# UNIVERSITA' DEGLI STUDI DI PADOVA

**DIPARTIMENTO DI SCIENZE ECONOMICHE ED AZIENDALI "M.FANNO"**

**DIPARTIMENTO DI MATEMATICA "TULLIO LEVI-CIVITA"**

**CORSO DI LAUREA MAGISTRALE IN ECONOMICS AND FINANCE**

**TESI DI LAUREA**

# PORTFOLIO OPTIMIZATION AND CLIMATE RISK: DECARBONIZATION OF THE S&P 500 INDEX

**RELATORE:**

**CH.MO PROF. CLAUDIO FONTANA**

**LAUREANDO: PIETRO ACERBI**

**MATRICOLA N. 2005814**

**ANNO ACCADEMICO 2021 – 2022**

Dichiaro di aver preso visione del "Regolamento antiplagio" approvato dal Consiglio del Dipartimento di Scienze Economiche e Aziendali e, consapevole delle conseguenze derivanti da dichiarazioni mendaci, dichiaro che il presente lavoro non è già stato sottoposto, in tutto o in parte, per il conseguimento di un titolo accademico in altre Università italiane o straniere. Dichiaro inoltre che tutte le fonti utilizzate per la realizzazione del presente lavoro, inclusi i materiali digitali, sono state correttamente citate nel corpo del testo e nella sezione 'Riferimenti bibliografici'.

*I hereby declare that I have read and understood the "Anti-plagiarism rules and regulations" approved by the Council of the Department of Economics and Management and I am aware of the consequences of making false statements. I declare that this piece of work has not been previously submitted – either fully or partially – for fulfilling the requirements of an academic degree, whether in Italy or abroad. Furthermore, I declare that the references used for this work – including the digital materials – have been appropriately cited and acknowledged in the text and in the section 'References'.*

Firma (signature) ..............................

# TABLE OF CONTENTS

# INTRODUCTION

In the investment process there are two crucial phases, the asset allocation choices and the management of the risks related to the investment decisions. The classical portfolio theory (Markowitz (1952)) states that there is an inseparable trade-off between the risk and the economic return of a portfolio. Therefore, all the situations and circumstances that might affect the volatility of an asset must be managed and, when possible, hedged. In recent years the extensive array of risks for the investments has included the climate risk, this type of risk is related to climate change. The rising of seas' level, the desertification, the exorbitant increasing number of natural catastrophes, the extinction of several animal species due to the harmful alteration of their habitats are just some examples of the devastating effects of climate change. However, climate change has not only affected the process of risk management of investments but also the decisions of asset allocation.

The fight against climate change has become one of the most compelling tasks in the agenda of supranational and national authorities, governments, regulatory institutions, international organizations and, in a lower extent, firms. Since climate change and global warming are strictly related, and global warming is due to the greenhouse effect, the first step is to reduce the emissions of carbon dioxide ($CO_2$). All the policies introduced in the last years aim to favor a transition towards a green economy, namely a new economic equilibrium which is sustainable from the point of view of the resources employed in the production and without the intensive use of carbon fossil fuels. Therefore, in recent years the tastes of the investors are tilting towards assets that are considered more environmentally sustainable and that might suffer less the negative outcomes of climate change. As demonstrated by the model of Pàstor et al. (2020), green stocks are becoming more appealing to investors at the expense of the stocks of carbon intensive firms. Firms that implement policies in accordance with ESG factors generate positive externalities for society. There are multiple dimensions of sustainability, thus agents derive utility in different ways. Agents care about firms' aggregate social impact, so they care about climate risk. Moreover, agents derive utility from financial wealth, Ardia et al. (2021) empirically test the model of Pàstor and demonstrate that green stocks outperform brow stocks when worries about climate change intensify unexpectedly.

The firms have to face the risk that the markets will more focus on companies that are less likely to be affected by the green transition, this type of risk is called, carbon risk. To manage that risk, it is possible to construct a decarbonized portfolio, starting from a reference benchmark some constraints on climate risk measures are added. Hence, it is obtained a portfolio that has the same assets of the benchmark but has a lower exposure to carbon risk. In financial literature, the contribution of Andersson et al. (2016) was fundamental. They implemented a model of portfolio decarbonization in which the tracking error with respect to the benchmark is minimized and the financial return is not sacrificed. The authors illustrated the notion of "free option on carbon" that investors hold when they invest in the decarbonized index. Once the policies of carbon emissions reduction become effective the low-carbon index is expected to outperform the benchmark. Among numerous climate-related risk measures, in my analysis I considered the main three, carbon emissions, carbon intensity and carbon beta. I focused more on describing how the carbon beta was estimated, mostly relying on the paper of Görgen et al. (2019).

The contribution of my thesis lays in the comparison of the three different climate risk metrics used to decarbonize the reference benchmark, that is the S&P 500 index, trying to figure out which one provides the best performances in absolute terms and with respect to the benchmark. To do this, I considered different approaches to estimate efficient frontiers and the related efficient portfolios, and different set of constraints. Therefore, the results I came up with are relative with respect to the data that I decided to use.

The thesis is structured as followed. The first chapter describes two types of risks due to climate change that cause financial instability, physical risk, and transition risk. Then, it is analyzed the difference between brown and green stocks and how the tastes of investors are changed. In the last section of the first chapter is reported the model of portfolio decarbonization developed by Andersson et al. (2016). The second chapter is focused on the description of the three climate risk measures that I decided to use for the portfolio decarbonization, I have given a specific relevance to the explanation of how to estimate the carbon beta. The third chapter is devoted to the practical implementation of portfolio decarbonization using the three climate risk measures. First are presented the efficient frontiers obtained with different constraints in order to establish the impact of climate constraints in a problem of portfolio optimization. Then, I tracked the performance records of some strategies and compared them both in absolute terms and with respect to the reference index. To conclude, in chapter four I propose some final remarks.

# CHAPTER 1: The Green Swan and its effect on asset management

The objective of this chapter is to highlight the consequences that climate change poses on the financial sector, specifically on asset management. First, I outline two causes of financial instability due to climate change, physical and transition risk. These risks, together with other variables, are used to sort between green and brown stocks. Therefore, by reviewing financial literature I present investors' preferences with respect to these types of stocks in sustainable investing. The last section of this chapter is devoted to deepening the theme of environmental sustainability in the asset management industry and to figure out if it is possible to hedge against climate risk in portfolio allocation.

## 1.1. CLIMATE CHANGE AND FINANCIAL INSTABILITY

"Over the 40 years of my career in finance, I have witnessed a number of financial crises and challenges — the inflation spikes of the 1970s and early 1980s, the Asian currency crisis in 1997, the dot-com bubble, and the global financial crisis […] Even when these episodes lasted for many years, they were all, in the broad scheme of things, short-term in nature. Climate change is different." This statement has been claimed by BlackRock's CEO Larry Fink in 2020. Climate change crisis is different from past crises because it has devastating impacts on all the aspects of our lives, and it casts serious doubts about the survival of future generations.

Some of the crises cited by the number one of the world's largest asset managers have been anticipated by some black swan events[1], such as a terrorist attack or a disruptive technology. These events exhibit a large skewness relative to that of normal distribution. Since they typically fit fat tailed probability distributions, they cannot be predicted by relying on backward-looking probabilistic approaches assuming normal distributions.

The presence of black swans requires the adoption of different epistemologies of risk. To hedge, at least partially, against the risks related to black swans the counterfactual reasoning is an option that can help. Counterfactuals are thoughts about alternatives to past events (Epstude and Roese (2008)). Such an epistemic position can provide a partial hedging against extreme risks but not make them disappear (Bolton et al. (2020)).

[1] *From Nassim Nicholas Taleb (1960 – present), who proposed the theory that A black swan is an unpredictable event that is beyond what is normally expected of a situation and has potentially severe consequences (source: www.investopedia.com).*

The worsening of climate conditions in recent years might lead to the occurrence of green swans, or “climate black swans” (Bolton et al. (2020)). Climate-related risks present many features of usual black swans, namely fat-tailed distributions. Both physical and transition risks are characterised by deep uncertainty and nonlinearity, their chances of occurrence are not reflected in past data, and the possibility of extreme values cannot be excluded (Weitzman (2009, 2011)). In this context, future climate-related risks cannot be assessed by traditional approaches to risk management consisting in extrapolating historical data and on assumptions of normal distributions.

However, green swans differ from black swans for three aspects. First, although the impacts of climate change are highly uncertain, “there is a high degree of certainty that some combination of physical and transition risks will materialize in the future” (NGFS (2019a), p 4). That is, the need for ambitious actions is certain despite prevailing uncertainty regarding the timing and nature of impacts of climate change. Second, climate catastrophes are even more serious than most systemic financial crises: they could pose an existential threat to humanity. The last aspect is about the complexity related to climate change, which is of a higher order than for black swans, the intricate chain reactions and cascade effects associated with both physical and transition risks could generate fundamentally unpredictable environmental, geopolitical, social, and economic dynamics.

It would be a terrible mistake assuming that climate change will be a concern just in the future: it has already started to transform human and non-human life on Earth, although the worst impacts are yet to come. Crop yields and food supply are already affected by climate change in many places across the globe (Ray et al (2019)). The number of severe water crises has steadily increased in last 40 years. IUCN (International Union for Conservation of Nature) in its Red List of 2017 has reported that almost half (47%) of the mammalian species monitored and almost a quarter of the bird species (24.4%) are negatively impacted by climate change, you are talking of a total of almost 700 species.

UN Paris Agreement of 2015 (UNFCCC (2015)) has set the goal of keeping global warming well below 2°C and as close as possible to 1.5°C above pre-industrial levels (defined as the climate conditions experienced during 1850–1900). To comply with that objective, we should reduce emissions to almost zero by 2050. However, the special report of the IPCC on the 1.5°C goal (IPCC (2018)) shows that the gap between current trends and emission reduction targets set by countries through their nationally determined contributions (NDCs) – which were already insufficient to limit global warming to 2°C – is widening and leading to somewhere between 3°C and 4°C of warming.

The impacts on economic output due to the excessive carbon emissions will be substantial if no actions are taken. In any case, both the demand side and the supply side are affected by gradual global warming and extreme weather events (Table 1).

*Table 1 - Climate change-related shocks and their effects*

| | Type of shock | From gradual global warming | From extreme weather events |
|---|---|---|---|
| **Demand** | Investment | Uncertainty about future demand and climate risks | Uncertainty about climate risk |
| | Consumption | Changes in consumption patterns, eg more savings for hard times | Increased risk of flooding to residential property |
| | Trade | Changes in trade patterns due to changes in transport systems and economic activity | Disruption to import/export flows due to extreme weather events |
| **Supply** | Labour supply | Loss of hours due to extreme heat. Labour supply shock from migration | Loss of hours worked due to natural disasters, or mortality in extreme cases. Labour supply shock from migration |
| | Energy, food and other inputs | Decrease in agricultural productivity | Food and other input shortages |
| | Capital stock | Diversion of resources from productive investment to adaptation capital | Damage due to extreme weather |
| | Technology | Diversion of resources from innovation to adaptation capital | Diversion of resources from innovation to reconstruction and replacement |

*Sources: NGFS (2019b), adapted from Batten (2018).*

Demand-side shocks are those that affect aggregate demand, such as private (household) or public (government) consumption demand and investment, business investment and international trade. Consumption patterns could experience a reduction that might be due to the necessity of people to save more for hard times. Even investments could be reduced by the uncertainty about future demand and climate risks. Gradual global warming is altering transport systems and economic activity, so international trade flows are changing too. Even less exposed economies can have extensive interactions with global markets and be affected by extreme climate shocks.

Supply-side shocks could affect the economy's productive capacity, acting through the components of potential supply: labour, physical capital, capital stock and technology. Gradual global warming is increasingly reducing agricultural productivity. Shocks that regard labour supply are due to massive population movements that are primarily triggered by climate change. To well react to supply shocks capital stock and technology resources must be diverted towards adaptation capital. Damages to assets affect the longevity of physical capital through an increased speed of capital depreciation (Fankhauser and Tol (2005)). Even if the relevant capital stocks might survive, efficiency might be reduced, and some areas might have to be abandoned (Batten (2018)).

These climate-change related shocks can progressively fuel stark financial instability through two transmission channels, physical and transition risk.

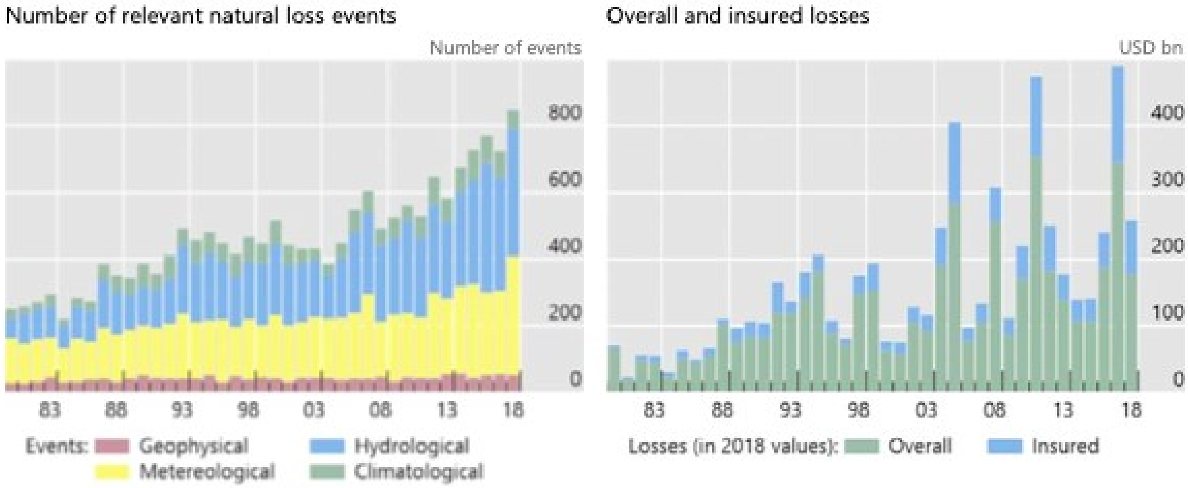
### 1.1.1. PHYSICAL RISK

Physical risks are "those risks that arise from the interaction of climate-related hazards [...] with the vulnerability of exposure to human and natural systems" (Batten et al (2016)). The economic costs and financial losses could arise from both an increase in the frequency and severity of weather events, and gradual climate change.

There might be a wide range of economic impacts due to physical risk. For what concerns people life, labour productivity might fall, while mortality and morbidity could increase thanks to changes in temperature extremes. Climate change and natural catastrophes destroy properties and infrastructure, and resources divert into reconstruction and replacement. Therefore, physical capital is eroded by physical risk. Natural capital is also aggressively affected by climate change due to disruption to agriculture (e.g. from crop failure) and other ecosystem services (e.g. from shifts in the productivity and distribution of fish stocks). A reallocation of household financial wealth could be induced by the destruction of capital and the decline in profitability of firms exposed to this risk. For instance, rising sea levels could lead to abrupt repricing of real estate (Bunten and Kahn (2014)) in some exposed regions, causing large negative wealth effects that may affect demand and prices through second-round effects. Climate-related physical risks can have effects on the expectation of future losses, which in turn may modify current risk preferences. For instance, homes exposed to sea level rise already sell at a discount relative to observationally equivalent unexposed properties equidistant from the beach (Bernstein et al (2019)).

Solvency of households, businesses and governments, and therefore financial institutions is seriously threatened by the increase of non-insured losses (Figure 1) linked to natural catastrophes worldwide (which represent 70% of weather-related losses (IAIS (2018)). On the other end, insured losses may place insurers and reinsurers in a situation of instability as claims for damages keep increasing (Finansinspektionen (2016)). More broadly, catastrophic climatic events affect the longevity of physical capital since they cause more damages to assets and capital depreciation (Fankhauser and Tol (2005)).

*Figure 1 – Increase in the number of extreme weather events and their insurance, 1980-2018*




*Includes copyrighted material of Munich Re and its licensors.*
*Source: MunichRe (2018).*

From Figure 1 the main highlight is that in the last 40 years the number of catastrophic events is almost quadruplicated, specifically the trend had a sharp surge from the beginning of 2000s onwards. This pattern is confirmed by the presence of spikes of the overall losses in the last 20 years.

Moreover, the fat-tailed probability distributions of many climate variables are such that the possibility of extreme values cannot be ruled out (Weitzman (2009, 2011)). Therefore, financial institutions might not have sufficient capital to absorb climate-related losses. In turn, the exposure of financial institutions to physical risks can prompt contagion and asset devaluations propagating throughout the financial system (Bolton et al. (2020)).

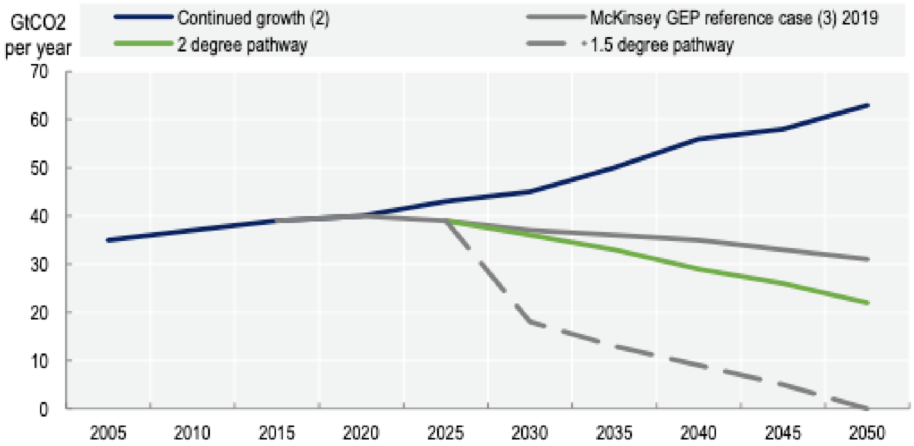
### 1.1.2. TRANSITION RISK

Transition risk is a consequence of meeting the Paris Agreement[2] targets and transitioning to low-carbon economies that will require a fundamental shift in energy and land use that will

[2] *The Paris agreement is legally binding treaty on climate change entered into force on 4 November 2016. It was adopted by 196 Parties at COP 21 in Paris. Its goal is to limit global warming well below 2, preferably to 1.5 degree Celsius, compared to pre-industrial levels (source: United Nations Climate Change).*

affect every sector of the economy. The transition could either be ordered or, if coordinated action is delayed, disordered. The latter would require sudden or rapid policy changes to achieve the declines in $CO_2$ emissions needed to meet a 2-degree target by the end of the century (Figure 2).

*Figure 2 – Projected global $CO_2$ emissions per scenario, metric gigatons of $CO_2$ per year*




*Source: McKinsey & Company (2020).*

In both circumstances there are transition risks for financial markets, though these would be significantly more evident in the case of a disorderly transition. Transition risks could lead to some of the existing capital stock being 'stranded' and labour market frictions as the economy shifts towards lower, and ultimately, net-zero emissions activities (OECD (2021)).

Transition away from fossil fuels will have an impact on market losses. Therefore, the policies used to mitigate climate transition risks will inevitably influence the way in which investors will manage losses throughout the financial system. These choices will influence how remarkably the transition contributes to sharp changes in asset price valuations, including book and market values. In the instance an orderly transition occurs, changes in asset prices can lead to losses, if the latter can be absorbed throughout the financial system, they do not set any threat to financial stability. Rather, they could represent price adjustments based on efficient financial markets, in a well-functioning financial system, that addresses investment towards low-carbon or carbon-neutral investments. This could occur even considering market failures due to the underpricing of externalities associated with carbon emissions. However, an unexpected change in public policy or technology relevant to transition can trigger a disorderly transition that could produce unpredicted price changes and heighten volatility due to uncertainty and risk aversion.

Stranded assets are fossil fuel dependent assets that suffer from unanticipated or premature write-downs, devaluations, or conversion to liabilities. The shift from these assets towards opportunities for growth has the potential to be orderly over an extended period if predictable policies and efficient and well-functioning markets are implemented. This is because obsolete investments in productive assets are replaced by greener and more efficient ways of generating economic output. However, there are many factors related to the transition to a low-carbon economies that may influence the reduction or the increase of market valuations (OECD (2021)).

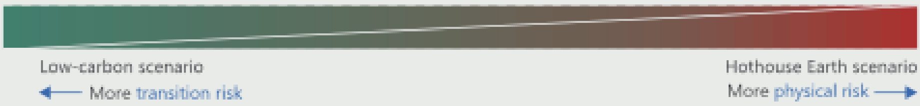
Stranded assets are one of the key drivers of downward pressure on market valuations. Another driver that decreases market valuations are higher operating costs, that are due to increased carbon pricing, unexpected shifts in energy costs and increased production costs. Revenues might be reduced by stigmatisation of carbon intensive sectors and reputational risks, as well as by higher cost of capital for firms unable to comply with the low carbon transition. The last aspects that lower valuations are uncertainty about the market and sudden policy changes that could lead to abrupt repricing of assets or securities valuations (OECD (2021)).

Rises in market valuations can occur due to a multitude of factors that improve market expectations of future cash flows or reduce the cost of capital. Gains on any assets that become in greater demand due to the rising consumption of various renewables. Using climate-related technologies and green products can increase revenues. Transitioning firms by employing potentially cheaper and more efficient production and distribution processes can increase production capacity and reduce costs. In low-carbon economies there is the possibility to access to new markets and assets, bringing opportunities for increased returns. Also, any policies that support green transition by further penalising fossil fuel usage and $CO_2$ emissions, reducing fossil fuel subsidies where they exist, or incentivising renewable energy and technologies could contribute to the transition (OECD (2021)).

Physical and transition risks are usually assessed separately, given the complexity involved in each case. However, they should be considered as part of the same framework, Figure 3 shows the interconnection between the two risks. A strong and immediate action to mitigate climate change would increase transition risks and limit physical risks, but those would remain present. In contrast, delayed and weak action to mitigate climate change would lead to higher and potentially catastrophic physical risks, without necessarily entirely removing transition risks (e.g. some climate policies are already enforced and more may soon be).

Delayed actions followed by strong actions to catch up would probably lead to high both physical and transition risks (Bolton et al. (2020)).

*Figure 3 – Framework for physical and transition risk*




| Scenario | Rapid Transition | Two-degree | Business-as-intended | Business-as-usual |
|---|---|---|---|---|
| Corrective transition response | Very strong | Strong | Substantial | Limited |
| Change in temperature, 2100 vs pre-industrial era | 1.5°C | 2°C | 3°C | 4°C |

*Source: adapted from Oliver Wyman (2019); Bolton et al. elaboration.*

## 1.2. SUSTAINABLE INVESTING

As described above, climate change will not spare the financial sector. For this reason, in recent years people are increasingly prompt to make sustainable investments[3]. Sustainable investing is an investment approach that considers not only financial but also environmental, social and governance (ESG) objectives.

The model of investing of Pàstor et al. (2020), that I present below, is based on environmental, social and governance (ESG) principles. The authors sort firms by the sustainability of their activities. "Green" firms generate positive externalities for society, "brown" firms impose negative externalities, and there are different shades of green and brown. Model's predictions are about the CAPM alphas, that in equilibrium are negative for green assets whereas positive for brown ones. Green assets' negative alphas stem from investors' preference for green holdings and from green stocks' ability to hedge climate risk. Green assets can nevertheless outperform brown ones during good performance of the ESG factor, which embeds changes in customers' tastes for green products and investors' tastes for green holdings.

Agents have different preferences for sustainability, or "ESG preferences," which have multiple dimensions. First, agents derive utility from holdings of green firms and disutility from holdings

[3] *According to the 2020 Global Sustainable Investment Review, global sustainable investment reach USD 35.3 trillion in five major markets and a 15% in the past two years (2018-2020). Sustainable investment assets under management make up a total of 35.9% of total assets under management, up from 33.4% in 2018.*

of brown firms. Second, agents care about firms' aggregate social impact. In a model extension, agents additionally care about climate risk. Naturally, agents also care about financial wealth.

Asset prices are affected by agents' tastes for green holdings. The greener the firm, the lower is its cost of capital in equilibrium. Since green assets have negatives alphas, agents with stronger ESG preferences, whose portfolios tilt more toward green assets and away from brown assets, earn lower expected returns. However, these agents are satisfied because they earn utility from their holdings and not just from expected returns.

The model implies that sustainable investing leads to a positive social impact. Social impact is defined by the authors as the product of a firm's ESG characteristic and the firm's operating capital. Agents' tastes for green holdings rise firms' social impact, through two channels. First, firms choose to become greener because greener firms have higher market values, this is also due to the transition risk. Second, real investment shifts from brown to green firms, due to shifts in firms' cost of capital (higher for brown firms, lower for green firms). A positive aggregate social impact is obtained even if agents have no direct preference for it, shareholders do not participate with management, and managers simply maximize market value.

The authors extend the baseline model to include climate in investor's utility function. Expected returns depends not only on market betas and investors' tastes, but also on climate betas, which measure firms' exposures to climate shocks. Evidence suggests that brown assets have higher climate betas than green assets (Engle et al. (2019)), for this reason in the model brown assets' have higher expected returns. The idea is that investors dislike unexpected deteriorations in the climate. If the climate worsens unexpectedly, brown assets lose value relative to green assets (e.g., due to new government regulation that penalizes brown firms). Because brown firms lose value in states of the world investors dislike, they are riskier, so they must offer higher expected returns. Lower CAPM alphas for green stocks are driven not only by investors' tastes for green holdings, but also by their ability to better hedge climate risk.

The extension of the authors about the theoretical treatment of climate risk is related to recent empirical works about the implications of such risk in asset pricing. Hong et al. (2019) analyses the response of food stock prices to climate risks. Bolton and Kacperczyk (2020) argue that investors demand compensation for exposure to carbon risk in the form of higher returns on carbon-intensive firms. Ilhan et al. (2019) show that firms with higher carbon emissions exhibit more tail risk and more variance risk. Engle et al. (2019) develop a procedure to dynamically hedge climate risk by constructing mimicking portfolios that hedge innovations in climate news series obtained by textual analysis of news sources. Bansal et al. (2016) model climate change as a long-run risk factor. Krueger et al. (2019) find that institutional investors consider climate risks to be important investment risks. Ardia et al. (2021) tested the prediction of the authors showing that green firms tend to outperform brown firms when concerns about climate change increase, using data for S&P 500 companies from January 2010 to June 2018.

### 1.2.1. THE BASELINE MODEL

The model of Pàstor et al. (2020) considers a single period, from time 0 to time 1, in which there are $N$ firms, n = 1,.. . , $N$. Let $\tilde{r}_n$ denote the return on firm $n$'s shares in excess of the riskless rate, $r_f$, and let $\tilde{r}$ be the $N \times 1$ vector whose nth element is $\tilde{r}_n$. We assume $\tilde{r}$ is normally distributed:

$$\tilde{r} = \mu + \tilde{\epsilon}\,, \tag{1}$$

where $\mu$ contains equilibrium expected excess returns and $\tilde{\epsilon} \sim N\ (0, \Sigma)$. In addition to financial payoffs, firms produce social impact. Each firm n has an observable "ESG characteristic" $g_n$, which can be positive (for "green" firms) or negative (for "brown" firms). Firms with $g_n > 0$ have positive social impact, meaning they generate positive externalities (e.g., cleaning up the environment). Firms with $g_n < 0$ have negative social impact, meaning they generate negative externalities (e.g., polluting the environment).

There is a continuum of agents who trade firms' shares and the riskless asset. The riskless asset is in zero net supply, whereas each firm's stock is in positive net supply. Let $X_i$ denote an $N \times 1$ vector whose $n$th element is the fraction of agent $i$'s wealth invested in stock $n$. Agent $i$'s wealth at time 1 is $W_{1i} = W_{0i}(1 + r_f + X'_i\tilde{r})$, where $W_{0i}$ is the agent's initial wealth. Agents derive utility also from holding green stocks and disutility from holding brown stocks. Each agent $i$ has an exponential (CARA) utility function

$$V\left(\widetilde{W}_{1i}, X_i\right) = -e^{-A_i\widetilde{W}_{1i} - b'_i X_i}\,, \tag{2}$$

$A_i$ is the agent's absolute risk aversion and $b_i$ is an $N \times 1$ vector of nonpecuniary benefits that the agent derives from her stock holdings. Holding the riskless asset brings no benefit.

This exponential utility function is an extended version of the one used by Arrow (1965) to elaborate the concept of absolute risk aversion. The extension lies in the presence of the term $b'_i X_i$, in the paper of Arrow it was assumed that the agent derives utility only from wealth and not from other nonpecuniary benefits. The benefit vector has agent-specific and firm-specific components:

$$b_i = d_i g \,, \tag{3}$$

where $g$ is an $N \times 1$ vector whose $n$th element is $g_n$ and $d_i \geq 0$ is a scalar measuring agent $i$'s "ESG taste." Agents with higher values of $d_i$ have stronger tastes for the ESG characteristics of their holdings.

Due to their infinitesimal size, agents take asset prices as given when choosing their optimal portfolios, $X_i$ , at time 0. To derive the first-order condition for $X_i$, it is computed the expectation of agent $i$'s utility in equation (2) and differentiate it with respect to $X_i$. Agent $i$'s portfolio weights are

$$X_i = \frac{1}{a_i} \Sigma^{-1} \left( \mu + \frac{1}{a_i} b_i \right), \tag{4}$$

where $a_i \equiv A_i W_{0i}$ is agent $i$'s relative risk aversion. For tractability, it is assumed that $a_i = a$ for all agents. We define $w_i$ to be the ratio of agent $i$'s initial wealth to total initial wealth: $w_i = \frac{W_{0i}}{W_0}$, where $W_0 = \int_i W_{0i} di$. The market-clearing condition requires that $x$, the $N \times 1$ vector of weights in the market portfolio, satisfies

$$x = \frac{1}{a} \sum{}^{-1} \mu + \frac{\bar{d}}{a^2} \sum{}^{-1} g \,, \tag{5}$$

$\bar{d} = \int_i w_i d_i di \geq 0$ is the wealth-weighted mean of ESG tastes di across agents. Note that $\bar{d} > 0$ unless the mass of agents who care about ESG is zero. Solving for $\mu$ gives

$$\mu = a \Sigma x - \frac{\bar{d}}{a} g \,. \tag{6}$$

Multiplying by $x'$ gives the market equity premium, $\mu_M = x'\mu$:

$$\mu_M = a\sigma_M^2 - \frac{\bar{d}}{a}x'g\ , \tag{7}$$

$\sigma_M^2 = x'\sum x$ is the variance of the market return. In general, the equity premium depends on the average of ESG tastes, $\bar{d}$, through $x'g$, which is the overall "greenness" of the market portfolio. If the market is net green ($x'g > 0$) then stronger ESG tastes (i.e., larger) reduce the equity premium. If the market is net brown ($x'g < 0$), stronger ESG tastes increase the premium. For simplicity, it is assumed that the market portfolio is ESG-neutral,

$$x'g = 0\ , \tag{8}$$

so that the equity premium is independent of agents' ESG tastes. In this case, equation (7) implies $a = \frac{\mu_M}{\sigma_M^2}$ . Combining this with equation (6) and noting that the vector of market betas is $\beta = \left(\frac{1}{\sigma_M^2}\right)\sum x$, it is obtained the equation for the expected excess returns in equilibrium

$$\mu = \mu_M\beta - \frac{\bar{d}}{a}g\ . \tag{9}$$

Equation (9) shows that the expected excess returns deviate from the values postulated by the CAPM (Sharpe (1964)), $\mu_M\beta$, because this model incorporates ESG tastes for holding green stocks.

### 1.2.2. INCLUDING CLIMATE RISK

Global climate change (part of "E" in ESG) is one of the main concerns that motivates people to sustainably invest. Many experts expect climate change to impair quality of life, essentially lowering utility of the typical individual beyond what is captured by climate's effect on wealth. Unforeseen climate changes present investors with an additional source of risk. Therefore, the authors have included climate in agents' utility function to also capture this type of risk.

Let $\tilde{C}$ denote climate at time 1, which is unknown at time 0. The utility function for individual $i$ in equation (2) is modified to include $\tilde{C}$ as follows:

$$V(\widetilde{W}_{1i}, X_i, \tilde{C}) = -e^{-A_i\widetilde{W}_{1i} - b'_i X_i - c_i\tilde{C}}\,, \tag{10}$$

where $c_i \geq 0$, so that agents dislike low realizations of $\tilde{C}$. Define $\bar{c} \equiv \int_i w_i c_i di$, the wealth-weighted mean of climate sensitivity across agents. It is assumed is $\tilde{C}$ normally distributed, and without loss of generality we set $\mathrm{E}\{\tilde{C}\} = 0$ and $\mathrm{Var}\{\tilde{C}\} = 1$.

The equation for the expected excess returns in equilibrium by considering climate risk is different from (9). In this circumstance, the expected excess returns are given by

$$\mu = \mu_M \beta - \frac{\bar{d}}{a} g + \bar{c}(1 - \rho_{MC}^2)\psi\,, \tag{11}$$

where $\psi$ is the $N \times 1$ vector of "climate betas," that is slope coefficients on $\tilde{C}$ in a multivariate regression of $\tilde{\epsilon}$ on both $\tilde{\epsilon}_M$ and $\tilde{C}$, and $\rho_{MC}$ is the correlation between $\tilde{\epsilon}_M$ and $\tilde{C}$. Even this equilibrium expected excess return differs from the one hypothesised by the CAPM (Sharpe (1964)).

Expected returns depend on climate betas, $\psi$, which represent firms' exposures to non- market climate risk. To understand the regression defining $\psi$, recall that $\tilde{\epsilon}$ is an $N \times 1$ vector of unexpected stock returns from equation (1) and $\tilde{\epsilon}_M$ is the unexpected market return. A firm's climate beta is its loading on $\tilde{C}$ after controlling for the market return.

Compared to equation (9), expected excess returns contain an additional component given by the last term on the right-hand side of equation (11). Stock $n$'s climate beta, $\psi_n$, positively affects expected return. Thus, a stock with a negative $\psi_n$, which provides investors with a climate-risk hedge, has a lower expected return than it would in the absence of climate risk. Vice versa, a stock with a positive $\psi_n$, which performs particularly poorly when the climate worsens unexpectedly, has a higher expected return.

Green stocks seem more likely than brown stocks to hedge climate risk. This hedging asymmetry can be motivated through two channels, the customer one and the investor one. The first channel is related to the unexpected worsening of the climate which can heighten consumers' climate concerns, prompting greater demands for goods and services of greener providers. These demands can arise not only from consumers' choices but also from government regulation. Negative climate shocks can lead governments to adopt regulations that favour green providers or penalize brown ones. By considering the investor channel, unexpected worsening of the climate can strengthen investors' preference for green holdings (i.e., increase $\bar{d}$). For example, Choi et al. (2019) show that retail investors sell carbon-intensive firms in extremely warm months, consistent with $\bar{d}$ rising in such months.

Evidence also suggests that greener stocks are better climate hedges. There are many empirical studies confirming that returns on green (brown) stocks have positive (negative) correlations with adverse climate shocks. For example, Choi et al. (2019) show that green firms, as measured by low carbon emissions, outperform brown firms during months with abnormally warm weather, which the authors argue alert investors to climate change. Ardia et al. (2021) finds that when concerns about climate change increase unexpectedly, green firms' stock prices increase, while brown firms' decrease. Further, they conclude that climate change concerns affect returns both through investors updating their expectations about firms' future cash flows and through changes in investors' preferences for sustainability. Hence, both studies show that a high-minus-low $g_n$ stock portfolio is a good hedge against climate risk, indicating that $g_n$ is negatively correlated with $\psi_n$ across firms.

In the special case where this negative correlation is perfect, so that

$$\psi_n = -\xi g_n \,, \tag{12}$$

where $\xi > 0$ is a constant, equation (11) simplifies to

$$\mu = \mu_M \beta - \left[\frac{\bar{d}}{a} + \bar{c}(1 - \rho_{MC}^2)\xi\right] g \,. \tag{13}$$

Stock n's CAPM alpha is then given by

$$\alpha_n = -\left[\frac{\bar{d}}{a} + \bar{c}(1 - \rho_{MC}^2)\xi\right] g_n \,. \tag{14}$$

Greener stocks have lower CAPM alphas not only because of investors' tastes for green holdings, but also because of greener stocks' ability to better hedge climate risk. Therefore, climate risk represents a further motivation to expect green stocks to underperform brown ones over the long run. For the same reason, green stocks have a lower cost of capital than brown stocks relative to the CAPM.

Climate risk implies that is possible to construct a climate-hedging portfolio whose agent $i$'s equilibrium portfolio weights are given by

$$X_i = x + \frac{\delta_i}{a^2}\left(\sum^{-1} g\right) - \frac{\gamma_i}{a}\left(\sum^{-1} \sigma_{\epsilon C}\right), \tag{15}$$

$\gamma_i \equiv c_i - \bar{c}$ and $\sigma_{\epsilon C}$ is an $N \times 1$ vector of covariances between $\tilde{\epsilon}_n$ and $\tilde{C}$. The weights of this climate-hedging portfolio are proportional to $\sum^{-1} \sigma_{\epsilon C}$. Agents with $\gamma_i > 0$, whose climate sensitivity is above average, short the hedging portfolio, whereas agents with $\gamma_i < 0$ go long.

The climate-hedging portfolio, $\sum^{-1} \sigma_{\epsilon C}$, is a natural mimicking portfolio for $\tilde{C}$. To see this, note that the $N$ elements of $\sum^{-1} \sigma_{\epsilon C}$ are the slope coefficients from the multiple regression of $\tilde{C}$ on $\tilde{\epsilon}$. Therefore, the return on the hedging portfolio has the highest correlation with $\tilde{C}$ among all portfolios of the $N$ stocks. Investors in the model hold this maximum-correlation portfolio, to various degrees determined by their $\gamma_i$, to hedge climate risk.

The climate-hedging portfolio is likely to favour green stocks over brown. This is because green stocks are generally better climate hedges than brown stocks. However, the climate-hedging portfolio is not necessarily simply long green stocks and short brown ones. Even if $\sigma_{\epsilon C}$ were perfectly correlated with $g$ across firms, $\sigma_{\epsilon C}$ in equation (15) is multiplied by $\sum^{-1}$, which in general makes the climate-hedging portfolio weights imperfectly aligned with stocks' ESG characteristics.

## 1.3. HEDGING CLIMATE RISK IN ASSET MANAGEMENT

The model of Pàstor et al. (2020) is a theoretical framework useful to interpret the new trends in the asset management industry. Public and private investors have invested almost USD 200 billion per year since 2015 in renewables asset finance, with several indices and products being made available to investors on the secondary market (OECD (2021)). The 2021 Financial Markets and Climate Transition report released by the OECD signals that there has been a growth in dedicated renewables investment, including climate transition indices and portfolios, and dedicated climate transition funds and exchange traded funds (ETFs). Climate transition indices and portfolios adopt climate-related risk and impact screening methodologies to implement holistic or sector-specific investment strategies with the goal of addressing specific carbon reduction objectives or to support carbon-neutral strategies. Typically, in these products will be included stocks of renewable energy companies (green firms), or undertaken specific low-carbon, conservation, or renewables projects. Beyond indices and benchmarks, broader asset managers' and asset owners' initiatives are shaping expectations about engagement with corporate issuers and transitions to net-zero climate paths (OECD (2021)). In December 2020 the largest 30 asset managers[4], that manage USD 9 trillion in assets, committed to a Net Zero Asset Managers Initiative that aims to use stewardship engagement with their invested firms to smooth the path toward net zero emissions by 2050.

Climate change may result in a new economic order because of the transition to a low-carbon economy. In this context, the main challenge is to understand how the financial system is resilient to climate-related risks. Therefore, managing, and hedging these risks have become the primary concerns in asset management industry.

In financial literature it is possible to find numerous papers that propose different approaches of hedging against climate risk. These methodologies show how to keep the tracking error with respect to a benchmark index at a negligible level. Andersson et al. (2016) develop a dynamic investment strategy that allows long-term passive investors to hedge climate risk without sacrificing financial returns. In their paper, Jondeau et al. (2021) build portfolios with decreasing carbon footprint which passive investors can use as new Paris-consistent (PC) benchmarks, meaning that are consistent with the targets of the Paris agreement, and have the same risk- adjusted returns as business as usual (BAU) benchmarks. Bolton et al. (2021) outline

[4] *Currently the number of asset managers involved in the Net Asset Mangers Initiative is increased to 273, with USD 61.3 trillion in assets (source: www.netzeroassetmanagers.org).*

a simple and robust approach, that works for both passive and active managers, to align portfolios with a science-based, carbon budget consistent with maintaining a temperature rise below 1.5 °C with 83% probability.

The common objective of all these studies is to construct a low-carbon index starting from a reference index and reducing the index exposure to carbon-intensive stocks while increasing the position to low-carbon ones. This investment approach is called “decarbonization” of an index. In the next paragraphs I briefly outline the investment strategy implemented by Andersson et al. (2016).

### 1.3.1. PURE - PLAY CLEAN ENERGY INDEXES vs. DECARBONIZED INDEXES

Long-term investors have to manage the uncertainty with respect to the timing of climate mitigation policies, this uncertain condition is one of the main challenges they must face. An analogy with financial crises might be helpful to explain the situation, it is extremely risky for a fund manager to short an asset class that is perceived to be overvalued and subject to a speculative bubble because the fund could be forced to close in response to massive redemptions before the bubble has burst. Similarly, an asset manager looking to hedge climate risk by disinvesting from stocks with high carbon footprints bears the risk of underperforming his benchmark for as long as climate mitigation policies are postponed and market expectations about their introduction are low.

Several “green” financial indexes have existed for many years. These indexes fall into two broad groups. First, pure-play (PP) indexes that focus on renewable energy, clean technology, and/or environmental services. Second, “decarbonized” indexes, whose basic construction principle is to take a standard benchmark, such as the S&P 500 or NASDAQ 100 and remove or underweight the companies with relatively high carbon footprints. The pure-play indexes offer no protection against the timing risk of climate change mitigation policies. On the other hand, an investor holding a decarbonized index is hedged against the timing risk of climate mitigation policies, which are expected to disproportionately hit carbon-intensive companies, since the decarbonized indexes are structured to maintain a low tracking error with respect to the benchmark indexes.

*Table 2 – Pure-Play Clean Energy Indexes vs. Global Indexes*

| | S&P 500 | NASDAQ 100 | PP 1 | PP 2 | PP 3 | PP 4 | PP 5 |
|---|---|---|---|---|---|---|---|
| Annualized return | 4.79% | 11.40% | 5.02% | –8.72% | 2.26% | –8.03% | –1.89% |
| Annualized volatility | 22.3 | 23.6 | 24.1 | 39.3 | 30.2 | 33.8 | 37.3 |

*Sources: Amundi and Bloomberg (1 September 2015).*

Table 2 compares the annualized returns and volatilities of two global indexes, S&P 500, and NASDAQ 100, with the ones of five pure- play clean energy indexes[5]. As we can notice, since the inception of the financial crisis in 2007-2008, global market benchmarks have greatly overperformed PP indexes both in terms of financial returns and riskiness.

The composition of pure-play green funds is very limited because they invest only in a couple of subsectors and, in any case, cannot serve as a basis for building a core equity portfolio for institutional investors. Instead, climate risk–hedging strategy that uses decarbonized indexes go beyond the simple disinvestment of high-carbon-footprint stocks, this is just the first key step of the investment strategy. The second one is to optimize the composition and weighting of the decarbonized index to minimize the tracking error (TE) with the reference benchmark index.

The investment strategy of Andersson et al. is based on a central underlying premise, financial markets underprice carbon risk, so for the investors is like holding a "free option on carbon". As long as the introduction of significant limits on $CO_2$ emissions is postponed, they can obtain the same returns as on a benchmark index. But from the day $CO_2$ emissions are priced meaningfully and consistently and limits on $CO_2$ emissions are introduced, the decarbonized index should outperform the benchmark, thus it is like they automatically exercise the option. Moreover, authors' fundamental belief is that is that eventually financial markets will begin to price carbon risk.

This premise stands in contrast with the papers of Pàstor et al. (2020) and of Bolton and Kacperczyk (2020), that claim investors already ask a premium to hold carbon-intensive stocks meaning that carbon risk is correctly embedded in stock prices. A possible explanation of these apparently contradictory results is that they reflect the transition in investors' preferences. As investors switch their preferences toward environment-friendly firms, there is a short-term selling pressure and low-carbon firms outperform the market.

### 1.3.2. DECARBONIZED INDEX OPTIMIZATION PROBLEM

The three authors opt for the dual formulation of the optimization problem. This formulation is divided in two steps, imposing a constraint on maximum allowable tracking error with the

[5] *Table 2 gives the financial returns of several ETFs that track leading clean energy pure-play indexes. Pure Play 1 refers to Market Vectors Environmental Services ETF, Pure Play 2 to Market Vectors Global Alternative Energy ETF, Pure Play 3 to PowerShares Cleantech Portfolio, Pure Play 4 to PowerShares Global Clean Energy Portfolio, and Pure Play 5 to First Trust NASDAQ Clean Edge Green Energy Index Fund. Annualized return and volatility were calculated using daily data from 5 January 2007 to the liquidation of Pure Play 1 on 12 November 2014.*

benchmark index and then, subject to this constraint, excluding and reweighting composite stocks in the benchmark index to maximize the green index's carbon footprint reduction.

The optimization problem is performed by following two alternatives amid the many possible formulations of the constrained optimization problem for the construction of a decarbonized index that trades off exposure to carbon, tracking error, and expected returns.

It is supposed that there are $N$ constituent stocks in the benchmark index and that the weight of each stock in the index is given by $w_i^b = \left[\frac{\text{Mkt cap }(i)}{\text{Total mkt cap}}\right]$. Then, each constituent company is ranked in decreasing order of carbon intensity, $q_l^i$, with company $l = 1$ having the highest carbon intensity and company $l = N$ the lowest (each company is thus identified by two numbers $[i, l]$, with the first number referring to the company's identity and the second to its ranking in carbon intensity).

In the first problem, the green portfolio can be constructed by choosing new weights, $w_i^g$, for the constituent stocks to solve the following minimization problem:

$$\text{MinTE} = sd(R^g - R^b)\,,$$

where

$w_l^g = 0$ for all $l = 1, \dots k$

$w_l^g \geq 0$ for all $l = k+1, \dots N$

$sd =$ standard deviation

The decarbonized index is constructed by first excluding the $k$ worst performers in terms of carbon intensity and reweighting the remaining stocks in the green portfolio to minimize TE (tracking error). This optimization problem follows transparent rules of exclusion, whatever the threshold $k$.

In the second problem formulation, the first set of constraints ($w_l^g = 0$ for all $l = 1, \dots k$) is replaced by the constraint that the green portfolio's carbon intensity must be smaller than a given threshold: $\sum_{l=1 \dots N} q_l w_l^g \leq Q$. The second problem potentially does not exclude any constituent stocks from the benchmark index and seeks only to reduce the carbon intensity of the index by reweighting the stocks in the green portfolio.

For both problem formulations, the authors estimated the $ex\ ante$ TE by using a multifactor model of aggregate risk. This multifactor model significantly reduces computations, and the decomposition of individual stock returns into a weighted sum of common factor returns and specific returns provides a good approximation of individual stocks' expected returns. More formally, under the multifactor model the TE minimization problem has the following structure:

$$\text{Min}\left[\sqrt{(W^p - W^b)'\left(\boldsymbol{\beta}\boldsymbol{\Omega}_f\boldsymbol{\beta}' + \Delta^{AR}\right)(W^p - W^b)}\right],$$

where

$w_l^g = 0$ for all $l = 1, \dots k$

$w_l^g \geq 0$ for all $l = k{+}1, \dots N$

$(W^p - W^b) =$ the vector of the difference in portfolio weights of the decarbonized portfolio and the benchmark

$\boldsymbol{\Omega}_f =$ the variance – covariance matrix of factors

$\boldsymbol{\beta} =$ the matrix of factor exposures

$\Delta^{AR} =$ the diagonal matrix of specific risk variances

To figure out the potential advantages of achieving a bounded tracking error, the authors ran several simulations with the pure optimization methodology and determined a TE–carbon efficiency frontier for a decarbonized index constructed from the MSCI Europe Index.

Figure 4 illustrates that achieving a nearly 100% reduction in the MSCI Europe carbon footprint would come at the price of a huge tracking error of more than 3.5%. Even in a good scenario whereby the decarbonized index is expected to outperform the benchmark because of climate mitigation policies, such a large TE would expose investors in the decarbonized index to significant financial risk relative to the benchmark.

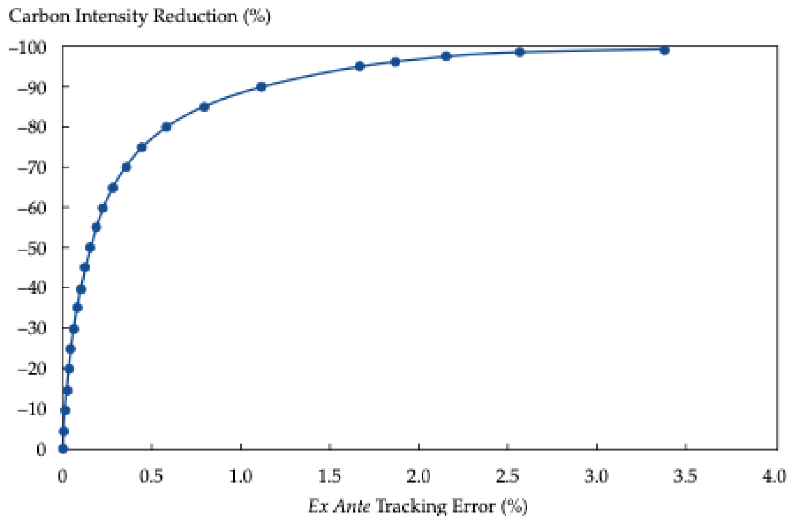



*Figure 4 – Carbon Frontier on the MSCI Europe Index*

*Source: Amundi (30 June 2015)*

Figure 5 depicts how a risk of a large TE can be mitigated by lowering the TE. The authors propose a first scenario where the expected yearly return of the green index is 2.5% higher than that of the benchmark and show (with a confidence interval of two standard deviations) that a 3.5% TE could expose investors to losses relative to the benchmark in the negative scenario. But, by lowering the TE of the decarbonized index from 3.5% to 1.2%, the decarbonized index generates returns at least as high as those of the benchmark even in the worst-case scenario.

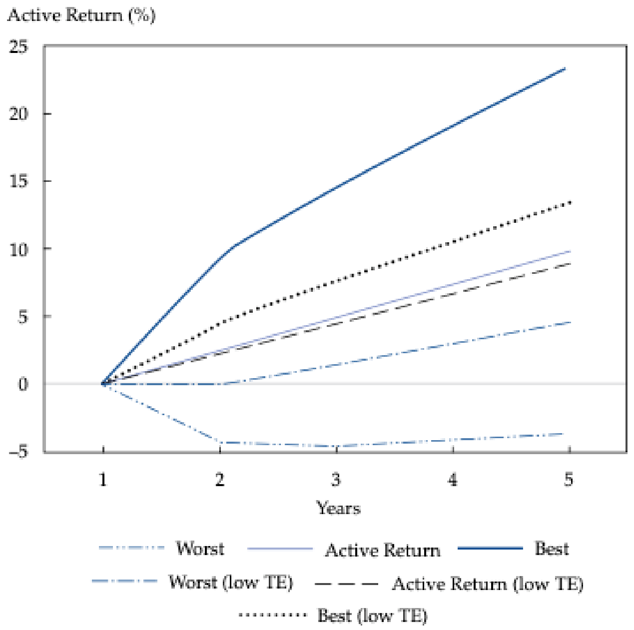



*Figure 5 – Returns and Risk with Low Tracking Error*

*Source: Amundi*

To sum up, this decarbonized index investment strategy stands on its own as a simple and effective climate risk– hedging strategy for passive long-term institutional investors, but it is also an important complement to climate change mitigation policies. Indeed, the authors argue that the boost of climate risk hedging can have real effects on reducing GHG emissions even before climate change mitigation policies are introduced.

# CHAPTER 2: Climate risk metrics

As seen before by presenting the portfolio decarbonization problem, it is fundamental to classify firms as green or brown according to their climate riskiness. Climate-related risk can be assessed by using different metrics. In the first section I present two measures of climate risks that gauge carbon footprint of firms, carbon emission and carbon intensity. The second section is devoted to the presentation of an approach to estimate the carbon beta of a firm. This value estimates the sensitivity of firm's stock returns to an acceleration in the transition process towards a low carbon economy.

## 2.1. CARBON FOOTPRINT

Carbon footprint is a generic term used to define the total greenhouse gas (GHG) emissions caused by a given system, activity, company, country, or region. Greenhouse gases are made up of water vapor ($H_2O$), carbon dioxide ($CO_2$), methane ($CH_4$), nitrous oxide ($N_2O$), Ozone ($O_3$), etc. Absorbing and emitting radiation energy, they cause the greenhouse effect. This effect was a crucial factor for the development of human life on Earth (Le Guenedal and Roncalli (2022)). Indeed, without the greenhouse effect, the average temperature of Earth's surface would be about −18°C. With the greenhouse effect, the current temperature of Earth's surface is about +15°C. Nevertheless, global warming is primarily due to the increasing concentration of some GHGs in the atmosphere. It mainly concerns carbon dioxide, and to a lesser extent, methane, and nitrous oxide (Le Guenedal and Roncalli (2022)).

Carbon dioxide equivalent ($CO_2e$) is the general unit of measure of carbon footprint, which is a term for describing different GHGs in a common unit. In this framework, a quantity of GHG is expressed as $CO_2e$ by multiplying the GHG amount by its global warming potential (GWP) (Le Guenedal and Roncalli (2022)). The GWP of a gas is the amount of $CO_2$ that would warm the earth equally. For instance, the IPCC's 4th Assessment Report has used the following rules: 1 kg of methane corresponds to 25 kg of $CO_2$ and 1 kg of nitrous oxide corresponds to 298 kg of $CO_2$. Thanks to a common unit, it is possible to compare properly two companies. However, if the two companies have different sizes their carbon emissions cannot be fairly compared. Therefore, it can be useful to adopt normalized metrics by transforming carbon emissions into carbon intensities.

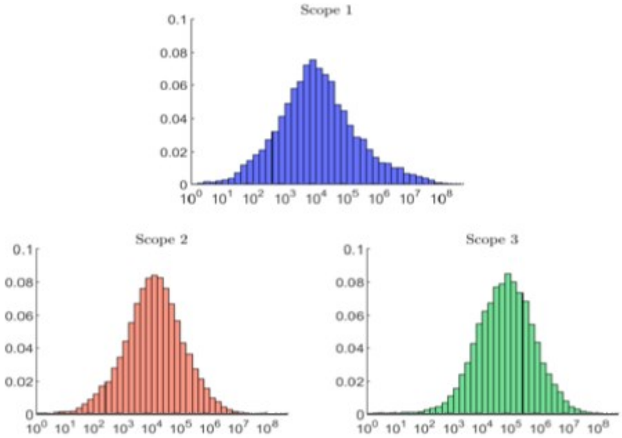
### 2.1.1. CARBON EMISSIONS

Since emissions are not physically material, GHG is an uncertain indicator at the company level. To provide a common accounting basis that can be used by states and companies, and to limit the measurement gap, the Greenhouse Gas Protocol (GHGP) classifies a company's greenhouse gas emissions in three scopes[6]:

- Scope 1 denotes direct GHG emissions occurring from sources that are owned or controlled by the company.
- Scope 2 corresponds to the indirect greenhouse gas emissions from consumption of purchased electricity, heat, or steam. Scope 2 emissions can be computed using the energy mix of the country (location-based) or the energy mix of the utility company supplying the electricity.
- Finally, Scope 3 are other indirect emissions, such as the extraction and production of purchased materials and fuels, transport-related activities in vehicles not owned or controlled by the reporting entity, electricity-related activities not covered in Scope 2. Scope 3 upstream emissions include the indirect emissions that come from the supply side, while scope 3 downstream emissions are mostly associated with the product sold by the entity.

In their paper, Le Guenedal and Roncalli (2022) distinguish these three carbon emission measures by introducing the notations $\mathcal{CE}_1$, $\mathcal{CE}_2$ and $\mathcal{CE}_3$. In carbon emission databases, carbon scopes can be self-reported or estimated and are generally expressed in tons of carbon dioxide equivalent or $tCO_2e$. The authors retrieve emissions data, for the year 2019, of about 15700 corporations by using S&P Trucost database. When the data are not available, they provide estimated values. Figure 6 reports the probability distribution of the carbon emission for the three scopes.

*Figure 6 – Histogram of carbon emission (log scale, $tCO_2e$)*




*Source: Trucost reporting year 2019 & Le Guenedal and Roncalli's calculations*

[6] *The standards can be found at www.ghgprotocol.org.*

Summing up all the observations, the total carbon emissions (expressed in $GtCO_2e$) for each scope are the following: $\mathcal{CE}_1 = 15.57$, $\mathcal{CE}_2 = 2.45$ and $\mathcal{CE}_3 = 10.17$. It follows that the total emissions of these corporate firms are about 28.2 $GtCO_2e$, which represents more than 75% of the 36 $GtCO_2e$ global emissions.

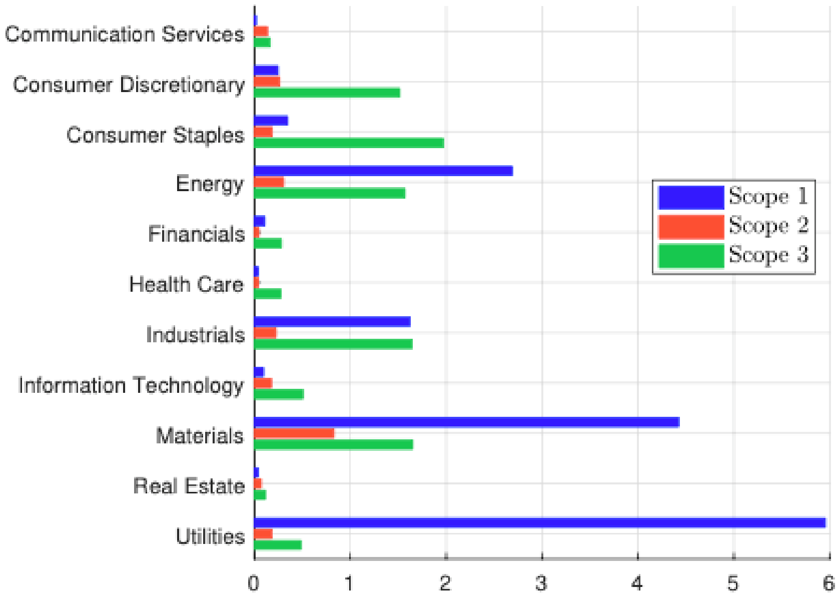



*Figure 7 – Total absolute scopes per GICS sector in $GtCO_2e$*

*Source: Trucost reporting year 2019 & Le Guenedal and Roncalli's calculations*

Figure 7 depicts the breakdown of these GHG emissions by GICS sector[7]. It is evident that the direct emissions (Scope 1) are concentrated in few sectors: Utilities, Materials, Energy and to some extent Industrials. Scope 2 emissions are more uniformly distributed. On the other hand, every sector's scope 2 emissions are already accounted for in the scope 1 emissions of the Utilities sector. This double-counting problem becomes particularly challenging when trying to build the carbon footprint of a portfolio. Finally, scope 3 emissions are discriminant for Information Technology, Health Care, Consumer Staples, Consumer Discretionary and Financials. However, scope 3 emissions are still uncertain to be estimated. Another important issue is the distribution of the absolute emissions within GICS sectors. In contrast with commonly used centered scores, the emissions are concentrated on very few actors. Le Guenedal and Roncalli filter the outliers[8] to obtain the quantile plot (QQ-plot) of the values (Figure 8). Since there are multiple scaling issues (sector, size, country), absolute emissions are extremely difficult to use in portfolio construction.

[7] *In 1999, MSCI and S&P Dow Jones Indices developed the Global Industry Classification Standard (GICS), seeking to offer an efficient investment tool to capture the breadth, depth and evolution of industry sectors (source: www.msci.com).*

[8] *The authors exclude the companies below the $20^{th}$ percentile and above the $80^{th}$ percentile.*

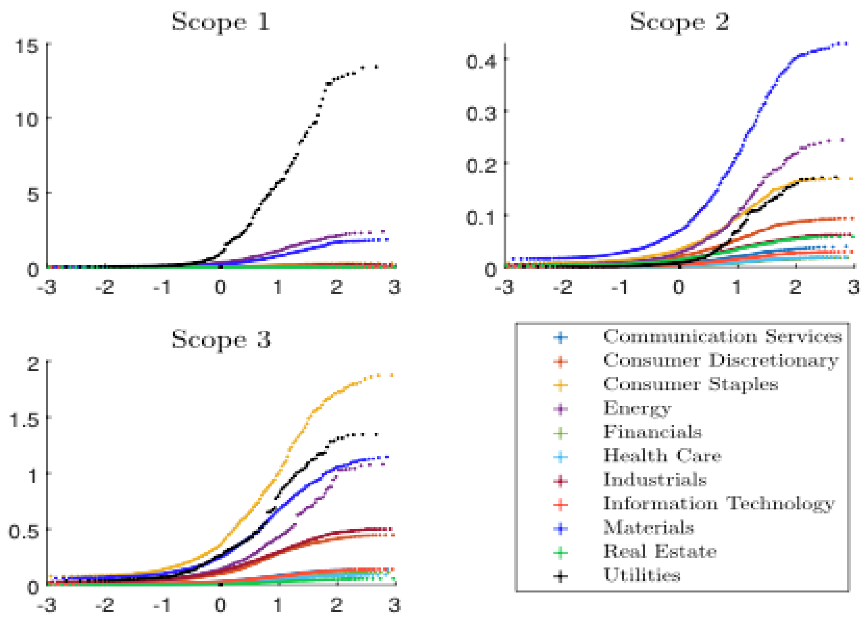



*Figure 8 – QQ-plot of carbon scopes per GICS sector in $MtCO_2e$*

*Source: Trucost reporting year 2019 & Le Guenedal and Roncalli's calculations*

### 2.1.2. CARBON INTENSITY

The carbon intensity of company $i$ with respect to scope $j$ is a normalization of the carbon emissions:

$$CI_{i,j} = \frac{C\mathcal{E}_{i,j}}{Y_i}$$

$C\mathcal{E}_{i,j}$ is the company's absolute scope $j$ emissions and $Y_i$ is an output indicator measuring its activity. In general, revenues (expressed in \$) are used to compute carbon intensities. For some major sectors, it is possible to find intensities per production unit. For instance, for a company from the Utilities sector, a carbon intensity in $CO_2e/kWh$ is more informative in terms of carbon efficiency than a carbon intensity in $CO_2e/\$$. In Table 3 are reported some examples of carbon intensities estimated by Le Guenedal and Roncalli (2022) by using revenues as normalization factor.

| Company | Emission (in $tCO_2e$) | | | Revenue | Intensity (in $tCO_2e$/\$ mn) | | |
|---|---|---|---|---|---|---|---|
| | Scope 1 | Scope 2 | Scope 3 | (in \$ mn) | Scope 1 | Scope 2 | Scope 3 |
| Alphabet | 74 462 | 5 116 949 | 7 166 240 | 161 857 | 0.460 | 31.614 | 44.275 |
| Amazon | 5 760 000 | 5 500 000 | 20 054 722 | 280 522 | 20.533 | 19.606 | 71.491 |
| Apple | 50 463 | 862 127 | 27 618 943 | 260 174 | 0.194 | 3.314 | 106.156 |
| BP | 49 199 999 | 5 200 000 | 103 840 194 | 276 850 | 177.714 | 18.783 | 375.077 |
| Danone | 722 122 | 944 877 | 28 969 780 | 28 308 | 25.509 | 33.378 | 1023.365 |
| Enel | 69 981 891 | 5 365 386 | 8 726 973 | 86 610 | 808.016 | 61.949 | 100.762 |
| Juventus | 6 665 | 15 739 | 35 842 | 709 | 9.401 | 22.198 | 50.553 |
| LVMH | 67 613 | 262 609 | 11 853 749 | 60 083 | 1.125 | 4.371 | 197.291 |
| Microsoft | 113 414 | 3 556 553 | 5 977 488 | 125 843 | 0.901 | 28.262 | 47.500 |
| Nestle | 3 291 303 | 3 206 495 | 61 262 078 | 93 153 | 35.332 | 34.422 | 657.647 |
| Netflix | 38 481 | 145 443 | 1 900 283 | 20 156 | 1.909 | 7.216 | 94.277 |
| Total | 40 909 135 | 3 596 127 | 49 831 487 | 200 316 | 204.223 | 17.952 | 248.764 |
| Volkswagen | 4 494 066 | 5 973 894 | 65 335 372 | 282 817 | 15.890 | 21.123 | 231.016 |

*Table 3 – Examples of carbon emissions and intensity*

*Source: Trucost reporting year 2019*

From Table 3 is possible to observe the heterogeny of the economic sizes of the companies, so the comparison of their carbon emissions does not make any sense. This explains why in portfolio management the carbon intensity measure is more popular and efficient than the carbon emission measure. These examples also illustrate how interconnected the three scopes are. Thus, if a company considerably outsources the manufacturing of its products, it reduces its scope 1 but increases its scope 3 emissions. Moreover, it is possible to deal with more disaggregated data, for example by making the distinction between upstream and downstream emissions resulting from the entire supply chain. However, this disaggregation is not straightforward since scope 3 is not accurately assessed as the other scopes.

The authors recognize another advantage of carbon intensity, it can reduce the skewness of the distribution compared to absolute emissions. Figure 9 shows the probability distribution of the carbon intensity when the normalization variable is revenues in dollars.

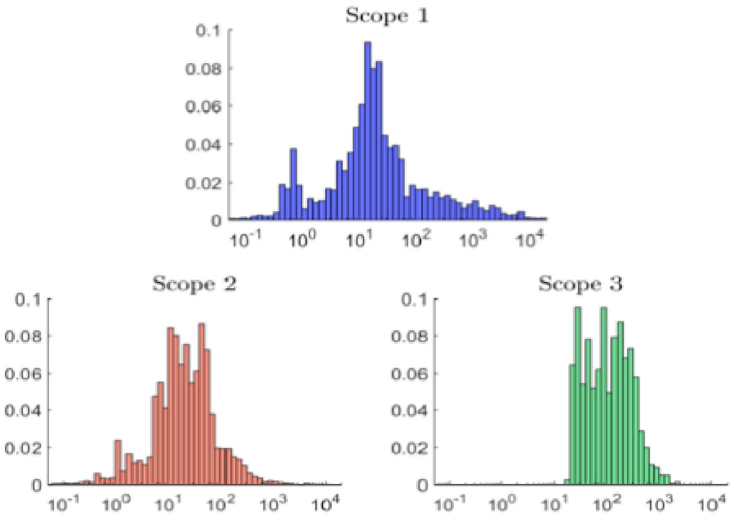



*Figure 9 – Histogram of carbon intensity (log scale, $tCO_2e/\$$ mn)*

*Source: Trucost reporting year 2019 & Le Guenedal and Roncalli's calculations*

In Table 4 are reported some statistics computed by the authors (average, median, 95% percentile and maximum) where they verify the skewness reduction. They provide an example, by computing the ratio between $Q$ (95%) and the median, it takes the value 233, 43 and 37 for the three scopes when are considered carbon emissions. These figures become 69, 9 and 5 when the ratio is computed by considering carbon intensity.

| Scope | Emission (in $10^6 \cdot tCO_2e$) | | | | Intensity (in $10^3 \cdot tCO_2e/\$$ mn) | | | |
|---|---|---|---|---|---|---|---|---|
| | Avg. | Med. | $Q$ (95%) | Max. | Avg. | Med. | $Q$ (95%) | Max. |
| 1 | 0.992 | 0.010 | 2.28 | 587.1 | 0.277 | 0.016 | 1.14 | 207.4 |
| 2 | 0.156 | 0.012 | 0.53 | 99.1 | 0.053 | 0.021 | 0.19 | 11.9 |
| 3 | 0.648 | 0.067 | 2.50 | 137.5 | 0.170 | 0.099 | 0.51 | 2.0 |

*Table 4 – Statistics of carbon emissions and intensity*

*Source: Trucost reporting year 2019 & Le Guenedal and Roncalli's calculations*

Figure 10 depicts the Spearman correlations between the several carbon metrics. Because of the economic size effect, it is verified that carbon emissions are more correlated than carbon intensities (80% vs. 55% on average). Regarding these latter measures, it is observed a high correlation of 66% between $CI_1$ and $CI_3$. Nevertheless, this figure is primarily explained by the sector effect. Indeed, the authors compute rank correlations by sector (Table 5) and observe that correlations are lower except for some specific sectors such as Utilities.

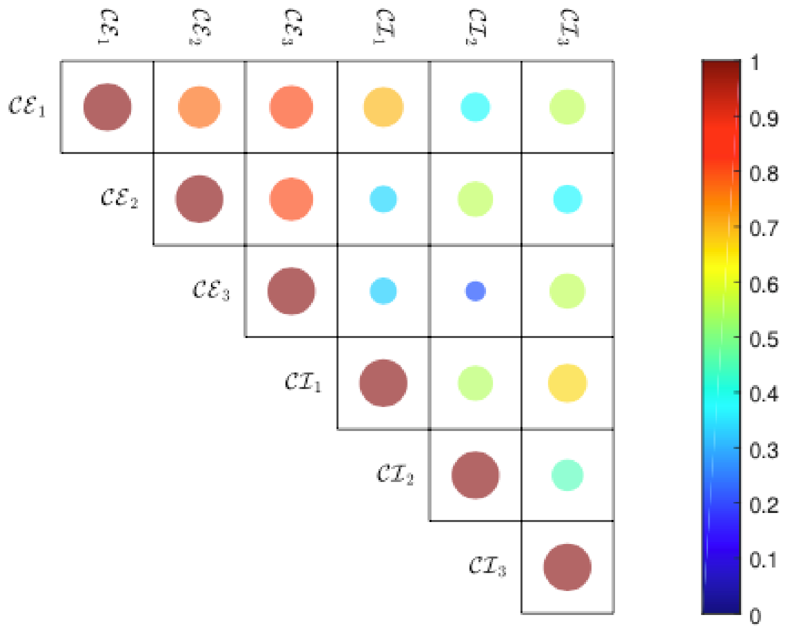



*Figure 10 – Rank correlations matrix of carbon metrics*

*Source: Trucost reporting year 2019 & Le Guenedal and Roncalli's calculations*

| Sector | $\mathcal{CE}_1$ $\mathcal{CE}_2$ | $\mathcal{CE}_2$ $\mathcal{CE}_3$ | $\mathcal{CE}_1$ $\mathcal{CE}_3$ | $\mathcal{CI}_1$ $\mathcal{CI}_2$ | $\mathcal{CI}_2$ $\mathcal{CI}_3$ | $\mathcal{CI}_1$ $\mathcal{CI}_3$ |
|---|---|---|---|---|---|---|
| Communication Services | 89.2 | 91.8 | 89.2 | 36.3 | 27.1 | 11.6 |
| Consumer Discretionary | 80.4 | 76.2 | 80.3 | 34.8 | 7.7 | 29.1 |
| Consumer Staples | 74.1 | 81.3 | 76.0 | 39.4 | 30.4 | 56.6 |
| Energy | 69.6 | 75.8 | 72.9 | 35.6 | 0.7 | 9.6 |
| Financials | 83.7 | 87.5 | 87.1 | 26.4 | 58.3 | −4.3 |
| Health Care | 93.9 | 94.7 | 94.3 | 26.4 | −12.4 | 25.7 |
| Industrials | 69.4 | 82.2 | 69.5 | 27.9 | 42.3 | 12.7 |
| Information Technology | 83.1 | 92.2 | 81.7 | 66.2 | 72.9 | 55.4 |
| Materials | 79.3 | 79.7 | 73.4 | 50.1 | 4.5 | 28.9 |
| Real Estate | 76.5 | 69.2 | 79.9 | 9.5 | −27.3 | 29.9 |
| Utilities | 34.7 | 55.9 | 83.1 | −14.0 | −4.8 | 68.5 |

*Table 5 – Rank correlations per sector*

*Source: Trucost reporting year 2019 & Le Guenedal and Roncalli's calculations*

## 2.2. CARBON BETA

In their article, Roncalli et al. (2020) defines carbon risk from a financial point of view and consider that the carbon risk of equities corresponds to the market risk priced in by the stock market. They claim the carbon beta to be the carbon-related systematic risk of a stock. Therefore, they elaborate a market-based approach that allows to manage this market risk and, since it is market-based, to mitigate the issue of a lack of climate change-relevant information. The framework they use as a starting point is the seminal paper of Görgen et al. (2019).

The objective of the carbon risk management (Carima) project, developed by Görgen et al. (2019), was to implement “a quantitative tool in order to assess the opportunities of profits and the risks of losses that occur from the transition process”. Görgen et al. (2019) extended the Fama-French-Carhart model by including a brown-minus-green (or BMG) risk factor. Relying on the technique of sorted portfolios promoted by Fama and French (1992), they build a factor-mimicking portfolio based on a scoring model. Then, they defined the carbon financial risk of a stock using its price sensitivity to the BMG factor (carbon beta).

Roncalli et al. (2020) enrich the original approach of Görgen et al. (2019) by estimating a time-varying model to analyze the dynamics of the carbon risk. Moreover, they postulate the distinction between relative and absolute carbon risk. Relative carbon risk might be considered as forward-looking measure of the carbon footprint, where the objective is to be more exposed to green firms than to brown firms. In this case, this is equivalent to encourage holding stocks with a negative carbon beta over stocks with a positive carbon beta. Absolute carbon risk deems that both large positive and negative carbon beta values have a financial risk that must be reduced.

### 2.2.1. BMG FACTOR

To build the BMG factor Görgen et al. (2019) employ a large amount of climate-relevant information provided by different databases. The methodology to develop this new common risk factor required two steps.

The first step is devoted to the development of a brown-green score (BGS) to assess if a firm is green, neutral, or brown. This scoring system uses four ESG databases over the period from 2010 to 2016: Thomson Reuters ESG, MSCI ESG Ratings, Sustainalytics ESG ratings and the

Carbon Disclosure Project (CDP) climate change questionnaire. Overall, 55 carbon risk proxy variables are retained. Then, Görgen et al. (2019) classified the variables into three different dimensions that may affect the firm's stock value in the event of unforeseen shifts towards a low carbon economy:

1. Value chain, that is the impact of a climate policy or a cap-and-trade[9] system on the different activities of a firm, such as inbound logistics and supplier chain, manufacturing production and sales;
2. Public perception, that is the external environmental image of a firm, such as ratings, controversies and disclosure of environmental information;
3. Adaptability, that is the capacity of the firm to shift towards a low carbon strategy without substantial efforts and losses.

The dimension of the value chain mainly deals with current emissions while the adaptability dimension regards potential future emissions determined by emission reduction targets and spending in environmental R&D. The Carima project considers the browner the firm the higher the value of the variable. Therefore, each variable (except the dummies) is converted into a dummy derived with respect to the median, meaning that 1 corresponds to a brown value and 0 corresponds to a green value. Then, three scores are created and correspond to the average of all variables contained in each dimension: the value chain VC, the public perception PP, and the non-adaptability NA. It follows that each score has a range between 0 and 1. Görgen et al. (2019) used the following equation to define the brown-green score (BGS):

$$\mathrm{BGS}_i(t) = \frac{2}{3}\left(0.7 \cdot \mathrm{VC}_i(t) + 0.3 \cdot \mathrm{PP}_i(t)\right) + \frac{\mathrm{NA}_i(t)}{3}\left(0.7 \cdot \mathrm{VC}_i(t) + 0.3 \cdot \mathrm{PP}_i(t)\right) \tag{1}$$

The brownness of the firm is higher the higher is the BGS value. In case of unexpected changes in the transition process the value chain and public perception directly impact stock prices. However, Görgen et al. (2019) considered that the impact of the value chain score is more significant than the impact of the public perception score. The equity value is influenced by the adaptability in a different way. Indeed, adaptability moderates the upward or downward impacts of the other two dimensions.

[9] *The cap-and-trade is a system designed to reduce carbon emissions of firms. The cap on greenhouse gas emissions is a firm limit on pollution. The trade part is a market for companies to buy and sell allowances that let them emit only a certain amount, as supply and demand set the price. Trading gives companies a strong incentive to save money by cutting emissions in the most cost-effective ways (source: www.edf.org).*

The less adaptable a firm is, more severe is the impact of an unexpected acceleration in the transition process. The second step consists in the construction of a BMG carbon risk factor. Carima project considers an average BGS for each stock that corresponds to the mean value of the BGS over the study period considered by the authors, from 2010 to 2016. The construction of the BMG factor exploits the methodology of Fama and French (1992, 1993), which consists in splitting the stocks into six portfolios:

| | Green | Neutral | Brown |
|---|---|---|---|
| Small | SG | SN | SB |
| Big | BG | BN | BB |

where the classification is based on the terciles of the aggregating BGS and the median market capitalization. Then, the return of the BMG factor is defined as follows:

$$R_{bmg}(t) = \frac{1}{2}\big(R_{SB}(t) + R_{BB}(t)\big) - \frac{1}{2}\big(R_{SG}(t) + R_{BG}(t)\big) \quad (2)$$

where the returns of each portfolio are value-weighted by market capitalization. The BMG factor can be integrated as a new common risk factor into a multi-factor model. Figure 11 shows the historical cumulative performance of the BMG factor. According to the graph, from 2010 to the end of 2012 brown firms slightly outperform green firms. Because of the unexpected path in the transition process towards a low carbon economy, the cumulative return fell by almost 35% in the next three years after 2012. From 2016 to the end of the study period, brown firms created a slight excess performance. Overall, the best-in-class green stocks outperform the worst-in-class green stocks over the study period with an annual return of 2.52%.


*Figure 11 – Cumulative performance of the BMG factor*

*Source: Görgen et al. (2019)*

### 2.2.2. STATIC ANALYSIS

Roncalli et al. (2020) follows the analysis of Görgen et al. (2019) to assess the relevance of the BMG factor during the study period. To do this, they consider the stocks that were present in the MSCI World index during the 2010-2018 period. As a result, their investment universe has less than 2 000 stocks, this is restricted with respect to the universe of 39 500 stocks considered by Görgen et al. (2019). Indeed, the computation of a market beta is not straightforward for some small and micro stocks because of OTC pricing and low trading activity. Therefore, calculating a carbon beta is even more arduous for such equities.

Roncalli et al. (2020) compare different common factor models to measure the information gain related to the carbon risk factor. The first considered model is the CAPM model introduced by Sharpe (1964) which is defined by:

$$R_i(t) = \alpha_i + \beta_{mkt,i} R_{mkt}(t) + \varepsilon_i(t) \tag{3}$$

$R_i(t)$is the return of asset $i$, $\alpha_i$ is the alpha of the asset $i$, $R_{mkt}(t)$ is the return of the market factor, $\beta_{mkt,i}$ is the systematic risk, the market beta, of stock $i$ and $\varepsilon_i(t)$ is the idiosyncratic risk. The authors also consider the case in which the risk is multi-dimensional with the model developed by Fama and French (1992):

$$R_i(t) = \alpha_i + \beta_{mkt,i} R_{mkt}(t) + \beta_{smb,i} R_{smb}(t) + \beta_{hml,i} R_{hml}(t) + \varepsilon_i(t) \tag{4}$$

$R_{smb}(t)$ is the return of the size (or small minus big) factor, $\beta_{smb,i}$ is the SMB sensitivity (or the size beta) of stock $i$, $R_{hml}(t)$ is the return of the value (or high minus low) factor and $\beta_{hml,i}$ is the HML sensitivity (or the value beta) of stock $i$. Even the four-factor model (4F) developed by Carhart (1997) is considered. This model corresponds to the following equation:

$$R_i(t) = \alpha_i + \beta_{mkt,i} R_{mkt}(t) + \beta_{smb,i} R_{smb}(t) + \beta_{hml,i} R_{hml}(t) + \beta_{wml,i} R_{wml}(t) + \varepsilon_i(t) \tag{5}$$

$R_{wml}(t)$ is the return of the momentum, or winners minus losers (WML), factor and $\beta_{wml,i}$ is the WML sensitivity of stock $i$.

However, all these three models do not include the carbon risk. Roncalli et al. (2020) extend the previous equations by adding the BMG factor, $R_{bmg}(t)$ is the return of the carbon risk factor and $\beta_{bmg,i}$ is the BMG sensitivity of stock $i$. Positive values of $\beta_{bmg,i}$ means that the stock $i$ is negatively affected by an acceleration in the transition towards a green economy. Therefore, the CAPM model becomes the MKT+BMG model:

$$R_i(t) = \alpha_i + \beta_{mkt,i}R_{mkt}(t) + \beta_{bmg,i}R_{bmg}(t) + \varepsilon_i(t) \quad (6)$$

The extended Fama-French (FF+BMG) model:

$$R_i(t) = \alpha_i + \beta_{mkt,i}R_{mkt}(t) + \beta_{smb,i}R_{smb}(t) + \beta_{hml,i}R_{hml}(t) + \beta_{bmg,i}R_{bmg}(t) + \varepsilon_i(t) \quad (7)$$

Equation 8 refers to the five-factor model (4F+BMG):

$$R_i(t) = \alpha_i + \beta_{mkt,i}R_{mkt}(t) + \beta_{smb,i}R_{smb}(t) + \beta_{hml,i}R_{hml}(t) + \beta_{wml,i}R_{wml}(t) + \beta_{bmg,i}R_{bmg}(t) + \varepsilon_i(t) \quad (8)$$

Estimates of risk factor model were performed on single stocks during the 2010-2018 period. Table 6 reports a comparison between the common factor models and their nested models by computing the average difference of the adjusted $\mathfrak{R}^2$ and the proportion of stocks for which the Fisher test is significant at 10%, 5% and 1%.

*Table 6 – Comparison of cross section regressions (%)*

| | Adjusted $\mathfrak{R}^2$ difference | $F$-test 10% | 5% | 1% |
|---|---|---|---|---|
| CAPM vs FF | 1.74 | 34.6 | 25.5 | 13.5 |
| CAPM vs MKT+BMG | 1.74 | 21.2 | 15.6 | 9.2 |
| FF vs FF+BMG | 1.73 | 22.5 | 17.5 | 9.7 |
| FF vs FF+WML | 0.22 | 6.6 | 3.0 | 0.8 |
| 4F vs 4F+BMG | 1.76 | 23.6 | 18.6 | 10.0 |

*Source: Roncalli et al. (2020)*

According to the first two tests, the authors observe that the Fama-French and MKT+BMG models significantly increase the explanatory power compared to the CAPM model. This surge is significant for almost 26% and 16% of the stocks respectively for Models (4) and (6) at the threshold of 5%. Nevertheless, the difference between the two models for the Fisher test declines when the threshold significance decreases to 1% and the effect on the explanatory power remains at the same level for the SMB and HML factors together and the BMG factor alone. The following two tests study the relevance of the carbon factor against the momentum factor (WML) when they are added to the Fama-French model. Looking at the table it is remarked that the sensitivity of the stock returns to the carbon factor is higher than the sensitivity of the stocks returns to the momentum factor. The last test confirms the relevance of the BMG factor when the BMG factor is added to the four-factor Carhart model. Overall, Roncalli et al. confirm the original results obtained by Görgen et al. (2019), meaning that the carbon factor is essential in the variation of stock returns.

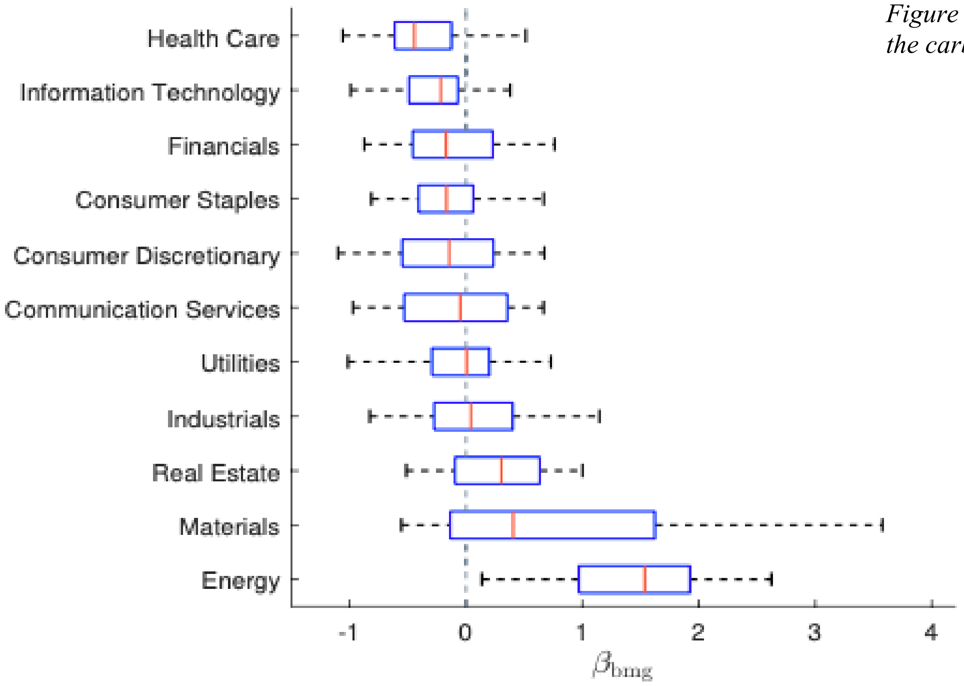



*Figure 12 – Box plots of the carbon sensitivities*

*Source: Roncalli et al. (2020)*

Figure 12 reports the GICS sector analysis of the carbon beta $\beta_{bmg,i}$ estimated with MKT+BMG model (6). The box plots provide the median, the quartiles and the 5% and 95% quantiles of the carbon beta. The energy, materials, real estate and, to a lesser extent, industrial sectors are negatively impacted by an unexpected acceleration in the transition process towards a green economy. The main reason is because these four sectors are responsible for a large part of greenhouse gas emissions (GHG). Indeed, the energy and the materials sectors have a huge scope 1 mainly because of oil and gas drilling and refining for the former and the extraction and processing of raw materials for the latter. Overall, the energy sector is the most sensitive to an unanticipated quickening in the transition process, but the materials sector has the widest carbon beta range, which indicates a high heterogenous risk for this sector. The latter is mostly influenced by the growth in material demand per capita. For what concerns the industrial sector, construction and transport are responsible for much of global final energy consumption, which leads to a high carbon risk for this sector. In the real estate sector, the firms have a large scope 2 and energy efficiency can be improved in many cases. If long-run investment is considered, a transition process that curbs climate change can protect households from physical risks like climate risk. Nevertheless, a short-term vision supposes that a climate policy negatively impacts households that over-consume. Therefore, real estate investment trusts are highly affected to climate-related policies. One unexpected result involves utility firms which do not have a substantial positive carbon beta though their scope 1 is on average the largest of any sector. This overall neutral carbon sensitivity for utilities is explained by their carbon emissions management and efforts to cut carbon exposure.

Health care, information technology and consumer staples due to their low GHG emissions are the sectors that are positively affected by an unexpected shift towards a green economy. Financials belongs to this group, but the interpretation of the carbon risk differs. Indeed, the carbon risk of financial institutions is less connected to their GHG emissions, rather it is related to their investments and financing programs. The greener a financial institution's investment, the lower its carbon beta. The low value of the median beta might be explained by two reasons, financial firms integrate carbon risk into their investment strategies or financials are not significantly disadvantaged by the relative carbon risk.

### 2.2.3. DYNAMIC ANALYSIS

The value added of the article of Roncalli et al. (2020) is that the authors consider the case where the risks are time-varying. For instance, carbon risk may change with the introduction of a climate-related policy, a change in the firm's environmental strategy or a greater integration of carbon risk into portfolio strategies. Therefore, they use the MKT+BMG dynamic model:

$$R_i(t) = R(t)^{\top} \beta_i(t) + \varepsilon_i(t) \tag{9}$$

$R_i(t) = \left(1, R_{mkt}(t), R_{bmg}(t)\right)$ is the vector of factor returns. Although the authors consider the CAPM+BMG model, an analogous dynamic analysis might be conducted even for the multi factor models presented before. $\beta_i(t)$ is the vector of factor betas of the MKT+BMG model (the beta estimates are based on the state space model (SSM) and the Kalman filter algorithm):

$$\beta_i(t) = \begin{pmatrix} \alpha_i(t) \\ \beta_{mkt,i}(t) \\ \beta_{bmg,i}(t) \end{pmatrix} \tag{10}$$

and $\varepsilon_i(t)$is a white noise. The authors assume that the state vector $\beta_i(t)$ follows a random walk process:

$$\beta_i(t) = \beta_i(t-1) + \eta_i(t) \tag{11}$$

$\eta_i(t) \sim \mathcal{N}(\mathbf{0}_3, \sum_{\beta,i})$is the white noise vector and $\sum_{\beta,i}$ is the covariance matrix of the white noise. Several specifications of $\sum_{\beta,i}$ may be used, but it is assumed that $\sum_{\beta,i}$is a diagonal matrix. As previously, the time-varying risk factor model is used on single stocks during the 2010-2018 period. The authors provide the average of two forecast error criteria between the OLS (ordinary least squares) model and the SSM model:

| Model | OLS | SSM |
|---|---|---|
| MAE (mean absolute error) | 4.95% | 4.63% |
| RMSE (root mean squared error) | 6.45% | 6.01% |

The time-varying risk (SSM) factor model reduces the forecast error. On average, the monthly return error is equal to 4.95% in the OLS model while it is equal to 4.63% in the SSM model.

Table 7 reports the proportion of firms for which the t-student test of the estimation of the covariance matrix $\sum_{\beta,i}$ is significant at 10%, 5% and 1% confidence levels. The coefficients of the covariance matrix are significant for a substantial number of firms implying that between 10% and 15% of stocks present time-varying market and carbon risks.

| Factor | 10% | 5% | 1% |
|---|---|---|---|
| $\alpha$ | 7.97 | 4.10 | 0.84 |
| $\beta_{mkt}$ | 15.95 | 10.22 | 3.93 |
| $\beta_{bmg}$ | 10.00 | 5.90 | 1.85 |

*Table 7 – Significance test frequency for the white noise covariance matrix (%)*

*Source: Roncalli et al. (2020)*

Figure 13 depicts the variation of the average carbon beta by region. For the whole study period, the carbon beta $\beta_{bmg,\mathcal{R}}(t)$ is positive in North America, which implies that American stocks are negatively influenced by an acceleration in the transition process towards a green economy. The average carbon beta is always negative in the Eurozone and, most of the time, in Japan.

Overall, the Eurozone has always a lower average carbon beta than the world as a whole, whereas the opposite is true for North America. Nevertheless, the sensitivity of European equity returns to carbon risk dramatically increases over the study period and the BMG betas of North America and the Eurozone are getting closer. In Europe ex EMU, the BMG beta is higher than in the Eurozone, but their trends are almost the same. Regarding the Japanese firms, the trend has followed the world level since 2013 but with a lower carbon beta.

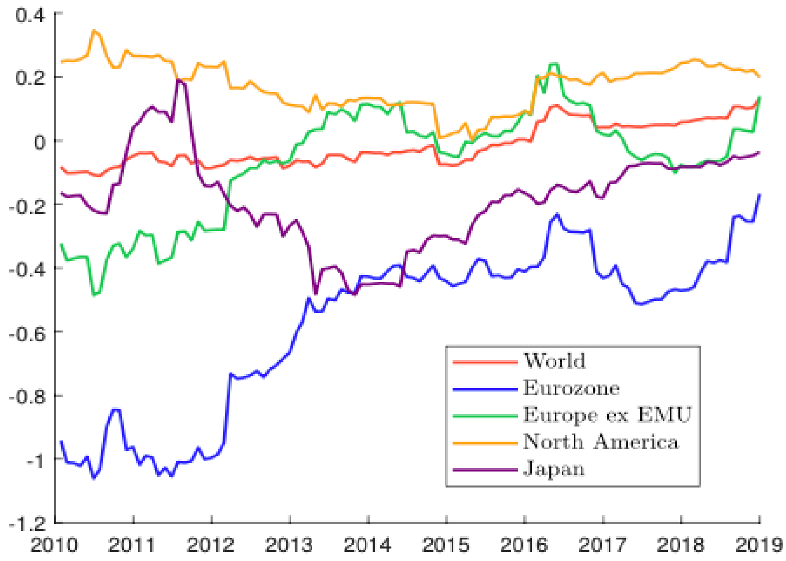



*Figure 13 – Dynamics of the average relative carbon risk* $\beta_{bmg,\mathcal{R}}(t)$ *by region*

*Source: Roncalli et al. (2020)*

Roncalli et al. (2020) notice that the carbon risk in the short term is not driven by climate agreements. For instance, the 2030 climate and energy framework, which includes EU-wide targets and policy objectives for the period from 2021 to 2030 does not impact the average European carbon beta in 2014 surely because of the lack of binding commitments. The 2015 Paris Climate Agreement is an alternative example, the agreement does not include any fiscal pressure mechanisms. Since there are deep differences between expectations and constraints, the Paris Climate Agreement has not been followed by a significant increase in the carbon beta and has been associated with the outperformance of brown stocks over green ones. It is possible that global surges of carbon beta are due to sector-specific effect. For instance, in February 2016 the materials sector has largely outperformed because of a substantial increase in gold, silver and zinc prices whereas the market index has decreased.

In Figure 14 are provided the time evolution of the average absolute carbon beta $|\beta|_{bmg,\mathcal{R}}(t)$ for each region $\mathcal{R}$. The higher the value of $|\beta|_{bmg,\mathcal{R}}(t)$, the greater the impact (positive or negative) of carbon risk on stock returns. Interestingly, the integration of carbon risk in the financial market decreases over time. There is a substantial reduction in 2012 and then a stabilization of the global average absolute carbon beta[10]. The Eurozone was the region with the highest sensitivity to carbon risk, but this slumped by almost 44% between 2010 and 2018. However, the authors observe that all the regions converge except Japan. The convergence of absolute sensitivities between large geographical regions suggests that investors consider carbon risk as a global concern.

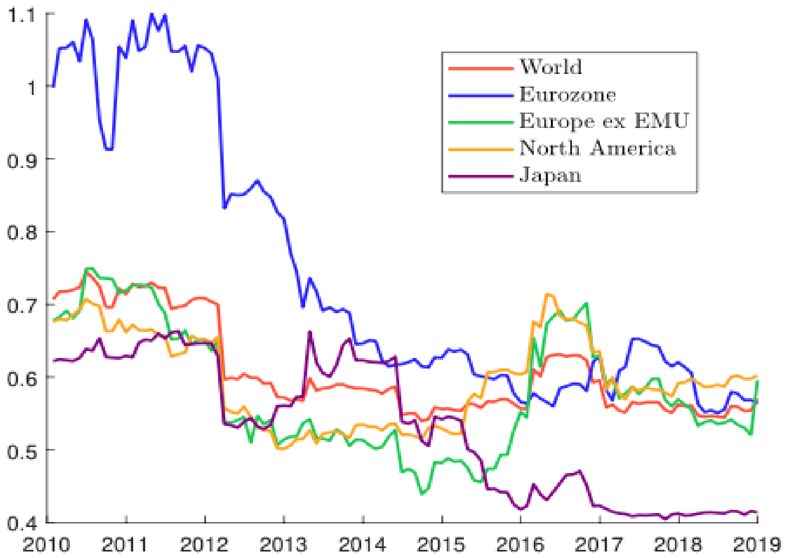



*Figure 14 – Dynamics of the average absolute carbon risk* $\beta_{bmg,\mathcal{R}}(t)$ *by region*

*Source: Roncalli et al. (2020)*

Recalling results of the static analysis (see figure 12, section 2.2.2), energy, materials, and real estate sectors were the most negatively impacted by an unexpected acceleration in the transition process, whereas the opposite was true for the health care, information technology and consumer staples sectors. Figures 15 and 16 provide the trends in the median carbon beta $|\beta|_{bmg,S}(t)$ for the sector $S$ at time $t$, which is defined as follows:

$$|\beta|_{bmg,S}(t) = \text{median}_{i \in S}\beta_{bmg,i}(t)$$

The energy sector is the most sensitive sector to the carbon factor. The stock price of the latter is increasingly negatively influenced by the movements of the carbon factor. The materials and real estate sectors have a much more moderate positive sensitivity of stock price to carbon factor. Sectors with a neutral or a low negative sensitivity to the carbon factor are industrials, utilities, communication services, consumer discretionary, consumer staples, financials, and information technology sectors. The health care sector is the only with a modest negative

[10] *The decrease of* $|\beta|_{bmg,\mathcal{R}}(t)$ *in March 2012 is not due to a climate-related policy but to green stocks far outperforming (see Figure 11). At the same time, the European market declined while the American market increased. Therefore, the increase of carbon beta for green European stocks was driven mostly by the European market's return rather than their carbon return. In a similar way, the decrease of carbon beta for brown American stocks was determined mainly by the American market's return rather than their carbon return (source: Roncalli et al. (2020)).*

sensitivity to the carbon factor. However, this sector is getting closer to the carbon risk-neutral sectors over time, even though it is still visible a gap.

By looking at Figure 15, the materials and real estate sectors started with a similar median carbon beta. However, the spread has been increasing between the two sectors since 2016, because the median carbon risk is stable in the case of the real estate sector whereas it is growing for the materials sector. This gap might be persistent in the long run, implying that the materials sector may be increasingly affected by carbon risk. The industrials sector was mostly negatively influenced by the BMG factor, but it has become a carbon risk-neutral sector. Overall, for investors sector diversity is more important than geographical breakdown since market-based carbon risks converge both in absolute and relative values at the geographical level.

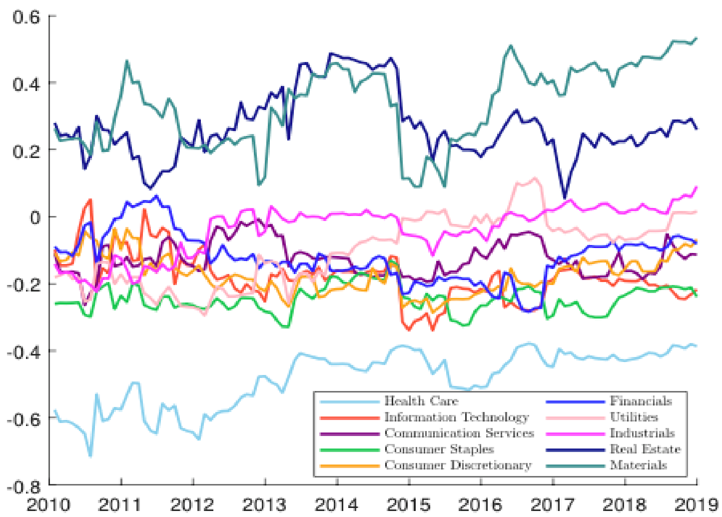



*Figure 15 – Dynamics of the median carbon risk $\beta_{bmg,s}(t)$ by sector*

*Source: Roncalli et al. (2020)*


*Figure 16 – Dynamics of the median carbon risk $\beta_{bmg,s}(t)$ by energy sector*

*Source: Roncalli et al. (2020)*

The advantage of this dynamic analysis is to assume that common risks are time-varying. In Figure 17, Roncalli et al. (2020) have described the density of the monthly variations $\beta_{bmg,i}(t) - \beta_{bmg,i}(t-1)$. Since fat tails have a significant number of extreme variations, the distribution is far from being a Gaussian distribution. Hence, it is possible to deduce that the time-varying model allows to consider some extreme changes in carbon risk. However, in the event of environmental debates[11], an analysis on the behavior of individuals shows that the model is not able to significantly change the carbon risk in the short term. The extreme changes are better explained by regional or sector-related effects. Therefore, the $\beta_{bmg,i}(t)$ is more a low-frequency systematic measure than a high-frequency idiosyncratic measure of the carbon risk.

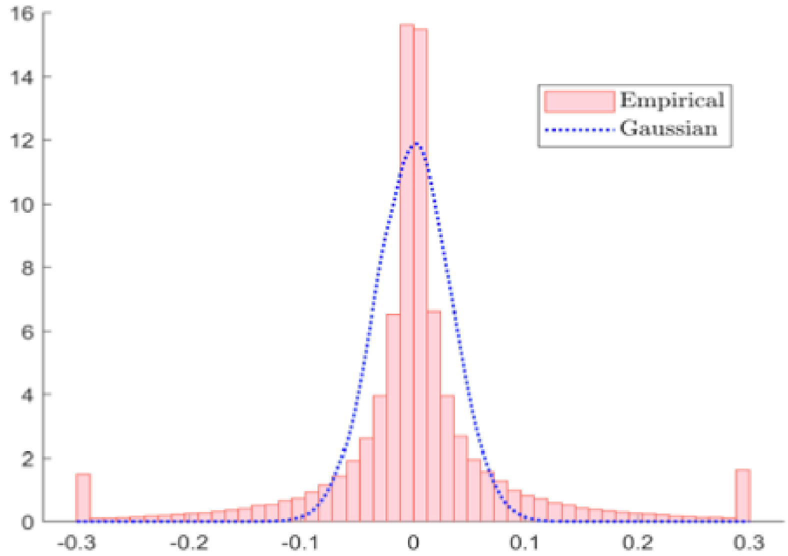



*Figure 17 – Density of the carbon risk first difference*

*Source: Roncalli et al. (2020)*

[11] *This is the case of some famous controversial events, e.g. Volkswagen, Bayer, etc. Whatever the variation of the carbon factor, the firm's stock return decreases in the case of an environmental controversy (source: Roncalli et al. (2020)).*

# CHAPTER 3: Portfolio decarbonization

In this chapter I perform the decarbonization of the S&P 500 index by imposing climate constraints on the three climate risk metrics presented in the previous chapter. The idea that lies behind decarbonization process is to construct a portfolio that tracks the benchmark index but with a lower climate risk exposure. The first section briefly outlines the theoretical framework of the portfolio optimization problem. In the second section, it is reported how climate and financial data have been retrieved and some descriptive analyses of them. Third and fourth sections are devoted to the description of the results obtained from the decarbonization problem using the three metrics. The last section is intended to compare some performance measures of the portfolios obtained previously attempting to establish which carbon risk metrics ensures the best performances.

## 3.1. THEORETICAL FRAMEWORK

The reference model I use for portfolio optimization is the classical Markowitz model (Markowitz, 1952). The basic assumption in this model is that portfolio's returns distribution is defined only by the first two moments, mean and variance. Thus, the standard optimization problem can be written as:

$$x^* = \arg\min \frac{1}{2} x^\top \Sigma x$$

$$\text{s.t.} \begin{cases} x^\top \mu = m \\ \mathbf{1}_n^\top x = 1 \\ x \in \Omega \end{cases}$$

where $x = (x_1, \dots., x_n)$ is the active portfolio, $\Sigma$ is the covariance matrix of asset returns, $\mu$ is the vector of expected returns and $\Omega$ is the set of constraints. For instance, one of the most used constraints is the long only constraint, meaning that the weights of the assets making up the portfolio must be non-negative. The Markowitz framework deals with the trade-off between the portfolio variance $\sigma^2(x) = x^\top \Sigma x$ and the average return $\mu(x) = x^\top \mu$. The efficient frontier is then obtained by minimizing the portfolio variance while fixing the average return. However, it possible to do the opposite, maximizing the expected return by fixing the portfolio variance.

Since portfolio decarbonization regards the reduction of climate related risks, the constraints are imposed to limit the portfolios' carbon intensity, carbon emissions and carbon beta.

Therefore, the new climate constraint can be written as follows:

$$\Omega = \left\{ x : C(x) = \sum_{i=1}^{n} x_i C_i \leq C^{+} \right\}$$

$C(x)$ is the carbon risk measure that it is wanted to limit, $C^{+}$ is the maximum level of the carbon risk measure that an efficient portfolio can attain. By considering different values of $C^{+}$ it possible to construct a three-dimension efficient frontier: expected return, standard deviation (volatility) and carbon risk measure.  In the third section I will explain the way in which $C^{+}$ has been defined.

## 3.2. DATASET

To address a problem of portfolio decarbonization other than financial data it is fundamental to recover climate data. The entire dataset, financial and climate data, has been retrieved from Thompson Reuters financial database, Refinitiv Eikon.

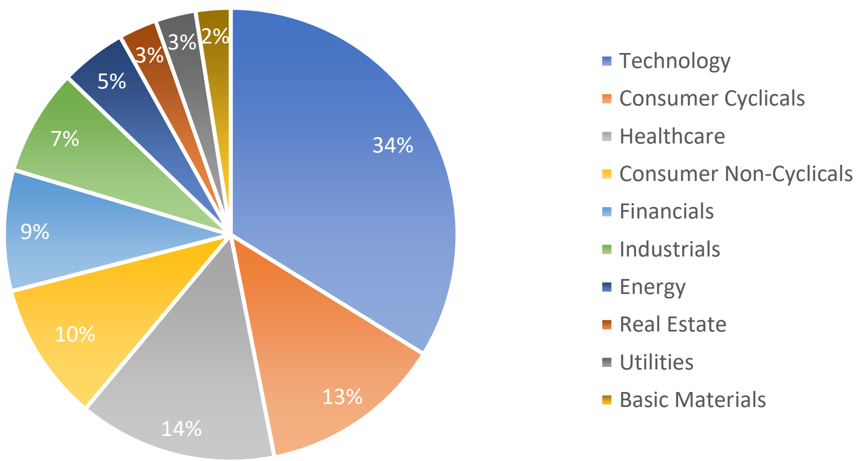
### 3.2.1. THE BENCHMARK

The first step in the construction of the dataset regards the choice of the benchmark index to decarbonize. I choose the Standard and Poor's 500 (S&P 500), in my decision I considered both financial features and climate related characteristics of the index. From a financial point of view, the S&P 500 is one of the most widely quoted American indexes because it represents the largest publicly traded corporations in the U.S. and it is considered as one of the best measures of American equities' performance, and by extension, that of the stock market overall. The S&P 500 focuses on the U.S. market's large-cap sector and it is a market-capitalization-weighted index made up of 500 publicly traded companies listed in the U.S. This means that some of the companies that constitutes the index are among the biggest polluters in the world, so they are more likely to be negatively affected by climate related risks, especially carbon risk. Therefore, decarbonizing the S&P 500 might be a preferable investment strategy in the long run thanks to its hedging power against climate risk.

The S&P 500 adopts a market-cap weighted function to compute the weights of the constituents, meaning that attaches a higher percentage allocation to companies with the largest market capitalizations. The market cap of the companies is calculated by multiplying the free-floating shares of the companies by their current stock price. Then, the weighting of each company in the index is computed by dividing the company's market cap for the total market cap of the index. The S&P adjusts each company's market cap to compensate for new share issues or company mergers.

There are 10 sectors in which is possible to divide the companies: Technology, Consumer Cyclicals, Healthcare, Consumer Non-Cyclicals, Financials, Industrials, Energy, Real Estate, Utilities and Basic Materials. Figure 18 shows the breakdown of the S&P 500 by the market cap of each sector at the end of April 2022.

*Figure 18 – S&P 500 breakdown by market cap sector*



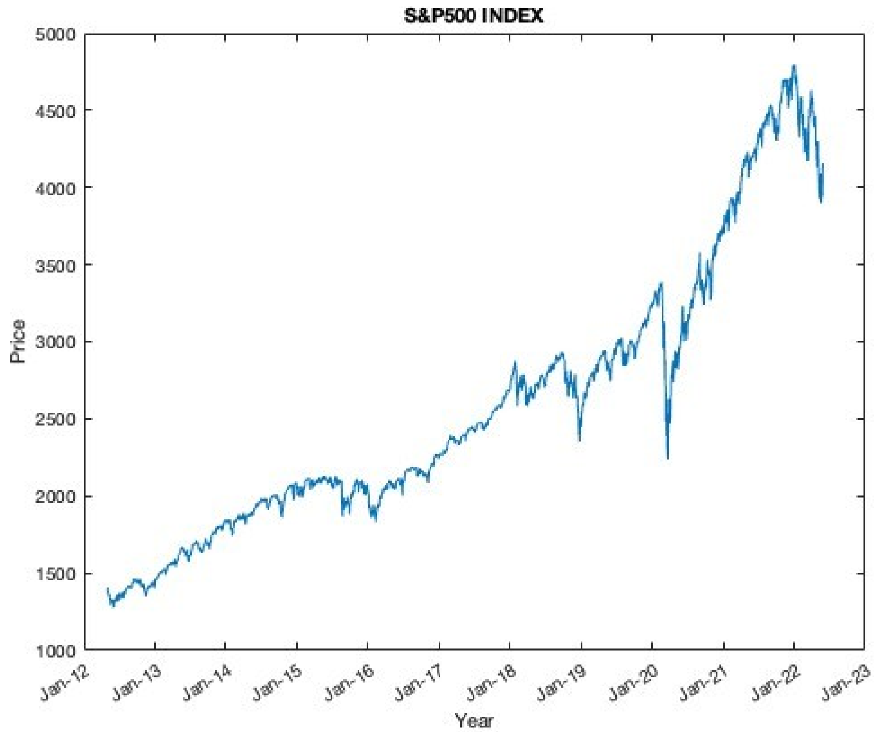

*Source: Refinitiv Eikon*

As expected, over one third of the total market cap of the index is made up by tech companies. Indeed, the world’s most valuable companies by market cap operate in this sector belongs to the S&P 500, including Apple which has been the first company in the world to reach a market capitalization of one trillion of dollars. At the second place there is the heartcare sector which has experienced a sudden growth in market cap with the outbreak of COVID19 crisis, especially those companies that accomplish to develop an approved vaccine like Johnson and Johnson and Pfizer Inc. With 13% of market cap there are the consumer cyclicals companies, even among them there are some of the biggest and well-known firms in the world, such as the absolute leader of e-commerce Amazon.com Inc. and the first electric car maker in the world Tesla.

Slightly under the 10% we find the consumer non-cyclicals sector, the financial sector, and the industrial sector. The remaining portion of the pie chart is divided almost equally between the sectors of energy, real estate, utilities, and basic materials. The value of the index is calculated by summing the market caps of each company and dividing the result by a divisor. Originally, the divisor was a number that the index creator chose to ensure that the index started at an arbitrary initial value. Divisors now change only when necessary to guarantee that the value of an index does not modify when the creator adds or deletes index components. In the case of the S&P 500 its divisor must increase when a high capitalization stock replaces a low capitalization stock. The divisors of value- weighted indexes do not have to change when stocks split, because splits do not change total capital values.

The time window I took in consideration when I retrieved financial data is from May 1st, 2012, to May 27th, 2022. Figure 19 depicts the evolution of the daily price of the S&P 500 across the time coverage I considered.

*Figure 19 – S&P 500 Index daily price evolution*




*Source: Refinitiv Eikon*

The value of the S&P 500 index increased steadily over time, reaching its peak at the end of 2021. However, we can notice a sharp slump in the first months of 2020 exactly in concomitance of the outbreak of COVID19 global crisis.

### 3.2.2. THE ASSETS

Of course, the assets used to construct a decarbonized portfolio are the same of the benchmark index to decarbonize. However, some of the companies that make up the S&P 500 does not have a price history that covers the 10-year window from May 2012 to May 2022 because their IPOs happened in recent years. For instance, the biotech company Moderna went public only in December 2018. Constellation Energy Corporation the largest provider of clean energy across the United States after the separation from the parent company, Exelon Corporation, started to trade its shares in January 2022. There are other examples of spin-offs that implied stock splits, such as the case of the chemical company DowDuPont that separated in two different companies in 2019.

To have a complete dataset, I computed the mean price of the companies that did not have the complete price history using the historical values and used this mean value as historical value for all the days for which there were not a price. I did not discard the assets that did not have a sufficient number of historical values because that might have a significant negative effect on the composition of the decarbonized portfolio. Then, I sorted the assets according to their sector and I created an index for each sector simply by calculating the average daily prices of the stock prices of the companies that belongs to the sector. The formula used to compute the daily price of each index is written as follows:

$$daily\ price\ of\ sector\ index\ _j(t) = \frac{1}{N}\sum_{i=1}^{N} price_{i,j}(t)$$

where $j$ refers to sector and $price_{i,j}(t)$ is the price at time $t$ of the company $i$ that belongs to sector $j$.

Therefore, I came up with 10 indexes made up of the assets that compose the S&P 500. There are two main reasons why I implemented this type of operations, first because it is more straightforward working with 10 assets instead of 500. Nevertheless, the second one is the fundamental motivation because I did not have the possibility to compute or to find the carbon beta values for all the companies of the S&P 500 but just for the sectors.

Figure 20 reports the daily prices evolution of the sector indexes by considering the same time coverage of the benchmark index. The index of the Consumer Non-Cyclicals sector is the one

with the highest level of price over the entire period, followed by the Technology index. The general trend of the indexes is of a constant increase in the prices, with an exception.
The index of the Energy sector decreased its value between 2014 and 2020, just after the plunge due to the COVID19 this index started to overturn the trend and increased its value.

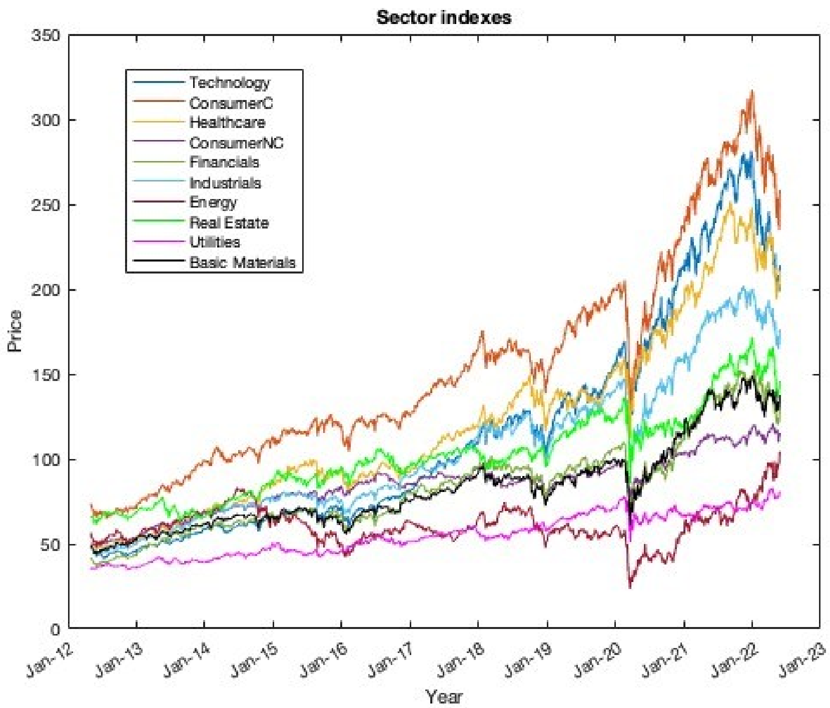



*Figure 20 – Sector indexes daily prices evolution*

*Source: Refinitiv Eikon and own calculations*

From the daily prices of the indexes, I obtained the daily returns by employing the following formula:

$$daily\ return = \left(\left(\frac{daily\ price_t}{daily\ price_{t-1}}\right) - 1\right) \times\ 100$$

the logarithm $\ln\left(\frac{p_t}{p_{t-1}}\right) = \ln(p_t) - \ln\ (p_{t-1})$ can be approximated to the previous formula.

Table 8 provides a brief descriptive analysis of daily returns of the benchmark and of the assets. The two indexes that have the highest mean returns are the Technology and the Healthcare. Real Estate, Utilities and Consumer Non-Cyclicals indexes have the lowest value of mean return. On the other hand, the latter has the lowest standard deviation which is the measure of the volatility. Moreover, the benchmark is less risky than all the other indexes. About the skewness, the Utilities index is the one with the most symmetric distribution, while Real Estate and Energy are the most asymmetric.

*Table 8 – Descriptive analysis of daily returns*

| | Mean | Median | StDev | Min | Max | Skew | Kurt |
|---|---|---|---|---|---|---|---|
| S&P500 | 0,048 | 0,064 | 1,064 | -11,984 | 9,383 | -0,630 | 20,723 |
| Technology | 0,067 | 0,116 | 1,266 | -12,150 | 9,298 | -0,438 | 12,660 |
| Consumer Cyclicals | 0,058 | 0,114 | 1,226 | -15,006 | 13,752 | -0,576 | 26,207 |
| Healthcare | 0,064 | 0,096 | 1,133 | -10,482 | 7,772 | -0,373 | 11,132 |
| Consumer Non-Cyclicals | 0,037 | 0,066 | 0,895 | -9,698 | 7,723 | -0,322 | 19,501 |
| Financials | 0,053 | 0,095 | 1,305 | -13,947 | 12,392 | -0,489 | 18,773 |
| Industrials | 0,058 | 0,101 | 1,171 | -12,276 | 11,795 | -0,519 | 19,015 |
| Energy | 0,042 | 0,051 | 1,893 | -24,255 | 17,206 | -0,877 | 21,239 |
| Real Estate | 0,038 | 0,073 | 1,168 | -15,629 | 8,701 | -1,013 | 22,884 |
| Utilities | 0,038 | 0,093 | 1,089 | -11,020 | 12,691 | 0,037 | 24,025 |
| Basic Materials | 0,048 | 0,083 | 1,239 | -12,374 | 11,616 | -0,496 | 15,117 |

### 3.2.3. CLIMATE DATA

Refinitiv Eikon provides a wide range of data and scores related to ESG factors. For the decarbonization problem the data required are the carbon emissions of the companies of the S&P 500 expressed in tons of $CO_2$. As explained in section 2.1.1 the greenhouse gases (GHG) are divided in three scopes, scope 1 measures the direct emissions, scope 2 are the indirect emissions from consumption of purchased electricity, heat, or steam and scope 3 are all the other indirect emissions. Since the scope 3 emissions are the most challenging to estimate and not all the companies disclosed this type of data, I retrieved a complete dataset only for scope 1 and 2 emissions for the year 2019 and 2020. By summing up the two scopes I obtained the total emissions for each company. Then, I divided the emissions by the total revenues expressed in millions of dollars to obtain the carbon intensity.

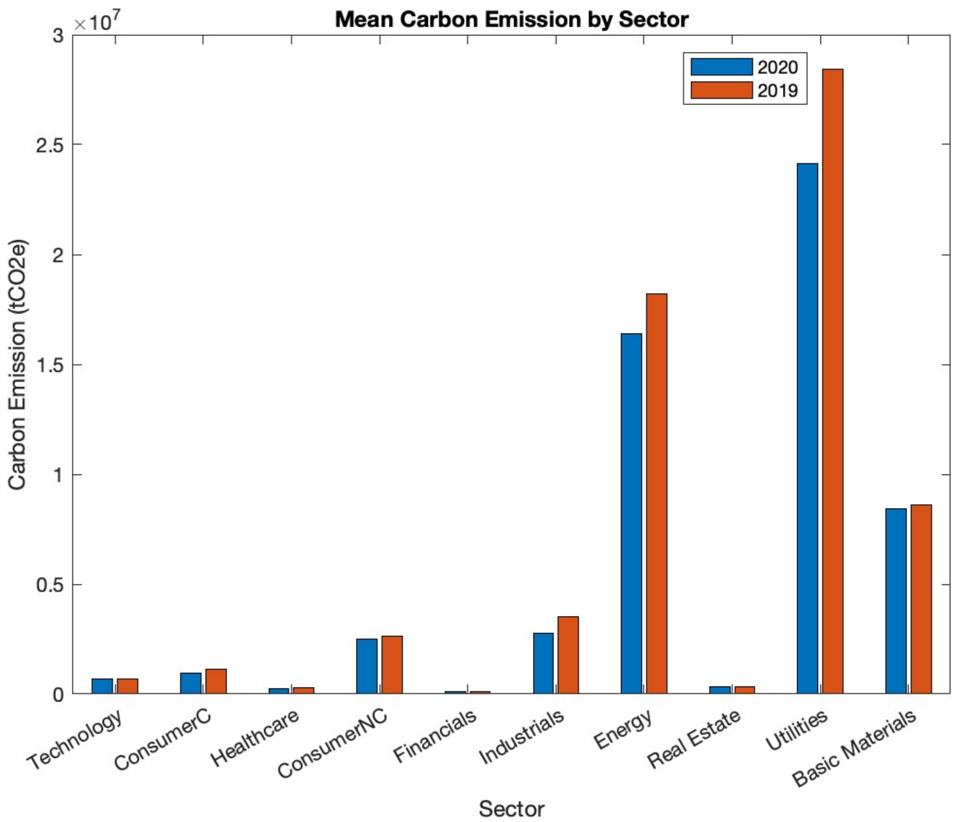
*Table 9 – Descriptive analysis of carbon emissions and carbon intensity*

| | Carbon emissions | | Carbon intensity | |
|---|---|---|---|---|
| | 2019 | 2020 | 2019 | 2020 |
| Mean | 3988651,19 | 3498677,62 | 260,97 | 261,66 |
| Median | 245260,00 | 216259,00 | 24,63 | 22,13 |
| Min | 0,32 | 0,29 | 0,000085 | 0,000076 |
| Max | 118000000,00 | 111000000,00 | 5149,95 | 5509,99 |

*Source: Refinitiv Eikon and own calculations*

Table 9 confirms something that we would have expected, the mean carbon emissions of the companies of the S&P 500 are significantly lower in 2020 with respect to 2019. The pandemic had an enormous impact on the economy, an incredible number of firms and companies had to slow down or really shut down the production forcing a sudden reduction of $CO_2$ emissions in 2020. When we consider the carbon intensity it comes up that the mean value to some extent increased in 2020. This might be explained by looking at the denominator, the total revenues on average might have been decreased with a lower rate than those of reduction of carbon emissions. However, it might be useful to investigate the mean values of carbon emissions and carbon intensity by considering the different sectors.

*Figure 21 – Mean Carbon Emission by sector*

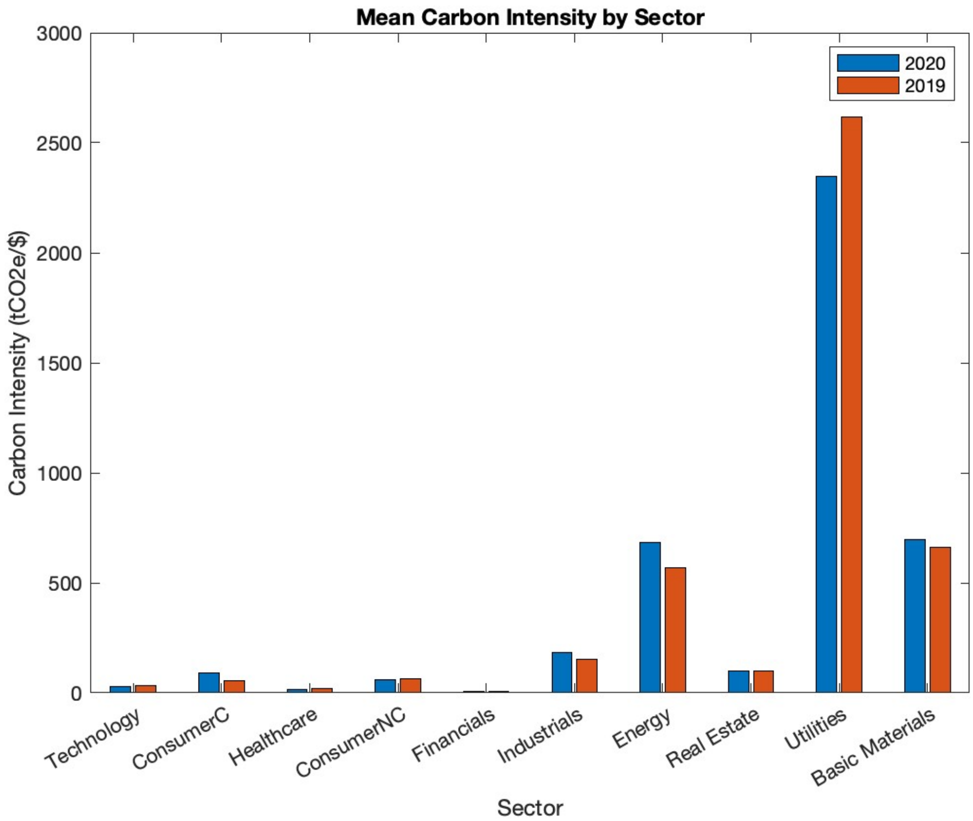



*Source: Refinitiv Eikon and own calculations*

In Figure 21 it is possible to observe that all the mean carbon emissions in 2019 (red bars) have higher values than in 2020 (blue bars). The companies that operate in Utility sector emit the largest number of tons of $CO_2$, while the companies of the financial sector pollute the less together with the Healthcare companies. The sectors for which there have been the more significant decrease between 2019 and 2020 are the Utilities and the Energy, this might have been due to the COVID19 crisis.

Considering the carbon intensity (Figure 22) some of the sectors have higher values in 2020 than 2019, for example Energy, Consumer Cyclicals, Basic Materials, and Industrials. It is interesting to notice that despite the companies of the energy sector are the second for carbon emissions, their carbon intensity in 2019 was lower than the one of the Basic Materials. This means that total revenues of the energy companies were much higher than those of the Basic Materials.

*Figure 22 – Mean Carbon Intensity by sector*

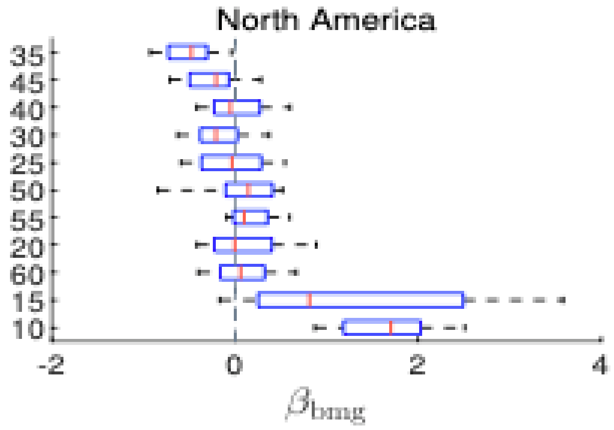



*Source: Refinitiv Eikon and own calculations*

Carbon emissions and carbon intensity are just two of the climate risk metrics employed to decarbonize the S&P 500 index, the third one is the carbon beta. These data have not been recovered from Eikon but from the paper of Roncalli and LeGuenedal (2020), they estimate the carbon beta by sector and region.

Since the companies in the S&P 500 are North American, Figure 23 reports the carbon sensitivities of the sectors in this region. The carbon beta measures how a sector will be impacted by a transition towards a green economy, a positive value means that it will be negatively affected a negative value the opposite. The sectors with a negative carbon beta are Healthcare, Information Technology, Financials, Consumer Non-Cyclicals and Consumer Cyclicals.

*Figure 23 – Box plots of carbon sensitivities by sector in North America*




*Legend: energy (10), materials (15), industrials (20), consumer cyclicals (25), consumer non cyclicals (30), health care (35), financials (40), information technology (45), communication services (50), utilities (55) and real estate (60)*

*Source: Roncalli e LeGuenedal (2020)*

## 3.3. STRATEGIC ASSET ALLOCATION

Portfolio decarbonization is nothing else than a problem of portfolio optimization with constraints on carbon risk measures. Portfolio optimization requires to estimate the mean return and the covariance matrix of the assets, I gauged both with the single factor Capital Asset Pricing Model (CAPM) using the historical prices of the sector indexes presented in the previous section. I proceeded using the CAPM model to obtain results comparable to the paper of Le Guenedal and Roncalli (2022) in which they adopted that model to estimate portfolio moments.

After the assessment of portfolio moments, I constructed the efficient frontiers according to different sets of constraints. First, I imposed the no short sale constraints, the constraint that does not allow negative weights of the assets (long-only constraint) and the constraint for which the weights of the assets must sum to 1. Then, I designed climate constraints for the three climate risk measures, carbon emissions, carbon intensity and carbon beta. I preferred to use climate data of 2019, because the pandemic crisis had heterogeneous effects on the sectors.
Therefore, the optimization problem can be written as follows:

$$x^* = \arg\min \frac{1}{2} x^\top \Sigma x$$

$$\text{s.t.} \begin{cases} \mathbf{1}_n^\top x = 1 \\ x^\top \mu = m \\ x \geq \mathbf{0}_n \\ \sum_{i=1}^{n} x_i C_i \leq C^+ \end{cases}$$

$C^+$ is the maximum level of climate risk measure that a portfolio can attain. To understand how changes the efficient portfolios with climate constraints I do not consider a single value of $C^+$ but I change it to have a three-dimension efficient frontier. I defined a grid of 20 target values of $C^+$ which are enclosed between the minimum and the maximum assets' values of $C$ such that:

$$\min C = \text{target}C(1) \leq \cdots \leq \text{target}C(20) = \max C$$

$\min C$ corresponds to the first target and it is the minimum value of a certain climate risk measure $C$ among the assets, while $\max C$ is the last target and the maximum value of the same climate risk measure among the assets.

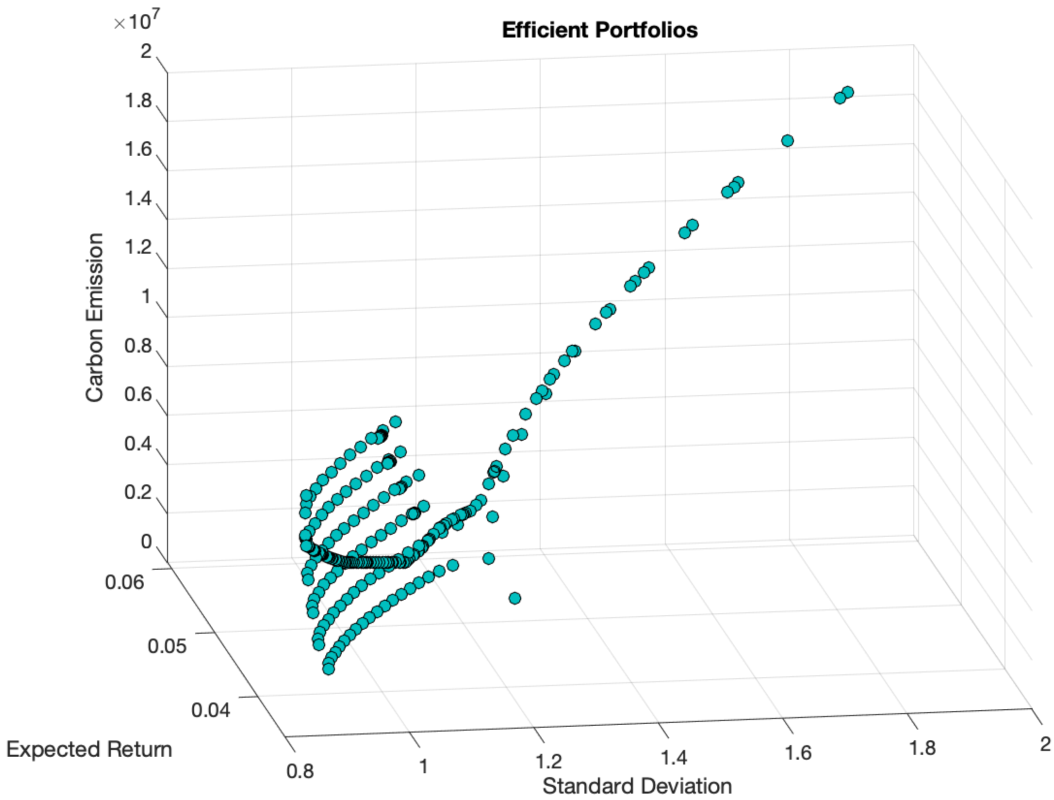
For each target of $C^+$, I found the efficient risk-return frontier. The $\max C$ should be changed to ensure that the returned portfolios achieve at most the target $C^+$. This method returns the weights of the portfolios on the mean-variance efficient frontier that have a $C$ of at most the target $C^+$(i). Using the weights obtained for each target $C^+$, compute the portfolios' expected return, risk, and level of $C$.

In addition to climate constraints, I considered the case in which also group constraints are imposed. These constraints are added to limit the exposure on certain assets. The first group constraints are on Technology, Healthcare and Financials since they are the indexes with the lowest carbon emissions and carbon intensity. Therefore, this constraint ensured that the weights of these assets were at least 15%. The second constraint regards all the other seven assets, I decided to bound their weights at most to 10%. Giving the new constraints, I had to reevaluate the grid of target values of $C^+$. In the next paragraphs, I report the different efficient frontiers for all the set of constraints.

### 3.3.1. PORTFOLIO DECARBONIZATION WITH CARBON EMISSIONS

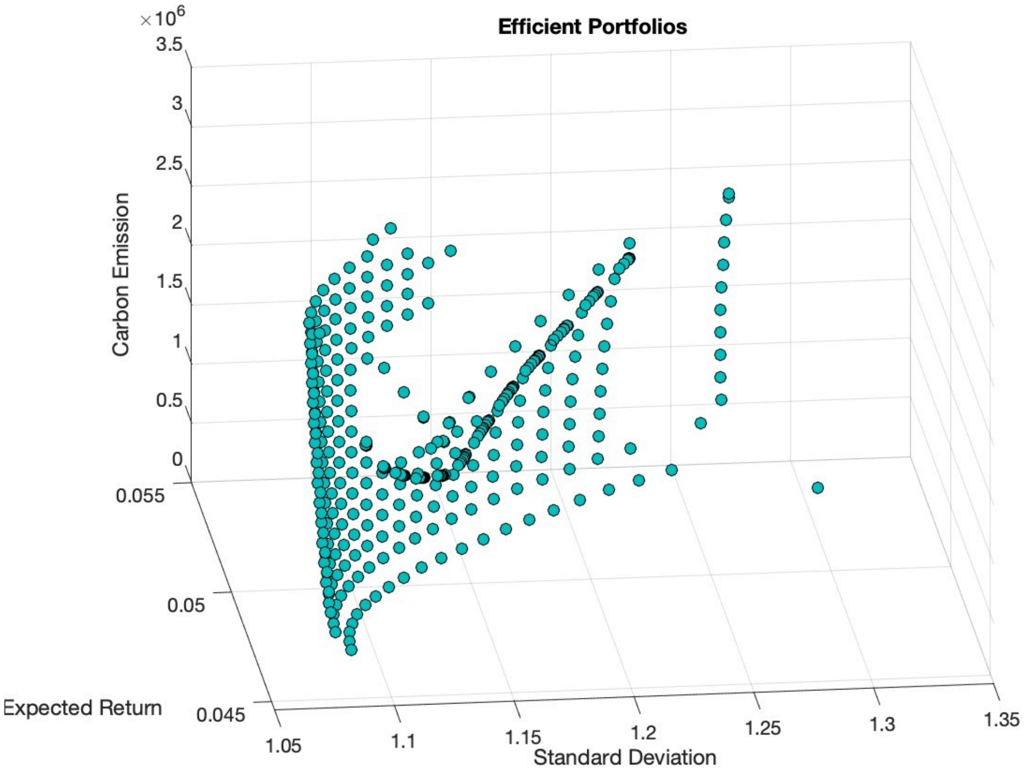
The first carbon risk measure used to decarbonize is the carbon emission. Figure 24 shows the efficient portfolios with climate constraints but without the group constraints. As predicted by classical portfolio theory expected returns of the portfolios increased together with the standard deviation. The levels of carbon emissions are less predictable, they tend to decrease till a standard deviation of 1.2, after that they increase with the rise of the portfolios' riskiness. The maximum level of carbon emissions that an efficient portfolio reaches almost $2 \times 10^7$ tons of $CO_2$ which is lower than the maximum value of carbon emissions among the assets.

*Figure 24 – Efficient portfolios without group constraints*



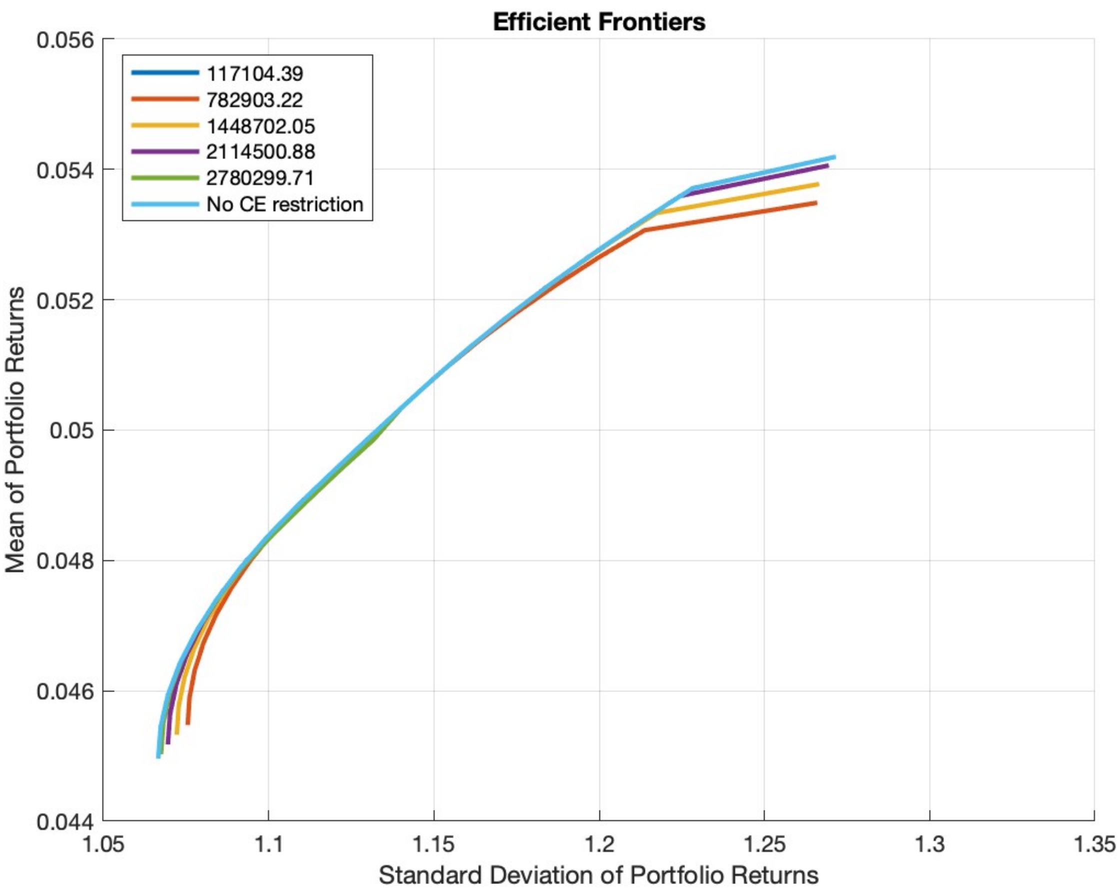

By adding the group constraints, the grid of target values of $C^+$, so the composition of the efficient frontier is expected to be different. Figure 25 confirms what expected, the efficient portfolios are more scattered than in Figure 24. The standard deviation is much lower with group constraints than without, the maximum level is less than 1.3. The values of expected returns are almost similar to those of Figure 24, meaning that group constraints limit more the volatility than the returns. However, what it is fundamental to highlight are the values of carbon emissions that are ten times lower with the presence of group constraints.

*Figure 25 – Efficient portfolios with group constraints*




To visualize the tradeoff between a portfolio with carbon emissions and group constraints and the traditional mean-variance efficient frontier, I computed a set of line plots for some carbon emissions target.

*Figure 26 – Efficient Frontiers with group constraints and target values of carbon emissions*



The light blue line is the efficient frontier without climate constraints, all the other lines are efficient frontiers with different levels of constraints on carbon emissions. It is important to notice that between a standard deviation of 1.1 and 1.2 the efficient frontier almost overlapped, meaning that at the same level of risk is associated the same return regardless the carbon emissions restrictions. However, we can see that the efficient frontiers with stricter levels of carbon emissions provide lower returns at the same risk. Indeed, the orange line is located below all the others and is the one with the lower target value, while the green one is almost totally overlapped to the one without restrictions.

### 3.3.2 PORTFOLIO DECARBONIZATION WITH CARBON INTENSITY

Carbon intensity is the second climate risk measure that I considered. However, I used the weighted average carbon intensity (WACI) of each sector. First, I computed the carbon intensity for all the companies, then I assessed the weight of each company by dividing its market cap for the total market cap of the companies that belongs to the same sector. To obtain the WACI of each sector I multiplied the weight of each company for its carbon intensity and made a summary of them.

$$CI_i = \frac{CE_i}{Y_i}$$

$$w_{i,j} = \frac{\text{market cap}_i}{\text{total market cap}_j},$$

where

$CI_i$ is the carbon intensity of the company $i$

$CE_i$ is the carbon emission of the company $i$

$Y_i$ is the total revenue of the company $i$

$i$ refers to the company

$j$ refers to the sector

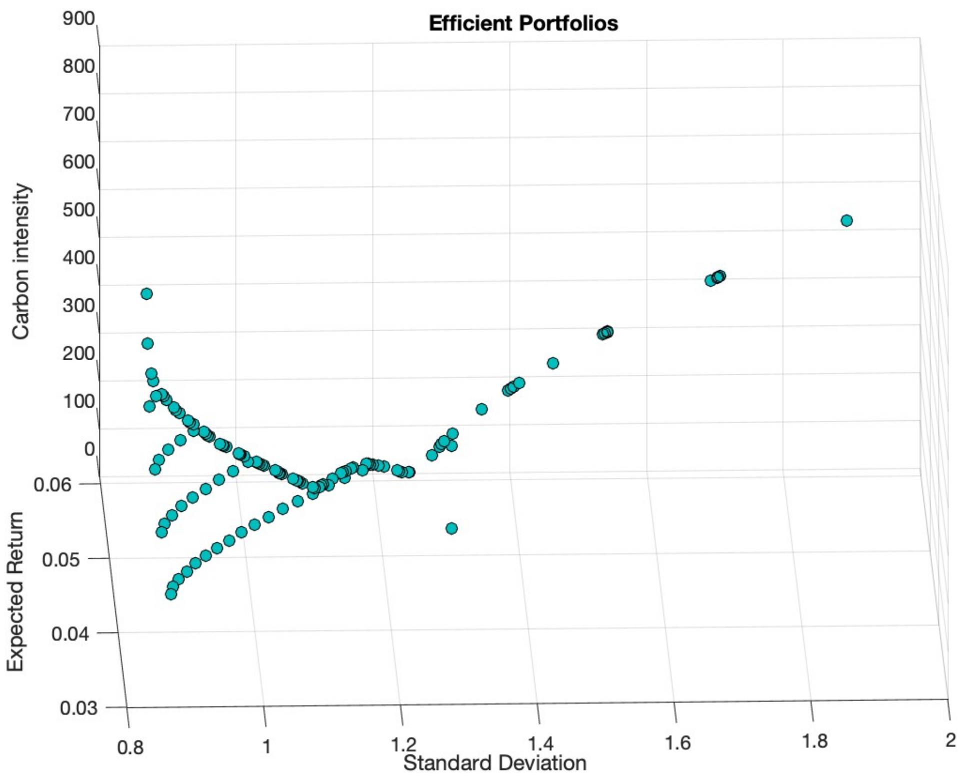
$w_{i,j}$ is the weight of the company $i$ with respcet to the sector $j$

$$\text{WACI}_j = \sum_{i}^{N} w_{i,j} \times CI_i$$

In figure 27 are shown the efficient portfolios with climate constraints but without group constraints. The efficient frontier is considerably similar to the one with carbon emissions.

Carbon intensity decreases with volatility until it reaches 1.2, from this value of standard deviation onwards the two quantities move in the same direction. Even the expected returns are almost identical. This result is not surprising since carbon intensity is directly related to carbon emissions.

*Figure 27 – Efficient portfolios without group constraints*




As for the case with carbon emissions, the presence of group constraints (Figure 28) offers strong differences in the composition of the efficient frontier with respect to the one constructed without group constraints. The efficient portfolios are grouped around lower values of carbon intensity, the standard deviation is lower than without the group constraints. In this figure we can observe that the tendency of carbon intensity to rise after a certain level of standard deviation is less evident than in the case with carbon emissions. Indeed, carbon intensity declines sharply up to a standard deviation of 1.15, but then it surges slightly, and it reaches lower values.

*Figure 28 – Efficient portfolios with group constraints*

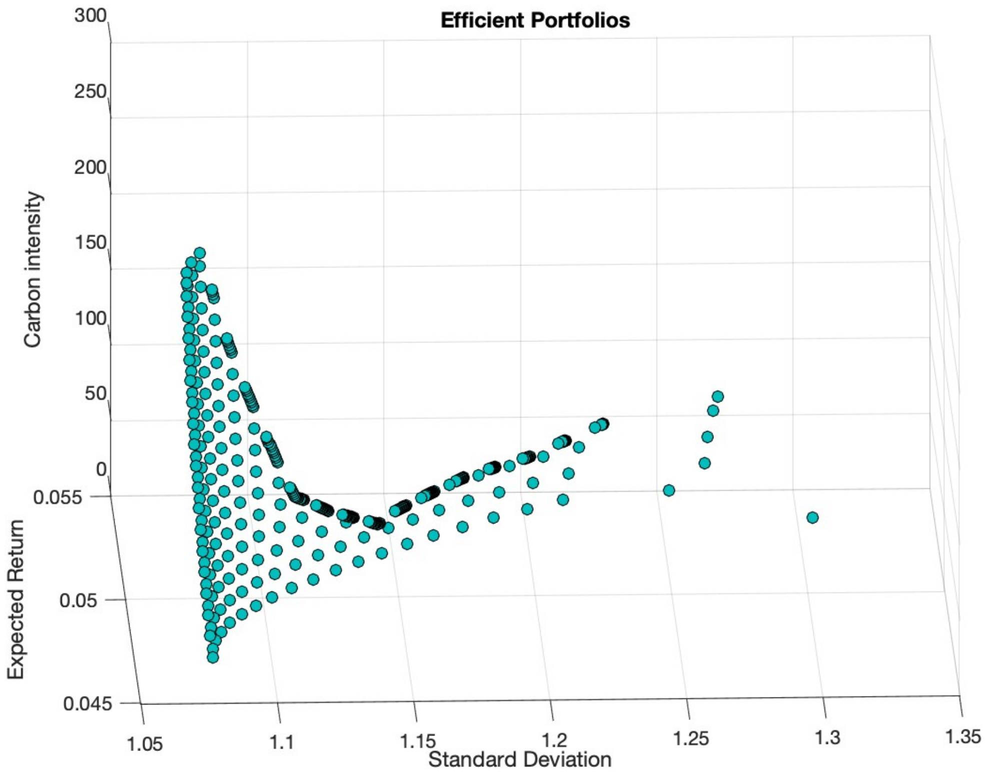



Figure 29 presents the same characteristics of Figure 26, but the the efficient frontiers are overlapped between a larger gap, namely between 1.1 and almost 1.25. The only efficient frontier visible for higher values of volatility is the one with a carbon intensity level of 62.83 (orange line), all the other are observable only for lower standard deviations. This is the major difference with respect to the efficient frontiers with restrictions on carbon emissions.

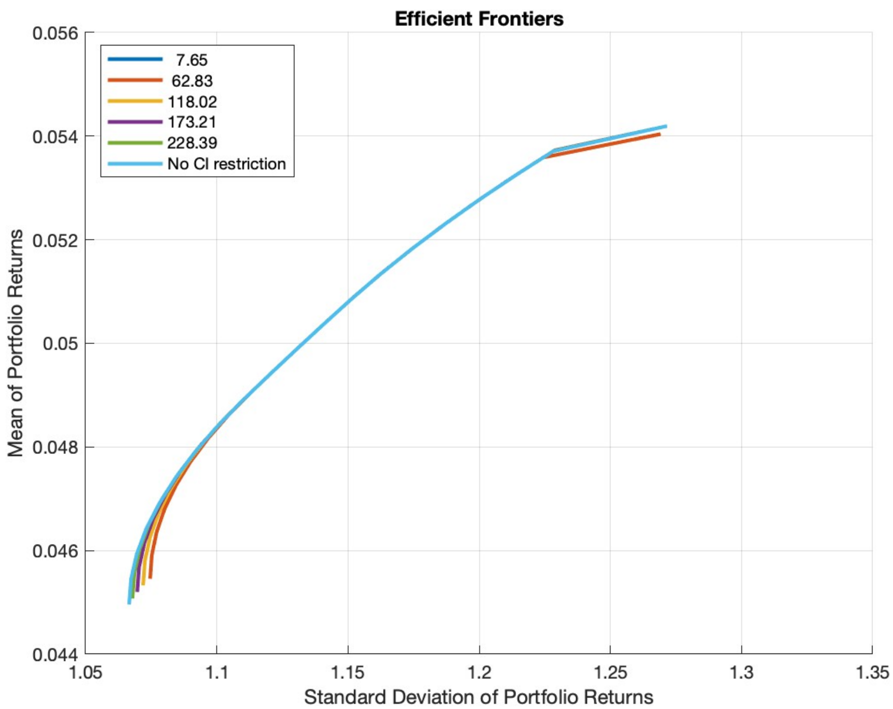


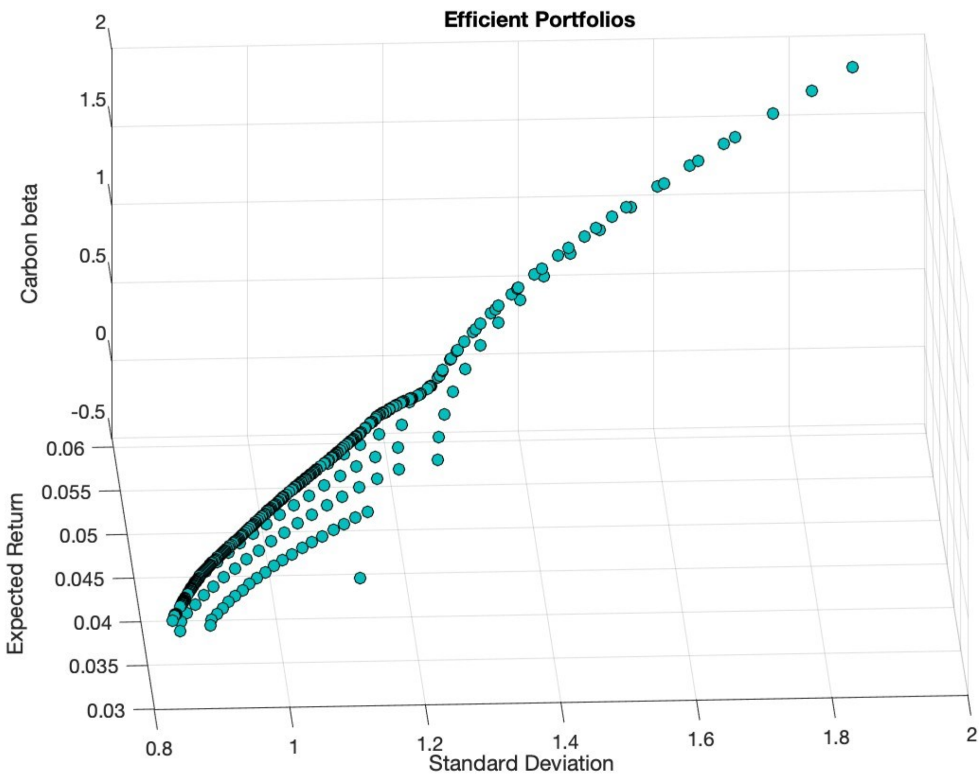

*Figure 29 – Efficient Frontiers with group constraints and target values of carbon intensity*

### 3.3.3. PORTFOLIO DECARBONIZATION WITH CARBON BETA

Carbon beta is a climate risk measure that presents different features with respect to carbon emissions and carbon intensity. This quantity does not offer a measure of how much a company pollute therefore it is difficult to compare directly with the other two climate risk measures. However, it is possible to observe the behavior of the efficient portfolios with carbon beta constraints when the set of constraints are changed and attempt to assess if there are similar patterns to the previous two measures.

Figure 30 is the efficient frontier when there are only the climate constraints. The level of carbon beta of the efficient portfolios is increasing with respect to their variance. This trend does not follow what we have observed in Figure 24 and 27 in which the climate variable first declines and then, after a certain level of volatility, rises.

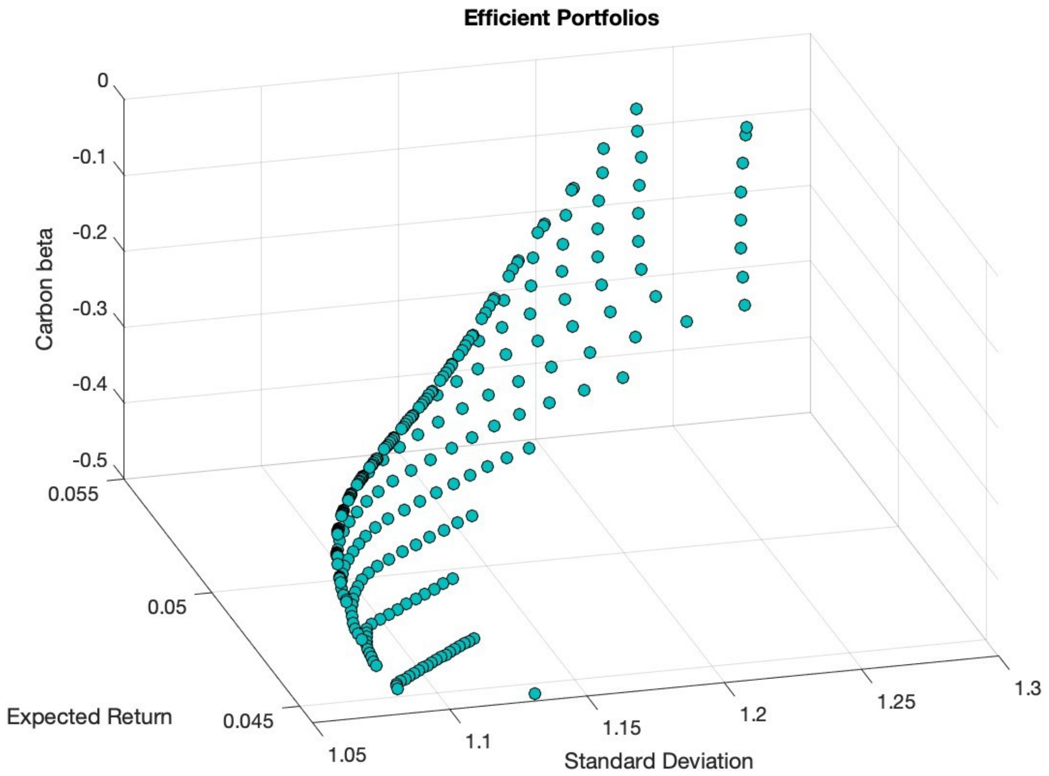
*Figure 30 – Efficient portfolios without group constraints*




By looking at Figure 31, we notice that the efficient portfolios have higher variability in carbon beta values with group constraints than without. Indeed, remarkable point to underline is that the values of the carbon beta now are all negative, this means that all the efficient portfolios now will be positively affected by an acceleration of the transition towards a less carbon intensive economy. Moreover, as with carbon emissions and carbon intensity the standard deviation of the portfolios is lower with group constraints.

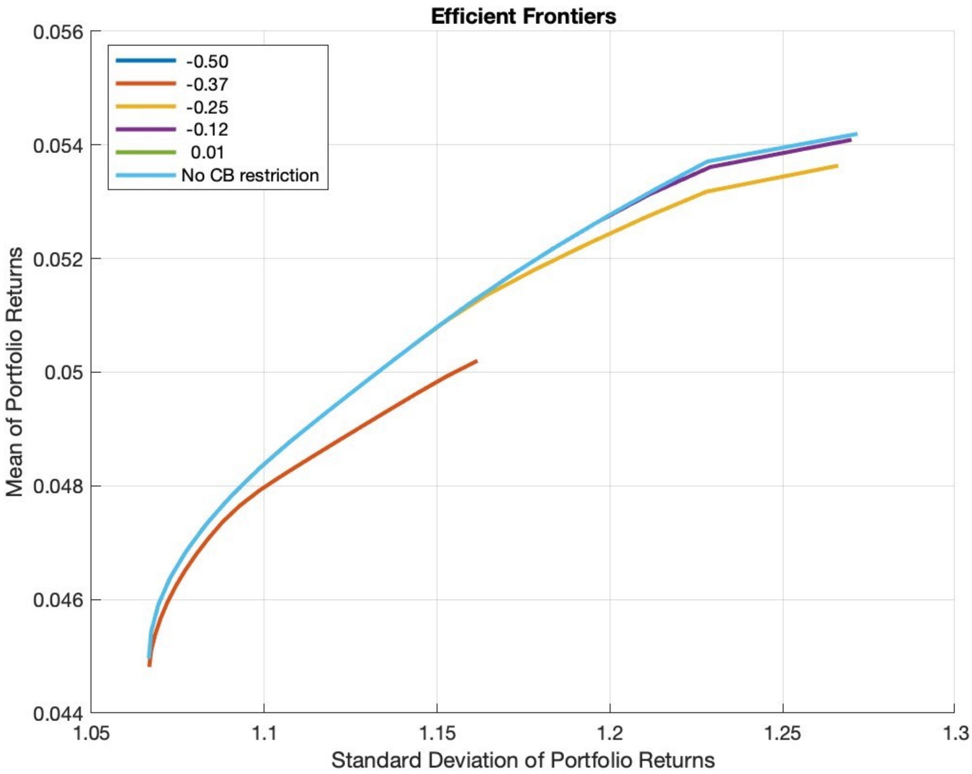
The characteristic of the increase of the carbon beta level together with volatility is preserved even when group constraints are added.

*Figure 31 – Efficient portfolios with group constraints*




Among all the figures related to the portfolio decarbonization with carbon beta the most interesting to analyze is Figure 32, it reports the efficient frontiers with different target values of carbon beta. Unlike the others it possible observe that the efficient frontier with a carbon beta target value of -0.37 (orange line) is completely visible and not overlapped with the one without restrictions (light blue line). The fact that the efficient frontiers with restrictions on carbon beta are located well below the light blue line means that at the same standard deviation they provide lower expected returns. However, since they have negative values of carbon beta, they are better hedged against the carbon risk. Therefore, when the volatility is fixed there is a trade-off between expected returns and carbon risk. For instance, when the standard deviation is equal to 1.2, the efficient frontier with a target carbon beta of -0.25 (yellow line) provides a mean return of 0.0523, while the one provided by the efficient frontier with no carbon beta restriction (light blue line) is equal to 0.053. In the figure, the efficient frontier with a carbon beta target of 0.01 (green line) is exactly overlapped to the one without carbon beta restriction, this is because the target is not so strict. Moreover, the efficient frontier with a target of -0.50 (blue line) is not represented in the figure, since that constraint is so strict that does not allow to find efficient portfolios.

*Figure 32 – Efficient Frontiers with group constraints and target values of carbon beta*




## 3.4. TRACK RECORDS OF ALLOCATION STRATEGIES

After the estimations of the efficient frontiers with different set of constraints, I made a comparison between the different strategies, trying to assess which one provides the best performances. To do that, I assessed the portfolio allocation with the constraints presented in the previous section and recovered the track records of the strategies. Specifically, I compared the evolution of portfolio composition, the realized portfolio returns, and some performance indicators.

First, I estimated portfolio moments, expected returns and covariance matrix, with different approaches. The expected returns were computed using the sample estimator and the CAPM (equilibrium return), the covariance matrix using the sample estimator as well and the exponential weighted moving average (EWMA) with a smoothing factor of 0.9. In the EWMA approach, the weights are decreasing over time, they are higher for recent observations and lower for observations far in the past. All the inputs, the realized returns and the weights of the strategies considered in my analysis were estimated by implementing a loop. That loop used a window for the estimation with a size of 600 historical daily returns of the assets and of the benchmark.

The equations used to estimate the expected returns and the covariance matrices in the different approaches are the following:

$$ErS = mean(rI)$$

$ErS$ is the expected return estimated with the sample estimator, and $rI$ are the daily returns of the assets over the study period.

$$ErE = \beta_{mkt} \times mean(rB)$$

$ErE$ is the expected return (equilibrium return) computed with the CAPM, $\beta_m$ is the market beta and $rB$ are the daily returns of the benchmark over the study period.

$$EvS = covariance(rI)$$

$EvS$ is the covariance matrix assessed with the sample estimator, obtained by computing the covariances between the returns of the assets.

$$EvW = (1 - \lambda) \times rI \times rI' + \lambda \times EvW_0$$

$EvW$ is the covariance matrix figured out with the EWMA method, $\lambda$ is the smoothing factor which can be between 0.9 and 0.99, $EvW_0$ is the covariance matrix in period 0.

Then, I built efficient frontiers using two combinations of portfolio moments. As first combination I used the sample estimator both for the expected returns and for the covariance matrix. In the second combination I estimated portfolio moments using different approaches; the CAPM for the expected returns, and EWMA for the covariance matrix.

For each combination of inputs, I considered three cases which differed only for the climate risk measure used as climate constraint. Since my focus was on the performances of efficient portfolios with different climate risk measures, in all the cases I maintained the default constraints and the group constraints defined in the previous section.

Therefore, in Table 10 are sum up the six strategies made up of different combination of portfolio moments and climate constraints:

*Table 10 – Combinations of portfolio moments and climate risk measures*

| Strategy | Expected returns | Covariance Matrix | Climate risk measure |
|---|---|---|---|
| SI | Sample Estimator | Sample Estimator | Carbon Intensity |
| SE | Sample Estimator | Sample Estimator | Carbon Emissions |
| SB | Sample Estimator | Sample Estimator | Carbon Beta |
| EI | CAPM | EWMA | Carbon Intensity |
| EE | CAPM | EWMA | Carbon Emissions |
| EB | CAPM | EWMA | Carbon Beta |

For each strategy I opted for a specific allocation method, the maximum trade-off. Therefore, I computed the portfolios with the Maximum Sharpe (MS) ratio for each strategy. The Sharpe ratio is an indicator that measures the trade-off between risk and return, the MS portfolio is the portfolio that maximizes this risk- return trade-off. However, I decided to set the risk-free rate equal to zero, thus I obtained the risk-return ratio as follows:

$$Risk - Return\ Ratio = \frac{R_p}{\sigma_p}$$

$R_p$ is the return of the portfolio and $\sigma_p$ is the volatility of the portfolio.

### 3.4.1. EVOLUTION OF PORTFOLIO COMPOSITION

The following figures display the evolution of the weights of the Max Sharpe (MS) portfolios of the six strategies I considered in my analysis. In Figure 33 are reported the evolution of portfolio composition of the strategies that have the portfolio moments estimated with the sample estimator. The portfolio with the carbon intensity as climate risk measure presents the most heterogenous composition, while the one with carbon beta constraint has a constant composition over time and it is made up of just two assets, Healthcare and Energy. In the first strategy the assets with greater weights are Healthcare until the last moths of 2016 and Technology between 2017 and the first quarter of 2022. Besides these two assets, all the other assets appear in portfolio composition but with minor weights and in different periods, such as Financials at the beginning of the study period, Utilities and Real Estate from 2016 to 2020, and Energy in the first semester of 2022. Even the strategy with carbon intensity constraint presents a changing composition though just until 2016. Indeed, after October 2015 the portfolio composition of the second strategy is constant, with a weight of 0.9 for the Healthcare asset and the remaining 0.1 for the Energy asset. As highlighted before, the SB strategy has a constant composition, the same of the SE strategy from October 2015 onwards. The reason for which the Technology and the Healthcare assets have the higher weights is due to the group constraints that do not limit their weights.

*Figure 33 – Evolution of portfolio composition of the strategies estimated with sample moments*

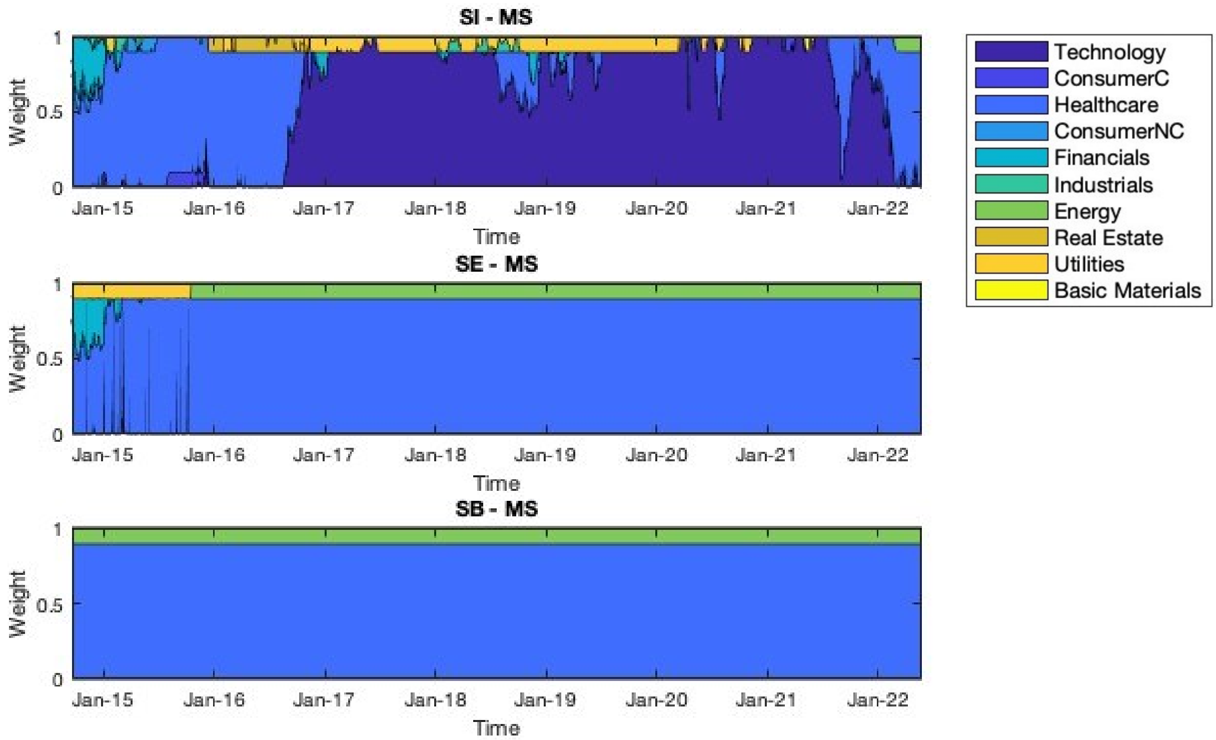
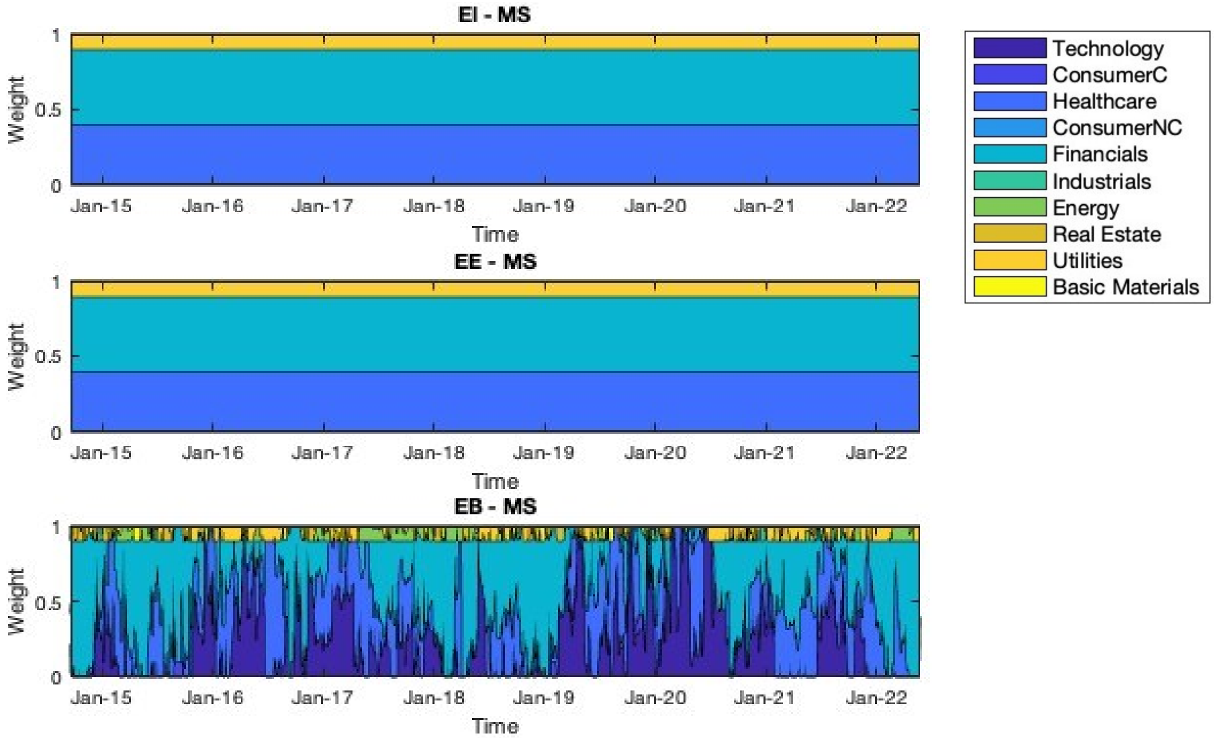



Figure 34 displays the MS portfolios for which the portfolio moments were estimated by CAPM and EWMA. First, we can notice that the strategies with carbon intensity and carbon emissions constraints have the same MS portfolio composition and that this is constant over the entire study period. These portfolios are made up of just three assets, Utilities has a weight of 0.1, Financials of 0.5, and the Healthcare asset accounts for the remaining 0.4. Instead, the strategy with carbon beta constraint presents am extremely variegate composition, the assets with the highest weights are, Financials, Utilities, Technology and Healthcare.

It is possible to make a comparison between these MS portfolios and the previous ones. The first issue to pinpoint is that the portfolio composition of the strategies is extremely different according to the approach used to estimate portfolio moments. For instance, the EI strategy is made of just three assets that have constant weights over time, conversely in the SI strategy are present all the assets with weights that change according to the period. By considering the strategies with carbon beta constraint, we observe the pattern thing but at the opposite, the strategy with the equilibrium returns and the EWMA covariance matrix presents an heterogenous portfolio composition and the one with sample moments a two-assets constant composition. The SE and EE strategies show deep differences in the composition but not so deep in the evolution of the composition, indeed even the strategy with sample moments has constant weights after October 2015. To conclude, it is possible to observe that the strategies with sample moments have different predominant assets with respect to the strategies estimated with CAPM and EWMA. Technology, Healthcare and Energy for the formers, and Utilities and Financials for the latter.

*Figure 34 – Evolution of portfolio composition of the strategies estimated with CAPM returns and EWMA covariance matrix*



### 3.4.2. CUMULATED RETURNS

Besides the evolution of portfolio composition, I compared the cumulated returns of the strategies. Therefore, starting from the realized returns computed with the loop described in section 3.4, I calculated the cumulated returns of each strategy with the following equation:

$$R_t = \left[\prod_{i=m+1}^{t} (1 + r_i)\right] - 1$$

$R_t$ is the cumulated return at time $t$ and $r_i$ is the realized return of a certain portfolio/strategy and it is indexed from time $m + 1$ since the returns used for the estimation window are not considered.

Other than the cumulated returns of the six strategies, I assessed the cumulated returns of the benchmark (S&P 500 index) and of a naive allocation approach, the equally weighted (EW) portfolio, in which all the assets have the same weight, $w_i = \frac{1}{N}$.

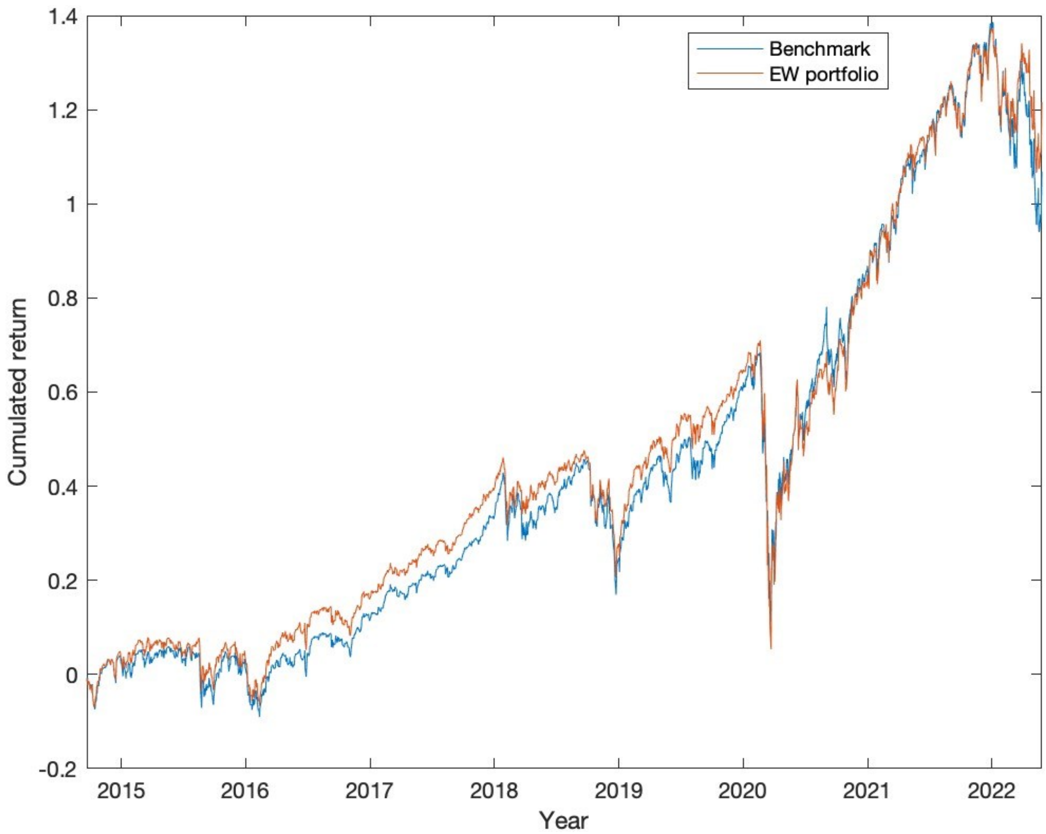



*Figure 35 – cumulated returns of the benchmark and equally weighted portfolio*

In Figure 35, I compared the cumulated returns of the benchmark with the ones of the EW portfolio over the study period. The cumulated returns of both portfolios show a similar trend, at the beginning of the study period the cumulated returns have values that swing around zero and sometimes are negative. From 2016 they have a steady increase until reaching the peak at the end of 2021, nevertheless there is sharp drop in the first months of 2020 due to the eruption of COVID19 crisis. Although the similarity in the evolution of cumulated returns, in certain period their values differ, indeed until 2020 the EW portfolio (orange line) has higher values than the benchmark, meaning that in terms of economic return it outperforms the benchmark. In general, after 2020 the cumulated returns of the portfolios are almost overlapped, with some exceptions, such as in the second semester of 2020 in which the cumulated returns of the benchmark are above the ones of the EW portfolio.

Figure 36 depicts the cumulated returns of the strategies computed with sample moments, the benchmark, and the EW portfolio. First, we notice that the all the three strategies provide higher cumulated returns than the benchmark and the EW portfolio, with the one with carbon intensity constraint (blue line) that deliver the highest returns from 2017 onwards. The cumulated returns of the other two strategies (orange and yellow line) have values that are quite similar but not identical, indeed the SB strategy (yellow line) outperform the SE over the entire study period. Although the higher values, the trends of cumulated returns of the strategies follow the ones of the benchmark and the EW portfolio. Even for the three strategies the peak is reached at the end of 2021 and after that the returns start to decrease, it is also visible the fall of 2020 due to the epidemic crisis.

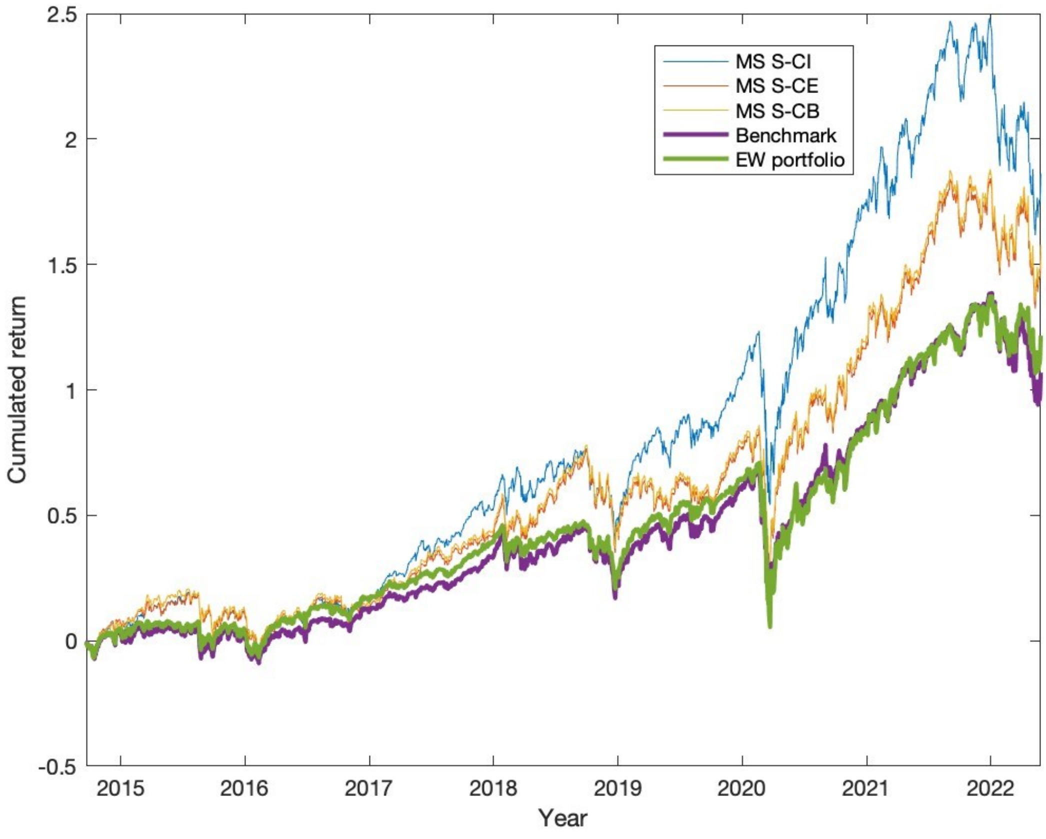



*Figure 36 – cumulated returns of the strategies estimated with sample moments*

In Figure 37 I examined the cumulated returns of the three strategies estimated with CAPM and EWMA. Although the different approaches to assess portfolio moments, the three strategies still present higher cumulated returns than the benchmark and the EW portfolio. However, the change of approaches has a great impact on the values of cumulated returns of the strategies, the strategy that provide highest returns is the one with carbon beta constraint (yellow line), conversely the EI strategy (blue line) experiences an evident decline in cumulated returns with respect to the SI strategy. Instead, the strategy with carbon emissions constraint (orange dotted line) does not suffer a significant change in its cumulated returns with different approaches of estimation of portfolio moments. In addition, we observe that the cumulated returns of the EI and EE strategies seem to be overlapped but they are not. This feature is explained by the fact that the two portfolios have the same composition over the entire study period (see Figure 34).

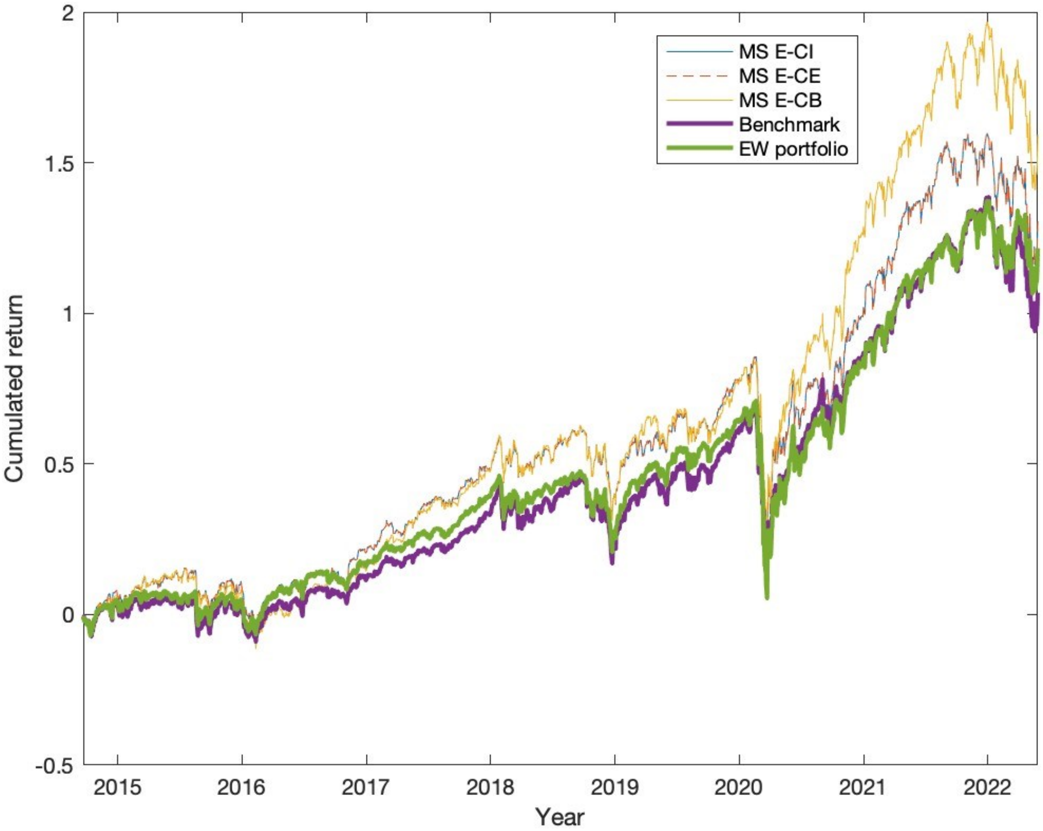



*Figure 37 – cumulated returns of the strategies estimated with CAPM returns and EWMA covariance matrix*

### 3.4.3. PERFORMANCE INDICATORS

From the daily returns I constructed the most common performance measures that then were used to rank the strategies. These measures can be divided in two groups, the first group regards the absolute performance indicators for each strategy, the second one is about the relative performance of the strategies with respect to the benchmark.

Below, I report the equations used to compute all the absolute performance measures:

$$Sh = \frac{E[R_t]}{V[R_t]}$$

the first performance measure is the Sharpe ratio which has been already explained at the end of the section 3.4, this ratio indicates the expected return per unit of total risk.

$$So = \frac{E[R_t]}{V[R_t I(R_t < 0)]}$$

this one is the Sortino ratio which is used to compute the expected return per unit of downside risk, that is the risk the return is negative.

$$Tr = \frac{E[R_t]}{\beta}$$

the Treynor ratio figures out the expected return per unit of systematic risk. All these ratios have two common elements, they are ratios of a reward index and of a risk measure.

$Var(\alpha)$ is the Value-at-Risk and it is the quantile of order 1- $\alpha$ of the empirical distribution of the returns and must satisfy the equation above, $\alpha$ is the probability of observing returns below the $Var(\alpha)$. In other words, the $Var(\alpha)$ is the maximum loss the portfolio can suffer with a probability $1 - \alpha$, and with a probability of $\alpha$ that the loss will be larger than the Value-at-Risk.

The $VaR$ presents the lack of the so-called sub-additivity property of a risk measure, the $VaR$ of a portfolio should be smaller than the combination of the $VaR$ of the underlying assets. The Expected Shortfall ($ES$) was proposed to overcome this limit, this quantity is a conditional expectation and equals the mean of returns below the $VaR$.

The equation to compute the Expected Shortfall is the following:

$$ES\,(R_t, \alpha) = E[R_t | R_t < VaR(\alpha)]$$

The other two measures I considered are associated with the quantity called Drawdown ($D$), whose sequence is graphically examined to evaluate the largest losses of a strategy and the time to recover them. Better strategies present smaller losses and quick recovery time. At a given starting point, time 1, the Drawdown is set equal to zero, $D_1 = 0$ and then the evaluation follows this equation:

$$D_t = \min(0, (1 + D_{t-1}) \times (1 + R_t) - 1)$$

The Calmar ratio[12] is one of the two gauges that use Drawdown to be computed. This ratio measures the return per unit of extreme risk where this is set equal to the largest Drawdown in absolute value. Analytically, the Calmar ratio is equal to

$$Cal = \frac{E[R_t]}{|\max Drawdown\,|}$$

The Sterling ratio[13] is analogous to the Calmar one, the difference is that the extreme risks are set to be equal to the absolute value of the average of the largest $k$ Drawdowns (with $k$ being small).

$$Ste = \frac{E[R_t]}{\frac{1}{k}\sum_{i=1}^{k} |\max Drawdown_i|}$$

To make the comparison of the performance measures of the strategies and of the benchmark more straightforward and effective I created a composite index. First, for each performance indicator I ranked the strategies from the best, 1, to the worst, 7. Then, I summed the ranks of each strategy and obtained a composite index where the best strategy is the one with the lowest sum of the ranks, meaning that on average it ranks in the first positions in all the performance gauges.

In table 11 are reported the rankings of the strategies for each performance measures and the related composite index (CI). The strategy with the best performances is the one with sample moments and carbon intensity constraint, it ranks first in all the performance indicators apart from the $VaR$ in which it is in the fourth place. However, the strategy with the same climate

*[12] This ratio was developed by the American economist Terry W. Young in 1991 (source: www.investopedia.com).*
*[13] The original definition of this ratio was proposed by the Dean Sterling Jones company (source: www.wikipedia.org).*

constraint but with the portfolio moments estimated with different approaches holds the penultimate place. The climate risk measure that provides better performances is the carbon beta, indeed the strategies (SB and EB) occupy the second and the third place of the composite index. Conversely, the strategies with carbon emissions constraint are the ones with the worst performances, the EE strategy ranks last in all the performance indicators. In general, it seems to be a correlation between the approaches used to estimate portfolio moments and the performance measure. Indeed, portfolios estimated with sample estimators have better performances than the ones estimated with CAPM and EWMA.

*Table 11 – Rankings of performance measures*

| Strategy | CI | Sh | So | Tr | VaR | ES | Cal | Ste |
|---|---|---|---|---|---|---|---|---|
| SI-MS | 10 | 1 | 1 | 1 | 4 | 1 | 1 | 1 |
| SB-MS | 17 | 3 | 2 | 2 | 1 | 3 | 3 | 3 |
| EB-MS | 18 | 2 | 3 | 4 | 3 | 2 | 2 | 2 |
| SE-MS | 25 | 4 | 4 | 3 | 2 | 4 | 4 | 4 |
| EI-MS | 35 | 5 | 5 | 5 | 5 | 5 | 5 | 5 |
| EE-MS | 42 | 6 | 6 | 6 | 6 | 6 | 6 | 6 |

As anticipated before, besides the absolute performance measures of a strategy there are also some indicators that assess the performance of that strategy with respect to a benchmark. These measures are computed starting from the deviations of the strategy returns from those of the benchmark (S&P 500 index), these deviations are called Tracking errors.

The first two measures are the moments of the Tracking error, average Tracking Error (TE) and Tracking Error Volatility (TEV):

$$TE = E[R_t - R_t^B]$$
$$TEV = V[R_t - R_t^B]$$
$$IR = TE/TEV$$

$R_t$ is the return of the strategy and $R_t^B$ is the return of the benchmark. The ratio $TE/TEV$ is called Information Ratio (IR) and it is comparable to a Sharpe ratio computed with Tracking errors without the risk-free asset. It is possible to compute the $TEV$ only on downside deviations, as for the Sortino ratio, and obtain the Semi-TEV.

Table 12 is constructed following the same method of Table 11, therefore the strategies with the best performances relative to the benchmark are those with the lowest value of the composite index (CI). The major issue to highlight is that the Si strategy is the best performer even relative to the benchmark, it ranks first in the TE but just fourth in the TEV. On the other hand, the strategy with the lowest Tracking error volatility is the EI. The strategies with carbon beta constraint maintain the second and the third positions even in relation to the benchmark but they are swapped, here the Eb strategy is above the one with sample moments. Another relevant issue to underline is that the SE and EE strategies have the same value of composite index and are the worst strategies with respect to the benchmark. Therefore, the carbon emissions constraint provides poor performances both in absolute and relative terms.

*Table 12 – Rankings of the strategies with respect to the benchmark*

| Strategy | CI | TE | TEV | Semi-TEV | IR |
|---|---|---|---|---|---|
| SI-MS | 10 | 1 | 4 | 3 | 2 |
| EB-MS | 11 | 3 | 3 | 4 | 1 |
| SB-MS | 14 | 2 | 6 | 1 | 5 |
| EI-MS | 15 | 5 | 1 | 6 | 3 |
| SE-MS | 17 | 4 | 5 | 2 | 6 |
| EE-MS | 17 | 6 | 2 | 5 | 4 |

# CONCLUSIONS

It is clear that is impossible to imagine a sustainable future for future generations if there will not be taken serious actions aim to drastically reduce carbon emissions. The latter are the main causes of global warming and of climate change. Economically speaking, both demand side and supply side are extremely influenced by the catastrophic events due to climate change. Consumption shrink since consumers might prefer to save more to face the uncertainty about future demand and climate risks. For those reasons, are expected to decline event the investments and the international trades. On the other hand, the productive capacity of the economy is affected by supply shocks. Physical capital and capital stock are the components of potential supply that suffers more the shock related to climate change. Physical assets are damaged by natural calamities, these latter throw serious threat also to the longevity of physical capital through an increased speed of capital depreciation. Therefore, efficiency might experience a decrease although there is a possibility that the relevant capital stocks might survive. As explained in the first chapter, financial instability due to climate-change related shocks can be transmitted two ways, physical and transition risk. Physical risk refers to the possibility of economic costs and financial losses that could arise from both an increase in the frequency and severity of weather events, and gradual climate change. The Paris Agreement of 2015 set relevant carbon emissions targets that aim to limit global warming and to favour transitioning to low-carbon economies. Thus, the "green" transition implies some of the existing carbon intensive assets being stranded, meaning that their market values will drastically reduce in the future. Physical and transition risk are interconnected and should be measured as part of the same framework.

Consumers' concerning about transition risk is increasing in recent years, therefore their tastes are changing. They prefer to invest in firms that comply with environmentally sustainable policies, those firms can be defined as "green". However, classical portfolio theory would not suggest constructing a portfolio made up of just green stocks since there might be a lack of diversification. To overcome this problem, it is possible to decarbonize a benchmark index. Through this process it is possible to hedge climate risk by imposing constraints on climate risk measures while maintaining a certain degree of portfolio diversification.

To perform the decarbonization of a benchmark, it is fundamental to define which climate risk measure to use. In many of the papers that treat this topic are used carbon emissions and carbon intensity, this latter is useful to compare firms that present evident dissimilarities especially in terms of size. Nevertheless, Görgen et al. (2019) developed another climate risk measure that quantify the carbon risk of a company. This risk is related to the transition risk; indeed, the carbon risk measures the sensitivity of the stock price of a company to the transition process. Therefore, the carbon beta is similar to the carbon beta in the CAPM, companies with a positive carbon beta are negatively affected by the transition towards a low-carbon economy. The study of Görgen has been expanded in the paper of Roncalli et al. (2020) that added a dynamic analysis of the carbon beta. Their contribution stems in the idea that climate risks are not constant over time but are time-varying and are subject to the policies that are implemented. Roncalli et al. (2020) estimated the dynamics of the median carbon beta by sector, and they noted that the energy sector is the one most at risk in the event of a green transition.

What described above is the framework in which it fits my analysis. I compared the results of portfolio decarbonization performed with three different climate risk measures, carbon emissions, carbon intensity and carbon beta. The choice of the benchmark to decarbonize was driven by the necessity to find an index for which there was a complete carbon emissions' dataset. The S&P 500 index being one the leading indexes of the global stock market, presents that characteristic.

My analysis was divided in four steps. First, I created three portfolios made up of the same assets of the benchmark, then I decarbonized each portfolio by using one of the three climate risk measures. The second step involved the comparison between the efficient frontiers of the portfolios with and without group constraints. In all the cases the presence of group constraints implied an increase in the variability of the values of climate risk measure of the efficient portfolios. Graphically, the efficient portfolios with group constraints looked more scattered in the three-dimension efficient frontier. In the third step, I maintained the same climate and group constraints of the second step, but I computed the efficient frontiers by estimating the portfolio moments with different approaches. The reason to construct efficient frontiers with different constraints and different portfolio moments was to observe which strategy would have provided the best performances. Therefore, I ended up with six strategies (see Table 10), for each strategy I computed the Maximum Sharpe portfolio. The last step consisted in the comparison of the portfolio compositions, cumulated returns, and performance measures of the six strategies. The performance indicators were assessed both in absolute and relative terms with respect to the benchmark. Table 11 and 12 report the results, and it emerges that the portfolio that provides the best performance is the one decarbonized with carbon intensity and portfolio moments estimated with sample estimators. Even in relative terms this strategy results to be the best.

The results of my analysis do not imply that carbon intensity is unequivocally the best climate risk measure for portfolio decarbonization. However, it is still possible to draw some final remarks. Climate change is modifying the tastes of the investors, they are shifting towards sustainable investments, and the asset management industry is trying to develop product that meet those new “green” tastes. Portfolio decarbonization is one of them, and it might become a viable option since it combines good financial performances with a hedging function against climate risk.

# APPENDIX A

## MATLAB CODES

### A.1. PRELIMINARY ANALYSIS

The following lines of code are used to perform a preliminary analysis of sectors' carbon emissions, carbon intensity and of the daily returns of the assets and of the benchmark. In the last lines I estimate the relevant quantities of the CAPM, then we compute the expected returns and the covariance matrix of the assets. All the climate data are retrieved from the excel file "Dataset".

```
% loading dataset from the Excel file 'Dataset'

CE = readmatrix('Dataset','Sheet','CO2 emissions','Range','C4:D503'); % carbon emissions
CI = readmatrix('Dataset','Sheet','CO2 emissions','Range','E4:F503'); % carbon intensity
input = readtable('Dataset','Sheet','CO2 emissions','ReadVariableNames', false, 'TreatAsEmpty',{'NA'});
input=input(3:502,1); % I only consider the names of the firms belonging to
S&P500

CE20=CE(:,1); % carbon emissions of 2020
CE19=CE(:,2); % carbon emissions of 2019
CI20=CI(:,1); % carbon intensity of 2020
CI19=CI(:,2); % carbon intensity of 2019

% descriptive analysis of carbon emissions and returns of S&P500
MeanCE19=mean(CE19)';
MeanCE20=mean(CE20)';
MedianCE19=median(CE19)';
MedianCE20=median(CE20)';
StDevCE19=sqrt(var(CE19))';
StDevCE20=sqrt(var(CE20))';
MinCE19=min(CE19)';
MinCE20=min(CE20)';
MaxCE19=max(CE19)';
MaxCE20=max(CE20)';
SkewCE19=skewness(CE19)';
SkewCE20=skewness(CE20)';
KurtCE19=kurtosis(CE19)';
KurtCE20=kurtosis(CE20)';
TMCE=table(MeanCE19,MeanCE20,MedianCE19,MedianCE20,StDevCE19,StDevCE20,MinCE19,MinCE20,
MaxCE19,MaxCE20,SkewCE19,SkewCE20,KurtCE19,KurtCE20);

MeanCI19=mean(CI19)';
MeanCI20=mean(CI20)';
MedianCI19=median(CI19)';
MedianCI20=median(CI20)';
StDevCI19=sqrt(var(CI19))';
StDevCI20=sqrt(var(CI20))';
MinCI19=min(CI19)';
MinCI20=min(CI20)';
MaxCI19=max(CI19)';
MaxCI20=max(CI20)';
```

```
SkewCI19=skewness(CI19)';
SkewCI20=skewness(CI20)';
KurtCI19=kurtosis(CI19)';
KurtCI20=kurtosis(CI20)';
TMCI=table(MeanCI19,MeanCI20,MedianCI19,MedianCI20,StDevCI19,StDevCI20,MinCI19,MinCI20,MaxCI
19,MaxCI20,SkewCI19,SkewCI20,KurtCI19,KurtCI20);

% Sectors' analysis
% Technology
inputTech = input(2:86,1);
CITech = readmatrix('Dataset','Sheet','Sectors','Range','E20:F104');
CETech = readmatrix('Dataset','Sheet','Sectors','Range','G20:H104');
MeanCETech=mean(CETech);
MeanCITech=mean(CITech);

% Consumer Cyclicals
inputConsumerCycles = input(2:75,10);
CIConsC = readmatrix('Dataset','Sheet','Sectors','Range','N20:O93');
CEConsC = readmatrix('Dataset','Sheet','Sectors','Range','P20:Q93');
MeanCEConsC=mean(CEConsC);
MeanCIConsC=mean(CIConsC);

% Healthcare
inputHealthcare = input(2:63,19);
CIHealthcare = readmatrix('Dataset','Sheet','Sectors','Range','W20:X81');
CEHealthcare = readmatrix('Dataset','Sheet','Sectors','Range','Y20:Z81');
MeanCEHealthcare=mean(CEHealthcare);
MeanCIHealthcare=mean(CIHealthcare);

% Consumer Non-Cyclicals
inputConsNC = input(2:41,28);
CIConsNC = readmatrix('Dataset','Sheet','Sectors','Range','AF20:AG59');
CEConsNC = readmatrix('Dataset','Sheet','Sectors','Range','AH20:AI59');
MeanCEConsNC=mean(CEConsNC);
MeanCIConsNC=mean(CIConsNC);

% Financials
inputFinancials = input(2:62,37);
CIFinancials = readmatrix('Dataset','Sheet','Sectors','Range','AO20:AP80');
CEFinancials = readmatrix('Dataset','Sheet','Sectors','Range','AQ20:AR80');
MeanCEFinancials=mean(CEFinancials);
MeanCIFinancials=mean(CIFinancials);

% Industrials
inputIndustrials = input(2:71,46);
CIIndustrials =readmatrix('Dataset','Sheet','Sectors','Range','AX20:AY89');
CEIndustrials =readmatrix('Dataset','Sheet','Sectors','Range','AZ20:BA89');
MeanCEIndustrials=mean(CEIndustrials);
MeanCIIndustrials=mean(CIIndustrials);

% Energy
inputEnergy = input(2:24,55);
CIEnergy = readmatrix('Dataset','Sheet','Sectors','Range','E424:F446');
CEEnergy = readmatrix('Dataset','Sheet','Sectors','Range','G424:H446');
MeanCEEnergy=mean(CEEnergy);
MeanCIEnergy=mean(CIEnergy);

% Real Estate
inputRE = input(2:31,64);
CIRE = readmatrix('Dataset','Sheet','Sectors','Range','BP20:BQ49');
CERE = readmatrix('Dataset','Sheet','Sectors','Range','BR20:BS49');
MeanCERE=mean(CERE);
MeanCIRE=mean(CIRE);
```

```
% Utilities
inputUtilities = input(2:30,73);
CIUtilities = readmatrix('Dataset','Sheet','Sectors','Range','BY20:BZ48');
CEUtilities = readmatrix('Dataset','Sheet','Sectors','Range','CA20:CB48');
MeanCEUtilities=mean(CEUtilities);
MeanCIUtilities=mean(CIUtilities);

% Basic Materials
inputBM = input(2:27,82);
CIBM = readmatrix('Dataset','Sheet','Sectors','Range','CH20:CI45');
CEBM = readmatrix('Dataset','Sheet','Sectors','Range','CJ20:CK45');
MeanCEBM=mean(CEBM);
MeanCIBM=mean(CIBM);

% creating sectors' labels
lab={'S&P500','Technology','ConsumerC','Healthcare','ConsumerNC','Financials','Industrials','Energy','Real
Estate','Utilities','Basic Materials'};

% tables of carbon emissions (2020 and 2019) and carbon intensities
% (2010 and 2019)
TMMeanCE=table(MeanCETech,MeanCEConsC,MeanCEHealthcare,MeanCEConsNC,MeanCEFinancials,Mea
nCEIndustrials,MeanCEEnergy,MeanCERE,MeanCEUtilities,MeanCEBM;

TMMeanCI=table(MeanCITech,MeanCIConsC,MeanCIHealthcare,MeanCIConsNC,MeanCIFinancials,MeanCI
Industrials,MeanCIEnergy,MeanCIRE,MeanCIUtilities,MeanCIBM;

writetable(TMMeanCE,'Dataset.xlsx','Sheet','Descriptive analysis','Range','C16');
writetable(TMMeanCI,'Dataset.xlsx','Sheet','Descriptive analysis','Range','C19');

figure;
bar(1:10,[MeanCITech' MeanCIConsC' MeanCIHealthcare' MeanCIConsNC' MeanCIFinancials'
MeanCIIndustrials' MeanCIEnergy' MeanCIRE' MeanCIUtilities' MeanCIBM']);
set(gca,'Xtick',1:10,'Xticklabel',lab(2:11));
xlabel('Sector');
ylabel('Carbon Intensity (tCO2e/$)');
title('Mean Carbon Intensity by Sector');
legend('2020','2019');

bar(1:10,[MeanCETech' MeanCEConsC' MeanCEHealthcare' MeanCEConsNC' MeanCEFinancials'
MeanCEIndustrials' MeanCEEnergy' MeanCERE' MeanCEUtilities' MeanCEBM']);
set(gca,'Xtick',1:10,'Xticklabel',lab(2:11));
xlabel('Sector');
ylabel('Carbon Emission (tCO2e)');
title('Mean Carbon Emission by Sector');
legend('2020','2019');

% loading historic prices from the Excel file 'Dataset'
datapr=readmatrix('Dataset.xlsx','Sheet','Daily Prices Sectors','Range','B3');
inputpr = readtable('Dataset.xlsx','Sheet','Daily Prices
Sectors','ReadVariableNames',false,'TreatAsEmpty',{'NA'});
inputpr=inputpr(:,1);

a=datapr(:,1);
plot(dD(2:end,:),a)
datetick('x','mmm-yy')
title('S&P500 INDEX')
xlabel('Year')
ylabel('Price')

b=datapr(:,2:11);
plot(dD(2:end,:),b(:,1),'Color',[0 0.4470 0.7410])
hold on
plot(dD(2:end,:),b(:,2),'Color',[0.8500 0.3250 0.0980])
```

```
hold on
plot(dD(2:end,:),b(:,3),'Color',[0.9290 0.6940 0.1250])
hold on
plot(dD(2:end,:),b(:,4),'Color',[0.4940 0.1840 0.5560])
hold on
plot(dD(2:end,:),b(:,5),'Color',[0.4660 0.6740 0.1880])
hold on
plot(dD(2:end,:),b(:,6),'Color',[0.3010 0.7450 0.9330])
hold on
plot(dD(2:end,:),b(:,7),'Color',[0.6350 0.0780 0.1840])
hold on
plot(dD(2:end,:),b(:,8),'Color',[0 1 0])
hold on
plot(dD(2:end,:),b(:,9),'Color',[1 0 1])
hold on
plot(dD(2:end,:),b(:,10),'Color',[0 0 0])
datetick('x','mmm-yy')
title('Sector indexes')
xlabel('Year')
ylabel('Price')
legend(lab(2:11))

% transforming date to datetime format
dD=table2array(inputpr);

% computing daily returns
rD=((datapr(2:(size(datapr,1)),:)./datapr(1:(size(datapr,1)-1),:))-1)*100;

% descriptive analysis of returns
Mean=mean(rD)';
Median=median(rD)';
StDev=sqrt(var(rD))';
Min=min(rD)';
Max=max(rD)';
Skew=skewness(rD)';
Kurt=kurtosis(rD)';
TM=table(Mean,Median,StDev,Min,Max,Skew,Kurt,'RowNames',lab);

% returns of the assets
r_asset=rD(:,2:11);

% returns of the benchmark/market (S&P 500)
r_bench=rD(:,1);

% estimating the CAPM and storing relevant quantities
% pre-allocating matrices
alpha=zeros(size(r_asset,2),1);                    % alpha
beta=zeros(size(r_asset,2),1);                     % beta
r2=zeros(size(r_asset,2),1);                       % r-squared
eqret=zeros(size(r_asset,2),1);                    % equilibrium return
resid=zeros(size(r_asset,1),size(r_asset,2));

% the market risk premium used is the mean return of the benchmark using the full sample
rmeq=mean(rD(:,1));

% the market variance is the variance of the benchmark using the full sample
vmeq=var(rD(:,1));
riskmeq=sqrt(vmeq);

for i=1:size(r_asset,2)
    out=regstats(r_asset(:,i),r_bench,'linear',{'beta','r','rsquare','tstat'});
    % storing
    alpha(i,1)=out.beta(1);
```

```
    beta(i,1)=out.beta(2);
    r2(i,1)=out.rsquare;
    resid(:,i)=out.r;
    eqret(i,1)=out.beta(2)*(rmeq);
end

% computing the mean return of the assets with the equilibrium moments method
MM=eqret';
figure
bar(1:10,MM) % plotting the returns in a bar chart
set(gca,'Xtick',1:10,'Xticklabel',lab(2:11));
xlabel('Asset');
ylabel('Mean return');
title('Mean return of assets');

% computing the covariance matrix of the assets with the equilibrium moments method
MV=beta*(beta')*vmeq+diag(diag(cov(resid)));
figure
bar(1:10,sqrt(diag(MV)))
set(gca,'Xtick',1:10,'Xticklabel',lab(2:11));
xlabel('Asset');
ylabel('Volatility');
title('Volatilities of assets');
```

## A.2. PORTFOLIO OPTIMIZATION

The second part of the code is devoted to portfolio optimization with constraints on climate risk measures. For each type of measure, I optimize the portfolio made up of the assets computed in the previous section with and without the presence of group constraints. To conclude, I plot some efficient frontiers with different target level of carbon risk measures, I do this with a local function.

```
%% CARBON INTENSITY PORTFOLIO OPTIMIZATION WITH GROUP CONSTRAINTS

% computing the 'No short selling' efficient frontier with equilibrium moments
p=Portfolio('AssetMean',MM,'AssetCovar',MV,'AssetList',lab(2:11));
p=p.setDefaultConstraints(10);
% setting group constraints
G=[1 0 1 0 1 0 0 0 0 0; % Tech, Healthcare and Financials
0 1 0 1 0 1 1 1 1 1]; % all the other sectors
LowG=[0.15;0];
UpG=[inf;0.1];
p=setGroups(p,G,LowG,UpG);

% Transform default constraints
lb = zeros(p.NumAssets,1); % Long-only constraint
ub = [];                   % No explicit weight upper bounds
Aeq = ones(1,p.NumAssets); % Weights must sum to 1
beq = 1;

%Transform group constraints
% GroupMatrix * x <= UpperGroup
% -GroupMatrix * x <= -LowerGroup
A = [G; -G];
b = [UpG; -LowG];
```

```
% Get rid of unbounded inequality constraints
ii = isfinite(b);
A = A(ii,:);
b = b(ii);

% loading CI19 for each sector
MCI19=readmatrix("Dataset.xlsx","Sheet","Sectors","Range","F5:F14");

% Find the portfolio with the minimum CI with the group constraints.
[wgt_minCI19,minCI19] = linprog(MCI19,A,b,Aeq,beq,lb,ub);
% Find the portfolio with the maximum CI with the group constraints.
[wgt_maxCI19,fvalI] = linprog(-MCI19,A,b,Aeq,beq,lb,ub);
maxCI19 = -fvalI;

% Define a grid of CI such that minCI19 = targetCI19(1) ≤…≤ targetCI19(N) = maxCI19.
N = 20; % Size of grid
targetCI19 = linspace(minCI19,maxCI19,N);

% set inequality
Ain = MCI19';
bin = maxCI19; % start with the highest CI
p = setInequality(p,Ain,bin);

% Compute risks and returns
prsk = cell(N,1);
pret = cell(N,1);
pCI19 = cell(N,1);
for i = 1:N
    p.bInequality = targetCI19(i);
    pwgt = estimateFrontier(p,N);
    [prsk{i},pret{i}] = estimatePortMoments(p,pwgt);
    pCI19{i} = pwgt'*MCI19;
end

% 3-D plot
scatter3(cell2mat(prsk),cell2mat(pret),cell2mat(pCI19),'MarkerEdgeColor','k','MarkerFaceColor',[0 .75 .75])
title('Efficient Portfolios')
xlabel('Standard Deviation')
ylabel('Expected Return')
zlabel('Carbon intensity')
% visualizing the tradeoff between a portfolio's Carbon Intensity and the traditional mean-variance efficient frontier
nC = 5; % Number of contour plots
minContour = max(pCI19{1}); % CI values lower than this
                            % return overlapped contours.

% Plot contours
plotContour(p,minContour,maxCI19,nC,N);

function [] = plotContour(p,minCI19, maxCI19,nContour,nPort)

% Set of CI levels for contour plot
contourCI19 = linspace(minCI19,maxCI19,nContour+1);

% Compute and plot efficient frontier for each value in
% contourCI.
figure;
hold on
labels = strings(nContour+1,1);
for i = 1:nContour
    p.bInequality = contourCI19(i);
    pwgt = estimateFrontier(p,nPort);
    [prsk,pret] = estimatePortMoments(p,pwgt);
```

```
    plot(prsk,pret,'LineWidth',2);
    labels(i) = sprintf("%6.2f% CI",contourCI19(i));
end

% Plot the "original" mean-variance frontier, i.e., the
% frontier without CI requierements
p.AInequality = []; p.bInequality = [];
pwgt = estimateFrontier(p,nPort);
[prsk,pret] = estimatePortMoments(p,pwgt);
plot(prsk,pret,'LineWidth',2);
labels(i+1) = "No CI restriction";
title('Efficient Frontiers')
xlabel('Standard Deviation of Portfolio Returns')
ylabel('Mean of Portfolio Returns')
legend(labels,'Location','northwest')
grid on
hold off
end

%% CARBON EMISSIONS PORTFOLIO OPTIMIZATION WITH GROUP CONSTRAINTS

% computing the 'No short selling' efficient frontier with equilibrium moments
% setting tracking portfolio (S&P500)
pE=Portfolio('AssetMean',MM,'AssetCovar',MV,'AssetList',lab(2:11));
pE=pE.setDefaultConstraints(10);
% setting gourp constraints
GE=[1 0 1 0 1 0 0 0 0 0; % Tech, Healthcare and Financials
0 1 0 1 0 1 1 1 1 1]; % all the other sectors
LowGE=[0.15;0];
UpGE=[inf;0.1];
pE=setGroups(pE,GE,LowGE,UpGE);

% Transform default constraints
lbE = zeros(pE.NumAssets,1); % Long-only constraint
ubE = [];                    % No explicit weight upper bounds
AeqE = ones(1,pE.NumAssets); % Weights must sum to 1
beqE = 1;


%Transform group constraints
% GroupMatrix * x <= UpperGroup
% -GroupMatrix * x <= -LowerGroup
AE = [GE; -GE];
bE = [UpGE; -LowGE];

% Get rid of unbounded inequality constraints
iiE = isfinite(bE);
AE = AE(iiE,:);
bE = bE(iiE);

% loading CE19 for each sector
MCE19=readmatrix("Dataset.xlsx","Sheet","Sectors","Range","H5:H14");

% Find the portfolio with the minimum CE with the group constraints.
[wgt_minCE19,minCE19] = linprog(MCE19,AE,bE,AeqE,beqE,lbE,ubE);

% Find the portfolio with the maximum CE with the group constraints.
[wgt_maxCE19,fvalE] = linprog(-MCE19,AE,bE,AeqE,beqE,lbE,ubE);
maxCE19 = -fvalE;

% Define a grid of CE such that minCE19 = targetCE19(1) ≤…≤ targetCE19(N) = maxCE19.
N = 20; % Size of grid
targetCE19 = linspace(minCE19,maxCE19,N);
```

```
% set inequality
AinE = MCE19';
binE = maxCE19; % start with the highest CE
pE = setInequality(pE,AinE,binE);
% compute and plot Efficient Frontier
prskE = cell(N,1);
pretE = cell(N,1);
pCE19 = cell(N,1);
for i = 1:N
    pE.bInequality = targetCE19(i);
    pwgtE = estimateFrontier(pE,N);
    [prskE{i},pretE{i}] = estimatePortMoments(pE,pwgtE);
    pCE19{i} = pwgtE'*MCE19;
end

% 3-D plot
scatter3(cell2mat(prskE),cell2mat(pretE),cell2mat(pCE19),'MarkerEdgeColor','k','MarkerFaceColor',[0 .75 .75])
title('Efficient Portfolios')
xlabel('Standard Deviation')
ylabel('Expected Return')
zlabel('Carbon Emission')

% visualizing the tradeoff between a portfolio's Carbon Emission and the traditional mean-variance efficient frontier
nC = 5; % Number of contour plots
minContourE = max(pCE19{1}); % CE values lower than this
                             % return overlapped contours.

% Plot contours
plotContours(pE,minContourE,maxSCE,nC,N);

function [] = plotContours(pE,minCE19,maxCE19,nContour,nPort)

% Set of CE levels for contour plot
contourCE19 = linspace(minCE19,maxCE19,nContour+1);

% Compute and plot efficient frontier for each value in
% contourCE.
figure;
hold on
labels = strings(nContour+1,1);
for i = 1:nContour
    pE.bInequality = contourCE19(i);
    pwgtE = estimateFrontier(pE,nPort);
    [prskE,pretE] = estimatePortMoments(pE,pwgtE);
    plot(prskE,pretE,'LineWidth',2);
    labels(i) = sprintf("%6.2f% CE",contourCE19(i));
end
% Plot the "original" mean-variance frontier, i.e., the
% frontier without CE requierements
pE.AInequality = []; pE.bInequality = [];
pwgtE = estimateFrontier(pE,nPort);
[prskE,pretE] = estimatePortMoments(pE,pwgtE);
% plot figure
plot(prskE,pretE,'LineWidth',2);
labels(i+1) = "No CE restriction";
title('Efficient Frontiers')
xlabel('Standard Deviation of Portfolio Returns')
ylabel('Mean of Portfolio Returns')
legend(labels,'Location','northwest')
grid on
hold off
end
```

```
%% CARBON BETA PORTFOLIO OPTIMIZATION WITH GROUP CONSTRAINTS

% computing the 'No short selling' efficient frontier with equilibrium moments
% setting tracking portfolio (S&P500)
pB=Portfolio('AssetMean',MM,'AssetCovar',MV,'AssetList',lab(2:11));
pB=pB.setDefaultConstraints(10);

% setting gourp constraints
GB=[1 0 1 0 1 0 0 0 0 0; % Tech, Healthcare and Financials
0 1 0 1 0 1 1 1 1 1]; % all the other sectors
LowGB=[0.15;0];
UpGB=[inf;0.1];
pB=setGroups(pB,GB,LowGB,UpGB);

% Transform default constraints
lbB = zeros(pB.NumAssets,1); % Long-only constraint
ubB = [];                    % No explicit weight upper bounds
AeqB= ones(1,pB.NumAssets); % Weights must sum to 1
beqB = 1;

%Transform group constraints
% GroupMatrix * x <= UpperGroup
% -GroupMatrix * x <= -LowerGroup
AB = [GB; -GB];
bB = [UpGB; -LowGB];

% Get rid of unbounded inequality constraints
iiB = isfinite(bB);
AB = AB(iiB,:);
bB = bB(iiB);

% loading CB for each sector
MCB=readmatrix("Dataset.xlsx","Sheet","Sectors","Range","I5:I14");

% Find the portfolio with the minimum CB with the group constraints.
[wgt_minCB,minCB] = linprog(MCB,AB,bB,AeqB,beqB,lbB,ubB);

% Find the portfolio with the maximum CB with the group constraints.
[wgt_maxCB,fvalB] = linprog(-MCB,AB,bB,AeqB,beqB,lbB,ubB);
maxCB = -fvalB;

% Define a grid of CB such that minCB = targetCB(1) ≤…≤ targetCB(N) = maxCB.
N = 20; % Size of grid
targetCB = linspace(minCB,maxCB,N);

% set inequality
AinB = MCB';
binB = maxCB; % start with the highest CB
pB = setInequality(pB,AinB,binB);

% compute and plot Efficient Frontier
prskB = cell(N,1);
pretB = cell(N,1);
pCB = cell(N,1);
for i = 1:N
   pB.bInequality = targetCB(i);
   pwgtB = estimateFrontier(pB,N);
   [prskB{i},pretB{i}] = estimatePortMoments(pB,pwgtB);
   pCB{i} = pwgtB'*MCB;
end

% 3-D plot
scatter3(cell2mat(prskB),cell2mat(pretB),cell2mat(pCB),'MarkerEdgeColor','k','MarkerFaceColor',[0 .75 .75])
```

```
title('Efficient Portfolios')
xlabel('Standard Deviation')
ylabel('Expected Return')
zlabel('Carbon beta')

% visualizing the tradeoff between a portfolio's Carbon beta and the traditional mean-variance efficient frontier
nC = 5; % Number of contour plots
minContourB = max(pCB{1}); % CB values lower than this
                          % return overlapped contours.

% Plot contours
plotContours(pB,minContourB,maxCB,nC,N);

function [] = plotContours(pB,minCB,maxCB,nContour,nPort)

% Set of CB levels for contour plot
contourCB = linspace(minCB,maxCB,nContour+1);

% Compute and plot efficient frontier for each value in
% contourCB.
figure;
hold on
labels = strings(nContour+1,1);
for i = 1:nContour
   pB.bInequality = contourCB(i);
   pwgtB = estimateFrontier(pB,nPort);
   [prskB,pretB] = estimatePortMoments(pB,pwgtB);
   plot(prskB,pretB,'LineWidth',2);
   labels(i) = sprintf("%6.2f% CB",contourCB(i));
end

% Plot the "original" mean-variance frontier, i.e., the
% frontier without CB requierements
pB.AInequality = []; pB.bInequality = [];
pwgtB = estimateFrontier(pB,nPort);
[prskB,pretB] = estimatePortMoments(pB,pwgtB);
% plot figure
plot(prskB,pretB,'LineWidth',2);
labels(i+1) = "No CB restriction";
title('Efficient Frontiers')
xlabel('Standard Deviation of Portfolio Returns')
ylabel('Mean of Portfolio Returns')
legend(labels,'Location','northwest')
grid on
hold off

end

%% CARBON INTENSITY PORTFOLIO OPTIMIZATION WITHOUT GROUP CONSTRAINTS

% computing the 'No short selling' efficient frontier with equilibrium moments
p=Portfolio('AssetMean',MM,'AssetCovar',MV,'AssetList',lab(2:11));
p=p.setDefaultConstraints(10);

% Transform default constraints
lb = zeros(p.NumAssets,1); % Long-only constraint
ub = [];                   % No explicit weight upper bounds
Aeq = ones(1,p.NumAssets); % Weights must sum to 1
beq = 1;

%Transform group constraints
% GroupMatrix * x <= UpperGroup
% -GroupMatrix * x <= -LowerGroup
```

```
A = [];
b = [];

% Get rid of unbounded inequality constraints
ii = isfinite(b);
A = A(ii,:);
b = b(ii);

% loading CI19 for each sector
MCI19=readmatrix("Dataset.xlsx","Sheet","Sectors","Range","F5:F14");

% Find the portfolio with the minimum CI with the group constraints.
[wgt_minCI19,minCI19] = linprog(MCI19,A,b,Aeq,beq,lb,ub);

% Find the portfolio with the maximum CI with the group constraints.
[wgt_maxCI19,fval] = linprog(-MCI19,A,b,Aeq,beq,lb,ub);
maxCI19 = -fval;

% Define a grid of CI such that minCI19 = targetCI19(1) ≤…≤ targetCI19(N) = maxCI19.
N = 20; % Size of grid
targetCI19 = linspace(minCI19,maxCI19,N);

% set inequality
Ain = MCI19';
bin=max(targetCI19);
p = setInequality(p,Ain,bin);

% Compute risks and returns
prsk = cell(N,1);
pret = cell(N,1);
pCI19 = cell(N,1);
for i = 1:N
    p.bInequality = targetCI19(i);
    pwgt = estimateFrontier(p,N);
    [prsk{i},pret{i}] = estimatePortMoments(p,pwgt);
    pCI19{i} = pwgt'*MCI19;
end

% 3-D plot
scatter3(cell2mat(prsk),cell2mat(pret),cell2mat(pCI19),'MarkerEdgeColor','k','MarkerFaceColor',[0 .75 .75])
title('Efficient Portfolios')
xlabel('Standard Deviation')
ylabel('Expected Return')
zlabel('Carbon intensity')


%% CARBON EMISSIONS PORTFOLIO OPTIMIZATION WITHOUT GROUP CONSTRAINTS

% computing the 'No short selling' efficient frontier with equilibrium moments
pE=Portfolio('AssetMean',MM,'AssetCovar',MV,'AssetList',lab(2:11));
pE=pE.setDefaultConstraints(10);

% Transform default constraints
lbE = zeros(pE.NumAssets,1); % Long-only constraint
ubE = [];                    % No explicit weight upper bounds
AeqE = ones(1,pE.NumAssets); % Weights must sum to 1
beqE = 1;

%Transform group constraints
% GroupMatrix * x <= UpperGroup
% -GroupMatrix * x <= -LowerGroup
AE = [];
bE = [];
```

```
% Get rid of unbounded inequality constraints
iiE = isfinite(bE);
AE = AE(iiE,:);
bE = bE(iiE);

% loading CE19 for each sector
MCE19=readmatrix("Dataset.xlsx","Sheet","Sectors","Range","H5:H14");

% Find the portfolio with the minimum CE with the group constraints.
[wgt_minCE19,minCE19] = linprog(MCE19,AE,bE,AeqE,beqE,lbE,ubE);

% Find the portfolio with the maximum CE with the group constraints.
[wgt_maxCE19,fvalE] = linprog(-MCE19,AE,bE,AeqE,beqE,lbE,ubE);
maxCE19 = -fvalE;

% Define a grid of CE such that minCE19 = targetCE19(1) ≤…≤ targetCE19(N) = maxCE19.
N = 20; % Size of grid
targetCE19 = linspace(minCE19,maxCE19,N);

% set inequality
AinE = MCE19';
binE = maxCE19; % start with the highest CE
pE = setInequality(pE,AinE,binE);

% compute and plot Efficient Frontier
prskE = cell(N,1);
pretE = cell(N,1);
pCE19 = cell(N,1);
for i = 1:N
    pE.bInequality = targetCE19(i);
    pwgtE = estimateFrontier(pE,N);
    [prskE{i},pretE{i}] = estimatePortMoments(pE,pwgtE);
    pCE19{i} = pwgtE'*MCE19;
end

% 3-D plot
scatter3(cell2mat(prskE),cell2mat(pretE),cell2mat(pCE19),'MarkerEdgeColor','k','MarkerFaceColor',[0 .75 .75])
title('Efficient Portfolios')
xlabel('Standard Deviation')
ylabel('Expected Return')
zlabel('Carbon Emission')

%% CARBON BETA PORTFOLIO OPTIMIZATION WITHOUT GROUP CONSTRAINTS

% computing the 'No short selling' efficient frontier with equilibrium moments
% setting tracking portfolio (S&P500)
pB=Portfolio('AssetMean',MM,'AssetCovar',MV,'AssetList',lab(2:11));
pB=pB.setDefaultConstraints(10);

% Transform default constraints
lbB = zeros(pB.NumAssets,1); % Long-only constraint
ubB = [];                    % No explicit weight upper bounds
AeqB = ones(1,pB.NumAssets); % Weights must sum to 1
beqB = 1;
%Transform group constraints
% GroupMatrix * x <= UpperGroup
% -GroupMatrix * x <= -LowerGroup
AB = [];
bB = [];

% Get rid of unbounded inequality constraints
iiB = isfinite(bB);
AB = AB(iiB,:);
```

```
bB = bB(iiB);

% loading CE19 for each sector
MCB=readmatrix("Dataset.xlsx","Sheet","Sectors","Range","I5:I14");

% Find the portfolio with the minimum CB with the group constraints.
[wgt_minCB,minCB] = linprog(MCB,AB,bB,AeqB,beqB,lbB,ubB);

% Find the portfolio with the maximum CB with the group constraints.
[wgt_maxCB,fvalB] = linprog(-MCB,AB,bB,AeqB,beqB,lbB,ubB);
maxCB = -fvalB;

% Define a grid of CB such that minCB = targetCB(1) ≤…≤ targetCB(N) = maxCB.
N = 20; % Size of grid
targetCB = linspace(minCB,maxCB,N);

% set inequality
AinB = MCB';
binB = maxCB; % start with the highest CB
pB = setInequality(pB,AinB,binB);

% compute and plot Efficient Frontier
prskB = cell(N,1);
pretB = cell(N,1);
pCB = cell(N,1);
for i = 1:N
   pB.bInequality = targetCB(i);
   pwgtB = estimateFrontier(pB,N);
   [prskB{i},pretB{i}] = estimatePortMoments(pB,pwgtB);
   pCB{i} = pwgtB'*MCB;
end

% 3-D plot
scatter3(cell2mat(prskB),cell2mat(pretB),cell2mat(pCB),'MarkerEdgeColor','k','MarkerFaceColor',[0 .75 .75])
title('Efficient Portfolios')
xlabel('Standard Deviation')
ylabel('Expected Return')
zlabel('Carbon beta')
```

## A.3. TACTICAL CHOICES

The following code is used to estimate the portfolio moments with different approaches. The expected returns are computed with sample estimator and the CAPM, and the covariance matrix with sample estimator and the EWMA. Then, by the implementation of a loop that uses a window for estimation with a size of 600 observations, I calculate the expected returns and the portfolio weights of the Maximum Sharpe portfolio for each of the six strategies. The six strategies are summed up in the Table 10 in Section 3.4.

```
%% TACTICAL CHOICES

% computing inputs of Markowitz using three approaches: sample moments,
% equilibrium moments and EWMA

rI=rD(:,2:11);      % indexes returns
rB=rD(:,1);         % benchmark/market return
[r,c]=size(rI);     % row/column dimensions

% window size for estimation
w=600;
% estimation of expected returns
```

```
% loop for both methods, sample estimator and equilibrium returns
% pre-allocation for expected returns computation
ErSE=zeros(w,c);   % sample estimator
for j=w:r-1    % r-1 as we use data up to r-1 to allocate 1-step-ahead
   % sample estimator
   ErSE(j+1-w,:)=mean(rI(j-w+1:j,:));
end

ErE=zeros(r-w,c); % equilibrium return

  for j=w:r-1    % r-1 as we use data up to r-1 to allocate 1-step-ahead
   % equilibrium return - first estimate coefficients and then compute
   % expected returns and allocate
   Y=rI(j-w+1:j,:);
   X=[ones(w,1) rB(j-w+1:j,:)];
   B=(X'*X)\(X'*Y);
   ErE(j+1-w,:)=B(2,:)*mean(rB(j-w+1:j,:));
  end

% estimation of covariances
% loop for sample estimator
% pre-allocation for covariance computation
% we have now a three-dimension array!
EvSE=zeros(r-w,c,c);     % sample estimator
EvW=zeros(r-w,c,c);     % EWMA
l=0.9;                 % smoothing factor for EWMA
EvW0=cov(rI(1:w,:));    % initialization for EWMA
for j=w:(r-1)   % r-1 as we use data up to r-1 to allocate 1-step-ahead
   % sample estimator
   EvSE(j+1-w,:,:)=cov(rI(j-w+1:j,:));
end
for j=w:(r-1)   % r-1 as we use data up to r-1 to allocate 1-step-ahead
   % EWMA
   EvW(j+1-w,:,:)=l*squeeze(EvW0)+(1-l)*(rI(j,:)'*rI(j,:));
   EvW0=EvW(j+1-w,:,:);
end

%% SI: sample estimators
% loop for the evaluation of realized returns with sample estimators of
% both expected returns and covariance
% pre-allocation for returns
PortRetSI=zeros(r-w,1);
% pre-allocation for weights
PortWSI=zeros(1,r-w,c);

% creating portfolio objects
pSI=Portfolio;
pSI=setGroups(pSI,G,LowG,UpG);
pSI=pSI.setAssetList(lab(2:11));
pSI=pSI.setDefaultConstraints; % no short selling constraint
% define groups
GSI=[1 0 1 0 1 0 0 0 0 0; % Tech, Healthcare and Financials
0 1 0 1 0 1 1 1 1 1]; % all the other sectors
LowGSI=[0.15;0];
UpGSI=[inf;0.1];

% Transform default contraints
lbSI = zeros(pSI.NumAssets,1); % Long-only constraint
ubSI = [];                  % No explicit weight upper bounds
AeqSI = ones(1,pSI.NumAssets); % Weights must sum to 1
beqSI = 1;

%Transform group constraints
```

```
% GroupMatrix * x <= UpperGroup
% -GroupMatrix * x <= -LowerGroup
ASI = [GSI; -GSI];
bSI = [UpGSI; -LowGSI];

% Get rid of unbounded inequality constraints
iiSI = isfinite(bSI);
ASI = ASI(iiSI,:);
bSI = bSI(iiSI);

% Find the portfolio with the minimum CI with the group constraints.
[wgt_minSCI,minSCI] = linprog(MCI19,ASI,bSI,AeqSI,beqSI,lbSI,ubSI);

% Find the portfolio with the maximum CI with the group constraints.
[wgt_maxSCI,fvalSI] = linprog(-MCI19,ASI,bSI,AeqSI,beqSI,lbSI,ubSI);
maxSCI = -fvalSI;

% Define a grid of CI such that minCI19 = targetCI19(1) ≤…≤ targetCI19(N) = maxCI19.
N = 20; % Size of grid
targetSCI = linspace(minSCI,maxSCI,N);
AinSI = MCI19';
binSI = maxSCI; % start with the highest CI
pSI = setInequality(pSI,AinSI,binSI);

% set moments
for j=(w+1):r
pSI=pSI.setAssetMoments(ErS(j-w,:),squeeze(EvS(j-w,:,:)));
end

 % MaxSharpe
for j=(w+1):r
   pwgtMSI=estimateMaxSharpeRatio(pSI);
   PortWSI(1,j-w,:)=pwgtMSI';
   PortRetSI(j-w,1)=rI(j,:)*(pwgtMSI);
end

%% %% EI: loop for the evaluation of realized returns with equilibrium approach and covariance with EWMA
% pre-allocation for returns
PortRetEI=zeros(r-w,1);
% pre-allocation for weights
PortWEI=zeros(1,r-w,c);

% baseline object
pEI=Portfolio;
pEI=pEI.setAssetList(lab(2:11));
pEI=pEI.setDefaultConstraints; % no short selling portfolio

% define groups
GEI=[1 0 1 0 1 0 0 0 0 0; % Tech, Healthcare and Financials
0 1 0 1 0 1 1 1 1 1]; % all the other sectors
LowGEI=[0.15;0];
UpGEI=[inf;0.1];
% set group constraints
pEI=setGroups(pEI,GEI,LowGI,UpGEI);


% Transform default contraints
lbEI = zeros(pEI.NumAssets,1); % Long-only constraint
ubEI = [];                     % No explicit weight upper bounds
AeqEI = ones(1,pEI.NumAssets); % Weights must sum to 1
beqEI = 1;
%Transform group constraints
% GroupMatrix * x <= UpperGroup
```

```
% -GroupMatrix * x <= -LowerGroup
AEI = [GEI; -GEI];
bEI = [UpGEI; -LowGI];


% Get rid of unbounded inequality constraints
iiEI = isfinite(bEI);
AEI = AEI(iiEI,:);
bEI = bEI(iiEI);

% Find the portfolio with the minimum CI with the group constraints.
[wgt_minCEI,minCEI] = linprog(MCI19,AEI,bEI,AeqEI,beqEI,lbEI,ubEI);

% Find the portfolio with the maximum CI with the group constraints.
[wgt_maxCEI,fvalEI] = linprog(-MCI19,AEI,bEI,AeqEI,beqEI,lbEI,ubEI);
maxCEI = -fvalEI;

% Define a grid of CI such that minCI19 = targetCI19(1) ≤…≤ targetCI19(N) = maxCI19.
N = 20; % Size of grid
targetECI = linspace(minCEI,maxCEI,N);
AinEI = MCI19';
binEI = maxCEI; % start with the highest CI
pEI = setInequality(pEI,AinEI,binEI);

for j=(w+1):r
  % set moments
  pEI=pEI.setAssetMoments(ErE(j-w,:),squeeze(EvW(j-w,:,:)));
end

for j=(w+1):r
  % MaxSharpe
  testW=pEI.estimateFrontierLimits('Max');
  testRet=ErE(j-w,:)*testW;
  if testRet>0
    pwgtMSEI= estimateMaxSharpeRatio(pEI);
    PortWEI(1,j-w,:)=pwgtMSEI';
    PortRetEI(j-w,1)=rI(j,:)*(pwgtMSEI);
  else
    PortWEI(1,j-w,:)=pwgtMSEI';
    PortRetEI(j-w,1)=rI(j,:)*(pwgtMSEI);
  end
end

%% SE: sample estimators
% loop for the evaluation of realized returns with sample estimators of
% both expected returns and covariance
% pre-allocation for returns
PortRetSE=zeros(r-w,1);
% pre-allocation for weights
PortWSE=zeros(1,r-w,c);
% creating portfolio objects
pSE=Portfolio;
pSE=pSE.setAssetList(lab(2:11));
pSE=pSE.setDefaultConstraints; % no short selling constraint
% define groups
GSE=[1 0 1 0 1 0 0 0 0 0; % Tech, Healthcare and Financials
0 1 0 1 0 1 1 1 1 1]; % all the other sectors
LowGSE=[0.15;0];
UpGSE=[inf;0.1];
% set group constraints
pSE=setGroups(pSE,GSE,LowGSE,UpGSE);

% Transform default contraints
```

```
lbSE = zeros(pSE.NumAssets,1); % Long-only constraint
ubSE = [];                 % No explicit weight upper bounds
AeqSE = ones(1,pSE.NumAssets); % Weights must sum to 1
beqSE = 1;

%Transform group constraints
% GroupMatrix * x <= UpperGroup
% -GroupMatrix * x <= -LowerGroup
ASE = [GSE; -GSE];
bSE = [UpGSE; -LowGSE];

% Get rid of unbounded inequality constraints
iiSE = isfinite(bSE);
ASE = ASE(iiSE,:);
bSE = bSE(iiSE);

% loading CE19 for each sector
MCE19=readmatrix("Dataset.xlsx","Sheet","Sectors","Range","H5:H14");

% Find the portfolio with the minimum CE with the group constraints.
[wgt_minSCE,minSCE] = linprog(MCE19,ASE,bSE,AeqSE,beqSE,lbSE,ubSE);

% Find the portfolio with the maximum CE with the group constraints.
[wgt_maxSCE,fval] = linprog(-MCE19,ASE,bSE,AeqSE,beqSE,lbSE,ubSE);
maxSCE = -fval;

% Define a grid of CE such that minCE19 = targetCE19(1) ≤…≤ targetCE19(N) = maxCE19.
N = 20; % Size of grid
targetSCE = linspace(minSCE,maxSCE,N);
AinSE = MCE19';
binSE = maxSCE; % start with the highest CE
pSE = setInequality(pSE,AinSE,binSE);

% set moments
for j=(w+1):r
   pSE=pSE.setAssetMoments(ErSE(j-w,:),squeeze(EvSE(j-w,:,:)));
end

% MaxSharpe
for j=(w+1):r
   pwgtMSE=estimateMaxSharpeRatio(pSE);
   PortWSE(1,j-w,:)=pwgtMSE';
   PortRetSE(j-w,1)=rI(j,:)*(pwgtMSE);
end

%% %% EE: loop for the evaluation of realized returns with equilibrium approach and covariance with EWMA
% pre-allocation for returns
PortRetEE=zeros(r-w,1);
% pre-allocation for weights
PortWEE=zeros(1,r-w,c);

% baseline object
pEE=Portfolio;
pEE=pEE.setAssetList(lab(2:11));
pEE=pEE.setDefaultConstraints; % no short selling portfolio

% define groups
GEE=[1 0 1 0 1 0 0 0 0 0; % Tech, Healthcare and Financials
0 1 0 1 0 1 1 1 1 1]; % all the other sectors
LowGEE=[0.15;0];
UpGEE=[inf;0.1];
% set group constraints
pEE=setGroups(pEE,GEE,LowGEE,UpGEE);
```

```
% Transform default contraints
lbEE = zeros(pEE.NumAssets,1); % Long-only constraint
ubEE = [];                     % No explicit weight upper bounds
AeqEE = ones(1,pEE.NumAssets); % Weights must sum to 1
beqEE = 1;

%Transform group constraints
% GroupMatrix * x <= UpperGroup
% -GroupMatrix * x <= -LowerGroup
AEE = [GEE; -GEE];
bEE = [UpGEE; -LowGEE];

% Get rid of unbounded inequality constraints
iiEE = isfinite(bEE);
AEE = AEE(iiEE,:);
bEE = bEE(iiEE);

% Find the portfolio with the minimum CE with the group constraints.
[wgt_minCEE,minCEE] = linprog(MCE19,AEE,bEE,AeqEE,beqEE,lbEE,ubEE);
% Find the portfolio with the maximum CE with the group constraints.
[wgt_maxCEE,fval] = linprog(-MCE19,AEE,bEE,AeqEE,beqEE,lbEE,ubEE);
maxCEE = -fval;

% Define a grid of CE such that minCE19 = targetCE19(1) ≤…≤ targetCE19(N) =
maxCE19.
N = 20; % Size of grid
targetCEE = linspace(minCEE,maxCEE,N);
AinEE = MCE19';
binBEE = maxCEE; % start with the highest CE
pEE = setInequality(pEE,AinEE,binBEE);

% set moments
for j=(w+1):r
  pEE=pEE.setAssetMoments(ErE(j-w,:),squeeze(EvW(j-w,:,:)));
end
% MaxSharpe
for j=(w+1):r
  testW=pEE.estimateFrontierLimits('Max');
  testRet=ErE(j-w,:)*testW;
  if testRet>0
    pwgtMSEE= estimateMaxSharpeRatio(pEE);
    PortWEE(1,j-w,:)=pwgtMSEE';
    PortRetEE(j-w,1)=rI(j,:)*(pwgtMSEE);
  else
    PortWEE(1,j-w,:)=pwgtMSEE';
    PortRetEE(j-w,1)=rI(j,:)*(pwgtMSEE);
  end
end
%% SB: sample estimators
% loop for the evaluation of realized returns with sample estimators of
% both expected returns and covariance
% pre-allocation for returns
PortRetSB=zeros(r-w,1);
% pre-allocation for weights
PortWSB=zeros(1,r-w,c);

% creating portfolio objects
pSB=Portfolio;
pSB=pSB.setAssetList(lab(2:11));
pSB=pSB.setDefaultConstraints; % no short selling constraint
% define groups
GSB=[1 0 1 0 1 0 0 0 0 0; % Tech, Healthcare and Financials
0 1 0 1 0 1 1 1 1 1]; % all the other sectors
```

```
LowGSB=[0.15;0];
UpGSB=[inf;0.1];
% set group constraints
pSB=setGroups(pSB,GSB,LowGSB,UpGSB);

% Transform default contraints
lbSB = zeros(pSB.NumAssets,1); % Long-only constraint
ubSB = [];                 % No explicit weight upper bounds
AeqSB = ones(1,pSB.NumAssets); % Weights must sum to 1
beqSB = 1;

%Transform group constraints
% GroupMatrix * x <= UpperGroup
% -GroupMatrix * x <= -LowerGroup
ASB = [GSB; -GSB];
bSB = [UpGSB; -LowGSB];

% Get rid of unbounded inequality constraints
iiSB = isfinite(bSB);
ASB = ASB(iiSB,:);
bSB = bSB(iiSB);

% Find the portfolio with the minimum CB with the group constraints.
[wgt_minCSB,minSCB] = linprog(MCB,ASB,bSB,AeqSB,beqSB,lbSB,ubSB);
% Find the portfolio with the maximum CB with the group constraints.
[wgt_maxCSB,fvalSB] = linprog(-MCB,ASB,bSB,AeqSB,beqSB,lbSB,ubSB);
maxSCB = -fvalSB;

% Define a grid of CB such that minCB = targetCB(1) ≤…≤ targetCB(N) = maxCB.
N = 20; % Size of grid
targetSCB = linspace(minSCB,maxSCB,N);
AinSB = MCB';
binSB = maxSCB; % start with the highest CB
pSB = setInequality(pSB,AinSB,binSB);

% set moments
for j=(w+1):r
   pSB=pSB.setAssetMoments(ErS(j-w,:),squeeze(EvS(j-w,:,:)));
end
% MaxSharpe
for j=(w+1):r
   pSB=pSB.setAssetMoments(ErS(j-w,:),squeeze(EvS(j-w,:,:)));
   pwgtMSB=estimateMaxSharpeRatio(pSB);
   PortWSB(1,j-w,:)=pwgtMSB';
   PortRetSB(j-w,1)=rI(j,:)*(pwgtMSB);
end
%% %% EB: loop for the evaluation of realized returns with equilibrium approach and covariance with EWMA
% pre-allocation for returns
PortRetEB=zeros(r-w,1);
% pre-allocation for weights
PortWEB=zeros(1,r-w,c);
% baseline object
pEB=Portfolio;
pEB=pEB.setAssetList(lab(2:11));
pEB=pEB.setDefaultConstraints; % no short selling portfolio

% define groups
GEB=[1 0 1 0 1 0 0 0 0 0; % Tech, Healthcare and Financials
0 1 0 1 0 1 1 1 1 1]; % all the other sectors
LowGEB=[0.15;0];
UpGEB=[inf;0.1];
% set group constraints
pEB=setGroups(pEB,GEB,LowGEB,UpGEB);
```

```
% Transform default contraints
lbEB = zeros(pEB.NumAssets,1); % Long-only constraint
ubEB = [];                     % No explicit weight upper bounds
AeqEB = ones(1,pEB.NumAssets); % Weights must sum to 1
beqEB = 1;

%Transform group constraints
% GroupMatrix * x <= UpperGroup
% -GroupMatrix * x <= -LowerGroup
AEB = [GEB; -GEB];
bEB = [UpGEB; -LowGEB];

% Get rid of unbounded inequality constraints
iiEB = isfinite(bEB);
AEB = AEB(iiEB,:);
bEB = bEB(iiEB);

% Find the portfolio with the minimum CB with the group constraints.
[wgt_minECB,minECB] = linprog(MCB,AEB,bEB,AeqEB,beqEB,lbEB,ubEB);
% Find the portfolio with the maximum CB with the group constraints.
[wgt_maxECB,fvalEB] = linprog(-MCB,AEB,bEB,AeqEB,beqEB,lbEB,ubEB);
maxECB = -fvalEB;

% Define a grid of CB such that minCB = targetCB(1) ≤…≤ targetCB(N) = maxCB.
N = 20; % Size of grid
targetECB = linspace(minECB,maxECB,N);
AinEB = MCB';
binEB = maxECB; % start with the highest CB
pEB = setInequality(pEB,AinEB,binEB);

% set moments
for j=(w+1):r
   pEB=pEB.setAssetMoments(ErE(j-w,:),squeeze(EvW(j-w,:,:)));
end

% MaxSharpe
for j=(w+1):r
   testW=pEB.estimateFrontierLimits('Max');
   testRet=ErE(j-w,:)*testW;
   if testRet>0
      pwgtMSEB= estimateMaxSharpeRatio(pEB);
      PortWEB(1,j-w,:)=pwgtMSEB';
      PortRetEB(j-w,1)=rI(j,:)*(pwgtMSEB);
   else
      PortWEB(1,j-w,:)=pwgtMSEB';
      PortRetEB(j-w,1)=rI(j,:)*(pwgtMSEB);
   end
end
```

### A.4. TRACK RECORDS OF ALLOCATION STRATEGIES

In the last part of the code, I compute the evolution of portfolio composition of the six strategies, their cumulated returns, and some performance indicators. The evolutions of portfolio compositions are plotted to facilitate the comparison between the strategies, the same is done with the cumulated returns of the strategies, of the benchmark and of the equally weighted portfolio. Moreover, after having calculated the performance indicators, I create a composite index and construct a table that ranks the strategies according to their composite index.

```
%% TIME EVOLUTION OF THE PORTFOLIO COMPOSITION
% area plots of MS computed with sample moments

subplot(3,1,1)
area(dD(w+1:r),squeeze(PortWSI(1,:,:)),'Facecolor','flat')
datetick('x','mmm-yy')
xlim([dD(w+1) dD(r)])
xlabel('Time')
ylim([0 1])
ylabel('Weight')
title('SI - MS')
hold on
subplot(3,1,2)
area(dD(w+1:r),squeeze(PortWSE(1,:,:)),'FaceColor','flat')
datetick('x','mmm-yy')
xlim([dD(w+1) dD(r)])
xlabel('Time')
ylim([0 1])
ylabel('Weight')
title('SE - MS')
hold on
subplot(3,1,3)
area(dD(w+1:r),squeeze(PortWSB(1,:,:)),'FaceColor','flat')
datetick('x','mmm-yy')
xlim([dD(w+1) dD(r)])
xlabel('Time')
ylabel('Weight')
title('SB - MS')
legend(lab(2:11))

% area plots of MS computed with equilibrium returns and EWMA covariances
subplot(3,1,1)
area(dD(w+1:r),squeeze(PortWEI(1,:,:)),'Facecolor','flat')
datetick('x','mmm-yy')
xlim([dD(w+1) dD(r)])
xlabel('Time')
ylim([0 1])
ylabel('Weight')
title('EI - MS')
hold on
subplot(3,1,2)
area(dD(w+1:r),squeeze(PortWEE(1,:,:)),'FaceColor','flat')
datetick('x','mmm-yy')
xlim([dD(w+1) dD(r)])
xlabel('Time')
ylim([0 1])
ylabel('Weight')
title('EE - MS')
hold on
subplot(3,1,3)
area(dD(w+1:r),squeeze(PortWEB(1,:,:)),'FaceColor','flat')
datetick('x','mmm-yy')
xlim([dD(w+1) dD(r)])
xlabel('Time')
ylabel('Weight')
title('EB - MS')
legend(lab(2:11))
```

```
%% CUMULATED RETURNS

w=600;   % window used for estimation
% pre-allocation for returns of the equally weighted portfolio (EW)
PortRetEW=zeros(r-w,1);
% return computation of the EW portfolio
for j=(w+1):r
    PortRetEW(j-w,1)=sum(rI(j,:))/c;
end
% cumulated returns of the equally weighted portfolio
CRrEW=cumprod(PortRetEW/100+1)-1;
% cumulated returns of the benchmarket
CRrB=cumprod(rB(w+1:end,1)/100+1)-1;
% cumulated returns of the indexes
CRrI=cumprod(rI(w+1:end,:)/100+1)-1;
% cumulated returns of the strategies
CRSIMS=cumprod(PortRetSI(:,1)/100+1)-1;
CRSEMS=cumprod(PortRetSE(:,1)/100+1)-1;
CRSBMS=cumprod(PortRetSB(:,1)/100+1)-1;
CREIMS=cumprod(PortRetEI(:,1)/100+1)-1;
CREEMS=cumprod(PortRetEE(:,1)/100+1)-1;
CREBMS=cumprod(PortRetEB(:,1)/100+1)-1;

% plots
% benchmark
figure
plot(dD(w+3:end),CRrB)
xlabel('Year')
ylabel('Cumulated return')
hold on
plot(dD(w+3:end),CRrEW) % EW portfolio
xlabel('Year')
ylabel('Cumulated return')
legend('Benchmark','EW portfolio')

% MS of the 3 strategies with sample moments
figure
plot(dD(w+3:end),[CRSIMS CRSEMS CRSBMS])
hold on
plot(dD(w+3:end),CRrB,'LineWidth',2)
hold on
plot(dD(w+3:end),CRrEW,'LineWidth',2)
ylabel('Cumulated return')
xlabel('Year')
legend({'MS S-CI','MS S-CE','MS S-CB','Benchmark','EW portfolio'})
title('MS-sample')

% MS of the 3 strategies with equilibrium return and EWMA covariance
figure
plot(dD(w+3:end),CREIMS)
hold on
plot(dD(w+3:end),CREEMS,'LineStyle','--')
hold on
plot(dD(w+3:end),CREBMS)
hold on
plot(dD(w+3:end),CRrB,'LineWidth',2)
hold on
plot(dD(w+3:end),CRrEW,'LineWidth',2)
ylabel('Cumulated return')
xlabel('Year')
legend({'MS E-CI','MS E-CE','MS E-CB','Benchmark','EW portfolio'})
title('MS-eq return EWMA cov')
```

```
%% PERFORMANCE MEASURES

% returns of the portfolios
allret=[PortRetSI(:,1) PortRetSE(:,1) PortRetSB(:,1)...
     PortRetEI(:,1) PortRetEE(:,1) PortRetEB(:,1)];

% Sharpe ratio
pm1=mean(allret)'./sqrt(var(allret))';

% Sortino ratio
s2=zeros(size(allret,2),1);
for j=1:size(allret,2)

% compute semi-standard deviation
   s2(j)=sqrt(var(allret(allret(:,j)<0,j)));
end
pm2=mean(allret)'./ s2;

% Treynor ration
s3=zeros(size(allret,2),1);
for j=1:size(allret,2)

% compute beta on the Benchmark
%     Y=XB B=Y'/X'=X\Y
%     B=(X'*X)\(X'*Y); B=(Y'*X)/(X'*X);
% beta=cov(BNCH,Y)/var(BNCH)
   s3(j)= (((rB(w+1:end,1)-mean(rB(w+1:end,1)))')*(allret(:,j)-mean(allret(:,j))))...
      /((rB(w+1:end,1)-mean(rB(w+1:end,1)))'*(rB(w+1:end,1)-mean(rB(w+1:end,1))));
end
pm3=mean(allret)'./ s3;

% Value-at-Risk
alpha=0.05;
s4=quantile(allret,alpha);
pm4=mean(allret)'./ abs(s4)';

% Expected Shortfall
alpha=0.05;
s5=zeros(size(allret,2),1);
for j=1:size(allret,2)
   % compute conditional mean
   s5(j)=mean(allret(allret(:,j)<quantile(allret(:,j),alpha),j));
end
pm5=mean(allret)'./ abs(s5);

% DrawDown sequence
DD=zeros(size(allret,1),size(allret,2));
for i=1:size(allret,2)
   DD(1,i)=min(allret(1,i)/100,0);
   for j=2:size(allret,1)
      DD(j,i)=min(0,(1+DD(j-1,i))*(1+allret(j,i)/100)-1);
   end
end
s6=max(abs(DD))'*100;

% Calmar ratio
pm6=mean(allret)'./s6;

% Sterling ratio
k=5;
s7=zeros(size(allret,2),1);
for j=1:size(allret,2)
   % average of the largest DD
```

```
    [sDDj,~]=sort(abs(DD(:,j)),'descend');
    s7(j)=mean(sDDj(1:k))*100;
end
pm7=mean(allret)'./s7;

% summarizing results
allPM=[pm1 pm2 pm3 pm4 pm5 pm6 pm7 pm8];
Tab1=table({'SI-MS';'SE-MS';'SB-MS';'EI-MS';'EE-MS';...
    'EB-MS'},allPM(:,1),allPM(:,2),allPM(:,3),allPM(:,4),allPM(:,5),allPM(:,6),allPM(:,7),...
    'VariableNames',{'Strategy' 'Sh' 'So' 'Tr' 'VaR' 'ES' 'Cal' 'Ste'});

% ranking performance measures
[~,pm1r]=sort(pm1,'descend');
pm1r(pm1r)=1:length(pm1r);
[~,pm2r]=sort(pm2,'descend');
pm2r(pm2r)=1:length(pm1r);
[~,pm3r]=sort(pm3,'descend');
pm3r(pm3r)=1:length(pm1r);
[~,pm4r]=sort(pm4,'descend');
pm4r(pm4r)=1:length(pm1r);
[~,pm5r]=sort(pm5,'descend');
pm5r(pm5r)=1:length(pm1r);
[~,pm6r]=sort(pm6,'descend');
pm6r(pm6r)=1:length(pm1r);
[~,pm7r]=sort(pm7,'descend');
pm7r(pm7r)=1:length(pm1r);
allPMr=[pm1r pm2r pm3r pm4r pm5r pm6r pm7r];

% computing a composite index
CIpm=allPMr*ones(size(allPMr,2),1);


% summary table
Tab2=table({'SI-MS';'SE-MS';'SB-MS';'EI-MS';'EE-MS';...
    'EB-MS'},CIpm, allPMr(:,1),allPMr(:,2),allPMr(:,3),allPMr(:,4),allPMr(:,5),...
    allPMr(:,6),allPMr(:,7),...
    'VariableNames',{'Strategy' 'CI' 'Sh' 'So' 'Tr' 'VaR' 'ES' 'Cal' 'Ste'});

% Comparison relatively to the benchmark

allret1=[rB(w+1:end,1) PortRetSI(:,1) PortRetSE(:,1) PortRetSB(:,1)...
      PortRetEI(:,1) PortRetEE(:,1) PortRetEB(:,1)];

% compute tracking errors
allTE=allret1(:,2:end)-(allret1(:,1)*ones(1,size(allret1,2)-1));
TE=mean(allTE);
TEV=var(allTE);
SemiTEV=zeros(size(allTE,2),1);
for j=1:size(allTE,2)

% compute semi-standard deviation
    SemiTEV(j)=sqrt(var(allTE(allTE(:,j)<0,j)));
end
IR=TE'./TEV';

Tab3=table({'SI-MS';'SE-MS';'SB-MS';'EI-MS';'EE-MS';...
    'EB-MS'},TE',TEV',SemiTEV,IR,...
    'VariableNames',{'Strategy' 'TE' 'TEV' 'SemiTEV' 'IR'});

% creating transpose arrays of TE and TEV
transTE=TE';
transTEV=TEV';
```

```
% ranking tracking errors measures
[~,TEr]=sort(transTE,'descend');
TEr(TEr)=1:length(TEr);
[~,TEVr]=sort(transTEV,'ascend');
TEVr(TEVr)=1:length(TEVr);
[~,SemiTEVr]=sort(SemiTEV,'descend');
SemiTEVr(SemiTEVr)=1:length(SemiTEVr);
[~,IRr]=sort(IR,'descend');
IRr(IRr)=1:length(IRr);
allTEr=[TEr TEVr SemiTEVr IRr];

% computing a composite index
CIte=allTEr*ones(size(allTEr,2),1);

% summary table
Tab4=table({'SI-MS';'SE-MS';'SB-MS';'EI-MS';'EE-MS';...
   'EB-MS'},CIte, allTEr(:,1),allTEr(:,2),allTEr(:,3),allTEr(:,4),...
   'VariableNames',{'Strategy' 'CI' 'TE' 'TEV' 'SemiTEV' 'IR'});
```

# REFERENCES

Andersson, M., Bolton, P., and Samama, F. 2016. *Hedging Climate Risk*. Financial Analysts Journal, 72(3), pp. 13-32.

Ardia, D., Bluteau, K., Boudt, K., and Inghelbrecht, K. 2021. *Climate change concerns and the performance of green versus brown stocks*. National Bank of Belgium, Working paper No. 395.

Arrow, K. J. 1965. *The Theory of Risk Aversion. Aspects of the Theory of Risk Bearing*. Helsinki, Yrjo Jahnssonin Saatio.

Bansal, Ravi, Marcelo Ochoa, and Dana Kiku. 2016. *Climate change and growth risks*. Working paper.

Batten, Sandra, Rhiannon Sowerbutts, and Misa Tanaka. 2016. *Let's Talk about the Weather: The Impact of Climate Change on Central Banks*. Bank of England Staff Working Paper no. 603.

Batten, Sandra. 2018. *Climate Change and the Macro-Economy: A Critical Review.* Bank of England Working Paper no. 706.

Bernstein, Asaf, Matthew T. Gustafson, and Ryan Lewis. 2019. *Disaster on the Horizon: The Price Effect of Sea Level Rise.* Journal of Financial Economics 134 (2): 253–72.

Bolton, P. and Kacperczyk, M. 2020. *Do investors care about carbon risk?* NBER Working paper No. 26968. Forthcoming in Journal of Financial Economics.

Bolton, P., Despres, M., Pereira Da Silva, L. A., Samama, F., and Svartzman, R. 2020. *The Green Swan: Central banking and financial stability in the age of climate change.* Bank for International Settlements Working Paper.

Bolton, P., Kacperczyk, M., and Samama, F. 2021. *Net-zero carbon portfolio alignment*. Working Paper.

Bunten, Devin, and Matthew E. Kahn. 2014. *The Impact of Emerging Climate Risks on Urban Real Estate Price Dynamics.* National Bureau of Economic Research Working Paper Series No. 20018: 1–21.

Carhart, M.M. 1997. *On Persistence in Mutual Fund Performance*. Journal of Finance, 52(1), pp. 57-82.

Dal Bianco, C. *Track records of allocation strategies*. Computational Finance notes, Academic year 2021-22. University of Padua – Department of Economics and Management

Engle, Robert, F., Giglio, S., Kelly, B.T., Heebum Lee, and Stroebel, J. 2019. *Hedging climate change news*. Forthcoming Review of Financial Studies.

Epstude, Kai, and Neal J. Roese. 2008. *The Functional Theory of Counterfactual Thinking.* Personality and Social Psychology Review 12 (2): 168–92. https://doi.org/10.1177/1088868308316091.

Fama, E.F., and French, K.R. 1992. *The Cross-Section of Expected Stock Returns*. Journal of Finance, 47(2), pp. 427-465.

Fama, E.F., and French, K.R. 1993. *Common Risk Factors in the Returns on Stocks and Bonds*. Journal of Financial Economics, 33(1), pp. 3-56.

Fankhauser, Samuel, and Richard Tol. 2005. *On Climate Change and Economic Growth.* Resource and Energy Economics 27 (1): 1–17.

Finansinspektionen. 2016. *Climate Change and Financial Stability.* Finansinspektionen.

Fink, L. 2019. Annual letter to CEOs.

Görgen, M., Jacob, A., Nerlinger, M., Riordan, R., Rohleder, M., and Wilkens, M. 2019. *Carbon Risk*. SSRN, https://www.ssrn.com/abstract=2930897.

Gueant, Olivier and Peladan, Jean-Guillaume and Robert-Dautun, Alain and Tankov, Peter. 2021. *Environmental transition alignment and portfolio performance*. SSRN: https://ssrn.com/abstract=3876731 or http://dx.doi.org/10.2139/ssrn.3876731

Hong, Harrison, Frank Weikai Li, and Jiangmin Xu. 2019. *Climate risks and market efficiency*. Journal of Econometrics 208, 265-281.

https://www.borsaitaliana.it/notizie/sotto-la-lente/sp500.htm

https://www.investopedia.com/terms/s/sp500.asp

IAIS (International Association of Insurance Supervisors). 2018. *Issues Paper on Climate Change Risks to the Insurance Sector.* International Association of Insurance Supervisors c/o Bank for International Settlements.

Ilhan, Emirhan, Sautner, Z., and Vilkov, G. 2019. *Carbon tail risk*. Working paper.

IPCC. 2018. *"Summary for Policymakers. In: Global Warming of 1.5°C. An IPCC Special Report on the Impacts of Global Warming of 1.5°C above Pre-Industrial Levels and Related Global Greenhouse Gas Emission Pathways, in the Context of Strengthening the Global Response To the Threat of Climate Change."* Geneva, Switzerland: IPCC.

Jondeau, E., Mojon, B., and Pereira da Silva, L. A. 2021. *Building Benchmarks Portfolios with Decreasing Carbon Footprints*. Working Paper

Krueger, Philipp, Zacharias Sautner, and Starks, T. 2019. *The importance of climate risks for institutional investors.* Forthcoming Review of Financial Studies.

Le Guenedal, Théo and Roncalli, Thierry. 2022. *Portfolio Construction with Climate Risk Measures.* SSRN: https://ssrn.com/abstract=3999971

Markowitz, H. 1952. *Portfolio Selection.* The Journal of Finance, 7(1), 77-91.

NGFS. 2019a. *NGFS First Comprehensive Report. A Call for Action - Climate Change as a Source of Financial Risk.* April 2019.

NGFS. 2019b. *Macroeconomic and Financial Stability: Implications of Climate Change.* NGFS Technical Supplement to the First Comprehensive Report." July 2019.

OECD. 202. *Financial Markets and Climate Transition: Opportunities, Challenges and Policy Implications.* OECD Paris, https://www.oecd.org/finance/Financial-Markets-and-Climate-Transition-Opportunities-challenges-and-policy-implications.htm

Pastor, L., R. F. Stambaugh, and L. A. Taylor. 2020. *Sustainable investing in equilibrium.* Forthcoming Journal of Financial Economics.

Ray, Deepak K., Paul C. West, Michael Clark, James S. Gerber, Alexander V. Prishchepov, and Snigdhansu Chatterjee. 2019. *Climate Change Has Likely Already Affected Global Food Production.* PLoS ONE 14 (5).

Roncalli, T., and Sekine, T. 2020. Measuring and Managing Carbon Risk in Investment Portfolios. SSRN: https://ssrn.com/abstract=3681266

Roncalli, T., Le Guenedal, T., Lepetit, F., Roncalli, T., and Sekine, T. (2020), Measuring and Managing Carbon Risk in Investment Portfolios, SSRN, 3681266.

Sharpe, W.F. 1964. *Capital Asset Prices: A Theory of Market Equilibrium under Con- ditions of Risk*. Journal of Finance, 19(3), pp. 425-442.

Taleb, N. 2010. *Fat Tails, Asymmetric Knowledge, and Decision Making: Essay in Honor of Benoit Mandelbrot's 80th Birthday*. Willmott, 2010.
Taleb, Nassim N. 2007. *The Black Swan*. New York: Penguin Random House.

UNFCCC. 2015. *Paris Agreement.* United Nations.

Weitzman, Martin L. 2009. *On Modeling and Interpreting the Economics of Catastrophic Climate Change.* Review of Economics and Statistics 91 (1): 1–19.

Weitzman, Martin L. 2011. *Fat-Tailed Uncertainty in the Economics of Catastrophic Climate Change.* Review of Environmental Economics and Policy 5 (2): 275–92.