



UNIVERSITÀ DEGLI STUDI DI PADOVA

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**Experimental Studies of Nuclear Matrix Elements
for Neutrinoless Double Beta Decay via γ -ray
Spectroscopy**

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Abstract

Neutrinoless double beta decay ($0\nu\beta\beta$) is a hypothetical nuclear decay predicted by several extensions of the Standard Model (SM). Unlike two-neutrino double beta decay ($2\nu\beta\beta$), it proceeds without neutrino emission, violating lepton number conservation. Its observation would therefore provide evidence of physics beyond the SM and important insight into the nature of neutrinos. Precise estimations of the Nuclear Matrix Element (NME) associated with this decay are essential for current experimental searches; unfortunately, existing estimations tend to differ significantly from one another, and thus their reliability remains uncertain.

In some descriptions of $0\nu\beta\beta$ decay, the process can be interpreted as a sequence of two consecutive β decays, with the two neutrinos effectively annihilating each other, suggesting that relevant information could be extracted from the study of these transitions and from the level structure of the intermediate nucleus. However, the intermediate β decays are usually not energetically favoured and are therefore difficult to observe directly. An alternative approach relies on γ spectroscopy and studies the decay from the Isobaric Analogue State (IAS) of the mother nucleus to the intermediate nucleus, or from the IAS of the intermediate nucleus to the ground state of the daughter one. The corresponding β -decay amplitude can then be inferred from the γ -decay amplitude, which can be determined experimentally by measuring the intrinsic IAS width Γ together with the number N of observed γ decays.

In this thesis we studied the feasibility of such an experiment using the AGATA γ array at Laboratori Nazionali di Legnaro (LNL), together with the silicon detector SAURON. A Geant4 simulation of the setup was developed to investigate the factors affecting the determination of the intrinsic IAS width Γ from the observed γ -ray spectrum. In addition, a Monte Carlo simulation was performed to assess the achievable precision on Γ and N under realistic experimental conditions.

In this kind of experiment, in order to be sensitive to these small decay branches, a high coincidence rate is expected in the silicon detector, which may result in a degradation of its performance. For this reason, we also investigated the effects of radiation damage on SAURON and tested some experimental procedures aimed at restoring its original capabilities.

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1

Introduction

Following the discovery of neutrino oscillations at the end of the 20th century [1], double beta decay has become a subject of intense scientific interest. Neutrino oscillations indicate that neutrinos are massive particles; if they are also Majorana particles, then neutrinoless double beta decay ($0\nu\beta\beta$) may occur [2]. The experimental observation of this process would open new scenarios in both particle physics and astrophysics.

In this introductory chapter, we discuss the importance of observing $0\nu\beta\beta$ decay and the need to extract valuable information from two-neutrino double beta decay ($2\nu\beta\beta$). We then briefly outline the main research strategies, highlighting their advantages and limitations. Finally, we introduce the possibilities offered by γ spectroscopy, which constitutes the main focus of this thesis.

1.1 An overview of double beta decay

1.1.1 Neutrinoless double beta decay

$0\nu\beta\beta$ decay is a second-order weak process in which two neutrons (protons) are converted into protons (neutrons), with the contemporary release of two electrons (positrons). The formula for such a process is the following:

$${}^A_ZX \rightarrow {}^A_{Z\pm 2}X + 2e^\mp.$$

There are many proposals regarding the precise mechanism through which $0\nu\beta\beta$ decay can proceed. Some of the proposed mechanisms are the light Majorana mass, the heavy neutrino and the right-handed weak currents [2]. In particular, in the light Majorana mass hypothesis, which is one of the most followed, $0\nu\beta\beta$

1.1. An overview of double beta decay

decay can be thought as a virtual-neutrino emission from one decaying nucleon and its re-absorption into the other, as it is depicted in the Feynman diagram in Figure 1.1 (left).

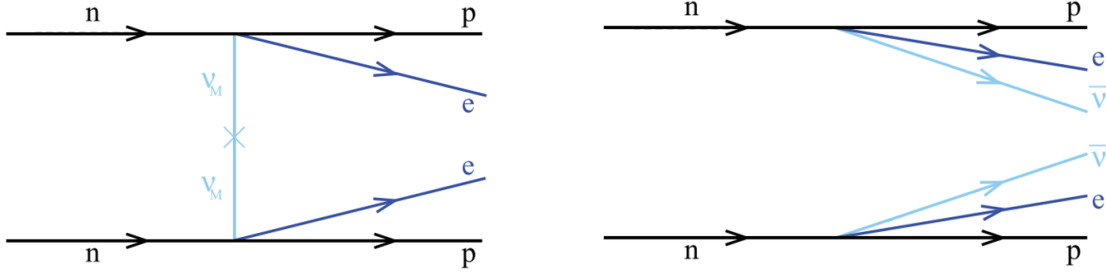


Figure 1.1: Feynman diagrams for $0\nu\beta\beta$ (left) and $2\nu\beta\beta$ decay (right) in the $\beta^-\beta^-$ direction. Annihilation of light neutrinos is showed for $0\nu\beta\beta$ decay with the "X" symbol. Taken from Ref. [3].

At present time, $0\nu\beta\beta$ decay has not yet been experimentally observed, despite the intense efforts that are put on its search. If the decay is really observed it would be one of the first clear evidence of physics beyond the Standard Model (SM) of particle physics. In particular, the observation of $0\nu\beta\beta$ decay would profoundly change our understanding of neutrinos. At present, neutrinos are generally assumed to behave as Dirac fermions, similarly to the other fermions of the SM, meaning that particles and anti-particles are distinct objects. However, if $0\nu\beta\beta$ decay exists, neutrinos must instead be Majorana particles, namely fermions that are identical to their own anti-particles [3]. The existence of Majorana neutrinos would also imply that neutrino masses originate from a Majorana mass term in the SM Lagrangian, which differs from the usual Dirac mass terms responsible for the masses of charged fermions. In addition, $0\nu\beta\beta$ decay would become the first process to explicitly violate lepton-number L conservation, suggesting a strong explanation for the matter-antimatter asymmetry that we see nowadays in our universe [2].

It must be underlined that, even if no experiment has succeeded in finding $0\nu\beta\beta$ decay, yet they have put several constraints on the half-life for such process in many nuclear species. ^{76}Ge for example, which is a potential candidate for $0\nu\beta\beta$ decay, has a lower limit on the $0\nu\beta\beta$ half-life of 1.8×10^{26} y (90% C.L.) [4]. Similarly, actual lower limits on other $0\nu\beta\beta$ candidates lie far above the lifetime of our universe, in the range 10^{17} – 10^{26} y [5]. Some of the state-of-the-art experiments which are currently investigating $0\nu\beta\beta$ decay are LEGEND-200 [6], CUORE [7] and KamLAND-Zen [8], which are focusing respectively on ^{76}Ge , ^{130}Te and ^{136}Xe . They are all expected to set new lower limits in $0\nu\beta\beta$ half-life in the order of 10^{27} y.

Other experiments, like CUPID [9] and nEXO [10], are going to join the research in the next years. A complete review of present and near future experiments on $0\nu\beta\beta$ decay can be found in Ref. [5].

1.1.2 $0\nu\beta\beta$ decay and Nuclear Matrix Elements

The $0\nu\beta\beta$ half-life depends on a well-known phase-space factor $G^{0\nu}$, the Nuclear Matrix Element (NME) $M^{0\nu}$ and the effective neutrino mass $m_{\beta\beta}$, which is a parameter that encodes the masses of all neutrinos. The formula is

$$\left(T_{1/2}^{0\nu}\right)^{-1} = G^{0\nu} |M^{0\nu}|^2 m_{\beta\beta}^2 . \quad (1.1)$$

Among all the terms, the NME contains all the relevant physical information regarding the dynamics of the process; thus, to calculate its value, a strong knowledge on $0\nu\beta\beta$ decay and on the nuclear medium in which it is happening is required. As shown in Ref. [3], the NME is usually written as

$$M^{0\nu} = M_{\text{GT}}^{0\nu} - \left(\frac{g_V}{g_A}\right)^2 M_{\text{F}}^{0\nu} + M_{\text{T}}^{0\nu} , \quad (1.2)$$

where $M_{\text{GT}}^{0\nu}$, $M_{\text{F}}^{0\nu}$ and $M_{\text{T}}^{0\nu}$ are the Gamow-Teller (GT), Fermi (F) and tensor NMEs and g_V/g_A is the ratio between the vector and axial-vector weak couplings. In the already mentioned hypothesis of light Majorana mass, each of these terms can be further decomposed into a sum over the intermediate states that are virtually populated during the process. Following Ref. [2], the GT NME is for example written as

$$M_{\text{GT}}^{0\nu} = \sum_k \langle t_{\pm} \sigma h_{\text{GT}}(r_{12}, E_k) t_{\pm} \sigma \rangle , \quad (1.3)$$

where h_{GT} is the GT neutrino potential (for its expression, see again Ref. [3]), with E_k the energy of the intermediate state k and r_{12} the relative distance between the two nucleons involved in the decay; $t_{\pm}$ and σ , on the other hand, are respectively the isospin raising/lowering operator and the Pauli spin operator. Among all the terms appearing in Equation 1.2, $M_{\text{GT}}^{0\nu}$ gives the greatest contribution.

Estimating the value of $0\nu\beta\beta$ NMEs is crucial for multiple reasons. We outline some of them:

1. In an ideal experiment with zero background, the amount of material that is needed in order to be sensitive to any particular $m_{\beta\beta}$ is proportional to $0\nu\beta\beta$ half-life, and so to the inverse square of the NME.

1.1. An overview of double beta decay

2. The choice of the material to be used in $0\nu\beta\beta$ decay searches is a compromise between experimental advantages and the value of the NME, which determines the expected decay rate. Knowing better estimates for NMEs would help in the choice of the isotope to be studied.
3. We would need NMEs to obtain information about $m_{\beta\beta}$ once $0\nu\beta\beta$ decay is observed and its half-life determined.

Despite their importance, $0\nu\beta\beta$ NMEs are poorly known: the various models that are used in the calculations give results that can differ by a factor greater than 3 and theoretical uncertainties are practically unknown [11]. The main models that have been used through the years are Nuclear Shell Model (NSM), Quasiparticle Random Phase Approximation (QRPA), Interacting Boson Model (IBM) and Energy Density Functional theory (EDF); they were also used to evaluate NMEs for single β decays and $2\nu\beta\beta$, which were later compared with experimental results. The problem is that these models describe nucleon-nucleon interactions in a simplified way, in order to reduce the complexity of the exact problem and make it computationally solvable. Each model uses different assumptions, and this usually leads to different final estimations; consequently, the results obtained from such models can be imprecise and of limited value if there are no experimental data for comparison. Figure 1.2 shows a collection of results obtained using various models, concerning $0\nu\beta\beta$ NMEs and the corresponding half-lives for some isotopes of interest.

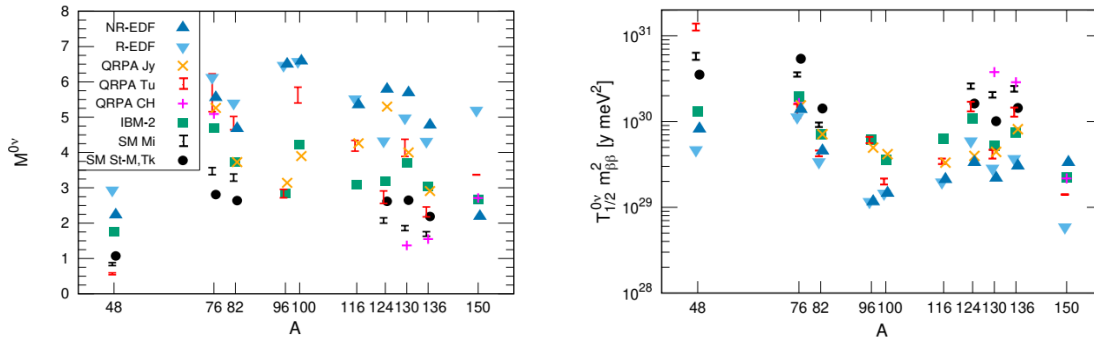


Figure 1.2: Comparison of $M^{0\nu}$ (left) and $T_{1/2}^{0\nu} m_{\beta\beta}^2$ (right) produced by NSM [12, 13, 14], different variants of QRPA [15, 16, 17, 18], IBM [19] and EDF [20, 21, 22]. Estimations are given as a function of the mass A of the parent nucleus. Adapted from Ref. [3].

1.1.3 Two-neutrino double beta decay

$2\nu\beta\beta$ decay is the counterpart of $0\nu\beta\beta$ in which also two neutrinos are emitted. The process is expressed as

$${}^A_ZX \rightarrow {}^A_{Z\pm 2}X + 2e^\mp + 2\bar{\nu}_e(\nu_e) .$$

$2\nu\beta\beta$ decay has already been observed and it is for sure much faster than $0\nu\beta\beta$ decay, being the half-lives in the range 10^{18} – 10^{24} y. For example, ${}^{76}\text{Ge}$ has a $2\nu\beta\beta$ half-life of $(1.89 \pm 0.08) \times 10^{21}$ y [23]. Nuclei that undergo $2\nu\beta\beta$ decay are the same that can undergo $0\nu\beta\beta$ decay, and those in which the process has already been observed are several, including ${}^{48}\text{Ca}$ [24], ${}^{96}\text{Zr}$ [25], ${}^{100}\text{Mo}$ [26] and ${}^{130}\text{Te}$ [27]. The process has been observed in both directions, i.e. $\beta^-\beta^-$ and $\beta^+\beta^+$, but $\beta^-\beta^-$ is easier to observe thanks to its greater Q -value.

Similarly to Equation 1.1 and to Equation 1.2, we can write the corresponding equations for $2\nu\beta\beta$ decay (again from Ref. [3]):

$$\left(T_{1/2}^{2\nu}\right)^{-1} = G^{2\nu} |M^{2\nu}|^2 ,$$

$$M^{2\nu} = M_{\text{GT}}^{2\nu} - \left(\frac{g_V}{g_A}\right)^2 M_{\text{F}}^{2\nu} ,$$

where $M_{\text{GT}}^{2\nu}$ and $M_{\text{F}}^{2\nu}$ are respectively the GT and F NMEs. Because isospin is a good quantum number in nuclei, $M_{\text{F}}^{2\nu}$ is negligibly small [28]. The general Feynman diagram for $2\nu\beta\beta$ decay can be seen in Figure 1.1 (right).

1.1.4 Correlations between $0\nu\beta\beta$ and $2\nu\beta\beta$ decay

Theoretical models have recently pointed out that there might be some important correlations between $0\nu\beta\beta$ and $2\nu\beta\beta$ NMEs. In Ref. [11], for example, Jokiniemi et al. calculated decay half-lives and corresponding NMEs for several nuclei ranging from calcium to xenon using NSM and proton-neutron QRPA (pnQRPA), observing good linear correlations between $0\nu\beta\beta$ and $2\nu\beta\beta$ decays for both models. These correlations can be combined with measured $2\nu\beta\beta$ half-lives to estimate $0\nu\beta\beta$ NMEs. Some examples of the results that they found are shown in Figure 1.3.

Consistently with the fact that the GT part dominates in the $2\nu\beta\beta$ NME, similar correlations have been found between the $0\nu\beta\beta$ NME and Double Gamow-Teller (DGT) transitions, as reported for example in Ref. [28]. DGT refers to general

1.2. Investigating $2\nu\beta\beta$ decay

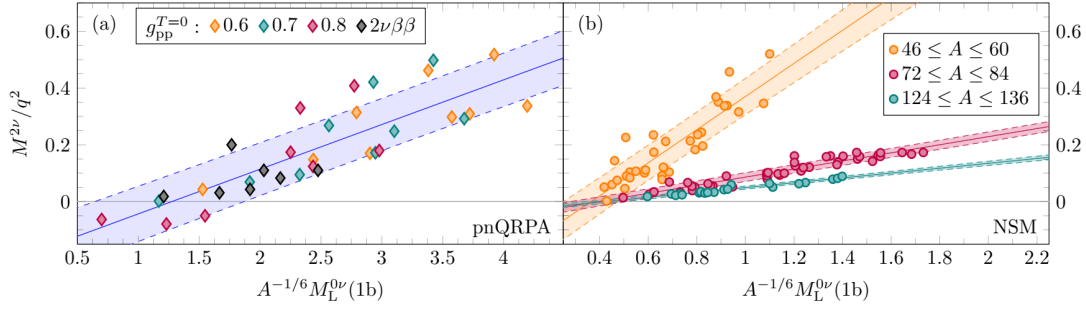


Figure 1.3: Correlation between $0\nu\beta\beta$ and $2\nu\beta\beta$ NMEs obtained using pnQRPA (left) and NSM (right). $0\nu\beta\beta$ NMEs (here denoted as $M_L^{0\nu}$) are multiplied by $A^{-1/6}$, while $M^{2\nu}$ values are divided by the squared axial coupling quenching factor q^2 . $g_{pp}^{T=0}$ denotes the isoscalar pairing parameter used in pnQRPA. Figure taken from Ref. [11].

processes where two GT operators, i.e. $t_{\pm}\sigma$, are consecutively applied, as in the case of the GT part of $2\nu\beta\beta$: it can be shown that, assuming the closure approximation, i.e. weighting all intermediate states equally, $M_{GT}^{2\nu}$ and the DGT NME are the same, apart for a geometrical factor $\frac{1}{\sqrt{3}}$.

These important correlations point to a promising line of research: a systematic and high-precision study of $2\nu\beta\beta$ decay and of its GT component as a means to probe nuclear models normally used to calculate NMEs and constrain their predictions. Significant information may come from accurate measurements of $2\nu\beta\beta$ half-lives and from new studies on the individual $\beta^{\pm}$ branches connecting the parent or the daughter nucleus to the intermediate one. In this way, improved experimental constraints from $2\nu\beta\beta$ decay can significantly reduce theoretical uncertainties and enhance the reliability of $0\nu\beta\beta$ NMEs predictions.

1.2 Investigating $2\nu\beta\beta$ decay

1.2.1 The two legs in $2\nu\beta\beta$ decay and Single State Dominance

$2\nu\beta\beta$ decay can be described as two consecutive β decays, occurring through an intermediate virtual state. We call *first leg* the first β decay that connects the initial state to the intermediate one, and *second leg* the second β decay that connects the same intermediate state to the final one. Since the dynamics of the reaction can be seen in this way, the NME can be decomposed into a sum over all the possible intermediate states that can be populated in this two-legs sequence. We have already mentioned that the F contribution is negligible in $2\nu\beta\beta$, thus we can just

focus on the GT part and decompose it as

$$M_{\text{GT}}^{2\nu} = \sum_m \frac{M_m(\text{GT}^+)M_m(\text{GT}^-)}{\frac{1}{2}Q_{\beta\beta} + E_m - E_0}. \quad (1.4)$$

In the above equation, taken from Ref. [29], m is the index that denotes the intermediate state, so that the sum must be carried over all the states in the intermediate nucleus that can be populated; so, $M_m(\text{GT}^+)$ and $M_m(\text{GT}^-)$ are the NMEs for the second and the first leg respectively, while $Q_{\beta\beta}$ and $E_m - E_0$ are the Q -value of the $\beta\beta$ decay and the energy difference between the intermediate nuclear state and the initial one. Note that the denominator introduces a strong dependence on the intermediate state energy, a dependence that is much stronger than in the $0\nu\beta\beta$ counterpart, Equation 1.3.

For a given $\beta\beta$ nucleus, the intermediate NMEs are generally unknown, and it is still unclear which of them are carrying the biggest contribution to the final NME. In particular, there is a situation, known as Single State Dominance (SSD), in which just one intermediate state gives almost all the contribution to the whole $2\nu\beta\beta$ NME. SSD is very important because, if it persists also for $0\nu\beta\beta$ decay, then experimental information on the dominant state can strongly constrain the $0\nu\beta\beta$ NME. At present, some theoretical models, such as Fermi Surface Quasiparticle (FSQP) model (see for example Ref. [30]), are able to explain SSD, and so to predict with a certain confidence whether it is realized or not for a given nucleus. Anyway, even if SSD is not realized, information on the intermediate decay legs would still give essential spectroscopic information (e.g. occupation numbers, density of states) about the nuclei involved: this has already been shown to be key in testing the validity of the nuclear models used in the description of $2\nu\beta\beta$ and $0\nu\beta\beta$ decays [31].

The greatest attention is put on β decays that connect the 0^+ Ground State (GS) of either the initial or the final nucleus to one 1^+ state (usually the lowest 1^+ among all those available) in the intermediate nucleus. This is done because $0^+ \rightarrow 1^+$ (or vice versa) transitions are usually the GT transitions with the greatest NME. The GS for initial and final nuclei in $\beta\beta$ decays are always 0^+ , since these are all even-even nuclei; on the other hand, the intermediate nuclei are always odd-odd nuclei, and therefore their GS is usually different from 0^+ . Thus, it is believed that transitions involving the lowest energy 1^+ state in the intermediate nucleus should carry the greatest contribution to $2\nu\beta\beta$ NMEs, also because of their rather small energy denominator.

1.2. Investigating $2\nu\beta\beta$ decay

Actually, Thies et al. [29, 32] have claimed to have verified SSD for ^{96}Zr and ^{100}Mo , with respect to the lowest-energy intermediate 1^+ state. Nevertheless, in these two cases it remains uncertain whether additional intermediate states could still contribute significantly, with their effects partially suppressed by relative phase cancellations. In contrast, for another well-studied $\beta\beta$ nucleus, ^{76}Ge , SSD has been excluded [33].

1.2.2 Studying single β decays

One way to experimentally investigate the two legs of $\beta\beta$ decay is to observe the corresponding single β decays. This is only possible when single β decay is a competitive process with respect to $\beta\beta$ decay, as in ^{48}Ca and ^{96}Zr . In these nuclei, the single β decay half-life is about one order of magnitude longer than that of $2\nu\beta\beta$ decay [2]. The case of ^{96}Zr is shown in Figure 1.4 (left). The problem with this strategy is that usually the intermediate state that we are interested in, mainly the first 1^+ , lies above the energy of the initial nucleus and thus the first leg β decay is energetically impossible. This is exactly the case of ^{96}Zr .

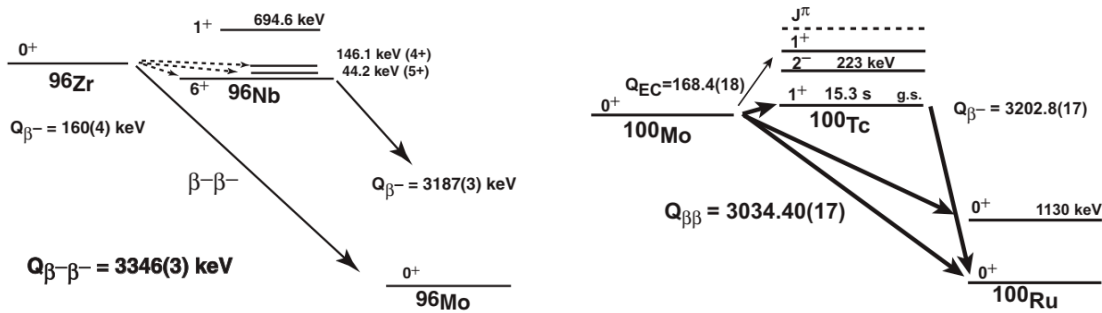


Figure 1.4: Sketch of $\beta\beta$ and single β decays in ^{96}Zr (left) and ^{100}Mo (right). The most important energy levels for the transitions are shown. Taken from Refs. [29, 32].

An alternative strategy consists in populating through an appropriate reaction the intermediate nucleus: then, we look for transitions which start from the intermediate nucleus and end either in the initial or in the final one. In this way, the first leg becomes a β^+ decay to the initial nucleus, and the second leg a β^- decay to the final one. If they are GT transitions, then they are termed respectively GT^+ and GT^- , as in Equation 1.4. The apparent reversed process in the first leg is not a problem, since crossing symmetry establishes that the NME in either direction must be the same. Anyway, this kind of study cannot be done for all $\beta\beta$ systems, since in most of them the intermediate 1^+ state is an excited state that first decays γ to the GS, and only then decays β . One favorable case is ^{100}Tc (see Figure 1.4,

right), which is the intermediate nucleus in the $^{100}\text{Mo} \rightarrow ^{100}\text{Ru}$ $\beta\beta$ decay: the GS in ^{100}Tc is exactly a 1^+ state. But this is rather the exception, not the rule. For example, ^{96}Nb , which is the intermediate nucleus in the $^{96}\text{Zr} \rightarrow ^{96}\text{Mo}$ transition, has a 6^+ GS, with the first 1^+ state lying at about 694.6 keV. This is the same case of ^{76}As , the intermediate nucleus in the $^{76}\text{Ge} \rightarrow ^{76}\text{Se}$ transition, which has a 2^- GS and the first 1^+ state at 44.4 keV.

1.2.3 Charge Exchange Reactions

Several alternative approaches are being studied to estimate NMEs when a more direct method is ineffective or not feasible. The main strategy consists in studying analogous processes that involve the same initial and final states. The analogy is based on the mathematical structure: the operator associated with the reaction of interest must have a similar form to that of the reaction being observed. Therefore, by measuring the cross section of the analogous reaction and determining the corresponding NME, it is possible to estimate the NME of the process of interest.

Charge Exchange Reactions (CERs) are one of the most used techniques to indirectly investigate weak processes. They are mediated by the strong force, and so by π , ρ and other mesons, which are different from those bosons that mediate weak interactions. Anyway, they still have similar operators, i.e. t , σ and multipole, and thus similar NMEs. Their main advantages are related to the usually large cross sections (10^{-26} – 10^{-28} cm²/str, which is 10^{15} – 10^{20} times larger than those for usual weak processes [2]) and the wide range of excited states that can be populated with medium energy incoming beams. Typical CERs used to mimic GT^- weak processes are (p, n) and $(^3\text{He}, t)$, while typical CERs for GT^+ are (n, p) , $(d, ^2\text{He})$ and $(t, ^3\text{He})$. It is generally preferable to use CERs where both the projectile and the ejectile are charged particles, as they can be more easily accelerated to the required energies and more efficiently detected.

Also CERs involving muons constitutes a valid alternative, above all Ordinary Muon Captures (OMCs). The formula for such reactions is

$$^A_Z\text{X} + \mu^- \rightarrow ^A_{Z-1}\text{X} + \nu_\mu .$$

Strictly speaking, these are still weak processes. Their possible application to double beta decay is discussed for example in Ref. [34].

To give the idea of how CERs are used, we may cite again the work of Thies et al. concerning ^{96}Zr [29]. What they did was to populate the intermediate nucleus ^{96}Nb via the CER $^{96}\text{Zr}(^3\text{He}, t)^{96}\text{Nb}$ and reconstruct its excitation spectrum by

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detecting the ejected tritons; then, they identified all the populated 1^+ states and calculated the respective cross sections; finally, from the cross sections they calculated the GT strength $B(\text{GT})$ and estimated the NME for the GT^- using $B_m(\text{GT}^-) = |M_m(\text{GT}^-)|^2$. The same procedure was applied to the second leg by Dohmann et al. [35]. The key difference is that, since they had to move in the opposite direction, from the stable ^{96}Mo back to ^{96}Nb , this was a GT^+ process; thus another CER, namely $^{96}\text{Mo}(d, {}^2\text{He})^{96}\text{Nb}$, had to be used. By comparing his own results with those from Dohmann, Thies et al. realized that just one element in the summation of Equation 1.4, i.e. the first 1^+ excited state, was giving almost exactly the $2\nu\beta\beta$ NME that can be extracted from the measured half-life. This is how SSD for ^{96}Zr , already mentioned in subsection 1.2.1, was established.

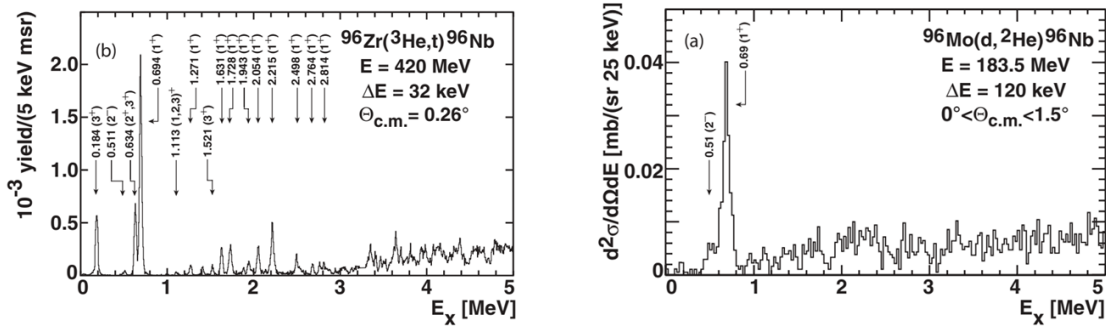


Figure 1.5: Excitation spectra of ^{96}Nb obtained by CERs mimicking the first (left) and the second leg (right) of ^{96}Zr $\beta\beta$ decay. The greater energy resolution for the first leg CER can be appreciated. Adapted from Refs. [29, 35].

Even if CERs have proved to be useful in the past, they show some important drawbacks:

1. Concerning the two steps in $\beta\beta$ decays, CERs tend to be more effective for GT^- transitions, so for the first leg, whereas for GT^+ they tend to be more complicated and less precise. This is due to the features of the typical reactions that are used. The $(d, {}^2\text{He})$ CER, for example, produces a ${}^2\text{He}$ in the final state, which is not a bound nucleus but rather a system of two independent protons. This results in more experimental difficulties and in a worse final resolution (see for example Figure 1.5, concerning the experiments by Thies et al. and Dohmann et al.).
2. Additionally, there are competing reactions, such as compound nucleus reactions, that may happen together with CERs, starting from the same initial state and producing the same final nuclei. The probability for this to occur is greater for heavier projectiles, that's why light projectiles are

preferable. Nevertheless, all CER results need to be somehow renormalized in order to take into account these competing reactions, and this usually introduces greater uncertainties in the final estimations for NMEs.

An additional method to make insightful comparisons is thus necessary, and it must be based on a different reaction mechanism.

1.3 $\beta\beta$ decays and γ spectroscopy

1.3.1 Weak processes and γ transitions

The main class of processes used alongside CERs, which can help extend their results, are electromagnetic ones, in particular γ transitions from excited nuclear states. These can be used to mimic β decay and other weak processes because the operators that mediate β decay, i.e. the vector GT and the axial-vector F operators, are similar to the electric and magnetic operators that mediate γ transitions. In the following, we show their formulas (adapted from Ref. [36]) to understand the principle of this comparison.

Vector and axial-vector weak operators are

$$T(VL) = g_V t^i r^L Y_L ,$$

$$T(AVL) = g_A t^i [\boldsymbol{\sigma} \times r^{L-1} Y_{L-1}]_L ,$$

while electric and magnetic operators are

$$T(EL) = g_{EL} r^L Y_L ,$$

$$T(ML) = g_S [\boldsymbol{\sigma} \times r^{L-1} Y_{L-1}]_L + g_L [\mathbf{j} \times r^{L-1} Y_{L-1}]_L .$$

In the above equations, L denotes the multipolarity of the interaction, g_V and g_A are the already mentioned vector and axial-vector weak couplings, and g_{EL} is the electromagnetic coupling; on the other hand, g_S and g_L can be rewritten in terms of better known quantities:

$$g_S = \frac{e\hbar}{2Mc} [L(2L+1)]^{1/2} \left[\frac{g_s}{2} - \frac{g_l}{L+1} \right] \quad \text{and} \quad g_L = \frac{2g_l}{L+1} ,$$

where g_s and g_l are respectively the spin and orbital g factors. The similarity between $T(VL)$ and $T(EL)$ and between $T(AVL)$ and $T(ML)$ (in the simplification

1.3. $\beta\beta$ decays and γ spectroscopy

of removing the g_L term, which usually gives a smaller contribution [36]) is evident.

A key difference concerning γ spectroscopy, if compared for example to CERs, is that here it is impossible to have the exact same initial and final states as in the corresponding weak process: in β decay the nuclear element changes, while in a γ transition just its excitation energy is modified. To avoid the problem, the initial state $|i\rangle$ is substituted with its Isobaric Analogue State (IAS), which then can decay via γ transition to the correct final state $|f\rangle$. This is possible because IASs conserve the inner structure of the corresponding state with just a rotation in the isospin space, i.e. a substitution of neutrons with protons. So, we end up with expressions such as

$$\langle f | g m^\beta | i \rangle \simeq \frac{g}{e} K \langle f | e m^\gamma | \text{IAS} \rangle ,$$

where m^β and m^γ are analogous weak and electromagnetic operators (e.g. $T(V1)$ and $T(E1)$), g and e are the corresponding weak and electromagnetic coupling constants and $K = (2T)^{1/2}$ is a normalization constant. IAS is obtained from the initial state by applying the isospin ladder operators, i.e. $|\text{IAS}\rangle = K^{-1} T^\pm |i\rangle$. An example of such processes is illustrated in Figure 1.6.

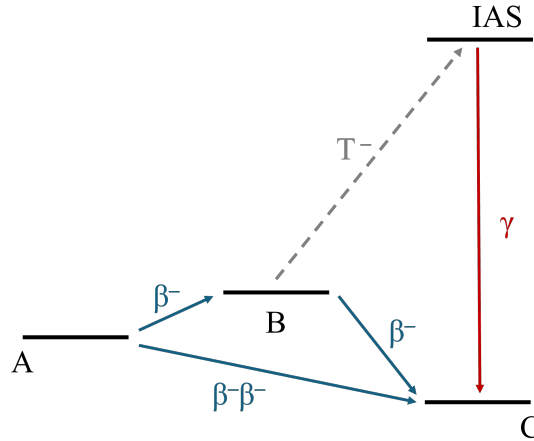


Figure 1.6: Schematic example of a γ transition used to study a β^- decay. Nucleus A decays via $\beta\beta$ to nucleus C , passing through the intermediate nucleus B . The second leg $B \rightarrow C$ can be studied by looking at the γ decay from the IAS to the GS in nucleus C . Isospin lowering operator T^- connects the intermediate state in nucleus B to the IAS.

Instead of starting with the IAS, it is possible to start directly with the final state; this one can be excited to the desired energy level using a photon beam, thus populating the IAS. This reaction, which is the mirror of a γ transition, is called

photo-nuclear reaction. The NME is usually extracted from the probability with which the IAS is populated. Anyway, photo-nuclear reactions tend to be more complicated to realize, because the laser beam that is used to excite the nucleus needs to be very precise in energy. If the energy of the laser is not precise, then giant resonances, which are usually the underlying structure that stands behind these high-energy IASs, might be populated, creating a background that is almost impossible to disentangle from the IAS population itself.

Gamma spectroscopy constitute a very promising field to study $2\nu\beta\beta$ and $0\nu\beta\beta$ decays, since they can be used to shed light on both legs of those processes. On a broader perspective, they can be used to validate the nuclear models that are applied in this field to calculate cross sections and NMEs: results obtained with this technique can be compared with theoretical predictions, so to discard those that are far by several order of magnitudes.

1.3.2 Experimental implementation

We can summarize the operative paradigm for this type of γ experiments. First of all, a reaction will be chosen to populate the IAS of interest. In the kind of reactions that we are focusing on, a projectile collides with a target, producing a light ejectile and a heavy recoil. The ejectile can be detected to fully reconstruct the reaction kinematics (for example, using a silicon detector or a split-pole spectrometer, as done for example in Refs. [37, 38]), while the recoil typically stops inside the target and is therefore not detected. The recoil is emitted in an excited state with a certain probability (with the IAS among the possible excited states); it then de-excites by emitting one or more γ rays, which can be detected using a γ detector. Actually, the IAS usually lies above proton and neutron separation energies, so also particle emission channels are open.

Ejiri et al. in Ref. [39] carefully explain what is the procedure to follow in these experiments. First of all, we rely on the following formula

$$\frac{d\sigma_{\text{IAS}}(\alpha')}{d\Omega} = \frac{d\sigma_{\text{IAS}}}{d\Omega} \frac{\Gamma_{\text{IAS}}(\alpha')}{\Gamma(T)}, \quad (1.5)$$

where $\frac{d\sigma_{\text{IAS}}(\alpha')}{d\Omega}$ is the differential cross section for the consecutive IAS population and its γ decay, while $\frac{d\sigma_{\text{IAS}}}{d\Omega}$ is the differential cross section for just IAS population; $\frac{\Gamma_{\text{IAS}}(\alpha')}{\Gamma(T)}$ is the γ -Branching Ratio (BR), with $\Gamma_{\text{IAS}}(\alpha')$ being the α' mode width and $\Gamma(T)$ the total IAS width. $\alpha' = M1, E1, \dots$ denotes type and multipolarity of the γ decay. As can be seen in Figure 1.7, $\Gamma(T)$ for IASs of nuclei involved in double beta decays are in the range 20–80 keV, but lower values cannot be excluded. Since

1.3. $\beta\beta$ decays and γ spectroscopy

there seems to be a correlation between $\Gamma(T)$ and the third component of the isospin T_z , the IAS width can be roughly estimated even if the state has not yet been observed.

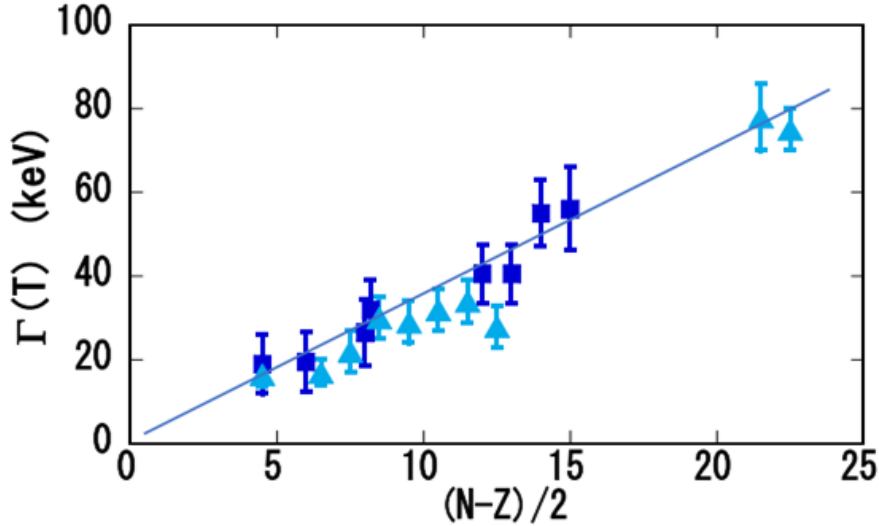


Figure 1.7: Experimental IAS widths as a function of the isospin $T_z = (N - Z)/2$. Blue squares denotes measurements taken by Ejiri et al.; light blue triangles by Ref. [40]. Taken from Ref. [39].

So, the procedure is the following:

1. Reconstruct the recoil excitation spectrum using the information got from the silicon detector or the split-pole spectrometer (which from now on we will call *ancillary detector*) and thus identify the IAS peak. From its FWHM and the number of events, $\Gamma(T)$ and $\frac{d\sigma_{IAS}}{d\Omega}$ can be extracted.
2. Count the number of γ s of the desired energy detected in coincidence with the ejectile in the ancillary detector and estimate $\frac{d\sigma_{IAS}(\alpha')}{d\Omega}$.
3. Invert Equation 1.5 and use the values just obtained to calculate $\Gamma_{IAS}(\alpha')$. $\Gamma_{IAS}(\alpha')$ is proportional to the NME for the γ transition and can be used, together with equations in subsection 1.3.1, to estimate the NME of the weak process.

The strategy outlined by Ejiri suggests what are the main requirements for the experimental apparatus that we are using. The population of the IAS might be rare or not, depending on the kind of reaction and on the specific nucleus under study; its γ decay, however, usually has a low probability, since decay via particle emission dominates. Consequently, a large number of beam–target interactions is required to obtain sufficient statistics for the analysis. For this reason, the ancillary

and the γ detector need to have as high efficiency as reasonably possible. In this sense, silicon detectors might be preferable over split-pole spectrometers, which on the other hand usually cover smaller solid angles. In addition, and this is particularly true for the ancillary detector, they must have a good durability, i.e. they must keep their performances after long irradiation.

Also energy resolution is an important parameter. The resolution that we need, however, depends strongly on the IAS width $\Gamma(T)$ that we have to measure, and on the relative precision that we aim to reach. A reasonable relative uncertainty is around 15 %, as the correlation between the GT component of the β -decay strength and the corresponding γ -decay branch can itself be determined with an accuracy of about 10–15 %, according to Ref. [39].

From the point of view of energy resolution, split-pole spectrometers perform better than silicon detectors; thus, for $\Gamma(T)$ around 50 keV silicon detectors can still be used, while in case of lower values split-pole spectrometers must be preferred. When $\Gamma(T)$ gets too small, i.e. below 30 keV, the strategy must be slightly revised if we still want a relative uncertainty of 15 %; thus, instead of using the excitation energy spectrum to evaluate $\Gamma(T)$, we should use the γ energy spectrum, that is produced by the γ detector and which usually have a much greater resolution, especially if we are using an array of High-Purity Germanium (HPGe) detectors.

1.3.3 The goals of this thesis

This thesis consists in a feasibility study, performed with a simulation, and in an experimental detector test. The work was carried out at Laboratori Nazionali di Legnaro (LNL), where AGATA is available for the type of experiment outlined in the previous section. AGATA is a state-of-the-art γ -ray tracking array composed of highly segmented HPGe detectors, designed to achieve excellent energy resolution and precise γ -ray reconstruction [41]. It has also a good efficiency, which may be further increased at high energies thanks to a new tracking technique, the so-called Bubble Tracking algorithm [42]. On the other hand, SAURON is a highly segmented Double Sided Silicon Strip Detector (DSSSD), developed by Micron Semiconductor Std. [43], that can be employed as an ancillary detector to complement AGATA measurements.

The present thesis focuses on some relevant aspects connected to the feasibility of such an experiment using AGATA and SAURON. More precisely, our goals are the following:

1. We want to understand whether AGATA is able to measure $\Gamma(T)$, with its value ranging from few keV to 100 keV, and how many events are needed in

1.3. $\beta\beta$ decays and γ spectroscopy

order to get an estimation within a desired level of uncertainty. In particular, we want to understand what are the parameters of the apparatus which are affecting the most the precision of our measurement. To do that, we will use a simulation of the proposed apparatus written in Geant4. This will be the topic of chapter 3.

2. Then, we want to characterize some of SAURON features and its performances, such as energetic resolution as a function of the bias voltage and charge-sharing between adjacent channels. In addition, we want to understand if its resolution, which usually degrades after intense irradiation, can be restored and, if so, to what extent. This will be the topic of chapter 4.

Before continuing, in chapter 2 we will briefly study the main features of AGATA and SAURON.

An overview of AGATA and SAURON

AGATA [41, 44, 45], acronym that stands for Advanced GAMMA Tracking Array, is an array of γ -tracking HPGe detectors. Initially proposed in 2001 [46], AGATA is considered the state-of-the-art of γ spectroscopy. Presently, it is operative at LNL, where it is involved in experiments of nuclear and particle physics. SAURON (Silicon AnnUlar stRipped iON detector), on the other hand, is a silicon detector produced by Micron Semiconductor Std. [43]. It can be used as ancillary to detect charged particles.

In this chapter we briefly introduce these two detectors, giving some information concerning their structure and mechanics. This is a preliminary discussion, as in the following we will study the possibility of performing the experiment outlined in subsection 1.3.2 with these two detectors.

2.1 AGATA

2.1.1 The AGATA array

AGATA (see Figure 2.1) is an array of HPGe detectors. “Array” means that the detector is made up of more sensitive elements, arranged one next to the other to form a compact spherical structure. In its complete 4π geometry, AGATA is made up of 180 germanium crystals.

HPGe crystals

The HPGe crystals are the building blocks of AGATA. They are made of n-type germanium, have a tapered hexagonal geometry and extends for a thickness of 9 cm. Each crystal has a 36-fold segmentation, which is exploited in the γ -tracking;

2.1. AGATA

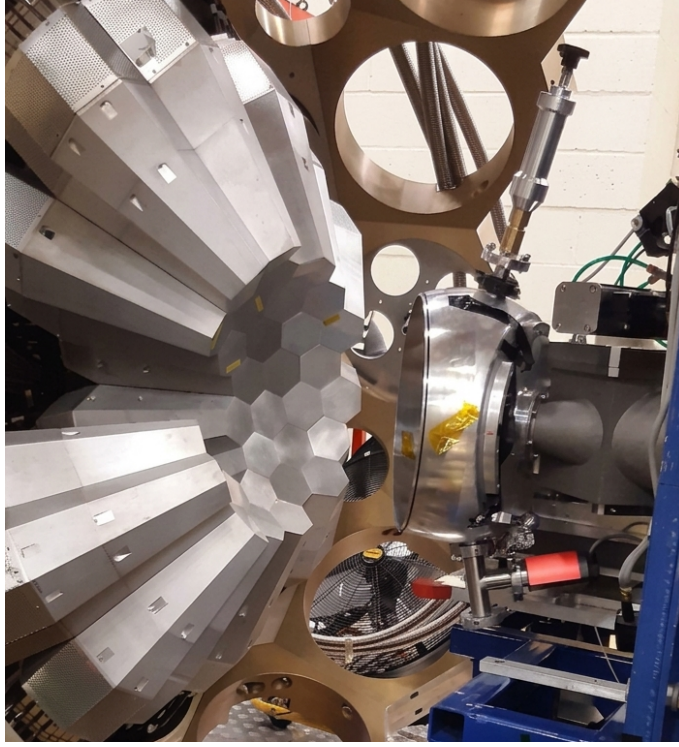


Figure 2.1: Picture of AGATA taken at LNL during the setup of an experiment (November 2025).

additionally, the total deposited energy is collected in the core, into a high- and a low-gain channel, for a total of 38 output channels. The segmentation is obtained by dividing each crystal in 6 longitudinal slices (named with numbers 1–6) and in 6 angular sectors (named with letters A–F), as shown in Figure 2.2 (left). The thickness of longitudinal segments is greater for sectors that are far from the centre of the array, so to provide a uniform distribution of γ -ray interactions and an optimised pulse-shape sensitivity.

Photons striking inside the detector create electron-hole pairs. Germanium, as a semiconductor, has a low electron-hole creation energy (~ 2.96 eV at low temperatures [47]), so that many pairs can be created once a photon interacts in the detector. Electrons are collected in the core contact of each crystal, since it is positively polarized, while holes are collected in the segment. Thus, electrons are responsible for the signal concerning total energy deposition in the crystal, while holes are responsible for the signal concerning energy deposition in each segment.

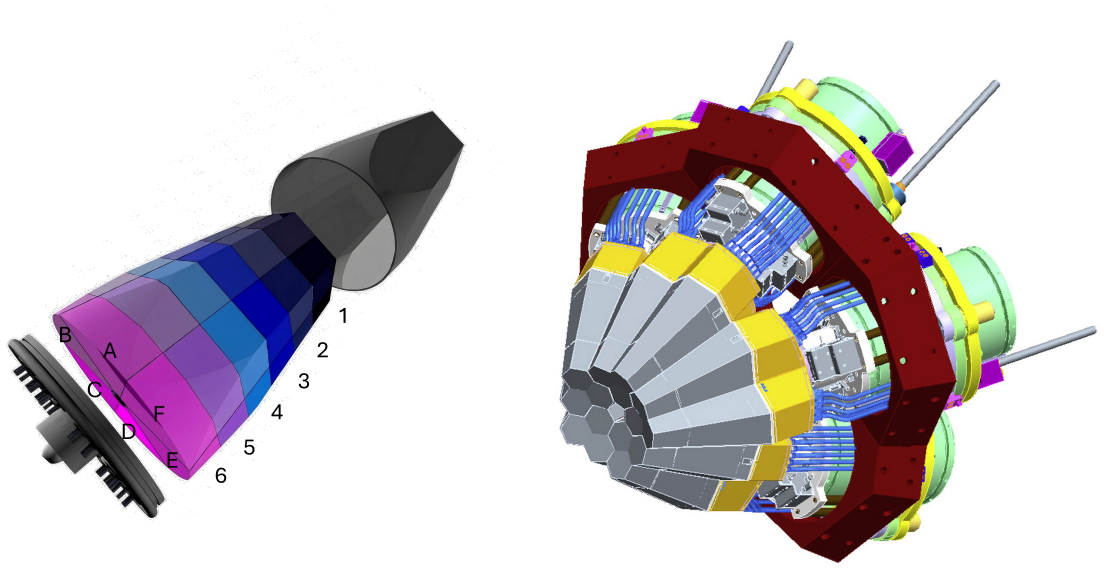


Figure 2.2: Left: structure of a single AGATA HPGe crystal. The aluminum protective capsule is also shown. Adapted from Ref. [44]. Right: AGATA sub-array with 5 ATCs mounted in the honeycomb. Taken from Ref. [48].

Encapsulation, vacuum and refrigeration

Every crystal is encapsulated inside a hermetically sealed aluminum canister, in order to protect the surface of the crystal from mechanical damage and to isolate it from the outside. Capsules are then collected in groups of three, to form AGATA Triple Clusters (ATCs). Each capsule in the ATC has hexagonal faces with different sides, to perfectly fill a spherical surface once all ATCs are assembled together.

Each ATC is thermally isolated from the outside by a vacuum with a pressure below 1×10^{-6} mbar. AGATA cryostats then provide the proper liquid nitrogen refrigeration to the detectors and to the cold part of the preamplifier electronics, which is mounted close to the detectors and cooled in order to minimize its noise contribution. Thanks to a separated cooling scheme for each component, crystals are cooled to 90 K, while the cold part of the preamplifiers is cooled to 130 K, where its noise is minimal. The liquid nitrogen used in the refrigeration of each ATC is contained inside a Dewar with a capacity of 4.5 l, which allows about 8 h of continuous operation. Liquid nitrogen filling level and temperature of each component are continuously monitored.

2.1.2 Infrastructure and Front-End Electronics

The AGATA infrastructure comprises its external mechanical structure and the Detector Support System (DSS). The latter provides the detectors with the cryogenic

2.1. AGATA

cooling, the low- and high-voltage power supplies and constant monitoring. The Front-End Electronics (FEE) digitises the signals from each crystal and process them in real time to extract useful information.

The honeycomb

ATCs are mounted together on a spherical structure, called *honeycomb* because of the hexagonal shape of its individual modules. The honeycomb is made up of 60 identical flanges, one for each ATC, which are located one next to the other with high precision (the maximum tolerated deviation from the nominal position is $200\text{ }\mu\text{m}$). Usually, AGATA does not work in its complete configuration, but with a smaller number of flanges and ATCs, so that the honeycomb is not complete and the array just cover a fraction of 4π (see Figure 2.2, right). In such cases, we say that AGATA is in a sub-array configuration. The honeycomb, be it complete or not, is then mounted on another frame, which allows AGATA to be moved in space.

AGATA support system

AGATA DSS is composed by the autofill system and the low- and high-voltage power supplies. The autofill system controls the filling cycle, operating all the associated valves, and monitors the temperature of every detector element and the amount of liquid nitrogen in each detector Dewar. Low-voltage supplies provide power to the preamplifiers, the digitizers, the high-voltage modules, the liquid nitrogen level monitoring system and the PROFIBUS Decentralized Peripherals, which are used to control power supplies. High-voltage supplies are instead used to provide power to each detector module.

Additionally, DSS comprises an Uninterruptable Power Supply (UPS), which regulates the power-off procedure in case of a power failure, and a Programmable Logic Controller (PLC), which acts as a bridge between the user interface and each element in the DSS.

Front-End Electronics

AGATA requires a high count-rate capability in order to exploit its high geometrical efficiency. For this reason, preamplifiers used in AGATA have been developed precisely for this array. They are divided into a cold (130 K) and a warm part (room temperature), which are separated by 15 cm cabling inside the vacuum. All

the signals coming from a single ATC are delivered to the digitizers by 21 MDR high-speed cables.

AGATA 100-MHz 14-bit digitizers work on the signal from each crystal. The signal is digitised, and then processed in order to establish the time at which the crystal detected a γ ray and the amount of energy delivered in each segment and in the core of the crystal. The signal in the core can be used as a trigger for the crystal, and thus allows coincidences measurements with ancillary detectors. Signal output from digitizers is then converted into optical signal and delivered through optical fibers to the Global Gigabit Processor (GGP) boards, which can sustain an incoming data rate of 2 Gbps per channel. Optical fibers are used in order to achieve the necessary high data-transmission rates.

2.1.3 Pulse-shape analysis

Relevant information concerning photon interactions in AGATA include the energy released in each photon interaction point, the time at which it occurred and its position. AGATA extracts this information using Pulse-Shape Analysis (PSA).

PSA identifies with high precision the location of the various interactions points in the detector volume, as well as the energy release. An accuracy in the location determination better than 5 mm Full Width at Half Maximum (FWHM) is desirable to ensure effective subsequent tracking. PSA works by comparing in real time the detected pulse shapes in each segment of germanium crystals with a calculated reference basis. This reference basis is obtained by simulation, and is called AGATA Data Library (ADL). Once the best reference signal has been determined, energy, time and three dimensional position of the interaction are provided. This procedure can be performed by a set of different algorithms, which are optimised for different types of events and are usually a compromise between resolution and computational time. Among them, the grid-search algorithm is considered the standard one by AGATA collaboration, due to its simplicity and robustness (for further information on the grid-search algorithm, see Ref. [49]).

More precisely, PSA relies on the following mechanism. A photon interacting in a crystal segment creates a *net-charge* signal in a segment, and induces some *transient* signals also in the neighbouring ones. This is shown in Figure 2.3. Since the detector is not perfect, but we have to account for impurities and different charge carriers mobilities, there is a spatial dependence in the shape of transient signals: PSA uses exactly this dependence to determine the position of photon interactions. This determination is achieved using χ^2 minimization over the various alternatives in the ADL, which usually means a resolution of ~ 4 mm [51].

2.1. AGATA

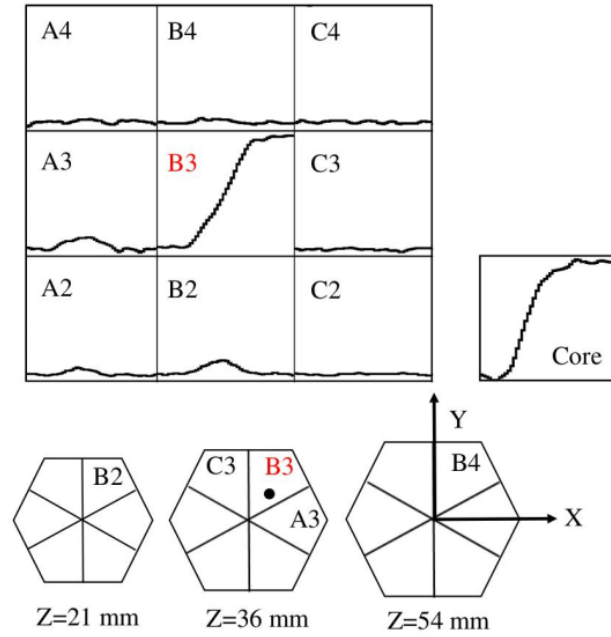


Figure 2.3: Example of net-charge and transient signals generated by a photon striking in segment B3 of a AGATA HPGe crystal. The net-charge signal is generated in the segment where the interaction is occurring; transient signals are generated in the neighbouring segments. Core signal is also shown. Taken from Ref. [50].

2.1.4 Tracking

AGATA is expected to study photons from few keV to about 20 MeV. Photons in this energy range can interact with matter in four different ways: photoelectric effect, Compton scattering, Rayleigh scattering and pair creation [44]. In germanium, the dominant process between 150 keV and 10 MeV is Compton scattering: this means that usually γ rays do not get absorbed in a single hit, but need several interaction points, in which a part of their initial energy is dissipated. In the final interaction point, γ rays interact by photoelectric effect and get totally absorbed. It is thus clear that photons in AGATA can create complicated tracks of events, which need to be correctly characterized in order to get valuable physical information. If the total energy of the initial photon is not reconstructed, then the γ spectrum would be dominated by the so-called Compton continuum; additionally, if the first interaction point is misassigned, then the trajectory of the photon would be wrong.

For these reasons, AGATA performs an accurate event characterization that is called *tracking*: once each vertex has been recognized thanks to PSA, tracking establishes the correct sequence of interaction points and reconstruct the complete

trajectory of each individual photon. In this way, by grouping together vertices and identifying the first event, it is possible to determine the energy and the direction of the initial photon.

As in PSA, also in tracking there are various alternative algorithms. In all of them, photon trajectories are tracked relying on the Compton scattering formula and on ranges and cross-sections for each of the photon interactions in matter. Alternative algorithms are usually divided in two categories: back-tracking [52] and forward-tracking algorithms [53]. Back tracking works by reconstructing the complete trajectory starting from the final interaction point, which always corresponds to a photoelectric interaction; forward tracking, on the other hand, groups together vertices by creating clusters in the (θ, ϕ) space, and reconstructs the trajectory starting from the first interaction point. Forward tracking have been showed to be more efficient than back tracking [54]. Still, old tracking algorithms are continuously improved and new ones are getting developed. This research has the goal of increasing AGATA efficiency and Peak-to-Total (P/T) ratio [41].

2.2 SAURON

2.2.1 General features of silicon strip detectors

SAURON is the name used at LNL to call the S1 silicon detector, produced by Micron Semiconductor Std. [43]. As all silicon and other semiconductor detectors, SAURON relies on the p-n junction, where we can create a depletion region by applying an appropriate reverse bias. Particles hitting in the depletion region generate electron-hole pairs, which then migrate, generating an electrical signal: electrons migrate towards the n-side, while holes migrate towards the p-side. When the reverse bias is sufficiently high, the depletion region extends for all the thickness of the detector, so that practically all the charges that are released during the interaction of a particle are collected.

Silicon detectors hold a lot of advantages, which make them a valuable choice in many particle physics experiments, especially with high rate and radiation levels. The main advantage is the fact that they have very low electron-hole creation energy, in the order of few eV. Silicon just requires about 3.6 eV to generate a single pair, which can be compared, for example, to the 30 eV required for gas chambers. Thanks to this low value, a large number of charge carriers is generated, and thus a good energy resolution can be achieved. Additionally, particles travelling in silicon have a high energy loss, thanks to its high density,

2.2. SAURON

and thus can be stopped by a thin layer of material. Silicon high density also reduces the range of energetic secondary electrons, allowing for good spatial resolution [55].

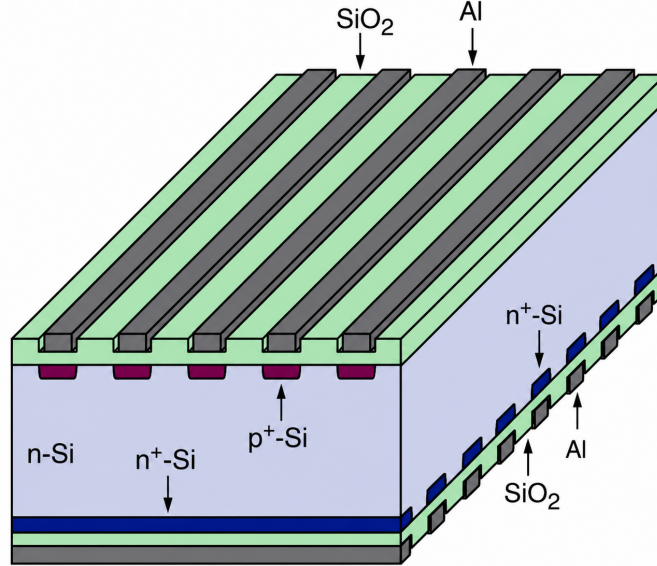


Figure 2.4: Scheme of a typical DSSSD. p^+ and n^+ strips are shown in purple and blue, respectively; on the other hand, aluminum and SiO_2 layers are shown in grey and light green. Taken from Ref. [56].

Silicon detectors can be produced with complex segmentation, in pixels or strips of various shapes and dimensions. Segmentation can be then used to recognize the position of the particle interaction, with a resolution that depends on the dimension of the specific segments and on the pitch (the distance between individual pixels). When segments have the shape of thin strips, we call the detector a Silicon Strip Detector (SSD). Typical SSDs are built as in Figure 2.4. They are designed starting from a low-doped n-type silicon wafer, which is then doped and segmented in the p^+ side (also called *junction* side) in a series of various strips. These strips are covered by aluminum or other conductive elements so to connect them to the read-out circuit. Interstrip regions, which usually have a thickness of some μm , are covered and insulated by a layer of SiO_2 .

On the opposite, n^+ side (also called *ohmic* side) it is possible to add another segmentation, for example by creating strips which are oriented in an orthogonal direction with respect to those in the junction side. In this way we create a Double Sided Silicon Strip Detector (DSSSD), which can be used to track the interaction point of incident particles, with a precision that depends on the dimensions of the strips. This can be done by looking at the strips in the two sides that are

detecting the signal. Strips on the two sides do not need to have the same shape, but different and more complicated shapes can be chosen to fit to the specific experimental requirements.

2.2.2 SAURON: geometry and resolution

SAURON is an example of DSSSD or, more precisely, a double-sided silicon *annular* detector, due to the annular shape of its sensitive strips. There exist various models of SAURON detectors, which basically just differ in the thickness of the silicon wafer. The SAURON detector used in the simulation in chapter 3 has a thickness of $1500\text{ }\mu\text{m}$, while those detectors studied in chapter 4 have a thickness of $500\text{ }\mu\text{m}$ and $1500\text{ }\mu\text{m}$.

All SAURON models are divided into 64 annular strips in the junction side (conventionally considered the front side) and 16 strips in the ohmic side (conventionally the rear side). The front side is divided into 4 quadrants; in each of them there are 16 annular strips with a width of 1.5 mm . They span a range which goes from 2.4 cm , taken from the center of the detector, to 4.8 cm . On the opposite side, there are 16 angular sectors, called *pads*, with an opening angle of 22.5° . Each quadrant in the front side, with its 16 strips, corresponds to 4 pads in the rear side. SAURON geometry can be appreciated in Figure 2.5.

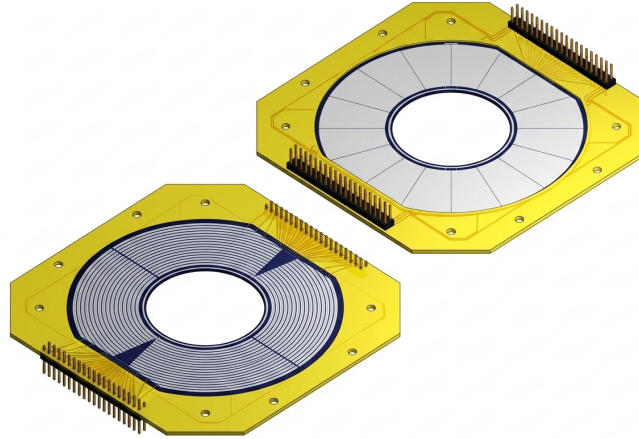


Figure 2.5: The two sides of SAURON. The geometrical shape of strips and pads is shown, as well as the position of the lateral pin-out connections. Taken from Ref. [43].

Read-out is performed by 80 pin-out connections located in two opposite sides of the detector. On the junction side two triangular aluminum regions connects the strips to the pin-out connections: this results in a little reduction of the total

2.2. SAURON

sensitive area in the junction side. Also, the total sensitive area of SAURON does not have a perfect circular shape, since its external borders are partly cut to leave space for the pin-out connections.

SAURON can be used as an ancillary detector in various experiments to detect charged particles, determining their energy and trajectory. Its energy resolution is declared to be 55 keV FWHM in both sides by the manufacturer [43], even though recent test studies have shown that brand new detectors can achieve a better energy resolution, around 30 keV [57]. We have to keep in mind, however, that radiation damage can spoil the energy resolution of each strip, and this deterioration can be different from one strip to the other. Position resolution, on the other hand, is determined by the dimension of strips and pads since, as in all DSSSDs, particle interaction point is determined by looking at which strip and which pad have together detected a signal.

3

Feasibility study by simulation

In subsection 1.3.2 we have outlined the kind of experiment that we need to perform in order to determine $\Gamma_{\text{IAS}}(\alpha')$. In the first stage of our study, we work with a Geant4 simulation that reproduces such an experiment with AGATA and SAURON. Our goal is to understand which sources of uncertainties give the greatest contribution to the final γ -ray energy resolution. The width of the IAS peak is a combination of the precision of the measurement and its intrinsic width $\Gamma(T)$, that is one of the parameters that we need to measure. We expect that a worse resolution in the γ -ray energy spectrum will result also in a less precise estimation of $\Gamma(T)$. Thus, it is important to know what instruments limits the resolution in our experiment and, given the actual resolution that we can achieve, if it is still possible to measure $\Gamma(T)$ with a reasonable precision.

In this chapter, after having described the Geant4 simulation that we use, we explain how we carry out this analysis. We also explain how we extract from the IAS peak the number N of events, which is another parameter that this experiment should be able to measure and which we need in order to calculate $\frac{d\sigma_{\text{IAS}}(\alpha')}{d\Omega}$.

3.1 Description of the Geant4 simulation

3.1.1 The physics of the simulation

The Geant4 simulation used in this work is based on a previous simulation, which was developed for a study of Inverse Beta Decay (IBD) in ^{71}Ga , reported in Ref. [58]. We can use the same simulation in our work for two main reasons:

1. This IBD experiment is based on the same fundamental principle as our study: investigating a weak interaction by analyzing experimental data

3.1. Description of the Geant4 simulation

from a γ decay. Accordingly, the experiment is designed in the same way as outlined in subsection 1.3.2, with AGATA and SAURON as main detectors.

2. At this stage, we are not focusing on a specific nucleus, but just on the main experimental constraints affecting the quality of the results. Therefore, although the nuclei involved in the simulation are not $0\nu\beta\beta$ candidates, they can still be used for the purposes of our study.

The physics of the setup is illustrated in Figure 3.1. The reaction under investigation is the neutron-stripping reaction $^{72}\text{Ge}(p, d)^{71}\text{Ge}^*$, with the ^{71}Ge recoil nucleus left in an excited state. In the original experiment, the main focus was on the IAS of ^{71}Ga , located at 8.91 MeV above the GS of ^{71}Ge [59].

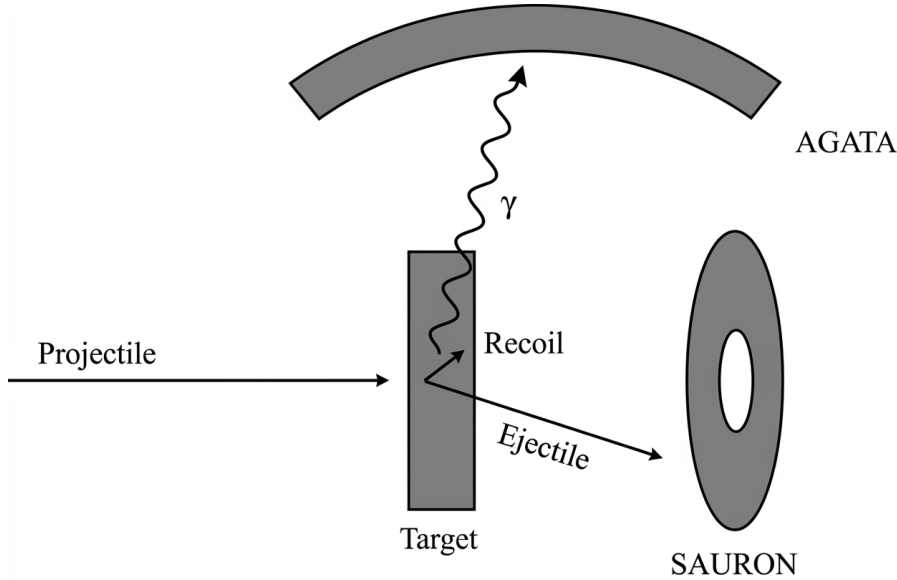


Figure 3.1: Schematic representation of the experimental apparatus in our simulation. Dimensions are not in scale.

The projectile, i.e. the proton beam, is emitted at an energy of 25 MeV, value that represents a compromise between several experimental factors. First of all, since the reaction is endoergic, the beam energy must be chosen above its specific Q -value, $Q = -16.40$ MeV, which accounts also for the final recoil excitation energy. Secondly, it is important to consider the cross section for the reaction as a function of energy, in order to identify the energy at which it happens with the greatest probability. Finally, at LNL proton beams are produced by the TANDEM accelerator, which can accelerate protons up to about 28 MeV [60]. This limit sets another constrain on the projectile initial energy, which drives to the final, more conservative choice of 25 MeV.

The target is made of ^{72}Ge , and has transversal dimensions $20\text{ mm} \times 20\text{ mm}$ and a thickness of 1.00 mg/cm^2 (which corresponds to $1.87\text{ }\mu\text{m}$). The latter is actually the relevant parameter, because its value is usually the result of another compromise: if the target is too thin, the reaction rate would be small; if it is too thick, the ejectile would loose a consistent amount of energy in the target, thus spoiling the kinematical reconstruction. Eventually, if the thickness is too big, the ejectile could even stop inside the target without being detected. If we consider deuteron with a kinetic energy of about 8 MeV , which is its typical emission energy in this experiment (see subsection 3.1.3 below), then its range in germanium, calculated using the software LISE++ [61], is 100 mg/cm^2 : this means that the chosen thickness should be good to let these particles pass. In the hypothesis that the reaction occurs in the first layers of the target, so that deuteron has to travel accross all of it, its energy loss in 1.00 mg/cm^2 of ^{72}Ge is about just 0.05 MeV , less than 0.75% of its emission energy.

After the reaction has occured, the ejectile is detected in SAURON, which is located at a distance of 5 cm from the target; considering SAURON dimensions (see section 2.2), this means that the detector is covering the forward angles in the range $25^\circ\text{--}44^\circ$, in the laboratory frame of reference. This angular range is chosen in such a way to avoid very forward angles, which are usually associated to high-intensity background reactions, such as elastic collisions. In these way, we avoid excessive background in our data, and also keep SAURON safe from intense irradiation.

The ^{71}Ge recoil, on the other hand, stops inside the target, since its range in germanium is only 0.028 mg/cm^2 . It decays from the excited state by emitting a γ ray, which is detected by AGATA. AGATA is operated in a sub-array configuration with 13 ATCs, covering a solid angle of about 1π above the reaction chamber.

3.1.2 The structure of the simulation

The Geant4 simulation that reproduces this experiment can be divided into three sequential scripts. Each of them generates the input file for the next script. We present them:

1. The `agata` component provides the physical simulation itself, as outlined in subsection 3.1.1. It reads a set of input files describing the relevant physical parameters for the simulation (such as the energy levels in ^{71}Ge and their decay properties) and generates a chosen number of events, whose data are stored in an output file. The saved data include the kinematical parameters

3.1. Description of the Geant4 simulation

of the particles (such as energy and momentum), as they are measured by SAURON and AGATA. At this stage, detectors are modeled with infinite resolution, thus providing exact information on the detected particles. The only exception is AGATA position resolution, whose impact on the final results is expected to be negligible.

2. The `tracking_simulated` component reads the output file produced by `agata` and performs AGATA γ tracking. It produces a further output file in which all γ events in AGATA are grouped together when they are associated, by the tracking procedure, to the same initial emitted photon.
3. The `ReadSimu.C` macro performs the kinematical reconstruction and applies Doppler correction to the γ rays detected in AGATA. In addition, it accounts for the finite detectors resolution by smearing selected final particle parameters: in particular, it incorporates SAURON finite resolution in both energy and position, and AGATA finite resolution in energy. Finally, `ReadSimu.C` produces the output ROOT file on which the subsequent analysis is performed.

3.1.3 Kinematics of the reaction

We now study the kinematics of the reaction, providing an estimate of the order of magnitude of some relevant variables. The following considerations are carried out in the laboratory reference frame, using the LISE++ software.

The ejectile can be emitted in any direction, from 0° to 180° . Its total kinetic energy K , which is a function of the angle, is in the range 7.4–8.6 MeV, with a velocity of 0.088–0.097 (in units of c). In contrast, the recoil is emitted within a more limited angular range, 0° – 55° , with an energy between 0.011 and 1.1 MeV. Its velocity thus ranges from 5.7×10^{-4} to 5.7×10^{-3} .

In our experiment we can see only ejectiles emitted within the angular range 25° – 44° , which corresponds to the angular coverage of SAURON. Restricting the analysis to this range, we find that the ejectile total kinetic energy is approximately 8.4–8.5 MeV, corresponding to a velocity range of 0.094–0.095. For the recoil, the corresponding angular range is 53° – 55° . In this interval, its total kinetic energy varies between 0.086 and 0.18 MeV, and its velocity between 1.4×10^{-3} and 1.8×10^{-3} .

These values are calculated under the assumption that the reaction takes place at the middle of the target. At that point, the projectile has lost a negligible fraction of its initial energy, namely less than 0.03 %, which is about 7 keV.

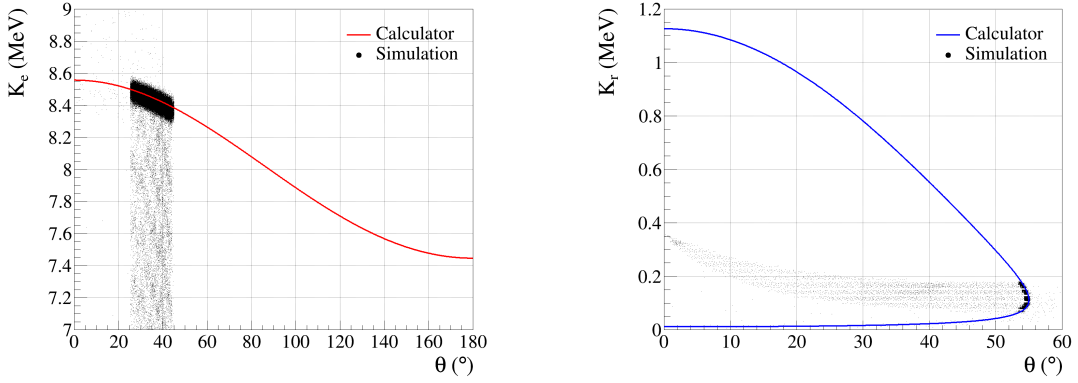


Figure 3.2: Predicted kinematical lines K vs θ_{lab} for the ejectile (left) and the recoil (right), obtained with LISE++ [61]. They are superimposed with the results of our simulation.

In Figure 3.2 results from our simulation are compared with LISE++ predictions. Our results are shown as black dots, while predictions are depicted as continuous lines. Our results follow the predicted kinematical lines, with a spread that is due to the finite resolution of SAURON. Its finite resolution, both in energy and space, affects the results for both the ejectile and the recoil, since kinematical variables for the latter are calculated from those of the ejectile. Points that deviate from the expected trend come from ejectiles interacting in the interstrip regions of SAURON, where a thin SiO_2 layer absorbs part of the ejectile energy.

3.1.4 Relevant spectroscopic input data

In our simulation, the input data describing the level scheme of the recoil are particularly important, as they directly determine the spectrum under investigation. For this experiment, we chose to exclude all events not relevant to our purposes. Accordingly, we simplified the recoil level structure by considering a single high-energy state, namely an excited state at 7.87 MeV. In addition, this state is assumed to be populated with 100 % probability, in order to minimize the number of peaks in the spectra and maximize the number of useful events. This approach is justified since we are interested only in the properties of a single, high-energy peak, making any information on additional peaks unnecessary. Furthermore, particle emission, which is an alternative decay channel typically dominant in decays from IASs, is neglected in the simulation. As a consequence, the selected state, populated with 100 % probability, is also assumed to decay exclusively via γ emission, with 100 % probability. Since no other states are included, the decay proceeds directly to the GS.

3.1. Description of the Geant4 simulation

The lifetime of the state is estimated from Figure 1.7, which shows that IASs typically have an intrinsic width in the range 10–100 keV. Recalling Heisenberg's uncertainty principle, $\Delta E \Delta t \simeq \frac{\hbar}{2}$, we obtain $\tau \simeq \frac{\hbar}{2\Gamma} \sim 10^{-20}$ s. This is the value adopted in the simulation. This lifetime is extremely short: for a recoil emitted with $\beta \simeq 0.002$, the distance traveled in this time is only about 6×10^{-3} μm , corresponding to a negligible energy loss in the target. Therefore, the recoil decays while still in motion, and the energy of the emitted γ ray is Doppler-shifted. For this reason, a Doppler correction must be applied to the γ -ray energy measured by AGATA.

It is important to underline here that the physical intrinsic width of the state is just used to estimate τ , but is not implemented within the Geant4 simulation; so, for what concerns the γ -ray energy spectrum, the width of the peak that we are going to measure is determined exclusively by the resolution of our instruments.

3.1.5 Kinematic reconstruction and Doppler correction

In our experiment, SAURON is needed in order to perform kinematic reconstruction, i.e. to determine all the kinematical variables of the particles involved in the reaction. The kinematical variables of the projectile are known with a certain precision, as they are controlled by the accelerating device; similarly, the parameters of the target nuclei can be considered perfectly known since, neglecting thermal motion, they are essentially at rest within the target. In contrast, the ejectile parameters must be measured: this is done by SAURON, which measures its kinetic energy and position.

Knowing the ejectile mass and assuming that it is not emitted in an excited state, these measurements allow us to reconstruct its 4-momentum. By combining this information with that of the projectile and the target, and applying 4-momentum conservation, we can determine the 4-momentum of the recoil, and thus its velocity β . The uncertainty in the final result is therefore driven by the uncertainty in the ejectile parameters, i.e. by the resolution of SAURON, both in energy and position.

The knowledge of the recoil velocity is essential for performing Doppler correction. The formula for Doppler correction, for an arbitrary angle θ between the direction of motion of the emitter and the direction of the γ ray, is the following:

$$E_{\gamma}^{cm} = E_{\gamma} \frac{1 - \beta \cos \theta}{\sqrt{1 - \beta^2}}, \quad (3.1)$$

where E_γ^{cm} is the Doppler-corrected photon energy, measured in the center-of-mass frame, E_γ is the photon energy in the laboratory (in other words, measured by AGATA), and β is the velocity of the emitter.

In nuclear physics, Doppler effect is responsible for the so-called Doppler broadening, a phenomenon in which the energy resolution of the detected γ rays is degraded. This occurs because, in the absence of Doppler correction, the measured energy depends on the angle θ , with a stronger dependence for larger values of β . This effect in our simulation is illustrated in Figure 3.3: on the left, the same peak is shown before and after Doppler correction; on the right, the angular dependence of the γ -ray energy E_γ is displayed, which can subsequently be removed by applying Doppler correction.

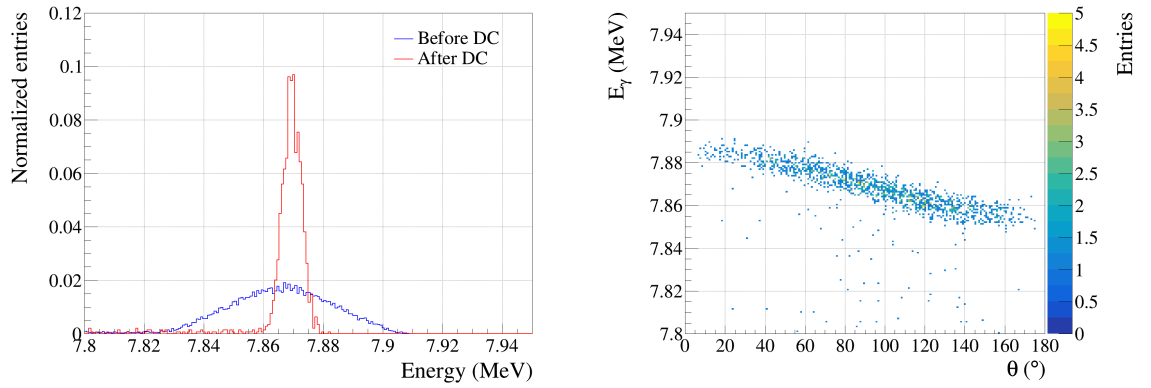


Figure 3.3: Left: comparison of 7.87 MeV peak before (blue) and after (red) Doppler correction. Data are obtained with realistic AGATA and SAURON resolutions and a zero natural width. FWHM is reduced from 40 keV to 7 keV thanks to Doppler correction. Right: angular dependence of E_γ before Doppler correction. Notice the cosinusoidal behaviour, predicted by special relativity formulas.

3.2 Analysis of the experimental resolution

3.2.1 SAURON and AGATA resolution

The position and energy resolutions of SAURON and AGATA are introduced at a later stage, specifically in the ReadSimu.C macro. They are implemented by applying a smearing to the simulated values, i.e. by modifying them according to a statistical distribution whose parameters depend on the actual detector resolutions. In the following, we describe in more detail how the smearing is applied to each relevant variable.

3.2. Analysis of the experimental resolution

SAURON

Regarding SAURON energy resolution, we smear measured values using a Gaussian function with $\text{FWHM} = 55 \text{ keV}$. If two or more values are measured, when the particle interacts in the interstrip region, energy smearing is applied to all before summing them together to recover a single energy value.

Concerning SAURON position resolution, we remind that the detector does not directly track the particle position, but only provides information on which pad and strip have recorded the signal. Therefore, the precision of the position estimate is given by the size of the pad–strip overlap region, whose area increases with the distance from the detector center. This region has the shape of an annular sector, with radial extent given by the strip height of 1.5 mm and angular aperture given by the pad opening angle of 22.50° .

The position smearing is implemented as follows. First, the exact interaction points are extracted. If multiple interactions associated with the same ejectile are recorded, their positions are averaged. The radius R and angle ϕ of the resulting position are then determined, i.e. the position on SAURON in cylindrical coordinates, neglecting the z coordinate. A uniform smearing is applied to both variables, yielding the final position stored in the ROOT tree. The width of the two smearing distributions are $\Delta R = 0.75 \text{ mm}$ (half the strip thickness) and $\Delta \phi = 11.25^\circ$ (half the angular aperture of each pad).

AGATA

For AGATA energy resolution, we use data from Ref. [41], where it is shown that for high-energy photons AGATA percentage resolution scales approximately as $\sim E^{-1/2}$ (see Figure 3.4). In the simulation, we implement this behaviour using a function $\Gamma = \sqrt{a + bE}$. With this approach, we obtain a FWHM at 7.87 MeV of 6.6 keV , corresponding to a percentage resolution of 0.083% . It is important to note that the smearing is applied to E_γ , i.e. to the laboratory-frame energy, which is Doppler-shifted.

As already mentioned in section 3.1, AGATA spatial resolution is directly implemented within the `agata` script and is therefore not accessible from `ReadSimu.C`. Its effect is expected to be negligibly small compared to the other sources of uncertainty discussed above.

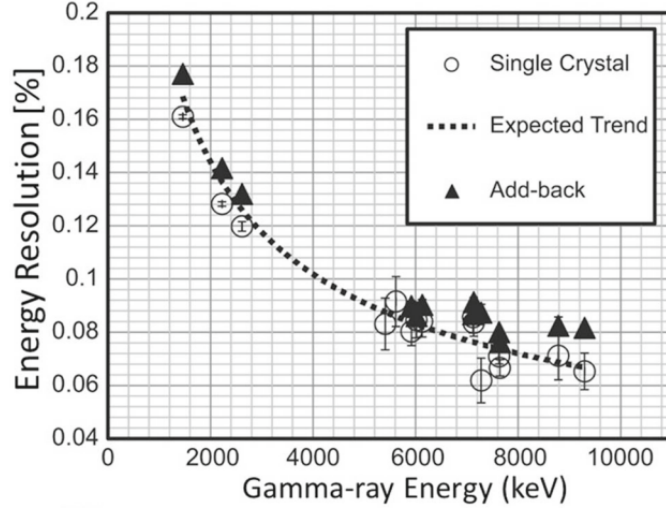


Figure 3.4: Experimental AGATA energy resolution up to 9 MeV. Empty black circles represent data concerning single crystals, while black triangles represent data for the add-back procedure; the dashed black line indicates the expected $E^{-1/2}$ trend. Taken from Ref. [41].

3.2.2 Doppler graphs

Doppler correction in AGATA can be performed thanks to its PSA and γ -tracking capabilities. Since the final peak that we observe results from this Doppler correction, the uncertainty on its energy is essentially given by the propagation of the uncertainties on all the parameters appearing in Equation 3.1. This implies that the final uncertainty is given by

$$\sigma_{E_\gamma^{cm}} = \sqrt{\left(\frac{\partial E_\gamma^{cm}}{\partial E_\gamma}\right)^2 \sigma_{E_\gamma}^2 + \left(\frac{\partial E_\gamma^{cm}}{\partial \theta}\right)^2 \sigma_\theta^2 + \left(\frac{\partial E_\gamma^{cm}}{\partial \beta}\right)^2 \sigma_\beta^2}, \quad (3.2)$$

where σ_{E_γ} , σ_θ and σ_β are the statistical uncertainties in E_γ , θ and β respectively. In this formula the different broadening sources are considered as statistically independent, neglecting for simplicity any possible correlation between them. The partial derivatives are:

$$\begin{aligned} \frac{\partial E_\gamma^{cm}}{\partial E_\gamma} &= \frac{1 - \beta \cos \theta}{\sqrt{1 - \beta^2}}, \\ \frac{\partial E_\gamma^{cm}}{\partial \theta} &= E_\gamma \frac{\beta \sin \theta}{\sqrt{1 - \beta^2}}, \\ \frac{\partial E_\gamma^{cm}}{\partial \beta} &= E_\gamma \frac{\beta - \cos \theta}{(1 - \beta^2)^{3/2}}. \end{aligned} \quad (3.3)$$

3.2. Analysis of the experimental resolution

It is clear that the FWHM value can be extracted from Equation 3.2 using the relation $\text{FWHM} = 2\sqrt{2\ln 2} \sigma$, valid for Gaussian-like distributions. In the following, however, when we will talk about the width of the γ peak, we will prefer to use σ instead of FWHM.

To align with the discussion in Refs. [41, 62], we refer to the three contributions to the final resolution in Equation 3.2 as the *intrinsic*, *opening*, and *recoil* contributions, respectively. An example of these contributions as a function of θ is shown in Figure 3.5, where the total final resolution is also displayed. In this example, it can be seen that all three sources provide a comparable contribution to the final resolution.

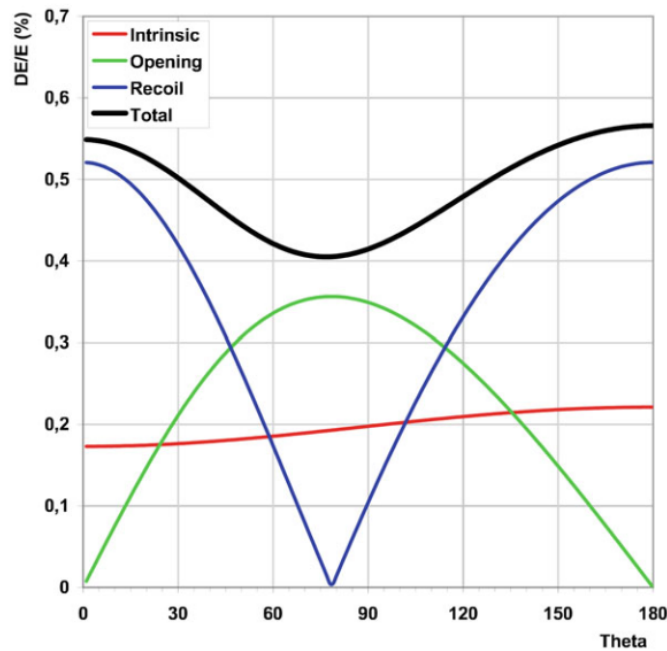


Figure 3.5: Examples of contributions of the different Doppler broadening sources to the final E_γ^{cm} , as a function of the relative angle between recoil velocity and photon direction. The intrinsic contribution is obtained using a photon energy of 1 MeV and the typical energy resolution of a germanium detector; the recoil contribution is obtained by assuming a recoil velocity $\beta = 20.0\%$ and an uncertainty $\Delta\beta = 0.5\%$; finally, the opening contribution comes from an uncertainty of $\Delta\theta = 1^\circ$ in the recoil direction. Taken from Ref. [62].

Our goal is to produce a graph similar to Figure 3.5 in order to study the hierarchy among these contributions in our simulation. Thus, let us briefly analyze the origin of the terms appearing in Equation 3.2 within the context of our experiment:

1. σ_{E_γ} (the intrinsic contribution) arises exclusively from AGATA energy resolution, which is implemented as a smearing in the ReadSimu.C macro.

2. σ_θ (the opening contribution) is mainly determined by the spatial resolution of both SAURON and AGATA. Since θ is the relative angle between the recoil velocity vector and the photon direction, its uncertainty depends on the precision with which both quantities are measured. In particular, the recoil direction is inferred from the ejectile direction, which is measured by SAURON, so this contribution depends mainly on the SAURON spatial resolution. Similarly, the photon direction is determined from the first interaction point in AGATA, and thus also depends on AGATA spatial resolution. Anyway, due to AGATA greater precision, we can assume the SAURON contribution to be the most relevant one.
3. Finally, σ_β (the recoil contribution) is mainly determined by SAURON energy resolution, since the recoil energy, and therefore its velocity, is obtained through kinematical reconstruction from the ejectile 4-momentum.

To be precise, since the recoil 4-momentum is derived through kinematical reconstruction from the ejectile parameters, we expect a cross-dependence in energy and position. In other words, the recoil angle of emission (eventually σ_θ) might depend also on the energy of the ejectile, and so on SAURON energy resolution; similarly, the recoil velocity (eventually σ_β) might depend also on the ejectile angle, and so on SAURON position resolution. However, in this experiment it can be verified that these cross-dependences are actually negligible, so that we can relate σ_θ just to SAURON (and AGATA) position resolution, and σ_β just to SAURON energy resolution.

Thus, our study is carried out by analyzing the same simulation dataset (provided by `tracking_simulated`) multiple times, each time using different input parameters in the `ReadSimu.C` macro. In particular, we introduce the various resolution effects one at a time, setting all other contributions to zero, and include all of them simultaneously only in the final case. More specifically, in the first case we keep a realistic value only for AGATA energy resolution; in the second case, only for SAURON position resolution; in the third case, only for SAURON energy resolution; and finally, in the last case, we consider all contributions together. If our assumptions are correct, the first three cases should reproduce the intrinsic, the opening and the recoil contribution, respectively.

We then produce 2D histograms of E_γ^{cm} as a function of θ . We study how the vertical width of the distribution, which is related to the resolution in E_γ^{cm} , varies with θ , as in Figure 3.5. The results for each of the four resolution configurations are shown in Figure 3.6.

3.2. Analysis of the experimental resolution

Since the dataset contains a finite number of events, the distribution must be divided into angular intervals of finite width. For each interval, the corresponding events are projected onto a 1D histogram, from which the width can be evaluated. The range of the angular intervals must be chosen to ensure sufficient statistics in each of them, and therefore cannot be too small. At the same time, we aim to maximize the number of intervals in order to obtain more points in the final plots. In the end, we choose angular intervals of 20° , resulting in 9 intervals covering the range from 0° to 180° .

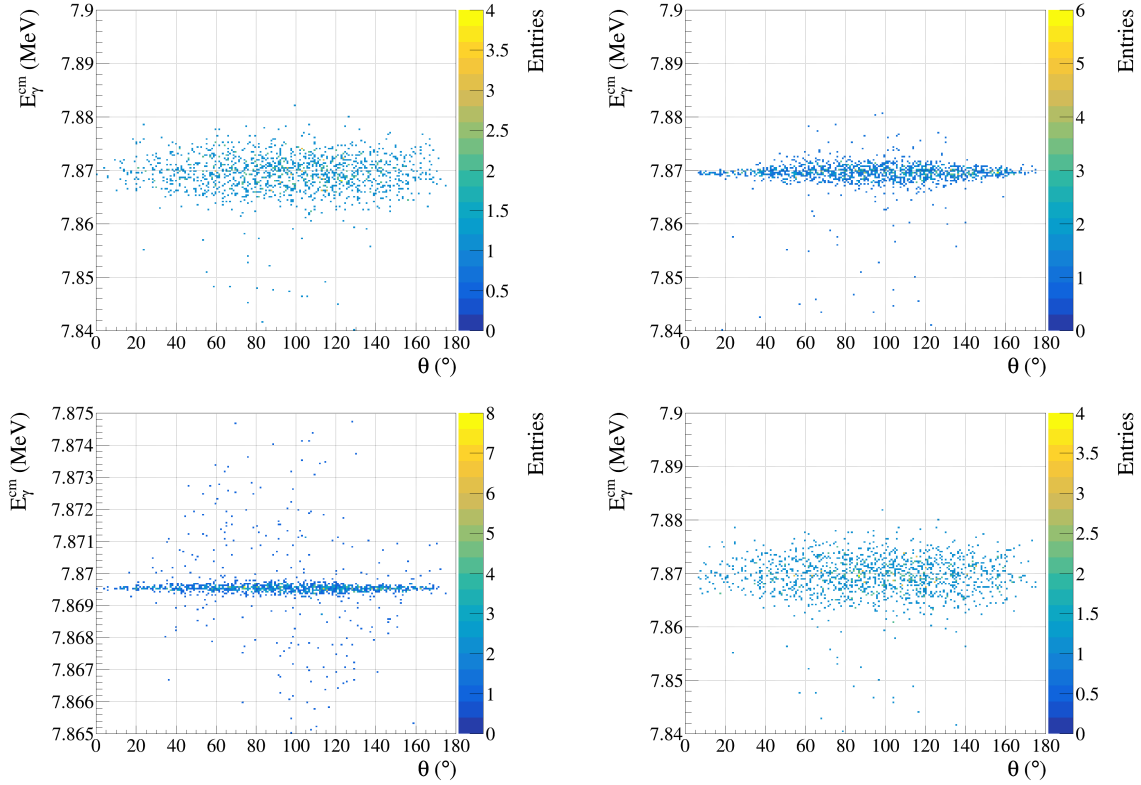


Figure 3.6: 2D histograms E_γ^{cm} vs θ to study the various contributions to E_γ^{cm} resolution: intrinsic (top-left), opening (top-right) and recoil (bottom-left). In the bottom-right panel we show the same 2D histogram with all the contributions. Notice the different energy scale used for the recoil contribution.

3.2.3 Main results

Results for the analysis outlined in the previous section can be seen in Figure 3.7. It can be immediately seen that the dominant contribution in our case is the intrinsic one: it remains almost constant over the entire angular range and is always compatible with the total resolution. Its mean value is around 3.0 keV, which corresponds to a FWHM of 7.0 keV, slightly higher than the value extracted

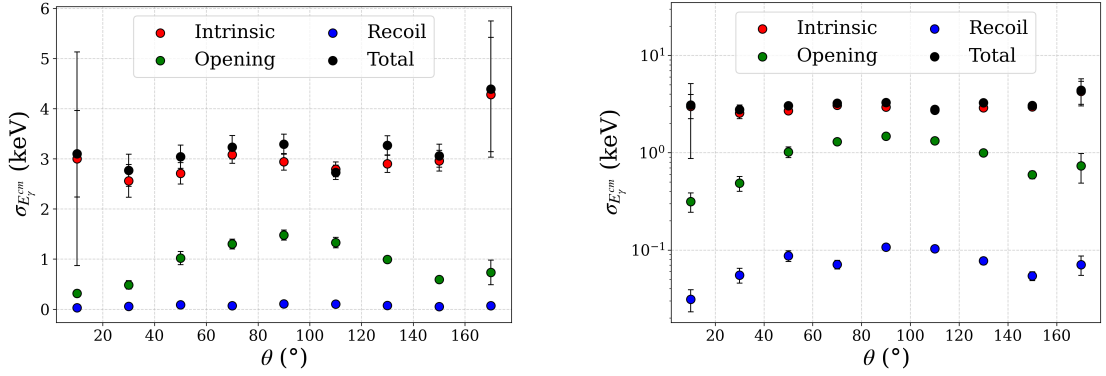


Figure 3.7: Contributions to the final E_{γ}^{cm} resolution as a function of θ in linear (left) and logarithmic scale (right). The logarithmic scale is included to show the behaviour of the small recoil contribution.

from Figure 3.4. It can also be observed that the largest uncertainties in the $\sigma_{E_{\gamma}^{cm}}$ estimates occur in the 0° – 20° and 160° – 180° ranges, which are those with the lowest statistics.

The next most relevant contribution is the opening term, which is a factor of 2 to 10 smaller than the intrinsic one. Its behaviour is similar to that shown in Figure 3.5, resembling a sine-like trend, with a maximum contribution of 1.5 keV in the central range 80° – 100° , and a minimum of 0.32 keV in the leftmost range 0° – 20° . The value in the rightmost range, 160° – 180° , appears to deviate from the expected trend, possibly due to its relatively large uncertainty.

The smallest contribution is the recoil term, which is almost two orders of magnitude lower than the intrinsic one, with values in the range 0.031–0.11 keV. Interestingly, this contribution does not follow the expected behaviour shown in Figure 3.5, but instead shows a sinusoidal trend. This can be observed in the logarithmic plot shown on the right of Figure 3.7.

The key result of this analysis is that the precision in the determination of $\Gamma(T)$ is ultimately limited by the intrinsic energy resolution of AGATA. This conclusion holds at the beam energy used in this experiment (25 MeV) and is expected to remain valid also at 40 MeV, where the reaction cross section reaches its maximum [63]. This can be understood by noting that the ion velocity β , which governs the coefficient of the next most relevant contribution to the resolution, i.e. the opening term, increases sublinearly with beam energy. As a consequence, assuming an increase of less than 60 % in the opening contribution and looking again at Figure 3.7, we can see that the AGATA intrinsic resolution still remains the dominant term.

3.2. Analysis of the experimental resolution

A similar conclusion applies to heavier nuclei, for which the ion velocity is lower even at higher beam energies. In this case as well, the relative importance of the opening contribution does not become sufficient to overcome the dominant effect of the AGATA energy resolution.

The fact that the precision on $\Gamma(T)$ is constrained by AGATA means that no significant contributions will be given by σ_β and σ_θ , and so by SAURON. Therefore, to further investigate how we can determine the natural width of the state, we do not need a simulation that covers all the apparatus: it is sufficient to concentrate on AGATA. Experimentally, this means that we do not need to invest into a higher-resolution ancillary detector, as better performances would not produce appreciable improvements. This is already a key result of our analysis.

3.2.4 Other observations

Even if our main concern is related to the absolute scales of the various contributions, still there are some observations, regarding their precise behaviour, that deserve to be pointed out.

Fitting the results

In subsection 3.2.2 we have made the reasonable assumption that AGATA resolution just affects the intrinsic contribution, SAURON position resolution just the opening one and SAURON energy resolution just the recoil one. If we further assume that σ_{E_γ} , σ_θ , σ_β and also β are constant with respect to θ , points in Figure 3.7 should follow the trend given by Equation 3.3. Thus we can perform some fits with the appropriate functions. In particular, points from the intrinsic data series are fitted using $f(x) = a(1 - b \cos x)$; the opening series is instead fitted using $f(x) = c \sin x$; finally, the recoil series, since it is clearly not following the expected trend, has been fitted using another sinusoidal function, $f(x) = d \sin x$.

Results, together with residue graphs, are illustrated in Figure 3.8. We see that the points follow the fitting functions pretty well. Residue graphs seem to report a small correlation on the deviations, with points that are following an oscillating trend with respect to the fitting function. Still, errorbars intersect the fitting line in most of the cases. The cosinusoidal trend in the intrinsic contribution is only slightly observable, but this is probably connected to the low value of β , which is actually the parameter that controls the cosinusoidal behaviour; on the other hand, the sine function in the opening and the recoil contributions is followed more clearly.

Parameters from the fit, as defined above, are:

$$\begin{aligned} a &= (2.88 \pm 0.07) \text{ keV} , \\ b &= 0.03 \pm 0.05 , \\ c &= (1.33 \pm 0.04) \text{ keV and} \\ d &= (1.01 \pm 0.02) \times 10^{-1} \text{ keV} . \end{aligned}$$

These values can be compared with Equation 3.3, to check the values of some quantities that are appearing there. For example, since in our experiments $\beta \ll 1$, then $\sqrt{1 - \beta^2} \simeq 1$ and a in the intrinsic fitting function should correspond to σ_{E_γ} . This is indeed the case, since 2.88 keV corresponds to a FWHM of 6.77 keV, consistently with what we have seen in subsection 3.2.1. Similarly, b should correspond to β . Anyway we see that b is greater than the expected value $\sim 1.5 \times 10^{-3}$ (see subsection 3.1.3) by one order of magnitude; still the relative uncertainty in b is huge, more than 100 %, so this estimation is probably not reliable. Concerning c , we can invert Equation 3.3 and use the correct value of β to estimate σ_θ , which cannot be deduced easily from known quantities: we get $(0.106 \pm 0.003) \text{ rad}$, that corresponds to $(6.1 \pm 0.1)^\circ$. Parameter d will be treated later on.

The assumption that β is constant is a small approximation: we have seen that its value is in the $1.4 \times 10^{-3} - 1.8 \times 10^{-3}$ range, which corresponds to a variability of 25 %. It follows that above calculations can be used just to make order-of-magnitude estimations.

Similarly, assuming that the σ values in Equation 3.2 are constant is not exactly correct. This should be true, at least in principle, for the intrinsic contribution, since AGATA resolution, which does not depend on the relative angle, affects directly σ_{E_γ} ; anyway, this is not the case for the angular and the recoil contributions, because σ_θ and σ_β depend in a more complicated way on SAURON resolution. These uncertainties might depend on the relative angle because of the kinematic reconstruction that we are performing.

The true nature of the recoil contribution

The deviation of the recoil contribution from expectations is nevertheless too sharp to be explained by simply the variability of σ_β . We have to assume that the real recoil contribution is somehow hidden behind some other effects that we have not appropriately considered, so that what we are actually seeing is the result of those effects.

3.2. Analysis of the experimental resolution

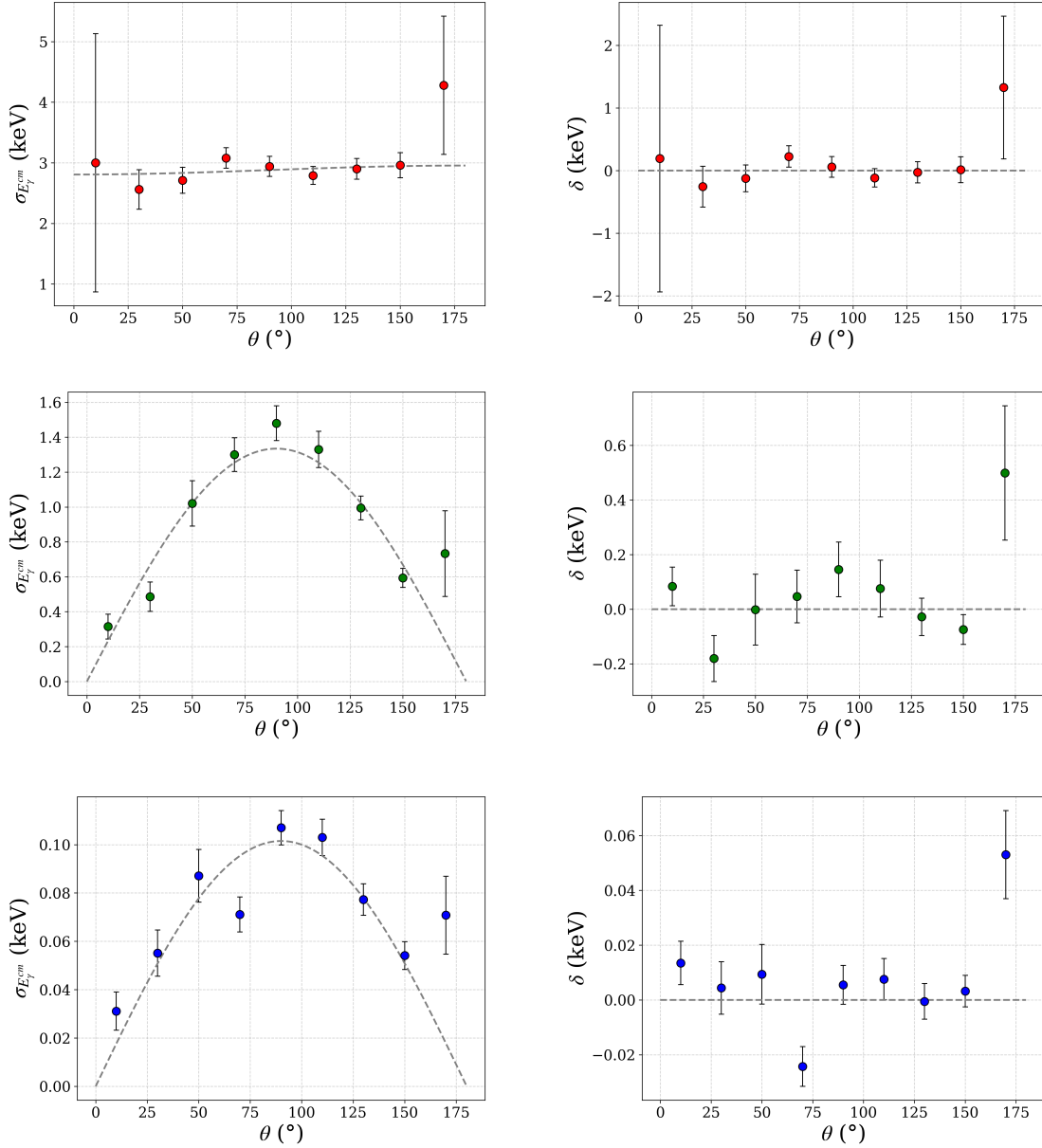


Figure 3.8: Resulting functions obtained by fitting results in Figure 3.7 (left) and corresponding residue graphs (right). From top to down: intrinsic, opening and recoil contribution.

First of all, we prove that what we are seeing is indeed not coming from the resolution of β . To do that, we set to 0 MeV SAURON energy resolution in the ReadSimu.C macro and look for differences in the Doppler graph. After this change, we obtain a result that is very similar to the previous one, both in the behaviour of the function and in its absolute scale, as can be seen in Figure 3.9, where we superimpose the old series with the new one (blue and purple series, respectively).

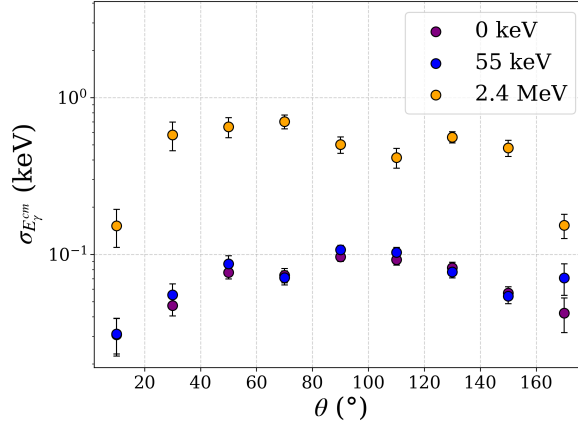


Figure 3.9: So-called recoil contributions with 0 (purple series) and 2.4 MeV (orange series) SAURON energy resolution (FWHM). They are superimposed to the recoil contribution with realistic SAURON resolution (blue series), the same appearing in Figure 3.7.

What we are seeing in Figure 3.7 is therefore not the real recoil contribution. There is in fact a contribution on the simulation side that has not been set to zero, namely AGATA spatial resolution, which was originally neglected because it was assumed to be smaller than all other contributions. However, if this contribution, although small, was larger than the recoil contribution, we would expect it to affect the energy resolution in a way very similar to the opening contribution, i.e. with the same sinusoidal trend but reduced amplitude. This is because the spatial uncertainty in AGATA contributes to the determination of θ , effectively introducing an additional opening-like contribution.

As we have seen in chapter 2, AGATA PSA has to reconstruct interaction vertices with a precision below 5 mm (FWHM); if we consider that AGATA is located at a distance of about 20 cm from the centre of the reaction chamber, then it means that the angular uncertainty of AGATA has $\text{FWHM} = 1.3 \times 10^{-2}$ rad, thus $\sigma = 5.3 \times 10^{-3}$ rad. We can check whether this is consistent with what we actually obtain: if we extract from d the value of σ_θ , as done for c , we get $(7.8 \pm 0.2) \times 10^{-3}$ rad. The order of magnitude is therefore compatible, and we can thus conclude that what we had previously identified as a recoil contribution is in fact another opening contribution.

The importance of SAURON

We thus increase to an unrealistic value SAURON energy resolution ($\sigma = 1.0$ MeV, corresponding to a $\text{FWHM} \simeq 2.4$ MeV), to seek for the real recoil contribution.

3.3. The Monte Carlo simulation for AGATA

The result that we get after this change is depicted in orange in Figure 3.9. We see that both the absolute scale and the behaviour have changed: the scale has increased of about one order of magnitude, and the points around 100° show a decrease that reminds something of a cosinusoidal function, as the one expected from Figure 3.5. Still we see that the contribution is slightly smaller than the opening contribution, and on average one order of magnitude less than AGATA intrinsic contribution.

This fact outlines a crucial property of our experiment: large variations in the ejectile energy have only a minor impact on the recoil velocity, which is the parameter that determines the recoil contribution in the Doppler correction. This is a consequence of the masses involved, since in the present simulation ^{71}Ge is much heavier than the deuteron. This explains why, even with a bad measurement of deuteron energy, the recoil contribution never clearly emerges.

3.3 The Monte Carlo simulation for AGATA

3.3.1 Description of the new simulation

We have seen that ion velocity β and relative angle θ are important to perform Doppler correction, but their contribution to the final γ peak resolution is sub-leading with respect to the intrinsic AGATA resolution. To see if we are able to measure the IAS total width $\Gamma(T)$ and the number N of emitted γ s we just have to focus on AGATA, and thus we do not need a complicated Geant4 simulation of all the apparatus. We therefore design a much simpler simulation, implemented as a ROOT macro, which we refer to as a *Monte Carlo simulation*, and which can be interpreted as a statistical pseudo-experiment.

In this simulation, we take into account both the intrinsic detector resolution, which is expected to have a Gaussian-like shape, and the natural width of the state, which follows a Breit–Wigner (often referred to as “Lorentzian”) distribution. Since both effects contribute, the shape of the generated peak is given by the convolution of these two functions. The convolution of a Gaussian and a Breit–Wigner is commonly known as a Voigt profile [64]. A Voigt profile is characterized by four parameters: the scaling factor A , the Gaussian width σ , the Breit–Wigner FWHM Γ , and the centroid μ of the distribution, as can be seen in its definition:

$$V(x; A, \sigma, \Gamma, \mu) = A \int_{-\infty}^{+\infty} \frac{e^{-\frac{(t-\mu)^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma} \frac{\Gamma}{\pi [\Gamma^2 + (x-t)^2]} dt .$$

In the simulation, we first set up the parameters of the Voigt distribution; then, we generate the random events and we fit the resulting energy distribution to estimate the relevant parameters. In the following we outline the details.

Choosing the values for Γ and N

First of all, we define the parameters to be simulated. The value $\sigma = 3 \text{ keV}$ is derived from AGATA energy resolution (corresponding to a FWHM of 7 keV , as above), while $\mu = 7.87 \text{ MeV}$ represents the energy of the detected photon (this parameter only introduces a shift in the profile, and is therefore not relevant for the purposes of our analysis). The parameter Γ is varied within the range $1\text{--}100 \text{ keV}$, which appears to be a reasonable interval for the natural width of the IAS we aim to measure, as suggested by Figure 1.7.

Once a value of Γ is selected, events are generated using these parameters. Recalling Equation 1.5 for the notation, if we assume that the γ -decay width $\Gamma_{\text{IAS}}(\alpha')$ remains constant, then the number N of γ decays is inversely proportional to $\Gamma(T)$, which we are denoting here as Γ . This follows from the fact that N is proportional to the γ BR, given by $\frac{\Gamma_{\text{IAS}}(\alpha')}{\Gamma(T)}$. Therefore, for example, doubling Γ results in halving N .

We still need to account for the statistical fluctuations of N , which are not automatically included in the Voigt profile. The number of decay events observed in a given time interval follows a Poisson distribution. If N is the expected number of events, any measurement is affected by an uncertainty of $\sqrt{N}$, i.e. the Poissonian uncertainty. Therefore, once a value of N is chosen (for instance, corresponding to the number of events expected for a given beam intensity and time) the actual number of events generated in the simulation is drawn from a Poisson distribution with mean N and standard deviation $\sqrt{N}$.

Fitting the distribution

Once the events have been generated and collected in a histogram, we can perform a fit using a Voigt profile, as shown in Figure 3.10. In the fitting function, we fix the parameters σ and μ , and determine A and Γ . Since the width of the distribution depends on both σ and Γ , the former parameter plays a crucial role in the correct determination of Γ . By fixing σ , we are assuming that, in a real experimental situation, the resolution of the photon detector is known with very high precision. This is typically the case, as the detector resolution at a given energy can be accurately measured through preliminary calibrations using peaks close in energy to the one of interest.

3.3. The Monte Carlo simulation for AGATA

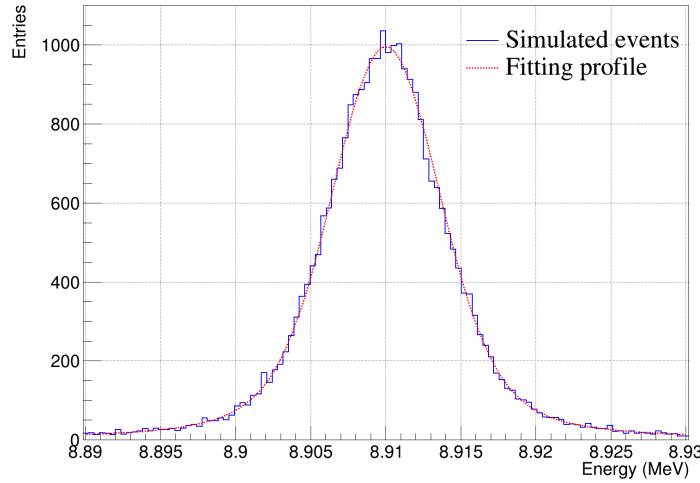


Figure 3.10: Example of a fit with a Voigt profile, with $\Gamma = 3$ keV and $N = 30000$.

The best estimators for A and Γ , namely $\hat{A}$ and $\hat{\Gamma}$, together with their uncertainties, are obtained from the fit results. Concerning $\hat{A}$, a further clarification is needed, since the quantity of interest is actually the effective number of events N . This can be estimated through the relation $\hat{N} = \hat{A}/\Delta x$, where Δx is the bin width of the histogram. This relation holds exactly only if the extremes of the distribution extend to infinity; however, in a simulation finite limits must be imposed for computational reasons. In this case, the number of events is slightly overestimated, although this effect becomes negligible as the limits are extended further away. In our simulation, the overestimation never exceeds 0.02 % of N , which is far smaller than the Poisson uncertainty and is therefore not relevant.

Generating bigger ensembles

Every distribution that we generate is different; similarly, every fit yields different results. This variability is not unexpected, but purely statistical. Nevertheless, if we want to study our results in a robust way, without being biased by the uniqueness of a single distribution and its corresponding fit, it is preferable to consider many distributions and fits collectively.

For this reason, once the true values of Γ and N are fixed, we generate 300 distributions and perform the corresponding fits, thus obtaining a dataset of 300 estimates $\hat{\Gamma}$ and $\hat{N}$. The choice of 300 is a reasonable compromise: it is small enough to keep computational times manageable, while still being large enough to be statistically significant. For convenience, we will refer to these datasets, each

consisting of 300 estimates and generated with the same input values of Γ and N , as *ensembles*.

3.3.2 Preliminary analysis

Before performing a complete study on how fits can produce accurate results, we perform a preliminary analysis on the best fitting algorithm. ROOT allows to perform fits using two main different algorithms, Least Squares (χ^2) and Maximum Likelihood (ML) [65], which usually give different results. It is thus convenient to understand which of the two is more appropriate to our situation, i.e. gives the greatest accordance between results and input parameters.

Once we have selected the values of Γ that we want to measure, and also a corresponding number N of generated events, following the logic outlined in the previous section, we study the quality of the average fit results in each ensemble by using χ^2 and ML. We study this quality by looking at three relevant parameters: the average compatibility $\langle\lambda_{\hat{\Gamma}}\rangle$ between the input Γ and the estimated one, the average uncertainty $\langle\sigma_{\hat{\Gamma}}\rangle$ and the (absolute) average residue $|\langle\delta_{\hat{\Gamma}}\rangle|$. Notice that the $\langle\cdots\rangle$ notation means average on the ensemble.

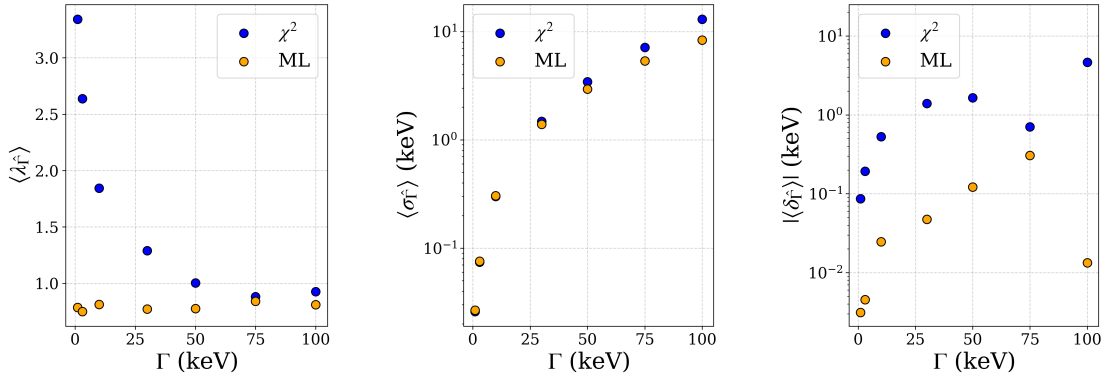


Figure 3.11: Average compatibility $\langle\lambda_{\hat{\Gamma}}\rangle$ (left), uncertainty $\langle\sigma_{\hat{\Gamma}}\rangle$ (center) and residue $|\langle\delta_{\hat{\Gamma}}\rangle|$ (right) as a function Γ obtained using χ^2 and ML algorithms. Γ is taking the values 1, 3, 10, 30, 50, 75 and 100 keV.

Results for such analysis can be seen in Figure 3.11. We see that ML gives better results. λ -compatibility is around 0.8 for all Γ s, while for χ^2 λ increases with decreasing Γ , getting to the average value of 3.34 for $\Gamma = 1$ keV. The better compatibility for ML is due to a real better agreement with the true value, and not by greater uncertainties, since actually $\langle\sigma_{\hat{\Gamma}}\rangle$ is always smaller for ML than for χ^2 , as can be seen in the central panel of Figure 3.11. A similar information is shown in the right panel, where we can see that the absolute residue from the real value

3.3. The Monte Carlo simulation for AGATA

is always smaller, sometimes even of more than one order of magnitude, for ML than for χ^2 .

In Appendix A, we report another study that shows the superiority of the ML in our situation. More precisely, we look at the pull distributions [66] for both fitting methods, to check for the presence of systematics in our estimated parameters. This additional study shows that results obtained with the ML method follow a good Gaussian distribution around the true value, as we expect if the procedure is perfectly unbiased; results obtained with the χ^2 method, on the other hand, fail to reproduce such a behaviour.

For all these reasons, in the analysis that follows we use the ML method.

3.3.3 Feasibility study and results

We perform our feasibility study for $\Gamma = 1, 3, 10, 30, 50, 75$ and 100 keV, as done in subsection 3.3.2. For each value of Γ we acquire a high-statistics ensemble and a low-statistics one, that means one with a big input value of N and another with a smaller value. The number of events is varied inversely proportionally with the value of Γ , as explained in subsection 3.3.1. In Table 3.1 we illustrate more precisely the value of N chosen for each Γ in high and low statistics. In addition, for $\Gamma = 3, 10, 50, 75$ and 100 keV, we try to explore additional values of N , in order to see how our results vary as a function of the number of the simulated events.

Γ (keV)	High statistics	Low statistics
1	30000	3000
3	10000	1000
10	3000	300
30	1000	100
50	600	60
75	400	40
100	300	30

Table 3.1: Number of events N generated in high- and low-statistics ensembles.

For each ensemble, we calculate the average relative uncertainties $\langle \varepsilon_{\hat{\Gamma}} \rangle$ and $\langle \varepsilon_{\hat{N}} \rangle$, which we use to evaluate the quality of our estimations. We also calculate the same parameters introduced in subsection 3.3.2, i.e. $\langle \lambda_{\hat{\Gamma}} \rangle$, $\langle \sigma_{\hat{\Gamma}} \rangle$ and $\langle \delta_{\hat{\Gamma}} \rangle$, and the equivalent values for N , so $\langle \lambda_{\hat{N}} \rangle$, $\langle \sigma_{\hat{N}} \rangle$ and $\langle \delta_{\hat{N}} \rangle$. We also calculate the pull distributions for both Γ and N (pull data can be found in Appendix A). In the following we focus on $\langle \varepsilon_{\hat{\Gamma}} \rangle$ and $\langle \varepsilon_{\hat{N}} \rangle$.

Average relative uncertainty in high and low statistics

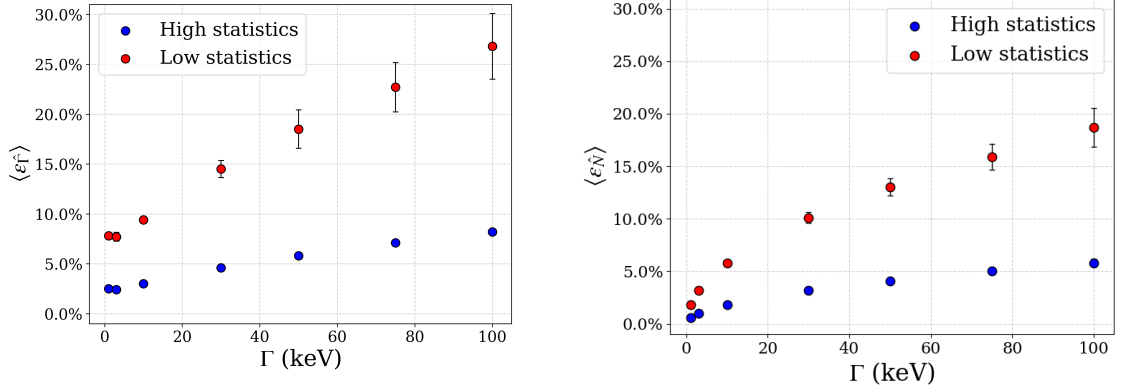


Figure 3.12: Average relative uncertainties $\langle \varepsilon_{\hat{\Gamma}} \rangle$ (left) and $\langle \varepsilon_{\hat{N}} \rangle$ (right) for high and low statistics.

In the two graphs in Figure 3.12 we can see a comparison of the relative uncertainties in the high and low statistics ensembles, concerning estimations $\hat{\Gamma}$ and $\hat{N}$. Data are plot as a function of Γ , so we can also see what happens for low and high IAS widths. Errorbars denote standard deviations of $\varepsilon_{\hat{\Gamma}}$ and $\varepsilon_{\hat{N}}$ in the ensemble.

As expected, we see that with a higher statistics results are always better, i.e. the average relative error is always smaller, both for $\hat{\Gamma}$ and $\hat{N}$. For example, the maximum value of $\langle \varepsilon_{\hat{\Gamma}} \rangle$ for the high statistics ensembles is $(8.2 \pm 0.3) \%$, while for the low statistics ones is around $(27 \pm 3) \%$. As absolute scale, we see that relative errors tend to be bigger for $\hat{\Gamma}$ than for $\hat{N}$, both in high and low statistics. If we compare low statistics ensembles, we see that the worst value for $\langle \varepsilon_{\hat{N}} \rangle$ is $(19 \pm 2) \%$, which is better than the $(27 \pm 3) \%$ for $\langle \varepsilon_{\hat{\Gamma}} \rangle$ that we have just seen.

All relative uncertainties tend to increase as a function of Γ : as we will see, this must be considered as an effect of the scaling of the number of events, which is reducing as Γ is increasing.

Average relative uncertainty as a function of N

We plot in Figure 3.13 $\langle \varepsilon_{\hat{\Gamma}} \rangle$ and $\langle \varepsilon_{\hat{N}} \rangle$ as a function of N . Each graph has a fixed value of Γ , so we can analyze just the dependence on N . We see now clearly that relative uncertainties decrease as we increase the number of detected events. This decrease is sharper for $\langle \varepsilon_{\hat{\Gamma}} \rangle$ than for $\langle \varepsilon_{\hat{N}} \rangle$. Notice also that $\langle \varepsilon_{\hat{N}} \rangle$ is always given by $1/\sqrt{\langle \hat{N} \rangle} \simeq 1/\sqrt{N}$, that is the Poissonian statistical uncertainty, as expected.

These graphs show clearly the importance of having a great number of events in order to have precise results. We can focus for example on the behaviour of $\langle \varepsilon_{\hat{\Gamma}} \rangle$

3.3. The Monte Carlo simulation for AGATA

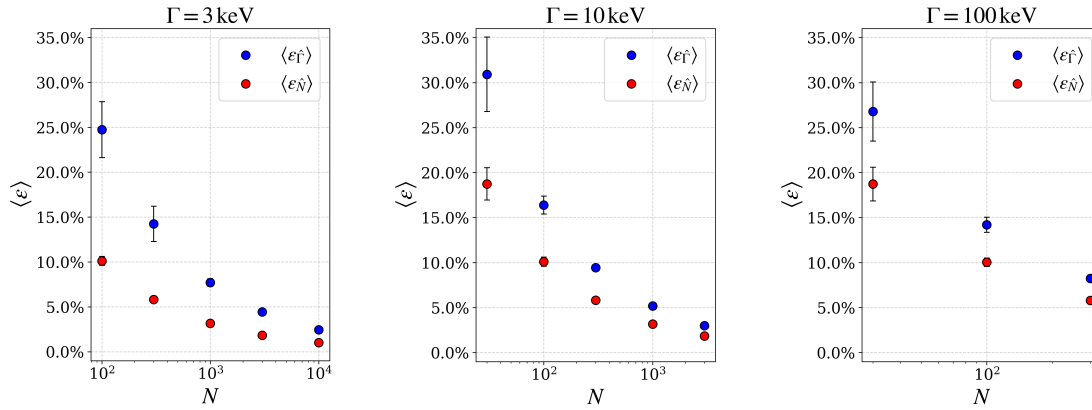


Figure 3.13: Average relative uncertainties $\langle \varepsilon_{\hat{\Gamma}} \rangle$ and $\langle \varepsilon_{\hat{N}} \rangle$ as a function of N for $\Gamma = 3, 10, 100$ keV.

for $\Gamma = 3$ keV. With a poor statistics of 100 events, we have a quite large relative uncertainty, $(25 \pm 3) \%$; on the other hand, with a bigger statistics of 10000 events, the relative uncertainty reduces to $(2.43 \pm 0.04) \%$. Actually, we see that to get a precision 10 times better, we need to increase the number of events by two orders of magnitude. A similar behaviour is followed by $\langle \varepsilon_{\hat{N}} \rangle$ and is valid also for other values of Γ .

Average relative uncertainty as a function of Γ

Finally, in Figure 3.14 we can look at the behaviour of $\langle \varepsilon_{\hat{\Gamma}} \rangle$ and $\langle \varepsilon_{\hat{N}} \rangle$ as a function of Γ and with a fixed value of N . Intuitively, once we fix N , we expect that it is easier to determine Γ with a low relative uncertainty if its value is big; on the other hand, since the uncertainty in $\hat{N}$ is purely Poissonian, we expect no dependence for it on Γ . Actually, in the graphs we see that our intuition is perfectly respected, with $\langle \varepsilon_{\hat{\Gamma}} \rangle$ decreasing as a function of Γ , and with $\langle \varepsilon_{\hat{N}} \rangle$ practically constant.

In the two bottom panels of Figure 3.14 we also see that $\langle \varepsilon_{\hat{\Gamma}} \rangle$ does not just decrease, but it seems to reach an asymptotic value as Γ increases. This limit is slightly greater than the Poissonian uncertainty in $\hat{N}$, and it is smaller for bigger N , as we can expect: for $N = 300$ the limit is around 8 %, while for $N = 100$ the limit is around 14 %.

3.3.4 Experimental perspectives

A relative uncertainty of 15 % can be regarded as a reasonable target for both $\Gamma(T)$ and N . This choice is motivated by the fact that the correlation between the GT component of the β -decay strength and the corresponding γ -decay branch

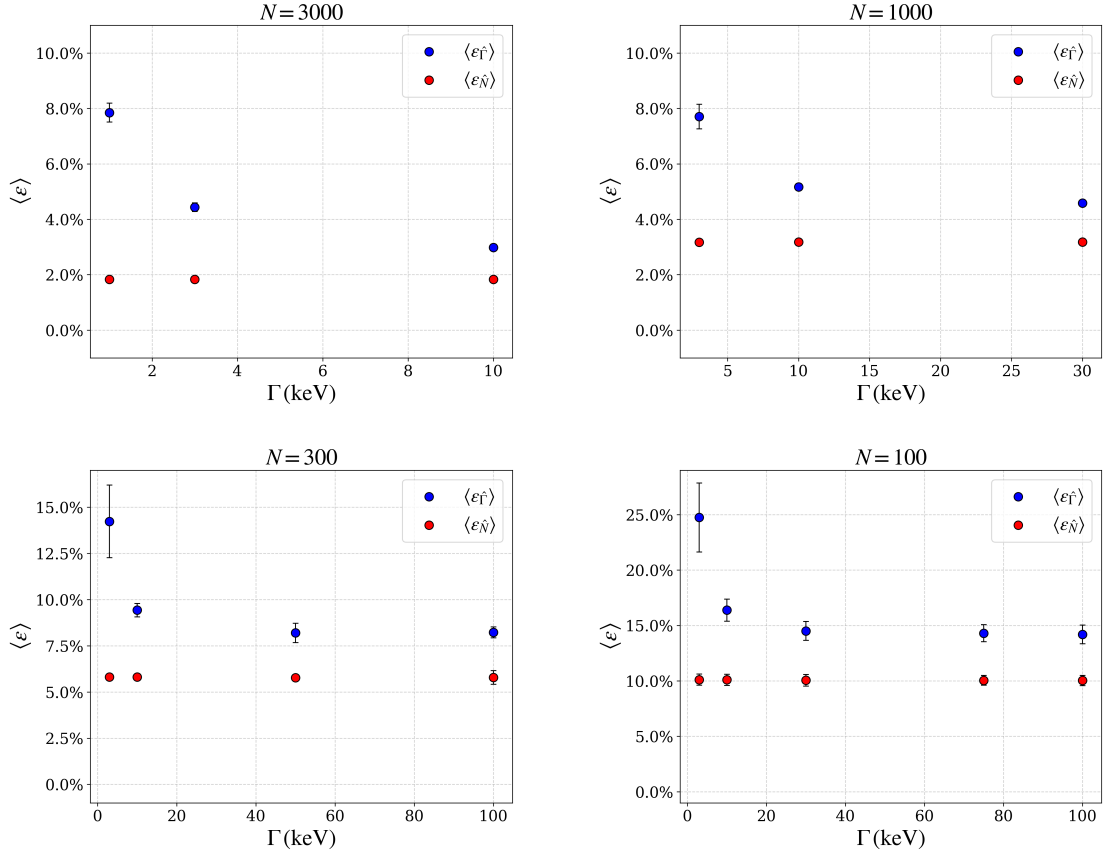


Figure 3.14: Average relative uncertainties $\langle \varepsilon_{\Gamma} \rangle$ and $\langle \varepsilon_N \rangle$ as a function of Γ for $N = 3000, 1000, 300, 100$.

can itself be determined with an accuracy of about 10–15 %, according to Ref. [39]. Therefore, there is little benefit in achieving a substantially smaller statistical uncertainty, as the overall precision would ultimately remain limited by this systematic contribution.

As shown in Figure 3.12, for $\Gamma(T) < 30$ keV, which is the region in which the total IAS width cannot be reconstructed through charged-particle measurements and must instead be inferred from the γ -ray spectrum, the relative uncertainties on both $\Gamma(T)$ and N remain below 15 %, even in the low-statistics scenario. In this regime, the required statistics are relatively modest, amounting to roughly one hundred detected events (for instance, $N = 100$ was assumed for $\Gamma(T) = 30$ keV).

It is therefore worth considering whether such statistics can be realistically achieved in an experiment. Assuming a proton beam current $I = 10$ pA, a target thickness $w = 1$ mg/cm², an AGATA efficiency of 2 %, and an integrated reaction cross section $\sigma = 1$ mb, approximately 15 days of beam time would be required

3.3. The Monte Carlo simulation for AGATA

to collect 100 γ -ray events. This duration is fully compatible with a standard nuclear-physics experimental campaign.

It should be noted that this estimate is based on a particularly unfavorable case, namely the γ -decay branch of ^{71}Ge , whose branching ratio is of the order of 10^{-5} [39]. Several other nuclei of interest for $0\nu\beta\beta$ studies exhibit significantly larger branching ratios, suggesting that comparable statistics could be accumulated in shorter measurement times.

Concerning the scenario with $\Gamma(T) > 30\text{ keV}$, the total IAS width can be measured through charged-particle measurements, so that in the end bigger statistics are easier to obtain.

4

Analysis of SAURON performances

In chapter 3, we verified that the AGATA energy resolution limits the precision of the measurement of the IAS width Γ . Nevertheless, SAURON still plays a crucial role in the experiment, as it enables both the kinematic reconstruction and the Doppler correction. Therefore, a detailed understanding of its properties is essential. In the present case, we are interested in the effects of radiation damage, a phenomenon in which the performance of a detector is degraded due to prolonged exposure to intense irradiation [67]. Our interest is motivated by the fact that the various SAURON detectors are used at LNL since 2023 and have been already used in many experiments.

In this chapter, our goal is to understand how deeply radiation damage affects SAURON performance, in particular its energy resolution. We will do so by experimentally testing the detector with a triple- α source. More precisely, we will study and characterize the structures that appear in strip-strip, pad-pad, and strip-pad correlation matrices, in order to identify anomalous structures and potential crosstalk. Then, we will investigate the possibility of restoring SAURON resolution using the position-resolved calibration technique presented in Ref. [68]. We will also examine how the energy resolution varies as a function of the applied bias voltage.

We will carry out this analysis using two different SAURON detectors: one with a thickness of 500 μm , which we will refer to as SAURON-500, and one with a thickness of 1500 μm , which we will refer to as SAURON-1500.

4.1. Experimental setup and preliminary studies

4.1 Experimental setup and preliminary studies

4.1.1 Experimental setup

To perform the experimental test we use a triple- α source of ^{239}Pu , ^{241}Am and ^{244}Cm , which emits α particles with energy of 5.244, 5.578 and 5.901 MeV, respectively [69]. The source is located at a distance of about 10 cm in front of the ohmic SAURON side (the one with the pads). Both the source and SAURON are located inside a vacuum chamber, which is connected to an Agilent TriScroll 300 pump [70] to create vacuum inside. Signal is collected from SAURON by using 5 appropriate connectors, which are responsible for 16 channels each: 4 connectors are used to collect the signal from the strips (one for each board), while the remaining one is used to collect the signal from the pads. In this way, the signal is delivered to the preamplifiers. The same connectors are responsible for delivering the voltage bias, which is used to create the depletion region in the silicon detector.

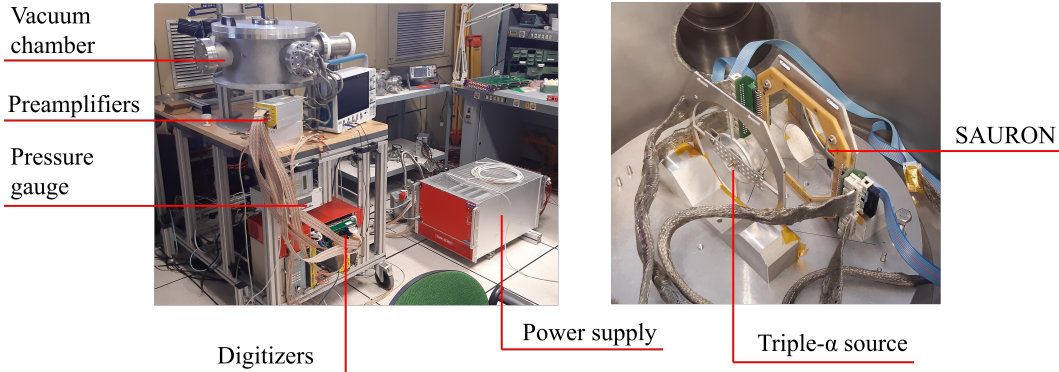


Figure 4.1: Left: picture of the experimental setup. Some of the main components are explicitly indicated. Right: detail of the interior of the vacuum chamber.

To correctly operate SAURON, we need a medium vacuum condition, with a pressure inside the vacuum chamber below 2.0×10^{-2} mbar. The scroll-chamber system is regulated by two valves: one connects the pump to the chamber, the other connects the chamber to the outside. In the first stage of vacuum creation, the pressure can be checked by using a Bourdon tube pressure gauge; when pressure gets below ~ 20 mbar, it can be checked by using a digital gauge (Pfeiffer Vacuum MaxiGauge [71]).

The bias voltage required for the operation of SAURON is provided by a CAEN Universal Multichannel Power Supply System, SY4527 [72]. The bias is applied to the detector through the preamplifiers and is generated by supplying a positive

voltage to the pad side of the detector. Preamplifiers (model MPR-64, produced by Mesytec [73]) are also responsible for initial signal amplification. We use two such preamplifiers in our experiment, one for the signal coming from the strips, and the other for the signal coming from the pads. The preamplified signal is then digitized by two Caen VX2740 digitizers [74], mounted on a Caen VME8004X mini crate [75], and sent to the DAQ system. The whole setup can be seen in Figure 4.1.

To conclude, in Figure 4.2, the SAURON lookup table, showing the numbering of the various segments and their positions on the detector, is presented. Strips are numbered from 0 to 63, while pads are numbered from 0 to 15. Boards are labeled with letters from A to D. Both panels are intended to represent the detector as viewed from the same side.

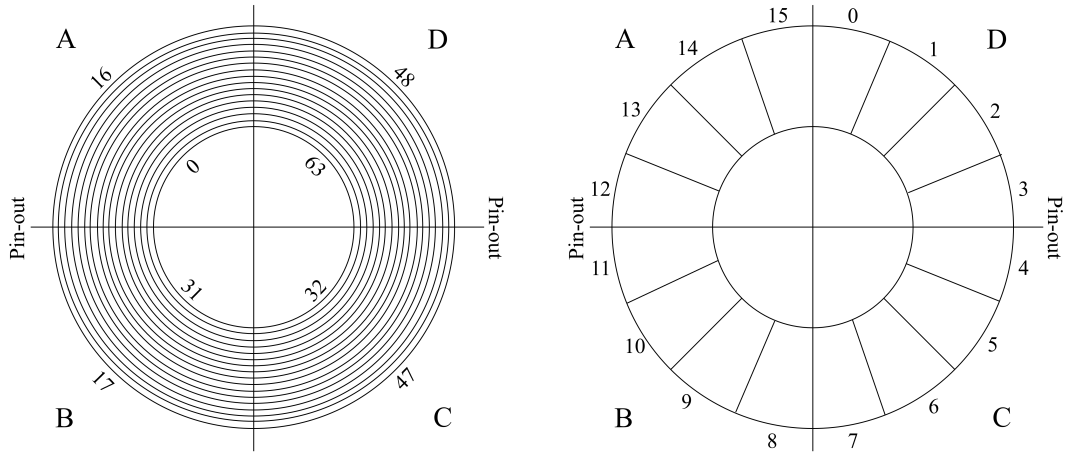


Figure 4.2: Lookup table for both SAURON-500 and SAURON-1500. Left panel: junction side with 64 strips. Just the first and the last strip in each board are explicitly numbered, while the others are meant to be numbered in sequential order. Right panel: ohmic side with 16 pads. Boards are labeled with letters from A to D. Pin-out connections are also shown. The two panels represent the detector as viewed from the same side, so that, for example, strip 0 in the junction side corresponds to pads 12 – 15 in the ohmic side.

4.1.2 Leakage current

When a bias voltage is applied to a silicon detector, a leakage current is always present, causing part of the applied voltage to be dissipated. Leakage current in silicon detectors can originate from several mechanisms. The main contributions arise from the diffusion of minority charge carriers within the depletion region, the thermal generation of electron–hole pairs due to excitation of electrons into the conduction band, and the tunneling of charge carriers into the conduction band

4.1. Experimental setup and preliminary studies

through quantum mechanical effects (see Ref. [76] for further details). Therefore, before performing our measurements, we investigated the leakage current of the detectors. The results of this analysis are shown in Figure 4.3, for SAURON-500 (left) and SAURON-1500 (right), as a function of the supplied voltage V .

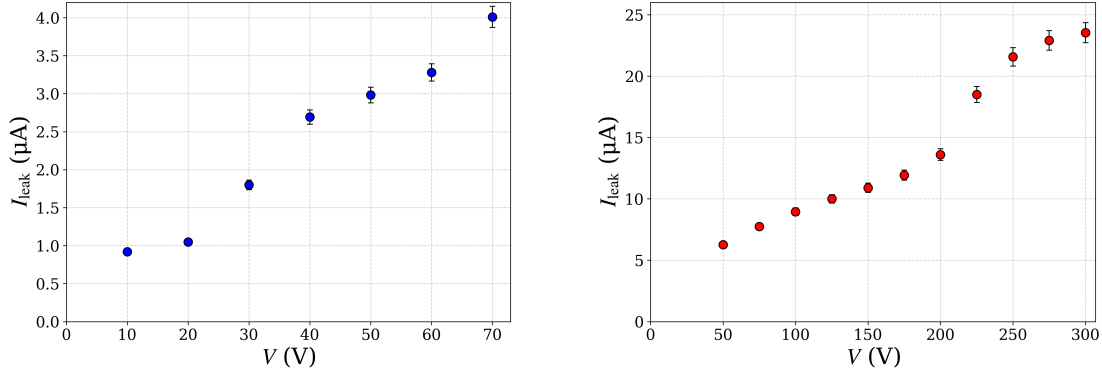


Figure 4.3: Leakage current I_{leak} as a function of the supplied voltage V for SAURON-500 (left) and SAURON-1500 (right).

Leakage current may fluctuate even when the supplied voltage is constant, with bigger fluctuations at bigger currents. After having analysed such fluctuations at some sample voltages, we calculated a relative statistical uncertainty of 3.5%, which we assumed to be constant at any voltage for both detectors. In this way errorbars in Figure 4.3 are calculated.

A direct consequence of the presence of a leakage current is that the effective bias V_{eff} is smaller than the externally supplied voltage V . Given the known internal resistance of the circuit, approximately $1.625 \text{ M}\Omega$, it is possible to determine the effective bias corresponding to each applied voltage. The values for the bias voltages most frequently used throughout the following analysis are reported in Table 4.1. The statistical uncertainty in V_{eff} is given by the propagation of the uncertainty on I_{leak} and of the sensitivity of our voltage measurements, which is 0.1 V and which corresponds to a statistical uncertainty of 0.03 V.

SAURON-500		SAURON-1500	
V (V)	V_{eff} (V)	V (V)	V_{eff} (V)
40.00 ± 0.03	35.6 ± 0.2	225.00 ± 0.03	195 ± 1
50.00 ± 0.03	45.2 ± 0.2	250.00 ± 0.03	215 ± 1
60.00 ± 0.03	54.7 ± 0.2	275.00 ± 0.03	238 ± 1
70.00 ± 0.03	63.5 ± 0.2	300.00 ± 0.03	262 ± 1

Table 4.1: Effective bias voltages V_{eff} for SAURON-500 and SAURON-1500.

4.1.3 Fully depletion condition

Fully depletion in a silicon detector is achieved once the depletion region extends over all the detector thickness. To meet this condition, the bias needs to be increased to an appropriate value V_{dep} . If the detector is not fully depleted, the charges that are generated when a particle interacts in the detector are not fully collected, and this results in reduced signal amplitudes [76].

Fully depletion condition can be verified by looking at the signal risetime, which is the duration in time of the leading edge of the signal, as it is measured by digitizers. We expect charges to be more rapidly collected when the bias is higher and the p-n junction more depleted, so we expect the signal risetime to decrease with increasing bias. The decrease should be sharper far from the fully depletion condition; once the condition is reached, risetime should reach a constant value.

To look at risetime we use CoMPASS, that is the DAQ software developed by CAEN [77]. Differences in risetime between different signals and between different strips and pads are within $\sim 10\%$. Results in Figure 4.4 show that risetime has not reached a limiting value, despite being close to that point. The decrease is sharper at lower voltages, then it almost flattens, around 150 ns for SAURON-500 and around 300 ns for SAURON-1500.

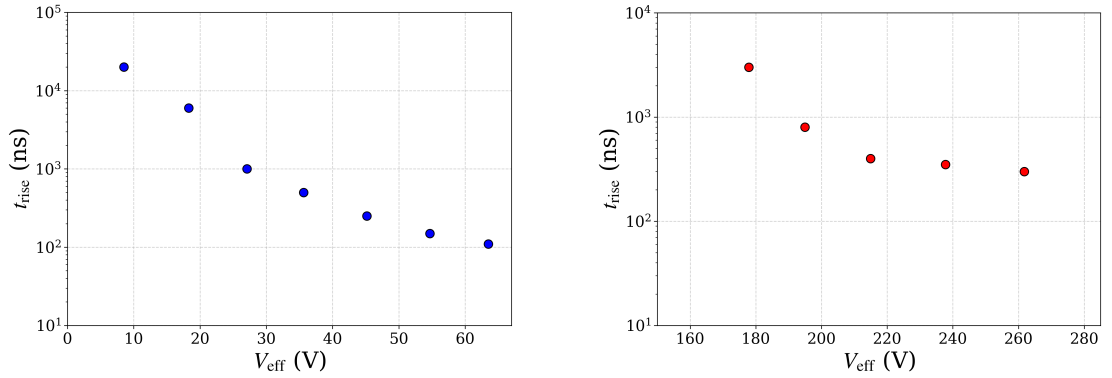


Figure 4.4: Risetime t_{rise} as a function of the effective bias V_{eff} , for SAURON-500 (left) and SAURON-1500 (right).

Another parameter that is related to detector depletion is charge collection efficiency, which is the amount of collected charge after a particle interacts in the detector. This quantity is directly proportional to the position of the three peaks of the triple- α source before calibration (so in arbitrary units, a.u.). This value is expected to rise as a function of the bias, until it reaches a limiting value when the detector is fully depleted.

4.2. Characterization of correlated signals

In Figure 4.5 we see the results for the voltages that we reported in Table 4.1, in strip 8 (left panels) and pad 13 (right panels). For pads in SAURON-500 (top-right panel), there are no data at $V_{\text{eff}} = (63.5 \pm 0.2) \text{ V}$, probably due to a cable misconnection. The position of the three peaks is still increasing in all panels, so that it must be concluded, also in accordance with data on risetime, that detectors were operating in a slightly underdepleted condition. Silicon detectors are usually employed in this condition to optimize PSA capabilities [78].

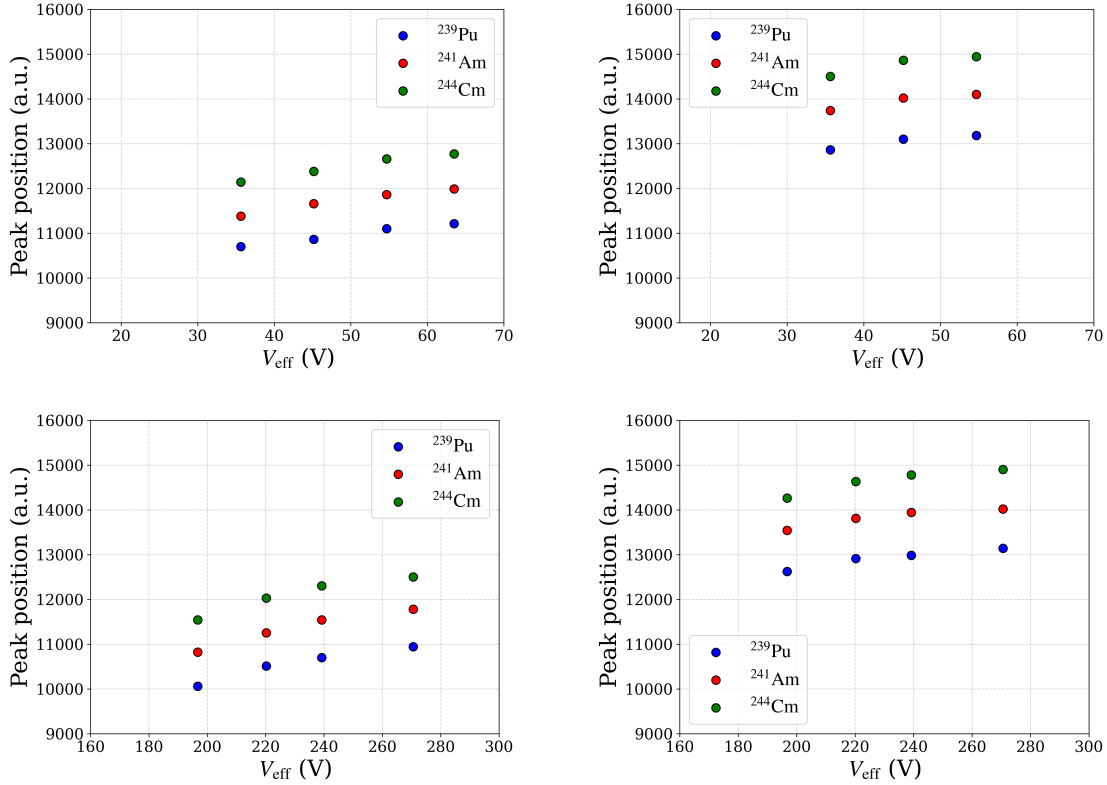


Figure 4.5: Position of the three α peaks in SAURON-500 (top panels) and in SAURON-1500 (bottom panels), as a function of V_{eff} . Strip 8 in the left panels, pad 13 in the right ones.

4.2 Characterization of correlated signals

4.2.1 Strip and pad spectra

Knowing the correlation between different detector segments is important in order to understand the effects of radiation damage on a silicon detector. Anomalous structures have been reported to emerge in correlation matrices between different channels after long irradiation [79]. In the following we look at the

strip-strip, pad-pad and strip-pad correlation matrices to study and interpret the structures that emerge. Since it was acquired with a much greater statistics, we focus on SAURON-1500 at the provided voltage $V = 250$ V, corresponding to $V_{\text{eff}} = (215 \pm 1)$ V. No significant differences are found at different voltages or in SAURON-500.

Since it would be useful to understand some of the structures in the correlation matrices, we first compare in Figure 4.6 the spectrum of a strip (strip 8) with that of a pad (pad 13). In the strip spectrum, the three peaks can still be appreciated quite clearly. Still, they show low-energy tails which degrade the resolution: the FWHM of the three peaks is around 210 keV, which is far worse than the declared value of 55 keV. The Valley-to-Peak (V/P) ratio, calculated considering the central peak and the valley between the first and the second peak, is around 0.13. The pad spectrum, on the other hand, exhibits even longer energy tails and double peaks. If we consider just the three rightmost peaks, we have a FWHM of 230 keV (for the central peak) and a V/P ratio of 0.41 (calculated as defined for the strip case).

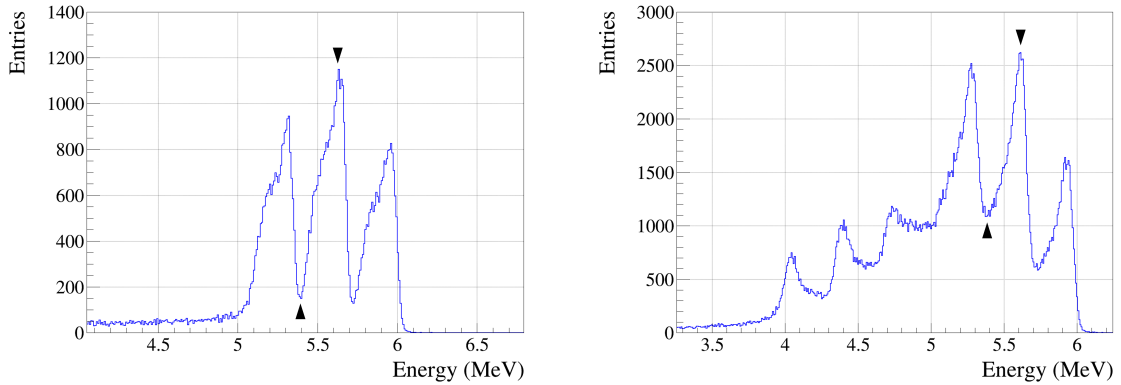


Figure 4.6: Spectra of strip 8 (left) and pad 13 (right) in SAURON-1500, with $V_{\text{eff}} = (215 \pm 1)$ V. Black arrows indicate the peaks and the valleys that are used to calculate the V/P ratio.

4.2.2 Strip-strip correlation matrix

We have verified that significant structures emerge just in correlation matrices that involve nearby strips in the radial direction. No significant correlation, on the other hand, is found between further strips, or between strips that are close but belong to different boards. So, as an example, in Figure 4.7 we study the correlation matrix between strip 56 and strip 57. In the picture, we identify each relevant structure with a specific letter:

4.2. Characterization of correlated signals

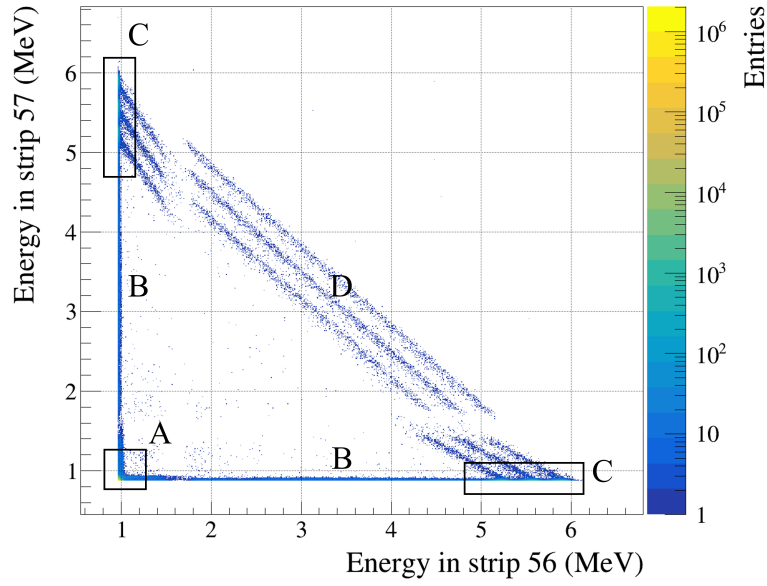


Figure 4.7: Correlation matrix of two neighboring strips in SAURON-1500, namely strip 56 and strip 57, with $V_{\text{eff}} = (215 \pm 1)$ V. The meaning of the letters is explained in the text.

A Electronic noise in both channels. Also called *pedestal*.

B Partial-energy signal in one channel, electronic noise in the other.

C Full-energy signal in one channel, electronic noise in the other.

D Charge sharing between the two channels.

When a particle impinges in the detector but strips 56 and 57 are not involved, they can still produce a signal due to electronic noise. When this happens in both channels we have the pedestal A. No anomalous structures emerge in the pedestal, and it can be noticed that it comprises the majority of the total events.

If a particle impinges exactly in one of the two strips, then the involved one detects the full energy of the particle. In the correlation matrix, this corresponds to the two structures marked with C, where the three peaks corresponding to the α energies can be distinguished.

When a particle impinges in the interstrip, the generated charge is partly collected by one strip, and partly by the other one, so that both detect a partial-energy signal: charge is actually shared between the strips, and we talk about charge sharing. If this happens between strips 55 and 56, or between strips 57 and 58, so that strips 56 and 57 are not involved at the same time, in the correlation matrix we have the lines marked with B; if, on the other hand, this happens in the interstrip between strips 56 and 57, we have the three diagonals marked with D.

According to Ref. [68], the slope of the B lines is important in order to establish the presence of crosstalk in the detector circuitry. Crosstalk is the phenomenon in which the signal generated in one channel produces a small signal in another channel due to an undesired electronic coupling between the circuit elements that are responsible for collecting the signal. In the present case, crosstalk should appear as a scissor-like closure of B lines in the correlation matrix. Since in our case B lines are perpendicular to each other, we can conclude that there is no significant crosstalk between the channels.

Diagonals marked with D should connect the three peaks of the two C structures. This is not exactly the case, as the three diagonals are divided in three segments each by two discontinuities. The slope of the central segments is 45° , and if prolonged they connect the peaks in one channel to those in the other. This is what we expect if all the charge generated by the α particle is collected by the strips. In this way, the sum of the two signals (after calibration) is equal to the total α energy.

The segments that are closer to the peaks, on the other hand, do not have a 45° slope, and if prolonged they do not match the peaks in the other channel. The observed discontinuity and slope change are most likely due to a transition in the strip readout behavior near the trigger threshold. In charge-sharing events, sub-threshold signals may still be recorded if another strip provides the trigger, leading to a reconstruction performed under different timing and validation conditions compared to self-triggered events. Once the strip exceeds its effective threshold, it is processed as an independent trigger, which can introduce a change in gain or offset and produce a kink in the diagonal.

The pronounced gap indicates additional selection effects, such as energy skim, event selector, peak-valid or pile-up rejection, coincidence-window losses, or an offline low-energy cut, which suppress events in the transition region.

4.2.3 Pad-pad correlation matrix

Similarly to the strip-strip case, we verify that pad-pad correlation matrices show interesting structures just when we compare nearby pads. There seem to be no differences if we compare pads that belong to different boards. In Figure 4.8 we study the correlation matrix between pad 0 and pad 1. The structures that emerge are very similar to the ones appearing in the strip-strip case, thus we name them using the same letters. Pedestal A does not show anomalous structures and lines B are orthogonal, so that we can exclude crosstalk between pad channels.

4.2. Characterization of correlated signals

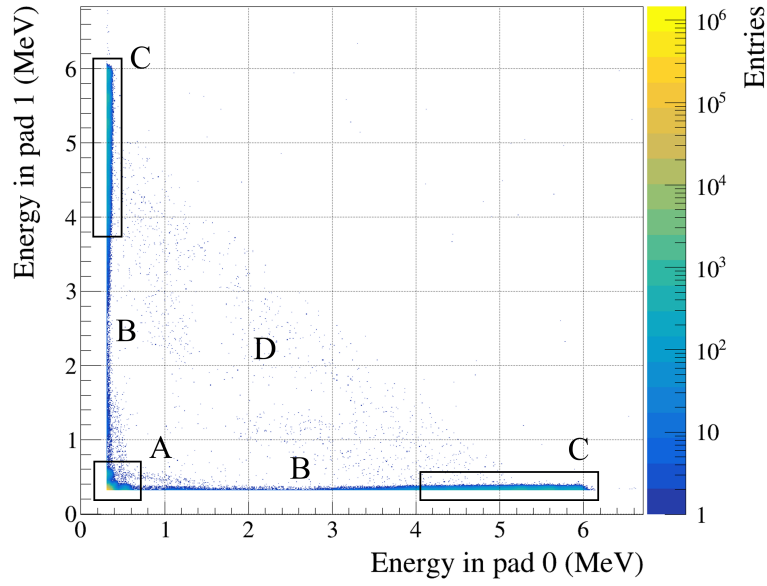


Figure 4.8: Correlation matrix of two neighboring pads in SAURON-1500, namely pad 0 and pad 1, with $V_{\text{eff}} = (215 \pm 1) \text{ V}$.

Concerning structures C and D there are some differences with respect to the strip–strip case that are worth mentioning. Concerning structure C, while in the strip–strip case we can clearly appreciate the three peaks, here they are less evident. This is related to the typical worse resolution in pads, as it was already observed in Figure 4.6.

Concerning D, on the other hand, we see here that the diagonal is far less populated than in the strip–strip case; in addition, there is not a clear three-line pattern, and the diagonal seems to be shifted towards the origin, so towards lower energy values. The smaller number of events can be explained by considering that the interstrip region is much smaller if compared to the dimension of the two pads, so that, in proportion, we expect a very small number of events in the diagonal.

On the other hand, the position of the diagonal and the missing three-line pattern is due to the orientation of SAURON, with its ohmic (pad) side towards the α source. This means that when a particle impinges in the interstrip between two pads, it loses energy in the SiO_2 layer that covers the interstrip (see section 2.2): this results in structure D to be shifted towards the origin. In addition, due to different impact angles and so different energy losses, the three lines are no longer appreciable.

4.2.4 Strip-pad correlation matrix

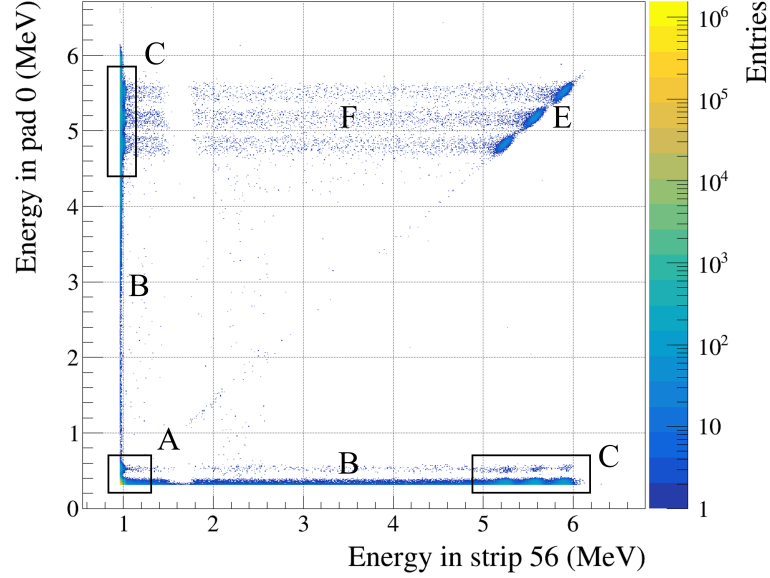


Figure 4.9: Correlation matrix between strip 56 and pad 0 in SAURON-1500, with $V_{\text{eff}} = (215 \pm 1) \text{ V}$. The meaning of the letters is explained in the text.

In the last case, we consider strip-pad coincidences. Interesting structures emerge only when we consider strips and pads that overlaps, thus in Figure 4.9 we consider for example strip 56 and pad 0. Again, we use the same letters to denote structures similar to the ones observed above. The new structures that emerge are the following:

E Full-energy signal in both channels.

F Partial-energy signal in the strip, full-energy signal in the pad.

Structure E is created by particles that are impinging exactly in the strip-pad coincidence region, so that the charges that they create are all collected by the strip and the pad that we are studying. Structure F, on the other hand, is created by particles impinging on the interstrip on the strip side, so that the charge is fully collected by the pad, and shared between two strips. We see a discontinuity in lines belonging to structure F, whose origin can be related to the discontinuities seen in strip-strip correlation matrices in structure D. If we focus on the signal detected by strip 56, we see that the discontinuity appears at the same energy value in both Figure 4.7 and Figure 4.9, around $1500 \text{ a.u.} \approx 1.6 \text{ MeV}$.

The symmetric of structure F, which should arise from particles impinging in the interstrip between two pads and inside the strip, is not visible, due to the

4.3. Radiation damage and resolution

geometrical smallness of such region and the partial energy absorption in the SiO_2 layer, which further distributes the points in the matrix.

If we look at the horizontal B line, which corresponds to partial energy deposition in the strip, and the corresponding three peaks in structure C, which correspond to full energy deposition in the strip, we see that these structures are doubled. This happens because the pad registers a slightly higher signal in coincidence with the events in the strip, around 500 a.u. ≈ 0.5 MeV. This signal is still very small, thus its origin is probably electronic.

No other anomalous structures emerge and B lines are orthogonal to each other, so that crosstalk can be neglected.

4.3 Radiation damage and resolution

4.3.1 Radiation damage in SAURON

Silicon detectors get damaged by intense irradiation. This damage can be bulk damage, if it involves the silicon crystal, or surface and interface-related damage, if it involves its surface or the boundaries, such as the Si-SiO₂ one. Bulk damage usually plays a major role for lifetime limitations of silicon detectors. It mainly comes from Non Ionizing Energy Loss (NIEL), so it is caused by particles which collide with the silicon atoms and displace them, creating defects in the lattice. Since defects can act as trapping centers, bulk damage results in a reduction of charge collection efficiency, and so in a worse energy resolution. Additional consequences include an increase in the leakage current and a reduction of the effective doping concentration, which increases the bias needed to reach full depletion. Further details on radiation damage in silicon detectors can be found in Ref. [67].

In the following we will try to analyse radiation damage in SAURON, for strips and pads, and then we will try to restore the detector initial performances using the technique outlined in Ref. [68], namely the position-resolved calibration. As done above, we will outline our analysis using SAURON-1500 at $V_{\text{eff}} = (215 \pm 1)$ V. The discussion for different voltages and also for SAURON-500 is exactly the same. Both SAURON detectors have been operated at LNL since 2023 and have been employed in a series of experimental campaigns, thus providing a good case study of detector performance degradation after long routine operating conditions.

Radiation damage in silicon detectors is generally non-uniform, since the irradiation profile in nuclear physics experiments depends on the angular distribution

of the particles impinging on the detector. This distribution is governed by the relevant differential cross sections of the underlying reactions. If the detector is positioned symmetrically around the beam line, regions at small polar angles are typically exposed to higher particle fluxes. In particular, elastic scattering often constitutes a dominant background contribution in many nuclear physics experiments [47]. The Rutherford differential cross section exhibits a strong enhancement at low scattering angles, with a dependence $\frac{d\sigma}{d\Omega} \propto 1/\sin^4(\theta/2)$, leading to a forward-peaked emission pattern. Consequently, in direct kinematics experiments, detector elements covering small angles are subject to the highest irradiation levels.

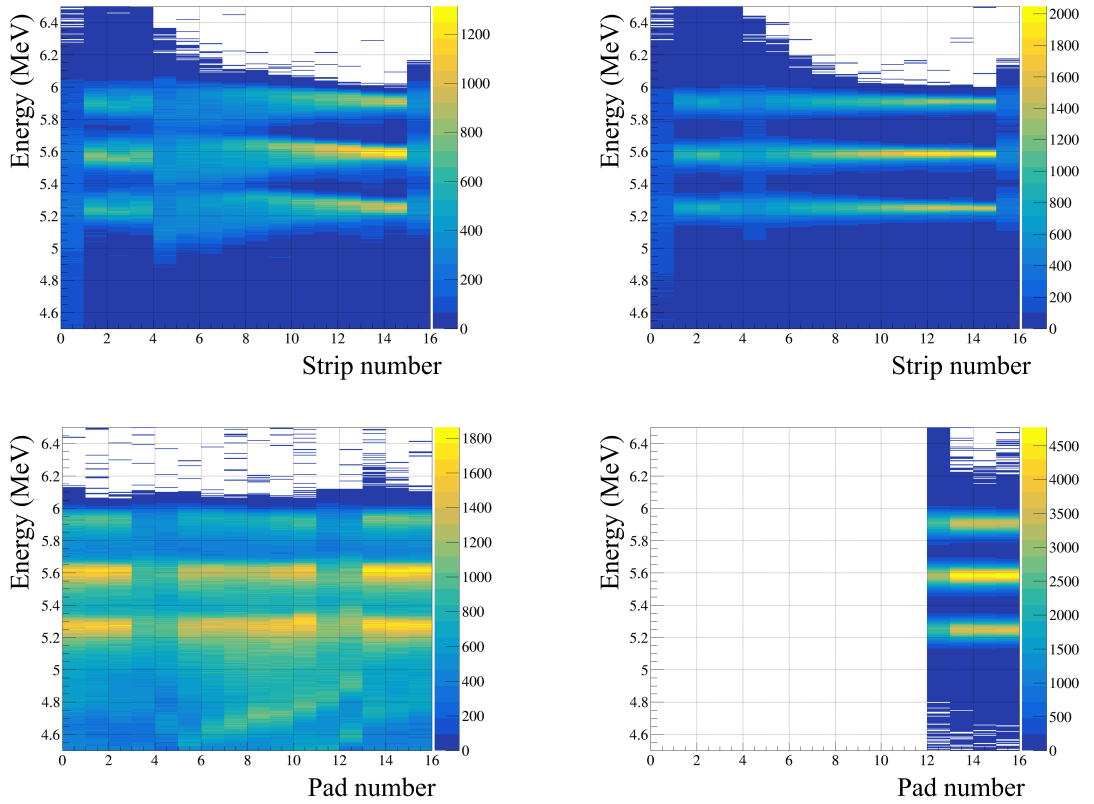


Figure 4.10: Calibrated energy spectra for strips in board 0 (top panels) and pads (bottom panels), in SAURON-1500. Spectra on the left have been calibrated using standard linear calibration on the individual strip/pad. Spectra on the right have been calibrated using position-resolved calibration. In bottom-right panel just four pads are shown, since we performed position-resolved calibration just in board 0.

In Figure 4.10 we evaluate radiation damage in SAURON. Signals coming from strips in board 0 and from all pads (see top-left and bottom-left panels, respectively) have been calibrated to the correct α energies. The worse spectra

4.3. Radiation damage and resolution

are observed for strips 0, 4, and 15: they appear significantly blurred, making the three peaks difficult to distinguish. Strips 1–3 exhibit comparable resolution, whereas strips 5–14 follow the expected trend of progressively worsening resolution toward the detector center, with the outermost strips providing the best peak separation. Concerning pad spectra, they all exhibit long low-energy tails, which were already recognized in Figure 4.6 (right). The pads with the worst resolution are pads 4, 5, 11 and 12, which are those closer to the pin-out connections.

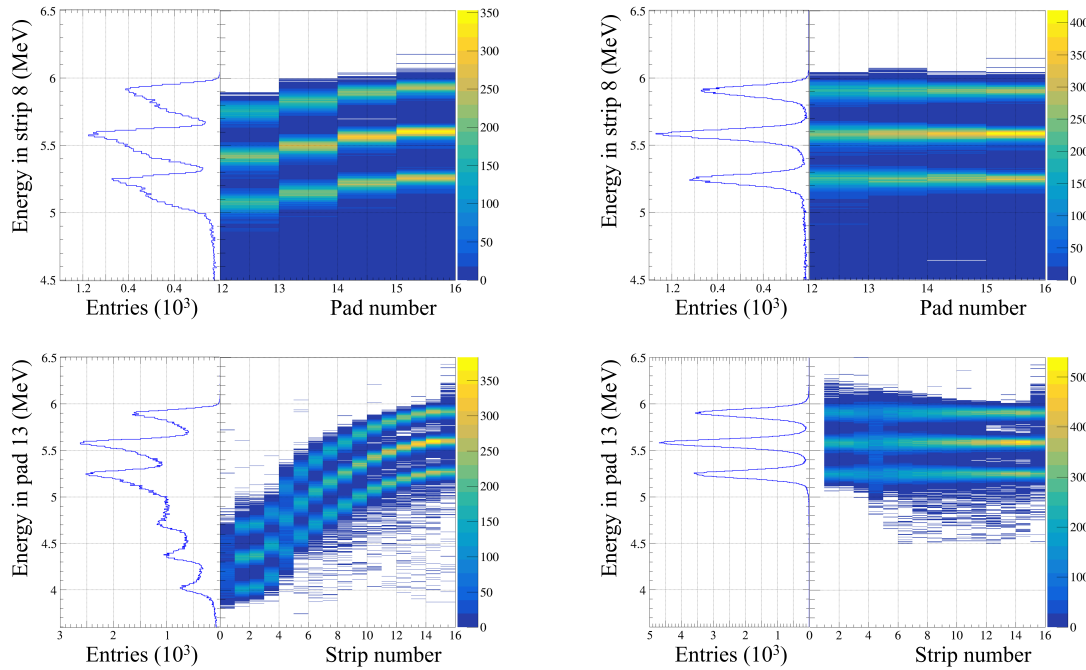


Figure 4.11: Calibrated energy spectra for strip 8 (top panels) and pad 13 (bottom panels) as a function of the strip/pad on the opposite side that detects a coincident signal. Spectra in the left panels have been calibrated using standard linear calibration on the whole strip/pad; spectra on the right have been calibrated using position-resolved calibration. Notice in the bottom-right picture that the signal in the first column, corresponding to the innermost strip, could not be properly calibrated because the original spectrum was too blurred.

In Figure 4.11 we compare the signal produced in a specific strip (pad) as a function of the pad (strip) that is detecting a signal in coincidence. In this way, we can study the resolution as a function of the particle position inside the specific strip or pad. Concerning strips (top-left panel) we see that the signal changes magnitude as a function of the corresponding 4 pads behind, which results in a shift of the peaks position and thus in a worse resolution once data are summed together in the same spectrum (as it is done on the left side of the panel). This effect is even sharper in the pad case (bottom-left panel), where fluctuations and

a decrease in the signal are observed, especially for coincident innermost strips. Reduced signals result from a loss in charge collection efficiency, which is a major consequence of bulk radiation damage. Long low-energy tails and double peaks are thus explained.

4.3.2 Position-resolved calibration

The insights that we get from Figure 4.11 can be used in order to design a calibration technique that should be able to partly recover SAURON resolution. Following Ref. [68], we will call this technique position-resolved calibration. The idea behind this technique is to calibrate not single strips and single pads, but instead to calibrate each virtual pixel generated by the strip-pad intersection. Then, once the signal in each virtual pixel is calibrated, all the signals coming from the same individual strip or pad can be collected together in the same spectrum.

To calibrate the spectrum of a single strip using this technique, we need to calibrate 4 spectra, one for each corresponding pad behind that strip; similarly, for each pad we need 16 calibrations, one for each corresponding strip. Eventually, to calibrate an entire board we need to calibrate 128 spectra; to calibrate the whole SAURON we need to calibrate 512 spectra.

In Figure 4.10 (top-right) we see the spectra of the 16 strips in board 0 after position-resolved calibration. Even if external strips still have a better resolution than internal ones, all spectra have an improved resolution with respect to the left panel, and the three lines corresponding to the three α peaks are much more definite. There are important improvements also concerning pads (bottom-right panel), since low-energy tails and double peaks are completely removed. Because we concentrated just on board 0 to perform position-resolved calibration, we cannot display the corrected spectra for all the 16 pads. Still, we expect to obtain similar results if we extend our analysis to the whole detector.

In Figure 4.11 (top-right) we show the decomposed signal inside the specific strip after position-resolved calibration. As desired, we now have aligned peaks, and thus a much better resolution in the whole spectrum. A similar result is achieved with the signal inside the specific pad (bottom-right panel). Here, the spectrum in the first column, corresponding to the innermost virtual pixel, is too damaged to be effectively corrected. Also, we see that external virtual pixels still have a better resolution than internal ones.

After position-resolved calibration, resolution of the central peak in strip 8 changes from 210 keV to 95 keV (FWHM), and the V/P ratio (calculated considering the central peak and the valley between the first and the second peak, as in

4.3. Radiation damage and resolution

subsection 4.2.1) changes from 0.13 to 0.04. Similarly, resolution in pad 13 changes from 230 keV to 110 keV, and the V/P ratio from 0.41 to 0.02.

4.3.3 Resolution as a function of bias

Using position-resolved calibration, we can study SAURON resolution as a function of the effective bias. Precisely, we focus again on strip 8 and on pad 13, whose spectra have three clearly recognizable peaks also at low biases. This analysis is carried out considering the central peak (^{241}Am), for both SAURON-500 and SAURON-1500. Results are in Figure 4.12.

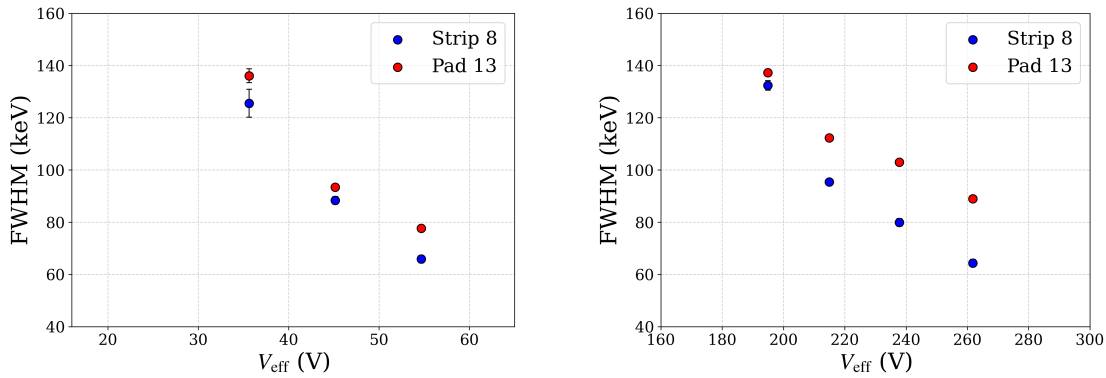


Figure 4.12: Resolution (FWHM) of the central peak (^{241}Am) in strip 8 and pad 13 after position-resolved calibration, as a function of the applied voltage V_{eff} : SAURON-500 (left) and SAURON-1500 (right).

Globally, we see that resolution gets better as the voltage is increased, as expected since charge collection efficiency is also increasing. More precisely, resolution (FWHM) in SAURON-500 goes from (126 ± 5) keV to (66 ± 1) keV for strip 8, and from (136 ± 3) keV to (78 ± 1) keV for pad 13; similarly, resolution in SAURON-1500 goes from (132 ± 2) keV to (64 ± 1) keV for strip 8, and from (137 ± 1) keV to (89 ± 1) keV for pad 13. Comparable results can be obtained by looking at the other two peaks. Thus, if we further increase the bias voltage, reaching the full depletion condition, we expect to recover the nominal energy resolution of 55 keV (FWHM), at least for the majority of strips and pads.

4.3.4 Summary

In this chapter, the performance of the detectors SAURON-500 and SAURON-1500 was systematically evaluated. Correlation matrices involving adjacent strips, neighboring pads, and strip-pad pairs were analyzed in detail. Although the

effects of the intense irradiation accumulated during previous experimental campaigns are clearly visible in the individual energy spectra, no unexpected features were observed in the correlation plots, and no evidence of significant channel-to-channel crosstalk was found. Furthermore, the application of a position-dependent calibration procedure resulted in a substantial improvement of the energy resolution. We expect to recover the nominal energy resolution of 55 keV (FWHM) once the bias is sufficiently high to reach the full depletion condition. For all these reasons, SAURON is a good candidate to perform this kind of low-event searches.

5

Conclusions

In this thesis, we carried out experimental studies within the broader framework of constraining $0\nu\beta\beta$ NMEs through γ -ray spectroscopy. The final goal is to study the γ decay from the IAS of the parent $\beta\beta$ nucleus or of the intermediate one, so to evaluate the strength of the corresponding single β decay. Following the approach discussed in Ref. [39], we considered the type of experiment required to access the relevant nuclear-structure information and identified AGATA and SAURON as suitable detector systems for such measurements. The work presented here focused in particular on the characterization of the experimental setup and its achievable performance.

More precisely, the aim of the first part of this work was to identify the features of the experimental setup that limit the precision of the estimations of the intrinsic IAS width $\Gamma(T)$, which is measured by looking at the peak width in the γ -ray spectrum. This was achieved through a Geant4 simulation of the apparatus involving both AGATA and SAURON. In a second stage, we aimed to assess more thoroughly the level of precision that can be achieved, as well as the number of events required to obtain a reliable estimate. This was carried out using a Monte Carlo simulation designed to reproduce the possible outcome of a real experiment.

Consistently with these goals, we sum up the main conclusions that we have reached:

1. In our apparatus and with the reaction $^{72}\text{Ge}(p, d)^{71}\text{Ge}$, the precision in the measurement of $\Gamma(T)$ is mainly given by AGATA resolution. Despite the need of an ancillary detector to perform Doppler correction, higher precision in β and θ is not required, since the energy resolution of the γ detector is the dominant contribution to the final precision. Better performances than those provided by SAURON (at least, nominally) are therefore not required

at the beam energy that we considered, 25 MeV, which is roughly the energy at which the reaction cross section is maximized.

2. A relative uncertainty of about 15 % on both the total width $\Gamma(T)$ and the number N of detected γ s can be achieved even in the most challenging regime, $\Gamma(T) < 30$ keV, where the total IAS width must be inferred from the γ -ray spectrum. In order to achieve this uncertainty, the required statistics are relatively modest, of the order of 10^2 detected γ rays, corresponding to beam times compatible with a standard nuclear-physics experiment even in unfavorable cases such as ^{71}Ge . For other nuclei of interest, larger γ branching ratios are expected to further reduce the required measurement time. In the case $\Gamma(T) > 30$ keV, the width can be determined through charged-particle measurements, allowing the measurement to be performed under more favorable statistical conditions, since larger event samples can be accumulated more easily.

SAURON is also essential to select the correct reaction channel and produce a clean γ spectrum. In the second part of this work, we investigated some characteristics of SAURON and analysed its performance after heavy irradiation occurred in previous experiments at LNL. In particular, we studied correlations between signals from different segments, in order to identify possible anomalous structures arising from radiation damage, and to assess potential crosstalk between the channels. In addition, we investigated whether a position-resolved calibration could restore the original energy resolution of the detector and, if so, to what extent. To address these questions, we performed an experimental test of the detector using a triple- α source. We worked with two SAURON detectors, SAURON-500 and SAURON-1500, with thicknesses of 500 μm and 1500 μm , respectively.

The main results of this study are summarized below:

1. Even if the detector performances are clearly degraded due to the intense irradiation occurred in previous experiments, as can be verified by looking at the single spectra, no anomalous structures emerge in the correlation matrices. In addition, crosstalk between different channels is still negligible.
2. Position-resolved calibration has been successfully applied, with a sharp improvement of energy resolution. This technique seems to fail just in the inner segments of the detector, where radiation damage is particularly high. Energy resolution improves as a function of the effective detector bias, as expected, so that, if we reach the proper full depletion voltage and

apply position-resolved calibration, we expect to restore its value to the one provided by the manufacturer.

Overall, the results presented in this thesis indicate that the proposed experimental approach is technically viable and that no major limitations have emerged from either the simulation studies or the detector characterization. The two parts of the work addressed complementary aspects of the problem: the first one assessed the capability of the setup to extract the nuclear-structure information of interest with the required precision, while the second one verified that the ancillary detector system can operate under conditions compatible with such measurements. Although further investigations will be required before performing a dedicated experiment, the present study provides a basis for future measurements of γ decays from IASs, which are relevant for obtaining experimental constraints on the NMEs for $0\nu\beta\beta$ decay.



Pull distributions from the Monte Carlo Simulation

We report here complete data concerning pull distributions from the Monte Carlo Simulation, section 3.3. Pulls are a powerful statistical tool in the context of parameter estimation and hypothesis testing, which can be used to check that the results are not affected by systematic biases. For a deep characterization of pull distributions, see for example Ref. [66].

In our study, we can calculate the pull distribution for $\hat{\Gamma}$ inside an ensemble by using the formula

$$g_{\Gamma} = \frac{\hat{\Gamma} - \Gamma}{\sigma_{\hat{\Gamma}}} ,$$

where g_{Γ} is a new variable, namely the pull, which we expect to follow a standard Gaussian distribution with $\mu_{g_{\Gamma}} \simeq 0$ and $\sigma_{g_{\Gamma}} \simeq 1$. We can fit each pull distribution with a standard Gaussian to check $\mu_{g_{\Gamma}}$ and $\sigma_{g_{\Gamma}}$: if they are different from 0 and 1, then we have to conclude that the statistical algorithm that we are using in our fit is giving biased results. We can do the same with N , thus calculating g_N .

For these standard Gaussians we can calculate also the reduced χ^2 , namely χ^2/NDF where NDF stands for Number of Degrees of Freedom. When this value is close to 1, it means that the fitting function is a good approximation to our distribution, thus that the variability in our ensembles has a statistical Gaussian-like origin. Conversely, if it is different from 1, then there must be a mistake in our assumptions. More specifically, if $\chi^2/\text{NDF} \gg 1$, it means that the fitting function is not appropriate for describing the data; on the other hand, if $\chi^2/\text{NDF} \ll 1$, then it may indicate an error (usually an overestimation) in the assignment of the uncertainties of the measured variables [80].

In subsection 3.3.2 pull distributions have been used as an additional tool to demonstrate that, in the context of our study, ML is giving better results than χ^2 . As can be seen here in Table A.1 and in Table A.2, pull distributions produced by the χ^2 procedure are not following a standard Gaussian, since $\mu_{g\Gamma}$ and $\sigma_{g\Gamma}$ are usually incompatible with 0 and 1; conversely, in the ML procedure $\mu_{g\Gamma} \simeq 0$ and $\sigma_{g\Gamma} \simeq 1$. This is enough to prefer ML over χ^2 . χ^2/NDF values for the two procedures are comparable: they tend to be slightly less than 1, indicating a possible overestimation of uncertainties.

In Table A.3 and in Table A.4 we report results on the pull distributions concerning the ensembles that have been used in the analysis of subsection 3.3.3, using the ML algorithm for both Γ and N . Here we see that $\mu_{g\Gamma}$ and μ_{gN} are both compatible with 0; similarly, $\sigma_{g\Gamma}$ and σ_{gN} are both compatible with 1, thus pointing towards the conclusion that our results are not biased. As above, the value of χ^2/NDF , which is always below 1, suggests that uncertainties have been probably overestimated.

Γ (keV)	N	$\mu_{g\Gamma}$	λ_μ (with 0)	$\sigma_{g\Gamma}$	λ_σ (with 1)	χ^2/NDF
1	30000	-3.34 ± 0.06	52.07	1.11 ± 0.05	2.44	0.20
3	10000	-2.64 ± 0.07	37.01	1.23 ± 0.05	4.66	0.27
10	3000	-1.80 ± 0.07	25.02	1.25 ± 0.05	4.86	0.37
30	1000	-1.02 ± 0.07	14.22	1.24 ± 0.05	4.69	0.18
50	600	-0.57 ± 0.07	8.51	1.17 ± 0.05	3.49	0.33
75	400	-0.03 ± 0.06	0.44	1.10 ± 0.04	2.22	0.28
100	300	0.18 ± 0.06	2.80	1.12 ± 0.05	2.67	0.48

Table A.1: Pull distributions and reduced χ^2 for Γ estimations obtained using the χ^2 algorithm. From subsection 3.3.2.

Γ (keV)	N	$\mu_{g\Gamma}$	λ_μ (with 0)	$\sigma_{g\Gamma}$	λ_σ (with 1)	χ^2/NDF
1	30000	0.11 ± 0.06	1.90	0.96 ± 0.04	0.92	0.76
3	10000	0.02 ± 0.06	0.40	1.01 ± 0.04	0.17	0.50
10	3000	0.06 ± 0.06	1.03	1.00 ± 0.04	0.03	0.27
30	1000	-0.08 ± 0.06	1.35	0.97 ± 0.04	0.89	0.40
50	600	0.00 ± 0.06	0.04	0.99 ± 0.04	0.34	0.31
75	400	-0.12 ± 0.06	1.98	1.05 ± 0.04	1.12	0.58
100	300	-0.10 ± 0.06	1.61	1.02 ± 0.04	0.59	0.29

Table A.2: Pull distributions and reduced χ^2 for Γ estimations obtained using the ML algorithm. From subsection 3.3.2.

Γ (keV)	N	$\mu_{g\Gamma}$	λ_μ (with 0)	$\sigma_{g\Gamma}$	λ_σ (with 1)	χ^2/NDF
1	30000	0.06 ± 0.05	1.02	0.95 ± 0.04	1.23	0.14
1	3000	-0.02 ± 0.05	0.36	0.92 ± 0.04	2.06	0.13
3	10000	0.02 ± 0.06	0.29	1.01 ± 0.04	0.30	0.13
3	3000	0.02 ± 0.06	0.39	0.98 ± 0.04	0.44	0.38
3	1000	-0.02 ± 0.06	0.43	0.96 ± 0.04	0.93	0.30
3	300	0.00 ± 0.06	0.03	0.99 ± 0.04	0.23	0.18
3	100	0.00 ± 0.05	0.04	0.95 ± 0.04	1.32	0.75
10	3000	-0.08 ± 0.06	1.32	0.99 ± 0.04	0.30	0.44
10	1000	-0.10 ± 0.05	1.97	0.90 ± 0.04	2.76	0.27
10	300	0.06 ± 0.06	1.13	0.98 ± 0.04	0.52	0.23
10	100	0.07 ± 0.05	1.39	0.91 ± 0.04	2.50	0.39
10	30	-0.09 ± 0.06	1.61	1.01 ± 0.04	0.28	0.49
30	1000	-0.09 ± 0.06	1.56	0.99 ± 0.04	0.13	0.36
30	100	0.06 ± 0.05	1.21	0.92 ± 0.04	2.21	0.30
50	600	-0.11 ± 0.06	1.93	0.99 ± 0.04	0.13	0.29
50	300	0.08 ± 0.06	1.31	1.04 ± 0.04	0.85	0.31
50	60	-0.05 ± 0.06	0.87	1.01 ± 0.04	0.27	0.23
75	400	0.08 ± 0.06	1.32	1.02 ± 0.04	0.53	0.14
75	100	0.05 ± 0.06	0.89	0.97 ± 0.04	0.75	0.27
75	40	-0.06 ± 0.06	1.01	0.99 ± 0.04	0.18	0.48
100	300	0.07 ± 0.06	1.27	0.97 ± 0.04	0.66	0.31
100	100	0.05 ± 0.05	0.99	0.92 ± 0.04	2.03	0.55
100	30	-0.10 ± 0.06	1.68	1.00 ± 0.04	0.12	0.32

Table A.3: Pull distributions and reduced χ^2 for Γ estimations. From subsection 3.3.3.

Γ (keV)	N	μ_{g_N}	λ_μ (with 0)	σ_{g_N}	λ_σ (with 1)	χ^2/NDF
1	30000	-0.03 ± 0.06	0.60	1.00 ± 0.04	0.07	0.37
1	3000	0.07 ± 0.06	1.18	1.01 ± 0.04	0.24	0.17
3	10000	0.09 ± 0.06	1.65	1.00 ± 0.04	0.09	0.19
3	3000	0.02 ± 0.06	0.32	1.05 ± 0.04	1.06	0.31
3	1000	0.12 ± 0.06	1.87	1.08 ± 0.04	1.91	0.44
3	300	-0.11 ± 0.06	1.98	0.98 ± 0.04	0.51	0.32
3	100	-0.14 ± 0.06	2.37	0.99 ± 0.04	0.24	0.44
10	3000	-0.12 ± 0.06	2.10	0.99 ± 0.04	0.23	0.18
10	1000	-0.06 ± 0.05	1.22	0.92 ± 0.04	2.20	0.27
10	300	-0.08 ± 0.06	1.30	1.02 ± 0.04	0.51	0.49
10	100	-0.11 ± 0.06	1.94	1.00 ± 0.04	0.05	0.48
10	30	-0.17 ± 0.06	2.84	1.04 ± 0.04	0.84	1.36
30	1000	-0.10 ± 0.06	1.82	0.97 ± 0.04	0.86	0.42
30	100	-0.03 ± 0.06	0.53	1.03 ± 0.04	0.77	1.05
50	600	0.02 ± 0.06	0.35	0.98 ± 0.04	0.52	0.20
50	300	0.05 ± 0.06	0.82	0.96 ± 0.04	1.03	0.63
50	60	-0.07 ± 0.06	1.32	0.97 ± 0.04	0.68	0.95
75	400	0.07 ± 0.06	1.27	0.97 ± 0.04	0.65	0.22
75	100	-0.09 ± 0.05	1.76	0.87 ± 0.04	3.63	0.30
75	40	-0.02 ± 0.06	0.30	0.97 ± 0.04	0.80	0.68
100	300	-0.07 ± 0.06	1.26	1.02 ± 0.04	0.41	0.58
100	100	0.02 ± 0.05	0.36	0.92 ± 0.04	2.00	0.36
100	30	-0.16 ± 0.06	2.67	1.06 ± 0.04	1.47	3.15

Table A.4: Pull distributions and reduced χ^2 for N estimations. From subsection 3.3.3.

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List of Acronyms

χ^2	Least Squares.
$0\nu\beta\beta$	neutrinoless double beta decay.
$2\nu\beta\beta$	two-neutrino double beta decay.
ADL	AGATA Data Library.
ATC	AGATA Triple Cluster.
BR	Branching Ratio.
CER	Charge Exchange Reaction.
DGT	Double Gamow-Teller.
DSS	Detector Support System.
DSSSD	Double Sided Silicon Strip Detector.
EDF	Energy Density Functional theory.
F	Fermi.
FEE	Front-End Electronics.
FSQP	Fermi Surface Quasiparticle.
FWHM	Full Width at Half Maximum.
GGP	Global Gigabit Processor.
GS	Ground State.
GT	Gamow-Teller.
HPGe	High-Purity Germanium.

List of Acronyms

IAS	Isobaric Analogue State.
IBD	Inverse Beta Decay.
IBM	Interacting Boson Model.
LNL	Laboratori Nazionali di Legnaro.
ML	Maximum Likelihood.
NIEL	Non Ionizing Energy Loss.
NME	Nuclear Matrix Element.
NSM	Nuclear Shell Model.
OMC	Ordinary Muon Capture.
P/T	Peak-to-Total.
PLC	Programmable Logic Controller.
pnQRPA	proton-neutron QRPA.
PSA	Pulse-Shape Analysis.
QRPA	Quasiparticle Random Phase Approximation.
SM	Standard Model.
SSD	Silicon Strip Detector.
SSD	Single State Dominance.
UPS	Uninterruptable Power Supply.
V/P	Valley-to-Peak.

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