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The Super-Higgs Mechanism in non-linear Supersymmetry

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Introduction

The Standard Model is, so far, the most successful theory that describes elementary particles and their interactions. It fits perfectly with the experimental data but it leaves some problems unsolved as the hierarchy problem in the Higgs mass, the gap between the running coupling constants in Grand Unification theories and the presence of dark matter. Supersymmetry had a great impact when introduced because it could give an answer to these problems. It provided a dark matter candidate, the neutralino. It removed the gap in Grand Unifications theories, the three running coupling constant of the standard model intersect perfectly at $10^{16} GeV$. Also, if supersymmetry were linearly realized at the TeV scale then it would solve also the Higgs hierarchy problem. Every boson s,interacting with the Higgs field, would have a fermionic superpartner f, which also interacts with the Higgs and at energies above the supersymmetry breaking scale the one loop contributions from these interactions would cancel. The Higgs mass would then be compatible with the standard model expectations for a supersymmetry breaking scale around the TeV. Supersymmetry can also be seen as a low energy theory of the String Theory and so the enthusiasm about supersymmetry was well motivated by phenomenological and theoretical reasons.

It is now clear that the supersymmetry breaking scale is not around the TeV scale but it is much higher. However, even if supersymmetry is broken at a very high energy scale, it can still be used to constrain effective lagrangians, in fact, when a symmetry is broken, we still have a non-linear realization of such symmetry on the effective degrees of freedom [1]. A generic consequence of supersymmetry breaking is a mass splittings in the spectrum, where the heavy states can be close to or higher than the supersymmetry breaking scale and therefore might be integrated out. The effective theory for the remaining states is then constrained by a non-linearly realized supersymmetry.

If global non linear supersymmetry is exact then the fermionic goldstone modes are massless. This is the most common scenario, but, starting from Volkok-Akulov work [2], non-linear supersymmetry were also used to study light fermions as pseudo-goldstone modes of an approximate supersymmetry. More recently inflationary theories in supergravity, [3–13], and brane supersymmetry breaking scenarios, [14–17], were described using non-linearly realized supersymmetry.

Non linear realizations of supersymmetry were initially described using the component fields formulation and then they were implemented in superspace. In superspace methods various properties of supersymmetry are manifest even when the spectrum is not supersymmetric anymore. Among these methods, an interesting approach is to describe non-linear supersymmetry trough constrained superfields [18–21].

This thesis is a review about constrained superfields and eventually it focuses on a open question about the unitarity bounds in inflationary models. In [13] the authors stated that there is no problem with unitarity thanks to the effective cutoff $\Lambda = (V + 3m_{3/2}^2)^{1/4}$ that they believed universal. The authors of [6] instead found that the energy range of validity in inflationary models is constrained by unitarity. In this work we will show that, for a simple model with constrained superfields in supergravity, Λ is not the only relevant scale and so the absence of problems linked to unitarity bound was a feature only of models analogous to those studied in [13].

This work starts with a discussion about supersymmetry and its linear realizations. The superfields formalism is introduced and a general way to construct invariant lagrangians under supersymmetry transformations is developed. Our interest in effective lagrangians in which supersymmetry is spontaneously broken leads us to the description of non-linear realizations and the constrained superfields formalism allows us to obtain them in a efficient way. This formalism is extended also to supergravity and this is useful in order to have a set-up compatible with inflationary models. The breaking of local supersymmetry is well described by the super-Higgs mechanism. This mechanism is analyzed first in linear supersymmetry and then in a simple model with constrained superfields. Eventually the interaction terms of this model are computed and it is shown that Λ is not the only relevant scale.

Chapter 1

Linear Supersymmetry Realizations

This chapter is a brief review about supersymmetry and its linear realizations based on [22]. After a brief discussion on supermultiplets there is a focus on superfields and on their properties. The aim of this chapter is to highlight how the superfields formalism can simplify the construction of supersymmetric lagrangians and to derive them for vector and chiral superfields.

1.1 Supermultiplets

Coleman and Mandula showed that the most general symmetry, compatible with the Poincaré symmetry group, that can be realized in a local QFT is the direct product of the Poincaré group and an an internal symmetry group [23]. It was then natural to define particles as unitary irreducible representations of the Poincaré algebra and to label them with the possible values of the two Poincaré Casimirs, P^2 and W^2. The first one is the square-mass operator while W^2 is related to the the spin (helicity) operator. Mass and spin (helicity) values describe completely a particle and they are invariant under the action of both the Poincaré group and the internal symmetry group. Coleman and Mandula considered only bosonic symmetries. Haag, Łopuszański and Sohnius investigated also fermionic symmetries and they found that most general symmetry compatible with the Poincaré group is given by the product of the Super-Poincaré group, an extension of the Poincaré group that contains the supersymmetry generators Q^I_α, and an internal symmetry group [24]. The irreducible representations of the Super-Poincaré algebra can be written as a collection of irreducible representations of the Poincaré algebra and they are called supermultiplets.

Supermultiplets play in a supersymmetry-invariant theory the same role that particles play in a Lorentz-invariant one. It is of great importance then to highlight some fundamental properties:

- Particles in the same supermultiplet have the same mass but different spin. W^2 can not be a Casimir of the Super-Poincaré algebra because particles in the same supermultiplet are related by the action of the supersymmetry generators that change the spin by half a unit. P^2 instead is still a Casimir because P^μ commutes with the supersymmetry generators.

- In every supermultiplet the bosonic and fermionic d.o.f. are the same. The operator $(-1)^{2s}$, where s is the spin, acts on bosonic and fermionic states as

$$(-1)^{2s}|B\rangle = |B\rangle \ , (-1)^{2s}|F\rangle = -|F\rangle \ . \tag{1.1}$$

This operator anti-commutes with Q_α and so

$$\begin{aligned} 0 =& \mathrm{Tr}\big(-Q_\alpha(-1)^{2s}\bar{B}_{\dot\beta} + (-1)^{2s}\bar{Q}_{\dot\beta}Q_\alpha\big) \\ =& \mathrm{Tr}\big((-1)^{2s}\{Q_\alpha, \bar{Q}_{\dot\beta}\}\big) = 2\sigma^\mu_{\alpha\dot\beta}\mathrm{Tr}\big[(-1)^{2s}\big]P_\mu \ . \end{aligned} \tag{1.2}$$

Choosing $P_\mu \neq 0$ it follows that $\mathrm{Tr}(-1)^{2s} = 0$, namely that $n_B = n_F$.

- Every state has positive energy. From the positive metric assumption [24]

$$0 \leq \sum_\alpha \sum_{\dot\alpha} \langle\phi|\{Q_\alpha, \bar{Q}_{\dot\alpha}\}|\phi\rangle = \mathrm{Tr}(2\sigma^\mu)\langle\phi|P_\mu|\phi\rangle = 4\langle\phi|P_0|\phi\rangle \ .$$

1.1.1 N=1 Supersymmetry Supermultiplets

In this work we are interested in $N = 1$, or minimal, supersymmetry. This means that the supersymmetry generators are given by a complex Weyl fermion Q_α. As seen before, in a supermultiplet there are particles related by the action of this generator. Unitary irreducible representations od the Poincaré group were derived acting on a Clifford vacuum with creation operators. A similar procedure can be developed in order to create unitary irreducible representations of the super-Poincaré group.

Massless Supermultiplets

The first step for building massless supermultiplets is finding creation and annihilation operators. By evaluating $\{Q, \bar{Q}\} = \sigma^\mu P_\mu$ in the rest frame, it follows that only Q_2 is non trivial. From this generator it is possible to define

$$a \equiv \frac{1}{\sqrt{4E}} Q_2 \ , \quad a^\dagger \equiv \frac{1}{\sqrt{4E}} \bar{Q}_{\dot{2}} \ , \tag{1.3}$$

such that they satisfy the anti-commutator relation for creation and annihilation operators:

$$\{a, a^\dagger\} = 1 \ . \tag{1.4}$$

When acting on some state, the operators a and $a^\dagger$ respectively lower and rise the helicity of $\frac{1}{2}$.

The second step is to define a Clifford vacuum $|E, \lambda_0\rangle$, where λ_0 is the helicity, such that

$$a\,|E, \lambda_0\rangle = 0 \ . \tag{1.5}$$

The full supermultiplet is obtained acting with $a^\dagger$ on $|E, \lambda_0\rangle \sim |\lambda_0\rangle$:

$$a^\dagger\,|\lambda_0\rangle = |\lambda_0 + \frac{1}{2}\rangle \ . \tag{1.6}$$

The last step is imposing CPT-invariance by doubling the supermultiplet, namely by adding its CPT conjugate.

The useful massless supermultiplets for this work are:

- Matter or chiral multiplet

$$\lambda_0 = 0 \to \left(0, +\frac{1}{2}\right) \underset{CPT}{\oplus} \left(-\frac{1}{2}, 0\right) \ . \tag{1.7}$$

  There are two bosonic degrees of freedom from a complex scalar and two fermionic from a Weyl fermion. This multiplet is also known as Wess-Zumino multiplet.

- Gauge or vector multiplet

$$\lambda_0 = \frac{1}{2} \to \left(+\frac{1}{2}, +1\right) \underset{CPT}{\oplus} \left(-1, -\frac{1}{2}\right) . \tag{1.8}$$

There are two bosonic degrees of freedom from a massless vector and two fermionic from a Weyl fermion.

- Graviton multiplet

$$\lambda_0 = \frac{3}{2} \to \left(+\frac{3}{2}, +2\right) \underset{CPT}{\oplus} \left(-2, -\frac{3}{2}\right) . \tag{1.9}$$

This multiplet contains a graviton and a gravitino.

Massive Supermultiplets

The procedure for constructing massive supermultiplets is similar to the previous one. The main differences are that there are not vanishing generators and that now the spin is taken in account rather than the helicity. The creation and annihilation operators

$$a_{1,2} \equiv \frac{1}{\sqrt{2m}} Q_{1,2} \ , \quad a^{\dagger}_{1,2} \equiv \frac{1}{\sqrt{2m}} \bar{Q}_{\dot{1},\dot{2}} \ , \tag{1.10}$$

respectively lower and raise the spin j by half unit. Starting from different Clifford vacua $|j_0\rangle$ it is possible to construct:

- Matter multiplet:

$$j_0 = 0 \to \left(-\frac{1}{2}, 0, 0', +\frac{1}{2}\right) ; \tag{1.11}$$

this multiplet is made of a massive complex scalar and a massive Majorana fermion;

- Gauge or vector multiplet:

$$j_0 = \frac{1}{2} \to \left(-1, \mathbf{2} \times -\frac{1}{2}, \mathbf{2} \times 0, \mathbf{2} \times +\frac{1}{2}, 1\right) , \tag{1.12}$$

the degrees of freedom are those of one massive vector, one massive Dirac fermion and one massive real scalar.

1.1.2 Superfields as Supermultiplets

Until now the focus was on unitary irreducible representations. For the Poincaré group there are also finite dimensional irreducible representations. They are labelled by a couple of numbers (m, n) and their dimension is $d = (2m+1)(2n+1)$. They are very useful because in a relativistic quantum field theory all the fields belong to one of these representations. It is possible to develop a similar formalism also in Super-Poincaré? The next section will provide a detailed answer but here a useful first attempt in this direction is made using the procedure illustrated above. For simplicity let's start from a complex scalar field $\phi(x)$ such that

$$[\bar{Q}_{\dot{\alpha}}, \phi(x)] = 0 \ . \tag{1.13}$$

Thanks to this constraint if the field were real it would be a constant. The action of Q_α on ϕ gives:

$$[Q_\alpha, \phi(x)] \equiv \psi_\alpha(x) \ . \tag{1.14}$$

A new field ψ_α is defined by the action of Q_α on ϕ. In the same multiplet now there are a complex scalar and a Weyl fermion. Acting again with the generators on ψ_α:

$$\{Q_\alpha, \psi_\beta(x)\} = F_{\alpha\beta}(x) \ ; \tag{1.15}$$

$$\{\bar{Q}_{\dot{\alpha}}, \psi_\beta\} = X_{\dot{\alpha}\beta}(x) \ . \tag{1.16}$$

After some calculations one gets:

$$X_{\dot{\alpha}\beta} \sim \partial_\mu \phi \ ; \tag{1.17}$$

$$F_{\alpha\beta}(x) = \epsilon_{\alpha\beta} F(x) \ . \tag{1.18}$$

$F(x)$ is a new scalar field that must be added to the field multiplet. No new fields are introduced with a further step and so all the fields that appear in the supermultiplet constructed starting from ϕ are

$$(\phi, \psi, F) \ . \tag{1.19}$$

This object is called chiral or Wess-Zumino multiplet. This multiplet starts with a complex scalar, whose associated state can be represented by $|0\rangle$. The action of Q_α gave a Weyl fermion, ψ_α. This operations is analogous to the action of $a^\dagger$ on $|0\rangle$ that creates $|1/2\rangle$. The first two fields correspond exactly to the particle states of a chiral supermultiplet. A problem arises when a third field, F, is generated. The degrees of freedom of the collection of fields generated with this procedure are four bosonic, two from ϕ, and two from F and four fermonic from ψ_α. The equivalence of the degrees of freedom is still valid but they are not the ones of the chiral supermultiplet. That multiplet was on-shell. The Weyl fermion looses two d.o.f. thanks to the Dirac equation. Also the bosonic number is diminished of two units, as will be shown later, because F is an auxiliary field. The on-shell numbers are then

$$n_F = n_B = 2 \ . \tag{1.20}$$

A collection of fields related by supersymmetry is called superfield. When on-shell superfields are considered there is a perfect correspondence with the superparticle states.

1.2 Superfields

Superfields are defined as functions of superspace coordinates $(x^\mu, \theta_\alpha, \bar{\theta}_{\dot{\alpha}})$. The Grassmann variables θ and $\bar{\theta}$ have been introduced in order to transform the graded supersymmetry algebra in a Lie algebra with generators:

$$Q_\alpha \to \theta Q \ , \tag{1.21}$$

$$\bar{Q}_{\dot{\alpha}} \to \bar{\theta}\bar{Q} \ . \tag{1.22}$$

Since θ and $\bar{\theta}$ are Grassmann variables, the most general superfield $\mathbf{Y}(x,\theta,\bar{\theta})$ has the following expansion

$$\begin{aligned}\mathbf{Y}(x,\theta,\bar{\theta}) =& f(x) + \theta\psi(x) + \bar{\theta}\bar{\chi}(x) + \theta\theta m(x) + \bar{\theta}\bar{\theta}n(x) \\ &+ \theta\sigma^{\mu}\bar{\theta}v_{\mu}(x) + \theta\theta\bar{\theta}\bar{\lambda}(x) + \bar{\theta}\bar{\theta}\theta\rho(x) + \theta\theta\bar{\theta}\bar{\theta}d(x)\ . \end{aligned} \tag{1.23}$$

The $\mathbf{Y}$ superfield has this name because it is a collection of ordinary fields. A supersymmetry transformation on $\mathbf{Y}$ with parameters $(\epsilon_{\alpha}, \epsilon_{\dot{\alpha}})$ is defined as

$$\mathbf{Y}(x+\delta x, \theta+\delta\theta, \bar{\theta}+\delta\bar{\theta}) = e^{-i(\epsilon\mathcal{Q}+\bar{\epsilon}\bar{\mathcal{Q}})}\mathbf{Y}(x,\theta,\bar{\theta})e^{-i(\epsilon\mathcal{Q}+\bar{\epsilon}\bar{\mathcal{Q}})}\ . \tag{1.24}$$

In this notation $\mathcal{Q}$ is the abstract operator. From the superfield variation is possible to derive the explicit expression for the coordinate variations:

$$\delta x^{\mu} = i\theta\sigma^{\mu}\bar{\epsilon} - i\epsilon\sigma^{\mu}\bar{\theta}\ ; \tag{1.25}$$

$$\delta\theta^{\alpha} = \epsilon^{\alpha}\ ; \tag{1.26}$$

$$\delta\bar{\theta}^{\dot{\alpha}} = \bar{\epsilon}^{\dot{\alpha}}\ . \tag{1.27}$$

The supersymmetry variation of $\mathbf{Y}$ can be written as

$$\delta_{\epsilon,\bar{\epsilon}}\mathbf{Y} = (i\epsilon Q + i\bar{\epsilon}\bar{Q})Y\ , \tag{1.28}$$

with Q the differential supersymmetry operator:

$$Q_{\alpha} = -i\partial_{\alpha} - \sigma^{\mu}_{\alpha\dot{\beta}}\bar{\theta}^{\dot{\beta}}\partial_{\mu}\ , \tag{1.29}$$

$$\bar{Q}_{\dot{\alpha}} = +i\bar{\partial}_{\dot{\alpha}} + \theta^{\beta}\sigma^{\mu}_{\beta\dot{\alpha}}\partial_{\mu}\ . \tag{1.30}$$

In (1.23) there is the general expression for a superfield. The supermultiplets derived above had less component than $\mathbf{Y}$. They can be obtained by imposing some supersymmetric constraint. The most common superfields are the chiral and the vector ones.

1.2.1 Chiral Superfields

In N=1 supersymmetry covariant derivatives D_α and $\bar{D}_{\dot{\alpha}}$ can be constructed in order to anti-commute with Q_α and $\bar{Q}_\beta$ defined above.

$$D_\alpha = \partial_\alpha + i\sigma^\mu_{\alpha\dot{\beta}}\bar{\theta}^{\dot{\beta}}\partial_\mu \; ; \tag{1.31}$$

$$\bar{D}_{\dot{\alpha}} = -\bar{\partial}_{\dot{\alpha}} - i\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu \; . \tag{1.32}$$

Since D and $\bar{D}$ anti-commute with Q and $\bar{Q}$ they also commute with the variation $\delta_{\epsilon,\bar{\epsilon}}$:

$$\delta_{\epsilon,\bar{\epsilon}}(D_\alpha Y) = D_\alpha(\delta_{\epsilon,\bar{\epsilon}} Y) \; . \tag{1.33}$$

This implies that if $\mathbf{Y}$ is a superfield, then also $D_\alpha\mathbf{Y}$ is a superfield. The constraints

$$\bar{D}_{\dot{\alpha}}\mathbf{\Phi} = 0 \; , \tag{1.34}$$

$$D_\alpha\bar{\mathbf{\Phi}} = 0 \; , \tag{1.35}$$

are supersymmetric invariant. A field $\mathbf{\Phi}$ that satisfies the first constraint is called *chiral* while a field $\bar{\mathbf{\Phi}}$ that satisfies the second *anti-chiral*. Finding a general expression for $\mathbf{\Phi}$ is quite simple with the following coordinates redefinition

$$y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta} \; . \tag{1.36}$$

With this coordinates

$$\bar{D}_{\dot{\alpha}}y^\mu = \bar{D}_{\dot{\alpha}}\theta = 0 \; . \tag{1.37}$$

The general expression for a chiral superfield is

$$\mathbf{\Phi} = A(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y) \; . \tag{1.38}$$

In the x^μ coordinates it becomes

$$\begin{aligned}\mathbf{\Phi} =& A(x) + i\theta\sigma^\mu\bar{\theta}\partial_\mu A(x) + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\Box A(x) + \theta\theta F(x) \\ &+ \sqrt{2}\theta\psi(x) - \frac{i}{\sqrt{2}}\theta\theta\partial_\mu\psi(x)\sigma^\mu\bar{\theta} \; .\end{aligned} \tag{1.39}$$

Analogously for an anti-chiral superfield, defining $\bar{y}^\mu = x^\mu - i\theta\sigma^\mu\bar{\theta}$,

$$\bar{\mathbf{\Phi}} = A^*(\bar{y}) + \sqrt{2}\bar{\theta}\bar{\psi}(\bar{y}) + \bar{\theta}\bar{\theta}F^*(\bar{y}) \tag{1.40}$$

$$= A^*(x) - o\theta\sigma^\mu\bar{\theta}\partial_\mu A^*(x) + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\Box A^*(x) + \bar{\theta}\bar{\theta}F^*(x)$$
$$+ \sqrt{2}\bar{\theta}\bar{\psi}(x) + \frac{i}{\sqrt{2}}\bar{\theta}\bar{\theta}\theta\sigma^\mu\partial_\mu\bar{\psi}(x) \,. \tag{1.41}$$

This superfield is equivalent to the field multiplet in (1.19).

A chiral superfield, under supersymmetry transformations, transforms as:

$$\delta_{\epsilon,\bar{\epsilon}}\mathbf{\Phi}(y,\theta) = (i\epsilon Q + i\bar{\epsilon}\bar{Q})\mathbf{\Phi}(y,\theta) \,. \tag{1.42}$$

Q and $\bar{Q}$ have to be expressed in term of the new coordinates:

$$Q = -i\partial_\alpha \,, \tag{1.43}$$

$$\bar{Q} = i\bar{\partial}_\alpha + 2\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\frac{\partial}{\partial y^\mu} \,. \tag{1.44}$$

Plugging these definitions in (1.42) the transformation becomes

$$\delta_{\epsilon,\bar{\epsilon}}\mathbf{\Phi}(y,\theta) = \sqrt{2}\epsilon\psi + \sqrt{2}\theta\left(+\sqrt{2}\epsilon F + \sqrt{2}i\sigma^\mu\bar{\epsilon}\frac{\partial}{\partial y^\mu}A\right) + \theta\theta\left(i\sqrt{2}\bar{\epsilon}\bar{\sigma}^\mu\frac{\partial}{\partial y^\mu}\psi\right) \,. \tag{1.45}$$

The supersymmetry transformations for each component of the chiral superfield multiplet are:

$$\delta_\epsilon A = \sqrt{2}\epsilon\psi \,, \tag{1.46}$$

$$\delta_\epsilon\psi = i\sqrt{2}\sigma^\mu\bar{\epsilon}\partial_\mu A + \sqrt{2}\epsilon F \,, \tag{1.47}$$

$$\delta_\epsilon F = i\sqrt{2}\bar{\epsilon}\bar{\sigma}^\mu\partial_\mu\psi \,. \tag{1.48}$$

The fermionic nature of supersymmetry is easily seen in these transformations. The variations of the scalars A and F are proportional to the spinor ψ while the variation of ψ is proportional to the scalars. From these transformation it is clear why superfields are linear realizations of the supersymmetry. Every field variation depends linearly on the other fields.

1.2.2 Vector Superfields

Vector superfields are defined imposing the condition:

$$\mathbf{V} = \bar{\mathbf{V}}\ . \tag{1.49}$$

Their power series expansion in θ and $\bar{\theta}$ is:

$$\begin{aligned}\mathbf{V}(x,\theta,\bar{\theta}) =& C(x) + i\theta\chi(x) - i\bar{\theta}\bar{\chi} + \frac{i}{2}\theta\theta[M(x)+iN(x)]\\ &- \frac{i}{2}\bar{\theta}\bar{\theta}[M(x)-iN(x)] - \theta\sigma^\mu\bar{\theta}v_\mu(x) + i\theta\theta\bar{\theta}\left[\bar{\lambda}(x) + \frac{i}{2}\bar{\sigma}^\mu\partial_\mu\chi(x)\right]\\ &- i\bar{\theta}\bar{\theta}\left[\lambda(x) + \frac{i}{2}\sigma^\mu\partial_\mu\bar{\chi}(x)\right] + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}\left[D(x) + \frac{1}{2}\Box C(x)\right]\ .\end{aligned} \tag{1.50}$$

The fields C, D, M, N and v_μ are real. Under the following supersymmetric generalization of a gauge transformation,

$$\mathbf{V} \to \mathbf{V} + \mathbf{\Phi} + \bar{\mathbf{\Phi}}\ , \tag{1.51}$$

where $\mathbf{\Phi}$ and $\bar{\mathbf{\Phi}}$ are respectively a chiral and an anti-chiral superfield, the component fields transform as

$$C \to C + A + A^*\ , \tag{1.52}$$

$$\chi \to \chi - i\sqrt{2}\psi\ , \tag{1.53}$$

$$M + iN \to M + iN - 2iF\ , \tag{1.54}$$

$$v_\mu \to v_\mu - i\partial_\mu(A - A^*)\ , \tag{1.55}$$

$$\lambda \to \lambda\ , \tag{1.56}$$

$$D \to D\ . \tag{1.57}$$

$$\tag{1.58}$$

It is possible to choose a gauge, often called the Wess-Zumino gauge, in which C, χ, M and N are all zero. Only a vector field, with a usual gauge transformation, a spinor and a scalar remain:

$$\mathbf{V} = -\theta\sigma^\mu\bar{\theta}v_\mu(x) + i\theta\theta\bar{\theta}\bar{\lambda}(x) - i\bar{\theta}\bar{\theta}\theta\lambda(x) + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D(x)\ . \tag{1.59}$$

The supersymmetry transformations for the component fields are:

$$\delta_\epsilon v^\mu = -i\bar{\lambda}\bar{\sigma}^\mu\epsilon + i\bar{\epsilon}\bar{\sigma}^\mu\lambda\ , \tag{1.60}$$

$$\delta_\epsilon\lambda = \sigma^{\mu\nu}\epsilon(\partial_\mu v_\nu - \partial_\nu v_\mu) + i\epsilon D\ , \tag{1.61}$$

$$\delta_\epsilon D = -\epsilon\sigma^\mu\partial_\mu\bar{\lambda} - \partial\lambda\sigma^\mu\bar{\epsilon}\ . \tag{1.62}$$

Also these transformations, as the chiral ones, are linear in the fields.

1.3 Invariant Actions

The real strength of the superfields formalism is linked to the possibility of building supersymmetric actions in a simple way. If only a set of fields, transforming as in (1.46)-(1.48), were taken in account, it would be problematic to construct an invariant lagrangian. Every time a new term is added at least another one must be taken in account in order to compensate its variation. With superfields it becomes quite easy because if $\mathbf{Y}$ is a superfield, then

$$\mathcal{S} = \int d^4x\, d^2\theta\, d^2\bar{\theta}\mathbf{Y}(x,\theta,\bar{\theta})\ , \tag{1.63}$$

is supersymmetric invariant:

$$\begin{aligned}\delta_{\epsilon,\bar{\epsilon}}\mathcal{S} &= \int d^4x\, d^2\theta\, d^2\bar{\theta}\delta_{\epsilon,\bar{\epsilon}}\mathbf{Y} \\ &= \int d^4x\, d^2\theta\, d^2\bar{\theta}[\epsilon^\alpha\partial_\alpha\mathbf{Y} + \bar{\epsilon}_{\dot{\alpha}}\bar{\partial}^{\dot{\alpha}}\mathbf{Y} + \partial_\mu[-i(\epsilon\sigma^\mu\bar{\theta} - \theta\sigma^\mu\bar{\epsilon})\mathbf{Y}]] = 0\ .\end{aligned} \tag{1.64}$$

The first equality holds because the integral in the Grassmann variables is translational invariant by construction while the last holds because the terms with ∂_α and $\partial^{\dot{\alpha}}$ don't have enough θs and $\partial_\mu[\dots]$ is a total derivative. The integral in the full superspace of a superfield always gives a supersymmetric action, if it is also suitably defined then, by integrating it over the Grassmann coordinates, it is possible to get a Lagrangian density of dimension four, which is real and transforms as a scalars.

$$\mathcal{S} = \int d^4x\, d^2\theta\, d^2\bar{\theta}\mathbf{A}(x,\theta,\bar{\theta}) = \int d^4x \mathcal{L}(\phi(x), \psi(x), A_\mu(x), \dots) . \quad (1.65)$$

1.3.1 Matter actions

Starting from a set of chiral superfields $\mathbf{\Phi}^i$, the combination $\bar{\mathbf{\Phi}}^i\mathbf{\Phi}_i$ has the right dimension for giving a four-dimension lagrangian $\mathcal{L}$. The only contribution comes from the $\theta\theta\bar{\theta}\bar{\theta}$-component of the integrated superfield.

$$\bar{\mathbf{\Phi}}^i\mathbf{\Phi}_i = F^{*i}F_i + \frac{1}{4}A^{*i}\Box A_i + \frac{1}{4}\Box A^{*i}A_i - \frac{1}{2}\partial_\mu A^{*i}\partial^\mu A_i + \frac{i}{2}\partial_\mu\bar{\psi}^i\bar{\sigma}^\mu\psi_i - \frac{i}{2}\bar{\psi}_i\bar{\sigma}^\mu\partial_\mu\psi_i . \quad (1.66)$$

Up to total derivatives, $\mathcal{L}$ is

$$\mathcal{L} = i\partial_\mu\bar{\psi}^i\bar{\sigma}^\mu\psi_i + A^{*i}\Box A_i + F^{*i}F_i . \quad (1.67)$$

This lagrangian contains the canonical kinetic term for a complex scalar and a Weyl spinor. Another kind of contribution, when considering chiral superfields may come from

$$\int d^4x\, d^2\theta\, \Sigma(x,\theta,\bar{\theta}) = \int d^4y\, d^2\theta\, \Sigma(y,\theta) . \quad (1.68)$$

Σ is chiral and the equation holds because $x^\mu = y^\mu$ up to total derivatives. These contributions were not included in the integral in the full superspace because, even if

$$\int d^4x\, d^2\theta\, d^2\bar{\theta}\mathbf{Y} = \int d^4x\, d^2\theta\, \bar{D}^2\mathbf{Y} , \quad (1.69)$$

it is not true that all the integrals in $d^2\theta$ can be written as integrals in $d^2\theta d^2\bar{\theta}$.

Every holomorphic function P of a chiral superfield $\mathbf{\Phi}$, namely a function that satisfies $\frac{\partial P}{\partial \bar{\mathbf{\Phi}}} = 0$, is chiral

$$\bar{D}_{\dot{\alpha}}P(\mathbf{\Phi}) = \frac{\partial P}{\partial \mathbf{\Phi}}\bar{D}_{\dot{\alpha}}\mathbf{\Phi} + \frac{\partial P}{\partial \bar{\mathbf{\Phi}}}\bar{D}_{\dot{\alpha}}\bar{\mathbf{\Phi}} = 0 . \quad (1.70)$$

The contribution to the lagrangian is

$$\mathcal{L}_{int} = \int d^4x\, d^2\theta\, P(\mathbf{\Phi}^i) + \int d^4x\, d^2\bar{\theta}\, \bar{P}(\bar{\mathbf{\Phi}}^i) = +\frac{\partial P}{\partial \mathbf{\Phi}^i} F^i - \frac{1}{2}\frac{\partial^2 P}{\partial \mathbf{\Phi}^i \partial \mathbf{\Phi}^j}\psi^i\psi^j + \text{h.c.}\,. \tag{1.71}$$

The derivatives are evaluated at $\mathbf{\Phi}^i = A^i$. The superpotential P has to satisfy some simple properties. First, as said before it has to be holomorphic, then it can not contain covariant derivatives since $D_\alpha\Phi$ is not chiral. Finally, P has to have dimension three in order to have a lagrangian of dimension four. This means that, for having renormalizable theories, P can be at most cubic in Φ^i. The lagrangian for matter fields is given by

$$\mathcal{L} = \int d^4x\, d^2\theta\, d^2\bar{\theta}\, \bar{\mathbf{\Phi}}^i \mathbf{\Phi}_i + \int d^4x\, d^2\theta\, P(\mathbf{\Phi}^i) + \int d^4x\, d^2\bar{\theta}\, \bar{P}(\bar{\mathbf{\Phi}}^i) \tag{1.72}$$

$$= i\partial_\mu \bar{\psi}^i \bar{\sigma}^\mu \psi_i + A^{*i}\Box A_i + F^{*i}F_i + \left(\frac{\partial P}{\partial A^i}F^i - \frac{1}{2}\frac{\partial^2 P}{\partial A^i \partial A^j}\psi^i\psi^j + \text{h.c.}\right). \tag{1.73}$$

This lagrangian is invariant, up to total derivatives, under the transformations (1.46)-(1.48). Furthermore there are not derivatives of the F fields so they are auxiliary fields and they can be integrated out:

$$F^{*i} = -\frac{\partial P}{\partial A_i}\,, \qquad F^i = -\frac{\partial \bar{P}}{\partial A_i^*}\,. \tag{1.74}$$

The on-shell Lagrangian obtained thanks to the F^i equations of motion is

$$\mathcal{L} = i\partial_\mu \bar{\psi}^i \bar{\sigma}^\mu \psi_i + A^{*i}\Box A_i - \left(\frac{1}{2}\frac{\partial^2 P}{\partial A^i \partial A^j}\psi^i\psi^j + \text{h.c.}\right) - V(A^i, A^{*i})\,. \tag{1.75}$$

The scalar potential $V(A^i, A^{*i})$ is

$$V = \left|\frac{\partial P}{\partial A^i}\right|^2\,. \tag{1.76}$$

It is important to underline that the interactions that are present in the on-shell lagrangian are due to the equation of motion for F. With a vanishing F-term there would be no interactions and masses but only the kinetic terms.

1.3.2 Vector Actions

The vector superfield V can be seen as a generalization of the Yang-Mills potential. The generalization of the field strength can be defined as:

$$W_\alpha = -\frac{1}{4}\bar{D}\bar{D}D_\alpha V \; , \qquad \bar{W}_{\dot{\alpha}} = -\frac{1}{4}DD\bar{D}_{\dot{\alpha}}V \; . \tag{1.77}$$

These superfields are chiral and gauge invariant:

$$\bar{D}_{\dot{\alpha}}W_\alpha = 0 \; , \; D_\alpha \bar{W}_{\dot{\alpha}} = 0 \; , \tag{1.78}$$

$$W_\alpha \to -\frac{1}{4}\bar{D}\bar{D}D_\alpha(V + \mathbf{\Phi} + \bar{\mathbf{\Phi}}) = W_\alpha - \frac{1}{4}\bar{D}\{\bar{D}, D_\alpha\}\mathbf{\Phi} = W_\alpha \; . \tag{1.79}$$

In the Wess-Zumino gauge they have a simple expression:

$$W_\alpha = -i\lambda_\alpha(y) + \left[\delta_\alpha{}^\beta D(y) - \frac{i}{2}(\sigma^\mu\bar{\sigma}^\nu)_\alpha{}^\beta(\partial_\mu v_\nu(y) - \partial_\nu v_\mu(y))\right]\theta_\beta + \theta\theta\sigma_{\alpha\dot{\alpha}}{}^\mu\partial_\mu\bar{\lambda}^{\dot{\alpha}}(y) \; ; \tag{1.80}$$

$$\bar{W}_{\dot{\alpha}} = -i\bar{\lambda}_{\dot{\alpha}}(\bar{y}) + \left[\epsilon_{\dot{\alpha}\dot{\beta}}D(\bar{y}) + \frac{i}{2}\epsilon_{\dot{\alpha}\dot{\gamma}}(\bar{\sigma}^\mu\sigma^\nu)^{\dot{\gamma}}{}_{\dot{\beta}}(\partial_\mu v_\nu(\bar{y}) - \partial_\nu v_\mu(\bar{y}))\right]\bar{\theta}^{\dot{\beta}} - \epsilon_{\dot{\alpha}\dot{\beta}}\bar{\theta}\bar{\theta}\bar{\sigma}^{m\dot{\beta}\alpha}\partial_\mu\lambda_\alpha(\bar{y}) \; . \tag{1.81}$$

In the chiral superfields W_α and $\bar{W}_{\dot{\alpha}}$ there are only the gauge invariant fields, λ_α and D, and the gauge invariant field strength $v_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu$. Since W_α is chiral an invariant action can be obtained from the integration in $d^2\theta$ of $W^\alpha W_\alpha$:

$$\mathcal{S} = \int d^4x\mathcal{L} = \frac{1}{4}\int d^4x\left(\int d^2\theta W^\alpha W_\alpha + \int d^2\bar{\theta}W_{\dot{\alpha}}W^{\dot{\alpha}}\right) \; ; \tag{1.82}$$

$$= \int d^4x\frac{1}{2}D^2 - \frac{1}{4}v^{\mu\nu}v_{\mu\nu} - i\lambda\sigma^\mu\partial_\mu\bar{\lambda} \; . \tag{1.83}$$

In this lagrangian there are the kinetic terms for a gauge vector, v_μ, and for a spinor, λ_α. The lagrangian above can be written as an integral in $d^2\theta d^2\bar{\theta}$ thanks to the definition and the chirality of W_α:

$$\mathcal{L} = \int d^2\theta d^2\bar{\theta}\frac{1}{4}(W^\alpha D_\alpha V + \bar{W}_{\dot{\alpha}}\bar{D}^{\dot{\alpha}}V) \; . \tag{1.84}$$

Mass term can be added but they aren't gauge invariant and so they ca not be expressed in the Wess-Zumino gauge.

$$\mathcal{L}_m = \int d^2\theta d^2\bar{\theta} V^2 \,. \tag{1.85}$$

1.3.3 Vector-Matter Interactions

In the standard model, when the matter lagrangian is globally invariant under some symmetry group, a set of vectors must be introduced in order to have local symmetry and these vectors interacts with the matter fields. The procedure for gauging chiral superfields is analogous. If a chiral lagrangian is invariant under the global action of a group with generators $\{T^a\}$ then introducing a vector superfields V_a this invariance can be preserved also locally. The gauge invariant lagrangian is:

$$\begin{aligned}\mathcal{L} =& \frac{1}{16kg^2}\left(H_{ab}\int d^2\theta W^a W^b + \text{h.c.}\right) + \int d^2\theta d^2\bar{\theta}\, \bar{\mathbf{\Phi}} e^V \mathbf{\Phi} \\ &+ \xi_A \int d^2\theta\, d^2\bar{\theta} V^A + \int d^4x d^2\theta\, P(\mathbf{\Phi}^i) + \int d^4x d^2\bar{\theta}\, \bar{P}(\bar{\mathbf{\Phi}}^i) \,. \end{aligned}\tag{1.86}$$

The contribution proportional to ξ_A is the Fayet-Iliopulos term and it is present only for the abelian factors, H_{ab} is a holomorphic function of $\mathbf{\Phi}$ and obviously P has to be gauge invariant.The explicit off-shell lagrangian, with $H_{ab} = \delta_{ab}$, is:

$$\begin{aligned}\mathcal{L} =& -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - i\bar{\lambda}^a\bar{\sigma}^\mu \mathscr{D}_\mu \lambda^a + \frac{1}{2}D^a D_a - \mathscr{D}_\mu A^\dagger \mathscr{D}^\mu A - i\bar{\psi}\bar{\sigma}^\mu \mathscr{D}_\mu \psi \\ &+ F^\dagger F + i\sqrt{2}g(A^\dagger T^a \psi\lambda^a - \bar{\lambda}^a T^a A\bar{\psi}) + gD_a A^\dagger T^a A + g\xi_A D^A \\ &+ \left(\frac{\partial P}{\partial A^i}F^i - \frac{1}{2}\frac{\partial^2 P}{\partial A^i \partial A^j}\psi^i\psi^j + \text{h.c.}\right) \,. \end{aligned}\tag{1.87}$$

where

$$\mathscr{D}_\mu A = \partial_\mu A + igv^a_\mu T^a A \,, \tag{1.88}$$

$$\mathscr{D}_\mu \psi = \partial_\mu \psi + igv^{(a)}_\mu T^{(a)} \psi \,, \tag{1.89}$$

$$\mathscr{D}\lambda^{(a)} = \partial_\mu \lambda^{(a)} - gt^{abc}v^{(b)}_\mu \lambda^{(c)} \,, \tag{1.90}$$

$$F^{(a)}_{\mu\nu} = \partial_\mu v^{(a)}_\nu - \partial_\nu v^{(a)}_\mu - gt^{abc}v^{(b)}_\mu v^{(c)}_\nu \,. \tag{1.91}$$

The transformation laws for the components of the superfield multiplets are linear and their expressions are:

$$\delta_\epsilon A = \sqrt{2}\epsilon\psi \ , \tag{1.92}$$

$$\delta_\epsilon \psi = i\sqrt{2}\sigma^\mu \bar{\epsilon}\mathscr{D}_\mu A + \sqrt{2}\epsilon F \ , \tag{1.93}$$

$$\delta_\epsilon F = i\sqrt{2}\bar{\epsilon}\bar{\sigma}^\mu \mathscr{D}_\mu \psi + i2gT^{(a)}A\bar{\epsilon}\bar{\lambda}^{(a)} \ , \tag{1.94}$$

$$\delta_\epsilon v_\mu^{(a)} = -i\bar{\lambda}^{(a)}\bar{\sigma}^\mu \epsilon + i\bar{\epsilon}\bar{\sigma}^\mu \lambda^{(a)} \ , \tag{1.95}$$

$$\delta_\epsilon \lambda^{(a)} = \sigma^{\mu\nu}\epsilon v_{\mu\nu}^{(a)} + i\epsilon D^{(a)} \ , \tag{1.96}$$

$$\delta_\epsilon D^{(a)} = -\epsilon\sigma^\mu \mathscr{D}_\mu \bar{\lambda}^{(a)} - \mathscr{D}\lambda^{(a)}\sigma^\mu \bar{\epsilon} \ . \tag{1.97}$$

The equations of motion for the auxiliary fields are:

$$F = -\frac{\partial \bar{P}}{\partial A^\dagger} \ , \tag{1.98}$$

$$F^\dagger = -\frac{\partial P}{\partial A} \ , \tag{1.99}$$

$$D^a = -gA^\dagger T^a A - g\xi^a \ . \tag{1.100}$$

The ξ_a contribution to D_a is present only for the Abelian factors. The on-shell lagrangian is

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - i\bar{\lambda}^a\bar{\sigma}^\mu \mathscr{D}_\mu \lambda^a - \mathscr{D}_\mu A^\dagger \mathscr{D}^\mu A - i\bar{\psi}\bar{\sigma}^\mu \mathscr{D}_\mu \psi \\ &+ i\sqrt{2}g(A^\dagger T^a \psi\lambda^a - \bar{\lambda}^a T^a A\bar{\psi}) - \left(\frac{1}{2}\frac{\partial^2 P}{\partial A^i \partial A^j}\psi^i\psi^j + \text{h.c.}\right) - V \ ,\end{aligned} \tag{1.101}$$

where the scalar potential V is:

$$V = \left|\frac{\partial P}{\partial A}\right|^2 + \frac{g^2}{2}|A^\dagger T^a A + \xi^a|^2 \ . \tag{1.102}$$

The scalar potential is always positive in a supersymmetry invariant gauge theory.

1.3.4 Kähler Chiral Models

In the previous description only $\bar{\mathbf{\Phi}}^i \mathbf{\Phi}_i$ appeared in the full superspace integral. This can be generalized integrating an analytic functions of the superfields $K(\mathbf{\Phi}^i, \bar{\mathbf{\Phi}}^j)$. This function has to satisfy all the condition necessary to give a meaningful lagrangian and it has an important property:

$$\int d^4x\, d^2\theta\, d^2\bar{\theta}\, K(\mathbf{\Phi}^i, \bar{\mathbf{\Phi}}^j) = \int d^4x\, d^2\theta\, d^2\bar{\theta} [K(\mathbf{\Phi}^i, \bar{\mathbf{\Phi}}^j) + \Lambda(\mathbf{\Phi}) + \bar{\Lambda}(\bar{\mathbf{\Phi}})] \,, \tag{1.103}$$

where Λ is a chiral superfield that depends only on $\mathbf{\Phi}$. This relation is true because the $\theta^2\bar{\theta}^2$ term of the variation is a total derivative. This kind of transformation is called Kähler transformation and $K(a^i, a^{*i})$ is called Kähler potential. It describes a manifold with metric:

$$g_{ij^*} = \frac{\partial K}{\partial a^i \partial a^{j*}} \,. \tag{1.104}$$

Obviously the metric is hermitian, positive defined and invariant under the Kähler transformations:

$$K(a^i, a^{*i}) \to K(a^i, k^{*i}) + F(a^i) + \bar{F}(a^{*i}) \,. \tag{1.105}$$

The only non vanishing Christoffel symbols in a Kähler geometry are:

$$\Gamma^k_{ij} = g^{kl^*} \frac{\partial}{\partial a^i} g_{jl^*} \,, \qquad \Gamma^{k^*}_{i^*j^*} = g^{lk^*} \frac{\partial}{\partial a^{i^*}} g_{lj^*} \,. \tag{1.106}$$

The covariant derivative on this manifold is defined as

$$\nabla_i V_j = \partial_i V_j - \Gamma^k_{ij} V_k \,. \tag{1.107}$$

The curvature of a Kähler metric is defined as

$$[\nabla_i, \nabla_{j^*}] V_k = R^l_{ij^*k} V_l \,. \tag{1.108}$$

The explicit expression for the curvature is

$$R_{ij^*kl^*} = g_{ml^*} \frac{\partial}{\partial a^{*j}} \Gamma^m_{ik} \,. \tag{1.109}$$

By using this formalism for integrating $K(\mathbf{\Phi}^i, \bar{\mathbf{\Phi}}^j)$ in $d^2\theta d^2\bar{\theta}$, the resulting lagrangian is:

$$\begin{aligned}\mathcal{L} =& g_{ij^*}F^iF^{*j} + \frac{1}{4}g_{ij^*,kl^*}\chi^i\chi^k\bar{\chi}^j\bar{\chi}^l - F^i\left\{\frac{1}{2}g_{im^*}\Gamma^{m^*}_{j^*k^*}\bar{\chi}^j\bar{\chi}^k - \frac{\partial P}{\partial A^i}\right\}\\ &- F^{*i}\left\{\frac{1}{2}g_{mi^*}\Gamma^m_{jk}\chi^j\chi^k - \frac{\partial P^*}{\partial A^{*i}}\right\} - g_{ij^*}\partial_\mu A^i\partial^\mu A^{*j} - ig_{ij^*}\bar{\chi}^j\bar{\sigma}^\mu D_\mu\chi^i\\ &- \frac{1}{2}\frac{\partial^2 P}{\partial A^i\partial A^j}\chi^i\chi^j - \frac{1}{2}\frac{\partial^2 P^*}{\partial A^{*i}\partial A^{*j}}\bar{\chi}^i\bar{\chi}^j\ . \end{aligned} \tag{1.110}$$

Here $D_\mu\chi^i = \partial_\mu\chi^i + \Gamma^i_{jk}\partial_\mu A^j\chi^k$. The equations of motion for F^i are

$$g_{ij^*}F^i - \frac{1}{2}g_{kj^*}\Gamma^k_{ml}\chi^m\chi^l + \frac{\partial P^*}{\partial A^{*j}} = 0\ . \tag{1.111}$$

The on-shell lagrangian becomes

$$\begin{aligned}\mathcal{L} =& - g_{ij^*}\partial_\mu A^i\partial^\mu A^{*j} - ig_{ij^*}\bar{\chi}^j\bar{\sigma}^\mu D_\mu\chi^i + \frac{1}{4}R_{ij^*kl^*}\chi^i\chi^k\bar{\chi}^j\bar{\chi}^l\\ &- \frac{1}{2}D_iD_jP\chi^i\chi^j - \frac{1}{2}D_{i^*}D_{j^*}P^*\bar{\chi}^i\bar{\chi}^j - g^{ij^*}D_iPD_{j^*}P^*\ ,\end{aligned} \tag{1.112}$$

where

$$D_iP = \frac{\partial}{\partial A^i}P\ , \tag{1.113}$$

$$D_iPD_jP = \frac{\partial^2}{\partial A^i\partial A^j}P - \Gamma^k_{ij}\frac{\partial}{\partial A^k}P\ . \tag{1.114}$$

Kähler isometries

Since we are dealing with a Kähler manifold there can be analytic isometries that have to be gauged including vector fields. These isometries are generated by holomorphic Killing vectors,

$$X^{(b)} = X^{i(b)}(a^j)\frac{\partial}{\partial a^i}\ , \tag{1.115}$$

$$X^{*(b)} = X^{*i(b)}(a^{*j})\frac{\partial}{\partial a^{*i}}\ . \tag{1.116}$$

Here $(b) = 1, \ldots, d$, where d is the dimension of the isometry group. The Killing equations, for a Kähler manifold, imply the existence of d real scalar function $D^{(a)}(a, a^*)$ such that

$$g_{ij^*} X^{*j(a)} = i \frac{\partial}{\partial a^i} D^{(a)} \ , \tag{1.117}$$

$$g_{ij^*} X^{i(a)} = -i \frac{\partial}{\partial a^{*j}} D^{(a)} \ . \tag{1.118}$$

The killing potentials $D^{(a)}$ are defined modulo a constant $c^{(a)}$. The Killing vectors are a representation of the isometry group:

$$[X^{(a)}, X^{(b)}] = -\ f^{abc} X^{(c)} \ ; \tag{1.119}$$

$$[X^{*(a)}, X^{*(b)}] = -\ f^{abc} X^{*(c)} \ ; \tag{1.120}$$

$$[X^{(a)}, X^{*(b)}] = 0 \ . \tag{1.121}$$

Also $D^{(a)}$ can be chosen to transform in the adjoint representation of the isometry group and this fixes the $c^{(a)}$ for non-Abelian group. For abelian $U(1)$ factors the constants $c^{(a)}$ are undetermined. These constants are related to the Fayet-Iliopulos terms. The variations of the Kähler potential and of the superpotential are:

$$\delta K = [\epsilon^{(a)} X^{(a)} + \epsilon^{*(a)} X^{*(a)}] K \ ; \tag{1.122}$$

$$\delta P = \epsilon^{(a)} X^{(a)} P \ . \tag{1.123}$$

Since the action must be invariant the variation of P has to vanish. The function $F^{(a)} = X^{(a)} K + i D^{(a)}$ satisfy $\partial_{j^*} F^{(a)} = 0$ and so

$$\delta K = \epsilon^{(a)} F^{(a)} + \epsilon^{*(a)} F^{*(a)} - i(\epsilon^{(a)} - \epsilon^{*(a)}) D^{(a)} \tag{1.124}$$

is a Kähler transformation for real parameters $\epsilon^{(a)}$. There is no need to make this term vanish because the action is invariant under Kähler transformations. By promoting the global symmetry to a local symmetry the parameters $\epsilon^{(a)}$ become chiral fields $\Lambda^{(a)}$ that are complex. The K variation is not a Kähler transformation anymore because of the term proportional to $D^{(a)}$ and so a new terms must be added for canceling it. The counterterms involve a vector superfield $V = V^{(a)} T^{(a)}$, where the $T^{(a)}$ are the generators of the isometry group. The $D^{(a)}$ components of the $V^{(a)}$ superfields are exactly the Killing potential defined above. After some calculation the explicit expression for a Kähler gauge invariant model is

$$\begin{aligned}\mathcal{L} =& - g_{ij^*}\mathscr{D}_\mu A^i \mathscr{D}^\mu A^{*j} - i\lambda^{(a)}\sigma^\mu \mathscr{D}_\mu \bar{\lambda}^{(a)} - \frac{1}{2}g^2 D^2 - ig_{ij^*}\chi^i\sigma^\mu\mathscr{D}_\mu\bar{\chi}^j \\ & - \frac{1}{2}F^{(a)}_{\mu\nu}F^{\mu\nu(a)} + g\sqrt{2}g_{ij^*}[X^{i(a)}\bar{\chi}^j\bar{\lambda}^{(a)} + X^{*j(a)}\chi^i\lambda^(a)] - \frac{1}{2}D_iD_jP\chi^i\chi^j \\ & - \frac{1}{2}D_{i^*}D_{j^*}P^* - g^{ij^*}D_iPD_{j^*}P^* + \frac{1}{4}R_{ij^*kl^*}\chi^i\chi^k\bar{\chi}^j\bar{\chi}^l \ . \end{aligned} \tag{1.125}$$

where

$$\mathscr{D}_\mu A^i = \partial_\mu A^i - g v^{(a)}_\mu X^{i(a)} \ , \tag{1.126}$$

$$\mathscr{D}_\mu \chi^i = \partial_\mu \chi^i + \Gamma^i_{jk}\mathscr{D}_\mu A^j \chi^k - g v^{(a)}_\mu \frac{\partial X^{i(a)}}{\partial A^j}\chi^j \ , \tag{1.127}$$

$$\mathscr{D}\lambda^{(a)} = \partial_\mu \lambda^{(a)} - g f^{abc} v^{(b)}_\mu \lambda^{(c)} \ , \tag{1.128}$$

$$D_i P = \frac{\partial}{\partial A^i}P \ , \tag{1.129}$$

$$D_i P D_j P = \frac{\partial^2}{\partial A^i \partial A^j}P - \Gamma^k_{ij}\frac{\partial}{\partial A^k}P \ . \tag{1.130}$$

The transformation laws are:

$$\delta A^i = \epsilon^{(a)} X^{i(a)} \ , \tag{1.131}$$

$$\delta \chi^i = \epsilon^{(a)}\frac{\partial X^{i(a)}}{\partial A^j}\chi^j \ , \tag{1.132}$$

$$\delta \lambda^{(a)} = f^{abc}\epsilon^{(b)}\lambda^{(c)} \ , \tag{1.133}$$

$$\delta v^{(a)}_\mu = g^{-1}\partial_\mu \epsilon^{(a)} + f^{abc}\epsilon^{(b)} v^{(c)}_m \ . \tag{1.134}$$

1.4 Supersymmetry breaking

In order to be compatible with experimental data supersymmetry must be broken at least at the TeV scale. Spontaneous symmetry breaking is a scenario in which the theory is supersymmetric but the scalar potential admits a supersymmetry breaking vacuum. On a vacuum, respecting Lorentz invariance, all the fields but the scalars have vanishing VEV and VEV's derivatives and so the transformation laws are

$$\begin{aligned}&\delta\langle\phi^i\rangle = 0\ , \qquad \delta\langle F^i\rangle = 0\ , \qquad \delta\langle\psi^i\rangle \sim \epsilon\langle F^i\rangle\ ,\\ &\delta\langle F^a_{\mu\nu}\rangle = 0\ , \qquad \delta\langle D^a\rangle = 0\ , \qquad \delta\langle\lambda^a\rangle \sim \epsilon\langle D^a\rangle\ .\end{aligned} \tag{1.135}$$

If the F and D expectation values vanish, the vacuum is supersymmetric, otherwise it breaks supersymmetry. Since the scalar potential is positive defined

$$V = \bar{F}F + \frac{1}{2}D^2\ , \qquad \bar{F}^i = \frac{\partial W}{\partial\phi^i}\ , \qquad D^a = -g(\bar{\phi}^i (T^a)^i{}_j \phi^j + \xi^a)\ , \tag{1.136}$$

then supersymmetric vacua are global minima of the potential and V vanishes on them. In a supersymmetry breaking vacuum $V \neq 0$ and the potential VEV is related to the supersymmetry breaking scale. The contributions from the F and D terms to the masses of the particles of the theory are:

- vector mass matrix

$$[(\mathcal{M}_1)^2]^{ab} = 2g^2\langle A^\dagger T^a T^b A\rangle = 2\langle D^a_i\rangle\langle D^{bi}\rangle\ , \tag{1.137}$$

  where $D^a_i = \partial D^a/\partial A^i$;

- fermionic mass matrix

$$\mathcal{M}_{1/2} = \begin{pmatrix} \langle F_{ij}\rangle & \sqrt{2}i\langle D^b_i\rangle \\ \sqrt{2}i\langle D^a_j\rangle & 0 \end{pmatrix}\ , \tag{1.138}$$

  where $F_{ij} = \partial F/\partial A^i \partial A^j$;

- scalar mass matrix

$$(\mathcal{M}_0)^2 = \begin{pmatrix} \langle\frac{\partial^2 V}{\partial A^i \partial A^{*j}}\rangle & \langle\frac{\partial^2 V}{\partial A^i A^l}\rangle \\ \langle\frac{\partial^2 V}{\partial A^{*j}\partial A^{*k}}\rangle & \langle\frac{\partial^2 V}{\partial A^{*j}\partial A^l}\rangle \end{pmatrix}\ . \tag{1.139}$$

These matrices satisfy the supertrace mass formula

$$\mathrm{STr}\mathcal{M}^2 = -2g\langle D^a\rangle \mathrm{Tr}T^a\ . \tag{1.140}$$

Every time supersymmetry is broken the fermionic mass matrix has a zero eigenvalue:

$$\mathscr{M}_{1/2} \begin{pmatrix} \langle F^i \rangle \\ -\frac{i}{\sqrt{2}} \langle D^a \rangle \end{pmatrix} = \begin{pmatrix} \langle F_{ij} \rangle \langle F^j \rangle + \langle D_i^b \rangle D_b \\ \langle D_j^a \rangle \langle F^j \rangle \end{pmatrix} = \begin{pmatrix} \frac{\partial V}{\partial A^i} \\ \delta W^a \end{pmatrix} = 0 \,. \tag{1.141}$$

The goldstino ψ^G can be defined as the massless fermion:

$$\begin{pmatrix} \psi^i \\ \lambda^a \end{pmatrix} = \begin{pmatrix} F^i \\ -\frac{i}{\sqrt{2}} D^a \end{pmatrix} \psi^G + \psi^{G\perp} \,. \tag{1.142}$$

Its explicit form is $\psi^G \sim F^i + \frac{i}{\sqrt{2}} D^a$.

The Goldstone theorem implies the existence of a massless scalar in the spectrum every time a symmetry generator is broken. Since supersymmetry generators are fermionic, the extension of the Goldstone theorem to supersymmetry breaking implies the existence of a massless spinor, the goldstino.

Chapter 2

Non-Linear SUSY and Constrained Superfields

In this chapter we describe three different approaches to construct models at energies much below the supersymmetry breaking scale. Some properties of linearly realized supersymmetry, such as the equivalence of the bosonic and fermionic numbers, the mass degeneracy for the fields in the same multiplet and the Kähler geometry of the scalar σ-model, are lost in these models but the superfields formalism is still present in two of them and it will be clear that constraining superfields is the easiest way to obtain non-linear representations.

2.1 Why Non-Linear Realizations?

There is no experimental evidence of linearly realized supersymmetry and so the main focus is on effective supersymmetric lagrangians. In order to have a better comprehension of what happens with supersymmetry it is useful to discuss briefly non linear realizations of a simple bosonic global symmetry. Let us consider a model with four real scalars ϕ^i with $SO(4)$ symmetry:

$$\mathcal{L} = -\frac{1}{2}\partial_\mu \phi_i \partial^\mu \phi^i - \frac{\mu^2}{2}\phi_i\phi^i - \frac{\lambda}{4}(\phi_i\phi^i)^2 \; . \tag{2.1}$$

If $\mu^2 < 0$ the vacuum choice breaks the symmetry $SO(4) \to SO(3)$. The choice of the vacuum is arbitrary and we consider the following vacuum:

$$\langle \phi_{1,2,3} \rangle = 0 \ , \quad \langle \phi_4 \rangle = \sqrt{-\frac{\mu^2}{\lambda}} = v \ . \tag{2.2}$$

With the following parametrization

$$\phi_i = \Pi_i(v + \rho(x)) \ , \quad \Pi_i \Pi^i = 1 \to \Pi_4 = \sqrt{1 - \Pi_I^2} \ , \tag{2.3}$$

with $I = 1, 2, 3$. Under the initial $SO(4)$ symmetry the scalar fields Π^I transform as

$$\Pi'_I = \Lambda_I^J \Pi_J + \Lambda_I^4 \sqrt{1 - \Pi_K^2} \ . \tag{2.4}$$

Their transformation laws are non-linear. The lagrangian, after the redefinitions, becomes

$$\mathcal{L} = -\frac{1}{2}[(v+\rho)^2 g_{IJ} \partial_\mu \Pi^I \partial^\mu \Pi^J + \partial_\mu \rho \partial^\mu \rho] - \lambda v^2 \rho^2 - \lambda v \rho^3 - \frac{\lambda}{4}\rho^4 \ . \tag{2.5}$$

The expression for g_{IJ} is

$$g_{IJ} = \delta_{IJ} + \frac{\Pi_I \Pi_J}{1 - \Pi_k^2} \ . \tag{2.6}$$

The only massive scalar is ρ, $m_\rho = \sqrt{2\lambda} v$, and if $v \to \infty$, then ρ can be integrated out. The equations of motion for ρ in the vacuum are satisfied for $\rho = 0$. After the substitution $\Pi_I \to \Pi_I / v$ the effective lagrangian becomes

$$\mathcal{L} = -\frac{1}{2}\left[\delta_{IJ} - \frac{1}{v^2}\frac{\Pi_I \Pi_J}{1 - \frac{\Pi_K^2}{v^2}}\right] \partial_\mu \Pi^I \partial^\mu \Pi^J \ . \tag{2.7}$$

This lagrangian can be expanded in a series is $1/v^2$ for $v \to \infty$. The first terms are:

$$\mathcal{L} = -\frac{1}{2}\partial_\mu \Pi_I \partial^\mu \Pi^I + \frac{1}{2v^2}(\Pi_I \partial_\mu \Pi^I)^2 + \dots \ . \tag{2.8}$$

This lagrangian can have infinite contributions from the expansion as a power series of $1/v^2$. All the contributions are $SO(3)$ invariant. The rescaled Π_I transform under the original $SO(4)$ symmetry group as:

$$\Pi'_I = \Lambda_I^J \Pi_J + v\Lambda_I^4 \sqrt{1 - \frac{\Pi_K^2}{v^2}} \ . \tag{2.9}$$

After the symmetry breaking the action of $SO(4)$ is not linear on the remaining fields and this is a general feature when a symmetry is broken. The lagrangian (2.7) is invariant under the non-linear $SO(4)$ transformations in (2.9). The approach in this section starts from a symmetry that is broken in a vacuum and it shows that integrating out a massive field gives an effective lagrangian with infinite contributions from the massless fields. A different approach is to consider the non-linear transformation laws and to build an invariant lagrangian starting from them. In the model considered in this section the effective lagrangian has to be invariant under linear $SO(3)$ and non-linear $SO(4)$. All the terms that satisfy this constraints can be inserted in the effective lagrangian.

2.2 An Historical approach

The first attempt to write an effective lagrangian in which supersymmetry was not-linearly realized was done following the general method illustrated by Callan, Coleman, Wess and Zumino in [1]. This approach is different from the one in the previous section because the effective lagrangian is built starting from non-linear supersymmetry transformations. This procedure starts considering the supersymmetry coordinates transformations:

$$x' = x + i(\theta\sigma\bar{\epsilon} - \epsilon\sigma\bar{\theta}) \ , \tag{2.10}$$

$$\theta' = \theta + \epsilon \ , \tag{2.11}$$

$$\bar{\theta}' = \bar{\theta}' + \bar{\epsilon} \ . \tag{2.12}$$

Introducing an arbitrary spinor field $\lambda(x)$ analogous to θ such as $\theta = k\lambda$ the transformations above become:

$$\lambda'(x') = \lambda(x) + \frac{1}{k}\epsilon \ , \tag{2.13}$$

$$\bar{\lambda}'(x') = \bar{\lambda}(x) + \frac{1}{k}\bar{\epsilon}\,. \tag{2.14}$$

The variation of λ at the same point is

$$\delta_\epsilon \lambda^\alpha = \lambda'^\alpha(x) - \lambda^\alpha(x) = \frac{1}{k}\epsilon^\alpha - ik(\lambda\sigma^\mu\bar{\epsilon} - \epsilon\sigma^\mu\bar{\lambda})\partial_\mu\lambda^\alpha\,. \tag{2.15}$$

Since

$$(\delta_\eta\delta_\epsilon - \delta_\epsilon\delta_\eta)\lambda^\alpha = -2i(\eta\sigma^\mu\bar{\epsilon} - \epsilon\sigma^\mu\bar{\eta})\partial_\mu\lambda^\alpha\,, \tag{2.16}$$

then the transformation law above realizes non-linearly the supersymmetry algebra. In (2.15) there are the non-linear transformation for λ we were interested in. Now we have to construct an effective lagrangian for λ that is invariant under that transformation. By using differential forms the coordinates transformations can be written as:

$$dx'^\mu = dx^\mu + id\theta\sigma^\mu\bar{\epsilon} - i\epsilon\sigma^\mu d\bar{\theta}\,; \tag{2.17}$$

$$d\theta'^\alpha = d\theta^\alpha\,; \tag{2.18}$$

$$d\bar{\theta}'_{\dot{\alpha}} = d\bar{\theta}_{\dot{\alpha}}\,. \tag{2.19}$$

The following combinations of differentials are invariant under the above transformations

$$e^\mu = dx^\mu - id\theta\sigma^\mu\bar{\theta} + i\theta\sigma^\mu d\bar{\theta}\,, \tag{2.20}$$

$$e^\alpha = d\theta^\alpha\,, \tag{2.21}$$

$$e_{\dot{\alpha}} = d\bar{\theta}_{\dot{\alpha}}\,. \tag{2.22}$$

In terms of λ, e^μ becomes

$$e^a \to dx^\mu[\delta^a_\mu - ik^2\partial_\mu\lambda\sigma^a\bar{\lambda} + ik^2\lambda\sigma^a\partial_\mu\bar{\lambda}] = dx^\mu A^a_\mu\,. \tag{2.23}$$

By considering the expression for the invariant quantity e^a, an invariant lagrangian under the non-linear transformation for λ may be:

$$\mathscr{L} = -\frac{1}{2k^2}\det A\,. \tag{2.24}$$

This lagrangian, known as Volkov-Akulov lagrangian [2], describes a massless spinor:

$$\mathcal{L} = -\frac{1}{2k^2} - \frac{i}{2}(\lambda\sigma^\mu\partial_\mu\bar{\lambda} - \partial_\mu\lambda\sigma^\mu\bar{\lambda}) + [\text{interactions}]\,. \tag{2.25}$$

The constant term is related to a non vanishing scalar potential in a linear realization of supersymmetry and so supersymmetry is spontaneously broken for non-linear realization.

2.3 Low Energy Lagrangians

In this section the most common way for obtaining effective lagrangian for supersymmetric theories is described trough some simple examples. The procedure is analogous to the one introduced for the $SO(4) \to SO(3)$ symmetry breaking discussed above. Expanding around a non-supersymmetric vacua some fields acquire mass and they can be integrated out leading to an effective lagrangian. The remaining fields transform non-linearly under the original supersymmetry action.

2.3.1 One Chiral Field

The first model describes one chiral field $\mathbf{X}$ with the following Kähler potential and superpotential:

$$K = \bar{\mathbf{X}}\mathbf{X} - \frac{1}{\Lambda^2}(\bar{\mathbf{X}}\mathbf{X})^2, \qquad W = f\mathbf{X}. \tag{2.26}$$

For simplicity $\Lambda, f \in \mathbb{R}$. Λ is a very high energy scale and f is related to the supersymmetry breaking energy scale as will be shown later and so 1TeV$< \sqrt{f} \ll \Lambda$. The metric and the Christoffel symbols at the first order in $1/\Lambda^2$ are:

$$g_{x\bar{x}} = 1 - \frac{4}{\Lambda^2}\bar{x}x, \quad \Gamma^x_{xx} = -\frac{4}{\Lambda^2}\bar{x}, \quad \Gamma^{\bar{x}}_{\bar{x}\bar{x}} = -\frac{4}{\Lambda^2}x. \tag{2.27}$$

The lagrangian of this model is

$$\mathcal{L} = \left(1 - \frac{4}{\Lambda^2}\bar{x}x\right)F^x\bar{F}^x - \frac{1}{\Lambda^2}\chi\chi\bar{\chi}\bar{\chi} - F^x\left(-\frac{2}{\Lambda^2}x\bar{\chi}\bar{\chi} - f\right) - \bar{F}^x\left(-\frac{2}{\Lambda^2}\bar{x}\chi\chi - f\right) - \left(1 - \frac{4}{\Lambda^2}\bar{x}x\right)\partial_\mu\bar{x}\partial^\mu x - i\left(1 - \frac{4}{\Lambda^2}\bar{x}x\right)\bar{\chi}\bar{\sigma}^\mu\partial_\mu\chi + \frac{4i}{\Lambda^2}\bar{x}\partial_\mu x\bar{\chi}\bar{\sigma}^\mu\chi. \tag{2.28}$$

This lagrangian is invariant under the following supersymmetry transformations:

$$\delta_\xi x = \sqrt{2}\xi\chi, \tag{2.29}$$

$$\delta_\xi \chi = i\sqrt{2}\sigma^\mu\bar{\xi}\partial_\mu x + \sqrt{2}\xi F^x, \tag{2.30}$$

$$\delta_\xi F^x = i\sqrt{2}\bar{\xi}\bar{\sigma}^\mu\partial_\mu\chi. \tag{2.31}$$

Since F^x is an auxiliary field, it can be integrated out in order to give the on-shell lagrangian

$$\mathcal{L} = -\left(1 - \frac{4}{\Lambda^2}\bar{x}x\right)\partial_\mu\bar{x}\partial^\mu x - i\left(1 - \frac{4}{\Lambda^2}\bar{x}x\right)\bar{\chi}\bar{\sigma}^\mu\partial_\mu\chi + i\frac{4}{\Lambda^2}\bar{x}\partial_\mu x\bar{\chi}\bar{\sigma}^\mu\chi - \frac{2f}{\Lambda^2}\bar{x}\chi\chi - \frac{2f}{\Lambda^2}x\bar{\chi}\bar{\chi} - \frac{1}{\Lambda^2}\chi\chi\bar{\chi}\bar{\chi} - V, \tag{2.32}$$

where V is the following scalar potential:

$$V = f^2\left(1 + \frac{4}{\Lambda^2}\bar{x}x\right). \tag{2.33}$$

The scalar potential never vanishes and it has a minimum for $x = 0$ in which $V = f^2$. Supersymmetry is spontaneously broken and the breaking scale is $\sqrt{f}$. As expected there is a massless fermion, χ, namely the goldstino, while the scalar acquires a mass, $m_x = 2f/\Lambda$. If the energy scale given by m_ϕ is much higher than the scale we are interested in, than x can be integrated out. Considering only zero-momenta contributions, the equation of motion for x is

$$x = -\frac{\chi\chi}{2f}. \tag{2.34}$$

Lagrangian (5.14), considering the substitution and taking the limit $\Lambda \to \infty$ becomes:

$$\mathcal{L} = +\frac{\bar{\psi}\bar{\psi}\Box(\psi\psi)}{4f^2} - i\bar{\chi}\bar{\sigma}^\mu\partial_\mu\chi - f^2. \tag{2.35}$$

This lagrangian is equivalent to (2.25), derived following the historical approach. The supersymmetry transformation laws now are non-linear and equivalent to (2.15).

$$\delta_\xi\chi = -\frac{i}{2f}\sqrt{2}\sigma^\mu\bar{\xi}\partial_\mu(\chi\chi) - \sqrt{2}f\xi. \tag{2.36}$$

2.3.2 Two Chiral Superfields

The second model describes two chiral superfields, **A** and **B**, with the following Kähler potential K and prepotential W:

$$K = \bar{\mathbf{X}}\mathbf{X} + \bar{\mathbf{Y}}\mathbf{Y} - \frac{1}{\Lambda^2}(\bar{\mathbf{X}}\mathbf{X})^2 - \frac{1}{\Lambda^2}\bar{\mathbf{X}}\mathbf{X}\bar{\mathbf{Y}}\mathbf{Y}, \qquad W = f\mathbf{X}. \tag{2.37}$$

The lagrangian is given by

$$\mathcal{L} = \int d^2\theta d^2\bar{\theta} K(\mathbf{X}, \bar{\mathbf{X}}, \mathbf{Y}, \bar{\mathbf{Y}}) + \left(\int d^2\theta W(\mathbf{X}, \mathbf{Y}) + \text{h.c.}\right). \tag{2.38}$$

This lagrangian is invariant under the usual linear supersymmetry transformations. The metric $g_{i\bar{j}}$ of the Kähler manifold is

$$\begin{pmatrix} g_{x\bar{x}} & g_{x\bar{y}} \\ g_{y\bar{x}} & g_{y\bar{y}} \end{pmatrix} = \begin{pmatrix} 1 - \frac{4}{\Lambda^2}x\bar{x} - \frac{1}{\Lambda^2}y\bar{y} & -\frac{1}{\Lambda^2}\bar{x}y \\ -\frac{1}{\Lambda^2}x\bar{y} & 1 - \frac{1}{\Lambda^2}\bar{x}x \end{pmatrix}. \tag{2.39}$$

The inverse metric $g^{i\bar{j}}$ is

$$\begin{pmatrix} g^{x\bar{x}} & g^{x\bar{y}} \\ g^{y\bar{x}} & g^{y\bar{y}} \end{pmatrix} = \begin{pmatrix} 1 + \frac{4}{\Lambda^2}x\bar{x}\frac{1}{\Lambda^2}y\bar{y} & +\frac{1}{\Lambda^2}x\bar{y} \\ +\frac{1}{\Lambda^2}\bar{x}y & 1 + \frac{1}{\Lambda^2}\bar{x}x \end{pmatrix}. \tag{2.40}$$

The only non vanishing, at first order in $1/\Lambda^2$, Christoffel symbols are

$$\Gamma^x_{xx} = -\frac{4}{\Lambda^2}\bar{x}, \qquad \Gamma^x_{xy} = -\frac{1}{\Lambda^2}\bar{y}, \qquad \Gamma^y_{xy} = -\frac{1}{\Lambda^2}\bar{x} \tag{2.41}$$

and their complex conjugates. The curvature $R_{i\bar{j}k\bar{l}}$ is

$$R_{x\bar{x}x\bar{x}} = -\frac{4}{\Lambda^2}, \qquad R_{x\bar{x}y\bar{y}} = -\frac{1}{\Lambda^2} = R_{x\bar{y}y\bar{x}}. \tag{2.42}$$

The scalar potential is:

$$V = f^2\left(1 + \frac{4}{\Lambda^2}x\bar{x} + \frac{1}{\Lambda^2}y\bar{y}_B\right). \tag{2.43}$$

There aren't supersymmetric vacua. There is a minimum for $x = y = 0$. The masses of the the two scalars are $m_x = 2f/\Lambda$ and $m_y = f/\Lambda$ while the fermions are massless. As seen before the scalars can be integrated out. The zero-momenta solution of the equation of motion for x and y are.

$$x = -\frac{\chi\chi}{2f}, \tag{2.44}$$

$$y = -\frac{\chi\psi}{f}. \tag{2.45}$$

The effective lagrangian, in the limit $\Lambda \to \infty$, is:

$$\begin{aligned}\mathcal{L} = &+ \frac{\bar{\chi}\bar{\chi}\Box\chi\chi}{4f^4} + \frac{\bar{\chi}\bar{\psi}\Box\chi\psi}{f^2} \\ &- i\bar{\chi}\bar{\sigma}^\mu\partial_\mu\chi - i\bar{\psi}\bar{\sigma}^\mu\partial_\mu\psi - f^2.\end{aligned} \tag{2.46}$$

The transformation law for the spinors are non-linear:

$$\delta_\xi\chi = -i\sqrt{2}\sigma^\mu\bar{\xi}\partial_\mu\left(\frac{\chi\chi}{2f}\right) - \sqrt{2}f\xi\ , \tag{2.47}$$

$$\delta_\xi\psi = -i\sqrt{2}\sigma^\mu\bar{\xi}\partial_\mu\left(\frac{\chi\psi}{f}\right)\ . \tag{2.48}$$

Also this lagrangian has the form of lagrangian [2] and the spinors carry a non-linear realization of supersymmetry. Both fields are massless but the goldstino is χ as can be seen by the f contribution to the χ transformation laws.

2.4 Constrained Superfields

The third method for obtaining effective lagrangians is constraining superfields. It is better than the historical one because it uses the superfields formalism that is the most efficient. It also is more convenient than the second because it eliminates the unwanted fields in a easier way. As usual a first description in given trough some simple examples.

2.4.1 One Chiral Constrained Superfield

The most simple model of supersymmetry with constrained superfield describes a superfield $\mathbf{X}$ that satisfies the constraint $\mathbf{X}^2 = 0$:

$$\begin{aligned} 0 &= x^2 + 2(\theta\chi)(\theta\chi) + 2\sqrt{2}x\theta\chi + 2x\theta\theta F_x \\ &= x^2 + 2\sqrt{2}x\theta\chi + \theta\theta(2xF_x - \chi\chi). \end{aligned} \tag{2.49}$$

The only non trivial solution is

$$x = \frac{\chi\chi}{2F_x}. \tag{2.50}$$

The most simple Kähler potential and superpotential that break supersymmetry are

$$K = \bar{\mathbf{X}}\mathbf{X}, \qquad W = f\mathbf{X}. \tag{2.51}$$

The lagrangian for $\mathbf{X}(x, \chi, F_x)$ is

$$\mathcal{L} = \bar{F}_x F_x - \partial_\mu x \partial^\mu x - i\bar{\chi}\bar{\sigma}^\mu \partial_\mu \chi + fF_x + f\bar{F}_x. \tag{2.52}$$

This lagrangian is invariant under the following supersymmetry transformations:

$$\delta_\xi x = \sqrt{2}\xi\chi, \tag{2.53}$$

$$\delta_\xi \chi = i\sqrt{2}\sigma^\mu \bar{\xi}\partial_\mu x + \sqrt{2}\xi F_x, \tag{2.54}$$

$$\delta_\xi F_x = i\sqrt{2}\bar{\xi}\bar{\sigma}^\mu \partial_\mu \chi. \tag{2.55}$$

By adding the constraint (2.50), lagrangian (2.52) becomes

$$\mathcal{L} = \bar{F}_x F_x + \frac{1}{4}\frac{\bar{\chi}\bar{\chi}}{\bar{F}_x}\Box\left(\frac{\chi\chi}{F_x}\right) - i\bar{\chi}\bar{\sigma}^\mu\partial_\mu\chi + fF_x + f\bar{F}_x. \tag{2.56}$$

The transformation laws now are not linear:

$$\delta_\xi\chi = i\sqrt{2}\sigma^\mu\bar{\xi}\partial_\mu\left(\frac{\chi\chi}{2F_x}\right) + \sqrt{2}\xi F_x, \tag{2.57}$$

$$\delta_\xi F_x = i\sqrt{2}\bar{\xi}\bar{\sigma}^\mu\partial_\mu\chi. \tag{2.58}$$

The equation of motion for F_x is

$$F_x = -f + \frac{1}{4}\frac{\bar{\chi}\bar{\chi}}{\bar{F}_x^2}\Box\left(\frac{\chi\chi}{F_x}\right). \tag{2.59}$$

This equation can be solved iteratively. This is possible because only finite combination of $\chi, \bar{\chi}$ and their derivatives contributes to F_x. The first step is $F_x = -f$. The second is

$$F_x = -f - \frac{\bar{\chi}\bar{\chi}\Box\chi\chi}{4f^3}. \tag{2.60}$$

The computation of $F_x^{-1} = A + B\bar{\chi}\bar{\chi}(\chi\chi) + C(\dots)$ is done imposing $F_x F_x^{-1} = 1$:

$$F_x^{-1} = -\frac{1}{f} + \frac{\bar{\chi}\bar{\chi}\Box(\chi\chi)}{4f^5}. \tag{2.61}$$

The third step is

$$F_x = -f + \frac{1}{4}\bar{\chi}\bar{\chi}\left(\frac{1}{f^2} - \frac{\chi\chi\Box(\bar{\chi}\bar{\chi})}{2f^6}\right)\Box\left[\chi\chi\left(-\frac{1}{f} + \frac{\bar{\chi}\bar{\chi}\Box(\chi\chi)}{4f^5}\right)\right]. \tag{2.62}$$

The solution is

$$F_x = -f - \frac{\bar{\chi}\bar{\chi}\Box(\chi\chi)}{4f^3} + \frac{3\bar{\chi}\bar{\chi}\chi\chi\Box(\bar{\chi}\bar{\chi})\Box(\chi\chi)}{16f^7}. \tag{2.63}$$

A fourth step is not necessary because the last contribution of (2.63) vanishes when inserted in (2.59).

With this expression for F_x the lagrangian (5.18) becomes

$$\mathcal{L} = -f^2 + \frac{\bar{\chi}\bar{\chi}\Box(\chi\chi)}{4f^2} - \frac{\bar{\chi}\bar{\chi}\chi\chi\Box(\bar{\chi}\bar{\chi})\Box(\chi\chi)}{16f^6} - i\bar{\chi}\bar{\sigma}^\mu\partial_\mu\chi. \tag{2.64}$$

This lagrangian is invariant under

$$\begin{aligned} \delta_\xi\chi = &+ i\sqrt{2}\sigma^\mu\bar{\xi}\partial_\mu\left[\frac{\chi\chi}{2}\left(-\frac{1}{f} + \frac{\bar{\chi}\bar{\chi}\Box(\chi\chi)}{4f^5}\right)\right] \\ &+ \sqrt{2}\xi\left(-f - \frac{\bar{\chi}\bar{\chi}\Box(\chi\chi)}{4f^3} + \frac{3\bar{\chi}\bar{\chi}\chi\chi\Box(\bar{\chi}\bar{\chi})\Box(\chi\chi)}{16f^7}\right). \end{aligned} \tag{2.65}$$

The two lagrangians (2.64) and (2.35) differ by a term with eight fermions. This contribution is suppressed by a factor k^4/f^6, where k is the momentum carried by the fermions. For momenta well below the supersymmetry breaking scale the two lagrangians coincide.

2.4.2 Two Chiral Constrained Superfields

The second model describes the two chiral fields introduced in the previous section but constrained by:

$$\mathbf{X}^2 = 0\,, \qquad \mathbf{XY} = 0\,. \tag{2.66}$$

The first constraint gives

$$x = \frac{\chi\chi}{2F_x}. \tag{2.67}$$

The second is equivalent to

$$0 = \frac{\chi\chi}{2F_x}y + \sqrt{2}\theta(\psi\frac{\chi\chi}{2F_x} + \chi y) + \theta^2(F_y\frac{\chi\chi}{2F_x} + F_x y - \chi\psi) \tag{2.68}$$

The θ^2-term vanishes for:

$$y = \frac{\chi\psi}{F_x} - \frac{F_y\chi\chi}{2F_x^2}. \tag{2.69}$$

This makes the scalar term to vanish trivially while the θ contribution becomes

$$\sqrt{2}\left((\theta\psi)\frac{(\chi\chi)}{2F_x} + (\theta\chi)\frac{(\chi\psi)}{F_x}\right) \tag{2.70}$$

Since $(\theta\chi)(\chi\psi) = -(\chi\chi)(\theta\psi)/2$ also this term vanish.

The two expressions (2.67) and (2.69) satisfy the constraints imposed and the lagrangian without the scalar fields is.

$$\begin{aligned}\mathcal{L} =& - i\bar{\chi}\bar{\sigma}^{\mu}\partial_{\mu}\chi - i\bar{\psi}\bar{\sigma}^{\mu}\partial_{\mu}\psi + \frac{\bar{\chi}\bar{\chi}}{2\bar{F}_x}\Box\left(\frac{\chi\chi}{2F_x}\right) \\ &+ \left(\frac{\bar{\chi}\bar{\psi}}{\bar{F}_x} - \frac{\bar{\chi}\bar{\chi}}{2\bar{F}_x^2}\bar{F}_y\right)\Box\left(\frac{\chi\psi}{F_x} - \frac{F_y\chi\chi_A}{2F_x^2}\right) \\ &+ \bar{F}_x F_x + \bar{F}_y F_y + fF_x + f\bar{F}_x. \end{aligned} \tag{2.71}$$

This lagrangian is invariant under the following non linear supersymmetry transformations

$$\delta_\xi\chi = i\sqrt{2}\sigma^{\mu}\bar{\xi}\partial_{\mu}\left(\frac{\chi\chi}{2F_x}\right) + \sqrt{2}\xi F_x\ , \tag{2.72}$$

$$\delta_\xi\psi = i\sqrt{2}\sigma^{\mu}\bar{\xi}\partial_{\mu}\left(\frac{\chi\psi}{2F_x} - \frac{\chi\chi}{2F_x^2}F_y\right) + \sqrt{2}\xi F_y\ , \tag{2.73}$$

$$\delta_\xi F_x = i\sqrt{2}\bar{\xi}\bar{\sigma}^{\mu}\partial_{\mu}\chi\ , \tag{2.74}$$

$$\delta_\xi F_y = i\sqrt{2}\bar{\xi}\bar{\sigma}^{\mu}\partial_{\mu}\psi. \tag{2.75}$$

The equations of motion for the fields F_x and F_y are

$$\begin{aligned} F_x =& -f + \frac{\bar{\chi}\bar{\chi}}{2\bar{F}_x^2}\Box\left(\frac{\chi\chi_A}{2F_x}\right) \\ &+ \left(\frac{\bar{\chi}\bar{\psi}}{\bar{F}_x^2} - \frac{\bar{\chi}\bar{\chi}}{\bar{F}_x^3}\bar{F}_y\right)\Box\left(\frac{\chi\psi}{F_x} - \frac{F_y\chi_A\chi}{2F_x^2}\right)\ , \end{aligned} \tag{2.76}$$

$$F_y = +\frac{\bar{\chi}\bar{\chi}}{2\bar{F}_x^2}\Box\left(\frac{\chi\psi}{F_x} - \frac{F_y\chi\chi}{2F_x^2}\right). \tag{2.77}$$

These equations can be solved iteratively. The first step is trivial

$$F_x = -f, \qquad F_y = 0. \tag{2.78}$$

The second step gives

$$F_x = -f - \frac{\bar{\chi}\bar{\chi}\Box\chi\chi}{4f^3} - \frac{\bar{\chi}\bar{\psi}\Box\chi\psi}{f^3}; \tag{2.79}$$

$$F_y = -\frac{\bar{\chi}\bar{\chi}\Box\chi\psi}{2f^3}. \tag{2.80}$$

The expression for F_x after the third step is

$$\begin{aligned} F_x =& - f - \frac{\bar{\chi}\bar{\chi}\Box\chi\chi}{4f^3} - \frac{\bar{\chi}\bar{\psi}\Box\chi\psi}{f^3} \\ &+ \frac{\bar{\chi}\bar{\chi}\chi\chi\Box\bar{\chi}\bar{\psi}\Box\bar{\chi}\bar{\psi}}{4f^7} + \frac{\bar{\chi}\bar{\chi}\chi\chi\Box\bar{\chi}\bar{\psi}\Box\chi\chi}{2f^7} \\ &+ 3\frac{\bar{\chi}\bar{\chi}\chi\chi\Box\bar{\chi}\bar{\chi}\Box\chi\chi}{16f^7} + \frac{\bar{\chi}\bar{\psi}\bar{\chi}\bar{\psi}\Box\chi\psi\Box\chi\psi}{f^7} \\ &+ \frac{\chi\psi\bar{\chi}\bar{\psi}\Box\bar{\chi}\bar{\chi}\Box\chi\chi}{4f^7} + 3\frac{\bar{\chi}\bar{\psi}\chi\psi\Box\bar{\chi}\bar{\psi}\Box\chi\psi}{f^7} \\ &+ \frac{\bar{\chi}\bar{\psi}\chi\chi\Box\bar{\chi}\bar{\chi}\Box\chi\psi}{2f^7} + \frac{\bar{\chi}\bar{\psi}\chi\chi\Box\bar{\chi}\bar{\chi}\Box\chi\psi}{4f^7} \\ &+ \frac{\bar{\chi}\bar{\chi}\chi\chi\Box\bar{\chi}\bar{\psi}\Box\chi\psi}{2f^7} + \frac{\bar{\chi}\bar{\psi}\chi\psi\bar{\chi}\bar{\psi}\Box^2\chi\psi}{f^7} \\ &+ (\text{ more than two } \Box). \end{aligned} \tag{2.81}$$

We stress that if we switch off the superfield $\mathbf{Y}$ we recover (2.63) as we expect.

With the third step F_y becomes

$$\begin{aligned} F_y =& - \frac{\bar{\chi}\bar{\chi}\Box\chi\psi}{2f^3} + \frac{\bar{\chi}\bar{\chi}\chi\psi\Box\bar{\chi}\bar{\chi}\Box\chi\chi}{8f^7} \\ &+ 3\frac{\bar{\chi}\bar{\chi}\chi\psi\Box\bar{\chi}\bar{\psi}\Box\chi\psi}{2f^7} + \frac{\bar{\chi}\bar{\chi}\chi\chi\Box\bar{\chi}\bar{\chi}\Box\chi\chi_A}{4f^7} \\ &+ \frac{\bar{\chi}\bar{\chi}\chi\chi\Box\bar{\chi}\bar{\chi}\Box\chi\psi}{8f^7} + (\text{ more than two } \Box). \end{aligned} \tag{2.82}$$

By substituting the expressions for F_x and F_y, the lagrangian (2.71) becomes

$$\begin{aligned} \mathcal{L} =& - i\bar{\chi}\bar{\sigma}^\mu\partial_\mu\chi - f^2 + \frac{\bar{\chi}\bar{\chi}\Box\chi\chi}{4f^2} - \frac{\chi\chi\bar{\chi}\bar{\chi}\Box\chi\chi\Box\bar{\chi}\bar{\chi}}{16f^6} \\ &- i\bar{\psi}\bar{\sigma}^\mu\partial_\mu\psi + \frac{\bar{\chi}\bar{\psi}\Box\chi\psi}{f^2} - \frac{\chi\psi\bar{\chi}\bar{\psi}\Box\chi\psi\Box\bar{\chi}\bar{\psi}}{f^6} \\ &- \frac{\bar{\chi}\bar{\chi}\chi\chi\Box\bar{\chi}\bar{\psi}\Box\chi\psi}{4f^6} - \frac{\bar{\chi}\bar{\psi}\bar{\chi}\bar{\psi}\Box\chi\psi\Box\chi\psi}{f^6} \end{aligned}$$

$$
\begin{aligned}
&- \frac{\bar\chi\bar\psi\chi\psi\bar\chi\bar\psi\Box^2\chi\psi}{f^6} - \frac{\bar\chi\bar\psi\chi\psi\Box\bar\chi\bar\chi\Box\chi\chi}{4f^6} \\
&- \frac{\bar\chi\bar\psi\chi\chi\Box\bar\chi\bar\chi\Box\chi\chi}{4f^6} + (\text{ more than two } \Box). \qquad (2.83)
\end{aligned}
$$

Considering $\psi = 0$, this lagrangian is identical to the lagrangian (2.64) found in the case of a single chiral field. By taking the low energy limit , or in other words by forgetting the two $\Box$ contribution, it is equivalent to the lagrangian (2.46) computed from supersymmetry breaking.

Most General Model

The two constraints seen until now can be used for constructing the most general non linear representation with two fermions. The constraints $\mathbf{X}^2 = 0 = \mathbf{XY}$ imply $\mathbf{Y}^3 = 0$:

$$
\mathbf{Y}^3 = y^3 + 3\sqrt{2}y^2\theta\psi + 6y(\theta\psi)^2 + 3y^2F_y\theta^2 \, . \qquad (2.84)
$$

y^3 and $y^2\psi$ vanish trivially. The non trivial part is the θ^2 coefficient

$$
y^2F_y - y\psi^2 = \frac{(\chi\psi)^2}{F_x^2}F_y + \frac{\chi\chi\psi\psi}{2F_x^2}F_y = 0. \qquad (2.85)
$$

The most general Kähler potential with the constraints above is

$$
K = \bar{\mathbf{X}}\mathbf{X} + \bar{\mathbf{Y}}\mathbf{Y} + a(\bar{\mathbf{X}}\mathbf{Y}^2 + \mathbf{X}\bar{\mathbf{Y}}^2) + b(\bar{\mathbf{Y}}\mathbf{Y}^2 + \mathbf{Y}\bar{\mathbf{Y}}^2) + c(\bar{\mathbf{Y}}\mathbf{Y})^2. \qquad (2.86)
$$

All the terms $f(\mathbf{X})$ or $g(\mathbf{Y})$ are swept away by a Kähler transformation while $d(\bar{\mathbf{X}}\mathbf{Y} + \bar{\mathbf{Y}}\mathbf{X})$ is absorbed using a linear combination of $\mathbf{X}$ and $\mathbf{Y}$.

The most general superpotential is

$$
W = f\mathbf{X} + g\mathbf{Y} + h\mathbf{Y}^2. \qquad (2.87)
$$

From the Kähler potential the metric $g_{i\bar{j}}$, with $i, j = \{\mathbf{X}, \mathbf{Y}\}$, can be derived

$$
g_{i\bar{j}} = \begin{pmatrix} 1 & 2a\bar{y} \\ 2ay & 1 + 2b(y + \bar{y}) + 4c(y\bar{y}) \end{pmatrix} . \qquad (2.88)
$$

The inverse is

$$g^{i\bar{j}} = \frac{1}{1+2b(y+\bar{y})+4(c-a^2)y\bar{y}} \begin{pmatrix} 1+2b(y+\bar{y})+4cy\bar{y} & -2ay \\ -2a\bar{y} & 1 \end{pmatrix} . \tag{2.89}$$

The non vanishing Christoffel symbols are

$$\Gamma^x_{yy} = \frac{2a(1+2b\bar{y})}{1+2b(y+\bar{y})+4(c-a^2)y\bar{y}} , \tag{2.90}$$

$$\Gamma^y_{yy} = \frac{2b+4(c-a^2)\bar{y}}{1+2b(\bar{y}+y)+4(c-a^2)y\bar{y}} . \tag{2.91}$$

The lagrangian of this model is

$$\begin{aligned} \mathcal{L} =& F_x\bar{F}_x + 2a\bar{y}F_x\bar{F}_y + 2ay\bar{F}_xF_y + (1+2b(y+\bar{y}))F_y\bar{F}_y \\ & - i\bar{\chi}\bar{\sigma}^\mu\partial_\mu\chi - i\bar{\psi}\bar{\sigma}^\mu\partial_\mu\psi - (\bar{F}_x(a\psi\psi - f)+ \\ & \bar{F}_y(b\psi\psi - g - 2hy) - h\psi\psi + \text{h.c.}) + (\geq 4 \text{ fermions}). \end{aligned} \tag{2.92}$$

The equations of motion for the auxiliary fields give

$$F_x = -f + F_x|_2 + (\geq 4 \text{ fermions}) , \tag{2.93}$$

$$F_y = -g + F_y|_2 + (\geq 4 \text{ fermions}) . \tag{2.94}$$

where $F_i|_2$ indicates the terms with two fermions.

In order to compute the masses of the fermions let's calculate

$$\begin{aligned} \mathcal{L}|_{\leq 2} =& f^2 - f(F_x|_2 + \bar{F}_x|_2) + 2afg(y+\bar{y}) + g^2(1+2b(y+\bar{y})) \\ & - g(F_y|_2 + \bar{F}_y|_2) + (fa\psi\psi - f^2 + f\bar{F}_x|_2 + \text{ h.c.}) \\ & (+bg\psi\psi - g^2 - 2hyg + g\bar{F}_y|_2 + \text{ h.c.}) \\ & - h\psi\psi - h\bar{\psi}\bar{\psi} - i\bar{\chi}\bar{\sigma}^\mu\partial_\mu\chi - i\bar{\psi}\bar{\sigma}^\mu\partial_\mu\psi. \end{aligned} \tag{2.95}$$

Eventually it becomes

$$\mathcal{L}|_{\leq 2} = - f^2 - g^2 - i\bar{\chi}\bar{\sigma}^\mu\partial_\mu\chi - i\bar{\psi}\bar{\sigma}^\mu\partial_\mu\psi - \frac{1}{2}\Big[-2\psi\psi(af+bg-h)$$

$$+ 2\frac{\chi\psi}{f}(2afg + 2bg^2 - 2hg) - 2\frac{\chi\chi}{f^2}g(afg + bg^2 - hg) + \text{h.c.}\Big] . \tag{2.96}$$

The scalar potential is f^2+g^2 and so the supersymmetry breaking scale is $\sqrt[4]{f^2+g^2}$.

The mass matrix is

$$M_{\frac{1}{2}} = \begin{pmatrix} -2\frac{g^2}{f^2}(af+bg-h) & 2\frac{g}{f}(af+bg-h) \\ 2\frac{g}{f}(af+gb-h) & -2(af+bg-h) \end{pmatrix} \tag{2.97}$$

One of the eigenvalues vanishes while the other is

$$m = 2\frac{(h-af-bg)(f^2+g^2)}{f^2} . \tag{2.98}$$

This eigenvalue can vanish if $h - af - bg = 0$. In the model seen before the two fermions were both massless because $h = g = a = b = 0$.

The zero eigenvalue corresponds to the goldstino, given by the linear combination $G = f\chi + g\psi$. The massive fermion $\tilde{G}$, is given by $\tilde{G} = g\chi - f\psi$. In this simple model both F_x and F_y concurs to the symmetry breaking and so the goldstino can not be only χ.

2.5 Constrained Superfields Theory

The aim of this section is to generalize the constraining procedure used until now following two recent articles, [25] and [26]. Every time supersymmetry is broken there is a massless fermion, the goldstino field interactions can always be described by means of a chiral superfield $\mathbf{X}$ satisfying $\mathbf{X}^2 = 0$. The supersymmetry breaking sector is described by the lagrangian

$$\mathcal{L}_X = \int d^2\theta d^2\bar{\theta}\bar{\mathbf{X}}\mathbf{X} + \left\{ f \int d^2\theta \mathbf{X} + \text{h.c.} \right\} . \tag{2.99}$$

With this lagrangian $F \neq 0$ and the explicit expression for $\mathbf{X}$ is

$$\mathbf{X} = \frac{\chi\chi}{2F} + \sqrt{2}\theta\chi + \theta^2 F . \tag{2.100}$$

When needed, the lowest component of a generic superfield Q_L, where L is an index labeling the Lorentz representation of the superfields, can be removed imposing:

$$\mathbf{X}\bar{\mathbf{X}}Q_L = 0\ . \tag{2.101}$$

The term that originates the mass for the lowest component of Q_L can be written as

$$-\frac{m_{Q_L}^2}{2f^2}\left\{\int d^2\theta d^2\bar{\theta}\bar{\mathbf{X}}\mathbf{X}Q_L\bar{Q}_L + \text{h.c.}\right\}. \tag{2.102}$$

Taking the formal limit $m_{Q_L} \to \infty$ for decoupling the lowest component of Q_L the action diverges. The divergent part cancels if and only if

$$\bar{\mathbf{X}}\mathbf{X}Q_L = 0\ . \tag{2.103}$$

Let's see this formalism applied to the model studied above with two chiral superfields. The constraint was

$$\mathbf{X}\mathbf{Y} = 0\ . \tag{2.104}$$

This is equivalent to imposing

$$\mathbf{X}\bar{\mathbf{X}}\mathbf{Y} = 0\ . \tag{2.105}$$

Ir is easily seen, remembering that $\bar{D}^2\bar{\mathbf{X}}$ never vanishes, from

$$\bar{D}^2(\mathbf{X}\bar{\mathbf{X}}\mathbf{Y}) = 0 \iff \mathbf{X}\bar{D}^2\bar{\mathbf{X}}\mathbf{Y} = 0 \iff \mathbf{X}\mathbf{Y} = 0\ . \tag{2.106}$$

The UV origin of this constraint comes from the mass term

$$-\frac{m_y^2}{f^2}\int d^2\theta d^2\bar{\theta}|\mathbf{X}|^2|\mathbf{Y}|^2\ . \tag{2.107}$$

This is the contribution considered also in (2.38). By taking the limit $m_y \to \infty$ the scalar fields are removed from the spectrum but the contribution written above diverges. It vanishes requiring (2.105).

The fermionic component of $\mathbf{Y}$ is removed imposing

$$\mathbf{X}\bar{\mathbf{X}}D_\alpha\mathbf{Y} = 0\ , \tag{2.108}$$

while the auxiliary field can be eliminated with the constraint

$$\mathbf{X}\bar{\mathbf{X}}D^2\mathbf{Y} = 0 \; . \tag{2.109}$$

These constraints are equivalent to

$$D_\alpha(\bar{\mathbf{X}}\mathbf{Y}) = 0 \; , \tag{2.110}$$

which removes both the fermion and the auxiliary fields.

Chiral Superfields Parametrization

Thanks to these constraints it's possible to parametrize an unconstrained chiral superfield $\mathbf{\Phi}$ with two constrained fields $\mathbf{X}$ and $\mathbf{S}$. Consider a simple supersymmetry breaking lagrangian with an additional term, suppressed by a scale $\Lambda > \sqrt{f}$, for generating masses for the scalar component.

$$\mathcal{L} = \int d^2\theta d^2\bar{\theta}\bar{\mathbf{\Phi}}\mathbf{\Phi} + f\left\{\int d^2\theta\mathbf{\Phi} + \text{h.c.}\right\} - \frac{1}{\Lambda^2}\int d^2\theta d^2\bar{\theta}\bar{\mathbf{\Phi}}^2\mathbf{\Phi}^2 \; . \tag{2.111}$$

For energies below $m_\phi \sim f/\Lambda$ the effective theory is described by the goldstino alone. It is natural splitting the degrees of freedom of $\mathbf{\Phi}$ into two constrained superfields $\mathbf{X}$ and $\mathbf{S}$ satisfying:

$$\mathbf{X}^2 = 0 \; , \qquad D_\alpha(\bar{\mathbf{X}}\mathbf{S}) = 0 \; . \tag{2.112}$$

The only surviving degrees of freedom are the goldstino and the auxiliary field from $\mathbf{X}$ and the scalar field from S. It is possible to define

$$\mathbf{\Phi} = \mathbf{X} + \mathbf{S} \; . \tag{2.113}$$

With this redefinition lagrangian (2.111) becomes

$$\mathcal{L} = \int d^2\theta d^2\bar{\theta}\left\{\bar{\mathbf{X}}\mathbf{X} + \bar{\mathbf{S}}\mathbf{S} - \frac{1}{\Lambda^2}(4\bar{\mathbf{X}}\mathbf{X}\bar{\mathbf{S}}\mathbf{S} + \bar{\mathbf{S}}^2\mathbf{S}^2)\right\} + f\left\{\int d^2\theta(\mathbf{X} + \mathbf{S}) + \text{h.c.}\right\} . \tag{2.114}$$

The zero momentum component equations are minimized by configuration in which the scalar component of $\mathbf{S}$ vanishes and this implies $\mathbf{S} = 0$. In the low energy limit the simple supersymmetry breaking model with a chiral constrained superfield is recovered:

$$\mathcal{L} = \int d^2\theta d^2\bar{\theta}\bar{\mathbf{X}}\mathbf{X} + f\left\{\int d^2\theta \mathbf{X} + \text{h.c.}\right\} . \tag{2.115}$$

Analogously for a model with two unconstrained chiral superfield it is possible to parametrize them with

$$\mathbf{\Phi} = \mathbf{X} + \mathbf{S} \, , \tag{2.116}$$

$$\mathbf{\Sigma} = \mathbf{Y} + \mathbf{H} \, . \tag{2.117}$$

$\mathbf{X}$ and $\mathbf{S}$ satisfy the same constraints imposed above while

$$\mathbf{XY} = 0 \, , \qquad D_\alpha(\bar{\mathbf{X}}\mathbf{H}) = 0 \, . \tag{2.118}$$

A simple supersymmetry breaking lagrangian is given by

$$\mathcal{L} = \int d^2\theta d^2\bar{\theta}\left(|\mathbf{\Phi}|^2 + |\mathbf{\Sigma}|^2 - \frac{|\mathbf{\Phi}|^4}{\Lambda^2} - \frac{|\mathbf{\Phi}|^2|\mathbf{\Sigma}|^2}{\Lambda^2}\right) + f\left(\int d^2\theta\mathbf{\Phi} + \text{h.c.}\right) . \tag{2.119}$$

As before this lagrangian can be expressed in term of the constrained fields and the low energy limit gives $\mathbf{S} = \mathbf{H} = 0$ recovering the lagrangian studied in the previous section for two constrained superfields.

Chapter 3

Constrained Superfields in Supergravity

In this chapter supergravity is introduced as local supersymmetry. The constrained superfields formalism is then extended to supergravity and in the last section it is applied also to the graviton multiplet.

3.1 Supergravity as Local Supersymmetry

This section will give an idea of how supergravity can be defined by imposing supersymmetry to be local. For simplicity the lagrangian for a chiral free superfield is considered:

$$\mathcal{L} = -i\bar{\chi}\bar{\sigma}^{\mu}\partial_{\mu}\chi - \partial_{\mu}x^{\dagger}\partial^{\mu}x \ . \tag{3.1}$$

This lagrangian is on shell and it is invariant under the following supersymmetry transformations:

$$\delta_{\epsilon}x = \sqrt{2}\epsilon\chi \ , \tag{3.2}$$

$$\delta_{\epsilon}\chi = i\sqrt{2}\sigma^{\mu}\bar{\epsilon}\partial_{\mu}x \ . \tag{3.3}$$

If we promote supersymmetry to be local $\epsilon \to \epsilon(x)$ the lagrangian variation is

$$\delta_{\epsilon}\mathcal{L} = \partial_{\mu}\bar{\epsilon}(\sqrt{2}\bar{\sigma}^{\mu}\sigma^{\nu}\bar{\chi}\partial_{\nu}x) + (\sqrt{2}\partial_{\nu}x^{\dagger}\chi\sigma^{\nu}\bar{\sigma}^{\mu})\partial_{\mu}\epsilon \ . \tag{3.4}$$

Since the lagrangian variation does not vanish some new terms have to be introduced to restore invariance. This procedure is somehow similar to the gauging in ordinary symmetry. Once a global symmetry is extended to a local one some new fields are required. Here the new field ψ must have a vectorial and a spinorial index ψ_μ^α and it sits in the $(1; \frac{1}{2})$ Lorentz representation. It has the right quantum numbers to be the gravitino. If its variations is:

$$\delta_\epsilon \psi_\mu^\alpha = 2\partial_\mu \epsilon^\alpha \ , \tag{3.5}$$

then the non-vanishing contribution of the previous lagrangian is deleted by:

$$\mathcal{L} = -\frac{1}{\sqrt{2}}\bar{\psi}_\mu(\bar{\sigma}^\mu\sigma^\nu\bar{\chi}\partial_\nu x) - \frac{1}{\sqrt{2}}(\partial_\nu x^\dagger \chi\sigma^\nu\bar{\sigma}^\mu)\psi_\mu \ . \tag{3.6}$$

Obviously the variation of the lagrangian above gives new terms that do not vanish:

$$\delta_\epsilon\mathcal{L} = -\frac{1}{\sqrt{2}}\bar{\psi}_\mu\delta_\epsilon(\bar{\sigma}^\mu\sigma^\nu\bar{\chi}\partial_\nu x) - \frac{1}{\sqrt{2}}\delta_\epsilon(\partial_\nu x^\dagger \chi\sigma^\nu\bar{\sigma}^\mu)\psi_\mu \ . \tag{3.7}$$

In order to delete them a similar path to the one illustrated above can be followed and it leads to the introduction of the vielbeins e_μ^a, which have two vectorial indexes, they sit in $(1; 1)$ Lorentz representation and they describe the graviton. By starting from the assumption of local supersymmetry both the gravitino and the graviton are recovered and they belong to the graviton multiplet.

A more formal discussion about supergravity can be found in [22]. Here some results are listed and briefly commented. The lagrangian for a chiral field in supergravity is

$$\mathcal{L} = \frac{1}{k^2}\int d^2\Theta 2\xi\left[\frac{3}{8}(\bar{\mathscr{D}}\bar{\mathscr{D}} - 8R)\exp\left\{-\frac{k^2}{3}K(\mathbf{\Phi}\bar{\mathbf{\Phi}})\right\} + k^2 P(\mathbf{\Phi})\right] + \text{h.c.} \ . \tag{3.8}$$

In this lagrangian $\mathbf{\Phi}^i = \{A^i, \psi^i, F^i\}$ are i off-shell chiral multiplet. R instead is the off-shell chiral graviton multiplet $R = \{M, b^a, \mathscr{R}(\psi_\mu^\alpha, e_\mu^a)\}$. The quantity ξ is defined as:

$$2\xi = e\{1 + i\Theta\sigma^\mu\bar{\psi}_\mu - \Theta\Theta[M^* + \bar{\psi}_\mu\bar{\sigma}^{\mu\nu}\bar{\psi}_\nu]\} \ . \tag{3.9}$$

It is analogous to the determinant of the metric in ordinary manifolds. In power expansion of $1/k^2$ the lagrangian becomes:

$$\mathcal{L} = -\frac{6}{k^2}\int d^2\Theta\xi R - \frac{1}{8}\int d^2\Theta 2\xi(\bar{\mathscr{D}}\bar{\mathscr{D}} - 8R)(K(\mathbf{\Phi}\bar{\mathbf{\Phi}}) + P(\mathbf{\Phi})) + \dots \, . \tag{3.10}$$

The first term gives the kinetic and interaction terms for the gravitino and the goldstino. The second is the chiral lagrangian extended for local supersymmetry. It is written as an integral only in $d^2\theta$ with the following substitution:

$$\theta \to \Theta \; ; \tag{3.11}$$

$$d^2\theta \to d^2\Theta\xi \; ; \tag{3.12}$$

$$\bar{\mathscr{D}}\bar{\mathscr{D}} \to (\bar{\mathscr{D}}\bar{\mathscr{D}} - 8R) \; . \tag{3.13}$$

The on-shell lagrangian is obtained by integrating out the auxiliary fields M, b^a and F^i.

One of the main differences between chiral fields in supersymmetry and in supergravity is the scalar potential. In supersymmetry it is always greater than zero and supersymmetric vacua satisfy $V = 0$. In supergravity instead it is defined as:

$$V = e^K[g^{ij*}(D_iP)(D_jP)^* - 3P^*P] \; . \tag{3.14}$$

Inside the square brackets there are two terms. The first one is exactly the one that appears in supersymmetry while the second is something new. First it depends by P and not only by its derivatives as all the P contribution in supersymmetry. This means that also the constant term in the P expression is meaningful. Second the scalar potential has a negative contribution and so it can be negative. Since the scalar potential can be related to the cosmological constant Λ, different space-times can be described:

$$\begin{cases} V > 0 \to \Lambda > 0 \to \text{De Sitter} \; ; \\ V = 0 \to \Lambda = 0 \to \text{Minkowski} \; ; \\ V < 0 \to \Lambda < 0 \to \text{Anti-De Sitter} \; . \end{cases} \tag{3.15}$$

A positive potential, and so an expanding universe, can be achieved only if supersymmetry is broken and if the supersymmetry breaking term $g^{ij*}D_iPD_jP^*$ is the dominant one.

In this section only chiral fields have been taken in account for simplicity, if also gauge invariant actions, and so vector fields, are introduced, then the previous comments for the scalar potential are still valid.

$$V = e^K[g^{ij*}(D_iP)(D_jP)^* - 3P^*P] + \frac{1}{2}g^2D^{(a)2} \, . \tag{3.16}$$

Supersymmetry has to be broken in the F or D term, or both, in order to have an expanding universe. The D term contribution is exactly the same for global and local supersymmetry.

3.2 Constrained Matter

As seen in the previous chapter, the constrained fields formalism is a useful tool to build effective lagrangians in which supersymmetry is non-linearly realized. In this section there is a review about different constraints in supergravity following the article [27].

3.2.1 Scalar-less models

A chiral superfield in supergravity satisfies $\bar{D}_{\dot{\alpha}}\mathbf{X} = 0$. Its expansion in the supergravity Θ variables is:

$$\mathbf{X} = x + \sqrt{2}\Theta\chi + \Theta^2F^x \, . \tag{3.17}$$

When supersymmetry is broken, χ plays the role of the goldstino, x of its superpartner, the sgoldstino, while F^x is proportional to the supersymmetry breaking scale. Effective theories without the sgoldstino can be described starting from the constraint:

$$\mathbf{X}^2 = 0 \, . \tag{3.18}$$

This constraint is trivially solved but it is useful for the next examples to introduce a general method for solving these constraints. The first step is to hit the constraint whit the highest possible number of covariant derivatives and then the $\Theta = \bar{\Theta} = 0$

projection has to be extracted:

$$\mathbf{X}^2 = 0 \to (D^2\mathbf{X}^2)|_{\Theta=\bar{\Theta}=0} = 2[\mathbf{X}D^2\mathbf{X}+D^\alpha\mathbf{X}D_\alpha\mathbf{X}]|_{\dots} = -8xF^x+4\chi\chi = 0 \,. \quad (3.19)$$

If supersymmetry is broken, and so $F^x \neq 0$, then the constraint is solved by $x = \chi\chi/2F^x$. The other constraints, $\mathbf{X}^2|_{\dots} = 0 = (D^\alpha\mathbf{X})|_{\dots}$ are simply consistency conditions. The solution for $\mathbf{X}^2 = 0$ is identical in form for global and local supersymmetry:

$$\mathbf{X} = \frac{\chi^2}{2F^x} + \sqrt{2}\Theta\chi + \Theta^2 F^x \,. \quad (3.20)$$

The lagrangian describing one chiral superfield in supergravity is

$$\mathcal{L} = \int d^2\Theta 2\xi \left[\frac{3}{8}(\bar{D}^2 - 8R)e^{-K/3} + P\right] + \text{h.c.} \,. \quad (3.21)$$

with

$$K = \mathbf{X}\bar{\mathbf{X}} \,, \qquad P = P_0 + f\mathbf{X} \,. \quad (3.22)$$

Thanks to the constraint $\mathbf{X}^2 = 0$ there can not be other contributions to the Kähler potential and to the superpotential. Since the goldstino is a pure gauge degree of freedom of the supersymmetry, $\delta\chi = \sqrt{2}\epsilon f$, then it can be removed from the final action fixing a gauge such that $\chi = 0$. In this gauge the component lagrangian is much simpler:

$$\begin{aligned} e^{-1}\mathcal{L} = &-\frac{1}{2}\mathscr{R} + \epsilon^{klmn}\bar{\psi}_k\bar{\sigma}_l\mathscr{D}_m\psi_n \\ &- P_0\psi_a\sigma^{ab}\psi_b - \bar{P}_0\bar{\psi}_a\bar{\sigma}^{ab}\bar{\psi}_b - f^2 + 3P_0^2 \,. \end{aligned} \quad (3.23)$$

In this lagrangian there are not explicit contributions from the fields in the chiral multiplet but they influence the gravitino mass $m_{3/2} = P_0$ and the cosmological constant $V = f^2 - 3P_0^2$.

When a second chiral superfield $\mathbf{Y}$ is introduced, the constraint $\mathbf{XY} = 0$ eliminates the scalar component of $\mathbf{Y}$ from the lagrangian. The solution for this constraint, as in the previous case, is the same for global and local supersymmetry:

$$\mathbf{Y} = \frac{\chi\psi}{F^x} - \frac{\chi^2}{2(F^x)^2}F^y + \sqrt{2}\Theta\psi + \Theta^2 F^y \,. \quad (3.24)$$

The most general Kähler potential and superpotential are:

$$K = \mathbf{X}\bar{\mathbf{X}} + \mathbf{Y}\bar{\mathbf{Y}} + a(\mathbf{X}\bar{\mathbf{Y}}^2 + \mathbf{Y}^2\bar{\mathbf{X}}) + b(\mathbf{Y}\bar{\mathbf{Y}}^2 + \mathbf{Y}^2\bar{\mathbf{Y}}) + c\mathbf{Y}^2\bar{\mathbf{Y}}^2 \; ; \tag{3.25}$$

$$P = P_0 + f\mathbf{X} + g\mathbf{Y} + h\mathbf{Y}^2 \; . \tag{3.26}$$

In this model the goldstino is the linear combination $f\chi + g\psi$ and so the more physical gauge choice would be $f\chi + g\psi = 0$. In the last chapter this model will be deeply described and it will be clear that imposing $\chi = 0$ as gauge fixing simplifies the computation of the interaction terms.

The next model we introduce has a single $U(1)$ vector and the interest is always in an effective theory without scalars. The field strength for the real superfield V is given by:

$$W_\alpha = -\frac{1}{4}(\bar{D}^2 - 8R)D_\alpha V \; . \tag{3.27}$$

The standard kinetic term for a vector superfield is:

$$\mathcal{L} = \frac{1}{4g^2}\int d^2\Theta 2W^2 + \text{h.c.} \; . \tag{3.28}$$

When supersymmetry is broken the gaugino acquires mass and it can be removed imposing

$$\mathbf{X}W_\alpha = 0 \; . \tag{3.29}$$

By acting with the highest number of covariant derivatives and by taking the $\Theta = \bar{\Theta} = 0$ projection the following expression is obtained:

$$\begin{aligned}\lambda_\alpha = &-\frac{i}{2}\tilde{\chi}^2\left[\sigma^c_{\alpha\dot{\beta}}\hat{D}_c\bar{\lambda}^{\dot{\beta}} - \frac{i}{2}(\lambda_\alpha\bar{M} + b^{\dot{\beta}}_\alpha\bar{\lambda}_{\dot{\beta}})\right] \\ &+ \frac{i}{\sqrt{2}}\tilde{\chi}^\beta\left[-2i\sigma^{ba\gamma}_\alpha\epsilon_{\gamma\beta}\hat{D}_b v_a + \epsilon_{\alpha\beta}D\right] \; .\end{aligned} \tag{3.30}$$

All the conventions can be read in [27]. Here only two features are highlighted. The first is that gaugini appear on both sides and so this equation has to be solved iteratively. The solution has not a simple expression as in the previous cases and it can be found in [27]. The second is the presence of the auxiliary fields M and b_a of R in the expression for λ_α and so the expressions for a vector superfields in local or global supersymmetry are deeply different. Also in this model, the gauge choice $\chi = 0$ simplifies the effective lagrangian:

$$\begin{aligned} e^{-1}\mathcal{L} =& -\frac{1}{2}\mathscr{R} + \epsilon^{klmn}\bar{\psi}_k\bar{\sigma}_l D_m\psi_n - \frac{1}{4g^2}F^{mn}F_{mn} \\ & - P_0\bar{\psi}_a\bar{\sigma}^{ab}\bar{\psi}_b - P_0\psi_a\sigma^{ab}\psi_b - f^2 + 3W_0^2 \ , \end{aligned} \tag{3.31}$$

where $F_{mn} = \partial_m v_n - \partial_n v_m$.

3.2.2 Models Constraining Fermions

These models in which the scalars are not completely removed are useful to describe inflationary scenarios in cosmology and the Higgs field in the standard model.

The first model contains the goldstino multiplet $\mathbf{X}$ and another chiral superfield

$$\mathbf{H} = h + \sqrt{2}\Theta\psi^H + \Theta^2 F^H \ . \tag{3.32}$$

By imposing

$$\bar{D}_{\dot{\alpha}}(\mathbf{X}\bar{\mathbf{H}}) = 0 \ , \tag{3.33}$$

both ψ^H and F^H are removed as in global supersymmetry. In supergravity fields of the graviton supermultiplet are present in the expression for ψ^H and F^H, as in the real vector model. The Kähler potential and the superpotential for a model with these two constrained superfields are:

$$K = |\mathbf{X}|^2 + \mathbf{X}P(\mathbf{H},\bar{\mathbf{H}}) + \bar{\mathbf{X}}P(\mathbf{H},\bar{\mathbf{H}}) + Z(\mathbf{H},\bar{\mathbf{H}}) \tag{3.34}$$

$$W = g(\mathbf{H}) + \mathbf{X}f(\mathbf{H}) \ . \tag{3.35}$$

In the $\chi = 0$ gauge the constrained superfields become

$$\mathbf{X} = \Theta^2 F^x \ , \qquad \mathbf{H} = h \ . \tag{3.36}$$

The lagrangian in component is

$$e^{-1}\mathcal{L} = -\frac{1}{2}\mathscr{R} + \epsilon^{klmn}\bar{\psi}_k\bar{\sigma}_l D_m\psi_n - Z_{h\bar{h}}\partial^m h\partial_m\bar{h} - e^{Z/2}(g\bar{\psi}_a\bar{\sigma}^{ab}\bar{\psi}_b + \bar{g}\psi_a\sigma^{ab}\psi_b)$$

$$+\frac{1}{4}\epsilon^{klmn}Z_h\psi_l\sigma_m\bar{\psi}_n\partial_k h+\frac{1}{4}\epsilon^{klmn}Z_{\bar{h}}\psi_l\sigma_m\bar{\psi}_n\partial_k\bar{h}-V\ , \tag{3.37}$$

where

$$V=e^Z(|f+gP|^2-3|g|^2)\ . \tag{3.38}$$

A more interesting model is the one in which only the real component of the scalar field survives and it can be identified with the inflaton in cosmological models. By starting with a chiral superfield

$$\mathbf{A}=a+i\Sigma+\sqrt{2}\Theta\psi^A+\Theta^2F^A\ , \tag{3.39}$$

the constraint

$$\mathbf{X}\mathbf{A}-\mathbf{X}\bar{\mathbf{A}}=0 \tag{3.40}$$

removes ψ^A, F^A and Σ. The most general coupling of $\mathbf{A}$ with the nilpotent field $\mathbf{X}$ in supergravity is described by the following Kähler potential and superpotential:

$$K=|\mathbf{X}|^2+Z(\mathbf{A},\bar{\mathbf{A}}) \tag{3.41}$$

$$W=g(\mathbf{A})+\mathbf{X}f(\mathbf{A})\ . \tag{3.42}$$

In the $\chi=0$ gauge the lagrangian in component becomes:

$$\begin{aligned}e^{-1}\mathcal{L}=&-\frac{1}{2}\mathscr{R}+\epsilon^{klmn}\bar{\psi}_k\bar{\sigma}_lD_m\psi_n-Z_{a\bar{a}}\partial^m a\partial_m a-e^{Z/2}(g\bar{\psi}_a\bar{\sigma}^{ab}\bar{\psi}_b+\bar{g}\psi_a\sigma^{ab}\psi_b)\\&+\frac{1}{4}\epsilon^{klmn}(Z_a-Z_{\bar{a}})\psi_l\sigma_m\bar{\psi}_n\partial_k a-V\ ,\end{aligned} \tag{3.43}$$

where

$$V=e^Z(|f|^2-3|g|^2)\ . \tag{3.44}$$

By choosing $Z=-\frac{1}{4}(\mathbf{A}-\bar{\mathbf{A}})$, $\bar{f}(z)=f(\bar{z})$ and $\bar{g}(z)=g(\bar{z})$ the model is described by the following lagrangian:

$$\begin{aligned}e^{-1}\mathcal{L}=&-\frac{1}{2}\mathscr{R}+\epsilon^{klmn}\bar{\psi}_k\bar{\sigma}_lD_m\psi_n\\&-\frac{1}{2}\partial^m a\partial_m a-g(a)(\bar{\psi}_a\bar{\sigma}^{ab}\bar{\psi}_b+\psi_a\sigma^{ab}\psi_b)-V(a)\ ,\end{aligned} \tag{3.45}$$

where

$$V = f(a)^2 - 3g(a)^2 \ . \tag{3.46}$$

This model describes a scenario in which the cosmological constant can vary and it is parametrized by two function of a scalar field.

3.3 Constrained Supergravity

While the previous section was about constrained superfields in supergravity, in this section the aim is to constrain the supergravity multiplet R itself [28]. In a supersymmetry breaking scenario there is a goldstino that can be described by the usual chiral superfield $\mathbf{X}$, constrained by $\mathbf{X}^2 = 0$. The aim of constraining supergravity is to remove the auxiliary fields M and b^a. Since M is the lowest component of R, which is a chiral superfield, it is removed by:

$$\mathbf{X}R = 0 \ . \tag{3.47}$$

The real superfield $B_{\alpha\dot{\alpha}}$ is introduced in order to remove b^a. It is related to R trough

$$D^\alpha B_{\alpha\dot{\alpha}} = D_{\dot{\alpha}} R \tag{3.48}$$

and it has b^a as its lowest components. The vector b^a can be removed imposing

$$\mathbf{X}\bar{\mathbf{X}} B_{\alpha\dot{\alpha}} = 0 \ . \tag{3.49}$$

The constraint $\mathbf{X}R = 0$ does not gives the usual form for the scalar potential, it is better to study the more general constraint

$$\mathbf{X}\left(R + \frac{c}{6}\right) = 0 \ . \tag{3.50}$$

Also this constraint removes the M auxiliary field. The most general Kähler potential and superpotential that can describe this model are

$$K = \mathbf{X}\bar{\mathbf{X}} \ , \qquad P = m_{3/2} + f\mathbf{X} \ . \tag{3.51}$$

The generic lagrangian for a chiral superfield in supergravity is:

$$\mathcal{L} = \frac{1}{k^2}\int d^2\Theta 2\xi\left[\frac{3}{8}(\bar{\mathscr{D}}\bar{\mathscr{D}} - 8R)\exp\left\{-\frac{k^2}{3}K(\mathbf{\Phi}\bar{\mathbf{\Phi}})\right\}+k^2P(\mathbf{\Phi})\right]+\text{h.c.}\,. \tag{3.52}$$

With $\mathbf{X}^2 = 0$, $\mathbf{X}R$ and $k = 1$ it reduces to:

$$\mathcal{L} = -6\int d^2\Theta\xi R + \int d^2\Theta\xi\mathbf{X}\left[-\frac{1}{4}(\bar{D}^2 - 8R)\right]\bar{\mathbf{X}} + \int d^2\Theta 2\xi P + \text{h.c.}\,. \tag{3.53}$$

The component lagrangian in the $\chi = 0$ gauge is

$$\begin{aligned} e^{-1}\mathcal{L} = &-\frac{1}{2}\mathscr{R} + \epsilon^{klmn}\bar{\psi}_k\bar{\sigma}_l D_m\psi_n + \frac{1}{3}b_a b^a \\ &- (m_{3/2}\bar{\psi}_a\bar{\sigma}^{ab}\bar{\psi}_b + \bar{m}_{3/2}\psi_a\sigma^{ab}\psi_b) - \Lambda\,, \end{aligned} \tag{3.54}$$

where

$$\Lambda = \frac{1}{3}|c|^2 + |f|^2 + m_{3/2}\bar{c} + \bar{m}_{3/2}c = \Lambda_S - 3|m_{3/2}|^2\,. \tag{3.55}$$

Λ_S is the effective supersymmetry breaking scale:

$$\Lambda_S = |f|^2 + \frac{1}{3}|c + 3m_{3/2}|^2\,. \tag{3.56}$$

There are three independent parameter: Λ, Λ_S and $m_{3/2}$. The lagrangian that describes this model depends only on two of them but in a more general scenario all of them will contribute. The cosmological constant depends on three parameters, f, $m_{3/2}$ and c. When $c = 0$, then $\Lambda = |f|^2$ and so pure de Sitter supergravity is obtained. If $c = -3m_{3/2}$, then $\Lambda_S = |f|^2$, so the effective supersymmetry breaking scale has the standard expression and also $\Lambda = |f|^2 - 3|m_{3/2}|^2$ the expression that would be expected in linearly realized supersymmetry.

In this construction the vector b_a is still present. It can be removed imposing $\mathbf{X}\bar{\mathbf{X}}B_{\alpha\dot{\alpha}} = 0$. After this operation the lagrangian in the unitary gauge is (3.54) without the $b^a b_a$ contribution. The consideration about the scalar potential are still true.

Chapter 4

The Super-Higgs Mechanism

For ordinary global symmetries, the Goldstone Theorem states that for every broken generator there is a massless scalar particle in the spectrum. The Higgs mechanism describes what happens when a global symmetry is gauged. The massless particles are "eaten" by gauge bosons which acquire a spin 0 polarization and become massive. In supersymmetry only one fermionic generator can be broken. In non-supersymmetric vacua a massless fermion appear, namely the goldstino. When local supersymmetry is considered the gravitino plays the role of the gauge vectors. It is the gravitino that "eats" the goldstino, it acquires a new spin 1/2 polarization and it becomes massive. This mechanism is called Super-Higgs mechanism. This chapter first describes what happens in a Minkowski vacuum when both F and D terms are present, then a cosmological constant is introduced and pure F term breaking is considered.

4.1 Super-Higgs in Minkowski

The Kähler invariant function $\mathscr{G} = K + \log |P|^2$ is useful in order to describe the Super-Higgs Mechanism. From $\mathscr{G}$ the following quantities can be derived:

$$\mathscr{G}_i \equiv \partial_i \mathscr{G} = \frac{D_i P}{P} \,, \tag{4.1}$$

$$D_i \mathscr{G}_j \equiv \partial_i \mathscr{G}_j - \Gamma^k_{ij} \mathscr{G}_k = \frac{\mathscr{D}_i D_j P}{P} - \mathscr{G}_i \mathscr{G}_j \,. \tag{4.2}$$

The scalar potential, written in term of $\mathscr{G}$, becomes:

$$V = e^K \left[g^{ij*}(D_i P)(D_{j*}\bar{P}) - 3|P|^2 \right] + \frac{1}{2} D_a D^a \tag{4.3}$$

$$= e^{\mathscr{G}} \left[g^{ij*}\mathscr{G}_i \mathscr{G}_{j*} - 3 \right] + \frac{1}{2} D_a D^a. \tag{4.4}$$

The derivative of the potential is:

$$\begin{aligned} \partial_j V = & e^{\mathscr{G}} \left[g^{ij*}\mathscr{G}_i \mathscr{G}_{j*} - 3 \right] \mathscr{G}_j + \\ & e^{\mathscr{G}} \left[\partial_j g^{ij*}\mathscr{G}_i \mathscr{G}_{j*} + g^{ij*}\partial_j \mathscr{G}_i \mathscr{G}_{j*} + g^{ij*}\mathscr{G}_i \partial_j \mathscr{G}_{j*} \right] + \\ & \frac{1}{2}(D_a D^a)_j \ . \end{aligned} \tag{4.5}$$

Since $\partial_j \mathscr{G}_{j*} = g_{jj*}$ and $\partial_j g^{ij*} = -\Gamma^i_{jk} g^{kj*}$, the scalar potential becomes:

$$\partial_j V = e^{\mathscr{G}} \left[\mathscr{G}_i \mathscr{G}^i \mathscr{G}_j - 2\mathscr{G}_j + G^i \mathscr{D}_j \mathscr{G}_i \right] + \frac{1}{2}(D_a D^a)_j \ . \tag{4.6}$$

In a Minkowski vacuum both the potential and its derivative have to vanish. By setting (4.4) to zero the following condition is found:

$$1 = \frac{1}{3}\mathscr{G}_i \mathscr{G}^i + \frac{e^{-\mathscr{G}}}{6} D_a D^a. \tag{4.7}$$

By multiplying $-2\mathscr{G}_n$ in (4.6) by the expression in (4.7), the derivative of the potential becomes:

$$\partial_j V = e^{\mathscr{G}} \left[\frac{1}{3}\mathscr{G}_i \mathscr{G}^i \mathscr{G}_j + \mathscr{G}^i \mathscr{D}_j \mathscr{G}_i \right] + \frac{1}{2}(D_a D^a)_j - \frac{1}{3}\mathscr{G}_j D_a D^a \ . \tag{4.8}$$

The mass term for the gravitino is given by [22]:

$$e^{-1}\mathcal{L} = -\, e^{K/2} \left\{ P^* \psi_m \sigma^{mn} \psi_n - \frac{i}{\sqrt{2}} D_{i*} P^* \psi_m \sigma^m \bar{\chi}^i \right\} - \frac{g}{2} D_a \psi_m \sigma^m \bar{\lambda}^a + \text{h.c.}\ . \tag{4.9}$$

This can be written as:

$$e^{-1}\mathcal{L} = -\, e^{K/2} P^* \psi_m \sigma^{mn} \left\{ \psi_n + \frac{2}{3}\sigma_n \bar{\zeta} \right\} + \text{h.c.}\,. \tag{4.10}$$

where

$$\bar{\zeta} = \frac{i}{\sqrt{2}} \mathscr{G}_{i*} \bar{\chi}^i - \frac{g}{2} \frac{e^{-K/2}}{P^*} \bar{\lambda}^a D_a \,. \tag{4.11}$$

If a new gravitino is defined as

$$\tilde{\psi}_m = \psi_n + \frac{1}{3}\sigma_n \bar{\xi} \,, \tag{4.12}$$

then the previous lagrangian becomes:

$$e^{-1}\mathcal{L} = -e^{K/2} P^* \left\{ \tilde{\psi}_m \sigma^{mn} \tilde{\psi}_n + \frac{2}{3} \bar{\zeta}\bar{\zeta} \right\} + \text{h.c.}\,. \tag{4.13}$$

With this redefinition the gravitino mass is decoupled from the spinor masses. The spinor mass contribution, written as a function of χ^i and λ^a, is

$$-\frac{2}{3} e^{K/2} P \zeta\zeta \to + e^{K/2} P \frac{1}{3} \mathscr{G}_i \mathscr{G}_j \chi^i \chi^j - \frac{g^2}{6} \frac{e^{-K/2}}{P} D_a D_b \lambda^a \lambda^b - \frac{ig\sqrt{2}}{3} \mathscr{G}_i D_a \chi^i \lambda^a \,. \tag{4.14}$$

The mass terms that are present in the lagrangian describing a gauge invariant model in supergravity can be written as

$$e^{-1}\mathscr{L} = -\begin{pmatrix} \chi^i & \lambda^a \end{pmatrix} \begin{pmatrix} \mathscr{M}_{ij} & \mathscr{M}_{ib} \\ \mathscr{M}_{ja} & \mathscr{M}_{ab} \end{pmatrix} \begin{pmatrix} \chi^j \\ \lambda^b \end{pmatrix} + \text{h.c.}\,. \tag{4.15}$$

Considering also the contributions that come from the redefinition of the gravitino the mass matrix elements are:

$$\mathscr{M}_{ij} = + \frac{e^{K/2} P}{2} \left(D_i \mathscr{G}_j + \frac{1}{3} \mathscr{G}_i \mathscr{G}_j \right) \,, \tag{4.16}$$

$$\mathscr{M}_{ib} = + \frac{ig}{3\sqrt{2}} \mathscr{G}_i D_b - \frac{i}{\sqrt{2}} g D_{b,i} + \frac{i}{4\sqrt{2}} g h_{cb,i} D^c \,, \tag{4.17}$$

$$\mathscr{M}_{ab} = \frac{e^{-K/2}}{6P}\left(g^2 D_a D_b - \frac{3}{2} e^{\mathscr{G}} \mathscr{G}^i h_{ab,i}\right) . \tag{4.18}$$

The spinor ζ is a combination of χ^i and λ^a:

$$\zeta = -\begin{pmatrix} \frac{i}{\sqrt{2}}\mathscr{G}_i & \frac{g}{2}\frac{e^{-K/2}}{P} D_a \end{pmatrix} \begin{pmatrix} \chi^i \\ \lambda^a \end{pmatrix} . \tag{4.19}$$

The spinors vector can be written as the sum of a part proportional to ζ and a part orthogonal to it:

$$\begin{pmatrix} \chi^i \\ \lambda^a \end{pmatrix} = \begin{pmatrix} a^i \\ b^a \end{pmatrix} \zeta + \begin{pmatrix} \zeta^{i\perp} \\ \zeta^{a\perp} \end{pmatrix} . \tag{4.20}$$

The condition

$$-\begin{pmatrix} \frac{i}{\sqrt{2}}\mathscr{G}_i & \frac{g}{2}\frac{e^{-K/2}}{P} D_a \end{pmatrix} \begin{pmatrix} a^i \\ b^a \end{pmatrix} = 1 \tag{4.21}$$

is solved by

$$\begin{pmatrix} a^i \\ b^a \end{pmatrix} = \begin{pmatrix} \frac{i\sqrt{2}}{3}\mathscr{G}^i \\ -\frac{e^{-K/2}}{3gP^*} D^a \end{pmatrix} . \tag{4.22}$$

The spinor ζ is the one "eaten" by the gravitino. If the Super-Higgs mechanism works as the standard Higgs one, then ζ has to be the massless goldstino:

$$\begin{pmatrix} \mathscr{M}_{ij} & \mathscr{M}_{ib} \\ \mathscr{M}_{ja} & \mathscr{M}_{ab} \end{pmatrix} \begin{pmatrix} a^j \\ b^b \end{pmatrix} = 0 . \tag{4.23}$$

After some calculations

$$\mathscr{M}_{ij} a^j + \mathscr{M}_{ib} b^b = \frac{i e^{-K/2}}{3\sqrt{2}P^*}(\partial_i V) = 0 , \tag{4.24}$$

$$\mathscr{M}_{ja} a^j + \mathscr{M}_{ab} b^b = -\frac{g}{3}(D_a - \mathscr{G}^i D_{a,i}) = 0 . \tag{4.25}$$

The ζ spinor is the massless goldstino that appears in supersymmetry breaking. The next step would be to verify that the goldstino also disappear in the kinetic part of the lagrangian. This can be done but it requires also a redefinition of the vielbeins. It is not done here where also vector superfields are involved but it is verified in the next section for chiral superfields in a non-Minkowski vacuum.

4.2 Super-Higgs with a Cosmological Constant

In this section the super-Higgs mechanism is described without the constraint of a Minkowski vacuum. De Sitter and anti-De Sitter configurations are taken in account in a model with chiral superfields. While in the previous section also vector superfields were considered, in this one they are not present. The approach followed in this section is the one of [29]. The lagrangian we start with is:

$$\begin{aligned} e^{-1}\mathcal{L} = & -\frac{1}{2}\mathscr{R} - g_{ij^*}\partial_m A^i \partial^m A^{*j} - i g_{ij^*}\bar{\chi}^j \bar{\sigma}^m \mathscr{D}_m \chi^i + \epsilon^{klmn}\bar{\psi}_k \bar{\sigma}_l \tilde{\mathscr{D}}_m \psi_n \\ & - e^{K/2}\left\{ P^* \psi_a \sigma^{ab} \psi_b + \frac{i}{\sqrt{2}} D_{i^*} P^* \bar{\chi}^i \bar{\sigma}^a \psi_a + \frac{1}{2}\mathscr{D}_i D_j P \chi^i \chi^j + \text{h.c.} \right\} \\ & - e^K [g^{ij^*}(D_i P)(D_j P)^* - 3PP^*] \,. \end{aligned} \tag{4.26}$$

Some useful redefinition are:

$$\bar{\zeta} = \frac{e^{K/2}}{\sqrt{2}} D_{i*} P^* \bar{\chi}^i = \frac{e^{\mathscr{G}/2}}{\sqrt{2}} \sqrt{\frac{P^*}{P}} \mathscr{G}_{i*} \bar{\chi}^i \,, \tag{4.27}$$

$$|m_{3/2}|^2 = e^K P^* P = e^{\mathscr{G}} \,, \tag{4.28}$$

$$m_{ij*} = e^{K/2} \mathscr{D}_i D_j P = e^{\mathscr{G}/2} \sqrt{\frac{P}{P^*}} [D_i \mathscr{G}_j + \mathscr{G}_i \mathscr{G}_j] \,, \tag{4.29}$$

$$V = e^K [g^{ij*}(D_i P)(D_j P)^* - 3PP^*] = e^{\mathscr{G}} [\mathscr{G}^i \mathscr{G}_i - 3] \,. \tag{4.30}$$

The derivative of the potential, that gives the only condition that can be imposed at the vacuum, can be written as:

$$\partial_\alpha V = e^{\mathscr{G}} [D_i \mathscr{G}_j \mathscr{G}^j + (\mathscr{G}^j \mathscr{G}_j - 2)\mathscr{G}_i] = 0 \,. \tag{4.31}$$

With these redefinitions the lagrangian above is:

$$\begin{aligned} e^{-1}\mathcal{L} = & -\frac{1}{2}\mathscr{R} - g_{ij^*}\partial_m A^i \partial^m A^{*j} - i g_{ij^*}\bar{\chi}^j \bar{\sigma}^m \mathscr{D}_m \chi^i + \epsilon^{klmn}\bar{\psi}_k \bar{\sigma}_l \tilde{\mathscr{D}}_m \psi_n \\ & - \left\{ m^*_{3/2} \psi_a \sigma^{ab} \psi_b + i\bar{\zeta} \bar{\sigma}^a \psi_a + \frac{1}{2} m_{ij*} \chi^i \chi^j + \text{h.c.} \right\} - V \,. \end{aligned} \tag{4.32}$$

In order to remove the mixing term contribution these substitutions are done:

$$\delta_\zeta \psi_m = \alpha \mathscr{D}_m \zeta + \frac{i}{2}\alpha m_{3/2}\sigma_m \bar{\zeta}\ , \tag{4.33}$$

$$\delta_\zeta e_m^a = \frac{i}{2}\alpha(\zeta\sigma^a\bar{\psi}_m + \bar{\zeta}\bar{\sigma}^a\psi_m)\ . \tag{4.34}$$

The gravitino kinetic term becomes:

$$\begin{aligned}\epsilon^{klmn}\bar{\psi}_k\bar{\sigma}_l\mathscr{D}_m\psi_n =& \epsilon^{klmn}(\bar{\psi}_k + \alpha\mathscr{D}_k\bar{\zeta} - \frac{i}{2}\alpha m_{3/2}^*\zeta\sigma_k)\bar{\sigma}_l\mathscr{D}_m(\psi_n + \alpha\mathscr{D}_n\zeta + \frac{i}{2}\alpha m_{3/2}\sigma_n\bar{\zeta}) \\ =& \epsilon^{klmn}\bar{\psi}_k\bar{\sigma}_l\mathscr{D}_m\psi_n + \alpha\epsilon^{klmn}(\bar{\psi}_k\bar{\sigma}_l\mathscr{D}_m\mathscr{D}_n\zeta + \text{h.c.}) \\ &+ 2\alpha[m_{3/2}^*\psi_m\sigma^{mn}\mathscr{D}_n\zeta + \text{h.c.}] + \alpha^2\epsilon^{klmn}\bar{\zeta}\bar{\sigma}_k\mathscr{D}_l\mathscr{D}_m\mathscr{D}_n\zeta \\ &+ \frac{3}{2}\alpha^2 i|m_{3/2}|^2\bar{\zeta}\bar{\sigma}^m\mathscr{D}_m\zeta - 2\alpha^2(m_{3/2}^*\zeta\sigma^{mn}\mathscr{D}_m\mathscr{D}_n\zeta + \text{h.c.})\ .\end{aligned} \tag{4.35}$$

The gravitino mass term is:

$$\begin{aligned}-m_{3/2}^*\psi_a\sigma^{ab}\psi_b =& - m_{3/2}^*\psi_a\sigma^{ab}\psi_b - 2m_{3/2}^*\alpha\psi_a\sigma^{ab}\mathscr{D}_b\zeta - \frac{3}{2}i\alpha|m_{3/2}|^2\bar{\zeta}\bar{\sigma}^a\psi_a \\ &+ \alpha^2 m_{3/2}^*\zeta\sigma^{ab}\mathscr{D}_a\mathscr{D}_b\zeta - \frac{3}{2}\alpha^2 m_{3/2}|m_{3/2}|^2\bar{\zeta}\bar{\zeta} - \frac{3}{2}|m_{3/2}|^2\alpha^2 i\bar{\zeta}\bar{\sigma}^a\mathscr{D}_a\zeta\ .\end{aligned} \tag{4.36}$$

The mixing mass term gives

$$-i\bar{\zeta}\bar{\sigma}^a\psi_a = - i\bar{\zeta}\bar{\sigma}^a\psi_a - i\alpha\bar{\zeta}\bar{\sigma}^a\mathscr{D}_a\zeta - 2\alpha m_{3/2}\bar{\zeta}\bar{\zeta}\ . \tag{4.37}$$

From the variation of e_m^a these terms arise:

$$\begin{aligned}-\frac{1}{2}e\mathscr{R} - eV =& - \frac{1}{2}e\mathscr{R} - eV - eV\left[\frac{i}{2}\alpha(\zeta\sigma^m\bar{\psi}_m + \bar{\zeta}\bar{\sigma}^m\psi_m)\right] \\ &+ ek\left(\mathscr{R}_a^m - \frac{1}{2}\mathscr{R}e_a^m\right)\left[\frac{i}{2}\alpha(\zeta\sigma^a\bar{\psi}_m + \bar{\zeta}\bar{\sigma}^a\psi_m)\right]\ .\end{aligned} \tag{4.38}$$

By imposing that the mixing terms have to vanish, this value of α is found:

$$-i\bar{\zeta}\bar{\sigma}^a\psi_a\left(1 + \frac{1}{2}\alpha V + \frac{3}{2}|m_{3/2}|^2\alpha\right) = 0 \rightarrow \alpha = -\frac{2}{V + 3|m_{3/2}|^2}\ . \tag{4.39}$$

By using

$$\epsilon^{klmn}\bar{\sigma}_l \mathscr{D}_m \mathscr{D}_n \zeta = \frac{i}{2} k \left(\mathscr{R}^k_a \bar{\sigma}^a - \frac{1}{2} \mathscr{R} \bar{\sigma}^k \right) \zeta \ . \tag{4.40}$$

the $\alpha\epsilon^{klmn}\bar{\psi}_k \bar{\sigma}_l \mathscr{D}_m \mathscr{D}_n \zeta$ term in the gravitino kinetic term is deleted by the last row in (4.38). By introducing a new vielbeins redefinition:

$$\delta e^a_m = \alpha^2 \left[\frac{1}{2} (i\bar{\zeta}\bar{\sigma}^a \mathscr{D}_m \zeta) + \frac{1}{8} e^a_m (m^*_{3/2} \zeta\zeta + \text{h.c.}) \right] \tag{4.41}$$

this additional terms have to be taken in account

$$\begin{aligned} -\frac{1}{2} e\mathscr{R} - eV = & -\frac{1}{2} e\mathscr{R} - eV - eV\alpha^2 \left[\frac{1}{2} (i\bar{\zeta}\bar{\sigma}^a \mathscr{D}_a \zeta) + \frac{1}{2} (m^*_{3/2} \zeta\zeta + \text{h.c.}) \right] \\ & + ek\alpha^2 \left[\frac{1}{2} \left(\mathscr{R}^m_a - \frac{1}{2} \mathscr{R} e^m_a \right) (i\bar{\zeta}\bar{\sigma}^a \mathscr{D}_m \zeta) - \frac{1}{8} \mathscr{R} (m^*_{3/2} \zeta\zeta + \text{h.c.}) \right] \end{aligned} \tag{4.42}$$

and the last row cancels the $\alpha^2[\epsilon^{klmn}\bar{\zeta}\bar{\sigma}_k \mathscr{D}_l \mathscr{D}_m \mathscr{D}_n \zeta - (m^*_{3/2} \zeta \sigma^{mn} \mathscr{D}_m \mathscr{D}_n \zeta + \text{h.c.})]$ contribution from the gravitino kinetic and mass terms. After all these substitutions the surviving terms are:

- $\epsilon^{klmn}\bar{\psi}^k \bar{\sigma}^l \mathscr{D}_m \psi_n$,
- $\left(\frac{3}{2}\alpha^2 |m_{3/2}|^2 - 3\alpha^2 |m_{3/2}|^2 - 2\alpha - \frac{1}{2} V\alpha^2\right) i\bar{\zeta}\bar{\sigma}^m \mathscr{D}_m \zeta = -i\alpha\bar{\zeta}\bar{\sigma}^m \mathscr{D}_m \zeta$,
- $-m^*_{3/2} \psi_a \sigma^{ab} \psi_b$,
- $-m_{3/2} \left(\frac{3}{2}\alpha^2 |m_{3/2}|^2 + 2\alpha + \frac{1}{2}\alpha^2 V\right) \bar{\zeta}\bar{\zeta} = -\alpha m_{3/2} \bar{\zeta}\bar{\zeta}$.

The lagrangian introduced at the beginning of this section, after these redefinitions, is:

$$\begin{aligned} e^{-1}\mathcal{L} = & -\frac{1}{2}\mathscr{R} - g_{ij^*} \partial_m A^i \partial^m A^{*j} - i g_{ij^*} \bar{\chi}^j \bar{\sigma}^m \mathscr{D}_m \chi^i + \epsilon^{klmn} \bar{\psi}_k \bar{\sigma}_l \tilde{\mathscr{D}}_m \psi_n - i\alpha\bar{\zeta}\bar{\sigma}^m \mathscr{D}_m \zeta \\ & - \left\{ m^*_{3/2} \psi_a \sigma^{ab} \psi_b + \frac{1}{2} m_{ij*} \chi^i \chi^j + \alpha m^*_{3/2} \zeta\zeta + \text{h.c.} \right\} - V \ . \end{aligned} \tag{4.43}$$

The spinors kinetic term can be written as:

$$-i g_{ij*} \bar{\chi}^j \bar{\sigma}^m \mathscr{D} \chi^i - i\alpha\bar{\zeta}\bar{\sigma}^m \mathscr{D}_m \zeta = -\left(g_{ij*} - \frac{\mathscr{G}_{j*} \mathscr{G}_i}{X} \right) i\bar{\chi}^j \bar{\sigma}^m \mathscr{D} \chi^i \ , \tag{4.44}$$

where $X = \mathscr{G}^k \mathscr{G}_k$. The spinors mass term as:

$$-\frac{1}{2} m_{ij*} \chi^i \chi^j - \alpha m^*_{3/2} \zeta\zeta = -\frac{1}{2} \left[\sqrt{\frac{P}{P^*}} e^{\mathscr{G}/2} \left(\mathscr{D}_i \mathscr{G}_j + \frac{X-2}{X} \mathscr{G}_i \mathscr{G}_j \right) \right] \chi^i \chi^j \ . \quad (4.45)$$

If ζ is the goldstino it has to be a zero mode of both the kinetic and mass terms. The spinors χ^i can be decomposed as:

$$\chi^i = a^i \zeta + \zeta^{i\perp} \ . \quad (4.46)$$

a^i is found imposing

$$a^i \frac{e^{\mathscr{G}/2}}{\sqrt{2}} \sqrt{\frac{P}{P^*}} \mathscr{G}_i = 1 \rightarrow a^i = \frac{\sqrt{2} e^{\mathscr{G}/2}}{V + 3e^{\mathscr{G}}} \sqrt{\frac{P^*}{P}} \mathscr{G}^i \ . \quad (4.47)$$

With this expression for a^i, the ζ kinetic term is proportional to

$$\left(g_{ij*} - \frac{\mathscr{G}_{j*} \mathscr{G}_i}{X} \right) \mathscr{G}^i = 0 \ ; \quad (4.48)$$

While the mass contribution to ζ is

$$e^{\mathscr{G}/2} \left(\mathscr{D}_i \mathscr{G}_j - \frac{X-2}{X} \mathscr{G}_i \mathscr{G}_j \right) \mathscr{G}^i = e^{\mathscr{G}/2} \left(\mathscr{G}^i \mathscr{D}_i \mathscr{G}_j + (X-2) \mathscr{G}_j \right) = \partial_j V = 0 \ . \quad (4.49)$$

The super-Higgs mechanisms with a cosmological constant works as the standard Higgs mechanism. The massless goldstino is "eaten" by the gravitino and it disappears from the lagrangian.

Chapter 5

Super-Higgs with Non-Linear Supersymmetry

The main aim of this work of thesis is to extend the Super-Higgs mechanism to non-linear realizations of supersymmetry. Constrained superfields will be used to achieve this goal. If only one chiral superfield $\mathbf{X}$ were considered, with the constraint $\mathbf{X}^2 = 0$, then the fermion χ, in the $\mathbf{X}$ supermultiplet, would be the goldstino and it would be "eaten" by the gravitino. This mechanism would lead to an effective lagrangian in which the only surviving fields would be a massive gravitino and the vielbeins. Considering two chiral superfields instead gives a more interesting scenario because also a massive fermion appears in the effective lagrangian and this is the model discussed here.

5.1 Constrained Superfields in a Non-Minkowski Vacuum

The Θ expansion of the two chiral superfields considered in this chapter is:

$$\mathbf{X} = x + \sqrt{2}\Theta\chi + \Theta^2 F^x \ , \tag{5.1}$$

$$\mathbf{Y} = y + \sqrt{2}\Theta\xi + \Theta^2 F^y \ . \tag{5.2}$$

The constraints that have to be imposed in order to remove the scalar degrees of freedom are:

$$\begin{cases} \mathbf{X} = 0 \\ \mathbf{XY} = 0 \end{cases} , \tag{5.3}$$

with solution:

$$x = \frac{\chi\chi}{2F^x} , \qquad y = \frac{\xi\chi}{F^x} - \frac{\chi\chi}{2(F^x)^2} F^y . \tag{5.4}$$

The most general Kähler potential and superpotential for a model with these cosntraints are

$$K = |\mathbf{X}|^2 + |\mathbf{Y}|^2 + a(\mathbf{X}\bar{\mathbf{Y}}^2 + \bar{\mathbf{X}}\mathbf{Y}^2) + b(\mathbf{Y}\bar{\mathbf{Y}}^2 + \bar{\mathbf{Y}}\mathbf{Y}^2) + c|\mathbf{Y}|^4 ; \tag{5.5}$$

$$P = P_0 + f\mathbf{X} + g\mathbf{Y} + h\mathbf{Y}^2. \tag{5.6}$$

The effective lagrangian can be computed starting from the off-shell lagrangian:

$$\mathcal{L} = \int d^2\Theta 2\xi \left[\frac{3}{8}(\bar{D}^2 - 8R)e^{-K/3} + P \right] + \text{h.c.} , \tag{5.7}$$

and substituting the scalar fields with their expression as function of F^i, χ and ξ:

$$x, y \to x(\chi, \xi, F^x), y(\chi, \xi, F^x) . \tag{5.8}$$

With the $\chi = 0$ gauge choice this operation would be trivial because all the scalar fields would vanish. This consideration will be useful in the next section. The on-shell lagrangian is obtained by solving the equations of motion for F^i, it can be very hard but a full expression for F^i is not necessary. The equations of motion can be written as:

$$F^x = -f + \tilde{F}^x(x, y, \chi, \xi, F^y) , \tag{5.9}$$

$$F^y = -g + \tilde{F}^y(x, y, \chi, \xi, F^x) . \tag{5.10}$$

The only constant terms are f and g while $\tilde{F}^i$ has to be scalar and so it has to be at least a fermion bilinear. Since x and y are F^i dependent these equations should be solved iteratively. The first step is to consider $F^x = -f$ and $F^y = -g$. The second gives more interesting contributions:

$$F^x = -f + \tilde{F}^x(x, y, \chi, \xi, F^y)|_{\{F^x=-f\}} \,, \tag{5.11}$$

$$F^y = -g + \tilde{F}^y(x, y, \chi, \xi, F^x)|_{\{F^y=-g\}} \,. \tag{5.12}$$

With these two steps all the two fermion contribution to F^i are found. From the third step only four ore more than four fermions terms are affected. The equations of motion for F^i are formally the same in the constrained and unconstrained case. The two fermions constrained lagrangian can be found starting from the unconstrained one and imposing:

$$x = -\frac{\chi\chi}{2f} \,, \qquad y = -\frac{\xi\chi}{f} + \frac{\chi\chi}{2f^2} g \,. \tag{5.13}$$

The unconstrained lagrangian is [22];

$$\begin{aligned} e^{-1}\mathscr{L} = &- \frac{1}{2}\mathscr{R} - g_{ij^*}\partial_m A^i \partial^m A^{*j} - i g_{ij^*}\bar{\chi}^j \bar{\sigma}^m \mathscr{D}_m \chi^i \\ &+ \epsilon^{klmn}\bar{\psi}_k \bar{\sigma}_l \tilde{\mathscr{D}}_m \psi_n - \frac{1}{\sqrt{2}} g_{ij^*}\partial_n A^{*j} \chi^i \sigma^m \bar{\sigma}^n \psi_m \\ &- \frac{1}{\sqrt{2}} g_{ij^*}\partial_n A^i \bar{\chi}^j \bar{\sigma}^m \sigma^n \bar{\psi}_m - \frac{1}{8}[g_{ij^*} g_{kl^*} - 2R_{ij^*kl^*}]\chi^i \chi^k \bar{\chi}^j \bar{\chi}^l \\ &+ \frac{1}{4} g_{ij^*}[i\epsilon^{klmn}\psi_k \sigma_l \bar{\psi}_m + \psi_m \sigma^n \bar{\psi}^m]\chi^i \sigma_n \bar{\chi}^j \\ &- e^{K/2}\Big\{ P^* \psi_a \sigma^{ab} \psi_b + P \bar{\psi}_a \bar{\sigma}^{ab} \bar{\psi}_b + \frac{i}{\sqrt{2}} D_i P \chi^i \sigma^a \bar{\psi}_a \\ &+ \frac{i}{\sqrt{2}} D_{i^*} P^* \bar{\chi}^i \bar{\sigma}^a \psi_a + \frac{1}{2}\mathscr{D}_i D_j P \chi^i \chi^j + \frac{1}{2}\mathscr{D}_{i^*} D_{j^*} P^* \bar{\chi}^i \bar{\chi}^j \Big\} \\ &- e^K [g^{ij^*}(D_i P)(D_j P)^* - 3PP^*] \,, \end{aligned} \tag{5.14}$$

where A^i and χ^i are the scalar and fermionic fields of i chiral superfields, $\mathscr{R}$ is the curvature given by the vielbeins e^a, ψ_a is the gravitino field, K is the Kähler potential and P is the superpotential. The covariant derivatives are defined as follows:

$$
\begin{aligned}
\mathscr{D}_m\chi^i &= \partial_m\chi^i + \chi^i\omega_n + \Gamma^i_{jk}\partial_m A^j\chi^k - \frac{1}{4}(K_j\partial_m A^j - K_{j^*}\partial_m A^{*j})\chi^i \, , \\
\tilde{\mathscr{D}}_m\psi_n &= \partial_m\psi_n + \psi_n\omega_m + \frac{1}{4}(K_j\partial_m A^j - K_{j^*}\partial_m A^{*j})\psi_n \, , \\
D_iP &= P_i + K_iP \, , \\
\mathscr{D}_iD_jP &= P_{ij} + K_{ij}P + K_iD_jP + K_jD_iP - K_iK_jP - \Gamma^k_{ij}D_kP \, . \qquad (5.15)
\end{aligned}
$$

For simplicity $\tilde{\mathscr{D}}_m\psi_n = \mathscr{D}_m\psi_n$ because the only difference is a term proportional to $\partial_m A^j$. With the constraints imposed above this is the derivative of a fermion bilinear. It leads to interaction terms that contain derivatives of fermions and we are not interested in them in this chapter.

For the model considered in this chapter the metric and the Christoffel symbols are:

$$
g_{ij^*} = \begin{pmatrix} 1 & 2a\bar{y} \\ 2ay & 1+2b(y+\bar{y}) \end{pmatrix} ; \qquad g^{ij^*} = \begin{pmatrix} 1 & -2ay \\ -2a\bar{y} & 1-2b(y+\bar{y}) \end{pmatrix} ; \qquad (5.16)
$$

$$
\Gamma^x_{yy} = 2a \, ; \qquad \Gamma^y_{yy} = 2b \, . \qquad (5.17)
$$

Only the contributions useful to the two-fermions lagrangian are considered. The kinetic and mass terms can be written as:

$$
e^{-1}\mathscr{L}\Big|_{2f,2g} = -\frac{1}{2}\mathscr{R} + \mathscr{L}_1(\chi,\xi,\psi_a) + \mathscr{L}_2(\chi\chi,\chi\xi) - V \, , \qquad (5.18)
$$

where

$$
\begin{aligned}
\mathscr{L}_1 = &- i\bar{\chi}\bar{\sigma}^m\mathscr{D}_m\chi - i\bar{\xi}\bar{\sigma}^m\mathscr{D}_m\xi + \epsilon^{klmn}\bar{\psi}_k\bar{\sigma}_l\mathscr{D}_m\psi_n \\
&- \Big\{ P_0\psi_a\sigma^{ab}\psi_b + P_0\bar{\psi}_a\bar{\sigma}^{ab}\bar{\psi}_b + \frac{i}{\sqrt{2}}(f\chi + g\xi)\sigma^a\bar{\psi}_a \\
&+ \frac{i}{\sqrt{2}}(f\bar{\chi} + g\bar{\xi})\bar{\sigma}^a\psi_a + (h - af + bg)(\xi\xi + \bar{\xi}\bar{\xi}) \Big\} \, , \qquad (5.19) \\
\mathscr{L}_2 = &- [-2afg(y+y^*) + 2hg(y+y^*) \\
&- 2bg^2(y+y^*) - 2f(x+x^*)P_0 - 2g(y+y^*)P_0] \, , \qquad (5.20)
\end{aligned}
$$

$$V = f^2 + g^2 - 3P_0^2 \ . \tag{5.21}$$

In these lagrangians the scalar fields x and y are meant as the function of χ and ξ in (5.13). The starting point of the Super-Higgs mechanism is to try to cancel the mass mixing term between the gravitino and the fermions. For this purpose the spinors can be redefined in this way

$$\zeta = \frac{f\chi + g\xi}{\sqrt{f^2 + g^2}} \ , \tag{5.22}$$

$$\hat{\chi} = \frac{f\xi - g\chi}{\sqrt{f^2 + g^2}} \ . \tag{5.23}$$

where ζ will be the goldstino. With these transformations the kinetic terms for the fermions are still canonical:

$$-i\bar{\chi}\bar{\sigma}^m \mathscr{D}_m \chi - i\bar{\xi}\bar{\sigma}^m \mathscr{D}_m \xi \rightarrow -i\bar{\hat{\chi}}\bar{\sigma}^m \mathscr{D}_m \hat{\chi} - i\bar{\zeta}\bar{\sigma}^m \mathscr{D}_m \zeta \ . \tag{5.24}$$

Following the steps of the previous chapter a supersymmetry transformation is performed for the gravitino and its superpartner:

$$\delta_\zeta \psi_m = \alpha \mathscr{D}_m \zeta + \frac{i}{2}\alpha P_0 \sigma_m \bar{\zeta} \ , \tag{5.25}$$

$$\delta_\zeta e_m^a = \frac{i}{2}\alpha(\zeta\sigma^a\bar{\psi}_m + \bar{\zeta}\bar{\sigma}^a\psi_m) \ . \tag{5.26}$$

The gravitino variation gives:

$$\begin{aligned} \epsilon^{klmn}\bar{\psi}_k\bar{\sigma}_l\mathscr{D}_m\psi_n = & \epsilon^{klmn}\bar{\psi}_k\bar{\sigma}_l\mathscr{D}_m\psi_n + \alpha\epsilon^{klmn}(\bar{\psi}_k\bar{\sigma}_l\mathscr{D}_m\mathscr{D}_n\zeta + \text{h.c.}) \\ & + 2P_0\alpha[\psi_m\sigma^{mn}\mathscr{D}_n\zeta + \text{h.c.}] + \alpha^2\epsilon^{klmn}\bar{\zeta}\bar{\sigma}_k\mathscr{D}_l\mathscr{D}_m\mathscr{D}_n\zeta \\ & + \frac{3}{2}\alpha^2 i P_0^2\bar{\zeta}\bar{\sigma}^m\mathscr{D}_m\zeta - 2\alpha^2 P_0(\zeta\sigma^{mn}\mathscr{D}_m\mathscr{D}_n\zeta + \text{h.c.}) \ , \end{aligned} \tag{5.27}$$

$$\begin{aligned} -P_0\psi_a\sigma^{ab}\psi_b = & - P_0\psi_a\sigma^{ab}\psi_b - 2P_0\alpha\psi_a\sigma^{ab}\mathscr{D}_b\zeta - \frac{3}{2}i\alpha P_0^2\bar{\zeta}\bar{\sigma}^a\psi_a \\ & + \alpha^2 P_0\zeta\sigma^{ab}\mathscr{D}_a\mathscr{D}_b\zeta - \frac{3}{2}\alpha^2 P_0^3\bar{\zeta}\bar{\zeta} - \frac{3}{2}P_0^2\alpha^2 i\bar{\zeta}\bar{\sigma}^a\mathscr{D}_a\zeta \ , \end{aligned} \tag{5.28}$$

$$-i\sqrt{\frac{f^2+g^2}{2}}\bar{\zeta}\bar{\sigma}^a\psi_a = -i\sqrt{\frac{f^2+g^2}{2}}\bar{\zeta}\bar{\sigma}^a\psi_a - i\alpha\sqrt{\frac{f^2+g^2}{2}}\bar{\zeta}\bar{\sigma}^a\mathscr{D}_a\zeta$$
$$- \alpha P_0\sqrt{2(f^2+g^2)}\bar{\zeta}\bar{\zeta}\;. \tag{5.29}$$

The variation of e_m^a leads to:

$$-\frac{1}{2}e\mathscr{R} - eV = -\frac{1}{2}e\mathscr{R} - eV - eV\left[\frac{i}{2}\alpha(\zeta\sigma^m\bar{\psi}_m + \bar{\zeta}\bar{\sigma}^m\psi_m)\right]$$
$$+ ek\left(\mathscr{R}_a^m - \frac{1}{2}\mathscr{R}e_a^m\right)\left[\frac{i}{2}\alpha(\zeta\sigma^a\bar{\psi}_m + \bar{\zeta}\bar{\sigma}^a\psi_m)\right]. \tag{5.30}$$

In order to cancel the new mixing terms from (5.28), (5.29) and (5.30) this constraint has to be imposed:

$$-\left(\frac{3}{2}\alpha P_0^2 + \sqrt{\frac{V+3P_0^2}{2}} + \frac{V}{2}\alpha\right)i\bar{\zeta}\bar{\sigma}^m\psi_m = 0\;. \tag{5.31}$$

where $f^2+g^2 = V+3P_0^2$ has been used. It is solved by:

$$\alpha = -\sqrt{\frac{2}{V+3P_0^2}} = -\sqrt{\frac{2}{f^2+g^2}}\;. \tag{5.32}$$

The relation (4.40) makes the second line of (5.30) to cancel with $\alpha\epsilon^{klmn}\bar{\psi}_k\bar{\sigma}_l\mathscr{D}_m\mathscr{D}_n\zeta$ in (5.27) and its hermitian conjugate. The relation (4.40) can be used also to make $\alpha^2[\epsilon^{klmn}\bar{\zeta}\bar{\sigma}_k\mathscr{D}_l\mathscr{D}_m\mathscr{D}_n\zeta - P_0(\zeta\sigma^{mn}\mathscr{D}_m\mathscr{D}_n\zeta + \text{h.c.})]$ vanish (from (5.27) and (5.28)) by introducing a new transformation for e_m^a as seen in the previous chapter:

$$\delta e_m^a = \alpha^2\left[\frac{1}{2}(i\bar{\zeta}\bar{\sigma}^a\mathscr{D}_m\zeta) + \frac{1}{8}e_m^a P_0(\zeta\zeta + \text{h.c.})\right]. \tag{5.33}$$

This variation gives:

$$-\frac{1}{2}e\mathscr{R} - eV = -\frac{1}{2}e\mathscr{R} - eV - eV\alpha^2\left[\frac{1}{2}(i\bar{\zeta}\bar{\sigma}^a\mathscr{D}_m\zeta) + \frac{1}{2}P_0(\zeta\zeta + \text{h.c.})\right]$$
$$+ ek\alpha^2\left[\frac{1}{2}\left(\mathscr{R}_a^m - \frac{1}{2}\mathscr{R}e_a^m\right)(i\bar{\zeta}\bar{\sigma}^a\mathscr{D}_m\zeta) - \frac{1}{8}\mathscr{R}P_0(\zeta\zeta + \text{h.c.})\right]. \tag{5.34}$$

The second row cancels the α^2 kinetic terms written above.

In (5.27) only $\bar{\zeta}\bar{\sigma}^a\mathscr{D}_a\zeta$ is still present in addition to the gravitino kinetic term. By summing the various contribution to it from (5.27), (5.28), (5.29) and (5.34) the result is:

$$i\left(+\frac{3}{2}\alpha^2P_0^2 - 3P_0^2\alpha^2 - \alpha\sqrt{2(f^2+g^2)} - \frac{V}{2}\alpha^2\right)\bar{\zeta}\bar{\sigma}^a\mathscr{D}_m\zeta = i\bar{\zeta}\bar{\sigma}^a\mathscr{D}_m\zeta\,. \tag{5.35}$$

This term cancel the one in (5.24) and so ζ disappear from the kinetic lagrangian.

The $\bar{\zeta}\bar{\zeta}$ term in (5.28) has contributions from (5.28), (5.29) and (5.34):

$$\left[-\frac{3}{2}\alpha^2P_0^3 - \alpha P_0\sqrt{2(f^2+g^2)} - \frac{V}{2}\alpha^2P_0\right]\bar{\zeta}\bar{\zeta} = P_0\bar{\zeta}\bar{\zeta}\,. \tag{5.36}$$

After all these calculation the following lagrangian is obtained:

$$\begin{aligned} -\frac{1}{2}\mathscr{R} + \mathscr{L}_1 = &-\frac{1}{2}\mathscr{R} - i\hat{\bar{\chi}}\bar{\sigma}^m\mathscr{D}_m\hat{\chi} + \epsilon^{klmn}\bar{\psi}_k\bar{\sigma}_l\mathscr{D}_m\psi_n \\ &- P_0(\psi_m\sigma^{mn}\psi_n + \text{h.c.}) + P_0(\bar{\zeta}\bar{\zeta} + \text{h.c.}) \\ &- (h - af + bg)(\xi\xi + \text{h.c.}) - V = -\frac{1}{2}\mathscr{R} + \tilde{\mathscr{L}_1}\,. \end{aligned} \tag{5.37}$$

In this lagrangian there is the canonical curvature term and the canonical kinetic terms for $\hat{\chi}$ and ψ_a. There is not a kinetic term for ζ but it is still present in a mass term. From now on $P_0 = m_{3/2}$ because it is the mass of the gravitino. Now the mass term for the spinors are taken in account with the hope that $\hat{\chi}$ and ζ diagonalize the mass matrix and that ζ is massless. The mass contributions for the spinors are:

$$\begin{aligned} \mathscr{L}_2 + m_{3/2}\zeta\zeta - (h - af + bg)\xi\xi = &-\frac{1}{2}\left[2\left(h - af - bg - \frac{g^2m_{3/2}}{f^2+g^2}\right)\xi\xi\right. \\ &- 4\frac{g}{f}\left(h - af - bg - \frac{g^2m_{3/2}}{f^2+g^2}\right)\chi\xi \\ &\left.+ 2\frac{g^2}{f^2}\left(h - af - bg - \frac{g^2m_{3/2}}{f^2+g^2}\right)\chi\chi\right]\,. \end{aligned} \tag{5.38}$$

The eigenvalues and corresponding eigenvectors of the fermionic mass matrix are:

$$\lambda = 0 \to \zeta = \frac{f\chi + g\xi}{\sqrt{f^2+g^2}} \,, \tag{5.39}$$

$$\lambda = 2\frac{(f^2+g^2)(h-af-bg)-g^2m_{3/2}}{f^2} = m_{1/2} \to \hat{\chi} = \frac{f\xi - g\chi}{\sqrt{f^2+g^2}} \,. \tag{5.40}$$

ζ and $\hat{\chi}$ diagonalize the mass matrix and ζ is the massless goldstino. The mass value for $\hat{\chi}$ is the same found in [27] with a Minkowski background. The main difference is that here f, g and $m_{3/2}$ are all independent parameters. The diagonalized two fermions lagrangian is:

$$\begin{aligned} e^{-1}\mathscr{L}\Big|_{2f,2g} = &-\frac{1}{2}\mathscr{R} - i\hat{\bar{\chi}}\bar{\sigma}^m\mathscr{D}_m\hat{\chi} + \epsilon^{klmn}\bar{\psi}_k\bar{\sigma}_l\mathscr{D}_m\psi_n \\ &- m_{3/2}(\psi_a\sigma^{ab}\psi_b + \text{h.c.}) - \frac{1}{2}m_{1/2}(\hat{\chi}\hat{\chi} + \text{h.c.}) - V \,. \end{aligned} \tag{5.41}$$

It is possible to see that ζ completely disappeared from the lagrangian and ψ became massive. The Super-Higgs mechanism has the same effect in linear and non-linear supersymmetry: a massless fermion, the goldstino, is "eaten" by the gravitino and it is removed from the lagrangian.

5.2 Four Fermion Interaction

The mass and kinetic terms of the model we are considering are well described in the previous section. Now the focus is on interactions. We are interested in low energy effective lagrangians and so we will not consider interactions that contain derivatives. Thanks to the Super-Higgs mechanism one spinor is removed and so the model we are describing contains only one spinor. The only meaningful non derivative interaction is therefore the four fermion interaction for the surviving spinor. Without considering a particular gauge it is very hard to compute interactions. A physical gauge is to choose $\zeta = 0$. The goldstino is automatically deleted from the lagrangian and the only fermion is $\hat{\chi}$. In this gauge the equations of motion for F^i are not trivial and so it is still difficult to compute the interaction terms. The best gauge choice is to set $\chi = 0$ because it makes the contributions from the scalars to vanish. F^i can be integrated out with the same equation of motion used in linear supersymmetry and so lagrangian (5.14) can be used also for computing interactions. Since the only surviving spinor is $\hat{\chi}$ all the fermions are proportional to it:

$$\xi = \frac{\sqrt{f^2+g^2}}{f}\hat{\chi}\,, \tag{5.42}$$

$$\zeta = \frac{g}{f}\hat{\chi}\,. \tag{5.43}$$

Lagrangian (5.14) becomes:

$$e^{-1}\mathscr{L} = -\frac{1}{2}\mathscr{R} + \mathscr{L}_1 + \mathscr{L}_i\,, \tag{5.44}$$

where $\mathscr{L}_1$ is the same defined in (5.19), $\mathscr{L}_2 = x = y = \chi = 0$, $\xi \sim \hat{\chi}$ and

$$\mathscr{L}_i = \frac{1}{4}[i\epsilon^{klmn}\psi_k\sigma_l\bar{\psi}_m + \psi_m\sigma^n\bar{\psi}^m]\xi\sigma_n\bar{\xi} - \frac{1}{8}[1-8(c-a^2-b^2)]\xi\xi\bar{\xi}\bar{\xi}\,. \tag{5.45}$$

With the variations introduced in the previous section with the aim of diagonalizing the two fermions lagrangian, the contributions to the $\hat{\chi}\hat{\chi}\hat{\bar{\chi}}\hat{\bar{\chi}}$ interaction come from:

$$\delta e\tilde{\mathscr{L}_1}\,, \qquad -ie\bar{\xi}\bar{\sigma}^n\xi\delta\omega_n\,, \qquad e\epsilon^{klmn}\delta\bar{\psi}_k\bar{\sigma}_l(\delta\psi_n\delta\omega_n)\,, \tag{5.46}$$

$$e(\mathscr{L}_i + \delta\mathscr{L}_i)\,, \qquad -\frac{1}{2}e\delta_\psi\mathscr{R}\,. \tag{5.47}$$

In $-\frac{1}{2}e\delta_\psi\mathscr{R}$, the notation δ_ψ means that the considered contribution are from the variation of ψ in $\mathscr{R}$ while, in all this work, the variation of $\mathscr{R}$ was always with respect to δe.

The first contribution taken in account is the one that comes from the interaction terms in (5.45):

$$\begin{aligned} e(\mathscr{L}_i + \delta\mathscr{L}_i) \to &-\frac{e}{8}\frac{(f^2+g^2)^2}{f^4}[1-8(c-a^2-b^2)]\hat{\chi}\hat{\chi}\hat{\bar{\chi}}\hat{\bar{\chi}} \\ &+\frac{e}{4}\Bigg[i\epsilon^{klmn}\left(+\frac{ig\alpha m_{3/2}}{2f}\sigma_k\hat{\bar{\chi}}\right)\sigma_l\left(-\frac{ig\alpha m_{3/2}}{2f}\hat{\chi}\sigma_m\right) \\ &+\left(\frac{ig\alpha m_{3/2}}{2f}\sigma_m\hat{\bar{\chi}}\right)\sigma^n\left(-\frac{ig\alpha m_{3/2}}{2f}\hat{\chi}\sigma^m\right)\Bigg]\frac{(f^2+g^2)}{f^2}\hat{\chi}\sigma_n\hat{\bar{\chi}} = \end{aligned}$$

$$= - e\left[\frac{1}{8}\frac{(f^2+g^2)^2}{f^4}[1-8(c-a^2-b^2)] + \frac{g^2 m_{3/2}^2}{f^4}\right]\hat{\chi}\hat{\chi}\hat{\bar{\chi}}\hat{\bar{\chi}} \,. \qquad (5.48)$$

A more difficult contribution to compute is the one that arise from$\epsilon^{klmn}\delta\bar{\psi}_n\bar{\sigma}_l(\delta\psi_n\delta\omega_m)$:

$$\begin{aligned}\epsilon^{klmn}\delta\bar{\psi}_k\bar{\sigma}_l(\delta\psi_n\delta\omega_m) =& + \frac{g^2}{4f^2}\alpha^2 m_{3/2}^2\epsilon^{klmn}(\hat{\chi}\sigma_k)\bar{\sigma}_l(\sigma_n\hat{\bar{\chi}}\delta\omega_m)\\ =& - i\alpha^2 m_{3/2}^2\frac{g^2}{f^2}\epsilon_{\beta\gamma}\epsilon^{\delta\mu}\hat{\chi}^{\alpha}\hat{\bar{\chi}}^{\dot{\alpha}}\delta\omega_{m\delta}{}^{\gamma}\\ &\left[-\frac{1}{2}(\sigma^m_{\alpha\dot{\alpha}}\delta^{\beta}_{\mu} - \bar{\sigma}^{m\dot{\gamma}\beta}\epsilon_{\alpha\mu}\epsilon_{\dot{\gamma}\dot{\alpha}})\right] \,. \end{aligned} \qquad (5.49)$$

Since $\delta\omega_{m\beta}{}^{\delta} \sim \delta\omega_{mln}(\sigma^{ln})_{\beta}{}^{\delta}$ that is traceless, the previous expression becomes:

$$\begin{aligned}\epsilon^{klmn}\delta\bar{\psi}_k\bar{\sigma}_l(\delta\psi_n\delta\omega_m) =& - i\alpha^2 m_{3/2}^2\frac{g^2}{2f^2}\epsilon_{\beta\gamma}\epsilon^{\delta\mu}\hat{\chi}^{\alpha}\hat{\bar{\chi}}^{\dot{\alpha}}\delta\omega_{m\delta}{}^{\gamma}\bar{\sigma}^{m\dot{\gamma}\beta}\epsilon_{\alpha\mu}\epsilon_{\dot{\gamma}\dot{\alpha}}\\ =& - i\alpha^2 m_{3/2}^2\frac{g^2}{2f^2}\hat{\bar{\chi}}_{\dot{\gamma}}\bar{\sigma}^{m\dot{\gamma}\beta}\epsilon_{\beta\gamma}\hat{\chi}^{\delta}\delta\omega_{m\delta}{}^{\gamma} \,. \end{aligned} \qquad (5.50)$$

The sum of the two contributions, $\epsilon^{klmn}\delta\bar{\psi}_k\bar{\sigma}_l(\delta\psi_n\delta\omega_m)$ and $-ie\frac{f^2+g^2}{f^2}\hat{\bar{\chi}}\bar{\sigma}^n\hat{\chi}\delta\omega_n$, gives:

$$-ie\frac{f^2+g^2}{f^2}\hat{\bar{\chi}}\bar{\sigma}^n\hat{\chi}\delta\omega_n + e\epsilon^{klmn}\delta\bar{\psi}_k\bar{\sigma}_l(\delta\psi_n\delta\omega_n) = -ie\left(\frac{f^2+g^2}{f^2} + \alpha^2 m_{3/2}^2\frac{g^2}{2f^2}\right)\hat{\bar{\chi}}\bar{\sigma}^n\hat{\chi}\delta\omega_n \,. \qquad (5.51)$$

With the notations in [22],

$$\hat{\chi}\delta\omega_n = \hat{\chi}^{\beta}\delta\omega_{n\beta}{}^{\alpha} = -\frac{1}{2}\hat{\chi}^{\beta}\delta\omega_{nml}(\sigma^{ml})_{\beta}^{\alpha} \,. \qquad (5.52)$$

The expression for ω_{nml} is:

$$\begin{aligned}\omega_{nml} =& \frac{1}{2}\Big\{-\frac{i}{2}(\psi_m\sigma_l\bar{\psi}_n - \psi_n\sigma_l\bar{\psi}_m) - \frac{i}{2}(\psi_n\sigma_m\bar{\psi}_l - \psi_l\sigma_m\bar{\psi}_n)\\ &+ \frac{i}{2}(\psi_l\sigma_n\bar{\psi}_m - \psi_m\sigma_n\bar{\psi}_l) + \ldots\Big\} \,. \end{aligned} \qquad (5.53)$$

Its contraction with σ^{ml} gives:

$$(\sigma^{ml})^{\alpha}_{\beta}\omega_{nml} = -\frac{i}{2}(\sigma^{ml})^{\alpha}_{\beta}(\psi_m\sigma_l\bar{\psi}_n + \psi_n\sigma_m\bar{\psi}_l + \psi_m\sigma_n\bar{\psi}_l)\,. \tag{5.54}$$

The useful contributions to the four fermions interaction come from $\delta\psi\sigma\delta\bar{\psi}$, because every $\delta\psi$ gives a $\hat{\chi}$ and two of them already multiply $\delta\omega$:

$$\begin{aligned}-ie\hat{\bar{\chi}}\bar{\sigma}^n\hat{\chi}\delta\omega_n =& - ie\hat{\bar{\chi}}_{\dot{\alpha}}\bar{\sigma}^{n\dot{\alpha}\alpha}\epsilon_{\alpha\beta}\chi^{\gamma}\delta\omega_{n\gamma}{}^{\beta}\\ =& - \frac{e}{8}\alpha^2 m_{3/2}^2\frac{g^2}{f^2}[\hat{\bar{\chi}}\sigma^{ml}\hat{\chi}\bar{\chi}\bar{\sigma}_m\sigma_l\bar{\chi} + (\hat{\bar{\chi}}\hat{\bar{\chi}})(\chi\sigma^{ml}\sigma_m\bar{\sigma}_l\hat{\chi}) - \hat{\chi}\sigma^{ml}\sigma_m\hat{\bar{\chi}}\hat{\chi}\sigma_l\hat{\bar{\chi}}]\end{aligned} \tag{5.55}$$

The first term vanishes because $\chi\sigma^m\bar{\sigma}^l\psi = \psi\sigma^l\bar{\sigma}^m\chi$ and $\sigma^{ml} = -\sigma^{lm}$, so $\hat{\chi}\sigma^{ml}\hat{\chi} = \hat{\chi}\sigma^{lm}\hat{\chi} = -\hat{\chi}\sigma^{ml}\hat{\chi} = 0$. The second gives $-6\hat{\chi}\hat{\chi}\hat{\bar{\chi}}\hat{\bar{\chi}}$ and the third $+3\hat{\chi}\hat{\chi}\hat{\bar{\chi}}\hat{\bar{\chi}}$. Eventually:

$$\begin{aligned}&\epsilon^{klmn}\delta\bar{\psi}_k\bar{\sigma}_l(\delta\psi_n\delta\omega_m) - ie\frac{f^2+g^2}{f^2}\hat{\bar{\chi}}\bar{\sigma}^n\hat{\chi}\delta\omega_n \to\\ &\quad + \frac{3e}{8}\alpha^2 m_{3/2}^2\frac{g^2}{f^2}\left(\frac{f^2+g^2}{f^2} + \alpha^2 m_{3/2}^2\frac{g^2}{2f^2}\right)\hat{\chi}\hat{\chi}\hat{\bar{\chi}}\hat{\bar{\chi}}\,.\end{aligned} \tag{5.56}$$

Now the contribution from $-\frac{1}{2}e\delta_{\psi}\mathscr{R}$ is taken in account. The curvature is:

$$\mathscr{R} = e^{ma}e^n_b\mathscr{R}_{mna}{}^b = \cdots + \omega_{nml}\omega^{mln} - \omega_m{}^{ml}\omega_{nl}{}^n\,. \tag{5.57}$$

The first term can be written as:

$$\omega_{nml}\omega^{mln} = -\frac{1}{4}[(\psi_m\psi^m)(\bar{\psi}_n\bar{\psi}^n) - (\psi_n\psi^m)(\bar{\psi}_m\bar{\psi}^n)]\,. \tag{5.58}$$

Its variation gives:

$$\begin{aligned}\delta(\omega_{nml}\omega^{mln}) =& -\frac{1}{4}\frac{\alpha^4 m_{3/2}^4}{16}\frac{g^4}{f^4}\{[(\sigma_m\hat{\bar{\chi}})^{\alpha}(\sigma^m\hat{\bar{\chi}})_{\alpha}][(\hat{\chi}\sigma_n)_{\dot{\alpha}}(\hat{\chi}\sigma^n)^{\dot{\alpha}}]\\ =& -\frac{3m_{3/2}^4}{4(f^2+g^2)^2}\frac{g^4}{f^4}\hat{\chi}\hat{\chi}\hat{\bar{\chi}}\hat{\bar{\chi}}\,.\end{aligned} \tag{5.59}$$

The second term in $\mathscr{R}$ is:

$$\omega_m{}^{ml}\omega_{nl}{}^n = \frac{i}{2}(-\psi_m\sigma^m\bar{\psi}^l + \psi^l\sigma^m\bar{\psi}_m)\omega_{nl}{}^n \, , \tag{5.60}$$

and its variation vanishes:

$$\delta(\omega_m{}^{ml}\omega_{nl}{}^n) = \frac{i}{8}\alpha^2 m_{3/2}^2 \frac{g^2}{f^2}(-(\sigma_m\hat{\bar{\chi}})^\alpha \sigma^m_{\alpha\dot{\alpha}}(\hat{\chi}\sigma^l)^{\dot{\alpha}} + (\sigma_l\hat{\bar{\chi}})^\alpha \sigma^m_{\alpha\dot{\alpha}}(\hat{\chi}\sigma_m)^{\dot{\alpha}})\delta\omega_{nl}{}^n = 0 \, . \tag{5.61}$$

The contribution from $\delta_\psi\mathscr{R}$ is:

$$-\frac{1}{2}e\delta_\psi\mathscr{R} \to +\frac{3m_{3/2}^4}{8(f^2+g^2)^2}\frac{g^4}{f^4}\hat{\chi}\hat{\chi}\hat{\bar{\chi}}\hat{\bar{\chi}} \, . \tag{5.62}$$

From $\delta e\tilde{\mathscr{L}}_1$ two kind of contributions are relevant, the first one is more intuitive than the second:

$$\delta e\left[-\frac{1}{2}m_{1/2}(\hat{\chi}\hat{\chi} + \text{h.c.})\right] \to -e\frac{m_{3/2}m_{1/2}g^2}{f^2(f^2+g^2)}\hat{\chi}\hat{\chi}\hat{\bar{\chi}}\hat{\bar{\chi}} \, . \tag{5.63}$$

The second comes from $-\delta eV$ and it is obtained considering two variations of δe^a_m when computing δe. Usually these contributions are forgotten because the transformations are infinitesimal but here they are finite. The full δe variation is:

$$\begin{aligned}\delta e =& \frac{1}{3!}\epsilon^{\mu\nu\rho\sigma}\epsilon_{abcd}e^a_\mu e^b_\nu e^c_\rho \delta e^d_\sigma + \frac{1}{4}\epsilon^{\mu\nu\rho\sigma}\epsilon_{abcd}e^a_\mu e^b_\nu \delta e^c_\rho \delta e^d_\sigma \\ &+ \frac{1}{3!}\epsilon^{\mu\nu\rho\sigma}\epsilon_{abcd}e^a_\mu \delta e^b_\nu \delta e^c_\rho \delta e^d_\sigma + \frac{1}{4!}\epsilon^{\mu\nu\rho\sigma}\epsilon_{abcd}\delta e^a_\mu \delta e^b_\nu \delta e^c_\rho \delta e^d_\sigma\end{aligned} \tag{5.64}$$

With this expression $-\delta eV$ is:

$$\begin{aligned}-\delta eV \to &- \frac{V}{4}\epsilon^{\mu\nu\rho\sigma}\epsilon_{abcd}e^a_\mu e^b_\nu \delta e^c_\rho \delta e^d_\sigma \\ =& - \frac{3eVm_{3/2}^2}{4(f^2+g^2)^2}\frac{g^4}{f^4}\hat{\chi}\hat{\chi}\hat{\bar{\chi}}\hat{\bar{\chi}}\end{aligned} \tag{5.65}$$

This is the only contribution proportional to V and it makes interactions in Minkowski different from interactions in De Sitter or anti-De Sitter.

By summing the contributions from (5.48), (5.56), (5.62), (5.63) and (5.65) the four fermions interaction is:

$$\begin{aligned}\mathscr{L}_{4f} =& - e\Big[\frac{1}{8}\frac{(f^2+g^2)^2}{f^4}[1-8(c-a^2-b^2)] + \frac{g^2 m_{3/2}^2}{4f^4} \\ &+ \frac{m_{3/2}m_{1/2}g^2}{f^2(f^2+g^2)} - \frac{3m_{3/2}^2}{8}\frac{g^4(m_{3/2}^2+2V)}{f^4(f^2+g^2)^2}\Big]\hat{\chi}\hat{\chi}\hat{\bar{\chi}}\hat{\bar{\chi}}\,. \qquad (5.66)\end{aligned}$$

The last two contributions are dumped by the supersymmetry breaking scale $f^2 + g^2 = V + 3m_{3/2}^2$. For inflationary models these contributions are not relevant. The first two terms instead depend on g/f and then if supersymmetry is mainly broken by F^y they are relevant even if a very energetic cosmological constant is present.

In a Minkowski background, $V = 0 \to (f^2 + g^2) = 3m_{3/2}^2$, the interaction term is:

$$\begin{aligned}\mathscr{L}_{4f} =& - e\Big[\frac{1}{8}\frac{(f^2+g^2)^2}{f^4}[1-8(c-a^2-b^2)] + \frac{g^2(f^2+g^2)}{12f^4} \\ &+ \frac{m_{1/2}g^2}{3m_{3/2}f^2} - \frac{g^4}{24f^4}\Big]\hat{\chi}\hat{\chi}\hat{\bar{\chi}}\hat{\bar{\chi}} \qquad (5.67)\end{aligned}$$

It is equal to the result in [27] but for the term proportional to $m_{1/2}$. This contribution comes from the variation δe, done in order to obtain a canonical kinetic term, and it is right to have it in the interaction.

$$\begin{aligned}e^{-1}\mathscr{L}\Big|_{2f,2g} =& -\frac{1}{2}\mathscr{R} - i\hat{\bar{\chi}}\bar{\sigma}^m\mathscr{D}_m\hat{\chi} + \epsilon^{klmn}\bar{\psi}_k\bar{\sigma}_l\mathscr{D}_m\psi_n \\ &- m_{3/2}(\psi_a\sigma^{ab}\psi_b + \text{h.c.}) - \frac{1}{2}m_{1/2}(\hat{\chi}\hat{\chi} + \text{h.c.}) - V\,. \qquad (5.68)\end{aligned}$$

The Super-Higgs mechanism for two constrained superfields leads to a very simple lagrangian:

$$\begin{aligned}e^{-1}\mathscr{L}\Big|_{2f,2g} =& -\frac{1}{2}\mathscr{R} - i\hat{\bar{\chi}}\bar{\sigma}^m\mathscr{D}_m\hat{\chi} + \epsilon^{klmn}\bar{\psi}_k\bar{\sigma}_l\mathscr{D}_m\psi_n \\ &- m_{3/2}(\psi_a\sigma^{ab}\psi_b + \text{h.c.}) - \frac{1}{2}m_{1/2}(\hat{\chi}\hat{\chi} + \text{h.c.}) - V \qquad (5.69) \\ &- \lambda_{4f}\hat{\bar{\chi}}\hat{\bar{\chi}}\hat{\chi}\hat{\chi} + \text{gravitino and derivative interactions}\,. \qquad (5.70)\end{aligned}$$

where

$$\lambda_{4f} = \frac{1}{8}\frac{(f^2+g^2)^2}{f^4}[1-8(c-a^2-b^2)] + \frac{g^2 m_{3/2}^2}{4f^4} + \frac{m_{3/2}m_{1/2}g^2}{f^2(f^2+g^2)} - \frac{3m_{3/2}^2}{8}\frac{g^4(m_{3/2}^2+2V)}{f^4(f^2+g^2)^2}\ . \tag{5.71}$$

This is an effective lagrangian for a fermion of mass $m_{1/2}$, with a four fermions interaction term in a background given by the potential V. The only constraint on the parameters of the theory is given by $V + 3m_{3/2}^2 = f^2 + g^2$.

Summary and Outlook

In this work the Super-Higgs mechanism has been analyzed when supersymmetry is non-linearly realized. We focused on a simple, but non-trivial, model that describes the interactions of two chiral superfields $\mathbf{X}$ and $\mathbf{Y}$, constrained by $\mathbf{X}^2 = 0 = \mathbf{XY}$, in supergravity. This model led to non-trivial results because supersymmetry is not only broken by F^x but also by F^y and this produces a goldstino that is a linear combination of the fermions in $\mathbf{X}$ and $\mathbf{Y}$. The other interesting point of this model is that, after the Super-Higgs mechanism, there is a surviving fermion and so fermionic interactions can be computed in the effective lagrangian. If only one chiral superfield were considered the Super-Higgs mechanism would eliminate the only fermion of the theory, the goldstino, and in the effective lagrangian only the gravitino and the graviton would appear.

Our main result is the computation of the four-fermions interaction in the effective lagrangian. As supposed in [13] there are terms proportional to $(V + 3m_{3/2}^2)^{-1} = (f^2 + g^2)^{-1}$which therefore put the unitarity threshold well above the energy scale of the cosmological constant V. However, if the contribution to supersymmetry breaking from F^y is relevant $g \sim f$, then there are also terms proportional to f^{-1}. These terms may create problems to unitarity because they can be relevant much below the supersymmetry breaking scale. If $g \gg f$ all the interaction terms may be relevant under the supersymmetry breaking scale. In order to restore unitarity the effective theory can not be valid for all the energies under the supersymmetry breaking scale and the new energy limit depends on f and g. This was a simple model but we suppose that in a more generic model every supersymmetry breaking contribution has to be taken in account in order to determine the validity of effective theories. In [13] supersymmetry was broken only by $f^2 = V + 3m_{3/2}^2$ and this is the reason why they had only one contribution.

In the Minkowski limit we recover the results of [27] but for a term proportional to $m_{1/2}$. This term was not considered by the authors of [27] who neglected the contribution coming from the redefinition of the vielbeins.

This model can be generalized by adding other constrained superfields. If the final effective lagrangian contains only more fermions we suppose the results will be similar to the ones we found. A more interesting generalization is adding a superfield in which the surviving field is a scalar. With this approach inflationary models can be well described and we can therefore address the issue of unitarity constraints on inflationary models from non-linear representations of supersymmetry.. We suppose that adding a scalar to the effective lagrangian does not change the energy scales taken in account.

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