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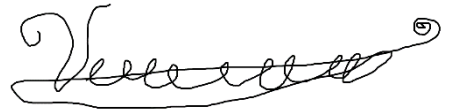
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## Introduction

One of the most important and challenging issues faced by the asset management industries is related to the optimal asset allocation decision. The goal of the investment managers is to achieve the highest return adjusted for risk, with the lowest value at risk (VaR) as possible. The probability of achieving a desired target return or minimising risk is chosen as the statistical measure that enforces optimal portfolio allocation by explicitly stating investment goals and downside boundaries. This process involves not only the analysis of the current perception of risk but also the way the risk and return evolves over time.

The financial market, despite the benefits and rewards, is a complexly volatile industry which requires a constant critical analysis to adequately evaluate risks relative to returns. Traditional asset allocation strategies, such as the classical 60/40, have failed to reward investors for the risk assumed. Moreover, the financial crisis have unveiled an important weakness of traditional portfolio strategies, the poor true diversification. Traditional strategic asset allocation theory is deeply rooted in the mean-variance portfolio optimization framework developed by Markowitz in 1952. However, the mean-variance optimization methodology is difficult to implement due to the challenges associated with estimating the expected return and covariances for asset classes with accuracy. The estimates are very often biased and investor can easily be trapped in undesired influences from the financial markets, resulting in underestimating risk or overestimating returns. As a consequence, after the 2008 turmoil in markets, new allocation approaches have emerged.

Among these new strategies, the ones which attracted the most interest from market practitioners and academics are certainly the risk-based approaches. These strategies are also called ‘heuristic’ because they do not rely on any formal equilibrium model of expected return.

Well known examples of such techniques are the equally weighted and minimum variance. The first one is a simple strategy method which allocates equally among assets in the investment universe,  $1/n$ . The second approach, in order to arrive at a unique solution, requires an optimization process which suffers from some drawbacks, such as overconcentration in low risk assets. The trade-off between these strategies have created the equally risk contribution portfolio. Keeping it simple, this portfolio mainly computes the risk contribution of each asset and allocates respectively to equalize them in the portfolio. In other words, no asset contributes

more than its peers to the total risk of the portfolio. Dealing with the risk contribution has created the branch of "risk budgeting", a strategy which allocates specific amount of risk contributions to assets in the investment universe.

From the same family of risk based approaches is also the maximum diversification portfolio, introduced by Choueifaty and Coignard (2008), which use an objective function that maximizes the ratio of weight and average asset volatilities to portfolio volatility. Like minimum variance, maximum diversification portfolios equalize each asset's marginal contributions, given a small change in the asset's weight. The objective function, of the maximum diversification portfolio, is motivated by maximizing the portfolio Sharpe ratio, where expected asset returns are assumed to be proportional to asset risk. Thus, the maximum diversification portfolio is the tangent portfolio on the efficient frontier if average asset returns increase proportionally with risk. On the other hand, if asset returns decrease with risk, the maximum diversification portfolio will be on the lower half of the traditional efficient frontier and clearly suboptimal (Clarke et al. 2013).

In this thesis all these strategies will be applied on European markets and back-tested. More specifically, we will analyse how well these portfolios performed in terms of risk adjusted returns for a period of 13 years, from 2005 to 2018. We will use an evaluation methodology that consider risk adjusted returns, maximum drawdowns, Value at Risk, turnover and risk contributions. The data subject to study are the historical components of the MSCI Sectorial Equity Indices along with the Citi Government and Corporate Indices.

In the first part of this thesis, a brief analysis of the recent macroeconomic environment will be presented along with the motivation and the research goals. The second part will focus on academic literature and on the most important theories behind the portfolio management. The chapter will continue with the methodologies and the techniques required for the risk based allocation strategies along with the performance measures. The last chapter will present the empirical observations of the back-testing results. All the strategies' performance will be deeply investigated and discussed. Finally, advantages and disadvantages of all portfolios will be presented in the conclusion remarks.



## 1.1 Background and Motivation

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Even though the 2008 financial crisis erupted in the United States, due to the ever increasing interconnection and integration in the global markets, European securities suffered the same negative outcome. The extreme losses in the portfolios made practitioners and academics to question the efficiency of the traditional approaches to asset allocation, such as the 60/40 strategy<sup>1</sup>. As a consequence, an endless list of solutions about how to mitigate and control risk have surged.

One of the optimization solutions that gained most interest from the asset management industry is the Risk Budgeting<sup>2</sup> (RB). The main principle of this approach is to allocate a specific amount of risk to each asset class in the portfolio. Because of the turmoil left behind during the financial crisis, investors started to emphasize not only the returns, but also the risk assumed, and so the risk based strategies flourished. Looking back at the historical performance of equity over bonds is easy to get the point.

*Table 1: Statistics of European Equity and Bond Indexes from 1987 to 2018.*

	<b>EU MSCI Equity Index</b>	<b>EU JPM Aggregate Bond Index</b>
Mean Return Ann. (%)	8.2	5.6
Standard Dev. Ann. (%)	17.08	3.9
Skewness Ann.	-1.33	0.15
Kurtosis Ann.	5.37	2.67
<b>Rolling Sharpe Ann.</b>	<b>0.27</b>	<b>0.59</b>

*Source: DataStream.*

As shown in the table 1, the EU MSCI Europe Equity Index outperformed the EU JPM Europe Bond Index by 2.6% annually on average from 1987 to 2018. However, the volatility of the equity index was 4.37 times higher than the volatility of the bond index. Moreover, negative skewness and high kurtosis statistics, which measure the “fat tails” in the returns distribution,

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<sup>1</sup> A traditional long term strategy which aims to allocate 60% of the capital to equities and 40% to bonds.

<sup>2</sup> The idea itself of Risk Budgeting is not new and has been used for some time in managing global, multi-asset portfolios. In 1996 the hedge fund ‘*Bridgewater*’ created the very first fund based on this strategy called ‘*All Weather Asset Allocation*’.

indicate large drawdowns in the equity prices. The analysis raise a fair question: Is the risk assumed worth for the 2.6% returns premium? The annual rolling Sharpe ratio makes evident that EU JPM Bond Index had a better performance in terms of risk adjusted returns. So, a strategy implemented to increase the allocation to fixed income securities, with a focus on underlying risk, would not only reduce the volatility of the portfolio but could also boost the performance.

From the start, must be pointed out that these new risk-based optimization techniques seem very appealing because of the better protection against substantial losses and because of a more cautious risk management but, are not immune to criticism. With the ECB official interest rates currently at the lowest level in history, one could argue that the long ‘‘bull market’’ in bonds is close to end and any long-term model that suggests higher allocation to fixed income should be carefully analysed.

In this macroeconomic framework it is clear why the proponents of risk parity<sup>3</sup> argue that the value of balancing risks between asset classes is realized only over long periods including periods of recessions, growth and higher inflation regimes. Historical analysis, as will be presented in this thesis, does provide some evidence of better performance than traditional portfolio allocation in growth and recessionary environments.

The fact that low risk assets, in which risk based strategies have the highest exposure, provide higher risk adjusted returns is not a revelation in the finance community. Black, Jensen, and Scholes (1972) already highlighted this point by indicating that the security market line which describes the relation between expected returns and risk is too flat relative to the CAPM. Other authors, like Frazzini and Pedersen (2014) recently argued that low risk assets do not only provide superior risk-adjusted returns, but also outperform high risk assets within several asset classes in absolute terms. The empirical evidence on the relationship between expected returns and volatility presented by Ilmanen (2012) shows that even though volatility and average returns in the long run are positively related across asset classes, the most volatile assets within each asset class, such as stocks with high volatility, tend to provide low returns and even lower risk-adjusted returns in the long-term.

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<sup>3</sup> The terms Risk Parity and Risk Budgeting will be interchangeable used in this thesis.

## 1.2 Objective and Research Questions

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The main goal of the thesis is to create a comprehensive overview of risk budgeting approach to portfolio optimal allocation framework. Multiple optimization strategies therefore are created and their performance is tracked to find out which asset allocation approach can deliver superior risk-adjusted returns in comparison to other traditional approaches, such as 60/40 or Equally Weighted (EW). Market frictions such as trading and borrowing costs are also examined. Additionally, this study estimates the performance of the strategies during different macroeconomic environments to recognize whether the risk parity could be a potential strategy for the an increasing interest rates context.

The empirical research contributes also to the existing literature by providing comparative evidence of different risk parity methods with a focus on European financial markets.

The key research questions that this study tries to respond are:

- Do risk parity strategies deliver higher risk-adjusted returns compared to other traditional allocation strategies?
- Does the Equal Risk Contribution (ERC) strategy outperform the Inverse Volatility (IV) strategy?
- Is there a room for Risk Budgeting in a rising rates market environment?
- How concentrated are the portfolio weights in risk based strategies relative to other traditional allocation strategies?

## 1.3 Macroeconomic Environment

In this section the most important macroeconomic episodes from 2006 to 2018 are very briefly presented. The aim of this section is to strengthen the belief of how uncertain and difficult to predict are the economic regimes, and therefore return estimates. The models that try to forecast returns or volatility based on historical observations or other techniques tend to have a very high probability of being completely wrong.

### 1.3.1 A New Europe

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With the introduction of the shared currency in the European countries in 1999, expectations of higher economic growth among investors increased considerably. While some regions benefited given and increased productivity and international trade, on the aggregate level the European economies struggled to deliver a sustainable growth. Moreover, the weaknesses of the monetary union's financial markets were tested in deep during the 2008 sub - prime bubble and sovereign debt crisis in 2011.

The rise of populism among European policymakers and a deflationary context created additional nerves for the investors. The financial markets continued to disappoint by the stagnation of developed economies, an increasingly large sovereign debt crisis together with political uncertainty and the continuing regulatory initiatives from the financial crisis. As a result, the markets are in an extended period of low market returns, high volatility and increased correlations across traditional asset classes. Even worse, the economic prospects depend even more than usual on highly uncertain events which, such as Brexit or trade wars. As a consequence and as many investors have realised, the economy enters a world where the old methods and approaches may not work anymore. New theories and ways of thinking are needed under the new economic regime. An increasing demand started to arise for the models which emphasize a more balanced risk management.

In response to the crisis and to the deflationary pressures, European Central Bank decreased interest rates considerably during last decade. Therefore, portfolio strategies that increased the exposure to fixed income securities performed very well in the last decade.

*Figure 1: ECB base interest rates from 1998 to 2018.*



Source: European Central Bank

Current yield levels are also without precedent, and some economists even subscribe the view that these low yields are a bond bubble. Other argue that the extreme economic circumstances of the past few years and the unprecedented central bank’s monetary operations explain this phenomenon.

*Figure 2: Historical Euro area 10-year Government Benchmark bond yield*



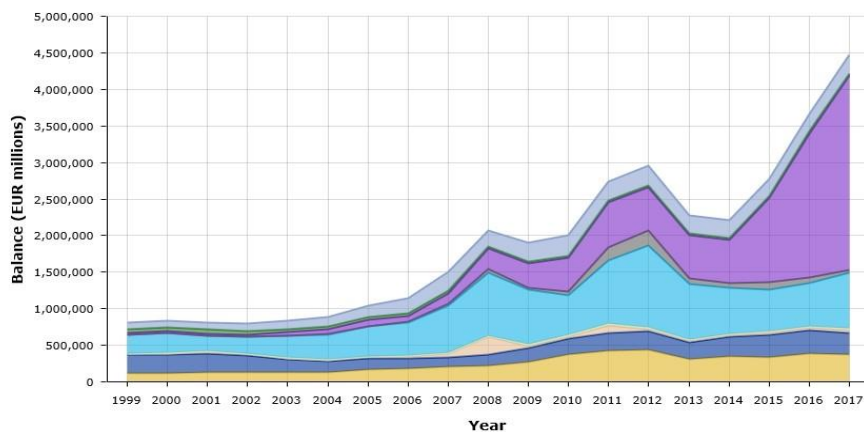
*Source: European Central Bank*

### 1.3.2 Super Mario

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That ‘whatever it takes’ promise achieved its aims – the euro still exists – but it did not revive European economies. The long battle against deflationary pressures pushed Mario Draghi to use the “bazooka” and buy government bonds each month for 60 billion euro. As a result the ECB’s balance sheet increased to the record levels, as Figure 3 shows.

*Figure 3: European Central Bank Balance Sheet*



*Source: European Central Bank*

The Central Bank's commitment to keep the funds rate near zero as long as the inflation will stay below the 2% target had a significant impact on long term yields. The expansive monetary policy, which started in 2011 with decreasing interest rates and became more aggressive in 2014 with the introduction of QE, pushed the yields to the record levels. The 10 year yield, which was 4.2%, fell by 120 bps to around 3% from 2011 to the end of 2013. Signals of a more aggressive policy expansive in 2014 pushed the yield to the record lows, close to 1%.

*Figure 4: Euro area 10-year Government Benchmark bond yield, 2011 -2018*

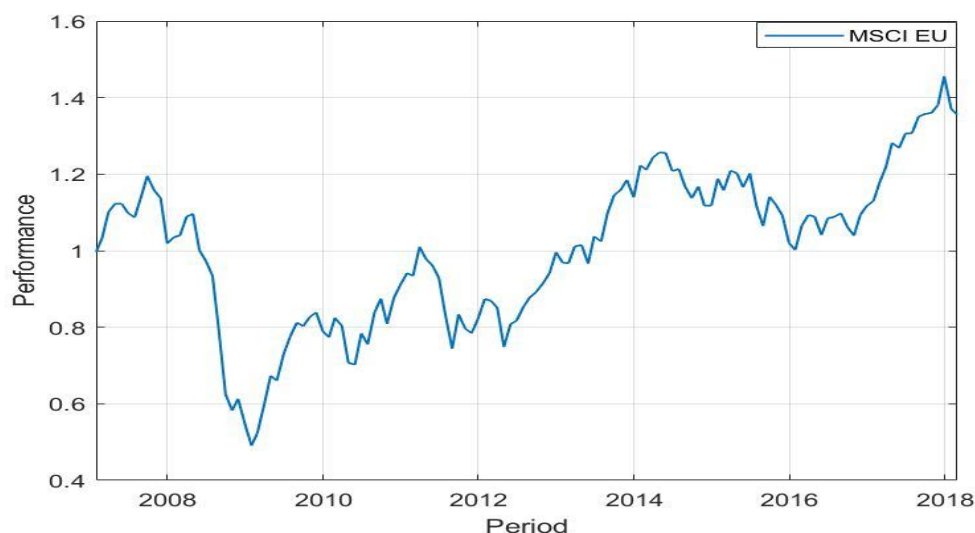


*Source: European Central Bank*

When long term yields are at such low levels historical speaking, the real interest rate, all else being equal, are turning negative. The level of the real interest rate conveys a large amount of information about pressures on asset prices in the past as well as in the future. A low real rate can be a reaction of either high inflation expectations or low nominal rates. This suggests an environment favourable to holding real assets at the expense of cash. Even with inflation expectations remaining well anchored as they are at the moment, unprecedentedly low nominal rates make the opportunity cost of holding cash much lower than it would be otherwise. At low or negative rates, investors are pressured into 'searching for yield' and taking on or advancing investment decisions as the opportunity costs declines. They are influenced to move their endowments to the risky assets, such as equities or High Yield Bonds.

But meanwhile in the US the same operation from the Federal Reserve boosted equity markets to all time high, in Europe the stock market struggled to gain momentum.

Figure 5: Performance of MSCI Europe Index from 2007 to 2018



Source: DataStream

### 1.3.3 After QE Asset Allocation Challenges

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The extraordinary stimulus programs from Global Central Banks increased the cross – correlation not only among securities from the same sector, but also among asset classes. This effect influenced investors to move to a more passive approach. As a result, index tracking investment products, such as Exchange Trade Funds (ETFs), have flourished in the last years by producing not few frictions in the markets. Investing by allocating to specific sectors or countries is increasing even more the correlations coefficients. In this new environment, where individual securities prices fluctuate far from their intrinsic value, is making the asset allocation process more challenged.

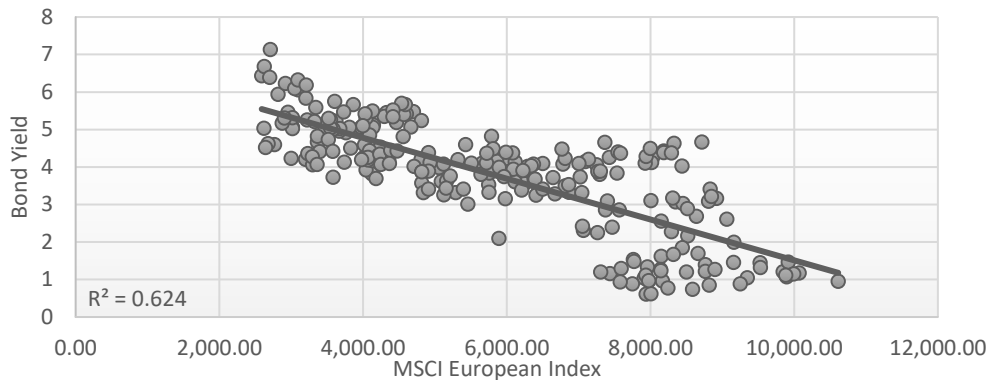
Figure 6 shows the regression analysis of the “10 Year Yield Benchmark” against “MSCI Europe Equity Index” for last 19 years. The downtrend in yields pushed equity markets to higher levels, as equity premiums<sup>4</sup> increased over time. Whether this trend will continue is hard to say. How stocks will trade when QE ends is a question on the minds of many investors. Unfortunately, there is no simple answer to this question. With QE expected to end at the end of 2018 and with first hike in interest rates in 2019 investors fear the reverse. Since equities are valued as the expected cash flows discounted back at the cost of capital, a reverse in yields will

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<sup>4</sup> A measure of equity risk computed as expected equity return minus risk free rate.

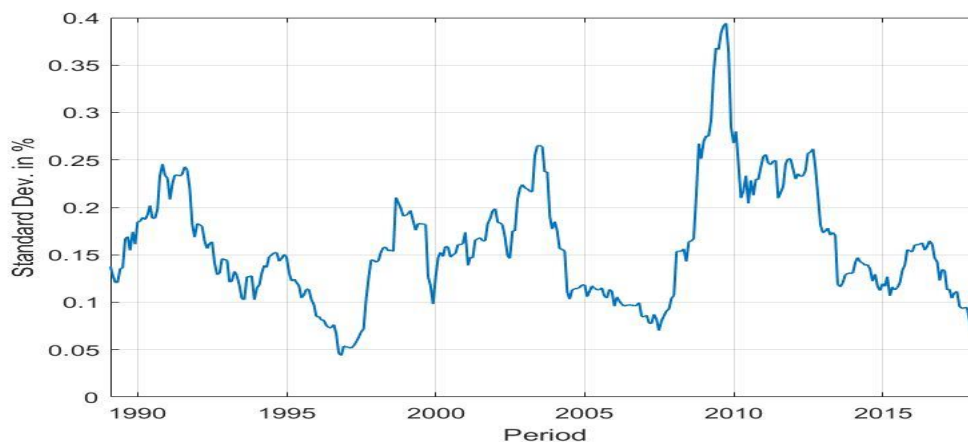
increase the discount rates and so will put pressure on equities. Moreover, a reverse in the yields trend may create also high levels of volatility.

Figure 6: Linear Regression of 10 Year Yields Benchmark Against MSCI Europe Index, 1997-2018



Source: DataStream for the MSCI data and European Central Bank for Bond Yield Benchmark data

Figure 7: Historical Volatility of MSCI Europe Index

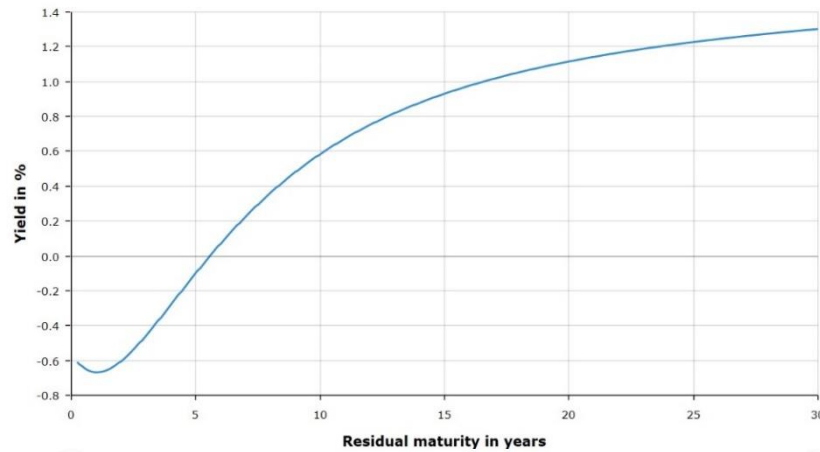


Source: DataStream. Note: Standard Deviation is computed on 12 months rolling base.

With the European Yield Curve very steep at the moment investors have no choice but to keep the allocation to higher risk assets in order to capture positive returns. How to optimally rebalance the portfolios and what strategies to use in a rising interest rates environment is the question that this thesis will try to answer in the conclusions.



Figure 8: Triple A Government Bonds yield curve<sup>5</sup>



Source: European Central Bank

### 1.3.4 Smart Beta Approaches

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Burton Malkiel (1995), in the paper ‘Returns from Investing in Equity Mutual Funds 1971 to 1991’ points out that investment funds, which tend to pick individual securities rather than allocating to the market portfolio, tend to underperform their benchmark portfolios after adjusting for both, management expenses and survivorship bias. The failure of most fund managers to achieve excess returns and to manage the risk exposures in the wake of distress markets, have shifted the investment industry versus a more passive approach. The growing momentum towards passive investment, observed after the 2008 financial crisis, increased the interest and the demand for new indexation forms, referred to as advanced or Smart Beta. The capitalization-weighted indices, used primary to represent market movements, proved to not be an efficient benchmark tool that can be used as a reference for an investor’s strategic allocation (Amenc et al. 2013). Smart Beta Indexation strategies, on the other side, reconcile the passive and active investment by providing investors access to ‘risk premia’ in a cheaper way, which were previously available only through expensive active strategies.

Moreover, by construction, a capitalized weighted index is a trend following strategy as it incorporates a momentum and growth bias, which leads to bubble exposure risk as weights of the best performers increase and weights of the worst performers decrease. In this environment,

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<sup>5</sup> The chart is based on aggregate level as per date 24<sup>th</sup> of May 2018.

alternative-weighted indexation has prompted great interest from both academic researchers and market practitioners. An alternative weighted index is defined as an index in which assets are weighted in a different way than those based on market capitalization and can be split in two families: fundamental indexation and risk-based indexation. The examples of risk based indexation, which are also used for the main analysis in this thesis, are Global Minimum Variance (GMV), equally weighted (EW), Most Diversified Portfolio (MDP), Maximum Sharpe Ratio (MSR), and Equally Risk Contribution (ERC). The main difference between fundamental and risk-based indexes is that the former promises alpha, whereas the latter promises diversification. In other words, the difference between the two methods comes from the opinion of modifying the risk-adjusted return ratio. In the case of fundamental indexes, one expects to have superior returns with respect to the capitalization index. In the case of risk-based indices, one expects to decrease the risk of the portfolio in either absolute or relative value. It is important to underline that in the risk-based investing, the key variable is the level of volatility reduction.

Investment in alternative weighting indices presupposes measurement of the systematic risk factors and integration of the factors, not only in absolute terms to evaluate the real risk adjusted performance created by better diversification of the benchmark, but also in relative terms to limit the tracking error risk and therefore the risk of underperformance in comparison with the cap-weighted index. As Richard and Roncalli (2015) points out, when the performance of stocks is high, it is better to invest in a more diversified portfolio than the capitalization weighting portfolio, but with a limited tracking error in order to fully benefit from the bull market. Conversely, in a bear market, a concentrated portfolio of low volatility stocks should do a better job.

The adoption of risk-based strategies is commonly justified by three principal arguments. First, risk-based strategies do not require any stock return forecasts, which eliminates the challenge of estimating them, and require only estimations of the variance-covariance matrix (Maillard et al., 2010). Second, risk-based strategies aim to improve the risk/return ratio by improving risk diversification. Third, when back-tested, risk-based strategies outperform the traditional CW investment strategy especially when crises occur (Maillard et al., 2010). Another interesting property is that these different alternative-weighted portfolios belong to the same optimization problem family. They are minimum variance portfolios and differ because of the implied constraint they consider. This thesis tries to analyse and evaluate the performance in terms of risk/returns of these allocation strategies on multi-asset classes. Thus the main goal is to compare the risk based approaches to other traditional investment strategies, such as 60/40 or

EW, and not to capitalization – indices. Another objective is to compare the risk performance among the same risk based strategies, IV, ERC, and MDP. Even though the allocation approaches discussed in this thesis are referred by the investment community as smart beta indices, we will analyse them as pure allocation strategies.

## Part II

### Academic literature and theoretical background

*"In investing, what is comfortable is rarely profitable."*

*Robert Arnott*

This chapter provides a general overview of the academic literature in portfolio management, along with the theories behind the risk based approaches.

#### 2.1 Theory Background

##### 2.1.1. The Diversification Call

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Investment portfolio theories guide the way an individual investor or financial planner allocates money and other capital assets within an investing portfolio. Economists often say that the only “free lunch” in investments is diversification, as it allows investors to reduce portfolio risk without sacrificing expected return or to increase expected return without accepting more risk. The pioneer of these theories is Harry Markowitz, an American economist that in the 1950s developed a theory of “portfolio choice,” which allows investors to analyse risk relative to their expected return. Markowitz’s theory is today known as the Modern Portfolio Theory (MPT). It is a theory of investment which attempts to maximize portfolio expected return for a given amount of portfolio risk, or equivalently minimize risk for a given level of expected return, by carefully choosing the proportions of various assets. These optimized portfolios compose the “efficient frontier,” a band of portfolios that dominate all other feasible portfolios in terms of their risk-return trade-off. MPT theory provides a quite broad framework for optimal asset allocation. It is the first, as are most mathematically based capital market models, independent of whether they are normative or positive a “one-period-model”. This means that the model can be used for theoretically every investment period or horizon.

Tobin (1958) expanded upon Markowitz’s mean-variance framework, showing that the introduction of a riskless asset implies that there is an optimal risky portfolio on the efficient

frontier whose selection is independent of the investor's risk aversion. The capital market line, which passes through the riskless return and the optimal risky ("tangency") portfolio, delineates the new set of efficient portfolios. Tobin's work led to the famous "separation theorem," the idea that portfolio selection is divided into two stages: first, an optimal sub-portfolio of risky assets is selected solely on the basis of the joint distribution of the returns of the risky and riskless assets; second, the investor divides wealth between the risky sub-portfolio and the riskless asset, choosing a portfolio from the capital market line on the basis of risk aversion or other factors.

The next larger step in modern portfolio theory was the development of the Capital-Asset-Pricing-Model by Sharpe (1964). CAPM was a further step for investors and academia to understand the connection between asset risk and asset returns. Specifically, the CAPM introduced the concept of distinguishing between two types of risk, namely the systematic risk (that is risk that cannot be diversified away in portfolio construction, no matter how much we diversify) and the non-systematic risk (that, on the other hand, is risk that can be eliminated by diversifying the portfolio). Sharpe also claimed that an investor may obtain a higher expected rate of return on his holdings only by incurring additional risk. The CAPM also introduced two other new concepts widely-used in finance: alpha and beta.

In contrast to the Sharpe CAPM model, mean-variance optimization takes into account the overall risk of securities (or asset classes), without separating out their systematic and idiosyncratic (unsystematic) components. Also, while in the Sharpe model securities correlate with one another through their relationship with the market return (Beta), in the Markowitz framework securities relate to one another more generally through the correlation matrix. So, as long as the correlation between asset classes is less than one, the variance of portfolio returns will be less than the weighted average of the variances of its constituent assets.

### **2.1.2 Risk and Returns**

---

CAPM model is very used by financial community because of the simplicity and intuitiveness it provides. However, it is not immune to criticism since it has too many unrealistic assumptions and fails on many dimensions. Above all, it does not capture all risk factors affecting a security's return. Accordingly, the model developed by Fama and French (1993) the return of an asset derives from 3 sources of risk, and particularly market risk, the outperformance of small versus

big companies, the outperformance of high book/market versus small book/market companies. That is, besides the market return, Fama and French identified size and value as major driving forces explaining an individual security's return.

All these models are trying to predict asset returns by measuring, in different ways and under distinct assumptions, the underlying risk. The usual practice for estimating the expected market return is to add the historical average realized excess market returns to the current observed interest rate. Merton (1980) however, argues that these models explicitly reflect the dependence of the market return on the interest rate, and fail to account for the effect of changes in the level of market risk.

Black and Litterman (1992) also argued that quantitative asset allocation models have not played the important role they should in global portfolio management. According to the authors, a good part of the problem is that such models are difficult to use and tend to result in portfolios that are badly behaved. Chopra et al. (1993) argues that the errors in means are over ten times as damaging as errors in variances, and over twenty times as damaging as errors in covariances. To overcome the issues related to difficulties in return estimation and to reduce the uncertainty of sample estimates, Black and Litterman suggested a model strategy that assumes that the optimal asset allocation is proportional to the market values of the available assets. Accordingly, equilibrium expected returns can be derived from observable security prices, and modified to represent the optimizations specific opinion about that assets future perspective.

### **2.1.3 Mean Variance Framework**

---

The economist Harry Markowitz in 1952 laid the foundation for portfolio optimization in his paper, *‘Portfolio Selection’* Markowitz (1952). The theory presented in the paper is often referred to as *‘mean-variance’* portfolio analysis, and investigate how wealth can be optimally invested in assets which differ in regard to their expected return and risk. Markowitz showed that under certain given conditions, the choice of a portfolio can be reduced to the expected return of the portfolio and its variance. And that it is possible to reduce risk through diversification such that the risk of the portfolio, measured as its variance, will depend not only on the individual variances of the return on different assets, but also on the pairwise covariances

of all assets. The practice of taking returns as well as risk into account when making investment decisions was well known before Markowitz, but he was the first to develop a rigorously formulated mathematical framework for portfolio optimization.

The ‘‘mean – variance’’ framework is at the heart of every portfolio construction process. In technical terminology, Mean (return) - variance (volatility) is the search for portfolio weights  $w_i$  that maximize the expected return  $\mathbb{E}(\mu_p)$  subject to  $\sigma_p$  to a portfolio  $p$ . This is known as the risk-return space that contains an investor's investment opportunity sets. These sets are all feasible pairs of  $\mathbb{E}(\mu_i)$  and  $\sigma_i$  from all portfolio resulting in different values of asset allocations. Considering a universe of  $n$  assets and a vector of weights in the portfolio  $x = (x_1, \dots, x_n)$  and assuming also that the portfolio is fully invested:

$$\sum_{i=1}^n x_i = 1^T x = 1 \quad (1)$$

and the vector of asset returns  $R = (R_1, \dots, R_n)$ , then the portfolio return is given by the

$$R_p = \sum_{i=1}^n x_i R_i = x^T R \quad (2)$$

By defining the vector of the expected values of the returns  $\mu = \mathbb{E}(R_i)$  we can now derive the variance – covariance matrix  $\Omega$

$$\Omega = \mathbb{E}[(R - \mu)(R - \mu)^T] \quad (3)$$

The expected return and the variance of the portfolio are defined respectively:

$$\mu(x) = \mathbb{E}[R(x)] = x^T \mu \quad (4)$$

$$\sigma^2(x) = \mathbb{E}[(R(x) - \mu(x))(R(x) - \mu(x))^T] = x^T \Omega x \quad (5)$$

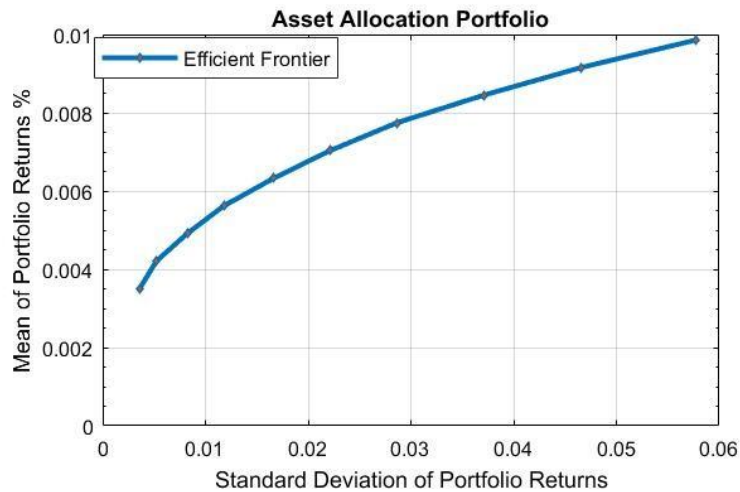
The classical Markowitz mean – variance optimization model becomes:

$$\begin{aligned} \min \quad & x^T \Omega x, \quad w. r. t. \\ & x^T \mu \geq \mathbb{E}^* \end{aligned} \quad (6)$$

Optimization problem (6) impose to minimize the variance subject to a lower limit on the expected return, where  $\mathbb{E}^*$  indicates a specified target. The optimal portfolio, according to the Markowitz theory, lays on the Efficient Frontier Curve (Figure 9), where the trade-off between risk and return is maximized. The optimal allocation point on the Efficient Frontier Curve is a choice related to the utility function, which incorporates the risk aversion of an investor. Maximizing the expected utility for a given  $\mu(x)$  or minimizing  $\sigma^2(x)$  is equivalent to maximizing the expected utility function.

Mean variance approach is very intuitive and useful model to introduce the risk – return trade-off and to observe the diversification benefits, which arise in case of negative correlation among assets.

*Figure 9: A Hypothetical Efficient Frontier Allocation*



## 2.2 Risk Parity Approach

---

The problems of the Markowitz’s approach for asset allocation (estimations errors and inconsistency) have given rise to numerous attempts from academics and practitioners to address them. Especially, the extreme events during the financial crisis which started in 2008 pressured asset management to reshape. Have been promoted several efforts to elaborate new approaches for construction that remove the mean – variance framework. The new models focus



on alternative allocation techniques, and more specifically on risk – based principles. In these approaches, portfolio weights are only function of specific risk properties of the constituents and do not include expected returns in the optimization equations. For this reason, risk based asset allocation approaches have been labelled as return free strategies. Some of the most popular of them (and on the main focus of this thesis) include:

- Equally -Weighted strategy (EW)
- Risk – Parity Strategy (RP)
- Global Minimum – Variance Strategy (GMV)
- The Most Diversified Portfolio Strategy (MDP)
- Inverse Volatility Strategy (IV)

The risk parity investment strategy started to gain attention from academics and practitioners especially after 2008 financial crisis. Observation that traditional asset allocation strategies, such as investing 60% in stocks and 40% in bonds is not sufficiently diversified when looking at risk contributions, triggered the demand for risk based approaches Asness et al. (2013). The 60/40 strategy does not sufficiently take risk into account, and simply suggest to invest 60% of the wealth in stocks and 40% of the wealth in bonds. Since stocks are significantly more volatile than bonds, and the nominal stock investments make up 60% of the portfolio in this strategy, the overall risk contribution from stocks is dominating. The 60/40 portfolio will therefore follow the movements in the stock market, since this is the major source of risk in the portfolios. Moreover, the conventional form that bond prices are negatively correlated with stocks prices can prove very erroneous in distressed times. In such situations, diversification between the two asset classes historically did not protect investors from huge losses, as the investors expected.

Academics and practitioners developed several other suggestions on how to cancel out and deal with the risk in portfolios. A traditional approach was to allocate funds to a wider range of investment categories, such as commodities and real estate. However, the dynamics of the correlations among asset classes are too volatile in order to construct a sustainable risk management approach. New heuristic solutions based on risk allocation, such as risk parity, are gaining interest from many investors that do not want to use input estimates. In a risk budgeting approach, the investor only decide on the risk repartition among the assets in the portfolio, without any consideration about expected returns.

This thesis will focus on the portfolio where all components contribute equally to the total risk in the portfolio and on the portfolio with ex-post defined risk budgets among asset classes.

### 2.2.1 Risk factors

---

To understand the risk parity portfolio first must be defined the marginal risk contribution and risk contribution of assets. According to Maillard et al. (2008) the marginal risk contribution is defined as the change in the total risk of the portfolio by an infinitesimal increase of  $x_i$ . By considering the total risk of a portfolio, defined earlier, the marginal risk contribution of an asset  $i$  as

$$MRC_i = \frac{\partial \sqrt{x^T \Omega x}}{\partial x_i} = \frac{(\Omega x)_i}{\sqrt{x^T \Omega x}} \quad (7)$$

Where  $(\Omega x)_i$  denotes the  $i$ th row of the vector from the product of  $\Omega$  with  $x$  (Maillard et al., 2008). The  $n$  marginal risk contributions in the vector can be collected

$$MRC = \frac{(\Omega x)}{\sqrt{x^T \Omega x}} \quad (8)$$

The total risk contribution from asset  $i$  is computed as the product of the allocation in asset  $i$  with its marginal risk contribution. Maillard et al. (2008) defines the risk contribution of asset  $i$  as the share of the total portfolio risk from that asset

$$RC_i = x_i \frac{(\Omega x)_i}{\sqrt{x^T \Omega x}} \quad (9)$$

Since the volatility is a homogeneous function of degree 1, it satisfies Euler's theorem and it can be written as the sum of its arguments multiplied by their first partial derivatives (Maillard

et al., 2008). By summarizing all the risk contributions from all assets, the total risk of the portfolio can be defined

$$TRC = \sum_{i=1}^n RC_i = x^T \frac{\Omega x}{\sqrt{x^T \Omega x}} = \sqrt{x^T \Omega x} \quad (10)$$

## 2.2.2 Equally Risk Contribution Allocation Strategy (ERC)

---

The ERC allocation strategy is a portfolio where no stock should contribute more to the risk of the portfolio than any other component. The weight  $x_i$  is therefore determined based on the single and joint risk contribution of an asset  $i$ . If all assets have the same volatility, then the ERC portfolio would be the equal to the equally-weighted portfolio. Is important to note that the minimum-variance portfolio also equalizes risk contributions, but only on marginal basis, and the total risk contributions from each asset in the portfolio is far from equal (Maillard et al. 2008).

The portfolio where the contribution of risk from all assets are equal must satisfy

$$RC_i = RC_j, \quad \text{for every } i, j \quad (10)$$

By following Maillard et al. (2008), there is the need to impose the restriction on short-sales, and on the weights, which must sum to one. It should be noted that the restriction of weights summing to one is not a necessity, but it works as a normalizing restriction and makes the ERC weights easier to compare with the other portfolio allocation strategies such as EW or IV. Under the above mentioned constraints, Maillard et al. (2008) formulate the problem of finding the risk-parity portfolio such as

$$x_{ERC} = \{x \in [0,1]^n: \sum x_i = 1, RC(x_i) = RC(x_j), \text{ for every } i \text{ and } j\} \quad (11)$$

With the  $MRC_i$  proportional to  $(\Omega x)_i$  the above equation can be rewritten as

$$x_{ERC} = \{x \in [0,1]^n: \sum x_i = 1, x_i(\Omega x)_i = x_j(\Omega x)_j, \text{ for every } i \text{ and } j\} \quad (12)$$

where  $(\Omega x)_i$  denotes the  $i$ th row of the vector issued from the product of  $\Omega$  with  $x$ .

In the special case where all the correlations are equal

$$\rho_{i,j} = \rho \quad \text{for every } i, j \quad (13)$$

the analytical solution to the ERC, Maillard et al. (2008) derive the analytical solution to be

$$x_i = \frac{\sigma_i^{-1}}{\sum_{j=1}^n \sigma_j^{-1}} \quad (14)$$

The weight allocated to each component  $i$  is given by the ratio of the inverse of volatility with the harmonic average of the volatilities. The higher (lower) the volatility of a component, the lower (higher) its weight in the ERC portfolio.

This solution is often used in practice when investors want to ignore the correlation coefficients. This is why this special case is sometimes referred to as a "naive risk parity" strategy or "inverse volatility".

When the correlations and asset volatility differ finding a solution requires the use of a numerical algorithm. Maillard et al. (2008) propose the following Sequential Quadratic Programming algorithm:

$$\begin{aligned} x_{ERC} &= \min f(x) \\ \text{u. c } 1^T x &= 1 \text{ and } 0 \leq x \leq 1 \end{aligned} \quad (15)$$

where

$$f(x) = \sum_{i=1}^n \sum_{j=1}^n (x_i(\Omega x)_i - x_j(\Omega x)_j)^2 \quad (16)$$

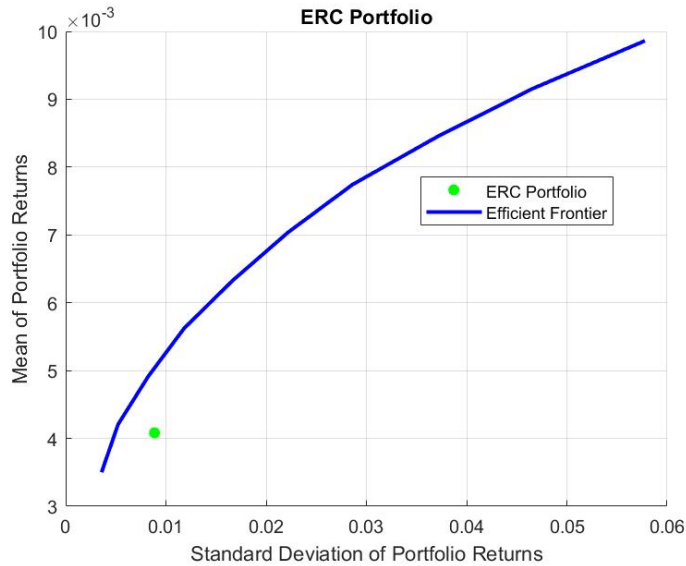
Basically, the program minimizes the variance of the (rescaled) risk contributions.

The ERC portfolio exist only when  $f(x) = 0$ , meaning that

$$x_i(\Omega x)_i - x_j(\Omega x)_j = 0, \text{ for every } j \text{ and } i \quad (17)$$

The ERC portfolio is therefore obtained by equalizing  $TRC$  from all the assets of the portfolio. The risk contribution is computed as the product of the asset weight with its  $MRC$ , the latter being given by the change in the total risk of the portfolio induced by an increase in holdings of the asset. The principle can be applied to different risk measures, however, the portfolio is restricted to the standard deviation of the portfolio as the only risk measure.

Figure 10: Equally Risk Contribution Portfolio position on the mean variance space



Note: The Efficient Frontier is plotted based on 5 years estimation period with data presented in the next chapter. All results are applied using Matlab. Source: Datastream

### 2.2.3 Risk Budgeting Approach (RB)

---

In the equal risk contribution (ERC) portfolio, the risk contribution from each portfolio asset is made equal. In a risk budgeting approach, the investor chooses the exact risk repartition to be allocated to each asset class or even to each asset.

Bruder and Roncalli (2012) are considering a set of risk budgets  $b_i = (b_1, \dots, b_n)$ , where  $b_i$  is the amount of risk allocated to the  $i$  asset. Therefore, risk budgeting portfolio is defined by the following constraints:

$$\begin{cases} RC_1(x_1, \dots, x_n) = b_1 \\ \vdots \\ RC_i(x_i, \dots, x_n) = b_i \\ \vdots \\ RC_n(x_1, \dots, x_n) = b_n \end{cases} \quad (18)$$

Since the system above may be too large to define a portfolio the authors prefer to define the portfolio in terms of weights and risk budgets in relative value. The RB long-only portfolio is thus specified by the following mathematical system:

$$\begin{cases} x_i(\Omega x)_i = b_i(x^T \Omega x) \\ b_i > 0 \\ x_i \geq 0 \\ \sum_{i=1}^n b_i = 1 \\ \sum_{i=1}^n x_i = 1 \end{cases} \quad (19)$$

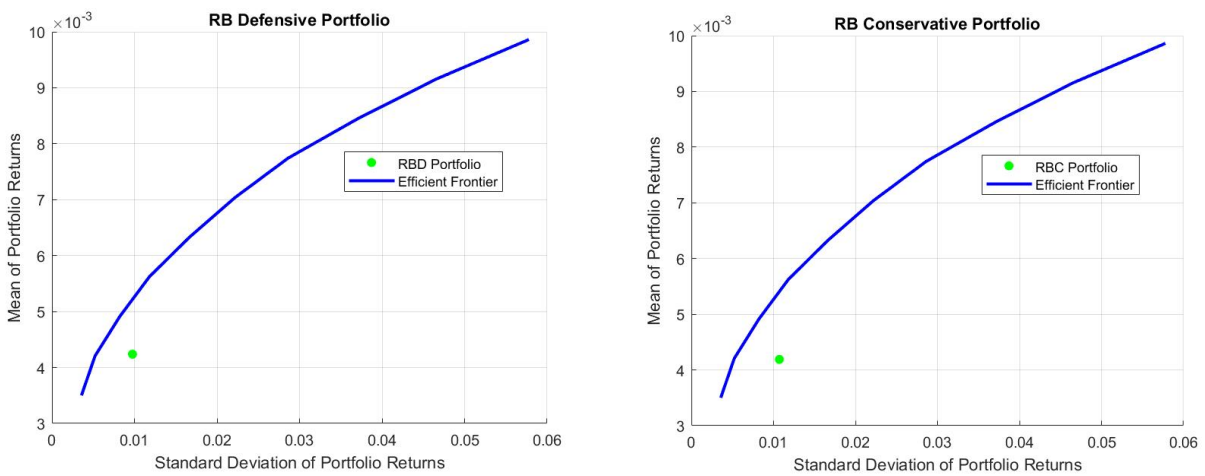
The constraint  $b_i > 0$  is necessary because if the investor sets one risk budget equal to zero, he would expect to not have the corresponding asset in his portfolio. Therefore, the investor will first have to reduce the universe of assets corresponding to these zero risk contribution before running the optimization problem.

For the computation purposes, as for the ERC portfolio, is used the following Sequential Quadratic Programming algorithm:

$$x_{RB} = \arg \min \sum_{i=1}^n \left( \frac{x_i(\Omega x)_i}{\sum_{j=1}^n x_j(\Omega x)_i} - b_i \right)^2 \quad (20)$$

$$u. c \ 1^T x = 1 \text{ and } 0 \leq x \leq 1$$

Figure 11: Risk Budgeting Defensive and Conservative Portfolios on mean variance space



*Note: The Efficient Frontier is plotted based on 5 years estimation period with data presented in the next chapter. All results are applied using Matlab. Source: Datastream*

## 2.2.4 The Global Minimum Variance Strategy (GMV)

Markowitz (1952) described the portfolio theory with risk averse investors having 2 choices:

- Increase returns as much as possible.
- Decrease the volatility as much as possible.

Global Minimum Variance is the portfolio strategy related to the second option. In other words, the GMV is the portfolio chosen by the most risk averse investors. To minimize the standard deviation, this strategy goes long on the assets with lowest variance and short on the assets with higher variance without taking into account expected returns. When short sales are not allowed the optimization process allocates zero, or close to zero, to the riskiest assets.

To construct the GMV strategy, which minimizes standard deviation under budget constraint, has to be computed the following algorithm

$$\begin{aligned}
 x^* &= \min f(x) \\
 f(x) &= \frac{1}{2} x^T \Omega x
 \end{aligned} \tag{21}$$

subject to *short selling constraint*

$$\begin{aligned}
 x &= \varepsilon[0; 1] \\
 x^T \mathbf{1} &= 1
 \end{aligned} \tag{22}$$

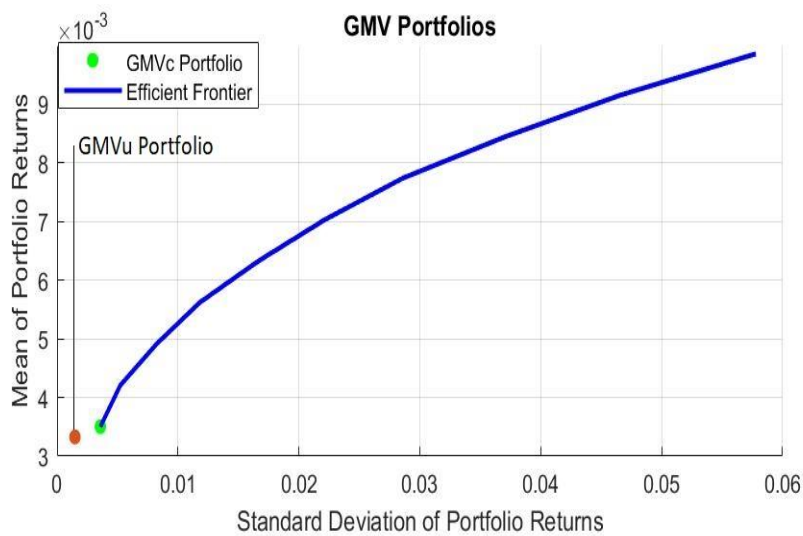
The unconstrained global minimum variance portfolio can be seen as a function of one parameter,  $\Omega$ , or more precisely,  $\Omega^{-1}$ . The inverse covariance matrix, also called the precision matrix, has a specific mathematical interpretation, the elements in  $\Omega^{-1}$  contains information about the partial correlation between variables. One can think of this as a measure of how correlated two assets are, given the influence of a set of other assets has been considered, and if the normal distribution is assumed as the distribution of the asset returns, then a partial correlation of 0 implies conditional independence.

The unconstrained strategy optimization problem can be solved in the following way

$$x_{uGMV}^* = \frac{\Omega^{-1} \mathbf{1}}{\mathbf{1}^T \Omega^{-1} \mathbf{1}} \tag{23}$$



Figure 12: Constrained and Unconstrained Global Minimum Variance Portfolios on mean variance space.



*Note: The Efficient Frontier is plotted based on 5 years estimation period with data presented in the next chapter. All results are applied using Matlab. Source: Datastream*

The GMVc portfolio, which lays on the leftmost point on the efficient frontier is efficient, in the sense that it has the best possible expected return for the level of risk. The unconstrained GMV portfolio is even more on the left because of the absence of constraints. Note that it not lays on the Efficient Frontier because the mean variance framework by definition doesn't allow short sales.

So, the Global Minimum-Variance portfolios, ex ante, are the portfolio with the lowest risk on the Efficient Frontier. In theory, these are also the portfolios with the lowest expected return.

### 2.2.5 Most Diversified Portfolio Strategy (MDP)

---

The Maximum Diversification Portfolio was introduced by Choueifaty and Coignard (2008) with the attempt to create the most diversified strategy by maximizing the distance between the weighted average volatility of each underlying in the assets portfolio and the overall portfolio volatility. By having the diversification benefits as the most important factor in order to immune the portfolio from external shocks, the authors introduce the diversification ratio (DR) which measure the gains from not having perfectly correlated assets. To obtain the MDP portfolio should be followed the same maximization techniques as in the mean variance framework. The

optimization process maximizes the DR which is defined as

$$DR(x) = \frac{x^T v}{\sqrt{x^T \Omega x}} \quad (24)$$

where  $v$  is the vector of standard deviation of asset returns.

The denominator of the equation is equal to the total portfolio volatility which takes into account the correlation between the underlying assets. The difference between the two is essentially the correlation term. To maximize the overall ratio, the denominator containing the correlations must be minimized. This allocation strategy attempts to select assets that minimize the correlation among the underlying assets and hence maximize diversification.

The optimization algorithm for the MDP becomes

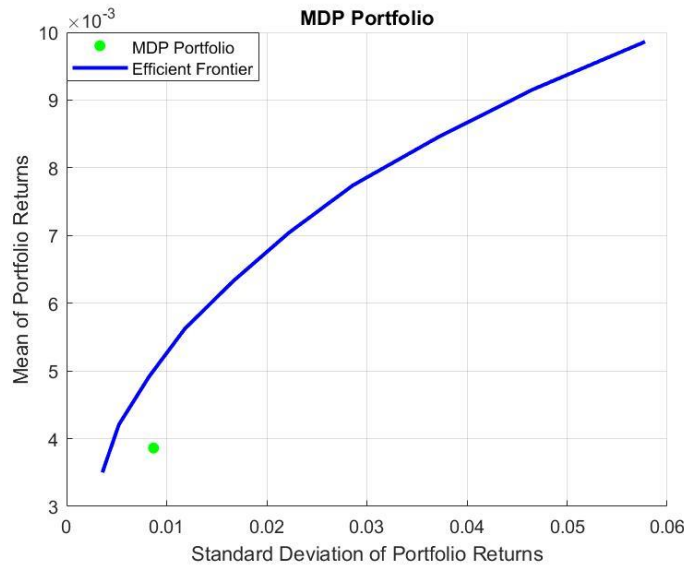
$$x_{MDP}^* = \max f(x)$$

$$f(x) = \frac{\sum_{i=1}^N x_i \sigma_i}{\sigma} = \frac{x^T \sigma}{\sqrt{x^T \Omega x}} \quad (25)$$

*under the constraints*

$$\begin{aligned} 1^T x &= 1 \\ 0 &\leq x \leq 1 \end{aligned} \quad (26)$$

Figure 13: Most Diversified Portfolio position on mean variance space.



Note: The *Efficient Frontier* is plotted based on 5 years estimation period with data presented in the next chapter. All results are applied using Matlab. Source: Datastream

From the figure can be seen how close the MDP is to the Efficient Frontier. This is because the GMV portfolio is optimized on the variance covariance matrix  $\Omega$  and the MDP is optimized based on the correlation matrix,  $\rho$ . and the final weights are retrieved by rescaling the intermediate weight vector with the standard deviations of the asset returns.

## 2.2.6 Maximum Sharpe Ratio Portfolio (MSR)

---

The “*Reward to Variability Ratio*”, (renamed in Sharpe ratio) was introduced as a fund performance measure by William Sharpe in 1966. It defines the maximum return achievable for the same amount of risk assumed.

The Maximum Sharpe Ratio Portfolio (MSR) follows the same logic that lays behind MDP. The MSR or Tangency Portfolio is a portfolio on the efficient frontier at the point where line drawn from the point (0, risk-free rate) is tangent to the efficient frontier. In other words, it corresponds to the second fund in the two-fund theorem (the first being the global minimum variance portfolio) and behaves as the market portfolio.

The optimization problem for the MSR portfolio becomes

$$x_{MDP}^* = \max f(x) \tag{27}$$

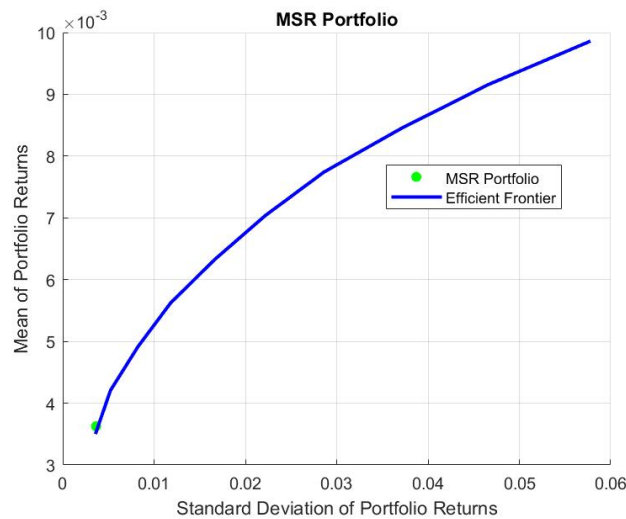
$$f(x) = \frac{\mu}{\sigma} = \frac{x^T \mu}{\sqrt{x^T \Omega x}}$$

under the constraints

$$\begin{aligned} 1^T x &= 1 \\ 0 &\leq x \leq 1 \end{aligned} \tag{28}$$

where  $\mu$  is the vector of expected returns.

Figure 14: The Maximum Sharpe Ratio Portfolio on mean variance space



Note: The Efficient Frontier is plotted based on 5 years estimation period with data presented in the next chapter. All results are applied using Matlab. Source: Datastream

The figure above shows that the MSR portfolio is very close the GMV portfolio. This is because the expected returns were computed as the average returns for the same period as the variance covariance matrix. As stated in previous paragraphs, the forecasting modelling of the expected asset returns is out of scope of this thesis and are used only for illustration purposes.

### 2.2.7 Inverse Volatility Portfolio (IV)

---

The Inverse Volatility, along with the Equally Weighted (EW), is a very intuitive and simple method for asset allocation. The strategy is implemented in a way that follows a volatility scheme such that the assets are weighted inversely to their volatility. There is no optimization process associated with the Inverse Volatility portfolio. Hence, the portfolio simply seeks to relative down weight more volatile assets. The optimal portfolio weights vector is given by

$$x_{IV}^* = \frac{1/\sigma_i}{\sum_n 1/\sigma_i} \quad (29)$$

(Note that the above equation is equal to the ERC strategy when equal correlations among assets is assumed)

Being called also Naive Risk Parity, the IV portfolio does not take into account the variance covariance matrix. This is the main drawback of this strategy since the assets may be penalized simply because of their relative higher volatility, while these may provide more diversification benefits should correlations also be considered.

## 2.3 Benchmark Portfolios

### 2.3.1 Equally Weighted Strategy (EW)

---

Equally Weighted Portfolio Strategy is probably the simplest way to allocate funds into different assets. Since it doesn't require an estimation of any kind, it is a useful tool for measuring different allocation strategies performances. In other words, it is a good benchmarking instrument.

The  $1/n$  strategy, also defined as a naive portfolio diversification was found by many studies to outperform other complex and sophisticated optimization techniques. DeMiguel et al. (2009) found that by dividing the wealth in equal  $n$  amounts, where  $n$  is the number of assets, and investing evenly regardless of risk, size or other factors, is a strategy that outperform most other techniques in terms of Sharpe Ratio. The authors claim that the results indicate that the gains from optimal diversification are more than offset by the estimation errors.

The appeal of the EW strategy is without doubt its implementation simplicity. By rebalancing the portfolio and keeping the weights fixed in time, investors sell winning stocks and buy losing stocks.

The weights of EW strategy are defined as

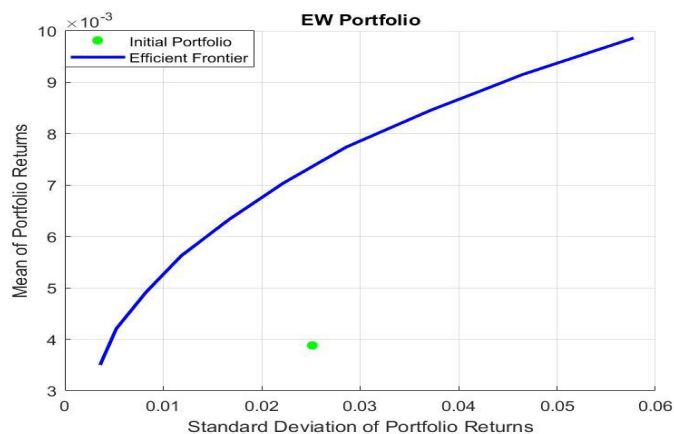
$$x_i = \frac{1}{n} \tag{30}$$

and the vector consisting of all portfolio weights as

$$x_{EW} = (1/n, \dots, 1/n)^T \tag{31}$$

The inefficiency of the EW portfolio in terms of mean variance optimization is pretty evident. The Figure 15 shows how far is the portfolio from the Efficient Frontier space. For a standard deviation of 2.5% the portfolio expected return is just 3.8%, while for the same volatility allocating on the efficient frontier the portfolio return would have been 7.4%. Given this framework one could expect low risk adjusted returns for EW strategy, but as we will later show, the story may be surprisingly different.

*Figure 15: Equally Weighted Portfolio Position on the mean variance space*



*Note: The Efficient Frontier is plotted based on 5 years estimation period with data presented in the next chapter. All results are applied using Matlab. Source: Datastream*

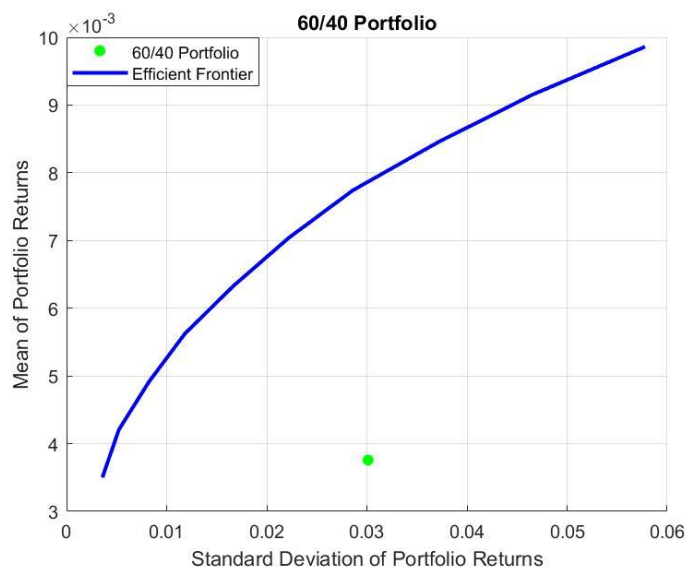
### 2.3.2 Traditional 60/40 Allocation Strategy

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For decades, the starting point of any conversation about asset allocation used to be the 60/40 portfolio – a basket of securities comprising 60% equities and 40% bonds. The strategy may sound diversified but in fact, over any period of time, equities will have accounted for between 80/90% of the volatility of the portfolio. Risk Parity was therefore introduced as a way to address this imbalance by emphasising balanced risk contribution among each asset class. While the solution to this disproportionate influence of the equity portfolio simply can be achieved by decreasing the equity exposure in favour of the bond weight, the problem with this approach is that the expected return would also decline.

On the Efficient Frontier space the portfolio position is far inefficient. An explanation for this extreme inefficiency may lay in high volatile and low return characteristics in the equity markets. Regardless of these critics, this portfolio is still applied among various institutional as well as private investors with a long-term perspective on their investments. This is why it is a good benchmark tool in order to evaluate the performance of the Risk Parity strategy.

*Figure 16: 60/40 Portfolio Position on the mean variance space*



*Note: The Efficient Frontier is plotted based on 5 years estimation period with data presented in the next chapter. All results are applied using Matlab. Source: Datastream*

## 2.4 Portfolio Performance Evaluation Methods

As a result of the empirical analysis, which will be described in the next chapter, different approaches to the performance evaluation are used. Besides the comparison with the benchmark portfolios, other traditional risk-returns measures have to be analysed.

To aid in the discussion of the strategies' return performances, three very popular measures are computed : Sharpe, Sortino and Treynor. For a more focused approach on risk dynamics are used: Drawdown, Expected Shortfall, Value at Risk, Calmar and Sterling Ratios.

### 2.4.1 Sharpe Ratio

---

The Sharpe ratio was introduced by Sharpe (1966), as a tool for measuring the risk-adjusted performances of investment funds. Since then, it became a very popular measure used by academics and practitioners. The ratio is based on the thought that the risk of the assets should be included when measuring return performance.

The excess return defined by Sharpe (1966) is given by

$$r_{ER} = r_p - r_f \quad (32)$$

where  $r_p$  is the return of the portfolio and  $r_f$  is the return from the risk free asset.

When the investor is re-investing returns in all periods, instead of the simple mean average, the geometric average is more appropriate

$$\bar{r}_{ER} = \left( \prod_{t=1}^T (1 + r_{ER,t}) \right)^{\frac{1}{T}} - 1 \quad (33)$$

By using the standard deviation definition from previous chapter can be obtained the Sharpe ratio:



$$\text{Sharpe Ratio} = \frac{\bar{r}_{ER}}{\sigma_p} \quad (34)$$

The ratio describes the average excess return over the volatility of the portfolio, or, in other words, how much an investor is rewarded for taking risk.

### 2.4.2 Sortino Ratio

---

Sortino and Price (1994) introduced an alternative way to consider risk and measure the performance of the portfolio. The idea behind the ratio is that the standard deviation is treating equally positive and negative returns volatility, therefore the authors propose to use only the downside volatility as a measure of risk.

The downside volatility is computed as following

$$\delta_D = \sqrt{\frac{1}{T} \sum_{n=1}^T \min(0, r_{i,t} - \bar{r}_{i,t})} \quad (35)$$

Therefore the Sortino ratio is given by

$$\text{Sortino Ratio} = \frac{\bar{r}_{ER}}{\delta_D} \quad (36)$$

### 3.4.3 Treynor Ratio

---

The Treynor ratio, named after the economist Jack Treynor, is a measurement of the returns earned in excess of that which could have been earned on an investment that has no diversifiable risk. Unlike Sharpe, Treynor uses beta in the denominator instead of the standard deviation. The beta measures only the portfolio's sensitivity to the market movement, while the standard deviation is a measure of the total volatility both upside as well as downside.

The beta of a portfolio is given by

$$\beta_p = \frac{Cov(r_p, r_M)}{Var(r_M)} \quad (37)$$

where  $r_M$  is the market return. Therefore the Treynor ratio is given by

$$Treynor\ Ratio = \frac{\bar{r}_{ER}}{\beta_p} \quad (38)$$

#### 2.4.4 The Maximum Drawdown

---

The Maximum Drawdown (MDD) is defined as a risk management measure to evaluate the maximum loss in value of portfolio over a specified time period

$$MDD_t = \frac{P - \max_{0 < s < t}(P_s)}{\max_{0 < s < t}(P_s)} \quad (39)$$

It should be taken into account that it only relates to the actual drop, and not the length of the drop or how many times there was an occurrence of a drawdown.

#### 2.4.5 Value at Risk and Expected Shortfall

---

The Value at Risk is a measure that estimates how much a set of investments might lose (with a given probability), given normal market conditions, in a set time period. In other words, it is a risk metric that summarizes the distribution of possible losses by a quantile, a point with a specified probability of greater losses. In other words, it answers the next question:

- *What is the most I can expect to lose, with 95% or 99% level of confidence, over the next time period?*

The VaR estimation is defined by Franke et al. (2004) in the following way

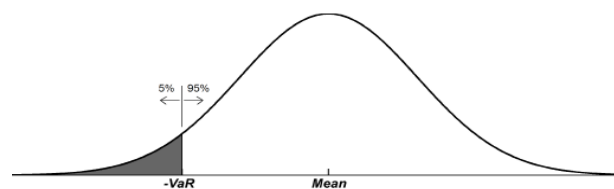
$$VaR_{q,t} = F_t^{-1}(q) \quad (40)$$

where  $F_t^{-1}$  represents the inverse function of the cumulative distribution of the underlying at

time  $t$  and confidence level  $q$ . Even though there are several ways to compute the VaR, this thesis will use mainly 2 approaches: the historical method and the Cornish – Fisher method.

The Historical method is the simplest way to proceed, where the historical rate of returns are organized in quantiles from worst to best in a histogram. For the 95% confidence level, the worst 5% of the outcomes are selected. Then can be concluded that the loss for a given period of time will not exceed this worst outcome with probability 95% . The weakness of the historical approach is that it relies on the assumption that history will repeat itself which is far from the reality.

*Figure 17: The 5% Value at Risk of a hypothetical profit-and-loss probability density function*



The Cornish-Fisher-Expansion, also called Modified VaR or Modified Cornish-Fisher VaR, is an alternative approach to calculate the Value at risk. If the return of a portfolio is not Gaussian distributed, then the classical VaR method is no longer an efficient measure of risk. In order to account for non-Gaussian one can use the Cornish-Fisher method which is accurate when returns are close to the Gaussian distribution. This method takes into account the higher moments, skewness and kurtosis. Skewness is the tilt of the returns and kurtosis is a measure for the fat-tails of the returns. The moments for a portfolio may be estimated either by using the historical returns of the portfolio or one can use multivariate estimate for the moments. When the returns have negative skewness or fat-tails (that is platykurtic) the Cornish-Fisher VaR will give a larger estimation for the loss than the usual VaR. On the other hand, when returns possess positive skewness or are leptokurtic, the loss estimation will be smaller than traditional VaR. When returns are Gaussian distributed this method converges to the usual parametric VaR.

The Cornish-Fisher formula for VaR is the following

$$mVaR = \mu(X) + \sigma(X)z_{cf} \quad (41)$$

$$z_{cf} = q_p + \frac{(q_p^2 - 1)S(X)}{6} + \frac{(q_p^3 - 3q_p)K(X)}{24} - \frac{(2q_p^3 - 5q_p)S^2(X)}{36}$$

where  $S(X)$  is skewness,  $K(X)$  is kurtosis and  $z_{cf}$  is the Cornish-Fisher critical value for the confidence level  $p$ .

The main difference between historical measures and Cornish-Fisher VaR is that while the former will not deviate from observed returns since it relies on historical returns, Cornish-Fisher VaR tries to estimate the shape of the tail for the returns mathematically even though extreme returns have not been observed yet.

An additional tool to measure the risk of extreme events is the Conditional Value-at-Risk or, Expected Shortfall (ES). VaR tries to compute the loss at a particular quantile  $q$  but tells nothing about the distribution below the  $q$ . ES on the other hand gives the average loss in the tail below  $q$ . A common definition of the ES is the following

$$ES_\alpha(X) = \frac{1}{1 - \alpha} \int_\alpha^1 VaR_\beta(X) d\beta \quad (42)$$

For a given portfolio can be proven that

$$ES_\alpha(X) \geq VaR_\alpha(X) \quad (43)$$

The ES and VaR are very good tools to measure the probabilities of extreme losses in the portfolio but high attention should be paid for the asymmetric dependence and non-normalities in the distribution of returns such as auto-regression, asymmetric volatility, skewness, and kurtosis.

In this thesis are computed the Value at Risk and Expected Shortfall metrics only at 95% confidence interval.

#### 2.4.6 Calmar and Sterling Ratios

---

Calmar and Sterling ratios are performance measures (very similar to the Sharpe ratio) that

consider the drawdown of a portfolio to assess the portfolio performance.

$$\text{Calmar/Sterling Ratio}_t = \frac{r_p}{MDD_t} \quad (44)$$

The only difference between the two ratios is the time period considered for estimation. Sterling ratio usually is used for annualized data meanwhile Calmar ratio is used for larger periods, such as 3 years.

## 2.4.7 Turnover and Transaction Costs

---

The last performance measure analysed is the portfolio turnover. Studies in the U.S have showed that on average, active equity managers historically underperformed the S&P 500 mainly because of transaction costs (Grinold and Kahn, 1999). Because of such findings, the transactions costs of the different investment strategies needs to be assessed.

The asset turnover is a volume-based measure of the amount of trading required to implement a particular portfolio strategy. For a given strategy  $k$  with  $N$  risky assets the turnover is computed in the following way

$$\text{Turnover}_t = \frac{1}{T-t} \sum_{t=1}^T -t \sum_{i=1}^N (w_{k,i,t+1} - w_{k,i,t}) \quad (45)$$

where  $w_{k,i,t}$  is the portfolio weight in asset  $i$  at time  $t$  according to strategy  $k$ . Should be noted that a high turnover rate does not necessarily imply high transaction costs. For instance, in the case of highly liquid markets and high asset concentration in the portfolio, trading costs can still be low. However, turnover is clearly a negative factor when considering larger universes with less liquid asset classes.

## Part III

### Data and Empirical Results

*"The four most dangerous words in investing are: 'this time it's different.'"*

*Sir John Templeton*

This chapter details the results obtained from the portfolio constructions methodologies, highlighted in the previous chapter, and investigates the benefits of the various asset allocation strategies' features such as return, risk, diversification and risk weighting. Data set and the investment universe along with a brief review of the recent European market environment will also be discussed.

#### 3.1 Methodology

##### 3.1.1 Terminology

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The empirical research is considering an institutional investor who invests all his endowment in the financial markets. In each of the optimizations models there are considered three periods of time. In general terms the investment is carried out at time  $t = 0$  based on the empirical data collected from time  $t = -1$ , with the returns of the investment is collected at  $t = 1$ . The investor has an initial wealth,  $W_0 > 0$ , and he invests all of this wealth at time  $t = 0$ . The investment universe consists of 20 assets that the investor can choose to invest in. The prices of these 20 assets at  $t = -1$  and  $t = 0$  are given by  $S_{i,t}$  for  $i = 1, \dots, 20$  with  $t = -1, 0, 1$ .

The prices from period  $t = -1$  to  $t = 0$  are known and the rate of return of every asset is defined by

$$r_i = \frac{S_{i,0} - S_{i,-1}}{S_{i,-1}} \quad (46)$$

with the vector of 20 assets return is defined by

$$r = (r_1, \dots, r_{20})^T \quad (47)$$

Note that the prices from period  $t = 0$  to  $t = 1$  are stochastic and unknown. This thesis, as early specified, does not try to forecast the expected returns and all allocation strategies are only risk-based.

The variance of asset  $i$ 's returns up to period  $T$  is defined as

$$\sigma_i^2 = \frac{1}{T} \sum_{n=1}^T (r_{i,t} - \bar{r}_{i,t})^2 ; \sigma_p^2 = x^T \Omega x \quad (48)$$

where  $\bar{r}_i$  is the mean return of asset  $i$ , and  $\Omega$  is the variance – covariance matrix

$$\bar{r}_i = \frac{1}{T} \sum_{n=1}^T r_{i,t} \quad (49)$$

The standard deviation of asset  $i$  return is

$$\sigma_i = \sqrt{\sigma_i^2}; \sigma_p = \sqrt{x^T \Omega x} \quad (50)$$

And the vector of all assets standard deviations  $v$  is given by

$$v = (\sigma_1, \dots, \sigma_{20}) \quad (51)$$

The covariance between 2 asset returns  $i$  and  $j$  is given by

$$\sigma_{i,j} = cov(r_i, r_j) = \sum_{n=1}^T \frac{(r_{i,t} - \bar{r}_i)(r_{j,t} - \bar{r}_j)}{T} \quad (52)$$

In matrix notation the matrix covariance matrix is defined as in Ledoit and Wolf (2004)

$$\Omega = \frac{1}{T} X(I - \frac{1}{T} II^T)X^T \quad (53)$$

and can be represented as a matrix with  $N \times N$  dimension, which is symmetric and quadratic

$$\Omega = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \ddots & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} & \cdots & \cdots & \sigma_n^2 \end{pmatrix} \quad (54)$$

The correlation between the returns  $i$  and  $j$  is the measure of the movement in standardized covariance, defined as

$$\text{corr}(r_i, r_j) = \rho_{i,j} = \frac{\sigma_{ij}}{\sigma_i \sigma_j} ; \rho_{i,j} = [-1; 1] \quad (55)$$

The investment strategies are considered the vectors of weights  $x = (x_1, \dots, x_{20})^T$  that satisfy the following equation

$$x^T \mathbf{1} = \sum_{i=1}^n \frac{a_i S_{i,0}}{W_0} = 1 \quad (56)$$

where  $a_i$  is the relative portfolio weight after the optimization process. The sum of all strategies must have the sum equal to 1.

### 3.1.2 Main Assumptions

---

Before proceeding with portfolio allocation analyses is important to point out the assumptions behind the mean - variance theory:

- The monthly returns are not autocorrelated in time, which means independent, identically and normally distributed.
- Assets are perfectly divisible.
- There are not market frictions such as illiquidity, variable transactions costs based on size, and taxes.



- All the portfolios are long only, therefore under short selling constraint.
- Investors are risk averse. In other words, the investor wants to allocate his endowment, to maximize his terminal wealth,  $W$ , and at the same time minimize the risk associated with his terminal wealth.
- Investors have a von Neumann-Morgenstern<sup>6</sup> utility function  $u(W)$  with the following properties: positive marginal utility of wealth  $u'(W) > 0$ ; negative marginal utility with an increase in wealth  $u''(W) < 0$  (properties which imply risk aversion). In other words, the investor's utility function increases by increased expected terminal wealth, and decreases with increasing risk measured in variance of the expected terminal wealth.

### 3.1.3 Data: Source, Methodology and Investment Universe.

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All data used for this thesis analysis is gathered exclusively from Datastream Data Base. Index prices are downloaded on a monthly basis starting from 1<sup>st</sup> January 1999 until 1<sup>st</sup> April 2018 resulting in 233 observations with all expressed in total return prices, which assume all dividends are re-invested at the adjusted closing price in the following way:

$$TRI_t = TRI_{t-1} \frac{S_t + D_t}{S_{t-1}} \quad (57)$$

where  $D_t$  is the dividend pay-out at time  $t$ .

The empirical analyses are based on rolling sample approach involving three steps on iteration:

- At time  $t = 0$ , the parameters, required for the portfolio optimization strategies, such as variance covariance matrix, are estimated, for a period of past 6 years, from 1999 to 2005, from 72 monthly observations.
- Next step implies the implementation of the optimization strategies based on the historical estimates computed in the previous step.
- The final step is to compute the portfolios analyses for the window  $t = 0$  to  $t + 1$ , thus from 2005 to 2018, based on the optimal set of asset weights at time  $t = 0$ .

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<sup>6</sup> For more information on the utility function see Neumann, John von and Morgenstern, Oskar, Theory of Games and Economic Behavior. Princeton, NJ. Princeton University Press, 1953.

The last tool of the back-testing setup is the portfolio rebalancing which is a paramount to investors that want to maintain their targeted asset allocation fixed. Rebalancing is also extremely important for a dynamic asset allocation, when for example, the increase in the risk aversion increases the exposure to the low risk assets. In this thesis monthly rebalancing is chosen and constant risk aversion is assumed, that is, the optimization strategies will not change the allocation on the factors other than  $\Omega$ ,  $\sigma$  and  $\mu$ .

The investment universe for the potential investor consist of 20 European Indices divided in Government Bonds, Corporate Bonds and Equities. Each asset class is further divided by its own main characteristics: government bonds by maturity, corporate bonds by rating, and equities by sectors. Even though there exist more categories in the bonds asset class, this thesis will focus only on the investment grade assets as the techniques applied correspond to an institutional investor which has the mandate to allocate the funds under certain constraints.

The indices are provided by Citigroup and Morgan Stanley Capital Indexes (MSCI) and are the following:

### **Government Bonds**

- Citi European Government TR Bond Index 1-3 Years Maturity
- Citi European Government TR Bond Index 3-5 Years Maturity
- Citi European Government TR Bond Index 5-7 Years Maturity
- Citi European Government TR Bond Index 7-10 Years Maturity
- Citi European Government TR Bond Index 10-15 Years Maturity
- Citi European Government TR Bond Index 30+ Years Maturity

### **Corporate Bonds**

- Citi European Corporate TR Bond Index AAA-Rating
- Citi European Corporate TR Bond Index AA-Rating
- Citi European Corporate TR Bond Index A-Rating
- Citi European Corporate TR Bond Index BBB-Rating

### **Equities**

- MSCI Europe Energy TR Index

- MSCI Europe Materials TR Index
- MSCI Europe Industrials TR Index
- MSCI Europe Consumer Discretionary TR Index
- MSCI Europe Consumer Staples TR Index
- MSCI Europe Health Care TR Index
- MSCI Europe Financials TR Index
- MSCI Europe Information Technology TR Index
- MSCI Europe Telecommunication Services TR Index
- MSCI Europe Utilities TR Index

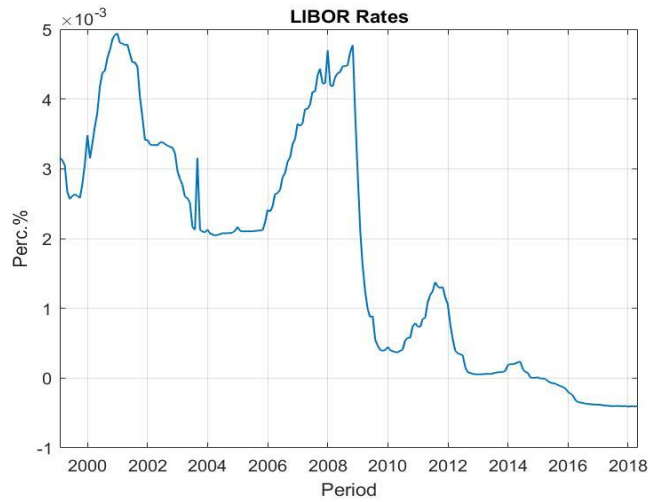
There are several reasons why considering these indices to be a good investment universe in Europe. First, all components in this index are listed in euro, so there will be no exchange rate risk for an investors that only allocate his wealth among these assets. Second, MSCI and Citi choose the components of the index to be balanced among 12 eurozone countries and 10 super-sectors. This secures a diversified universe across sectors and markets.

As for the benchmarks are used the following indices:

- MSCI Europe Index – benchmark for equity markets.
- JPM Europe Aggregate Euro Bond– benchmark for bond markets.

As for the risk free rate is used the London Interbank Offered Rate (LIBOR) since it is the world's most widely used benchmark for short-term interest rates. It serves as the primary indicator for the average rate, at which contributing banks may obtain short-term loans in the London interbank market.

Figure 18: LIBOR Rates from 1999 to 2018

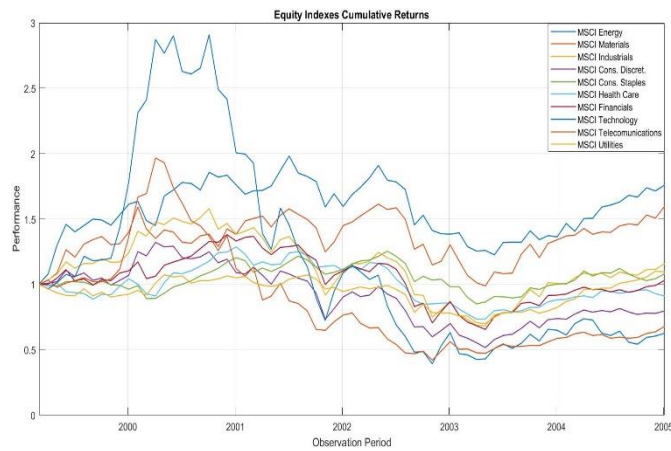


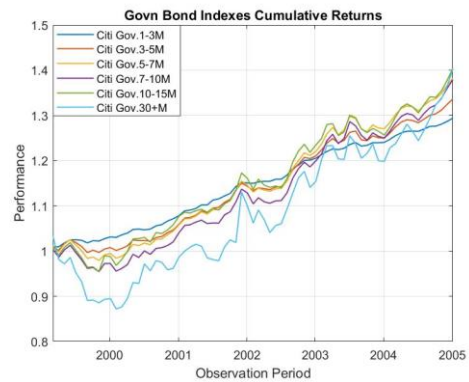
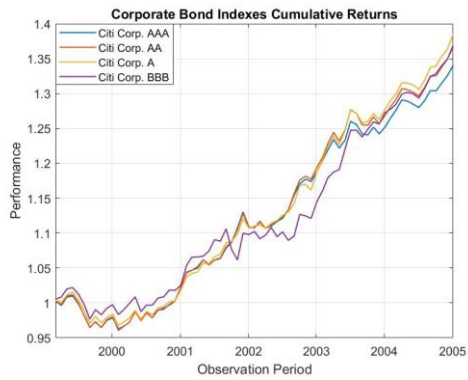
Note: All results are applied using Matlab Source: FRED

### 3.1.4 Facing the Allocation Puzzle

One of the main goal of this thesis is to analyse how different optimization strategies have performed from 2005 to 2018 relative to more simple or traditional allocation principles. As pointed earlier, the empirical analysis virtually turn back in time and, according to the historical performance and moment estimates of asset classes, constructs optimal allocation strategies. In January 2005 the financial picture in Europe was as following:

Figure 19: Performance of all asset classes from 1999 to 2005

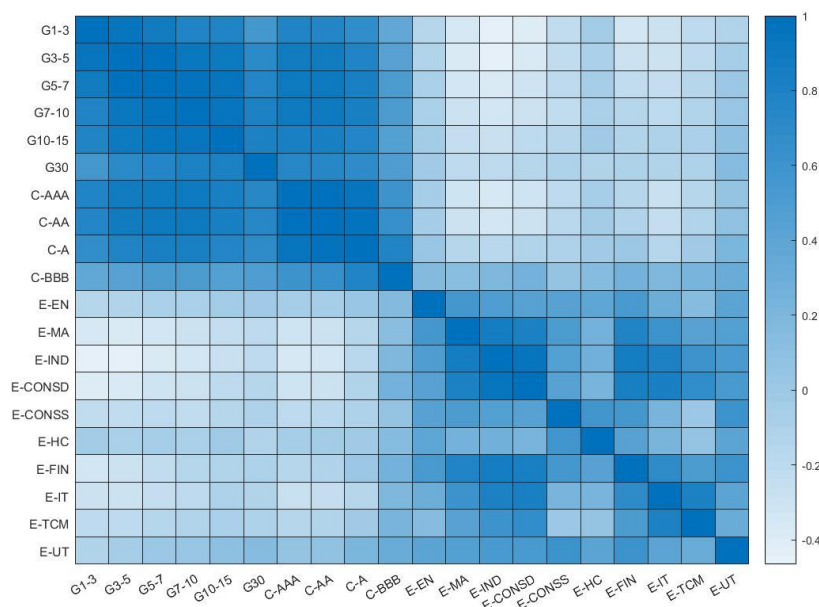




*Note: All results are applied using Matlab. Source: Datastream*

The bond markets performed pretty well during the 6 year period. Specifically, government bonds gained around 30%, and corporate bonds around 35%. Recall from the Figure 1 that during this period the ECB was following an accommodative monetary policy and decreased the interest rates from 4.5% to 2%. This environment was positive for fixed-income assets and boosted their returns, but at the same time, lower borrowing costs failed to remove pressure from the equity markets. The stock indices performed modestly during the same period relative to the bonds, with Financials, Technology, and Telecommunications indices experiencing significant losses.

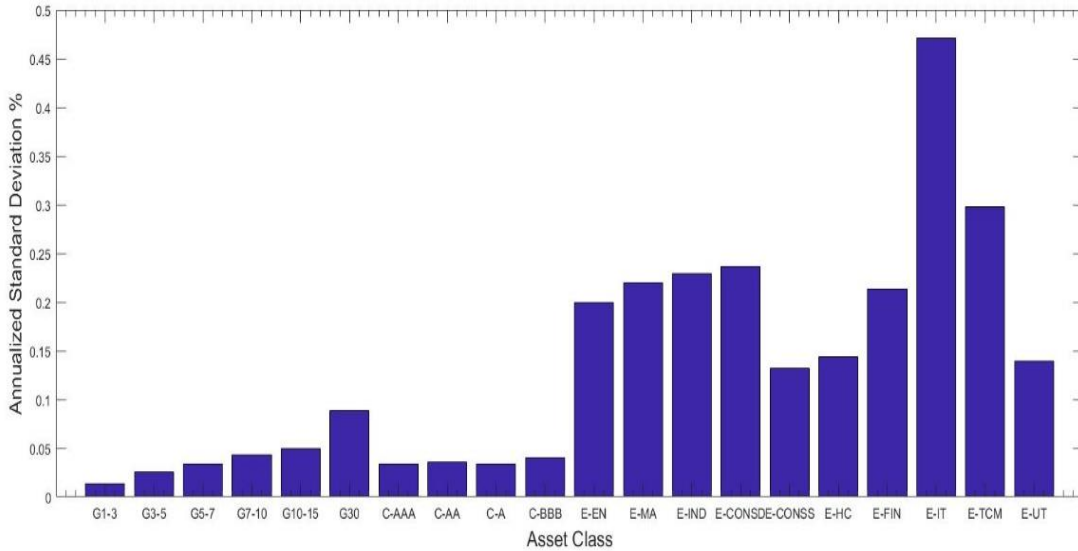
*Figure 20: Correlation Matrix Heatmap of Asset Classes, 1999-2005*



*Note: All results are applied using Matlab. Source: Datastream*

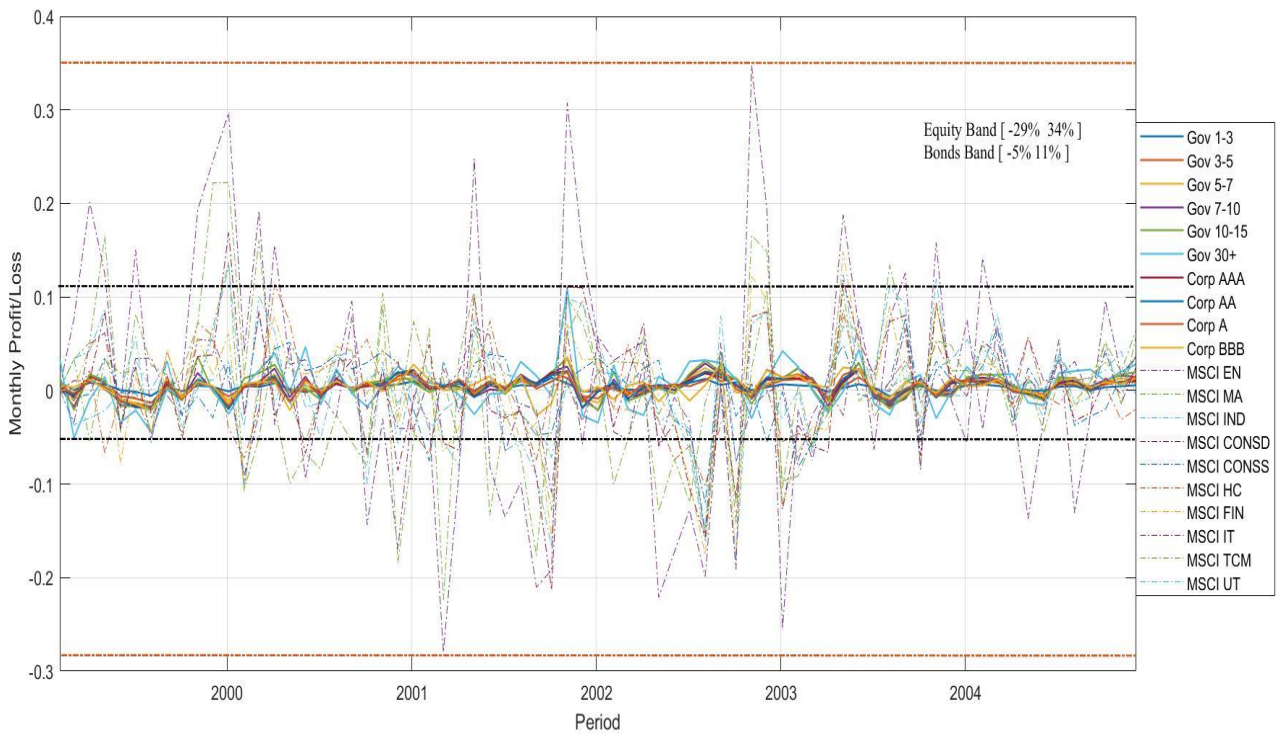
Looking at the relationship of price movements, correlation coefficients, as expected, proved to be positive inside the asset classes and negative among the 3 asset classes.

Figure 21: Standard Deviation of Asset Classes, 1999-2005



Note: All results are applied using Matlab. Source: Datastream

Figure 22: Monthly Returns Dynamics of Asset Classes, 1999-2005



Note: All results are applied using Matlab. Source: Datastream

The fact that correlations are not all the same, and even negative for the sample period, suggests that incorporating correlation estimates in the portfolio construction would be very useful. Moreover, the high volatility in equity returns demands optimization approaches which could take risk as a main allocation factor. Using the Risk Budgeting portfolio strategies to rank asset classes, strongly benefits in relying on correlation estimates which are not expected to change dramatically in different economic regimes.

The Figure 22 evidences how much volatile were stocks relative to bonds for the 6 years estimation period. While bonds' prices fluctuated within a narrow band with the maximum loss experienced of 5%, equities' prices were extremely more variable with some indices touching losses as much as 30%. Observing that the return moments are not equal and highly diverging between equity and fixed-income and also omitting any temptation to forecast volatility or returns, strengthens the path forward a strategic asset allocation.

### **3.2 Results Analyses and Portfolio Back-testing**

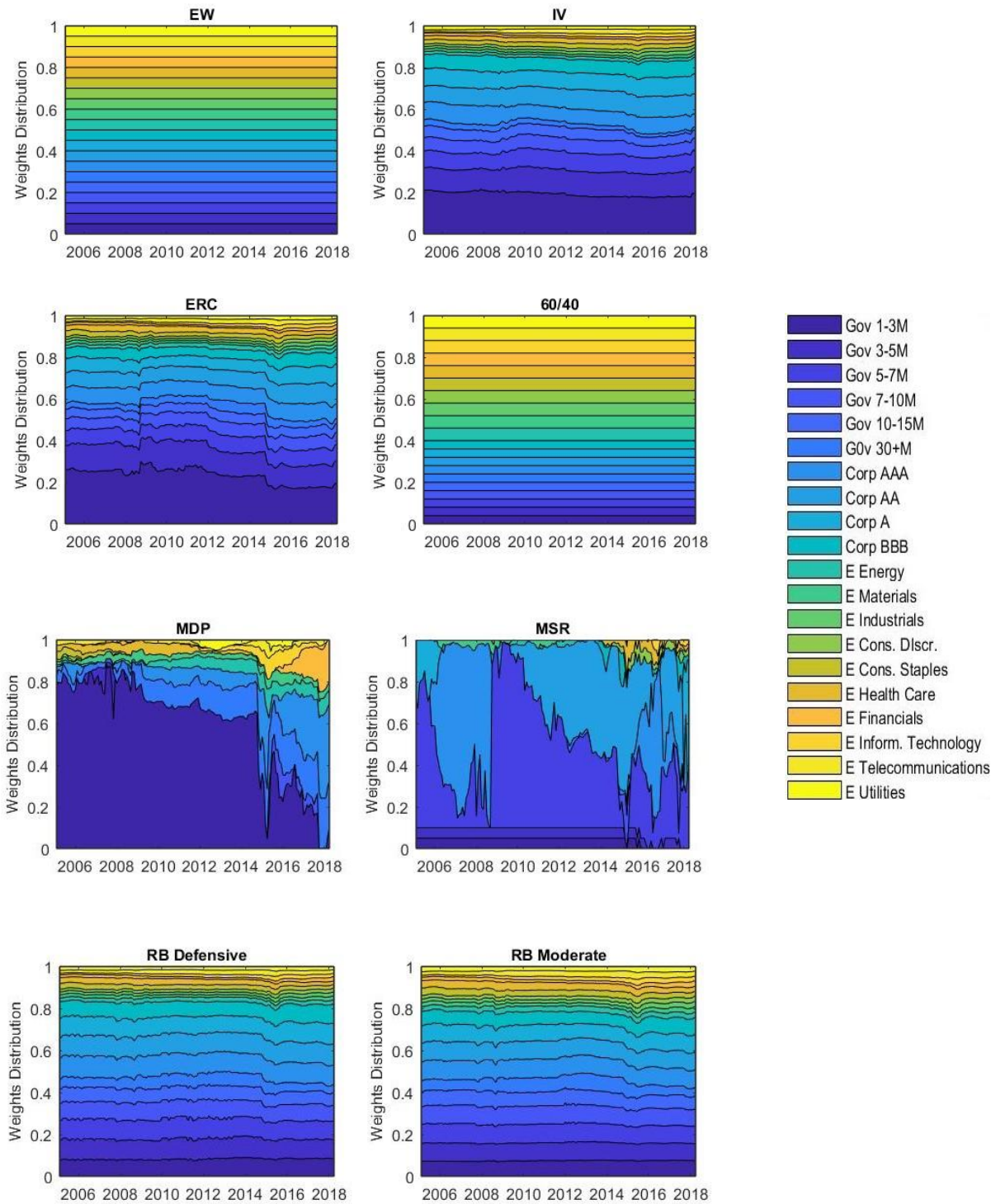
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In this session the main results of the empirical analyses are presented. The session is organized so that to answer the questions which were posed in the previous chapters along with the discussions for each of them. The result analysis will start with the asset weights optimization and their distribution in time. Second, the volatility and other risk measures will be very closely investigated, and finally, the portfolio returns estimates and investment performance ratios will be deeply evaluated.

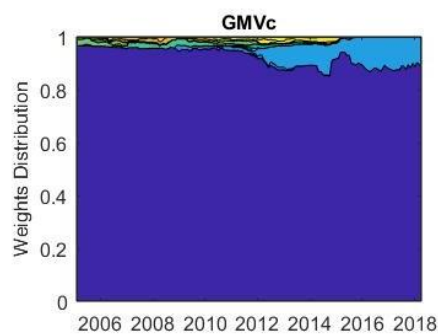
#### **How much concentrated are the risk based allocation strategies compared to traditional strategies?**

The portfolio strategies, as pointed earlier, are optimized on a monthly basis from an estimation

Figure 23: Weights Distribution as a result of optimization strategies







*Note: The weights optimization and distributions are computed on a monthly rebalancing and on a rolling estimation for a period of 72 months.*

*All results are applied using Matlab. Source: Datastream*

period of 6 years. The above figures present how many assets and in which amount these are included in each of the portfolio strategies. The GMVc portfolio, as highly expected, has the highest concentration of assets, with around 90% invested only in short term government bonds. Note that an upper bound of 5% was introduced in the MSR portfolio for the first 2 assets, Citi Government Bond 1-3 Maturity and Citi Government Bond 3-5 Maturity. This constraint was necessary in order to avoid that the optimization process over-weights the portfolio concentration in these assets and create a strategy without much sense<sup>7</sup>.

The MDP and especially MSR portfolios, started the allocation distribution with a high concentration in short term government bonds but gradually the optimization process initiated to include other assets and especially corporate bonds, which performance weren't affected so much during the "European Sovereign Debt Crisis".

The ERC and IV portfolios experienced a more stable allocation distribution in asset weights by having constantly above 80% of the portfolio endowment invested in fixed income assets. Moreover, the IV portfolio, which doesn't go through an optimization strategy, experienced a more balanced asset allocation compared to the ERC. Another interesting difference between the two portfolios is the allocation to the corporate bonds, in which ERC has invested no more than 20% of the endowment compared to 30% for IV portfolio. Obviously, this discrepancy is due to the highly positive correlation coefficients inside the corporate fixed income assets.

The Defensive RB portfolio, which allocates risk 40%, 30% and 30% to government bonds, corporate bonds and equities respectively, distributed the endowment more equally among

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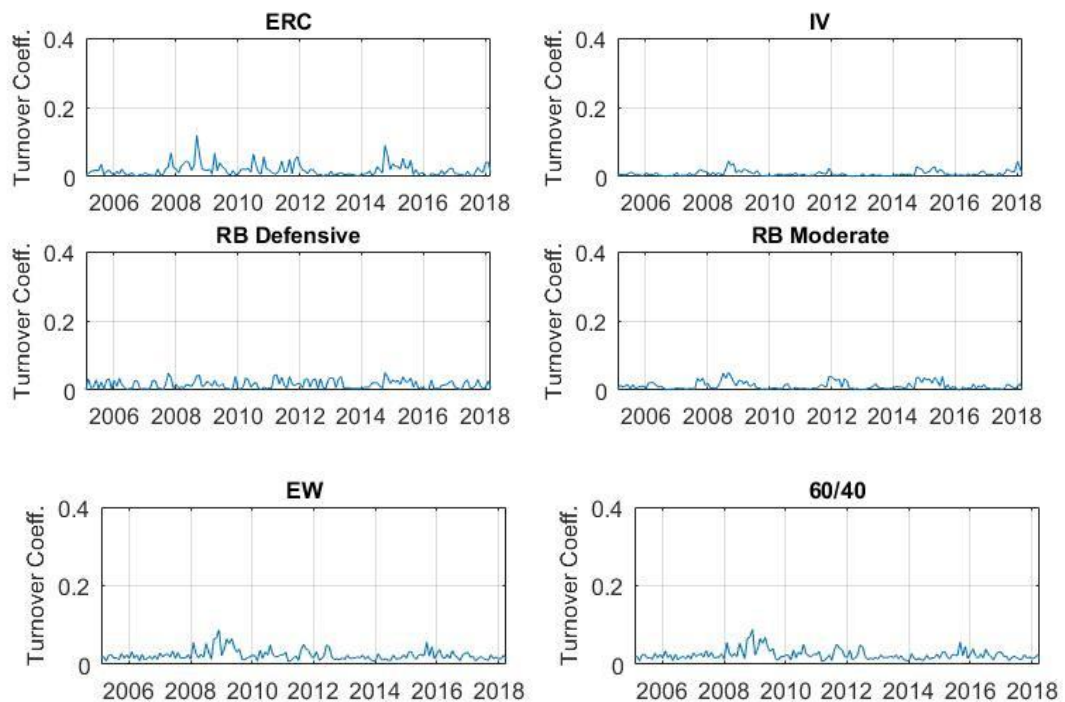
<sup>7</sup> For more information about the importance of the constraints on portfolio see "Understanding the Impact of Weights Constraints in Portfolio Theory" Roncalli (2011).

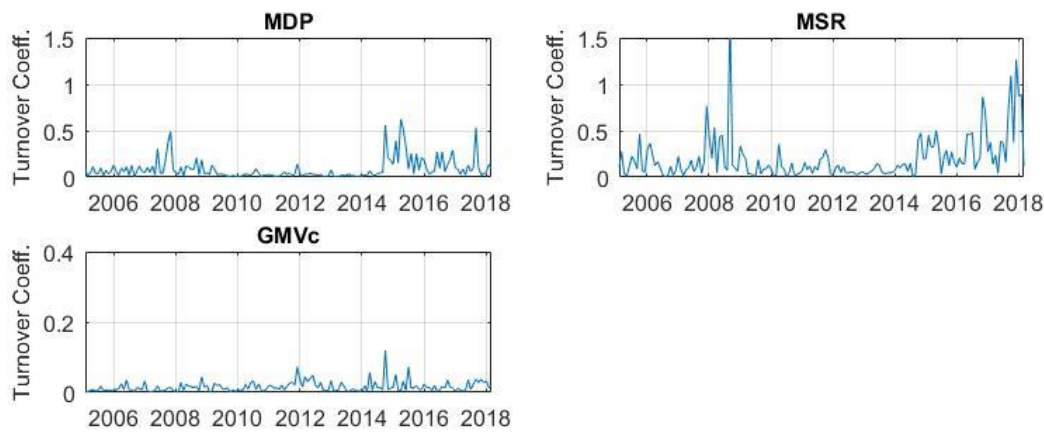
fixed income assets compared to the ERC, which allocated more than 20% only to the short term government bonds.

Important also is to outline how extreme was the concentration of endowment in a very few assets for GMVc, MDP, and MSR portfolios. Even though they try to maximize their objective function, these strategies raise fair questions regarding their robustness and ability to diversify risk.

Another indicator of stability and robustness of allocation strategies are given by the turnover rates, since they give an important information regarding the need and the frequency of rebalancing with the associated costs to maintain a strategy. From the figure can be noted that the ERC portfolio had the highest turnover coefficient among risk based strategies but anyway, didn't surpass 0.1% of the assets. The IV portfolio had a relevant lower rate of turnover than ERC, result which evidences IV to be a probable better alternative to the optimized strategies.

*Figure 24: Turnover Rates of allocation strategies*





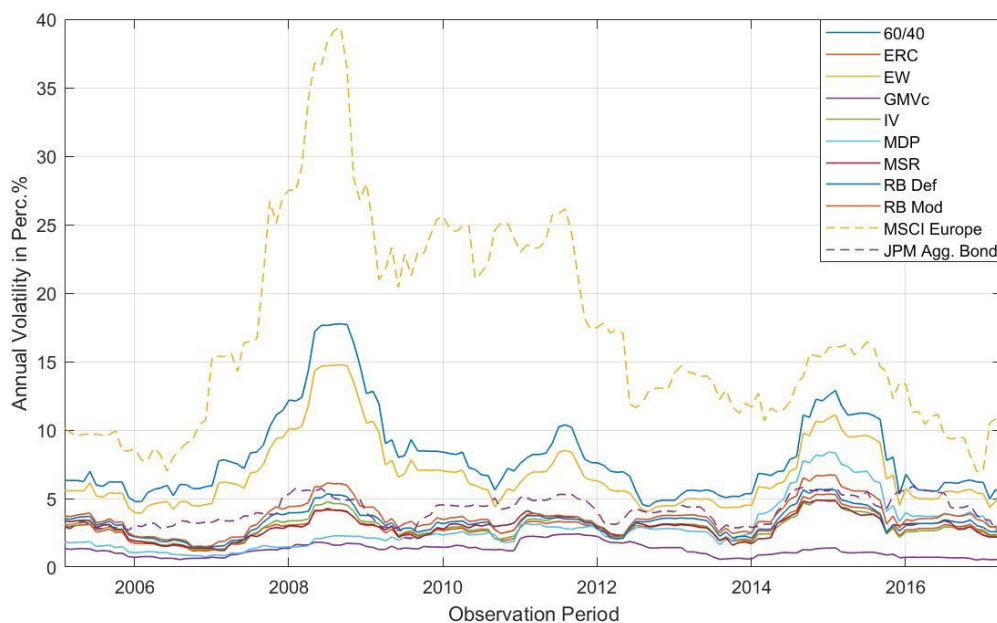
*Note: The rebalancing of the portfolios is executed at the end of each month based on optimization process. All results are applied using Matlab. Source: Datastream.*

The MDP and MSR strategies, on the other side, experienced very high turnover ratios reaching more than 100% in some distressed periods. Such extreme levels point out another disadvantage of these allocation strategies: intensive rebalancing which also induce higher transaction costs. EW and 60/40 strategies, which sell the winners and buy the losers, experienced relatively a low turnover for the estimation period showing some slightly increases after the subprime financial crisis.

### **How risk based allocation strategies manage risk/volatility compared to traditional strategies?**

The first, and probably the most important, measure of risk to compute and to analyse is the dispersion from the mean of the return observation. The next figure presents the dynamics of the annualized standard deviation of returns of all allocation strategies over the whole observation period.

Figure 25: Standard Deviation of Portfolio Allocation Strategies and Benchmarks



Note : The moments were computed on a 12 month rolling volatility and annualized by using the rule of “the square root of time”  $\sigma_T = \sqrt{T} \cdot \sigma_T$ . All results are applied using Matlab. Source: Datastream.

The allocation strategies that experienced the highest volatile returns proved to be those which were not optimized, 60/40 along with the EW. During the 2008 year, when the subprime crises caused a turmoil in global financial markets, the above mentioned portfolios experienced extreme levels of volatility with the 60/40 and EW reaching 18% and 14.5% respectively. The volatility of the risk based allocations, comparatively, increased only by 250 basis points during the same period. The MSR and MDP allocation strategies, due to their exposure to a very limited number of low risk assets, haven’t felt the meltdown in asset prices experiencing mostly flat dynamics. The second shock to the asset prices, according to the estimation period, happened in 2011. This event was characterized by the “European Sovereign Debt Crisis<sup>8</sup>”, a period in which the high level of government debt raised questions for institutional investor regarding whether the European countries will be able to sustain their huge amount of debt. The spike in government bonds yields produced high fluctuations in asset prices with the most sensitive allocation strategies proved to be one more time the 60/40 and EW.

Impossible to not note how extremely high was the volatility of the MSCI Europe Index relative to the allocation strategies. Such an observation evidences the importance of the inclusion of

<sup>8</sup> For more information see “The European Sovereign Debt Crisis” Lane (2012).

different asset classes in the allocation strategy. On the same time, the JPM Europe Aggregate Bond Index, featured a lower volatility with respect to the 60/40 and EW strategies but failed to compete with all other allocation strategies. Such a relatively high level of volatility observed for the fixed income basket of securities evidences one more time the weakness of a merely passive capitalization-index investing.

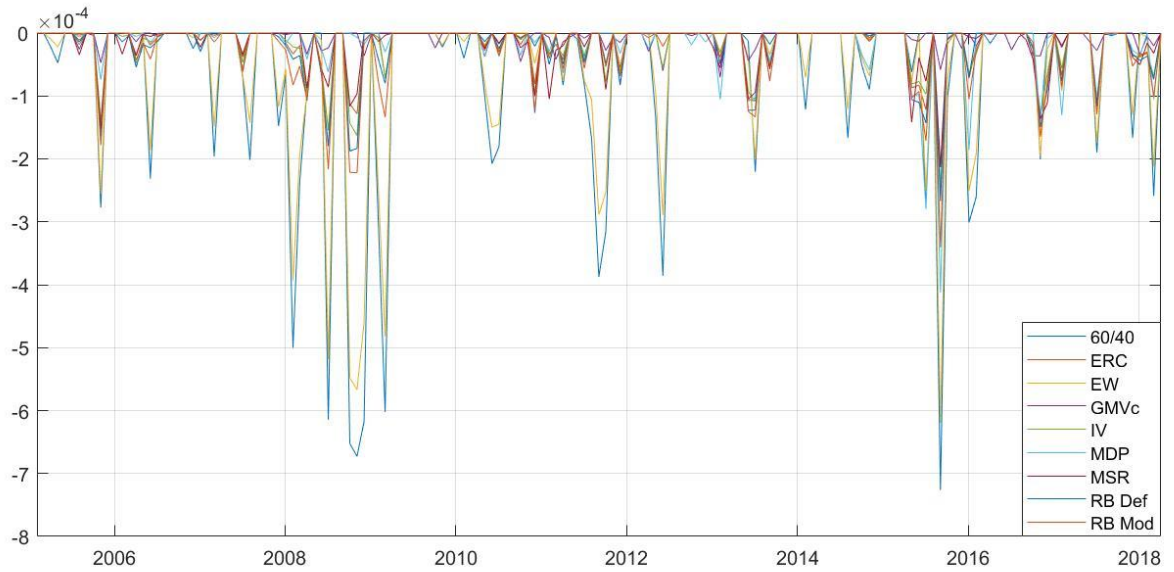
The Quantitative Easing monetary operation, applied by the ECB to fight the deflationary spiral in 2014, created another increase in the asset prices' volatility. Remarkable dynamics during this period experienced the MDP portfolio whose price' fluctuation increased by 600 basis points, from an average of 2% to reach more than 8% annually. Such an unexpected lance in volatility raise fair questions regarding the robustness of this optimization strategy. The QE monetary operations revealed very well the weaknesses of the MDP portfolio, which proved to be very sensitive in an environment where the correlations among asset classes tend to increase.

Another interesting evidence can be noted from the Figure 25. The volatility of the IV portfolio followed very closely the volatility of risk based optimized portfolios. Without taking into account the correlation coefficients in the allocation process, IV strategy proved to be a reliable tool to deal with unexpected shocks in financial markets.

The GMVc strategy experienced a relatively constant volatility for the whole estimation period, fluctuating within a narrow band without surpassing 2% annually.

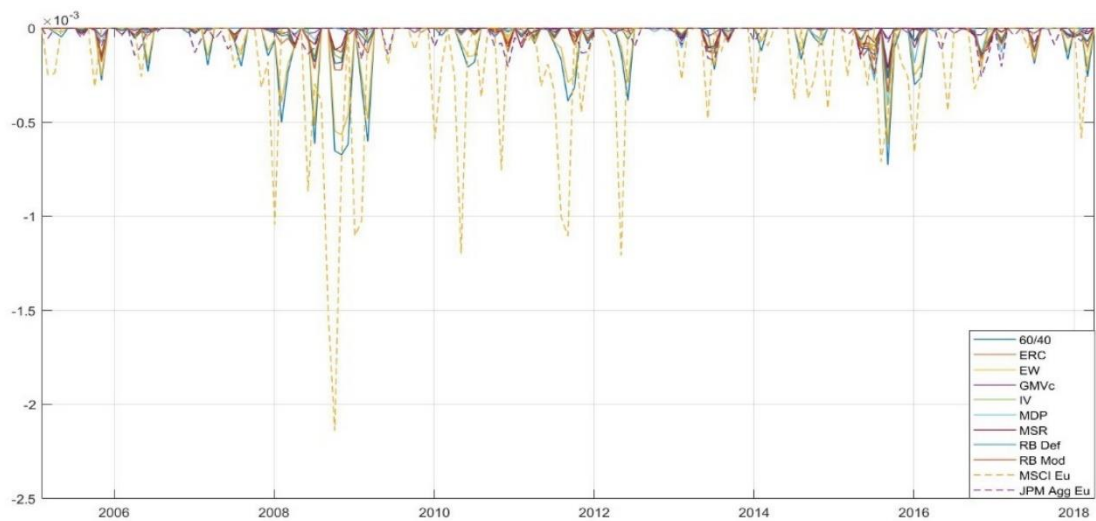
The Drawdown, presented in the Figure 26, tells the same story from a different corner. The portfolios that suffered the largest fall in prices are 60/40 and EW. For the estimation period, these allocation strategies experienced 5 drawdowns of more than 5% on a monthly basis, meanwhile the falls in prices of risk based strategies were all under 3%.

Figure 26: Monthly drawdowns of allocation strategies



Note: All results are applied using Matlab. Source: Datastream

Figure 27: Monthly drawdowns of allocation strategies relative to Benchmarks

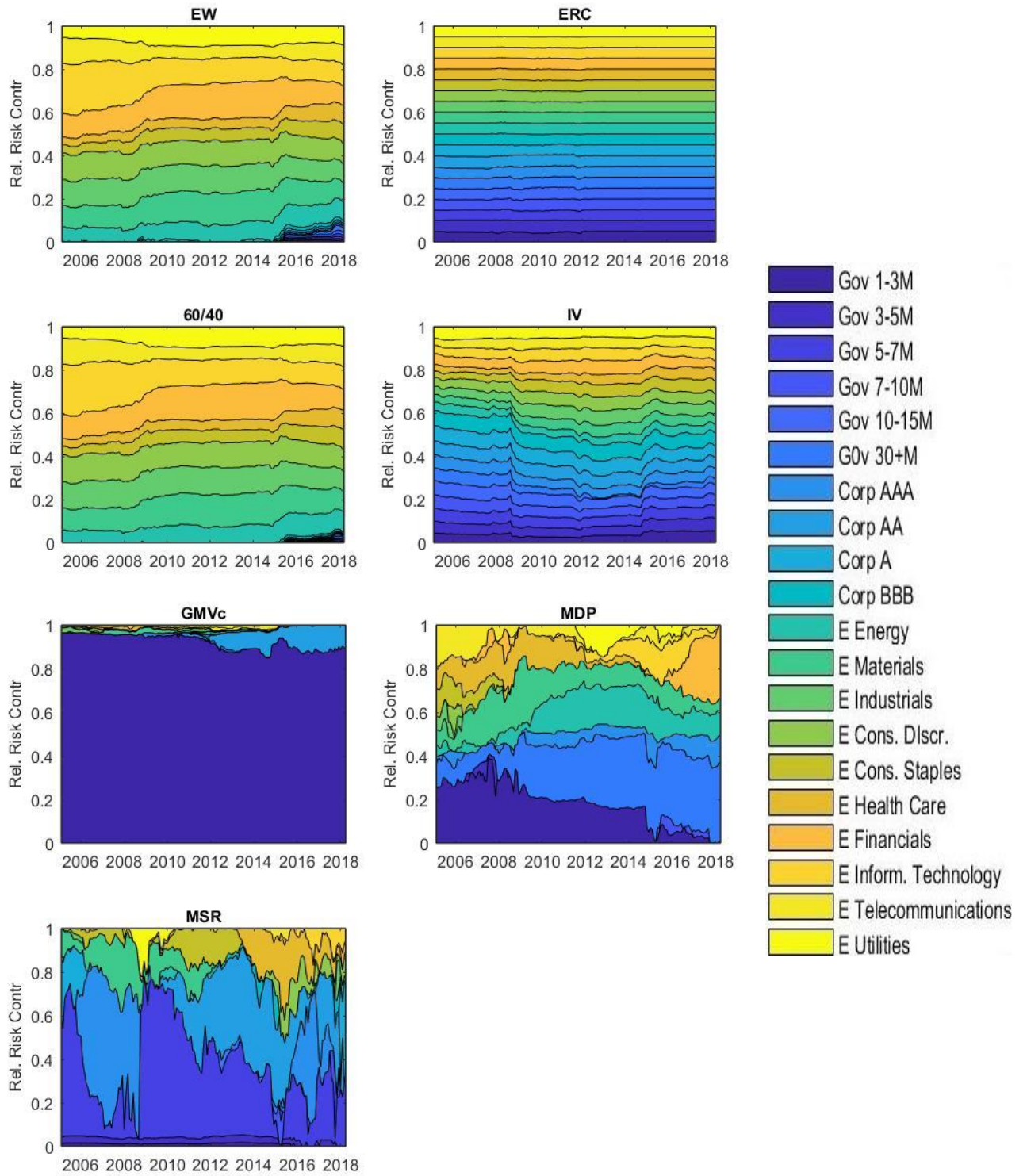


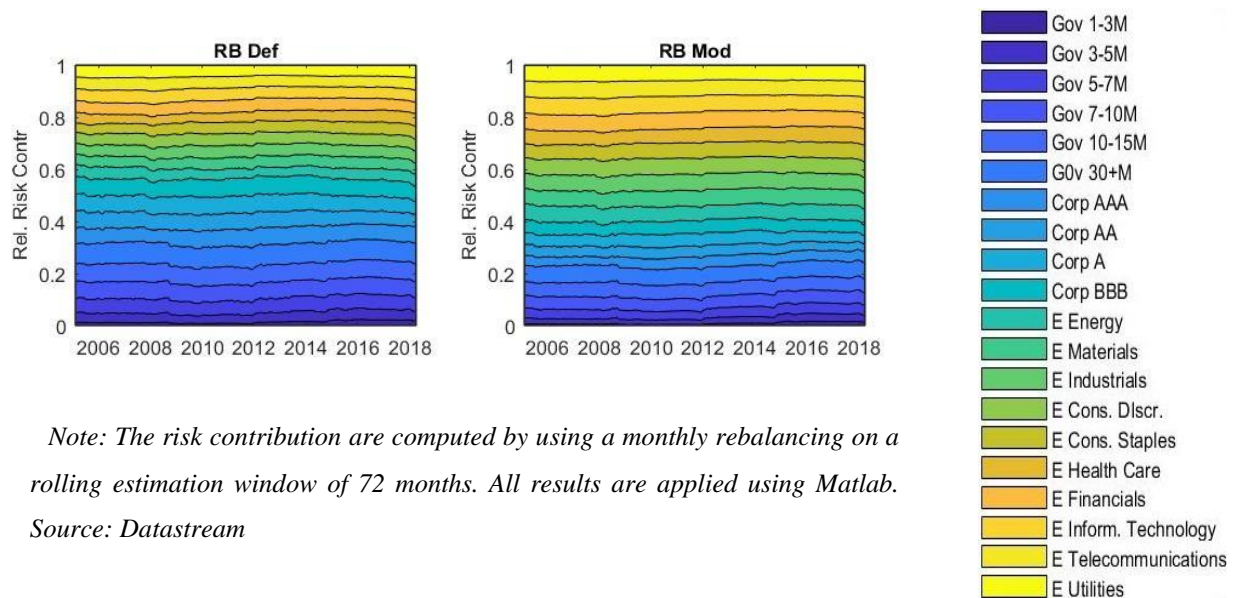
Note: All results are applied using Matlab. Source: Datastream

The Risk Contribution figures give an insightful outlook about what happens with risk inside the portfolios during the estimation period. Recalling the theoretical properties of the risk-based portfolios from the second chapter some useful remarks can be made.



Figure 28: Risk Contribution (RC) of assets in the allocation strategies





*Note: The risk contribution are computed by using a monthly rebalancing on a rolling estimation window of 72 months. All results are applied using Matlab.*

*Source: Datastream*

First, pursuing the traditional 60/40 and EW portfolios, which are diversified in terms of capital, one cannot question the fact that more than 95% of the risk in this portfolios occurs from equities. Secondly, the risk contributions inside MDP and MSR portfolios are extremely volatile for the whole estimation period. More specifically, in the MSR portfolio, government bonds' risk allocation decreased considerably, from contributing with more than 90% in 2005 and finishing with 20% in 2018 allocations. This result is due to the increased exposure to the corporate bonds and equities in the optimization process starting from 2012. The GMVc and GMVu portfolios, due to their extreme allocation exposure to a limited number of assets, proved to have a very high risk concentration.

Another interesting remark is the difference in risk contributions between ERC and EW portfolio. Allocating equally to each asset in the investment universe doesn't ensure that the risk exposure will also be equal. Indeed, the EW portfolio concentrates close to 95% of risk to equities and so is extremely exposed to the stock markets. Among risk based approaches, the IV portfolio captures risk contributions very close to the ERC, but with some exceptions, as it increases the risk from equities from 2010 to 2014 and squeeze the risk from long term government bonds for the same period.

### **How risk based allocation strategies managed extreme shocks compared to traditional strategies?**

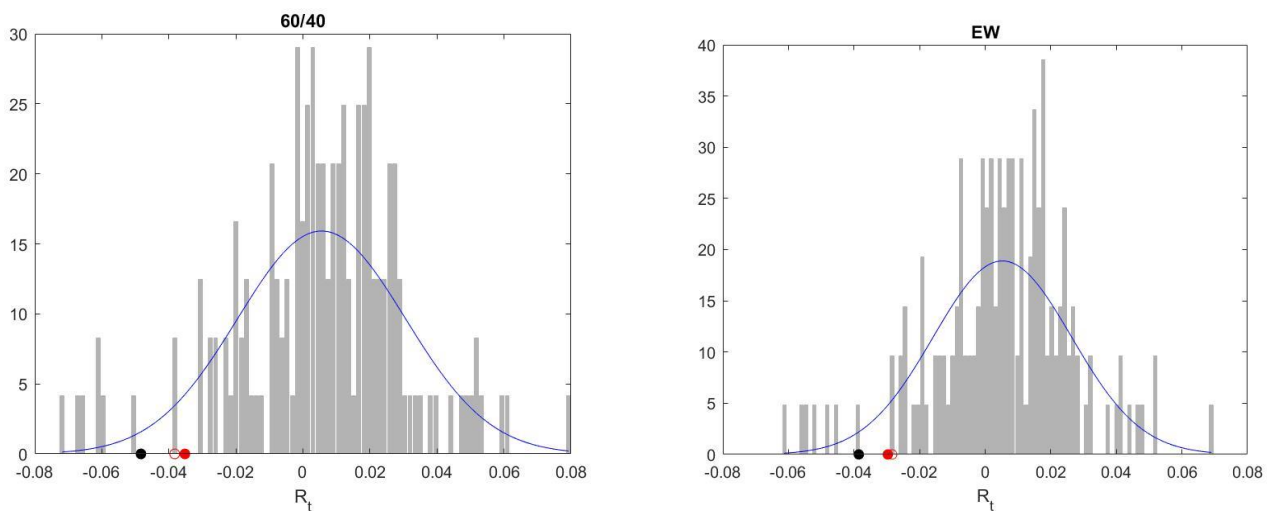
To capture more closely the specifics of the underlying risk of portfolio strategies, especially in case of extreme events, tools such as VaR and Expected Shortfall might be very useful. The Figure 29 presents the return distributions of all portfolio strategies along with the VaR and ES

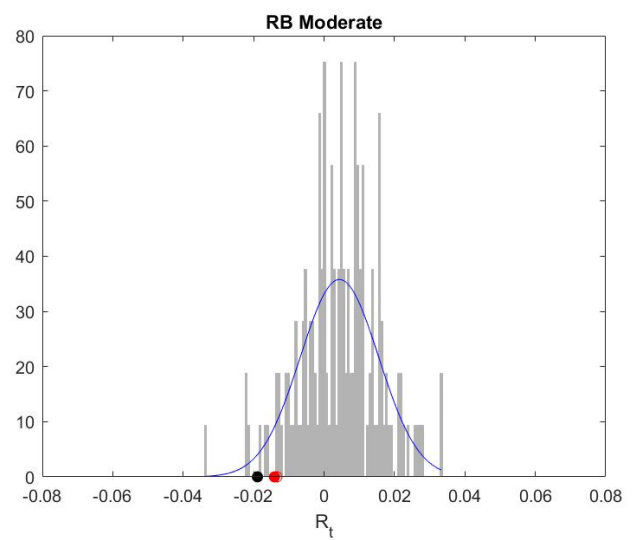
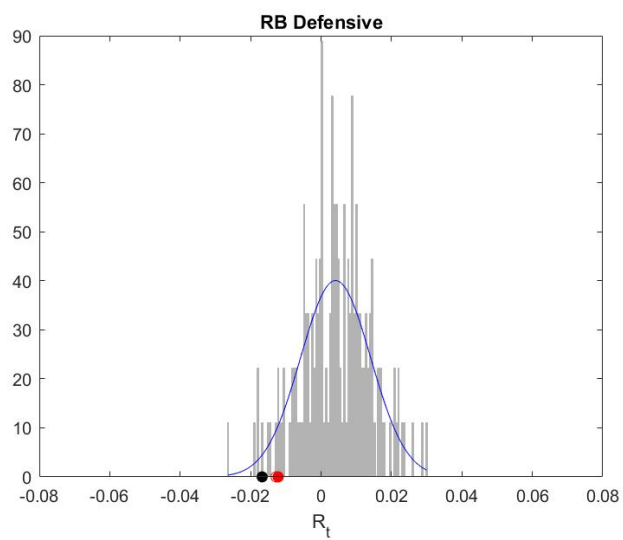
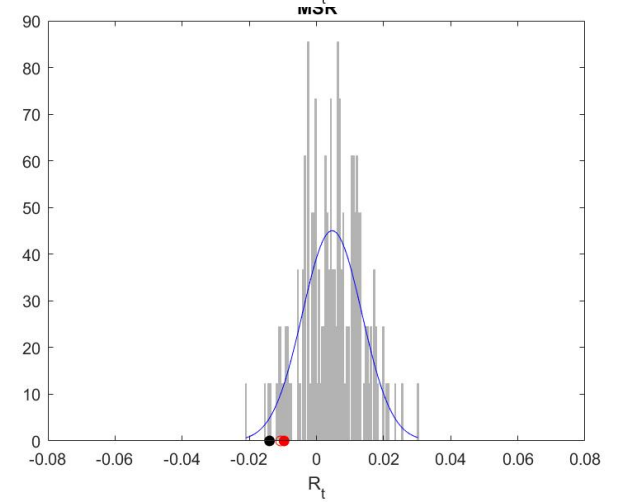
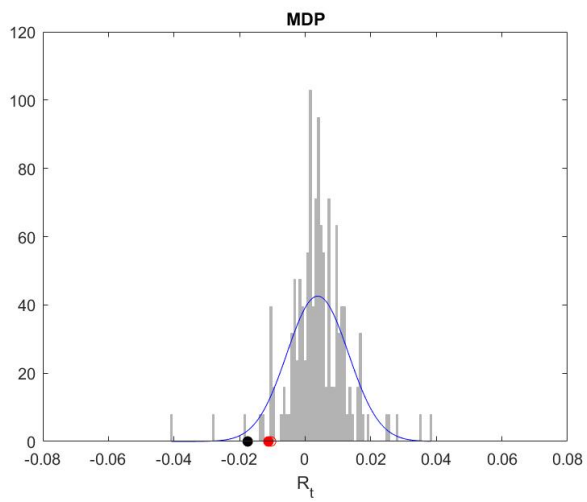
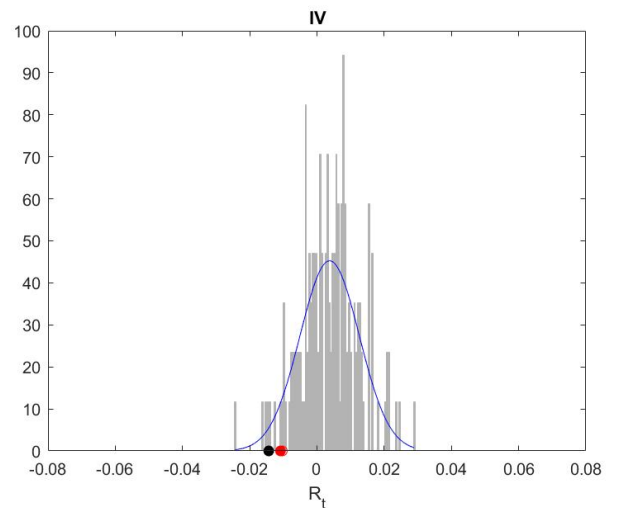
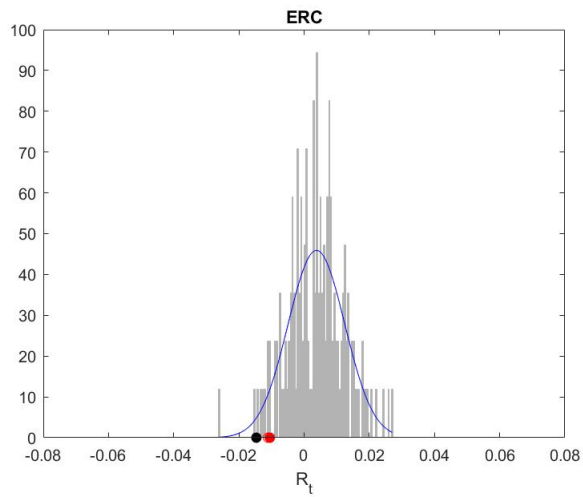


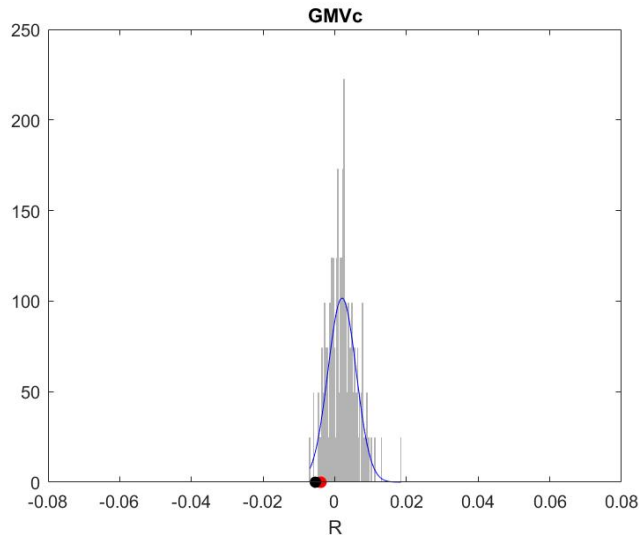
coefficients in red and black respectively. First thing to remark is, without doubt, the ‘‘non-normality’’ of portfolio returns. While these anomalous events are rare, we observe such extreme ‘‘non-normality’’ in real-world markets more frequently than current risk management approaches allow for. In other words, conventionally derived portfolios carry a higher level of downside risk than many investors believe, or current portfolio modelling techniques can identify.

The primary reason for this underestimation of risk lies in the conventional approach to applying mean-variance theory. A standard assumption in the mean-variance framework, and indeed many other holistic asset allocation frameworks, is that future asset class returns will be independent from period to period and normally distributed. The following results are compelling: ignoring empiric observations of non-normality in return distributions understates portfolio downside risk, in this case more than 30 basis points for the EW portfolio. Likewise, using only standard deviation, rather than more behaviourally attuned conditional value-at-risk measures, may in fact inadvertently increase rather than decrease downside risk, as can be observed for ERC portfolio. Such an over/underestimation of downside risk can have severe consequences for investors, even in extreme cases presenting a solvency risk.

*Figure 29: Portfolio Strategies’ Return Distribution with Var and ES coefficients in red and black respectively.*







*Note: The return distributions are computed from the 159 monthly observations. All results are applied using Matlab. Source: Datastream*

One can see that the observed return series are more peaked, have a higher density at the extreme left, and lean further to the right than the normal distribution. The rightward lean is its “negative skewness”. The consequence of this skewness is it has a longer tail, which indicates a greater magnitude of extreme negative events. Remarkable is the distribution of the 60/40 and EW strategies which experienced a relevant number of negative returns very far from their mean. These empirical density functions clearly indicate “fat” left tails relative to those implied by the normal distribution. The distribution of MSR portfolio’ returns also shows a platykurtic shape, which evidences the fat tails.

The numerical results of VaR, Modified VaR, and ES are presented in the next table. The highest Value at Risk belongs to, as highly expected, the 60/40 and EW strategies. Among the risk based strategies, the ERC present a higher coefficient than IV from the “Historical Model VaR” but reversed by the “Cornish-Fisher VaR”, which takes into account the kurtosis and skewness of returns.

*Table 2: VaR, mVaR and ES of portfolios strategies at 95% confidence level*

	<b>60/40</b>	<b>EW</b>	<b>ERC</b>	<b>IV</b>	<b>GMVc</b>	<b>MDP</b>	<b>MSR</b>	<b>RB Def.</b>	<b>RB Mod.</b>
<b>VaR</b>	-3.83%	-2.84%	-1.10%	-1.03%	-0.38%	-1.04%	-1.07%	-1.27%	-1.34%
<b>mVaR<sup>9</sup></b>	-3.84%	-3.15%	-1.06%	-1.07%	-0.36%	-1.15%	-1.01%	-1.26%	-1.44%
<b>ES</b>	-4.83%	-3.85%	-1.47%	-1.44%	-0.54%	-1.75%	-1.40%	-1.67%	-1.89%

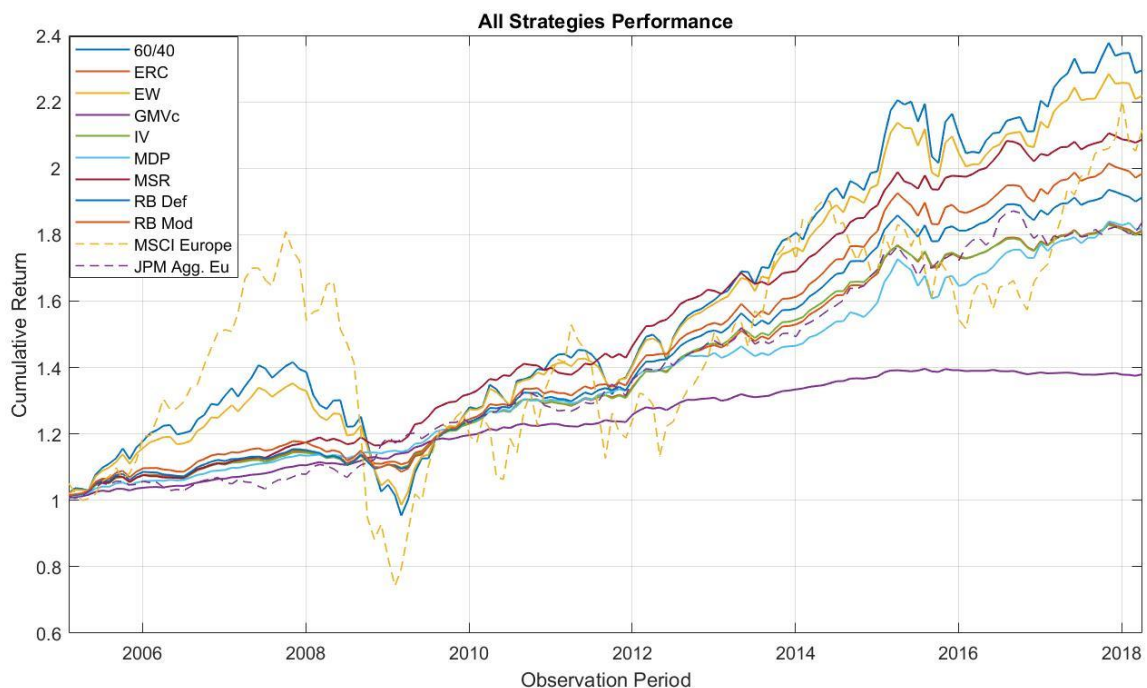
*Note: All results are applied using Matlab. Source: Datastream*

<sup>9</sup> mVaR stays for Modified Value at Risk by Cornish – Fisher.

## How risk based allocation strategies perform when re-investing returns compared to other traditional strategies?

The return performance is the last measure analysed in this empirical research. This presentation will include portfolio returns with Sortino, Sharpe, Calmar and Sterling along with ratios and return performances in terms of VaR and Drawdowns.

Figure 30: Cumulative Returns of Portfolio Allocation Strategies



Note: All results are applied using Matlab. Source: Datastream.

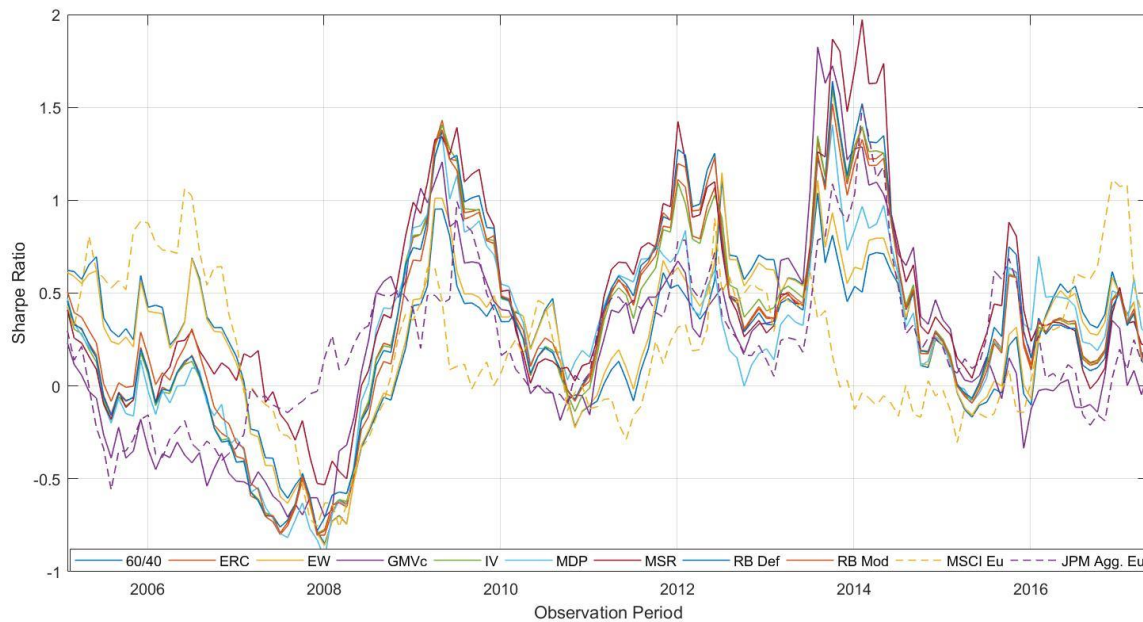
The above figure presents very interesting and insightful results. The portfolios that achieved the highest performance for the whole estimation period are those exposed to higher risk, 60/40 and EW, with the last outperforming the former, since relatively more exposed to equities. Among the risk based strategies, the RB Moderate, which splits the risk exposure 50% to equities and 50% to bonds, achieved the highest performance, outperforming the Defensive RB by 10% and ERC and IV by almost 20% for the whole estimation period. The constrained GMVc is the portfolio with the lowest performance, achieving only 37% from 2005 to 2018. The MSR strategy total return, below the 60/40 and EW, outperformed consistently all other strategies. The inclusion of more equity in the portfolio contributed significantly to the MSR

performance results. However, this is not the whole story. Even though outperforming all portfolio strategies, 60/40 and EW allocations strategies suffered extreme losses during the financial shocks by losing around 35% from 2008 to 2009 while for the same period, the risk based strategies have lost only 4%. Important to remark also that the return performance of 60/40 and EW strategies were equal to those of ERC and IV in 2013, which means that a long term investor would have been exposed for 7 years to very high levels of risk without being compensated. The same sad conclusion can be made regarding to the MSCI Europe Index, which performed extremely poorly relative to all other allocation strategies proving to be highly sensitive to market shocks.

In terms of financial shocks, the risk based strategies performed relatively well by passing smoothly through all major crises. The highest drawdown, interestingly, was experienced during the 2015 financial shock when all asset classes suffered losses. Anyway, the drawdown experienced by these strategies didn't surpass 4%, result clearly seen in the figure the Figure 26.

The rolling Sharpe Ratio figure presents a very volatile ratios for all strategies without a clear

*Figure 31: 12 Month Rolling Sharpe Ratio of All Allocation Strategies*



*Note: All results are applied using Matlab. Source: Datastream.*

winner. Remarkable is how the MSR portfolio, which aim is to maximize the ratio, actually fails to deliver a time consistent superior risk adjusted returns being overperformed by the ERC

and IV strategies consistently. The worst performer in term of the ratio turns out to be the GMVc. Such a result express an important message: minimizing the variance is not only costly in terms of asset turnover but also, and most importantly, in terms of lost return opportunity.

The rolling Diversification Ratio, presented in the below figure, gives other interesting results.

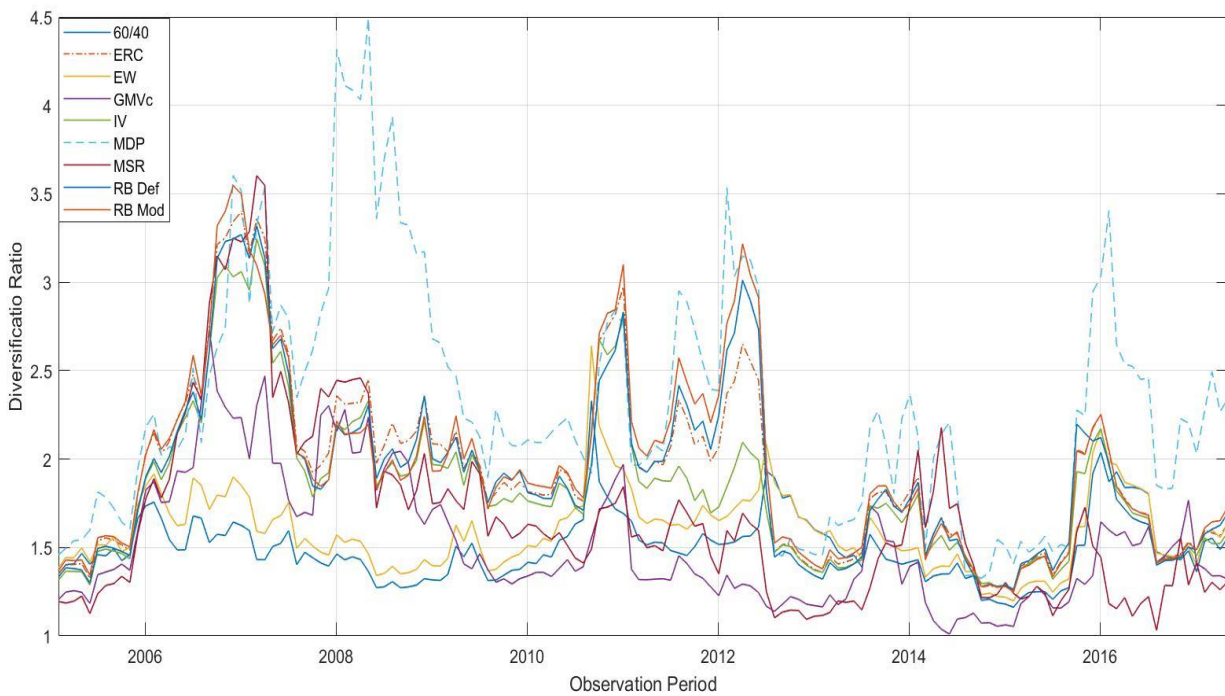
Recall from paragraph 2.2.5 Most Diversified Portfolio Strategy (MDP) that the Diversification Ratio measures how much a given portfolio is diversified, therefore the higher the Diversification Ratio, the more diversified the portfolio is. It's important to emphasize that holding a large number of assets or investments does not necessarily increase a portfolio's DR. Rather, for a portfolio to be characterized by a high DR it must be exposed to a diversified number of sources of risk.

The DR formula is the following:

$$DR(P) = \frac{\textit{Combination of the risks}}{\textit{Risk of the combination}} = \frac{x_1\sigma_1 + x_2\sigma_2 \dots x_i\sigma_i}{\sigma_p} \quad (58)$$

First, note the poor diversification of the 60/40 and even, EW portfolios. Even though these strategies try to allocate the capital in every asset to diversify the risk, are extremely failing to achieve their purpose. On the other side, the risk based strategies performed very well during distress times. The ERC portfolio, along with the RB Moderate were constantly keeping the ratio above all other portfolios. The IV, which does not include the covariance matrix in the optimization process, obviously struggled to keep the ratio close to those of the ERC' and RB'. The MSR portfolio also shows a very low diversification by touching from 2014 even coefficients close to 1, which means that is not capturing the benefits of diversification at all. Obviously, the MDP has the highest ratio since its optimization process is based on the maximization of the diversification.

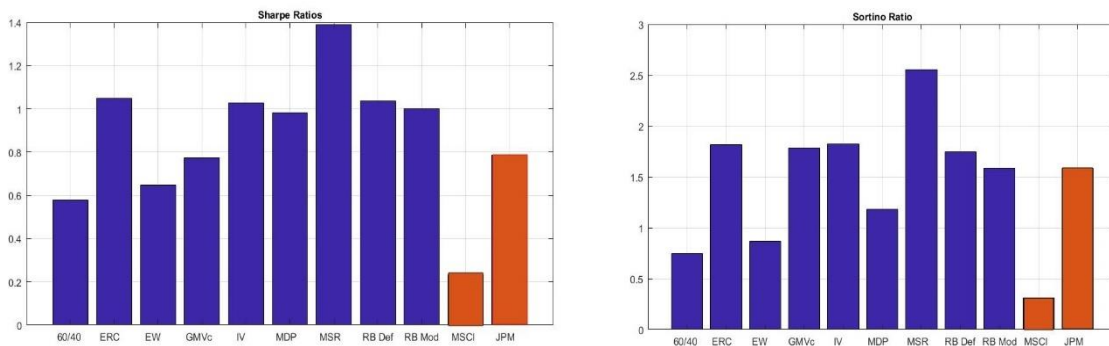
Figure 32: 12 Month Rolling Diversification Ratio of All Allocation Strategies

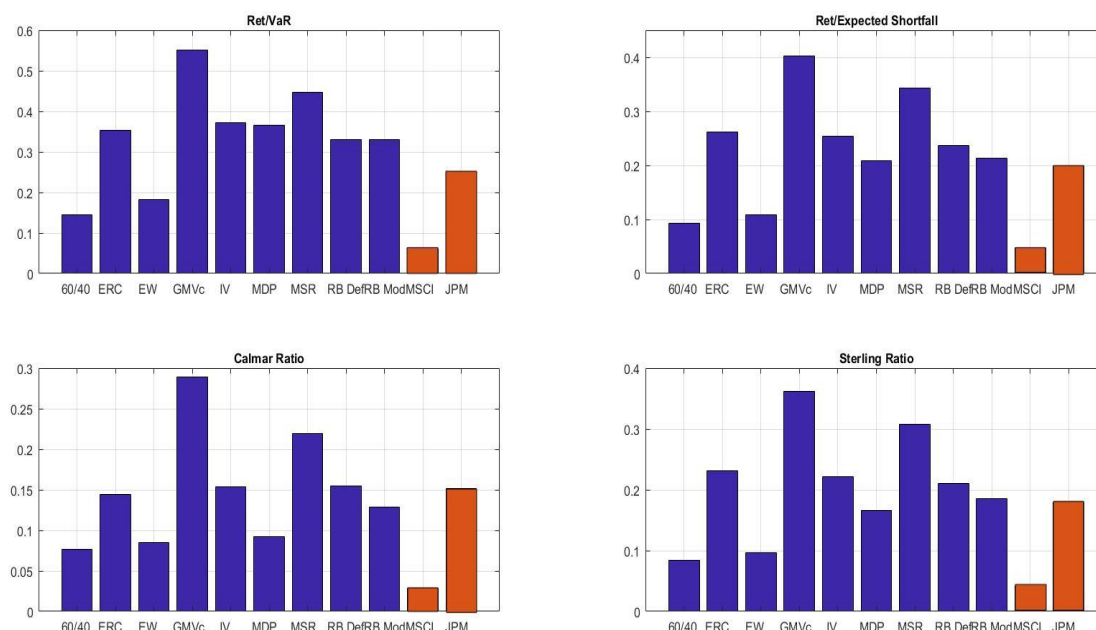


Note: All results are applied using Matlab. Source: Datastream.

The figure below summarizes all the return performance ratios of all portfolios based on their historical moments from 2005 to 2018. The most important remark is that the 60/40 and EW portfolios underperformed the ERC and IV portfolios in all ratios. The most striking divergence are on the Sortino Ratios, which measures the performance returns based on the downside risk. In terms of VaR and ES, GMVc and GMVu are the clear winners since these strategies have a very low tail risk.

Figure 33: Performance Ratios of All Allocations Strategies





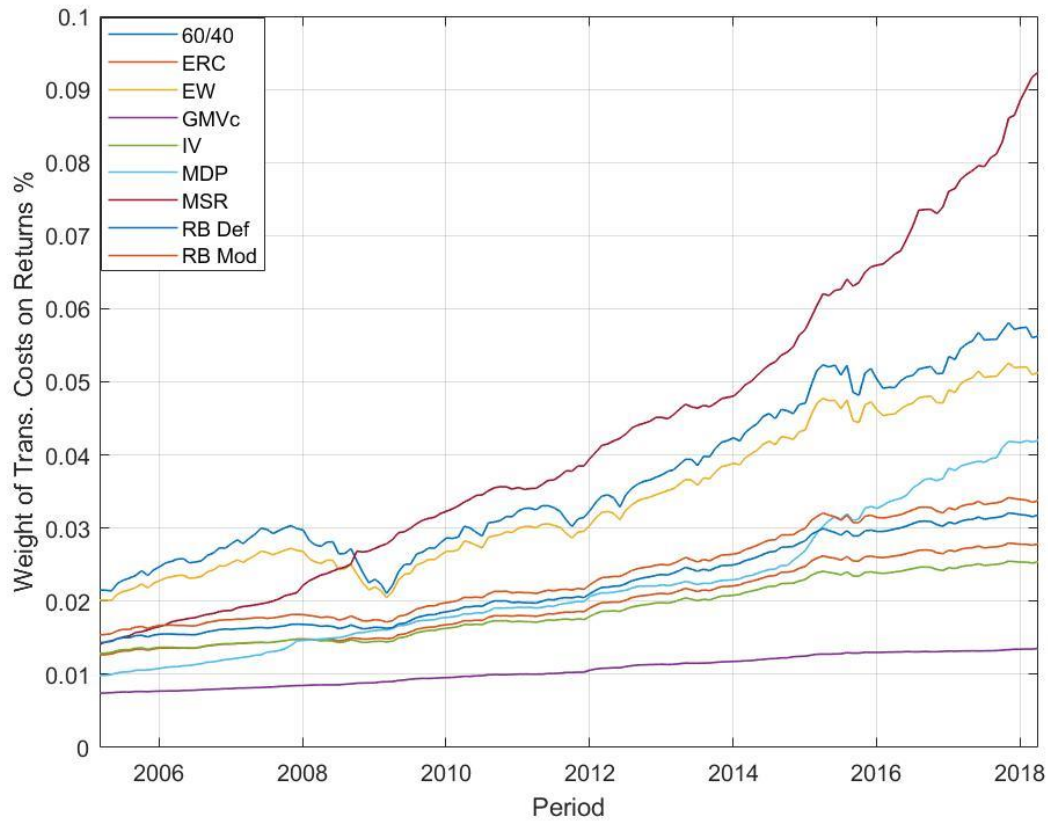
*Note: All ratios are computed based on historical portfolio returns from 2005 to 2018. All results are applied using Matlab. Source: Datastream.*

The Calmar, which computes the return in terms of the largest drawdown, and Sterling, which computes the return in terms of the average of all drawdowns, are highlighting also the GMVc, and MSR strategies. The Sharpe Ratio points out the ERC along with IV portfolios with the highest risk adjusted performance and strengthens considerably the belief of the efficiency of these strategies.

The last performance measure to analyse is the weight of transaction costs on the portfolio performance. For this analysis, as a proxy, are considered 10 basis points of return as the cost of transaction operations. Clearly, higher is the turnover of assets, higher will be the weight of the cost of the transactions on the overall portfolio returns. In the next figure are reported the difference between the Total Cumulated Return and After Transaction Cost Cumulated Returns of all allocation strategies. As observed also in the weight distribution analysis, the portfolio with the highest turnover, MSR, suffered the most also in terms of returns considering the transaction costs. Interesting to note that the second portfolio with the highest turnover is MDP, but in terms of lost value is only fourth. This happened because the opportunity cost of paying for the transaction cost was lower compared to the 60/40 or EW.



Figure 34: Weight of Transaction Costs on Cumulated Portfolio Returns



Note: All results are applied using Matlab. Source: Datastream.

In other words, since the return performance of the 60/40 and EW was higher relative to the MDP, losing money early on the stage weights more because of the lost return opportunity on that transaction costs. Note that this conclusion applies only if the investor would have invested in 2005 and would not made any redemption operation from the portfolio since then.

The tables provided starting from the next page present the summary results of all allocation strategies.

Table 3: Descriptive Statistics of Allocation Strategies

	<b>60/40</b>	<b>ERC</b>	<b>EW</b>	<b>GMVc</b>	<b>IV</b>	<b>MDP</b>	<b>MSR</b>	<b>RB Def</b>	<b>RB Mod</b>	<b>MSCI Europe</b>	<b>JPM EU Agg Bond</b>
<b>Mean Returns (Annually)</b>	6.87%	4.64%	6.48%	2.47%	4.62%	4.67%	5.76%	5.08%	5.39%	7.65%	4.71%
<b>Geomean of Excess Returns (Annually)</b>	5.00%	3.16%	4.74%	1.05%	3.14%	3.18%	4.26%	3.57%	3.86%	4.35%	3.19%
<b>Standard Deviation (Annually)</b>	8.68%	3.01%	7.31%	1.36%	3.05%	3.07%	2.01%	3.45%	3.86%	18.47%	4.14%
<b>Kurtosis</b>	4.286	3.479	4.264	4.268	3.331	3.037	4.457	3.207	3.583	16.149	10.992
<b>Skewness</b>	-0.497	-0.056	-0.434	0.589	-0.012	-0.077	-0.066	-0.124	-0.181	-2.205	0.438
<b>Value at Risk</b>	-3.83%	-2.84%	-1.10%	-1.03%	-0.26%	-1.04%	-1.04%	-1.27%	-1.34%	-10.07%	-1.57%
<b>MVaR</b>	-3.84%	-3.15%	-1.06%	-1.07%	-0.23%	-1.15%	-1.01%	-1.26%	-1.44%	-8.90%	-1.54%
<b>Expected Shortfall</b>	-4.83%	-3.85%	-1.47%	-1.44%	-0.40%	-1.75%	-1.40%	-1.67%	-1.89%	-12.86%	-1.96%
<b>MDD (Monthly)</b>	-7.26%	-2.63%	-6.19%	-0.70%	-2.46%	-4.12%	-2.13%	-2.67%	-3.41%	-21.39%	-2.58%
<b>Sharpe Ratio</b>	0.58	1.05	0.65	0.77	1.03	0.98	1.38	1.04	1.00	0.24	0.77
<b>Sortino Ratio</b>	0.75	1.82	0.87	1.79	1.82	1.18	2.55	1.75	1.59	0.31	1.55
<b>Sterling Ratio</b>	0.08	0.23	0.10	0.36	0.22	0.17	0.30	0.21	0.19	0.04	0.18
<b>Calmar Ratio</b>	0.08	0.14	0.08	0.29	0.15	0.09	0.21	0.15	0.13	0.03	0.15
<b>Diversification Ratio (Monthly)</b>	1.49	1.93	1.59	1.51	1.83	2.30	1.67	1.90	1.95	-	-
<b>Turnover (Monthly)</b>	2.43%	1.86%	2.33%	1.56%	0.87%	8.69%	19.89%	1.62%	1.22%	-	-

Table 4: Annual Cumulative Returns of Allocation Strategies

	<b>60/40</b>	<b>ERC</b>	<b>EW</b>	<b>GMVc</b>	<b>IV</b>	<b>MDP</b>	<b>MSR</b>	<b>RB Def.</b>	<b>RB Mod.</b>	<b>MSCI Europe</b>	<b>JPM EU Agg. Bond</b>
<b>2005</b>	17.74%	6.33%	15.24%	3.21%	6.12%	4.84%	5.80%	6.90%	8.05%	13.73%	4.08%
<b>2006</b>	11.29%	3.79%	9.47%	2.88%	3.60%	3.74%	4.15%	3.68%	4.38%	26.30%	-0.33%
<b>2007</b>	-1.59%	2.36%	-0.75%	3.86%	2.43%	3.42%	5.82%	1.97%	1.63%	3.89%	4.97%
<b>2008</b>	-22.95%	-2.41%	-18.91%	2.49%	-3.01%	1.21%	-0.41%	-3.72%	-5.45%	-52.64%	6.43%
<b>2009</b>	25.71%	10.91%	22.87%	5.34%	11.92%	8.05%	12.93%	12.79%	13.78%	57.81%	6.05%
<b>2010</b>	12.81%	4.37%	11.07%	2.51%	4.45%	4.63%	4.13%	4.85%	5.68%	21.67%	2.08%
<b>2011</b>	1.35%	5.91%	2.67%	3.40%	5.74%	6.29%	8.17%	6.88%	6.77%	-7.18%	8.84%
<b>2012</b>	11.27%	6.69%	10.59%	2.24%	7.35%	3.45%	8.28%	7.84%	8.18%	10.87%	6.24%
<b>2013</b>	9.98%	5.43%	9.00%	2.85%	5.72%	3.07%	5.53%	5.80%	6.32%	26.23%	3.97%
<b>2014</b>	17.42%	12.29%	16.81%	3.64%	11.56%	12.56%	12.94%	14.00%	15.19%	-2.85%	14.44%
<b>2015</b>	-2.40%	-0.06%	-1.85%	0.25%	-0.10%	-0.39%	2.15%	-0.13%	-0.34%	-15.67%	1.02%
<b>2016</b>	6.81%	2.37%	5.77%	-0.48%	2.16%	6.02%	3.25%	2.61%	3.05%	12.82%	0.26%
<b>2017</b>	7.46%	2.65%	6.36%	-0.33%	2.49%	4.84%	2.21%	2.90%	3.59%	21.24%	1.94%

Table 5: Annual Standard Deviation of Allocation Strategies

	<b>60/40</b>	<b>ERC</b>	<b>EW</b>	<b>GMVc</b>	<b>IV</b>	<b>MDP</b>	<b>MSR</b>	<b>RB Def.</b>	<b>RB Mod.</b>	<b>MSCI Europe</b>	<b>JPM EU Agg. Bond</b>
<b>2005</b>	6.89%	3.21%	6.10%	1.47%	3.35%	1.95%	3.57%	3.77%	4.01%	9.39%	3.02%
<b>2006</b>	4.89%	1.83%	4.14%	0.73%	1.99%	1.12%	1.99%	2.31%	2.41%	9.19%	2.93%
<b>2007</b>	5.97%	1.18%	4.60%	0.69%	1.26%	0.81%	1.48%	1.38%	1.42%	9.79%	3.40%
<b>2008</b>	9.21%	2.97%	7.74%	1.21%	3.04%	1.56%	2.55%	3.46%	3.81%	20.17%	4.18%
<b>2009</b>	19.20%	4.35%	15.94%	1.74%	4.91%	2.45%	4.44%	5.55%	6.44%	39.58%	5.34%
<b>2010</b>	7.43%	1.99%	6.18%	1.34%	2.03%	1.90%	2.02%	2.22%	2.60%	21.93%	2.10%
<b>2011</b>	6.44%	2.76%	5.55%	1.37%	2.92%	2.43%	3.22%	3.42%	3.61%	20.29%	4.84%
<b>2012</b>	8.96%	3.45%	7.52%	2.30%	3.66%	3.19%	4.01%	3.87%	4.01%	26.16%	5.31%
<b>2013</b>	7.13%	2.18%	5.67%	1.87%	2.45%	2.27%	2.44%	2.09%	2.26%	10.14%	3.24%
<b>2014</b>	6.03%	3.25%	5.36%	1.11%	3.19%	2.77%	3.06%	3.73%	3.99%	15.19%	4.74%
<b>2015</b>	6.82%	2.85%	5.93%	0.97%	2.59%	4.22%	2.25%	3.00%	3.59%	11.39%	2.97%
<b>2016</b>	12.62%	5.04%	10.85%	1.18%	4.67%	8.13%	4.76%	5.34%	6.42%	15.66%	5.02%
<b>2017</b>	4.64%	2.33%	3.95%	0.72%	2.22%	2.69%	3.30%	2.73%	2.85%	12.28%	5.89%

## Part IV

### Discussion and Conclusion Remarks

*In the long run, diversification wins.*

#### **4.1 Limitations of the Study**

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The mean variance framework, introduced for the first time by Markowitz in 1956, is based mainly on two dimensions, return and risk. The assumption of his theory is that investors need to base their portfolio construction solely on maximizing the relationship between expected return and the associated risk, assuming that the markets are frictionless and the transaction costs wouldn't harm the returns. However this framework has a well-known problem: the risk of the portfolios constructed in this context is under-estimated, leading to a discrepancies between ex-ante and ex-post risk of optimal portfolios.

The constraints on the optimization process, such as long only and upper/lower bounds, add another limitation to this study. However, allowing for long-short optimization can be a problem in measuring the risk contribution when the long and short positions are perfectly matched.

The constraint on the investment universe and estimation period is also not ignorable. The choice of selecting only European Equity Markets along with the Investment Grade Bonds could lead to a substantial data bias. The empirical results observed could be totally different when applied to other markets, such as US or Emerging Markets, or for shorter or longer estimation periods. Introducing other asset classes, such as commodities or real estate, could lead to very interesting and different results.

#### **4.2 Is there room for Risk Parity in a rising yields environment?**

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It is commonly said that the success of ERC portfolios is coming mostly from the high performance of bonds over the last 20 years. Is true that risk parity portfolios are more exposed to interest rate changes than traditional balanced stock and bond portfolios, but shouldn't be forgot that the goal of the ERC is to rely more equally on all asset classes. Hurst et. al (2013) in a research simulates risk parity portfolios in the US markets, starting from November 1947,

a post-war period characterised by high yields environment. They found that from 1947 to 1981 a simple risk parity strategy would have outperformed the 60/40 portfolio on both, risk and returns. The major threat to the ERC portfolios, in terms of bond risk exposure, is the speed in the spike in yields and the behaviour of equities in such environments. In the authors view, risk parity, offers a modest long-term edge over a traditional allocation which persists even in long-term periods of rising rates. To reap the potential long-term benefits of any investment strategy that has a real but modest edge requires investors to be disciplined and invest globally in different asset classes. There is no magic: if all asset classes go down, the ERC will also lose money.

### **4.3 Conclusion Remarks**

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The objective of the thesis was to analyse and to strengthen the understanding of the asset allocation in a Risk Budgeting framework. This was achieved by reviewing the main developments of the main portfolio optimization academic literature along with the recently emerged theories and developments in Risk Parity portfolio construction. The intent was to construct asset allocation strategies based on the theoretical framework and back-test them empirically.

We constructed and analysed 9 long-only asset allocation strategies: 60/40, EW, ERC, IV, MSR, MDP, GMVc, and Risk Budgeting Moderate/Defensive.

The 60/40 is a traditional and very popular among practitioners allocation strategy. The purpose is to allocate 60% of the portfolio to equities and 40% to bonds. Even though this strategy seem to be diversified in terms of capital it is far to be diversified in terms of risk. The EW portfolio is a very simple strategy which allocates equally among the assets in the investment universe. This method suffers from the same drawback as the 60/40 strategy.

Among the risk based approaches, the IV portfolio is the simplest, since it inverts each asset class' volatility, however not guaranteeing a balanced portfolio in terms of risk. In the case we take correlation coefficients into account we arrive at the ERC portfolio. Specifying the amount of risk exposures to each asset is the aim of Risk Budgeting strategy. MSR, MDP and GMVc

are portfolios that try to maximize, or minimize in the case of GMVc, a specific return/risk trade-off.

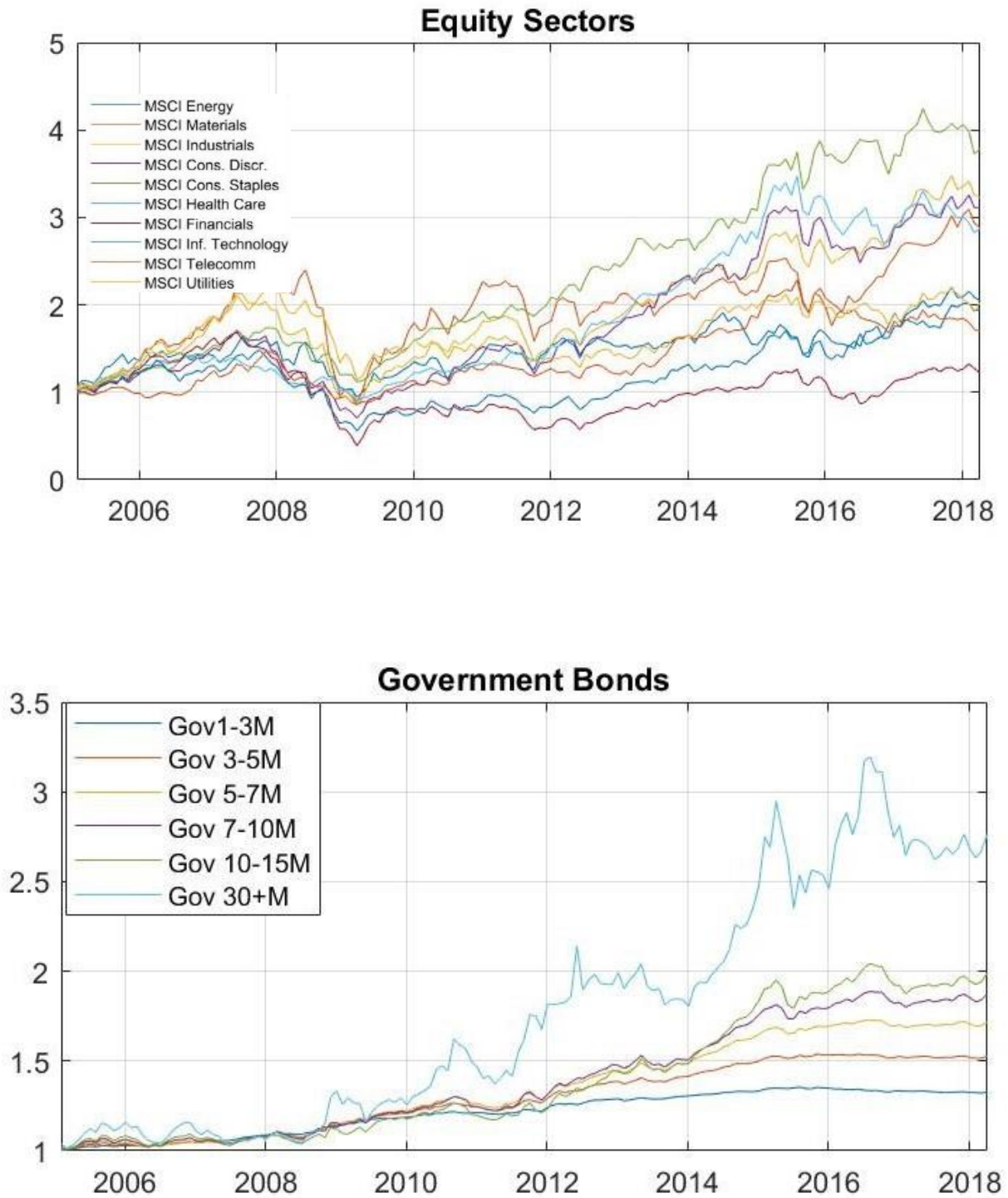
We applied these strategies specifically only to European Equity and Bond markets. Even if the results are sensitive to the investment universe, some interesting conclusion can be pronounced. First, the volatility of all risk-based strategies is considerably lower than that of the traditional 60/40 and EW portfolios. Further, the 60/40 and EW present extreme downward risks during distress markets compared to the risk based strategies. These results are punishing heavily the capital based allocations in terms of risk over returns. Further, the naive risk parity, IV, follows very closely the performance of the ERC, result that can raise some questions regarding the usefulness of the correlation estimates in the optimization process. The upper bounded MSR portfolio, despite having the highest turnover, outperforms all risk based strategies on performance ratios, and highlights the importance of constraints in the optimization process. The second portfolio in terms of Sharpe Ratio is the ERC, adjusting even for transaction costs and risk free rate.

The rolling Sharpe Ratio analysis concludes that there doesn't exist a perfect strategy since the performance depends highly on the economic environments. The GMVc and MDP portfolios perform very well during financial crises and so are useful to investors that do not always want to be exposed in all assets taking into account only the assets with lowest risk and negative correlation to achieve a superior risk adjusted performance. The ERC and IV portfolios truly diversifies risk but an investor must take into account that one could lose the opportunity of higher returns from equities during bull markets. The 60/40, despite being heavily criticized as a naïve and poor allocation strategy, is the portfolio with the highest cumulated return for the considered estimation period.

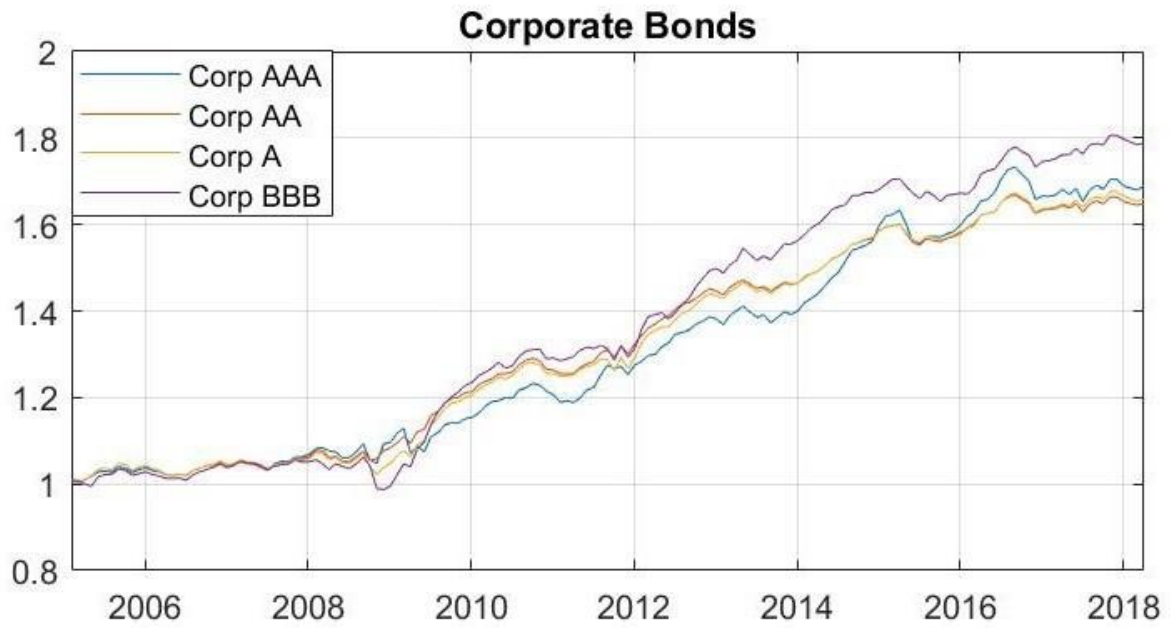
Finally, from our results, the risk parity approach seems a good way to obtain well diversified portfolios in terms of risk and to obtain competitive returns in the same time. Indeed, such good results attracted a lot of interest from the market practitioners. It is not a coincidence that the share of passive indices constructed on risk based methodologies, known as Smart Beta, in the investment portfolios around the world is highly increasing and creating a new bedrock in the asset allocation industry.

## Appendix

Figure 35: Cumulated Returns of all assets in the investment universe from 2005 to 2018.

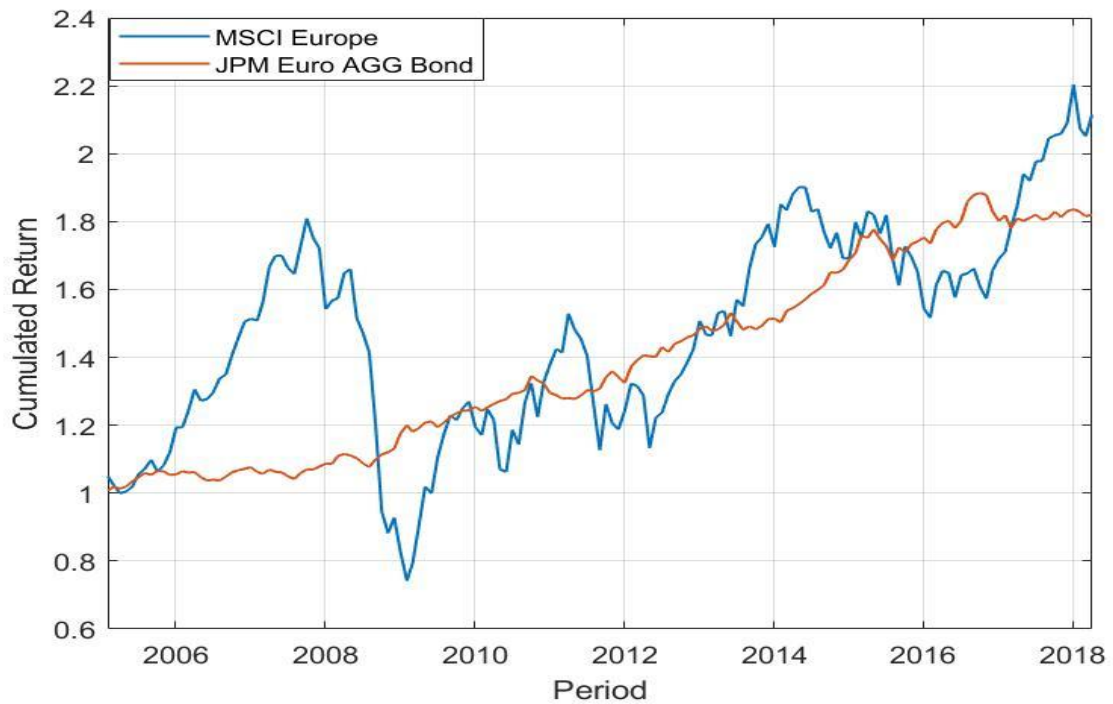






*Note: All results are applied using Matlab. Source: Datastream.*

*Figure 36: MSCI Europe and JPM Euro Aggregate Bond Indices Return Performances*





## Matlab Code

---

```
clear all;
close all;
clc;

%-----
% Portfolio management approaches within a risk budgeting
framework:...
%% evidence from the European markets.
%-----

%% Extracting Data and Main Analyses

filename = 'IndexDATA.xlsx';
T = readtable(filename);
symbol = T.Properties.VariableNames(2:end)'; % Extract Index Names
symbols = categorical(symbol);
date = table2array(T(2:73,1)); % Extract Date Column
date2 = table2array(T(74:end,1));
ALLMonthlyReturns = tick2ret(T(:,2:end)); % Compute Monthly Returns
from 01/99-12/04
MonthlyReturns05 = tick2ret(T{1:72,2:end}); % Compute Monthly
Returns from 01/99-12/04
MonthlyReturns18 = tick2ret(T{73:end,2:end});
MonthlyReturns05Tab =
array2table(MonthlyReturns05, 'VariableNames', symbol); % Monthly
Returns Table
CorrMatrix = corrcoef(MonthlyReturns05); % Correlation Matrix
PcumR18 = cumprod(MonthlyReturns18 + 1); % Cum Returns from 2005 to
2018
CumulativeReturns = cumprod(MonthlyReturns05 + 1); % Cum Returns from
1999 to 2005

%% Load Benchmark Data

filename2 = 'Benchmarks.xlsx';
T2 = readtable(filename2);
A1 = xlsread(filename2);
EqB = tick2ret(A1(73:end,2)); % MSCI Europe Total Return 2005-2018
ALibor = A1(74:end,1)/1000; % Libor rates from 2005 to 2018 (Risk
free Rate)
BondB = tick2ret(A1(73:end,3)); % JPM EU AGG Bond Index 2005-2018
date3 = table2array(T2(:,1));

%% Create Portfolios

%% Parameters and Boundary Conditions

[T,N] = size(ALLMonthlyReturns); % T = Numb. Observ, N = Investment
Universe
```

```

LB = zeros(1,N); % Lower bound
UB1 = ([0.05,0.05,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1]); % Upper
Bound for MSR
UB = ones(1,N); % Upper bound
beq = 1; % 100% Investment
Amat = ones(1,N);
b = -eye(N);
roll = 72;
options = optimset('Display','off','algorithm','sqp');
options2 = optimset('Display','off','algorithm','interior-point-
convex');
options3 = optimset('Display','off');

%%%pre allocation
covar = zeros(N,N,T-roll);
stdPr = zeros(N,T-roll);
meanPr = zeros(T-roll,N);
wGMVc = zeros(N,T-roll);
wEW = zeros(N,T-roll);
wERC = zeros(N,T-roll);
wMDP = zeros(N,T-roll);
wMSR = zeros(N,T-roll);
w60_40 = zeros(N,T-roll);
wIV = zeros(N,T-roll);
wRB = zeros(N,T-roll);
wRB2 = zeros(N,T-roll);
RRC_RB2 = ones(N,T-roll);
RRC_RB = ones(N,T-roll);
RRC_IV = ones(N,T-roll);
RRC_EW = ones(N,T-roll);
RRC_MDP = ones(N,T-roll);
RRC_MSR = ones(N,T-roll);
RRC_GMVu = ones(N,T-roll);
RRC_GMVc = ones(N,T-roll);
RRC60_40 = ones(N,T-roll);
% Risk contributions
PortRC_ERC = ones(N,T-roll);
PortRCr_ERC = ones(N,T-roll);
PortRC_EW = ones(N,T-roll);
PortRCr_EW = ones(N,T-roll);
PortRC_GMVc = ones(N,T-roll);
PortRCr_GMVc = ones(N,T-roll);
PortRC_60_40 = ones(N,T-roll);
PortRCr_60_40 = ones(N,T-roll);
PortRC_MSR = ones(N,T-roll);
PortRCr_MSR = ones(N,T-roll);
PortRC_RB = ones(N,T-roll);
PortRCr_RB = ones(N,T-roll);
PortRC_RB2 = ones(N,T-roll);
PortRCr_RB2 = ones(N,T-roll);
PortRC_MDP = ones(N,T-roll);
PortRCr_MDP = ones(N,T-roll);
PortRC_IV = ones(N,T-roll);
PortRCr_IV = ones(N,T-roll);

%% Budgets
%Defensive

```

```

w1 = ones(N,1);
budgets1 =
[(0.4*(w1(1:6,1))/6);(0.3*(w1(7:10,1))/4);(0.3*(w1(11:20,1))/10)];
%Moderate
w2 = ones(N,1);
budgets2 = [(0.5*(w2(1:10,1))/10);(0.5*(w2(11:20,1))/10)];

%% Loop
for i = 1:(T-roll)

%% Asset Moments

covar(:, :, i) = cov(ALLMonthlyReturns(i:i+roll, :));
meanPr(i, :) = mean(ALLMonthlyReturns(i:i+roll, :));
stdPr(:, i) = sqrt(diag(covar(:, :, i)));

%% Optimization Process 1 step ahead

%% ERC Equally Risk Contribution Portfolio

f1 = @(w1) ERC_FUNCTION(w1, covar(:, :, i));
w0 = 1/N*ones(N,1);
wERC(:, i) = fmincon(f1, w0, [], [], UB, beq, LB, UB', [], options);
PortRC_ERC(:, i) =
wERC(:, i).*(covar(:, :, i)*wERC(:, i))/sqrt((wERC(:, i)'*covar(:, :, i)*wE
RC(:, i))); % Risk Contribution
PortRCr_ERC(:, i) = PortRC_ERC(:, i)/sum(PortRC_ERC(:, i));

%% GMV Global Minimum Variance Portfolio (Constrained)

wGMVc(:, i) =
quadprog(covar(:, :, i), LB, b, LB', UB, beq, LB', UB', UB'/N, options2);
for j = 1:size(wGMVc,1)
for k = 1:size(wGMVc,2)
if wGMVc(j,k)<0 % No short selling allowed
wGMVc(j,k)=0;
end
end
end
PortRC_GMVc(:, i) =
wGMVc(:, i).*(covar(:, :, i)*wGMVc(:, i))/sqrt((wGMVc(:, i)'*covar(:, :, i)
*wGMVc(:, i))); % Risk Contribution
PortRCr_GMVc(:, i) = PortRC_GMVc(:, i)/sum(PortRC_GMVc(:, i));

%% Equally Weighted Portfolio(EW)

wEW(:, i) = (1/N);
PortRC_EW(:, i) =
wEW(:, i).*(covar(:, :, i)*wEW(:, i))/sqrt((wEW(:, i)'*covar(:, :, i)*wEW(:, i)
)); % Risk Contribution
PortRCr_EW(:, i) = PortRC_EW(:, i)/sum(PortRC_EW(:, i));

%% Inverse Volatility Portfolio(IV)

wIV(:, i) = 1./(stdPr(:, i))/sum(1./(stdPr(:, i)));
PortRC_IV(:, i) =
wIV(:, i).*(covar(:, :, i)*wIV(:, i))/sqrt((wIV(:, i)'*covar(:, :, i)*wIV(:, i)
)); % Risk Contribution

```

```

    PortRCr_IV(:,i) = PortRC_IV(:,i)/sum(PortRC_IV(:,i));

%% Max Sharpe Ratio Portfolio (MSR)

    f3 = @(w3) MSR_FUNCTION(w3,meanPr(i,:),covar(:, :, i));
    w0 = 1/N*ones(N,1);
    wMSR(:,i) = fmincon(f3,w0,[],[],UB1,beq,LB',UB1',[],options);
    PortRC_MSR(:,i) =
wMSR(:,i).*(covar(:, :, i)*wMSR(:,i))/sqrt(wMSR(:,i)'*covar(:, :, i)*wMS
R(:,i)); % Risk Contribution
    PortRCr_MSR(:,i) = PortRC_MSR(:,i)/sum(PortRC_MSR(:,i));

%% Traditional 60/40 Portfolio

    Equity60 = 0.06*ones(10,1); % 60% allocation to equities
    Bonds40 = 0.04*ones(10,1); % 40% allocation to bonds
    w60_40(:,i) = [Bonds40; Equity60];
    PortRC_60_40(:,i) =
w60_40(:,i).*(covar(:, :, i)*w60_40(:,i))/sqrt((w60_40(:,i)'*covar(:, :
,i)*w60_40(:,i))); % Risk Contribution
    PortRCr_60_40(:,i) = PortRC_60_40(:,i)/sum(PortRC_60_40(:,i));

    %% Risk Budgets Portfolio 40%B 30%CB 30%S

    f4 = @(w4) RiskB_function(w4,covar(:, :, i),budgets1);
    wRB(:,i) = fmincon(f4,w1,[],[],Amat,beq,LB,UB',[],options3);

% relative risk contributions and portfolio risk measure
    PortRC_RB(:,i) =
wRB(:,i).*(covar(:, :, i)*wRB(:,i))/sqrt((wRB(:,i)'*covar(:, :, i)*wRB(
,i)));
    PortRCr_RB(:,i) = PortRC_RB(:,i)/sum(PortRC_RB(:,i));

%% Risk Budgets Portfolio 50%B 50%S

    f5 = @(w5) RiskB_function(w5,covar(:, :, i),budgets2);
    wRB2(:,i) = fmincon(f5,w2,[],[],Amat,beq,LB,UB',[],options3);

% relative risk contributions and portfolio risk measure
    PortRC_RB2(:,i) =
wRB2(:,i).*(covar(:, :, i)*wRB2(:,i))/sqrt((wRB2(:,i)'*covar(:, :, i)*wR
B2(:,i)));
    PortRCr_RB2(:,i) = PortRC_RB2(:,i)/sum(PortRC_RB2(:,i));

%% Most Diversified Portfolio (MDP)

    f2 = @(w2) MDP_FUNCTION(w2,stdPr(:,i),covar(:, :, i));
    w0 = 1/N*ones(N,1);
    wMDP(:,i) = fmincon(f2,w0,[],[],UB,beq,LB',UB',[],options);
    PortRC_MDP(:,i) =
wMDP(:,i).*(covar(:, :, i)*wMDP(:,i))/sqrt(wMDP(:,i)'*covar(:, :, i)*wMD
P(:,i)); % Risk Contribution
    PortRCr_MDP(:,i) = PortRC_MDP(:,i)/sum(PortRC_MDP(:,i));

end
%% Returns

%%% pre allocation
PortRet_ERC = ones(1,length(MonthlyReturns18));

```

```

PortRet_GMVC = ones(1,length(MonthlyReturns18));
PortRet_EW = ones(1,length(MonthlyReturns18));
PortRet_MSR = ones(1,length(MonthlyReturns18));
PortRet_MDP = ones(1,length(MonthlyReturns18));
PortRet_RB = ones(1,length(MonthlyReturns18));
PortRet_RB2 = ones(1,length(MonthlyReturns18));
PortRet_IV = ones(1,length(MonthlyReturns18));
PortRet_60_40 = ones(1,length(MonthlyReturns18));

for i = 1:length(MonthlyReturns18)

PortRet_ERC(i) = MonthlyReturns18(i,:)*wERC(:,i);
PortRet_GMVC(i) = MonthlyReturns18(i,:)*wGMVC(:,i);
PortRet_EW(i) = MonthlyReturns18(i,:)*wEW(:,i);
PortRet_MSR(i) = MonthlyReturns18(i,:)*wMSR(:,i);
PortRet_MDP(i) = MonthlyReturns18(i,:)*wMDP(:,i);
PortRet_RB(i) = MonthlyReturns18(i,:)*wRB(:,i);
PortRet_RB2(i) = MonthlyReturns18(i,:)*wRB2(:,i);
PortRet_IV(i) = MonthlyReturns18(i,:)*wIV(:,i);
PortRet_60_40(i) = MonthlyReturns18(i,:)*w60_40(:,i);

end

%% Computing rolling annualized volatility

vrolltime = 12; % 1-year rolling base
%% pre allocation fro
Vol_MSCI = ones(1,(length(MonthlyReturns18)-vrolltime));
Vol_JPM = ones(1,(length(MonthlyReturns18)-vrolltime));
Vol_ERC = ones(1,(length(MonthlyReturns18)-vrolltime));
Vol_GMVC = ones(1,(length(MonthlyReturns18)-vrolltime));
Vol_EW = ones(1,(length(MonthlyReturns18)-vrolltime));
Vol_IV = ones(1,(length(MonthlyReturns18)-vrolltime));
Vol_MDP = ones(1,(length(MonthlyReturns18)-vrolltime));
Vol_60_40 = ones(1,(length(MonthlyReturns18)-vrolltime));
Vol_MSR = ones(1,(length(MonthlyReturns18)-vrolltime));
Vol_RB = ones(1,(length(MonthlyReturns18)-vrolltime));
Vol_RB2 = ones(1,(length(MonthlyReturns18)-vrolltime));

for i = 1:(length(MonthlyReturns18)-vrolltime)

Vol_MSCI(i) = std(EqB(i:i+vrolltime))*sqrt(12)*100;
Vol_JPM(i) = std(BondB(i:i+vrolltime))*sqrt(12)*100;
Vol_ERC(i) = std(PortRet_ERC(i:i+vrolltime))*sqrt(12)*100;
Vol_GMVC(i) = std(PortRet_GMVC(i:i+vrolltime))*sqrt(12)*100;
Vol_EW(i) = std(PortRet_EW(i:i+vrolltime))*sqrt(12)*100;
Vol_IV(i) = std(PortRet_IV(i:i+vrolltime))*sqrt(12)*100;
Vol_MDP(i) = std(PortRet_MDP(i:i+vrolltime))*sqrt(12)*100;
Vol_MSR(i) = std(PortRet_MSR(i:i+vrolltime))*sqrt(12)*100;
Vol_60_40(i) = std(PortRet_60_40(i:i+vrolltime))*sqrt(12)*100;
Vol_RB(i) = std(PortRet_RB(i:i+vrolltime))*sqrt(12)*100;
Vol_RB2(i) = std(PortRet_RB2(i:i+vrolltime))*sqrt(12)*100;

end

ALLVol = [Vol_60_40' Vol_ERC' Vol_EW' Vol_GMVC' Vol_IV' Vol_MDP'...
          Vol_MSR' Vol_RB' Vol_RB2'];

```

```

VolBench = [Vol_MSCI' Vol_JPM'];

%% Asset Portfolio Turnover

TO_ERC = sum(abs(diff(wERC')));
TO_GMVC = sum(abs(diff(wGMVC')));
TO_IV = sum(abs(diff(wIV')));
TO_MSR = sum(abs(diff(wMSR')));
TO_RB = sum(abs(diff(wRB')));
TO_RB2 = sum(abs(diff(wRB2')));
TO_MDP = sum(abs(diff(wMDP')));

% Equally Weighted and 60/40 Turnover
TO_EW =
(sum((abs(((MonthlyReturns18+1).*wEW')./(mean((MonthlyReturns18+1).
...
,2))) - wEW')),2))';
TO_60_40 =
(sum((abs(((MonthlyReturns18+1).*w60_40')./(mean((MonthlyReturns18+
1)...
,2))) - w60_40')),2))';

%% Transaction costs 10bps per transaction

AftPortRet_60_40 = PortRet_60_40 - (TO_60_40.*0.001);
AftPortRet_ERC = PortRet_ERC(2:end) - (TO_ERC.*0.001);
AftPortRet_EW = PortRet_EW - (TO_EW.*0.001);
AftPortRet_GMVC = PortRet_GMVC(2:end) - (TO_GMVC.*0.001);
AftPortRet_IV = PortRet_IV(2:end) - (TO_IV.*0.001);
AftPortRet_MDP = PortRet_MDP(2:end) - (TO_MDP.*0.001);
AftPortRet_MSR = PortRet_MSR(2:end) - (TO_MSR.*0.001);
AftPortRet_RB = PortRet_RB(2:end) - (TO_RB.*0.001);
AftPortRet_RB2 = PortRet_RB2(2:end) - (TO_RB2.*0.001);

%% save file

save TESI3.mat;
=====
load TESI3.mat;

%% Symbols and Returns

Psymbols = char('60/40','ERC','EW','GMVC','IV','MDP','MSR','RB
Def','RB Mod','MSCI','JPM');
ALL_PortRET =
[PortRet_60_40',PortRet_ERC',PortRet_EW',PortRet_GMVC',...
PortRet_IV',PortRet_MDP',PortRet_MSR',PortRet_RB',PortRet_RB2',EqB,B
ondB];
ALL_AFTPortRET =
[AftPortRet_60_40(2:end)',AftPortRet_ERC',AftPortRet_EW(2:end)',AftP
ortRet_GMVC',...
AftPortRet_IV',AftPortRet_MDP',AftPortRet_MSR',AftPortRet_RB',AftPor
tRet_RB2'];
ALL_Portcum = cumprod(ALL_PortRET+1); % cumulative returns
ALL_Excess_Ret = ALL_PortRET - ALibor;

```



```

ALL_AfTPort_cum = cumprod(ALL_AfTPortRET+1);
benchcum = cumprod([EqB BondB]+1);
benchExce = ([EqB BondB])-ALibor;
differ = ALL_Port_cum(2:end,1:9)-ALL_AfTPort_cum;
allmsci = ALLMonthlyReturns(73:end,11:end)*wEW(1:10,1); % create
equity benchmark
allmscic = cumprod(allmsci+1);

% Standard Deviation and Mean Returns

stdP_ALL = std(ALL_PortRET)*sqrt(12); % annualized
meanP_ALL = ((mean(ALL_PortRET) + 1).^12)-1;% annualized
geomeanP_ALL = ((geomean(ALL_Excess_Ret+1)).^12)-1;% annualized

% Sharpe, Sortino and Treynor
%Sharpe Ratio

sharpe = geomeanP_ALL'./(sqrt(var(ALL_PortRET))*sqrt(12))'; % based
on annualized data
sharpe = sharpe';
%Rolling Sharpe Ratio
t = 11;
for i = 1:(size(ALL_Excess_Ret)-t)

sharperoll(i,:) =
(mean(ALL_Excess_Ret(i:i+t,:)))/(std(ALL_Excess_Ret(i:i+t,:)));%
based on annualized data
end

% Rolling Diversification Ratio
index = 12:(size(ALL_PortRET));

for i = 1:size(ALL_PortRET)-t
    v = index(i);
    DR_60_40(i) =
(std(MonthlyReturns18(i:v,:))*w60_40(:,v))/std(PortRet_60_40(:,i:v));
    DR_ERC(i) =
(std(MonthlyReturns18(i:v,:))*wERC(:,v))/std(PortRet_ERC(:,i:v));
    DR_EW(i) =
(std(MonthlyReturns18(i:v,:))*wEW(:,v))/std(PortRet_EW(:,i:v));
    DR_GMVC(i) =
(std(MonthlyReturns18(i:v,:))*wGMVC(:,v))/std(PortRet_GMVC(:,i:v));
    DR_IV(i) =
(std(MonthlyReturns18(i:v,:))*wIV(:,v))/std(PortRet_IV(:,i:v));
    DR_MDP(i) =
(std(MonthlyReturns18(i:v,:))*wMDP(:,v))/std(PortRet_MDP(:,i:v));
    DR_MSR(i) =
(std(MonthlyReturns18(i:v,:))*wMSR(:,v))/std(PortRet_MSR(:,i:v));
    DR_RB(i) =
(std(MonthlyReturns18(i:v,:))*wRB(:,v))/std(PortRet_RB(:,i:v));
    DR_RB2(i) =
(std(MonthlyReturns18(i:v,:))*wRB2(:,v))/std(PortRet_RB2(:,i:v));
end

% Sortino Ratio

s2 = zeros(size(ALL_PortRET,2),1);

```

```

for j = 1:size(ALL_PortRET,2)
    % compute semi-standard deviation
    s2(j) = sqrt(var(ALL_PortRET(ALL_PortRET(:,j)<0,j)));
end

sortino = (geomeanP_ALL'./ ((s2)*sqrt(12)));

% Treynor Ratio on Equity Benchmark and Bond Benchmark
betaeq = zeros(size(ALL_PortRET,2),1);
betab = zeros(size(ALL_PortRET,2),1);

for j = 1:size(ALL_PortRET,2)
    % compute beta on the MSCI European market index
    betaeq(j) = (((EqB(:,1) - mean(EqB(:,1)))') * (ALL_PortRET(:,j) -
mean(ALL_PortRET(:,j)))) ...
                / ((EqB(:,1) - mean(EqB(:,1)))' * (EqB(:,1) -
mean(EqB(:,1)))));
    betab(j) = (((BondB(:,1) - mean(BondB(:,1)))') *
(ALL_PortRET(:,j) - mean(ALL_PortRET(:,j)))) ...
                / ((BondB(:,1) - mean(BondB(:,1)))' * (BondB(:,1) -
mean(BondB(:,1)))));
end
treynoreq = mean(ALL_Excess_Ret)'./ betaeq;
treynorb = mean(ALL_Excess_Ret)'./ betab;

%% Value-at-Risk

alpha = 0.05;
s4 = quantile(ALL_PortRET,alpha);
VaR = mean(ALL_PortRET)'./ abs(s4)';

%% Value at risk Cornesh-Fisher Model

for i = 1:9
    CF_Var(i) = CornFishvar(ALL_PortRET(:,i),alpha);
end

%% Expected Shortfall

alpha=0.05;
s5 = zeros(size(ALL_PortRET,2),1);
for j=1:size(ALL_PortRET,2)
    % conditional mean
    s5(j) = mean(ALL_PortRET(ALL_PortRET(:,j)<
quantile(ALL_PortRET(:,j),alpha),j));
end
ExSH = mean(ALL_PortRET)'./ abs(s5);

%% % DrawDown

```

```

DD = zeros(size(ALL_PortRET,1),size(ALL_PortRET,2));
for i = 1:size(ALL_PortRET,2)

    DD(1,i) = min(ALL_PortRET(1,i)/100,0);

    for j = 2:size(ALL_PortRET,1)

        DD(j,i) = min(0,(1+DD(j-1,i)/100)*(1+ALL_PortRET(j,i)/100)-
1);
    end
end

s6 = max(abs(DD))*100;

% Calmar ratio
calmar = mean(ALL_PortRET)'./ s6;

k = 5;
s7 = zeros(size(ALL_PortRET,2),1);

for j = 1:size(ALL_PortRET,2)

    % average of the largest DD
    [sDDj,~] = sort(abs(DD(:,j)),'descend');
    s7(j) = mean(sDDj(1:k))*100;

end

sterling = mean(ALL_PortRET)'./s7;

```

---

## Functions

### Risk Budgeting

```

function x = RiskB_function(w,Sigma,b)

% inputs
% w - portfolio weights
% Sigma - covariance of the assets
% b - risk budgets in percentages

% total risk
R=sqrt((w')*Sigma*w);

% un-standardized Marginal Risks
mR = Sigma*w;

% standardized Risk Contributions
RC= w.*mR/R;

% squared deviations between RC and fractions of total risk
SqRC=(RC - b*R).^2;

% computing criterion function
x =sum(SqRC);
end

```

## Maximum Sharpe Function

```
function f = MSR_FUNCTION(w3,mu,sigma)
f = -((mu*w3)/sqrt(w3'* sigma*w3));
end
```

## Maximum Diversification Function

```
function f = MDP_FUNCTION(w2,StandardDev,CovMatrix)
f = -(w2'*StandardDev)/(w2'*CovMatrix*w2)^(-.5);
end
```

## Equal Risk Contribution Function

```
function [x]=ERC_FUNCTION(w1,Sigma)
x = 0;
R = Sigma*w1;
for i=1:size(w1)
for j=1:size(w1)
x = x + (w1(i)*R(i)-w1(j)*R(j))^2;
end
end
x = x/(w1'*R);
```

## Cornesh-Fisher VaR Function

```
function CFVAR = CornFishvar>Returns,alpha)
%INPUT
%Rt: returns series
%alpha: VaR level
%OUTPUT:
%CFVAR: Cornish fisher VaR with alpha probability and time horizon as the
returns
%frequency
% calculate cornish fisher VaR
z = norminv(alpha,0,1); %
sigma2 = var>Returns);
skew = skewness>Returns);
kurt = kurtosis>Returns);
CFVAR = -(mean>Returns)+(z+(1/6)*(z^2-1)*skew+(1/24)*(z^3-3*z)*(kurt-3)-
(1/36)*(2*z^3-5*z)*skew^2)*sigma2^0.5);
```

=====

Mendeley Citations: (Franke et al. 2008),(Goetzmann, Grinold, and Kahn 1996),(Richard 2015)(MALKIEL 1995)(Chopra and Ziemba 1993)(Roncalli 2011)(Clarke, Silva, and Thorley 2013)

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