.

When one computes the covariance (or the associated error) for auto and for cross correlation Cℓ the covariance is the same and includes the extra contribution of the noise. In the absence of noise the covariance is non-zero: this is called "cosmic variance". At the high ℓ (so in the noise-dominated regime) the central limit theorem ensures a Gaussian likelihood and the cosmic variance contribution to the covariance is negligible and thus the covariance does not depend on the cosmological parameters. However at low ℓ (cosmic-variance-dominated regime) the Fisher matrix approach over-estimates errors by about a factor 2; in this case it is therefore preferable to go for the more numerically intensive option.

CALCULATIONS

We start by computing our theoretical power spectrum with CLASS ('Cosmic Linear Anisotropy Solving System'). CLASS is an accurate Boltzmann code, designed to offer a more user-friendly and flexible coding environment to cosmologists. It is very structured, easy to modify, and offers a rigorous way to control the accuracy of output quantities. Its purpose is to simulate the evolution of linear perturbations in the universe and to compute CMB and large-scale structure observables.

For our data set, as a starting point, we considered a universe's volume of $10\ Gpc^3$ with a number density of galaxies of 10^{-3}. We add UT corrections to the matter power spectrum generated by CLASS at redshift equal to 1, considering a magnification bias of 0.5 and assuming the following multi-tracers case:

$$b_A(z) = (2+z)^{0.5}, \qquad b_B(z) = (0.5+z)^{1.2}$$

$$\frac{dN_A}{dz} = \left(\frac{z}{0.54}\right)^4 e^{-\left(\frac{z}{0.54}\right)^{1.5}}, \qquad \frac{dN_B}{dz} = \left(\frac{z}{0.53}\right)^{1.1} e^{-\left(\frac{z}{0.53}\right)^{1.5}}$$

Firstly we plot the ET power spectrum:

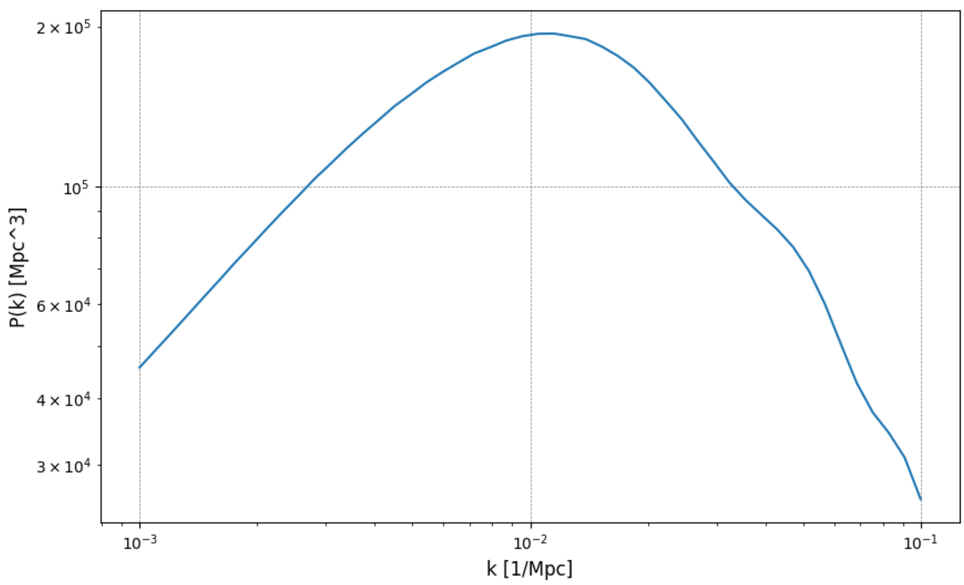

Figure 5 – Equal-time matter power spectrum at z equal to 1

Then we add the first order UT correction, given by the next plot.

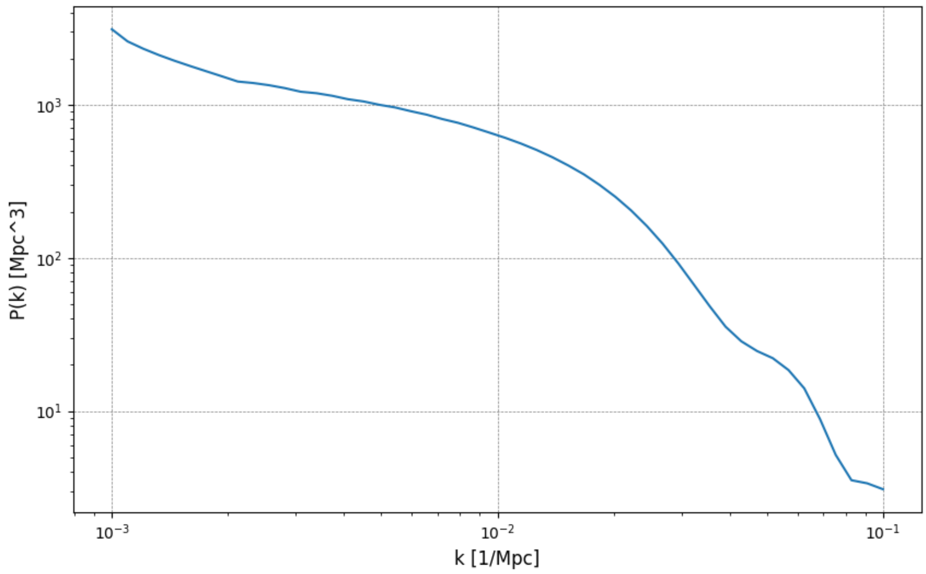

Figure 6 – First order Unequal-time corrections for the matter power spectrum at z equal to 1

By comparing the two, from the following plot we can see that these corrections can amount up to a few percent at small *k* (so at large scales).

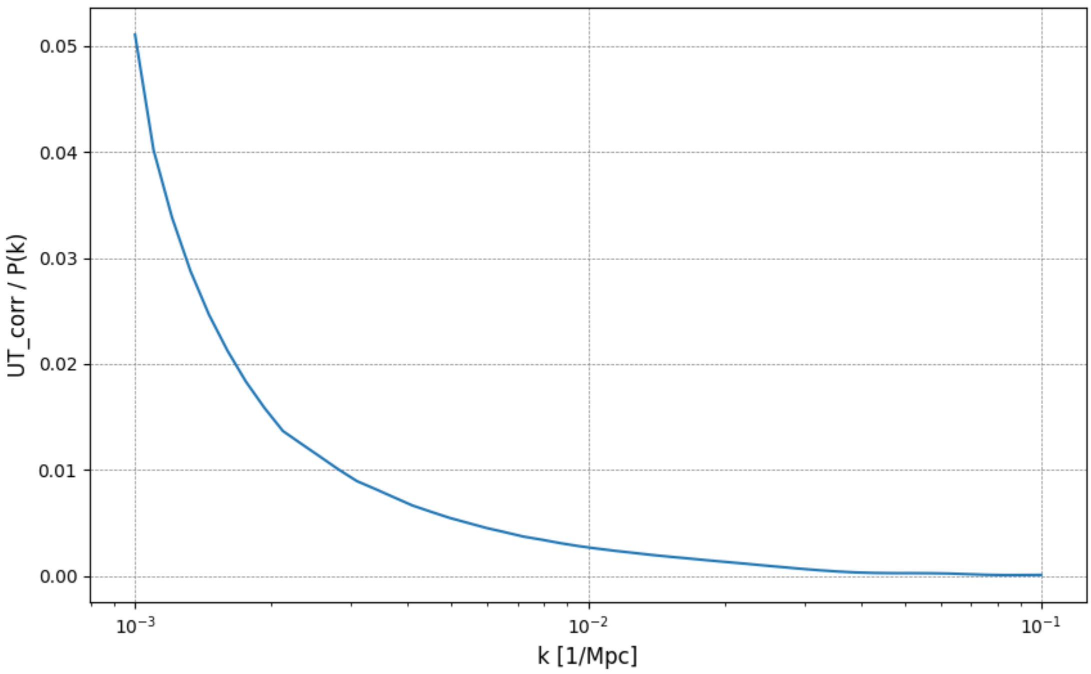

Figure 7 – First order Unequal-time corrections over the UT-corrected matter power spectrum at z equal to 1

Then we applied the Fisher matrix formalism to both ET and UT cases, considering as parameters:

$$\omega_{cdm},\ \ \omega_b,\ \ n_s,\ \ h,\ \ \gamma$$

Taking as fiducial values respectively:

$$0.1201075,\ \ 0.9660499,\ \ 0.0223828,\ \ 0.6781,\ \ 0.55$$

To study the cosmological parameters dependence, we compared the ET case with the UT one using the confidence ellipses. We obtained that the two cases nearly coincide:

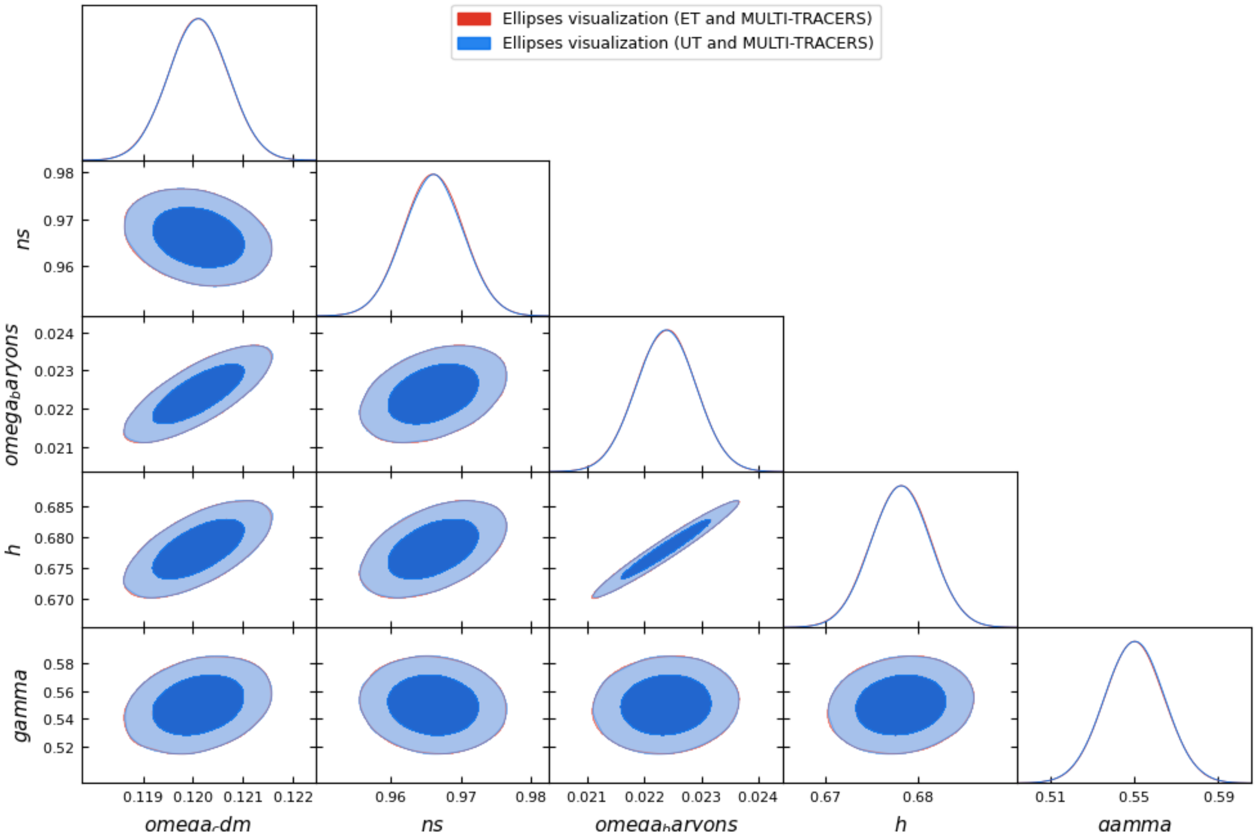

Figure 8 – Confidence ellipses for our cosmological parameters of interest. UT corrected ellipses nearly coincides with the ET ones

Using the following relation for the covariance:

$$Cov(k,z) = \frac{4\pi^2}{V\ k^2\ dk\ d\mu}\left(\left(P^{tot}_{obs,AA}(k,z)P^{tot}_{obs,BB}(k,z) + \left(P^{tot}_{obs,AA}(k,z)\right)^2\right)\right.$$

We also computed the Signal-to-Noise ratio as:

$$SNR^2(\bar{z}) = \int_{k,\mu} \frac{|D(\bar{z})^2 i\mu H(\bar{z})\partial_k\left[c_{1,AB}(k,\mu,\bar{z})Pk\right)]|^2}{Cov(\boldsymbol{k},\bar{z})}$$

And we obtained a value of about 0.91.

By increasing our k-FoV, in the next plot we can also see that the NG corrections introduce a new behaviour of P at low wavenumber.

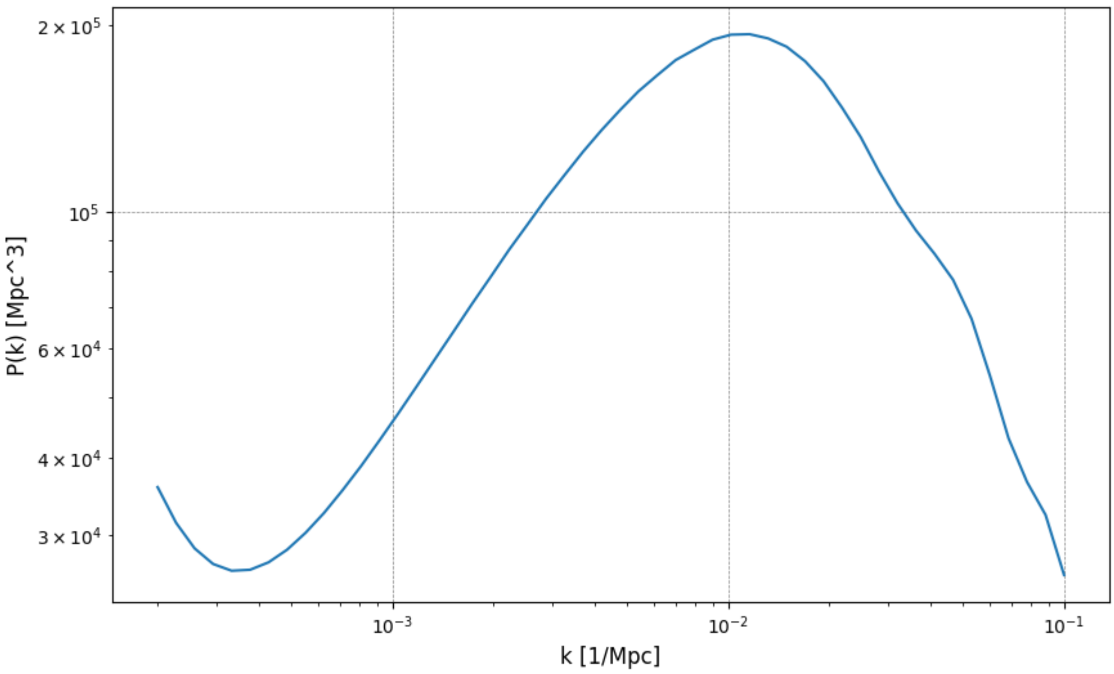

Figure 9 – Total matter power spectrum at z=1 at higher scales than the previous plot. At low k the PnG effect is visible.

We used a 1D Fisher matrix approach to calculate the error on the parameter f_{NL} and we obtained 0.1134 as the error for the ET case, and a bit lower value (0.1105) for the UT one.

CONCLUSIONS

In this work, we started from the theoretical framework of the background, moving to the definition of power spectrum up to the Baryon Acoustic Oscillations; then we introduced the concept of Linear Redshift Space Distortions, that leads us to our central topic: Unequal-Time Corrections.

Even if seems to be small, the impact of UT correction, especially at large scales, can be quite important (especially with the nowadays increasing instrumental precision). Every improvement in our theoretical modelling of the matter power spectrum, even if small, could allow us to distinguish between different cosmological models.

In our case, we found unexpectedly a significantly too small amplitude of the UT corrections; this might be due to an error in our implementation of these corrections.

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