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BEHAVIOR OF BUCKET BRIGADE IN AN ORDER-PICKING SYSTEM UNDER THE EFFECT OF FATIGUE

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To all the fragile people in the world, because we are all human beings

To all the people that made me smile at least once in my lifetime

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Riassunto esteso

Introduzione

Sempre più spesso in questi ultimi dieci anni scienziati e ricercatori hanno iniziato a considerare nei loro lavori la componente umana (*human factors*). Ciò ha permesso loro di avere risultati più vicini alla realtà aziendale e, di conseguenza, di predire con maggiore precisione il comportamento di tutti i sistemi industriali dove l'essere umano è il "motore". Questi risultati più accurati permettono di prendere più agevolmente e con maggiore velocità le decisioni aziendali e rendono il sistema più affidabile, veloce e preciso.

E' da qui che viene l'idea di considerare l'effetto della fatica degli operatori in un sistema di *picking* in un magazzino. In particolare, sarà studiato il tema del *bucket brigade* che verrà applicato ad un sistema di *order-picking*. Un *bucket brigade* è "un nuovo modo di coordinare i lavoratori che stanno progressivamente assemblando (prelevando) un prodotto da una linea (da uno scaffale), nel quale gli operatori sono in minoranza rispetto alle stazioni (alle postazioni di *picking*) (Bartholdi and Hackman, 2017).

Il lavoro inizia con una spiegazione sui concetti di base dei magazzini e sull'*order-picking* (capitolo 1); i successivi due capitoli sono dedicati a un riassunto dei principali articoli riguardanti il *bucket brigade*, prima sulle linee di assemblaggio (capitolo 2) e poi sull'*order-picking* (capitolo 3). Il successivo capitolo (capitolo 4) parla di una nuova funzione da me elaborata, che modellizza il rallentamento degli operatori durante un turno di lavoro di otto ore a causa della fatica muscolare. Nell'ultimo capitolo (capitolo 5), infine, vengono considerate quattro differenti tipologie di *bucket brigade*, facendo variare la velocità massima degli operatori e la velocità con cui questi si stancano a seconda del lavoro che devono svolgere. Tutti i risultati numerici, che sono stati ottenuti con simulazioni su MATLAB, sono presentati con grafici che spiegano il comportamento del sistema studiato. Per garantire la correttezza dei risultati, allo studio numerico è stato affiancato il calcolo analitico, eseguito con carta e penna.

Con questo lavoro si dimostrerà che il *bucket brigade* applicato all'*order-picking* funziona bene, anche considerando l'effetto che la fatica fisica ha sugli operatori. Lo scopo dell'elaborato è quello di studiare il comportamento di tutti i possibili tipi di *bucket brigade* che rientrano nelle ipotesi sopra citate e decidere quale di questi è il più performante. Inoltre, si studierà come l'effetto della fatica faccia rallentare il sistema e i risultati così ottenuti verranno confrontati con quelli ottenuti da Bartholdi and Eisenstein (1996a, 1996b), che non considerano l'affaticamento. Alla fine del lavoro, oltre a presentare (sia in maniera estesa che schematica) i risultati ottenuti, sono fornite anche delle istruzioni che il manager deve seguire per innalzare la performance di qualsiasi tipo di *bucket brigade*.

Capitolo 1 - Scienza del magazzino

Lo scopo del primo capitolo è quello di dare al lettore una conoscenza di base sui magazzini. Dopo avere presentato brevemente i differenti tipi di scorta e i costi del magazzino, l'attenzione passa sul flusso di materiale nel magazzino. Questo viene diviso in varie parti (ricevere, mettere via, stoccare, fare *picking*, impacchettare e spedire). La sezione che verrà maggiormente approfondita sarà quella sull'*order-picking*, perché è l'attività più critica nei magazzini. Alla fine del capitolo, sono spiegate alcune idee di base per fare *picking* con maggiore efficacia prima in un magazzino *low-volume*, poi in un magazzino *high-volume*.

Capitolo 2 - I *bucket brigade*

In questo capitolo il tema dei *bucket brigade* sulle linee di assemblaggio viene approfondito tramite la spiegazione dei più importanti articoli scritti dal 1996, partendo da quello di Bartholdi e Eisenstein, i primi a studiare questo sistema. Tramite questo articolo, viene spiegata la matematica di un sistema *bucket brigade*. Queste regole saranno valide sia per le applicazioni sulle linee di assemblaggio sia per quelle sull'*order-picking*. Il capitolo prosegue dando una rassegna di tutti i più importanti articoli riguardanti il *bucket brigade* dal 1996 ad oggi. In questi *papers* è spiegato come reagisce il sistema se sono modificate alcune ipotesi di base o se sono aggiunte ulteriori ipotesi.

Capitolo 3 - I *bucket brigade* in un sistema di *order picking*

In questo terzo capitolo è illustrato il tema del *bucket brigade* in un sistema di *order-picking*. La spiegazione è presentata seguendo come linea guida ciò che

Bartholdi e Eisenstein hanno scritto nel loro articolo *Bucket brigades: a self-balancing order-picking system for a warehouse* (1996b), dove hanno analizzato il fenomeno dell'*order-picking* nei magazzini di una catena di negozi. Dopo aver spiegato quali sono la scaffalatura e il sistema ottimale per lavorare con questa tipologia di magazzini, i due scienziati danno alcuni consigli per rendere più performanti i *bucket brigade* in un sistema di *order picking*. Successivamente, vengono mostrati i risultati ottenuti da Bartholdi e Eisenstein (1996b) nel loro articolo, partendo dall'ipotesi di lavoro esponenzialmente distribuito. Alla fine del capitolo, sono spiegati i vantaggi dell'utilizzo del *bucket brigade* in un sistema di *order-picking*.

Capitolo 4 - Componente umana nell'*order picking*: modelli di fatica ed ergonomia

Lo scopo di questo capitolo è quello di dare al lettore una conoscenza di base sulla fatica e su come la fatica abbia a che fare con i sistemi di *order picking*, in particolare con i sistemi *bucket brigade* nell'*order picking*. Il concetto di fatica è strettamente legato a quello di ergonomia, che può essere usata per migliorare l'efficienza di un sistema, riducendo la fatica. Alla fine del capitolo, è descritto un modello matematico che descrive come il livello di fatica cresca nel tempo in un sistema di *order-picking*.

Capitolo 5 - *Bucket brigade* e fatica

Lo scopo di questo capitolo è di connettere i precedenti articoli sul *bucket brigade* (capitoli 2 e 3) con i modelli di fatica (capitolo 4). E' presentato, prima di tutto, un nuovo modello di mia ideazione che descrive il rallentamento della velocità di *picking* nel tempo. Usando questo modello, è possibile descrivere matematicamente la dinamica di un *bucket brigade* di due operatori in un sistema di *order picking*, dove i lavoratori rallentano durante un turno di lavoro di otto ore. Utilizzando la matematica è stato possibile scrivere alcuni programmi in MATLAB: grazie a questi è stato possibile simulare il comportamento di vari *bucket brigade* in un turno lavorativo di otto ore. Nelle simulazioni vengono considerate diverse combinazioni di velocità massime degli operatori e di velocità con cui questi si stancano a seconda del lavoro che devono svolgere.

Risultati e possibili sviluppi

Il lavoro conferma che un *bucket brigade* composto da due operatori in un sistema di *picking* è efficace, anche considerando la componente umana. Sia analiticamente sia numericamente è stato dimostrato che gli effetti che si hanno considerando la fatica muscolare degli operatori sono molteplici. I più importanti sono una riduzione del *throughput* nelle otto ore del turno lavorativo (gli operatori rallentano) e uno spostamento della posizione di *hand-off* lungo la linea durante il turno (cambia il rapporto di velocità tra gli operatori). Inoltre, vengono forniti importanti consigli che un *manager* deve mettere in atto per migliorare la *performance* del *bucket brigade*.

Nel particolare, ho ideato una nuova funzione che modellizza l'affaticamento degli operatori durante le otto ore del turno di lavoro. Una volta fatto ciò, sono state eseguite diverse simulazioni con il software MATLAB. I risultati numerici sono stati confrontati con quelli teorici ottenuti con carta e penna. I risultati, infine, sono stati riassunti in una unica tabella (tabella 5.5). Tutti i risultati ottenuti rappresentano con ottima approssimazione ciò cha accade nella realtà, poiché si è partiti da ipotesi più precise, considerando, ad esempio, l' affaticamento.

Il lavoro svolto è il primo a collegare (analiticamente e numericamente) il *bucket brigade* con la componente umana. Per questa ragione, l'argomento risulta molto ampio e difficile da comprendere, così che non è stato possibile approfondire alcune parti del lavoro. La vastità del tema lascia aperti alcuni quesiti.

- Come si comporta il sistema se gli operatori sono più di due?
- Come cambiano i risultati se si considera tutto il magazzino e non solo un corridoio?
- Cosa succede se non si considera solo l'effetto della fatica, ma si considera anche quello dell'apprendimento?
- La funzione utilizzata per descrivere il rallentamento degli operatori è esponenziale. Alcuni autori, però, suggeriscono altri tipi di funzioni. Qual è la funzione che dà risultati più vicini alla realtà?
- Una delle ipotesi di lavoro è quella di considerare il lavoro ugualmente distribuito lungo le postazioni di *picking* e gli ordini tutti uguali. Cosa succede se il lavoro viene considerato non equamente distribuito e gli ordini tutti diversi?

- Alcuni importanti risultati sono stati ottenuti solamente in maniera numerica grazie a simulazioni con MATLAB. E’ possibile dimostrare matematicamente tutto ciò che è stato trovato?

Contents

More recently in these last ten years, many scientists and researchers are starting to consider human factors in their works and in their papers. This allows them to have results closer to reality and therefore to predict better the behavior of all the kinds of systems where the “engine” is the human being. Having more precise results facilitates the managerial work and makes the system more reliable, fast and flawless.

From here comes the idea of taking into account the effect of fatigue in order-picking system and, in particular, in order-picking bucked brigade systems. A bucket brigade is “a way of coordinating workers who are progressively assembling (picking) product along a flow line (aisle) in which there are fewer workers than stations” (Bartholdi and Hackman, 2017).

The work starts with an explanation of the basic principles of warehouses and order-picking (chapter 1) and a summary of the most important papers about bucket brigade both on assembly lines (chapter 2) and on order-picking (chapter 3). Then, the following chapter (chapter 4) talks about a new function that takes into account the slowdown of the pickers because of muscular fatigue. In the last chapter (chapter 5), four different kinds of bucket brigades are considered, each changing the maximum speed of the pickers and their level of effort. All the numerical results we obtained in the MATLAB simulations are presented with plots that clarify the behavior of the system. To have more guaranteed results, everything we obtained with the simulation is confirmed by analytical calculations.

With this work, we will prove that an order-picking bucket brigade system can perform well, even if the effect of fatigue is considered. Our aim is to compare the different kinds of bucket brigades we have studied and then to decide which one performs better. Moreover, we will show how this effect will change the results that the other researchers found without taking into account the effect of fatigue. At the end of the work, not only the results are presented, but also some advice that a manager can use to improve the performance of the system are given.

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Introduction

According to Bartholdi and Hackman (2017), "*order-picking* is the most labor-intensive activity in warehouses". It includes 55% of the whole warehouse operating costs. This cost can be divided further: traveling takes the 55% of the time (and so of the cost), searching takes the 15%, extracting the 10% and paperwork and other activities the 20%. Because of its paramount importance, order-picking is always an important topic to work on. Working and understanding the phenomena which regulates the behavior of the order-picking system can allow companies to fulfill costumers' orders faster, with better quality and, as a result, to gain more money, increasing the level of service offered to the customer.

An innovative way to deal with order-picking in high-volume distribution warehouses of a chain retailer[1] is *bucket brigade*. This assembly system was first invented by Bartholdi and Eisenstein in their paper "A production line that balances itself" (1996a) and generalized for order-picking in their paper "Bucket brigades: a self-balancing order-picking system for a warehouse" (Bartholdi and Eisenstein, 1996b). In the latter they analyzed the phenomenon of order-picking in chain retailers. In the two papers mentioned, they explain the basic principles of bucket brigade and how it works. Bucket brigade is a "new style of order-picking in which the work is reallocated by the independent movements of the workers. If the bucket brigade is configured properly, the order-pickers will balance the work amongst themselves and so eliminate bottlenecks. Moreover, this happens spontaneously, without intention or awareness of the workers. This means that the order-picking can be more effective than if planned by a careful engineer or manager" (Bartholdi and Hackman, 2017).

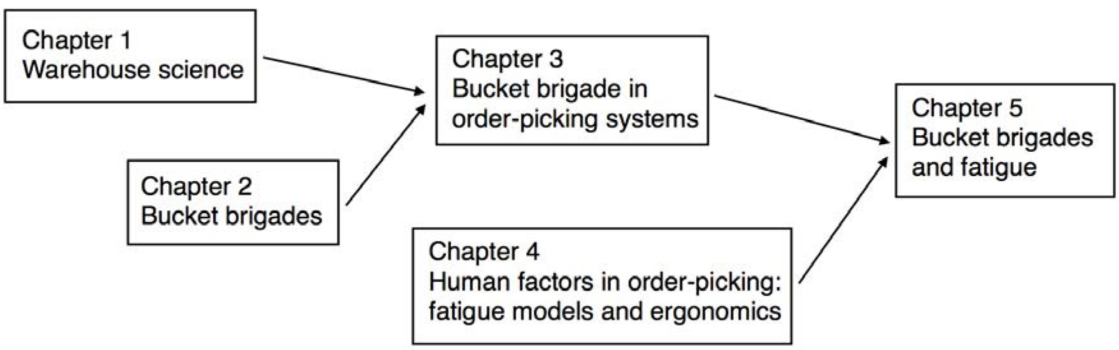
A lot of other papers about bucket-brigade have been written in the following years, both about assembly lines and order-picking systems. The dynamics of two and three operators in bucket brigade production lines has been deepened by Bartholdi, Bunimovic and Eisenstein in their paper "Dynamics of two- and three-worker bucket brigade production lines" (1999). Armbruster and Gel (2002) studied the behavior of a two workers bucket brigade, where one worker has a constant speed over the whole production line and the other is slower over the first portion and faster over the second portion of line. Bartholdi, Eisenstein and Foley (2001) studied the behavior of a bucket brigade system when the work is stochastic, which is in presence of variability in the work content. Bartholdi, Eisenstein and Lim (2003, 2009) proved that, under certain conditions, a bucket brigade systems can be chaotic, even if the starting data are deterministic.

[1] In a chain retailer a typical order consists in a lot of skus, but a small number each.

All of these papers consider the bucket brigade system to be a “perfect machine”, in which *fatigue* is not taken into consideration. Therefore, all the results are not exactly in accordance with what happens in a real warehouse, where the pickers get tired along the work shift. These last ten years have been used to obtain more precise results. In fact, researchers now consider the *human factors* in their paper, obtaining results that are closer to reality. According to Grosse, Glock and Neumann (2016), in fact, “human factors can have a great impact on the performance of the overall system” and, because of this, it is of paramount importance to consider them. From here comes the idea to include human factors in the mathematics of order-picking bucket brigade systems.

Then, the aim of this work is to find new mathematical and analytical formulae to model the behavior of a bucket brigade order-picking system. These formulae take into account the effect of muscular fatigue on the pickers. In particular, this work deals with all the different possible cases that a manager could face, considering the slow down of the pickers (thanks to a new formula) and considering different levels of work effort. All of these cases are also studied numerically with MATLAB. We used this software to confirm the correctness of the results and to find some other results that are impossible to find analytically. At the end of the work, all the different possible cases are compared to find which kind of bucket brigade performs best. Moreover, also some important strategies that the manager should use to improve the performance of the system are given.

Down here, a schematic diagram is used to describe the structure of the work and how the chapters are linked between each other.



Chapter 1

Warehouse science

The aim of this first chapter is to give the reader the basic knowledge on warehouses. After we have given a quick overview of the different kinds of stocks and of the costs in a warehouse, we will bring our attention on the flow of items in a warehouse, dividing the flow in different sections (receive, put-away, storage, pick, pack and ship). The section that we will deepen more is the one about order-picking, because it is the most labor-intensive activity in warehouses. At the end of the chapter, we will give some basic ideas to deal with low-volume and high-volume distribution warehouses.

1.1 Classification of stocks

The first question we have to answer is "why do we need a warehouse?". A warehouse requires labor, capital and information systems, all of which are expensive. Can we avoid this expense? The answer is no, because we need a warehouse to match supply with customer demand. The main problem is that demand can change quickly, but supply takes longer to change. This is one of the most challenging problems to solve in every factory and, to solve it, we can use warehouses, that allow us to respond quickly when demand changes.

From here on, in particular in this paragraph, we will explain deeply which kinds of stock exist and which are their aim; then, in the next paragraph, we will deepen the theme of warehouse costs.

To classify the different kinds of stock, we will divide them in six categories. For each category we will describe the reasons why to use that kind of stock, how the stock works and, at the end, which is the aim of the stock that we are taking into account. This list of stocks will give the reader a quick overview on the different kinds of stocks that it is possible to find. The six categories are presented deeply in the text De Toni, Panizzolo and Villa (2013) and in the notes from the lectures of the course "Organizzazione della Produzione e dei Sistemi Logistici" held by Panizzolo R. (2017).

The six stock categories are:

- Cycle stock: they are used when orders are bigger than costumer's demand. Using them, companies can take advantage of economies of scale. Some reasons why cycle stock exists are discounts when a lot of material is purchased, setups and fixed shipping costs. The aim of this stock is to use economic order quantity as much as possible.

- Decoupling stock: they are used when it is necessary to decouple different part of the production chain. Some examples are different speeds of consecutive machines, different criteria of order aggregation, bottlenecks. The aim of this stock is to maximize the efficiency of the productive factors.

- Transit or pipeline stock: they are linked with material handling and transportation between different areas of the production chain or between suppliers and customers. This kind of stock is used when distribution and supply times are long. The goal of this stock is to guarantee a high service level.

- Safety stock: they are used to avoid delays caused by uncertainty of demand and supplies; for example, if a customer orders more than it is expected, if there are delays in the production of if a line gets stuck. The aim of this stock is to protect from this uncertainty.

- Seasonal or anticipation stock: they are used for products which have a seasonal demand. In general, this kind of stock is used when a factory cannot produce enough during the season when there is a surplus of demand. In these cases, it is better to produce smoothly all year long, keeping stock when the demand is low and using the stock when the demand is higher. The goal of this stock is to balance capacity and load.

- Speculation stock: they are linked to expectations of rising costs of supply materials (for example gold, grain, …). Generally, the daily price of these supply materials is decided by a particular entity (for example the price of gold is decided in London). The aim of this stock is to minimize purchase costs.

Now that it is clear what are the advantages of a warehouse and why it is necessary to have it, we will discuss about its cost, which are of different nature.

1.2 Costs to maintain a warehouse

A good overview of the costs to maintain a warehouse is given in the book “Gestione della produzione” (De Toni, Panizzolo and Villa, 2013) and in the notes from the lectures of the course “Organizzazione della Produzione e dei Sistemi Logistici” (Panizzolo, 2017) and in the notes from the lectures of the course “Logistica Industiale” (Battini, 2018).

The costs of a warehouse are:

- Cost of issuing of the order: it is the cost that has to be payed when an order has been done. It can be divided in two main parts: cost to order raw materials from a supplier (10-80 €), composed by administrative costs + shipping cost and cost to order material needed for production, composed by preparation cost + setup cost (10-1000 € due to the stop of the machine, long times).

- Cost of maintenance: it is the cost that comes from the material which is in the warehouse; this cost is directly proportional to the size of the warehouse. It can be divided in cost of fixed assets in the warehouse (the fixed assets could be to invest in something more profitable; it is usually the 5% of the warehouse value), maintenance cost, insurance and tax costs, obsolescence or senescence, raw material depreciation cost. The cost of maintenance can be calculated as the 15-30% of the warehouse value (the most used value today is 20%); for example, if inside a warehouse there are materials which are worth 1000000 €, the cost of maintenance can be valued around 200000 €.

- Cost of stock-out: it is linked to the money that is not gained when a customer order a product missing in the warehouse. The cost is generated by the inability to supply the customer. Furthermore, the name of the brand is going to be damaged. The stock-out cost is very difficult to calculate.

The warehouse is necessary, but it costs a lot and, like all the costs, we have to reduce them. One of the main ideas of lean production is to reduce as much as possible the size of the warehouse to lower the costs. The aim of every company should be reducing the size of the warehouse as much as possible, but having a warehouse for all the reasons explained above.

1.3 Warehouse flow

1.3.1 Generalities

In general, warehouse reorganizes and repackages products. In a warehouse, the product typically arrives packaged and leaves the warehouse packaged, but in a smaller scale: a warehouse disassembles products together in smaller quantities, as it is shown in figure 1.1.

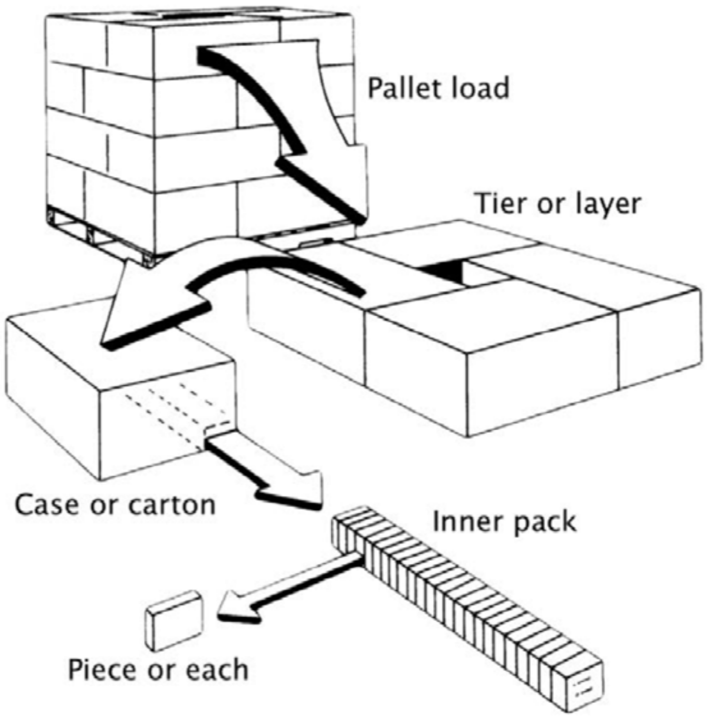


Fig 1.1 - Disassembly of the products in a warehouse. The products usually arrive in a warehouse together in a lot. Inside the warehouse they are separated in smaller lots, sometimes eaches, ready to be shipped. (from Bartholdi III J.J. and Hackman S., 2017)

For example, if a pallet is shipped in a warehouse, it will be divided in the warehouse and shipped out as eaches. The reason why the products arrive in a warehouse in lot is that it is faster and simpler to handle lots than eaches. A golden rule, suggested by Bartholdi III and Hackman (2017) is: “The smaller the handling unit, the greater the handling cost”. In fact, it is simpler to ship or handle products packed together than eaches. For example, if 1000 products has to be handled, it is faster and simpler (it is cheaper as well) to handle them in 100 lots of 10 products and it is even better to handle them in 20 lots of 50 products. Therefore, when it is possible, it is better to handle as many products together, because it costs less.

More or less in every warehouse, there is a common flow of materials: warehouses receive bulk shipments and stage them for quick retrieval; then, in response to a customer's order, products are picked automatically or by an operator and they are shipped to the customer as soon as possible. The flow of material in a warehouse can be summarised in two parts: inbound processes and outbound processes. In inbound processes the two main activities are receiving and put-away. In outbound processes the main activities are order-picking and checking, packaging, shipping. Between inbound and outbound processes there is storage, where products are stocked. This material flow is summarized in figure 1.2.

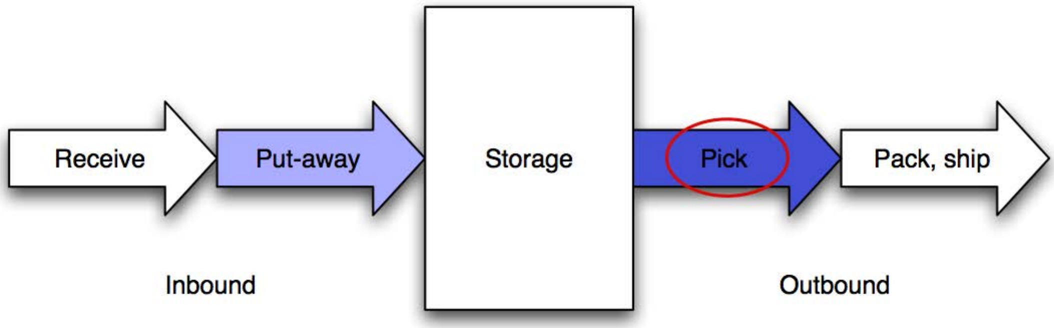


Fig 1.2 - Warehouse flow. After the materials are received and put away in the storage, they are picked and then packed and shipped depending on the order of the customer. Receive and put-away are inbound processes, while pick and pack, ship are outbound processes. In this work we will deepen in particular the picking part, circled in red. (from Bartholdi III J.J. and Hackman S., 2017)

A product must flow continuously along the process as fast as possible and without interruptions, because each time a product is put down, it means it has to be picked up again later: double-handling is a loss of time, energy and money. Transportation is one of the waste of lean production as and therefore it has to be eliminated. The effect of double-handling is wider if we think that we have to handle thousand of *skus*[2] per hour; that is another reason why it is better to handle materials in lots and not in eaches. In conclusion, it is possible to say that when it is possible to avoid double handling, it is better to do it to save money and therefore to gain more.

[2] A *stock keeping unit*, or *SKU*, is the smallest physical unit of a product that is tracked by an organization. For example, this might be a box of 100 Gem Clip brand paper clips. In this case the final customer will use a still smaller unit (individual paper clips), but the supply chain never handles the product at that tiny scale.

From here on, we will explain widely the four part of the flow we mentioned before. After that, we will focus more on picking, which is the most labor-intensive activity in most warehouses.

1.3.2 Receiving

Material is received after an order has been done. Receiving begins with a list, which shows the schedule of arrivals; this list lets the warehouse to know exactly when the trucks are arriving and in which order. Trucks usually arrive within 30-60 minutes time windows. As soon as a product arrives, it is registered in the database, it is checked and it is stocked. Products are usually shipped in pallets: it means they are held together on a platform 800x1200 (European pallet), 1016x1219 (American pallet) or 1165x1165 (Australian pallet); the main advantage to use pallets is that loads and unloads of trucks are faster. Along the flow these pallets will be disassembled is smaller groups of products.

The cost of receiving is around the 10% of the whole cost.

1.3.3 Put away

Put away is a very important issue in warehouses. Before doing it, it is very important to decide the location to stock pallets. The place where they are stocked determines how quickly products can be reached and the later cost of products handling. The location of products is essential to write the picking list, which shows the order-pickers or the machines where retrieving the product when a customer asks for it. As soon as a product is put away, it has to be registered on a software, which creates the picking list.

Put away typically accounts the 15% of the warehouse costs, but this cost can be reduced if the locations to stock pallets are chosen well.

1.3.4 Order-picking

Order-picking is the most labor-intensive activity in warehouses. It also determines the service seen by the customers. It must be flawless and fast. It can be done by a person or by a machine.

Once a customer orders some products, it is checked if these products are available in the warehouse; if they are, the order can be accepted. As soon as the order is accepted, a large software called *warehouse management system* (WMS) creates a picking list to guide order-pickers. The software produces all the shipping documentation and the shipping schedule and coordinates all the different activities in the warehouse.

Order-picking includes 55% of the whole warehouse operating costs. Therefore, as we said before, it is the most labor-intensive activity in the warehouse. This cost can be divided further: traveling takes the 55% of the time (and so of the cost), searching takes the 15%, extracting the 10% and paperwork and other activities the 20%. The division of the cost is shown in chart 1.1.

Chart 1.1 - Division of costs in order-picking. The costs of order-picking includes 55% of the whole warehouse operating costs and can be divided in traveling (55%), searching (15%), extracting (10%) and paperwork and other activities (20%). (from Bartholdi III J.J. and Hackman S., 2017)

Activity	% Order-picking time
Traveling	55%
Searching	15%
Extracting	10%
Paperwork and other activities	20%

The object that catches the eye immediately is the cost of traveling, which is the major cost. This means that, to reduce dramatically the cost of a warehouse, the first thing to do is to reduce the traveling cost, because it is the biggest one. To reduce the traveling cost it is important to optimize the layout of the warehouse, to reduce travel (problem of pick-path optimization) and then to have an efficient picking list. Is it also important to notice that picking is not an action which adds value to the product, because it is an action that is not requested by the customer. In other words, the customer is not willing to pay for transportation and picking. In lean production philosophy, all the actions that are not adding value to the product can be considered waste (*muda*) and they have to be reduced to a minimum or eliminated.

The part of the flow which concerns picking starts when a customer places an order: his order can be seen as a shopping list. The warehouse management system (WMS) collects all the orders and checks if the material is available in the warehouse. If it is, the WMS creates the picking list, taking into account the layout of the warehouse and the present operations. With this list, pickers know the number of products they have to pick, where to go and in which order pick products. Putting different orders together, it is possible to make the order-pickers concentrate themselves only on one area of the warehouse, so that they can reduce travel and be faster. The picking list is usually a piece of paper, but it can be also written on labels, communicated by lights, RF or vocal transmission.

The most labor-intensive type of picking is picking of less-than-cartoon quantities (*broken-case* or *split-case*), that means to pick products which are not held together by a cartoon or a box. This kind of picking is more difficult than *cartoon-picking* (picking full cartoons), because it requires handling of small units such as pieces or eaches. Broken-case picking cannot be automatized, because every each has a different shape and volume; on the contrary, cartoon-picking can, because of the uniformity in shape and dimension of cartons, which are almost always rectangular and equal. Collecting products in a carton can also be useful because cartons protect products from damages. At the end, also pallet can be moved; if they go directly from receiving to shipping, the operation is called crossdock. All the possible kind of picking are shown in figure 1.3.

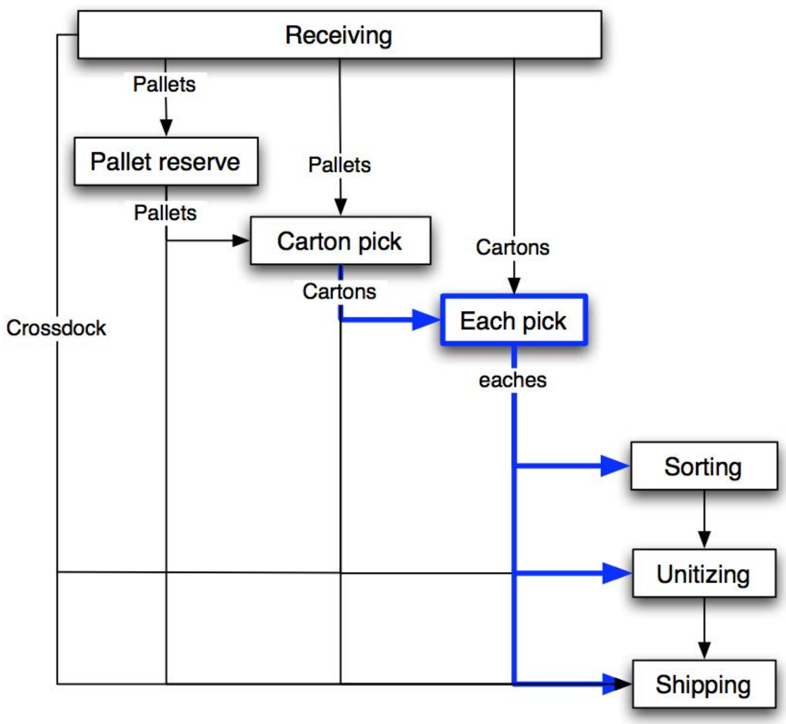


Fig 1.3 - Different kinds of picking. It is possible to pick pallets, cartoons or eaches, depending on what arrives from the receiving department. If eaches are picked, they should be put together in pallets to be shipped easily. If a pallet goes directly from receiving to shipping, the operation is called crossdock. (from Bartholdi III J.J. and Hackman S., 2017)

In conclusion, there are different kind of level of picking, depending on the dimension of the picking unit.

It can be useful to define some parameters which help to understand how is the picking in a warehouse. The first one is *sku density*, which counts the number of skus available per unit of area on the pick-face, which is the 2-dimensional surface from which skus are extracted. The second one is *pick density*, which is correlated with sku density. Pick density is the number of picks achieved per unit of area on the pick face. In general, if a warehouse has a high sku density it means it has a high pick density as well and therefore it means the travels are shorter. Then a good strategy to save money could be to have a high sku density and a high pick density. Another important and more useful parameter is *picks per unit of distance along the aisle traveled by an order-picker*. If it is high, it means the order does not require much travel per pick and it means picking is cheap, because we are paying only for retrieval and not for travel. If it is low, it means the order-picker has to travel for a long distance to reach all the products he needs, therefore the cost of picking is higher. Pick density is usually high for big orders and low for small orders. A huge advantage in picking can be obtained increasing pick density; but pick density depends on the order of the customer, so we can't raise it as we like. A good strategy, then, could be to ensure high sku density: as it is written before, if sku density raises, pick density will raise as well. There are a few ways to do it, but the most common is to store the most popular skus together, so that they can be reached at the same time and with shorter traveling distance; moreover, order-pickers can fulfill the customer's order faster because of the short travel. A second way to increase the pick density is to *batch orders*. It means to assign more than one order at the same time to an order-picker, so that he can retrieve many orders in one trip. Doing this, pick density is increased, but it creates some problems: more organization is needed and pickers must bring with them a container for each order. This slows down the process and gives the pickers more possibilities to make mistakes; furthermore, more space is required. A good trade-off could be batch single-line orders only: this means batch order only if they are in the same aisle.

The most difficult challenge is to accept medium-size orders: more than two picking lines are taken into account, but picking lines are too few to amortize the cost of walking. In their book Bartholdi III and Hackman (2017) give some general rules to decide how to fulfill orders:

- It is better to batch orders when the costs of work to separate the orders plus the cost of additional space are less than the extra walking incurred if orders are not batched.
- It is almost always better to batch single-line orders, because no sortation is required.
- It is never better to batch large order, because the pick density it is high.

- Decisions have to be taken time after time with medium size orders.

Another problem of order-picking is that the products which are picked must be replenished. Operators who are dedicated to the replenishment of the shelves (*restockers*) usually take lots and divide them in skus. Because of this, the number of restockers must be lower than the number of order pickers: the general rule is to have one restocker to every five pickers. The cost of replenishment is generally higher than the cost of picking, because restockers retrieve lots from bulk storage and they split them to obtain skus ready for picking.

The last important decision to take is how many pickers have to be dedicated to an order. There are three possibilities that can be chosen:

- One operator per order.
- Many operators per order, operators pick one at a time.
- Many operators per order, operators pick together.

The key factor to decide which kind of strategy is better to use is *flow time*. The question that has to be answered is: “How can we reduce at a minimum the flow time?”. The answer will give us the right way to proceed. Reducing flow time means that orders flow quickly and that the request of the customer can be fulfilled as fast as possible: this means that the level of service is also as high as possible. A strategy to shorten flow time is to create a *fast-pick area*, which is a “warehouse within the warehouse”: the most popular skus are stocked together in this area. This means that for most orders, traveling distances are reduced to a minimum, therefore the time to fulfill an order is shorter and the flow is fast. The disadvantage of this area is that it needs replenishment from bulk storage.

1.3.5 Checking and packing

In general, after all the products of an order have been picked, every order has to be checked to control if it is complete and accurate. Order accuracy is one of the most useful indicators to measure the level of service given to a customer. Inaccurate orders lead to problems: the customer can be annoyed and he could send the products back, generating a return, which is very expensive to handle (up to 10 times the cost of normal shipping). Because of this, it is very important to be sure that every order is perfect. If it is possible, then, it is always better to pack all the parts of an order together. The customer often requires it, because he can shorten the time of shipping, unloading and handling.

1.3.6 Shipping

When the products are ready and packed together, they can be shipped. In general, shipping works with larger units than picking, because all the items are consolidated in few containers (cases, pallets). Depending on the type of pallet and on the type of truck, a different number of pallet can be shipped. As soon as the truck leaves the factory, the departure is registered and the customer is warned about the departure.

1.4 Order-picking

1.4.1 Phases and categories of order-picking

As it has been told in the previous chapter, order-picking is the most labor-intensive activity in warehouses. It takes around the 55% of the whole warehouse operating costs and it also determines the level of service seen downstream by the customer; for all this reasons it must be flawless and fast.

According to Bartholdi and Hackman (2017), the action of picking can be divided in three phases:

- Travel to the storage location: the operator, thanks to the picking list, has to reach the right storage location; this is the most expensive action in terms of time and money and the activity is non-value-adding, so that it can be considered a waste.

- Local search: once the operator has reached the right location, he has to find the exact each or product. The smaller is the product, the more difficult is the operation, because it takes more time and requires more accuracy. That is the part of picking in which is simpler to make mistakes, so the operator has to pay a lot of attention not to pick the wrong sku. This activity is non-value-adding as well.

- Reach, grab and put: it is when the operator takes and put in the container the products requested by the customer. It is the only part of picking which is value-adding. These actions can be automatized to speed up the process.

Order-picking can be divided in two main categories:

- In *low-volume distribution* customer orders are small, urgent and different from each other. Therefore it is normal that an order-picker has to travel long distances to fulfill an order; there are a lot of choices about traveling routes.

The aim is to find the fastest way (the shortest way) to visit and pick all the requested items. The name of this problem is “problem of pick-path optimization”.

- In *high-volume distribution* customer orders are typically large and of similar products. Each order-picker makes a lot of picks in a short distance and it is typical that order-pickers follow a common path, such as along an aisle of flow rack. The challenge here is to find a way to balance the flow, keep it smooth and eliminate bottlenecks. A solution of this problem is *bucket brigade*, which is the main theme of this work and it will be deepened widely in the next chapters.

1.4.2 Low-volume distribution

As it has been told before, in high-volume distribution order-pickers have to travel long distances and becomes crucial the *problem of pick-path optimization* (traveling Salesman Problem - TSP), which is to find the shortest travel (shortest time) to pick all the items to fulfill an order. This has to be done, because travel time is a waste: it does not add value to the product.

The TPS problem is difficult to solve, because:

- For the general problem there is not a general solution yet.
- Even if the problem is small, the time to solve it could be very long.
- The optimal solution can be very hard to find.

Then, the difficulty to find the solution is due to the layout of the warehouse: it is pretty easy to find quickly a solution when the travel is constrained by aisles, but it is not when the warehouse has a more complicated layout. In general, WMSs do not support pick-path optimization, because they do not have the information of distances between locations where material is stocked. Then, even if a WMS supports pick-path optimization, it can tell the picker only in which order he has to visit locations, but not the path he has to follow; the operator has to decide which path is the fastest one and he can make the wrong choice, because he does not have a global vision of the warehouse, but only a limited one. The most developed WMS can tell the pickers which is the right path to follow, but they are very expensive and they require a lot of time to calculate the optimal solution. Because of the difficulty to find the optimal solution, it is preferred to use heuristic methods to find a good solution. With this kind of methods the solution is not the best one, but the time to find it is shorter. The idea of this kind of methods it to find a good global path, which visits all the locations of a warehouse and then shorten it depending on the order: if the global path is efficient, even the sub-path has to be efficient as well. For a simple layout, the global path could be the serpentine along the aisle, as shown in figure 1.4; every time the customer makes an order, it has to be evaluated if and how to change the general path.

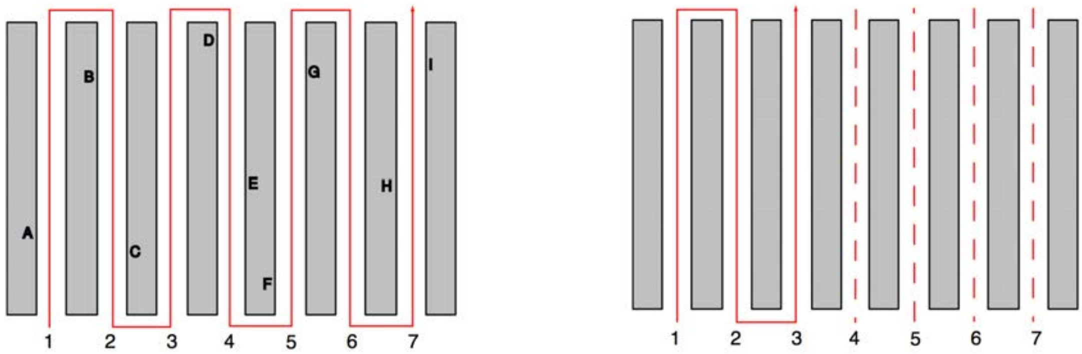


Fig 1.4 - Pick path optimization. The picture shows a heuristic method used to find a good solution. The basic idea under this heuristic method is to find a good global path, which visits all the locations of a warehouse and then shorten it depending on the order. For a layout of parallel aisles a good global path could be a serpentine. Every time the customer makes an order, some parts of the path can be cut. (from Bartholdi III J.J. and Hackman S., 2017)

One of the most used heuristic methods to optimize pick-path is due to Ratliff and Rosenthal (see Ratliff and Rosenthal, 1983). The algorithm generates near-optimal pick paths, with the constraints that the aisles cannot be revisited and that the aisles cannot be visited out of their natural sequence. Because of these restrictions the solution could be slightly longer than the optimal one. With this constraints, when a picker picks an item he has two choices: return back from the same way or continue his travel along the aisle. The WMS has the task of giving the operator the right instruction: the algorithm tells the picker what he has to do in every situation.

How much is optimization worth? If an order is composed by 1-3 items optimization is useless, because it is very simple to find the best traveling path. If an order is composed by a lot of items, it optimization does not worth as well, because order-pickers have to visit nearly every location of the warehouse. The only case in which pick-path optimization worths a lot it the case in which in a warehouse there are many slow-moving items and customer orders are medium-sized.

1.4.3 High-volume distribution

A well known *lean* principle is to reduce inventories as much as possible, in order to save money and underline and discover problems and inefficiencies of the system. Therefore, this need of a small inventory has led to more frequent shipments of smaller quantities. In high-volume distribution only a little travel is required, because pick density is very high. Because of this, the most non-value-adding activity is not anymore traveling as in low-volume distribution, but it is work due to local search. A common way to reduce the time due to common search is *pick-to-light*, which is a system mostly used America: a computer switches on a light to indicate to the order-picker where the right product is. The challenge of high-volume order-picking is to get workers to where they are most needed, so that everyone remains busy every time. To reach this aim, a lot of organization is needed. The best strategy to succeed is to use a self-organizing system that balances itself, without the need of a centralized authority, who coordinates the system. This self-organizing system is called *bucket brigade* and it will be the main topic of this thesis. According to Bartholdi and Hackman (2017), bucket brigades “can function as a self-organizing system that spontaneously achieves its own optimum configuration without conscious intention of the workers, without guidance from management, without any model of work content, indeed without any data at all. The system in effect acts as its own computer”.

Chapter 2

Bucket brigades

In this chapter, the theme of bucket brigade in assembly lines will be deepened through the explanation of the most important papers written since 1996, when Bartholdi and Eisenstein wrote the first paper. We will start from the basic rules and mathematics of bucket brigade: this rules will be valid for both bucket brigade in assembly lines and in order-picking. We will continue the chapter explaining what happens when some of the basic hypothesis are modified or some more hypothesis are added.

2.1 How does a bucket brigade work?

The first paper about bucket brigade is “A production line that balances itself”, written by Bartholdi and Eisenstein (1996a). The paper explains the basic principles of bucket brigade and how it works; it is mainly focused on bucket brigade in assembly lines.

According to Bartholdi and Eisenstein (1996a), traditional assembly lines are inflexible, because each worker has his workspace and he cannot move from it. In general, the number of stations is equal to the number of workers. There are two ways to change production rate: change number of the shift (only coarse adjustments) or redistribute tasks, tools and parts over different stations (expensive and disruptive).

Particularly in the last 20 years and always more, the production system has to be flexible, because of seasonalities or short life-cycles. A good strategy to increase the flexibility of an assembly line is to have less workers than stations and workers are allowed to walk along the stations to continue work on an item. There is no manager that tells the workers what to do, because the system balances itself. Furthermore, there are no buffers for work-in-process inventory. To obtain this, each worker independently follows a simple rule that determines what to do next, as suggested by Bartholdi and Eisenstein (1996a). In their paper, they use the example of “Toyota Sewn Products Management System” (TSS). Let’s see how TSS works.

Let’s consider a flow line with m stations as in figure 2.1.

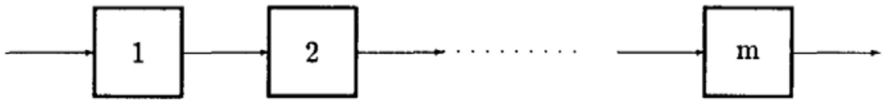


Fig 2.1 - Flow line. A simple flow line in which each item requires processing on the same sequence of workstation. The generic station is station n. (from Bartholdi and Eisenstein, 1996a)

A station can process at most one item per time and only one worker can work in one station. Each worker carries an item from station to station, processing it at each station, until passing it off (*take over, hand-off*) to a subsequent worker. The workers can be numbered from 1 to n, according with their sequence on the line (following the direction of the product flow).

The last worker, once finished with his task, walks back to the worker behind him and continues with his task. This worker does the same to the picker behind him. Finally, as this process continues, the first worker is reached and he must walk back to the depot to receive a new amount of work. Workers are not allowed to pass each other, so a worker can be blocked when he is faster than the one who is preceding him. The operator who is blocked can start working again only when the station is free.

Bartholdi and Eisenstein (1996a) proved that if the operators are allocated from the slowest to the fastest then, during the natural operation of the line, the work content of the product will be spontaneously reallocated among the workers to balance the line. The result, then, is a pure pull system without unattended work-in-process (WIP) between the stations. The throughput of the line depends only on the number of operators and their speed. The difference between a TSS and a bucket brigade is that in a bucket brigade the workers are ordered from slowest to fastest and in a TSS no ordering is imposed. This leads to a system which balances the workload of each worker automatically (Bartholdi et al. 1996a). Balance means that a stable partition of work has emerged, so that each worker performs the same portion of work content from item to item. For the TSS workload balance has to be enforced manually.

2.2 Mathematical model of bucket brigade

It is difficult to understand the behavior of a bucket brigade on paper, because it is a dynamic system which evolves through the time. Moreover, the speed of the operators are different. The easiest way to represent the system is the vector **x** of the worker's position. If the numbers of the workers is n, the vector will be $\mathbf{x} = \{(x_1, \dots, x_n)\}$, dove $0 \leq x_1 \leq \dots \leq x_n \leq 1$, because the operators cannot overtake each other and because the length of the line is normalized at 1.

Another important parameter is the instantaneous speed of each worker i, which is $v_i(x)$. To avoid complications we will consider the speed of the operators constant through the time and along the line and of finite value (not 0 and not ∞). Hence, it is possible to build the vector of workers velocities $\mathbf{v} = \{(v_1, \dots, v_n)\}$, which is constant through the time and along the line. Another important assumption is that the velocities of moving backwards can be considered ∞, because the time to walk back is much shorter than the time requested to assembly (pick) an item, so the time to walk backwards can be neglected. This leads to an important conclusion: the line resets itself at such an instant. It means that when the last worker finishes an item, then, at the same instant, worker n takes over from worker n-1, who takes over from n-2, …, who takes over from worker 1, who introduces a new item into the system.

All this simplification gives us the possibility to describe the behavior of the system considering only the hand-off positions. The only thing we have to do is now consider the vector of vectors $\{(\mathbf{x}^{(0)}, \mathbf{x}^{(1)}, \mathbf{x}^{(2)}, \dots, \mathbf{x}^{(t)}, \dots)\}$ of workers positions at the instant immediately after the line resets. The vector $\mathbf{x}^{(0)}$ is the vector composed by the initial positions of the workers. It is important to notice that in each vector $\mathbf{x}^{(t)}$ the first component $x_1^{(t)} = 0$ (everywhere except for the vector of starting position $\mathbf{x}^{(0)}$, where it is possible to have $x_1^{(0)} \neq 0$). Therefore, it is possible to study the behavior of a bucket brigade system studying the evolution of the vector $\mathbf{x}^{(t)}$, which depends on the starting vector $\mathbf{x}^{(0)}$ and on the speed vector **v**.

The bucket brigade system is a dynamic system. In terminology of dynamical system $\mathbf{x}^{(t)}$ is the t^{th} iterate of the system and the sequence of worker position is the *orbit* beginning at $\mathbf{x}^{(0)}$. For each time the system evolves following the function $\mathbf{x}^{(t+1)} = f(\mathbf{x}^{(t)})$. In conclusion, it is possible to study the behavior of a bucket brigade studying its orbits.

Bartholdi and Eisenstein (1996a) worked on bucket brigades starting from the following assumptions and restrictions.

Assumptions:

- Total ordering of workers by velocities: each worker is characterized by a distinct, constant work velocity v_i.
- Insignificant walking time: the total time to assemble a product is significantly greater than the time to walk the length of the (assembly) line.
- Smoothness and predictability of work: the nominal work content of the product is a constant (which is normalized to 1); and the work content is spread continuously and uniformly along the assembly line.

Restrictions:

- The workers are ordered from slowest to fastest along the flow line.
- The workers are not allowed to pass one another. If a worker is blocked by another worker, he must wait until the other worker is finished.

The model with this assumptions and restrictions is called *normative model*.

A dynamical system, and in particular a bucket brigade can be balanced or not. According to Bartholdi and Eisenstein (1996a), a bucket brigade production line is balanced if each worker repeats the same interval of work content on successive items. Moreover, a balanced line produces at a steady rate and each worker can concentrate on a subset of the work content. Bartholdi and Eisenstein (1996a) shown that in a bucket brigade line could exist a fixed point and it means that, under certain conditions, the system can be balanced after a few iterations. They proved that if workers are sequenced from the slowest to the fastest ($v_1 < \ldots < v_n$), the fixed point is unique and it does not depend on the starting position of the workers $\mathbf{x}^{(0)}$; furthermore, in this case there are no blocks. If workers are not sequenced from the slowest to the fastest, multiple fixed point could exist, so the system could not converge (see 2.2).

It is interesting to show how a bucket brigade evolves during the time. Let's consider a two operators bucket brigade line of normalized length 1, as in figure 2.2. The initial data we need are the speed vector $\mathbf{v} = (v_1, v_2)$ where $v_2 > v_1$ and the vector of starting position $\mathbf{x}^{(0)} = (x_1^{(0)}, x_2^{(0)})$, which gives the position of workers at time $t_0 = 0$.

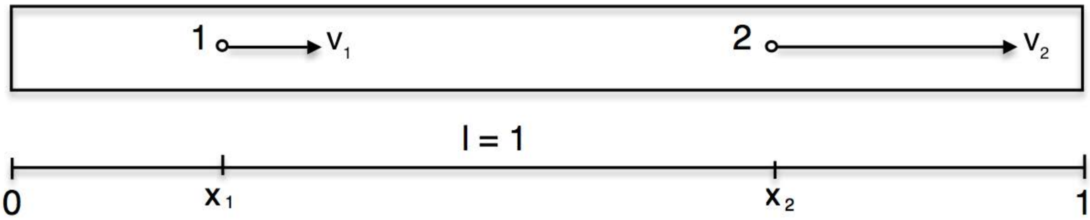


Fig 2.2 - Two workers bucket brigade assembly line. At the time t the 2 workers are in the positions x_1 and x_2 and their velocities are v_1 and v_2. The length of the production line is normalized at 1 unit ($l = 1$).

After the first step (at time t_1, after the first hand-off):

the dynamic of the first worker is $x_1^{(0)} + v_1 * t_1 = x_2^{(1)}$

the dynamic of the second worker is $x_2^{(0)} + v_2 * t_1 = 1$

and it is possible to obtain t_1 from the second equation: $t_1 = (1 - x_2^{(0)}) / v_2$

replacing t_1 in the first equation we obtain $x_2^{(1)} = x_1^{(0)} + v_1 * t_1$

then, we know that $x_1^{(1)} = 0$ and, more in general $x_1^{(t)} = 0 \ \forall t - \{t = 0\}$.

From here $\mathbf{x}^{(1)} = (x_1^{(1)}, x_2^{(1)}) = (0, x_2^{(1)})$ and t_1 come.

After the second step (after t_2 more, so immediately after the second hand-off):

$x_1^{(1)} + v_1 * t_2 = x_2^{(2)}$, where $x_1^{(1)} = 0$, so $v_1 * t_2 = x_2^{(2)}$

$x_2^{(1)} + v_2 * t_2 = 1$

and we know that $x_1^{(2)} = 0$.

From the second equation it is possible to obtain $t_2 = (1 - x_2^{(1)}) / v_2$ and replacing t_2 in the first equation it is possible to obtain $x_2^{(2)} = v_1 * t_2$.

From here $\mathbf{x}^{(2)} = (x_1^{(2)}, x_2^{(2)}) = (0, x_2^{(2)})$ and t_2 come.

After the third step (after t_3 more, so immediately after the third hand-off):

$x_1^{(2)} + v_1 * t_3 = x_2^{(3)}$, where $x_1^{(2)} = 0$, so $v_1 * t_3 = x_2^{(3)}$

$x_2^{(2)} + v_2 * t_3 = 1$

and we know that $x_1^{(3)} = 0$.

From the second equation it is possible to obtain $t_3 = (1 - x_2^{(2)}) / v_2$ and replacing t_3 in the first equation it is possible to obtain $x_2^{(3)} = v_1 * t_3$.

From here $\mathbf{x}^{(3)} = (x_1^{(3)}, x_2^{(3)}) = (0, x_2^{(3)})$ and t_3 come.

After the t^{th} step:

$x_1^{(t-1)} + v_1 * t_t = x_2^{(t)}$, where $x_1^{(t-1)} = 0$, so $v_1 * t_t = x_2^{(t)}$

$x_2^{(t-1)} + v_2 * t_t = 1$

and we know that $x_1^{(t)} = 0$.

From the second equation it is possible to obtain $t_t = (1 - x_2^{(t-1)}) / v_2$ and replacing t_t in the first equation it is possible to obtain $x_2^{(t)} = v_1 * t_t$.

From here $\mathbf{x}^{(t)} = (x_1^{(t)}, x_2^{(t)}) = (0, x_2^{(t)})$ and t_t come.

After the $t+1^{th}$ step:

$x_1^{(t)} + v_1 * t_{t+1} = x_2^{(t+1)}$, where $x_1^{(t)} = 0$, so $v_1 * t_{t+1} = x_2^{(t+1)}$

$x_2^{(t)} + v_2 * t_{t+1} = 1$

and we know that $x_1^{(t+1)} = 0$.

From the second equation it is possible to obtain $t_{t+1} = (1 - x_2^{(t)}) / v_2$ and replacing t_{t+1} in the first equation it is possible to obtain $x_2^{(t+1)} = v_1 * t_{t+1}$.

From here $\mathbf{x}^{(t+1)} = (x_1^{(t+1)}, x_2^{(t+1)}) = (0, x_2^{(t+1)})$ and t_{t+1} come.

The dynamics of the line is shown in figure 2.3.

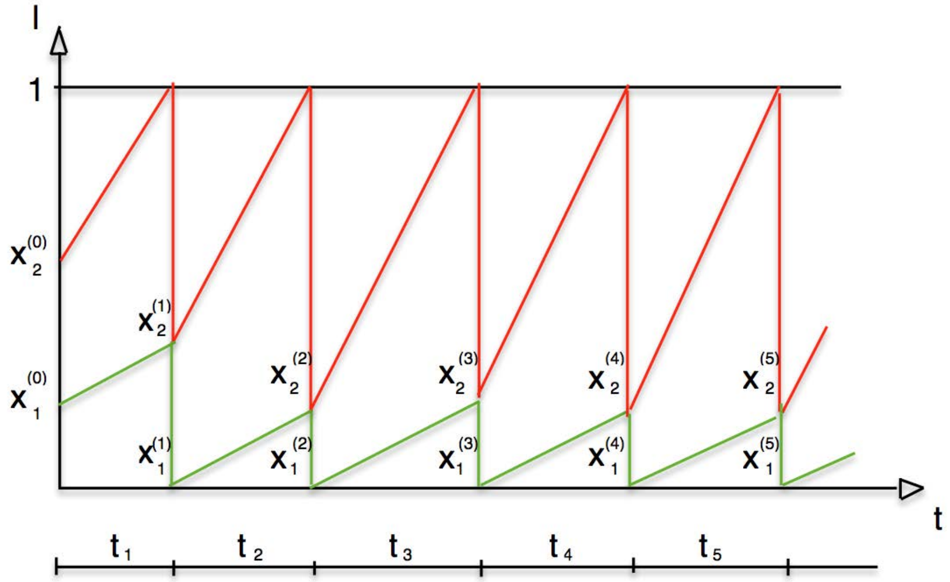


Fig 2.3 - Dynamics of a 2 workers bucket brigade. When the second worker reaches the end of the line (l = 1) he walks back and takes over the work of the first operator; simultaneously, the first operator returns at the beginning of the line and starts a new item. $x_1^{(0)}$ and $x_2^{(0)}$ are the starting positions of the workers, while their speed (constant during the time) is given from the slope of the function. It is possible to notice how the system converges after a few iterations.

Let's take into consideration the t^{th} step. After a few calculations, it is possible to write the succession of x_2. That is $(v_1/v_2) * (1-x_2^{(t-1)}) = x_2^{(t)}$; and from simple algebra it is possible to write: $x_2^{(t)} = v_1/v_2 - (v_1/v_2) * x_2^{(t-1)}$. It is easy to recognize that it is a fixed point equation $x = g(x)$. It is possible to find the fixed point with an iterative method $x_{k+1} = g(x_k)$. To demonstrate the existence of the solution it is possible to use Bolzano's theorem, while to demonstrate the uniqueness we have to prove that the first derivative of the function g(x) is < 0 or > 0 ∀x. Another important information we need is to have a method to understand when the fixed point converges.

THEOREM: Given $g(x) : I \rightarrow R$ and $\xi \in I$, $g(x) \in C^1$ (continuous and derivable).

If $\exists m$: $|g'(x)| \leq m < 1$ (sufficient condition, but not necessary),

then the fixed point method converges $\forall\ x_0 \in I$.

Proof: see appendices (A.1).

The bucket brigade function is $(v_1/v_2) * (1-x_2^{(t-1)}) = x_2^{(t)}$, where $g(x) = (v_1/v_2) * (1-x)$ and $x = x$. Hence, it is possible to write $(v_1/v_2) * (1-x) = x$ and, from here $f(x) = (v_1/v_2) * (1-x) - x = 0$. This equation has at least one solution because of Bolzano's theorem: $f(0) = v_1/v_2 > 0$, $f(1) = -1 < 0$. To demonstrate the uniqueness of the solution we have to calculate the first derivate: $f'(x) = -(v_1/v_2) - 1$, which is always negative, so the function is always decreasing and the fixed point (the solution) is unique. To demonstrate that the fixed point method converges, we have to prove that $\exists m$: $|g'(x)| \leq m < 1$; in bucket brigade systems $|g'(x)| = |-(v_1/v_2)| = v_1/v_2$ and because of the hypothesis $v_2 > v_1$, $v_1/v_2 < 1$ and it is possible to find a number m: $|g'(x)| \leq m < 1$, so the fixed point method converges and the solution is unique.

Practically, it means that in bucket brigade systems, if worker's velocities are constant, operators are ordered from the slowest to the fastest and if workers are never blocked, after some iterations, the line balances itself. It means that it converges exponentially fast to a unique fixed point at which worker i repeatedly executes the same interval of work in the same time.

Hence, at convergence $x_2^{(t-1)} = x_2^{(t)} = x_2^*$ and $t_{t-1} = t_t = t^*$.

It follows that:

$(v_1/v_2) * (1-x_2^*) = x_2^*$

and after easy algebra:

$x_2^* = v_1 / (v_1+v_2)$

It means that the point of hand-off depends only on the operators' speeds and, more in particular, from their ratio. Another fact that is important to notice is that the vector $\mathbf{x}^{(0)}$ of initial positions does not affect the final results when the bucket brigade system converges.

It is also possible to obtain t^*, which is the time between two consecutive hand-offs when the system converges. From the t^{th} step we obtained:

$t_t = (1 - x_2^{(t-1)}) / v_2$

at convergence:

$t^* = (1 - x_2^*) / v_2$

and substituting $x_2^* = v_1 / (v_1+v_2)$ in the formula, after a few simple algebra:

$t^* = 1 / (v_1+v_2)$

which is the cycle time (CT) of the bucket brigade system.

Therefore, the throughput (TR) of the bucket brigade is TR = 1/CT = v_1+v_2 .

More in general, Bartholdi and Eisenstein (1996a) proved that in a system with n operators, if workers velocities are constant with $v_1 < \ldots < v_n$ (from slowest to fastest) and there is no blockage, then the line converges exponentially fast to a unique fixed point. Moreover, every operator does always the same interval of work, the production rate is the summation of all the velocities $TR = v_1 + v_2 + \ldots + v_n$ and it is the largest possible, so the system automatically optimizes itself.

For two operators the solution is:

$TR = v_1 + v_2$; $\mathbf{x}^* = (0, v_1 / (v_1+v_2))$

For three operators the solution is:

$TR = v_1 + v_2 + v_3$; $\mathbf{x}^* = (0, v_1 / (v_1+v_2+v_3), (v_1+v_2) / (v_1+v_2+v_3))$

For five operators the solution is:

$TR = v_1 + v_2 + v_3 + v_4 + v_5$;

$\mathbf{x}^* = (0, v_1 / (v_1+v_2+v_3+v_4+v_5), (v_1+v_2) / (v_1+v_2+v_3+v_4+v_5), (v_1+v_2+v_3) / (v_1+v_2+v_3+v_4+v_5), (v_1+v_2+v_3+v_4) / (v_1+v_2+v_3+v_4+v_5))$

For n operators the solution can be obtained inductively:

- The production rate is the largest possible and it is:

$$TR = \sum_{j=1}^{n} vj$$

- Worker i repeatedly executes the interval of work content:

$$\left[\frac{\sum_{j=1}^{i-1} vj}{\sum_{j=1}^{n} vj}, \frac{\sum_{j=1}^{i} vj}{\sum_{j=1}^{n} vj} \right]$$

And it is easy to see that the results do not depend on the starting position of the workers $\mathbf{x}^{(0)}$.

For a simple two operators numerical example see appendix A.2.

What does it happen, if the workers are not sequenced from the slowest to the fastest? Bartholdi and Eisenstein (1996a) shown that if workers are not sequenced from slowest to fastest, there can be a structural tendency toward persistent imbalance in the line. The solution of the problem with 2-3 operators, for every different combination of worker's speed, has been given by Bartholdi, Bunimovich and Eisenstein (1999) (see paragraph 2.2).

If the workers are not order from the slowest to the fastest, the system could behave differently and in an anomalous way. An example is that adding a worker to the line can decrease the production rate, if workers are not sequenced from the slowest to the fastest; this anomalous behavior happens because the fastest worker can be blocked by slower operators. Another strange behavior of a system, in which slowest-to-fastest sequence is not respected, is that increasing the velocity of a worker can decrease the production rate; this is always due to blockage between operators. If workers are sequenced from the slowest to the fastest, complicated or anomalous behavior cannot be possible: in particular, adding or speeding up a worker will never decrease the production rate. In conclusion, given a certain set of operators, the maximum theoretical throughput is always obtained sequencing them from the slowest to the fastest.

Something more has to be said about workers' speed. In general, the speed of an operator to complete a task can vary significantly, because of the inevitable small noise, because of small variations and so on. The best procedure to follow is to take a lot of observations of an operator who is doing his task, always taking the time and, then, calculate the average of the measurement: the result will be the speed of that operator.

Another important problem to take into consideration is how to decide the ranking of speeds. The first paper on this theme has been written by Trego; according with him, workers can be ranked considering a single measure that will predict their productivity (Trego, 1981, 1989). Then, Bartholdi and Eisenstein (1996a) shown that it is always possible to define a speed ranking between operators and people that usually work together agree on the ranking. The best strategy to follow is to ask the operators to vote secretly about the ranking of their speeds and gather the results by secret ballot. Looking at the votes, it is possible to decide the speed ranking of the workers.

In conclusion, a bucket brigade line balances itself without the need of a manager. It means that if a worker takes a break, in a few cycle the bucket brigade system will find a new equilibrium point, because work will be reallocated among the remaining workers. The throughput can be varied working on the numbers of operators and their speed. If the workers are sequenced from the slowest to the fastest along the line, adding workers never reduces the production rate and removing workers never increases it. Then, the only data that is important to know to understand how a bucket brigade system will work is the relative speed between the workers, not even their value; knowing the values of the speeds, it is always possible to calculate the theoretical production rate of the line.

2.3 Dynamics of bucket brigade with two or three workers

The dynamics of two and three operators in bucket brigade production lines has been deepened by Bartholdi, Bunimovic and Eisenstein (1999). They decided to study two and three operators systems, because they are very common in both apparel manufacturing (Bartholdi and Eisenstein, 1996a) and distribution warehousing (Bartholdi and Eisenstein, 1996b). They studied a bucket brigade system where operators are not allowed to pass each other. The main idea of Bartholdi, Bunimovic and Eisenstein (1999) is to study the dynamics of a three workers line and, then, simplify the model to obtain results for two workers. This model could be extended to n workers, but with some more problems. They worked with the hypothesis of normative model (see 2.2): the work to assemble an item is deterministic and it is spread continuously and uniformly along the line (rather than concentrated in work stations).

The behavior of a three-worker bucket brigade can be explained with figure 2.4.

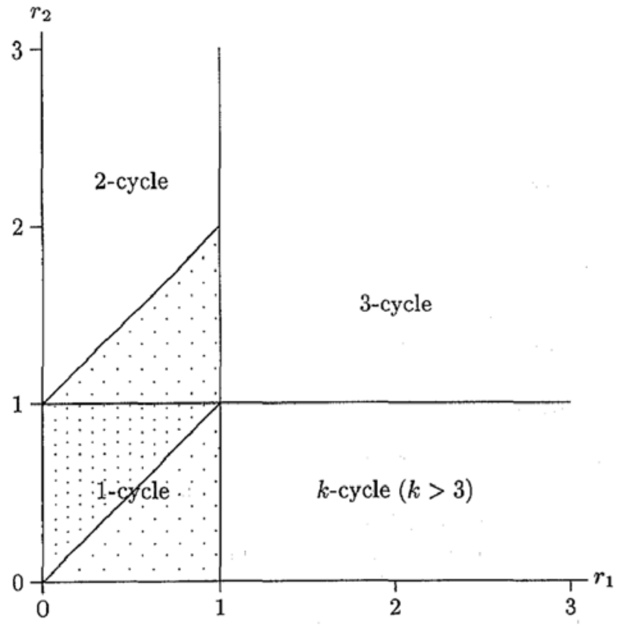


Fig 2.4 - Asymptotic behavior of a three workers bucket brigade line. This picture shows all the different combinations between workers speed in a three workers bucket brigade assembly line. The ratio $r_1 = v_1 / v_3$ is the ratio between the velocities v_1 and v_3, while the ratio $r_2 = v_1 / v_3$ is the ratio between the velocities v_2 and v_3. Each possible three workers bucket brigade can be described with a point on the chart, depending on the velocities of the three workers. For every different combination, the bucket brigade system has a different behavior. (from Bartholdi, Bunimovic and Eisenstein, 1999)

This figure classifies all three-worker lines and it is based on the relative velocities of the workers: $r_1 = v_1 / v_3$ is the ratio between v_1 and v_3, while $r_2 = v_2 / v_3$ is the ratio between v_2 and v_3. In general, $r_i = v_i / v_3$, where v_3 is the speed of the worker closest to the end of the line. Every single point (r_1, r_2) in the figure represents a different bucket brigade flow line. In general, in real assembly lines or picking systems the values of r_1 e r_2 are between 1/3 and 3, so the figure well summarize all the possible behaviors of a three-worker bucket brigade. As it has been written before, the figure also contains all possible behavior of two-workers line: to see it, it is enough to restrict the velocity of the first worker to be $v_1 = 0$, so that $r_1 = 0$. Therefore, it is possible to understand the behavior of a 2-worker line using only the y-axis of the Cartesian plane in figure.

Let x_i be the position of worker i (i = 1, 2, 3) immediately after walkback. The time between the completion of t^{th} and $(t+1)^{st}$ items is $(1 - x_3^{(t)}) / v_3$, and it does not depend on the speed order of the workers. Operator 1 and 2 proceed with their own speed or, if blocked, they will proceed with their successor's velocity, because passing is not allowed. This means that the dynamic function $x^{(t+1)} = f(x^{(t)})$ that describes the behavior of the system is piecewise-linear and its form depends on the relative speeds of workers, which can catch up and be blocked by their successor.

Bartholdi, Bunimovic and Eisenstein (1999) studied this problem simulating the different combinations between workers' speeds. They found that changing the ratio between the workers' velocities, the system can converge to one (1-cycle) or more (2-cycle, 3-cycle, k-cycle) fixed points: this means that there could be one, two, three, k possible positions of hand-off. Let's see it more precisely, taking into account figure 2.4.

In region 1 (shaded, $0 < r_1 < 1$ and $0 < r_2 < 1$) the system converges to a single fixed point (1-cycle) corresponding to a perfectly balanced line and an optimal production rate (TR = v1+v2+v3). In their first paper about bucket brigade (see 4.1), Bartholdi and Eisenstein (1996a) worked only on the heavily shaded region of the diagram, where $v_1 < v_2 < v_3$ ($r_1 < r_2 < r_3$). In this area, the system is solid; in other words, it means that the system responds quickly and well to every perturbation, without changing qualitative behavior. The novelty of the paper written by Bartholdi, Bunimovic and Eisenstein (1999) is that, in the whole shadowed surface in region 1, the system is balanced and there is only one fixed point; che convergence is slower than if $v_1 < v_2 < v_3$, but after more iteration the fixed point is going to be reached anyhow. Another thing to notice is that whenever worker 3 is the fastest one, the system converges to the fixed point, so the bucket brigade system balances itself. This last conclusion is not true if we have to deal with bucket brigade systems with more than three operators. In general, with more than 3 operators, the following condition is effective: if the last worker is faster than the first worker, the bucket brigade will be balanced.

In region 2 ($0 < r_1 < 1$, $r_2 > 1$) and 3 ($r_1 > 1$, $r_2 > 1$) the operators are not sequenced from the slowest to the fastest, so the faster workers tend to be blocked by slower workers. In region 2 the operators' speeds are $v_1 < v_3 < v_2$: after walkback the positions of operators can be $(0, r_1 / (r_1+r_3), 1)$ or $(0, 0, r_1 / (r_1+r_3))$, with the result of a suboptimal rate of production $TR = 2*(v_1+v_3)$. In region 2 the bucket brigade has two fixed points (2-cycle) or, in other words, there are two possible position of hand-off. On the other hand, in region 3 the speeds of workers are $v_3 < v_2 < v_1$ or $v_3 < v_1 < v_2$, so the slowest worker is always the one at the end of the line, therefore he will block the other two workers. The workers' positions after walkback can be (0, 1, 1), (0, 0, 1) and (0, 0, 0) and the production rate is $TR = 3*v_3$, so that everyone is reduced to the velocity of the slowest worker. In this case, the bucket brigade has 3 fixed points (3-cycle).

In region k ($r_1 > 1$, $0 < r_2 < 1$) the speeds of operators are $v_2 < v_3 < v_1$, so the first worker is the fastest and the second is the slowest. The behavior of the system, unlike the other regions, depends not only on the value of r_1 and r_2, but also on the initial positions of the workers $\mathbf{x}^{(0)}$. Consequently, the dynamic of the system in this region is very complicated, because the number k of cycle depends also on the initial positions of workers $\mathbf{x}^{(0)}$.

It could happen that the workers' speed changes during the work shift: in this case the behavior of the system can change heavily, in particular if the system goes from a region of figure 2.4 to another one. The result of this changing can be very different, depending on a lot of variables.

As it has been told before, thanks to the figure is it also possible to predict the behavior of a two operators bucket brigade line. It is possible to describe a two operators line like a three operators line by restricting the speed of the first worker to be $v_1 = 0$. Hence the two operators system is described by the y-axis of the figure. As we have already discussed, the system has one fixed point with the maximum throughput when the second operator is faster than the first, while the system has a period two orbit with suboptimal production rate if the second operator is slower than the first.

The higher is the number of the workers, the more difficult is to predict the behavior of the system. In general, a unique necessary and sufficient condition to say if a bucket brigade balances itself (converges to a 1-cycle) does not exist, but there is one necessary condition: the last worker must be faster than the first worker.

In conclusion, the best strategy is to deal with a bucket brigade system is to order the operators from the slowest to the fastest; then, a bucket brigade works better when composed by workers of a wide spectrum of velocities. Ordering operators from the slowest to the fastest means to have the maximum production rate and the greatest stability. It means that asymptotic behavior will assert itself more quickly and will be more resistant to disruption.

2.4 Bucket brigades when worker speed do not dominate each other uniformly

It could be interesting to see what happens if workers change their speed along the production line. Armbruster and Gel (2002) studied the behavior of a two workers bucket brigade, where one worker has a constant speed over the whole production line and the other is slower over the first portion and faster over the second portion of line. All the systems in which workers have varying levels of specialization at different tasks are taken into account with this paper. In particular, in all this systems the speed of the workers at a particular task depends more on the type of task, rather than the worker's skill profile. There are different systems in real life where workers have different speeds in different parts of the line: systems with stations that requires high specialization or training, systems with high labour turnover etc. Armbruster and Gel (2002) tested the performance of this system for both the passing case and the case in which workers are blocked. The main output of their research is the throughput of the system: if it is high the system performs well, if it is not the system does not perform well.

The modeling assumptions of Armbruster and Gel (2002) are similar to those made by Bartholdi and Eisenstein (1996a): workers are cross-trained, the task and the work are continuous along the line, the processing times are deterministic and workers walk back with infinite velocity (zero walk back time). The only difference is that the velocity of one worker does not dominate that of another at every point along the production line.

The bucket brigade proposed by Armbruster and Gel (2002) also converges to a self-balancing system although with more than one fixed hand-over point between the workers. In particular, they found that the bucket brigade always organizes itself; the bucket brigade may not always balance itself on one fixed point but may also self organize to a stable period-two orbit: it means that workers hand over jobs at exactly two fixed locations that they visit periodically.

2.5 Deterministic chaos in a model of discrete manufacturing

Some simple deterministic systems can generate surprisingly complicated behavior that has been called "chaotic". According to Bartholdi, Eisenstein and Lim (2003, 2009), under certain conditions, bucket brigade systems can be chaotic. A chaotic system is a system which has long-term behavior that can be hard to describe, to predict and even harder to simulate. Long term behavior of chaotic systems seems to be deeply connected to randomness (see Alligood, Sauer and Yorke, 1996; Devaney R.L., 1989; Martelli M., 1999).

In their paper, Bartholdi, Eisenstein and Lim (2003, 2009) expanded the normative bucket brigade model. As it has been explained before, the idea of bucket brigades is that workers have to follow this simple rule: each worker carries work forward, from work station to work station, until either completes an item or it is taken by a downstream colleague; then, he walks back to get more work, either from an upstream colleague or from a buffer at the start of the line. Till now, the hypothesis of insignificant walking time has been always considered, so that the walk back time of all the workers could be considered 0, according to the fact that the walk back velocity of each operator was $w_i = 0$. Bartholdi, Eisenstein and Lim (2003, 2009) considered both the forward velocity v_i and the backward velocity w_i arbitrary and constant. This model approximates well the behavior of low density picking systems, where workers may walk a lot between to picks, so the time between pick is comparable with the time to return back. Furthermore, in this case, workers are allowed to overtake each others both when walking forward and backward; then, when a worker is walking back, he can pass a worker who is working in the forward direction. Because of this possibility, blockage is no longer possible; therefore the production rate is as large as possible, regardless of how the workers are sequenced. We will say that a worker *overtakes* another worker when the first one catches up and passes another worker, when they are both walking forward or backward. We will say that a worker *passes* another worker when the first one, who is going backward to get more work, walks past a successor who is working forward.

Considering also the walk back velocities w_i, the pattern of hand-offs becomes more complex. The main consequence is that hand-offs are no longer contemporaneous, because of the finite velocities of walk back. Furthermore, there can be multiple completions before the next hand-off or before the next start. In same cases, it is possible to have no hand-offs at all, as when workers have velocities $v_1 = 2$, $w_1 = 1$, $v_2 = 1$, $w_2 = 2$: in this case workers work independently and there are no hand-offs.

It is easy to see that the effective production of each worker i of a n workers bucket brigade, considering the walk back velocities w_i, is $\psi_i = (1/v_i + 1/w_i)^{-1}$.

Therefore, considering all the workers together, the long term behavior of the system gives an average production rate of:

$$\sum_{i=1}^{n} \psi i$$

In their paper, Bartholdi, Eisenstein and Lim (2003, 2009) gave a convergence condition to know if the bucket brigade converges to a stable allocation of work.

Convergence condition: the workers on the bucket brigade assembly line should be indexed so that:

$$\frac{1}{v_1} - \frac{1}{w_1} > \frac{1}{v_2} - \frac{1}{w_2} > \cdots > \frac{1}{v_n} - \frac{1}{w_n}$$

In other words, workers should be ordered from the most slowed to the least slowed. It is surprising, because a worker who is slower in both directions could have a higher index of a worker who is faster in both directions. The convergence condition tells us to make the least slowed worker work more downstream than the most slowed worker. For example, the two workers described by $v_1 = 3$, $w_1 = 6$, $v_2 = 2$, $w_2 = 1$ satisfy the convergence condition.

According to Bartholdi, Eisenstein and Lim (2003, 2009) "the bucket brigade assembly line is balanced if each worker invests the same clock time and repeats the same interval of work content for each item produced and, moreover, those intervals are non-overlapping". Let the balance point at which worker i hands off work, given as a fraction of work content completed, be x_i^* and let $x^* = (x_1^*, x_2^*, \ldots, x_{n-1}^*)$.

They proved that:

- For any bucket brigade the point

$$x_i^* = \frac{\sum_{j=1}^{i} \psi_j}{\sum_{j=1}^{n} \psi_j} \quad \text{for } i = 1, \ldots, n-1$$

  is a fixed point with respect to the map that relates successive points of hand-off and is, moreover, the unique point of balance. This means that if the line is balanced, if there are no perturbations, the line remains balanced.

  NB: there could be other fixed points, but the point of balance (work is partitioned among the workers) is unique.

- If workers are sequenced on the assembly line from the most slowed to the least slowed (the convergence condition is verified) then x^* is an attractor. This means that when the bucket brigade is close enough to balance, then it will converge to balance. The point of balance is a local attractor: it means that the system restores his balance if there are some perturbations which are not too disruptive.

- If the convergence condition is true, the point of balance is an attractor: it means that after some iterations the system will always balance itself.
- A two operators bucket brigade could have a chaotic behavior under certain conditions. It means that the sequence of hand-off positions is chaotic, if the convergence condition fails to hold.

Let's focus on the last point and let's take into account a bucket brigade with the following velocities: $v_1 = 1$, $w_1 = 1/3$, $v_2 = 1$, $w_2 = 1$. This bucket brigade fails to satisfy the convergence condition and it is easy to prove that the dynamic function that shows the position of consecutive hand-offs is $x^{k+1} = 1 - (2x^k \bmod 1)$, where x^k is the position of the k^{th} hand-off. It is easier to understand the sequence of hand-offs looking at figure 2.5.

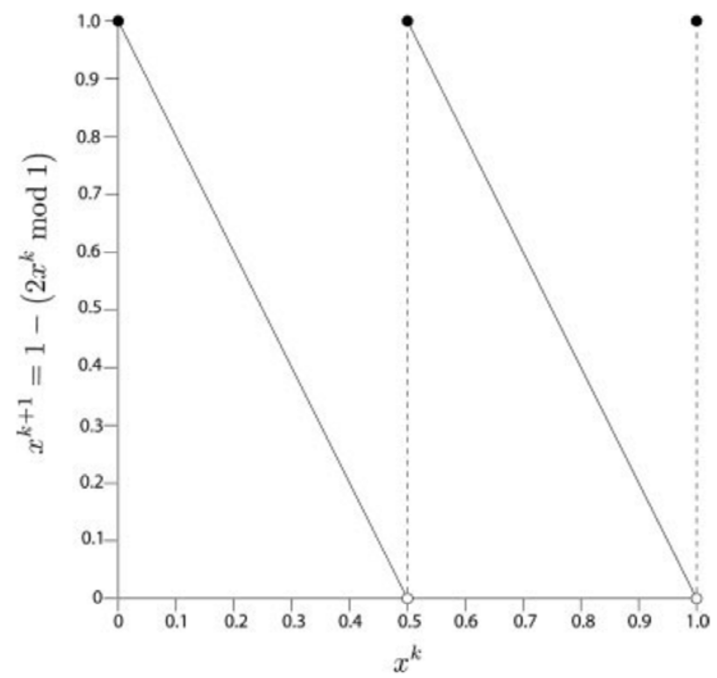


Fig 2.5 - The dynamics map of a chaotic bucket brigade. If work is handed off at position x^k, then the next hand-off will occur at position x^{k+1}. (from Bartholdi, Eisenstein and Lim, 2003, 2009)

The function showed in figure 2.5 is an *expanding* map; this means that it has slope of absolute value strictly greater than 1, where defined (it has discontinuities at 1/2 and 1). An expanding map is a map in which the orbit of all nearby starting points eventually separate (Devaney,1989): the system will remain stable only if we start exactly from the stable points; if not, the system is going to diverge chaotically. It is easy to notice that the function has two fixed points: the point 1/3 is the unique point of balance, but it is a *repelling* fixed point, which means that the system spontaneously avoids balance. The point 2/3 is another repelling fixed point.

According to Martelli (1999), the dynamic function $x^{k+1} = 1 - (2x^k \bmod 1)$ is very similar to the Bernoulli map $x^{k+1} = 2x^k \bmod 1$. Since Bernoulli map has a chaotic behavior (it is an expansive map), it is possible to conclude that the dynamic function $x^{k+1} = 1 - (2x^k \bmod 1)$ of the bucket brigade we described before has a chaotic behavior as well (it is an expansive map too).

A chaotic bucket brigade is capable of some strange behaviors. For example, its long term behavior depends a lot on the initial conditions of the system; it means that the long term behavior of the bucket brigade system depends mainly on the starting positions $\mathbf{x}^{(0)}$ of the workers. The conclusion is that when the convergence condition is not true, it is impossible to predict the future state of the bucket brigade, due to unavoidable inaccuracy of initial conditions. In other words, the computer cannot handle the non periodic numbers that comes out from the calculation, because it works in binary and it has to approximate necessarily, losing precision. This lack of precision becomes wider iteration by iteration, so the computer gives wrong results.

In figure 2.6, an example of two different bucket brigades is shown. The behavior of a stable bucket brigade is shown above; on the contrary, the behavior of a chaotic bucket brigade is shown under. In the first case (above) the convergence condition holds, so the system converges to a single point after a few iterations and products are produced at regular intervals. When the convergence condition does not hold (under), it is impossible to predict the behavior of the system, because it is completely chaotic: it means that the time between the production of two consecutive products is not constant and hand-offs are not distributed uniformly along the interval of work content.

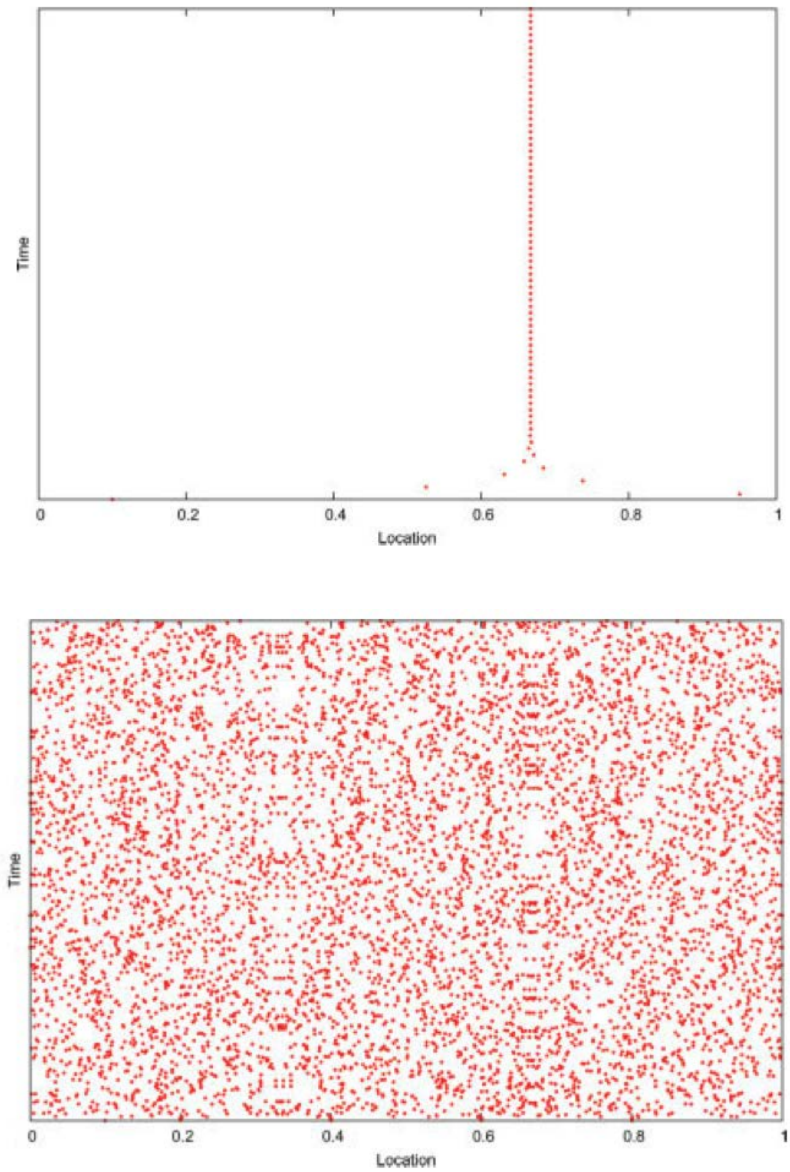


Fig 2.6 - Locations of hand-offs under a stable bucket brigade (above) and a chaotic one (under). When the convergence condition holds (above), the hand-off locations quickly converge to a single point and products are completed at regular intervals. When the convergence condition is violated (under), the hand-offs appear to be distributed uniformly throughout the interval of work-content and completion times are erratic. (Above, worker velocities are $v_1 = w_1 = 1$ and $v_2 = 1$, $w_2 = 1/3$. Under the workers are swapped). (from Bartholdi, Eisenstein and Lim, 2003, 2009)

In figure 2.6, a bucket brigade with velocities $v_1 = w_1 = 1$, $v_2 = 1$ and $w_2 = 1/3$ is shown above: the velocities satisfy the convergence condition, so the system balances itself after a few iterations; a bucket brigade with velocities $v_1 = 1$, $w_1 = 1/3$ and $v_2 = w_2 = 1$ is shown under: the velocities do not satisfy the convergence condition and the hand-offs seem to be randomly distributed in space and time.

In conclusion, it has been proved that a deterministic system could have a chaotic behavior under certain conditions. In particular, if the time to return back is not considered 0 and if the convergence condition fails to hold, the behavior of the bucket brigade can be chaotic. It is very uncomfortable to have a production system which behaves chaotically, because it will complete products randomly, even if the assembly line is completely deterministic. Therefore, the cost of production increases, because safety stock increases and downstream and upstream processes are more difficult to coordinate. Moreover, the fact that hand-offs can happen in every part of the line can slow down the learning process, because workers would not experience a stable assignment of work (see Muñoz and Villalobos, 2002; Armbruster, Gel, and Murakami, 2007 for discussions of bucket brigades under models of learning); because of this reason each worker has to learn every subinterval of work content. The possible chaotic behavior of deterministic systems is a new challenge that managers have to deal with. When possible, it is better to avoid chaotic systems, because a central goal of manufacturing systems control is the reduction of variability and chaotic system are not stable.

2.6 Performance of bucket brigade when the work is stochastic

Another important question that has to be answered is how bucket brigade works even in the presence of variability in the work content. Bartholdi, Eisenstein and Foley (2001) answered this question, extending previous analysis to a stochastic model of work content and shown that the dynamics and production rate will be similar to those of the deterministic model when there is "sufficient work" distributed among "sufficiently many" work stations. Because of the variability of customer orders, the work content may be imagined to be stochastic.

The main assumption to set up a bucket brigade are:

- Insignificant walking time: the total time to assemble a product is significantly greater than the time to walk the length of the flow line. Therefore all hand-offs occur, for all practical purposes, simultaneously, synchronized by item completions of the last worker.

- Total ordering of workers by velocity: each worker i=1, …, n is characterized by a distinct, constant work velocity v_i.

- Smoothness and predictability of work: the nominal work content of the product is a constant (which we normalize to 1), and the work content is spread continuously and uniformly along the flow line of normalized length 1.

Under these hypothesis the bucket brigade follows the *normative model* (see paragraph 2.2): these conditions are sufficient to guarantee that the bucket brigade achieves the maximum possible throughput.

Assumptions 1 and 2 are uncontroversial. Assumption 3 tends to be true for a lot of cases, but in some very important economic contexts, such as order-picking in a warehouse, this last assumption cannot be always true.

In their paper, Bartholdi, Eisenstein and Foley (2001) changed assumption 3 in “exponentially distributed work”: let the work to assemble a product consist of m discrete task primitives at m successive work stations. The nominal work-content at each station is independent and follows an exponential distribution with common mean normalized to 1.

This leads to the fact that the time required for the i^{th} worker to complete a task follows an exponential distribution with mean $1/v_i$. This means that there will be greater variance at each work station than one would expect to find in practice. This unrealistically large variance reduces the throughput of bucket brigades because it increases the chances of blockage. Therefore the theoretical solution of the problem will give as a result a lower throughput than the real one.

After working on the stochastic dynamics of the system, Bartholdi, Eisenstein and Foley (2001) proved that, as the number of stations increases, the moment-to-moment behavior of the stochastic line will increasingly resemble that of the normative model. Moreover, this resemblance will assert itself with great uniformity.

This hypothesis of exponentially distributed work is well suited to the context of order-picking in warehouses, where the work content is stochastic because of the variability of customer orders. In chapter 3, this topic will be explored.

Chapter 3

Bucket brigade in order-picking systems

In this third chapter, we will explore the theme of bucket brigade in order-picking systems, following what Bartholdi and Eisenstein wrote in their paper "Bucket brigades: a self-balancing order-picking system for a warehouse" (1996b), where they analyzed the phenomenon of order-picking in chain retailers. After speaking about flow rack and after explaining the differences between zone picking and bucket brigade, we give some hints to improve the performance of a bucket brigade in an order-picking system. After that, we show the results obtained by Bartholdi and Eisenstein (1996b), starting from the hypothesis of exponential distributed work. At the end, the advantages to use bucket brigade also in warehouse and, more in particular, in order-picking are explained.

3.1 A self-balancing order-picking system for a warehouse

According to Bartholdi and Eisenstein (1996b), bucket brigades are not only very efficient in assembly lines, but they are also a new way of sharing work among pickers in a warehouse. The result is increased pick rates, without conscious intention of the workers, without guidance from management, without any model of work content, without any computation, indeed without any data-gathering at all. The result of implementing order-picking by bucket brigade in the warehouses is that pick rates increased more than 30%. Moreover, bucket brigade can replace zone-picking as the standard method of picking orders in high-volume retail trade. All this advantages are cost-free.

All the ideas that we analyzed in the context of manufacturing (Bartholdi and Eisenstein, 1996a) could be translated into ideas to pick better in warehouses. Bartholdi and Eisenstein (1996b) analyzed the phenomenon of order-picking in chain retailers, where the space to stock material is severely limited, and the skus have to be replenished frequently and in small, less-than-caseload amounts. A typical store orders a lot of skus, but a small number each. Picking this many skus in small amount is very labor intensive.

The best layout to deal with these circumstances is, generally, *flow rack* (figure 3.1).

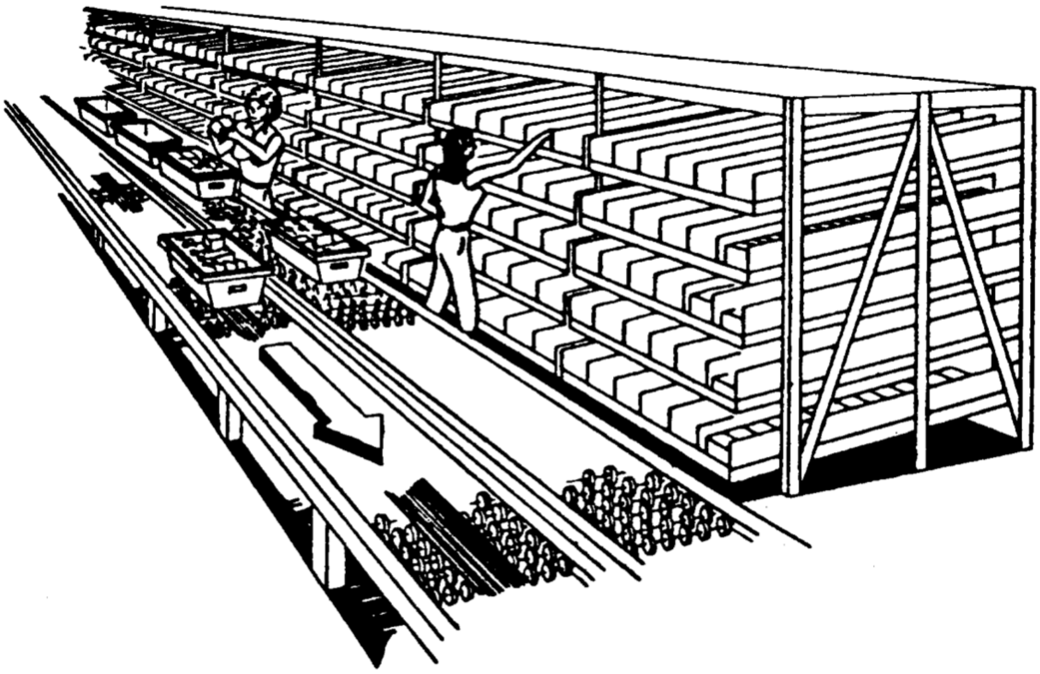


Fig 3.1 - Flow rack. A team picking from an aisle of flow rack to a conveyor (from “Warehouse Modernization and Layout Planning Guide,” Department of the Navy, Naval Supply Systems Command Publication 529, March 1985, p. 8–17). The “passive” conveyor (closer to pickers) holds partially completed orders. The powered “take-away” conveyor transports completed orders to the shipping department.

A picking team is picking products from an aisle of flow rack to a conveyor. The flow rack is unidirectional: the order-pickers pick products from one side and the flow rack is replenished by the opposite side. The racks are slanting and divided into *bays*; within each bay there are shelves with rollers, which help the product to slide down, because of the force of gravity. Thanks to this system each sku is can slide down towards the operator; skus are stored as a *lane* of cases, so the skus can be picked separately. Then, there are two conveyors: the passive one, which is the one closest to the pickers, holds partially completed orders; the active one transports completed orders to the shipping department.

According to Bartholdi and Eisenstein (1996b), an *order* is a list of skus for a single customer together with quantities to be picked. All the orders that have to be picked wait at the start of the aisle. In every list there is written how many skus of each type have to be picked and their position along the aisle. Once workers have the list with them, they move along the aisle, they pick products and they put them into *totes* (cartoons), which travel on the conveyor. The aisles work in parallel to pick the orders of a common set of customers. It is very important that the time of picking is synchronized with the time of departure of every truck. An important thing to remember, then, is to keep orders in sequence to have them in order, when they are at the shipping dock, where trucks are loaded in reverse order of delivery.

3.2 Sequential zone-picking vs bucket brigade

The most common way of organizing picking in warehouses is to divide the bays into contiguous zones and to restrict each picker to his zone. The picker in the first zone takes a new order and starts to fill the tote with the requested items, while pushing it on the passive conveyor. As soon as he reaches the end of the zone, he leaves the order on the passive conveyor for the next worker and returns back to start a new order. The second worker picks the requested item in his zone and pass the tote to the third worker when he reaches the end of the zone. The system goes on like this, till the last picker pushes the tote of a complete order onto the take away conveyor, which brings the tote to the shipping department. This kind of order-picking is called *sequential zone-picking*. The idea is that all workers will remain busy for the same amount of time, if their work is more or less the same. Zones are fixed in advance of picking and they are based on the distribution of the work along the flow rack; they require continuous readjustment to maintain balance, because:

- The work could be distributed in such a way that there is no perfect solution to divide the work between operators equally.
- The model of work content is unavoidably inaccurate: in addition to the number and location of the skus that has to be picked, we should consider also their weight, their shape and the height of their location. Then, we should consider the opening of new cases, empty cases, sealing totes, putting and pushing cases on the conveyors. In conclusion, there are to many inputs to take into consideration to calculate exactly an equal distribution in zones.

- Zones are static: they could balance only the total work, but they fail to maintain balance from order to order. So, it could happen that, at the end of the day, every picker has picked the same amount of work, but they were not fully utilized.

A lot of solutions to this problem have been tried, but none of this strategies overcome the inherent inefficiencies of zone-picking. In summary, zone-picking requires continuous supervision to reach balance, which is impossible to reach. The cost of this imbalance is reduced pick rates due to incompletely utilized pickers and due to work-in-process, which interferes with picking.

To solve these problems, Bartholdi and Eisenstein (1996b) suggest bucket brigades: each picker follows the rule "pick forward until someone takes over your work; then go back for more". When the last picker completes an order, he returns back and take over the order of his predecessor, who returns back and take over and so on, until the first picker starts a new order. The mathematics and the dynamics of the system are exactly the same as chapter 2, about bucket brigade in assembly lines. Workers are no more restricted to zones, but the zones are flexible. There are no buffers, so the only work-in-process inventory is that on the hands of the pickers. Pickers must maintain their sequence: no passing is allowed. If pickers are sequenced from the slowest to the fastest, pickers will spontaneously migrate where there is more work, pick rates will increase and no management will be required, because the system will balance itself. The idea of Bartholdi and Eisenstein (1996b) is to use the same ideas of bucket brigade in manufacturing (Bartholdi and Eisenstein, 1996a) and use them in warehousing, with some little differences. For example, a deterministic model of work is appropriate for assembly lines, while in order-picking the location and the amount of work vary from order to order. In both cases, if workers are sequenced from the slowest to fastest, the bucket brigade line will spontaneously balance itself, while improving its throughput.

It is easy to improve the picking system changing from zone picking to bucket brigade, because no special equipment and no changes to a typical warehouse management system are required. Moreover, no changes in the layout are required. Bartholdi and Eisenstein (1996b) support the idea that it is possible to configure a bucket brigade system in only one morning. In fifteen minutes the general idea is described to the workers, then they have to be ordered from the slowest to the fastest and, within half an hour, they can pick comfortably.

3.3 How to improve a bucket brigade system?

According to Bartholdi and Eisenstein (1996b), there are two main wastes under bucket brigade: time lost when a worker is blocked and time spent to walk back to get more work. As lean production teaches, the waste must be eliminated. The natural way to do this is increase the side of the bucket: it means that every worker carries a larger number of orders at the same time. Working with larger buckets means fewer walk back to start new orders and reduced variance in the amount and location of work; this makes possible to use the hypothesis of normative model, even if in warehouses work is not equally distributed. In other words, using larger bucket size makes the work more smooth along the aisles and, therefore, between the workers. Reduced variance also mean less opportunities for a faster but busy worker to block a slow one. If the bucket is too large, the conveyor will be congested. The workers could find their totes pushed downstream by those of their predecessors and so they have to walk to reach the right tote: the probability to make mistakes is higher and this walking reduces the pick rate and, so the throughput. To find the right size of a bucket brigade system has to be found considering the statistics of the order stream and differs from case to case.

Another big waste is the time lost by a worker to seek the item that has to be picked in a flow rack. A strategy to shorten the searching time is a pick-to-light system, by which a central computer switches on a light to point out the item which has to be picked by the operator. Thanks to this light the worker knows exactly where, which and how many items he has to pick. This reduces both searching time and paper-handling. Moreover, because all the locations to be picked within a bay are lighted at the same time, two workers can pick simultaneously, side-by-side, on the same order.

One other strategy to improve the functioning of bucket brigade in order-picking is to allow any picker who completes an order to push his order directly onto the take-away conveyor and go back to take the order of his predecessor. This allows to reduce inactivities, because no picker who has completed his order has to wait. Doing like this, orders will not maintain their sequence, so this modification is useful when order sequence is not important and when picks for each order are not well distributed along the aisle, but concentrated in only one spot.

It is possible to obtain another advantage from a bucket brigade picking system if workers are not allowed to pass each other. The advantage is that aisles could be narrowed to the width of a single cart. This gives the possibility to build the warehouse in a more compact area; because of this reason walking spaces are reduced and walking times are shortened as well: this leads to increased pick rates and higher throughput.

3.4 The effectiveness of bucket brigade

On their paper, Bartholdi and Eisenstein (1996b) considered a simple model of bucket brigade in picking, where the amount and location of works varies.

They started from 4 assumption:

- Total ordering of position: let's describe every discrete picking position with a number j between 1 and m (j = 1, …, m); the position could correspond to the bays of flow rack. For any given order, every pick of position j must be completed before all the picks which are in a position after (more towards the end of the line) j.
- Iid orders, exponentially distributed work: orders are independent and identically distributed random vectors, the components of which are independent; the j^{th} component of each vector represents the standard work at location j and it follows an exponential distribution with common mean 1/μ.
- Total ordering of the workers by velocity: each picker i (i = 1, …, n) has his own velocity v_i, so the time that a picker needs to complete a pick is exponentially distributed with mean 1 / (μ*v_i). This assumption does not work well for order-picking, because the pertinent skills are simply dexterity and motivation. According to Bartholdi and Eisenstein (1996a), it is easy to rank the workers velocities.
- Insignificant walking time: the time to pick an order is significantly grater then the time to walk along the whole aisle.

Starting from these hypothesis, Bartholdi and Eisenstein (1996b), described the behavior of a two pickers bucket brigade system in order-picking. They proved that the bucket brigade line perform at its best if the pickers are sequenced from the slowest to the fastest, as it happens in assembly line bucket brigade (Bartholdi and Eisenstein, 1996a). This is very intuitive, because ranking the workers in this way reduces the probabilities of blockage. Solving the equations of the system, it is possible to observe that the randomness of the orders does not qualitatively change the dynamics of a bucket brigade from those of the deterministic model analyzed by Bartholdi and Eisenstein (1996a) and Bartholdi, Bunimovic and Eisenstein (1999). The asymptotic behavior remains the same, but the variance of work among orders makes the system less predictable.

The distributions of hand-off positions are presented in figures 3.2 and 3.3. On one hand, in figure 3.2 the two pickers are ordered from the faster to the slower, on the other hand, in figure 3.3 the two pickers are ordered from the slower to the faster.

When pickers are sequenced from the faster to the slower, the faster one tends to be blocked repeatedly by the slower one and thus hand-offs alternate between positions near the end of the line or near the start of the line, as shown in figure 3.2.

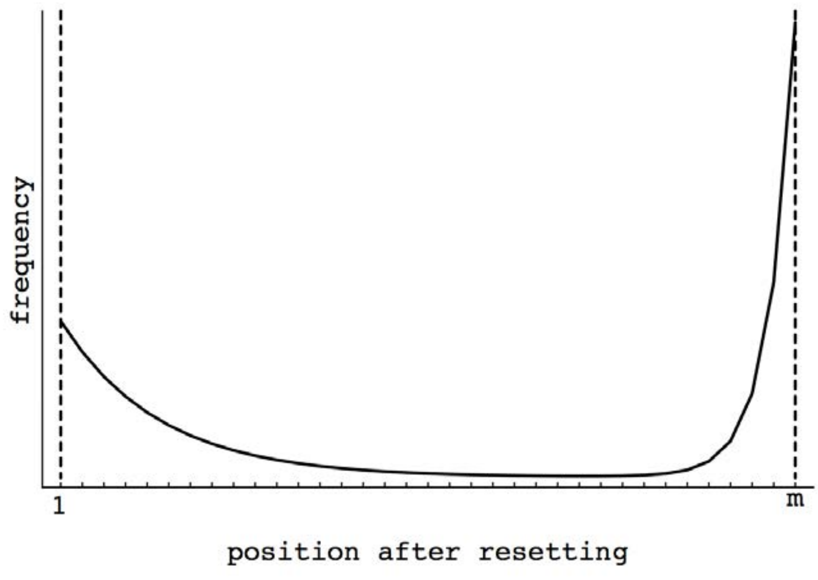


Fig 3.2 - Faster to slower: position of the second, slower, of two pickers immediately after walk back. In the deterministic model of work, the second picker alternates between the first and last position (dashed vertical lines). (from Bartholdi and Eisenstein, 1996b)

In figure 3.3, instead, shows the distribution of hand-offs when pickers are ordered from slower to faster.

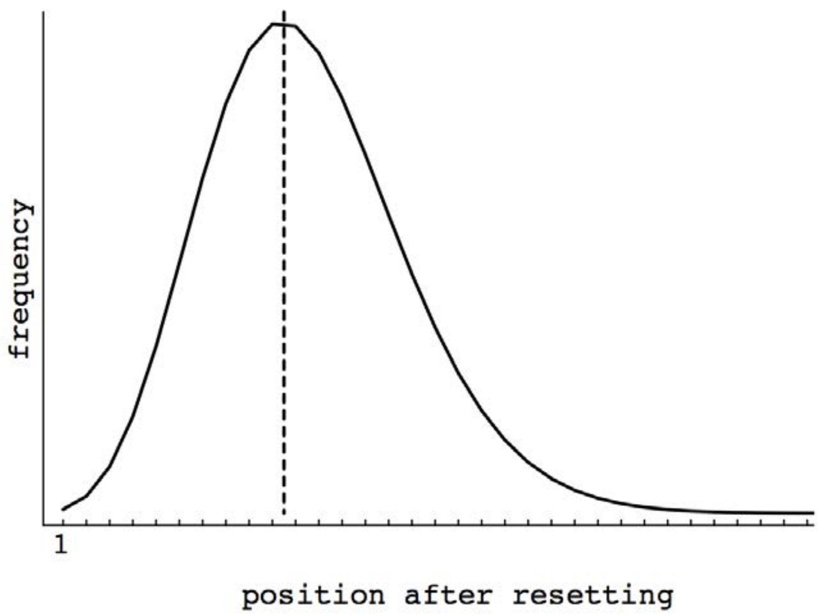


Fig 3.3 - Slower to faster: position of the second, faster, of two pickers immediately after walk back. In the deterministic model of work, the second picker repeatedly returns to the position marked by the dashed vertical line. (from Bartholdi and Eisenstein, 1996b)

How good is the performance of a bucket brigade picking system, generally speaking? Is it possible to keep always all the operators busy? The answer, if the work is probabilistic, is no; but strategies to improve the performance of the system can be found. The first strategy to set up to increase the performance of a bucket brigade picking system is to try to reduce as much as possible blockage between operators. Figure 3.4 shows all the possible behaviors (production rate) of a two pickers bucket brigade, taking into account all the different pair of workers velocities. The production capacity remains constant, so $v_1 + v_2 = 1$ and $\mu = m$, but the ratio between the two velocities changes along the x-axis, where the velocity of the second worker (the one which is working closer to the end of the line) is taken into account. With this hypothesis, the maximum production rate of the system is 1 (the horizontal dashed line).

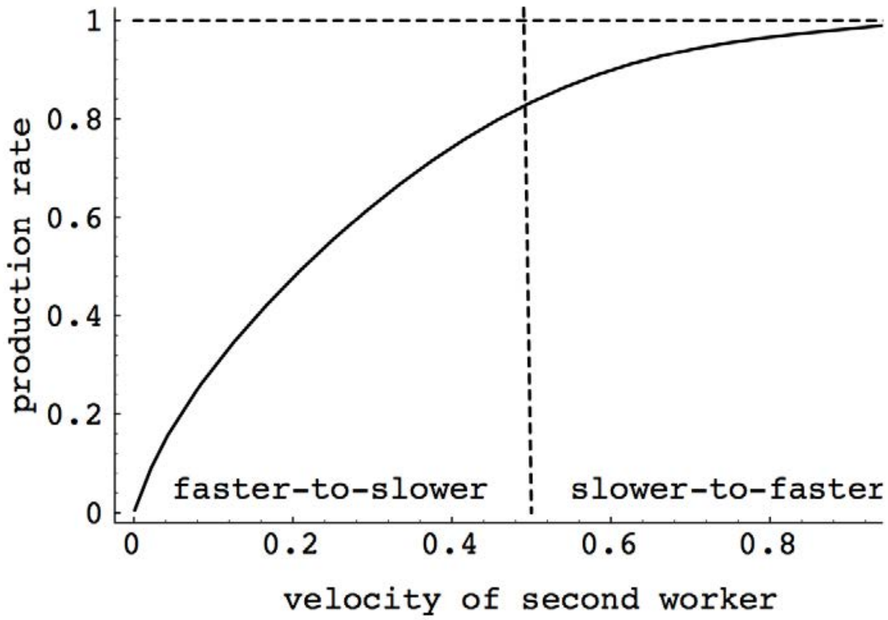


Fig 3.4 - Different behavior of a two operators bucket brigade under the hypothesis of exponentially distributed work. The production rate increases with the contribution of the second picker. The velocity of the team remains constant ($v_1 + v_2 = 1$), but the velocity of the second picker increases continuously from 0 to 100% of the total work velocity of the team. The most important thing to notice is that slower to faster order gives always the highest production rate, so the best result. (from Bartholdi and Eisenstein, 1996b)

Thanks to figure 3.4, it is easy to see that the production rate increases, if the velocity of the second worker v_2 increases; in particular, the system works better (higher production rate) when the pickers are ordered from slower to faster, because there is no blockage. In general, a bucket brigade picking system works at its best if the velocities of the workers are different between themselves. All this discussion is consistent with the deterministic model of Bartholdi, Bunimovic and Eisenstein (1999).

Another strategy it is possible to use to increase the effectiveness of bucket brigade picking systems is to reduce the variance of the work, because this leads to a minor chance of blockage. As the variance decreases, the behavior of the bucket brigade approaches that of the deterministic model described by Bartholdi and Eisenstein (1996a) and Bartholdi, Bunimovic and Eisenstein (1999), where the work is the same order by order, distributed uniformly and continuously along the space. It is possible to reduce the variance of the work by increasing the bucket size (aggregating groups of orders for batch picking).

A last advice to reduce the number of blockages is to reduce the number of pickers, so the velocities of workers will tend to be more different between them.

3.5 Conclusions

Also in order-picking, bucket brigades work well. According to Bartholdi and Eisenstein (1996b), picking bucket brigades are more productive than zone picking for a lot of reasons:

- The production rate of bucket brigade is higher (+30%) than production rate of zone picking (figure 3.5).

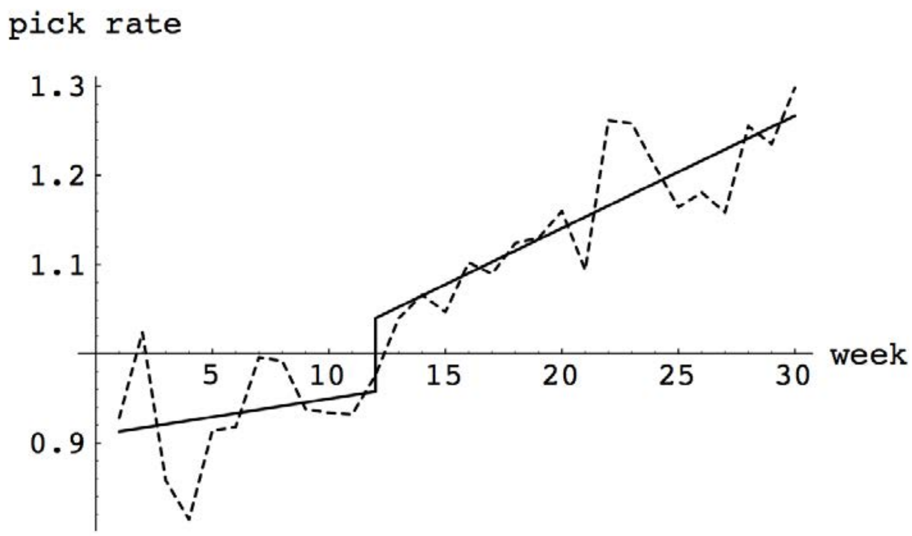


Fig 3.5 - Difference in pick rate between zone picking and bucket brigade. The average pick rate is shown as a fraction of standard work. Zone picking was replaced by bucket brigade in week 12. The continuous line represents best fit to weekly average pick rates before and after the introduction of bucket brigade. It is important to notice how bucket brigade performs better than zone picking. (from Bartholdi and Eisenstein, 1996b)

- Bucket brigades constantly and spontaneously seek balance.
- Balance is based on actual, realized work content and not on previsions made by a manager (no need for a management intervention).

- The bucket brigades are around 85% faster than order-picking and WIP in bucket brigade is around 50% less than in zone picking (figure 3.6).

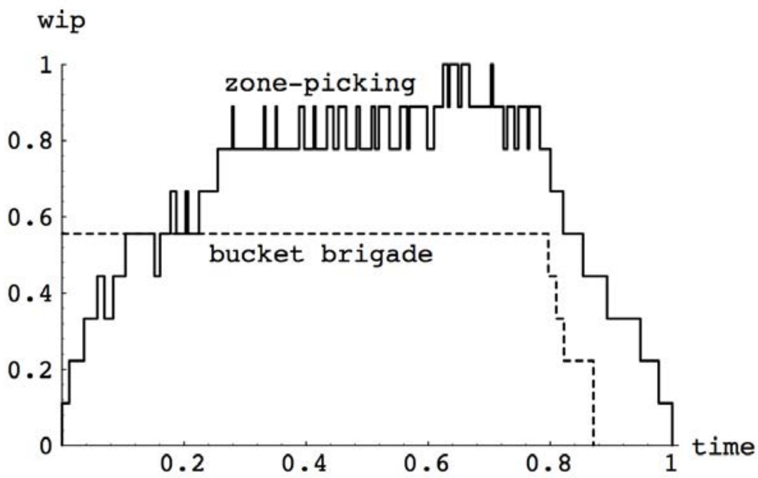


Fig 3.6 - Difference in WIP and time between zone picking and bucket brigade. The diagram is the result of a simulation by Bartholdi and Eisenstein (1996b). It shows that bucket brigade has in average less work in process and it is faster than zone piking. (from Bartholdi and Eisenstein, 1996b)

- Reducing the level of WIP, the mistakes are less, because there are less totes on the conveyor.
- The synchronization of multiple aisles becomes easier.
- The pickers are more satisfied because they prefer working in teams.

Chapter 4

Human factors in order-picking: fatigue models and ergonomics

The goal of this chapter is to give to the reader a basic knowledge about what is fatigue and how fatigue deals with order-picking systems, more in particular with order-picking bucket brigade. The concept of fatigue is strictly linked with ergonomics, which can be used to improve the efficiency of a system, reducing fatigue. At the end of the chapter, a mathematical model to describe how the fatigue level grows over time in an order-picking system is described.

4.1 What is fatigue?

To improve the performance of picking systems, it is important to consider more carefully human factors. One of the human factors that worths more is accumulation of fatigue during the work shift. Fatigue can have a bad long term impact on workers' physical conditions, reducing the productivity during the work shift and making absenteeism more frequent. According to Grosse, Glock and Neumann (2016), taking into account that order-picking is usually carried out by operators rather than robots, human factor can have a great impact on the performance of the overall system.

The first thing to understand is what fatigue is. According to Gawron, French and Funke (2001), *fatigue* is defined as "every loss caused by an effort, and it is classified in psychological and physiological fatigue". Psychological fatigue is the mental fatigue of an operator who is performing a task for a long time and it is very subjective. Physiological fatigue is the fatigue that comes out when a worker is doing a physical effort for a long time with a predetermined force; this kind of fatigue lead to a reduction in generating force or to an increased reaction time (Battini, Calzavara, Persona, Sgarbossa, Visentin, 2017).

Fatigue leads to tiredness and lack of energy, physical exertion, physical discomfort, lack of motivation and also sleepiness. All this factors contribute to create performance problems: the workers are slower, less productive and the possibility to have quality problems is higher. When fatigue become excessive or chronic, it reduces the quality of life of a worker and this can increase the percentage of absenteeism. When an operator performs a task, muscle force capacity is reduced over time, up to a threshold value (*maximum endurance time*) due to the muscle fatigue. This means that an operator can work until his muscle force capacity reaches its limit. The fatigue can be alleviated by a break or by the time that the worker spends to move from a place to another. In our work we will consider only physiological fatigue, because it is the easiest one to model.

Unfortunately, there is no study available that demonstrates the mathematical relation between worker fatigue and production outputs such as production time and volume, even if there are a lot of paper in which the authors tried to describe the function that describes the fatigue of workers during the time. Moreover, there are only a few papers in which the correlations between fatigue and order-picking is described. One of them, is a paper written by Grosse and Glock (2013), where they observed the presence of workers learning and fatigue in order-picking system through an experimental study. We will describe a new mathematical model to describe fatigue over time in order-picking bucket brigade in paragraph 4.4.

4.2 Human factors in order-picking bucket brigade

The next question to answer is why and how the activity of picking in a warehouse deals with fatigue. It is important to consider the effect of fatigue in our analysis to have results closer to reality, because neglecting the effects of fatigue can lead to a complete different set of results, which are often wrong.

Fatigue in order-picking exists because picking activities are typically repetitive and physically demanding. Different are the actions that a worker have to perform when he is working in an order-picking system and, more in particular, in a bucket brigade order-picking system. Down here, we will describe the most common physical[3] factors that can increase the level of fatigue of a worker during his work (see Grosse, Glock, Jaber and Neumann, 2015):

[3] In our work we will consider only physical aspects, because they are the easiest to model. In reality, also perceptual (ex. read pick list) and mental aspects (ex. understand and remember pick route) should be considered, but it is very difficult to build a model considering them as well.

- Set up: set up workstation.
- Travel: travel between depot and pick locations, carry items and push/pull trolleys.
- Search: neck flexion/extension.
- Pick: stretch, bend, reach for items and extract, grab, pick, put down items.

Excessive manual handling may manifest in muscle fatigue and discomfort, which may decrease performance and quality. Unfortunately, not a lot of papers have been written on this theme. The only authors who dealt with this theme were Landers, Beavers, Sadiq and Stuart (1994) and Gong and De Koster (2011), who noted a relationship between physical tasks in OP and performance. This authors mentioned, for example, that pick time can depend on the size, weight and number of items per pick and ergonomic issues such as accumulated fatigue, but did not elaborate how. As we already said, all this factors can decrease the performance of the system and can also make wrong the results of a simulation; therefore, considering human factors in order-picking is of paramount importance.

In addition to the small amount of paper about human factors and order-picking systems, there are no papers which links human factors and fatigue with bucket brigade in order-picking systems and that is what we are going to develop at the end of this chapter (4.4) and in chapter 5.

Now, it is important to understand if there are different human factors between a bucket brigade and a simple order-picking system and, if yes, what they are. It is easy to notice how an order-picking bucket brigade system requires less demanding work then a normal picking system, both physically and mentally. First of all, the fact that the workers work always on the same fraction of aisle makes the picker more comfortable with his work and this leads to less errors and higher quality. With bucket brigade, then, the WIP is reduced and this allows the operators to deals with less totes along the aisle. Speaking about the physical part, on the other hand, the conveyor helps the workers to handle easier and effortless the totes. Moreover, workers have to walk shorter distances than in a normal picking system, because each worker is limited to his “zone”; in addition to this, workers always travel along the aisle without carrying totes, because of the conveyor: the travel is no more an effort, but it is time to recover energy, in particular during the backward travel. In conclusion, for all this reasons, bucket brigade order-picking systems are less demanding than normal picking system.

Once we explained what are the reasons which make a worker tired, it is important to reduce at minimum his fatigue over the work shift, so that he can pick faster, without mistakes. It is possible to reduce the level of effort of the pickers, working on the ergonomics of the system. Some examples to make a bucket brigade order-picking system[4] works better, using ergonomics, are:

- Presence of two conveyors: it is possible to add a second conveyor which brings directly the finished orders into the shipping area. This second conveyor gives the operators the possibility to travel always without lifting and bringing totes. This reduces the number of back injuries and make the flow faster. (see paragraph 5.3 for more)
- Pick-to-light system: a central computer switches on a light to point out the item which has to be picked by the operator. Thanks to this light the worker knows exactly where, which and how many items he has to pick. This reduces the mental effort to search and identify items and to remember item locations; moreover it reduces the physical effort due to neck movements. (see paragraph 3.3 for more)

Ergonomics is the main strategy to reduce fatigue in a system, improving its general performance. For this reason, paragraph 4.3 is dedicated entirely on ergonomics.

4.3 Ergonomics

When an engineer is designing a work station or a warehouse, ergonomics is always of paramount importance. Both when assembling and picking, the worker has to be comfortable. According to Sgarbossa (2017), ergonomics extremely influences both production and picking rate. Working ergonomically and improving ergonomics is a win-win approach, because not only the operators produce better, but also they are less tired at the end of their work shift: this leads to a higher production rate, to higher quality and to less absenteeism. Hence,

[4] see paragraph 5.3 for ideas that can be used to improve order-picking in general

when an engineer designs a work station or a warehouse, he has to consider ergonomics as well.

The basic principles of ergonomics are:

- Worker's height: it is important to let the operator work on or pick products which are in the optimal area; to do this, it is possible to assembly the products on variable height desk, or use shelves of average height in picking.
- Picking range control: the operator should reach everything he needs with his hands, without making strange or unnatural movements.
- Optimization of pieces/products position and material flow: it is very important to refill the warehouse, stocking the products in the best location and to have a continuous flow within the warehouse or along the assembly line. A good strategy could be to add an operator who has to think only about this operations.
- Avoid picking and assembling above the height of the heart: it is very tiring to pick or work on something keeping the hands up. The working desk should be at the right heigh and the picking shelves should be lower than the heigh of the heart.
- Visual field: the worker should see the parts where he is working; the picker has to see all the products he needs to pick, to be faster and to lose less time.
- Light intensity: the operator has to see as well as possible the object he needs. In picking, as we have already said, the pick-to-light system, which uses a light to tell the operator where is the product he has to pick, is often used.
- Adjustment of working equipment: everything should be adjusted to make the operator as comfortable as possible.

In Italy, law 81-2008 chapter 6 speaks about manual handling of loads. Annex 33 tells how employers should behave in relation to ergonomics.

In addition, ISO speaks about:

- Lifting and transport.
- Pushing and towing.
- Handling of high frequency low weights.

To know if ergonomics has been well designed by engineering, there are a lot of methods to measure the effort of an operator; the effort is directly linked to the ergonomics of the system. The main methods are:

- Self assessment: ask the operator which is his effort (Borg scales).
- Observational methods: stare at the operator and think about how is his effort (OCRA, …).
- Simulation: study with a software on the PC the system.
- Direct methods: EMG, O2 consumption, PMES.
- Real time systems to evaluate the level of ergonomics: use of motion capture probe which are connected directly with the operator.

4.4 A mathematical model to describe fatigue

Now that it is clearer how fatigue is linked to bucket brigade order-picking systems and how it is possible to reduce fatigue with ergonomics, we will introduce a mathematical model used to describe fatigue in order-picking systems, that, in chapter 5, we will adapt to bucket brigade order-picking systems.

The most appropriate fatigue model we found is the one presented by Jaber, Givi and Neumann, in their paper “Incorporating human fatigue and recovery into the learning-forgetting process” (2013). In their paper, they gave a mathematical model for muscular fatigue, which is defined as the inability of the body muscles to sustain a specific posture or force level required to perform a task (Ma, Chablat, Bennis and Zhang, 2009). Jaber, Givi and Neumann (2013) developed a model to quantify the relation between worker’s fatigue and production outputs.

As it has been told before, not a lot of literature has been written about the mathematics of fatigue. Most of the paper we know assert that fatigue accumulates exponentially with time, but it has not been proved nether empirically nor experimentally (Lindstrom, Kadefors and Petersen, 1977; Bechtold and Sumners, 1988; Konz, 1998). For our work we will assume that fatigue accumulates exponentially as well.

In particular Jaber, Givi and Neumann (2013) used the formula[5]:

$F(t) = 1 - e^{-\lambda t}$

where F(t) is the fatigue which the operator accumulates by the time t and λ is the *fatigue accumulation index* (figure 4.1).

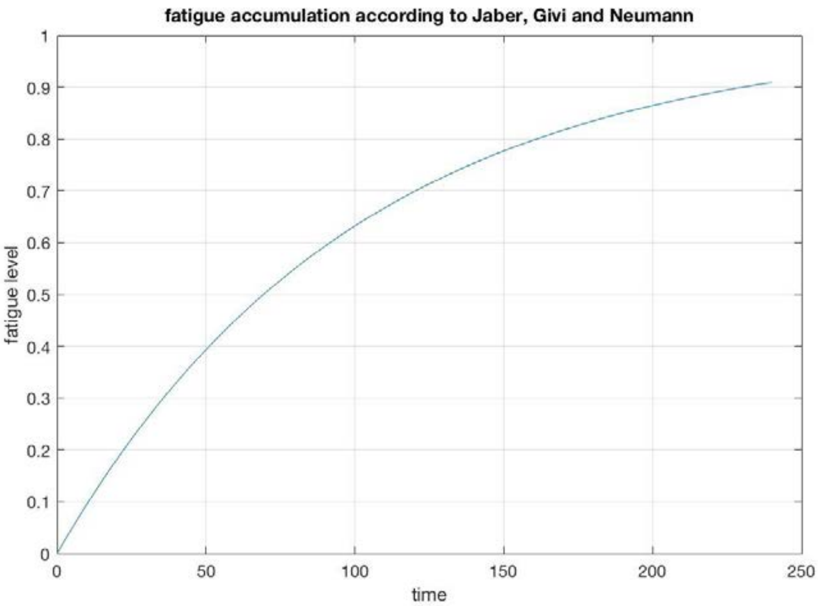


Fig 4.1 - Fatigue over time. The picture represents the exponential function that links the time with the fatigue level, according to the formula $F(t) = 1 - e^{-\lambda t}$, given by Jaber, Givi and Neumann (2013).

It is possible to calculate the first derivative F'(t) of the function:

$F'(t) = \lambda e^{-\lambda t}$

that is always positive, because λ is always positive end $e^{-\lambda t}$ is always positive. This means that the slope of the function is always positive, according to the fact that fatigue is always increasing over time, if rest are not considered[6].

[5] The formula of Jaber, Givi and Neumann is not the only one that can be used to describe the growth of the fatigue level over time. Other authors suggest that a worker reaches his maximum speed after 1-2 hours, because he needs a few time to "warm up".

[6] Rest is not considered in our work, so that we will consider a 8 hours work shift. Rest is not considered, because we are considering the backward travel (nothing to carry) as rest.

It is also possible to calculate the second derivative F''(t) of the function:

$F''(t) = -\lambda^2 e^{-\lambda t}$

that is always negative, because $-\lambda^2$ is always negative and $e^{-\lambda t}$ is always positive. This means that a worker gets tired faster at the beginning of his activity and the more the times goes on, the more his fatigue level grow slower. This is in accordance with the fact that an operator can work faster when he is less tired and therefore, working faster, he gets tired faster.

Another object which is very important is the fatigue accumulation index λ, that tells how fast an operator gets tired. If λ is low, it means slow fatigue accumulation, if λ is high, it means fast fatigue accumulation. Hence, the value of λ depends on the heaviness of the work. Jaber, Givi and Neumann (2013) set λ to three levels: slow, moderate and fast, corresponding respectively to λ = 0,01, λ = 0,03 and λ = 0,05. Setting this three different values it is possible to obtain the functions in figure 4.2, where it is shown how the functions change when the value of λ is varying.

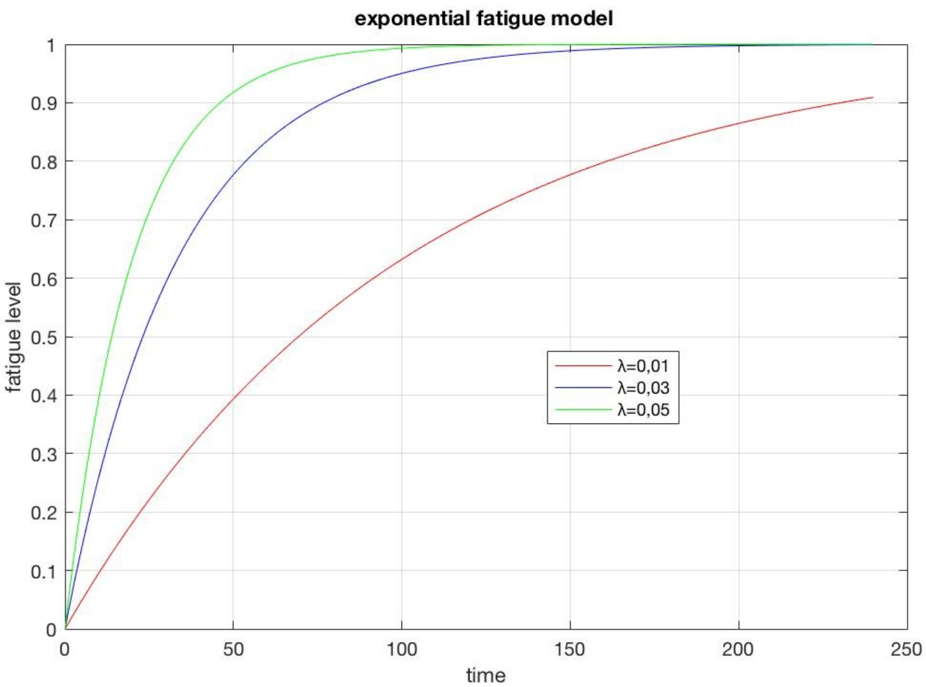


Fig 4.2 - Fatigue level with different values of λ. In this picture it is shown how the functions change their positions, varying the value of the fatigue accumulation index λ. The higher is the value of λ, the more the function goes faster to the maximum level of fatigue (1). In the picture, three levels of fatigue are considered: easy work (λ = 0,01), average work (λ = 0,03) and hard work (λ = 0,05).

It is interesting to see how is the level of fatigue of a worker after 8 hours (the duration of a work shift) in the three cases described before. This value will be obtained starting from the formula given by Jaber, Givi and Neumann (2013) in their paper. We will use the values that we will obtain to set the value of μ in the next chapter (chapter 5).

To find the values of F(t) for t = 8 hours is enough to substitute the value t = 8 in the three different formulas for each λ. The results are:

- λ=0,01 (easy work) —> $F(8) = 1 - e^{-0,01*8} = 0,07688$ —> 7,688%
- λ=0,03 (average work) —> $F(8) = 1 - e^{-0,03*8} = 0,21337$ —> 21,337%
- λ=0,05 (hard work) —> $F(8) = 1 - e^{-0,05*8} = 0,32968$ —> 32,968%

The results are confirmed by figure 4.3.

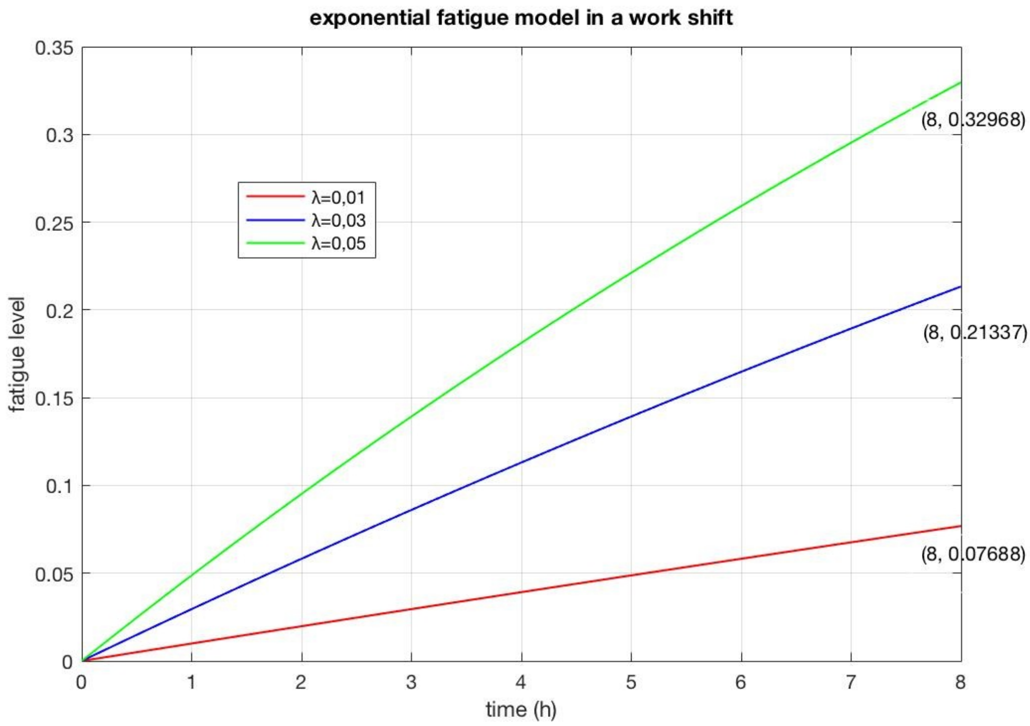


Fig 4.3 - Exponential fatigue model in a work shift. The figure shows how the fatigue level of one operator grows with different type of works: easy work (λ = 0,1), average work (λ = 0,3) and hard work (λ = 0,5). The higher is the value of λ, the more demanding is the work. After an eight hours work shift, the level of fatigue is 0,07688 for easy work, 0,21337 for average work and 0,32968 for hard work.

In a practical point of view and approximating, the level of fatigue after 8 hours is 10% for an easy work, 20% for an average work and 30% for a hard work[7]. We will use this values in chapter 5 to build our model of fatigue in an order-picking bucket brigade system.

4.5 Easy, average or hard work?

The next step is to find a way to decide if a work is easy, average or hard to decide the value of λ which is more appropriate[8]. To do this, we need to consider all the factors which could influence the fatigue level of a picker during his work shift. Down here, the most important are listed:

- Dimension of an item: items of middle dimension are easy to pick (low value of λ), but very big items or very small items require more effort to be picked (high value of λ).
- Weight of an item: the heavier is an item and the more effort it requires to be picked. Heavy items have a high value of λ, while light items have a low value of λ.
- Shape of an item: an item with regular shape (ex. parallelepiped) is easier to pick than an object with a strange shape, because it can be difficult to grab it or to handle it. In general, on one hand, the more an item has a regular shape, the more the value of λ is lower; on the other hand, the more an item has an irregular shape and the more the value of λ is higher.
- Number of items per pick: frequent and fast movements make workers tired faster. Because of this, if the number of items per pick is high, λ is high, if the number of items per pick is low, the value of λ is low.
- Ergonomics of the warehouse and, more in particular of the flow rack: as we discussed in paragraph 4.3, ergonomics is one of the most important factor to determine if a picking system works well or not. The more ergonomically the flow rack is designed (ex. it allows the workers to never pick items above the height of the heart), the slower the workers get tired and, so, the value of λ is

[7] In reality, there are not only three different kind of work effort (easy, average and hard work), but the effort of a work can vary from 0 effort (the fatigue level does not grow after 8 hours) to hard work (the level of fatigue grows to 30% after 8 hours) continuously. Therefore, λ can assume each value between $\lambda = 0$ and $\lambda = 0,5$.

[8] The same considerations are valid to find the most appropriate value of μ in chapter 7.

low. If the flow rack is designed without thinking about ergonomics, the value of λ is high, because the operators have to spend more energies on picking.

- Young / old worker: a young worker is considered to have more energy and resistance than an old worker. In general, young workers have a lower value of λ than old workers.
- Male / female worker: in general, men are stronger than women, so men get tired slower. For this reason it is usually true that male workers have a lower λ than female workers.
- Trained / not trained worker: a worker who is trained physically and mentally better than another gets tired slower. Trained workers have a low value of λ, while not trained workers have a high value of λ.
- Physical and mental tiredness at the beginning of the work shift: if a picker is tired at the beginning of the work shift, for example because he has slept bad the previous night or he did a hard physical activity the day before, he will have a higher value of λ than a picker who is completely rested at the beginning of the work shift.

There also are some papers that confirm the factors in the bulleted list. Two examples are the papers of Landers, Beavers, Sadiq and Stuart (1994) and Gong and De Koster (2011) who mentioned that pick time can depend on the size, weight and number of items per pick and ergonomic issues such as accumulated fatigue, but they did not elaborate how.

In conclusion, to choose the correct value of λ, we need to consider all this factors, balance them and, only after a careful analysis, we can choose the right value of λ.

Chapter 5

Bucket brigade and fatigue

The aim of this chapter is to link the previous papers about bucket brigade (see chapter 2 and 3) with fatigue models (see chapter 4). A new model to describe the slowdown of the pick rate over time is given. Using this model, it is possible to describe mathematically the dynamics of a two workers bucket brigade system in order-picking, when the workers slow down over time in a work shift. Thank to the mathematics, then, we will write some codes on MATLAB to simulate the behavior of a bucket brigade in a work shift (8 hours), considering different maximum workers' speeds and different levels of working effort. Interesting results will follow.

5.1 Taking into account fatigue in bucket brigade order-picking systems

It is interesting to see what happens to bucket brigades, if a worker changes his speed during the work shift. Unfortunately, this field has been not deeply explored in literature. The only literature we know about this theme is the paper "Bucket brigades when worker speeds do not dominate each other uniformly", written by Armbruster and Gel (2002), which speaks about a two operators bucket brigade line where the operators' velocities change with different constant value along the line; in particular they studied the case in which the first operator has a constant speed along the line and the second one is slower in the first part of the line and faster in the second. Armbruster and Gel (2002) studied the behavior of this system first in the case of passing and then in the case of blockage (see 2.4); they did not speak about bucket brigade in order-picking, but they spoke about bucket brigade in assembly lines. At the end of their paper, they gave some hints to extend their work; one of them is to see what it would happen, if workers changed their speed dynamically over time.

Our idea is to take into account the effect of fatigue in a bucket brigade order-picking system: this means that the pickers decrease their speed during the work shift. To study this theme, we will use the equation about fatigue described in chapter 4 and we will adapt it to the field of bucket brigade in order-picking systems.

5.2 Fatigue model to describe the slowdown of workers' pick rate

5.2.1 Mathematical formulation

The first step is to find an equation that describes how the speed of the workers changes over time. According to Jaber, Givi and Neumann (2013), the curve that describes muscular fatigue is

$F(t) = 1 - e^{-\lambda t}$

where F(t) is the fatigue level, λ is fatigue accumulation index and the t is the time. As shown in figure 5.1, the fatigue of a worker increases exponentially during the work shift. The maximum level of fatigue is 1: when F(t) = 0 (t = 0) the level of fatigue is the minimum and the worker can work at his maximum speed. When F(t) = 1 (t = ∞) the level of fatigue is maximum and the operator cannot work anymore.

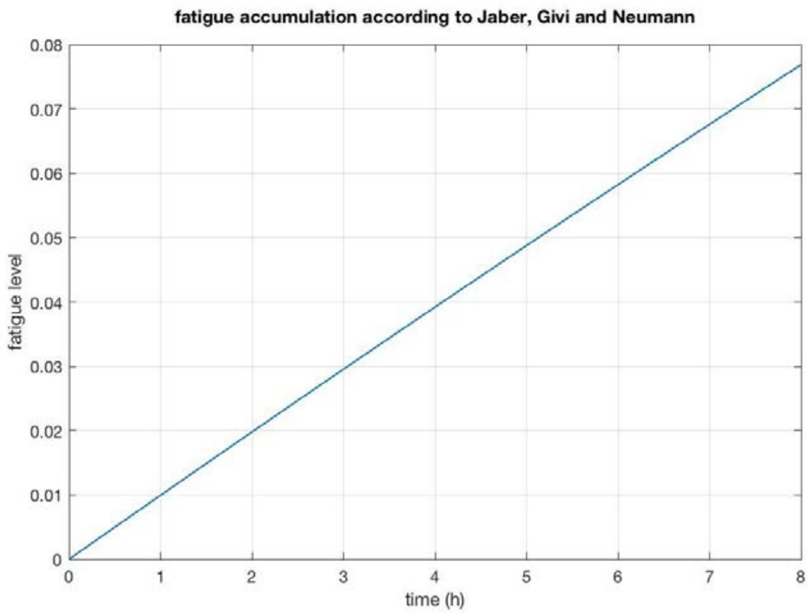


Fig 5.1 - Exponential fatigue accumulation in a work shift. According to Jaber, Givi and Neumann (2013), at the beginning of the work shift the fatigue level is 0 and is grows following the function $F(t) = 1 - e^{-\lambda t}$ during the work shift. The higher the fatigue level is and the more the slower workers produce.

The next step is to translate the function $F(t) = 1 - e^{-\lambda t}$ described by Jaber, Givi and Neumann (2013), in a new function, that describes the pickers' speed during their work shift in a bucket brigade order-picking system. The function that seems more appropriate to describe the evolution of the speed of a picker over time on a bucket brigade order-picking system is:

$v(t) = a * e^{-\mu t}$

where $v(t)$ is the speed of the operator[9] over time, a is a constant that we will define later, μ is the *fatigue constant* and t is the time in seconds[10] from the beginning of the work shift.

And we want that at time $t = 0$ the operator works at his maximum speed v_{max}, because at $t = 0$ he is not tired at all, so:

$v(0) = v_{max} = a * e^{-\mu * 0} = a * e^0 = a$

then:

$v_{max} = a$

The basic equation[11] that describes the changing of a worker's speed in a bucket brigade is, then:

$v(t) = v_{max} * e^{-\mu t}$

It is important to notice that for $t = \infty$ the speed of the operator is $v(\infty) = 0$, according to the fact that after a time ∞ a worker is so tired that he cannot pick anymore, so his speed is 0.

Also for this function, it is possible to calculate the first and the second derivative, to obtain some important properties.

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[9] The speed of the operator is measured in lines/s or aisles/s and, for the fact that we normalized the length of the aisle (line) to 1, the most appropriate unit of measurement for $v(t)$ is aisles/s or 1/s. For example, if the speed of a worker is 0,5 m/s and the aisle (line) is 100 m long, then the worker's speed is 0,5 / 100 = 0,005 aisles/s = 0,005 1/s.

[10] We will use seconds because we want that in the equation $v(t) = a * e^{-\mu t}$ the time has the same unit of measurement of time between hand-offs in a bucket brigade.

[11] To write this equation we took a cue from $F(t) = 1 - e^{-\lambda t}$, used by Jaber, Givi and Neumann (2013). The equation $v(t) = v_{max} * e^{-\mu t}$ is not the only one that can be used to describe the slowdown of the workers over a work shift. Other authors, for example, suggest that a worker reaches his maximum speed after 1-2 hours, because he needs some time to "warm up".

The first derivative F'(t) of the function is:

$v'(t) = - v_{max} * \mu * e^{-\mu t}$

that is always negative, because v_{max} and μ are always positive, $e^{-\mu t}$ is always positive and there is a minus. This means that the slope of the function is always negative, according to the fact that pickers pick items slower over time, if rests are not considered[12].

It is also possible to calculate the second derivative v''(t) of the function:

$v''(t) = v_{max} * \mu^2 * e^{-\mu t}$

which is always positive, because μ^2 is always positive and $e^{-\mu t}$ is always positive. This means that a worker gets tired faster at the beginning of his activity and the more the times goes on, the more his fatigue level grows slower. This is in accordance with the fact that an operator can work faster when he is less tired and therefore, working faster, he gets tired faster.

The behavior of the function $v(t) = v_{max} * e^{-\mu t}$ is confirmed by the plot in figure 5.2. According to practice, the worker reduces his speed over time, because of fatigue.

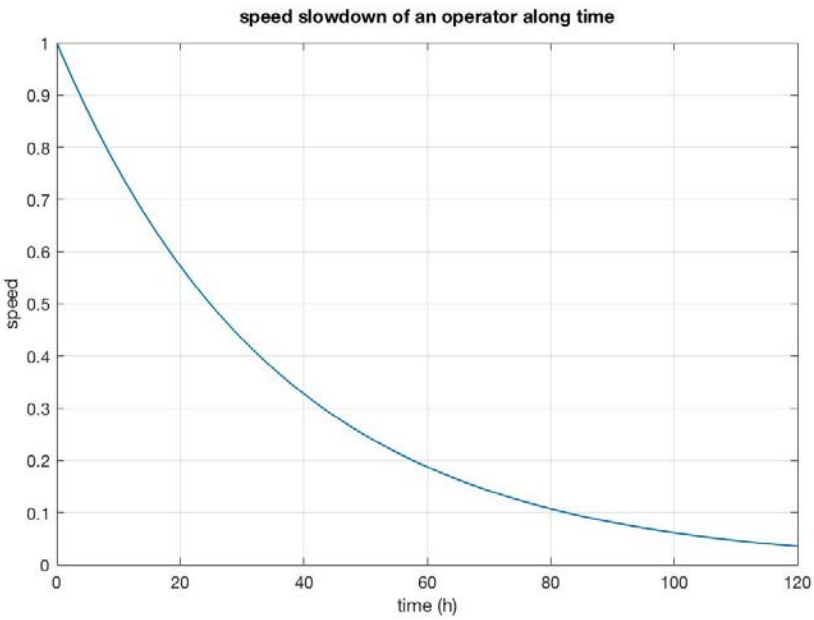

Fig 5.2 - Fatigue level in function of time. The picture shows the slowdown of a worker over time, according to the function $v(t) = v_{max} * e^{-\mu t}$ that we took as hypothesis. In this case, 1 is the maximum speed of the worker.

[12] As we already told we will not consider rest.

To use the function $v(t) = v_{max} * e^{-\mu t}$ we need to set the value of v_{max} and of μ, which are constant.

5.2.2 Setting the value of v_{max}

The value of v_{max} is the maximum picking speed of a worker. In the model that we described and that we will use, $v_{max} = v(0)$, which means that the maximum speed of a worker is reached only at the beginning of the work shift[13].

Before setting the value of v_{max}, we have to understand which are the reasons that makes a worker pick faster than another. The main reasons are:

- Experience of a worker: if a worker has more years of experience than another one, the former is, in general, faster than the latter. The worker with more experience has a higher value of v_{max} than the worker with less experience.
- Young / old worker: a young worker is considered to have more energy and resistance than an old worker. In general, young workers have a higher value of v_{max} than old workers.
- Male / female worker: in general, men are stronger than women, so men can pick faster. For this reason it is usually true that male workers have a higher v_{max} than female workers.
- Trained / not trained worker: a worker who is trained physically and mentally better than another can pick faster. Trained workers have a high value of v_{max}, while not trained workers have a low value of v_{max}.
- Physical and mental tiredness at the beginning of the work shift: if a picker is tired at the beginning of the work shift, for example because he has slept bad the previous night or he did a hard physical activity the day before, he will have a lower maximum speed v_{max}, than a picker who is completely rested at the beginning of the work shift.
- Ergonomics: sometimes is very difficult to design a warehouse taking into account all the principles that we mentioned in paragraph 4.3. For this reasons

[13] On the contrary, in the literature, it is possible to find also other models that support the fact that the maximum speed is reached only after 1-2 hours from the beginning of the work shift, because workers need some time to “warm up”.

the ergonomics of the warehouse could be good for some operators, but bad for someone else. For example, if there are two operators, one of average height and one very short, and the flow rack is of average height, the ergonomics of the warehouse facilitates the average height operator: in this case, in general, the average height operator has a higher value of v_{max}, because he can pick items more easily.

- Motivation: the more a picker is motivated and the higher his v_{max} will be.

In conclusion, to choose the correct value of v_{max}, exactly like in the case of λ (see paragraph 4.5), we need to consider all these factors, balance them and, only after a careful analysis, we can choose the right value of v_{max}.

To find the precise numerical value of v_{max} for each picker, it is enough to ask each picker to pick the same items written on a list[14], starting from the beginning to the end of the aisle, and time them with a stopwatch. Then, we have to convert the time t in seconds[15] and, knowing the length of the aisle l, we can obtain their maximum normalized (length of the aisle l = 1) speed, calculating $v_{max} = l / t$. The unit of measurement of v_{max} is aisles/s or 1/s.

In general, the values of v_{max} are very small, because the length of the aisle is normalized to l = 1: the unit of measurement of v_{max} is aisles/s and not m/s. Just to give an idea, let's take into account a 100 m long aisle in a warehouse and let's say that a picker travels along all the aisle, picking the requested items, in 4 min 30 s (= 270 s). Adding the 30%[15], the time of the picker is 351 s; now, we can calculate v_{max}:

$v_{max} = 100 / 351 = 0{,}285$ m/s $= 0{,}00285$ aisles/s

In our analysis we will use the value in l/s, because we will develop all the mathematics of bucket brigade starting from the hypothesis of normalized length of the aisle (l=1). In conclusion, the values of v_{max} are slightly small.

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[14] The order in the picking list should be an average order. By average order, we mean an order that requires an average time to be picked. In this way we are calculating the average v_{max}; it is impossible to calculate the real v_{max}, because the time t that a worker needs to pick an order depends on the number and kind of items requested by the customer: for this reason, also v_{max} depends on the number and kind of items requested by the customer in his order.

[15] In general is better to increase the time t of +20% or +30%, because the workers pick slower during a normal work shift than during a single pick that they have to perform to show their maximum speed.

In figure 5.3, the behavior of two functions with two different values of v_{max} is shown.

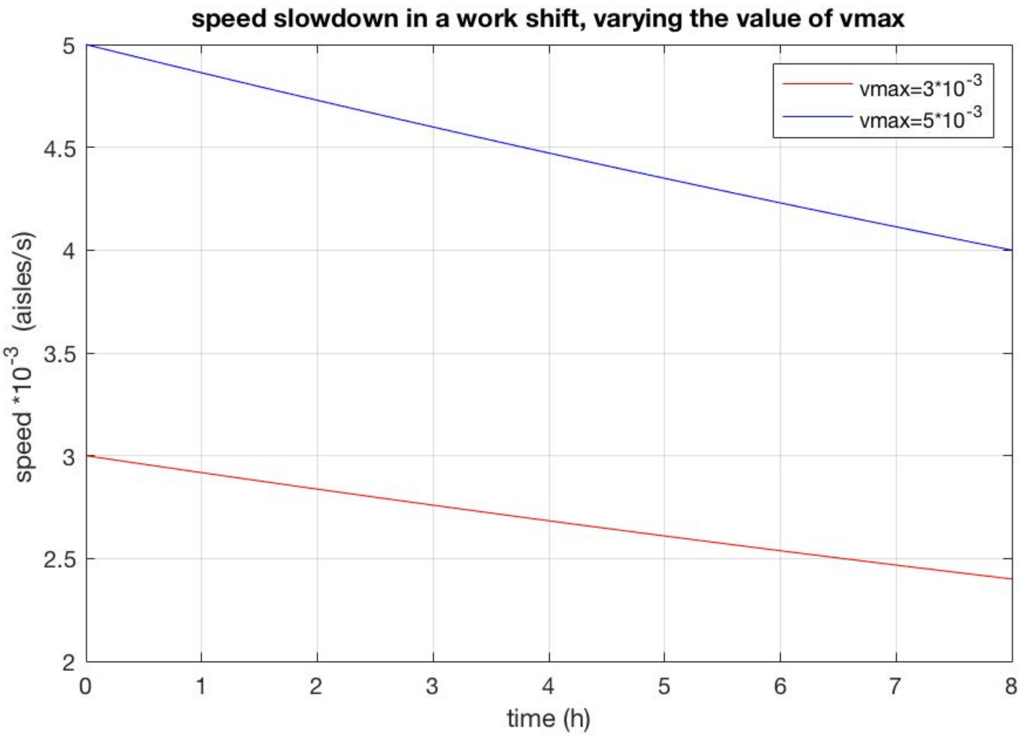

Fig 5.3 - Speed slowdown in a work shift, varying the value of v_{max}, with a constant value of μ=7,7480*10^{-6} (average work, see paragraph 7.2.3). The figure shows how the speed of one operator decreases during the work shift, according to the formula $v(t) = v_{max} * e^{-\mu t}$. The starting maximum speed of the two pickers are v_1 = 3*10^{-3} aisles/s and v_2 =5*10^{-3} aisles/s. It is important to notice that, even if the value of μ is the same for the two workers, the picker with the lower v_{max} slows down less than the picker with a higher v_{max}, because the same value of μ indicates that both operators have to lose the same percentage (20% in this case) of speed at the end of the work shift. This is consistent with the fact that, if an operator picks faster, he will get tired sooner.

All this calculation is possible only under the hypothesis of high sku density in a high-volume distribution warehouse of a chain retailer, so that we can consider valid the hypothesis written in paragraph 5.3.

It could be interesting to generalize the solutions of the problem considering the work not uniformly distributed along the aisle. An idea could be to consider the work exponentially distributed. In our work we will not deepen this theme.

5.2.3 Setting the value of μ

After we have defined the function $v(t) = v_{max} * e^{-\mu t}$ and that we have explained how to set the value of v_{max}, the next step is to find the values of μ, which indicates easy work, average work and hard work. We will consider three different level of effort in work:

- Easy work: it is the work that leads to a 10% speed slowdown after 8 hours (a work shift).
- Average work: it is the work that leads to a 20% speed slowdown after 8 hours (a work shift).
- Hard work: it is the work that leads to a 30% speed slowdown after 8 hours (a work shift).

Now, it is possible to calculate the value of μ for all the three levels of effort[16]. To do it, it is enough to invert the formula $v(t) = v_{max} * e^{-\mu t}$, finding the value of μ after 8 hours (8*60*60 = 28800 seconds) in the three different cases[17]:

- Easy work —> $v(8*60*60) = 90\% \ v_{max}$ —> $90 = 100*e^{-\mu*8*60*60}$ —> $\mu = 3{,}6584*10^{-6}$
- Average work —> $v(8*60*60) = 80\% \ v_{max}$ —> $80 = 100*e^{-\mu*8*60*60}$ —> $\mu = 7{,}7480*10^{-6}$
- Hard work —> $v(8*60*60) = 70\% \ v_{max}$ —> $70 = 100*e^{-\mu*8*60*60}$ —> $\mu = 1{,}2385*10^{-5} = 12{,}3850*10^{-6}$

[16] In reality, there are not only three different kind of work effort (easy, average and hard work), but the effort of a work can vary from 0 effort (the speed of the workers remains constant for 8 hours) to hard work (the speed of the workers decreases of 30% after 8 hours) continuously. Therefore, μ can assume each value between $\mu = 0$ and $\mu = 1{,}2385*10^{-5}$.

[17] The values of μ are calculated for a t in seconds, so it is possible to use this value of μ only if the calculations are in seconds. We made this choice because all the times in the mathematics of bucket brigade are expressed in seconds.
If the reader wants to obtain the values of μ in hours, he has to calculate v(8) in the three cases and not v(8*60*60) as we did.

The results we have just obtained with the calculation are represented in figure 5.4.

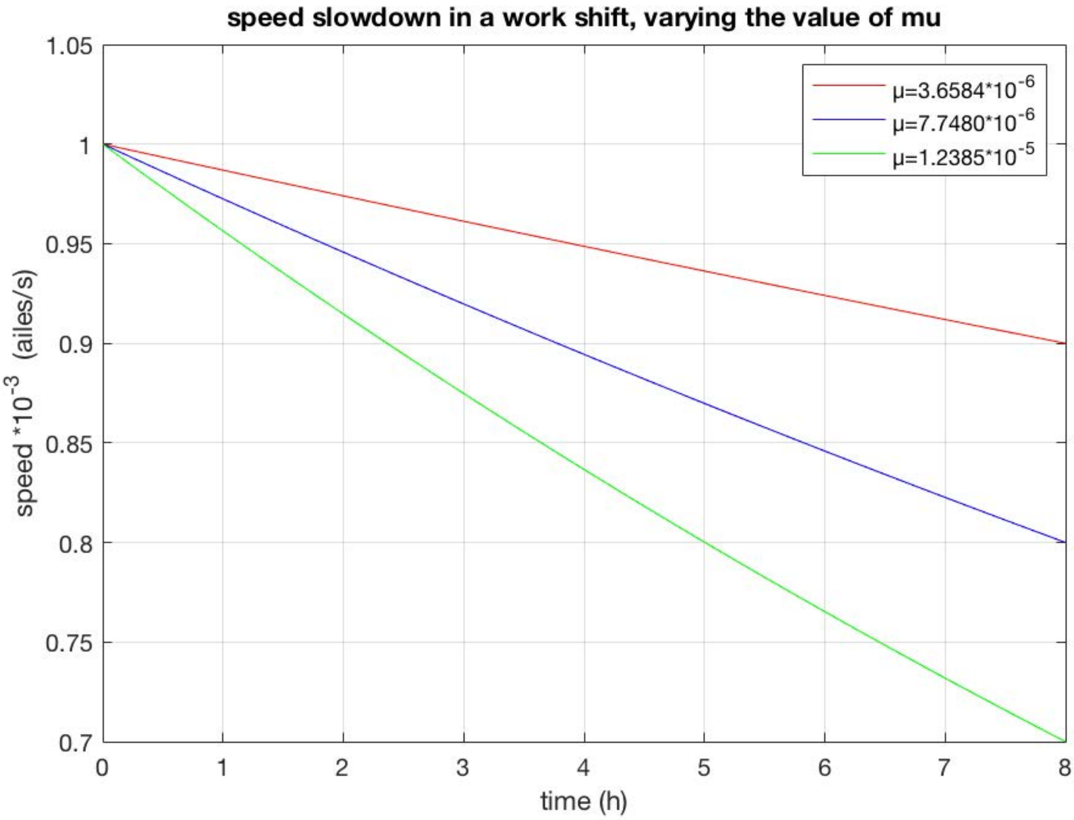

Fig 5.4 - Speed slowdown in a work shift, varying the value of μ, with a constant value of $v_{max} = 1*10^{-3}$. The figure shows how the speed of one operator decreases during the work shift, according to the formula $v(t) = v_{max} * e^{-\mu t}$. The speed of slowdown depends on the type of work: easy work ($\mu = 3{,}6584*10^{-6}$, in 8 hours the worker's speed decreases of 10%), average work ($\mu = 7{,}7480*10^{-6}$, in 8 hours the worker's speed decreases of 20%) and hard work (μ = 1,2385*10-5, in 8 hours the worker's speed decreases of 30%). The higher is the value of μ, the more demanding is the work.

To decide if a work is easy, average or hard, the procedure to follow is the same that the one we explained in paragraph 4.5: if the value of λ is low, also the value of μ will be low and if the value of λ is high, also the value of μ will be high.

It is important to notice that the shape of the exponential function that we are using in a work shift (8 hours) can be approximated with a straight line (see both figures 5.3 and 5.4. We will use this fact in paragraph 5.4 to simplify the mathematics of the system.

5.3 Working hypothesis

Now that we have explained how to use the formula $v(t) = v_{max} * e^{-\mu t}$ of our fatigue model and before talking about the mathematics of the system, it is appropriate to clarify the working hypothesis of a two operators bucket brigade in an order-picking system, where the pickers' speeds slow down, because of fatigue, following the function $v(t) = v_{max} * e^{-\mu t}$.

The first assumptions that come to mind are the assumption of the normative model, used by Bartholdi and Eisenstein (1996a) in their paper "A production line that balances itself", where they were studying bucket brigades in an assembly line (see paragraph 2.1 and 2.2). For reasons of clarity, we will write them again.

Assumptions:

- Total ordering of workers by velocities: each worker is characterized by a distinct, constant work velocity v_i.
- Insignificant walking time: the total time to assemble a product is significantly greater than the time to walk the length of the assembly line.
- Smoothness and predictability of work: the nominal work content of the product is a constant (which is normalized to 1); and the work content is spread continuously and uniformly along the assembly line.

Restrictions:

- The workers are ordered from slowest to fastest along the flow line.
- The workers are not allowed to pass one another. If a worker is blocked by another worker, he must wait until the other worker is finished.

To make these hypothesis fit our model, we need to change something. First of all, we are working in a warehouse and not in an assembly line: in fact, in our analysis, we are working on a high-volume distribution warehouse. The warehouse that we are taking into account is a warehouse of a chain retailer, where the space to stock material is severely limited and the skus have to be replenished frequently and in small, less-than-caseload amounts. A typical order consists in a lot of skus, but a small number each. Moreover, every order is different. The best layout to deal with these circumstances is, generally, flow rack, as suggested by Bartholdi and Eisenstein (1996b) in their paper “Bucket brigades: a self-balancing order-picking system for a warehouse” (see paragraph 3.1). Second, we are considering fatigue, which makes the pickers slow down during the work shift, so that the speeds of the pickers are not constant over time.

This reasoning suggests us to change the first and the third assumption: the first because the pickers’ speed decreases over time, so it is not constant and the third because in a warehouse the work content is not continuously spread along the aisle and all the customers’s orders are different. The second assumption and both restrictions can be considered still valid also in our case. This bring us towards the hypothesis that Bartholdi and Eisenstein (1996b) made in their paper “Bucket brigades: a self-balancing order-picking system for a warehouse”, where they considered a simple model of bucket brigade in picking, where the amount and location of works varies, but the speed is still constant. For the sake of clarity, we will write their assumption again (see paragraph 3.4):

- Total ordering of position: let’s describe every discrete picking position with a number j between 1 and m (j = 1, …, m); the position could correspond to the bays of flow rack. For any given order, every pick of position j must be completed before all the picks which are in a position after (more towards the end of the line) j.

- Iid orders, exponentially distributed work: orders are independent and identically distributed random vectors, the components of which are independent; the j^{th} component of each vector represents the standard work at location j and it follows an exponential distribution with common mean 1/μ.

- Total ordering of the workers by velocity: each picker i (i = 1, …, n) has his own velocity v_i, so the time that a picker needs to complete a pick is exponentially distributed with mean $1 / (\mu * v_i)$. This assumption does not work well for order-picking, because the pertinent skills are simply dexterity and motivation. According to Bartholdi and Eisenstein (1996a), it is easy to rank the workers velocities.

- Insignificant walking time: the time to pick an order is significantly grater then the time to walk along the whole aisle.

And in our case the third hypothesis cannot be used, because it doesn't take into account fatigue.

Starting from these assumptions, Bartholdi and Eisenstein (1996b) proved that the randomness of the orders does not qualitatively change the dynamics of a bucket brigade from those of the deterministic model analyzed by Bartholdi and Eisenstein (1996a) and Bartholdi, Bunimovic and Eisenstein (1999). The asymptotic behavior remains the same, but the variance of work among orders makes the system less predictable. Moreover, always working on the hypothesis of "exponential distributed work"[18], Bartholdi, Eisenstein and Foley (2001) proved that, as the number of stations increases, the moment-to-moment behavior of the stochastic line will increasingly resemble that of the normative model. Moreover, this resemblance will assert itself with great uniformity. Another reason to make the two models closer is that the variance of the work is very small in a high-volume distribution warehouse of a chain retailer. As the variance decreases, the behavior of the bucket brigade approaches the one of the deterministic model described by Bartholdi and Eisenstein (1996a) and Bartholdi, Bunimovic and Eisenstein (1999), where the work is the same order by order, distributed uniformly and continuously.

In other words, because of the fact that our model takes into account a high-volume distribution warehouse of a chain retailer, where a typical order consists in a lot of skus, but a small number each, we can consider, approximating, the work continuously and uniformly spread along the aisle, just like in the third assumption of normative model. We will consider, then, all the customers' order equal to further simplify the mathematics of the system.
The last problem is to fix the assumption of constant workers' speed.

After all this reasoning, it is possible to write the assumptions and the restrictions of our model[19].

[18] This assumption is well suited to the context of order-picking in warehouses, where the work content is stochastic, because of the variability of customer orders.

[19] Our starting hypothesis (both assumptions and restrictions) are valid both in the case of bucket brigade in assembly line and in bucket brigade in order-picking. This means that everything we will say from now on will be valid not only in warehouses, but also in assembly lines.

Assumptions:

- Velocities decreasing exponentially[20]: each picker is characterized by a distinct picking velocity v_i., which decreases over time, following the function $v_i(t) = v_{max\ i} * e^{-\mu t}$.

- Insignificant walking time: the total time to pick the item written in the picking list and ordered by a customer is significantly greater than the time to walk backwards the length of the aisle.

- Smoothness and predictability of work and orders: the nominal work content along the aisle is a constant (which is normalized to 1); and the work content is spread continuously and uniformly along the aisle. All the customers' orders are equal.

Restrictions:

- The pickers are ordered from slowest to fastest along the aisle.

- The pickers are not allowed to pass one another. If a picker is blocked by another picker, he must wait until the other picker finishes to pick items from a location of the flow rack.

5.4 Mathematics of the system

5.4.1 Pickers' speed varying with $v(t) = v_{max} * e^{-\mu t}$ over time

Once we have clarified the working hypothesis, it is possible to build the mathematical model. The dynamics of the system is nearly the same of the one studied by Bartholdi and Eisenstein (1996a), but we have to take into account the fact that the speed of the pickers slows down with the function $v(t) = v_{max} * e^{-\mu t}$ where t is the time from the beginning of the work shift. In this case, we expect the time between two consecutive hand-offs to grow over time, because of the slowdown of the pickers' speed.

---

[20] Considering the picking speed decreasing over time, it is not always possible to decide who is the faster picker. For example, it could happen that $v_{max\ 1} > v_{max\ 2}$, but $\mu 1 > \mu 2$: in this case, at the beginning, picker 1 is faster, but after a certain periof of time t° could happen that picker 2 becomes faster, because picker 1 gets tired faster (he has a higher value of $\mu 1$). In this case the two function $v_1(t) = v_{max\ 1} *e^{-\mu t}$ and $v_2(t) = v_{max\ 2} *e^{-\mu t}$ are crossing each other and it is possible to find a point of intersection.

According to Bartholdi and Eisenstein (1996a, 1996b), a bucket brigade system perform at its maximum level when the workers are sequenced from the slowest to the fastest along the line; this happens because the chance of blockage is reduced, so the throughput is the maximum that we can obtain. Also if the picking speed of the pickers decreases over time, it is always convenient to order the pickers from the slowest to the fastest to reduce the chance of blockage. Ordering the pickers from the slowest to the fastest and considering the slowdown of the pickers, the dynamics of the system is the one in figure 5.5.

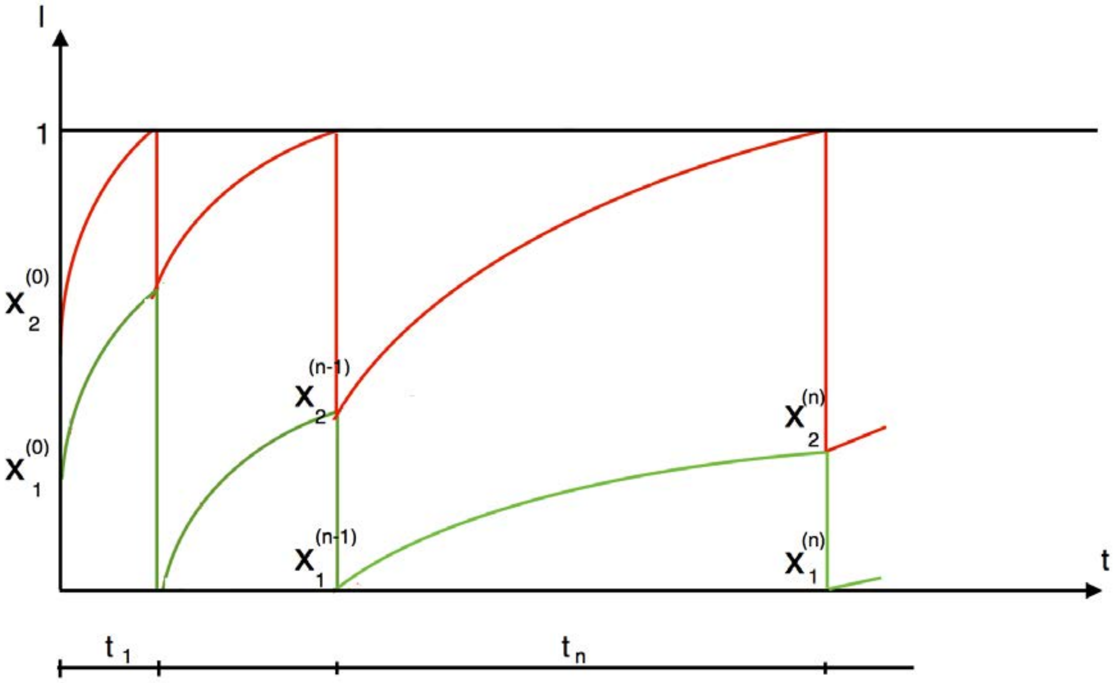

Fig 5.5 - Dynamics of a 2 operators BB system when workers' speed varies with $v(t) = v_{max} * e^{-\mu t}$ between two hand-offs. In the x axis the time is shown, while in the y axis the normalized length of the aisle is represented. The plot is similar to the one of figure 4.3, which represents the dynamics of a bucket brigade under the hypothesis of normative model. The difference between the two pictures is that in picture 4.3 the lines are straight and with the same slope (v is constant over time), while in this picture the lines are exponential, because v is not constant over time, but it decreases with the function $v(t) = v_{max} * e^{-\mu t}$. This leads to an enlargement of the time between hand-off over time: in general, after the first few iterations, when the system is balanced, we will have $t_{n+1} > t_n > t_{n-1}$. Moreover the slope of the line immediately before and immediately after every hand-off is the same, because x'(t) is a continuous function, even if x(t) is not.

The speed over time is not constant (the lines between hand-offs are not straight), so we need to integrate the function $v(t) = v_{max} * e^{-\mu t}$ in the interval between two consecutive hand-offs to find the function x(t), which tells us how the pickers move along the aisle.

$v = ds/dt$

$v_{max} * e^{-\mu t} = ds/dt$

$ds = v_{max} * e^{-\mu t} * dt$

$$\int_{si}^{sf} ds = \int_{ti}^{tf} v\max \cdot e^{-\mu t}\, dt$$

$$\int_{si}^{sf} ds = \int_{\sum_1^{n-1} t}^{\sum_1^{n} t} v\max \cdot e^{-\mu t}\, dt$$

$s_f - s_i = -(v_{max} / \mu) * e^{-\mu * \Sigma \text{ from 1 to n (t)}} + (v_{max} / \mu) * e^{-\mu * \Sigma \text{ from 1 to n-1 (t)}}$

$s_f - s_i = -(v_{max} / \mu) * (e^{-\mu * \Sigma \text{ from 1 to n (t)}} - e^{-\mu * \Sigma \text{ from 1 to n-1 (t)}}) =$

$= -(v_{max} / \mu) * (e^{-\mu * t_f} - e^{-\mu * t_i})$ (*)

and, more in general, $s_f = x^{(n)}$ and $s_i = x^{(n-1)}$.

The equation we found allows us to find the space travelled by a picker in the time interval between t_i and t_f (interval of time between two consecutive hand-offs).

It is interesting to find the formula which gives the final time t_f (time of the following hand-off), when the initial time t_i (time of a hand-off) is given. To obtain it, it is enough to work on the formula that we found before.

$s_f - s_i = -(v_{max} / \mu) * (e^{-\mu * \Sigma \text{ from 1 to n (t)}} - e^{-\mu * \Sigma \text{ from 1 to n-1 (t)}})$

$s_f - s_i = -(v_{max} / \mu) * (e^{-\mu * t_f} - e^{-\mu * t_i})$

In general, the space s_i is always $s_i = 0$, because for each iteration $x_1^{(n)} = 0$. Only for the first iteration it can be $s_i \neq 0$, because of the starting position of the first worker $x_1^{(0)} \neq 0$.

$- (sf - si) / (v_{max} / \mu) = e^{-\mu * t_f} - e^{-\mu * t_i}$

$- (sf - si) / (v_{max} / \mu) + e^{-\mu * t_i} = e^{-\mu * t_f}$

$\ln (- (sf - si) / (v_{max} / \mu) + e^{-\mu * t_i}) = -\mu * t_f$

$$tf = \frac{\ln\left(-\frac{sf-si}{v\max}\cdot\mu + e^{-\mu\cdot ti}\right)}{-\mu}$$

where t_f and t_i can be written as summations:

$$\sum_{1}^{n} t = \frac{\ln\left(-\frac{sf-si}{v\max}\cdot\mu + e^{-\mu\cdot\sum_{1}^{n-1} t}\right)}{-\mu} \qquad (**)$$

All the formulas that we found till now can be used for both the pickers. In particular, we will use directly (*) and (**).

Now it is possible to study the dynamics of the system.

In the first step:

$$t1 = \frac{\ln\left(-\frac{1-x2^{(0)}}{v\max 2}\cdot\mu 2 + e^{-\mu 2\cdot 0}\right)}{-\mu 2} = \frac{\ln\left(-\frac{1-x2^{(0)}}{v\max 2}\cdot\mu 2 + 1\right)}{-\mu 2}$$

$t_1 = t_1 - t_0 = t_1 - 0 = t_1$

$x_1^{(1)} = 0$

$$x2^{(1)} = x1^{(0)} - \frac{v\max 1}{\mu 1}\cdot\left(e^{-\mu 1\cdot t1} - e^{-\mu 1\cdot 0}\right) = x1^{(0)} - \frac{v\max 1}{\mu 1}\cdot\left(e^{-\mu 1\cdot t1} - 1\right)$$

In the second step:

$$t1 + t2 = \frac{\ln\left(-\frac{1-x2^{(1)}}{v\max 2}\cdot\mu 2 + e^{-\mu 2\cdot (t1)}\right)}{-\mu 2}$$

$t_2 = (t_1 + t_2) - t_1 = t_2$

$x_1^{(2)} = 0$

$$x2^{(2)} = x1^{(1)} - \frac{vmax1}{\mu 1}\cdot\left(e^{-\mu 1\cdot(t1+t2)} - e^{-\mu 1\cdot(t1)}\right) = -\frac{vmax1}{\mu 1}\cdot\left(e^{-\mu 1\cdot(t1+t2)} - e^{-\mu 1\cdot(t1)}\right)$$

In the third step:

$$t1+t2+t3 = \frac{\ln\left(-\frac{1-x2^{(2)}}{vmax2}\cdot\mu 2+e^{-\mu 2\cdot(t1+t2)}\right)}{-\mu 2}$$

$t_3 = (t_1 + t_2 + t_3) - (t_1 + t_2) = t_3$

$x_1^{(3)} = 0$

$$x2^{(3)} = x1^{(2)} - \frac{vmax1}{\mu 1}\cdot\left(e^{-\mu 1\cdot(t1+t2+t3)} - e^{-\mu 1\cdot(t1+t2)}\right) = -\frac{vmax1}{\mu 1}\cdot\left(e^{-\mu 1\cdot(t1+t2+t3)} - e^{-\mu 1\cdot(t1+t2)}\right)$$

In the n step:

$$\sum_1^n t = \frac{\ln\left(-\frac{1-x2^{(n-1)}}{vmax2}\cdot\mu 2+e^{-\mu 2\cdot\sum_1^{n-1} t}\right)}{-\mu 2}$$

$$tn = \sum_1^n t - \sum_1^{n-1} t$$

$x_1^{(n)} = 0$

$$x2^{(n)} = x1^{(n-1)} - \frac{vmax1}{\mu 1}\cdot\left(e^{-\mu 1\cdot\sum_1^n t} - e^{-\mu 1\cdot\sum_1^{n-1} t}\right)$$

In all these calculations the step (the number of the iteration) is indicated with the letter n, in particular with $^{(n)}$, and not with the letter t as in the previous chapters (chapter 2 in particular). This change has been done because in the formula v(t) = $v_{max} * e^{-\mu t}$, t is already used to indicate the time from the beginning of the work shift so we could not use the same letter for the steps.

5.4.2 Linear approximation of v(t) between two consecutive hand-offs

As we have already noticed at the end of paragraph 5.2.3, the shape of the exponential function is slightly linear in an 8 hours work shift (see figures 5.3 and 5.4). This suggests that the function $v(t) = v_{max} * e^{-\mu t}$ could be approximated with a straight line, more precisely with a piecewise linear function. The idea is to consider the speed of the pickers constant over the time between two consecutive hand-offs. The function $v(t) = v_{max} * e^{-\mu t}$ is used to determine the value of v at the beginning of the interval between two consecutive hand-offs; after each hand-off, a new value of v must be calculated, changing the value of t[21].

In other words, there are two ways to study the behavior of a two workers bucket brigade in an order-picking system, considering the slowdown of the pickers:

- Considering the pickers' speed varying with the function $v(t) = v_{max} * e^{-\mu t}$ between two hand-offs. This is the most precise way to calculate the final result. (see paragraph 5.4.1)
- Considering the pickers' speed constant between two hand-offs. Doing like this, the result will be approximated. (see paragraph 5.4.3)

Both procedures give very close and precise results. This happens because, as we have told before, the shape of the exponential function we are considering can be approximated with a straight line and because the time because two hand-off is small (seconds), while the work shift is a lot larger (hours). Moreover, the value of μ is small (around 10^{-5}, 10^{-6}) and this leads to the fact that the exponential function is very flat[22]. For these reasons it is possible to approximate the function $v(t) = v_{max} * e^{-\mu t}$ with small straight lines between hand-off. Therefore, the approximation that we are doing considering the pickers' speed constant between two hand-offs leads to a result very close to the perfect one, obtained considering the pickers' speed varying with the function $v(t) = v_{max} * e^{-\mu t}$ between two hand-offs. All the concepts that we have just explained are shown in figure 5.6.

[21] The value of t is the value of time from the beginning of the work shift.

[22] The function $v(t) = v_{max} * e^{-\mu t}$ is very flat because the very low value of μ. In fact, the first derivative of the function is $v'(t) = - v_{max} * \mu * e^{-\mu t}$ and both v_{max} and μ are very small, so also v'(t) is small and the function v(t) is flat.

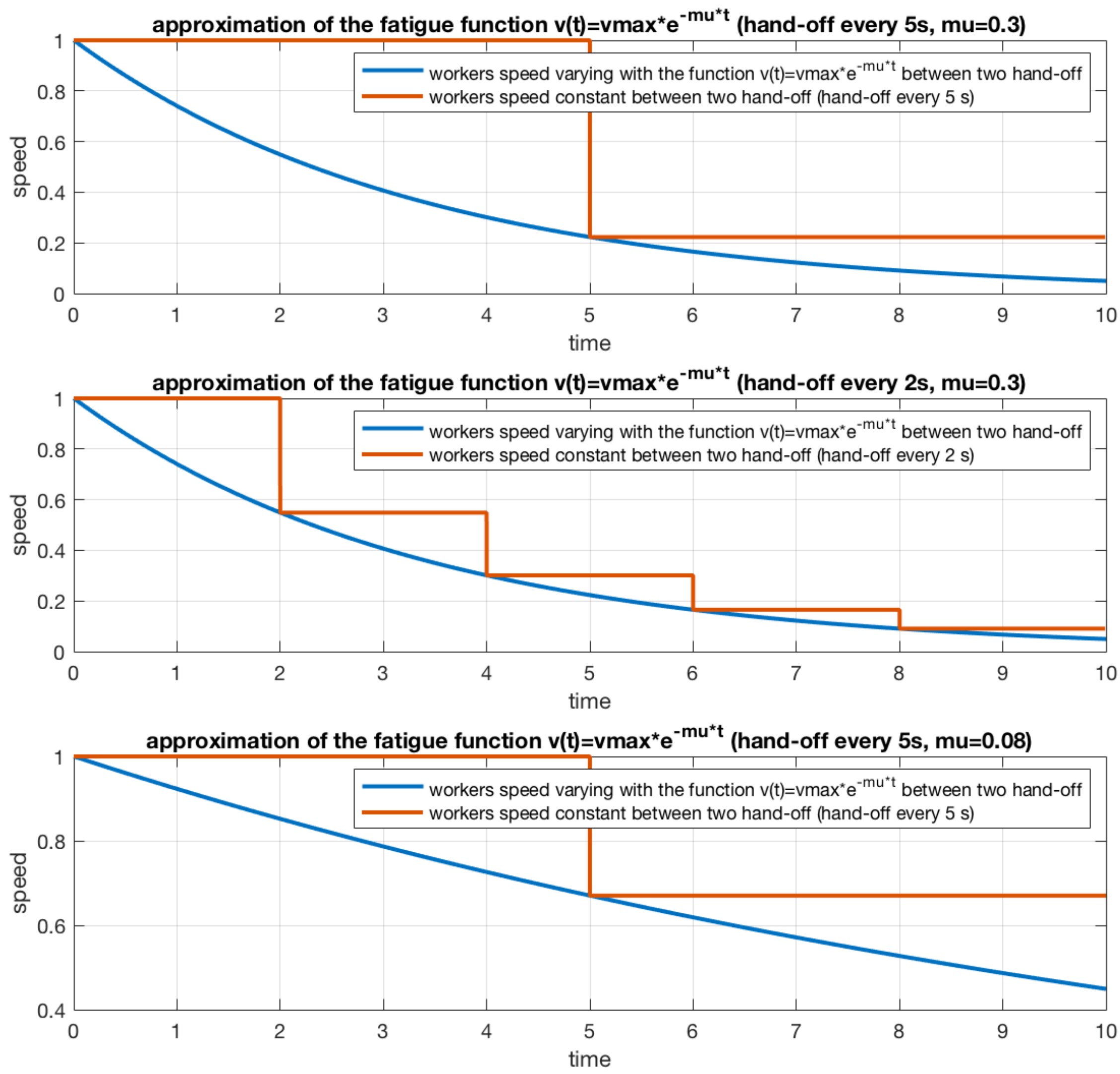

Fig 5.6 - Fatigue approximation. The picture shows why it is possible to approximate the exponential fatigue model with a piecewise linear function. The approximation is better if the time is divided in a lot of subintervals (the hand-off are a lot, so that the time between hand-offs is short) and the value of μ is low (the function is flat). In our case, the total time (8 hours) is divided, in general, in more than 100 intervals (in 8 hours there are more than 100 hand-offs) and the value of μ is between 10^{-5} and 10^{-7}: the approximation is nearly perfect. For the sake of simplicity, we have normalized the speed of the picker to 1.

It is interesting to notice that if the value of μ is μ = 0, then the worker is not tiring and we are returning to the case studied by Bartholdi and Eisenstein (1996a). In this case, the two ways to study the behavior of a two workers bucket brigade in an order-picking system give exactly the same results, because the speed of the workers is constant during the whole work shift and, therefore, between two hand-offs as well.

5.4.3 Pickers' speed constant between two consecutive hand-offs

For all the reasons we explained in the previous paragraph, it is possible to develop, with a very good approximation, the mathematics of the problem also considering the workers' speed constant between two consecutive hand-offs. As we have already explained, the idea of this approximation is to consider the speed of the pickers constant over the time between two consecutive hand-offs. The function $v(t) = v_{max} * e^{-\mu t}$ is used to determine the value of v at the beginning of the interval between two consecutive hand-offs: this value of v will remain constant for all the time between two consecutive hand-offs; after each hand-off, a new value of v must be calculated, changing the value of t[23]. Also in this case, as we already told at the beginning of 5.4.1, it is convenient to order the pickers from the slowest to the fastest to avoid blockage and, so, to maximize the throughput.

Here, we will show an example about how to calculate the speed of the pickers between two consecutive hand-offs, with the method of constant speed.

Let's say that after time t from the beginning of the work shift there is a hand-off. To calculate the constant speeds of the two pickers we have to calculate:

$v_1 = v_{max1} * e^{-\mu 1t}$

$v_2 = v_{max2} * e^{-\mu 2t}$

The speeds v_1 and v_2 we have just found are not function of time anymore, because we are considering the speeds constant between two consecutive hand-offs. We have to use the speeds that we found, considering them constant, until the next hand-off is done. When another hand off is done, we have to make the calculation again: the values of v_{max1}, v_{max2}, μ_1 and μ_2 remain the same[24], but the value of t changes, because some time is passed by. In particular, the new time t will be calculated adding the time between the two hand-offs to the old time t, so that the new time t will be always a bigger number than the old time t. Because of the fact that the new time t is bigger than the old time t, the speeds decrease their value every iteration, according to the fact that the pickers slow down over time, because of fatigue.

[23] The value of t is the value of time from the beginning of the work shift.

[24] All this values are constant over time.

If we order the pickers from the slowest to the fastest and consider their speeds slowing down over time in the work shift, we will obtain the dynamics of the system represented in figure 5.7.

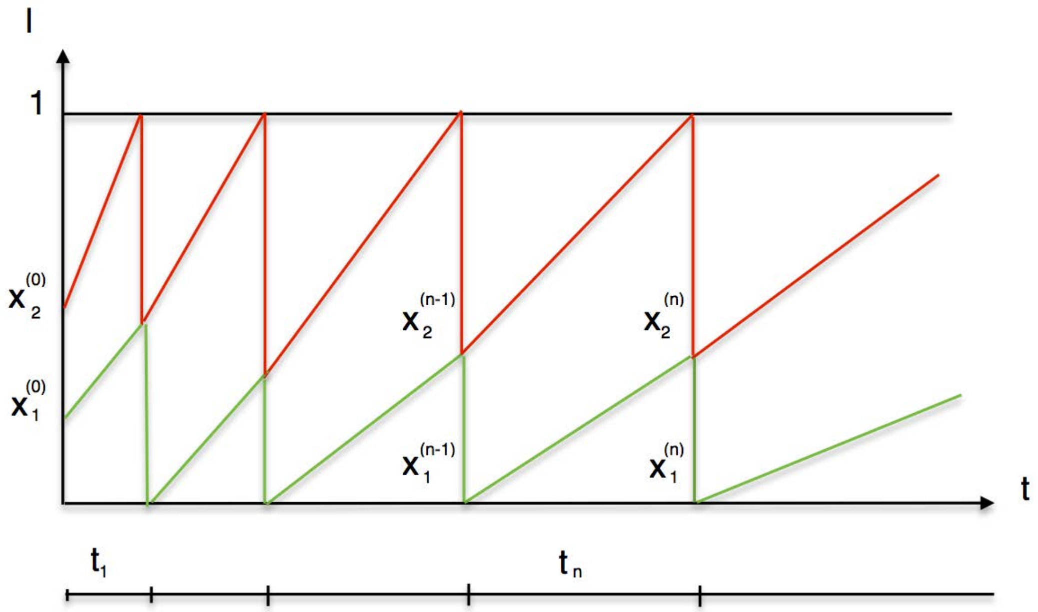

Fig 5.7 - Dynamics of a 2 operators BB line when workers' speed is constant between two hand-off. In the x axis the time is shown, while in the y axis the normalized length of the aisle is represented. The plot is similar to the one of figure 4.3, which represents the dynamics of a bucket brigade under the hypothesis of normative model. The difference between the two pictures is that in picture 4.3 the lines are straight and with the same slope (v is constant over time), while in this picture the lines are always straight, but the slope of the lines decreases over time, because v is not constant over time, but it decreases iteration by iteration. In particular we have to calculate the constant speed in each interval using the formula $v(t) = v_{max} * e^{-\mu t}$, where t is the time from the beginning of the work shift to the first of the two hand-offs that we are considering. This leads to an enlargement of the time between hand-offs over time: in general, after the first few iterations, when the system is balanced, we will have $t_{n+1} > t_n > t_{n-1}$. In this case neither x(t) nor x'(t) are continuous functions.

In this case we are considering that the speed of the operators decreases over time with the function $v(t) = v_{max} * e^{-\mu t}$, but not continuously. We will first calculate the speed $v(t_n)$ immediately after a hand-off at time t_n with the formula $v(t_n) = v_{max} * e^{-\mu t_n}$ and we will consider the speed $v(t_n)$, found with this procedure, constant for all the time between t_n and t_{n+1}. These calculations can be done for each interval of time between hand-offs, always considering the speed constant in the time interval.

In the first step[25]:

$v_1(t_0=0) = v_1^{(0)} = v_{1max} * e^{-\mu_1 * t_0} = v_{1max} * e^{-\mu_1 * 0} = v_{1max} * 1 = v_{1max}$

$v_2(t_0=0) = v_2^{(0)} = v_{2max} * e^{-\mu_2 * t_0} = v_{2max} * e^{-\mu_2 * 0} = v_{2max} * 1 = v_{2max}$

$t_1 = (1-x_2^{(0)})/v_2^{(0)} = (1-x_2^{(0)})/v_{2max}$

$x_1^{(1)} = 0 \qquad x_2^{(1)} = v_1^{(0)}*t_1+x_1^{(0)} = v_{1max}*t_1+x_1^{(0)}$

In the second step:

$v_1(t_1) = v_1^{(1)} = v_{1max} * e^{-\mu_1*(t_0+t_1)} = v_{1max} * e^{-\mu_1*t_1}$

$v_2(t_1) = v_2^{(1)} = v_{2max} * e^{-\mu_2*(t_0+t_1)} = v_{2max} * e^{-\mu_2*t_1}$

$t_2 = (1-x_2^{(1)})/v_2^{(1)}$

$x_1^{(2)} = 0$

$x_2^{(2)} = v_1^{(1)}*t_2+x_1^{(1)} = v_1^{(1)}*t_2$

In the third step:

$v_1(t_2) = v_1^{(2)} = v_{1max} * e^{-\mu_1*(t_0+t_1+t_2)} = v_{1max} * e^{-\mu_1(t_1+t_2)}$

$v_2(t_2) = v_2^{(2)} = v_{2max} * e^{-\mu_2*(t_0+t_1+t_2)} = v_{2max} * e^{-\mu_2(t_1+t_2)}$

$t_3 = (1-x_2^{(2)})/v_2^{(2)}$

$x_1^{(3)} = 0$

$x_2^{(3)} = v_1^{(2)}*t_3+x_1^{(2)} = v_1^{(2)}*t_3$

In the n step:

$v_1(t_{n-1}) = v_1^{(n-1)} = v_{1max} * e^{-\mu_1*\Sigma \text{ from 0 to n-1 } (t_n)} = v_{1max} * e^{-\mu_1*(t_1+t_2+\ldots+t_{n-1})}$

[25] As we already told at the end of 7.4.1, the step (the number of the iteration) is indicated with the letter n and not with t as in the previous chapters.

$v_2(t_{n-1}) = v_2^{(n-1)} = v_{2max} * e^{-\mu_2 * \Sigma \text{ from 0 to n-1 } (t_n)} = v_{2max} * e^{-\mu_2*(t_1+t_2+\ldots+t_{n-1})}$

$t_n = (1-x_2^{(n-1)})/v_2^{(n-1)}$

$x_1^{(n)} = 0$

$x_2^{(n)} = v_1^{(n-1)}*t_n+x_1^{(n-1)} = v_1^{(n-1)}*t_n$

And it is possible to compare the results we have just obtained with the results that we shown in chapter 2. In that chapter we found that after the t^{th} step:

$t_t = (1 - x_2^{(t-1)}) / v_2$

$x_1^{(t)} = 0$

$x_2^{(t)} = v_1 * t_t + x_1^{(t-1)} = v_1 * t_t$

The only difference between the two results is that in the result that we have just obtained, the speeds of the pickers v_1 and v_2 are changing (they slow down) over time during the work shift. We remind the reader another time, then, that from this chapter on we will use n (not t anymore) to indicate the number of an iteration (step).

After simple algebra, it is possible to show the successions of consecutive point of hand-off and the succession of times between hand-offs.

$$x_2^{(n)} = \frac{v_1^{(n-1)}}{v_2^{(n-1)}} \cdot \left(1-x_2^{(n-1)}\right) = \frac{v_{1max} \cdot e^{-\mu_1 \cdot (t_1+\ldots+t_{n-1})}}{v_{2max} \cdot e^{-\mu_2 \cdot (t_1+\ldots+t_{n-1})}} \cdot \left(1-x_2^{(n-1)}\right)$$

$$t_n = \frac{1-v_1^{(n-2)} \cdot t_{n-1}}{v_2^{(n-2)}} = \frac{1}{v_{2max} \cdot e^{-\mu_2 \cdot (t_1+\ldots+t_{n-1})}} - \frac{v_{1max} \cdot e^{-\mu_1 \cdot (t_1+\ldots+t_{n-2})}}{v_{2max} \cdot e^{-\mu_2 \cdot (t_1+\ldots+t_{n-1})}} \cdot t_{n-1}$$

And we found the succession that links consecutive hand-off positions (links $x_2^{(n)}$ and $x_2^{(n-1)}$) and the one that links the time between two consecutive hand-offs (links t_n and t_{n-1}). It is not easy to prove and understand when these equations lead to a convergence point or not and we will work on it, case by case, in paragraph 5.5, where we will give mathematical explanations to the results of the simulations, starting from this equations.

5.4.4 Errors of approximation

We have already explained why it is possible to approximate very well the behavior of a bucket brigade system considering pickers' speed varying with v(t) = $v_{max} * e^{-\mu t}$ over time with a system where pickers' speed is constant between two consecutive hand-offs. In this paragraph we will run some MATLAB simulations to prove this fact again. The results are presented in chart 5.1.

Chart 5.1 - Comparison between the results obtained with exponential speed and the results obtained with constant speed between two consecutive hand-offs. The comparison is done taking into account x_2 after 8 hours, the time between two consecutive hand-offs after 8 hours and the throughput after 8 hours. For all these three values a percentage error is calculated. The speeds v_1 and v_2 and the starting positions $\mathbf{x}^{(0)}$ are random.

starting data	**x_2 (8h) exp. speed / con. speed error %**	**t between h-o (8h) exp. speed / con. speed error %**	**TR (8h) exp. speed / con. speed error %**
v_1=0,0010 a/s v_2=0,0063 a/s μ_1=3,6584*10^{-6} μ_2=3,6584*10^{-6}	0,13364 / 0,13364 0%	152,23 / 152,19 -0,026%	201 / 201 0%
v_1=0,0049 a/s v_2=0,0096 a/s μ_1=3,6584*10^{-6} μ_2=12,3850*10^{-6}	0,39479 / 0,39467 -0,030%	90,38 / 90,33 -0,055%	366 / 366 0%
v_1=0,0042 a/s v_2=0,0092 a/s μ_1=7,7480*10^{-6} μ_2=3,6584*10^{-6}	0,29048 / 0,29052 0,014%	86,07 / 86,05 -0,023%	360 / 360 0%

Chart 5.1 shows that, as we were guessing before, the approximation is nearly perfect, in fact all the results obtained with constant speed differ no more than 0,1% from the perfect results obtained with exponentially decreasing speed. These simulations suggest that this error can be neglected. In particular, the throughput after 8 hours remains exactly the same, while there is a little error in x_2 and in time between hand-off. Moreover, if the speed constant, the time between hand-offs is always longer than the case with exponential speed, because the speed is constant and, so higher inside the interval (see picture 5.6). On the other hand, if we consider the speed constant, the position x_2 after 8 hours remains exactly the same if $\mu_1=\mu_2$, it is a little bit bigger if $\mu_1<\mu_2$ and it is a little bit smaller if $\mu_1>\mu_2$.

5.5 Simulations

5.5.1 Hypothesis to set up the simulations

In this paragraph we will simulate with the software MATLAB the behavior of a two operators bucket brigade in an order-picking system, considering the speeds of the pickers slow down over time following the equation $v(t) = v_{max} * e^{-\mu t}$. All the results obtained with the simulations will be supported by mathematical explanation.

Before starting with the simulations, we have to explain which are the starting hypothesis that we will use. Down here, a bulleted list is shown:

- Assumptions and restrictions we wrote in paragraph 5.3 are valid.
- The pickers are sequenced from the slowest to the fastest along the line to have the maximum throughput (TR). We will call "picker 1" the picker who has the minimum v_{max} (we will call his maximum speed v_{max1}) and "picker 2" the picker who has the maximum v_{max} (we will call his maximum speed v_{max2}).
- For every simulation, we will consider different levels of v_{max}: the maximum speed of the slowest picker will be v_{max1} and the maximum speed of the fastest picker will be v_{max2} ($v_{max1} < v_{max2}$).
- For every simulation, we will consider four different level of μ (0, $3.6584*10^{-6}$, $7.7480*10^{-6}$ and $12.3850*10^{-6}$).
- We will chose the vector of the starting point $\mathbf{x}^{(0)} = (x_1^{(0)}, x_2^{(0)})$ randomly in the interval [0, 1]; we will chose this interval because the length of the aisle is normalized to l=1. Moreover the restriction $x_1^{(0)} < x_2^{(0)}$ is important to avoid blockage.
- We will consider the pickers' speed constant between two hand-offs[26] (see 5.4.3). Doing like this, the result will be approximated, but very close to the perfect one, which can be obtained considering the pickers' speed varying with the function $v(t) = v_{max} * e^{-\mu t}$ between two hand-offs (see 5.4.1).

[26] It is possible to simulate everything also with the code that takes into account that the picking speed varies with $v(t) = v_{max} * e^{-\mu t}$ over time (see code C.3). In this case the results give exactly the same number of iterations and there is only a little mistake on the time between iterations (always less than 0,1%, see chart 5.1). We will consider the pickers' speed constant between two hand-offs because the mathematics to justify the results of the simulation is easier.

Starting from these hypothesis, we will study the behavior of the system in four different cases, always using the code “stepwise_function_mu.m” (see code C.2), that we run with the script MyScript_mu.m (see code C.1):

- Pickers with same v_{max} and same μ (5.5.2)
- Pickers with different v_{max} and same μ (5.5.3)
- Pickers with same v_{max} and different μ (5.5.4)
- Pickers with different v_{max} and different μ (5.5.5)

The aims of the simulations are a lot. First of all, we want to understand which case performs better and how to order the two pickers to obtain the maximum throughput. For each case, we will first give a mathematical explanation, trying to understand how the results could be, before running the simulations; then, running each simulation, the behavior of the system in the 8 hours work shift will become clear. In particular, we want to understand if the system is balanced or not. After that, for each simulation, we will provide the throughput of the system, the fixed points (if they exist) and finally, we will study how the system reacts to the changing of some starting data.

5.5.2 Pickers with same v_{max} and same μ

The easiest problem to solve is to simulate the behavior of the system where work is perfectly distributed along the warehouse, the orders are all equal and the operators are completely equivalent. This means that they get tired in the same way, so that it is possible to consider $v_{max1} = v_{max2} = v_{max}$ and $\mu_1 = \mu_2 = \mu$. In this case, the problem is trivial.

The equations obtained at the end of 5.4.3 can be simplified, because $v_{max1} = v_{max2} = v_{max}$ and $\mu_1 = \mu_2 = \mu$.

The succession of hand-off positions gives:

$x_2^{(n)} = 1-x_2^{(n-1)}$

that is an equation of fixed point with g(x) = 1-x; the first derivative of the function g(x) is g'(x) = -1 and |g'(x)| = 1 that is not |g'(x)|<1; therefore the succession does not converge to a fixed point and the system is not balanced. It is important to notice that the system does not converges regardless of the values of v_{max} and μ, which disappear in the simplification in the formula.

The same considerations can be done also for the succession of time between consecutive hand-offs. This succession does not converge as well. It is very difficult to prove it mathematically, so we will show the behavior of the system using a simulation, which will clarify everything. The common sense suggests that the time between two consecutive hand-offs grows over time, because the pickers slows down over time because of fatigue.

In this first simulation, the starting data are: v_{max} = 0,006 aisles/s (see figure 5.8), $\mu = 7,7480 \cdot 10^{-6}$ (average work), $\mathbf{x}^{(0)}$=(0,5797; 0,8693).

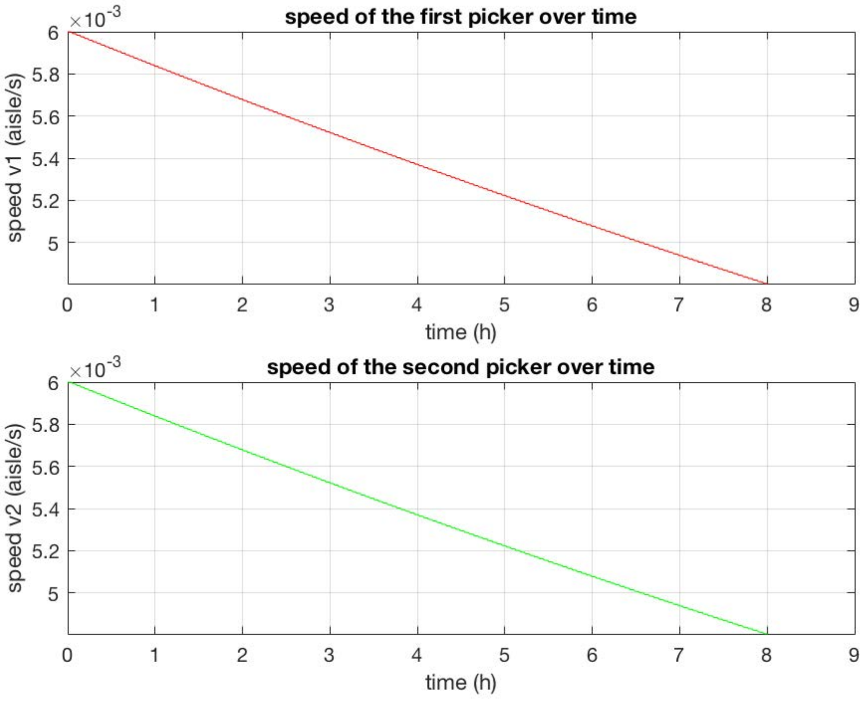

Fig 5.8 - Speed of the operators over time in a work shift in the case of $v_{max1} = v_{max2} = v_{max}$ = 0,006 aisles/s and $\mu_1 = \mu_2 = \mu = 7,7480 \cdot 10^{-6}$ (average work).

It is possible to notice that the speed of the pickers are exactly the same, because $v_{max1} = v_{max2}$ and $\mu_1 = \mu_2$. In other words, the pickers are equivalent.

The results are shown in figure 5.9 and 5.10.

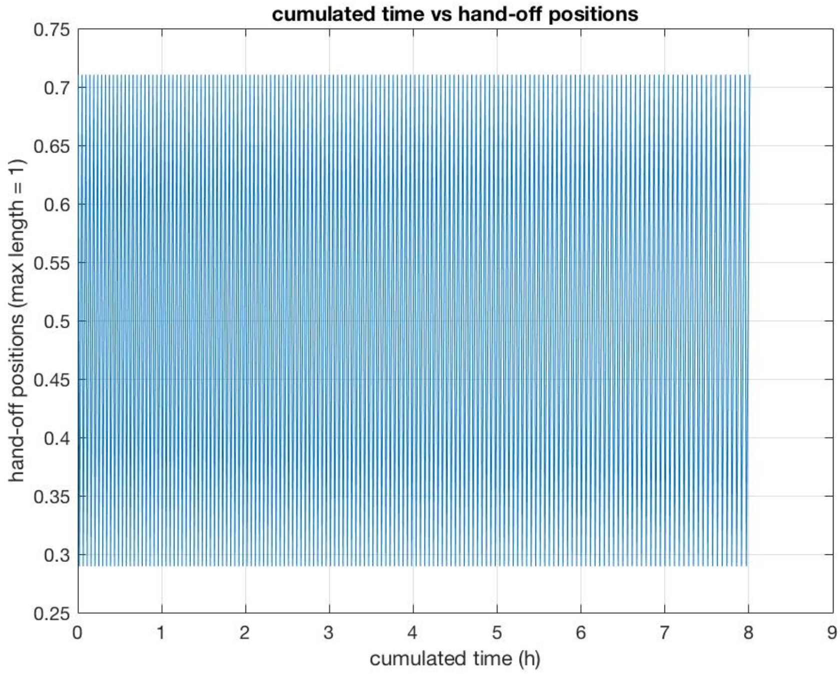

Fig 5.9 - Cumulated time vs hand-off positions when $v_{max1} = v_{max2} = v_{max} = 0{,}006$ aisles/s, $\mu_1 = \mu_2 = \mu = 7{,}7480 \cdot 10^{-6}$ and $\mathbf{x}^{(0)} = (0{,}5797; 0{,}8693)$.

The succession of the hand-off positions does not converge, as we proved before with the calculations. In this particular case the positions of hand-off alternate themselves between x=0,28959 and x=0,71041. To predict the hand-off positions in this kind of system it is enough to calculate the first two iterations and they will alternate for all the work shift. Another interesting fact to notice is that the average between the two positions is always 0,5 and this value does not depend from the starting data; in our case we have (0,28959+0,71041)/2 = 0,5.

It is possible to notice that 0,5 is exactly the value of x* that it is possible to obtain if the bucket brigade is working under the hypothesis of the normative model ($v = v_{max}$ = constant). The value of x2 depends, in fact, from the ratio v1/v2: the more the ratio is close to 1 and the more the hand-off position is close to 0,5; the more the ratio is close to 0 and the more the hand-off position shifts towards the beginning of the aisle.

$$x_2^{av} = v_1 / (v_1+v_2) = (v_{max1} * e^{-\mu1*t}) / ((v_{max1} * e^{-\mu1*t}) + (v_{max2} * e^{-\mu2*t}))$$

$$= (v_{max} * e^{-\mu*t}) / ((v_{max} * e^{-\mu*t}) + (v_{max} * e^{-\mu*t}))$$

$$= v_{max} / (v_{max}+v_{max}) = 1/2 = 0{,}5$$

The result is confirmed by figure 5.8: the ratio is 1, so $x_2^{av} = 0{,}5$.

The behavior of the time between hand-offs is different and it is shown in figure 5.10.

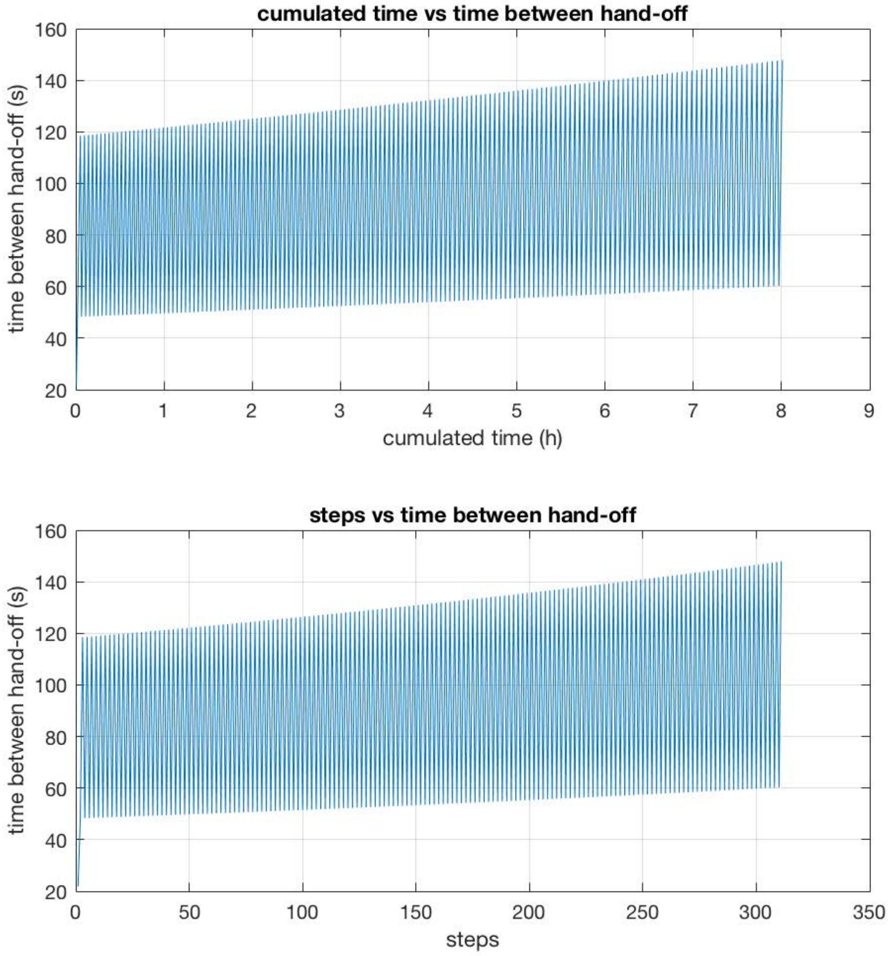

Fig 5.10 - Cumulated time and steps vs time between hand-offs when $v_{max1} = v_{max2} = v_{max} = 0{,}006$ aisles/s, $\mu_1 = \mu_2 = \mu = 7{,}7480 \cdot 10^{-6}$ and $\mathbf{x}^{(0)} = (0{,}5797; 0{,}8693)$. In 8 hours the system finishes 311 orders.

As we told before, also the succession of times between hand-offs does not converge. In an eight hours work shift, there are 311 hand-offs and this means that the pickers have finished picking 311 orders. It is important to notice how the average time between hand-offs grows over time: this is due to the slowdown of the pickers, in fact they are getting tired while picking and we considered picking as an average tiring work[27] ($\mu_1 = \mu_2 = \mu = 7{,}7480 \cdot 10^{-6}$).

Also with times it is possible to find a formula that can tell us the average time between two consecutive hand-offs after time t from the beginning of the work

[27] If we have considered $\mu_1 = \mu_2 = \mu = 0$ (no fatigue), also the time between two consecutive hand-offs would have swung between two well determined positions, which are easy to find with the first two iterations.

shift. The formula can be obtained working on the system like if it was a bucket brigade that works under the conditions of normative model.

$$t^{av} = 1 / (v_1+v_2) = 1 / ((v_{max1} * e^{-\mu 1 * t}) + (v_{max2} * e^{-\mu 2 * t}))$$

$$= 1 / ((v_{max} * e^{-\mu t}) + (v_{max} * e^{-\mu t}))$$

$$= 1 / (2v_{max} * e^{-\mu t})$$

With this formula we can calculate the average time between two hand-offs after a time t from the beginning of the work shift. It is important to notice that the result we have found does not depend on the starting positions of the pickers $\mathbf{x}^{(0)}$. It is easy to see that the higher is the value of μ, the longer is the value between two consecutive hand-offs. In accordance to the fact that the pickers get tired over time (t grows), the value of t^{av} grows over time.

In the case of our simulation, for example:

$t^{av}(4h) = 93{,}17$ s

$t^{av}(8h) = 104{,}16$ s

All these results are confirmed in figure 5.10.

A good strategy to make the system works better, reducing the instability of the system could be to work on the starting positions of the pickers. In particular, we noticed that the more $x_1^{(0)} + 0{,}5 = x_2^{(0)}$ is true and the more the system is completely stable from the beginning. In this case, the hand-off position will be always at 0,5 and the time between two consecutive hand-off will grow exponentially and without instabilities.

In conclusion, when the operators are completely equivalent ($v_{max1} = v_{max2} = v_{max}$ and $\mu_1 = \mu_2 = \mu$) the system does not behave well, because it is not balanced. It has been proved, then, that the behavior of the system is not chaotic, but the positions of hand-off can be predicted: in fact they swing between two precise positions, with average 0,5. Also the average time between two consecutive hand offs can be easily calculated and it grows over time, because the pickers slow down over time during the work shift. Even if the system is not balanced, its behavior is easy to predict. In particular, we found that if the starting point of the pickers are $x_1^{(0)} + 0{,}5 = x_2^{(0)}$ this kind of bucket brigade works very well.

5.5.3 Pickers with different v_{max} and same μ

The second step is to consider two pickers that get tired with the same speed, but with one operator faster than another. This means that $v_{max1} < v_{max2}$ and $\mu_1 = \mu_2 = \mu$. Following what Bartholdi and Eisenstein (1996a) proved in their paper, to have the maximum throughput the picker 2 has to be positioned closer to the end of the aisle. It is easy to notice that picker 2 will be always faster than picker 1, because the two pickers get tired with the same value of $\mu_1 = \mu_2 = \mu$: therefore there is no chance of blockage and the throughput is maximum if we make picker 2 work closer to the end of the line.

The equations obtained in 5.4.3 can be simplified, because $\mu_1 = \mu_2 = \mu$.

After a simple algebra, the succession of hand-off positions is:

$$x_2^{(n)} = \frac{v_{1max}}{v_{2max}} \cdot \left(1 - x_2^{(n-1)}\right)$$

that is an equation of fixed point with $g(x) = (v_{1max}/v_{2max})*(1-x)$; the first derivative of the function g(x) is $g'(x) = -v_{1max}/v_{2max}$ and $|g'(x)| = v_{1max}/v_{2max}$ that is always $|g'(x)|<1$, because we are in the case $v_{max1} < v_{max2}$; therefore the succession converges to a fixed point and the system is balanced. This means that after a few time the hand-off position will be always in the same place x_2^* and that the pickers will have to pick always in the same zone: the first picker in the interval $[0, x_2^*]$ and the second picker in the interval $[x_2^*, 1]$.

To find the convergence point is enough to set $x_2^{(n)} = x_2^{(n-1)} = x_2^*$.

$x_2^* = (v_{1max}/v_{2max})*(1-x_2^*)$

And solving the equation in x_2^*:

$x_2^* = v_{1max} / (v_{1max}+v_{2max})$

This is exactly the same results that Bartholdi and Eisenstein (1996a) found in their paper, under the hypothesis of normative model, considering the speed of the workers constant. This result tells that the fixed hand-off position does not depend on the value of $\mu_1 = \mu_2 = \mu$.

The conclusions are different if we speak about the succession of time between two consecutive hand-offs. In this case the succession of times between two consecutive hand-offs seems to converge, but it grows inevitably, because the pickers get tired over time. It is very difficult to prove it mathematically, so we will show it with some the simulations.

In this first simulation we will run the code with v_{2max} = 0,006 aisles/s, v_{1max} = 0,003 aisles/s (see figure 5.11), μ = 7,7480*10^{-6} (av. work), $\mathbf{x}^{(0)}$=(0,8147; 0,9058).

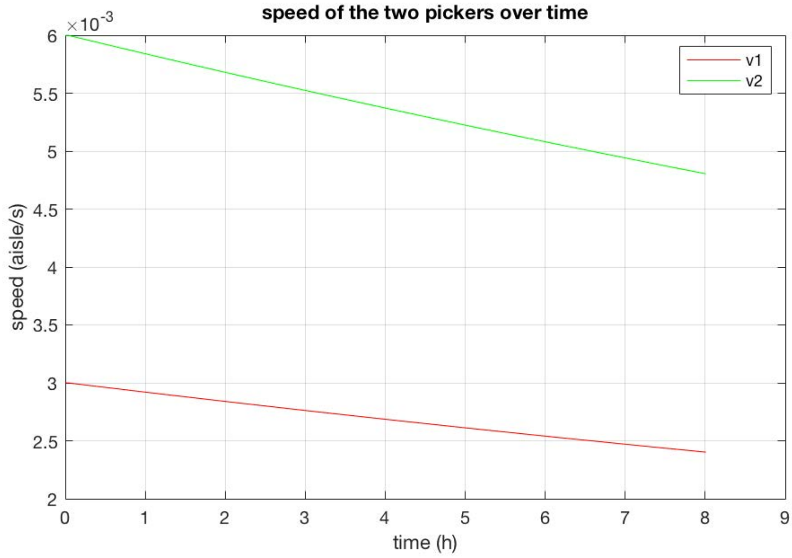

Fig 5.11 - Speed of the operators over time in a work shift in the case of v_{max1} = 0,003 aisles/s v_{max2} = 0,006 aisles/s and $\mu_1 = \mu_2 = \mu$ = 7,7480*10^{-6} (average work).

The results are shown in figure 5.12 and 5.13.

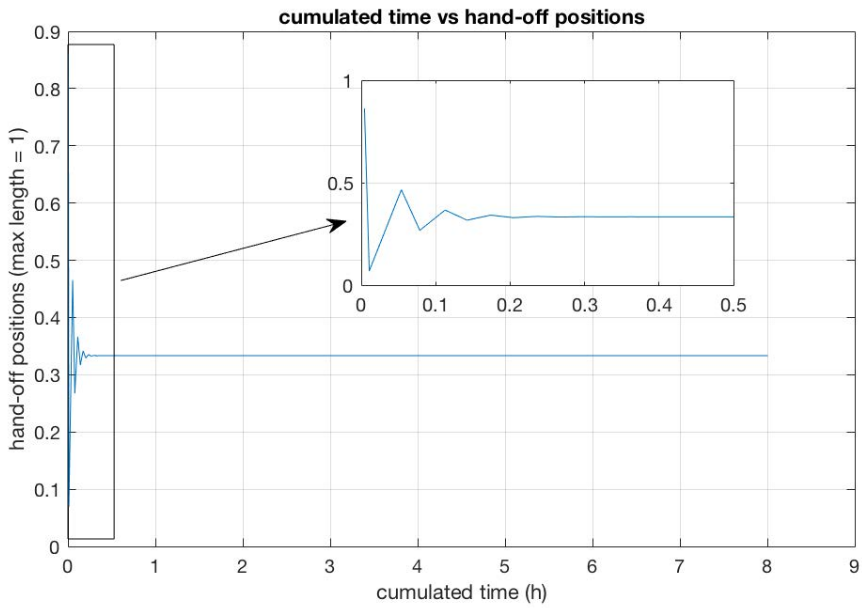

Fig 5.12 - Cumulated time vs hand-off positions when v_{max1} = 0,003 aisles/s, v_{max2} = 0,006 aisles/s, $\mu_1 = \mu_2 = \mu$ = 7,7480*10^{-6} and $\mathbf{x}^{(0)}$ = (0,8147; 0,9058).

As we expected from the mathematical calculation, the system is balanced after a few iterations. The simulation gives as fixed point $x_2^*= 0{,}333$, according to the formula $x_2^* = v_{1max} / (v_{1max}+v_{2max})$ that we found before. After a little bit more than 10 minutes the pickers have to work always in the same zone: picker 1 works in the interval [0; 0,333] and picker 2 works in the interval [0,333; 1].

The results, as the common sense suggests, are completely different with regard to the time between hand-offs (figure 5.13). In fact, because of the value of μ that is not 0, the pickers get tired and, inevitably, the more the pickers are tired, the more they slow down and the more the time between two consecutive hand-offs is longer; moreover, the throughput of the bucket brigade is lower.

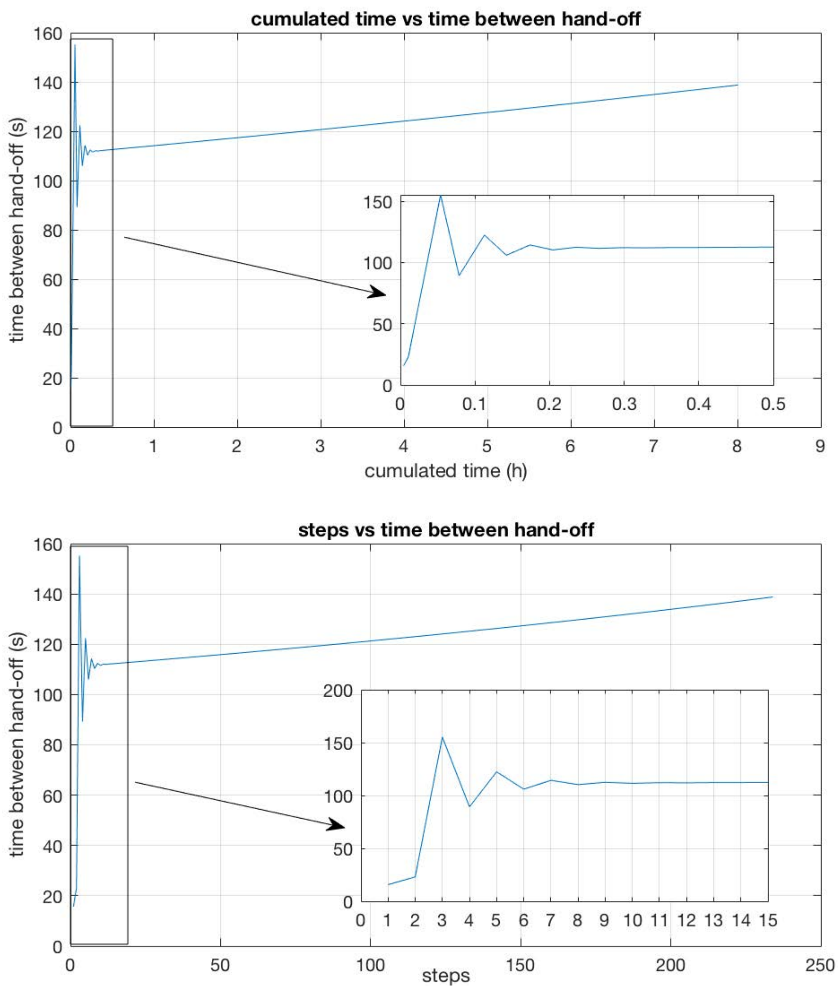

Fig 5.13 - Cumulated time and steps vs time between hand-offs when $v_{max1} = 0{,}003$ aisles/s, $v_{max2} = 0{,}006$ aisles/s, $\mu_1 = \mu_2 = \mu = 7{,}7480*10^{-6}$ and $\mathbf{x}^{(0)} = (0{,}8147; 0{,}9058)$. In 8 hours the system finishes 234 orders.

In an 8 hours work shift, there are 234 hand-offs and this means that the pickers have finished picking 234 orders.

Moreover, the results show that the plots cumulated time and steps vs time between hand-offs show a particular behavior. In a first moment, the time between two consecutive hand-offs seems to converge to a fixed point, but it is not like this. In fact, from picture 5.13, it is possible to notice that the time between two consecutive hand-offs grows exponentially with time, in accordance with the fact that pickers' speed decreases exponentially during the work shift.

Also in this case, it is possible to find an equation that gives as the value of t°, that is the time between two consecutive hand-offs after a time t from the beginning of the work shift[28]. It is possible to find the equation we are seeking, starting always from the results obtained under the hypothesis of normative model.

$$t° = 1 / (v_1+v_2) = 1 / ((v_{max1} * e^{-\mu t}) + (v_{max2} * e^{-\mu t}))$$

$$= 1 / (e^{-\mu t} * (v_{max1} + v_{max2}))$$

$$= t^* / e^{-\mu t}$$

So, to find the time between two consecutive hand-offs after a time t from the beginning of the work shift, it is enough to calculate the value $t^* = 1 / (v_{max1} + v_{max2})$ and then correct it with the factor $e^{-\mu t}$. The factor $e^{-\mu t}$ is always positive and $0 < e^{-\mu t} \leq 1$. Therefore, it is always true that $t° > t^*$, according to the fact that when the speeds (pick rate) of the workers slow down over time, the time between two consecutive hand-offs is longer as time passes by. Moreover, t° is not the result of a fixed point iteration anymore, but tells us the time between after consecutive hand-offs when the system is balanced (when the position of hand-off is constant along the line, over time) at time t: the value of t° varies iteration by iteration, depending on the time t from the start of the work shift[29].

In our simulation, for example:

$$t^* = 1 / (v_{max1} + v_{max2}) = 1 / (0,003 + 0,006) = 111,11 \text{ s}$$

$$t°(4 \text{ h}) = 111,11 / 0,894 = 124,28 \text{ s}$$

---

[28] We will call it t° and not t* because in this case the time between two consecutive hand-offs does not converge, but it grows over time.

[29] The formula we have found doesn't work for the first iterations, but it starts working only when the system is balanced, after a few iterations.

$t°(8\ h) = 111{,}11 / 0{,}800 = 138{,}89$ s

The results are confirmed by figure 5.13.

It is important to notice that the number 0,800 in the calculation t°(8 h) is exactly the factor of reduction for average work that we found before. 20% is the very factor of reduction of speed in average work, which is the kind of work that we are taking into account. If we had worked with easy work, the value of $e^{-\mu t}$ after 8 hours (t = 3600*8) would have been $e^{-\mu t} = 0{,}900$ that is linked to the 10% (easy work); if we had worked with hard work, $e^{-\mu t}$ after 8 hours (t = 3600*8) would have been $e^{-\mu t} = 0{,}700$ that is linked to the 30% (hard work). Summarizing, the factor $e^{-\mu t}$ tells us how much, after time t, the time between two consecutive hand-offs diverges from the value t*, that we can obtain starting from the hypothesis of the normative model (speeds constant over time). In particular, after t = 8 hours, $e^{-\mu t}$ is exactly 0,7 for hard work, 0,8 for average work and 0,9 for easy work, in accordance with what we found in paragraph 5.2.3.

In the second simulation the starting data are: $v_{2max} = 0{,}006$ aisles/s, $v_{1max} = 0{,}003$ aisles/s (see figure 5.14), $\mu = 3{,}6584 \cdot 10^{-6}$ (easy work), $\mathbf{x}^{(0)}=(0{,}0975; 0{,}6324)$.

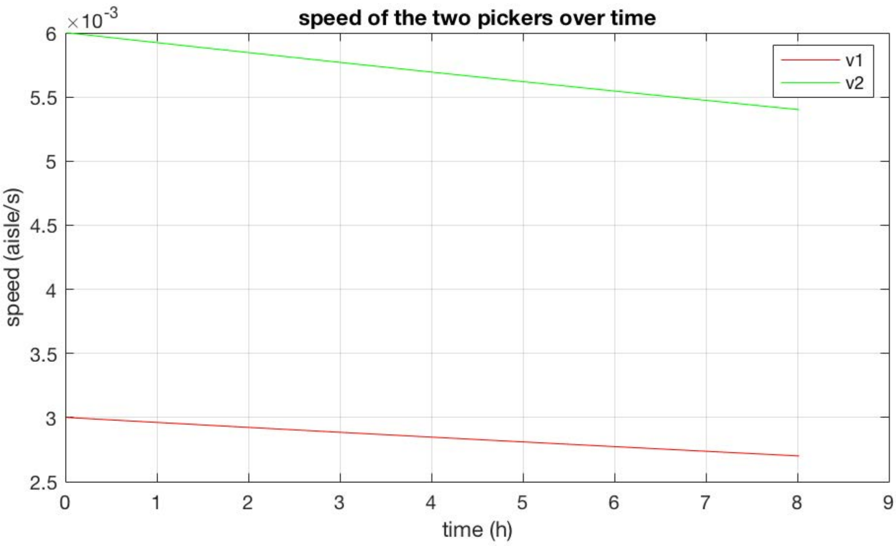

Fig 5.14 - Speed of the operators over time in a work shift in the case of $v_{max1} = 0{,}003$ aisles/s $v_{max2} = 0{,}006$ aisles/s and $\mu_1 = \mu_2 = \mu = 3{,}6584 \cdot 10^{-6}$ (easy work).

In this second simulation the starting speeds are the same, but the pickers get tired slower, because of the lower value of μ (easy work). The starting vector $\mathbf{x}^{(0)}$ is randomly generated, in fact, after a few iterations, the behavior of the system does not depend on the initial positions of the pickers.

The results are shown in figure 5.15 and 5.16.

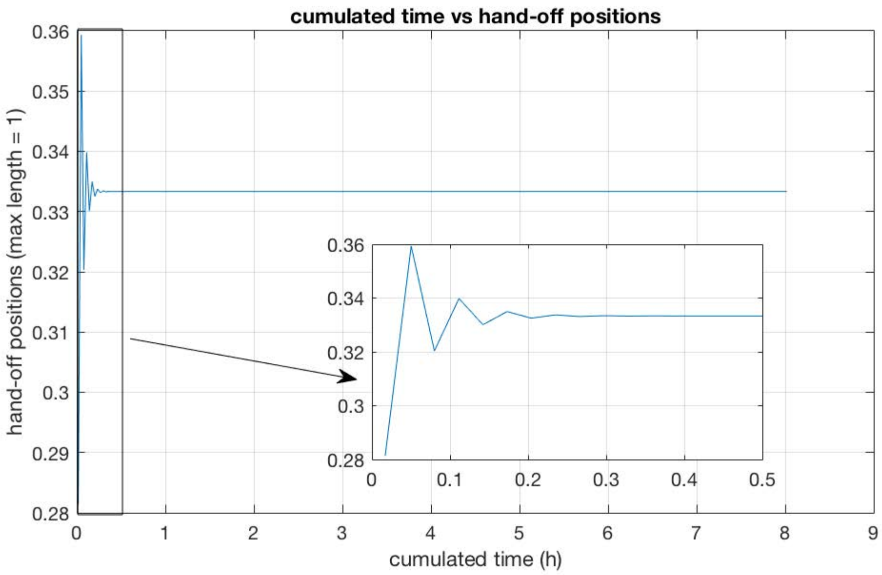

Fig 5.15 - Cumulated time vs hand-off positions when v_{max1} = 0,003 aisles/s, v_{max2} = 0,006 aisles/s, $\mu_1 = \mu_2 = \mu$ = 3,6584*10^{-6} and $\mathbf{x}^{(0)}$ = (0,0975; 0,6324).

As we were expecting, even if we change the values of $\mathbf{x}^{(0)}$ and μ, the results of x_2* is always the same x_2*=0,333. In fact, as we shown before, the value of x_2* depends only on the values of v_{max1} and v_{max2} and not on the values of $\mathbf{x}^{(0)}$ and μ.

The behavior of the system is easy to predict and each picker works always in the same zone for all the 8 hours work shift. The behavior of the system does not depend on the initial position of the operators and the strategy to give the pickers well determined initial positions can always be used, but it is not so efficient and important as in the case of paragraph 5.5.2.

The behavior of the system is different, if we focus on the succession of time between two consecutive hand-offs. Also in this case we have to expect that the time between two consecutive hand-offs grows over time, because the pickers get tired while picking items, so they slow down (μ>0).

The behavior of the system is shown in figure 5.16.

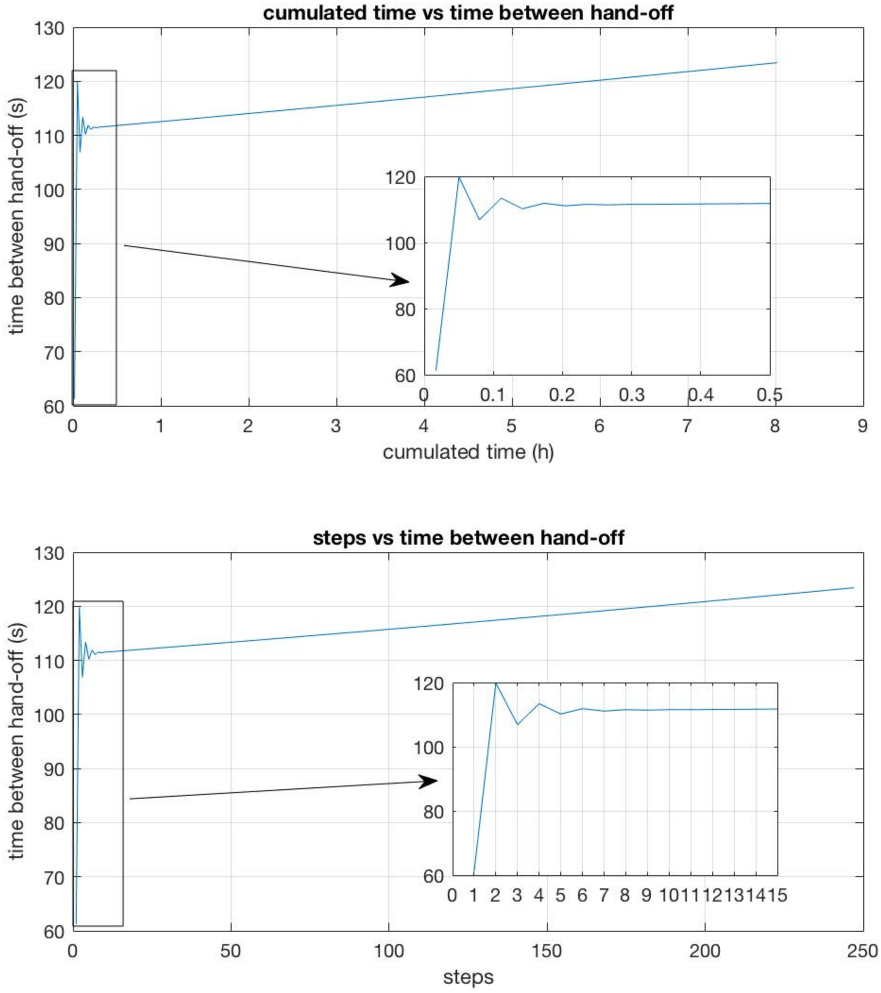

Fig 5.16 - Cumulated time and steps vs time between hand-offs when $v_{max1} = 0{,}003$ aisles/s, $v_{max2} = 0{,}006$ aisles/s, $\mu_1 = \mu_2 = \mu = 3{,}6584 \cdot 10^{-6}$ and $\mathbf{x}^{(0)} = (0{,}0975; 0{,}6324)$. In 8 hours the system finishes 247 orders.

The shape of the two functions is the same, but, after the system becomes balanced (x_2^* converges), the slope of the function is lower than the previous case, in accordance with the fact that the speed of the pickers slows down slower and, consequently, the time between hand-offs is shorter than when pickers are dealing with a harder work (average or hard). This reasoning leads to the fact that the pickers can pick faster and, so, the throughput of the system is higher: in an 8 hours work shift 247 orders are picked (234 with average work), so that the throughput of the system is 5,56% higher.

Also in this case it is possible to calculate the time t° between two consecutive hand-offs after a certain time t.

$t° = 1 / (e^{-\mu t} * (v_{max1} + v_{max2})) = t^* / e^{-\mu t}$

$t°(8\ h) = 111{,}11/0{,}9 = 123{,}45$

As we have already told the value 0,9 corresponds exactly to the factor of reduction in an easy work; moreover, the result is consistent with the result obtained from the simulation and shown in chart 5.2 (see next page). Another important fact to notice is that the value of t* depends only on the value of v_{max1} and v_{max2}, so it is the same for each value of μ, once that v_{max1} and v_{max2} have been chosen.

We ran also a third simulation with the following starting data: v_{2max} = 0,006 aisles/s, v_{1max} = 0,003 aisles/s, μ =12,3850*10^{-6} (easy work), $\mathbf{x}^{(0)}$ = (0,4854; 0,9572). Also in this case the system balances itself after a few iterations. The fixed point x_2* is always x_2* = 0,333, confirming another time that the fixed point x_2* depends only on v_{max1} and v_{max2} and not on the values of $\mathbf{x}^{(0)}$ and μ. In conclusion, in all the three cases, the value of x_2* is the same. The throughput of the system in an eight hours work shift is 220 orders picked, against 247 that we obtained with an easy work and 234 with an average work. This confirms what the common sense suggested before: the harder is the work and the smaller is the throughput of the system.

At the end, we ran a forth simulation with μ=0: the convergence point is always x2*=0,333, the time between the last two hand-offs after 8 hours and constant over all the 8 hours is t*=111,11 s [30] and the throughput in the work shift is 260 orders [31].

Down here we will build a chart to sum up the behavior of the bucket brigade system with the four different levels of μ, if the pickers have different v_{max}, but same μ. The chart is built for v_{2max} = 0,006 aisles/s, v_{1max} = 0,003 aisles/s and random values of $\mathbf{x}^{(0)}$.

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[30] In accordance with the formula $t^*=1/(v_1+v_2)$ found by Bartholdi and Eisenstein (1996a).

[31] In accordance with the formula $TR=v_1+v_2$ found by Bartholdi and Eisenstein (1996a).

Chart 5.2 - Behavior of the bucket brigade system when pickers have different v_{max} and same μ. The starting data to build the chart are v_{2max} = 0,006 aisles/s, v_{1max} = 0,003 aisles/s and random values of $\mathbf{x}^{(0)}$. The chart compares four different kinds of work: no effort, easy work, average work and hard work. The results given are the fixed point of the hand-off positions x_2*, the time between consecutive hand-offs after 8 hours t°(8) (time between the two last hand-offs) and the total throughput of the system in the work shift.

	x_2*	t°(8 h) (s)	throughput (orders in 8 h)
no effort (μ=0)	0,333	111,11	260
easy work	0,333	123,43	247
average work	0,333	138,80	234
hard work	0,333	158,49	220

Chart 7.2 confirms that the succession of hand-off positions converges, after a few iteration, to a value $x_2^* = v_{1max} / (v_{1max}+v_{2max})$, so this value does not depend on the starting vector of the initial positions $\mathbf{x}^{(0)}$. The time t°(8), which is the time between the last two hand-offs of the work shift, grows with the level of effort required by the work. In particular, from the value t*=111,11 s, the time t°(8) is obtained calculating t*/0,9 in case of easy work, t*/0,8 in case of average work and t*/0,7 in case of hard work. The throughput of the system, then, decreases with the level of effort that the picking requires.

In conclusion, if the bucket brigade has pickers with different v_{max} and same μ, the system performs very well, because it balances itself very fast and both the position of hand-off and the time between two consecutive hand-offs have a regular shape and are balanced over time. Under this kind of conditions it is simpler to organize the work, schedule the departure of the trucks and so fulfill the orders of the customer, because all the orders will be ready after the same amount of time.

5.5.4 Pickers with same v_{max} and different μ

The third step is to consider pickers with the same starting speed, but that get tired in different ways. This leads to $v_{max1} = v_{max2} = v_{max}$ and $\mu_1 < \mu_2$ or $\mu_1 > \mu_2$. According to Bartholdi and Eisenstein (1996a), to have the maximum throughput the workers should be ordered from the slowest to the fastest. In this case we decided that the worker who has the minimum value of μ will be the one closer to the end of the aisle. Immediately after the beginning of the work shift, the picker with the lower value of μ will be faster than the picker with the higher value of μ, because the former gets tired slower and so he decreases his speed slower. For this reason we will call "picker 2" the picker with the picker with lower μ and "picker 1" the picker with the higher μ, so that it will be always $\mu_1 > \mu_2$.

The equations obtained in 5.4.3 can be simplified, because $\mu_1 = \mu_2 = \mu$.

After a simple algebra, the succession of hand-off positions is:

$$x_2^{(n)} = \frac{e^{-\mu_1 \cdot t}}{e^{-\mu_2 \cdot t}} \cdot \left(1 - x_2^{(n-1)}\right)$$

$$x_2^{(n)} = \frac{e^{-\mu_1 \cdot \left(t_1 + \ldots + t_{n-1}\right)}}{e^{-\mu_2 \cdot \left(t_1 + \ldots + t_{n-1}\right)}} \cdot \left(1 - x_2^{(n-1)}\right)$$

And it is not so easy to demonstrate that the system has not a fixed point of convergence. The idea to follow to understand why the fixed point does not converge is the fact that the instant ratio between the speeds of the pickers v_1/v_2 is changing continuously over time. In particular v_1/v_2 is decreasing, because $\mu_1 > \mu_2$, so v_1 is decreasing faster than v_2. In conclusion we have to expect that the hand-off position will shift back towards the beginning of the line.

The succession of times between two consecutive hand-offs, which we could obtain working on the equation at the end of 5.4.3, is very complicated and it is very difficult to work on it and to find if it converges to a unique fixed point or not. The behavior that we expect is that the time between two consecutive hand-offs will grow over time, because the pickers slow down over time (μ>0).

In this first simulation we will run the code with $v_{1max} = v_{2max} = 0{,}006$ aisles/s (see figure 5.17), $\mu_1 = 3{,}6584 * 10^{-6}$ (easy work), $\mu_2 = 0$ (no effort), $\mathbf{x}^{(0)} = (0{,}1576; 0{,}9706)$.

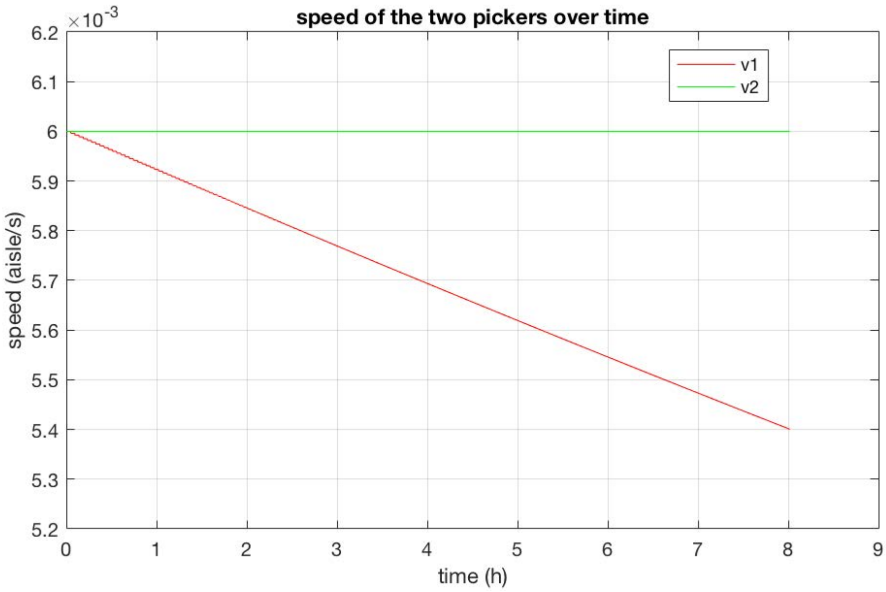


Fig 5.17 - Speed of the operators over time in a work shift in the case of $v_{max1} = v_{max2} = 0{,}006$ aisles/s, $\mu_1 = 3{,}6584 * 10^{-6}$ (easy work) and $\mu_2 = 0$ (no effort).

The results are shown in figure 5.18 and 5.19.

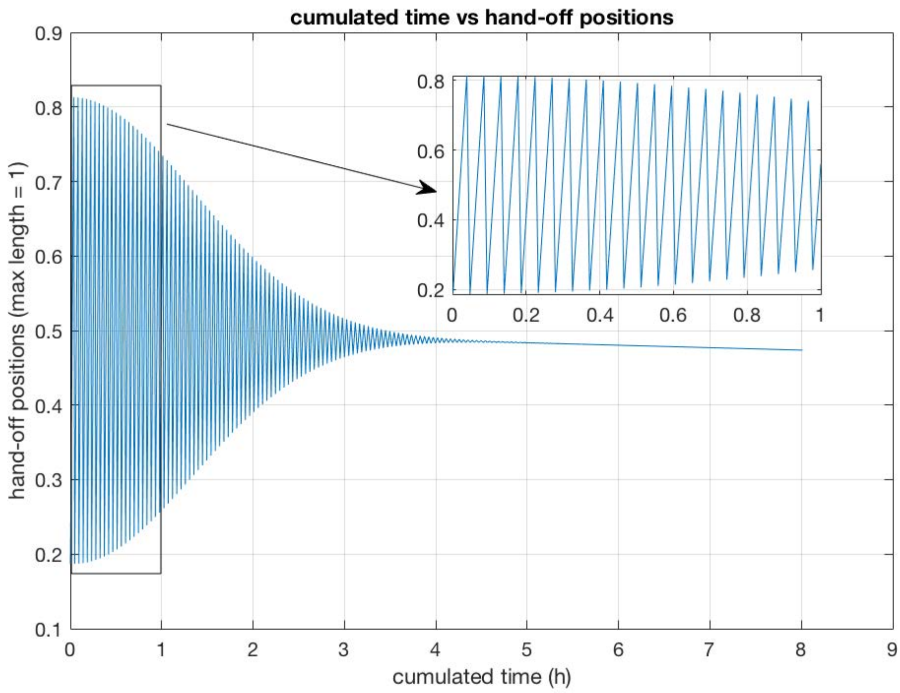


Fig 5.18 - Cumulated time vs hand-off positions when $v_{max1} = v_{max2} = 0{,}006$ aisles/s, $\mu_1 = 3{,}6584 * 10^{-6}$ (easy work), $\mu_2 = 0$ (no effort) and $\mathbf{x}^{(0)} = (0{,}1576; 0{,}9706)$.

The first fact that we notice in this plot is the scattering of the hand-off positions, in particular at the beginning of the work shift, due to the very close speeds of the pickers at the beginning of the work shift. At the beginning, in fact $v_{max1} = v_{max2}$, but after one iteration the pickers' speeds start to have different values, because of the different values of μ. At the beginning, the system behaves exactly like the bucket brigade that we studied in the first case[32] (see 5.5.2). After a few iterations, as we told, the speeds of the pickers become different, and the system seems to converge, even if it does not converge to a fixed point, but the time between two consecutive hand-offs goes down along the line over time. The hand-off positions seem to be well-ordered only after 4 hours.

As we saw in the previous paragraph, there is a formula to find the position of the hand-off $x_2°$ after a certain time t from the beginning of the work shift[33]. It is possible to find the equation we are seeking, starting always from the results obtained under the hypothesis of normative model.

$$x_2° = v_1 / (v_1+v_2) = (v_{max1} * e^{-\mu 1*t}) / (v_{max1} * e^{-\mu 1*t} + v_{max2} * e^{-\mu 2*t})$$

$$= (v_{max} * e^{-\mu 1*t}) / (v_{max} * e^{-\mu 1*t} + v_{max} * e^{-\mu 2*t})$$

$$= (v_{max} * e^{-\mu 1*t}) / (v_{max} * (e^{-\mu 1*t} + e^{-\mu 2*t}))$$

$$= e^{-\mu 1*t} / (e^{-\mu 1*t} + e^{-\mu 2*t})$$

As we could expect, $x_2°$ is always $0 < x_2° <1$, in accordance with the fact that the hand-off position is along the aisle of normalized length 1.

In our simulation, the function gives the following results, confirmed in figure 5.18:

$$x_2° = e^{-3,6584*10\text{^}-6*t} / (e^{-3,6584*10\text{^}-6*t} + 1)$$

$x_2°(0 \text{ h}) = 0,5$

$x_2°(1 \text{ h}) = 0,497$

$x_2°(4 \text{ h}) = 0,487$

[32] In fact the average of the data at the beginning of the work shift is again 0,5 as we proved in paragraph 5.5.2.

[33] We will call it $x_2°$ and not x_2* because in this case the hand-off position does not converge, but it shifts back along the line over time.

$x_2°(8\ h) = 0,474$

The results have some similarities even when we speak about time between two consecutive hand-offs, as shown in figure 5.19.

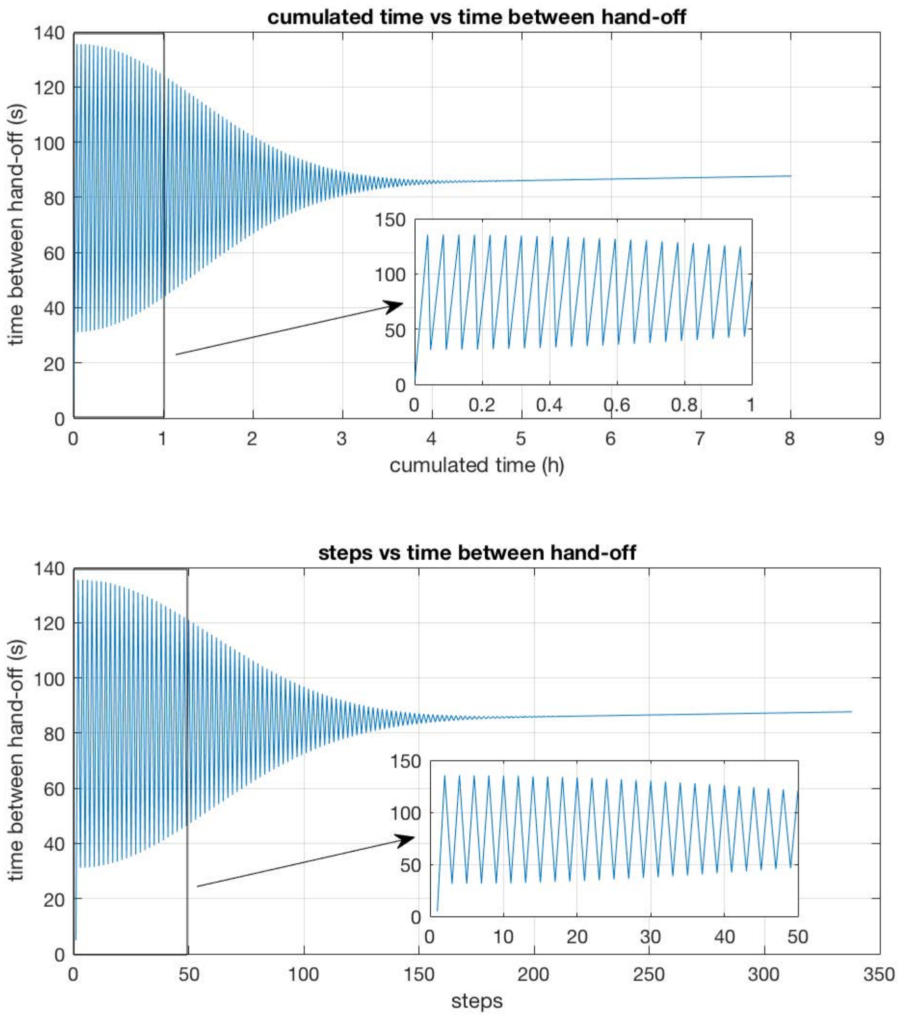


Fig 5.19 - Cumulated time and steps vs time between hand-offs when $v_{max1} = v_{max2} = 0,006$ aisles/s, $\mu_1 = 3,6584*10^{-6}$ (easy work), $\mu_2 = 0$ (no effort) and $\mathbf{x}^{(0)} = (0,1576; 0,9706)$. In 8 hours the system finishes 329 orders.

Also in the case of the time between two consecutive hand-offs, the system seems not to converge in the first three-four hours, because the speed of the pickers are very close and the behavior of the system is close to the case we described in 5.5.2. In this case, contrary to what we saw in the hand-off position, the time between two consecutive hand-offs is longer over time, in fact the slope of the plots is positive, so that the function is growing. This is in accordance with the fact that picker 1 slows down over time and so he picks slower: consequently the time he needs to pick is longer. After 8 hours, the throughput of the system is 329 finished orders.

Also in this case it is possible to find a function t° that gives the average time between two consecutive hand-offs after time t from the beginning of the work shift. To find it, we have to start again from the hypothesis of the normative model.

$t° = 1 / (v_1+v_2) = 1 / (v_{max1} * e^{-\mu1*t} + v_{max2} * e^{-\mu2*t})$

$= 1 / (v_{max} * e^{-\mu1*t} + v_{max} * e^{-\mu2*t})$

$= 1 / (v_{max} * (e^{-\mu1*t} + e^{-\mu2*t}))$

And also here it is possible to calculate t° for some values of t.

$t°(0h) = 1 / (v_{max1} + v_{max2}) = t* = 83{,}33$ s

$t°(1h) = 83{,}88$ s

$t°(4h) = 85{,}53$ s

$t°(8h) = 87{,}72$ s

In this second simulation we will run the code with $v_{1max} = v_{2max} = 0{,}006$ aisles/s (see figure 5.17), $\mu_1 = 12{,}3850*10^{-6}$ (hard work), $\mu_2 = 0$ (no effort), $\mathbf{x}^{(0)} = (0{,}1419; 0{,}4218)$. So the speeds are the same, but the value of μ_1 changes.

The results are shown in figure 5.20 and 5.21.

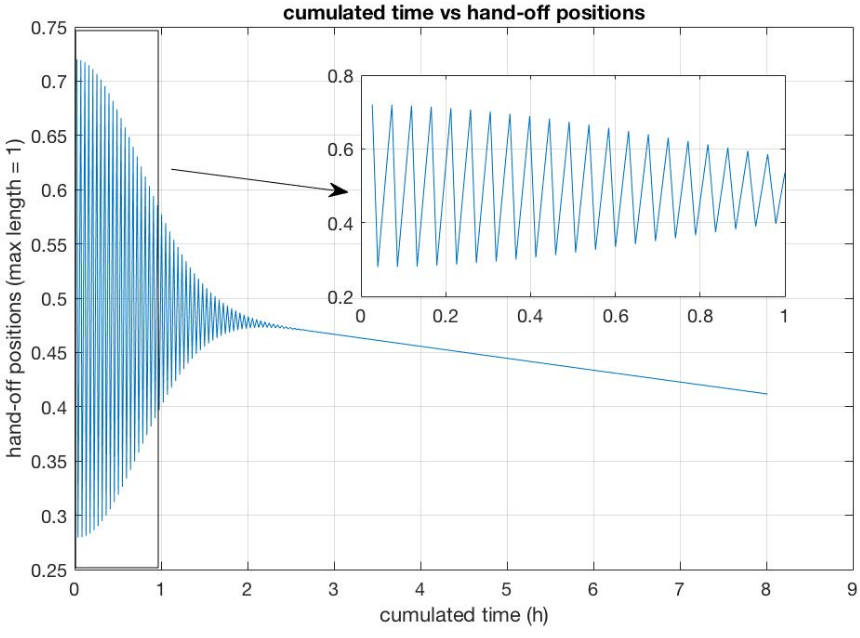
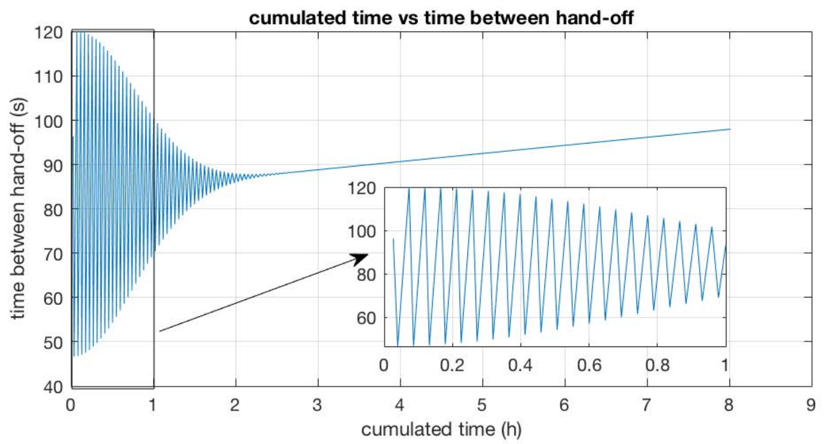


Fig 5.20 - Cumulated time vs hand-off positions when $v_{max1} = v_{max2} = 0{,}006$ aisles/s, $\mu_1 = 12{,}3850*10^{-6}$ (hard work), $\mu_2 = 0$ (no effort) and $\mathbf{x}^{(0)} = (0{,}1419; 0{,}4218)$.

In this simulation the system finds his balance after a little bit more than 2 hours, so it is faster than in the first simulation. This fact is due to the higher difference between the values of μ: 0 and $3,6584*10^{-6}$ in the first simulation, against 0 and $12,3850*10^{-6}$ in this one. If the value of μ are more different, this means that the speeds of the pickers will be different in a shorter time and this allows the system to find his balance sooner. Another fact that helps the system to find balance sooner or, at least, to have less scatter of hand-off positions at the beginning of the work shift, is that the starting positions of the pickers should be as much as possible $x_1^{(0)} + 0,5 = x_2^{(0)}$, but we will see this fact more in detail later, using some simulations. Another important fact to notice is that the slope of the function, when the system find its balance, is steeper than in the first simulation. This is always due to the bigger difference between the values of μ. In fact the more different are the values of μ, the more the speed of the pickers becomes different over time and the more the hand-off position shifts itself back towards the beginning of the aisle, because picker 2 is step by step faster than picker 1.

Some observations can be done also looking figure 5.21.



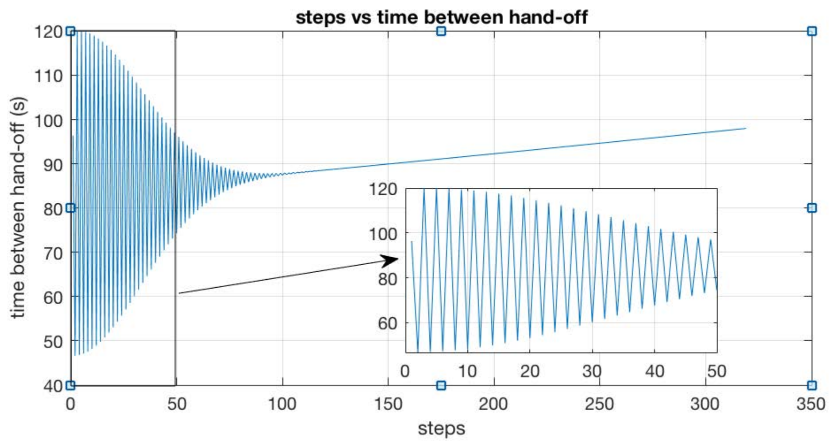


Fig 5.21 - Cumulated time and steps vs time between hand-offs when $v_{max1} = v_{max2} = 0,006$ aisles/s, $\mu_1 = 12,3850*10^{-6}$ (hard work), $\mu_2 = 0$ (no effort) and $\mathbf{x}^{(0)} = (0,1419;\ 0,4218)$. In 8 hours the system finishes 319 orders.

Also here the time the system needs to find its balance is less than in the first simulation. The cause of this is always the fact that the values of μ are very different. Then, the slope of the function is positive and steeper than in the first simulation, because the values of μ are higher: this means that pickers get tired faster, that they pick slower and consequently the time between two consecutive hand-offs grows over time. After 8 hours, the throughput of the system is 319 finished orders, according with the fact that, if a picker slows down more, the throughput is minor (319 here, against 329 in the first simulation).

Also here it is possible to calculate some values of t°, to show that t°(0h) is the same and that the function grows faster, because the values of μ are more distant.

t°(0h) = 1 / (v_{max1} + v_{max2}) = t* = 83,33 s (exactly like in the first simulation)

t°(1h) = 85,19 s (83,88 s in the previous simulation)

t°(4h) = 90,74 s (85,53 s in the previous simulation)

t°(8h) = 98,04 s (87,72 s in the previous simulation)

In this third simulation we will run the code with v_{1max} = v_{2max} = 0,003 aisles/s (half speed, see figure 5.22), μ_1 = 12,3850*10^{-6} (hard work), μ_2 = 0 (no effort), $\mathbf{x}^{(0)}$ = (0,0357; 0,6557).

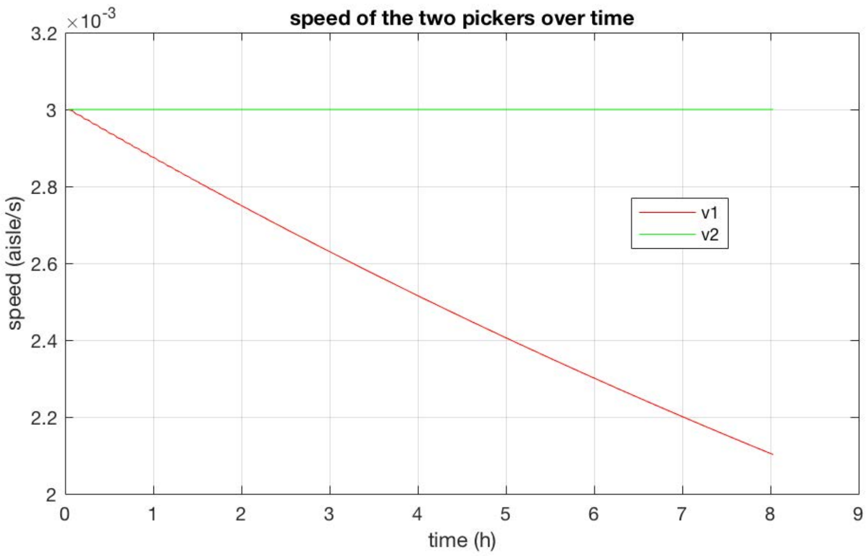


Fig 5.22 - Speed of the operators over time in a work shift in the case of v_{max1} = v_{max2} = 0,003 aisles/s, μ_1 = 12,3850*10^{-6} (hard work) and μ_2 =0 (no effort).

The values of μ are the same than in the second simulation, but the value of v_{max1} = v_{max2} = 0,003 aisles/s, so the starting speed of the two pickers is halved.

The results are shown in figure 5.23 and 5.24.

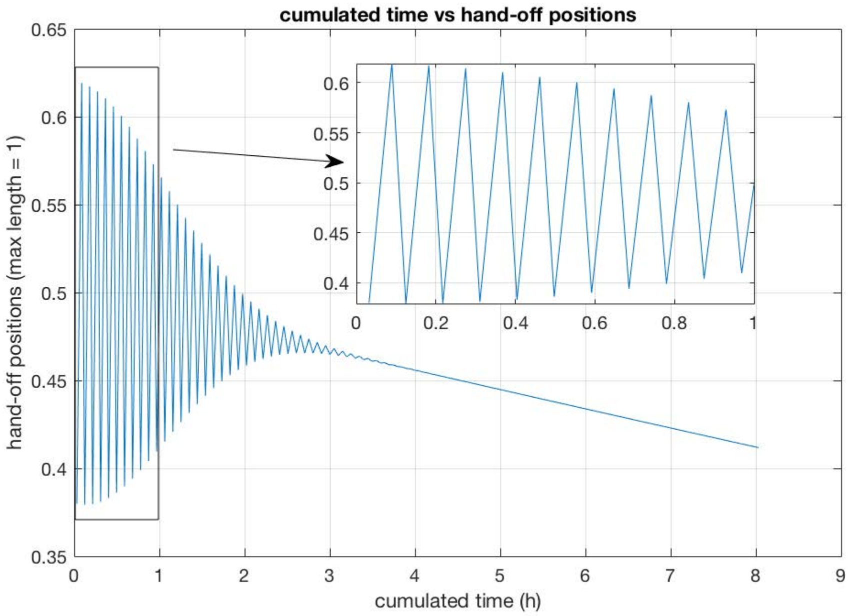


Fig 5.23 - Cumulated time vs hand-off positions when $v_{max1} = v_{max2}$ = 0,003 aisles/s, μ_1 = $12{,}3850 * 10^{-6}$ (hard work), μ_2 =0 (no effort) and $\mathbf{x}^{(0)}$ = (0,0357; 0,6557).

At the end of the 8 hours and more in particular in every moment during the work shift, the position $x_2°$ is the same in the second and in the third simulation: this means that $x_2°$ does not depends on the values of v_{max}, but only on the values of μ_1 and μ_2, in fact in the formula $x_2° = e^{-\mu 1 * t} / (e^{-\mu 1 * t} + e^{-\mu 2 * t})$ no speed is present. The system finds its balance after 3 hours (one hour more than in the first simulation), because the speeds are closer than in the second simulation; in fact the same slowdown in percentage (with the same percentage, because in the second and third simulations the values of μ are the same) is more effective on a higher initial speed (0,006 in the second simulation) then on a lower one (0,003 in the first simulation). Also here, the slope of the function is negative, because picker 1 slows down more than picker 2 over time. The last thing to notice is that the plot is less “dense” than in the previous simulations, because the speed of the pickers is smaller, so that there are less hand-offs in a work shift.

We cannot find the same results speaking about the time between consecutive hand-offs. The formula $t° = 1 / (v_{max} * (e^{-\mu 1 * t} + e^{-\mu 2 * t}))$ depends not only on the values of μ, but also on the value of v_{max}. Also in this case we have to expect that the average function will grow over time, because the pickers slow down (μ>0).

All of these results about time between two consecutive hand-offs find confirmation in figure 5.24.

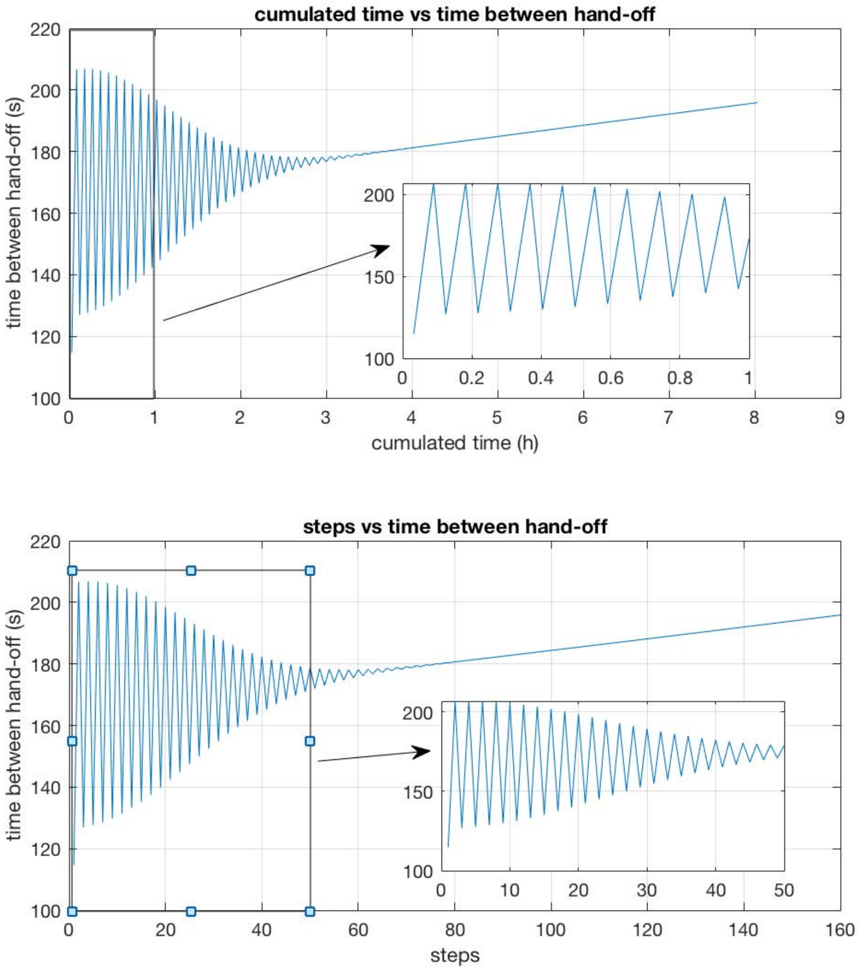


Fig 5.24 - Cumulated time and steps vs time between hand-offs when $v_{max1} = v_{max2} = 0,003$ aisles/s, $\mu_1 = 12,3850*10^{-6}$ (hard work), $\mu_2 = 0$ (no effort) and $\mathbf{x}^{(0)} = (0,0357; 0,6557)$. In 8 hours the system finishes 160 orders.

The slope of the function is always positive after the first period of unbalance. The slope is positive and it is the same as in the second simulation, according with the fact that the values of μ are the same. The most interesting facts to notice are that the time t° is exactly the double of the time t° in the second simulation, in accordance with the formula $t° = 1 / (v_{max} * (e^{-\mu 1*t} + e^{-\mu 2*t}))$: in this third simulation the speed is halved and the values of μ are the same. We can say the same about the throughput: in fact, after 8 hours, the throughput is 160, that is the half [34] of 319, the result that we obtained in the second simulation. This kind of observations are explained better in chart 5.3, at the end of this paragraph.

[34] It is not exactly the half because the different values of $\mathbf{x}^{(0)}$ introduce a little mistake.

As we have already discussed, the starting positions of the pickers can have an influence on the scatter of the data in the plot we represented. In particular, the more the starting positions of the pickers are in a relation $x_1^{(0)} + 0,5 = x_2^{(0)}$, and the more the scatter of data (both in hand-off positions and time between hand-offs) is reduced. In figure 5.25 and 5.26 this fact is shown: the starting data are the same as the previous simulation, but the starting vector of initial positions is different and it is $\mathbf{x}^{(0)} = (0,3028; 0,8326)$.

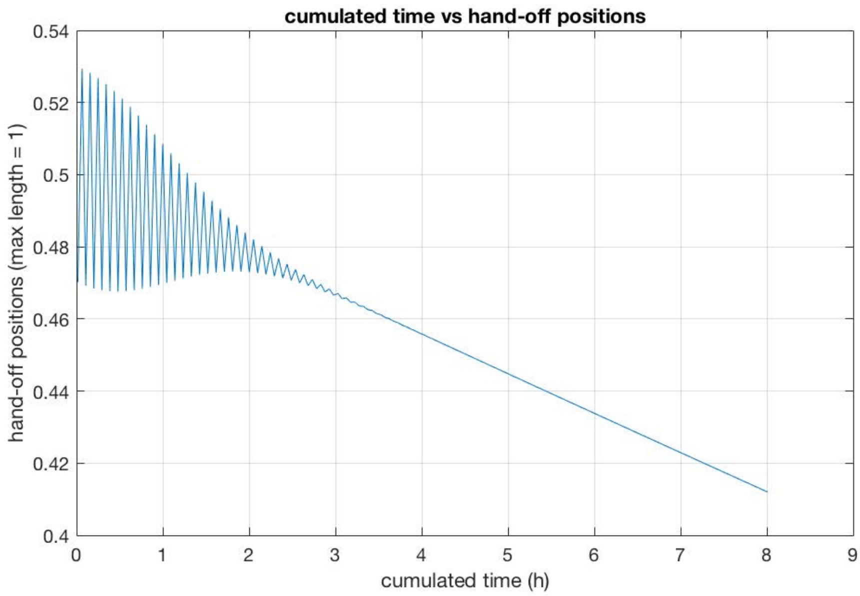


Fig 5.25 - Cumulated time vs hand-off positions when $v_{max1} = v_{max2} = 0{,}003$ aisles/s, $\mu_1 = 12{,}3850*10^{-6}$ (hard work), $\mu_2 = 0$ (no effort) and $\mathbf{x}^{(0)} = (0{,}3028; 0{,}8326)$.

In this case the relationship between the two starting positions is 0,5298, that is very close to 0,5. As it is possible to see in figure 5.25 the sequence of hand-off positions swings less than in the previous simulation, because the difference between the values of the two starting points is closer to 0,5. In particular, if the difference between the starting points is exactly 0,5 the system has no fluctuation, both in hand-off positions and in time between two consecutive hand-offs.

This fact leads to an important result: a manager can control the amplitude of the fluctuations of a bucket brigade system over time, only choosing in the right way the starting positions of the pickers. Taking this little decision the manager can balance the system immediately, from the first iteration. In conclusion, the manager can improve a lot the efficiency of the system only with a very simple decision.

In figure 5.26 , the behavior of the system in time between hand-off is shown.

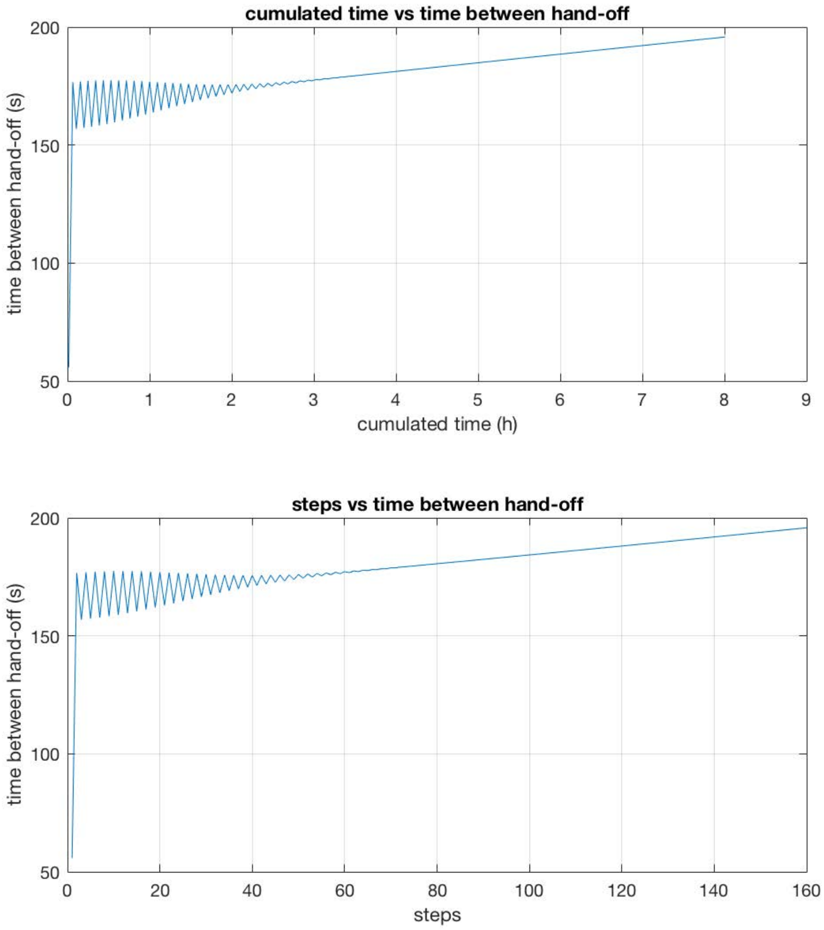


Fig 5.26 - Cumulated time and steps vs time between hand-offs when v_{max1} = v_{max2} = 0,003 aisles/s, μ_1 = $12,3850*10^{-6}$ (hard work), μ_2 =0 (no effort) and $\mathbf{x}^{(0)}$ = (0,0357; 0,6557). In 8 hours the system finishes 160 orders.

It is important to notice that all the results of $x_2°$ and t° are the same over time, even the throughput after 8 hours, in fact all these results depend on $x_2°$ and t°, which do not depend on the values of the starting points of pickers $\mathbf{x}^{(0)}$. The only difference is that the more $x_1^{(0)} + 0,5 = x_2^{(0)}$ and the less the system is unbalanced[35] at the beginning at the work shift (the fluctuation of data is reduced). This means that the system is less unbalanced and that it is simple to deal with it from a practical point of view. Therefore, when in a warehouse a manager has to deal with this kind of case of bucket brigade he should give to the pickers the starting positions in this relationship: $x_1^{(0)} + 0,5 = x_2^{(0)}$. This works for all the simulations in 5.5.2 as well.

[35] It does not mean that the system converges in a shorter time, it only means that at the beginning, when the system is unbalanced, the results in hand-off positions and time are closer between them, so that the behavior of the system is more regular (less fluctuations). Only if the number is exactly 0,5 the system converges immediately and has no fluctuations.

In the following chart (chart 5.3) we summarize all the results in 12 different cases. The starting data take into account all the possible combinations of μ (6 combinations for each level of speed), first with $v_{max1} = v_{max2} = v_{max} = 0{,}006$ aisles/s, then with $v_{max1} = v_{max2} = v_{max} = 0{,}003$ aisles/s. The initial positions $\mathbf{x}^{(0)}$ are random.

Chart 5.3 - Comparison between the 6 possible kinds of μ combinations (always under the hypothesis $\mu_1 > \mu_2$), considering first $v_{max1} = v_{max2} = v_{max} = 0{,}006$ aisles/s and then $v_{max1} = v_{max2} = v_{max} = 0{,}003$ aisles/s. The simulations with (*) are the one that we presented in this paragraph till now (see above).

	x_2°(8 h)	t°(8 h) (s)	throughput (orders in 8 h)
μ2=0, μ1=easy, v_{max}=0,006 a/s (*)	0,474	87,71	338
μ2=0, μ1=average, v_{max}=0,006 a/s	0,444	92,56	329
μ2=0, μ1=hard, v_{max}=0,006 a/s (*)	0,412	97,99	319
μ2=easy, μ1=average, v_{max}=0,006 a/s	0,473	98,00	320
μ2=easy, μ1=hard, v_{max}=0,006 a/s	0,437	104,13	311
μ2=average, μ1=hard, v_{max}=0,006 a/s	0,466	111,09	302
μ2=0, μ1=easy, v_{max}=0,003 a/s	0,474	175,35	169
μ2=0, μ1=average, v_{max}=0,003 a/s	0,444	185,01	165
μ2=0, μ1=hard, v_{max}=0,003 a/s (*)	0,412	195,88	160
μ2=easy, μ1=average, v_{max}=0,003 a/s	0,470	196,03	161
μ2=easy, μ1=hard, v_{max}=0,003 a/s	0,438	208,03	156
μ2=average, μ1=hard, v_{max}=0,003 a/s	0,467	221,69	150

The value of x_2° after 8 hours is different for each combination of μ, in fact x_2° depends on the ratio of the speeds v_1/v_2 in a certain moment of time t and this ratio varies over time because the speeds change over time, because of the different values of μ that the two pickers have ($x_2° = e^{-\mu1*t} / (e^{-\mu1*t} + e^{-\mu2*t})$). The more the value of μ_1 and μ_2 are similar over time and the more x_2° after 8 hours is going to be closer to the value 0,5[36]. Because of the fact that in both cases with $v_{max1} = v_{max2} = v_{max} = 0{,}006$ aisles/s and with $v_{max1} = v_{max2} = v_{max} = 0{,}003$ aisles/s

[36] value that we found also in paragraph 5.5.2, where both the v_{max} and the values of μ were the same

the ratio between speeds is 1, if the values of μ are the same, also the values of x2° after 8 hours are the same (0,412 both in the second and in the third simulation). Always from chart 5.3, we can notice that if the speed v_{max} is halved, the value of time t° after 8 hours is doubled (t° = 1 / (v_{max} * ($e^{-\mu 1*t}$ + $e^{-\mu 2*t}$))) and the throughput is halved, if the values of μ_1 and μ_2 are the same (always comparing the second and the third simulation). After all this conclusions, we can notice that all the three results x2°, t° and the throughput do not depend on the starting positions of the pickers. As we saw, working on the initial position of the pickers has a consequence only on the amplitude of the fluctuations, both in hand-off positions and in times between two consecutive hand-offs.

In conclusion, if the pickers have the same v_{max} and different μ, the system does not perform well, because the convergence is very slow (sometimes more than four hours) and it is difficult to predict the behavior of the system. Moreover, the worst problem is that the system does not provide a constant throughput over time, when the system is not balanced. This can cause a lot of problems, because the orders will be fulfilled not in regular interval of time, because the time between two consecutive hand-offs is very irregular. Luckily, we noticed that there is a way to solve this problem of slow convergence and that is to work on the starting positions of the pickers. The ideal starting positions of the operators should be in a relationship $x_1^{(0)} + 0,5 = x_2^{(0)}$. If this condition is verified, the system will be balanced from the first iterations and it will work without fluctuations in hand-off positions and time between two consecutive hand-offs. The intervention of a manager who makes the pickers start in the right position of the aisle is of paramount importance in this kind of bucket brigade system. After his intervention, in fact, the system can perform very well, even if the average hand-off positions x_2° will always shift from 0,5 towards the beginning of the aisle and the time between two consecutive hand-offs will always become longer because of the slowdown of the pickers.

5.5.5 Pickers with different v_{max} and different μ

The last step is to see what happens if the operators are completely different, both in maximum speed and in the way they get tired. This means that both the value of maximum speed v_{max} and μ are different. We will consider the cases where $v_{max1} < v_{max2}$ or $v_{max1} > v_{max2}$ and $\mu_1 < \mu_2$ or $\mu_1 > \mu_2$.

According to the hypothesis in 5.5.1, "picker 2" is the one who has the highest v_{max}: it means that we only have to study the case $v_{max1} < v_{max2}$. In this case, if $\mu_1 < \mu_2$, a problem could arise: if "picker 2" has a higher μ than "picker 1", it could happen that at a certain moment in the work shift, "picker 1" could be faster than "picker 2" and, after some other iterations, this could lead to blockage, with the result of a loss in throughput.

To check a priori if the pickers will be blocked or not, we have to follow the following steps:

- Compare the value of μ_1 and μ_2: if $\mu_1 < \mu_2$ the problem of blockage could be real (in this case read the next step), if $\mu_1 > \mu_2$ there is no problem.

- Intersecate the two functions of speed $v_1(t) = v_{1max}*e^{-\mu_1 t}$ and $v_2(t) = v_{2max}*e^{-\mu_2 t}$. To find the point of intersection we have to solve the system when $v_1(t) = v_2(t)$ and t are the same. Combining the two equation, it is possible to find t^:

$$t^\wedge = \frac{\ln\left(\frac{v_{max2}}{v_{max1}}\right)}{\mu_2 - \mu_1}$$

  If t^ < 8 h (28800 s), then the possibility of blockage could exist (in this case go on reading the next step); if t^ > 8 h (28800 s), then blockage is no longer possible. The system starts losing its exponential average behavior and diverges; the more the time passes by and the more the hand-off positions and the time between two consecutive hand-offs swing.

- See with the simulation if the system diverges "enough" or not[37]. It is possible to understand if the pickers get blocked or not looking at the plots and the numerical output of the code. If the plots are meaningless and in the

[37] We did not find an analytical way to find the time t from the beginning of the work shift, when the first blockage happens.

output data of the simulation there are hand-off positions $x_2 < 0$ or $x_2 > 1$ and the time between two consecutive hand-offs negative, the fastest picker is blocked. The blockage exists when some of these anomalies happen. In particular the exact moment when the fastest picker is first blocked is when the first one of these anomalies comes out.

Once we decided to call "picker 2" the picker with maximum v_{max} ($v_{max2} > v_{max1}$), there are three cases that we can study:

- $\mu_1 > \mu_2$ (divergent speeds): in this case we expect a fast convergence, with hand-off position shifting towards the beginning of the line over time and the time between two consecutive hand-offs growing over time.
- $\mu_1 < \mu_2$ and $\ln(v_{max2} / v_{max1}) / (\mu_2 - \mu_1) = t\hat{} > 8$ h (convergent, but not crossing speeds): in this case the system does not suffer from blockage and we expect a fast convergence as well, with the hand-off position shifting towards the end of the line over time and the time between two consecutive hand-offs growing over time.
- $\mu_1 < \mu_2$ and $\ln(v_{max2} / v_{max1}) / (\mu_2 - \mu_1) = t\hat{} < 8$ h (convergent and crossing speed): in this case the system could suffer from blockage. We expect a fast convergence at the beginning of the work shift and, after time $t\hat{}$, the slower picker becomes the faster and, after a few time more, the system starts to diverge again: it means that there is chance of blockage. Blockage exists if the system has enough time to find such an unbalance that leads to one of the anomalies that we described before (the plots become meaningless and in the output data of the simulation there are hand-off positions $x_2 < 0$ or $x_2 > 1$ and time between two consecutive hand-offs negative); this means that the more the pickers' speed cross each other early in the work shift and the higher the probability of blockage is. In other words we can divide this case in two sub-cases: the first one is when the system diverges, because of the crossing speeds, but not enough to create blockage and the second one is when the system diverges sufficiently to create blockage. In all the cases, the time between two consecutive hand-offs always grows over time.

We will run five simulations, one for each kind of situation that could happen.

In the first simulation, the starting data are v_{1max} = 0,003 aisles/s, v_{2max} = 0,006 aisles/s (see figure 5.27), μ_1 = 12,3850*10^{-6} (hard work), μ_2 = 3,6584*10^{-6} (easy work), $\mathbf{x}^{(0)}$ = (0,1869; 0,4898).

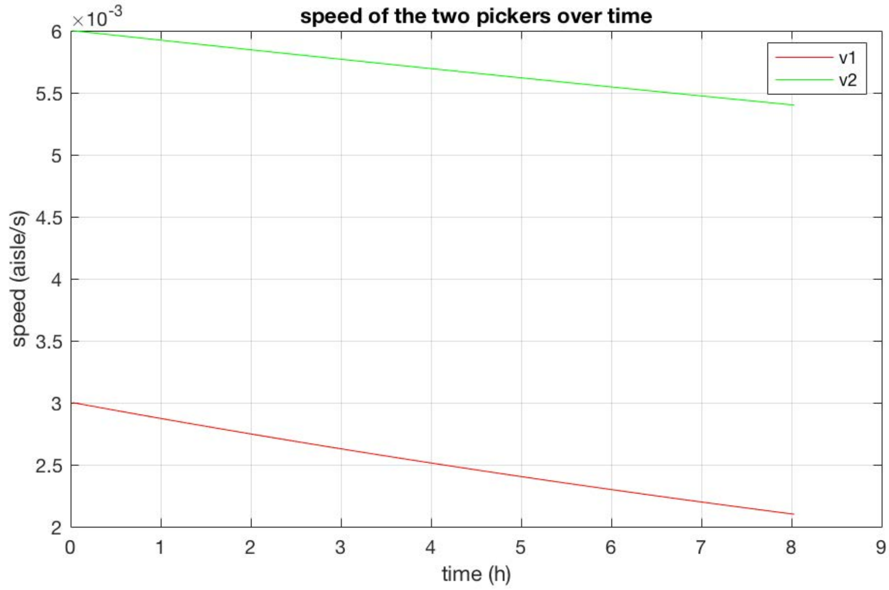


Fig 5.27 - Speed of the operators over time in a work shift in the case of v_{max1} = 0,003 aisles/s, v_{max2} = 0,006 aisles/s, μ_1 = 12,3850*10^{-6} (hard work) and μ_2 = 3,6584*10^{-6} (easy work).

In this simulation $\mu_1 > \mu_2$, so there is no possibility of blockage, because picker 2 is always faster than picker 1.

The results are shown in figure 5.28 and 5.29.

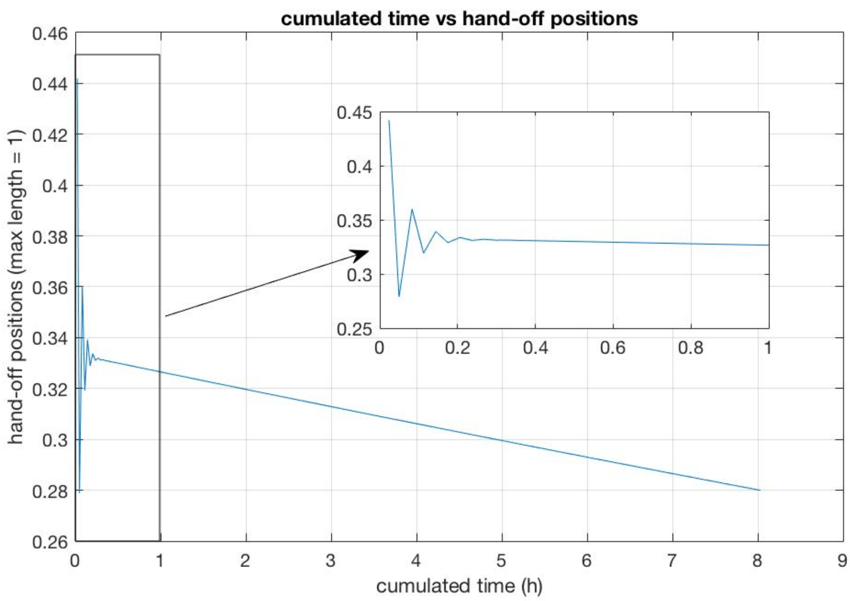


Fig 5.28 - Cumulated time vs hand-off positions when v_{max1} = 0,003 aisles/s, v_{max2} = 0,006 aisles/s, μ_1 = 12,3850*10^{-6} (hard work), μ_2 = 3,6584*10^{-6} (easy work) and $\mathbf{x}^{(0)}$ = (0,1869; 0,4898).

The speeds are diverging over time. This leads to the fact that the hand-off positions shift towards the beginning of the line over time, because the ratio v_1/v_2 is decreasing over time. The system converges very fast: after 10 minutes it has already balanced itself.

Also in the case of different maximum (starting) speeds v_{max} and different values of μ, it is possible to find a function that gives the average hand-off position after a period of time t from the beginning of the work shift.

$x° = v_1 / (v_1+v_2) = (v_{max1} * e^{-\mu 1*t}) / (v_{max1} * e^{-\mu 1*t} + v_{max2} * e^{-\mu 2*t})$

With this formula it is possible to verify that figure 5.27 is correct.

$x°(0) = v_{max1} / (v_{max1} + v_{max2}) = x^* = 0{,}333$

$x°(4) = 0{,}306$

$x°(8) = 0{,}280$

Speaking about the time between two consecutive hand-offs, the results are shown in figure 5.29.

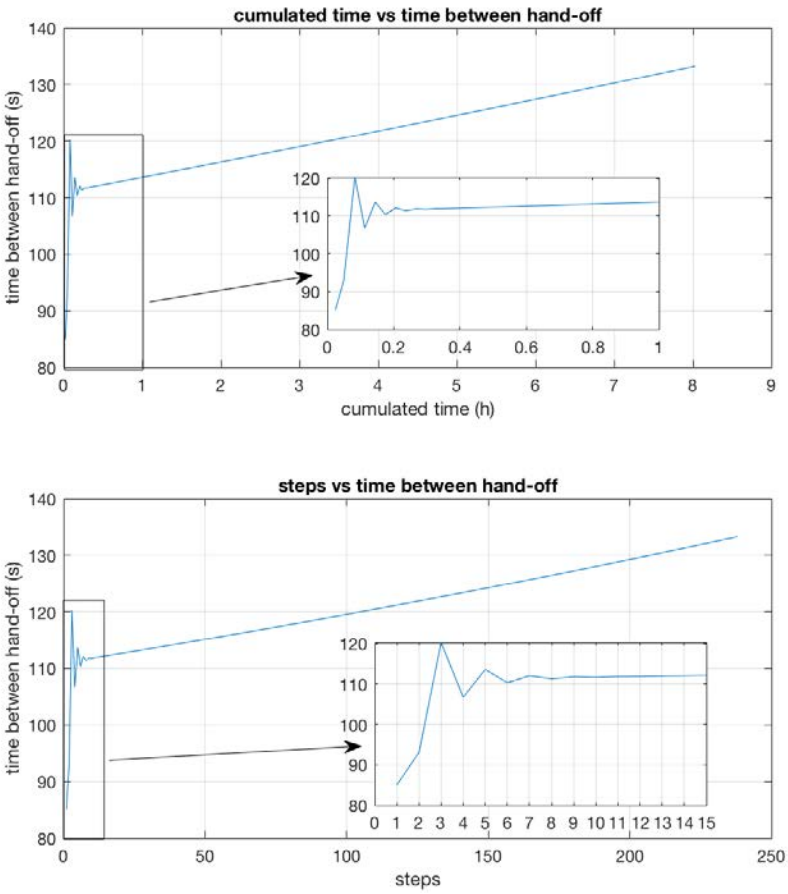


Fig 5.29 - Cumulated time and steps vs time between hand-offs when v_{max1} = 0,003 aisles/s, v_{max2} = 0,006 aisles/s, μ_1 = 12,3850*10^{-6} (hard work), μ_2 = 3,6584*10^{-6} (easy work) and $\mathbf{x}^{(0)}$ = (0,1869; 0,4898). In 8 hours the system finishes 238 orders.

The system converges very fast and it is very stable, since the beginning of the work shift. The time between two consecutive hand-offs is growing over time, because, as always, the pickers are slowing down over time because of fatigue ($\mu > 0$). In 8 hours the bucket brigade finishes 238 orders.

Also here it is possible to find a function that gives the average time between two consecutive hand-offs after time t from the beginning of the work shift.

$t° = 1 / (v_1+v_2) = 1 / (v_{max1} * e^{-\mu 1*t} + v_{max2} * e^{-\mu 2*t})$

With this function it is possible to verify the correctness of the plot (fig 5.29).

$t°(0) = 1 / (v_{max1}+v_{max2}) = 111{,}11$

$t°(4) = 121{,}92$

$t°(8) = 133{,}33$

In the second simulation, the starting data are $v_{1max} = 0{,}003$ aisles/s, $v_{2max} = 0{,}006$ aisles/s (see figure 5.30), $\mu_1 = 3{,}6584*10^{-6}$ (easy work), $\mu_2 = 12{,}3850*10^{-6}$ (hard work), $\mathbf{x}^{(0)} = (0{,}7094; 0{,}7547)$.

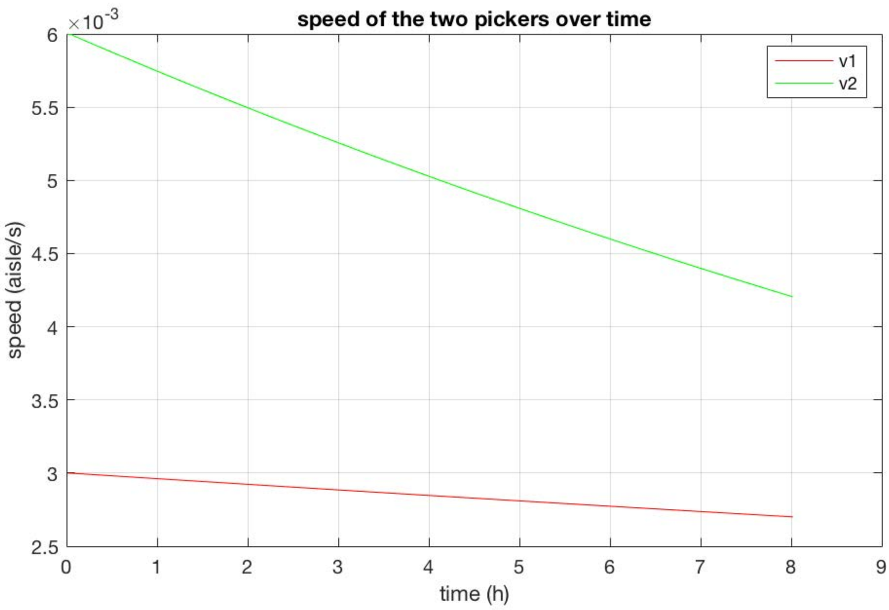


Fig 5.30 - Speed of the operators over time in a work shift in the case of $v_{max1} = 0{,}003$ aisles/s, $v_{max2} = 0{,}006$ aisles/s, $\mu_1 = 3{,}6584*10^{-6}$ (easy work) and $\mu_2 = 12{,}3850*10^{-6}$ (hard work).

In this simulation $\mu_1<\mu_2$, so there is possibility of blockage, because picker 2 slows down faster than picker 1. To check if the possibility of blockage is real or not we have to calculate the value of t^ = 22,06. Because of the fact that 22,06 > 8, there is no intersection point between the two speeds in 8 hours, so there is no possibility of blockage.

The results are shown in figure 5.31 and 5.32.

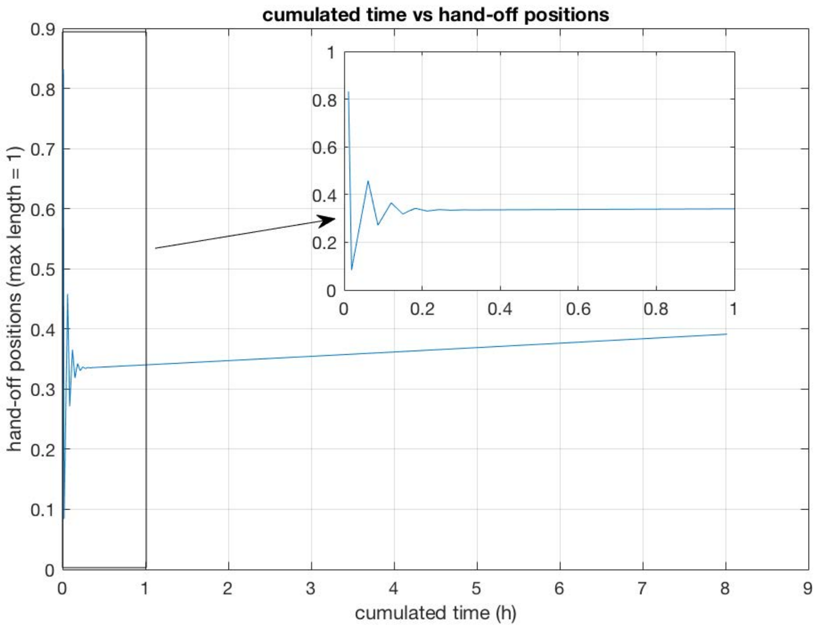


Fig 5.31 - Cumulated time vs hand-off positions when v_{max1} = 0,003 aisles/s, v_{max2} = 0,006 aisles/s, μ_1 = 3,6584*10^{-6} (easy work), μ_2 = 12,3850*10^{-6} (hard work) and $\mathbf{x}^{(0)}$ = (0,7094; 0,7547).

The speeds are converging over time. This leads to the fact that the hand-off positions are shifted towards the end of the line over time, because the ratio v_1/v_2 is growing over time. The system converges very fast: after 10 minutes it has already balanced itself. Notice that this is the first time that we are studying a case in which the hand-off position is shifting toward the end of the line and not towards the beginning.

Also here it is possible to calculate analytically the average of the function:

$x°(0) = v_{max1} / (v_{max1} + v_{max2}) = x^* = 0,333$ (as in the first simulation)

$x°(4) = 0,362$

$x°(8) = 0,391$

And x° is growing over time and not decreasing, as we were expecting.

Speaking about the time between two consecutive hand-offs, the results are shown in figure 5.32.

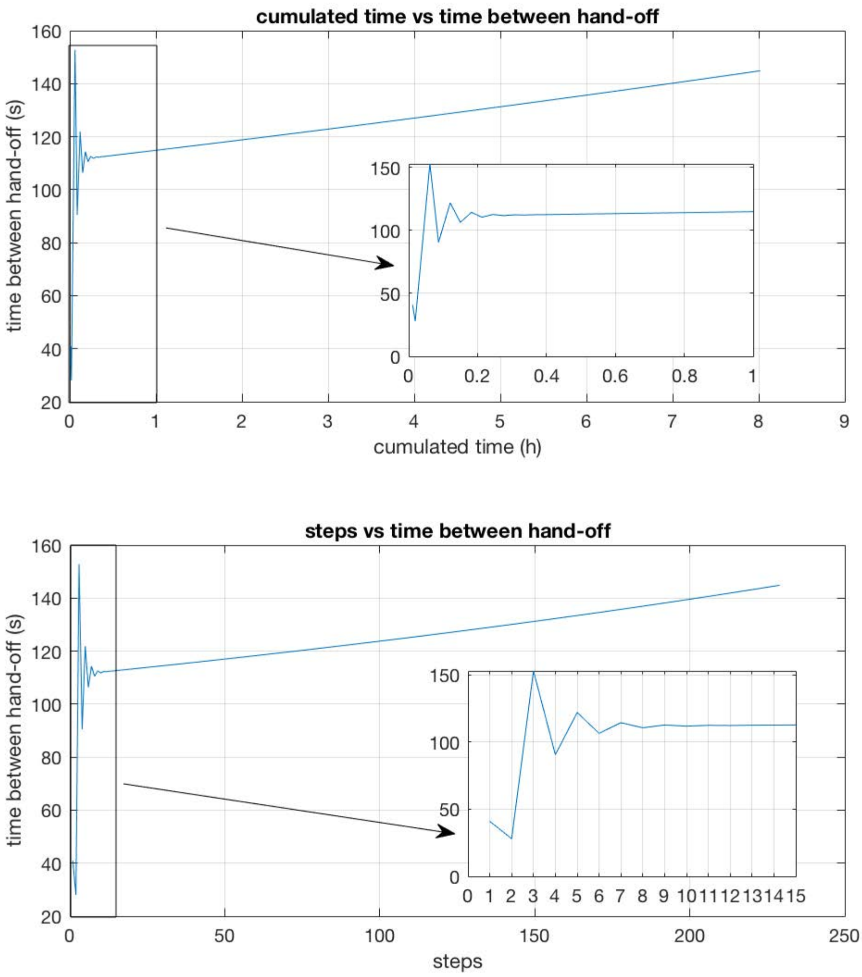


Fig 5.32 - Cumulated time and steps vs time between hand-offs when v_{max1} = 0,003 aisles/s, v_{max2} = 0,006 aisles/s, μ_1 = 3,6584*10^{-6} (easy work), μ_2 = 12,3850*10^{-6} (hard work) and $\mathbf{x}^{(0)}$ = (0,7094; 0,7547). In 8 hours the system finishes 229 orders.

The system converges very fast and it is very stable, since the beginning of the work shift. The time between two consecutive hand-offs is increasing, because, as always, the pickers are slowing down over time because of fatigue ($\mu > 0$). In 8 hours the bucket brigade finishes 229 orders: less then the simulation before, because the speeds are decreasing more, even if the values of μ are the same.

And also here we can check if the function we found is correct or not:

$t°(0) = 1 / (v_{max1}+v_{max2}) = 111{,}11$

$t°(4) = 127{,}13$

$t°(8) = 144{,}93$

In the third simulation, the starting data are v_{1max} = 0,005 aisles/s, v_{2max} = 0,006 aisles/s (see figure 5.33), $\mu_1 = 3{,}6584 \cdot 10^{-6}$ (easy work), $\mu_2 = 12{,}3850 \cdot 10^{-6}$ (hard work), $\mathbf{x}^{(0)}$ = (0,1190; 0,4984).

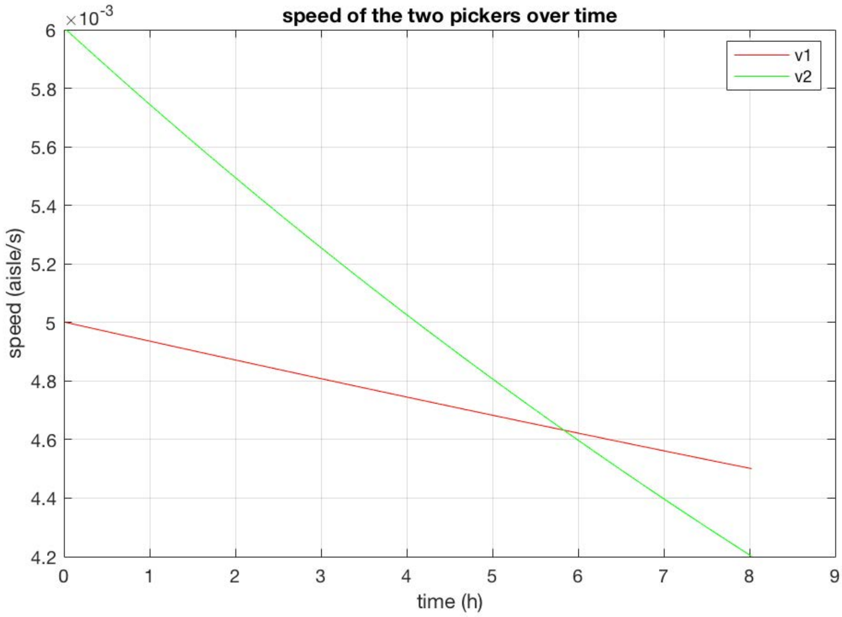


Fig 5.33 - Speed of the operators over time in a work shift in the case of v_{max1} = 0,005 aisles/s, v_{max2} = 0,006 aisles/s, $\mu_1 = 3{,}6584 \cdot 10^{-6}$ (easy work) and $\mu_2 = 12{,}3850 \cdot 10^{-6}$ (hard work).

In this simulation $\mu_1 < \mu_2$, so there is possibility of blockage, because picker 2 slows down faster than picker 1. To check if the possibility of blockage is real or not, we have calculated the value of t^ = 5,80 h.

$$t\hat{} = \frac{\ln\left(\frac{v_{max2}}{v_{max1}}\right)}{\mu_2 - \mu_1} = 20892{,}62 \text{ s} = 5{,}80 \text{ h} = 5\text{h } 48 \text{ min}$$

Because 5,80 < 8 there is an intersection point between the two speeds in 8 hours as shown in figure 5.33, so the possibility of blockage exists. In particular, because of the intersection of the speed, the system will diverge for sure. The question that we have to answer is: “Does the system have enough time to become this unbalanced to make the blockage happen?”. To understand if the blockage will happen or not, we have to look at the plots and at the output of the code. If the plots become meaningless and in the output data of the simulation there are hand-off positions $x_2 < 0$ or $x_2 > 1$ and times between two consecutive hand-offs negative, this means that the fastest picker will be blocked.

The results are shown in figure 5.34 and 5.35.

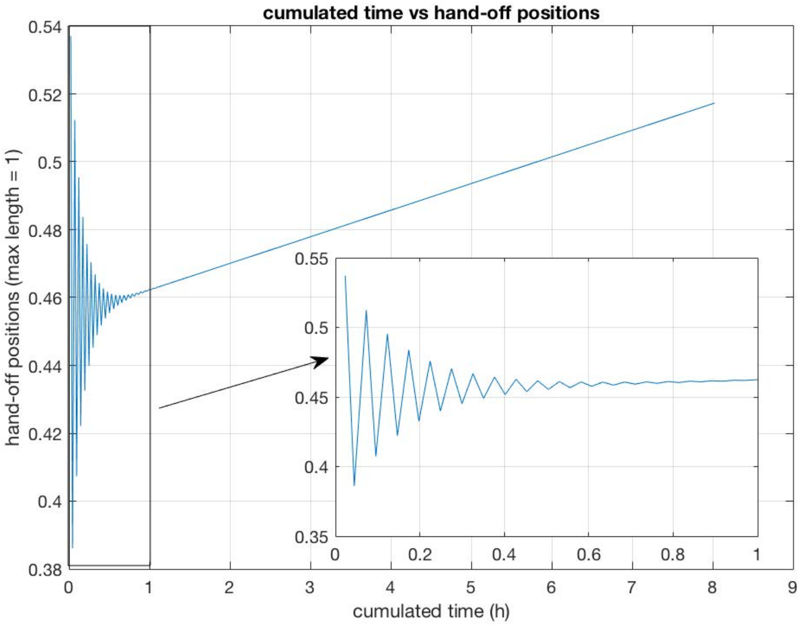


Fig 5.34 - Cumulated time vs hand-off positions when v_{max1} = 0,005 aisles/s, v_{max2} = 0,006 aisles/s, μ_1 = 3,6584*10^{-6} (easy work), μ_2 = 12,3850*10^{-6} (hard work) and $\mathbf{x}^{(0)}$ = (0,1190; 0,4984).

Also in this case, the speeds are converging over time. This leads to the fact that the hand-off positions are shifted towards the end of the line over time, because the ratio v_1/v_2 is growing over time. Because of the fact that the ratio v_1/v_2 is closer to 1 than the previous simulation, the positions of hand-off are more shifted towards the position 0,5. The position 0,5 is reached exactly after the time t^ that we calculated before, because in that moment the speeds are the same, the ratio between the two speeds is 1 and the hand-off position is 0,5. We can conclude that a sufficient and necessary condition to have the hand-off at a position 0,5 is that the speeds of the two pickers have to be the same. The system converges very fast, but slower than the previous simulations, because the starting speeds are very similar at the beginning and even more similar over time. Even if the pickers' speeds have a point of intersection in figure 5.33, the behavior of the system seems regular, so we can conclude that the pickers do not block each other. This happens because the speed intersection at a time t^ is close to the end of the work shift; because of this the system does not have enough time to become so unbalanced to create blockage. Even though it is impossible to see it in the picture, even zooming, the system, after t^, slightly diverges from the average balanced behavior.

Speaking about the time between two consecutive hand-offs, the results are shown in figure 5.35.

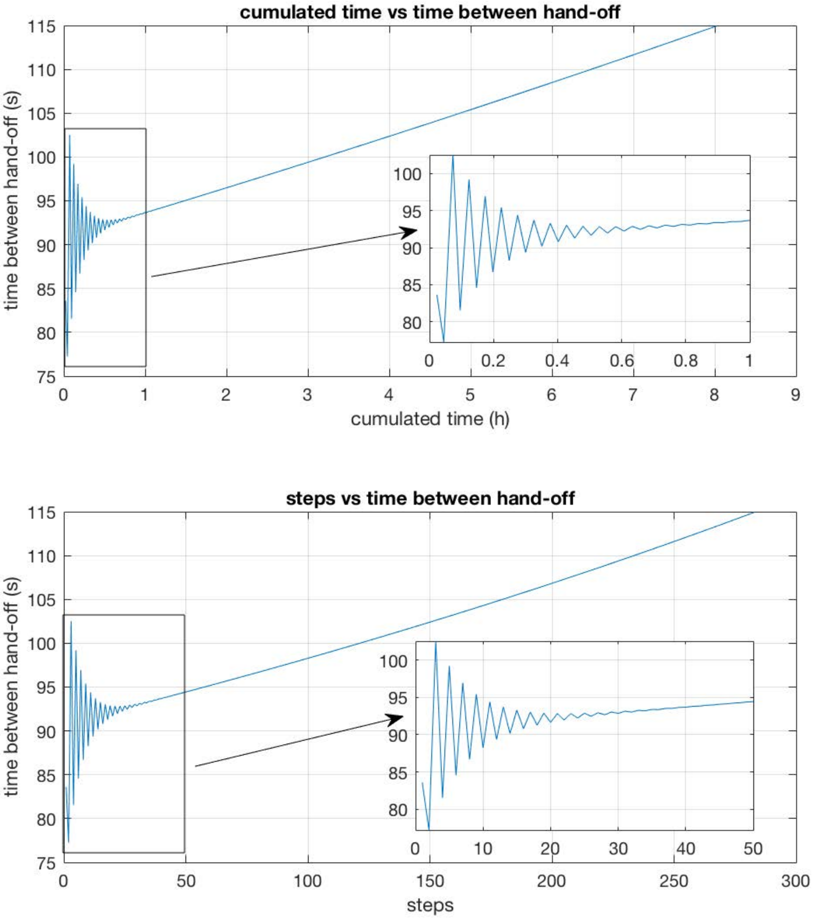


Fig 5.35 - Cumulated time and steps vs time between hand-offs when v_{max1} = 0,005 aisles/s, v_{max2} = 0,006 aisles/s, μ_1 = $3,6584*10^{-6}$ (easy work), μ_2 = $12,3850*10^{-6}$ (hard work) and $\mathbf{x}^{(0)}$ = (0,1190; 0,4984). In 8 hours the system finishes 283 orders.

The system converges pretty fast and it is very stable, since the beginning of the work shift. The time between two consecutive hand-offs is increasing, because, as always, the pickers are slowing down over time because of fatigue ($\mu > 0$). In 8 hours the bucket brigade finishes 283 orders: more then the simulation before, because the speed of picker 1 is 0,005 aisles/s and not anymore 0,006 aisles/s. As we noticed before the system behaves normally and seems not to suffer of blockage. As we told before, the system has not enough time to become this unbalanced to block the pickers. Also here, even if it is not visible in picture 5.35, after time t^ the system slightly diverges from the average balanced behavior.

In the fourth simulation, the starting data are v_{1max} = 0,005 aisles/s, v_{2max} = 0,006 aisles/s (see figure 5.36), $\mu_1 = 0$ (no effort), $\mu_2 = 12{,}3850 \cdot 10^{-6}$ (hard work), $\mathbf{x}^{(0)}$ = (0,5060; 0,6991).

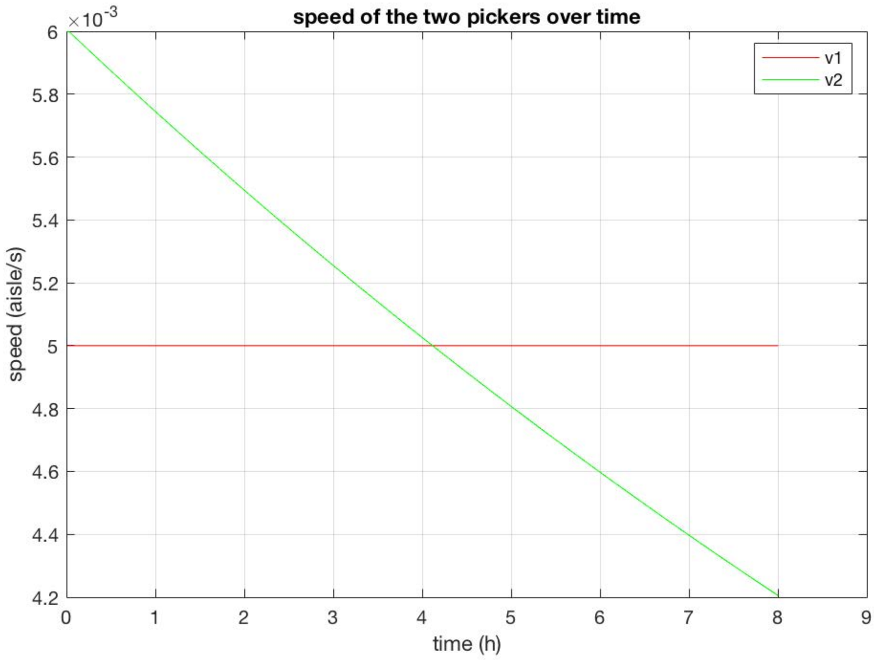


Fig 5.36 - Speed of the operators over time in a work shift in the case of v_{max1} = 0,005 aisles/s, v_{max2} = 0,006 aisles/s, $\mu_1 = 0$ (no effort) and $\mu_2 = 12{,}3850 \cdot 10^{-6}$ (hard work).

In this simulation $\mu_1 < \mu_2$, so there is a possibility of blockage, because picker 2 slows down faster than picker 1. To check if the possibility of blockage is real or not we have calculated the value of t^ = 4,09. Because 4,09 < 8 there is an intersection point between the two speeds in 8 hours, so there is a possibility of blockage. The fact that we found a time t^ < 8 hours means that the system, after t^ hours, starts to diverge from its average exponential balanced behavior, slowly at the beginning and faster step by step over time. It is important to notice that the time t^ is lower than the values we obtained in the previous simulation, so the system has more time to get unbalanced. The question that we have to ask ourselves is: “Does the system has enough time to become enough unbalanced to lead to a blockage?”. If yes, the fastest picker will be blocked from the slowest one and the results of the simulation will be meaningless. We remind the reader that a blockage leads to a loss in throughput. In conclusion, to check if the blockage exists or not, we have to look at the results of the simulations.

The results are shown in figure 5.37 and 5.38.

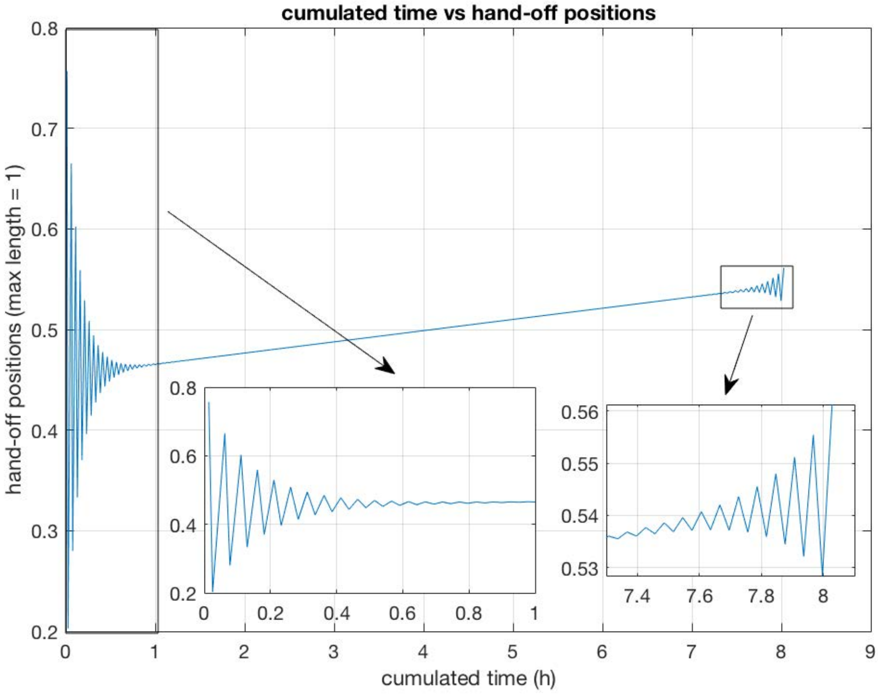


Fig 5.37 - Cumulated time vs hand-off positions when v_{max1} = 0,005 aisles/s, v_{max2} = 0,006 aisles/s, μ_1 = 0 (no effort), μ_2 = 12,3850*10^{-6} (hard work) and $\mathbf{x}^{(0)}$ = (0,5060; 0,6991).

The most important fact that it is easy to notice is that the system converges at the beginning of the work shift and diverges at the end of the work shift. This means that the system returns unbalanced. Here the unbalance is bigger than in the previous simulation, in fact it is possible to notice it even if we do not zoom the plot in figure 5.37. The general behavior of the system is the same that the one we had in the previous simulation: after time t^ the system starts to be unbalanced and the more the time goes by and the faster the unbalance grows. It is important to notice that, even though the system is unbalanced, in no one of the simulations till now the blockage exists. The only difference between simulation three and four (this one) is that in this simulation here the system remains unbalanced for a longer time, therefore it has more time to diverge and it is possible to see, without zooming, the plot diverging (see figure 5.37). Also here the slope of the function is increasing, because the ratio v_1/v_2 is increasing. At the beginning of the work shift the system converges fast, but not as fast as in the first two simulations, because in this simulation the speeds have closer values.

Speaking about the time between two consecutive hand-offs, the results are shown in figure 5.38.

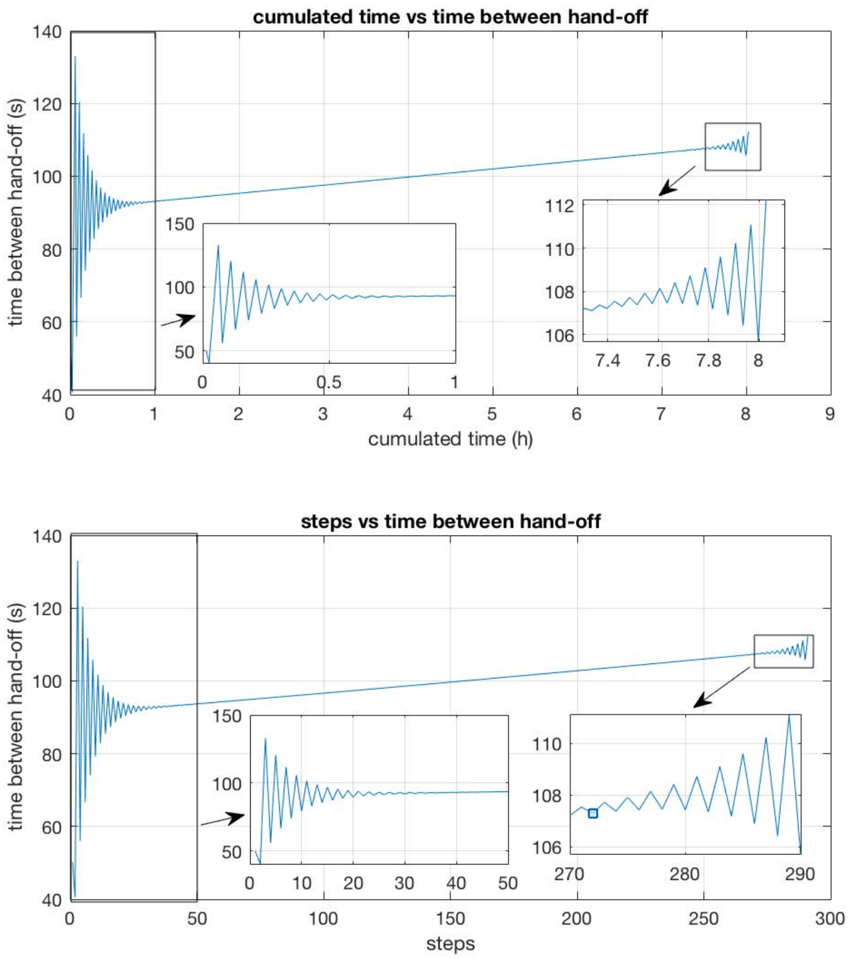


Fig 5.38 - Cumulated time and steps vs time between hand-offs when v_{max1} = 0,005 aisles/s, v_{max2} = 0,006 aisles/s, $\mu_1 = 0$ (no effort), μ_2 = 12,3850*10^{-6} (hard work) and $\mathbf{x}^{(0)}$ = (0,5060; 0,6991). In 8 hours the system finishes 291 orders.

Also here the system converges, and then starts to diverge again after t^ = 4,09 hours, because the pickers' speeds are crossing, so the system is not balanced anymore. The plot in figure 5.38 is not meaningless, the values of hand-off position are always in the interval $0 < x_2 < 1$ and the time between two consecutive hand-offs is always positive: the code is still working well and all this factors suggest us that there is no blockage inside the eight hours work shift. The slope of the function is increasing on average, according to the fact that the pickers get tired over time during the work shift, therefore the time between two consecutive hand-offs becomes longer over time.

In the fifth and last simulation, the starting data are v_{1max} = 0,0055 aisles/s, v_{2max} = 0,006 aisles/s (see figure 5.39), μ_1 = 0 (no effort), μ_2 = 12,3850*10^{-6} (hard work), $\mathbf{x}^{(0)}$ = (0,1869; 0,4898).

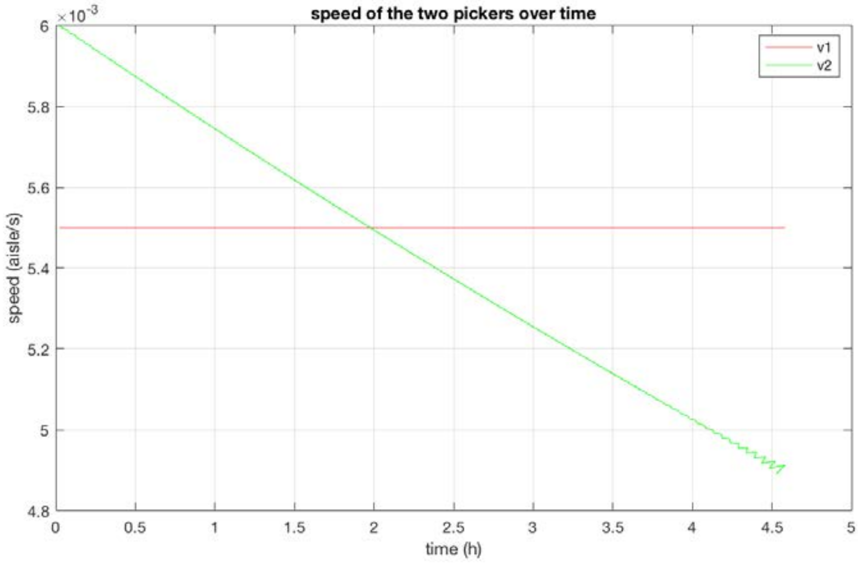


Fig 5.39 - Speed of the operators over time in a work shift in the case of v_{max1} = 0,0055 aisles/s, v_{max2} = 0,006 aisles/s, μ_1 = 0 (no effort) and μ_2 = 12,3850*10^{-6} (hard work). The plot takes into account only the first 4,5 hours.

In this simulation $\mu_1<\mu_2$, so there is possibility of blockage, because picker 2 slows down faster than picker 1. To check if the possibility of blockage is real or not, we calculated the value t^ = 1,95. Because of the fact that 1,95 < 8, there is an intersection point between the two speeds in 8 hours, therefore there is a possibility of blockage. From time t^ on, the system will be unbalanced and it will diverge always faster over time. Moreover, in this last simulation, the system has the time to become so unbalanced that picker 1 (the fastest picker) is blocked by picker 2 (the slowest picker). We can notice it immediately looking carefully at figure 5.39: the speed of picker 2 (the green one), after around 4,5 hours, has a strange behavior. This strange behavior suggests that the code is not working properly anymore and that is because of the fact that the fastest picker is blocked by the slowest one. It is important to notice that figure 5.39 does not consider the whole eight hours work shift; if the we had considered the whole work shift, the result would have been a meaningless doodle. To understand better the behavior of the system when the slowest picker blocks the fastest one, we have to focus our attention on the results we obtained with the simulation.

The results are shown in figure 5.40 and 5.41.

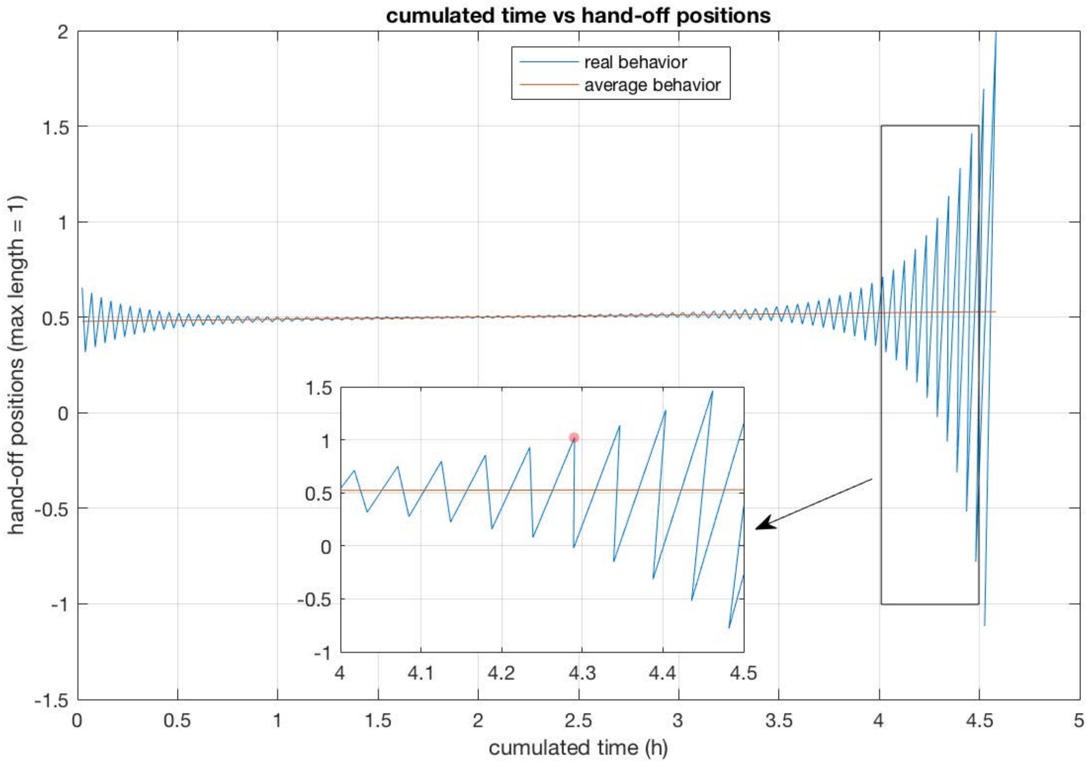


Fig 5.40 - Cumulated time vs hand-off positions when v_{max1} = 0,0055 aisles/s, v_{max2} = 0,006 aisles/s, μ_1 = 0 (no effort), μ_2 = 12,3850*10^{-6} (hard work) and $\mathbf{x}^{(0)}$ = (0,1869; 0,4898). In red the average behavior is shown. The red spot is the "breaking point": from that point on the code does not work properly anymore; this means that the fastest picker is blocked by the slowest one. The plot takes into account only the first 4,5 hours.

As we have already told before, in this case the fastest picker is blocked by the slowest one. This is suggested by a lot of anomalies in the results. First of all, if we had considered the whole work shift in figure 5.40, the result would have been a meaningless doodle. Moreover, always in figure 5.40, the plot of x_2, after a little bit less than 4,3 hours, goes above 1 (the code is not working properly anymore), but that is impossible in reality because 0 < x2 < 1 for hypothesis: the existence of this *breaking point* indicates the presence of blockage. In the simulation, in fact, at the 169th iteration (after a little bit less than 4,3 hours), the value of the hand-off position is x_2 = 1,021 > 1. The following iteration (170th), shows a value of time between hand-offs of -4,21 seconds, that is completely meaningless.

Speaking about the time between two consecutive hand-offs, the results are shown in figure 5.41.

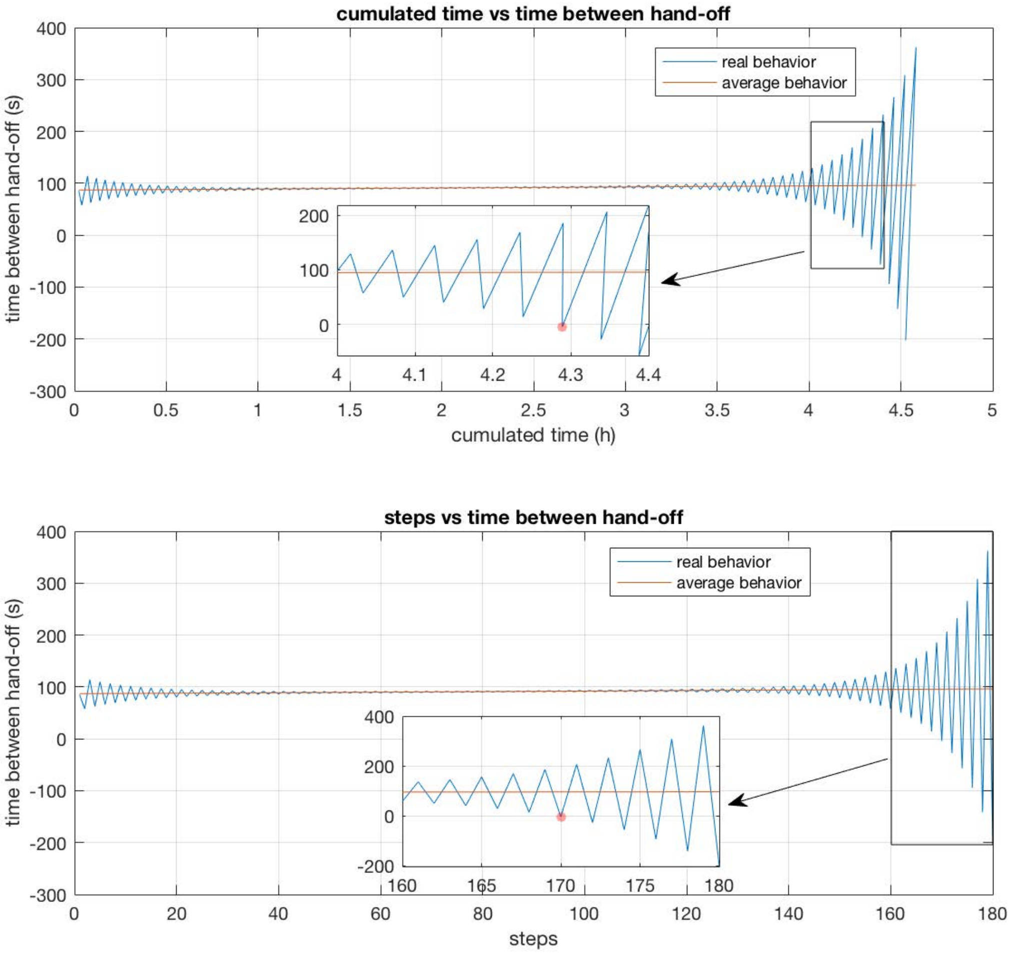


Fig 5.41 - Cumulated time and steps vs time between hand-offs when v_{max1} = 0,0055 aisles/s, v_{max2} = 0,006 aisles/s, μ_1 = 0 (no effort), μ_2 = 12,3850*10^{-6} (hard work) and $\mathbf{x}^{(0)}$ = (0,1869; 0,4898). In red the average behavior is shown. The red spots are the "breaking points": from that points on the code does not work properly anymore; this means that the fastest picker is blocked by the slowest one. The plot takes into account only the first 4,5 hours, corresponding to the first 180 steps (iterations).

Also in this case there are a lot of hints that suggest us that the fastest picker is blocked by the slowest one. At iteration number 170 (a little bit less than 4,3 hours) the code does not work properly anymore, in fact the time between two consecutive hand-offs goes below 0 (negative time). This breaking point corresponds to the moment when the fastest picker is blocked by the slowest one.

To summarize the results we obtained in paragraph 5.5.5 we will use chart 5.4.

Chart 5.4 - Comparison between five different behaviors that a bucket brigade system could have under the hypothesis of different v_{max} (always under the hypothesis $v_{max2} > v_{max1}$) and different values of µ. All the five simulations have been presented in this paragraph.

v_{max} (a/s)	µ	t^ (h)	divergence	blockage
$v_{max1} = 0{,}003$ $v_{max2} = 0{,}006$	$\mu_1 = 12{,}3850*10^{-6}$ $\mu_2 = 3{,}6584*10^{-6}$	x	no	no
$v_{max1} = 0{,}003$ $v_{max2} = 0{,}006$	$\mu_1 = 3{,}6584*10^{-6}$ $\mu_2 = 12{,}3850*10^{-6}$	22,06 > 8	no	no
$v_{max1} = 0{,}005$ $v_{max2} = 0{,}006$	$\mu_1 = 3{,}6584*10^{-6}$ $\mu_2 = 12{,}3850*10^{-6}$	5,80 < 8	yes, but not visible	no
$v_{max1} = 0{,}005$ $v_{max2} = 0{,}006$	$\mu_1 = 0$ $\mu_2 = 12{,}3850*10^{-6}$	4,08 < 8	yes and visible	no
$v_{max1} = 0{,}0055$ $v_{max2} = 0{,}006$	$\mu_1 = 0$ $\mu_2 = 12{,}3850*10^{-6}$	1,95 < 8	yes, visible and makes the plots meaningless	yes, after 169 iterations (a little bit less than 4,3 hours

As we told at the beginning of this paragraph, the behavior of this kind of system mainly depends on the combination of the values of v_{max} and µ. All the possible behaviors are summarized in figure 5.42.

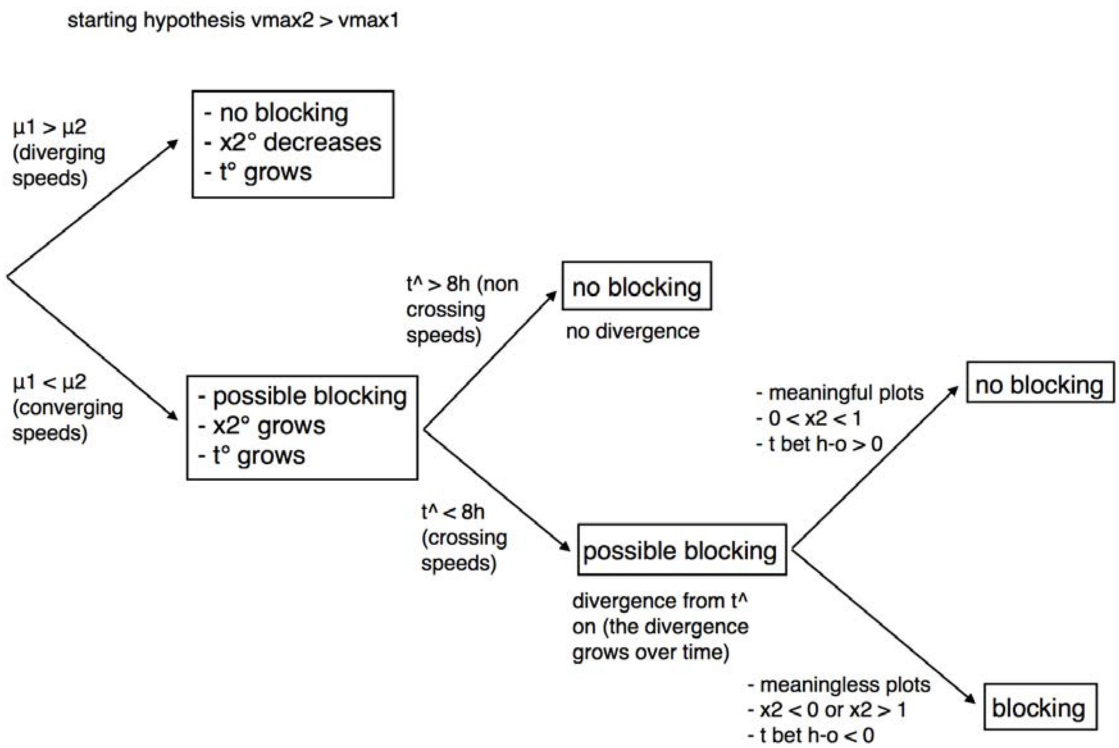


Fig 5.42 - Possible behaviors of the bucket brigade when $v_{max1} \neq v_{max2}$ and µ1 ≠ µ2.

In conclusion, there are a lot of possible behaviors that the bucket brigade system could have, depending on the relationships between v_{max1} and v_{max2}, μ_1 and μ_2. The system converges always pretty fast. In particular, if $\mu_1 > \mu_2$, the speeds are diverging, there is no chance of blockage, x2° decreases and t° grows; if $\mu_1 < \mu_2$, the speeds are converging, there is a chance of blockage, x2° grows over time and t° grows as well. To understand if the fastest picker can be blocked by the slowest one the number t^ = $\ln(v_{max2} / v_{max1}) / (\mu_2 - \mu_1)$ is very important: if t^ > 8 hours, the speeds are not crossing and the system will be balanced during the whole work shift; if t^ < 8 hours the speeds have an intersection in the work shift and, from time t^ on, the system will start to diverge, first slowly and then always faster over time. In this case, the system starts to be unbalanced and this leads to a major amplitude in the plots of hand-off positions and time between two consecutive hand-offs. In general, the more the time passes by and the more the system becomes unbalanced. If the bucket brigade remains unbalanced for enough time, the fastest picker will be blocked, after some time, by the slowest one and this will lead to a loss in throughput. It is possible to understand if the fastest picker is blocked by the slowest one looking at the plots and at the results of the simulation. In case of blockage the code does not work properly anymore and some anomalies will be visible; these anomalies could be:

- The plots become meaningless.
- In the output data of the simulation there are hand-off positions $x_2 < 0$ or $x_2 > 1$.
- The times between two consecutive hand-offs are negative.

To avoid this situation of blockage, we advice the managers to operate as follow: as soon as the speeds of the pickers cross each other after time t^, the manager should tell the pickers to switch their position. With this strategy the pickers will be working along the aisle always from the slowest to the fastest and blockage is no longer possible: the bucket brigade will maintain its balance and it will perform at its best.

We can say that also if v_{max} and μ are different, the system performs well, because it is possible to avoid blockage using the simple strategy which we described in the previous line.

5.6 Conclusions and techniques to improve

Once we have shown the behavior of the system in all these different cases we consider appropriate to use chart 5.5 to summarize all the results that we have found.

Chart 5.5 - Summary of the behavior of an order-picking bucket brigade system, when the speeds of the pickers are not constant over time. The chart takes into consideration all the possible behaviors that the system can have, considering all the different possible combinations of v_{max} and μ values. For each case the evolution of the succession of hand-off positions and times between two consecutive hand-offs is explained. At the end, some notes for each case are given.

	convergence x_2	**convergence t**	**notes**
v_{max} =, μ=	no $x_2^{av}=1/2$	no $t^{av}=1/(2*v_{max}*e^{-\mu t})$	no convergence, the hand-off position swings between two points with average 0,5; system easy to predict, but not good
v_{max} ≠, μ=	x_2 constant $x_2^*=v_{max1}/(v_{max1}+v_{max2})$	t up $t°=1/((v_{max1}+v_{max2})*e^{-\mu t})$	fast convergence (vmax different), good system
v_{max} =, μ≠	x_2 down $x_2°=e^{-\mu 1t}/(e^{-\mu 1t}e^{-\mu 2t})$	t up $t°=1/(v_{max}*(e^{-\mu 1t}+e^{-\mu 2t}))$	slow convergence (vmax close), bad system
v_{max} ≠, μ≠	x_2 up or down depending on μ_1 and μ_2 $x_2°=(v_{max1}*e^{-\mu 1t})/(v_{max1}*e^{-\mu 1t}+v_{max2}*e^{-\mu 2t})$	t up $t°=1/(v_{max1}*e^{-\mu 1t}+v_{max2}*e^{-\mu 2t})$	could exist divergence if the speeds are crossing, possibility of blockage, lot of factors to take into account; system to use carefully

As chart 5.5 shows, the bucket brigade can have a lot of different behaviors.

The hand-off position x_2 could remain constant, shift towards the end of the aisle or shift towards the beginning of the aisle. In every case, we could find a function $x_2°$ that tells us where the hand-off position is after a time t when the system is balanced. The hand-off position $x_2°$ over time depends only on the values of the speeds v_1 and v_2, that are functions of t. More precisely, it is possible to find the hand-off position $x_2°$ after a certain time t from the beginning of the work shift only knowing the ratio $r = v_1/v_2$ between the two speeds at time t. This observation is valid both when the speeds are constant and when the speeds slow down over time, because of fatigue. The formula can be obtained from the hypothesis of the normative model; moreover we have already found the formula we are seeking in paragraph 2.2 to calculate the value of x_2*. Working on x_2* = v_1 / (v_1+v_2), it is possible to obtain x_2* = r / (1+r).

Another way to write the formula that we found is:

$$x_2^* = \frac{\frac{v_1}{v_2}}{1+\frac{v_1}{v_2}} = r / (1+r)$$

In the formula only the values of speeds appear, in fact the position of x_2 depends only on the speeds of the workers and not on their initial positions.

If we consider the speeds changing over time, the ratio r = v_1/v_2 is not constant and the formula becomes: x2° = r(t) / (1+r(t)), where r(t) = v_1(t) / v_2(t).

We can plot the function, as in figure 5.40.

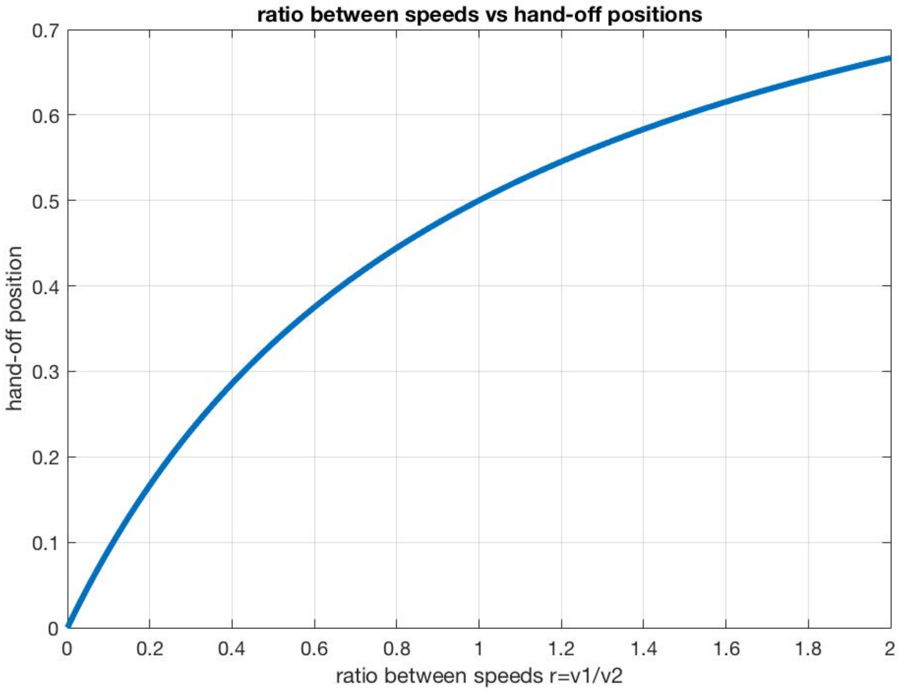


Fig 5.43 - Hand-off position vs instantaneous speed ratio r = v_1/v_2. The plot shows the function x_2° of the hand-off positions evolving in function of the instantaneous speed ratio v_1/v_2.

The function is a hyperbola: the more the ratio v_1/v_2 grows and the more the hand-off position is shifted towards the end of the line, the more the ratio v_1/v_2 decreases and the more the hand-off position is shifted towards the beginning of the line. It is important to notice that when the speeds assume the same value (v_1=v_2), the hand-off position is 0,5, so exactly in the middle of the aisle.

Moreover, according to Bartholdi and Eisenstein (1996a), a bucket brigade works well (it provides the maximum throughput) if the workers are ordered to the slowest to the fastest; also in our work, in fact, we always ordered the pickers with this rule, so that $v_1<v_2$ [38]. If $v_1<v_2$, the ratio $r = v_1/v_2$ is always $0 \leq r \leq 1$ and consequently the hand-off position is always in the first half of the line. The fact that the pickers' speeds are changing over time leads to a translation of the hand-off position along the aisle, following the function in figure 5.43. As last thing, we remind the reader that this function works only from the moment when the system is balanced and on; if the system is not balanced the formula and the plot are useful to find the average behavior of the system, not the instantaneous one.

On the other hand, the time between two consecutive hand-offs always grows over time, because of fatigue ($\mu>0$), that makes the pickers slow down. The harder the work is, the higher the values of μ are and, consequently, the time between two consecutive hand-offs becomes longer. The time between two consecutive hand-offs depends both on the pickers' speeds and on the values of μ, but it does not depend on the starting positions of the pickers.

Another interesting problem is to try to find a function to calculate the throughput of the bucket brigade after time t. According to Bartholdi and Eisenstein (1996a) and under the hypothesis of normative model, because of the constant values of the speeds, it was possible to calculate the throughput with $TR = 1/t^* = v_1+v_2$. Unfortunately, we could not find a closed formula to calculate the throughput of the system analytically, so we calculated all the throughput after 8 hours using a numerical approach, thanks to the simulations. However, to find the instantaneous throughput after time t, it is possible to use the formula $TR = v_1(t)+v_2(t)$, where v_1 and v_2 are decreasing over time, because of fatigue.

After all the simulations, we noticed that the best system to use is the case where pickers have different v_{max} and same μ (5.5.3), because the system converges very fast and the hand-off position is fixed over time: this fact guarantees a regular and predictable throughput during the work shift. Also the system with pickers working at different v_{max} and different μ (5.5.5) performs well: it ensures a rapid convergence, even if the hand-off position shifts along the aisle over time. In this case, blockage could be possible; if it exist, it creates losses in throughput. Then, bucket brigades with same v_{max} and same μ (5.5.2) seem to perform badly,

[38] Only in the case of pickers with different v_{max}, different μ and crossing speed this statement is not valid anymore.

because the system does not converge, even if the behavior of the system is easy to predict, because it does not diverge neither. At the end, systems with same v_{max} and different μ (5.5.4) converge very slowly, because the starting maximum speeds of the pickers are very close at the beginning of the work shift. This type of bucket brigade is the most difficult to deal with. Moreover, it is important to notice that it is impossible to have a constant time between two consecutive hand-offs because of fatigue: a consequence of this fact is that it is impossible to follow exactly the takt time imposed by the customer.

As we have just explained, the bucket brigade could perform in different ways, depending on the starting data. Anyway, we noticed that there are some strategies to improve the efficiency of the bucket brigade, taking simple managerial decisions. In particular, as we noticed, it could be very important to work on the initial positions of the pickers $\mathbf{x}^{(0)}$ to make the behavior of the system more regular and balanced during the work shift. In particular, the general strategy is to calculate the average hand-off position $x_2°(0h)$ at the beginning of the work shift and position the pickers in a starting position where $x_2^{(0)} = x_2°(0h) + x_1^{(0)}$, where $x_2°(0h) = v_{max1} / (v_{max1} + v_{max2})$. Using this foresight, the system is already balanced from the beginning of the work shift: the fluctuations of the system[39] are reduced and the system immediately converges in hand-off positions and time between two consecutive hand-offs. As the simulations confirm, even though the system is balanced, the hand-off positions could shift along the aisle or remain in the same place, depending on the ratio v_1/v_2. On the contrary, the time between to consecutive hand-off is always growing over time, because of fatigue ($\mu>0$). There are two ways to reduce the time between two consecutive hand off and so to increase the throughput. The first one is to increase the maximum speed of the pickers, working on all the factors that we mentioned in paragraph 5.2.2 (experience, training, ergonomics, motivation, …), while the second one is to reduce fatigue, reducing the value of μ, working on the factors that we mentioned in paragraph 4.5 (make the picking action easier, ergonomics, good rest, …).

We want to conclude the discussion of this chapter, making the reader notice that the most common case that a manager has to deal with is when pickers have different v_{max} and different μ, in accordance to the fact that all the human beings are different.

---

[39] In the plots the scatter of the data is reduced, so that there are no “vibrations” in the plot even during the first iterations.

Results

This thesis has demonstrated that a two workers order-picking bucket brigade system performs very well even when considering human factors. We showed analytically and numerically that the effects of considering muscular fatigue in an eight hours work shift are a decrement in throughput (due to the slowdown of the pickers) and a shift of the hand-off position along the line over time (due to the changing of the ratio between the speeds of the two pickers over time). Moreover, some important advice to improve the performance of the system are given.

In particular, a new function to model the slowdown of the pickers during the work shift over time has been invented. Then, we considered four different cases of bucket brigade:

- Pickers with same v_{max} and same μ.
- Pickers with different v_{max} and same μ.
- Pickers with same v_{max} and different μ.
- Pickers with different v_{max} and different μ.

After that, we analyzed the behavior of the four cases. First we analyzed them analytically and then numerically, showing the different results through plots. For a matter of clarity, we propose again chart 5.5 (see paragraph 5.6[40]), in which the main results are summarized.

	convergence x_2	**convergence t**	**notes**
v_{max} =, μ=	no $x_2^{av}=1/2$	no $t^{av}=1/(2*v_{max}*e^{-\mu t})$	no convergence, the hand-off position swings between two points with average 0,5; system easy to predict, but not good
v_{max} ≠, μ=	x_2 constant $x_2^*=v_{max1}/(v_{max1}+v_{max2})$	t up $t°=1/((v_{max1}+v_{max2})*e^{-\mu t})$	fast convergence (vmax different), good system
v_{max} =, μ≠	x_2 down $x_2°=e^{-\mu 1t}/(e^{-\mu 1t}e^{-\mu 2t})$	t up $t°=1/(v_{max}*(e^{-\mu 1t}+e^{-\mu 2t}))$	slow convergence (vmax close), bad system
v_{max} ≠, μ≠	x_2 up or down depending on μ_1 and μ_2 $x_2°=(v_{max1}*e^{-\mu 1t})/(v_{max1}*e^{-\mu 1t}+v_{max2}*e^{-\mu 2t})$	t up $t°=1/(v_{max1}*e^{-\mu 1t}+v_{max2}*e^{-\mu 2t})$	could exist divergence if the speeds are crossing, possibility of blockage, lot of factors to take into account; system to use carefully

[40] For more detailed results, we suggest the reader to consult paragraph 5.6.

All of these results are more precise than the results obtained by Bartholdi and Eisenstein (1996a, 1996b) and by the others researchers in their papers, because they did not consider human factors. Taking into account muscular fatigue allowed us to obtain results closer to what happens in a real warehouse, so that managers can predict better the behavior of the system they set up and they can improve or make it perform better just utilizing the advice we gave.

This thesis is the first to link (analytically and numerically) bucket brigades and human factors. Due to the fact that it is the first to link these topics, the theme is very wide and difficult to understand. For this reason, it is not possible to study deeply all the parts of the work. Then, because of the width of the theme, we decide to give more importance to the numerical results than to the analytical part. It is with the latter approach that we find the most interesting and useful results.

Even though the results are vast and very interesting, there are some unanswered questions. The answers to these questions will lead to a development of our work.

- What happens if the BB has more than two pickers? Could the results we obtained be generalized to n pickers?

- How would the results change if we considered not only an aisle, but all the warehouse?

- We found some formulae to model the function of the average hand-off position $x_2°$ and the function of the average time between two consecutive hand-offs t°. Is it possible to find a closed formula to calculate the throughput after time t from the beginning of the work shift TR°?

- What happens if we consider not only the effect of fatigue, but also the effect of learning?

- In our work we used an exponential function to describe the slowdown of the pickers over time. Some other authors suggest that fatigue is not exponential. Which model gives results that are closer to reality?

- One of the working hypothesis was to consider the work equally distributed along the line; we were allowed to do this because we were making the hypothesis to work in a high-volume distribution warehouse of a chain retailer, where a typical order consists in a lot of skus, but a small number each. Because of that, we could consider, approximately, the work continuously and uniformly spread along the aisle. What happens if the warehouse is low-volume and/or we consider the work exponentially distributed?

- The last one of the four cases studies the behavior of a system with different values of maximum speed and different values of work effort. The system is very complex to study. How can we explain better all the possible behaviors of the system? Is it possible to find an analytical formulation to predict after how much time the fastest picker is blocked by the slowest one?

- Some important results have been obtained only by simulations and it was too difficult to prove them. Is it possible to demonstrate mathematically everything we found?

Appendices

A.1 Proof of fixed point convergence theorem

$x_{k+1} = g(x_k)$

$x_{k+1} - \xi = g(x_k) - \xi$

$\varepsilon_{k+1} = g(x_k) - g(\xi)$

For Lagrange's theorem:

$\varepsilon_{k+1} = g'(\eta) * \varepsilon_k$

$|\varepsilon_{k+1}| = |g'(\eta)| * |\varepsilon_k| \leq m * |\varepsilon_k|$ where m is the error increase for every step

And everything works $\forall k$, therefore:

$|\varepsilon_1| \leq m * |\varepsilon_0|$

$|\varepsilon_2| \leq m * |\varepsilon_1| \leq m^2 * |\varepsilon_0|$

…

$|\varepsilon_k| \leq m^k * |\varepsilon_0|$

And because of the Squeeze theorem:

$0 \leq |\varepsilon_k| \leq m^k * |\varepsilon_0|$, where $0 \to 0$, $m^k \to 0$, then $|\varepsilon_k| \to 0$

That means the error $|\varepsilon_k|$ goes to 0 after a few iterations, therefore the fixed point method converges.

NB: $|g'(x)| \leq m < 1$ is only a sufficient condition, so if $|g'(x)| > 1$ the method can converge anyway.

A.2 Two operators bucket brigade numerical example

Now, let's have a look at a simple 2 operators numerical example to understand better the dynamics of the bucket brigade system. Passing is not allowed along the line.

The starting data of the problem are l = 1 (hypothesis of normalization of line length), $\mathbf{x}^{(0)}$ = (0,3; 0,6), $\mathbf{v}$ = (0,1; 0,2). The vector $\mathbf{v}$ of the speeds is constant along the line and over time.

Using the formulas which have been found above, the results presented in chart A2.1 can be obtained:

Chart A2.1 - Results of a two operators bucket brigade numerical example. In the chart the different iterations are shown; for each iteration the position of the first and second worker after the hand-off and the time between the t and the t-1 iteration are indicated.

step	0	1	2	3	4	5	6	7	8	9	10
$x_1^{(t)}$	0,3	0	0	0	0	0	0	0	0	0	0
$x_2^{(t)}$	0,6	0,5	0,25	0,375	0,313	0,344	0,328	0,336	0,332	0,334	0,333
t_t	-	2	2,5	3,75	3,125	3,438	3,281	3,359	3,320	3,340	3,330

It is possible to notice, that all the three lines converge to a number: it means that after a few iterations the line balances itself, converging to a single fixed point, as Bartholdi and Eisenstein (1996a) proved in their paper.

The results at 10th step are already very close to the convergence solution (*), which is:

$x_1^* = 0$

$x_2^* = v_1 / (v_1+v_2) = 1/3 \simeq 0{,}33333\ldots$

$t^* = 1 / (v_1+v_2) = 10/3 \simeq 3{,}33333\ldots$

$TR = 1/t^* = v_1 + v_2 = 0{,}3$ pcs/s = 1800 pcs/h

As it is possible to notice the results do not depend from the starting positions of operators $\mathbf{x}^{(0)}$, but only on their velocities $\mathbf{v}$.

Not only the calculations, but also figure A2.1 and figure A2.2 confirm the convergence of the system, both in time and in space.

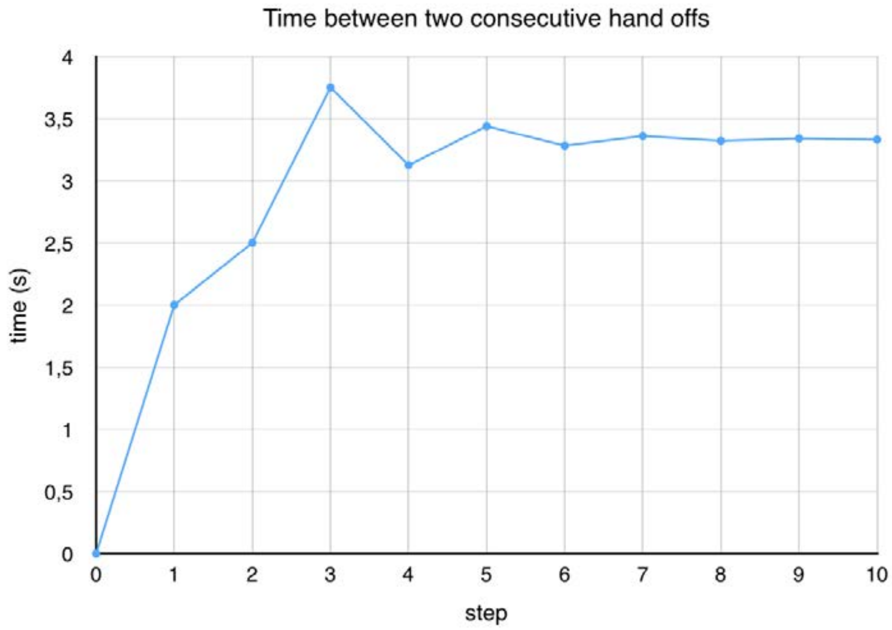


Fig A2.1 - Convergence of the time between two consecutive hand-offs in the numerical example. The diagram shows the time between two consecutive hand-offs in relation to the step (hand-offs).

Fig A2.2 - Convergence of the positions of workers during the hand-offs in the numerical example. The diagram shows the position of operator 1 and 2 immediately after the hand-off (at each step).

Codes in MATLAB

C.1 “MyScript_mu.m”

Script that provides the starting data (v_{max1}, v_{max2}, μ_1, μ_2) and calls one of the two functions that solve the problem.

```
%clears everything
clearvars;
close all;
clc;

%mu values: 0 no effort, 3.6584e-06 easy work, 7.7480e-06 average work, 12.3850e-06 hard work
mu1=3.6584e-06;
mu2=12.3850e-06;

%starting positions of the pickers
x_0=rand(1,2);

%x_0=[0.3028 0.8326];    %vector [x_0(1) x_0(2)]

if x_0(2)<x_0(1)
    aux=x_0(1);
    x_0(1)=x_0(2);
    x_0(2)=aux;
end

%starting (maximum) speeds of the pickers
%v=10e-3*rand(1,2);

v=[0.003 0.006];  %vector [vmax1 vmax2]

if v(2)<v(1)
    aux=v(1);
    v(1)=v(2);
    v(2)=aux;
```

```
end

%t intersection (to use only in the case of mu1<mu2 (fourth case)
t_intersection=(log(v(1,2)/v(1,1))/(mu2-mu1))/3600

%decide which function to call
stepwise_function_mu(x_0,v,mu1,mu2);
continuous_function_mu(x_0,v,mu1,mu2);
```

C.2 "stepwise_function_mu.m"

Function that solves the problem of a two pickers bucket brigade considering the speed of the pickers constant between two consecutive hand-offs, but decreasing exponentially over time (it is the file that we used to run all the simulations in this thesis; this code gives results that are slightly approximated).

```
function [] = stepwise_function_mu(x_0,v_0,mu1,mu2)
%   Bucket brigade with 2 operators
%   The program solves the problem of a two operators bucket brigade system
%   both when the speed of the operators is constant during the time (mu=0)
%   and when it decreases stepwise-exponentially during time (mu=/0)

%starting data
l=1; %normalized length of aisle
t=0;
x1=x_0(1,1);
x2=x_0(1,2);
v1=v_0(1,1);
v2=v_0(1,2);
fprintf('the starting data are: x1=%0.4f, x2=%0.4f, v1=%0.4f, v2=%0.4f \n',x_0(1,1),x_0(1,2),v_0(1,1),v_0(1,2));

ws_h=8;
ws_s=ws_h*3600;
```

```
%initialization of data
steps=[];
vett_t=[];
positions=[];
times=[];
positions_av=[];
times_h_av=[];
t_tot=0;
i=0;

%while cycle that calculates what happens step by step
while t_tot<ws_s

    v1=v_0(1,1)*exp(-mu1*t_tot);
    v2=v_0(1,2)*exp(-mu2*t_tot);

    i=i+1;

    t=(l-x2)/v2;
    x2=v1*t+x1;
    x1=0;


    %creation of the vectors that we need to create the plots
    steps(i,:)=i;

    positions(i,:)=x2;
    vett_t(i,:)=t;
    t_tot=sum(vett_t);
    times(i,:)=t_tot;
    times_h(i,:)=t_tot/3600;

    speeds1(i,:)=v1;
    speeds2(i,:)=v2;
```

```
    x_av=(v_0(1,1)*exp(-mu1*t_tot))/(v_0(1,1)*exp(-mu1*t_tot)+v_0(1,2)*exp(-
   mu2*t_tot));
   t_av=1/(v_0(1,1)*exp(-mu1*t_tot)+v_0(1,2)*exp(-mu2*t_tot));

   positions_av(i,:)=x_av;
   vett_t_av(i,:)=t_av;

    fprintf('iteration number %0.0d gives as results t=%0.5f, x1=%0.5f, x2=%0.5f
   \n',i,t,x1,x2)


end

t_tot=sum(vett_t);
t_tot_h=t_tot/3600;


fprintf('in %0.0f hours there are %0.0d iterations \n',t_tot_h,i)

 %creates the plot that shows the position of hand-offs over time
 figure;
 plot(times_h,positions);
 hold on;
 plot(times_h,positions_av);
 hold off;
 grid on;
 title('cumulated time vs hand-off positions');
 legend('real behavior','average behavior')
 xlabel('cumulated time (h)');
 ylabel('hand-off positions (max length = 1)');

 % creates the two plots that show the time between two consecutive hand-offs
 % over time
 figure;
 subplot(2,1,1);
 plot(times_h,vett_t);
```

```
hold on;
plot(times_h,vett_t_av);
hold off;
grid on;
title('cumulated time vs time between hand-off');
legend('real behavior','average behavior')
xlabel('cumulated time (h)');
ylabel('time between hand-off (s)');
subplot(2,1,2);
plot(steps,vett_t);
hold on;
plot(steps,vett_t_av);
hold off;
grid on;
title('steps vs time between hand-off');
legend('real behavior','average behavior')
xlabel('steps');
ylabel('time between hand-off (s)');

%creates the plot of the speeds over time
figure;
subplot(3,1,2);
plot(times_h,speeds1,'r');
grid on;
title('speed of the first picker over time');
xlabel('time (h)');
ylabel('speed v1 (aisle/s)');
subplot(3,1,3);
plot(times_h,speeds2,'g');
grid on;
title('speed of the second picker over time');
xlabel('time (h)');
ylabel('speed v2 (aisle/s)');
subplot(3,1,1);
plot(times_h,speeds1,'r');
hold on;
```

```
 plot(times_h,speeds2,'g');
 hold off;
 grid on;
 title('speed of the two pickers over time');
 xlabel('time (h)');
 ylabel('speed (aisle/s)');
 legend('v1','v2');

  fprintf('the starting data are: x1=%0.4f, x2=%0.4f, v1=%0.4f, v2=%0.4f \n',x_0(1,1),x_0(1,2),v_0(1,1),v_0(1,2));

end
```

C.3 "continuous_function_mu.m"

Function that solves the problem of a two pickers bucket brigade considering the speed of the pickers decreasing exponentially both between two consecutive hand-offs and in general over time (this is what happens exactly in reality; this code gives the perfect results).

```
function [] = continuous_function_mu(x_0,v_0,mu1,mu2)
% Bucket brigade with 2 operators
%   The program solves the problem of a two operators bucket brigade system
%   when the speed of the operators decreases exponentially during time

%starting data
l=1; %normalized length of aisle
t=0;
t_0=0;
x1=x_0(1,1);
x2=x_0(1,2);
v1=v_0(1,1);
v2=v_0(1,2);
fprintf('the starting data are: x1=%0.4f, x2=%0.4f, v1=%0.4f, v2=%0.4f \n',x_0(1,1),x_0(1,2),v_0(1,1),v_0(1,2));
```

```
ws_h=8;
ws_s=ws_h*3600;

%initialization of data
steps=[];
vett_t=[];
positions=[];
times=[];
positions_av=[];
times_h_av=[];
t_tot=0;
t_tot_prec=0;
i=0;

%while cycle that calculates what happens step by step
while t_tot<ws_s

    i=i+1;

    t_tot=(log((((-(1-x2)*mu2)/v_0(1,2))+exp(-mu2*t_tot_prec)))/-mu2;
    x2=x1-v_0(1,1)/mu1*(exp(-mu1*t_tot)-exp(-mu1*t_tot_prec));
    x1=0;

    t=t_tot-t_tot_prec;
    t_tot_prec=t_tot;

    %creation of the vectors that we need to create the plots
    steps(i,:)=i;

    positions(i,:)=x2;
    vett_t_tot(i,:)=t_tot;
    vett_t(i,:)=t;
    t_tot=sum(vett_t);
    times(i,:)=t_tot;
    times_h(i,:)=t_tot/3600;
```

```
    speeds1(i,:)=v1*exp(-mu1*t_tot);
    speeds2(i,:)=v2*exp(-mu2*t_tot);

     x_av=(v_0(1,1)*exp(-mu1*t_tot))/(v_0(1,1)*exp(-mu1*t_tot)+v_0(1,2)*exp(-
    mu2*t_tot));
    t_av=1/(v_0(1,1)*exp(-mu1*t_tot)+v_0(1,2)*exp(-mu2*t_tot));

    positions_av(i,:)=x_av;
    vett_t_av(i,:)=t_av;

     fprintf('iteration number %0.0d gives as results t=%0.5f, x1=%0.5f, x2=%0.5f
    \n',i,t,x1,x2)

end

t_tot_h=t_tot/3600;

fprintf('in %0.0f hours there are %0.0d iterations \n',t_tot_h,i)

 %creates the plot that shows the position of hand-offs over time
 figure;
 plot(times_h,positions);
 hold on;
 plot(times_h,positions_av);
 hold off;
 grid on;
 title('times vs hand-off positions');
 legend('real behavior','average behavior')
 xlabel('cumulated time (h)');
 ylabel('hand-off positions (max length = 1)');

 % creates the two plots that show the time between two consecutive hand-offs
 % over time
 figure;
 subplot(2,1,1);
```

```
plot(times_h,vett_t);
hold on;
plot(times_h,vett_t_av);
hold off;
grid on;
title('cumulated time vs time between hand-off');
legend('real behavior','average behavior')
xlabel('cumulated time (h)');
ylabel('time between hand-off (s)');
subplot(2,1,2);
plot(steps,vett_t);
hold on;
plot(steps,vett_t_av);
hold off;
grid on;
title('steps vs time between hand-off');
legend('real behavior','average behavior')
xlabel('steps');
ylabel('time between hand-off (s)');

%creates the plot of the speeds over time
figure;
subplot(3,1,2);
plot(times_h,speeds1,'r');
grid on;
title('speed of the first picker over time');
xlabel('time (h)');
ylabel('speed v1 (aisle/s)');
subplot(3,1,3);
plot(times_h,speeds2,'g');
grid on;
title('speed of the second picker over time');
xlabel('time (h)');
ylabel('speed v2 (aisle/s)');
subplot(3,1,1);
plot(times_h,speeds1,'r');
```

```
hold on;
plot(times_h,speeds2,'g');
hold off;
grid on;
title('speed of the two pickers over time');
xlabel('time (h)');
ylabel('speed (aisle/s)');
legend('v1','v2');

 fprintf('the starting data are: x1=%0.4f, x2=%0.4f, v1=%0.4f, v2=%0.4f \n',x_0(1,1),x_0(1,2),v_0(1,1),v_0(1,2));

end
```

NB: this code does not work if $\mu_1 = 0$ or/and $\mu_2 = 0$, because of the mathematics of the system. In this case, the results can be obtained with code C.2. In fact, if $\mu_1 = 0$ and $\mu_2 = 0$, it means that the speeds are constant during the whole work shift, so, as a consequence, they are constant also inside the intervals between two consecutive hand offs: that is exactly the idea that we used to build code C.2.

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