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MARKOV SWITCHING MODEL WITH TIME VARYING TRANSITION PROBABILITIES AND RANKING AMONG A CLUSTER: A NEW MODEL FOR SHARE RETURNS AND TRADING STRATEGIES

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Summary

Abstract

Nonlinear time series models are growing in importance for the description of how the returns of financial assets evolve over time. This is fundamental to perform a good forecast, on which being able to build a successful strategy for investing in the markets.

In this thesis, I studied the performances, throughout the period, 2001-2010, of 50 shares among the ones that were composing the Nasdaq 100, using a Markov switching model with time varying transition probabilities.

Then, I used the parameters estimated to forecast the return for the period 2011-2014, and, on these forecasts, I built two trading strategies based on the returns alone and on the Sharpe Ratio.

The results for the estimation were surprising: contrary to most literature, the model shows that the two regimes are defined by the volatility of the series rather than by the returns.

In the forecast, on the other hand, the strategies performed, in any of their specification, worse than a strategy of Buy-and-Hold the market, as the theory suggests.

1. Introduction

Modeling and forecasting time series of financial prices and returns has covered an important role in modern economic studies, and it is used by all type of agents operating in the financial markets (investors, investment banks, money managers and hedge funds among all).

The main theories started with Markowitz (1959), that, borrowing from the theories of the games of chance, formulated its idea of evolution of prices according to the martingale property, which states that the prices of tomorrow are equal to the prices of today given a particular information set

$$E(p_{t+1}|F_t) = p_t$$

A more restrictive formulation was afterward made as

$$\log p_{t+1} = \mu_t + \log p_t + \varepsilon_{t+1}$$

with μ being a constant drift term and ε_{t+1}being the error, independent and identically distributed as a normal with constant variance and zero mean.

Recently, these theories have been integrated with the theories of nonlinear time series.

In fact, although the linear models are for sure easier to calculate, they probably fail to detect special characteristics of the series.

To clarify, following the definition of Lee, White and Granger (1993), a series is linear if exists a vector X for which

$$Pr[E(y_t|X_t) = X'_t\theta^*] = 1$$

with θ^* being a vector of parameters.

In a recent work, Gonzalez-Rivera and Lee (2008) reviewed some of the nonlinear models utilized to forecast the conditional mean and the conditional variance of some financial items.

To start, Goyal and Welch (2006) studied the S&P 500 equity premium over the treasury bill rate, using variables taken from the overall economy (income ratio, wealth, consumption, inflation), interest rates (T-bills, long term yields and yields of corporate bonds) and stock indicators (Earnings/Price, Book-to-Market, Dividend yield), imposing a lower bound to the equity premium (because nobody is interested in a negative premium). Their forecasting results were good performance especially in periods of big crashes in the markets.

Utilizing the same type of predictors, Campbell and Thompson (2007), built a model in which they totally eliminated the forecasts in which the equity premium was negative and the variable which estimated parameters resulted of opposite sign with respect to the expected one.

This procedure, called "shrinkage", where you reduce the error variance, and therefore the mean squared error, but you increase the bias of the forecast, proven itself to be useful to improve the forecasted performances.

Another class of models is the threshold autoregressive (TAR) models, first described by Tong (1983).

In these models, it is assumed that the series follows different regimes, depending on the value of a variable called "conditioning or threshold".

In formula

$$y_t = \sum_{j=1}^{n} \left[\phi_0^{(j)} + \sum_{i=1}^{p_j} \phi_i^{(j)} y_{t-i} + \varepsilon_t^{(j)} \right] 1(r_{j-1} < x_t \leq r_j)$$

where the r_0 and r_n are $\pm\infty$.

If r (the conditioning or threshold variable) is the dependent variable itself (with some lag), we call the model a SETAR (self-exciting threshold autoregressive) model.

These models have been used to study various economical indexes, but the results where not superior to the ones obtained with linear, simpler models.

Terasvirta (1994) proposed a particular specification for the SETAR, in which there is a continuum of regimes.

These models are called STAR (Smooth Transition Autoregressive models), and one of their specifications is:

$$y_t = \phi_0 + \sum_{i=1}^{p} \phi_i y_{t-i} + \left(\theta_0 + \sum_{i=1}^{p} \theta_i y_{t-i} \right) F(y_{t-d}) + \varepsilon_t$$

where $F(y_{t-d})$ usually is in the form of a logistic or exponential function.

For an insight into TAR, SETAR and STAR, Enders (2015).

Another type of nonlinear specification is the one proposed by Hamilton (2001) that mixed a linear component with a random field component.

In particular, a random field is "a function $m(\omega, x): \Omega \times A \rightarrow R$ such that $m(\omega, x)$ is a random variable for each $x \in A, A \subseteq R^k$." (Gonzalez-Rivera and Lee, 2008).

Dahl and Gonzalez-Rivera (2003), on the other hand, implemented a similar model, but which tries to better detect the nonlinear elements of the model and the covariance function of the random field.

In the environment of the factor models (as, for example, the APT), Bai and Ng (2007) formulated a model where the returns are governed by a non linear link function g such that:

$$g(x_{it}) = \phi_i' J_t + v_{it}$$

with J_t being the factors affecting every asset, and ϕ_i' the transposed vector of parameters for the asset i.

The functional forms of the g studied are various, such as the squared principal components or the squared factors (for a deeper knowledge of the models, go to the original paper).

Gonzalez-Rivera and Lee (2008) presented various Artificial Neural Network models, that are inspired by the functioning of the human brain, and in which the input is connected to the output through some hidden layers.

Showing a model with a unique layer, the dependent variable y is calculated from the independent x as

$$y_t = x_t\beta + \sum_{j=1}^{q} \delta_j \psi(x_t \gamma_j) + \varepsilon_t$$

where γ_j is a parameter connecting the dependent variable to the hidden unit j, and δ_j is a parameter that regulates the strength with which the hidden unit j is connected to the output.

ψ, on the other hand, is the so called "squashing function", that, after the input has sent a signal to the intermediate hidden unit, regulates their activation, which brings a new signal towards the output.

Among all the authors writing about ANN models, Trippi and Turban (1992) made a review about their applications to investment and finance in general.

Finally, Gonzalez-Rivera and Lee (2008) reviewed the functional coefficient model of Cai, Fan and Yao (2000), in which a stationary process depends on a multiplicative effect of a scalar variable and the relative parameters, that results in an autoregressive model where the coefficients are time varying.

Anyhow, among the models presented in the paper, the ones that caught my attention and interest the most were the Markov-switching model and the varying cross sectional rank model.

In the first, formulated by Hamilton (1989), the process is thought to follow different regimes, but these states are non-observable and follow a Markov chain.

In the second one, the returns are modeled with a bimodal normal variance, where the mean is a function of the dependent variable lagged and of another variable built on the interaction between this lagged return and the lagged value of the returns of other assets belonging to the same cluster, and the variance (which is the same between the two distribution) is modeled as a Garch model.

Which of the two distribution is followed at a particular time t depends on a dummy variable constructed on the second variable defining the mean of the two different states, and it reminds, from a theoretical point of view, the idea of stochastic jumps described by many modern pricing model, but, instead of being unknown, it is known.

The probability that this "jump" variable takes value 1 is modeled as a particular hazard function.

Taking as a reference the variables used in this last model, I built a particular Markov switching model with time varying transition probabilities, where the means of the normal distributions depends on four variables and the probabilities of switching from a regime to the other depend on two of the them, trying in this way to account for both the linear and nonlinear effect of these variables on the process.

I applied this model to the returns of 50 shares composing the Nasdaq 100 index, in the period 2001-2010, and then applied the estimated parameters to the period 2011-2014 for an out of sample forecast, on which I built two different trading strategies, and I tested how good they performed with respect to a buy-and hold strategy of the index.

The thesis is structured as follows: in Chapter 2, I present the theoretical background behind the model I developed; Chapter 3 explains in details the model I studied; Chapter 4 presents and comments the results, and Chapter 5 concludes.

2. Theoretical background

2.1 Markov Switching Models

2.1.1 Markov Chains and Martingales

As explained by Ross (2006), starting from a set of non-negative integer values $\{0,1,2,\ldots\}$ that denotes a particular state of the process X, at time n, we call Markov chain a process for which the state of the process in n+1 depends only on the state in n and is independent of all the previous realization. In formula:

$P\{X_{n+1} = j | X_n = i, X_{n-1} = i_{n-1}, \ldots, X_1 = i_1, X_0 = i_0\} = P_{ij}$.

Hamilton (1994) adds that $p_{i1} + p_{i2} + \cdots + p_{iN} = 1$, and represents all the transition probabilities in an *(NxN)* matrix called *transition matrix*

$$\boldsymbol{P} = \begin{bmatrix} p_{11} & p_{21} & \cdots & p_{N1} \\ p_{12} & p_{22} & \cdots & p_{N2} \\ \vdots & \vdots & \ddots & \vdots \\ p_{1N} & p_{2N} & \cdots & p_{NN} \end{bmatrix}$$

It is to be noticed that the state of the process in n+1 is not known, and so the process follows what is described as a *hidden Markov chain*.

A martingale, on the other hand, is (as defined by Lamberton and Lapeyre (2008)) a measurable sequence of real-valued random variables *M*, that, having information up to time *t* (defined as filtration F_t),

$E(M_{t+1} | F_t) = M_t$

2.1.2 Mixture distribution

In every time t, the dependent variable (observable) y_t is thought to be drawn from a Gaussian distribution dependent on the state (regime) in which the process is in.

So, if there are N unobservable regimes *s* in the model, the density will be

$$f(y_t | s_t = j; \theta) = \frac{1}{\sqrt{2\pi}\sigma_j} exp\left\{\frac{-(y_t - \mu_j)^2}{2\sigma_j^2}\right\}$$

for j=1,2,…N and θ representing a vector of the parameters including $\mu_1, \ldots, \mu_N$ and $\sigma_1, \ldots \sigma_N$.

The unconditional probabilities that the unobserved state *s* will be equal to j at time t, are defined as

$P\{s_t = j; \theta\} = \pi_j$ for j=1,2,…,N and these probabilities are also included in the vector θ.

From here we can calculate the joint density distribution:

$$f(y_t, s_t = j; \theta) = \frac{\pi_j}{\sqrt{2\pi}\sigma_j} exp\left\{\frac{-(y_t - \mu_j)^2}{2\sigma_j^2}\right\}$$

and also the unconditional, by summing all the possible outcomes of j:

$$f(y_t; \theta) = \frac{\pi_1}{\sqrt{2\pi}\sigma_1} exp\left\{\frac{-(y_t - \mu_1)^2}{2\sigma_1^2}\right\} + \frac{\pi_2}{\sqrt{2\pi}\sigma_2} exp\left\{\frac{-(y_t - \mu_2)^2}{2\sigma_2^2}\right\} + \cdots + \frac{\pi_N}{\sqrt{2\pi}\sigma_N} exp\left\{\frac{-(y_t - \mu_N)^2}{2\sigma_N^2}\right\}$$

This last expression is vital to describe y_t, because the states are not observable.

Supposing that s_t is i.i.d. (independent and identically distributed), the log likelihood of the function will be

$$\mathfrak{L}(\theta) = \sum_{t=1}^{T} \log f\,(y_t; \theta)$$

maximized with respect to θ and subject to the constraints that the sum of the π must be equal to 1 and that every π must be bigger than or equal to 0.

By this algorithm, it is found $\hat{\theta}$, that is a vector constituted by the solutions of the following system of equations:

$$\hat{\mu}_j = \frac{\sum_{t=1}^{T} y_t P\{s_t = j | y_t; \hat{\theta}\}}{\sum_{t=1}^{T} P\{s_t = j | y_t; \hat{\theta}\}} \quad \text{for j=1,2,…,N}$$

$$\hat{\sigma}_j^2 = \frac{\sum_{t=1}^{T} (y_t - \hat{\mu}_j)^2 P\{s_t = j | y_t; \hat{\theta}\}}{\sum_{t=1}^{T} P\{s_t = j | y_t; \hat{\theta}\}} \quad \text{for j=1,2,…,N}$$

$$\hat{\pi}_j = T^{-1} \sum_{t=1}^{T} P\{s_t = j | y_t; \hat{\theta}\} \quad \text{for j=1,2,…,N}$$

Since we do not know whether at t we are in regime j or in another regime, Ps are always between 0 and 1 and every $\hat{\mu}_j$ is a weighted average of the outcomes, proportional on the likelihood that in t we will observe regime j.

The same goes for $\hat{\sigma}_j^2$,while $\hat{\pi}_j$ is just the number of periods in which we observe regime j on the total number of periods.

In order to calculate the parameters, an iterative algorithm is needed, so Hamilton (1994) uses the EM principle developed by Dempster, Laird, and Rubin (1997).

Starting from an initial value chosen arbitrarily for θ, one can find $P\{s_t = j; \theta\}$, and then, with this value, calculate $\hat{\mu}_j$, $\hat{\sigma}_j^2$, $\hat{\pi}_j$.

With these estimates, different from the first ones, estimate again $P\{s_t = j; \theta\}$, and with these new estimates, recalculate $\hat{\mu}_j$, $\hat{\sigma}_j^2$, $\hat{\pi}_j$.

The EM (which stands for Expectations and Maximization) consists in repeating this procedure until the difference between two estimations will be less than a particular threshold or, even better, the difference will be null.

2.1.3 Inference

Hamilton then explains the inference for the model in a more general case with respect to the i.i.d. case, in which s_t is determined just by y_t, so in a model where it depends on "all the observation available".

The author calls $\mathcal{Y}_t$ all the observation obtained up to time t, η_t a vector containing the conditional density in every time of the series, and $\xi_{t|t}$ a vector containing all the inferences about the regime at time t, made with observations collected up to the same time $P\{s_t = j|\mathcal{Y}_t; \theta\}$.

For the same reason, it is defined $\xi_{t+1|t}$ as a vector containing $P\{s_{t+1} = j|\mathcal{Y}_t; \theta\}$.

Assuming that x_t has no information about s_t once you controlled for $\mathcal{Y}_{t-1}$, we can state that the conditional joint distribution of y_t and s_t is

$$P\{s_t = j|x_t, \mathcal{Y}_{t-1}; \theta\} \cdot f(y_t|s_t = j, x_t, \mathcal{Y}_{t-1}; \theta) = p(y_t, s_t = j|x_t, \mathcal{Y}_{t-1}; \theta)$$

where f is also the jth element of η_t.

The density of y_t is

$$f(y_t|x_t, \mathcal{Y}_{t-1}; \theta) = 1'(\hat{\xi}_{t|t-1} \odot \eta_t)$$

where $\mathbf{1}$ is a column vector of ones, with a number of rows equal to the one of $\hat{\xi}_{t|t-1}$ (and this quantity be called N) and $\odot$ is the vector element by element multiplication.

Consequently, the distribution of s_t is

$$\frac{p(y_t, s_t = j|x_t, \mathcal{Y}_{t-1}; \theta)}{f(y_t|x_t, \mathcal{Y}_{t-1}; \theta)} = P\{s_t = j|y_t, x_t, \mathcal{Y}_{t-1}; \theta\} = P\{s_t = j|\mathcal{Y}_{t-1}; \theta\}$$

and so

$$P\{s_t = j|\mathcal{Y}_{t-1}; \theta\} = \frac{p(y_t, s_t = j|x_t, \mathcal{Y}_{t-1}; \theta)}{1'(\hat{\xi}_{t|t-1} \odot \eta_t)}$$

Noticing that $p(y_t, s_t = j|x_t, \mathcal{Y}_{t-1}; \theta)$ is an element of $(\hat{\xi}_{t|t-1} \odot \eta_t)$ and $P\{s_t = j|\mathcal{Y}_{t-1}; \theta\}$ is an element of $\xi_{t|t}$,

$$\hat{\xi}_{t|t} = \frac{(\hat{\xi}_{t|t-1} \odot \eta_t)}{1'(\hat{\xi}_{t|t-1} \odot \eta_t)}$$

while, to forecast ξ one step forward, we take the expectations conditional on $\mathcal{Y}_t$,

$$E(\xi_{t+1}|\mathcal{Y}_t) = \boldsymbol{P} \cdot E(\xi_t|\mathcal{Y}_t) + E(v_{t+1}|\mathcal{Y}_t)$$

with $\boldsymbol{P}$ representing the transition probability matrix.

Since v_{t+1} is a martingale, its expectations one period forward are zero, so the second term on the right hand side is null.

2.1.4 Smoothed Inference

The smoothed inference regarding the probabilities (called "smooth probabilities") for the regimes at a certain time t is defined as $\xi_{t|\tau}$ when $\tau > t$.

The algorithm to calculate these smoothed probabilities, as formulated by Kim (1994), is

$$\hat{\xi}_{t|T} = \hat{\xi}_{t|t} \odot \{\boldsymbol{P}'[\hat{\xi}_{t+1|T} (\div) \hat{\xi}_{t+1|t}]\}$$

with $\div$ being the element by element division.

To calculate the elements of this vector, you have to iterate the formula above from *t*=*T*, which value is just $\hat{\xi}_{T|T}$ obtained from the normal inference, moving backwards.

Necessary conditions in order for this algorithm to be true are:

- the regime follows a first order Markov chain;
- $f(y_t|s_t = j, s_{t-1} = i, \dots, s_1 = m; \theta) = f(y_t|s_t = j; \theta)$
- x_t is independent of s_τ $\forall\, t \wedge \tau$.

The probability p_{ij} is then calculated as:

$$p_{ij} = \frac{\sum_{t=2}^{T} P\{s_t = j, s_{t-1} = i | \mathcal{Y}_T; \hat{\theta}\}}{\sum_{t=2}^{T} P\{s_{t-1} = i | \mathcal{Y}_T; \hat{\theta}\}}$$

where $\hat{\theta}$ is the maximum likelihood full estimator and $\mathcal{Y}_T$ represents all the available information up to time *T*.

So, p_{ij} is just the ratio between the times the series was in state i and the number of times the series was in state j after being in state i, counted on the basis of the smooth probabilities.

To start the probabilities algorithm, there are many options described.

For example, you can set $\hat{\xi}_{1|0}$ equal to the unconditional probabilities vector; else, you can fix $\hat{\xi}_{1|0} = \rho$, where ρ can be a vector of nonnegative constants which sum is one; it can also be estimated with the maximum likelihood estimation, under the characteristic conditions of the probabilities.

With this smoothed inference, it becomes $\hat{\xi}_{1|T} = \hat{\rho}$.

Finally, the maximum likelihood at this point becomes

$$\sum_{t=1}^{T} \left(\frac{\partial \log \eta_t}{\partial \alpha'}\right)' \xi_{t|T} = 0$$

where the element inside the parenthesis is the derivative of the vector of the densities with respect to α, which is a vector containing all the parameters defining the densities.

The result is an (Nxk) matrix, where k are the number of elements in α.

Analysing the model containing also explanatory variables for the observed outcome,

$$y_t = z_t \beta_{s_t} + \varepsilon_t$$

with ε_t i.i.d. $\mathcal{N} \backsim (0, \sigma^2)$ and a different β for each regime of the model.

The vector of the probability density function is then

$$\eta_t = \begin{bmatrix} \frac{1}{\sqrt{2\pi\sigma^2}} exp\left\{\frac{-(y_t - z_t'\beta_1)^2}{2\sigma^2}\right\} \\ \vdots \\ \frac{1}{\sqrt{2\pi\sigma^2}} exp\left\{\frac{-(y_t - z_t'\beta_N)^2}{2\sigma^2}\right\} \end{bmatrix}$$

2.1.5 Forecast of the observable variable

For what concerns the forecast of the series, there are N different forecast of *y* at *t+1* conditional on the regime (with N being the number of states in the model).

Therefore, the forecast is simply the sum of all the possible distributions of the model, weighted by the probability that the series will be in that precise regime.

To show it in vector notations, calling *R* a *(Nx1)* vector with all the forecasts conditional on the regime, we have:

$$E(y_{t+1} | \mathcal{Y}_t; \theta) = R_t' \hat{\xi}_{t+1|t}$$

The reader should note that this forecast is not linear, since $\hat{\xi}_{t|t}$ is non linear on $\mathcal{Y}_t$.

2.2 Markov Switching Model with Time Varying Transition Probabilities

The Markov Switching Model with Time Varying Transition Probabilities is a model developed by Diebold, Lee and Weinbach (1994).

Even though praising the Hamilton model, the authors stated that the fact that the probabilities of switching are not changing over time is an heavy constraint; therefore, they postulated a model where they let the probabilities vary over time depending on some economic variable.

Using a notation similar to the Hamilton's one, $P(s_t = j|s_{t-1} = i, x_{t-1}; \beta_i)$ follows a logistic function defined as

$$p_t^{ij} = P(s_t = j|s_{t-1} = i, x_{t-1}; \beta_i) = \frac{exp(x'_{t-1}\beta_i)}{1 + exp(x'_{t-1}\beta_i)}$$

with x_{t-1} being a *(kx1)* vector containing economic variables affecting the state transition probabilities, and β_i being a *(Nkx1)* vector containing the parameters linked to the *k* variables and the *N* states.

To simplify the explanations of the model, as in Diebold, Lee and Weinbach (1994), I will take the case where there are just two states.

Also in this model, to perform the maximization of the likelihood function, an EM algorithm is performed.

Naming θ a vector containing all the parameters (that will be shortly explained), and starting from an arbitrarily chosen value for each of them, we find the probabilities that in s_1 the series is in a certain regime, and the probabilities that at every successive time the regime will be in state 0 or 1 after being in state 0 or 1 (4 probabilities for every *t*).

Having calculated these values, we build up the expectations of the log of the jointed distribution function of *s* and *y*, and we iterate the algorithm up to the convergence of the maximization.

The expectations formula is

$$\begin{aligned} E\left[\log f\left(\underline{y}_T, \underline{s}_T | \underline{x}_T; \theta^{(j-1)}\right)\right] \\ = \rho^{(j-1)} \cdot \left[\log f\left(y_1|s_1 = 1; \alpha_1^{(j-1)}\right) + \log \rho^{(j-1)}\right] + \left(1 - \rho^{(j-1)}\right) \\ \cdot \left[\log f\left(y_1|s_1 = 0; \alpha_0^{(j-1)}\right) + \log\left(1 - \rho^{(j-1)}\right)\right] \\ + \sum_{t=2}^{T} \left\{P\left(s_t = 1|\underline{y}_T, \underline{x}_T; \theta^{(j-1)}\right) \cdot \log f\left(y_t|s_t = 1; \alpha_1^{(j-1)}\right) + P\left(s_t \right.\right. \\ \left. = 0|\underline{y}_T, \underline{x}_T; \theta^{(j-1)}\right) \cdot \log f\left(y_t|s_t = 0; \alpha_0^{(j-1)}\right) + P\left(s_t = 1, s_{t-1} = 1|\underline{y}_T, \underline{x}_T; \theta^{(j-1)}\right) \\ \cdot \log(p_t^{11}) + P\left(s_t = 0, s_{t-1} = 1|\underline{y}_T, \underline{x}_T; \theta^{(j-1)}\right) \\ \cdot \log(1 - p_t^{11}) + P\left(s_t = 1, s_{t-1} = 0|\underline{y}_T, \underline{x}_T; \theta^{(j-1)}\right) \\ \left. \cdot \log(1 - p_t^{00}) + P\left(s_t = 0, s_{t-1} = 0|\underline{y}_T, \underline{x}_T; \theta^{(j-1)}\right) \cdot \log(p_t^{00})\right\} \end{aligned}$$

In this formula, the superscript *(j-1)* indicates the current best guess of the parameters, and α and ρ are, with β, the components of the vector θ.

In particular, α contains the parameters defining the probability distribution function, while ρ is the long run probability of $S_1 = s_1$. The underlining is the sign of "past history of the variable" from period 1 till the subscript (Diebold, Lee and Weinbach (1994)).

While in the stationary case it is straightforward to calculate ρ from β, in the nonstationary case, where there is no long run probability, it becomes another parameter to be estimated.

All the probabilities must be calculated starting first from the filtered joint state probabilities, from which you calculate afterwards the smoothed joint state probabilities that are in the expectation formula.

In particular, the filtered state probabilities are:

$$P\left(s_t, s_{t-1} | \underline{y}_t, \underline{x}_t; \theta^{(j-1)}\right) = \frac{f\left(y_t, s_t, s_{t-1} | \underline{y}_t, \underline{x}_t; \theta^{(j-1)}\right)}{f\left(y_t | \underline{y}_t, \underline{x}_t; \theta^{(j-1)}\right)}$$

where the numerator changes whether we are in the second period or in any other following period, and it is

$$f\left(y_2, s_2, s_1 | y_1, x_1; \theta^{(j-1)}\right) = f\left(y_2 | s_2; \alpha^{(j-1)}\right) \cdot P\left(s_2 | s_1, x_1; \beta^{(j-1)}\right) \cdot P(s_1)$$

$$f\left(y_t, s_t, s_{t-1} | \underline{y}_{t-1}, \underline{x}_{t-1}; \theta^{(j-1)}\right) = \sum_{s_{t-2}=0}^{1} f\left(y_t | s_t; \alpha^{(j-1)}\right) \cdot P\left(s_t | s_{t-1}, x_{t-1}; \beta^{(j-1)}\right) \cdot P\left(s_{t-1}, s_{t-2} | \underline{y}_{t-1}, \underline{x}_{t-1}; \theta^{(j-1)}\right)$$

(with the first two elements of the right hand side given by the previous iteration, while the third element given by the result of the previous calculus for the filtered state probabilities), and the denominator being

$$f\left(y_t | \underline{y}_{t-1}, \underline{x}_{t-1}; \theta^{(j-1)}\right) = \sum_{s_t=0}^{1} \sum_{s_{t-1}=0}^{1} f\left(y_t, s_t, s_{t-1} | \underline{y}_{t-1}, \underline{x}_{t-1}; \theta^{(j-1)}\right)$$

Once this calculation have been made for all the T time period, it is possible to calculate the smoothed joint state probabilities.

Setting $\tau = t+2, t+3, \ldots, T$, you start the algorithm for $\tau = t+2$ with

$$P\left(s_{t+1}, s_t, s_{t-1} | \underline{y}_{t+1}, \underline{x}_{t+1}; \theta^{(j-1)}\right) = \frac{f\left(y_{t+1} | s_{t+1}; \alpha^{(j-1)}\right) \cdot P\left(s_{t+1} | s_t, x_t; \beta^{(j-1)}\right) \cdot P\left(s_t, s_{t-1} | \underline{y}_t, \underline{x}_t; \theta^{(j-1)}\right)}{f\left(y_{t+1} | \underline{y}_t, \underline{x}_t; \theta^{(j-1)}\right)}$$

and then, for every other τ

$$P\left(s_\tau, s_{\tau-1}, s_t, s_{t-1} | \underline{y}_\tau, \underline{x}_\tau; \theta^{(j-1)}\right) = \frac{\sum_{s_{\tau-2}=0}^{1} f\left(y_\tau | s_\tau; \alpha^{(j-1)}\right) \cdot P\left(s_\tau | s_{\tau-1}, x_{\tau-1}; \beta^{(j-1)}\right) \cdot P\left(s_{\tau-1}, s_{\tau-2}, s_t, s_{t-1} | \underline{y}_{\tau-1}, \underline{x}_{\tau-1}; \theta^{(j-1)}\right)}{f\left(y_\tau | \underline{y}_{\tau-1}, \underline{x}_{\tau-1}; \theta^{(j-1)}\right)}$$

Calculate this algorithm up to reach $\tau = T$, for which

$$P\left(s_t, s_{t-1} | \underline{y}_T, \underline{x}_T; \theta^{(j-1)}\right) = \sum_{s_t=0}^{1} \sum_{s_{t-1}=0}^{1} P\left(s_T, s_{T-1}, s_t, s_{t-1} | \underline{y}_T, \underline{x}_T; \theta^{(j-1)}\right)$$

Repeat this procedure for every *t* from *t=3*, obtaining all the smoothed probabilities.

From these, it is possible to obtain straightforward the smoothed marginal state probabilities

$$P\left(s_t = 1 | \underline{y}_T, \underline{x}_T; \theta^{(j-1)}\right) = P\left(s_t = 1, s_{t-1} = 1 | \underline{y}_T, \underline{x}_T; \theta^{(j-1)}\right) + P\left(s_t = 1, s_{t-1} = 0 | \underline{y}_T, \underline{x}_T; \theta^{(j-1)}\right)$$

Having calculated these probabilities, one can start with the maximization step of the algorithm.

μ, σ (which are contained in α) and ρ calculations are straightforward, since their first order conditions are linear in the parameters.

$$\mu_i^{(j)} = \frac{\sum_{t=1}^{T} y_t P\left(s_t = i | \underline{y}_T, \underline{x}_T; \theta^{(j-1)}\right)}{\sum_{t=1}^{T} P\left(s_t = i | \underline{y}_T, \underline{x}_T; \theta^{(j-1)}\right)}$$

$$(\sigma_i^2)^{(j)} = \frac{\sum_{t=1}^{T} \left(y_t - \mu_i^{(j)}\right)^2 P\left(s_t = i | \underline{y}_T, \underline{x}_T; \theta^{(j-1)}\right)}{\sum_{t=1}^{T} P\left(s_t = i | \underline{y}_T, \underline{x}_T; \theta^{(j-1)}\right)}$$

$$\rho^{(j)} = P\left(s_1 = 1 | \underline{y}_T, \underline{x}_T; \theta^{(j-1)}\right)$$

For the βs, however, this is not possible, since the first order conditions are not linear.

(I will show just the functions of β_0. For β_1, just substitute the different indexations of β_0 with the same of β_1, and p_t^{00} with p_t^{11}).

$$\sum_{t=2}^{T} x_{t-1} \left\{ P\left(s_t = 0, s_{t-1} = 0 | \underline{y}_T, \underline{x}_T; \theta^{(j-1)}\right) - p_t^{00} P\left(s_{t-1} = 0 | \underline{y}_T, \underline{x}_T; \theta^{(j-1)}\right) \right\} = 0$$

So, Diebold, Lee and Weinbach suggest performing a first order Taylor approximation of the βs

$$p_t^{00}\left(\beta_0^{(j-1)}\right) \approx p_t^{00}\left(\beta_0^{(j-1)}\right) + \frac{\partial p_t^{00}(\beta_0)}{\partial \beta_0} |_{\beta_0 = \beta_0^{(j-1)}} \left(\beta_0 - \beta_0^{(j-1)}\right)$$

With these evaluations, the first order conditions become

$$\sum_{t=2}^{T} x_{t-1}\left\{P\left(s_t = 0, s_{t-1} = 0|\underline{y}_T, \underline{x}_T; \theta^{(j-1)}\right) - P\left(s_{t-1} = 0|\underline{y}_T, \underline{x}_T; \theta^{(j-1)}\right)\right.$$
$$\left.\cdot\left[p_t^{00}\left(\beta_0^{(j-1)}\right) + \frac{\partial p_t^{00}(\beta_0)}{\partial \beta_0}\cdot\left(\beta_0 - \beta_0^{(j-1)}\right)\right]\right\} = 0$$

Solving this equation leads to

$$\beta_0^{(j)} = \left(\sum_{t=2}^{T} x_{t-1} P\left(s_{t-1} = 0|\underline{y}_T, \underline{x}_T; \theta^{(j-1)}\right)\cdot\frac{\partial p_t^{00}(\beta_0)}{\partial \beta_0}\right)^{-1}$$
$$\cdot\left(\sum_{t=2}^{T} x_{t-1}\left\{P\left(s_t = 0, s_{t-1} = 0|\underline{y}_T, \underline{x}_T; \theta^{(j-1)}\right) - P\left(s_{t-1} = 0|\underline{y}_T, \underline{x}_T; \theta^{(j-1)}\right)\right.\right.$$
$$\left.\left.\cdot\left[p_t^{00}\left(\beta_0^{(j-1)}\right) - \frac{\partial p_t^{00}(\beta_0)}{\partial \beta_0}\cdot\beta_0^{(j-1)}\right]\right\}\right)$$

2.3 Hypothesis testing

For the evaluation of the parameters in a maximum likelihood estimation, various methods are used.

However, in order to perform them, two conditions have to be met.

First, the series must be stationary; second, the parameters and the true value must be both inside the allowable parameter space and not on a boundary (for example, if we are talking about a probability *p*, *p* must not be equal either to one or zero).

With large values for T, the distribution of the parameters can be approximated by

$$\hat{\theta} \approx \mathcal{N}(\theta_0, T^{-1}\mathcal{P}^{-1})$$

with θ_0 being the true parameter and $\mathcal{P}$ the so called information matrix.

To estimate the second derivative of the information matrix we have:

$$\hat{\mathcal{P}}_{2D} = -T^{-1}\frac{\partial^2 L(\theta)}{\partial\theta\cdot\partial\theta'}|_{\theta=\hat{\theta}}$$

In this way, we can approximate the variance covariance matrix by

$$E(\hat{\theta} - \theta_0)(\hat{\theta} - \theta_0)' \cong \left[-\frac{\partial^2 L(\theta)}{\partial\theta\cdot\partial\theta'}|_{\theta=\hat{\theta}}\right]^{-1}$$

Another approximation of the information matrix is the so called outer product

$$\hat{\mathcal{P}}_{OP} = T^{-1}\sum_{t=1}^{T}[q(\hat{\theta}, \mathcal{Y}_t)]\cdot[q(\hat{\theta}, \mathcal{Y}_t)]'$$

where $\mathcal{Y}_t$ contains all the information of the dependent variable up to time t.
$q(\hat{\theta}, \mathcal{Y}_t)$, following the definition of Hamilton (1994), is a "vector of derivatives of the log conditional density of the ith observation with respect to the elements of the parameter vector θ, with this derivative evaluated at the maximum likelihood estimate $\hat{\theta}$":

$$q(\hat{\theta}, \mathcal{Y}_t) = \frac{\partial \log f(y_t | y_{t-1}, y_{t-2}, \dots, \theta)}{\partial \theta} |_{\theta=\hat{\theta}}$$

and so, the variance covariance matrix becomes

$$E(\hat{\theta} - \theta_0)(\hat{\theta} - \theta_0)' \cong \left[\sum_{t=1}^{T} [q(\hat{\theta}, \mathcal{Y}_t)] \cdot [q(\hat{\theta}, \mathcal{Y}_t)]'\right]^{-1}$$

Hamilton (1994) also underlines the fact that, if it is not the case that every element off-the diagonal is zero, you have to calculate every element of the matrix in order to invert it and calculate the standard deviation.
The author also underlines that both matrixes obtained with the two different methods are just approximation of the real information matrix, and there is no clear guidance on which of the two is the best estimator: usually researchers use the easiest to calculate between the two.

2.4 Duration models

The studies concerning duration are spreading out in modern economical studies.
As defined by Wooldridge (2010), the duration is "the time elapsed until a certain event occurs".
In particular, Wooldridge (ibidem) focuses on the social sciences studies, explaining models constructed to describe, for example, the length of time the state of unemployment will persist for a given person, or how long will it be until a former prisoner will be arrested again.
This type of analysis is called survival analysis and is usually modeled by the hazard function, which, according to Wooldridge, "allows us to approximate the probability of exiting the initial state within a short interval, conditional on having survived up to the starting time of the interval" (where the initial state is, for example, being unemployed or being out of prison).
Calling T the times in which an individual leaves the "initial state", we define the survivor function, starting from a cumulative distribution function F, as $S(t) \equiv 1 - F(t) = P(T > t)$, where t is a certain value of T.
The hazard function, therefore, is

$$\lambda(t) = \lim_{q \to 0} \frac{P(t \leq T < t + q | T \geq t)}{q}$$

Moreover, we can find different shaped hazard functions, depending on the process they are describing.

If $\lambda(t)$ is constant, there is no dependency between how much the individual has spent in the initial state and the probability of exiting from it, and F is driven by an exponential distribution.

Otherwise , we will have duration dependence, which will be positive or negative if, respectively, the longer one stays in the initial states increases or decreases the probability of exiting from it; mathematically, if $d\lambda(t)/d(t)$ is positive, we will have positive duration dependence, if it is negative we will have negative duration dependence.

One of the most common functions used to describe the distribution of F is the Weibull distribution, for which:

$F(t) = 1 - \exp(-\gamma t^{\alpha})$

$f(t) = \gamma \alpha t^{\alpha-1} \exp(-\gamma t^{\alpha})$

$\lambda(t) = f(t)/S(t) = \gamma \alpha t^{\alpha-1}$

with α and γ parameters, both nonnegative.

The time dependency is given by the value of α, since, whether it is bigger, smaller or equal to one, we will have positive dependency, negative dependency or an exponential distribution (no time dependency).

For further information about duration models (for example , about data censoring or heterogeneity in the model), go to Chapter 22 of Wooldridge (2010).

2.5 The Autoregressive Conditional Hazard Model

In the paper of Gonzalez-Rivera, Lee & Mishra (2008) (that will later be explained in details), the model used to perform the analysis is the autoregressive conditional hazard (ACH) model by Hamilton and Jordà (2002), that was firstly used to forecast the probability of a change in the Federal funds rate target by the Fed.

The authors themselves started from the autoregressive conditional duration (ACD) model by Engle and Russel (1998), that "described the average interval of time between events" (Hamilton and Jordà, 2002).

The equation for the ACD (m, r) is:

$$\tilde{\psi}_i = \omega + \sum_{j=1}^{m} \alpha_j \tilde{u}_{i-j} + \sum_{j=1}^{r} \beta_j \tilde{\psi}_{i-j}$$

where $\tilde{u}_i$ is the length of time between the ith and the $(i+1)$th change in the Federal funds target rate and $\tilde{\psi}_i$ its expectations given its past observations.

Moving from this equation, Hamilton and Jordà rewrote it in order to index the variables by calendar time rather than by the cumulative number of events (changes in the target):

$$w_{1t} = tx_t + (1 - x_t)w_{1,t-1}$$

where $\{w_{1t}\}, t = 1, 2, \dots, T$ is the most recent event occurrence as of week t and x_t is a dummy variable which is equal to one if the event has occurred during week t.

In this way, w_{1t} is equal to t when the last event occurred at time t, and it remains the same up to the next event: for example, if the event, after occurring at t, occurs at time $t + s$, w_{1t} will be equal to t from t to $t + s$, when it will become equal to $t + s$.

Writing the equation in general term:

$$w_{jt} = x_t w_{j-1,t-1} + (1 - x_t)w_{j,t-1}$$

where w_{jt} is the date of the j^{th} most recent event as of date t.

Therefore, $w_{1t} - w_{2t}$, using Engle and Russel notation, is equal to the last $\tilde{u}_i$ completed as of date t.

In this prospective, the authors use ψ_t to represent the value of the $\tilde{\psi}_i$ of the ACD equation associated with date t, and wrote the ACD functions in "calendar time"

$$\psi_t = \omega + \sum_{j=1}^{m} \alpha_j (w_{j,t-1} - w_{j+1,t-1}) + \sum_{j=1}^{r} \beta_j \psi_{w_{j,t-1}}$$

Next, the authors define the hazard rate d_t as the conditional probability of an event occurrence at time t given the information as of $t - 1$ (Υ_{t-1}).

$$d_t = P(x_t = 1 | \Upsilon_{t-1})$$

It is clear that, if Υ_{t-1} only represents the dates of the previous events, the hazard rate will remain unchanged until the next event. Therefore, the expected time till the next event will be:

$$\sum_{j=1}^{\infty} j(1-d_t)^{j-1} d_t = {}^{1}\!/_{d_t}$$

and, consequently, for the ACD, $d_t = {}^{1}\!/_{\psi_t}$.

The authors then assume, for the viability of the likelihood algorithm, that intervals are such that duration cannot be smaller than 1 and so h_t is always between 0 and 1.

As a generalisation

$$d_t = \frac{1}{\psi_t + \boldsymbol{\delta}'\boldsymbol{z}_{t-1}}$$

where $z_{t-1}$is a vector of known variables at t-1.

For calculation purposes, they set the first component of z_{t-1} as a constant and normalise the relative parameter to 1 and, in the last equation for ψ, they normalise ω to 0.

Calling the results of this equation q_t (in place of ψ_t), we can see that its expected value will not be the expected interval between events ($\bar{u}$), but instead:

$$\bar{q} = \frac{\sum_{j=1}^{m} \alpha_j \bar{u}}{1 - \sum_{j=1}^{r} \beta_j}$$

with starting values for the recursion:

$$q_t = \bar{q} \quad \text{for t=0,-1,…}$$

$$w_{j,0} - w_{j+1,0} = \bar{u} \quad \text{for j=1,…, m}$$

To start the calculation, the authors take a value for $\bar{u}$ equal to the average duration, and calculate $\bar{q}$ from the previous equation.

Then, they iterate q_t and calculate

$$d_t = \frac{1}{1 + q_t + \delta' z_{t-1}}$$

Setting $v_t = q_t + \delta' z_{t-1}$, it is noticed that this quantity cannot be negative if we want d_t never to be bigger than one, but still the function must be differentiable, and so, continuous.

To ensure that this will be the case, Hamilton and Jorda use the following sigmoidal function:

$$\ell(v_t) = \begin{cases} 0.0001 & v_t \le 0 \\ 0.0001 + {}^{2\Delta_0 v_t^2}\!/_{(\Delta_0^2 + v_t^2)} & 0 < v_t < \Delta_0 \\ 0.0001 + v_t & v_t \ge \Delta_0 \end{cases}$$

and calculate that the optimal value for Δ_0 is 0.1.

So, the ACH(r,m) specification becomes

$$d_t = \frac{1}{1 + \ell\{q_t + \delta' z_{t-1}\}}$$

After calculating the hazard, and knowing that the probability of the event occurrence given Y_{t-1} is

$$g(x_t | Y_{t-1}; \theta_1) = (d_t)^{x_t}(1 - d_t)^{1-x_t}$$

the log likelihood is

$$L_1(\theta_1) = \sum_{t=1}^{T} \{x_t \log(d_t) + (1 - x_t) \log(1 - d_t)\}$$

that as to be maximised with respect to θ_1, with $\theta_1 = (\delta', \alpha', \beta')'$.

2.6 The Varying Cross-Sectional Rank model and the Trading Rule by Gonzalez-Rivera, Lee and Mishra

Gonzalez-Rivera, Lee and Mishra (2008) developed a nonlinear model that creates a particular trading rule, which performed even better than a buy-and-hold the market strategy during a particular period; this rule takes into account the weekly returns of a stock, and the rank of this particular return among a set of many other firms' share.

For simplicity, as in the paper, I will call this set *market.*

Being *M* the number of firms in the market, *i* the index for a particular firm and *y* its weekly return, the ranking of the firm i at time t is

$$z_{i,t} \equiv M^{-1} \sum_{j=1}^{M} 1\left(y_{j,t} \leq y_{i,t}\right)$$

where the one represents the indicator function.

$z_{i,t}$ is defined as the *varying cross sectional rank (VCR)* of firm i within the market, and its value is always between 0 (excluded) and 1 (included).

Therefore, the rank of a particular share is dependent both on its returns and on the returns of all the other shares in the market.

For example, if a firm experiences an increase in its returns, but is outperformed by its peers, we will see its ranking diminish; at the same time, if the firm has decreasing returns, but its peers are performing worse, it will see an increase in its ranking.

The authors, then define the jump in the ranking $J_{i,t}$ as

$$J_{i,t} \equiv 1\left(|z_{i,t} - z_{i,t-1}| \geq 0.5\right)$$

So, J is a dummy variable that takes the value one when the ranking of the share experience an upward or downward movement of a median of the market.

The authors specified also that the value of 0.5 is chosen arbitrarily, in order to make the jumps not so frequent but also to prevent the possibility that in some period the probability of the occurrence of a jump would be 0 (if the threshold is settled at 0.7 and $z_{i,t} = 0.4$, it is impossible that a jump will happen next period).

The joint distribution of the jumps and the returns is given by

$$f(y_{i,t}, J_{i,t}|F_{t-1}) = f_1(J_{i,t}|F_{t-1}) \cdot f_2(y_{i,t}|J_{i,t}, F_{t-1})$$

with F_{t-1} being all the information available (filtration) up to time *t-1*.

After controlling for any linear dependency of ARMA type between the rankings, in the paper it is detected also whether there is any temporal relationship that requires some ARMA-GARCH modeling; in both cases, it is found that there is no evident dependency among the data, although, for what concerns the linear dependency, some of the shares have tendency to remain among the upper or the lower ranks broadly speaking.

Calling θ_1 and θ_2 respectively the vector of parameters needed to estimate the first and the second function (and at the same time dropping the indexation for every firm), the optimization of the model is calculated via log-likelihood

$$\sum_{t=1}^{T} \log f(y_t, J_t|F_{t-1}; \theta) = \sum_{t=1}^{T} \log f_1(J_{i,t}|F_{t-1}; \theta_1) + \sum_{t=1}^{T} \log f_2(y_{i,t}|J_{i,t}, F_{t-1}; \theta_2)$$

Since there is no loss of efficiency, the estimation of the two elements in the right hand side of the last expression (which we will call $L_1(\theta_1)$ and $L_2(\theta_2)$) can be made separately.

Regarding the first log likelihood, it can be easily seen that J is a Bernoulli variable, and so, for its distribution and log factorization,

$$L_1(\theta_1) = \sum_{t=1}^{T} \log f_1(J_{i,t}|F_{t-1}; \theta_1) = \sum_{t=1}^{T} [J_t \log p_t (\theta_1) + (1 - J_t) \log(1 - p_t(\theta_1))]$$

The probability of a jump next period is treated according to the ACH model by Hamilton and Jorda (2002), as

$$p_t = [\Psi_{N(t-1)} + \delta' X_{t-1}]^{-1}$$

In particular, $\Psi_{N(t-1)}$ has the same notations and characteristics as q_t in Hamilton and Jorda, while $\delta' X_{t-1}$ is

$$\delta' X_{t-1} = \delta_1 + \delta_2 y_{t-1} 1(z_{t-1} \leq 0.5) + \delta_3 y_{t-1} 1(z_{t-1} > 0.5)$$

The second log likelihood, on the other hand, is treated as if there are two different distributions, and the one that is followed at a particular t depend on the occurrence of a jump

$$L_2(\theta_2) = \sum_{t=1}^{T} \log\left[\frac{J_t}{\sqrt{2\pi\sigma_{1,t}^2}} exp\left\{-\frac{1}{2}\left(\frac{y_t - \mu_{1,t}}{\sigma_{1,t}}\right)^2\right\} + \frac{1 - J_t}{\sqrt{2\pi\sigma_{0,t}^2}} exp\left\{-\frac{1}{2}\left(\frac{y_t - \mu_{0,t}}{\sigma_{0,t}}\right)^2\right\}\right]$$

So, the distributions are treated just as normal, without taking into account any skewness or kurtosis (which presences were not checked by the authors).

The authors tested whether there is statistical difference about the distributions, so if the two means and the variances are different one from the other.

It resulted that, while the means where statistically different, the null hypothesis of no difference between the variances was not rejected.

So, to calculate them, the authors specify

$$\mu_{it} \equiv E(y_t|F_{t-1}, J_t = i) = v_i + \gamma_i y_{t-1} + \eta_i z_{t-1}$$

$$\sigma_{1t}^2 = \sigma_{0t}^2 = \sigma_t^2 = \omega + \rho\varepsilon_{t-1}^2 + \tau\sigma_{t-1}^2$$

The variance is therefore treated as being one regardless the state, and follows a GARCH (1,1) process

The part described until this very point is the one concerning the estimation in sample of the model, so the ones in which the log likelihoods are optimized with respect to the parameters.

Afterwards, Gonzalez-Rivera, Lee and Mishra start explaining the out-of-sample evaluation of the model, where they built their trading strategies, based on the one step ahead forecast of the shares' returns, calculated as

$$\hat{y}_{i,t+1}(\hat{\theta}_t) = p_{t+1}(\hat{\theta}_{1,t}) \cdot \hat{\mu}_{1,t+1}(\hat{\theta}_{2,t}) + \left(1 - p_{t+1}(\hat{\theta}_{1,t})\right) \cdot \hat{\mu}_{0,t+1}(\hat{\theta}_{2,t})$$

On the basis of these estimations, the rankings are calculated, and then the investment decisions are taken according to different evaluations criteria and investment preferences.

In details, in every period are chosen the K best performing shares (with K chosen arbitrarily, in the paper K is equal to 5) according to the forecast and the trading strategy chosen.

Three different trading rules are compared: the *VCR mixture trading rule*, which is the one explained up to now, the *VCR trading rule*, where the means of the two different states are taken as being the same, and the *Buy-and-Hold the Market trading rule*.

The trading strategies analyzed are six.

The first ones is based just on the returns, and it is called 'mean trading returns':

$$MTR = P^{-1} \sum_{t=R}^{T-1} \pi_{t+1}$$

where R is the last period of the in-sample period, $P = T - R$ and π_{t+1} is the yield of the portfolio (which composition can change every week according to the selection of the K best performing shares), calculated as

$$\pi_{t+1} = K^{-1} \sum_{j=1}^{M} y_{j,t+1} \cdot 1\left(\hat{z}_{j,t+1} \geq \hat{z}_{t+1}^{K}\right)$$

The second one is based on the Sharpe Ratio (to have a better knowledge of the Sharpe Ratio, Berk and DeMarzo (2011) and Elton and ot. (2014))

$$SR = P^{-1} \sum_{t=R}^{T-1} \frac{\left(\pi_{t+1} - r_{f,t+1}\right)}{\sigma_{t+1}^{\pi}(\hat{\theta}_t)}$$

The third is similar to the second, and consists of a modified Sharp Ratio, where the excess return is not weighted by the volatility, but by the VaR (to see in details The VaR, Hull (2012) and Resti and Sironi(2007))

$$MSR = P^{-1} \sum_{t=R}^{T-1} \frac{\left(\pi_{t+1} - r_{f,t+1}\right)}{VaR_{t+1}^{\alpha}(\hat{\theta}_t)}$$

where α is the tail coverage probability.

The last three models are VaR based, so they evaluate the allocation of capital to optimize the losses in case of unlikely events.

For the calculation of the VaR, while for the Buy-and-Hold and in the VCR trading rule, the calculation is straightforward ($VaR_{t+1}^{\alpha}(\hat{\theta}_t) = \mu_{t+1}^{\pi}(\hat{\theta}_t) + \Phi_{t+1}^{-1}(\alpha)\sigma_{t+1}^{\pi}(\hat{\theta}_t)$), for the VCR mixture, since it is composed by a mixture of normal distributions, a more complex calculation method is needed, and so the authors implement the analytical Monte Carlo method of Wang (2001).

So, the three defining equations are

$$V_1 \equiv P^{-1} \sum_{t=R}^{T-1} MRC_{t+1}^{\alpha}(\hat{\theta}_t) \cong P^{-1} \sum_{t=R}^{T-1} VaR_{t+1}^{\alpha}(\hat{\theta}_t)$$

$$V_2 \equiv P^{-1}\left[-2\left(L(\alpha) - L(\hat{\alpha})\right)\right]$$

$$= P^{-1} \sum_{t=R}^{T-1} 2\left[1\left(\pi_{t+1} < VaR_{t+1}^{\alpha}(\hat{\theta}_t)\right) \log\frac{\hat{\alpha}}{\alpha} + 1\left(\pi_{t+1} > VaR_{t+1}^{\alpha}(\hat{\theta}_t)\right) \log\frac{1-\hat{\alpha}}{1-\alpha}\right]$$

$$V_3 \equiv P^{-1} \sum_{t=R}^{T-1} \left(\pi_{t+1} - VaR_{t+1}^{\alpha}(\hat{\theta}_t)\right)\left[\alpha - 1\left(\pi_{t+1} < VaR_{t+1}^{\alpha}(\hat{\theta}_t)\right)\right]$$

where MRC is the minimum required capital as set by Basel (see Resti and Sironi (2007)), while $L(\alpha) - L(\hat{\alpha})$ is "the difference between the nominal and empirical lower tail probability" (Gonzalez-Rivera, Mishra and Lee (2008)).

The best trading rule will be the one that minimize the values of the three equations (while for the formers the best trading rule was the one that maximized them).

In detail, V_1 looks for the portfolio that minimizes the capital to put aside, the second detects which one has the "predicted tail coverage ability", while the last one ("tick function") gives the best quantile forecast.

To test whether this trading rule is really better performing with respect to the others (and it is not just a matter of case) the authors performed the "reality check" by White (2000) as modified by Hansen (2005).

This check, starting from a benchmark, detects the differences between this benchmark and "the values of evaluation produced by the other trading rules" (Gonzalez-Rivera, Lee, Mishra, 2008).

If the null hypothesis is rejected, it means that one trading rule is producing a better value than the benchmark (to see the complete explanations of the tests, go to the original papers).

The results are strongly in favor of the VCR mixture trading rule, which outperforms the other two trading rule in five of the six trading strategies (being at most as efficient as the Buy-and-Hold), with a p-value that is equal to one in the White test and over 0.9 in the Hansen test. On the other hand, in the V_3 trading strategies, the results are not in complete favor of the VCR mixture over the Buy-and-Hold, but still the null hypothesis of the benchmark not being outperformed is not rejected.

Finally, the authors provided an analysis of the effective returns of the portfolio: while the Buy-and-Hold strategy does not require particular transaction costs, the VCR mixture trading rule constructed portfolio has a composition that can potentially change on a weekly basis, with a high turnover degree.

So, supposing a 100% turnover degree (which may not be far from the real percentage) and the cases where the transaction costs are 0.1% and 0.2%, they calculate the net return as

$$r_{it}^{net} = \log P_{it}\,(1-c) - \log P_{it-1}\,(1+c) = r_{it} + \log\frac{1-c}{1+c} \cong r_{it} - 2c$$

It resulted that the VCR mixture trading rule is concretely outperformed just with c = 0.2% and in periods of bull market.

But, since these transaction costs, on a compounded calculation, are 10.95% per year, Gonzalez-Rivera, Mishra and Lee stated that this level of costs are “exorbitant by any industry standard”, and so that they shouldn’t be a deterrent to exploit the trading rule.

3. Description of the model developed

The model I developed to describe the returns of the 50 shares in the in-sample period and to build a trading rule in the out-of-sample period via a static forecast, is a Markov switching model with time-varying transition probabilities similar to the one of Diebold, Lee and Weinbach (1994).

On the other hand, I used (to construct the mean of the log-likelihood function) independent variable that were similar, or even identical, to the ones used by Gonzalez-Rivera, Mishra and Lee (2008).

I performed the study on a dataset of 50 shares included in the Nasdaq100 index, which is a weighted index of the 100 biggest non financial firms quoted on the Nasdaq market; the weights in the index are based on the capitalization on the market and on some other rules accounting for the influences of the major components.

The shares chosen are the ones that had remained the most in the index in the period 2001-2010 and that were already listed as January 1st, 2001.

The equation characterizing the model is

$$y_{i,j,t} = \beta_{1i,j} z_{j,t-1} + \beta_{2i,j} J_{j,t} + \beta_{3i,j} D_{j,t} + \beta_{4i,j} NDQ_{t-1} + \varepsilon_t$$

where the indicators j and i stand for, respectively, the firm considered and the state i in which the shares is at time t.

All the data are taken on a weekly basis.

The model has two different regime, which, according to the literature, can be thought of as a bull and a bear period for each share.

y_t is the return that the share experienced between week t-1 and week t; taking the data from Yahoo Finance, every week value of the share is calculated as the average of the closing prices of the days of the week, and then the return is calculated as the log of the ratio between this average and the average of the previous week.

On the value of y_t obtained in this way are calculated the values of z_t, that are the ranking of the shares among the market, and are obtained in the same way in which Gonzalez-Rivera, Mishra and Lee calculate them, and equally for J_t, a binary variable which is 1 if between t-1 and t the value of z increased or decreased by 0.5 or more.

D_t, on the other hand, is the distance at t from the last J, so if $J_{t-1} = 1$, then D_t=1, and, if we don't experience a jump in the ranking at t, D_{t+1}=2, and so on until the next jump.

Since their values are bigger than the realizations of the dependent variable, to perform the optimization, I scaled both z_t and D_t, dividing the first one by 10 and the second by 1000.
Lastly, NDQ_t is the log weekly return between week t-1 and t (calculated as y_t) of the Nasdaq composite index (.IXIC).
Including this last variable makes the model resemble a little bit the APT (Arbitrage Pricing Theory) model of Ross (1976).
Anyway, the APT includes in the regression all market indices (called “factors”) which are common to all the shares, so the realizations of the explanatory variables of the regressions are the same.
On the contrary, in this model the values of the independent variables are different for every share.
Moreover, since none of the other three variable affects the return of the market index, it was no necessary to net the return of the index from the influences given by the other regressors (to have a deeper knowledge of the APT model, Elton and ot. (2014)).
The probabilities are calculated, as in Diebold, Lee and Weinbach (1994), with a logit model, with two independent variables, which are z_t and D_t.
So, these two variables have both a linear and a nonlinear effect on the calculation of the yields.
In a paper where he analyses the conditions necessary to develop a well specified Markov switching model with time varying transition probabilities, Filardo (1998) explains also that there are no misspecification connected to including the variables both in the regression and in the switching part.
The computational part is made through the usage of the software Matlab, using the script of Perlin (2010) as modified by Ding (2012) to include the time varying transition probabilities.
The function used to estimate the model is MS_Regress_Fit_tvtp.
This function, however, has a drawback: in the Estimation part of the EM algorithm, the smoothed transition probabilities are not calculated, and so all the model is based on the filtered transition probabilities; the smoothed transition probabilities are calculated just at the end, to plot the probabilities throughout the in-sample period.
This is because the script is not following precisely the model of Diebold, Lee and Weinbach, but the one in Perez-Quiros and Timmermann (2000), which, on their own, took it from Gray (1996), and these models stop the estimation part at the filtered probabilities.

Even though in Perez-Quiros and Timmermann the study is made on firms (so it could be the case that this model is more suited to study shares' returns), a further study could be performed to see how the results change including the smoothed transition probabilities.

The estimated parameters are then tested using the script getvarMatrix_MS_Regress_tvtp, which leaves you the choice to select whether the variance-covariance matrix will be calculated with the Hessian matrix, or the Outer product matrix.

I always selected the Hessian matrix, which is the default method in the script.

In case the script couldn't find a proper optimal solution (so in the cases in which the standard deviations were either null or infinite), I deleted the variables in question from the model of the particular asset, by either substituting them with a constant or not including it at all.

Anyway, the cases were a few, and I could compute more than 90% of the shares with the complete equation.

After finding the best estimates for all the parameters, I calculated the forecast, on which values I based the trading strategies.

The strategies were made on a weekly basis, thinking as to made the decision about the investment during the weekend, buying them at the opening on Monday morning and selling them at the closing of Friday.

The forecast performed is static, using the parameters estimated in the in-sample period and changing each time the independent variable realization observed at the relative time.

In this way, there were no computational drawbacks about z and Ndq, since the values in the model were calculated with a lag.

There was no much difference also regarding D, since, even though the values are taken at the same time of the return realization, you already know with one week in advance its value: if you calculate that the previous week a jump in the ranking has happened, you know that next week D will be equal to one; otherwise, its value will be the previous value plus one.

On the opposite, for J, since its realization is registered at the same time of y, you don't know at t-1 which value will it take at t.

So, in the out-of sample period, I used a proxy for it, which was the inverse of the average duration registered in the in-sample period, calculated as the number of jumps observed divided by the number of weeks in the in-sample period; therefore, it worked as a constant throughout the forecast period.

This study was performed with the apposite script of the Ding package, MS_Regress_For_tvtp.

The script gives as output the forecasted mean and standard deviation for the following period.
As in Gonzalez-Rivera, Mishra and Lee (2008), I constructed the portfolio to invest in choosing the best five performing assets according to the one period forecast and based on two criteria: the highest returns and the highest Sharpe ratio.
Since in the out-of sample period (2011-2014) the risk free interest rates were almost zero, to compute the Sharpe ratio I used an approximation and calculate it just as the ratio between the yield and the standard deviation.
Also the portfolio returns are calculated in the same way as in Gonzalez-Rivera, Mishra and Lee (2008) as the arithmetical average of the yields realized by the assets chosen.
The difference is essentially the percentage of assets chosen on the total dataset analyzed.
In fact, the study I performed is just on 50 shares, and so my method chooses the best 10%, against a choice of almost the best percentile performed by the other authors (5 over almost 500).

Since we have two different regimes, so two different distributions, I would expect the variances of every state to have a small value compared to the returns themselves.
Looking at the literature, for the ranking and the Nasdaq, I would also expect values of the same order of the returns (0.1), and for the first one at most one of the two values negative (in the “bull” state it must be positive, since the returns are positive, and the ranking is always a positive number).
On the other hand, I would expect negative or very low values for the parameters related to the duration, since the longer one share is stuck in the same ranking, or doesn’t change it much, the more I will expect a contrary movement.
For what concerns the jumps, I would expect a positive value for the “bull” state, and a negative value for the “bear” state: this is because the jumps used are at the same time frame of the yields, and so a jump in the ranking would mean that the asset has decreased of a median its position if it was among the best, or that it has increased if it was among the worst.
I do not know what to expect from the coefficients related to the variables that affect the probabilities, because there are, logically, contradicting signs related to the effect of an increasing in the variables on the probability of a switch from a regime to the other.
In the next chapter, I will present the result of the study I performed.

4. Results

I performed the study on a time frame of 521 weeks, so calculated on 520 returns (on the first week, I calculated just the average of the adjusted closing prices, which served as the denominator of the logarithm for the first return).

As already underlined, to get a consistent outcome, I scaled the variables to make them consistent with the yields. So I divided the values of the ranking and of the duration, respectively, for 10 and for 1000.

Therefore, the real effect of the variables on the mean or on the probabilities is obtained by dividing the parameters for the same amount.

In the Appendix II, I will present the full results, with the values of the parameters for each of the shares in the study.

Here, I will show the mean and the median value for each parameter.

I tried to keep all the variables in the study for every asset in the sample, but unfortunately, it was not possible for each asset, because sometimes the optimization could not find an optimum value for one particular parameter.

In these cases, I tried first to substitute the relative variable with a constant (that, for the variables affecting the mean, works as an intercept), and, when even this solution was not viable, I totally cancelled the variable from the estimation.

For, this reason, in the resuming table here below, I showed also how many times the variable was used for the estimation, and in the parenthesis how many times it was substituted with a constant.

	Table 1				
	Average Value	Median Value	Presence (as a constant)	Number of significant parameters	with both states significant
State 0 Distrib	0,001142026	0,00081111	50	50	50
State 1 Distrib	0,003893714	0,0031752		50	
z State 0	0,16859253	0,141655	50	40	16
z State 1	0,15397436	0,16074		22	
J State 0	-0,0049177 (-0,026625)	-0,0044725	49(1)	27(1)	22(1)
J State 1	0,02931818 (-0,0094198)	0,014375		26(1)	
D State 0	-0,4297	-0,302425	50	14	5
D State 1	-2,7606	-1,75565		17	
Ndq State 0	0,02282498	0,0623475	50	12	4
Ndq State 1	0,24849988	0,180805		19	
z Px State 0	87,39228	16,86500	50	26	12
z Px State 1	-77	-9,9159		27	
D Px State 0	502,329655 (2,2975)	76,011	48(1)	19(1)	4(0)
D Px State 1	-121,32584 (0,59387)	-46,536		11(0)	

Table 1: Details of the parameters estimated

Talking about the inference, the values calculated are at the 5% level. In the appendix, I formatted the cells of the inference for which the values were under the confidence level with green background and dark green numbers.

The different variances were significant for each asset in the sample, showing strong results in favor of a double regime structure for the study of assets' returns.

The variable that had at least one significant parameter for almost all of the shares (46 out of 50) was the ranking; on the other hand, it was significant for both of them in less than one third of the sample (16).

The jumps variable, on the contrary, was the one with the highest amount of parameters significant for both states (22), a percentage really high considering that the shares for which it has at least one parameter significant are 31.

The duration parameters, among the cluster, are the least significant: just one asset out of two is significant in at least one state, and just five are significant in both states.

There are also not much Nasdaq-related parameters significant: 27 assets have at least one state parameter significant, and 4 (the least among the four variables) are significant in both states.

This is surprising to me, because many models are based on the relationship between the market and the specific asset (in the CAPM in particular, but also the APT that may use, among the indicators, the market yield netted from the effects of the other indicators).

To recap, the variable with the highest number of assets that had at least one parameter significant is the ranking, while the one with the highest number of both parameters significant is the jump.

Talking about the probabilities-related variables, the quantity of significant variables is quite similar to the number of the independent variables significant, just a little bit lower.

However, just in three cases for the ranking and no one for the duration, if both parameters are significant for the linear part, are also both significant for the probabilities; such a thing questions the model for what concerns the account for both linear and nonlinear effects of the variables on the model.

Moreover, just respectively 37 and 10 times, when a particular parameter is significant in one state for the mean, it is also significant for the probabilities in the same state (37 out of 100 and 10 out of 96, because it is calculated for each of the two regimes of every one of the 50 shares for the ranking, while for the duration two assets are not included since in the probability part is either calculated as a constant or totally excluded).

To test whether the fact that the variables with the highest significance are the first one among the independent variables (and so MATLAB maybe tries with the first, and then fixes it to calculate the second, and so on), I tried on some of the shares to recalculate the model changing the order of them, but the results do not vary, so I can say that the variables that are more significant in general are truly the first two.

It is also interesting to see that the assets for which both the ranking and the jumps have the two parameters significant are 12, so more than one fifth of the sample.

On the other hand, none of the fifty assets had all the parameters relative to the logistic function significant, and just in some cases we find that three out of four are.

The script that I used to calculate the model also plotted the yield of the asset, the standard deviation and the smoothing probabilities (so the probabilities that at t we will be in state 0 or in state 1).

I will not show all the results, but just the most significant ones.

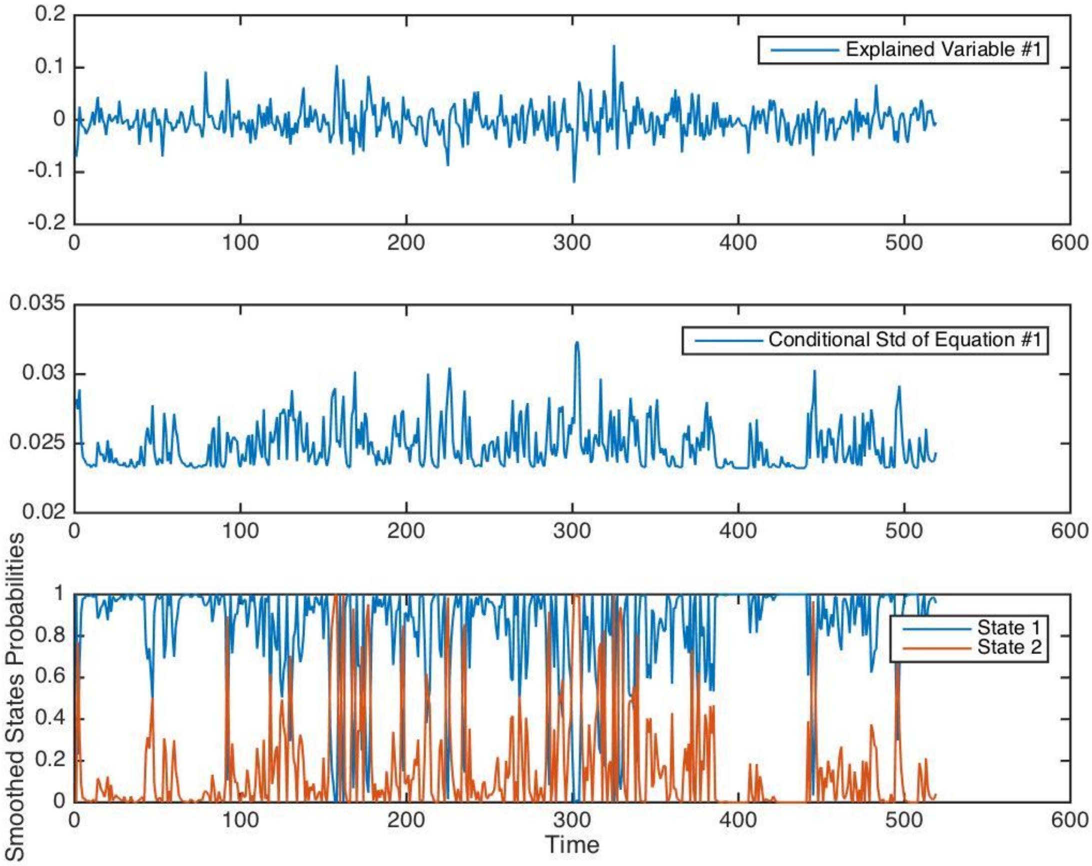

Fig. 1: Plot for ORCL (Oracle)

This is the plot coming from the script for the share of Oracle, which, among the fifty, is the asset with the highest number of significant parameters (12 out of 14).

The returns fluctuate around the zero, with some peak that do not reach the positive or negative threshold of 0.2.

The standard deviation also fluctuates, around a value of 0.025, never falling under 0.02 and never reaching 0.035.

The smoothed states probability are never flat, and have the most noisiest plot in correspondence of the highest values for the standard deviation.

There are two possible explanation for this phenomenon. The first is that, when the variability is high, there are higher chances that the process will switch regime, with respect to moments when the standard deviation is low, and so it is expected that the series will remain stable in the same regime.

On the other hand, it seems that the probability of being in state 1 reaches its maximum in the exact moment in which the volatility is higher, and so this could be a sign that, instead of being driven by the different values in the return, the regimes are driven by the volatility of the asset.

This would make all the assumptions made on the parameters meaningless.

To see whether this is the case, I will present other assets' plots and compare them.

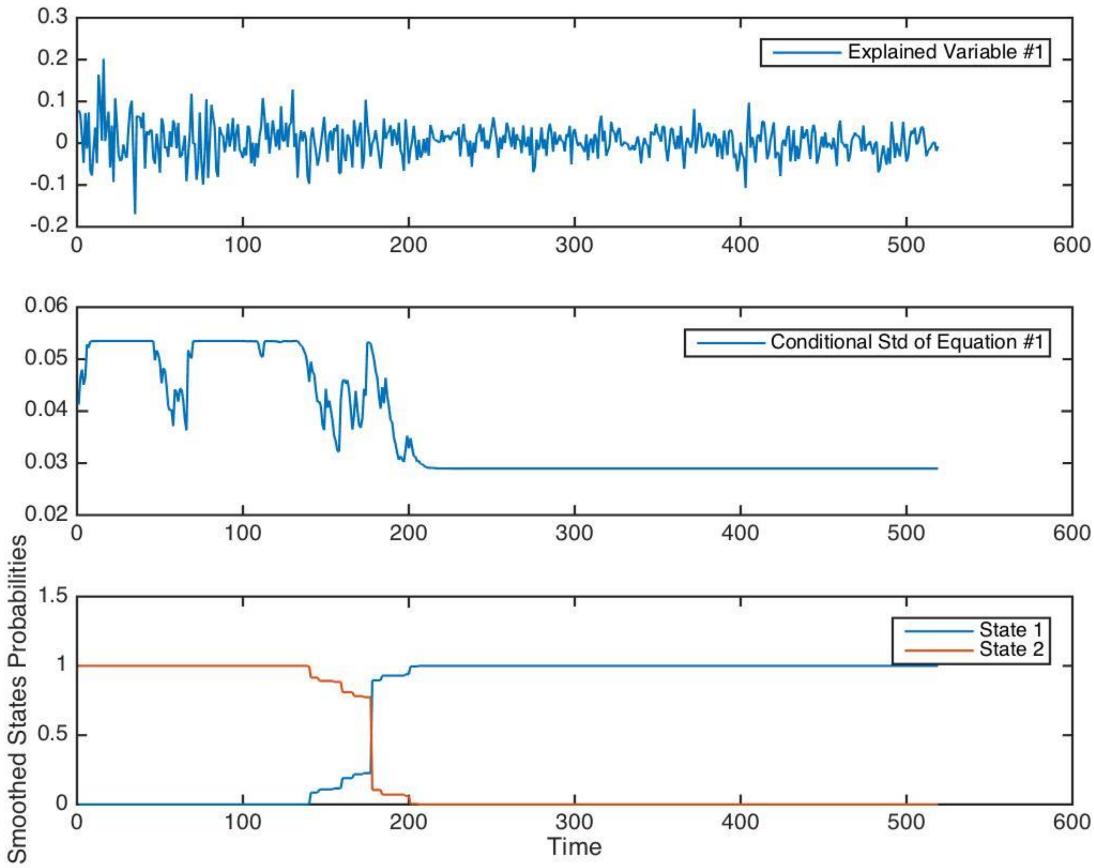

Fig.2 Plot for GILD (Gilead Science)

The plot over here shows the three characteristic elements for Gilead Science, which was computationally the most "challenging", and for which, to reach a complete optimization, I had to net from the variables the duration present in the logistic function, and so I have just one variable affecting the probability of switching.

The plots shows a totally different picture with respect to the one of Oracle.

Here, the yields are also fluctuating around the zero, but we can divide a first period, in which the returns are moving a lot, from a second period, in which they are much more stable.

This is shown also by the standard deviation, where at first the series takes a value of approximately 0.05, while in the second part it is stable around 0.03.

On the smoothed state probabilities, this is surprisingly reflected by a stable scheme, with high probability of being in state 2 (the one called in the tables state 1) in the first period, and high probability of being in state 1 (state 0) in the second period.

In particular, looking at the values for the parameters, also the variance for state 1 is more than three times the variance for state 0.

So, for this asset, it seems that the two states are defined by the volatility rather than by the value of the returns, and so if we are in state 1, we are facing periods of high volatility for the asset.

Looking at the inference of the other parameters, there are just three out of ten significant, but both the parameters related to the ranking are significant (one of the two even at the 1% level), which could indicate that the model is not specified in a wrong manner.

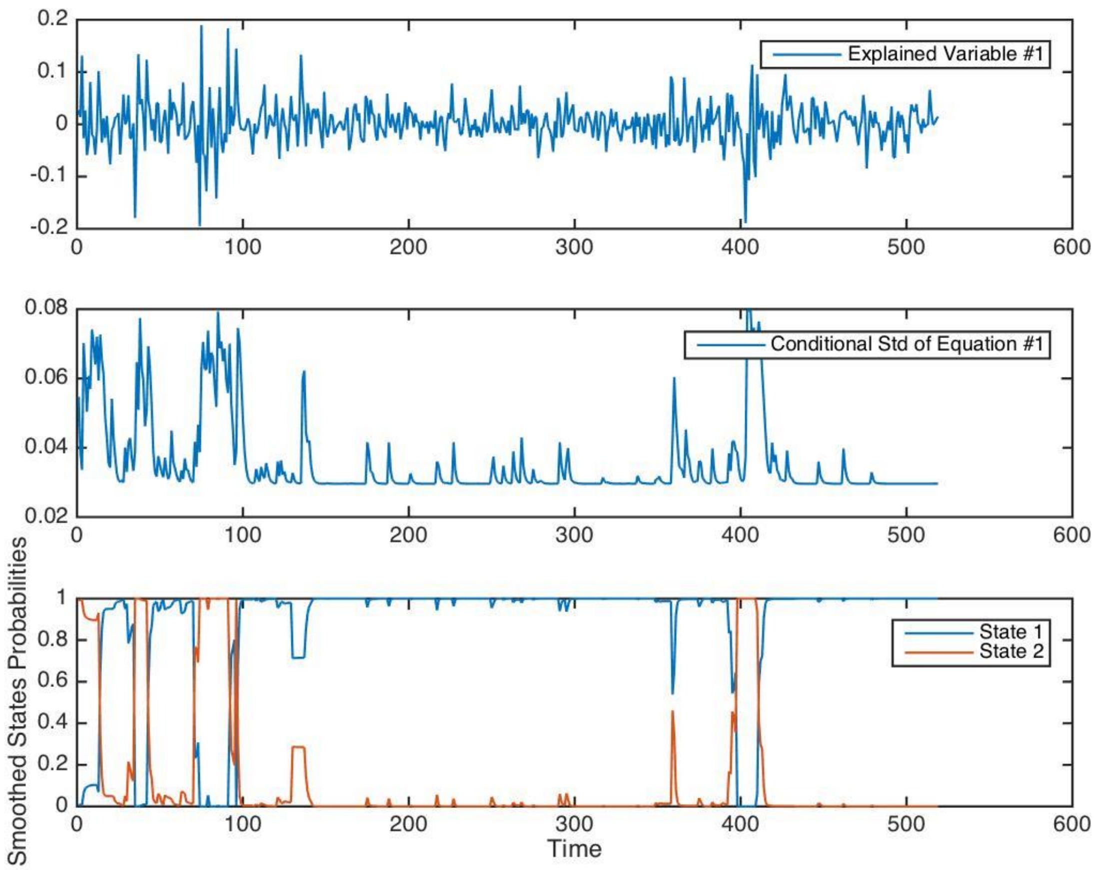

Fig. 3 Plot for SPLS (Staples)

Over here, it is shown the plot for Staples, which is the asset with less parameters significant, which are just the two variances.

As can be seen both by the returns and by the standard deviations plot, there is a long time frame of “calm” period, broken two times by moments of high volatility.

Looking at the smoothed state probabilities plot, as in the one of Gilead Science, the values are quite flat, and the two states are much more defined by the volatility rather than by the returns.

To have another proof of the fact that the regime are volatility-driven, I will also plot the results for a well known company, Apple.

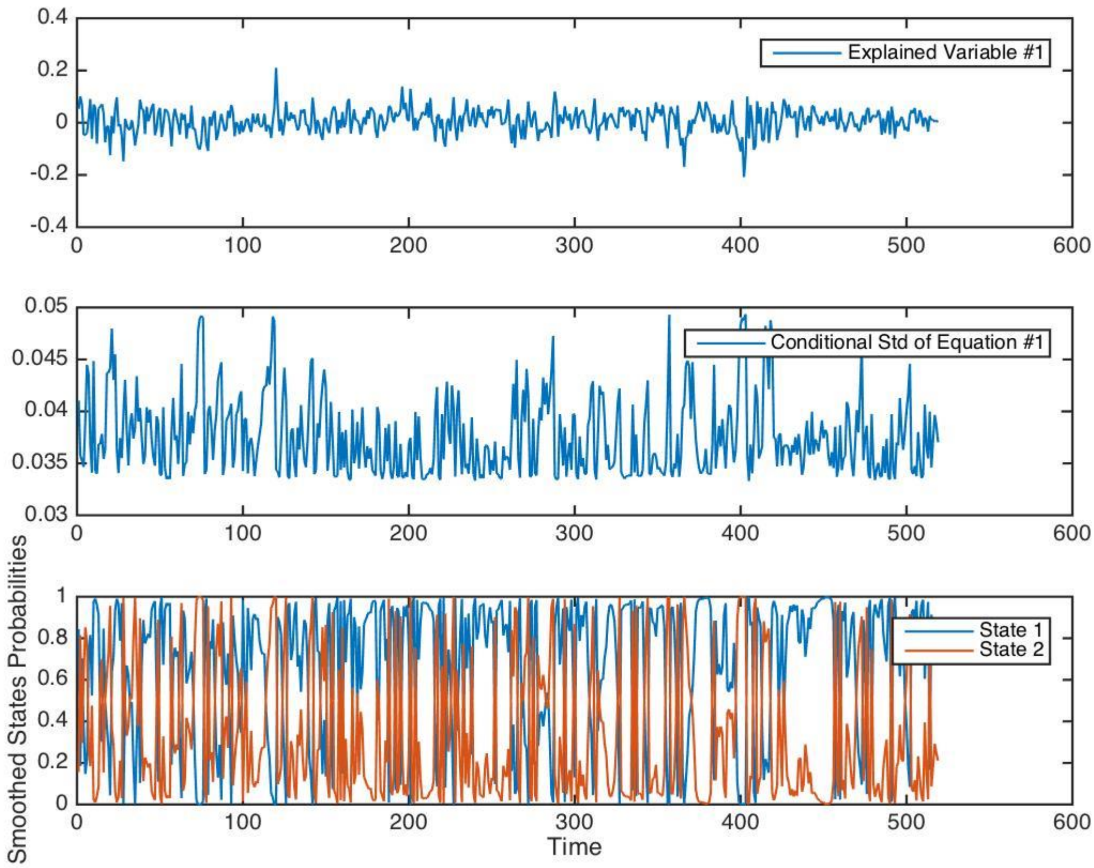

Fig. 4 Plot for AAPL

For this asset, the returns are less stable, and the variance fluctuates a lot.

What can be seen is that, even for Apple, the moments in which, the value for the probability that the process is in state 1 (state 2 in the figure) is close to one or is equal to one are the ones in which the volatility is high.

This is valid both for moments when the returns are high (as in the point close to the peak around t=130) and moments when the returns are strongly negative (as in the point close to the lowest value of the series, with t ~405).

I can conclude that, contrary to what the literature postulates, the two states of the Markov Switching are not in the sense of “bull” and “bear” period, but in the sense of “volatile” and “calm” periods.

Having enlightened this, it is also easier to analyze all the parameters.

All the variance parameters are significant because, in fact, they are the ones defining the regimes.

Moreover, states 1, so the one for period with high volatility, as a mean value in the sample which is three times bigger than the mean value for state 0.

The average parameters for both the ranking and the market are, on average, not so different one from the other.

Obviously, there are no cases for which the ranking related parameters are both negative, and rarely one of them is.

Even though in many cases the difference between the parameters is proportionally big, on the average this is not shown because in some cases the highest parameter is the one related to the volatility regime, and in other is the state 0 related one.

The Nasdaq related parameters, on the other hand, are often either close values or the parameter related to state 0 is negative, showing that, in times of low volatility, there is a negative correlation between the market and the assets. Just in few cases (as for Oracle), the correlation is negative in times of high volatility, while positive in the other regime.

The average values for the jump parameters are also quite low.

Therefore, a lot of times the parameters are significant for both states is due to the fact the jump dummy is itself an indicator of volatility.

Most of the shares have at least one of the duration-related parameters negative, especially the state 1 parameters.

This is a sign that, the longest one share does not make a big shift among the ranking, the highest will be the negative impact on its return in case the asset is in a high volatility period: therefore, if the stock does not move with respect to the cluster, but it is facing high volatility period in the market, we will expect lower returns.

Looking at the values of the parameters affecting the probabilities, we see that the means and the average are both negative for state 1 and positive for state 0.

The meaning of these results is that the highest values of both the ranking and the duration, the highest the probability, if we are in state 0, that the regime the next period will still be state 0; on the opposite, the highest they are, if we are in state 1, the highest the regime will switch to state 0 the next period.

Anyhow, looking carefully at the results for each share, we can see that some results are going on the opposite direction of the average results.

To test more carefully these results, I performed the calculation of mean just on the parameters which are significant at the 5% level. Obviously, I did not report the standard deviation values, since they are all significant.

These are the results:

Table 2	Average Value
z State 0	0,19065615
z State 1	0,323389091
J State 0	-0,007730862
J State 1	0,045805103
D State 0	-0,6990
D State 1	-5,2908
Ndq State 0	0,116981667
Ndq State 1	0,483390526
z Px State 0	68,61058
z Px State 1	-150
D Px State 0	266,6004737
D Px State 1	-324,1048182

Table 2: Average value of the significant parameters

Looking at the independent variables, we can see that the spread between values is wider.

The ranking parameters are both bigger, but the increase in the high volatility state is larger; therefore, an increase in the ranking always has a positive effect on the mean, but in period of high volatility this effect is bigger.

For the Nasdaq parameters, the increase in the spread is caused both by a smaller average parameter for state 0 and a bigger for state 1 (which is almost doubled), but both are positive, and so in any case an increase in the market yields cause, on average, an increase in the asset's returns.

Also for the average jumps variable the reasons for the increase in the spread are the same, but in this case the state 0 is a negative value; so, when a jump in the ranking occurs, there will be an expected reduction in the returns if we are in a low volatility period, and an increase if we are in a high volatility moment.

On the other hand, the duration parameters are both smaller, but the bigger decrease is registered by the already lower value for the high volatility regime, which is almost doubled: therefore, a longer period without jumps in the ranking will in any case, on average, effect in a negative way the returns, but in moments of high volatility, the expected decrease will be higher.

Turning to the probabilities related parameters, their values, on average, decreased. This means that, looking just at the significant parameters, on average, the persistency effect of the variables in the state 0 is lowered, and the switching effect for state 1 is widened: an increase in the ranking or in the duration will lead to an higher probability of remaining in state 0 or switching to state 0 (depending on which state we are right now), but, with respect to the total model average effect, the average effect of the significant parameters makes less likely to remain in state 0 and more likely to switch from state 1 to state 0.

Nonetheless, we still find, even in the "restricted" group of parameters, some that have a contrary effect with respect to the most of them.

This could either mean that there is no general rule for describing the class of assets, or that there are some mistakes nested in the model.

But in my opinion, the first case is more likely: just thinking about the CAPM model, some assets have a positive and some other a negative beta, depending on how the stock reacts to a movement of the market; so, it is not surprising to see some negative parameters for the Nasdaq variable.

Also, if a share sees brief period of high volatility alternated by brief period of low volatility, both parameters in the logistic function related to the duration should be negative, and that is also what happens in the Apple case.

Looking at the possible improvement of the model, the standard deviations are calculated just as parameters, as it happens in a normal likelihood.

In most part of the recent literature, it is proven that the calculation of the volatility for the shares return is best performed with a GARCH (1,1) model, as in Gonzalez-Rivera, Mishra and Lee (2008) (for a study on various Garch models and their predicting ability, Franses and Van Dijk (1996)).

So, implementing a model where the variance follows a GARCH (1,1) could be a good further test to implement a variation of the model.

4.1 Results for the strategies based on the forecast

After finding the parameters for each asset, I performed the forecast for the returns.

These forecasts are static, so the parameters are the optimal one from the model, and the variables takes not the estimated value, but the value they effectively assumed in the relative period.

There were no problems for any variable except for the jump. In fact, the jumps are calculated in the model at the same lag of the dependent variable, but in the forecast period, you do not know in advance the value of J.

To solve this problem, I used a constant throughout all the out of sample period, which is the inverse of the average duration between two jumps, or the probability that a jump will occur.

The forecast are always one step ahead both for the returns and for the standard deviation.

Starting from this forecast, I used two different strategies to select on which shares to invest.

Among all the shares studies, I selected the best 5 with the highest value according to these evaluations.

The first strategy selects the assets just on the base of the forecasted return: the five assets with the highest forecasted returns are chosen to form the portfolio for the next week.

The second is based on the ratio forecasted mean over forecasted standard deviations of the asset, so a Sharpe Ratio with zero risk free yields (an assumption which is not far from the truth in the years on which the forecasts are performed).

Anyhow, the calculation of the standard deviations is made just asset by asset, and the correlations between assets are not accounted for in the calculation of the standard deviations, so the "Sharpe Ratio" strategy selects the five assets that per se have the best ratio.

To compare the results of these strategies, I compared them with a Buy and Hold strategy performed on the Nasdaq 100 index, to which all the stocks have belonged for long periods in the time frame analyzed (2001-2014).

To show how the series evolved through time, I calculated week per week the returns of the three strategies, and showed them in Appendix III, together with the value of the capital invested.

The strategies I performed are based on a full week holding of the assets; therefore the returns are calculated as if I bought the assets on the market opening on Monday morning and sold them at the close of Friday.

On the other hand, since for the Buy and Hold strategy the index (and so, in reality, a passive managed portfolio mimicking the returns of the Nasdaq 100) is kept throughout the whole period, the returns are calculated as the closing on Friday over the opening of Monday for the first week (January 3rd-January 7th, 2011), and then as the closing price of Friday over the closing price of Friday of the previous week: so, in the calculation of the returns, there is a little difference caused by the difference in the strategies themselves. Anyhow, numerically, they are all calculated in the logarithmic form.

To be consistent with the in sample period, the values of the independent variables used to perform the forecast (ranking, jumps, duration, the Nasdaq Composite index returns), are all computed on the logarithm of the weekly average of the closing price over the same average of the previous week for each asset.

Therefore, there is a difference between the calculation of the effectively realized returns and the variables that are used to select the asset on which to invest.

All in all, these strategies are based on variables calculated on general trends more than on oscillation of the prices, and then try to invest based on these general-trend following statistics, but are subject to price oscillation due to the rule of investing-disinvesting in particular moment of the week.

This is a drawback of the strategy as it is here calculated.

Obviously, a more accurate study on the results could lead to form prediction on the expected returns (maybe on the level of the single asset), and therefore allow creating more complex strategies, with cap to disinvest when the asset is thought to have reached its weekly peak and floors to disinvest if the loss is becoming too heavy.

The portfolios built on the forecast from the Markov switching are equally weighted, so every asset is worth 0.2 of the whole portfolio. Therefore, the weekly returns (as in Gonzalez-Rivera, Mishra and Lee (2008)) are just the arithmetic averages of the weekly returns of the single assets.

This is not so unrealistic, given the elevated number of possibilities given by all the financial instruments present in the markets. On the other hand, these types of possibilities usually come with a cost, which are not accounted for here.

In the calculation of the value of the portfolio, and so the capital invested, the numbers are relative to an initial investment, as of January 3rd 2011, of 1 million of dollars, and the data in Annex III and in the schematic tables, are all expressed in million of dollars.

Here below, I am showing some schematic results of the three strategies

	Table 3		
	Nasdaq	Best Returns	Best Sharpe
Total Return	79.52%	20.12%	22.69%
Annualized Return	15.75%	4.69%	5.25%
Avg Weekly Return	0.31%	0.12%	0.13%
Highest Invest. Value	1.8396	1.2426	1.2618
Lowest Invest. Value	0.8984	0.9242	0.8338
Highest Weekly Ret	7.40%	7.94%	7.10%
Lowest Weekly Ret	-7.40%	-10.03%	-9.86%
Avg Win	1.75%	1.90%	1.68%
Avg Loss	-1.64%	-2.13%	-2.31%
Returns >5%	6	6	6
Returns <-5%	3	7	8
Weeks with Inv<Init. Inv	18	36	104
Weeks with Ret<0	89	92	81
Value Strategy>Value Nasdaq		40	18
Return Strategy>Return Nasdaq		93	97
Weeks where every Ret>0		26	27
Weeks where every Ret<0		23	21
Value<1 but >Nasdaq Value		6	2
Returns<0 but >Nasdaq Ret		24	24

Table 3: Results of the investment period

Applying the strategies, not all the assets in the study were invested in.

In particular, Cintas, Intel, Microsoft and Staples were never selected by anyone of the strategies; looking at each strategy, on the other hand, the Return based strategy never invests in Cisco, Expeditors, Gilead Science, Oracle, Paychex and Dentsply, while the Sharpe based strategy never picks Adobe nor Biogen.

Therefore, of the all cluster of assets, the first strategy selects at least one time just 40 stocks, while the second strategy 44.

The overall return of the Nasdaq 100 is between the three and the four time bigger than the return of the other two strategies, which slightly differ in favor of the Sharpe strategy.

Once annualized, the difference is even more evident between the first column and the other two.

The difference between the Returns and the Sharpe strategy is given by the final value itself rather than by the history of the two strategies, as can be easily spotted by the difference in the average returns.

The two values are almost identical (they differ of 0.01%), while, even in this category, they are dominated by the buy-and-hold the market, which has an average returns almost three times bigger.
Just looking at the highest and lowest investment value, it seems that the Return strategy is the safest one, having a higher minimum point and a lower maximum point. But paying attention to the highest and lowest weekly returns, is easy to notice that the other two strategies neither have such good (7,94%) nor bad (-10,03%) results.
Therefore, the fact that the value of the investment in the Return strategy is not as volatile as the other two is probably given just by the particular sequence of positive and negative results.
The Nasdaq 100 is perfectly symmetric with respect to the 0 for the higher and lower returns (+/- 7.40%), and it is performing better than the Sharpe strategy in both categories.
To sum up, the highest returns are not so different among the three strategies, while the lowest are, with the Nasdaq that has a gap of more than 2% with the best of the two strategies (the Sharpe), which, in any case, is not that far from the worst one.
Looking at the average returns in case these are positive or negative, it is noticeable that the main difference is made by the average loss.
Between the two strategies based on the Markov switching, the Return strategy outperforms the Sharpe in both categories, and in the wins is even more efficient than the Nasdaq.
But on the loss side, the Nasdaq has a value which is almost 0.5% and 0.7% higher respectively than the Return and the Sharpe strategies.
This is likely due to the higher diversification of the total index with respect to the portfolio constructed by the two strategies.
A sign of the low diversification of the portfolio is also given by the number of weeks in which the returns of the assets in the portfolio are all either negative or positive.
As you can see, almost one fourth of the times there is no compensation of the results. And even if this is eventually what we would like to happen on the positive side (26 and 27 times), since what we are looking for is an higher return, on the other hand when 23 and 21 times it happens on the negative side, we are just losing money.
In Appendix III, I wrote with character red the worst value both for the capital and the returns, while the best are colored in green.
Every strategy reaches its peak value on the same date, November 28th, 2014, while the worst are all in the second half of 2011, with the Buy-and-Hold being the first one (August 19th) and

the Sharpe being the last one (December 16^{th}, anyhow just three weeks from the Return strategy).

On the return side, instead, all the strategies have their worst week on August 8^{th}, 2011, while the best results are all different, with the Nasdaq and the Return strategy that registered their best week respectively in October and September of the same year, while to see the Sharpe strategy one we have to go to August 2014; the fact that the best results happens late in the sample may be one of the main reasons why the Sharpe strategy, at the end, has a better return than the Return strategy.

To control for the weights of the "rare" event, I counted for the weeks in which the returns are very high and the weeks in which the returns are very low. This is because sometimes the overall results are just determined by really positive or really negative trade, which undermine a good strategy or bust the results of a negative one.

I took as a threshold a level of positive and negative 5%, which is really high for a weekly return on the market.

The results showed that the big positive events happened an equal number of times for all the three investments, while the negative ones for the Buy-and-Hold are less than a half with respect to the ones of the other strategies.

Even this result is likely to be given by the higher degree of diversification, but if a negative event thought to be rare happens two times more in one strategy with respect to another, it is a clear signal that the first one is not really efficient.

Analyzing the negative returns, all three strategy had a high number of weeks in which they lost money, but at the same time, all of them were positive more than half of the weeks in the sample.

In this category, the best one is the Sharpe strategy, which had more or less ten weeks negative less (and so ten positive more) than the other two strategies.

This is probably the main reason for which the Sharpe strategy, in the end, is performing better than the Return strategy, also because, looking at the number of weeks in which the capital is below the initial investment, the results for the Sharpe strategy are dramatic.

In fact, if the investor had randomly chosen a week in which to withdraw the investment, there would have been 50% of probability that he would have lost some money, since 104 weeks over 209 the level of the investment is below the threshold of 1 million.

Also in this category, the Buy-and-Hold strategy is the best performing one, with just 18 weeks under the level of the initial investment. The Return strategy, with its 36 weeks, has two times the number of losing weeks with respect to the Buy-and-Hold, but ranks well above the Sharpe strategy (almost one third its quantity).

I performed also some statistics on the comparison of the two strategy based on the one period ahead forecast of the Markov switching with time varying transition probabilities with the But-and-Hold strategy, keeping this last one as the benchmark.

First, I compared the weeks in which the value of the investment of the two strategies was bigger than the value of the investment in the Nasdaq 100.

Of the whole period of 209 weeks, the Return strategy capital was higher than the Buy-and-Hold strategy result just 40 times, which is anyway better than the Sharpe strategy one, that had a higher value just 18 weeks.

Looking carefully at the values, we can see that the value of the capital invested in the strategies is higher than the capital invested in the Nasdaq just in the first year, more precisely in the first 40 weeks of the out of sample period: therefore, the time frame in which the forecast performs better is the one just following the end of the in sample period.

Especially for the Return strategy (that, at the beginning, performs better in each one of the first forty periods), this should not be a random outcome.

As stated by Granger and Terasvirta (1993), even if the model is a good fit in the in-sample period, it will produce a good forecast (in the book it is compared with normal linear model, so it would be a better predictor than a forecast based on a linear model) only if the nonlinear characteristics of the series are still present in the forecasted period, reducing in this way the forecast errors.

Therefore, if the nonlinear elements are differently characterized or shaped, the model won't be anymore a good predictor for the series.

In this sense, to improve the results of the forecast (and therefore of the investment) it would be useful to test an out of sample period of just one year after an in sample period of 10 years (in my case, to perform the 2012 forecast with the in sample period 2002-2011, forecast for the year 2013 with the in sample 2003-2012, and so on).

The two strategies outperform the Buy-and-Hold strategy (in terms of returns) 93 and 97 times, so not far from half of the weeks their performance are better than the index ones.

Of these quantities, more or less just one fourth of the times (24 for both) the strategies are performing better when they are negative (so they are losing money, but they are losing less than the Buy-and Hold strategy); the rest of the times, they are either gaining while the Nasdaq has a negative return, or their results are better with respect to the Buy-and Hold strategy when this is positive.

Finally, I tested how many times, with a negative overall return on the investment, the capital of the two strategies was worth more than the capital invested in the Nasdaq: the results were that the Return strategy was higher in 6 weeks (over the 40 weeks in which it was worth more in general), while the Sharpe strategy was higher just in 2 cases (over 18).

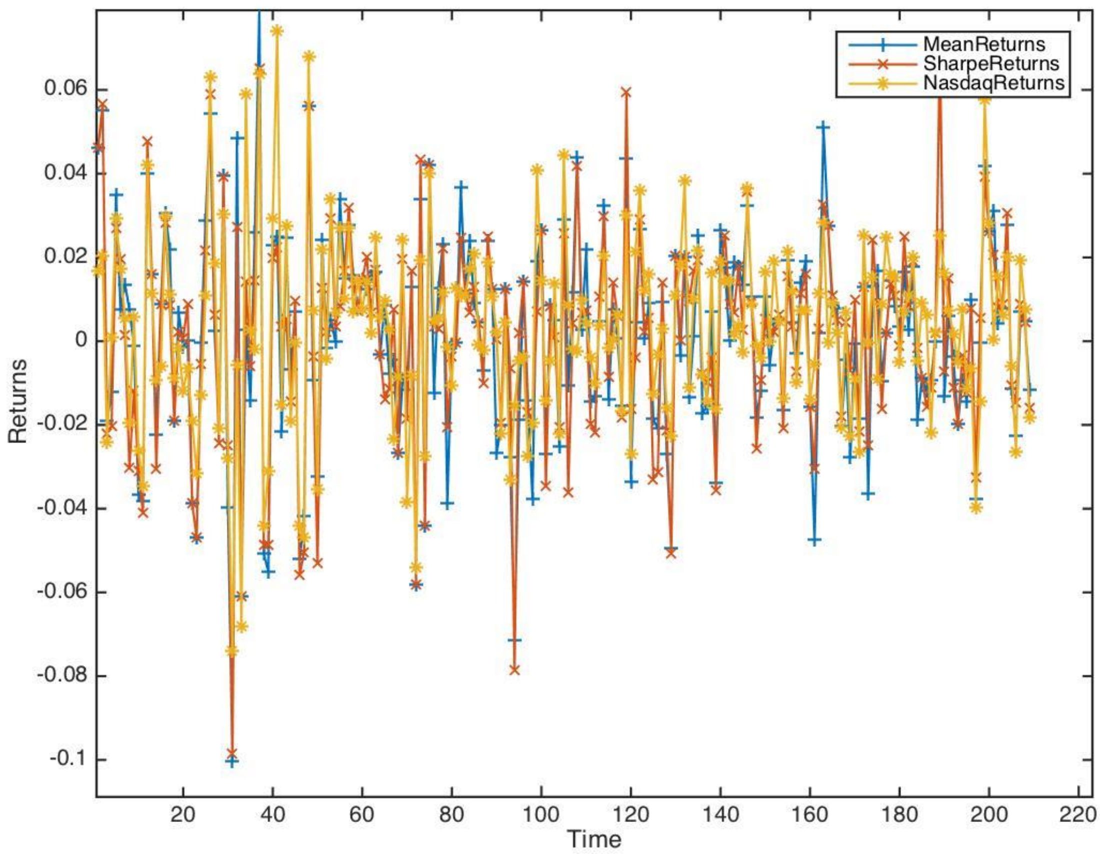

Fig.5 Returns of the three strategies

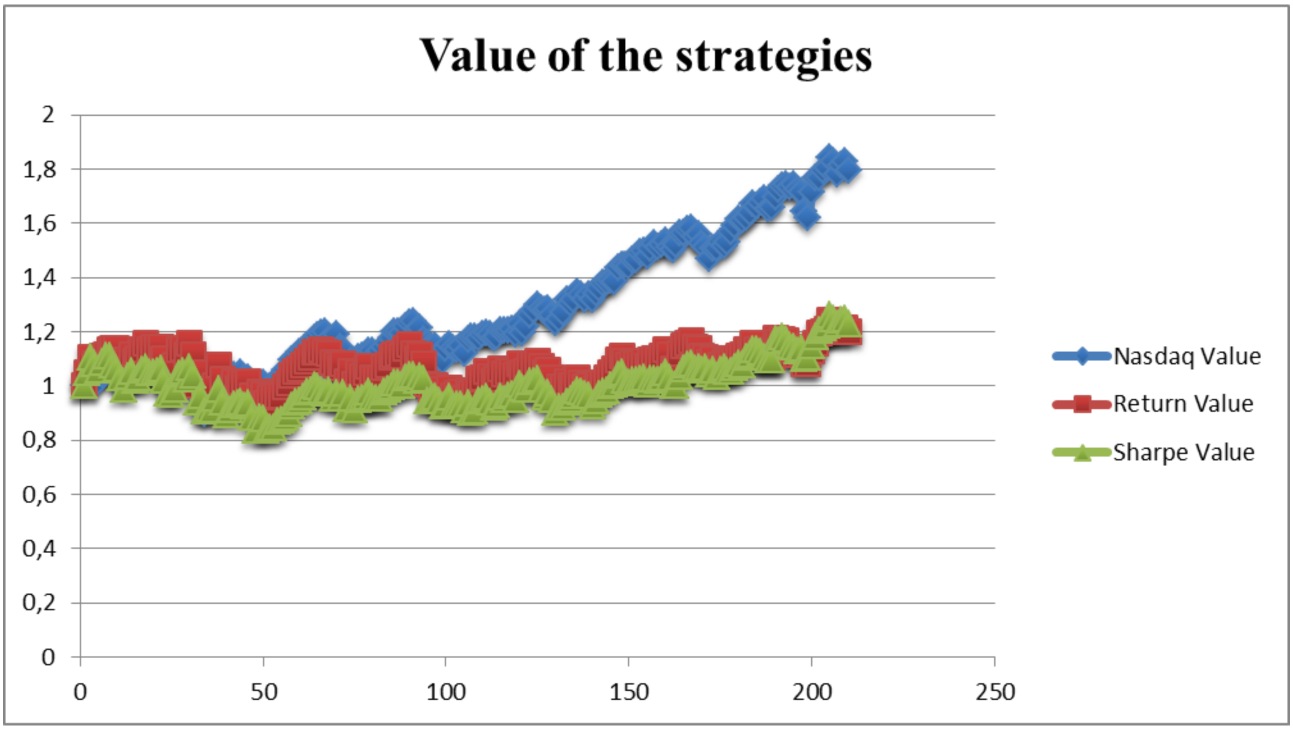

Fig.6 The Value of the strategies over the forecast period

Over here, I showed the plots for the weekly returns of the strategies and the value of the invested capital, printed out respectively with Matlab and Excel.

The first one is very confused, and it is not useful to understand how the returns evolve exactly over time, but allows spotting clearly the outliers in the returns for each strategy.

The second one, on the other hand, permits to grasp clearly the various trends for each one of the strategies.

After the first year, when the capital invested with the Return strategy is slightly higher than the other two, follows a second year, in which the capital kept invested on the Nasdaq 100 is worth more than the other two, but still close to them, with the Return strategy still giving higher return than the Sharpe strategy.

The turning point happens around the starting of the third year, when the Nasdaq invested capital starts a positive trend, while the other two strategies still fluctuates around the initial value of one million.

They start a positive trend just at the half of the last year, with the Nasdaq that keeps on growing with a steeper line.

The Returns and the Sharpe capitals start to grow at the same rate just at the end of the fourth year, especially the Sharpe invested capital, that, after having had a lower value almost throughout all the period, at the end is worth more than the Return invested capital.

For all the results showed until this point, in the strategies I did not take into account the commission costs due to brokers for buying and selling share on the market.
Even without them, the results for the Buy-and-Hold strategy are better than the ones for the other strategy.
Anyhow, for completeness, I calculated the results taking into account the transaction costs. Their calculation was made as in Gonzalez-Rivera, Mishra and Lee (2008), taking a commission of 0.1%.
The commissions are calculated on the returns of the overall portfolio rather than on the return of the single asset since, computationally, there is no difference between the two calculation methods.
The results are shown in Table 3 of Annex III.
They are dramatic: the strategies, after registering some good performances at the beginning of the investment period, fall below the threshold of the initial investment, and are unable to recover, ending with a loss of approximately 20% each.
Besides the suggestion of the calculation of an out of sample of 1 year after a ten year in sample period, another way to optimize the investment would be to apply, after having selected the five assets in which invest, a study following the Markowitz approach, and so the covariance between the elements composing the portfolio other than their singular variances and expected returns, and select in this way the weight of each asset in the portfolio.
This is valid especially for the Sharpe strategy, because selecting the portfolio with the highest returns with Markowitz approach just makes you invest all your capital in the asset with the highest expected return, or maybe investing more than you have in that asset by assuming a short position in some other assets.
To perform a similar analysis, I computed a last test and tried to invest just in the asset that had the best one step ahead forecast of the whole sample for both the strategies.
In this way, I tried to test the effect of the assets that were on the best ten percentiles but were not the best on the overall results. Investing just in the best expected asset, which represents the best 2% of the sample, makes this ratio closer to the one used by Gonzalez-Rivera, Mishra and Lee (2008) (that selected 5 out of 500 assets, so 1%), but on the other hand makes the investment more vulnerable to volatility, since there is no diversification at all.
The results are represented in Table 4 of Annex III.
The capital invested with the Return-based strategy performs well over almost the first two years, than fall under the initial investment level in the last quarter of 2012, fluctuates around

that level for some weeks, and then starts falling down again, concluding the period having lost more than one third of the amount invested.

On the other hand, the Sharpe-based strategy works better, also performing well in the first two years (in June 2012 it also doubled the invested capital), than has a turning point at the end of the second year, where it lose a consistent amount, but manages to recover and ends the period with an overall return of 71,60%, which is anyhow lower than the return from the Buy-and-Hold strategy (and the spread gets wider if we consider the commissions).

In detail, the turning point for both strategy is the week of October 19th, 2012: here, both strategies indicate, according to their valuations, that the best asset in that week would have been Apollo Education Group (APOL). Unfortunately, the stock in that week lost more than 30%, leading the Return capital under the initial investment level, and the Sharpe capital to lose almost 70% of the initial value.

This is a clear example of why, in the financial markets, nobody should ever “put all the eggs in one basket”: diversifying, if, on one hand, you reduce a possible gain (none of the previous portfolio doubled the initial capital), on the other you reduce the chances of high losses, which are likely to happen sooner or later.

To clearly show how the strategies that pick just one asset work, I’ll show a picture down here of how the capital invested evolved throughout the investment period.

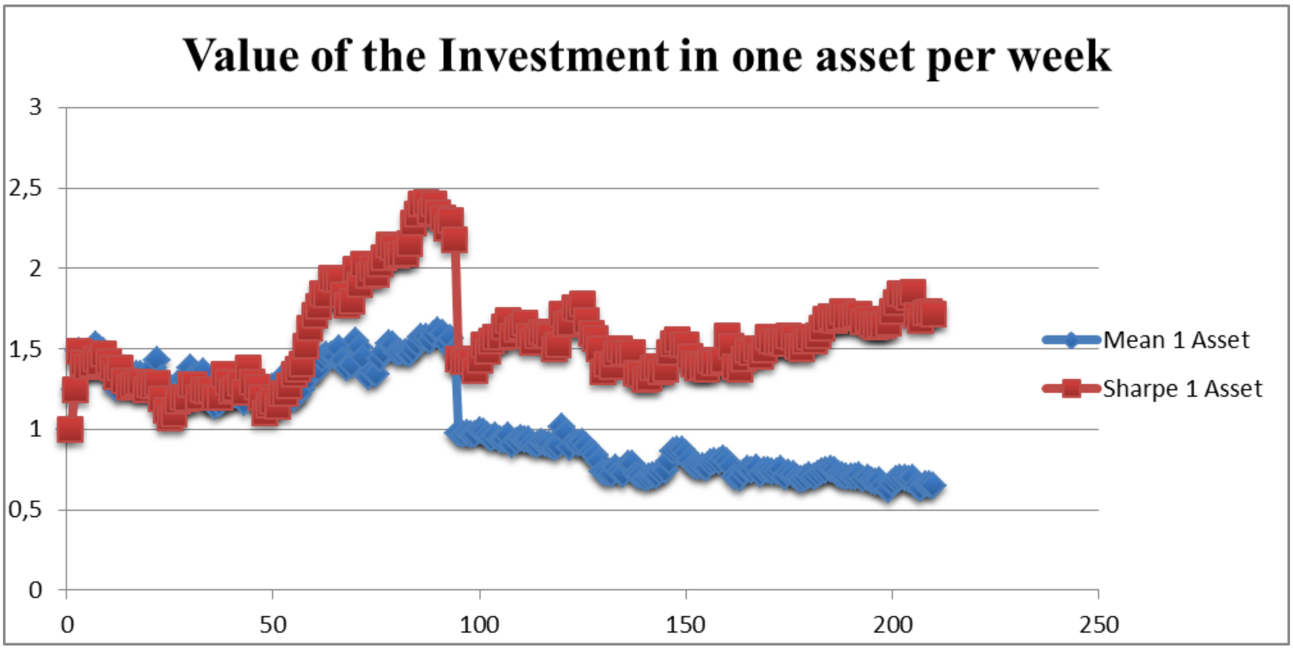

Fig.7 Value of the Investment in one asset per week

5. Conclusions

I performed a study on the 50 shares that in the period 2001-2010 were included the most in the Nasdaq 100 index, being already listed at the date of January 1st, 2001, and never being delisted in the period considered.

Considering the in sample period 2001-2010, I applied the Markov switching model with time varying transition probabilities, and used as variable for the estimation of the returns and for the calculation of the probability of switching, some index based on the returns of the specific asset compared with the returns of the other shares, and other variables based on this one.

On the contrary of what most of the literature suggests, the two regimes of the model are driven by the volatility of the period, and so I found that every asset follows a regime for the low volatility period and another one for the high volatility period, rather than by the level of the returns, so a “bear” and a “bull” period.

I was able to perform the model in its complete formulation for most of the shares in the sample, and just in three of them I had to exclude a variable or substitute it with a constant.

In every asset, both the variances were significant, and for most of the assets each variable was significant at the 5% level for at least one of the two regimes. In particular, for some assets the model worked very well, with just a couple of parameters not significant, while in other assets just the variances are significant.

Then, for each stock, I performed a static forecast based on the parameters estimated by the model in the in sample period and on the values of the independent variables realized in the out of sample period. On these results, I built two trading strategies, that selected the best five assets on the basis of the one step ahead forecast of the returns itself and of the Sharpe ratio, and compared the results with a Buy and Hold strategy of the Nasdaq 100 itself.

The strategies performed worse than the index without accounting for the commissions, and once you include them in the calculation, both the strategies were losing money.

After that, I also tried to see how the strategies worked selecting just the best performing asset according to the forecasts (always without accounting for the commissions). While the Return alone strategy lost more than one third of the initial investment, the Sharpe ratio strategy gained almost as much as the Buy-and-Hold strategy (gap that, anyhow, would have been much bigger accounting for the commissions).

Based on these results, some further specification on the parameters or on the variables could maybe be helpful to improve the estimation. Furthermore, specifying some other criteria for the strategies, as cap, floor, or even other type of strategies (for example, VAR based strategies) might improve the performances of the investment on the market.

Appendix

Shares analised

Source of the data: [Yahoo finance]

Shares analyzed (Symbols)

Apple (AAPL)

Adobe(ADBE)

Applied Materials (AMAT)

Amgen (AMGN)

Amazon (AMZN)

Apollo Group (APOL)

Bed Bath & Beyond(BBBY)

Biogen(BIIB)

Check Point Software Technologies (CHKP)

CH Robinson Worldwide (CHRW)

Costco (COST)

Cisco (CSCO)

Cintas (CTAS)

Citrix (CTXS)

Dish Network (DISH)

Electronic Arts (EA)

EBay (EBAY)

Electronic Scripts (ESRX)

Expeditors (EXPD)

Fastenal (FAST)

Fiserv (FISV)

Flextronics (FLEX)

Gilead Science (GILD)

Garmin (GRMN)

Hologic (HOLX)

IAC Interactive (IAC)

Intel (INTC)

Intuit (INTU)

Juniper (JNPR)

Kla-Tencor (KLAC)

Linear Technology (LLTC)

Lam Research (LRCX)

Microchip Technology (MCHP)

Monster Beverage (MNST)

Microsoft (MSFT)

Nvidia (NVDA)

Oracle (ORCL)

Paychex (PAYX)

Paccar (PCAR)

Patterson (PDCO)

Qualcomm (QCOM)

Starbucks (SBUX)

Staples (SPLS)

Stericycle (SRCL)

Symantec (SYMC)

Teva Pharmaceutical Industries Limited (TEVA)

Verisign (VRSN)

Xilinx (XLNX)

Dentsplay (XRAY)

Yahoo (YHOO)

Appendix II

	AAPL			ADBE			AMAT			AMGN		
	value	SD	p-value	value	SD	p-value	value	SD	p-value	value	SD	p-value
State 0 Distrib	0.0010529	0.00014703	2.85E-12	0.0011538	0,000098962	0.00E+00	0.00096862	0.00017638	6.32E-08	0.00030998	0,000045869	3.87E-12
State 1 Distrib	0.0024645	0.00032165	9.44E-14	0.0055214	0.00062896	0.00E+00	0.0029625	0.00042561	1.06E-11	0.0017901	0.00021614	1.11E-15
Indep 1 State 0	0.34307	0.049708	1.55E-11	0.13453	0.043629	0.0021577	0.041615	0.072842	0.56804	0.12036	0.040913	0.0034121
Indep 1 State 1	0.30262	0.20868	0.14764	0.07201	0.13011	0.58021	0.40794	0.21947	0.063643	0.20396	0.079977	0.011057
Indep 2 State 0	-0.051021	0.0077388	1,09E-10	0.0055755	0.0047274	0.23879	-0.022241	0.0067592	0.00107	0.017915	0.0051436	0.0005389
Indep 2 State 1	0.051987	0.0090582	1.64E-08	-0.03068	0.013698	0.025539	0.063025	0.01274	1.03E-06	-0.011685	0.0079176	0.14062
Indep 3 State 0	0.081913	0.42693	0.84792	-0.7486	0.39228	0.056914	0.29097	0.32535	0.37158	-0.21606	0.10201	0.034666
Indep 3 State 1	-4.4057	1.08160	5.38E-05	0.4359	1.3976	0.75525	-3.6273	1.2248	0.0032052	-1.6904	0.75635	0.025856
Indep 4 State 0	-0.15136	0.093991	0.10794	0.1193	0.10334	0.24883	-0.095261	0.09824	0.33267	-0.1037	0.067078	0.12275
Indep 4 State 1	0.41788	0.1751	0.017378	0.10413	0.13712	0.44796	0.67625	0.26024	0.0096348	0.41208	0.11924	0.00059451
Px 1 State 0	16.731	4.3762	0.00014814	470.8	178.4	0.0085703	8.5726	5.7278	0.1351	-8.3409	6.16430	0.17663
Px 1 State 1	16.2990	6.1226	0.0080129	-14.413	20.394	0.48006	23.102	5.6336	0.0187	-3.2442	6.7503	0.63101
Px 2 State 0	-7.7876	29.716	0.79338	-194.05	101.19	0.055712	13.29	29.505	0.43399	75.321	23.473	0.0014176
Px 2 State 1	-180.6	46.87	0.00013166	-835.46	500.31	0.095565	-21.832	29.645	0.46181	10.819	31.516	0.73154

	AMZN			APOL			BBBY			BIIB		
	value	SD	p-value	value	SD	p-value	value	SD	p-value	value	SD	p-value
State 0 Distrib	0.0018292	0.00022017	8.88E-16	0.00081846	0,000072887	0.00E+00	0.00068989	0,000070781	0.00E+00	0.00067497	0,000057547	0.00E+00
State 1 Distrib	0.0053782	0.00088668	2.58E-09	0.0032758	0.00044365	6.40E-13	0.0044371	0.00079452	3.83E-08	0.0057853	0.00060964	0.00E+00
Indep 1 State 0	0.38611	0.069678	4.84E-08	0.25273	0.032846	7.53E-14	0.11438	0.034343	0.00093065	0.15528	0.034982	1.11E-05
Indep 1 State 1	0.82755	0.28018	0.0032867	-0.6072	0.3173	0.056227	0.52011	0.22434	0.020826	0.14554	0.11471	0.20512
Indep 2 State 0	-0.064877	0.0075085	0.00E+00	-0.034538	0.0051547	5.57E-11	-0.018833	0.0051917	0.00031526	-0.0049548	0.0039195	0.20676
Indep 2 State 1	0.072655	0.012053	3.21E-10	0.047745	0.010998	1.71E-05	0.078645	0.021438	0.0002699	-0.011045	0.012105	0.36197
Indep 3 State 0	-2.3934	1.5581	0.12514	-0.32681	0.27559	0.23624	-0.26699	0.15443	0.084444	-0.45078	0.24863	0.070419
Indep 3 State 1	-5.4069	1.6388	0.0010375	-1.0988	1.68960	0.51577	-3.9698	1.71210	0.020809	-0.50866	0.90914	0.57607
Indep 4 State 0	0.18253	0.11122	0.10139	-0.10988	0.059745	0.066476	-0.037338	0.060925	0.54025	0.093304	0.079321	0.24004
Indep 4 State 1	0.20452	0.22887	0.37194	0.015072	0.13351	0.91016	0.08417	0.18359	0.6468	0.17453	0.14004	0.21324
Px 1 State 0	20.839	7.85850	0.0082595	17.508	3,71020	3.0733E-06	26.003	6,26360	0,000038795	117.47	31.559	0.00021957
Px 1 State 1	47.835	22.479	0.033822	84.866	43.569	0.051983	4.4066	4.7412	0.35311	-239.35	77.053	0.0020007
Px 2 State 0	-120.71	153.63	0.43239	56.044	27.458	0.041759	46.681	20.454	0.022889	-100.49	53.183	0.059395
Px 2 State 1	-488.61	213.61	0.022589	-789.98	366.49	0.031595	-94.202	45.403	0.03851	270.16	112.14	0.016347

	CHKP			CHRW			COST			CSCO		
	value	SD	p-value	value	SD	p-value	value	SD	p-value	value	SD	p-value
State 0 Distrib	0.00072978	0,000062189	0.00E+00	0.00030983	0,000078948	9.90E-05	0.00032206	0,0000292	0.00E+00	0.00074774	0,000062251	0.00E+00
State 1 Distrib	0.0055913	0.00064607	0.00E+00	0.0012758	0.00012748	0.00E+00	0.0013975	0.00012466	0.00E+00	0.0042817	0.00045673	0.00E+00
Indep 1 State 0	0.12827	0.040057	0.0014498	0.16978	0.091385	0.063774	0.13645	0.028141	1.66E-06	0.075255	0.038288	0.049904
Indep 1 State 1	0.20589	0.13404	0.12516	0.24152	0.050291	2.0686E-06	0.074802	0.059773	0.21136	0.064353	0.12173	0.59728
Indep 2 State 0	-0.0044725	0.0043814	0.30784	0.020681	0.0049095	2.99E-05	0.0023436	0.0037867	0.53627	0.0014355	0.0043439	0.74118
Indep 2 State 1	-0.0088439	0.011711	0.4505	-0.026502	0.0069313	0.00014802	-0.003151	0.0057008	0.58069	0.013201	0.012419	0.28831
Indep 3 State 0	-0.40378	0.33161	0.22392	-0.18641	0.2759	0.49957	-0.35383	0.14547	0.015348	-0.26292	0.19843	0.18576
Indep 3 State 1	-2.7087	1.71890	0.11568	-1.0147	0.39686	0.010858	-0.63522	0.57308	0.2682	-1.8209	1.11340	0.10258
Indep 4 State 0	0.18601	0.075551	0.014145	0.11902	0.11369	0.29566	0.064708	0.066757	0.33285	0.23785	0.081822	0.0038109
Indep 4 State 1	0.15279	0.14857	0.30424	0.10945	0.079667	0.17012	0.013122	0.059356	0.82513	0.202	0.1252	0.10729
Px 1 State 0	-6.9812	12.416	0.57418	-15.463	14.609	0.29034	-62.487	78.468	0.42621	324.27	134.54	0.016302
Px 1 State 1	-33.236	27.233	0.22287	-19.101	7.5671	0.011901	-59.591	34.272	0.082682	-115.71	52.611	0.028312
Px 2 State 0	2079.1	888.06	0.01961	107.12	67.282	0.11201	6252.4	6797.5	0.35811	-133.46	84.054	0.11296
Px 2 State 1	-717.64	308.06	0.020222	2.9206	26.189	0.91125	-389.93	290.85	0.18064	-114.48	125.16	0.36079

	CTAS			CTXS			DISH			EA		
	value	SD	p-value	value	SD	p-value	value	SD	p-value	value	SD	p-value
State 0 Distrib	0.00050922	0,000047164	0.00E+00	0.00057339	0,000089943	4.13E-10	0.0040644	0.00046276	0.00E+00	0.0011003	0,000088313	0.00E+00
State 1 Distrib	0.0029365	0.0005103	1.51E-08	0.0040835	0.00042598	0.00E+00	0.00071286	0,000075714	0.00E+00	0.0022163	0.00047264	3.54E-06
Indep 1 State 0	0.06204	0.032116	0.05395	0.02471	0.078088	0.7518	0.39237	0.14812	0.0083256	0.11311	0.034759	0.001214
Indep 1 State 1	0.16615	0.14235	0.2437	0.51214	0.10215	7.4145E-07	0.22282	0.065181	0.00068056	-0.33871	0.37187	0.36282
Indep 2 State 0	0.002492	0.0048625	0.60852	0.031338	0.0060419	3.10E-07	-0.026625	0.010378	0.010585	-0.0078519	0.0051889	0.13085
Indep 2 State 1	-0.017848	0.018828	0.34361	-0.049876	0.012005	3.83E-05	-0.0094198	0.0042163	0.025911	0.04288	0.015427	0.0056479
Indep 3 State 0	-0.13882	0.09093	0.12746	1.0308	0.44999	0.022387	0.6451	1.2344	0.60148	-0.20198	0.224	0.36764
Indep 3 State 1	-0.65239	0.75961	0.39083	-4.1591	1.1314	0.00026213	0.013685	0.26936	0.9595	-2.5596	1.32140	0.053304
Indep 4 State 0	0.09388	0.057078	0.10064	-0.04014	0.12055	0.73929	0.17064	0.11892	0.15193	-0.12515	0.058648	0.033323
Indep 4 State 1	0.22781	0.12628	0.071837	0.2731	0.12884	0.034518	0.14033	0.11333	0.21621	1.5786	0.34436	5.7508E-06
Px 1 State 0	16.9040	7.8712	0.03222	-4.02580	4.76190	0.39828	10.69500	13.24500	0.41978	22.81200	5.18700	0,000013342
Px 1 State 1	-7.6474	8.439	0.36527	-12.108	4.1531	0.0037106	-78.263	36.0010	0.030178	26.777	18.1090	0.13986
Px 2 State 0	103.88	43.668	0.017736	76.701	29.071	0.0085876	815.32	370.17	0.028079	17.484	27.017	0.51783
Px 2 State 1	-62.461	31.276	0.046353	37.39	30.217	0.21652	-182.24	163.56	0.26571	37.9740	60.653	0.53155

	EBAY			ESRX			EXPD			FAST		
	value	**SD**	**p-value**	**value**	**SD**	**p-value**	**value**	**SD**	**p-value**	**value**	**SD**	**p-value**
State 0 Distrib	0.0019319	0.00017527	0.00E+00	0.00096236	0,000081862	0.00E+00	0.00074715	0,000073594	0.00E+00	0.000752	0,000086872	0.00E+00
State 1 Distrib	0.0012207	0.00015882	7.95E-14	0.0043213	0.00097616	1.17E-05	0.0030785	0.00041952	8.71E-13	0.0033348	0.0005124	1.84E-10
Indep 1 State 0	0.31523	0.099872	0.0016929	0.13431	0.036634	0.00027223	0.1669	0.038606	1.85E-05	0.24437	0.036588	6.37E-11
Indep 1 State 1	0.2908	0.064695	8.6394E-06	0.46762	0.39283	0.23446	-0.062433	0.10927	0.56801	-0.96403	0.35854	0.0074087
Indep 2 State 0	0.025278	0.0068661	0.00025678	-0.010935	0.0042763	0.010843	-0.004944	0.0043509	0.25637	-0.046359	0.0065865	6.38E-12
Indep 2 State 1	-0.05111	0.0080129	4.05E-10	0.049673	0.025865	0.055363	0.029164	0.011695	0.012961	0.049559	0.013394	0.00023921
Indep 3 State 0	-1.7927	0.45396	0.000089623	0.3316	0.38548	0.39008	-0.26571	0.21778	0.22299	-0.36861	0.24248	0.12909
Indep 3 State 1	-1.2051	1.40760	0.39236	-13.1850	6.83220	0.054185	-1.5654	1.64440	0.34159	1.5019	1.12700	0.18323
Indep 4 State 0	-0.16063	0.093229	0.085517	-0.050318	0.058406	0.38936	0.060155	0.069872	0.38968	-0.081968	0.059367	0.16798
Indep 4 State 1	0.56819	0.10335	0.00061189	0.93555	0.37739	0.0135	0.10147	0.11789	0.38981	0.46821	0.21043	0.026523
Px 1 State 0	-72.88400	42.80800	0.089259	13.72600	4.47960	0.0023003	15.55300	13.62500	0.2542	16.47600	4.86570	0.00076409
Px 1 State 1	-102.98	53.2340	0.053603	18.972	19.2640	0.32518	-53.689	26.2250	0.041153	46.706	26.9920	0.084184
Px 2 State 0	612.2	303.49	0.044201	142.4	61.919	0.021869	491.02	385.38	0.20321	23,158	25.323	0.36088
Px 2 State 1	1880.4	995.89	0.059572	236.99	234.44	0.31256	-43,8860	120.5	0.71586	-202.82	150.97	0.17974

	FISV			FLEX			GILD			GRMN		
	value	**SD**	**p-value**	**value**	**SD**	**p-value**	**value**	**SD**	**p-value**	**value**	**SD**	**p-value**
State 0 Distrib	0.00096363	0,000076907	0.00E+00	0.010506	0.0012479	4.44E-16	0.00083841	0,000076909	0.00E+00	0.00078778	0.00011324	1.09E-11
State 1 Distrib	0.00024337	0,000087018	0.005358	0.0016186	0.00012087	0.00E+00	0.0028623	0.00033217	0.00E+00	0.0031757	0.00031182	0.00E+00
Indep 1 State 0	0.10366	0.03726	0.0056027	0.14686	0.19968	0.4624	0.096788	0.036068	0.0075239	0.39724	0.11744	0.00077376
Indep 1 State 1	0.20022	0.099554	0.044834	0.15617	0.052518	0.0030836	0.1965	0.093707	0.036494	0.40349	0.072178	3.717E-09
Indep 2 State 0	-0.0056388	0.0048336	0.24393	-0.0064224	0.017987	0.7212	-0.0014662	0.0036544	0.68844	0.040299	0.0040319	0.00E+00
Indep 2 State 1	0.10453	0.014113	5.46E-13	-0.001735	0.0051304	0.73537	-0.008388	0.0097396	0.38952	-0.052947	0.0094867	3.90E-08
Indep 3 State 0	-0.4363	0.14278	0.0023643	-3.2479	1.3422	0.015876	-0.42808	0.33481	0.20163	-0.81893	0.38973	0.036112
Indep 3 State 1	0.10888	0.17487	0.53381	-0.68308	0.51464	0.18501	-0.28898	0.71346	0.68562	-3.9630	1.22560	0.0013024
Indep 4 State 0	0.26917	0.063819	0,000029246	0.38503	0.2154	0.074459	0.1619	0.066543	0.015321	0.078553	0.081654	0.3365
Indep 4 State 1	-0.58722	0.090387	1.9702E-10	0.14653	0.11061	0.18585	-0.075648	0.11245	0.50143	0.25536	0.1264	0.043886
Px 1 State 0	17.38200	5.83870	0.0030507	177.9	114.77	0.12178	2474.8	155090	0.98727	-46.52	35.49800	0.19062
Px 1 State 1	14.089	9.0742	0.12114	-179.85	104.59	0.086137	-256.68	103.55	0.013511	-18.824	8,1133	0.020733
Px 2 State 0	10.575	11.305	0.35003	296.11	329.8	0.36969	NaN	NaN	NaN	249.46	158.17	0.11537
Px 2 State 1	-32.74	17.504	0.061994	-342.39	281	0.2236	NaN	NaN	NaN	201.98	70.723	0.0044669

	HOLX			IAC			INTC			INTU		
	value	SD	p-value	value	SD	p-value	value	SD	p-value	value	SD	p-value
State 0 Distrib	0.0009232	0.00019327	2.34E-06	0.00086692	0.00010177	2.22E-16	0.00067379	0,000067338	0.00E+00	0.0007799	0,000079252	0.00E+00
State 1 Distrib	0.0056716	0.00086506	1.37E-10	0.0048498	0.00090747	1.38E-07	0.0027816	0.00027287	0.00E+00	0.0047171	0.0010802	1.53E-05
Indep 1 State 0	0.25844	0.058173	1.09E-05	0.20868	0.042751	1.38E-06	0.10256	0.037431	0.0063638	0.10738	0.028218	0.00015905
Indep 1 State 1	0.27066	0.12419	0.029761	-0.06809	0.28118	0.79955	-0.054375	0.084517	0.52028	0.086739	0.21119	0.68146
Indep 2 State 0	-0.022859	0.012385	0.065523	-0.0184	0.0059871	0.0020933	-0.0025436	0.0044916	0.57144	-0.004595	0.0049806	0.35666
Indep 2 State 1	0.018676	0.01173	0.11195	0.036972	0.015905	0.019714	0.023563	0.011897	0.048173	0.030054	0.028583	0.29355
Indep 3 State 0	-1.4219	0.60589	0.019323	-1.0699	0.48604	0.027945	-0.26664	0.16413	0.10489	0.0079345	0.087048	0.92741
Indep 3 State 1	-2.0985	1.37370	0.12723	-2.6729	1.60270	0.10072	0.045063	0.2988	0.88018	-0.95593	0.95977	0.31973
Indep 4 State 0	0.071225	0.1564	0.64901	0.06454	0.066287	0.33025	0.097055	0.098998	0.32737	-0.044888	0.06877	0.51423
Indep 4 State 1	0.38106	0.18606	0.041074	0.16521	0.21092	0.42364	0.27712	0.091881	0.0026902	0.031612	0.22996	0.89071
Px 1 State 0	21.06700	6.28850	0.00086857	42.49500	8.73210	1.521E-07	87.84600	44.63100	0.049581	44.94600	16.20900	0.005761
Px 1 State 1	4.4349	6.3961	0.48839	22.855	8.7057	0.0089192	-91.883	36.1810	0.011398	-8.7588	8.7093	0.31505
Px 2 State 0	25.17	37.74	0.50511	0.41503	20.682	0.984	38.913	51.369	0.4491	-21.49	24.017	0.37133
Px 2 State 1	-405.88	277.33	0.14395	-986.2	475.34	0.038517	-6.8360	45.158	0.87974	24.4030	36.021	0.49843

	JNPR			KLAC			LLTC			LRCX		
	value	SD	p-value	value	SD	p-value	value	SD	p-value	value	SD	p-value
State 0 Distrib	0.0018508	0.00014579	0.00E+00	0.00058068	0.00010568	0.00E+00	0.00084311	0,000088971	0.00E+00	0.001923	0.00018862	0.00E+00
State 1 Distrib	0.019247	0.0025226	1.18E-13	0.0027412	0.00026175	2.02E-11	0.0035012	0.00065632	1.45E-07	0.0035011	0.00037787	0.00E+00
Indep 1 State 0	0.17071	0.054681	0.0019004	0.16318	0.090001	0.070412	0.0056965	0.048408	0.90637	0.34498	0.061025	2.64E-08
Indep 1 State 1	0.21383	0.33046	0.51787	0.19643	0.069909	0.0051488	0.38639	0.19454	0.047555	0.13119	0.23905	0.58339
Indep 2 State 0	-0.0050044	0.0049401	0.31154	0.039881	0.0059603	5.90E-11	-0.017647	0.0065586	0.0073661	-0.069896	0.0073338	0.00E+00
Indep 2 State 1	0.0015269	0.027174	0.95521	-0.027475	0.0091821	0.0029047	0.07633	0.019679	0.00011887	0.061916	0.0083647	5.65E-13
Indep 3 State 0	-0.62251	0.62979	0.32341	-0.9889	0.50524	0.050865	0.081808	0.13043	0.53081	-1.8595	1.3237	0.1607
Indep 3 State 1	-4.3683	4.66910	0.34994	-2.5834	0.86197	0.0028593	-2.2114	1.37280	0.10784	-2.6410	1.14920	0.021962
Indep 4 State 0	0.085482	0.10835	0.4305	-0.34794	0.12973	0.0075572	-0.039074	0.084948	0.64573	0.2462	0.1032	0.017419
Indep 4 State 1	0.094107	0.30877	0.76066	0.47402	0.10348	5.842E-07	0.80111	0.22096	0.00031746	0.35121	0.15866	0.027307
Px 1 State 0	-51.74100	52.67700	0.32646	-21.62700	14.49500	0.1363	7.06790	4.61500	0.12627	83.65400	26.17700	0.0014821
Px 1 State 1	9.446	10.3350	0.36118	-13.406	5.3833	0.013082	9.8469	4.4619	0.02777	60.575	29.46	0.04028
Px 2 State 0	6309.5	4845	0.19342	290.04	137.17	0.034969	70.208	32.826	0.032931	-1221.5	417.65	0.0036028
Px 2 State 1	-1982.3	777.5	0.01108	-49.1860	31.386	0.11771	-14.7230	76.362	0.84719	-401.62	205.26	0.050937

	MCHP			MNST			MSFT			NVDA		
	value	**SD**	**p-value**	**value**	**SD**	**p-value**	**value**	**SD**	**p-value**	**value**	**SD**	**p-value**
State 0 Distrib	0.00068115	0,000061811	0.00E+00	0.0015861	0.00016818	0.00E+00	0.00036119	0,000032512	0.00E+00	0.0024137	0.0002432	0.00E+00
State 1 Distrib	0.003401	0.00032247	0.00E+00	0.0063989	0.0011153	1.66E-08	0.0018574	0.00020119	0.00E+00	0.0063087	0.00073677	2.22E-16
Indep 1 State 0	0.16555	0.041865	8.77E-05	0.12252	0.049048	0.012807	0.074204	0.029237	0.011449	0.54508	0.065126	6.66E-16
Indep 1 State 1	0.036081	0.078083	0.64422	1,3321	0.34346	0.00011899	0.012204	0.063783	0.84834	-0.83582	0.57377	0.14582
Indep 2 State 0	-0.00012958	0.012394	0.99166	-0.021734	0.005897	0.00025278	0.0060165	0.0040959	0.14247	-0.087452	0.0084521	0.00E+00
Indep 2 State 1	0.022708	0.0087434	0.0096739	0.10348	0.023782	1.64E-01	-0.014528	0.0094619	0.12531	0.11185	0.013046	2.22E-16
Indep 3 State 0	-0.25143	0.15049	0.095379	1.3213	0.62968	0.03637	-0.090913	0.12839	0.47922	-0.504	0.92712	0.58695
Indep 3 State 1	-1.3329	0.6892	0.053684	-19.9820	6.83790	0.0036315	-0.082721	0.17037	0.62751	-7.5177	2.37	0.0016056
Indep 4 State 0	-0.068446	0.088126	0.43771	0.21693	0.082992	0.0092206	0.16258	0.066136	0.014296	0.015399	0.11136	0.89007
Indep 4 State 1	-0.068445	0.12439	0.5824	0.39054	0.36081	0.27959	0.11835	0.07624	0.12121	0.75517	0.21527	0.0004919
Px 1 State 0	263.46	157.29	0.094567	21.78	4.51540	1.8713E-06	-6.39670	11.43600	0.57618	21.90900	4.87860	8.7987E-06
Px 1 State 1	-133.3	61.0460	0.029452	5.0835	3.8804	0.19078	-115.48	55.5440	0.038119	94.441	54.1720	0.081881
Px 2 State 0	-92.006	75.481	0.22344	23.897	49.564	0.62991	2.29750	0.62845	0.00028321	15.094	54.484	0.78186
Px 2 State 1	-75.52	101.23	0.45599	30.1110	94.265	0.74954	0.59387	0.75113	0.42953	-1206.9	624.04	0.053673

	ORCL			PAYX			PCAR			PDCO		
	value	**SD**	**p-value**	**value**	**SD**	**p-value**	**value**	**SD**	**p-value**	**value**	**SD**	**p-value**
State 0 Distrib	0.00053973	0,000047674	0.00E+00	0.00058084	0,000004704	0.00E+00	0.00034808	0,000058089	3.94E-09	0.0005347	0,000048037	0.00E+00
State 1 Distrib	0.0011028	0.0002121	2.91E-07	0.0021022	0.00028416	5.80E-14	0.0022142	0.00022623	0.00E+00	0.0019058	0.00033058	1.42E-08
Indep 1 State 0	0.076879	0.03591	0.032761	0.11511	0.032985	0.00052581	0.22727	0.045984	1.05E-06	0.16753	0.028673	9.21E-09
Indep 1 State 1	0.62651	0.18609	0.00081868	0.11863	0.097386	0.22375	0.16176	0.075893	0.033541	-0.42706	0.2488	0.086683
Indep 2 State 0	-0.0080164	0.0036748	0.029613	0.0063851	0.0050015	0.20232	0.036909	0.0054157	2.69E-12	-0.015502	0.0042642	0.00030613
Indep 2 State 1	0.054694	0.011224	1.4755E-06	-0.0033313	0.012523	0.79034	-0.025054	0.0083822	0.002936	0.033474	0.010716	0.0018879
Indep 3 State 0	-0.98716	0.26123	0.00017638	-0.22326	0.14101	0.11398	-0.41835	0.19703	0.034216	-0.27804	0.21942	0.20568
Indep 3 State 1	-7.3836	2.08250	0.00042811	-1.4589	0.53079	0.0061994	-0.66213	0.67646	0.32814	-1.5879	0.69203	0.022169
Indep 4 State 0	0.10623	0.049178	0.03124	0.15197	0.067242	0.024243	-0.12291	0.073711	0.096031	0.056166	0.053207	0.29166
Indep 4 State 1	-0.18876	0.22515	0.40223	-0.077278	0.094869	0.4157	0.27105	0.10011	0.0070116	0.33932	0.16857	0.044658
Px 1 State 0	31.45200	13.21800	0.017702	61.03100	26.43400	0.021357	-6.10010	6.67520	0.36123	22.25	4.82730	5.1263E-06
Px 1 State 1	12.494	5.7279	0.029622	-108.08	54.0000	0.045879	-11.073	6.2616	0.077595	26.232	12.9670	0.043611
Px 2 State 0	128.42	56.567	0.023611	81.508	42.672	0.056686	115.95	38.287	0.0025836	13.977	21.479	0.51552
Px 2 State 1	-213.07	118.94	0.073813	28.0830	54.583	0.60712	-14.4770	34.62	0.676	-155.37	97.107	0.11022

	QCOM			SBUX			SPLS			SRCL		
	value	SD	p-value	value	SD	p-value	value	SD	p-value	value	SD	p-value
State 0 Distrib	0.00093918	0,000069343	0.00E+00	0.00080376	0,000007708	0.00E+00	0.00087781	0,000067022	0.00E+00	0.0002454	0,000049099	8.00E-07
State 1 Distrib	0.0058138	0.00077936	3.81E-13	0.0031747	0.00050516	7.11E-10	0.0063673	0.0012956	1.21E-06	0.0012603	0.00011791	0.00E+00
Indep 1 State 0	0.15513	0.033513	4.68E-06	0.14901	0.034503	1.89E-05	0.047049	0.036487	0.19783	0.19688	0.066875	0.0033897
Indep 1 State 1	0.097922	0.17457	0.57508	0.18883	0.11827	0.11099	0.048535	0.21326	0.82006	0.15972	0.049696	0.0013932
Indep 2 State 0	-0.00060604	0.0040831	0.88206	-0.0025603	0.0043897	0.55999	0.00448	0.0036781	0.22379	0.032727	0.0040025	2.44E-15
Indep 2 State 1	-0.0036677	0.016071	0.81957	0.014686	0.013402	0.27369	0.014375	0.022543	0.52396	-0.015762	0.0058877	0.007669
Indep 3 State 0	-0.48421	0.17355	0.005469	-0.37549	0.20068	0.061906	-0.021871	0.24066	0.92762	-0.47682	0.27139	0.07954
Indep 3 State 1	-3.6531	2.82920	0.19722	-1.5781	1.35670	0.24529	-2.8192	3.94100	0.47473	-0.82693	0.60172	0.16997
Indep 4 State 0	0.073103	0.072691	0.31505	-0.041919	0.071626	0.55864	0.021708	0.06205	0.72659	-0.071949	0.095718	0.4526
Indep 4 State 1	0.12108	0.15897	0.44661	0.22888	0.11929	0.055573	0.07526	0.21553	0.72709	-0.070474	0.07789	0.36601
Px 1 State 0	35.13	43.81	0.42299	128.8	86.26200	0.13601	-30.45700	24.32300	0.21107	-6.61540	4.90640	0.17816
Px 1 State 1	-179.1	87.6080	0.041435	-99.894	47.2980	0.035176	-5.1026	8.3026	0.53911	-32.069	9.5987	0.0008966
Px 2 State 0	869.91	657.2	0.18622	183.99	232.44	0.42899	3635.4	2090.9	0.0827	68.904	27.726	0.01327
Px 2 State 1	-42.0970	130.52	0.74719	56.9420	119.4	0.63363	-391.29	230.15	0.089714	158.26	85.369	0.064355

	SYMC			TEVA			VRSN			XLNX		
	value	SD	p-value	value	SD	p-value	value	SD	p-value	value	SD	p-value
State 0 Distrib	0.0010275	0,000094151	0.00E+00	0.00043402	0,000004987	0.00E+00	0.0013191	0.00011556	0.00E+00	0.00093615	0.00015752	5.22E-09
State 1 Distrib	0.0056817	0.0011121	4.61E-07	0.001043	0,000090421	0.00E+00	0.012935	0.0015961	4.00E-15	0.0028847	0.00029816	0.00E+00
Indep 1 State 0	0.13216	0.040524	0.0011835	0.049795	0.067837	0.46327	0.1687	0.042437	8.05E-05	0.20415	0.087048	0.019397
Indep 1 State 1	0.74907	0.34597	0.030847	0.12849	0.045634	0.005059	-0.035333	0.21257	0.86805	0.29218	0.087173	0.0008632
Indep 2 State 0	-0.015501	0.0052829	0.0034954	0.027954	0.0037282	2.93E-14	-0.0013883	0.0045538	0.76059	0.046552	0.0052343	0,00E+00
Indep 2 State 1	0.077132	0.030673	0.012224	-0.020772	0.0052575	8.90E-05	0.0048576	0.018905	0.79732	-0.071027	0.011308	7.26E-06
Indep 3 State 0	0.13818	0.27105	0.61042	-0.21587	0.27852	0.43866	-0.41106	0.31997	0.19949	-0.89056	0.26766	0.00094117
Indep 3 State 1	-13.2610	6.25590	0.034518	-0.2429	0.75399	0.74747	-2.3834	3.55930	0.50341	-1.8989	0.98436	0.054277
Indep 4 State 0	0.07945	0.073815	0.28229	-0.060968	0.065411	0.35174	0.007719	0.066587	0.90776	-0.14225	0.11127	0.20167
Indep 4 State 1	0.33183	0.24289	0.17249	0.10206	0.078489	0.1941	0.092956	0.22247	0.67625	0.53636	0.14474	0.00023398
Px 1 State 0	15.17500	6.19580	0.014656	-199.66	132.95	0.1338	258.46	117.71	0.028572	-25.93700	9.12190	0.0046449
Px 1 State 1	2.0658	6.4945	0.75056	-95.036	28.2460	0.00082476	-296.63	124.7	0.017739	-15.939	7.8530	0.042913
Px 2 State 0	193.91	78.496	0.013828	1011.8	663.01	0.12763	46.217	137.83	0.73752	128.48	40.252	0.0015011
Px 2 State 1	-70.7920	114.16	0.53548	1264.7	362.34	0.00052453	33.569	127.87	0.79302	195.76	111.5	0.079752

	XRAY			YHOO		
	value	**SD**	**p-value**	**value**	**SD**	**p-value**
State 0 Distrib	0.00044054	0.000037174	0.00E+00	0.0012472	0.00016431	1.55E-13
State 1 Distrib	0.001777	0.00034195	2.95E-07	0.0074817	0.0011758	4.44E-10
Indep 1 State 0	0.12291	0.028038	1.42E-05	0.092655	0.051232	0.071116
Indep 1 State 1	0.022353	0.14496	0.87751	0.14994	0.15846	0.34449
Indep 2 State 0	-0.017584	0.0036408	1.82E-06	0.006741	0.0053253	0.20615
Indep 2 State 1	0.026044	0.015129	0.085775	0.0033091	0.015634	0.83246
Indep 3 State 0	-0.17667	0.10995	0.10871	-0.070879	0.41296	0.86379
Indep 3 State 1	-0.78711	1.2943	0.54337	-2.4046	2.18830	0.27235
Indep 4 State 0	0.13803	0.047608	0.0039022	-0.038807	0.11045	0.72548
Indep 4 State 1	0.10629	0.12813	0.40718	0.18708	0.17527	0.28631
Px 1 State 0	16.8260	6.68690	0.012171	3.05940	9.70370	0.75268
Px 1 State 1	5.1349	6.2707	0.41325	-34.853	23.0070	0.13043
Px 2 State 0	139.15	56.492	0.014102	1144.2	466.53	0.014517
Px 2 State 1	-116.34	75.925	0.12606	-337.77	192.82	0.080433

	Table 1: Total Value		
	Nasdaq Value	Mean Value	Sharpe Value
Starting Value	1	1	1
07/01/11	1,0168	1,0462	1,0462
14/01/11	1,0375	1,1039	1,1054
21/01/11	1,0126	1,0829	1,081
28/01/11	1,0136	1,0699	1,0592
04/02/11	1,0434	1,1074	1,0879
11/02/11	1,0615	1,1159	1,1092
18/02/11	1,0674	1,1311	1,1106
25/02/11	1,0466	1,1395	1,077
04/03/11	1,0527	1,1383	1,0641
11/03/11	1,0252	1,0966	1,0311
18/03/11	0,98977	1,0547	0,98886
25/03/11	1,0314	1,0968	1,0362
01/04/11	1,0431	1,1143	1,0528
08/04/11	1,0334	1,0895	1,0208
15/04/11	1,0273	1,0991	1,0298
21/04/11	1,0579	1,1327	1,059
29/04/11	1,0697	1,1576	1,0682
06/05/11	1,0604	1,1356	1,0479
13/05/11	1,0586	1,1435	1,0503
20/05/11	1,0462	1,1419	1,0511
27/05/11	1,0394	1,1422	1,0604
03/06/11	1,0197	1,0979	1,0193
10/06/11	0,98751	1,0464	0,97149
17/06/11	0,97492	1,0462	0,96625
24/06/11	0,98557	1,0764	0,98716
01/07/11	1,0477	1,1349	1,0454
08/07/11	1,0673	1,1376	1,0521
15/07/11	1,0452	1,1139	1,0266
22/07/11	1,077	1,158	1,0668
29/07/11	1,0471	1,1121	1,0403
05/08/11	0,96963	1,0006	0,93775
12/08/11	0,96417	1,049	0,9634
19/08/11	0,89843	0,98513	0,90468
26/08/11	0,95138	0,98787	0,91751
02/09/11	0,95396	0,97401	0,91205
09/09/11	0,95212	0,99931	0,95253
16/09/11	1,0128	1,0786	0,98557
23/09/11	0,96827	1,0239	0,93751

Appendix III

	Nasdaq Value	Mean Value	Sharpe Value
30/09/11	0,93811	0,9676	0,89179
07/10/11	0,96559	0,98986	0,90947
14/10/11	1,037	1,0146	0,92981
21/10/11	1,0212	0,99259	0,93298
28/10/11	1,0494	1,0172	0,93799
04/11/11	1,0295	1,0104	0,92445
11/11/11	1,0293	1,0175	0,93341
18/11/11	0,9838	0,96445	0,88129
25/11/11	0,93775	0,92423	0,83682
02/12/11	1,0014	0,97605	0,88374
09/12/11	1,0087	0,9669	0,88039
16/12/11	0,97301	0,93557	0,83377
23/12/11	0,99425	0,95832	0,84444
30/12/11	0,99001	0,9568	0,84635
06/01/12	1,0235	0,96161	0,87111
13/01/12	1,0303	0,96143	0,87421
20/01/12	1,0582	0,994	0,88157
27/01/12	1,0689	1,0089	0,89628
03/02/12	1,0978	1,037	0,92485
10/02/12	1,1056	1,0532	0,93758
17/02/12	1,1215	1,0612	0,94442
24/02/12	1,1302	1,0769	0,95834
02/03/12	1,1463	1,0902	0,97763
09/03/12	1,1485	1,107	0,98426
16/03/12	1,1768	1,1252	1,001
23/03/12	1,1836	1,1217	0,99788
30/03/12	1,1952	1,1313	0,98392
05/04/12	1,1983	1,1226	0,97285
13/04/12	1,1704	1,1176	0,98017
20/04/12	1,1604	1,0877	0,95396
27/04/12	1,1884	1,0777	0,97267
04/05/12	1,1427	1,0653	0,95464
11/05/12	1,1332	1,0792	0,97072
18/05/12	1,072	1,0166	0,9144
25/05/12	1,0928	1,0511	0,95408
01/06/12	1,0629	1,0047	0,91196
08/06/12	1,1054	1,0469	0,95031
15/06/12	1,1106	1,0339	0,95362
22/06/12	1,1167	1,0471	0,9565
29/06/12	1,1297	1,0713	0,9777

	Nasdaq Value	Mean Value	Sharpe Value
06/07/12	1,1282	1,0298	0,95754
13/07/12	1,1164	1,029	0,95409
20/07/12	1,1306	1,0286	0,95374
27/07/12	1,143	1,0663	0,9773
03/08/12	1,1554	1,0785	0,98845
10/08/12	1,1755	1,1043	0,99524
17/08/12	1,2	1,1143	1,0082
24/08/12	1,1991	1,1192	1,0125
31/08/12	1,1966	1,1115	1,0022
07/09/12	1,2192	1,138	1,0271
14/09/12	1,2321	1,1523	1,04
21/09/12	1,2349	1,1214	1,0403
28/09/12	1,2076	1,0988	1,0199
05/10/12	1,2131	1,1124	1,0325
12/10/12	1,1728	1,0817	1,0257
19/10/12	1,1547	1,0045	0,94511
26/10/12	1,1493	0,98553	0,94706
02/11/12	1,1451	0,99952	0,96051
09/11/12	1,1136	0,98528	0,94433
16/11/12	1,0919	0,94826	0,92584
23/11/12	1,1364	0,96642	0,93227
30/11/12	1,1527	0,99215	0,95709
07/12/12	1,1365	0,96531	0,92397
14/12/12	1,1312	0,97367	0,9316
21/12/12	1,1468	0,97852	0,93242
28/12/12	1,1214	0,9539	0,91335
04/01/13	1,1711	0,98168	0,93676
11/01/13	1,1813	0,97128	0,90294
18/01/13	1,1792	0,98262	0,90686
25/01/13	1,1764	1,0258	0,94471
01/02/13	1,188	1,0285	0,95123
08/02/13	1,193	1,051	0,95816
15/02/13	1,1883	1,0357	0,93924
22/02/13	1,1765	1,0221	0,91881
01/03/13	1,181	1,0269	0,92856
08/03/13	1,2049	1,0601	0,95626
15/03/13	1,2029	1,0454	0,94814
22/03/13	1,2035	1,0534	0,96138
28/03/13	1,2112	1,0542	0,96725
05/04/13	1,1908	1,038	0,9468

	Nasdaq Value	Mean Value	Sharpe Value
12/04/13	1,2267	1,0834	1,0061
19/04/13	1,1936	1,047	0,98972
26/04/13	1,2191	1,0516	0,98585
03/05/13	1,263	1,0798	1,0145
10/05/13	1,2785	1,0806	1,0168
17/05/13	1,2989	1,0905	1,0224
24/05/13	1,2825	1,0702	0,98874
31/05/13	1,2786	1,0478	0,95771
07/06/13	1,2825	1,0578	0,97105
14/06/13	1,2621	1,0292	0,95041
21/06/13	1,2335	0,97832	0,90226
28/06/13	1,247	0,99832	0,9207
05/07/13	1,2698	0,99479	0,92076
12/07/13	1,3185	1,0149	0,93938
19/07/13	1,3038	1,0014	0,94838
26/07/13	1,3171	1,0026	0,96422
02/08/13	1,3457	1,0279	0,98285
09/08/13	1,3349	1,0102	0,97533
16/08/13	1,3157	0,99466	0,96566
23/08/13	1,3371	1,0016	0,96179
30/08/13	1,3153	0,9677	0,92751
06/09/13	1,3405	0,99328	0,94404
13/09/13	1,3596	1,0108	0,96791
20/09/13	1,3793	1,0111	0,9765
27/09/13	1,3817	1,0302	0,98313
04/10/13	1,3869	1,0486	1,0007
11/10/13	1,3832	1,0628	1,0034
18/10/13	1,4336	1,0974	1,0391
25/10/13	1,4464	1,109	1,049
01/11/13	1,4446	1,0888	1,022
08/11/13	1,4391	1,0759	1,0124
15/11/13	1,4627	1,0873	1,0178
22/11/13	1,4625	1,0812	1,0201
29/11/13	1,4903	1,0856	1,0224
06/12/13	1,4973	1,0912	1,0288
13/12/13	1,4768	1,0733	1,0072
20/12/13	1,5084	1,0942	1,0229
27/12/13	1,5266	1,0991	1,0265
03/01/14	1,5114	1,0958	1,0189
10/01/14	1,5226	1,1113	1,0306

	Nasdaq Value	Mean Value	Sharpe Value
17/01/14	1,5338	1,1325	1,0471
24/01/14	1,5123	1,1147	1,0306
31/01/14	1,504	1,0619	0,99921
07/02/14	1,521	1,0639	1,0023
14/02/14	1,5639	1,1183	1,0351
21/02/14	1,5633	1,1492	1,0637
28/02/14	1,5776	1,1587	1,0753
07/03/14	1,5807	1,1679	1,0807
14/03/14	1,5481	1,1441	1,0612
21/03/14	1,5588	1,1391	1,066
28/03/14	1,5236	1,1076	1,0584
04/04/14	1,5099	1,1069	1,0688
11/04/14	1,4699	1,0865	1,0457
17/04/14	1,5068	1,1006	1,0593
25/04/14	1,5062	1,0606	1,033
02/05/14	1,5293	1,0746	1,0581
09/05/14	1,5156	1,0927	1,068
16/05/14	1,5289	1,0823	1,0507
23/05/14	1,5669	1,0845	1,0529
30/05/14	1,592	1,1001	1,0673
06/06/14	1,6164	1,1092	1,079
13/06/14	1,6083	1,113	1,0778
20/06/14	1,6198	1,1314	1,1048
27/06/14	1,6375	1,1345	1,1141
03/07/14	1,6707	1,1548	1,1355
11/07/14	1,6628	1,1333	1,1335
18/07/14	1,6778	1,1232	1,1235
25/07/14	1,6885	1,1099	1,1058
01/08/14	1,6517	1,0997	1,0935
08/08/14	1,6553	1,0997	1,0955
15/08/14	1,6971	1,1774	1,1733
22/08/14	1,7246	1,1618	1,1647
29/08/14	1,7372	1,169	1,1822
05/09/14	1,7404	1,1648	1,1692
12/09/14	1,7315	1,1417	1,1461
19/09/14	1,7446	1,131	1,1421
26/09/14	1,7248	1,1148	1,1279
03/10/14	1,7135	1,126	1,1367
10/10/14	1,6456	1,0835	1,0996
17/10/14	1,6219	1,0831	1,1056

	Nasdaq Value	Mean Value	Sharpe Value
24/10/14	1,7154	1,1285	1,1492
31/10/14	1,7641	1,1582	1,1794
07/11/14	1,765	1,1942	1,2037
14/11/14	1,7922	1,1994	1,2135
21/11/14	1,8033	1,2089	1,2243
28/11/14	1,8396	1,2426	1,2618
05/12/14	1,8285	1,2286	1,2484
12/12/14	1,7802	1,2009	1,23
19/12/14	1,8149	1,2095	1,241
26/12/14	1,8285	1,2153	1,2468
31/12/14	1,7952	1,2012	1,2269

	Table 2:Weekly Returns		
	Nasdaq Returns	Mean Returns	Sharpe Returns
07/01/11	0.01685	0.046218	0.046218
14/01/11	0.020318	0.055154	0.056595
21/01/11	-0.024005	-0.019038	-0.02209
28/01/11	0.00096498	-0.012008	-0.020179
04/02/11	0.029377	0.035061	0.0271
11/02/11	0.017362	0.0076915	0.019556
18/02/11	0.0055831	0.013574	0.0013202
25/02/11	-0.019491	0.0074742	-0.0303
04/03/11	0.0058093	-0.0010776	-0.011961
11/03/11	-0.026057	-0.036645	-0.030998
18/03/11	-0.034598	-0.038218	-0.040989
25/03/11	0.042008	0.039925	0.047846
01/04/11	0.011401	0.015933	0.016074
08/04/11	-0.0093223	-0.022263	-0.030439
15/04/11	-0.0058763	0.0088152	0.0088224
21/04/11	0.029766	0.030561	0.028343
29/04/11	0.011202	0.022015	0.0086921
06/05/11	-0.0087316	-0.018962	-0.018962
13/05/11	-0.0016546	0.0069016	0.0022806
20/05/11	-0.011757	-0.0013681	0.00079095
27/05/11	-0.006545	0.00022631	0.00877
03/06/11	-0.018919	-0.038746	-0.038746
10/06/11	-0.031562	-0.046881	-0.046881
17/06/11	-0.012746	-0.00023097	-0.0053958
24/06/11	0.01093	0.028859	0.021634
01/07/11	0.063068	0.054324	0.059046
08/07/11	0.018669	0.002438	0.0063947
15/07/11	-0.02067	-0.020849	-0.024296
22/07/11	0.030436	0.039537	0.039158
29/07/11	-0.027834	-0.039618	-0.024828
05/08/11	-0.073952	-0.10028	-0.098556
12/08/11	-0.0056347	0.048436	0.02735
19/08/11	-0.068188	-0.060906	-0.060951
26/08/11	0.058943	0.0027813	0.014186
02/09/11	0.0027069	-0.014031	-0.0059578
09/09/11	-0.0019255	0.02598	0.014453
16/09/11	0.063752	0.079359	0.065223
23/09/11	-0.043983	-0.050745	-0.048767
30/09/11	-0.031148	-0.054972	-0.048773

	Nasdaq Returns	Mean Returns	Sharpe Returns
07/10/11	0.029289	0.023007	0.019832
14/10/11	0.073997	0.024993	0.022366
21/10/11	-0.015298	-0.021689	0.0034011
28/10/11	0.027596	0.024746	0.0053709
04/11/11	-0.018905	-0.006638	-0.014427
11/11/11	-0.00022921	0.0069942	0.0096868
18/11/11	-0.044188	-0.05211	-0.055839
25/11/11	-0.046807	-0.041704	-0.05046
02/12/11	0.067919	0.056069	0.056069
09/12/11	0.0072023	-0.0093773	-0.0037855
16/12/11	-0.035335	-0.032403	-0.052955
23/12/11	0.021827	0.024323	0.0128
30/12/11	-0.0042669	-0.0015885	0.0022515
06/01/12	0.033814	0.0050237	0.029263
13/01/12	0.0066877	-0.0001811	0.0035526
20/01/12	0.027051	0.033875	0.0084279
27/01/12	0.010105	0.014964	0.016679
03/02/12	0.027011	0.027859	0.031875
10/02/12	0.0071507	0.015636	0.013767
17/02/12	0.01439	0.0076291	0.0073007
24/02/12	0.0076979	0.01473	0.01473
02/03/12	0.014248	0.012395	0.020135
09/03/12	0.001993	0.015432	0.0067801
16/03/12	0.024604	0.016414	0.017027
23/03/12	0.0057964	-0.0031352	-0.0031352
30/03/12	0.0097451	0.0085789	-0.013993
05/04/12	0.0026206	-0.0077125	-0.011251
13/04/12	-0.023258	-0.0044385	0.0075224
20/04/12	-0.0085395	-0.026733	-0.026733
27/04/12	0.024109	-0.0091772	0.019607
04/05/12	-0.038456	-0.01151	-0.018533
11/05/12	-0.0083519	0.012989	0.016845
18/05/12	-0.053973	-0.058019	-0.058019
25/05/12	0.019387	0.03394	0.043389
01/06/12	-0.027367	-0.044149	-0.044149
08/06/12	0.040013	0.042059	0.042059
15/06/12	0.0046858	-0.012414	0.0034774
22/06/12	0.0055462	0.012778	0.0030257
29/06/12	0.011609	0.023101	0.022158
06/07/12	-0.0013121	-0.038715	-0.020615

	Nasdaq Returns	Mean Returns	Sharpe Returns
13/07/12	-0.010513	-0.0007947	-0.0036052
20/07/12	0.012712	-0.00036571	-0.00036571
27/07/12	0.011012	0.036613	0.024704
03/08/12	0.010885	0.011407	0.011407
10/08/12	0.017396	0.023909	0.0068687
17/08/12	0.020839	0.0091389	0.012995
24/08/12	-0.00080959	0.0043984	0.0042607
31/08/12	-0.0020936	-0.0069065	-0.01015
07/09/12	0.018892	0.023859	0.024902
14/09/12	0.010605	0.012521	0.012521
21/09/12	0.0022425	-0.026796	0.0003171
28/09/12	-0.022065	-0.020155	-0.019646
05/10/12	0.0045445	0.012354	0.012354
12/10/12	-0.033191	-0.027623	-0.0065421
19/10/12	-0.015494	-0.071351	-0.078607
26/10/12	-0.0046743	-0.018867	0.00206
02/11/12	-0.0035888	0.014204	0.014204
09/11/12	-0.027549	-0.014254	-0.016848
16/11/12	-0.019515	-0.037566	-0.019579
23/11/12	0.040761	0.019151	0.0069455
30/11/12	0.014402	0.026621	0.026621
07/12/12	-0.014042	-0.027057	-0.0346
14/12/12	-0.0047261	0.0086652	0.0082554
21/12/12	0.013823	0.0049839	0.00087616
28/12/12	-0.022126	-0.02516	-0.02045
04/01/13	0.044327	0.029116	0.025625
11/01/13	0.0086867	-0.010596	-0.036093
18/01/13	-0.0018283	0.011675	0.0043366
25/01/13	-0.0023759	0.043959	0.041741
01/02/13	0.0099007	0.0026393	0.0069018
08/02/13	0.0041881	0.021812	0.0072785
15/02/13	-0.0039313	-0.014529	-0.019739
22/02/13	-0.0099565	-0.013147	-0.021756
01/03/13	0.0038177	0.0047252	0.010615
08/03/13	0.020304	0.03237	0.029827
15/03/13	-0.0016776	-0.013883	-0.0084859
22/03/13	0.00050003	0.0075995	0.013957
28/03/13	0.0063635	0.00079012	0.0061074
05/04/13	-0.016793	-0.015368	-0.01816
12/04/13	0.030111	0.043727	0.059409

	Nasdaq Returns	Mean Returns	Sharpe Returns
19/04/13	-0.026974	-0.033594	-0.016282
26/04/13	0.021381	0.0044098	-0.0039129
03/05/13	0.035972	0.026829	0.029034
10/05/13	0.012296	0.00071347	0.0022664
17/05/13	0.015954	0.0091314	0.0055681
24/05/13	-0.012605	-0.018552	-0.032954
31/05/13	-0.0031007	-0.020908	-0.031388
07/06/13	0.0030506	0.0094749	0.013937
14/06/13	-0.015843	-0.026985	-0.021262
21/06/13	-0.022647	-0.049467	-0.050662
28/06/13	0.010941	0.020438	0.020438
05/07/13	0.018261	-0.0035335	0,00067715
12/07/13	0.038351	0.02022	0.02022
19/07/13	-0.01115	-0.013336	0.0095803
26/07/13	0.010227	0.0011954	0.0167
02/08/13	0.021638	0.025293	0.019323
09/08/13	-0.0079686	-0.017233	-0.0076512
16/08/13	-0.014424	-0.01539	-0.0099122
23/08/13	0.01625	0.0070058	-0.0040114
30/08/13	-0.016283	-0.03387	-0.035634
06/09/13	0.019194	0.026427	0.017814
13/09/13	0.014228	0.017674	0.025284
20/09/13	0.014509	0.00024012	0.0088809
27/09/13	0.0017258	0.01894	0.0067902
04/10/13	0.0037912	0.017844	0.017844
11/10/13	-0.002699	0.013573	0.0027189
18/10/13	0.036451	0.032483	0.0356
25/10/13	0.0088904	0.010611	0.009484
01/11/13	-0.0012035	-0.01821	-0.025754
08/11/13	-0.0038301	-0.011846	-0.0093778
15/11/13	0.01642	0.010592	0.0053586
22/11/13	-0.00016365	-0.0056652	0.002258
29/11/13	0.019046	0.0040765	0.0022573
06/12/13	0.0047025	0.005187	0.0062292
13/12/13	-0.013752	-0.016355	-0.020921
20/12/13	0.021407	0.019466	0.015548
27/12/13	0.012056	0.0044395	0.0034951
03/01/14	-0.0099231	-0.0029658	-0.0073672
10/01/14	0.0074186	0.014101	0.011425
17/01/14	0.0073138	0.019115	0.016031

	Nasdaq Returns	Mean Returns	Sharpe Returns
24/01/14	-0.013956	-0.015787	-0.015787
31/01/14	-0.0055384	-0.047318	-0.030412
07/02/14	0.011291	0.0018929	0.0030777
14/02/14	0.028226	0.051093	0.032692
21/02/14	-0.00034936	0.02765	0.02765
28/02/14	0.0091049	0.0082617	0.010913
07/03/14	0.0019731	0.007913	0.0050419
14/03/14	-0.020606	-0.020338	-0.018064
21/03/14	0.0069222	-0.0043789	0.0045183
28/03/14	-0.022585	-0.027661	-0.0070774
04/04/14	-0.0090313	-0.00065469	0.0097698
11/04/14	-0.026491	-0.018447	-0.021549
17/04/14	0.02512	0.012992	0.012992
25/04/14	-0.00040464	-0.036318	-0.024812
02/05/14	0.015319	0.013175	0.024268
09/05/14	-0.0089426	0.016882	0.0093836
16/05/14	0.00882	-0.0095269	-0.016208
23/05/14	0.024815	0.0020358	0.0020358
30/05/14	0.016048	0.014414	0.01366
06/06/14	0.015336	0.0082285	0.010972
13/06/14	-0.0050224	0.0034061	-0.0010618
20/06/14	0.0071468	0.016553	0.025008
27/06/14	0.010932	0.0027817	0.0084308
03/07/14	0.020231	0.017886	0.019213
11/07/14	-0.004709	-0.018656	-0.0017394
18/07/14	0.0090025	-0.0088678	-0.0088678
25/07/14	0.0063959	-0.011888	-0.015754
01/08/14	-0.021799	-0.0091767	-0.011094
08/08/14	0.002168	-0,32098	0.0018739
15/08/14	0.025249	0.070717	0.071004
22/08/14	0.016229	-0.013238	-0.0073595
29/08/14	0.0073286	0.0061497	0.015034
05/09/14	0.0018011	-0.0035777	-0.010965
12/09/14	-0.0050716	-0.019835	-0.019835
19/09/14	0.0075551	-0.0093275	-0.0034737
26/09/14	-0.011374	-0.014342	-0.012394
03/10/14	-0.0065363	0.0099826	0.0077616
10/10/14	-0.039622	-0.037724	-0.032605
17/10/14	-0.014413	-0.0003204	0.0054506
24/10/14	0.057681	0.041866	0.039388

	Nasdaq Returns	Mean Returns	Sharpe Returns
31/10/14	0.02834	0.026344	0.026286
07/11/14	0.00055058	0.031039	0.020637
14/11/14	0.015382	0.004373	0.0081356
21/11/14	0.0062125	0.0079369	0.0089387
28/11/14	0.020136	0.027845	0.030598
05/12/14	-0.0060629	-0.011221	-0.010629
12/12/14	-0.026389	-0.02252	-0.014735
19/12/14	0.019456	0.0070792	0.0089754
26/12/14	0.0075176	0.0048051	0.0046372
31/12/14	-0.018201	-0.01157	-0.015949

Table 3:Weekly Returns with commissions		
	Mean Value	Sharpe Value
07/01/11	1,0442	1,0442
14/01/11	1,0997	1,1012
21/01/11	1,0766	1,0747
28/01/11	1,0615	1,0509
04/02/11	1,0966	1,0772
11/02/11	1,1028	1,0962
18/02/11	1,1156	1,0954
25/02/11	1,1217	1,06
04/03/11	1,1183	1,0452
11/03/11	1,075	1,0107
18/03/11	1,0318	0,96729
25/03/11	1,0709	1,0116
01/04/11	1,0859	1,0259
08/04/11	1,0595	0,99259
15/04/11	1,0667	0,99936
21/04/11	1,0972	1,0257
29/04/11	1,1192	1,0326
06/05/11	1,0957	1,0109
13/05/11	1,1011	1,0112
20/05/11	1,0974	1,01
27/05/11	1,0954	1,0168
03/06/11	1,0508	0,97538
10/06/11	0,99942	0,9277
17/06/11	0,99719	0,92084
24/06/11	1,024	0,93892
01/07/11	1,0776	0,99248
08/07/11	1,078	0,99684
15/07/11	1,0534	0,97063
22/07/11	1,0929	1,0067
29/07/11	1,0475	0,97969
05/08/11	0,94031	0,88118
12/08/11	0,98398	0,90351
19/08/11	0,92208	0,84664
26/08/11	0,9228	0,85695
02/09/11	0,90801	0,85013
09/09/11	0,92978	0,86072
16/09/11	1,0017	0,91514
23/09/11	0,94887	0,86868

	Mean Value	Sharpe Value
30/09/11	0,89481	0,82457
07/10/11	0,91361	0,83928
14/10/11	0,93462	0,85637
21/10/11	0,91248	0,85757
28/10/11	0,93323	0,86046
04/11/11	0,92517	0,84633
11/11/11	0,92979	0,85283
18/11/11	0,87948	0,80351
25/11/11	0,84104	0,76135
02/12/11	0,88652	0,80252
09/12/11	0,87643	0,79788
16/12/11	0,84628	0,75403
23/12/11	0,86517	0,76217
30/12/11	0,86207	0,76236
06/01/12	0,86467	0,78315
13/01/12	0,86279	0,78436
20/01/12	0,89029	0,78941
27/01/12	0,90183	0,80099
03/02/12	0,92515	0,82492
10/02/12	0,93777	0,83463
17/02/12	0,94305	0,83905
24/02/12	0,95505	0,84974
02/03/12	0,96498	0,86515
09/03/12	0,97794	0,86928
16/03/12	0,99204	0,88234
23/03/12	0,98694	0,87781
30/03/12	0,99344	0,86377
05/04/12	0,98379	0,85233
13/04/12	0,97745	0,85704
20/04/12	0,94937	0,83241
27/04/12	0,93876	0,84707
04/05/12	0,92607	0,82967
11/05/12	0,93625	0,84199
18/05/12	0,88006	0,79146
25/05/12	0,90817	0,82421
01/06/12	0,86626	0,78618
08/06/12	0,90096	0,81767
15/06/12	0,88797	0,81888
22/06/12	0,89754	0,81972
29/06/12	0,91648	0,83624

	Mean Value	Sharpe Value
06/07/12	0,87917	0,81733
13/07/12	0,87671	0,81275
20/07/12	0,87464	0,81083
27/07/12	0,90491	0,82924
03/08/12	0,91342	0,83704
10/08/12	0,93343	0,84111
17/08/12	0,9401	0,85036
24/08/12	0,94235	0,85228
31/08/12	0,93396	0,84193
07/09/12	0,95438	0,86121
14/09/12	0,96442	0,87027
21/09/12	0,93665	0,8688
28/09/12	0,91589	0,85
05/10/12	0,92538	0,8588
12/10/12	0,89796	0,85146
19/10/12	0,8321	0,78283
26/10/12	0,81473	0,78288
02/11/12	0,82468	0,79243
09/11/12	0,81127	0,77749
16/11/12	0,77917	0,76072
23/11/12	0,79254	0,76448
30/11/12	0,81205	0,7833
07/12/12	0,78845	0,75463
14/12/12	0,79371	0,75935
21/12/12	0,79608	0,7585
28/12/12	0,77446	0,74147
04/01/13	0,79546	0,75899
11/01/13	0,78544	0,73008
18/01/13	0,79304	0,73178
25/01/13	0,82631	0,76086
01/02/13	0,82684	0,76459
08/02/13	0,84322	0,76863
15/02/13	0,82928	0,75192
22/02/13	0,81672	0,73406
01/03/13	0,81895	0,74038
08/03/13	0,84382	0,76098
15/03/13	0,83042	0,753
22/03/13	0,83507	0,76201
28/03/13	0,83406	0,76514
05/04/13	0,81957	0,74971

	Mean Value	Sharpe Value
12/04/13	0,85377	0,79275
19/04/13	0,82338	0,77826
26/04/13	0,82536	0,77366
03/05/13	0,84586	0,79457
10/05/13	0,84477	0,79478
17/05/13	0,85079	0,79762
24/05/13	0,83331	0,76974
31/05/13	0,81422	0,74404
07/06/13	0,8203	0,75292
14/06/13	0,79653	0,73541
21/06/13	0,75553	0,69668
28/06/13	0,76946	0,70952
05/07/13	0,76521	0,70815
12/07/13	0,77915	0,72106
19/07/13	0,7672	0,72652
26/07/13	0,76658	0,7372
02/08/13	0,78444	0,74997
09/08/13	0,76935	0,74273
16/08/13	0,75597	0,73389
23/08/13	0,75976	0,72948
30/08/13	0,7325	0,70202
06/09/13	0,7504	0,71312
13/09/13	0,76216	0,72973
20/09/13	0,76082	0,73475
27/09/13	0,77371	0,73827
04/10/13	0,78596	0,74997
11/10/13	0,79506	0,75051
18/10/13	0,8193	0,77572
25/10/13	0,82635	0,78153
01/11/13	0,80965	0,75984
08/11/13	0,79844	0,75119
15/11/13	0,8053	0,75372
22/11/13	0,79913	0,75391
29/11/13	0,80079	0,7541
06/12/13	0,80334	0,75729
13/12/13	0,78859	0,73994
20/12/13	0,80237	0,74996
27/12/13	0,80433	0,75108
03/01/14	0,80033	0,74405
10/01/14	0,81002	0,75106

	Mean Value	Sharpe Value
17/01/14	0,82388	0,7616
24/01/14	0,80923	0,74805
31/01/14	0,76932	0,72381
07/02/14	0,76923	0,72459
14/02/14	0,807	0,74682
21/02/14	0,8277	0,76598
28/02/14	0,83288	0,77281
07/03/14	0,83781	0,77516
14/03/14	0,81909	0,75961
21/03/14	0,81387	0,76152
28/03/14	0,78972	0,75461
04/04/14	0,78763	0,76047
11/04/14	0,77152	0,74256
17/04/14	0,78	0,75072
25/04/14	0,75012	0,73059
02/05/14	0,7585	0,74686
09/05/14	0,76979	0,75238
16/05/14	0,76091	0,73868
23/05/14	0,76094	0,73871
30/05/14	0,77039	0,74732
06/06/14	0,77519	0,75402
13/06/14	0,77628	0,75171
20/06/14	0,78757	0,76901
27/06/14	0,78819	0,77396
03/07/14	0,80071	0,78728
11/07/14	0,78417	0,78433
18/07/14	0,77565	0,77581
25/07/14	0,76487	0,76204
01/08/14	0,75633	0,75206
08/08/14	0,75479	0,75196
15/08/14	0,80666	0,80385
22/08/14	0,79436	0,79633
29/08/14	0,79766	0,80671
05/09/14	0,79321	0,79625
12/09/14	0,77589	0,77886
19/09/14	0,7671	0,7746
26/09/14	0,75457	0,76345
03/10/14	0,76059	0,76785
10/10/14	0,73038	0,74128
17/10/14	0,72868	0,74384

	Mean Value	Sharpe Value
24/10/14	0,75773	0,77165
31/10/14	0,77618	0,79039
07/11/14	0,79872	0,80512
14/11/14	0,80061	0,81006
21/11/14	0,80537	0,81568
28/11/14	0,82618	0,839
05/12/14	0,81526	0,82841
12/12/14	0,79527	0,81454
19/12/14	0,79931	0,82023
26/12/14	0,80155	0,82239
31/12/14	0,79067	0,80763

	Table 4:Investment in just 1 asset	
	Mean Value	Sharpe Value
Starting Val	1	1
07/01/11	1,2471	1,2471
14/01/11	1,4839	1,4839
21/01/11	1,4154	1,4154
28/01/11	1,3865	1,3865
04/02/11	1,4914	1,4028
11/02/11	1,5249	1,4343
18/02/11	1,5025	1,4133
25/02/11	1,4878	1,4672
04/03/11	1,4379	1,4181
11/03/11	1,3458	1,3806
18/03/11	1,2571	1,3283
25/03/11	1,3075	1,3816
01/04/11	1,3507	1,3019
08/04/11	1,3065	1,2593
15/04/11	1,3224	1,2746
21/04/11	1,3404	1,2919
29/04/11	1,3396	1,2912
06/05/11	1,2924	1,2457
13/05/11	1,3196	1,2744
20/05/11	1,3848	1,266
27/05/11	1,4268	1,2821
03/06/11	1,3211	1,1871
10/06/11	1,2237	1,1171
17/06/11	1,1683	1,0665
24/06/11	1,1951	1,091
01/07/11	1,2744	1,1748
08/07/11	1,3003	1,1986
15/07/11	1,2888	1,188
22/07/11	1,3835	1,2753
29/07/11	1,3314	1,2758
05/08/11	1,248	1,1958
12/08/11	1,3595	1,2454
19/08/11	1,246	1,2375
26/08/11	1,1836	1,2429
02/09/11	1,1422	1,1994
09/09/11	1,1575	1,1973
16/09/11	1,2983	1,3429

	Mean Value	Sharpe Value
23/09/11	1,2537	1,2968
30/09/11	1,1942	1,2352
07/10/11	1,215	1,2568
14/10/11	1,2189	1,2827
21/10/11	1,1647	1,2257
28/10/11	1,3144	1,3832
04/11/11	1,34	1,2901
11/11/11	1,3068	1,2582
18/11/11	1,2409	1,1948
25/11/11	1,1424	1,0999
02/12/11	1,2347	1,15
09/12/11	1,2611	1,1747
16/12/11	1,226	1,1419
23/12/11	1,3075	1,2179
30/12/11	1,2906	1,2236
06/01/12	1,2613	1,2712
13/01/12	1,1881	1,3376
20/01/12	1,2127	1,3701
27/01/12	1,244	1,4054
03/02/12	1,3396	1,5134
10/02/12	1,3165	1,6249
17/02/12	1,366	1,7091
24/02/12	1,3759	1,7607
02/03/12	1,4375	1,8395
09/03/12	1,457	1,8386
16/03/12	1,4645	1,9394
23/03/12	1,4566	1,9305
30/03/12	1,5006	1,9297
05/04/12	1,4275	1,8358
13/04/12	1,3791	1,7734
20/04/12	1,3987	1,7952
27/04/12	1,551	1,9906
04/05/12	1,5064	1,896
11/05/12	1,4456	2,0299
18/05/12	1,3285	1,9506
25/05/12	1,361	1,9984
01/06/12	1,3372	1,9634
08/06/12	1,4364	2,0281
15/06/12	1,4805	2,0649
22/06/12	1,5362	2,1426

	Mean Value	Sharpe Value
29/06/12	1,5278	2,1309
06/07/12	1,4729	2,0808
13/07/12	1,4789	2,0893
20/07/12	1,4769	2,0865
27/07/12	1,4697	2,1549
03/08/12	1,4957	2,279
10/08/12	1,5351	2,3392
17/08/12	1,5742	2,3986
24/08/12	1,5752	2,4049
31/08/12	1,5513	2,3522
07/09/12	1,591	2,4035
14/09/12	1,6143	2,3478
21/09/12	1,5929	2,3166
28/09/12	1,5326	2,249
05/10/12	1,5671	2,2996
12/10/12	1,4832	2,1764
19/10/12	0,97768	1,4347
26/10/12	0,96943	1,4226
02/11/12	0,97065	1,4244
09/11/12	0,96908	1,422
16/11/12	0,96567	1,3624
23/11/12	0,99123	1,4301
30/11/12	0,9838	1,5256
07/12/12	0,95883	1,4869
14/12/12	0,94586	1,549
21/12/12	0,96042	1,5728
28/12/12	0,93434	1,5691
04/01/13	0,9347	1,6361
11/01/13	0,95529	1,6722
18/01/13	0,90281	1,5803
25/01/13	0,9267	1,6221
01/02/13	0,94504	1,6323
08/02/13	0,9328	1,6575
15/02/13	0,93139	1,5763
22/02/13	0,91037	1,5407
01/03/13	0,90375	1,5777
08/03/13	0,91796	1,6025
15/03/13	0,91423	1,596
22/03/13	0,90313	1,5766
28/03/13	0,88496	1,5

	Mean Value	Sharpe Value
05/04/13	0,89712	1,5206
12/04/13	1,0131	1,7172
19/04/13	0,9751	1,6528
26/04/13	0,89052	1,6624
03/05/13	0,91922	1,7585
10/05/13	0,91456	1,7496
17/05/13	0,92185	1,7782
24/05/13	0,89662	1,676
31/05/13	0,8534	1,5952
07/06/13	0,83635	1,5634
14/06/13	0,8007	1,4967
21/06/13	0,73462	1,3551
28/06/13	0,72578	1,401
05/07/13	0,7197	1,3892
12/07/13	0,76033	1,4677
19/07/13	0,73561	1,5004
26/07/13	0,72308	1,4748
02/08/13	0,78187	1,4742
09/08/13	0,78209	1,4746
16/08/13	0,71743	1,3527
23/08/13	0,71269	1,3437
30/08/13	0,69317	1,3069
06/09/13	0,70025	1,3203
13/09/13	0,71446	1,3804
20/09/13	0,70816	1,3923
27/09/13	0,7332	1,3594
04/10/13	0,74202	1,3758
11/10/13	0,79766	1,479
18/10/13	0,86046	1,5354
25/10/13	0,87058	1,5535
01/11/13	0,86835	1,5268
08/11/13	0,84437	1,5244
15/11/13	0,80607	1,4553
22/11/13	0,77728	1,4033
29/11/13	0,76694	1,3847
06/12/13	0,76971	1,3897
13/12/13	0,76078	1,3735
20/12/13	0,79077	1,4277
27/12/13	0,80246	1,414
03/01/14	0,80199	1,4132

	Mean Value	Sharpe Value
10/01/14	0,81196	1,4308
17/01/14	0,79648	1,5794
24/01/14	0,76536	1,5177
31/01/14	0,70028	1,3887
07/02/14	0,69251	1,3733
14/02/14	0,72553	1,4387
21/02/14	0,75322	1,4936
28/02/14	0,74311	1,4736
07/03/14	0,7581	1,5033
14/03/14	0,72362	1,4551
21/03/14	0,74177	1,4916
28/03/14	0,74713	1,5721
04/04/14	0,73679	1,5504
11/04/14	0,73689	1,5396
17/04/14	0,7497	1,5485
25/04/14	0,70159	1,5318
02/05/14	0,7254	1,5838
09/05/14	0,72031	1,5727
16/05/14	0,69366	1,5145
23/05/14	0,6884	1,503
30/05/14	0,69964	1,5443
06/06/14	0,71234	1,5723
13/06/14	0,70134	1,548
20/06/14	0,71652	1,5815
27/06/14	0,73695	1,6266
03/07/14	0,74661	1,6835
11/07/14	0,75492	1,7022
18/07/14	0,74357	1,6766
25/07/14	0,71971	1,6891
01/08/14	0,71453	1,7289
08/08/14	0,70234	1,6994
15/08/14	0,70308	1,7012
22/08/14	0,70473	1,7052
29/08/14	0,70963	1,717
05/09/14	0,68912	1,6674
12/09/14	0,69645	1,6851
19/09/14	0,68123	1,6545
26/09/14	0,67416	1,6374
03/10/14	0,68076	1,6534
10/10/14	0,64721	1,6423

	Mean Value	Sharpe Value
17/10/14	0,6234	1,6605
24/10/14	0,66568	1,7325
31/10/14	0,68805	1,7908
07/11/14	0,69636	1,8422
14/11/14	0,69499	1,8386
21/11/14	0,68642	1,8159
28/11/14	0,69853	1,848
05/12/14	0,64527	1,7071
12/12/14	0,63262	1,6736
19/12/14	0,65534	1,7225
26/12/14	0,6544	1,7331
31/12/14	0,64797	1,716

Appendix IV

I copied here the script I wrote in Matlab to make all the calculation. Some passages, as calculating the average of the weekly adjusted close prices, are omitted since the calculations have been made in Excel.

```
%After having downloaded and managed all the data with Excel, I put all
%the adjusted closing price in a matrix called 'Closed', where every column
%represents an asset.

%calculation of the log returns
YRatio=zeros(size(Closed,1),size(Closed,2));
for i=2:length(YRatio)
    for j=1:size(YRatio,2)
        YRatio(i,j)=(Closed(i,j))/(Closed(i-1,j));
    end
end
YRatio(1,:)=[];

Yields=zeros(length(YRatio),size(YRatio,2));
for i=1:size(Yields,1)
    for j=1:size(Yields,2)
        Yields(i,j)=log(YRatio(i,j));
    end
end

%repeat the same procedure for the return of the Nasdaq, managed as the
%others and that I will need as a variable for the model
NRatio=zeros(length(Nasdaq),1);
for i=2:length(Nasdaq)
    NRatio(i,1)=Nasdaq(i,1)/Nasdaq(i-1);
end

NRatio(1,:)=[];
NasdaqLog=zeros(length(NRatio),1);
for i=1:length(NasdaqLog)
    NasdaqLog(i,1)=log(NRatio(i,1));
end

%calculation of the ranking
for i=1:size(Yields,1)
    for j=1:size(Yields,2)
        r=Yields(i,:);
        r=r(r<Yields(i,j));
        Ranking(i,j)=(size(r,2)/50) + 1/50;

    end
end

%calculation of the jumps dummy variable
mdiff=zeros(729,50);
for i=2:size(mdiff,1)
    for j=1:size(mdiff,2)
        mdiff(i,j)=Ranking(i,j)-Ranking(i-1,j);
    end
```

```
end

mdiffA=abs(mdiff);

jumps=zeros(729,50);
for i=1:size(mdiffA,1)
    for j=1:size(mdiffA,2)
        if mdiffA(i,j)>=0.5
            jumps(i,j)=1;
        else
            jumps(i,j)=0;
        end
    end
end

%reorganization of the data divided per asset. Since I used the same
%formula for every asset of the sample, I will show the name of the various
%matrixes as "Asset" and the corresponding column number as N
Asset=zeros(729,10);
Asset(:,1)=Yields(:,N);
Asset(:,2)=Ranking(:,N);
Asset(:,3)=mdiff(:,N);
Asset(:,4)=mdiffA(:,N);
Asset(:,5)=jumps(:,N);

%calculation of the duration variable (divided per asset)
for ib=2:size(Asset,1)
    if Asset(ib-1,5)==1
        Asset(ib,6)=1;
    else
        Asset(ib,6)=Asset(ib-1,6)+1;
    end
end

%setting of the variables with the proper name and launch of the script
insperiod=519;

dep=Asset(2:insperiod+1,1);
indep=ones(length(dep),4);
px=ones(length(dep),2);
indep(:,1)=Asset(1:insperiod,2)/10;
indep(:,2)=Asset(2:insperiod+1,5);
indep(:,3)=Asset(2:insperiod+1,6)/1000;
indep(:,4)=NasdaqLog(1:519,1);
px(:,1)=Asset(1:insperiod,2)/10;
px(:,2)=Asset(2:insperiod+1,6)/1000;
k=2;
S=[1 1 1 1 1];

[Spec_Out]=MS_Regress_Fit_tvtp(dep,indep,px,k,S);

%the results are inserted in a structured composed by elements in format
%cells. I converted them into numeric matrixes and then convert them into
%csv files to organizing them in tables utilizing Excel
Tab_els=Spec_Out.param;
filename='in_sample_params.xlsx';
xlswrite(filename,Tab_els,1,'C3:C16')
Errori_standard=cell2mat(Spec_Out.Coeff_SE.S_Param);
Errori_standard(1,3:4)=Errori_standard(2,1:2);
Errori_standard(1,5:6)=Errori_standard(3,1:2);
```

```
Errori_standard(1,7:8)=Errori_standard(4,1:2);
Errori_standard(2:4,:)=[];
Errori_standard=Errori_standard';
filename='in_sample_Sde';
xlswrite(filename,Errori_standard,1,'D5:D12');
Errori_stati=cell2mat(Spec_Out.Coeff_SE.covMat);
Errori_stati=Errori_stati';
filename='in_sample_Sde_stati';
xlswrite(filename,Errori_stati,1,'D3:D4');
Errori_prob=cell2mat(Spec_Out.Coeff_SE.pa);
Errori_prob(1,3:4)=Errori_prob(2,1:2);
Errori_prob(2,:)=[];
Errori_prob=Errori_prob';
filename='in_sample_prob';
xlswrite(filename,Errori_prob,1,'D13:D16');
PVal_Indep=cell2mat(Spec_Out.Coeff_pValues.S_Param);
PVal_Indep(1,3:4)=PVal_Indep(2,1:2);
PVal_Indep(1,5:6)=PVal_Indep(3,1:2);
PVal_Indep(1,7:8)=PVal_Indep(4,1:2);
PVal_Indep(2:4,:)=[];
PVal_Indep=PVal_Indep';
filename='in_sample_Pindep';
xlswrite(filename,PVal_Indep,1,'E5:E12');
PVal_stati=cell2mat(Spec_Out.Coeff_pValues.covMat);
PVal_stati=PVal_stati';
filename='in_sample-Pstati';
xlswrite(filename,PVal_stati,1,'E3:E4');
PVal_prob=cell2mat(Spec_Out.Coeff_pValues.pa);
PVal_prob(1,3:4)=PVal_prob(2,1:2);
PVal_prob(2,:)=[];
PVal_prob=PVal_prob';
filename='in_sample_Pprob';
xlswrite(filename,PVal_prob,1,'E13:E16');

%after having saved the plots automatically printed in the script, save
%them as jpg in order to insert them in the Word file
figName='Asset.fig';
outName='Asset.jpg';
h=openfig(figName,'new','invisible');
saveas(h,outName,'jpg')
close(h);

%calculating the probability inverse of the average duration for the
%forecast, setting the variables to perform it and its launch
AvgJD=(sum(indep(:,2)))/(length(indep));
YForec=zeros(209,1);
SigmaForec=zeros(209,1);
SharpeForec=zeros(209,1);
JxForec=ones(length(YForec),1);
JxForec(:,1)=AvgJD;

newIndepData=ones(209,4);
newpxData=ones(209,2);
newIndepData(:,1)=Asset(520:728,2)/10;
newIndepData(:,2)=JxForec(:,1);
newIndepData(:,3)=Asset(521:729,6)/1000;
newIndepData(:,4)=NasdaqLog(520:728,1);
newpxData(:,1)=Asset(520:728,2)/10;
newpxData(:,2)=Asset(521:729,6)/1000;

[meanFor,stdFor]=MS_Regress_For_tvtp(Spec_Out,newIndepData,newpxData);
```

```
%conversion to numeric array of the values and calculation of the ratio
ForecM=cell2mat(meanFor);
ForecS=cell2mat(stdFor);
ForecSharpe=zeros(1,size(ForecM,2));

for i=1:size(ForecM,2)
    ForecSharpe(1,i)=ForecM(1,i)/ForecS(1,i);
end

%rearranging the arrays and writing saving them in a csv file
Forec_Eval=[ForecM;ForecS;ForecSharpe];
Forec_Eval=Forec_Eval';
filename='oos_Asset';
xlswrite(filename,Forec_Eval,1,'C4:E212')

%after having saved all the ForecM and the ForecSharpe in two matrixes
%called FutMean and FutSharpe, I classified them to choose the best five in
%which invest for each strategy
FutRank1=zeros(size(FutMean,1),size(FutMean,2));
for i=1:size(FutMean,1)
    for j=1:size(FutMean,2)
        r=FutMean(i,:);
        r=r(r<FutMean(i,j));
        FutRank1(i,j)=(size(r,2)/50) + 1/50;
    end
end

FutRank2=zeros(size(FutSharpe,1),size(FutSharpe,2));
for i=1:size(FutSharpe,1)
    for j=1:size(FutSharpe,2)
        r=FutSharpe(i,:);
        r=r(r<FutSharpe(i,j));
        FutRank2(i,j)=(size(r,2)/50) + 1/50;
    end
end

%changing the values of the matrixes making them 0/1, where 1 indicates in
%which asset to invest (they are ordered in alphabetical order)
for i=1:size(FutRank1,1)
    for j=1:size(FutRank1,2)
        if FutRank1(i,j)>0.9
            FutRank1(i,j)=1;
        else
            FutRank1(i,j)=0;
        end
    end
end

for i=1:size(FutRank2,1)
    for j=1:size(FutRank2,2)
        if FutRank2(i,j)>0.9
            FutRank2(i,j)=1;
        else
            FutRank2(i,j)=0;
        end
    end
end

%After having seen which asset had to be selected, I downloaded the data
```

```
%from Yahoo Finance and reorganized them in couples of weekly opening and
%closing prices. Then imported them in Matlab with the names VarName
%followed by the number of the column. Then calculated the returns
%effectively happened.
for i=1:length(FutRank1)
    FirstMean(i,1)=log(VarName3(i,1)/VarName2(i,1));
end

for i=1:length(FutRank1)
    SecondMean(i,1)=log(VarName6(i,1)/VarName5(i,1));
end

for i=1:length(FutRank1)
    ThirdMean(i,1)=log(VarName9(i,1)/VarName8(i,1));
end

for i=1:length(FutRank1)
    FourthMean(i,1)=log(VarName12(i,1)/VarName11(i,1));
end

for i=1:length(FutRank1)
    FifthMean(i,1)=log(VarName15(i,1)/VarName14(i,1));
end

%calculation of the return of the portfolio
LogMeanYield=(FirstMean+SecondMean+ThirdMean+FourthMean+FifthMean)/5;

%calculation of the level of capital (starting value 1) and clearing the
%useless arrays
LogMeanRet=zeros(length(LogMeanYield),1);

LogMeanRet(1,1)=1+LogMeanYield(1,1);
for i=2:length(LogMeanRet)
    LogMeanRet(i,1)=(1+LogMeanYield(i,1))*LogMeanRet(i-1,1);
end

clear  VarName11 VarName12 VarName14 VarName15...
    VarName2 VarName3 VarName5 VarName6 VarName8 VarName9

%same calculation for the Sharpe strategy (after the importation of the
%data)
for i=1:length(FutRank1)
    FirstSharpe(i,1)=log(VarName3(i,1)/VarName2(i,1));
end

for i=1:length(FutRank1)
    SecondSharpe(i,1)=log(VarName6(i,1)/VarName5(i,1));
end

for i=1:length(FutRank1)
    ThirdSharpe(i,1)=log(VarName9(i,1)/VarName8(i,1));
end

for i=1:length(FutRank1)
    FourthSharpe(i,1)=log(VarName12(i,1)/VarName11(i,1));
end

for i=1:length(FutRank1)
    FifthSharpe(i,1)=log(VarName15(i,1)/VarName14(i,1));
```

```
end

LogSharpeYield=(FirstSharpe+SecondSharpe+ThirdSharpe+FourthSharpe+...
    FifthSharpe)/5;

LogSharpeRet=zeros(length(LogSharpeYield),1);
LogSharpeRet(1,1)=1+LogSharpeYield(1,1);
for i=2:length(LogSharpeRet)
    LogSharpeRet(i,1)=(1+LogSharpeYield(i,1))*LogSharpeRet(i-1,1);
end

clear VarName2 VarName3 VarName5 VarName6 VarName8 VarName9...
    VarName11 VarName12 VarName14 VarName15

%calculation of the returns for the Nasdaq 100 (which data are not on two
%different arrays, but on the same one) after having imported them as
%VarName15
for i=2:length(VarName15)
    NasdRet(i,1)=log(VarName15(i,1)/VarName15(i-1,1));
end
NasdRet(1,:)=[];
NasdYield=NasdRet;
NasdRet=zeros(length(NasdYield),1);
NasdRet(1,1)=1+NasdYield(1,1);
for i=2:length(NasdRet)
    NasdRet(i,1)=(1+NasdYield(i,1))*NasdRet(i-1,1);
end

%Annualized returns for the three strategies
AnRet=LogMeanRet(length(LogMeanRet),1)^(1/4);
AnRetNas=NasdRet(length(NasdRet),1)^(1/4);
AnRetShar=LogSharpeRet(length(LogSharpeRet),1)^(1/4);

%calculation of the number of times in which the level of the capital of
the
%Markov-switching strategies is higher than the level for the Buy-and-Hold
%strategy
BestStr=LogMeanRet-NasdRet;
for i=1:length(BestStr)
    if BestStr(i,1)>0
        EffectStr(i,1)=1;
    else
        EffectStr(i,1)=0;
    end
end
BestW=sum(EffectStr);

BestStr1=LogSharpeRet-NasdRet;
for i=1:length(BestStr1)
    if BestStr1(i,1)>0
        EffectStr1(i,1)=1;
    else
        EffectStr1(i,1)=0;
    end
end
BestW1=sum(EffectStr1);

%converting the arrays in a csv file in order to table them and plot
%them via Excel
filename='SharpeRet';
```

```
xlswrite(filename,LogSharpeRet,1,'C4:C212');
filename='SharpeYield';
xlswrite(filename,LogSharpeYield,1,'C4:C212');
filename='MeanRet';
xlswrite(filename,LogMeanRet,1,'C4:C212');
filename='MeanYield';
xlswrite(filename,LogMeanYield,1,'C4:C212');
filename='NasdRet';
xlswrite(filename,NasdRet,1,'C4:C212');
filename='NasdYield';
xlswrite(filename,NasdYield,1,'C4:C212');

%calculate how many times the assets in the portfolios are all either
%positive or negative
for i=1:length(FirstMean)
    if FirstMean(i,1)<0 && SecondMean(i,1)<0 && ThirdMean(i,1)<0 &&...
            FourthMean(i,1)<0 && FifthMean(i,1)<0
        AllNegative(i,1)=1;
    elseif FirstMean(i,1)>0 && SecondMean(i,1)>0 && ThirdMean(i,1)>0 &&...
            FourthMean(i,1)>0 && FifthMean(i,1)>0
        AllPositive(i,1)=1;
    else
        AllNegative(i,1)=0;
        AllPositive(i,1)=0;
    end
end
SumAllNegative=sum(AllNegative);
SumAllPositive=sum(AllPositive);

for i=1:length(FirstSharpe)
    if FirstSharpe(i,1)<0 && SecondSharpe(i,1)<0 && ThirdSharpe(i,1)<0
&&...
            FourthSharpe(i,1)<0 && FifthSharpe(i,1)<0
        AllNegative1(i,1)=1;
    elseif FirstSharpe(i,1)>0 && SecondSharpe(i,1)>0 && ThirdSharpe(i,1)>0
&&...
            FourthSharpe(i,1)>0 && FifthSharpe(i,1)>0
        AllPositive1(i,1)=1;
    else
        AllNegative1(i,1)=0;
        AllPositive1(i,1)=0;
    end
end
SumAllNegative1=sum(AllNegative1);
SumAllPositive1=sum(AllPositive1);

%calculate how many weeks the Markov switching strategies perform better
%than the Buy and Hold
BestYtr=LogMeanYield-NasdYield;
for i=1:length(BestYtr)
    if BestYtr(i,1)>0
        EffectYtr(i,1)=1;
    else
        EffectYtr(i,1)=0;
    end
end
BestY=sum(EffectYtr);

BestYtr1=LogSharpeYield-NasdYield;
for i=1:length(BestYtr1)
    if BestYtr1(i,1)>0
```

```
        EffectYtr1(i,1)=1;
    else
        EffectYtr1(i,1)=0;
    end
end
BestY1=sum(EffectYtr1);

%calculate how many weeks the Return and the Sharpe have an higher overall
%return even though losing money from the initial investment
for i=1:length(EffectStr)
    if EffectStr(i,1)==1 && LogMeanRet(i,1)<1
        EffectNegR(i,1)=1;
    else
        EffectNegR(i,1)=0;
    end
end
TotEffNeg=sum(EffectNegR);

for i=1:length(EffectStr1)
    if EffectStr1(i,1)==1 && LogSharpeRet(i,1)<1
        EffectNegR1(i,1)=1;
    else
        EffectNegR1(i,1)=0;
    end
end
TotEffNeg1=sum(EffectNegR1);

%check how many weeks the strategies have negative returns but still higher
%than the Nasdaq
for i=1:length(EffectYtr)
    if EffectYtr(i,1)==1 && LogMeanYield(i,1)<0
        EffectNegY(i,1)=1;
    else
        EffectNegY(i,1)=0;
    end
end
TotEffNeg2=sum(EffectNegY);

for i=1:length(EffectYtr1)
    if EffectYtr1(i,1)==1 && LogSharpeYield(i,1)<1
        EffectNegY1(i,1)=1;
    else
        EffectNegY1(i,1)=0;
    end
end
TotEffNeg3=sum(EffectNegY1);

%check for particularly high and particularly low weekly returns
NasdOver=0;
for i=1:length(NasdYield)
    if NasdYield(i,1)>0.05
        NasdOver=NasdOver+1;
    end
end

NasdUnder=0;
for i=1:length(NasdYield)
    if NasdYield(i,1)<-0.05
        NasdUnder=NasdUnder+1;
    end
```

```
end

LogMeanOver=0;
for i=1:length(LogMeanYield)
    if LogMeanYield(i,1)>0.05
        LogMeanOver=LogMeanOver+1;
    end
end

LogMeanUnder=0;
for i=1:length(LogMeanYield)
    if LogMeanYield(i,1)<-0.05
        LogMeanUnder=LogMeanUnder+1;
    end
end

LogSharpeOver=0;
for i=1:length(LogSharpeYield)
    if LogSharpeYield(i,1)>0.05
        LogSharpeOver=LogSharpeOver+1;
    end
end

LogSharpeUnder=0;
for i=1:length(LogSharpeYield)
    if LogSharpeYield(i,1)<-0.05
        LogSharpeUnder=LogSharpeUnder+1;
    end
end

%count how many weeks the capital invested is under the initial value
NasdRetUnder=0;
for i=1:length(NasdRet)
    if NasdRet(i,1)<1
        NasdRetUnder=NasdRetUnder+1;
    end
end

LogMRUnder=0;
for i=1:length(LogMeanRet)
    if LogMeanRet(i,1)<1
        LogMRUnder=LogMRUnder+1;
    end
end

LogSRUnder=0;
for i=1:length(LogSharpeRet)
    if LogSharpeRet(i,1)<1
        LogSRUnder=LogSRUnder+1;
    end
end

%count the number of negative weekly returns
NasdMeanUnder=0;
for i=1:length(NasdYield)
    if NasdYield(i,1)<0
        NasdMeanUnder=NasdMeanUnder+1;
    end
end
```

```
LogMYUnder=0;
for i=1:length(LogMeanYield)
    if LogMeanYield(i,1)<0
        LogMYUnder=LogMYUnder+1;
    end
end

LogSYUnder=0;
for i=1:length(LogSharpeRet)
    if LogSharpeYield(i,1)<0
        LogSYUnder=LogSYUnder+1;
    end
end

%calculate maximum and minimum value for the returns and the capital in the
%three cases, and the average weekly returns
MaxDownN=min(NasdYield);
MaxDownM=min(LogMeanYield);
MaxDownS=min(LogSharpeYield);

MaxUpN=max(NasdYield);
MaxUpM=max(LogMeanYield);
MaxUpS=max(LogSharpeYield);

MaxDownNR=min(NasdRet);
MaxDownMR=min(LogMeanRet);
MaxDownSR=min(LogSharpeRet);

MaxUpNR=max(NasdRet);
MaxUpMR=max(LogMeanRet);
MaxUpSR=max(LogSharpeRet);

AvgMY=mean(LogMeanYield);
AvgNY=mean(NasdYield);
AvgSY=mean(LogSharpeYield);

%Average positive and negative weekly return for each strategy
counting=1;
for i=1:length(LogMeanYield)
    if LogMeanYield(i,1)<0
        MeanLosses(counting,1)=LogMeanYield(i,1);
        counting=counting+1;
    end
end
AvgMeanLoss=mean(MeanLosses);

counting=1;
for i=1:length(LogMeanYield)
    if LogMeanYield(i,1)>0
        MeanWins(counting,1)=LogMeanYield(i,1);
        counting=counting+1;
    end
end
AvgMeanWins=mean(MeanWins);

counting=1;
for i=1:length(LogSharpeYield)
    if LogSharpeYield(i,1)<0
        SharpeLosses(counting,1)=LogSharpeYield(i,1);
        counting=counting+1;
```

```
    end
end
AvgSharpeLoss=mean(SharpeLosses);

counting=1;
for i=1:length(LogSharpeYield)
    if LogSharpeYield(i,1)>0
        SharpeWins(counting,1)=LogSharpeYield(i,1);
        counting=counting+1;
    end
end
AvgSharpeWins=mean(SharpeWins);

counting=1;
for i=1:length(NasdYield)
    if NasdYield(i,1)<0
        NasdLosses(counting,1)=NasdYield(i,1);
        counting=counting+1;
    end
end
AvgNasdLoss=mean(NasdLosses);

counting=1;
for i=1:length(NasdYield)
    if NasdYield(i,1)>0
        NasdWins(counting,1)=NasdYield(i,1);
        counting=counting+1;
    end
end
AvgNasdWins=mean(NasdWins);

%plot the yield over time of the three strategies and convert the image in
%jpg
figure
x=linspace(1,209,209);
p=plot(x,LogMeanYield,x,LogSharpeYield,x,NasdYield);
xlabel('Time');
ylabel('Returns')
legend('MeanReturns','SharpeReturns','NasdaqReturns');
p(1).Marker='+';
p(2).Marker='x';
p(3).Marker='*';

figName='StratReturns.fig';
outName='StratReturns.jpg';
h=openfig(figName,'new','invisible');
saveas(h,outName,'jpg')
close(h);

%calculation of the returns and the capital level accounting for the
%commissions in the Markov switching strategies, and conversion of the
%arrays in a csv file to table them
commissions=ones(209,1)*0.002;
LogMYcomm=LogMeanYield-commissions;
LogMRcomm=zeros(209,1);
LogMRcomm(1,1)=1+LogMYcomm(1,1);
for i=2:length(LogMRcomm)
    LogMRcomm(i,1)=(1+LogMYcomm(i,1))*(LogMRcomm(i-1,1));
end
```

```
LogSYcomm=LogSharpeYield-commissions;
LogSRcomm=zeros(209,1);
LogSRcomm(1,1)=1+LogSYcomm(1,1);
for i=2:length(LogSRcomm)
    LogSRcomm(i,1)=(1+LogSYcomm(i,1))*(LogSRcomm(i-1,1));
end

filename='MeanRetcomm';
xlswrite(filename,LogMRcomm,1,'C4:C212');

filename='SharpeRetcomm';
xlswrite(filename,LogSRcomm,1,'C4:C212');

%calculation to find just the best asset on which invest in the case of one
%single asset investment. Then, showing which one is with respect to the
%five early selected
for i=1:size(FutMean,1)
    for j=1:size(FutMean,2)
        r=FutMean(i,:);
        r=r(r<FutMean(i,j));
        FutRank3(i,j)=(size(r,2)/50) + 1/50;
    end
end

for i=1:size(FutRank3,1)
    for j=1:size(FutRank3,2)
        if FutRank3(i,j)==1
            FutRM(i,j)=1;
        else
            FutRM(i,j)=0;
        end
    end
end

for i=1:size(FutSharpe,1)
    for j=1:size(FutSharpe,2)
        r=FutSharpe(i,:);
        r=r(r<FutSharpe(i,j));
        FutRank4(i,j)=(size(r,2)/50) + 1/50;
    end
end

for i=1:size(FutRank4,1)
    for j=1:size(FutRank4,2)
        if FutRank4(i,j)==1
            FutRS(i,j)=1;
        else
            FutRS(i,j)=0;
        end
    end
end

for i=1:length(FutRank3)
    for j=1:size(FutRank3,2)
        if FutRank3(i,j)==1 && FutRM(i,j)==1
            Fut1Best(i,j)=1;
        elseif FutRank3(i,j)>0.9 && FutRM(i,j)==0
            Fut1Best(i,j)=-1;
        else
            Fut1Best(i,j)=0;
```

```
        end
    end
end

for i=1:length(FutRank4)
    for j=1:size(FutRank4,2)
        if FutRank4(i,j)==1 && FutRS(i,j)==1
            Fut1BSharpe(i,j)=1;
        elseif FutRank4(i,j)>0.9 && FutRS(i,j)==0
            Fut1BSharpe(i,j)=-1;
        else
            Fut1BSharpe(i,j)=0;
        end
    end
end

%having found which is the chosen one for every week, import the data from
%the Excel sheet and then calculate yields and capital level.
MY1Asset=zeros(length(LogMeanYield),1);
MY1Asset=log(VarName3./VarName2);
MR1Asset=zeros(length(MY1Asset),1);
MR1Asset(1,1)=1+MY1Asset(1,1);
for i=2:length(MR1Asset)
    MR1Asset(i,1)=(1+MY1Asset(i,1))*(MR1Asset(i-1));
end

SY1Asset=zeros(length(LogSharpeYield),1);
SY1Asset=log(VarName8./VarName7);
SR1Asset=zeros(length(SY1Asset),1);
SR1Asset(1,1)=1+SY1Asset(1,1);
for i=2:length(SR1Asset)
    SR1Asset(i,1)=(1+SY1Asset(i,1))*(SR1Asset(i-1));
end

filename='Mean1Asset';
xlswrite(filename,MR1Asset,1,'C4:C212');

filename='Sharpe1Asset';
xlswrite(filename,SR1Asset,1,'C4:C212');
```

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