

UNIVERSITÀ DEGLI STUDI DI PADOVA

DIPARTIMENTO DI INGEGNERIA DELL'INFORMAZIONE

MASTER DEGREE IN ICT FOR INTERNET AND MULTIMEDIA

Title

Game theoretic analysis of Age of Information
in multi-source scenarios

SUPERVISOR:
LEONARDO BADIA

CANDIDATE:
EMILIJA DOKANOVIC
2055577

ACADEMIC YEAR 2023-2024

Abstract

Scheduling updates from sensors is an important task for various network systems, especially for Internet of Things (IoT) scenarios where resources are scarce. The freshness of received data is often described using the mathematical concept of Age of Information (AoI). Therefore, this research represents nodes of networks as players in a game-theoretic framework to find an optimal schedule and achieve equilibrium, which is the optimized value of Age of Information (AoI). The players have a common goal to decrease the average AoI by transmitting over the network, but they cannot communicate with each other, and they are not sure if one of them has updated the information. We investigate what happens in the system when N players transmit with or without coordination. Further, we apply Harsanyi's equilibrium selection principle to identify strategies that collectively minimize AoI in the network. We evaluate the Price of Anarchy, which quantifies the inefficiency of selfish management of the sources. We also propose practical implementations to improve the distributed management of status updates by multiple IoT nodes.

Acknowledgements

I extend my deepest gratitude to Professor Leonardo Badia, whose unwavering support and enthusiasm for game theory have profoundly shaped my journey. His generosity in sharing knowledge, along with his invaluable guidance, patience, and expert advice, have been pivotal throughout my research. Professor Badia's insights and feedback have not only enriched this work but have also kept me focused and inspired.

I am profoundly grateful for the opportunity to study at the prestigious University of Padua, to be part of its rich history, and to contribute to the scientific community through this endeavor. This experience has been nothing short of transformative.

Special thanks are owed to my family—my father and sisters, whose unconditional support and belief in my abilities encouraged me to strive for excellence and to continually grow. Their unwavering faith in my success has been a source of strength and motivation.

I would also like to express my heartfelt appreciation to my boyfriend, whose support and encouragement have been invaluable during my master's degree journey. My friends deserve a special mention for their understanding and patience, which have been a great comfort to me during the preparation of this work.

Lastly, I am thankful to everyone who has played a role in my nearly two-decade-long educational journey. Your contributions have shaped me into the person I am today, and I am hopeful and excited for the learning experiences that lie ahead.

Contents

Abstract	III
Acknowledgements	V
1 Introduction	3
1.1 Applications of Internet of Things	5
1.2 Challenges of IoT with growth of Artificial Intelligence	6
1.3 Motivation	7
2 Background	9
2.1 Age of Information	9
2.1.1 AoI in queue theory	10
2.1.2 Threshold-based scheduling policies and AoI	11
2.1.3 AoI in zero-wait policy	13
2.1.4 AoI in networks with limited transmission opportunities	14
2.2 Game Theory for Networks	14
2.2.1 Different forms for modeling Network Scenarios	15
2.2.2 Network Security and Game Theory	16
2.2.3 Resouce Management and Game Theory	17
2.2.4 AoI in game-theoretic framework	18
2.3 Harsanyi’s theory for equilibrium selection for games with complete information	19
2.3.1 Definition of the solution of the game in terms of theoretical probabilities	22
2.4 Related work and contribution	23
3 Implementation	25
3.1 Model	25
3.1.1 Equilibrium in pure strategies	28
3.1.2 Equilibrium in mixed strategies	29

3.2	Price of Anarchy	30
3.3	Finding Nash equilibrium in pure and mixed strategies	32
3.4	Equilibrium selection in practice	33
3.4.1	Possible scenarios in the game	34
3.4.2	Adjustment following the rest of horizon	37
3.4.3	Adjustment following the initial interval	37
3.4.4	Adjustment to prevent no update case with delay	38
3.4.5	Adjustment to prevent no update case without delay	38
3.4.6	Adjustment to prevent no update case with non-evenly spread intervals	41
3.4.7	Adjustment designed to facilitate an ideal scenario within unevenly spread intervals with a cost parameter β	41
4	Results	43
4.1	Results in pure strategies	43
4.2	Results in mixed strategies	44
4.3	PoA results	46
4.4	Results for implementing horizon shifting when no update occurs	48
4.4.1	Mechanism following the rest of the horizon	48
4.4.2	Mechanism following the initial interval	49
4.4.3	Comparative analysis of the first group adjustment mechanisms	50
4.5	Results for mechanisms that prevent cases of non - transmission	51
4.5.1	Mechanism to prevent no update case with delay	51
4.5.2	Mechanism to prevent no update case without delay	52
4.5.3	Mechanism to prevent no update case with non-evenly spread intervals	54
4.5.4	Comparative analysis of the second group mechanisms	55
4.6	Results for a mechanism designed to facilitate an ideal scenario within un- evenly spread intervals, considering a cost parameter β	56
4.7	Comparative analysis of proposed adjustments	56
5	Conclusions and Future Work	61
	Bibliography	62

List of Figures

1.1	Number of Internet of Things (IoT) connected devices worldwide from 2019 to 2023, with forecasts from 2024 to 2030 (in billions) [Graph], Transforma Insights, Exploding Topics, July 1, 2023. [Online]. Available: https://www.statista.com/statistics/1111111/connected-devices-worldwide/	4
3.1	System model for N players over finite horizons with τ_N possible chance to update	26
3.2	Optimal AoI in pure strategies when Nash equilibrium is reached	28
3.3	Possible scenarios of the number of updates in the slots during the game	34
3.4	Illustration for the AoI evolution over time. In the specific scenario, no updates were initiated by the players during the third temporal slot; however, a transmission occurred in the subsequent fourth slot. Failure to capitalize on the opportunity resulted in an escalation of the AoI, symbolized by the delineation of a dashed surface	35
3.5	Illustration for the AoI evolution over time. Within the context under examination, multiple updates occurred during the third temporal slot, resulting in the forfeiture of an opportunity for transmission until the end of the observational horizon. Consequently, the AoI experiences an increase as depicted by the dashed surface.	36
4.1	Average value of AoI in pure strategies	44
4.2	Value of average AoI in pure, mixed, and uniform strategies	45
4.3	Value of average AoI in pure, mixed, and uniform strategies	46
4.4	PoA for different number of players	47
4.5	AoI after applying mechanism adjustment according rest of horizon shift	49
4.6	Difference between optimal and actual Nash equilibrium	49
4.7	AoI after applying mechanism adjustment according initial interval shift	50

4.8	Difference between optimal and actual Nash equilibrium	50
4.9	AoI after applying mechanism for shifting slots when no update occurs	51
4.10	AoI after applying mechanism to force transmission with delay in the case when no update occurs	52
4.11	Difference between optimal and actual Nash equilibrium	52
4.12	AoI after applying mechanism avoid no transmission state	53
4.13	Difference between optimal and actual Nash equilibrium	53
4.14	AoI after applying mechanism avoid no transmission state	54
4.15	AoI after applying mechanism to prevent no update state	55
4.16	AoI after applying mechanism to apply preplay communication with different β factor	57
4.17	Efficiency ratio choosing different adaptive machanisms	58
4.18	Simplified presentation of efficiency ratios for different strategies	58
4.19	Efficiency ratio for the first group of adaptive mechanisms	59
4.20	Efficiency ratio for the second group of adaptive mechanisms	59
4.21	Efficiency ratio for the third group of adaptive mechanisms	60

Chapter 1

Introduction

The growth of the Internet from its humble origins as a research network to a ubiquitous global infrastructure serving billions of users over the past five decades has been remarkable. Now, with the continued miniaturization and cost reduction of electronic components, the Internet is expanding into a new realm: the Internet of Things (IoT). In this paradigm, everyday physical objects are enhanced by small electronic devices, allowing them to connect to the digital world. These smart objects serve as cyber-physical systems, bridging the gap between physical entities and the information world by processing sensor data and establishing wireless connections to the Internet.

As the Internet continues to rapidly expand, the global shift towards the Internet of Things (IoT) is poised to significantly alter our daily lives soon [1, 2, 3]. Beyond traditional devices like computers, laptops, smartphones, and tablets, a diverse array of smart devices is now connecting to the Internet and interconnecting with each other. From household appliances such as refrigerators and microwave ovens to large-scale industrial machinery, virtually everything is becoming "smart." This transformation has been dubbed by some as the "Next Digital Revolution" or the "Next Generation of Internet," signifying its profound impact on technology and society [2].

The graph presented in Figure 1.1 illustrates the global count of connected devices, representing the IoT, spanning from 2019 to 2023. Beyond this period, projections indicate an anticipated trend for IoT devices from 2024 to 2030. Forecasts suggest a doubling of connected devices by the end of 2030 compared to the figures observed in 2023, which stood at approximately 15 billion devices.

The development and adoption of the IoT are propelled by several key drivers. Firstly, advancements in technology, particularly in the miniaturization and cost reduction of electronic

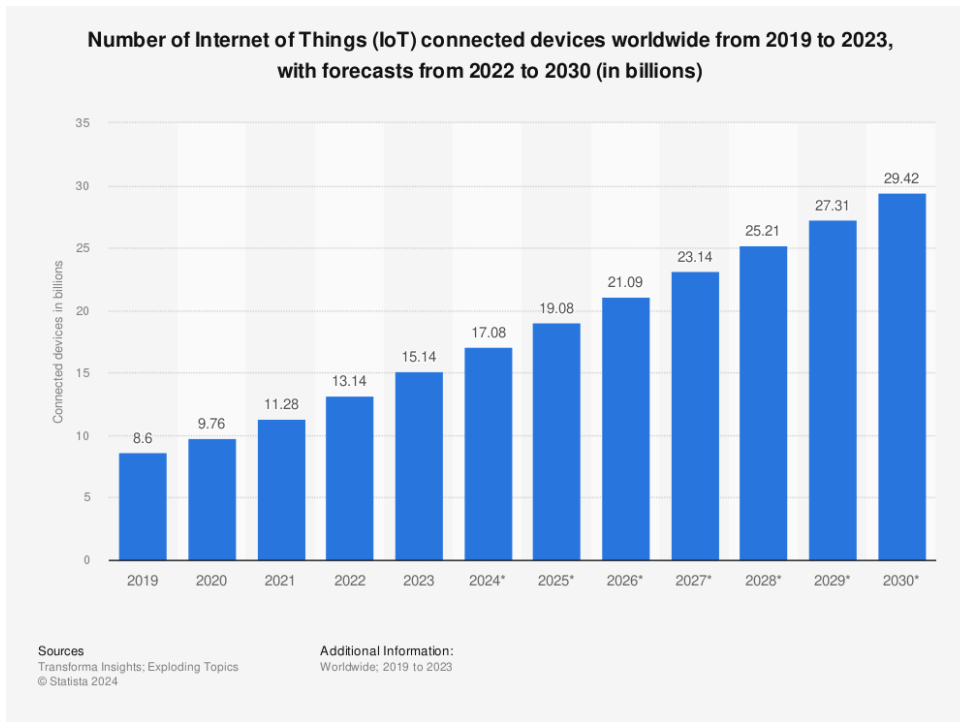


Figure 1.1: Number of Internet of Things (IoT) connected devices worldwide from 2019 to 2023, with forecasts from 2024 to 2030 (in billions) [Graph], Transforma Insights, Exploding Topics, July 1, 2023. [Online]. Available: <https://www.statista.com/statistics/1183457/iot-connected-devices-worldwide/>

devices [2], have enabled the embedding of computational capabilities into everyday objects. This has ushered in a new era where objects can connect to the Internet and interact with each other seamlessly. Moreover, the emergence of low-power wireless communication technologies has facilitated wireless connectivity between smart objects and the Internet. Physical objects are being outfitted with RFID tags or other electronic barcodes. These tags can be scanned by smart devices such as smartphones or small embedded RFID scanners. Each object has a unique identity, and specific information related to that object is stored within the RFID tags [1, 3].

Further, the widespread availability of GPS signals plays a crucial role in enabling smart objects to determine their location and time [2], thereby enhancing context awareness and enabling location-based services. Additionally, IoT devices can leverage domain-specific knowledge bases and reasoning capabilities to autonomously navigate and operate within specific application domains. This empowers smart objects to make informed decisions and perform tasks efficiently. Furthermore, the autonomic management and self-organization capabilities of IoT networks empower smart objects to dynamically adapt to environmental changes [3, 2]. This enables them to optimize their behavior without requiring human intervention[4].

These factors, combined with elements like data mash-up and information fusion, as well as the anticipated impact on the global economy and society, are driving the continuous ex-

pansion of IoT. This trajectory is shaping a future where interconnected smart objects will revolutionize various facets of our lives [1, 3, 2].

1.1 Applications of Internet of Things

In our modern world, smart devices have become ubiquitous, permeating almost every aspect of our lives. It is challenging, if not impossible, to identify an area without the application of various Internet-connected devices. Among the myriad applications of IoT, a few key areas stand out for their widespread application and impact:

- **Smart Homes:** IoT technologies have transformed home automation, leading to the emergence of smart homes equipped with a variety of connected devices including thermostats, lighting systems, security cameras, and appliances [5, 3, 1]. These devices offer remote control capabilities via smartphone apps or voice commands, enhancing convenience, energy efficiency, and security for homeowners.
- **Medical IoT:** IoT has transformed healthcare delivery through the development of wearable health monitoring devices, remote patient monitoring systems, and smart medical devices [1, 6]. These technologies enable continuous monitoring of vital signs, early detection of health issues, and remote consultation with healthcare professionals, improving patient outcomes and reducing healthcare costs.
- **Transportation:** IoT plays a crucial role in optimizing transportation and logistics operations through vehicle telematics, asset tracking systems, and smart traffic management solutions. These technologies enhance fleet efficiency, route optimization, real-time monitoring of goods in transit, and traffic flow management [7], leading to reduced congestion, fuel consumption, and emissions.
- **Smart Cities:** IoT is instrumental in the development of smart cities by integrating various infrastructure components such as smart streetlights, waste management systems, public transportation networks, and environmental monitoring sensors [3, 1]. These interconnected systems enable efficient resource management, enhanced public safety, improved urban mobility, and sustainable urban development. Projections suggest a significant rise in smart home device adoption, with an estimated 1.4 billion devices expected by 2026[3].

- **Industrial IoT (IIoT):** In the industrial sector, IoT technologies are driving the adoption of Industry 4.0 initiatives, leading to the creation of smart factories and intelligent manufacturing processes [1, 2, 8]. IIoT solutions enable predictive maintenance, asset tracking, remote monitoring of equipment, and real-time data analytics, resulting in increased productivity, reduced downtime, and optimized resource utilization.

Various examples of IoT device usage demonstrate common positive effects from the user's perspective. Utilizing connected devices that enable real-time communication, users experience benefits such as increased energy efficiency through smart home technologies, enhanced safety measures, heightened security measures, and improved product quality [3].

1.2 Challenges of IoT with growth of Artificial Intelligence

As the number of connected devices continues to expand and Artificial Intelligence advances, the Internet of Things (IoT) encounters various challenges, both present and anticipated. The proliferation of billions or even trillions of connected smart objects presents novel technical and societal hurdles. These challenges encompass authentic identification, autonomous network management, diagnostics and maintenance, context awareness, and privacy intrusion, as highlighted in [2]. Moreover, key challenges outlined in [1] shed light on additional obstacles faced by IoT:

- **Naming and Identity Management:** With billions of objects connecting to the IoT, an efficient system for assigning and managing unique identities is essential to ensure seamless communication and service delivery [2].
- **Information Privacy:** As IoT devices collect and share vast amounts of data, protecting privacy becomes paramount. Proper measures must be implemented to safeguard sensitive information and prevent unauthorized access[6, 2].
- **Objects Safety and Security:** With the proliferation of IoT devices, ensuring the physical safety and security of these objects against intrusions and tampering is imperative to prevent potential damage or disruptions [3, 6].
- **Data Confidentiality and Encryption:** Strong encryption mechanisms are essential to protect data integrity and confidentiality as it is transmitted and processed within the IoT ecosystem, safeguarding sensitive information from unauthorized access.

- **Network Security:** Securing the transmission network against data loss [3], external interference, and unauthorized monitoring is crucial for maintaining the integrity and reliability of IoT communications.
- **Spectrum Management:** Efficient allocation of spectrum resources is necessary to accommodate the communication needs of billions of IoT devices, requiring dynamic cognitive spectrum allocation mechanisms to mitigate congestion and ensure reliable connectivity [4].
- **Interoperability and Standardization:** Standardizing IoT technologies is crucial for ensuring compatibility and interoperability among devices from different manufacturers, thus facilitating seamless integration and communication.
- **Greening of IoT:** Given the escalating energy consumption associated with IoT infrastructure, adopting energy-efficient technologies [4, 9] and practices is essential to mitigate the environmental impact and ensure sustainability in the long term.

1.3 Motivation

The proliferation of IoT promises numerous benefits, including enhanced efficiency and convenience across various domains [2, 1, 3, 5, 6, 7]. However, it also brings about concerns regarding power dynamics [4] and information control, particularly concerning data access [2]. Many IoT sensors operate with constrained battery capabilities and are often situated in hard-to-reach locations [9, 10]. Optimizing transmissions to obtain fresh information while conserving energy is a significant challenge, particularly in critical applications like medical IoT [6], where up-to-date information is vital.

Efforts to tackle these challenges have spurred the development of diverse techniques aimed at minimizing Age of Information (AoI) and optimizing resource utilization in constrained device networks [10, 11]. A notable research direction involves minimizing AoI over finite horizons, recognizing the paramount importance of conserving energy resources while ensuring information freshness [9, 12].

This research endeavors to apply concepts from game theory to analyze AoI in multi-source communication scenarios which corresponds to many IoT networks [13, 12]. Its objectives include understanding AoI dynamics in such environments, identifying strategies to minimize AoI, and assessing how the strategic behavior of information sources influences information freshness in the network [14, 13, 15].

The implications of this research are significant for devising effective information management strategies in complex IoT networks. Leveraging game theory offers a deeper understanding of AoI dynamics and aids in identifying optimal strategies for minimizing AoI in multi-source scenarios. By contributing valuable insights, this research informs future development efforts aimed at enhancing information freshness in communication networks.

The structure of the work is outlined as follows: In Chapter 2, we delve into the concept of the Age of Information, explore various applications of game theory in computer networks, and discuss Harsanyi's theory of equilibrium selection. Additionally, we provide an overview of relevant literature, highlighting our unique contribution to the field. Chapter 3 constitutes the central part of our work, where we develop the model. This chapter encompasses the formulation and presentation of our model, detailing its components and underlying principles. Following the model development, Chapter 4 is dedicated to the discussion of results. Here, we analyze and interpret the findings obtained from our model, providing insights into its implications and potential applications. Finally, Chapter 5 serves as a summary of the most significant results obtained throughout the study, encapsulating key findings and their significance. Additionally, we outline avenues for future research, identifying areas for further studies and development in the field.

Chapter 2

Background

In this chapter, we will delve into the concept of the Age of Information, a metric that has recently captured the interest of many researchers due to its relevance in modern networked systems. We will provide an overview of this metric, discussing its significance and implications in various contexts.

Following our discussion on the age of information, we will shift our focus to the application of game theory in networks. We will explore some of the most typical examples where game theory has been utilized to model strategic interactions among network entities, shedding light on the dynamics of decision-making and resource allocation in such environments.

Next, we will provide an account of Harsanyi's theory of equilibrium selection, a foundational framework in game theory that offers insights into how rational agents make decisions in non-cooperative settings. We will discuss the principles underlying this theory and its relevance to our understanding of strategic behavior in networked systems.

Finally, we will refer to the relevant literature that has informed our discussion throughout this chapter. We will highlight key papers and studies that have contributed to the advancement of research in the areas of information age metrics and game theory in networks. Additionally, we will discuss our own contribution to this body of work, emphasizing the insights and findings that we have brought to the field.

2.1 Age of Information

Efforts to minimize the AoI in communication networks have garnered significant attention in recent years [14, 8, 16, 17, 18], particularly in the context of the generation of time-sensitive data [6]. AoI is a metric used to quantify the freshness or timeliness of information in a communication system. It measures the elapsed time from when a piece of information is generated

or updated at its source until it is received or observed by the intended recipient. In other words, AoI indicates how outdated the latest received information is compared to the most recent available data. AoI is a more suitable metric for remote sensing applications [9, 13] rather than traditional metrics such as throughput, delay, or latency. Researchers have explored various techniques and methodologies to mitigate AoI and enhance the freshness of information delivery, addressing the unique challenges posed by constrained devices [12].

2.1.1 AoI in queue theory

In addition to investigating AoI dynamics, researchers have turned to queue theory to dissect the complexities of constrained device networks [19, 20, 21]. In the study by Kaul et al. [19], queueing models were leveraged to explore optimal update policies tailored for energy-constrained devices, aiming to strike a delicate balance between information update frequency and energy consumption. The research encompassed diverse assumptions regarding arrival and service processes, as well as the queue discipline of first-come-first-served (FCFS), ultimately emphasizing the importance of mitigating packet waiting times while maximizing server utilization.

Similarly, Yates et al. [16] extended this investigation to FCFS M/M/1 queues, expanding the analysis to accommodate multiple independent sources. Additionally, in their study [22], Kaul et al. further extended their exploration to address the same problem within Last Come First Served (LCFS) queues. This extension contributed to a deeper understanding of optimal update strategies in resource-constrained communication environments, highlighting the nuances of queueing dynamics in optimizing information update processes.

In [20], the authors delve into the tradeoff between the frequency of status updates and queueing delay within a system featuring heterogeneous users, modeled as a multi-class M/G/1 queue. Specifically, the study considers systems where only one packet can be kept at a time, reflecting a single packet management system denoted as M/G/1/1. The research scenario involves entities generating status messages with varying lengths, characterized by diverse service time distributions. Notably, the authors depart from the conventional focus on AoI and instead emphasize optimizing the Peak of Age of Information (PoAoI). This shift is motivated by recognizing that different entities may have distinct service requirements for their status updates. By centering the analysis on PoAoI, which captures the maximum AoI experienced by any entity at any given time, the study aims to better capture the performance dynamics of systems with heterogeneous user needs. This approach sheds light on the nuanced interplay

between update frequency, queueing delay, and the specific service requirements of individual entities, contributing to a deeper understanding of system optimization in such environments.

In [21], the focus lies on packet management at the source node, particularly in scenarios where the source node receives random updates but retains the ability to control which samples are transmitted through the network, potentially discarding certain samples before transmission. The study employs models resembling $M/M/1/1$ and $M/M/1/2$ queue scenarios, where only one packet can be in service at a time, with the latter allowing for the possibility of a single sample to be kept in a queue. In these models, samples arrive according to a Poisson process, and the time taken for packet transmission follows an exponential distribution.

Similar to the findings in [23], the authors observed that the average age can be improved, particularly with high sampling rates, by discarding packets that encounter a busy source node rather than storing them in a queue for later transmission. In their work, [18] investigated the impact of various packet management policies on the average values of both AoI and PAoI within an $M/M/1$ queuing model. This insight highlights the importance of strategic packet management strategies in optimizing AoI dynamics, particularly in scenarios where resources are limited and packet processing capabilities are constrained.

These endeavors underscore the pivotal role of queue theory in elucidating the intricacies of device networks, offering insights into effective update policies and resource management strategies.

2.1.2 Threshold-based scheduling policies and AoI

In one group of works, AoI is integrated as a threshold constraint within scheduling frameworks, giving rise to diverse formalizations of optimization problems. These problems are approached from various perspectives, including linear programming techniques, alongside the development of practical policies such as greedy or consecutive scheduling strategies.

In the realm of real-world IoT systems, where the dynamics and characteristics of each physical process can vary significantly, researchers have delved into optimizing the AoI by incorporating various constraints and policies. One notable avenue is the introduction of threshold constraints, such as packet deadlines, to regulate scheduling and processing times. For instance, [23] examined the impact of different packet deadlines on the average AoI in queuing systems, distinguishing between fixed and random exponential deadlines. Their analysis was conducted within an $M/M/1/2$ queue framework similar to what is done in [21]. If a packet remains in the queue beyond the deadline, it is discarded from the system adding the con-

straint on time sensitivity where packets must be processed within a certain time window. The authors revealed that an optimal deadline exists, with its value inversely proportional to the arrival rate. Overall, their work demonstrates that incorporating packet deadlines can offer a new dimension for optimizing age performance, ensuring the freshness of information in real-time applications. This work underscores the importance of time sensitivity in processing packets within prescribed timeframes to maintain their relevance.

The authors of [24] focused on understanding how scheduling policies affect AoI dynamics within energy harvesting systems. Their investigation centered on devising optimal on-line status update policies for an energy harvesting source across various battery sizes in continuous-time settings. Building upon the assumptions of instantaneous package generation and transmission if sufficient energy is available at the source, akin to previous works [25, 17].

By leveraging insights from prior research, particularly the work by [26], [24] crafted distinct (asymptotically) optimal sensing policies based on battery size considerations. For scenarios involving infinite battery capacity, they advocated for a best-effort uniform status update policy, demonstrating its efficacy in minimizing long-term average AoI. Conversely, in cases of finite battery capacity, they proposed an energy-aware adaptive status update policy, showcasing its asymptotic optimality as battery size tends towards infinity.

In scenarios where battery size is constrained to a single unit, [24], introduced a novel threshold-based update policy. This policy mandates that upon energy influx into an empty battery if the AoI falls below a predefined threshold, the sensor defers update transmission until the threshold is met; otherwise, it proceeds with an immediate update. Other authors also have done studies related to the minimization of AoI in the energy harvesting systems, for example, [27] highlights the significance of adopting a cautious approach in energy-harvesting wireless networks, advocating for updates to be dispatched only when the server is available to mitigate queuing delays. Contrary to conventional wisdom, a proactive strategy of updating immediately upon system idleness proves less efficient. Instead, a 'lazy' update policy, introducing inter-update delays, demonstrates superior performance. Despite these insights, the quest for identifying the optimal update policy persists, even within these circumstances. Furthermore, [17] delves into an exploration of threshold-based update policies in energy harvesting networks, establishing their optimality under specific contextual conditions.

Study [28] employed a threshold-ALOHA approach, wherein terminals defer transmission until the AoI of their status updates surpasses a predefined threshold Γ . Once this threshold is reached, terminals initiate transmission with a constant probability τ per time slot, akin to

standard-slotted ALOHA. The investigation focused on analyzing the time-average expected AoI achieved by this policy and exploring its scalability with network size, denoted as n . Similarly, [29] introduced a threshold-based lazy variant of Slotted ALOHA. In this approach, each node transmits with a certain probability once its age exceeds the threshold, aiming to minimize the overall AoI in the system.

Furthermore, researchers have sought to determine a Maximum AoI Threshold (MAT) to address scheduling challenges at the network edge, as explored by Li et al. [30]. The primary objective of this endeavor is to assess the schedulability of a vector containing MATs for the source nodes and, if feasible, identify an appropriate scheduler capable of meeting these requirements. These studies collectively contribute to the advancement of threshold-based transmission strategies in optimizing AoI dynamics and addressing scheduling complexities in networked systems.

In essence, these collective efforts underscore the critical role of incorporating threshold constraints and adaptive policies to optimize AoI in different network scenarios with information criticality offering new dimensions for enhancing information freshness and system performance.

2.1.3 AoI in zero-wait policy

Ensuring timely updates to a destination regarding a remote system is a complex task, distinct from maximizing communication system utilization or minimizing update delay, as noted by Kosta et al. [31]. One potential approach to mitigate delay is adjusting the update rate to alleviate congestion within the communication system. However, reducing the update rate introduces the risk of providing outdated status information to the destination, as updates may not occur frequently enough to capture real-time changes in the system.

The zero-wait policy, where a fresh update is submitted immediately after the delivery of the previous update, is often considered reasonable as it maximizes throughput and minimizes delay. However, it is noteworthy that this policy does not always minimize the AoI, as also observed by Kadota et al. [15]. They point out that policies optimized for throughput and delay may not necessarily minimize AoI. Interestingly, they find that average-age optimal scheduling policies often coincide with throughput-optimal ones, but the reverse is not always true.

This counterintuitive observation has prompted further investigation into optimal control strategies for information updates to maintain data freshness. Researchers, such as [32], seek to understand the conditions under which the zero-wait policy is optimal, shedding light on

the intricate trade-offs between system throughput, delay, and data freshness.

2.1.4 AoI in networks with limited transmission opportunities

In practical scenarios, sensing devices often face constraints that can limit their functionality, such as finite battery life or component deterioration over time. A significant area of research focuses on minimizing the AoI over a finite horizon in the presence of energy-constrained devices [10, 9]. Given that these devices rely on limited batteries, optimizing AoI performance becomes crucial for extending network lifetime while ensuring information freshness. In [9], the authors investigated the role of discounted AoI in energy-constrained networks, highlighting the trade-offs between information freshness and energy consumption. Similarly, [12] considered energy harvesting with finite-size batteries, collecting status updates from multiple heterogeneous information sources, and examined how combinations of cost and age distribution impact average AoI. In a related study [33], different policies were explored to minimize AoI in setups with finite batteries and energy harvesting networks, addressing the trade-off between information freshness and energy cost.

Numerous works have explored AoI performance in energy harvesting networks [17, 34, 35]. Authors. [24] examined the impact of different battery horizons on AoI performance, providing insights into optimal update strategies under energy constraints. These studies collectively contribute to understanding and optimizing AoI performance in energy-constrained environments, with implications for enhancing the efficiency and longevity of IoT networks.

2.2 Game Theory for Networks

While game theory traditionally analyzes social dynamics and conflicts between participants, its relevance extends to computer networks and telecommunications. In these domains, a *game* refers to situations where the outcome for each participant depends not just on their own choices but also on those of others. Networks often mirror such scenarios, with nodes acting as players competing or collaborating to improve their service quality. This approach is driven by the significant interdependence among network actions, such as resource utilization, with wireless interference serving as a prime example.

While a player in a game can encompass various entities such as machines, programs, persons, or even molecules, it is crucial to recognize that a game primarily serves as a mathematical construct designed to model and analyze interactive scenarios. Despite being a repre-

sentation of real systems, mathematical modeling, particularly through game theory, proves to be a potent tool in network applications. It effectively portrays various conflict scenarios involving system performance and associated costs, offering insight into system defense mechanisms against unwanted attacks and optimizing the utilization of shared resources.

2.2.1 Different forms for modeling Network Scenarios

Study [36] applied game theory to various digital signal processing scenarios, highlighting the distinction between strategic and coalition-form modeling approaches. The authors illustrated this difference through two wireless sensor dilemmas: the classical Prisoner's Dilemma, where sensors decide whether to share information at a cost, and the Cognitive Radio Dilemma, where nodes must choose between transmitting over narrow or wide frequency bands independently and simultaneously.

Strategic-form representations typically address non-cooperative scenarios, assuming players act selfishly without cooperation or communication exchange [13, 10, 36]. However, many signal processing applications necessitate cooperation among players, such as in cooperative networking where devices may route packets collectively. In such cases, players may form coalitions to improve their position. While players within coalitions still select strategies, the focus shifts to analyzing coalition formation and considering communication possibilities.

In coalition-form games [36], two key features for solutions are stability and fairness. The solution must ensure that formed coalitions resist individual or subgroup deviations while also ensuring fairness in utility division among coalition members. Achieving a balance between fairness and stability is challenging and relies heavily on factors like the value function structure, player goals, and the specific application under study.

Indeed, while both static and dynamic forms of game models have their merits, for modeling network security games, the dynamic form playing Bayesian game emerges as the superior choice [37, 38, 39]. This dynamic approach aligns more closely with scenarios involving defenders and attackers in network security. Dynamic game models, particularly Bayesian games [38], offer a more realistic representation of the evolving nature of security threats and defenses in dynamic network environments. By accounting for uncertainty and information asymmetry among players, Bayesian games provide a flexible framework to model and analyze complex interactions, making them well-suited for dynamic security scenarios where both defenders and attackers continuously adapt their strategies based on observed outcomes and new information

2.2.2 Network Security and Game Theory

The work by [40] addresses the shortcomings of traditional solutions for network security, particularly focusing on Intrusion Detection Systems (IDSs). These systems monitor network or computer events to identify potential attacks using methods like attack signature identification and statistical analysis. The authors provide a survey of game-theoretic approaches applied to enhance network security, emphasizing the role of IDSs in security game modeling. They highlight that the accuracy of IDSs influences whether the security game should be modeled as one with perfect, in cases when IDS is error-free, or imperfect information. The existing game models for security encounter several limitations, like scalability where the game is represented as a two-player game, simplifying scenarios with multiple attackers and defenders into a single player for each side. They criticized static model representation, because many scenarios involve dynamic interactions between attackers and defenders, yet existing models often adopt static frameworks. In their works [37] they agreed that these scenarios are better to be modeled as dynamic games, therefore, in addition to static representation they provided a dynamic form of the game. The proposed approach [37] introduces a Bayesian hybrid detection strategy for the defender, combining both lightweight and heavyweight monitoring systems. The lightweight system serves to estimate the opponent's actions, while the heavyweight system acts as a final line of defense. Through dynamic game analysis, the strategy yields energy-efficient monitoring tactics for the defender, enhancing the overall effectiveness of hybrid detection. Additionally, [40] argued about zero-sum game assumptions. Some stochastic game models consider attacking and defending as zero-sum games, which may not accurately capture the dynamics of real-world scenarios. A more realistic approach involves considering general-sum games.

In paper [38], authors study the jamming problem in underwater acoustic sensor networks, where nodes try to communicate despite a jammer's interference. In these scenarios, nodes lack advanced signal processing, so jamming just increases noise. They placed the problem as a Bayesian zero-sum game where the sensor network tries to maximize transmission capacity while the jammer minimizes it. Authors were interested in how sensor and jammer placements impact the game, considering Bayesian methods due to imperfect knowledge about the network structure. They did a similar setup in [41] for the radio network.

Puzzle-based defense mechanisms have emerged as a potential solution against flooding denial-of-service (DoS) attacks in networks. This paper [42] employs game theory to propose optimal puzzle-based strategies for combating increasingly sophisticated flooding attack sce-

narios. Utilizing the Nash equilibrium solution concept, the defender's role is to craft an optimal defense against rational attackers. The study addresses distributed attacks from unknown sources by modeling the interaction between attackers and defenders as an infinitely repeated game of discounted payoffs. Four defense mechanisms are proposed: PDM1, PDM2, PDM3, and PDM4, each tailored to different attack scenarios. These mechanisms leverage open-loop and closed-loop solution concepts to effectively counter single-source and distributed attacks, including cases where the size of the attack coalition is unknown..

2.2.3 Resource Management and Game Theory

In order to ensure quality of service for users required by the delay-sensitive and bandwidth-intensive multimedia data, authors [43] utilized bargaining methods to distribute bandwidth equitably and optimally among multiple collaborative users. Specifically, they were investigating two bargaining solutions: the Nash bargaining solution (NBS) and the Kalai-Smorodinsky bargaining solution (KSBS). The NBS is aimed at maximizing system utility, while the KSBS ensures that all users experience a similar utility penalty relative to the maximum achievable utility. These strategies are implemented through a resource manager within the network, taking into account application-specific distortion for bandwidth allocation. They demonstrate that these bargaining solutions adhere to crucial properties and propose criteria for determining bargaining powers, considering factors such as visual quality impact and spatiotemporal resolutions. Additionally, they assess the complexity of these solutions and evaluate their performance across various scenarios.

The authors proposed a game theory model with multi-agent games to address resource allocation challenges in radio networks with device-to-device (D2D) communication [44]. In their article, they discussed various game models tailored for D2D direct communication and D2D local area networks (LANs), categorizing them based on game types. For D2D direct communication, noncooperative and auction game models were identified as suitable for resource allocation. In contrast, cooperative game models like coalition formation games were deemed appropriate for D2D LANs, where collaboration among mobile devices is necessary. They elaborated on an auction model and a coalitional game model, presenting them in detail. Furthermore, they outlined potential research directions for developing game-theoretic models to address key radio resource management challenges in D2D communication.

The authors of the study [45] investigated resource allocation in cloud infrastructure under uncertain task specifications, aiming to minimize wastage while configuring services preemp-

tively before actual requests occur. They adopted a coalition form game model, where servers act as agents with varying resource capabilities based on compute, memory, and storage capacities. Host machines have the flexibility to participate in multiple coalitions as long as they adhere to specified maximum participation limits. Additionally, coalitions are not constrained to a predefined number of members, allowing for dynamic composition based on resource requirements and availability.

2.2.4 AoI in game-theoretic framework

In recent years, many authors have been interested in the study of AoI optimization in the context of game theory related to different network scenarios, for example [14, 4, 46, 39]. In his work [14], the author analyzes a system comprising two independent players who make individual decisions regarding transmission without coordination. In this system, sources are required to periodically transmit updates, each associated with an individual cost. However, the overarching goal is to optimize the global benefit of the receiver's AoI. This scenario reflects a decentralized communication environment where each player must weigh the trade-offs between the cost of transmitting updates and the resultant improvement in the receiver's AoI.

In their recent study [13], the authors employ a game theory approach to demonstrate that even uncoordinated contention-based access protocols can exhibit a degree of efficiency. They show that a relatively efficient Nash equilibrium (NE) can be achieved when players' individual objectives combine their local AoI with a transmission cost term. While their work primarily focuses on slotted ALOHA, a protocol known for its low access efficiency due to collisions leading to wasted transmission slots, they introduce a novel aspect. In addition to the previous work [47], slotted ALOHA is analyzed with *capture effect* that refers to the enhancement in the probability of successful transmission due to stronger signals surviving collisions and being correctly decoded despite interference from other signals. Together, these studies shed light on the potential efficiency of contention-based access protocols, particularly in dense network environments. Other authors also conducted research of selfish behavior with game theory framework in ALOHA protocol [48, 49, 50].

Despite plenty of work done with games with complete information, there is interesting work regarding scenarios with a lack of communication and coordination. In their work [39], authors evaluated the performance of a queuing system with multiple strategic servers with Bayesian game-theoretic formulation in which players can be of different types, and each type has its utility. Having different types is common in a network with a large number of

nodes, which are often heterogeneous and have severe constraints in terms of power and computational capabilities. This is a way to capture that the players may behave in different ways and it represents a Bayesian game with incomplete information.

Authors [46] did research on game-theoretic approaches for non-cooperative resource allocation in relay-assisted interference channels and they focused on optimizing physical layer energy efficiency. In the context of the Internet of Medical Things (IoMT), the study by [6] tackled the challenge of efficient resource allocation for health monitoring networks. Specifically, for intra-wireless Body Area Networks (WBANs), they devised a cooperative game aimed at allocating wireless channel resources effectively. However, for networks extending beyond WBANs, where individual rationality and potential selfishness come into play, they proposed a decentralized non-cooperative game. This approach was designed to address the complexities arising from the diverse nature of devices and the need to balance resource allocation with individual incentives and behaviors in IoMT environments. Researchers in the study [4] tackle the challenges of resource allocation in large Internet of Things (IoT) networks using game theory. They introduce non-conventional game theory models such as specific approaches such as mean-field games or minority games, tailored to better fit the characteristics of these setups.

2.3 Harsanyi's theory for equilibrium selection for games with complete information

Harsanyi's theory of equilibrium selection [51], developed by Nobel laureate John Harsanyi, addresses the issue of how players in a game with complete information can select among multiple Nash equilibria to coordinate on a particular outcome.

Harsanyi's new theory [51] introduces a departure from the traditional approach to equilibrium selection in non-cooperative games. Instead of focusing on bilateral risk comparisons between pairs of equilibria, as in his previous work with Selten [52], he proposes the concept of multilateral risk dominance. The essence of multilateral risk dominance lies in determining the equilibrium that minimizes risk across all equilibria with relevant properties. By comparing the risks associated with various equilibria, players aim to identify the one that offers the lowest level of strategic risk. In this context, strategic risk pertains to the likelihood that a player's chosen strategy will not yield the best possible outcome given the strategies chosen by other players. While it may be impossible for players to entirely eliminate strategic risk,

they can mitigate it significantly by selecting strategies that lead to equilibria with the highest theoretical probability of realization. In other words, players seek strategies that are robust and resilient against deviations by other players, thereby increasing the likelihood of achieving favorable outcomes. The equilibrium that emerges as the least risky choice, is considered the solution of the game according to Harsanyi's new theory.

Harsanyi's new theory challenges the notion of subgame consistency by highlighting how pre-subgame moves can compel players to deviate from strategies that would otherwise be consistent within subgames. Subgame consistency posits that strategies chosen at any stage of a game should remain consistent with optimal play in subsequent subgames. However, pre-subgame moves, made before reaching a specific subgame, can influence players' decisions and lead them to adopt strategies that are not subgame-consistent.

Building upon this rejection of subgame consistency, Harsanyi introduces base dominance and inferiority relations between strategies at the level of the game as a whole. These relations enable players to evaluate the relative strength of different strategies in achieving favorable outcomes across the entire game, rather than solely within specific subgames. Base dominance refers to the superiority of one strategy over another in achieving better overall outcomes across the entire game. Conversely, inferiority relations identify strategies that are weaker or less effective compared to others in achieving favorable results. By focusing on base dominance and inferiority relations, Harsanyi's theory [51] provides a comprehensive framework for evaluating strategies based on their performance throughout the entire game, rather than being limited to subgame considerations.

Harsanyi involves the modification of the equilibrium selection criterion from a combination of payoff and risk dominance to solely relying on risk dominance. This adjustment is made in response to the nature of non-cooperative games and aligns with Aumann's theory [53], which emphasizes the limitations of achieving payoff dominant equilibria in such contexts.

In non-cooperative games, players act independently and pursue their own interests without coordinated agreements or communication. Aumann's theory [53] underscores that even with the possibility of *preplay* communication, players cannot reliably enforce agreements to achieve payoff-dominant equilibria. This is because there is no mechanism to ensure compliance with agreements, and players may have incentives to deviate from agreed-upon strategies to maximize their individual gains.

Given these constraints, Harsanyi's theory [51] opts to focus solely on risk dominance as the criterion for equilibrium selection. Risk dominance prioritizes equilibrium choices that

minimize strategic risk and offer more stable outcomes, making them more feasible and realistic in the context of non-cooperative games.

Further, Harsanyi incorporates tie-breakers to address disparities between different equilibria. Prior theory [52], suggests that for symmetric games, should be selected a symmetric equilibrium which often leads to suboptimal outcomes characterized by low stability and poor payoffs. Consequently, in the updated theory[51], Harsanyi advocates for an alternative approach. At least in scenarios where *preplay communication* is permitted, he argues that the optimal solution should manifest as a *correlated equilibrium*. This correlated equilibrium reflects a probability distribution across all possible Nash equilibria, allowing for a more nuanced representation of strategic interactions and potentially yielding improved outcomes.

In scenarios where *preplay communication* is not feasible, achieving a correlated equilibrium becomes impossible. In such cases, players can strive for what Harsanyi terms a "quasi solution." Harsanyi illustrates this concept with an example from the Battle of the Sexes game, which features three equilibria: two in pure strategies and one in mixed strategies.

Traditionally, under the previous theory, the solution would lean towards the mixed strategy equilibrium, primarily due to its symmetry. However, Harsanyi's theory suggests a different approach. He proposes that the solution should reflect a correlated equilibrium that combines the strengths of two persistent equilibria, the equilibrium in pure strategies. This equilibrium yields to higher outcomes compared to symmetric equilibria, making it a more appealing choice despite its asymmetry.

Indeed, both of Harsanyi's theories, as outlined in his works from 1988 and 1995, maintain Nash properties for nondegenerate unanimity games. These properties ensure that equilibria possess certain stability and optimality characteristics crucial for strategic decision-making.

Harsanyi's approach involves an assumption that players will strategically limit their choices to equilibria demonstrating acceptable stability properties. Specifically, he emphasizes equilibria categorized as proper and persistent, which he refers to as eligible equilibria. Proper equilibria [54] represent strategic outcomes where no player has an incentive to unilaterally deviate from their chosen strategy. Persistent equilibria [55] further reinforce this stability by ensuring that players continue to adhere to their strategies even under various perturbations or changes in the game's environment.

2.3.1 Definition of the solution of the game in terms of theoretical probabilities

In Harsanyi's theory, the set of all eligible equilibria is denoted as S^* . The theory offers a solution only for games in which S^* is a finite set, indicating a limited number of equilibria to consider. Harsanyi defines the solution of the game, denoted as s^* , as the eligible equilibrium with the highest theoretical probability of occurrence.

If S^{**} represents the set of all eligible equilibria s^* , then three distinct cases can be distinguished:

- If S^{**} contains only one equilibrium s^* , s^* will be defined as a solution.
- If S^{**} contains two or more equilibria, and the game allows *preplay communication* among players, then the solution is defined as the correlated equilibrium representing an equal-probability mixture s^{**} of all equilibria s^* in S^* .
- If S^* contains two or more equilibria, and the game does not permit *preplay communication*, then the solution is left undefined. However, a unique Nash equilibrium s chosen by the tracing procedure when the centroid of set S^{**} is used as the starting point will be defined as a quasi-solution for the game. The term "quasi-solution" refers to a solution that is not formally defined but serves as a practical approximation or benchmark.

In games with complete information, players have full knowledge of the game structure, including the payoffs associated with each possible combination of strategies chosen by the players. Despite this complete information, there may still be multiple Nash equilibria, each representing a possible outcome where no player has an incentive to unilaterally deviate from their chosen strategy.

In summary, while Harsanyi's theory aligns with the standard theory in its foundation within noncooperative games and its consideration of Nash equilibria, it diverges by emphasizing a stronger concept of strategic rationality. Moreover, it shares certain aspects with cooperative solution theories by focusing on fundamental game parameters, thereby providing a more comprehensive framework for analyzing strategic interactions and selecting equilibrium solutions.

2.4 Related work and contribution

Given the extensive attention directed towards the minimization of the AoI, we aim to contribute to this vibrant field with innovative insights. While numerous scholars have explored various scheduling policies to reduce data age within networks, only a handful have approached this challenge through the lens of game theory.

In our research, we tackle the problem of minimizing the AoI by framing it as a static game with complete information, building upon previous work such as [14]. However, we extend the analysis to scenarios involving multiple players and nodes, addressing the critique from [40] that many network scenarios are better represented as multi-agent games rather than just two-player games. Inspired by the limited transmission opportunities in constrained devices discussed in [9], we model the scenario as a game in the finite horizon.

Unlike previous studies that consider various information processing times such as exponentially [16, 20, 19], we simplify our model by assuming instantaneous processing, similar to [9]. Additionally, we focus on collision-free scenarios, unlike [8, 13, 50], and therefore, we do not introduce the influence of feedback in case transmission fails, as discussed in [9]. Furthermore, we neglect the cost of transmission to isolate the impact of non-cooperative behavior on the AoI, unlike [14].

Our work stands out for its integration of Harsanyi's equilibrium selection theory with the well-established problem of minimizing data age. Harsanyi's framework, rooted in the selection of equilibria in games with complete information, directly aligns with a subsection of our research problem. In our investigation, we leverage this theory by introducing preplay communication mechanisms. This strategic maneuver enables us to move beyond the confines of symmetric equilibria, which often exhibit poor stability and yield low payoffs. Instead, we explore the realm of correlated equilibria, which offer more desirable properties and lead to superior payoffs, ultimately resulting in a reduction in the AoI.

To our knowledge, our study is the first to connect the concept of minimizing the AoI with Harsanyi's equilibrium selection theory in non-cooperative games. This novel approach sheds new insights into strategies for improving information freshness in networks, opening avenues for future research in both game theory and information age optimization.

Chapter 3

Implementation

This chapter will present a game-theoretic framework that models the network scenario. The aim is to simulate the real scenario, representing it as a game with N players with the common goal of keeping the information as fresh as possible. Therefore, at the beginning, we are going to represent the basic model to find Nash equilibrium in pure strategies and the domain of mixed strategies. Later, we will introduce some model adjustments aimed at attaining more efficient equilibria.

3.1 Model

The initial game is formulated as an N -player game operating over a finite horizon, where each player corresponds to an independent information source. Within this framework, each player is allowed to transmit within one of N slots over the game horizon. Consequently, each player is constrained to a single transmission during the game. Given that all players share the common objective of minimizing the AoI, with each transmission resetting this metric to zero, the overarching goal is to collectively reduce the freshness of information as much as possible.

This game bears resemblance to the concept of *Public goods games* in game theory, albeit with a notable difference: here, players lack any incentive to abstain from transmitting. Unlike traditional Public goods games [56] where participants may withhold contributions to reap individual benefits, the absence of transmission holds no advantage for either the system or the players themselves in this scenario. Furthermore, the absence of any associated costs for updates further diminishes any rational reason for players to remain idle indefinitely.

All players possess an equal capacity to initiate an update within any of the N available slots, but they are also aware that sending multiple updates is inefficient in terms of resource

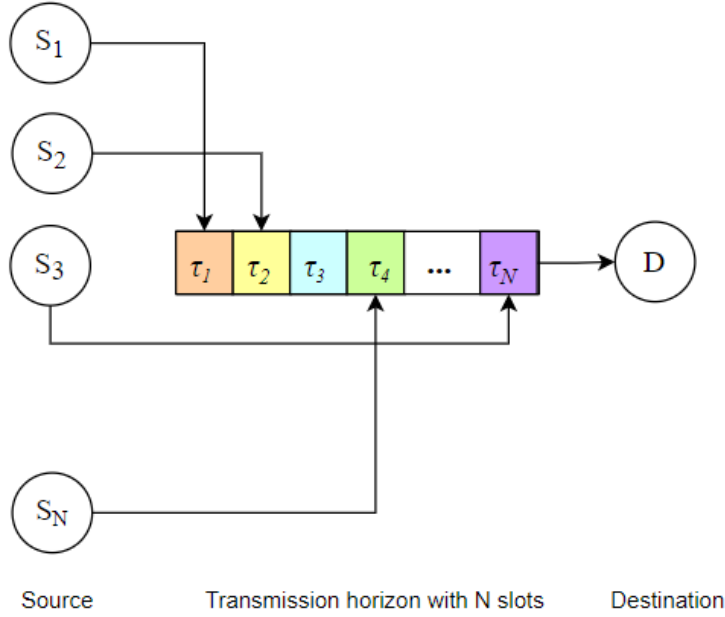


Figure 3.1: System model for N players over finite horizons with τ_N possible chance to update

utilization (Fig.3.1). The utility of the game is presented by the AoI, which is a metric to measure the freshness of information in the network system. This aspect distinguishes the game from conventional game theoretic paradigms, as the optimal outcome is attained when the average AoI is minimized [14]. Furthermore, all players receive identical payoffs, incentivizing them to strive for minimal AoI values. Consequently, their behavior mirrors that of a static game with complete information, where players are not coordinated but rather synchronized [8].

For the N players, we can represent the problem by defining *a static game of complete information* denoted as follows:

$$\mathcal{G} = \{\mathcal{N}, \mathcal{S}, \mathcal{U}\} \quad (3.1)$$

In the game scenario, \mathcal{N} represents the set of N independent sources, where each source is a player in the game. Mathematically, $\mathcal{N} = \{1, 2, \dots, N\}$. We model the game scenarios with three up to eight homogeneous agents.

$$\mathcal{S} = \{\pi_j\}, \quad 0 \leq j \leq N \quad (3.2)$$

Another crucial component is \mathcal{S} which denotes the set of possible strategies of players. In this context, \mathcal{S} encompasses the probabilities associated with a player selecting a particular

slot for transmission. In the realm of pure strategies, the value π_j is equal to 1 for the j -th slot in which the player updates, while for all other slots, this value is equal to 0. Therefore, we can transform it to a set of actions A : $\mathcal{A} = \{0, 1\}$.

A single player's action space \mathcal{A} comprises only two options: 0, representing the player's decision to remain idle, and 1, representing the choice to update. Each player independently selects their action in the current slot τ_j , $0 \leq j \leq i$, and i depends on N number of players.

The last component, denoted as \mathcal{U} encompasses the payoffs associated with games involving varying numbers of players $n \in \mathcal{N}$, contingent upon the actions chosen by the agents. The overarching objective of the defined model is to minimize the average AoI across the finite horizon, which serves as the payoff metric for different game scenarios. In assessing data freshness, we define the instantaneous AoI [9] for a given information source at time t as $A(t) = t - \tau(t)$, we calculate the time difference between the present moment t and the most recent slot where the update is received by the receiver. Our focus is on understanding the average AoI within a specific time window, i.e.

$$\Delta := \frac{1}{N} \int_0^N A(t) dt \quad (3.3)$$

In Fig 3.5 and Fig.3.4, we observe a potential timeline illustrating the evolution of the AoI. Notably, Δ can be calculated as the area beneath the AoI curve $A(t)$ within the interval $[0, N]$, with normalization relative to the overall time horizon.

A key constraint in our model is the maximum updates in the single slot denoted as M , which may be smaller than the total number of available updates ($M \leq N$) and depends on the updates that occurred in the slots before the current one. Therefore, we can say that we model a *dynamical system* that depends on the previous states [57]. In total, the maximum capacity of updates in the whole network can not exceed N . In the model implementation, this constraint arises because our players independently decide when to update. Consequently, there are instances where multiple updates occur within a single time slot, resulting in missed opportunities for updates in other slots.

The minimization of the average AoI value over the finite horizon can be represented by the modified formula, as follows:

$$\Delta y = \frac{1}{N} \sum_{i=0}^M \left[\frac{y_i^2}{2} + \sum_{j=i+1}^M y_i y_j (1-p)^{j-i} \right], \quad (3.4)$$

where the first contribution within brackets represents the area of the triangle with side y_i ,

which is always associated with the i -th update. The subsequent summation captures the area of parallelograms with sides y_i and y_j that contribute to the AoI only in case of zero updates in i -th (and possibly subsequent) transmissions (see Fig. 3.4) slot or when occurs multiple updates and till the end of the horizon we do not have any chance to transmit (see Fig.3.5).

3.1.1 Equilibrium in pure strategies

Pure strategies refer to strategies where players make specific, predetermined choices without any randomness or uncertainty involved. These strategies represent a single, definite course of action chosen by each player in a game. In the model that has been developed, each participant is presented with a binary choice: they can either choose to transmit or to remain idle.

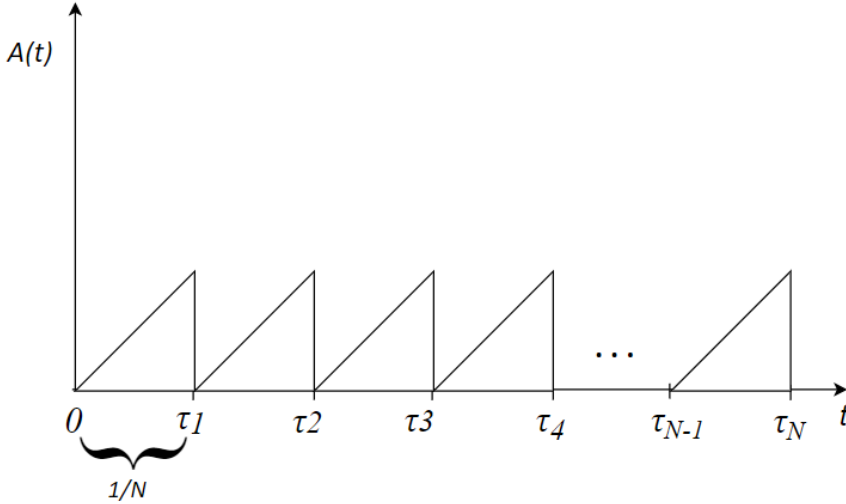


Figure 3.2: Optimal AoI in pure strategies when Nash equilibrium is reached

Definition 1. An Nash equilibrium NE of the game $\mathcal{G} = (N, (S_n)_{n \in N}, (u_n)_{n \in N})$ is a strategy profile $s^{\text{NE}} = (s_1^{\text{NE}}, \dots, s_n^{\text{NE}}) = (s_n^{\text{NE}}, s_{-n}^{\text{NE}})$ such that

$$\forall n \in N, \forall s_n \in \mathcal{S}_n, u_n(s_n^{\text{NE}}, s_{-n}^{\text{NE}}) \geq u_n(s_n, s_{-n}^{\text{NE}}). \quad (3.5)$$

The minimization of AoI is realized when each player chooses to transmit in distinct slots Fig 3.2. This essentially means that the players are in a state of anti-coordination. In other words, Nash equilibrium, a state of optimal strategy for all players, is only reached when each player selects a unique slot for transmission. This scenario reflects the dynamics observed in the classic game theory example, the *Battle of the Sexes* [56].

When players optimally play a game, the above formula is transformed into:

$$\Delta(y) = \frac{1}{N} \sum_{i=0}^N \left[\frac{y_i^2}{2} \right] \quad (3.6)$$

3.1.2 Equilibrium in mixed strategies

Since we have defined the game with the final number of strategies, we need to find a solution in mixed strategies:

Theorem 1. [58] *In a strategic-form game $\mathcal{G} = (N, (S_n)_{n \in N}, (u_n)_{n \in N})$, if N is finite and S_n is finite for every n , then there exists at least one NE, possibly involving mixed strategies.*

In game theory, when agents play a game with mixed strategies they have to apply the *indifference theorem* [56] that the player's decision probabilities should be such that the other players are indifferent between their own set of actions. This concept is crucial in higher-order game theory, where agents think about the actions of other agents.

Definition 2.: Player n 's mixed strategy $\pi_n \in \Delta(S_n)$ is a distribution that assigns a probability $\pi_j(s_n)$ to each strategy s_n , such that $\sum_{s_n \in S_n} \pi_n s_n = 1$. For mixed strategies, the joint probability distribution over the strategy profile s is, by definition, the product of the marginals $\pi_n, n \in N$.

The set of actions \mathcal{S} is defined for all players in terms of their update probability, π_j which represents the likelihood of a player making an update in the j -th slot. Owing to the symmetry of the game, all players share identical probabilities when choosing the j -th slot to update. An additional implication of this game symmetry is that $\pi_j = \pi_{n-j}$ effectively reduces the size of the set by half.

Definition 3. A mixed strategy of the game $\mathcal{G} = (N, (S_n)_{n \in N}, (u_n)_{n \in N})$ is a *mixed strategy profile* $\pi^{\text{NE}} = (\pi_1^{\text{NE}}, \dots, \pi_n^{\text{NE}}) = (\pi_n^{\text{NE}}, \pi_{-n}^{\text{NE}})$ such that

$$\forall n \in N, \forall \pi_n \in \Delta \mathcal{S}_n, \tilde{u}_n, (\pi_n^{\text{NE}}, \pi_{-n}^{\text{NE}}) \geq \tilde{u}_n(\pi_n, \pi_{-n}^{\text{NE}}). \quad (3.7)$$

where

$$\tilde{u}_n(\pi_n, \pi_{-n}) = \mathbb{E}(u_n) = \sum_{s \in \mathcal{S}} \left(\prod_{j \in N} \pi_j(s_j) \right) u_n(s) \quad (3.8)$$

is the expected utility of player n when selecting the mixed strategy π_n , and $S = S_1 \dots S_n$.

The significance of mixed strategies lies in their generality and their ability to capture more complex decision-making scenarios [59]. In many games, players may benefit from introducing randomness into their choices to achieve better outcomes or to make their strategies less

predictable to opponents. Note that playing a game in pure strategies is just a specific case of mixed strategies where the player chooses one strategy with certainty, assigning one to a particular strategy and zero to all others.

One key advantage of mixed strategies is the availability of existence results for mixed Nash equilibria. A mixed NE occurs when each player's mixed strategy is optimal given the mixed strategies chosen by all other players. In other words, no player can unilaterally deviate from their chosen mixed strategy to improve their payoff. However, this symmetrical NE also has some undesirable properties as lower payoffs and instability, in our particular case, mixed NE results in higher average AoI. Existence results for mixed Nash equilibria provide theoretical guarantees that in many games, there is at least one equilibrium where players randomize their strategies. This is particularly important because it assures analysts that a solution to a game exists, even if it involves players using randomization in their decision-making.

Now we can redefine our objective function as :

$$\Delta y = \arg \min_{s_n \in \mathcal{S}_n} u_n(s_n, s_{-n}). \quad (3.9)$$

The Principle of Indifference, also known as the Principle of Insufficient Reason, suggests that if there are n possible outcomes and there is no reason to view one as more likely than another, then each should be assigned a probability of $\frac{1}{N}$ [59]. However, we have strong beliefs that our probabilities are not uniformly distributed and that "central" probabilities $\pi_{n/2}$ are slightly higher than π_1 or π_n .

3.2 Price of Anarchy

Price of Anarchy (PoA) is a concept in economics and game theory that quantifies the decline in system efficiency caused by the self-interested actions of its participants [60, 61]. It is an idea applicable to various systems and definitions of efficiency. Various interpretations of Nash equilibrium give rise to different understandings of the PoA, including Pure PoA (deterministic equilibria), Mixed PoA (for equilibria involving randomness), and Bayes-Nash PoA (relevant to games with incomplete information) [60]. Authors, Elias Koutsoupias and Christos Papadimitriou introduced the term PoA, but the idea of measuring the inefficiency of equilibrium is older.

The PoA captures the inefficiency that arises due to the lack of coordination among independent, selfish agents. It is a measure that describes how efficient a system could be if

Algorithm 1 An algorithm for implementing Indifference theorem

Require: N random probability vectors p_j with the same length

$temp_{dev} = 100$ \triangleright Initialize a temporary deviation variable with a high value

Ensure:

$\sum_{j=1}^N p_j = 1$ \triangleright Constraint that sum of all probabilities must be one
 $p_j \geq 0$

$p_j = p_{n-1}$ \triangleright Ensure symmetry

$p_j \leq p_{j+1} \leq \dots \leq p_{N/2}$, where $0 \leq j \leq N$ \triangleright Constraint that requires that subsequent slots probabilities must be greater than previous

while Loop over the length of random probability vector **do**

 Calculate the coefficients of the polynomial using probabilities

 Calculate expected values for different outcomes

$u_1(1, coef) = u_1(2, coef) = \dots = u_1(N/2, coef)$ \triangleright Due symmetry we have to

 calculate only $N/2$ expected values

 Calculate the average value of expected payoffs

$dev = abs(u_1(1, coef) - average)$

if $temp_{dev} \geq dev$ **then**

$temp_{dev} = dev$

$results = [u_1(1, coef), p]$ \triangleright If the condition is satisfied, then keep results of the

 expected value and probabilities

end if

end while

its agents could be coordinated by a central authority, as opposed to each agent acting in its self-interest[61]. The PoA is the ratio between the worst Nash Equilibrium versus the payoff for the players in the optimal scenario[56], [62]. One of the key factors driving the relevance of the PoA in computer networks is the inherent tension between individual and collective interests. Each node or participant in a network typically seeks to optimize its own objectives, whether it be maximizing throughput, minimizing latency, or conserving resources. However, in doing so without regard for the overall network's well-being, these self-interested actions can lead to suboptimal outcomes for the network as a whole.

$$PoA = \frac{\max_{s \in S} \sum_{n \in N} u_n(s)}{\min_{s \in S^{NE}} \sum_{n \in N} u_n(s)}, \quad (3.10)$$

where S^{NE} represents sets of all NE in the game.

In our model all players are the same utility function; minimal AoI in the network is the common value for all of them, therefore we can simplify the above function representing PoA as:

$$PoA = \frac{\max_{s \in S} u_n(s)}{\min_{s \in S^{NE}} u_n(s)} \quad (3.11)$$

where $n \in \mathcal{N}$.

Networks, particularly dynamic ones like the Internet, are seldom perceived as static models. Their inherent volatility means that patterns within them can undergo rapid and substantial changes, particularly in the event of failing routers or other network disruptions [57] as a result of selfish routing. PoA is frequently utilized to signify the cost incurred due to the absence of cooperation in dynamic network flows [63]. In the realm of routing algorithms, selfish routing strategies can result in increased congestion, longer transmission delays, and inefficient resource utilization. Research [64] emphasizes the importance of considering the PoA when designing routing protocols and network topologies. Their work underscores how the PoA provides valuable insights into the trade-offs between individual incentives and collective welfare in decentralized network environments.

The introduction of this concept into our model aims to quantify the extent to which the "anarchy" resulting from the absence of coordination can adversely impact information freshness. It will furnish us with invaluable insights into system efficiency and highlight the derivation between perfect coordination, characterized by all players updating in distinct slots, and spontaneous coordination, which may lead to multiple updates and an increase in data age within the channel. Models are inherently static representations, while the real world is dynamic [57].

3.3 Finding Nash equilibrium in pure and mixed strategies

After introducing the concepts of equilibrium in both, pure and mixed strategies, as well as the price of anarchy, we will delve a bit deeper into the methodology behind these results. For solving a game in pure strategies, we are going to introduce the concept of the *Best response* (BR) [58]:

Definition 4. Player n 's best response $BR_n(s_{-n})$ to the vector of strategies s_{-n} is the set-valued function

$$BR_n(s_{-n}) = \operatorname{argmax}_{s_n \in S} u_n(s_n, s_{-n}). \quad (3.12)$$

Definition 5. Let set $\mathcal{G} = (N, (S_n)_{n \in N}, (u_n)_{n \in N})$ be a strategic-form game. A strategy profile s^{NE} is an NE if and only if

$$s^{NE} \in BR(s^{NE}). \quad (3.13)$$

In pure strategies, our equilibrium is determined by each player selecting the best response given the strategies chosen by others. In our scenario, this equilibrium occurs when each router transmits in a different time slot, resulting in the lowest AoI value being achieved.

On the other hand, in mixed strategies, players randomly select strategies to ensure their opponents are indifferent between their available choices. If players mix their strategies correctly, opponents receive equal payoffs regardless of which strategy they choose, *Indifference theorem* [56]. In our case, since all players have the same payoff structure, mixing strategies entails choosing a combination of strategies that yield equal payoffs across all options.

After identifying the equilibria in both pure and mixed strategies, we utilize the concept of the PoA to quantify the inefficiency resulting from the lack of coordination within the model. This measurement helps us understand whether the system requires enhancements to optimize the transmission schedule. Given that we have a cost of anarchy of approximately 1.5 for a three-player game (which means we lose 50% in efficiency), and that inefficiency tends to grow with an increasing number of players, it justifies enhancements to the model based on these findings. Moreover, Harsanyi's theory [51] suggests that the outcome of a non-cooperative game should ideally surpass that of the mixed strategy equilibrium if preplay communication is allowed. Therefore, these results further support the justification for implementing improvements in our model to optimize performance.

3.4 Equilibrium selection in practice

Following the inefficiency results derived from the PoA, we aim to introduce several modifications to our model to explore the effects of different initial conditions. These adjustments are particularly meaningful in the context of a finite horizon, as our model is defined within such constraints. This finite horizon aligns with scenarios involving numerous energy-constrained devices, as highlighted in studies such as [9, 10].

One way to improve efficiency and decrease PoA is to transform the game, which is known as *mechanism design* [65]. It consists of applying different transformations to utility functions to obtain NE which is more efficient than the one considered in the original game. Another possibility to improve efficiency is to keep the game unchanged but to modify the solution concept. This may be a *correlated equilibrium* (CE) which Harsanyi has discussed in his theory as a solution for non-cooperative games with *preplay* communication [51]. A CE is a joint distribution over the possible actions or pure strategy profiles of the game from which no player has an interest in deviating unilaterally. More formally, we have the following definition.

Definition 6.[58]: Correlated Equilibrium CE is a joint probability distribution $q^{CE} \in \Delta(S)$, which verifies

$$\forall n \in \mathcal{N}, \forall \sigma_n, \sum_{s \in \mathcal{S}} q^{CE}(s_n, s_{-n}) u_n(s_n, s_{-n}) \geq \sum_{s \in \mathcal{S}} q^{CE}(s_n, s_{-n}) u_n(\sigma_n(s_n), s_{-n}), \quad (3.14)$$

where $\sigma_n : S_n \mapsto S_n$ can be any mapping, and $S_n = S_1 \times \dots \times S_{n-1} \times S_{n+1} \times \dots \times S_N$.

In our model, the introduced modifications entail the capability for routers to listen to a channel within the network and react accordingly based on the prevailing situation. This setup aligns with Aumann’s concept of an “exogenous public signal,” where such a signal allows the game to attain new equilibria within the convex hull of the set of mixed Nash equilibria of the game [58]. Here, “public” denotes that all players can observe this signal, while “exogenous” signifies that the signal is unrelated to the players’ actions.

Throughout these discussions, we will explore several enhancements aimed at approximating the optimal solution, especially within the realm of pure strategies. While achieving Nash equilibrium in pure strategies stands as the most efficient solution, it proves challenging in static games played in a single shot. Consequently, we establish these two equilibria as benchmarks for adaptive strategies, thereby setting the lower and upper bounds of our analysis.

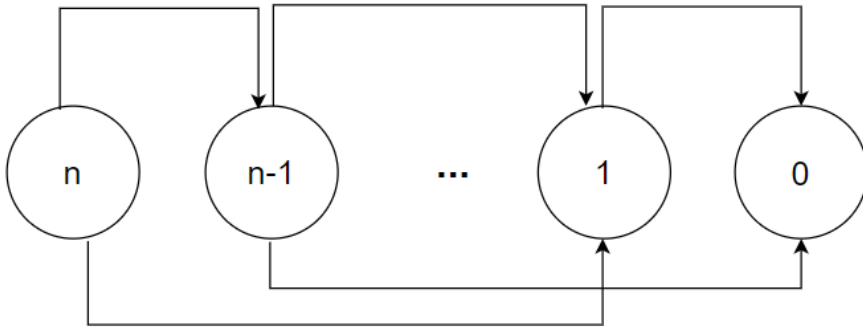


Figure 3.3: Possible scenarios of the number of updates in the slots during the game

3.4.1 Possible scenarios in the game

Since we are talking now about practical strategies, we will define “milestone” as discrete moments where players have the option to update or stay idle. A milestone is nothing less than the beginning of an interval where players make their decisions with the assumption

that players can only transmit in them. From now on, we will refer to these milestones in the context of our work. At the outset, all milestones are initially set at intervals of $\frac{1}{n+1}$, where n represents the number of players in the game. Since we have n milestones and $n + 1$ intervals, this division ensures uniform spacing. In an ideal scenario, we have one transmission over time which corresponds to *Round Robin division* [58].

In our model, individual players autonomously select their update schedules. However, this method may lead to redundant updates for certain slots while leaving other slots without updates (see Fig. 3.3). Unfortunately, this leads to an increase in the average AoI, which is contrary to our desired outcome. To address this issue, we propose implementing mechanisms to prevent AoI escalation. Some adaptations focus on optimizing the current situation, while others aim to proactively avoid undesired states. However, adjustments are necessary based on the transmission behavior:

1. **No Update/Transmission:** In cases where no updates occur, we need to shift the milestones which are time scheduled for transmission. This adjustment ensures that the remaining milestones align with the number of players, making the shifting that results in setting the next milestone earlier than planned.

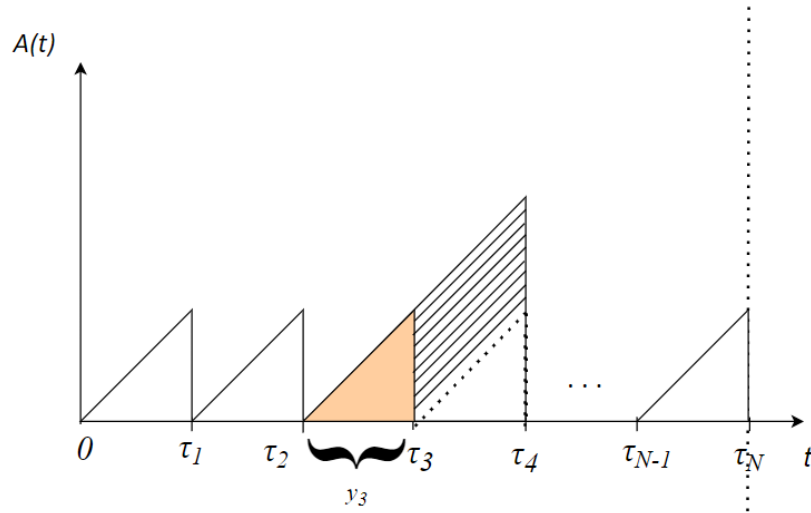


Figure 3.4: Illustration for the AoI evolution over time. In the specific scenario, no updates were initiated by the players during the third temporal slot; however, a transmission occurred in the subsequent fourth slot. Failure to capitalize on the opportunity resulted in an escalation of the AoI, symbolized by the delineation of a dashed surface

In the event of the worst-case scenario where no update occurs, continuous shifting is not a viable solution. Instead, we adjust the interval to the right until reaching the final slot, which corresponds to the $\frac{1}{n+1}$ of the initial interval. This signifies the final opportunity for an update, and if reached, transmission must occur.

2. **Single Transmission:** If only one update is transmitted per milestone, we maintain the existing milestones without modification. The desired objective is to encounter this specified scenario throughout the game. At the game's outset, our probabilities align with the outcomes of the *indifference theorem* implementation. Upon entering this scenario, we adjust the current probabilities in the following manner:

$$p_j = \frac{p_j}{1 - p_i} \quad \text{where } i = t, \quad i < j \leq n$$

where p_i corresponds to the probability of the current slot where an update has occurred, and p_j represents the probabilities of subsequent slots until the end of the horizon.

3. **Multiple Updates:** When multiple updates are transmitted, some opportunities for updates have been missed. To rectify this, we adjust the milestones to create uniform intervals from the remaining opportunities. This involves widening the intervals to accommodate the available slots.

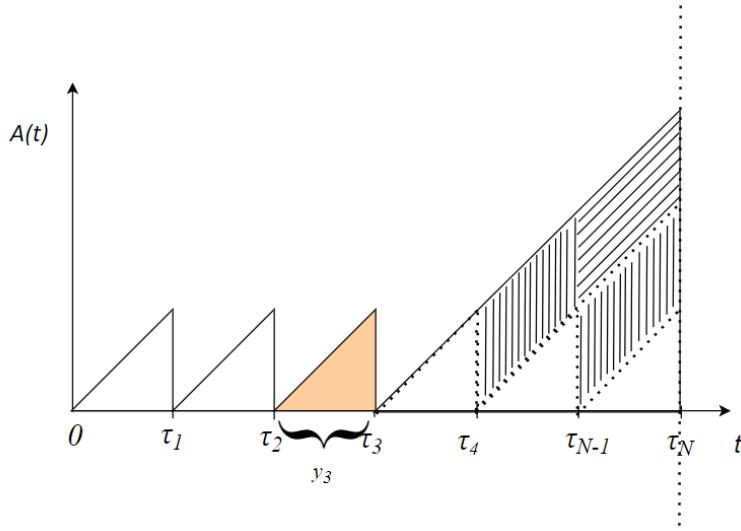


Figure 3.5: Illustration for the AoI evolution over time. Within the context under examination, multiple updates occurred during the third temporal slot, resulting in the forfeiture of an opportunity for transmission until the end of the observational horizon. Consequently, the AoI experiences an increase as depicted by the dashed surface.

Not only do we need to update the intervals, but we also need to adjust the probabilities of future updates. In cases of multiple updates, these probabilities are recalculated based on the corresponding scenario of a reduced $n - k$ players' game. For instance, if we initially play a game with six players and encounter a situation with only one update in the first slot, followed by multiple updates of three transmissions in the second slot, we are left with only two remaining opportunities to transmit. Consequently, we adjust both the horizon and the probabilities to match the scenario of a game with four players.

We take the probabilities p_3 and p_4 in the game of four players and use their values to determine \dot{p}_3 and \dot{p}_4 and in our game.

3.4.2 Adjustment following the rest of horizon

In this model enhancement, our players adapt to situations where an update is missed in the current time slot t , which corresponds to cases with zero transmission. This information is shared among all players listening to the channel, enabling them to adjust their planning horizon. The adjustment is based on the remaining length of the horizon. Specifically, we calculate the shift and reschedule the rest of the intervals and define the factor to postpone schedule *adjust* as follows:

$$adjust = \frac{1 - horizon[t]}{(n + 1)}$$

This shift is applied from the current until the end of the horizon but without a change at the end of the last interval which remains 1. Players continue to play the game until they do not reach the end.

3.4.3 Adjustment following the initial interval

An alternative approach to enhance the average AoI in instances where transmission is absent in the current time slot t is to introduce adjustments for rescheduling transmission slots based on the initial interval divided by the number of players augmented by one. The change in transmission schedule when an update is missed, introduced in this model, can be expressed as follows by changing the planned schedule for :

$$adjust = \frac{1}{(n + 1)^2}$$

Subsequently, this resulting adjustment is applied from the current interval until the conclusion of the planning horizon, excluding the final interval, which retains its original value of 1.

Nevertheless, akin to its predecessor, this method fails to preclude scenarios where no transmission occurs or where multiple updates take place. While it does signify an enhancement, it does not ensure immunity from these states within the models.

3.4.4 Adjustment to prevent no update case with delay

The preceding adjustments mentioned lack an assurance that transmission will indeed take place in the newly adjusted slot shifted to the right. Consequently, there arises a necessity to devise a mechanism aimed at compelling transmission within the slot should it remain vacant. Upon the realization by other participants of the absence of transmission in the current epoch, a rescheduling process ensues, wherein precisely one player transmits in the current slot after the adjustment. The adaptation of slots involves shifting the current slot accordingly *forced transmission delay*:

$$T_d = t_{\text{detection}} + t_{\text{decision}} + t_{\text{initiation}} \quad (3.15)$$

However, this adaptive mechanism does not prevent us from the scenario when we have multiple updates and waste resources.

Algorithm 2 Pseudocode for adjusting horizon when transmission does not occur

Require:

horizon

n - number of players

k - current milestone

rest number of players

▷ all of these arguments as input

Calculating adjustment according to different approaches:

adjust = *criterion*

i = milestone

Adjust the horizon starting from the current milestone where transmission did not occur until the end, except the last one which stays 1

while $i \leq \text{length}(\text{horizon}) - 1$ **do**

$\text{horizon}[i] += \text{adjust}$

end while

if $\text{horizon}[i] \geq (n/(n+1))$ **then** ▷ we are checking if the last interval is reached; in case it is, we need to transmit because it is the last chance

if $i == 0$ **then**

$\text{aoi} = 0.5 \cdot \text{horizon}[i]^2 + n \cdot 0.5 \cdot (\text{adjust})^2$

else

$\text{aoi} = 0.5 \cdot (\text{horizon}[i] - \text{horizon}[i-1])^2 + (\text{rest}) \cdot 0.5 \cdot (\text{adjust})^2$

end if

end if

return aoi

3.4.5 Adjustment to prevent no update case without delay

This approach bears a striking resemblance to the previous one, with a notable distinction: the absence of any delay. Remarkably, our players possess the capacity to discern, before its

Algorithm 3 Pseudocode for scenarios when transmission happened with horizon and probabilities adjustment in case of multiple updates, and calculating AoI

Require:

horizon

n - number of players

k - current milestone

p - represents the probabilities

rest number of players

updates - how many updates have occurred in the current slot

▷ all of these arguments as input

$i = k$

if $i == 0$ **then**

$aoi = 0.5 \cdot horizon[i]^2$

else

$aoi = 0.5 \cdot (horizon[i] - horizon[i - 1])^2$

end if

▷ Transmission has happened, therefore we need to calculate AoI

In case more than one update happens, we need to re-adjust the horizon, because we face the case when we are left with $n - num_{updates}$ and $n - k$ slots, we have fewer chances to update, therefore, we need to spread evenly rest of the horizons divided with rest opportunities augmented with 1

adjust

Adjust the horizon starting from the next milestone because we had more than one update in the current milestone

if $updates > 1$ *and* $rest > 1$ **then** ▷ check condition if we are left with more than one opportunity and if more than one update happened

for $j = k; j \leq length(horizon) - rest; j++$ **do**

$horizon[j + 1] += adjust$

end for

Recalculate probabilities, for example, if we are left after the first milestone with three opportunities more, then we behave as if we are playing game with four players

end if

if $update == 1$ **then**

$$p_j = \frac{p_j}{1 - p_i} \quad \text{where} \quad i = slot_{current}, \quad i < j \leq n$$

end if

If we are left with only one opportunity or if without any after the current slot, then we need to calculate AoI and finish the game

if $rest == 0$ **then**

$aoi+ = 0.5 \cdot (1 - horizon[i])^2$

end if

if $rest == 1$ **then**

$$adjust = \frac{(1 - horizon[i])}{2} \qquad aoi+ = adjust^2 \qquad i+ = 1$$

end if

Algorithm 4 The main function for adaptive strategies

Require:

p - represents the probabilities

▷ take probabilities as input

$n = \text{len}(p)$

▷ it is equal to the number of players

$\text{rest} = n$

▷ on the beginning we have all players/possibilities to transmit

$\text{interval} = 1/(n + 1)$

▷ initial interval depending on the number of players

n

$\text{horizon} = [\text{interval} + i \cdot \text{interval} \text{ for } i \text{ in range}(n + 1)]$

▷ initial horizon

$i = 0$

▷ we initialize current milestone to be zero

$\text{aoi} = 0$

▷ initialize aoi, our goal to calculate

while ($i < n$ and $\text{rest} > 0$) **do**

We ask each player if it wants to transmit and store it in

updates

▷ how many players want to transmit in the current

slot

if $\text{updates} > 0$ **then**

$\text{rest} - = \text{updates}$

call the function for transmission and calculate AoI

▷ Algorithm 3

$\text{aoi} + = \text{Algorithm 3}$

$i + = 1$

▷ increase milestone

end if

if above condition is not met **then**

call the function for no transmission

▷ Algorithm 2

end if

end while

return aoi

occurrence, the absence of intent to transmit in a specific slot. Consequently, one player, and only one, undertakes the decision to update. This simplified model, in contrast to its predecessor, retains the horizon unchanged, while endowing players with additional insights into the game's dynamics. Thus, we enforce the transmission obligation on a single player should the situation warrant, ensuring the prevention of scenarios characterized by zero updates.

3.4.6 Adjustment to prevent no update case with non-evenly spread intervals

In this effort to enhance the model, we aim to avoid situations where a state with zero updates attempts, compensating it by using intervals depending on the factor α instead of evenly distributed ones.

$$x \cdot \alpha^i, \quad \text{where } 0 \leq i \leq n$$

$$x \cdot \sum_{i=0}^n \alpha^i = 1 \tag{3.16}$$

from where we get x value that depends on sum of geometric serie of α

$$x = \frac{1}{\sum_{i=0}^n \alpha^i} \tag{3.17}$$

While we acknowledge the possibility of multiple updates occurring within a slot, our focus lies in investigating the potential influence of the factor α on such occurrences. We aim to modulate the α factor to observe its impact on multiple updates, thereby altering the distribution of intervals to be either more dispersed initially and denser later, or vice versa.

3.4.7 Adjustment designed to facilitate an ideal scenario within unevenly spread intervals with a cost parameter β

In the latest proposed mechanism, our objective is to enforce precisely one update per time slot, commencing from the onset of the horizon. Through the implementation of this adjustment, we aim to mitigate instances of both zero transmissions and multiple transmissions. This strategy endeavors to instigate comprehensive coordination among participants, albeit at a discernible cost, given the inherent challenge of attaining complete coordination. Thus, we introduce a parameterized cost determined by the factor β using geometric sequences of the

form

$$x \cdot \beta^i \quad \text{where } 0 \leq i \leq n.$$

By modulating the cost with the factor β our intention is to foster full coordination while striving to ascertain the optimal price to pay, enabling us to approach the optimal solution as closely as feasible.

Chapter 4

Results

This section will present the outcomes derived from the model implementation outlined in Chapter 3. Initially, we generated results pertaining to the basic game model, wherein we identified Nash equilibrium points both in pure and mixed strategies. Subsequently, we enriched the model by introducing several adaptive strategies. The first two enhancements focused on modifying the slot by shifting it to the right if no transmission occurred. Following this, we introduced two adaptations where transmission is compelled to occur after a certain delay, either with evenly spread intervals or without delay and with non-evenly spread intervals; both adaptations do not incorporate mechanisms to prevent multiple updates. The final adjustment aimed to foster coordination among players by stipulating that only one transmission is permitted per slot, thereby eliminating scenarios involving no transmission or multiple transmissions, with a cost of β imposed in the latter case.

4.1 Results in pure strategies

To derive results concerning pure strategies, we employed (3.6), which computes the surface area of equilateral triangles with sides measuring $\frac{1}{(n+1)}$, corresponding to the initial interval size, where n denotes the number of players participating in the game. In the scenario of pure strategies, there exist $n + 1$ such triangles. By summing their individual surface areas, we determined the average AoI for a system comprising n agents. Thus, the acquisition of results depicted in Fig. 4.1 for any given number of players, n , is facilitated through this straightforward computation.

In the realm of pure strategies, the Nash equilibrium signifies a set of actions taken by individual players, representing a collection of distinct chosen slots. Within this framework, none of the players possess any incentive to deviate from this equilibrium. In essence, players

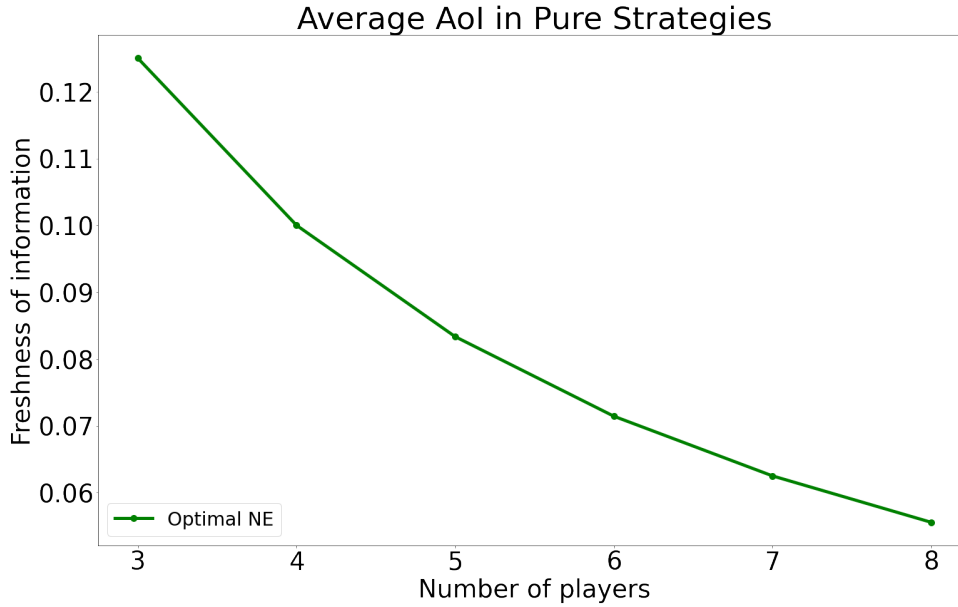


Figure 4.1: Average value of AoI in pure strategies

attain the optimal outcome only when all opt to transmit in unique slots, achieving perfect coordination, or rather, *anti-coordination*. From the insights gleaned from Fig. 4.1, it becomes evident that as the system comprises a greater number of fully coordinated players adhering to the optimal scenario, the overall freshness of information diminishes correspondingly. However, a significant question arises concerning how to come close to this optimal state, because achieving outcomes equivalent to an NE in pure strategies without any form of coordination in a static game is exceedingly challenging, if not impossible in certain contexts[36]. Without coordination mechanisms or communication among players, at least *preplay*, reaching such a state where each player independently selects their optimal strategy can be highly unlikely, particularly in complex environments like networks [48].

4.2 Results in mixed strategies

Implementing the *Indifference theorem* with more than two players in the game presents a more challenging task [56]. This challenge arises because, as the number of players increases, the polynomial degree and the number of variables also increase. Specifically, the polynomial takes the form $(p_1 + p_2 + \dots + p_n)^{(n-1)}$, where p_i represents the probability of a player choosing a certain strategy i , and n is the number of players. Initially, we assumed that players do not distribute their probabilities equally across distinct slots; rather, they are more likely to choose slots closer to the center of the horizon. As a consequence of game symmetry, we have the following equality $p_i = p_{n-i}$, respecting that $\sum_{i=0}^n p_i = 1$.

Our initial approach begins with uniformly distributed probabilities [59], which are subsequently adjusted by decreasing outer probabilities and increasing inner ones. Before advancing to the next stage of probability enhancement, we assess whether the value of $u_1(1, p)$ aligns with the average value of other utilities. If this criterion is not met, we proceed with further iterations, continuing to refine the probabilities. Conversely, if the condition is satisfied, we revert to the previous probabilities and compute the average AoI value in mixed strategies across varying numbers of players.

In practice, probabilities are not uniformly distributed; players show preferences for certain slots over others. However, despite this deviation from uniformity when speaking about probabilities, the average AoI closely approximates the value obtained when utilizing uniformly distributed probabilities from the outset which can be seen in Fig. 4.2. Therefore, we justified that players have preferences among certain slots.

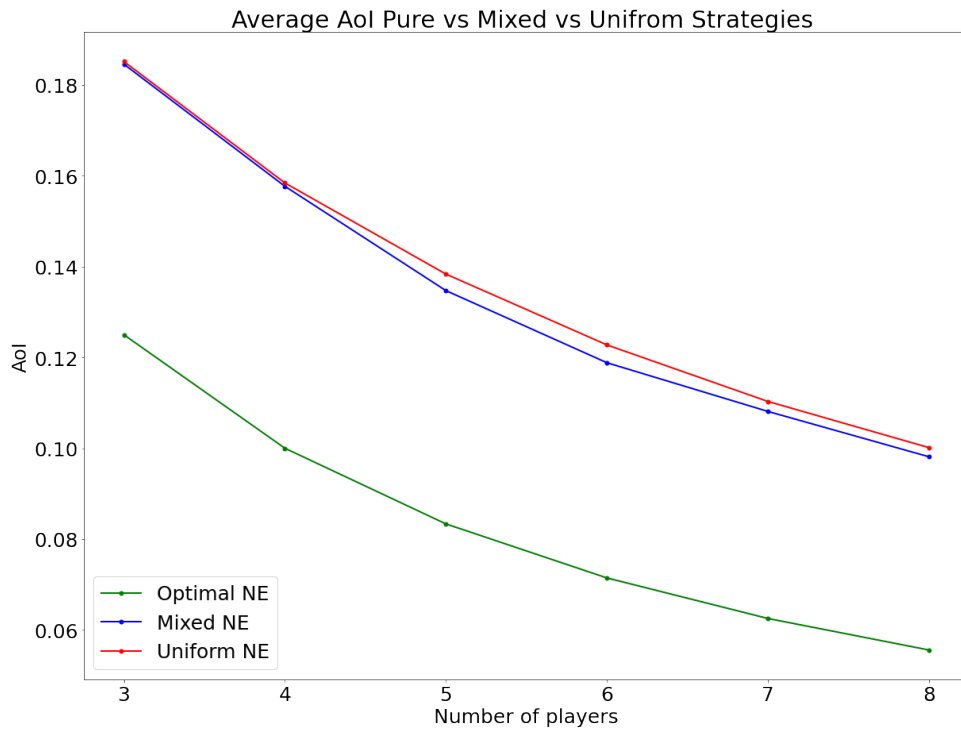


Figure 4.2: Value of average AoI in pure, mixed, and uniform strategies

Figure?? demonstrates the resulting average AoI in scenarios where players are perfectly coordinated and coordinated with probabilities corresponding to the mixed equilibrium across varying numbers of players. The blue curve, representing the Nash equilibrium in mixed strategies, exhibits a similar trend to the optimal solution: as the number of players increases, the average information freshness decreases. However, it diverges from the optimal solution depicted by the grey surface. Equilibria in mixed strategies are known not to be persistent, and in our case, they yield unfavorable values for average information freshness.

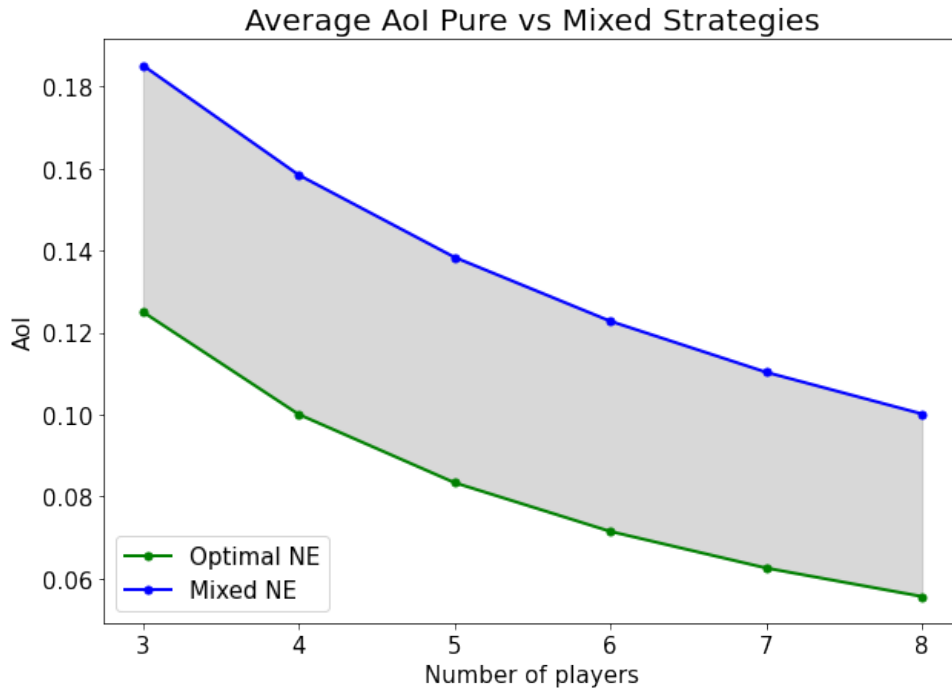


Figure 4.3: Value of average AoI in pure, mixed, and uniform strategies

To avoid symmetrical solutions in the mixed strategies which are more likely in real-life scenarios, introducing *preplay* communication can lead to a more attractive solution. This approach results in a probability mixture of pure Nash equilibria [51], defining a solution known as correlated equilibrium (3.14) [66]. Such a solution is expected to yield a lower value of the AoI compared to that in mixed strategies.

4.3 PoA results

As discussed, PoA refers to the inefficiency that can arise when users in a network act selfishly to optimize their objectives without considering the overall network performance. It quantifies the degradation in network efficiency caused by selfish behavior, compared to an idealized scenario where users cooperate for the common good [60]. Specifically, it examines how network performance metrics AoI which is used in our study is impacted (Fig.4.4) when users independently make decisions, such as selecting slots based solely on their decision not taking into account the actions of other players. In our context, the PoA serves as a metric to quantify the extent to which coordination among players benefits the system. It aids in the development of mechanisms aimed at incentivizing coordination or mitigating its adverse effects through the establishment of *preplay* agreements. These agreements are designed to encourage strategic alignment among players, thereby fostering more favorable outcomes for the system as a whole.

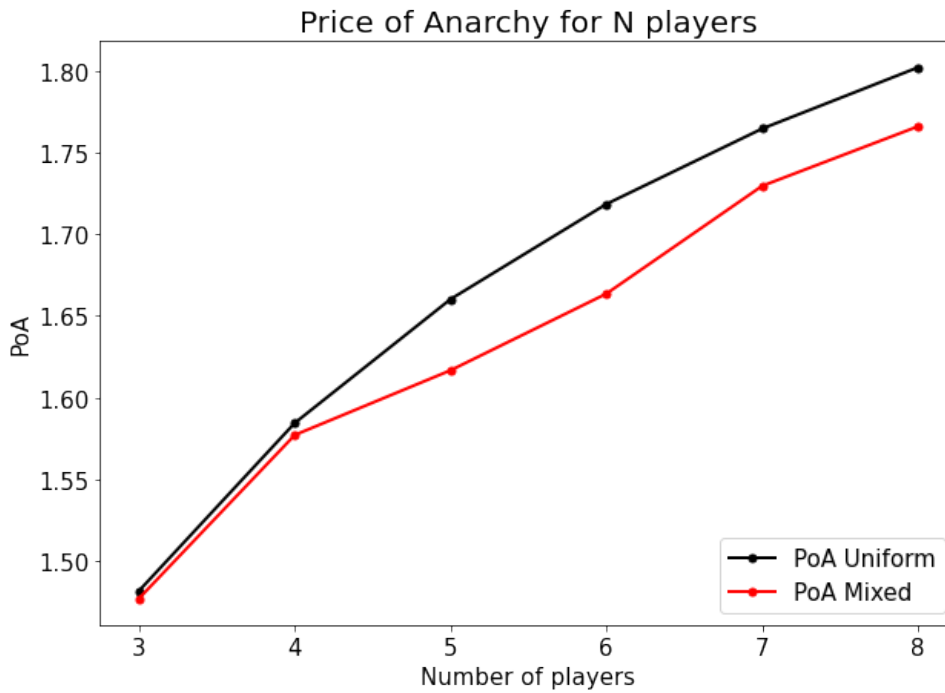


Figure 4.4: PoA for different number of players

Figure 4.4 studies the impact on the network performance metric AoI in the absence of any coordination mechanism, in both, equal and mixed probabilities. Although we acknowledge that players may not engage in the game with equal probabilities, it is crucial to demonstrate the inefficiency of playing the game with equal probabilities. Hence, we also present results when players choose strategies with probabilities of $\frac{1}{n}$. The penalty for non-cooperative behavior escalates with an increasing number of players in the system, which aligns with expectations. In systems with a higher number of players, the likelihood of anarchy arising also increases. Consequently, the upward trend observed in the PoA curve in Fig. 4.4 is anticipated; the penalty incurred for eight players without a coordination mechanism surpasses that of scenarios involving only three players. This observation underscores the escalating inefficiency stemming from individualistic decision-making in larger networks.

Moreover, the trend suggests that as the network scales up, the disparity between optimal and non-cooperative outcomes becomes more pronounced. This indicates the importance of promoting cooperation and coordination among network participants to mitigate the detrimental effects of selfish behavior. Additionally, the results confirm our beliefs that it is necessary to introduce some changes in the model to approach the optimal solution.

4.4 Results for implementing horizon shifting when no update occurs

Firstly, we will delve into the outcomes derived from implementing mechanisms for horizon shifting in scenarios where no transmission occurs. We introduced adjustments dictating how players should adapt to this situation. Our observation suggests that players should not wait until the next milestone to transmit; rather, transmission should occur earlier. The extent to which this shift should occur compared to the planned milestone varies, and we explored two different mechanisms to address this.

In the first scenario, we shifted the horizon according to the remaining time divided by the total number of opportunities increased by one. In the subsequent scenario, we increased the horizon by the initial interval, which corresponds to $\frac{1}{(n+1)}$, and augmented the remaining opportunities by one.

4.4.1 Mechanism following the rest of the horizon

At the onset of each slot, we modeled the players' decision-making process regarding whether to update or not based on their preferences. Subsequently, we implemented various scenarios: no transmission has occurred, only one occurred, or multiple updates. In the scenario where updates have occurred, either one or more, we calculated the AoI and adjusted the number of players accordingly, updating the selection probabilities for subsequent slots. In scenarios where multiple transmissions occurred, we also modified the horizon making it evenly spread after we were left with fewer opportunities to update. Our adjustment mechanism is used to shift the horizon to the right according to the rest of it to achieve better results than the one we have achieved in mixed strategies, playing without any additional protocol.

Upon implementing the first algorithm and conducting 10,000 iterations for different numbers of players, we got the following results:

The yellow curve in Fig. 4.5 illustrates the value achieved after incorporating the possibility for players to adjust the next milestones when no transmission occurs in the current slot. As anticipated, we observe progress following the integration of this adjustment into the model. However, it remains closer to the solution in mixed strategies than to the optimal one depicted in Fig. 4.6. Undoubtedly, by shifting the current horizon to the right and enabling the next transmission to take place there, we witness improvements in the system's performance.

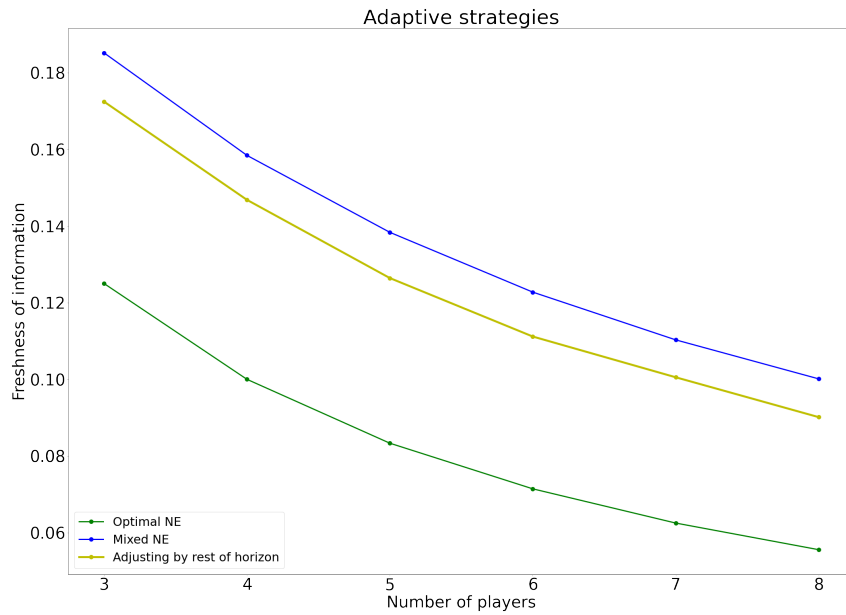
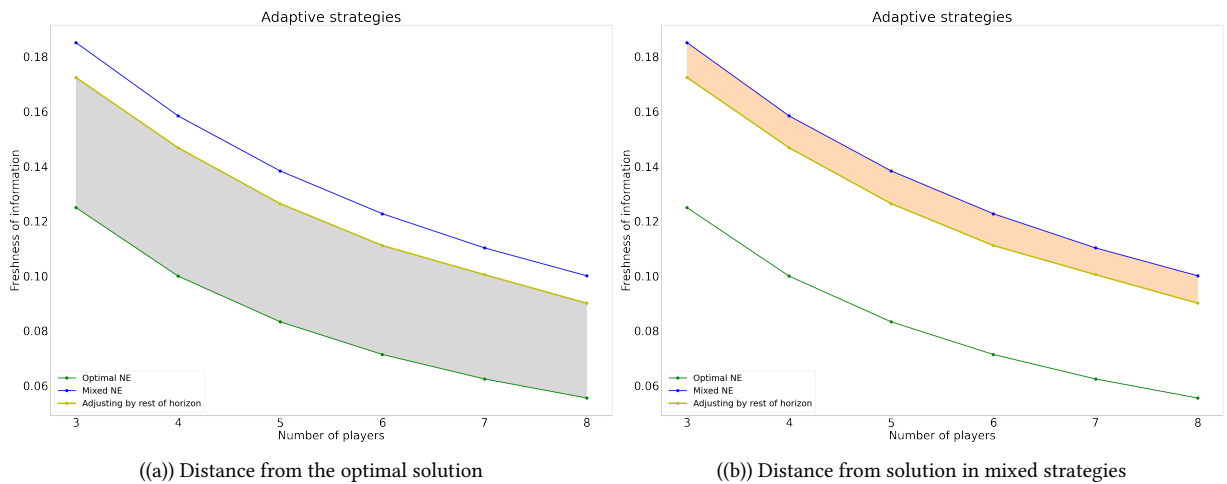


Figure 4.5: AoI after applying mechanism adjustment according rest of horizon shift



((a)) Distance from the optimal solution

((b)) Distance from solution in mixed strategies

Figure 4.6: Difference between optimal and actual Nash equilibrium

4.4.2 Mechanism following the initial interval

After introducing the next improvement, which aims to reorganize the horizon when transmission does not occur, such that the opportunity for the next update is shifted by the initial interval divided by the total number of players increased by 1, we maintained the same behavior in scenarios where multiple or only one update occurs. Upon running the code for 10,000 iterations, we got the following results:

In Fig. 4.9, we observe a clear improvement with the introduction of the adjustment in the model, positioning the performance almost midway between the upper bound, represented by mixed NE, and the lower bound, an optimal NE. However, despite adapting the model to react in this manner, we find ourselves much closer to the worst-case scenario depicted by the red surface in Fig. 4.8(a) than to the optimal one. If the cost of implementing this adaptive

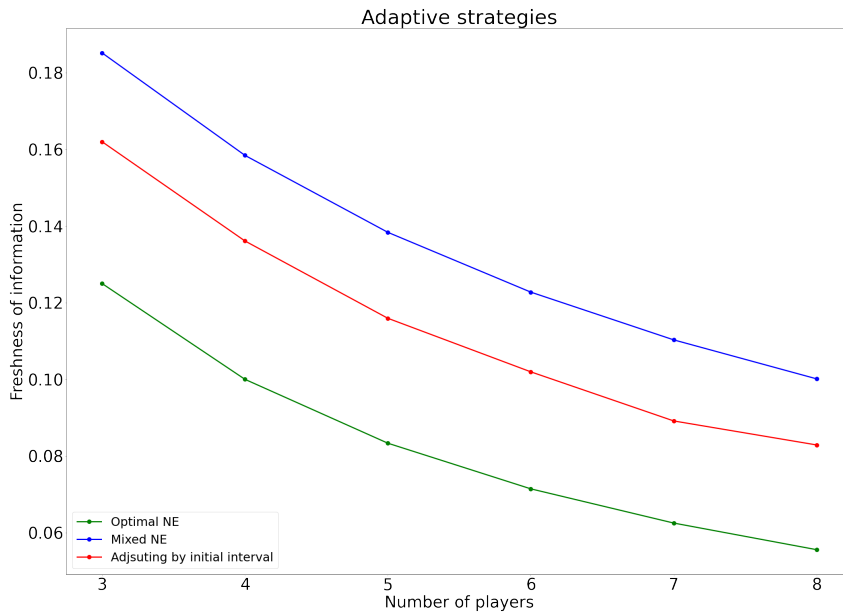


Figure 4.7: AoI after applying mechanism adjustment according initial interval shift

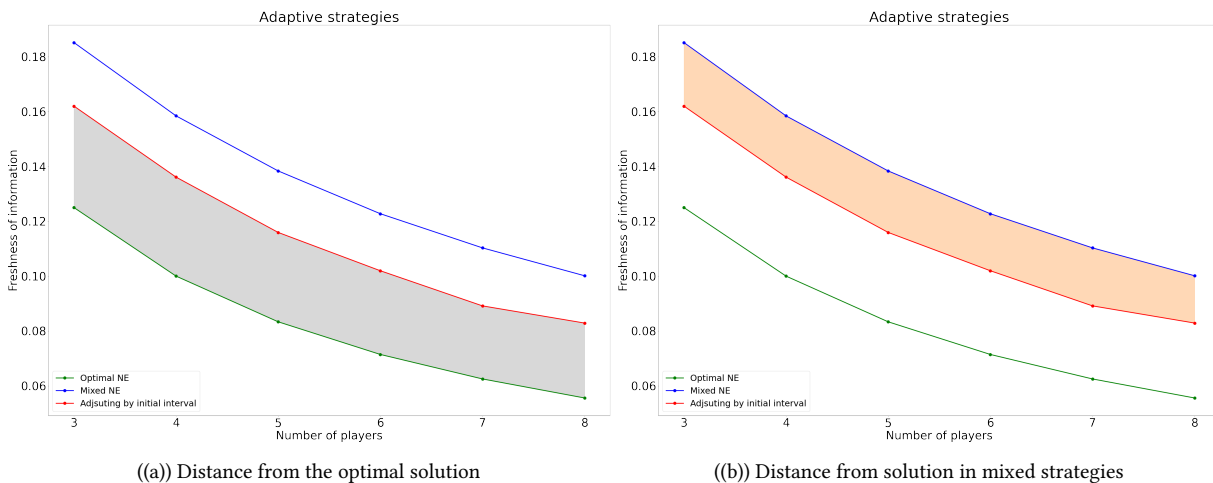


Figure 4.8: Difference between optimal and actual Nash equilibrium

behavior in the model is not prohibitive, it still presents a promising solution.

4.4.3 Comparative analysis of the first group adjustment mechanisms

Both introduced mechanisms exhibit improvements, but adjusting by taking the initial interval and dividing it by $(n + 1)$ proves to be a significantly better approach. Shifting the next possibility to adjust just slightly before the next slot may not be the most effective mechanism to apply. However, if no better alternative is available, it should still be considered. With the other adjustment, we find average data freshness closer to the middle of the bounds. If the only agreed-upon action at the outset of the game is to react in case transmission does not occur by recalculating new slots for potential updates, then it becomes imperative to consider alternative mechanisms. However, it remains of interest to explore whether further mechanisms

can enhance these results or not.

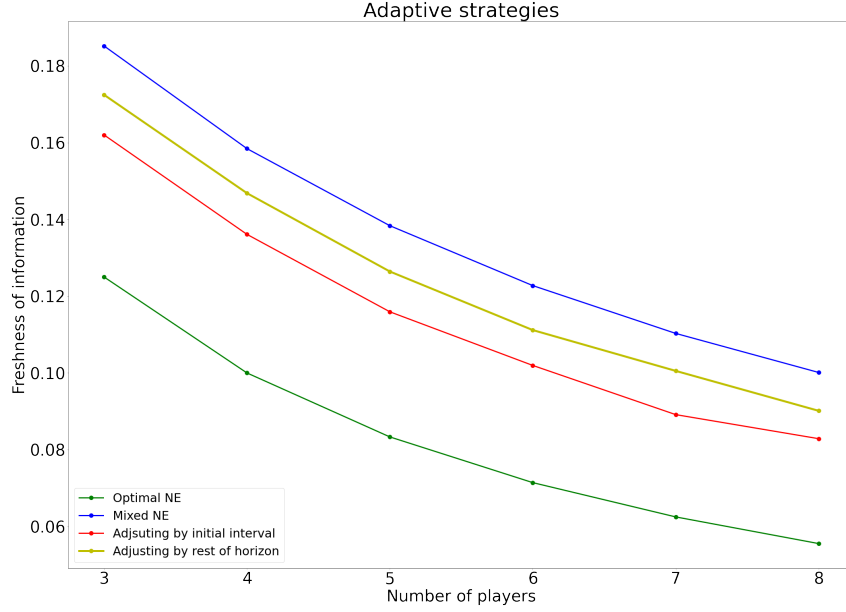


Figure 4.9: AoI after applying mechanism for shifting slots when no update occurs

4.5 Results for mechanisms that prevent cases of non - transmission

Within this category of mechanisms, the aim is to avoid the potential for transmission failure meaning slots without any update, thereby ensuring that every time slot is actively utilized. The sole admissible states within this framework encompass either singular or multiple updates occurring within the allocated slot. Various iterations of these models have been introduced, each entailing distinct adaptations of scheduling algorithms aimed at mitigating instances of idle time slots.

4.5.1 Mechanism to prevent no update case with delay

The initial approach entails adjusting the temporal scope to accommodate the *forced transmission time*, which encompasses detection, decision-making, and initialization intervals, followed by the computation of information age. For experimental validation, a delay of 0.01 was designated. Upon executing the algorithm for 10,000 iterations, the ensuing outcomes were recorded and analyzed.

After implementing this approach, we observe notable progress 4.10. The results now fall within the range between the upper and lower limits, but they are closer to the lower limit,

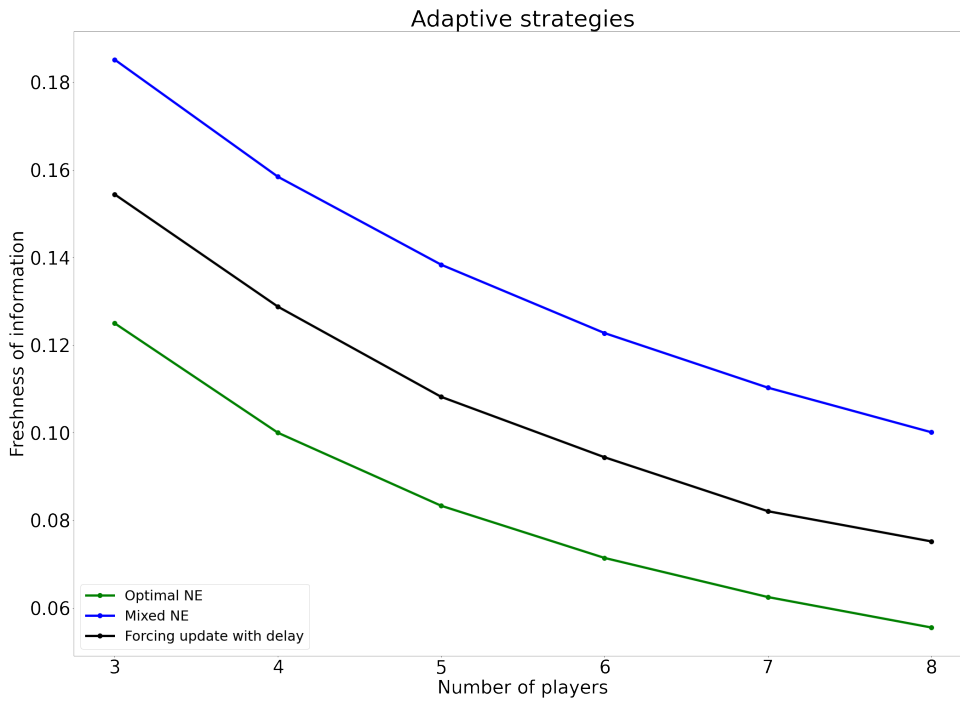


Figure 4.10: AoI after applying mechanism to force transmission with delay in the case when no update occurs

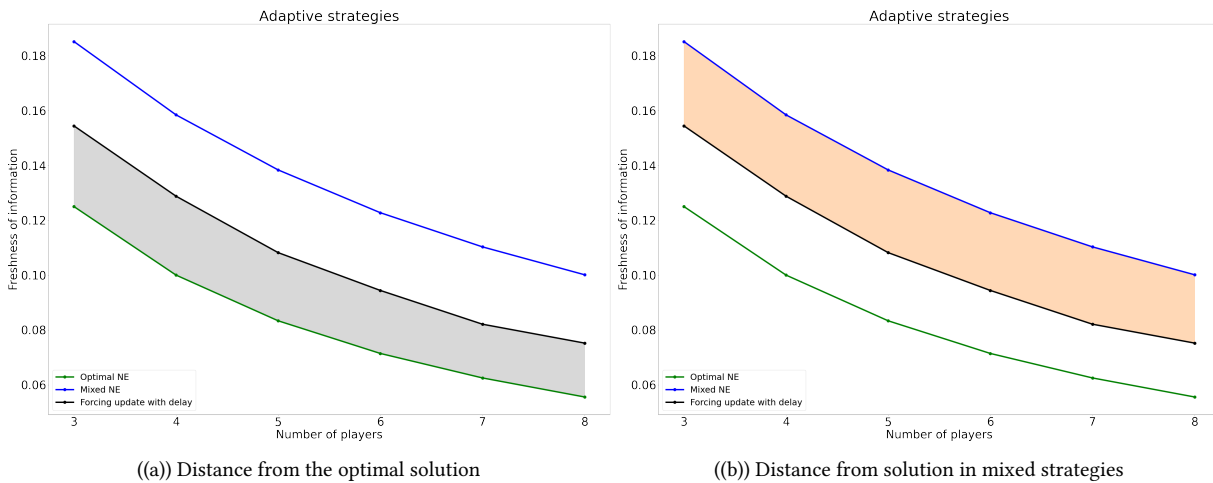


Figure 4.11: Difference between optimal and actual Nash equilibrium

which corresponds to the optimal result. This inference is also supported by the shaded areas depicted in the graphs Fig. 4.13. We can see if the *forced delay time* remains sufficiently low and allows players to react in the event of non-transmission, thereby enabling the selection of a singular player willing to transmit, this approach proves advantageous. Facilitating communication among players and establishing coordination in these scenarios, lowers average AoI.

4.5.2 Mechanism to prevent no update case without delay

In this adaptation, we circumvent the occurrence of a transmission-less scenario by employing a mechanism that iteratively prompts players to update until a participant opts to do so. While

this strategy effectively precludes the loss of transmission opportunities due to empty slots, it lacks resilience against the occurrence of multiple updates within the same interval. After 10 000 we have the following results:

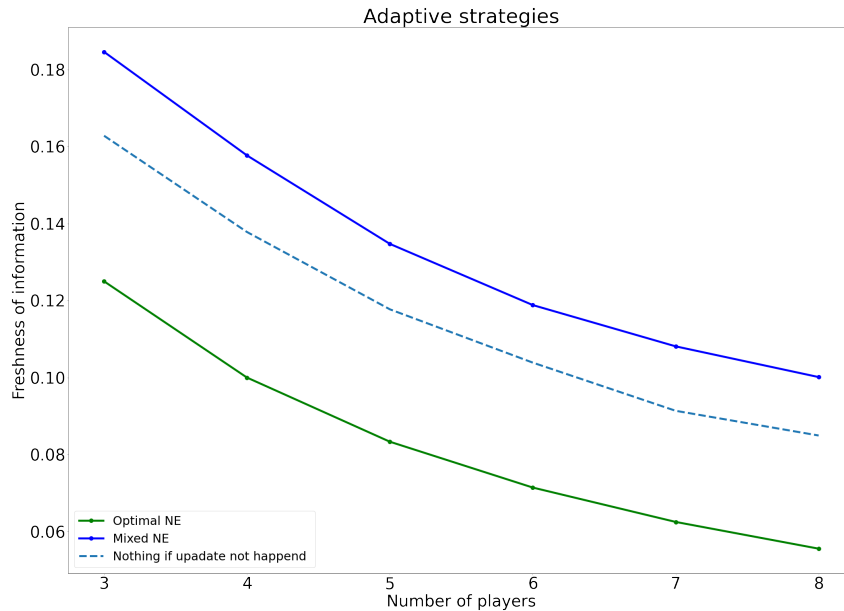


Figure 4.12: AoI after applying mechanism avoid no transmission state

We can deduce that the introduced mechanism while demonstrating an enhancement over a model lacking responsiveness, is not the optimal solution for minimizing the average age of data. Despite observing an improvement compared to a non-responsive model, the average age (Fig. 4.12) of data computed using this mechanism tends to align more closely with the upper bound rather than achieving optimal performance. In comparison to the preceding mechanism, this approach yields inferior results.

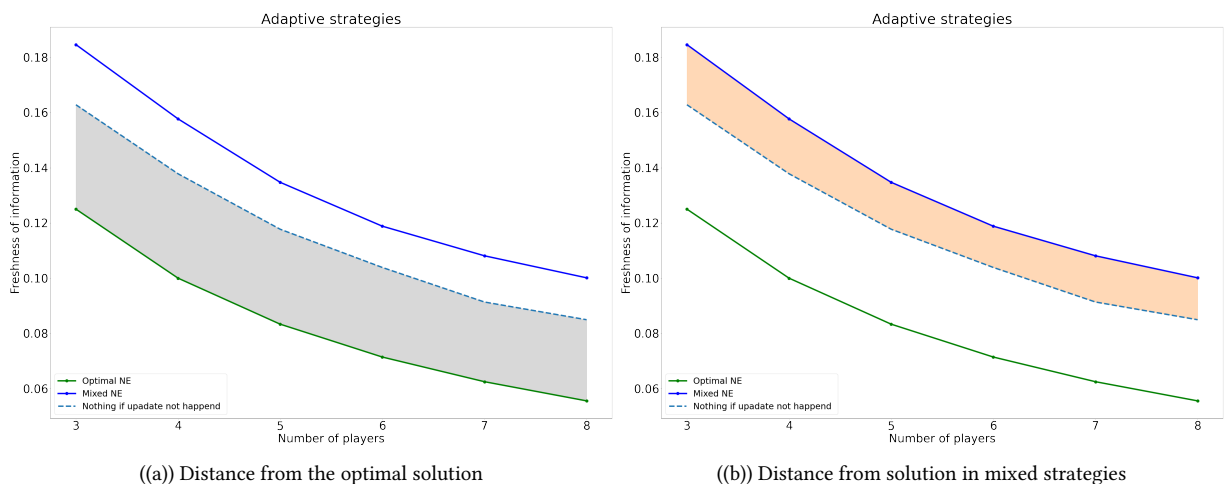


Figure 4.13: Difference between optimal and actual Nash equilibrium

4.5.3 Mechanism to prevent no update case with non-evenly spread intervals

This mechanism endeavors to address the absence of transmission scenarios by introducing unequally distributed transmission intervals. The degree of unevenness in the interval distribution is contingent upon the alpha factor, which, in practical terms, may signify the trade-off incurred by implementing this mechanism. Specifically, in network contexts characterized by limited energy resources, where energy is a scarce commodity, the alpha factor may encapsulate the additional energy consumption associated with this approach. Thus, a pertinent inquiry arises regarding the threshold values of the alpha factor deemed acceptable in exchange for achieving coordination.

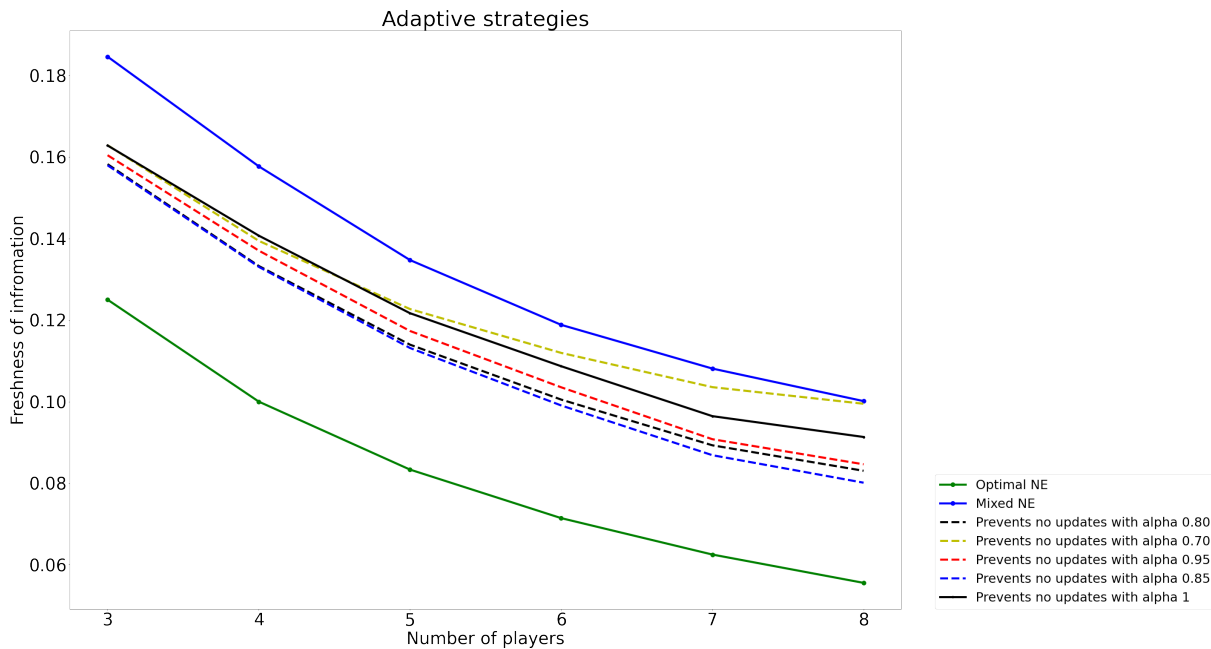


Figure 4.14: AoI after applying mechanism avoid no transmission state

Observing the graphical representation Fig. 4.14, it becomes evident that the alpha factor significantly influences the degree of improvement attained. Upon integrating this adaptive mechanism, optimal enhancements are observed at alpha values of 0.85 and 0.80. For larger disparities in the alpha factor ($\alpha=0.70$), it becomes evident that the results converge towards values similar to those observed in mixed strategies for more players in the game. Interestingly, optimal outcomes are achieved at an alpha factor of 0.80 compared to 0.95. Furthermore, when no alpha price is introduced (corresponding to $\alpha = 1$), and intervals are not distributed evenly as more transmissions occur, the results deteriorate compared to scenarios where a penalty factor of $\alpha = 0.95$ or 0.80 is applied. However, excessive deviation in the distribution of intervals leads to less favorable outcomes. These observations highlight

the delicate choice in price alpha we are supposed to pay for achieving this preplay agreement.

4.5.4 Comparative analysis of the second group mechanisms

Upon comparing all mechanisms within this group, it becomes apparent that the most favorable outcomes are achieved by implementing a forced transmission delay in instances without transmission, coupled with precisely one update occurring within each time slot. In contrast, the approach of prompting players for updates until at least one participant commits to transmission proves to be less effective, regardless of whether it involves no delay introduction, equal interval distribution, or the incorporation of an alpha-dependent pricing mechanism dictating uneven interval settings. Among the latter two mechanisms, superior performance is observed when the alpha factor exhibits minimal deviation, particularly at values of 0.80 or 0.85. Thus, it is evident that the strategy of awaiting player decisions for updates is comparatively less efficient (Fig. 4.15), underscoring the advantage of mechanisms that enforce precisely one transmission with a specified delay.

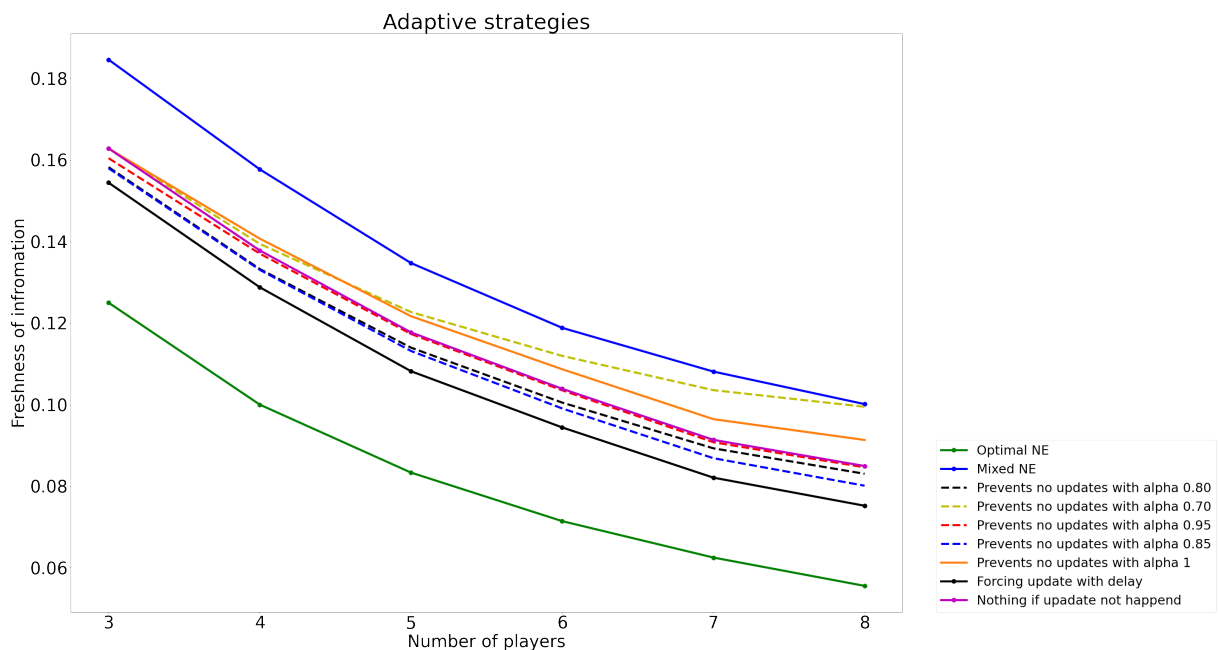


Figure 4.15: AoI after applying mechanism to prevent no update state

4.6 Results for a mechanism designed to facilitate an ideal scenario within unevenly spread intervals, considering a cost parameter β

In the last introduced mechanism, the objective is to preempt all unfavorable scenarios, specifically those involving zero or multiple updates, by instating coordination from the outset of the game. Even in the scenario of complete coordination, or what we term *anti-coordination* in our context, which represents the ideal gameplay approach, a cost is incurred for enforcing this predefined rule, denoted by the parameter β . The beta factor dictates the degree of unevenness in interval distribution, where values below one indicate a prevalence of evenly distributed intervals at the outset. Conversely, values exceeding one signify a scenario where intervals are more densely concentrated initially. When β assumes an exact value of one, we depart from the ideal scenario, albeit acknowledging that the beta factor necessitates a deviation from unity to elicit distinct behavioral patterns within the model.

Observing the impact of varying beta values on the system's average AoI, we note that for a marginal deviation from unity, such as $\beta = 0.95$, indicative of minimal departure from equally distributed intervals, the resultant average age closely approximates the ideal value. However, as the deviation increases, exemplified by $\beta = 0.85$ or $\beta = 1.15$ – representing equivalent deviations but in opposing directions—we observe slightly superior outcomes with $\beta = 1.15$. This suggests that, for a consistent degree of deviation, the mechanism performs better when initial intervals are more densely distributed, followed by narrower subsequent intervals. This trend becomes more pronounced with larger deviations, such as $\beta = 0.70$ and $\beta = 1.30$, where the discrepancy in results intensifies due to significantly unequal intervals, with wider initial intervals followed by substantially narrower subsequent ones, or vice versa in the case of $\beta = 1.30$. Consequently, we can infer that this mechanism yields favorable outcomes when beta exhibits minimal deviation, and the gameplay entails slightly broader initial intervals followed by marginally narrower subsequent ones (Fig. 4.16).

4.7 Comparative analysis of proposed adjustments

To facilitate comparative analysis across all mechanisms, we introduce an efficiency ratio, comparing the performance of each adaptive approach against the optimal solution. The initial graphical representation delineates the efficacy of various strategies. Notably, leveraging

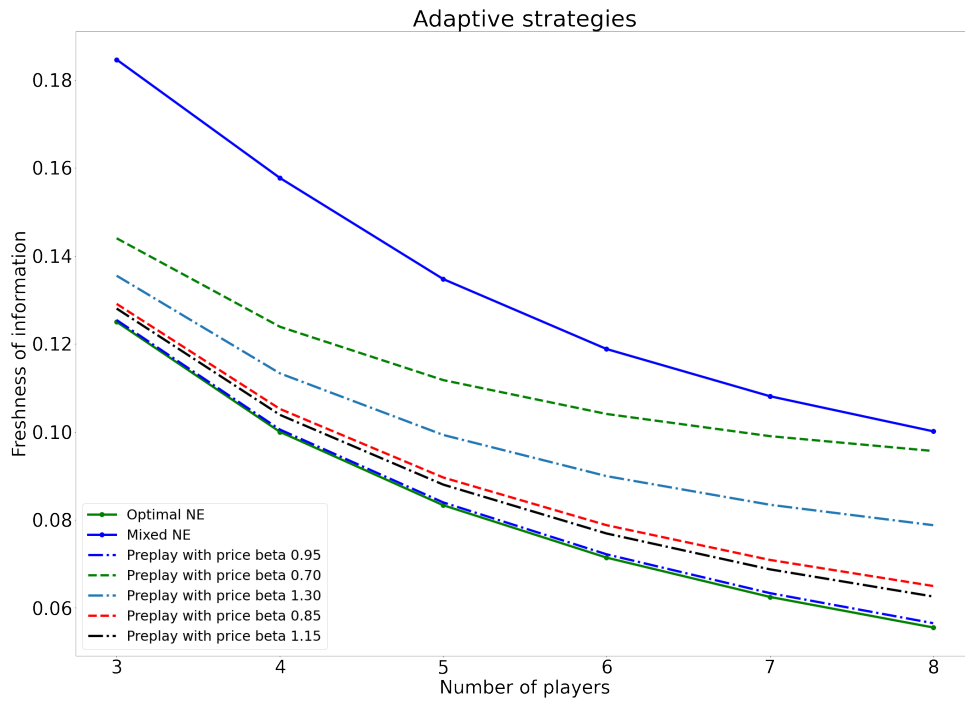


Figure 4.16: AoI after applying mechanism to apply preplay communication with different β factor

pre-game communication opportunities [51] exhibits the potential for enhancing the average AoI. The introduction of diverse tiebreaker mechanisms yields varying degrees of efficiency (Fig.4.17). Particularly, the incorporation of a beta factor set at beta=0.95 emerges as the closest approximation to the optimal solution, thus constituting a preferred choice if feasible for implementation. However, constrained by network limitations, alternative beta factor values such as 1.15 or 0.85 warrant consideration. Subsequently, the mechanism enforcing transmission with slight delay in the absence thereof presents a viable intermediate solution. While other mechanisms demonstrate varying degrees of efficiency relative to the mixed strategies solution, they generally gravitate towards this benchmark rather than the optimal one. The efficacy of mechanisms contingent upon the alpha and beta factors hinges significantly on the prudent selection of these parameters. Hence, careful consideration is imperative in selecting these factors, ensuring that the associated costs lead to improved rather than compromised solutions.

The figures (Fig. 4.19, 4.20, 4.21) show the efficiency ratio by the groups of introduced mechanisms.

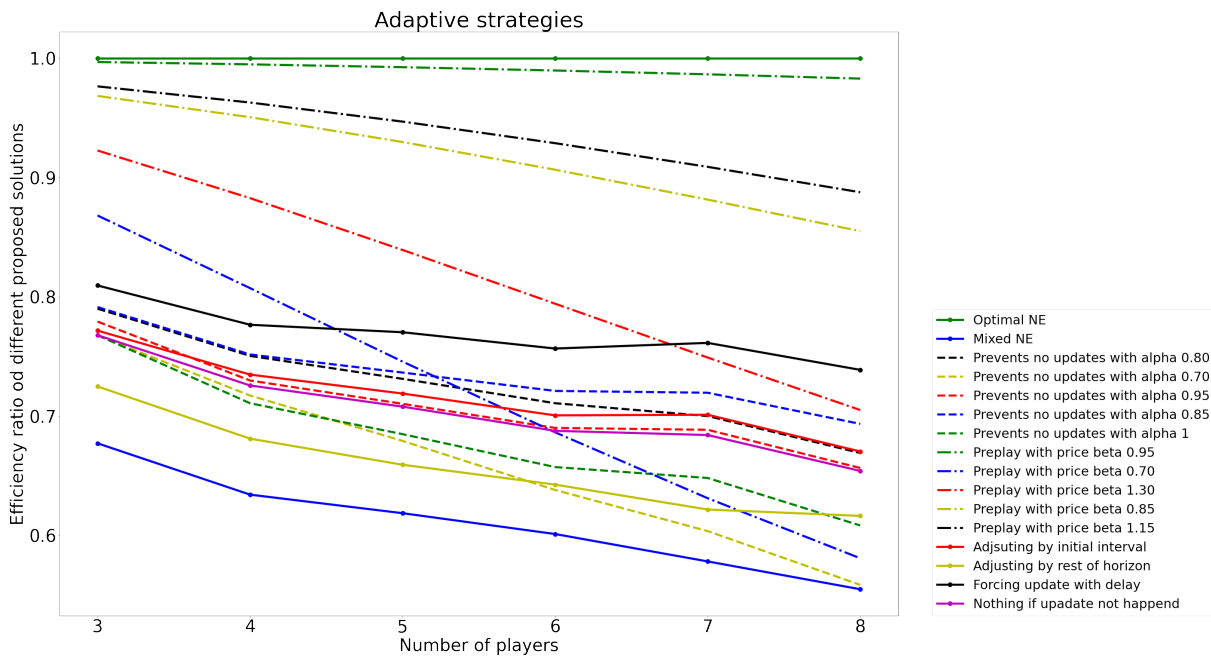


Figure 4.17: Efficiency ratio choosing different adaptive machanisms

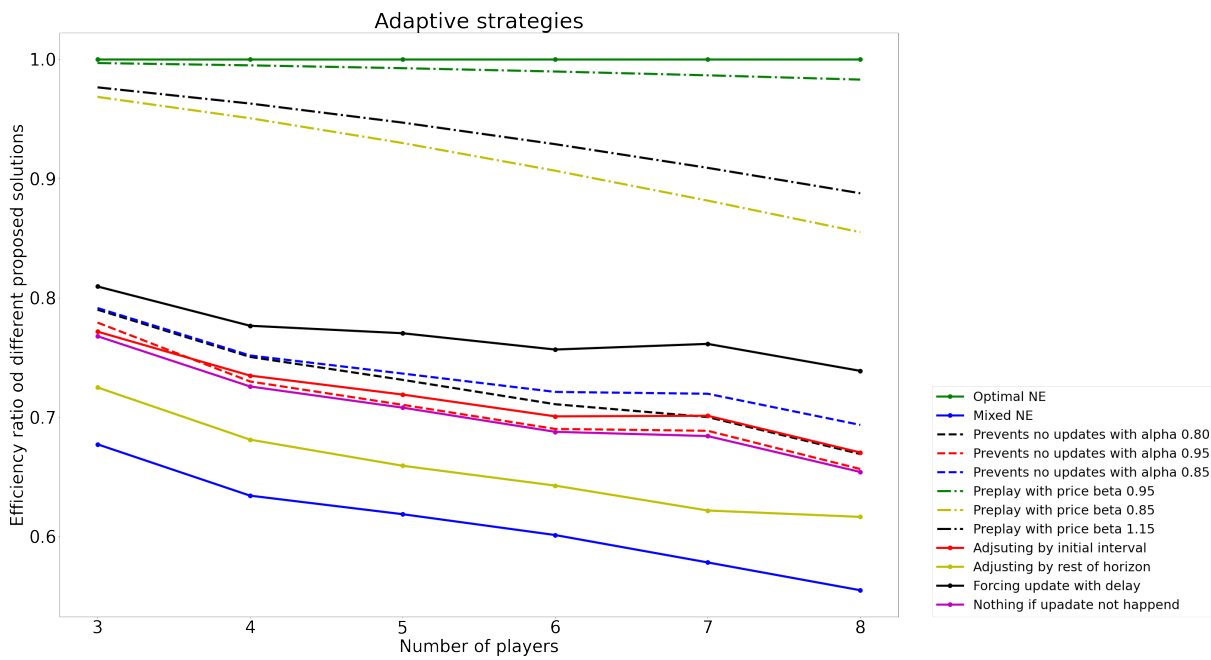


Figure 4.18: Simplified presentation of efficiency ratios for different strategies

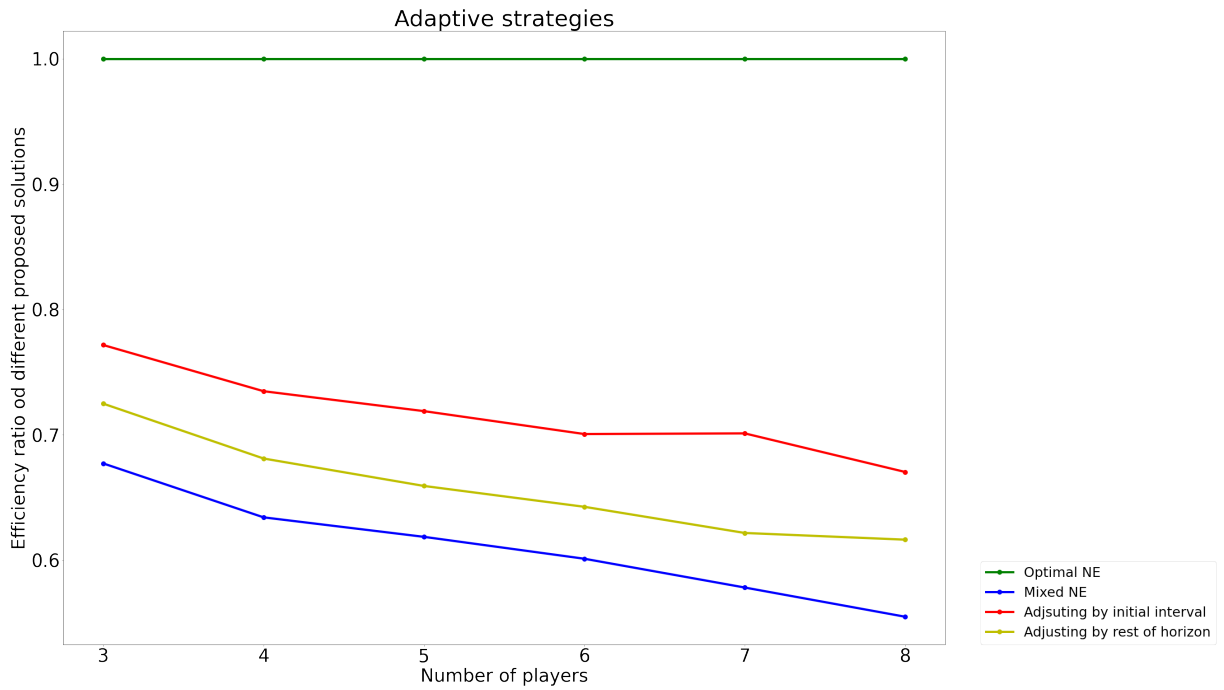


Figure 4.19: Efficiency ratio for the first group of adaptive mechanisms

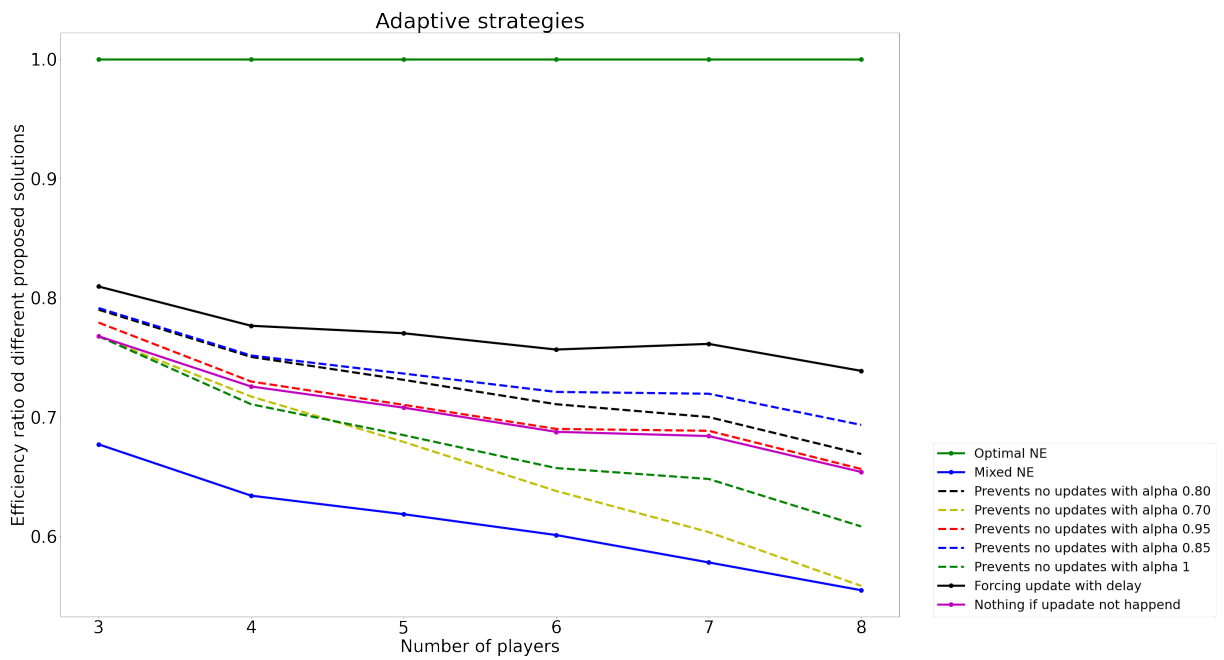


Figure 4.20: Efficiency ratio for the second group of adaptive mechanisms

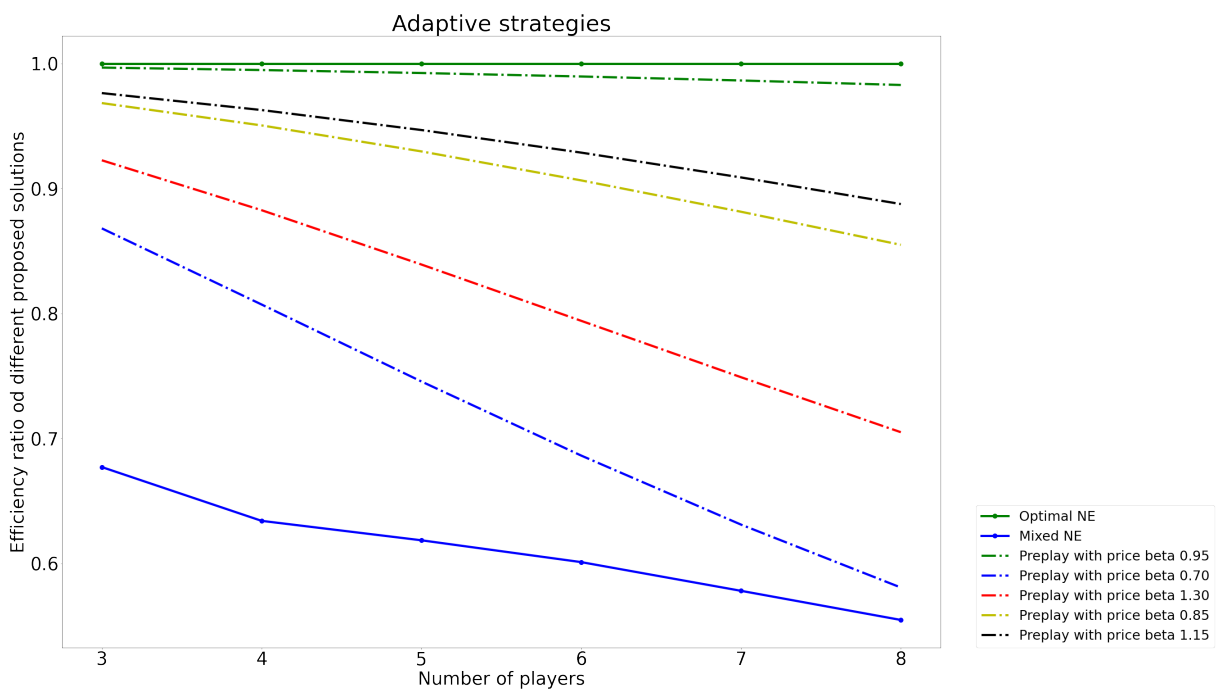


Figure 4.21: Efficiency ratio for the third group of adaptive mechanisms

Chapter 5

Conclusions and Future Work

Our analysis has revealed valuable insights into the effectiveness of various adaptive mechanisms for improving the average age of information in communication systems. We have demonstrated that leveraging pre-game communication opportunities holds promise for enhancing performance and finding more *persistent* equilibrium [51]. Obtain results have shown that adaptation mechanisms yield more favorable outcomes compared to scenarios devoid of pre-game agreements. This observation resonates with Harsanyi's theory, which underscores the significance of strategic coordination in achieving optimal results.

Three distinct categories of mechanisms were introduced: first, those rescheduling horizons to react on cases without transmission; second, those mandating at least one transmission per slot, either with a minimal delay or through iterative player prompting; and finally, those enforcing singular transmissions per slot while modulating interval distribution via the beta factor which represents the cost of this solution. Our findings underscore the efficacy of these mechanisms, particularly noting the favorable outcomes observed with a beta factor of 0.95.

However, it is imperative to acknowledge the importance of addressing network constraints and associated costs when parameterizing these mechanisms. Prudent calibration of alpha and beta factors is indispensable for optimal performance, as significant deviations may yield suboptimal results. Notably, the mechanism enforcing delayed transmission has shown promising results, warranting further exploration across a spectrum of delay values.

Future research avenues include investigating additional mechanisms, such as those inducing full coordination with random number selection among players, while carefully managing scenarios prone to multiple updates. Additionally, exploring alternative probability adjustment techniques, perhaps leveraging machine learning or reinforcement learning approaches[36], is warranted, especially in scenarios where transmission probabilities vary across slots.

Furthermore, incorporating constraints such as collisions or unsuccessful updates[9, ?], as well as considering decision costs associated with transmission [47], could enrich our understanding of system behavior. Additionally, exploring the impact of exponential packet processing [16, 19, 20] compared to instantaneous processing would offer valuable insights.

In conclusion, our study underscores the significance of pre-game communication in scenarios lacking established coordination. By enabling the sources to monitor the channel and proactively respond to observed conditions, our proposed mechanisms demonstrate significant improvements, resulting in a lower AoI. Mathematically, these outcomes exhibit greater stability and yield lower objective function values than the symmetric solution in mixed strategies when coordination or pre-agreement is absent. Despite inherent limitations, our findings offer valuable insights into the design and optimization of communication systems, emphasizing the critical role of *preplay* communication in non-cooperative scenarios to uphold data freshness within the network.

Bibliography

- [1] Rafiullah Khan, Sarmad Ullah Khan, Rifaqat Zaheer, and Shahid Khan. Future internet: the internet of things architecture, possible applications and key challenges. In *2012 10th international conference on frontiers of information technology*, pages 257–260. IEEE, 2012.
- [2] Hermann Kopetz and Wilfried Steiner. Internet of things. In *Real-time systems: design principles for distributed embedded applications*, pages 325–341. Springer, 2022.
- [3] Neha Sharma, Madhavi Shamkuwar, and Inderjit Singh. The history, present and future with iot. *Internet of things and big data analytics for smart generation*, pages 27–51, 2019.
- [4] Prabodini Semasinghe, Setareh Maghsudi, and Ekram Hossain. Game theoretic mechanisms for resource management in massive wireless iot systems. *IEEE Communications Magazine*, 55(2):121–127, 2017.
- [5] Alberto Zancanaro, Giulia Cisotto, and Leonardo Badia. Tackling age of information in access policies for sensing ecosystems. *Sensors*, 23(7):3456, 2023.
- [6] Zhaolong Ning, Peiran Dong, Xiaojie Wang, Xiping Hu, Lei Guo, Bin Hu, Yi Guo, Tie Qiu, and Ricky YK Kwok. Mobile edge computing enabled 5g health monitoring for internet of medical things: A decentralized game theoretic approach. *IEEE Journal on Selected Areas in Communications*, 39(2):463–478, 2020.
- [7] Umberto Michieli and Leonardo Badia. Game theoretic analysis of road user safety scenarios involving autonomous vehicles. In *2018 IEEE 29th Annual International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC)*, pages 1377–1381. IEEE, 2018.
- [8] Leonardo Badia and Andrea Munari. A game theoretic approach to age of information in modern random access systems. In *2021 IEEE Globecom Workshops (GC Wkshps)*, pages 1–6. IEEE, 2021.

- [9] Andrea Munari and Leonardo Badia. The role of feedback in aoi optimization under limited transmission opportunities. In *GLOBECOM 2022-2022 IEEE Global Communications Conference*, pages 1972–1977. IEEE, 2022.
- [10] Leonardo Badia and Andrea Munari. Discounted age of information for networks of constrained devices. In *2022 20th Mediterranean Communication and Computer Networking Conference (MedComNet)*, pages 43–46. IEEE, 2022.
- [11] Elif Tuğçe Ceran, Deniz Gündüz, and András György. Average age of information with hybrid arq under a resource constraint. *IEEE Transactions on Wireless Communications*, 18(3):1900–1913, 2019.
- [12] Elvina Gindullina, Leonardo Badia, and Deniz Gündüz. Age-of-information with information source diversity in an energy harvesting system. *IEEE Transactions on Green Communications and Networking*, 5(3):1529–1540, 2021.
- [13] Leonardo Badia, Andrea Zanella, and Michele Zorzi. Game theoretic analysis of age of information for slotted aloha access with capture. In *IEEE INFOCOM 2022-IEEE Conference on Computer Communications Workshops (INFOCOM WKSHPS)*, pages 1–6. IEEE, 2022.
- [14] Leonardo Badia. Age of information from two strategic sources analyzed via game theory. In *2021 IEEE 26th International Workshop on Computer Aided Modeling and Design of Communication Links and Networks (CAMAD)*, pages 1–6. IEEE, 2021.
- [15] Igor Kadota, Elif Uysal-Biyikoglu, Rahul Singh, and Eytan Modiano. Minimizing the age of information in broadcast wireless networks. In *2016 54th Annual Allerton Conference on Communication, Control, and Computing (Allerton)*, pages 844–851. IEEE, 2016.
- [16] Roy D Yates and Sanjit Kaul. Real-time status updating: Multiple sources. In *2012 IEEE International Symposium on Information Theory Proceedings*, pages 2666–2670. IEEE, 2012.
- [17] Baran Tan Bacinoglu and Elif Uysal-Biyikoglu. Scheduling status updates to minimize age of information with an energy harvesting sensor. In *2017 IEEE international symposium on information theory (ISIT)*, pages 1122–1126. IEEE, 2017.
- [18] Maice Costa, Marian Codreanu, and Anthony Ephremides. On the age of information in status update systems with packet management. *IEEE Transactions on Information Theory*, 62(4):1897–1910, 2016.

- [19] Sanjit Kaul, Roy Yates, and Marco Gruteser. Real-time status: How often should one update? In *2012 Proceedings IEEE INFOCOM*, pages 2731–2735. IEEE, 2012.
- [20] Longbo Huang and Eytan Modiano. Optimizing age-of-information in a multi-class queueing system. In *2015 IEEE international symposium on information theory (ISIT)*, pages 1681–1685. IEEE, 2015.
- [21] Maice Costa, Marian Codreanu, and Anthony Ephremides. Age of information with packet management. In *2014 IEEE International Symposium on Information Theory*, pages 1583–1587. IEEE, 2014.
- [22] Sanjit K Kaul, Roy D Yates, and Marco Gruteser. Status updates through queues. In *2012 46th Annual conference on information sciences and systems (CISS)*, pages 1–6. IEEE, 2012.
- [23] Clement Kam, Sastry Kompella, Gam D Nguyen, Jeffrey E Wieselthier, and Anthony Ephremides. On the age of information with packet deadlines. *IEEE Transactions on Information Theory*, 64(9):6419–6428, 2018.
- [24] Xianwen Wu, Jing Yang, and Jingxian Wu. Optimal status update for age of information minimization with an energy harvesting source. *IEEE Transactions on Green Communications and Networking*, 2(1):193–204, 2017.
- [25] Baran Tan Bacinoglu, Elif Tugce Ceran, and Elif Uysal-Biyikoglu. Age of information under energy replenishment constraints. In *2015 Information Theory and Applications Workshop (ITA)*, pages 25–31. IEEE, 2015.
- [26] Jing Yang, Xianwen Wu, and Jingxian Wu. Optimal online sensing scheduling for energy harvesting sensors with infinite and finite batteries. *IEEE Journal on Selected Areas in Communications*, 34(5):1578–1589, 2016.
- [27] Roy D Yates. Lazy is timely: Status updates by an energy harvesting source. In *2015 IEEE International Symposium on Information Theory (ISIT)*, pages 3008–3012. IEEE, 2015.
- [28] Orhan Tahir Yavascan and Elif Uysal. Analysis of slotted aloha with an age threshold. *IEEE Journal on Selected Areas in Communications*, 39(5):1456–1470, 2021.
- [29] Doga Can Atabay, Elif Uysal, and Onur Kaya. Improving age of information in random access channels. In *IEEE INFOCOM 2020-IEEE Conference on Computer Communications Workshops (INFOCOM WKSHPS)*, pages 912–917. IEEE, 2020.

- [30] Chengzhang Li, Shaoran Li, Yongce Chen, Y Thomas Hou, and Wenjing Lou. Aoi scheduling with maximum thresholds. In *IEEE INFOCOM 2020-IEEE Conference on Computer Communications*, pages 436–445. IEEE, 2020.
- [31] Antzela Kosta, Nikolaos Pappas, Vangelis Angelakis, et al. Age of information: A new concept, metric, and tool. *Foundations and Trends® in Networking*, 12(3):162–259, 2017.
- [32] Yin Sun, Elif Uysal-Biyikoglu, Roy D Yates, C Emre Koksal, and Ness B Shroff. Update or wait: How to keep your data fresh. *IEEE Transactions on Information Theory*, 63(11):7492–7508, 2017.
- [33] Elvina Gindullina, Leonardo Badia, and Deniz Gündüz. Average age-of-information with a backup information source. In *2019 IEEE 30th Annual International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC)*, pages 1–6. IEEE, 2019.
- [34] Ahmed Arafa, Jing Yang, and Sennur Ulukus. Age-minimal online policies for energy harvesting sensors with random battery recharges. In *2018 IEEE international conference on communications (ICC)*, pages 1–6. IEEE, 2018.
- [35] Ahmed Arafa, Jing Yang, Sennur Ulukus, and H Vincent Poor. Age-minimal transmission for energy harvesting sensors with finite batteries: Online policies. *IEEE Transactions on Information Theory*, 66(1):534–556, 2019.
- [36] Giacomo Bacci, Samson Lasaulce, Walid Saad, and Luca Sanguinetti. Game theory for networks: A tutorial on game-theoretic tools for emerging signal processing applications. *IEEE Signal Processing Magazine*, 33(1):94–119, 2015.
- [37] Yu Liu, Cristina Comaniciu, and Hong Man. A bayesian game approach for intrusion detection in wireless ad hoc networks. In *Proceeding from the 2006 workshop on Game theory for communications and networks*, pages 4–es, 2006.
- [38] Valentina Vadori, Maria Scalabrin, Anna V Guglielmi, and Leonardo Badia. Jamming in underwater sensor networks as a bayesian zero-sum game with position uncertainty. In *2015 IEEE Global Communications Conference (GLOBECOM)*, pages 1–6. IEEE, 2015.
- [39] Anna V Guglielmi and Leonardo Badia. Bayesian game analysis of a queueing system with multiple candidate servers. In *2015 IEEE 20th International Workshop on Computer Aided Modelling and Design of Communication Links and Networks (CAMAD)*, pages 85–90. IEEE, 2015.

- [40] Xiannuan Liang and Yang Xiao. Game theory for network security. *IEEE Communications Surveys & Tutorials*, 15(1):472–486, 2012.
- [41] Maria Scalabrin, Valentina Vadori, Anna V Guglielmi, and Leonardo Badia. A zero-sum jamming game with incomplete position information in wireless scenarios. In *Proceedings of European Wireless 2015; 21th European Wireless Conference*, pages 1–6. VDE, 2015.
- [42] Mehran Fallah. A puzzle-based defense strategy against flooding attacks using game theory. *IEEE transactions on dependable and secure computing*, 7(1):5–19, 2008.
- [43] Hyunggon Park and Mihaela van der Schaar. Bargaining strategies for networked multimedia resource management. *IEEE Transactions on Signal Processing*, 55(7):3496–3511, 2007.
- [44] Lingyang Song, Dusit Niyato, Zhu Han, and Ekram Hossain. Game-theoretic resource allocation methods for device-to-device communication. *IEEE Wireless Communications*, 21(3):136–144, 2014.
- [45] Parvathy S Pillai and Shrisha Rao. Resource allocation in cloud computing using the uncertainty principle of game theory. *IEEE Systems Journal*, 10(2):637–648, 2014.
- [46] Alessio Zappone, Stefano Buzzi, Eduard Jorswieck, and Michela Meo. A survey on game-theoretic approaches to energy-efficient relay-assisted communications. In *2013 24th Tyrrhenian International Workshop on Digital Communications–Green ICT (TIWDC)*, pages 1–6. IEEE, 2013.
- [47] Leonardo Badia. Impact of transmission cost on age of information at nash equilibrium in slotted aloha. *IEEE Networking Letters*, 4(1):30–33, 2021.
- [48] Allen B MacKenzie and Stephen B Wicker. Selfish users in aloha: a game-theoretic approach. In *IEEE 54th Vehicular Technology Conference. VTC Fall 2001. Proceedings (Cat. No. 01CH37211)*, volume 3, pages 1354–1357. IEEE, 2001.
- [49] Andrea Munari. Modern random access: An age of information perspective on irregular repetition slotted aloha. *IEEE Transactions on Communications*, 69(6):3572–3585, 2021.
- [50] Leonardo Badia, Andrea Zanella, and Michele Zorzi. A game of ages for slotted aloha with capture. *IEEE Transactions on Mobile Computing*, 2023.

- [51] John C Harsanyi. A new theory of equilibrium selection for games with complete information. *Games and Economic Behavior*, 8(1):91–122, 1995.
- [52] John C Harsanyi, Reinhard Selten, et al. A general theory of equilibrium selection in games. *MIT Press Books*, 1, 1988.
- [53] Robert Aumann. Nash equilibria are not self-enforcing. *Economic decision making: Games, econometrics and optimisation*, pages 201–206, 1990.
- [54] Roger B Myerson. Refinements of the nash equilibrium concept. *International journal of game theory*, 7:73–80, 1978.
- [55] Ehud Kalai and Dov Samet. Persistent equilibria in strategic games. *International Journal of Game Theory*, 13:129–144, 1984.
- [56] Steven Tadelis. *Game theory: an introduction*. Princeton university press, 2013.
- [57] Tim Roughgarden. *Selfish routing and the price of anarchy*. MIT press, 2005.
- [58] Samson Lasaulce and Hamidou Tembine. *Game theory and learning for wireless networks: fundamentals and applications*. Academic Press, 2011.
- [59] Itzhak Gilboa. *Theory of decision under uncertainty*, volume 45. Cambridge university press, 2009.
- [60] Elias Koutsoupias and Christos Papadimitriou. Worst-case equilibria. *Computer science review*, 3(2):65–69, 2009.
- [61] George Christodoulou and Elias Koutsoupias. The price of anarchy of finite congestion games. In *Proceedings of the thirty-seventh annual ACM symposium on Theory of computing*, pages 67–73, 2005.
- [62] Allen B MacKenzie and Luiz A DaSilva. *Game theory for wireless engineers*. Springer Nature, 2022.
- [63] José Correa, Andrés Cristi, and Tim Oosterwijk. On the price of anarchy for flows over time. In *Proceedings of the 2019 ACM Conference on Economics and Computation*, pages 559–577, 2019.
- [64] Jose R Correa, Christian Scheideler, and Thim Frederik Strothmann. A new perspective on competitive caching. In *Proceedings of the 31st ACM Symposium on Parallelism in Algorithms and Architectures*, pages 249–259, 2019.

- [65] SN Durlauf and L Blume. *The new palgrave dictionary of economics*. palgrave macmillan. *New York*, 2008.
- [66] Robert J Aumann. *Game and Economic Theory: Selected Contributions in Honor of Robert J. Aumann*. University of Michigan Press, 1995.