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"BANK RISK AGGREGATION: ANALYSIS OF THEORETICAL AND OPERATIONAL APPROACHES"

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INTRODUCTION

In my thesis, I discuss on bank risk aggregation, analyzing theoretical and operational approaches to identify, measure and aggregate different types of risks, in order to determine the economic capital in a coherent way.

In Chapter I, I talk about Basel Accords, describing how requirements are changed during the time, and about the history of risk management and its application in determining the economic capital.

Chapter II is about risk categories. I define the principal risks financial institutions face, i.e. credit, market and operational risk, showing how to measure them and their relative distributions. I discuss also on the relevance of credit, market and operational risk with respect to the trading book, stating that the interactions between different types of risks are to be taken into consideration.

Once identified and measured, there are different ways to aggregate risks.

In Chapter III, top-down approaches are described. Within top-down approaches, the calculation of economic capital is first computed for individual risk types, and then aggregated to obtain the total economic capital of the bank. These approaches always reference an institution as a whole, and assume that market and credit risk are separable and then aggregated in some way.

The main methods are simple summation, Var - Covar and copulas. They are shown theoretically and then through examples from the literature.

Chapter IV is about bottom - up approaches, which allow to model and measure simultaneously different risk types. First, take into consideration interactions between market and credit risk, comparing examples from the literature that use top - down and bottom - up methods. Then, I elaborate on interest rate risk in particular, as regard to its interaction with credit risk.

CHAPTER I: COMMERCIAL BANK RISK MANAGEMENT AND CAPITAL ADEQUACY

I.1: The economic background and Basel Accords

Basel Committee on Banking Supervision was established by the central bank governors of the G-10 Countries in 1974 (Belgium, Canada, France, Germany, Italy, Japan, Netherlands, Sweden, Switzerland, United Kingdom, United States) and meets at the Bank for International Settlements in Basel. Its objective is to enhance understanding of key supervisory issues and improve the quality of banking supervision worldwide. The Basel Committee formulates broad supervisory standards and guidelines in order to control how much capital banks need to guard against the financial risks. The Committee intends not to be a formal supranational supervisory authority and its conclusions do not have legal force.

In 1988 the Basel Committee on banking Supervision introduced the Basel I Capital Accord, which intent was to establish the total amount of bank capital in order to reduce the risk of bank insolvency and the costs to depositors of a bank failure. Basel I was introduced after the deregulation period after 1980 that led to an increase of the international presence of banks, to the decline in capital ratios and to the increase of riskiness in the banking system.

The Basel I accord focused on the determination of the total amount of bank capital in the G-10 countries: they should have to hold capital equal to at least 8 percent of group of assets measured in different ways, according to their level of risk.

$$\frac{Total capital}{Credit risk RWA + Market risk RWA} = The\ bank's\ capital\ ratio \geq 8\% \qquad (1)$$

Capital had the goal to protect deposits and it was defined into two tiers: Tier 1, or Core Capital, was shareholders' equity and retained earnings; so it was composed by stock issues and disclosed reserves, which means funds to absorb potential future losses. Tier 2, or supplementary capital, was additional internal and external resources available to the bank, such as perpetual securities, undisclosed reserves, subordinated debt with maturity longer than five years, and shares redeemable at the option of the issuer. We have to consider that long term debt has a junior status relative to deposits, so it could been used to protect depositors and the deposit insurer. In addition, Basel I stated that the bank had also to hold half of its measured

capital in Tier 1 form[1]. Under Basel I, exposures were classified only into five categories: sovereign, domestic public-sector entities, banks, mortgage lending and other loans to the private sector).

The intent of Basel I was to give more emphasis to banks' own internal control and management and on the supervisory review process, in order to ensure more safety of the financial system: banks could not build business volume without adequate capital backing. However, Basel I had some limitations in reality: setting the level of claims at 8 percent, banks had the incentive to move assets off the balance – sheet, reducing the quality of bank loan portfolios. Basel I also did not consider actual economic risk setting risk weights; there was also an inadequate recognition of advanced credit risk mitigation techniques, such as securitization and guarantees. Thus, it was necessary a more risk sensitive framework that considers also operational risk, eliminates regulatory arbitrage by getting risk weights right and aligns regulation with best practices in risk management providing banks with incentives to enhance risk measurement and management capabilities. Basel I accords did not protect big banks against an hypothetical financial crisis because they supposed that the international financial system was much more stable than it really was. Banks needed more qualitative capital and it was fundamental to eliminate the divergence between Basel I risk weights and the actual economic risk, and the regulatory arbitrage by getting risk weights right.

In June 2004 it was published the Basel II Accord, which introduced the use of credit ratings and gave much more importance to the quality of credit, estimating it through risk drivers, such as the probability of default (PD) and the loss given default (LGD). The new accord stated that it was important to hold capital also for operational risk[2]. Two methodologies were used to compute the Risk Weighted Assets and, thus, the capital requirement for credit risk: the Standardized Approach and the Internal Rating Based Approach.

The first one is based on external risk assessments produced by rating agencies; risk weights are set by the committee as a function of the external rating and taking only discrete values. Exposures are assigned to 16 different classes based on the nature of counterparty, the technical

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[1]Gallati Reto R., *Risk management and capital adequacy*, McGraw-Hill, 2003, Ch. 5, pp. 343-344

[2]Hull John C., *Risk Management and Financial Institutions*

characteristics of the transaction and the manner in which it is carried out. Then, different risk weights are assigned to each portfolio, considering the ratings assigned by ECAIs and ECAs to each individual country. This system has the pros to be simple and attractive for small banks too, to not require extensive modeling, and to be uniform across banks. However, less flexibility means less pragmatism.

The Internal Rating Based Approach is based on banks' internal credit risk systems and uses risks weights obtained by applying the risk weight function set by the Committee, giving rise to a range of values for risk weights.[3] To implement the IRB Approach, credits are categorized into classes of assets with different underlying risk characteristics (corporate, sovereign, banks, retail, equity). Exposures are then distinguished between small and medium size (firms with annual sales lower than 50 million Euros), and larger firms. The formers (SMEs) are categorized either in the retail class, if the size of the exposure is smaller than 1 million Euros, or in the corporate class; the latter are always categorized in the corporate class.

The approach considers four main risk components: the PD (Probability of Default), the LGD (Loss Given Default), which is the expected value of the ratio between the loss due to default; the amount of the exposure at the time of default (EAD), which is the value off on-balance-sheet and off-balance-sheet exposures; and the Maturity (M), which is the average, for a given exposure, of the residual contractual maturities of the payments due, each weighted by its amount.

As regard to the risk weight function, there are two different functions: one for sovereign, corporate and bank exposures, and another one for retail exposures.

Sovereign – corporate – bank exposures function:

$$K = \left\{ LGD * N\left[\left(\frac{1}{1-R}\right)^{0,5} NI(PD) + \left(\frac{R}{1-R}\right)^{0,5} NI(0,999) \right] - LGD * D \right\} \left\{ \frac{1+(M-2,5)*b(PD)}{1-1,5b(PD)} \right\} * 1,06 \qquad (2)$$

Where $\boldsymbol{R} = 0,12 \frac{1-e^{-50PD}}{1-e^{-50}} + 0,24 \left[1 - \frac{1-e^{-50PD}}{1-e^{-50}} \right] - 0,04 \left[1 - \frac{S-5}{45} \right]$

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[3]Antão Paula, Lacerda Ana, *Capital requirements under the credit risk-based framework*, Journal of Banking & Finance, n° 35, 2011, pp. 3.

S = function of annual sales of the firm concerned, expressed in millions of Euros

M = maturity of the exposure, expressed in years

$b(PD) = [0,11852 - 0,05478\ ln(PD)]^2$

N = standard normal distribution

NI = inverse of the standard normal cumulative distribution

PD = probability of default

LGD = loss given default

The sales adjustment (third term on the R definition) applies only to corporate exposures.

S= annual sales in millions of Euros if annual sales are between 5 and 50 million Euros, equal to 5 if annual sales are smaller than or equal to 5 million Euros, and equal to 50 if annual sales are higher than or equal to 50 million Euros.

So, we can say that capital requirements are positively related with PD, LGD, M and R, dependently on the loss given default and on the level of sales. In fact, a change in the maturity of the credit has an higher impact on capital requirements for higher values of S and LGD. R is the correlation coefficient representing the degree of co-movement in credit risk of all exposures in the portfolio. The factor 1,06 is an ad-hoc adjustment introduced in 2004 by the Basel Committee.

Retail exposures function:

$$K = \left\{ LGD * N \left[\left(\frac{1}{1-R} \right)^{0,5} NI(PD) + \left(\left(\frac{R}{1-R} \right)^{0,5} \right) NI(0,999) \right] - LGD * PD \right\} \tag{3}$$

Where

$$R = 0,03 \frac{1 - e^{-35PD}}{1 - e^{-35}} + 0,16 \left[1 - \frac{1 - e^{-35PD}}{1 - e^{-35}} \right]$$

This risk function is relevant as exposures lower than one million Euros to SMEs will be classified as retail, because capital requirements are not dependent on the maturity of the credit

as well as on the level of annual sales. Thus, the correlation R is smaller than the one for corporate exposures[4].

Regarding the estimation of risk parameters, the Committee made two approaches available: the Foundation Approach, under which banks are required to use their own loss probability models but uses prescribed estimates of Loss Given Default; and the Advanced Approach, under which banks must use their own estimates of PD, LGD, EAD and M. The two approaches are applied to all credit classes with the exception of retail exposures; for this type of exposures, the IRB method is applied.

Finally, we can deduce that costs are higher for implementing the IRB approach, but banks could have an advantage because it better reflects the risk of each credit exposure, and it could be translated in a competitive advantage for those banks that present better credit risk drivers, and, as a consequence, lower capital requirements.

In the first Pillar of Basel II, the Committee stated the minimum capital requirements also for market and operational risk, that will be treated in the second chapter.

The second Pillar of Basel II regarded the Supervisory review. Banks should have a process for assessing their overall capital adequacy in relation to their risk profile and a strategy for maintaining their capital levels; then the supervisors should review and evaluate banks' internal capital adequacy assessments and strategies and ensure their compliance. Supervisors should also expect banks to operate above the minimum regulatory capital ratios and should have the ability to require banks to hold capital in excess of the minimum; than they should intervene at an early stage to prevent capital from falling below the minimum levels and requiring rapid remedial action if capital is not maintained or restored.

The third Pillar of Basel II regarded the market discipline and introduced rules on the information banks are required to publish. It greatly increased the disclosures that the banks must make and the transparency in their reporting, allowing marketplace participants to better reward well' managed banks and penalize poorly managed ones.

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[4]Antão Paula, Lacerda Ana, *Capital requirements under the credit risk-based framework*, Journal of Banking & Finance, n° 35, 2011

Basel II had some limitations: the total required capital was the sum of the capital for three different risks (credit, market and operational), and this implicitly assumed that the risks are perfectly correlated, but they are not. In addition, Basel II did not allow a bank to use its own credit risk diversification calculations when setting capital requirements for credit risk within the banking book.

I.2: Basel III: structure, proposals and impacts on risk management

After the financial crisis of 2008 – 2009, the Basel Committee on Banking Supervision reviewed the existing capital adequacy standards in order to respond to the deficiencies in financial regulation revealed by the crisis itself. The intent was to strengthen bank capital requirements by increasing bank liquidity and decreasing bank leverage.

In addition, Basel III introduced requirements on liquid asset holdings and funding stability, thereby seeking to mitigate the risk of a run on the bank.

Basel III had these proposals: to increase the quality and the quantity of capital, to reduce the leverage through introduction of a backstop leverage ratio, to increase the long-term balance sheet funding, and, finally, to strengthen risk capture, notably counterparty risk.[5]

Basel III is structured into three pillars: Pillar I is about capital, risk coverage and containing leverage; Pillar II concerns risk management and supervision; and Pillar III contains the market discipline.

In the first Pillar, the Committee wanted to improve loss-absorption capacity in both going concern and liquidation scenarios. The Accord states that common equity and retained earnings should be the predominant component of Tier 1 capital, instead of debt-like instruments; the requirements of Tier 2 are harmonized and simplified with explicit targets and it has been provided full deduction of capital components with loss-absorption capacity, such as minority interests, holdings in other financial institutions, Deferred Tax Assets; thus, banks were encouraged to rise significantly their capital, along with retention of profits and reduced dividends. Another implication is that national regulators has less flexibility to allow capital instruments to be included in Tier. As regard to the level of capital held, the minimum common

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[5]*Basel III: Issues and Implications*, kpmg.com

equity Tier 1 has been increased from 2.0% to 4.5%, the plus capital conservation buffer is set at 2.5%, and the bringing total common equity requirements at 7.0%. As a consequence, banks faced a significant additional capital requirement, to be raised as common equity or by retaining dividends.

In Pillar I, the Committee tries also to strengthen risk capture, notably counterparty credit risk through more stringent requirements for measuring exposure, capital incentives for banks to use central counterparties for derivatives, and higher capital for inter-financial sector exposures. It has been modeled the Internal Model Methods (IMM) to stressed periods. The correlation in the IRB formula for certain financial institutions has been increase to reflect the experience of the recent crisis and the committee set new capital charges for Credit Valuation Adjustments and wrong-way risk. The right approach to use, in order to encourage the use of central counterparties (CCPs) for standardized derivatives, is "Carrot and sticks", but the control and quality of the CCPs risk management is critical because risk is focused on central bodies. The counterparty risk management standards have been improved also in the areas of collateral management and stress testing. The cost of dealing with financial counterparties need to be priced into the business, leading to a review of the business model. As regard to risk coverage, it has been also required banks to conduct more rigorous credit analyses of externally rated securitization exposures. As regard to the trading book, it is set a significantly higher capital for trading and derivatives activities, as well as complex securitization held in the trading book. The new accord introduces a stressed value-at-risk framework to help mitigate pro-cyclicality and a capital charge for incremental risk that estimated the default and migration risks of un-securitized credit products and takes liquidity into account.

Another objective of Basel III is to reduce the leverage through the introduction of a non-risk based leverage ratio that included off-balance sheet exposures and served as a backstop to the risk-based capital requirement. The accord set the leverage limit at 3 percent, so the bank's total assets (on- and off-balance-sheet) should not be more than 33 times bank capital. The Committee introduces the leverage because it could reduce landing and to incentive banks to strengthen their capital position and to focus on higher-risk/higher-return lending. It is also increased the short-term liquidity coverage, to strengthen the liquidity framework by developing two standards for funding liquidity. The 30-day Liquidity Coverage Ratio is intended to promote short-term resilience to potential liquidity disruptions. The LCR has the goal to ensure that global banks had sufficient high-quality assets to withstand a stressed

funding scenario specified by supervisors. For the LCR the stock of high quality liquid assets is compared with expected cash outflows over a 30-day stress scenario and the expected cash outflows are to be covered by sufficiently liquid, high-quality assets. Assets have a liquidity-based weighting varying from 100 percent for governments bonds and cash to weightings of 0 percents-50 percent for corporate bonds. These changes have the implication to reduce the impact from a bank-run and to improve the overall stability of the financial sector. The introduction of the LCR has required banks to hold significantly more liquid, low-yielding assets to meet the LCR, which will may have a negative impact on profitability. Another implication is that banks could change their funding profile, with will lead to more demand for longer-term funding.

Basel III also increases stable long-term funding, designing the Net Stable Funding Ratio (NSFR), in order to encourage and incentivize banks to use stable sources to fund their activities to reduce the dependency on short-term wholesale funding. Banks should increase the proportion of wholesale and corporate deposits with maturities greater than one year, but it could lead to higher funding costs. Stronger banks with a higher NSFR will be able to influence market pricing of assets. Weaker banks will see their competitiveness reduced, which will potentially decrease the level of competition.

Pillar II is about risk management and supervision and has the aim of capturing the risk of off-balance sheet exposures and securitization activities, of managing risk concentrations, and of providing incentives for banks to better manage risk and returns over the long term.

Pillar III contains the market discipline: the requirements introduced relate to securitization exposures and sponsorship of off-balance sheet vehicles. Enhanced disclosures on the detail of the components of regulatory capital and their reconciliation to the reported accounts will be required, including a comprehensive explanation of how a bank calculates its regulatory capital ratios.

In addition to meeting the Basel III requirements, global systemically important financial institutions (SIFIs) must have higher loss absorbency capacity to reflect the greater risks that they pose to the financial system. The Committee has developed a methodology that includes both quantitative indicators and qualitative elements to identify global systemically important banks (SIBs). The additional loss absorbency requirements are to be met with a progressive Common Equity Tier 1 (CET1) capital requirement ranging from 1% to 2.5%, depending on a

bank's systemic importance. For banks facing the highest SIB surcharge, an additional loss absorbency of 1% could be applied as a disincentive to increase materially their global systemic importance in the future. A consultative document was published in cooperation with the Financial Stability Board, which is coordinating the overall set of measures to reduce the moral hazard posed by global SIFIs.

Jordi Gual in his paper titled *"Capital Requirements under Basel III and their impact on the banking industry"* (2011) discussed on the relationship between capital ratios and risk taking and on whether increased capital ratios mean greater costs for the banks. In his paper, Gual first broke down the regulatory capital ratio, *"E/RWA (in which (E) = equity and (RWA) = risk-weighted assets), into its two individual components: equity in relation to total assets (E/TA) and the (RWA/TA) ratio, which measures the level of risk attached to the institution's total assets. Thus, (E/RWA) = (E/TA)/(RWA/TA)."*[6] As a consequence, under perfect regulatory conditions, an increase in risk uptake, i.e. an increase in (RWA/TA), should be matched by an increase in the non-weighted capital ratio (E/TA), or a decrease in leverage (TA/E) and would not entail greater risk but it would result in a higher capital. After Basel II, the problem was that the risk calculated by banks through the Risk Weighted Asset method did not reflect the actual one and there was divergence between the risk actually taken and the risk calculated. Gual in his paper demonstrates that a greater proportion of equity in an institution's financing structure does not bring a lower assumption of risk. In his opinion banks attempt to offset the increased cost of equity resulting from greater solvency requirements by assuming greater risk in order to maintain the return on their equity. Gual states also that increased capital ratios meant greater financing costs for banks, in contrast to what the Basel Committee said and to the Modigliani-Miller theorem stated. In his opinion, following an increase in capital requirements, the cost of financing institutions could rise moderately over the long term, though this rise will be more substantial in the short term or during an accelerated transition period. To conclude, Gual states that the new regulation set by Basel III could have effects on both the cost of financing and the availability of capital, because it does not face the asymmetric information problems which exacerbate the valuation of the quality of capital and influence the market. In addition, Basel III does not take care of the main problem of bank financing, which

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[6]Gual Jordi, Capital requirements under Basel III and their impact on the banking industry, "la Caixa" economic papers, n°07, December 2011

is the explicit and implicit underwriting of banking debt, and, in practice, banks' liabilities do not act as buffers for the absorption of losses.

In January 2013, the Basel Committee on Banking Supervision published the BCBS 239 paper: "*Principles for effective risk data aggregation and risk reporting*". The impact of this is significant for Global Systemically Important Banks (G-SIBs), as it defines strong requirements in terms of data management. The objective of this regulation was to ensure that data used for risk calculation and reporting have the appropriate level of quality and that the published risk figures can be trusted. This implies that not complying with these principles would jeopardize the trust of regulators, which could lead to capital add-ons. At this stage, only G-SIBs are concerned, but it is strongly recommended that regulators apply the same rules for local systemically important banks, which may lead to wider scope of application. The timeline for expected implementation was set the beginning of 2016. This new constraint was also an opportunity for banks to improve their operational excellence and increase revenues.[7]

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[7] Basel Committee on Banking Supervision, Principles for effective risk data aggregation and risk reporting, Bank for International Settlements, January 2013

Figure 1: Enterprise data management framework
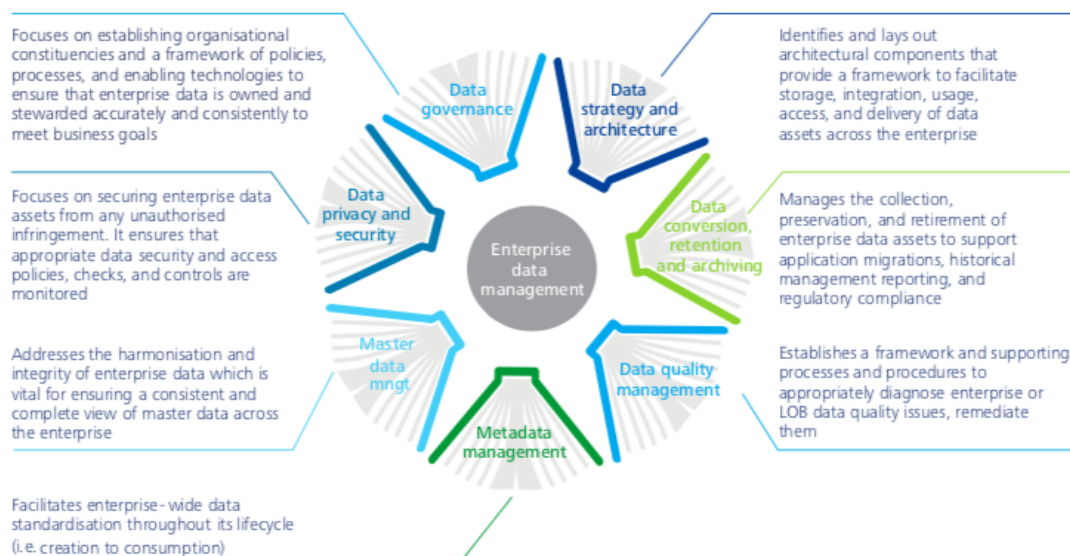
Source: BCBS (2013)

I.3: Risk Management: what it is, history and approaches, banking economic capital

What is "*risk*"? There are many definitions. The Oxford English Dictionary defines risk as *"the exposure to the possibility of loss, injury, or other adverse or unwelcome circumstance; a chance or situation involving such a possibility".* In finance, we can say that risk is the possibility that an actual return on an investment will be lower than the expected return; so, it is a condition in which there exists an exposure to adversity. Adversities can be booth internal or external. The term *"risk"* is linked with probability. We cannot measure the degree of risk but we can assign a percentage between 0 and 100 to the probability of the adverse outcome[8].

Over the past decades, investors, regulators, and industry self-regulatory bodies have forced banks, other financial institutions, and insurance companies to develop organizational structures and processes for the management of the different types of risks.

The function of Risk Management is to identify, evaluate and prioritize risks in order to applicate resources to minimize, monitor and control the impact of unfortunate events and to

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[8]Gallati Reto R., *Risk management and capital adequacy*, McGraw-Hill, 2003, pp. 7 - 8

maximize the realization of opportunities. "*Managers need risk measures to direct capital to activities with the best risk/reward ratios. They need estimates of the size of potential losses to stay within limits imposed by readily available liquidity, by creditors, customers, and regulators. They need mechanisms to monitor positions and create incentives for prudent risk-taking by divisions and individuals.*

Risk management is the process by which managers satisfy these needs by identifying key risks, obtaining consistent, understandable, operational risk measures, choosing which risks to reduce and which to increase and by what means, and establishing procedures to monitor the resulting risk position."[9]

It is fundamental to develop an overall risk management approach able to quantify the overall risk exposures of the company.

Risk management is an evolving concept and has been used since the dawn of human society. It has its roots in the corporate insurance industry, where managers were employed at the turn of the twentieth century by the first giant companies, the railroads and steel manufacturers. As capital investment in other industries grew, insurance contracts became an increasingly significant line item in the budgets of firms in those industries, as well. However, risk management emerged as an independent approach signaling a dramatic, revolutionary shift in philosophy and methodology, occurring when attitudes toward various insurance approaches shifted. In the literature, the first risk management concept appeared in 1956 in the *Harvard Business Review, w*here Russell Gallagher proposed the revolutionary idea that someone within the organization should be responsible for managing the organization's pure risk: *"The aim of this article is to outline the most important principles of a workable program for "risk management"—so far so it must be conceived, even to the extent of putting it under one executive, who in a large company might be a full-time "risk manager."*

Within the insurance industry, managers had always considered insurance to be the standard approach to dealing with risk. Though insurance management included approaches and techniques other than insurance (such as non-insurance, retention, and loss prevention and

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[9]Pyle David H., *Bank Risk Management: Theory*, Research Program in Finance, Working Paper RPF-272, July 1997

control). In 1950s there was a revolution in the risk management concept, and it was given more emphasis to cost-benefit analysis, expected value, and a scientific approach to decision making under uncertainty. The development from insurance management to risk management occurred over a period of time and paralleled the evolution of the academic discipline of risk management. During World War II, there was the origination of operations research because scientists were engaged in solving logistical problems, developing methodologies for deciphering unknown codes, and assisting in other aspects of military operations. The development happened simultaneously in the industry and in the academic discipline

New courses such as operations research and management science emphasized the shift in focus from a descriptive to a normative decision theory.

Figure 2: *Enterprise data management framework*
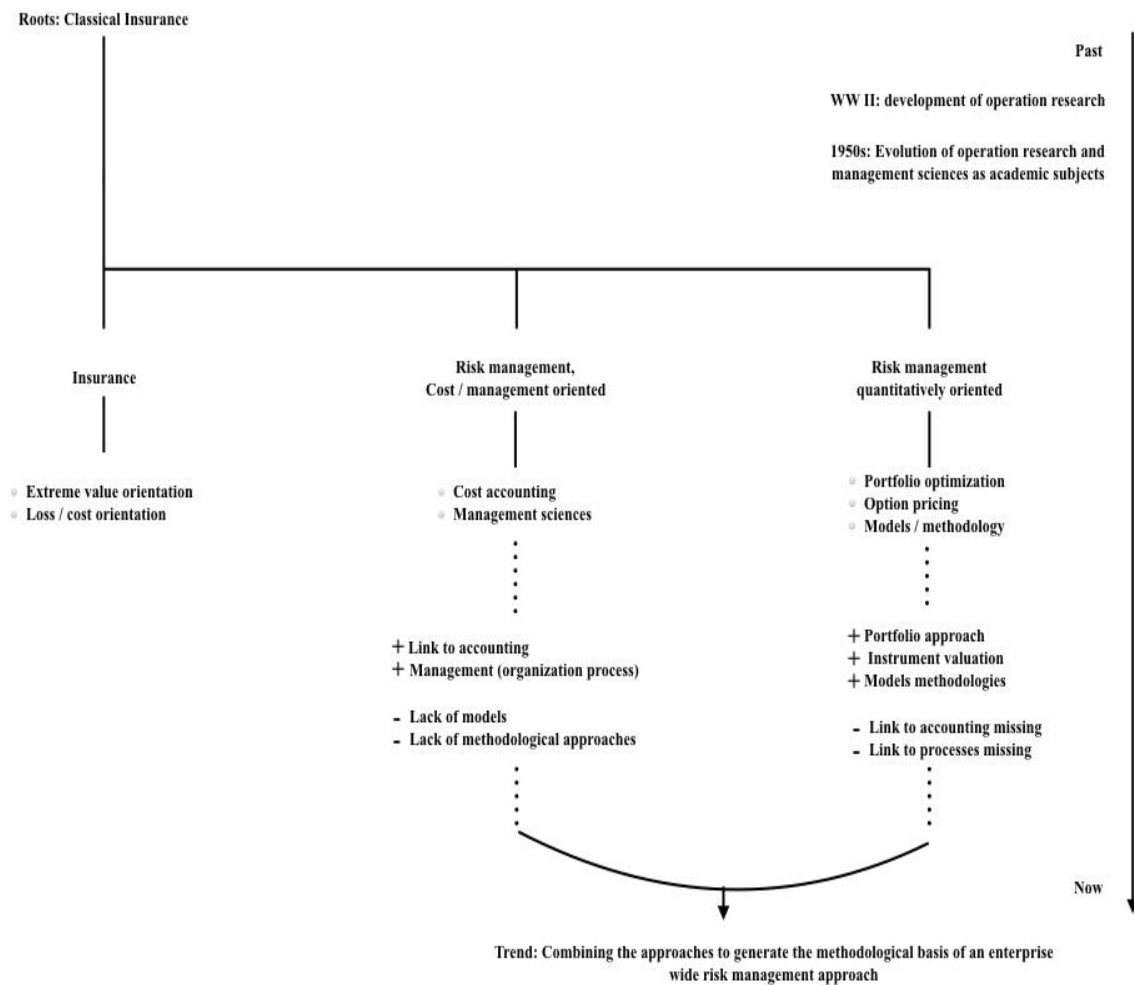
Source: Pyle (1997)

Markowitz was the first financial theorist to explicitly include risk in the portfolio and diversification discussion. He linked terms such as return and utility with the concept of risk, combining approaches from operations research and mathematics with his new portfolio theory.[10] This approach became the modern portfolio theory, and was followed by other developments, such as Fischer Black's option-pricing theory, which is considered the foundation of the derivatives industry. In the early 1970s, Black and Scholes made a

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[10]Harry M. Markowitz, *Portfolio Selection*, Journal of Finance 7 (March 1952), pp. 77–91

breakthrough by deriving a differential equation which must be satisfied by the price of any derivative instrument dependent on a non-dividend stock. This approach has been developed further and is one of the driving factors for the actual financial engineering of structured products[11].

The current trend in risk management is a convergence of the differing approaches, and almost all leading consulting practices have developed value-at-risk concepts for enterprise-wide risk management.

Figure 3: *Development levels of different risk categories*
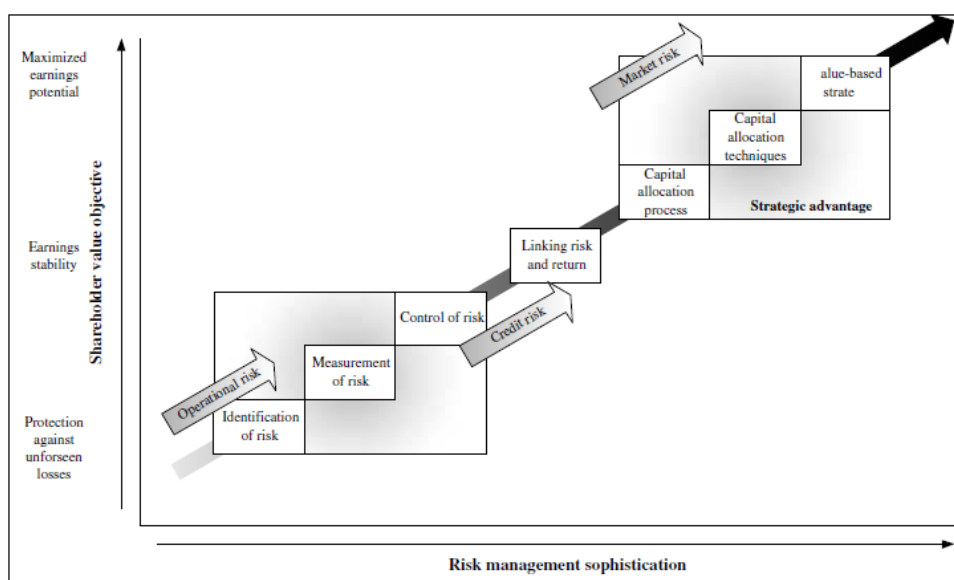
Source: Pyle (1997)

Risk management is basically a scientific approach to the problem of managing the pure risks faced by individuals and institutions. The concept of risk management evolved from corporate insurance management and has as its focal point the possibility of accidental losses to the assets and income of the organization. So we can define risk management, in a broad sense, as "*the process of protecting one's person or organization intact in terms of assets and income*" and in

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[11] Fischer Black and Myron Scholes, *The Pricing of Options and Corporate Liabilities*, Journal of Political Economy 81 (May–June 1973), pp. 637–654.

a narrow sense as *"the managerial function of business, using a scientific approach to dealing with risk"[12]*.

Modigliani and Miller thought that the firm risk profile could be changed through the use of debt financing (1958) or the distribution of dividends (1961)[13],and not through the use of financial derivative securities. However, in the real world we have to face to transaction costs and lack of transparency. Over the past decades, the risk management strategy proposed by Modigliani and Miller could not been able to prevent disasters caused by speculation, fraud, or money laundering, that left investors with dramatically devalued investments or even in bankruptcy because shareholders were unable to take any action to offset or mitigate the risks.

As a consequence, regulators became more active in considering credit, market and operational risks, forcing financial organizations to invest in their infrastructure, processes, and knowledge bases. The objective of both management and the regulators is to build and enforce an integrated risk management framework. We have to consider that managers and regulators have completely different strategies to reach the same objective: management seeks to protect clients' assets at the lowest possible cost by avoiding losses and by increasing the value of the shareholders' investment through business decisions that optimize the risk premium. Regulators seek to protect the clients' assets without regard to cost, maintaining market stability and protecting the financial market by excluding systemic risk.

Risk management has to serve both purposes and thus has to be structured, built, and managed in such a way that it can answer these different needs simultaneously. The models and approaches used in the different risk categories must give statements about the risk exposures and allow aggregation of risk information across different risk categories: market risk, credit risk, operational risk.

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[12]Pyle David H., *Bank Risk Management: Theory*, Research Program in Finance, Working Paper RPF-272, July 1997, p. 14

[13] M. H. Miller and F. Modigliani, *Dividend Policy, Growth and the Valuation of Shares*, Journal of Business34 (1961), pp. 411–433; *The Cost of Capital, Corporation Finance and the Theory of Investment*, American Economic Review48 (1958), pp. 261–29

The approach that implement firm-wide concepts aggregating market, credit and operational risk is the Total Risk Management, or Integrated Risk Management, which is "*the development and implementation of an enterprise-wide risk management system that spans markets, products, and processes and requires the successful integration of analytics, management, and technology"*.[14]

At present, many risk management programs attempt to provide a level of assurance that the most significant risks are identified and managed. However, they frequently fall short in aggregating and evaluating those risks across the enterprise from a strategic perspective.

Commercial banks are in the risk business: providing financial services, they assume various kind of financial risks. Banking risk management relies on a sequence of steps to implement a risk management system. The *sine qua non* for banks, is to underwrite standards and reports, in order to evaluate and rate the exposures, and to understand the risks in the portfolio, trying to mitigate them. The standardization of financial reporting is fundamental, too. A second technique banks use for internal control is to pose limits and minimum standards for participators, to increase their quality. Another technique is to state investment guidelines and strategies in order to hedge themselves against systematic risk of a particular type. Banks use securitization and derivative activity to reduce their exposures. The management can also enter incentive schemes, such as position posting, risk analysis and the allocation of costs.[15]

To explain how these techniques of risk management are employed by the commercial banking sector, we have first to describe what are the risks banks face: systematic or market risk, credit risk, operational risk, and others (like counterparties risk, liquidity risk, legal risk).

Banks should correctly identify and measure risks, in order to determine the economic capital. It is first calculated for individual risk types, most prominently for credit, market and operational risk, and then the stand - alone figures are aggregated to obtain the total economic capital of the bank.

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[14]Pyle David H., *Bank Risk Management: Theory*, Research Program in Finance, Working Paper RPF-272, July 1997, p. 15

[15] Santomero Anthony M., *Commercial Bank Risk Management: An Analysis of the Process*, Journal of Financial Services Research, 1997

CHAPTER II: RISK CATEGORIES

The identification and quantification of risks are fundamental for financial institutions: their focus is on calculating the economic capital. In this part of chapter II, I will explain what are the main risks that banks have to face and how to correctly measure them, taking into consideration their relative distributions. Once quantified and measured, risks should be aggregated in order to determine a coherent economic capital, but I will discuss it in the following two chapters.

The main three risks banks should take into consideration are credit, market and operational risk.

II.1: Credit Risk: measures and relative distribution

Credit risk is the loss that a firm can occur when there are changes in the credit standing of any counterparties. The capital required to cover credit risk should be sufficient for losses due to unexpected defaults or deterioration of the credit standing of counterparties. Credit risk is usually calculated for each counterpart and then aggregated to arrive at the total credit risk for the insurer[16].

Credit risk could be defined also as changes in portfolio value due to the failure of counterparties to meet their obligations or due to changes in the market's perception of their ability to continue to do so. Traditionally, banks have used a number of methods, such as credit scoring, ratings, credit committees, to assess the creditworthiness of counterparties[17].

So, credit risk is the risk of loss due to default on the part of borrowers. Banks are exposed to different types of risks due to loans and receivables, to the financial securities such as holding

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[16]Corrigan Joshua, De Decker Jethro, Hoshino Takanori, Van Delft Lotte, Verheugen Henny, *Aggregation of Risks and Allocation of Capital,* Milliman Research Report, September 2009

[17] Pyle David H., *Bank Risk Management: Theory*, Research Program in Finance, Working Paper RPF-272, July 1997

bonds, to future obligations assumption, such as the guarantees they give, and to the trading activities.

A "loan" is a financial asset resulting from the delivery of cash or other asset by a lender to a borrower in return for an obligation to repay on a specified date or dates, or on demand, usually with interest.

Loans include the following instruments: consumer installments, overdrafts and credit card loans, residential mortgages, non-personal loans (such as commercial mortgages, project finance loans, and loans to businesses, financial institutions, governments, and their agencies), direct financing leases. Loan impairment represents deterioration in the credit quality of one or more loans such that it is probable that the bank will be unable to collect, or there is no longer reasonable assurance that the bank will be able to collect, all amounts due according to the contractual terms of the loan agreements. As regard to the methodologies used to compute the Risk Weighted Assets and, thus, the capital requirement for credit risk, I have described them in Chapter I: the Standardized Approach and the Internal Rating Based Approach.

In the management of credit risk, it is fundamental the quantification of losses. Losses are made by two components: Expected Losses (*EL*), which are manageable by adequate reserves, and Unexpected Losses (*UL*).

$$EL = E(T_1) \times \left(1 - E(T_r)\right) \tag{4}$$

Where

EL = expected loss rate

$E(T_1)$ = expected default rate (*PD*)

$E(T_r)$ = expected recovery rate in case of default (*RR*)

There are two different methodological approaches to assign the credit risk: the Default Mode (*DM*) and the Market to Market (*MtM*). In the former, the default is the only event that generates losses; in the latter, losses are measured also considering the downgrading of the creditworthiness of the borrowers.

In the Default Mode Approach, the process consists in evaluating the probability of default of the borrower, the bank's exposure, and then estimating the amount of the retrievable credit portion.

Thus, the event that generate losses, is default only. The credit loss is calculated as the difference between the bank's credit exposure, which is the amount due to the bank at the moment of default, and the present value of future net recoveries. The current and the future values of the credit are defined in the default-mode paradigm based on the underlying two-state (default versus non-default) notion of the credit loss. The current value is typically calculated as the bank's credit exposure (e.g., book value). The future value of the loan is uncertain, and it would depend on whether the borrower defaults during the defined time horizon. In the case of non-default, the credit's future value is calculated as the bank's credit engagement at the end of the time horizon. In the case of a default, the future value of the credit is calculated as the credit minus loss rate given default (LGD). The higher the recovery rate following default, the lower the LGD. Applying a default-mode credit risk model for each individual credit contract (e.g., loan versus commitment versus counter- party risk), the financial institution must define or estimate the joint probability distribution between all credit contracts with respect to these following three types of random variables: the bank's associated credit exposure; a binary zero/one indicator denoting whether the credit contract defaults during the defined time horizon; and, in case of default, the associated LGD.

Expected Loss = p x *LGD* x *exposure*   (5)

To calculate the unexpected loss, some assumptions regarding the default probability distributions and recoveries have to be made: recovery rates are fixed and are independent of the distribution of probabilities. As the borrower either defaults or does not default, the default probability can, most simply, be assumed to be binomially distributed with a standard deviation of:

$$\sigma = \sqrt{p(1-p)} \tag{6}$$

The unexpected loss on the loan, given a fixed recovery rate and exposure amount is:

$$\textit{Unexpected Loss} = \sqrt{p(1-p)} \text{x } \textit{LGD} \text{ x } \textit{Exposure} \tag{7}$$

Applying the *DM* approach to the credit portfolio, we have that:

$$EL_p = \sum_{i=1}^{n} EL_i \text{ and } LGD_p = \sum_{i=1}^{n} \frac{V_i \times LGD_i}{n} \tag{8}$$

Where

V_i = value of the *i*-th exposure

n = amount of portfolio's exposures

$$UL_p = \sqrt{\sum_{i=1}^{n}\sum_{j\neq 1}^{n} UL_i UL_j p_{ij}} \tag{9}$$

Where

$UL_{i,j}$ = unexpected losses on the *i*-th and *j*-th exposure

ρ_{ij} = correlation between the default event on the *i*-th exposure and on the *j*-th exposure

The mark-to-market (MTM) paradigm treats all credit contracts under the assumption that a credit loss can arise over time, deteriorating the asset's credit quality before the end of the planned time horizon. All credit contracts are instruments of a portfolio being marked to market

at the beginning and at the end of the defined time horizon. The credit loss reflects the difference of the valuation at the beginning and at the end of the time horizon. This approach considers changes in the asset's creditworthiness, reflecting events that occur before the end of the time horizon and must incorporate the probabilities of credit rating migrations through the rating transition matrix, reflecting the changes in creditworthiness. The first step is to calculate migration paths for each credit position in the portfolio using the rating transition matrix and Monte Carlo simulation. This migration is then used to mark the position to market at the end of time horizon. We can use two approaches to estimate the current and future values of the credit contracts: the discounted contractual cash flow (DCCF) approach and the risk-neutral valuation (RNV) approach.

The discounted contractual cash flow (DCCF) approach is commonly associated with J. P. Morgan's CreditMetrics methodology. The current value of a non-defaulted loan is calculated as the present discounted value of its future contractual cash flows. The current value is treated as known, because the future value of the credit depends on the uncertain end-of-period rating and the market-determined term structure of credit spreads associated with the specific rating. The future value of the credit is subject to changes in creditworthiness or in the credit spreads according to the market-determined term structure. In the event of default, the future value of a credit would be given by its recovery value, calculated as the credit minus loss rate given default.

One limitation of this approach is that it assigns the same discount rates to all contracts with the same rating. Thus, for all contracts not defaulted within the defined time horizon, the future value does not depend on the expected loss rate given default, as they are not defaulted. Modern finance theory holds that the value of an asset depends on the correlation of its return with that of the market. So, borrowers in different market segments, exposed to different business cycles and other risk factors.

To avoid this, Robert Merton developed a structural approach imposing a model of firm value and bankruptcy. A company defaults when the value of its underlying assets falls beneath the level required to serve its debt. The risk- neutral valuation (RNV) approach discounts contingent payments instead of discounting contractual payments. This approach considers a credit as a set of derivative contracts on the underlying value of the borrower's assets. If a payment is contractually due at date t, the payment actually received by the lender will be the contractual amount only if the firm has not defaulted by date t. The lender receives a portion of the credit's face amount equal to the credit minus loss rate given default (similarly to DM Approach). The value of the credit equals the sum of the present values of these derivative contracts. The difference from the discount rates used for the discounted contractual cash flow approach is that the discount rate applied to the contracts' contingent cash flows is determined using a risk-free term structure of interest rates and the risk-neutral pricing measure. The risk-neutral pricing measure can be regarded as an adjustment to the probabilities of borrower default at each horizon t, which incorporates the market risk premium associated with the borrower's default risk. The magnitude of the adjustment depends on the expected return and volatility of the borrower's asset value. Returns modeled consistently with the capital asset pricing model (CAPM) can be expressed in terms of the market expected return and the firm's correlation beta (β) with the market. This approach combines pricing of the credits with the respective credit losses.

What are the differences between DCCF and RNV approaches? The DCCF assumes a nonparametric approach to estimate discount factors. The public debt issuers are grouped into rating categories, and the credit spreads on the issuers are then averaged within each rating "bucket." The RNV approach is more complex: each credit is simultaneously modeled in an individual framework and the modeling of the market risk premium for each credit is typically referenced to credit spreads from the debt market. The two approaches will, in general, assign different credit losses to any given loan.

There are the following four models, according to the two methodological approaches *DM* and *MtM*, they can be used to measure credit risk: *CreditMetrics,* the is a *MtM* model; *CreditRisk+*, that is a *DM* model; *Credit Portfolio View* and *KMV,* that could be implemented with both the approaches.

Table 1*: Comparison of different approaches*

Dimensions for Comparison	**CreditMetrics (J. P. Morgan)**	**CreditPortfolio-View (McKinsey)**	**CreditRisk+ (CSFP)**	**KMV**
Definition of risk	MTM	MTM or DM	DM	MTM or DM
Risk drivers	Asset values	Macrofactors	Expected default rates	Asset values
Volatility of credit events	Constant	Cyclical	Variable	Variable
Correlations of credit events	Multivariate normal asset returns	Factor loadings correlation of residual risk	Conditional independence assumption or correlation with expected default rate	Multivariate normal asset returns
Recovery rates	Random	Random	Constant with buckets	Constant or random
Numerical approach	Simulation or analytic (one-period VaR)	Simulation (one-period VaR)	Closed-form solution	Closed-form solution

Gallati Reto R., *2003*

In particular, KMV model, that is an unconditional model that links the process of estimating the asset values, rates of return, and volatility to current equity prices, which are information-efficient and incorporate all information available in the market. This approach is comparable to the arbitrage price theory (APT) and the multi-factor models. The drawbacks of these models are timing and parameterization, that might underestimate losses as the credit cycle enters a downturn and overestimate losses as the cycle bottoms out.

The KMV looks at the bank's lending problem from the viewpoint of the borrowing firm's equity holders and considers the loan repayment incentive problem. To solve the two unknown variables, E and σ_E, the system uses the structural relationship between the market value of a firm's equity and the market value of its assets and the relationship between the volatility of a firm's assets and the volatility of a firm's equity. After values of these variables are derived, an expected default frequency (EDF) measure for the borrower can be calculated. A firm borrows OB and the end-of-period market value of the firm's assets is OA_2 (where $OA_2 > OB$). The firm will then repay the loan, and the equity owners will keep the residual value of the firm's assets $(OA_2 - OB)$. The larger the market value of the firm's assets at the end of the loan period, the greater the residual value of the firm's assets to the equity holders (borrowers). However, if the firm's assets fall below OB (e.g., are equal to OA_1), the borrowers of the firm will not be able to repay the loan. They will be economically insolvent, will declare bankruptcy, and will turn the firm's assets over to the bank. The pay-off to the equity holder of a leveraged firm has a limited downside and a long-tailed upside. The market-value position of equity holders in a borrowing firm E as isomorphic to holding a call option on the assets of the firm A.

Equity can be valued as:

$$E = h\,(A,\ \sigma_A,\ r,\ B,\ T) \tag{10}$$

In equation (10), the observed market value of a borrowing firm's equity depends on the same five variables as per the Black-Scholes-Merton model for valuing a call option. It is necessary to solve two unknown variables, A and σ_A, from one equation where the other variables are observable. Thus, we can observe the relationship between the observable volatility of a firm's equity value σ_E and the unobservable volatility of a firm's asset value σ_A.

$$\overline{\sigma}_E = g(\sigma_a) \tag{11}$$

KMV uses an option-pricing Black-Scholes-Merton-type model that allows for dividends. B, the default exercise point, is taken as the value of all short-term liabilities, plus half the book

value of outstanding long-term debt. While the KMV model uses a framework similar to that of Black-Scholes-Merton, the actual model implemented, which KMV calls the *Vasicek-Kealhofer model,* makes a number of changes in order to produce usable results. These modifications include defining five classes of liabilities, reflecting cash payouts such as dividends, handling convertible debt, assuming the default point is an absorbing barrier, and relying on an empirical distribution to convert distance to default into a default probability. The precise strike price or default boundary has varied under different generations of the model.

After they have been calculated, the A and σ_A values can be employed, along with assumptions about the values of B and T, to generate a theoretically based EDF score for any given borrower.

Empirical EDF = Number of borrowers that defaulted within a year with asset values of 2σ from B at the beginning of the year/Total population of borrowers with asset values of 2σ from B at the beginning of the year.

The EDFs have a tendency to rise before the credit quality deterioration is reflected in the agency ratings. This greater sensitivity of EDF scores, compared to both accounting-based and rating-based systems, comes from the direct link between EDF scores and stock market prices. As new information about a borrower is generated, its stock price and stock price volatility will react, as will its implied asset value A and standard deviation of asset value σ_A. Changes in A and σ_A generate changes in EDFs. Because an EDF score reflects information signals transmitted from equity markets, it might be argued that the model is likely to work best in highly efficient equity market conditions and might not work well in many emerging markets. This argument ignores the fact that many thinly traded stocks are those of relatively closely held companies. Thus, major trades by insiders, such as sales of large blocks of shares (and thus major movements in a firm's stock price), may carry powerful signals about the future prospects of a borrowing firm.

The weakness of this system are the following ones: the construction of theoretical EDFs is difficult without the assumption of normality of asset returns; it does not differentiate between different types of long-term bonds according to their seniority, collateral, covenants, or convertibility; the private firms' EDFs can be estimated only by using some comparability analysis based on accounting data and other observable characteristics of the borrower and thus are subject to the same criticisms regarding subjectivity and consistency as are the expert systems; the Merton model cannot capture the financial behavior of those firms that seek to maintain a constant or target leverage ratio (debt to equity) across time[18].

As regard to Credit risk distribution, the best is Beta Distribution.

$L_c \sim Beta(p,q)$, where L_c is the credit loss rate.

The probability density function of L_c is:

$$f(x) = \frac{\Gamma(p-q)}{\Gamma(p)\Gamma(q)} x^{p-1}(1-x)^{q-1}, 0 < x < 1 \tag{12}$$

Where p and q are the two parameters of beta distribution and Γ denotes Gamma function.

The Beta distribution depends on the parameters p and q. If we know p and q, we can determine the credit loss distribution, because we know the mean $E(L_c)$ and the variance $Var(L_c)$ of credit loss rate. As a consequence, we can derive p and q from the mean and the variance[19]:

$$\begin{cases} p = \frac{E(L_c)^2}{Var(L_c)(1-E(L_c))} - E(L_c) \\ q = \frac{p}{E(L_c)} - p \end{cases} \tag{13}$$

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[18]Gallati Reto R., *Risk management and capital adequacy*, McGraw-Hill, 2003, pp. 129 - 265

[19]Li, J., Zhu, X., Lee, C., Wu, D., Feng, J., Shi, Y., *On the aggregation of credit, market and operational risks*, Review of Quantitative Finance and Accounting, Volume 44, Issue 1: 161-189, 2015

II.2: Market risk: measures and relative distribution

The Bank for International Settlement (BIS) defines market risk as *"the risk of losses in on- and off-balance-sheet positions arising from movements in market prices"*.

The main factors contributing to market risk are equity, interest rate, foreign exchange, and commodity risk. There are also residual risks that can influence the price of financial instruments. They are the following ones: spread risk, which is the potential loss due to changes in spreads between two or more instruments; basis risk, that is the loss due to the different pricing between equivalent instruments; specific risk, which refers to the issuer; and volatility risk, that is the loss due to fluctuations in volatilities. The total market risk is the aggregation of all risk factors.

There are three approaches for the quantitative measurement of market risk.

The first one is the standardized measurement method, under which the capital adequacy requirements are preset, depending on the risk factor categories, that are interest-rate and equity-price risks in the trading book; and currency, precious metals, and commodity risks in the entire organization.

The capital adequacy requirements are calculated for each individual position and then added to the total capital requirement for the institution. The different types of market risks are defined as specific risk or general market risk. The former includes the risk that and individual debt or equity can move in day-to-day trading and event risk. It is connected to the volatility of positions, and it depends on the issuer of specific instruments and not on general market factors. Price changes reflect changes in the rating of the acquiring or merging partner. On the contrary, the latter is associated to the volatility of positions depending on general market factors.

As regard to interest-rate risk, if it is measured only in the trading book, it represents a specific kind of market risk. For this type of risk there is a set of maturity bands, within which net positions are identified across all on- and off-balance sheet items. A duration weight is then assigned to each of the 13 bands, varying from 0.20 percent for positions under 3 months to 12.50 percent for positions over 20 years. Then, all weighted net positions are summed to obtain an overall interest-rate-risk indicator.

For currency and equity risk, the market risk capital charge is essentially 8 percent of the net position; for commodities, the charge is 15 per-cent. All of these capital charges apply to the trading books of commercial banks, except for currency risks, which apply to both trading and banking books.

The framework for measurement of market risks and the capital calculation to support market risks has to ensure that banks and securities dealers have adequate capital to cover potential changes in value (losses) caused by changes in the market price. Not including derivatives, which usually exhibit nonlinear price behavior, the potential loss based on the linear relationship between the risk factors and the financial instruments corresponds to the product of position amount, sensitivity of the position value regarding the relevant risk factors, and potential changes in the relevant risk factors.

The following equation is the basis for the calculation ok market risk and capital requirements under the standardized approach:

$$\Delta_\omega = \omega \cdot s \cdot \Delta_f \tag{14}$$

where:

Δ_ω = change in value of the position

ω = value of the position

s = sensitivity

Δ_f = change in the price-relevant factor

Thus, the direction of the change of the relevant risk factors is less important than the change, because the method is based on the assumption that the long and short position are influenced by the same risk factors, which causes a loss on the net position. The extent of the potential changes of the relevant risk factors has been defined by BIS such that the computed potential losses, which would have to be supported by capital, cover approximately 99 percent of the value changes that have been observable over the last 5 to 10 years with an investment horizon of 2 weeks.

Table 2: *Capital Adequacy requirements with the Standardized Measurement Method*

Instrument	Risk Decomposition
Interest-rate-sensitive position	General market risk: duration or maturity method
	Specific market risk: net position by issuer per weight factor, depending on the instrument class
Equity instruments	General market risk: 8% of the net position per national market
	Specific market risk: 8% of the net position per issuer
Precious metals	10% of the net position
Currencies	10% of all net long positions or all net short positions, whichever is greater
Commodities	20% of the net position per commodity group plus 3% of the brutto position of all commodity group

Source: Gallati Reto R. (2003)

The framework of the standard approach is based on the building- block concept, which calculates interest rate and equity risks in the trading book and currency, precious metals, and commodity risks in the entire institution separate from capital requirements, which are subsequently aggregated by simple addition. The building-block concept is also used within the risk categories. As with equity and interest-rate risks, separate requirements for general and specific market risk components are calculated and aggregated. That implies that correlations between the movements, which are the changes in the respective risk factors, are not included in the calculation and aggregation. With movements in the same direction, a correlation of +1 between the risk factors is assumed, and with movements in opposite directions, a correlation of −1 is assumed. The standard approach is thus a strong simplification of reality, as the diversification effect based on the correlations between the risk factors is completely neglected,

which results in a conservative risk calculation and in a higher capital requirement, with respect to the internal model approach.

The second approach is the Internal Model Approach, presented in 1995 by the Basel Committee in order to give banks the possibility of using their own risk management models. To use this approach, banks have to satisfy qualitative requirements and regular review.

The regular risk charge on any day t is:

$$MRC_t = max\left(K - \frac{1}{60}\sum_{i=1}^{60} Var_{t-i}, Var_{t-1}\right) \tag{15}$$

Where k is the multiplication factor determined by the supervisory authority, which can be set higher than its minimum of 3 if the supervisor is not satisfied with the bank's internal risk model.

To obtain total capital adequacy requirements, banks add their credit risk charges to their market risk charges applied to trading operations. The local supervisory authority can authorize an institution to compute the capital requirements for market risks by means of risk aggregation models specific to each institution; they are statistical processes used to determine the potential changes in the value of portfolios on the basis of changes in the factors that determine such risks. In this connection, value at risk (VaR) is defined as that value which represents the maximum potential change in value of the total position, given a certain confidence level during a pre-determined period of time. The equity requirements for interest-rate and equity price risks in the trading book, and for foreign-exchange and commodity risks throughout an institution, result from the aggregation of VaR-based capital charges and any applicable additional requirements for specific risks on equity and interest-rate instruments.

Using the Internal Model, the computation of VaR shall be based on a set of uniform quantitative inputs, using a horizon of 10 trading days, or two calendar weeks. It is required a 99 percent confidence interval, and an observation period based on at least a year of historical

data and updated at least once a quarter. Correlations can be recognized in broad categories (such as fixed income) as well as across categories (e.g., between fixed income and currencies). The capital charge shall be set as the higher of the previous day's VaR or the average VaR over the last 60 business days, times a multiplication factor. The exact value of this factor is to be determined by the local regulators, subject to an absolute floor of 3. This factor is intended to provide additional protection against environments that are much less stable than historical data would lead one to believe. A penalty component shall be added to the multiplication factor if back-testing reveals that the bank's internal model incorrectly forecasts risks. The purpose of this factor is to give incentives to banks to improve the predictive accuracy of the models and to avoid overly optimistic projections of profits and losses due to model fitting. As the penalty factor may depend on the quality of internal controls at the bank, this system is designed to reward internal monitoring, as well as to develop sound risk management systems. This method was particularly criticized by the ISDA (International Swaps and Derivatives Association), because the multiplication factor was thought to be too large. The association demonstrated tax it was enough to set it equal to 1. Another criticism of the model, is that the capital requirement under it is higher than what prescribed by the Committee with the standard model.

There is a third method to assess market risk: the Pre-commitment approach. Under that, the bank would pre-commit to a maximum trading loss over a designated horizon. This loss would become the capital charge for market risk. The supervisor would then observe a quarterly reporting period, whether trading losses exceeded the limit. If so, the bank would be penalized, which might include a fine, regulatory discipline, or higher future capital charges. Violations of the limits would also bring public scrutiny to the bank, which provides a further feedback mechanism for good management. The main advantage of this "incentive-compatible" approach is that the bank itself chooses its capital requirement. Regulators can then choose the penalty that will induce appropriate behavior. This proposal was welcomed by the ISDA

because it recognizes the links between risk management practices and firm-selected deployment of capital. Critics, in contrast, pointed out that quarterly verification is very slow in comparison to the real-time daily capital requirements of the Basel proposals. Others worried that dynamic portfolio adjustments to avoid exceeding the maximum loss could exacerbate market movements.[20]

As regard to market risk distribution, it is used the normal one to describe market loss rate L_m:

$$L_m \sim \Phi\left(\mu_m, \sigma_m\right) \tag{16}$$

The probability density function of L_m is:

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma_m} exp\left\{-\frac{(x-\mu_m)^2}{2\sigma_m^2}\right\} \tag{17}$$

Where μ_m and σ_m are the mean and the standard deviation of market loss rate, respectively. Thus, market risk loss rate depends on μ_m and σ_m. If they are known, the market risk loss rate distribution can be determined.[21]

II.3: Operational risk: measures and relative distribution

Operational risk is "*the risk of loss resulting from failed operational activities. Such failures arise from application of the firm's productive inputs, such as natural resources, labor, and capital, to the process of the production of output goods and services.*" Thus, it is the "*risk of*

[20]Gallati Reto R., *Risk management and capital adequacy*, McGraw-Hill, 2003;

[21] Li, J., Zhu, X., Lee, C., Wu, D., Feng, J., Shi, Y., *On the aggregation of credit, market and operational risks*, Review of Quantitative Finance and Accounting, Volume 44, Issue 1: 161-189, 2015;

loss resulting from inadequate or failed internal processes, people, and systems, or from external events".[22]

According to the BIS, operational risk is connected with breakdowns in internal controls and in the corporate governance, which may lead to financial losses through errors, fraud, or failure to perform obligations. It is released to the behavior of officers and staff members that could conduct business in a risky manner. Operational risks include failure of information technology systems o events such as fires or disasters. Thus, operational risk arises both from the influence of internal and external events.[23]

There are three methods of calculation of operational risk: the basic indicator approach, the standardized approach, the advanced measurement approach (AMA).

The first one, the BIA approach, is designed for less sophisticated and smaller banks, and allocates risk capital based on a single indicator of operational risk, which is the gross income.

$$Capital\ charge = 15\% \text{ x } \sum_{t=1}^{3} max(0, GI_i)/n \qquad (18)$$

Where GI_i is the annual gross income over the *i*-th previous year, and *n* is the number of the previous three years for which gross income is positive.[24]

[22] Corrigan Joshua, Luraschi Paola, *Operational risk modelling framework*, Milliman Research Report, February 2013

[23] Gallati Reto R., *Risk management and capital adequacy*, McGraw-Hill, 2003

[24] Corrigan Joshua, Luraschi Paola, *Operational risk modelling framework*, Milliman Research Report, February 2013

The second approach used is the Standardized one. Using it, banks should map their own business units into a standard set of business units defined by the regulator. Then, each of these standard units is associated with a financial indicator, which level defines the capital charge.[25]

This approach divides the total gross income between eight business lines:

$$Capital\ charge = 15\%\ \text{x}\ \sum_{i=1}^{3} max(0, GI_i)/n \tag{19}$$

Where GI_i is the annual gross income over the *i-th* previous year and *n* is the number of the previous three years for which gross income is positive.

The total gross income is divided between eight business lines:

$$Capital\ charge = \sum_{j=1}^{8} \beta_j \times \sum_{i=1}^{3} max(0, G_{i,j})/3 \tag{20}$$

Where is a fixed percentage set for the *j-th* business line and $GI_{i,j}$ is the annual gross income over the *i-th* previous year for the business line.

The β_j coefficients are set to:

- 18% for the corporate finance, trading, sales, payment, and settlements

- 15% for the commercial banking and agency services

- 12% for the retail banking, asset management, and retail brokerage business line.

At a national supervisor's discretion, the bank may be allowed to use the alternative standardized approach, which basis its calculations for the retail and commercial banking on 3,5% of total outstanding loans and advances, instead of the gross income.

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[25]Gallati Reto R., *Risk management and capital adequacy*, McGraw-Hill, 2003

The third approach used is called Advanced Measurement Approach (AMA), and it allows the exact formula for required capital to be specified by the financial institution itself, if it is approved by the supervisors. It should aim at holding enough capital to cover operational losses in one-year-horizon with 99,9% probability. Once adopted, it is expected that risk allocation techniques will be constantly developed. Before a bank can use AMA, its model is subject to initial monitoring of its appropriateness by the supervisor.[26]

As regard to operational risk distribution, it is necessary to notice that losses related to operational risk could be either small or very large. So, the distribution is characterized by sharp peaks and fat tails.

Cruz, Coleman and Salkin, in their study "*Modeling and measuring operational risk*" (1998), presented a quantitative operational risk measurement model based on extreme value theory. Given a distribution of operational losses for a certain business, they observed the behavior of the extreme of the distribution, in order to measure operational risk. They used the Generalized Extreme Value Distribution (GEV), to study the behavior of the extremes. This model is very similar to VaR methodologies used for market and credit risk.[27]

Another reasonable distribution is the log-normal one:

$L_0 \sim$ log normal (μ_0, σ_0)

The probability density function is:

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma_0 x} exp\left\{-\frac{(\ln x - \mu_0)^2}{2\sigma_0^2}\right\} \tag{21}$$

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[26] Corrigan Joshua, Luraschi Paola, *Operational risk modelling framework*, Milliman Research Report, February 2013

[27] Cruz M., Coleman R., Salkin G., *Modeling and measuring operational risk*, Imperial College, London, 1998

Where μ_o and σ_o are two parameters of lognormal distribution, respectively the mean and the standard deviation of logarithmic operational loss. The distribution fully depends on μ_o and σ_o and if they are known, we can determine the operational loss rate distribution. However, μ_o and σ_o are not provided directly.

Let $Q_0(\theta)$ be the quantile of operational loss rate at $\theta(0 < \theta < 1)$ and $m(L_0)$ denote the mode, i.e. the most frequent value of operational loss rate.

If $Q_0(\theta)$ and $m(L_0)$ are known, μ_o and σ_o can be calculated by solving this function:

$$\begin{cases} \exp(\mu_0 - \sigma_0^2)^2 = m(L_0) \\ \exp\left(\mu_0 + \sigma_0 \phi^{-1}(\theta)\right) = Q_0(\theta) \end{cases} \tag{22}$$

If we know the mean $E(L_0)$ and the variance $Var(L_0)$, we can compute μ_o and σ_o:

$$\begin{cases} \mu_0 = lnE(L_0) - \frac{1}{2}\ln\left(1 + \frac{Var(L_0)}{E(L_0)^2}\right) \\ \sigma_0^2 = \ln\left(1 + \frac{Var(L_0)}{E(L_0)^2}\right) \end{cases} \tag{23}$$

II. 4: The fundamental review of trading book: the relevance of credit, market and operational risks for banking and trading book with reference to Basel reform

In 2012, the Basel Committee started a revision on the trading book capital requirements, because it believed that the definition of the regulatory boundary between the trading book and banking book has been a source of weakness. "*A key determinant of the boundary has been banks' self-determined intent to trade. Trading intent has proven to be an inherently subjective criterion that is difficult to police and insufficiently restrictive from a prudential perspective in some jurisdictions. Coupled with large differences in capital requirements against similar types of risk on either side of the boundary, the overall capital framework proved susceptible to*

arbitrage before and during the crisis." [28] Thus, the Committee gave two different definitions for the boundary, a trading evidence-based approach and a valuation-based approach, and it provided example instruments which have to be assigned either to the banking book or to the trading book. The Committee has also agreed on a range of documentation that banks would need to make available to supervisors, as part of new valuation and evidence-based reporting requirements for all trading book positions. This would facilitate a better understanding of the types of activity that are within the scope of trading book capital requirements. The intent of the Committee was to reduce the risk of arbitrage and the differences in capital requirements against different types of risk on either side of the boundary. It investigated also the development of Pillar 1 charges for interest rate and credit spread risk in the banking book. Since the Credit Related products were the main source of losses during the 2009 financial crisis, the BIS Committee has agreed to bring the trading book requirements closer to the banking book. In addition, the Securitized and non-securitized products are treated differently:

- As regard to securitization Exposure, **i**nstead of allowing banks to use their "Internal Models" approach, the committee required banks to use a "Revised Standardized Approach". This is also applicable to "Correlation Trading activities" in the trading books.

- As regard to Non-Securitization exposures, in order to justify the Non Securitized products to match the right capital requirements, taking into account the default risk and spread risk, the committee proposed a separate incremental default charge (IDR), that will increase the capital requirements for the trading books.

Additionally, the BIS Committee proposed to keep that the CVA (Credit Valuation Adjustments) separate from the Market Risk calculations.

The Committee proposed to calibrate the capital framework to a stressed market condition time frame and to move from VaR to Expected Shortfall method because VaR does not capture the

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[28] Basel Committee on Banking Supervision, *Fundamental Review on the trading book: a revised market risk framework,* October 2013

tail risk. The new FRTB rules proposed to capture the average of the expected risk in the tail, with a 97.5 percentile confidence interval.

Under Basel 2.5, a "*liquidity Horizon*" was introduced and it was defined as "*the time required to execute transactions that extinguish an exposure to a risk factor, without moving the price of the hedging instruments, in stressed market conditions*". It formed the input to the Incremental Risk Charge (IRC) and the Comprehensive Risk Measure (CRM). Under FRTB (so called Basel IV FRTB), Banks' risk factors were assigned five liquidity horizon categories, ranging from 10 days to one year, to ensure consistency in capital outcomes, and in balancing the trade-off between simplicity and risk sensitivity.

The Committee pointed out that trading books benefit in capital reduction by hedging their portfolios and by incorporating diversification in their portfolios. However, in times of stress, diversification benefits go away and the spread risk increases for the hedging, leading to huge loss that have not been incorporated in the capital calculation. To mitigate this risk, FRTB regulation proposed the following two main changes:

- For "**Internal Models based approach**", the diversification effects will be recognized with some constraints

- For the "**revised Standardized approach**", the recognition of hedging and diversification will be significantly increased relative to the current approach.

From empirical evidence, it became clear that there is a very large difference in capital calculation by banks when they use the internal models vs. standardized approaches. The BIS tried to bridge this gap and working on proposals that will bring the models based calculation closer to the standardized approach calculations by recommending the following three steps:

- Establish the link between capital calculated by the two approaches

- Require mandatory calculation of capital using standardized approach by the banks

- Making the Standardized Capital as the floor for the capital requirements or introduce a surcharge on the models based approach.

The Basel recommendations of 1996 of calculating the Credit Risk or Market Liquidity Risk over a 10-day period proved insufficient during the stressed period of 2009. Keeping this in mind, FRTB proposed to break the Model approval process into smaller steps and a set of quantitative tools to measure the performance of models. First, a P&L attribution process that provides an assessment of how well a desk's risk management model captures risk factors that drive its P&L. Second, an enhanced daily back - testing framework for reconciling forecasted losses with actual losses. Where a trading desk fails these tests, the bank would be required to calculate capital requirements for that desk using the standardized approach.

The Revised Standardized approach provides a method for calculating capital requirements for banks with business models that do not require a more sophisticated measurement of market risk; in addition, it provided a credible fall - back in the event that a bank's revised market risk framework (FRTB) internal market risk model is deemed inadequate, including its potential use as a surcharge or floor to an internal models-based capital charge. The approach should facilitate transparent, consistent and comparable reporting of market risk across banks and jurisdictions.

The FRBT was finalized in January 2019, and banks are required to report under the new standards by 2020.

CHAPTER III: TOP - DOWN APPROACHES FOR BANK RISK AGGREGATION

III.1: Top - down aggregation methods: simple summation, Var - Covar and copulas

Within top-down approaches, the calculation of economic capital is first computed for individual risk types, and then aggregated to obtain the total economic capital of the bank. These approaches always reference an institution as a whole, and assume that market and credit risk are separable and then aggregated in some way.

The first top-down aggregation approach I will discuss on, is the simple summation approach. It consists in calculating single risks and then aggregating them by just adding them up.

The Value At Risk is used to calculate single risks; then, single VaRs are aggregated into total VaR.

The Value at Risk at a specific confidence level $\alpha \in (0,1)$ is defined as the smallest number l such that the probability of loss L exceeding l is not larger than $(1 - \alpha)$:

$$VaR = inf\{l: P(L \leq l) \leq (1 - \alpha)\}S \tag{24}$$

Figure 4: VaR of loss distribution at confidence level α
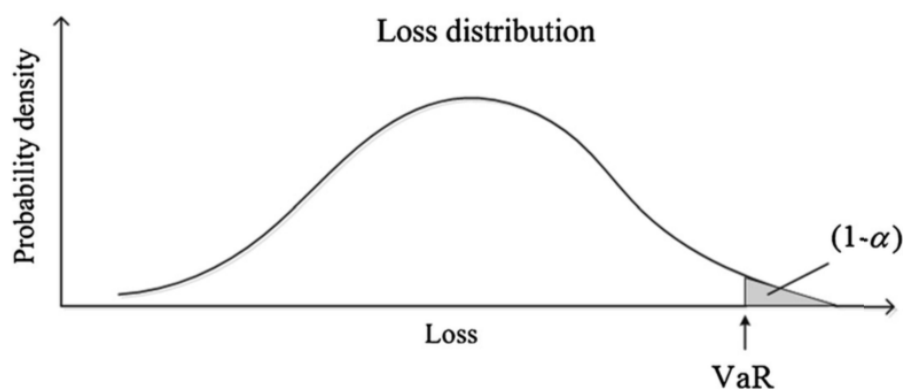
Source: Li, J., Zhu, X., Lee, C., Wu, D., Feng, J., Shi, Y. (2015)

VaR can be calculated from the probability distribution of losses during time T and is equal to the loss at the α-th percentile of the distribution. According to the distributions of credit, market, and operational risk described in Chapter II, the single VaRs can be calculated. The simple summation approach aggregates the risks by simply adding the individual VaRs of different risks:

Total VaR = VaR (credit) + VaR (market) + VaR (operational)   (25)

This approach assumes that all the inter-risk correlation coefficients are equal to one and does not matters potential diversification benefits. Thus, it assumes that the worst-case scenarios always happen simultaneously.

The simple summation approach risks to be too much conservative because it does not consider the imperfect correlations between different types of risks. However, the approach is often considered as the benchmark for calculating diversification coefficient because it ignores potential diversification benefits and produces an upper bound to the true economic capital figure.

If we want to consider the correlations between different types of risks, the basic method is the Var-Covar one. It introduces a correlation matrix to combine the single VaRs and to calculate the total VaR:

$$TotalVar = \sqrt{\begin{pmatrix} Var(credit) \\ Var(market) \\ Var(operational) \end{pmatrix}^T \begin{pmatrix} 1 & \rho_{cm} & \rho_{co} \\ \rho_{cm} & 1 & \rho_{mo} \\ \rho_{co} & \rho_{mo} & 1 \end{pmatrix} \begin{pmatrix} Var(credit) \\ Var(market) \\ Var(operational) \end{pmatrix}} \quad (26)$$

This approach is very popular because of its simplicity. Within this approach, the diversification benefits exist. As regard to the correlation coefficient (R), the smaller it is, the smaller the total VaR is. The approach assumes that all risks are jointly normally distributed, and it is a restrictive assumption.[29]

A more flexible approach is the Copula one. Copulas are functions which combine univariate marginal distributions to a multivariate joint distribution. Sklar's (1959) theorem states that a*n-*

[29]Li, J., Zhu, X., Lee, C., Wu, D., Feng, J., Shi, Y., *On the aggregation of credit, market and operational risks*, Review of Quantitative Finance and Accounting, Volume 44, Issue 1: 161-189, 2015

dimensional joint distribution function $F(x)$ evaluated at $x = (x_1, x_2, \ldots, x_n)$ may be expressed in terms of the joint distribution's copula C and its marginal distributions $F_1, F_2, \ldots, F_n$ as

$$F(x) = C\big(F_1(x_1), F_2(x_2), \ldots, F_n(x_n)\big), x \in R^n \tag{27}$$

The copula function C is itself a multivariate distribution with uniform marginal distributions on the interval $U_1 = [0 - 1], C: U_1^n \rightarrow U_1$.

Reformulating the previous formula, we obtain:

$$C(u) = F(F_1(u_1)_1^{-1} F_2(u_2)^{-1}, \ldots, F_n(u_n)^{-1}), u \in U_1^n \tag{28}$$

Where $u = (u_1, u_2, \ldots, u_n) = \big(F_1(x_1), F_2(x_2), \ldots, F_n(x_n)\big)$ are the respective univariate marginal distributions.

Thus, a copula-based approach allows a decomposition of a joint distribution into its marginal distributions and its copula. The marginal distributions may be combined into a joint distribution assuming a specific copula. The approach allows for a separate modeling of the marginal distributions and of the dependence structure (i.e. the copula).[30]

VaR is then derived from the joint risk distribution.

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[30] Cech Christian, *Copula-based top-down approaches in financial risk aggregation*, Working Paper Series by the University of Applied Sciences of bfi Vienna, n° 32, December 2006;

Figure 5: *Risk aggregation using the copula approach*
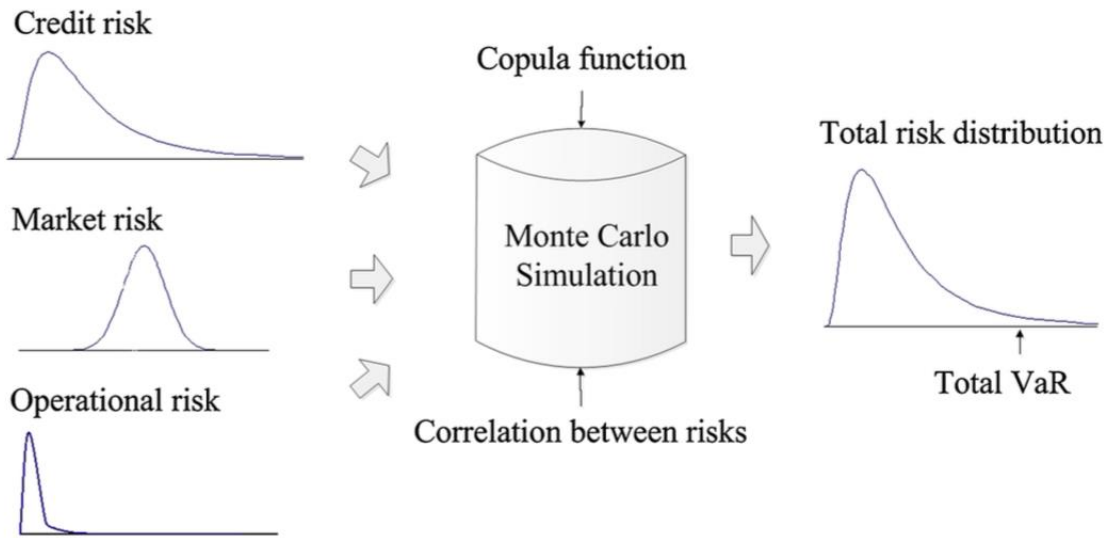
This approach involves the following five steps:

- Determining the loss rate distribution of credit, market, and operational risk;

- Chose a proper copula function;

- Using Monte Carlo simulation in order to simulate joint credit, market and operational loss rates based on copula function and the three individual loss rate distributions;

- Calculating the total loss rate by adding simulated joint credit, market and operational loss rate;

- Calculating the total VaR according to the total loss rate distribution.

Let F_c, F_m and F_o denote the cumulative distributions of credit, market and operational loss rate, L_c, L_m and L_o, respectively. Let C denotes the copula function and F denotes the joint loss rate distribution. Then, the joint loss rate distribution F can be derived by using the copula function to link these marginal cumulative distributions together:

$$F(L_c, L_{m,} L_o) = C(F_c(L_c), F_m(L_m), F_o(L_o)) \tag{29}$$

Analytical expression of multivariate distribution of loss rate does not always exist. In this case, Monte Carlo simulation is used to derive the total loss rate distribution. Monte Carlo simulation uses conditional sampling to construct multiple correlated variables. Let (u_c, u_m, u_o) denotes a joint sample of uniform random variables from the multivariate distribution specified by the copula. Then a sample of joint loss rate is

$$F_c^{-1}(u_c), F_m^{-1}(u_m), F_o^{-1}(u_o), \tag{30}$$

where $F_c^{-1}(u_c)$ is the simulated credit loss rate, $F_m^{-1}(u_m)$ is the simulated market loss rate and $F_o^{-1}(u_o)$ is the simulated operational loss rate.

L is the total loss rate, defined as follows:

$$L = F_c^{-1}(u_c) + F_m^{-1}(u_m) + F_o^{-1}(u_o) \tag{31}$$

Assume the simulation is conducted n times. Then n simulated total loss rates are derived. As the number of simulations increases, the total loss rates L_1, ..., L_n converge to the total loss rate distribution. When the number of simulations reaches a certain threshold, this empirical distribution and the actual distribution are very close, so it is reasonable to derive the total VaR from the empirical distribution. Total VaR is equal to the α-th percentile of the total loss rate distribution. Joint distribution of risk loss rates is decomposed into marginal distribution and dependence structure by the copula function. Therefore, copula functions can be much more flexible in describing dependence structure of different risks. Besides, in contrast to the simple summation approach and Var-Covar approach, the copula approach here uses the entire credit, market and operational risk distributions as the inputs for the risk aggregation process, rather than VaRs of the three risks. Accordingly, the outputs include the total risk distribution rather than just a total VaR, which is very valuable.

There are many Copula functions that can be used. The most used are the elliptical one, such as Gaussian copulas and Student-t copulas, and the Gumbel, Clayton and Frank copulas from the Archimedean family.

The Gaussian Copula is presented as:

$$C_{Gaussian}(u_1, \ldots, u_n | R) = \phi_R\left(\phi^{-1}(u_1), \ldots, \phi^1(u_n)\right) \qquad (32)$$

Where $u_1, u_2, \ldots, u_n$ follow univariate standard uniform distribution, ϕ_r denotes the standard multivariate Gaussian distribution function with correlation coefficient matrix R, ϕ^{-1} is the inverse function of standard univariate Gaussian distribution. Gaussian copula is very easy to use but it does not allow for tail dependence.

The t-Copula is presented as:

$$C_t(u_1, \ldots, u_n | v, R) = t_{v,R}\left(t_v^{-1}(u_1), \ldots, t_v^{-1}(u_n)\right) \qquad (33)$$

Where $t_{v,R}$ denotes the t distribution function with degree of freedom v and correlation coefficient matrix R, and t_v^{-1} denotes the inverse function of t distribution function. The t-Copula has the ability to incorporate tail dependence. The lower the degree of freedom, the heavier is the tail dependence for a t copula. Gaussian copula is in fact, a limiting case of t copula as the degree of freedom approaching ∞. By contrast, t-Copula exhibits the heaviest tail dependence as the degree of freedom approaching 1.

The Gumbel Copula is presented as:

$$C_{Gumblel}(U_1, \ldots, U_n | \alpha) = exp\left\{-[(-ln u_1)^\alpha + \cdots + (-ln u_n)^\alpha]^{\frac{1}{2}}\right\}, \alpha \in [1, \infty) \qquad (34)$$

Where the larger α is, the heavier the dependence is. $\alpha = 1$ implies the independence copula and as α approaches ∞ we obtain the perfect dependence copula. Gumbel copula exhibits greater dependence in the right tail than in the left.

The Clayton copula is presented as:

$$C_{Clayton}(u_1, \ldots, u_n | \theta) = \left(u_1^{-\theta} + \cdots + u_n^{-\theta} - 1\right)^{-\frac{1}{\theta}}, \theta \in (0, \infty) \qquad (35)$$

Where the larger θ is, the heavier the dependence is. As θ approaches 0, we obtain the independence copula, whereas as θ approaches ∞ we obtain the perfect dependence copout. Clayton copula exhibits greater dependence in the left tail than in the right.

Frank Copula is presented as:

$$C_{Frank}(u_1, \dots, u_n | \lambda) = -\frac{1}{\lambda} ln \left(1 + \frac{(e^{-\lambda u_1} - 1) \dots (e^{-\lambda u_n} - 1)}{e^{-\lambda} - 1} \right), \lambda \in (-\infty, \infty) \tag{36}$$

Where Frank copula displays perfect negative dependence for λ equal to $-\infty$, perfect positive dependence for λ equal to ∞, and independence for λ equal to 0. The type of copula can only capture symmetric dependence and is sensitive to neither left nor right tail dependence.

Table 3: *Overall comparison of the five frequently-used copulas*

Copula Type	Elliptical Copula		Archimedean Copula		
	Gaussian	**t**	**Gumbel**	**Clayton**	**Frank**
Ease of simulation	Easy	Easy	Difficult	Difficult	Difficult
Tail dependence	No	Yes	Yes	Yes	No
Tail simmetry	Symmetric	Symmetric	Asymmetric	Asymmetric	Symmetric
Negative dependence	Yes	Yes	No	No	Yes
Structure simmetry	Asymmetric		Symmetric		

Source: BCBS (2010)

Concluding, we can say that there is no single copula that can be applied to all situations, so it could be used a mixture of copulas. In fact, Nelson (1999) demonstrates that mixtures of copulas are copulas too. With a mixture of copulas, we can generate a dependence structure that does not belong to one particular copula family and so we can better capture the interactions between credit, market and operational risk. Considering that *t*-Copula is the most suitable in modeling tail dependence, including the negative one, and in allowing asymmetric interactions between risks, and that the Gumbel one, on the contrary, is highly symmetric and sensitive to right tail dependence, a mixture of the two Copulas can be used.

$$C_{mixture}(u_1, \dots, u_n | v, R, \alpha, w) = w C_t(u_1, \dots, u_n | v, R) + (1 - w) C_{Gumbel}(u_1, \dots, u_n | \alpha) \tag{37}$$

Where *w* denotes the weight of *t* - copula. This mixture copula exhibits heavier right tail dependence and allows asymmetric interactions and negative dependence between risks.

Since the Clayton copula captures the left tail dependence, a mixture of *t*-Copula and Clayton one could show heavier left tail dependence.

Mixture of copulas are more complex but able to reflect more flexibility and to involve more parameters

It is also possible to extend this aggregation model that operates in a single-period framework developing a multi-period extension. As Brockmann and Kalkbrener (2010) demonstrated, the first setup does not work well for all risk types that are illustrated by a comparison between banking book vs risks in the trading book.[31] Since banking positions are not actively traded and more difficult to hedge than the trading one, it seems justified to use a buy-and-hold assumption over the planning period of one year in the economic capital calculations for the banking book. The same assumption would clearly overestimate the risk in the trading book: the higher liquidity of trading book positions facilitates the implementation of more effective risk management strategies. Therefore, Brockmann and Kalkbrener suggested that multi-period models provide the natural framework for the specification of portfolio strategies for liquid positions. In this setup, the planning period is split into time intervals determined by the liquidity horizon of the underlying portfolio The rollover at the end of each period is specified by a portfolio strategy. The level of risk is kept constant in this portfolio strategy by rebalancing the portfolio at the end of each period. This strategy is in contrast with the assumption of holding the same position for the entire planning period of one year. Another basic model assumption is the independence of loss distributions in non-overlapping time periods. This is a common assumption in risk modeling, which is additionally supported by the fact that rebalancing of the portfolio reduces potential autocorrelations, and an asset is typically replaced if its credit quality has changed at the end of a liquidity period. Thus, the autocorrelations of rating migrations are not relevant for the aggregation of the loss distributions across non-overlapping time periods: in order to obtain a cumulative loss over one year, the loss distribution at the liquidity horizon is calculated and then re-sampled for the following time periods. This rollover model is then extended to the aggregation of risk types with different liquidity horizons.

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[31] Brockmann Michael, Kalkbrener Michael, *On the Aggregation of Risk*, Risk Analytics & Instruments, Deutsche Bank, Frankfurt, 2010

III.2: The interactions between different types of risk

Determining the economic capital is the core part of financial institution and its calculation is the focus risk management strategies. For many reasons, both historical and practical, especially market and credit risks have often been treated as if they are unrelated sources of risk. The development of credit risk transfer markets and the moves to mark-to-market accounting for portions of held-to-maturity banking book positions, however, have blurred distinctions between market and credit risk and raise questions regarding approaches that treat the two types of risk separately. Market participants have argued that there are significant diversification benefits to be reaped from the integrated measurement and management of market and credit risks. It is demonstrated that non - linear interactions between market and credit risk could lead to compounding effects, which are not captured in standalone risk assessments for the different risks.[32]

The recent financial crisis has also illustrated how the two risks may reinforce each other and generate large losses if not managed jointly.

As discussed by Grundke (2010), the interaction between different types of risks was clear in the context of the subprime crisis, too.[33]

In the USA, interest rates were low for the first part of the decade. The funding rate of the Federal Reserve Board was 1% in June 2003 and then started to slowly increase in June 2004 until it was 5.25% in June 2006. Rising interest rates were a problem, in particular, for those homeowners who accepted an adjustable rate mortgage (ARM) loan. In 2005 and 2006, the most common subprime loans were of the so-called ''short-reset'' type. These ARM loans had a relatively low fixed interest rate for the first two or three years (teaser rate), which was then reset semiannually to an index plus a considerable margin for the remaining period. As from mid-2004 onwards short-term interest rates and sometime later also mortgage rates began to rise, debt service burdens for ARM loans also increased, which caused financial distress for

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[32] Brockmann Michael, Kalkbrener Michael, *On the Aggregation of Risk*, Risk Analytics & Instruments, Deutsche Bank, Frankfurt, 2010

[33] Grundke Peter, *Top-down approaches for integrated risk management: How accurate are they?*, European Journal of Operational Research, n° 203, 2010

many homeowners. This was not very surprising because subprime borrowers are typically not very creditworthy and are often highly levered with high debt service-to-income ratios. Furthermore, the mortgages extended to them have a large loan -to - value ratio, which implies that the equity stake of the borrowers is very small. During 2005 and 2006, subprime borrowers could even finance 100% of the purchase price of their homes by taking out two mortgages on their homes. These events clearly demonstrate that there is indeed a relationship between interest rate risk and default risk and, thus, that there is a need for integrated risk management approaches that are able to consider these relationships.

Rising default rates for ARM subprime loans were accompanied by decreasing home prices. Thus, the value of the collateral also decreased and the loss given default of the ARM loans increased. Of course, rising default rates of homeowners and decreasing home prices were not completely independent of each other. On the one hand, an increasing number of liquidations of the collateral of those homeowners who were no longer able to service their debts was at least responsible for a further decline in home prices. On the other hand, decreasing home prices made it impossible for many homeowners with difficulties in servicing their debts, to repay their loan prematurely, which caused further defaults.

The financial crisis also illustrated the significant role that illiquidity can play in such stress situations. Liquidity conditions interact with market risk and credit risk through the horizon over which assets can be liquidated. In particular, deteriorating market liquidity often forces banks to lengthen the horizon over which they can execute their risk management strategies. As this time horizon lengthens, overall risk exposures generally increase, as does the contribution of credit risk relative to market risk. The liquidity of traded products can vary substantially over time and in unpredictable ways. Such liquidity fluctuations, all else equal, should have a larger impact on prices of products with greater credit risk. Conversely, as the current financial crisis illustrates, valuation uncertainties or other shocks that enhance actual or perceived credit risks can have adverse effects on liquidity and put in motion a downward spiral between market prices and liquidity of traded credit products. Banks' exposures to market and credit risk depend on their risk management strategies. Because many strategies rely on liquid markets for hedging, or for unwinding positions to limit losses on exposures that cannot be hedged, asset market liquidity is an important determinant of banks' overall risk profile. Additionally, since liquidity is time varying, and markets typically become less liquid when

risk increases appreciably, it became clear that liquidity interacts with other sources of risk.[34] As a consequence, it is suggested to adjust valuation methods in order to take endogenous liquidity risk into account (such as computing VaR integrating liquidity risk)[35].

III.3: Examples of top down approaches from the literature

Rosenberg and Schuermann in 2006 conducted a top-down analysis of a representative large, and internationally active bank that uses copulas to construct the joint distribution of losses. This allows to incorporate realistic marginal distributions that capture essential skewness and fat-tails. The aggregation of market, credit and operational risk requires knowledge of the marginal distributions of the risk components as well as their relative weights. Rosenberg and Schuermann assign inter-risk correlations and specify a copula, such as the Student-t copula, which captures tail dependence as a function of the degrees of freedom. They impose correlations of 50% for market and credit risk, and 20% for the other two correlations with operational risk. They find several interesting results, such as that changing the inter- risk correlation between market and credit risk has a relatively small impact on total risk compared to changes in the correlation of operational risk with the other risk types. The authors examine the sensitivity of their risk estimates to business mix, dependence structure, risk measure, and estimation method. Overall, they find that "*assumptions about operational exposures and correlations are much more important for accurate risk estimates than assumptions about relative market and credit exposures or correlations*"[36]. Comparing their VaR measures for the 0.1% tail to the sum of the three different VaR measures for the three risk types, they find

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[34] Basel Committee on Banking Supervision, *Findings on the interactions of market and credit risk*, Working paper n. 16, Bank for International Settlements, May 2009

[35] Basel Committee on Banking Supervision, *Messages from the academic literature on risk measurement for the trading book*, Working Paper n. 19, Bank for International Settlements, January 2011

[36]Rosemberg Joshua V., Schuermann Til, *A general approach to integrated risk management with skewed, fat-tailed risks,* Journal of Financial Economics, n° 79, 2006

diversification benefits in all cases. Their results suggest values ranging from 0.42 to 0.89. They found similar results when the expected shortfall (ES) measure is used.

Rosenberg and Schuermann authors stated that the sum of the separate risk measures is always the most conservative and overestimates risk, "*since it fixes the correlation matrix at unity, when in fact the empirical correlations are much lower*". While the statement of imposing unit correlation is mathematically correct, it is based on the assumption that the risk categories can be linearly separated. If that assumption were not correct the linear correlations could actually be greater than one and lead to risk compounding[37].

Inanoglu and Jacobs in their paper "*Models for risk aggregation and sensitivity analysis: an application to bank economic capital*" (2009) estimated and compared alternative frameworks. First, they extended the scope of the analysis to liquidity and interest rate risk. Second, they utilized actual data representative of five major banking institutions' loss experience, extracted from call reports and submitted by banks to supervisory agencies, in order to explore the impact of business mix and inter-risk correlations on total risk. Third, they estimated alternative copula models and established a framework for capturing realistic distributional features of different risk types (e.g., non-normality) and cohesively combining such, on the same data-set. Then, they compared the models to several conventional approaches and they applied the goodness-of-fit (GOF) tests to the various copula models. They observed a wide divergence in measured VaR, diversification benefits as well as the sampling variation across different risk aggregation methodologies and types of institutions and they observed that, contrary to asymptotic theory, empirical copula simulation (ECS) tends to produce the highest absolute magnitudes of VaR with respect to standard copula formulations (e.g. Gaussian copula simulation) on the order of about 20% to 30%, while the Var-Covar approximation tends to understate risk. The proportional diversification benefits, measured by the relative VaR reduction with the assumption of perfect correlation, exhibit wide variation across banks and aggregation techniques. The ECS generally yields the highest values relative the other methodologies (127% to 243%), while the Archimadean Gumbel copula (AGCS) is the lowest (10-21%). They concluded that while the ECS may overstate absolute risk, proportional diversification benefits

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[37] Basel Committee on Banking Supervision, *Messages from the academic literature on risk measurement for the trading book*, Working Paper n° 19, Bank for International Settlements, January 31st, 2011

may be understated by standard methodologies on the order of about 15% to 30%. Through differences observed across the five largest banks, they failed to find the effect of business mix to exert a directionally consistent impact on total integrated diversification benefits. In the GOF tests, they found mixed results: in many cases most of the copula methods exhibit poor fit to the data relative to the ECS, with the Archimadean copulas fitting worse than the Gaussian or Student-*t* copulas. They found also that the variability of the VaR is significantly lower for the ECS and higher for Var-Covar, as compared to other copula formulations. Given the conservatism and stability of the ECS methodology, the poor performance of the standard methodologies in GOF analysis, and the lack of consensus upon the best copula to use, they concluded that the ECS method is most robust.

Elsinger and Lehar in 2006 proposed an approach to assess systemic financial stability of the Austrian banking industry using a network model of interbank loans, using a cross section of all 881 Austrian reporting banks in September 2002 from MAUS database. They performed an aggregation of the same types of risks of all the 881 banks and figured out aggregate losses due to credit, market and operational risk respectively. Li et Al. Used this study to perform a higher level of risk aggregation: the aggregated credit, market and operational risk to calculate the total risk of Austrian banking industry using the different approaches I discussed above.[38]

These are the data extracted from Elsinger and Lehar (2006), showing mean values and standard deviation of loss due to credit, market and operational risks, in million Euros and in percentage of total assets. The loss rate is loss divided by the corresponding risk exposure.

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[38]Li, J., Zhu, X., Lee, C., Wu, D., Feng, J., Shi, Y., *On the aggregation of credit, market and operational risks*, Review of Quantitative Finance and Accounting, Volume 44, Issue 1: 161-189, 2015

Table 4: *Statistics of loss of the entire Austrian banking industry*

	Credit risk loss		Market risk loss		Operational risk loss		Total assets
	Mean	**SD**	**Mean**	**SD**	**Mean**	**SD**	
MillionEuros	971	800	-36	2,805	883	30	575,000
% of total assets	0.17	0.14	-0.01	0.49	0.15	0.01	

In this table, the time horizons of loss due to credit and operational risk are 1 year and the time horizon of loss due to market risk is 10 days. % of total assets denotes the loss rate which equals loss divided by total assets. The data of operational risk loss are from e-companion of Elsinger and Lehar (2006).

SD = standard deviation

Source: Elsingeret Al.(2006)

Credit, market and operational loss rates are assumed to follow respectively beta, normal and lognormal distributions.

Table 5: *Parameters of credit, market and operational risk loss rate distributions*

Credit risk			Market risk			Operational risk		
Distribution	**Parameters**		**Distribution**	**Parameters (%)**		**Distribution**	**Parameters**	
Beta	p	1.47	Normal	μ_m	-0.01	Log-normal	μ_o	-6.51
	q	863.24		σ_m	0.49		σ_o	0.07

Source: Li et Al. (2015)

Figure 6: *Credit, market and operational loss rate distributions*
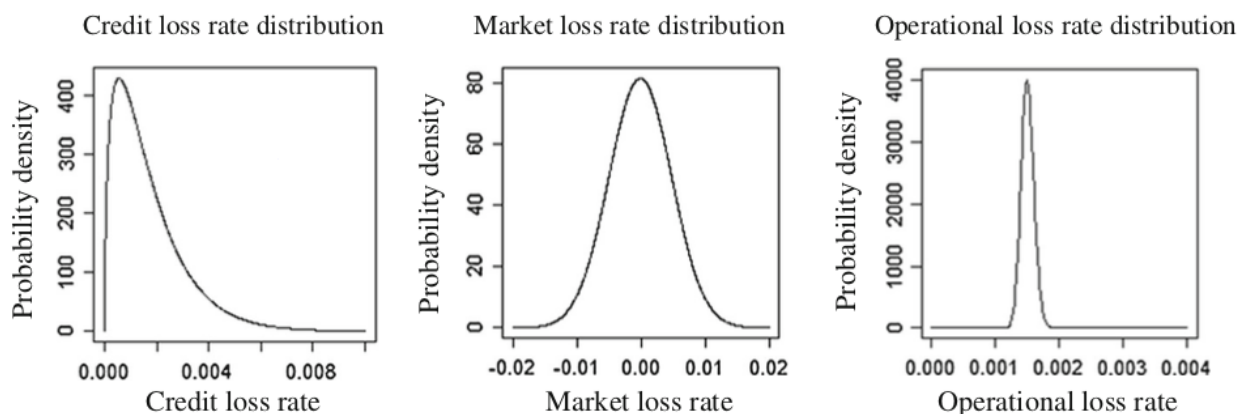
Source: Li et Al. (2015)

Table 6: *VaRs of credit, market and operational risk at different confidence levels*

Confidencel evel	90%	95%	99%	99,5%	99,9%	99,97%
Credit risk	2,046	2,562	3,725	4,217	5,345	6,179
Market risk	3,553	4,577	6,497	7,200	8,649	9,611
Operational risk	937	960	1,005	1,022	1,057	1,082

Source: Li et Al. (2015)

The correlation matrix used in the Li et Al. study is the IFRI/CRO[39] one:

$$\begin{pmatrix} 1 & 0,66 & 0,3 \\ & 1 & 0,3 \\ & & 1 \end{pmatrix}$$

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[39]International Financial Risk Institute Foundation and Chief Risk Officer Forum, *Insights from the joint IFRI/CRO*, Forum Survey on economic capital practice and applications, KPMG Business Advisory Services, 2007

where 0,66 is the correlation coefficient of credit and market risk; 0,3 is the correlation coefficient between credit and operational risk; 0,3 is the correlation coefficient between market risk and operational risk.

The following tables, show the total VaRs estimated respectively using the simple summation, Var-Covar and Copula approaches.

Table 7: *Total VaR estimated by using simple summation approach*

Confidence level	90%	95%	99%	99,5%	99,9%	99,97%
Simple summation	6,537	8,099	11,227	12,438	15,051	16,051

Unit: million Euros

Table 8: *Total VaR estimated by using the Var-Covar approach*

Confidence level	90%	95%	99%	99,5%	99,9%	99,97%
Var-Covar	5,517	6,930	9,757	10,851	13,206	14,845

Unit: million Euros

Table 9: *Copulas approach: copulas used and parameter assumption*

Copula types	Parameters	Features
Gaussian copula	R	Not any tail dependence
t - copula (1 df)	$v = 1, R$	Explicit tail dependence
t - copula (10 df)	$v = 10, R$	Weak tail dependence
Mixture copula	$v = 1, \alpha = 5, w = 0,5, R$	Heavier right tail dependence

Mixture copula consists of *t* - copula and Gumbel copula, *R* is the IFRI/CRO correlation matrix.
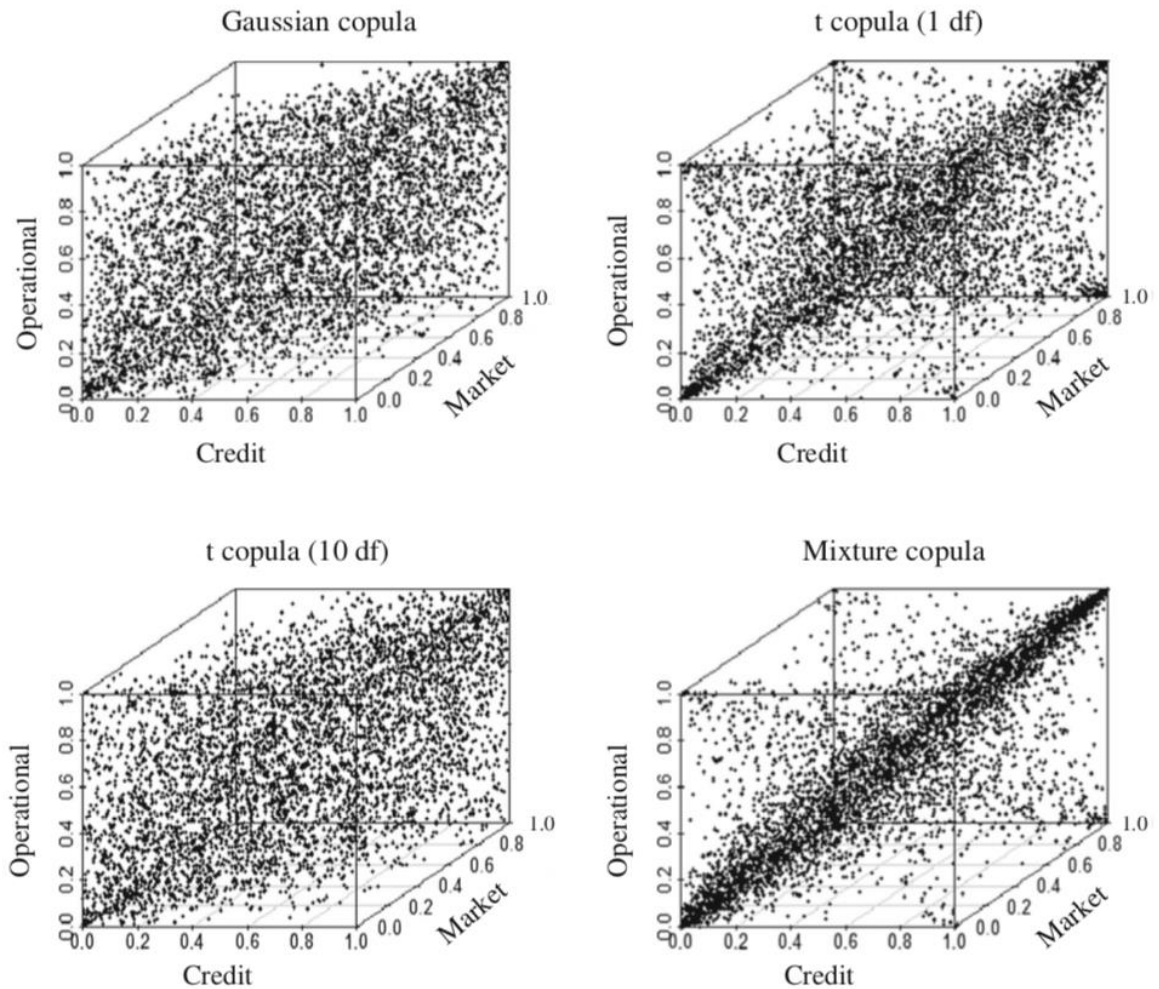
Source: Li et Al. (2015)

We can observe that with the simple summation approach and the Var-Covar approach, as the confidence level increases, the total VaR becomes larger.

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In the Copula approach, the total risk distribution can be derived by Monte Carlo Simulation. In their study, Li et Al. use four types of copula functions, as described in Table 7.

The following figure describes the 3-dimensional scatter plot of these four copulas:

Figure 7: *3-Dimensional scatter plot of different copulas*

Source: Li et Al. (2015

We can notice that Gaussian copula doesn't show any tail dependence. The t copula (1 df) shows a clear view of symmetric tail dependence. The t copula (10 df) is very close to Gaussian copula. Compared with t copula (1 df), mixture copula exhibits a heavier right tail dependence. The four copulas give four typical cases of risk dependence.

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For each of the four copula functions, 1000000 observations are generated by Monte Carlo simulation, consuming about 20 s. After the joint loss rates are obtained based on these observations and marginal loss rate distributions, the total loss rate can be calculated by adding joint loss rates. Then, 1000000 total loss rates are derived for each copula. Based on the total loss rate distribution, finally the total VaR can be calculated. Different copulas lead to different total loss rate distributions and mainly affect tail shapes. Consequently, different tail shapes result in different total VaRs.

Table 10: Total VaR estimated by using different copula approaches

Confidence level	90%	95%	99%	99,5%	99,9%	99,97%
Gaussian copula	6,177	7,555	10,291	11,363	13,585	15,239
t - copula (1 df)	6,155	7,554	10,361	11,486	13,942	15,667
t - copula (10 df)	6,050	7,542	10,639	11,860	14,564	16,413
Mixture copula	6,281	7,815	10,941	12,196	14,844	16,696

Unit: million Euros

Source: Li et Al. (2015)

Looking at the same copula, as the confidence level increases, total VaR becomes larger. Therefore, if high confidence level is required by the regulatory commission, VaR is larger and so is the required economic capital. At the same confidence level, total VaRs from different copulas are different. The heavier the tail dependence of the copula is, the larger is the total VaR. This can be explained by the fact that extreme losses are more likely to occur simultaneously and frequently based on heavy tail dependence, which finally leads to more extreme total loss rate in general.

In the following table, Li et Al. (2015) have calculated VaR ratios by dividing the total VaRs of different approaches by the total assets.

Table 11: *VaR ratios (unit = %)*

Confidence level	90%	95%	99%	99,5%	99,9%	99,97%
Simple summation	1,14	1,41	1,95	2,16	2,62	2,93
Var-Covar	0,96	1,21	1,70	1,89	2,30	2,58
Gaussian copula	1,07	1,31	1,79	1,98	2,36	2,65
t - copula (1 df)	1,07	1,31	1,80	2,00	2,42	2,72
t - copula (10 df)	1,05	1,31	1,85	2,06	2,53	2,85
Mixture copula	1,09	1,36	1,90	2,12	2,58	2,90

Source: Li et Al. (2015)

By comparing the Total VaRs, we can conclude that:

• As the confidence level increases, the VaR ratio becomes larger. Thus, banks should be cautious in choosing proper confidence level: a high confidence level may be too conservative, and a low one could not cover regular risks. BCBS recommends 99,9% as a proper confidence level.

• As the confidence level increases, the differences between VaR ratios based on different approaches become greater. At high confidence levels of 99.9 and 99.97 %, different approaches affect the aggregation results significantly and at lower confidence levels, differences between aggregation results from different approaches are relatively small. As mentioned above, 99.9 % is the confidence level specified by BCBS, so the banks should be prudent in choosing the appropriate aggregation approach since if the chosen aggregation approach is not suitable for the banks, the aggregation results can be very inaccurate.

- VaR ratios based on simple summation approach are the largest at all the six confidence levels. Simple summation is the briefest approach to aggregate the different types of risks by simply adding them up. Because it assumes the correlations between the risks are perfect, it ignores potential diversification benefits and is generally perceived as a conservative approach. In summary, simple summation approach imposes an upper bound on the true economic capital requirement.

- VaR ratios based on Var-Covar approach are significantly the smallest. Var-Covar approach only use a matrix of linear correlations which provides a poor metric for capturing tail dependence between risks. Copulas, by contrast, are capable of specifying a full dependence structure, not only linear dependence. So, it is natural that the lines of all copulas are above Var-Covar approach.

- All VaR ratios based copulas are between VaR ratios based on Var-Covar approach and simple summation approach and slightly different from each other. Simple summation is too conservative for assumption of perfect correlation and Var-Covar is always considered as too simplified for assumption of linear correlation. In contrast to them, copula approach provides a more flexible way to describe dependence and offers many choices for different types of bank. At 95 % or higher confidence level, the larger the degree of freedom is, the smaller the VaR ratio from a t copula is. The degree of freedom of t - copula ranges from 1 to ∞, which indicates t - copula is able to provide a series of dependence with fine distinctions. The VaR ratio from mixture copula is merely next to simple summation approach. So if t - copula (1 df) is still not conservative enough, the mixture of t - copula and Gumbel copula is a good choice.

CHAPTER IV: BOTTOM - UP APPROACHES FOR BANK RISK AGGREGATION

IV.1: Top-down vs Bottom-up approaches: the interactions between market and credit risk in the literature

In Chapter III we have seen that within the top-down approach the marginal distributions of losses resulting from the different risk types are determined separately and then aggregated.

Traditionally, credit and market risk are treated independently, and it is thought that credit risk is mainly relevant for the banking book, and market risk for the trading book. The separation between credit and market risk mimics the traditional organization within banks into credit departments and a market investment department. Top-down approaches assume that under a sub-additive risk measure, the risk of the total portfolio will be smaller than / almost equal to the sum of the risk of the banking book and of the trading book, and that credit risk is only relevant for the banking book and market risk for the banking book. As a consequence, under all sub additive risk measures, total risk will be smaller or almost equal to the sum of credit and market risk. Is it always true? In many situations, a split into credit and market risk is not possible because some positions will simultaneously depend on market and credit factors. Enforcing a separation between two portfolios could lead to a wrong assessment of the true risk.

As we have seen in the previous chapters, standard credit models are constructed from four parameters: the probability of default (PD), the exposure at default (EAD), the loss given default (LGD) and default correlations. In this scenario, market risk derived from market price movements, plays no role. Breuer et Al. [40] discussed the ways market and credit risk interact with each other. In their opinion, the PD may depend on market prices, like interest rates, exchange rates and commodity prices. If the payment obligation of the counterparty increases, it is much more likely to default. The EAD may depend on market prices, too, because the discounted value of a loan depends on the interest rate at which future payments are discounted, and if the loan is in a foreign currency, the value depends also on the exchange rate. As regard

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[40]Breuer T., Jandačka M., Rheinberger K., Summer M., *Compounding effects between Market and Credit Risk: the case of variable rate loans*, London, 2008

to the LGD, it depends on market risk factors too: for example, the LDG of mortgages increases if the real estate price drop. The authors noticed that there is a connection between PD and LGD: if banks liquidate collaterals because their PD increase, this could lead to an increase supply of the collaterals that will reduce their value. In addition, defaults may increase exposure to market risk; in fact, positions hedged against market risk might suddenly be exposed because on side of the hedge defaults.

Considering all these interactions between market and credit risk, it is possible to use bottom - up approaches instead of the top - down ones, because they allow to model and measure simultaneously different risk types and there is no need for a later aggregation of risk-specific economic capital numbers.

Grundke (2008)[41] compared the two approaches, top-down vs bottom up (developed by extending CreditMetrics) and find out that the previous one can underestimate the true risk measure of economic capital. In addition, the performance of top-down approaches, in his opinion, could be influenced by the accuracy of the marginal loss distributions, of the employed copula function and of the loss definitions.

Breuer, Jandača and Rheinberger (2010)[42] proposed an analysis of hypothetical loan portfolios for which the impact of market and credit risk fluctuations are not separable. They argue that changes in aggregate portfolio value caused by market and credit risk fluctuations in isolation should sum up to the integrated change incorporating all risk interactions very rarely.
They suppose that market and credit risk are computed for a common time horizon, while market risk is usually computed over a ten days-time-horizon and credit risk over one year horizon.
How market and credit risk may interact? In credit risk analysis PDs may depend on market prices, which are linked to interest rates or exchange rates. If these types of risks move adversely, there could implicate default. Another interaction between market and credit risk

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[41]Grundke Peter, *Top-Down versus Bottom-Up Approaches in Risk Management,* 2008

[42]Breuer T., Jandačka M., Rheinberger K., *Does adding up of economic capital for market and credit risk amount to conservative risk assessment?*, Journal of Banking and Finance, April 2010

over the same time horizon is given by the fact that if collaterals are Market to Market, the Loss Given Default may depend on market risk factor, too. Breuer et Al. In their paper talk about the "wrong way exposure" problem, which is defined as the risk that occurs when the exposure to a counterparty is adversely correlated with the credit quality of the counterparty itself. It arises when default risk and credit exposure increase together, that is when there are high PDs and high exposure at the same time. The authors try to capture this "wrong way exposure" via the payment obligation process, which captures the dependence between PD and exposure and between macro risk factors.

If on one hand PDs may depend on market prices, on the other hand PDs can affect them: if PDs increase and banks have to liquidate collaterals, their value would decrease. As a consequence, it may exist such a loop from credit to market risk and back to credit risk. Breuer et al. (2010) used the following approach: they analyzed all the risk factors, without considering if they are market or credit risks. Simply adding up market and credit risk in determining the economic capital, could lead to an underestimation, in case of "*malign*" interaction between the two risks, or an overestimation, in case of "*benign*" interaction. The authors first analyzed the logic of risk underestimation in theory through an example on foreign currency loans, and then they extend the model to the real world, proposing an integrated market and credit risk model with a stochastic default barrier defined by the payment obligation of the customer, which depends on both the two risks and captures the relation between interest rate and exchange rate fluctuations, and between PD and LGD. An increase in default risk trigged by adverse interest rate and exchange rate moves is the principal risk of Foreign Currency Loans (FX loans). This risk cannot be captured by a pure credit risk or market risk model, nor by a simple integrated model.

The model presented by Breuer et al. (2010) is the following one. The portfolio of loans has N obligors indexed by $I = 1, ..., N$. The loans are underwritten at the initial time $t = 0$. An obligor takes a loan of $l_i/f(0)$ units in a foreign currency in order to receive the home currency amount l_i. The bank borrows $l_i/f(0)$ units of the foreign currency on the interbank market. At time $t = 1$ (i.e. after one year) the loan expires and the bank repays the foreign currency on the interbank market with an interest rate r_f and it claims a home currency amount from the customer which is exchanged at the rate $f(1)$ to the foreign currency amount $(l_i/f(0))(1 + r + s_f)$. This is the initial loan plus interest r_f rolled over from four quarters

plus a spread s_f. The customer's payment obligation to the bank at time 1 in the home currency is:

$$o_i = l_i(1 + r_f)f(1)/f(0) + l_i s_f f(1)/f(0) \tag{38}$$

The first term on the right hand is what the bank has to repay on the interbank market, the second term is the spread profit of the bank. For a home currency loan the payment obligation would be $o_i = l_i(1 + r_h + s_h)$, where r_h is the interest rate in home currency and s_h is the spread to be paid by the customer on a home currency loan. Whether an obligor will be able to meet this obligation depends on his payment ability a_i. He will default if his payment ability is smaller than his payment obligation, i.e. if $a_i < o_i$. In this case the customer pays a_i instead of o_i.

The profit of the bank with obligor I is:

$$V_i = min(a_i, o_i) - l_i(1 + r_f)f(1)/f(0) \tag{39}$$

Where $f(0)$ is the known exchange rate at time $t = 0$, $f(1)$ and r are random variables. In the profit function (39), the first term is what the obligor repays and the second term is what the bank has to pay on the interbank market.

In this model, the payment ability of the obligator is modeled as a function of macroeconomic conditions and an idiosyncratic risk component. At the final time $t = 1$, it is:

$$a_i(1) = a_i(0) \cdot \frac{GDP(1)}{GDP(2)} \cdot \in,$$

$$log(\in) \sim N(\mu, \sigma) \tag{40}$$

Where $a(0)$ is a constant, and $\mu = \frac{-\sigma^2}{2}$ ensuring $E(\in) = 1$. For different obligors the realizations $\in_i$ are independent of each other and of GDP.

GDP (0) is the known GDP at time $t = 0$, *GDP (1)* is a random variable. The distribution of $\in_i$ reflects obligor specific random events, like losing or changing job. The support of $\in_i$ is $(0, \infty)$ because a_i cannot be less than 0: in that case the obligor have no lines of credit open with the bank.

Since the expected value of $\in_i$ is 1 and $\in$ is independent of GDP, the expectation of $a_i(1)$ is $a_i(0)$ times the expectation of $\frac{GDP(1)}{GDP(0)}$.

Figure 8: *Plots of density function of the payment ability distribution, with GDP equal to its expected value (solid line), and GDP equal to $\pm$ 3 standard deviations.*
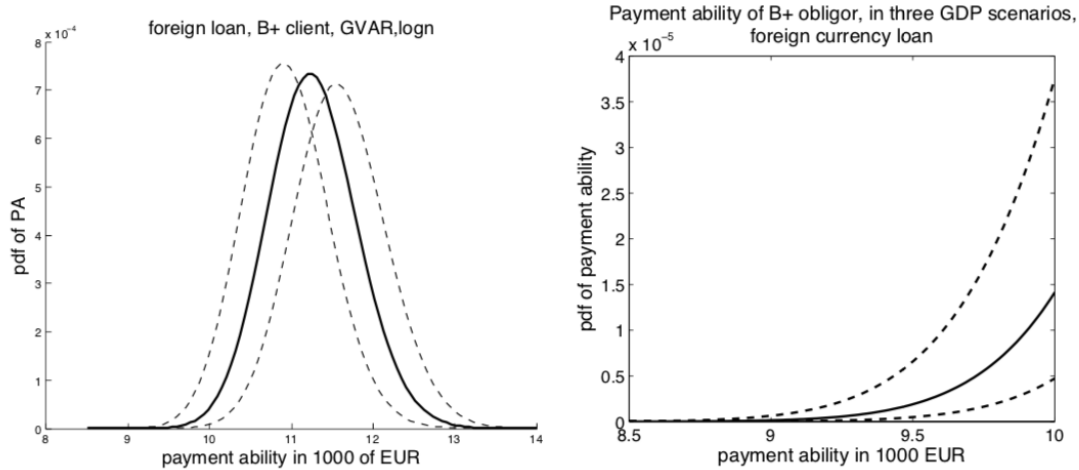
Source: Breuer T., Jandača M., Rheinberger K. (2010)

In this figure, we notice that a GDP increase lead to an increase in all quantiles, and shifts the payment ability distribution to the right. It increases distance to default and reduces default probabilities.

The independency between $\in_i$ implies that customers default are independent, too.

The initial payment ability $a_i(0)$ is determined in the loan approval procedure. Assume that a bank, for instance, gives loans only if they have $a_i(0)$ equal to 1,2 times the loan amount. This is an extra margin that is considered in the rating. Banks use the rating system in order to determine the default probability p_i of customers. The payment ability distribution satisfies this condition:

$$p_i = P[a_i(1) < o_i] \tag{41}$$

Where $a_i(1)$ is a function of σ and o_i is a function of the spreads, that are set to achieve some target expected profit for each loan (42):

$$E\big(V_i(\sigma, s)\big) = EP_{target} \tag{42}$$

Where V_i is the profit with obligor i and EP_{target} is some target expected profit. The two free parameters σ and *s* are determined from these two conditions.

The authors show then how market and credit risk interact in this mode.

At *t = 1* the bank has met its obligation at the interbank market, and has a net foreign currency position $s_f l_i f(1)/f(0)$ for obligor *i*. For this part of the position, current regulation requires market risk capital. On the other hand, default risk is determined by the probability that payment ability falls below payment obligation, and, as a consequence, it depends on both interest rate and exchange rate. Thus, default risk is a function of market risk factors and an integrated risk analysis is required.

The authors use the GVAR time series model of Pesaran et al. (2000)[43], that is an error correction model that allows a parsimonious modeling of economic interdependence between countries or regions.

It involves exchange rate, interest rates and macroeconomic interactions between Austria and Switzerland because it allows the global model to be built from separately estimated country models with foreign variables trade weighted averages. Country models can be estimated separately and stacked into a global model without re-estimating parameters. The authors estimate a GVAR model for Switzerland and Austria and include their three most important trading partners (Germany, Italy and France) and the most important trading partner of Germany, that is the USA. They consider the following variables for each country: real GDP, that is the three months LIBOR interest rate, and the exchange rate to the US dollar. The model is based on quarterly data from 1980q1 and 2005q4, and the authors use the above equations to simulate the distribution for the loan portfolio. They use Monte Carlo simulation of 100000 draws from the distribution of *f (1)*, *GDP*, and *r*, in order to calculate the distribution of profit *V*. They calculate also defaults of customers' payment ability from the distribution of the payment ability process.

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[43] Pesaran H., Shin Y., Smith R., *Structural analysis of vector error correction models with exogenous i(1) variable,* Journal of Econometrics, 97:293-343, 2000

Table 12: *Estimated values for means and covariates of logarithm of the macro risk factors*

	GDP	r_{EUR}	r_{CHF}	CHR/EUR
Mean	5,446	1,246	0,556	0,423
Std. dev.	0,097	1,870	6,301	0,387
Correlations				
GDP	1,000	0,291	0,217	-0,040
r_{EUR}		1,000	0,519	0,140
r_{CHF}			1,000	0,007
CHR/EUR				1,000

Source: Breuer et al. (2010)

If the portfolio has $N = 100$ loans of $l_i = €\ 10000$ taken out in CHF by customers in the rating class B+, corresponding to a default probability of $p_i = 2\%$, or in rating class BBB+, corresponding to a default probability of $p_i = 0,1\%$. As said before, the bank gives loans to customers with $a_i(0)$ equal to 1,2 times the loan amount.

The authors compare the sum of market plus credit risk capital to capital required to cover integrated risk in order to quantify the interactions between market and credit risk. The common time horizon is set at 1 year, under the assumption that all loans are underwritten at time 0 and expire at time 1. The spreads s_f and s_h for each rating class are set to have an expected profit on each loan of € 160, which amounts to a 20% return on an assumed capital charge of € 800 for a loan of € 10000.

Table 13: *Spreads of rating classes*

Rating	Loan Type	σ	Spread [bp]
BBB+	Home	0,0491	160,15
B+	Home	0,0736	165,62
BBB+	Foreign	0,0363	162,29
B+	Foreign	0,0755	168,97

Source: Breuer et al.. (2010)

We have to notice that in the same rating class spread for FX loans are higher than for home currency loans, and that a customer with a given standard deviation σ in his payment ability, will be in a higher rating class for a home currency loan, and in a lower class for a FX loan.

Market risk factor are $e := \left(r_f - f(1) \right)$ for foreign currency loans and $e := (r_h)$ for home currency loans. Credit risk factors are $a := (GDP, \in_i)_{i=1...,N}$, and the portfolio value function is

$$V = \sum_{i=1}^{N} V_i. \tag{43}$$

$e_0 := E(e)$ are the expected values of the market risk factors and $a_0 := (E(GDP), \in_i = \infty)_{i=1,...,N}$, which implies that no obligor defaults. The authors compare the distribution of integrated risk $\Delta V(a, e)$ to the sum $\Delta C_a + \Delta M_e$ of the distributions of market and credit risk by their Expected Shortfall at different quantiles α[4]. For a profit distribution, risk capital is calculated as:

$$RC_\alpha(x) := E(X) - ES_\alpha(X) \tag{44}$$

Table 14: *Risk capital for market, credit, and integrated risks of the FX portfolio.*

rating	α	MR no CR RC(ΔM)		CR no MR RC(ΔC)		Integrated MR&CR RC(ΔV)		risk interaction I
BBB+	10%	1 059	(3)	0	(0)	1 193	(32)	1.13
BBB+	5%	1 234	(4)	0	(0)	1 522	(64)	1.23
BBB+	1%	1 576	(8)	0	(0)	3 056	(32)	1.94
BBB+	0.5%	1 698	(10)	1	(0)	4 641	(637)	2.73
BBB+	0.1%	1 951	(21)	3	(2)	16 076	(3137)	8.22
B+	10%	1 102	(4)	795	(4)	2 711	(49)	1.43
B+	5%	1 285	(5)	1 022	(6)	4 420	(94)	1.92
B+	1%	1 641	(8)	1 523	(14)	11 201	(388)	3.54
B+	0.5%	1 768	(11)	1 730	(19)	15 658	(713)	4.48
B+	0.1%	2 032	(22)	2 257	(45)	32 568	(2921)	7.59

Source: Breuer T., Jandača M., Rheinberger K. (2010)

The table shows malign risk interaction for all quantiles α and in both the rating classes. Integrated risk capital is higher than the sum of credit and market risk capital. Thus, separate analysis underestimates true risk. In addition, holding separate risk capital for market and credit risk is not sufficient to capture the main risk for FX loans, that is the danger of increased defaults trigged by adverse exchange rate moves.

Table 15: *Risk capital for market, credit, and integrated risks of the home currency loan portfolio*

rating	α	MR no CR RC(ΔM)		CR no MR RC(ΔC)		Integrated MR&CR RC(ΔV)		risk interaction I
BBB+	10%	0	(0)	48	(1)	132	(2)	2.75
BBB+	5%	0	(0)	102	(2)	238	(3)	2.33
BBB+	1%	0	(0)	310	(5)	462	(6)	1.49
BBB+	0.5%	0	(0)	396	(8)	556	(8)	1.40
BBB+	0.1%	0	(0)	589	(17)	774	(17)	1.31
B+	10%	0	(0)	961	(5)	1 257	(6)	1.31
B+	5%	0	(0)	1 222	(7)	1 582	(9)	1.29
B+	1%	0	(0)	1 805	(16)	2 299	(19)	1.27
B+	0.5%	0	(0)	2 052	(21)	2 594	(26)	1.26
B+	0.1%	0	(0)	2 601	(47)	3 260	(55)	1.25

Source: Breuer T., Jandača M., Rheinberger K. (2010)

There is negative risk interaction also for home currency loans, because default rates depends on the home interest rate, that are reflected in payment obligation changes. Also in this case, separate analysis underestimates true risk factors. The negative risk interaction for home currency loans is smaller than for foreign currency loans, because the payment obligation of home currency loans is less sensitive on market factor changes.

Breuer et al. (2010) concluded that often it is not possible to separate a portfolio between a market and a credit component, because portfolio positions depend simultaneously on both risks, just think about the exchange rate in FX loans portfolio, that is both a market and a credit risk factor. If for such a portfolio market risk and credit risk are calculated separately, this could lead to a wrong assessment of true portfolio risk.

In a previous study of 2008, Breuer et al. used a similar analytical framework to examine variable rate loans as regard to the interactions between market and credit risk. They modeled the dependence of credit risk factors, such as the loans' default probabilities (PD), exposure at default (EAD), and loss-given-default (LGD), on the interest rate environment. A key risk of variable rate loans is the danger of increased defaults triggered by adverse rate moves. For these loans, market and credit risk factors cannot be readily separated, and their individual risk measures cannot be readily aggregated back to a unified risk measure. The authors conducted a simulation study based on portfolios of 100 loans of equal size by borrowers rated B+ or BBB+ over a one-year horizon using the expected shortfall measure at various tail percentiles. They find that the ratio of unified expected shortfall to the sum of the separate expected shortfalls is slightly greater than one, suggesting that risk compounding effects can occur. Furthermore, these compounding effects are more pronounced for lower-rated loans and higher loan-to-value ratios.[44]

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[44]Breuer T., Jandačka M., Rheinberger K., Summer M., *Compounding effcts between Market and Credit Risk: the case of variable rate loans*, London, 2008

Barnhill et al (2000)., in their study *"Measuring Integrated Market and Credit Risk in Bank Portfolios: An Application to a set of Hypothetical Banks Operating in South Africa"* [45] presented a numerical solution based on a simulation model that links changes in the relevant variables that characterize the financial environment and the distribution of possible future bank capital ratios, measuring both market and credit risk together. The authors develop a model for an application to various hypothetical banks operating in the South African financial environment and assess the correlated market and credit risks associated with business lending, mortgage lending, asset and liability maturity matches, foreign lending and borrowing, and direct equity, real estate, and gold investments. It is shown to produce simulated financial environments (interest rates, exchange rates, equity indices, real estate price indices, commodity prices, and economic indicators) that match closely the assumed parameters, and generate reasonable credit transition probabilities and security prices. The credit quality and diversification characteristics of the loan portfolio, asset and liability maturity mismatches, and financial environment volatility, interact to determine bank risk levels. They find that the credit quality of a bank's loan portfolio is the most important risk factor, and show the risk reduction benefits of diversifying the loan portfolio across various sectors and regions of the economy and the importance of accounting for volatility shocks that occur periodically in emerging economies. Banks with high credit risk and concentrated portfolios are shown to have a high risk of failure during periods of financial stress. Alternatively, banks with lower credit risk and broadly diversified loan portfolios across business and mortgage lending are unlikely to fail even during very volatile periods.

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[45]Barnhill T. B., Panagiotis P., Schumacher L., *Measuring Integrated Market and Credit Risk in Bank Portfolios: An Application to a set of Hypothetical Banks Operating in South Africa*, IMF Working paper #2000-212

Figure 9: *Bank risk levels under typical and high credit risk*
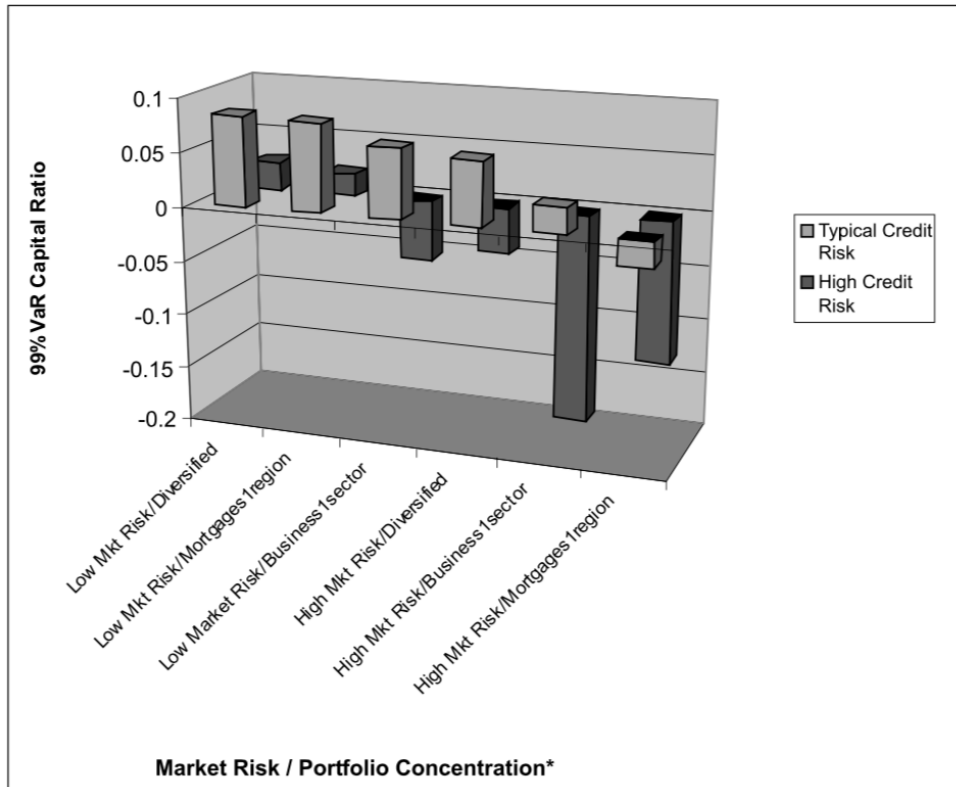
Source: Barnhill et Al. (2002)

They demonstrated also that asset and liability maturity gaps generally increase bank risk levels. However, because credit losses are positively correlated with interest rate increases, banks with high credit risk may reduce overall risk levels by holding liabilities with longer maturities than their assets. This occurs because rising net interest rate income resulting from rising interest rates offsets rising credit losses.

Figure 10: *Bank risk levels under zero, positive, and negative maturity gaps for a high market risk environment*
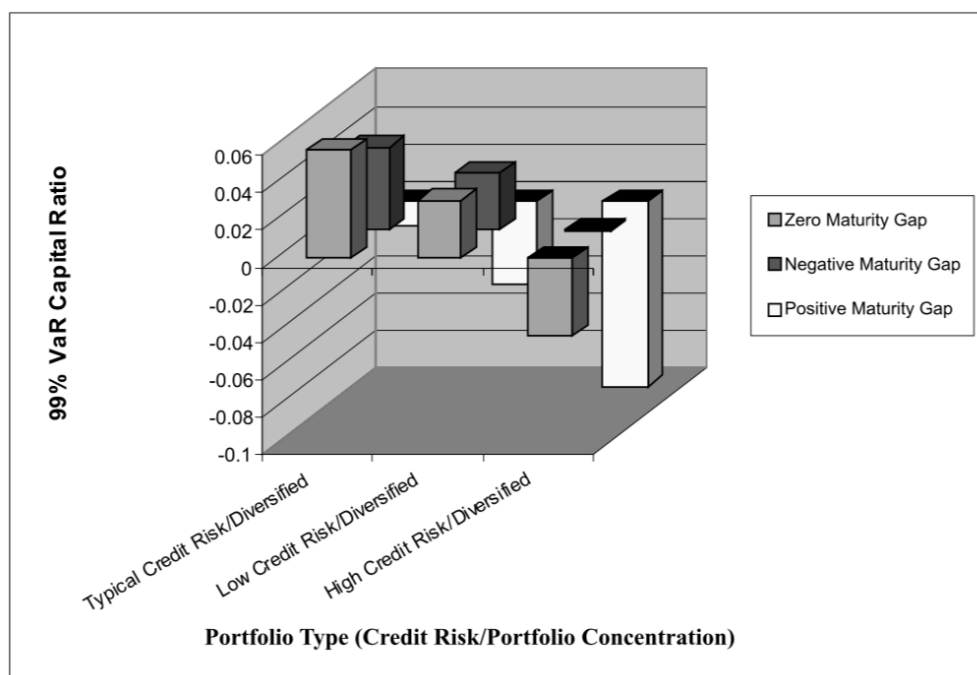
Source: Barnhill et Al. (2002)

The authors concluded that risk assessment methodologies which measure market and credit risk separately do not capture these various interactions and thus misestimate overall risk levels: single risks are not additive and need to be evaluated as a set of correlated risks.

Kupiec (2007)[46] proposes a single-factor, migration-style credit risk model, measuring the market risks of the non-defaulting credits in an asymptotic portfolio. The MTM value distribution are derived and then calibrated using historical data on the market yields of alternative credit quality instruments. The model incorporates correlations between portfolio default rates, credit migration probabilities and credit-quality specific market yields. The integrated exposure distribution of the model is used to examine capital allocations at various thresholds. These integrated capital allocations are compared to the separated assessments. The results show that capital allocations derived from a unified risk measure importantly alter the estimates of the minimum capital needed to achieve a given target solvency margin. The capital

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[46]Kupiec P. H., *An integrated structural model for portfolio market and credit risk,* Federal Deposits Insurance Corporation, 2007

amount could be larger or smaller than capital allocations estimated from compartmentalized risk measures. Regarding specifically the Basel II AIRB approach, the author argues that the results show that no further diversification benefit is needed for banking book positions since no market risk capital is required. Thus, Basel II AIRB capital requirements fall significantly short of the capital required by a unified risk measure.

Numerically speaking, the risk measure used in this study is the amount of capital that the unified and the compartmentalized capital approaches generate as the appropriate value to assure funding costs of a certain magnitude calibrated to historical funding rates for specific credit ratings. The hypothetical portfolios of interest are corporate loans with various rating categories represented in proportion to historical data. The author examines a wide variety of alternative separated approaches with which to calculate economic capital measures, ranging from three different alternative credit risk models to several methods for measuring market risk. Correspondingly, the range of inter-risk diversification index values is quite wide for the AAA- and BBB-rated portfolios, ranging from about 0,60 to almost 4,00. In summary, the author's capital calculations show that capital allocations derived from a unified market and credit risk measure can be larger or smaller than capital allocations that are estimated from aggregated compartmentalized risk measures.

IV.2: How interest rate risk may interact with credit risk

The studies discussed above examine the different risk implications of a unified risk measurement approach relative to a compartmentalized approach for specific portfolios, taking into consideration credit risk and market risk.

Several authors consider Interest Rate risk as the most important source of market risk for commercial banks. Thus, there are papers in which the interactions between credit and interest rate risk are specifically treated.

Drehmann et al. (2010)[47] propose a general framework to measure the riskiness of banks which are subject to correlated interest rate and credit shocks, incorporating the integrated impact of the two risks. They model a whole portfolio of banks, in which assets, liabilities, and off-

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[47]Drehmann M., Sorensen S., Stringa M., *The integrated impact of credit and interest rate risk on banks: an economic value and capital adequacy perspective*, Working paper no. 339, Bank of England, January 2008

balance-sheet items are included; they take into account the repricing structure of the portfolio, too. Their analysis is based on two conditions: one on economic value and one on capital adequacy. The former is based on risk-adjusted discounting cash flows and involve a framework which takes into account the repricing structure of the portfolio and capture the interactions between credit and interest rate risk. In fact, interest rate risk is the cause of the repricing mismatch: assets and liabilities are repriced to reflect changes in the free-risk yield curve; in addition, credit spreads could be adjusted to reflect changes in the banks' or borrowers' credit risk. The authors capture these two effects modeling the bank's net interest income.

The latter conditions imply to calculate whether the economic value of assets falls below the face value of liabilities, that is, in other words, whether a bank would be sufficiently well capitalized in the short to medium term. They propose a framework that captures the impact of credit risk on the whole portfolio, the interest rate risk stemming from the repricing mismatch between assets, liabilities and off balance sheet positions as well as basis and yield curve risk, and the interdependence between credit and interest rate risk.

The framework is a severe macro-stress scenario, which implies a sharp rise in the risk-free yield curve, in order to assess the exposure to credit and interest rate risk of a hypothetical bank.

The simulation made, confirm that interest rate risk and credit risk must be assessed jointly for the whole portfolio to gauge overall risk correctly. To capture the combined impact of these two risks, they employ a bottom-up approach linking macroeconomics factors to the risk-free yield curve and PDs of companies and households.
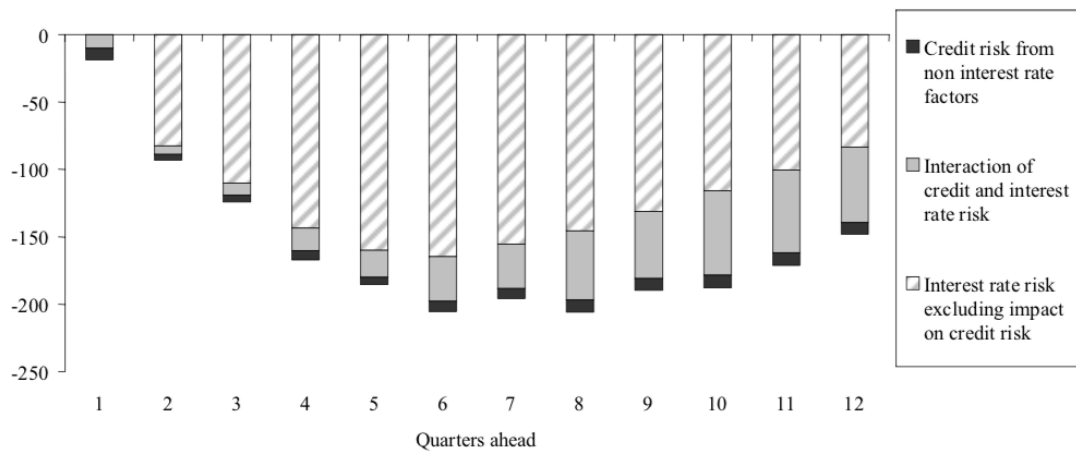
Figure 11: *Interaction of interest and credit risk - the impact on write-offs*
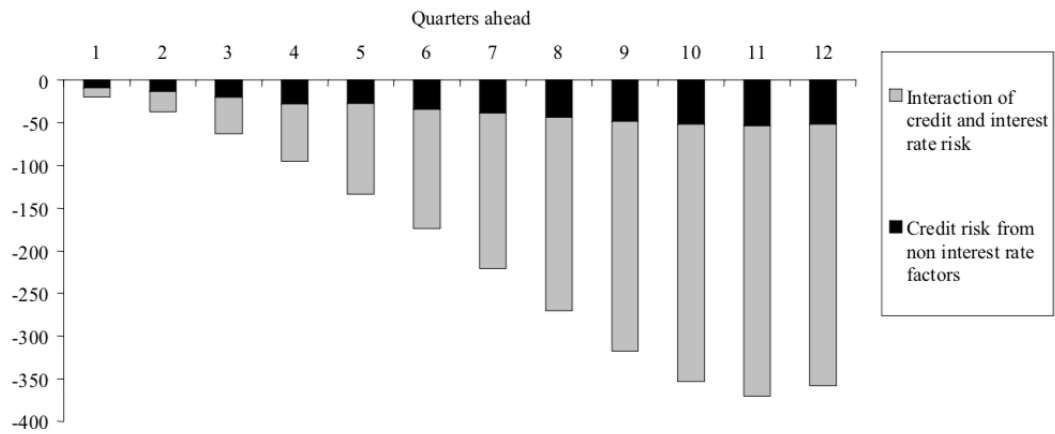
Figure 11: *Interaction of interest and credit risk - the impact on net interest income*
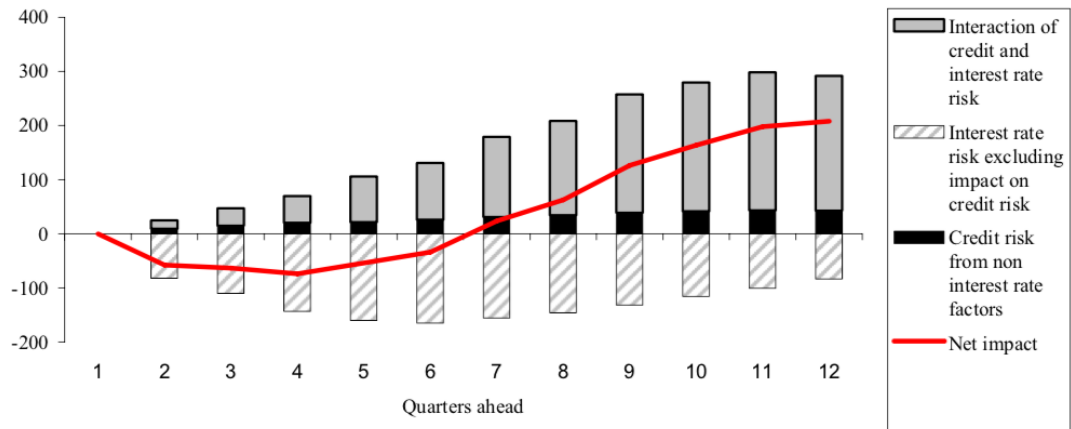
Figure 12: *Interaction of interest and credit risk - the impact on net profits*

Source: Drehman et Al. (2008)

As seen in the three figures above, in comparison to other macroeconomic factors, interest rates are the key drivers of the rise in credit risk. The increase in credit risk has two opposing effects on net interest income: higher write-offs decrease net interest income as borrowers default on coupon payments and the bank's exposures decline over time; on the other hand, there is a positive impact of credit risk on net interest income because, over time, banks adjust the credit spread on loans that are repriced. Looking at the overall impact on profits (Figure 12) it is evident that the rise in interest rates is the main cause of the fall in net profits as it drives both the squeeze in net margins and the rise in write-offs. It seems clear that credit and interest rate risk have to be assessed jointly and simultaneously for the whole portfolio. Were the bank focus only on the impact of credit risk on write-offs (Figure 10) without taking net interest income into account, it would overestimate the overall negative impact of the scenario on net profits by around 25%. The effects are not symmetric over time. In the first year, focusing only on write-offs would lead to underestimate the negative impact on net profits by over 50% as the decrease in net interest income (red line in Figure 11) is not taken into account. However, by the third year, the bank has repriced a large proportion of its assets leading to an increase in net interest income. Therefore, a bank focusing solely on write-offs would ignore this positive effect and overestimate the negative impact on net profits by nearly 100% in the third year. Conversely, were the bank to assess the impact of higher interest rates on its book by purely undertaking a sensitivity analysis based on its repricing mismatch (shaded area in Figure 11), it would underestimate the negative impact of the shock by around 30% over the three-year period.

Starting from this study, Alessandri and Drehmann (2010) develop an integrated economic capital model that jointly accounts for credit and interest rate risk in the banking book, i.e. where all exposures are not market - to - market but held to maturity, and interest rate risk arises due to volatility in the bank's net income. They explicitly examine repricing mismatches (and thus market and credit risks) that typically arise between a bank's assets and liabilities.

Figure 13: *A view of economic capital*

	Confidence Level		
	95%	99%	99.9%
EC_{CR}	1,348	3,412	7,493
EC_{NI}	424	611	797
EC_{RNI}	429	612	809
$EC_{CR}+ EC_{RNI}$	1,777	4,024	8,302
EC_{NP}	0	112	4,183
M_{EC}	100.00%	97.21%	49.62%
$E(NP)-VaR_{NP}$	1,372	3,516	7,586
M_2	22.76%	12.62%	8.62%

Note: in millions. EC_{CR} is the economic capital against credit risk; EC_{NI} is the economic capital against changes in net interest income excluding the impact of defaults on coupon payments; EC_{RNI} is the economic capital against changes in net interest income including the impact of defaults on coupon payments; EC_{NP} is the economic capital against changes in net profits. M_{EC} is the ratio of $[(EC_{CR}+ EC_{RNI})- EC_{NP}]$ over $(EC_{CR}+ EC_{RNI})$. $E(NP)$ are expected net profits. VaR_{NP} is the VaR of net profits at confidence interval (1-y) where y is the confidence level stated in the table. M_2 is the ratio of $[(EC_{CR}+ EC_{RNI}) – (E(NP)-VaR_{NP})]$ over $(EC_{CR}+ EC_{RNI})$.

Source: Alessandri P., Drehmann M. (2009)

As shown in Figure 13, for a hypothetical, average UK bank with exposures to only the UK and US, the difference between simple and aggregated economic capital, derived over a one-year horizon, is often significant: simple economic capital is an upper bound. The difference depends on various bank features, such as the granularity of assets, the funding structure or bank pricing behavior.

For credit and interest rate risk, they define unexpected losses and thus economic capital as the difference between VaR at the specified 99% confidence level and expected losses. Their measures of economic capital for just credit risk and just interest rate risk do not fully disentangle these risks as the credit risk measure incorporates the effects of higher interest rates on default probabilities and the latter the effect of higher credit risk on income. The key point is that the framework represents a plausible description of how current capital models for the banking book capture these risks. The authors examine the ratio of unified economic capital to the sum of the component measures at three VaR quantiles. For the 95% percentile of portfolio

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losses, unified capital measure is near zero, and thus the ratio is nearly zero as well. For the 99% percentile, the ratio is quite small at 0.03, but the ratio rises quickly to just over 50% for the 99.9% percentile. This result still suggests that the compartmentalized approach is more conservative than the unified approach.

The authors examine certain modifications of their assumptions, such as infinitely fine-grained portfolios to increase the correlation of portfolio credit risk with the macroeconomic factors, banking funding scenarios from all short- term debt that is frequently repriced to all long-term debt that is repriced only on a yearly basis, and find some interesting difference with the base case scenario. However, the lower integrated capital charge holds. On balance, these authors conclude that the bank's capital is mis-measured if risk interdependencies are ignored. In particular, the addition of economic capital for interest rate and credit risk derived separately provides an upper bound relative to the integrated capital level. Two key factors determine this outcome. First, the credit risk in this bank is largely idiosyncratic and thus less dependent on the macroeconomic environment; and second, bank assets that are frequently repriced lead to a reduction in bank risk. Given that these conditions may be viewed as special cases, the authors recommend that "*as a consequence, risk managers and regulators should work on the presumption that interactions between risk types may be such that the overall level of capital is higher than the sum of capital derived from risks independently.*" [48]

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[48]Alessandri P., Drehmann M., *An economic capital model integrating credit and interest rate risk in the banking book*, ECB Working Paper no. 1041, April 2009

CONCLUSIONS

In my thesis, I primarily explicated what the capital requirements are, taking into account the changes given by the Basel Accords. Banks, and financial institutions in general, should correctly identify and measure risks, in order to determine the economic capital in a coherent way.

The main three risks banks face are credit risk, market risk, and operational risk.

I discussed on risk measures and, then, on different approaches to calculate and aggregate risks, making a distinction between top-down and bottom-up methods.

As regard to risk measures, in literature they are usually defined as random variable, i.e. portfolio losses or returns. I focused on VaR as a standard measure used in financial risk management, due to its conceptual simplicity, computational facility and ready applicability. However, VaR has some problems: it measures only quantiles of losses and does not regard any loss beyond the VaR level. VaR is criticized for not being a coherent risk measure because it may violate the sub-additivity criterion, in contrast with the idea that diversification could reduce risk.

As regard to the aggregation of risks, I examined different approaches, dividing them into two macro-groups: top–down Vs bottom–up methods.

Within Top-down approaches, the calculation of economic capital is first computed for individual risk types, and then aggregated to obtain the total economic capital for the bank. These approaches assume that market and credit risk are separable and look at the institution as a whole. The approaches used are simple summation, Var-Covar and Copulas. The simple summation approach seems to be too much conservative, because it does not consider the inter-risk correlation and does not matters potential diversification benefits. Thus, it is often considered as an upper bound to the true economic capital, tending to overestimate true risk. As regard to Var-Cover method, it is the simplest approach that consider correlations between risks, but it assumes that all risks are jointly normally distributes, and this is a restrictive assumption. In many examples from the literature, VaR ratios based on Var-Covar approaches seem to be the smallest, while those based on simple summation approaches the largest. As regard to Copula approach, it allows a decomposition of a joint distribution into its marginal ones and its copulas. Combining marginal distributions into a joint one, once defined a specific

copula, the approach allows for a separate modeling of the marginal distributions and of the dependence structure. Copula functions are much more flexible than simple summation approach and Var-Cover approach, because they use the entire credit, market and operational risk distributions. A mixture of *t*- Copula and Gumbel Copula seems to be the best choice to have a coherent VaR ratio.

As mentioned above, top - down approaches assume that risks are separable. However, it is usually not possible to divide the overall portfolio of a bank into sub-portfolios purely consisting of market, credit and operational risk, i.e. a sub-portfolio of risk factors. It is therefore incorrect to think of the banking book as a sub-portfolio of the overall bank portfolio for which only credit risk is relevant, and, at the same time, it is incorrect to view the trading boos as another sub-portfolio related solely on market risk. In fact, I discussed on how interest rate risk, for example, could affect both credit and market risk.

Paper using the top-down approach assume the splitting up of the bank portfolio into sub-portfolios, according to different types of risks, without considering that there could be compounding effects between the banking and trading book, between credit, market and operational risk.

Using Bottom - up approaches, it is possible to model and measure simultaneously different risk types and there is no need for a later aggregation.

Comparing results obtained using top-down and bottom-up approaches, many authors find out that separate analysis could underestimate risk, and does not capture interactions between market and credit risk, such as interest rate risk and exchange rate risk.

Concluding, we can say that bank's capital is mis-measured if risk interdependencies are ignored, and that an aggregated measure is necessary.

REFERENCES

Aas Kjersti, Dimakos Xeni K., Øksendal Anders, *Risk Capital Aggregation*, Norsk Regnesentral, Note n° SAMBA/40/05, December 2005;

Acerbi Carlo, Tasche Dirk, *On the coherence of expected shortfall*, Journal of Banking & Finance, n° 26, 2002;

Alessandri Piergiorgio, Drehmann Mathias, *An economic capital model integrating credit and interest rate risk in the banking book,* Journal of Banking & Finance, n° 34, 2010;

Alexander Carol, Pézier Jaques, *On the Aggregation of Market and Credit Risks*, ISMA Centre Discussion Paper in Finance 2003-2013, The University of Reading, October 1st, 2003;

Amihud Yakov, *Illiquidity and stock returns: cross-section and time-series effects*, Journal of Financial Markets, n° 5, 2002;

Antão Paula, Lacerda Ana, *An assessment of capital requirements under Basel II: the Portuguese case,* Financial Stability Report, Banco de Portugal, 2008;

Antão Paula, Lacerda Ana, *Capital requirements under the credit risk-based framework*, Journal of Banking & Finance, n° 35, 2011;

Arbenz Philipp, Hummel Christoph, Mainik Georg, *Copula based hierarchical risk aggregation through sample reordering*, Insurance: Mathematics and Economics, 51, 2012;

Bakiciol Tamer, Cojocaru-Durand Nicolas, Lu Dongxu, *Basel II,* Slides, Princeton;

Barnhill T., Maxwell W., *Modelling correlated interest rate, exchange rate, and credit risk in fixed income portfolios*, Journal of banking and finance, vol. 26, pp. 347 - 374, 2002;

Barnhill T. B., Panagiotis P., Schumacher L., *Measuring Integrated Market and Credit Risk in Bank Portfolios: An Application to a set of Hypothetical Banks Operating in South Africa*, IMF Working paper #2000-212, 2000;

Basel Committee on Banking Supervision, *Developments in Modelling Risk Aggregation*, Bank for International Settlements, October 2010;

Basel Committee on Banking Supervision, *Findings on the interactions of market and credit risk*, Working paper n° 16, Bank for International Settlements, May 2009;

Basel Committee on Banking Supervision, *International Convergence of Capital Measurement and Capital Standards*, Bank for International Settlements, June 2006;

Basel Committee on Banking Supervision, *Messages from the academic literature on risk measurement for the trading book*, Working Paper n° 19, Bank for International Settlements, January 31st, 2011;

Basel Committee on Banking Supervision, *Principles for effective risk data aggregation and risk reporting*, Bank for International Settlements, January 2013;

Basel Committee on Banking Supervision, *Progress in adopting the principles for effective risk data aggregation and risk reporting*, Bank for International Settlements, December 2013;

Basel Committee on Banking Supervision, *Trends in risk integration and aggregation*, Bank for International Settlements, August 2003;

Basel III: Issues and Implications, kpmg.com;

Bluhm Christian, Overbeck Ludger, Wagner Cristoph, *An introduction to credit risk modeling*, Chapman & Hall/CRC Financial Mathematics Series, 2003;

Böcker Klaus, Hillebrand Martin, *Interaction of market and credit risk: an analysis of inter-risk correlation and risk aggregation*, Discussion Paper Series 2: Banking and Financial Studies n° 11, 2008;

Bouyé Eric, Durrleman Valdo, Nikeghbali Ashkan, Riboulet Gaël, Roncalli Thierry, *Copulas for Finance A Reading Guide and Some Applications*, 2000;

Breuer T., Jandačka M., Rheinberger K., Summer M., *Compounding effcts between Market and Credit Risk: the case of variable rate loans*, London, 2008;

Breuer T., Jandačka M., Rheinberger K., *Does adding up of economic capital for market and credit risk amount to conservative risk assessment?*, Journal of Banking and Finance, April 2010;

Breuer Thomas, Jandačka Martin, Rheinberger Klaus, Summer Martin, *Regulatory capital for market and credit risk interaction: is current regulation always conservative?*, Discussion Paper Series 2: Banking and Financial Studies, n° 14, 2008;

Brockmann Michael, Kalkbrener Michael, *On the Aggregation of Risk*, Risk Analytics & Instruments, Deutsche Bank, Frankfurt;

Brodeur André, Pergler Martin, *Top-down ERM: A Pragmatic Approach to Managing Risk from the C-Suite*, McKinsey Working Papers on Risk, n°22, August 2010;

Cech Christian, *Copula-based top-down approaches in financial risk aggregation*, Working Paper Series by the University of Applied Sciences of bfi Vienna, n° 32, December 2006;

Chapman Chris, Ward Stephen, *Project Risk Management*, *Processes, Techniques and Insights*, John Wiley & Sons, Ltd., 2003;

Chavez-Demoulin V., Embrechts P., Nešlehová J., *Quantitative models for operational risk: Extremes, dependence and aggregation*, Journal of Banking & Finance, n° 30, 2006;

Chen Yi-Hsuan, Tu Anthony H., *Estimating hedged portfolio value-at-risk using the conditional copula: An illustration of model risk*, International Review of Economics and Finance, n° 27, 2013;

Cherubini Umberto, Luciano Elisa, Vecchiato Walter, *Copula Methods in Finance*, Willey Finance Series, 2004;

Coqueret Guillaume, *Second order risk aggregation with the Bernstein copula*, Insurance: Mathematics and Economics, n°58, 2014;

Corrigan Joshua, De Decker Jethro, Hoshino Takanori, Van Delft Lotte, Verheugen Henny, *Aggregation of Risks and Allocation of Capital,* Milliman Research Report, September 2009;

Corrigan Joshua, Lurasch Paola, *Operational risk modeling framework*, Milliman Research Report, February 2013;

Cruz M., Coleman R., Salkin G., *Modeling and measuring operational risk*, Imperial College, London 1998;

De Raaji Gabriella, Raunig Burkhard, *A Comparison of Value at Risk Approaches and Their Implications for Regulators*, Focus on Austria, n° 4, 1998;

Demarta Stefano, McNeil Alexander J., *The t Copula and Related Copulas*, Department of Mathematics Federal Institute of Technology Zurich, May 2004;

DimakosXeni K., Aas Kjersti, *Integrated Risk Modeling*, NR Report, December 18th, 2003;

Drehmann Mathias, Sorensen Steffen, Stringa Marco, *The integrated impact of credit and interest rate risk on banks: An economic value and capital adequacy perspective,* Bank of England, Working Paper n° 339, January 2008;

Drehmann Mathias, Sorensen Steffen, Stringa Marco, *The integrated impact of credit and interest rate risk on banks: A dynamic framework and stress testing application*, Journal of Banking & Finance, n°34, 2010;

El Arif Fatima Zahra, Hinti Said, *Application of the approaches Top-down and Bottom-up for the construction of a Risk Mapping of an insurance company*, IOSR Journal of Business and Management, Volume 16, Issue 3. Ver. I, PP 60-67, March 2014;

Elsinger Helmut, Lehar Alfred, Summer Martin, *Risk Assessment for Banking Systems* (2006);

Embrechts Paul, FurrerHansjörg, Kaufmann Roger, *Quantifying regulatory capital for operational risk;*

Faccincani Lorenzo, *Il capitale e la redittività nelle banche: i possibili impatti di Basilea 3*;

Forum Joint, *Risk managenent practices and regulatory capital, cross sectoral comparison,* Bank for International Settlements, Basel (2010);

Frachot A., Georges P., Roncalli T., *Loss Distribution Approach for operational risk*, April 25th, 2001;

Gallati Reto R., *Risk management and capital adequacy*, McGraw-Hill, 2003;

Giacometti Rossella, RachevSvetlozar, Chernobai Anna, Bertocchi Marida, *Aggregation Issues in Operational Risk*;

Grundke Peter, *Computational aspects of integrated market and credit portfolio models*, OR Spectrum, n° 29, 2007;

Grundke Peter, *Importance sampling for integrated market and credit portfolio models*, European Journal of Operational Research, n° 194, 2009;

Grundke Peter, *Top-down approaches for integrated risk management: How accurate are they?*, European Journal of Operational Research, n° 203, 2010;

Grundke Peter, *Top-Down versus Bottom-Up Approaches in Risk Management, 2008*;

Gual Jordi, *Capital requirements under Basel III and their impact on the banking industry*, "la Caixa" economic papers, n°07, December 2011;

Hull John C., *Risk Management and Financial Institutions*;

Inanoglua Hulusi , Jacobs Michael, Jr., *Models for Risk Aggregation and Sensitivity Analysis: an Application to Bank Economic Capital*, Journal of Risk and Financial Management, n°2, 2009;

Jorion Philippe, *Value at Risk: The New Benchmark for Managing Financial Risk*, McGraw-Hill, 2001;

Kretzschmar Gavin, McNeil Alexander J., Kirchner Axel, *Integrated models of capital adequacy – Why banks are undercapitalized*, Journal of Banking & Finance, n° 34, 2010;

Kruz Marcelo, Coleman Rodney, Salkin Gerry, *Modeling and measuring operational risk*, Fall 1998;

Kupiec P. H., *An integrated structural model for portfolio market and credit risk,* Federal Deposits Insurance Corporation, 2007;

Kuritzkes Andrew, Schuermann Til, Weiner Scott M., *Risk Measurement, Risk Management and Capital Adequacy in Financial Conglomerates*, Financial Institutions Center, 2002;

Li Jianping, Feng Jichuang, Wu Dengsheng, Lee Cheng-Few, *Integration of Credit, Market and Operational Risk: A Comparative Analysis of Copula and Variance/Covariance Approach*

Maissin Jean-Pierre, Vander Elst Ronan, Saint Ghislain Loic, *Principles for effective risk data aggregation and risk reporting*, Basel III;

Nelson RB, *An introduction to copulas*, Springer, New York, 1999;

Pesaran H., Shin Y., Smith R., *Structural analysis of vector error correction models with exogenous i(1) variable,* Journal of Econometrics, 97:293-343, 2000;

Peziér Jacques, *Application-Based Financial Risk Aggregation Methods,* ISMA Centre Discussion Paper in Finance 2003-2011, The University of Reading, September 10th, 2003;

Protiviti, *What is the path toward effective risk data aggregation and risk reporting?*, 2013.

Pyle David H., *Bank Risk Management: Theory*, Research Program in Finance, Working Paper RPF-272, July 1997;

Rhys David,*Safety Risk Aggregation: The Bigger Picture*, Partner Safety Assurance Services Ltd.;

Rosemberg Joshua V., Schuermann Til, *A general approach to integrated risk management with skewed, fat-tailed risks,* Journal of Financial Economics, n° 79, 2006;

Santomero Anthony M., *Commercial Bank Risk Management: An Analysis of the Process*, Journal of Financial Services Research, 1997;

Sempi Carlo, *An introduction to Copulas*, Slides, The 33rd Finnish Summer School on Probability Theory and Statistics, June 6th – 10th, 2011;

Shevchenko P. V., *Loss Distribution Approach*, Modelling Operational Risk Using Bayesian Inference, 2011;

Shimko David C., Went Peter, *The International Certificate in Banking Risk and Regulation (ICBRR),* GARP (Global Association of Risk Professionals);

Stan Uryasev Ursula A. Theiler Gaia Serraino, *Risk-return optimization with different riskaggregation strategies*, The Journal of Risk Finance, Vol. 11 Iss 2 pp. 129 – 146, 2010;

Van Malleghem Jonathan, Blard Frederic, *Principles for effective Risk Data Aggregation and Risk Reporting*, Avantage Reply, June 2014;

Wilson Paul W., *Detecting Influential Observation in Data Envelopment Analysis*, The Journal of Productivity Analysis, n° 6, 1995;

Yang Yimin, *Practical Methods for Aggregating Bank's Economic Capital*, Paper submitted for the: Enterprise Risk Management Symposium, Chicago, April 22nd – 24th, 2013.

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