UNIVERSITÀ DEGLI STUDI DI PADOVA

Dipartimento di Fisica e Astronomia “Galileo Galilei”

Master Degree in Physics

Final Dissertation

Quantum to Classical transition in the early Universe: models and observational signatures

Thesis supervisor
Prof. Nicola Bartolo
Thesis co-supervisor
Prof. Sabino Matarrese

Candidate
Aoumeur Daddi Hammou

Academic Year 2019/2020

Contents

Introduction

Inflation theory was conceived to solve the shortcomings of of the standard Hot Big-Bang model, as the flatness of universe and horizon problem. Further, it was realized that it provides us also with a mechanism which explains the origin of all the anisotropies and inhomogeneities we are observing nowadays, namely CMB anisotropies and Large scale structures (LSS), where it traces them back to a tiny primordial quantum fluctuations (of one scalar field or more) during the early universe which got stretched to astrophysical scales by the accelerated expansion and amplified by gravitational instability; similarly, inflation tends to predict through the same mechanism a relic background of primordial gravitational waves (or gravitons) [30, 44]. As a consequence of this mechanism and as confirmed by data, inflation leads with a striking success, toward a quasi-scale invariant primordial power spectra of CMB anisotropies with presence of acoustic peaks as characteristic signature of adiabatic initial conditions that can be set by inflation [2]. Hopefully, observations will confirm soon the same quasi-scale invariant power spectrum for tensor perturbations through the detection of the B mode polarization of CMB. However, this convincing scenario is still facing two puzzles which make, within the suggested solutions, the subject of the thesis.

It has been shown that the inflationary accelerated expansion drives the initial vacuum coherent state of curvature and tensor fluctuations into a strongly squeezed state [35] which is a step toward classicalization of those fluctuations. Since in the strong squeezing limit, the quantum expectations values of a product of the perturbations fields operators are indistinguishable from ensemble averages over products of classical stochastic fields [39, 66, 6], at least up to two point functions. In other words, although produced by different mechanisms, both the classical and quantum fluctuations are normalized to give rise to the same equal-time correlation functions. Unfortunately this implies that it is extremely difficult to observe genuine quantum effects on CMB, since in this strong squeezing limit, the measurement of power spectrum alone could not be used to infer the quantum origin of the primordial perturbations. Nevertheless, at the end, there must be either a classical or quantum origin for the primordial perturbations, so how could we prove one scenario and rule out the other? In other words, we are looking for an observable that could distinguish between the two scenarios, and get different predictions which could, subsequently, be confronted with future experiments. To this end, many efforts have been devoted, and we will present the main ones. One of them was inspired from the work of J.Bell where, as is famously known, he formulated an inequality which must be respected by any local classical (hidden variable) theory and violated by any non local quantum theory, and thanks to this inequality it was possible through the experiments carried out since then to rule out any possibility for a local hidden variable theory. So inspiring from this success we ask ourselves whether is there any possibility for such an experiment in cosmological context with, of course, different approaches and ingredients? the answer turns out to be yes, and we will present the ideas suggested which imply, at least in principle, the possibility of having models that contains an observable subject to bell inequalities, where J.Maldaecena [46] has devised a baroque model based on existence of two scalar fields alongside the inflaton filed which helped to construct a cosmological Bell operator. Another approach, which is more concrete, was adopted by J.Martin et al and also by Choudhury et al [49, 50, 51, 24] where they constructed dichotomic operators from the Mukhanov Sasaki (MS) variable, that is a continuous filed operator, to get a cosmological Bell operator. However, this approach is still facing some problems due to the impossibility of measuring the extremely suppressed decaying mode of perturbations, in addition to the non commutativity of some of them with MS variable. So we conclude that the implementation of bell inequalities in cosmology is still an active research area.

Adopting the scenario of a quantum origin for the universe leads to a confusion that needs to be cleaned away in order to have a rigid formalism for the derivation of primordial perturbations [66, 65]. We observe certainly a classical universe rather than a quantum one, where we do not observe any of the weird properties of quantum mechanics, such as superposition of states implied by the squeezed state. So how did quantum superposition among the quantum state of perturbations was destroyed, and how it collapsed into a single outcome which represents the inhomogeneous observed universe; besides, why it collapsed in one basis rather than another. Those last questions get even more serious in the context of Copenhagen interpretation that attributes the collapse to the act of an external observer which for sure is not the case for the primordial universe that is considered a single system with no external classical domain. All the aforementioned questions boil down to the quantum measurement problem consisting of three aspects that , namely, are the non observability of interference, the problem of preferred basis and the problem of outcomes, and in order to tackle this questions many models have been suggested, and cosmology gives the chance to test and constrain those models in a completely different physical scales compared to our laboratories experiments. Among the many phenomenological models devised to solve the quantum measurement we find dynamical collapse models and decoherence as the leading ones and the most adequate to apply in cosmology.

The Dynamical, or objective, collapse models are based on modifying Schrodinger equation by coupling the quantum system to an external stochastic classical field called noise, which is supposed to induce the collapse of the wavefunction. It is worth to mention that collapse models are not formulated yet in a relativistic context so their application to cosmology, where quantum field theory on curved spacetime governs, should be taken carefully [9, 56]. But the justification to take risk and apply them in the study of primordial perturbations context is based on the fact that at linear order the different Fourier modes evolve independently and they do not interact, thus at this level we could pass the need for a quantum field description. Collapse models are divided into several types, we will consider two of them, quantum mechanics with universal position localization model (QMUPL) and Continuous spontaneous localization model (CSL). Applying those models in a cosmological context and being guided by the high accurate data at our disposal, especially the well confirmed quasi scale invariant power spectrum, will lead to constraints on the values of their free parameters which could subsequently be confronted with the values obtained in laboratory experiments [56, 29, 54, 55].

Decoherence is based on the fact that a quantum system is no more a closed but rather an open system interacting with its environment and this idea leads to solve the first two aspects of measurement problem without the need to an external observer, but whether it solves the third aspect or not is still a matter of debate[68] . Through applying decoherence we are considering our whole perturbations spectrum as open quantum system. Therefore, we study the effects of self interactions of large and short modes of primordial fluctuations, or their interactions with an environment consisting of the rest degrees of freedom in universe (which could be standard model fields that populate post inflation era, the beyond standard model fields,..etc). Decoherence implementation will lead to corrections to the two and higher point correlation functions of primordial curvature (density) perturbations and gravitational waves. Where, non vanishing higher-order correlators (i.e primordial non-Gaussianity) has emerged in the last ten years as a precision tests of inflation and early universe physics [8, 3]. Most interestingly, J.Martin et al [53, 52] showed that considering a very massive scalar field as environment for the primordial scalar perturbations will induce a scale invariant corrections to the two point and four point functions for some value in parameter space of the model they devised. Their computations make their model in agreement with CMB data, especially regarding the power spectrum which is well measured. However, due to the type of the interactions considered, J.Martin et al obtained a vanishing three point function, so our idea was to adopt a different approach and show that for a different type of interaction operator, there is a possibility of having non vanishing three point function and we gave its explicit expression. Through our approach we succeeded to generalize the previous results of J.Martin et al, but also to shed light on new insights and corrections to their work. Another important result of the thesis, is that we succeeded to generalize the model to include the primordial tensor perturbations and compute decoherence induced corrections to their power spectrum; the correction was found to be blue tilted which represents an interesting result. Also combining the corrections to tensor and curvature perturbations one, we computed decoherence induced correction to the tensor to scalar ratio. As a conclusion for this part of thesis, we notice that implementing decoherence in inflationary context and upon confronting the results obtained for the different point correlation functions with data will lead to bounds on the strength of interaction between system-environment and this could reveal some properties of inflation era through constraining the possible environments. Another consequence of considering decoherence is the improvement of fit for some inflation models with data as power law inflation, and ruling out others as natural inflation model.

All in all, no one denies the success of inflation in making many strong predictions. However, inflation formalism regarding the quantum origin of primordial cosmological perturbations still does not answer all questions; in particular, the question of quantum to classical transition and the breakdown of homogeneity and isotropy in the early universe. The answer to this two questions could solve some anomalies and bring new predictions into light, in addition to constraining any possible new physics that could be probed in the early universe.

Chapter 1

Quantum fluctuations in the inflating universe

The aim of this first chapter is to discuss the primordial cosmological perturbation under the frame of inflation theory. In contrast to other models, inflation consider our highly inhomogeneous and anisotropic universe to be seeded by primordial tiny quantum fluctuations, of the gravitational and inflaton field(s), superposed on top of a highly homogeneous and isotropic universe. We try to convince the reader through this chapter that there is a missing chapter in the inflationary scenario of the origin of universe. The chapter is supposed to answer to a crucial question, invoked by many leading cosmologists, and its answer could bring us to a very important observational constraints and predictions, because it could open the exciting possibility of observing for the first time a genuine quantum gravitational effect. The question could be expressed as follow[1]

The success of the inflationary picture about the quantum origin of CMB anisotropies and large scale structures requires the primordial quantum fluctuations to be converted into classical spatial variations in the energy density, whose amplitude varies stochastically as one moves from one Hubble patch to another within the Universe during the much later recombination epoch [18]. Therefore, Starting from a universe considered as closed quantum system that undergoes a pure unitary evolution, and observing a purely classical universe around us necessitates an understanding of how did this quantum to classical transition took place?

To grasp fully the previous question we need to remember that an inflationary period is supposed to erase all memory of initial conditions, leading to a flat, homogeneous and isotropic early universe

[1] In [12] it was argued that the question we are about to express is actually two distinct questions: 1) the first is about classicalization of primordial quantum fluctuations and 2) second is about the breakdown of homogeneity and isotropy. However, we do not see the point of such separation as we will see clear from our explanation below, namely, because any mechanism that is responsible for classicalization is non unitary process that could, also, induce a breaking of the initial symmetries.

with tiny quantum fluctuations superposed on top of it. However, we should be careful not to confuse the quantum fluctuations, that are a mere intrinsic quantum uncertainties on the filed state[2], with a statistical physical fluctuations of the field. In other words, we know that in quantum mechanics and for given system S, we cannot argue that S has a definite value of an observable $\hat{O}$ prior to its measurement, which takes place once the system wave function Ψ collapses into an eigenstate of $\hat{O}$. Before the collapse, the system state undergoes a unitary evolution that preserves the symmetries of initial state[3], it is only after the collapse, that is a non unitary operation, the initial symmetries of the state get broken. Having said that, we conclude that the state of the universe is still isotropic and homogeneous until the wavefunctional of our quantum fluctuations collapse and give rise to statistical and physical fluctuations. Subsequently, those physical fluctuations evolve classically and give rise, at late time, to the highly inhomogeneous and anisotropic universe. Therefore, inflation scenario, implicitly, assumes a transition from a symmetric quantum state to an essentially classical non-symmetric one[4], so how this transition did took place? and how a single field configuration was chosen out of superposition of configuration? [63]

The previous two questions shows that, indeed, there is a missing chapter in the inflationary scenario of the origin of universe, so the current chapter serves to discuss in more details the two questions, and the rest chapters of thesis will attempt to be the missing one!

1.1 Introduction to Inflation

A universe that is perfectly homogeneous and isotropic singles out a unique form of spacetime geometry which is described by the maximally symmetric Friedmann-Lemaitre-Robertson-Walker (FLRW) metric written as

$$\mathrm{d}s^2 = a^2(\eta)\left[-\mathrm{d}\eta^2 + \frac{\mathrm{d}r^2}{1-\kappa r^2} + \mathrm{r}^2\mathrm{d}\Omega^2\right], \tag{1.1}$$

where $a(\eta)$ is the scale factor encoding the expansion of universe, η is the conformal time related to the comic time through

$$\mathrm{d}\eta = \frac{\mathrm{d}t}{a(t)}, \tag{1.2}$$

[2]These could be understood clearly by remembering that the quantum fluctuations are consequence of Heisenberg uncertainty principle.

[3]In our case inflaton is initially in Bunch-Davis vacuum that is homogeneous and isotropic state, and it evolves into a squeezed state which is still a homogeneous and isotropic state. So there must be a non unitary evolution that would break the symmetry and give rise to physical fluctuations.

[4]We will see later on this chapter that an evolution governed by Schrodinger equation could not account by itself for such transition.

(t, r, θ, ϕ) are the spherical comoving coordinates measured by an observer comoving with cosmic fluid[5], and $\mathrm{d}\Omega^2 = \mathrm{d}\theta^2 + \sin^2(\theta)\,\mathrm{d}\phi^2$ is the infinitesimal solid angle and the curvature κ takes the values

$$\kappa = \begin{cases} +1 & \text{spatially closed} \\ 0 & \text{spatiallyflat} \\ -1 & \text{spatially open} \end{cases}, \tag{1.3}$$

According to our current observations, the universe is very close to be spatially flat, so to a good a approximation our universe could be described by the metric

$$\mathrm{d}s^2 = a^2(\eta)\left[-\mathrm{d}\eta^2 + \mathrm{d}\mathbf{x^2}\right], \tag{1.4}$$

where we used Cartesian coordinates this time, with $\mathrm{d}\mathbf{x^2} = \delta_{ij}\mathrm{d}x^i\mathrm{d}x^j$. Using Einstein equations derived from (1.1), we obtain the Friedman equations which govern the evolution of scale factor a as function of matter content of the universe [10, 30]

$$\begin{aligned} H^2 &= \tfrac{\rho}{3M_{pl}^2} - \tfrac{\kappa}{a^2} \Longleftrightarrow (a')^2 = \tfrac{\rho a^4}{3M_{pl}^2} - a^2\kappa \\ \tfrac{\ddot{a}}{a} &= -\tfrac{1}{6M_{pl}^2}(\rho + 3P) \Longleftrightarrow \tfrac{a''}{a^3} = -\tfrac{1}{6M_{pl}^2}(\rho - 3P) \end{aligned}, \tag{1.5}$$

where $H = \frac{a'}{a^2}$ is Hubble expansion rate, with a prime denoting derivative with respect to η and a dot denoting a derivative with respect to t. In addition, ρ and P are the background density and pressure, respectively, that are linked through the constant equation of state $P = \omega\rho$, where ω depend on the energy content considered

$$\begin{cases} \omega = 0 & Dust\,or\,pressureless\,matter \\ \omega = \frac{1}{3} & Radiation \\ -1 & Cosmological\,constant\,\Lambda \end{cases}, \tag{1.6}$$

Another useful quantity is the so called particle horizon that quantifies the maximum comoving

[5]Multiplying them by the scale factor gives rise to coordinates. For a closed universe $0 \leq r \leq 1$

distance that a light signal could travel between two times η_i and η[6][7]

$$d_H(t) = \eta - \eta_i = \int_{t_i}^{t} \frac{\mathrm{d}t}{a(t)} = \int_{\ln a_i}^{\ln a} (aH)^{-1}\, d\ln a\,, \tag{1.7}$$

we see that particle horizon is proportional to the comoving Hubble radius[8] defined by $(aH)^{-1}$. Using the constant equation of state mentioned before, It could shown that

$$(aH)^{-1} = H_0^{-1} a^{\frac{1}{2}(1+3\omega)}\,, \tag{1.8}$$

so for a finite particle horizon we have

$$d_H(t) = \frac{2}{(1+3\omega)} (aH)^{-1}\,, \tag{1.9}$$

notice from second Freedman equation in (1.5) that as long as $\omega > -\frac{1}{3}$ we have a decelerating universe, therefore, all familiar matter contents satisfying strong energy condition $1+3\omega > 0$ implies an increasing of the Hubble radius $(aH)^{-1}$. This last remark tells us that if two patches of the universe get into causal contact with each other at some time t, then for all $t' < t$ they were for sure causally disconnected since $(aH)^{-1}$ increases only. However, observations tells us that the cosmic microwave background[9] (CMB) is very isotropic, where, we can observe regions that share the same statistical properties (in particular the same temperature T, up to very tiny fluctuations $\frac{\delta T}{T} \approx 10^{-5}$), without having been in causal connection ever before, because they are separated by distances that are much larger than the largest distance traveled by light in all the history of the universe. In particular, detecting two photons which come from two different directions that are separated by more than one degree on celestial sphere implies that they were never in causal contact[10], see figure (1.1), and this makes us wonder how they share almost the same temperature though they have never interacted with each other? This question sheds light on the horizon problem.

An intuitive solution for horizon problem is to assume that the causally disconnected patches observed in universe at some time were, actually, during the very early universe in causal contact but a rapid acceleration phase of the universe expansion caused them to get causally disconnected, since an accelerated expansion implies a decreasing of Hubble radius $(aH)^{-1}$. After this acceleration phase, the expansion starts to decelerate and the two patches renter the horizon of each other at late times, or at least our horizon. This early exponential accelerated phase of expansion of universe is called

[6] We set the speed of light $c = 1$.

[7] It could also be understood as the maximum coordinate distance between two points that have been in causal contact at some point since t_i and until time t.

[8] Roughly speaking, the Hubble radius measures the distance that a light signal could cross as the scale factor doubles.

[9] It is electromagnetic radiation that was emitted when electrons recombined with protons to form neutral Hydrogen atoms. Those last could not scatter the thermal radiation, therefore the universe became transparent to radiation and photons started their free streaming journey since then until we detect them with our satellites.

[10] This means that their past light cones do not overlap.

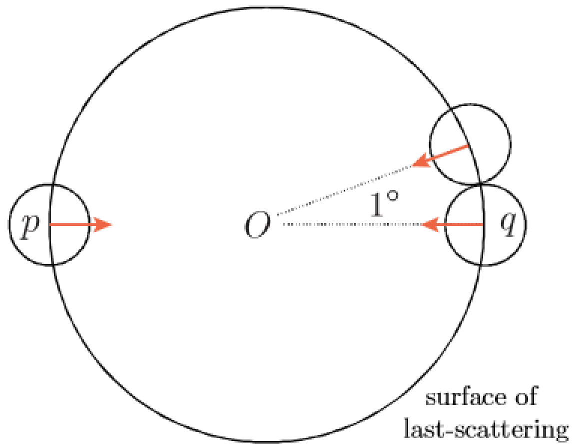

Figure 1.1: Illustration of horizon problem: Any two points on the surface of last-scattering that are separated by more than 1 degree, appear never to have been in causal contact. We illustrated this for opposite points on the sky labeled p and q. [11]

inflation, which solves also other shortcomings of standard Hot Big-Bang model as Flatness problem and unwanted relics [44]. . It is worth to mention that an exponential acceleration phase is also called de-Setter phase which corresponds to a universe energy density that is dominated by a kind of cosmological constant. However, since we want inflation to last for a limited period and give rise, at the end, to radiation era, we are actually looking for a quasi de-Sitter phase.

On mathematical level, we can see from second Freedman equation in (1.5) that an accelerated expansion $\ddot{a} > 0$ requires

$$\ddot{a} > 0 \Rightarrow\Rightarrow \omega < -\frac{1}{3}\,, \tag{1.10}$$

but since the energy density could only be positive,i.e $\rho > 0$, we infer from the previous equation that in order to have inflation we need a matter source with negative pressure $P < 0$. For sure ordinary matter could not, since they satisfy the strong energy condition, while from (1.10) we see the need for a fluid that violates strong energy condition, i.e $1 + 3\omega < 0$. The constraint (1.10) could also be rexpressed as

$$\ddot{a} > 0 \Rightarrow \epsilon_1 < 1\,, \tag{1.11}$$

where $\epsilon_1 = -\frac{\dot{H}}{H^2}$ is called the first slow roll parameter for a reason that will be clear in am moment. But an accelerated phase of expansion alone is not enough to solve Horizon problem! we must be sure that such accelerated lasted enough time to allow for, at least, the scale that we can probe today to

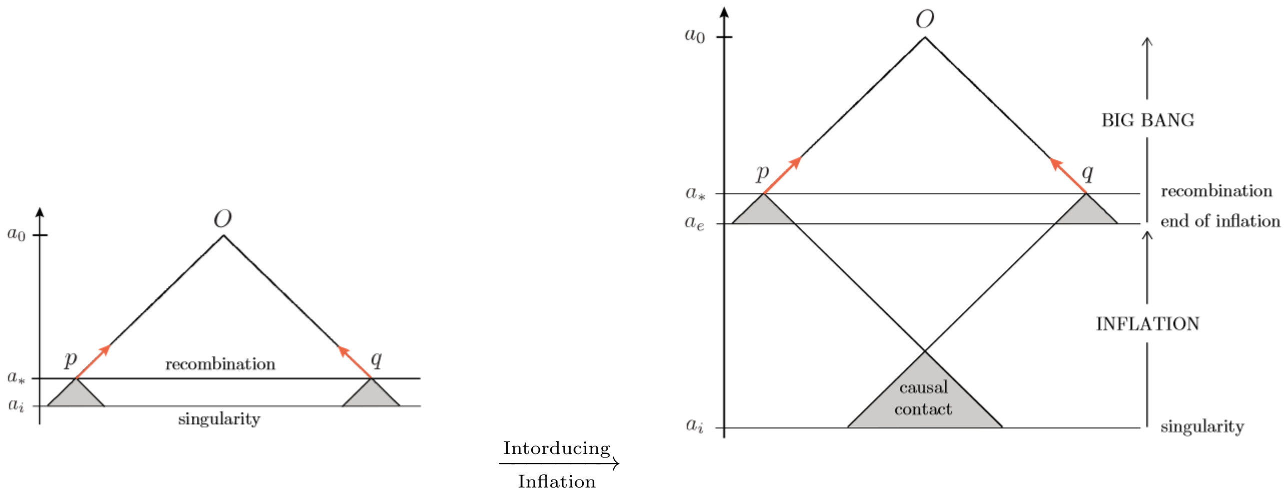

Figure 1.2: Illustration of how inflation does solve horizon problem. Left: The intersection of our past light cone with the spacelike slice labeled “recombination" corresponds to the “surface of last-scattering". We see that without inflation, most of points on CMB do not have overlapping past light conses. Right: While introducing inflation we, somehow, give those points enough time to interact in their past, and we send the Big bang time singularity to the infinite past.[11]

be under the horizon when inflation took place i.e

$$(a_0 H_0)^{-1} \leq (a_i H_i)^{-1} \ , \tag{1.12}$$

where $(\cdots)_0$ refers to today's values, and $(\cdots)_i$ to the values at beginning of inflation. Defining the number of e-folds N by

$$N = \ln\left(\frac{a_f}{a_i}\right) \ , \tag{1.13}$$

we can show that in order for inflation to solve the horizon problem we need $N \simeq 60 - 70$, said differently, the early universe should had been expanded by a factor of 10^{26}. It could be shown that similar values of N could solve the other problems of standard Hot Big-Bang model .

There is a natural question that one can ask: What caused this accelerated expansion? As mentioned previously, it could not be the ordinary matter. Actually, it could be shown that a scalar field φ, called inflaton, could cause inflation within some requirements to be fulfilled[11]. The full action of our early universe with an inflaton could be written as

$$S_{tot} = S_{HE} + S_\varphi + S_m = \frac{M_{pl}^2}{2} \int \mathrm{d}^4 x \sqrt{-g} \left[R - \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi) \right] + S_m \,, \tag{1.14}$$

[11]The basic idea behind a scalar field, is that it could mimic an effective, positive, cosmological constant and is given by $\Lambda_{eff} = 8\pi G \langle \rho \rangle$, where $\langle \cdots \rangle$ refers to vacuum expectation value, and ρ is energy density of our field.

where S_{HE} is the Hilbert Einstein action, S_φ is the inflaton action, and S_m is action of possible other fields present along inflaton. In the last equation, g is the determinant of the metric tensor$g_{\mu\nu}$, R is the curvature scalar, and $V(\varphi)$ is the potential that possibly has driven inflation, and it is crucial for the understanding of the physics of inflation. There are different forms of $V(\varphi)$ depending on the model we are considering, we will give some examples in chapter 4. Minimizing the above action with respect to φ, i.e $\frac{\delta S}{\delta \varphi}=0$, yields the field equation satisfied by[12] φ

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\varphi\right)-V'(\varphi)=0\,, \tag{1.15}$$

then minimizing the action with respect to the metric yields the stress energy tensor

$$T_{\mu\nu}=-\frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}}=\left(-\frac{1}{2}g^{\alpha\beta}\partial_\alpha\varphi\partial_\beta\varphi-V(\varphi)\right)-V'(\varphi)\,. \tag{1.16}$$

Which enters into Einstein equation

$$G_{\mu\nu}=\frac{1}{M_{Pl}^2}T_{\mu\nu}\,, \tag{1.17}$$

with $G_{\mu\nu}$ being the Einstein tensor. In case of FLRW metric, the stress energy tensor has the components

$$\begin{array}{ll} T_{\mu\nu} & =\left(\frac{1}{2}\dot{\varphi}^2-V(\varphi)\right)\delta_{\mu\nu}-\delta_\mu^0\delta_\nu^0\dot{\varphi}^2 \\ \Rightarrow & \rho=\frac{1}{2}\dot{\varphi}^2+V(\varphi)\,,\;P=\frac{1}{2}\dot{\varphi}^2-V(\varphi) \end{array}\,, \tag{1.18}$$

a very crucial remark is that P could be negative even for a positive potential V. The first Friedman equation[13] and field equation (1.15) now become

$$\begin{array}{ll} H^2 & =\frac{1}{3M_{pl}^2}\left(\frac{1}{2}\dot{\varphi}^2+V(\varphi)\right) \\ \ddot{\varphi} & +3H\dot{\varphi}+V'(\varphi)=0 \end{array}\,. \tag{1.19}$$

We discuss, now, the conditions that inflaton should verify in order to realize the early acceleration phase of universe expansion. If we impose the constraint (1.10) on the inflaton field, then from (1.18) we find

$$\omega<-\frac{1}{3}\Longleftrightarrow P<-\frac{\rho}{3}\Rightarrow\dot{\varphi}^2\ll V\,, \tag{1.20}$$

therefore, Inflation occurs if the the potential term V dominates over the kinetic term $\dot{\varphi}^2$, and this

[12]Prime here denotes taking a derivative with respect to the scalar field φ. This should not be confused with derivative with respect to the conformal time η; potential V is a function of φ only.

[13]For a flat universe, i.e $\kappa=0$.

could be done by a sufficiently flat potential. Such inflation is called the slow roll inflation[14] and is well compatible with data [44, 4, 3]. The constraint (1.20) is called first slow roll condition, and in order to assure that inflation lasts for enough time we need to introduce a second slow roll condition that is defined by

$$\ddot{\varphi} \ll 3H\dot{\varphi}\,. \tag{1.21}$$

After some straight forward calculations exploiting (1.19, 1.20, 1.21), we can show that the first slow roll parameter we defined in (1.11) is given by

$$\epsilon_1 = -\frac{\dot{H}}{H^2} \simeq \frac{3}{2}\frac{\dot{\varphi}^2}{V}\,, \tag{1.22}$$

form where we can understand its name, therefore, ϵ_1 could be seen as the ratio between kinetic energy and potential[15]. Using now second slow roll condition (1.21) we can define a second slow roll parameter that is defined by[16]

$$\epsilon_2 = -\frac{\ddot{\varphi}}{H\dot{\varphi}} \ll 1\,. \tag{1.23}$$

There are various potentials which fulfill the two slow roll conditions, more details could found in [44, 10].

During inflation the observed universe can become extremely homogeneous and isotropic. However, small inhomogeneities always exist due to quantum fluctuations[17]. Therefore, we may ask several questions: How to describe those fluctuations and compute their evolution? Since they will be quantized and in the matter of predictive power and computations easiness, is there an advantage in using Heisenberg picture over Schrodinger one ? Or they are totally equivalent? Our next section carries the answers to those questions.

1.2 Heisenberg or Schrodinger picture ?

Since we are talking about quantum perturbations in inflation then we should choose the picture to work with, Heisenberg or Schrodinger. Even though it is well known that they are equivalent in their physical implications, still one picture could be more convenient to work with than the other, due to difference

[14] Fortunately, it could be shown that slow-roll solution is an attractor: systems with wildly different initial conditions tend towards it as the field value changes.

[15] It could also be shown that $\epsilon_1 \simeq \epsilon_1^V = \frac{1}{16\pi G}\left(\frac{V'}{V}\right)^2$,so in this sense ϵ_1^V quantifies the flatness of potential. ϵ_1^V is called first potential slow roll parameter.

[16] Similarly, It could also be shown that $\epsilon_2 = \epsilon_2^V - \epsilon_1$, with $\epsilon_2^V = \frac{1}{16\pi G}\left(\frac{V'}{V}\right)^2$. So in this sense ϵ_2^V constrains the flatness of potential to last enough time. ϵ_2^V is called second potential slow roll parameter.

[17] It is worth remarking that the theory was not engineered to produce these fluctuations, but that their origin is instead a natural consequence of treating inflation quantum mechanically. [11]

between the two in the language and parameters used to describe the state of a quantum system [66]. On the one hand, Heisenberg picture is featured by the use of mode functions and Bogolubov transformations for creation and annihilation operators, where the study of primordial perturbations in this picture usually ends up by considering the mode functions as classical variables with stochastic Gaussian amplitudes, and some cosmologists see this answers to the of question classicaliztion of perturbations [39, 6, 35], but we will see that this is not enough! [73, 63]. On the other hand, the Schrodinger picture is featured by the use of the wave function and Schrodinger equation to study its evolution, and this picture is more practical to implement decoherence and collapse models in the study of inflationary perturbations [56, 29, 53]. We will see throughout this chapter how the squeezed state formalism links the two pictures together in the cosmological context following [66, 6]. Let us now translate the above discussion into equations.

1.2.1 Heisenberg picture and squeezing formalism

Since we are about to discuss inflationary fluctuations we need to go beyond the homogeneous and isotropic FRW metric, therefore superposing small perturbations on top of FLRW metric gives rise to the perturbed one, which including all type of metric perturbations, scalar, vector, tensor, could be written as[18] [67, 1, 58]

$$\begin{aligned} \mathrm{d}s^2 = & \quad a^2(\eta)\left\{-\left(1+2\sum_n^{+\infty}\frac{\psi^{(n)}}{n!}\right)\mathrm{d}\eta^2+2\sum_n^{+\infty}\frac{\omega_i^{(n)}}{n!}\mathrm{d}x^i\mathrm{d}\eta+\left[\left(1-2\sum_n^{+\infty}\frac{\phi^{(n)}}{n!}\right)\delta_{ij}+\sum_n^{+\infty}\frac{\chi_{ij}^{(n)}}{n!}\right]\mathrm{d}x^i\mathrm{d}x^j\right\} \\ \omega_i = & \quad \partial_i\omega^{\shortparallel}+\omega_i^{\perp} \\ \chi_{ij} = & \quad D_{ij}\chi^{\shortparallel}+2\partial_{(i}\chi_{j)}^{\perp}+\chi_{ij}^{T} \end{aligned} , \tag{1.24}$$

where n stands for the order of perturbations. ψ, ϕ, $\omega^{\shortparallel}$,$\chi^{\shortparallel}$ represents metric scalar perturbations, $\omega_i^{\perp}$, $\chi_i^{\perp}$, $\chi_j^{\perp}$the pure transverse (i.e divergence-free) vector parts $\left(\partial^i\omega_i^{\perp}=0,\, \partial^i\chi_i^{\perp}=0\right)$, and χ_{ij}^T is transverse trace-free pure tensor parts[19] $\left(\partial^i\chi_{ij}^T=0, \chi_i^{iT}=0\right)$, finally $D_{ij}$is the trace free operator. We will not go into details here, since our aim in this chapter is to discuss the quantized fields in flat FLRW background , however more details on the relativistic cosmological perturbations and the gauge problem could be found in many of the standard books as [44, 30]. In what follows we will focus on scalar perturbations up to first order since they are the dominant in CMB and are the only ones observed so far, in addition, to the fact that they decouple from the tensor ones[20], so (1.24) becomes

[18] Now on we put ourselves in a flat universe, and switch to Cartesian coordinates.

[19] Of course all those perturbations are spacetime dependent since we are considering an inhomogeneous and anisotropic universe.

[20] We will discuss the tensor perturbations in the chapter devoted to decoherence.

$$\mathrm{d}s^2 = a^2(\eta)\left\{-(1+2\psi)\,\mathrm{d}\eta^2 + 2\omega_i \mathrm{d}x^i \mathrm{d}\eta + \left[(1-2\phi)\,\delta_{ij} + D_{ij}\chi^{\shortparallel}\right]\mathrm{d}x^i \mathrm{d}x^j\right\}\,, \tag{1.25}$$

due to the gauge freedom the four scalar degrees of freedom could be reduced into only two[7, 61] which by turn could be combined into single gauge invariant (GI)quantity, namely the Bardeen potential [7]

$$\Phi_B(\eta, \boldsymbol{x}) = -\phi - \frac{1}{6}\nabla^2\chi^{\shortparallel} + \mathcal{H}\omega^{\shortparallel} - \frac{1}{2}\mathcal{H}\left(\chi^{\shortparallel}\right)\,, \tag{1.26}$$

primes denote derivatives with respect to η, gauge invariance formalism is well suited to study the evolution of vacuum fluctuations as is the case in inflationary scenario.

Having presented the geometric part of perturbations we turn now to matter sector. The inflaton is decomposed into classical background $\varphi_0(\eta)$ plus a fluctuation $\delta\varphi(\eta, \boldsymbol{x})$ to be quantized later, this last, i.e $\delta\varphi(\eta, \boldsymbol{x})$ can be treated as a massless scalar field, which is an excellent approximation when the inflaton field satisfies the slow-roll conditions [39]

$$\varphi(\eta, \boldsymbol{x}) = \varphi_0(\eta) + \delta\varphi(\eta, \boldsymbol{x})\,, \tag{1.27}$$

similarly to (1.26), we can combine inflaton fluctuation, included in $\delta\rho$, with scalar metric perturbation into single gauge invariant quantity

$$\delta\rho^{gi} = \delta\rho + \rho_0'\left(\omega^{\shortparallel} - \chi^{\shortparallel\prime}\right), \tag{1.28}$$

but thanks to the Einstein perturbed equations, the Bardeen potential and $\delta\rho^{gi}$ could be combined into single gauge invariant variable called Mukhanov Sasaki (MS) variable that is given, in the spatially flat gauge, by [41, 60]

$$v(\eta, \boldsymbol{x}) = a\left(\delta\varphi(\eta, \boldsymbol{x}) + \frac{\varphi_0'}{\mathcal{H}}\phi\right)\,, \tag{1.29}$$

where $\mathcal{H}$ represents the conformal Hubble parameter $\mathcal{H} = \frac{a'}{a}$. We emphasize on the fact that v characterizes fully the scalar sector. The MS variable is related to the so called comoving curvature ζ through [10]

$$v(\eta, \boldsymbol{x}) = \frac{a\varphi_0'}{\mathcal{H}}\zeta\,, \tag{1.30}$$

Curvature perturbations ζ have the advantage of being conserved on super-horizon scales until they renter horizon at the radiation or matter era, thus, they are not affected by the reheating era complications. We will see in 1.3 that ζ is very important to link late time observations with early universe predictions.

The goal now is to derive the action S_{pert} governing v and subsequently its equation of motion.

Obviously the starting point is expanding the action of inflaton

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V\left(\varphi\right)\right] , \tag{1.31}$$

up to second order of perturbations which yields [61]

$$S^{(1)}_{pert} = \frac{1}{2}\int \mathrm{d}^4x\left[\left(v'\right)^2 - \left(\nabla v\right)^2 + \frac{z''}{z}v^2\right] , \tag{1.32}$$

with $z \equiv a\sqrt{2\epsilon_1}M_{pl}$, $\epsilon_1 = 1 - \frac{\mathcal{H}'}{\mathcal{H}^2}$ is the first slow roll parameter and $M_{pl}$is the reduced Planck mass. Notice that (1.32) represents the action of a massive scalar field with time dependent effective mass given by[21] $m^2_{\text{eff}} = \frac{z''}{z}$. It is important to mention that we are considering, for simplicity, the intrinsic mass of the inflaton m_φ to be zero, otherwise we would have $m^2_{\text{eff}} = \frac{z''}{z} - m^2_\varphi$. Anyway, the inflation mass is always $m_\varphi << H$ or $m_\varphi \simeq H$, since, having $m_\varphi \gg H$ would lead to the suppression of the primordial fluctuations.

The action $S^{(1)}_{pert}$ will be useful in the Schrodinger picture, while its equivalent one $S^{(2)}_{pert}$[22], up to total derivative, will be more convenient to work with in the Heisenberg picture especially for the analysis of the squeezing of modes [6, 29]

$$S^{(2)}_{pert} = \frac{1}{2}\int \mathrm{d}^4x\left[\left(v'\right)^2 - \left(\nabla v\right)^2 + -2\frac{z'}{z}vv' + \left(\frac{z'}{z}\right)^2 v^2\right] , \tag{1.33}$$

We will consider that the slow roll parameters vary negligibly with time, therefore $\frac{z''}{z}=\frac{a''}{a}$and $\frac{z'}{z} = \frac{a'}{a}$, we can proceed now with the standard quantization and the study of squeezing modes.

First, we need to get the Hamiltonians describing v as a classical field, and subsequently we will quantize it along its canonical conjugate momentum defined for each of the two equivalent actions 1 and 2, by

$$\begin{aligned} p^{(1)} = \quad & \frac{\partial\mathcal{L}^{(1)}}{\partial v'} = v' \\ p^{(2)} = \quad & \frac{\partial\mathcal{L}^{(2)}}{\partial v'} = v' - \frac{a'}{a}v \end{aligned} , \tag{1.34}$$

$\mathcal{L}^{(i)}$ is the Lagrangian density that can be derived from from the action $S^{(i)}_{pert}$ (1.33). The computations get simpler by working in Fourier space because we are working at linear order of perturbation theory, hence, all modes evolve independently and do not interact with each other [56] so decomposing the

[21] Or, equivalently, it represents the action of parametric harmonic oscillator, i.e a harmonic oscillator with time dependent mass.

[22] By (1) and (2)we just refer to the action type 1 or two 2 and not to perturbative order up to which we are expanding the total action of our system, since it is clear that this last was expanded up to second order as mentioned above.

fields (v,p) into their Fourier modes

$$\begin{aligned} v(\eta,\boldsymbol{x}) = & \frac{1}{(2\pi)^{3/2}}\int d^3\boldsymbol{k} v_{\boldsymbol{k}} e^{i\boldsymbol{k}.\boldsymbol{x}} \\ p(\eta,\boldsymbol{x}) = & \frac{1}{(2\pi)^{3/2}}\int d^3\boldsymbol{k} \hat{p}_{\boldsymbol{k}} e^{i\boldsymbol{k}.\boldsymbol{x}} \end{aligned}, \tag{1.35}$$

with $\hat{v}^*_{\boldsymbol{k}} = \hat{v}_{-\boldsymbol{k}}$ since $v(\eta,\boldsymbol{x})$ is real. Then, the Hamiltonians () are given by

$$\begin{aligned} H^{(i)} = & \int \mathrm{d}^3x \mathcal{H}^{(i)} = \int \mathrm{d}^3x \left[pv' - \mathcal{L}^{(i)}\right] \\ \xrightarrow{FT} H^{(1)} = & \frac{1}{2}\int d^3\boldsymbol{k}\left[p_{\boldsymbol{k}}p^*_{\boldsymbol{k}} + v_{\boldsymbol{k}}v^*_{\boldsymbol{k}}\left(k^2 - \frac{a''}{a}\right)\right] \\ \xrightarrow{FT} H^{(2)} = & \frac{1}{2}\int d^3\boldsymbol{k}\left[p_{\boldsymbol{k}}p^*_{\boldsymbol{k}} + k^2 v_{\boldsymbol{k}}v^*_{\boldsymbol{k}} + \frac{a'}{a}\left(v_{\boldsymbol{k}}p^*_{\boldsymbol{k}} + v^*_{\boldsymbol{k}}p_{\boldsymbol{k}}\right)\right] \end{aligned}, \tag{1.36}$$

the integral over $\boldsymbol{k}$ is only over half of Fourier space due to $v^*_{\boldsymbol{k}} = v_{-\boldsymbol{k}},$. From (1.2.1) we can easily see why $S^{(1)}_{pert}$, or equivalently $H^{(1)}$, is suitable to the Schrodinger picture, where $H^{(1)}$ represents a collection of parametric oscillators (one per each mode) with time dependent frequency given by[23]

$$\omega^2(\eta,k) = k^2 - \frac{a''}{a}, \tag{1.37}$$

and the similarity between parametric and simple harmonic oscillators in some regimes [24] makes it easier to solve it, and that will be important to show the equivalence of Schrodinger and Heisenberg picture, but also for the implementation of CSL later on. Notice also that the dependence of ω^2on a and a''implies that different inflation models (i.e potentials) give rise to different ω, therefore to different behaviors of $v_{\boldsymbol{k}}$[56], which result in observational consequences since as we will see power spectrum is $\propto v^*_{\boldsymbol{k}}v_{\boldsymbol{k}}$.

The modes $v_{\boldsymbol{k}}$ satisfy the Mukhanov Sasaki equation which obviously could be derived from either of the actions above

$$v''_{\boldsymbol{k}} + \omega^2(\eta,k)\, v_{\boldsymbol{k}} = 0 \tag{1.38}$$

from this equation we can see that indeed each mode evolves independently. In order to solve the previous equation fully, we need to specify the inflation model, thus the potential $V(\varphi)$, in addition to the initial conditions.

[23] It is well known that a field in flat space-time can be interpreted as an infinite collection of harmonic oscillators, each oscillator corresponding to a given Fourier mode. Likewise, a scalar field living in a cosmological, curved, space-time can be viewed as an infinite collection of parametric oscillators, the fundamental frequency of each oscillator becoming a time-dependent function because of cosmic expansion.[54]

[24] As in sub Hubble regime $-k\eta \gg 1$.

Quantizing the fields (v,p) in the Heisenberg picture where $(\hat{v},\hat{p})$ are time dependent and expressing them as function of creation and annihilation operators $\left(\hat{a}_{\boldsymbol{k}},\hat{a}^{\dagger}_{-\boldsymbol{k}}\right)$ defined as usual by

$$a_{\boldsymbol{k}}(\eta)=\frac{1}{\sqrt{2}}\left(\sqrt{k}\hat{v}_{\boldsymbol{k}}(\eta)+i\frac{1}{\sqrt{k}}\hat{p}_{\boldsymbol{k}}(\eta)\right)\ , \tag{1.39}$$

with suitable normalization, leads to

$$\begin{aligned}\hat{v}_{\boldsymbol{k}}(\eta)= &\ \tfrac{a_{\boldsymbol{k}}(\eta)+a^{\dagger}_{-\boldsymbol{k}}(\eta)}{2k}\\ \hat{p}_{\boldsymbol{k}}(\eta)= &\ -i\sqrt{\tfrac{k}{2}}\left(\hat{a}_{\boldsymbol{k}}(\eta)-\hat{a}^{\dagger}_{-\boldsymbol{k}}(\eta)\right)\end{aligned}\ , \tag{1.40}$$

those last equalities could easily be reversed to express $\left(\hat{a}_{\boldsymbol{k}},\hat{a}^{\dagger}_{-\boldsymbol{k}}\right)$ as function of $(\hat{v}_{\boldsymbol{k}},\hat{p}_{\boldsymbol{k}})$. The canonical conjugate variables $(\hat{v}(\eta,\boldsymbol{x}),\hat{p}(\eta,\boldsymbol{x}))$,$(\hat{v}_{\boldsymbol{k}}(\eta),\hat{p}_{\boldsymbol{k}}(\eta))$, $\left(\hat{a}_{\boldsymbol{k}},\hat{a}^{\dagger}_{-\boldsymbol{k}}\right)$ satisfy the canonical commutation relations

$$\begin{aligned}\left[\hat{v}(\eta,\boldsymbol{x}),\hat{p}(\eta,\boldsymbol{y})\right]= &\ i\delta^{(3)}(\boldsymbol{x}-\boldsymbol{y})\\ \left[\hat{v}_{\boldsymbol{p}}(\eta),\hat{p}^{\dagger}_{\boldsymbol{k}}(\eta)\right]= &\ i\delta^{(3)}(\boldsymbol{p}-\boldsymbol{k})\\ \left[\hat{a}_{\boldsymbol{p}}(\eta),\hat{a}^{\dagger}_{\boldsymbol{k}}(\eta)\right]= &\ \delta^{(3)}(\boldsymbol{p}-\boldsymbol{k})\end{aligned}\ , \tag{1.41}$$

notice from (1.40) that $(\hat{v}_{\boldsymbol{k}}(\eta),\hat{p}_{\boldsymbol{k}}(\eta))$ are not hermitian and they mix different modes $(\boldsymbol{k},-\boldsymbol{k})$, but we can easily construct a pair of hermitian operators $(\hat{q}_{\boldsymbol{k}}(\eta),\hat{\pi}_{\boldsymbol{k}}(\eta))$ that will turn out to be useful when we discuss Bell inequalities in the next chapter,

$$\begin{aligned}\hat{q}_{\boldsymbol{k}}(\eta)= &\ \tfrac{a_{\boldsymbol{k}}(\eta)+a^{\dagger}_{\boldsymbol{k}}(\eta)}{2k}\\ \hat{\pi}_{\boldsymbol{k}}(\eta)= &\ -i\sqrt{\tfrac{k}{2}}\left(\hat{a}_{\boldsymbol{k}}(\eta)-\hat{a}^{\dagger}_{\boldsymbol{k}}(\eta)\right)\end{aligned}\ , \tag{1.42}$$

$(\hat{v}_{\boldsymbol{k}}(\eta),\hat{p}_{\boldsymbol{k}}(\eta))$ and $(\hat{q}_{\boldsymbol{k}}(\eta),\hat{\pi}_{\boldsymbol{k}}(\eta))$ are related by[25]

$$\begin{aligned}\hat{v}_{\boldsymbol{k}}= &\ \tfrac{1}{2}\left[\hat{q}_{\boldsymbol{k}}+\hat{q}_{-\boldsymbol{k}}+\tfrac{i}{k}\left(\hat{\pi}_{\boldsymbol{k}}-\hat{\pi}_{-\boldsymbol{k}}\right)\right]\\ \hat{p}_{\boldsymbol{k}}= &\ \tfrac{1}{2i}\left[k\left(\hat{q}_{\boldsymbol{k}}-\hat{q}_{-\boldsymbol{k}}\right)+i\left(\hat{\pi}_{\boldsymbol{k}}+\hat{\pi}_{-\boldsymbol{k}}\right)\right]\end{aligned}\ , \tag{1.43}$$

[25]In order to avoid heavy notations we will omit the time dependence from operators from now on since it is clear that we are working in the Heisenberg picture.

Using (1.40) the classical Hamiltonian $H^{(2)}$ gives rise to the two mode Hamiltonian operator

$$\hat{H}^{(2)} = \frac{1}{2}\int d^3\boldsymbol{k}\left[k\left\{\hat{a}_{\boldsymbol{k}}\hat{a}^{\dagger}_{\boldsymbol{k}} + \hat{a}^{\dagger}_{-\boldsymbol{k}}\hat{a}_{-\boldsymbol{k}}\right\} - i\frac{a'}{a}\left\{\hat{a}_{\boldsymbol{k}}\hat{a}_{-\boldsymbol{k}} - \hat{a}^{\dagger}_{\boldsymbol{k}}\hat{a}^{\dagger}_{-\boldsymbol{k}}\right\}\right] , \tag{1.44}$$

which could be written in a more illuminating way as

$$\hat{H}^{(2)} = \frac{1}{2}\int_{R^3} d^3\boldsymbol{k}\hat{H}_{free}\left(\boldsymbol{k}\right) + \lambda\left(\eta\right)\frac{1}{2}\int_{R^3} d^3\boldsymbol{k}\hat{H}_{int}\left(\boldsymbol{k}\right) , \tag{1.45}$$

with $\lambda\left(\eta\right) = a'/\left(2a\right)$ is a time dependent coupling constant and

$$\hat{H}_{free}\left(\boldsymbol{k}\right) = \frac{k}{2}\left(\hat{a}_{\boldsymbol{k}}\hat{a}^{\dagger}_{\boldsymbol{k}} + \hat{a}^{\dagger}_{-\boldsymbol{k}}\hat{a}_{-\boldsymbol{k}}\right) , \tag{1.46}$$

is the free evolution piece the Hamiltonian that describes a collection of free harmonic oscillators, while the squeezing piece

$$\hat{H}_{int}\left(\boldsymbol{k}\right) = -i\left(\hat{a}_{\boldsymbol{k}}\hat{a}_{-\boldsymbol{k}} - \hat{a}^{\dagger}_{\boldsymbol{k}}\hat{a}^{\dagger}_{-\boldsymbol{k}}\right) , \tag{1.47}$$

represents the interaction of quantum perturbations with the classical background which will be crucial in our discussion of squeezing and how this last phenomenon contribute to the classicalization of primordial quantum fluctuations. Notice that in case of non dynamical background, i.e Malinowskian, then $\lambda\left(\eta\right) = 0$. [54]

To study the evolution of our quantum scalar fluctuations we have to define their initial state. To this end, we assume that all the modes of interest (i.e. the modes on subhorizon scales today) are well within the horizon at the initial time η_{in} so that $k\eta_{in} \gg 1$ therefore $\lambda\left(\eta_{in}\right) \ll H^{-1}$ and (1.45) reduces to the free Hamiltonian which leads to choose for the initial state the ground state of the free Hamiltonian, i.e. the Poincare invariant vacuum state[6], which is known in inflation literature as Bunch-Davies vacuum state [17],

$$\left|0\right\rangle_{in} = \bigotimes_{\boldsymbol{k}}\left|0_{\boldsymbol{k}}\right\rangle , \tag{1.48}$$

with $\hat{a}_{\boldsymbol{k}}\left(\eta_{in}\right)\left|0_{\boldsymbol{k}}\right\rangle = 0$, notice that in (1.48) the different modes are assumed to be uncorrelated initially. The choice we have just made is a key factor for the empirical success of inflationary paradigm [54]. However, it would be interesting to see what would happen if we start from an excited states based on the BD state, or starting from non BD vaccum state; another possibility is to consider an entangled initial state, either between the modes of the same degree freedom or with another degree of freedom. For the last case we present briefly some of the results obtained in [5, 14, 13], because this possibility will be related to our work on decoherence where we will assume an uncorrelated initial state of our system with the environment, we will discuss this point in more details later. For the case of an initial excited state we refer the reader to [47, 28, 36, 26, 27, 21].

Having chosen the initial conditions, let us now write the equations of motion of operators $\left(\hat{a}_{\boldsymbol{k}}, \hat{a}^{\dagger}_{-\boldsymbol{k}}\right)$

and their solutions, using Heisenberg equation

$$\frac{\mathrm{d}\hat{a}_{\boldsymbol{k}}}{\mathrm{d}\eta} = i\left[\hat{H}^{(2)}_{\boldsymbol{k}}, \hat{a}_{\boldsymbol{k}}\right] , \tag{1.49}$$

leads to the system

$$\begin{pmatrix} \hat{a}'_{\boldsymbol{k}} \\ \\ \hat{a}^{\dagger\prime}_{-\boldsymbol{k}} \end{pmatrix} = \begin{pmatrix} -ik & \frac{a'}{a} \\ \\ \frac{a'}{a} & ik \end{pmatrix} \begin{pmatrix} \hat{a}_{\boldsymbol{k}} \\ \\ \hat{a}^{\dagger}_{-\boldsymbol{k}} \end{pmatrix} \tag{1.50}$$

a general solution to this coupled equations is given by Bogolubov transformation[26]

$$\begin{aligned} \hat{a}_{\boldsymbol{k}}(\eta) = &\quad \mathrm{u}_k(\eta)\,\hat{a}_{\boldsymbol{k}}(\eta_0) + \mathrm{v}_k(\eta)\,\hat{a}^{\dagger}_{-\boldsymbol{k}}(\eta_0) \\ \hat{a}^{\dagger}_{-\boldsymbol{k}}(\eta) = &\quad \mathrm{u}^*{}_k(\eta)\,\hat{a}^{\dagger}_{-\boldsymbol{k}}(\eta_0) + \mathrm{v}^*_k(\eta)\,\hat{a}_{\boldsymbol{k}}(\eta_0) \end{aligned} , \tag{1.51}$$

which physically means that particles are produced in pairs with opposite momenta, this phenomenon is also called squeezing Schrodinger picture [39] . It is worth to mention that this phenomenon of Bogolubov transformation and large squeezing is induced by the interaction with the expanding universe in addition the existence of Hubble radius [66]. In quantum field theory, this is a common situation and typical examples are the dynamical Schwinger effect [16]. The canonical commutation relations (1.41), preserved under unitary time evolution, are satisfied by (1.51) provided that

$$|\mathrm{u}_k(\eta)|^2 - |\mathrm{v}_k(\eta)|^2 = 1 , \tag{1.52}$$

which suggest substituting (1.51) into (1.50) leads to equations of motion for mode functions $\mathrm{u}_k(\eta)$, $\mathrm{v}_k(\eta)$

$$\begin{aligned} \mathrm{u}'_k = &\quad -ik\mathrm{u}_k + \tfrac{a'}{a}\mathrm{v}^*_k \\ \mathrm{v}'_k = &\quad -ik\mathrm{v}_k + \tfrac{a'}{a}\mathrm{u}^*_k \end{aligned} . \tag{1.53}$$

The MS variable could be decomposed in terms of mode functions $f_k(\eta)$ as

$$\hat{v}_{\boldsymbol{k}}(\eta) = f_k(\eta)\,\hat{a}_{\boldsymbol{k}}(\eta_{in}) + f^*_k(\eta)\,\hat{a}^{\dagger}_{-\boldsymbol{k}}(\eta_{in}) , \tag{1.54}$$

imposing the canonical commutation relations relations again leads to constrain the conserved Wronskian[27] to satisfy

$$W = f_k f^{*\prime}_k - f^*_k f'_k = i , \tag{1.55}$$

[26] Here v_k is the Bogolubov transformation factor, or mode function, and not the MS variable $v_{\boldsymbol{k}}(\eta)$.

[27] It easy to show that W is conserved $W' = 0$ using (1.56).

substituting (1.54) into Ms equation (1.38) gives the Euler Lagrange equation satisfied by mode function

$$f_k'' + \omega^2(\eta, k) f_k = 0\,, \tag{1.56}$$

it is important to notice that both f_k and f_k^* satisfy the same equation and represent two linearly independent solutions as is clear from the non zero Wronskian. Comparing (1.40) and (1.54) results in an important relation between mode function f_k and Bogolubov factors u_k and v_k which will be useful later to link Heisenberg and Schrodinger pictures

$$f_k = \frac{\mathrm{u}_k + \mathrm{v}^*{}_k}{\sqrt{2k}}\,, \tag{1.57}$$

doing the same for the momentum modes, then

$$\hat{p}_{\boldsymbol{k}}(\eta) = -i\left[g_k(\eta)\,\hat{a}_{\boldsymbol{k}}(\eta_{in}) - g_k^*(\eta)\,\hat{a}^\dagger_{-\boldsymbol{k}}(\eta_{in})\right]\,, \tag{1.58}$$

and is related to u_k and v_k by

$$g_k = \sqrt{\frac{k}{2}}\left(\mathrm{u}_k - \mathrm{v}^*{}_k\right)\,. \tag{1.59}$$

We will pause now to discuss another formalism that is equivalent to Heisenberg and Schrodinger one, but most importantly it will help to link them together. We can find different starting points to introduce it in litterateurs, either we start from the constraint (1.52) or more fundamentally from the Hamiltonian (1.44). We will present both approaches, so let us start by the former, where from (1.52) we can infer that a possible parameterization of u_k and v_k is

$$\begin{aligned} \mathrm{u}_k(\eta) &= \mathrm{e}^{-i\theta_{\boldsymbol{k}}}\cosh r_{\boldsymbol{k}} \\ \mathrm{v}_k(\eta) &= \mathrm{e}^{i\theta_{\boldsymbol{k}} + 2i\phi_{\boldsymbol{k}}}\sinh r_{\boldsymbol{k}} \end{aligned}\,, \tag{1.60}$$

where the three functions $r_{\boldsymbol{k}}(\eta)$, $\phi_{\boldsymbol{k}}(\eta)$,$\theta_{\boldsymbol{k}}(\eta)$ are called the squeezing parameters, squeezing angle and rotation angle, respectively [29, 56]. Equations (1.57) and (1.60) give explicitly the relation between the mode functions f_k which are typically used in the Heisenberg approach and the squeezing parameters characteristic for the Schrodinger approach[66]. In order to get a taste of the physics behind the squeezing parameters, we should introduce them through linking the squeezed state formalism to that of particles creation in an external field [35, 6].

We start from (1.44) and derive the corresponding evolution operator $\hat{\mathcal{U}}_{\mathcal{H}_{\boldsymbol{k}}}(\eta, \eta_0)$ which could be written as product of squeezing operator $\hat{\mathcal{S}}(r_{\boldsymbol{k}}, \phi_{\boldsymbol{k}})$ and rotation operator $\hat{\mathcal{R}}(\theta_{\boldsymbol{k}})$,

$$\hat{\mathcal{U}}_{\mathcal{H}_{\boldsymbol{k}}}(\eta, \eta_0) = \hat{\mathcal{S}}(r_{\boldsymbol{k}}, \phi_{\boldsymbol{k}})\,\hat{\mathcal{R}}(\theta_{\boldsymbol{k}})\,, \tag{1.61}$$

with

$$\hat{\mathcal{R}}\left(\theta_{\boldsymbol{k}}\right)=\exp\left[-i\theta_{\boldsymbol{k}}\left(\hat{a}_{\boldsymbol{k}}\hat{a}_{\boldsymbol{k}}^{\dagger}+\hat{a}_{-\boldsymbol{k}}^{\dagger}\hat{a}_{-\boldsymbol{k}}\right)\right]\,, \tag{1.62}$$

and

$$\hat{\mathcal{S}}\left(r_{\boldsymbol{k}},\phi_{\boldsymbol{k}}\right)=\exp\left[\frac{r_{\boldsymbol{k}}}{2}\left(\mathrm{e}^{-2i\phi_{\boldsymbol{k}}}\hat{a}_{-\boldsymbol{k}}\hat{a}_{\boldsymbol{k}}-h.c\right)\right]\,, \tag{1.63}$$

such decomposition is a general property of momentum preserving quadratic Hamiltonians [70, 69], notice that time dependence in (1.62) and (1.63) is solely through squeezing parameters while creation and annihilation operators are fixed at η_{in}. From (1.62) and (1.63) we see that $r_{\boldsymbol{k}}$ gives us a measure of the excitation of the state, while $\phi_{\boldsymbol{k}}$ measures the sharing of the excitation between the canonical variables, and $\theta_{\boldsymbol{k}}$contributes just by a phase [6]. Applying such decomposed evolution operator to the initial vacuum state chosen above would reveal us the reason behind the nomination assigned to $\mathcal{S}$ and $\mathcal{R}$. The action of this last on the vacuum state (1.48) produces an irrelevant phase thus the name of $\mathcal{R}$

$$\hat{\mathcal{R}}\left(\theta_{\boldsymbol{k}}\right)|0\rangle_{in}=\mathrm{e}^{-i\theta_{\boldsymbol{k}}}|0\rangle_{in}\,, \tag{1.64}$$

while the action of $\mathcal{S}$ evolves the vacuum state into a two mode squeezed state[28][70, 69, 35]

$$|\Psi_{2sq}\rangle=\hat{\mathcal{S}}\left(r_{\boldsymbol{k}},\phi_{\boldsymbol{k}}\right)|0\rangle_{in}=\frac{1}{\cosh r_{\boldsymbol{k}}}\sum_{n}\left(\mathrm{e}^{-2i\phi_{\boldsymbol{k}}}\tanh r_{\boldsymbol{k}}\right)^{n}|n,\boldsymbol{k};n,-\boldsymbol{k}\rangle\,, \tag{1.65}$$

where

$$|n,\boldsymbol{k};n,-\boldsymbol{k}\rangle=\frac{1}{n!}\left(\hat{a}_{\boldsymbol{k}}^{\dagger}\hat{a}_{-\boldsymbol{k}}^{\dagger}\right)|0\rangle_{in}\,, \tag{1.66}$$

is the two mode occupation number state. Usually, this squeezing which amplifies the initial vacuum fluctuations by creating momentum conserving pairs of quanta is thought to be sufficient to explain the quantum to classical transition in early universe, particularly, for the superhorizon modes. This last statement is justified by knowing that, on the one hand, the number of created particles is given by

$$n_{\boldsymbol{k}}=\langle\Psi_{2sq}|\,\hat{N}_{\boldsymbol{k}}\,|\Psi_{2sq}\rangle=\sinh^{2}r_{\boldsymbol{k}}\,, \tag{1.67}$$

on the other hand, and as we will show in a moment, the value of the squeezing parameter $r_{\boldsymbol{k}}$ gets very large values for superhorizon modes ($k\eta\to0$), i.e the modes k probed in the CMB. Where the cosmological typical values of $r_{\boldsymbol{k}}$ is of the order $\approx 10^{2}$, and this value is much larger than what can be achieved in the laboratory [56]. However, as we will discuss in details towards the end of this chapter, large squeezing is not enough to explain the quantum to classical transition in early universe, as first hint of that is to notice that the state (1.65) is an entangled one. It is therefore reasonable to conclude that the quantum state $|\Psi_{2sq}\rangle$is a highly non-classical state.

Before of deriving the evolution equations of of squeezing parameters, it is worth to link the

[28] Notice that here we switched temporarily to Schrodinger picture since we are talking about the evolution of states, and this is for the purpose of linking this picture to Heisenberg as we will do soon.

Bogolubov transformations and squeezed state formalism through

$$\hat{a}_{\boldsymbol{k}}(\eta) = \hat{\mathcal{R}}^{\dagger}_{\boldsymbol{k}} \hat{\mathcal{S}}^{\dagger}_{\boldsymbol{k}} \hat{a}_{\boldsymbol{k}}(\eta_{in}) \hat{\mathcal{S}}_{\boldsymbol{k}} \hat{\mathcal{R}}_{\boldsymbol{k}} \,, \tag{1.68}$$

so that, now, we see explicitly equivalence of Schrodinger and Heisenberg pictures.

The dynamical equations satisfied by the squeezing parameters could be obtained by substituting (1.60) into (1.53) and using the identity satisfied by hyperbolic functions $\cosh^2 r_{\boldsymbol{k}} - \sinh^2 r_{\boldsymbol{k}} = 1$

$$\begin{aligned} r'_{\boldsymbol{k}} &= \frac{a'}{a}\cos 2\phi_{\boldsymbol{k}} \\ \phi'_{\boldsymbol{k}} &= -k - \frac{a'}{a}\sin 2\phi_{\boldsymbol{k}} \coth 2r_{\boldsymbol{k}} \quad , \\ \theta'_{\boldsymbol{k}} &= k + \frac{a'}{a}\sin 2\phi_{\boldsymbol{k}} \tanh 2r_{\boldsymbol{k}} \end{aligned} \tag{1.69}$$

notice that in the superhorizon limit which has $r_{\boldsymbol{k}} \gg 1$, $(\phi_{\boldsymbol{k}} + \theta_{\boldsymbol{k}})'_{r_{\boldsymbol{k}} \gg 1} \to 0$, thus in this limit $\phi_{\boldsymbol{k}} + \theta_{\boldsymbol{k}} \to \delta_{\boldsymbol{k}}$and the mode function (1.57) could be written as

$$f_k\,|_{r_{\boldsymbol{k}} \gg 1} \to \mathrm{e}^{-i\delta_{\boldsymbol{k}}} \mathrm{e}^{r_{\boldsymbol{k}}} \cos\phi_{\boldsymbol{k}} \,, \tag{1.70}$$

this freezing of phases on superhorizon scales in standard inflation is related the appearance of acoustic peaks in the CMB; we will see later, the implementation of collapse models does not spoil this distinguished feature of standard inflation[29].

The solution of evolution modes equations is given by Bessel function, and upon using Bunch Davies vacuum as initial state leads to

$$\begin{aligned} f_k &= \frac{1}{\sqrt{2k}}\left(1 - \frac{i}{k\eta}\right)\mathrm{e}^{-i\boldsymbol{k}\eta} \\ g_k &\equiv i\left(f'_k - \frac{a'}{a} f_k\right) = \sqrt{\frac{k}{2}}\mathrm{e}^{-i\boldsymbol{k}\eta} \end{aligned} \quad , \tag{1.71}$$

now using (1.57, 1.58, 1.60), and by simple manipulations we obtain the following expressions of squeezing parameters[29]

$$\begin{aligned} r_{\boldsymbol{k}} &= \sinh^{-1}\left(\frac{1}{2k\eta}\right) \\ \phi_{\boldsymbol{k}} &= \frac{\pi}{4} - \frac{1}{2}\tan^{-1}\left(\frac{1}{2k\eta}\right) \quad , \\ \theta_{\boldsymbol{k}} &= k\tau + \tan^{-1}\left(\frac{1}{2k\eta}\right) \end{aligned} \tag{1.72}$$

[29]In those solutions we are considering $\eta \equiv |\eta|$.

we can see from the first equation above that in the superhorizon limit $k|\eta| \to 0$ we indeed have $r_{\boldsymbol{k}} \gg 1$ as we were stating before. While in the subhorizon limit, the squeezing parameters oscillate [6, 66, 35].

One of the key predictions of inflation is the quasi scale invariant power spectrum of CMB anisotropies, therefore as a direct way to show the equivalence of the two pictures is to compute it in both and show the that we get the same result. The Heisenberg picture is, usually, the one used in standard inflation literature to compute power spectrum so we will reproduce it here. The power spectrum of MS variable is defined as the two point correlation function of this fluctuations

$$
{}_{in}\langle 0|\, \hat{v}(\eta, \boldsymbol{x})\, \hat{v}(\eta, \boldsymbol{x})\, |0\rangle_{in} = \int \frac{\mathrm{d}k}{k} \mathcal{P}_v(k) \ , \tag{1.73}
$$

where here $\mathcal{P}_v(k)$ is the dimensionless power spectrum related to dimensional one by

$$
\mathcal{P}_v(k) = \frac{k^3}{2\pi^2} P_v(k) \ , \tag{1.74}
$$

and $P_v(k)$is given by

$$
{}_{in}\langle 0|\, \hat{v}^*_{\boldsymbol{k}}(\eta)\, \hat{v}_{\boldsymbol{k'}}(\eta)\, |0\rangle_{in} = (2\pi)^3 P_v(k)\, \delta(\boldsymbol{k'} + \boldsymbol{k}) \ , \tag{1.75}
$$

using (1.54) we obtain

$$
\mathcal{P}_v(k) = \frac{k^3}{2\pi^2} |f_k|^2 \ , \tag{1.76}
$$

therefore the power spectrum of curvature perturbations is obtained using (1.30),

$$
\mathcal{P}_\zeta(k) = \left(\frac{\mathcal{H}}{a\varphi_0'}\right)^2_* \mathcal{P}_v(k) \equiv A_s k^{n_s - 1} \,, \tag{1.77}
$$

where $(\cdots)_*$ means the quantity is computed at horizon crossing $k = a_* \mathcal{H}_*$. A_s determines the amplitude of power spectrum, and the spectral index n_s encodes the scale dependence of power spectrum which is predicted by inflation to be $n_s \simeq 1$, where in a quasi de Sitter spacetime

$$
\mathcal{P}_\zeta(k) = \frac{\mathcal{H}_*^2}{2\pi^2} \left(\frac{\mathcal{H}^2}{\varphi_0^{2\prime}}\right)_* \left(\frac{k}{a\mathcal{H}}\right)^{3-2\nu} \ , \tag{1.78}
$$

with $\nu = \frac{3}{2} - 3\epsilon_1^V + \epsilon_2^V$, and ϵ_1^V, ϵ_2^V are the first and second potential slow roll parameters. Indeed, by the observation of CMB spectra this prediction was confirmed by Planck satellite[2]

$$
n_s = 0.968 \pm 0.006 \quad (95\%\,\mathrm{C.L}) \ , \tag{1.79}
$$

1.2.2 Schrodinger picture and squeezing formalism

In the Schrodinger approach, the quantum state of the system is described by a wavefunctional, $\Psi[v(\eta,x)]$, and since we work in Fourier space, In addition to the fact that the theory is still free in the sense that it does not contain terms with power higher than two in the Hamiltonian, then each mode $\boldsymbol{k}$ evolves independently and so do the real and imaginary parts of their wavefunctional), the wavefunctional can also be factorized into mode components [56]

$$\Psi\left[v\left(\eta,x\right)\right]=\prod_{\boldsymbol{k}}\Psi\left[v_{\boldsymbol{k}}^{R},v_{\boldsymbol{k}}^{I}\right]=\prod_{\boldsymbol{k}}\Psi_{\boldsymbol{k}}^{R}\left(v_{\boldsymbol{k}}^{R}\right)\Psi_{\boldsymbol{k}}^{I}\left(v_{\boldsymbol{k}}^{I}\right)\ , \tag{1.80}$$

with

$$\hat{v}_{\boldsymbol{k}}=\frac{\hat{v}_{\boldsymbol{k}}^{R}+i\hat{v}_{\boldsymbol{k}}^{I}}{\sqrt{2}}\quad , \tag{1.81}$$

similarly for $\hat{p}_{\boldsymbol{k}}$. The canonical commutation relations of $\left(\hat{v}_{\boldsymbol{k}}\left(\eta\right),\hat{p}_{\boldsymbol{k}}\left(\eta\right)\right)$ admit the representation

$$\hat{v}_{\boldsymbol{k}}\Psi=v_{\boldsymbol{k}}\Psi,\ \ \hat{p}_{\boldsymbol{k}}\Psi=-i\frac{\partial\Psi}{\partial v_{\boldsymbol{k}}}\,. \tag{1.82}$$

The common point between the two pictures from which we will start the computations is the initial state of the field amplitude v that was chosen to be vacuum at some initial time,

$$\hat{a}_{\boldsymbol{k}}\left(\eta_{in}\right)\left|0\right\rangle_{in}\ , \tag{1.83}$$

this state corresponds to Gaussian and time evolution preserves its Gaussianity since it is evolved through a quadratic Hamiltonian. The most important trick which will enable us to translate the Heisenberg picture results into Schrodinger one is to reexpress the above relation using (1.54) and (1.58) as follow

$$\left\{\hat{v}_{\boldsymbol{k}}\left(\eta\right)+i\Omega_{k}^{-1}\left(\eta\right)\hat{p}_{\boldsymbol{k}}\left(\eta\right)\right\}\left|0,\eta_{in}\right\rangle_{H}=0\,, \tag{1.84}$$

the time dependent function $\Omega_k\left(\eta\right)$ could be inferred through (1.57) and (1.59)

$$\Omega_{k}=k\frac{\mathrm{u}_{k}^{*}-\mathrm{v}_{k}}{\mathrm{u}_{k}^{*}+\mathrm{v}_{k}}\equiv\frac{g_{k}^{*}}{f_{k}^{*}}=-i\frac{f_{k}^{*\prime}}{f_{k}^{*}}+i\frac{a'}{a}\,. \tag{1.85}$$

Translating (1.84) into Schrodinger picture by applying (1.61) to (1.83) as follow

$$\mathcal{S}\hat{a}_{\boldsymbol{k}}\left(\eta_{in}\right)\mathcal{S}^{-1}\left|0,\eta\right\rangle_{S}=0\quad\Longleftrightarrow\quad\mathcal{S}\left\{\hat{v}_{\boldsymbol{k}}\left(\eta\right)+i\Omega_{k}^{-1}\left(\eta\right)\hat{p}_{\boldsymbol{k}}\left(\eta\right)\right\}\mathcal{S}^{-1}\left|0,\eta\right\rangle_{S}=0\ \ , \tag{1.86}$$

leads to

$$\left\{\hat{v}_{\boldsymbol{k}}\left(\eta_{in}\right)+i\Omega_{k}^{-1}\left(\eta\right)\hat{p}_{\boldsymbol{k}}\left(\eta_{in}\right)\right\}\left|0,\eta\right\rangle_{S}=0\,, \tag{1.87}$$

notice the similar structure between (1.84) and (1.87). However, since the sate has evolved in time and due to to creation of particles the state will be annihilated by a more complicated operator[39]. Using the representation of operators $\left(\hat{v}_{\boldsymbol{k}}\left(\eta\right),\hat{p}_{\boldsymbol{k}}\left(\eta\right)\right)$ in (1.82), then multiplying (1.87) from the left by the basis vector[30]$\left|v_{\boldsymbol{k}}\left(\eta\right)\right\rangle$ and solve the equation we can easily see that outcome is a Gaussian functional

$$\Psi\left[v_{\boldsymbol{k}}\left(\eta_{in}\right),v_{-\boldsymbol{k}}\left(\eta_{in}\right)\right]=N_{k}\left(\eta\right)\exp\left(-\Omega_{k}v_{\boldsymbol{k}}\left(\eta_{in}\right)v_{-\boldsymbol{k}}\left(\eta_{in}\right)\right)\,, \tag{1.88}$$

decomposing it into real and imaginary part

$$\Psi_{\boldsymbol{k}}^{R,I}\left[\eta,v_{\boldsymbol{k}}^{R,I}\left(\eta_{in}\right)\right]=\sqrt{N_{k}\left(\eta\right)}\exp\left(-\frac{\Omega_{k}\left(\eta\right)}{2}\left(v_{\boldsymbol{k}}^{R,I}\right)^{2}\right)\,, \tag{1.89}$$

the functions $N_k\left(\eta\right)$ and $\Omega_k\left(\eta\right)$ do not carry indices R/I because they are the same for real and imaginary parts. Notice that the wavefunctional (1.89) is completely known once the time dependence $\Omega_k\left(\eta\right)$ is determined, and this will be done through Schrodinger equation. It is worth to notice that role of Schrodinger equation here is not to get the wavefunctional form since this last was already determined by the initial state constraint (1.87), however, the role is to get the equations governing the evolution of $N_k\left(\eta\right)$ and $\Omega_k\left(\eta\right)$.

For each mode $\boldsymbol{k}$, the real and imaginary parts of the wavefunctional $\Psi_{\boldsymbol{k}}^{R,I}$ satisfy the functional Schrodinger equation,

$$i\frac{\partial\Psi_{\boldsymbol{k}}^{R,I}}{\partial\eta}=\hat{\mathcal{H}}_{\boldsymbol{k}}^{R,I}\Psi_{\boldsymbol{k}}^{R,I}\,, \tag{1.90}$$

where now we use $H^{(1)}$due its similarity to that of the harmonic oscillator, in Fourier space it could be written as

$$\hat{\mathcal{H}}_{\boldsymbol{k}}=\hat{\mathcal{H}}_{\boldsymbol{k}}^{R}+\hat{\mathcal{H}}_{\boldsymbol{k}}^{I}\,, \tag{1.91}$$

and now the conjugate momentum is given by

$$p_{\boldsymbol{k}}=\frac{\partial\mathcal{L}}{\partial v_{\boldsymbol{k}}'}=\overset{*}{v}_{\boldsymbol{k}}{}'\,, \tag{1.92}$$

therefore using (1.82) we find

$$\hat{\mathcal{H}}_{\boldsymbol{k}}^{R,I}=-\frac{1}{2}\frac{\partial^{2}}{\partial v_{\boldsymbol{k}}^{R,I\,2}}+\frac{1}{2}\omega^{2}\left(v_{\boldsymbol{k}}^{R,I}\right)^{2}\,, \tag{1.93}$$

[30]choosing the field amplitude eigenstates $\left|v_{\boldsymbol{k}}\left(\eta\right)\right\rangle$ is motivated by the fact that the they are the pointer basis as we will discuss in the chapter of decoherence. But, it could also be seen through (1.77) and (1.108) where we see that is the field v not conjugate momentum p that is related to observation.

where the frequency ω expression was given (1.37). and by substituting (1.89) into (1.90) we get the equations we are looking for

$$i\tfrac{N_k'}{N_k} = \quad \Omega_k \quad , \tag{1.94}$$

$$\Omega_k' = \text{-i}\Omega_k^2 + i\omega^2 \,, \tag{1.95}$$

but notice that with the Hamiltonian $H^{(1)}$, Ω_k is given now by

$$\Omega_k = k\frac{\mathrm{u}_k^* - \mathrm{v}_k}{\mathrm{u}_k^* + \mathrm{v}_k} \equiv \frac{g_k^*}{f_k^*} = -i\frac{f_k^{*\prime}}{f_k^*} \,, \tag{1.96}$$

needless to say that substituting this in (1.95) would result in equation for f_k that is exactly the same one given by (1.56), where by doing such substitution we have reduced the Ricatti equation into second order linear differential equation. We can see (1.96) as a solution to (1.95) while the solution of (1.94) is given by

$$|N_k| = \left(\frac{\Re e\Omega_k}{\pi}\right)^{1/2} \quad , \tag{1.97}$$

$$\int \mathrm{d}v_{\boldsymbol{k}}^{R,I}\Psi_{\boldsymbol{k}}^{*R,I}\Psi_{\boldsymbol{k}}^{R,I} = 1 \,, \tag{1.98}$$

which assures proper normalization of the wavefunctional. Before moving forward, it is important to mention that now f_k is a mere mathematical parameter which would help one to simplify commutations and determine the functional form of Ω_k in Schrodinger picture, while in the Heisenberg picture it was representing a mode function that is a physical observable [29].

Let us now see what are the initial conditions that could be obtained for $N_k(\eta)$ and $\Omega_k(\eta)$ through imposing the state of quantum field v to be initially in the vacuum state. This last choice was chosen, as explained previously, due to the inflation assumption that all modes of interest toady were inside the horizon at the beginning inflation i.e $\omega^2(\eta, k) \to k^2$, so we are exactly in the case of an harmonic oscillator, instead of the parametric one. In this regime the solution of (1.56) is given by

$$f_k(\eta) = A_k \mathrm{e}^{ik\eta} + B_k \mathrm{e}^{-\mathrm{i}k\eta} \,, \tag{1.99}$$

therefore (1.96) gives

$$\Omega_k = -\frac{k}{2}\frac{A_k^*\mathrm{e}^{-ik\eta} - B_k^*\mathrm{e}^{\mathrm{i}k\eta}}{A_k^*\mathrm{e}^{-ik\eta} + B_k^*\mathrm{e}^{\mathrm{i}k\eta}} \,, \tag{1.100}$$

and in order for (1.89) to represents the ground $\Rightarrow \Omega_k = k/2$, which implies from (1.100) that $A_k = 0$. Having now the expression of f_k we can Compute the Wronskian and get $W = 2ik\,|B_k|^2$, so imposing (1.55) leads to $B_k = 1/\sqrt{2k}$, therefore by imposing the initial state of fluctuations to be the vacuum has completely fixed the initial conditions [56]. Before moving forward, we can easily obtain the corresponding probability to the wavefunctional (1.89)

$$P\left(v_{\boldsymbol{k}}\left(\eta_{in}\right), v_{-\boldsymbol{k}}\left(\eta_{in}\right)\right) \propto \exp\left(-2\Re e\Omega_k \left|v_{\boldsymbol{k}}\left(\eta_{in}\right)\right|^2\right) . \tag{1.101}$$

Having established the formalism describing perturbations in Schrodinger picture, we are able now to compute the power spectrum in this picture and compare it to the one obtained in Heisenberg picture. We will follow closely [56, 29], starting by the two point correlation function definition in Schrodinger picture

$$\langle\Psi|\,\hat{v}\left(\eta,\boldsymbol{x}\right)\hat{v}\left(\eta,\boldsymbol{x}\right)|\Psi\rangle = \int \mathrm{d}^3p\mathrm{d}^3q\prod_{\boldsymbol{k}}\mathrm{d}v^R_{\boldsymbol{k}}\mathrm{d}v^I_{\boldsymbol{k}}\Psi^*_{\boldsymbol{k}}\left[v^R_{\boldsymbol{k}}, v^I_{\boldsymbol{k}}\right]\hat{v}\left(\eta,\boldsymbol{x}\right)\hat{v}\left(\eta,\boldsymbol{x}\right)\Psi_{\boldsymbol{k}}\left[v^R_{\boldsymbol{k}}, v^I_{\boldsymbol{k}}\right] . \tag{1.102}$$

Fourier transforming the MS variables in the above expression yields

$$\langle\Psi|\,\hat{v}\left(\eta,\boldsymbol{x}\right)\hat{v}\left(\eta,\boldsymbol{x}\right)|\Psi\rangle = \frac{1}{\left(2\pi\right)^3}\int \mathrm{d}^3\boldsymbol{p}\mathrm{d}^3\boldsymbol{q}\mathrm{e}^{i(\boldsymbol{p}+\boldsymbol{q}).\boldsymbol{x}}\prod_{\boldsymbol{k}}\left(\frac{\Re e\Omega_k}{\pi}\right)\int\prod_{\boldsymbol{k}}\mathrm{d}v^R_{\boldsymbol{k}}\mathrm{d}v^I_{\boldsymbol{k}}\mathrm{e}^{-\sum_{\boldsymbol{k}}\Re e\Omega_k\left[\left(v^R_{\boldsymbol{k}}\right)^2+\left(v^I_{\boldsymbol{k}}\right)^2\right]}v_{\boldsymbol{p}}v_{\boldsymbol{q}} , \tag{1.103}$$

since the the integral over is weighted over a Gaussian then it vanishes

- If we consider $\boldsymbol{p} \neq \pm\boldsymbol{q}$, since the integrand becomes linear in $v^{R.I}_{\boldsymbol{p}}$ or $v^{R.I}_{\boldsymbol{q}}$.
- If we consider $\boldsymbol{p} = \boldsymbol{q}$, then the only non linear term in $v_{\boldsymbol{p}}v_{\boldsymbol{q}}$ is given by $\left[\left(v^R_{\boldsymbol{p}}\right)^2 - \left(v^I_{\boldsymbol{p}}\right)^2\right]/2$ and each term contributes equally therefore the difference gives vanishing result.

so the only choice remaining is $\boldsymbol{p} = -\boldsymbol{q}$, because through $v_{-\boldsymbol{p}} = v^*_{\boldsymbol{p}}$, we can see that $v_{\boldsymbol{p}}v_{\boldsymbol{q}} =\left[\left(v^R_{\boldsymbol{p}}\right)^2 + \left(v^I_{\boldsymbol{p}}\right)^2\right] / 2$ and this obviously gives non vanishing result

$$\langle\Psi|\,\hat{v}\left(\eta,\boldsymbol{x}\right)\hat{v}\left(\eta,\boldsymbol{x}\right)|\Psi\rangle = \frac{1}{\left(2\pi\right)^3}2\int \mathrm{d}^3\boldsymbol{p}\prod_{\boldsymbol{k}}\left(\frac{\Re e\Omega_k}{\pi}\right)\int\prod_{\boldsymbol{k}}\mathrm{d}v^R_{\boldsymbol{k}}\mathrm{d}v^I_{\boldsymbol{k}}\mathrm{e}^{-\sum_{\boldsymbol{k}}\Re e\Omega_k\left[\left(v^R_{\boldsymbol{k}}\right)^2+\left(v^I_{\boldsymbol{k}}\right)^2\right]}\left(v^R_{\boldsymbol{p}}\right)^2 , \tag{1.104}$$

the factor two comes from the fact that $\left(v^R_{\boldsymbol{p}}\right)^2$and $\left(v^I_{\boldsymbol{p}}\right)^2$ contributes equally. Carrying out the integrals above using the well known expressions for $\int \mathrm{d}x\mathrm{e}^{-\alpha x^2}$ and $\int \mathrm{d}x x^2\mathrm{e}^{-\alpha x^2}$ we end up with

$$\langle\Psi|\,\hat{v}\left(\eta,\boldsymbol{x}\right)\hat{v}\left(\eta,\boldsymbol{x}\right)|\Psi\rangle = \frac{1}{\left(2\pi\right)^3}\int \mathrm{d}^3\boldsymbol{p}\frac{1}{4\Re e\Omega_k} , \tag{1.105}$$

using (1.96). then it is easy to get

$$\langle\Psi|\,\hat{v}\left(\eta,\boldsymbol{x}\right)\hat{v}\left(\eta,\boldsymbol{x}\right)|\Psi\rangle = \frac{1}{2\pi^2}\int\frac{\mathrm{d}k}{k}k^3\left|f_k\right|^2 , \tag{1.106}$$

therefore we conclude[31]

$$\mathcal{P}_v(k) = \frac{k^3}{2\pi^2} |f_k|^2 \,, \tag{1.107}$$

which is exactly the result obtained in Heisenberg picture (1.76).

1.3 From quantum fluctuations to CMB anisotropies

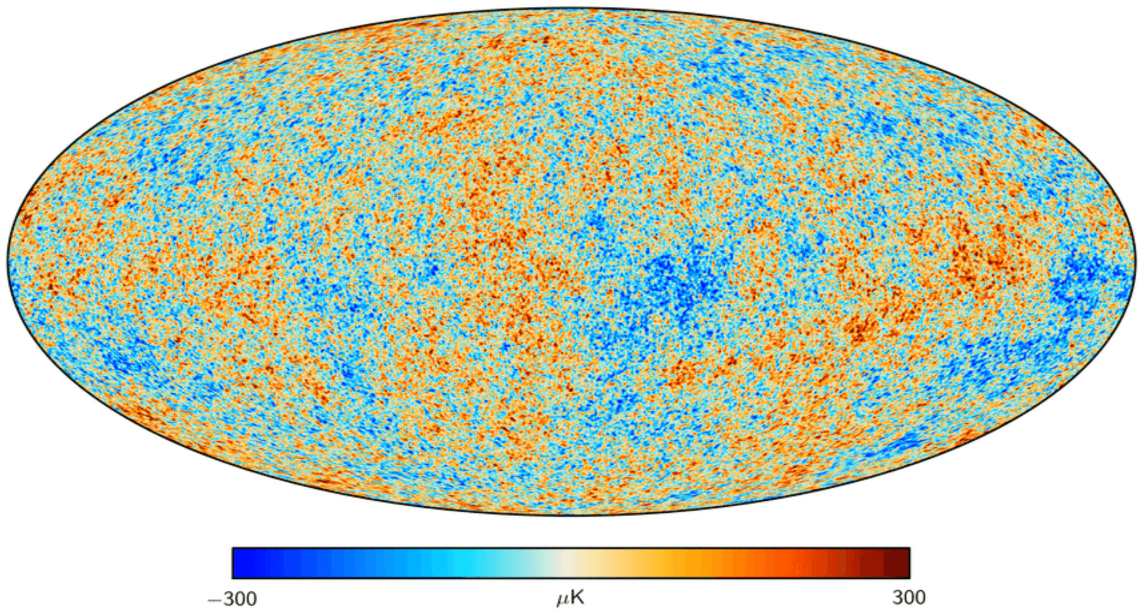

Figure 1.3: CMB anisotropies probed by Planck satellite

The importance of curvature perturbations lays on their conservation on superhorizon scale and being immune against the complications of post inflationary phase, especially the reheating phase [75, 71, 42], until they renter horizon in the radiation or matter dominated era and evolve according to the laws of standard cosmology[48, 45], and this evolution is captured by appropriate transfer functions in the Sachs-Wolfe effect [30]. Therefore, the primordial quantum fluctuations in the inflaton and gravitational fields could have many observational implications, on top of them we have the existence of Cosmological microwave background (CMB) anisotropies, where by measuring those last we can probe primordial fluctuation. The two types of perturbations, temperature along the direction $\mathbf{e}$ in the sky and curvature, are related at the last scattering surface by Sachs-Wolfe effect

$$\left(\frac{\delta T}{T}\right)(\mathbf{e}) = \frac{1}{5}\zeta\left[\eta_{lss}, -\mathbf{e}\left(\eta_{lss} - \eta_0\right) + \boldsymbol{x}_0\right] \,, \tag{1.108}$$

[31] We changed variable $p \to k$.

where T represents the averaged background temperature $T \approx 2.73K$, η_{lss} and η_0 the conformal time at last scattering surface (i.e time of emission) and today, respectively, $\boldsymbol{x}_0$ is the position of the earth from where the fluctuations are observed. Since for a cosmological perturbations of quantum origin the MS variable has been quantized $\hat{v}$, then we quantize also the curvature perturbation $\hat{\mathcal{R}}$ and temperature perturbation $\frac{\delta \hat{T}}{T}$. The temperature anisotropies live in the celestial sphere, therefore their corresponding operator is expanded over spherical harmonics according to [30, 56]

$$\left(\frac{\delta \hat{T}}{T}\right)(\mathbf{e}) = \sum_{l=2}^{\infty} \sum_{m=-l}^{m=l} \hat{a}_{lm} Y_{lm}(\theta, \phi) \ , \tag{1.109}$$

the angles (θ, ϕ) define the direction of $\mathbf{e}$. The angular two point correlation function could be expressed in terms of multipole moments C_l as

$$\langle \Psi | \, \hat{a}_{lm} \hat{a}^{*}_{l'm'} \, | \Psi \rangle = C_l \delta_{ll'} \delta_{mm'} \ , \tag{1.110}$$

therefore the two point correlation function of temperature anisotropies operator could be expressed as

$$\langle \Psi | \, \frac{\delta \hat{T}}{T}(\mathbf{e}_1) \frac{\delta \hat{T}}{T}(\mathbf{e}_2) \, | \Psi \rangle = \frac{1}{4\pi} \sum_{l=2}^{\infty} (2l+1) \, C_l P_l(\mathbf{e}_1 . \mathbf{e}_2) \ , \tag{1.111}$$

where P_l denotes the Legendre polynomials. Now it comes the step where we can see directly how the primordial quantum fluctuations determine the CMB anisotropies, to this end we express the multipole moments in terms of cosmological fluctuations power spectrum using (1.108 and 4.245)

$$\hat{a}_{lm} = \frac{1}{(2\pi)^{3/2}} \int \mathrm{d}\Omega_{\mathrm{e}} \mathrm{d}^3 \boldsymbol{k} \frac{\hat{\zeta}_{\boldsymbol{k}}(\eta_{lss})}{5} \mathrm{e}^{-i\boldsymbol{k}.[\mathbf{e}(\eta_{lss}-\eta_0)-\boldsymbol{x}_0]} \ , \tag{1.112}$$

from this expression and using

$$\langle \Psi | \, \hat{\zeta}_{\boldsymbol{k}} \hat{\zeta}^{*}_{\boldsymbol{p}} \, | \Psi \rangle = \frac{1}{2a^2 M_{pl} \epsilon_1} \frac{2\pi^2}{k^3} \mathcal{P}_v(k) \, \delta(\boldsymbol{k} - \boldsymbol{p}) \ , \tag{1.113}$$

we can show that

$$C_l = \frac{1}{2a^2 M_{pl} \epsilon_1} \frac{4\pi}{25} \int \frac{dk}{k} j_l^2 \left[k(\eta_0 - \eta_{lss})\right] \mathcal{P}_v(k) \ , \tag{1.114}$$

here j_l represents the Bessel function which could be considered here as transfer function that translates the three dimensional spatial frequency $\boldsymbol{k}$ into two dimensional frequency on celestial sphere. Notice that this last result is valid only on large scales for which we supposed that the transfer function accounting for the subsequent evolution after horizon reentry T_ζ is $T_\zeta(k \to 0) = 1$. [56]

1.4 Why do cosmological perturbations look classical?

1.4.1 Squeezing and Wigner function

We mentioned previously that under the momentum conserving quadratic Hamiltonian, the quantum fluctuations initial state, i.e vacuum state, evolves into a strongly squeezed state which is an important piece of the answer to the question asked above, so let us discuss it in more detail.

The fact that in the super-Hubble regime the the squeezing parameter reaches very large values is seen in the literature as an implication of the classicalization of the quantum fluctuations, and therefore justifying the standard scenario of structure formation dealing with classical fluctuations [66, 34, 39, 43]. But, what does the classicalization mentioned in those references means? is it the same one we used to discuss in the standard quantum mechanics ? or, does it refer to something else?

To answer the above questions, we use the winger function that could be seen as classical probability distribution function whenever it has positive values everywhere, which is expected to be the case here because we are dealing with a Gaussian wavefunctional. Wigner function is our tool to study the nature of mode functions on superhorizon scales weather are classical or quantum, where the function recognizes the correlation between the the canonical conjugate operators, in our case $(\hat{v}, \hat{p})$ [56, 29] . Wigner function is defined by

$$W\left(v_{\boldsymbol{k}}^R, v_{\boldsymbol{k}}^I; p_{\boldsymbol{k}}^R, p_{\boldsymbol{k}}^I\right) = \frac{1}{(2\pi)^2}\int \mathrm{d}x\mathrm{d}y \Psi_{\boldsymbol{k}}^*\left(v_{\boldsymbol{k}}^R - \frac{x}{2}, v_{\boldsymbol{k}}^I - \frac{y}{2}\right) \mathrm{e}^{-ip_{\boldsymbol{k}}^R.\boldsymbol{x} - ip_{\boldsymbol{k}}^I.\boldsymbol{y}} \Psi_{\boldsymbol{k}}\left(v_{\boldsymbol{k}}^R + \frac{x}{2}, v_{\boldsymbol{k}}^I + \frac{y}{2}\right), \tag{1.115}$$

substituting the wavefunctional found for the system (1.89) we get

$$\begin{aligned} W\left(v_{\boldsymbol{k}}^R, v_{\boldsymbol{k}}^I; p_{\boldsymbol{k}}^R, p_{\boldsymbol{k}}^I\right) &= \Psi^*\Psi \tfrac{1}{\pi \Re e\Omega_k} \exp\left[-\tfrac{1}{\Re e\Omega_k}\left(p_{\boldsymbol{k}}^R + \Im m\Omega_k v_{\boldsymbol{k}}^R\right)\right] \exp\left[-\tfrac{1}{\Re e\Omega_k}\left(p_{\boldsymbol{k}}^I + \Im m\Omega_k v_{\boldsymbol{k}}^I\right)\right] \\ &= \tfrac{1}{\pi^2} \exp\left[-\Re e\Omega_k\left(v_{\boldsymbol{k}}^{R\,2} + v_{\boldsymbol{k}}^{I\,2}\right)\right] \exp\left[-\tfrac{1}{\Re e\Omega_k}\left(p_{\boldsymbol{k}}^R + \Im m\Omega_k v_{\boldsymbol{k}}^R\right)\right] \exp\left[-\tfrac{1}{\Re e\Omega_k}\left(p_{\boldsymbol{k}}^I + \Im m\Omega_k v_{\boldsymbol{k}}^I\right)\right] \end{aligned}, \tag{1.116}$$

it is worth to note that in case we used the Hamiltonian $H^{(2)}$ instead of $H^{(1)}$ in the Schrodinger picture computations then we would have found the same Wigner function at the end.

The function (1.116) is product of four Gaussians, the first two have standard deviation given by $\sqrt{2/\Re e\Omega_k}$with zero mean value while the last two Gaussians have have standard deviation given $\sqrt{\Re e\Omega_k/2}$ with mean value $\Im m\Omega_k v_{\boldsymbol{k}}^R$ and $\Im m\Omega_k v_{\boldsymbol{k}}^R$ respectively. The nature of the Wigner function is determined by the values of variances which using definitions (1.57) and (1.96) are given by

$$\begin{aligned} \Re e\Omega_k &= \tfrac{k}{\cosh(2r_k)+\cos(2\phi_k)\sinh(2r_k)} \\ \Im m\Omega_k &= \tfrac{k\sin(2\phi_k)\sinh(2r_k)}{\cosh(2r_k)+\cos(2\phi_k)\sinh(2r_k)} \end{aligned}, \tag{1.117}$$

from (1.72) we see that on superhorizon limit $|r_k| \to \infty$ and $\phi_k \to \frac{\pi}{4}$ therefore

$$\mathfrak{Re}\Omega_k \to 0,\ \mathfrak{Im}\Omega_k \to \infty\,, \tag{1.118}$$

in this limit (1.116) could be written as

$$W\left(v^R_{\boldsymbol{k}}, v^I_{\boldsymbol{k}}; p^R_{\boldsymbol{k}}, p^I_{\boldsymbol{k}}\right) = \frac{1}{\pi^2}\exp\left[-\mathfrak{Re}\Omega_k\left(v^{R\,2}_{\boldsymbol{k}} + v^{I\,2}_{\boldsymbol{k}}\right)\right]\delta\left(p^R_{\boldsymbol{k}}\right)\delta\left(p^I_{\boldsymbol{k}}\right)\,, \tag{1.119}$$

thus the Wigner function acquires the cigar-like shape distinguishing the Wigner function of highly squeezed state. In contrast to coherent states whose Wigner function is centered around one physical trajectory in phase space with minimum uncertainty, it comes with an infinite number of classical trajectories. (see figure 1.4).

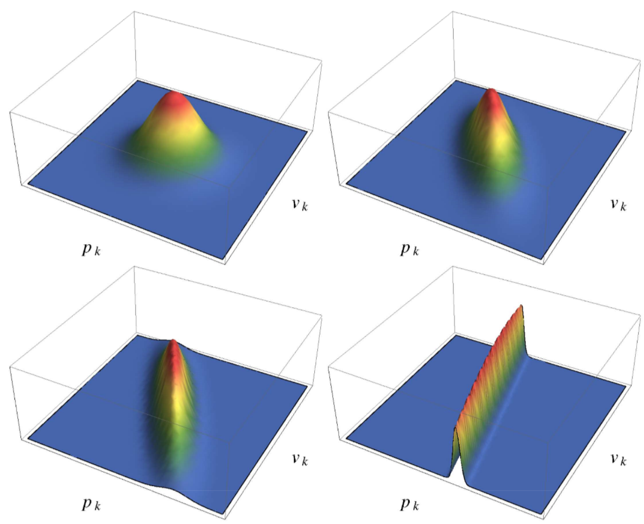

Figure 1.4: Plot of Wigner function for variables $\left(v^R_{\boldsymbol{k}}, p^R_{\boldsymbol{k}}\right)$ at different times which encoded are through different values of r_k. The upper left figure is for $r_k = 0.0005$, the right upper figure $r_k = 0.48$, the left bottom panel $r_k = 0.88$, the right bottom $r_k = 2.31$. We the squeezing in the direction of momentum[56].

The result obtained for Wigner function does carry a part of the answer to the question asked in the title of this section, where we see that in high squeezing limit one of the variances does vanish. But unfortunately the vanished variance is the one related to the measurement of conjugate variable p, or filed momentum, while we saw in the previous section that the field amplitude v is the one linked to observation of CMB anisotropies observables, but v measurements come along enormous uncertainty as seen through[32] (1.118) and (1.119). Therefore from this last statement we conclude that the cosmological perturbations are highly non classical, which is totally expected since they are placed in a highly squeezed state which described as highly non classical state, in contrast to coherent states that are considered the closest to classical states.

We are not over yet, there is one more option to use to overcome the non classicality of states. The aim is to be able, at least, to make predictions without the need of having rigorous mechanism or model that would induce the transition from quantumness to classicality. But such a model is still crucial and could have observational implications and corrections to current predictions3.

The option mentioned above consist in exchanging quantum expectation values by averages over a classical stochastic field through considering the quantum field v as a classical stochastic one, however the price to pay is the lost of any possibility to test the quantum origin of universe, since with this approach a set of classical perturbations would just lead to the same predictions. But how can we justify this approach?

The starting point is to observe that in the strong squeezing limit we have

$$\langle\Psi| \left(\hat{p}^R_{\boldsymbol{k}}\right)^2 |\Psi\rangle = \frac{\left(\Im\mathfrak{m}\Omega_k\right)^2}{\Re e\Omega_k} \,, \tag{1.120}$$

and the result could be obtained if we use the Wigner function, instead,

$$\int \mathrm{dv}^{\mathrm{R}}_{\boldsymbol{k}} \mathrm{dp}^{\mathrm{R}}_{\boldsymbol{k}} W_{r_k \to \infty} \left(v^R_{\boldsymbol{k}}, p^R_{\boldsymbol{k}}\right) \left(p^R_{\boldsymbol{k}}\right)^2 = \frac{\left(\Im\mathfrak{m}\Omega_k\right)^2}{\Re e\Omega_k} \,, \tag{1.121}$$

it may seem a surprising result and that it justifies the exchange of quantum expectation values with classical averages; but actually this is neither a surprising result nor justification of the approach mentioned, simply because there is a theorem which states that exact the Wigner function, and not necessarily the squeezing limit of it, satisfies for an arbitrary operator $\hat{A}\left(v^R_{\boldsymbol{k}}, p^R_{\boldsymbol{k}}\right)$

$$\langle\Psi| \hat{A}\left(v^R_{\boldsymbol{k}}, p^R_{\boldsymbol{k}}\right) |\Psi\rangle = \int \mathrm{dv}^{\mathrm{R}}_{\boldsymbol{k}} \mathrm{dp}^{\mathrm{R}}_{\boldsymbol{k}} W\left(v^R_{\boldsymbol{k}}, p^R_{\boldsymbol{k}}\right) A\left(v^R_{\boldsymbol{k}}, p^R_{\boldsymbol{k}}\right) \,, \tag{1.122}$$

where we see that (1.120) and (1.121) is a particular case [66].

The particularity that makes the approach justified, is the fact in the large squeezing limit all quantum expectation values could be reproduce if one assumes that the system always follow classical laws but has random initial conditions [38, 40, 39, 56], where in those last references they considered an example of free particle and the measured observable was position, first as quantum expectation value, then as classical average over an ensemble of classical particles. Therefore to sum up, the classicality discussed in literature is usually in the sense that the cosmological quantum fluctuations

[32] We will see later that an appropriate CSL model would cure that an rend the cigar like shape of Wigner function along the field amplitude v instead of momentum p.

could be described, or mimicked, by classical stochastic process and is this consideration what made inflation predictive for some cosmological observables, as CMB anisotropies. Another way to see this kind of classicality is to understand it in the following sense: if we assign to each point in phase-space $(v(k), p(k))$ the probability given by (1.101), then it will evolve in time according to the classical Hamiltonian equations [66]. But with such pragmatic approach [39] and because of the observational indistinguishability between classical and quantum correlation functions in this closed system approach, inflation is unable to provide a direct test of the quantum origin of the universe that could not be mimicked by non-inflationary cosmological models in which perturbations are assumed to be classical from the beginning on. In order to have such test, we need to implement some other tools as Bell inequalities, collapse models or decoherence, where the first one serves just to test the quantum origin, while the last two[33] provides also a mechanism to explain the transition from a quantum to classical universe. Before concluding it is useful to compute the density matrix

$$\hat{\rho}_{\boldsymbol{k}}^{R,I} = \left|\Psi_{\boldsymbol{k}}^{R,I}\right\rangle \left\langle\Psi_{\boldsymbol{k}}^{R,I}\right| , \tag{1.123}$$

corresponding to the wavefunctional (1.89) which is nothing but the Fourier transform of Wigner function, working in the filed operator $\hat{v}_{\boldsymbol{k}}$ eigenbasis $|\hat{v}_{\boldsymbol{k}}\rangle$

$$\left\langle \hat{v}_{\boldsymbol{k}}^{R,I\,\prime}\right| \hat{\rho}_{\boldsymbol{k}}^{R,I} \left|\hat{v}_{\boldsymbol{k}}^{R,I}\right\rangle = \left(\frac{\Re e\Omega_k}{\pi}\right)^{1/2} \boldsymbol{e}^{-\Re e\Omega_k \left[\left(\hat{v}_{\boldsymbol{k}}^{R,I\,\prime}\right)^2 + \left(\hat{v}_{\boldsymbol{k}}^{R,I}\right)^2\right]} \boldsymbol{e}^{-i\Im m\Omega_k \left[\left(\hat{v}_{\boldsymbol{k}}^{R,I\,\prime}\right)^2 - \left(\hat{v}_{\boldsymbol{k}}^{R,I}\right)^2\right]} , \tag{1.124}$$

from this last equation we can see the that off diagonal elements, or interference terms, $\hat{v}_{\boldsymbol{k}}^{R,I} \neq \hat{v}_{\boldsymbol{k}}^{R,I\,\prime}$oscillate rapidly and they do not vanish, which is totally expected because we are considering pure state. Therefore in order for the decoherence defined as the suppression of interference terms to take place we need to consider the interaction of our system with an environment [56], and this will make the subject of a whole chapter in this thesis, but before getting there, we still have many stations worth to stop by!

1.4.2 Decoherence without decoherence

We turn now to discuss another motivation that enable us to consider the cosmological perturbations as classical in the standard model of cosmology; regardless of whether they were, initially, originated through a quantum or classical process. We go back to the solution of MS equation (1.38) and write the field amplitude solution (1.54) in more illuminating way [39, 66]

$$v_{\boldsymbol{k}}\left(\eta\right) = \sqrt{2k} f_{k_1}\left(\eta\right) v_{\boldsymbol{k}}\left(\eta_{in}\right) - \sqrt{\frac{2}{k}} f_{k_2} p_{\boldsymbol{k}}\left(\eta_{in}\right) , \tag{1.125}$$

[33]Those consider an open system approach.

where $f_{k_1} = \mathfrak{Re} f_k$ and $f_{k_2} = \mathfrak{Im} f_k$. Similarly for the field momentum

$$p_{\boldsymbol{k}}(\eta) = \sqrt{\frac{2}{k}} g_{k_1} p_{\boldsymbol{k}}(\eta_{in}) + \sqrt{2k} g_{k_2}(\eta) v_{\boldsymbol{k}}(\eta_{in}) \ , \tag{1.126}$$

now the question is: after quantizing the variables (v, p), to which extent they will remain classical as universe evolve?

Notice that in case f_{k_2} and g_{k_1}vanish (or equivalently f_{k_1} and g_{k_2}), then the non commutativity of the variables (v, p) is no more relevant since the two are related by

$$p_{\boldsymbol{k}}(\eta) \equiv p_{\boldsymbol{k}}^{cl}(v_{\boldsymbol{k}}(\eta)) \simeq \frac{g_{k_2}(\eta)}{f_{k_1}(\eta)} v_{\boldsymbol{k}}(\eta) \ , \tag{1.127}$$

where by $p_{\boldsymbol{k}}^{cl}(v_{\boldsymbol{k}}(\eta))$ we refer to the fact that once an expectation value of $\hat{v}_{\boldsymbol{k}}(\eta)$ is obtained, then $\hat{p}_{\boldsymbol{k}}(\eta)$is fixed and is equal to the classical momentum corresponding to the value $\langle \hat{v}_{\boldsymbol{k}}(\eta) \rangle$, therefore we can consider the quantum system to be effectively equivalent to a classical random system. Actually, this value $p_{\boldsymbol{k}}^{cl}(v_{\boldsymbol{k}}(\eta))$ is the value we see in (1.4) around which the Wigner function is peaked in large squeezing limit.

Another way to see the classical limit considered above, i.e vanishing f_{k_2} and g_{k_1}, is to compare to the ratio of growing mode to that of decaying mode in f_k, where this last could be written as

$$f_k(\eta) = f_1(k) a + f_2(k) a \int_{\infty}^{\eta} \frac{\mathrm{d}\eta'}{a^2(\eta')} \ , \tag{1.128}$$

similarly g_k is written as

$$g_k(\eta) = O\left(iC_1(k) k^2 a\eta\right) + i\frac{C_2(k)}{a} \ , \tag{1.129}$$

using the expressions obtained for f_k and g_k in (1.71) then

$$f_1(k) = \frac{H_k}{\sqrt{2k^3}}, \quad f_2(k) = -i\frac{k^{3/2}}{\sqrt{2}H_k} \ , \tag{1.130}$$

the first term in (1.128) is the growing mode, also called the quasi isotropic mode, and it gives rise to constant value of inflaton perturbation $\delta\varphi$; similarly, it gives rise to the leading term of scalar, adiabatic, metric perturbations in synchronous gauge [66]. As the name reveals, the decaying mode becomes quickly suppressed and quantum coherence between growing and decaying mode gets lost, where the coherence is described by the correlation relation

$$f_{k_1} g_{k_1} + f_{k_2} g_{k_2} = \frac{1}{2} \ , \tag{1.131}$$

and the ratio ratio between the decaying mode to growing one is given by

$$\frac{f_{k_2}}{f_{k_2}} \propto \mathbf{e}^{-2|r_k|}\,, \tag{1.132}$$

form this last expression we can see that large squeezing implies the suppression of decaying mode and, consequently, suppression of coherence. This last observation gives rise to the notion of "decoherence without decoherence" [66], which refers to achieving a classical limit without considering any external environment or concrete decoherence process[34]. Therefore this notion of "decoherence without decoherence" is independent of the interaction with other fields in the theory and relies solely on the fact that the solutions of the Heisenberg equations of motion for a minimally coupled scalar field feature a growing and a decaying mode in the long time limit after the particular physical wavelength has become super-Hubble. This feature does not apply to either fermionic fields (which are never classical) nor to massless conformally coupled scalar fields (at least in absence of interactions) [15]. However, this **pragmatic view** about classicalization of primordial perturbations has many shortcomings and is only a way to understand the reason behind the observational success of the predictions based on using initially pure classical perturbations in the study of large scale structures formation, CMB anisotropies, and generally the evolution of the universe [73]. So we still in a need for concrete model to understand properly and rigorously the quantum to classical transition[35]. We will discuss the shortcomings we just mentioned in the coming section, but as quick argument against the notion of "decoherence without decoherence" is sufficient to notice in the density matrix expression (1.124) that, even in large squeezing limit, the interference terms do not vanish but oscillate rapidly, while usually the disappearance of interference terms is a considered a key element of classical limit.

Before closing this discussion, we prefer to explain better the role of decoherence without decoherence in the success of standard cosmology predictions. As we mentioned previously, the field amplitude v is the one directly related to observations, therefore we put ourselves in the $\hat{v}_{\boldsymbol{k}}$ eigenbasis $|v_{\boldsymbol{k}}\rangle$, and try to compute the expectation value of an operator $\hat{A}\left(\hat{v},\hat{p}\right)$[74]. From (1.127), we see that $|v_{\boldsymbol{k}}\rangle$ is an

[34]consisting in an almost vanishing commutator between conjugate variables (v, p), therefore they could be seen classical stochastic fields.

[35]It is worth to mention that some physicists who believe on the universality of quantum mechanics do not agree with this sentence, for example in [73] we read "most people would agree that there are no classical or quantum regimes. The fundamental description ought to be always a quantum description. However, there exist regimes in which certain quantities can be described to a sufficient accuracy by their classical counterparts represented by the corresponding expectation values. All this depends, of course, on the physical state, the underlying dynamics, the quantity of interest, and the context in which we might want to use it".

eigenbasis of both operators $(\hat{v}, \hat{p})$, therefore

$$\begin{aligned} \left\langle \hat{A}\left(\hat{v},\hat{p}\right)\right\rangle &= Tr\left(\hat{A}\left(\hat{v},\hat{p}\right)\hat{\rho}\right) \\ &= \int dv_{\boldsymbol{k}} \left\langle v_{\boldsymbol{k}} \left| \hat{A}\left(\hat{v}, \frac{g_{k_2}(\eta)}{f_{k_1}(\eta)}\hat{v}_{\boldsymbol{k}}\right)\hat{\rho}\right| v_{\boldsymbol{k}}\right\rangle \quad , \\ &= \int dv_{\boldsymbol{k}} \left\langle v_{\boldsymbol{k}} \left|\hat{\rho}\right| v_{\boldsymbol{k}}\right\rangle A\left(v_{\boldsymbol{k}}, \frac{g_{k_2}(\eta)}{f_{k_1}(\eta)} v_{\boldsymbol{k}}\right) \end{aligned} \tag{1.133}$$

so expectation values of operators can be calculated just as classical stochastic avrages with a phase space probability distribution given by $\left\langle v_{\boldsymbol{k}} \left|\hat{\rho}\right| v_{\boldsymbol{k}}\right\rangle \delta\left(v_{\boldsymbol{k}} - \frac{g_{k_2}(\eta)}{f_{k_1}(\eta)} v_{\boldsymbol{k}}\right)$, consequently, we think that a more appropriate term is "Semi-classicalisation without decoherence".

1.5 Shortcomings of the pragmatic view

The basis of the pragmatic approach to the quantum-to-classical transition discussed above, for the primordial fluctuations, lays on the approximate coincidence between quantum and classical expectation values shown in (1.133). However, as was pointed out, this is not sufficient and there must be more fundamental way to deal with such transition. Indeed, before enumerating the arguments in favor of a more rigorous way of dealing with quantum to classical transition it would be useful to quote from some literature and show that cosmologists do acknowledge that there seems to be something unclear at this point, we will present those cited in [73]

- T. Padmanabhan in [64] indicates that one must work with *certain classical objects mimicking the quantum fluctuations*, and that this is not easy to do and to justify.
- S.Weinberg in [76] states "... *the field configurations must become locked into one of an ensemble of classical configurations with ensemble averages given by quantum expectation values... It is not apparent just how this happens....*"
- S.Mukhanov acknowledges in [59] that the problem is not resolved simply by invoking decoherence: " .. *However decoherence is not enough to explain the breakdown of translational invariance..*", Actually this last argument will be discussed in the conclusion, when we will come to discuss weather decoherence or collapse models accounts better for quantum to classical transition. However, it could be inferred, at least, that the author agrees with the fact that there is an unclear part concerning standard treatment of primordial perturbations.

From those statements and others in the different literature on cosmology, we realize that indeed there is a gap, or a missing chapter in our understanding of the origin of the universe and that the pragmatic approach is unable to provide us with convincing explanation to feel the gap. But let us be more specific and ask: what are precisely the shortcomings of pragmatic approach that any quantum to classical transition model is supposed to address and clean away? To answer this question, we will enumerate the most important missing points in the pragmatic approach or "decoherence without decoherence", so those last do not explain:

1. Given a system whose initial quantum state has a given symmetry (in the situation at hand the symmetry is homogeneity and isotropy), there is no mechanism by which the standard unitary evolution governing the system, would result in a state lacking that symmetry, as long as the dynamics governing the evolution respects the symmetry [73]. How the transition from a homogeneous and isotropic state to an inhomogeneous and anisotropic state took place?

2. Starting from a highly non classical and pure state given by the entangled and squeezed state in (1.65), how we ended up with a mixed state that mimics a classical system?

3. Why the universe is localized in the field amplitude eigenbasis $\hat{v}$ rather than field momentum $\hat{p}$ one?

4. Why we have specific CMB map rather than another? In other words, we have seen that CMB anisotropies are related to quantum field amplitude $\hat{v}$ which is supposed to be in superposition of states, or values, so how a single value is obtained rather than another one?

5. Finally, we ask an interesting question related to the other quantum fields coexisting with inflaton, for example, the fields of standard model of particles that dominate post inflation era. If some modes of quantum inflaton field become equivalent to their classical counterpart at the end of inflation, then, how about the other quantum fields, do they have a classical part also? we will not tackle this question in the current thesis, but a more detailed discussion of it could be found in [73].

Even though all the points listed above are related, but the points 2), 3), 4) boil down to the quantum measurement problem on which we will spend few words in a moment. But before doing so, the point 1) is worth to receive a special attention, so following closely [22, 54] we will consider a simple example where point 1) is involved.

We saw that the temperature anisotropies $\frac{\delta T}{T}(\mathbf{e})$ probed by different satellites, COBE, WMAP, and Planck, correspond to operators[36] $\frac{\hat{\delta T}}{T}(\mathbf{e})$ that are observable quantities. According to basics of quantum mechanics, the CMB map must be an eigenstate $|Planck\rangle$ of the observables $\frac{\hat{\delta T}}{T}(\mathbf{e})$ in the following way

$$\frac{\hat{\delta T}}{T}(\mathbf{e})\,|Planck\rangle_{(\mathbf{e})} = \frac{\delta T}{T}(\mathbf{e})\,|Planck\rangle_{(\mathbf{e})}\ , \tag{1.134}$$

[36] One operator for each direction $(\mathbf{e})$in the sky.

but the squeezed state (1.65) could not be the eigenstate $|Planck\rangle_{(\mathbf{e})}$we are looking for. This last statement could be proven either by direct computation or by just physical and intuitive argument based on symmetry considerations, and it is this last method which will highlight the point 1) in the list of shortcomings.

For the sake of simplifying the argument, we use the fact that curvature perturbations could be seen as a massless scalar field on top of FLRW background with an action

$$S = -\frac{1}{2}\int \mathrm{d}^4\boldsymbol{x}\sqrt{-g}g^{\mu\nu}\partial_\mu\zeta\partial_\nu\zeta\,, \tag{1.135}$$

and we define the four momentum operator by

$$\hat{P}_\mu = -\int \mathrm{d}^3\boldsymbol{x}\sqrt{{}^{(3)}g}\hat{T}^{0}_{\ \mu}\,, \tag{1.136}$$

where $\hat{T}_{\mu\nu}$ is the quantized stress energy tensor that could be calculated from the action given above using[37]

$$\hat{T}_{\mu\nu} = \partial_\mu\hat{\xi}\partial_\nu\hat{\xi} - g_{\mu\nu}g^{\alpha\beta}\partial_\alpha\hat{\zeta}\partial_\beta\hat{\zeta}\,, \tag{1.137}$$

and ${}^{(3)}g$ in is the determinant of the spatial part of the metric. In cosmic time, the operator $\hat{P}_0$ corresponds to the generator of time evolution of the system i.e the the Hamiltonian, and the operator

$$\hat{P}_i = a\int \mathrm{d}^3\boldsymbol{x}\dot{\hat{\zeta}}\partial_i\hat{\zeta}\,, \tag{1.138}$$

corresponds to spatial translations along x_i, using (1.30) and (1.40) we can express the $\hat{P}_i$ in terms of creation and annihilation operators as

$$\hat{P}_i \propto \int \mathrm{d}^3\boldsymbol{k}k_i\hat{a}^\dagger_{\boldsymbol{k}}\hat{a}_{\boldsymbol{k}}\,, \tag{1.139}$$

and by acting on initial vacuum state we obviously have

$$\hat{P}_i\,|0\rangle_{in} = 0\,, \tag{1.140}$$

and same result would be obtained for the rotation generator i.e angular momentum $\hat{L}_i$

$$\hat{L}_i\,|0\rangle_{in} = 0\,, \tag{1.141}$$

those last two results express the fact that vacuum state is homogeneous and isotropic, which reflects the symmetries of FLRW background. Having shown that, we want now to investigate whether the

[37]Notice that we are following semi classical approach here, where we quantized the matter sector as could be inferred from $\hat{T}_{\mu\nu}$, while the geometric part $g_{\mu\nu}$is still classic i.e unquantized.

unitary time evolution through Schrodinger equation would break those symmetries and allow for CMB anisotropies to show up. As stated above, the the time evolution is generated by the Hamiltonian (1.45) so using the explicit expression of the two pieces, $\hat{H}_{free}$ and $\hat{H}_{int}$, in terms of creation and annihilation operators it could be shown that

$$\left[\hat{P}_i, \hat{H}_{free}\right] = 0, \ \left[\hat{P}_i, \hat{H}_{int}\right] = 0\,, \tag{1.142}$$

similarly

$$\left[\hat{L}_i, \hat{H}_{free}\right] = 0, \ \left[\hat{L}_i, \hat{H}_{int}\right] = 0\,, \tag{1.143}$$

therefore the unitary time evolved state by Schrodinger equation, through $\hat{H} \propto \hat{H}_{free} + \hat{H}_{int}$, is still an eigenstate of $\hat{P}_i$ and $\hat{L}_i$ i.e

$$\hat{P}_i \left|\Psi_{2sq}\right\rangle = 0\,, \tag{1.144}$$

$$\hat{L}_i \left|\Psi_{2sq}\right\rangle = 0\,, \tag{1.145}$$

therefore the squeezed state is still describing an homogeneous and isotropic state and so it could not be $\left|Planck\right\rangle_{(\mathbf{e})}$ because this last is characterized by

$$\hat{P}_i \left|Planck\right\rangle_{(\mathbf{e})} \neq 0\,, \tag{1.146}$$

and

$$\hat{L}_i \left|Planck\right\rangle_{(\mathbf{e})} \neq 0\,, \tag{1.147}$$

therefore a serious question arises is how the transition from a homogeneous and isotropic state to an inhomogeneous and anisotropic state took place?

Actually it could be that $\left|\Psi_{2sq}\right\rangle$ is superposition of inhomogeneous and anisotropic states, but the superposition makes the state $\left|\Psi_{2sq}\right\rangle$ homogeneous and isotropic. Then, at some point the superposition breaks and single outcome is picked out, and here comes the discussion of points 2), 3), 4) which ,as mentioned above, refer to the well known quantum measurement problem[38]. However, in the cosmological context this problem is more subtle and gets exacerbated compared to laboratory [73] for the following reason:

Based on quantum mechanics postulates a measurement takes place upon the collapse of the system wavefunction into an eigenstate of the observable measured. Let us project the content of this postulate on our case and specify the ingredients building up the measurement problem: The system for us is the primordial quantum fluctuations, the operator (or observable) to be measured is the CMB anisotropies $\frac{\hat{\delta T}}{T}(\mathbf{e})$, the eigenvalue and the eigenstate we get after measurement is the values $\frac{\delta T}{T}(\mathbf{e})$ and the CMB map $\left|Planck\right\rangle_{(\mathbf{e})}$, respectively. Having said that and bearing in mind that CMB was emitted at recombination era, when the fluctuations started to grow and give rise to the structures

[38] Is also called "macro-objectivation" problem.

we are observing nowadays, including ourselves!; we conclude that there was not an observer at the time of emission of CMB, nor before, therefore how could we fit the postulate mentioned above to CMB case. More precisely, though no external observer was there to induce the collapse, how did the wavefunction collapsed into the CMB map eigenvalue and eigenstate? ; who or what decided the basis on which the collapse is supposed to take place ? and how a specific outcome of measurement arises rather than another one?

To answer those questions we can either [54]

- Look for different interpretation of quantum mechanics[39] and leave the formalism unmodified.
- Add new degrees of freedom to the mathematical formalism which is done usually to rend quantum theory deterministic, as example de Broglie-Bohm theory.
- Leave quantum theory unmodified and consider that our system is a part of bigger one. We consider an appropriate interaction between the two and study its effects on the system, decoherence falls within this option.
- Consider quantum mechanics as an approximate theory of a more universal theory that is valid at all scales.

The last two options are the closest to experimental tests and so are falsifiable, because they lead to different predictions from that of conventional quantum mechanics. In this thesis we will see how the application of collapse models and decoherence in the inflationary context lead to interesting predictions that differ from the ones obtained from standard quantum mechanics.

[39] For example: Copenhagen, many worlds, ...etc.

Chapter 2

Signatures of a quantum universe

In this chapter we discuss briefly various possible probes of the origin of the universe that had been suggested so far. We will focus on the cosmological Bell inequalities implementation and the obstacles facing those attempts, and whether there are other alternatives to probe the origin of universe? To answer this last question in the affirmative, we summarize the results obtained in a recent work about using non Gaussianities to confirm whether the universe originated quantumly or classically.

2.1 Bell inequalities in quantum mechanics

Bell inequalities were devised by J.Bell as a mean of testing whether or not particles connected through quantum entanglement communicate information faster than the speed of light. Where, the violation of those inequalities confirm that no theory of local hidden variables can account for all of the predictions of quantum mechanics. Before presenting the models that had been suggested so far to implement those inequalities in cosmology, we prefer to show how they are performed in laboratory and the needed ingredients. Our aim, later, is to find for each laboratory experiment ingredient a counterpart in the cosmological context.

The elements composing Bell experiment are [46]:

- Two separate spatial places, causally disconnected, where measurements could be performed, we call them Alice's and Bob's location.
- An entangled quantum st state with components at these two locations.
- At each location we should be able to perform two possible measurements that are described by two non-commuting operators. Call them $\hat{A}$ and $\hat{A}'$ for Alice's location and $\hat{B}$, $\hat{B}'$ for Bob's location, with $\left[\hat{A}, \hat{A}'\right] \neq 0$ and $\left[\hat{B}, \hat{B}'\right] \neq 0$.

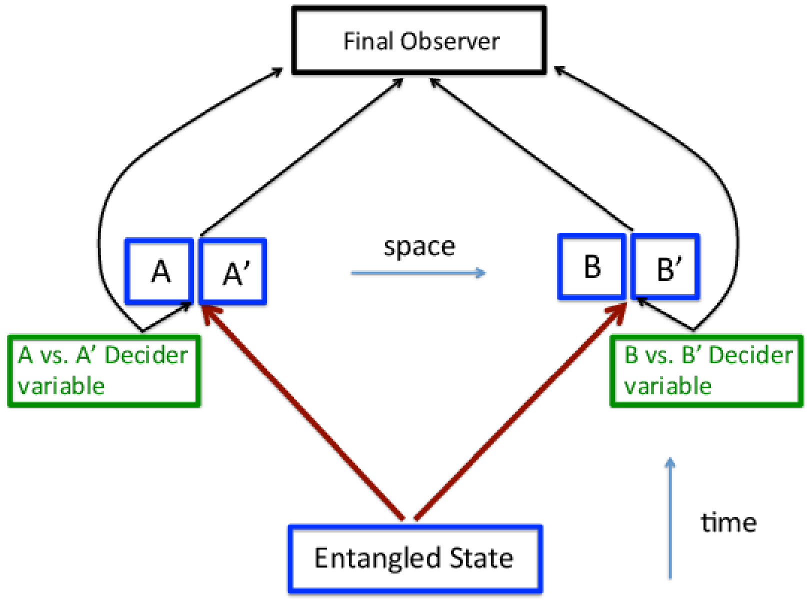

Figure 2.1: A generic Bell experiment setup, the black lines refer to classical channels of information transmission.[46]

- Alice should have the "free will" to select randomly between the $\hat{A}$ and $\hat{A}'$. The same holds for Bob for his choice of $\hat{B}$, or $\hat{B}'$. These choices are made locally and are uncorrelated with each other. These choices are made by physics outside the quantum system under consideration. In practice this is done by looking at local random variables that are assumed to be independent of the quantum system in question.
- We should have a quantum measurement of these operators with definite answers.
- We classically transmit the results of these measurements to a central location where we correlate the results.

The simplest example to see all those elements enter the experimental protocol is to consider a bipartite system under Clauser, Horne, Shimony and Holt (CHSH) setup, where the bipartite systems Hilbert space is written as $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$ [51, 24]. The entangled state corresponds to a pair of spins belonging to the particles A and B whose spin along z direction are correlated, as seen in figure (2.2), and the state of the system is assumed to be

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \left(|+-\rangle - |-+\rangle \right) , \tag{2.1}$$

where $|\pm\rangle$ are eigenstates of $\hat{S}_z$with eigenvalues ± 1. The particle A is sent to Alice location and

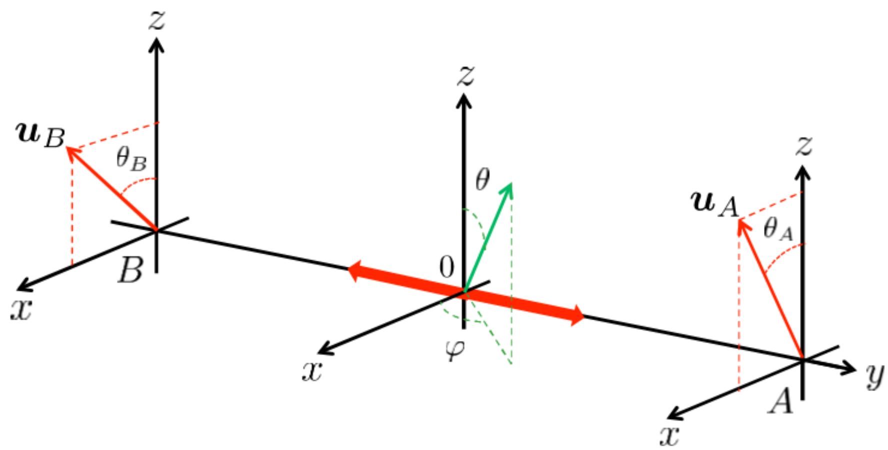

Figure 2.2: Standard set up of Bell experiment using a spin singlet.[51]

similarly B to Bob location, such that Alice and Bob will act on the state by one of a two non commuting operators that correspond to different directions i.e Alice has $\hat{A} = \overrightarrow{n}_A.\overrightarrow{S}_A$ and $\hat{A}' = \overrightarrow{n}'_A.\overrightarrow{S}_A$ such that $\left[\hat{A}, \hat{A}'\right] \neq 0$, similarly for Bob $\left[\hat{B}, \hat{B}'\right] \neq 0$. Alice and Bob's choice of the either of the operators should be independent from each other. The direction $\overrightarrow{n}$ is given by $\overrightarrow{n} = (\sin\theta\cos\varphi, \sin\theta\sin\theta\varphi, \cos\theta)$, so the eigenstates of $\overrightarrow{n}.\overrightarrow{S}$ with eigenvalues $+$ and $-$ are, respectively,

$$\begin{cases} |+_{\overrightarrow{n}}\rangle = \cos(\theta/2)\,\mathrm{e}^{-\mathrm{i}\varphi/2}\,|+\rangle + \sin(\theta/2)\,\mathrm{e}^{i\varphi/2}\,|-\rangle \\ \\ |+_{\overrightarrow{n}}\rangle = \cos(\theta/2)\,\mathrm{e}^{-\mathrm{i}\varphi/2}\,|+\rangle + \sin(\theta/2)\,\mathrm{e}^{i\varphi/2}\,|-\rangle \end{cases} . \tag{2.2}$$

We introduce Bell operator [25]

$$\left\langle \hat{\mathcal{B}}_{CHSH}\left(\hat{A}, \hat{B}\right)\right\rangle = \left\langle \hat{A}\hat{B}\right\rangle + \left\langle \hat{A}'\hat{B}\right\rangle + \left\langle \hat{A}\hat{B}'\right\rangle - \left\langle \hat{A}'\hat{B}'\right\rangle, \tag{2.3}$$

that is a particular linear combination of expectation values ± 1.

We want now to show how the Bell operator could distinguish between quantum mechanics and a local hidden variable theory. According to this last, for each value of the hidden variable we have well defined answer for each of the two possible measurements at each location, and since there is no possibility for a causal contact between them, then measuring A or A' does not influence the measurement of B and B'. Therefore, we can have for each value of hidden variable $B = B'$ and $B = -B'$ so either the first two terms cancel (2.3), or the last two ones, which set an upper bound

$$\left|\left\langle \hat{\mathcal{B}}_{CHSH}\left(\hat{A},\hat{B}\right)\right\rangle\right| \leq 2 \tag{2.4}$$

We come now to quantum mechanics prediction, we will see that actually the upper bound on $\left|\left\langle \hat{\mathcal{B}}_{CHSH}\left(\hat{A},\hat{B}\right)\right\rangle\right|$ is bigger. The starting point is the fact that $\hat{A}^2 = \hat{A}^{2\prime} = 1\hat{I}$, then, $\hat{\mathcal{B}}_{CHSH}\left(\hat{A},\hat{B}\right)$ could be written us

$$\hat{\mathcal{B}}^2_{CHSH} = 4 - \left[\hat{A},\hat{A}'\right]\left[\hat{B},\hat{B}'\right]\,, \tag{2.5}$$

we see that the product of commutators could make the bell operator bigger than 2, from which we understand why any violation of (2.4) would rule out any local hidden variable theory. Since $\left|\left[\hat{A},\hat{A}'\right]\right| \leq 2$ where the maximum could be reached for example for $\hat{A} = \hat{\sigma}_x$ and $\hat{A}' = \hat{\sigma}_y$, then we can see that in quantum mechanics we have

$$\left|\left\langle \hat{\mathcal{B}}_{CHSH}\left(\hat{A},\hat{B}\right)\right\rangle\right| \leq 2\sqrt{2}\,, \tag{2.6}$$

it is worth to mention that any violation this last inequality, i.e $2\sqrt{2} < \left|\left\langle \hat{\mathcal{B}}_{CHSH}\left(\hat{A},\hat{B}\right)\right\rangle\right|$, would single out another theory to take over standard quantum mechanics!

Going back to our two particles example and choosing $\theta_A - \theta_B = \frac{\pi}{4}$, $\theta_A - \theta'_B = \theta'_A - \theta_B = -\frac{\pi}{4}$ and $\theta'_A - \theta'_B = -3\pi/4$ leads to $\left\langle \hat{\mathcal{B}}_{CHSH}\left(\hat{A},\hat{B}\right)\right\rangle = -2\sqrt{2}$, so quantum mechanics win. It is well known that experiments proved the violation of Bell inequalities which rules out any possible local hidden variable theory.

2.2 Bell inequalities tests on CMB

We have seen in the previous section how Bell experiments were decisive in singling out quantum theory to describe nature, at least up to some approximation. Therefore, as we are facing the puzzle of origin of universe and weather it is classical or quantum, we ask ourselves if there is any possibility to perform a Bell experiment, but this time at a cosmological scale. The role of such experiment is to reveal us the quantum, or classical, nature of origin of universe.

The first attempt to perform a Bell experiment on the sky is to do as follow, see figure (2.3),

- Divide the celestial sphere into causally disconnected regions, at least back to recombination, and each region gets also divided into two disconnected sub-regions to play the role of Alice and Bob locations.
- The two non commuting observables to be measured could be the the field amplitude and momentum conjugate variables $(\hat{v},\hat{p})$.

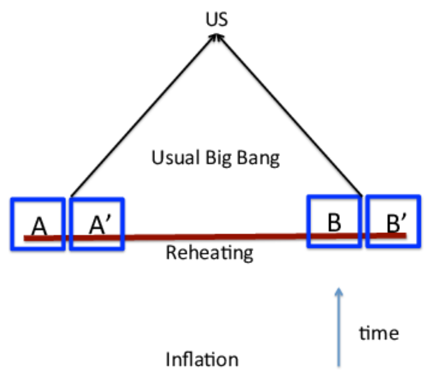

Figure 2.3: a) An unsuccessful set up for a cosmological Bell inequality. [46]

But, unfortunately, such set up is unlikely to work due to the fact that the field momentum $\hat{p}$ is proportional to the decaying mode which makes the commutator extremely suppressed, $k^3\left[\hat{v},\hat{p}\right] \propto a^{-3} \approx \mathrm{e}^{-3\mathrm{N_k}}$ where N_k is the number of e folds spent outside horizon for the mode k. Therefore, for scales of observational interest that has $N_k \approx 30-40$ we need a precision greater than 10^{-90} which, even for a theorist, looks impossible [46]. Therefore, we cannot rely on $(\hat{v},\hat{p})$ to a perform a Bell experiment and we need more creativity!

The analysis just made was quite qualitative, so it would be advantageous to show, in a concrete way, how does decaying mode prevent us from performing a Bell experiment on CMB. A first step toward answering this question is to build out of the operators $(\hat{v},\hat{p})$, that have continuous spectrum, a set of dichotomic operators, which have discontinuous spectrum. Those last will mimic the spin operators seen in previous section, therefore we will call them pseudo spin operators.

There are various ways of getting the pseudo spin operators out of $(\hat{v},\hat{p})$, we will adopt the Gour-Khanna-Mann-Revezen (GKMR) spin operators that are defined as function of

$$\begin{array}{rcl} |\mathcal{E}_{\boldsymbol{k}}\rangle & = & \frac{1}{\sqrt{2}}\left(|q_{\boldsymbol{k}}\rangle + |-q_{\boldsymbol{k}}\rangle\right) \\ |\mathcal{O}_{\boldsymbol{k}}\rangle & = & \frac{1}{\sqrt{2}}\left(|q_{\boldsymbol{k}}\rangle - |-q_{\boldsymbol{k}}\rangle\right) \end{array}, \tag{2.7}$$

where the $|q_{\boldsymbol{k}}\rangle$ is the eigenstate of the operator $\hat{q}_{\boldsymbol{k}}$defined in (1.42) which plays the role of position in the subspace $\mathcal{E}_{\boldsymbol{k}}$. So using (2.7) we obtain the pseudo spin operators which satisfy the $SU\left(2\right)$ algebra,

$[\hat{s}_m, \hat{s}_n] = 2i\epsilon_{mnl}\hat{s}_l$,

$$\begin{aligned} \hat{s}_x &= \int \mathrm{d}q_{\boldsymbol{k}} \left(|\mathcal{E}_{\boldsymbol{k}}\rangle \langle \mathcal{O}_{\boldsymbol{k}}| + |\mathcal{O}_{\boldsymbol{k}}\rangle \langle \mathcal{E}_{\boldsymbol{k}}| \right) \\ \hat{s}_y &= \int \mathrm{d}q_{\boldsymbol{k}} \left(|\mathcal{O}_{\boldsymbol{k}}\rangle \langle \mathcal{E}_{\boldsymbol{k}}| - |\mathcal{E}_{\boldsymbol{k}}\rangle \langle \mathcal{O}_{\boldsymbol{k}}| \right) \quad , \\ \hat{s}_z &= \int \mathrm{d}q_{\boldsymbol{k}} \left(|\mathcal{E}_{\boldsymbol{k}}\rangle \langle \mathcal{E}_{\boldsymbol{k}}| - |\mathcal{O}_{\boldsymbol{k}}\rangle \langle \mathcal{O}_{\boldsymbol{k}}| \right) \end{aligned} \tag{2.8}$$

the other definitions of the pseudo spin operators could be found in [49, 50, 51]. Having found our pseudo spin operators, the next step is to define the Bell operator as follow

$$\begin{aligned} \hat{\mathcal{B}}_{GKMR}(\boldsymbol{k}, -\boldsymbol{k}) &= \hat{A}\hat{B} + \hat{A}'\hat{B} + \hat{A}\hat{B}' - \hat{A}'\hat{B}' \\ &= \vec{n}.\hat{s}(\boldsymbol{k}) \otimes \vec{m}.\hat{s}(-\boldsymbol{k}) + \vec{n}.\hat{s}(\boldsymbol{k}) \otimes \vec{m}'.\hat{s}(-\boldsymbol{k}) \quad . \\ &+\vec{n}'.\hat{s}(\boldsymbol{k}) \otimes \vec{m}.\hat{s}(-\boldsymbol{k}) - \vec{n}'.\hat{s}(\boldsymbol{k}) \otimes \vec{m}'.\hat{s}(-\boldsymbol{k}) \end{aligned} \tag{2.9}$$

Now, the difference between our case and the standard Bell experiment discussed in previous section, is that in cosmological case we compute the mean value $\left\langle \hat{\mathcal{B}}_{GKMR} \right\rangle$ of Bell operator with respect to the two mode squeezed state (1.65)

$$\left\langle \hat{\mathcal{B}}_{GKMR} \right\rangle = \langle \Psi_{2sq}| \hat{\mathcal{B}}_{GKMR} |\Psi_{2sq}\rangle \ . \tag{2.10}$$

For the sake of simplifying computations, we assume that all azimuthal angles vanish so that $\vec{n}.\hat{s} = \sin\theta_n \hat{s}_x + \cos\theta_n \hat{s}_z$, and all mean values of cross terms, as $\langle \hat{s}_x(\boldsymbol{k})\, \hat{s}_z(-\boldsymbol{k})\rangle$, vanish. In addition, choosing $\theta_n = 0$, $\theta_{n'} = \pi/2$, and $\theta_{m'} = -\theta_m$ with the optimal choice of $\theta_m$which maximizes the violation Bell inequalities one on obtains [51]

$$\begin{aligned} \left\langle \hat{\mathcal{B}}_{GKMR} \right\rangle &= 2\sqrt{\langle \Psi_{2sq}| \hat{s}_z(\boldsymbol{k})\, \hat{s}_z(-\boldsymbol{k}) |\Psi_{2sq}\rangle^2 + \langle \Psi_{2sq}| \hat{s}_x(\boldsymbol{k})\, \hat{s}_x(-\boldsymbol{k}) |\Psi_{2sq}\rangle^2} \\ &= 2\sqrt{1 + \langle \Psi_{2sq}| \hat{s}_x(\boldsymbol{k})\, \hat{s}_x(-\boldsymbol{k}) |\Psi_{2sq}\rangle^2} \end{aligned} \quad , \tag{2.11}$$

in the large squeezing limit, $r_k \to \infty$ and $\phi_k \to -\frac{\pi}{2}$, we get the Cirel'son bound saturation i.e

$$\left\langle \hat{\mathcal{B}}_{GKMR} \right\rangle = 2\sqrt{2}\,. \tag{2.12}$$

It could be shown that the same result is obtained for other definitions of pseudo spin operators, but the question now is the following: having shown, theoretically, thst we do have Bell violation, then, could we measure such a violation? or the previous qualitative analysis regarding the decaying

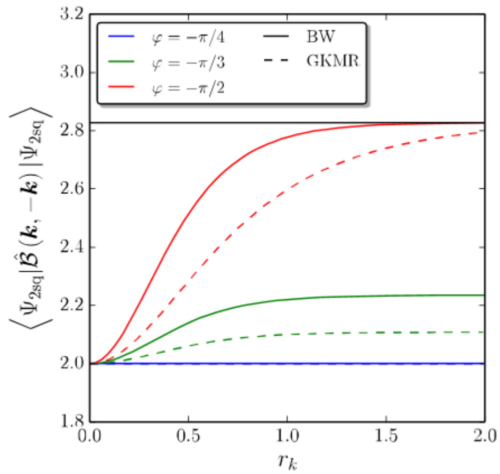

Figure 2.4: Mean value of Bell operator as function of squeezing parameters (the GKMR operator is with dashed lines).[51]

mode will show up again to obstruct us from achieving such goal?

Since curvature perturbations are directly related the temperature anisotropies through Sachs-Wolf effect, our starting point is reexpress the operator $\hat{q}_{\boldsymbol{k}}$ as function of curvature perturbation operator and its conjugate momentum, so using (1.43) and (1.30) we get

$$\hat{q}_{\boldsymbol{k}} = \frac{z}{2}\left(\hat{\zeta}_{\boldsymbol{k}} + \hat{\zeta}_{-\boldsymbol{k}}\right) + \frac{z}{2k}\left(\hat{\zeta}'_{\boldsymbol{k}} - \hat{\zeta}'_{-\boldsymbol{k}}\right) . \tag{2.13}$$

From this last equation we notice that the knowledge of $\hat{q}_{\boldsymbol{k}}$ requires the knowledge of momentum conjugate $\hat{\zeta}'_{\boldsymbol{k}}$ proportional to decaying mode. However, if we neglect the decaying mode, since it is extremely suppressed, then we have

$$\hat{q}_{\boldsymbol{k}} = \hat{q}_{-\boldsymbol{k}} \Rightarrow \hat{s}_x\left(\boldsymbol{k}\right) = \hat{s}_x\left(-\boldsymbol{k}\right) \Rightarrow \left\langle \Psi_{2sq} \right| \hat{s}_x\left(\boldsymbol{k}\right)\hat{s}_x\left(-\boldsymbol{k}\right)\left| \Psi_{2sq} \right\rangle = 1 \, , \tag{2.14}$$

therefore we sill have a maximal violation of Bell inequalities, i.e

$$\left\langle \hat{\mathcal{B}}_{GKMR} \right\rangle = 2\sqrt{2} \, , \tag{2.15}$$

which is quite remarkable result. Because in our previous qualitative analysis we argued that the suppressed decaying mode is an obstacle toward Bell experiment on CMB, however with our current quantitative analysis we realized that even if we neglect the decaying mode, we still have a maximal violation of Bell inequalities. Actually, the real obstacle comes from the fact that with the eigenvalues of a single operator, namely $\hat{q}_{\boldsymbol{k}} \propto \hat{\zeta}_{\boldsymbol{k}} \propto \left(\frac{\delta \hat{T}}{T}\right)(\mathbf{e})$ we have to infer the values of, at least, two of the pseudo spin operators introduced in (2.8), and this is possible if and only if we have at least two pseudo spin operators commute with $\hat{q}_{\boldsymbol{k}}$. Unfortunately, this is not the case neither for GKMR operators nor for the other possible definitions, where in our case we have only $\hat{s}_x$ that commutes with $\hat{q}_{\boldsymbol{k}}$

$$\langle q_{\boldsymbol{k}}| \left[\hat{s}_x(\boldsymbol{k}), \hat{q}_{\boldsymbol{k}}\right] |q'_{\boldsymbol{k}}\rangle = 0 , \tag{2.16}$$

while $\hat{s}_y$ and $\hat{s}_z$ do not commute. Therefore, we have shown with a concert example that it is impossible to use CMB as an arena of a Bell experiment by extending the standard Bell experiment set up. But there still a possibility to implement a CMB experiment on CMB if we design a method which makes use of only one pseudo-spin component, indeed, using Leggett-Garg inequality based on measuring the same spin component at different times we can hope for a CMB bell experiment, since it has been show with Leggett-Garg proposal, that for squeezed states we do have a violation of Bell inequality.

2.3 Cosmological baroque model of bell inequalities

While we are waiting to find a way of implementing a Bell experiment on CMB, we can look for other alternatives and hope nature to be on our side. We ask ourselves: what if the universe already carried out a Bell experiment during its evolution, specifically before the end of inflation, and the outcomes of the experiment are right there waiting us to collect them. Let us follow this ambitious idea and see weather there is a possibility for it.

We will work with a single field, and consider that its quantum fluctuations are the source of entangles states subject to Bell experiment. The measurement should be some process which depends on the quantum state of one of the pieces of the entangled sate. We split our fluctuations into long wavelengths which are of observations interest to us, and the short wavelengths which will act, both, as the decider of measuring $\hat{A}$ ($\hat{B}$) or $\hat{A}'(\hat{B}')$ for Alice (Bob), and as the measuring apparatus. The choice of short wavelengths to act as the free decider assures that choice is made locally and independently between Alice and Bob locations. The outcomes of measurement will be left as imprints on the classical fluctuations background that would be transmitted to us. However the set up we just described suffer from a sever problem, which is the fact that the measuring apparatus is "more" quantum than the measured system. Where we have seen that there is a strong argument in favor of the classicalization of primordial fluctuations as they exit horizon, therefore the short wavelengths could not act as classical measuring apparatus.

To come out of this conundrum, J.Maldecena proposed a baroque model of inflation to show that an inflation Bell friendly model is possible, at least conceptually, though nature is unlikely to choose such multifield model. In What follows we will present the main points of Maldecena model, and more details could be found in [46].

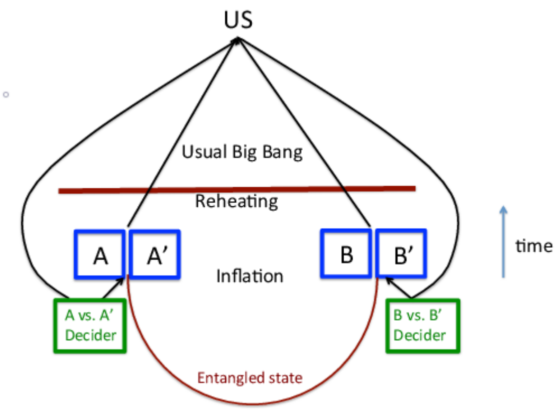

Figure 2.5: A diagram of a more successful set up where the whole process occurs during inflation.[46]

As mentioned previously, the model is based on three scalar fields, 1) the inflaton φ, 2) a massive scalar field ψ, 3) an axion field θ, the last two are very crucial for the Bell experiment set up. In order to make the presentation relatively simple we will list the Bell experiment ingredients presented previously and discuss their counterparts in Maldecena model .

- The quantum system: The massive scalar field is supposed to create pairs of entangled scalar particles that have inflaton dependent mass, $m(\varphi)$. Their mass is very heavy, $m(\varphi) \gg H$, except for a value $\varphi_0$where they become relatively light $m(\varphi_0) \sim H$ and they get created. The massive particles should not be created with very large numbers, but only with enough abundance to be observed leave distinguishable signatures for post inflation observers. We also require the distance between the elements of each pair to be large enough to allow for a local measurement that is independent of that of the other particle. The particles have an isospin degree of freedom and are created in spin singlet, where the masses of particles have an isospin dependent contribution. It is exactly this last point that enables the measurement to be observable for post inflation observers. where different masses will result in hot spots and hotter spots.

- The free decider : We assume an axion field θ that has an inflaton dependent decay constant $f_a(\varphi) \to f_a(\eta)$
$$S = \int \mathrm{d}^4\boldsymbol{x}\frac{f_a^2(\eta)}{H^2}\frac{\left[(\partial_\eta\theta)^2 - (\partial_i\theta)^2\right]}{\eta^2}, \tag{2.17}$$
where this last is very large $f_a > H$ except around $\varphi \simeq \varphi_1$where it becomes relatively small $f_a(\varphi_1) > H$ which creates fluctuations in the axion field at distances $x \sim |\eta_1|$. as result of this fluctuations it is important that θ picks different values values around each member of the scalar particles in order to act as free decider, see figure (2.6); needless, to say that we need to have $\varphi_0 \lesssim \varphi_1$. The axion fluctuations induce isocurvature fluctuations in the dark matter

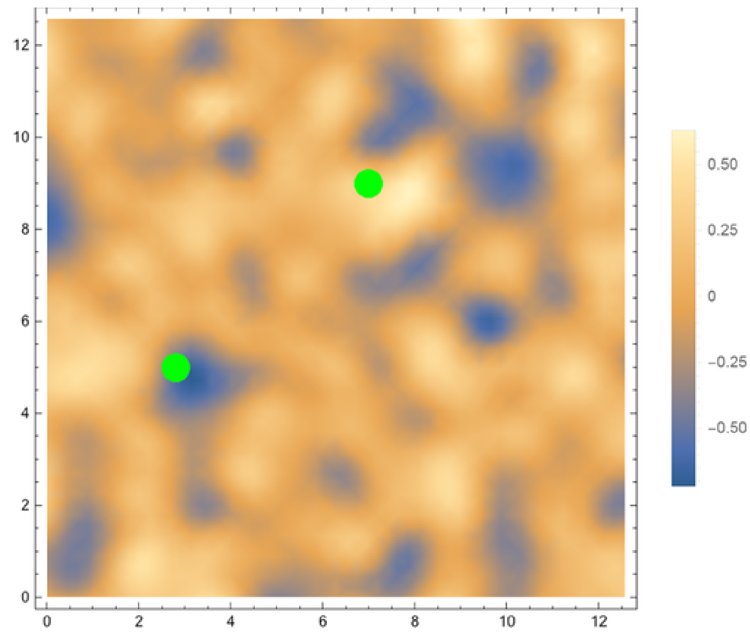

Figure 2.6: The axion field profile, where we see that each member of the pair sees different value of the field.[46]

component, however they are subdominant, so by measuring their size we can determine the initial amplitude of θ in the different regions of the sky, therefore we get the "Alice" and "Bob" choice of the measured operator.

- Measurement: The measurement is induced through the breaking of isospin symmetry, where the massive particles are coupled to inflaton and their mass has a component that depends on the isospin projection along an axis that depends on the axion field. So expressing the massive scalar field as isospin doublet $\psi = (\psi_1, \psi_2)$, then the interaction describing the measurement could be written as

$$\begin{aligned} & m_1^2(\varphi)\,\psi^\dagger\psi + \lambda_2(\varphi)\,\psi^\dagger\left(\sigma_x \cos \mathrm{n}\theta + \sigma_y \sin \mathrm{n}\theta\right)\psi \\ &= m_1^2(\varphi)\left[|\psi_1|^2 + |\psi_2|^2\right] + \left[\lambda_2(\varphi)\,\mathrm{e}^{\mathrm{i}\mathrm{n}\theta}\psi_1^*\psi_2 + c.c\right] \end{aligned}, \tag{2.18}$$

with σ being Pauli matrices. The diagonalization of previous interaction leads to

$$m_\pm = \sqrt{m_1^2(\varphi) \pm |\lambda_2(\varphi)|}, \tag{2.19}$$

at early times λ_2 must be small so that the particles pair are created in spin singlet, then at late times λ_2 increases such that $\lambda_2 \approx m_1$ and $m_\pm \sim M_{pl}$ to make the two masses observable and distinguishable.

- Late time observation: As stated before, the primordial axion fluctuations that survive to the post inflation era and which represent isocurvature fluctuations will inform us about the free "decider choice" made in Alice and Bob location. Then, the outcome of measurement will show up as very hot, corresponding to m_- and super hot spots for m_+, superposed on top of a Gaussian distributed scalar fluctuations. In other words, the outcomes of Bell measurement will be stored in the late time, effectively, classical probability distribution. It is worth to mention that the coupling of the massive particles to inflaton induce modification of the fluctuations of inflaton, where the pairs delay locally the end of inflation which by turn results in a modified power spectrum of curvature fluctuations with [46]

$$\langle \zeta_{part}\left(x\right) \rangle = \frac{m\left(\eta = -\left|x\right|\right)}{2\sqrt{2\epsilon}M_{pl}} \times \left(\frac{1}{2\pi\sqrt{2\epsilon}}\frac{H}{\varphi}\right), \tag{2.20}$$

where the factor $\frac{m(\eta=-|x|)}{2\sqrt{2\epsilon}M_{pl}}$ captures the modification induced by presence of massive particles. In order to make this modifications visible for post inflation observers, it must stand out among quantum fluctuations, therefore we require

$$\frac{m\left(\eta = -\left|x\right|\right)}{2\sqrt{2\epsilon}M_{pl}} > 1\,, \tag{2.21}$$

so for $m \sim M_{pl}$ we need $\epsilon \approx 10^{-3}$, this analysis is summarized in figure (2.7).

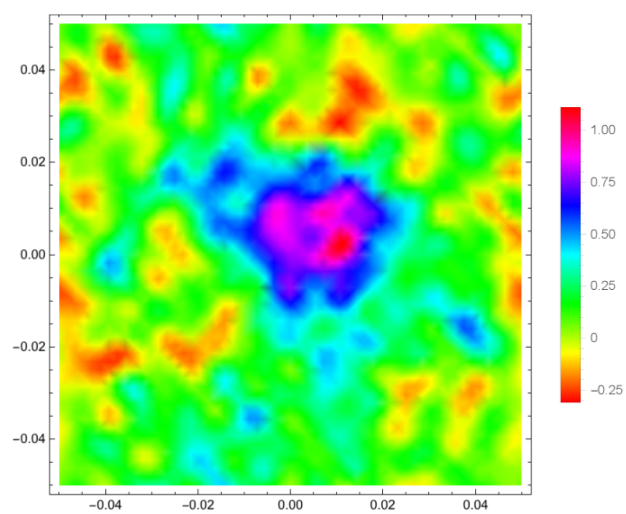

Figure 2.7: We see hot spots induced by massive particles superposed on top of the quantum fluctuations .[46]

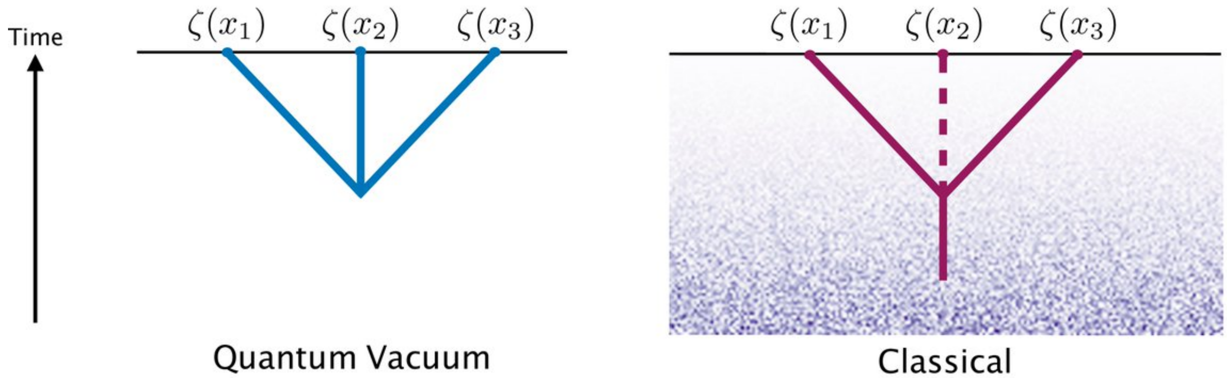

Figure 2.8: Left: quantum fluctuation resulting in creation of three correlated particles from vacuum, with no physical poles. Right: Classical fluctuations of a state containing physical particles, leading to net creation of particles with possibility of decay (dashed line) that gives rise to physical poles.[33]

As stated at the beginning of this section, the model just discussed serves just to show that there is a possibility for a Bell experiment to be arranged spontaneously by the universe itself during its evolution, and that the outcomes of it are right there waiting us to collect them with our instruments. There could be a closer Bell experiment model to the currently successful slow roll single field inflation but is waiting us to find its ingredients and implement it!

2.4 quantum signatures in non Gaussianities

Fortunately, Bell experiments are not the only tool to probe the origin of the universe, but there are other tools among which we find higher order correlation functions of the density fluctuations, namely non Guassianities . A recent work showed that three point functions could carry the signature of a quantum or classical universe, and here we will summarize the arguments presented there[33].

The main idea is that the origin of fluctuations, the non local quantum or the local classical theory, gives rise to different analytical structures for the shapes of non Guassianities, specifically the type of poles in the correlator expression[1]. This difference is due to the fact that in the case of a quantum origin, the long range correlations arise from vacuum fluctuations resulting through their local non linear evolution into creation of virtual particles, without possibility of decay, since we are talking about vacuum; see figure (2.8) . Therefore, the absence of decay processes lead to the absence of physical momenta as poles in non Gaussianities, because of possibility for tiny violation of energy

[1]We are considering a Gaussian initial conditions, and that non Gaussianities arise from non local evolution of perturbations.

conservation in an expanding universe $\triangle t \sim H^{-1}$. As case in point, we adopt a cubic interaction

$$H_{\mathrm{int}} = -\frac{\lambda}{3}\dot{\zeta}^3\,, \tag{2.22}$$

Using in-in formalism we get for bispectrum

$$\begin{aligned} \langle \zeta_{\boldsymbol{k}_1}\zeta_{\boldsymbol{k}_2}\zeta_{\boldsymbol{k}_3}\rangle'_q &= i\int \mathrm{d}\tau' \left\langle \left[H_{\mathrm{int}}\left(\tau'\right), \zeta_{\boldsymbol{k}_1}\zeta_{\boldsymbol{k}_2}\zeta_{\boldsymbol{k}_3}\left(0\right)\right]\right\rangle \\ &= \frac{4\lambda H^{-1}\triangle_\zeta^6}{(k_1+k_2+k_3)^3 k_1k_2k_3} \end{aligned}\,, \tag{2.23}$$

where by the primed $\langle\ldots\rangle'$ we refer to a bispectrum up to a momentum conserving δ function; notice that the pole is given by total momentum $k_t = k_1+k_2+k_3$. No physical momenta poles because there are no real particles to scatter off in the vacuum, but only a creation of the three virtual particles giving rise to late time observed long correlations.

When it comes to initial classical fluctuations, then, we are talking about real particles which could decay and annihilate through various on shell process, as $k_1 \to k_2 + k_3$, which gives rise to poles in Folded limit, indeed, the bispectrum is given by

$$\langle \zeta_{\boldsymbol{k}_1}\zeta_{\boldsymbol{k}_2}\zeta_{\boldsymbol{k}_3}\rangle'_c = \frac{\lambda H^{-1}\triangle_\zeta^6}{3k_1k_2k_3}\left[\frac{3}{k_t^3} + \frac{1}{(k_1+k_2-k_3)^3} + \frac{1}{(k_1-k_2+k_3)^3} + \frac{1}{(k_2-k_1+k_3)^3}\right]\,, \tag{2.24}$$

So comparing (2.8) and (2.8), we conclude that quantum mechanics is the only way we can guarantee a non-Gaussian signal without violations of locality/causality, while avoiding the existence of poles at physical momenta. A direct comparison between the quantum and classical computation of bispectrum gives for an interaction $H_{\mathrm{int}} = -\frac{\lambda}{3!}\prod_l \left(\hat{D}_{l=1,2,3}\zeta\right)$, where $\hat{D}_l$ is a local differential operator with respect to to $\boldsymbol{x}_l$,

$$\begin{gathered} \left\langle \zeta\left(\boldsymbol{x}_1,\tau\right)\zeta\left(\boldsymbol{x}_2,\tau\right)\zeta\left(\boldsymbol{x}_3,\tau\right)\right\rangle_q - \left\langle \zeta\left(\boldsymbol{x}_1,\tau\right)\zeta\left(\boldsymbol{x}_2,\tau\right)\zeta\left(\boldsymbol{x}_3,\tau\right)\right\rangle_c = \\ \frac{i\lambda}{24}\sum_\sigma \int_{-\infty}^{\tau} \mathrm{d}^3\boldsymbol{x}'\mathrm{d}\tau' a^4\left(\tau'\right)\left[\zeta\left(\boldsymbol{x}_1,\tau\right), \hat{D}_{\sigma(1)}\zeta\left(\boldsymbol{x}',\tau'\right)\right]\left[\zeta\left(\boldsymbol{x}_2,\tau\right), \hat{D}_{\sigma(2)}\zeta\left(\boldsymbol{x}',\tau'\right)\right]\left[\zeta\left(\boldsymbol{x}_3,\tau\right), \hat{D}_{\sigma(3)}\zeta\left(\boldsymbol{x}',\tau'\right)\right] \end{gathered}\,, \tag{2.25}$$

where σ is a permutation of $(1,2,3)$ .The integral over commutators reveals the difference between the classical and quantum case, where it goes to zero in the limit of vanishing commutators.

Chapter 3

Dynamical collapse models in cosmology

The Dynamical, or objective, collapse models are based on modifying Schrodinger equation by coupling the quantum system to an external stochastic classical field called noise. On mathematical level it consists in adding non linear, stochastic and non unitary terms to Schrodinger equation, where non linearity lead to the breakdown of superposition during measurement, stochasticity is needed to generate random outcomes and non unitary evolution (but norm preserving) allows stochastic evolution to cause all but one outcome to decay exponentially [9, 20]. With this properties we solve two aspects of the measurement problem and remain the one of preferred basis unsolved in explicit way, where the collapse operator is to be chosen "by hand". In addition to solving the measurement problem, the dynamical collapse models allow to derive the Born rule rather than being postulated as is done in standard quantum mechanics. However, the new added terms to Schrodinger equation should be efficient, only, at the macroscopic level in order not to spoil the success of standard quantum mechanics at the microscopic level. Indeed, this constraint is captured by the amplification mechanism resulting from the mass density dependence of the collapse parameter which gets very large only for macroscopic objects inducing an efficient and quick collapse. It is worth to mention that collapse models are not formulated yet in a relativistic context so their application to cosmology, where quantum field theory on curved spacetime governs, should be taken carefully [9, 56, 20]. However, the justification to take risk and apply them in the inflationary perturbations context lays on the fact that, at linear order, the different Fourier modes evolve independently and they do not interact, therefore, at this level we could overpass the need for a quantum field description.

Collapse models are divided into several types, we will consider two of them, quantum mechanics with universal position localization model (QMUPL) and Continuous spontaneous localization model (CSL). They differ by the choice of collapse operator in addition to the type of noise implemented.

QMUPL adopts position as collapse operator and the noise is, solely, time dependent so this model has one free parameter, collapse rate. For CSL model, the collapse operator is the mass (or energy) density[1], while, the noise is time and space dependent, therefore, it has two free parameters: collapse rate and collapse length. Applying those models in cosmological context and being guided by the high accurate data at our disposal, especially the well confirmed quasi scale invariant power spectrum, will lead to constraints on the values of their free parameters which could be, subsequently, confronted with the values obtained in laboratory experiments. [56, 29, 54, 55].

3.1 QMUPL model

The basic idea behind dynamical collapse models is to consider the continuous interaction between the quantum system and an external classical stochastic field, that we call noise; the nature of this last is still a matter of research. The interaction between the system and the noise induces spontaneous and random wave function collapses that occur all the time for all particles, whether isolated or interacting and whether they are forming a microscopic, mesoscopic or macroscopic system [9, 29].

The external white noise, that is solely time dependent in the context of QMUPL, is encoded by a stochastic classical function of time W_t, which is nothing but the usual Wiener process (Brownian motion). The coupling parameter, called collapse parameter, between the system and the noise is given by γ, and in the standard QMUPL is proportional to the mass of the object[2] m i.e $\gamma \equiv \gamma(m)$, such that it gets very large values for large objects, resulting in efficient collapse and localization for those objects. Having said that, and combing all those ingredients together result in the following modified Schrodinger equation

$$\mathrm{d}\Psi_{\mathrm{t}} = \left[-i\hat{H}\mathrm{d}t + \sqrt{\gamma}\left(\hat{x} - \langle\hat{x}\rangle\right)\mathrm{d}W_t - \frac{\gamma}{2}\left(\hat{x} - \langle\hat{x}\rangle\right)^2\mathrm{d}t\right]\Psi_{\mathrm{t}}\,, \tag{3.1}$$

with $\mathbb{E}\left(\mathrm{d}W_t\mathrm{d}W_{t'}\right) = \delta\left(t - t'\right)\mathrm{d}t$; localization is supposed to take place in position space $\hat{x}$ as required by the definition of QMUPL. We notice that under collapse models we have two types of averages

1. A stochastic average $\mathbb{E}\left(\ldots\right)$ refers to ensemble average over the system final states which resulted from the evolution, of initial state, through the non unitary, stochastic, part of (3.1).

2. A quantum average, or expectation value, over the system final states resulted from evolution through unitary part of (3.1).

the existence of this two types average, at the same time, would result in an ambiguity in defining the power spectrum of curvature perturbations under the framework of collapse models.

[1]This choice creates a problem in cosmology since we face the so called gauge problem resulting from the invariance of Einstein equations under diffeomorphism, thus, in order to implement CSL in cosmology we need to pick up a gauge invariant variable as the ones introduced by J.Bardeen[54].

[2]For example $\frac{\gamma}{\gamma_0} = \frac{m}{m_0}$, with m_0 being the mass the nucleon or particle composing the object.

When it comes to the application of QMUPL in inflation, specifically to the primordial curvature fluctuations, we have two aims to fulfill, first, we would like to rend the Wigner function squeezed in the field amplitude v direction, rather than field momentum p as was obtained in standard inflation formalism, see equation (1.119). Second, we wold like to obtain quasi scale invariant correction to the curvature perturbations power spectrum. However, in order to implement collapse models in inflationary context, we need to clean away two ambiguities:

First one concerns the collapse operator, we ask what is the equivalent of position operator $\hat{x}$ in our case of curvature perturbations? To answer this question, we need to remember that the MS operator $\hat{v}$, see equation(1.114), is the one related directly to observation, therefore it would be legitimate to assume that the perturbations wave functional Ψ_t will be localized in the eigenbasis $|v\rangle$ of $\hat{v}$ in order to allow for well definite outcomes to our cosmological experiments. Actually, this last choice is consistent with previous approaches aimed at studying decoherence of cosmological perturbations where the pointer basis is often assumed to be precisely the Mukhanov-Sasaki operators [37], we will give more details on this point in the next chapter.

The second ambiguity consists in the cosmological scale that would capture the amplification mechanism, where in standard QMUPL the scale was the object mass. To clean this ambiguity away we need to remember from the first chapter, that the modes k were getting squeezed as they cross the Hubble radius $k\eta \to 0$, and that squeezing leads toward the equivalence between quantum expectation values of operators and classical stochastic averages of complex variables, which could be considered a semi-classicalization of our quantum fluctuations. Therefore, this give us a hint to adopt a collapse parameter that depends on the modes scale as follow [29]

$$\gamma = \frac{\gamma_0\left(k\right)}{\left(-k\eta\right)^{\alpha}}\,. \tag{3.2}$$

It is worth to mention that [56] adopted a constant γ, which leaded them to unsatisfactory results as we will show briefly, in a moment.

So having agreed on using the MS operator as collapse operator and remembering that we are interested in working in Fourier space, then, it could be shown that the fluctuation wavefunctional $\Psi\left[v,\eta\right]$ is still decomposable into real and imaginary parts such that each component satisfy

$$\mathrm{d}\left|\Psi_{\boldsymbol{k}}^{R,I}\right\rangle = \left[-i\hat{H}_{\boldsymbol{k}}^{R,I}\mathrm{d}t + \sqrt{\gamma}\left(\hat{v}_{\boldsymbol{k}}^{R,I} - \left\langle\hat{v}_{\boldsymbol{k}}^{R,I}\right\rangle\right)\mathrm{d}W_{\eta} - \frac{\gamma}{2}\left(\left(\hat{v}_{\boldsymbol{k}}^{R,I} - \left\langle\hat{v}_{\boldsymbol{k}}^{R,I}\right\rangle\right)\right)^{2}\mathrm{d}t\right]\left|\Psi_{\boldsymbol{k}}^{\mathrm{R,I}}\right\rangle, \tag{3.3}$$

notice that the dimension of γ depends on the dimension of scale factor, and whether we adopt constant γ, or scale dependent one as in (3.2). The above equation could be written as[20]

$$\mathrm{d}\Psi_{\boldsymbol{k}}^{R,I} = -i\left[\hat{H}_{\boldsymbol{k}}^{R,I}\mathrm{d}t + i\hat{H}_{\boldsymbol{k},collapse}^{R,I}\right]\Psi_{\boldsymbol{k}}^{\mathrm{R,I}}\,, \tag{3.4}$$

where the collapse Hamiltonian is given by

$$\hat{H}^{R,I}_{\boldsymbol{k},collapse} = \sqrt{\gamma}\left(\hat{v}^{R,I}_{\boldsymbol{k}} - \left\langle \hat{v}^{R,I}_{\boldsymbol{k}} \right\rangle\right) \mathrm{d}W_\eta - \frac{\gamma}{2}\left(\hat{v}^{R,I}_{\boldsymbol{k}} - \left\langle \hat{v}^{R,I}_{\boldsymbol{k}} \right\rangle\right)^2 \mathrm{d}\eta\,, \tag{3.5}$$

Obviously, the main part of this section is to solve the modified Schrodinger equation (3.3), and compute the various quantities of interest, namely the Wigner function and the dimensionless power spectrum $\mathcal{P}_v \propto k^{n_s-1}$. [3] Since, the initial state of fluctuations, i.e Bunch Davies vacuum, is of Gaussian shape and since both $\hat{H}^{R,I}_{\boldsymbol{k}}$ and $\hat{H}^{R,I}_{\boldsymbol{k},collapse}$ are quadratic, then, we can consider that the most general solution of (3.3) assumes a Gaussian shape that is given by

$$\Psi^{R,I}_{\boldsymbol{k}}\left(\hat{v}^{R,I},\eta\right) = \sqrt{|N_{\boldsymbol{k}}(\eta)|}\exp\left[-\frac{\Re e\Omega_{\boldsymbol{k}}(\eta)}{2}\left(v^{R,I}_{\boldsymbol{k}} - \bar{v}^{R,I}_{\boldsymbol{k}}\right)^2 + i\sigma^{R,I}_{\boldsymbol{k}}(\eta) + i\chi^{R,I}_{\boldsymbol{k}}(\eta)\, v^{R,I}_{\boldsymbol{k}} - i\frac{\Im m\Omega_{\boldsymbol{k}}(\eta)}{2}\left(v^{R,I}_{\boldsymbol{k}}\right)^2\right], \tag{3.6}$$

where $\bar{v}^{R,I}_{\boldsymbol{k}}$, $\sigma^{R,I}_{\boldsymbol{k}}(\eta)$, and $\chi^{R,I}_{\boldsymbol{k}}(\eta)$ are stochastic real functions which if we set equal to zero, then, we recover the Gaussian functional adopted in the standard Schrodinger formalism.

3.1.1 Constant collapse parameter γ

In this subsection we will summarize the main results obtained in [56] by considering a constant collapse parameter $\gamma(k) \equiv \gamma$. Substituting the suggested solution (3.6) in (3.3) and by using Itô calculus we derive the equations satisfied by the various functions parameterizing the solution (3.6), we will write only those of $\Re e\Omega_{\boldsymbol{k}}(\eta)$and $\Im m\Omega_{\boldsymbol{k}}(\eta)$ since those last are the ones involved in the Wigner function expression and power spectrum definition [56]

$$\begin{aligned} \frac{|N_k|'}{N_k} &= \frac{1}{4}\frac{(\Re e\Omega_{\boldsymbol{k}})'}{\Re e\Omega_{\boldsymbol{k}}} \\ (\Re e\Omega_{\boldsymbol{k}})' &= \gamma + 4\left(\Re e\Omega_{\boldsymbol{k}}\right)\left(\Im m\Omega_{\boldsymbol{k}}\right) \qquad , \\ (\Im m\Omega_{\boldsymbol{k}})' &= -2\left(\Re e\Omega_{\boldsymbol{k}}\right)^2 + 2\left(\Im m\Omega_k\right)^2 + \tfrac{1}{2}\omega^2(\eta,k) \end{aligned} \tag{3.7}$$

with $\omega^2(\eta,k)$being given by (1.37), we quoted also the equation of $N_k$because of its role in renormalizing $\Psi^{R,I}_{\boldsymbol{k}}\left(\hat{v}^{R,I},\eta\right)$ and its link to $\Re e\Omega_{\boldsymbol{k}}$. The first equation in (3.7) implies the conservation of the norm of the wavefunctional in case it was initially normalized as

$$|N_k| = \left(\frac{\Re e\Omega_{\boldsymbol{k}}}{\pi}\right)^{1/2}. \tag{3.8}$$

Next, by combining the second and third equation of (3.7) we arrive at the Ricatti equation for

[3] It is worth to mention that in [20] the authors made a distinction between the collapse operator with respect to which the wavefunctional will get localized, $\hat{v}$in our case, and the "focus operator" that represents the observable we want measure.

$\Re e\Omega_{\boldsymbol{k}} + i\Im m\Omega_{\boldsymbol{k}}$

$$\Omega'_{\boldsymbol{k}} = -i\Omega^2_{\boldsymbol{k}} + i\tilde{\omega}^2 \,, \tag{3.9}$$

notice the similarity of this equation with the one obtained from standard theory (1.95), except that now we have $\tilde{\omega}^2(\eta, k) = \omega^2 - 2i\gamma = \kappa^2 - \frac{a''}{a}$ with $\kappa^2 = k^2 - 2i\gamma$. It is important to note that under the framework of Collapse models modified theory we do not have anymore the corresponding Heisenberg picture, because of their lack for for a Lagrangian formulation. Therefore, in using the trick $\Omega_{\boldsymbol{k}} = -i\frac{\bar{f}'_k}{\bar{f}_k}$, the function f_k is a mere parameter that serve to simplify computations and has nothing to do with Heisenberg modes functions. The equation of motion for f_k is given by

$$f''_k + \left(\kappa^2 - \frac{a''}{a}\right) f_k = 0 \,, \tag{3.10}$$

notice that now f^*_kdoes not satisfy the same equation due to the fact that κ is a complex number now, therefore, we define $\bar{f}_k$ by

$$\bar{f}_k \equiv f^*_k \,\text{with}\, \gamma \to -\gamma \,, \tag{3.11}$$

it is easy to see check that $\bar{f}_k$ satisfy the same equation as f_k

$$\bar{f}''_k + \left(\kappa^2 - \frac{a''}{a}\right) \bar{f}_k = 0 \,, \tag{3.12}$$

so they both represent a linearly two independent solutions. Adopting the same notation as in the squeezing formalism, and exchanging $\mathrm{u}_k \to \bar{\mathrm{u}}_k$ in the definition of $\Omega_{\boldsymbol{k}}$as

$$\Omega_{\boldsymbol{k}} \equiv -i\frac{\bar{f}'_k}{\bar{f}_k} = \kappa\frac{\bar{\mathrm{u}}_k - \mathrm{v}_k}{\bar{\mathrm{u}}_k + \mathrm{v}_k} \,. \tag{3.13}$$

Following the same steps as was done in the Schrodinger picture to solve (3.12) and adopting the same limits, namely super horizon limit $k\eta \to 0$ and equivalently large squeezing $r_k \gg 1$, will lead us to two different expressions of $\Re e\Omega_{\boldsymbol{k}}$ as function of the scale[4] $\frac{\gamma}{k^2}$, in case large modes $\frac{\gamma}{k^2} \gg 1$ then

$$\Re e\Omega_{\boldsymbol{k}} \approx \frac{2\gamma}{k}\left(-k\eta\right) \,, \tag{3.14}$$

while for the shorter modes $\frac{\gamma}{k^2} \ll 1$ we get

$$\Re e\Omega_{\boldsymbol{k}} \approx 2k\left(-k\eta\right)^2 \,, \tag{3.15}$$

Notice that in both cases we do have $\Re e\Omega_{\boldsymbol{k}} \to 0$ as $-k\eta \to 0$, so from (1.116) we can see that the Wigner function is still squeezed along the filed momentum p direction, as was obtained in stan-

[4]For more details on the steps toward the expression of $\Re e\Omega_{\boldsymbol{k}}$, see [56, 29].

dard inflation theory, and this is not the desired result, sought for, which could explain the macro-objectification of the inflationary quantum fluctuations.

On the other hand and concerning the power spectrum we see through equation (1.105), that the scale dependence of curvature power spectrum $\mathcal{P}_\zeta$ comes from $\left(a^2\Re e\Omega_{\boldsymbol{k}}\right)^{-1}$ and since $\Re e\Omega_{\boldsymbol{k}}$ depends on time we will evaluate at the end of inflation inspired by standard inflation formalism prediction of the freeze out of large modes until horizon reentry. Therefore, having

$$-k\eta = \frac{k}{k_0}\mathrm{e}^{-\triangle N}\,, \tag{3.16}$$

with k_0 being the comoving mode that is at horizon today, i.e $k_0 = a_0 H_0$, we get for power spectrum

$$\mathcal{P}_\zeta = \begin{cases} \frac{1}{16\pi^2\epsilon M_{pl}} & \text{for } \frac{\gamma}{k^2} \ll 1 \\ \\ \frac{k^3}{16\pi^2\epsilon M_{pl}\gamma} & \text{for } \frac{\gamma}{k^2} \gg 1 \end{cases}\,, \tag{3.17}$$

we see that the power spectrum corresponding to larger modes are still scale dependent, so it was suggested in [56] the existence of a transition scale $k_\gamma$such that

$$\frac{k_\gamma}{k_0} \ll 1\,, \tag{3.18}$$

this last constraint rends the modes for which we obtain a scale dependent power spectrum, called QMUPL branch, to be much larger than current Hubble radius. Their ratio was estimated to be

$$\frac{l_H}{l_\gamma} \ll 10^{-13}\,, \tag{3.19}$$

But this could arise a confusion regarding the role of implementing collapse models in the study of inflationary perturbations, where we see that the observed modes which we were expecting to be affected the most, in order to have quantum to classical transition model, were unfortunately the less affected. Therefore, we need to look for loophole in our previous treatment and try to follow another approach. Indeed, Das et al in [29] noticed that the missing piece was the key feature of collapse models, namely the amplification mechanism. Where with a constant collapse parameter we were putting the different scales on the same footing, so the idea is to a complicate a bit the formalism.

However before doing so, some comments are in order here:

- Notice that our suggested solution of the modified Schrodinger equation (3.6) consisted of single Gaussian, therefore what we were doing was the study of its localization, the decrease of its spread in a given eigenbasis, namely, that of the collapse operator. However to study collapse of the wavefunctional, we need to consider two, or more, Gaussians and study the evolution of our wavefunctional. In this last case, there are two possibilities, either the wavefunctional will collapse into one of the Gaussians or the Gaussians will merge into a single one. It was shown in [56] that collapse takes place much faster than merging phenomenon.

- The choice of MS operator $\hat{v}$ as collapse operator is certainly not a unique choice, since we could have associated a function of scale factor to it as $h(a)\,\hat{v}$, this arbitrariness is called the temporal gauge problem. Indeed, by associating a function $h(a)$ to the collapse operator will lead to some results which are quite similar to a case of collapse parameter given by (3.2). In particular, for an adequate choice of $h(a)$ we can get a Wigner function localized in the field amplitude direction, but, unfortunately, regardless of the choice of $h(a)$ there is no possibility to get scale independent power spectrum.

- In [56] the authors adopted the power spectrum definition

$$\mathcal{P}_v(k) = \mathbb{E}\left(\left\langle \hat{v}_k^2 \right\rangle\right) - \mathbb{E}\left(\left\langle \hat{v}_k \right\rangle^2\right) , \tag{3.20}$$

however, it was argued by D.Sudarsky that such a definition is not fully accurate, simply, because such power spectrum does not vanish in the limit $\gamma \to 0$. While it would be logical that in this limit, where our wavefunctional did not collapse, we are still in homogeneous and isotropic universe. Therefore, D.Sudarsky suggested, instead, the following definition that does vanish in the aforementioned limit

$$\mathcal{P}_v(k) = \mathbb{E}\left(\left\langle \hat{v}_k^2 \right\rangle\right) - \mathbb{E}^2\left(\left\langle \hat{v}_k \right\rangle\right) , \tag{3.21}$$

as we mentioned previously such ambiguity arises whenever we have two types averages, namely, a quantum and classical stochastic averages in our case. Where, the quantum expectation value $\langle \hat{v}_k \rangle$is no more a number but stochastic quantity so we need to make the stochastic average $\mathbb{E}\left(\langle \hat{v}_k \rangle\right)$ to end up with a meaningful quantity.

3.1.2 Scale dependent collapse parameter γ

In order to cure the shortcomings of the previous implementation of collapse models in cosmology, we make the ansatz

$$\gamma = \frac{\gamma_0(k)}{(-k\eta)^{\alpha}} , \tag{3.22}$$

with $\alpha > 0$ so that amplification mechanism effect increases as the modes cross the horizon. Using (3.2), then the evolution equation of f_k becomes

$$f_k'' + \left(\kappa^2 - \frac{a''}{a}\right) f_k , \tag{3.23}$$

where κ^2 is given now by

$$\kappa^2 = k^2 - 2i\gamma_0\,(k)\,(-k\eta)^{-\alpha}\ , \tag{3.24}$$

if$\alpha > 2$ the term proportional to γ in (3.23) dominates the dynamics at the end of inflation, when $k\eta$ goes to 0, in and one can expect the power spectrum scale invariance to be destroyed. Therefore we are left with the cases $0 < \alpha \leq 2$.

With a scale dependent collapse operator we obtain at leading order in $|k\eta|$, and for superhorizon limit $k\eta \to 0$, the following expression of $\Re e\Omega_{\boldsymbol{k}}$ [29]

$$\Re e\Omega_{\boldsymbol{k}} \approx \frac{k}{2}\,(-k\eta)^{1-\alpha}\left(\frac{2\gamma_0\,(k)}{k^2}\right)\ . \tag{3.25}$$

In order to have a localization in the direction of field amplitude v we need $\Re e\Omega_{\boldsymbol{k}} \to \infty$, therefore, from the last equitation we can see that for $1 < \alpha \leq 2$ we do have localization in filed amplitude, see figure (3.1), which is a quite interesting result.

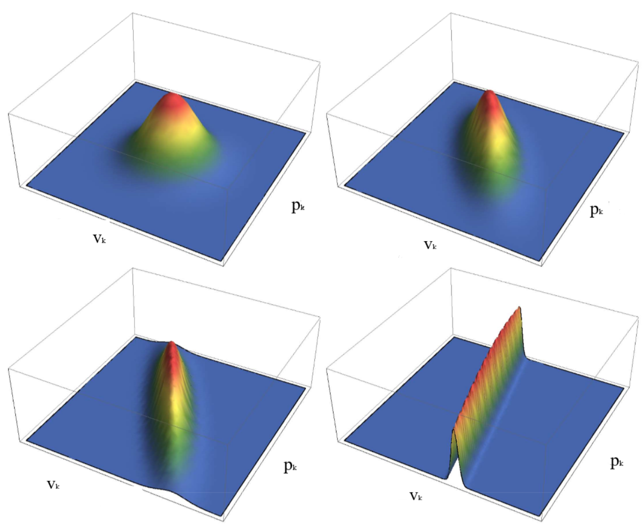

Figure 3.1: A schematic representation of a Wigner function localized in field amplitude direction.

In order to compute the correction to power spectrum we restrict ourselves to the range $1 < \alpha \leq 2$, so using (3.25) and (3.16) , then, the curvature power spectrum is proportional to [29]

$$\mathcal{P}_\zeta \propto \frac{k^3}{a^2\Re e\Omega_{\boldsymbol{k}}} \approx \left(\frac{\gamma_0\,(k)}{k^2}\right)^{-1}\left(\frac{k}{k_0}\right)^{1+\alpha} \mathrm{e}^{-(1+\alpha)\triangle N}\ , \tag{3.26}$$

so if γ_0 is independent of k then we get scale dependent power spectrum, however we can take advantage of the remaining freedom in our ansatz (3.2) and adopt the following form [29]

$$\gamma_0(k) = \tilde{\gamma}_0 \left(\frac{k}{k_0}\right)^{\beta}, \tag{3.27}$$

with $\tilde{\gamma}_0$ being a constant. Therefore, combining the last two equations yield

$$\mathcal{P}_\zeta \propto k^{3+\alpha-\beta}, \tag{3.28}$$

so if we we set $\beta = 3+\alpha$, then, we obtain a scale independent power spectrum. For $1 < \alpha \leq 2$, the parameter β is constrained to the range $4 < \beta < 5$. But it is important to remember that there still the problem of time dependence of power spectrum which rends any comparison of it to that of recombination a highly non trivial task due to the complications of pre- and reheating phase [56, 29].

Remark. A similar work of implementing collapse models in the study of primordial perturbations was done in [20], however there was several differences in their approach with respect to that used in [56, 29], which consequently leaded to a different conclusion . We summarize the main differences in the following:

- The authors in [20] adopted a semi classical approach where metric perturbations were remained classical while inflaton ones were quantized, while the authors in [56, 29] adopted the standard approach of quantizing both, metric and matter, perturbations. Therefore, in the former case the observable related to CMB measurements was power spectrum that is proportional to momentum conjugate of matter perturbations i.e $\langle \hat{p}^2 \rangle (k)$, while in the case of [56, 29] the observable was power spectrum proportional to MS variable i.e $\langle \hat{v}^2 \rangle (k)$. Using the terms of [20] then we can say that the "focus operator" in the two approaches was $\hat{p}$ and $\hat{v}$, respectively.

- The aim of [20] from implementing collapse models was, solely, to account from the quantum to classical transition and get a scale independent power spectrum, but there was not a need to rend the Wigner function squeezed in field amplitude direction, because their focus operator was the momentum conjugate field along which the Wigner function was already squeezed. However, in [56, 29] the focus operator was MS variable, so the aim was twofold, first, rend the Wigner function squeezed along of it, and second to obtain a scale independent power spectrum.

- The constraints obtained in [20] on the collapse operator γ to get a scale independent power spectrum depended on the choice of collapse operator

$$\begin{cases} \gamma \propto k^{-1} & \text{if the collapse operaor was } \hat{p} \\ \\ \gamma \propto k & \text{if the collapse operaor was } \hat{y} \end{cases}, \tag{3.29}$$

where $\hat{y}$ is the operator associated to the matter perturbation. On the other hand, we saw that [56, 29] obtained a scale independent power spectrum in case we adopt the ansatz

$$\gamma = \tilde{\gamma}_0 \left(\frac{k}{k_0}\right)^{\beta} (-k\eta)^{-\alpha} \, , \tag{3.30}$$

for

$$\begin{cases} 1 < \alpha \leq 2 \\ \\ 4 < \beta \leq 5 \end{cases} . \tag{3.31}$$

As final comment, we would like to mention that unfortunately the constraints obtained on collapse parameter through cosmological QMUPL could not be confronted with those obtained thorough laboratory QMUPL where the collapse operator is chosen to be position. In other words, each time one considers different "collapse operators", this leads to different collapse parameters with different mass dimension, so it is meaningless to compare them. What could be done is to consider the CSL theory where the "collapse operator" is usually taken to be the averaged density operator $\delta\rho\,(\eta, \boldsymbol{x})$ which is unfortunately not uniquely defined in cosmology due to the gauge problem. Therefore, we need to complicate a bit the computations and consider a gauge invariant definition of $\delta\rho\,(\eta, \boldsymbol{x})$ to be able to compare the cosmological constraints on γ to those of laboratory, such work was done in [54, 55] and we will summarize the main results in next section.

3.2 CSL model

As mentioned previously the CSL model considers the mass/energy density operator as a collapse operator, so the modified Schrodinger equation adapted to cosmological perturbations is given in comoving coordinates [54] [5]by

$$\begin{aligned} \mathrm{d}\Psi\,(\boldsymbol{x}, t) \; &= \Big[-i\hat{H}\mathrm{d}t + \tfrac{1}{m_0}\sqrt{\tfrac{\gamma}{a^3}} \int \mathrm{d}^3\boldsymbol{x} a^3 \left(\hat{C}\,(\boldsymbol{x}) - \left\langle \hat{C}\,(\boldsymbol{x})\right\rangle\right) \mathrm{d}W_t\,(\boldsymbol{x}) \\ &\quad - \tfrac{\gamma}{2m_0^2} \int \mathrm{d}^3\boldsymbol{x} a^3 \left(\hat{C}\,(\boldsymbol{x}) - \left\langle \hat{C}\,(\boldsymbol{x})\right\rangle\right)^2 \mathrm{d}t\Big] \,\Psi\,(\boldsymbol{x}, t) \end{aligned} \, , \tag{3.32}$$

[5]The comoving coordinate $\boldsymbol{x}$ and physical ones $\boldsymbol{x}_p$ are related by $\boldsymbol{x}_p = a\boldsymbol{x}$. Notice also that the equation is written as function of cosmic time.

if we write the energy density as $\hat{\rho} = \bar{\rho} + \delta\hat{\rho}$, where $\bar{\rho}$is the background energy density that remains classical, while $\delta\hat{\rho}$ is the fluctuation that gets quantized and is involved as collapse operator through

$$\hat{C}(\boldsymbol{x}) = \bar{\rho}\frac{\delta\hat{\rho}}{\bar{\rho}}\bigg|_{cg}(\boldsymbol{x}) = 3M_{pl}^2\frac{\mathcal{H}^2}{a^2}\,\frac{\delta\hat{\rho}}{\bar{\rho}}\bigg|_{cg}(\boldsymbol{x})\,, \tag{3.33}$$

where we used the first Friedman equation. In the previous equation we coarse grained $\frac{\delta\hat{\rho}}{\bar{\rho}}\Big|_{cg}(\boldsymbol{x})$ over the localization distance r_c using the a Gaussian coarse-gaining

$$\frac{\delta\hat{\rho}}{\bar{\rho}}\bigg|_{cg}(\boldsymbol{x}) = \left(\frac{a}{r_c}\right)^3\frac{1}{(2\pi)^{3/2}}\int \mathrm{d}^3\boldsymbol{y}\frac{\delta\hat{\rho}}{\bar{\rho}}(\boldsymbol{x}+\boldsymbol{y})\,\mathrm{e}^{-\frac{|\boldsymbol{y}|^2a^2}{2r_c^2}}\,, \tag{3.34}$$

As we mentioned previously and due to the gauge problem in the relativistic study of cosmological fluctuations, there is no a unique definition of $\frac{\delta\hat{\rho}}{\bar{\rho}}$ and each choice would lead to a different cosmological CSL theory. However [54] considered that a well physical motivated choice consists in measuring the energy density relative to the hypersurface which is as close as possible to a "Newtonian" time slicing, and is given by

$$\frac{\delta\rho}{\bar{\rho}} = \epsilon_1\zeta - \epsilon_1\left(1+\epsilon_1\mathcal{H}^2\partial^{-2}\right)\zeta'/(3\mathcal{H})\,, \tag{3.35}$$

for a universe dominated by a scalar field. Notice also that, now and in contrast to QMUPL, the wiener process $W_t(\boldsymbol{x})$ is function of both space and time and satisfies $\mathbb{E}\left[\mathrm{d}W_t(\boldsymbol{x})\,\mathrm{d}W_{t'}(\boldsymbol{x}')\right] = \delta(\boldsymbol{x}-\boldsymbol{x}')\,\delta(t-t')\,\mathrm{d}t^2$.

Even though we are still using a quadratic Hamiltonian, we may worry that the stochastic part of (3.32) would induce interaction between the different modes in Fourier space. However and fortunately this not the case, where it was shown in [54] each mode satisfy an individual equation, that is given by

$$\mathrm{d}\Psi_{\boldsymbol{k}}^{R,I}(t) = \left[-i\hat{H}_{\boldsymbol{k}}^{R,I}\mathrm{d}t + \frac{1}{m_0}\sqrt{\gamma a^3}\left(\hat{C}_{\boldsymbol{k}}^{R,I} - \left\langle\hat{C}_{\boldsymbol{k}}^{R,I}\right\rangle\right)\mathrm{d}W_t^{R,I} - \frac{\gamma a^3}{2m_0^2}\left(\left(\hat{C}_{\boldsymbol{k}}^{R,I} - \left\langle\hat{C}_{\boldsymbol{k}}^{R,I}\right\rangle\right)\right)^2\mathrm{d}t\right]\Psi_{\boldsymbol{k}}^{R,I}(t)\,, \tag{3.36}$$

where

$$\delta\hat{\rho}_{cg}^s(\boldsymbol{k}) = \alpha_{\boldsymbol{k}}\hat{v}_{\boldsymbol{k}}^s + \beta_{\boldsymbol{k}}\hat{p}_{\boldsymbol{k}}^s\,, \tag{3.37}$$

with

$$\begin{aligned}\alpha_{\boldsymbol{k}} &\equiv \tfrac{M_{pl}^2H^2\epsilon_1}{z}\mathrm{e}^{-\frac{k^2r_c^2}{2a^2}}\left[4+\tfrac{\epsilon_2}{2} - 3\left(\tfrac{aH}{k}\right)^2\epsilon_1(1+\epsilon_2)\right]\\ \beta_{\boldsymbol{k}} &\equiv \tfrac{M_{pl}^2H^2\epsilon_1}{az}\mathrm{e}^{-\frac{k^2r_c^2}{2a^2}}\left[3\epsilon_1\left(\tfrac{aH}{k}\right)^2 - 1\right]\end{aligned}\,, \tag{3.38}$$

where $\epsilon_2 \equiv \mathrm{d}\ln\epsilon_1/\mathrm{d}\ln a$ is the second slow roll parameter. So the presence of the extra stochastic and non-linear terms does not destroy the property that the modes still evolve separately. It is worth to mention that form the last two equations we can see explicitly from the exponential term that CSL effects become efficient only after the mode wavelength exceeds the localization distance i.e $\frac{k}{a} < r_c^{-1}$. This last property is important, since it implies that at early time, when $\frac{k}{a} > r_c^{-1}$ the standard theory applies, which implies. By turn, that one of the great advantages of inflation, namely the possibility to choose well-defined initial conditions in the Minkowski limit (the so-called Bunch-Davies vacuum state), is preserved.

Substituting again the most general stochastic Gaussian (3.6) in (3.36), and defining the dimensionless power spectrum by [54]

$$\mathcal{P}_v(k) = \frac{k^3}{2\pi^2}\left\{\mathbb{E}\left(\left\langle \hat{v}_k^{s2}\right\rangle\right) - \mathbb{E}\left[\left\langle \left(\hat{v}_k^s - \bar{v}_k^s\right)^2\right\rangle\right]\right\} \quad s \equiv R \text{ } or \text{ } I\,, \tag{3.39}$$

will lead us to different expressions of power spectrum depending on weather the collapse of the wavefunctional took place during inflation or during radiation dominated era. Knowing that the collapse of the wavefunctional is quantified through the parameter D defined as

$$D = \frac{\mathbb{E}\left[\left\langle \left(\hat{v}_k^s - \bar{v}_k^s\right)^2\right\rangle\right]}{\mathbb{E}\left(\bar{v}_k^{s2}\right)}\,, \tag{3.40}$$

which measures the ratio of the width of $\Psi_{\boldsymbol{k}}^{R,I}(t)$ relative to the typical dispersion of its mean $\bar{v}_k^s$, therefore, an efficient collapse corresponds to $D \ll 1$. From (3.6) we can see that the wavepacket is centered around $\bar{v}_k^s$ with a variance $\left\langle \left(\hat{v}_k^s - \bar{v}_k^s\right)^2\right\rangle = \left(4\Re e\Omega_{\boldsymbol{k}}\right)^{-1}$, and it could be shown that $\mathbb{E}\left(\bar{v}_k^{s2}\right) = \left(4\Re e\Omega_{\boldsymbol{k}}|_{\gamma=0}\right)^{-1}$, this last two pieces of information will be useful in the discussion of power spectrum expression. Beofre turning on to the discussion of CSL effect on power spectrum and for the sake of comparison with QMUPL, we quote here the equation of motion satisfied by $\Omega_{\boldsymbol{k}}$

$$\Omega_{\boldsymbol{k}}' = \frac{4i\gamma a^4\alpha_{\boldsymbol{k}}\beta_{\boldsymbol{k}}}{m_0^2}\Omega_{\boldsymbol{k}} - 2\left(i + \frac{2\gamma a^4\beta_{\boldsymbol{k}}^2}{m_0^2}\right)\Omega_{\boldsymbol{k}}^2 + \frac{\gamma a^4\alpha_{\boldsymbol{k}}^2}{m_0^2} + i\frac{\omega^2(k,\eta)}{2}\,, \tag{3.41}$$

we can, easily, see that (3.41) reduces to an equation very similar to (3.9) for $\alpha_{\boldsymbol{k}} = 1$, $\beta_{\boldsymbol{k}} = 0$, and $m_0 = 1$, and this could understood form (3.37) where we see that for the aforementioned values of $\alpha_{\boldsymbol{k}}, \beta_{\boldsymbol{k}}$ the collapse operator becomes MS operator just as in QMUPL.

The evaluation of $\mathcal{P}_v(k)$ depends on weather r_c was crossed during inflation or during radiation dominated era. In case the collapse took place during inflation, then

$$\mathcal{P}_v(k) = \frac{k^3}{2\pi^2}\frac{1}{4\Re e\Omega_{\boldsymbol{k}}|\,\gamma = 0}\left[1 + \frac{3\gamma}{2m_0^2}\epsilon_1^3\bar{\rho}_{\text{inf}}\left(\frac{k}{aH}\right)_{\text{end}}^{-1} - \frac{\Re e\Omega_{\boldsymbol{k}}|_{\gamma=0}}{\Re e\Omega_{\boldsymbol{k}}}\right]\,, \tag{3.42}$$

where $\bar{\rho}_{\text{inf}} = 3H_{\text{inf}}^2M_{pl}^2$, and $\Re e\Omega_{\boldsymbol{k}}|_{\gamma=0}$ refers to the value of $\Re e\Omega_{\boldsymbol{k}}$ obtained in standard inflation, namely the $\Re e\Omega_{\boldsymbol{k}}$ present in equation (1.88). Let us now discuss the previous expression of power spectrum as function of collapse parameter γ:

- For $\gamma < \gamma_{\min}$ the power spectrum vanishes and this could be understood from the discussion we had previously on the definition of power spectrum, where for $\gamma = 0$ there is no collapse so our state is still homogeneous and isotropic which implies a vanishing power spectrum. Notice that the J.Martin et al adopted a different definition of power spectrum in [54] with respect to that they were adopted in [56].

- For $\gamma_{\min} < \gamma < \gamma_{\max}$ the collapse takes place so the last term in (4.67) vanishes, and if the second term remains negligible, since it is proportional to ϵ_1^3, then we get a scale independent power spectrum.

- If $\gamma_{\max} < \gamma$, then the second term will dominate and the power spectrum acquires a spectral index $n_s = 0$ which is excluded by data. Therefore, this last observation will help us to constrain the value of collapse parameter. Using $\frac{k}{aH}\big|_{\text{end}} = \mathrm{e}^{-\triangle N}$ with $\triangle N \approx 50$ being the number of e-folds spent by a mode between Hubble radius crossing during inflation and the end of inflation, then we get an upper bound on $\gamma_{\max}$ as follow

$$\frac{3\gamma_{\max}}{2m_0^2}\epsilon_1^3\bar{\rho}_{\text{inf}}\left(\frac{k}{aH}\right)^{-1}_{\text{end}} \ll 1 \Rightarrow \gamma_{\max} \ll m_0^2\left(448\epsilon_1^3\bar{\rho}_{\text{inf}}\right)^{-1}\mathrm{e}^{-\triangle N}\,. \tag{3.43}$$

  On the other hand requiring the collapse to take place before the end inflation, i.e $H_{\text{end}}r_c < \mathrm{e}^{\triangle \mathrm{N}}$, would provide us with lower bound on γ

$$D \ll 1 \Rightarrow \gamma_{\min} > m_0^2\left(1152\bar{\rho}_{\text{inf}}\right)^{-1}\mathrm{e}^{-\triangle N}\,, \tag{3.44}$$

  so we conclude that if the collapse took place during inflation then the collapse parameter is constrained to be

$$m_0^2\left(1152\bar{\rho}_{\text{inf}}\right)^{-1}\mathrm{e}^{-\triangle N} < \gamma \ll m_0^2\left(448\epsilon_1^3\bar{\rho}_{\text{inf}}\right)^{-1}\mathrm{e}^{-\triangle N}\,, \tag{3.45}$$

Combining the last constraints with those of the case of a collapse taking place during radiation dominated era result in the figure (3.2), which compares the cosmological constraints with those obtained from different laboratory experiments. We notice that there is no overlap between the cosmological constraints and the others, therefore, we conclude that for the choice we made for $\frac{\delta\rho}{\bar{\rho}}$, CSL theory is ruled out. However, it is important to remember that there could other choices of $\frac{\delta\rho}{\bar{\rho}}$, for which there exist an agreement between cosmological CSL and laboratory CSL [54, 55].

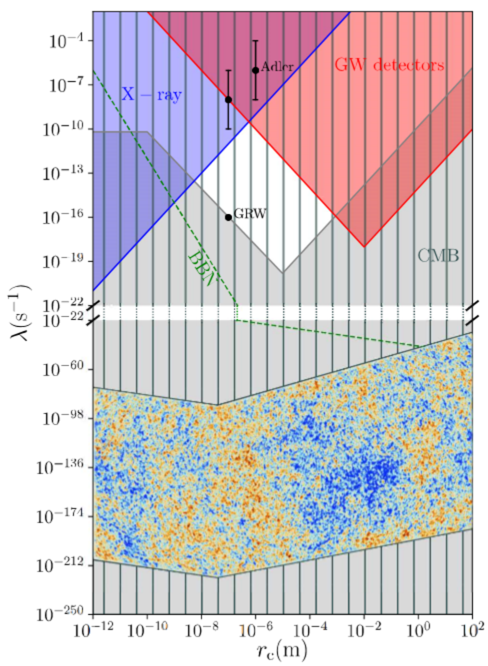

Figure 3.2: Observational constraints on the two parameters r_c and $\lambda = \gamma\ /\left(8\pi^{3/2}r_c^3\right)$ of the CSL model. The white region is allowed by laboratory experiments while the unbarred region is allowed by CMB measurements (one uses ΔN = 50 for the pivot scale of the CMB, $H_{\text{inf}} = 10^{-5}M_{pl}$, and ϵ_1= 0.005). The two allowed regions are incompatible[54].

Chapter 4

Decoherence of primordial perturbations

In the chapter one we saw the notion of "decoherence without decoherence" which was derived by studying the intrinsic properties of primordial perturbations, in particular their evolution into squeezed states and the effect of squeezing on their classicalization. However, by doing this we were actually studying how much the states were classical, so now the aim is to focus on the quantum coherence properties of the states, and understand how coherence gets lost to give rise to classical perturbations that seeded CMB anisotropies and large scale structures. Decoherence is a powerful tool to investigate such question, not only from theoretical and foundational perspective, but also and most importantly from observational point of view as we will see. In addition, decoherence implementation in the study of cosmological primordial perturbation is due to the fact that those last compose an open system interacting, at least gravitationally, with the other degrees of freedom present in the early universe. Therefore it is unavoidable to study the effect of those extra degrees of freedom on the evolution of primordial perturbations, to this end we should derive the Lindblad equation describing our system evolution and how the interaction with the environment modifies it. Then, we use this equation to study the statistical properties of cosmological perturbations and compare the results with standard ones that are well constrained by the measurement of CMB anisotropies.

4.1 Derivation of the Lindblad equation

4.1.1 Free system

We reserved the appendix B to discuss the formalism of decoherence and how it contributes to solve the measurement problem[1], so in this chapter we focus solely on its application to cosmology.

The quantum system, we are interested in, is described by the wavefunctional (1.80), where in the free case the time evolution induced by the free Hamiltonian or intrinsic system Hamiltonian $\hat{H}^{(1)} \equiv \hat{H}_S$ will preserve the initial factorized form of system state i.e

$$\Psi\left[v\left(\eta,x\right)\right] = \prod_{\boldsymbol{k}\in R^{3+}} \Psi\left[v_{\boldsymbol{k}}^R, v_{\boldsymbol{k}}^I\right] = \prod_{\boldsymbol{k}} \Psi_{\boldsymbol{k}}^R\left(v_{\boldsymbol{k}}^R\right)\Psi_{\boldsymbol{k}}^I\left(v_{\boldsymbol{k}}^I\right) . \tag{4.1}$$

However in case there was non linear interactions, then, the evolved state could no more be in factorized form due to coupling between modes, therefore we will work with $\Psi\left[v\left(\eta,x\right)\right]$. Since our main goal is to study quantum to classical transition, then it is more practical to use the density matrix operator

$$\hat{\rho}_{sys}\left(\eta\right) = \left|\Psi\left[v\right]\right\rangle\left\langle\Psi\left[v\right]\right| , \tag{4.2}$$

through which we can see the transition of our initial pure state into a mixed one as result of suppression of the interference terms. As long as we are in the free theory, i.e no mode coupling, $\hat{\rho}_v$ could be factorized as

$$\hat{\rho}_{sys}\left(\eta\right) = \prod_{\boldsymbol{k}\in R^{3+}}\prod_{s=R,I} \hat{\rho}_{\boldsymbol{k}}^s\left(\eta\right) , \tag{4.3}$$

the evolution of the system is controlled by the Schrodinger equation or equivalently the Liouville-von Neumann equation

$$\frac{\mathrm{d}\hat{\rho}_v}{\mathrm{d}\eta} = -i\left[\hat{H}_S, \hat{\rho}_v\right] , \tag{4.4}$$

We move to Fourier space where the left hand side could be written as

$$\frac{\mathrm{d}\hat{\rho}_{sys}}{\mathrm{d}\eta} = \int_{R^{3+}} \mathrm{d}^3\boldsymbol{k}\left(\frac{\mathrm{d}\hat{\rho}_k^R}{\mathrm{d}\eta}\hat{\rho}_{\boldsymbol{k}}^I + \hat{\rho}_{\boldsymbol{k}}^R\frac{\mathrm{d}\hat{\rho}_{\boldsymbol{k}}^I}{\mathrm{d}\eta}\right)\prod_{\boldsymbol{k}'\neq\boldsymbol{k}}\prod_{s=R,I}\hat{\rho}_{\boldsymbol{k}'}^s , \tag{4.5}$$

then using

$$\hat{H}_{sys} = \int_{R^{3+}} \mathrm{d}^3\boldsymbol{k}\sum_{s=R,I}\hat{\mathcal{H}}_{\boldsymbol{k}}^s , \tag{4.6}$$

[1]In appendix B it is explained why decoherence does not solve the whole measurement problem and how to cure that.

the commutator in (4.4)could be written as

$$\begin{aligned}\left[\hat{H}_{sys}, \hat{\rho}_v\right] &= \int_{R^{3+}} \mathrm{d}^3\boldsymbol{k} \sum_{s=R,I} \left[\hat{\mathcal{H}}^s_{\boldsymbol{k}}, \hat{\rho}_v\right] \\ &= \int_{R^{3+}} \mathrm{d}^3\boldsymbol{k} \left(\left[\hat{\mathcal{H}}^R_{\boldsymbol{k}}, \hat{\rho}^R_{\boldsymbol{k}}\right]\hat{\rho}^I_{\boldsymbol{k}} + \hat{\rho}^R_{\boldsymbol{k}}\left[\hat{\mathcal{H}}^s_{\boldsymbol{k}}, \hat{\rho}^I_{\boldsymbol{k}}\right]\right) \quad \prod_{\boldsymbol{k}'\neq\boldsymbol{k}} \prod_{s=R,I} \hat{\rho}^s_{\boldsymbol{k}'}\end{aligned} \quad , \tag{4.7}$$

therefore from (4.5) and (4.5) we get

$$\frac{\mathrm{d}\hat{\rho}^s_{\boldsymbol{k}}}{\mathrm{d}\eta} = -i\left[\hat{\mathcal{H}}^s_{\boldsymbol{k}}, \hat{\rho}^s_{\boldsymbol{k}}\right] , \tag{4.8}$$

so the Liouville-von Neumann equation decouples into a set of equations governing the evolution of each Fourier mode, independently of the others, as result of the absence of non linear interactions. This property will remain valid also when we consider a linear interaction between the system and the environment. Notice also that that the density matrix is not a usual operator, in the sense that it follows the von Neumann equation (4.8) where the sign is opposite to the standard Heisenberg equation.

4.1.2 Interacting system

A truly closed gravitational system is a practical impossibility (unless one considers the totality of the universe to constitute the system as in, for example, quantum cosmology) [57]. Therefore one should rather consider the primordial perturbations to be an open system interacting with an environment that could be composed of all other degrees of freedom of other fields as cosmological perturbations outside our causal horizon, physics beyond the UV or IR cutoffs of the theory...etc [53]. To be more precise about a possible environment we can consider the reheating phase implications. Where, after the end of inflation and during reheating, an excitation of some degrees of freedom is supposed to take place and give rise to radiation dominated era, and this implies the existence of a coupling between inflaton and those degrees of freedom. Based on physical grounds, Such interaction could not switch on only at the end of inflation but it was present even during inflationary stages [15], therefore those fields (fermionic and scalar) composing radiation era could be an environment that decohere the primordial quantum fluctuations.

The total Hamiltonian of the composite system i.e system+environment, living in the Hilbert space $\mathcal{E} = \mathcal{E}_S \otimes \mathcal{E}_E$, is given by

$$\hat{H} = \hat{H}_0 + \hat{H}_{int} = \hat{H}_{sys} \otimes \hat{I}_{env} + \hat{I}_{sys} \otimes \hat{H}_{env} + g\hat{H}_{int} , \tag{4.9}$$

where $\hat{H}_{sys}$ is the intrinsic Hamiltonian of the system acting on the Hilbert space $\mathcal{E}_S$, $\hat{H}_{env}$ is the free evolution Hamiltonian of the environment acting on $\mathcal{E}_E$ that could be left unspecified, $\hat{H}_{int}$ is the interaction Hamiltonian and g is a dimensionless constant. We specialize to the case of interest for field theory, where interactions are local. Suppose, then, that system and environment interact through interactions of the local form,$\hat{H}_{int}$ could be written as

$$\hat{H}_{int}(\eta) = \int d^3\boldsymbol{x}\hat{A}(\eta,\boldsymbol{x}) \otimes \hat{R}(\eta,\boldsymbol{x}) \ , \tag{4.10}$$

where $\hat{A}$ denotes a local functional of the fields describing the system sector, and similarly for $\hat{R}$ that belongs to the environment sector. The pointer, or preferred, basis choice is dictated by the system sector part of $\hat{H}_{int}(\eta)$ where, as was explained in Appendix B, the preferred basis is the eigenstates basis of $\hat{A}(\eta,\boldsymbol{x})$due to the non commutativity criteria. However, in our case we will do things in the opposite way, where based on the fact that CMB map is localized in the field amplitude $\hat{v}$ basis (look at equation (1.114)) we conclude that $|v_{\boldsymbol{k}}\rangle$ constitute the pointer basis [37] and that $\hat{A}(\eta,\boldsymbol{x})$ involves only the $\hat{v}$ i.e

$$\hat{A} \equiv \hat{A}[\hat{v}^n] \ , \tag{4.11}$$

with n an integer, however, the computations that will be made all throughout this chapter could also be applied for the case where $\hat{A}$ is function of momentum field $\hat{p}$ as well [53]. But still there is an other reason behind neglecting a dependence of $\hat{A}$ on $\hat{p}$, where as could be seen through equation (1.129), the momentum field is proportional to the decaying mode, therefore any contribution from p would be subdominant compared to that of $\hat{v}$. We give more details on this point when we discuss the case of a system and environment made of same degree of freedom and the horizon plays the role of cutoff such that long wavelength modes of observation interest makes the system, and short wavelengths act as environment [57, 15, 18].

We come now to derive the Lindblad equation that will turn out to be crucial for all the results obtained in this chapter, we will follow closely [53, 15, 57, 18]. We will get a quantum master equation up to second order in the coupling[2].

We adopt the local form of interaction given by (4.10), we also assume no initial correlation, or entanglement, between system and environment

$$\hat{\rho}(\eta_{in}) = \hat{\rho}_{sys}(\eta_{in}) \otimes \hat{\rho}_{env}(\eta_{in}) \ , \tag{4.12}$$

the presence of the interaction $\hat{H}_{int}$ generates correlations between the two sectors as the system evolves; these correlations make a general description of their further evolution difficult. However, a great simplification is possible if some conditions are satisfied as we will see in a moment. It is worth to mention that many interesting results emerge when we consider our system to be initially entangled

[2]The linear order in g gives vanishing contribution to Lindblad equation due to the assumption 3 in the below list.

with its environment, while the total system evolve according to

$$\hat{H} = \hat{H}_{sys} \otimes \hat{I}_{env} + \hat{I}_{sys} \otimes \hat{H}_{env}\,, \tag{4.13}$$

where no interaction term in the total Hamiltonian and both components evolve freely, we will summarize the main results of such model later on this chapter. Besides, it could be a good idea to investigate the case where we combine both of previous models, namely the case of having an initial correlation between the system and environment but they still interacting with each other during their evolution.

The total density matrix $\hat{\rho}$ obeys the unitary Lioiville von Neumann equation

$$\frac{\mathrm{d}\hat{\rho}}{\mathrm{d}\eta} = -i\left[\hat{\mathcal{H}}, \hat{\rho}\right]\,, \tag{4.14}$$

since we assumed an uncorrelated initial state then the important physics is coming from the evolution induced by $\hat{H}_{int}$, therefore we can factor out the time dependence of $\hat{\rho}$ due to $\hat{\mathcal{H}}_0$by moving into interaction picture, so having the unitary time evolution operator of the free theory

$$\hat{U}\left(t\right) = \mathrm{e}^{-\mathrm{i}\int_0^{\mathrm{t}} \mathrm{H}_0\left(\mathrm{t}'\right)\mathrm{dt}'} \;\; with\ \hat{U}\left(0\right) = 1\,, \tag{4.15}$$

governed by the equation

$$i\frac{\mathrm{d}\hat{U}}{\mathrm{d}\eta} = \hat{H}_0\left(t\right)\hat{U}\,, \tag{4.16}$$

we obtain

$$\tilde{\rho}\left(t\right) = \hat{U}^{\dagger}\rho\left(t\right)\hat{U}\,, \tag{4.17}$$

$$\tilde{H}_{int} = \hat{U}^{\dagger}\hat{H}_{int}\hat{U}\,, \tag{4.18}$$

where $(\tilde{\cdots})$refers to operators in interaction picture. As consequence, the evolution equation is

$$\frac{\mathrm{d}\tilde{\rho}}{\mathrm{d}\eta} = -i\left[\tilde{H}_{int}, \tilde{\rho}\left(t\right)\right]\,, \tag{4.19}$$

whose formal solution is

$$\tilde{\rho}\left(t + \triangle t\right) = \tilde{\rho}\left(t\right) - ig\int_t^{t+\triangle t} dt'\left[\tilde{H}_{int}\left(t'\right), \tilde{\rho}\left(t'\right)\right]\,, \tag{4.20}$$

this expression gives rise to a Born expansion in g of the solution of (4.19) , Indeed, this solution is inserted back into (4.19) leading to the iterative equation

$$\begin{aligned} \tilde{\rho}(t+\triangle t)-\tilde{\rho}(t) &= -ig\int_t^{t+\triangle t}dt'\left[\tilde{H}_{int}(t'),\tilde{\rho}(t)\right]-g^2\int_t^{t+\triangle t}dt'\int_t^{t+\triangle t}dt'' \\ &\times\left[\tilde{H}_{int}(t'),\left[\tilde{H}_{int}(t''),\tilde{\rho}(t)\right]\right] \end{aligned}, \quad (4.21)$$

notice that in the second term of this last equation the density matrix is evaluated at time t, but evaluating it at another time between t and $t+\triangle t$ would just lead to a correction of order g^3, so for future convenience we evaluate it at[3] t''.

Since we are interested on the study of the system and the influence of the environmental fields on it, we need the system reduced density matrix and the quantum master equation that describes its time evolution which is the ultimate goal of this subsection. A firs step toward that is to consistently trace over the environmental degrees of freedom

$$\tilde{\rho}_{sys}(t)=Tr_{env}\left[\tilde{\rho}(t)\right], \quad (4.22)$$

so acting by this trace on (4.21) yields

$$\begin{aligned} \tilde{\rho}_{sys}(t+\triangle t)-\tilde{\rho}_{sys}(t) &= -ig\int_t^{t+\triangle t}dt'Tr_{env}\left[\tilde{H}_{int}(t'),\tilde{\rho}(t)\right]-g^2\int_t^{t+\triangle t}dt'\int_t^{t+\triangle t}dt'' \\ &\times Tr_{env}\left[\tilde{H}_{int}(t'),\left[\tilde{H}_{int}(t''),\tilde{\rho}(t'')\right]\right] \end{aligned}, \quad (4.23)$$

same line of thoughts apply in case we were interested in the studying environment, where its density matrix is given by

$$\tilde{\rho}_{env}(t)=Tr_{sys}\left[\tilde{\rho}(t)\right], \quad (4.24)$$

knowing that generally even if (4.12) holds, but generally the evolved density matrix is given by

$$\tilde{\rho}(t)=\tilde{\rho}_{sys}(t)\otimes\tilde{\rho}_{env}(t)+g^p\tilde{\rho}_{correl}(t), \quad (4.25)$$

where $\tilde{\rho}_{correl}(t)$ describes the correlation between the system and environment; it satisfies $Tr_{sys}\left[\tilde{\rho}_{correl}(t)\right]=0$ and $Tr_{env}\left[\tilde{\rho}_{correl}(t)\right]$. In our case, $\tilde{\rho}_{correl}(t)$ appears if and only if $\hat{H}_{int}$ is switched on, simply because we started from uncorrelated density matrices (4.12), and this fact justifies the appearance of

[3] We did this to be able to use equal time commutators.

g^p in front of it in (4.25) with p being an unknown natural integer. Plugging (4.25) in (4.23) leads to

$$\begin{aligned}\tilde{\rho}_{sys}(t+\triangle t)-\tilde{\rho}_{sys}(t) &= -ig\int_t^{t+\triangle t}dt' Tr_{env}\left[\tilde{H}_{int}(t'),\tilde{\rho}_{sys}(t)\otimes\tilde{\rho}_{env}(t)\right]\\ &-ig^{p+1}\int_t^{t+\triangle t}dt' Tr_{env}\left[\tilde{H}_{int}(t'),\tilde{\rho}_{correl}(t)\right]\\ &-g^2\int_t^{t+\triangle t}dt'\int_t^{t'}dt'' Tr_{env}\left[\tilde{H}_{int}(t'),\left[\tilde{H}_{int}(t''),\tilde{\rho}_{sys}(t'')\otimes\tilde{\rho}_{env}(t'')\right]\right]\\ &-g^{p+2}\int_t^{t+\triangle t}dt'\int_t^{t'}dt'' Tr_{env}\left[\tilde{H}_{int}(t'),\left[\tilde{H}_{int}(t''),\tilde{\rho}_{correl}(t'')\right]\right]\end{aligned}\quad, \tag{4.26}$$

The aim now is determine the terms that contribute the most to the time evolution of $\tilde{\rho}_{sys}(t)$, to this end, we need to specify the form of $\hat{H}_{int}$ so we assume that it is of the form

$$\hat{H}_{int}=A(t)\otimes R(t)\ , \tag{4.27}$$

then we will generalize our results to the interaction form (4.10). Based on (4.15) and (4.27), we see that the evolution operator $\hat{U}$could be factorised as

$$\hat{U}=\hat{U}_{sys}\otimes\hat{U}_{env}\,, \tag{4.28}$$

as consequence

$$\begin{aligned}\tilde{H}_{int}(t) &= \left(\hat{U}^{\dagger}_{sys}\otimes\hat{U}^{\dagger}_{env}\right)(A(t)\otimes R(t))\left(\hat{U}_{sys}\otimes\hat{U}_{env}\right)\\ &= \left(\hat{U}^{\dagger}_{sys}A(t)\hat{U}_{sys}\right)\otimes\left(\hat{U}^{\dagger}_{env}R(t)\hat{U}_{env}\right)\\ &= \tilde{A}(t)\otimes\tilde{R}(t)\end{aligned}\quad, \tag{4.29}$$

we now use this result to evaluate the first term of (4.26) and get

$$Tr_{env}\left[\tilde{H}_{int}(t'),\tilde{\rho}_{sys}(t)\otimes\tilde{\rho}_{env}(t)\right]=Tr_{env}\left[\tilde{R}(t')\tilde{\rho}_{env}(t)\right]\left[\tilde{A}(t'),\tilde{\rho}_{sys}(t)\right]\ , \tag{4.30}$$

to make it further in the computations we must adopt some some crucial assumptions regarding the environment which will give it its properties that could help to constrain the set of possible environments,

1. Since the system is supposed to be small, then we assume that the environments state evolution

is negligibly affected by its interaction with the system, theretofore

$$\tilde{\rho}_{env}(t) \simeq \tilde{\rho}_{env}(0) \equiv \tilde{\rho}_{env}\,, \tag{4.31}$$

is constant in time in the interaction picture, but still, we could have $\hat{\rho}_{env}(t)$.

2. We assume that the intrinsic environment Hamiltonian is time independent, i.e environment is in stationary state so

$$\hat{U}_{env} = \mathrm{e}^{-iH_{env}t}\,, \tag{4.32}$$

in addition, we assume

$$[\tilde{\rho}_{env}, H_{env}] = 0 \Rightarrow \left[\tilde{\rho}_{env}, \hat{U}_{env}\right] = 0\,, \tag{4.33}$$

then from (4.17),

$$\rho_{env}(t) = \hat{U}\tilde{\rho}_{env}\hat{U}^{\dagger}\,, \tag{4.34}$$

we see that $\rho_{env}(t)$ is itself time independent, i.e $\rho_{env} = \tilde{\rho}_{env}$. Consequently,

$$[\rho_{env}, H_{env}] = 0\,, \tag{4.35}$$

and the environment density operator can be written as

$$\tilde{\rho}_{env} = \sum_n p_n |n\rangle\langle n|\,, \tag{4.36}$$

where $|n\rangle$are eigenstates of H_E with eigenvalues E_n, and p_n are constant real coefficients. This second assumption called **factorization** in [15], or together with previous assumption they are called **Born approximation** in [68], could be written as

$$\tilde{\rho}(t) = \tilde{\rho}_{sys}(t) \otimes \tilde{\rho}_{env}(0)\,, \tag{4.37}$$

and as argued in this last reference, the factorization assumption is an ubiquitous approximation. [62, 19]

3. Finally, we assume that the mean value of $\hat{R}(t)$ vanishes, namely [4]

$$\left\langle \hat{R}(t) \right\rangle = Tr_{env}\left[\hat{R}(t)\,\tilde{\rho}_{env}\right] = 0\,, \tag{4.38}$$

the trace is taken with respect to environmental initial states in Hilbert space $\mathcal{E}$. This condition

[4] This condition could be a achieved by a simple redefinition of system Hamiltonian and interaction Hamiltonian while the total one remains unchanged, namely we perform the transformation $\hat{H}_{sys} \to \hat{H}_{sys} + Tr_{env}\left[\hat{R}\tilde{\rho}_{env}\right]$ and $\hat{H}_{int} \to A(t) \otimes R(t) - Tr_{env}\left[\hat{R}\tilde{\rho}_{env}\right] \otimes I_{env}$.

could be achieved by a simple redefinition of system Hamiltonian and interaction Hamiltonian while the total one remains unchanged, namely we perform the transformation $\hat{H}_{sys} \rightarrow \hat{H}_{sys} + Tr_{env}\left[\hat{R}\tilde{\rho}_{env}\right]$ and $\hat{H}_{int} \rightarrow A(t) \otimes R(t) - Tr_{env}\left[\hat{R}\tilde{\rho}_{env}\right] \otimes I_{env}$. The same assumption was adopted in [15], where he assumed an interaction Hamiltonian normal ordered in $\hat{R}$ i.e

$$\hat{H}_{int} = A(t) \otimes \left(R(t) - \langle R(t)\rangle_{env}\right) , \tag{4.39}$$

where $\langle R(t)\rangle_{env}$ refer to the expectation value with respect to the initial density matrix $\tilde{\rho}_{env}$. Notice that we have in (4.30) $Tr_{env}\left[\tilde{R}(t')\tilde{\rho}_{env}\right]$, so how to relate this last to the assumption in (4.38)?

using the cyclic property of the trace in addition to (4.33), then

$$\begin{aligned} Tr_{env}\left[\tilde{R}(t')\tilde{\rho}_{env}(t)\right] &= Tr_{env}\left[\hat{U}_E^{\dagger}\hat{R}\hat{U}_E\tilde{\rho}_{env}\right] = Tr_{env}\left[\hat{U}_E^{\dagger}\hat{R}\tilde{\rho}_{env}\hat{U}_E\right] = Tr_{env}\left[\hat{U}_E\hat{U}_E^{\dagger}\hat{R}\tilde{\rho}_{env}\right] \\ &= Tr_{env}\left[\hat{R}\tilde{\rho}_{env}\right] = 0 \end{aligned} , \tag{4.40}$$

therefore the first term in (4.26), and this result will enable us to constrain the value of p. Since in the absence of interaction, i.e $\hat{H}_{int} = 0$, the reduced system density matrix in interaction picture $\tilde{\rho}_{sys}$ does not evolve we conclude that the left hand side of (4.26) is proportional to g^{p+1}. While the right hand side of (4.26) has terms proportional to g^2, g^{p+1}, g^{p+2}, so identifying the two sides constrain the value of p to 2 which gives the dominant contribution, therefore (4.26) reduces now to

$$\tilde{\rho}_{sys}(t+\triangle t) - \tilde{\rho}_{sys}(t) = -g^2\int_t^{t+\triangle t} dt' \int_t^{t'} dt'' Tr_{env}\left[\tilde{H}_{int}(t'), \left[\tilde{H}_{int}(t''), \tilde{\rho}_{sys}(t'') \otimes \tilde{\rho}_{env}(t'')\right]\right] , \tag{4.41}$$

cutting the other terms induces a fourth implicit assumption that is

4.The interaction modifies the dynamics of the system in the perturbative regime only, so (4.41) is valid only at leading order in g and this last statement will be important when we come to evaluate the different correlation functions, where it will dictate the terms of Lindblad which could contribute.

The Lindblad equation, or more precisely the real and non unitary Lindblad term, we are looking for will result from (4.41) so a first step toward its derivation is to develop the double commutator[5],

$$\begin{aligned} Tr_{env}\left[\tilde{H}_{int}(t'), \left[\tilde{H}_{int}(t''), \tilde{\rho}_{sys}(t'') \otimes \tilde{\rho}_{env}(t'')\right]\right] &= \left[\tilde{A}(t'), \tilde{A}(t'')\tilde{\rho}_{sys}(t'')\right] C_R(t'-t'') \\ &- \left[\tilde{A}(t'), \tilde{\rho}_{sys}(t'')\tilde{A}(t'')\right] C_R(t''-t') \end{aligned} , \tag{4.42}$$

[5]More details on the computations could be found in Appendix A of [53].

where C_R refers to the environment two point correlation function

$$C_R\left(t',t\right) = Tr_{env}\left[\tilde{\rho}_{env}\left(t\right)\tilde{R}\left(t\right)\tilde{R}\left(t'\right)\right]\,, \tag{4.43}$$

Since the environment is in stationary state it could be shown that $C_R\left(t',t\right)$is in fact a function of $\tau = t - t'$, where we just need to use the cyclic property of the trace in addition to

$$\tilde{R}\left(t\right) = \mathrm{e}^{iH_{env}t}\tilde{R}\left(0\right)\mathrm{e}^{-iH_{env}t}\,, \tag{4.44}$$

so

$$C_R\left(t',t\right) \equiv C_R\left(\tau\right) = C_R\left(t'-t\right)\,, \tag{4.45}$$

If we use the expansion (4.36) then we can obtain a more explicit from of $C_R\left(\tau\right)$

$$C_R\left(\tau\right) = \sum_{n,m} p_n \mathrm{e}^{i\left(E_n - E_m\right)\tau}\left|\langle n|\,\tilde{R}\left(0\right)|m\rangle\right|\,, \tag{4.46}$$

with

$$C_R\left(-\tau\right) = C_R^*\left(\tau\right)\,, \tag{4.47}$$

we see that $C_R\left(\tau\right)$ is sum of exponentials oscillating at Bohr frequencies of environment [6], so in case of large environment with an almost continuous set of energy levels, then destructive interferences occurs quickly within a characteristic time t_c

$$C_R\left(\tau\right) \simeq C_R\left(0\right)\mathrm{e}^{-|\tau|/t_c}\,, \tag{4.48}$$

Before substituting (4.42) in (4.41) and carry out the integral, we perform a change of the integration domain into the parameters $t'$and $\tau = t' - t''$ so that

$$\int_t^{t+\triangle t} dt' \int_t^{t'} dt'' = \int_0^{\triangle t} d\tau \int_{t+\tau}^{t+\triangle t} dt'\,, \tag{4.49}$$

notice that τ is comprised

$$0 \leq \tau \leq \triangle t\,, \tag{4.50}$$

so once we fix τ then integration along $t'$is bounded from below by $t+\tau$, the original integration domain is displayed in blue in figure (4.1). But due to the presence of $C_R\left(\tau\right)$and $C_R\left(-\tau\right)$ in (4.42) the integrand vanishes for $|\tau| \gg t_c$, so the support of $\int_0^{\triangle t} d\tau$ is limited by the pale green strip in figure (4.1).

[6]Since the energy levels refer to states of environment.

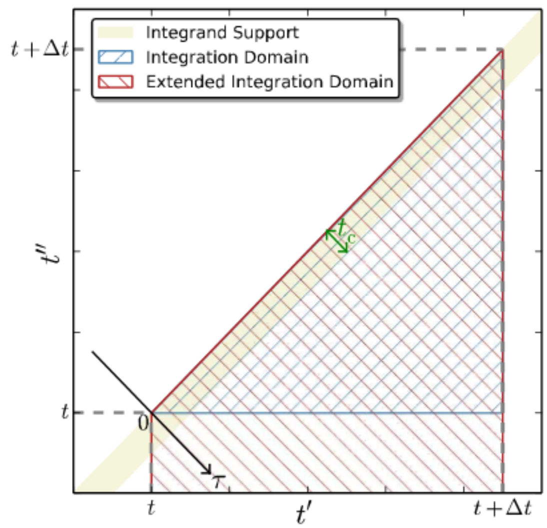

Figure 4.1: Original Integration domain (hatched blue surface). In the limit where $t_C \ll \Delta t$, the extended integration domain (hatched red surface) almost coincides with the initial one when restricted to the region where the integrand is not vanishingly small (pale green surface).[53]

In order to simplify the integration we consider an extended integration domain

$$\int_0^{\infty} d\tau \int_t^{t+\triangle t} dt' \, , \tag{4.51}$$

where the upper bound on τ has been extended to ∞, and the lower bound on t' to t, this extension adds to original integration domain two regions. The first is outside integrand support and therefore is very suppressed, while the second one is the small triangle that is inside the integrand support but its contribution would be very suppressed if

$$t_c \ll \triangle t \, , \tag{4.52}$$

so we add a fifth assumption into the four previous ones and suppose that

5. We follow the evolution of the reduced density matrix for the system on time scales Δt much larger than the typical correlation time of the environment. In this case the dynamics of system over times $t \gg t_c$ does not retain any memory of the correlations with environment, since these only survive for much shorter times. This allows us to treat the evolution of system in the presence of environment as a Markov process.

Taking into account all this transformations in addition to the fifth assumption, then substituting (4.42) in (4.41) yields

$$\begin{aligned} \tilde{\rho}_{sys}(t+\triangle t)-\tilde{\rho}_{sys}(t) &= -g^2\int_0^\infty d\tau\int_t^{t+\triangle t}dt'\left\{\left[\tilde{A}(t'),\tilde{A}(t'-\tau)\tilde{\rho}_{sys}(t'-\tau)\right]C_R(\tau)\right. \\ &\left.-\left[\tilde{A}(t'),\tilde{\rho}_{sys}(t'-\tau)\tilde{A}(t'-\tau)\right]C_R(-\tau)\right\} \end{aligned}, \quad (4.53)$$

the time derivative of $\tilde{\rho}_{sys}(t)$ could be approximately obtained by dividing the left hand side by $\triangle t$. Notice that since the time variation of $\tilde{\rho}_{sys}(t)$ is proportional to g^2 then, as we are interested in the leading order evolution of it, we can approximate $\tilde{\rho}_{sys}(t'-\tau)$ in the right hand side by $\tilde{\rho}_{sys}(t)$, this approximation is called the second Markov approximation [15]. A final assumption is

6.The time scale $\triangle t$ is much smaller than the time scale by which the system interaction operator $\hat{A}$varies, i.e $\tilde{A}(t')\simeq\tilde{A}(t)$and $\tilde{A}(t'-\tau)\simeq\tilde{A}(t-\tau)$. This automatically implies that $\hat{A}$ must vary on time scales much larger than environmental autocorrelation time t_c.

With those considerations the integral over $t'$is now trivial and gives a factor of $\triangle t$

$$\begin{aligned} \frac{\triangle\tilde{\rho}_{sys}}{\triangle t} &= -g^2\int_0^\infty d\tau\left\{\left[\tilde{A}(t),\tilde{A}(t-\tau)\tilde{\rho}_{sys}(t)\right]C_R(\tau)\right. \\ &\left.-\left[\tilde{A}(t'),\tilde{\rho}_{sys}t\tilde{A}(t-\tau)\right]C_R(-\tau)\right\} \end{aligned}, \quad (4.54)$$

the above equation describes a Markovian process, since the evolution of the system is dictated only by its current state. In order to compare it to the mater equation obtained in other papers for a similar system, i.e cosmological perturbations and especially scalar ones, we define

$$\begin{aligned} \hat{L}_1 &\equiv g^2\int_0^\infty d\tau C_R(\tau)\tilde{A}(t-\tau) \\ \hat{L}_2 &\equiv g^2\int_0^\infty d\tau C_R(-\tau)\tilde{A}(t-\tau)=g^2\int_0^\infty d\tau C_R^*(\tau)\tilde{A}(t-\tau)=L_1^\dagger(t) \end{aligned}, \quad (4.55)$$

the last equality holds if $\tilde{A}$ is hermitian. using $L_1$and L_2 then (4.54) it could be written

$$\frac{\triangle\tilde{\rho}_{sys}}{\triangle t}=\left[\tilde{A}(t),\tilde{\rho}_{sys}(t)L_2\right]-\left[\tilde{A}(t),\tilde{\rho}_{sys}(t)L_1\right], \quad (4.56)$$

this equation could be simplified further by computing the explicit expressions of $L_1$and L_2, so using (4.48) and (4.52) in addition to the fact that $\tilde{A}(t-\tau) \simeq \tilde{A}(t)$ then

$$\hat{L}_1 = g^2 \int_0^\infty d\tau C_R(\tau) \tilde{A}(t-\tau) \simeq g^2 \int_0^\infty d\tau C_R(0) \mathrm{e}^{-|\tau|/t_c} \tilde{A}(t) = g^2 C_R(0) t_c \tilde{A}(t) \ , \tag{4.57}$$

and same expression for $\hat{L}_2$, therefore (4.56) becomes

$$\frac{\mathrm{d}\tilde{\rho}_{sys}}{\mathrm{d}t} = -g^2 C_R(0) t_c \left[\tilde{A}(t), \left[\tilde{A}(t), \tilde{\rho}_{sys}\right]\right] \ , \tag{4.58}$$

so going back to standard picture

$$\frac{\mathrm{d}\hat{\rho}_{sys}}{\mathrm{d}t} = i\left[\hat{\rho}_{sys}, \hat{H}_{sys}\right] - g^2 C_R(0) t_c \left[\hat{A}(t), \left[\hat{A}(t), \tilde{\rho}_{sys}\right]\right] \ , \tag{4.59}$$

we obtained the standard Lindblad equation, that was also obtained in the various references mentioned above. As intermediate step toward the Lindblad equation describing the local interaction (4.10) we consider

$$\hat{H}_{int} = \sum_i \hat{A}_i(t) \otimes \hat{R}_i(t) \ , \tag{4.60}$$

associated to the the environmental two point correlation function

$$C_{R,ij}(t,t') \equiv Tr_E \left[\tilde{\rho}_{env}(t) \tilde{R}_i(t) \tilde{R}_j(t')\right] \ , \tag{4.61}$$

redoing the steps leaded to (4.59) yield

$$\frac{\mathrm{d}\hat{\rho}_{sys}}{\mathrm{d}t} = i\left[\hat{\rho}_{sys}, \hat{H}_{sys}\right] - g^2 \sum_{i,j} C_{R,ij}(0) t_{c,ij} \left[\hat{A}_i(t), \left[\hat{A}_j(t), \tilde{\rho}_{sys}\right]\right] \ , \tag{4.62}$$

where $t_{c,ij}$ is the characteristic time of correlation functions $C_{R,ij}$and they all must be less than $\triangle t$. Now it is easy to see how to generalize the previous equation in case we promoted the discrete indices (i,j) into continuous ones $(\boldsymbol{x},\boldsymbol{y})$ so that $\hat{H}_{int}$ is now given by (4.10) and (4.62) becomes

$$\frac{d\hat{\rho}_{sys}}{d\eta} = i\left[\hat{\rho}_{sys}, \hat{H}_{sys}\right] - \frac{\gamma}{2}\int d^3\boldsymbol{x} d^3\boldsymbol{y} C_R(\boldsymbol{x},\boldsymbol{y}) \left[\left[\hat{\rho}_{sys}, \hat{A}(\boldsymbol{x})\right], \hat{A}(\boldsymbol{y})\right] \ , \tag{4.63}$$

where $\gamma = 2g^2 t_c$, knowing that any possible dependence of t_c on $(\boldsymbol{x},\boldsymbol{y})$is absorbed in $C_R(\boldsymbol{x},\boldsymbol{y})$ that represents the same time correlation function of $\hat{R}$ in the environment. In addition, the parameter γ is generally time dependent so we adopt for it a power law dependence in scale factor [53]

$$\gamma = \gamma_* \left(\frac{a}{a_*}\right)^p \ , \tag{4.64}$$

where p represents a free parameter, and $*$ refers to reference time that is taken to be the Hubble time crossing of the pivot scale $k_* = 0.051 Mpc^{-1}$, i.e $k_* = a_* H$. We need also to adopt a convention for the correlator $C_R(\boldsymbol{x}, \boldsymbol{y})$, so assuming the environment to be in statistically homogeneous, i.e $C_R(\boldsymbol{x}, \boldsymbol{y}) \propto \boldsymbol{x} - \boldsymbol{y}$, and isotropic configuration, i.e $C_R(\boldsymbol{x}, \boldsymbol{y}) \propto |\boldsymbol{x} - \boldsymbol{y}|$, in addition, if we assume also that it is characterized by a correlation, physical, length scale l_E then $C_R(\boldsymbol{x}, \boldsymbol{y})$ must be a function of [7] $a|\boldsymbol{x} - \boldsymbol{y}|/l_E$, and for convenience we assume it to be top hat function [53]

$$C_R(\boldsymbol{x}, \boldsymbol{y}) = \bar{C}_R \Theta\left(\frac{a|\boldsymbol{x} - \boldsymbol{y}|}{l_E}\right), \tag{4.65}$$

with

$$\Theta(x) = \begin{cases} 1 & if\, x < 1 \\ \\ 0 & \text{otherwise} \end{cases}, \tag{4.66}$$

and $\bar{C}_R$ is constant. The correlation function will be left unspecified in the following, but it could computed explicitly once a specific model for the environment is considered, some examples could be found in [15, 53, 18].

Before moving to discuss the important consequences of (4.63) we summarize briefly the conditions under which this equation holds, essentially are [52]

1. The environment evolves on a time scale that is much smaller than that of the system.
2. The backreaction of the system on the environment is negligible.
3. The influence of the environment on the system, that is here clearly crucial, can be treated perturbatively.

4.1.3 Transition from pure to mixed state

A key feature of transition from quantum to classical state is the transition from a pure state

$$\hat{\rho}_{sys} = |\Psi\rangle\langle\Psi|, \tag{4.67}$$

for which there exist a vector state $|\Psi\rangle$ encoding all the information about the system, into a mixed state which could not be built from a vector state as in (4.67), and for which the off diagonal elements of $\hat{\rho}_{sys}$ are suppressed. Our aim now is to show how the non unitary, real, Lindblad term in (4.63) is responsible for transition, and that such suppression of non diagonal elements of $\hat{\rho}_{sys}$ is controlled by the correlation function $C_R(\boldsymbol{x}, \boldsymbol{y})$, we will follow closely [18].

[7] Remember that $(\boldsymbol{x}, \boldsymbol{y})$ are comoving coordinates.

Let us pick up the field amplitude $\hat{v}$ eigenbasis $|v\rangle$ as pointer basis, such that $\hat{v}\,|v\rangle = v\,|v\rangle$, therefore the system density matrix elements in this basis are given by

$$\rho_{sys}\left[v', v\right] = \langle v'|\, \tilde{\rho}_{sys}\, |v\rangle \ , \tag{4.68}$$

and the action of $\hat{A}\left(\boldsymbol{x}\right)$ on $|v\rangle$is given by

$$\hat{A}\left(\boldsymbol{x}\right)|v\rangle = A\left(\boldsymbol{x}\right)|v\rangle \quad and\ \hat{A}\left(\boldsymbol{x}\right)|v'\rangle = A'\left(\boldsymbol{x}\right)|v'\rangle \ , \tag{4.69}$$

where for simplicity we supposed that $\hat{A}\left(\boldsymbol{x}\right)$ are functions of $\hat{v}$. So taking into account that the eigenbasis vectors are fixed in time , then from (4.63) we have

$$\begin{aligned} \frac{d\rho_{sys}\left[v',v\right]}{d\eta} &= i\,\langle v'|\left[\hat{\rho}_{sys}, \hat{H}_{sys}\right]|v\rangle - \tfrac{\gamma}{2}\int d^3\boldsymbol{x}d^3\boldsymbol{y}C_R\left(\boldsymbol{x},\boldsymbol{y}\right)\langle v'|\left[\left[\hat{\rho}_{sys}, \hat{A}\left(\boldsymbol{x}\right)\right], \hat{A}\left(\boldsymbol{y}\right)\right]|v\rangle \\ &= i\,\langle v'|\left[\hat{\rho}_{sys}, \hat{H}_{sys}\right]|v\rangle - \tfrac{\gamma}{2}\rho_{sys}\left[v',v\right]\int d^3\boldsymbol{x}d^3\boldsymbol{y}C_R\left(\boldsymbol{x},\boldsymbol{y}\right)\left[A\left(\boldsymbol{x}\right) - A'\left(\boldsymbol{x}\right)\right]\left[A\left(\boldsymbol{y}\right) - A'\left(\boldsymbol{y}\right)\right] \end{aligned} \ . \tag{4.70}$$

Notice that upon integration the first term in right hand side describes a Hamiltonian evolution which could not generate a mixed state from an initial pure state, so focusing on the second term then

$$\rho_{sys}\left[v', v\right]_{\eta} = \rho_{sys}\left[v', v\right]_{\eta_{in}} \mathrm{e}^{-\Gamma} \ , \tag{4.71}$$

with

$$\Gamma = \int_{\eta_{in}}^{\eta} d\eta' \frac{\gamma}{2}\int d^3\boldsymbol{x}d^3\boldsymbol{y}C_R\left(\boldsymbol{x},\boldsymbol{y}\right)\left[A\left(\boldsymbol{x}\right) - A'\left(\boldsymbol{x}\right)\right]\left[A\left(\boldsymbol{y}\right) - A'\left(\boldsymbol{y}\right)\right] \ , \tag{4.72}$$

if we adopt a local form of correlation function[8] i.e

$$C_R\left(\boldsymbol{x},\boldsymbol{y}\right) \equiv Tr_{env}\left[\tilde{\rho}_{env}\left(t\right)\tilde{R}\left(\boldsymbol{x}\right)\tilde{R}\left(\boldsymbol{y}\right)\right] = C_R\left(\boldsymbol{x}\right)\delta^{(3)}\left(\boldsymbol{x}-\boldsymbol{y}\right) \ , \tag{4.73}$$

then we can easily see that the Lindblad term causes the system density matrix to take the form of classical Gaussian distribution in in $A\left(\boldsymbol{x}\right)$,with a time dependent width controlled by $\gamma C_R\left(\boldsymbol{x}\right)$. Therefor if this width shrinks at late times then the $\rho_{sys}\left[v', v\right]_{\eta}$ evolves into diagonal one with probabilities set by the initial density matrix

$$P_{\eta}\left[v\right] \equiv \rho_{sys}\left[v, v\right]_{\eta} = \rho_{sys}\left[v, v\right]_{\eta_{in}} = \left|\Psi\left[v\left(\eta_{in}, x\right)\right]\right|^2 \ . \tag{4.74}$$

[8]Actually this is strong condition which is taken here just to show the pure to mixed states transition, however, in the rest of thesis we will work with (4.65) where $C_R\left(\boldsymbol{x},\boldsymbol{y}\right)$depends of the speration.

These solutions describe the decoherence of the initial state into the classical stochastic ensemble for the variables $\{v\}$ that diagonalize the interactions with the decohering environment for a decoherence induces preferred basis $|v\rangle$.

4.1.4 What if the system and environment initial states were correlated?

Among the key assumptions in the derivation of Lindblad equation was assuming that our system and its environment states were uncorrelated, or not entangled, as reflected by equation (4.12), so we would like to ask what if they were correlated?

The **partial** answer to this question is available in several papers, and we will briefly summarize the main results obtained in [5] which could easily be compared to the result presented in this thesis. In [5] the authors assumed the state of primordial scalar perturbations to be entangled with an environment made of an external scalar field, minimally coupled to gravity and with no other interactions. This last point is what makes their work provide only a partial answer to our question, because their system and its environment evolve freely, i.e there are no interactions in the Hamiltonian between them. Therefore, it would be interesting to study a model that contains both cases, namely an initial correlated states which continue to interact as they evolve.

The action of the two scalar fields φ and ψ that make our system and environment, respectively, is given by

$$S \quad = -\tfrac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\left[g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi+m_\varphi^2\varphi^2+g^{\mu\nu}\partial_\mu\psi\partial_\nu\psi+M^2\psi^2\right] \quad , \tag{4.75}$$

since we will study our system in Schrodinger picture, then, we will need the Hamiltonian to solve the Schrodinger functional equation later on,

$$H = \frac{1}{2}\int \mathrm{d}^3\boldsymbol{x}\left[\frac{P_\varphi^2}{a^2}+\frac{P_\psi^2}{a^2}+a^2\left((\nabla\varphi)^2+a^2m_\varphi^2\varphi^2\right)+a^2\left((\nabla\psi)^2+a^2M^2\psi^2\right)\right] , \tag{4.76}$$

where the conjugate momenta $P_\varphi$and P_ψ are defined by

$$P_\varphi = \frac{\partial\mathcal{L}}{\partial\varphi'},\ P_\psi = \frac{\partial\mathcal{L}}{\partial\psi'} , \tag{4.77}$$

The sate of the total system is described by the wave functional $\Psi\left[\{\varphi_{\boldsymbol{k}}\}\{\psi_{\boldsymbol{k}}\};\eta\right]$ where the fields are Fourier transformed similarly to (1.35), we will adopt the following representation of field momentum operators

$$\hat{P}_\varphi = -i\frac{\delta}{\delta\varphi_{-\boldsymbol{k}}},\ \hat{P}_\psi = -i\frac{\delta}{\delta\psi_{-\boldsymbol{k}}} , \tag{4.78}$$

and since we have

$$\hat{H} = \int \mathrm{d}^3\boldsymbol{k}\left(\hat{H}_{\varphi} + \hat{H}_{\psi}\right), \tag{4.79}$$

$$\begin{aligned} \hat{H}_{\varphi,\boldsymbol{k}} &= \tfrac{\hat{P}_{\varphi,\boldsymbol{k}}\hat{P}_{\varphi,-\boldsymbol{k}}}{2a^2} + a^2\left(k^2 + a^2 m_{\varphi}^2\right)\hat{\varphi}_k\hat{\varphi}_{-k} \\ \hat{H}_{\psi,\boldsymbol{k}} &= \tfrac{\hat{P}_{\psi,\boldsymbol{k}}\hat{P}_{\psi,-\boldsymbol{k}}}{2a^2} + a^2\left(k^2 + a^2 m_{\varphi}^2\right)\hat{\psi}_k\hat{\psi}_{-k} \end{aligned} , \tag{4.80}$$

then we see that the different Fourier modes evolve interdependently and do not interact with each other so we can write

$$\Psi\left[\{\varphi_{\boldsymbol{k}}\}\{\psi_{\boldsymbol{k}}\};\eta\right] = \prod_k \Psi_k\left[\hat{\varphi}_k, \hat{\psi}_k; \eta\right], \tag{4.81}$$

in spite of no interactions in $\hat{H}$ between φ and ψ, we cannot factorize $\Psi_k\left[\hat{\varphi}_k, \hat{\psi}_k; \eta\right]$ into two pieces depending only on $\hat{\varphi}_k$ and $\hat{\psi}_k$ separately because we want to consider an initial entangled state. Since $\hat{H}$ is a quadratic Hamiltonian, then, an intuitive choice of the state of the total system is the nearest to a Gaussian state

$$\begin{aligned} \Psi_k\left[\hat{\varphi}_k, \hat{\psi}_k; \eta\right] &= N_k(\eta)\exp\Big[-\tfrac{1}{2}\Big(\Omega_k^{\varphi}(\eta)\,\varphi_{\boldsymbol{k}}(\eta_{in})\,\varphi_{-\boldsymbol{k}}(\eta_{in}) + \Omega_k^{\psi}(\eta)\,\psi_{\boldsymbol{k}}(\eta_{in})\,\psi_{-\boldsymbol{k}}(\eta_{in})\Big) \\ &\quad + C_k(\eta)\left(\varphi_{\boldsymbol{k}}(\eta_{in})\,\psi_{-\boldsymbol{k}}(\eta_{in}) + \psi_{\boldsymbol{k}}(\eta_{in})\,\varphi_{-\boldsymbol{k}}(\eta_{in})\right)\Big] \end{aligned} , \tag{4.82}$$

it is interesting to notice the similar structure of this state with the one considered in (1.88),where the state above would correspond to a four mode ($\pm k$ for each field) squeezed state, generalizing the usual two mode squeezed state description of a single field in de Sitter space given by (1.88). The parameter $C_k(\eta)$ is the one that serves to impose initial correlated state between the two fields through imposing $C_k(\eta_{in}) \neq 0$. Following the same steps done in (1.2.2) we can obtain the equations of motion for the various parameters $N_k(\eta)$, $\Omega_k^{\varphi}(\eta)$, $\Omega_k^{\psi}(\eta)$,$C_k(\eta)$. Obviously we expect to get modified equations with respect to the ones obtained in (1.2.2) due to the presence of entanglement parameter. Therefore a modified expression for $\Omega_k^{\varphi}(\eta)$would imply a modification of the power spectrum since they are proportional as could be seen through equation (1.105).

The modified power spectrum obtained in [5]

$$\mathcal{P}_{\varphi} = \triangle_{\varphi}^2 \equiv \frac{k^3}{2\pi^2}\left\langle \varphi_{\boldsymbol{k}}\varphi_{-\boldsymbol{k}}\right\rangle, \tag{4.83}$$

exhibited an oscillatory behavior which is justified by the fact that the initial state of inflaton was deviated from the Bunch Davies vacuum due to the initial entanglement. This last puts the inflaton in a mixture of energy eigenstates which resulted in oscillations, see figure(4.2), therefore, the size of those last is controlled by the amount of deviation from Bunch Davies vacuum.

It is worth to mention that the authors treated the same problem in a different paper [14], but this

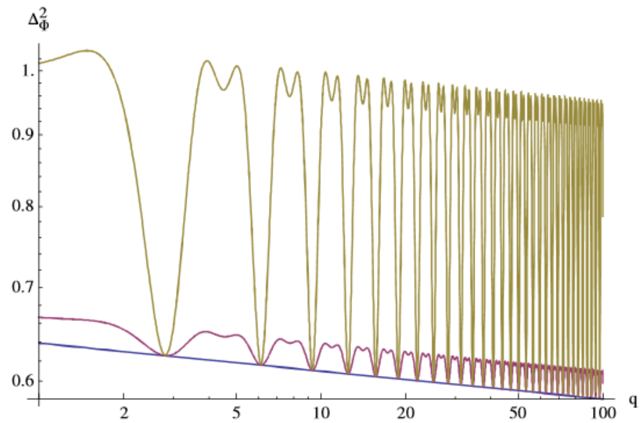

Figure 4.2: The power spectrum $\triangle^2_\varphi$ versus q, for different values of the entanglement parameter $\lambda_q = 0, 0.1, 0.3$using φ nearly massless and ψ massless. The non-entangled curve is straight, and increasing the entanglement introduces increasing amplitudes of oscillatory behavior on top of the straight piece.[5]

time the environment was made of the primordial tensor fluctuations and they obtained interesting results which could explain some CMB anomalies, namely the large scale anomalies.

4.2 Decoherence of scalar perturbations

As mentioned previously, an efficient way of verifying, or falsifying, an early universe model is by computing their various correlation functions and confront them with the excellent data collected so far which made cosmology enters the precision era. The excellent observational constraints on power spectrum of primordial scalar perturbations serve to constrain any decoherence induced correction, and this will help us by turn to constrain the set of possible environments and reveal important hints about early universe physics. Same line of thoughts apply to higher order correlation functions, namely non guassianities, but unfortunately the experimental bounds on them are not yet good enough to serve to constrain cosmological decoherence models.

In the computation of decoherence induced corrections to the various scalar correlation functions there are two approaches distinguished by the form of system interaction operator $\hat{A}$ considered in (4.63). The first is the one adopted by J.martin et al in [53, 52] where they considered either linear or quadratic interaction operator i.e $\hat{A} = \hat{v}$ or $\hat{A} = \hat{v}^2$, while our approach is based on considering the operator $\hat{A} = \sum_n \alpha^{n-1}\hat{v}^n$ up to certain order[9]. Our approach does not only generalize the work of

[9]During computations with our approach, we will stop at the leading order that gives non vanishing correction to the

J.Martin et al, but it leads also to important new results as we will see.

To compute the correlation functions there are two methods:

1. Either, we solve the evolution equation (4.63) and obtain an explicit expression of the system density matrix $\hat{\rho}_{sys}$, then we use it to compute the correlation function $O = f[\hat{v}, \hat{p}]$ through obtaining its expectation value

$$O \equiv \left\langle \hat{O} \right\rangle = Tr_{sys}\left(\hat{O}\hat{\rho}_{sys}\right) . \tag{4.84}$$

However, in most of the cases it is highly challenging to solve (4.63) exactly and the only case that this is doable with reasonable easiness is the linear case $\hat{A} \propto \hat{v}$. Where, in that case, the Lindblad term is quadratic in v just as the free Hamiltonian is, and we obtain Gaussian density matrix with a width controlled by the environment.

2. But apart from the linear case it is better to pursue the second method in computing the correlation functions, which consists in solving directly the equation of motion governing $\left\langle \hat{O} \right\rangle$. Therefore, using (4.63) and (4.84) we obtain

$$\frac{d\left\langle \hat{O} \right\rangle}{d\eta} = \left\langle \frac{\partial \hat{O}}{\partial \eta} \right\rangle - i\left\langle \left[\hat{O}, \hat{H}_v\right] \right\rangle - \frac{\gamma}{2}\int d^3\boldsymbol{x}d^3\boldsymbol{y}C_R(\boldsymbol{x},\boldsymbol{y})\left\langle \left[\left[\hat{O}, \hat{A}(\boldsymbol{x})\right], \hat{A}(\boldsymbol{y})\right] \right\rangle . \tag{4.85}$$

For the linear case $\hat{A} \propto \hat{v}$ we will compute the power spectrum using both of previous methods and show that they are equivalent, however for higher order correlation functions we will use solely the second method. Please notice that by the word **"method"** we refer to the way of computing the correlation functions, while the word **"approach"** refers to the choice adopted regarding the system interaction operators $\hat{A}(\boldsymbol{x})$, either that of J.Martin et al or our choice.

4.2.1 J.Martin et al approach

4.2.1.1 Computation of power spectrum with linear interaction

First Method Considering a linear interaction with environment $\hat{A}(\boldsymbol{x}) = \hat{v}(\boldsymbol{x})$ then (4.63) becomes

$$\frac{d\hat{\rho}_{sys}}{d\eta} = i\left[\hat{\rho}_{sys}, \hat{H}_{sys}\right] - \frac{\gamma}{2}\int d^3\boldsymbol{x}d^3\boldsymbol{y}C_R(\boldsymbol{x},\boldsymbol{y})\left[\left[\hat{\rho}_{sys}, \hat{v}(\boldsymbol{x})\right], \hat{v}(\boldsymbol{y})\right] , \tag{4.86}$$

defining the Fourier transform

$$\hat{v}(\boldsymbol{x}) = \frac{1}{(2\pi)^{3/2}}\int \mathrm{d}^3\boldsymbol{k}\hat{v}_{\boldsymbol{k}}e^{i\boldsymbol{k}.\boldsymbol{x}} , \tag{4.87}$$

correlation function considered, except for the case of power spectrum we will consider also next leading order to show that the computations done by J.martin et all were missing a contribution.

$$C_R(\boldsymbol{x},\boldsymbol{y}) = \frac{1}{(2\pi)^{3/2}} \int \mathrm{d}^3\boldsymbol{k}\widetilde{C}_R(|\boldsymbol{k}|)\, e^{i\boldsymbol{k}.(\boldsymbol{x}-\boldsymbol{y})}\,, \tag{4.88}$$

in this last we used the fact that due to homogeneity and isotropy assumption about environment we have

$$C_R(\boldsymbol{x},\boldsymbol{y}) = C_R(|\boldsymbol{x}-\boldsymbol{y}|)\ . \tag{4.89}$$

Now substituting (4.65) for the environmental correlation function gives

$$\widetilde{C}_R(|\boldsymbol{k}|) = \sqrt{\frac{2}{\pi}}\frac{\bar{C}_R}{k^3}\left[\sin\left(\frac{kl_E}{a}\right) - \frac{kl_E}{a}\cos\left(\frac{kl_E}{a}\right)\right]\,, \tag{4.90}$$

this last can itself be be approximated by a top hat function

$$\widetilde{C}_R(|\boldsymbol{k}|) = \sqrt{\frac{2}{\pi}}\frac{\bar{C}_R l_E}{k^3 a^3}\Theta\left(\frac{kl_E}{a}\right)\,, \tag{4.91}$$

We have shown that in the free case the evolution equation of $\hat{\rho}$ gets decoupled into a set of equations (4.8) governing the evolution of each Fourier mode independently of others, redoing the same steps with the Lindblad non linear term

$$-\frac{\gamma}{2}(2\pi)^{3/2}\int d^3\boldsymbol{k}\widetilde{C}_R(|\boldsymbol{k}|)\left\langle\left[\left[\hat{O},\hat{v}_{\boldsymbol{k}}\right],\hat{v}_{-\boldsymbol{k}}\right]\right\rangle\,, \tag{4.92}$$

using (4.3), in addition to$\hat{v}_{\boldsymbol{k}} = \left(\hat{v}^R_{\boldsymbol{k}} + i\hat{v}^I_{\boldsymbol{k}}\right)/\sqrt{2}$ with $\hat{v}^R_{\boldsymbol{k}} = \hat{v}^R_{-\boldsymbol{k}}$ and $\hat{v}^I_{-\boldsymbol{k}} = -\hat{v}^I_{-\boldsymbol{k}}$, one obtains [10]

$$\begin{aligned}\int d^3\boldsymbol{x}d^3\boldsymbol{y}C_R(\boldsymbol{x},\boldsymbol{y})\left[\left[\hat{\rho}_{sys},\hat{v}(\boldsymbol{x})\right],\hat{v}(\boldsymbol{y})\right] &= (2\pi)^{3/2}\int \mathrm{d}\boldsymbol{k}\widetilde{C}_R(|\boldsymbol{k}|)\left[\left[\hat{\rho}_{sys},\hat{v}_{\boldsymbol{k}}\right],\hat{v}_{\boldsymbol{k}}\right]\\ &= (2\pi)^{3/2}\int \mathrm{d}\boldsymbol{k}\widetilde{C}_R(|\boldsymbol{k}|)\left(\left[\left[\hat{\rho}^R_{\boldsymbol{k}},\hat{v}^R_{\boldsymbol{k}}\right],\hat{v}^R_{\boldsymbol{k}}\right]\hat{\rho}^I_{\boldsymbol{k}} + \hat{\rho}^R_{\boldsymbol{k}}\left[\left[\hat{\rho}^I_{\boldsymbol{k}},\hat{v}^R_{\boldsymbol{k}}\right],\hat{v}^R_{\boldsymbol{k}}\right]\right)\,,\\ &\times \prod_{\boldsymbol{k}'\neq\boldsymbol{k}}\prod_{s=R,I}\hat{\rho}^s_{\boldsymbol{k}'}\end{aligned} \tag{4.93}$$

so combining this last result with (4.8) one obtains

$$\frac{d\hat{\rho}^s_{\boldsymbol{k}}}{d\eta} = -i\left[\hat{H}_{\boldsymbol{k}},\hat{\rho}^s_{\boldsymbol{k}}\right] - \frac{\gamma}{2}(2\pi)^{3/2}\widetilde{C}_R(|\boldsymbol{k}|)\left[\left[\hat{\rho}^s_{\boldsymbol{k}},\hat{v}^s_{\boldsymbol{k}}\right],\hat{v}^s_{\boldsymbol{k}}\right]\,, \tag{4.94}$$

Notice that a particular comoving scale appears in the interaction term, indeed, in order for (4.160) (or equivalently (4.165)) to have the correct dimension, $\gamma\widetilde{C}_R(|\boldsymbol{k}|)$ must be homogeneous to the square

[10] In what follows we omit the sub index"sys" in Fourier space to lighten up the notation.

of a comoving wavenumber which we define it as

$$k_\gamma = \sqrt{\frac{2}{\pi}\bar{C}_R\frac{\gamma_* l_E^3}{3a_*^3}}\,, \tag{4.95}$$

this scale will be very important to constrain the interaction strength between the system and environment through the comparison of decoherence induced corrections with observations.

The next move is to project (4.94) on the eigenvecotrs $|v_{\boldsymbol{k}}^s\rangle$ of $\hat{v}_{\boldsymbol{k}}^s$ and solve the equation. Using the Hamiltonian expression (1.93)

$$\begin{aligned}\frac{d\left\langle v_{\boldsymbol{k}}^{s,(1)}\middle|\hat{\rho}_{\boldsymbol{k}}^s\middle|v_{\boldsymbol{k}}^{s,(2)}\right\rangle}{d\eta} &= \left\{\frac{i}{2}\left[\frac{\partial^2}{\partial v_{\boldsymbol{k}}^{s,(1)2}} - \frac{\partial^2}{\partial v_{\boldsymbol{k}}^{s,(2)2}} +\right] - \frac{i}{2}\omega^2\left[v_{\boldsymbol{k}}^{s,(1)^2} - v_{\boldsymbol{k}}^{s,(2)^2}\right]\right. \\ &\left. -\frac{\gamma}{2}(2\pi)^{3/2}\widetilde{C}_R(|\boldsymbol{k}|)\left[v_{\boldsymbol{k}}^{s,(1)} - v_{\boldsymbol{k}}^{s,(2)}\right]^2\right\}\left\langle v_{\boldsymbol{k}}^{s,(1)}\middle|\hat{\rho}_{\boldsymbol{k}}^s\middle|v_{\boldsymbol{k}}^{s,(2)}\right\rangle\end{aligned}\,, \tag{4.96}$$

if the matrix element $\left\langle v_{\boldsymbol{k}}^{s,(1)}\middle|\hat{\rho}_{\boldsymbol{k}}^s\middle|v_{\boldsymbol{k}}^{s,(2)}\right\rangle$ is seen as function of $v_{\boldsymbol{k}}^{s,(1)}$, $v_{\boldsymbol{k}}^{s,(2)}$ and η then the above equation is a linear second order partial differential equation which could be transformed into set of first order partial differential equations by the change of variable

$$X = v_{\boldsymbol{k}}^{s,(1)} - v_{\boldsymbol{k}}^{s,(2)}\ \text{and}\ Y = v_{\boldsymbol{k}}^{s,(1)} + v_{\boldsymbol{k}}^{s,(2)}\,, \tag{4.97}$$

the details of this transformation in addition to their solution through the use of method of characteristics could be found in appendix C of [53]. The solution of (4.96) is given by

$$\begin{aligned}\left\langle v_{\boldsymbol{k}}^{s,(1)}\middle|\hat{\rho}_{\boldsymbol{k}}^s\middle|v_{\boldsymbol{k}}^{s,(2)}\right\rangle &= \frac{(2\pi)^{-1/2}}{\sqrt{|v_{\boldsymbol{k}}|^2+\mathcal{J}_k}}\exp\left\{-\frac{v_{\boldsymbol{k}}^{s,(1)^2}+v_{\boldsymbol{k}}^{s,(2)^2}+i|v_{\boldsymbol{k}}|^{2'}\left[v_{\boldsymbol{k}}^{s,(2)^2}-v_{\boldsymbol{k}}^{s,(1)^2}\right]}{4\left(|v_{\boldsymbol{k}}|^2+\mathcal{J}_k\right)}\right\} \\ &\times\exp\left\{-\frac{\left[v_{\boldsymbol{k}}^{s,(2)}-v_{\boldsymbol{k}}^{s,(1)}\right]^2}{2\left(|v_{\boldsymbol{k}}|^2+\mathcal{J}_k\right)}\left(\mathcal{I}_k\mathcal{J}_k - \mathcal{K}_k^2 + |v_{\boldsymbol{k}}'|^2\mathcal{J}_k + |v_{\boldsymbol{k}}|^2\mathcal{I}_k - |v_{\boldsymbol{k}}|^{2'}\mathcal{K}_k\right)\right. \\ &\left. - \frac{i\mathcal{K}_k}{2\left(|v_{\boldsymbol{k}}|^2+\mathcal{J}_k\right)}\left[v_{\boldsymbol{k}}^{s,(2)^2} - v_{\boldsymbol{k}}^{s,(1)^2}\right]\right\}\end{aligned}\,, \tag{4.98}$$

the prime in previous equation refers to derivative with respect to η. The quantities $\mathcal{I}_k$, $\mathcal{J}_k$ and $\mathcal{K}_k$

are defined as

$$\begin{aligned}
\mathcal{I}_k(\eta) &= 4(2\pi)^{3/2}\int_{-\infty}^{\eta} d\eta'\gamma(\eta')\,\widetilde{C}_R(|\boldsymbol{k}|,\eta')\,\mathfrak{Im}^2\left[\mathrm{v}_{\mathbf{k}}(\eta')\,\mathrm{v}_{\mathbf{k}}^{*'}(\eta)\right] \\
\mathcal{J}_k(\eta) &= 4(2\pi)^{3/2}\int_{-\infty}^{\eta} d\eta'\gamma(\eta')\,\widetilde{C}_R(|\boldsymbol{k}|,\eta')\,\mathfrak{Im}^2\left[\mathrm{v}_{\mathbf{k}}(\eta')\,\mathrm{v}_{\mathbf{k}}^{*}(\eta)\right] \\
\mathcal{K}(\eta) &= 4(2\pi)^{3/2}\int_{-\infty}^{\eta} d\eta'\gamma(\eta')\,\widetilde{C}_R(|\boldsymbol{k}|,\eta')\,Im^2\left[\mathrm{v}_{\mathbf{k}}(\eta')\,\mathrm{v}_{\mathbf{k}}^{*'}(\eta)\right]\mathfrak{Im}\left[\mathrm{v}_{\mathbf{k}}(\eta')\,\mathrm{v}_{\mathbf{k}}^{*'}(\eta)\right]
\end{aligned} \quad , \tag{4.99}$$

needless to remind that $v_{\mathbf{k}}(\eta)$in the previous equations is the solution of Mukhanov-Sasaki equation with initial conditions set in Bunch Davies vacuum. By using (4.3) the equation (4.98) would represent a full solution of Lindblad equation. Notice that as consequence of considering linear interaction the state is still Gaussian which reduces to the two mode squeezed state in the limit $\gamma = 0$ i.e no interaction with environment

$$\left\langle v_{\boldsymbol{k}}^{s,(1)}\right|\hat{\rho}_{\boldsymbol{k}}^{s}\left|v_{\boldsymbol{k}}^{s,(2)}\right\rangle = \Psi_{\boldsymbol{k}}^{s}\left(v_{\boldsymbol{k}}^{s,(1)}\right)\Psi_{\boldsymbol{k}}^{s*}\left(v_{\boldsymbol{k}}^{s,(2)}\right) , \tag{4.100}$$

with the wavefunction $\Psi_{\boldsymbol{k}}^{s}(v) \propto \mathrm{e}^{i\frac{v_{\boldsymbol{k}}'}{2v_{\boldsymbol{k}}}v^2}$. A final comment is to notice that the diagonal elements are affected by the environment since they involve $\mathcal{J}_k(\eta)$, therefore we predict that the correction induced by decoherence to the power spectrum will be $\propto \mathcal{J}_k(\eta)$.

Having found the explicit expression of density matrix, we turn now to compute the power spectrum of field amplitude v that is defined by

$$P_{vv}(k) = \left\langle |\hat{v}_k|^2\right\rangle = \left\langle (\hat{v}_k^s)^2\right\rangle = Tr_{sys}\left[(\hat{v}_k^s)^2\,\hat{\rho}_{sys}\right] = \int \mathrm{d}\hat{v}_k^s\,\langle v_{\boldsymbol{k}}^s|\,\hat{\rho}_{\boldsymbol{k}}^s\,|v_{\boldsymbol{k}}^s\rangle\,(\hat{v}_k^s)^2 , \tag{4.101}$$

this last integral is Gaussian and could be performed easily to yield

$$P_{vv}(k) = |\hat{v}_k|^2 + \mathcal{J}_k(\eta) , \tag{4.102}$$

we recover the standard result, $P_{vv}(k) = |\hat{v}_k|^2$, in case of absence of environment $\mathcal{J}_k(\eta) = 0$. The dimensionless curvature power spectrum $\mathcal{P}_\zeta$, that is of interest for us, is obtained from (4.102) by using (1.30) which leads to

$$\mathcal{P}_\zeta = \frac{k^3}{2\pi^2}\frac{1}{2a^2M_{pl}\epsilon_1}P_{vv}(k) = \mathcal{P}_\zeta\,|_{standard}\left(1 + \triangle\mathcal{P}_{\boldsymbol{k}}\right) , \tag{4.103}$$

with

$$\triangle\,\mathcal{P}_k = \frac{\mathcal{J}_k}{|\hat{v}_k|^2} , \tag{4.104}$$

Second method We now compute the power spectrum using the alternative method based on solving the equation (4.85); this method will be the standard one for computing correlation functions for quadratic interactions, with environment, and beyond.

Fourier transforming (4.85) and considering the various two point correlation functions $\left\langle \hat{O}_{\boldsymbol{k}_1}\hat{O}_{\boldsymbol{k}_2}\right\rangle$ with $\hat{O}_{\boldsymbol{k}_i}=\hat{v}_{\boldsymbol{k}_i}$or $\hat{p}_{\boldsymbol{k}_i}$ we obtain

$$\begin{aligned}
\frac{d\left\langle \hat{v}_{\boldsymbol{k}_1}\hat{v}_{\boldsymbol{k}_2}\right\rangle}{d\eta} &= \left\langle \hat{v}_{\boldsymbol{k}_1}\hat{p}_{\boldsymbol{k}_2}\right\rangle + \left\langle \hat{p}_{\boldsymbol{k}_1}\hat{v}_{\boldsymbol{k}_2}\right\rangle \\
\frac{d\left\langle \hat{v}_{\boldsymbol{k}_1}\hat{p}_{\boldsymbol{k}_2}\right\rangle}{d\eta} &= \left\langle \hat{p}_{\boldsymbol{k}_1}\hat{p}_{\boldsymbol{k}_2}\right\rangle - \omega^2\left(k_2\right)\left\langle \hat{v}_{\boldsymbol{k}_1}\hat{v}_{\boldsymbol{k}_2}\right\rangle \\
\frac{d\left\langle \hat{p}_{\boldsymbol{k}_1}\hat{v}_{\boldsymbol{k}_2}\right\rangle}{d\eta} &= \left\langle \hat{p}_{\boldsymbol{k}_1}\hat{p}_{\boldsymbol{k}_2}\right\rangle - \omega^2\left(k_1\right)\left\langle \hat{v}_{\boldsymbol{k}_1}\hat{v}_{\boldsymbol{k}_2}\right\rangle \\
\frac{d\left\langle \hat{p}_{\boldsymbol{k}_1}\hat{p}_{\boldsymbol{k}_2}\right\rangle}{d\eta} &= -\omega^2\left(k_2\right)\left\langle \hat{p}_{\boldsymbol{k}_1}\hat{v}_{\boldsymbol{k}_2}\right\rangle - \omega^2\left(k_1\right)\left\langle \hat{v}_{\boldsymbol{k}_1}\hat{p}_{\boldsymbol{k}_2}\right\rangle + \gamma\left(2\pi\right)^{3/2}\widetilde{C}_R\left(\left|\boldsymbol{k}_1\right|\right)\delta\left(\boldsymbol{k}_1+\boldsymbol{k}_2\right)
\end{aligned} \quad , \tag{4.105}$$

even though the environment affects only $\left\langle \hat{p}_{\boldsymbol{k}_1}\hat{p}_{\boldsymbol{k}_2}\right\rangle$ through the term $\propto\gamma$, but the equations of various correlation functions in (4.105) are coupled together, so $\left\langle \hat{v}_{\boldsymbol{k}_1}\hat{v}_{\boldsymbol{k}_2}\right\rangle$ is also affected. In addition, the presence of $\delta\left(\boldsymbol{k}_1+\boldsymbol{k}_2\right)$ in the extra term induced by interaction with environment implies that this last preserves the statistical homogeneity, and since environmental correlator preserves isotropy, then the above system admits homogeneous and isotopic solutions $P_{oo'}\left(k\right)$ that are solely functions of modulus k. The equations (4.105) form a closed system of equations that could be reduced into a single third order differential equation

$$p_{vv}''' + 4\omega^2 p_{vv}' + 4\omega\omega' p_{vv} = S_1\,, \tag{4.106}$$

where S_1 is a source function given by

$$S_1\left(k,\eta\right) = 2\left(2\pi\right)^{3/2}\gamma\widetilde{C}_R\left(k\right)\,, \tag{4.107}$$

indeed we can check that the solution obtained by the first method (4.102) does satisfy (4.106). In case we considered $\hat{A}\left(\boldsymbol{x}\right)=\hat{p}\left(\boldsymbol{x}\right)$ instead of $\hat{v}$, then equation (4.106) would remain valid but with a different source function

$$S_1\left(k,\eta\right) = \left(2\pi\right)^{3/2}\left[\left(\gamma\widetilde{C}_R\right)'' + 2\omega^2\gamma\widetilde{C}_R\right]\,, \tag{4.108}$$

The decoherence induced correction $\triangle\mathcal{P}_k$ The final step in our analysis is the computation of $\triangle\mathcal{P}_k$ given by (4.104), with $\mathcal{J}_k$ defined in (4.99). We adopt the slow roll regime solutions concerning mode functions that are involved in $\mathcal{J}_k$, and are given in terms of Bessel functions with index $\nu=$

$\frac{3}{2}+\epsilon_1^*+\epsilon_2^*$, where the first and second slow roll parameters (ϵ_1^*, ϵ_2^*) are computed Horizon exit of pivot scale k_*. The details of computing $\mathcal{J}_k$ could be found in appendix C of [53], and here will just state two approximations which were adopted by the authors to get the corrections that we will write in a moment. We will also use them to compute the various correlation functions with our approach for the scalar case, in addition to tonsorial case. The approximations consist in

- The first limit uses the Markovian approximation that requires the environment autocorrelation time t_c to be very short compared to the typical time scale over which the system evolves $\sim H^{-1}$. Assuming the environment correlation time t_c and length l_E to be of the same order $t_c \sim l_E$ then

$$Hl_E \ll 1\,, \tag{4.109}$$

- The second limit is to evaluate the corrections at the end of inflation i.e $-k\eta \to 0$, when the modes of observational interest today are outside horizon.

Therefore, using these two approximations and remembering that the coupling system-environment was conventionally assumed to be of the form (4.64), we obtain the corrections $\triangle\mathcal{P}_k$ as function of p that gives the dominant contribution. We, first, write the most general form of it[11]

$$\triangle\,\mathcal{P}_k\,|_i\,\simeq \mathcal{A}_i\,(k)\left[1+\mathcal{B}_i\epsilon_1^*+\mathcal{C}_i\epsilon_2^*+(\mathcal{D}_i\epsilon_1^*+\mathcal{E}_i\epsilon_1^*)\ln\left(\frac{k}{k_*}\right)\right]\,, \tag{4.110}$$

1. If $p>3+\frac{2+2\nu}{1+\epsilon_1^*}$

$$\begin{aligned}\mathcal{A}_1\,(k) &= \left(\tfrac{k_\gamma}{k_*}\right)^2\left(\tfrac{k}{k_*}\right)^3\left(\tfrac{\eta}{\eta_*}\right)^{2+2\nu-(p-3)(1+\epsilon_1^*)}\frac{2}{(p-2)(p-5)(p-8)}\\ \mathcal{B}_1 &= 2\gamma_E+\ln(4)-7+\tfrac{1}{2-p}+\tfrac{3}{8-p}+\tfrac{2}{5-p}\\ \mathcal{C}_1 &= \gamma_E+\ln(2)-2+\tfrac{6}{(p-2)(p-8)},\ \mathcal{D}_1=2,\ \mathcal{E}_1=1\end{aligned}\quad . \tag{4.111}$$

[11] Notice that this the slow roll expanded solution of the exact one, within the two limits discussed above.

2. If $3+\frac{1}{1+\epsilon_1^*}<p<3+\frac{2+2\nu}{1+\epsilon_1^*}$

$$\begin{aligned}&\mathcal{A}_2(k)=\left(\frac{k_\gamma}{k_*}\right)^2\left(\frac{k}{k_*}\right)^{p-5}\frac{(6-p)\pi}{2^{6-p}(p-2)\sin(\pi p/2)\Gamma(p-3)}\\&\mathcal{B}_2=-2\frac{(p-1)(p-3)}{(p-4)(p-2)}-\frac{1}{2}(p-5)\psi\left(4-\frac{p}{2}\right)-\psi\left(-2+\frac{p}{2}\right)\\&-\frac{1}{2}(p-3)\psi\left(-\frac{3}{2}+\frac{p}{2}\right)\\&\mathcal{C}_2=\gamma_E+\ln(2)-2+\frac{6}{(p-2)(p-8)},\ \mathcal{D}_2=p-3,\ \mathcal{E}_2=0\end{aligned}\quad. \tag{4.112}$$

3. Finally if $p<3+\frac{1}{1+\epsilon_1^*}$

$$\begin{aligned}&\mathcal{A}_3(k)=\left(\frac{k_\gamma}{k_*}\right)^2\left(\frac{k}{k_*}\right)^{p-5}\frac{(H_*l_E)^{p-4}}{2(p-4)}\\&\mathcal{B}_3=3-p+\frac{1}{4-p}+\ln(H_*l_E)\\&\mathcal{C}_3=0,\ \mathcal{D}_3=2,\ \mathcal{E}_3=0\end{aligned}\quad, \tag{4.113}$$

where $\gamma_E=0.5777$ is the Euler-Masheroni constant and $\psi(z)$is the digamma function. Notice that the second and third corrections settle to stationary values at late times, while the first one is time dependent and continues to grow on large scales at late times as is revealed from the factor

$$\left(\frac{\eta}{\eta_*}\right)^{2+2\nu-(p-3)(1+\epsilon_1^*)}, \tag{4.114}$$

in $\mathcal{A}_1(k)$.

The accuracy of the two limits taken above, namely $Hl_E\ll1$ and the limit $-k\eta\to0$, could be checked by integrating numerically the correction $\triangle\mathcal{P}_k=\frac{\mathcal{J}_k}{|\hat{v}_k|^2}$. As could be seen in (4.3), the two results fit perfectly.

The high accurate CMB measurements revealed a quasi scale independent power spectrum, while we see a scale dependent corrections, except for $p=5$, so we need to constraint the scale dependent branch of power spectrum to be beyond our observational scales. To this end, we define a transition scale k_t that marks the breakdown of scale independence and having correction of order unity i.e $\mathcal{A}_i(k_t)\sim1$, Therefore form (4.111, 4.112, 4.113) we get

$$
\begin{aligned}
\frac{k_t}{k_*}\Big|_1 &\simeq \left(\frac{k_\gamma}{k_*}\right)^{-\frac{2}{3}} \exp\left\{\frac{\triangle N_*}{3}\left[p-3-\frac{2+2\nu}{1+\epsilon_1^*}\right]\right\} \\
\frac{k_t}{k_*}\Big|_2 &\simeq \left(\frac{k_\gamma}{k_*}\right)^{-\frac{2}{p-5}} \qquad\qquad\qquad\qquad , \\
\frac{k_t}{k_*}\Big|_2 &\simeq \left(\frac{k_\gamma}{k_*}\right)^{-\frac{2}{p-5}} \left(H_* l_E\right)^{-\frac{p-4}{p-5}}
\end{aligned}
\tag{4.115}
$$

where $\triangle N_*$corresponds to number of e folds between the Hubble crossing of pivot scale $k_*$and the end of inflation i.e $N_{end} - N_*$.

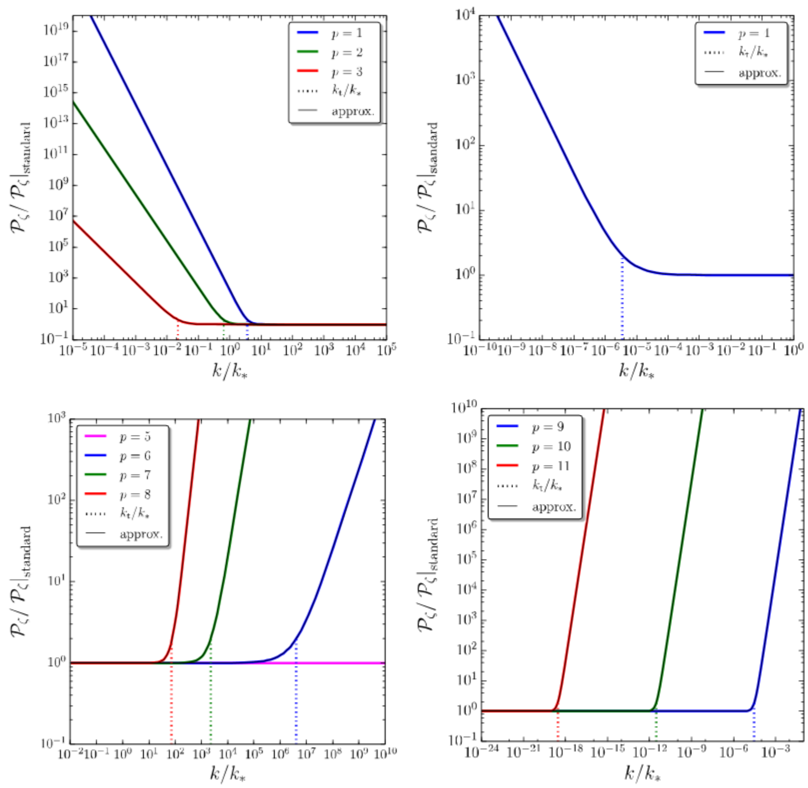

Figure 4.3: Comparison of exact and approximated corrections to power spectrum for different values of p (black lines represent the exact results and colored lines represent the approximated ones). The vertical dotted lines refer to the position of k_t. [53]

Depending on whether the corrections grow for large values of k or small ones, we will require in this last case $\frac{k_t}{k_*} \gg 1$, while for the former we require $\frac{k_t}{k_*} \ll 1$. For example, in case one $\mathcal{A}_1\left(k\right) \propto k^3$,therefore we want this scale dependent correction to be outside observable window, thus

$$\frac{k_\gamma}{k_*}|_1 \ll \mathrm{e}^{-\frac{1}{2}(\mathrm{p}-8+3\epsilon_1^*-\epsilon_2^*)\triangle \mathrm{N}_*}\,, \tag{4.116}$$

Similarly for the cases 2 and 3

$$\frac{k_\gamma}{k_*}|_2 \ll 1\,, \tag{4.117}$$

$$\frac{k_\gamma}{k_*}|_3 \ll (H_* l_E)^{\frac{4-p}{2}}\ , \tag{4.118}$$

Those upper constraints on k_γ are very important, since they constrain, through (4.95), the interaction of system with environment to be very small, we will obtain also lower constraints on k_γ later on by requiring decoherence of cosmological perturbations to take place.

The only case where no constraint on k_γ was obtained is for $p = 5$, where the correction to power spectrum is scale independent. We will show below that having a massive scalar field as environment gives for linear interaction exactly this value, $p = 5$, which is an interesting observation.

4.2.1.2 J.Martin model of a heavy massive scalar field as environment

Interesting results emerge for this type of environment. Indeed, we get scale independent corrections for power spectrum in case of linear, or quadratic, interaction with system i.e $\hat{A} = \hat{v}$ or $\hat{A} = \hat{v}^2$ for $p = 5$ and $p = 3$, respectively. In addition, considering such example will enable us to see how the approximations and the parameters appearing in Lindblad equation can related to microphysical quantities. Let us consider the action describing a scalar field φ, our system, interacting with heavy massive scalar field ψ representing an environment

$$S = -\int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + V(\varphi) + \lambda\mu^{4-n-m}\langle\psi^m\rangle_{st}\varphi^n + \frac{1}{2}g^{\mu\nu}\partial_\mu\psi\partial_\nu\psi + \frac{M^2}{2}\psi^2 + \lambda\mu^{4-n-m}\varphi^n\left(\psi^m - \langle\psi^m\rangle_{st}\right)\right], \tag{4.119}$$

we can define an effective potential for φ as

$$V_{eff}(\varphi) = V(\varphi) + \lambda\mu^{4-n-m}\langle\psi^m\rangle_{st}\varphi^n\,. \tag{4.120}$$

The reason behind adding adding and subtracting $\lambda\mu^{4-n-m}\langle\psi^m\rangle_{st}\varphi^n$ is to satisfy the condition (4.38) in the derivation of Lindblad equation, namely the quantum mean value of the interacting term vanishes in the stationary configuration of the environment, where in our case

$$\hat{H}_{int} = \lambda\mu^{4-n-m}a^4\int \mathrm{d}^3\boldsymbol{x}\varphi^n\left(\psi^m - \langle\psi^m\rangle_{st}\right)\,. \tag{4.121}$$

Defining $v(\eta, \boldsymbol{x}) = a(\eta)\varphi((\eta, \boldsymbol{x}))$, and assuming $V_{eff}(\varphi) = m^2\varphi^2/2$,then in Fourier space

$$S_{\varphi} = \frac{1}{2}\int \mathrm{d}\eta \frac{1}{2}\int d^3\boldsymbol{k}\left[\hat{v}'_{\boldsymbol{k}}\hat{v}^{*\prime}_{\boldsymbol{k}} + \left(k^2 - \frac{a''}{a} + m^2a^2\right)\hat{v}_{\boldsymbol{k}}\hat{v}^{*}_{\boldsymbol{k}}\right], \tag{4.122}$$

so if $m = 0$ we get the action of curvature perturbations in case we ignore metric perturbations, so by identifying $v(\eta, \boldsymbol{x})$ with Mukhanov Sasaki variable in uniform gauge then we can identify φ with inflaton and the system be curvature perturbations. So in terms of MS variable (4.121) could be rewritten as

$$\hat{H}_{int} = \lambda\mu^{4-n-m}a^{4-n}\int \mathrm{d}^3\boldsymbol{x}v^n\left(\psi^m - \langle\psi^m\rangle_{st}\right), \tag{4.123}$$

making an indentification with the $\hat{H}_{int}$ adopted in (4.10) to derive Lindblad equation we get

$$\begin{aligned} \hat{A} &= \hat{v}^n \\ \hat{R} &= \hat{\psi}^m - \left\langle\hat{\psi}^m\right\rangle_{st}, \\ g &= \lambda\mu^{4-n-m}a^{4-n} \end{aligned} \tag{4.124}$$

we see that indeed the coupling is time dependent as claimed in adopting the ansatz

$$\gamma = \gamma_*\left(\frac{a}{a_*}\right)^p. \tag{4.125}$$

Given that in the derivation of Lindblad equation we defined $\gamma = 2g^2\tau_c$, where $\tau_c = t_c/a$, then (4.125) is satisfied if

$$\gamma_* = 2t_c\lambda^2\mu^{8-2n-2m}a_*^{7-2n}, \tag{4.126}$$

$$p = 7 - 2n, \tag{4.127}$$

from this last we see that for $n = 1$ we get $p = 5$ which is exactly the value that gives a scale independent correction to power spectrum, similarly, for quadratic interaction $n = 2$ we will see that the correction is scale independent for $p = 3$ which is again satisfied by (4.127).

Remarque: It is important to mention that with our choice $\hat{A} = \sum_n \alpha^{n-1}\hat{v}^n$, we will show in the next section that the scale independent corrections are not exclusively related to having a massive scalar field as environment but is related to the order n in α from which the correction is obtained. Consequently, there is a possibility to have scale independent corrections of power spectrum for environments other than massive scalar field.

4.2.1.3 D.Boyanovsky model of a massless scalar field as environment

Before proceeding with the implications of considering a massive scalar field as environment in the model adopted by J.Martin et al, it is worth to mention that a similar work was done by D.Boyanovsky in [15]. In this work he considered the action for φ representing the system, inflaton, and ψ representing the environment

$$\begin{aligned} S \quad &= -\int \mathrm{d}^4x\sqrt{-g}\left[\tfrac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + \tfrac{1}{2}\left(m_\varphi^2 + \xi_\varphi\mathcal{R}\right)\varphi^2\right. \\ &\left. +\tfrac{1}{2}g^{\mu\nu}\partial_\mu\psi\partial_\nu\psi + \tfrac{1}{2}\left(M^2 + \xi_\psi\mathcal{R}\right)\psi^2 + \lambda\varphi\left(\psi^2 - \left\langle\psi^2\right\rangle_{st}\right)\right] \end{aligned} , \tag{4.128}$$

where $\mathcal{R}$ is the Ricci scalar and $\xi_{\varphi/\psi} = 0, \frac{1}{6}$ represent the minimal and conformal coupling respectively. Following the same previous steps to derive the Lindblad equation including the assumptions made along the way, Boyanovsky obtained a master equation that enabled him to study the effect of external scalar field on curvature power spectrum using the second method above to derive the equations of motion for P_{vv}. However, there are some differences with respect to the previous model of J.Martin et al in terms of computations and final results :

- Boyanovsky expanded the system interaction operator $\hat{A}$ in terms of creation and annihilation operators $\left(\hat{a}_{\boldsymbol{k}}, \hat{a}^\dagger_{-\boldsymbol{k}}\right)$ in order to study to the evolution of number operators

  $$\hat{N}_{\boldsymbol{k}} = \left\langle \hat{a}^\dagger_{\boldsymbol{k}}\hat{a}_{\boldsymbol{k}}\right\rangle = Tr_{sys}\left(\hat{\rho}_{sys}\hat{a}^\dagger_{\boldsymbol{k}}\hat{a}_{\boldsymbol{k}}\right) , \tag{4.129}$$

  and

  $$\hat{M}_{\boldsymbol{k}} = \left\langle \hat{a}_{\boldsymbol{k}}\hat{a}_{-\boldsymbol{k}}\right\rangle = Tr_{sys}\left(\hat{\rho}_{sys}\hat{a}_{\boldsymbol{k}}\hat{a}_{-\boldsymbol{k}}\right) , \tag{4.130}$$

  which reflect the particle production and and production of correlated pairs of particles. This production is coming purely from interaction with environment and independently from the production we saw in first chapter that is coming from interaction with classical spacetime as reflected by the Hamiltonian of the system seen in (1.44). Particle production is sign of classicality, where highly populated fields are considered macroscopic fields, and the particles populating them could easily turn into classical rather than quantum since it is hard to preserve quantum coherence between large number of particles.

- The second and main difference, is that Boyanovsky considered an explicit expression for the environment correlator by assuming ψ to be massless conformally couple scalar filed to gravity. Where the correlator expression he obtained for $C_R\left(\eta, \boldsymbol{x}; \eta', \boldsymbol{y}\right)$ was made of two parts: a local one, as for J.Martin et all, and a non local one. It is exactly this non local part of the environment correlator which causes the power spectrum to decay at late times, where he obtained for its expression

$$\mathcal{P}_\zeta(k,\eta) = \frac{H^2}{8\pi^2\epsilon_1^* M_{pl}^2} \mathrm{e}^{\frac{\lambda^2}{6\pi^2 H^2}\left[\ln(-k\eta_0)\ln(-k\eta) - \frac{1}{2}\ln^2(-k\eta)\right]}, \tag{4.131}$$

with $-k\eta \to 0$. Though it seems from a first glance that there is strong decay, we have to remember that Lindblad equation was derived perturbatively, so the fact that $\frac{\lambda^2}{H^2} \ll 1$ is enough to make the decay marginally observable for for scales of cosmological interest today. For $\frac{\lambda}{H} = 0.1$, $\ln(-k\eta_0) \approx 50$ and $\ln(-k\eta_f) = 10$, the power spectrum suppression is $\lesssim 10\%$ which is very small. For the usefulness of comparing the results of the two models, we reproduce the brief conclusion drawn by Boyanovsky after this result: *"Therefore, although the power spectrum decays as a consequence of the interaction with the environmental degrees of freedom, it is likely that these corrections are of marginal observational relevance, at least within the model studied here. However, this important observational fact notwithstanding, there is the noteworthy and fundamental aspect that the amplitude of the perturbation does not freeze out but decays after crossing the Hubble radius. These results ... also point(s) out that not only the power spectrum does not freeze-out after "horizon crossing" but that the time dependence is associated with a violation of scale invariance even when in absence of interactions the power spectrum is exactly scale invariant."*We notice that, also, in the case of the results obtained under the model of J.Martin et al we found regimes where the power spectrum corrections were time dependent and/or scale dependent, which seems to be a general feature of the interaction of our perturbations with their environment.

4.2.1.4 Decoherence induced Corrections to observables n_s and r

Among the most important observables that CMB measurements tend to obtain are the spectral index n_s and tensor to scalar ratio r. Where we remind that the former measures the deviation from scale independence in curvature power spectrum $\mathcal{P}_\zeta \propto k^{n_s-1}$, while the second measures ratio between tensor and curvature power spectra $r = \frac{\mathcal{P}_h}{\mathcal{P}_\zeta}$. The measurement of the two parameters serve to verify, or falsify, the different inflationary potentials, in addition, the parameter r helps to set the energy scale at which inflation took place which would be a huge step toward and beyond standard model theory.

Naturally we would be curious to ask how decoherence could affect this two parameters. Therefore, to answer this question we adopt the value $p = 5$ which gives a scale independent correction to power spectrum with a massive scalar field as environment for[12] $p = 5 - 6m\epsilon_1^*$. Considering the standard

[12]We remind here that m is the power of ψ in (4.123).

power spectrum expanded in slow roll parameters

$$\mathcal{P}_\zeta\,|_{standard} \quad = \frac{H_*^2}{8\pi^2\epsilon_1^* M_{pl}^2}\left[1-2\left(C+1\right)\epsilon_1^* - C\epsilon_2^* - \left(2\epsilon_1^*+\epsilon_2^*\right)\ln\left(\frac{k}{k_*}\right)\right] \quad , \tag{4.132}$$

where $C = \gamma_E + \ln 2 - 2 \simeq -0.7296$, so considering this expression in addition to (4.103) and (4.112) one obtains

$$\mathcal{P}_\zeta \quad = \frac{H_*^2\left(1+\frac{\pi}{6}\frac{k_\gamma^2}{k_*^2}\right)}{8\pi^2\epsilon_1^* M_{pl}^2}\left\{1-F\left(\frac{k}{k_*},\frac{k_\gamma}{k_*},\epsilon_1^*,\epsilon_2^*\right)\right\} \quad , \tag{4.133}$$

where $F(\ldots)$ is a function of its arguments, whose explicit expression could be found in [53]. The new expression has two distinct features, first it differs from the standard one by the additional dependence on $\frac{k_\gamma}{k_*}$. In [53] it was assumed that if the tensor perturbations were not affected by the environment then the standard expression of r, namely $r\,|_{standard} = 16\epsilon_1^*$, would be modified into

$$r = \frac{r\,|_{standard}}{1+\frac{\pi}{6}\frac{k_\gamma^2}{k_*^2}}\,, \tag{4.134}$$

this expression will have important consequences in the limit $\frac{k_\gamma}{k_*}$ not negligibly small, we remind that in the case $p \simeq 5$ there are no upper constraints on $\frac{k_\gamma}{k_*}$ because the correction to power spectrum is scale independent. Before discussing how n_s is also modified, we want to mention that the assumption that tensor modes are not affected by the environment will turn out to be not accurate, where will show later that the environment does modify the tensor power spectrum and we will provide a detailed computations regarding that. But for the moment let us stick to the assumption made by the authors in [53] and see what they obtained. Using the definition of $n_s \equiv 1 + \frac{\mathrm{d}\ln\mathcal{P}_\zeta}{\mathrm{dk}}$ and the modified power spectrum we obtain the modification to the standard $n_s\,|_{standard} = 1 - 2\epsilon_1^* - \epsilon_2^*$ as

$$n_s = n_s\,|_{standard} - \frac{\frac{\pi}{6}\frac{k_\gamma^2}{k_*^2}}{1+\frac{\pi}{6}\frac{k_\gamma^2}{k_*^2}}\left(6m-2\right)\epsilon_1^*\,, \tag{4.135}$$

for $\frac{k_\gamma}{k_*} \gg 1$

$$n_s = n_s\,|_{standard} - \left(6m-2\right)\epsilon_1^*\,, \tag{4.136}$$

Having found the modified n_s and r, we turn now to see how some inflationary models predictions could be modified with respect to data, the results could be seen in figure (4.4). We see that Starobinsky model, Higgs potential

$$V\left(\varphi\right) \propto \left[1 - exp\left(-\sqrt{2/3}\varphi/M_{pl}\right)\right]^2\,, \tag{4.137}$$

predictions are not affected that much be decoherence, because this potential already predicts small

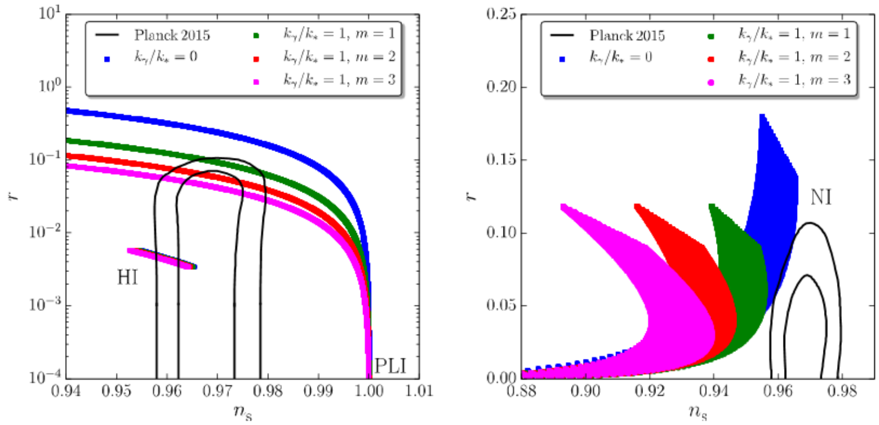

Figure 4.4: Spectral index and tensor to scalar ratio for different potentials, Higgs inflation (HI), Power law inflation (PLI), and Natural inflation (NI). The blue color refers to standard results and the other colors to modified values as function of m for $\frac{k_\gamma}{k_*} = 1$. the black lined represent the one and two sigma contours from Planck 2015 data.[53]

ϵ_1^* so modifications are very small. Concerning natural inflation potential

$$V(\varphi) \propto 1 + \cos\left(\frac{\varphi}{f}\right) , \tag{4.138}$$

decoherence worsen its predictions which already disfavored by data, since this model predicts too small spectral index. Finally, the potential which decoherence works in its favor, the most, is the power law one

$$V(\varphi) \propto \exp\left(-\alpha\varphi/M_{pl}\right) , \tag{4.139}$$

where this model was disfavored by data, but thanks to decoherence we see that the strongest the interaction with environment is, then the best potential has chances to fit data for some values of its free parameters.

4.2.1.5 Computation of power spectrum with quadratic interaction

In the case of quadratic interactions we are obliged to adopt the second method in computing decoherence induced corrections to power spectrum, simply because for $\hat{A} = \hat{v}^2$ the different modes will be interacting and the Lindblad equation cannot be decoupled into a set of equations, one for each mode as in (4.94). Therefore, it could not be solved entirely to have explicit expression of system density matrix $\hat{\rho}_{sys}$.

Using

$$\hat{v}^2(\boldsymbol{x}) = \frac{1}{(2\pi)^3}\int \mathrm{d}^3\boldsymbol{k'}\mathrm{d}^3\boldsymbol{p}\hat{v}_{\boldsymbol{k'}}\hat{v}_{\boldsymbol{p}-\boldsymbol{k'}}e^{i\boldsymbol{p}.\boldsymbol{x}}\,, \tag{4.140}$$

to Fourier transform (4.85) gives

$$\frac{d\left\langle\hat{O}\right\rangle}{d\eta} = \left\langle\frac{\partial\hat{O}}{\partial\eta}\right\rangle - i\left\langle\left[\hat{O},\hat{H}_v\right]\right\rangle - \frac{\gamma}{2\,(2\pi)^{3/2}}\int d^3\boldsymbol{k}d^3\boldsymbol{p}_1 d^3\boldsymbol{p}_2\widetilde{C}_R(|\boldsymbol{k}|)\times\left\langle\left[\left[\hat{O},\hat{v}_{\boldsymbol{p}_1}\hat{v}_{\boldsymbol{k}-\boldsymbol{p}_1}\right],\hat{v}_{\boldsymbol{p}_2}\hat{v}_{-\boldsymbol{k}-\boldsymbol{p}_2}\right]\right\rangle\,, \tag{4.141}$$

with this equation we can obtain a set of equations similar to what we did in linear case

$$\begin{aligned}
\frac{d\langle\hat{v}_{\boldsymbol{k}_1}\hat{v}_{\boldsymbol{k}_2}\rangle}{d\eta} &= \langle\hat{v}_{\boldsymbol{k}_1}\hat{p}_{\boldsymbol{k}_2}\rangle + \langle\hat{p}_{\boldsymbol{k}_1}\hat{v}_{\boldsymbol{k}_2}\rangle \\
\frac{d\langle\hat{v}_{\boldsymbol{k}_1}\hat{p}_{\boldsymbol{k}_2}\rangle}{d\eta} &= \langle\hat{p}_{\boldsymbol{k}_1}\hat{p}_{\boldsymbol{k}_2}\rangle - \omega^2(k_2)\langle\hat{v}_{\boldsymbol{k}_1}\hat{v}_{\boldsymbol{k}_2}\rangle \\
\frac{d\langle\hat{p}_{\boldsymbol{k}_1}\hat{v}_{\boldsymbol{k}_2}\rangle}{d\eta} &= \langle\hat{p}_{\boldsymbol{k}_1}\hat{p}_{\boldsymbol{k}_2}\rangle - \omega^2(k_1)\langle\hat{v}_{\boldsymbol{k}_1}\hat{v}_{\boldsymbol{k}_2}\rangle \qquad , \\
\frac{d\langle\hat{p}_{\boldsymbol{k}_1}\hat{p}_{\boldsymbol{k}_2}\rangle}{d\eta} &= -\omega^2(k_2)\langle\hat{p}_{\boldsymbol{k}_1}\hat{v}_{\boldsymbol{k}_2}\rangle - \omega^2(k_1)\langle\hat{v}_{\boldsymbol{k}_1}\hat{p}_{\boldsymbol{k}_2}\rangle \\
&\quad + \frac{4\gamma}{(2\pi)^{3/2}}\int d^3\boldsymbol{k}\widetilde{C}_R(|\boldsymbol{k}|)\langle\hat{v}_{\boldsymbol{k}+\boldsymbol{k}_1}\hat{v}_{-\boldsymbol{k}+\boldsymbol{k}_2}\rangle
\end{aligned} \tag{4.142}$$

notice that even if we do not see the delta function $\delta(\boldsymbol{k}_1+\boldsymbol{k}_2)$, which indicates that Lindblad contribution is still preserving statistical homogeneity, we have to remember that Lindblad equation is valid only at leading order in γ. Therefore, we have to use the standard power spectrum derived from the free theory which is proportional to $\delta(\boldsymbol{k}_1+\boldsymbol{k}_2)$. This guarantees that the solution that is obtained at the first iteration is statistically homogeneous. Since it sources the equation at the second iteration, the solution is again statistically homogeneous, and so on up to higher orders.

Combining the above equations into a single third order differential equation gives

$$p'''_{vv} + 4\omega^2 p'_{vv} + 4\omega\omega' p_{vv} = S_2\,, \tag{4.143}$$

where S_2 is the source function given by

$$S_2(k,\eta) = \alpha^2\frac{8\gamma}{(2\pi)^{3/2}}\int d^3\boldsymbol{k'}\widetilde{C}_R(k')\,P_{vv}(|\boldsymbol{k'}+\boldsymbol{k}|)\,, \tag{4.144}$$

form this last we see that, indeed, the modes are coupled, so it is difficult to solve it in full generality, however, at leading order in γ we use the free theory to compute S_2 which rends it a fixed function in time and the equation (4.144) could be solved. In order to find the solution, we go back to the linear case to get a hint; indeed, we notice that the source function S_1 enters the solution in special way as could be seen through second equation in (4.99). Therefore, by analogy we trust our intuition and suggest that the following function

$$\mathcal{S}_k\left(\eta\right) = -\frac{2}{W^2}\int\limits_{\eta_0}^{\eta} d\eta' S_2\left(k,\eta'\right)\mathfrak{Im}^2\left[\mathrm{v}_{\mathbf{k}}\left(\eta'\right)\mathrm{v}^*_{\mathbf{k}}\left(\eta\right)\right], \tag{4.145}$$

does solve the differential equation (4.144), where W is the Wronskian, $v'_{\mathbf{k}}\left(\eta\right)v^*_{\mathbf{k}}\left(\eta\right) - v_{\mathbf{k}}\left(\eta\right)v^{*\prime}_{\mathbf{k}}\left(\eta\right) = i$. It is worth to mention that the solution obtained is unique and independent of the mode function $v_{\boldsymbol{k}}\left(\eta\right)$ choice, because those last are distinguished by the initial conditions[13]while is easy to see that $\mathcal{S}_{\boldsymbol{k}}\left(\eta_0\right) = \mathcal{S}'_{\boldsymbol{k}}\left(\eta_0\right) = \mathcal{S}''_{\boldsymbol{k}}\left(\eta_0\right) = 0$. We can check that the the suggested solution (4.145) does indeed solve (4.143) by using the Mukhanov Sasaki equation $v''_{\mathbf{k}}+\omega^2 v_{\mathbf{k}} = 0$ in addition to $v_k = \left(v_k^R + iv_k^I\right)/\sqrt{2}$.

The full solution is the obtained after adding the solution of the homogeneous equation $p'''_{vv} + 4\omega^2 p'_{vv} + 4\omega\omega' p_{vv} = 0$ which gives the standard solution, so that the final result is

$$P_{vv}\left(k\right) = \left|\hat{v}_k\right|^2 + \mathcal{S}_k\,, \tag{4.146}$$

in this solution we use the Bunch Davies normalized mode functions. Calculating the source term and substituting it in previous equation, and using again equation (4.103) to define $\triangle\mathcal{P}_k$ but now

$$\triangle\,\mathcal{P}_k = \frac{\mathcal{S}_k}{\left|\hat{v}_k\right|^2}\,, \tag{4.147}$$

we obtain[14] the following corrections as function of p

- For $p > 6$

$$\Delta\mathcal{P}_k = \frac{1}{9\pi^2}k_\gamma^2\left(\frac{k}{k_*}\right)^3\left(\frac{\eta}{\eta_*}\right)^{6-p}\left[\frac{1}{p^2} - \frac{2}{\left(p-3\right)^2} + \frac{1}{\left(p-6\right)^2} + 8\frac{18}{p\left(p-3\right)\left(p-6\right)}\ln\left(\frac{\eta_{IR}}{\eta}\right)\right], \tag{4.148}$$

η_{IR} is an IR cutoff in the integral (4.145) and k_* refers to a pivot scale; $\ln\left(\frac{\eta_{IR}}{\eta}\right) = N - N_{IR}$ gives the number of e-folds elapsed since the beginning of inflation. We notice that in this regime the power spectrum correction scales as k^3, in addition to be not frozen on large scales and continues to increase, leading to a very large enhancement of the correction to the standard power spectrum at late time.

[13]Solving Mukhanov Sasaki equation to get $v_{\boldsymbol{k}}\left(\eta\right)$, requires to fix initial conditions which for us is Bunch Davies vacuum.

[14]More details on computation steps could be found in [53].

- For $2 < p < 6$

$$\begin{aligned}\Delta\mathcal{P}_k &= \tfrac{2^{p+1}\sqrt{2}(p-4)}{8\pi\Gamma(p-1)sin(\pi p/2)}k_\gamma^2\left(\tfrac{k}{k_*}\right)^{p-3}\left[\ln\left(\tfrac{\eta_{IR}}{\eta_*}\right)+\tfrac{1}{p-4}-\tfrac{2(p-1)}{p(p-2)}\right.\\ &\left.-\tfrac{\pi}{2}cot\left(\tfrac{\pi p}{2}\right)+ln\left(2\right)-\psi\left(p-2\right)+\ln\left(\tfrac{k}{k_*}\right)\right]\end{aligned}, \tag{4.149}$$

  ψ is the digamma function. In this case we obtain a scale invariant correction for $p = 3$, which again gives a possibility for a massive scalar field to be the environment, though probably not the only one that could give $p = 3$.

- For $p < 2$

$$\Delta\mathcal{P}_k = \frac{(H_* l_E)^{p-2}}{2\pi^2\left(2-p\right)}\beta^2 k_\gamma^2\left(\frac{k}{k_*}\right)^{p-3}\left[\frac{1}{2-p}+N_*-N_{IR}+ln\left(H_* l_E\right)+\ln\left(\frac{k}{k_*}\right)\right], \tag{4.150}$$

  notice that in this case the power spectrum freezes on small scales,

- For $p = 2$ and $p = 6$ which are singular we have

$$\begin{aligned}\Delta\mathcal{P}_k\,|_{p=2} &= \tfrac{1}{48\pi^2}\beta^2 k_\gamma^2\left(\tfrac{k}{k_*}\right)^{-1}\left[12-\pi^2+12C\left(2+C\right)-12\ln^2\left(H_* l_E\right)+24\left[C+1-\ln\left(H_* l_E\right)\right]\right.\\ &\left.\times\left[2\left(N_*-N_{IR}\right)+\ln\left(\tfrac{k}{k_*}\right)\right]\right]\\ \Delta\mathcal{P}_k\,|_{p=6} &= \tfrac{1}{432\pi^2}\beta^2 k_\gamma^2 k_\gamma^2\left(\tfrac{k}{k_*}\right)^{3}\left[2\pi^2-21-12C\left(1+2C\right)-12\left(3+4C\right)\left(N-N_{IR}\right)+12\left(1+4C\right)\left(N-N_*\right)\right.\\ &\left.+24\left(N-N_*\right)\left[2\left(N-N_{IR}\right)-\left(N-N_*\right)\right]-12\ln\left(\tfrac{k}{k_*}\right)\left[1+4\left(C+N_*-N_{IR}\right)2\ln\left(\tfrac{k}{k_*}\right)\right]\right]\end{aligned}, \tag{4.151}$$

  where C is a constant

By requiring the scale dependent corrections to be outside our observation window so we get an upper bound on the interaction strength with environment, as in the linear case.

4.2.1.6 Decoherence before the end of inflation

By requiring decoherence to take place before the end of inflation we could obtain lower bounds on k_γ, but we emphasize that this requirement is not mandatory. In other words, decoherence could take place after end of inflation, but it must occur before recombination to allow for a classical CMB fluctuations. But how can we impose such constraint?

In order to impose decoherence of primordialperturbations by the end on inflation we need to define a parameter which quantifies how much decoherence took place. To this end, we define the purity of state

$$\begin{gathered} Tr\left(\hat{\rho}_k^{s2}\right) = \int_{-\infty}^{+\infty} \mathrm{d}v_k^{s,(1)} \int_{-\infty}^{+\infty} \mathrm{d}v_k^{s,(2)} \left|\left\langle v_k^{s,(1)} \left|\hat{\rho}_k^s\right| v_k^{s,(2)} \right\rangle\right|^2 = \frac{1}{\sqrt{1+4\delta_k}} \\ \delta_k \equiv \mathcal{I}_k \mathcal{J}_k - \mathcal{K}_k^2 + \left|v_{\boldsymbol{k}}\right|^2 \mathcal{I}_k + \left|v_{\boldsymbol{k}}'\right|^2 \mathcal{J}_k - \left|v_{\boldsymbol{k}}\right|^{2\prime} \mathcal{K}_k \end{gathered} , \qquad (4.152)$$

so if $\delta_k \ll 1$ then $Tr\left(\hat{\rho}_k^{s2}\right) \simeq 1$ and the state is still pure, while for $\delta_k \gg 1$ we have $Tr\left(\hat{\rho}_k^{s2}\right) \ll 1$ and the state is highly mixed. To get (4.152) we used the explicit expression of $\hat{\rho}_k^s$ that was obtained only in linear case, so how can we find δ_k in case of quadratic interactions and higher orders, where no explicit expression of $\hat{\rho}_k^s$ is available?

The detailed argument and computations could be found in [53], but the idea, briefly, is to take advantage of the Lindblad equation. As case in point in linear case

$$\frac{dTr\left(\hat{\rho}_k^{s2}\right)}{d\eta} = 2Tr\left(\hat{\rho}_k^s \frac{d\hat{\rho}_k^s}{d\eta}\right) = -\gamma 2\left(2\pi\right)^{3/2} \gamma \widetilde{C}_R\left(k\right) Tr\left(\hat{\rho}_k^s\left[\hat{v}_k^s,\left[\hat{v}_k^s,\hat{\rho}_k^s\right]\right]\right) , \qquad (4.153)$$

with further considerations this last equation could be integrated and an expression for δ_k could be obtained using

$$Tr\left(\hat{\rho}_k^{s2}\right) = \frac{1}{\sqrt{1+4\delta_k}} \qquad (4.154)$$

Going back to the linear case and computing the explicit expression of δ_k and requiring it to be large by the end of inflation leads to lower bounds on k_γ/k_* which is necessary for decoherence to occur before the end of inflation. as follow

$$\frac{k_\gamma}{k_*} \gg \begin{cases} \left(H_* l_E\right)^{\frac{1-(p-3)\left(1+\epsilon_1^*\right)}{2}} & \text{if } p < 3 + \frac{2-2\nu}{1+\epsilon_1^*} \\ \mathrm{e}^{\left(\frac{1-\nu}{1+\epsilon_1^*} - \frac{p-3}{2}\right)\triangle N_*} & \text{if } p > 3 + \frac{2-2\nu}{1+\epsilon_1^*} \end{cases}$$

Combining those lower bounds with the upper bounds obtained previously by requiring the scale independence of the various corrections, we find the range of possible values for $\frac{k_\gamma}{k_*}$ which satisfy both requirements, the results are summarized in figure (4.5)

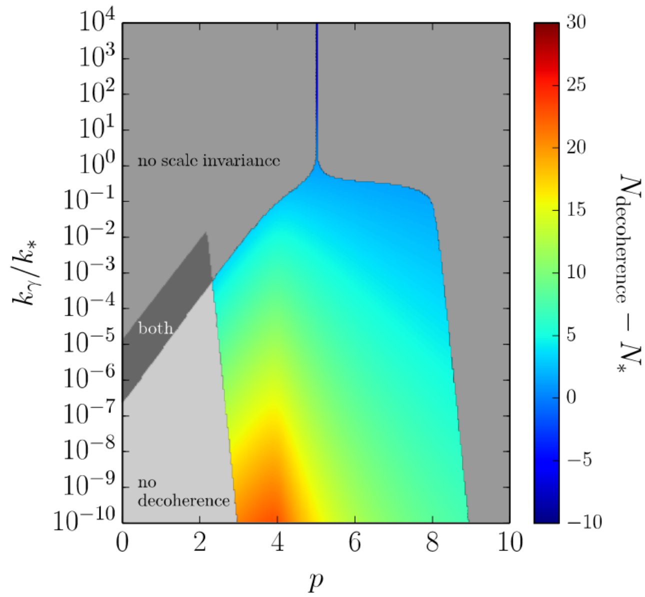

Figure 4.5: Regions in parameter space $(p, \frac{k_\gamma}{k_*})$ where decoherence and quasi scale invariance can or cannot be realized. The light grey region corresponds to values of p and $\frac{k_\gamma}{k_*}$ where the interaction strength with the environment, parameterized by $\frac{k_\gamma}{k_*}$ is too small to lead to decoherence. The medium grey region is where it is too large to preserve quasi scale invariance, and the dark grey region is where both problems occur (no decoherence and scale invariance breaking). The colored region corresponds to parameters where perturbations decohere and scale invariance is preserved. The color code, indicated by the vertical bar, quantifies how many e-folds since Hubble crossing it takes before complete decoherence is reached. Here decoherence is supposed to occur for $\delta_k \geq 10$, and scale independence is preserved if $|n_s - \bar{n}_s| < 5\sigma_{n_s}$, where $\bar{n}_s = 0.96$ and $\sigma_{n_s} \simeq 0.006$ are the mean value and standard deviation of the Planck spectral index measurement.[53]

The special thing in this Figure is the thin vertical line around $p = 5$, where this line reflects that the correction for $p = 5$ is scale invariant and k_γ/k_*could take large values.

4.2.1.7 Computation of bispectrum

Cosmology has entered the precision era! we are aiming now to measure non guassianities in CMB, or at least constrain them, in order to be able to test some degenerate inflationary models regarding their predictions of power spectrum. Having said that, we will naturally be interested in quantifying how decoherence could affect those non guassianities, but surprisingly we find out that decoherence could by itself generate non guassianities even if they were zero in the free theory which is a very interesting observation. We will be interested in bispectrum but we can also find a computations of trispectrum in [52].

Using the Lindblad equation (4.85), and choosing a linear interaction $\hat{A} = \hat{v}$ we can easily see that no bispectrum correction could arise from the Lindblad term, and generally non Guassianities from the free theory remain unaffected. Simply, because, as we showed previously, for linear interactions the the system density matrix $\hat{\rho}_{sys}$ remain Gaussian (see equation (4.98)), we can also see the reason behind the vanishing bispectrum thorough

$$-\frac{\gamma}{2}\int d^3\boldsymbol{k}\widetilde{C}_R(k)\left\langle\left[\left[\hat{O},\hat{v}_{\boldsymbol{k}}\right],\hat{v}_{-\boldsymbol{k}}\right]\right\rangle, \tag{4.155}$$

where if we consider three point correlator with less than two filed momentum $\hat{p}_{\boldsymbol{k}}$, as $\langle\hat{v}_{\boldsymbol{k_1}}\hat{v}_{\boldsymbol{k_2}}\hat{v}_{\boldsymbol{k_3}}\rangle$, or $\langle\hat{v}_{\boldsymbol{k_1}}\hat{v}_{\boldsymbol{k_2}}\hat{p}_{\boldsymbol{k_3}}\rangle$, then the term gives zero because of the vanishing commutator $[\hat{v}_{\boldsymbol{k}},\hat{v}_{\boldsymbol{k}'}] = 0$. While if we consider correlators with two momentum filed or three, i.e $\langle\hat{v}_{\boldsymbol{k_1}}\hat{p}_{\boldsymbol{k_2}}\hat{p}_{\boldsymbol{k_3}}\rangle$ and $\langle\hat{p}_{\boldsymbol{k_1}}\hat{p}_{\boldsymbol{k_2}}\hat{p}_{\boldsymbol{k_3}}\rangle$,then we will end up wit either $\langle\hat{v}_{\boldsymbol{k}}\rangle$or $\langle\hat{p}_{\boldsymbol{k}}\rangle$, both of which gives zero due to $\left\langle\hat{a}_{\boldsymbol{k}} \pm \hat{a}^{\dagger}_{-\boldsymbol{k}}\right\rangle = 0$. Regarding the trispectrum and when consider the connected terms, we find out that the corrections cancel out and we end up with unmodified trispectrum.

Now, we need to consider more complicated interactions, so choosing the quadratic interaction $\hat{A} = \hat{v}^2$ and implement it again in (4.85 we get a set of eight equations that represent the various $\left\langle\hat{O}_{\boldsymbol{k_1}}\hat{O}_{\boldsymbol{k_2}}\hat{O}_{\boldsymbol{k_3}}\right\rangle$ which could be built from $\hat{O}_{\boldsymbol{k}} = \hat{v}_{\boldsymbol{k}}$or $\hat{p}_{\boldsymbol{k}}$. But the correlators which get modified by the Lindblad term, marked by $\underbrace{()}$, are proportional to the initial bispectrum coming from the free theory , which is zero in our case[15], for example

$$\begin{aligned}\frac{d\langle\hat{v}_{\boldsymbol{k_1}}\hat{p}_{\boldsymbol{k_2}}\hat{p}_{\boldsymbol{k_3}}\rangle}{d\eta} &= \langle\hat{p}_{\boldsymbol{k_1}}\hat{p}_{\boldsymbol{k_2}}\hat{p}_{\boldsymbol{k_3}}\rangle - \omega^2(k_2)\langle\hat{v}_{\boldsymbol{k_1}}\hat{v}_{\boldsymbol{k_2}}\hat{p}_{\boldsymbol{k_3}}\rangle - \omega^2(k_3)\langle\hat{v}_{\boldsymbol{k_1}}\hat{p}_{\boldsymbol{k_2}}\hat{v}_{\boldsymbol{k_3}}\rangle \\ &+ \underbrace{\frac{4\gamma}{(2\pi)^{3/2}}\int \mathrm{d}^3\boldsymbol{k}\widetilde{C}_R(|\boldsymbol{k}|)\langle\hat{v}_{\boldsymbol{k_1}}\hat{v}_{\boldsymbol{k_2}-\boldsymbol{k}}\hat{v}_{\boldsymbol{k}+\boldsymbol{k_3}}\rangle},\end{aligned}, \tag{4.156}$$

on the other hand, the Lindblad equation is valid only at leading term in γ, therefore, we cannot use in the previous equation the bispectrum generated by decoherence since we will be at γ^2order where we can no more rely on Lindblad equation.

Having found a vanishing bispectrum we pass to trispectrum which does receive corrections as was shown in detail in [52]. This last result leaded the authors to conclude saying:

> "It is interesting to notice that decoherence is one of the rare examples where the bispectrum is perturbatively suppressed compared to the trispectrum, which therefore contains the relevant signal"
>
> However, we will show that their conclusion was not accurate, where using their model of decoherence with a different choice of system interaction operator $\hat{A}$we will obtain a bispectrum which is not only different from zero, but is dominant with respect trispectrum!

[15]Because we are considering quadratic Hamiltonian (1.2.1) that does not contain cubic and higher corrections.

4.2.2 Our approach

As mentioned previously, the results obtained J.Martin et al were based on restrict choices of $\hat{A}$, namely of of monomial type $\hat{v}$ and $\hat{v}^2$ or generally $\hat{v}^n$. Their choice leaded them to interesting results but still restrict, so we have extended their approach such that their model becomes more interesting. But our extension implied also that some of their general conclusions need to be scrutinized again under the light of results obtained by our approach.

The approach we will adopt is to consider a system interaction operator of the from

$$\hat{A} = \sum_n \alpha^{n-1} \hat{v}^n \,, \tag{4.157}$$

up to certain order n. There are several physical processes which could give rise to such form of interaction. For the bispectrum we will stop at the order $n = 2$ that represents the leading order for which bispectrum does receive non vanishing corrections. After dealing with bispectrum, we will go back to the power spectrum and show that the computations made by J.Martin et al for quadratic interaction were missing a contribution in the Lindblad term which does not show up, except if we consider more general choice of $\hat{A}$, as we did. This last observation is applicable also to their trispectrum computations.

4.2.2.1 Computation of bispectrum

First let us discuss briefly how does our choice of $\hat{A}$ reproduce and generalize the results obtained by J.Martin et al in [53][52], and most importantly the reason behind obtaining non vanishing bispectrum in contrast to them.

Considering again the previously derived Lindblad equation

$$\frac{d\left\langle \hat{O} \right\rangle}{d\eta} = \left\langle \frac{\partial \hat{O}}{\partial \eta} \right\rangle - i \left\langle \left[\hat{O}, \hat{H}_v \right] \right\rangle - \frac{\gamma}{2} \int d^3\boldsymbol{x} d^3\boldsymbol{y} C_R\left(\boldsymbol{x}, \boldsymbol{y}\right) \left\langle \left[\left[\hat{O}, \hat{A}\left(\boldsymbol{x}\right) \right], \hat{A}\left(\boldsymbol{y}\right) \right] \right\rangle \,, \tag{4.158}$$

but now with our choice of system interaction operator as

$$\hat{A}\left(\boldsymbol{x}\right) = \hat{v} + \alpha \hat{v}^2 \tag{4.159}$$

where α is coupling constant with dimension $[momntum]^{-1}$ which is introduced for the sake of dimensions homogeneity . Substituting (4.159) in (4.158) gives

$$\begin{aligned}
\frac{d\langle\hat{O}\rangle}{d\eta} &= \left\langle\frac{\partial\hat{O}}{\partial\eta}\right\rangle - i\left\langle\left[\hat{O},\hat{H}_v\right]\right\rangle - \frac{\gamma}{2}\int d^3\boldsymbol{x}d^3\boldsymbol{y}C_R\left(\boldsymbol{x},\boldsymbol{y}\right)\left\langle\left[\left[\hat{O},\hat{v}\left(\boldsymbol{x}\right)+\alpha\hat{v}^2\left(\boldsymbol{x}\right)\right],\hat{v}\left(\boldsymbol{y}\right)+\alpha\hat{v}^2\left(\boldsymbol{y}\right)\right]\right\rangle \\
&= \left\langle\frac{\partial\hat{O}}{\partial\eta}\right\rangle - i\left\langle\left[\hat{O},\hat{H}_v\right]\right\rangle - \frac{\gamma}{2}\int d^3\boldsymbol{x}d^3\boldsymbol{y}C_R\left(\boldsymbol{x},\boldsymbol{y}\right)\left\{\left\langle\left[\left[\hat{O},\hat{v}\left(\boldsymbol{x}\right)\right],\hat{v}\left(\boldsymbol{y}\right)\right]\right\rangle\right| \\
&+ \alpha^2\left\langle\left[\left[\hat{O},\hat{v}^2\left(\boldsymbol{x}\right)\right],\hat{v}^2\left(\boldsymbol{y}\right)\right]\right\rangle + \underbrace{\alpha\left|\left\langle\left[\left[\hat{O},\hat{v}\left(\boldsymbol{x}\right)\right],\hat{v}^2\left(\boldsymbol{y}\right)\right] + \left[\left[\hat{O},\hat{v}^2\left(\boldsymbol{x}\right)\right],\hat{v}\left(\boldsymbol{y}\right)\right]\right\rangle\right\}}
\end{aligned},$$

(4.160)

from this equation we can see how our interaction contains all results derived in [52, 53]. In order to obtain their results, it is sufficient for their linear case to set $\alpha = 0$, where for this type of interaction only power spectrum receives correction while all the other correlation functions remain untouched. Then, to get the pure quadratic contribution for power spectrum correction we can add dimensionless constant in front of $\hat{v}$ in $\hat{A}\left(\boldsymbol{x}\right)$ and set it to zero for that purpose, though in perturbative approach it is more legitimate to consider the quadratic order within linear one, since this last usually gives the dominant contribution[16]. In case of trispectrum we do not need this constant since there is no correction to it from linear order, and same for the last term of the real, non unitary, part of (4.160)

$$\propto \alpha\left\langle\left[\left[\hat{O},\hat{v}\left(\boldsymbol{x}\right)\right],\hat{v}^2\left(\boldsymbol{y}\right)\right] + \left[\left[\hat{O},\hat{v}^2\left(\boldsymbol{x}\right)\right],\hat{v}\left(\boldsymbol{y}\right)\right]\right\rangle, \tag{4.161}$$

marked with $\underbrace{\cdots}$, since it gives corrections proportional to to the initial bispectrum which is zero in our case[17]. However, it is this last term (4.143) that makes our bispectrum receives non vanishing correction at leading order in γ, and it is there thanks to the new choice of interaction operator $\hat{A}$ which enables cross terms of the type $\left[\left[\hat{O},\hat{v}^k\left(\boldsymbol{x}\right)\right],\hat{v}^l\left(\boldsymbol{y}\right)\right]$ to show up with $k \neq l$. We remind that in [52] it was found that the last term in (4.160) referring to pure quadratic interaction does give corrections to bispectrum but they are proportional to the initial bispectrum which is zero, while our interaction operator gives corrections proportional to the power spectrum which is obviously different from zero in the free theory. Having said that, and since we are focusing on bispectrum in this section we will keep only the new term in Lindblad equation so that we start the Fourier transform operation

[16] We will see in the next section that adopting an interaction operator $\hat{A}\left(\boldsymbol{x}\right) = \hat{v} + \alpha\hat{v}^2 + \alpha^2\hat{v}^3$, will induce a correction to both, the power spectrum and trispectrum, of the same order in α as that of pure pure quadratic contribution $\alpha^2\left\langle\left[\left[\hat{O},\hat{v}^2\left(\boldsymbol{x}\right)\right],\hat{v}^2\left(\boldsymbol{y}\right)\right]\right\rangle$, namely of order α^2. Those additional corrections come from $\alpha^2\left[\left[\hat{O},\hat{v}^3\left(\boldsymbol{x}\right)\right],\hat{v}\left(\boldsymbol{y}\right)\right]$ and $\alpha^2\left[\left[\hat{O},\hat{v}\left(\boldsymbol{x}\right)\right],\hat{v}^3\left(\boldsymbol{y}\right)\right]$ and were not considered in [53] because they show up only if we consider the most general form of system interaction operator mentioned above, thus we must consider also those extra corrections to get a complete and accurate result, we will come back to this point soon.

[17] It is worth to remind that we we are considering the free Hamiltonian, thus there are no initial non guassianities.

from

$$\frac{d\left\langle\hat{O}\right\rangle}{d\eta}=\left\langle\frac{\partial\hat{O}}{\partial\eta}\right\rangle-i\left\langle\left[\hat{O},\hat{H}_v\right]\right\rangle-\frac{\gamma\alpha}{2}\int \mathrm{d}^3\boldsymbol{x}\mathrm{d}^3\boldsymbol{y}C_R\left(\boldsymbol{x},\boldsymbol{y}\right)\left\{\left\langle\left[\left[\hat{O},\hat{v}\left(\boldsymbol{x}\right)\right],\hat{v}^2\left(\boldsymbol{y}\right)\right]+\left[\left[\hat{O},\hat{v}^2\left(\boldsymbol{x}\right)\right],\hat{v}\left(\boldsymbol{y}\right)\right]\right\rangle\right\}\,. \tag{4.162}$$

Fourier transforming this last using (4.140) and (4.88), leads to

$$\begin{aligned}\frac{d\langle\hat{O}\rangle}{d\eta} &= \left\langle\frac{\partial\hat{O}}{\partial\eta}\right\rangle-i\left\langle\left[\hat{O},\hat{H}_v\right]\right\rangle-\frac{\gamma\alpha}{2}\int \mathrm{d}^3\boldsymbol{k}\mathrm{d}^3\boldsymbol{p}\widetilde{C}_R\left(|\boldsymbol{k}|\right)\left\{\left\langle\left[\left[\hat{O},\hat{v}_{\boldsymbol{k}}\right],\hat{v}_{\boldsymbol{p}}\hat{v}_{\boldsymbol{k}-\boldsymbol{p}}\right]\right.\right.\\ &+ \left.\left.\left[\left[\hat{O},\hat{v}_{\boldsymbol{p}}\hat{v}_{\boldsymbol{k}-\boldsymbol{p}}\right],\hat{v}_{-\boldsymbol{k}}\right]\right\}\right\rangle\end{aligned}\quad, \tag{4.163}$$

making the variable change $-\boldsymbol{k}\to\boldsymbol{k}$ in the first term inside integral which does not affect the environment correlation function since it depends only on modulus, and then using the commutators property for A, B, C being three operators

$$\left[\left[A,B\right],C\right]=\left[\left[A,C\right],B\right]\quad if\quad \left[B,C\right]=0\,, \tag{4.164}$$

equation becomes(4.163)

$$\frac{d\left\langle\hat{O}\right\rangle}{d\eta}=\left\langle\frac{\partial\hat{O}}{\partial\eta}\right\rangle-i\left\langle\left[\hat{O},\hat{H}_v\right]\right\rangle-\gamma\alpha\int \mathrm{d}^3\boldsymbol{k}\mathrm{d}^3\boldsymbol{p}\widetilde{C}_R\left(|\boldsymbol{k}|\right)\left\langle\left[\left[\hat{O},\hat{v}_{\boldsymbol{p}}\hat{v}_{\boldsymbol{k}-\boldsymbol{p}}\right],\hat{v}_{-\boldsymbol{k}}\right]\right\rangle\,, \tag{4.165}$$

this last is the main equation that will enable us to get the differential equations satisfied by each bispectrum which amounts to be eight equations, before giving those last notice that a particular comoving scale appears in the interaction term, indeed, in order for (4.160) (or equivalently (4.165)) to have the correct dimension, $\gamma\widetilde{C}_R\left(|\boldsymbol{k}|\right)$ must be homogeneous to the square of a comoving wavenumber which we define it as

$$k_\gamma=\sqrt{\frac{2}{\pi}\bar{C}_R\frac{\gamma_* l_E}{3a_*^3}}\,, \tag{4.166}$$

Using the commutation relation derived before

$$\left[\hat{v}_{\boldsymbol{p}},\hat{p}_{\boldsymbol{k}}\right]=i\delta^{(3)}\left(\boldsymbol{p}+\boldsymbol{k}\right)\,, \tag{4.167}$$

, and after some straightforward computation we obtain

$$
\begin{aligned}
\frac{d\langle \hat{v}_{\boldsymbol{k}_1}\hat{v}_{\boldsymbol{k}_2}\hat{v}_{\boldsymbol{k}_3}\rangle}{d\eta} &= \langle \hat{v}_{\boldsymbol{k}_1}\hat{v}_{\boldsymbol{k}_2}\hat{p}_{\boldsymbol{k}_3}\rangle + \langle \hat{v}_{\boldsymbol{k}_1}\hat{p}_{\boldsymbol{k}_2}\hat{v}_{\boldsymbol{k}_3}\rangle + \langle \hat{p}_{\boldsymbol{k}_1}\hat{v}_{\boldsymbol{k}_2}\hat{v}_{\boldsymbol{k}_3}\rangle \\
\frac{d\langle \hat{v}_{\boldsymbol{k}_1}\hat{v}_{\boldsymbol{k}_2}\hat{p}_{\boldsymbol{k}_3}\rangle}{d\eta} &= \langle \hat{v}_{\boldsymbol{k}_1}\hat{p}_{\boldsymbol{k}_2}\hat{p}_{\boldsymbol{k}_3}\rangle + \langle \hat{p}_{\boldsymbol{k}_1}\hat{p}_{\boldsymbol{k}_2}\hat{v}_{\boldsymbol{k}_3}\rangle - \omega^2\left(k_3\right)\langle \hat{v}_{\boldsymbol{k}_1}\hat{v}_{\boldsymbol{k}_2}\hat{v}_{\boldsymbol{k}_3}\rangle \\
\frac{d\langle \hat{v}_{\boldsymbol{k}_1}\hat{p}_{\boldsymbol{k}_2}\hat{v}_{\boldsymbol{k}_3}\rangle}{d\eta} &= \langle \hat{v}_{\boldsymbol{k}_1}\hat{p}_{\boldsymbol{k}_2}\hat{p}_{\boldsymbol{k}_3}\rangle + \langle \hat{v}_{\boldsymbol{k}_1}\hat{p}_{\boldsymbol{k}_2}\hat{p}_{\boldsymbol{k}_3}\rangle - \omega^2\left(k_2\right)\langle \hat{v}_{\boldsymbol{k}_1}\hat{v}_{\boldsymbol{k}_2}\hat{v}_{\boldsymbol{k}_3}\rangle \\
\frac{d\langle \hat{p}_{\boldsymbol{k}_1}\hat{v}_{\boldsymbol{k}_2}\hat{v}_{\boldsymbol{k}_3}\rangle}{d\eta} &= \langle \hat{p}_{\boldsymbol{k}_1}\hat{p}_{\boldsymbol{k}_2}\hat{v}_{\boldsymbol{k}_3}\rangle + \langle \hat{p}_{\boldsymbol{k}_1}\hat{p}_{\boldsymbol{k}_2}\hat{v}_{\boldsymbol{k}_3}\rangle - \omega^2\left(k_1\right)\langle \hat{v}_{\boldsymbol{k}_1}\hat{v}_{\boldsymbol{k}_2}\hat{v}_{\boldsymbol{k}_3}\rangle \\
\frac{d\langle \hat{v}_{\boldsymbol{k}_1}\hat{p}_{\boldsymbol{k}_2}\hat{p}_{\boldsymbol{k}_3}\rangle}{d\eta} &= \langle \hat{p}_{\boldsymbol{k}_1}\hat{p}_{\boldsymbol{k}_2}\hat{p}_{\boldsymbol{k}_3}\rangle - \omega^2\left(k_2\right)\langle \hat{v}_{\boldsymbol{k}_1}\hat{v}_{\boldsymbol{k}_2}\hat{p}_{\boldsymbol{k}_3}\rangle - \omega^2\left(k_3\right)\langle \hat{v}_{\boldsymbol{k}_1}\hat{p}_{\boldsymbol{k}_2}\hat{v}_{\boldsymbol{k}_3}\rangle \\
&+ 2\gamma\alpha\left(\widetilde{C}_R\left(|\boldsymbol{k}_2|\right) + \widetilde{C}_R\left(|\boldsymbol{k}_3|\right)\right)\langle \hat{v}_{\boldsymbol{k}_1}\hat{v}_{\boldsymbol{k}_2+\boldsymbol{k}_3}\rangle \\
\frac{d\langle \hat{p}_{\boldsymbol{k}_1}\hat{v}_{\boldsymbol{k}_2}\hat{p}_{\boldsymbol{k}_3}\rangle}{d\eta} &= \langle \hat{p}_{\boldsymbol{k}_1}\hat{p}_{\boldsymbol{k}_2}\hat{p}_{\boldsymbol{k}_3}\rangle - \omega^2\left(k_1\right)\langle \hat{v}_{\boldsymbol{k}_1}\hat{v}_{\boldsymbol{k}_2}\hat{p}_{\boldsymbol{k}_3}\rangle - \omega^2\left(k_3\right)\langle \hat{p}_{\boldsymbol{k}_1}\hat{p}_{\boldsymbol{k}_2}\hat{v}_{\boldsymbol{k}_3}\rangle \qquad , \\
&+ 2\gamma\alpha\left(\widetilde{C}_R\left(|\boldsymbol{k}_1|\right) + \widetilde{C}_R\left(|\boldsymbol{k}_3|\right)\right)\langle \hat{v}_{\boldsymbol{k}_2}\hat{v}_{\boldsymbol{k}_1+\boldsymbol{k}_3}\rangle \\
\frac{d\langle \hat{p}_{\boldsymbol{k}_1}\hat{p}_{\boldsymbol{k}_2}\hat{v}_{\boldsymbol{k}_3}\rangle}{d\eta} &= \langle \hat{p}_{\boldsymbol{k}_1}\hat{p}_{\boldsymbol{k}_2}\hat{p}_{\boldsymbol{k}_3}\rangle - \omega^2\left(k_1\right)\langle \hat{v}_{\boldsymbol{k}_1}\hat{p}_{\boldsymbol{k}_2}\hat{v}_{\boldsymbol{k}_3}\rangle - \omega^2\left(k_2\right)\langle \hat{p}_{\boldsymbol{k}_1}\hat{v}_{\boldsymbol{k}_2}\hat{v}_{\boldsymbol{k}_3}\rangle \\
&+ 2\gamma\alpha\left(\widetilde{C}_R\left(|\boldsymbol{k}_1|\right) + \widetilde{C}_R\left(|\boldsymbol{k}_2|\right)\right)\langle \hat{v}_{\boldsymbol{k}_3}\hat{v}_{\boldsymbol{k}_1+\boldsymbol{k}_2}\rangle \\
\frac{d\langle \hat{p}_{\boldsymbol{k}_1}\hat{p}_{\boldsymbol{k}_2}\hat{p}_{\boldsymbol{k}_3}\rangle}{d\eta} &= -\omega^2\left(k_1\right)\langle \hat{v}_{\boldsymbol{k}_1}\hat{p}_{\boldsymbol{k}_2}\hat{p}_{\boldsymbol{k}_3}\rangle - \omega^2\left(k_2\right)\langle \hat{p}_{\boldsymbol{k}_1}\hat{v}_{\boldsymbol{k}_2}\hat{p}_{\boldsymbol{k}_3}\rangle - \omega^2\left(k_3\right)\langle \hat{p}_{\boldsymbol{k}_1}\hat{p}_{\boldsymbol{k}_2}\hat{v}_{\boldsymbol{k}_3}\rangle \\
&+ 2\gamma\alpha\widetilde{C}_R\left(|\boldsymbol{k}_1|\right)\left[\langle \hat{p}_{\boldsymbol{k}_2}\hat{v}_{\boldsymbol{k}_1+\boldsymbol{k}_3}\rangle + \langle \hat{p}_{\boldsymbol{k}_3}\hat{v}_{\boldsymbol{k}_1+\boldsymbol{k}_2}\rangle\right] + 2\gamma\alpha\widetilde{C}_R\left(|\boldsymbol{k}_2|\right) \\
&\times \left[\langle \hat{p}_{\boldsymbol{k}_1}\hat{v}_{\boldsymbol{k}_2+\boldsymbol{k}_3}\rangle + \langle \hat{p}_{\boldsymbol{k}_3}\hat{v}_{\boldsymbol{k}_1+\boldsymbol{k}_2}\rangle\right] + 2\gamma\alpha\widetilde{C}_R\left(|\boldsymbol{k}_3|\right)\left[\langle \hat{p}_{\boldsymbol{k}_1}\hat{v}_{\boldsymbol{k}_2+\boldsymbol{k}_3}\rangle + \langle \hat{p}_{\boldsymbol{k}_2}\hat{v}_{\boldsymbol{k}_1+\boldsymbol{k}_3}\rangle\right]
\end{aligned}
\tag{4.168}
$$

combining those eight educations into single equation for $\langle \hat{v}_{\boldsymbol{k}_1}\hat{v}_{\boldsymbol{k}_2}\hat{v}_{\boldsymbol{k}_3}\rangle$ yields an equation of order eight which is not too illuminating, so to get a simpler equation of lower order we adopt the equilateral configuration limit in which all the momenta $\boldsymbol{k}_1$, $\boldsymbol{k}_2$, $\boldsymbol{k}_3$ have the same modulus $k = |\boldsymbol{k}|$ so $\omega^2\left(k_i\right) \equiv \omega^2(k)$ and $\widetilde{C}_R\left(|\boldsymbol{k}_i|\right) = \widetilde{C}_R\left(|\boldsymbol{k}|\right) = \widetilde{C}_R\left(k\right)$, doing so, the order of equation could be reduced into an equation of order four as we will see now. For the rest we will adopt the notation $\langle \hat{v}_{\boldsymbol{k}_1}\hat{v}_{\boldsymbol{k}_2}\hat{v}_{\boldsymbol{k}_3}\rangle \equiv B_{vvv}$, $\langle \hat{v}_{\boldsymbol{k}_1}\hat{v}_{\boldsymbol{k}_2}\hat{p}_{\boldsymbol{k}_3}\rangle = B_{vvp}$...etc, since we are interested in the bispectrum of curvature perturbations B_{vvv} then the strategy now is to differentiate this expression and use the other equations in (4.168) in order to obtain a closed differential equation in B_{vvv}, let us show how it works[18]

$$
\begin{aligned}
\tfrac{d^2 B_{vvv}}{d\eta^2} &= 2\left[B_{vpp}+B_{pvp}+B_{ppv}\right]-3\omega^2 B_{vvv}\\
\tfrac{d^3 B_{vvv}}{d\eta^3} &= 6B_{ppp}-4\omega^2\left[B_{vvp}+B_{vpv}+B_{pvv}\right]-3\omega^2\tfrac{dB_{vvv}}{d\eta}-6\omega\omega' B_{vvv}\\
&+4\gamma\alpha\widetilde{C}_R\left(k\right)\left[\langle\hat{v}_{\boldsymbol{k}_1}\hat{v}_{\boldsymbol{k}_2+\boldsymbol{k}_3}\rangle+\langle\hat{v}_{\boldsymbol{k}_2}\hat{v}_{\boldsymbol{k}_1+\boldsymbol{k}_3}\rangle+\langle\hat{v}_{\boldsymbol{k}_3}\hat{v}_{\boldsymbol{k}_1+\boldsymbol{k}_2}\rangle\right]\\
\tfrac{d^4 B_{vvv}}{d\eta^4} &= -14\omega^2\left[B_{vpp}+B_{pvp}+B_{ppv}\right]-8\omega\omega'\left[B_{vvp}+B_{vpv}+B_{pvv}\right]\\
&+12\omega^2 B_{vvv}-12\omega\omega'\tfrac{dB_{vvv}}{d\eta}-3\omega^2\tfrac{d^2B_{vvv}}{d\eta^2}-6\omega'^2 B_{vvv}-6\omega\omega'' B_{vvv}\\
&+24\gamma\alpha\widetilde{C}_R\left(k\right)\left[\langle\hat{p}_{\boldsymbol{k}_1}\hat{v}_{\boldsymbol{k}_2+\boldsymbol{k}_3}\rangle+\langle\hat{p}_{\boldsymbol{k}_2}\hat{v}_{\boldsymbol{k}_1+\boldsymbol{k}_3}\rangle+\langle\hat{p}_{\boldsymbol{k}_3}\hat{v}_{\boldsymbol{k}_1+\boldsymbol{k}_2}\rangle\right]\\
&+4\alpha\left(\gamma\widetilde{C}_R\left(k\right)\right)'\left[\langle\hat{v}_{\boldsymbol{k}_1}\hat{v}_{\boldsymbol{k}_2+\boldsymbol{k}_3}\rangle+\langle\hat{v}_{\boldsymbol{k}_2}\hat{v}_{\boldsymbol{k}_1+\boldsymbol{k}_3}\rangle+\langle\hat{v}_{\boldsymbol{k}_3}\hat{v}_{\boldsymbol{k}_1+\boldsymbol{k}_2}\rangle\right]\\
&+4\gamma\alpha\widetilde{C}_R\left(k\right)\tfrac{d}{d\eta}\left[\langle\hat{v}_{\boldsymbol{k}_1}\hat{v}_{\boldsymbol{k}_2+\boldsymbol{k}_3}\rangle+\langle\hat{v}_{\boldsymbol{k}_2}\hat{v}_{\boldsymbol{k}_1+\boldsymbol{k}_3}\rangle+\langle\hat{v}_{\boldsymbol{k}_3}\hat{v}_{\boldsymbol{k}_1+\boldsymbol{k}_2}\rangle\right]
\end{aligned}
\quad , \tag{4.169}
$$

notice from the the first equation in (4.169) that

$$
B_{vpp}+B_{pvp}+B_{ppv}=\frac{1}{2}\left(\frac{d^2 B_{vvv}}{d\eta^2}+3\omega^2 B_{vvv}\right)\,, \tag{4.170}
$$

so substituting it in in fourth derivative equation of B_{vvv} in addition to using the first equation in (4.168) we get our final differential equation

$$
\frac{d^4 B_{vvv}}{d\eta^4}+10\omega^2\frac{d^2 B_{vvv}}{d\eta^2}+20\omega\omega'\frac{dB_{vvv}}{d\eta}+\left[9\omega^2+6\omega'^2+6\omega\omega''\right]B_{vvv}=S\left(k,\eta\right)\,, \tag{4.171}
$$

[18] In those equations primes denote derivatives with respect to conformal time, we adopted this convention to lighten up equations.

where at leading order in γ, the source function $S(k,\eta)$ is given by

$$\begin{aligned} S(k,\eta) = \quad & 24\gamma\alpha\widetilde{C}_R(k)\left[\mathfrak{Re}\left\langle \hat{p}_{\boldsymbol{k}_1}\hat{v}_{\boldsymbol{k}_2+\boldsymbol{k}_3}\right\rangle + \mathfrak{Re}\left\langle \hat{p}_{\boldsymbol{k}_2}\hat{v}_{\boldsymbol{k}_1+\boldsymbol{k}_3}\right\rangle + \mathfrak{Re}\left\langle \hat{p}_{\boldsymbol{k}_3}\hat{v}_{\boldsymbol{k}_1+\boldsymbol{k}_2}\right\rangle\right] \\ & +4\alpha\left(\gamma\widetilde{C}_R(k)\right)'\left[\left\langle \hat{v}_{\boldsymbol{k}_1}\hat{v}_{\boldsymbol{k}_2+\boldsymbol{k}_3}\right\rangle + \left\langle \hat{v}_{\boldsymbol{k}_2}\hat{v}_{\boldsymbol{k}_1+\boldsymbol{k}_3}\right\rangle + \left\langle \hat{v}_{\boldsymbol{k}_3}\hat{v}_{\boldsymbol{k}_1+\boldsymbol{k}_2}\right\rangle\right] \qquad , \\ & +4\gamma\alpha\widetilde{C}_R(k)\,\frac{d}{d\eta}\left[\left\langle \hat{v}_{\boldsymbol{k}_1}\hat{v}_{\boldsymbol{k}_2+\boldsymbol{k}_3}\right\rangle + \left\langle \hat{v}_{\boldsymbol{k}_2}\hat{v}_{\boldsymbol{k}_1+\boldsymbol{k}_3}\right\rangle + \left\langle \hat{v}_{\boldsymbol{k}_3}\hat{v}_{\boldsymbol{k}_1+\boldsymbol{k}_2}\right\rangle\right] \end{aligned} \tag{4.172}$$

An important remark is that the term involving γ is not explicitly proportional to $\delta(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)$, where the presence of delta function ensures the three Fourier modes of the bispectrum closes triangle, but remember that the system is solved through a perturbative expansion in γ, during the first iteration the Lindblad term contains the correlators $\left\langle \hat{p}_{\boldsymbol{k}_1}\hat{v}_{\boldsymbol{k}_2+\boldsymbol{k}_3}\right\rangle$ ...,$\left\langle \hat{v}_{\boldsymbol{k}_1}\hat{v}_{\boldsymbol{k}_2+\boldsymbol{k}_3}\right\rangle$ ... evaluated in the free theory, which are proportional to $\delta(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)$thus we retrieve our delta function, thus the correlators become $\left\langle \hat{p}_{\boldsymbol{k}}\hat{v}_{-\boldsymbol{k}}\right\rangle, \left\langle \hat{v}_{\boldsymbol{k}}\hat{v}_{-\boldsymbol{k}}\right\rangle$ so (4.172) becomes

$$S(k,\eta) = \quad 72\gamma\alpha\widetilde{C}_R(k)\,\mathfrak{Re}\left\langle \hat{p}_{\boldsymbol{k}}\hat{v}_{-\boldsymbol{k}}\right\rangle + 12\alpha\left(\gamma\widetilde{C}_R(k)\right)'\left\langle \hat{v}_{\boldsymbol{k}}\hat{v}_{-\boldsymbol{k}}\right\rangle + 12\gamma\alpha\widetilde{C}_R(k)\,\frac{d}{d\eta}\left\langle \hat{v}_{\boldsymbol{k}}\hat{v}_{-\boldsymbol{k}}\right\rangle \quad , \tag{4.173}$$

. Our result (4.171) endorses the conjecture made in [52] stated as “One can even conjecture that any correlator must obey a linear differential equation with a source term that describes the interaction with the environment”.

The analogy between our equation (4.171) and the one of power spectrum and trispectrum derived in [53, 52], makes us suggest that n exact solution of it is given by

$$B_{vvv} = \frac{4}{3W^3}\int_{-\infty}^{\eta} d\eta' S(k,\eta')\,\mathfrak{Im}^3\left[\mathrm{v}_{\mathbf{k}}(\eta')\,\mathrm{v}^*_{\mathbf{k}}(\eta)\right], \tag{4.174}$$

indeed using the Mukhanov Sasaki equation $v''_{\mathbf{k}}+\omega^2 v_{\mathbf{k}}=0$ in addition to $v_k=\left(v_k^R+iv_k^I\right)/\sqrt{2}$, we can check in straightforward but lengthy computation that (4.174) provides an exact solution to (4.171). It is worth to mention that the bispectrum we obtained is purely caused by decoherence, because we started with the quadratic Hamiltonian for the system which induces vanishing non guassianities.

Computing the integral (4.174) will lead us to discuss the non linearity-parameter f_{NL} characterizing the amplitude of bispectrum, which is defined as the ratio between the bispectrum in the equilateral configuration to the square of power spectrum of curvature perturbation [8, 23]

$$f_{NL} = \frac{5}{18}\frac{B_{\zeta\zeta\zeta}\left(k^3\right)}{P_{\zeta}^2(k)}\,, \tag{4.175}$$

where $P_\zeta(k)=\frac{H^2}{4\epsilon M_{pl}^2 k^3} = \frac{P_{vv}}{2\epsilon M_{pl}^2 a^2}$ is the dimensionless power spectrum of the curvature perturbation related to MS variable by $\zeta = \frac{v}{\sqrt{2\epsilon}M_{pl}a}$, therefore our f_{NL} could expressed as

$$f_{NL} = \frac{5}{18}\frac{\sqrt{2\epsilon}M_{pl}aB_{vvv}\left(k^3\right)}{P_{vv}^2(k)} = \frac{10}{27W^3}\frac{\sqrt{2\epsilon}M_{pl}a}{\Delta_v^2(k)}\int\limits_{-\infty}^{\eta} d\eta' S\left(k,\eta'\right)\mathfrak{Im}^3\left[\mathrm{v}_{\mathbf{k}}\left(\eta'\right)\mathrm{v}_{\mathbf{k}}^*\left(\eta\right)\right], \tag{4.176}$$

Let us remind some definitions which will be needed to evaluate (4.174), first we will put ourselves in the de-Sitter limit thus the scale factor is given by a =-1/$(H\eta)$ and $v_{\mathbf{k}}\left(\eta\right)$ by

$$v_{\mathbf{k}}\left(\eta\right) = \frac{e^{-ik\eta}}{\sqrt{2k}}\left(1-\frac{i}{k\eta}\right)\,, \tag{4.177}$$

suing this last we get

$$\begin{aligned}
&\mathfrak{Im}^3\left[\mathrm{v}_{\mathbf{k}}\left(\eta'\right)\mathrm{v}_{\mathbf{k}}^*\left(\eta\right)\right] = \tfrac{\cos\left(k\left(\eta-\eta'\right)\right)}{2k}\left(\tfrac{1}{k\eta}-\tfrac{1}{k\eta'}\right)+\tfrac{\sin\left(k\left(\eta-\eta'\right)\right)}{2k}\left(1+\tfrac{1}{k^2\eta\eta'}\right)\\
&\left\langle\hat{v}_{\boldsymbol{k}}\hat{v}_{-\boldsymbol{k}}\right\rangle = \tfrac{1}{2k}\left[1+\tfrac{1}{(-k\eta)^2}\right] \qquad ,\\
&\mathfrak{Re}\left\langle\hat{p}_{\boldsymbol{k}}\hat{v}_{-\boldsymbol{k}}\right\rangle = \tfrac{1}{2(-k\eta)^3}
\end{aligned} \tag{4.178}$$

reminding also

$$\tilde{C}_R\left(k\right) = \sqrt{\frac{2}{\pi}}\bar{C}_R\frac{l_E}{3a^3}\Theta\left(\frac{kl_E}{a}\right)\,, \tag{4.179}$$

$$\gamma = \gamma_*\left(\frac{a}{a_*}\right)^p\,, \tag{4.180}$$

from using $k_* = a_* H_*$and the fact that $H \simeq H_* \simeq constant$ then (4.173) could be written as

$$B_{vvv} = B_{vvv}^{(1)} + B_{vvv}^{(2)} + B_{vvv}^{(3)}\,, \tag{4.181}$$

where [19]with the change of variable $z = k\eta'$ we get

$$
\begin{aligned}
B^{(1)}_{vvv} = & \; 7\alpha \tfrac{k_\gamma^2}{k_*^4} \left(\tfrac{k}{k_*}\right)^{p-7} \textstyle\int_{-\infty}^{k\eta} dz \left\{ \Theta\left(-zHl_E\right)\left(-z\right)^{-p} \right. \\
& \left. \times \left[\cos\left(k\eta - z\right)\left(\tfrac{1}{k\eta} - \tfrac{1}{z}\right) + \sin\left(k\eta - z\right)\left(1 + \tfrac{1}{k\eta z}\right)\right]^3 \right\} \\
B^{(2)}_{vvv} = & \; \alpha \tfrac{k_\gamma^2}{k_*^4} \left(\tfrac{k}{k_*}\right)^{p-7} Hl_E \textstyle\int_{-\infty}^{k\eta} dz \left\{ \delta\left(1 + zHl_E\right)\left(-z\right)^{3-p}\left(1 + \tfrac{1}{(-z)^2}\right) \right. \\
& \left. \times \left[\cos\left(k\eta - z\right)\left(\tfrac{1}{k\eta} - \tfrac{1}{z}\right) + \sin\left(k\eta - z\right)\left(1 + \tfrac{1}{k\eta z}\right)\right]^3 \right\} \\
B^{(3)}_{vvv} = & \; \alpha \tfrac{k_\gamma^2}{k_*^4} \left(\tfrac{k}{k_*}\right)^{p-7} (p-3) \textstyle\int_{-\infty}^{k\eta} dz \left\{ \Theta\left(-zHl_E\right)\left(-z\right)^{2-p}\left(1 + \tfrac{1}{(-z)^2}\right) \right. \\
& \left. \times \left[\cos\left(k\eta - z\right)\left(\tfrac{1}{k\eta} - \tfrac{1}{z}\right) + \sin\left(k\eta - z\right)\left(1 + \tfrac{1}{k\eta z}\right)\right]^3 \right\}
\end{aligned}
\quad , \tag{4.182}
$$

Notice that due to the presence of the $\Theta\left(-zHl_E\right)$ in the first and third integral sets a lower bound on our variable z ($z_{min} =- \left(Hl_E\right)^{-1}$) and this ensures that the those integrals are finite, it is also obvious that the presences of delta function in second integral is due to the derivative of Heaviside function. The above integrals are not exactly analytically computable, but the fact that we are interested on super Horizon modes $-k\eta \to 0$ will help to simplify them, the behavior of the correlators can be obtained by identifying the region in the integration domain from where the integral receives its main contribution.

1-For $B^{(1)}_{vvv}$ we see that for[20] $0 < p$the main contribution is always coming from the upper bound where $-k\eta \ll 1$ and $-z \ll 1$ (with $Hl_E \ll 1$) thus we can expand the integrand in this limit and $B^{(1)}_{vvv}$ becomes

$$
B^{(1)}_{vvv} = \; -7\alpha \tfrac{k_\gamma^2}{k_*^4} \left(\tfrac{k}{k_*}\right)^{p-7} \int_{-(Hl_E)^{-1}}^{k\eta} dz \frac{\left(z - k\eta\right)^3}{\left(-z\right)^p} \quad , \tag{4.183}
$$

[19]Notice that the in $B^{(2)}_{vvv}$ we have $\delta\left(1 + zHl_E\right)$ instead of $\delta\left(zHl_E\right)$ and this is due to our definition of top hat function as being non zero if $-zHl_E < 1$, therefore we shifted the argument and used the definition of the derivative of Heaviside function.

[20]Supposing p positive is due the fact that classicalization of our perturbations becomes more efficient as our modes exit horizon, which is equivalent to saying that decoherence becomes more efficient through the increasing coupling. (this argument needs to be discussed with professors).

For $p \neq 1,\, 2,\, 3,\, 4$, which are singular cases to be computed apart, the above integral gives for $4 > p$

$$\begin{aligned} B^{(1)}_{vvv} &= -7\alpha \tfrac{k_\gamma^2}{k_*^4} \left(\tfrac{k}{k_*}\right)^{p-7} (-k\eta)^{4-p} \left[\tfrac{1}{p-4} - \tfrac{3}{p-3} + \tfrac{3}{p-2} - \tfrac{1}{p-1}\right] \\ &= -7\alpha \tfrac{k_\gamma^2}{k_*^4} \left(\tfrac{k}{k_*}\right)^{p-7} (-k\eta)^{4-p} \tfrac{3p-4}{(p-4)(p-3)(p-2)(p-1)} \end{aligned} , \tag{4.184}$$

- For $p = 1$, (4.183) gives

$$B^{(1)}_{vvv} = 7\alpha \frac{k_\gamma^2}{k_*^4} \left(\frac{k}{k_*}\right)^{-6} (k\eta)^3 \left(\ln\left(|k\eta|\right) - \frac{11}{6}\right) , \tag{4.185}$$

- For $p = 2$, (4.183) gives

$$B^{(1)}_{vvv} = 21\alpha \frac{k_\gamma^2}{k_*^4} \left(\frac{k}{k_*}\right)^{-5} (k\eta)^2 \left(\ln\left(|k\eta|\right) - \frac{1}{2}\right) , \tag{4.186}$$

- For $p = 3$, (4.183) gives

$$B^{(1)}_{vvv} = +21\alpha \frac{k_\gamma^2}{k_*^4} \left(\frac{k}{k_*}\right)^{-4} (k\eta) \left(\ln\left(|k\eta|\right) + \frac{1}{2}\right) , \tag{4.187}$$

- For $p = 4$, (4.183) gives

$$B^{(1)}_{vvv} = 7\alpha \frac{k_\gamma^2}{k_*^4} \left(\frac{k}{k_*}\right)^{-3} \left(\ln\left(|k\eta|\right) + \frac{11}{6}\right) , \tag{4.188}$$

  we see that in in the singular cases and in super horizon limit the first term $\propto \ln\left(|k\eta|\right)$ gives the largest contribution , we notice also that in all cases, singular and non singular, $B^{(1)}_{vvv} \propto k^{-3}$.

2- For $B^{(2)}_{vvv}$ the delta function rends the integration task simple where considering only the dominant terms for in the limit $-k\eta \ll 1$ and $(Hl_E)^{-1} \gg 1$ we get

$$B^{(2)}_{vvv} = \quad \alpha \tfrac{k_\gamma^2}{k_*^4} \left(\tfrac{k}{k_*}\right)^{p-7} \tfrac{1}{(k\eta)^3} \left(Hl_E\right)^{p-2} \cos^3\left(Hl_E\right)^{-1} \quad , \tag{4.189}$$

3- For $B^{(3)}_{vvv}$ in the case $p > 2$ and we find ourselves again in the limit where the main contribution comes from the upper bound thus for $-k\eta \ll 1$, $-z \ll 1$ $B^{(3)}_{vvv}$ could be written as

$$B^{(3)}_{vvv} = -\alpha\frac{k_\gamma^2}{k_*^4}\left(\frac{k}{k_*}\right)^{p-7}(p-3)\int_{-(Hl_E)^{-1}}^{k\eta} dz\frac{(z-k\eta)^3\left(z^2+1\right)}{(-z)^p} \quad , \tag{4.190}$$

which gives

$$\begin{aligned} B^{(3)}_{vvv} = & -\alpha\frac{k_\gamma^2}{k_*^4}\left(\frac{k}{k_*}\right)^{p-7}(p-3)\left\{(k\eta)^{6-p}\left[\frac{1}{p-6}-\frac{3}{p-5}+\frac{3}{p-4}-\frac{1}{p-3}\right]\right. \\ & \left. +\,(k\eta)^{4-p}\left[\tfrac{1}{p-4}-\frac{3}{p-3}+\frac{3}{p-2}-\frac{1}{p-1}\right]\right\} \end{aligned} \quad , \tag{4.191}$$

simplifying this last we get

$$B^{(3)}_{vvv} = -6\alpha\frac{k_\gamma^2}{k_*^4}\left(\frac{k}{k_*}\right)^{p-7}\left[(-k\eta)^{6-p}\frac{1}{(p-6)(p-5)(p-4)}+(-k\eta)^{4-p}\frac{1}{(p-4)(p-2)(p-1)}\right], \tag{4.192}$$

as in the previous case we got some singular cases which need to be computed apart

- For $p=4$ (4.190) gives

$$\begin{aligned} B^{(3)}_{vvv} &= -\alpha\frac{k_\gamma^2}{k_*^4}\left(\frac{k}{k_*}\right)^{-3}\int_{-(Hl_E)^{-1}}^{k\eta} dz\frac{(z-k\eta)^3\left(z^2+1\right)}{z^4} \\ &= -\alpha\frac{k_\gamma^2}{k_*^4}\left(\frac{k}{k_*}\right)^{-3}\left[(k\eta)^2\left(3\ln\left(|k\eta|\right)-\tfrac{5}{2}\right)+\ln\left(|k\eta|\right)-\tfrac{17}{6}\right] \end{aligned} \quad , \tag{4.193}$$

- For $p=5$ (4.190) gives

$$\begin{aligned} B^{(3)}_{vvv} &= 2\alpha\frac{k_\gamma^2}{k_*^4}\left(\frac{k}{k_*}\right)^{-3}\int_{-(Hl_E)^{-1}}^{k\eta} dz\frac{(z-k\eta)^3\left(z^2+1\right)}{z^5} \\ &= -2\alpha\frac{k_\gamma^2}{k_*^4}\left(\frac{k}{k_*}\right)^{-3}\left[3\left(k\eta\right)\left(\tfrac{7}{2}+\ln\left(|k\eta|\right)\right)+\tfrac{1}{4k\eta}\right] \end{aligned} \quad , \tag{4.194}$$

- For $p=6$ (4.190) gives

$$\begin{aligned} B^{(3)}_{vvv} &= -3\alpha\frac{k_\gamma^2}{k_*^4}\left(\frac{k}{k_*}\right)^{-3}\int_{-(Hl_E)^{-1}}^{k\eta}dz\frac{(z-k\eta)^3\left(z^2+1\right)}{z^6} \\ &= 3\alpha\frac{k_\gamma^2}{k_*^4}\left(\frac{k}{k_*}\right)^{-3}\left[\ln\left(|k\eta|\right)+\frac{3}{2}+\frac{17}{60}\frac{1}{(k\eta)^2}\right] \end{aligned} , \tag{4.195}$$

Now we turn to the cases $p=2\,,1$

$p=2$ given by

$$\begin{aligned} B^{(3)}_{vvv} &= -\alpha\frac{k_\gamma^2}{k_*^4}\left(\frac{k}{k_*}\right)^{-5}\int_{-(Hl_E)^{-1}}^{k\eta}dz\left\{\left(1+\frac{1}{(-z)^2}\right)\right. \\ &\times\left.\left[\cos\left(k\eta-z\right)\left(\frac{1}{k\eta}-\frac{1}{z}\right)+\sin\left(k\eta-z\right)\left(1+\frac{1}{k\eta z}\right)\right]^3\right\} \end{aligned} , \tag{4.196}$$

we see two contributions to the above integral $B^{(3)}_{vvv}=B^{(3)}_{vvv}|_1+B^{(3)}_{vvv}|_2$, the first is dominated by the upper bound therefore expanding the integrand again around $-k\eta\ll 1$, $-z\ll 1$ leads to

$$\begin{aligned} B^{(3)}_{vvv}|_1 &= \alpha\frac{k_\gamma^2}{k_*^4}\left(\frac{k}{k_*}\right)^{-5}\int_{-(Hl_E)^{-1}}^{k\eta}dz\frac{(z-k\eta)^3}{z^2} \\ &= -3\alpha\frac{k_\gamma^2}{k_*^4}\left(\frac{k}{k_*}\right)^{-5}(k\eta)^2\left[\ln\left(|k\eta|\right)-\frac{1}{2}\right] \end{aligned} , \tag{4.197}$$

second contibution is given by

$$B^{(3)}_{vvv}|_2 = \alpha\frac{k_\gamma^2}{k_*^4}\left(\frac{k}{k_*}\right)^{-5}\int_{-(Hl_E)^{-1}}^{k\eta}dz\left(\cos\left(z-k\eta\right)\left(\frac{1}{k\eta}-\frac{1}{z}\right)-\sin\left(z-k\eta\right)\left(1+\frac{1}{zk\eta}\right)\right)^3 , \tag{4.198}$$

this last could be computed exactly, and since the antiderivative is too involved we will pick only the leading terms

$$B^{(3)}_{vvv}|_2 = \alpha\frac{k_\gamma^2}{k_*^4}\left(\frac{k}{k_*}\right)^{-5}\left[\frac{3}{4}\frac{Si\left(-(Hl_E)^{-1}\right)}{(k\eta)^3}-\frac{11}{16}\frac{Ci(-k\eta)}{(k\eta)^3}\right] , \tag{4.199}$$

where $Ci\left(x\right)$ and $Si\left(x\right)$are the CosIntegral and SinIntegral, respectively, Thus we see that in the limit

$k\eta \to 0$ the $B^{(3)}_{vvv}\,|_1$ is subdominant with respect to $B^{(3)}_{vvv}\,|_2$, therefore we may safely conclude that

$$B^{(3)}_{vvv} = \alpha \frac{k_\gamma^2}{k_*^4}\left(\frac{k}{k_*}\right)^{-5}\left[\frac{3}{4}\frac{Si\left(-(Hl_E)^{-1}\right)}{(k\eta)^3} - \frac{11}{16}\frac{Ci(-k\eta)}{(k\eta)^3}\right] \quad . \tag{4.200}$$

Finally we compute $B^{(3)}_{vvv}$ for $p = 1$ which is given by

$$\begin{aligned} B^{(3)}_{vvv} &= 2\alpha \frac{k_\gamma^2}{k_*^4}\left(\frac{k}{k_*}\right)^{-6} \int_{-(Hl_E)^{-1}}^{k\eta} dz \left\{ \left(z - \frac{1}{z}\right) \right. \\ &\times \left. \left[\cos(k\eta - z)\left(\frac{1}{k\eta} - \frac{1}{z}\right) + \sin(k\eta - z)\left(1 + \frac{1}{k\eta z}\right)\right]^3 \right\} \end{aligned} \quad , \tag{4.201}$$

the first integral could be computed exactly, but again for simplicity we write down the leading terms

$$\begin{aligned} &\int_{-(Hl_E)^{-1}}^{k\eta} dz z \left[\cos(k\eta - z)\left(\frac{1}{k\eta} - \frac{1}{z}\right) + \sin(k\eta - z)\left(1 + \frac{1}{k\eta z}\right)\right]^3 \\ \simeq\ & \frac{16}{9}\frac{\cos(Hl_E)^{-1}}{(k\eta)^3} - \frac{5}{6}\frac{(Hl_E)^{-1}\sin(Hl_E)^{-1}}{(k\eta)^3} \end{aligned} \quad , \tag{4.202}$$

the second integral receives its main contribution from the upper bound thus as usual expanding the integrand in the limit $-k\eta \ll 1, -z \ll 1$ we end up with

$$\begin{aligned} &\int_{-(Hl_E)^{-1}}^{k\eta} dz \frac{(z-a)^3}{z} \\ \simeq\ & (k\eta)^3 \left[\frac{5}{3} - \ln(|k\eta|)\right] \end{aligned} \quad , \tag{4.203}$$

so in total, and considering only the leading term in (4.203), namely the one $\propto \ln(|k\eta|)$, which becomes now subdominant it the total $B^{(3)}_{vvv}$

$$B^{(3)}_{vvv} = 2\alpha \frac{k_\gamma^2}{k_*^4}\left(\frac{k}{k_*}\right)^{-6}\left[\frac{16}{9}\frac{\cos(Hl_E)^{-1}}{(k\eta)^3} - \frac{5}{6}\frac{(Hl_E)^{-1}\sin(Hl_E)^{-1}}{(k\eta)^3} - (k\eta)^3\ln(|k\eta|)\right] \quad , \tag{4.204}$$

Having finished the computations of of the different $B^{(i)}_{vvv}$, we substitute the results found for the different values of p in (4.176) in order to obtain the decoherence induced f_{NL}.

- $p = 1$, in this case the contribution coming from $B^{(1)}_{vvv}$ is negligible compared to the other two

$$f_{NL} = \frac{10}{9}\sqrt{2\epsilon}\alpha k_\gamma^2 \frac{M_{pl}}{H}\left(\frac{k}{k_*}\right)^{-3}\left[\frac{5}{6}\frac{(Hl_E)^{-1}\sin(Hl_E)^{-1}}{(k\eta)^3} - \frac{16}{9}\frac{\cos(Hl_E)^{-1}}{(k\eta)^3} - (Hl_E)^{-1}\cos^3(Hl_E)^{-1}\right], \tag{4.205}$$

- $p = 2$, in this case the contribution coming from $B^{(1)}_{vvv}$ is negligible compared to the other two

$$f_{NL} = \frac{10}{9}\sqrt{2\epsilon}\alpha k_\gamma^2 \frac{M_{pl}}{H}\left(\frac{k}{k_*}\right)^{-2}\left[\cos^3(Hl_E)^{-1} + \frac{3}{4}Si\left(-(Hl_E)^{-1}\right) - \frac{11}{16}Ci(-k\eta)\right], \tag{4.206}$$

- $p = 3$, in this case the contribution coming from both $B^{(1)}_{vvv}$ and $B^{(3)}_{vvv}$ is negligible compared to the $B^{(2)}_{vvv}$, thus we get

$$f_{NL} = -\frac{10}{9}\sqrt{2\epsilon}\alpha k_\gamma^2 \left(\frac{k}{k_*}\right)^{-1}\frac{M_{pl}}{H}Hl_E\cos^3(Hl_E)^{-1}, \tag{4.207}$$

- $p = 4$, in this case also $B^{(2)}_{vvv}$ gives the dominant contribution

$$f_{NL} = -\frac{10}{9}\sqrt{2\epsilon}\alpha k_\gamma^2 \frac{M_{pl}}{H}(Hl_E)^2\cos^3(Hl_E)^{-1}, \tag{4.208}$$

  it is quite remarkable that we obtained a scale invariant bispectrum for $p = 4$ which was an expected result for the following reason:

The scale invariance was also obtained in [53, 52] for both power spectrum and trispectrum. Where it was obtained for a value $p = 5$ in case of linear interaction thus the term of Lindblad equation (4.158)

$$\frac{\gamma}{2}\int d^3\boldsymbol{x}d^3\boldsymbol{y}C_R(\boldsymbol{x},\boldsymbol{y})\left\langle\left[\left[\hat{O},\hat{A}(\boldsymbol{x})\right],\hat{A}(\boldsymbol{y})\right]\right\rangle, \tag{4.209}$$

what gives the corrections was $\propto \left(\hat{A}(\boldsymbol{x})\right)^2 = \hat{v}^2$, while they obtained the scale invariance for $p = 3$ for the case of pure quadratic interaction thus (4.209) is $\propto \left(\hat{A}(\boldsymbol{x})\right)^2 = \hat{v}^4$, while we in our case we considered $\hat{A}(\boldsymbol{x}) = \hat{v} + \alpha\hat{v}^2$, and we noticed that the correction to bispectrum was coming from a term in (4.209) that is $\propto \hat{v}^3$as could be seen in (4.165), thus comparing the powers of $\hat{v}$ and values of p we see it a logical and expected to get this scale invariance in our case for $p = 4$. Notice also that f_{NL}^{local} is suppressed by the slow roll parameter ϵ, in addition to $Hl_E \ll 1$ and possibly also by by the coupling α which is supposed to be small.

- $p = 5$

$$f_{NL} = -\frac{10}{9}\sqrt{2\epsilon}\alpha k_\gamma^2 \frac{M_{pl}}{H}\frac{k}{k_*}(Hl_E)^3\cos^3(Hl_E)^{-1}, \tag{4.210}$$

- $p = 6$

$$f_{NL} = -\frac{10}{9}\sqrt{2\epsilon}\alpha k_\gamma^2 \frac{M_{pl}}{H}\left(\frac{k}{k_*}\right)^2\left[(Hl_E)^3\cos^3(Hl_E)^{-1} - \frac{7}{60}\eta + \frac{17}{20}\left(\frac{k}{k_*}\right)^{-1}\eta\right], \tag{4.211}$$

for $p > 6$, taking into account only the leading terms we get

$$\begin{aligned} f_{NL} =& -\frac{10}{9}\sqrt{2\epsilon}\alpha k_\gamma^2 \frac{M_{pl}}{H} k^6\eta^3\left(\frac{k}{k_*}\right)^{p-7}\left[\frac{1}{(k\eta)^3}(Hl_E)^{p-2}\cos^3(Hl_E)^{-1}\right. \\ &\left. + - (-k\eta)^{4-p}\frac{3p-4}{(p-4)(p-3)(p-2)(p-1)} - 6(k\eta)^{4-p}\frac{1}{(p-4)(p-2)(p-1)}\right], \end{aligned} \tag{4.212}$$

4.2.2.2 Additional correction to power spectrum

As we already mentioned in the previous section J.Martin et al in [52, 53] considered the corrections induced by decoherence for two cases of system interaction operator, linear $\hat{A}(\boldsymbol{x}) = \hat{v}$ and pure quadratic $\hat{A}(\boldsymbol{x}) = \hat{v}^2$, and when it comes to the power spectrum, P_{vv}, they obtained non zero corrections for both cases. However, as we stated before, we should consider a system interaction operator which is expanded perturbatively in $\hat{v}$ with an expansion coupling α, and by doing so we realize that actually there is a correction which must be considered along the pure quadratic one in order to have a consistent and accurate result, because they are both of the same order in α, namely second order, in what follows we show this last point and compute the additional correction that must be added to the ones already obtained in [53].

Since we are interested in corrections up to order α^2, we adopt the system interaction operator

$$\hat{A}(\boldsymbol{x}) = \hat{v} + \alpha\hat{v}^2 + \alpha^2\hat{v}^3 \tag{4.213}$$

and (4.158) becomes up to second order in α

$$\begin{aligned}
\frac{d\langle\hat{O}\rangle}{d\eta} &= \left\langle\frac{\partial\hat{O}}{\partial\eta}\right\rangle - i\left\langle\left[\hat{O},\hat{H}_v\right]\right\rangle - \frac{\gamma}{2}\int d^3\boldsymbol{x}d^3\boldsymbol{y}C_R\left(\boldsymbol{x},\boldsymbol{y}\right)\left\langle\left[\left[\hat{O},\hat{v}\left(\boldsymbol{x}\right)+\alpha\hat{v}^2\left(\boldsymbol{x}\right)+\alpha^2\hat{v}^3\left(\boldsymbol{x}\right)\right],\hat{v}\left(\boldsymbol{y}\right)+\alpha\hat{v}^2\left(\boldsymbol{y}\right)+\alpha^2\hat{v}^3\left(\boldsymbol{y}\right)\right]\right\rangle \\
&= \left\langle\frac{\partial\hat{O}}{\partial\eta}\right\rangle - i\left\langle\left[\hat{O},\hat{H}_v\right]\right\rangle - \frac{\gamma}{2}\int d^3\boldsymbol{x}d^3\boldsymbol{y}C_R\left(\boldsymbol{x},\boldsymbol{y}\right)\Big\{\left\langle\left[\left[\hat{O},\hat{v}\left(\boldsymbol{x}\right)\right],\hat{v}\left(\boldsymbol{y}\right)\right]\right\rangle + \\
&\quad \alpha\left\langle\left[\left[\hat{O},\hat{v}\left(\boldsymbol{x}\right)\right],\hat{v}^2\left(\boldsymbol{y}\right)\right]+\left[\left[\hat{O},\hat{v}^2\left(\boldsymbol{x}\right)\right],\hat{v}\left(\boldsymbol{y}\right)\right]\right\rangle \\
&+ \quad \alpha^2\left(\left\langle\left[\left[\hat{O},\hat{v}^2\left(\boldsymbol{x}\right)\right],\hat{v}^2\left(\boldsymbol{y}\right)\right]\right\rangle + \underbrace{\left\langle\left[\left[\hat{O},\hat{v}\left(\boldsymbol{x}\right)\right],\hat{v}^3\left(\boldsymbol{y}\right)\right]+\left[\left[\hat{O},\hat{v}^3\left(\boldsymbol{x}\right)\right],\hat{v}\left(\boldsymbol{y}\right)\right]\right\rangle}\right)\Big\} \\
&+
\end{aligned} \tag{4.214}$$

from this last equation we can see in the last line that the pure quadratic interaction term $\left\langle\left[\left[\hat{O},\hat{v}^2\left(\boldsymbol{x}\right)\right],\hat{v}^2\left(\boldsymbol{y}\right)\right]\right\rangle$ is accompanied by a cross term coming from a combination of the linear and cubic interaction $\left\langle\left[\left[\hat{O},\hat{v}\left(\boldsymbol{x}\right)\right],\hat{v}^3\left(\boldsymbol{y}\right)\right]+\left[\left[\hat{O},\hat{v}^3\left(\boldsymbol{x}\right)\right],\hat{v}\left(\boldsymbol{y}\right)\right]\right\rangle$. It is this last term that was missing in the computations done in [53], so now we want to compute the correction induced by it to P_{vv}. Fortunately enough that all what we will need to do at the end is to add this correction to linear and quadratic ones, because as we will see in a moment the solution to the equation of evolution of P_{vv} is linear in sources functions which by turn are promotional within a numerical factor to those real terms in (4.214).

Before doing the computations we need to Fourier transform (4.214) using again (4.88) and for cubic terms we use

$$\hat{v}^3\left(\boldsymbol{x}\right) = \frac{1}{\left(2\pi\right)^{9/2}}\int d^3\boldsymbol{p}d^3\boldsymbol{p}_1d^3\boldsymbol{p}_2d^3\hat{v}_{\boldsymbol{p}_1}\hat{v}_{\boldsymbol{p}_2}\hat{v}_{\boldsymbol{p}-\boldsymbol{p}_1-\boldsymbol{p}_2}e^{i\boldsymbol{p}.\boldsymbol{x}} \tag{4.215}$$

using also (4.164) and noticing that the term linear in α in (4.214) gives vanishing contribution to the correction of P_{vv}, then the Fourier transform is given by

$$\begin{aligned}
\frac{d\langle\hat{O}\rangle}{d\eta} &= \left\langle\frac{\partial\hat{O}}{\partial\eta}\right\rangle - i\left\langle\left[\hat{O},\hat{H}_v\right]\right\rangle - \frac{\gamma}{2}\left(2\pi\right)^{3/2}\int d^3\boldsymbol{k}\widetilde{C}_R\left(|\boldsymbol{k}|\right)\left\langle\left[\left[\hat{O},\hat{v}_{\boldsymbol{k}}\right],\hat{v}_{-\boldsymbol{k}}\right]\right\rangle - \alpha^2\frac{\gamma}{2(2\pi)^{3/2}}\int d^3\boldsymbol{k}d^3\boldsymbol{p}_1d^3\boldsymbol{p}_2\widetilde{C}_R\left(|\boldsymbol{k}|\right) \\
&\times \quad \left(\left\langle\left[\left[\hat{O},\hat{v}_{\boldsymbol{p}_1}\hat{v}_{\boldsymbol{k}-\boldsymbol{p}_1}\right],\hat{v}_{\boldsymbol{p}_2}\hat{v}_{-\boldsymbol{k}-\boldsymbol{p}_2}\right]\right\rangle + 2\left\langle\left[\left[\hat{O},\hat{v}_{\boldsymbol{k}}\right],\hat{v}_{\boldsymbol{p}_1}\hat{v}_{\boldsymbol{p}_2}\hat{v}_{-\boldsymbol{k}-\boldsymbol{p}_1-\boldsymbol{p}_2}\right]\right\rangle\right)
\end{aligned}, \tag{4.216}$$

the next step is to derive the equations governing the various two-point correlators

$$\begin{aligned}
\frac{d\left\langle \hat{v}_{\boldsymbol{k}_1}\hat{v}_{\boldsymbol{k}_2}\right\rangle}{d\eta} &= \langle \hat{v}_{\boldsymbol{k}_1}\hat{p}_{\boldsymbol{k}_2}\rangle + \langle \hat{p}_{\boldsymbol{k}_1}\hat{v}_{\boldsymbol{k}_2}\rangle \\
\frac{d\left\langle \hat{v}_{\boldsymbol{k}_1}\hat{p}_{\boldsymbol{k}_2}\right\rangle}{d\eta} &= \langle \hat{p}_{\boldsymbol{k}_1}\hat{p}_{\boldsymbol{k}_2}\rangle - \omega^2\left(k_2\right)\langle \hat{v}_{\boldsymbol{k}_1}\hat{v}_{\boldsymbol{k}_2}\rangle \\
\frac{d\left\langle \hat{p}_{\boldsymbol{k}_1}\hat{v}_{\boldsymbol{k}_2}\right\rangle}{d\eta} &= \langle \hat{p}_{\boldsymbol{k}_1}\hat{p}_{\boldsymbol{k}_2}\rangle - \omega^2\left(k_1\right)\langle \hat{v}_{\boldsymbol{k}_1}\hat{v}_{\boldsymbol{k}_2}\rangle \\
\frac{d\left\langle \hat{p}_{\boldsymbol{k}_1}\hat{p}_{\boldsymbol{k}_2}\right\rangle}{d\eta} &= -\omega^2\left(k_2\right)\langle \hat{p}_{\boldsymbol{k}_1}\hat{v}_{\boldsymbol{k}_2}\rangle - \omega^2\left(k_1\right)\langle \hat{v}_{\boldsymbol{k}_1}\hat{p}_{\boldsymbol{k}_2}\rangle + \gamma\left(2\pi\right)^{3/2}\widetilde{C}_R\left(|\boldsymbol{k}_1|\right)\delta\left(\boldsymbol{k}_1+\boldsymbol{k}_2\right) + \alpha^2\frac{\gamma}{(2\pi)^{3/2}} \\
&\times\left[4\int d^3\boldsymbol{k}\widetilde{C}_R\left(|\boldsymbol{k}|\right)\langle \hat{v}_{\boldsymbol{k}+\boldsymbol{k}_1}\hat{v}_{-\boldsymbol{k}+\boldsymbol{k}_2}\rangle + \underbrace{3\left(\widetilde{C}_R\left(|\boldsymbol{k}_1|\right)+\widetilde{C}_R\left(|\boldsymbol{k}_2|\right)\right)\int d^3\boldsymbol{k}\,\langle \hat{v}_{\boldsymbol{k}}\hat{v}_{\boldsymbol{k}_1+\boldsymbol{k}_2-\boldsymbol{k}}\rangle}\right]
\end{aligned}, \tag{4.217}$$

the term marked with $\underbrace{\quad}$ is the new contribution to power spectrum correction which was not considered in [53]. It worth to mention that though decoherence effect appears only in the correlator $\langle \hat{p}_{\boldsymbol{k}_1}\hat{p}_{\boldsymbol{k}_2}\rangle$but it affects all the correlators since they are governed by coupled system of equations as is obvious from (4.217). Another point to mention, which we already discussed it in the case of bispectrum, is the appearance of $\delta\left(\boldsymbol{k}_1+\boldsymbol{k}_2\right)$ in the linear contribution to $\langle \hat{p}_{\boldsymbol{k}_1}\hat{p}_{\boldsymbol{k}_2}\rangle$ which insures that the interaction with the environment preserves statistical homogeneity, but we don not see the delta function in the last two terms $\propto \alpha^2$, however remember that this term is also $\propto \gamma$ and the system is solved through a perturbative expansion in γ, during the first iteration the last two Lindblad terms contain the correlators $\langle \hat{v}_{\boldsymbol{k}+\boldsymbol{k}_1}\hat{v}_{-\boldsymbol{k}+\boldsymbol{k}_2}\rangle$and $\langle \hat{v}_{\boldsymbol{k}}\hat{v}_{\boldsymbol{k}_1+\boldsymbol{k}_2-\boldsymbol{k}}\rangle$, respectively, in the free theory which are proportional to $\delta\left(\boldsymbol{k}_1+\boldsymbol{k}_2\right)$ This guarantees that the solution that is obtained at the first iteration is statistically homogeneous. In addition we adopted the case $\widetilde{C}_R\left(\boldsymbol{k}\right) \equiv \widetilde{C}_R\left(k\right)$ i.e an isotropic environmental correlation function. thus the system for isotropic solutions then reduces to

$$\begin{aligned}
\frac{dP_{vv}(k)}{d\eta} &= P_{vp}(k) + P_{pv}(k) \\
\frac{dP_{vp}(k)}{d\eta} &= \frac{dP_{pv}(k)}{d\eta} = P_{pp}(k) - \omega^2(k) P_{vv}(k) \\
\frac{dP_{pp}(k)}{d\eta} &= -\omega^2(k)\left(P_{vp}(k) + P_{pv}(k)\right) + \gamma (2\pi)^{3/2} \widetilde{C}_R(k) + \alpha^2 \frac{\gamma}{(2\pi)^{3/2}} \\
&\times \left[4 \int d^3\boldsymbol{k'} \widetilde{C}_R(k') P_{vv}(|\boldsymbol{k'}+\boldsymbol{k}|) + 6\widetilde{C}_R(k) \int d^3\boldsymbol{k'} P_{vv}(|\boldsymbol{k'}|)\right]
\end{aligned} , \tag{4.218}$$

as was done in the bispectrum case, one can combine the above equations in order to get a single differential equation for p_{vv} only, this time of third order, and it is given by

$$p_{vv}''' + 4\omega^2 p_{vv}' + 4\omega\omega' p_{vv} = S_1 + S_2 + S_3 \,, \tag{4.219}$$

where S_1 is a source function coming from the linear contribution to the correction and is given by

$$S_1 = 2(2\pi)^{3/2} \gamma \widetilde{C}_R(k) \,, \tag{4.220}$$

while S_2 and S_3 are source functions belonging the pure quadratic interaction correction and mixed term between linear and cubic interaction, respectively, and are functions of time that involve the power spectrum p_{vv} itself evaluated at all scales, namely

$$\begin{aligned}
S_2(k,\eta) = &\quad \alpha^2 \frac{8\gamma}{(2\pi)^{3/2}} \int d^3\boldsymbol{k'} \widetilde{C}_R(k') P_{vv}(|\boldsymbol{k'}+\boldsymbol{k}|) \\
S_3(k,\eta) = &\quad \alpha^2 \frac{12\gamma}{(2\pi)^{3/2}} \widetilde{C}_R(k) \int d^3\boldsymbol{k'} P_{vv}(|\boldsymbol{k'}|)
\end{aligned} , \tag{4.221}$$

again using the intuition we have cherished based on previous computations, both those in [53, 52] and that of bispectrum, we introduce the following functions which will constitute a solution to (4.219)

$$p_{vv}^{(i)} = \quad -\frac{2}{W^2} \int_{-\infty}^{\eta} d\eta' S_i(k,\eta') \,\mathfrak{Im}^2\left[\mathrm{v}_{\mathbf{k}}(\eta') \mathrm{v}_{\mathbf{k}}^*(\eta)\right] \,, \tag{4.222}$$

with $i = 1, 2, 3$. So the general solution to (4.219) is given by

$$p_{vv} = v_{\boldsymbol{k}}(\eta)\, v^*_{\boldsymbol{k}}(\eta) + p^{(1)}_{vv} + p^{(2)}_{vv} + p^{(3)}_{vv}\,, \tag{4.223}$$

where the first term is the standard result, i.e without decoherence; $p^{(1)}_{vv}$ and $p^{(2)}_{vv}$ have already been computed in [53], and the aim now is to compute $p^{(3)}_{vv}$ which represents the new correction we obtained however before starting it is useful to make a comparison between $p^{(2)}_{vv}$and $p^{(3)}_{vv}$computations since they belong to same order in α. We have seen that the authors in [53] when they came to computation of $p^{(2)}_{vv}$, which differs from $p^{(3)}_{vv}$ by the fact that in the former's source function is inside integral while in the last is outside of it, they made an approximation in the limit $k\eta \to 0$ and $Hl_E \ll 1$ which we remind quickly here[21]

$$\begin{aligned} S_2(k,\eta) = \quad & \int d^3\boldsymbol{k'} \widetilde{C}_R(k')\, P_{vv}(|\boldsymbol{k'}+\boldsymbol{k}|) \\ & \simeq 4\pi\alpha^2 \tfrac{8\gamma}{(2\pi)^{3/2}} \widetilde{C}_R(k) \int_{-\frac{1}{\eta_{IR}}}^{\frac{1}{\eta}} dk' k'^2 P_{vv}(k') \\ & = \alpha^2 \tfrac{8\gamma}{(2\pi)^{1/2}\eta^2} \widetilde{C}_R(k) \ln\left(\tfrac{\eta_{IR}}{\eta}\right) \\ & = \tfrac{3}{2} S_3(k,\eta) \end{aligned} \quad , \tag{4.224}$$

where η_{IR} is an IR cutoff to make the integral finite. Thus from (4.224) we deduce that there is no need to compute $p^{(3)}_{vv}$ it is related to $p^{(2)}_{vv}$, therefore the overall correction at order α^2 is given by $\frac{5}{2}p^{(2)}_{vv}$ so this factor of $5/2$ represents the correction brought by considering the system interaction operator (4.213) instead of a pure quadratic one. The second important fact to be deduced from (4.224) is that $p^{(3)}_{vv}$ will represent a scale independent correction to standard power spectrum for[22] $p = 3$ since it was shown in [53] that $p^{(2)}_{vv}$ is scale independent for this value of p, and this endorses once again the analysis made after the result (4.208), where this time despite that $p^{(2)}_{vv}$ and $p^{(3)}_{vv}$ are originated from different terms in (4.216), $\alpha^2 \left\langle \left[\left[\hat{O}, \hat{v}_{\boldsymbol{p_1}} \hat{v}_{\boldsymbol{k}-\boldsymbol{p_1}}\right], \hat{v}_{\boldsymbol{p_2}} \hat{v}_{-\boldsymbol{k}-\boldsymbol{p_2}}\right]\right\rangle$ and $\alpha^2 \left\langle \left[\left[\hat{O}, \hat{v}_{\boldsymbol{k}}\right], \hat{v}_{\boldsymbol{p_1}} \hat{v}_{\boldsymbol{p_2}} \hat{v}_{-\boldsymbol{k}-\boldsymbol{p_1}-\boldsymbol{p_2}}\right]\right\rangle$, respectively, but both are $\propto v^4$ thus should share the same value of p for which they become scale independent. We can take this analysis further and claim that for correction of the order α^n induced by the the Lindblad term

$$-\frac{\gamma}{2} \int d^3\boldsymbol{x} d^3\boldsymbol{y} C_R(\boldsymbol{x},\boldsymbol{y}) \left\langle \left[\left[\hat{O}, \hat{A}(\boldsymbol{x})\right], \hat{A}(\boldsymbol{y})\right]\right\rangle\,, \tag{4.225}$$

[21] All those details of computation concerning $p^{(1)}_{vv}$and $p^{(2)}_{vv}$would be already discussed and reproduced in full detail in a chapter prior to this one in the thesis, thus there is no need to reproduce them here since the aim of those notes is just to keep track of computations.

[22] we remind here that p is the free parameter introduced in (4.180) .

in $\frac{d\langle\hat{O}\rangle}{d\eta}$, then the corresponding correction is scale invariant for

$$p = 5 - n\,, \tag{4.226}$$

this last claim is valid, at least, in all the computations made so far, namely power spectrum, bispectrum and trispectrum.

Notice that the same line of thoughts should be applied to the case of trispectrum which receives its first non vanishing correction at the order α^2, so in addition the pure quadratic interaction computed in [52], we must add the new term marked with $\underbrace{\quad}$ in (4.214).

4.2.3 Conclusion of the scalar part

Finally, we conclude by emphasizing on the fact that all the correlation functions receive corrections at leading order in γ in contrast to conclusion drawn in [52][53]. In addition, we attract the attention to the fact that the highest the order of correlation function is, then the the highest is the order of α at which it receives its first non vanishing corrections, which by turn implies their suppression, i.e corrections, as we increase the order of correlation functions. In particular, we saw in this work that the first non vanishing correction to power spectrum, bispectrum, and trispectrum was found at the order α^0, α^1, and α^2, respectively. Finally, we found that the scale independence of the decoherence induced corrections is related to the order in α from which it receives its correction, and having a massive scalar field as environment doe not give necessarily a scale independent correction as could be seen clearly through the case of bispectrum.

4.3 Decoherence of tensor perturbations

The aim of this sections is to compute the decoherence induced corrections to tensor power spectrum, and according to our knowledge this is the first time to consider the effects of decoherence on tensor modes. Since the ultimate goal is to compare the effect of the interaction with environment on the scalar and tensor perturbations, in addition, to the computation of corrected tensor to scalar ratio, we will apply the same Lindblad equation (4.158) to the tensor perturbations which represent, now, our system. The Lindblad equation (4.158) was derived in full generality regardless of the nature of the system, however, we should bear in mind that the environment should satisfy some properties and conditions encoded in the correlation function

$$C_R\left(\boldsymbol{x},\boldsymbol{y}\right) = Tr_E\left(\rho_E R\left(\boldsymbol{x}\right) R\left(\boldsymbol{y}\right)\right)\,. \tag{4.227}$$

We introduce the following Fourier expansion of of tensor perturbations $h_{ij}(\eta,\boldsymbol{x})$, which will be useful in later computations,

$$h_{ij}\left(\boldsymbol{x}\right)=\frac{1}{\left(2\pi\right)^{3/2}}\int d^{3}\boldsymbol{k}h_{ij}\left(\boldsymbol{k}\right)e^{i\boldsymbol{k}.\boldsymbol{x}}\,, \tag{4.228}$$

with

$$h_{ij}\left(\boldsymbol{k}\right)=\sum_{\lambda=+,\times}h_{\lambda}\left(\boldsymbol{k}\right)e_{ij}^{\lambda}\ with\ e_{ij}^{\lambda}e_{ij}^{\lambda'}=2\delta^{\lambda\lambda'}\,, \tag{4.229}$$

where we adopted the well-known case that $\boldsymbol{k}$ is aligned along z-direction i.e $\boldsymbol{k}\equiv k(0,0,1)$ and the polarization tensors are given by

$$e^{+}=\begin{pmatrix}1&0&0\\0&-1&0\\0&0&0\end{pmatrix}\,and\,e^{\times}=\begin{pmatrix}0&1&0\\1&0&0\\0&0&0\end{pmatrix}\,, \tag{4.230}$$

and for canonical normalization, we introduce

$$v_{\lambda}=\frac{aM_{pl}}{\sqrt{2}}h_{\lambda}\,. \tag{4.231}$$

Now,we have to choose the $\hat{H}_{int}$ between the tensor perturbations and the environment where we adopt the following of form of $\mathcal{H}_{int}$

$$\hat{H}_{int}=\int d^{3}\boldsymbol{x}A\left(\eta,\boldsymbol{x}\right)\otimes E\left(\eta,\boldsymbol{x}\right)\,. \tag{4.232}$$

- The first obvious choice is to have linear interaction in tensor perturbations since it is expected to give dominant contribution thus

  $$A\left(\eta,\boldsymbol{x}\right)=h_{ij}\left(\eta,\boldsymbol{x}\right)\,, \tag{4.233}$$

  in that case and in order the contract the spatial indices in $h_{ij}$the environment interaction operator should be of the form

  $$E\left(\eta,\boldsymbol{x}\right)\equiv E_{ij}\left(\eta,\boldsymbol{x}\right)\,, \tag{4.234}$$

  to make contact with previous computations we take the case where indices refer to derivatives[23] thus let us put it as [24]

  $$E_{ij}\left(\eta,\boldsymbol{x}\right)=\partial_{i}\partial_{j}R\left(\eta,\boldsymbol{x}\right)\,, \tag{4.235}$$

[23] Note that by doing so we are not excluding an evironment made of tensor field since we can for example $E_{ij}\left(\eta,\boldsymbol{x}\right)=\partial_{i}\partial_{j}\left(\chi_{kl}\chi^{kl}\right)$

[24] Of course there is an implicit summation in indices $i$and j.

or

$$E_{ij}(\eta, \boldsymbol{x}) = \partial_i R_1(\eta, \boldsymbol{x}) \partial_j R_2(\eta, \boldsymbol{x}) \ , \tag{4.236}$$

as case in point if we consider a scalar field φ as environment[25] then possible interactions are

$$\mathcal{H}_{int} \propto h_{ij}\partial_i\partial_j\varphi,\ h_{ij}\varphi\partial_i\partial_j\varphi,\ h_{ij}\partial_i\varphi\partial_j\varphi ...etc. \tag{4.237}$$

We adopt this convention remembering that the environment states with respect to which we are taking trace are usually considered to be the Bunch Davies vacuum[15, 32]which are homogeneous and isotropic. Using the linearity of trace, then we can exchange between trace and derivatives in

$$C_E(\boldsymbol{x}, \boldsymbol{y}) = Tr_E(\rho_E E(\boldsymbol{x}) E(\boldsymbol{y})) \ , \tag{4.238}$$

thus[26]

$$C_E(\boldsymbol{x}, \boldsymbol{y}) = \partial^x{}_i\partial^x_j\partial^y_m\partial^y_n Tr_E(\rho_E R(\boldsymbol{x}) R(\boldsymbol{y})) = \partial^x_i\partial^x_j\partial^y_m\partial^y_n C_R(\boldsymbol{x}, \boldsymbol{y}) \, , \tag{4.239}$$

where we will preserve the convention adopted in [53] that is given by (4.65).

We will show in a moment that at linear order in h_{ij} the tensor power spectrum does not receive a correction due to fact that h_{ij} is transversal i.e $\partial^i h_{ij} = 0$.

- Since the linear order gives vanishing correction to power spectrum, we consider the second choice of $A(\eta, \boldsymbol{x})$ consisting in a quadratic interaction

$$A(\eta, \boldsymbol{x}) = h_{ij}(\eta, \boldsymbol{x}) h^{ij}(\eta, \boldsymbol{x}) \ , \tag{4.240}$$

therefore in that case by relabeling (4.238) we have

$$C_E(\boldsymbol{x}, \boldsymbol{y}) \equiv C_R(\boldsymbol{x}, \boldsymbol{y}) \ , \tag{4.241}$$

we will see that in this case the results will differ slightly from the curvature power spectrum case. However, it is still important to emphasize on the fact that curvature power spectrum receives corrections already for an $A(\eta, \boldsymbol{x})$ linear in Mukhanov Sasaki variable while for the tensor case it starts to receive corrections only at quadratic order. This last remark implies that tensor corrections are suppressed with respect to those of curvature case, the same as the standard tensor power spectrum is suppressed with respect to curvature one.

[25] Which is the case we have mostly in mind but, still, there is possibility of having tensor field as environment with an interaction operator of the form shown in previous footnote.

[26] By ∂^x_i we mean $\frac{\partial}{\partial x_i}$,and same for others.

4.3.1 Linear interaction

For $A(\eta, \boldsymbol{x}) = h_{ij}(\eta, \boldsymbol{x})$and using our convention in (4.239), then (4.158) is given by [27]

$$\frac{d\left\langle\hat{O}\right\rangle}{d\eta} = \left\langle\frac{\partial\hat{O}}{\partial\eta}\right\rangle - i\left\langle\left[\hat{O}, \hat{H}_h\right]\right\rangle - \frac{\gamma}{2}\int d^3\boldsymbol{x}d^3\boldsymbol{y}\partial_i^x\partial_j^x\partial_k^y\partial_l^y C_R(\boldsymbol{x}, \boldsymbol{y})\left\langle\left[\left[\hat{O}, h_{ij}(\boldsymbol{x})\right], h_{kl}(\boldsymbol{y})\right]\right\rangle, \quad (4.242)$$

where for a tensor perturbations system $\hat{H}_v$ is given by

$$\hat{H}_h = \frac{M_{pl}^2 a^2}{8}\int d^3\boldsymbol{x}\left[h_{ij}'^2 + (\nabla h_{ij})^2\right], \quad (4.243)$$

As in scalar case, we prefer to work in Fourier space so transforming this last equation using (4.229) leads to

$$\frac{d\left\langle\hat{O}\right\rangle}{d\eta} = \left\langle\frac{\partial\hat{O}}{\partial\eta}\right\rangle - i\left\langle\left[\hat{O}, \hat{H}_h\right]\right\rangle - \frac{\gamma}{2}\int d^3\boldsymbol{k}k_ik_jk_mk_n\boldsymbol{p}\widetilde{C}_R(|\boldsymbol{k}|)\left\langle\left[\left[\hat{O}, h_{ij}(\boldsymbol{k})\right], h_{mn}(-\boldsymbol{k})\right]\right\rangle, \quad (4.244)$$

however, since $\partial^i h_{ij} = 0 \xrightarrow[space]{in\,Fourier} k^i h_{ij} = 0$, therefore we see that the real, non unitary, part of Lindblad equitation induced by interaction with environment vanishes so the tensor power spectrum remains unchanged.

4.3.2 Quadratic interaction

The result obtained in previous section pushes us toward next order in $h_{ij}(\eta, \boldsymbol{x})$, so having (4.240) and (4.241), Lindblad equation is given by

$$\frac{d\left\langle\hat{O}\right\rangle}{d\eta} = \left\langle\frac{\partial\hat{O}}{\partial\eta}\right\rangle - i\left\langle\left[\hat{O}, \hat{H}_h\right]\right\rangle - \frac{\gamma\xi^2}{2}\int d^3\boldsymbol{x}d^3\boldsymbol{y}C_R(\boldsymbol{x}, \boldsymbol{y})\left\langle\left[\left[\hat{O}, h_{ij}h^{ij}(\boldsymbol{x})\right], h_{mn}h^{mn}(\boldsymbol{y})\right]\right\rangle, \quad (4.245)$$

where ξ^2 is an expansion constant will serve later to set up the right dimensions. Fourier transforming last equation and using therefore (4.245) is now given by

$$\frac{d\left\langle\hat{O}\right\rangle}{d\eta} = \left\langle\frac{\partial\hat{O}}{\partial\eta}\right\rangle - i\left\langle\left[\hat{O}, \hat{H}_v\right]\right\rangle - \beta^2\frac{\gamma}{2(2\pi)^{3/2}}\sum_{\lambda\lambda'}\int d^3\boldsymbol{k}d^3\boldsymbol{p}_1d^3\boldsymbol{p}_2\widetilde{C}_R(|\boldsymbol{k}|)\left\langle\left[\left[\hat{O}, \hat{v}_{\boldsymbol{p}_1}^{\lambda}\hat{v}_{\boldsymbol{k}-\boldsymbol{p}_1}^{\lambda}\right], \hat{v}_{\boldsymbol{p}_2}^{\lambda'}\hat{v}_{-\boldsymbol{k}-\boldsymbol{p}_2}^{\lambda'}\right]\right\rangle, \quad (4.246)$$

where now

$$\hat{H}_v = \frac{1}{2}\sum_{\lambda}\int d^3\boldsymbol{k}\left[\hat{P}_{\boldsymbol{k}}^{\lambda}\hat{P}_{-\boldsymbol{k}}^{\lambda} + \omega^2\hat{v}_{\boldsymbol{k}}^{\lambda}\hat{v}_{-\boldsymbol{k}}^{\lambda}\right] \quad with\,\omega^2 = k^2 - \frac{a''}{a}, \quad (4.247)$$

[27] The indices of h_{ij} are raised and lowered by δ_{ij} so we do not sharply distinguish the upper and lower indices of h_{ij}

we defined also

$$\beta = \frac{2\xi}{M_{pl}^2}\,, \tag{4.248}$$

where β is coupling constant with dimension $[momntum]^{-1}$ so that we can preserve the definition made in (4.166) for k_γ which has a dimension of momentum. It may seem that there is a missing factor of a^{-4} in the real part of (4.246) but actually it has been absorbed in the definition of γ given in (4.180). This last step was made because, on the one hand, we want to compute the decoherence corrected tensor to scalar ratio, r, and for this purpose we need to adopt the same conventions of scalar case regarding parameters, and on the other hand for scalar case we considered an interaction system operator $A(\eta, \boldsymbol{x})$ directly proportional to Mukhanov Sasaki variable rather than inflaton fluctuation or metric scalar fluctuation so the a relating the two was already absorbed in the definition (4.180). (this point will be more clear in the thesis, because, again, to grasp it fully it is needed to see the full derivation of Lindblad equation done in [53]).

Having derived the Lindblad equation governing the expectation values of our system observables, all what remains to do is to derive the equations obeyed by the different correlators, as was done in (4.217). However, this time we should add the polarization indices to (4.167) thus we use the relation $\left[\hat{v}^s_{\boldsymbol{p}}, \hat{p}^\lambda_{\boldsymbol{k}}\right] = i\delta^{s\lambda}\delta^{(3)}(\boldsymbol{p}+\boldsymbol{k})$, so following the same steps as before we obtain the following system of equations

$$\begin{aligned}
\frac{d\left\langle \hat{v}^s_{\boldsymbol{k}_1}\hat{v}^s_{\boldsymbol{k}_2}\right\rangle}{d\eta} &= \left\langle \hat{v}^s_{\boldsymbol{k}_1}\hat{p}^s_{\boldsymbol{k}_2}\right\rangle + \left\langle \hat{p}^s_{\boldsymbol{k}_1}\hat{v}^s_{\boldsymbol{k}_2}\right\rangle \\
\frac{d\left\langle \hat{v}^s_{\boldsymbol{k}_1}\hat{p}^s_{\boldsymbol{k}_2}\right\rangle}{d\eta} &= \left\langle \hat{p}^s_{\boldsymbol{k}_1}\hat{p}^s_{\boldsymbol{k}_2}\right\rangle - \omega^2(k_2)\left\langle \hat{v}^s_{\boldsymbol{k}_1}\hat{v}^s_{\boldsymbol{k}_2}\right\rangle \\
\frac{d\left\langle \hat{p}^s_{\boldsymbol{k}_1}\hat{v}^s_{\boldsymbol{k}_2}\right\rangle}{d\eta} &= \left\langle \hat{p}^s_{\boldsymbol{k}_1}\hat{p}^s_{\boldsymbol{k}_2}\right\rangle - \omega^2(k_1)\left\langle \hat{v}^s_{\boldsymbol{k}_1}\hat{v}^s_{\boldsymbol{k}_2}\right\rangle \qquad , \\
\frac{d\left\langle \hat{p}^s_{\boldsymbol{k}_1}\hat{p}^s_{\boldsymbol{k}_2}\right\rangle}{d\eta} &= -\omega^2(k_2)\left\langle \hat{p}^s_{\boldsymbol{k}_1}\hat{v}^s_{\boldsymbol{k}_2}\right\rangle - \omega^2(k_1)\left\langle \hat{v}^s_{\boldsymbol{k}_1}\hat{p}^s_{\boldsymbol{k}_2}\right\rangle \\
&\quad +\beta^2\frac{4\gamma}{(2\pi)^{3/2}}\int d^3\boldsymbol{k}\widetilde{C}_R(|\boldsymbol{k}|)\left\langle \hat{v}^s_{\boldsymbol{k}+\boldsymbol{k}_1}\hat{v}^s_{-\boldsymbol{k}+\boldsymbol{k}_2}\right\rangle +
\end{aligned} \tag{4.249}$$

apart from the polarization indices and the expansion constant (β VS α in scalar case), the previous system of equations does not differ from the scalar case, therefore, all what we need to do is to exchange α by β in the solutions of scalar case and multiply the result by 2 to account for the two possible polarizations $(+, \times)$. The tensor power spectrum $p_{vv} = \sum_s \left\langle \hat{v}^s_{\boldsymbol{k}_1}\hat{v}^s_{\boldsymbol{k}_2}\right\rangle$, is governed by

$$p'''_{vv} + 4\omega^2 p'_{vv} + 4\omega\omega' p_{vv} = S(\boldsymbol{k}, \eta)\ , \tag{4.250}$$

with

$$S(\boldsymbol{k},\eta)=\beta^2\frac{16}{(2\pi)^{3/2}}\int d^3\boldsymbol{k'}\widetilde{C}_R(k')\,P_{vv}(|\boldsymbol{k'}+\boldsymbol{k}|)\;, \tag{4.251}$$

which admits as solution.

$$p_{vv}=\sum_s v^s_{\mathbf{k}}(\eta')\,v^{s*}_{\mathbf{k}}(\eta)+2\sum_s\int\limits_{-\infty}^{\eta}d\eta' S(k,\eta')\,\mathfrak{Im}^2\left[\mathrm{v}^{\mathrm{s}}_{\mathbf{k}}(\eta')\,\mathrm{v}^{\mathrm{s}*}_{\mathbf{k}}(\eta)\right], \tag{4.252}$$

where the first term is the the standard result while the second term is the correction induced by interaction with environment. Since we are interested in the power spectrum of tensor perturbations h and not the variable v, and in order to facilitate the task of computing the corrected tensor to scalar ratio r then, as was done for scalar case in [53], we adopt the following notation for the dimensionless tensor power spectrum $\mathcal{P}_T$[28]

$$\mathcal{P}_T=\frac{k^3}{2\pi^2}\frac{2p_{vv}}{M_{pl}^2a^2}=\mathcal{P}_T\,|_{standard}\left[1+\Delta\mathcal{P}_T\right]\;, \tag{4.253}$$

with

$$\begin{cases}\Delta\mathcal{P}_T=\frac{2\sum_s\int_{-\infty}^{\eta}d\eta' S(k,\eta')\mathfrak{Im}^2\left[\mathrm{v}^{\mathrm{s}}_{\mathbf{k}}(\eta')\mathrm{v}^{\mathrm{s}*}_{\mathbf{k}}(\eta)\right]}{\sum_s v^s_{\mathbf{k}}(\eta')v^{s*}_{\mathbf{k}}(\eta)}\\ \mathcal{P}_T\,|_{standard}=\frac{8}{M_{pl}^2}\left(\frac{H}{2\pi}\right)^2\left(\frac{k}{aH}\right)^{-2\epsilon}\simeq\frac{8}{M_{pl}^2}\left(\frac{H}{2\pi}\right)^2\end{cases}, \tag{4.254}$$

$$\Delta\mathcal{P}_T=\frac{2\sum_s\int_{-\infty}^{\eta}d\eta' S(k,\eta')\,\mathfrak{Im}^2\left[\mathrm{v}^{\mathrm{s}}_{\mathbf{k}}(\eta')\,\mathrm{v}^{\mathrm{s}*}_{\mathbf{k}}(\eta)\right]}{\sum_s v^s_{\mathbf{k}}(\eta')\,v^{s*}_{\mathbf{k}}(\eta)}\;, \tag{4.255}$$

The computation of the integral in (4.252) was already done in [53] for curvature perturbations, so we need just to make the modifications already mentioned above to get the results corresponding to tensor case [29]. We will report the expressions $\Delta\mathcal{P}_T$ as function of parameter p defined in (4.180). We have three regimes in addition to two singular cases,

- for $p>6$

$$\Delta\mathcal{P}_T=\frac{1}{9\pi^2}\beta^2k_\gamma^2\left(\frac{k}{k_*}\right)^3\left(\frac{\eta}{\eta_*}\right)^{6-p}\left[\frac{1}{p^2}-\frac{2}{(p-3)^2}+\frac{1}{(p-6)^2}+8\frac{18}{p(p-3)(p-6)}\ln\left(\frac{\eta_{IR}}{\eta}\right)\right] \tag{4.256}$$

η_{IR} is an IR cutoff in the integral (4.252) and k_* refers to a pivot scale; $\ln\left(\frac{\eta_{IR}}{\eta}\right)=N-N_{IR}$ gives the number of e-folds elapsed since the beginning of inflation. We notice that in this regime the power spectrum correction scales as k^3, in addition to be not frozen on large scales and continues to increase, leading to a very large enhancement of the correction to the standard power spectrum at late time.

[28] By total we mean the standard result plus the correction induced y decoherence.

[29] as I told you last time, the thesis will be written from scratch and the scalar case will be treated fully before tensors case . So the reader of thesis will be more comfortable passing through the tensor case.

- For $2 < p < 6$

$$\begin{aligned}\Delta\mathcal{P}_T &= \tfrac{2^{p+1}\sqrt{2}(p-4)}{8\pi\Gamma(p-1)sin(\pi p/2)}\beta^2 k_\gamma^2 \left(\tfrac{k}{k_*}\right)^{p-3}\Big[\ln\left(\tfrac{\eta_{IR}}{\eta_*}\right)+\tfrac{1}{p-4}-\tfrac{2(p-1)}{p(p-2)} \\ &\quad -\tfrac{\pi}{2}cot\left(\tfrac{\pi p}{2}\right)+ln\left(2\right)-\psi\left(p-2\right)+\ln\left(\tfrac{k}{k_*}\right)\Big]\end{aligned} , \qquad (4.257)$$

ψ is the digamma function. In this case we obtain a scale invariant correction for $p = 3$ which was also the case for the correction of curvature power spectrum coming from the quadratic interaction i.e $p_{vv}^{(2)} + p_{vv}^{(3)}$ in (4.223). However, it is worth to notice that, in contrast to tensor case, the dominant correction to curvature power spectrum is coming from the linear interaction[30] that is scale invariant for $p = 5$. Therefore at leading order, the scale invariant corrections to the scalar and tensor power spectra correspond to different values of p. (I think when it comes to compute the tensor to scalar ratio, I will pick up only the scale invariant corrections even if they belong to different orders and different values of p)

- For $p < 2$

$$\Delta\mathcal{P}_T = \frac{\left(H_* l_E\right)^{p-2}}{2\pi^2\left(2-p\right)}\beta^2 k_\gamma^2\left(\frac{k}{k_*}\right)^{p-3}\left[\frac{1}{2-p}+N_*-N_{IR}+ln\left(H_* l_E\right)+\ln\left(\frac{k}{k_*}\right)\right], \qquad (4.258)$$

notice that in this case the power spectrum freezez on small scales,

- For $p = 2$ and $p = 6$ which are singular we have

$$\begin{aligned}\Delta\mathcal{P}_T\left|_{p=2}\right. &= \tfrac{1}{48\pi^2}\beta^2 k_\gamma^2\left(\tfrac{k}{k_*}\right)^{-1}\Big[12-\pi^2+12C\left(2+C\right)-12\ln^2\left(H_* l_E\right)+24\left[C+1-\ln\left(H_* l_E\right)\right] \\ &\quad \times\Big[2\left(N_*-N_{IR}\right)+\ln\left(\tfrac{k}{k_*}\right)\Big]\Big] \\ \Delta\mathcal{P}_T\left|_{p=6}\right. &= \tfrac{1}{432\pi^2}\beta^2 k_\gamma^2 k_\gamma^2\left(\tfrac{k}{k_*}\right)^{3}\Big[2\pi^2-21-12C\left(1+2C\right)-12\left(3+4C\right)\left(N-N_{IR}\right)+12\left(1+4C\right)\left(N-N_*\right) \\ &\quad +24\left(N-N_*\right)\left[2\left(N-N_{IR}\right)-\left(N-N_*\right)\right]-12\ln\left(\tfrac{k}{k_*}\right)\Big[1+4\left(C+N_*-N_{IR}\right)2\ln\left(\tfrac{k}{k_*}\right)\Big]\Big]\end{aligned} , \qquad (4.259)$$

[30]We have showed above that the linear interaction gives vanishing correction to tensor power spectrum.

where C is a constant.

4.3.3 Decoherence induced Corrections to the observable r

We presented previously the computations made by J.Martin et al to get the decoherence induced corrections to n_s and r. However, in their computations of r they assumed the tensor power spectrum $\mathcal{P}_h$ to remain unaffected by the environment which is an inaccurate assumption according to the computations we just carried out. Therefore, our aim now is to give the decoherence corrected expression of r taking into account our results regarding the effect of environment on tensor modes.

The problem that we can face once we come to compute the corrected r, is the fact that the scale independence of the leading terms of decoherence induced corrections to the scalar and tensor power spectra correspond to different values of p, namely $p = 5$and $p = 3$, respectively. However, it is important to remember that scalar corrections are dominant with respect to tensor ones, since those last receive their first non vanishing correction only at quadratic level. Therefore, if we restrict ourselves to linear interactions, then, the decoherence corrected r is still given by (4.134) that we reproduce here

$$r = \frac{r\,|_{standard}}{1 + \frac{\pi}{6}\frac{k_\gamma^2}{k_*^2}}\,. \tag{4.260}$$

However if we consider the leading corrections of both power spectra, scalar and tensor, regardless of their order, then, the previous equation will be modified. Since scalar power spectrum has been well confirmed to be scale independent, at least up to certain sensitivity, then ,we choose the value for which its correction is also scale independent[31], namely $p = 5$. Therefore, for the chosen value of p, the tensor power spectrum correction is blue titled with a spectral index[32] $n_T \approx 2$, as seen from (4.257), and is given up to leading order by

$$\Delta\mathcal{P}_T \quad \approx \beta^2 k_\gamma^2 \left(\frac{k}{k_*}\right)^2 \left[\ln\left(\frac{\eta_{IR}}{\eta_*}\right) + \ln\left(\frac{k}{k_*}\right)\right] \quad , \tag{4.261}$$

knowing that a blue tilted tensor power spectrum could be of observation interest for us, though research is centered around an $n_T \approx 0.5$. But we need to remember that (4.261) represents a correction of second order in β, therefore, the the spectral index $n_T \approx 2$ could

[31] This reasoning could be not fully accurate, since the measured scale independent scalar power spectrum could be merely the leading order and if we increase the measurements sensitivity we will detect weak scale dependence which reflects the decoherence effect for $p \neq 5$.

[32] Notice that with a value of $p = 5$ we get also a scale dependent Bispectrum as could be seen form (4.210), where we have $f_{NL} \propto \frac{k}{k_*}$

be perceived as smaller. Substituting (4.261) in (4.253) yields

$$\mathcal{P}_T \approx \mathcal{P}_T\left|_{standard}\right. \left[1+\frac{4\sqrt{2}}{3}\beta^2 k_\gamma^2\left(\frac{k}{k_*}\right)^2\left[\ln\left(\frac{\eta_{IR}}{\eta_*}\right)+\ln\left(\frac{k}{k_*}\right)\right]\right], \tag{4.262}$$

Through this last equation we can infer that decoherence rends the tensor to scalar ratio, weakly, scale dependent; weakly because the correction is proportional to β^2 which is assumed to be a small perturbative expansion parameter, the same as is α in the scalar case (see equation (4.157)). So combining (4.133) with (4.262) leads us to the decoherence corrected r

$$r = r\left|_{standard}\right. \frac{1+\beta^2 k_\gamma^2\left(\frac{k}{k_*}\right)^2\left[\ln\left(\frac{\eta_{IR}}{\eta_*}\right)+\ln\left(\frac{k}{k_*}\right)\right]}{1+\frac{\pi}{6}\frac{k_\gamma^2}{k_*^2}}, \tag{4.263}$$

a high precise measurement of, either or both , the tensor power spectrum $\mathcal{P}_T$ and tensor to scalar ratio r will induce tight constraints on interaction strength, encoded in k_γ, between the primordial fluctuations and their environment. Those constraints could, subsequently, be compared with those obtained from the scalar correlation functions, both, power spectrum and bispectrum.

Conclusion

In this thesis we discussed possible signatures of a quantum origin of the universe and addressed the question of the quantum to classical transition in the early universe. In particular, we aimed to the identification of observational connections between the quantum initial state and the classical universe we observe today, in addition, we aimed to the complete inflation formalism with a model that accounts for the quantum to classical transition of the primordial quantum fluctuations.

In chapter 1 we motivated the problem addressed by the thesis, namely, the lack of a rigid understanding for the quantum to classical transition in the early universe. We, first, showed the equivalence of Heisenberg and Schrodinger picture in the study of primordial fluctuations and how squeezing formalism relates the two pictures. Then we explained why the cosmological observables measured up to date are insensitive to weather the universe originated quantumly or classically. In particular, we explained how the squeezing of modes, along considering stochastic initial conditions, can justify the exchange of quantum expectations values with classical stochastic averages. However, we argued that this two ingredients are insufficient, neither, to explain how the transition from quantum fluctuations into classical ones took place in the early universe, nor, to explain how an inhomogeneous and anisotropic state, i.e our universe, can emerge out of an initial homogeneous and isotropic state. We came out of this chapter with the conclusion that, indeed, there is a missing chapter in our understanding of the early universe. The filling of this missing chapter requires some cosmological observables that could probe the origin of universe and remove degeneracy among early universe models. In addition, it requires a model which could explain quantum to classical transition in the early universe.

Chapter 2 served to address the first part of the missing chapter, where, we tried to investigate the possibility of implementing a Bell experiment on a cosmological level. First, we aimed at taking advantage of CMB and see weather there is a possibility of building an experimental protocol that contains all the ingredients of a Bell set up. We showed that, even with a vanishing decaying mode, the Cosmological counterpart of Bell inequalities built out of GKMR pseudo spin operators could be maximally violated, Equ (2.15). However, such violation could not be measured due to the impossibility of measuring the extremely suppressed decaying mode which, by turn, implies that out of a single observable, in our case is $\hat{q}_k$, we cannot infer the values of, at least, two pseudo spins. where, no matter how the pseudo spins were defined we get only one of them commuting with $\hat{q}_k$. The hope now is to find a way to realize a cosmological experiment of Leggett-Garg inequality proposal, since this last makes use of only one pseudo-spin component which is measured at different times. But, fortunately, we can find other alternatives have cosmological Bell experiments, where we summarized a baroque model, suggested by Maldecena, showing that the universe could have carried out a Bell experiment during its evolution and the outcomes are registered somewhere in the fluctuations of a primordial field that is still there in the universe. Maldecena Model is unlikely to, really, describe what happened in the early universe. However, it shows, at least in principle, the possibility for a self realization of Bell experiment in early universe. We concluded this chapter by discussing another alternative for probing the origin of universe, namely, non Gaussianities. Where we saw that weather the universe originated quantumly or classically would leave prints on the shape of three point functions. In particular, the absence of physical momenta, as poles, in bispectrum expression for the case of a quantum origin of the universe.

Adopting the inflationary scenario of a quantum origin for the universe, we discussed in chapter 3 dynamical collapse models as, first, possible alternative to have a better understanding of how the transition from a primordial quantum fluctuations into classic ones took place. As their name reveals, there are several models under the hat of collapse models, we presented the work done so far in promoting two of them in cosmological context. The first consisted in QMUPL, where we saw that considering a constant collapse parameter does not produce insightful results due to the lack of amplification mechanism which is a key feature of collapse models. Then, we adopted scale dependent collapse parameter, given by the ansatz (3.22 and 3.27), through which we succeeded to get a Wigner function localized in the filed amplitude direction, in addition to a scale independent power spectrum provided that the model free parameters are subject to the constraints (3.31). However, any cosmological constraints on collapse parameter from cosmological QMUPL could not be compared to those obtained from laboratory experiments due to mass dimension differences in the two cases. In order to overcome such obstacle, there was a need for a cosmological CSL model that uses mass/energy density operator as collapse operator. We saw that adopting one of the gauge invariant definitions of such operator, leaded to a cosmological CSL model whose prediction clash with the high precise cosmological data. Therefore cosmology ruled out such CSL model, but there could be others which fit the data.

The final chapter contained the original and most important results of the thesis, where we discussed decoherence in cosmological context. We presented the derivation of Lindblad equation (4.63) that represents the corner stone of this chapter; we saw how general it was apart from some assumptions made along the way which constrained the environment properties. Using the aforementioned equation, J.Martin et al derived the decoherence induced corrections to scalar power spectrum and higher order correlation functions, where they considered , solely, a linear (or quadratic) interaction with environment. In what follows we summarize their main results and conclusions which will, subsequently, be confronted with the conclusions we obtained through our own approach regarding the form of interaction with environment. J.Martin used the corrected scalar power spectrum to extract the new corrected scalar spectral index n_s and tensor to scalar ration r, assuming tensor modes to be unaffected by environment, their results are given by Eqs (4.134) and (4.135), respectively. Through the new observables (n_s, r) they showed that power law inflation model could be back into agreement with data, as could be seen in figure (4.4). Another important result by J.Martin et al, is their conclusion that getting a scale independent spectra, both power spectrum and trispectrum, is related to having a massive scalar field as environment. Their final important result was obtaining vanishing bispectrum but non vanishing trispectrum, which made them conclude that decoherence is one of the rare cases where the former is suppressed with respect to the last. This last result motivated us to adopt the new interaction operator given by (4.157) which leaded us to following new results:

1. First of all, through (4.157) we are able not only to reproduce all the results obtained by J.Martin et all, as could be seen thorough (4.160), but we also showed in 4.2.2.2 that there was missing terms in their computations of power spectrum and trispectrum due to the restricted form of interactions considered by them, .

2. We showed in 4.2.2.1 that decoherence does induce non vanishing bispectrum, moreover, it is dominant with respect to trispectrum in contrast to the conclusion drawn by J.Martin et all. In particular, we concluded that the highest the order of correlation function is, then the the highest is the order of α at which it receives its first non vanishing corrections which by turn implies their suppression, i.e corrections, as we increase the order of correlation functions.

3. We conjectured that obtaining a scale independent decoherence induced corrections of a given correlation function is related to the order in α from which the correction is received, independently from having a massive scalar field as environment. Moreover, a massive scalar field as environment does not give necessarily a scale independent correction as could be seen clearly through the case of bispectrum.

Another important original part of the thesis consisted in computing decoherence effects on primordial tensor perturbations, which could be the first computations of its kind. We have shown, in 4.3.1, that by considering a linear interaction, decoherence does not affect tensor fluctuations. Where we saw that transversality property causes decoherence corrections to vanish. Therefore, we passed to the next order and considered a quadratic interaction which indeed modifies tensor power spectrum. This last modification implies by turn that the tensor to scalar ratio r needs to be recomputed, taking this time into account the aforementioned tensor modification. However, we saw that scalar and tensor power spectra corrections were scale independent for different values of the free parameter p. So adopting the value that makes scalar correction scale independent leads to a blue titled tensor correction, therefore, we get a weakly scale dependent tensor to scalar ratio given by (4.263).

The work done in this thesis shows that the question of quantum to classical transition in the early universe is not a mere foundational question, but its answer could contain important observational signatures which could bring new insights into our current standard predictions.

Future prospects

The work done in this thesis could be extended in various ways, and here we list main possible extensions:

- We discussed in chapter 3 the corrections induced by collapse models to scalar power spectrum. A first possible extension would be to investigate the effect of collapse models on tensor perturbations and see how the tensor to scalar ration r would be modified, then after, comparing it with the one derived through considering decoherence effects, given by (4.263). A second possible extension, is to compute the corrections induced by collapse models into higher order correlation functions, both scalar and tensor. To this end, we can take advantage of the method adopted in [54] to compute the corrections to power spectrum where the starting point is to cast the CSL equation into a Lindblad equation given by [72]

$$\frac{d\hat{\rho}}{d\eta} = i\left[\hat{\rho}, \hat{H}\right] - \frac{\gamma}{2m_0^2}\int d^3\boldsymbol{x} a^3 \left[\left[\hat{\rho}, \hat{C}\left(\boldsymbol{x}\right)\right], \hat{C}\left(\boldsymbol{x}\right)\right], \tag{4.264}$$

  with $\hat{C}\left(\boldsymbol{x}\right)$ being the collapse operator. Notice the similarity between this last equation and equation (4.63) that we used to compute decoherence induced corrections to the various correlation function. Therefore, we can inspire from computations done 4.2.2.1 in addition to those in [52] to tackle non Guassianities under the framework of collapse models.

- We computed in this thesis decoherence corrections to tensor power spectrum, so an obvious possible extension is to compute also the corrections induced to higher order correlation functions, especially the bispectrum.

- We can use all the aforementioned possible results to constrain the interaction strength between system and environment in the case of decoherence, or to constrain the collapse parameter in case of collapse models.

- Another quite interesting extension, that already we mentioned in 4.1.4, is to assume an initial state of primordial perturbations that is entangled with an environment which is made of some other degrees of freedom, for example we can readopt the state (4.82)

$$\begin{aligned}\Psi_k\left[\hat{\varphi}_k,\hat{\psi}_k;\eta\right] &= N_k\left(\eta\right)\exp\left[-\tfrac{1}{2}\left(\Omega_k^{\varphi}\left(\eta\right)\varphi_{\boldsymbol{k}}\left(\eta_{in}\right)\varphi_{-\boldsymbol{k}}\left(\eta_{in}\right)+\Omega_k^{\psi}\left(\eta\right)\psi_{\boldsymbol{k}}\left(\eta_{in}\right)\psi_{-\boldsymbol{k}}\left(\eta_{in}\right)\right)\right. \\ &\quad \left.+C_k\left(\eta\right)\left(\varphi_{\boldsymbol{k}}\left(\eta_{in}\right)\psi_{-\boldsymbol{k}}\left(\eta_{in}\right)+\psi_{\boldsymbol{k}}\left(\eta_{in}\right)\varphi_{-\boldsymbol{k}}\left(\eta_{in}\right)\right)\right],\end{aligned} \tag{4.265}$$

in case of a massive scalar field as environment; obviously, we can consider other possibilities. However and in contrast to the work we presented in 4.1.4 that was done in [5, 14] , we should add interactions between the two parts in their total Hamiltonian so that they continue to interact as they evolve, namely,

$$\hat{H}_{tot}=\hat{H}_{\varphi}+\hat{H}_{\psi}+\hat{H}_{\text{int}}\,. \tag{4.266}$$

This kind of extension merges the work in [5, 14, 13] with a part of the work done in chapter 4.

Appendix

The aim of this appendix is twofold, first, to give some more details on the derivation of CSL equation (3.32) and, second, to introduce some concepts of decoherence that we used in chapter 4.

The CSL master equation

We reproduce the arguments and assumptions made in [54, 55] to promote the standard CSL equation into a cosmological context. We start with the standard modified Schrodinger equation that describes a non relativistic system[31]

$$\begin{aligned} \mathrm{d}\Psi\left(\boldsymbol{x},t\right) &= \Bigg[-i\hat{H}\mathrm{d}t + \tfrac{1}{m_0}\tfrac{\sqrt{\gamma}}{m_0}\sum_{\mathrm{i}}\left(\hat{C}_i - \left\langle\hat{C}_i\right\rangle\right)\mathrm{d}W_i\left(t\right) \\ &\quad - \tfrac{\gamma}{2m_0^2}\sum\left(\hat{C}_i - \left\langle\hat{C}_i\right\rangle\right)^2\mathrm{d}t\Bigg]\Psi\left(\boldsymbol{x},t\right) \end{aligned} , \tag{4.267}$$

where $\hat{C}_i$ is the collapse operator to be chosen with three components $i = x, y, z$, and m_0 is a reference mass usually taken to be the mass of a nucleon; the dimension of collapse parameter γ depends on the choice of $\hat{C}_i$. The stochastic noise $\mathrm{d}W_i\left(t\right)$ satisfies $\mathbb{E}\left[\mathrm{d}W_i\left(t\right)\mathrm{d}W_j\left(t'\right)\right] = \delta_{ij}\delta\left(t-t'\right)$, where $\mathbb{E}\left[\cdots\right]$ refers to stochastic average.

As first for promoting previous equation into cosmology, we consider field a $v\left(\boldsymbol{x}\right)$ that is described by a wavefunctional $\Psi\left[v\left(\boldsymbol{x}\right)\right]$, then, in the absence of a fully relativistic version for (4.267) we present some plausible assumptions which would lead us into an cosmological extension of (4.267).

We first assume that the Hamiltonian in is simply the Hamiltonian in (1.2.1) obtained from the theory of relativistic perturbations. Next, to make a direct contact between (4.267 and a field version of it, we can view space-like sections as an infinite grid of discrete points. In this case, the functional can be interpreted as an ordinary function of an infinite number of variables ζ_i, $\Psi\left[\cdots, v_i, v_j, \cdots\right]$, where $v_i = v\left(\boldsymbol{x}_i\right)$ is the value of the field at each point of the grid. Therefore, instead of dealing with a three-dimensional index i as before, we now deal with an infinite-dimensional one. As a consequence, we can write an equation similar to (4.267) for $\Psi\left[v_i\right]$ where, now, the operators $\hat{H}$and $\hat{C}$ are functions of field amplitude $\hat{v}$ and it conjugate "momentum" $\hat{p}_i = -i\partial/\partial\zeta_i$. Then, taking the continuous limit $\sum\limits_{\mathrm{i}} \to \int \mathrm{d}^3\boldsymbol{x}_p$ and passing from the physical coordinates $\boldsymbol{x}_p$ to the comoving ones $\boldsymbol{x}$, related by $\boldsymbol{x}_p = a\boldsymbol{x}$, we arrive at

$$\begin{aligned} \mathrm{d}\Psi\left(\boldsymbol{x},t\right) &= \Big[-i\hat{H}\mathrm{d}t + \tfrac{1}{m_0}\sqrt{\tfrac{\gamma}{a^3}}\int \mathrm{d}^3\boldsymbol{x}a^3\left(\hat{C}\left(\boldsymbol{x}\right) - \left\langle\hat{C}\left(\boldsymbol{x}\right)\right\rangle\right)\mathrm{d}W_t\left(\boldsymbol{x}\right) \\ &\quad - \tfrac{\gamma}{2m_0^2}\int \mathrm{d}^3\boldsymbol{x}a^3\left(\hat{C}\left(\boldsymbol{x}\right) - \left\langle\hat{C}\left(\boldsymbol{x}\right)\right\rangle\right)^2\mathrm{d}t\Big]\Psi\left(\boldsymbol{x},t\right) \end{aligned} , \tag{4.268}$$

where the stochastic functions in the two coordinate systems are related by $\mathrm{d}W_t\left(\boldsymbol{x}\right) = a^{-3/2}\mathrm{d}W_t\left(\boldsymbol{x}_p\right)$ as could be seen through

$$\mathbb{E}\left[\mathrm{d}W_t\left(\boldsymbol{x}_p\right)\mathrm{d}W_{t'}\left(\boldsymbol{x}'_p\right)\right] = \delta^{(3)}\left(a\boldsymbol{x} - a\boldsymbol{x}'\right)\delta\left(t-t'\right)\mathrm{d}t^2 = a^{-3}\delta\left(\boldsymbol{x}-\boldsymbol{x}'\right)\delta\left(t-t'\right)\mathrm{d}t^2\,, \tag{4.269}$$

We still need to show that, indeed, in Fourier space each mode obeys the equation we used in the main text (3.36). As we mentioned previously working with at linear order of perturbations and with quadratic Hamiltonian rend Fourier space a very helpful tool to simplify computations. However, the only worry that we may have, now, comes from the non linear and stochastic terms in (4.268) which could cause the modes to couple. But no concern about that, indeed, if we recall that Hamiltonian could be decomposed as in (1.91) with

$$\hat{\mathcal{H}} = \int_{R^{3+}} \mathrm{d}^3\boldsymbol{k}\sum_{s=R,I}\hat{\mathcal{H}}_{\boldsymbol{k}}^s\,, \tag{4.270}$$

in addition, we use the Fourier transform

$$\hat{C}\left(\boldsymbol{x}\right) = \left(2\pi\right)^{-3/2}\int \mathrm{d}\boldsymbol{k}\hat{C}\left(\boldsymbol{k}\right)e^{-i\boldsymbol{k}.\boldsymbol{x}}\,, \tag{4.271}$$

then this leads to

$$\mathrm{d}\Psi_{\boldsymbol{k}}^{R,I}\left(t\right) = \left[-i\hat{H}_{\boldsymbol{k}}^{R,I}\mathrm{d}t + \frac{1}{m_0}\sqrt{\gamma a^3}\left(\hat{C}_{\boldsymbol{k}}^{R,I} - \left\langle\hat{C}_{\boldsymbol{k}}^{R,I}\right\rangle\right)\mathrm{d}W_t^{R,I} - \frac{\gamma a^3}{2m_0^2}\left(\left(\hat{C}_{\boldsymbol{k}}^{R,I} - \left\langle\hat{C}_{\boldsymbol{k}}^{R,I}\right\rangle\right)\right)^2\mathrm{d}t\right]\Psi_{\boldsymbol{k}}^{R,I}\left(t\right)\,, \tag{4.272}$$

which is the CSL equation we used in the main text. In order to get a hint of how stochastic term do not induce mode coupling we adopt a simpler version and consider $\hat{H} = \hat{H}\left(\boldsymbol{x},\boldsymbol{p}\right) = \hat{H}\left(x_1,p_1\right) + \hat{H}\left(x_2,p_2\right) + \hat{H}\left(x_3,p_3\right)$in addition to $\hat{C}_i = C_i\left(\hat{\boldsymbol{x}},\hat{\boldsymbol{p}}\right) = C_i\left(\hat{x}_i,\hat{p}_i\right)$, notice in this last that the dependence of $\hat{C}_i$ is, solely, on $\left(\hat{x}_i,\hat{p}_i\right)$ and not $\left(\hat{x}_j,\hat{p}_j\right)$ for $j \neq i$. Then, using the decomposition $\Psi = \prod\limits_i \Psi_i\left(x_i\right)$we can show that

$$\begin{aligned} \mathrm{d}\Psi_i &= \left[-i\hat{H}_i dt + \tfrac{1}{m_0}\sqrt{\tfrac{\gamma}{a^3}}\int \mathrm{d}^3\boldsymbol{x} a^3 \left(\hat{C}_i - \left\langle \hat{C}_i \right\rangle\right) \mathrm{d}W_i\left(t\right)\right. \\ &\left. - \tfrac{\gamma}{2m_0^2}\int \mathrm{d}^3\boldsymbol{x} a^3 \left(\left(\hat{C}_i - \left\langle \hat{C}_i \right\rangle\right)\right)^2 \mathrm{d}t\right] \Psi_i \end{aligned} , \tag{4.273}$$

where we used the fact that

$$\left\langle \hat{C}_i \right\rangle \equiv \left\langle \Psi_i \left| \hat{C}_i \right| \Psi_i \right\rangle = \left\langle \prod_j \Psi_j \left| \hat{C}_i \right| \prod_k \Psi_k \right\rangle = \left\langle \prod_{j\neq i} \Psi_j \middle| \prod_{k\neq i} \Psi_k \right\rangle \left\langle \Psi_i \left| \hat{C}_i \right| \Psi_i \right\rangle = \left\langle \Psi_i \left| \hat{C}_i \right| \Psi_i \right\rangle . \tag{4.274}$$

We see that we can have independent equation for each Ψ_i.

Useful concepts of quantum decoherence

We used in the main text several concepts upon which the thesis generally, and particularly chapter 4, are heavily based. Therefore, we will briefly present those notions under the framework of quantum decoherence, we will follow closely [68].

Decoherence basic idea to consider that the openness of quantum systems, i.e., their interaction with the environment, is essential to explaining how quantum systems [33] become effectively classical. There are two main consequences for the interaction with environment

1. The disappearance of QM coherence which is the source of QM effects we observe such as the interference effect from which the name decoherence was coined.
2. The dynamical definition of physical observable properties of the system, the selection of a set of robust preferred states, formally observable of the system.

It is worth to mention that decoherence is neither an extraneous theory distinct from quantum mechanics itself nor something that we could freely choose to include or neglect. Decoherence is a ubiquitous effect in nature, with far-reaching and fascinating consequences that must be taken into account in order to arrive at a realistic description of physical systems. From this last remark we can see why decoherence must be considered in the physics of early universe.

We shall adopt the widely accepted notion that a quantum state vector (expressed, for example, as a ket $|\psi_i\rangle$in the standard Dirac notation) provides a complete description of the physical state of an individual system. To reflect the "completeness" of such quantum states, they are commonly called pure in contrast to the so-called mixed states which are simply classical ensembles of pure states. We

[33] Assuming that quantum mechanics is a universal physical theory, all systems.

can also define the density operator $\hat{\rho}$ corresponding to such a pure state $|\Psi\rangle$ as

$$\hat{\rho} = |\Psi\rangle \langle\Psi| \;, \tag{4.275}$$

which is simply the projection operator onto the state $|\Psi\rangle$. So if we express the state as superposition of states $|\Psi\rangle = \sum_i c_i |\psi_i\rangle$ then the density operator is given by

$$\hat{\rho} = \sum_{ij} c_i c_j^* |\psi_i\rangle \langle\psi_j| \;, \tag{4.276}$$

the terms $i \neq j$ embody the coherence between the different components. Accordingly, they are usually referred to as interference terms, or off-diagonal terms (since these terms correspond to the off-diagonal elements in the matrix representation of $\hat{\rho}$ in the basis $\{|\psi_i\rangle\}$). Now, having an operator $\hat{O}$ with eigenstates given by $|o_i\rangle$, then

$$Tr\left(\hat{\rho}\hat{O}\right) = \sum_i \langle o_i| \hat{\rho}\hat{O} |o_i\rangle = \sum_i o_i \left|\langle \Psi |o_i \rangle\right|^2 = \left\langle \hat{O} \right\rangle \;, \tag{4.277}$$

which represents the expectation value of the operator O. This correspondence between the mathematical rule of the trace and the expectation value is known as the trace rule.

We may also describe our system by a mixed state. A mixed state expresses insufficient information about the state of the system, in the sense that the system is (before the measurement) in one of the pure states $|\psi_i\rangle$ (which do not need to be orthogonal) but the observer simply does not know in which. Therefore we can only ascribe probabilities $0 \leq p_i$ to each of the states $|\psi_i\rangle$. The density matrix associated to a mixed state is given by

$$\hat{\rho} = \sum_i p_i |\psi_i\rangle \langle\psi_i| \;, \tag{4.278}$$

We can view this density matrix as a classical probability distribution of pure-state density matrices $\hat{\rho}_i = |\psi_i\rangle \langle\psi_i|$ with probabilities given by p_i.

It is important to notice that a mixed state must is clearly distinct from a pure-state, writing this as superposition of the form

$$|\Psi\rangle = \sum_i \sqrt{p_i}\, |\psi_i\rangle \;, \tag{4.279}$$

we have yields

$$\hat{\rho} = |\Psi\rangle \langle\Psi| = \sum_i p_i |\psi_i\rangle \langle\psi_i| + \sum_{i\neq j} \sqrt{p_i p_j}\, |\psi_i\rangle \langle\psi_j| \;, \tag{4.280}$$

the last term distinguishes the density matrix of a mixed state from that of a superposition of pure-states, which represents the interference between the different components. In addition, a pure state satisfies

$$\hat{\rho} = \hat{\rho}^2 \, , \tag{4.281}$$

this last relation gives us a hint on how to quantify the purity of a given state, or its mixedness. Where, a simple and commonly used measure is the so-called *purity* of the density matrix, defined as

$$\xi = Tr\left(\hat{\rho}^2\right) \, , \tag{4.282}$$

where if $\hat{\rho}$ represents a pure state then $\xi = 1$ by virtue of (4.281). While for a mixed state we have

$$\xi < 1 \, . \tag{4.283}$$

Another a criterion to distinguish a pure and mixed state is the Von Neumann entropy

$$S\left(\hat{\rho}\right) = -Tr\left(\hat{\rho} log \hat{\rho}\right) \, , \tag{4.284}$$

but in chapter 4 we used the former definition, namely (4.282).

Having two systems S and $\mathcal{E}$ which are entangled and the state of the composite system could well be pure one so it is completely known but the observer has only access to the system S, then the object that contains, exhaustively and correctly, all information (i.e., all measurement statistics) that can be extracted by the observer of system S is the reduced density matrix defined by

$$\hat{\rho}_S = Tr_{\mathcal{E}} \hat{\rho} \, , \tag{4.285}$$

Here the subscript "$\mathcal{E}$" means that the trace is to be performed using an orthonormal basis of the Hilbert space $H_{\mathcal{E}}$ of $\mathcal{E}$ only. Accordingly, the operation "$Tr_{\mathcal{E}}$" is also referred to as the partial trace over $\mathcal{E}$ and may be interpreted as an "averaging" over the degrees of freedom of the unobserved system $\mathcal{E}$. Obviously, for our case the system S represents the primordial fluctuations while $\mathcal{E}$ refers to their environment.

Bibliography

[1] Viviana Acquaviva, Nicola Bartolo, Sabino Matarrese, and Antonio Riotto. Second order cosmological perturbations from inflation. *Nucl. Phys. B*, 667:119–148, 2003.

[2] N. Aghanim et al. Planck 2018 results. VI. Cosmological parameters. *Astron. Astrophys.*, 641:A6, 2020.

[3] Y Akrami, F Arroja, M Ashdown, J Aumont, C Baccigalupi, M Ballardini, AJ Banday, RB Barreiro, N Bartolo, S Basak, et al. Planck 2018 results-ix. constraints on primordial non-gaussianity. *Astronomy & Astrophysics*, 641:A9, 2020.

[4] Y. Akrami et al. Planck 2018 results. X. Constraints on inflation. *Astron. Astrophys.*, 641:A10, 2020.

[5] Andreas Albrecht, Nadia Bolis, and R Holman. Cosmological consequences of initial state entanglement. *Journal of High Energy Physics*, 2014(11):93, 2014.

[6] Andreas Albrecht, Pedro Ferreira, Michael Joyce, and Tomislav Prokopec. Inflation and squeezed quantum states. *Physical Review D*, 50(8):4807, 1994.

[7] James M Bardeen. Gauge-invariant cosmological perturbations. *Physical Review D*, 22(8):1882, 1980.

[8] Nicola Bartolo, Eiichiro Komatsu, Sabino Matarrese, and Antonio Riotto. Non-gaussianity from inflation: Theory and observations. *Physics Reports*, 402(3-4):103–266, 2004.

[9] Angelo Bassi, Kinjalk Lochan, Seema Satin, Tejinder P Singh, and Hendrik Ulbricht. Models of wave-function collapse, underlying theories, and experimental tests. *Reviews of Modern Physics*, 85(2):471, 2013.

[10] Daniel Baumann. Tasi lectures on inflation. *arXiv preprint arXiv:0907.5424*, 2009.

[11] Daniel Baumann. Primordial Cosmology. *PoS*, TASI2017:009, 2018.

[12] Javier Berjon, Elias Okon, and Daniel Sudarsky. Debunking prevailing explanations for the emergence of classicality in cosmology. 9 2020.

[13] Nadia Bolis, Andreas Albrecht, and R Holman. Non-gaussianity from entanglement during inflation. *Journal of Cosmology and Astroparticle Physics*, 2019(07):021, 2019.

[14] Nadia Bolis, Andreas Albrecht, and Rich Holman. Modifications to cosmological power spectra from scalar-tensor entanglement and their observational consequences. *Journal of Cosmology and Astroparticle Physics*, 2016(12):011, 2016.

[15] D Boyanovsky. Effective field theory during inflation: Reduced density matrix and its quantum master equation. *Physical Review D*, 92(2):023527, 2015.

[16] Edouard Brézin and Claude Itzykson. Pair production in vacuum by an alternating field. *Physical Review D*, 2(7):1191, 1970.

[17] Timothy S Bunch and Paul CW Davies. Quantum field theory in de sitter space: renormalization by point-splitting. *Proceedings of the Royal Society of London. A. Mathematical and Physical Sciences*, 360(1700):117–134, 1978.

[18] Cliff P. Burgess, R. Holman, and D. Hoover. Decoherence of inflationary primordial fluctuations. *Phys. Rev. D*, 77:063534, 2008.

[19] P. Zoeller C. Gardiner. *Quantum Noise*. Springer-Verlag, Berlin, 2010.

[20] Pedro Cañate, Philip Pearle, and Daniel Sudarsky. Continuous spontaneous localization wave function collapse model as a mechanism for the emergence of cosmological asymmetries in inflation. *Phys. Rev. D*, 87(10):104024, 2013.

[21] Daniel Carney, Willy Fischler, Sonia Paban, and Navin Sivanandam. The inflationary wavefunction and its initial conditions. *Journal of Cosmology and Astroparticle Physics*, 2012(12):012, 2012.

[22] Mario Castagnino, Sebastian Fortin, Roberto Laura, and Daniel Sudarsky. Interpretations of quantum theory in the light of modern cosmology. *Foundations of Physics*, 47(11):1387–1422, 2017.

[23] Xingang Chen. Primordial non-gaussianities from inflation models. *Advances in Astronomy*, 2010, 2010.

[24] Sayantan Choudhury, Sudhakar Panda, and Rajeev Singh. Bell violation in the sky. *The European Physical Journal C*, 77(2):60, 2017.

[25] John F Clauser, Michael A Horne, Abner Shimony, and Richard A Holt. Proposed experiment to test local hidden-variable theories. *Physical review letters*, 23(15):880, 1969.

[26] Hael Collins and R Holman. Renormalization of initial conditions and the trans-planckian problem of inflation. *Physical Review D*, 71(8):085009, 2005.

[27] Hael Collins and Richard Holman. The renormalization of the energy-momentum tensor for an effective initial state. *Physical Review D*, 74(4):045009, 2006.

[28] Ulf H Danielsson. Note on inflation and trans-planckian physics. *Physical Review D*, 66(2):023511, 2002.

[29] Suratna Das, Kinjalk Lochan, Satyabrata Sahu, and TP Singh. Quantum to classical transition of inflationary perturbations: Continuous spontaneous localization as a possible mechanism. *Physical Review D*, 88(8):085020, 2013.

[30] Scott Dodelson. *Modern cosmology*. Elsevier, 2003.

[31] Gian Carlo Ghirardi, Philip Pearle, and Alberto Rimini. Markov processes in hilbert space and continuous spontaneous localization of systems of identical particles. *Physical Review A*, 42(1):78, 1990.

[32] Jinn-Ouk Gong and Min-Seok Seo. Quantum non-linear evolution of inflationary tensor perturbations. *Journal of High Energy Physics*, 2019(5):21, 2019.

[33] Daniel Green and Rafael A. Porto. Signals of a Quantum Universe. *Phys. Rev. Lett.*, 124(25):251302, 2020.

[34] LP Grishchuk et al. Best unbiased estimates for the microwave background anisotropies. *Physical Review D*, 56(4):1924, 1997.

[35] LP Grishchuk and Yu V Sidorov. Squeezed quantum states of relic gravitons and primordial density fluctuations. *Physical Review D*, 42(10):3413, 1990.

[36] Nemanja Kaloper, Matthew Kleban, Albion Lawrence, and Stephen Shenker. Signatures of short distance physics in the cosmic microwave background. *Physical Review D*, 66(12):123510, 2002.

[37] Claus Kiefer, Ingo Lohmar, David Polarski, and Alexei A Starobinsky. Pointer states for primordial fluctuations in inflationary cosmology. *Classical and Quantum Gravity*, 24(7):1699, 2007.

[38] Claus Kiefer and David Polarski. Emergence of classicality for primordial fluctuations: Concepts and analogies. *Annalen der Physik*, 7(3):137–158, 1998.

[39] Claus Kiefer and David Polarski. Why do cosmological perturbations look classical to us? *Advanced science letters*, 2(2):164–173, 2009.

[40] Claus Kiefer, David Polarski, and Alexei A Starobinsky. Quantum-to-classical transition for fluctuations in the early universe. *International Journal of Modern Physics D*, 7(03):455–462, 1998.

[41] Hideo Kodama and Misao Sasaki. Cosmological perturbation theory. *Progress of Theoretical Physics Supplement*, 78:1–166, 1984.

[42] Lev Kofman, Andrei Linde, and Alexei A Starobinsky. Towards the theory of reheating after inflation. *Physical Review D*, 56(6):3258, 1997.

[43] Julien Lesgourgues, David Polarski, and Alexei A Starobinsky. Quantum-to-classical transition of cosmological perturbations for non-vacuum initial states. *Nuclear Physics B*, 497(1-2):479–508, 1997.

[44] David H Lyth and Andrew R Liddle. *The primordial density perturbation: Cosmology, inflation and the origin of structure*. Cambridge University Press, 2009.

[45] DH Lyth. Large-scale energy-density perturbations and inflation. *Physical Review D*, 31(8):1792, 1985.

[46] Juan Maldacena. A model with cosmological bell inequalities. *Fortschritte der Physik*, 64(1):10–23, 2016.

[47] Jerome Martin and Robert H Brandenberger. Trans-planckian problem of inflationary cosmology. *Physical Review D*, 63(12):123501, 2001.

[48] Jerome Martin and Dominik J Schwarz. Influence of cosmological transitions on the evolution of density perturbations. *Physical Review D*, 57(6):3302, 1998.

[49] Jérôme Martin and Vincent Vennin. Bell inequalities for continuous-variable systems in generic squeezed states. *Physical Review A*, 93(6):062117, 2016.

[50] Jérôme Martin and Vincent Vennin. Leggett-garg inequalities for squeezed states. *Physical Review A*, 94(5):052135, 2016.

[51] Jérôme Martin and Vincent Vennin. Obstructions to bell cmb experiments. *Physical Review D*, 96(6):063501, 2017.

[52] Jérôme Martin and Vincent Vennin. Non gaussianities from quantum decoherence during inflation. *Journal of Cosmology and Astroparticle Physics*, 2018(06):037, 2018.

[53] Jérôme Martin and Vincent Vennin. Observational constraints on quantum decoherence during inflation. *Journal of Cosmology and Astroparticle Physics*, 2018(05):063, 2018.

[54] Jérôme Martin and Vincent Vennin. Collapse models and cosmology. *arXiv preprint arXiv:1912.07429*, 2019.

[55] Jérôme Martin and Vincent Vennin. Cosmic microwave background constraints cast a shadow on continuous spontaneous localization models. *Physical Review Letters*, 124(8):080402, 2020.

[56] Jerome Martin, Vincent Vennin, and Patrick Peter. Cosmological inflation and the quantum measurement problem. *Physical review D*, 86(10):103524, 2012.

[57] Patrick Martineau. On the decoherence of primordial fluctuations during inflation. *Classical and quantum gravity*, 24(23):5817, 2007.

[58] Sabino Matarrese, Silvia Mollerach, and Marco Bruni. Second order perturbations of the Einstein-de Sitter universe. *Phys. Rev. D*, 58:043504, 1998.

[59] Viatcheslav Mukhanov. *Page 348 of "Physical foundations of cosmology"*. Cambridge university press, 2005.

[60] Viatcheslav F Mukhanov and GV Chibisov. Quantum fluctuations and a nonsingular universe. *JETP Letters*, 33(10):532–535, 1981.

[61] Viatcheslav F Mukhanov, Hume A Feldman, and Robert Hans Brandenberger. Theory of cosmological perturbations. *Physics Reports*, 215(5-6):203–333, 1992.

[62] F. Petruccione N. P. Breuer. *The theory of open quantum systems.* 2007.

[63] Elias Okon and Daniel Sudarsky. Less decoherence and more coherence in quantum gravity, inflationary cosmology and elsewhere. *Foundations of Physics*, 46(7):852–879, 2016.

[64] Thanu Padmanabhan. *Section 10.4, page 364 of Structure formation in the universe*. Cambridge university press, 1993.

[65] Alejandro Perez, Hanno Sahlmann, and Daniel Sudarsky. On the quantum origin of the seeds of cosmic structure. *Classical and Quantum Gravity*, 23(7):2317, 2006.

[66] David Polarski and Alexei A Starobinsky. Semiclassicality and decoherence of cosmological perturbations. *Classical and Quantum Gravity*, 13(3):377, 1996.

[67] Antonio Riotto. Inflation and the theory of cosmological perturbations. *arXiv preprint hep-ph/0210162*, 2002.

[68] Maximilian A Schlosshauer. *Decoherence: and the quantum-to-classical transition*. Springer Science & Business Media, 2007.

[69] Bonny L Schumaker. Quantum mechanical pure states with gaussian wave functions. *Physics Reports*, 135(6):317–408, 1986.

[70] Bonny L Schumaker and Carlton M Caves. New formalism for two-photon quantum optics. ii. mathematical foundation and compact notation. *Physical Review A*, 31(5):3093, 1985.

[71] Yu Shtanov, J Traschen, and R Brandenberger. Universe reheating after inflation. *Physical Review D*, 51(10):5438, 1995.

[72] Andrea Smirne and Angelo Bassi. Dissipative continuous spontaneous localization (csl) model. *Scientific reports*, 5(1):1–9, 2015.

[73] Daniel Sudarsky. Shortcomings in the understanding of why cosmological perturbations look classical. *International Journal of Modern Physics D*, 20(04):509–552, 2011.

[74] Eemeli Tomberg. Decoherence in inflation, master thesis, 2016.

[75] Jennie H Traschen and Robert H Brandenberger. Particle production during out-of-equilibrium phase transitions. *Physical Review D*, 42(8):2491, 1990.

[76] Steven Weinberg. *Page 476 "Cosmology"*. Oxford university press, 2008.