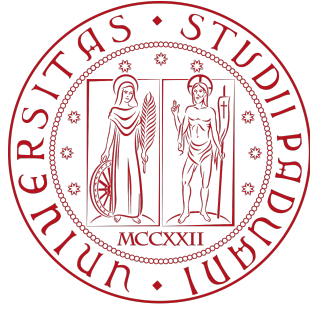




UNIVERSITÀ DEGLI STUDI DI PADOVA
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Final Dissertation

Classical Control for a Compact Squeezed Vacuum Source

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Abstract

Advanced gravitational-wave interferometric detectors, such as Advanced LIGO and Advanced Virgo, are quantum-noise limited over a significant portion of their detection band. To overcome this limitation, special states of light, known as squeezed states, are employed to minimize the quantum noise in the signal quadrature at the expense of the orthogonal one. With this application in mind, several squeezed-light sources have been developed over the past few decades, achieving up to 15dB of noise suppression in the signal quadrature compared to of the “conventional” light. These devices are highly complex and delicate, typically requiring densely populated optical benches.

In this thesis, we investigate a novel approach, referred to as a hybrid squeezer, in which the light sources used for the generation and control of the squeezed state are implemented in a fiber-optic circuit, while the actual squeezing process takes place in a free-space Optical Parametric Oscillator (OPO). This approach is expected to simplify the optical setup required for the generation of squeezed light, making it more accessible to a broader range of users and easier to replicate. The work presented in this thesis primarily focuses on the characterization of the fiber-optic sources, with the aim of assessing their compliance with the requirements for achieving 15dB of squeezing at 1550nm . In particular, the work focuses on the generation of the second-harmonic signal used to pump the OPO, the generation of the Quantum Control Field (QCF) required for the stabilization of the squeezed angle through an Acousto Optic Modulators (AOM), and the characterization of the phase noise of the laser employed in the setup conducted using an unbalanced Mach-Zehnder interferometer.

These preliminary characterizations provide crucial bounds on residual classical seed contamination in the QCF, the available pump power and its stability, the level of phase noise introduced by the laser which ultimately determine the performance of squeezed-state generation.

Contents

1	Introduction	3
2	Theory	5
2.1	Gravitational-Wave Detectors	5
2.2	Quantum noise	6
2.2.1	The Standard Quantum Limit	7
2.3	Reducing quantum noise with squeezed states	8
2.3.1	Vacuum and Coherent States	8
2.3.2	The Two Field Quadratures	8
2.3.3	A Simplified Detector	9
2.3.4	From a Single Mirror to an Interferometer	10
2.3.5	Squeezed Vacuum States	11
2.4	Squeezing formalism	13
2.4.1	Field quantization and sideband operators	13
2.4.2	Quadratures in the sideband picture	13
2.4.3	The squeezing operator	14
2.5	The squeezing angle	14
2.5.1	Detection of a squeezed state	15
2.6	The Optical Parametric Oscillator	15
2.6.1	Squeezed vacuum generation via parametric down-conversion	15
2.6.2	Action of the OPO on the field	16
2.7	The Coherent Control	16
2.7.1	The Quadrature Control Field	17
2.7.2	First error signal: locking the QCF to the pump phase	18
2.7.3	Second error signal: locking the pump to the local oscillator	18
2.8	Limitations of squeezed vacuum states	18
2.8.1	Optical losses	20
2.8.2	Phase noise	21
3	Setup description	23
3.1	The Laser	24
3.2	The AOM	25
3.3	The SHG	26
3.4	The EOM	27

4	Setup characterization	28
4.1	Quadrature Control Field	28
4.1.1	Motivation	28
4.1.2	Output Power as a function of RF voltage and RF input frequency	30
4.1.3	AOM unshifted frequency contamination: estimation method 1 . .	32
4.1.4	AOM unshifted frequency contamination: estimation method 2 . .	32
4.1.5	AOM unshifted frequency contamination: estimation method 3 . .	34
4.2	Laser Phase Noise	37
4.2.1	Reference data calibration	37
4.2.2	Setup 1	38
4.2.3	Setup 2	46
4.3	SHG	56
4.3.1	Power VS Temperature	57
4.3.2	Power Stability	58
5	Conclusion	60

Chapter 1

Introduction

Advanced-generation gravitational-wave interferometers, such as Advanced Virgo and Advanced LIGO, have enabled the direct observation of gravitational waves, opening a new observational window onto the Universe. The sensitivity of these instruments is determined by the combined contribution of numerous noise sources, which dominate in different frequency ranges. Among these contributions, quantum noise plays a unique role, as it is the only noise source that remains significant across the entire detection band. It dominates both at low and high frequencies and represents one of the main noise contribution also in the central region, from hundred Hz to a few kHz , where the detector reaches its highest sensitivity.

Quantum noise has a fundamental origin, arising from the quantum-mechanical nature of the light used for the measurement, and manifests itself in the form of shot noise and radiation pressure noise. The trade-off between these two contributions defines the so-called Standard Quantum Limit (SQL). However, this limit does not represent an insurmountable obstacle: quantum squeezing techniques make it possible to redistribute the quantum fluctuations of the electromagnetic field asymmetrically between its two quadratures, reducing the noise below the vacuum level in one quadrature at the expense of increased fluctuations in the other. Injecting squeezed vacuum states into the interferometer can therefore reduce quantum noise and surpass the SQL. This technique is now routinely employed in current gravitational-wave detectors and is expected to play an increasingly central role in next-generation instruments.

The practical implementation of squeezed states nevertheless presents significant technical challenges. In particular, the stability of the squeezed state must be continuously maintained, since any uncontrolled phase drift rapidly degrades the benefit provided by quantum squeezing. For this reason, the active and stable control of the squeezed state represents one of the main theoretical and technological challenges associated with the implementation of squeezed states in gravitational-wave detectors. Control schemes which are based on the injection of a weak coherent signal at the carrier frequency, introduce a level of classical noise that is incompatible with the sensitivity requirements in the audio band. It is therefore necessary to develop coherent control schemes based on auxiliary fields appropriately frequency-shifted (Quadrature Control Field - QCF) so as not to directly interfere with the squeezed field.

This thesis focuses on the preliminary experimental design and characterization of a compact squeezed-vacuum source with a fiber-based coherent control scheme. The primary objective is to evaluate the technical feasibility and noise performance of this architecture.

The core of the dissertation is structured as follows:

- Chapter 2 (Theory): Reviews the theoretical foundations of quantum noise in interferometers, the formalism of squeezed states, and the mechanics of degenerate optical parametric oscillation. The coherent control scheme and the primary physical degradation mechanisms - namely optical losses and laser phase noise - are detailed.
- Chapter 3 (Setup Description): Describes the experimental setup designed for $1550nm$ squeezed-vacuum generation, focusing on the cascaded Acousto-Optic Modulator (AOM) pair used to synthesize the $+5MHz$ shifted QCF beam and the Second-Harmonic Generation (SHG) pump stage.
- Chapter 4 (Setup Characterization): Presents the experimental results divided into three main measurement campaigns:
 1. AOM Characterization: Evaluation of diffraction efficiency, frequency response, and carrier-suppression ratio via direct power measurements, single-laser beat-notes, and dual-laser offset-locking schemes to bound unshifted carrier contamination.
 2. Laser Phase Noise: Independent measurements of the laser source's phase noise using an unbalanced Mach-Zehnder interferometer and an I/Q beat-note demodulation setup, benchmarked against recalibrated manufacturer specifications.
 3. SHG Characterization: Measurement of the second-harmonic generation phase-matching profile through its characteristic sinc^2 temperature response curve, along with long-term output power stability measurements to verify the reliability of the green pump beam.

Chapter 2

Theory

2.1 Gravitational-Wave Detectors

Gravitational waves are perturbations of spacetime predicted by General Relativity, generated by non-spherically symmetric mass distributions with a time-varying mass quadrupole moment, as typically occurs in the coalescence of compact binary systems. As a gravitational wave passes, the relative separations between freely falling masses are alternately stretched and compressed. This effect is described by the dimensionless quantity known as *strain*, $h(t)$, which characterizes the relative deformation induced by the gravitational wave; in a simplified description, it is associated with the relative change in distance $\Delta L/L$ between freely falling masses. For the relevant astrophysical sources observed so far, $h(t)$ has extremely small amplitudes, of order 10^{-21} or smaller. Detecting such signals therefore requires an instrument capable of measuring extremely small changes in length.

The principle underlying interferometric gravitational-wave detectors is that of the Michelson interferometer. A laser beam is split by a beamsplitter into two beams that travel along two orthogonal arms of length L , each terminated by a mirror. The mirrors are suspended so as to behave, at the frequencies of interest, as freely falling masses; they therefore act as the test masses whose relative separation is perturbed by the passage of the wave. In current detectors, each arm also forms a Fabry-Perot cavity, which causes the beam to traverse the same arm multiple times before exiting, thereby increasing the phase response for a given ΔL and consequently improving the sensitivity of the instrument.

After travelling along their respective arms, the two beams are recombined at the beamsplitter. The interferometer is maintained in a condition of destructive interference: in the absence of a signal, essentially all of the laser power returns towards the source through what is known as the *bright port* (or *common port*), while the remaining port, directed towards the photodetector, is ideally free of carrier light - the so-called *dark port* (or *differential port*). When a gravitational wave changes the separation between the test masses, the symmetry between the two optical paths is broken: a small optical field appears at the dark port, with an amplitude proportional to $h(t)$. It is therefore the optical signal emerging from the dark port, appropriately detected and calibrated, that carries the information about the gravitational-wave signal.

2.2 Quantum noise

The sensitivity of a gravitational-wave detector is not limited by a single source of noise, but by the combination of many contributions of different origins, which dominate in different frequency bands. Figure 2.1 shows the expected noise budget for the Advanced Virgo detector during the next observing run (O5): at low frequencies, Newtonian noise (due to fluctuations of the local gravitational field) and suspension thermal noise dominate; at intermediate frequencies, the thermal noise of the mirror coatings is the dominant contribution; residual gas noise and substrate thermal noise contribute to a lower extent, but over a large part of the detection band.

Quantum noise (purple curve), on the other hand, is the only contribution that remains significant across the entire detection band: it dominates at both low and high frequencies, while remaining comparable to the total noise curve (black) also in the central region, between a few hundred Hz and a few kHz, where the detector is most sensitive. This is why quantum noise plays a central role in the design of current gravitational-wave detectors, and why it is the type of noise on which this thesis focuses.

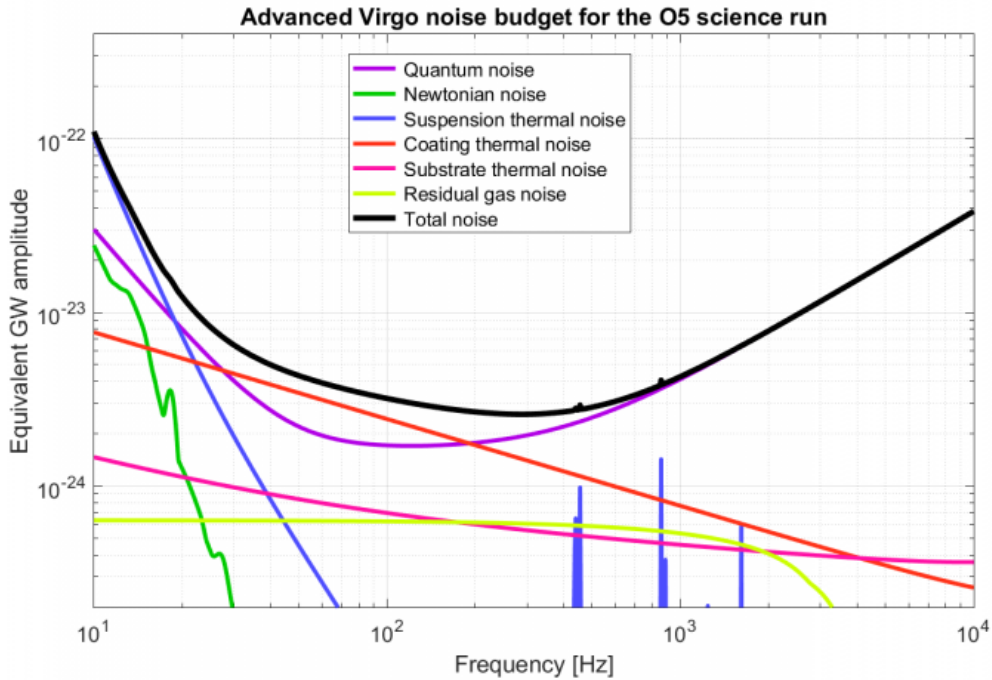


Figure 2.1: *Noise budget for the O5 science run of the Advanced Virgo gravitational-wave interferometer (from <https://www.virgo-gw.eu/science/detector/sensitivity/>). Note that, in this figure, the quantum noise has already been suppressed by 6dB through the injection of frequency-dependent squeezing and, despite this, it remains comparable to the noise budget..*

Unlike the other noise sources listed above, many of which can in principle be reduced through technological improvements, quantum noise has a fundamental origin: it arises from the quantum nature of the light used to perform the measurement, and manifests itself in two distinct ways.

Shot noise. For a laser beam with power P_0 and angular frequency ω_0 , the average number of photons collected during a measurement time τ is $N = P_0\tau/(\hbar\omega_0)$. Since photon counting follows Poisson statistics, the fluctuation in this number is $\Delta N = \sqrt{N}$: this statistical fluctuation translates into an uncertainty in the estimated phase,

$$\delta\phi_{\text{sh}} \sim \frac{1}{\sqrt{N}}, \quad (2.1)$$

which in turn translates into an uncertainty in the measurable arm length difference, and therefore in the strain $h(t)$. This contribution is known as *shot noise*. Increasing the laser power increases the number of collected photons and, consequently, improves the signal-to-noise ratio: calculating the amplitude spectral density of the strain induced by shot noise, one finds that it scales as $P_0^{-1/2}$:

$$S_{h,sn}^{1/2} = \frac{1}{L} \sqrt{\frac{4\pi\hbar c\lambda_l}{P_0}}, \quad (2.2)$$

where λ_l is the laser wavelength and L is the arm length of the interferometer.

Radiation pressure noise. The same random photon counting has a second consequence. Each photon that strikes a mirror transfers an impulse $\hbar k$, corresponding to a force $2\hbar k/\tau$; since the number of photons arriving during a given time interval fluctuates ($\Delta N = \sqrt{N}$), the force exerted on the mirrors also fluctuates, imparting a random motion around their equilibrium position. This effect, known as *radiation pressure noise*, becomes increasingly important as the laser power increases: unlike shot noise, the equivalent strain induced by radiation pressure grows with the laser power, scaling as $P_0^{+1/2}$:

$$S_{h,rp}^{1/2} = \frac{1}{mL\Omega^2} \sqrt{\frac{2\hbar\omega_l P_0}{c^2}}, \quad (2.3)$$

where the response function of a free mass in the frequency domain, $-1/m\Omega^2$, has been used, with m denoting the reduced mass of the mirrors and Ω the angular frequency of the gravitational-wave signal.

2.2.1 The Standard Quantum Limit

The two contributions therefore have opposite dependencies on the laser power: shot noise decreases, while radiation pressure noise increases. For each frequency of interest, there is therefore an optimal laser power that minimizes the sum of the two contributions. The noise level corresponding to this optimal power at each frequency defines the *standard quantum limit* (SQL):

$$S_{h,SQL}^{1/2}(\Omega) \sim \frac{1}{\Omega L} \sqrt{\frac{\hbar}{m}}, \quad (2.4)$$

which cannot be further reduced simply by changing the laser power.

The fact that shot noise and radiation pressure noise cannot be reduced simultaneously without limit is not a technological limitation, but a consequence of the quantum nature of light and the Heisenberg uncertainty principle. However, this does not mean that the standard quantum limit constitutes an insurmountable fundamental limit to the sensitivity of the detector. Through techniques that allow the quantum fluctuations of light to be modified and controlled, such as quantum squeezing, it is possible to achieve a sensitivity below the SQL.

2.3 Reducing quantum noise with squeezed states

In the previous section, we saw that the SQL arises from a trade-off between shot noise and radiation-pressure noise, both originating from quantum fluctuations of the electromagnetic field. In order to understand how this limit can be circumvented, it is necessary to examine more precisely the behaviour of these fluctuations and their impact on interferometric measurements. To this end, we first introduce vacuum states and coherent states and subsequently discuss the field quadratures.

2.3.1 Vacuum and Coherent States

In a quantum description of the electromagnetic field, the *vacuum state* $|0\rangle$ is the lowest-energy state of the field: it contains no photons and has zero mean field amplitude. This does not mean, however, that the field is identically zero at every instant: as for any quantum oscillator, the ground state also exhibits zero-point fluctuations around its mean value.

An ideal laser beam is instead described by a *coherent state* [1], which has a non-zero mean amplitude $A \propto \sqrt{P_0}$ and can equivalently be regarded as a vacuum state displaced around this classical mean value. This equivalence is crucial: the fluctuations of a coherent state around its mean amplitude have exactly the same statistical properties as the fluctuations of the vacuum state around zero.

2.3.2 The Two Field Quadratures

The electromagnetic field in a given mode, apart from a normalization factor, can be written in the form [2]:

$$\hat{E}(t) = [A + \hat{a}_1(t)] \cos \omega_0 t + \hat{a}_2(t) \sin \omega_0 t \approx A \left[1 + \frac{\hat{a}_1(t)}{A} \right] \cos \left[\omega_0 t - \frac{\hat{a}_2(t)}{A} \right], \quad (2.5)$$

where A represents the classical field amplitude (proportional to $\sqrt{P_0}$) and $\hat{a}_1, \hat{a}_2$ are two Hermitian operators, referred to as *quadratures*, which describe the field fluctuations in phase and in quadrature with respect to the carrier oscillation, respectively. A variation of $\hat{a}_1$ corresponds to a fluctuation in the field amplitude, while a variation of $\hat{a}_2$ corresponds to a fluctuation in its phase. The two quadratures are conjugate observables: they do not commute and therefore obey an uncertainty relation of the form $\Delta^2 \hat{a}_1 \cdot \Delta^2 \hat{a}_2 \geq 1/4$. In a vacuum state, as well as in any coherent state, the two variances are equal, $\Delta^2 \hat{a}_1 = \Delta^2 \hat{a}_2 = 1/2$, and the two quadratures are uncorrelated: the quantum noise is distributed symmetrically (see Figure 2.2).

A rigorous treatment of these operators, starting from the quantization of the electromagnetic field, will be given in Chapter 2; here, we limit ourselves to this intuitive picture, which is sufficient to understand the role of squeezing.

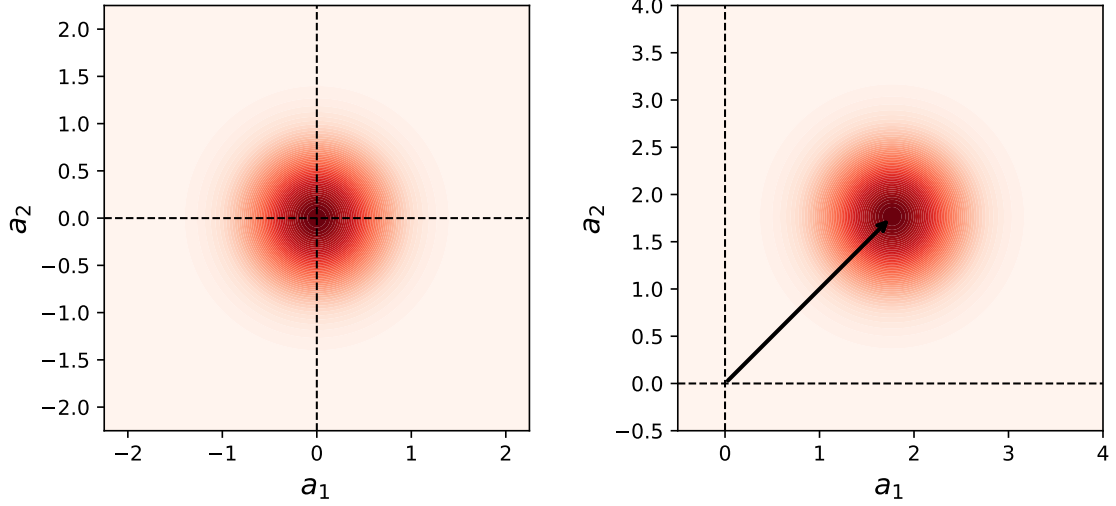
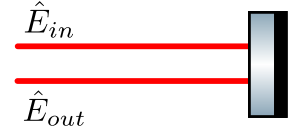


Figure 2.2: *Phasor diagrams for a vacuum state (left) and a bright coherent state (right). Note that the two figures are essentially identical, except that in the vacuum state the uncertainty circle is centred at the origin*

2.3.3 A Simplified Detector

To understand concretely how the two quadratures enter the measurement, let us first consider the simplest possible case [2]: a single mirror, free to move along the direction of the beam, which reflects the incident field (Figure 2.3). We can write the input field as described above in Equation 2.5:



$$\hat{E}_{in}(t) = \left[\sqrt{\frac{2I_0}{\hbar\omega_0}} + \hat{a}_1(t) \right] \cos \omega_0 t + \hat{a}_2(t) \sin \omega_0 t$$

The output field is given by $\hat{E}_{out}(t) = \hat{E}_{in}(t - 2\tau - \hat{x}/c)$, where $\hat{x}$ is the signal-carrying displacement of the mirror, while $\tau \equiv L/c$ is the time required by the beam to travel along the arm of length L . We can therefore rewrite $\hat{E}_{out}(t)$ using the formalism introduced in equation 2.5, defining the quadratures $\hat{b}_1$ and $\hat{b}_2$:

Figure 2.3: *Model of a single mirror, free to move along the beam direction, illuminated by the coherent field $\hat{E}_{in}$. A displacement $\hat{x}$ of the mirror imprints a phase shift on the reflected field $\hat{E}_{out}$.*

$$\hat{E}_{out}(t) = \left[\sqrt{\frac{2I_0}{\hbar\omega_0}} + \hat{b}_1(t) \right] \cos \omega_0 t + \hat{b}_2(t) \sin \omega_0 t. \quad (2.6)$$

Considering that the displacement $\hat{x}$ of the mirror introduces a small phase shift in the reflected field ($\Delta\phi = 2\omega_0\hat{x}/c \ll 1$), we can obtain the relations linking the input and output fields:

$$\hat{b}_1(t) = \hat{a}_1(t - 2\tau), \quad \hat{b}_2(t) = \hat{a}_2(t - 2\tau) - 2\sqrt{\frac{2I_0}{\hbar\omega_0}} \frac{\omega_0}{c} \hat{x}(t - \tau). \quad (2.7)$$

From these two expressions, we can see that the signal due to the mirror displacement appears exclusively in the phase quadrature $\hat{b}_2$. This is why shot noise, which is simply the noise associated with the $\hat{a}_2$ quadrature of the incoming field, directly limits the measurement precision. At the same time, however, the amplitude quadrature $\hat{a}_1$ of the incoming field is not harmless: through radiation pressure, it exerts a fluctuating force on the mirror, causing it to move and therefore introducing fluctuations into $\hat{x}(t)$, and consequently into $\hat{b}_2(t)$, through the radiation-pressure noise discussed above.

2.3.4 From a Single Mirror to an Interferometer

We now observe that the same type of relation can be obtained, with some additional complications, for a complete Michelson interferometer. The only difference is that the displacement $\hat{x}$ is replaced by the differential displacement $\hat{x}_d$ between the two arms, i.e. the quantity directly affected by the gravitational-wave signal.

Consider the simplified scheme of a Michelson interferometer shown in Figure 2.4 [2], in which all propagating optical fields are represented. The laser is injected into the interferometer through the common port, while no radiation is injected through the differential port. We can therefore write the two input fields at ports c and d using the formalism of equation 2.5 as follows:

$$\hat{E}_{in}^c(t) = \left[\sqrt{\frac{2I_0}{\hbar\omega_0}} + \hat{c}_1(t) \right] \cos \omega_0 t + \hat{c}_2(t) \sin \omega_0 t$$

$$\hat{E}_{in}^d(t) = \hat{a}_1(t) \cos \omega_0 t + \hat{a}_2(t) \sin \omega_0 t$$

Applying the same procedure described in Section 2.3.3, we get

$$\hat{E}_{out}^d(t) = \frac{E_{out}^B - E_{out}^A}{\sqrt{2}} = \hat{b}_1(t) \cos \omega_0 t + \hat{b}_2(t) \sin \omega_0 t, \quad (2.8)$$

and obtain the relations:

$$\hat{b}_1(t) = \hat{a}_1(t - 2\tau), \quad \hat{b}_2(t) = \hat{a}_2(t - 2\tau) - \sqrt{\frac{2I_0}{\hbar\omega_0}} \frac{\omega_0}{c} \hat{x}_d(t - \tau). \quad (2.9)$$

As anticipated, these relations are identical to 2.7, apart from $2\hat{x} \rightarrow \hat{x}_d$, and it can be shown that the result is analogous when considering a realistic interferometer, including all the Fabry-Perot cavities, instead of the simplified model shown in Figure 2.4. There is, however, an important conceptual point that is not highlighted by the single-mirror treatment and instead emerges from this latter analysis: in a balanced interferometer, the field fluctuations that constitute the quantum noise of the instrument are those entering through the dark port of the interferometer.

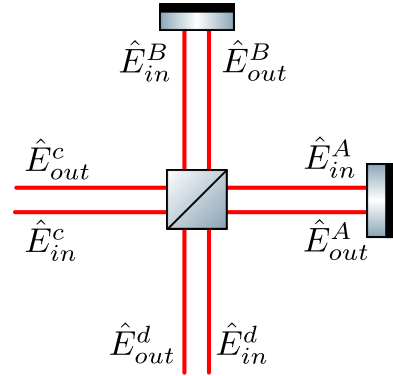


Figure 2.4: *Simplified scheme of a Michelson interferometer. The laser field enters through the common port c ; the fields $\hat{E}^A$, $\hat{E}^B$ travel along the two arms and are reflected by the test-mass mirrors; the output field at the differential port d carries the gravitational-wave signal and is detected.*

This result has a direct and important consequence: in order to modify the quantum noise of an interferometer, the relevant contribution can be traced back to the vacuum entering through the dark port. Under ordinary conditions, this port is simply occupied by the electromagnetic vacuum, with its symmetric fluctuations $\Delta^2 \hat{a}_1 = \Delta^2 \hat{a}_2 = 1/2$. Replacing this ordinary vacuum with a vacuum state in which the fluctuations are redistributed asymmetrically between the two quadratures therefore makes it possible to directly modify the quantum noise of the instrument, without changing the laser power.

2.3.5 Squeezed Vacuum States

The state described at the end of the previous section is referred to as a *squeezed vacuum state*: like the ordinary vacuum state it has zero mean field, but its fluctuations are distributed unequally between the two quadratures,

$$\Delta^2 \hat{a}_1 = \frac{1}{2} e^{2r}, \quad \Delta^2 \hat{a}_2 = \frac{1}{2} e^{-2r}, \quad (2.10)$$

where $r \geq 0$ is the *squeezing parameter*. The product of the two variances remains $\Delta^2 \hat{a}_1 \cdot \Delta^2 \hat{a}_2 = \frac{1}{4}$, which is the minimum value allowed by the uncertainty principle. Squeezing therefore makes it possible to reduce the fluctuations below the vacuum level in one quadrature, at the expense of a corresponding increase in the fluctuations of the other.

Considering the power spectral density of quantum noise referred to the input [2],

$$S_h(\Omega) = \left[\frac{S_{\hat{a}_2}(\Omega)}{k(\Omega)} + k(\Omega) S_{\hat{a}_1}(\Omega) \right] \frac{h_{SQL}^2(\Omega)}{2}, \quad (2.11)$$

- where $k(\Omega)$ is the optomechanical coupling factor, which encodes the coupling between the laser beam and the mirrors, and $h_{SQL}(\Omega)$ is the strain sensitivity corresponding to the standard quantum limit - the injection of a squeezed vacuum state into the dark port, assuming that the squeezing ellipse is aligned with the quadratures defined above (i.e. squeezing angle $\phi = 0$), leads to

$$S_h(\Omega) = \left[\frac{e^{-2r}}{k(\Omega)} + k(\Omega) e^{2r} \right] \frac{h_{SQL}^2(\Omega)}{2}. \quad (2.12)$$

In general, it is possible to choose the direction in quadrature space along which the noise reduction is achieved; this is determined by the *squeezing angle* ϕ (see Figure 2.5). For a generic ϕ , the rotated quadratures acquire nonzero cross-correlations, and the expression for $S_h(\Omega)$ contains an additional term proportional to $\sinh(2r) \sin(2\phi)$, so that the simple form above is recovered only in the special case $\phi = 0$.

This is particularly important when aiming to reduce quantum noise over the entire detection bandwidth. Indeed, if the squeezing angle ϕ is fixed (*frequency independent squeezing*), this cross term vanishes for $\phi = 0$ or $\phi = \pi/2$, so that the effect is simply analogous to changing the laser power. For $\phi \rightarrow \pi/4$, instead, the cross term is maximal and can partially cancel the sum of the two diagonal contributions, allowing $S_h(\Omega)$ to drop below $h_{SQL}^2(\Omega)$. However, since this cancellation is optimal only where the frequency-dependent coupling $k(\Omega)$ matches a specific value, a fixed ϕ can satisfy this condition only in a narrow neighborhood of a single frequency, inevitably degrading the performance over the remainder of the spectrum. Conversely, by varying the squeezing angle as a function of frequency (*frequency dependent squeezing*), it is possible to keep the cross

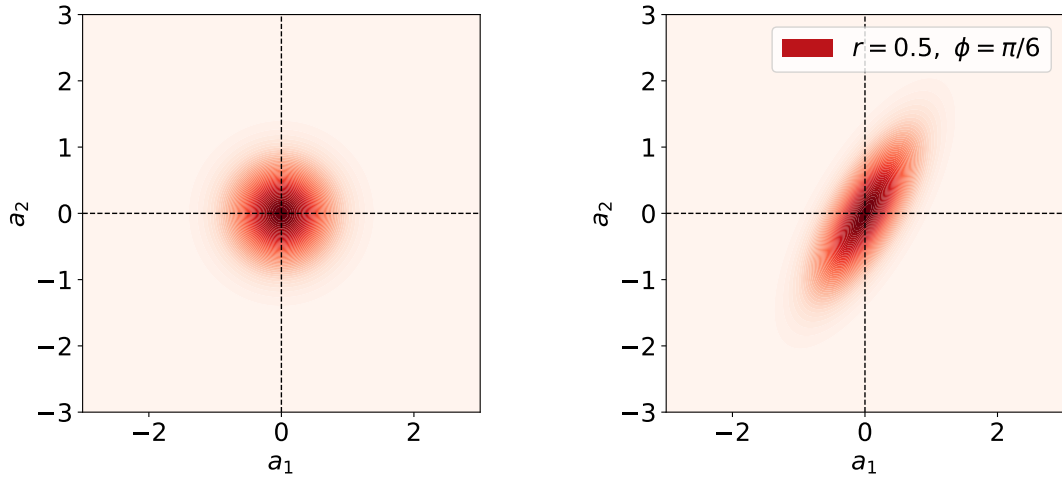


Figure 2.5: *Uncertainty region for a vacuum state (left) and a squeezed vacuum state (right) in phase space*

term optimally aligned with $k(\Omega)$ at every frequency, allowing the Standard Quantum Limit to be surpassed over a broad bandwidth and overcoming the limitations imposed by individual fixed configurations, as shown in Figure 2.6.

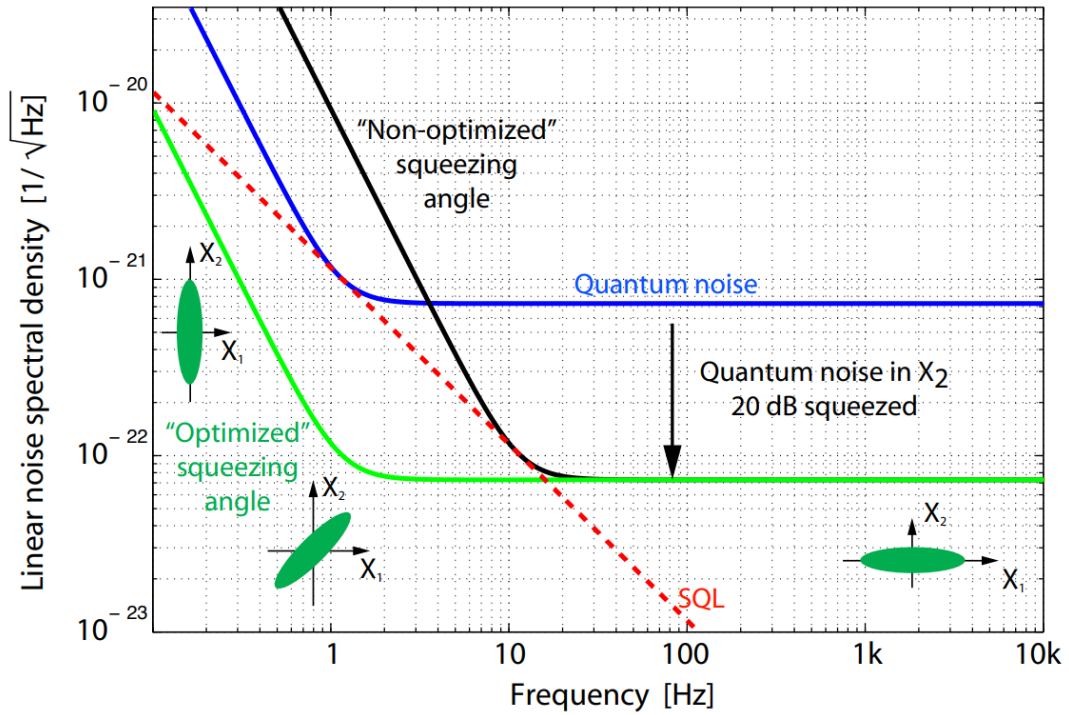


Figure 2.6: *The effect of squeezing on a Michelson interferometer. The blue curve shows the quantum noise of the detector. The black curve shows the effect of injecting frequency-independent squeezing with $\phi = 0$: the quantum noise is reduced at high frequencies at the expense of low frequencies, producing a shift of the curve along the SQL (red curve). The green curve shows the effect of injecting frequency-dependent squeezing: the sensitivity can be improved over the complete detection bandwidth. Figure from [3]*

2.4 Squeezing formalism

In the previous sections we introduced, in an intuitive way, the concepts of vacuum state, coherent state, and field quadratures, showing how replacing the ordinary vacuum entering the dark port of the interferometer with a squeezed vacuum state makes it possible, in principle, to reduce quantum noise below the standard quantum limit. In the next sections, this treatment is made rigorous and is completed.

2.4.1 Field quantization and sideband operators

Let's return to the electromagnetic field introduced in Section 2.3.2, and place it on a rigorous footing. In the quantum mechanical description of light, the electromagnetic field is expressed in terms of creation and annihilation operators. For a single-mode field at frequency ω_0 , the electric field operator can be written as

$$\hat{E}(t) \propto \hat{a}_{\omega_0} e^{-i\omega_0 t} + \hat{a}_{\omega_0}^\dagger e^{+i\omega_0 t}, \quad (2.13)$$

where $\hat{a}_{\omega_0}$ and $\hat{a}_{\omega_0}^\dagger$ are the annihilation and creation operators, satisfying the commutation relation $[\hat{a}_{\omega_0}, \hat{a}_{\omega_0}^\dagger] = 1$. These operators act on states $|n\rangle$ - eigenstates of the photon number operator $\hat{n} = \hat{a}_{\omega_0}^\dagger \hat{a}_{\omega_0}$ - according to $\hat{a}_{\omega_0}|n\rangle = \sqrt{n}|n-1\rangle$ and $\hat{a}_{\omega_0}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$.

As discussed in Section 2.1, the quantity that actually carries information in an interferometric measurement - a gravitational-wave strain $h(t)$, or more generally any signal that phase-modulates the carrier - oscillates at some frequency Ω much smaller than ω_0 . This phase modulation generates sideband fields at frequencies $\omega_0 \pm \Omega$, which carry the gravitational wave signal. It is therefore natural to decompose the field into upper and lower sideband operators

$$\hat{a}_+ \equiv \hat{a}_{\omega_0+\Omega}, \quad \hat{a}_- \equiv \hat{a}_{\omega_0-\Omega}, \quad (2.14)$$

so that the full field operator becomes

$$\hat{E}(t) \propto \hat{a}_{\omega_0} e^{-i\omega_0 t} + \hat{a}_+ e^{-i(\omega_0+\Omega)t} + \hat{a}_- e^{-i(\omega_0-\Omega)t} + h.c. . \quad (2.15)$$

2.4.2 Quadratures in the sideband picture

This description of the electromagnetic field is particularly convenient because it allows one to redefine the quadratures we introduced in Chapter 2.3.2:

$$\hat{a}_1 = \frac{\hat{a}_+ + \hat{a}_-^\dagger}{\sqrt{2}}, \quad \hat{a}_2 = \frac{\hat{a}_+ - \hat{a}_-^\dagger}{i\sqrt{2}}. \quad (2.16)$$

These operators satisfy the commutation relation $[\hat{a}_1, \hat{a}_2] = i$, which is the formal expression of the Heisenberg uncertainty principle between the two quadratures, and recover the bound $\Delta^2 \hat{a}_1 \cdot \Delta^2 \hat{a}_2 \geq \frac{1}{4}$ introduced in Section 2.3.2. The variances of the two quadratures can be computed directly from the definitions above (Equations 2.16): expanding $\Delta^2 \hat{a}_i = \langle \hat{a}_i^2 \rangle - \langle \hat{a}_i \rangle^2$ and using the commutation relations, one finds

$$\Delta^2 \hat{a}_1 = \frac{1}{2} \left[1 + \langle \hat{a}_- \hat{a}_+ \rangle + \langle \hat{a}_+^\dagger \hat{a}_-^\dagger \rangle \right] \quad (2.17)$$

$$\Delta^2 \hat{a}_2 = \frac{1}{2} \left[1 - \langle \hat{a}_- \hat{a}_+ \rangle - \langle \hat{a}_+^\dagger \hat{a}_-^\dagger \rangle \right] \quad (2.18)$$

These expressions reveal a key aspect: the noise properties of the field are governed by the correlation terms $\langle \hat{a}_- \hat{a}_+ \rangle$ and $\langle \hat{a}_+^\dagger \hat{a}_-^\dagger \rangle$, which describe correlations between the upper and lower sideband modes. For a coherent state or a vacuum state, these correlations vanish, and one recovers $\Delta^2 \hat{a}_1 = \Delta^2 \hat{a}_2 = \frac{1}{2}$. When the sideband modes are correlated, however, the two variances become unequal: noise is reduced in one quadrature and increased in the other. This is precisely the condition for squeezing.

2.4.3 The squeezing operator

The action that generates such sideband correlations is described by the *squeezing operator* [4]:

$$S(r, \phi) = \exp \left[r \left(\hat{a}_+ \hat{a}_- e^{-2i\phi} - \hat{a}_+^\dagger \hat{a}_-^\dagger e^{2i\phi} \right) \right] \quad (2.19)$$

where $r \geq 0$ is the *squeezing parameter* and ϕ is the *squeezing angle* that were mentioned in Chapter 2.3.5. Notice that:

- the squeezing parameter r controls the amount of noise redistribution: the variance along the squeezed quadrature is reduced by a factor e^{-2r} relative to the shot noise level, while the variance along the anti-squeezed quadrature is increased by e^{+2r} ;
- the squeezing angle ϕ determines the orientation of the squeezed quadrature in phase space - that is, which linear combination of $\hat{a}_1$ and $\hat{a}_2$ has reduced noise.

2.5 The squeezing angle

Figure 2.2 highlighted the strong analogy that exists between a bright coherent state and a vacuum state. However, the fact that the former is identical to the latter except for a translation given by the amplitude A of the coherent state hides a fundamental detail: a phase is defined for the coherent state, but not for the vacuum state. Indeed, the noise circle is isotropic, so the phase is a property of the mean field, not of the fluctuations. A phasor with a short mean length is “buried” inside its own noise circle, and only for $|A| \gg 1$ does the phase approach a well-defined classical quantity. For the vacuum itself, where $|A| = 0$, no phase can be defined at all: the phasor has zero length, and the isotropic noise circle simply surrounds the origin with no preferred direction.

The situation changes qualitatively for a squeezed vacuum state. Applying the squeezing operator $S(r, \phi)$ to the vacuum still gives a state with zero mean field - so there is still no “classical” phase in the sense of a coherent amplitude - but the noise circle is deformed into an ellipse, whose orientation is set by the squeezing angle ϕ . Even though the state itself carries no field to point in any direction, the shape of its noise does: rotating the ellipse by ϕ changes which quadrature is quiet and which is noisy. This angle is a well-defined physical quantity even for a state with exactly zero mean amplitude - in sharp contrast to the coherent-state phase discussed above, which becomes meaningless precisely in that limit. In this sense we can state that “the vacuum has no phase, but a squeezed vacuum does”.

2.5.1 Detection of a squeezed state

Once the squeezing angle has been defined as a characteristic of a squeezed vacuum state, let us look at how a state of this type can be measured. The standard tool for this is the *balanced homodyne detector* [5]: as shown in Figure 2.7 a signal field is combined with a strong auxiliary field, the *local oscillator*, on a 50:50 beamsplitter, and the two resulting beams are detected on separate photodiodes whose photocurrents are subtracted. Because the local oscillator is strong, it can be treated as a classical field with a well-defined phase ϕ_{LO} ; the signal field, by contrast, must be treated quantum-mechanically and is naturally decomposed into its two quadratures. It can be shown that the difference photocurrent is proportional to a combination of the signal quadratures [5],

$$i_1 - i_2 \propto \cos(\phi_{LO})\hat{a}_1 + \sin(\phi_{LO})\hat{a}_2, \quad (2.20)$$

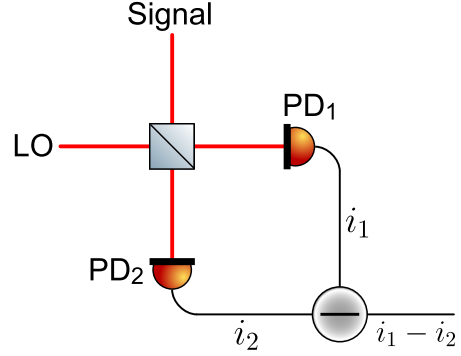


Figure 2.7: The balanced homodyne detector consists of a 50:50 beam splitter and two photodiodes whose photocurrents are subtracted, producing an output ($i_1 - i_2$). The signal field and the Local Oscillator (LO) enter the two input ports of the beam splitter.

i.e. the balanced homodyne detector measures exactly the quadrature of the signal field that is in phase with the local oscillator. By scanning ϕ_{LO} , one can effectively rotate the measurement axis around the phase-space origin and probe the noise along any direction.

2.6 The Optical Parametric Oscillator

The discussion of Section 2.4.2 showed that squeezed states arise when correlations between the upper and lower sideband modes of the field are established - that is, when the terms $\langle \hat{a}_- \hat{a}_+ \rangle$ and $\langle \hat{a}_+^\dagger \hat{a}_-^\dagger \rangle$ are non-zero. The question is then how to generate such correlations in practice. The most successful approach is optical parametric oscillation (OPO): a nonlinear optical process in which a pump field drives a crystal with second-order susceptibility $\chi^{(2)}$, creating correlations between pairs of sideband modes at $\omega_0 \pm \Omega$. In the following we first describe the physical mechanism underlying this process and then show explicitly how the OPO transforms an input field - establishing the connection between the parametric interaction and the squeezing operator $S(r, \phi)$.

2.6.1 Squeezed vacuum generation via parametric down-conversion

The generation of squeezed vacuum states relies on the process of spontaneous parametric down-conversion in a crystal with second-order nonlinear susceptibility $\chi^{(2)}$. When such a crystal is driven by a strong pump field at frequency $2\omega_0$, the nonlinear interaction can split a pump photon into two photons at ω_0 , subject to energy and momentum conservation. Crucially, the two photons are generated simultaneously and their quantum fluctuations are therefore correlated - if one photon carries a fluctuation in a given quadrature, the other is aware of it through the shared interaction. In the sideband picture of Section 2.4, this corresponds precisely to establishing non-zero correlations $\langle \hat{a}_- \hat{a}_+ \rangle \neq 0$ between the upper

and lower sideband modes at $\omega_0 \pm \Omega$ - the condition identified in Section 2.4.2 as the origin of squeezing.

In a single pass through the crystal, however, this interaction is weak and the resulting correlations are negligible. To enhance the effect, the crystal is placed inside an optical resonator - the optical parametric oscillator (OPO) - which recycles the field at ω_0 and allows it to interact repeatedly with the pump, accumulating parametric gain on each round trip. The strength of this gain is controlled by the pump power P : as P increases towards the oscillation threshold P_{thr} , the gain per round trip approaches the round-trip losses and the correlations between the sideband modes grow stronger, producing increasingly squeezed vacuum states. In this work the OPO is always operated below threshold, so no coherent field builds up at ω_0 .

2.6.2 Action of the OPO on the field

The description above covers the case of no injected signal, which is the one relevant for generating the squeezed vacuum field itself. It is useful, however, to describe the action of the OPO more generally, for an arbitrary field entering the cavity.

Consider an input field with coherent amplitudes α_+ , α_- in the upper and lower sidebands, respectively (the vacuum-only case of the previous Section corresponds to $\alpha_+ = 0$, $\alpha_- = 0$). It can be demonstrated [6] that we can use the transformation of the squeezing operator derived in Chapter 2.4.3,

$$S(r, \phi) \hat{a}_+ S^\dagger(r, \phi) = \hat{a}_+ \cosh(r) + \hat{a}_-^\dagger e^{2i\phi} \sinh(r), \quad (2.21)$$

$$S(r, \phi) \hat{a}_- S^\dagger(r, \phi) = \hat{a}_- \cosh(r) + \hat{a}_+^\dagger e^{2i\phi} \sinh(r), \quad (2.22)$$

in order to get the mean sideband amplitudes at the output of the OPO:

$$\langle \hat{b}_+ \rangle = \alpha_+ \cosh(r) + \alpha_-^* e^{2i\phi} \sinh(r), \quad (2.23)$$

$$\langle \hat{b}_- \rangle = \alpha_- \cosh(r) + \alpha_+^* e^{2i\phi} \sinh(r). \quad (2.24)$$

Physically, the OPO couples each sideband to the conjugate of the other: part of the amplitude injected in one sideband is converted into the other sideband, with a conversion efficiency set by the parametric gain $g = e^r$, and a relative phase set by 2ϕ . Together with the carrier at ω_0 , these two sideband amplitudes fully determine the output field $E_{out}(t)$.

This general relation contains, as special cases, both situations of interest for this thesis: setting $\alpha_+ = \alpha_- = 0$ removes any coherent output and recovers the pure squeezed-vacuum case, in which only the fluctuations - not the mean field - carry the signature of g and ϕ ; conversely, injecting a coherent seed in only one of the two sidebands generates a coherent output field, whose relative amplitude and phase both depend on the OPO gain and squeezing angle.

2.7 The Coherent Control

As we saw in Chapter 2.5.1, the practical use of squeezed states requires active and precise control of the phase relative to the Local Oscillator field used in the detection stage. This control is crucial: without continuous and stable phase alignment, the benefits of quantum noise reduction are rapidly nullified by environmental fluctuations and optical

drifts. For this reason, the stabilization of squeezing angle represents the main theoretical and technical challenge for their applications in gravitational wave detectors.

The most common control scheme involves injecting into the OPO a weak, phase-modulated seed field at the carrier frequency. In this configuration, the seed field, being coherent with the interferometer's main laser, passes through the nonlinear crystal and is amplified or deamplified depending on its relative phase with respect to the OPO pump field. The phase modulation impressed on the seed generates, at the output of the device, suitable sidebands that can be detected using demodulation techniques. From the demodulated signal, an error signal is obtained, proportional to the phase deviation between the squeezed field and the optical reference. This signal is then used to continuously correct the phase of the pump field, of the local oscillator, or of other optical elements of the system, keeping the orientation of the squeezed quadrature stable during operation.

However, this technique presents strong limitations: it has been shown that even minimal carrier powers introduce an excessive level of classical laser noise in the audio frequency band, completely compromising the squeezing effect and nullifying the desired noise reduction [7].

The solution adopted in this thesis - following [8] - is to control the squeezing angle using an auxiliary field called *Quadrature Control Field* (QCF) that is coherent with, but frequency-shifted away from, the carrier, so that it never interferes with the squeezed vacuum field itself and cannot contaminate it with classical noise.

2.7.1 The Quadrature Control Field

The QCF is generated from the same laser used for the local oscillator, frequency-shifted by Ω so that, before entering the OPO, it populates only the upper sideband at $\omega_0 + \Omega$, with a real amplitude α_Ω : in the notation of Section 2.6.2, this corresponds to

$$\alpha_+ = \alpha_\Omega, \quad \alpha_- = 0. \quad (2.25)$$

Since the QCF does experience the OPO nonlinearity, the general input-output relation derived in Section 2.6.2 applies directly, and leads to

$$\langle \hat{b}_1(\Omega) \rangle = -\frac{i\alpha_\Omega}{\sqrt{2}} \sin(2\phi) \sinh(r) + \frac{\alpha_\Omega}{\sqrt{2}} [\cosh(r) + \cos(2\phi) \sinh(r)] \quad (2.26)$$

$$\langle \hat{b}_2(\Omega) \rangle = \frac{\alpha_\Omega}{\sqrt{2}} \sin(2\phi) \sinh(r) - \frac{i\alpha_\Omega}{\sqrt{2}} [\cosh(r) - \cos(2\phi) \sinh(r)] \quad (2.27)$$

i.e. the OPO generates a lower sideband at $\omega_0 - \Omega$ that was entirely absent at the input, purely as a consequence of the parametric interaction. Writing $g \equiv e^r$ for the parametric gain, the corresponding output field takes the form

$$E^{\text{QCF}}(t) \propto \frac{1+g}{\sqrt{2g}} \alpha_\Omega \cos(\omega_0 t + \Omega t) - \frac{1-g}{\sqrt{2g}} \alpha_\Omega \cos(\omega_0 t - \Omega t - 2\phi) \quad (2.28)$$

This expression reveals the essential action of the OPO: a field that entered the cavity with only one sideband at $\omega_0 + \Omega$ exits with two sidebands, one at $\omega_0 + \Omega$ and one at $\omega_0 - \Omega$. The missing lower sideband has been generated by the parametric interaction inside the cavity. The amplitudes of the two sidebands are unequal and depend on the gain g : the upper sideband is amplified by a factor $(1+g)/\sqrt{2g}$, while the lower sideband has

amplitude $(1 - g)/\sqrt{2g}$. The relative phase between the two sidebands is 2ϕ , set by the phase of the pump field. It is precisely this asymmetric two-sideband structure that encodes the squeezing angle and allows the generation of the error signals needed for coherent control.

2.7.2 First error signal: locking the QCF to the pump phase

The first control loop stabilizes the relative phase ϕ between the QCF and the OPO pump field. It is obtained by detecting the outgoing QCF with a single photodiode, whose photocurrent is

$$I^{QCF} \propto \frac{\alpha_\Omega^2}{2g} [(1 + g) \cos(\omega_0 t + \Omega t) - (1 - g) \cos(\omega_0 t - \Omega t - 2\phi)]^2. \quad (2.29)$$

This contains a beat term between the two sidebands oscillating at 2Ω . Demodulating at 2Ω and low-pass filtering isolates this beat and yields an error signal directly proportional to $\sin(2\phi)$

$$S_{err}^{QCF-P} \propto -\frac{(1 - g^2)\alpha_\Omega^2}{4g} \sin(2\phi). \quad (2.30)$$

Once this loop is closed, ϕ (the squeezing angle itself) is stabilized with respect to the OPO pump.

2.7.3 Second error signal: locking the pump to the local oscillator

Stabilizing ϕ with respect to the pump is not yet sufficient: what ultimately matters is the alignment between the squeezed quadrature and the readout quadrature (Section 2.5.1), i.e. the relative phase θ between the OPO pump and the local oscillator field of the downstream homodyne detector,

$$E^{LO}(t) \propto \alpha_{LO} e^{-i\omega_0 t} e^{-i\theta} + \text{c.c.} \quad (2.31)$$

The second error signal is obtained by overlapping the outgoing QCF with the local oscillator at the homodyne beamsplitter and taking the difference of the two homodyne photocurrents. Because the QCF sidebands sit at $\omega_0 + \Omega$ while the local oscillator sits exactly at ω_0 , this beat note oscillates at Ω rather than 2Ω ; after low-pass filtering, demodulating at Ω gives

$$S_{err}^{QCF-LO} \propto \alpha_{LO} \alpha_\Omega \frac{(-1 + g)}{g} \sin(2\phi + \theta). \quad (2.32)$$

This error signal depends on both ϕ and θ , but since the first loop has already locked ϕ (Eq. 2.30) to a known, fixed value, S_{err}^{QCF-LO} becomes, for all practical purposes, a clean error signal for θ alone, usable to lock the local-oscillator phase.

2.8 Limitations of squeezed vacuum states

The discussion so far has described squeezed states in an idealized setting. In any real experimental implementation, however, a number of physical imperfections degrade the squeezing performance. It is useful to establish a quantitative framework for these limitations, as they directly motivate the design choices of the setup characterized in this

thesis. In this section, it is convenient to adopt the notation of [9] and [10] rather than the quadrature-variance notation $\Delta^2 \hat{a}_1, \Delta^2 \hat{a}_2$ used in previous sections. We denote by V_- and V_+ the variances of the squeezed and antisqueezed quadratures, respectively.

We begin by considering the squeezing produced by an OPO below threshold in the ideal case [9]. The variances of the squeezed and antisqueezed quadratures as a function of the pump power P and sideband frequency f are given by

$$V_{\pm} = 1 \pm \eta \frac{4\sqrt{P/P_{thr}}}{\left(1 \mp \sqrt{P/P_{thr}}\right)^2 + 4(2\pi f \kappa^{-1})^2}, \quad (2.33)$$

where P_{thr} is the oscillation threshold power, κ is the cavity decay rate, and η is the total detection efficiency, which accounts for all the optical losses affecting the squeezed field, from its generation inside the OPO cavity to its detection. In the limit of low sideband frequencies compared to the cavity linewidth, $f \ll \kappa/2\pi$, the frequency-dependent term in the denominator becomes negligible. Setting furthermore $\eta = 1$ - as we are considering the ideal case - and introducing the normalized pump amplitude $x = \sqrt{P/P_{thr}}$, the expression simplifies to

$$V_{\pm} = \left(\frac{1 \pm x}{1 \mp x} \right)^2. \quad (2.34)$$

Notice that, as $x \rightarrow 1$, i.e. as the pump power approaches threshold, $V_- \rightarrow 0$ and $V_+ \rightarrow \infty$, so the squeezing diverges in the ideal case. It is straightforward to verify that this expression is consistent with the squeezing operator formalism of Section 2.4.3 by computing the squeezing ratios

$$R_- = \sqrt{\frac{V_-}{V_{vac}}} = e^{-r} = \frac{1-x}{1+x} \quad (2.35)$$

$$R_+ = \sqrt{\frac{V_+}{V_{vac}}} = e^r = \frac{1+x}{1-x} \quad (2.36)$$

which confirms that the parametric gain $g = e^r$ grows monotonically with pump power and diverges at threshold. The ideal squeezing and antisqueezing ratios as a function of normalized pump power are shown in Figure 2.8.

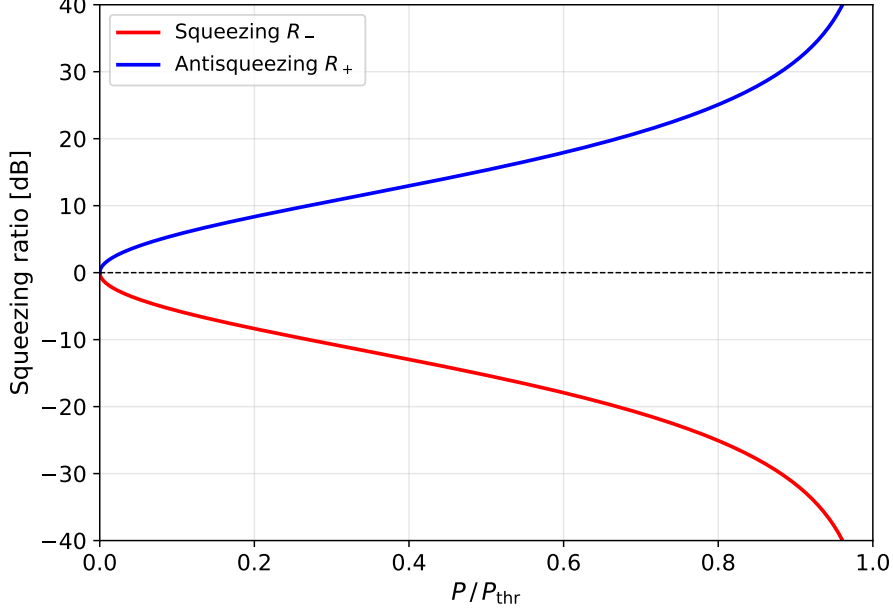


Figure 2.8: *Ideal squeezing and antisqueezing ratios as a function of the normalized pump power P/P_{thr} , for the ideal lossless case $\eta = 1$. The red curve shows the squeezed quadrature, and the blue curve the antisqueezed quadrature. As $P/P_{thr} \rightarrow 1$, the squeezing diverges towards $-\infty$, while the antisqueezing diverges towards $+\infty$.*

2.8.1 Optical losses

In any real optical setup, propagation losses are unavoidable. Absorption, scattering and imperfect mode matching between the squeezed field and the local oscillator contribute to an effective total loss l . Each of these mechanisms can be modelled as a beamsplitter that partially replaces the squeezed field with ordinary vacuum, coupling in unsqueezed fluctuations from the open port [10]. For a total power loss l , the variances of the output state are

$$V_{\pm}^l = (1 - l) V_{\pm} + l V_{vac}$$

which in terms of squeezing ratios becomes

$$R_{\pm}^l = (1 - l) R_{\pm} + l$$

The physical interpretation is immediate: losses mix the squeezed state with vacuum noise, pushing both quadrature variances towards the vacuum level $R = 1$. Since $R_- < 1$ and $R_+ > 1$, this mixing always degrades the squeezing while partially reducing the antisqueezing. This effect is illustrated in Figure 2.9: optical losses cause the squeezing to saturate rapidly as the pump power increases, while the antisqueezing continues to grow largely unaffected. Notice that in case of multiple independent loss mechanisms, the total efficiency is simply the product of the individual efficiencies $\eta_{tot} = \prod_i \eta_i$, where $\eta_i = 1 - l_i$.

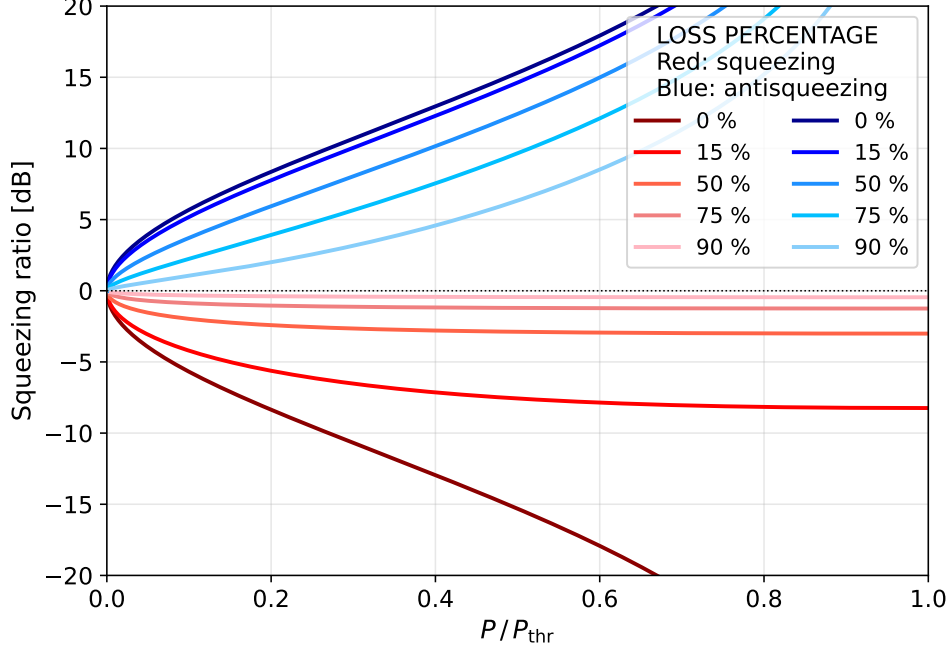


Figure 2.9: *Effect of optical losses on squeezing and antisqueezing ratios as a function of normalized pump power P/P_{thr} . Increasing the total loss pulls both quadratures towards the vacuum level (0dB): the squeezing saturates at a loss-dependent maximum value well before threshold is reached, while the antisqueezing - always the less-affected quadrature, since $R_+ > 1$ is only partially reduced towards unity - keeps growing with pump power over a much wider range.*

2.8.2 Phase noise

A second important source of degradation is phase noise - residual fluctuations of the angle between the squeezed quadrature and the readout quadrature [10]. Measuring a quadrature rotated by an angle θ with respect to the squeezed quadrature yields a variance:

$$V_{\pm}^{\theta} = V_{\pm} \cos^2 \theta + V_{\mp} \sin^2 \theta$$

In practice, the squeezing angle is actively stabilized by a servo control loop - this is precisely the role of the coherent control scheme described in Section 2.7. However, even with the best active control, residual fast fluctuations remain that cannot be corrected. Modelling these as normally distributed around zero with RMS deviation $\tilde{\theta}_{rms}$, the time-averaged observed variance is

$$V_{\pm}^{\tilde{\theta}_{RMS}} \approx V_{\pm} \cos^2 \tilde{\theta}_{RMS} + V_{\mp} \sin^2 \tilde{\theta}_{RMS}$$

The key difference with respect to optical losses is the presence of the $V_{\mp} \sin^2 \tilde{\theta}_{RMS}$ term, which couples the antisqueezed quadrature into the squeezing measurement. As the pump power increases, V_+ grows without bound in the ideal case, and even a small but fixed $\sin^2 \tilde{\theta}_{RMS}$ multiplies an increasingly large number. Beyond a certain pump power, this term dominates over the squeezed contribution, and the observed squeezing begins to worsen rather than improve. This behaviour is illustrated in Figure 2.10: unlike losses, phase noise produces a non-monotonic dependence of the observed squeezing on pump

power, with an optimal pump level that depends on $\tilde{\theta}_{\text{RMS}}$. This has an important practical consequence, qualitatively different from the effect of optical losses discussed above. Losses degrade squeezing *monotonically*: increasing the pump power can never make the observed squeezing worse, only cause its improvement to saturate, so operating at high gain is at worst wasteful, never actively harmful. Phase noise, by contrast, makes overdriving the OPO actively counterproductive: beyond the optimal pump power set by $\tilde{\theta}_{\text{RMS}}$, a further increase in P couples more antisqueezed noise into the measurement than it removes squeezed noise, and the observed squeezing gets worse rather than better. In other words, the residual phase noise does not merely set a ceiling on the best squeezing attainable, but it also fixes the pump power (equivalently, the parametric gain $g = e^r$) at which the experiment must actually be operated to reach that ceiling.

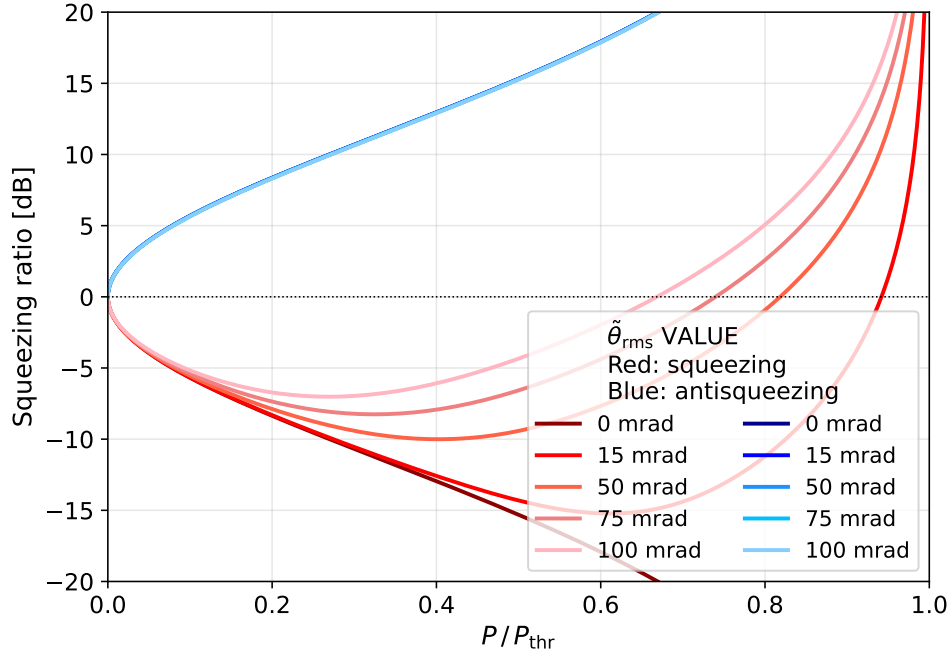


Figure 2.10: *Effect of phase noise on squeezing and antisqueezing ratios as a function of normalized pump power P/P_{thr} . For any $\tilde{\theta}_{\text{rms}} \neq 0$, the observed squeezing (red) improves with pump power only up to an optimal point, beyond which the growing antisqueezed contribution $V_+ \sin^2(\tilde{\theta}_{\text{rms}})$ dominates and the observed squeezing degrades*

Chapter 3

Setup description

The purpose of this chapter is to describe the experimental setup designed to produce a compact vacuum squeezer at 1550nm , and to provide the physical tools necessary to understand the characterizations of the different components reported in Chapter 4. The simplified scheme of the setup is shown in Figure 3.1, and this thesis focuses predominantly on the fiber-based part of the setup, which is actually the innovative part of the project that is use to provide the beams that will produce the squeezing in the external OPO cavity.

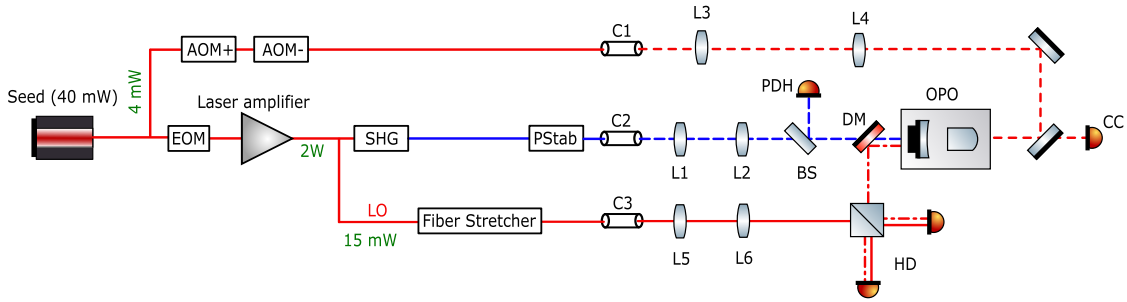


Figure 3.1: *Scheme of the coherent control setup. The output of a single laser is split into three branches. In the upper branch, the frequency-shifted Quadrature Control Field (QCF) is generated and sent into the OPO cavity; in the central branch, the seed is sent to a fiber amplifier and then to a second-harmonic generation stage (SHG) in order to generate the pump for the OPO; in the lower branch, the remaining seed light serves as the Local Oscillator (LO) for the balanced homodyne detector, where it is combined with the squeezed field exiting the OPO*

The starting point is a single laser, whose output is split into three distinct branches, each of which performs a different function: the generation of the pump field for the OPO, the generation of the Quadrature Control Field (QCF), and the generation of the Local Oscillator (LO) for the homodyne detector.

The central branch - pump generation. Most of the seed power is sent to a fiber amplifier and subsequently passes through a second-harmonic generation (SHG) stage. This converts part of the power from the fundamental frequency ω_0 to the doubled frequency $2\omega_0$. The $2\omega_0$ field thus generated, after a power stabilization stage, is injected into the OPO cavity: this is the pump field that drives the parametric process described in Section 2.6,

and which determines the parametric gain $g = e^r$ and therefore the amount of squeezing generated.

The upper branch - QCF generation. A small fraction of the seed power is extracted before the amplifier and sent to a pair of AOMs in series (AOM+ and AOM-). As discussed in Section 2.7.1, the QCF must be a field coherent with the main laser but frequency-shifted by Ω with respect to the carrier, so as to populate exclusively the upper sideband at $\omega_0 + \Omega$ before entering the OPO, without directly interfering with the squeezed vacuum field generated by the OPO itself. The use of two cascaded AOMs, with opposite frequency shifts, rather than a single AOM is motivated, on the one hand, by the need to obtain a shifted beam as free as possible from contamination by the undiffracted light at the original frequency ω_0 - an effect that, if not controlled, would degrade the purity of the QCF and therefore the quality of the error signals derived from it - and, on the other hand, by the possibility of implementing frequency shifts of the order of $\Delta f \sim 5 \text{ MHz}$ - as required for the implementation on interferometers - which are very low values for conventional AOMs.

The lower branch - Local Oscillator generation. The remaining beam at ω_0 exiting the amplifier is sent, passing through a fiber stretcher, to the homodyne detector. As discussed in Section 2.5.1, the Local Oscillator must have a well-defined phase θ with respect to the pump field, since it is this phase relationship that determines which quadrature is actually measured by the homodyne detector. The fiber stretcher therefore acts as the actuator for the control loop described in Section 2.7.3, while the error signal, i.e. the phase difference between the LO and the QCF, is provided by the RF channel of the homodyne detector.

3.1 The Laser

The first element of the setup in Figure 3.1 is the laser itself, whose phase stability constitutes, as discussed in Section 2.8.2, one of the limitations in the generation of squeezed vacuum.

A laser essentially consists of an active medium enclosed within an optical resonant cavity. The active medium is “pumped”, i.e. brought into a condition of population inversion between two energy levels, typically by supplying energy through an external source. Under this condition, a photon passing through the medium can stimulate the emission of a second photon identical to the first - same frequency, phase, direction, and polarization - through the process of stimulated emission. The resonant cavity selects the longitudinal modes of the field that can oscillate stably within it and provides the feedback necessary for stimulated emission to overcome the losses, leading to laser oscillation. The result is, ideally, a coherent and monochromatic electromagnetic wave at the frequency ω_0 of the active medium, whose output power depends on the balance between the gain provided by the pumping and the cavity losses:

$$E(t) = \text{Re}\{E_0 \exp[-i(\omega t + \varphi_0)]\}. \quad (3.1)$$

In physical reality, the emitted radiation deviates from this ideal description due to the coupling of the laser system (active medium and resonant cavity) to the surrounding

environment [11]. This introduces stochastic fluctuations that modulate both the amplitude and the phase of the electromagnetic field:

$$E(t) \rightarrow \text{Re}\{[E_0 + e_N(t)] \exp[-i(\omega t + \varphi_0 + \delta\varphi(t))]\} \quad (3.2)$$

where $e_N(t)$ represents *amplitude noise* and $\delta\varphi(t)$ represents *phase noise*.

As analyzed in the previous sections, phase noise plays a significantly more critical role in our experiment because any uncontrolled phase fluctuation inevitably leads to a degradation of the measured squeezing.

From a microscopic and theoretical point of view, phase noise has a fundamental origin and is related to several factors. In particular, spontaneous emission within the gain medium adds photons with completely random phase to the cavity field. Since there is no restoring force constraining the phase to a fixed value, the latter undergoes a random walk. This phase diffusion process determines the fundamental limit of the laser linewidth, known as the *Schawlow-Townes limit*. Another source of phase noise is related to the intrinsically stochastic processes in the pumping mechanism (e.g. fluctuations in the flow of charge carriers or Poisson statistics in the pumping) and to random cavity losses, which induce continuous quantum fluctuations in the population inversion and polarization. Finally, in addition to the fundamental quantum limits, technical noise sources, typically much larger in amplitude, are added in experimental practice. These include temperature fluctuations, microphonics and mechanical vibrations of the cavity, fluctuations in the refractive index, and noise from the laser power-supply electronics.

In order to limit this phenomenon, we selected a laser source with a very low phase noise [12] provided by *NKT Photonics*¹

3.2 The AOM

We now turn to the AOMs, which are used for QCF generation due to their ability to shift the frequency of the incoming beam.

An AOM exploits the interaction between an acoustic wave and a light wave that both propagate within the same transparent medium, typically a crystal [11]. A piezoelectric transducer, bonded to one end of the crystal and driven by an RF electrical signal, excites a pressure wave inside it that propagates as an acoustic wave at frequency Ω . This wave generates a periodic compression and rarefaction of the material and therefore a periodic modulation of its refractive index, with the same spatial and temporal period as the acoustic wave itself. In practice, the sound wave creates a moving optical diffraction grating inside the crystal, whose periodicity is given by the acoustic wavelength. At the end of the crystal opposite the transducer, an acoustic absorber is typically placed, whose purpose is to prevent reflection of the sound wave and the consequent formation of a standing wave, which would complicate the behavior of the device.

An optical beam passing through this moving grating is diffracted. If the thickness of the medium traversed by the light is sufficiently large compared with the acoustic grating period, the light beam is diffracted in the regime known as *Bragg diffraction*: in this regime, the optical energy is efficiently directed into a single diffraction order, provided that the beam enters the crystal at a precise angle with respect to the acoustic wavefront

¹<https://lf.hamamatsu.com/>

(the Bragg angle). Furthermore, since the grating is not static but moving, the diffracted light undergoes a frequency shift equal to the frequency of the acoustic wave itself.

At the output of the AOM, two beams are therefore generally observed: the transmitted beam, which passes through the crystal without being diffracted and retains its original direction and frequency, and the first-order diffracted beam, which is deflected by a small angle and frequency-shifted.

3.3 The SHG

Let's now consider the second-harmonic generation (SHG) stage, whose purpose is to convert part of the power of the seed field at the fundamental frequency ω_0 into the pump field at $2\omega_0$ required to drive the OPO.

Second-harmonic generation is a second-order nonlinear optical process. When an intense optical field at frequency ω_0 propagates through a medium lacking inversion symmetry - such as a nonlinear crystal - the induced polarization of the medium is no longer simply proportional to the field, but acquires a second-order term

$$P(t) = \epsilon_0 [\chi^{(1)} E(t) + \chi^{(2)} E(t)^2 + \dots] , \quad (3.3)$$

where $\chi^{(1)}$ is the usual linear susceptibility and $\chi^{(2)}$ is the second-order nonlinear susceptibility, a material-dependent quantity that is non-zero only in crystals without inversion symmetry [13]. This nonlinear polarization acts as a source term at the doubled frequency, generating a new field E_{SHG} at $2\omega_0$ - the second-harmonic field that ultimately provides the pump for the OPO.

Solving the wave equation with this nonlinear source term [13] shows that the amplitude of the generated second-harmonic field grows along the propagation direction z inside the crystal proportionally to the square of the fundamental field amplitude and to the nonlinear coefficient $d_{eff} = \chi^{(2)}/2$, but only under a specific condition: the fundamental and the harmonic wave must remain in phase with each other as they propagate. This condition is quantified by the phase mismatch

$$\Delta k = k_{2\omega} - 2k_{\omega} = \frac{2\omega_0}{c}(n_{2\omega} - n_{\omega}) , \quad (3.4)$$

which depends on the difference between the refractive indices experienced by the fundamental and the harmonic field. Because of the normal dispersion of any real material, $n_{2\omega} \neq n_{\omega}$ in general, so that $\Delta k \neq 0$: the two waves progressively dephase as they propagate, and the conversion efficiency oscillates rather than growing monotonically with the crystal length.

In the regime of weak conversion - appropriate here, since the fundamental wave is only slightly weakened - the intensity of the generated second harmonic at the output of a crystal of length L can be shown to follow [13]

$$I_{SHG} \propto I_{FUN}^2 L^2 \text{sinc}^2 \left(\frac{\Delta k L}{2} \right) \quad \text{sinc}(x) = \frac{\sin(x)}{x} . \quad (3.5)$$

Efficient conversion is therefore obtained only when $\Delta k L \approx 0$, a condition known as *phase matching*. Away from this condition, the SHG output oscillates and rapidly drops as $|\Delta k L|$ increases, tracing out the characteristic sinc^2 lineshape.

3.4 The EOM

The last element to be introduced is the electro-optic modulator (EOM), which is used throughout the setup - both as the actuator of the feedback loops described in Chapter 2.7 and, in the characterization measurements of Chapter 4, to inject calibrated phase modulations for transfer-function measurements.

An EOM exploits the electro-optic (Pockels) effect: in crystals lacking inversion symmetry, an applied electric field induces a change in the refractive index that is linear in the field strength. In a typical configuration, discussed in [14], a longitudinal field U is applied along the crystal, inducing two new principal axes along which the ordinary index is shifted by equal and opposite amounts, proportional to U . A beam polarized along one of these axes therefore accumulates a voltage-dependent phase as it propagates through the crystal.

For a sinusoidal driving voltage $U(t) = U_0 \sin(\Omega t)$, this results in a pure phase modulation of the output field,

$$\Phi(t) = \Phi_0 + m \sin(\Omega t) \quad (3.6)$$

where the modulation index m is proportional to U_0 and to the material's electro-optic coefficient. The key parameter used to characterize a given EOM, the one relevant for this work, is the half-wave voltage V_π , i.e. the voltage required to produce a phase shift of π .

Chapter 4

Setup characterization

4.1 Quadrature Control Field

Unlike traditional methods in which the QCF is generated by an auxiliary laser offset frequency locked to the main squeezer laser, in this project we chose to use a branch of the main laser frequency, shifted using an Acousto-Optical Modulator (AOM). Based on the experience gained with the Virgo Detector, the ideal frequency offset is about $5MHz$. As anticipated in Chapter 3, single AOMs with these specifications are not available, so the plan is to use two AOMs in series: one providing a frequency shift at $82.5MHz$ and the other $-77.5MHz$. In this way, a total frequency shift of $+5MHz$ is achieved, as required. In this section, I will summarize the measurements relevant to our application, carried out on two AOMs with center frequency $\pm 80MHz$.

4.1.1 Motivation

Before entering into the detailed characterization of the two AOMs, it is useful to motivate the specific measurements presented in this chapter. As discussed in Sections 3.2, the two AOMs in series are used to generate the QCF, a field that is meant to populate exclusively the sideband shifted by $+\Omega$ before entering the OPO cavity. In practice, however, each AOM diffracts only a fraction of the incident power: part of the beam is transmitted unshifted, at the original carrier frequency ω_0 . This residual carrier component travels together with the QCF along the same optical path and is therefore injected into the OPO cavity as well.

Although strongly attenuated with respect to the diffracted beam, this residual carrier cannot be neglected a priori, since even a very weak coherent field entering the OPO can significantly degrade the generated squeezing. Following the treatment of [7], [15], [16], the power spectral density of the squeezed and antisqueezed quadratures detected at the homodyne output can be written, in the presence of a coherent seed field circulating in the cavity, as the sum of two contributions:

$$S_{sqz}^{\pm}(\Omega) = \frac{C_s(\Omega)}{|D^{\pm}(\Omega)|^2} S_s^{\pm}(\Omega) + \alpha^2 \frac{C_p(\Omega)}{|D^{\pm}(\Omega)|^2} S_p^{\pm}(\Omega), \quad (4.1)$$

where “s” is for the seed, “p” for the pump and

$$D^{\pm}(\Omega) = i\Omega + \kappa_{1550} + \alpha^{\pm} \frac{\epsilon^2 a_s^2}{2\kappa_{775}} \mp \epsilon a_p \quad \alpha^+ = 3 \quad \alpha^- = 1 \quad (4.2)$$

$$C_s = 4\kappa_{in-1550} \cdot \kappa_o \quad C_P = 4\kappa_o \cdot \kappa_{in-775} \cdot \frac{\epsilon}{k_{1550}} \quad (4.3)$$

with the cavity decay rate κ , defined for a $1550nm$ wavelength as

$$\kappa_{in-1550} = FSR \cdot T_{i-1550} \quad \kappa_o = FSR \cdot T_o \quad \kappa_{1550} = \kappa_{in-1550} + \kappa_o \quad (4.4)$$

and for a $775nm$ wavelength as

$$\kappa_{in-775} = FSR \cdot T_{i-775} \quad \kappa_o = FSR \cdot T_o \quad \kappa_{775} = \kappa_{in-775} + \kappa_o \quad (4.5)$$

and a_s , a_p the average number of photons stored in the cavity at the seed angular frequency ω_s and the pump angular frequency ω_p , T_{i-1550} and T_{i-775} the transmissivities of the input couplers for the respective wavelengths and T_o the transmissivity of the outcoupling mirror. In terms of the input power, they can be written as

$$a_s = \frac{P_{seed-cav}}{\hbar\omega_s(T_{i-1550} + T_o)} \quad a_p = \frac{P_{seed-cav}}{\hbar\omega_p(T_{i-775} + T_o)} \quad (4.6)$$

with $P_{seed-cav}$ and $P_{pump-cav}$ being the intracavity power of the seed (1550 nm) and the pump (775 nm) respectively and FSR the Free Spectral Range of the cavity.

Notice that in Equation 4.1 one term is associated with the vacuum fluctuations of the seed mode itself, and one is associated with the classical (technical) amplitude and phase noise carried by the seed. In the absence of any seed, only the first term survives and the usual ideal (anti)squeezed variances (e^{2r}) e^{-2r} are recovered. If, instead, a coherent seed is present, its technical noise is added on top of the vacuum fluctuations, and it can partially or completely spoil the observed squeezing - an effect that is particularly severe at low frequencies, where the classical noise of the laser is largest. In [7] it is shown that a seed power of order $P_{seed} \approx 1pW$, injected through the OPO output coupler, is already sufficient to visibly degrade the squeezing at low frequency.

To translate this requirement into a bound on the power that can be tolerated at the output of the AOM chain, it is necessary to relate the power incident on the OPO to the corresponding intracavity power, which depends on which mirror the field enters through. For a field of power P_{seed} injected through a mirror of power transmissivity T_x , with the cavity output mirror having transmissivity T_o , the intracavity power is

$$P_{seed-cav} = P_{seed} \frac{T_x}{\left[1 - \sqrt{(1 - T_x)(1 - T_o)}\right]^2}, \quad (4.7)$$

with $T_x = T_{i-1550}$ if the field enters through the front (input-coupler) mirror, or $T_x = T_o$ if it enters from the back of the crystal, i.e. through the output coupler itself. For the hemilithic OPO cavity design that will be adopted for the setup, reproduced from [17], the relevant parameters are $T_{i-1550} = 0.1$, $T_{i-775} = 0.2$, $T_o < 1000ppm$ and $FSR = 3.44GHz$.

Since the QCF - and with it, the residual unshifted carrier from the AOM chain - is injected into the OPO from the back of the crystal, the relevant transmissivity in Eq.4.7 is T_o , rather than T_{i-1550} . This yields an estimated tolerance of order $P_{seed} \leq 1pW$ for the residual unshifted power that can be allowed at the output of the AOM chain, without significantly compromising the squeezing performance.

The second term in Equation 4.1 introduces a contribution that couples the pump noise to a possible seed field a_s present in the cavity. This term is therefore identically zero in

the absence of a seed, but becomes non-negligible whenever $a_s \neq 0$, even if the pump is quantum-noise limited. Using Equation 4.1, the parameters listed above, and assuming an ideal squeezing factor of 15dB at zero seed power (given by the first term alone), it is possible to estimate the effect of the seed on the squeezing level, optimistically assuming a quantum-noise-limited pump. The result is shown in Figure 4.1: the squeezing starts to degrade once the seed power reaches the pW level. Note that this plot is obtained under the assumption of a quantum-noise-limited pump, so the situation in the audio band is expected to be worse.

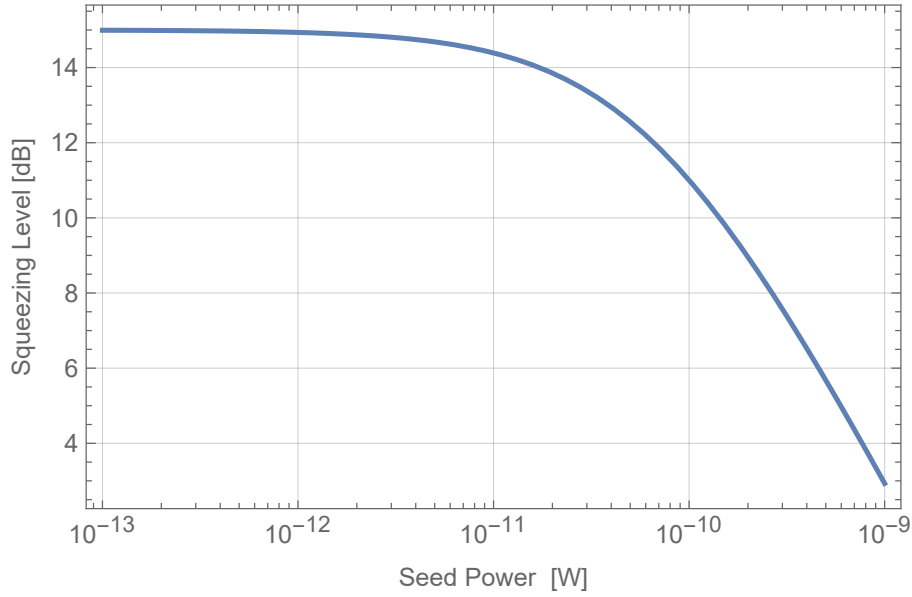


Figure 4.1: Squeezing level (absolute value) as a function of the seed power entering from the output-coupler side of the OPO crystal. A degradation of the initial 15dB squeezing level begins to appear at the pW level. Note that this plot is obtained assuming a quantum-noise-limited pump, so the situation in the audio band is expected to be worse.

4.1.2 Output Power as a function of RF voltage and RF input frequency

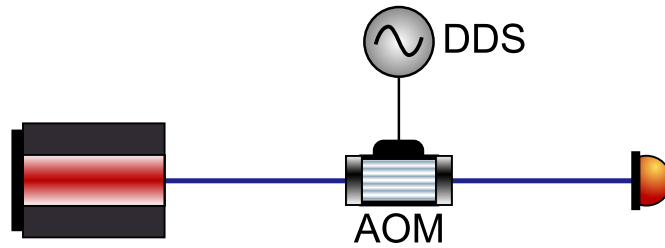


Figure 4.2: Setup used to characterize the diffraction efficiency of a single AOM as a function of the RF driving voltage and frequency. The beam from the seed laser is diffracted by the AOM, driven by a Direct Digital Synthesizer operating at 80MHz , and the diffracted output power is measured with a power meter

As a first step, the response of the two AOMs was observed as a function of the input RF signal voltage, generated by a home made low noise Direct Digital Syntetizer (DDS). The nominal shift frequency of 80MHz was set, and the output power was measured with a power meter as a function of the AOM RF driver voltage. The setup is shown in Figure 4.2. The components used are:

- Laser: *NKT - Koheras BASIK E15*
- AOM: *Brimrose - AMF-80-5-50-1550-2FP-PM*
- Power Meter: *Thorlabs - S145C*

The results are shown in Figure 4.3 and show a saturation region starting from about 7V , reaching a maximum efficiency of around 58% .

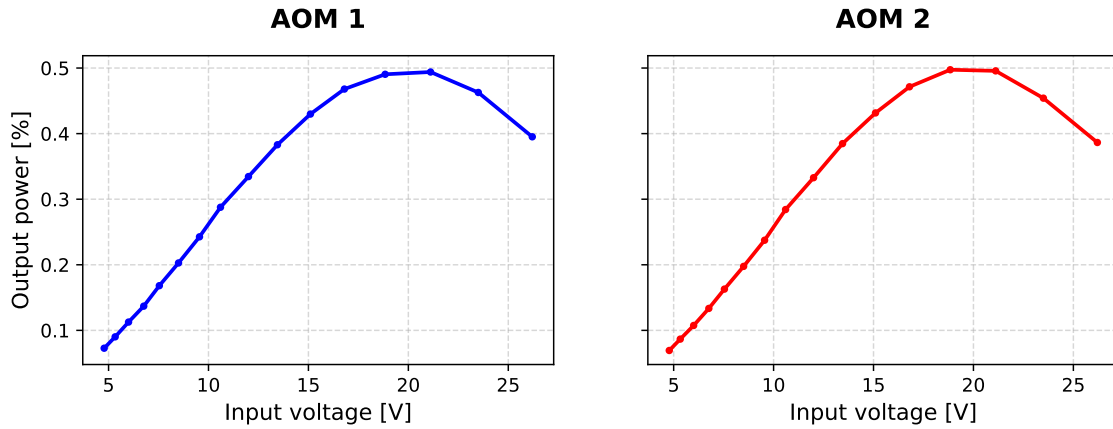


Figure 4.3: *Output power of AOM 1 (left) and AOM 2 (right) as a function of the RF driving voltage, measured with the setup of Figure 4.2 at a fixed shift frequency of 80MHz*

Next, the response of the AOMs was measured by applying a constant RF voltage and varying its frequency. The result is shown in Figure 4.4 and confirms the maximum nominal response of 80MHz for AOM 1, while for AOM 2 it reveals a maximum response at 80.5MHz .

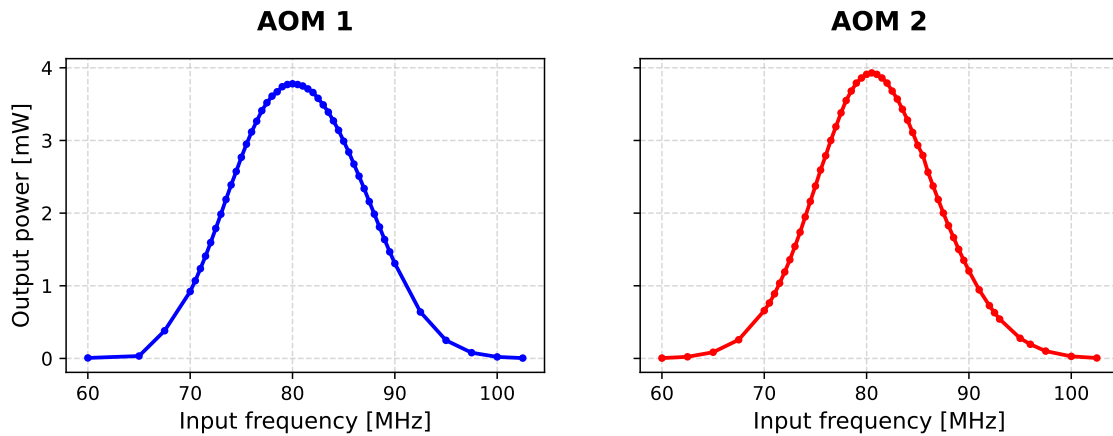


Figure 4.4: *Output power of AOM 1 (left) and AOM 2 (right) as a function of the RF driving frequency, measured with the setup of Figure 4.2 at fixed RF voltage*

Figure 4.4 shows the typical resonance response of the AOMs. However the resonance is sufficiently broad to allow the two AOMs to be operated at 77.5MHz and 82.5MHz respectively, without a significant signal attenuation.

4.1.3 AOM unshifted frequency contamination: estimation method 1

As discussed in Section 3.2, not all the power incident in the AOM is frequency shifted. A certain fraction remains at the original frequency, forming the primary beam. However, within the AOM, the 2 beams follow different optical paths, allowing the unshifted beam to be damped before the output, so that the output consists predominantly of the frequency shifted beam. The purpose of the measurements presented in this section is to estimate the amount of unshifted light remaining in the AOM output beam. This parameter is extremely important because it determines the level of seed field reaching the OPO. The higher the seed field, the greater the degradation of the measured squeezing level due to technical noise from sources (see Section 4.1.1)

To do this, a first basic approach was used, employing the setup in Figure 4.2, with a more sensitive photodiode than the *Thorlabs - S145C*, namely the *Thorlabs - S155C*, sensitive down to a hundredth of a nanoWatt. The ratio between the power transmitted by the AOM with the RF signal off and with the RF signal on (at 80MHz) was measured, assuming that, with the signal off, all the observed power actually came from the unshifted part of the input beam. The result obtained is shown in Table 4.1.

AOM 1		AOM2	
on	off	on	off
$(16.5 \pm 0.8)\text{mW}$	$(0.27 \pm 0.01)\text{nW}$	$(16.5 \pm 0.8)\text{mW}$	$(1.33 \pm 0.07)\text{nW}$

Table 4.1: Comparison between the output powers of the AOMs with the RF signal on and off

From this result it is possible to estimate the fractional presence of unshifted light in the AOM output beam on the order of $\sim 10^{-7} - 10^{-8}$.

4.1.4 AOM unshifted frequency contamination: estimation method 2

To obtain further confirmation of the result, the two AOMs were connected in series as shown in the setup in Figure 4.5, consisting of:

- Laser: *NKT - Koheras BASIK E15*
- AOMs: *Brimrose - AMF-80-5-50-1550-2FP-PM*
- Collimator: *Schäfter+Kirchhoff - 60FC-4-A6.2S-03*
- Photodiode: *Thorlabs - PDA05CF2*

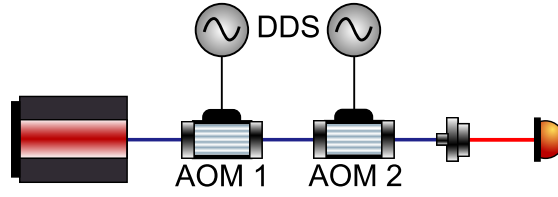


Figure 4.5: Setup used to search, via a beat-note measurement, for residual unshifted light at the output of the two AOMs connected in series. The beam from the seed laser passes through both AOMs, is collimated, and is detected by a photodiode connected to a spectrum analyzer, which looks for the beat note between the twice-shifted beam and the residual unshifted carrier transmitted through both AOMs

The collimated beam output from AOM 2 was detected by the photodiode, which in turn was connected to a spectrum analyzer (*Rhode&Schwarz - FPL1003*). The idea is to observe on the spectrum analyzer the peak related to the beat note between the beam shifted twice by the two AOMs and the unshifted beam transmitted by both. A similar scheme had previously been employed using a single AOM driven at 80MHz . In this case, in an unshifted residual component was present in the output beam, an 80MHz beat note would be expected at the photodiode output. Indeed an 80MHz signal was visible but it turned out that it was caused by electronic pickup from RF signal generated by the DDS, which masked the tiny optical beat note signal. Using the two AOMs in series instead, the peaks generated by electronic interference are located at frequencies different from the one at which the beat note is expected. The two AOMs were in fact driven at 82.5MHz and 77.5MHz so as to obtain a beat note at 5MHz , and the result obtained is shown in Figure 4.6.

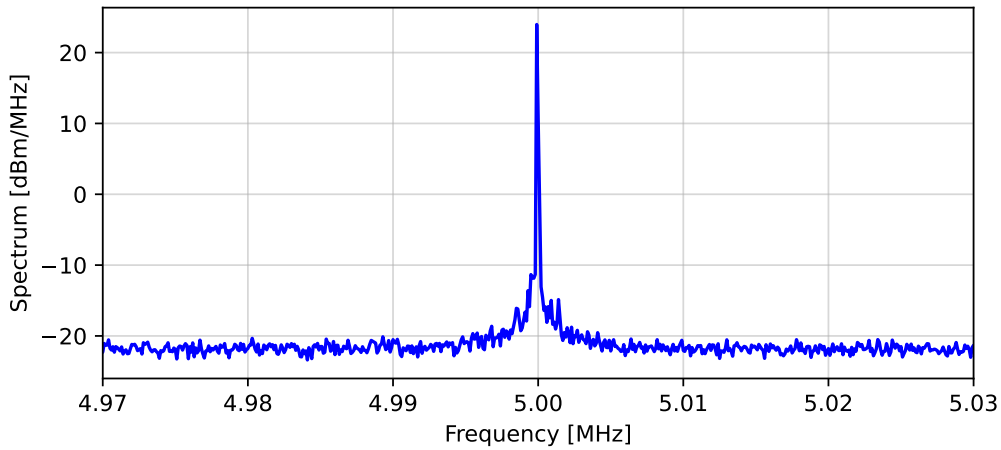


Figure 4.6: Beat-note spectrum measured with the setup of Figure 4.5, with the two AOMs driven at 82.5MHz and -77.5MHz so that the beat note between the doubly-shifted beam and the residual unshifted carrier is expected at 5MHz . The clear peak observed at 5MHz , together with the DC photocurrent measured separately with a multimeter, was used to estimate the carrier-to-diffracted power ratio P_C/P_{AOM}

Note that the photodiode sums the fields E_C of the unshifted carrier and E_{AOM} of the shifted beam and squares them. The signal shown in Figure 4.6 therefore contains only the

information relative to the oscillating part of a signal of the form:

$$I(t) \propto P_C + P_{AOM} + 2\sqrt{P_C \cdot P_{AOM}} \cdot \cos(\Delta\omega t + \varphi) \quad (4.8)$$

where $\Delta\omega = 2\pi \cdot 5MHz$. The DC part associated with the first two terms was then measured with a multimeter (*FLUKE-115*), which returned a value of $(7.78 \pm 0.06)V$. From this information, and by integrating the peak in Figure 4.6, it was possible to estimate the value of $P_C/P_{AOM} \approx 1.1 \cdot 10^{-5}$. This result is inconsistent with the result obtained in the previous section, which would have predicted P_C/P_{AOM} to be on the order of 10^{-8} .

4.1.5 AOM unshifted frequency contamination: estimation method 3

The results obtained using the 2 methods described above do not appear to be consistent to each other. Therefore, to further investigate this fact, the setup in Figure 4.7 was built. The method uses two identical lasers that are frequency locked with an offset f_Δ .

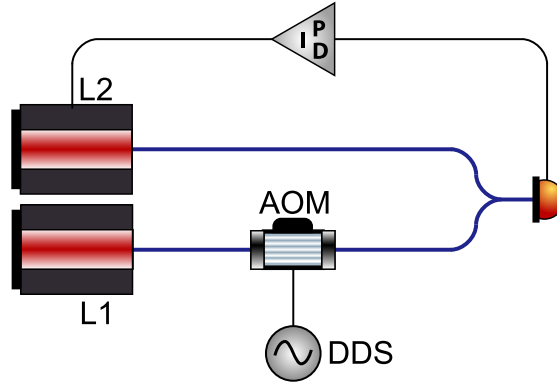


Figure 4.7: Setup used to measure the carrier-suppression ratio of a single AOM, using two frequency-locked lasers offset by f_Δ . The beam from Laser 1 (L1) passes through the AOM before being combined, on a photodiode, with the beam from Laser 2 (L2). A offset Phase Locking Loop (PLL) was engaged to maintain a stable frequency offset f_Δ between the two lasers to compensate for thermal drift and keep f_Δ - and hence the beat-note frequencies on the photodiode - stable over time

this consists of:

- Laser 1: *NKT - Koheras BASIK E15*
- Laser 2: *NKT - Koheras BASIK X15*
- AOM: *Brimrose - AMF-80-5-50-1550-2FP-PM*
- Photodiode: *Thorlabs - PDA05CF2*
- PID loop: *Liquid Instruments - Moku:Lab*

The underlying idea is that with this configuration, two beat notes should be generated on the photodiode: one, the main one, at frequency $f_{AOM} + f_\Delta$, where $f_{AOM} = 80MHz$ is the frequency of the beam shifted by the AOM, while f_Δ is the frequency difference between the two lasers, while a second weaker beat note should be generated by the beam transmitted by the AOM interfering with that of the other laser, thus producing a peak at f_Δ . The PLL loop was added because of the thermal drift of the lasers, which causes the

peak to shift over time and prevents it from being measured by averaging over multiple acquisitions. The loop instead allows locking onto the main beat note and sending a voltage to the laser that compensates for its drift. In this way, the frequency difference between the two lasers is kept constant and the peaks are stable. In Figure 4.8 the two peaks observed can be seen, driving the AOM at 80MHz and setting the wavelengths of the two lasers so as to have a frequency difference of about 34MHz .

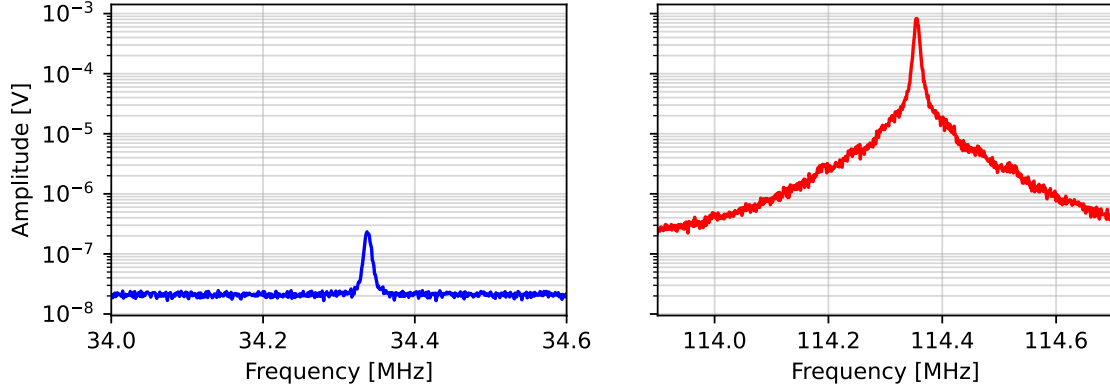


Figure 4.8: *Beat-note spectra obtained with the setup of Figure 4.7, with the AOM driven at $f_{AOM} = 80\text{MHz}$ and the two lasers locked to a frequency difference $f_{\Delta} \approx 34.35\text{MHz}$. The main peak (right panel), at $f_{AOM} + f_{\Delta} \approx 114.35\text{MHz}$, is the beat note between the diffracted, frequency-shifted beam and Laser 2; the much weaker secondary peak (left panel), at $f_{\Delta} \approx 34.35\text{MHz}$, is the beat note between the residual unshifted carrier transmitted through the AOM and Laser 2*

To obtain the ratio between the power transmitted and that diffracted by the AOM, the ratio between the maxima of the peaks obtained was then computed. This is $2.83 \cdot 10^{-4}V$, which, when squared, gives $P_C/P_{AOM} \approx 8.01 \cdot 10^{-8}$. The same measurement was repeated by sending the AOM an RF signal at frequencies of 77.5MHz and 82.5MHz to observe any variations in the system's response, but the results obtained are fully comparable with the one obtained at 80MHz : $8.07 \cdot 10^{-8}$ and $7.56 \cdot 10^{-8}$ respectively.

The result obtained above is therefore consistent with that obtained using Method 1. The possibility therefore remains that the discrepancy reported in Section 4.1.4 is related to the use of two AOMs in series rather than a single AOM. To test this hypothesis, we modified the setup described above by introducing two AOMs in series, as shown in Figure 4.9.

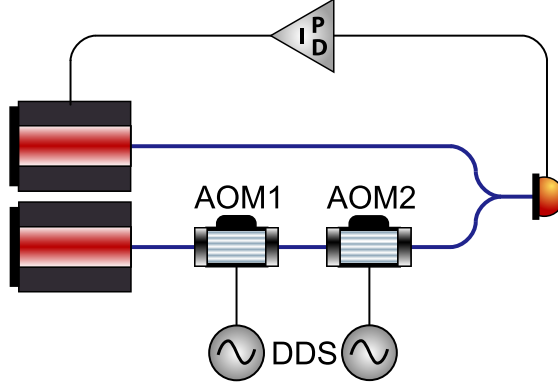


Figure 4.9: Setup used to extend the two-laser beat-note measurement of Figure 4.7 to the two AOMs connected in series, in order to probe the carrier suppression of the combined $+82.5\text{MHz} - 77.5\text{MHz}$ shift. In this configuration the expected secondary beat note associated with the residual unshifted carrier would appear 5MHz from the main peak at $f_{\text{AOM}} + f_{\Delta}$, rather than 80MHz away as in the single-AOM case

In this case, as shown in the figure 4.10, the number of observed resonances is higher than expected. The reason for this behavior is not yet fully understood, and an investigation is currently underway to clarify its origin.

Nevertheless, it can be observed from the figure that the peak at 5MHz is approximately six orders of magnitude lower than the main peak. The ratio between the heights of these peaks should correspond to the fraction of the field that remains unshifted in frequency by each individual AOM. This indicates that, after passing through two AOMs, the fraction of the beam power that propagates through both AOMs without being frequency-shifted is certainly below 10^{-10} - 10^{-11} . This allows us to meet and exceed the requirements of our setup, which are at least one order of magnitude less stringent.

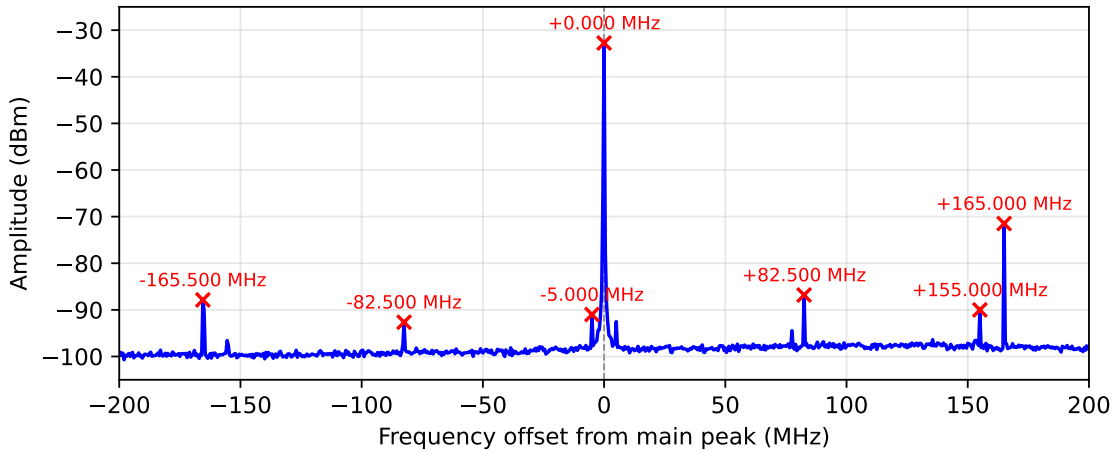


Figure 4.10: Beat-note spectrum measured with the setup of Figure 4.9, with the two AOMs driven at $+82.5\text{MHz}$ and -77.5MHz . Besides the main peak at 0MHz and the expected peak at 5MHz associated with the residual unshifted carrier, several additional peaks appear at frequencies corresponding to combinations of the two AOM drive frequencies (e.g. $\pm 82.5\text{MHz}$, $\pm 165\text{MHz}$).

4.2 Laser Phase Noise

As discussed in Section 2.8.2, phase noise is one of the most critical limitations for achieving a large squeezing degree. Since the laser is the primary source of this phase noise (Section 3.1), an experimental characterization of its performance is a necessary step in assessing the feasibility of the coherent control scheme presented in this thesis.

Beyond degrading the achievable squeezing level, laser phase noise also directly constrains the performance of the coherent-control scheme itself. Since the QCF, the pump, and the local oscillator are all derived from the same laser but travel along different optical paths before being recombined at the homodyne detector, any path length difference ΔL between the squeezed field generated in the OPO and the local-oscillator field introduces an effective delay τ between the two arrival time at the HD. Laser phase fluctuations $\phi_n(t)$ accumulated over this delay translate into a residual differential phase noise $\Delta\phi(t)$ between them, with power spectral density

$$S_{\Delta\phi\Delta\phi}(\omega) = (\omega\tau)^2 S_{\phi_n\phi_n}(\omega), \quad (4.9)$$

which, integrated over the detection bandwidth, yields the corresponding residual RMS phase noise

$$\sigma_{\Delta\phi}^2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_{\Delta\phi\Delta\phi}(\omega) d\omega. \quad (4.10)$$

For reference, the best coherent-control demonstrations reported in the literature achieve a residual (closed-loop) phase noise of about 5 mrad RMS [17]. The phase noise data presented in the following are therefore used, at the end of this Section, to estimate whether the laser employed in this work can, in principle, support a comparable performance.

As a first step (Section 4.2.1), the phase noise data published by *NKT Photonics* for the laser model used in this setup are recalibrated. These data, provided as a reference by the manufacturer, do not represent a direct measurement of the laser's intrinsic phase noise $S_{\phi\phi}$, but rather the output of a specific setup used by *NKT* to characterize the laser. A calibration procedure is therefore required to convert the published values into a phase noise spectral density that can be meaningfully compared with our own measurements. Furthermore, these data cannot be used to estimate Equation 4.10, since the frequency range over which they are provided is too narrow to obtain a reliable value of $\sigma_{\Delta\phi}^2$. Subsequently, two independent experimental setups developed during this thesis work are presented (Sections 4.2.2 and 4.2.3 respectively). For each of them, the working principle, the alignment procedure, the calibration of the acquired data, and the resulting conclusions are discussed.

4.2.1 Reference data calibration

For the setup described in Chapter 3, we used an *NKT Photonics - Koheras BASIK E15* laser source. *NKT Photonics* characterizes the phase noise of its laser products using a homodyne technique based on an unbalanced Mach-Zehnder interferometer¹, in which one arm includes an optical path mismatch of the order of 100 m . The quantity actually accessed by this measurement is not the phase noise of the laser field itself, but the phase

¹The measurement procedure followed by *NKT* is described in the application note available at the link <https://www.nktphotonics.com/wp-content/uploads/2025/03/phasenoise-appnote-20240930.pdf>, while the corresponding numerical data can be found at <https://lf.hamamatsu.com/products/single-frequency-fiber-lasers/koheras-basik/>

difference accumulated between the two interferometer arms as a consequence of the path mismatch $x_1 - x_2$:

$$\Delta\phi = k(x_1 - x_2) = \frac{2\pi}{\lambda}(x_1 - x_2) = \frac{2\pi\nu}{c}(x_1 - x_2), \quad (4.11)$$

where ν is the instantaneous optical frequency of the laser. Consequently, the spectral density reported by *NKT* as “phase noise” is related to the frequency noise of the laser, $S_{\nu\nu}$, by

$$S_{\Delta\phi\Delta\phi} = \left(\frac{2\pi}{c}\right)^2 (x_1 - x_2)^2 S_{\nu\nu}, \quad (4.12)$$

rescaled for the path mismatch of the interferometer, $x_1 - x_2 = 1m$. In the datasheet, this quantity is displayed on a logarithmic scale, $S_{\Delta\phi\Delta\phi}^{log} = 10\log(S_{\Delta\phi\Delta\phi})$.

In order to obtain the intrinsic phase noise of the laser, $S_{\phi\phi}$, and make it directly comparable with the measurements presented in this thesis, the published data were recalibrated as follows. First, the logarithmic values were converted back to linear units,

$$S_{\Delta\phi\Delta\phi} = 10^{\frac{S_{\Delta\phi\Delta\phi}^{log}}{10}}, \quad (4.13)$$

from which the frequency noise spectral density was recovered by inverting the relation above,

$$S_{\nu\nu} = \left(\frac{c}{2\pi}\right)^2 S_{\Delta\phi\Delta\phi}. \quad (4.14)$$

Finally, using the standard relation between frequency and phase noise in the Fourier space (frequency being the time derivative of phase), the laser phase noise was obtained as

$$S_{\phi\phi} = \frac{S_{\nu\nu}}{f^2}. \quad (4.15)$$

The result is again expressed in logarithmic form, $S_{\phi\phi}^{log} = 10\log(S_{\phi\phi})$, so that it can be plotted, and directly compared, on the same $dB(rad^2/Hz)$ scale used for the measurements described in the following sections. This recalibrated curve constitutes the reference benchmark against which the phase noise measured with the two experimental setups developed in this thesis is compared.

4.2.2 Setup 1

A first implementation of the laser phase noise was realized using the free-space unbalanced Mach-Zehnder interferometer which is shown in Figure 4.11.

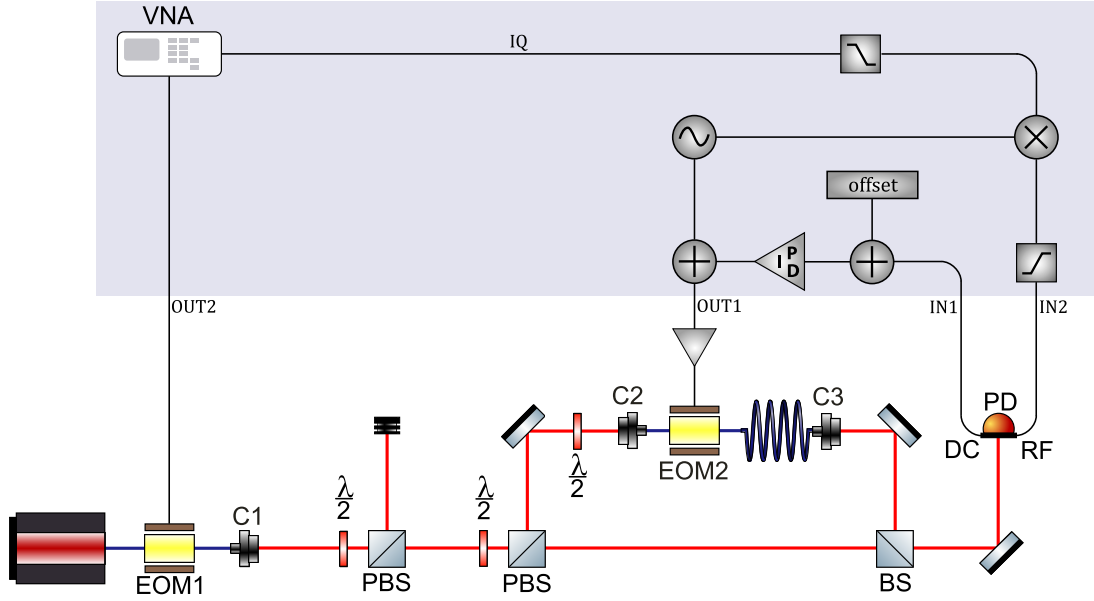


Figure 4.11: Scheme of an experimental setup used to characterize the laser phase noise, based on an unbalanced Mach-Zehnder interferometer. The laser output is split by a polarizing beamsplitter (PBS) into a short arm, and a long arm containing a 10m fiber delay-line and an electro-optic modulator (EOM), used both to calibrate the setup and as the actuator of the locking loop. The two beams are recombined on a beamsplitter (BS) and detected by a photodiode (PD). The shaded region shows the RedPitaya-based control electronics: the DC output of the PD is compared to a reference offset and fed back, through a PID controller, to the delay-line EOM (OUT1) to keep the interferometer locked at mid-fringe; the RF output of the PD is demodulated (IQ channel) to read out the phase noise signal. A second EOM (EOM1) placed right after the laser, driven by the Vector Network Analyzer (VNA) function of the RedPitaya, is used to measure the transfer function phase versus PD voltage calibration.

One useful outcome of this implementation was the possibility of experimentally verifying the nominal half-wave voltage of the fiber EOMs used throughout this work, $V_\pi = 3.5V$, as quoted by the manufacturer. This was obtained from the fringe pattern recorded while driving the delay-line EOM with a triangular voltage ramp (Figure 4.12) in order to visualize and maximise the visibility on the photodiode: the observed behaviour was found to be in good agreement with $V_\pi = 3.5V$, the value subsequently used both in this section and in Section 4.2.3.

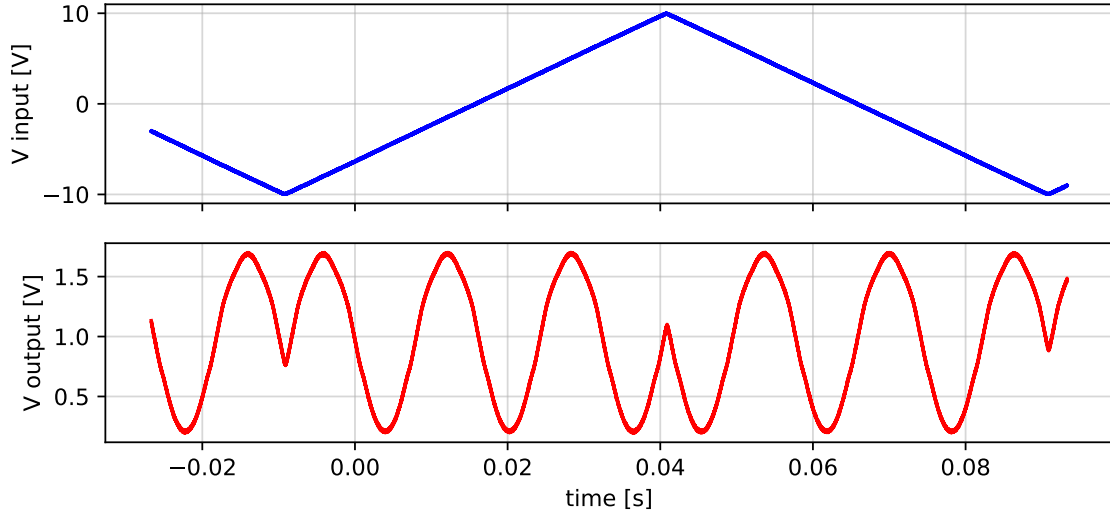


Figure 4.12: The blue curve shows the voltage ramp signal applied to EOM2 in Figure 4.11, while the red curve represents the signal measured by the photodiode. This measurement was used to increase the visibility at the photodiode by adjusting the optical components along the delay line and to optimize the laser power reaching the photodiode. The figure also shows the onset of photodiode saturation, which can be identified from the distortions appearing at the upper part of the red curve. The optical power was therefore adjusted to a level below the saturation threshold.

An alternative setup, based on the same operating principle, was also developed by replacing the free-space Mach-Zehnder interferometer with an equivalent fiber-optic configuration. Since more time was devoted to the optimization of this setup, the results presented in this work refer to this configuration.

Setup Description. The scheme of the fiber-based setup is shown in Figure 4.13.

The components are:

- Laser: *NKT - Koheras BASIK E15*
- EOMs: *EXAIL - MPX-LN-0.1*
- First beamsplitter: *DK Photonics - PMTC-102-15-2-F-20-1-90-10-FA*
- Second beamsplitter (recombination): *Thorlabs - PN1550R5A2*
- Photodiode: *modified custom Thorlabs FGA01FC-pigtailed photodiode*
- DC output board: *Texas Instruments - INA826EVM*
- Control electronics: *Red Pitaya*

As anticipated, the setup consists of an unbalanced Mach-Zehnder interferometer, conceptually analogous to the one employed by *NKT Photonics* for the phase noise characterization discussed in Section 4.2.1, with the addition of an electro-optic modulator (EOM) in one of the two arms, used as the actuator of a feedback loop.

The laser output is split into two beams that travel along two different paths before being recombined and detected by a photodiode. One arm is short and simply redirects the beam onto the recombining beamsplitter; the other arm contains a 10m long optical fiber delay line, introducing an additional propagation length that corresponds to a time delay τ between the two arms.

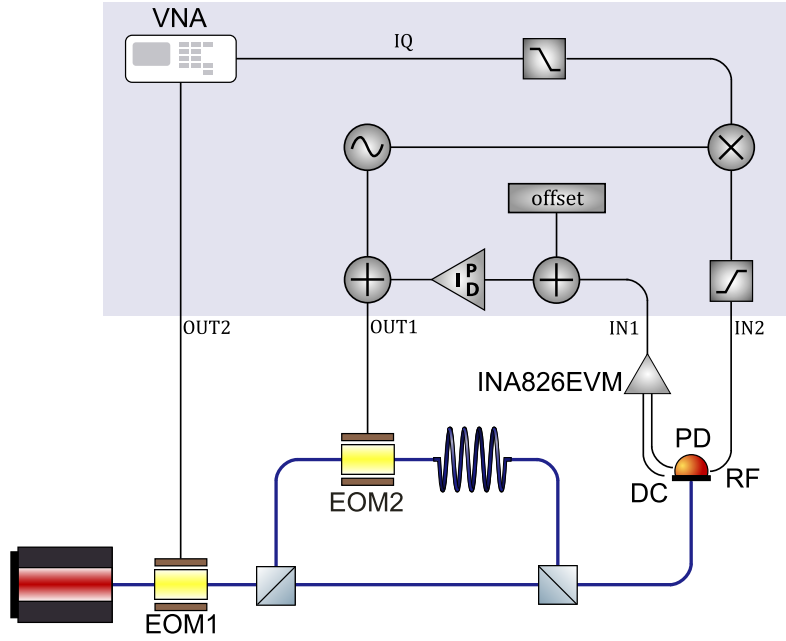


Figure 4.13: Scheme of the fully fiber-based unbalanced Mach-Zehnder interferometer used to characterize the laser phase noise. This is a revised version of the setup shown in Figure 4.11. The laser output is split by a fiber splitter into a short arm and a long arm containing a 10m fiber delay line and a fiber-based EOM (EOM2), used as the actuator of the locking loop. The two beams are recombined by a second fiber beamsplitter and detected by a photodiode. Its differential DC output is converted into a single-ended signal by an INA826EVM board before being digitized by the RedPitaya-based control electronics (shaded region). The DC signal is compared with a reference offset and fed back through a PID controller to EOM2 (OUT1) to keep the interferometer locked at the half-fringe position, while the RF output of the photodiode is demodulated (IQ channel) to measure the phase noise signal. A second EOM (EOM1), placed immediately after the laser and driven by the RedPitaya VNA function, is used to inject a calibration signal and measure the transfer function of the setup.

Writing the optical field emitted by the laser, including its phase noise $\phi(t)$, as

$$E(t) = E_0 e^{i[\omega_0 t + \phi(t)]}, \quad (4.16)$$

the two fields reaching the beamsplitter can be written as $E_1(t) \propto E(t)$, corresponding to the short arm, and $E_2(t) \propto E(t - \tau)$, corresponding to the delayed arm. The intensity measured by the photodiode is proportional to the squared modulus of the sum of the two fields,

$$I(t) \propto |E_1(t) + E_2(t)|^2 \propto 1 + \cos[\omega_0 \tau + \Delta\phi(t)], \quad (4.17)$$

where $\Delta\phi(t) \equiv \phi(t) - \phi(t - \tau)$ is the differential phase noise accumulated over the delay time τ .

By acting on the DC EOM2 voltage, the interferometer is set at the mid-fringe operation, that is such that $\omega_0 \tau \equiv \frac{\pi}{2}$. In this configuration, $\cos(\frac{\pi}{2} + \Delta\phi(t)) \approx -\Delta\phi(t)$ for $\Delta\phi(t) \ll 1$, so that the intensity fluctuations detected by the photodiode become, to first order, directly proportional to the phase noise:

$$I(t) \propto 1 - \Delta\phi(t). \quad (4.18)$$

The mid-fringe lock is essential to obtain a linear and maximally sensitive transduction of phase noise into an intensity (and hence photocurrent) signal. This locking condition is maintained by a *RedPitaya*-based control system (shaded region in Figure 4.13), which performs two main functions.

The first, as anticipated, is to keep the interferometer locked at the mid-fringe point. A phase fluctuation with respect to $\pi/2$ produces an error signal at the output of the photodiode, which is proportional to the phase deviation. This signal is digitalized by an analog-to-digital converter (ADC integrated in the *RedPitaya*) and processed by a PID controller, whose output (OUT1) is fed back to the EOM in the delay-line. Since the EOM directly controls the accumulated optical phase, this feedback loop continuously compensates for slow drifts of the interferometer's operating point - due to thermal, mechanical, or acoustic disturbances - keeping the system locked at mid-fringe over the timescale of the measurement.

The second function is to enable the measurement of the phase noise in the high-frequency region, where lower electronic noise is expected. In particular, an RF signal at $\Omega = 3\text{MHz}$ is applied to EOM2 using the *Red Pitaya*. Consequently, an additional term $\beta \cos(\Omega\tau)$ is added to the phase of the optical field in Equation 4.16, so that the intensity on the photodiode becomes:

$$\begin{aligned} I(t) &\propto 1 + \cos[\omega_0 \tau + \Delta\phi(t) + \beta \cos(\Omega\tau)] = \\ &\approx 1 + \cos[\omega_0 \tau + \Delta\phi(t)] - \sin[\omega_0 \tau + \Delta\phi(t)] \cdot \beta \cos(\Omega\tau). \end{aligned}$$

This way a high frequency term comes out so that, after demodulation, the phase noise $\Delta\phi(t)$ can be obtained. The purpose of this demodulation stage is to shift the frequency band in which the phase noise is measured away from DC, where the readout electronics is expected to be larger. By demodulating the RF signal at a suitably chosen frequency, the measurement is effectively moved to a cleaner spectral region, identified by the “IQ channel” label.

Setup configuration. The laser output is split into two paths at the first beamsplitter and later recombined on the second one, with the photodiode measuring the resulting interference. The splitting ratio is 90:10 rather than 50:50 because the long arm, which contains both the 10m fiber delay line and EOM2, introduces a large insertion loss (not independently characterized, but found to be substantial), so that routing 90% of the input power into this arm was found to be necessary to obtain a comparable optical power - and therefore good interference visibility - at the recombination beamsplitter.

Because the DC output of the photodiode is fully differential, it is first converted to a single-ended signal by the *INA826EVM* board before being digitized by the *Red Pitaya*. Setting the correct operating power levels required some care: the current available at the output of the *INA826EVM* board was found to be insufficient to directly drive a 50 Ω -terminated attenuator without clipping the signal. The optical power reaching the photodiode was therefore reduced instead, by inserting a fiber-coupled neutral filter and by further lowering the laser output power, until the DC signal at the *Red Pitaya* input spanned the full 0 – 1V range without saturating the photodiode. In this configuration, the laser was operated with a power setpoint of 30mW (out of a maximum of 40mW), the maximum optical power at the output of the interferometer was 7mW, and an in-fiber attenuator with 15dB of attenuation was inserted ahead of the photodiode.

Data Calibration. Figure 4.13 shows an EOM, placed immediately after the laser, before the beam is split. This element is not part of the interferometer proper, and plays no role in the phase noise measurement itself. Rather, it is used together with the VNA function of the *Red Pitaya* (represented by the VNA block feeding OUT2 and receiving the IQ output) to inject a known signal in the laser line with the purpose of calibrating the setup. This transfer function is precisely what is needed to calibrate the acquired data and make them directly comparable with the recalibrated NKT reference curve.

The calibration of the phase noise spectra acquired with this setup relies on the transfer function of the interferometer, measured by driving the EOM1 with a swept-frequency signal generated by the VNA function of the *Red Pitaya*, and recording the corresponding response at both the DC and the IQ outputs of the photodiode. Since the phase noise spectra were acquired on both output ports, this transfer function was measured separately for each of them, defining $H = V_{out}/V_{in}$, where V_{in} is the voltage applied to the EOM and V_{out} is the corresponding voltage measured at the DC or IQ output. The two transfer functions obtained in this way are shown in Figure 4.14 and 4.15. A clear overall trend can be identified in the amplitude response over the explored frequency range.

Notice that, since a given voltage applied to the EOM corresponds to a well-defined phase modulation, determined by the EOM's half-wave voltage $V_{\pi} = 3.5V$, the measured transfer function - initially expressed in V_{out}/V_{in} - can be converted into a transfer function expressed in V_{out}/rad by multiplying it by V_{π}/π .

To obtain a continuous transfer function from the noisy raw data, a linear fit was performed on the transfer function $H^{dB(V/V)}$ expressed in $dB(V/V)$ as a function of $\log_{10}(f)$, separately for the DC and IQ ports, i.e.

$$H^{dB(V/V)} = a \cdot \log_{10}(f) + b, \quad (4.19)$$

with a and b the fit parameters. This provides, for any frequency of interest, an estimated value of $H^{dB(V/V)}$, from which the transfer function in linear units was recovered as

$$H^{V/V} = 10^{H^{dB(V/V)}/20}, \quad (4.20)$$

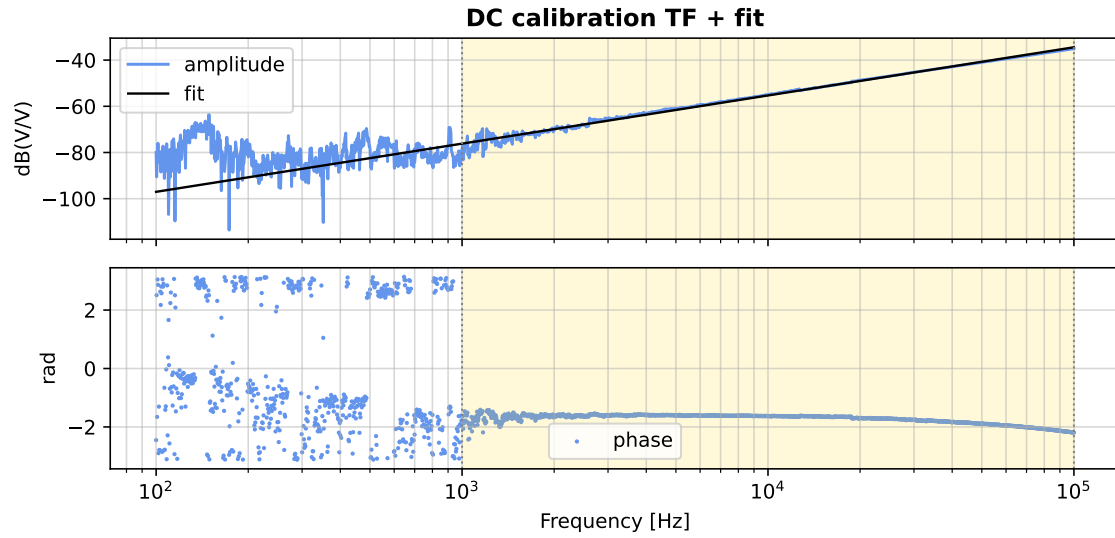


Figure 4.14: Amplitude (top) and phase (bottom) of the transfer function of the DC output of the setup in Figure 4.13, measured by driving the EOM placed right after the laser with a frequency-swept signal from the RedPitaya VNA and recording the corresponding response at the DC port of the photodiode. This transfer function is used to convert the phase noise spectra measured at the DC port into a calibrated phase noise spectral density.

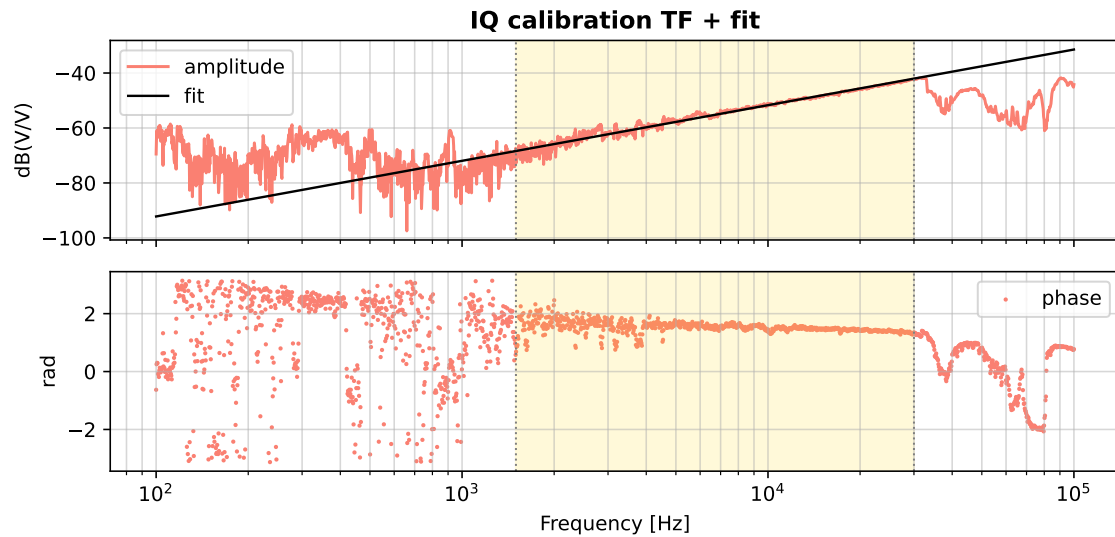


Figure 4.15: Amplitude (top) and phase (bottom) of the transfer function of the IQ output of the setup in Figure 4.13, measured by driving the EOM placed right after the laser with a frequency-swept signal from the RedPitaya VNA and recording the corresponding response at the IQ port of the photodiode. This transfer function is used to convert the phase noise spectra measured at the IQ port into a calibrated phase noise spectral density.

and subsequently converted into units of V/rad as

$$H^{V/\text{rad}} = H^{V/V} \cdot \frac{V_\pi}{\pi}. \quad (4.21)$$

This fitted transfer function was then used to calibrate the phase noise spectra measured at the DC and IQ outputs, $S^{V^2/\text{Hz}}$, by dividing them by the squared modulus of $H^{V/\text{rad}}$ at the corresponding frequency:

$$S^{\text{rad}^2/\text{Hz}} = \frac{S^{V^2/\text{Hz}}}{|H^{V/\text{rad}}|^2}. \quad (4.22)$$

The resulting spectra are then converted to the logarithmic scale $10 \log_{10}(S^{\text{rad}^2/\text{Hz}})$ for direct comparison with the calibrated NKT reference data of Section 4.2.1.

Results. Figure 4.16 shows the result of the measurement described above. It is immediately apparent that both measurements (DC in blue and IQ in red) lie above the reference level of the curve provided by NKT. In particular, the curve corresponding to the IQ channel exhibits a very high dark level, which is reached by the phase noise curve at around 200Hz , while maintaining a relatively low clearness even at lower frequencies. The curve observed at the DC port, on the other hand, exhibits greater clearness; however, the dark level of this channel is found to be substantially comparable to the phase noise level that one would ideally like to observe. The measured curve therefore lies at a higher level. This could be explained by excess noise picked up by the optical fibers, by nonlinear effects in the case where $\Delta\phi$ is not $\ll 1$ (the approximation $\sin(\Delta\phi) \approx \Delta\phi$ was used), or by laser amplitude noise mimicking phase noise.

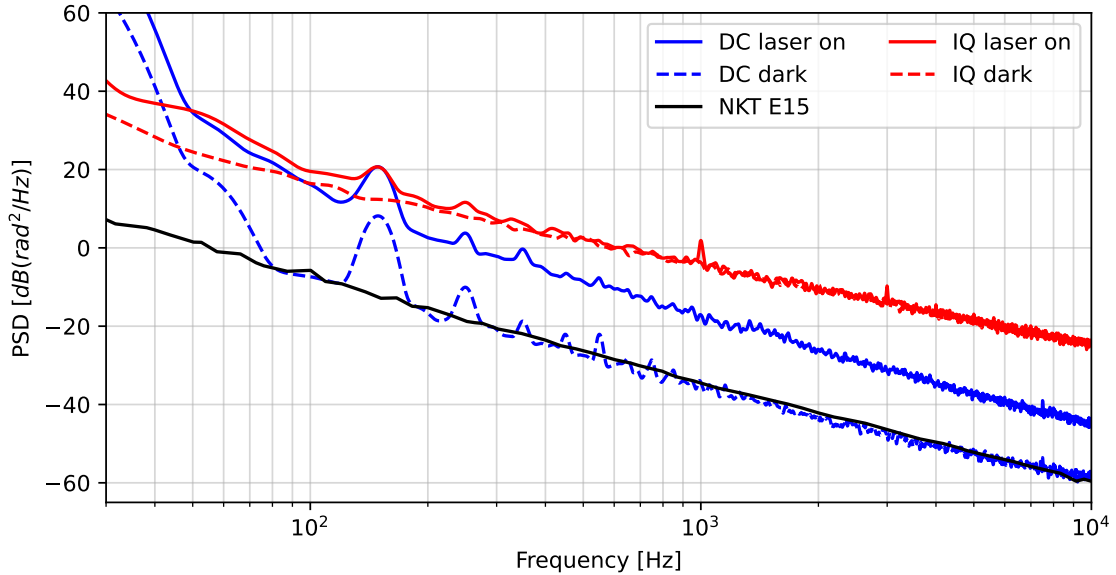


Figure 4.16: *phase noise Power Spectral Density measured with the setup in Figure 4.13, using an NKT Koheras BASIK E15 laser. The blue and red solid curves show the phase noise measured at the DC and IQ outputs of the interferometer, respectively, with the laser on and the loop locked at mid-fringe; the corresponding dashed curves show the same measurements with the laser off, giving the dark noise floor of each readout channel. The black curve is the reference phase noise spectrum of the E15 laser, recalibrated from the NKT datasheet as described in Section 4.2.1.*

For this reason, a second setup was developed in order to measure the phase noise. As will be described in the following section, this setup doesn't require $\Delta\phi \ll 1$, it allows the phase noise to be measured independently of the amplitude noise and, furthermore, does not require a control loop to lock the setup at the half-fringe position.

4.2.3 Setup 2

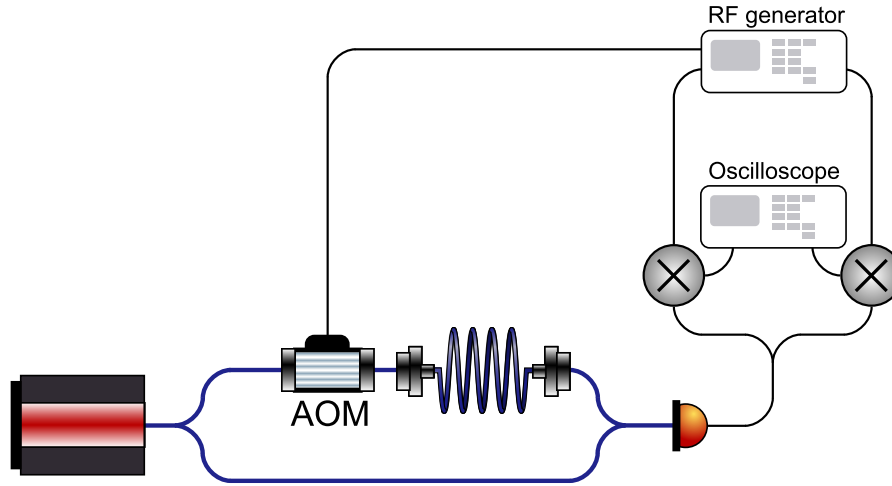


Figure 4.17: Scheme of an experimental setup used to characterize the laser phase noise, based on an I/Q demodulation of a beat-note signal. The laser beam is split by a first beamsplitter into a direct arm and an arm passing through an AOM, driven at $\Omega/2\pi = 80\text{MHz}$, and (optionally) a 10m fiber delay line; the two beams are recombined on a second beamsplitter and detected by a single photodiode, producing a beat note at 80MHz that carries the differential phase noise accumulated over the delay. The RF output of the photodiode is split by a power splitter and sent to two mixers, driven by DDS-generated local-oscillator tones at 80MHz mutually shifted by 90° , producing the I and Q quadrature signals that are acquired on the oscilloscope and used to reconstruct the phase $\phi(t) = \arctan(Q/I)$.

The second setup developed to characterize the laser phase noise is shown in Figure 4.17. The components are:

- Laser: *NKT - Koheras BASIK E15 / X15*
- First Beam Splitter: *DK Photonics - PMTC-102-15-2-F-20-1-90-10-FA*
- AOM: *Brimrose - AMF-80-5-50-1550-2FP-PM*
- Second Beam Splitter: *Thorlabs - PN1550R5A2*
- Photodiode: *Thorlabs DET01CFC/M*
- Power Splitter: *Mini-Circuits - ZX10-2-12-S+*
- Mixer 1: *Mini-Circuits - ZX05-1-S+*
- Mixer 2: *Mini-Circuits - ZX05-1MHW-S+*
- Oscilloscope: *Siglent - SDS7304A H12*

Unlike the setup described in Section 4.2.2, which relies on a Mach-Zehnder interferometer, this configuration is based on a detection scheme, in which the phase noise is extracted through an I/Q demodulation of a beat-note signal.

The laser beam is split by a beamsplitter into two arms. One arm reaches the photodiode directly, while the other passes through an acousto-optic modulator (AOM), driven at a frequency $\Omega/2\pi = 80\text{MHz}$, and subsequently through a 10m fiber delay line, introducing a time delay τ between the two arms (measurements were taken both with and without this delay line, in order to isolate its contribution). The two beams are then recombined and detected on a single photodiode.

Writing the optical field emitted by the laser as $E(t) = E_0 e^{i[\omega_0 t + \phi(t)]}$, the field in the direct arm reaching the photodiode is simply $E_1(t) = E_0(t)$, while the field in the AOM arm, having been frequency-shifted by Ω and delayed by τ , is

$$E_2(t) = E_0 e^{i[(\omega_0 + \Omega)(t - \tau) + \phi(t - \tau)]}. \quad (4.23)$$

Since the two fields now oscillate at different optical frequencies, ω_0 and $\omega_0 + \Omega$, their superposition on the photodiode produces a beat note at the AOM frequency Ω . The photocurrent is proportional to $|E_1(t) + E_2(t)|^2$, whose oscillating part is

$$I_{PD}(t) \propto \cos[\Omega(t - \tau) + \Delta\phi(t)], \quad (4.24)$$

where $\Delta\phi(t) \equiv \phi(t) - \phi(t - \tau)$ is again the differential phase noise accumulated over the delay τ . Moreover, the phase information is encoded in the phase of an RF beat note oscillating at Ω , exactly like the IQ signal of the previous setup.

To extract $\Delta\phi(t)$ from this beat note, the RF output of the photodiode is split by a power splitter and sent to two separate mixers, each fed with a local oscillator signal at the same frequency $\Omega/2\pi = 80\text{MHz}$ generated by the DDS, but with the two LO signals mutually shifted in phase by 90° . Denoting the beat note as $I_{PD}(t) \propto \cos(\Omega t + \theta(t))$, with $\theta \equiv \Delta\phi(t) - \Omega\tau$, multiplying it by the two references $\cos(\Omega t)$ and $\sin(\Omega t)$, and low-pass filtering to remove the resulting components at 2Ω , yields the two quadrature signals

$$I(t) \propto \cos \theta(t), \quad Q(t) \propto \sin \theta(t), \quad (4.25)$$

which are acquired on the oscilloscope. This pair of signals fully determines $\theta(t)$, and hence $\Delta\phi(t)$ up to the constant offset $-\Omega\tau$, through

$$\theta(t) = \arctan \left[\frac{Q(t)}{I(t)} \right]. \quad (4.26)$$

Once the differential phase noise time series is reconstructed sample by sample from the acquired I and Q traces, its power spectral density is computed via a Fast Fourier Transform.

As anticipated, the main advantages of this setup compared to the one described in Section 4.2.2 are:

1. Since the I and Q terms in Equation 4.25 are proportional to the square root of the laser power, their dependence on the laser power is cancelled out when taking the ratio in Equation 4.26. Therefore, the measurement is independent of laser amplitude fluctuations, and the only quantity being measured is the laser phase noise.
2. In this case, no feedback control loop is required to keep the setup at the half-fringe point.
3. The condition $\Delta\phi \ll 1$ is not required for this estimation

IQ ellipse calibration. Although the setup was carefully optimized to obtain two RF signals feeding the mixers with a relative phase shift as close as possible to 90° , achieving perfect quadrature in practice is extremely difficult, since even small asymmetries between the two RF paths - for instance slightly different cable lengths - introduce a residual deviation from the ideal 90° phase relationship. This effect is not due to phase imbalance alone. In addition to the residual deviation from the ideal 90° phase relationship, the I and Q channels are also affected by an amplitude imbalance, arising from the non-ideal 50:50 splitting ratio of the beamsplitters used in the demodulation chain, as well as from differences in the gain of the amplification and demodulation electronics along the two paths. Together, these two effects - phase and amplitude imbalance - combine to produce, instead of the expected circle, an ellipse in the I/Q plane, in general rotated with respect to the I - Q axes and displaced from the origin.

This effect is directly visible in the data. Figure 4.18 shows the raw I and Q data: rather than lying on an arc of a circle centered at the origin, as would be expected for an ideal 90° phase-quadrature demodulation, the points trace out an arc which is visibly better described by a portion of a rotated ellipse.

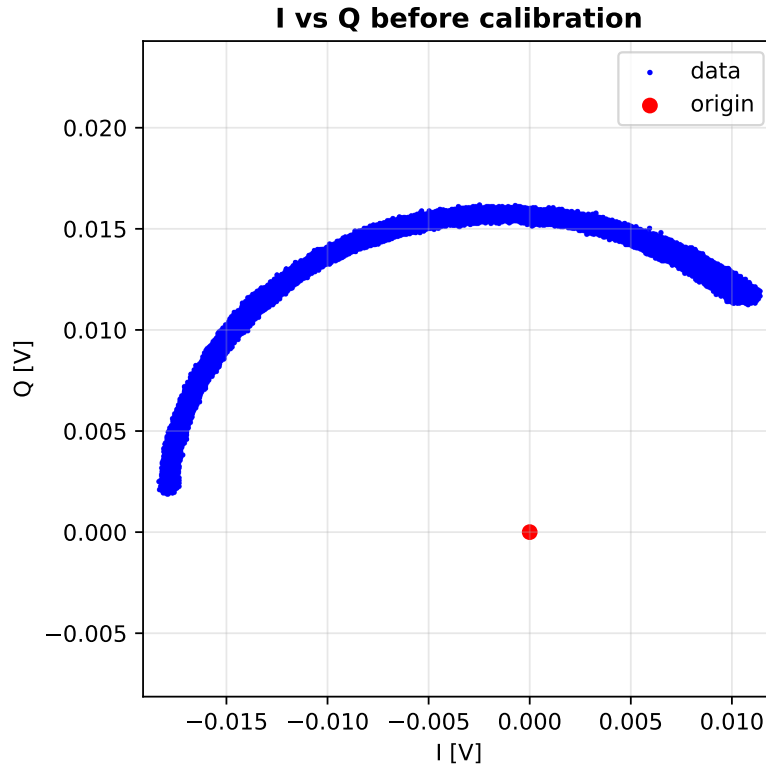


Figure 4.18: *Raw I and Q signals acquired with the setup in Figure 4.17, plotted against each other. Instead of tracing an arc of a circle centered at the origin, as expected for an ideal 90° phase-quadrature demodulation, the points trace an arc of a rotated, off-centered ellipse, revealing a residual phase imbalance between the two local-oscillator paths feeding the mixers.*

In order to characterize and subsequently correct for this imperfection, a calibration measurement was performed. The RF signal driving the AOM was detuned by 10kHz with respect to the local oscillator frequency used for the demodulation. This detuning

causes the phase to sweep continuously and rapidly through all values between 0 and 2π , effectively rotating the (I, Q) point around the underlying ellipse at a rate of 10kHz and thereby tracing out its full contour, rather than the short arc obtained under normal operating conditions. The result of this measurement is shown in Figure 4.19: the acquired points clearly trace a complete ellipse, confirming the presence of the phase imbalance discussed above.

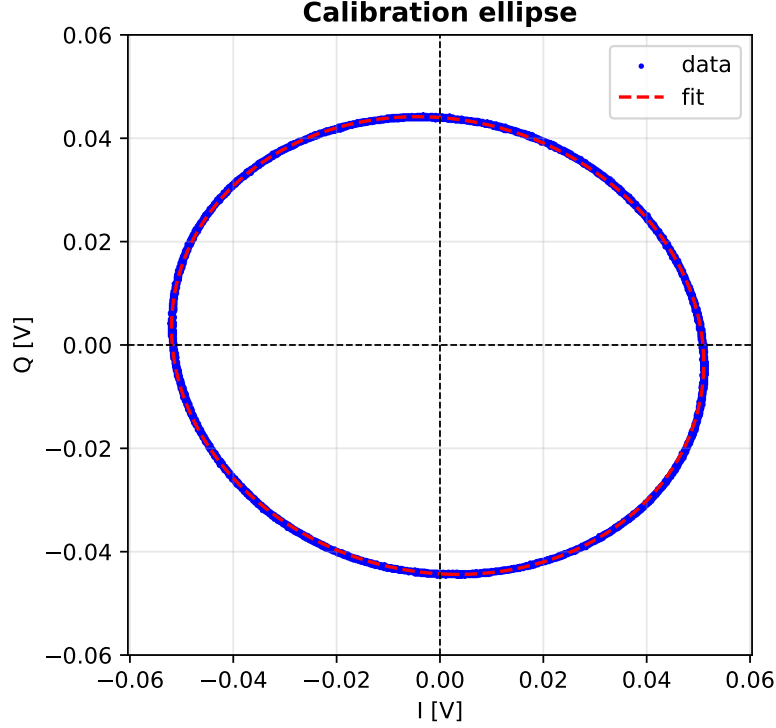


Figure 4.19: Full calibration ellipse (blue points) obtained with the setup in Figure 4.17, by detuning the RF signal driving the AOM by 10kHz with respect to the demodulation local-oscillator frequency. The dashed red curve is the fitted ellipse, whose center, rotation angle, and axis ratio characterize the systematic distortion introduced by the imperfect 90° quadrature and are then used to correct all I/Q phase-noise data.

An ellipse was then fitted to this dataset, yielding its characteristic parameters: the coordinates of the center, the rotation angle of its major axis with respect to the $I - Q$ axes, and the ratio between the lengths of the semi-major and semi-minor axes. These parameters, extracted once from the dedicated 10kHz -detuned calibration dataset, fully characterize the systematic distortion introduced by the imperfect quadrature demodulation and are independent of the specific phase noise measurement being calibrated. In particular, the rotation angle of the fitted ellipse accounts for the residual phase imbalance, while the ratio between its semi-major and semi-minor axes provides the amplitude correction factor needed to compensate for the unequal gains of the I and Q channels.

The fitted parameters were then used to correct the raw I and Q data acquired during the actual phase noise measurements, transforming the ellipse arc into a circle arc. The result of this procedure, applied to the same dataset shown in Figure 4.18, is displayed in Figure 4.20. The calibrated datasets are the one subsequently used to reconstruct the phase $\theta = \arctan(Q/I)$ and, from it, the phase noise spectral density.

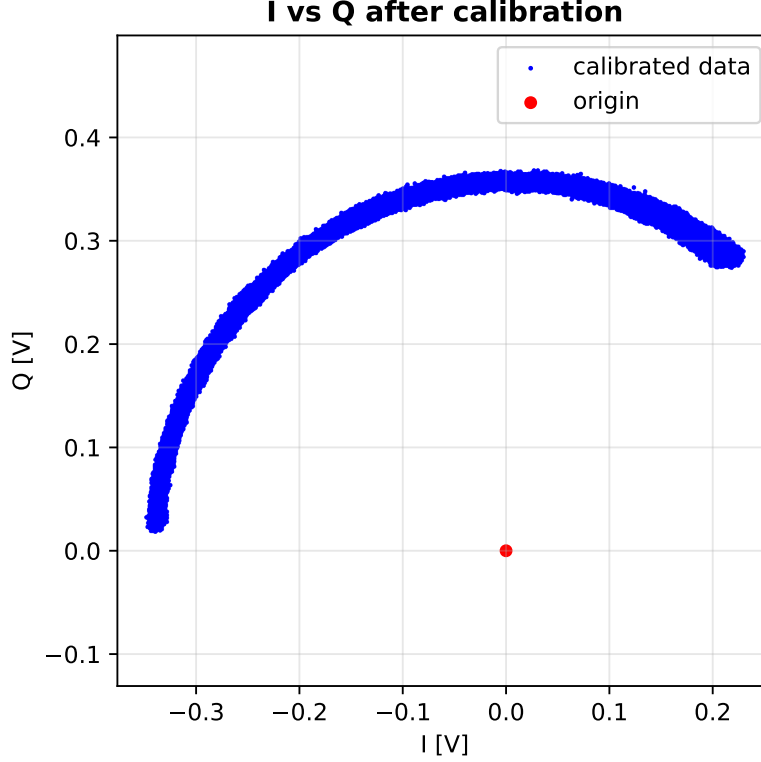


Figure 4.20: I and Q data of Figure 4.18, after correction for the ellipse distortion using the parameters extracted from the calibration ellipse of Figure 4.19. The corrected points now trace an arc of a circle centered at the origin, as expected for an ideal quadrature demodulation.

Data calibration. Once the raw I and Q signals have been corrected for the ellipse distortion described above, a second calibration step is needed to convert the resulting phase spectra, measured at the output of the interferometer, into the actual phase noise of the laser output. This requires measuring the transfer function of the setup itself, i.e. its overall response to a known phase modulation.

For this purpose, an EOM was inserted between the laser and the first beamsplitter of the interferometer (analogous to the calibration EOM used in Section 4.2.2), and driven with a series of sinusoidal tones at fixed frequencies, spanning from 10kHz to 2MHz , at a constant driving voltage of 1V . Each of these tones injects a known, well-defined phase modulation of amplitude $\beta_{in} = \pi \cdot V_{in}/V_{\pi} \approx 0.898\text{rad}$ at the corresponding frequency, which propagates through the setup exactly as any other phase fluctuation of the laser field, and can therefore be used as a calibration reference.

For each injected frequency, $\theta(t) = \arctan[Q(t)/I(t)]$ was computed from the calibrated I and Q traces. Its power spectrum was then calculated via a Fourier transform. In each of these spectra, a clear peak appears at the corresponding injected frequency, superimposed on the underlying phase noise floor of the setup; these peaks were identified and isolated for each of the injected tones (see the peaks shown in Figure 4.21, obtained with and without the fiber delay line).

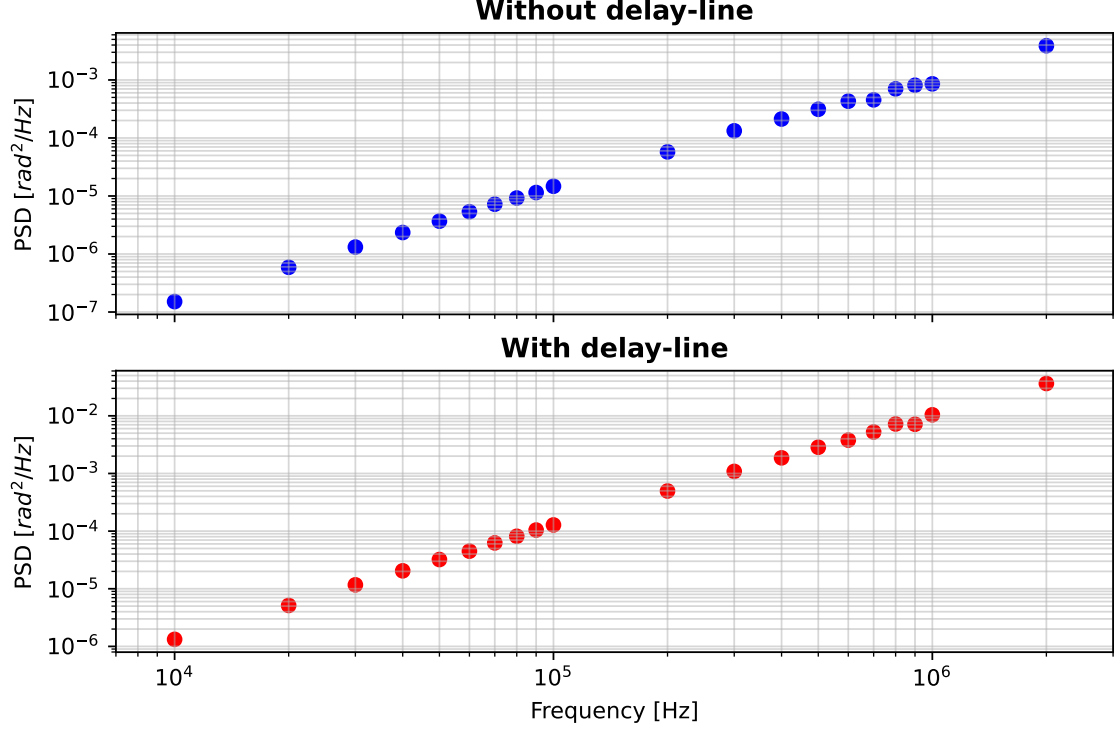


Figure 4.21: *Peaks amplitude of the Power Spectral Densities of the phase $\theta(t) = \arctan(Q/I)$ reconstructed from the setup in Figure 4.17, measured without (top) and with (bottom) the 10m fiber delay line, for a series of sinusoidal phase-modulation tones injected via a calibration EOM at fixed frequencies between 10kHz and 2MHz with input amplitude $\beta_{in} \approx 0.898\text{rad}$. The estimated RMS peak amplitude from the PSD is used to determine the setup's transfer function shown in Figure 4.22.*

Each isolated peak was numerically integrated over a narrow frequency window around its central frequency, after subtracting a local noise floor estimated from the surrounding bins, in order to obtain the power P associated with that specific tone (notice that the notation “power P ” comes from the Power Spectral Density which is integrated, even if the integral is expressed in rad^2). From this integrated power, the corresponding phase amplitude at the output of the setup was obtained as $A_{out} = \sqrt{2P}$. The (pointwise) transfer function of the system, $H^{\text{rad}_{out}/\text{rad}_{in}}$, was then computed at each injected frequency as the ratio between this output amplitude and the known input phase amplitude,

$$H^{\text{rad}_{out}/\text{rad}_{in}}(f) = \frac{A_{out}(f)}{\beta_{in}}. \quad (4.27)$$

The resulting values of $H^{\text{rad}_{out}/\text{rad}_{in}}(f)$, obtained separately for the measurements taken with and without the fiber delay line, are shown in Figure 4.22. Moreover, in order to obtain a continuous, rather than pointwise, transfer function, a linear fit was performed on $\log_{10}(H)$ as a function of $\log_{10}(f)$, yielding, for each configuration, a relation of the form $\log_{10}(H) = m \log_{10}(f) + q$.

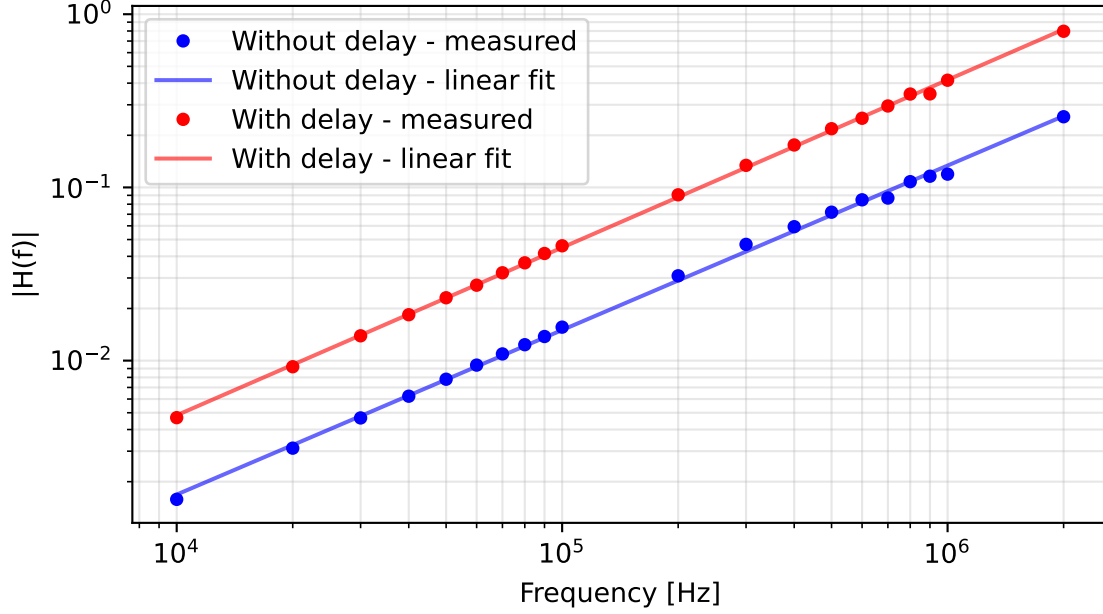


Figure 4.22: Measured transfer function of the setup in Figure 4.17, obtained from the calibration peaks of Figure 4.21, without (blue) and with (red) the 10m fiber delay line, together with the corresponding linear fits (solid lines). This transfer function is used to calibrate the raw phase-noise spectra.

This measured transfer function also provides a useful consistency check on the setup. In the limit $\omega\tau \ll 1$, a time delay τ is expected to produce a transfer function of the form $|H(\omega)| = \omega\tau$, i.e. a linear response in ω (equivalently, a unitary slope on the log-log plot). From the relation

$$\log_{10}(H) = m \log_{10}(f) + m \log_{10}(2\pi) + \log_{10}(\tau), \quad (4.28)$$

where the parameter m is free to vary in order to estimate its observational value, the value of the delay time τ can be extracted from the fit parameters. Using the slope m actually obtained from the fit ($m \approx 0.95$ without delay-line and $m \approx 0.97$ with the delay-line), the corresponding values are $\tau \approx 47ns$ and $\tau \approx 109ns$, for the setup without delay and with delay respectively. These estimates are consistent with the expected propagation delays of the setup, providing a sanity check on the calibration procedure.

Once the fitted transfer function $H^{rad_{out}/rad_{in}}(f)$ was obtained, it was used to calibrate the measured phase noise spectra, $S^{rad^2_{out}/Hz}$, recovering the phase noise referred to the input of the setup as

$$S^{rad^2_{in}/Hz} = \frac{S^{rad^2_{out}/Hz}}{|H^{rad_{out}/rad_{in}}|^2}. \quad (4.29)$$

Results. The results of the phase noise measurements obtained with this setup are shown in Figure 4.23-4.26. In particular, the first two figures show the results obtained using a laser source of the *NKT - Koheras Basik E15* model, while the last two show the results obtained with the *NKT - Koheras Basik X15* model, for which a lower phase noise is tabulated. As anticipated, for both models measurements were acquired both with and without the 10m optical fiber delay line, in order to observe its effect on the measured

phase noise. Moreover, for each of these measurements, 3 different laser power levels reaching the photodiode were used (photodiode saturation power equal to $5.5mW$). These values are reported in Table 4.2. All the data were then compared with the reference curve obtained from the calibration of the data provided by *NKT Photonics* for the two laser models, as described in Section 4.2.1.

	E15	X15
Low P	$0.90mW$	$0.66mW$
Mid P	$1.83mW$	$1.35mW$
High P	$3.62mW$	$2.70mW$

Table 4.2: Optical power levels reaching the photodiode in the setup of Figure 4.17, used for the phase noise measurements shown in Figures 4.23-4.26, for the two laser models NKT Koheras BASIK E15 and X15. Three power levels (Low P, Mid P, High P) were used for each laser, all well below the photodiode saturation power of $5.5mW$.

The first thing that can be observed is that, for all configurations, as the power on the photodiode increases, the estimated phase noise decreases. This pattern does not depend on whether the power is reduced directly at the source, by acting on the power emitted by the laser, or by means of attenuators placed between the fiber output of the setup and the photodiode. Indeed, the results obtained with both methods are fully consistent with each other. This trend can be explained analytically by extending the treatment of Equation 4.25 to include the additive electronic noise of the photodiode and mixers. Denoting by $\mathcal{A}$ the amplitude of the beat-note signal at the mixer outputs, proportional to the square root of the optical power reaching the photodiode ($\mathcal{A} \propto \sqrt{P_{laser}}$), the quadrature signals can be more completely written as

$$I(t) = \mathcal{A} \cos \theta(t) + I_n, \quad Q(t) = \mathcal{A} \sin \theta(t) + Q_n, \quad (4.30)$$

where I_n and Q_n are the additive noise contributions arising from the electronic noise and the shot noise, with typically $I_n = Q_n$. Propagating this noise through $\theta = \arctan(Q/I)$ gives

$$\delta\theta = \frac{\partial\theta}{\partial I} \cdot I_n + \frac{\partial\theta}{\partial Q} \cdot Q_n = \frac{I \cdot Q_n - Q \cdot I_n}{\mathcal{A}^2} \approx \frac{I - Q}{\mathcal{A}^2} I_n, \quad (4.31)$$

from which the additive noise contribution to the estimate of $\Delta\theta_n$ due to the electronics is

$$S_{\delta\theta\delta\theta} = \left(\frac{\partial\theta}{\partial I}\right)^2 S_{Q_n Q_n} + \left(\frac{\partial\theta}{\partial Q}\right)^2 S_{I_n I_n} = \frac{[\sin^2(\theta)\mathcal{A}^2 + \cos^2(\theta)\mathcal{A}^2] S_{I_n I_n}}{\mathcal{A}^4} = \frac{S_{I_n I_n}}{\mathcal{A}^2}. \quad (4.32)$$

Since $\mathcal{A}^2 \propto P_{laser}$, the electronic-noise floor contributes to the measured phase noise with a term inversely proportional to the optical power reaching the photodiode. Therefore, the measurements of Figures 4.23-4.26 show that our estimate is biased by the presence of electronic noise. A more accurate estimate of the phase noise could be obtained by using lower-noise photodetectors, or photodetectors capable of handling higher optical powers. Alternatively, the signal due to the laser phase noise could be increased by extending the length of the delay line.

The phase noise measurements reported in this section are significantly lower than those obtained using the method presented in the previous section. We believe this is due to the fact that, in this case, the laser amplitude noise does not contribute to the estimate,

nor does any additional noise introduced by the fringe stabilization loop, which is not required in this configuration. Furthermore, a lower-noise readout was used in this case.

Finally, using the spectra that are reported in this chapter and Equation 4.10, we estimated the expected phase noise between the squeezed beam and the LO, assuming a path-length mismatch of $\Delta L \approx 5m$ as $S_{\phi_n\phi_n}(f)$, and assuming a path-length mismatch $\Delta L \approx 5m$ between the squeezed-light and local-oscillator paths. The integral yields a residual RMS phase noise of

$$\sigma_{\Delta\phi} \approx 1.7mrad, \quad (4.33)$$

comfortably below the $\approx 5mrad$ residual phase noise reported for state-of-the-art coherent-control implementations [17]. This result is encouraging: it suggests that the phase noise performance of the laser characterized in this work is not, in principle, a limiting factor for achieving a coherent-control performance comparable to that of existing setups. The value obtained is, in fact, lower than the reference value, despite including an electronic noise contribution, as discussed previously. It should be noted, however, that this estimate only accounts for the phase noise contribution associated with the laser source. The additional phase noise introduced, for example, by the optical fiber paths is not taken into account. Therefore, the presence of coherent control is still necessary to stabilize the squeezing.

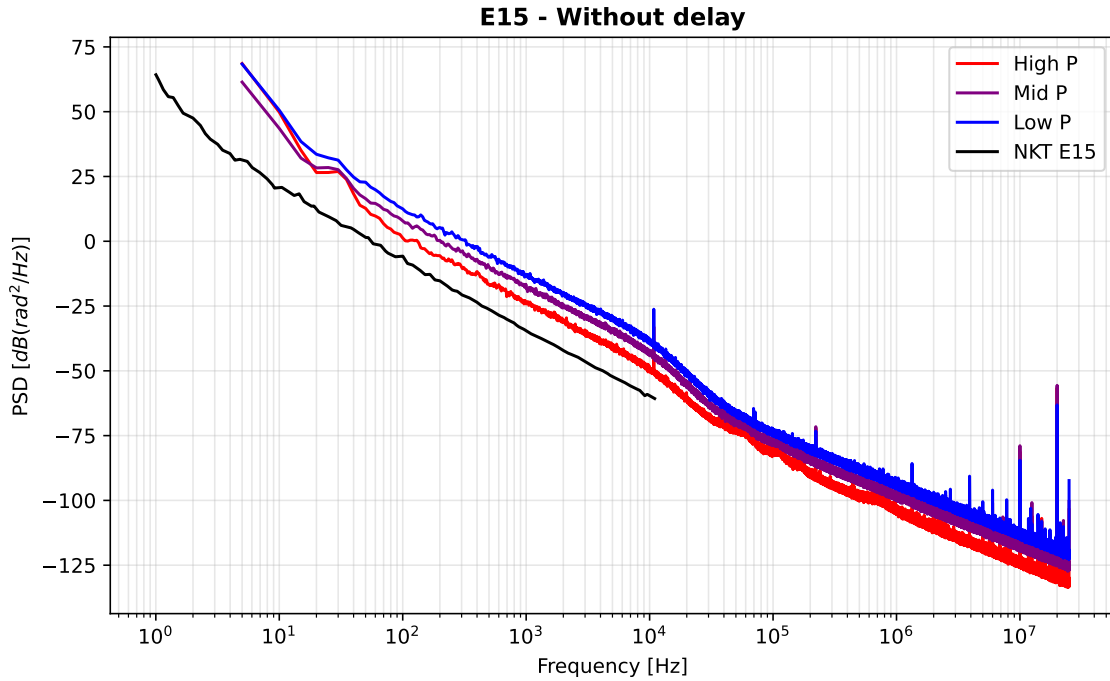


Figure 4.23: Calibrated phase noise power spectral density measured with the setup in Figure 4.17, without the 10m fiber delay line, using an NKT Koheras BASIK E15 laser, at three different optical power levels reaching the photodiode. The black curve is the reference phase noise spectrum of the E15 laser, recalibrated from the NKT datasheet.

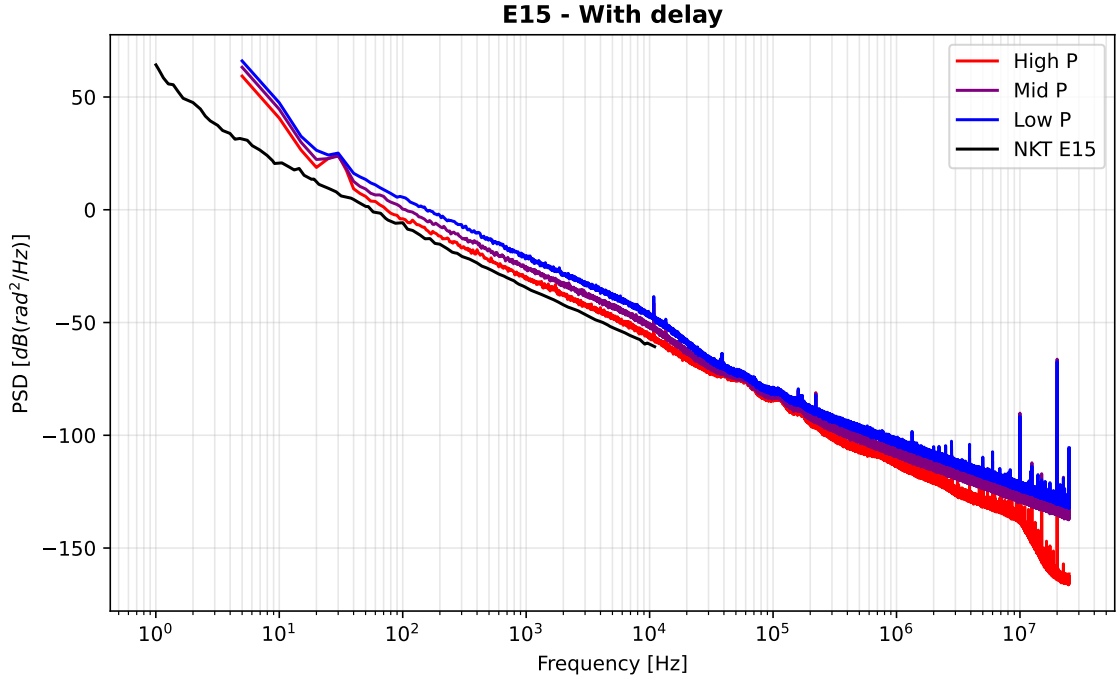


Figure 4.24: Calibrated phase noise power spectral density measured with the setup in Figure 4.17, with the 10m fiber delay line, using an NKT Koheras BASIK E15 laser, at three different optical power levels reaching the photodiode. The black curve is the reference phase noise spectrum of the E15 laser, recalibrated from the NKT datasheet.

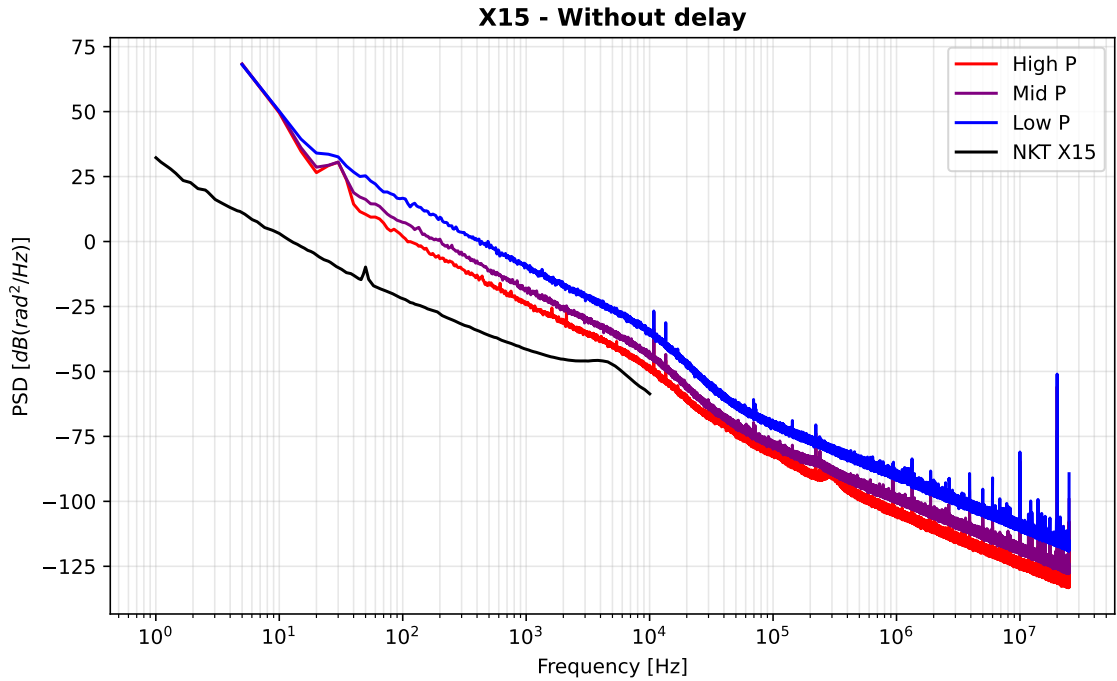


Figure 4.25: Calibrated phase noise power spectral density measured with the setup in Figure 4.17, without the 10m fiber delay line, using an NKT Koheras BASIK X15 laser, at three different optical power levels reaching the photodiode. The black curve is the reference phase noise spectrum of the X15 laser, recalibrated from the NKT datasheet.

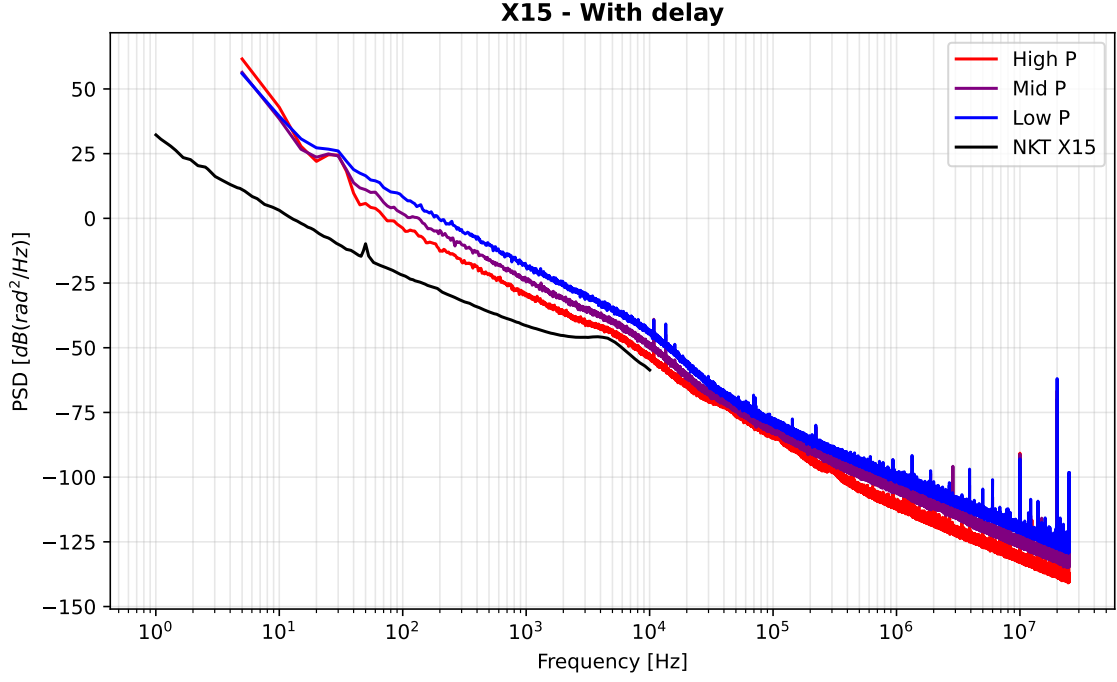


Figure 4.26: Calibrated phase noise power spectral density measured with the setup in Figure 4.17, with the 10m fiber delay line, using an NKT Koheras BASIK X15 laser, at three different optical power levels reaching the photodiode. The black curve is the reference phase noise spectrum of the X15 laser, recalibrated from the NKT datasheet.

4.3 SHG

Before proceeding to the detailed characterization of the phase-matching behaviour, a preliminary measurement was carried out to verify that the second-harmonic power generated by the SHG stage was compatible with the requirements of the subsequent OPO stage. To this end, the SHG output power was measured as a function of the seed power injected at the fundamental wavelength, with the crystal kept at its optimal operating point. The results are reported in Figure 4.27, which shows the scaling of the generated second-harmonic power with the seed power, indicating that the process was still far from saturation over the explored range. Most importantly, the measurement confirmed that the SHG output power reaches values of the order of a few hundred *mW*, thus satisfying the power requirement needed to efficiently pump the OPO.

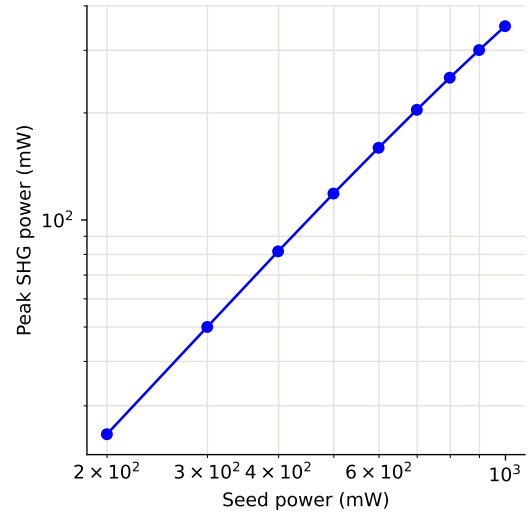


Figure 4.27: Peak second-harmonic power generated by the SHG stage as a function of the seed power at the fundamental wavelength

4.3.1 Power VS Temperature

As discussed in Section 3.3, efficient second-harmonic generation requires the phase-matching condition $\Delta k = 0$ to be satisfied. In this setup, this condition is achieved by tuning the temperature of the nonlinear crystal, such as $\Delta k \equiv \Delta k(T)$. The first characterization of the SHG stage therefore consists in reproducing the expected sinc^2 -shaped dependence of the generated second-harmonic power on the crystal temperature, in order to verify the phase-matching behaviour of the crystal and to identify its optimal operating point.

The measurement was performed by recording, with a power meter (*Thorlabs - S145C*) placed at the SHG (*HCP Photonics - QO2406060003*) output, the photocurrent generated by the second-harmonic beam while scanning the crystal temperature. According to the manufacturer's specifications, for an input wavelength of $\lambda = 1550\text{nm}$ the phase-matching peak is expected at around $T = 41^\circ\text{C}$.

A first, preliminary scan (blue curve in Figure 4.28) confirmed the expected qualitative growth of the SHG signal with temperature, but showed that, at $T = 41^\circ\text{C}$ the generated current was still rising, with no sign of having reached the expected maximum. Rather than further increasing the crystal temperature - which would have thermally stressed the crystal - the phase-matching condition was instead approached by slightly shifting the fundamental wavelength: the laser wavelength was reduced by $\Delta\lambda = 350\text{pm}$ with respect to the wavelength that was used for the first reference measurement. Since Δk depends on both temperature and wavelength through the dispersion of the crystal, this shift is equivalent, for phase-matching purposes, to an additional increase in temperature, but without the associated thermal stress.

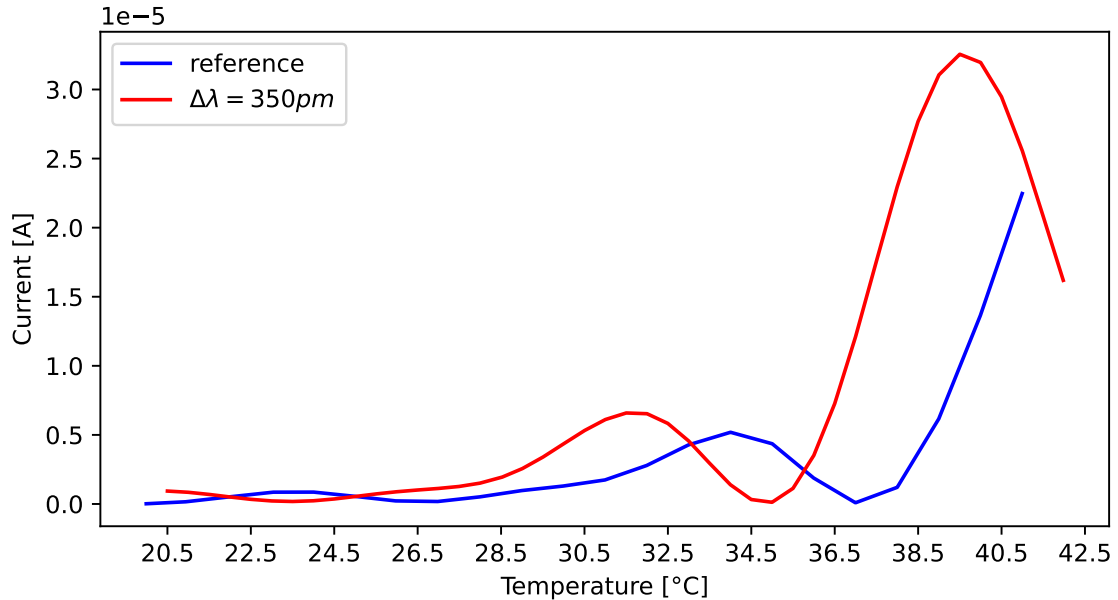


Figure 4.28: *Second-harmonic current generated at the output of the SHG stage as a function of the nonlinear crystal temperature, measured with a power meter while performing a temperature scan. The blue curve shows a preliminary scan. The red curve shows the scan obtained after reducing the fundamental laser wavelength by $\Delta\lambda = 350\text{pm}$, thereby shifting the phase-matching point without introducing additional thermal stress on the crystal.*

With this adjusted wavelength (red curve in Figure 4.28), the peak of the SHG output was successfully recovered at a temperature of approximately $T \approx 39^\circ\text{C}$, followed by the expected decrease on both sides of the resonance. The overall shape of this curve reproduces the dependence expected from Equation 3.5, confirming both the correct phase-matching behaviour of the crystal and the reliability of temperature tuning.

4.3.2 Power Stability

An analysis of the response as a function of temperature for different wavelengths showed that the response at 775nm reaches its maximum at 42°C . Then, once the SHG stage was set to the operating point of 42°C , a long-term measurement was performed to characterize the stability of the generated second-harmonic power. The crystal temperature was held at the fixed setpoint and the SHG output power was continuously monitored with a power meter over a period of 5 hours, while the temperature readback from the controller was logged simultaneously.

The results are shown in Figure 4.29. The top-left panel shows the temperature readback over the course of the measurement: the discrete, step-like pattern reflects the finite resolution of the temperature controller rather than any genuine thermal instability, and confirms that the setpoint was maintained throughout the measurement. The bottom-left panel shows the corresponding SHG output power as a function of time, together with its rolling mean, while the right panel shows the histogram of the power values collected over the full 5-hour trace.

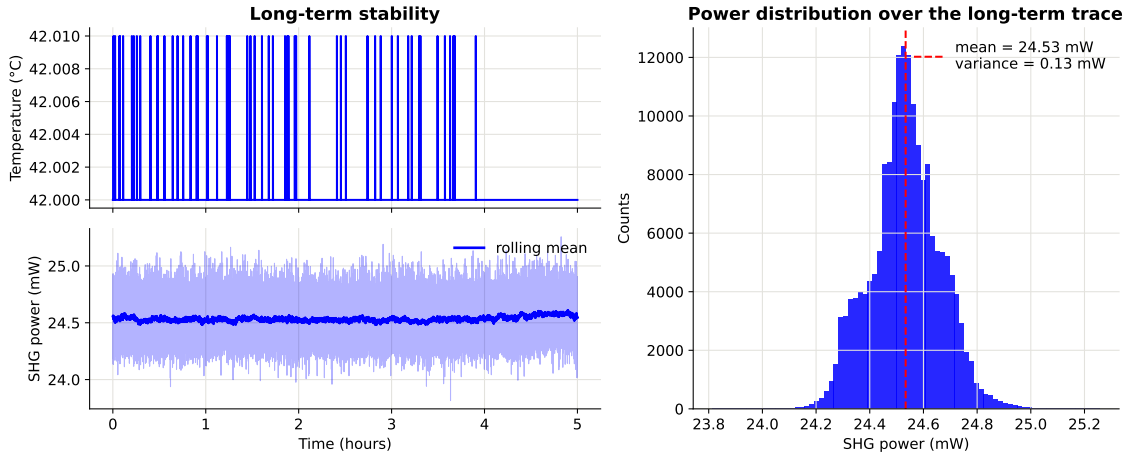


Figure 4.29: Long-term stability measurement of the SHG output power, taken over 5 hours with the crystal held at the fixed operating temperature of 42°C . Top left: temperature readback from the controller. Bottom left: SHG output power as a function of time (light blue), together with its rolling mean (dark blue). Right: histogram of the SHG power values collected over the full 5-hour trace, with the mean and variance indicated.

The stability of the SHG power is a fundamental requirement for achieving a stable squeezing level over time, since fluctuations in the pump power directly translate into fluctuations of the squeezing. In our case, over the 5-hour measurement, it was observed a relative standard deviation of 0.53%, together with a slow linear drift of the mean power of $0.024\%/h$. Therefore, the SHG output power can be considered relatively stable and the long-term drift negligible. This level of stability is expected to improve further once

the power-stabilization stage (PStab), shown in the full setup scheme of Figure 3.1, will be implemented downstream of the SHG.

Chapter 5

Conclusion

The work presented in this thesis addressed the preliminary experimental characterization of a compact, hybrid squeezed-vacuum source at $1550nm$. The goal was to assess whether the components of the fiber-based architecture can meet the requirements for achieving up to $15dB$ of squeezing, and the results obtained provide encouraging, though not yet conclusive, evidence in this direction.

The characterization of the Quadrature Control Field generation chain showed that a single AOM leaves a residual unshifted carrier at the level of $P_C/P_{AOM} \sim 10^{-7} - 10^{-8}$, consistently confirmed by two independent measurement methods (direct power ratio and heterodyne beat-note with offset-locked lasers). When two AOMs are cascaded, as required to obtain the target $+5MHz$ shift, this contamination is further suppressed to below $10^{-10} - 10^{-11}$, exceeding by at least one order of magnitude the tolerance of $P_{seed} \lesssim 1pW$ estimated in Section 4.1.1 for the residual carrier power injected into the OPO cavity. This indicates that the double-AOM scheme is well suited to generate a QCF field sufficiently free of classical carrier contamination not to compromise the squeezing performance. An unexplained excess of resonances was however observed in the two-AOM beat-note spectrum (Figure 4.10), and its origin remains to be clarified in future work.

The characterization of the laser phase noise, carried out using two independent experimental setups, highlighted the importance of separating genuine phase fluctuations from spurious contributions. The unbalanced Mach-Zehnder setup (Setup 1) proved to be limited by its sensitivity to amplitude noise and by the dark noise of the DC and IQ readout channels, yielding phase noise spectra systematically above the recalibrated NKT reference curve. The I/Q demodulation setup based on a frequency-shifted beat note (Setup 2) overcame these limitations, being intrinsically insensitive to amplitude fluctuations and not requiring a fringe-locking loop; the measured spectra were correspondingly closer to the manufacturer's specifications, with the residual excess attributable to electronic and shot noise in the photodetection chain, whose contribution was shown to scale as the inverse of the optical power reaching the photodiode. Propagating these measured phase noise spectra to the phase difference expected between the squeezed field and the local oscillator, for a plausible path-length mismatch of $\Delta L \approx 5m$, yields an estimated residual RMS phase noise of $\sigma_{\Delta\phi} \approx 1.7mrad$, comfortably below the $\approx 5mrad$ benchmark reported for state-of-the-art coherent-control demonstrations. This result suggests that the laser source selected for this setup is not, in principle, expected to be a limiting factor for the coherent control performance, although this estimate accounts only for the laser contribution and excludes additional phase noise sources such as the fiber paths themselves.

Finally, the characterization of the second-harmonic generation stage confirmed the expected sinc^2 phase-matching behaviour of the nonlinear crystal as a function of temperature. The long-term power stability measurement, performed over five hours at the chosen operating temperature of 42°C , showed a relative standard deviation of 0.53% and a negligible long-term drift of $0.024\%/h$, indicating that the SHG stage can provide a stable pump field for the OPO, that will be further stabilized by the planned power-stabilization stage (PStab) downstream implemented. Moreover, it was demonstrated that up to $\sim 100\text{mW}$ can be generated at 775nm , as required for the generation of 15dB squeezed vacuum at 1550nm .

Taken together, these results indicate that the individual fiber-based building blocks of the proposed hybrid squeezer - the QCF generation chain, the laser source, and the SHG pump stage - are compatible, within the limits of the present characterization, with the requirements for high-level squeezed-vacuum generation at 1550nm . The natural continuation of this work is the integration of these sub-systems with the OPO cavity, the closure of the full coherent-control loop and the direct measurement of the generated squeezing level through the homodyne detector, which will provide the ultimate test of the feasibility of this compact, fiber-based approach to squeezed-light generation for gravitational-wave detectors.

Bibliography

- [1] “The Laser”. In: *Optics, Light, and Lasers*. John Wiley & Sons, Ltd, 2017. Chap. 8, pp. 249–284. ISBN: 9783527685486. DOI: <https://doi.org/10.1002/9783527685486.ch8>. URL: <https://onlinelibrary.wiley.com/doi/abs/10.1002/9783527685486.ch8>.
- [2] Haixing Miao. “Exploring Macroscopic Quantum Mechanics in Optomechanical Devices”. Dissertation. Perth: The University of Western Australia, 2010. 218 pp.
- [3] Henning Vahlbruch. “Squeezed light for gravitational wave astronomy”. Dissertation. Hannover: Gottfried Wilhelm Leibniz Universität Hannover, 2008. 196 pp.
- [4] Carlton M. Caves and Bonny L. Schumaker. “New formalism for two-photon quantum optics. I. Quadrature phases and squeezed states”. In: *Phys. Rev. A* 31 (5 May 1985), pp. 3068–3092. DOI: [10.1103/PhysRevA.31.3068](https://doi.org/10.1103/PhysRevA.31.3068). URL: <https://link.aps.org/doi/10.1103/PhysRevA.31.3068>.
- [5] Mark Fox. *Quantum Optics: An Introduction*. Oxford University Press, Apr. 2006. ISBN: 9780198566724. DOI: [10.1093/oso/9780198566724.001.0001](https://doi.org/10.1093/oso/9780198566724.001.0001). URL: <https://doi.org/10.1093/oso/9780198566724.001.0001>.
- [6] Ling-An Wu, Min Xiao, and H. J. Kimble. “Squeezed states of light from an optical parametric oscillator”. In: *J. Opt. Soc. Am. B* 4.10 (Oct. 1987), pp. 1465–1475. DOI: [10.1364/JOSAB.4.001465](https://doi.org/10.1364/JOSAB.4.001465). URL: <https://opg.optica.org/josab/abstract.cfm?URI=josab-4-10-1465>.
- [7] Kirk McKenzie et al. “Squeezing in the Audio Gravitational-Wave Detection Band”. In: *Phys. Rev. Lett.* 93 (16 Oct. 2004), p. 161105. DOI: [10.1103/PhysRevLett.93.161105](https://doi.org/10.1103/PhysRevLett.93.161105).
- [8] Henning Vahlbruch et al. “Coherent Control of Vacuum Squeezing in the Gravitational Wave Detection Band”. In: *Phys. Rev. Lett.* 97 (1 July 2006), p. 011101. DOI: [10.1103/PhysRevLett.97.011101](https://doi.org/10.1103/PhysRevLett.97.011101). URL: <https://link.aps.org/doi/10.1103/PhysRevLett.97.011101>.
- [9] Moritz Mehmet et al. “Squeezed light at 1550 nm with a quantum noise reduction of 12.3 dB”. In: *Optics Express* 19 (Dec. 2011), pp. 25763–25772. DOI: [10.1364/OE.19.025763](https://doi.org/10.1364/OE.19.025763).
- [10] Emil Schreiber. “Gravitational-wave detection beyond the quantum shot-noise limit: The integration of squeezed light in GEO 600”. PhD thesis. Leibniz University Hannover, 2018. DOI: [10.15488/3773](https://doi.org/10.15488/3773).

- [11] “Laser Dynamics”. In: *Optics, Light, and Lasers*. John Wiley & Sons, Ltd, 2017. Chap. 9, pp. 285–317. ISBN: 9783527685486. DOI: <https://doi.org/10.1002/9783527685486.ch9>. URL: <https://onlinelibrary.wiley.com/doi/abs/10.1002/9783527685486.ch9>.
- [12] Fabian Meylahn and Benno Willke. “Characterization of Laser Systems at 1550 nm Wavelength for Future Gravitational Wave Detectors”. In: *Instruments* 6.1 (2022). ISSN: 2410-390X. DOI: 10.3390/instruments6010015. URL: <https://www.mdpi.com/2410-390X/6/1/15>.
- [13] “Nonlinear Optics I: Optical Mixing Processes”. In: *Optics, Light, and Lasers*. John Wiley & Sons, Ltd, 2017. Chap. 15, pp. 457–483. ISBN: 9783527685486. DOI: <https://doi.org/10.1002/9783527685486.ch15>. URL: <https://onlinelibrary.wiley.com/doi/abs/10.1002/9783527685486.ch15>.
- [14] “Light Propagation in Matter: Interfaces, Dispersion, and Birefringence”. In: *Optics, Light, and Lasers*. John Wiley & Sons, Ltd, 2017. Chap. 3, pp. 83–120. ISBN: 9783527685486. DOI: <https://doi.org/10.1002/9783527685486.ch3>. URL: <https://onlinelibrary.wiley.com/doi/abs/10.1002/9783527685486.ch3>.
- [15] Sheon S. Y. Chua. “Quantum Enhancement of a 4km Laser Interferometer Gravitational Wave Detector”. PHD Thesis. Canberra: Australian National University, 2013. 218 pp.
- [16] Xiaocong Sun et al. “Dependence of the squeezing and anti-squeezing factors of bright squeezed light on the seed beam power and pump beam noise”. In: *Opt. Lett.* 44.7 (Apr. 2019), pp. 1789–1792. DOI: 10.1364/OL.44.001789. URL: <https://opg.optica.org/ol/abstract.cfm?URI=ol-44-7-1789>.
- [17] Fabian Meylahn, Benno Willke, and Henning Vahlbruch. “Squeezed States of Light for Future Gravitational Wave Detectors at a Wavelength of 1550 nm”. In: *Phys. Rev. Lett.* 129 (12 Sept. 2022), p. 121103. DOI: 10.1103/PhysRevLett.129.121103. URL: <https://link.aps.org/doi/10.1103/PhysRevLett.129.121103>.

