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Controllo Wave-Based di Modelli Flessibili Multi-GdL per Razzi Aerospaziali

Wave-Based Control of Multi-DoF Flexible Models for Aerospace Launchers

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MASTER THESIS

Wave-Based Control of Multi-DoF Flexible Models for Aerospace Launchers

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ABSTRACT

In the aerospace structures, as well as robotics, strategies to reduce weight are sought. Lighter structures require less power to move and, if well controlled, allow to have better performances. But lighter weight means more flexibility, considering the same material and the same geometrical dimensions. Vibrations could arise while trying to move the structure, making motion control challenge much more complicated.

This thesis affords vibrational problems during motion of a multi degree of freedom flexible system, with particular focus on long-arm manipulators and aerospace launchers, considering, for each case, its own physical limitation and external environment. Wave Based Control strategy is applied to both of them, with some modifications in order to satisfy the required specifics for each case of study. Precision and high dynamic are required for the robotic arm, with a specific tuning of the controller coefficients. Robustness to system parameters changing and disturbance rejection, instead, is the priority for launchers control system.

WBC is a relatively new approach to the problem of controlling flexible systems. It is based on the theory of mechanical waves propagating in elastic mechanical structures and provides an elegant and simple solution to combine motion control with active vibration damping. It allows to suppress vibrations, while moving the structure to the target position. Wave Based concepts have been already tested in many applications, for example 1-D and 2-D mass-spring array and light aerospace structures like satellites.

In the current project, 2-D lumped elastic models are implemented to make numerical simulations about the two case of study and test WBC applicability to both of them. WBC proves its capability to deal

with this kind of flexible models and their specific actuators, providing good performance and fulfilling required specifications for each situation, after some improvements. In specific terms, it will be modified to deal with long movements on the plane, cross-coupling of motion, actuators physical limits and in order to manage 3 DoF or 2 DoF actuator, depending on the case. Modifications concern also the possibility to cope well with simulated external disturbances of different kind and fluid sloshing.

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INTRODUCTION

1.1 STUDY AND CONTROL VIBRATIONS IN A FLEXIBLE ROCKET STRUCTURE

Figure 1: Arianespace Vega rocket for the European Space Agency

In the last decades, needs of energy efficiency together with higher dynamic performances have grown. In several fields, from robotics to spacecraft, frames and components have become lighter, preserving or even increasing their dimension, with the same target on mind: the less the weight is, the less the power to achieve the identical performances is. Unfortunately this thought is not completely true if one consider the same dimensions and similar materials properties. In fact, this mass reduction makes the structure more flexible. This requires a more careful motion control than before, which often reduce dynamic performance, in terms of maximum allowable accelerations, in order to prevent exciting vibrations in the structure.

In some cases flexibility is sought, for example for safety reason, to prevent damage when a robotic arm impacts on something or on humans. Another example is the robots designed for surgery, to allow a surgeon to move within organs to reach remote areas inside the body. [1, 12]

In the aerospace field the need for reducing weight of the vehicles is present since the beginning. Lighter structures require less power to move them, are easy to manage and, using the same motors, allow to have a bigger payload. But, again, using the same material and with the same geometrical dimensions, lighter weight means more flexible structure, which could bring to oscillation during its movements.

Vibrations could be increased also by other effect, like and external impact, for example due to an unwanted collision between two stages when they detach each other. Or the sloshing of fluid fuel inside the tank of the launcher.

The ultimate target of this thesis is to analyse vibrational problems in a flexible launcher structure during its motion, considering the problems above and other typical launcher aspects, like thruster limitation in dynamic and force generations, required trajectories, external disturbances of different kind, errors on measuring sensors and other similar aspects. In particular, the difference between a traditional actuator like an electric motor and the thrusters, will required to modify the control system, designing some specific modifications.

1.1.1 *Challenge in dealing with flexible structure*

The more the structure is flexible, the more oscillations and deflections are important to be taken into account. Vibrations can cause instability of the system, if they are not well controlled, requiring the decreasing of the dynamic in order to avoid critical oscillations. In structures which require high movement precision, like robots, vibrations must be avoided or, at least, dampened very fast.

When it is required to deal with vibrations in flexible structure, whether it is to avoid, damp or produce them, three important tasks are to be taken into account:

- to build a model for the flexible system which could simulate main vibrations on it;
- to design a specific controller to deal with these vibrations, to maintain or to damp them;
- to test the designed controller on the model.

1.1.2 *Modelling flexible structure*

The model of the flexible system has two functions: the first one is to allow to study vibrations in a virtual system, on a numerical simulation for example. The second one is to test the effect of the controller law action on the virtual system.

The model is not often easy to get, because some parameters could be not well known and the structure could be very complicate to build an accurate model of it. Engineering approach often tries to decompose a complicate case of study in different parts easier to study. This means that, in order to study vibrations, a complete model of the flexible structure is not required, but can be replaced with a simpler model. It can simulate the same vibrational modes of the structure, without trying to get an exact numerical reproduction of it, which could contemplate other problems to study separately.

This assumption does not mean that nowadays it is not possible to get such a realistic model: a lot of simulation software allow to obtain high level results in modelling. However, it could be not so easy to deal with such a high detailed models, like FEM, while designing and testing the controller. Therefore a simpler model is sought, i.e. a model whereby it is easy to understand relation between its parameters and its dynamic behaviour. This allows, for example, to make the first vibrational mode 2 time higher than before, just changing one coefficient of the model and test how the same controller deals with both of them.

Lumped models are the easiest way to model flexible mechanical systems. The inertia of the structure is split into point rigid masses, connected by massless springs. Springs represent elasticity of the structure. Setting the number of masses and springs and their values allows to establish the number of degree of freedom and oscillatory modes of the model. Damping effects can be included as well, by adding viscous friction effect where each spring acts. The dynamics of such a model is described by Newton's Law, applied to each mass of the system. [12]

Lumped models fit very well to system which can be naturally divided into discrete elements, like robot arm with flexible joint and some space structure like solar panel arrays. When the system flexibility is more distributed, like a light single piece robot arm, or a single light stage of a launcher, lumped model makes an important simplification of the overall dynamics, but it still provides an adequate model for reproducing main vibrational modes, particularly useful to design control law. [12]

For these reasons a simple masses and springs lumped system is chosen to model rocket structure. The develop of the model and the related control system start from a beam two-dimensional case of study, analysed by Hossein Habibi during his PhD, under the supervision of Doctor William J. O'Connor, and described in his PhD thesis [1, chap. 3].

In this thesis, Habibi's beam model is modified and improved, passing through a robot arm case of study and develop a specific control system for such a specific application, which needs high dynamic and path following precision. In a second stage, the robot arm lumped system is modified and adapted to model a flexible launcher single stage, basically changing parameters of masses and springs.

This capability of the lumped model to adapt to both the robot arm and the launcher shows the power of such a model to be able to be applied to and to describe different systems, but, simultaneously the model shows its weak points to be too poor in describe the details. However, the model reveals to be able to reproduce main significant aspects about their vibrations, so it is useful enough to test the controller action for both the systems.

1.1.3 *Control vibrations in a flexible structure*

There are several possible control systems to be apply to a flexible mechanical structure to control it. However, in a book published by Sandia National Laboratories (USA), summarising their work over decades on this problem, the authors make the following general remarks:

> 'to date a general solution to the control problem [of flexible systems] has yet to be found. One important reason is that computationally efficient (real-time) mathematical methods do not exist for solving the extremely complex sets of partial differential equations and incorporating the associated boundary conditions that most accurately model flexible structures.' (Robinett et al 2001, p.165)

Another book, completely dedicated to the same problem, makes the same considerations:

> 'Many issues are not resolved yet, and simple, effective and reliable controls of flexible manipulators still remain an open quest.' (Wang and Gao, 2003).

In 1998, a research carried out by O'Connor and Lang put the first stone to the new Wave-Based Control (WBC) technique, describing a new approach in which the actuator is considered to launch a wave into the system as well as to absorb the (previously launched) wave coming back from this system. Aspects of these two actions happen simultaneously. To justify their approach, they also presented a new way to model a uniform lumped flexible system based on a loop of wave transfer functions (WTFs). [1]

WBC provides a generic approach to the difficult problem of flexible mechanical structures control, giving an answer to the *open quest*. It considers actuator motion as launching a mechanical wave into the flexible system while absorbing the return one. The launching and absorbing proceed simultaneously. This simple, intuitive idea leads to robust, generic, highly efficient, precise, adaptable controllers, allowing rapid repositioning of the system and suppressing the vibrations, using only sensors collocated at the actuator-system interface. These wave-based ideas have already been shown to work on simple systems such as mass-spring strings, systems of Euler-Bernoulli beams, 2-D mass-spring arrays, and flexible aerospace structures. [10]. This thesis applies and test Wave-Based Control theory to aerospace launchers. In Chapter 2 theoretical concept about WBC will be explained.

1.2 MODELLING AND SIMULATION INSTRUMENTS

The 2D lumped models and the control systems for both robot arm and rocket single stage frame are developed and simulated using the commercial software MATLAB and Simulink. In Appendix A and Appendix B there are the most important concepts about the implementation of the numerical models and the control systems tested on it.

1.3 CHAPTERS AND TOPICS

Here there is a brief summary of the topics afforded in this thesis, divided by chapters.

- In Chapter 2 it is explained the main and the most important instrument used to control both motion and vibrations together in a flexible structure. It is the *Wave Based Control*,
- 2D lumped masses and springs model was already implemented by Hossein Habibi. It is explained in Chapter 3. Habibi developed the model for lattice and beam model. To model rocket structure, it needs modification and improvements.
- In Chapter 4 Habibi model is improved and adapted to describe a flexible robot arm. These modifications are functional to model rocket structure. In particular because the modifications in this chapter make the model more general and useful for many particular cases.
- Finally in Chapter 5 it is explained rocket structure model and control. Simulations have been done in a lot of configurations, with one or two thrusters and different environment parameters and adding sloshing of the fuel.

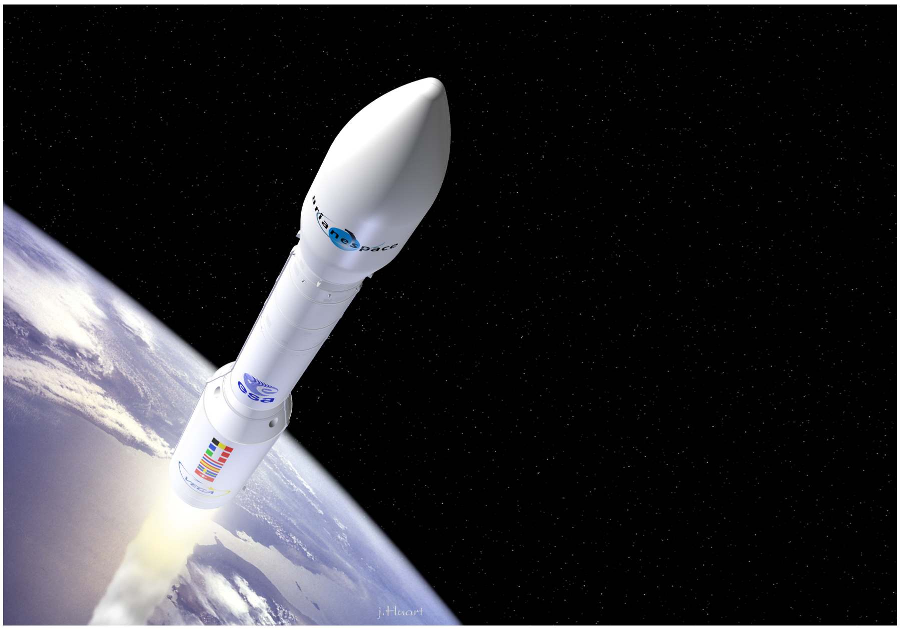


Figure 2: Arianespace Vega rocket for the European Space Agency

2

WAVE BASE CONTROL THEORY

2.1 INTRODUCTION TO WBC

The first publication about Wave Based Control was by O'Connor and Lang in 1998 [8]. The flexible structure dynamics is modelled as the superposition of two waves travelling in opposite directions, namely an outgoing and a returning wave.

For the simplest case of a system with a single actuator at one of its side, the wave going out from the actuator, by its motion, into the system, is called *launching wave*, while the superposed wave coming back from the system to the actuator is called *returning wave*.

The control law is achieved by launching a specified displacement wave into the system, while simultaneously absorbing the returning waves by suitable motion of the actuator. The absorption of the returning wave actively damps the vibrations in the flexible system. To quantify the waves in order to calculate the launching one, different methods can be used. One of these methods will be explained in Section 2.2.1.

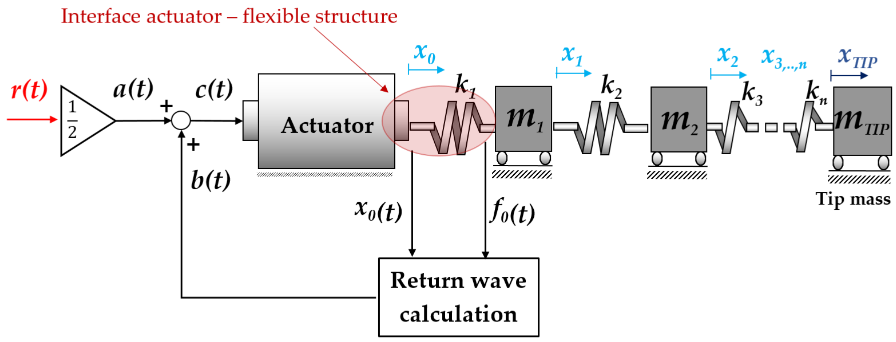


Figure 3: Wave Based Control applied to a one-dimensional lumped array

The originality of such a control system is the combination of position control and active vibration damping in a single actuator, by simultaneously launching and absorbing motion in a controlled way. The control law removes vibrations and accurately repositions the system. For rest-to-rest motion to a target displacement, the reference for the actuator is set to half the target displacement. During the absorption of the returning wave, considering no external disturbing forces, the actuator will move through an additional net displacement equal to that of the half reference, thus bringing the system to rest at the target reference.

This technique has many advantages, solving the difficulties associated with other approaches. The most important one is that the controller does not require a good model of the system, therefore it can work even if system parameters are not perfectly known or if they change during motion. It will be seen that controller has also a good disturbance rejection and robustness in general, even when it becomes more complicate or when considering actuators with bandwidth limitation, saturation and more than one degree of freedom. It does not need sensors throughout the flexible system, but it can have all the sensors where the actuator meets the flexible system. It is inherently fast, and can do rest-to-rest manoeuvres in times which are close to the theoretical minimum time of time-optimal control. All these performance are justified in published research papers. Some of them will be confirmed in this thesis, for the two cases of study which will be considered.

The idea was applied to a gantry crane problem at first [3, 4]. Accurate rest-to-rest movement of the crane load with a speed limited gantry trolley was achieved in a finite time, with robustness to unknown system parameters. Further researches about WBC have been published in O'Connor (2006), O'Connor (2007) [6], McKeown (2009) [2], O'Connor and Fumagalli (2009) [9], O'Connor (2011) [7]. Much of this work was tested using computer simulation and numerical models, but experimental verification was also reported. For example, the ideas were applied to a very light and flexible arm, driven by a DC motor, to re-position a tip mass, supported on an almost frictionless air table, to a target position in an horizontal plane (O'Connor et al 2009) [1, 10, 12].

2.2 THEORETICAL CONCEPTS ABOUT WBC

Wave-Based Control (WBC) of under-actuated flexible systems consists of a directly-controlled actuator which is indirectly controlling an attached flexible system. To move the system through a target displacement from rest-to-rest, the requested motion input to the actuator, $c(t)$, is set to be the sum of a *launch* displacement $a(t)$ of half the reference displacement, $\frac{1}{2}r(t)$, and a measured *return* displacement, $b(t)$. The returning motion component $b(t)$ provides active vibration damping, while also causing a net displacement which, in the absence of external disturbances, equals the second half of the target displacement, $\frac{1}{2}r(\infty)$. Thus active vibration damping and accurate, rest-to-rest position control are combined in a single actuator movement and they reach their completion at the same time, which is necessary to avoid further disturbances. The reference displacement, $r(t)$, can have any desired shape, including step, ramp, or s-shaped (double parabola), provided it settles at the target rest displacement, $r(\infty)$.

The returning wave $b(t)$ is determined from two interface measurements, here taken as the actuator position, $x(t)$, and the force, $f(t)$, which the actuator applies to the flexible system. It provides what could be described as real-time system identification, as it gives the actuator control system the required information about the system dynamics. As these dynamics change (e.g. change of mass, or system geometry) the returning wave changes, while the control strategy and control law remain unchanged. This is partly the reason why the control system is very robust. [10]

The actuator could have its own sub-controller or could be directly controlled. In every case, it is important to take into account that it has its own dynamic limitations, while thinking to a real system. It is shown in the literature that this dynamic does not affect WBC performance so much, so, in a first stage, it is possible to explain WBC theory considering the actuator to be ideal. In other words, the actuator is considered to be able to do everything WBC ask for, without any delay or loss in gain, in terms of force generation or position settling, depending on the application. If the actuator is ideal, then $x_0(t) = c(t)$. Assuming to have an ideal actuator is not a requirement of WBC, but it can help in giving a simpler explanation of how WBC works [1]. Under this assumption:

$$c(t) = x_0(t) = \frac{1}{2}r(t) + b(t) \tag{1}$$

2.2.1 *Calculation of returning wave* $b(t)$

The returning wave calculation is the core part of WBC, since controller law depends on it. Indeed, once the reference is chosen, $a(t)$ is fixed, so the launch wave is function of the returning wave and the damping effect depends just on it. There are different ways to determine $b(t)$, which choice is a matter of convenience. In this thesis, just the impedance method was considered, revealing very easy to manage situations whereby flexible system becomes more complex and Wave-Base Control law requires modification to work well.

2.2.2 *Impedance WBC*

The simplest way to determine *returning* wave $b(t)$ is using force-impedance method. It is based on a time integral of the interface force and on a fixed value of mechanical or wave impedance. The returning wave is again determined from two independent interface measurements: the actuator position, x(t), and the force, f(t), which the actuator applies to the flexible system.

$$
\begin{aligned}
c(t) &= a(t) + b(t); && (2)\\
a(t) &= \frac{1}{2} r(t); && (3)\\
b(t) &= \frac{1}{2}\left[x_0(t) - \frac{1}{Z}\int f_0(t)dt\right] && (4)
\end{aligned}
$$

The mechanical impedance term Z is a constant. The full justification for this control law will not be considered here. Nevertheless two important points will be noted. By differentiating c(t) = a(t) + b(t) with respect to time, it can be seen that the b(t) component of the actuator motion is providing a viscous damping effect for the returning motion, that is, a velocity proportional to the force in the returning wave, with damping coefficient equal to Z. In other words, it causes the actuator to provide active vibration damping with an appropriate damping coefficient. Secondly, for rest-to-rest manoeuvres, when the initial and final moments are zero, the force integral in b will return to zero, so the final position of x must equal the final value of r. This effect is independent of the value of Z, and its value is not critical to the controller behaviour. For a lumped system is $Z = \sqrt{km}$, where k is the spring stiffness at the interface and m the first mass element.

For rotational motion, the variables $x(t)$, $a(t)$ and $b(t)$ will correspond to angular displacements and $f(t)$ to torque. The main point to note is that moments of forces should not be taken about the actuator rotation axis if this axis moves, but either about a fixed point in the space (such as the initial position) or about the system mass centre CM (which would then need to be calculated and updated continually). This aspect will be explained and evaluated in Section 4.1.4.

This control strategy has been tested on flexible systems of many kinds, sizes and flexibilities, with different kinds of motion, including 1-D and 2-D translation, rotation, and simultaneous translation and rotation. In the latter case, the actuator has three degrees of freedom, each of which can be controlled by a WBC control loop, with all three acting in parallel, simultaneously [10]. These considerations will be explained in depth in Chapter 3.

Considering Laplace transformations, useful in the next sections, it is obtained:

$$C(s) = A(s) + B(s); \tag{5}$$

$$A(s) = \frac{1}{2}R(s); \tag{6}$$

$$B(s) = \frac{1}{2}\left[X_0(s) - \frac{1}{Z}\frac{1}{s}F_0(s)\right] \tag{7}$$

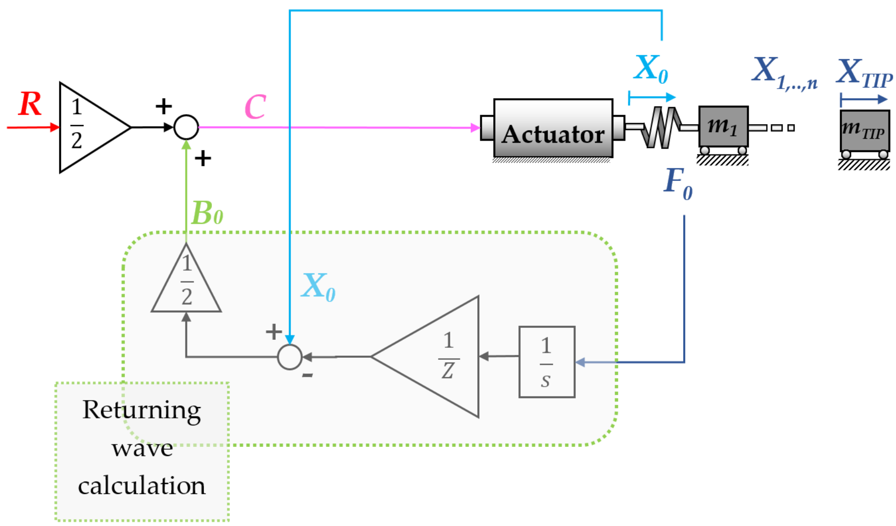


Figure 4: Impedance WBC of type 2.

$x_0(t)$ is the physic quantity (position or force) produced by the actuator, i.e. the *zero* going wave in the system, between actuator and the first mass, passing throw the first spring which connects the actuator to that mass. If it is a position, it is basically the position reached by the actuator. If the actuator is ideal, $c(t) = x_0(t)$. Under this hypothesis, which could be considered valid also if actuator is not ideal, accepting some approximations, it is possible to write:

$$x_0(t) = c(t) = \frac{1}{2}r(t) + \frac{1}{2}x_0(t) - \frac{1}{2}\frac{1}{Z}\int f_0(t)dt \quad (8)$$

$$x_0(t) = r(t) - \frac{1}{Z}\int f_0(t)dt \quad (9)$$

2.2.3 *Different versions of WBC*

Three version of WBC have been developed so far. Each one is suitable for some specific system. In Table 1 diffent kinds of WBC are compared.

	Directly controlled variable	Measured variable
WBC1	actuator position x_0	First mass position x_1
WBC2	actuator position x_0	Actuator-interface force f_0
WBC3	actuator input force f_c	actuator position x_0

Table 1: Different types of WBC

For robotics arm, WBC 1 or 2 is very simple to use, because actuator position and exchanged force with the system are easy to measure. In floating structure like spacecraft and rockets, is preferred to specify launching wave in terms of force or torque, so WBC 3 is the best choice.

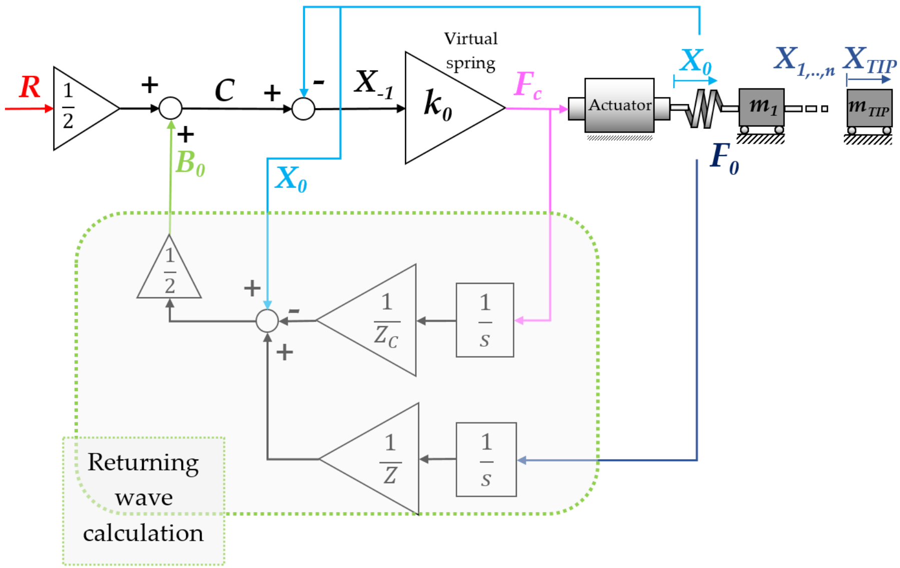


Figure 5: Impedance WBC 3.

3

WAVE BASED CONTROL OF 2D FLEXIBLE STRUCTURE

3.1 INTRODUCTION

Wave base control theory was originally developed considering lumped masses and springs in series, constituting a 1-dimensional chain-like array. This model can represent a lot of study cases, for example antennas, elastic transmission and, virtually, all the mechanical systems which are flexible just along one dimension. [1]

In his PhD Thesis, Hossein Habibi affords the topic about extending WBC to 2 dimensions mechanical structures. He considers 2 type of structures, a general 2-D lumped masses and springs grid, extended on both directions, and a simpler beam lumped model, which is a simplification of the grid (or lattice) model. In this case the 2-D structure extends mainly in one direction and it will be the starting point to develop the two case of study which this thesis concerns.

3.2 2D MASSES AND SPRINGS ARRAY - MECHANICAL MODEL OF THE STRUCTURE

In Figure 6 and Figure 7 one can see lumped masses and springs constituting respectively a lattice and beam model, developed by Hossein Habibi. Masses are connected in both directions and diagonal springs are added in order to have some shear stiffness for the structure itself. Otherwise, the structure would collapse on itself. These two models could represent different engineering structure, in which there are two important directions of flexibility, such as manipulators and robot arms or large space structures.

It is clear that this model is a discretisation of the real system, like it was already done with good result in a lot of case of study, modelling flexible 1-D structure to test 1-D WBC. Such a system is quite different from the chain-like one dimensional array and it is not obvious if it is possible or not to apply WBC to it. There are many degree of freedom with many undamped vibration modes and natural frequencies. The general main target is to control this structure using just one actuator, connected to a few masses (at least two). This one tries to control 2 directions and the rotation on the structure plane, so basically it has 3 DoF and it needs 3 references that should be set by 3 WBC loops, as one can see in Figure 6.

Unlike the 1-D array, there are more than one path for the waves to go to the system boundaries and to come back. So it is not clear whether or not WBC theory could work with this kind of structures and, if it does, how well it works. Two possible problems could arise: the dispersion and trapping of the waves (also present in the 1-D system, but often negligible) and cross coupling between transverse and longitudinal motion [1]. Considerations about these problems will be done in the following sections.

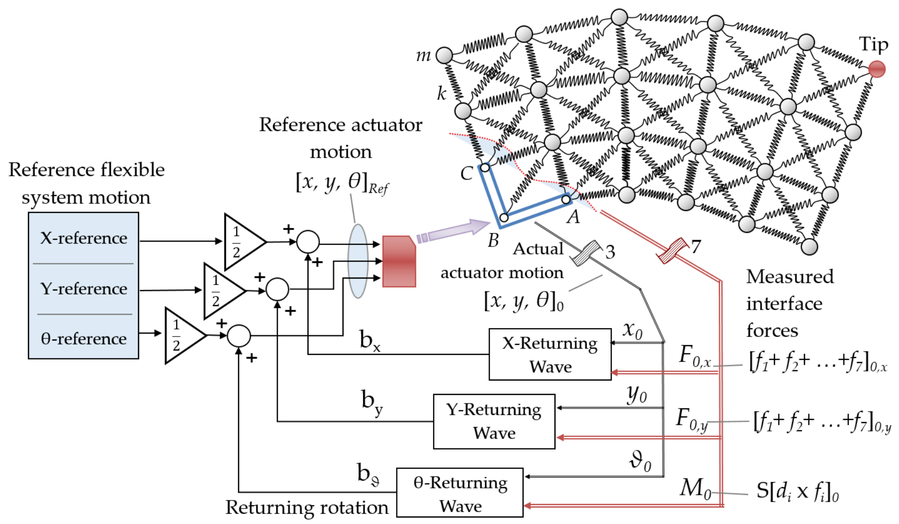


Figure 6: Lattice model and 3 DoF WBC controller - Habibi

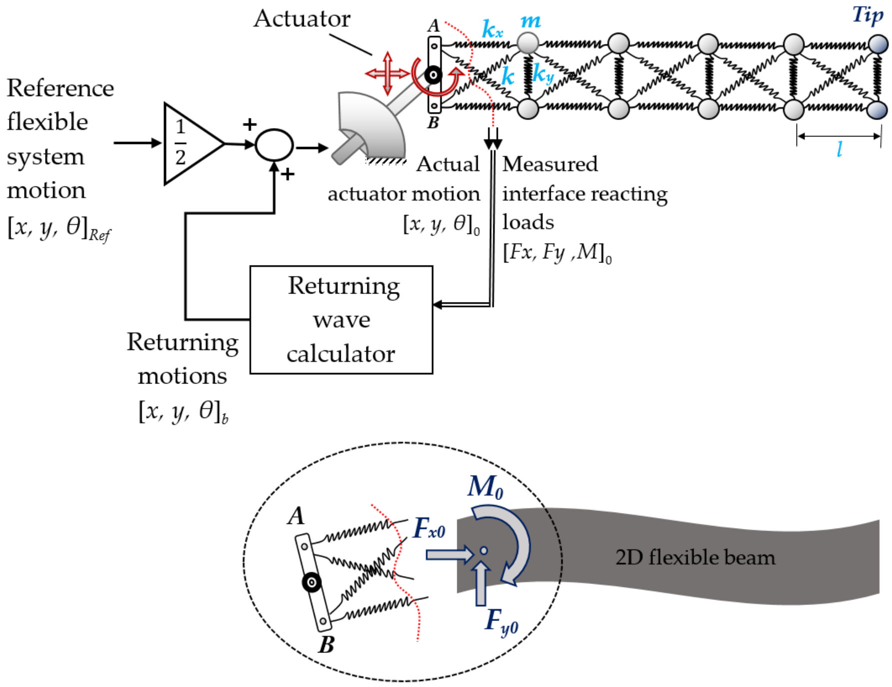


Figure 7: Beam model and 3 DoF WBC controller - Habibi

3.2.1 *Implementation of the 2-D flexible structure model*

In his PhD project, Hossein Habibi builds a numeric computational system in order to simulate the lumped 2-D structure. The scheme uses simple mechanical laws, such as Newton's second law of motion and Hooke's law.

Basically, the computational system calculates forces acting on each mass, produced by each spring. Summing all the forces acting on each mass, it is possible to calculate acceleration using Newton law and, therefore, velocity and position, setting the desired initial integral condition on each physical quantity. Having each mass position allows to know springs compressions and calculate forces exchanged between masses. Moreover, he also implements the damping on each spring, in order to simulate viscous friction inside the structure.

All the parameters are settable (masses, springs stiffness, damping coefficient), so one can tune the coefficients to simulate a particular case of study. Habibi sets damping coefficient to zero, in order to simulate the system in the worst case, leaving to WBC the task of damping all the vibrations which could have origin during the simulation. And, if not different specified, all the results in the following chapters will be presented without any mechanical damping.

In Appendix A the physical formulas used to implement the 2D computational model are explained. He develops the project using the commercial software MATLAB, designing the mechanical model in Simulink. This means that all the physical quantities are managed as signals. In Appendix B there are some explanations about the implementation on Simulink.

Habibi considers both ideal actuator (with neither dynamic nor saturations limits) and real one, showing differences between the two situations. In the cases of study afforded in this thesis, however, the actuators will be redrawn to model the specific system analysed.

3.2.2 *Modelling 2-D flexible structure: consideration about lumped method*

This thesis, as well as Hossein one, is not about comparison of mechanical system models and the model itself is thought as a test system for WBC, rather than to be an accurate description of the mechanical system. Anyway, Hossein research proves that the lumped model gives results comparable to classical approaches to modelling distributed systems, and so this model should be more than sufficient for testing WBC ideas [1, p. 82].

Lumped model has a lot of advantages. First of all it is easier to implement and modify if parameters change. In particular, it is possible to use the same numerical model for different kind of systems, just changing parameters in order to match the physical characteristics of the structure considered, in term of mass, stiffness and vibrational modes.

During his project, Habibi didn't find any literature about flexible beam controlling, which combines motion control with 2 translation directions and rotation, together with active vibration damping. Furthermore, the results he obtained suggest that the same technique could extend to the control of 3-D flexible systems, where the actuator has 6 degrees of freedom (3 translational and 3 rotational), in the same way it was done from 1-D systems to 2-D. However he did not test an 3-D flexible system, because it was not in the target of his project [1, p. 82].

3.3 CONTROL OF 2D FLEXIBLE STRUCTURE WITH 3 DOF ACTUATOR

Controlling the 2-D flexible structure with WBC is basically not so different from controlling 1-D structures, because the main idea is to replicate the standard WBC loop 3 times, leaving them independent each other. 3 DoF actuator requires 3 references and each WBC loop provides one, managing the interface between the flexible system and the actuator. This interface is a gateway, which forces and torques pass through.

Actuator can be considered to have its own motion control system, so WBC is just required to set the references. Problems about modelling the dynamic of the actuator and how to deal with WBC loop applied to real actuators will be afforded in the following chapters.

In his project, Habibi uses WBC of type 2, i.e. forces and torques coming from the structure to the actuator are measured and constituting the returning waves. The launch waves are the position references for the actuator control system. In Figure 7 one can see how the reacting load is modelled for the beam structure. In Figure 6 the three impedance Wave-Based Control loops are shown.

3.4 2-D BEAM LUMPED STRUCTURE TO MODEL REAL CASE OF STUDY

The aim of this thesis is to adapt Habibi 2-D beam model at two case of study: flexible robot arm and aerospace flexible launchers. In order to model these structures, beam lumped numerical system shall be modified and improved. It is possible to summarize here some important aspects required to generalize the 2-D lumped model and its WBC from a simple extreme actuated beam to a 2-D structure which can potentially move in an infinite 2-D plane, subjected to some external environmental conditions and, eventually, some modifications in the shape of the structure itself.

- actuator could move in 2 dimensions with no limits, plus rotation in that plane;
- Need to redefine spatial reference systems and deal with translation and rotation of a extended NOT-rigid body;
- possible cross coupling problems;
- gravity and other possible external disturbances, such as impact forces and viscous friction;
- actuator dynamic modelling;
- possible modification in the structure: move or add masses and springs in order to model particular characteristic of the mechanical system (for example sloshing of fluid).

In the following chapters all of these problems, and others more, will be afforded, building step by step a generalized 2-D lumped structure and adapting it to some specific engineering applications.

4 WBC FOR ROBOT ARM CONTROL

4.1 Modifications and improvements to 2D beam lumped model

In this chapter the modifications made in the 2-D beam model are explained. These modifications have been done in order to generalize and adapt the model to a flexible robot arm and aerospace flexible launchers, controlled by a 3 DoF actuator. Robot arm case is functional to build the model for launchers, which is the ultimate target of this thesis.

The difference between these two models are basically related to the parameters of the structure and to the type of actuators considered, but both models rely on a 2-dimensions lumped structure, which target is to represent the main vibrational modes on the plane, considering that robot arm and aerospace rocket have two significant dimensions of flexibility to model and control.

During the project it was noticed that 2-D beam model needed a lot of adjustments before it could simulate a rocket structure. Therefore the research work passed through a case of study in the middle between beam and rocket, with the idea to have a mechanical structure very similar to the beam, with parameters of the same magnitude order, but with a completely different motion demand and, as a consequence, the need of different type of actuator and control system. In the meantime, this functional middle stage of the research shows to be interesting from itself as well, so it was developed autonomously with the idea to simulate a flexible robot arm and the related problems of controlling it during its motion.

Involved movements could be virtually thought to be in an infinite 2-D plane, quite different from small amplitude rest-to-rest motion which is demanded to the beam. Moreover, structure could be subjected to some external disturbances like gravity, impact and any other kind of external forces.

Considering that the robot arm model is functional to rocket stage model, some modifications involved also the structure itself and, on specific terms, the starting geometric conditions, changed from horizontal to vertical orientation, i.e. the common launchers starting position, with the longitudinal axis orthogonal to the ground base line (see Figure 8)

These modifications involve the reference systems first of all, which need to be carefully managed. One of the first challenge, indeed, was to understand how the high number of reference systems works together, managing absolute and relative system and dealing with transformation (translation and rotation) of them and of the flexible structure. Some problems arose when it was asked to the structure to achieve both translational and rotational movements. Other problems were found trying to make big movements in relation with the dimensions of the structure itself.

In a nutshell, the main target of this chapter is to modify the numeric computational model of the physical structure and its controller, to make it general and suitable to any kind of flexible structure which has substantial elasticity in 2 dimensions and is potentially required to move on a infinite extension plane.

Moreover, just to fulfil robot arm task, WBC will be improved to get as much motion accuracy and high dynamic as possible.

4.1.1 *Transformations and reference systems*

As it was explained in Chapter 3, the numerical model of 2D flexible structure is made by lumped masses, each one implemented like a

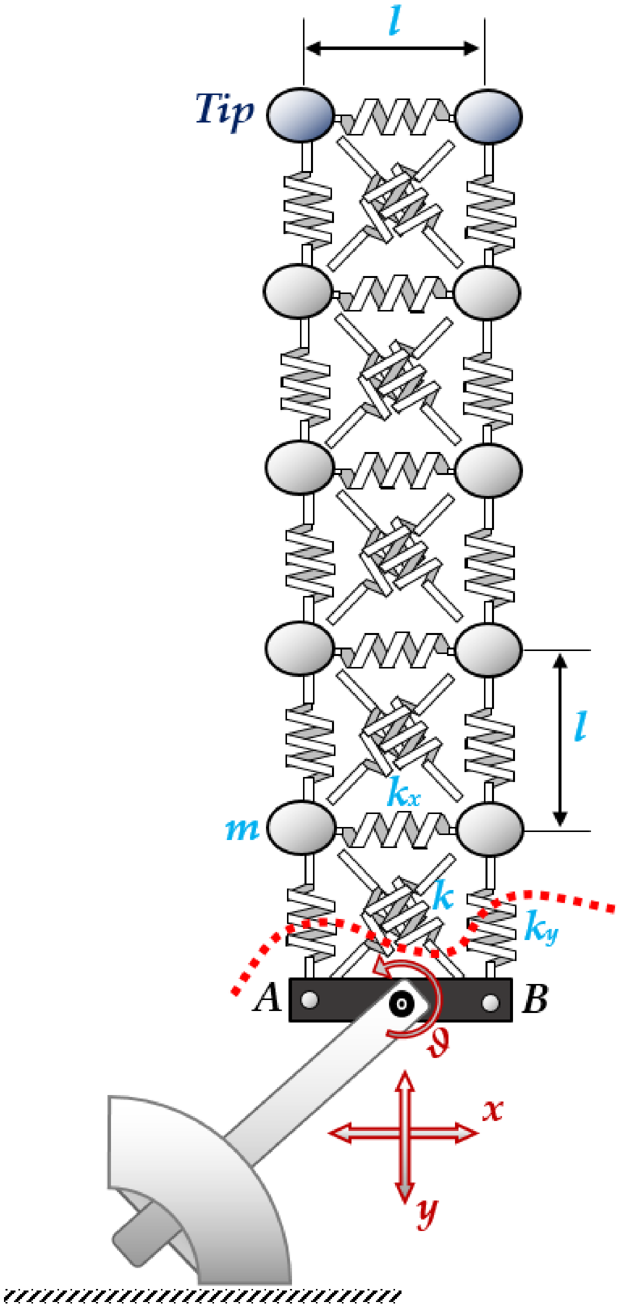


Figure 8: Robot arm lumped model and its 3 DoF actuator.

block which receives as input the positions of the masses above. The block calculates the forces acting on the mass and gives as output the position of that mass. It is necessary to define in a strict way all the reference systems for each mass, relatives on actuator or absolute.

Each mass has its own reference system, which has origin in the starting position of that mass. When the mass starts to move, the position signal calculated is referred to that particular reference system. This means that it is necessary to transform each position considering the same reference system, which could be fixed (grounded) or moving (for example actuator real time position). This is an obvious consideration if one thinks about rigid bodies moving on a plane, but it becomes less clear if the structure is not rigid and its shape change during the motion.

For this reasons, in order to get the absolute measurements necessary to make calculations in the simulation, some masses positions are transformed considering a grounded reference system which has origin on the actuator middle point starting position. This is particularly important if it is required to calculate instant centre of mass, as described in Section 4.1.4.

In other words, there are traditional transformations due to translation and rotation of the structure considered as a rigid body, which oscillation movements are summed to.

4.1.2 *Adaptation of the reference considering rotation of the structure*

Like position measurements need transformation in some case to deal with, reference positions signals need to be transformed as well. Since the target of WBC is to move the flexible structure with as minimum vibration as possible, the reference for it will be calculated considering to have a rigid body structure to rotate and translate. So it is necessary to take into account the transformations happening on the reference rigid body when there will be the comparison between reference signals and output signals (tip mass position and orientation).

If reference signals ask the structure tip mass to follow a certain path in term of displacement x and y and, at the same time, in term of angle orientation of the top of the structure, translation and rotation of the rigid body are mixed together with oscillations. Considering just the ideal rigid body, the position reference for the tip mass shall be transformed, adding rotation displacement to the translation and rotation required by the original reference. In figure Figure 9 it is shown the results of the original references and the calculated transformed references, using Equation 4.1.2. It is possible to see the offset between the original and the transformed references on x and y, due to the rotation of the tip masses shown in the third graphic.

$$\begin{aligned} x_{ref}^{transf} &= x_{ref} - H \sin\theta \\ y_{ref}^{transf} &= y_{ref} + H \cos\theta - H \end{aligned} \tag{10}$$

That is a very simple implementation consideration, but, without it, the simulation gives results which seems not to match the references. The solution to this problem is just instantly adapting the references with actuator angle movements. This consideration is not only necessary to make simulation results more human friendly, but it will be very important if the tip mass is required to follow the references with high dynamic and no steady-state errors, especially in the presence of cross-coupling or external disturbances, as explained in the following sections.

4.1.3 *The need of Wave based Control*

It is important to compare results whereby Wave Based controllers have been already applied to, with results from system without WBC, in order to understand the improvements WBC brings. The Figure 10 compares the output of the same system with and without WBC.

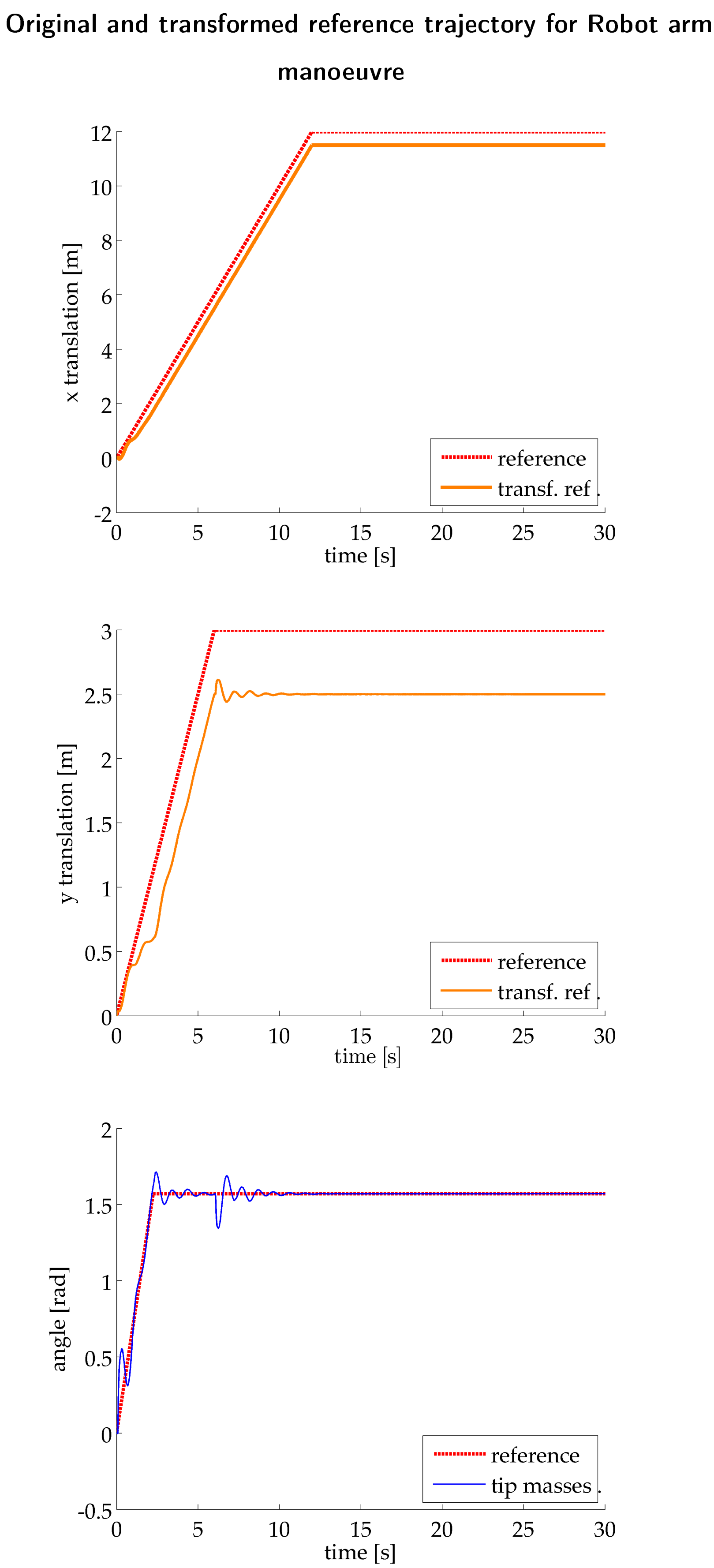


Figure 9: To test WBC applied to robot arm, 3 saturated ramps are considered as references for x, y and θ. References on x and y axes need adaptations, to take into account the rotation of the structure. The angle displacement of the structure is obtained using WBC.

Since the system in not internally damped (or under-damped), it requires the active vibration damping supplied by WBC.

4.1.4 *Calculation of moment considering mass centre of structure*

In Figure 11 it is possible to see the instability problem which comes out when it was asked to the structure to have big movements. Initially, these problems seemed to be due to the cross-coupling of motion, but, trying to simulate controlling just only one degree of freedom, the problem did not disappear. This meant that the problem was not due to WBC itself, but from something located in the structure model. The first stuff to check was the calculation of the moment.

In his project, Habibi used a grounded fixed point as fulcrum to calculate the arms of the forces acting from the structure to the actuator. This calculation is good as long as the beam is stressed by an actuator which movements were limited in distance and time.

But, in a general situation, it is necessary to move the structure with wide movements. In this case, if the moment were calculated by a grounded fulcrum, forces arms would increase too much, giving a excessively high value of moment, sensible to numerical errors, and making WBC controller unstable.

Therefore, as Habibi suggested in his thesis [1], it is possible to calculate the moment on the centre of mass instead of calculating it on a fixed point. If the moment is calculated considering the centre of mass of the whole structure, forces arms increasing problem disappears.

4.1.5 *Estimation of the centre of mass*

It is necessary to take into account that this method adds more numerical load during simulation and, thinking about a real implementation, it is impossible to measure the exact position of all masses in

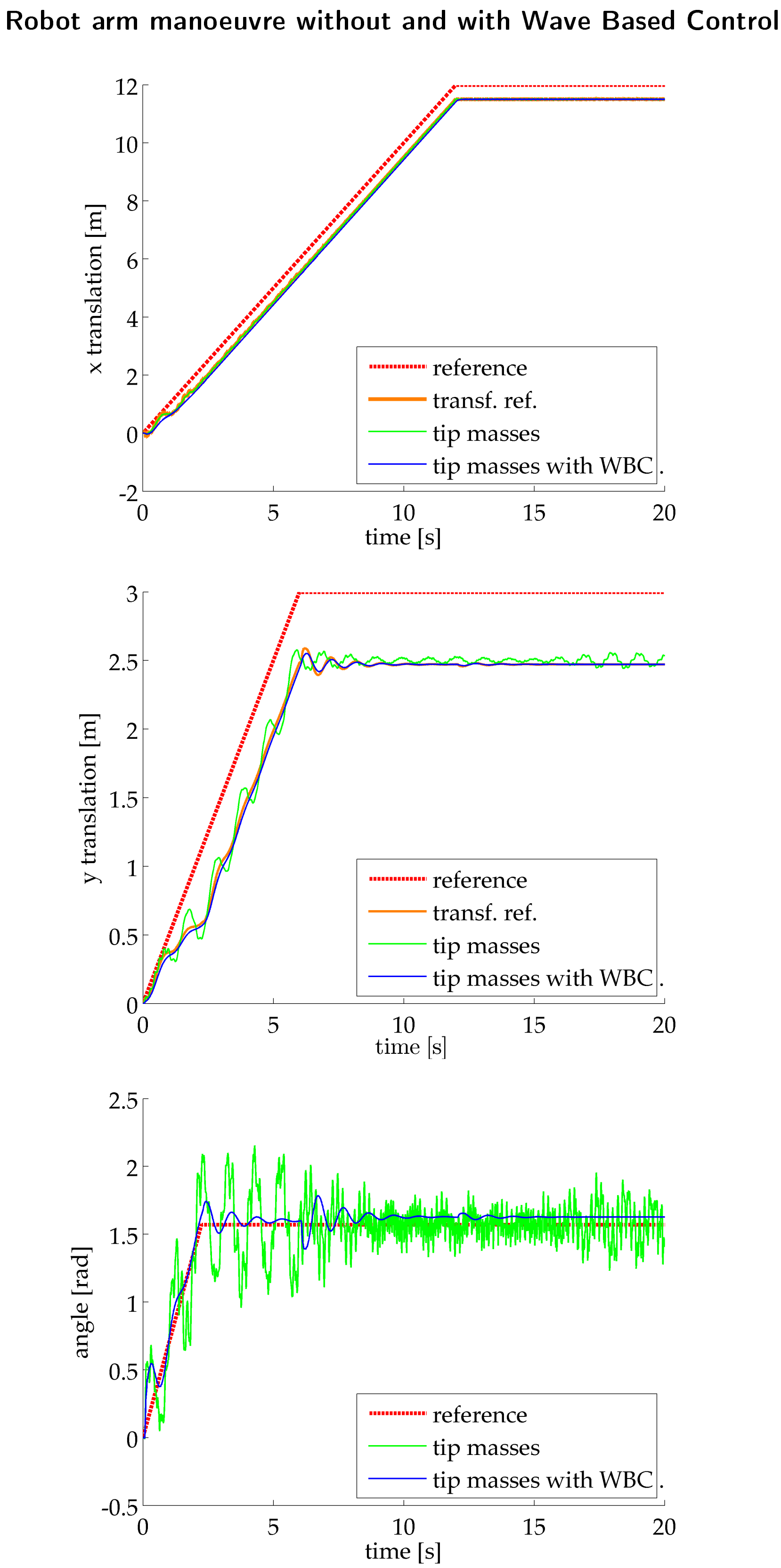


Figure 10: Simulation results of robot arm rest-to-rest manoeuvre made with saturated ramps as references and without WBC. It is possible to observe oscillations are not damped in the second case (green signals).

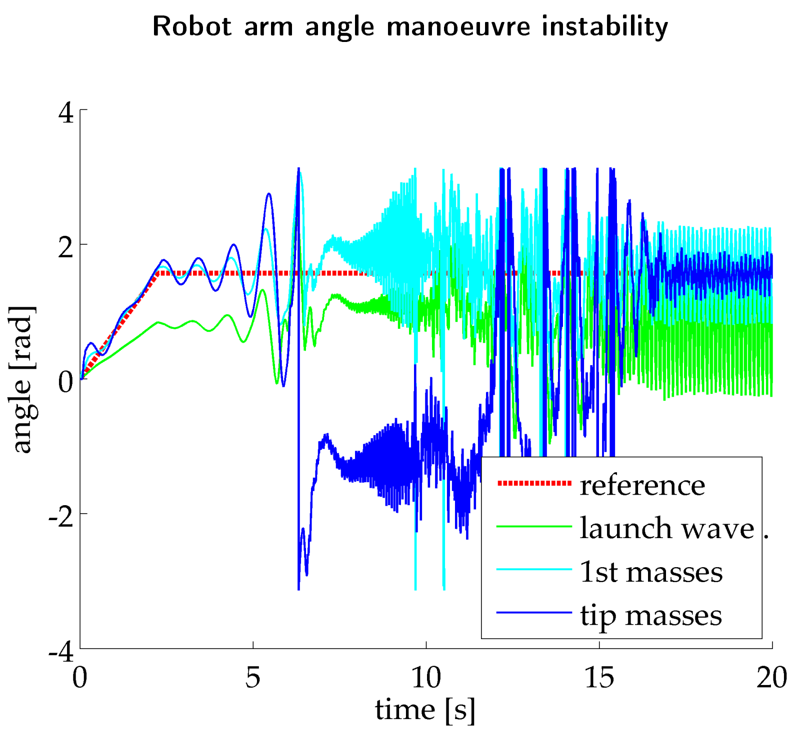


Figure 11: Robot arm manoeuvre using WBC 2. The instability is due to a problem in the calculation of the torque load acting on the actuator. It was calculated considering a fixed point as fulcrum.

order to calculate analytically the centre of mass. A simple solution is estimate the centre of mass.

The simpler way to do this is to think about the structure as a rigid body, made by lumped masses which cannot move each other. So the centre of mass is fixed and it is possible to obtain its position if all other masses are known. Once the centre of mass position is known before the structure starts to move, it is possible to predict where it is going to be, just considering rotation and translation of the structure.

Changing from the analytical calculation to the estimation, makes the simulation more relevant and applicable to a real case of study, because it is not required to know the exact positions of all the masses of the structure. The results obtained with this method are very similar to the previous case, almost difficult to distinguish each others, despite the structure is flexible.

4.1.6 *Cross-coupling of motions*

While the structure was controlled, it was possible to observe a cross-coupling and dynamic coupling of motions. This events happen when actuator makes an acceleration or deceleration at least in one degree of freedom.

For example, if the structure is asked to follow three saturated ramps, one for each actuator degree of freedom, it is possible to see an oscillation effect on the other two axes, when one reaches steady-state condition, i.e. when there is a deceleration from a certain velocity to zero. This effect looks like an external disturbance, and it is similar to the effects caused by external impact forces, considered in Section 4.2.1.

In Figure 13 for example, it is possible to see the effect on the angular movements. When each axis (y and than x) settles to steady-state, an angular oscillation occurs in the transversal tip axis of the struc-

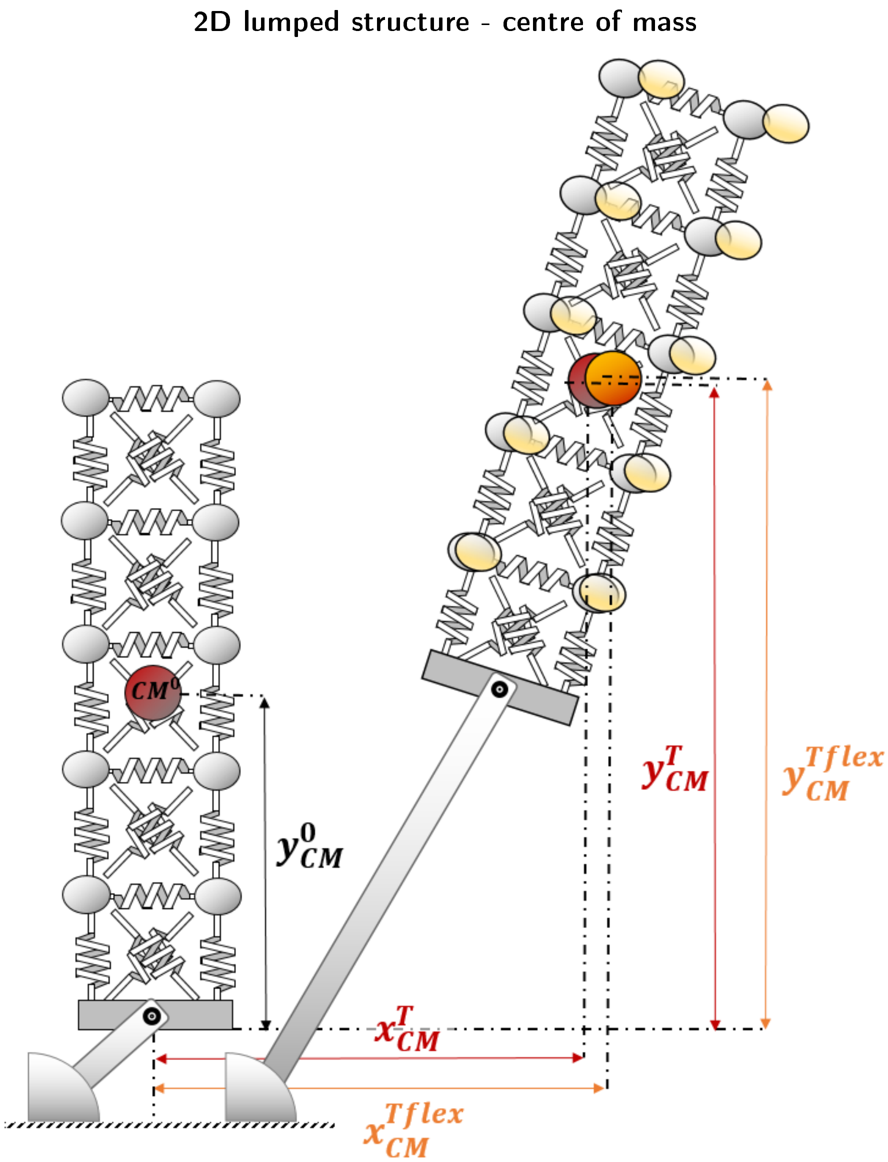


Figure 12: Centre of mass positions of the 2D lumped structure. It is possible to see the difference between the hypothetical transformed rigid body (structure on the right with grey masses) and the flexible system (masses in yellow) is not so high in this case of study.

ture, unsettling it from its steady-state condition. Moreover, a small steady-state error is present on the angle.

It is possible to explain this result considering inertia of the structure related with acceleration/deceleration on one axis. This inertia acts like a disturbance on the other axes in the moment which a change on velocity happens in that axis. This hypothesis is supported considering a different motion reference, for example making the saturated ramp smoother. This modification reduces deceleration when the ramp reaches its limitation and therefore the inertial force is less than in the previous case. Having less inertial force reduces the effect of 'external disturbance' on the other axes, thus cross-coupling problem becomes less evident.

In Figure 13 it is possible to observe that tip masses follow the transformed reference on x and y, which seems to be disturbed as well. This result does not surprise because transformed reference is calculated with the measured tip angle, so it contains all the oscillations which occur in the angle. But there is an important consideration to infer: the transformation of reference was done to take into account the geometrical rotation of the ideal rigid body representing the structure in its geometrical shape. So, if now one would like to make the rotation function of a time oscillating angle, time oscillating result will be obtained, just considering a *rigid* body. This means that if tip masses follow the oscillating transformed reference calculated on the measured angle, these oscillations are not due to problems on x and y WBC control loops, because controllers are perfectly following what they are asked to do. In other words, WBC is able to damp all vibrations on x and y, but not the oscillations on the angle.

Summarizing, cross-coupling is present but it does not create problems on the stability of the system. It acts like a disturbance on angle control loop. Modifications to make the controller able of disturbance rejection will be useful also to solve cross-coupling problems. In the following sections these improvements will be explained.

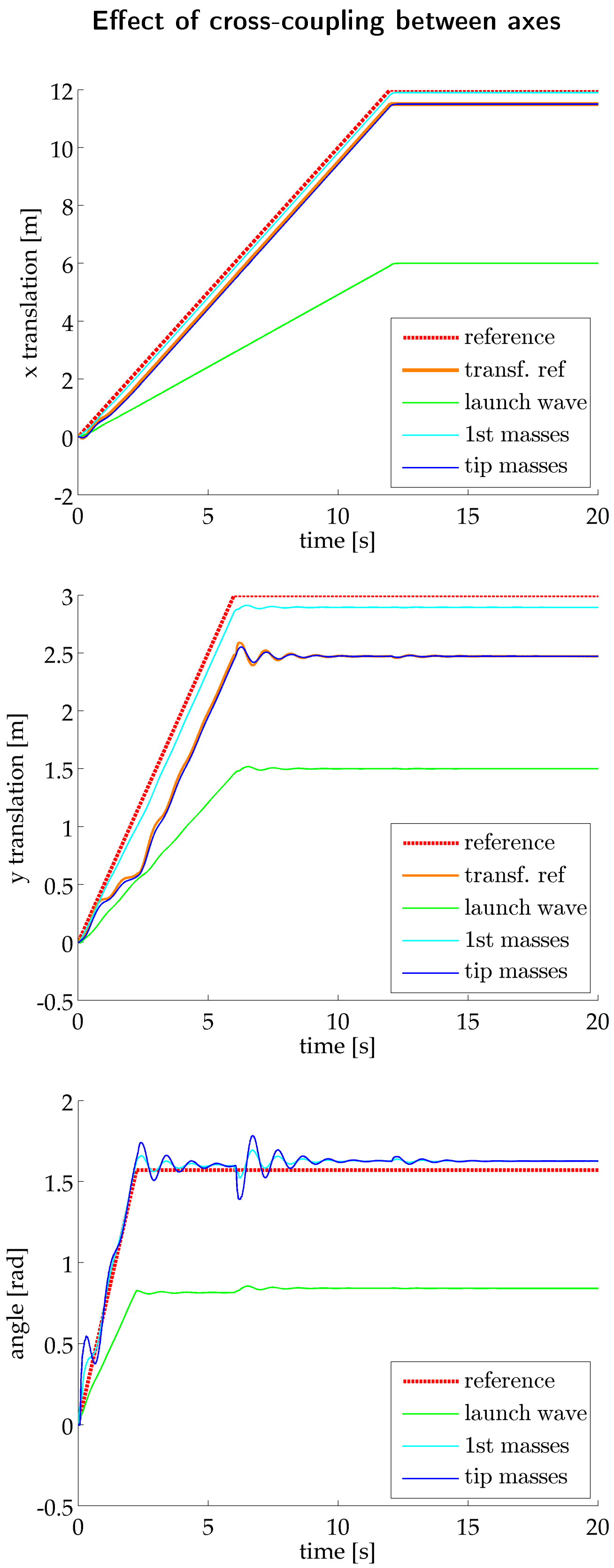


Figure 13: Results of simulation of robot arm control with a rest-to-rest manoeuvre made with saturated ramps as references. It is possible to observe oscillations and steady-state error on the angle, due to cross-coupling of motions

4.1.7 *Considering constraints due to actuators*

In order to have a significant model of the whole system, it is mandatory to consider also that between WBC calculations and the flexible structure there are some actuators, one for each axis. The actuators are the interface between the control virtual world and the mechanical world model. This means that, when control signal enters in the actuator, setting its reference, physical constraints must be taken into account.

It is possible to consider ideal actuators during numerical simulations, i.e. supposing the signal calculated by the controller could go directly into the structure as it is, just adapting it for geometrical and physical reasons. However this is an unrealistic model, too much simplified from reality, which could hide some problems about control the system during the simulation.

For these reasons, actuators have to be modelled considering their own physical constraints, basically related to the force and torque limitations and inertia that they have. Actuators implementation depends on the signal produced by the control law, therefore it changes turning from WBC 2 to WBC 3. In the first case the controller supplies a signal position to the actuator and the actuator sets the required position on the structure. If nothing about actuators dynamic and limitation is taken into account, it is supposing they are able to set the positions which WBC asks for, not only regardless the force limitations but also regardless inertia they have.

In a WBC of type 2 controlled system, actuator is a position to position block, so it can be considered and modelled as a position feedback controlled loop. It is reasonable to consider the actuator has its own position control system, which WBC sets the reference for. It is possible to model it as a first order transfer function, with unitary gain and a pole which corresponds to the position bandwidth of the actuator (see Figure 14).

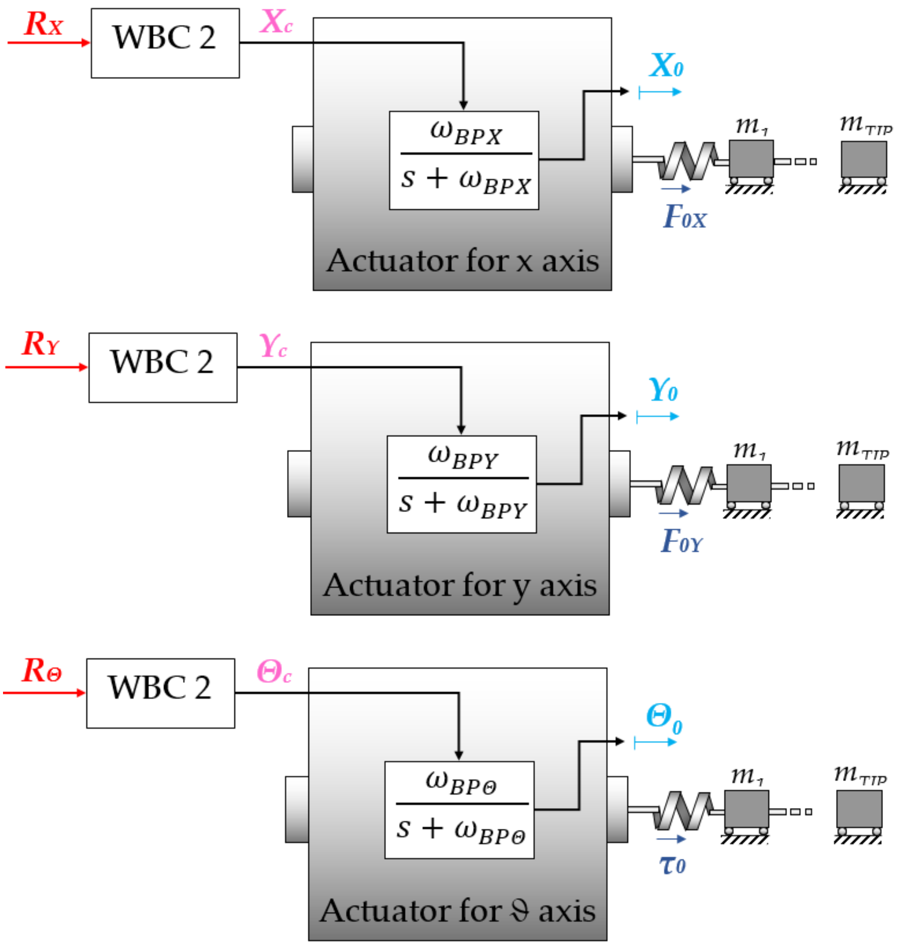


Figure 14: Real actuators model. It is considered the dynamic of the actuator to generate the WBC required positions. Feedback for returning wave is not shown.

In a system controlled by type 3 WBC, actuators are driven by the force (or torque) signals calculated by the controller and give the position set on the load (the flexible structure) as output. So in this case, actuators need to be modelled in a different way, not just with a simple low pass filter, but considering separately the force generation (which depends on the type of actuator) and the mechanical side, i.e. the inertia of the actuator moving part and the load acting on it. In Section 4.3.1 and Section 4.4 the model adopted for WBC3-driven actuators and some simulation results are shown.

The ultimate target of this thesis is to test the controller and his applicability, so it is not particularly important to set accurate parameters for the actuator in this stage of the project, but the main idea is to put the system in its worst condition for the controller, so, if the controller works in the worst case, it is going to work in all the other cases. In terms of actuator parameters, this means to set the lowest bandwidth in it, to which the system remains controllable with an adequate dynamic.

At the beginning of the analysis with both WBC 2 and 3, during the modifications and improvements in the control system, no filters were put between WBC calculated signals C and actuator output α_0 or F_0. Simulations were made without considering actuators constraints in the generation of the physical magnitude required by the controller, thinking to be in a higher level and trying to solve all the other problems faced.

When these problems were solved, actuators limitations were introduced. It was verified that these dynamics did not create big problems. In Section 4.4 some results for WBC 2 and 3 are shown.

4.2 IMPROVEMENTS ON THE CONTROL SYSTEM FOR POSITION-DRIVEN ACTUATORS

Once the model is modified and improved to simulate robot flexible 2D arm, it is possible to concentrate on the controller for the system. The passages described in the previous sections solve several problems, which had seemed caused by the wave-base control incapability to manage this kind of 2d flexible model, when it was required to move the structure with a more demanding trajectory (big amplitude). With those improvements and corrections of the model, WBC already works quite well in controlling the system, considering ideal conditions and no gravity applied to the system.

The aim now is to improve the controlled system response, in terms of robustness, steady-state errors and dynamic. It was already said that in the robot arm case the interest is concentrated on getting high dynamic and accuracy. For the rocket controller instead, the main target is robustness to structure parameters and environment disturbances, more than wide bandwidth and precise controlling in general. These specifications will be taken into account during controller design.

4.2.1 *Modelling and coping with external disturbances*

2D flexible model is so far considered to be in an ideal environment, without gravity and any other kind of external disturbance. The target is now to test WBC capability to cope with external forces of any kind applied to 2D flexible structure: gravity first of all, viscous friction and bumping forces.

The model is modified to simulate this kind of external forces, simply adding, instant by instant, the sum of external forces acting on each mass. A specific block is created in order to generate these forces, allowing to produce, potentially, any kind of desirable force and, easily, to change the parameters of the forces to generate.

Keeping impedance WB controller of type 2 as before, the system became uncontrollable under external forces action, in particular with gravity (see Figure 15). The solution to this problem was solved by O'Connor and Habibi (in the same thesis), considering 1D flexible system and 2D beam-like structure.

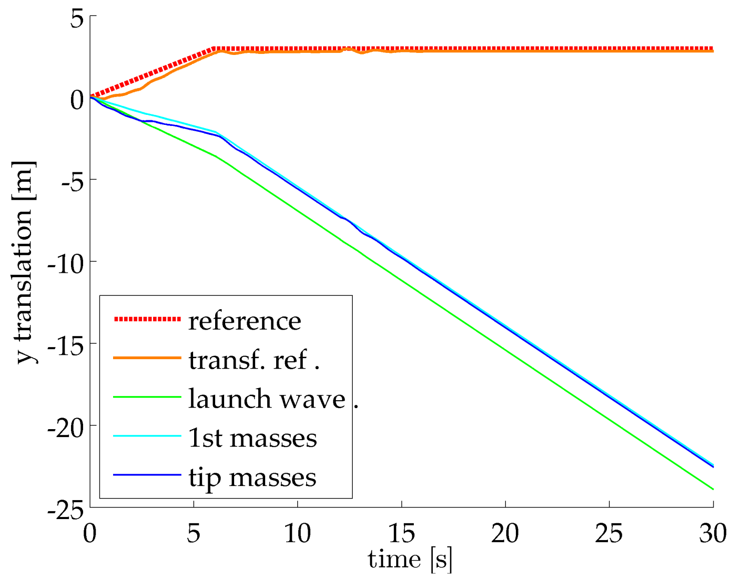


Figure 15: Effect of gravity on the system, with no modification to the WB controller

Variable external forces are often negligible, in comparison with the inertial forces of the system, but not always. If they are significant, they change the measured return wave, leading errors in the system response. O'Connor and Habibi developed a strategy in order to modify WBC traditional loop to cope with this problem.

The strategy divides the external forces in two main categories: enduring, non-impulsive external forces and transient external forces. The aim is to detect the disturbance and modify WBC action without interfere with the active vibration damping aspect of the controller.

In order to get these results for the enduring disturbances like gravity, Habibi and O'Connor found that the best solution is to modify

Equation 4 by subtracting an estimate of the continuous force obtained as a integral average of the measured returning force wave, calculated as:

$$f_{DC} = \frac{1}{T_1} \int_{t-T_1}^{t} f(t) dt \tag{11}$$

Where $f(t)$ is the interface force between the actuator and the flexible system, already considered in the traditional impedance WBC. T_1 is a time constant. Considering ideal actuators ($c(t) = x_0(t)$) and theory of impedance WBC in Section 2.2.2, one can write:

$$a(t) = \frac{1}{2} r(t) \tag{12}$$

$$c(t) = x_0 = a(t) + b(t) \tag{13}$$

$$c(t) = r(t) - \frac{1}{Z} \int_0^t \left(f(t) - \frac{1}{T_1} \int_{t-T_1}^{t} f(t) \right) dt \tag{14}$$

[10]

$a(t)$ remain unchanged from the WBC standard formulation. $b(t)$ provides active vibration damping like in the other cases, measuring $f(t)$, according to the standard impedance WBC theory. But now it is required to identify the continuous force component, in order to delete it from the return wave, before using it to create the launch wave. So in $b(t)$ the new integral term, which identifies the DC component of the force, is subtracted. In this way $b(t)$ contains just the AC component of the wave, i.e. the necessary information for active vibration damping.

The averaging time T_1 should be long enough to smooth residual oscillations in f and short enough to track longer term variation of DC component. In practice, a time slightly higher than the period of the first harmonic of the structure works well.

In order to implement as Simulink blocks, it is required to transform Equation 14 in Laplace domain. It is possible to write:

$$\int_{t-T_1}^{t} f(\tau)d\tau = f^{\int}\Big|_{\tau=t-T_1}^{\tau=t} = f^{\int}(t) - f^{\int}(t-T_1) \quad (15)$$

$$\mathcal{L}\left[f^{\int}\Big|_{\tau=t-T1}^{\tau=t}\right] = \frac{1}{s}F(s) - \frac{1}{s}F(s)e^{-T_1 s} = \frac{1}{s}\left(F(s) - F(s)e^{-T_1 s}\right) \quad (16)$$

Therefore, considering Equation 14, launch wave can be calculated in this way:

$$C(s) = R(S) - \frac{1}{Z}\frac{1}{s}\left(F(s) - \frac{1}{T_1}\frac{1}{s}\left(F(s) - F(s)e^{-T_1 s}\right)\right) \quad (17)$$

so, according to Equation 7, return wave has this formula:

$$B(s) = \frac{1}{2}\left[C(s) - \frac{1}{Z}\frac{1}{s}\left(F(s) - \frac{1}{T_1}\frac{1}{s}\left(F(s) - F(s)e^{-T_1 s}\right)\right)\right] \quad (18)$$

where it has been taken into account to approximate actuator output position $X_0(s)$ with actuator input $C(s)$. There is no approximation if actuator is ideal and, if not, good result was obtained as well.

Results also show that the DC filtering works well even when combining x,y and angular movement. But this modification is not enough to deal with transient and impulsive external forces, which could originate during impact of the structure with and external body. For example, in the robot arm case, it could be a force transmitted to the arm, originated in the pincers during the clamp of an object. In the rocket case, instead, it could originate from the impact of an appendage detached from the main body during the mission.

Thinking of transient forces, one important example is external viscous damping (not to be confused with the natural inner damping of the structure) or other hydrodynamic forces such as aerodynamic friction. A robot could work in a fluid like water and the rocket is subject to aerodynamic friction as long as it is inside the atmosphere.

When one of these forces acts on the system, the steady-state error is again not negligible applying Equation 14. If external impulsive forces act on the system, they make an extra contribution to the average integral term, causing an apparent settling in the wrong place,

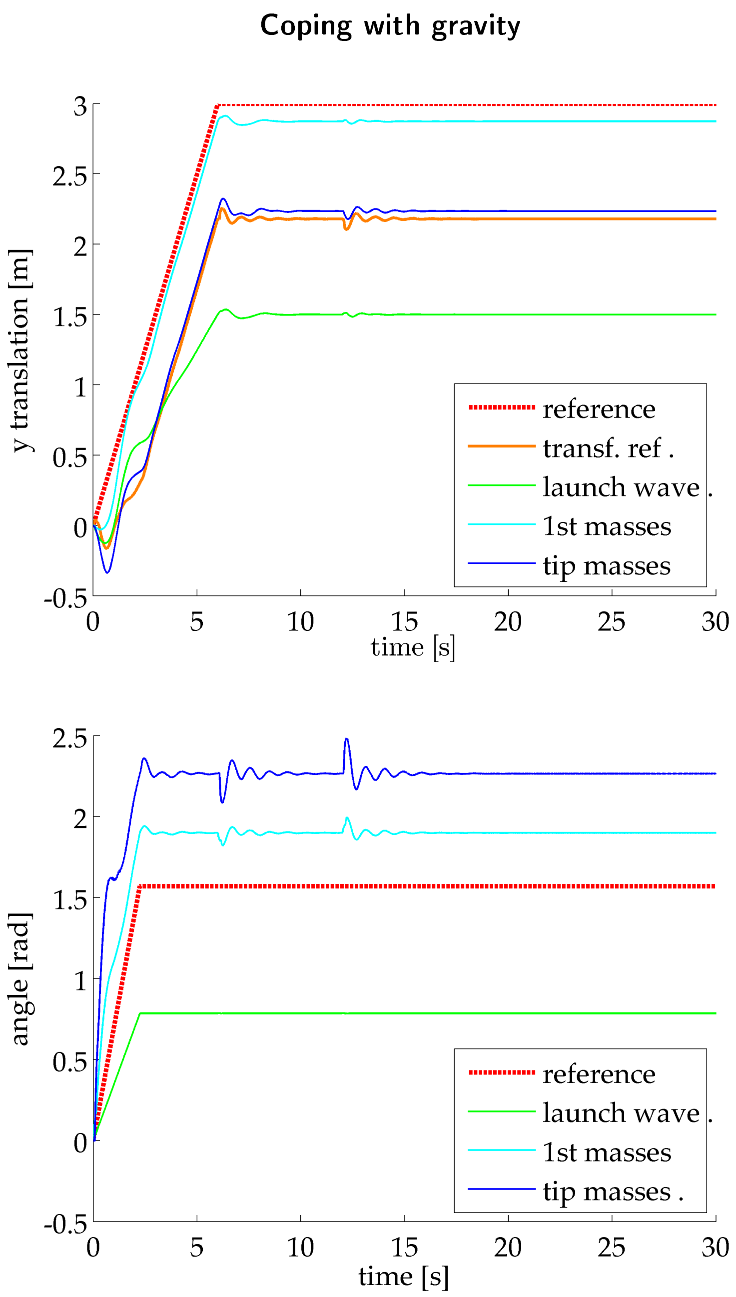


Figure 16: Behaviour of the system under gravity, after modification to the WB controller of type 2

requiring long time to reach the final value. So $b(t)$ needs another modification. O'Connor and Habibi found that the best solution is to delete the whole integral term after a certain time. So the formula is:

$$c(t) = r(t) - \frac{1}{Z}\int_0^t \left(f(t) - \frac{1}{T_1}\int_{t-T_1}^t f(t) \right) dt + \quad (19)$$

$$+ \frac{1}{T_2}\int_{t-T_2}^t \left[\frac{1}{Z}\int_0^t \left(f(t) - \frac{1}{T_1}\int_{t-T_1}^t f(t) \right) dt \right] dt \quad (20)$$

The second averaging time T_2 is not critical and can be similar or identical to T_1. At steady-state the two integral terms delete each other regardless any accumulated values in the force integral, so $c(inf) = r(inf)$. This result is achieved without interfering with the traditional vibration damping action of WBC and filtering action works even if disturbing forces act during steady state.

With the same method used before, it is possible to write $C(s)$ in terms of Laplace transformations. In order to simplify the equation, it is possible to call $B1(s)$ the first part of the return wave, already modify to filter DC components:

$$B_1(s) = \frac{1}{Z}\frac{1}{s}\left(F(s) - \frac{1}{T_1}\frac{1}{s}\left(F(s) - F(s)e^{-T_1 s}\right) \right) \quad (21)$$

and therefore, considering $c(t)$ formulation:

$$C(s) = R(S) - B_1(s) + \frac{1}{T_2}\frac{1}{s}\left(B_1(s) - B_1(s)e^{-T_2 s}\right) \quad (22)$$

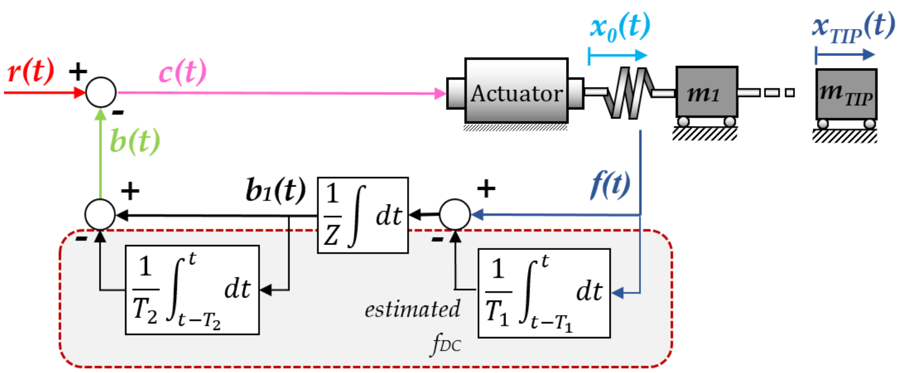


Figure 17: Impedance WBC 2 loop, with disturbing forces filtering highlighted with red dash line. Signals are expressed in time domain.

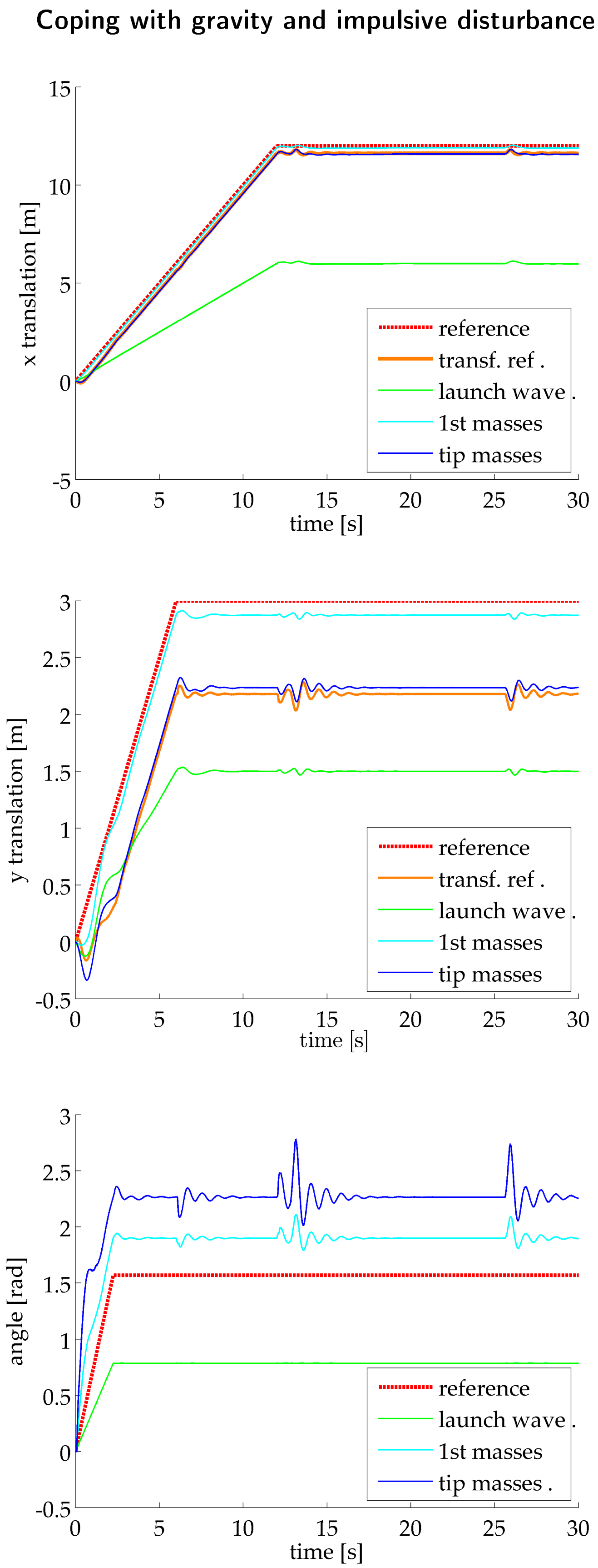


Figure 18: Behaviour of the system under gravity and external periodic impulsive force, with filters in WB controller, applying modifications to the controller

From the considerations explained in this section, it is possible to infer that the return wave contains a lot of interesting information, useful for system identification and environment monitoring. The modified returning wave can be used to say whether or not an external force acted on the system, whether or not it continues at the end and if the waveform is impulsive or continuous.

The usual WBC claims that no system model is needed, is still valid. All the parameters, indeed, are easy to estimate, but, more important, not critical for the controller, which is able to manage the system to stability even with wide variation of them from the optimal point (paying in worse dynamic, but keeping system stable).

Applying this formula to each WBC loop for the robot arm control with 3 DoF actuator, the results are very interesting, showing the high disturbance rejection and robustness to system parameter changing. It is proved also, that delay parameters are of no particular concern, as it is possible to choose them in a wide range of values.

4.2.2 *PI controller for getting zero steady-state error on tip mass*

After this improvement of the controller, the system works very well, even in not ideal conditions. It is possible to move it far away from its origin, combining two translations and a rotation on the plane, with some limits in the dynamic. It is possible to control the vibrations and damping them in a rest-to-rest motion.

But still a problem remains unsolved. If the gravity (or other enduring non negligible external forces) acts on the system, there is a constant steady-state error on the tip-mass position. This error is due to the deflection of the structure, which creates a different angle between the actuator and the tip of the structure.

Moreover, the target now is to improve the dynamic response of the system, to manage a possible manipulator flexible arm. The most effective solution found is to alter the half-reference for each of the 3

WBC loop, with the aim to obtain zero error between the measured tip-mass position and modified reference calculated in Section 4.1.2. In order to obtain this target, it is considered a PI controller for each loop, which elaborates the error, producing the adapted signal to add to the WBC launch wave.

From a physical point of view, this could be explained in a modification of the actuator settling position, which takes into account the steady-state deflection due to gravity. So, for example, if it is required to set the structure horizontally, the actuator (base of the structure) orientation will be some degrees less than 90° such that the tip of the structure will be exactly at 90°.

Of course, to do this, it is suppose to have tip position measurements or its estimation available (i.e. a not co-located sensor measurement).

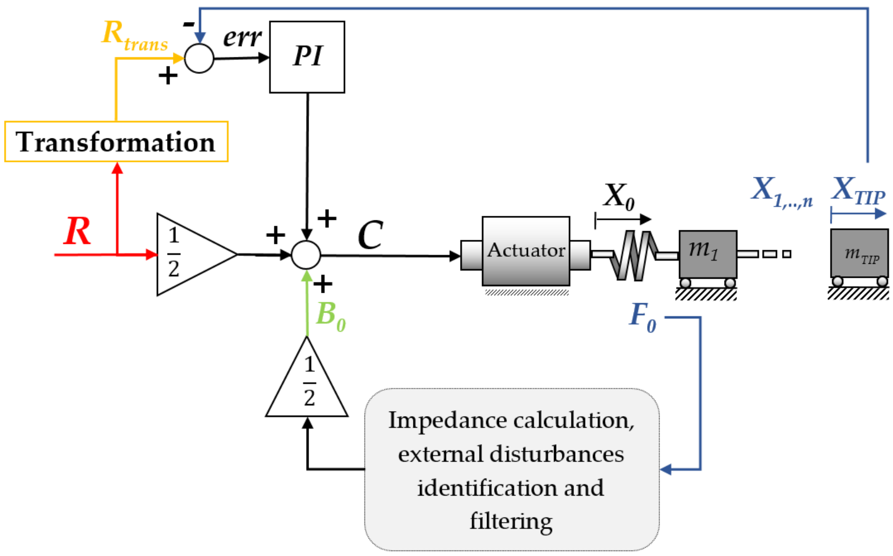


Figure 19: Controller scheme with PI correction and external forces filtering - generic axis controlled by impedance WBC 2

Tuning the PI controller with such a complex system is not so easy to do with traditional algebraic instruments, like gain and phase margin or similar ideas. In particular, the complication is due to the WBC loop which interacts between the PI controller and the system. In this case, indeed, PI controller is just a block which filters the error and adapt it to improve WBC dynamic.

Therefore, the simpler strategy adopted was to tune the PI parameter with *trial and error* method, finding the solution which allows to get the best response of the system. This method, however, has some inconvenient, first of all that if system parameters change a lot, PI action could degrade the response, instead of improving it, bringing it to instability in some cases.

This outcome is exactly what WBC theory try to overwhelm. In other words, adding a PI controller to the main WBC controller, in general creates more difficulties and problems than what it solves. This is the reason why PI is rarely used in combination with WBC, and why it will be not used in the rocket control system developed in Chapter 5.

The target of this chapter, instead, is precisely to explore a possible application of PI to WBC trying to improve WBC dynamic, with the price of a decreasing in robustness and generality of the controller. PI controller reveals to be quite useful to improve system response, aiding to decrease the characteristic delay of WBC and, above all, it allows to reduce to zero the steady-state error due to deflection and other possible disturbances.

However, as already forecast, PI controller performances depend on system parameters, on the contrary of WBC. Changing proportional and integral parameters modify the over-elongation and settling time in the angular position loop.

4.3 IMPEDANCE WBC 3 AND FORCE-DRIVEN ACTUATORS

Impedance WBC used till now drives the system producing a position reference for the position control system of the actuator, which could be thought as an internal control loop closed and untouchable. This loop receives the right reference from WBC and produces the

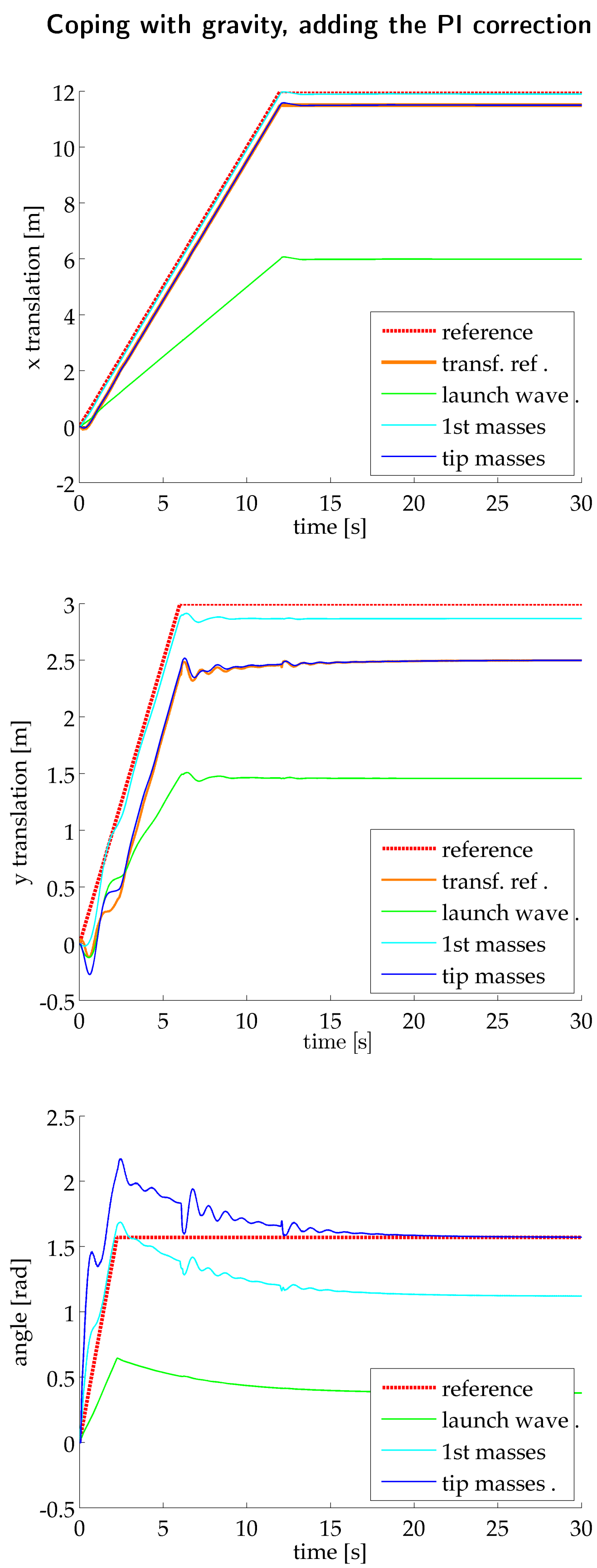


Figure 20: Behaviour of the system under gravity with filters in WB controller and PI correction

movement for the actuator. In the computational model the dynamic of the inner untouchable loop is represented like a simple first order transfer function, which bandwidth is settable by modifying the pole and gain of the first order filter, to model the actuator control system in each particular case.

This WBC idea (WBC type 2) is not exactly the best if it is required to control the actuator directly without any other controller interposition. In some cases it is mandatory to interface WBC just setting actuator references, because it is physically impossible (or inconvenient) to throw away the original controller of a certain device and rebuild a specific WBC controller.

However, this thesis wants to explore also new possible applications and systems designed to be controlled with WBC from the beginning. In the specific, if a person thinks about rockets controlling, which actuators are devices just able to work with reaction forces (mainly thrusters), it makes no sense to think in terms of inner pre-existent position control systems. But also thinking to manipulators, one can think to drive electric motors directly by force, instead to apply position reference to electric motor pre-existent control system. Therefore this section is going to deal with WBC of type 3, which allows to set directly the reference for the force in the actuator. In Section 2.2.3 it is explained the theoretical characteristics of WBC 3 and the difference from the type 1 and 2.

Moreover, WBC 3 formulation is more appropriate when the actuators are not grounded like in aerospace structures. For a floating actuator on a spacecraft it is particularly important to control input force or moment acting on it in order to get the desired motion and vibration damping.

4.3.1 *Physics and model of the force-driven actuators*

For all of these reasons, it was decided to apply and test WBC of type 3, which basically produces a force reference for the actuator, i.e works directly on actuator heart and physics. It does not matter if actuator is a pneumatic system, an electric motor or a thruster. In the electric motor case, the force will be the current to set in it. In the thrusters case, it will be the intensity of the propulsion.

Considering the second dynamic principle, the generic equation for each controlled axis, valid either for linear or for rotational case:

$$\begin{aligned} m_{act}\ddot{x} + f_x\dot{x} &= F_M^x - F_0^x \\ m_{act}\ddot{y} + f_y\dot{y} &= F_M^y - F_0^y \\ J_{act}\ddot{\theta} + f_\theta\dot{\theta} &= \tau_M - \tau_0 \end{aligned} \tag{23}$$

Therefore, the produced forces (or torque) in the actuator (F_M) will be physically detracted from the forces load of the flexible structure (F_0) and the resulting force will be double integrated to get the position of the actuator x, y or θ (viscous friction f is considered negligible). In Figure 21 it is shown the blocks to model the physic of the actuator. This model scheme already takes into account the main not ideal behaviours of the actuator, i.e. inertia and viscous friction.

In the first step, it is possible to consider a simple unitary gain for the *Actuator force dynamic* block, ignoring the physical limit of the actuator in the generation of force, in term of bandwidth and saturation limits. In Section 4.4 it will be analysed the effect of a not ideal dynamic for the generation of the force.

WBC calculates the position going wave (X_{-1}) like in the previous case. However this signal does not drive the actuator directly, but a virtual spring modelled by a gain block, which receives the position signal from the WB controller and gives the force signal (F_C) acting on the virtual spring. So the going wave is calculated like the virtual forces in a virtual spring under a virtual compression or extension (see Section 2.2.3 for more references).

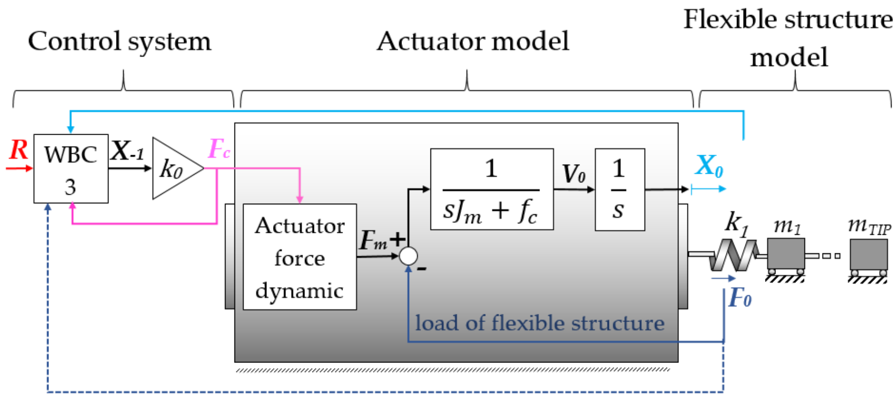


Figure 21: Physic model of the Actuator, driven by impedance WBC 3. Depending on flexible system and actuator parameters, the measure of F_0 could be necessary or not. With robot arm parameter is not necessary

In Figure 21 it is shown one of the three WBC loop, in which it is possible to see the virtual spring modelled with a settable gain block. The interesting point is that just actuator position is required as system measure for the controller. In his thesis, Habibi showed that it is not necessary to use measured returning forces like in WBC 2. He found that this is possible when the inertia ratio between actuator and the structure is very high. This result is not unexpected, indeed if one considers Equation 4.3.1, it is clear that if resistant forces (or torques) are much higher than actuator inertia (inertia of the rotor in case of electric motor), this means that inertial term is negligible and therefore, considering viscous friction negligible as well, it is obtained $F_m = F_0$. This implies the possibility to identify the resistance force of the structure by calculating required actuator force with WBC, with no need of measure it. The more the actuator inertia increases, the less is the possibility to identify the resistance force using just actuator inertia.

In Figure 23 results obtained with different inertia ratios are compared. It is considered to turn the structure of 90° with no translations on x and y. Therefore it is possible to compare system responses to a saturated ramp in the third WB controlled axis (angle axis). In this

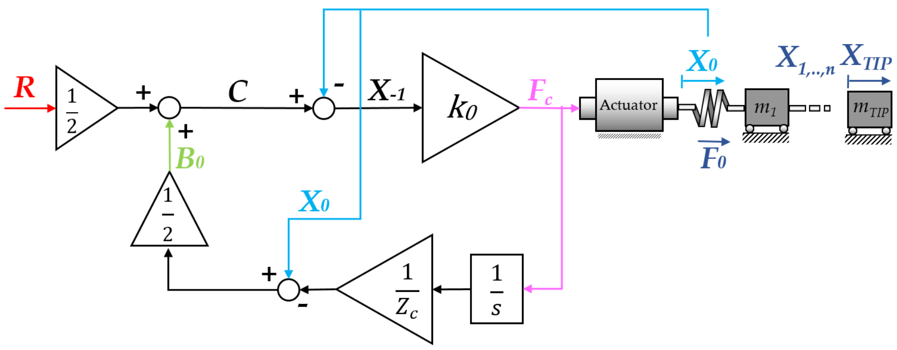


Figure 22: One of the three axis of the flexible 2D structure controlled by impedance WBC 3. With high inertia ratio between actuator and the flexible structure, there is no need of measure and consider the load F_0 to calculate the back wave B_0.

project it is considered to have the lumped masses of the structure of weight $m_1 = m_2 = ... = m_10 = m$ and each actuator moving part of weigh $3m$, so the inertia ratio is $r_m = \frac{m_0}{m_1} = \frac{3m}{m} = 3$. So the result related with the parameters of this case of study is the graph b of Figure 23.

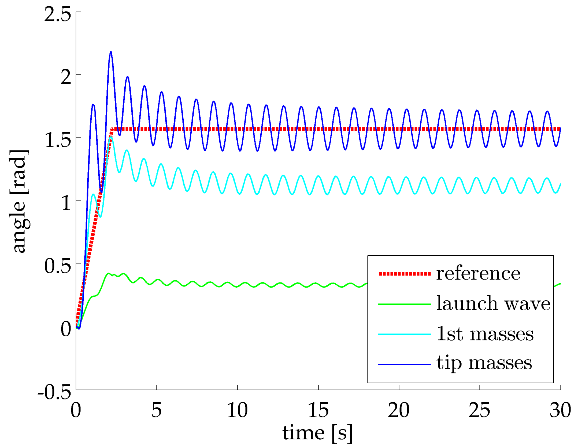
Considering robot arm case, the inertia of the actuator is all the mass of the moving rigid part of the actuator, which is connected to the rest of the structure by some springs. Those springs transmitted the forces between the actuator and the flexible structure.

4.3.2 *External disturbances rejection with WBC of type 3*

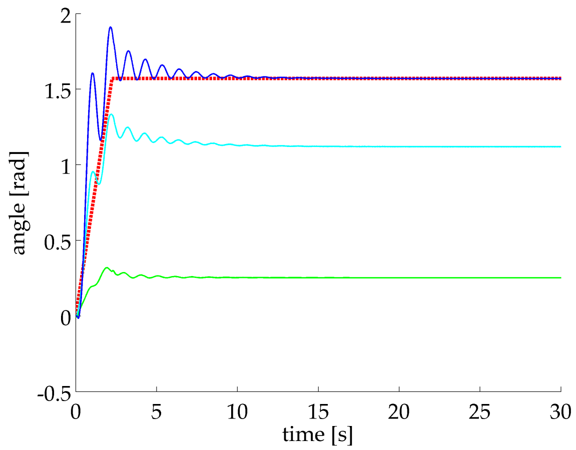
Problems faced and solved in Section 4.2.1 for WBC 2 controller present again themselves in WBC of type 3. Now it is not possible to use the same solution as for WBC 2 because, if it is considered the simplified WBC 3 scheme, it is not possible to identify the disturbances without measuring the force which come back from the structure.

Therefore another strategy is looked for. Habibi proposed the following modification to the control system, in order to modify the back wave, adding the information about external disturbances.

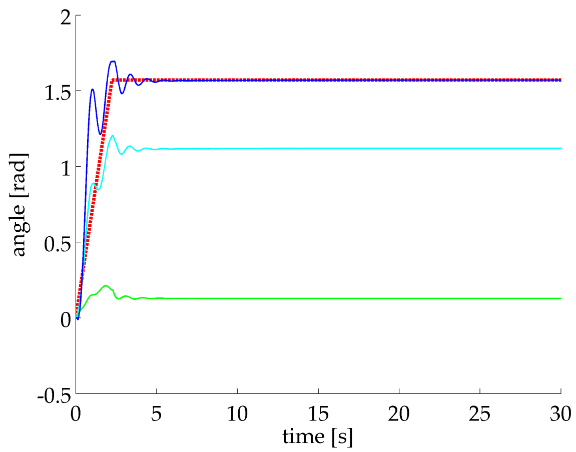
Angles movements of the trasversal axes of the structure



(a) $m_0 = 10m$



(b) $m_0 = 3m$



(c) $m_0 = m$

Figure 23: Results with different values of actuator mass, turning the structure of 90° with null translations on x and y. It is possible to see that when actuator mass decreases (and so the inertia ratio), the response of the system improves.

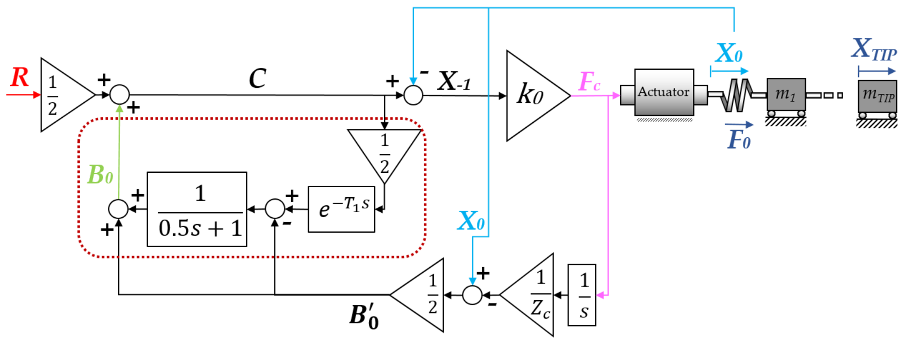


Figure 24: Impedance WBC 3 with external disturbances filter, highlighted with red dash line

The interesting aspect is that information are taken both from the launching and return waves, which contain all the possible steady-state error due to external disturbances and cross-coupling. The theoretical idea is very similar to that one used into WBC 2 control system: to make a delayed version of the signal, subtracting the original back wave and filter the result with a transfer function, which cuts high frequency. The result is added to the original return wave, obtaining the new return wave.

The difference is that now delayed information are taken from the launching wave, compared with return wave and the result filtered. As for WBC 2, delay time and low pass filter parameters are not of big concern. A simple first-order low pass filter is ok and can be $\frac{1}{0.5s+1}$. The 1/2 block gain is required because b_α is half the reference at steady-state, so it is mandatory to adapt the signal c_α to b_α.

The formula for the new return wave is therefore the following:

$$B' = LPF(s)[C(s)e^{T_1 s} - B(s)] + B(s) \tag{24}$$

where $LPF(s)$ is the low pass filter.

It is possible to apply also to WBC of type 3 the same method used for impedance WBC 2, in order to get zero steady-state error. Comparing the reference with the measured position of the tip mass, it is possible to modify the reference for WBC using a PI controller, with the same consideration made in Section 4.2.2. In Figure 25 it is possible to see the modifications introduced to each of the three impedance WBC 3 control loops.

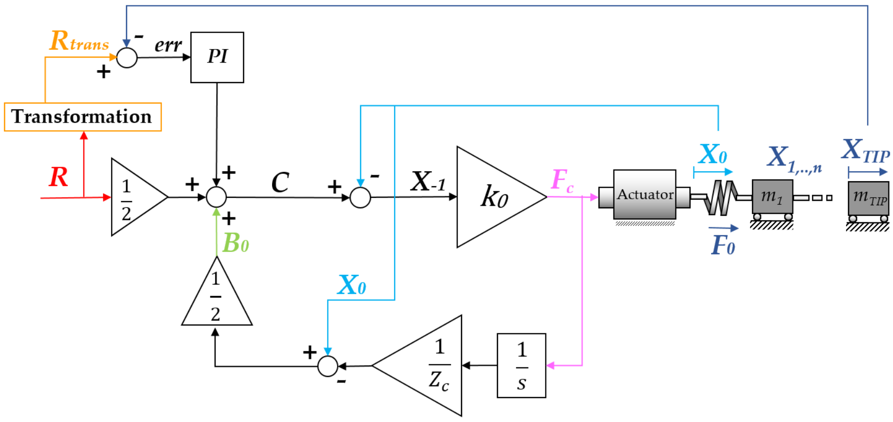


Figure 25: One of the three axis of impedance WBC 3 with PI correction in order to get zero steady-state error on the tip mass.

4.4 REAL ACTUATORS

In Section 4.1.7 problems about actuators constraints are explained and a simple low pass filter model for WBC of type 2 is shown. After the modifications and improvements on the structure model and control system, it is necessary to take into account other possible problems that actuators limitations could add in the system. In this chapter some results about considering actuators dynamics are presented, for both WBC 2 and 3.

For WBC 2 the model is very simple, just a first order transfer function for each actuator on x, y, θ axes. (see Figure 14).

In the WBC 3 case, the model is a little complicated, because actuators need to transform force command into position output. In the last section the mechanical side of the actuator required for WBC 3 was already modelled. The inertia and friction of the actuator were already taken into account during the implementation of the model, considering Newton second dynamic law in order to convert force into position. However, it was supposed so far that the actuators could generate the force (or torque) exactly as asked by WBC of type 3. This consideration is clearly unrealistic, because each actuator has its own limitation in the production of the power, characterized by saturation and response bandwidth.

4.4.1 *Modelling real force-driven actuators*

In order to model actuator physic limitations, it is possible to add a low pass filter with steady-state unitary gain to simulate the limited bandwidth and a saturation block to cut WBC requests if they are too high in comparison with the capability of the real actuator considered (see Figure 26).

The bandwidth to consider (ω_{BF}) is related with the type of actuator to deal with. Considering robotic arm, the more common type is electric motor, one for each axis to control (x, y, θ). From a general point of view, bandwidth depends on the way the force is produced. So, for electric motor, it is the current loop dynamic, usually very high ($500 - 1000\mathrm{Hz}$). If actuator is pneumatic, the dynamic decreases, becoming very low in reaction motors, in which it is difficult to set the intensity of the driving force very fast. In Chapter 5 it will be taken into account the problems related with this kind of actuators, both for modelling and for controlling the system.

Making the simulations with these considerations, with a typical electric motor current bandwidth, showed the WBC is still able to control the flexible system with good performances. In the following section, there are the main results obtained with different configurations.

4.4.2 *Control of the structure using WBC applied to real actuator*

As one can forecast, putting a low pass filter where the launching wave passes through, makes impossible to damp high frequency os-

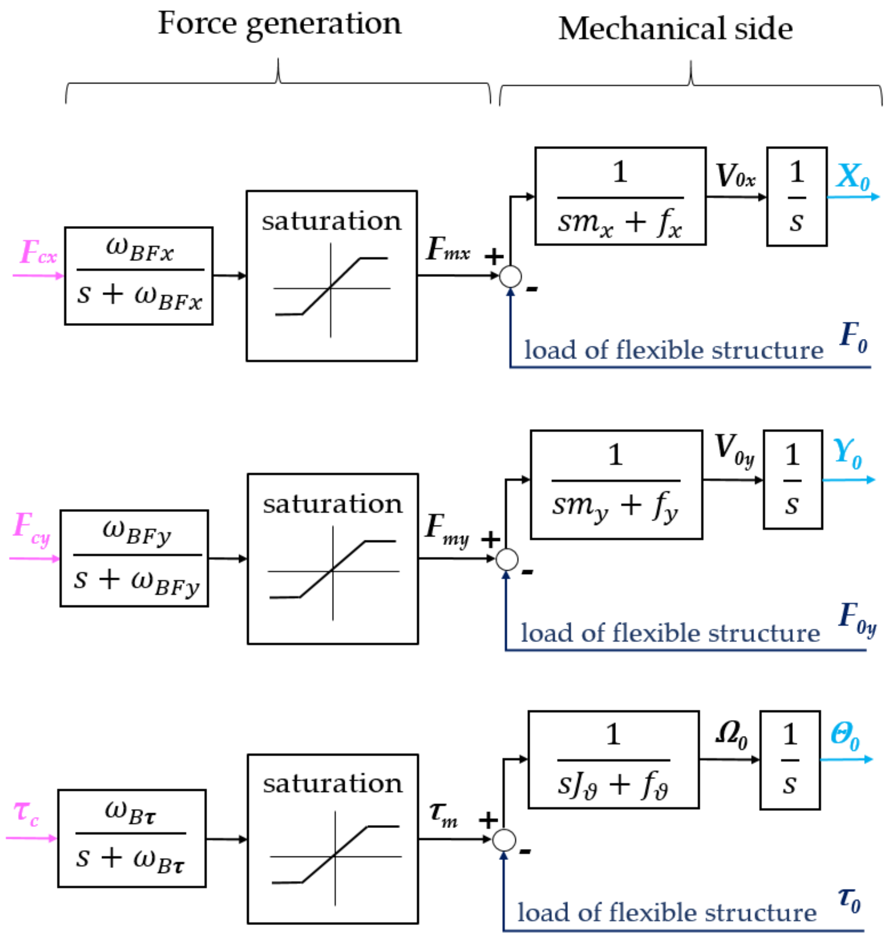


Figure 26: Real actuators model. It is considered the dynamic of the actuator to generate the WBC required force and the mechanical side, where the force is transformed in a position of the moving part of the actuator.

cillations in the system, because the high frequency damping effect elaborated by WBC are cut by the filter. It could happen if one considers a first or second order transfer function to model the bandwidth limitation of a real actuator. This could be a problem, and makes the system unstable, if there is absolutely no damping in the flexible structure. However, such a not-damped structure does not exist in the real world, because every material has some intrinsic damping in it. Even if one imagines to build a real lumped masses and springs structure, this kind of system has a minimal damping, localized on the steel of the springs, due to hysteresis.

There is no physical point in trying to model a real actuator, which cuts WBC demands of high frequency dynamic, and an ideal zero-damped structure. But, of course, if it is possible to control the zero-damped structure, it will be possible to obtain better result with partial damped structure, because damping aids to cut high frequency dynamic of the whole system.

If real actuators are considered, it has to be taken into account also that in a real flexible structure there is some damping and this aids to cut high frequency dynamic of the whole system. WBC has again proved to easily cope with this actuator limitations, as it is possible to observe in Figure 27, where, just putting a damping coefficient $\xi = 0.01$, the system is well controlled even considering a 80Hz band limitation for the forces. In Figure 28 the simulation is done considering 3 filtered ramps. This expedient solves the problem of dynamic-coupling, because decelerations are softer.

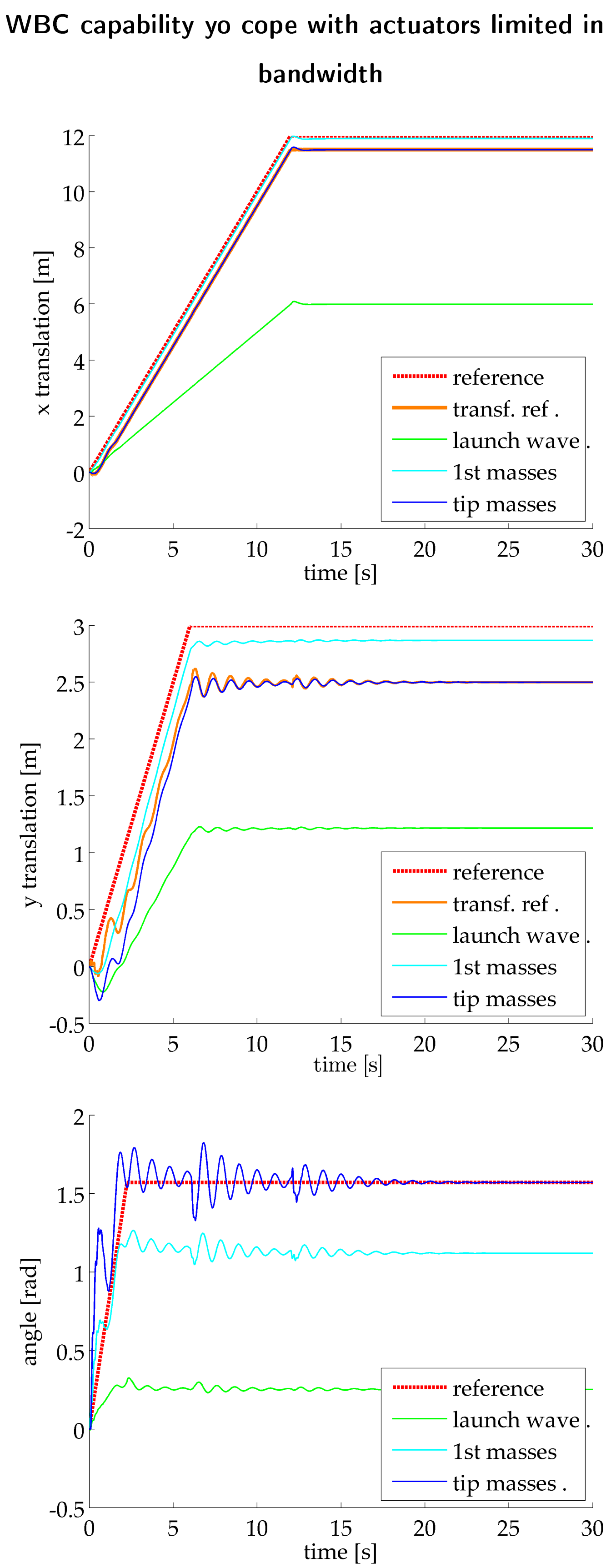


Figure 27: Simulation results using WBC 3, with actuator limited in force bandwidth to 80 Hz.

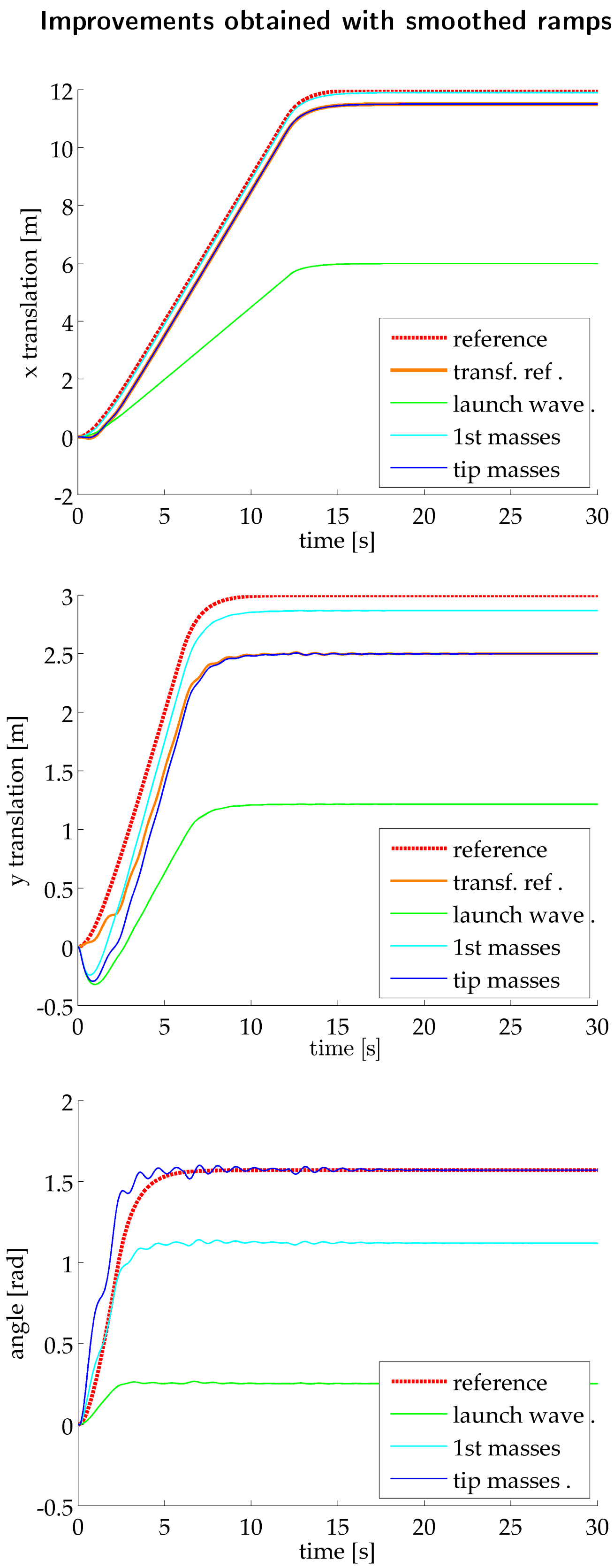


Figure 28: Simulation results using WBC 3, with actuator limited in force bandwidth to 80 Hz and smoothed saturated ramps.

5

ROCKET STRUCTURE CONTROL WITH WBC

Figure 29: Arianespace Vega rocket for the European Space Agency

5.1 MODIFICATION TO MODEL ROCKET STRUCTURE

In this chapter the 2D model of the flexible robot arm will be modified in order to represent the main vibrational modes of an aerospace rocket stage. The reference launcher is the Arianespace Vega first stage, which is moved by a single thruster on its bottom. All the required technical data of the spacecraft are taken from *Vega User Manual* [11]. In Figure 29 there is the Vega launcher used by the European Space Agency. The first stage considered, called P80, is the thick one in the left of the figure, which has attached the main thruster.

The target of this project is not to make an accurate model of Vega spacecraft, but just to simulate a structure which could represent the flexibility of the rocket with an adequate approximation, in order to have similar dynamic behaviour in term of inertia and vibrational modes. In Figure 30 there is the 2D lumped flexible model considered, with 2 different configurations for the actuator: two thrusters and one thruster. In the following sections those two models will be explained and compared.

5.1.1 *Typical parameters of the rocket*

For this reason, information about the magnitude order of the mass and dimensions are taken from the *Vega User Manual*, considering P80 stage, the first part of the rocket [11], chapter 1, page 6.

Once the values of the masses are fixed, the springs stiffness are tuned to have 2Hz as the main vibrational mode on the lateral axis of the structure. This frequency is typical of launchers of the same type of the Vega, which however are lighter and therefore more flexible in the transversal direction. The assumption is made in order to use a worst case model to test WBC capability of dealing with it.

Comparing with the robot arm case of study, the two dimensions structure is now stiffer in longitudinal direction and softer in transversal direction. This modification is made considering a possible real rocket structure, which has low frequency vibrational modes in the transversal direction and much higher in the longitudinal one. This fact implicates the need of a different kind of trajectory to test oscillations of the system and find out how WBC deals with them. In Section 5.1.2 it will be explained different references considered for the control system.

A specific simulation was made to test vibrational modes of the structure. A chirp signal was exerted directly on the actuator without using WBC. System response becomes higher when input frequency matches the structure main vibrational mode. In this way was possible to tune springs stiffness and masses to get the required resonance frequency, using parameter comparable to *Vega* first stage.

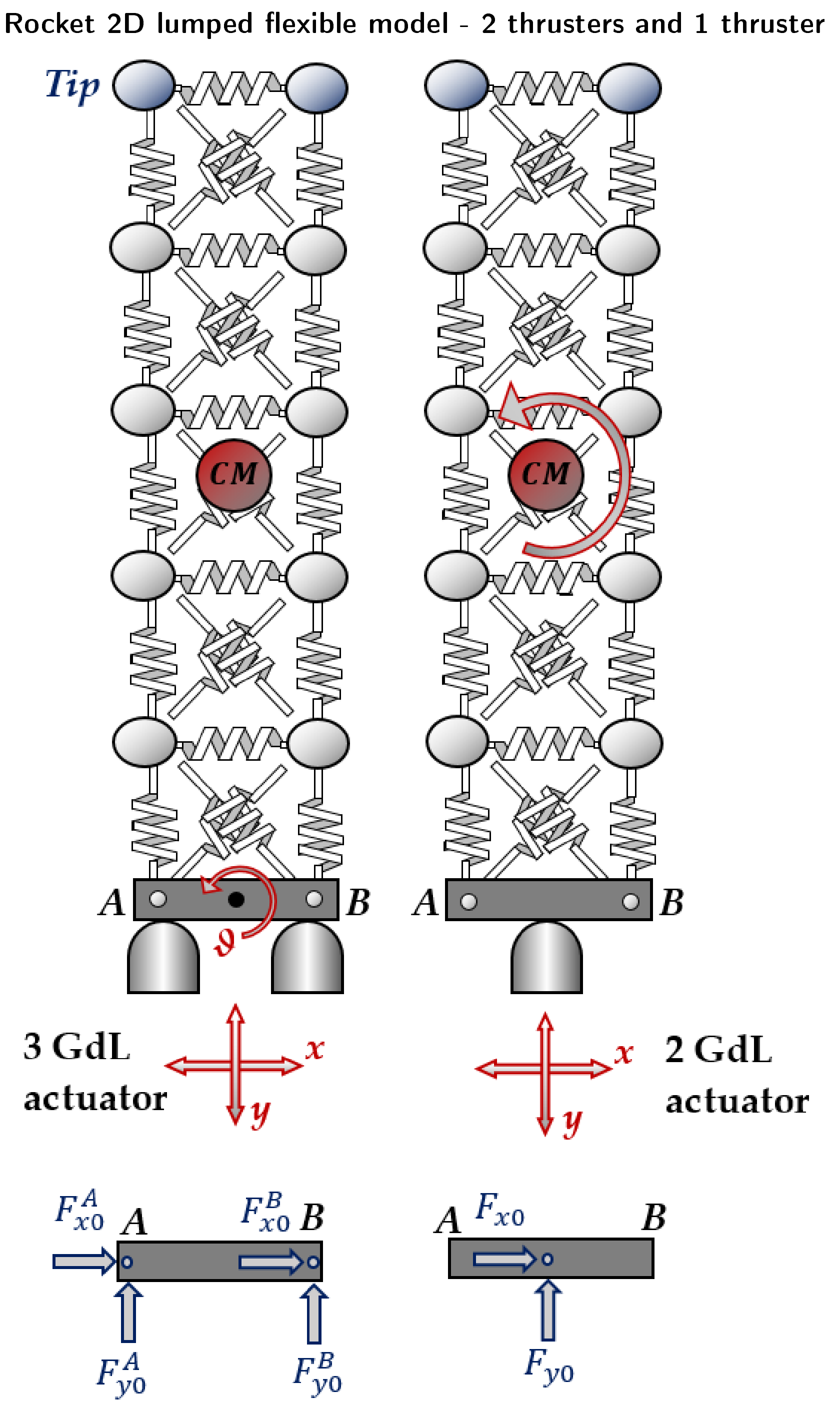


Figure 30: 2D lumped flexible model for the launcher. Two thrusters in the left and one thruster configuration on the right. It is shown the centre of mass of the structure, taking into account the actuator mass. Two thrusters configuration as 3 DoF x, y, θ and one thruster configuration 2 DoF x, y.

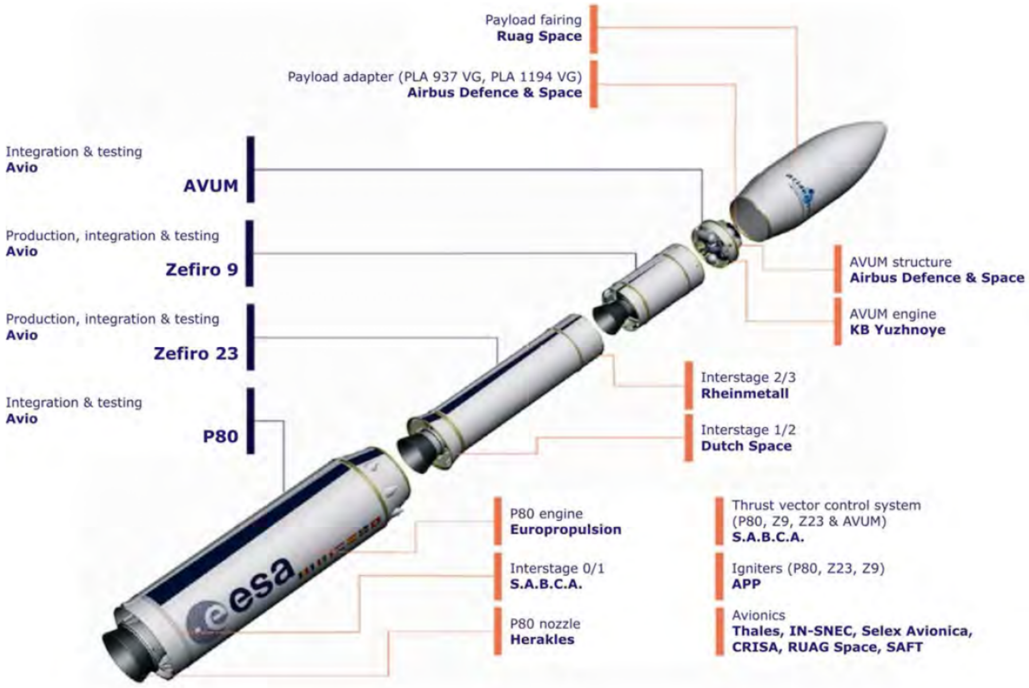


Figure 31: Stages and components of Arianespace Vega rocket

In Figure 33 it is possible to see result of the simulation with a chirp signal which drives the actuator of the structure. Red line represents how the value of the frequency of the sinusoidal signal changes during simulation, from 0.5Hz to 3Hz. Green line represents the time whereby input sinusoidal signal has the frequency of 2Hz, so time whereby it is desired to have the resonance of the structure. One can observe that the wider amplitude of the tip oscillations is around the wanted frequency, marked by green line.

5.1.2 *Trajectories*

Changing the model from robot arm to aerospace launcher requires to modify reference trajectory as well, for three reasons: the first one is related with making reasonable simulations, using a trajectory coherent with a launcher. The second reason concerns the physical limits of such a heavy and big structure like this one and the third reason is

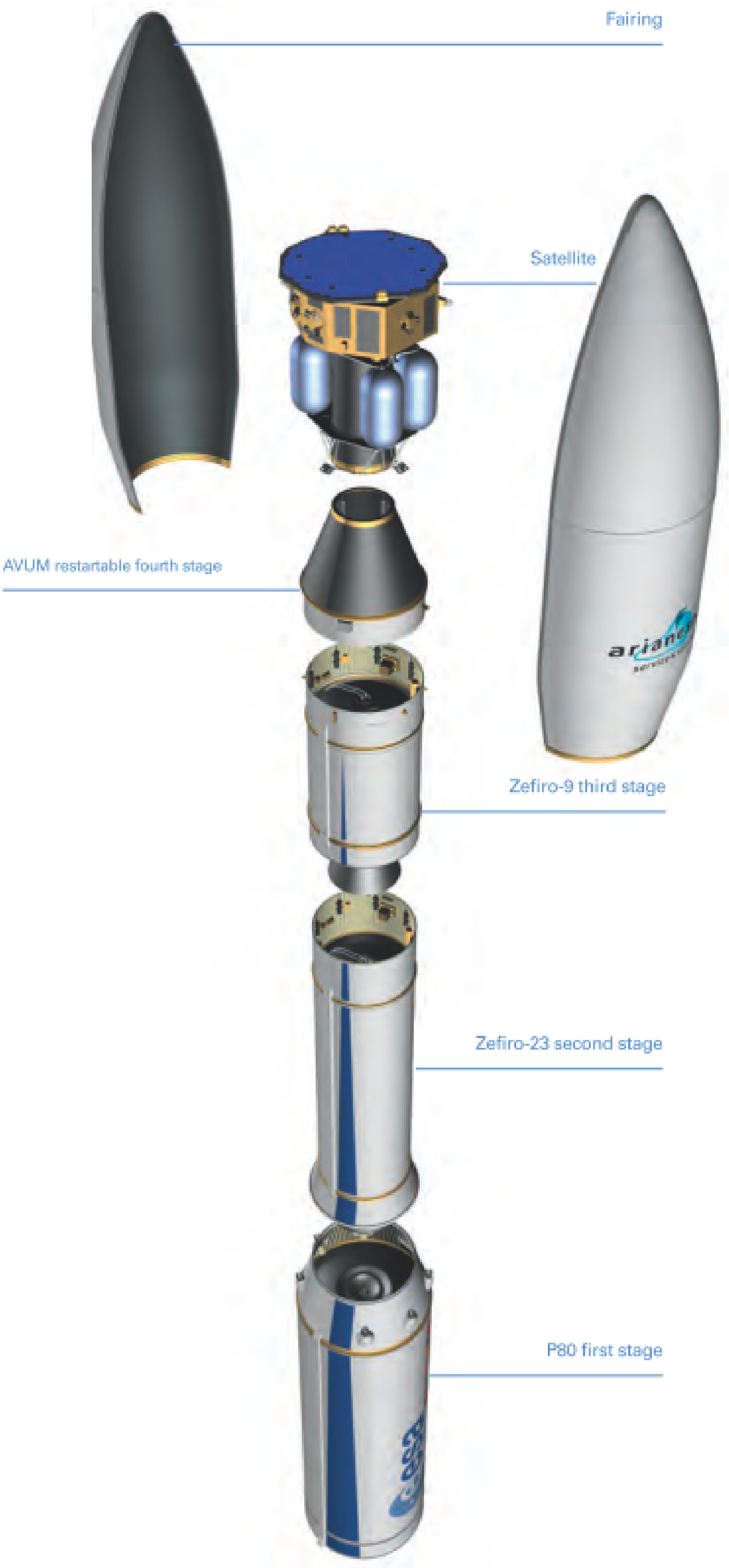


Figure 32: Stages of Arianespace Vega rocket

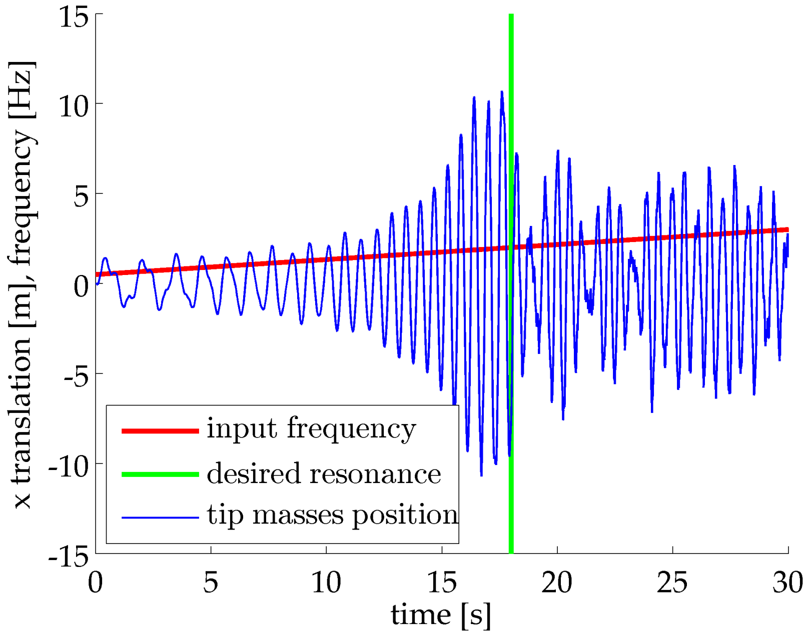


Figure 33: Simulation of rocket structure driven by a chirp signal and without control. Red line represents the value of the frequency (in Hz) of the sinusoidal input signal and green one the desired resonance of the structure.

due to the flexible model considered for the launcher, which is softer in transversal direction and stiffer in the longitudinal one.

A lot of possible trajectories could be considered to simulate a launcher and its movements. Again it is repeated that the target of this thesis is not to make an accurate simulation of a spacecraft rocket and its trajectory, but to test WBC capability to control such a system with comparable shape and similar mechanical parameters of the structure and even considering it more flexible, adding more challenge to control it. For all these reasons, just trajectories useful to test the structure in the worst condition are taken into account.

The third reason explained in the first paragraph implies that there is no point in simulating a trajectory with a direction parallels to longitudinal axis of the structure, as a rocket launching could be thought. In other words, simulating vertical launching of the rocket does not give any interesting result to test WBC, because no significant vibrational modes are involved.

The first reference considered was the saturated position ramp for x, y and θ, used for the robot arm. However this set of trajectories was proved to be not useful to test and control the structure. There is no point in moving in y direction, because just transversal axis of the structure has low frequency to damp. The target is to excite the low frequency vibrational mode and trajectories are chosen with this aim. This is obtained by moving in the direction parallel to the transversal axis, i.e. x axis.

A simple one dimensional x-axis rest-to-rest movement revealed very good to test the rocket structure driven by double thruster, with disturbances, parameter variations and other possible modifications to environment and structure model. The target in this case is to keep the rocket in the vertical position, while gravity and other disturbances are acting.

Another trajectory used to test the controller with a single thruster is a trapezoidal position movement on the angle. This reference puts the system in a non equilibrium position, useful to see if the modified WBC is able to control and stabilize the system.

Thinking of a real aerospace launcher, this trajectories could be considered as the path that one could see being on the rocket, watching a relative movement of the structure, from the structure itself. So it is possible to imagine that rocket is following its own wide trajectory on space. It has been just introduced a modification in this trajectory or external disturbances, in order to excite vibrations.

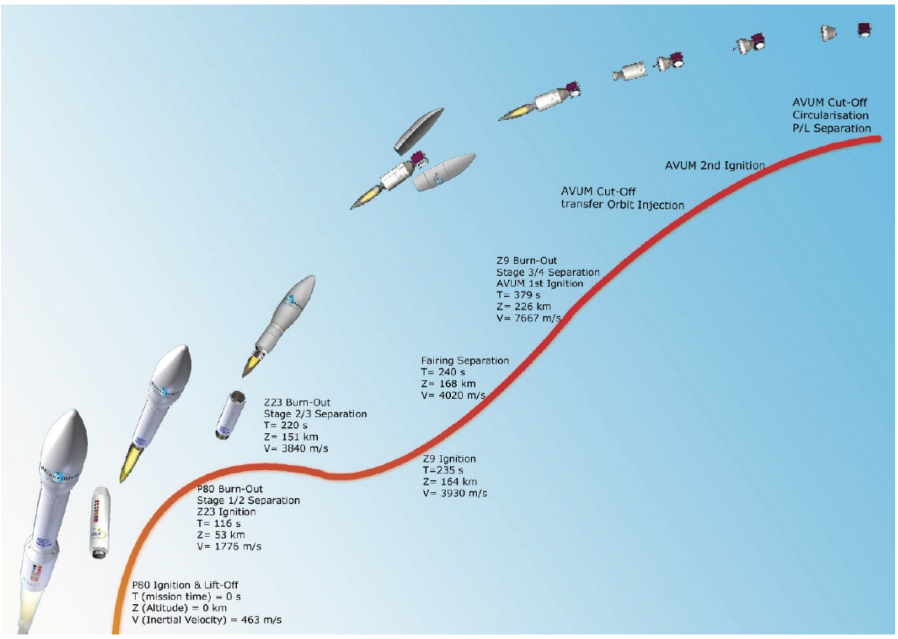


Figure 34: Graphic of the Vega launch. It can be observed that P80 stage detaches in the first part of the launching, remain almost vertical.

5.2 TYPICAL ROCKET ACTUATORS: THRUSTERS

In Figure 35 launchers family used by European Space Agency are shown. It is possible to observe that different numbers and types of thrusters are used in a rocket. As this chapter tries to control a 2D flexible structure similar to Vega, which has just one central thruster on the bottom, final target is to apply WBC to a single centred actuator on the bottom side of the lumped model.

Figure 35: Family of launchers used by the European Space Agency. It can be observed different kind and number of thruster on their bottoms.

However there are also rockets with two thrusters and it is possible to model these actuators in the same way as it was done for the robot arm model. In this case it is possible to imagine to have the same 3 DoF actuator like in Chapter 4. This assumption is made for the first simulations, using a control system very similar to one used in that chapter.

If we consider to have just 1 orientable thruster in the middle point of the base below the structure, like in the Vega P80 stage, it is required to figure out how to produce the necessary torque below the flexible system, to control it, because force applied to this position has no arm on the base, but it could have arm on the structure, producing a torque on it. In Section 5.6 this problem will be afforded.

5.3 MODIFICATION IN THE CONTROL SYSTEM

The control system is based on the impedance WBC 3 designed to control robot arm, just considering to scale the parameters of the controller, in relation with the parameters of the structure. There is no need of big modifications, or rather there is need of simplification in the controller. The specifications on path following and disturbance rejections are lower than in robot arm case. Here stability and robustness are much more important than high dynamic.

For this reason, the PI controller designed in the previous case of study is deactivated, since it reduces the stability margins. Indeed, as explained in Chapter 4, PI controller has high dependence on system parameters (which are more unknown in this case). Moreover, the specifications of the problem do not require a perfect path following, but, rather, robustness on parameters changing and other not ideal behaviours of the system, considered in the following paragraphs.

Except for PI correction, the controller remains basically the same used for the robot arm if a similar 3 DoF actuator is used. That is a 3 impedance WBC loops, one for each actuator. Each loop has the filter for external disturbances (gravity first of all), which eventually solves also cross-coupling problems.

In Figure 36 the model for a flexible not-damped rocket structure is moved without applying WBC. Vibrations are not damped in such a not naturally damped structure. Applying WBC to the same system, vibrations decrease rapidly, as one can see in Figure 37. In Figure 38, Wave-Based controller is tuned with different coefficients, making virtual springs on x and y axes softer. With this tuning, vibrations are dampened very fast, and a negligible steady-state error appears in y.

In the end, when just one thruster will be considered, the 3 WBC loops will need to be changed as well, in order to deal with a different kind of force and torque generation. In Section 5.6 the problem of controlling the structure with one thruster will be afforded: the new challenge will be the impossibility to generate a torque on the base of the structure, as it was done till now to control the torque load of the structure. So WBC output shall be changed in order to fulfil what this new kind of actuator needs as reference.

5.4 ROBUSTNESS OF THE WBC TO PARAMETERS VARIATIONS AND EXTERNAL DISTURBANCES

In this thesis it was already showed the WBC capability to deal with to 2 dimension lumped structure, driven by a three degree of freedom actuator. It was observed that there is a small cross-coupling of motion but WBC is able to manage it and, after some improvements to the control system, even the rejection to external disturbances is very high.

Until now it was supposed to have perfect measurement of all the required physic values (forces, torques, positions). However this is unrealistic because all real measures contain some types of noise, not always negligible. Moreover some measures could not be available on some particular systems.

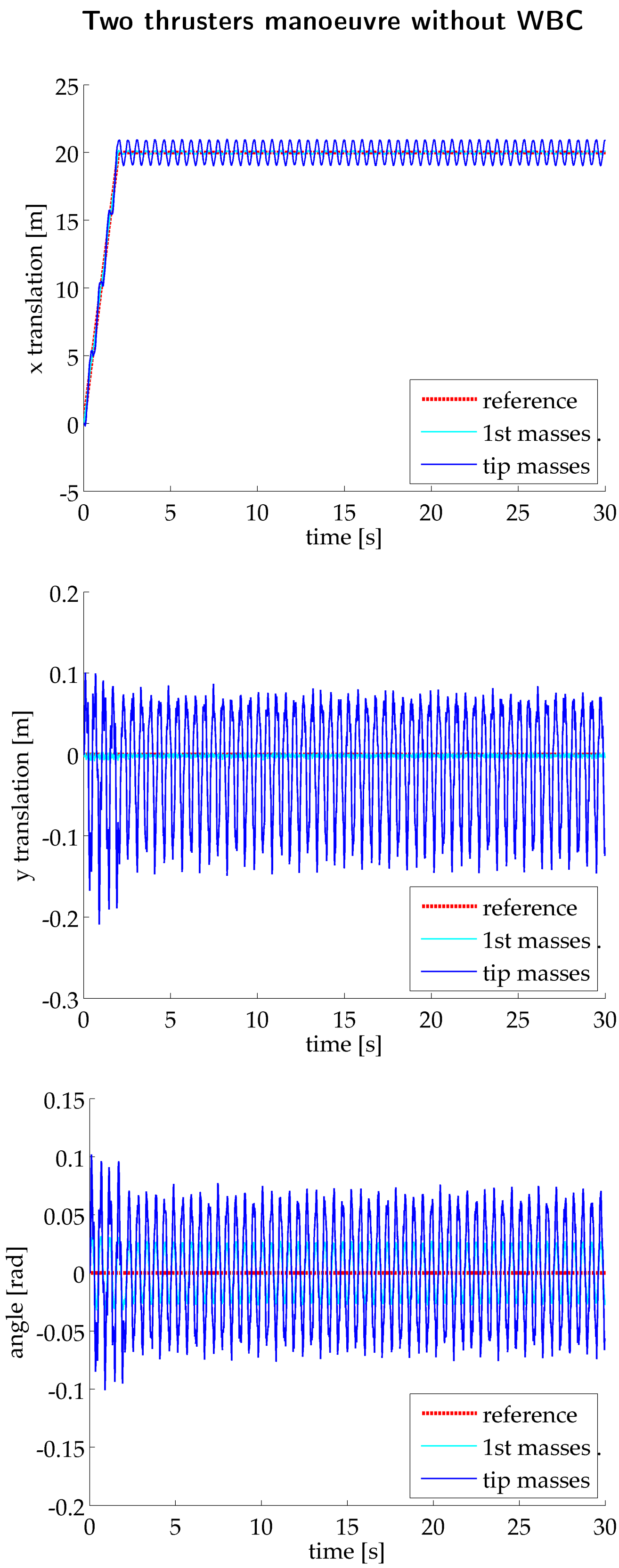


Figure 36: Two thrusters launcher manoeuvre, without Wave Based Control. Reference trajectories are put directly into actuator control system. The structure has no passive damping.

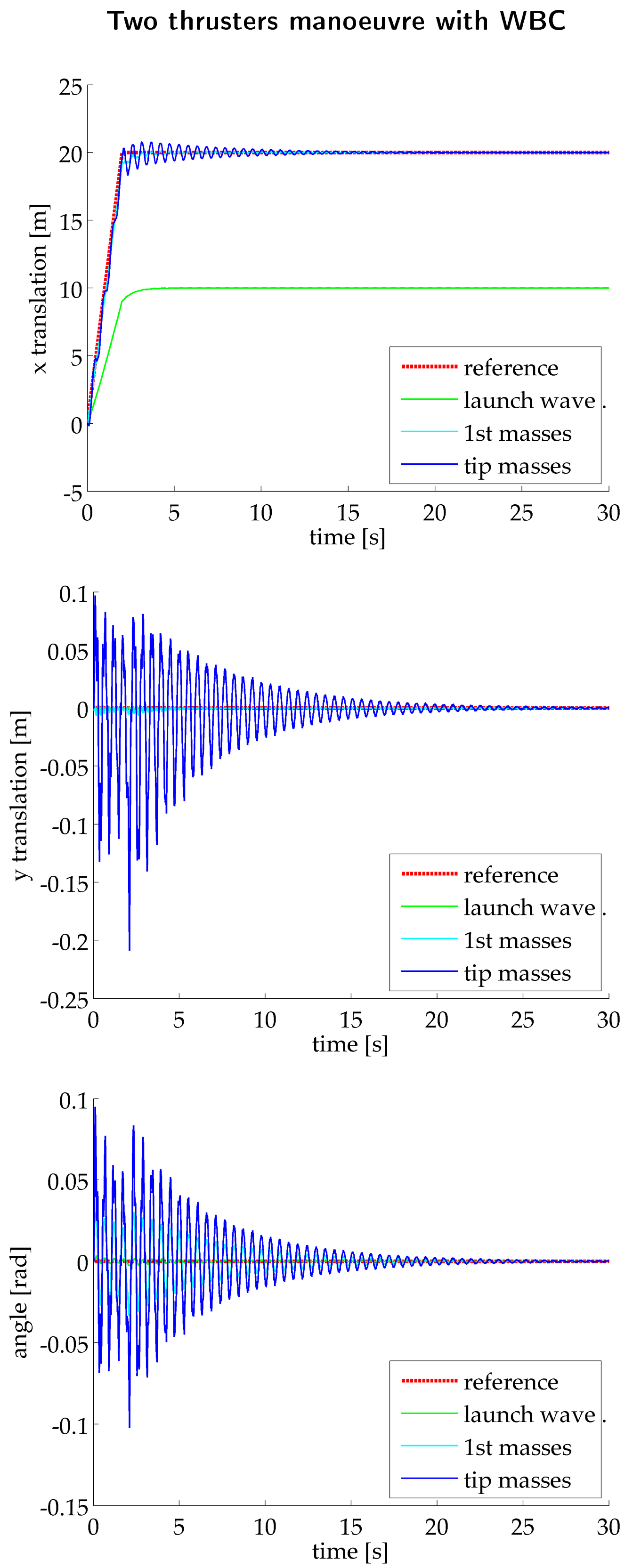


Figure 37: Two thrusters launcher manoeuvre, using impedance WBC of type 3. The structure has no passive damping.

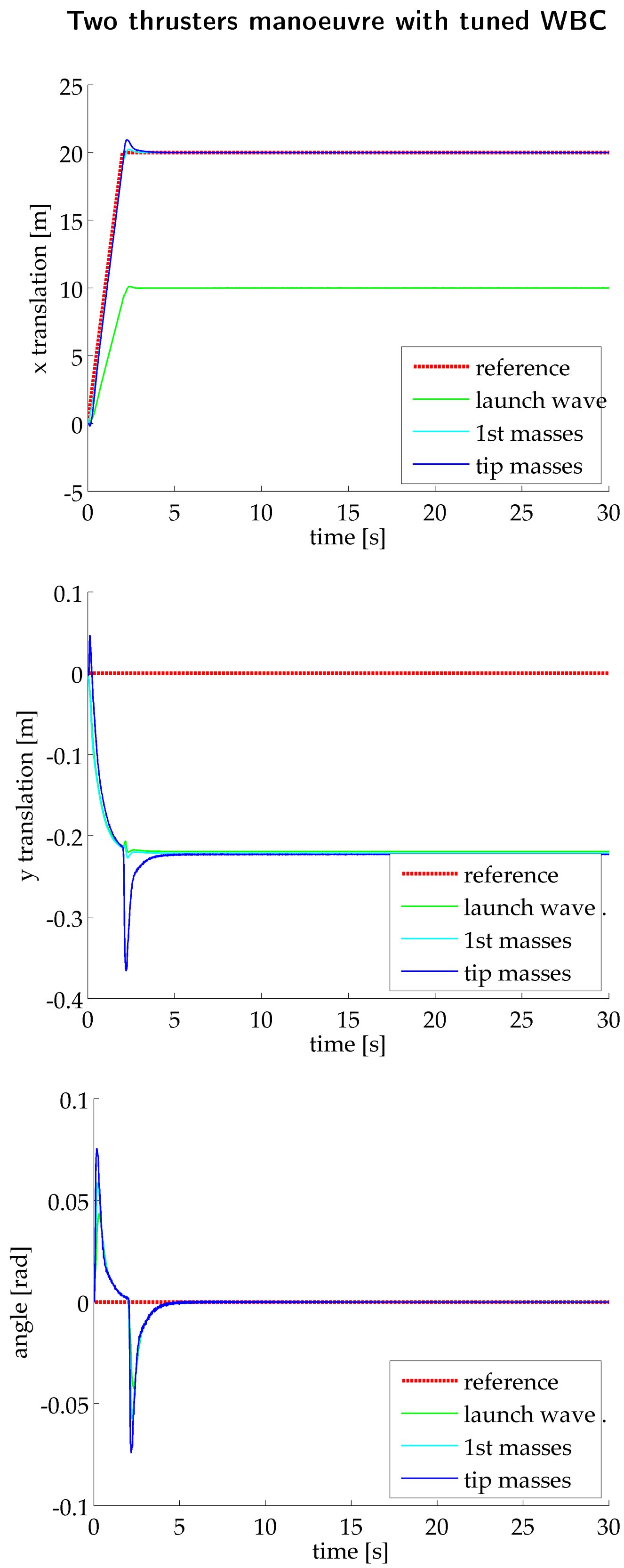


Figure 38: Two thrusters launcher manoeuvre, using impedance WBC of type 3. WB controller coefficients are tuned in different way, decreasing virtual spring stiffness on x and y axes. x-response improve but there is a small steady state error on y position (negligible comparing with rocket dimensions). The structure has no passive damping.

A strength of WBC is the capability to control a mechanical structure without needing a lot of parameters and measurements, just knowing the return wave. Therefore it is sought to analyse WBC behaviour in a system where minimum number of sensors are present and, moreover, in which there are some measurement noises.

5.4.1 *Parameters uncertainty*

It is now supposed some parameters on the structure are not measured well or under/over estimated or even they change during manoeuvre. WBC already showed to be very robust to this changing in one-dimensional system. Now the robustness is tested on a 2D rocket-like structure.

The results obtained prove again the WBC capability to manage systems which parameters are not completely known. The simulations of such a case of study were made considering to have WB controllers tuned to the supposed parameters of the structure (same of the previous case) and changing structure parameters while keeping controller tuning fixed.

For example, considering the unitary mass 2 time as before, controller still works and remains stable, just with performances decreasing. Trying to keep the same mass unit and increasing springs stiffness, system response improves, as one can figure out because the structure becomes more rigid, so vibration problems become less important. In Figure 39 and Figure 40, respectively, two values of masses and two values of springs stiffness are compared, having the same tuning of WBC and all the other structure parameters equal to Figure 37.

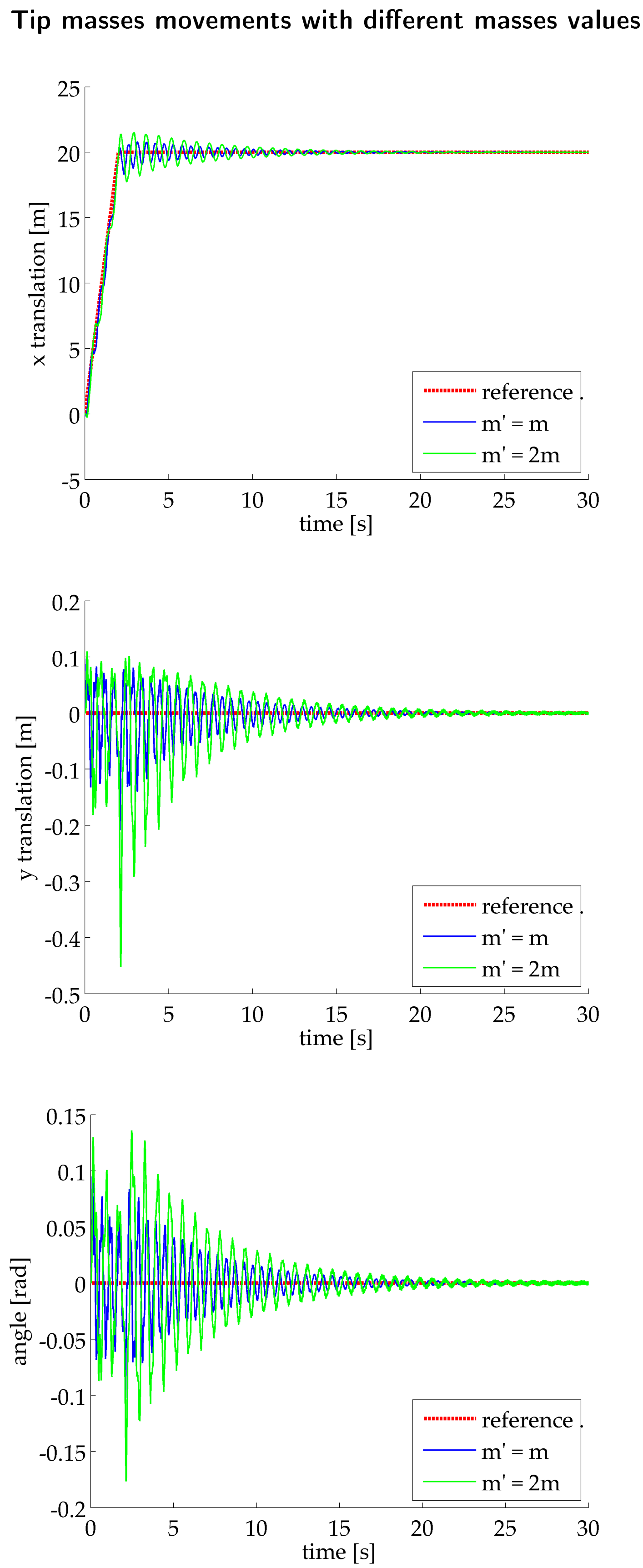


Figure 39: Two thrusters launcher manoeuvre, using impedance WBC of type 3, considering each mass in the system the double of before. WB controller is tuned on the previous masses values. The structure has no passive damping.

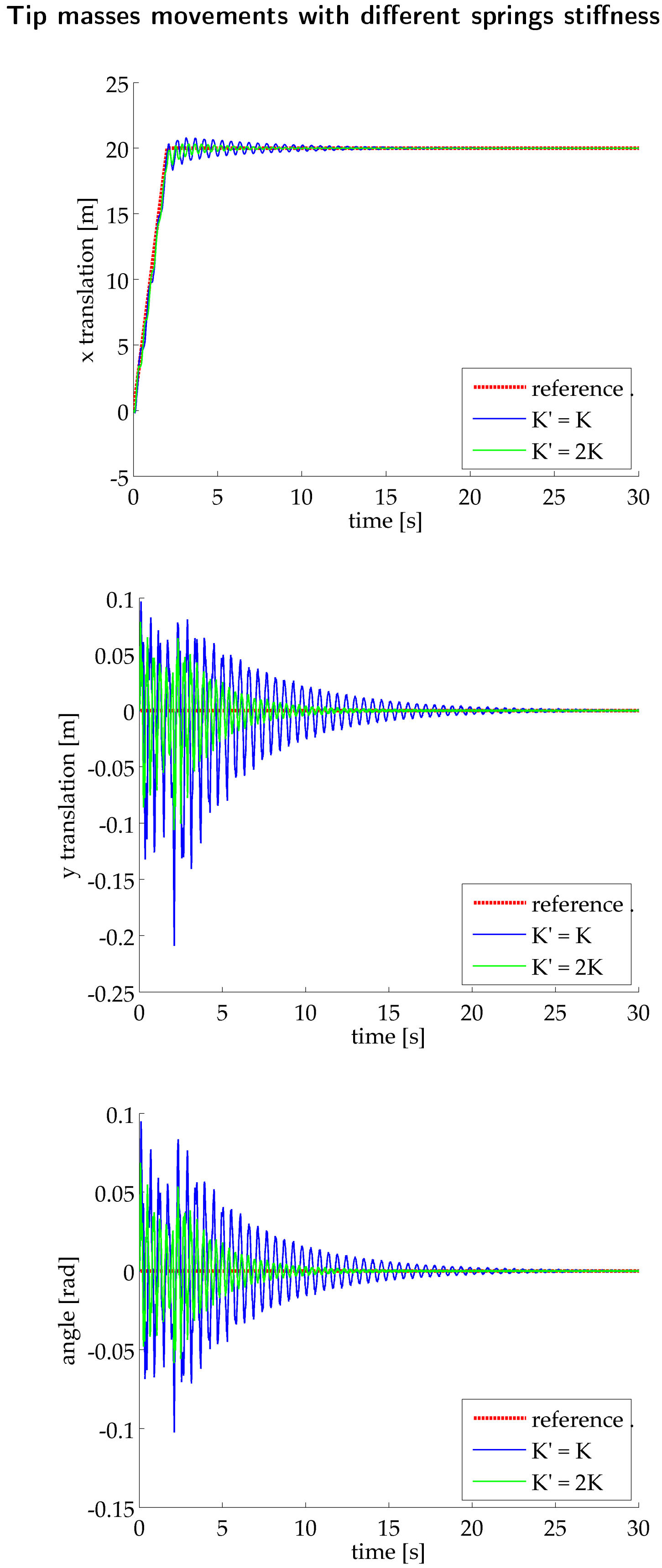


Figure 40: Two thrusters launcher manoeuvre, using impedance WBC of type 3, considering each spring stiffness in the system the double of before. WB controller is tuned on the previous springs values. The structure has no passive damping.

5.4.2 *Measuring errors*

It is now required to test system behaviour considering to have some noises in the measurements. The way to simulate noise is very simple and consists in adding the desired noise error to the clear signal. The only measure the Wave-Based impedance controller of type 3 needs in this configuration is the actuator position on each axis (x, y, θ). In the previous simulations just clear signals, originated from numerical calculations, was used as the positions measures WBC needed.

The measuring error is simulated by a source of band-limited white noise, which output pass through a low pass filter in order to simulate the noise signal due to quantization. Each position has a different noise power, which depends on the range of measure on that dimension. It is considered a range of $20\,\mathrm{m}$ for x, $10\,\mathrm{m}$ for y and $\pi\,\mathrm{rad}$ for θ. Two different quantizers are taken into account: 8 bits and 12 bits. WBC is able to deal quite well with the 8 bits quantizer, showing just some low amplitude vibrations at steady-state (Figure 41), which does not compromise the stability and easily damped in a real system. Applying 12 bits one, the system behaves with good performances (Figure 42). Figure 41 and Figure 42 show result obtained with a x-ramp saturated at $10\,\mathrm{m}$ instead of $20\,\mathrm{m}$, so it is possible to see better the effect of the quantizer at steady-state. It is just a geometric scale strategy to have a zoom on the signal. It does not affect the output, since the deceleration at the end of the slope is the same as before.

5.4.3 *External disturbances*

In this section external forces are applied to the structure to see how WBC deals with them. Gravity is already considered during previous simulations and Wave-Based controller of type 3 already has the filter introduced for the robot arm case in Section 4.3.2.

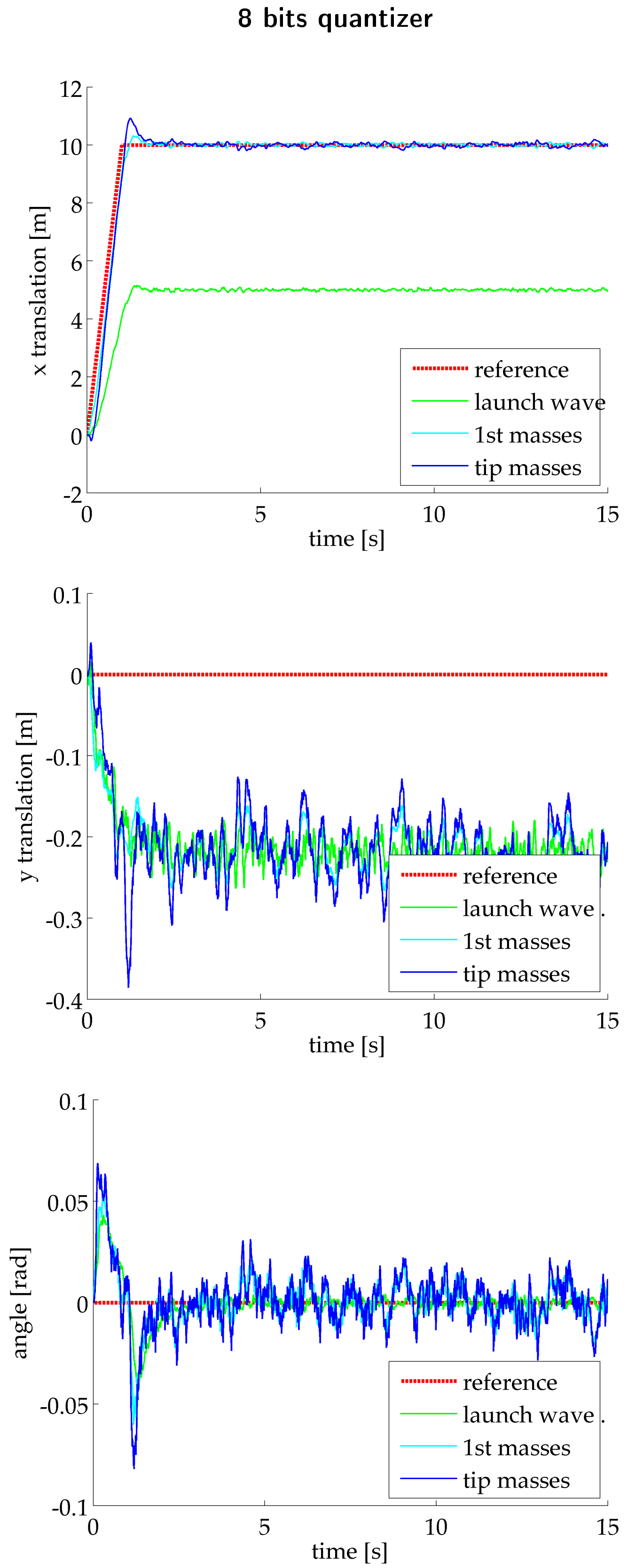


Figure 41: Two thrusters launcher manoeuvre, using impedance WBC of type 3 and considering a 8 bits quantizer on the actuator position measurement. WBC parameters are the same used in Figure 38. x-trajectory is limited to 10m in order to see the small steady-state vibrations that are not dampened with this ADC.

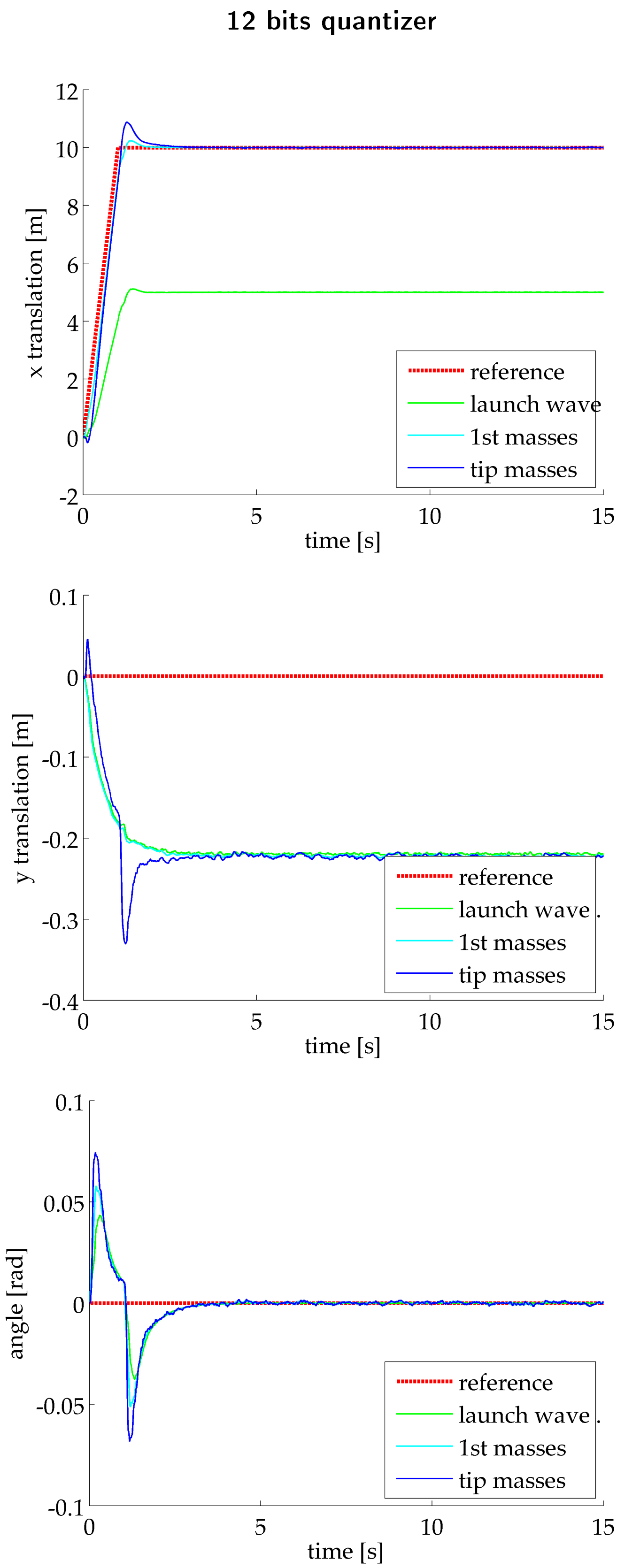


Figure 42: Two thrusters launcher manoeuvre, using impedance WBC of type 3 and considering a 12 bits quantizer on the actuator position measurement. WBC parameters are the same used in Figure 38. x-trajectory is limited to 10m in order to see that there are no steady-state vibrations with a 12 bits ADC.

A similar test done for the robotic arm (Figure 18) is considered here. A sequence of external impulsive forces is applied to the top left mass of the lumped model, with an intensity 10 time higher than the weight force of the whole structure. In figure Figure 43 are shown results without considering WBC: it is possible to see the external periodic force increases or decreases the undamped vibration of the structure, depending on the instant it acts on the system.

In Figure 44 Wave-Based controller is applied. It copes very well with the disturbance, fast damping the vibrations.

5.5 LIQUID PROPELLANT: SLOSHING OF FLUID IN ROCKET TANK

This project wants to add more challenges in controlling rocket stage. Some launchers use liquid propellant, as a difference from Vega P80 stage, which uses solid propellant [11]. Using liquid fuel for thrusters has different effects, which are of no concern on this thesis. Just one of these effects is taken into account, because it affects the dynamic behaviour of the structure, increasing the difficulties which control system has to cope with.

The presence of a tank of fuel inside the rocket can create problems when the structure moves with some accelerations, because the fluid inside the tank oscillates. This behaviour is called sloshing and can cause problems on the control of the system, because it adds disturbances to the structure.

The target of this section is to model the fluid sloshing in the launcher, by simulating disturbance effects on the structure and testing WBC capability to cope with them.

5.5.1 *Model of the sloshing*

Sloshing dynamics needs to be modelled to see the effects it has on the lumped structure. The model chosen to represent sloshing is com-

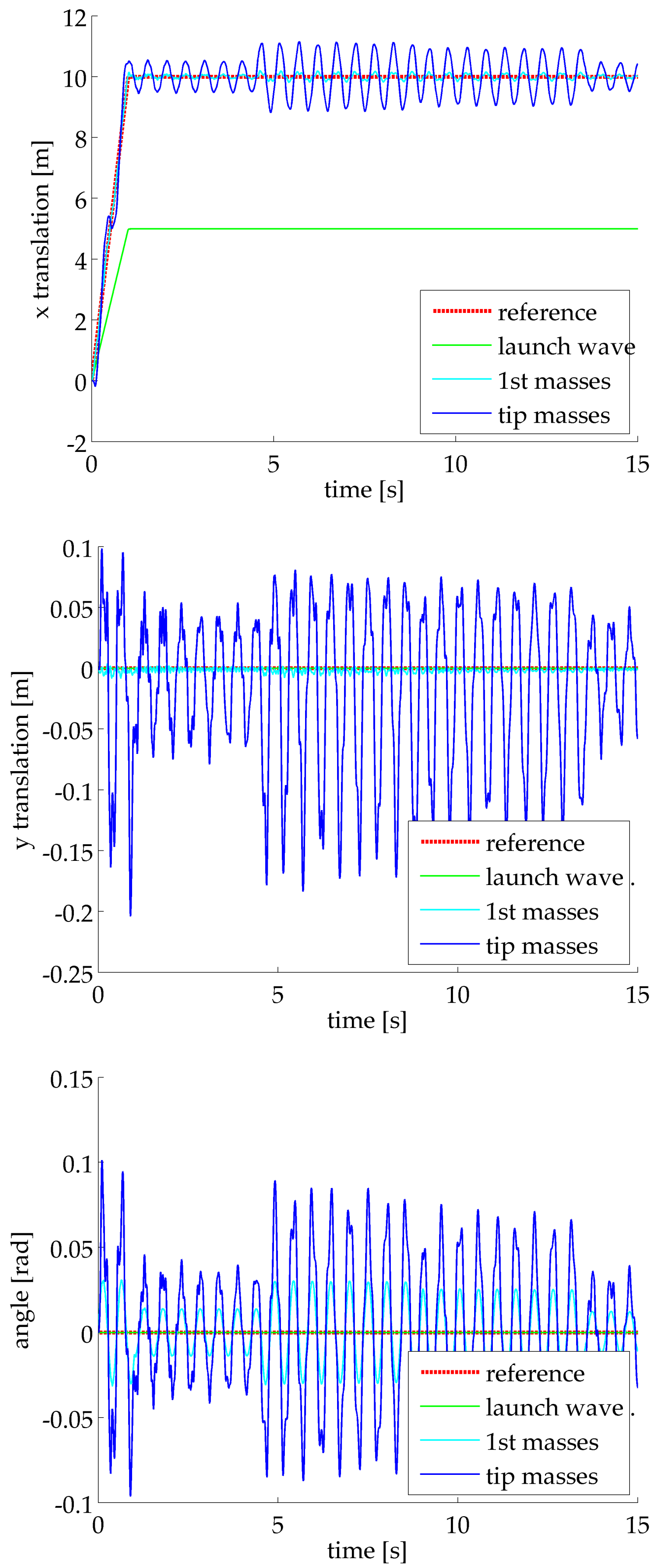


Figure 43: Two thrusters launcher manoeuvre with an impulsive periodic external force acting on the top left mass of the lumped structure. WBC is not applied here.

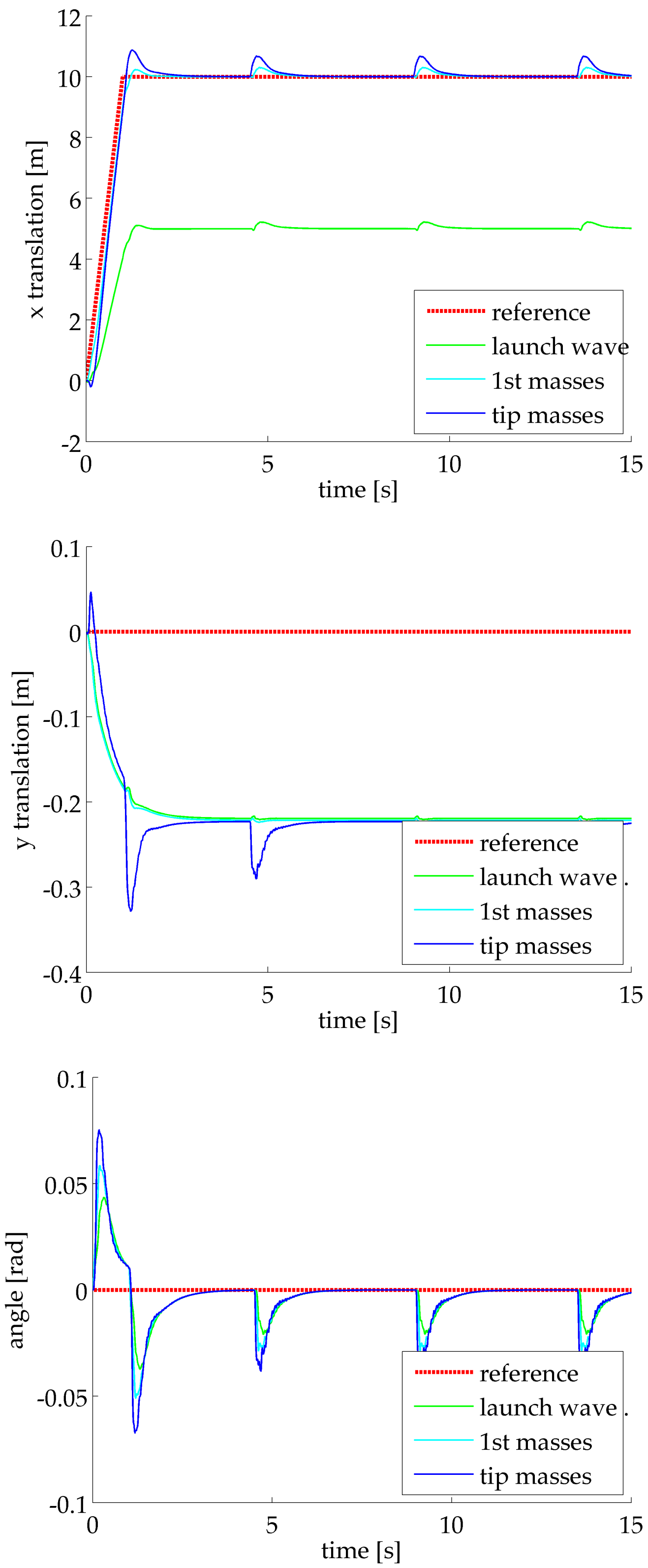


Figure 44: Two thrusters launcher manoeuvre with an impulsive periodic external force acting on the top left mass of the lumped structure. WBC of type 3 controls the structure. WBC parameters are the same used in Figure 38.

posed by one mass and two horizontal springs. This is a very simple model but it is enough to simulate the effect that the sloshing fluid produces on the structure, allowing to change easily parameters of the fluid, that are the mass m_s and the natural frequency of sloshing f_s. It is supposed the load acts only in the transversal axis direction, that is the horizontal axis in the simulations for this project (rocket is in vertical position).

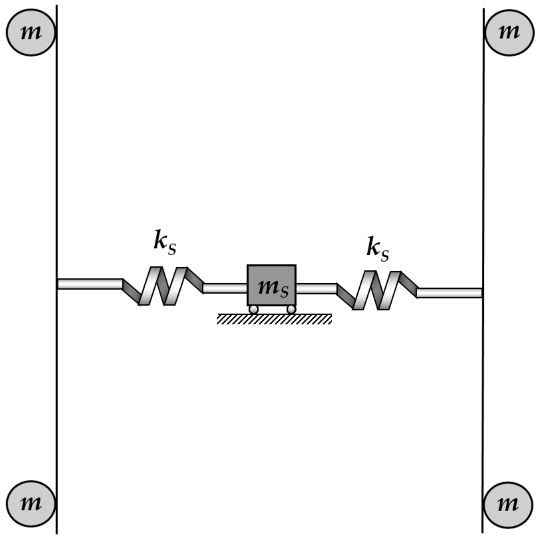


Figure 45: Model of the fluid sloshing. Exchanged forces are applied to the neighbouring four masses of the lumped structure. Lumped structure mass values are m and sloshing mass is m_s. Springs stiffness of the sloshing model (k_s) are chosen in order to set the required frequency.

Sloshing load is equally distributed on two masses on the left and two symmetrical masses on the right. So four masses of the structure lumped model are involved, and this could represent a tank which has the length equal to the distance between two masses, when springs are not under stress. This representation does not mean the four masses involved are rigidly connected in some way, but they can move in the same way as before.

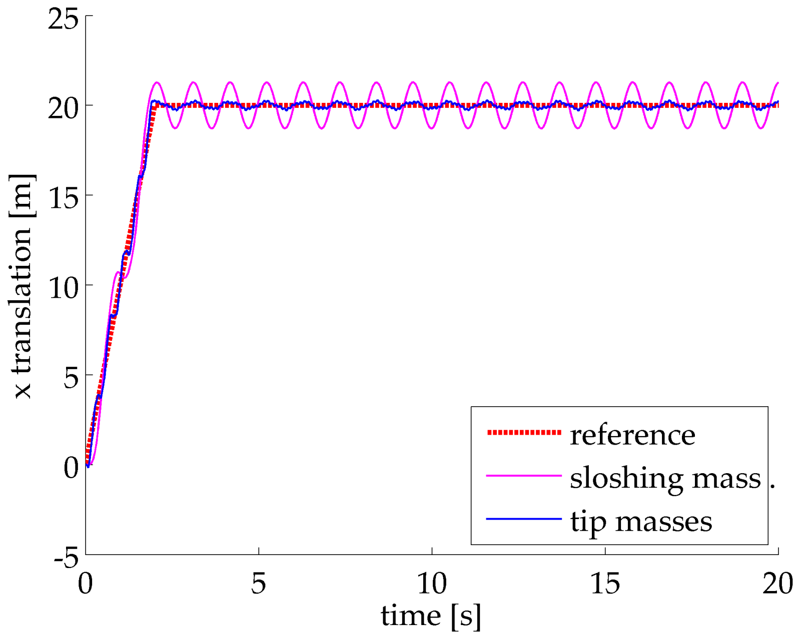
In *Vega User Manual* [11] the gross weight of the P80 stage and its propellant weight are written. It is relevant to observe that the second one is more than 90% of the gross mass (stage structure plus propellant). It does not matter the exact values of the masses, but it is important to take into account, during simulation, that fuel in launchers makes up the most considerable part of the structure weight. Two different weights are considered for fuel mass: $1/3$ and 1 time the mass of the lumped model.

Sloshing elastic forces can be thought as external disturbances which are related with the movement of the structure. The elastic model of the sloshing can be easily set to the desired natural frequency, just setting spring stiffness, once fluid mass value is chosen. To have an idea of the natural frequency the fluid sloshing can have in a rocket tank, it is possible to use the following formula:

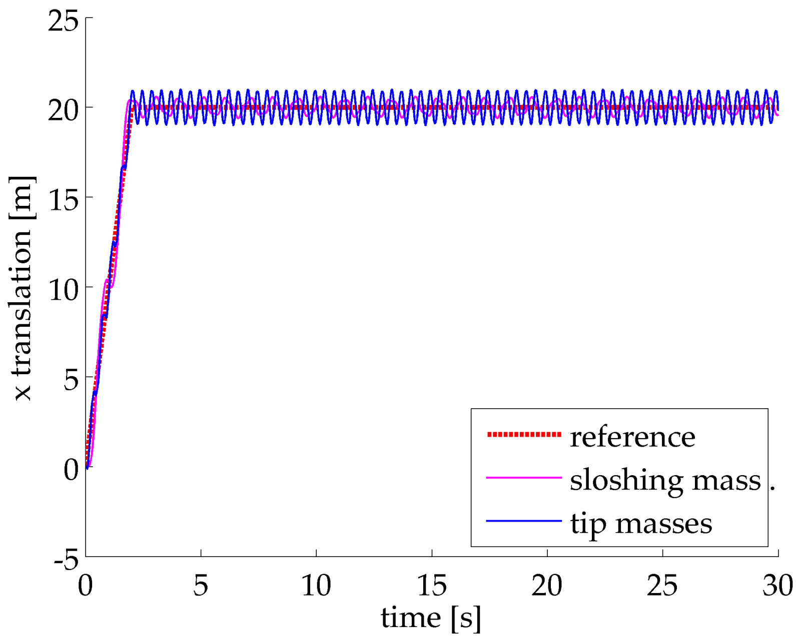
$$f_s = \frac{1}{2\pi}\sqrt{\frac{g}{R}1.841 \tanh\left(1.841\frac{h}{R}\right)} \tag{25}$$

Considering a tank with a radius of $0.5m$, partially filled with fluid up to $3m$, the natural frequency is $f_s = 0.96Hz$. It is possible to think also of a bigger tank with $R = 1m$, which could be the maximum for Vega P80 stage. With this dimension, one obtains $f_s = 0.68Hz$. Simulations are done considering a sloshing frequency of $1Hz$.

In Figure 46 a sloshing mass of $1/3$ the mass of the structure is compared with a sloshing mass value equals to the mass of the structure. Wave-Based Control is not applied there. It is possible to observe that in the first case the oscillation of the sloshing mass is higher but does not affect the oscillations of the structure so much. In the second case it is the opposite: sloshing mass oscillations are lower, but they disturb more the structure. This effect can be explained considering that, having the same natural frequency, but lower mass, springs stiffness are higher, so transmitted forces are higher than in the first case.



(a) $m_s = M_{struct}$



(b) $m_s = \frac{1}{3} M_{struct}$

Figure 46: Two thrusters launcher manoeuvre with fuel sloshing and without applying WBC. Comparison between different fuel sloshing mass values.

5.5.2 *Control the structure with sloshing*

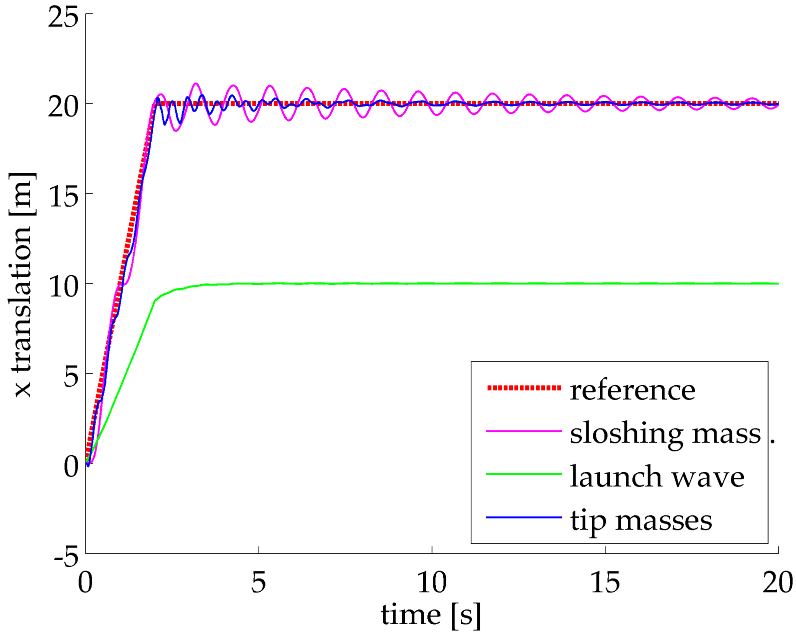
One important fact to consider is that the controller target is to stabilize the structure, regardless what could happen on the fluid. Since disturbance force waves due to the sloshing are linked with forces waves passing through the lumped system, if a controller tries to damps the second ones, also the first ones will be damped, but generally with different results difficult to predict, which will depend on the parameters of the system to control.

WBC revealed to deal well with sloshing in the cases considered, without any need of changing its coefficient. Considering lower frequency case, sloshing mass has a high excursion difficult to damp, but this does not affect the control of the structure more than how 1Hz frequency case does.

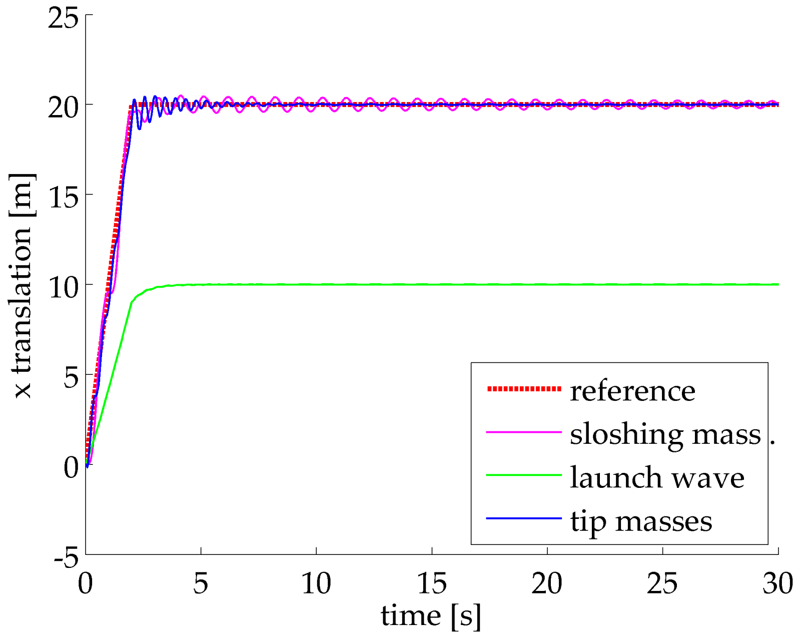
In Figure 48 results of control with WBC well tuned are shown. Sloshing mass oscillation is damped very fast.

5.6 CONTROL ROCKET WITH ONE THRUSTER

This section applies WBC to the same rocket lumped model, considering to have just one thruster as actuator, with its limitations in angle orientation and bandwidth. First of all, it is necessary to take into account that it is not possible to produce torque on the actuator base (i.e the base of the structure), because thruster acts on the medium point of the base segment. In other words, the actuator has lost one degree of freedom. This does not mean that it is not possible to generate torque in order to rotate the structure and balance the load. Indeed, the force vector produced by the propulsion and applied on the base has a moment arm about the centre of mass fulcrum. This torque allows to control the orientation of the structure. And that is the reason why some real rockets can be controlled just by one thruster.

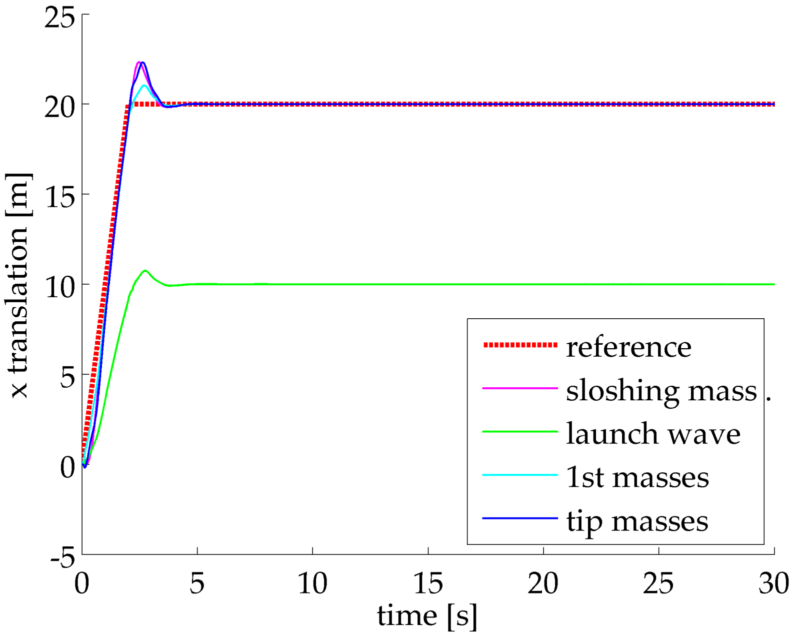


(a) $m_s = M_{struct}$

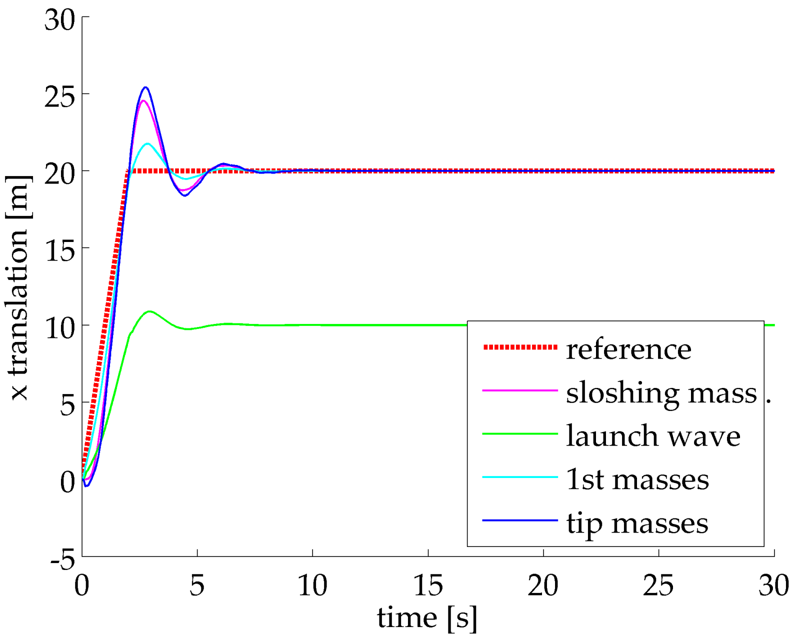


(b) $m_s = \frac{1}{3}M_{struct}$

Figure 47: Two thrusters launcher manoeuvre with fuel sloshing, applying WBC. Comparison between different fuel sloshing mass values.



(a) $f_s = 1\text{Hz} \quad m_s = M_{struct} \quad k_0^x = kx \quad k_0^y = ky \quad k_0^\theta = 100k$



(b) Same parameters but $k_0^\theta = 10k$

Figure 48: Two thrusters launcher manoeuvre with fuel sloshing, applying WBC. The mass of the fluid is equal to the mass of the structure. Comparison between different WBC tuning.

The problem is about how to control the structure with WBC applied to this kind of actuator. How can it be controlled the displacement and vibrations on x and y, together with angle orientation, if it is not possible to produce torque on actuator base, using angle launch wave to set actuator torque reference? In other words, WBC3 controller sets 3 launch waves (two forces F_x, F_y and one torque) but it is not possible to produce the torque on the base by the thruster. This does not mean that it is impossible to use torque information to control the structure angle. Indeed one can manage this information in order to modify or to generate the right reference for the thruster.

The solution depends on the limitation set for the actuator and, especially, on the required trajectory. Considering the idea about simulation of relative movements, described in Section 5.1.2, there is no need of movements on x or y. Or, much better, it does not matter motion control on x and y. It is mandatory just to stabilize the structure, i.e, no residual steady-state vibration, neither inside the structure (no trapped waves which moves the masses) nor oscillation of the structure itself. Moreover, as in the previous cases, it is required to find out the best configuration and tuning of the controller in order to have the best response, in term of delay, over-elongation and settling time.

Now, a different trajectory is taken into account. It has a trapezoidal shape for the angle position reference, which is useful to test if the modified Wave-Based controller is able to control the structure with a 2 DoF actuator.

WBC loops need some modifications to deal with a 2 DoF actuator. Now it is not possible to set a torque reference on the actuator, because actuator is not able to produce a torque. It is necessary to take advantage of the other two degree of freedom of the actuator to launch a wave which contains information for angle control. The best solution found is to use the torque reference produced by WBC angle controller loop as it is, transforming the calculated launching torque wave into a force reference, which the actuator is able to generate. In other words, it is desired to calculate the force that actuator shall produce in order to generate the required torque about the mass centre of the structure.

In Figure 49 there is the modification to the WBC loops in order to implement this idea. The torque launching wave calculated by WBC of type 3 (τ_c) is divided by the force arm about the mass centre (y_{CM}). The angle between actuator x-force and the arm of the force (θ_0) is considered, dividing the by the cosine of this one. In this way a reference x-force (F_c^x) for the actuator is obtained. Reference force for y-axis (F_c^y) is used as it is. Those two reference enter in the *Actuator force dynamic* block, where physical and geometrical constraints of the thruster are modelled. The following simulations are done considering no particular constraints for the thruster. In Section 5.6.2 thruster limitation will be introduced.

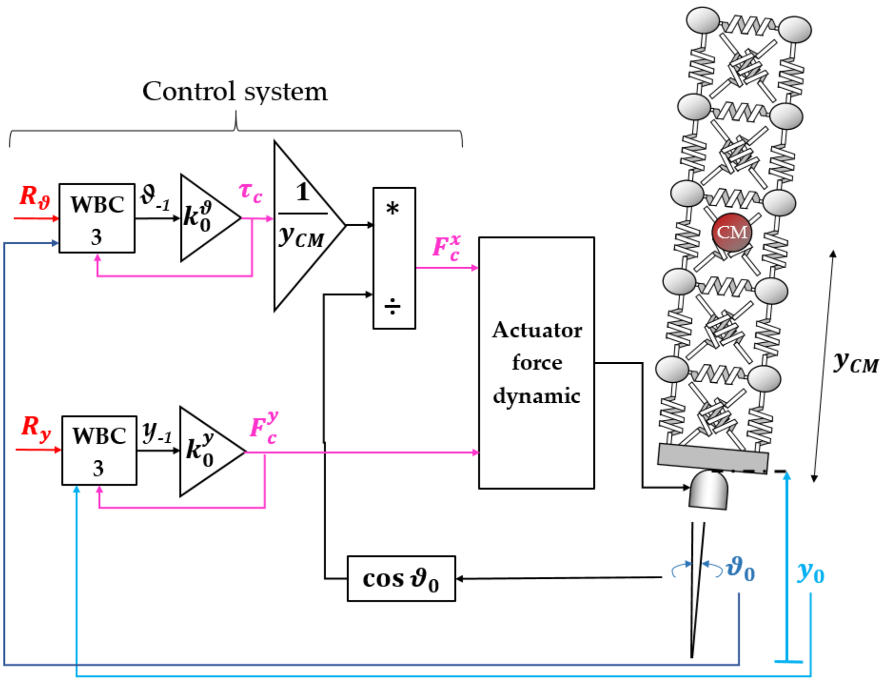


Figure 49: Modification of WBC control loops to manage 2DoF actuator.

In the following pictures, the trajectory introduced in the paragraph above is used: references are zero for x and y, instead a trapezoidal movement is asked to the angle. In Figure 50 there are the results obtained moving the structure with two thrusters, as in the previous sections, without WBC. It is possible to observe vibrations at steady-state.

In figure Figure 51 the same references are used, but now considering to have one thruster and applying WBC of type 3 with the modifications of Figure 49. It is possible to see that vibrations are damped at steady-state, but an unwanted movement on x axis is obtained. This is not a problem of concern, because the target is to stabilize the structure to have no vibration at steady state. This effect is due to physical reasons, because when the thruster wants to turn the structure, it apply a x-force which generates an acceleration on x axis.

Figure 52 and Figure 53 show results obtained with the same configurations as before, but setting a non-equilibrium position for the sloshing mass as starting condition.

5.6.1 *Different tuning of virtual spring and impedance for WBC 3*

Impedance Wave-Based controller has two important coefficients to tune, which basically depend on the masses and springs values in the axis to control. The first one is the virtual spring stiffness k_0, which generally could be similar or equal to springs stiffness in that axis. The second one is the impedance z_c, which transforms the launch force wave in part of the returning position wave.

In Figure 54 and Figure 55 some results obtained respectively with different k_0 and z_c are shown. The parameter k stays for diagonal spring stiffness and z_c' is the natural spring stiffness between virtual spring and actuator rotational inertia J_0: $z_c' = \sqrt{k_0^\theta J_0}$.

An important consideration can be done considering results obtained. It is an analogy with PI controller: k_0 acts like a gain in the controller. If it increases, the system becomes more ready but it is more sensible to external disturbances and parameter variation. The system could easily fall into instability. Indeed, if it is increased more than 20k, the lumped structure becomes uncontrolled.

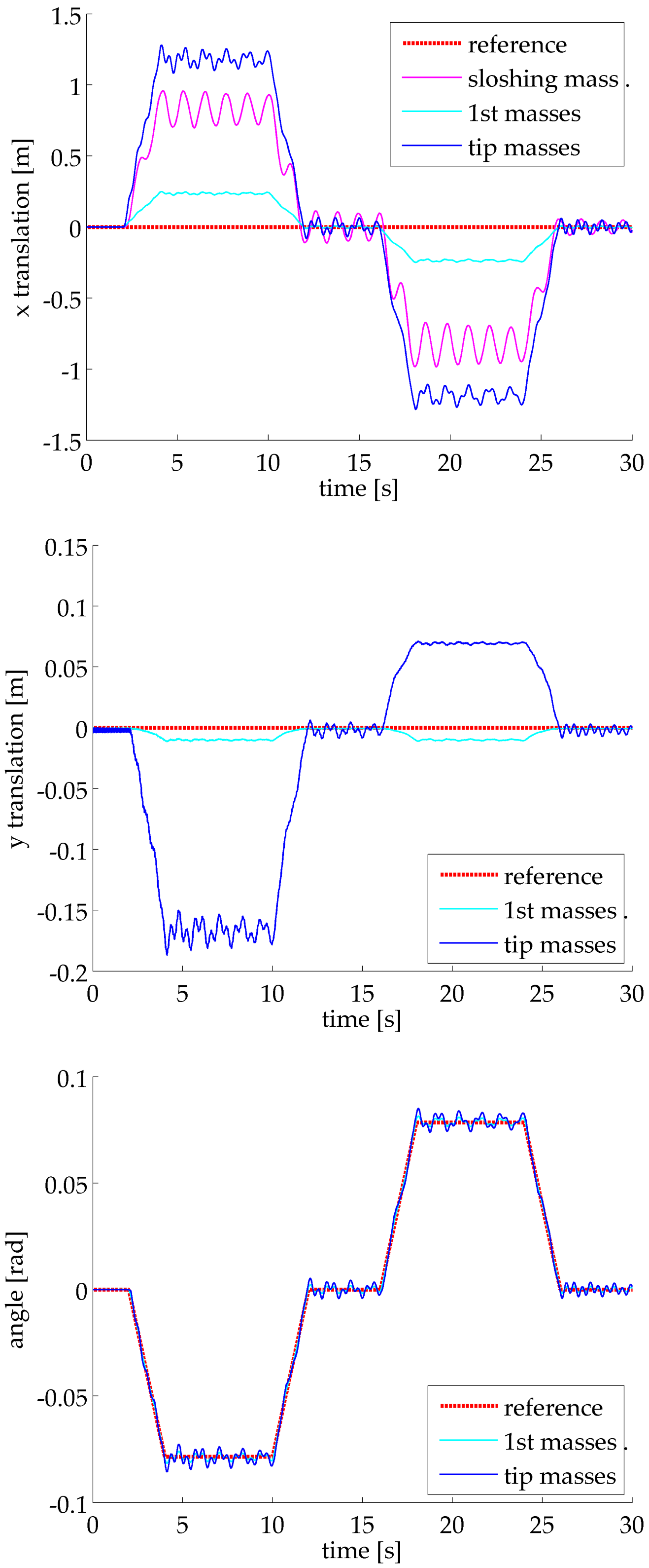


Figure 50: Two thrusters launcher manoeuvre, without WBC control. Reference for x and y are set to zero.

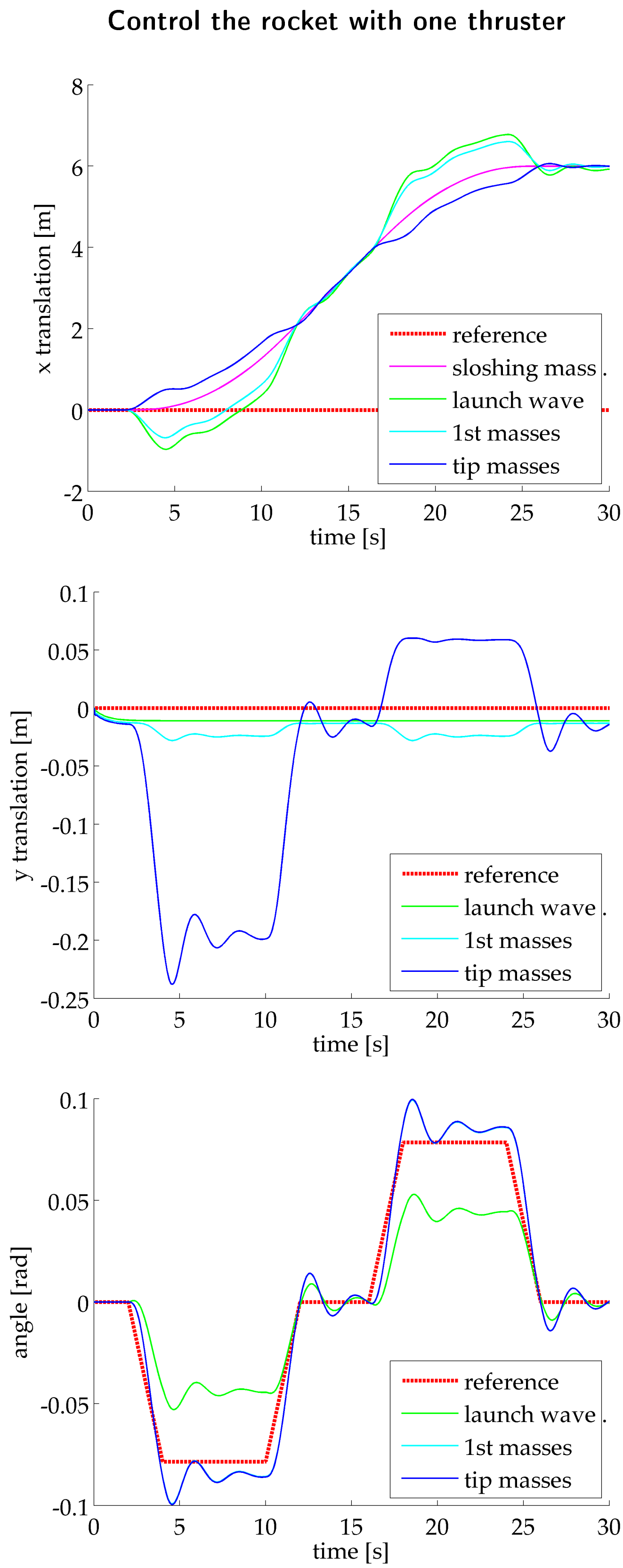


Figure 51: One thruster launcher manoeuvre, using impedance WBC of type 3. Reference for x and y are set to zero.

Rocket with sloshing mass in a not-equilibrium position

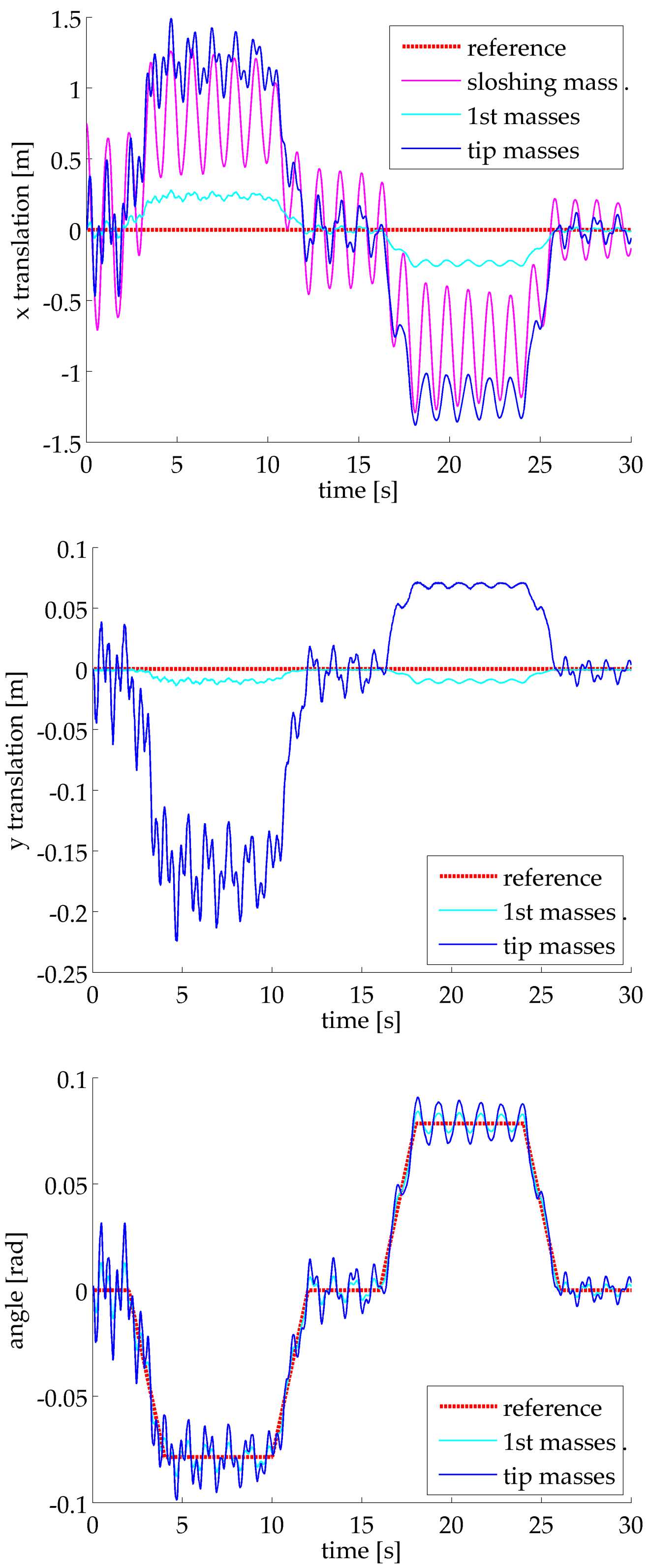


Figure 52: Two thrusters launcher manoeuvre, without impedance WBC of type 3. Reference for x and y are set to zero. Sloshing mass starts in a not equilibrium position.

Control the rocket with one thruster and sloshing mass in a not-equilibrium position

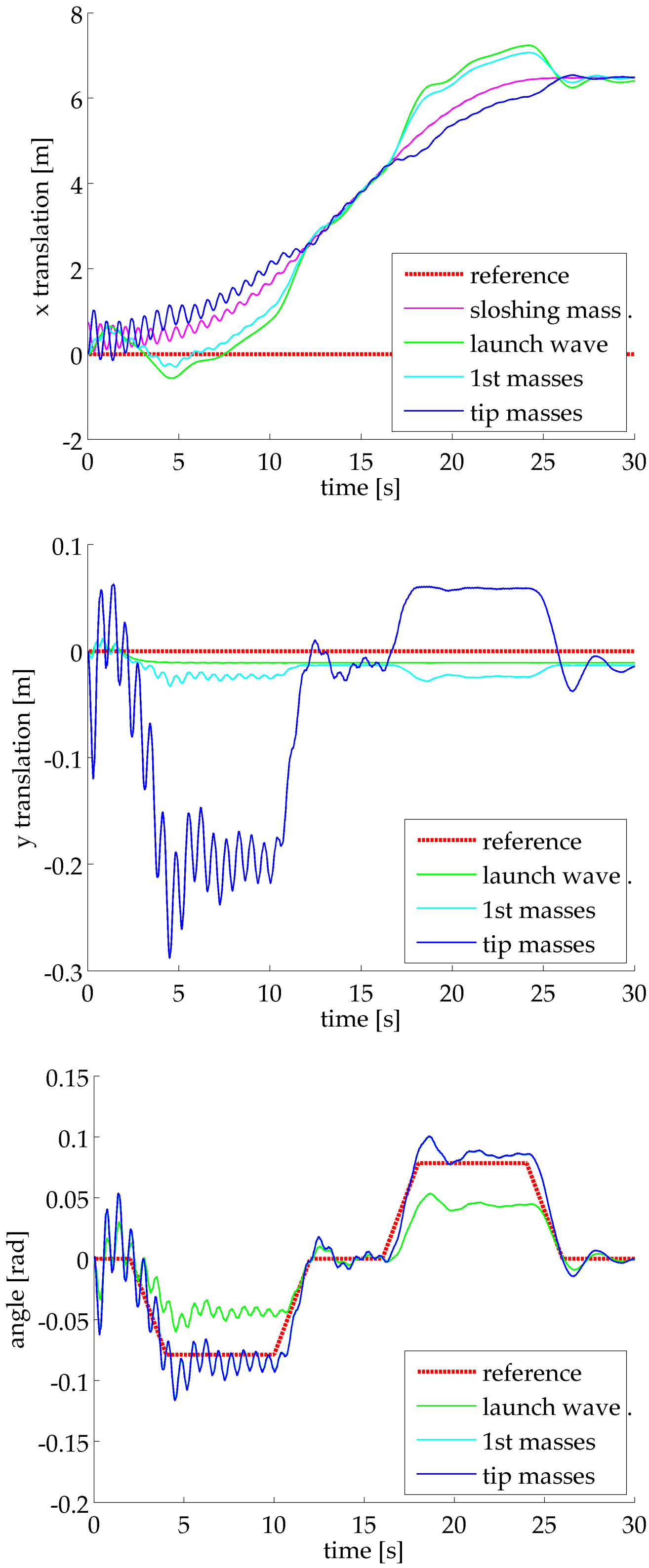
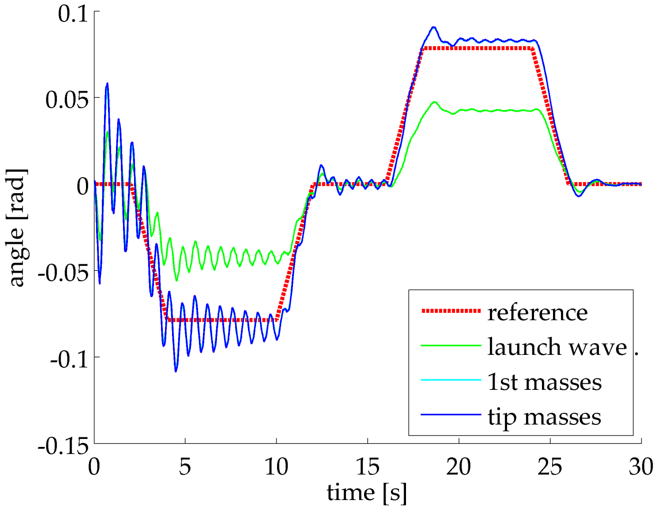
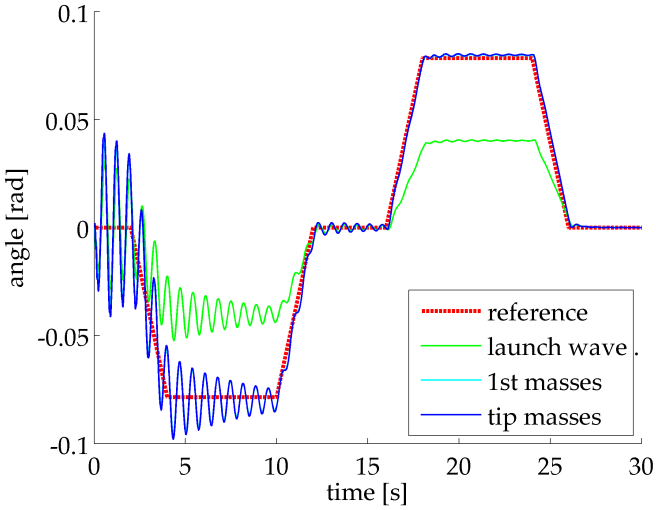


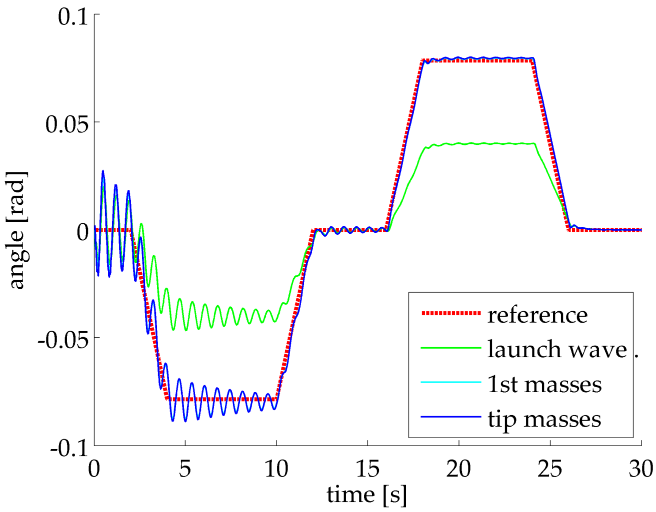
Figure 53: One thruster launcher manoeuvre, using impedance WBC of type 3. Reference for x and y are set to zero. Sloshing mass starts in a not equilibrium position.



(a) $k_0^\theta = k$

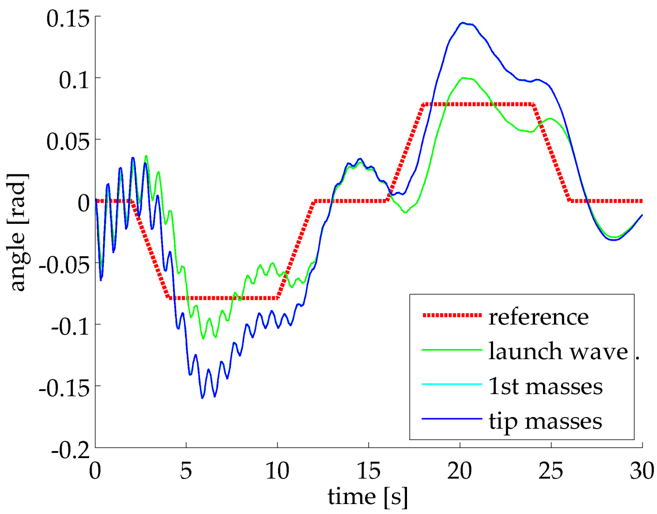


(b) $k_0^\theta = 10k$

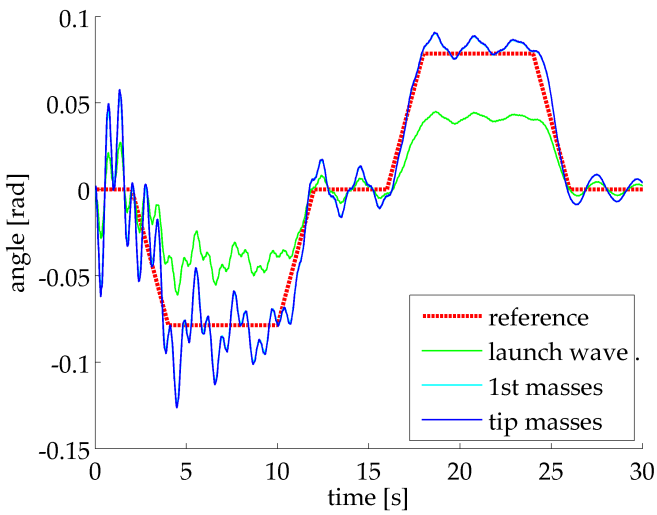


(c) $k_0^\theta = 20k$

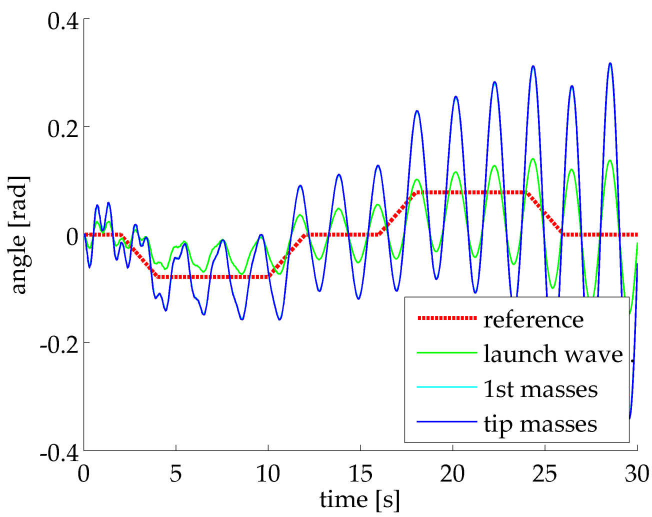
Figure 54: One thruster launcher manoeuvre with sloshing, using impedance WBC of type 3. Different tuning of the virtual spring k_0^θ, using $z_c = 5z'_c$.



(a) $z_c^\theta = z_c'$



(b) $z_c^\theta = 10 z_c'$



(c) $z_c^\theta = 20 z_c'$

Figure 55: One thruster launcher manoeuvre with sloshing, using impedance WBC of type 3. Different tuning of the impedance of the angle loop z_c^θ, using $k_0^\theta = 0.5k$.

z_c instead acts like the integral coefficient, correcting the error going to steady-state condition. If the parameter increases to much, oscillation are increased instead of being damped.

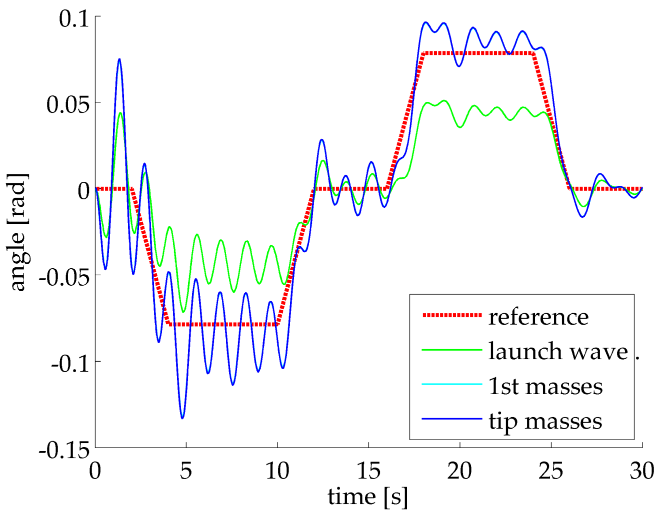
In Figure 56, there are the results obtained with different choice of k_0, supposing to have a sloshing frequency of 0.5 Hertz, like it could happen when launcher is far away from Earth, because gravity is lower (see Equation 5.5.1). For example, with a gravity acceleration of $3m/s^2$, the sloshing natural frequency is around half than before, comparing with the case considered, that is a tank with the radius of $0.5m$ and the fluid height of $3m$. So the natural frequency is $f_s = 0.53Hz$ instead of $0.96Hz$.

From the results obtained in Figure 54 and Figure 56 one can infer that $k_0 = 20k$ is the best option to chose, together with z_c around $10z_c'$. With the configuration tested till now this is right, but such a high gain will be too much when actuator bandwidth and saturation limitation will be added (see Section 5.6.2).

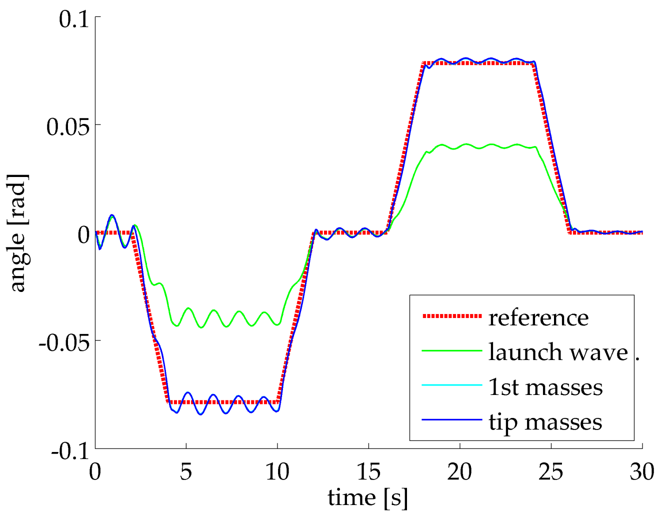
5.6.2 *Introducing dynamic limitations for the single thruster*

As it was moved up, it is required to consider actuator constraints due to thruster physics. Technical data about them are collected from *Vega User Manual* [11]. The important constraints of the thruster are the thrust propulsion of $2261kN$ and the attitude control, obtained with an electro actuator, which gives a gimbaled angle of $6.5°$. These parameters give an idea of the physical limitation of the thruster to be taken into account.

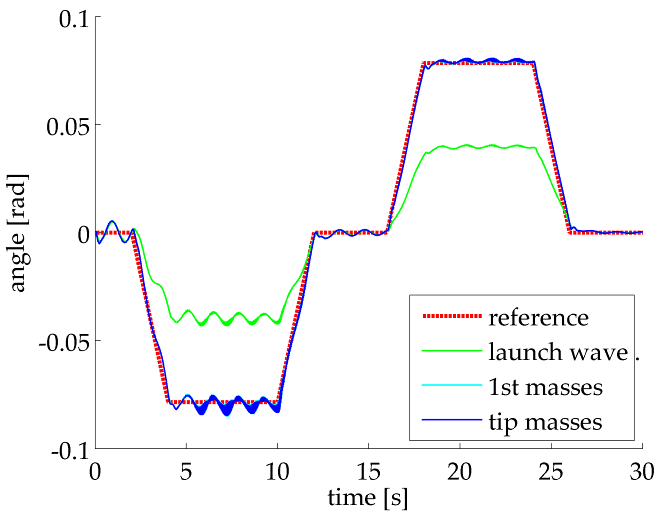
To model thrust propulsion limit, just a saturation block on the modulus of the force vector requested by WBC is enough. Limitation on angle is more difficult to introduce, because WBC gives x and y components of the force to generate. So the idea is to set some constraints on the ratio between these two components, in order to get a force vector with an angle not outside the range $\pm 6.5°$ from the



(a) $k_0^\theta = 0.5k$



(b) $k_0^\theta = 20k$



(c) $k_0^\theta = 40k$

Figure 56: One thruster launcher manoeuvre with sloshing, supposing to have a gravitational acceleration of $3m/s^2$, controlling with impedance WBC of type 3. Different tuning of the virtual spring k_0^θ, using $z_c = 5z_c'$.

vertical axis. Moreover y component can not be negative for obvious reasons. In mathematical terms, considering $F_y > 0$ this means:

$$\arctan\left(\frac{F_x}{F_y}\right) \in (-7^\circ, +7^\circ) \tag{26}$$

This, in general terms, gives a limitation in the ratio between F_x and F_y. There are different ways to stay inside the limit, but some of them would alter control performance, bringing to instability. It is important to understand that after this limitation will have been introduced on the actuator, system will not be able to follow trajectories which involves high acceleration on x axis with low one in the y axis. This assumption is consistent with a launcher motion control, which requires much more acceleration on y axis than in x and this is the reason why launchers have already worked since decades with this kind of thrusters.

The limit set in Equation 26 means, roughly, that y module component of the force should be at least 10 time the x, while considering $F_y > 0$. Formalizing:

$$\begin{aligned} F_x > 0 \quad &\Rightarrow \quad F_y > 10F_x \\ F_x < 0 \quad &\Rightarrow \quad F_y > -10F_x \end{aligned} \tag{27}$$

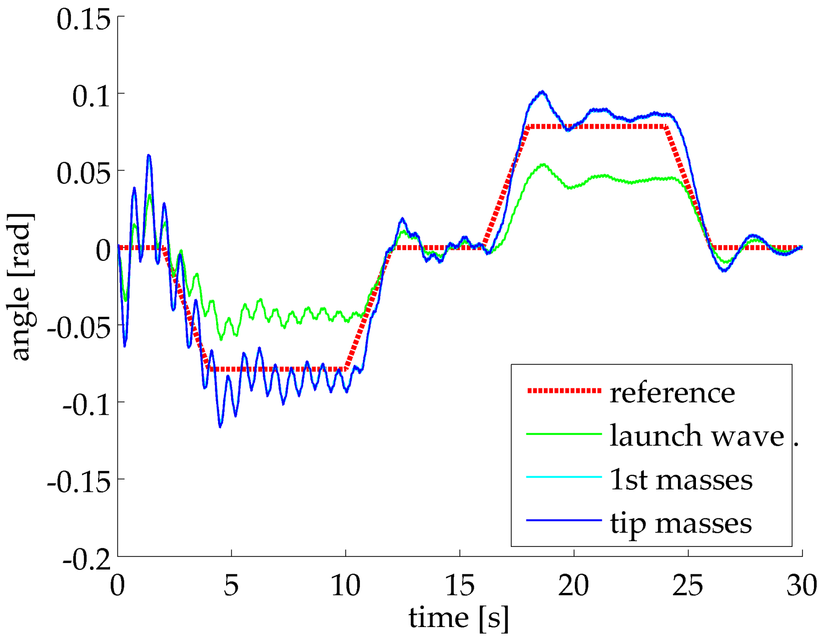
This is usually easy to obtain if the system is subject to gravity, because the big part of thrust propulsion is generated for balance the weight force of the structure. But gravity could be lower in the space or required acceleration on x quite high in some cases. In order to manage this situations, it is necessary to alter WBC request, accepting not to have a perfect control on x and y axis, but, again, with the target to keep the system stable, in vertical position and without steady-state vibrations, even if external disturbances or sloshing is acting. These specifications are reasonable for such a rocket system, where not accurate motion control is sought but a strict need of stabilisation and controller robustness is mandatory.

The introduction on the model of the constraints in Equation 27 is obtained setting some limitations on the F_y launch wave, leaving F_x launch wave as calculated by WBC. This is done because F_x launch wave now contains information about angle control, which are the most important in this system (see Figure 49). Summarizing, now the x WBC loop is not considered anymore and y loop has a saturation limit which depends on the calculated value of F_x. This last value originates from angle WBC loop, which acts via the *x-force* actuator, because the single thruster is not able to produce torque on the base of the structure.

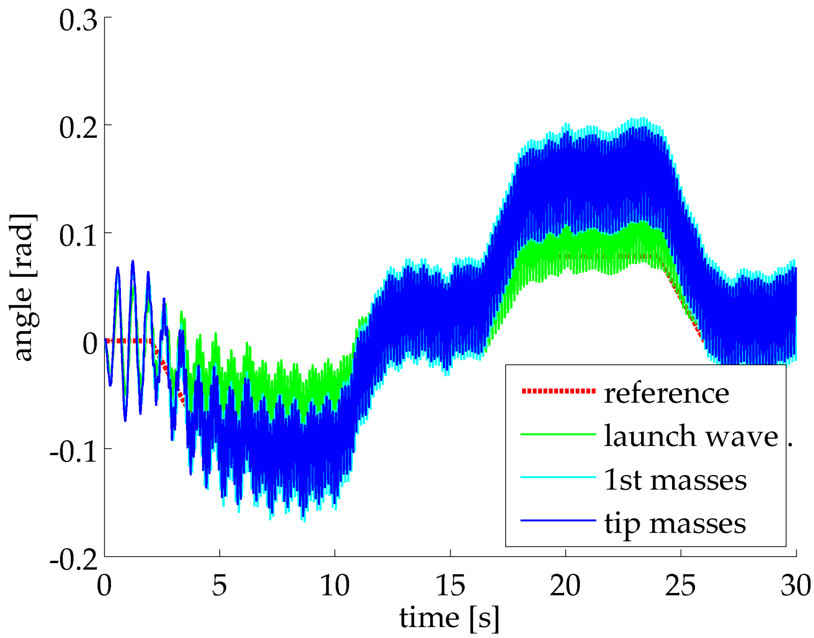
It is also possible to implement thrust propulsion and dynamic bandwidth limitation. Once the modified Cartesian components F_x and F_y are calculated, they are transformed in polar coordinates, that are module and angle. So the module will have a low bandwidth limitation, set at 10 rad/s and a saturation limit set to the same magnitude order of thrust propulsion limits written in *Vega User Manual*. The geometrical angle limitation has been already taken into account, but now also a low pass filter is added to model the dynamic, with a bandwidth set to 50 rad/s.

The two bandwidth limitations are chosen thinking about a possible real thruster, which generally has a very low dynamic, comparing with electric motors taken into account for the robotic arm. With these parameters, the system deals well with the trapezoid trajectory for the angle.

It is possible to understand here what was moved up in Section 5.6.1, that is the problem about setting an high k_0^θ parameter. It is possible to see that, even with $k_0^\theta = 10k$, the system becomes unstable, whereas in the results shown in Section 5.6.1 it seemed to be a better choice than $k_0^\theta = 0.5k$. Therefore, in the end, the best configuration identified is to set $k_0 = 0.5k$ and $z_c = 5z_c'$. This choice guarantees a well performance with sloshing fluid and it demonstrates to be quite robust, even to external impulsive disturbances. For example, in Figure 58 there is the result obtained in controlling the angle with the



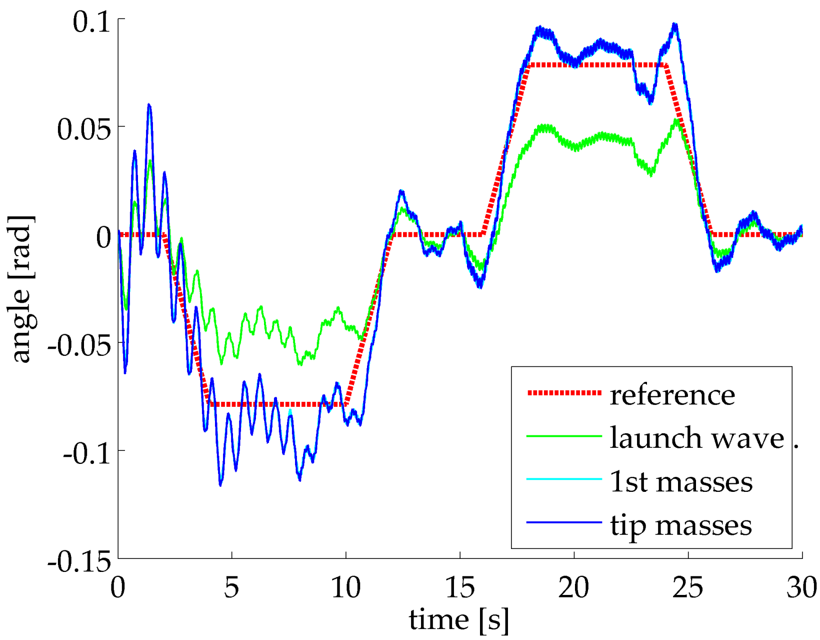
(a) $k_0^\theta = 0.5k$



(b) $k_0^\theta = 10k$

Figure 57: One thruster launcher manoeuvre with sloshing, supposing to have dynamic and propulsion constraints on the thruster, controlling with impedance WBC of type 3. Different tuning of the virtual spring k_0^θ, using $z_c = 5z_c'$.

same configuration as before, but with an external periodic impact disturbance, with an amplitude of the same magnitude order of the weight force of the structure.



(a) $k_0^\theta = 0.5k$

Figure 58: One thruster launcher manoeuvre with sloshing, supposing to have dynamic and propulsion constraints on the thruster and an external impulsive periodic force acting on the left tip mass of the lumped structure.

CONCLUSIONS

This thesis has analysed a simple lumped flexible model for a rocket single stage frame, supposing it has under-damped frequency vibrational modes, lower than in those nowadays launchers, in the light of possible future weight reductions.

The framework considered is the Wave Based Control, developed in its first formulation in 1998 by O'Connor and Lang and than improved and tested in numerical simulation and in some real applications, proved to be very efficient in coping with multi degrees of freedom flexible under-actuated systems.

The project started considering Hossein Habibi's PhD thesis and his 2-D lumped model and for a flexible beam, whereby it was applied 3 WBC loops acting in parallel and simultaneously. Habibi's model was improved and modified in order to make it more general and be able to describe different cases of study, which have 2 important directions of flexibility.

The work passes through robot arm case of study, which is basically composed by a floating beam in a plane, driven by a 3 degree of freedom actuator on its side. Interesting results were obtained, considering typical specifications for a robot arm application. For example high precision both during transitory and steady-state conditions, making some modification to the traditional impedance WBC, accepting to reduce robustness and generality of the controller, but gaining in precision and fast response.

Finally the rocket stage was modelled, starting from the robot arm lumped model, changing parameters to described the launcher stage in term of weight, flexibility and vibrational modes. Specific actuators of the aerospace rocket were considered, that are the thruster(s). Two cases had been afforded: control with two and one thrusters. For both the examples, force saturation and limited bandwidth were taken into account.

In the first instance, actuator has three degree of freedom, like in the robotic arm case. In the second one, rotation degree of freedom is lost. In this case, the challenge in controlling the structure increased. A very simple solution was found and it reveals to work quite well, even considering all the possible limitation a real thruster could have.

In launcher applications control specifications are completely different: robustness is much more important than following the trajectory with high precision. Therefore the controller was modified in order to obtain robustness and generality, but losing a little bit in precision and dynamic, compared with the robot arm system.

A simple but effective model for the sloshing of the fluid was added to the lumped structure and WBC shows to be able to control it without particular modifications, but just using the best setting founded before.

WBC proved again to be a very powerful control tool for these kind of flexible structures, allowing to obtain motion control together with vibrations suppressing.

One possible future development could be an improvement on the numerical implemented lumped model, in order to increase typical rocket details, like other objects inside, or to model and control two or more stages together.

Another interesting part to think about could be the simulation of a complete rocket launching, contemplate the huge absolute movement of the launcher seen from the Earth, following its trajectory, and the vibration control obtained by WBC. Is it possible to use just WBC, perhaps with some modifications, to control all the motions of a launcher, form the take-off to the landing?

APPENDIX

A

NUMERICAL IMPLEMENTATION OF LUMPED MASSES-SPRINGS MODEL

Lattice and *beam* models are implemented in a numerical simulation, considering Newton's Second Law of dynamics and Hooke's Law. Each point mass is connected to its neighbours through 8 springs. For each mass, all the elastic and, possibly, viscous friction forces acting on it are calculated separately for each spring connection, algebrically added to obtain resultant of forces. Then the result is divided by the mass value considered, obtaining acceleration, and then double integrated to get the position, supposing to have null initial conditions on velocity and position. If not, it is possible to add easily an initial condition of position for example, to contemplate the initial deflection due to weight or to simulate a not-equilibrium position, as it was done in the last simulations of the rocket with sloshing, where sloshing mass was supposed to be in a not equilibrium state, just to see if WBC was able to compensate an already sloshing fluid.

In this appendix, all masses are m and all springs have the same stiffness k, with unstretched length L, which projection on x and y axes is l. Viscous damper elements with damping coefficient c are placed in parallel with each spring (not shown). Since the masses are considered as points without area or orientation, the springs exert no torque on the masses.

In the model each mass takes the positions of its eight neighbouring masses as inputs and gives its own displacement as output [1, Appendix A].

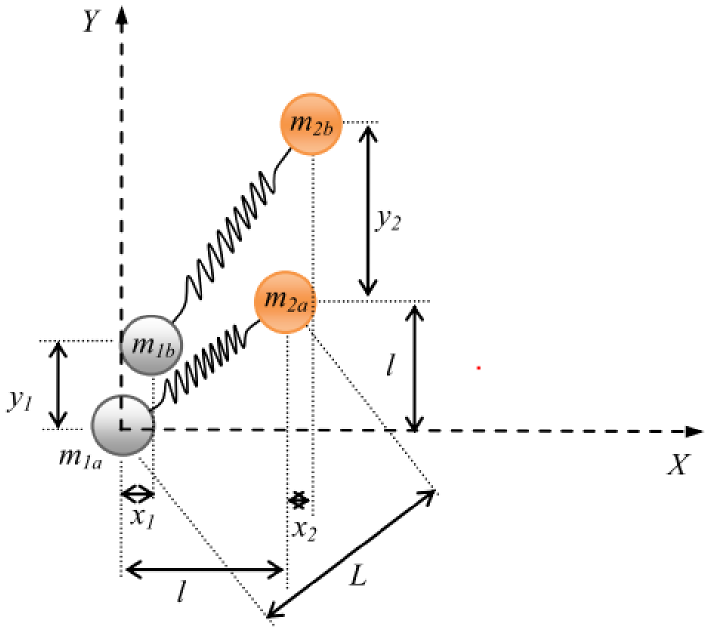


Figure 59: Implementation of the masses-springs lumped model. Displacement of two masses and their spring connection - Habibi

$$F = k\left[\sqrt{((x_2+l)-x_1)^2+((y_2+l)-y_1)^2}-L\right]+ + c\frac{d}{dt}\left[\sqrt{((x_2+l)-x_1)^2+((y_2+l)-y_1)^2}-L\right] \quad (28)$$

Since springs are free to change directions on the plane when masses moves, the numerical model must consider this attitude, calculating, instant by instant, the angle for force projections.

$$F_x = F\cos\theta \qquad F_y = F\sin\theta \quad (29)$$

where

$$\cos\theta = \frac{(x_2+l)-x_1}{\sqrt{((x_2+l)-x_1)^2+((y_2+l)-y_1)^2}}$$

$$\sin\theta = \frac{(y_2+l)-y_1}{\sqrt{((x_2+l)-x_1)^2+((y_2+l)-y_1)^2}} \quad (30)$$

B

SIMULINK IMPLEMENTATION OF THE 2-D MECHANICAL LUMPED MODEL

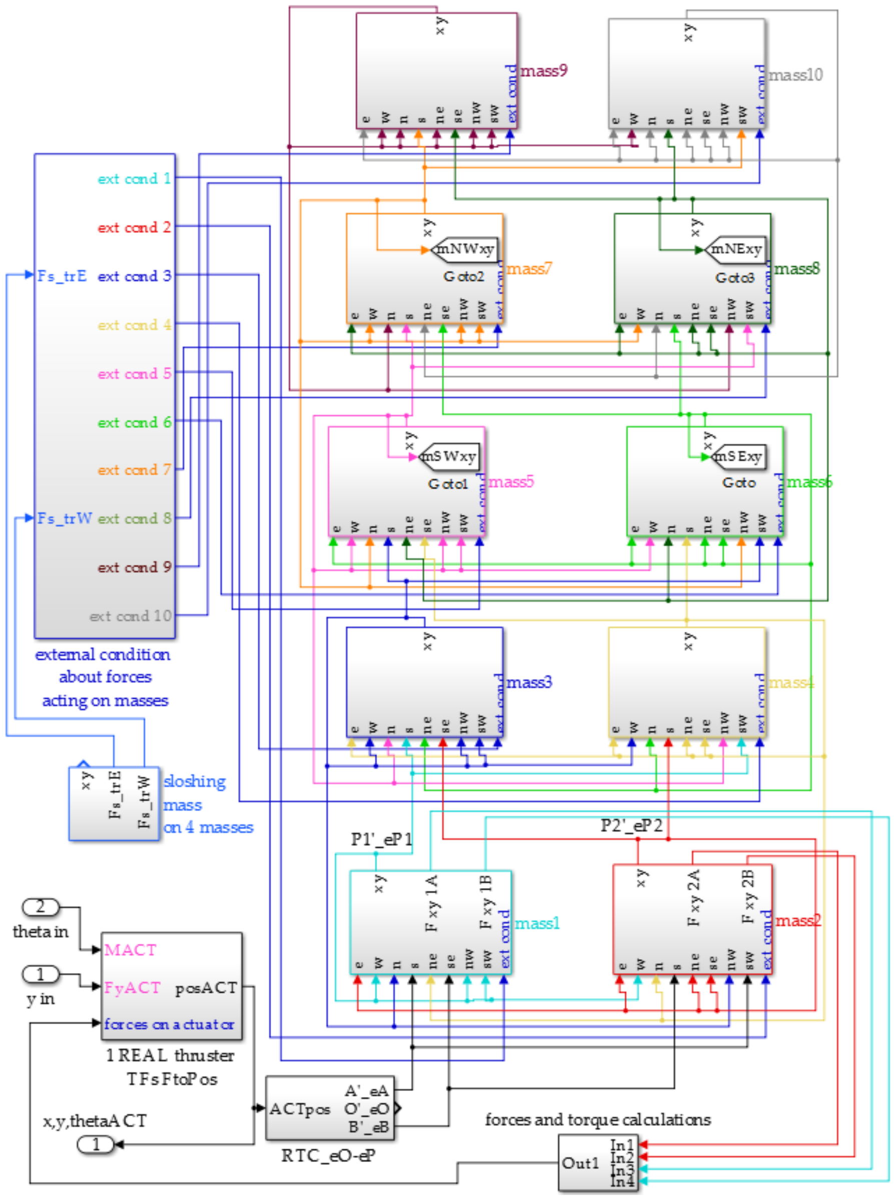


Figure 60: Simulink implementation of the 2-D mechanical lumped model for the launcher with one thruster, external disturbances and sloshing fuel.

The formulas explained in Appendix A are implemented in the commercial software *Simulink*. Each mass is realized by a single block where positions of all the neighbouring masses are taken as input and the position of the mass itself is given as output. Inside each mass block, forces due to spring compression or extension are calculated, considering viscous friction if required.

The actuator is attached to the bottom end of the lumped structure, and it could be thought like a rigid bar with two extreme points where the first four springs are attached. The RTC_{eO-eP} block considers rigid rotation and translation of this bar, calculating instantly position of its two side A' and B'. The input is given by actuator position output asked by WBC, which depends on actuator dynamics and features. In Figure 60 actuator block is that one which implements one thruster limitations, so it receives a torque requirement from WBC angle control loop and a y component of force requirement from WBC y-axis control loop (see Section 5.6). Considering simplest situation like robot arm modelling, actuator block receives as input also the x-axis WBC requirement.

The up-left block simulates all the possible required external disturbances, giving as output the total force disturbance x and y components for each mass. Sloshing effect are included here as input to this block. Sloshing block receives as input the four masses positions whereby fluid sloshing acts and gives as output the disturbance forces it produces on the lumped system.

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