



UNIVERSITÀ DEGLI STUDI DI PADOVA

Dipartimento di Fisica e Astronomia “Galileo Galilei”

Master Degree in Physics

Final Dissertation

Experimental and numerical performance assessment of
the Plasma Position Reflectometer of RFX-mod2

Thesis supervisor
Prof. Matteo Zuin

Candidate
Lorenzo Barone

Thesis co-supervisors
Dr. Gianluca De Masi
Dr. Federico Ruffini
Dr. Filipe da Silva

Academic Year 2025/2026

Abstract

The thesis work aims at assessing the performance of the RFX-mod2 Plasma Position Reflectometry (PPR) system through both experimental and numerical procedures. Frequency-Modulated Continuous Wave (FMCW) reflectometry is a well-established diagnostic technique [1] that enables the measurement of the plasma electron density profile by exploiting its reflective properties on probing electromagnetic waves. Similar to Density Profile Reflectometry (DPR), which relies on the full reconstruction of the density profile, PPR aims to probe only its outer portion along various lines of sight (LoS) to provide accurate real-time control of the plasma boundary inside the vessel. This technique, currently under development, is crucial for monitoring plasma magnetic confinement and is being considered for future applications in large fusion devices.

RFX-mod2 will benefit from an advanced PPR system involving four LoS. The upper, lower, and low-field side (LFS) are equipped with standard reflectometric set-ups, including pyramidal horn antennas and linear waveguides. The development and integration of a reflectometric system for the high-field side (HFS) that can accurately measure the nanosecond-scale delay shifts of reflected waves and reconstruct the position within a 1-*cm* target accuracy are the crucial focuses of the present discussion. A complex system of hoghorn antennas and bent waveguides is required to meet the severe spatial constraints imposed by the absence of a diagnostic port in this section of the machine. At the same time, it is necessary to avoid power losses and internal reflections as much as possible [2].

The purpose of the thesis is twofold: in the first part, the performance assessment of the HFS LoS antennas and waveguides is experimentally carried out by exploiting a simple plane metallic mirror configuration as a plasma surrogate. From the corresponding group delay measurements, the actual effects of antennas and waveguides on the propagation of microwaves are investigated, and the experimental results are compared with expectations. The second part, instead, is dedicated to a numerical performance assessment of the same subsystem. The wave propagation is simulated through the Finite-Difference Time-Domain (FDTD) REFMUL3 code to investigate the effects introduced by the RFX-mod2 geometry and to obtain a comprehensive description of the physics that occur within. Initially, the diagnostic performance is assessed by replicating the mirror set-up, whose results are then compared with the experimental ones. Finally, to investigate the system's capability of measuring the RFX-mod2 edge density profile, a set of expected RFP plasmas has been included in the simulations. As a main result, a good accordance between experimental and numerical mirror reconstruction capabilities is found, featuring a millimetre-scale accuracy well below the acceptable 1-*cm* one and demonstrating the correct integration of the diagnostic into the simulated system, as well as the overall validity of the current approach.

Contents

1	Introduction to Nuclear Fusion	1
1.1	Existing energy options	2
1.2	Nuclear fission: advantages and disadvantages	2
1.3	Fusion energy as a promising alternative	4
1.4	Fusion energy: state of the art and road map	8
1.5	The Padova contribution: RFP and the RFX-mod2 experiment	10
1.5.1	RFP configuration	11
1.5.2	RFX-mod2 outline	12
1.6	Structure of the thesis	14
2	Principles of Reflectometry	15
2.1	Basics of Plasma Two-fluid model	15
2.1.1	Plasma and gyro frequencies	15
2.1.2	Plasma Dynamical theory	16
2.2	EM waves in a magnetized uniform plasma	18
2.3	Measurement principles of reflectometry	22
2.3.1	Profile reconstruction	23
2.3.2	Applications of reflectometry: DPR and PPR	26
2.4	Reflectometry: set-up and detection principles	27
2.4.1	Reflectometer set-up	28
2.4.2	Detection techniques	28
2.4.3	Group delay extraction	30
2.4.4	Sources of error	31
2.4.5	STFT and resolution	32
3	Theory of antennas and waveguides	35
3.1	Basics of Antenna theory	35
3.1.1	Circuit representation	36
3.1.2	Radiation pattern	37
3.1.3	Field regions	39
3.1.4	Rectangular aperture antennas	40
3.2	Modelling of Rectangular Waveguides	41
3.2.1	TE waves in rectangular waveguides	43
3.2.2	TE _{1,0} -mode	45
4	Reflectometry at RFX-mod2	47
4.1	System overview	47
4.2	HFS subsystem	48
4.2.1	HFS Antenna design and tests	49
4.2.2	Waveguides design and tests	50

4.2.3	Machine integration	50
4.3	Operational scenarios	51
4.3.1	Tokamak scenarios	53
4.3.2	RFP scenarios	53
5	Experimental results	55
5.1	Overview of the experimental set-up	55
5.2	Mirror position reconstruction	60
5.2.1	Dispersion subtraction	62
5.2.2	Method 1: calibration	65
5.2.3	Method 2: differential analysis	68
5.3	Effect of antenna tilt	69
5.4	Effect of incidence angle	72
6	Numerical results	77
6.1	REFMUL3 basics	78
6.1.1	FDTD approach for REFMUL* EM simulation	78
6.1.2	Preliminary work: model implementation	80
6.1.3	Outputs	85
6.2	Mirror simulation	86
6.2.1	Effects of the simulation set-up	87
6.2.2	Dispersion Subtraction and position reconstruction	90
6.3	Plasma simulation	93
6.3.1	SHAx state manipulation	93
6.3.2	SHAx numerical implementation	97
6.3.3	SHAx simulation results	98
6.3.4	SHAx translation	104
6.3.5	Profile reconstruction	107
7	Discussion and final remarks	111
7.1	Experimental results	111
7.2	Numerical results	112
8	Conclusions and future perspectives	115

Chapter 1

Introduction to Nuclear Fusion

It is well established that the standard of living in society is directly proportional to the amount of energy employed to provide all the services and facilities necessary for human development. Furthermore, it is also clear how energy consumption has increased dramatically over the last few years [3]: the growth of the world population, the increase in industrialization, and the high energy demand in rapidly industrializing countries make the problem of energy procurement one of the main challenges of the 21st century, as well as one of the major topics of discussion in the international arena. A critical issue driving the supply problem concerns the evidence that greenhouse gases (GHG) have a significant negative impact on climate change; thus, energy sources that are not only affordable but also clean, stable, and sustainable are the types of supplies that capture global interest to ensure high living standards and reduce GHG emissions at the same time.

Among the future options, *nuclear fusion* is one of the more promising; however, it still must cope with several technical and engineering challenges that require significant research work.

This chapter provides a standard introduction to nuclear fusion, starting with a global overview of the current energy sources employed to address the consumption problem (Section 1.1). It will be followed by a brief description of nuclear technologies, from current nuclear fission (Section 1.2) to future nuclear fusion (Section 1.3), and the respective advantages and limitations are specified. Section 1.4 provides more details on the current state-of-the-art of fusion technologies. Finally, an in-depth description of nuclear fusion research in Padova can be found in Section 1.5.

1.1 Existing energy options

The energy sources can be subdivided into three categories: fossil fuels, renewable energy sources (RES) and nuclear fission. Globally, fossil fuels are still the most commonly employed energy supplies, even though, in terms of sustainability, they are obviously the worst choice when considering the limited natural reserves and the high level of GHG emissions. It is thus necessary to include both renewables and nuclear in the energy mix to meet the increasing energy demand while achieving the environmental objectives necessary to cope with climate change. As a matter of fact, the capital investments and political decisions are proceeding towards this target, with an increasing share of low-emission sources (RES and nuclear) recorded in the past five years and a further growth forecast by 2030 (Figure 1.1), where they are expected to cover 50% of electricity demand [4]. The main issues with RES are the non-uniform availability, the seasonal fluctuations, and the strong dependence on external factors (both climatic and geopolitical), which make them random producers of electricity. There are two alternatives to cope with this problem: the deployment of batteries and advanced storage systems, or the implementation of nuclear capacity as a backup to RES.

Global electricity generation by source, 2019-2030

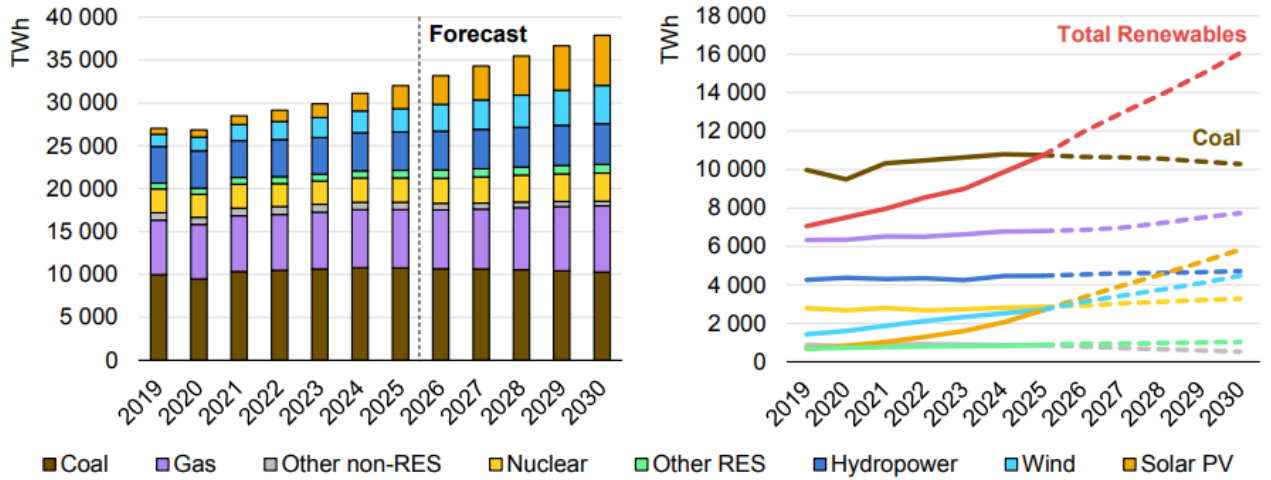


Figure 1.1: Global electricity generation by source from 2019 and forecast up to 2030 [4].

1.2 Nuclear fission: advantages and disadvantages

Nuclear fission is one of the alternative technologies of major interest globally for stable and clean energy generation. Its contribution is still limited, with a global share for electricity generation of about 8.9% in 2025, which is much lower than that of fossil fuels and RES. However, nuclear generation is expected to increase

by an average of 2.8% over the forecast period, more than double the 1.3% rate in 2021-2025 (Figure 1.1). The gain is related to the lifetime extension of some existing plants and the realization of new plants, mainly in China and India. Furthermore, the advent of new technologies such as *Small Modular Reactors* (SMR) and *Advanced Modular Reactors* (AMR) has increased interest in nuclear research and capital investment [4].

The main advantages of nuclear energy are thus listed.

- 1) **Flexibility**: it provides a continuous and flexible energy source.
- 2) **Low emissions**: it causes one of the lowest levels of GHG emissions among all energy sources (Figure 1.2).
- 3) **Energy density**: uranium is one million times more energetically dense than fossil fuels [5], thus requiring a lower volume (and hence land) for the plants, a lower amount of uranium ore to extract and carry, and less waste.
- 4) **Safety**: it is one of the safest energy sources, causing the least number of deaths (direct and indirect) per TWh of energy produced (Figure 1.2).

Together with these positive aspects, nuclear power has some disadvantages.

- 1) **Scarcity**: although convenient, nuclear fuel is not sustainable in the long term. However, some advanced techniques and special mixed fuels that allow for the recovery of part of the spent fuel are currently under study.
- 2) **Waste**: one of the major concerns of the nuclear industry is the maintenance of the long-lived high-level radioactive isotopes that are produced by the decay of fission products. However, several solutions to this problem have been explored, from permanent storage to more sophisticated transmutation and reprocessing techniques.
- 3) **Cost**: the complexity of the reactor and all the facilities required to handle the radioactive products make a nuclear reactor an expensive investment. The fuel accounts for only 5% of the total cost of electricity.

New technologies like SMR and AMR, along with new types of fuel, can improve many of the weak aspects of the nuclear industry, achieving a lower level of waste, increasing inherent safety and reducing the capital cost thanks to their smaller dimensions, thus enhancing their short term competitiveness.

All things considered, the risks and challenges of producing energy by fission have fostered the exploration of another nuclear process, namely the fusion reaction.

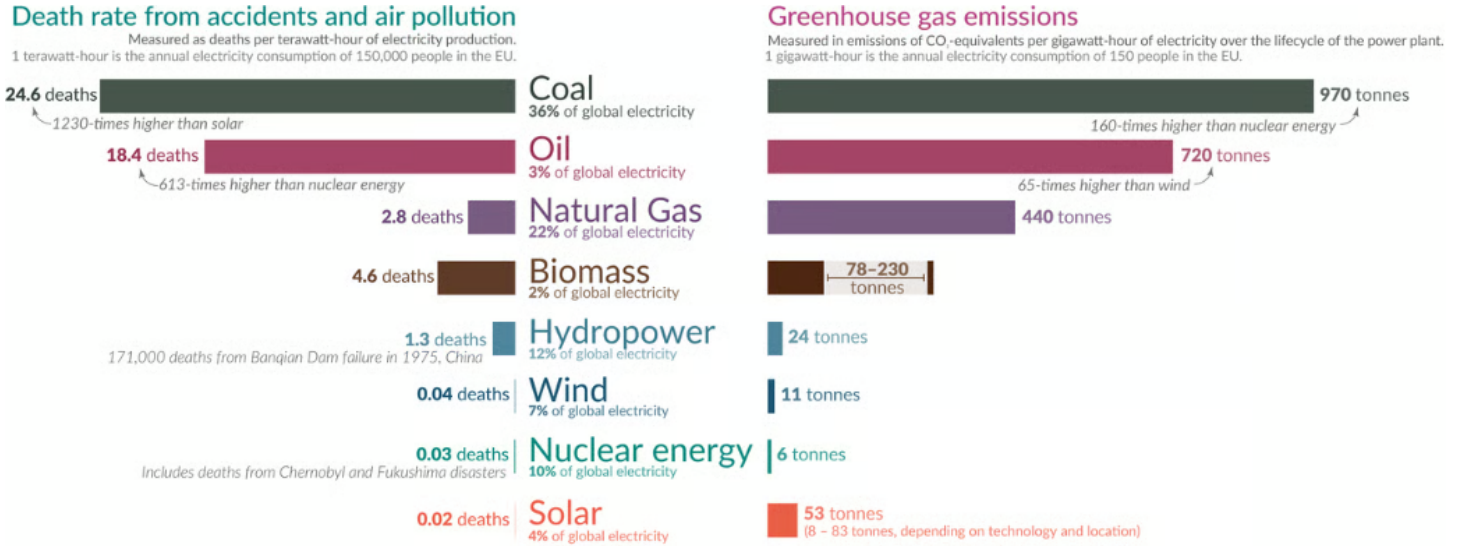


Figure 1.2: Number of death casualties (left) per energy source and global emissions (right) by each source ($\text{g CO}_2/\text{GWh}$) [6].

1.3 Fusion energy as a promising alternative

Nuclear energy production relies on the fission of heavy elements. An alternative process that is under investigation is nuclear fusion, which, by contrast, involves the merging of light nuclei into a heavier one and releases energy in the form of the kinetic energy of the compound nucleus itself, plus other particles (such as neutrons) produced during the reaction.

Since both nuclear reactions involve a change in the structure of the nuclei, they provide an energy gain only when the nuclear binding energy of the final state is higher than that of the initial state, implying a decrease in mass. Looking at Figure 1.3, which displays the behaviour of the binding energy as a function of the mass number [7], it is clear how fission is more likely to involve heavier elements, while fusion occurs on the left side of the plot, exploiting lighter nuclei. Although conceptually similar, fission and fusion are physically very different and require specific conditions and precise set-up geometries to be triggered in the proper way. Fission employs particular unstable isotopes, such as ^{235}U , which easily split after neutron bombardment, releasing energy and, on average, 2.4 further neutrons that, in turn, can trigger additional reactions. This chain reaction requires particular control systems to avoid reactor meltdown and a protective layer (namely the containment vessel) to screen against accidental runaways.

On the contrary, fusion is much more challenging to realize. The only example of spontaneously occurring nuclear fusion occurs in stars, so reproducing the physical conditions of such giants is really hard and sets many scientific and engineering constraints. First of all, the fusion process is very rare compared to other types of processes like elastic Rutherford scattering. Secondly, even if one manages to create a beam of energetic particles and shoot it against a target to maximize

fusion probability, the energy required to accelerate such particles is far greater than the energy obtained. To overcome this problem, scientists figured out that a convenient way to generate a fusion reaction could be to exploit the properties of plasmas. A plasma is an ionized gas that is heated to a temperature of about 150 million K, hotter even than the centre of the Sun, and kept at low pressure (typical densities are $\sim 10^{20}$ particles per cubic meter). If this hot ionized gas is well confined in a proper geometry for a sufficient time, then the ions have the possibility of undergoing a fusion reaction without being lost due to elastic collisions. The ultimate goal is to exploit part of the heat generated by fusion to balance the power losses, thus reaching the so-called *ignition condition* where the discharge is self-sustained and fusion continuously occurs. Satisfying the ignition condition means achieving a sufficiently high value of the triple product of density, confinement time and temperature ($n \cdot \tau_E \cdot T$), which is one of the key parameters in fusion research.

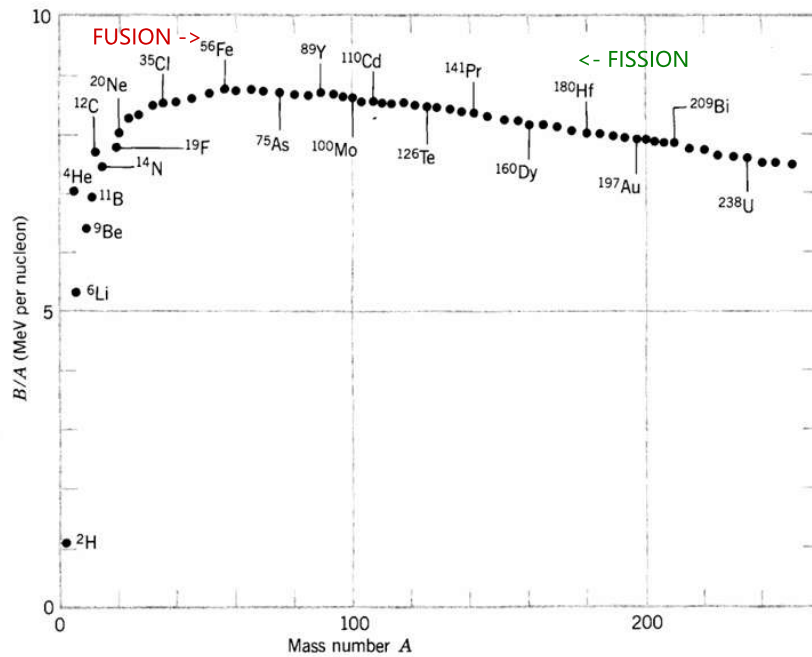


Figure 1.3: Nuclear binding energy per nucleon [7].

Table 1.1 lists a series of fusion fuels that have actually been employed in past experiments. The criteria for choosing the best fuel rely not only on the energy released but also on the fusion cross-section of the specific process in the specific temperature operation range. Typically, such reactions involve hydrogen isotopes like deuterium (D) and tritium (T), although sometimes helium isotopes are employed. From Figure 1.4, it is clear that the D-T fusion reaction is the most prominent one, and in fact, 65% of worldwide fusion programs are focusing on achieving fusion with D-T fuels [8].

Reactant 1	Reactant 2	Products
D	D	${}^3\text{He}$ (0.8 MeV) + n (2.45 MeV)
D	D	T (1.01 MeV) + p (3.02 MeV)
D	T	${}^4\text{He}$ (3.5 MeV) + n (14.1 MeV)
D	${}^3\text{He}$	${}^4\text{He}$ (3.6 MeV) + p (14.7 MeV)
D	${}^6\text{Li}$	2 ${}^4\text{He}$ + 22.4 MeV
T	T	${}^4\text{He}$ + 2n + 11.3 MeV
${}^3\text{He}$	T	${}^4\text{He}$ + p + n + 12.1 MeV
${}^3\text{He}$	T	${}^4\text{He}$ (4.8 MeV) + D (9.5 MeV) + p (11.9 MeV)
${}^3\text{He}$	T	${}^4\text{He}$ (0.5 MeV) + n (1.9 MeV) + p (11.9 MeV)
${}^3\text{He}$	${}^3\text{He}$	${}^4\text{He}$ + 2p + 12.86 MeV
${}^3\text{He}$	${}^6\text{Li}$	2 ${}^4\text{He}$ + p + 16.9 MeV
p	${}^6\text{Li}$	${}^4\text{He}$ (1.7 MeV) + ${}^3\text{He}$ (2.3 MeV)
p	${}^{11}\text{B}$	3 ${}^4\text{He}$ + 8.7 MeV

Table 1.1: List of nuclear fusion fuels that are actually employed in experiments [8].

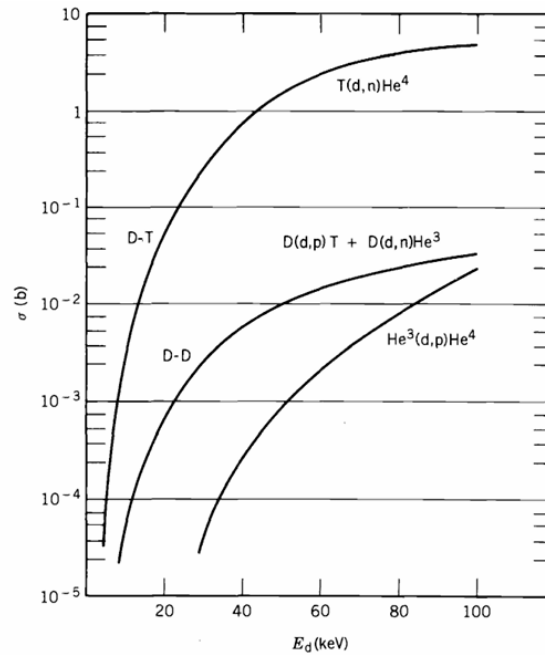


Figure 1.4: Fusion cross-section for three different processes: D-D, D-T and D- ${}^3\text{He}$.

All things considered, one might wonder what significant advantages make the fusion challenge worth pursuing compared to fission. These are listed below.

- **More energy per nucleon:** the fission of ${}^{235}\text{U}$ releases 206 MeV, which corresponds to 0.88 MeV/A, while the D-T fusion outcome amounts to 3.52 MeV/A. In other words, one million tons of fossil fuels are equivalent to 0.8 tons of uranium and 0.14 tons of deuterium and tritium [5].

- **Larger fuel reserves:** deuterium is very abundant and can be easily extracted from seawater, while tritium is a radioactive element with a half-life of 12 years. It is possible to produce tritium from lithium inside the blanket that reacts with the neutrons generated by the fusion plasma until D-D fusion becomes technologically viable.
- **Less environmental impact:** the fusion process produces neither GHG emissions nor significant radioactive waste, thus requiring less stringent containment measures. The only harmful output relies on the neutron activated structural materials, but their half-life is on the order of 100 years, much shorter than fission waste. The overall radiotoxicity of the plant after shutdown, shown in Figure 1.5, is thus well below the level of fission plants and, in most cases, lower than coal plants [9].
- **Safety:** the nuclear fuel is continuously fed into the reactor at a rate such that the density is low enough to maintain the plasma state. The very low mass of fuel present in the reactor at any instant of time makes a meltdown impossible.

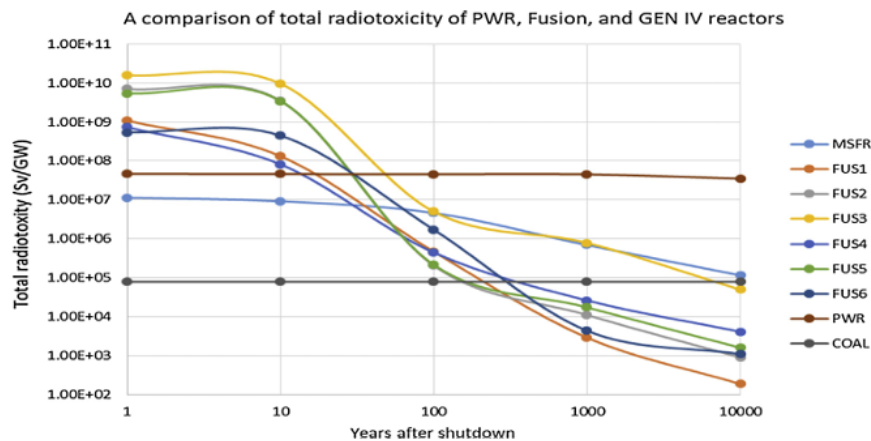


Figure 1.5: Radiotoxicity after shutdown of various nuclear plants: a III generation PWR (Pressurized Water Reactor), a IV generation MSFR (Molten Salt Fast Reactor) and six DEMO fusion plants [9].

Besides the clear advantages, there are still several engineering constraints that must be addressed.

- **Tritium breeding:** the T isotope is rare and has a half-life of about 12 years. It is thus necessary to develop a method to *breed* T at the level of the reactor walls by exploiting the large neutron fluxes. The feasibility of tritium breeding in a dedicated structure (the so-called *blanket*) is one of the main goals of fusion research (see Section 1.4).
- **Large heat and particle fluxes:** the D-T fusion process inside a reactor releases a significant amount of neutrons and α particles. These are highly

energetic and can damage the surrounding structures, which can become radioactive due to neutron activation. The study of low-activation materials capable of withstanding large neutron fluxes is a key research topic in the fusion framework.

- **Energy and particle confinement:** as already mentioned, the plasma must be confined within the vacuum vessel for a sufficient time. This is done by developing a suitable confinement mechanism (see Section 1.4). The investigation of new reactor configurations and operational regimes to improve particle confinement is another crucial point to tackle within fusion research.
- **Heating:** a proper heating system needs to be developed to reach the working temperature. This is another non-trivial engineering task that requires intensive work.

The overall fusion goal is quite complex and requires a major research and development programme, as well as large capital investments.

1.4 Fusion energy: state of the art and road map

The combined requirement of confining a sufficient quantity of plasma for a sufficiently long time at a sufficiently high temperature to achieve net fusion power has been the focus of the World's fusion research program for the past 70 years (Figure 1.6a). Recently, the increasing number of public-private partnerships has enhanced scientific research and industrial investment from companies concerned with the economic viability of fusion energy [8]. As already stated, the main goal is to achieve a steady state well-confined fusion plasma that is capable of producing a net energy gain. In fusion engineering, this is translated into the energy gain Q factor (Figure 1.6b) defined as the ratio of power released to that injected; clearly, the previously quoted ignition condition corresponds to $Q \rightarrow \infty$.

Many different fusion concepts have been investigated throughout history and are summarized in Figure 1.6a, reporting all the records of triple product values reached by any configuration (until 6 October 2025 [10]). They can be subdivided into three main categories: Magnetic Confinement Fusion (MCF), Inertial Confinement Fusion (ICF) and Magneto-Inertial Fusion (MIF).

Current statistics [11] show that among 146 fusion devices around the world, 59% are actually in operation, 10% are under construction, and 31% are planned. One third of the investments are funded by industrial companies, showing a growing private involvement in the challenge and strong confidence in the market potential, while the remaining two thirds are under public ownership. The most promising and advanced concept is MCF, featuring significant involvement from all over the World, especially from Europe. In *Magnetic Confinement* configurations, the par-

ticles are confined in an appropriate geometry thanks to the effect of a magnetic field. A plasma is produced by heating a neutral gas to sufficiently high temperatures until it is fully ionised and the generated charged particles are trapped by the Lorentz force. The choice of geometry, which is usually toroidal, is crucial in determining the reactor steady state performance and stability. The main MCF devices are *Tokamak*, *Spheromak*, *Stellarator*, and *Reversed Field Pinch* (RFP).

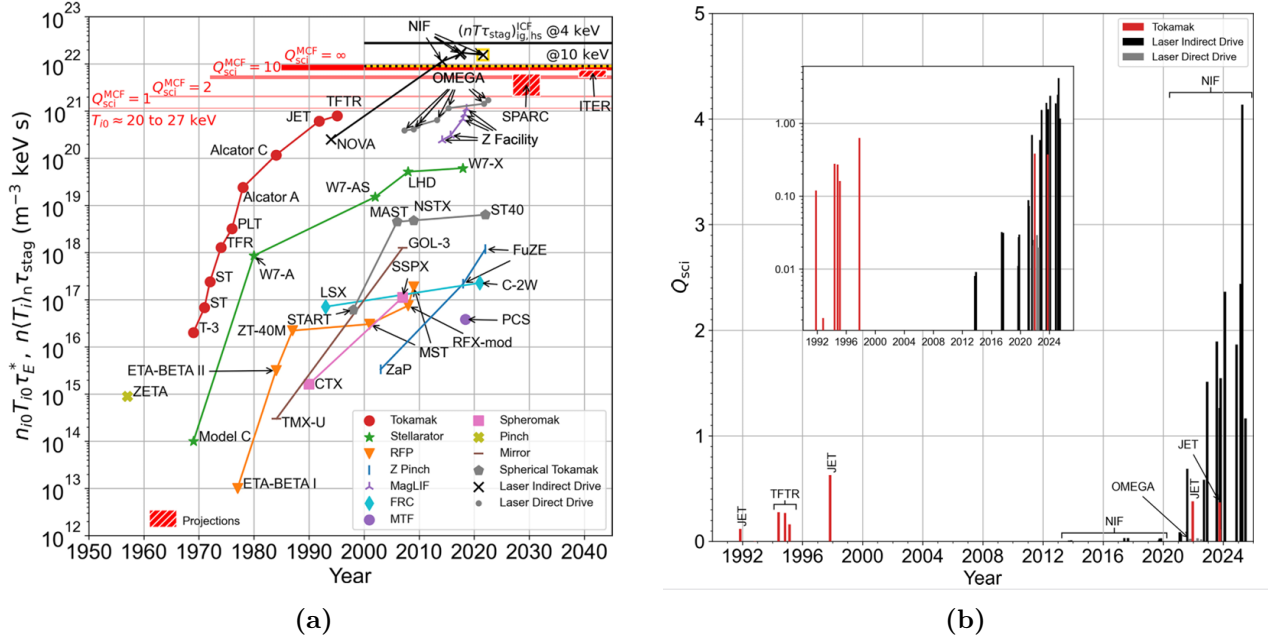


Figure 1.6: (a) Triple products that set a record for a given concept versus achieving year (updated 6 October 2025). (b) Q -values reached vs year by various experiments [10].

The *Tokamak* (Figure 1.7) is the most popular configuration and exploits a toroidally-symmetric chamber to confine the plasma. The magnetic field is helically torn due to a combination of the external coils and the internal current drive. Its spherical variant (spherical Tokamak or *Spheromak*) is also investigated for its confinement properties. Among the future Tokamak experiments, it is worth mentioning one of the greatest international experiments, namely *the International Thermonuclear Experimental Reactor* (ITER). Under construction in Cadarache (France), it is currently the largest Tokamak ever designed and involves 35 different nations, including those in the European Union (which will cover 45.6% of construction costs), China, India, Japan, South Korea, Russia, and the US. Its primary objective is to investigate and demonstrate D-T plasma burning while reducing the relative contribution of external heating with respect to alpha-particle self-heating, with a target Q of around 10 and a discharge flat-top duration of around 400 seconds. Several other technical challenges are addressed, such as the employment of auxiliary heating and superconducting coils (limitations that were found in the previous largest machine, the *Joint European Torus* (JET) experiment), the study of energetic particles, the characterisation of trans-

port phenomena, the demonstration of tritium breeding viability, the mitigation of power exhaust on the divertor and many others.

The results gained from ITER will then bridge the gap towards a further step, known as the *DEMONstration power plant* (DEMO), which is expected to translate the fusion experiment into electricity generation and thus pave the way for the future commercialisation of power plants. DEMO is planned to be operational by 2060, roughly 20 years after the high-power D-T burning operations carried out at ITER (expected by the end of 2040), and will include tritium breeding for fusion fuel, suitable materials for neutron handling, and safety and sustainability requirements.

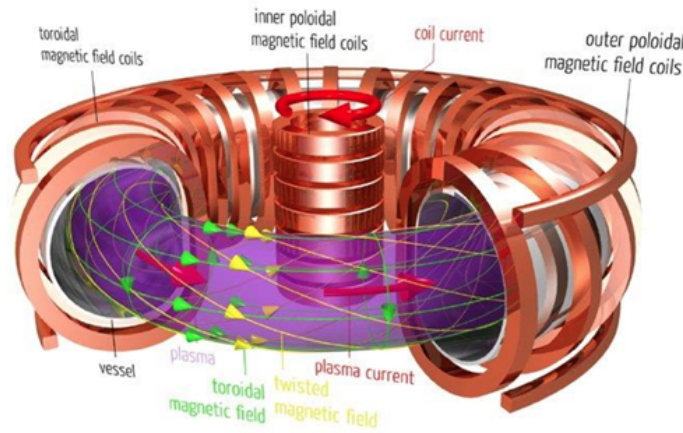


Figure 1.7: Scheme of the Tokamak configuration.

Although the Tokamak with D-T fuel is still considered the most established experimental fusion design, a growing interest in alternative configurations is being recorded worldwide. The *Stellarator* is highly investigated for its complex coil design that is capable of producing the magnetic field entirely without the need to drive a current. Finally, the *Reversed field pinch* configuration (Section 1.5.1) exploits the same Tokamak structure but works in a completely different regime. Indeed, its high current drive and its peculiar magnetic equilibrium make the RFP a valid alternative to the Tokamak thanks to its potentially improved confinement properties.

1.5 The Padova contribution: RFP and the RFX-mod2 experiment

The research work on plasma physics and fusion energy in Padova started in 1958 with the first studies on plasma discharge and has always been largely dedicated

to RFP configuration experiments. A brief general outline of the latter is provided in Section 1.5.1, followed by a more specific description of RFX-mod2 features and improvements with respect to its previous versions (Section 1.5.2).

1.5.1 RFP configuration

The previously mentioned Tokamak plasma, the most utilized set-up for fusion devices, exploits the toroidal shape of the vessel (Figure 1.7) to reach an axisymmetric equilibrium (i.e., symmetric with respect to the toroidal angle ϕ), provided by a low toroidal plasma current I_p and a high magnetic field. While the former is limited by stability concerns, the latter must be high enough to ensure confinement, even though it is limited by the external coils.

The challenge consists in reaching an equilibrium configuration where the magnetic field lines and the current density lines inside the plasma are organized into a set of *magnetic flux surfaces*. The theoretical nature of these surfaces relies on the so-called *Magneto-HydroDynamic* (MHD) picture, which, together with Maxwell's equations, provides an analytic description of plasma equilibrium. Thanks to the axisymmetric nature of Tokamaks, it is enough to provide a description of such surfaces in the poloidal plane. One of the major concerns of nuclear fusion is the capability of preserving such equilibrium for a sufficiently long period. Particular attention is paid to a class of instabilities, namely MHD instabilities, which might lead to fast discharge terminations, commonly known as *disruptions*. The build-up of such instabilities poses a limit on the maximum value of the plasma current in Tokamak discharges.

In this context, the RFP follows straightforwardly. The RFP configuration [12], [13], alongside the Tokamak, is obtained in axisymmetric devices driving a toroidal plasma current and a toroidal magnetic field. The main difference is that the previously quoted limit of I_p is completely overcome, allowing several unstable modes. This configuration, known as *Multiple Helicity* (MH) state, is not stationary: the unstable modes grow and saturate, leading to a loss of axisymmetry and the reversal of the toroidal magnetic field at the edge (Figure 1.8a). At this point, the magnetic field shows a peculiar tendency to self-organize into a helical equilibrium with conserved beam-shaped magnetic flux surfaces (Figure 1.8b), in which only one unstable mode is present. This theoretically predicted configuration, defined as *Single Helicity* (SH) state, helps achieve fusion conditions with a lower externally applied magnetic field, eliminating the need for a further heating mechanism (other than the already present Ohmic one) and providing a general improvement in confinement properties. From an experimental point of view, a *quasi-SH* (namely QSH) has been obtained, with a single mode dominating over the small (but still present) secondary ones and showing a *double axis* (DAX)

structure. Increasing I_p leads to an increasing persistence of the QSH, until the spontaneous transition towards an even better configuration, known as *Single Helical Axis* (SHAx) state, is observed.

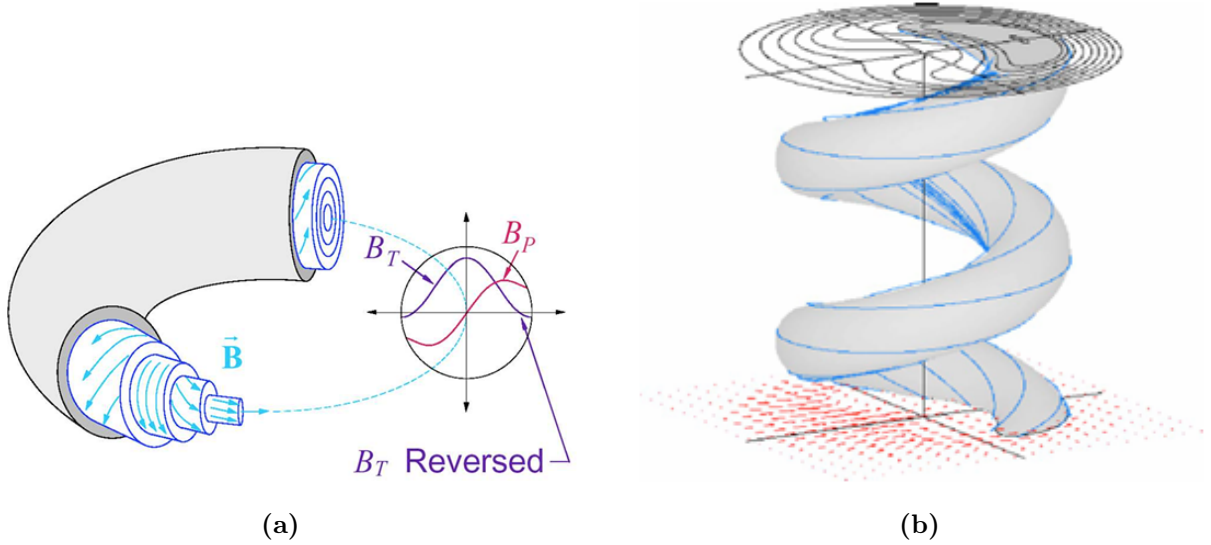


Figure 1.8: (a) Schematic picture of RFP configuration [14]. (b) Theoretical SH state flux surface in 3D and in 2D poloidal plane [15].

1.5.2 RFX-mod2 outline

From 1970 to 1980, several experimental results were obtained by Eta-Beta I and Eta-Beta II until Consorzio RFX was established in 1996. The name RFX is inherited from the *Reversed Field eXperiment* realized in 1992, while its *modified* version (RFX-mod) operated from 2004 to 2016. RFX-mod is a toroidal machine that can operate both in Tokamak and RFP configurations, provided the plasma current is properly adjusted. A further improvement of the machine is expected to be completed by 2027, leading to the RFX-mod2 experiment.

The first evidence of the SHAx transition was obtained in RFX-mod [12], demonstrating the validity of the theoretical predictions and paving the way for future experiments. The investigation of low- q regimes, once considered prohibited for the wide range of MHD resistive instabilities, has finally become feasible thanks to a control system made up of both passive and active elements. The passive stabilizing effect is provided by a copper shell placed around the vacuum vessel, while the active system involves 192 saddle coils. The control of MHD instabilities, especially when interacting with the first wall, enabled the setting of I_p values up to 2 MA (SHAx is observed at $I_p > 1$ MA). The peculiarity of RFX-mod was its ability to operate also in Tokamak configuration if $I_p < 150$ kA.

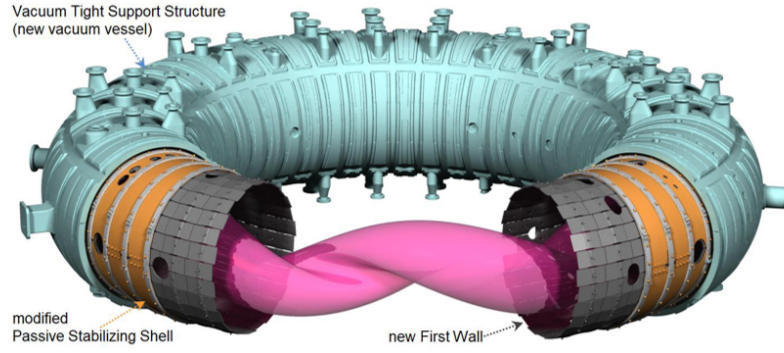


Figure 1.9: Schematic view of RFX-mod2 [16].

The design of the forthcoming RFX-mod2 [16] involves a general improvement of the machine parameters in order to confirm the RFP configuration as a valid choice. Due to the higher level of instabilities and the absence of a divertor, the first wall is made of graphite tiles in order to absorb the outgoing energy and particle flux arising from transport losses. Unlike RFX-mod, the resistive vacuum chamber has been removed, making the plasma closer to the copper passive stabilizing shell and thus reducing the overall resistivity of the structure. The decrease in the *shell-plasma proximity* (Figure 1.10) will ensure several advantages.

1. Reduction of plasma wall interaction.
2. Lower magnetic chaos.
3. Reduction of localized power deposition.
4. Improvement of the self-organized helical plasma regime.

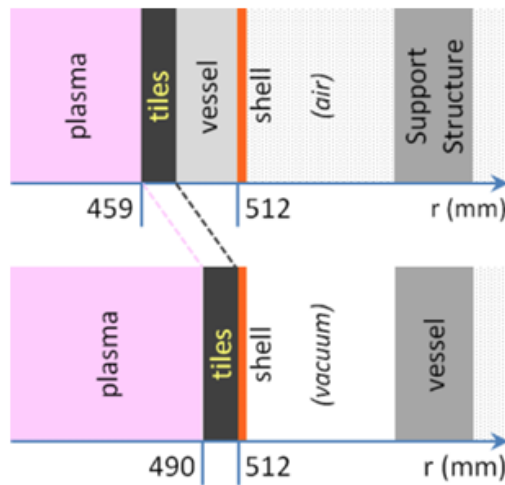


Figure 1.10: Schematic picture of the field proximity. A comparison between RFX-mod and RFX-mod2 is presented [16].

The shell is thus suitably modified to directly support the graphite wall and it is enclosed in the new vacuum vessel, which, in turn, corresponds to the previous RFX-mod toroidal support structure. Finally, the last feature that makes RFX-mod2 more reliable is the sophisticated system of diagnostics, which ensures a high degree of control of plasma and magnetic field properties. Several efforts were dedicated to the development of magnetic sensors, interferometric systems, and spectroscopic apparatuses.

Among these, RFX-mod2 will be equipped with an advanced reflectometric system. Microwave reflectometry [1] is a widely employed diagnostic that exploits the reflection of microwaves against the plasma to reconstruct its density profile. Besides this application, which will be referred to as *Density Profile Reflectometry* (DPR), a further application concerns the use of a more sophisticated set-up designed for plasma position control purposes. The *Plasma Position Reflectometry* (PPR) technique is gathering growing interest in fusion applications and will be intensively investigated through the RFX-mod2 system.

1.6 Structure of the thesis

The main focus of the thesis is a characterisation of the PPR apparatus by means of experimental and numerical results to assess its quality in the most comprehensive way. A general introduction on reflectometry can be found in Chapter 2 from a physical and practical point of view, followed by an overview of the main components of a reflectometric set-up (antennas and waveguides) in Chapter 3. Chapter 4 is instead focused on RFX-mod2 and provides an in-depth description of its *High-Field Side* (HFS) equatorial plane subsystem, which features the most complex layout. A general summary of previous tests and experiments can be found, together with a brief description of the main operational scenarios with which PPR and DPR can be integrated. After these mainly descriptive chapters, the core of the experimental and numerical work on this subsystem is described in Chapters 5 and 6, respectively. The former provides an overview of the bench tests that have been carried out, along with a description of the main experimental procedures to extract information and the most relevant experimental results. The latter describes the numerical tools employed to integrate the HFS subsystem and simulate a semi-realistic RFP scenario. Finally, Chapter 7 summarises and discusses the results, comparing the numerical outputs to the previously obtained experimental ones.

Chapter 2

Principles of Reflectometry

Before starting the practical description of reflectometry, it is worth mentioning some interesting properties of the matter under study, the plasma, and how it behaves under the action of electromagnetic perturbations. The term *plasma* was employed for the first time by Langmuir in 1928 to denote a collection of electrons and ions held together in an overall charge-neutrality condition. The easiest way to do that is to heat the gas to a sufficiently high temperature to allow the electrons to break their energetic bonds. From this definition, it is clear that the plasma cannot follow the same physical description as a neutral gas since the long-range collective Coulomb interactions dominate over the single particle contributions. The ***quasi-neutrality*** and the **low-collisionality** regime are responsible for many interesting properties, such as high electrical conductivity and collectivity. The latter, together with the assumption of local thermodynamic equilibrium, sets the basis of the plasma fluid model.

The following chapter is organised as follows: Section 2.1 introduces the previously mentioned dynamical theory. The discussion then moves on to the propagation of electromagnetic waves in a plasma (Section 2.2), which is the physical phenomenon of interest for reflectometric applications. Section 2.3 exploits the latter physical framework to introduce the measurement principle of reflectometry by analytically extracting the density profile. Finally, Section 2.4 provides an overall description of the experimental detection and data processing techniques to extract the most relevant information.

2.1 Basics of Plasma Two-fluid model

2.1.1 Plasma and gyro frequencies

A simplified dynamical description of a plasma requires specific assumptions that depend on the physical phenomena of interest. In any case, it is worth identifying

the characteristic temporal scales that regulate the dynamics of a particle. The latter, being subject to EM fields, defines two characteristic motions: one related to the electric field and one to the magnetic field.

The former temporal scale is defined by the inverse of the *plasma frequency*, and it is associated with the motion of a charged particle under fast varying electric fields:

$$\omega_{p,s} = \sqrt{\frac{e^2 n_s}{m_s \epsilon_0}} \quad (2.1)$$

Depending on the mass m_s of the particle species s , it is clear how this parameter differs for electrons and ions and quantifies the ability of each species to restore *quasi*-neutrality after a fast charge displacement. Having much lower inertia, electrons are certainly faster than ions, making the electronic plasma frequency $\omega_{p,e} = \omega_p$ the typical reference value. Physically, the plasma frequency can be interpreted as the capability of shielding fast-varying AC fields: if the frequency of the applied field is lower than ω_p , the electrons have enough time to shield out the field. Otherwise, if the field oscillates too rapidly, they can barely move before the field has already changed sign, making shielding impossible. The transition between these two regimes is exactly ω_p .

The second temporal scale concerns the fast gyro motion of charged particles moving along a magnetic field line. From a simple calculation involving the Lorentz force, the expression for the electron gyro (cyclotron) frequency follows:

$$\omega_c = \frac{eB}{m_e} \quad (2.2)$$

2.1.2 Plasma Dynamical theory

To provide a description of the phenomena that occur in a plasma, it is necessary to build a self-consistent model. The most accurate approach concerns the development of a kinetic theory for plasma systems in a similar way to how it is done for gases and fluids in general, involving an ensemble of N particles and defining the density of the ensemble points $\rho_n(\mathbf{x}_1, \mathbf{u}_1, \dots, \mathbf{x}_n, \mathbf{u}_n, t)$ in the phase space. The subtle point is that, unlike neutral gases, the long-range Coulomb interaction plays a crucial role in this analysis, making the full problem very challenging. One way to simplify this description is to neglect collisions between particles, yielding the following equation:

$$\frac{\partial f_1}{\partial t} + \mathbf{u}_1 \cdot \nabla_{\mathbf{x}_1} f_1 + \frac{\mathbf{F}}{m} \cdot \nabla_{\mathbf{u}_1} f_1 = 0 \quad (2.3)$$

which is known as *Vlasov equation* and $f_1 = f_1(\mathbf{x}_1, \mathbf{u}_1, t)$ is the single particle distribution function. Physically, this means taking into account only the average force acting on a particle that results from the smooth distribution of all

the other particles. This is perfectly coherent with plasma behaviour, where low collisionality and long-range collective interactions are the dominant effects. It is necessary to develop a theoretical framework capable of describing all those phenomena that are sufficiently fast to assume a collision-less approach. A smart strategy is represented by the *two-fluid model* [17], which relies on subdividing the medium of interest into a large number of fluid volume elements and averaging the main physical microscopic quantities over the particles contained inside. This procedure shifts the focus to the macroscopic properties of the fluid elements, such as the average density $n = \int f d\mathbf{u}$ and the average velocity $\mathbf{v} = \int \mathbf{u} f d\mathbf{u} / n$, without the necessity of employing the microscopic distribution function to track single particle motion. All things considered, only those phenomena that involve high frequency perturbations, where collisions can be neglected and thermal equilibrium is not significantly perturbed, and that primarily affect the perpendicular motion can be properly described by the two fluid model. In this frame, (2.3) holds for both electrons and ions. Taking the moments of the Vlasov equation (i.e., multiplying both sides of (2.3) by a quantity $\chi(\mathbf{u})$ and integrating over the velocity space):

$$\chi = m_s \quad \rightarrow \quad \frac{\partial n_s}{\partial t} + \nabla \cdot (n_s \mathbf{v}_s) = 0 \quad (2.4)$$

$$\chi = m_s \mathbf{u}_s \quad \rightarrow \quad m_s n_s \left[\frac{\partial \mathbf{v}_s}{\partial t} + (\mathbf{v}_s \cdot \nabla) \mathbf{v}_s \right] = -\nabla p_s + q_s n_s (\mathbf{E} + \mathbf{v}_s \times \mathbf{B}) \quad (2.5)$$

where p_s is the pressure and q_s is the charge of the particle species s . Equation (2.4) is known as *continuity equation*, while (2.5) is the *momentum equation*. A third equation that relates pressure to density is required to achieve closure. For the purpose of the thesis, the main application involves the propagation of E.M. waves, whose speed is much greater than the thermal velocity of both electrons and ions and the frequency of the induced perturbation is usually faster than the plasma frequency, or at least of the same order. In this case, it is useful to neglect the thermal motion of the particles by setting $T_s = 0$ and hence $p_s = 0$, thus adopting the *cold plasma* approximation to describe the main physical processes. Finally, to achieve a complete dynamical theory, it is necessary to include Maxwell's equations in the description:

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \quad (\text{Gauss's Law}) \quad (2.6)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (\text{Gauss's Law for Magnetism}) \quad (2.7)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (\text{Faraday's Law}) \quad (2.8)$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \quad (\text{Ampère-Maxwell Law}) \quad (2.9)$$

2.2 EM waves in a magnetized uniform plasma

Once the theoretical framework has been set, the propagation of electro-magnetic (EM) waves can be introduced [17], [18]. As already mentioned, the plasma is a complex system of charged particles that can react under field perturbations, restoring charge neutrality. Its two-fluid description relies, as stated, on two main assumptions: the absence of collisions and local thermal equilibrium. It is then possible to build a self-consistent description of the propagation of EM waves, provided the frequency is high enough to neglect collisions and the intensity is low enough to avoid causing significant perturbations, thus maintaining local thermal equilibrium (a fundamental condition for (2.3)). Since $c \gg v_{th,e}, v_{th,i}$, the cold plasma approximation can be employed. The ion dynamics are thus neglected (they are a sort of fixed background), while the electrons are treated in the cold plasma approximation, with vanishing thermal velocity ($p_e = 0$). Only the perturbed motion induced by the wave is retained. This allows to neglect also (2.4) since the density of the electrons is no longer an unknown in (2.5). The motion of interest is only that induced by the wave as a perturbation over the relatively slow thermal motion. In this framework, it is convenient to interpret a generic quantity χ as the sum of a 0^{th} order term χ_0 which accounts for the unperturbed dynamics and a 1^{st} order term χ_1 that encodes the perturbation. It is then possible to write the density as $n_0 + n_1$ and let v_1 , E_1 and $B_0 + B_1$ respectively denote the instantaneous velocity, the electric field and the magnetic field. Linearizing (2.5), it is possible to get the following equation:

$$m_e n_0 \frac{\partial \mathbf{v}_1}{\partial t} = -e n_0 (\mathbf{E}_1 + \mathbf{v}_1 \times \mathbf{B}_0) \quad (2.10)$$

The linear perturbation theory described above and the assumption of a homogeneous, uniform plasma allow for the Fourier analysis of the perturbed quantity of interest, for example:

$$\mathbf{E}_1(\mathbf{x}, t) = \frac{1}{(2\pi)^4} \int \mathbf{E}_1(\mathbf{k}, \omega) e^{i(\mathbf{k} \cdot \mathbf{x} - \omega t)} d^3 k d\omega$$

where the linearity ensures that each Fourier mode behaves independently. In particular, Ohm's law holds between electric fields and currents *Ohm's law* :

$$\mathbf{j}_1(\mathbf{k}, \omega) = \boldsymbol{\sigma}(\mathbf{k}, \omega) \cdot \mathbf{E}_1(\mathbf{k}, \omega) \quad (2.11)$$

where $\boldsymbol{\sigma}$ is the electrical conductivity of the plasma. In the general case, it has a tensorial shape since the medium can be anisotropic. It is shown below that this is exactly the case of a magnetized plasma, since the presence of B_0 breaks the isotropy of the dynamics.

Taking the z axis in the direction $\mathbf{B}_0$, equation (2.10) yields the following components for a single Fourier mode:

$$\begin{aligned} -m_e i\omega v_{1,x} &= -eE_{1,x} - eB_0 v_{1,y}, \\ -m_e i\omega v_{1,y} &= -eE_{1,y} + eB_0 v_{1,x}, \\ -m_e i\omega v_{1,z} &= -eE_{1,z}. \end{aligned}$$

We may readily solve for v in terms of E :

$$\begin{aligned} v_{1,x} &= \frac{-ie}{\omega m_e} \frac{1}{1 - \omega_c^2/\omega^2} \left(E_{1,x} - i\frac{\omega_c}{\omega} E_{1,y} \right), \\ v_{1,y} &= \frac{-ie}{\omega m_e} \frac{1}{1 - \omega_c^2/\omega^2} \left(i\frac{\omega_c}{\omega} E_{1,x} + E_{1,y} \right), \\ v_{1,z} &= \frac{-ie}{\omega m_e} E_{1,z}, \end{aligned} \tag{2.12}$$

where the expression (2.2) for the cyclotron frequency ω_c has been introduced. In this cold plasma approximation, all electrons move alike, and the current density is:

$$\mathbf{j} = -en_0 \mathbf{v}_1 = \boldsymbol{\sigma} \cdot \mathbf{E}_1, \tag{2.13}$$

where

$$\boldsymbol{\sigma} = \frac{in_0 e^2}{m_e \omega} \cdot \frac{1}{1 - \omega_c^2/\omega^2} \begin{bmatrix} 1 & -i\omega_c/\omega & 0 \\ i\omega_c/\omega & 1 & 0 \\ 0 & 0 & 1 - \omega_c^2/\omega^2 \end{bmatrix}. \tag{2.14}$$

This is the conductivity tensor we require. Now considering (2.8) and (2.9) in their linearized form, for a single Fourier mode:

$$\nabla \times \mathbf{E}_1 = i\omega B_1 \tag{2.15}$$

$$\nabla \times \mathbf{B}_1 = \mu_0 \mathbf{j} - i\mu_0 \epsilon_0 \omega E_1 \tag{2.16}$$

Substituting (2.12) into (2.16), it is possible to find:

$$\nabla \times \mathbf{B}_1 = -i\omega \mu_0 \epsilon_0 \boldsymbol{\epsilon} \mathbf{E}_1$$

where $\boldsymbol{\epsilon}$ is the generic dielectric tensor defined as follows:

$$\boldsymbol{\epsilon} = \left[\mathbf{1} + i \frac{\boldsymbol{\sigma}}{\epsilon_0 \omega} \right] \tag{2.17}$$

Expanding $\nabla \times \mathbf{B}_1$ with (2.15) and using the vectorial identity $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$:

$$\left(\mathbf{k}\mathbf{k} - k^2 \mathbf{1} + \frac{\omega^2}{c^2} \boldsymbol{\epsilon} \right) \cdot \mathbf{E}_1 = 0 \tag{2.18}$$

Now, equation (2.18) is a matrix equation that establishes the dispersion relation between $\mathbf{k}$ and ω . This is obtained by setting the determinant to zero:

$$\det \left(\mathbf{k}\mathbf{k} - k^2 \mathbf{1} + \frac{\omega^2}{c^2} \boldsymbol{\epsilon} \right) = 0 \quad (2.19)$$

The corresponding eigenvector, which represents the solution $\mathbf{E}_1(\mathbf{k}, \omega)$ associated with a particular eigenvalue, determines the characteristic polarization of the wave.

This approach can be applied to any medium, provided the uniformity and linearity, with the properties of the specific medium encoded into the dielectric tensor (or equivalently the conductivity). A suitable form of $\boldsymbol{\sigma}$ is given by (2.14), so everything is set to continue the analysis. First, it is convenient to write the following quantities in a more schematic way:

$$X = \omega_p^2/\omega^2, \quad Y = \omega_c/\omega, \quad N = kc/\omega.$$

where ω_p is given by (2.1), ω_c by (2.2) and N is the refractive index of the medium. Substituting $\boldsymbol{\sigma}$ from (2.14) into (2.19) and defining k in a way such that it is fully determined by the angle θ between $\mathbf{k}$ and $\mathbf{B}_0 \parallel \hat{z}$:

$$\mathbf{k} = k \cdot (0, \sin \theta, \cos \theta)$$

it is possible to write the full matrix equation as:

$$\begin{vmatrix} -N^2 + 1 - \frac{X}{1 - Y^2} & \frac{iXY}{1 - Y^2} & 0 \\ -\frac{iXY}{1 - Y^2} & -N^2 \cos^2 \theta + 1 - \frac{X}{1 - Y^2} & N^2 \sin \theta \cos \theta \\ 0 & N^2 \sin \theta \cos \theta & -N^2 \sin^2 \theta + 1 - X \end{vmatrix} = 0$$

The final form of dispersion relation is a well-known result found by Appleton (1927) and Hartree (1931) [19] while studying the propagation of radio waves in the ionosphere:

$$N^2 = 1 - \frac{X(1 - X)}{1 - X - \frac{1}{2}Y^2 \sin^2 \theta \pm [(\frac{1}{2}Y^2 \sin^2 \theta)^2 + (1 - X)^2 Y^2 \cos^2 \theta]^{1/2}}. \quad (2.20)$$

that will be addressed as *Appleton-Hartree* formula for the refractive index in the cold plasma approximation. From (2.20) it is clear how, given the frequency, there exist two different solutions for N that correspond to two characteristic waves propagating into the plasma. These are usually known as *fast* and *slow* waves (with $N_{fast} > N_{slow}$) and the corresponding eigenvectors show polarization states orthogonal to each other. For what concerns reflectometry, only the case with $\theta = \frac{\pi}{2}$ (perpendicular propagation), is of major interest. The two characteristic waves are linearly polarized and the corresponding expressions for the refractive index read:

$$\begin{aligned} N^2 &= (1 - X) & O - Mode \\ N^2 &= 1 - \frac{X(1 - X)}{1 - X - Y^2} & X - Mode \end{aligned}$$

A brief explanation of the two modes is provided hereafter.

- **Ordinary mode** (O-Mode): $E_x = E_y = 0$. The wave is linearly polarized along the z axis, and therefore the electrons oscillate parallel to the magnetic field. Thus, the dispersion relation is independent of B_0 , which is the main advantage of this mode (check Figure 2.1a).
- **Extraordinary mode** (X-Mode). The wave is linearly polarized in the x - y plane, and thus the electron dynamics occur perpendicularly. The dispersion relation shows a clear dependence on the cyclotron frequency (in the Y factor), which encodes the influence of the magnetic field (see Figure 2.1b).

Finally, it is worth discussing the basic assumption on which this entire analysis has been built: the uniformity of the plasma. In practical cases, there exists no plasma that is capable of satisfying this condition due to the high degree of complexity of the system. On the other hand, neglecting uniformity leads to the impossibility of performing a Fourier decomposition since a coupling between different modes might arise (for example, consider a change in the refractive index that can cause a partial reflection of the wave). A good compromise relies on the idea that the properties of the plasma vary sufficiently slowly in space, ensuring local uniformity. Putting this into mathematical formulas, the condition that must be satisfied is:

$$|\nabla k| / k \ll 1$$

i.e., the spatial variation of the wavevector has to be much lower than the modulus of the vector itself. In this context, there is locally a well defined k and hence refractive index N that solves the homogeneous dispersion relation for a given frequency, thus corresponding to the local values of the plasma parameters. Therefore, the propagation of a wave along the direction $\mathbf{x}$ is not described by a

single mode phase $\mathbf{k} \cdot \mathbf{x}$ but rather by the sum of the phases that the wave acquires while propagating due to the small variations of N along $\mathbf{x}$:

$$E \approx \exp \left(i \int \mathbf{k} \cdot d\mathbf{x} - \omega t \right) \quad (2.21)$$

This is a common approach for solving partial differential equations in slowly varying media, known as the *Wentzel-Kramers-Brillouin-Jeffreys* (WKBJ) approximation. As will be mentioned in the next section, the relevant concept to emphasize is the possibility of expressing the phase of the wave as:

$$\int \mathbf{k} \cdot d\mathbf{x} = \int N(x) \frac{\omega}{c} dx = \frac{\omega}{c} \int N(x) dx \quad (2.22)$$

provided $\mathbf{k} \parallel d\mathbf{x}$. This is the starting point of the reflectometric analysis.

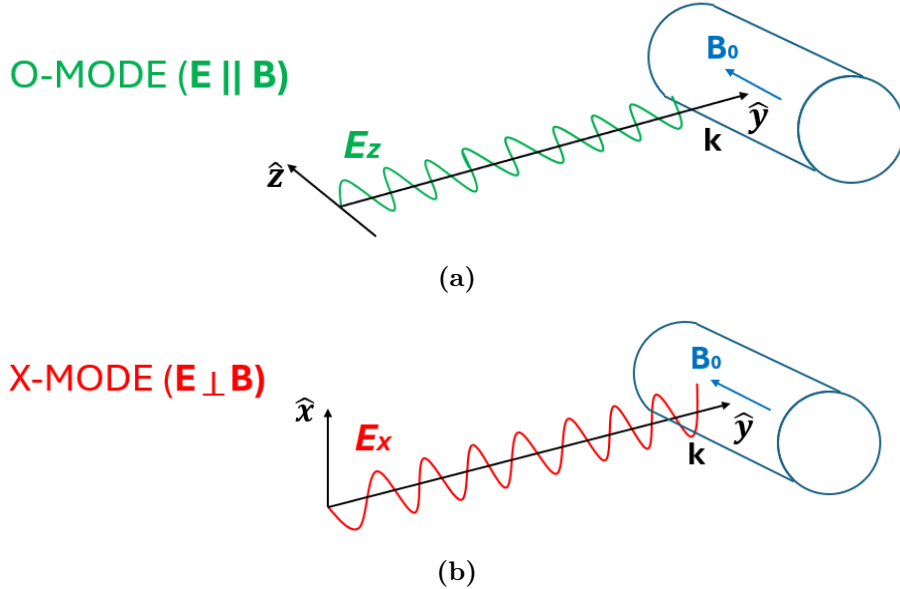


Figure 2.1: Scheme of (a) O-mode and (b) X-mode wave propagation in a plasma with respect to the magnetic field $B_0 \parallel \hat{z}$.

2.3 Measurement principles of reflectometry

Once the theoretical framework is set, it is possible to carry on the discussion by developing an analytical picture of reflectometry. The main principle is rather simple: a wave with frequency ω is injected into the plasma and propagates inside; if the frequency of the wave is properly chosen, it may reach a layer beyond which N^2 becomes negative, and thus N is purely imaginary. In this situation, the wave is evanescent and is reflected back (Figure 2.2). The measurement of this reflected radiation provides interesting information about the n_e , since the reflection occurs in a point where the density of the plasma is high enough to make it optically

thick. Such density value is known as *cut-off density* n_c and it is directly related to ω via the specific dispersion relation that holds locally in the plasma. Thus, detecting the reflected wave means ensuring that anywhere inside the plasma there must be a layer in which $n_e = n_c(\omega)$, but one might wonder whether it is also possible to know where such a layer is located, thus reconstructing a full density spatial profile $n_e(r)$. This is possible, as will be shown, by measuring the phase delay of the reflected wave with respect to the incident one. More specifically, it is the *group delay* that is the most appropriate observable employed to reconstruct the profile. Considering that n_c is a function of ω , a series of frequencies must be applied to cover the largest possible range of density values and reconstruct the widest portion of the profile, with each frequency providing information about one density layer. For this reason, the most used technique is the *Frequency-Modulated Continuous Wave* (FMCW) broadband reflectometry, where the frequency continuously changes over an established range.

2.3.1 Profile reconstruction

The density profile reconstruction method [20] allows a full measurement of the plasma density profile by means of reflectometry, provided the frequency range is large enough. To perform this task, a simple 1-D slab geometry is assumed, as shown in Figure 2.2, defining the direction of propagation of both the incident and reflected waves as the coordinate r , the cut-off position as r_c , and assuming the density profile $n_e(r)$ (and hence the plasma frequency $\omega_p(r)$) varies along r . The generic phase delay is denoted by φ , while the cut-off condition of vanishing N is defined by the polarization of the propagating wave. Narrowing the analysis to perpendicular propagation, it is worth distinguishing again between the ordinary and extraordinary cases and identifying their cut-off conditions for propagation.

- *O – Mode*. The cut-off condition shows a dependence solely on the plasma frequency:

$$N^2 = \left(1 - \frac{\omega_p^2}{\omega^2}\right) = 0 \quad \Leftrightarrow \quad \omega = \omega_p \quad (2.23)$$

which is directly translated into the condition $n_e = n_c$, where $n_c = \frac{\omega m_e \epsilon_0}{c^2}$ is the cut-off density defined by (2.1).

- *X – Mode*. The cut-off condition is a function of a combination of density and magnetic field:

$$N^2 = 1 - \frac{\frac{\omega_p^2}{\omega^2}(1 - \frac{\omega_p^2}{\omega^2})}{1 - \frac{\omega_p^2}{\omega^2} - \frac{\omega_e^2}{\omega^2}} = 0 \quad \Leftrightarrow \quad \omega = \frac{1}{2} \left(\sqrt{\omega_c^2 + 4\omega_p^2} \pm \omega_c \right) \quad (2.24)$$

where the $\pm$ denotes the *upper* and *lower* X-mode cut-off frequencies.

While the X-mode is much more difficult to figure out and requires numerical inversion methods, from the O-mode propagation picture, it is already possible to understand the core idea of reflectometry since the reflection occurs when the density reaches the critical value n_c , which, in turn, is completely determined by the injecting wave.

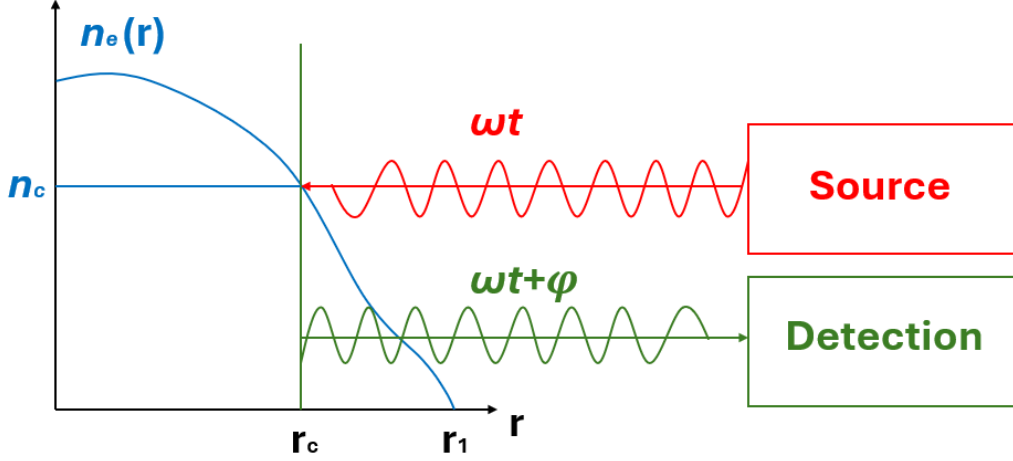


Figure 2.2: Scheme of 1-D reflectometric measurement. The probing signal is reflected once the cut-off position r_c is reached (adapted from [20]).

Applying the WKBJ approximation for the phase delay (2.22) and assuming that the density gradient is nearly constant near the reflection point, it is possible to derive the analytic form of the phase delay by simply treating the reflecting layer as a mirror.

$$\varphi = \frac{2\omega}{c} \int_{r_c}^{r_1} N dr - \frac{\pi}{2} \quad (2.25)$$

where $\frac{\pi}{2}$ accounts for the reflection and r_1 is the plasma edge (i.e., where the wave is injected). (2.25) allows for the extraction of information only about an average of the density values over the region $[r_c, r_1]$. However, as mentioned before, it is worth combining the phase delay coming from a series of different probing frequencies to obtain complete information about the profile. Suppose it is possible to construct a function $\varphi(\omega)$ with a frequency that varies continuously over a chosen range of interest. Differentiating (2.25) with respect to ω and expressing N in the O-Mode configuration:

$$\frac{d\varphi}{d\omega} = \frac{2}{c} \int_{\omega}^{\infty} \frac{\omega d\omega_p}{\sqrt{\omega^2 - \omega_p^2}} \frac{dr}{d\omega_p} = \frac{2}{c} \int_{\lambda}^{\infty} \frac{\lambda_p d\lambda_p}{\sqrt{\lambda_p^2 - \lambda^2}} \frac{dr}{d\lambda_p} \quad (2.26)$$

where first the change of variable $r \rightarrow \omega_p$ and $r_c \rightarrow \omega$ is performed, and then everything is expressed in terms of $\lambda = 2\pi c/\omega$.

The problem now consists of recovering the actual information about the profile

(i.e. $\frac{dr}{d\lambda_p}$) from the measured quantity (i.e. $\frac{d\varphi}{d\omega}$). To do that, it is necessary to introduce a mathematical method known as *the Abel transform*, which enables the extraction of a quantity $f(r)$ from its chordal integral through the plasma. Looking at Figure 2.3 and assuming cylindrical symmetry (a property that many plasmas share), the chordal integral is written as:

$$F(y) = \int_A^B f(r) dx = 2 \int_y^a f(r) \frac{r dr}{\sqrt{r^2 - y^2}} \quad (2.27)$$

The inversion procedure, performed by Abel in the nineteenth century, yields:

$$f(r) = -\frac{1}{\pi} \int_r^a \frac{dF}{dy} \frac{dy}{\sqrt{y^2 - r^2}} \quad (2.28)$$

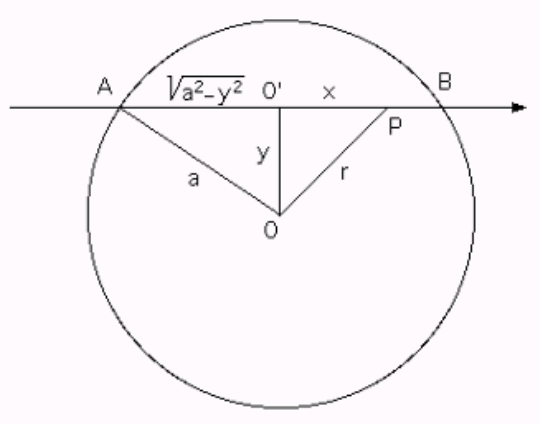


Figure 2.3: Chordal measurement in a cylinder [18].

In this specific case, it is possible to recognize in (2.26) the same structure as (2.27) with $F(y) \rightarrow \frac{d\varphi}{d\omega}$, $f(r) \rightarrow \frac{1}{c} \frac{dr}{d\lambda_p}$, $y \rightarrow \lambda$, $a \rightarrow \infty$, and $r \rightarrow \lambda_p$, leading to the following inversion:

$$r_c(\omega) = r_1 - \frac{c}{\pi} \int_0^\omega \frac{d\varphi}{d\omega'} \frac{d\omega'}{\sqrt{\omega^2 - \omega'^2}} \quad (2.29)$$

It is clear now that in order to extract the cut-off position $r_c(\omega)$ for a certain frequency, the knowledge of the derivative of φ with respect to ω is required for all frequencies less than ω . For this reason, the probing signal is a *frequency-modulated continuous wave* (FMCW) spanning a certain frequency interval that depends on the particular application of reflectometric measurements. This derivative is precisely the *group delay* accumulated by the wave travelling across the plasma:

$$\tau_g(\omega) = 2 \int_{r_c}^{r_1} \frac{1}{v_g(r)} dr = 2 \int_{r_c}^{r_1} \frac{dk}{d\omega} dr = \frac{2}{c} \int_{r_c}^{r_1} \frac{d}{d\omega} (\omega N) dr = \frac{d\varphi}{d\omega} \quad (2.30)$$

where $v_g(r)$ is the group velocity of the wave packet. A one-to-one relationship between the cut-off position r_c and the cut-off density n_c , both of which are functions of ω ($[r_c(\omega), n_c(\omega)]$), is then established. In the case of DPR, a broad spectrum of frequencies has to be swept in order to fully reconstruct the profile.

2.3.2 Applications of reflectometry: DPR and PPR

The DPR and PPR techniques rely on the same exact FMCW principle but exploit two different approaches. DPR is the standard profile reconstruction approach and employs a large bandwidth to reconstruct the profile as much as possible. PPR, instead, is not designed to fully reconstruct the density profile but aims to achieve real-time control of the plasma boundary by inferring the position and shape of the *Last Closed Flux Surface* (LCFS), under the assumption of isodensity flux surfaces [2]. The idea behind PPR is to exploit the measurement of solely the edge portion of the profile (thus requiring a narrower range of frequencies with respect to DPR) along a proper number of lines of sight and then reconstruct the LCFS by means of numerical algorithms (Figure 2.4). In Tokamaks, the position is usually determined with the knowledge of the separatrix density (i.e., the LCFS for Tokamaks) provided by other diagnostics (such as interferometry) or obtained directly through profile shape analysis. The O-mode is preferred due to the fact that the profile reconstruction is independent of the magnetic field, while X-mode allows for the reconstruction of the lower density portion of the profile, which is not accessible by O-mode reflectometry.

The reason why PPR is gathering interest is the possibility of achieving control of plasma position even in the presence of extreme environmental conditions due to high neutron fluxes and long pulse duration, where the actual availability of magnetic sensors is still a critical point and requires thinking about alternative solutions that prove less sensitive [21], [22].

Besides these main aspects, PPR is widely appreciated for several other features: the compact and non-invasive nature of the system, which consists of a set of poloidally distributed antennas and waveguides, excellent temporal resolution ($\sim \mu s$), good spatial accuracy ($\sim cm$) in position reconstruction, and minimal demand in terms of access to the machine [21].

PPR has been initially tested on ASDEX-Upgrade [23], while a partial PPR system is also being designed on DTT [24] in view of a full application on DEMO [22], [25]. Despite the potential advantages of PPR, few open issues still remain unsolved. Indeed, the LCFS reconstruction is far from trivial and requires precise knowledge of edge plasma dynamics, usually obtained via complementary diagnostic measurements. Moreover, the ideal situations in which PPR works properly concern the perfect normal incidence of the probing wave on the flux surfaces.

This condition is hardly realized close to the boundary due to the presence of several MHD instabilities, which introduce local curvatures that might downgrade the optimal performance of the reflectometric system.

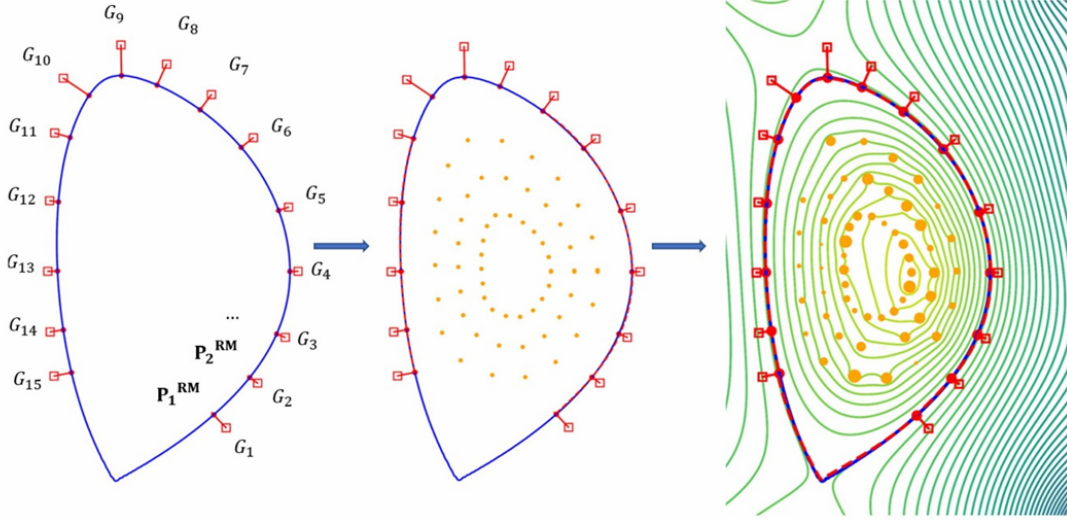


Figure 2.4: Example of LCFS reconstruction (from [25]). The G3C algorithm integrates PPR measurements from different LoS distributed in the poloidal plane (left) to reconstruct the LCFS (blue) on the HFS and LFS (red dashed lines in the middle). A series of Plasma Equivalent Currents (orange dots) are optimized to get a match between the magnetic flux and the reconstructed boundary (right) to reconstruct the LCFS entirely (red dashed line).

2.4 Reflectometry: set-up and detection principles

Once the theoretical framework has been established and $\tau_g = \frac{d\varphi}{d\omega}$ has been identified as the quantity of interest, it is worth developing a system to extract this information from the reflected wave. A generic reflectometer is composed of a microwave source (Section 2.4.1) that generates the FMCW and processes its reflected counterpart, a transmission line with antennas (Section 3.1) and waveguides (Section 3.2) to carry and collect the signal, and complementary hardware and software components for data extraction and processing (Section 2.4.2 and Chapter 5). Similar to radars, a reflectometer can exploit a single antenna that works as both an emitter and receiver (*monostatic* configuration) or consist of a system in which two antennas are employed separately (*bistatic* configuration). The former is optimal for the ideal condition in which the plasma is perfectly perpendicular to the probing direction and minimizes space occupation, while the latter avoids the problem of internal reflections and can be employed in non-optimal conditions by tuning the angle of emission of the two antennas.

2.4.1 Reflectometer set-up

A generic reflectometric unit is shown in Figure 2.5 and is adapted to a bistatic system. The probing modulated wave (usually referred to as TX) is generated by the reflectometer and carried to the transmitting antenna. The sweeping procedure is not run continuously; rather, it is externally controlled and synchronized with the data acquisition system by means of a trigger signal. Once the latter is provided, a Voltage Controlled Oscillator (VCO) generates a linear voltage ramp that, in turn, linearly modulates the output signal frequency. The reflected signal (denoted as RX) is collected by the receiving terminal of the reflectometer and mixed with a reference signal, which is a copy of TX. The final output contains information about the corresponding phase shift and is thus fed to the Data Acquisition System (DAQ) to be processed and analysed. The reference TX is conveniently delayed to counterbalance the long path of the RX through waveguides, antennas, and free-space and to avoid high beating frequencies (see Section 2.4.2) that might exceed the sampling rate of the DAQ.

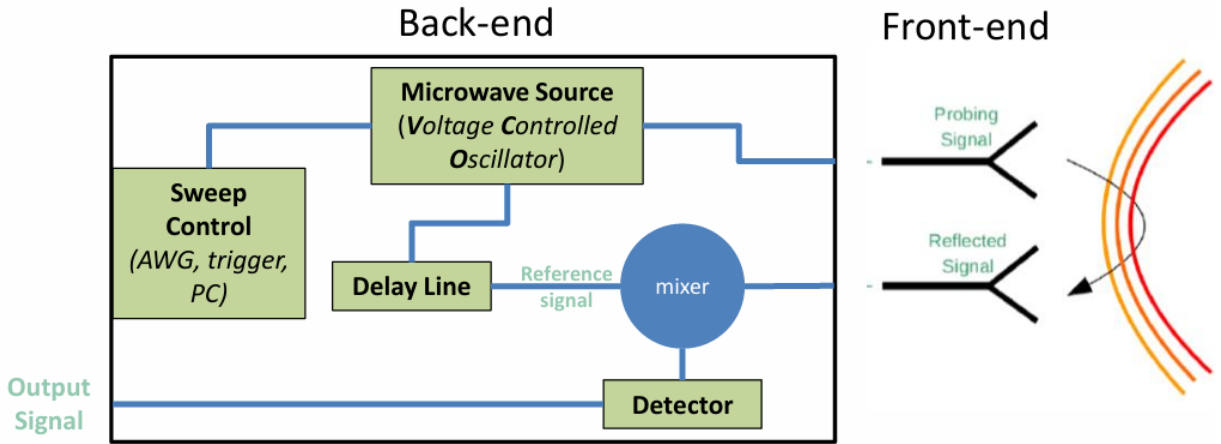


Figure 2.5: block diagram of a generic reflectometric unit and its components [2].

2.4.2 Detection techniques

Two main detection schemes are employed: homodyne and heterodyne [26]. The group delay is then extracted from the detected signal, as described in Section 2.4.3 [27]. Homodyne detection is based on measuring the cosine of the phase shift between the emitted and injected waves. The experimental apparatus is shown in Figure 2.6a for a bi-static configuration in which the reference probing signal is denoted as:

$$S_{TX}(t) = A_{TX}(t)\cos[\phi_{TX}(t)] \quad (2.31)$$

where $\phi_{TX}(t) = \omega t$ is its phase. The reflected signal, instead, is defined as:

$$S_{RX}(t) = A_{RX}(t)\cos[\phi_{RX}(t)] \quad (2.32)$$

with $\phi_{RX}(t) = \omega t + \varphi(t)$. To extract the phase shift $\varphi(t)$ from S_{RX} , the reflected signal is mixed with a copy of S_{TX} . A square-law mixer is thus employed to carry out the product of the two signals, yielding:

$$S_{mix}(t) = S_{TX}(t) \cdot S_{RX}(t) = \frac{1}{2} A_{TX}(t) A_{RX}(t) \cos[\varphi(t)] + \frac{1}{2} A_{TX}(t) A_{RX}(t) \cos[2\omega t + \varphi(t)]$$

By applying a low-pass filter, the high frequency component is rejected, leaving only information about the part proportional to $S_{mix,LP} \simeq \cos[\varphi(t)]$. This method does not allow one to reconstruct the amplitude of the detected signal and provides only information about the modulus of the phase, leaving the ambiguity of its sign. To overcome these limitations, heterodyne detection enables the measurement of the absolute amplitude and phase of the reflected signal. As pointed out in Figure 2.6b, it introduces a third signal corresponding to a $\pi/2$ -phase shifted copy of $S_{TX}(t)$. By employing two mixers and low-pass filtering both outputs, it is possible to extract the I and Q signals:

$$I(t) = \frac{1}{2} A_{TX}(t) A_{RX}(t) \cos[\varphi(t)]$$

$$Q(t) = \frac{1}{2} A_{TX}(t) A_{RX}(t) \sin[\varphi(t)]$$

and the amplitude and phase of the reflected signal are defined (with $A_{TX} = 1$):

$$A_{RX}(t) = 2\sqrt{I(t)^2 + Q(t)^2} \quad (2.33)$$

$$\varphi(t) = \arctan\left(\frac{Q(t)}{I(t)}\right) \quad (2.34)$$

This method is usually implemented to increase the dynamic range of reflectometer and the signal-to-noise (SNR) ratio.

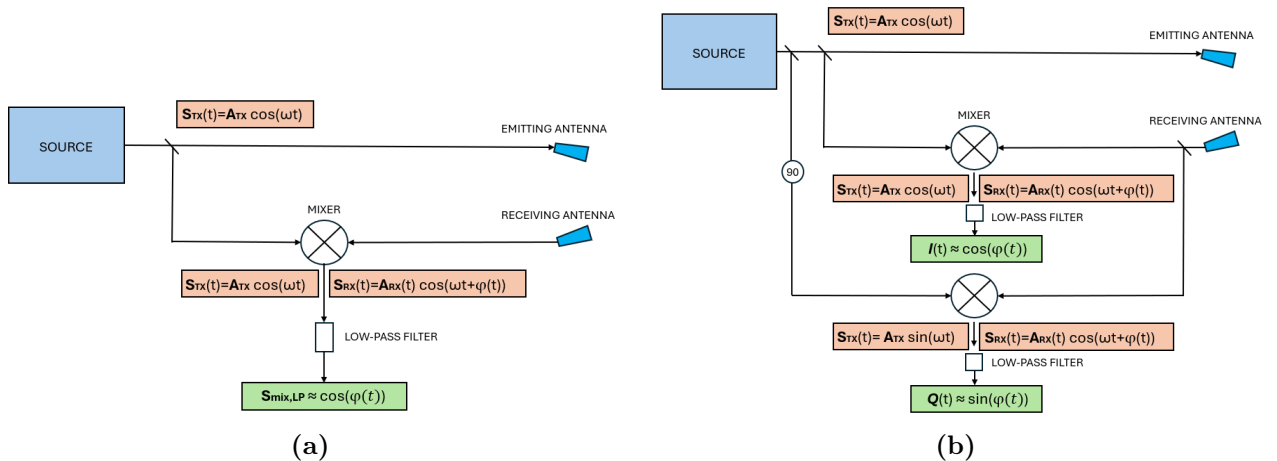


Figure 2.6: Scheme of (a) homodyne and (b) heterodyne bistatic apparatuses (adapted from [27]).

2.4.3 Group delay extraction

The next step concerns the extraction of group delay from the measured phase shift $\varphi(t)$. As mentioned before, typical reflectometry applications involve the use of a frequency sweep to cover the largest possible density range. The most used technique is Frequency-Modulated Continuous Wave (FMCW) broadband reflectometry, where the frequency continuously changes over an established range $[\omega_{min}, \omega_{max}]$ according to a linear temporal law:

$$\omega(t) = \omega_{min} + k_s \cdot t \quad (2.35)$$

where $k_{sw} = \frac{\omega_{max} - \omega_{min}}{T_{sw}}$ is defined as the sweeping rate and T_{sw} as the sweeping time. Typical values of T_{sw} are around the μs scale, while the range of frequencies spans between 10^2 and 10^3 GHz. The frequency ranges that are employed are called microwave (MW) bands and are named according to the portion of the MW spectrum they cover (some of them are reported in Table 2.1).

The generic expression for φ is given by (2.25) and depends on the spatial path travelled by the wave and its specific propagation. A subtle point relies on the fact that the emission and measurement times are not perfectly coincident: a tiny delay, which corresponds to the group delay, develops, and the frequencies associated with the two signals slightly differ according to (2.35), making the expression of φ a complex function of the group delay. However, provided the time-of-flight of each wave is much lower than T_{sw} , it is possible to approximately associate amplitude, phase, and group delay to the same time t and hence to the same instantaneous frequency of the sweep:

$$\phi_{RX}(t) = \omega(t - \tau_g) \cdot t + \varphi(t) \approx \omega(t) \cdot t + \varphi(t) \quad (2.36)$$

This simplification allows for establishing a direct correspondence between phase and group delay for each instant of time t (or frequency ω):

$$\varphi(t) = \phi_{RX}(t) - \phi_{TX}(t) \approx \omega(t) \cdot t + \varphi(t) - \omega(t) \cdot t \quad (2.37)$$

and thus, the group delay can be extracted as follows:

$$\tau_g(t) = \frac{\partial \varphi(t)}{\partial \omega} = \frac{\partial \varphi(t)}{\partial t} \cdot \left(\frac{d\omega}{dt} \right)^{-1} = \frac{\omega_b(t)}{k_{sw}} \quad (2.38)$$

where $\omega_b(t) = \frac{\partial \varphi(t)}{\partial t}$ is the instantaneous beating frequency of the output signal of the mixer, regardless of the detection technique. While the heterodyne set-up allows for the direct computation of the time derivative of the phase, the standard procedure to extract the group delay within the homodyne method usually involves the time-varying frequency distribution of the output filtered signal, using

techniques such as the Sliding Fast Fourier Transform, where the instantaneous beating frequency as a function of time (or equivalently, the probing frequency ω) can be computed (see Section 2.4.5).

At this point, the measured group delay $\tau_{g,m}$ is the result of many contributions introduced by the microwave circuit (waveguides, antennas, delay line, cables, etc.) and by the experimental conditions (the distance between the mouth of the antenna and the first wall, the plasma-wall distance, etc.). The inversion procedure is numerically implemented once the contributions of the circuit $\tau_{g,c}$ are subtracted, leaving only the actual physical delay:

$$\tau_g = \tau_{g,m} - \tau_{g,c} \quad (2.39)$$

Usually, this is done by placing a reflecting mirror in front of the set-up in order to subtract the unwanted component [27], as in Chapter 5 and 6. The distance of the mirror must be well-known to correct for the group delay accumulated by the wave in the portion of space comprised between the mirror and the antenna mouth.

Band	Frequency range (GHz)
K	16-26.5
K_a	26.5-40
Q	33-50
U	40-60
V	50-75
W	75-110

Table 2.1: MW bands and their corresponding frequency range.

2.4.4 Sources of error

The description provided up to now is based on the picture described by the cold plasma and the WKB approximations, where the plasma response is linear, the wave propagates as a plane wave, and the profile is assumed to be one-dimensional and monotonic. In reality, these conditions are not fully satisfied, leading to unavoidable measurement errors. Possible sources of error [27] are listed below.

- **Deviation from ideal propagation in the plasma:** the curvature of isodensity surfaces, the presence of turbulence and instabilities, and strong density gradients with variable directions can seriously affect the propagation of the waves and their probing efficiency.

- **Radiation pattern irregularities:** the presence of secondary lobes, the size of the beam, and its imperfect alignment with the magnetic field can affect the probing beam (see Section 3.1 for more details).
- **Plasma-wall reflections:** the presence of the first wall can induce multiple reflections of the probing waves, resulting in a systematic shift of the measured group delay that can be a multiple of the true value (Section 6.3.3).
- **Inversion algorithm:** the reconstruction of the cut-off position results from the integral of the group delay value from $\omega = 0$ up to the corresponding cut-off frequency. In O-Mode configuration, the inversion algorithm requires an initialization procedure since the minimum frequency of the lower band defines the minimum measurable group delay. A non-optimal initialization can lead to a significant error (Section 6.3.4).
- **LoS geometry:** the reference plasma position depends on the density profile along the LoS of interest, which usually coincides with the probing beam direction. However, the same measured group delay can provide different position reconstructions depending on the chosen LoS.

The two quantities to monitor in order to assess the quality of a PPR system are the position of the LCFS and the amplitude of the reflected signal. To validate the latter, there is a further figure-of-merit defined as *round-trip power loss*:

$$P_L = 20 \cdot \log_{10} \left(\frac{A_{RX}}{A_{TX}} \right) \quad (2.40)$$

This provides the power loss of the detected signal with respect to the probing one, and it is thus useful for characterising the proper compatibility of the emitting and receiving terminals, especially for bistatic systems.

2.4.5 STFT and resolution

One of the most useful tools for extracting and processing the group delay data is the *Short-Time Fourier Transform*, which consists of a progressive FT spectrogram that provides an analysis of the frequency content of the data over the sweep. The spectrogram allows us to extract an instantaneous beating frequency and to check its evolution as a function of time. This picture can be substituted by an equivalent one where the instantaneous group delay is evaluated from (2.38) and the probing frequency is taken as a temporal reference provided by (2.35). A set of spectrogram parameters must be introduced.

- f_{samp} : it is the sampling frequency of the signal and typically matches the sampling rate of the acquisition system.

- w_s : it sets the number of points for the temporal window in which the FT is performed. The choice of the size is a trade-off between frequency resolution (high w_s) and time resolution (low w_s).
- w_o : it is the overlap of the sliding window. The closer the overlap is to w_s , the better it smooths out the transitions between adjacent time slices in the spectrogram.
- N_{fft} : it is the number of FT frequency points. It can be set higher than w_s to make the FT spectrum smoother.

On one hand, good frequency resolution is necessary for the τ_g extraction, but having high time resolution allows for an increase in the number of segments and thus in the number of measured group delays. Setting a high overlap is a good strategy to increase time resolution, but it comes at the cost of increasing computational time. Another common practice is to extract the measured beating frequency (or group delay) within each segment by identifying the maximum peak, since relying on a Gaussian fit would again introduce higher computational effort. The necessity of keeping the latter low is justified by real-time applications where fast data processing is preferred.

Figure 2.7 shows an example of an experimental spectrogram with $f_{samp} = 1.25 \cdot 10^9 \text{ Hz}$, $w_s = 2500$, $w_o = 2400$, and $N_{fft} = 25000$. The blue dots are the maxima of each FT spectrum.

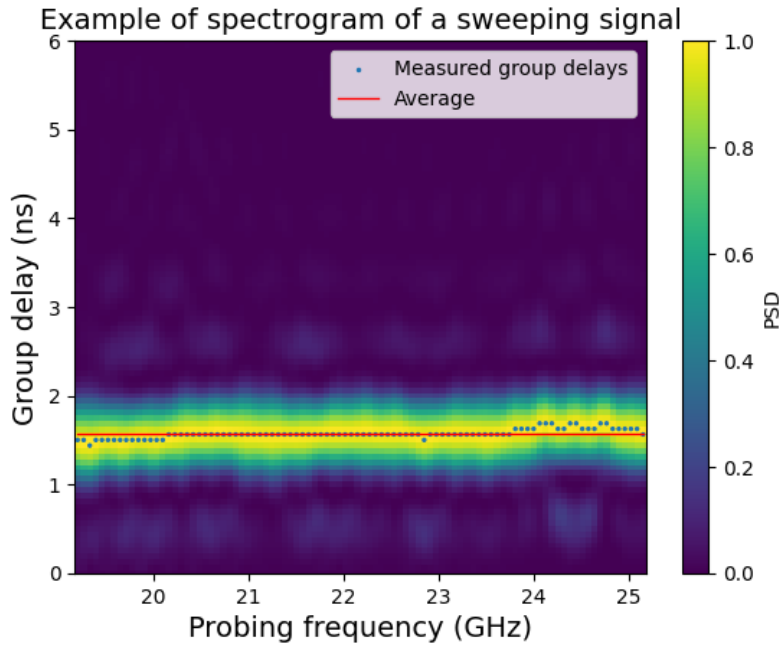


Figure 2.7: Example of a spectrogram for a sweeping signal in a fixed dispersion-less mirror configuration.

The theoretical spatial resolution of the reflectometric measurement (i.e., the ability to separate two close targets) can be evaluated from the MW bandwidth B .

For a full K band sweep ($B = 10.5 \text{ GHz}$), the resolution is computed as [20]:

$$\Delta R = \frac{c}{2B} \simeq 1.4 \text{ cm} \quad (2.41)$$

The accuracy of a reflectometric measurement can be much better than its resolution and depends on the signal to noise ratio of the detected signal. The evaluation of the accuracy is the main focus of Chapter 5 and 6 where the STFT is largely applied. As for Chapter 5, the spectrogram settings are those specified in this section, so the previously defined resolution serves as a reference value for the experimental characterization of the system.

Chapter 3

Theory of antennas and waveguides

As already mentioned in Section 2.4, the essence of any reflectometric diagnostic measurement relies on the design and implementation of a suitable microwave (MW) circuit to carry, launch, and collect the wave components. This is fulfilled by a system of antennas and waveguides capable of transporting and directing the MW sweeping beam towards the region of interest, thus acting as a transmission line between the reflectometric unit and the target. Again, Section 2.4 points out how these hardware components affect the final group delay measurement and that having a comprehensive understanding of how the waves interact with them is crucial for improving the quality of the diagnostic output.

For this reason, this chapter will introduce some relevant concepts concerning the physical behaviour of antennas and waveguides. It does not aim to cover all the complex aspects of the underlying theory but rather to discuss their impact on the actual reflectometric analysis. Section 3.1 provides a few insights into antenna theory [28], introducing the most relevant figures-of-merit and listing the overall features of aperture antennas, a specific configuration employed for reflectometry. Section 3.2 is instead focused on the characterisation of rectangular waveguides [29] and provides interesting details about their effects on MW propagation.

3.1 Basics of Antenna theory

A definition of "antenna" is provided by Webster's Dictionary as "a usually metallic device (as a rod or wire) for radiating or receiving radio waves". It can thus be interpreted, in the context of transmission lines, as a transitional structure between free-space and a guiding device. The latter is employed to transfer electromagnetic energy either from the source to the antenna (*transmitting antenna*) or all the way back from the antenna to the detector (*receiving antenna*).

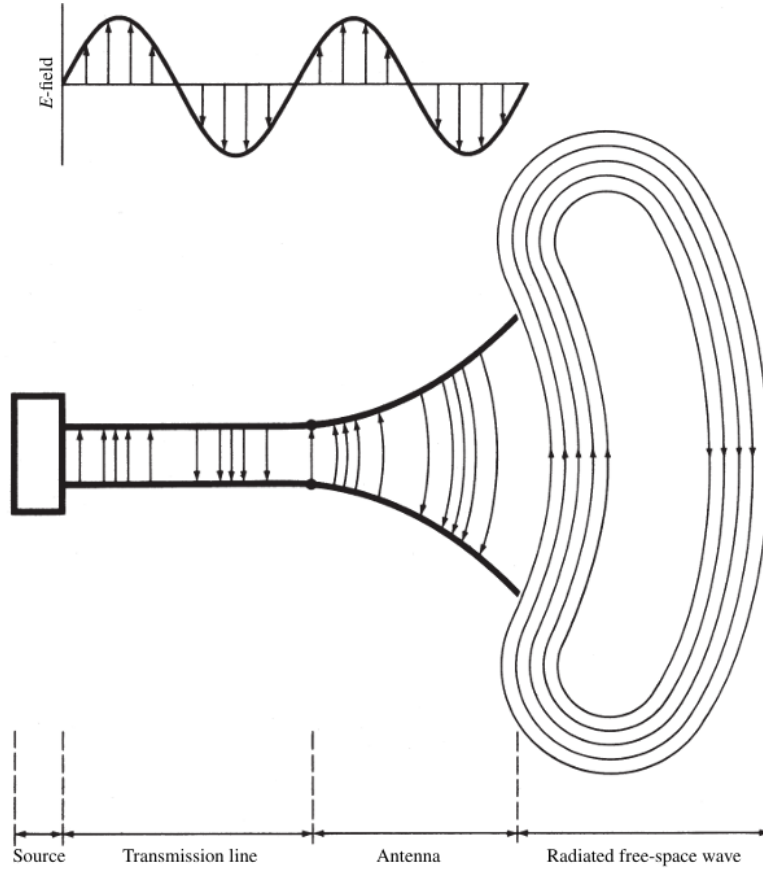


Figure 3.1: Scheme of a generic free-space transmitting antenna and its transmission line [28].

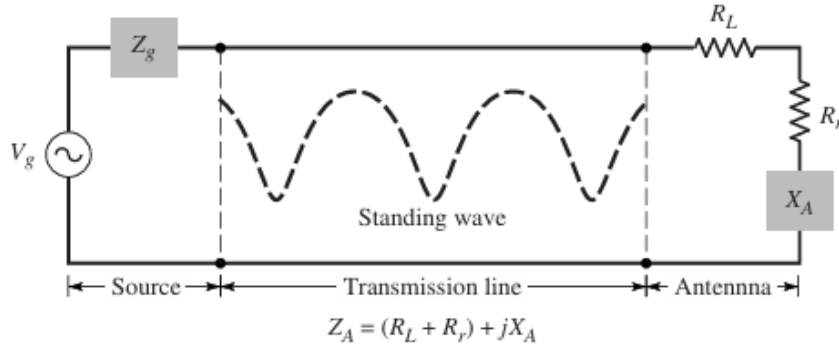


Figure 3.2: Transmission line Thevenin-equivalent of a generic free-space transmitting antenna [28].

3.1.1 Circuit representation

Figure 3.1 shows the generic scheme of a transmitting antenna set-up, including all its components (source, transmission line, antenna), while Figure 3.2 depicts the Thevenin-equivalent circuit of the same configuration.

Without entering into details, note that the antenna acts as a load for the transmission line, featuring an impedance defined as $Z_A = (R_L + R_r + jX_A)$. This gives a first interpretation of some practical physical effects.

- R_L is the loss resistance that accounts for conduction (e_c) and dielectric (e_d) losses associated with the antenna structure.

- R_r and X_A are the real and imaginary parts of Z_A , with R_r representing the emitted radiation and X_A representing the reactance (i.e., the energy stored in the reactive near field).
- V_g and Z_g are the source voltage and impedance. A mismatch between the load and the source impedances can lead to internal reflections (e_r).

The overall efficiency of the antenna is thus the product of the three previously mentioned effects:

$$e_0 = e_{cd} \cdot e_r \quad (3.1)$$

where $e_{cd} = e_c \cdot e_d$. Tracking these effects is a relevant exercise for any reflectometric measurement, making the choice of the bistatic system more suitable to avoid spurious components.

3.1.2 Radiation pattern

The term *Radiation pattern* refers to the spatial distribution of the radiated energy, both in terms of E.M. field intensity (*field pattern*) and power (*power pattern*). Figure 3.3 shows a generic field pattern generated by a transmitting antenna. The typical structure of the emitted beam is composed of the so-called *radiation lobes*. The *major lobe* defines the direction of maximum radiation and indicates how directional the antenna behaves. A series of *minor lobes* is typically observed together with the main one and must be minimized to avoid secondary beam components towards undesired directions.

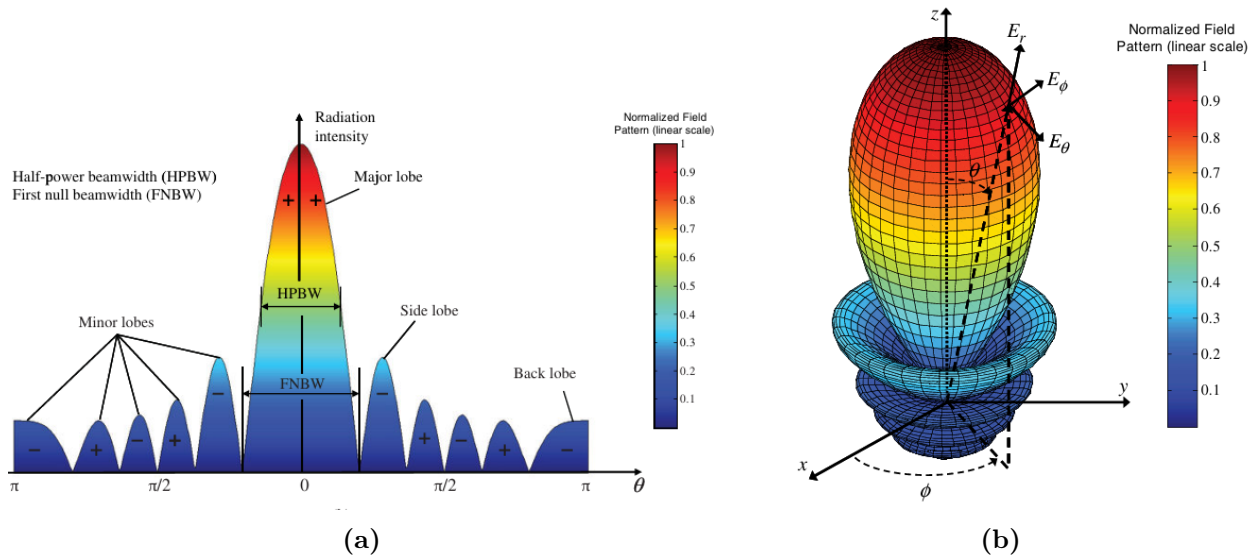


Figure 3.3: (a) 2D polar visualization of lobes and beamwidths of an antenna field pattern. (b) 3D Cartesian visualization of the same pattern [28].

The characterisation of the radiation pattern enables the extraction of several figures-of-merit to assess the performance of the antenna. Three relevant concepts are introduced.

- **Principal pattern:** for a linearly polarized antenna, performance is often described in terms of its principal E-plane and H-plane patterns. The E-plane is defined as the plane containing the electric-field vector and the direction of maximum radiation, while the magnetic field vector substitutes the electric one for the H-plane definition.

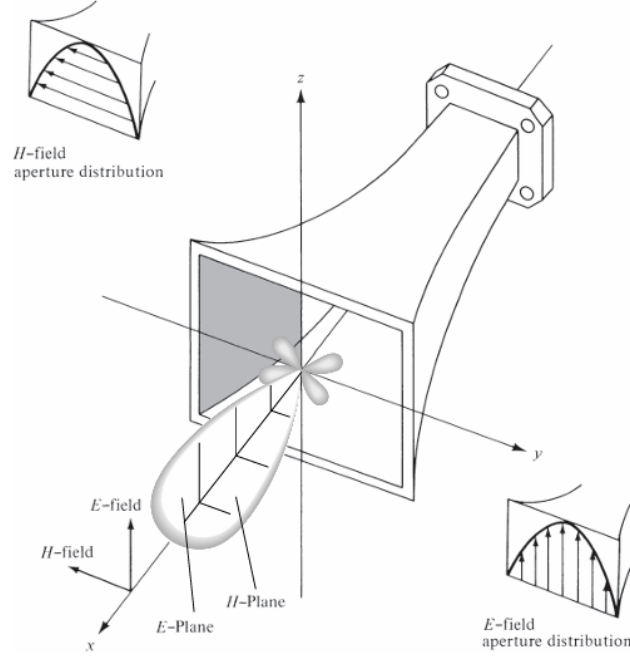


Figure 3.4: E and H planes for a pyramidal horn antenna [28].

- **Beamwidth:** it is the angular separation between two identical points on opposite sides of the major lobe. Typically, it is convenient to refer to the *Half-Power Beamwidth* (HPBW), defined as the angular separation between the two directions where the power is half its maximum value, or the *First-Null Beamwidth* (FNBW), which is defined by the two first nulls of the same pattern (see Figure 3.4). The beamwidth is an interesting property of the antenna's beam that characterizes its directional trend, the presence of side lobes (the narrower the beam is, the greater they are) and its resolution capability.
- **Directivity:** it is defined as the ratio of the intensity in a given direction to the intensity averaged over all directions. When the direction is not specified, it refers to the one in which the radiation intensity is maximum:

$$D_{max} = \frac{4\pi U_{max}}{P_{rad}} \quad (3.2)$$

where U_{max} is the radiation intensity (i.e., power per unit solid angle) at the maximum while P_{rad} is the overall irradiated power. The directivity

measures how concentrated the radiation is in a given direction and can be approximately linked to the beamwidth according to the following relationship:

$$D_0 \simeq \frac{4\pi}{\theta_1 \cdot \theta_2} \quad (3.3)$$

where θ_1 and θ_2 are the HPBW in two perpendicular planes.

3.1.3 Field regions

The space surrounding the antenna can be subdivided into three main regions (Figure 3.5):

1. **Reactive near-field region:** it coincides with the portion of space close to the mouth of the antenna, where the reactive field dominates. It features a spread pattern in which the main lobe structures are difficult to identify. In terms of distance, it can be approximately confined within the radius:

$$R < R_1 = 0.62 \sqrt{\frac{D^3}{\lambda}} \quad (3.4)$$

where D is the largest dimension of the antenna. It is thus necessary to avoid performing measurements in this region.

2. **Near-field (Fresnel) region:** the pattern starts to gain its shape but continues to depend on the distance from the antenna. Its boundaries are estimated by the following:

$$R_1 < R < R_2 = 2 \frac{D^2}{\lambda}$$

In the case in which $D > \lambda$ is not fulfilled, this region may not exist.

3. **Far-field (Fraunhofer) region:** the pattern here is independent of the distance, and the fields here are completely transverse, meaning that the radial components (E_r and H_r) can be neglected and that the fields are approximately described by the E-plane and H-plane. The lower limit of the region is defined by:

$$R > R_2 = 2 \frac{D^2}{\lambda} \quad (3.5)$$

provided $D > \lambda$. The pattern is well defined, and the directivity is established, thus making the far-field the most suitable region for any measurement.

It is thus fundamental to ensure that the reflecting target is located in the antenna far-field region to better exploit its directional properties.

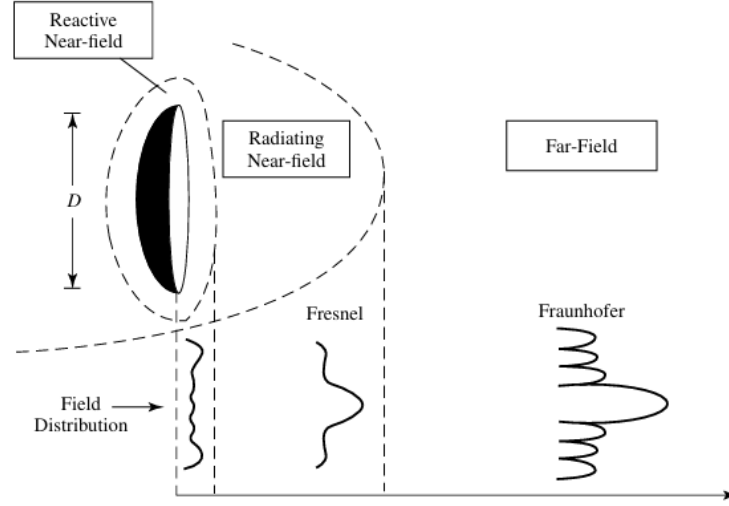


Figure 3.5: Visualization of the three regions of space with the corresponding pattern evolution [28].

3.1.4 Rectangular aperture antennas

As a conclusion to this introduction, a few more details about rectangular aperture antennas are presented. A complete analytical description is rather complex and requires a method to construct the equivalent current densities of the physical situation (Huygens Principle [28], [29]). These are then employed to solve the equivalent problem and compute the far-field radiation pattern.

In the case of a free-space rectangular aperture with dimensions a (H-plane) and b (E-plane) with $a > b$, the pattern (Figure 3.6) shows a narrow peak structure with a set of multiple lobes that appear when the aperture dimensions are greater than λ . The number of lobes increases as the dimensions exceed λ , meaning that a higher number is also expected for higher frequencies.

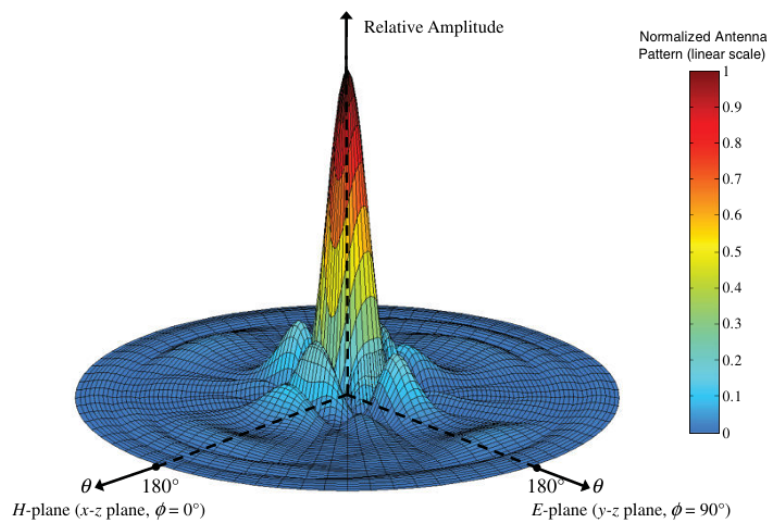


Figure 3.6: Visualization of the pattern for a free-space rectangular aperture [28].

The pattern of an aperture antenna depends on its design. A set of approximate

figures-of-merit is reported to check the main parameters affecting the quality of the probing beam.

- **Beamwidth:** the values of the beamwidth in both the E and H-planes are inversely proportional to the aperture size and the frequency:

$$HPBW(^{\circ}) \sim \begin{cases} \frac{50.8}{b}\lambda & \text{E-plane} \\ \frac{50.8}{a}\lambda & \text{H-plane} \end{cases}$$

$$FNBW(^{\circ}) \sim \begin{cases} \frac{114.6}{b}\lambda & \text{E-plane} \\ \frac{114.6}{a}\lambda & \text{H-plane} \end{cases}$$

- **Side lobe level:** the level of the side lobe power with respect to the maximum can also be computed and amounts to:

$$SLL(dB) \sim \begin{cases} -13.26 & \text{E-plane} \\ -13.26 & \text{H-plane} \end{cases}$$

- **Directivity:** finally, the directivity of the beam is directly proportional to the area of the aperture and on the frequency:

$$D_0 \sim \frac{4\pi}{\lambda^2} a \cdot b \quad (3.6)$$

To summarize these results in view of the reflectometric applications, it is important to stress that, for lower wave frequencies, a broader beam is expected, featuring fewer minor lobes, while the opposite trend should be observed as frequency increases

3.2 Modelling of Rectangular Waveguides

A comprehensive description of waveguide theory is necessary to understand the nature of wave propagation within reflectometric applications. In particular, the dispersive effects introduced by waveguides will seriously affect both the experimental (Chapter 5) and numerical (Chapter 6) STFT spectrograms, making the group delay extraction far from a trivial exercise.

The propagation of a wave through any guiding structure arises either from a boundary condition on the E.M. fields of the wave itself or from some reflection constraint at the guides' edges. These conditions allow only a discrete set of waves, called *modes*, to survive and propagate inside, featuring a dominant component (i.e., *the fundamental mode*) that depends on the specific size of the guiding structure and the frequency band to be carried, and a set of minor components (i.e.,

secondary modes) that might arise during propagation. The so-called *mode conversion* is one of the main physical effects occurring inside a waveguide, especially when the latter features bends and curved portions.

The general formulation of the guided wave problem starts from the solution of Maxwell's curl equations under specific boundary conditions. Assuming a plane wave with a time harmonic form propagating in the z -axis direction:

$$E \sim e^{j(\omega t - \beta z)} \quad (3.7)$$

where $j\beta$ is the generic propagation constant¹. Maxwell's equations can be reduced by substituting (3.7), thus obtaining the Helmholtz equation [29] for phasor fields:

$$\nabla^2 \mathbf{E} = -k^2 \mathbf{E}, \quad \nabla^2 \mathbf{H} = -k^2 \mathbf{H}$$

The three-dimensional ∇^2 can be broken into two parts, one of which accounts for the propagation direction and the other for the transverse dynamics:

$$\nabla^2 \mathbf{E} = \nabla_t^2 \mathbf{E} + \frac{\partial^2 \mathbf{E}}{\partial z^2}$$

With the assumed propagation function $e^{-j\beta z}$ in the axial direction, the system of equations now reads:

$$\frac{\partial^2 \mathbf{E}}{\partial z^2} = -\beta^2 \mathbf{E} \quad (3.8)$$

$$\nabla_t^2 \mathbf{E} = -(k^2 - \beta^2) \mathbf{E} \quad (3.9)$$

$$\nabla_t^2 \mathbf{H} = -(k^2 - \beta^2) \mathbf{H} \quad (3.10)$$

The usual procedure is to find two components of the fields (E_z and H_z) that satisfy the wave equations (3.9) and (3.10) and the boundary conditions, and

¹Note that, unlike Chapter 2, where the imaginary unit was denoted by i , the engineering convention j is adopted here to be consistent with [28] and [29].

then find the other field components by using Maxwell's curl equations.

$$\nabla_t^2 E_z = -k_0^2 E_z \quad (3.11)$$

$$\nabla_t^2 H_z = -k_0^2 H_z \quad (3.12)$$

$$E_x = -\frac{1}{k_0^2} \left(j\beta \frac{\partial E_z}{\partial x} + j\omega\mu \frac{\partial H_z}{\partial y} \right) \quad (3.13)$$

$$E_y = \frac{1}{k_0^2} \left(-j\beta \frac{\partial E_z}{\partial y} + j\omega\mu \frac{\partial H_z}{\partial x} \right) \quad (3.14)$$

$$H_x = \frac{1}{k_0^2} \left(j\omega\varepsilon \frac{\partial E_z}{\partial y} - j\beta \frac{\partial H_z}{\partial x} \right) \quad (3.15)$$

$$H_y = -\frac{1}{k_0^2} \left(j\omega\varepsilon \frac{\partial E_z}{\partial x} + j\beta \frac{\partial H_z}{\partial y} \right) \quad (3.16)$$

where $k_0^2 = k^2 - \beta^2$. The propagation constant β is a complex number encoding both the actual wave propagation along z ($Re(\beta)$) and the attenuation ($Im(\beta)$) associated with conduction current losses across the dielectric medium and on the metallic walls of the waveguide. Introducing the boundary conditions that the waves must fulfil, it is possible to identify three main categories of propagating modes:

<i>Transverse E.M. modes (TEM)</i>	$\rightarrow E_z = H_z = 0$
<i>Transverse magnetic modes (TM)</i>	$\rightarrow H_z = 0$
<i>Transverse electric modes (TE)</i>	$\rightarrow E_z = 0$

3.2.1 TE waves in rectangular waveguides

For the purposes of the thesis, the discussion will focus only on the TE modes for rectangular waveguides, meaning that boundary conditions on 4 sides must be included. The equation of interest (3.12) has a generic solution of the form:

$$H_z = [A \sin(k_x x) + B \cos(k_x x)] \cdot [C \sin(k_y y) + D \cos(k_y y)] \quad (3.17)$$

with $k_0 = k_x^2 + k_y^2$. The electric field components $E_x = -\frac{j\mu\omega}{k_0^2} \frac{\partial H_z}{\partial y}$ and $E_y = \frac{j\mu\omega}{k_0^2} \frac{\partial H_z}{\partial x}$ arising from (3.13) and (3.14) are subjected to specific boundary conditions, requiring the former to be 0 at $y = 0$ and $y = b$, the latter at $x = 0$ and $x = a$, according to the coordinate system in Figure 3.7a.

The expression for H_z thus reduces to:

$$H_z = K \cos(k_x x) \cos(k_y y) \quad (3.18)$$

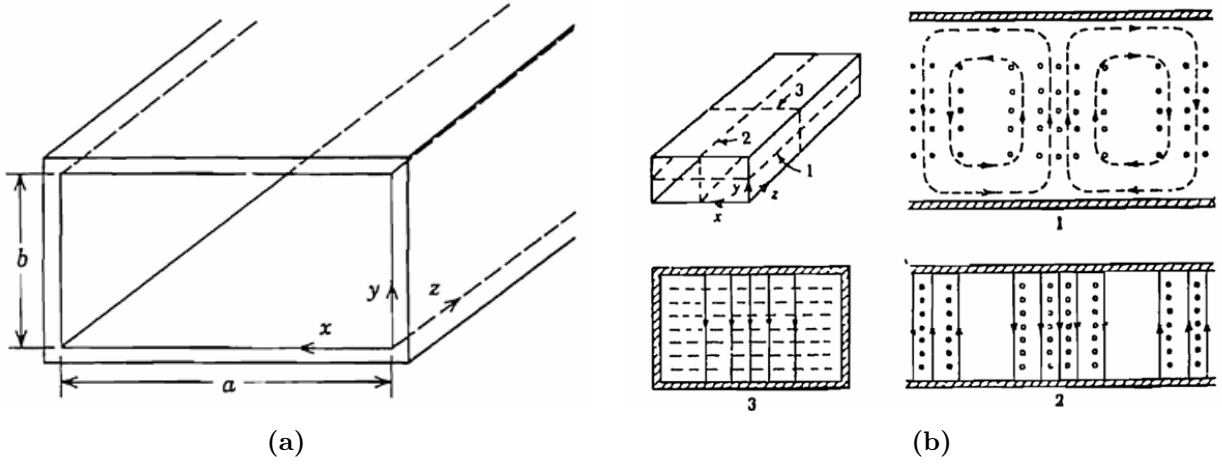


Figure 3.7: (a) Coordinate system for rectangular guide. (b) Scheme of the $TE_{1,0}$ -mode fields inside the rectangular guide [29].

with $K = B \cdot D$, $k_x = m\frac{\pi}{a}$ and $k_y = n\frac{\pi}{b}$. The indices m and n introduce the previously mentioned *mode discretization*, ensuring that only a discrete set of waves, characterized by an integer couple (m, n) , can survive inside a waveguide. The corresponding electric and magnetic field components are thus listed:

$$\begin{aligned} E_x &= \frac{j\omega\mu k_y}{k_{0m,n}^2} K \cos k_x x \sin k_y y \\ E_y &= -\frac{j\omega\mu k_x}{k_{0m,n}^2} K \sin k_x x \cos k_y y \\ H_x &= \frac{j\beta k_x}{k_{0m,n}^2} K \sin k_x x \cos k_y y \\ H_y &= \frac{j\beta k_y}{k_{0m,n}^2} K \cos k_x x \sin k_y y \end{aligned}$$

where $k_{0m,n} = \sqrt{\left(m\frac{\pi}{a}\right)^2 + \left(n\frac{\pi}{b}\right)^2}$. The expression for $k_{0m,n} = \omega_{0m,n}\sqrt{\mu\epsilon}$ allows for the study of the propagation and attenuation inside the guiding structure. Recovering the expression for β :

$$\beta = \omega\sqrt{\mu\epsilon}\sqrt{1 - \frac{\omega_{0m,n}^2}{\omega^2}} \quad (3.19)$$

It is straightforward to check how this parameter is real when $\omega > \omega_{0m,n}$, thus identifying $\omega_{0m,n}$ as the *cut-off frequency* for the (m, n) -mode:

$$\omega_{0m,n} = \frac{1}{\sqrt{\mu\epsilon}}\sqrt{\left(m\frac{\pi}{a}\right)^2 + \left(n\frac{\pi}{b}\right)^2} \quad (3.20)$$

The condition for propagation requires the frequency of the wave to be greater than the cut-off and is strictly dependent on the aperture sizes of the guide, thus

enabling proper tuning of its dimensions to select any specific mode.

Finally, it is possible to derive the group velocity of the waves by exploiting the expression for β :

$$v_g(\omega) = \frac{d\omega}{d\beta} = v \sqrt{1 - \frac{\omega_{0m,n}^2}{\omega^2}} \quad (3.21)$$

where $v = 1/\sqrt{\mu\epsilon}$ is the speed of light in a dielectric medium. The dependence of v_g on ω clearly indicates a dispersive effect of the guides on the waves propagating inside, provided their velocity assumes a different value according to their frequency. The corresponding group delay can be evaluated as:

$$\tau_g(\omega) = \frac{d\phi}{d\omega} = \Delta z \cdot \frac{d\beta}{d\omega} = \frac{\Delta z}{v} \cdot \frac{1}{\sqrt{1 - \frac{\omega_{0m,n}^2}{\omega^2}}} \quad (3.22)$$

Equation (3.22) is a fundamental result of the present discussion and will be taken as a reference to model the dispersion contribution in Chapters 5 and 6.

3.2.2 TE_{1,0}-mode

The end of the chapter is dedicated to the discussion of the TE_{1,0} mode. This is the fundamental mode typically employed in many cases, featuring the lowest cut-off frequency achievable for a guide. By substituting $m = 1$ and $n = 0$ into (3.20), one obtains:

$$\omega_{01,0} = \frac{\pi}{a} \frac{1}{\sqrt{\mu\epsilon}} \iff \lambda = 2a \quad (3.23)$$

In general applications, a waveguide is designed to host only the TE_{1,0}-mode by selecting an operating frequency band close enough to the cut-off. This strategy ensures that only one mode can propagate, avoiding signal distortions caused by multimode propagation and limiting the higher attenuation encountered at increasing frequencies. On the other hand, it is also convenient to work not too close to the cut-off to minimize dispersion as much as possible. The specific guide design is the result of a trade-off between the necessity of avoiding higher-order mode excitations and the effort of limiting the TE_{1,0}-mode attenuation. On one side, from (3.23) it is clear how $\omega_{01,0}$ depends only on the higher dimension of the guide (a), while the lower one (b) can be made small enough to prevent any higher order mode excitation by increasing the separation between TE_{1,0} and TE_{0,1} cut-off frequencies (with the latter depending only on b , by symmetry). On the other side, a larger attenuation is observed for small b/a ratios because of the high ratio of surface to cross-sectional area. However, the latter is not excessive when compared with other wave types in guides of similar size [29].

The specific E.M. fields for TE_{1,0}-mode are the following:

$$H_z = K \cos k_x x$$

$$E_y = -\frac{j\omega\mu K}{k_x} \sin k_x x$$

$$H_x = \frac{j\beta K}{k_x} \sin k_x x$$

and the corresponding visualization is provided by Figure 3.7b. The electric field is completely vertically polarized, while the magnetic field lies in horizontal planes and surrounds the E-field lines.

The TE_{1,0}-mode propagation inside rectangular waveguides is the standard experimental design employed in reflectometric applications. In this specific case, RFX-mod2 will feature an advanced PPR system made of rectangular waveguides and exploiting TE_{1,0} to obtain a vertically (or poloidally) polarized field to ensure X-mode propagation inside the plasma. Understanding the physical nature of the probing waves and how they are transported throughout the various sections of the machine is of fundamental importance to provide a correct interpretation of the extracted data, as will be made clear in Sections 5 and 6.

Chapter 4

Reflectometry at RFX-mod2

The structural changes of RFX-mod2 depicted in Section 1.5.2 involve the design of an advanced poloidally distributed reflectometric diagnostic consisting of both DPR and PPR systems. In particular, the former is designed mainly for RFP scenarios, while the latter will be operated only in the Tokamak configuration. The integration of the PPR turned out to be quite challenging due to severe constraints linked to the design of the reflectometric system (antennas plus waveguides), especially in the *High Field Side* (HFS) region, where space is rather limited. In this chapter, a description of the main aspects of RFX-mod2 reflectometry is provided, starting from an overview of the systems (Section 4.1), and then moving to a more technical description of the HFS subsystem (Section 4.2), [2], [22], [30]. Finally, the possible applications under various operational scenarios are discussed in Section 4.3 [2], [31].

4.1 System overview

A broadband O-mode reflectometric diagnostic has been implemented on the previous RFX-mod experiment [31], [22]. It consisted of an ultrafast sweeping reflectometric system with five bands (from 26.5 GHz to 110 GHz) covering a large density range ($\simeq 9 \cdot 10^{18}$ to $\simeq 1.5 \cdot 10^{20} \text{ m}^{-3}$) and a homodyne detection scheme (Section 2.4.2). It was designed to perform density measurements along a single line of sight in the *Low Field Side* (LFS) equatorial plane to characterize the edge plasma dynamics in RFP discharges and its relationship with the edge magnetic topology. The forthcoming RFX-mod2 will instead be equipped with an advanced system including both PPR and DPR. In particular, the PPR diagnostic, as shown in Figure 4.1, will be installed in 4 different poloidal locations, two of which lie in the vertical LoS (top and bottom), while the remaining two cover the two sides of the equatorial plane (HFS and LFS). Each reflectometric system is built in a bistatic configuration with two antennas that handle transmission and reception separately. The top, bottom and LFS apparatuses feature standard pyramidal

horn antennas, while the signal is delivered by a copper WR-42 linear waveguide system. Table 4.1 summarises the main parameters of interest for the two set-ups. The PPR system will exploit the K band upper X-Mode cut-off frequency (2.24) to probe the edge portion of the density profile, from which the position of the LCFS is reconstructed using some of the procedures introduced in Section 2.3.2. The DPR system will instead be located at the LFS and will operate from K to U band in O-mode configuration to measure a larger portion of the density profile. Although its application is mainly addressed to RFP scenarios, the DPR can also be employed in X-mode in the same way as PPR by changing the plasma configuration to Tokamak.

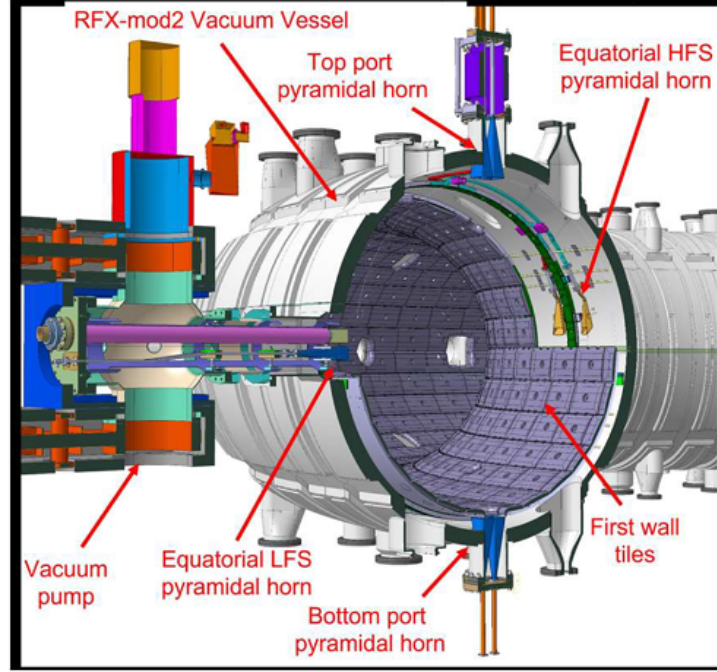


Figure 4.1: Scheme of the 4 reflectometric systems integrated in RFX-mod2 [22].

System	Scenario	Propagation mode	Frequency range (GHz)	Density range ($10^{18} \cdot m^{-3}$)
DPR	RFP	O-mode	16 - 60	$\simeq 3 - 45$
PPR	Tokamak	X-mode	16 - 26.5	$\simeq 0.5 - 3.5$

Table 4.1: Summary of DPR and PPR parameters.

4.2 HFS subsystem

The highest complexity is found in the HFS region since, due to severe space limitations, neither pyramidal antennas nor straight waveguides would fulfil the machine integration constraints. Instead, the waveguides will enter the vacuum vessel from the top port and will follow a curved trajectory along the toroidal

passive stabilizing shell. At the level of the inner mid-plane, two hoghorn antennas with a parabolic reflecting surface are employed as the launcher and receiver, ensuring a compact design and allowing for space optimization.

These constraints pose several complications and the system required a series of verification tests to assess its proper performance.

4.2.1 HFS Antenna design and tests

The design of the HFS antennas is the result of meticulous work devoted to coping with the limited available space while ensuring proper efficiency. Indeed, a *Metal Additive Manufacturing process* (MAM) is necessary to realise a hoghorn design that is compatible with the space constraints. In this specific case, stainless steel antennas (AISI 316) are chosen, and further surface treatment (electropolishing and rhodium/silver coating) is applied to reduce the surface roughness introduced by the MAM. The specific design of the antenna has been chosen after a proper analysis with the COMSOL Multiphysics Radio Frequency module [30]. Four early versions of the hoghorn antennas were developed, and their radiation patterns were characterised, along with the evaluation of the corresponding directivity. The chosen prototype provided the most uniform and directional response in the expected plasma boundary region (from 5-10 cm to 15-20 cm antenna-plasma distance) as clearly shown in Figure 4.2.

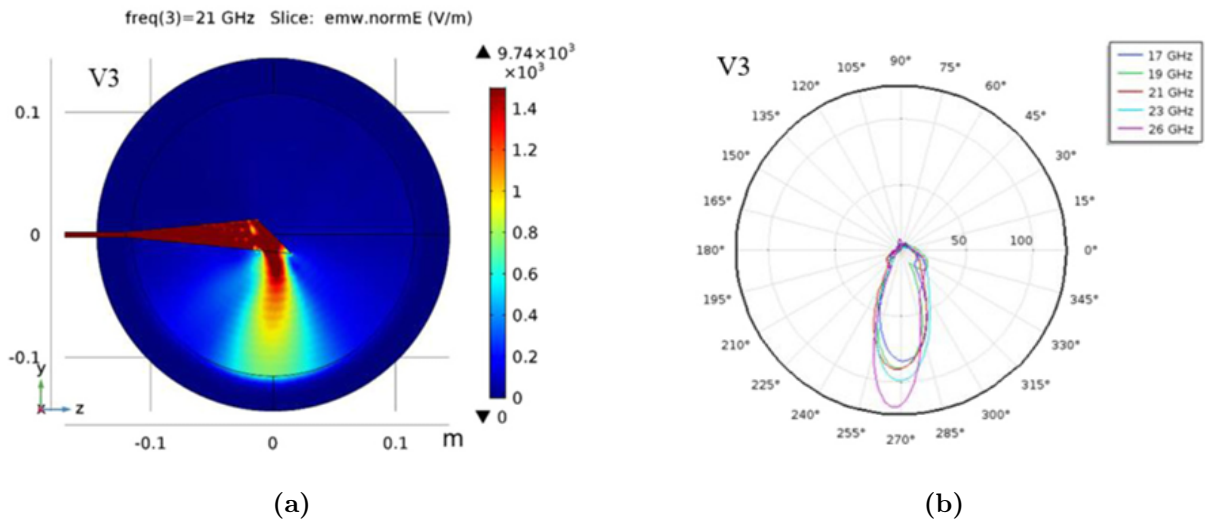


Figure 4.2: Results of antennas' pattern analysis. (a) Radiation pattern for a 21 GHz wave. (b) Far-field polar plot at various frequencies [30].

To further assess the performance of the hoghorn antenna, a set of preliminary measurements has been performed [22], [32], employing both experimental bench tests, numerical modelling using COMSOL, and theoretical calculations by means of the Huygens principle. A discrepancy between theoretical and experimental/numerical results was found due to the partial limitation of the parabolic reflector in tilting the radiation entirely. This led to interference effects between

the reflected and unaltered components of the electromagnetic field. Despite this limitation, an overall validation of the MAM antennas' performance with respect to the traditional ones was achieved.

4.2.2 Waveguides design and tests

As for waveguide manufacturing, there are three types of issues that might attenuate and downgrade signal delivery: ohmic losses, higher-order mode excitations, and internal reflections [33]. While the first two are marginal for K band applications, for which neither higher order surviving modes nor relevant ohmic dissipations are expected, the third effect is the main concern of the analysis of [22]. The guide bending represents a delicate step, and its effects are far from trivial and it might cause the rise of internal reflections due to the deformation of the waveguide's section. Figure 4.3 shows a comparison of the received signal resulting from a straight waveguide prototype and that from two curved prototypes. The signature of reflections is highlighted by the presence of periodic structures in the received signal amplitude, as occurs for the WG-1 waveguide.

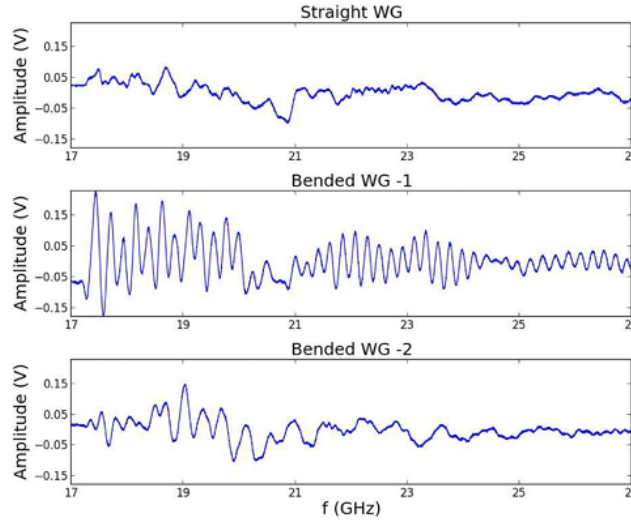


Figure 4.3: Signal received for different waveguide configurations [22].

4.2.3 Machine integration

The integration of the hoghorn antennas in the HFS subsystem requires the set-up to cope with the limited available space, forcing the waveguides to be anchored to one of the copper shell stiffening rings (Figure 4.4). The two antennas are thus placed at a toroidal distance of $2\Delta \simeq 13 \text{ cm}$, making them more susceptible to possible performance downgrades under strong edge turbulence. To overcome

this issue, a reciprocal tilt angle $\theta_t \simeq 13^\circ$ between them is introduced to improve their relative coupling. For this reason, a specific twisting element (Figure 4.5) is mounted between the antenna and the waveguide.

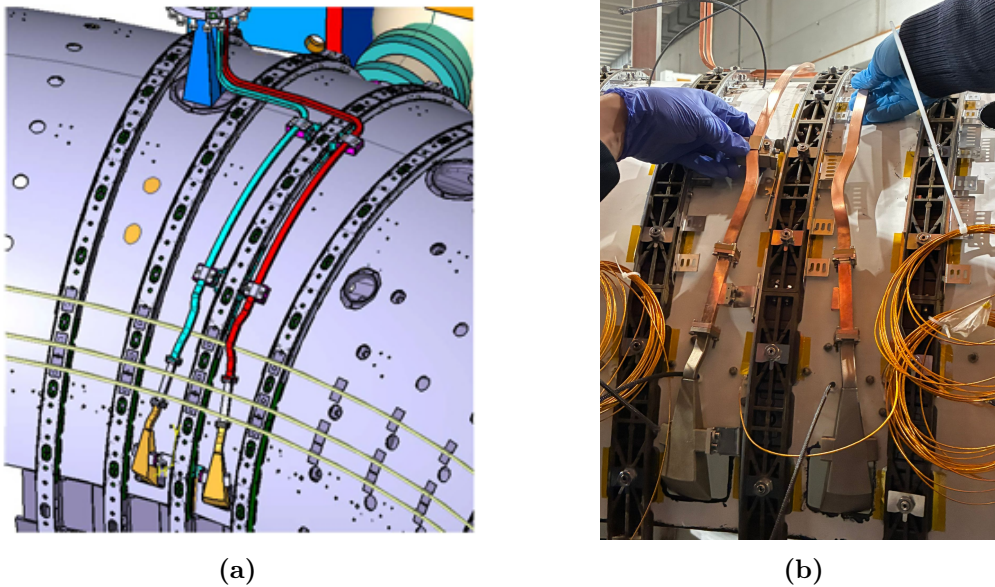


Figure 4.4: HFS systems. (a) CAD scheme of the HFS waveguides and antennas [2]. (b) Picture of HFS system mounted on RFX-mod2.

As a result of the tilt, the HFS LoS features a focal antenna-plasma distance $L_f \simeq 28.2 \text{ cm}$ at which the coupling between the two antennas should be optimized. This is consistent with the expected Tokamak scenarios (Section 4.3.1) where the separatrix of a D-shaped plasma is expected to be located at $\simeq 20\text{-}25\text{cm}$ from the first wall. The following Table 4.2 summarizes several technical features of the machine and HFS subsystem. Given the impossibility of performing direct measurements on the machine's set-up, these values are gathered either from theoretical tabulated sources or from the RFX-mod2 CAD model (see Section 6.3.2) using *Autodesk Fusion* (AF) measurement tools, and thus they are all reported without uncertainty. The ones coming from the AF are indicated with the same acronym inside Table 4.2. Of particular importance for the following chapters are the height of the antennas' LoS (z_{LoS}) and the antenna-wall distance (d_{AW}), both coming from AF.

4.3 Operational scenarios

The previously described advanced reflectometric set-up (Figure 4.1) will take advantage of the extreme flexibility of RFX-mod2 for operating both in Tokamak and RFP magnetic configurations. In particular, the PPR system is specifically optimized to provide plasma position measurements in Tokamak operations [2], while the DPR can be reliably operated in both types of discharge.

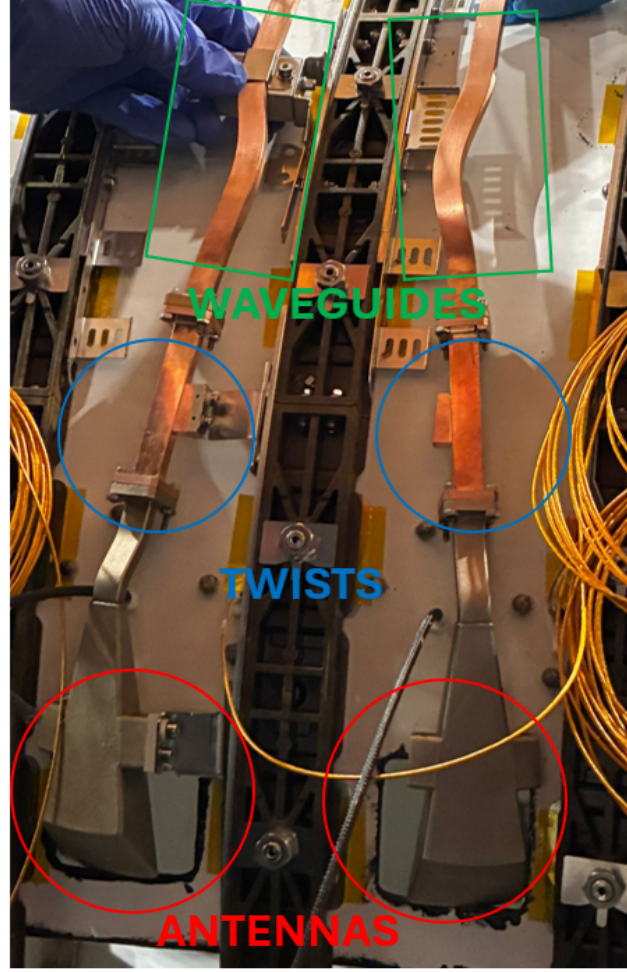


Figure 4.5: Picture of the HFS components: antennas (red), twists (blue) and waveguides (green).

Component	Value
Major radius R_0 [22]	$\simeq 2 \text{ m}$
Minor radius a [22]	$\simeq 0.49 \text{ m}$
Distance of antennas 2Δ [22]	$\simeq 13 \text{ cm}$
Antenna a aperture A_a [AF]	$\simeq 39.8 \text{ mm}$
Antenna b aperture A_b [AF]	$\simeq 22.2 \text{ mm}$
Antennas near field (3.4)	$\simeq 3.8 \text{ cm} - 4.6 \text{ cm}$
Antennas far field (3.5)	$\simeq 19 \text{ cm} - 28 \text{ cm}$
Tilt angle θ_t [22]	$\simeq 13^\circ$
Focal distance L_f	$\simeq 28.2 \text{ cm}$
Antenna-wall distance d_{AW} [AF]	$\simeq 3.3 \text{ cm}$
LoS height z_{LoS} [AF]	$\simeq 5.8 \text{ cm}$

Table 4.2: Various components and their measures.

4.3.1 Tokamak scenarios

The Tokamak applications of RFX-mod2 can serve as a test framework for demonstrating, on smaller scales, the applicability of reflectometry for horizontal and vertical position control purposes [2]. It is thus necessary to investigate PPR performance under non-ideal magnetic structures and obtain results that can address some of the issues related to the development of a reflectometry-based plasma control and evaluate the robustness of this technique in a flexible environment. On one hand, varying the plasma shape (Figure 4.6) affects the performance of the PPR system in the upper and lower (X-point) regions, where the curvature of the magnetic surfaces is more significant. On the other hand, the complexity and additive manufacturing of the HFS system, which represent the main focus of the previous section, pose several technical challenges that need to be tested further involving real discharge applications. Finally, a calibration procedure is required to account for several group delay contributions associated with the microwave circuit, while the evaluation of separatrix position (Figure 4.7) requires more comprehensive information on the edge dynamics. These requirements can be made more realistic by integrating the reflectometric measurements with additional edge diagnostics available on RFX-mod2 to provide the best possible description of the edge dynamics. For example, Thomson scattering can serve as a calibration technique for reflectometry, while interferometry can be employed for plasma position control purposes, as in ASDEX-Upgrade [23].

These investigations will provide useful information in light of the similar problems that must be faced in future experiments, such as DTT and DEMO.

4.3.2 RFP scenarios

As concerns RFP applications, ultrafast sweeping reflectometry is crucial to investigate the mechanism that regulates the physics of the edge. The internal transport barrier, the helical rotating structure, and the high level of fluctuations characterizing an MH state can seriously affect the main edge physical phenomena: plasma-wall interactions, impurity influx, and wall conditioning are some of the several challenges for which fast control of the edge is strongly required. To deal with this complexity, an ultrafast microwave reflectometer has already been developed for RFX-mod [31], and the measurements with Ka-band have been successfully obtained for the first time for an RFP plasma, paving the way for future applications on RFX-mod2. In particular, the interplay between the external $m = 0$ mode and the internal $m = 1$ mode during a QSH state can be investigated (Figure 4.8a). DPR, through the O-mode broadband reflectometry approach, is crucial for understanding the relationship between edge density dynamics and local magnetic topology and for identifying the global parameters that regulate outer plasma behaviour (Figure 4.8b) and the persistence of QSH. This

can be implemented in RFX-mod2 activities once the machine is operational.

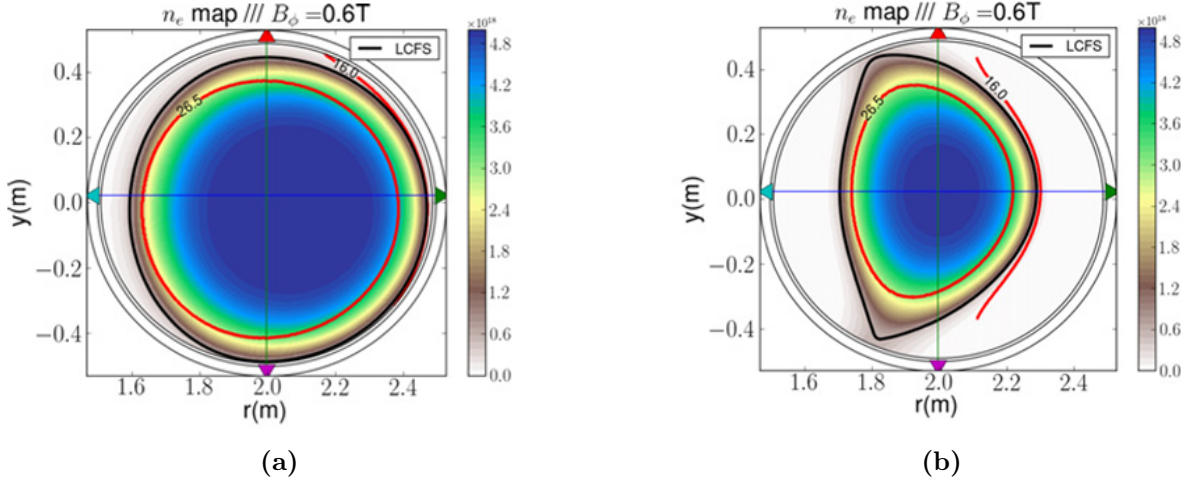


Figure 4.6: Examples of RFX-mod2 Tokamak scenarios with the corresponding separatrix and frequency range: (a) circular plasma and (b) D-shaped plasma [2].

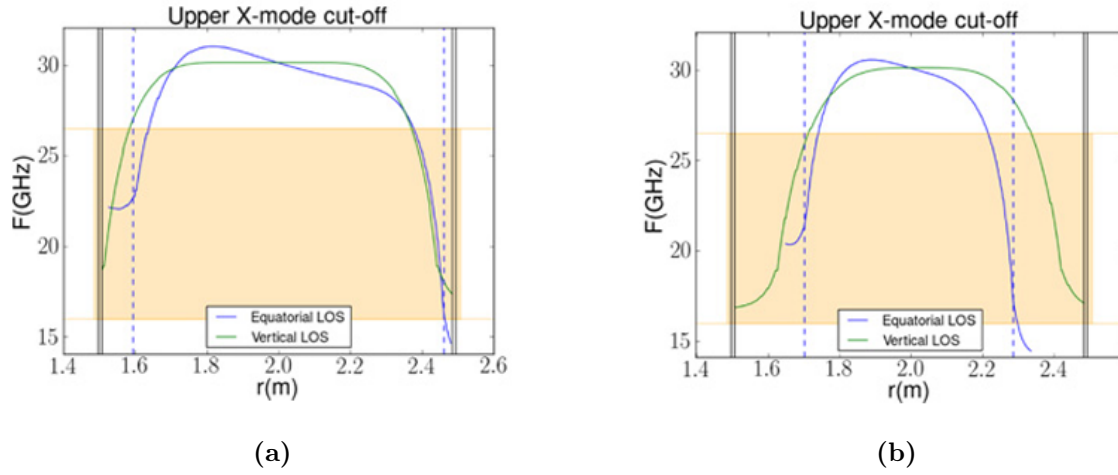


Figure 4.7: Profile reconstruction for equatorial (blue) and vertical (green) LoS with the corresponding frequency range and the evaluated equatorial separatrix for: (a) circular and (b) D-shaped plasma [2].

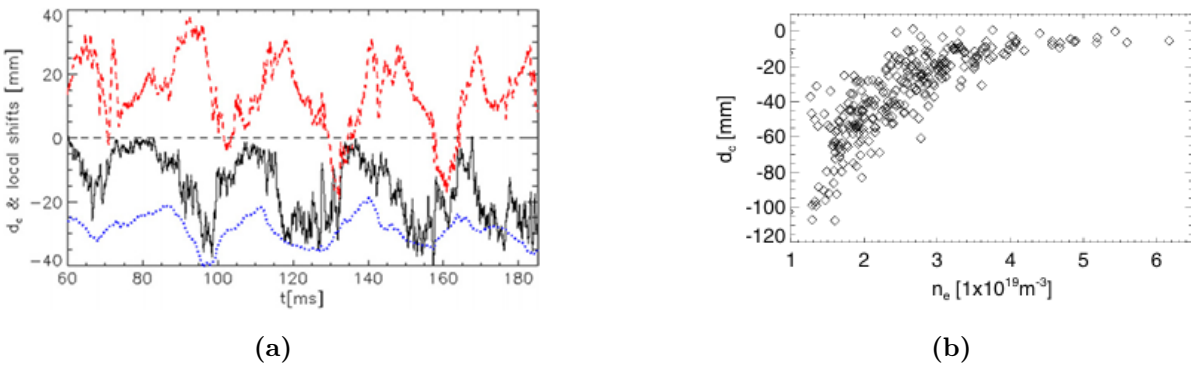


Figure 4.8: (a) Time evolution of local edge magnetic topology featuring the plasma-wall distance d_c (black) the $m = 0$ displacement (red) and the $m = 1$ displacement (blue). (b) Average d_c as a function of the line-averaged density n_e [31].

Chapter 5

Experimental results

The work aims to demonstrate the system’s capability of reconstructing the position of a reflecting plane mirror by detecting the group delay accumulated by the reflected wave with respect to the probing one (Section 2.3). Once the performance is validated in this ideal configuration, the same analysis is carried out under more unfavourable conditions to investigate the limitations of the reflectometric measurement set-up and to characterize the main sources of error.

The chapter is thus organised as follows: after introducing the experimental set-up (Section 5.1), the first part of the analysis is presented in Section 5.2. The antenna-mirror distance is varied, and the corresponding position is reconstructed to assess the linearity of the system’s response and characterise its accuracy. In the second part, an analysis of the performance is carried out by investigating the effect of the tilted geometry (Section 5.3) and by changing the mirror deviation angle (Section 5.4) to look for possible downgrades due to the non-optimal perpendicularity of the reflecting surface.

5.1 Overview of the experimental set-up

The experimental characterisation of the RFX-mod2 PPR system is performed with a series of bench measurements. Figure 5.1 shows the experimental set-up: the waveguides are held by a supporting structure (Figure 5.1a) to simulate the spatial geometry of RFX-mod2, and a twisting element applied between the waveguide and the antenna allows for the 13° tilting (Figure 5.1b). A metallic mirror is placed in front of the antennas to simulate the reflecting behaviour of the plasma.

The reflectometric unit (Section 2.4.1) follows the generic scheme reported in Figure 2.5. The ends of the two waveguides are connected to the probing (TX) and receiving (RX) ports of the LC Technology Sweep Reflectometer 0496 through RF Cable Assembly SF104/11SMA/11SMA77A (Figure 5.2). The *Putty* software installed on a computer allows for configuring the reflectometer through the Telnet

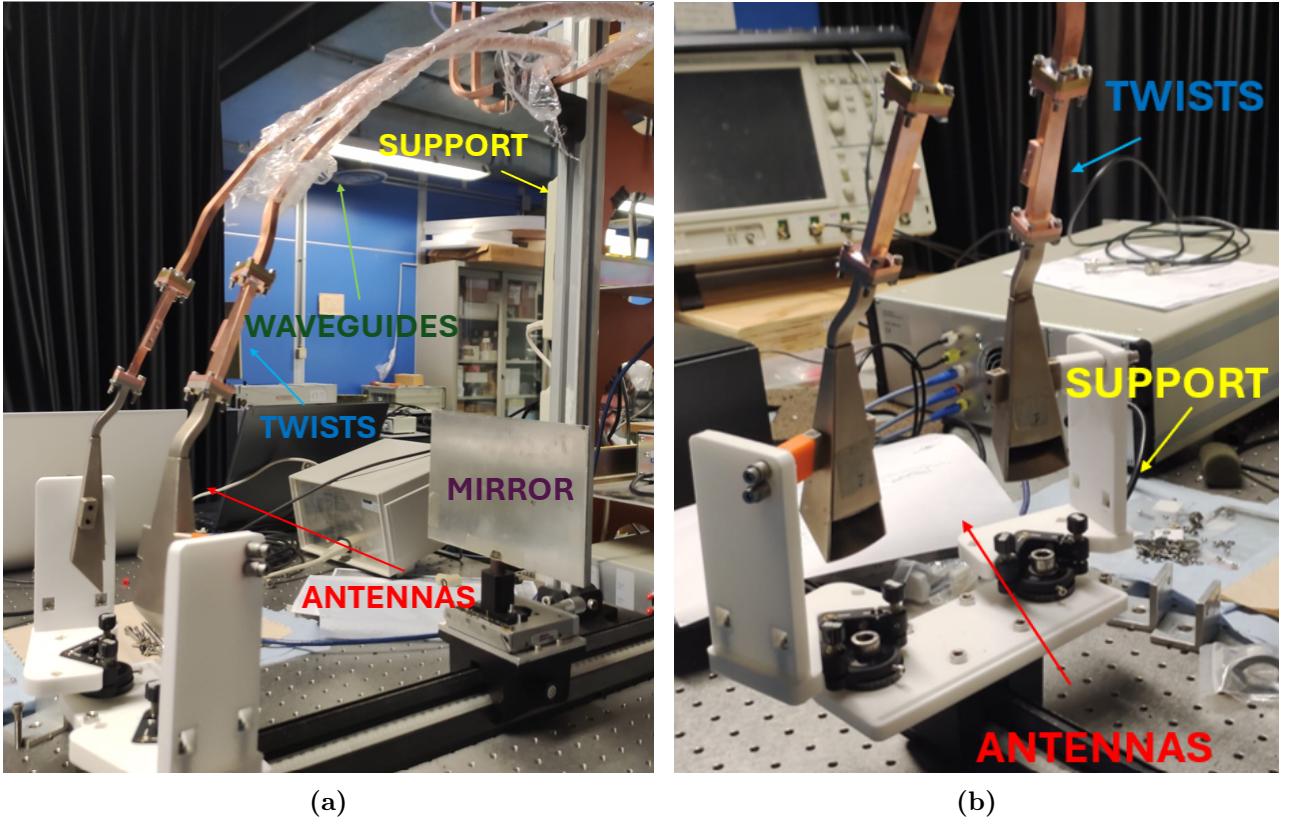


Figure 5.1: HFS subsystem mounted on the optical bench, reproducing the RFX-mod2 geometry: (a) overall view showing the supporting structure that simulates the spatial layout of the machine; (b) close-up of the two hoghorn antennas with the twisting elements introducing the 13° tilt.

protocol, setting an FMCW in the range $[18, 26]$ GHz , corresponding to the MW K band, and $T_{sw} = 10 \mu s$. The latter is mixed with the reference signal and low-pass filtered to extract $S_{mix,LP}$ according to the homodyne detection scheme (Section 2.4.2). The auxiliary components of the reflectometer set-up are as follows.

- **Pulse generator:** it generates a $\simeq 8 V$ square pulse with $100 \mu s$ of repetition time and $10 \mu s$ of width that serves as the trigger for both the VCO and the DAQ (Figure 5.3a).
- **Picoscope 6000E series:** it serves as the DAQ system for the mixer signal, allowing data to be stored directly on the user's computer thanks to the *Picoscope 7 Software* and the *pypicosdk* Python library (Figure 5.3b).
- **Delay line:** it induces a $2 m$ delay in the reference signal (Figure 5.2b).
- **Attenuator:** it is applied to avoid saturating the voltage range of the mixer (Figure 5.2b).

To test the response of the system at varying target–antenna distances L (and therefore varying free-space round-trip group delay τ_g), the mirror apparatus mounted on a translation stage (Figure 5.1a) is the best choice for providing

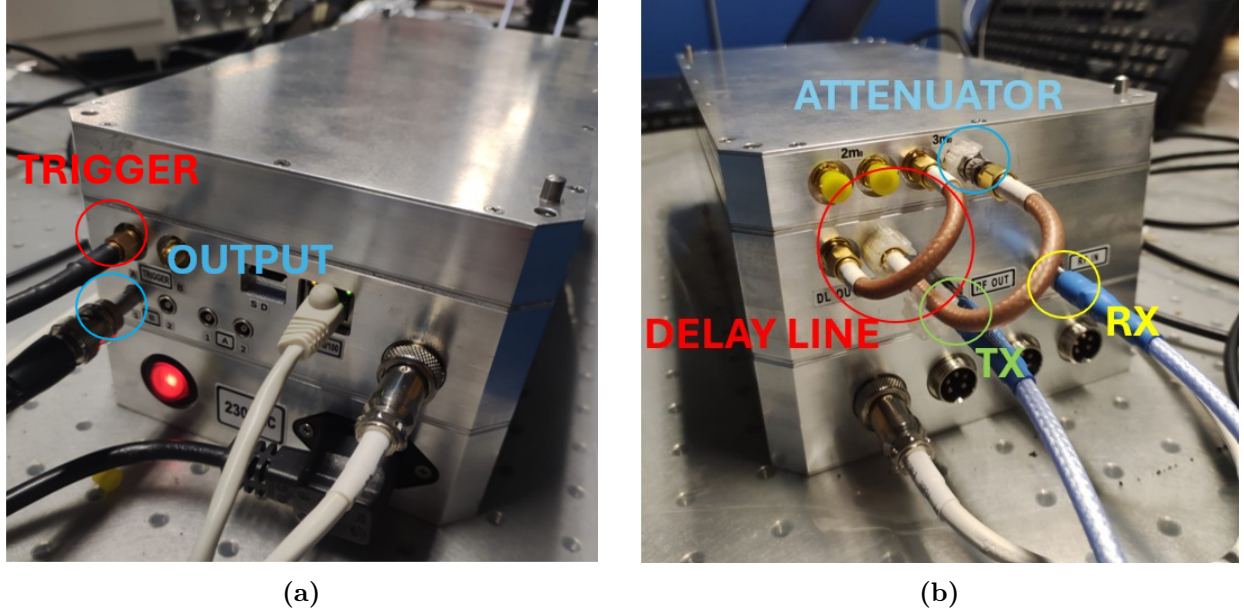


Figure 5.2: Reflectometer interface and trigger logic: (a) output port and trigger signal provided by the Pulse Generator that synchronises the start of the FMCW sweep; (b) RF connection between the two waveguide ends and the TX/RX ports of the LC Technology Sweep Reflectometer 0496 via the SF104 SMA cable assembly; the delay line and attenuator are also indicated.

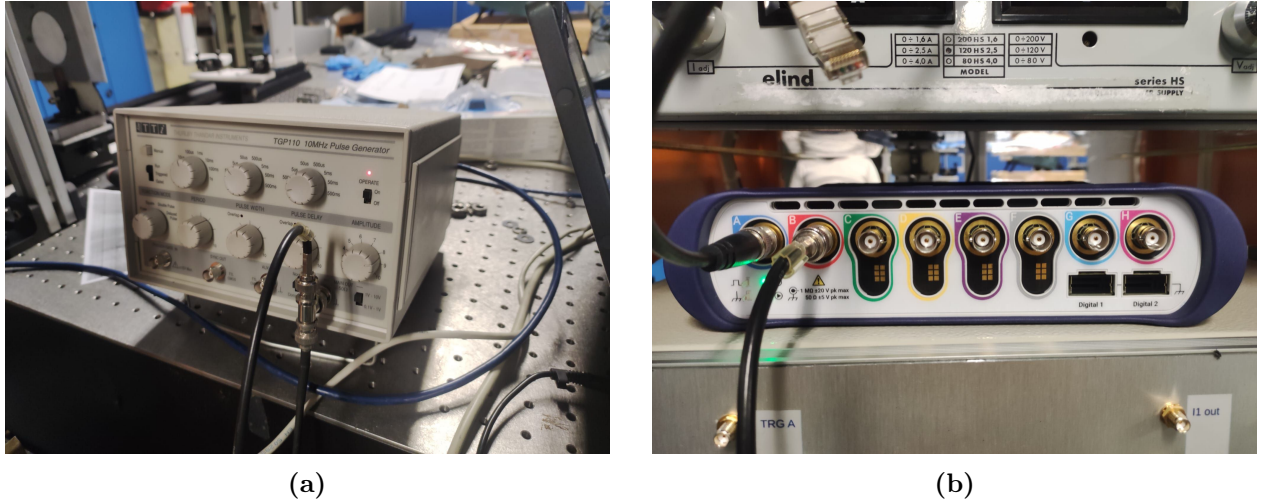


Figure 5.3: (a) Pulse generator that generates the trigger synchronising the reflectometer's sweep and the DAQ. (b) Picoscope 6000E series for multichannel data processing.

an adjustable reflecting target for each wave of the sweep. This configuration can thus prove very effective for testing the various components, checking for physical effects capable of downgrading its performance, and calibrating the whole set-up. The experimental procedure (Section 5.2) consists in simply varying the antenna-mirror distance and recording the change in the measured group delay, thus simulating the shift of the plasma column in a hypothetical RFX-mod2 ideal operational scenario. A linear correlation between the true position of the mirror and the measured time is expected, and subsequent interpolation leads to a calibration function (Section 5.2.2) that can be employed to reconstruct the position of the mirror. The experimental apparatus is reported in Figure 5.4, while the

design specifications in Table 5.1. The tilt has two important implications:

1. The effective path of the wave is not defined by the mirror-antenna distance L , but occurs diagonally along $L_c = \sqrt{L^2 + \Delta^2}$ (see Figure 5.4).
2. The convergence of the lines of sight defines a focal distance L_f in which the coupling of the two antennas is optimal (see Table 5.1).

It is reasonable to expect the best performance around L_f and an investigation of this effect outside of the focus (Section 5.2.3) is needed.

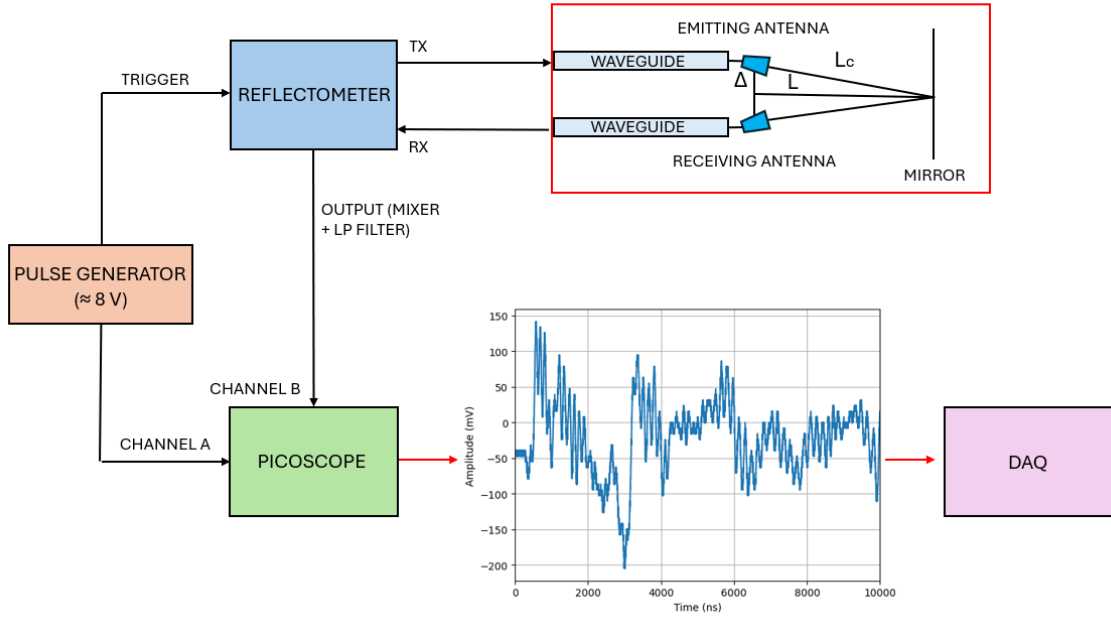


Figure 5.4: Scheme of the bench mirror set-up: the two hoghorn antennas (separation $2\Delta = 13.90 \text{ cm}$), the twist elements producing the 13° tilt, and the mirror mounted on a translation stage allowing controlled axial displacement L . The two LoS converge at the focal distance $L_f \simeq (30.2 \pm 0.1) \text{ cm}$ (Table 5.1). The diagonal optical path $L_c = \sqrt{(L^2 + \Delta^2)}$ is also represented.

Component	Value
Antennas separation 2Δ	$(13.90 \pm 0.03) \text{ cm}$
Tilt angle θ_t	$(13 \pm 1)^\circ$
Focal distance L_f	$(30.2 \pm 0.1) \text{ cm}$
Twist length l_{tw}	$(8.00 \pm 0.03) \text{ cm}$
Directional coupler length l_{dc}	$(22.00 \pm 0.03) \text{ cm}$
Cut-off frequency f_0	14.051 GHz

Table 5.1: Geometrical and electrical parameters of the set-up reproducing the RFX-mod2 HFS subsystem.

Once the various components are introduced, it is useful to evaluate the expected group delay for the specific configuration. As already pointed out by (2.39), each component of the circuit introduces a delay, so distinguishing the τ_g associated with the free-space propagation between the antenna and the mirror is the crucial task of this pre-analysis. Recalling Section 2.4.3, it is possible to build an approximate model for the three main components contributing to the final $\varphi(t)$.

- **Mirror (Plasma) contribution:** the free-space propagation is affected by the presence of the plasma density layers. In this specific case, the plasma density is modelled as a 1-D Heaviside step function, mimicking the presence of one cut-off density layer that reflects all the probing waves:

$$n_e(r) = n_c \Theta(r_1 - r_c - r) \quad (5.1)$$

where the coordinates recall Figure 2.2. From (2.25), it follows that:

$$\varphi_{\text{mirr}} = \frac{2\omega}{c} \int_{r_1-L_c}^{r_1} \sqrt{1 - \frac{n_c e^2}{m_e \epsilon_0 \omega^2} \Theta(r_1 - r_c - r)} dr - \pi = \frac{2\omega L_c}{c} - \pi \quad (5.2)$$

which results in:

$$\tau_{g,\text{mirr}} = \frac{2L_c}{c} \quad (5.3)$$

thus recovering the expected free-space outcome for the mirror configuration. Note that this time the reflection is represented by a π term, which accounts for the hard reflection introduced by the 1-D Heaviside step discontinuity, distinct from the $\frac{\pi}{2}$ phase term for the WKB model. However, no practical consequences are found in the group delay term.

- **Waveguides contribution:** the long journey of the waves across the MW circuit introduces a non-negligible group delay. While the behaviour of the antennas is difficult to predict and hence omitted from the present discussion, the dispersion effects on the propagation of TE_{1,0}-mode inside the waveguides have a well-known theory. Recalling equation (3.22), it is easy to show that:

$$\tau_{g,wg} = \frac{L_{wg,1} + L_{wg,2}}{c \sqrt{1 - \left(\frac{\omega_0}{\omega}\right)^2}} \quad (5.4)$$

where $\omega_0 = 2\pi f_0$, while $L_{wg,1}$ and $L_{wg,2}$ denote the lengths of waveguides 1 (the shortest) and 2 (the longest).

- **Delay line and antenna contribution:** the local reference signal is delayed, according to the reflectometer set-up, by making the wave travel a distance of $\simeq 2m$. Its exact contribution is difficult to establish, as in the case of the antennas. Thus, a generic term L_0 is introduced to address all

the remaining effects:

$$\tau_{g,0} = \frac{L_0}{c} \quad (5.5)$$

L_0 is identified as the "equivalent zero of the system" and is usually determined by the previously mentioned linear interpolation to perform a calibration of the system.

Taking into account all the terms, it is possible to develop a model of the expected group delay as a function of the probing frequency:

$$\tau_g = \frac{|2L_c - L_0|}{c} + \frac{L_{wg,1} + L_{wg,2}}{c\sqrt{1 - \left(\frac{\omega_0}{\omega}\right)^2}} \quad (5.6)$$

Once the details and expectations for the experimental procedure are set, it is possible to move on to the analysis and check the results.

5.2 Mirror position reconstruction

The first part of the analysis is dedicated to the validation of the system response under different plane mirror configurations. In particular, the mirror is placed at $L \simeq L_f$, and then its position is varied by a quantity $\Delta L = 1 \text{ cm}$ (comparable with the measurement resolution (2.41)) forward and backward from the starting spot (from $L \approx 15 \text{ cm}$ to $L \approx 35 \text{ cm}$). For each L , the $S_{mix,LP}$ from the mixer is acquired by the DAQ, processed, and analysed to extract the corresponding group delay. This series of operations is identified as a *Run*. A set of 20 Runs is taken, corresponding to 20 different antenna-mirror distances and, hence, 20 different group delay measurements.

The starting operation of a Run involves the proper filtering of $S_{mix,LP}$. As shown by the example in Figure 5.5, which corresponds to the signal for the mirror almost at L_f (Run 16 at $L = (30.5 \pm 0.3) \text{ cm}$), the raw signal is significantly distorted by several spurious reflections that the wave might experience when hitting nearby metallic components. To solve this issue, it is sufficient to perform a background run by simply removing the mirror (red plot in Figure 5.5) and subtracting it from the raw component. The results are reported in Figure 5.6 and show much more regularity.

Applying the STFT to the filtered signal (Figure 5.6) over time (with the settings described in Section 2.4.5) enables the direct identification of the group delay. Its trend is modelled by (5.6) and takes into account the dispersive effects of the waveguides (Section 3.2) and the propagation of the wave through the MW components (twists, antennas, delay line).

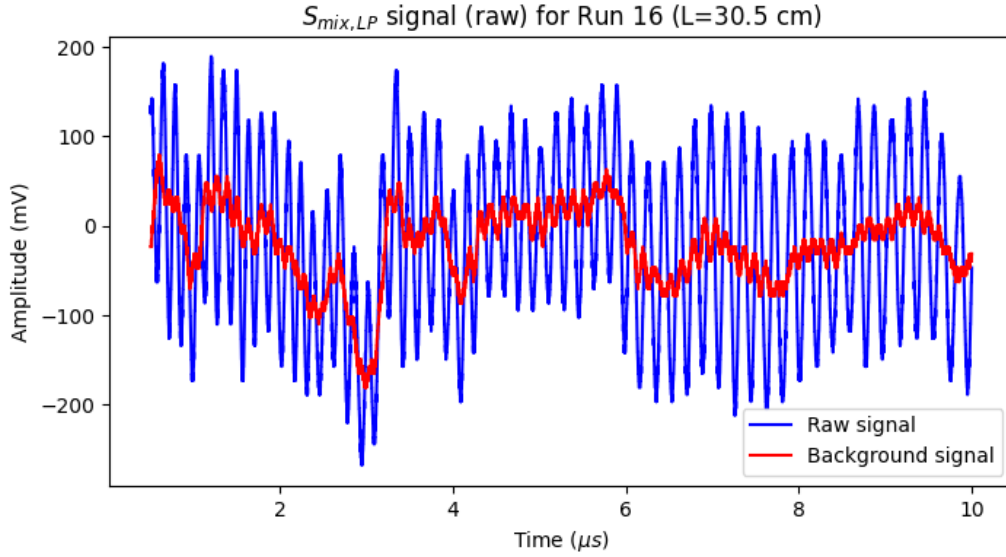


Figure 5.5: Raw mixer signal $S_{mix,LP}$ for Run 16 (mirror at $L = (30.5 \pm 0.3)$ cm, the closest approach to the focal distance L_f), shown together with the corresponding background trace acquired with the mirror removed (red). The spurious oscillations in the raw signals are caused by reflections off nearby metallic components and are removed by background subtraction (Figure 5.6).

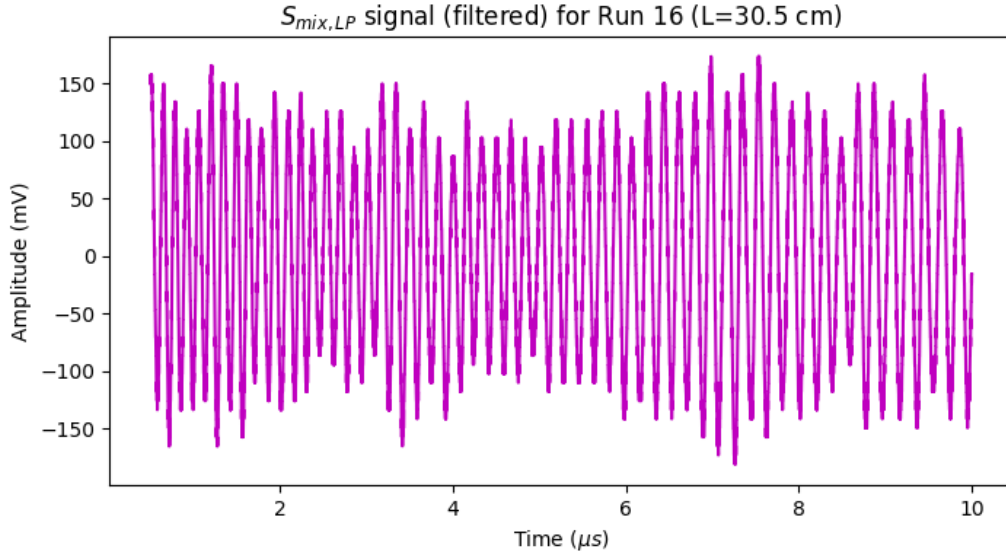


Figure 5.6: Background-subtracted mixer signal $S_{mix,LP}$ for Run 16. The signal features a regular trend throughout the 10 μ s sweep and a slightly fluctuating amplitude.

An example of the spectrogram is reported in Figure 5.7 for the same Run 16. The maxima distribution shows the typical dispersive trend of the group delay as a function of the probing frequency (or time), demonstrating how the waves travel faster at the beginning of the sweep. This introduces an intrinsic difficulty in the τ_g extraction that cannot rely on an average but results in a more complex

interpolation according to (5.6):

$$\tau_g = A + \frac{B}{\sqrt{1 - \left(\frac{\omega_0}{\omega}\right)^2}} \quad (5.7)$$

where $A = |2L_c - L_0|/c$ is the offset absorbing the non-dispersive contributions of the directional coupler signal path, and $B = (L_{wg,1} + L_{wg,1})/c$ is the dispersion amplitude proportional to the waveguides length.

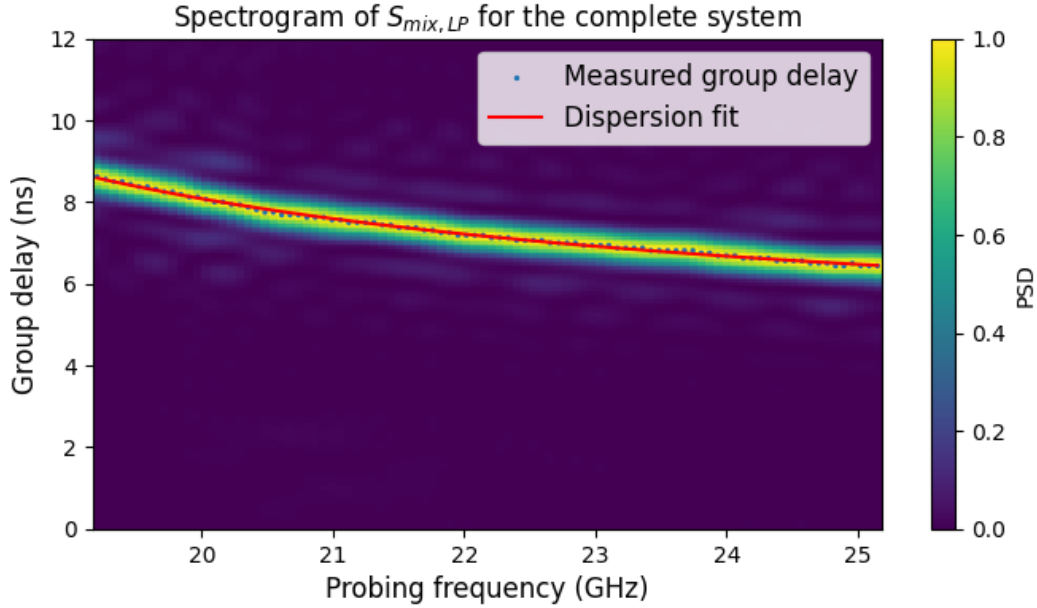


Figure 5.7: Spectrogram of $S_{mix,LP}$ for Run 16 (mirror at $L \simeq L_f$) with the complete microwave circuit. The strong frequency dependence of τ_g reflects the TE_{10} dispersion of the waveguides of Eq. (5.6), with higher-frequency waves travelling faster. The red curve is the fit of Eq. (5.7) used to model this dispersion.

5.2.1 Dispersion subtraction

Although the fitting function (red line in Figure 5.7) properly follows the measured τ_g points, its parameter A is not able to provide a reliable estimate of the effective group delay induced by the free-space back and forth propagation. Indeed, (5.7) is only an approximate function for the group delay and does not take into account minor contributions associated with the propagation across the antennas and cables.

For these reasons, the best strategy consists of isolating and removing the waveguide spectrogram from that of the signal. To perform this task, a directional coupler (Figure 5.8a) is employed. The working principle of a directional coupler [34] is explained in Figure 5.8b: a series of holes placed at a distance of $\lambda/4$ (where λ is the characteristic wavelength of the propagating waves) connects the primary waveguide (where the wave enters) to a secondary one and creates a copy

of the signal itself. The specific structure of the holes defines a specific travelling path for the secondary waves that makes them add in the forward direction (Port 4) and cancel out in the backward direction (Port 3). In this specific context, the TX is connected to Port 1 and the RX to Port 3 (as shown in Figure 5.8a), while Port 2 joins the directional coupler with the waveguide. If a mirror is placed at the other extremity of the same waveguide, the signal returns, propagating through it and reaching Port 2 for the second time. In this way, when the waves are sent from the TX, nothing is detected by the RX terminal, and the wave just enters the waveguide as usual. However, as the waves come back, the situation in Figure 5.8b is reversed, and the RX can detect them. The phase shift of the reflected signal can be extracted and accounts for the waveguide contribution twice. By repeating the procedure with the other waveguide, it is possible to obtain the two spectrograms reported in Figure 5.9. A fitting procedure is performed by applying (5.7) to the spectrogram points (red lines in Figure 5.9 and Table 5.2).

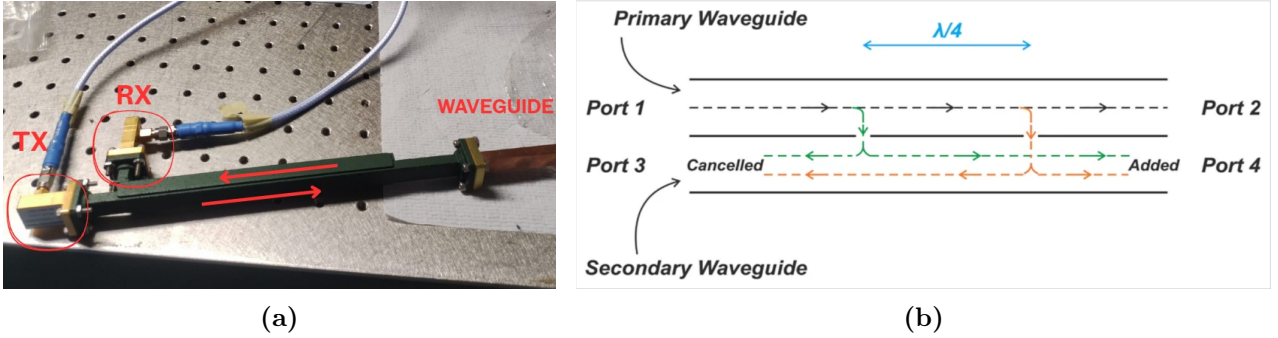


Figure 5.8: Directional coupler scheme for isolating the dispersion contribution. (a) The unit as integrated in the laboratory chain (TX $\rightarrow$ Port 1, RX $\rightarrow$ Port 3, waveguide $\rightarrow$ Port 2). (b) Working principle (from [34]): $\lambda/4$ -spaced holes between the primary and secondary guides cause forward-direction signals to add constructively at Port 4 and to cancel at Port 3, while return signals couple to Port 3.

Waveguide	A (ns)	B (ns)
1	-6.10 ± 0.08	8.87 ± 0.06
2	-6.08 ± 0.08	9.63 ± 0.06

Table 5.2: Fit parameters of the dispersion function Eq. (5.7), for the two waveguides measured via the directional coupler scheme (Figure 5.8). Parameter A is an offset absorbing the non-dispersive contributions of the signal path; parameter $B = 2L_{wg}/c$ is proportional to the doubled waveguide length.

Finally, the fitting functions can be subtracted from the spectrogram in Figure 5.7. Since the waveguides contribute twice, the fitting dispersion functions are divided by two and summed. A further correction on τ_g is applied to account for the dispersion introduced by the directional coupler, whose length l_{dc} is not negligible. The result of the subtraction procedure is reported in Figure 5.10 for Run 16.

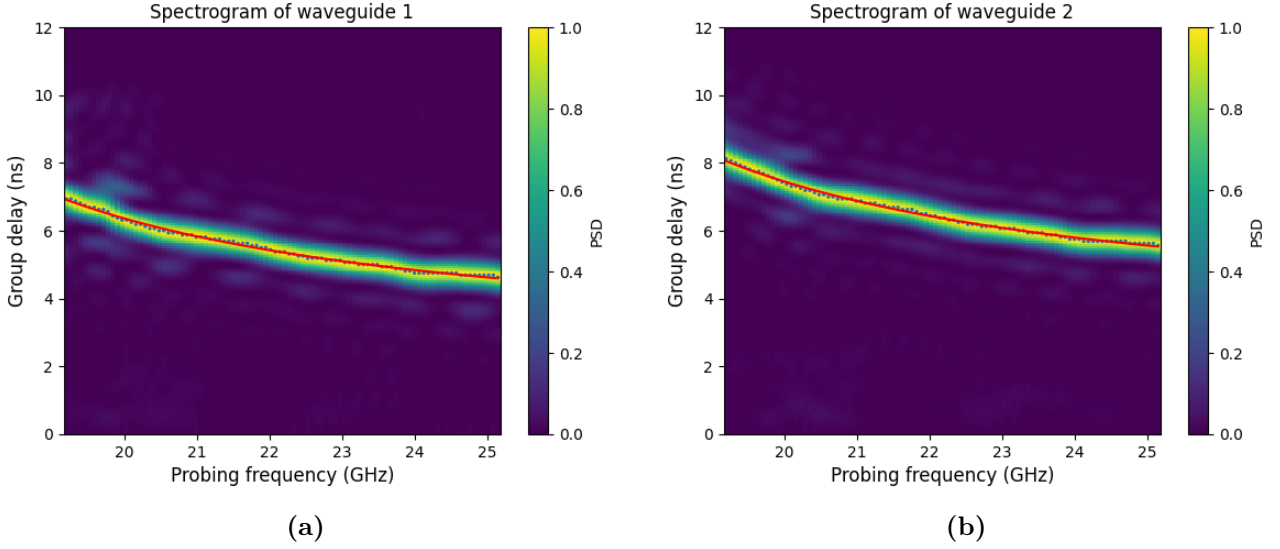


Figure 5.9: Spectrograms of (a) waveguide 1 and (b) waveguide 2, each acquired via the directional coupler scheme (Figure 5.8). The red curves are the fits of the dispersion function (Eq. (5.7)) used to characterise waveguide group delay contributions. The fitted parameters A and B are reported in Table 5.2.

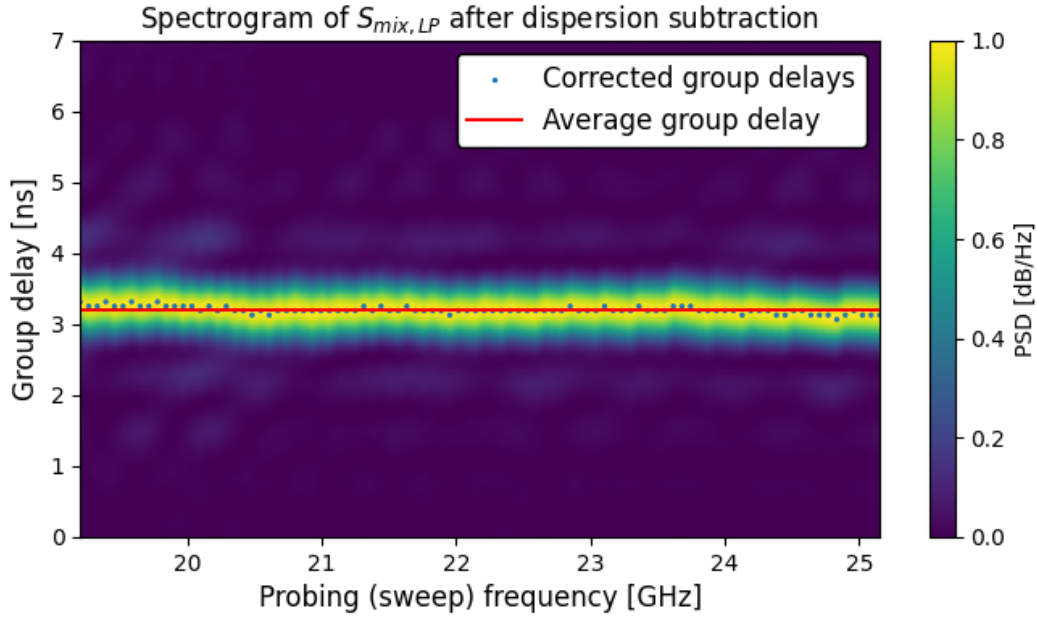


Figure 5.10: Spectrogram of $S_{mix,LP}$ for Run 16 (mirror at $L \simeq L_f$) after subtraction of the waveguide dispersion contribution obtained from the directional-coupler measurements (Figure 5.9, Table 5.2). The previously strong frequency dependence of τ_g (Figure 5.7) is now flattened to the constant baseline expected for the free-space round trip and the residual antenna contribution. The red line denotes the average τ_g .

After the dispersion is subtracted, the spectrogram maxima in Figure 5.10 feature a flat distribution, and each segment identifies the same τ_g ; thus, a more precise measurement can be extracted by simply averaging the instantaneous group delays throughout the sweep (red line in Figure 5.10). This last piece of information encodes only the physical free-space propagation of the reflected waves (from (5.6)):

$$\tau_g = \frac{|2L_c - L_0|}{c} \quad (5.8)$$

where, again, L_0 recalls the definition (5.5) and denotes the residual group delay contributions. The latter includes the propagation across the antennas, which cannot be isolated either with modelling or by means of experimental subtraction.

Two different approaches are employed to avoid the antennas contribution and to identify the actual antenna-mirror group delay: the first one relies on the calibration procedure of the system (Section 5.2.2) by exploiting the information coming from the 20 Runs. The linear correlation between the mirror position and the measured group delay, according to (5.8), enables the validation of the proper response of the system, and the intercept of the linear fit provides information on L_0 . This latter parameter can thus be employed to extract L_c , reconstruct the mirror-antennas distance L_m and compare the measured positions with the theoretical ones (L_{th}) by means of the corresponding accuracy $\delta = L_m - L_{th}$. The second method (Section 5.2.3) consists of the differential analysis of consecutive Runs: by taking the differences of the reconstructed $2L_c - L_0$ for consecutive mirror shifts, the L_0 is subtracted away, and the measured variations $\Delta L_{c,m}$ can be compared to the theoretical ones ($\Delta L_{c,th}$). A separation between the first ($\approx 15 - 25$) and the last ($\approx 25 - 35$) 10 Runs is then provided to investigate the accuracy of the system when moving away from the focal point. As a general note, the subscripts m and th denote measured and theoretical quantities, respectively.

5.2.2 Method 1: calibration

The calibration procedure establishes a direct correlation between the antenna-mirror distance and the measured group delay to validate the system's response and isolate the L_0 contribution. This is done by employing the data from all 20 runs to ensure statistical validity. Each Run is associated with the nominal mirror-antenna distance L_{th} measured by a standard meter. The uncertainty on L_{th} is mainly affected by the imperfect identification of the antenna mouth, and it is reasonable to suppose the existence of a systematic error ϵ_L that overwhelms the meter's millimetre-scale resolution. Because the antenna mouth is tilted at θ_t with respect to the probing direction, its higher aperture spans a range $A_a \cdot \sin(\theta_t)$ along that direction, creating ambiguity in the reference point at which the antenna mouth starts. To determine an approximate value for this systematic contribution, the previously mentioned projection of the antenna's higher aperture over the probing direction is taken, and the error is estimated as the standard deviation of a uniform distribution over the latter:

$$\epsilon_L = \frac{A_a \cdot \sin(\theta_t)}{\sqrt{12}} \simeq 3 \text{ mm} \quad (5.9)$$

where $A_a \simeq 39.8 \text{ mm}$ is the antenna's higher aperture (Table 4.2) and $1/\sqrt{12}$ follows from the standard deviation of a uniform distribution of width W ($\sigma = W/\sqrt{12}$). This uncertainty is then propagated to $L_c = \sqrt{L^2 + \Delta^2}$.

The correlation between L_c and τ_g is shown in Figure 5.11, while the results of the fit are collected in Table 5.3. Since the uncertainties on the path lengths are larger ($\simeq 1\%$) compared to those on the group delays ($\simeq 0.1\%$), the fit is performed with L_c on the y-axis:

$$L_c = a \cdot \tau_g + b$$

where $a = c/2$ and $b = L_0/2$. A brief discussion of the fit parameters is presented.

- The linear coefficient provides an estimate of the free-space speed of light $c_{exp} = (3.02 \pm 0.01) \cdot 10^8 \text{ m/s}$, which can be compared to its actual value to assess the quality of the linear fit. A discrepancy of $\simeq 2.2 \sigma$ ensures acceptable consistency of the measured trend with respect to the ideal one.
- $L_0 = (-34, 0 \pm 0.3) \text{ cm}$ encodes the additional path that the waves travel across the two antennas. The length of the single antenna (which amounts to $\simeq 12 \text{ cm}$) can thus be roughly evaluated from the coefficient b , which turns out to be quite reasonable if one considers that this estimate does not include the specific propagation of waves inside the antenna.
- The reduced χ^2 for 18 degrees of freedom (d.o.f) reported in Table 5.3 is much less than its optimal value $\chi^2 \simeq 1$ and seems highly unreasonable. A possible explanation relies on the overestimated position uncertainties, which include the previously assumed systematics (5.9).

d.o.f	$a \text{ (cm/ns)}$	$b \text{ (cm)}$	$c_{exp} \text{ (} 10^8 \text{m/s)}$	$L_0 \text{ (cm)}$	χ^2
18	15.10 ± 0.05	-17.0 ± 0.1	3.02 ± 0.01	-34.0 ± 0.3	0.11

Table 5.3: Results of the linear calibration fit $L_c = a \cdot \tau_g + b$ (Figure 5.11). The slope a yields the experimental speed of light $c_{exp} = 2a$ while the intercept b corresponds to half the equivalent zero of the system $L_0 = 2b$. The reduced χ^2 of 0.11 over 18 d.o.f. is marginal relative to the expected value of unity and is discussed in the text together with the c_{exp} vs c comparison.

Figure 5.12 displays the distribution of the discrepancies $\delta = L_m - L_{th}$ between the measured (L_m) and theoretical (L_{th}) positions. Note that these are systematically below 0, indicating, as expected, the presence of a systematic error in the theoretical length. The corresponding average $\bar{\delta} = (-0.32 \pm 0.02) \text{ cm}$ provides an estimate of the systematic accuracy associated with the position reconstruction, proving to be consistent with the previously estimated ϵ_L from (5.9). The standard deviation $\sigma_\delta \simeq 0.09$ quantifies the average stochastic uncertainty on the reconstructed position.

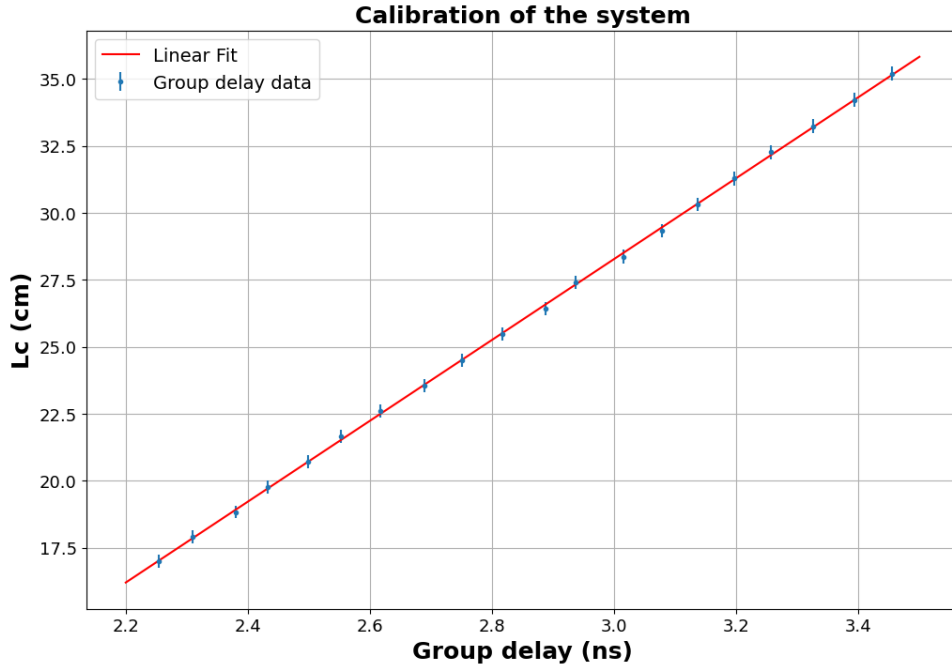


Figure 5.11: Linear calibration fit $L_c = a \cdot \tau_g + b$ applied to the 20 mirror runs. The slope a yields the experimental free-space speed of light c_{exp} ; the intercept b corresponds to the equivalent zero of the system L_0 (Table 5.3). The reduced χ^2 and the c_{exp} vs c comparison are discussed in the text.

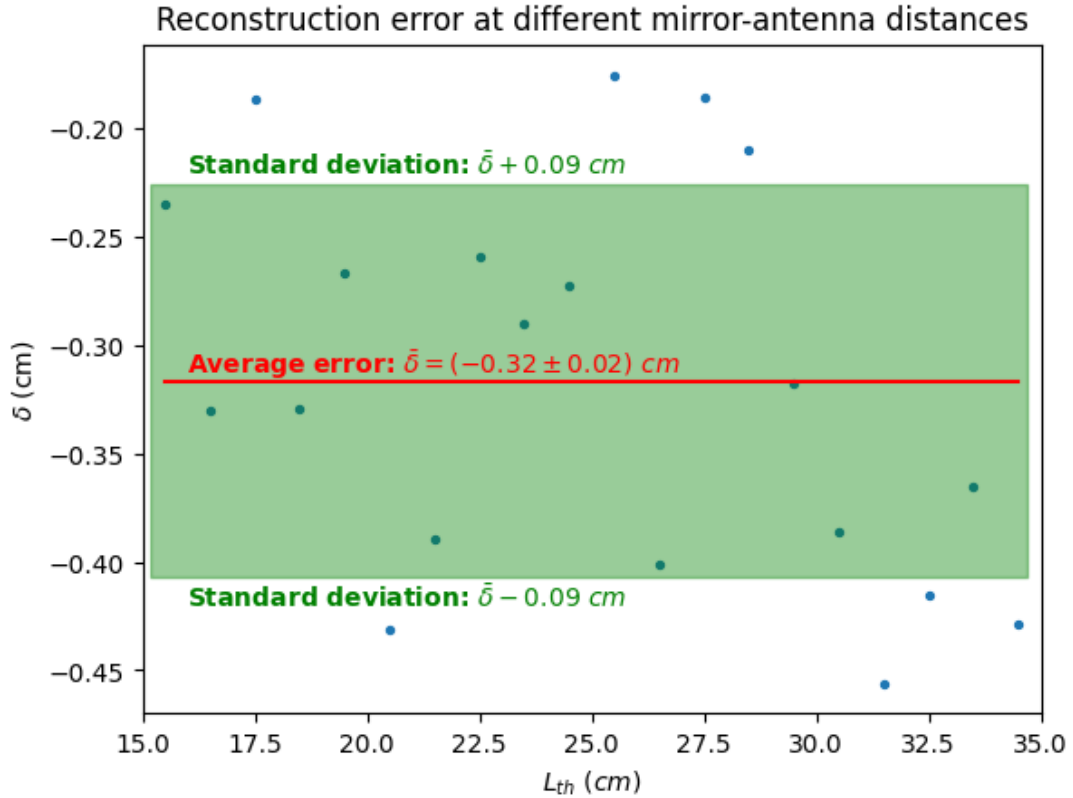


Figure 5.12: Distribution of the position-reconstruction errors $\delta = L_m - L_{th}$ over the 20 runs. The mean $\bar{\delta} = (-0.32 \pm 0.02) \text{ cm}$ is an estimate of the residual systematic, consistent with ϵ_L of Eq. (5.9). The spread $\sigma_\delta \simeq 0.09 \text{ cm}$ quantifies the average stochastic uncertainty on the reconstructed position.

σ_δ can be interpreted as the average error of the reconstruction procedure using the calibration method. Its value, well below the 1 *cm*-target accuracy for reflectometry, is probably associated with several minor stochastic contributions, such as mechanical vibrations of the set-up, pattern irregularities (broadening, secondary lobes...), and small deviations from the ideal propagation path (the reflection can occur in any point on the mirror, not only in its exact centre).

5.2.3 Method 2: differential analysis

A second approach consists in validating the system's capability in recording a series of consecutive 1 *cm*-shifts regardless of the initial mirror-antenna distance. This differential analysis allows for the reconstruction of mirror displacement while avoiding the systematic contribution encountered in Section 5.2.2, since it affects consecutive measurements in the same way. Moreover, Runs 1-10 (set around 20 *cm*) are separated from Runs 11-20 (set around L_f) to check whether significant differences are recorded when the distance from the focal position is considerable. Tables 5.4 and 5.5 display the results for Runs 1-10 and Runs 11-20, respectively.

Runs	$\Delta\tau_g$ (ns)	$\Delta L_{c,th}$ (cm)	$\Delta L_{c,m}$ (cm)	δ (cm)
1 $\rightarrow$ 2	0.055 ± 0.006	0.92 ± 0.04	0.83 ± 0.09	-0.09
2 $\rightarrow$ 3	0.070 ± 0.006	0.93 ± 0.04	1.06 ± 0.09	0.13
3 $\rightarrow$ 4	0.053 ± 0.007	0.93 ± 0.04	0.80 ± 0.10	-0.14
4 $\rightarrow$ 5	0.066 ± 0.006	0.94 ± 0.04	1.00 ± 0.10	0.06
5 $\rightarrow$ 6	0.053 ± 0.007	0.94 ± 0.04	0.79 ± 0.10	-0.16
6 $\rightarrow$ 7	0.066 ± 0.006	0.95 ± 0.04	0.99 ± 0.10	0.04
7 $\rightarrow$ 8	0.072 ± 0.006	0.95 ± 0.04	1.08 ± 0.09	0.12
8 $\rightarrow$ 9	0.062 ± 0.006	0.96 ± 0.04	0.93 ± 0.10	-0.03
9 $\rightarrow$ 10	0.065 ± 0.007	0.96 ± 0.04	0.98 ± 0.10	0.02

Table 5.4: Differential reconstruction of the consecutive mirror shifts for Run 1-10 ($L_{th} \simeq 15 - 25$ *cm*, far from L_f). For each consecutive pair of Runs the measured $\Delta\tau_g$, the expected $\Delta L_{c,th}$ from the meter, the reconstructed $\Delta L_{c,m}$ and the discrepancy $\delta = \Delta L_{c,m} - \Delta L_{c,th}$ are reported.

From Tables 5.4 and 5.5, most of the reconstructed position shifts $\Delta L_{c,m}$ are compatible with expectations $\Delta L_{c,th}$ within their 1 σ uncertainties (which derive from the statistical spread of the spectrogram measurement). The two average values $\bar{\delta}$ are consistent with zero, indicating no detectable systematic offset between the two range sub-samples. The average stochastic errors σ_δ reported in Table 5.6 are consistent with that of Section 5.2.2 and are well below the 1 *cm*-threshold, thus demonstrating the reliability of the differential analysis, the local sensitivity of the

Runs	$\Delta\tau_g$ (ns)	$\Delta L_{c,th}$ (cm)	$\Delta L_{c,m}$ (cm)	δ (cm)
11 $\rightarrow$ 12	0.050 ± 0.007	0.97 ± 0.04	0.75 ± 0.11	-0.22
12 $\rightarrow$ 13	0.078 ± 0.007	0.97 ± 0.04	1.18 ± 0.10	0.21
13 $\rightarrow$ 14	0.063 ± 0.007	0.97 ± 0.04	0.95 ± 0.10	-0.02
14 $\rightarrow$ 15	0.058 ± 0.007	0.97 ± 0.04	0.87 ± 0.10	-0.11
15 $\rightarrow$ 16	0.061 ± 0.007	0.97 ± 0.04	0.91 ± 0.10	-0.07
16 $\rightarrow$ 17	0.061 ± 0.007	0.98 ± 0.04	0.91 ± 0.10	-0.07
17 $\rightarrow$ 18	0.068 ± 0.007	0.98 ± 0.04	1.02 ± 0.10	0.04
18 $\rightarrow$ 19	0.068 ± 0.007	0.98 ± 0.04	1.03 ± 0.10	0.05
19 $\rightarrow$ 20	0.061 ± 0.007	0.98 ± 0.04	0.92 ± 0.11	-0.06

Table 5.5: Differential reconstruction of the consecutive mirror shifts for Runs 11–20 ($L \simeq 25 - 35$ cm, around L_f). Columns as in Table 5.4.

Runs	$\bar{\delta}$ (cm)	σ_δ (cm)
1-10	-0.005 ± 0.033	0.11
11-20	-0.03 ± 0.04	0.12

Table 5.6: Mean discrepancy $\bar{\delta}$ and standard deviation σ_δ for the two ranges of mirror positions. Both averages are consistent with zero within 1σ , indicating no detectable systematic offset. The comparable σ_δ values indicate that the focalisation effect on the stochastic reconstruction error is minimal.

set-up and the coherence of the two methods. Moreover, no significant differences are found between the two cases (far and close to the focal point), showing the minimal influence of focussing on position detection.

5.3 Effect of antenna tilt

A complementary analysis is dedicated to the characterisation of the effect introduced by the tilt of the antennas. In this context, the waveguides are removed, leaving only the mirror, the antennas and the delay line contributions. Two sets of 10 data Runs are taken, as in Section 5.2: the former includes the twisting elements, while the latter has the tilted configuration removed (Figure 5.13).

Since the waveguides are taken away, the dispersion does not affect wave propagation (the twists introduce a slight dispersion that can be neglected in this framework), and the group delay trend in the STFT (Figure 5.14) recalls (5.8).

The τ_g measurements are reported in Figure 5.15a and show good correlation with the oblique mirror-antenna distance L_c , featuring a reversed sign due to the presence of the delay line (the reference signal arrives after the RX). The results

of the linear fits in Table 5.7 are consistent with the previous calibration (Section 5.2.2), and the same discussion can be made for c_{exp} and the reduced χ^2 . No significant discrepancies are found between the two configurations.

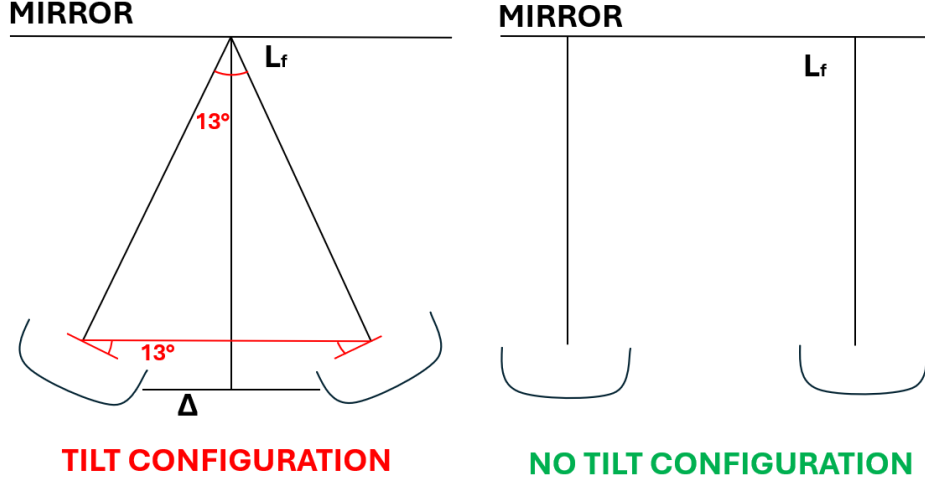


Figure 5.13: Schematic view of the two configuration employed to investigate the effect of the tilt: tilted (left) and non-tilted (right) antenna-mirror set-ups.

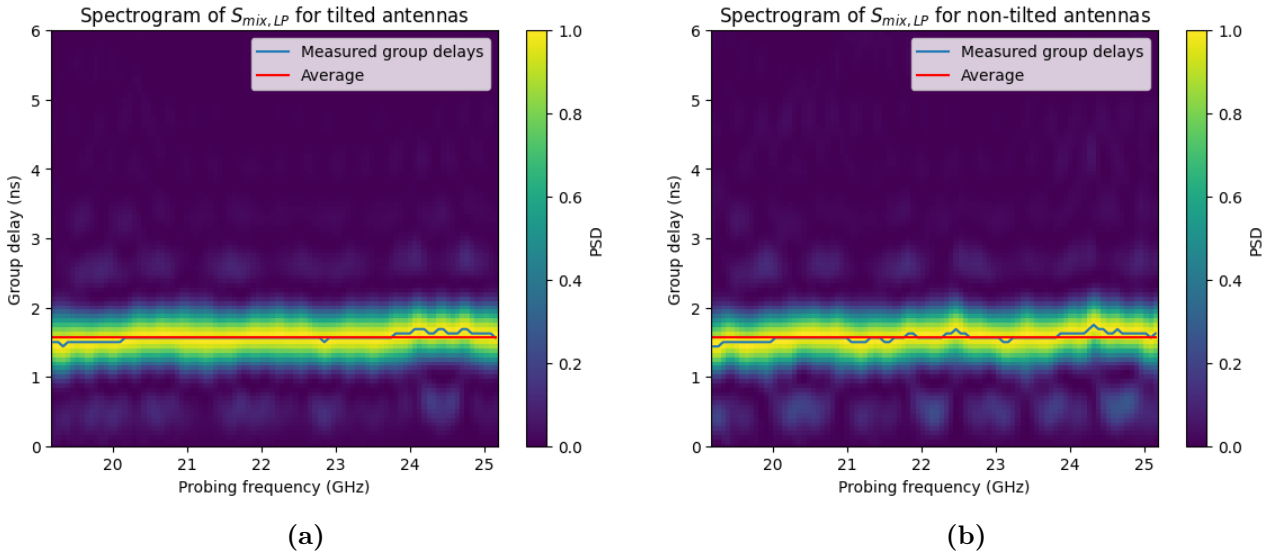


Figure 5.14: Spectrogram of the background-subtracted $S_{mix,LP}$ for Run 6 ($L \simeq L_f$) in (a) the tilted and (b) the non-tilted antenna configuration. The red line is the average group delay τ_g , extracted from the per-segment peak maxima of the STFT. Both configurations recover essentially the same delay.

An evident difference is recorded, as expected, in the amplitude values, which are significantly lower for the non-tilted configuration. A simple estimate of the amplitude can be obtained by taking the average of $|S_{mix,LP} - \bar{S}_{mix,LP}|$, where $\bar{S}_{mix,LP}$ denotes the mean output signal over time. Figure 5.15b shows the round-trip power loss (from (2.40)) by comparing the detected average amplitude for the non-tilted configuration with that for the tilted case at varying mirror positions.

A significant power loss is thus recorded when the tilt is removed and is more evident at lower mirror-antenna distances, while it mitigates as the latter increases.

case	d.o.f	a (cm/ns)	b (cm)	c_{exp} ($10^8 m/s$)	L_0 (cm)	χ^2
tilt	8	-15.1 ± 0.1	55.2 ± 0.1	3.03 ± 0.02	110.5 ± 0.3	0.04
no tilt	8	-14.9 ± 0.1	55.0 ± 0.2	2.98 ± 0.03	110.0 ± 0.4	0.1

Table 5.7: Results of the linear calibration fit $L_c = a \cdot \tau_g + b$ for both the tilted and non-tilted configuration (Figure 5.15a) as in Table 5.3.

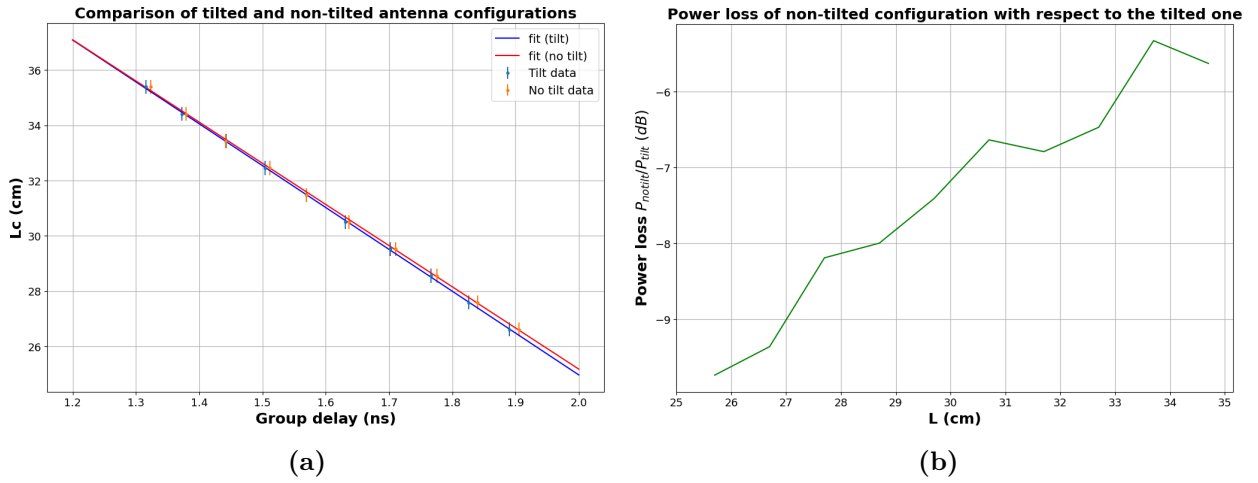


Figure 5.15: Comparison between tilted and non-tilted antenna configurations: (a) group delay linear trend as in Figure 5.11 showing a good accordance between the two cases (Table 5.7); (b) round-trip power loss P_{notilt}/P_{tilt} evaluated with (2.40) of the non-tilted signal with respect to the tilted one.

Finally, the capability of the system in reconstructing the position is validated for both configurations with the differential analysis (Section 5.2.3). The corresponding $\Delta\tau_g$ is affected only by the shift in the mirror position, with the L_0 term of (5.8) being constant. The results are shown in Table 5.8 and 5.9 for titled and non-tilted cases, respectively. The tables also display the discrepancy $\delta = \Delta L_{c,m} - \Delta L_{c,th}$ between the measured and expected shifts.

For both configurations, the reconstructed position shifts are all consistent with expectations (within 1σ). Table 5.10 provides a final comparison between the two configurations by reporting the mean and the standard deviation of the δ values over the Runs ($\bar{\delta}$ and σ_δ , respectively), leading to the same conclusions of Section 5.2.3 and demonstrating again the marginal impact of the tilted structure in ideal plane mirror reflectometric measurements.

Runs	$\Delta\tau_g$ (ns)	$\Delta L_{c,th}$ (cm)	$\Delta L_{c,m}$ (cm)	δ (cm)
1 $\rightarrow$ 2	0.064 ± 0.007	0.97 ± 0.04	0.97 ± 0.11	0.0002
2 $\rightarrow$ 3	0.059 ± 0.008	0.97 ± 0.04	0.89 ± 0.12	-0.08
3 $\rightarrow$ 4	0.064 ± 0.008	0.97 ± 0.04	0.97 ± 0.12	-0.004
4 $\rightarrow$ 5	0.071 ± 0.008	0.97 ± 0.04	1.07 ± 0.12	0.09
5 $\rightarrow$ 6	0.061 ± 0.008	0.97 ± 0.04	0.92 ± 0.12	-0.06
6 $\rightarrow$ 7	0.066 ± 0.009	0.98 ± 0.04	0.99 ± 0.13	0.01
7 $\rightarrow$ 8	0.061 ± 0.010	0.98 ± 0.04	0.92 ± 0.16	-0.06
8 $\rightarrow$ 9	0.070 ± 0.010	0.98 ± 0.04	1.06 ± 0.15	0.08
9 $\rightarrow$ 10	0.056 ± 0.010	0.98 ± 0.04	0.85 ± 0.14	-0.13

Table 5.8: Differential reconstruction of the consecutive mirror shifts (tilted case) as in Table 5.4.

Runs	$\Delta\tau_g$ (ns)	$\Delta L_{c,th}$ (cm)	$\Delta L_{c,m}$ (cm)	δ (cm)
1 $\rightarrow$ 2	0.065 ± 0.008	0.97 ± 0.04	0.98 ± 0.11	0.01
2 $\rightarrow$ 3	0.056 ± 0.007	0.97 ± 0.04	0.98 ± 0.11	0.008
3 $\rightarrow$ 4	0.064 ± 0.008	0.97 ± 0.04	0.97 ± 0.12	-0.004
4 $\rightarrow$ 5	0.074 ± 0.010	0.97 ± 0.04	1.12 ± 0.14	0.14
5 $\rightarrow$ 6	0.066 ± 0.009	0.97 ± 0.04	1.00 ± 0.14	0.02
6 $\rightarrow$ 7	0.059 ± 0.009	0.98 ± 0.04	0.88 ± 0.13	-0.1
7 $\rightarrow$ 8	0.069 ± 0.010	0.98 ± 0.04	1.04 ± 0.14	0.06
8 $\rightarrow$ 9	0.063 ± 0.011	0.98 ± 0.04	0.84 ± 0.16	-0.04
9 $\rightarrow$ 10	0.056 ± 0.010	0.98 ± 0.04	0.84 ± 0.16	-0.14

Table 5.9: Differential reconstruction of the consecutive mirror shifts (non-tilted case) as in Table 5.4.

case	$\bar{\delta}$ (cm)	σ_δ (cm)
tilt	-0.02 ± 0.02	0.07
no tilt	-0.005 ± 0.027	0.08

Table 5.10: Final comparison between the tilted and non-tilted configurations, as in Table 5.6. The two configurations are consistent within their uncertainties.

5.4 Effect of incidence angle

To further investigate the system performance, the mirror is designed to rotate around its horizontal axis, mimicking the curvature of plasma isodensity surfaces in the poloidal plane. The goal of this Section is to check the performance down-

grade when the ideal condition of perfect orthogonality is lost (see Figure 5.16). The system is mounted in its full version (Section 5.2) and the previous calibration (Section 5.2.2) can be employed to identify the error in position reconstruction due to the beam divergence for different mirror configurations.

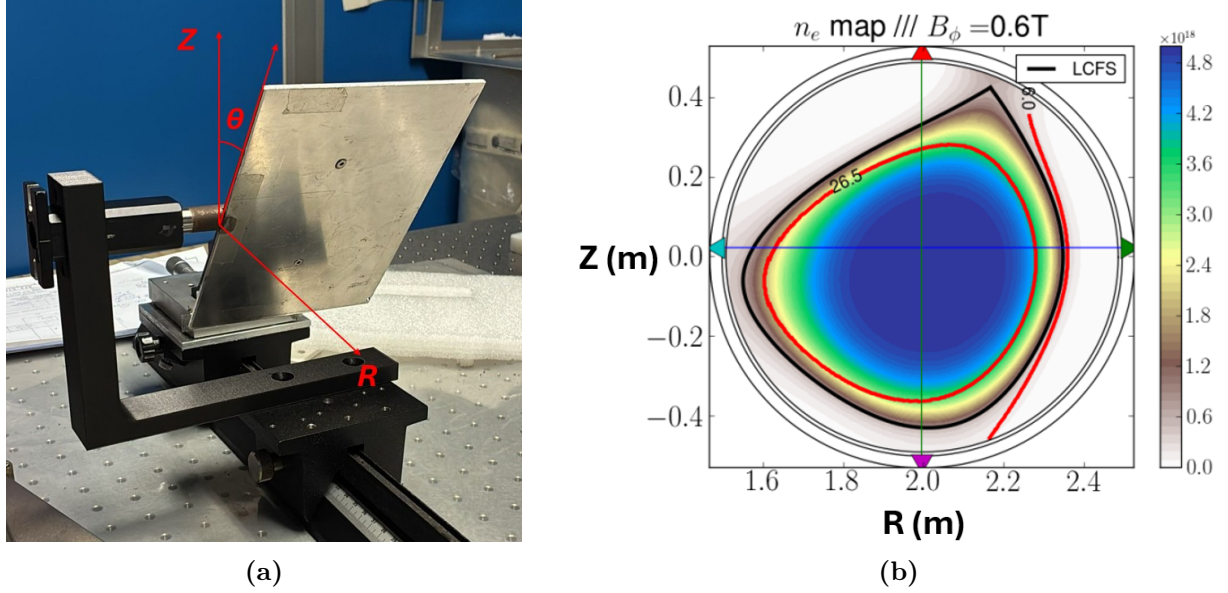


Figure 5.16: Inclined mirror configuration. (a) picture of the inclined mirror in the corresponding $R - Z$ poloidal coordinates (see Section 6.3) to simulate an irregular plasma shape. Sign convention: positive θ when the upper half of the mirror rotates away from the antennas, negative when it rotates towards them. (b) Example of an irregular plasma shape (negative triangularity single-null configuration [35], [36]) where the plasma boundary does not fall orthogonally with respect to the PPR LoS.

A first analysis is carried out by keeping the mirror in a fixed position ($L_{th} = (30.1 \pm 0.3) \text{ cm}$) and varying only the inclination with respect to its mid-plane horizontal axis. The sign of the angle is positive when the upper half of the mirror rotates "away" from the antennas and negative when it moves towards them. Table 5.11 summarizes the results, indicating $\delta = L_m - L_{th}$ the shifts of the reconstructed positions with respect to the actual one, while Figure 5.17 shows the worsening of the filtered $S_{mix,LP}$ at different inclination angles. The uncertainty in the angular values is taken larger to account for the instability of the goniometric measures, which do not rely on a fixed-centred scale integrated into the set-up but require an external aid that is subject to the user's ability to maintain the goniometer still. A $\pm 2^\circ$ error, twice the goniometer's notch, can reasonably represent this difficulty.

Both Table 5.11 and Figure 5.17 clearly indicate a performance downgrade as the angle is set above $\theta \simeq 15^\circ$, while keeping reasonable behaviour for low angles. In particular, the best performance is obtained for $\theta \simeq -5^\circ$, even though it can be

Run	θ ($^\circ$)	τ_g (ns)	L_m	δ (cm)
1	0 ± 2	3.177 ± 0.005	29.8 ± 0.3	-0.29
2	5 ± 2	3.158 ± 0.005	29.5 ± 0.3	-0.58
3	10 ± 2	3.120 ± 0.005	28.9 ± 0.3	-1.17
4	15 ± 2	3.080 ± 0.007	28.3 ± 0.4	-1.78
5	20 ± 2	3.057 ± 0.007	28.0 ± 0.4	-2.14
6	25 ± 2	3.011 ± 0.012	27.2 ± 0.5	-2.86
7	30 ± 2	3.015 ± 0.013	27.3 ± 0.5	-2.80
8	-5 ± 2	3.188 ± 0.005	30.0 ± 0.3	-0.13
9	-10 ± 2	3.184 ± 0.005	29.9 ± 0.3	-0.19
10	-15 ± 2	3.147 ± 0.008	29.3 ± 0.4	-0.75
11	-20 ± 2	3.348 ± 0.026	32.4 ± 0.9	2.34
12	-25 ± 2	3.638 ± 0.030	36.9 ± 1.0	6.76
13	-30 ± 2	3.836 ± 0.055	39.9 ± 1.7	9.78

Table 5.11: Mirror-position reconstruction at fixed mirror–antenna distance $L = (30.1 \pm 0.3)$ cm and variable mirror inclination angle θ . Sign convention: positive θ when the upper half of the mirror rotates away from the antennas, negative when it rotates towards them. For each inclination, the measured τ_g , the reconstructed L_m and the discrepancy $\delta = L_m - L_{th}$ are reported. Reconstruction degrades above $|\theta| \simeq 15^\circ$ (see body text).

affected by the previously discussed systematic error. This is consistent with the antenna pattern (Section 4.2.1), which features a maximum directivity slightly above the mid-plane and thus can operate under optimal conditions for forward-inclined reflective surfaces. The general trend shows an underestimate of the position for almost all the angles, clearly illustrated by the prevalence of negative δ values, which can be reasonably explained by the widening of the beam section at larger distances. This causes a larger fraction of waves to be reflected earlier against the closest portion of the mirror, regardless of the directivity. Instead, for the three larger negative angles (-20° , -25° , -30°), a strong overestimate of the distance is recorded, probably associated with the reflection of the waves on the metallic table that forces them to travel a longer path to rejoin the antennas.

A further analysis is dedicated to evaluating the reconstruction error δ error by inducing a change in both the angle and the mirror–antennas distance. This allows for the creation of a 2D matrix in which the absolute shift for each (L, θ) configuration is shown. Figure 5.18 provides a proper 2D visualization of the absolute error of position reconstruction and the corresponding projections in the top (error as a function of distance for different angles) and right (error as a function of

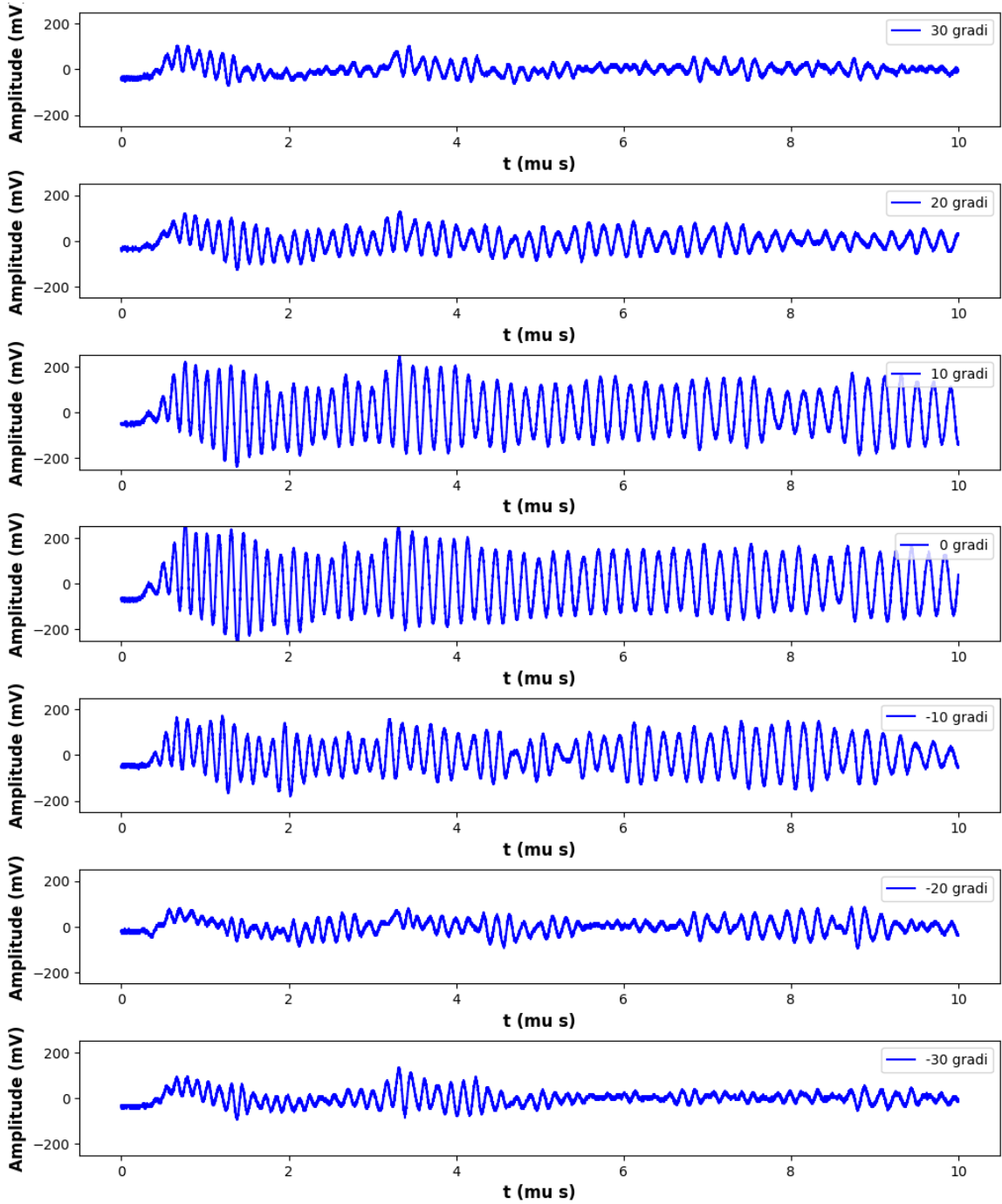


Figure 5.17: Worsening of the background subtracted mixer signal $S_{mix,LP}$ at increasing inclination angle (from $+30^\circ$ to -30°), consistent with Table 5.11.

angle for different distances) panels. It is clear how the error is well restrained within the $\simeq 1\text{ cm}$ limit in the range $[-15, 15]^\circ$, showing a significant detriment only at $\pm 20^\circ$, with significant values reached at $\theta \simeq -20^\circ$ for larger distances when the wave has a higher probability of being reflected by the metallic table. The analysis demonstrates how a non-orthogonal reflecting surface has a strong effect on the quality of the measurement, thus representing the most relevant source of error for the reflectometric system.

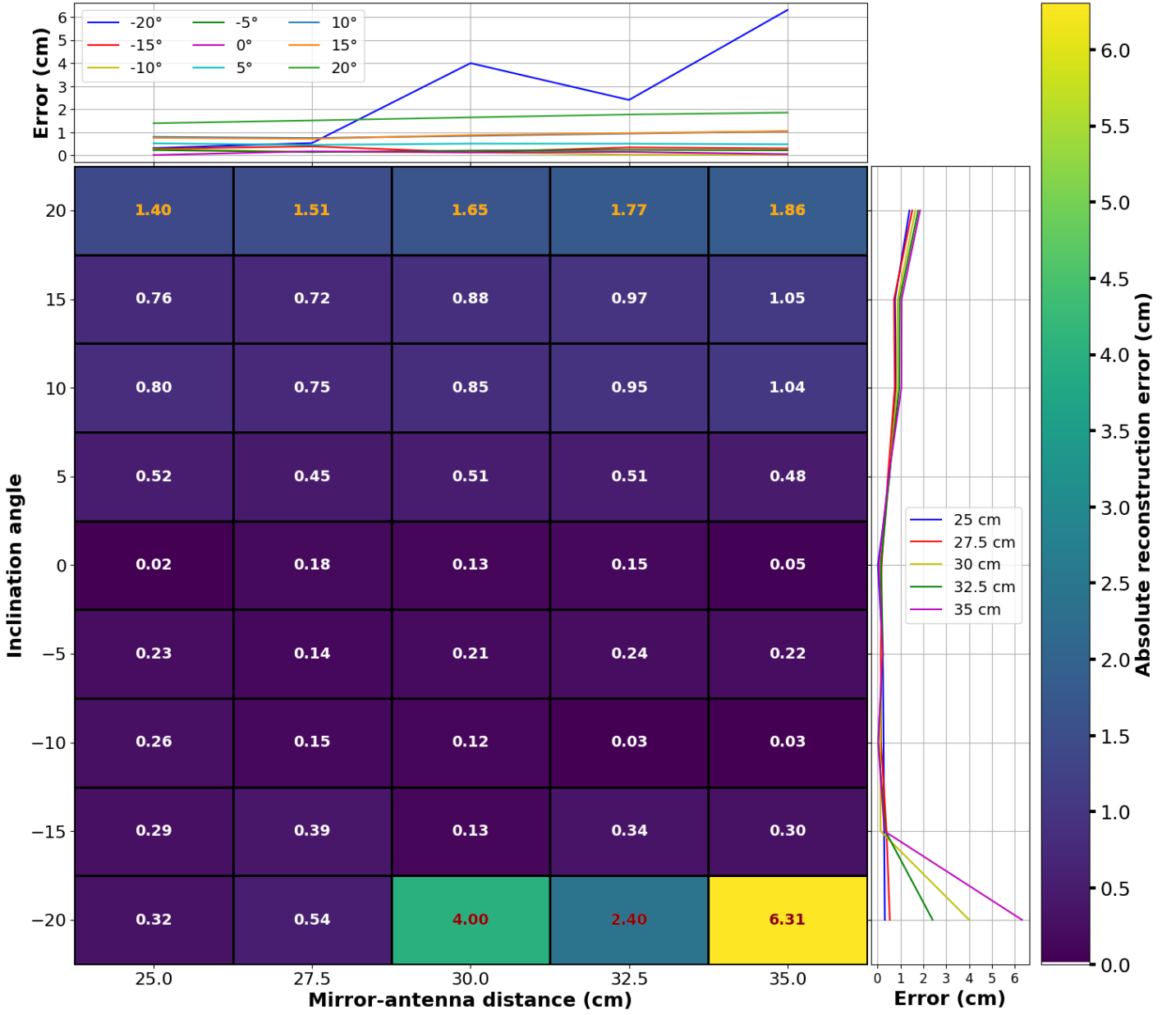


Figure 5.18: Absolute position reconstruction error $|\delta|$ as a function of the mirror–antenna distance L and the mirror inclination angle θ . The error is contained within $\simeq 1$ cm in the central region $[-15^\circ, 15^\circ]$ across the full L range, and degrades sharply at $\theta = \pm 20^\circ$, with the largest errors occurring at $\theta = -20^\circ$ and large L , where waves are increasingly reflected off the metallic table beneath the mirror.

A summary and discussion of the experimental results can be found in Chapter 7. In the next Chapter 6, the same configuration is replicated numerically with the aid of REFMUL3, enabling a direct experimental–numerical comparison on the mirror set-up, an investigation of the effect of the RFX-mod2 first wall, and an extension to a semi-realistic plasma scenario where bench measurements are not possible.

Chapter 6

Numerical results

The numerical performance assessment carried out in the second half of the work consists of two parts. On one hand, it provides the numerical counterpart of the mirror experimental procedure (Chapter 5) to quantify the ability of the HFS PPR system when integrated into the RFX-mod2 layout. On the other hand, the same system is subjected to an RFP-like plasma scenario to validate its performance in a more realistic application. The numerical assessment relies on the *Finite-Difference Time-Domain* (FDTD) REFMUL3 code [37], [38], which can solve the 3D Maxwell equations in a previously established environment and simulate microwave propagation into the plasma in O-mode configuration. This contrasts with RFX-mod2 PPR settings, which are designed to exploit X-mode propagation inside a Tokamak plasma. However, since the waves oscillate in the poloidal direction, an RFP SHAx state plasma is employed by taking advantage of the almost poloidal structure of the magnetic field due to the edge reversal of the toroidal one (Figure 6.1). This justifies assuming O-mode propagation in the edge portion of the profile and thus simulating the PPR system performance by employing REFMUL3 in a more realistic way. To ensure more consistency with the optimal Tokamak scenarios (Section 4.3.1), whose LCFS can be located significantly farther from the wall ($\simeq 20\text{-}25\text{ cm}$), the radial shift of the plasma column is the best strategy to investigate PPR performance in more realistic conditions. The chapter is organised as follows: after a general explanation of the code (Section 6.1.1), the preliminary work is introduced (6.1.2), which has been conducted to set the basis for the mirror (6.2) and the RFP SHAx state plasma (6.3) simulations. A manipulation of the latter was necessary to adapt the density profile to the K band measurement range (6.3.1) and thus make it suitable for the simulation box (6.3.2). Finally, the translation of the density profile is performed to check the HFS system response under the expected geometric conditions (6.3.4). As an additional step, the full profile reconstruction is attempted (6.3.5), demonstrating its strong dependence on the initialisation procedure.

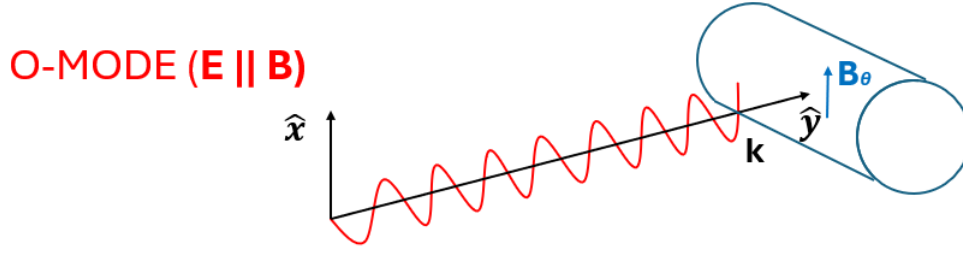


Figure 6.1: O-mode wave propagation in a RFP plasma: the electric field of the wave oscillates parallelly to the poloidal magnetic field B_θ which represents the dominant edge contribution for this kind of scenarios. In comparison with Figure 2.1b, the wave maintains its polarisation, while the magnetic field rotates of 90° .

6.1 REFMUL3 basics

The REFMUL* code family consists of a set of Finite-Difference Time-Domain (FDTD) computational methods that allow the set-up of synthetic systems to assess the behaviour of reflectometry diagnostics [37]. Their employment is necessary to account for the main propagation mechanisms of EM waves inside a plasma, where the complexity and inhomogeneities of the medium make the physical framework far from the ideal situation. The family includes REFMUL, a 2D O-mode code, REFMULX/REFMULXp for 2D X-mode simulations, and the two most recent versions, REFMULF and REFMUL3. REFMULF allows for 2D full-polarization wave simulations, including both O-mode and X-mode, and is able to deal with an external magnetic field. REFMUL3, on the other hand, is a 3D full wave parallel code that relies on the extension of the previous 2D REFMULF over a third coordinate. Its 3D nature makes the simulations highly computationally demanding, so a parallel approach is needed to cope with the computational burden, and multiple CPU nodes are employed on the PITAGORA HPC cluster at CINECA Italy.

6.1.1 FDTD approach for REFMUL* EM simulation

REFMUL* code solves Maxwell's equations by coupling them to the linear cold plasma model [27], recalling the results obtained with equation (2.10):

$$\frac{\partial \mathbf{J}}{\partial t} = \epsilon_0 \omega_p^2(\mathbf{r}) \mathbf{E} + \omega_c(\mathbf{r}) [\mathbf{b} \times \mathbf{J}] \quad (6.1)$$

where $\mathbf{b}$ is a unit vector that accounts for the direction of the magnetic field. The simulation relies on the FDTD Maxwell solver in which time and space are discretised, and Maxwell's equations are substituted with an equivalent set of *finite-difference scalar equations* according to the Yee scheme [38]. A spatial grid

is defined together with a suitable coordinate system to perform the discretization procedure and must fulfil three conditions, depending on the geometry of the simulated model and the MW band under consideration.

- The **boundary conditions** need to be specified by assuming perfectly conducting surfaces. These are introduced by setting the tangential electric field and the normal magnetic field to 0 on the conductive surface. The latter is approximated by a collection of polygons whose sides are parallel to the cells' coordinate axes.
- The **spatial discretisation** is taken to be uniform along the N axes ($N = 2$ for 2D simulation, $N = 3$ for 3D), featuring a common step $\delta_r = \delta_x = \delta_y = \delta_z$. This must be low enough to ensure that the EM field does not significantly change over one increment. This condition is strongly dependent on the specific band and is implemented by taking N_r cells per wavelength at a chosen reference frequency f_{ref} , yielding:

$$\delta_r = \frac{\lambda_{ref}}{N_r} \quad (6.2)$$

with $\lambda_{ref} = c/f_{ref}$. Typically, $N_r \geq 20$ and the reference frequency is set to $f_{ref} = 30 \text{ GHz}$ for the K band, slightly above its maximum sweep frequency (26.5 GHz), so the resolution criterion is satisfied with a margin.

- The *Courant-Friedrichs-Lewy* condition [39] for the time step δ must also be fulfilled. This ensures the numerical stability of the FDTD scheme by imposing an upper limit on the ratio:

$$c \cdot \delta t \leq \frac{\delta r}{\sqrt{N}} \iff S = \frac{c \cdot \delta t}{\delta r} \leq \frac{1}{\sqrt{N}} = S_{max} \quad (6.3)$$

where $S_{max} = 1/\sqrt{2}$ for a 2D simulation, $1/\sqrt{3}$ for a 3D one.

The FDTD method requires the integration of a simulation set-up where the physical phenomena of interest are investigated. In the case of REFMUL*, this structure is divided into four components [27].

1. **Metallic structure:** it includes all the metallic components of the MW circuit (waveguides, antennas) and the first wall (graphite tiles, copper shell) that can influence the wave's behaviour. These are modelled as perfect conductors and are easily implemented by a binary conductivity tensor mask σ_s that is 0 at the grid points lying on a metallic surface and 1 elsewhere.
2. **Plasma:** it is represented by a 3D density tensor n_e over a selected region of the simulated volume.
3. **Vacuum region:** it hosts the free-space propagation of EM waves in the absence of any reflective component.

4. **Perfectly Matched Layer (PML)**: it consists of a covering layer placed around the FDTD grid to completely absorb the outgoing waves.

The definition of the simulation set-up is a fundamental step to move on to the actual simulation process in which Maxwell's equations are consistently solved. The 2D approach of REFMULF consists in identifying a set of planes (typically (x, y) planes) as a reference domain where the EM fields are solved, avoiding the dependence on the z coordinate. This simplifies the system of equations, which can be decoupled into two subsystems and solved on two sets of staggered grids: one for TE waves ($E_z = B_x = B_y = 0$) and one for TM waves ($B_z = E_x = E_y = 0$). The major difference from classic 2D codes is that a generic external magnetic field $\mathbf{B}_0 = B_{0,x}\mathbf{e}_x + B_{0,y}\mathbf{e}_y + B_{0,z}\mathbf{e}_z$ is now allowed. The coupling between TE and TM planes is performed through the $\mathbf{b} \times \mathbf{J}$ term in Equation (6.1) [37].

The 3D REFMUL3 code is obtained from the extension of the 2D set-up by overlaying N TE planes with $N - 1$ TM planes, coupling them through the equation for $\mathbf{J}$ and enabling the variation along the z -axis of the field components [39]. For stability purposes, only the O-mode propagation has been implemented so far, relying on a simpler version of (6.1) with the $\mathbf{b} \times \mathbf{J}$ term equal to zero.

The following analysis has been carried out via REFMUL3. This has been a necessary choice to investigate the RFX-mod2 reflectometric system: the large toroidal separation of the antennas and their relative tilt make a 2D poloidal picture not sufficiently satisfactory to account for all the physical processes that occur in a 3D volume. On the other hand, the constraint of O-mode propagation introduces, as already mentioned, the most relevant limitation on the numerical assessment, as it is not consistent with the PPR configuration to be installed on RFX-mod2. However, the employment of O-mode reflectometry simulations in an RFP scenario can still provide interesting results that are worth discussing.

6.1.2 Preliminary work: model implementation

The correct employment of REFMUL3 requires a specific framework to facilitate the input definition process, automate the generation of discrete models, and perform simulations and data processing in the most efficient way. This framework relies on a well-defined pipeline where a set of scripts is run to build the simulation set-up. The user's contribution is focused on defining the input configuration to provide to those scripts and creating an organised folder structure for extracting the relevant information. Figure 6.2 illustrates the five phases of the simulation procedure [27]. In phase 1, the user defines the input data of the simulated set-up (region of interest, working band, geometry, plasma profile, etc.), leading to the creation of the Simulation Directory (SD) in phase 2, where the necessary

ingredients are gathered. The simulation can thus be run (phase 3), and the data processing scripts extract and store the output information from the simulated scenario (phase 4). In the last phase, this information can be further analysed via user-defined processing scripts.

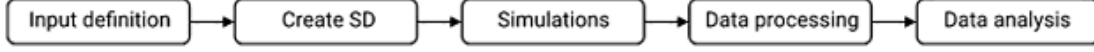


Figure 6.2: Five-phase pipeline for a REFMUL3 simulation (adapted from [27]): (i) user-defined inputs (CAD model, band, plasma scenario); (ii) generation of the simulation directory and FDTD grid via the script chain; (iii) submission of the simulation; (iv) data processing (mixer signal, low-pass filtering, STFT); (v) physical analysis (spectrograms, dispersion fit, group delay evaluation).

A preliminary task is thus necessary to perform phases 1 and 2 before moving on to the actual simulation procedure. The first ingredient is a realistic 3D CAD representation of the RFX-mod2 PPR system, detailed enough to include all the components of the set-up that can affect the diagnostic measurement. Note that the waveguides in the CAD are not exactly coincident with the real ones but are adapted to be compatible with the simulation. Starting from the model (typically a *.step* file), a series of additional operations are necessary to integrate the RFX-mod2 set-up inside the REFMUL3 simulation box.

1. The CAD model is created using the *Autodesk Fusion* program (Figure 6.3) and is stored in a dedicated directory. It is transformed into a mesh by running the *step2mesh* script. The resulting mesh (Figure 6.4a) is an ensemble of tetrahedra in which the structure of the model is subdivided.
2. The vertices of the mesh are converted into a coordinate text list by the *mesh2ptlist* code: both the point coordinates (*plist.dat*) and their corresponding tetrahedra list (*tlits.dat*) are provided.
3. The set of coordinates must now be constrained by a suitable domain (Figure 6.4b), which sets a reference for the simulation components and limits its output. This is specified by a text file (*RFXmod2_HFS_PPR_axis_simbox*) that contains the origin, axis, and dimension of the parallelepiped domain.
4. Finally, the *mkrefmul_geo_models* script gathers all this information and generates the FDTD grid associated with the chosen geometry.

The output result is a *.h5* file that is fed to REFMUL3 for undertaking the simulation. It consists of a matrix of 0s and 1s, depending on whether a point is located inside a metal structure or not. The size of the grid is established by the simulated MW band and by the number of points per wavelength N_r , according to equation (6.2). In this case, $N_r = 20$ points for $f_{ref} = 30 \text{ GHz}$ define the grid

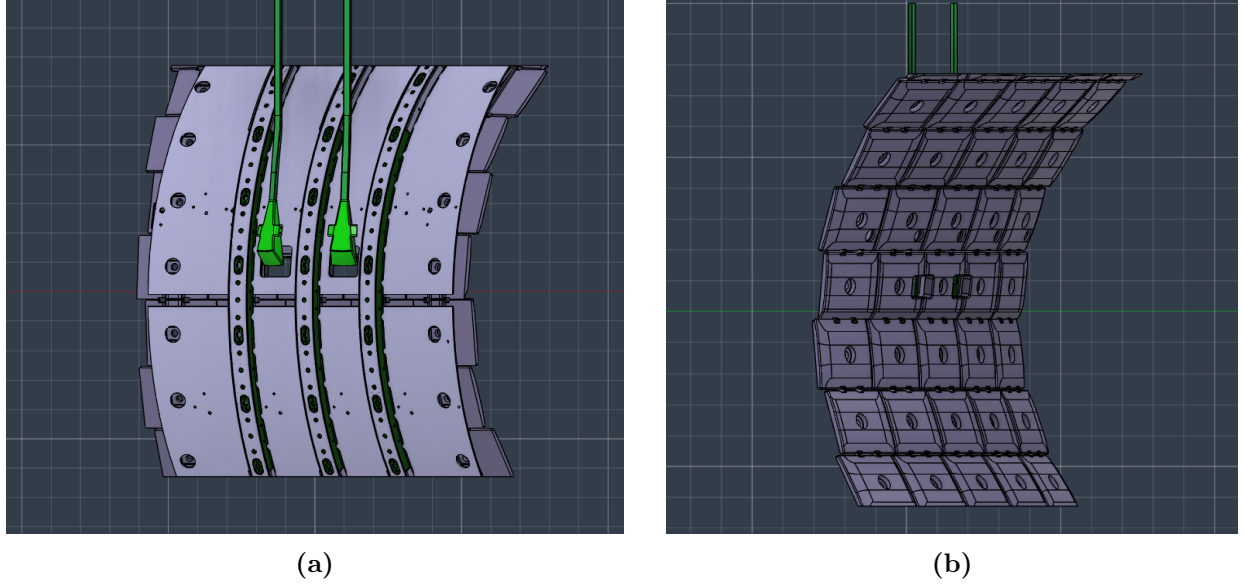


Figure 6.3: CAD model of the HFS PPR subsystem in Autodesk Fusion: (a) external view of the waveguides + twists + hoghorn antennas mounted on the supporting structure; (b) cut-away view exposing the internal geometry in contact with the plasma.

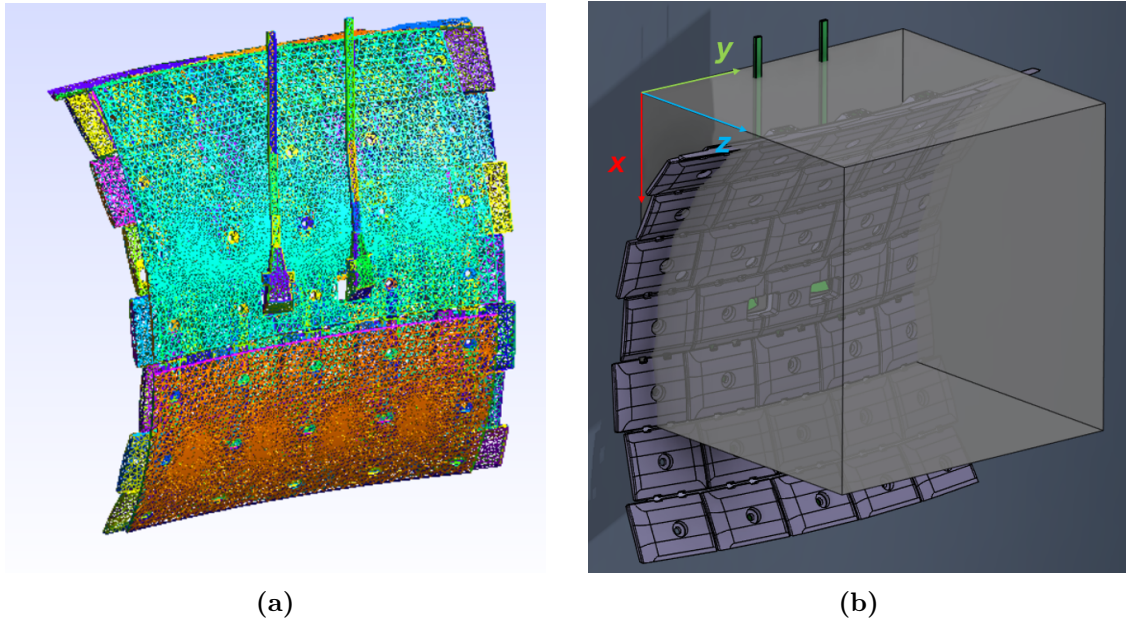


Figure 6.4: (a) Tetrahedral mesh generated from the CAD model by the *step2mesh* script. (b) Simulation parallelepiped (Eqs. (6.4)–(6.6)) containing the HFS subsystem.

step $\delta_r \simeq 5 \cdot 10^{-4} m$, which corresponds to approximately two cells for each *mm* of the domain (Figure 6.5a). This information, combined with the three dimensions of the parallelepiped defined by the user, sets the number of points for each axis:

$$l_x \simeq 543 \text{ mm} \quad N_x \simeq 1086 \quad (6.4)$$

$$l_y \simeq 540 \text{ mm} \quad N_y \simeq 1080 \quad (6.5)$$

$$l_z \simeq 500 \text{ mm} \quad N_z \simeq 1000 \quad (6.6)$$

Now that the FDTD grid is built, further work is dedicated to the implementation of the simulated wave. This concerns a second series of steps in which the

fundamental parameters are defined.

1. The injection (TX) and receiving (RX) points of the waves must be specified. This is easily checked by a dedicated python script (*txrxver.py*), which takes the .h5 file and opens a graphic interface to visualize it in the y-z plane (Figure 6.5). This is done on the Consorzio RFX's RAT4 local server. These points are set inside the probing waveguide at the very top of the domain and must coincide with a perfectly centred cell (Figure 6.5a). If this condition is not fulfilled, a suitable tiny shift on the coordinate axis must be introduced.
2. Besides the (y-z) coordinates of TX and RX, a set of parameters of the simulated geometry is gathered. From the RAT4 interface, it is possible to check the dimensions of the domain, the size of the waveguides, the probing axis (z) and the coordinates of the antenna mouth (Figure 6.5b), which will serve as a reference for simulations.
3. The temporal duration and discretization need to be specified. This is done starting from the definition of a total number of iterations $N_{it} = 150000$ (30000 of which are an offset) and a time sampling $\delta_t = 1/(40 \cdot f_{ref}) \simeq 8.33 \cdot 10^{-13}$ s, with $f_{ref} = 30$ GHz. This enables the direct evaluation of the total duration of the physical time as:

$$T_{tot} = N_{it}\delta_t \simeq 0.125 \mu s \quad (6.7)$$

Note that (6.3) is satisfied, giving $S \simeq 0.5 < 1/\sqrt{3}$ for any 3D wave.

4. The temporal duration is subdivided into a series of segments in order to reach the starting frequency and set the time sweep. The first 20000 iterations (**NPPRE**) set the start of the sweep at the corresponding frequency: the first 5000 (**NPR**) serve as ramps in which the wave ranges from 0 to $f_{min} = 18$ GHz, while the remaining 15000 iterations are needed to let the wave oscillate with frequency f_{min} . Then, 80000 iterations (**NPRAMP**) correspond to the actual sweep $f_{min} \rightarrow f_{max}$. Finally, 20000 iterations (**NPPOS**) end the sweep: again, for 15000 of them the wave oscillates at $f_{max} = 26.5$ GHz, while the last 5000 (**NPF**) let the wave decay from f_{max} to 0. **NPRAMP** defines T_{sw} as:

$$T_{sw} = \mathbf{NPRAMP} \cdot \delta_t \simeq 66.7 \text{ ns} \quad (6.8)$$

much lower than the experimental case (Section 5).

5. Finally, the cube sampling must be specified to establish at which iterations the .h5 field matrices should be generated. In this case, the programme starts from 20000 iterations, stops at 120000, and saves data every 40000.

After all these parameters are defined, they are fed to a SLURM script that generates the command line for the REFMUL3 code and creates the output folder

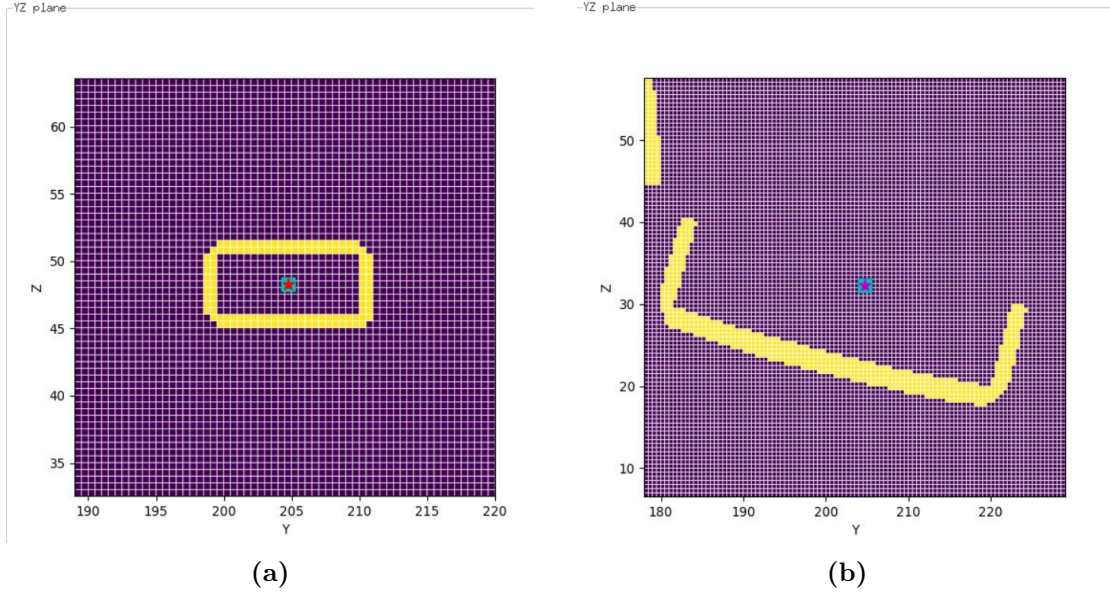


Figure 6.5: Interactive identification of (a) the TX wave injection point and (b) the antenna mouth reference cell on the RAT4 visualisation server. The selected coordinates for TX and RX must coincide with a centred Yee cell along the antenna axis to ensure clean mode launching and detection; small axis shifts are applied when necessary.

to host the simulation results. The parallel nature of the code requires a certain number of CPU nodes to run the simulation within an acceptable time. The nodes are provided by PITAGORA HPC upon request, and each project has a limited number of available hours. The SLURM implements a series of *sbatch* instructions to request a certain number of nodes, define a limit for the total amount of hours, select the name of the simulation, and set the structure of the command line. The options that can be selected for the latter are listed.

- **Programming model (PMODE):** defines how memory is shared among the nodes. It can be pure MPI or hybrid (HYB) MPI+OpenMP.
- **Structure (STRUCT):** allows inclusion (WALL) or removal (NOWALL) of the first wall (a .h5 matrix for the latter case needs to be integrated).
- **RMOD:** selects the type of physics to simulate. It can perform a calibration (CAL), simulate wave propagation in vacuum (VAC), insert a mirror at a fixed distance (MIRR), or include a plasma scenario (PLS).
- **Sweep mode (SWP):** defines the type of sweep of the wave. In this specific case, the frequency modulated (FM) option is selected.
- **Band (BND):** selects the working band (from K to W).
- **Plasma scenario (PLS_SCE):** specifies the folder that hosts the plasma model (details on plasma implementations will be given in Section 6.3).

The final step that precedes the actual simulations is the calibration of the system, which is launched by the following command:

```
sbatch -export=ALL, 'PMODE=HYB', 'STRUCT=WALL', 'RMOD=CAL', 'SWP=FM', 'BND=K', 'PLS_SCE='
```

This is fundamental to identify the TX and RX terminals of the reflectometric measurements. Indeed, when the electromagnetic field is locally excited to generate a probing signal, it propagates in all possible directions. The excitation of the fundamental mode in the waveguide generates two waves propagating equally in opposite directions. To specify the correct direction of propagation, it is necessary to implement a *unidirectional transparent source* (UTS) [27], [40] to enable signal emission in one direction and signal detection in the other, making it ideal for reflectometry simulations. The calibration Run serves as a unidirectional source to properly identify TX and RX for the system under study, especially when employed in a monostatic configuration.

6.1.3 Outputs

Once a simulation is complete, a series of output files is available to visualize and extract the main physical information. REFMUL3 provides two types of output: a *.dat* format for data processing and a *.pvti* extension for visualization. The 8 *.dat* files are listed below.

- **Ezsrc.dat** and **Eysrc.dat** correspond to the TX electric field signals measured at the source level (z and y components, respectively)
- **Ezrx.dat** and **Eyrx.dat** correspond to the RX electric field components measured at the receiving antenna (useful for bistatic measurements).
- **Ezm1src.dat** and **Eym1src.dat** coincide with the residual electric field components that return to the emitting antenna (useful for monostatic measurements).
- **ezref.dat** and **ezrefq.dat** are two reference oscillators that serve as I and Q signals for the filtering procedure (both for homodyne and heterodyne detection schemes).

It is important to note that REFMUL3 outputs enable working with both bistatic and monostatic configurations, as well as with both homodyne and heterodyne schemes, thus ensuring a variety of physical phenomena to investigate.

The *.pvti* files are 3D matrices that allow for the complete visualization of the propagating fields. Due to the high memory demand, typically only a slice of the 3D geometry can be depicted. This is done through the *Paraview* multi-platform data analysis programme that can be connected to the PITAGORA server to access the simulation files.

6.2 Mirror simulation

The first numerical analysis addresses a replica of the experimental mirror set-up, both including and removing the first wall to check for possible effects induced by the latter. It also compares the actual bistatic set-up with its monostatic counterpart. The mirror distance is set to $L_{th} = 30 \text{ cm}$ inside the SLURM script to be consistent with the experimental conditions, where 30 cm is taken as a reference for being close to the RFX-mod2 focal length L_f (Table 4.2). Figure 6.6 displays a visualization of the simulated electric field starting inside the probing waveguide and then propagating and oscillating in the poloidal plane towards the reflecting mirror. Figure 6.7 shows the same situation projected in the antennas mid-plane.

As for Chapter 5, the goal of this section is to obtain an estimate of the reconstructed mirror position L_m and compare it with the theoretical L_{th} . Due to the complexity of each simulation, only the $L_{th} = 30 \text{ cm}$ is considered, and neither calibration nor differential analysis is performed. As a general note, the subscripts m and th denote measured and theoretical quantities, respectively.

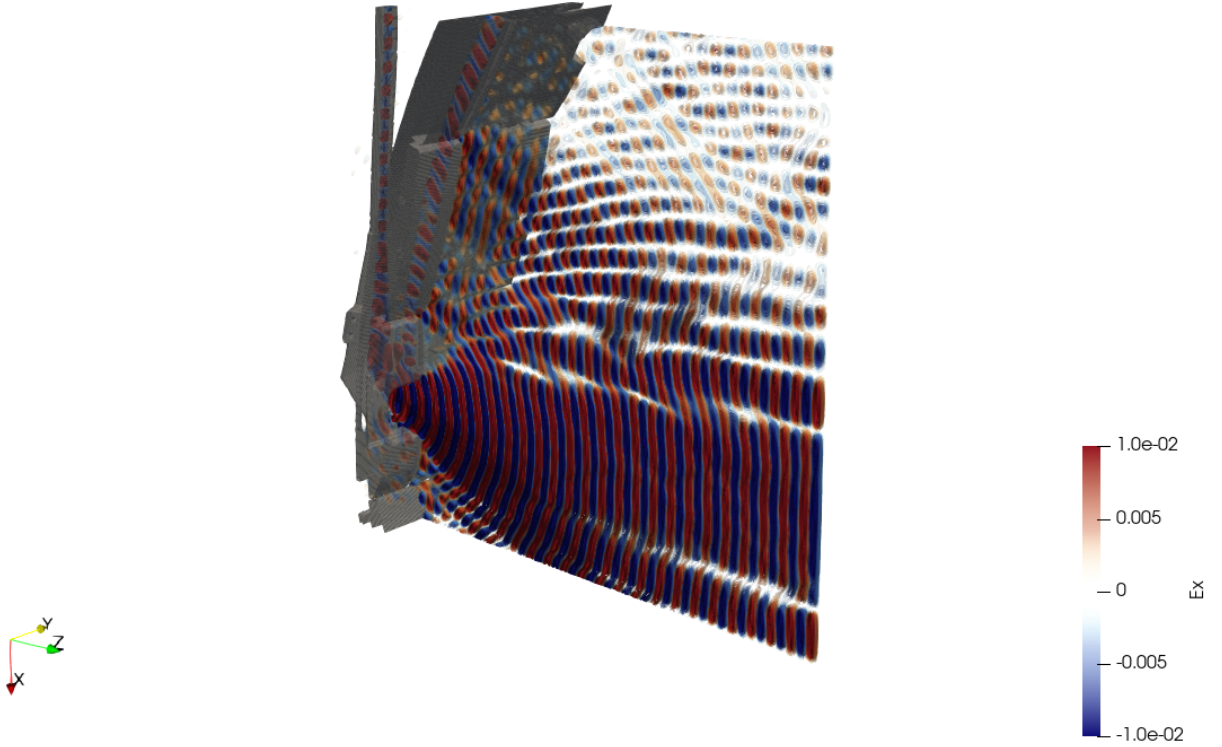


Figure 6.6: Paraview poloidal cut rendering of the simulated E field for the mirror configuration ($L \simeq 30 \text{ cm}$, K band sweep). The wave is launched from the probing waveguide, traverses the twist element and hognhorn antenna, and is reflected from the metallic mirror back into the receiving antenna.

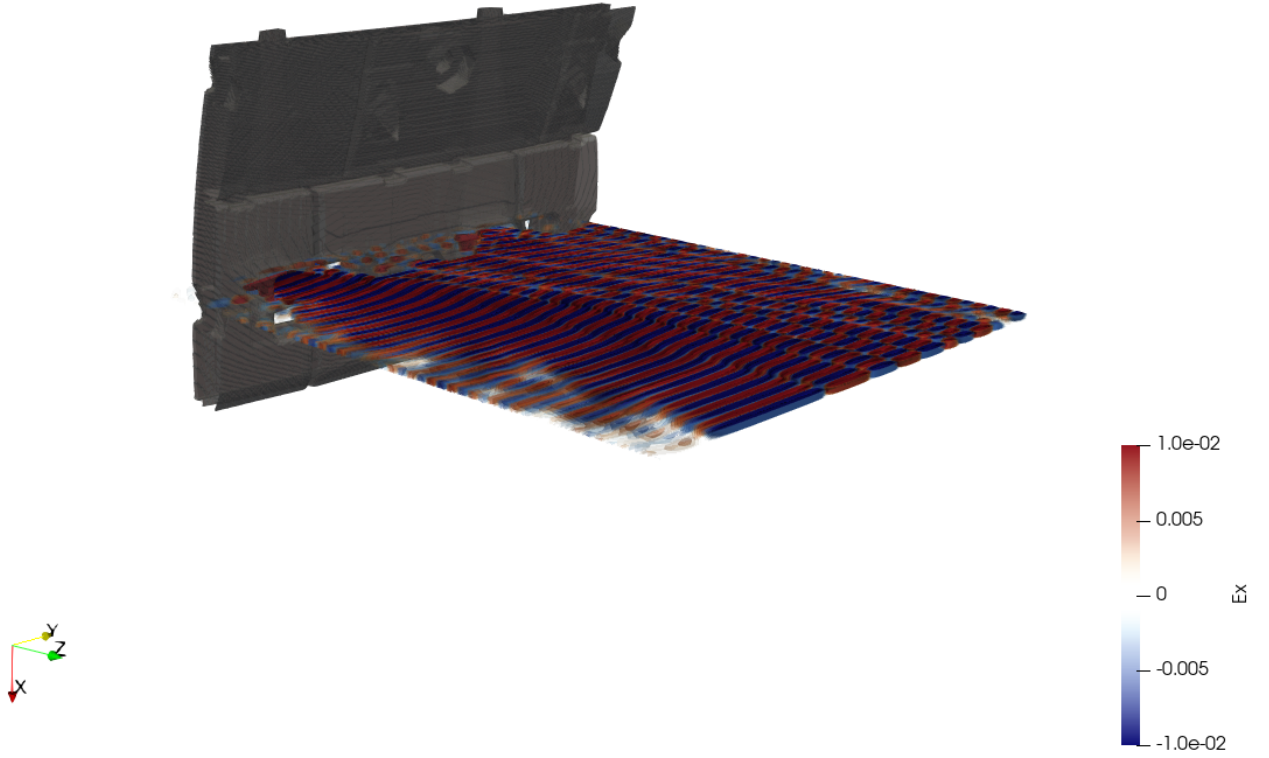


Figure 6.7: Same simulation as Figure 6.6 showing the mid-plane cut between the two antennas. The convergence of the probing beam and the reflection off the mirror are both visible.

6.2.1 Effects of the simulation set-up

Some preliminary investigations are needed to assess the quality and consistency of the output signal. These are performed starting from the simpler waveguides+antennas configuration that replicates the experimental set-up by running the simulation with the "NOWALL" instruction.

The signal processing involves a series of steps that correspond to the homodyne operations of the reflectometric unit: the raw RX signal (*Ezrx.dat*) is multiplied by the reference I signal (*ezref.dat*), and then a Butterworth digital filter of order 8 is applied for low-pass filtering. Figure 6.8 shows the raw beating signal (S_{mix}) together with its LP filtered counterpart ($S_{mix,LP}$), their Fourier Transform, and the round-trip power loss (2.40) of the RX signal with respect to the reference oscillator (typically implemented with unitary amplitude). The $S_{mix,LP}$ spectrogram in logarithmic scale is shown in Figure 6.9 and the maxima in the group delay spectrum resemble the typical dispersion trend. The logarithmic scale highlights the presence of other components that interfere with the pure mirror signal and might deteriorate the performance of the system. It is difficult to obtain a comprehensive understanding of all these effects, whose nature might be due to the physical properties of the system or to computational limitations. An attempt to identify the spurious wave component is provided (see Figure 6.9).

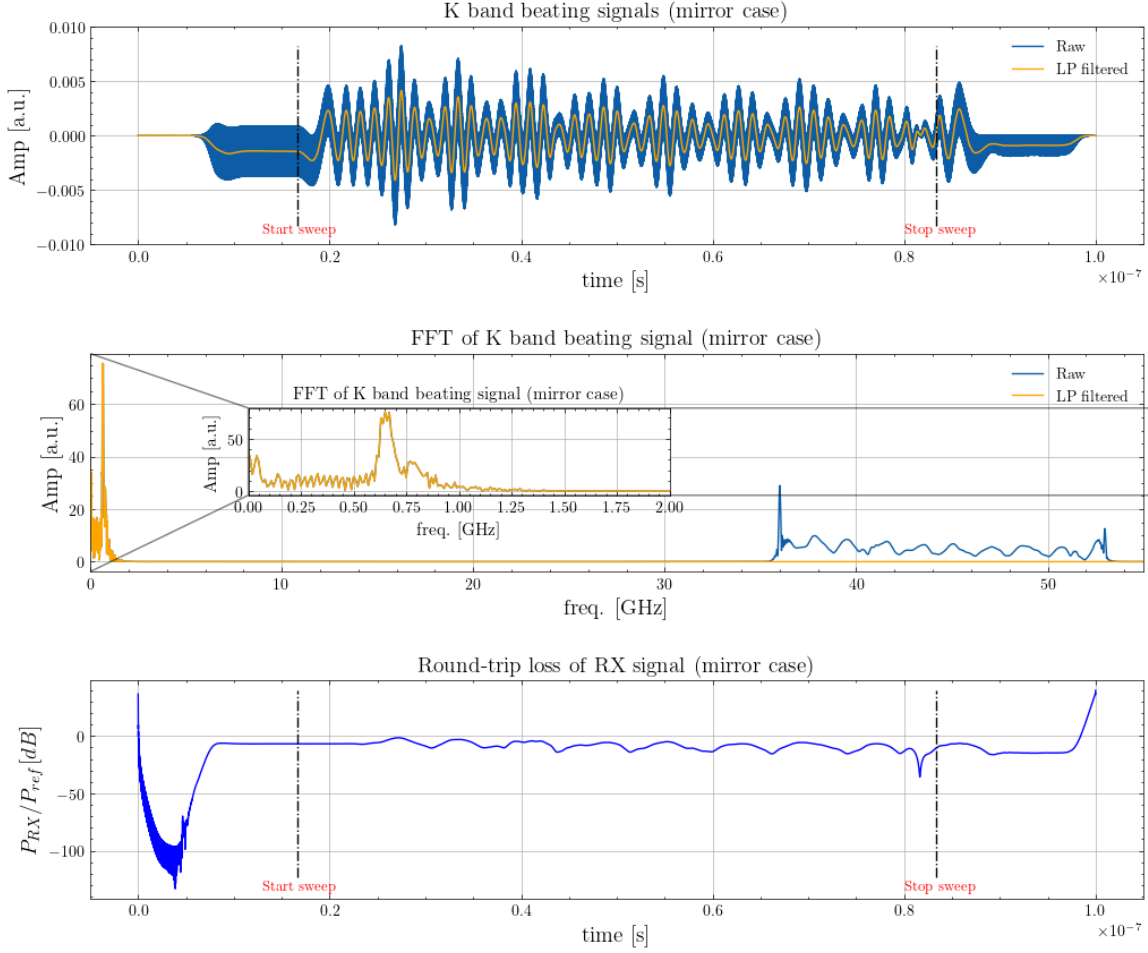


Figure 6.8: Signal-processing chain for the mirror simulation (NOWALL bistatic): raw RX field $E_{z,rx}$ multiplied by the I reference oscillator $ezref$ and the low-pass filtered $S_{mix,LP}$ after Butterworth filter of order 8 (top); FFT of both signals (middle); round-trip power loss P_L (eq. (2.40)) vs time (bottom).

1. **Short sweep duration:** the sharp fall at the beginning of the sweep defines a transient region of null delay. This is probably associated with the short sweep duration, for which the wave propagation time is not completely negligible. To overcome this issue, the probing frequency vector is conveniently translated to remove the transient part.
2. **Large beamwidth:** a spread of the group delay measurement is observed at the beginning of the sweep, making the measurement less reliable. This might be associated with the large beamwidth (see Section 3.1) that characterizes the lower frequencies and might increase the noise level.
3. **Antenna cross-talk:** a series of faint lines is observed at lower group delays, showing an equally spaced pattern. This is probably related to direct cross-talk between the two antennas, possible through their relative angle ($\simeq 13^\circ$).
4. **Multiple reflections:** a further line is present for higher group delays. This effect is due to multiple reflections of the waves between the mirror and the

antennas' surface, for which the free-space path is greater and the group delay is reasonably upward shifted.

5. **Minor lobes:** a spread of the group delay is partially observed at the end of the sweep, together with a "step" in the dispersion curve. This might be related to the presence of secondary lobes, which become more relevant for higher frequencies.

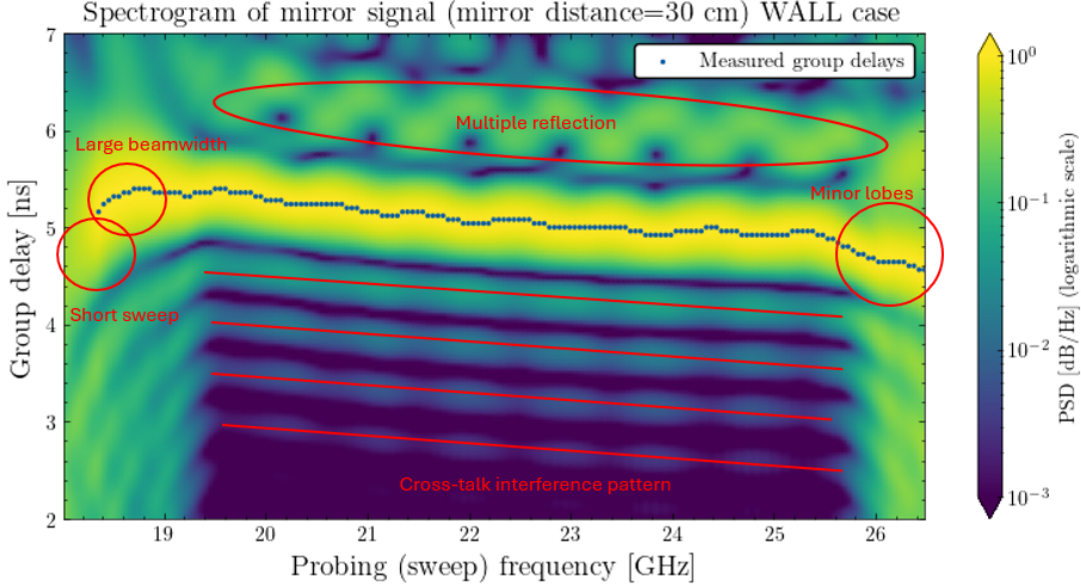


Figure 6.9: STFT spectrogram of the NOWALL bistatic mirror simulation (mirror at $L \simeq 30\text{cm}$). The dominant τ_g track at $\simeq 5.5\text{ ns}$ corresponds to the mirror reflection. Five spurious features are visible: the sharp fall at the beginning of the sweep (1) the larger beamwidth at low frequencies (2), the interference pattern at low τ_g (3), reflections near 6.5 ns (4), and the higher frequency secondary lobes (5).

Regarding computational effects, a possible detriment can be associated with the misalignment of the twisted waveguide structure relative to the computational grid, which can lead to higher order mode excitations. Verifying this effect is outside the scope of the present discussion, but it is worth acknowledging that some spurious features in the spectrograms may have this origin in addition to the previously offered explanations.

Once these elements are discussed, it is possible to develop a recipe for the mirror analysis. The probing frequency vector is translated according to the waves' propagation time and the initial and final portions of the spectrograms are cut away. The remaining frequency segment $[18.5, 25]\text{ GHz}$ is then analysed and fitted with the usual dispersion function (5.7) to assess its quality. Figure 6.10 shows the adjusted spectrograms for NOWALL (6.10a) and WALL (6.10b) configurations. Table 6.1 shows the results of the dispersion fit for both cases and allows for a direct identification of the antenna length l_{ant} and the waveguide extension L_{wg}

(with the subscripts *ant* and *wg* referring to antennas and waveguides, respectively). In this case, A encodes the free-space and antenna propagation components ($L_c + l_{ant}$), while B is a measure of the length of the waveguides included in the simulation (L_{wg}). The latter is extracted and compared with the expectations to show the consistency of the fit. Note that the expected $L_{wg,AF} \simeq 25$ cm is evaluated with Autodesk Fusion (the same subscript is employed for Table 4.2) and starts from the selected injection point. Considering the oblique wave path $L_c = \sqrt{L^2 + \Delta^2} \simeq 30.7$ cm (the same definition as in Figure 5.4 but with Δ defined in Table 4.2), it is also possible to nearly recover the true antenna length $l_{ant} \simeq 12$ cm from the measured $L_c + l_{ant}$. This represents another reasonable result demonstrating the proper implementation of the model.

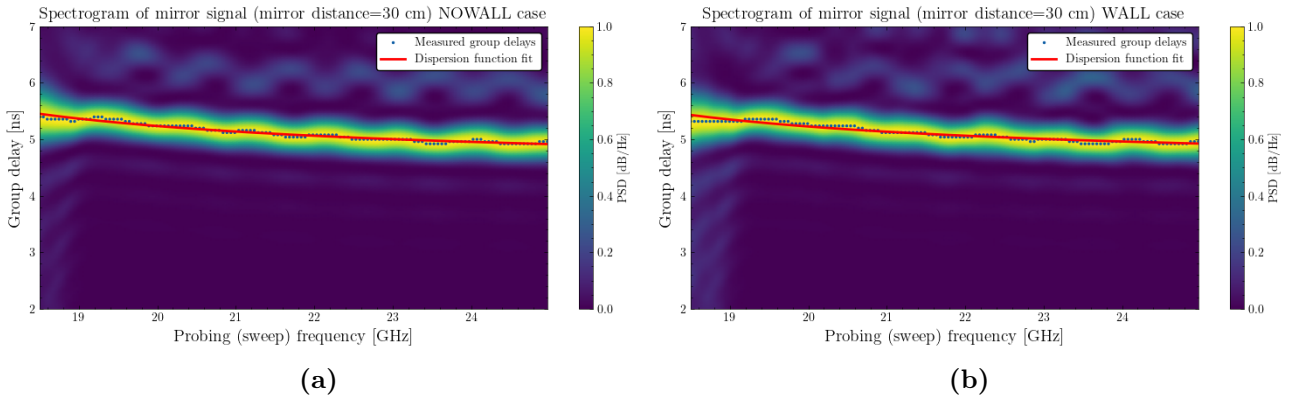


Figure 6.10: Mirror spectrograms after the analysis recipe of 6.2.1 (probing-frequency-vector translation + crop to $[18.5, 25]$ GHz), with the red curve showing the fit of Eq. (5.7) used to extract the dispersion parameters of Table 6.1: (a) NOWALL bistatic; (b) WALL bistatic.

case	A (ns)	B (ns)	$L_c + l_{ant}$ (cm)	L_{wg} (cm)	l_{ant} (cm)
NOWALL	2.94 ± 0.04	1.64 ± 0.03	44.0 ± 0.6	24.5 ± 0.5	13.3 ± 0.6
WALL	3.05 ± 0.05	1.55 ± 0.04	45.8 ± 0.8	23.2 ± 0.6	15.1 ± 0.8

Table 6.1: Dispersion fit parameters of the function (5.7) applied to the mirror spectrograms of Figure 6.10 ($L_{th} = 30$ cm), in the NOWALL and WALL bistatic configurations. Parameter A captures the non-dispersive contributions (free-space round trip to the mirror plus residual antenna terms); parameter B is proportional to the doubled waveguide length traversed. The recovered $L_c + l_{ant}$ are consistent with the SLURM-imposed mirror distance + the nominal antenna length $l_{ant} \simeq 12$ cm; the recovered L_{wg} match the expected value $L_{wg,AF} \simeq 25$ cm from the Autodesk Fusion CAD model.

6.2.2 Dispersion Subtraction and position reconstruction

The next step concerns the subtraction of the dispersion of the MW circuit. The strategy is to run a second simulation in which the mirror is set closer to the antennas' mouth ($L_{th} \simeq 3.5$ cm). The corresponding dispersion function is subtracted from the previous mirror spectrogram in both configurations, thus removing the group delay contribution of waveguides and antennas. The final measured group

delays are corrected with the 3.5 cm free-space delay of the subtracted mirror to recover the actual antenna-mirror distance. Since the mirror is too close to the antennas, the dispersion spectrogram is obtained in monostatic configuration with the waves travelling backward through the emitting antenna. This relies on the assumption that the two antennas share the same dispersion properties, which is reasonable if one considers that, unlike the experimental case, the two halves of the MW circuit are coincident in the CAD model. It is also necessary to subtract the vacuum monostatic contribution to remove internal reflections that might occur inside the waveguides or at the aperture of the emitting antenna. Figure 6.11 shows the dispersion for both NOWALL and WALL cases. It is immediately clear how the presence of the first wall introduces multiple reflections that deteriorate the signal with several spurious components. For this reason, the NOWALL case is chosen as a reference for the identification of the dispersion, and it is employed in both configurations to account for this effect.

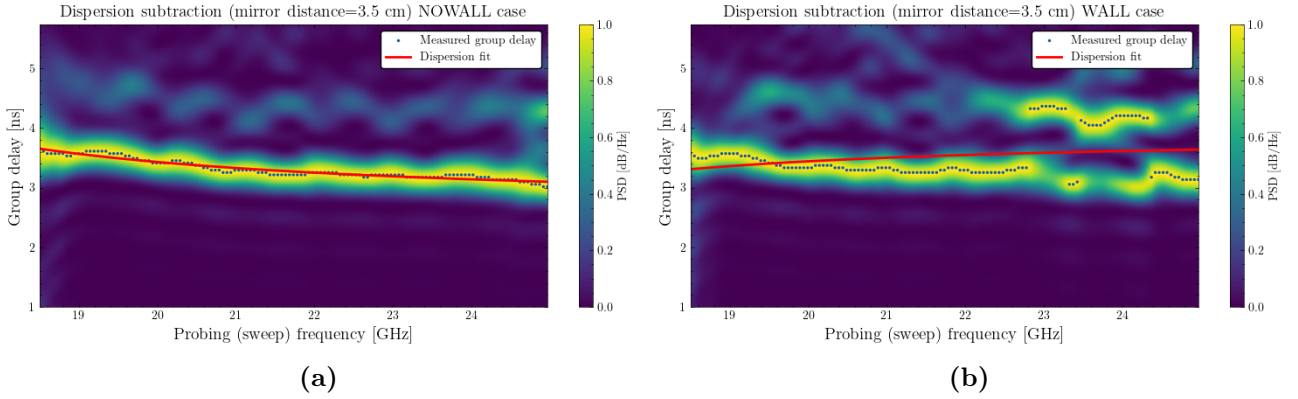


Figure 6.11: Mirror spectrograms at $L \simeq 3.5$ cm for dispersion subtraction (same procedure as in Figure 6.10): (a) NOWALL bistatic, (b) WALL bistatic.

case	A (ns)	B (ns)	$L_c + l_{ant}$ (cm)	L_{wg} (cm)	l_{ant} (cm)
NOWALL	1.05 ± 0.06	1.69 ± 0.04	15.7 ± 0.9	25.4 ± 0.6	12.2 ± 0.9
WALL	4.9 ± 0.5	-1.0 ± 0.4	73 ± 7	-15 ± 5	69 ± 7

Table 6.2: Same results as in Table 6.1 for the NOWALL and WALL bistatic mirror spectrogram at $L_{th} \simeq 3.5$ cm.

The fit results in Table 6.2 show how the NOWALL case is consistent with the one obtained in Table 6.1 and with the real dimensions of the machine, while the WALL case is highly unreasonable.

Finally, it is possible to reconstruct the mirror position once the dispersive component is subtracted. For the bistatic configuration, the results are shown in Figure 6.12 and in Table 6.3, where the measured group delay $\tau_{g,m}$ and position L_m are compared with the expectations. The same analysis is also presented for the monostatic system, yielding almost the exact same result.

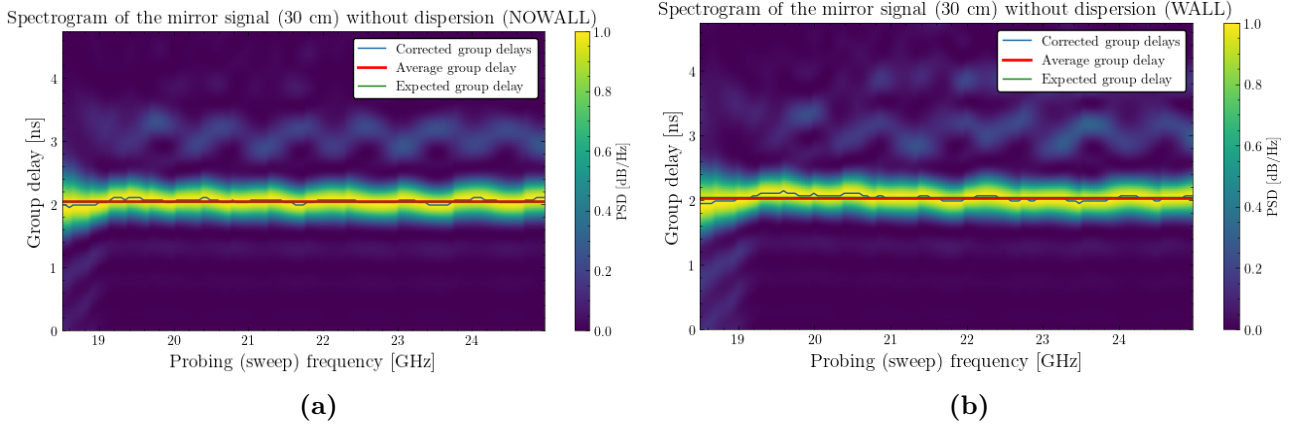


Figure 6.12: Mirror spectrograms after subtraction of the waveguide dispersion contribution, with mirror at $L_{th} \simeq 30$ cm: (a) NOWALL bistatic; (b) WALL bistatic. The τ_g is now flattened to the constant baseline expected for the free-space round trip. Reconstructed positions are reported in Table 6.3.

case	$\tau_{g,m}$ (ns)	$\tau_{g,th}$ (ns)	L_m (cm)	L_{th} (cm)	δ (mm)
BISTATIC					
NOWALL	2.044 ± 0.003	$\simeq 2.048$	29.94 ± 0.05	$\simeq 30$	-0.6
WALL	2.032 ± 0.004	$\simeq 2.048$	29.76 ± 0.05	$\simeq 30$	-2.4
MONOSTATIC					
NOWALL	1.986 ± 0.003	$\simeq 2.001$	29.77 ± 0.05	$\simeq 30$	-2.3
WALL	1.988 ± 0.004	$\simeq 2.001$	29.80 ± 0.05	$\simeq 30$	-2.0

Table 6.3: Mirror position reconstruction after dispersion subtraction (Figure 6.12), for the NOWALL and WALL configurations both in bistatic and monostatic case. The measured group delay $\tau_{g,m}$, the expected $\tau_{g,th}$ from the geometrical model, the reconstructed position L_m , the theoretical position L_{th} and the discrepancy $\delta = L_m - L_{th}$ are reported. All four cases yield position-reconstruction errors below 3 mm, well within the 1 cm accuracy target.

The analysis shows that the system is capable of evaluating the correct mirror position with high accuracy (less than the 1 cm-target) once the set-up is numerically implemented. The recorded discrepancy $\delta \simeq -0.6$ mm for the bistatic NOWALL case (the replica of 5.4) is consistent with the observed experimental uncertainty ($\sigma_\delta \simeq O(mm)$) discussed in Section 5.2. Although the simulation is deterministic, the numerical δ captures the statistical spread of the $\tau_{g,m}$ extraction across the spectrogram, probably associated with pattern irregularities and small deviations from the expected propagation path (the mirror is extended throughout the entire (x, y) plane). The WALL bistatic simulation still provides a satisfactory $\delta \simeq -2.4$ mm accuracy, demonstrating the limited impact of the first wall and thus emphasising the correct integration of the diagnostic in the HFS sector. It will be taken as the reference dispersion signal to subtract from the plasma one (Section 6.3). Finally, the monostatic measurement provides consistent results, demonstrating the feasibility of this configuration in ideal applications.

6.3 Plasma simulation

The final part of the numerical analysis is dedicated to the numerical implementation of a realistic plasma scenario in the simulation set-up. As already mentioned, an RFP *Single-Helical Axis* (SHAx) state has been chosen to ensure more regularity in the edge portion of the profile, with almost concentric isodensity surfaces. It refers to an old RFX-mod discharge, so preliminary work on the profile needs to be performed to extend it to the RFX-mod2 configuration. A further manipulation is necessary to adapt the profile to the K band-accessible density range (Section 6.3.1), making it semi-realistic. The resulting RFP-like plasma is then integrated into the simulation (6.3.2), and further analysis is performed by translating the density profile far away from the first wall to check the system response (6.3.4). The RFX-mod profiles have been obtained with the help of Dr. Marco Veranda and Prof. Fulvio Auriemma, but only one has been selected. The chosen profile corresponds to RFX-mod shot #1590, provided by Prof. Fulvio Auriemma via a *.sav* file and manipulated through a user-made python script.

6.3.1 SHAx state manipulation

The SHAx state refers to a peculiar magnetic equilibrium that corresponds to the closest approach to the theoretical SH (Section 1.5.1). It is thus an improved version of the MH state, where one MHD tearing mode (specifically, the one with $m = 1$, $n = 7$, where m and n are defined as poloidal and toroidal mode numbers) dominates over the others, featuring a higher degree of confinement and greater resilience to chaos. The equilibrium surfaces of a SHAx [41] can thus be described by a superposition of a 0^{th} order axisymmetric equilibrium and a 1^{st} order helical (1-7) perturbation. In this specific case, the amplitude of the latter is small compared to the axisymmetric field to ensure a regular circular shape at the edge, thus setting the best possible working condition for the reflectometric diagnostic. The isodensity magnetic surfaces of the SHAx are identified by the contours of the *helical flux function* χ . Narrowing the look within the poloidal plane (at a fixed toroidal angle), an effective radius ρ can be defined from the normalized helical flux to tag these surfaces:

$$\rho = \sqrt{\frac{\chi - \chi_0}{\chi_a - \chi_0}} \quad (6.9)$$

where χ_0 is the flux on the helical axis, while χ_a is on the Last-Closed Flux Surface (LCFS). The density profile $n_e(\rho)$ allows for the reconstruction of the 3D density map, even when axisymmetry is lost. The contours of the normalized helical flux ρ for shot #1590 are shown in Figure 6.13 in the poloidal plane. The latter is

defined by the (R,Z) coordinate system:

$$R = R_0 + r \cdot \cos(\theta) \quad (6.10)$$

$$Z = r \cdot \sin(\theta) \quad (6.11)$$

where r and θ are the usual polar coordinates for the vessel's circular section. The coordinate boundaries are already adapted to the RFX-mod2 geometry, which features a higher minor radius ($a_{mod2} = 0.49 \text{ m}$) with respect to RFX-mod ($a_{mod} = 0.459 \text{ m}$). The effective radius ranges from $\rho = 0$ at the axis to $\rho = 1$ at the LCFS, reaching $\rho_{max} > 1$ at the first wall. For $\rho > 1$, the density is set to 0 up to the first wall.

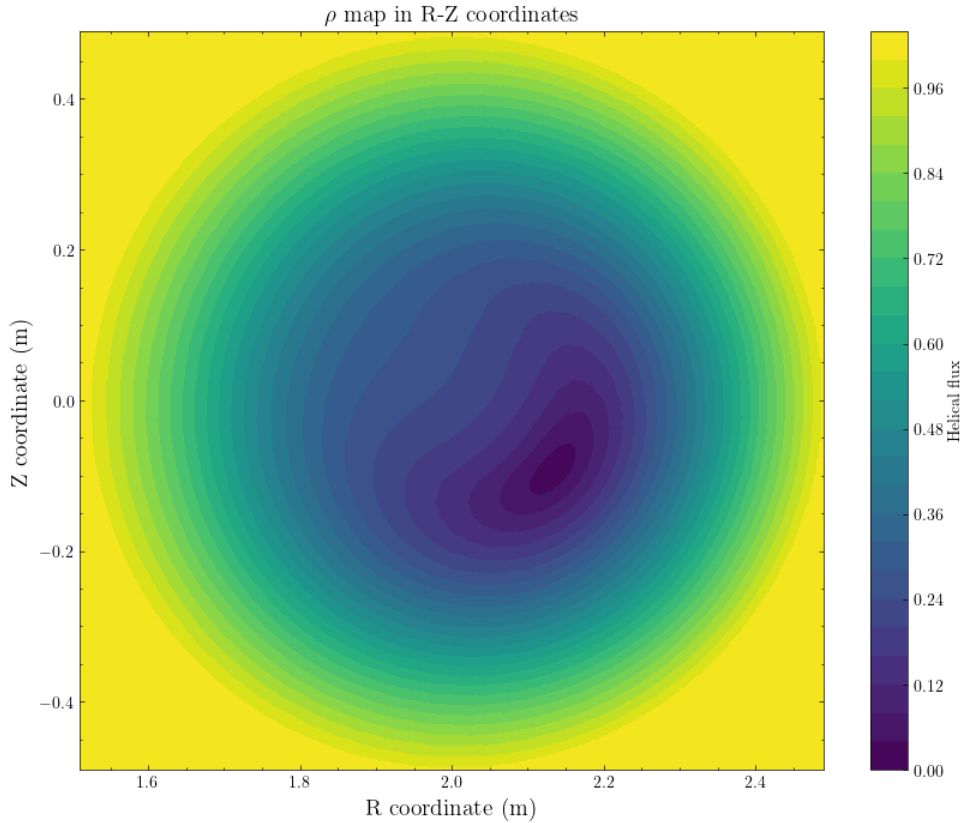


Figure 6.13: Normalised helical flux ρ (Eq. (6.9)) in the (R, Z) poloidal plane for RFX-mod shot #1590. The contour $\rho = 0$ marks the helical magnetic axis; $\rho = 1$ marks the LCFS. The coordinate system is adapted to RFX-mod2.

The 2D density map is provided as well, and the corresponding $n_e(\rho)$ profile is obtained by identifying the LoS that passes through the axis and spans all the values of ρ . The result (Figure 6.14) is a hollow profile with large density gradients and a high edge value ($n_{edge} \simeq 0.77 \cdot 10^{19} \text{ m}^{-3}$).

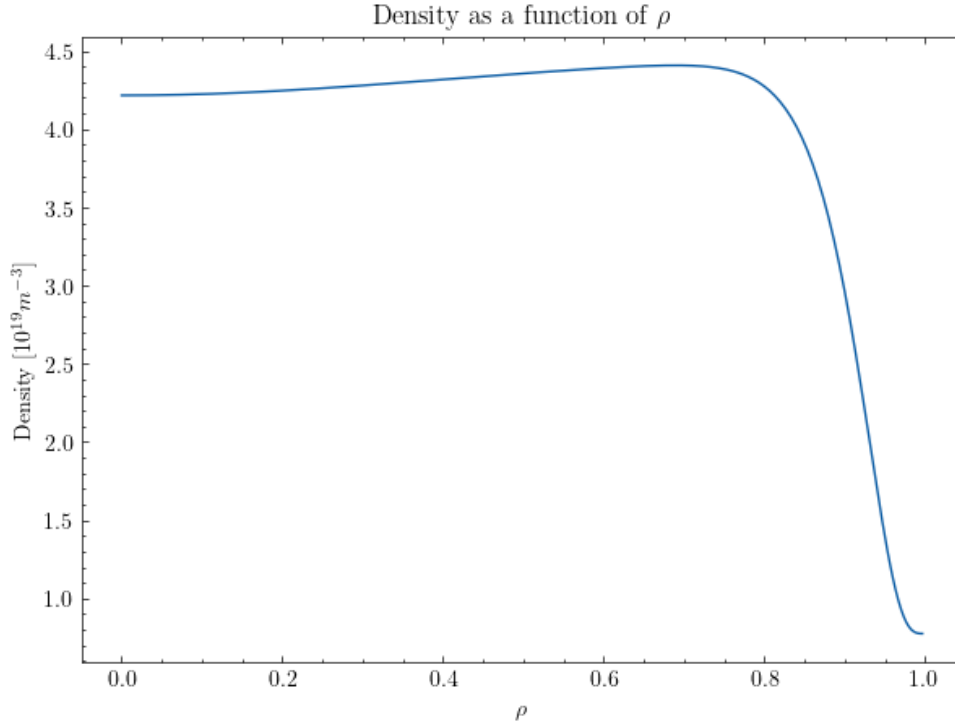


Figure 6.14: Original RFX-mod density profile $n_e(\rho)$ for shot #1590, as a function of the effective radius ρ . The profile is hollow (n_e increases outward from the helical axis) with edge density $n_{edge} \simeq 0.77 \cdot 10^{19} \text{ m}^{-3}$.

The major concern at this point is the incompatibility of this plasma scenario with the K band PPR set-up. Indeed, as already mentioned, the edge density value is still too high to be accessed by the reflectometric measurement, since the K band frequency range ($[18.5, 25] \text{ GHz}$) corresponds to an accessible density interval $[n_{e,min}, n_{e,max}] \simeq [4.2, 7.8] \cdot 10^{18} \text{ m}^{-3}$ for the O-mode propagation (2.23). To solve this issue, a suitable profile extension is performed to lower the edge density to a measurable limit. The strategy consists of a series of steps.

- The radial density profile for RFX-mod is reconstructed (Figure 6.15) by identifying a one-to-one relationship between the radial coordinate r and the flux coordinate ρ along the previously selected LoS. The profile ranges from the axis position (which is negative) to the HFS edge value at $r = a_{mod}$.
- An exponential fit is performed on a selected region of the HFS edge radial profile (Figure 6.15). The fitting function is extended up to a_{mod2} , and the density values are modified accordingly (Figure 6.16 on the left).
- Recalling the relationship between r and ρ , the extended profile $n_e(\rho)$ can be reconstructed (Figure 6.16 on the right).

Note from Figure 6.16 that the edge density value is significantly lower ($n_{edge} \simeq 0.65 \cdot 10^{18} \text{ m}^{-3}$) and is more compatible with the nature of the O-mode K band. It is also important to stress that even the $\rho > 1$ portion is included, meaning that the edge density point coincides with the first wall.

An updated version of the 2D poloidal density map is shown in Figure 6.17a. The actual profile seen by the reflectometric set-up is recovered from the latter, considering the height of the Antennas' LoS $z_{LoS} \simeq 5.8 \text{ cm}$ from the equatorial mid-plane (Figure 6.17b).

It is important to note that this procedure is rather artefact and does not correspond to a rigorous profile extension. Nevertheless, the modified profile can be functional for the scope of the analysis while maintaining the overall structure and the fundamental features of a typical SHAx state.

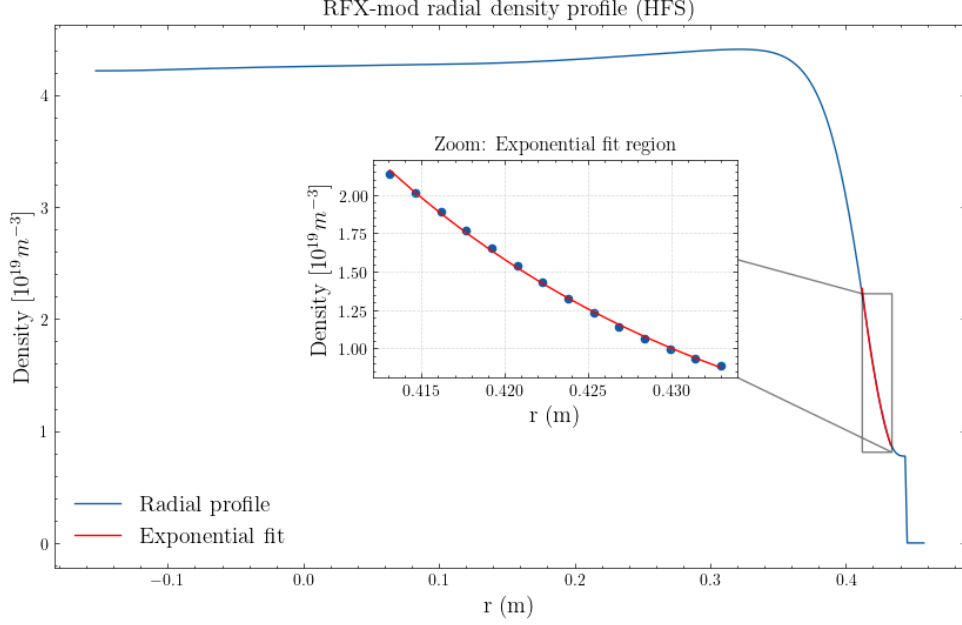


Figure 6.15: Radial density profile $n_e(r)$ along the LoS through the helical axis (blue), with the exponential fit performed on the HFS edge region (red). The fitting function is extended outward to the RFX-mod2 minor radius $a_{mod2} = 0.49 \text{ m}$, providing the modified density values in Figure 6.16.

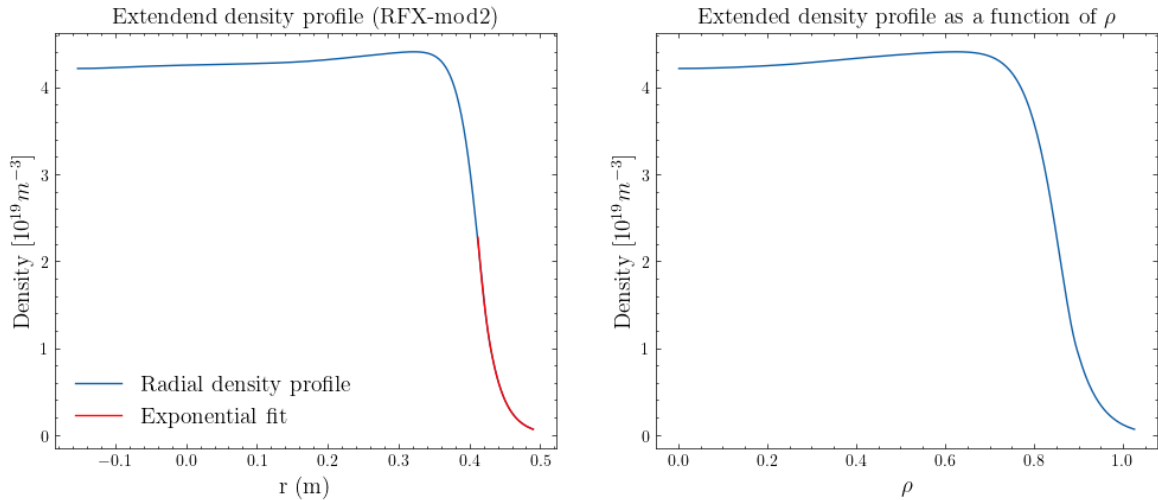


Figure 6.16: Modified density profile (a) as a function of radial position r and (b) as a function of the helical flux coordinate ρ , after the exponential extension on the HFS edge. The new edge density $n_{edge} \simeq 0.65 \cdot 10^{18} \text{ m}^{-3}$ is compatible with the K band O-mode probed-density range $[4.2, 7.8] \cdot 10^{18} \text{ m}^{-3}$.

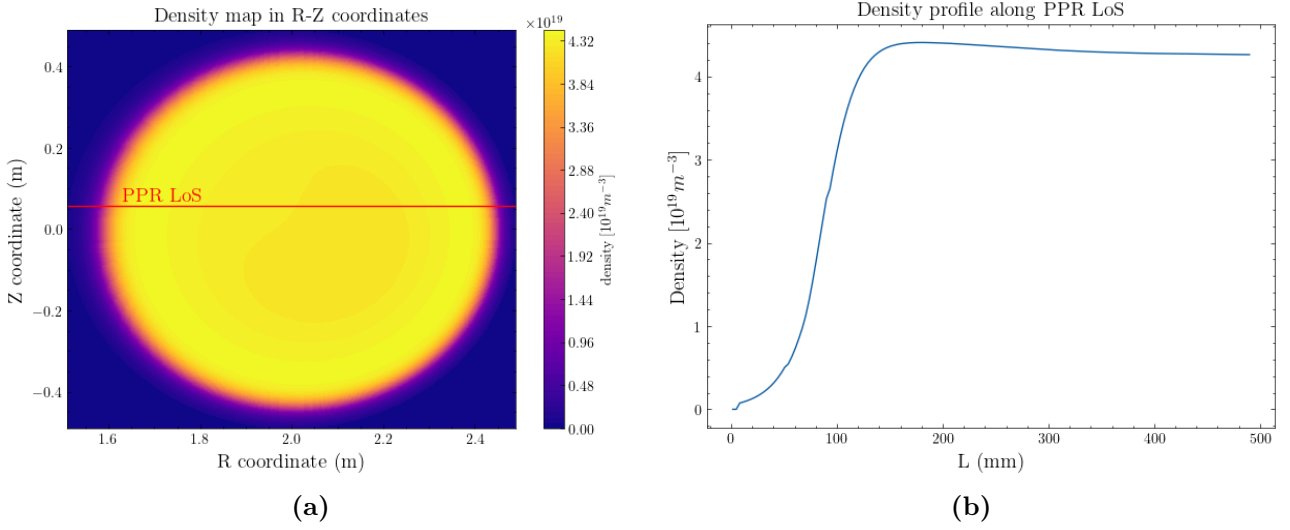


Figure 6.17: Results of the SHAx profile extension to the RFX-mod2 minor radius. (a) 2D poloidal density map in the (R, Z) plane built from the extended $n_e(\rho)$ and the helical flux map of Figure 6.13. (b) Density profile along the PPR LoS at $z_{LoS} \simeq 5.8$ cm above the mid-plane, plotted against the radial coordinate from the antenna mouth.

6.3.2 SHAx numerical implementation

Once the physical and geometrical details about the plasma SHAx scenario are gathered, a series of steps is required to integrate it into the simulation set-up. To simulate the plasma, REFMUL3 needs a model in which the poloidal density map as a function of the vessel coordinates is specified. The plasma will then be associated with the CAD, and the code will be able to include it in the simulation.

The information is synthesized into 4 files.

- **R.dat**: is the column with the R coordinate (from $R_0 - a$ to $R_0 + a$).
- **Z.dat**: is the column with the Z coordinate (from $-a_{mod2}$ to $+a_{mod2}$).
- **rho.dat**: is the helical flux matrix with the values of the effective radius ρ in the $(R-Z)$ plane.
- **rhone.dat**: is a two column text file where each ρ is associated with a corresponding $n_e(\rho)$.

The plasma 3D matrix is built starting from the 2D poloidal one by exploiting the axisymmetry along the toroidal section included in the domain. This is not completely realistic for a SHAx state, which does not feature axisymmetric behaviour, but it can be considered a good approximation in the present case since the outer magnetic flux surfaces are almost concentric, as occurs for Tokamak scenarios.

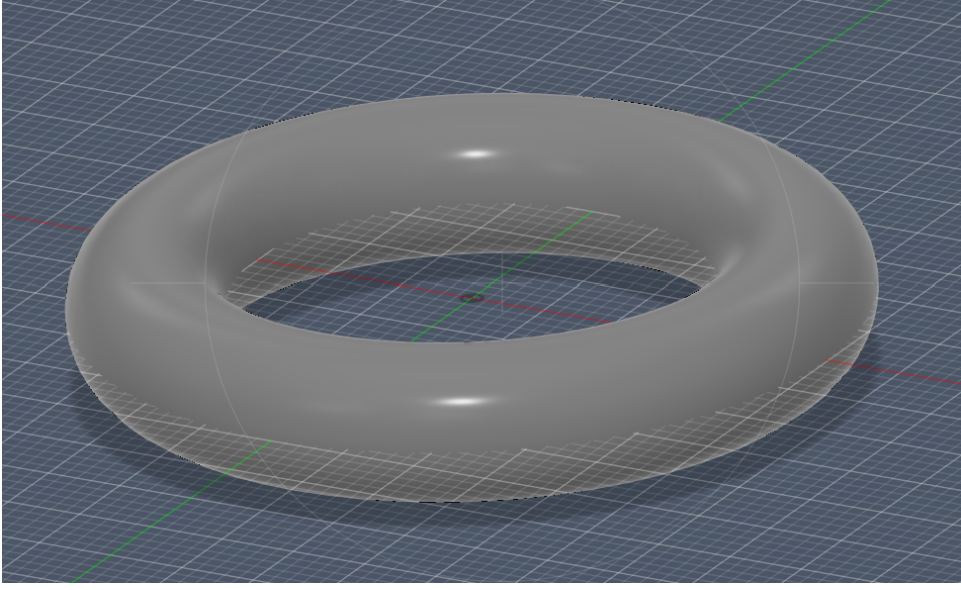


Figure 6.18: CAD model of the toroidal plasma column inserted into the RFX-mod2 vessel geometry. The torus is sliced by *mk_lim_slice* to generate the 2D poloidal reference section, and the corresponding mask is built by *mkMASK* and *mkrefmul_pla_models* scripts.

The starting point for the plasma implementation relies on the definition of a *Plasma mask*, i.e., a portion of the space where the 2D plasma will be inserted. This is done first by integrating a CAD model of a torus (Figure 6.18), ensuring its coherence and compatibility with the RFX-mod2 set-up. Then, the *mk_lim_slice* script is launched to generate the 2D section of the torus that will serve as the poloidal plane of reference. Finally, the mask for the specific scenario is generated by the *mkMASK* script. Finally, the *mkrefmul_pla_models* script is applied after specifying the working band, the dimensions of the simulation domain, and its axis of reference.

6.3.3 SHAx simulation results

Figure 6.19 and 6.20 show the wave propagation from the MW circuit to the plasma reflecting layers, both in the poloidal plane and in the antennas' mid-plane. The features of the implemented plasma scenario and its integration into the machine are highlighted in Figure 6.21, where the poloidal projection of the wave propagation is made clearer. The wave propagation starts from the antenna mouth and occurs along the corresponding LoS, located at $z_{LoS} \simeq 5.8 \text{ cm}$ from the mid-plane. The expected plasma-antenna distance d_{AP} must take into account the antenna-wall spacing $d_{AW} \simeq 3.3 \text{ cm}$ (Table 4.2) and the plasma-wall distance d_{PW} , whenever the plasma column is located far away from the first wall:

$$d_{AP} = d_{AW} + d_{PW} \quad (6.12)$$

Recalling Section 6.3.2, it is clear how $d_{PW} = 0 \text{ cm}$ applies in this specific case, and d_{AP} is completely defined by d_{AW} .

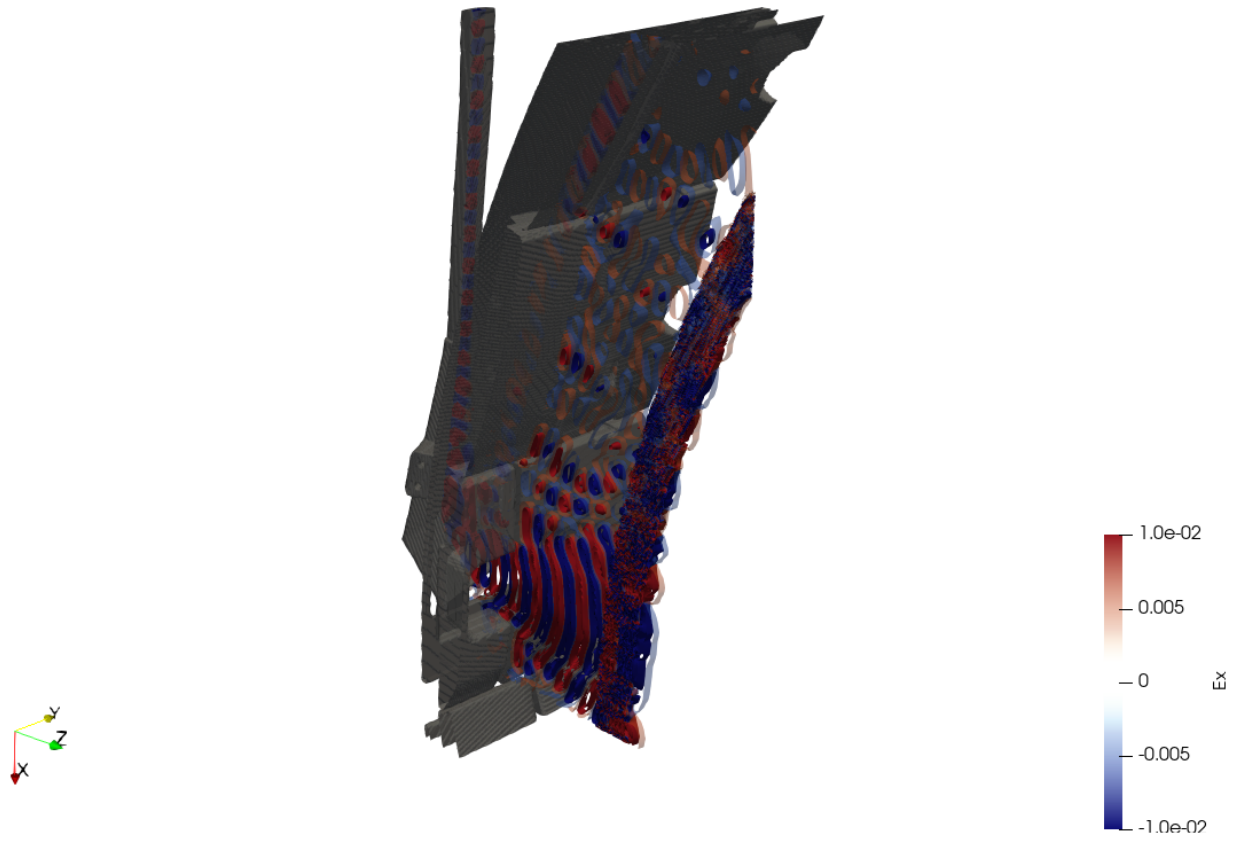


Figure 6.19: Poloidal cut of the SHAx plasma simulation ($d_{PW} = 0$, K band sweep): wave propagation from the HFS subsystem into the plasma column up to the reflection layer.

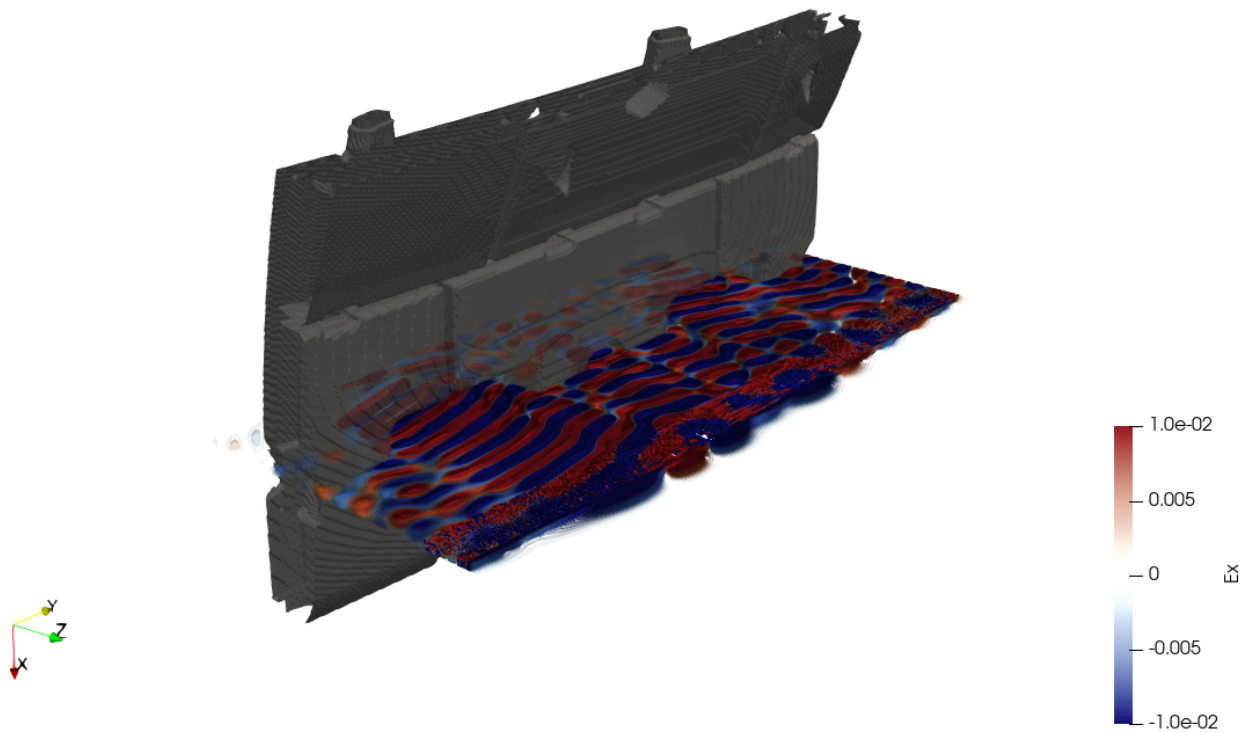


Figure 6.20: Same simulation as Figure 6.19, mid-plane cut between the two antennas. The probing beam converges toward the plasma and is reflected.

The corresponding density values identified by the probing beam range from $n_{e,min} \simeq 4.2 \cdot 10^{18} \text{ m}^{-3}$ to $n_{e,max} \simeq 7.8 \cdot 10^{18} \text{ m}^{-3}$. Since the plasma starts from a null density, the reflection occurs further away from the edge and the wave must travel an additional length before being reflected. The distance of the first reflecting layer is defined as $d_{ref} > d_{AW}$ and shown in Figure 6.21.

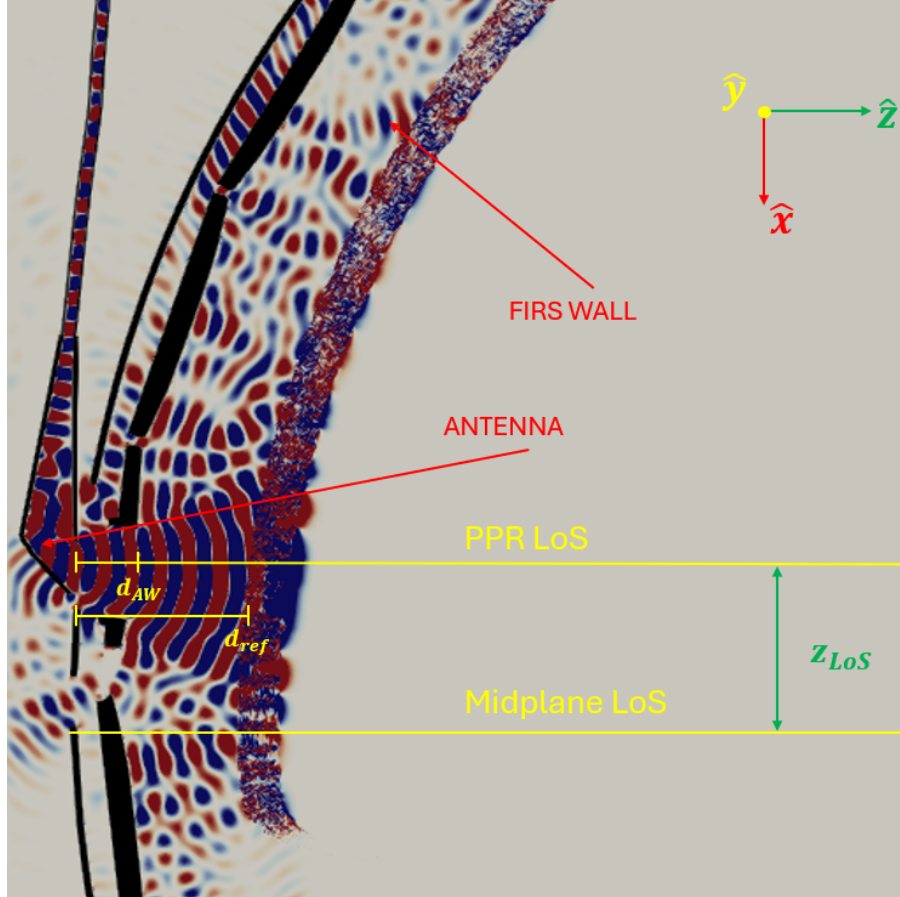


Figure 6.21: Paraview rendering of the simulated wave propagation in the poloidal plane towards the implemented SHAx scenario ($d_{PW} = 0$ case). The Figure shows the geometry of the set-up (antenna, first wall), indicating $d_{AW} = d_{AP}$, d_{ref} , the PPR LoS and its height (z_{LoS}) from the mid-plane.

The choice of the LoS is fundamental for the identification of the actual profile seen by the wave, which, in turn, can be compared with the reconstructed one. The major concern of this analysis relies, once again, on the tilt of the antennas, which causes the waves to propagate along oblique lines. This makes the simple straight-line LoS inadequate for the present discussion and necessitates a more comprehensive analysis of the exact wave propagation according to the position of the target. However, to average the effects of oblique propagation, a proper profile is chosen along the LoS that lies in the middle of the two antennas. The latter is shown in Figure 6.22 in the antennas' mid-plane.

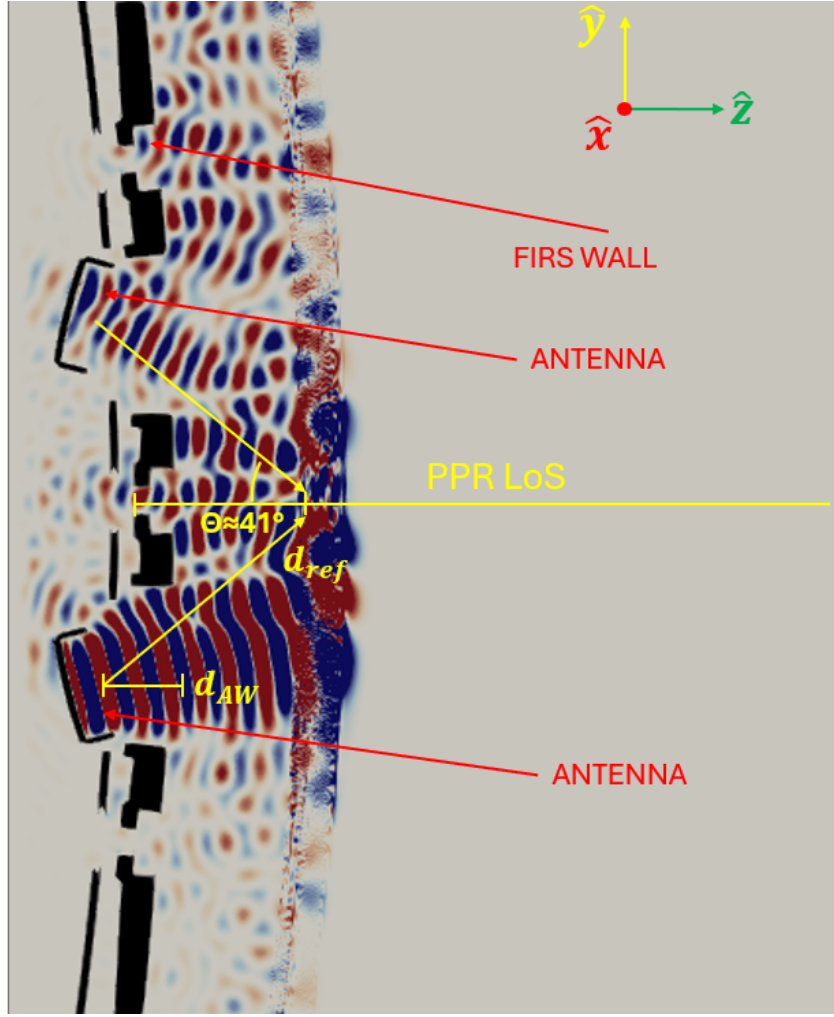


Figure 6.22: Paraview rendering of the simulated wave propagation in the antennas' mid-plane towards the implemented SHAx scenario ($d_{PW} = 0$ case). The Figure shows the geometry of the set-up (antennas, fir wall), indicating $d_{AW} = d_{AP}$, d_{ref} and the average LoS in the middle of the two antennas. Due to the proximity of the plasma, the coupling angle per beam relative to normal incidence is $\theta \simeq 41^\circ$, too far from the antenna tilt design value $\theta_t \simeq 13^\circ$, thus motivates the $\eta = 1/2$ correction discussed in the text.

The analysis of the plasma RX signal follows the same steps as the previous mirror configuration: the signal is processed to generate the $S_{mix,LP}$ component, from which the group delay is extracted by means of the STFT method. The dispersion contribution is subtracted from the $S_{mix,LP}$ spectrogram, similarly to what is done for the mirror. The dispersion function is evaluated from the bistatic mirror spectrogram placed at $L \simeq 30 \text{ cm}$ in WALL configuration, and the final τ_g is corrected for the mirror free-space propagation. This choice is motivated by the attempt to subtract the dispersion contribution of both waveguides without relying on the assumption that they are exactly equal. Moreover, the proximity of the mirror to the focal point ensures high reliability and regularity. The resulting measured group delay is then compared with the one generated by the expected profile. The latter is obtained from a Paraview plot along the chosen PPR LoS

(Figure 6.22) and implemented in a python script to be converted into delay.

Figure 6.23 offers a comparison between the measured delay of the plasma signal and the validated delay from the expected profile, computed via equation (2.25). The corresponding absolute error is also discussed. It is possible to observe a significant discrepancy between the two, with the experimental group delay strongly exceeding the expected amount. A reason for this behaviour can be identified by looking at Figure 6.22: the high proximity of the plasma to the wall makes the reflectometric system work under non-optimal conditions. The coupling angle θ , defined as the angle of the optimal oblique path (yellow) with respect to normal incidence, is far from the tilt angle $\theta_t \simeq 13^\circ$ for which the antennas are designed. In this case, an angle of $\simeq 41^\circ$ ensures the best coupling of the two antennas. In this situation, the probing beam is more likely to be reflected twice by hitting the first wall after the initial interaction with the plasma. This double plasma-wall reflection can increase the path of the wave by a factor of 2 and raise the effective group delay. To account for this, a correction $\eta = 1/2$ is implemented, defining the corrected group delay $\tau_{g,corr} = \eta \cdot \tau_{g,m}$. The result is shown in Figure 6.24.

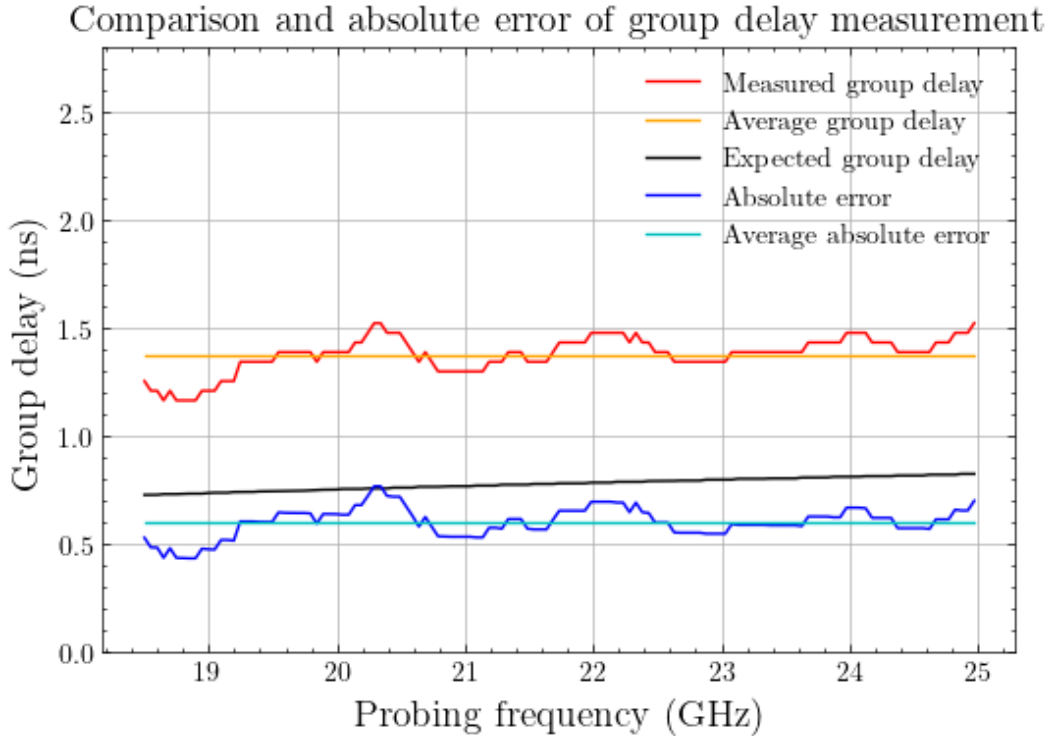


Figure 6.23: Comparison between the measured $\tau_{g,m}$ (red) extracted from the SHAx simulation ($d_{PW} = 0$) and the expected $\tau_{g,th}$ (black) computed from the implemented density profile along the PPR LoS (Eq. (2.25)). The blue curve shows the relative discrepancy. The systematic excess in $\tau_{g,m}$ of roughly a factor of two motivates the $\eta = 1/2$ correction applied in Figure 6.24.

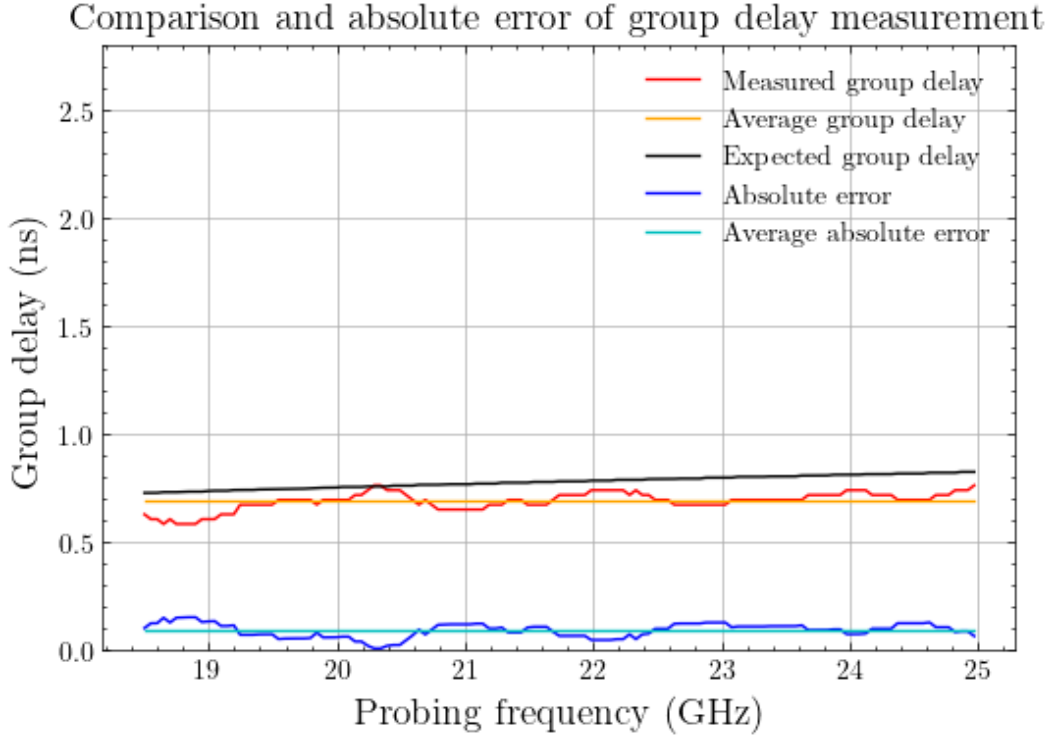


Figure 6.24: Same as Figure 6.23, with the $\eta = 1/2$ correction applied to $\tau_{g,m}$ to account for the double plasma–wall reflection. The relative error $\epsilon_\tau = 13\%$ demonstrates that the correction is partially successful.

Despite the rough nature of the correction, the comparison looks more reasonable. Figure 6.23 and 6.24 show the analysis of the discrepancy between the measured and expected group delays, the results of which are reported in Table 6.4. The average measured group delay $\bar{\tau}_{g,m}$ gives an idea of the overall delay introduced by the plasma, while the discrepancy $|\delta\bar{\tau}_g|$ corresponds to the mean absolute error. To assess the quality of the measurement, the relative error $\epsilon_\tau = \frac{|\delta\bar{\tau}_g|}{\bar{\tau}_{g,m}}$ on the average group delay is also reported, thus showing significant improvements in the corrected case with respect to the previous one. However, the residual $\simeq 13\%$ relative error suggests that this correction works only partially and that a more detailed model must be implemented to account for the complex wave propagation in this scenario.

CASE	$d_{PW}(cm)$	$d_{ref}(cm)$	$ \theta - \theta_t (^{\circ})$	$\bar{\tau}_{g,m}(ns)$	$ \delta\bar{\tau}_g (ns)$	ϵ_τ
NO-CORR	0	$\simeq 8$	$\simeq 28.1$	1.37 ± 0.08	0.599 ± 0.006	$\simeq 44\%$
CORR	0	$\simeq 8$	$\simeq 28.1$	0.69 ± 0.04	0.089 ± 0.003	$\simeq 13\%$

Table 6.4: Group-delay analysis for the SHAx scenario at $d_{PW} = 0$, before and after the $\eta = 1/2$ correction for the double reflection of Figure 6.22. For both cases, the Table shows the nominal plasma–wall distance d_{PW} , the reflection position d_{ref} , the mismatch $|\theta - \theta_t|$, the average measured group delay $\bar{\tau}_{g,m}$, the mean absolute error $|\delta\bar{\tau}_g|$ and relative error ϵ_τ . The correction brings the latter from $\simeq 44\%$ to $\simeq 13\%$.

6.3.4 SHAx translation

To assess the quality of the simulation across the RFX-mod2 spatial domain, a further manipulation of the profile has been implemented by translating the poloidal flux map outward to increase the plasma-wall distance d_{PW} . In this way, it is possible to obtain more realistic plasma configurations that reflect the PPR working conditions for Tokamak scenarios. Varying d_{PW} , the reflectometry system might behave differently, given its large tilt angle and the imperfect perpendicularity of the plasma along the probing LoS. A comprehensive analysis of the translated plasma is presented in terms of group delay measurement to check the system response and linearity and to address possible detriments. The translation is implemented by shifting the ρ map in Figure 6.13 away from the HFS and associating the same $n_e(\rho)$. An example is shown in Figure 6.25, where the density map features a 10 cm shift (6.25a) and the PPR LoS profile changes accordingly (6.25b).

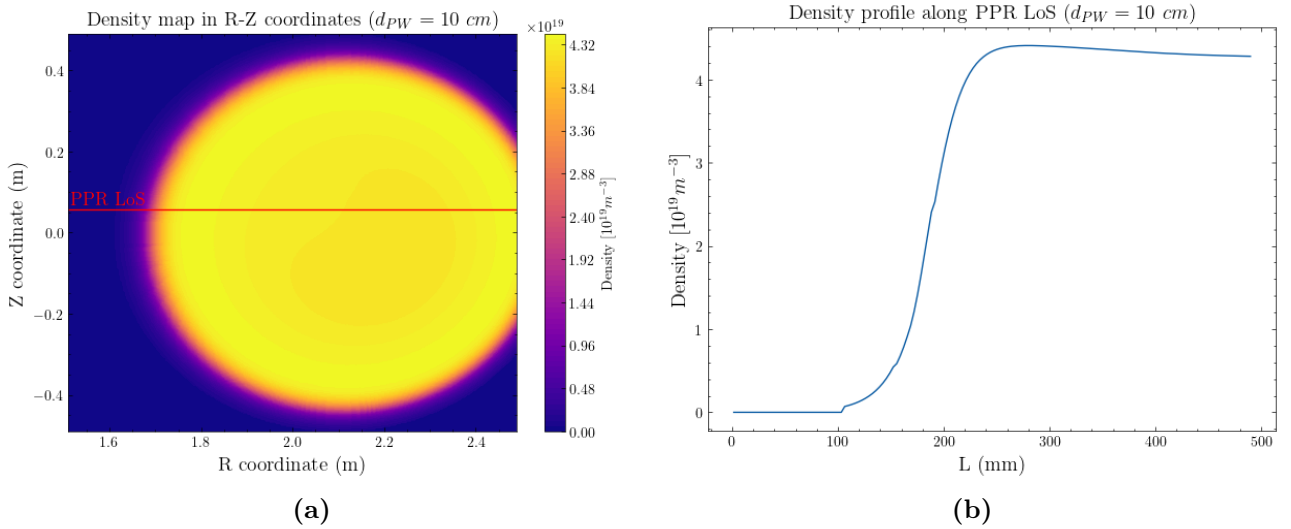


Figure 6.25: Example of SHAx profile translation ($d_{PW} = 10$ cm) by shifting the helical flux map outward while preserving $n_e(\rho)$: (a) translated 2D poloidal density map; (b) corresponding profile along the PPR LoS. The procedure is repeated for $d_{PW}=10, 15, 20, 25, 30$ cm to scan the system response (Table 6.5).

Figure 6.26 shows the poloidal projection of the wave propagation inside the RFX-mod2 layout, and a direct comparison with Figure 6.21 is straightforward.

The increasing distance of reflection (d_{ref}) of the wave against the plasma column has an impact on the round-trip power loss for the RX signal with respect to the reference one, which is reported in Figure 6.27. As expected, a downgrade in the amplitude of the detected signal is recorded at higher distances. At lower d_{ref} (0-10 cm), a positive gain is sporadically observed, probably resulting from a superposition of multiple reflected components.

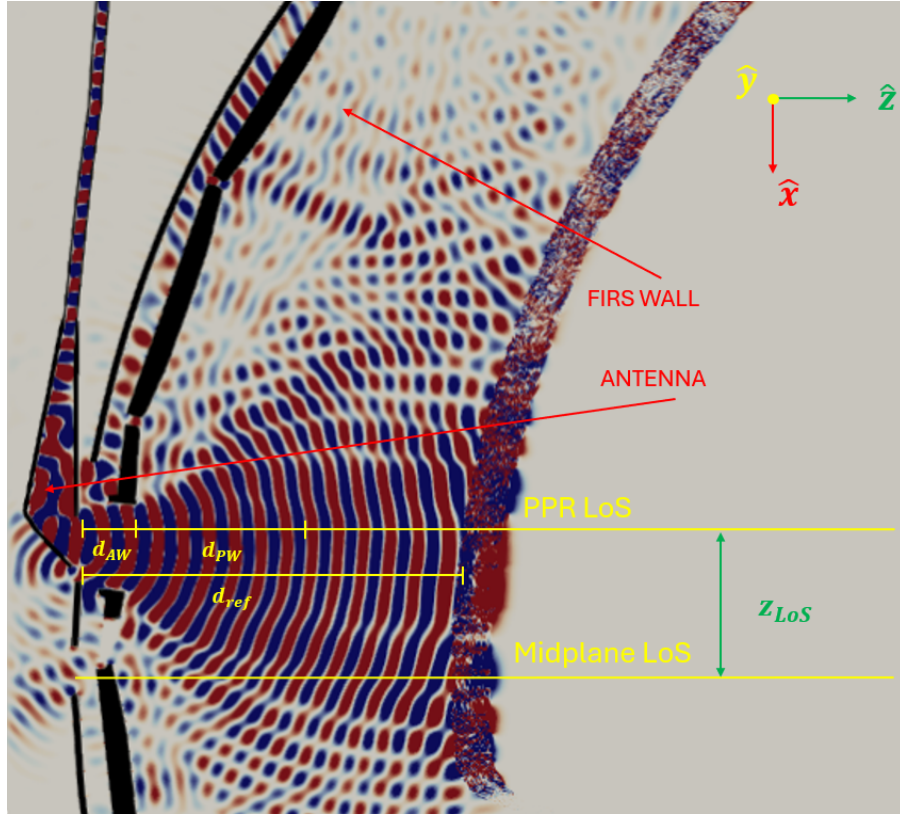


Figure 6.26: Paraview rendering of the simulated wave propagation in the poloidal plane towards the implemented SHAx scenario ($d_{PW} = 10$ cm). The Figure shows the geometry of the set-up (antenna, first wall), indicating d_{AW} , d_{PW} , d_{ref} , the PPR LoS and its height (z_{LoS}) from the mid-plane.

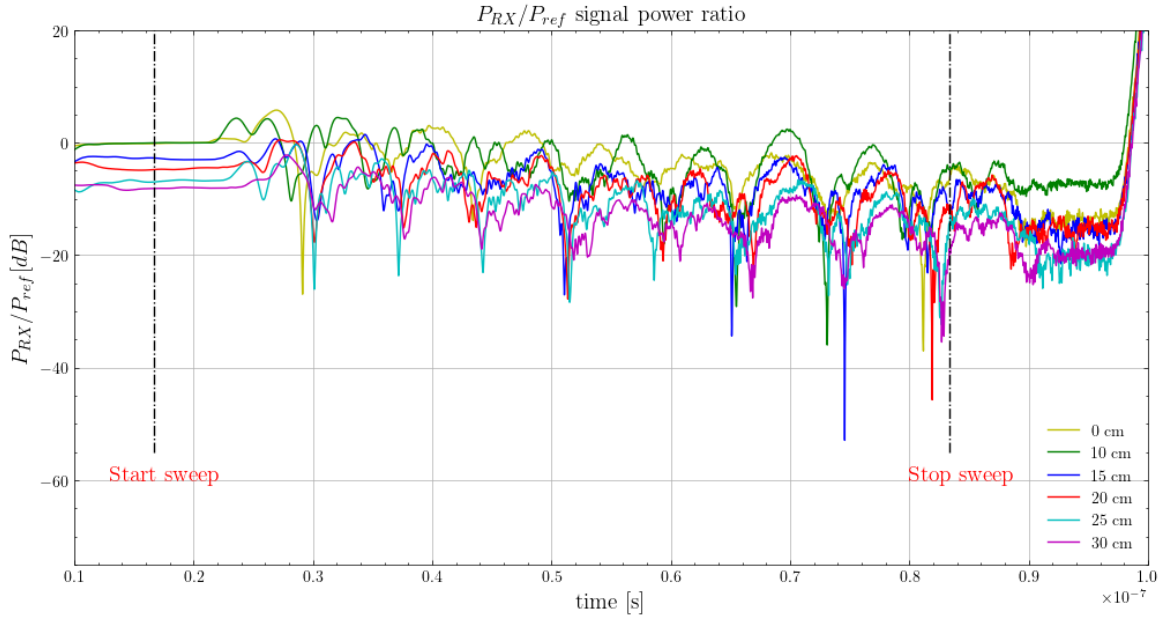


Figure 6.27: Comparison of RX round-trip power loss (eq. (2.40)) for the six plasma-wall distances (d_{PW}). The loss increases with d_{PW} , reflecting the propagation through a longer free-space path. Sporadic positive gains observed at the lowest d_{PW} are attributed to the superposition of multiple reflected components.

The summary of the translation analysis is reported in Figure 6.28, where a comparison between measured ($\tau_{g,m}$) and expected ($\tau_{g,th}$) group delays is provided. Table 6.5 gathers all the extracted information: the nominal d_{PW} and the ef-

fective d_{ref} at which the first reflecting layer is located, the difference between the coupling and the optimal angle ($|\theta - \theta_t|$), and the average measured group delay ($\bar{\tau}_{g,m}$), together with the mean and the relative absolute error ($|\delta\bar{\tau}_g|$ and $\epsilon_\tau = \frac{|\delta\bar{\tau}_g|}{\bar{\tau}_{g,m}}$). The system features a good linear response, proving capable of correctly identifying the plasma-induced delay under the shift of the target position. As expected, moving the plasma column away from the wall produces an improvement in performance, providing a lower relative error in all five cases with respect to the 0 cm-plasma scenario. No significant results are recorded in the optimal geometrical configuration (the $d_{PW} = 20$ cm case) where the reflection occurs at the closest distance to $L_f \simeq 28.2$ cm (Table 4.2).

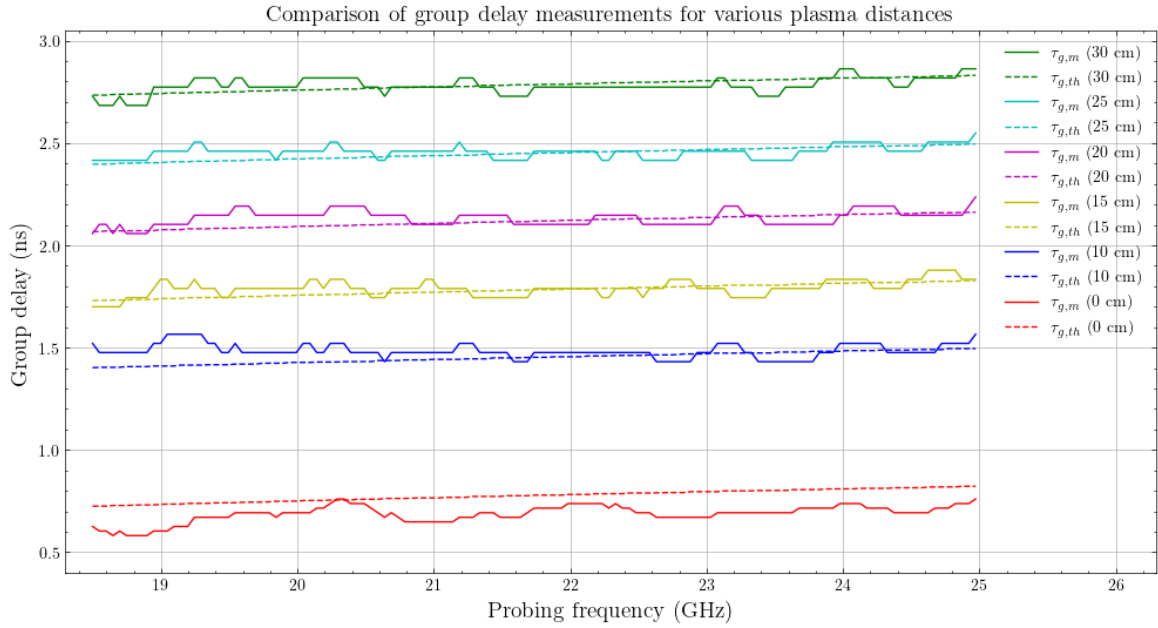


Figure 6.28: Comparison between measured $\tau_{g,m}$ and expected $\tau_{g,th}$ across the five plasma–wall distances. The linear response of the system is visible: $\tau_{g,m}$ tracks $\tau_{g,th}$ faithfully across the 10–30 cm range, with the relative error ϵ_τ remaining below 3.1 % everywhere (Table 6.5).

$d_{PW}(cm)$	$d_{ref}(cm)$	$ \theta - \theta_t (^{\circ})$	$\bar{\tau}_{g,m}(ns)$	$ \delta\bar{\tau}_g (ns)$	ϵ_τ
10	$\simeq 18$	$\simeq 7.9$	1.48 ± 0.03	0.046 ± 0.003	$\simeq 3.1\%$
15	$\simeq 23$	$\simeq 3.7$	1.79 ± 0.04	0.030 ± 0.002	$\simeq 1.7\%$
20	$\simeq 28$	$\simeq 0.8$	2.13 ± 0.04	0.035 ± 0.002	$\simeq 1.7\%$
25	$\simeq 33$	$\simeq 1.3$	2.46 ± 0.03	0.029 ± 0.002	$\simeq 1.2\%$
30	$\simeq 38$	$\simeq 2.6$	2.78 ± 0.04	0.030 ± 0.002	$\simeq 1.1\%$

Table 6.5: Group-delay analysis under SHAx plasma translation across $d_{PW} = 10, 15, 20, 25, 30$ cm as in Table 6.4. The relative error ϵ_τ remains within 3.1 %, decreasing monotonically with increasing d_{PW} .

6.3.5 Profile reconstruction

The final part of the analysis can be dedicated to an attempt at reconstructing the density profile. This is done by inverting the measured group delay $\tau_g(\omega)$ employing the technique explained in Section 2.3. The inversion algorithm for O-mode reflectometry is provided analytically by the Abel integral (2.29), and the resulting $r_c(\omega)$ requires knowledge of the group delay for any frequency below ω . The lack of $\tau_g(\omega)$ data for frequencies below 18.5 GHz represents the main limitation introduced by O-mode propagation, preventing a rigorous profile reconstruction from being discussed and analysed. To partially solve this issue, a proper initialisation procedure is necessary to get information across the entire frequency spectrum. This is usually accomplished with the aid of additional edge diagnostics (such as Thermal Helium Beam) that can provide an edge density characterisation and improve the accuracy of the reconstruction.

For the scope of the thesis, a brief discussion is presented to investigate the effect of the initialisation on the final profile reconstruction. As a first step, a rough initialisation is defined by a simple linear ramp between the initial τ_g value, corresponding to the minimum delay at the expected plasma starting position, and the measured value at the beginning of the sweep. Figure 6.29 provides an example of such a rough initialization (at 10 cm), and for the same situation, Figure 6.30 shows the corresponding profile reconstruction (left) and the discrepancy of the evaluated position with respect to the expected one (right).

The employment of a rough initialization procedure introduces an inevitable discrepancy between the two profiles (Figure 6.29) for probing frequencies below 18.5 GHz , leading to a non-negligible error contribution to the final profile reconstruction (Figure 6.30) that exceeds the 1 cm -target accuracy for a reflectometric system. This source of error arises purely from the inversion algorithm and does not rely on any stochastic physical mechanism related to wave propagation, nor on any geometric systematic contribution attributable to the set-up.

The same situation is then subject to an ideal initialisation, defined by imposing the lower group delay to perfectly match the expected one (Figure 6.31). The reconstruction (Figure 6.32) features a significant reduction in the corresponding error, which is now negligible in the transparent region (as expected), but retains a relatively low value also in the probed region, thus demonstrating the sensitivity of the inversion procedure to the choice of initialization. This confirms the significant improvement in the results with the optimal initialisation and shows how a proper choice of the latter can be crucial to keep the accuracy below the required threshold.

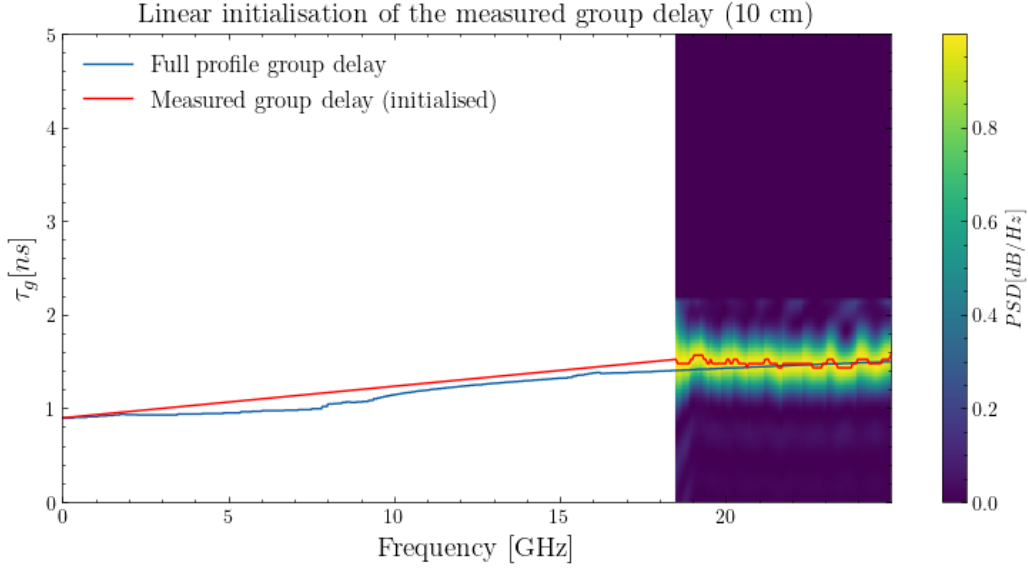


Figure 6.29: Linear initialisation procedure for the τ_g inversion at $d_{PW} = 10$ cm: the measured τ_g (red) is filled in below 18.5 GHz by a linear ramp between the minimum-delay value at the expected plasma starting position and the first measured τ_g value of the sweep. The expected τ_g (blue) from the profile inversion is shown for comparison.

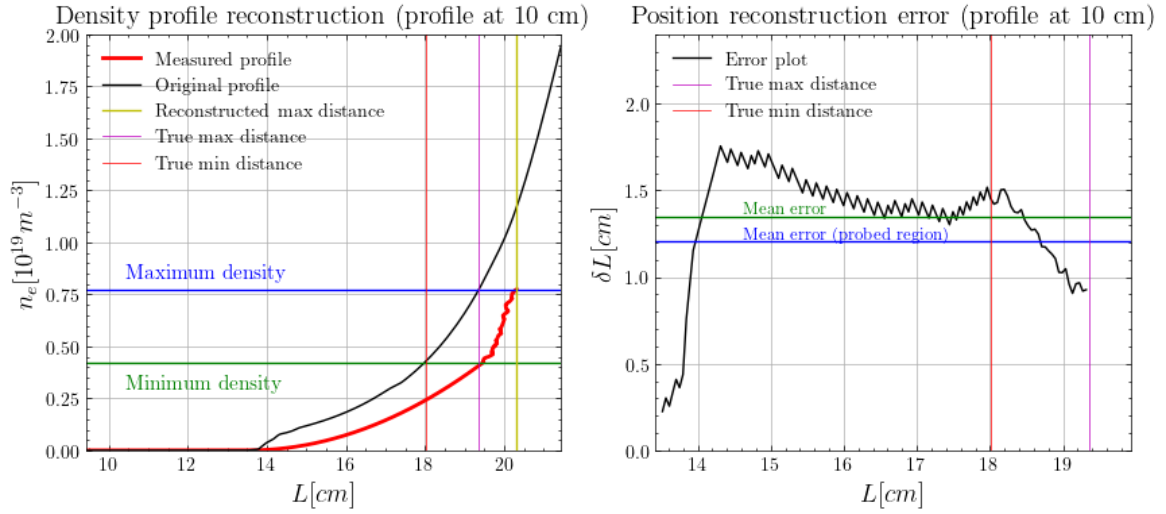


Figure 6.30: Profile reconstruction with linear initialisation at $d_{PW} = 10$ cm: (left) the recovered $n_e(L)$ (red) compared with the implemented profile (black); (right) the absolute position error $\delta L(L)$ across the profile. The error is large below the K band-accessible region (the initialisation region) and smaller, though non-negligible, in the probed region $[n_{e,min}, n_{e,max}]$. The plot shows the mean error $\delta \bar{L} = (1.35 \pm 0.03)$ cm throughout the whole reconstruction and the mean error $\delta \bar{L}_{pr} = (1.21 \pm 0.04)$ cm in the probed region.

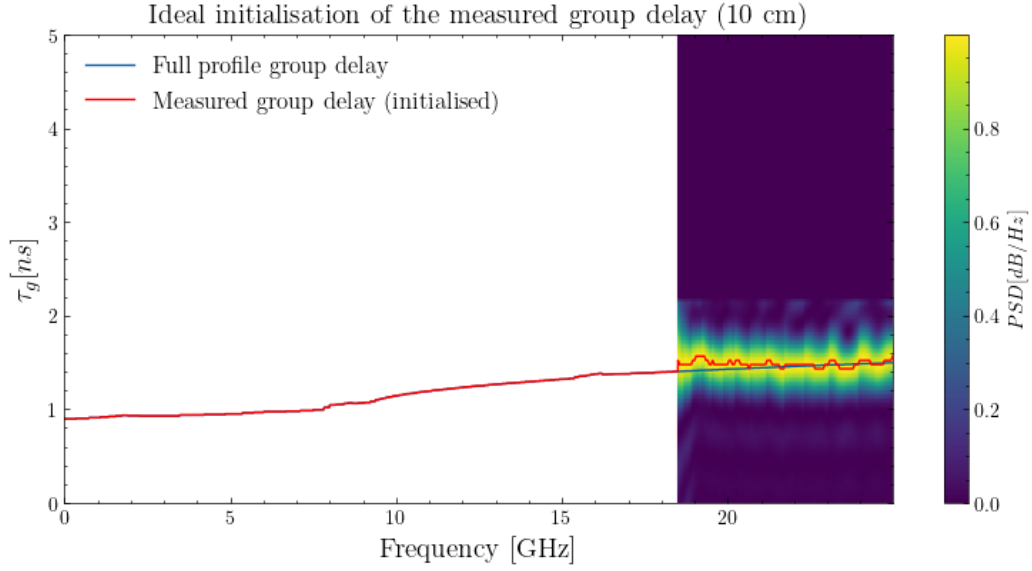


Figure 6.31: Ideal initialisation procedure for the τ_g inversion at $d_{PW} = 10$ cm: the measured τ_g (red) below 18.5 GHz are set equal to the expected τ_g values (blue) from the profile inversion. This is unrealistic and is used here only to isolate the sensitivity of the inversion algorithm to the initialisation choice.

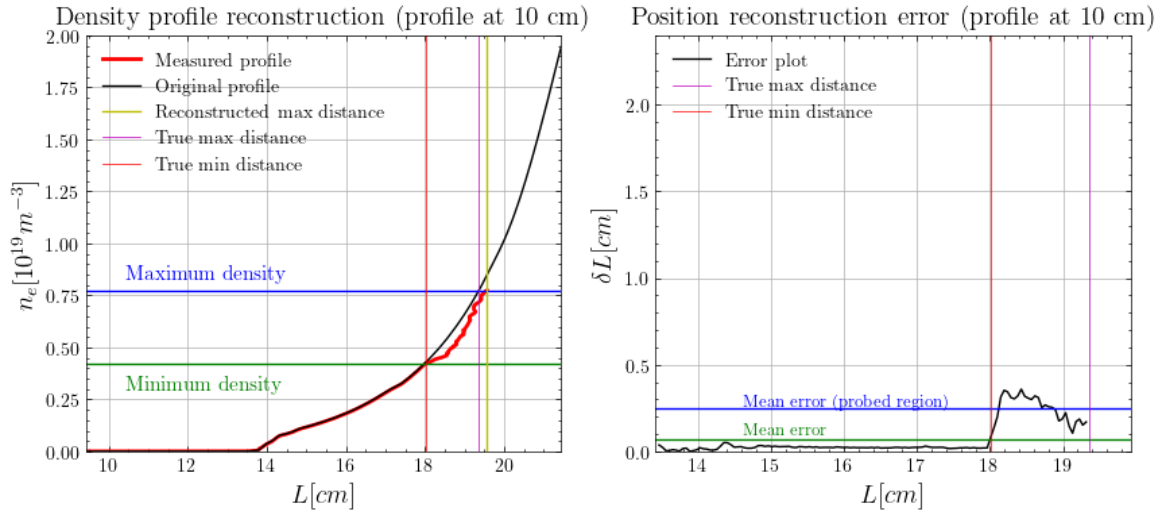


Figure 6.32: Profile reconstruction with ideal initialisation at $d_{PW} = 10$ cm: same plots as Figure 6.30 but using the ideal τ_g values below 18.5 GHz. The error in the initialisation region is now negligible ($\delta\bar{L} = (0.07 \pm 0.01)$ cm), and the error in the probed region drops significantly $\delta\bar{L}_{pr} = (0.25 \pm 0.01)$ cm.

As a final positive note, Figure 6.33 shows the best profile reconstruction among the various attempts ($d_{PW} = 15$ cm). From the analysis of the errors, it is possible to see how the discrepancy is kept below the 1-cm threshold across the reconstructed profile. The result obtained at 15 cm does not reflect any particular physical behaviour, and it is not robust; however, it suggests that even with a rough initialization, the edge density profile can still be partially reconstructed, thus confirming the adequate integration of the diagnostic despite the clear limitations of the O-mode configuration.

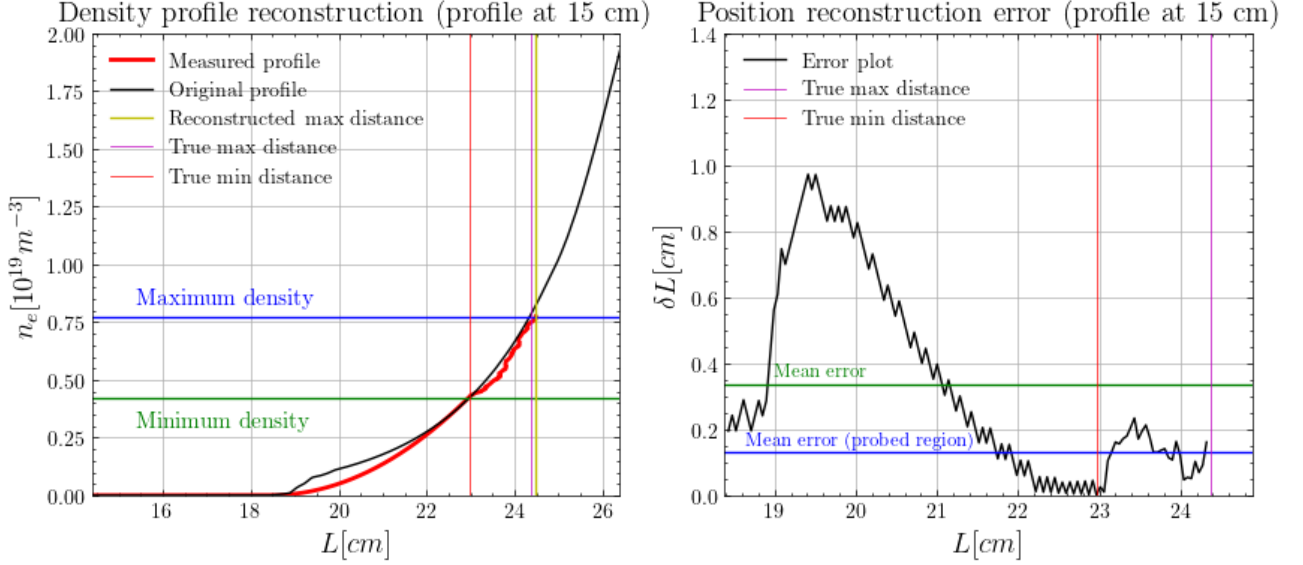


Figure 6.33: Best reconstruction performance with the rough linear initialisation ($d_{PW} = 15$ cm): the image shows the recovered profile (red) vs implemented (black) on the left, with absolute position error δL well below 1 cm across the whole profile on the right.

Chapter 7

Discussion and final remarks

The measurement of the group delay is coherent with expectations, and the system is capable of evaluating the correct mirror position with high accuracy ($< 1\text{ cm}$) both experimentally and numerically, thus validating the proper performance of RFX-mod2 PPR in the expected geometrical conditions. This chapter provides a summary and a brief discussion of the main experimental and numerical results obtained throughout the thesis. Alongside the most relevant findings, in the next Chapter 8, future perspectives and possible improvements are also provided in view of the forthcoming start of RFX-mod2, foreseen for the next few years.

7.1 Experimental results

The experimental characterization of the system proved successful: the waveguide dispersive contribution was properly isolated by means of a directional coupler, modelled and subtracted from the original signal (Section 5.2.1), thus recovering a flat group delay trend expected in the absence of dispersion. The measurement of the group delay in Section 5.2 showed excellent linear correlation with the mirror position and enabled a proper calibration procedure. The reconstruction of the mirror-antennas distance yielded optimal results with millimetre-scale errors that fell well under the $\simeq 1\text{ cm}$ accuracy threshold required for a reflectometry system. These were mainly characterised by a $\simeq 3\text{ mm}$ accuracy (Section 5.2.2) comparable with the assumed systematic uncertainty (5.9) on the antenna mouth reference position. The residual stochastic contribution fell within $\simeq 1\text{ mm}$ and is consistent with the error obtained from the differential analysis (Section 5.2.3) for consecutive mirror shifts. No significant discrepancies are found either in terms of deviation from the focal position (Section 5.2.3) or between the tilted and non-tilted configurations (Section 5.3), confirming the minimal impact of antenna geometry for plane mirror applications. The most severe detriments were observed for large mirror inclination angles ($\simeq \pm 20^\circ$ and $\pm 30^\circ$), mimicking large plasma surface curvatures. This demonstrates how perpendicular beam propaga-

tion represents, as expected, the most relevant factor influencing the reflectometric measurements and the most critical limitation for applications in fusion devices, since the imperfect geometrical alignment causes the beam to diverge. Nevertheless, an error $< 1\text{ cm}$ is recorded for the $[-15^\circ, 15^\circ]$ angular interval, thus ensuring a partial resilience of the system for modest distortions.

7.2 Numerical results

The numerical assessment of the system provided interesting results: the mirror simulation (Section 6.2) showed good agreement between the measured and expected group delays once the effects of the waveguides and the antennas were subtracted. The error in position reconstruction was well below the 1 cm -target accuracy and on the order of $\simeq 1\text{ mm}$, consistent with the stochastic error contribution in the experimental counterpart (Section 5.2.2, 5.2.3). This was recorded both without and with the first wall ($\simeq -0.6\text{ mm}$ and $\simeq -2.4\text{ mm}$, respectively), thus demonstrating the proper performance of the system even when the complex wall structure is integrated and the rise of spurious reflections is more likely (especially when the mirror was placed very close to the antennas). A similar result was obtained by employing a monostatic configuration ($\simeq 2.3\text{ mm}$ and $\simeq 2.0\text{ mm}$), which can represent a valid alternative for plane mirror applications.

The optimal comparison between experimental and numerical outcomes is a remarkable result, demonstrating that a simplified diagnostic model integrated into the simulation framework can still provide a reliable investigation of the occurring physical effects, a consistent identification of the dominant sources of error, and an overall validation of the signal processing pipeline (signal extraction, low-pass filtering, dispersion subtraction). The consistency between NOWALL and WALL configurations represents another fundamental result of the thesis, confirming the correct integration of the diagnostics in the HFS sector of RFX-mod2.

As for the plasma part (Section 6.3), Tokamak scenarios require the PPR to be employed in X-mode, and a complete numerical assessment of the profile reconstruction cannot be obtained in this case. On the other hand, O-mode reflectometry simulations are more realistic for the RFP case. For these reasons, considering the latter scenario in the expected Tokamak geometric conditions, REFMUL3 simulations can still provide interesting results.

The manipulated SHAx-like profile configuration allowed for the extraction of a measured group delay, thanks to its K band-accessible adapted density. The first scenario, located at a plasma-wall distance $d_{PW} = 0\text{ cm}$, was rather close to the first wall and proved not to be optimal for the bistatic system, yielding a group delay measurement that was dominated by double plasma-wall reflection. To ac-

count for this effect, a $\eta = \frac{1}{2}$ rough correction factor was applied, and a significant improvement (from $\simeq 44\%$ to $\simeq 13\%$ relative discrepancy) was recorded. To be more consistent with the optimal Tokamak plasma scenarios, which are expected to be significantly farther from the wall, the horizontal shift of the plasma column has turned out to be the best strategy to investigate PPR performance in more realistic conditions. This was done by simply increasing the distance to values comparable to the Tokamak ones ($d_{PW} \simeq 20\text{-}30\text{ cm}$). As in the mirror case, the system showed a consistent response at various plasma-wall distances and proved capable of correctly evaluating the group delay associated with the reflection layer, featuring a relative discrepancy below 3%.

A final investigation concerned an attempt at profile reconstruction (Section 6.3.5), which is limited by the O-mode-specific lack of group delay data for probing frequencies below 18.5 GHz , leading to a non-negligible error contribution to the final profile reconstruction. To quantify and discuss this effect, two different initialisation procedures were applied, and the corresponding final reconstructions were compared. First, a linear ramp was introduced, leading in the worst case ($d_{PW} = 10\text{ cm}$) to a reconstruction error above the 1 cm threshold. Then, an ideal initialisation was performed by imposing the group delay at lower frequencies to perfectly match the true profile, and a significant reduction in the reconstruction error was recorded. This latest result is highly unphysical, but it demonstrates the strong sensitivity of the reconstruction procedure to the choice of the initialisation and the need for additional edge density data from further diagnostics. However, it was shown that the system is still partially capable of reconstructing the profile, yielding some interesting results despite the clear limitations of the O-mode configuration.

Chapter 8

Conclusions and future perspectives

The thesis provides a comprehensive performance assessment of the HFS reflectometric system for the forthcoming RFX-mod2. The validity of the combined experimental and numerical approach is validated and can serve as a benchmark for future investigations.

The HFS PPR system's performance has been validated and ensures optimal working behaviour under the most promising conditions, where the probing beam is normally incident on the reflecting surface. At larger surface inclinations, the tilted nature of the bistatic HFS set-up still grants good resilience for lower inclination angles ($[-15^\circ, 15^\circ]$), thus demonstrating its flexibility in accepting a low degree of distortion.

Once RFX-mod2 is operational, it is necessary to investigate PPR performance under real Tokamak scenarios featuring non-ideal magnetic structures. This allows for the gathering of interesting results that can address some of the issues related to the development of a reflectometry-based plasma control, as well as evaluate the robustness of such a technique in a flexible environment.

The integration of other diagnostics for calibration and LCFS identification (Thomson scattering, interferometry, etc.) is another open challenge to be tackled in real plasma discharges. Finally, the development of a real-time architecture will represent the last step in fully exploiting the PPR system's potential.

The Tokamak applications of RFX-mod2 can thus serve as a test framework for demonstrating, on a smaller scale, the applicability of reflectometry for horizontal and vertical position control purposes in view of future applications on larger machines (DTT, DEMO).

For RFP applications, ultrafast sweeping reflectometry is crucial to investigate the mechanism that regulates the physics of the edge. The HFS system can, in principle, be applied in O-mode configuration for DPR measurements, but it re-

quires additional edge diagnostic data (for example, edge density measurements from the Thermal Helium Beam diagnostics) to provide a proper initialisation procedure and improve the quality of the profile reconstruction.

In summary, the RFX-mod2 reflectometric system, together with the flexible nature of the machine, represents a huge opportunity to perform studies on several edge physical phenomena and to tackle some of the most relevant open problems associated with PPR, in light of similar challenges that will be faced in future experiments.

Acknowledgements

As a final and true conclusion of my thesis, I would like to thank all the people who supported me throughout the entire internship period.

First of all, I am sincerely grateful to the Consorzio RFX for allowing me to carry out my work and have direct contact with fusion research. It has been an outstanding opportunity not only for learning purposes but also for building personal experience that has led me to overcome my limits and clarify my future ambitions, strengthening and motivating my interest in fusion research.

I would like to thank my academic supervisor, Prof. Matteo Zuin, who demonstrated a serious interest in my work and gave me several pieces of advice to help me orient myself in the research framework.

Many thanks to my co-supervisor, Dr. Gianluca De Masi, for all the time he has dedicated to me throughout these months and for always being available for any suggestions, feedback, and practical help.

A huge acknowledgement is dedicated to Dr. Federico Ruffini for all his help, especially in the numerical part, his cooperation with most of the results of the thesis, and his suggestions concerning the research and my future opportunities.

Finally, the last big acknowledgement goes to Dr. Filipe da Silva for his help and cooperation with REFMUL and for his significant feedback, especially in the last part of the internship.

A general thanks is dedicated to all researchers at Consorzio RFX, in particular: Dr. Roberto Cavazzana for his help with some critical tasks, Dr. Marco Veranda, and Prof. Fulvio Auriemma for their cooperation on the plasma density profiles.

I would like to thank Prof. Maurizio Giacomini and Prof. Lidia Piron for their plasma physics courses, which gave me the opportunity to get a first flavour of the subject and really motivated me to pursue a thesis at Consorzio RFX.

A special thank you is also dedicated to all the PhD students I met for their suggestions and support during the thesis work, and to all the students who were completing their theses like I was for sharing plans, thoughts, and opinions.

Finally, I would like to thank Prof. Giovanna Montagnoli and Prof. Alberto Stefanini for allowing me to carry out my first experimental campaign at Legnaro National Laboratories during the Advanced Laboratory course, which I consider one of the major turning points of my academic career for having strengthened my motivation towards experimental physics.

Moving now to the more personal side, I would like to thank all the people I love who have kept me up during these months.

I would like to thank my university friends, in particular, Michelangelo, Dalle, Lorenzo, Nicolò, Arman, Jacopo, Tiziano, Tommaso, and Maria, for all the thoughts and feelings we have shared in these last years and for the motivation they have given me during the most difficult choices.

Then, I want to thank all my friends from my town: Ste, Ale, Giacomo, Marco, Stefano, my room-mates Luca and Gaia, Davide, Eugenio, Paolo, and Pamela for all the stories and moments we have lived together since we were children and for the massive support they have always given me.

I would like to thank all my family, in particular my grandma, uncles, and cousins, for all the affection and consideration they have always shown me.

A massive thank you to Mum and Dad for everything they have given me in terms of love and support for my wishes and ambitions.

Finally, I would like to thank my love, Sophie, for what she gives me every time. I am lucky to have her by my side, especially in this crucial point in my life.

Bibliography

- [1] E. Mazzucato. Microwave reflectometry for magnetically confined plasmas. *Review of Scientific Instruments*, 69(6):2201–2217, 1998. <https://doi.org/10.1063/1.1149121>.
- [2] G. De Masi, R. Cavazzana, D. Abate, M. Bernardi, G. Marchiori, M. Moresco, A. Tiso, and S. Peruzzo. Design of a new reflectometric system for real time plasma position control on the RFX-mod2 device. *Journal of Instrumentation*, 17(01):C01071, 2022. <https://doi.org/10.1088/1748-0221/17/01/C01071>.
- [3] IEA. World Energy Outlook 2025. <https://www.iea.org/reports/world-energy-outlook-2025>. Licence: CC BY 4.0 (report); CC BY NC SA 4.0 (Annex A).
- [4] IEA. Electricity 2026. <https://www.iea.org/reports/electricity-2026>, 2026. Licence: CC BY 4.0.
- [5] J. P. Freidberg. *Plasma Physics and Fusion Energy*. Cambridge University Press, 2008.
- [6] H. Ritchie. What are the safest and cleanest sources of energy? *Our World in Data*, 2020. <https://archive.ourworldindata.org/20260305-101825/safest-sources-of-energy.html>.
- [7] K. S. Krane. *Introductory Nuclear Physics*. John Wiley & Sons, 1988.
- [8] M. Mohamed, N. D. Zakuan, T. N. A. Tengku Hassan, S. S. M. Lock, and A. Mohd Shariff. Global Development and Readiness of Nuclear Fusion Technology as the Alternative Source for Clean Energy Supply. *Sustainability*, 16(10,4089), 2024. <https://www.doi.org/10.3390/su16104089>.
- [9] M. Zucchetti, L. Candido, V. Khripunov, B. Kolbasov, and R. Testoni. Fusion power plants, fission and conventional power plants. Radioactivity, radiotoxicity, radioactive waste. *Fusion Engineering and Design*, 136:1529–1533, 2018. <https://doi.org/10.1016/j.fusengdes.2018.05.049>.

-
- [10] S. E. Wurzel and S. C. Hsu. Continuing progress toward fusion energy breakeven and gain as measured against the Lawson criteria. *Physics of Plasmas*, 32(11):112106, 2025. <https://doi.org/10.1063/5.0297357>.
- [11] IAEA. *IAEA World Fusion Outlook 2025*. Non-serial Publications. IAEA, Vienna, 2025. <http://www.iaea.org/publications/15935/iaea-world-fusion-outlook-2025>.
- [12] R. Lorenzini, E. Martines, P. Piovesan, D. Terranova, P. Zanca, M. Zuin, A. Alfier, D. Bonfiglio, F. Bonomo, A. Canton, S. Cappello, L. Carraro, R. Cavazzana, D. Escande, A. Fassina, P. Franz, M. Gobbin, P. Innocente, L. Marrelli, and P. Martin. Self-organized helical equilibria as a new paradigm for ohmically heated fusion plasmas. *Nature Physics*, 5:570–574, 2009. <https://doi.org/10.1038/nphys1308>.
- [13] M. E. Puiatti et al. Helical equilibria and magnetic structures in the reversed field pinch and analogies to the tokamak and stellarator. *Plasma Physics and Controlled Fusion*, 51(12):124031, 2009. <https://doi.org/10.1088/0741-3335/51/12/124031>.
- [14] B. Momo, E. Martines, and P. Martin. Investigation of the magnetic topology of helical RFP plasmas. *Padua Research Archive - PhD Thesis*, 2012. <https://hdl.handle.net/11577/3422500>.
- [15] S. Cappello, D. Bonfiglio, and D. F. Escande. Magnetohydrodynamic dynamo in reversed field pinch plasmas: Electrostatic drift nature of the dynamo velocity field. *Physics of Plasmas*, 13(5):056102, 2006. <https://doi.org/10.1063/1.2177198>.
- [16] S. Peruzzo et al. The new vessel complex for the RFX-mod2 experiment: An effective synergy between fusion research and technological development. *Fusion Engineering and Design*, 194:113890, 2023. <https://doi.org/10.1016/j.fusengdes.2023.113890>.
- [17] A. R. Choudhuri. *The Physics of Fluids and Plasmas*. Cambridge University Press, 2012. <https://doi.org/10.1017/CBO9781139171069>.
- [18] I. H. Hutchinson. *Principles of Plasma Diagnostics*. Cambridge University Press, second edition, 2002.
- [19] M. Taylor. The Appleton-Hartree formula and dispersion curves for the propagation of electromagnetic waves through an ionized medium in the presence of an external magnetic field part 1: curves for zero absorption. *Proceedings of the Physical Society*, 45(2):245, 1933. <https://doi.org/10.1088/0959-5309/45/2/310>.

- [20] C. Laviron, A. J. H. Donné, M. E. Manso, and J. Sanchez. Reflectometry techniques for density profile measurements on fusion plasmas. *Plasma Physics and Controlled Fusion*, 38(7):905, 1996. <https://doi.org/10.1088/0741-3335/38/7/002>.
- [21] G. Marchiori, G. De Masi, R. Cavazzana, A. Cenedese, N. Marconato, R. Moutinho, and A. Silva. Study of a Plasma Boundary Reconstruction Method Based on Reflectometric Measurements for Control Purposes. *IEEE Transactions on Plasma Science*, 46(5):1285–1290, 2018. <https://doi.org/10.1109/TPS.2018.2797549>.
- [22] G. De Masi, R. Cavazzana, F. Ruffini, G. Marchiori, M. Moresco, R. Agnello, L. Cordaro, M. Bernardi, E. Giroto, A. Tiso, and S. Peruzzo. Technology challenges and integration of the plasma position reflectometer in RFX-mod2. *Fusion Engineering and Design*, 201:114257, 2024. <https://doi.org/10.1016/j.fusengdes.2024.114257>.
- [23] J. Santos, L. Guimarães, M. Zilker, W. Treutterer, M. Manso, and the ASDEX Upgrade Team. Reflectometry-based plasma position feedback control demonstration at ASDEX Upgrade. *Nuclear Fusion*, 52(3):032003, 2012. <https://doi.org/10.1088/0029-5515/52/3/032003>.
- [24] F. Ruffini, G. De Masi, R. Cavazzana, G. Marchiori, J. Santos, F. da Silva, A. Silva, S. Heuraux, T. Ribeiro, D. Mascali, G. Torrissi, G. Emma, and A. Belpane. Performance assessment and integration of the High Field Side Plasma Position Reflectometry system on DTT. *Journal of Instrumentation*, 20(05):C05030, 2025. <https://doi.org/10.1088/1748-0221/20/05/C05030>.
- [25] F. Ruffini, G. Marchiori, G. De Masi, R. Cavazzana, G. Tartaglione, M. Ariola, A. Pironti, J. Santos, F. da Silva, and A. Silva. G3C: a non magnetic, reflectometry-based plasma boundary reconstruction algorithm for control purposes. *Plasma Physics and Controlled Fusion*, 68(2):025002, 2026. <https://doi.org/10.1088/1361-6587/ae3ad7>.
- [26] E. Ricardo, F. da Silva, S. Heuraux, A. Silva, and J. Santos. Optimizing plasma position reflectometry systems: the DEMO case and future perspectives. *Plasma Physics and Controlled Fusion*, 68(1):015014, 2026. <https://doi.org/10.1088/1361-6587/ae2e3e>.
- [27] E. Ricardo, F. da Silva, S. Heuraux, A. Silva, and J. Santos. Simulation and data processing techniques to design optimized PPR systems on plasma fusion devices. *Computer Physics Communications*, 294:108945, 2024. <https://www.sciencedirect.com/science/article/pii/S0010465523002904>.

-
- [28] C. A. Balanis. *Antenna Theory - Analysis and design*. John Wiley & Sons, fourth edition, 2016.
- [29] S. Ramo, J. R. Whinnery, and T. Van Duzer. *Fields and waves in communication electronics*. John Wiley & Sons, third edition, 1994.
- [30] G. Marchiori, R. Cavazzana, G. De Masi, and M. Moresco. Design of the high field side antenna of the new reflectometric system for plasma position estimate in RFX-mod2. *Journal of Instrumentation*, 17(01):C01002, 2022. <https://doi.org/10.1088/1748-0221/17/01/C01002>.
- [31] G. De Masi, R. Cavazzana, A. Fassina, E. Martines, B. Momo, and M. Moresco. Edge density characterization in the RFX-mod experiment using the ultrafast reflectometry technique. *Nuclear Fusion*, 51(5):053016, 2011. <https://doi.org/10.1088/0029-5515/51/5/053016>.
- [32] F. Ruffini, M. Zuin, R. Cavazzana, and G. De Masi. Performance assessment of the new plasma position reflectometer antennas for the RFX-mod2 device. *Padua Thesis and Dissertation Archive*, 2023. <https://thesis.unipd.it/browse?type=author&authority=st939043>.
- [33] B. Gonçalves, P. Varela, A. Silva, F.e Silva, J. Santos, E. Ricardo, A. Vale, R. Luís, Y. Nietiadi, A. Malaquias, J. Belo, J. Dias, J.and Ferreira, T. Franke, W. Biel, S. Heuraux, T. Ribeiro, G. De Masi, O. Tudisco, R. Cavazzana, G. Marchiori, and O. D’Arcangelo. Advances, Challenges, and Future Perspectives of Microwave Reflectometry for Plasma Position and Shape Control on Future Nuclear Fusion Devices. *Sensors*, 23(8:3926), 2023. <https://doi.org/10.3390/s23083926>.
- [34] Steve Arar. Introduction to the Directional Coupler for RF applications. *Journal of Instrumentation*, Technical articles, 2024.
- [35] D. Abate, G. Marchiori, P. Bettini, and F. Villone. Modelling of RFX-mod2 tokamak equilibria with DEMO-like shape conditions and negative triangularity. *Plasma Physics and Controlled Fusion*, 62(8):085001, 2020. <https://doi.org/10.1088/1361-6587/ab93a5>.
- [36] I. Predebon, D. Abate, and L. Pigatto. Positive and negative triangularity in RFX-mod2: a comparative analysis. *Nuclear Fusion*, 62(6):066039, 2022. <https://doi.org/10.1088/1741-4326/ac54ce>.
- [37] F. da Silva, S. Heuraux, E. Ricardo, A. Silva, and T. Ribeiro. Modelling reflectometry diagnostics: finite-difference time-domain simulation of reflectometry in fusion plasmas. *Journal of Instrumentation*, 14(08):C08003, 2019. <https://doi.org/10.1088/1748-0221/14/08/C08003>.

- [38] K. Yee. Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media. *IEEE Transactions on Antennas and Propagation*, 14(3):302–307, 1966. <https://doi.org/10.1109/TAP.1966.1138693>.
- [39] F. da Silva, S. Heuraux, T. Ribeiro, E. Ricardo, J. Santos, A. Silva, J. Ferreira, J. Vicente, G. De Masi, O. Tudisco, R. Cavazzana, G. Marchiori, R. Luís, and Y. Nietiadi. An overview of the evolution of the modeling of reflectometry diagnostics in fusion plasmas using finite-difference time-domain codes. *Fusion Engineering and Design*, 202:114354, 2024. <https://doi.org/10.1016/j.fusengdes.2024.114354>.
- [40] F. da Silva, S. Heuraux, S. Hacquin, and M.E. Manso. Unidirectional transparent signal injection in finite-difference time-domain electromagnetic codes –application to reflectometry simulations. *Journal of Computational Physics*, 203(2):467–492, 2005. <https://doi.org/10.1016/j.jcp.2004.09.002>.
- [41] E. Martines, R. Lorenzini, B. Momo, D. Terranova, P. Zanca, A. Alfier, F. Bonomo, A. Canton, A. Fassina, P. Franz, and P. Innocente. Equilibrium reconstruction for single helical axis reversed field pinch plasmas. *Plasma Physics and Controlled Fusion*, 53(3):035015, 2011. <https://doi.org/10.1088/0741-3335/53/3/035015>.